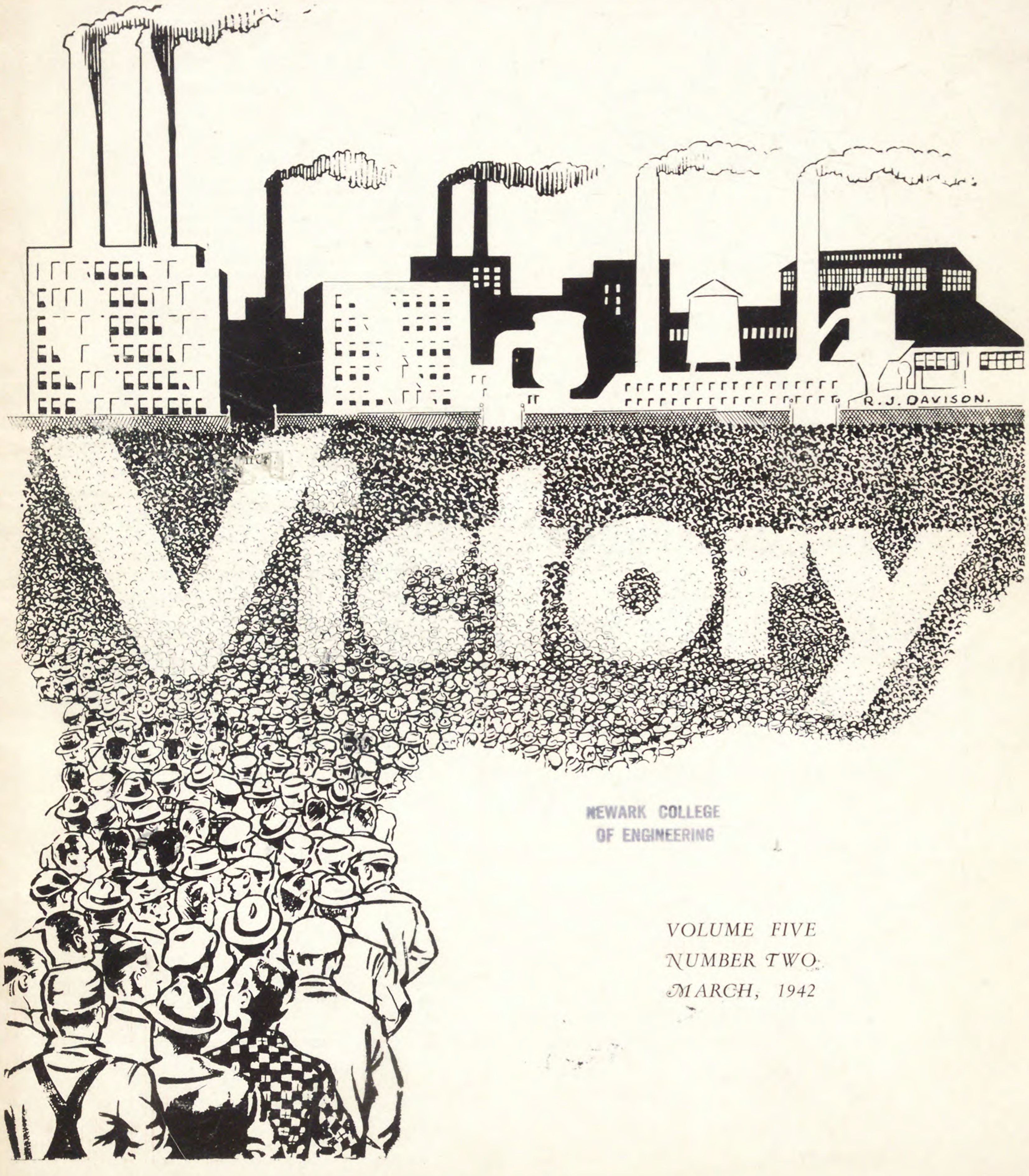
Newark ENGINEERING NOTES



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THE PRESIDENT'S DIARY

February 1st

I think I mentioned in my last diary about the great necessity of having the units in this defense machinery of ours sound, in having the individuals competent and willing to sacrifice and give all the energy possible to this defense effort. I think this point of view is sound and vital, but unfortunately it isn't enough. After we have cleared our own decks for action, declared war ourselves, gotten into the spirit of help and sacrifice and more sacrifice and still more sacrifice, we can't do the thing individually. We have got to do it collectively. While, as I have said, the individuals are vital and their soundness is vital, it is also vital to have an organization which can put together and arrange these individuals into a machine so that all acting together they can do the things that have to be done to ensure success and to ensure victory.

So that after we have done the very important job of getting ourselves into the proper frame of mind, after we have created internally the proper morale, there then comes the problem of putting and getting us all together so that we can act, not as separate units bumping each other without rhyme or reason, but as a unified whole so as not to get in each other's way and hinder rather than help, as some of us do even when we have the best intentions.

This is the function of organization, the function of planning, the function of engineering as we understand it. After all, engineering is nothing more than arranging things so they work, just as an electrical engineer arranges a generator so that it will work. This we call design. We design a locomotive or a bridge in the same way. We take a lot of different things and put them together so that they do the thing that we want done. After we have the strong materials, the individuals—let us say, the iron bars, the rivets, the concrete and the reinforcing—we design by putting those things together so that they work. This is a technical job, an engineering job, or perhaps we ought to say a management job.

Now one thing in America that we haven't paid too much attention to up until the last ten or fifteen years has been organizing people into machines so that they, together, would do things that we wanted them to. To be sure, we have heard of political machines, and while they have been sometimes pretty effective, we haven't taken much trouble to build up machines made up of human parts until quite lately. One of the most important things that modern industry has done, that modern engineering has done, is to teach certain ways of developing and arranging human beings so that they will do the thing that is wanted. Great organizations, like for instance the American Telephone & Telegraph Company, are splendid examples of organization, of the organization of human as well as other material.

In this defense effort, all the way from our local defense councils, through our state and into our national councils, the problem of organization is paramount. It isn't a very easy problem for people who are not trained and practiced in it simply because they are not in the habit of thinking in this way. The average American for many generations has been an individualist. The highest thing that he sought was the ability to do what he wanted when he wanted and as he wanted. He rebelled against organization unless it was absolutely necessary because it kept him from doing the thing that he wanted to do at the time he wanted to do it. Organization told him, perhaps, that staggered hours would be a good thing, and yet he didn't want to stagger hours. He wanted to get up and go to bed as he pleased, as he had been in the habit of doing—and he is still doing it. We wanted to do as we pleased—and most of us are still doing it. I am afraid there is going to come an end to that, and I am afraid that we are all going to do some things we don't please in a way we don't please to do them, at times when it is extremely inconvenient for us to do them.

There has been, perhaps, a mixing of the word "regimentation" and the word "organization"; but I would rather give myself up to an organization under the rules and practices and traditions of America—where these rules and practices and traditions still hold—where I could choose my own organizer, than have somebody sail in either from Japan or Germany or both and regiment me in the way that they wanted it done.

I believe thoroughly that the next step in this great process of national defense is to give ourselves up wholeheartedly into the hands of an organizer of our own choosing. What else can we do and win? After all, it seems so simple. If one-half of us disagree and fight with the other half, we have a civilian army composed of half our people, who are fighting against America.

February 10th

It is with regret and only after much thought that I make the announcement that this will be the last issue of the NEWARK ENGINEERING NOTES until after the war. It has been with much interest and not a little pride that I have followed the evolution and growth of the magazine from its first issue in April, 1938, to the present. I believe that you all will agree that it has served a very useful purpose. It has not only kept our alumni in touch with the College but has served as a medium for acquainting our many friends in industry and education with the progress of the College.

We shall all have to make sacrifices. We shall have to do without some of those things which we have enjoyed. The preparation of a magazine such as the NEWARK ENGINEERING NOTES requires the expenditure of much time, thought and energy, not only by the editor and the members of his staff, but also by each contributor. Our every effort at this time should be directed to the successful prosecution of the war. Anything which may give any indication of interfering with the expenditure of the maximum effort by each of us toward this end must be eliminated. So! The NOTES must be discontinued for the present so that all who have given so generously to its successful development may be free to give even more generously of their energy and time to their Government. I sincerely hope that it will not be too long before the NOTES will again be published.

ALLAN R. CULLIMORE

INDUCTIVELY-FILTERED RECTIFIER CIRCUITS

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Rectification is, inherently, what might be called a "recurrent transient" phenomenon. Electronic rectifier tubes can be considered as automatic voltage- and current-operated switches which function in such a manner as to open during the negative half-cycles of an alternating wave and to close during the positive portion. If the resultant predominantly positive pulsating current can be smoothed out sufficiently by the filter circuit, a direct current results.

Each time that one of the rectifier anodes starts or ceases to conduct, the phenomenon is equivalent to that of the opening or closing of a switch at that instant. Each time that a switch is opened or closed in a circuit containing inductance, a readjustment of the energy distribution must occur in the circuit. These readjustments occur as transient currents and voltages. A mechanically-operated switch may be opened at any point of the alternating-current cycle; if it is not opened when the current is instantaneously zero, an additional transient, arcing at the switch points, results. In the electronic rectifier, however, conduction ceases only when the current wave is passing through zero.

After a rectifier circuit has been operating for a period long enough so that equilibrium has been established, the switching operations will occur regularly, that is, at corresponding points of each successive cycle of the input voltage wave. It is, therefore, logical to attack the mathematical solution from the standpoint of periodically recurring transients. By this means a rigorous solution is obtained which must check as closely in practice as the analyzed equivalent circuit checks with the actual circuit.

It is the purpose of this paper* to develop formulas for basic parallel and bridge electronic rectifier circuits having inductive filters. The equations will be developed in a form showing in particular how the characteristics depend on the ratio of inductance to load resistance, and on the number of phases employed. Formulas will be obtained which express the following relations, important from the standpoint of rectifier tube design and selection:

1. Peak tube current
2. Average tube current
3. Peak inverse tube voltage

and the following values, important from the standpoint of circuit operation:

1. Average load current
2. Average load voltage
3. Amount of ripple
4. Voltage regulation
5. Maximum voltage at no load
6. Input power and power factor

The following assumptions are made concerning the properties of the circuit:

1. Tube drop during conduction, and tube back current during non-conduction, are negligible;
2. The circuit has reached equilibrium;
3. The supply has balanced phase voltages and negligible impedance;
4. Capacitance in the circuit is negligible.
5. Inductance in the circuit is constant.

The resultant equations and characteristic curves are expressed entirely in terms of the values of:

E_m, the maximum instantaneous value of the input phase voltage;

I_m, the maximum instantaneous value of the sinusoidal current which would flow if the filter and load were to be connected directly across the phase voltage, with no rectifier interposed;

θ, the phase angle of the current and voltage under the above steady-state conditions;

n, the number of phases ($n = 1$ for a half-wave rectifier, $n = 2$ for a full-wave circuit).

Half-Wave Rectifier

Although of little practical importance, the half-wave rectifier has a circuit which offers a simple introduction to the method of analysis employed throughout. The circuit *(Figure 1)* is a simple series circuit in which conduction by the rectifier tube commences as soon as the voltage e becomes positive and greater than zero. The current i, starting at the time $\omega t = 0$, tends to approach in time phase the steady-state current i_s, (shown dotted) which would flow if there were no rectifier. By the time the current again becomes zero and conduction ceases, the angular time difference between i and i_s is small so that β very nearly equals $\pi + \theta$. It will be noted that conduction continues for a time after e has become negative.

The sinusoidal voltage has the familiar form

$$e = E_m \sin \omega t \qquad (1)$$

in which E_m is the peak value of the voltage and $\omega = 2\pi f$ radians per second. Now it is known that the current can be represented by the sum of two terms, so that at any instant

$$i = i_s + i_t \qquad (2)$$

where i_s is the instantaneous value of the steady-state current, shown dotted in the diagram, and given by

$$i_s = I_m \sin (\omega t - \theta) \qquad (3)$$

in which I_m is computed by the familiar series-circuit formulas

$$I_m = \frac{E_m}{\sqrt{R^2 + (\omega L)^2}} \qquad (4)$$

$$\theta = \arctan \frac{\omega L}{R} \qquad (5)$$

where θ is the phase angle between steady-state voltage and current. It is known that the transient component of the current is of the form

$$i_t = K\varepsilon^{-\frac{R}{L}t} \qquad (6)$$

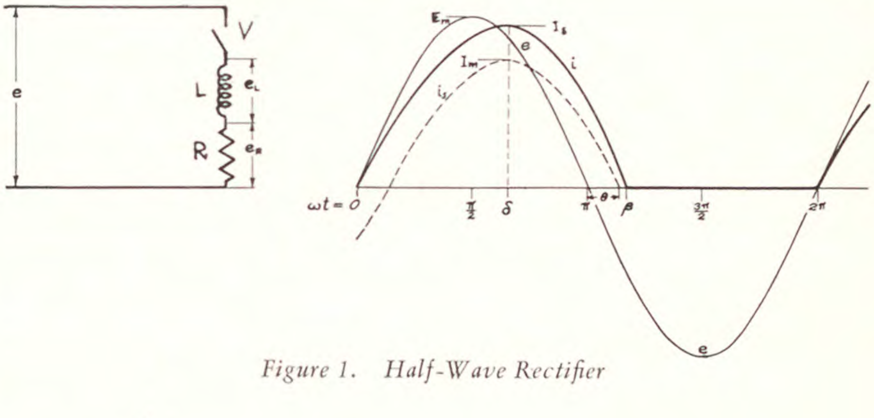

Figure 1. Half-Wave Rectifier

*Condensed from portions of a thesis entitled "Analysis of Electronic Rectifier Circuits" submitted to the Newark College of Engineering by the author in June, 1939, for the degree of Electrical Engineer.

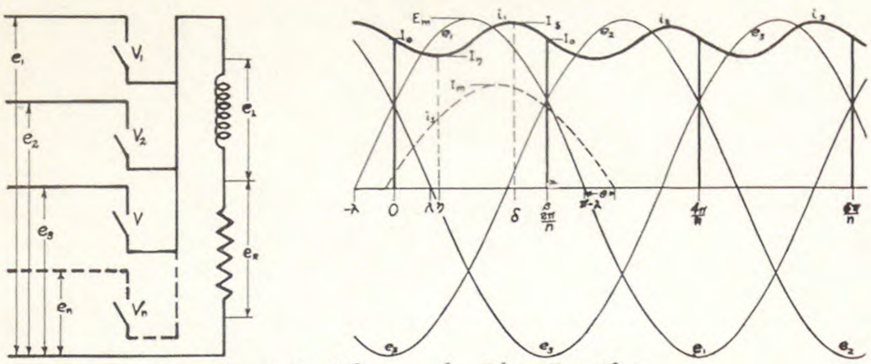

Figure 2. Three- and n*-Phase Rectifiers*

Therefore, the total current

$$i = I_m \sin(\omega t - \theta) + K\varepsilon^{-\frac{R}{L}t} \qquad (7)$$

To evaluate the constant of integration K, let $t = 0$, at which time, since the circuit is open, $i = 0$. Solving thus for K in equation 7, we get

$$i = I_m\left[\sin(\omega t - \theta) + \varepsilon^{-\omega t \cot\theta}\sin\theta\right] \qquad (8)$$

Conduction will, therefore, occur from $\omega t = 0$ to $\omega t = \beta$, where β is the angle at which $i = 0$ in equation 8, whence

$$\sin(\beta - \theta) + \varepsilon^{-\beta\cot\theta}\sin\theta = 0 \qquad (9)$$

where $\beta < \pi + \theta$ if $\theta \neq 0$, or $\beta = \pi$ if $\theta = 0$.

It will be seen from equation 9 that β is dependent only on θ, and numerical solutions may readily be computed and a curve of β vs. θ drawn. The curves of *Figure 3* were drawn to indicate quantitatively the difference between the point at which the steady-state current would reach zero and the point at which the actual current reaches zero (see *Figure 1*). The former would, of course, be zero at $\omega t = \pi + \theta$. So the difference between this value and the value of β was plotted against the phase angle θ. Examination of the curves shows that for phase angles less than 40 degrees the transient current has very nearly coincided with the steady-state sine wave by the time the cutoff point is reached.

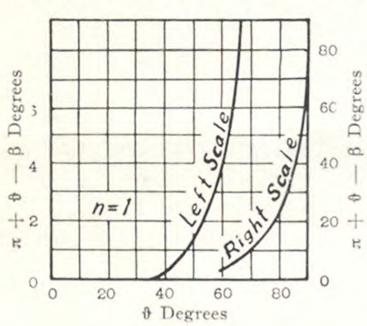

Figure 3
Angle of Commutation for Half-Wave Rectifier

The average and rms values of the load current can be computed by integration of equation 8 between the limits 0 and β. This will be done later for a more general circuit. The peak current, I_δ, will occur at some angle δ, which must be found by differentiation of equation 8. If the derivative be set equal to zero and we substitute δ for ωt, the resultant equation for δ in terms of θ is

$$\cos(\delta - \theta) - \varepsilon^{-\delta\cot\theta}\sin\theta\cot\theta = 0 \qquad (10)$$

When solved for δ, which can readily be done numerically, this angle represents the point at which the peak tube and load current occurs:

$$\frac{I_\delta}{I_m} = \sin(\delta - \theta) + \varepsilon^{-\delta\cot\theta}\sin\theta \qquad (11)$$

The average load voltage at no load, from equation 1, is

$$E_{avg} = \frac{E_m}{2\pi}\int_0^\pi \sin\omega t\, d(\omega t) = \frac{E_m}{\pi} \qquad (12)$$

So the voltage regulation at any load is

$$\frac{E_m}{\pi} - RI_{avg}\text{ volts} \qquad (13)$$

Full-Wave Rectifier

For the full-wave rectifier, the diagrams will be similar to those of *Figure 2* if $n = 2$, that is, there are two tubes V_1 and V_2 and two sine-wave voltages e_1 and e_2 which intersect at zero volts at the angle π. Zero time has been chosen at the point where the current shifts from V_2 to V_1. Consider first this shift of current, at $\omega t = \pi$. At this instant, assume that current has been flowing through V_1 according to the equation 8, derived for the half-wave rectifier. This current would normally cease at $\omega t = \beta$. But before this point is reached, at the point of the intersection of the voltages e_1 and e_2, the voltage of both anodes is instantaneously identical (in this case zero). Immediately thereafter, the anode of V_2 is at a higher potential than its cathode; hence it starts to conduct.

When the anodes of both tubes are conducting, they are electrically connected and must, therefore, be at the same potential, if the tube impedances are neglected. Also, if source impedance is neglected, there is now no inductance in the circuit through V_1 to V_2, hence the current in V_1 will immediately become in phase with the voltage. Since this voltage is zero, the current i_1 in V_1 will instantly drop to zero, and conduction through V_1 will cease. Furthermore, at this same instant e_2 is becoming greater that e_1, and if the two tubes are conducting, a short-circuit current will tend to flow. This current travels in a direction opposite to the current through V_1. If there is no source impedance this short-circuit current will rise immediately, thus bringing the current through V_1 to zero and leaving V_2 carrying the entire current. During this short-circuit of V_1-V_2, the load circuit is also shorted through the zero impedance source and the tubes. Because of the inertia of the inductive circuit, the current in this circuit cannot change instantly. Hence, since commutation occurs instantly, the load current through V_2 after commutation will be identical with that through V_1 prior to commutation.

In short, with an impedanceless source, commutation of the load current from tube to tube occurs instantaneously, with no angle of overlap. Let the value of the current i at the instant of commutation be I_o. Conditions at that point obtain which are equivalent to the switching of a circuit when the current is not zero over to an a-c supply voltage which is instantaneously zero at the time of switching.

The equation of the current will be identical with that of equation 7. However, the value of the constant will differ, for when $t = 0$, $i = I_o$ (not zero, as before). Then

$$K = I_o - I_m\sin(-\theta) \qquad (14)$$

and

$$i = I_m\left[\sin(\omega t - \theta) + \left(\frac{I_o}{I_m} + \sin\theta\right)\varepsilon^{-\omega t\cot\theta}\right] \qquad (15)$$

This equation can be applied to either half cycle, if ωt is taken as zero when the voltage is zero.

To determine the value of I_o, consider the fact that when equilibrium has been reached in the circuit, the values of i at the start ($\omega t = 0$) and conclusion ($\omega t = \pi$) of each half-cycle will be identical and equal to I_o. Substitution of these values for the variable in equation 15 yields two equations which when solved simultaneously by subtraction yield

$$\frac{I_o}{I_m} = \frac{2\sin\theta}{1 - \varepsilon^{-\pi\cot\theta}} - \sin\theta \qquad (16)$$

This equation gives the commutating current in terms of the circuit parameters. A convenient method used throughout is to express all currents as ratios with respect to I_m, and this ratio is plotted in *Figure 4*.

As in the half-wave circuit, the point at which the maximum current occurs can be solved for by differentiation of equation 15. This gives for δ, the angle at which the maximum current occurs:

$$\cos(\delta - \theta) - \frac{\frac{I_o}{I_m} + \sin\theta}{\tan\theta}\,\varepsilon^{-\delta\cot\theta} = 0 \qquad (17)$$

A plot of this equation is shown on the curve labeled $n = 2$ in *Figure 5*. Then substituting back, the maximum current is readily found by letting $\omega t = \delta$ in equation 15 (*Figure 6*). As seen in *Figure 2*, there exists in this circuit a *minimum* current which is not zero, and which occurs a short time after commutation. Let this point be the angle η, which will be the minimum value given by the differentiation which led to equation 17. Then the angle at which the minimum current occurs will be given by equation 17 with η substituted for ωt, and the minimum current I_η will be given by substitution of the angle η in equation 15. These relationships are graphed in *Figure 7* and *Figure 8*.

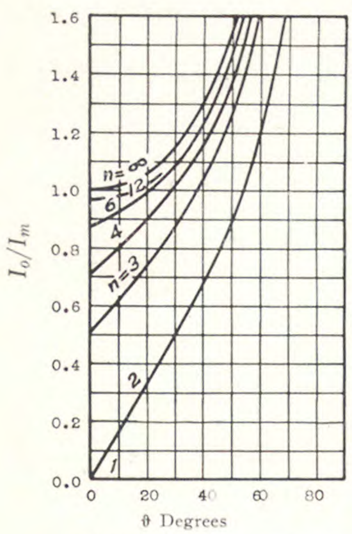

Figure 4. Relative Commutating Current

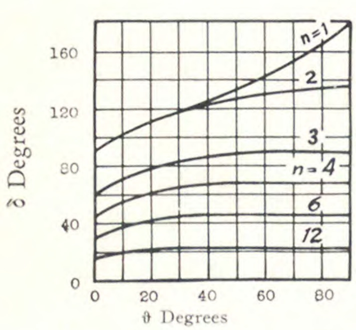

Figure 5
Angle of Peak Current

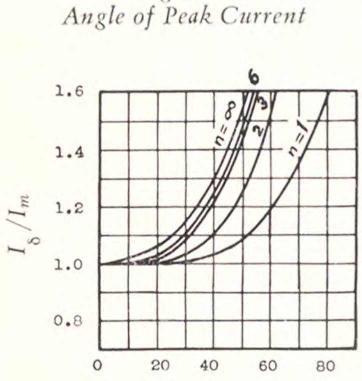

Figure 6
Relative Peak Current

Three-Phase Rectifier

For the three-phase circuit, shown in *Figure 2*, the conditions which obtain at commutation (intersection of the adjacent positive loops of the phase-voltage waves) are the same as for the full-wave rectifier except that the phase voltage is not zero. The current would, therefore, not of itself fall to zero in the tube about to cease conduction, but is brought to zero by the circulating short-circuit current as described in the previous section. Here, again, with an impedanceless source, there will be no angle of overlap during which both tubes conduct simultaneously. The equation of the current during conduction will be the same as equation 15. The period of conduction is shorter, and the voltage is not zero at the start of conduction.

We take $\omega t = 0$ at the point of commutation, as before, but we now have a new angle to consider, λ, the angle at which the voltage is zero, which previously was β (see *Figures 1* and *2*). For a three-phase circuit, by inspection of the wave diagram, $\lambda = \pi/6$ and $\beta = 2\pi/3$. The points of commutation will occur at $2\pi/3$, $4\pi/3$, and 2π. Due to the shift in the point of zero time with respect to the phase voltage, the steady-state current becomes

$$i_s = I_m \sin(\omega t + \lambda - \theta) \qquad (18)$$

and in the other equations where previously the term $\omega t - \theta$ occurred, we will hereafter have instead $\omega t + \lambda - \theta$. Instead of writing out the equations for this case alone, we shall continue with the general case in detail.

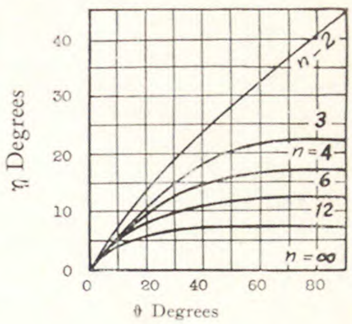

Figure 7
Angle of Minimum Current

The n-Phase Rectifier

All the circuits analyzed above and all other multiphase circuits of the type shown in *Figure 2*, and bridge circuits of any number of phases, may be grouped under the general term of n-phase rectifiers. It will be shown that one set of equations may be derived which will apply to any of these circuits when inductively filtered. For the half-wave circuit, $n = 1$; for the full-wave and the ordinary bridge circuit, $n = 2$; for the three-phase parallel (*Figure 2*) and the three-phase bridge, $n = 3$, etc. for multi-phase circuits. The equations derived below will, therefore, cover anything that has been developed above, and the method of attack will be the same.

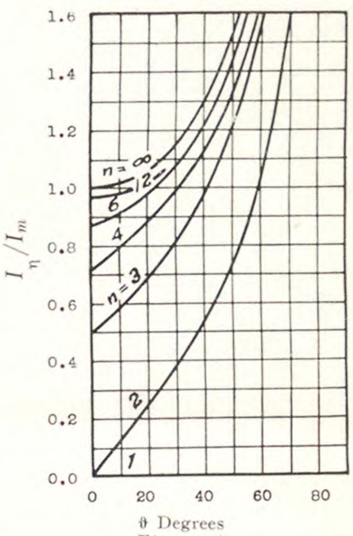

Figure 8
Relative Minimum Current

The independent variables will be I_m (equation 4), θ (equation 5), and n (the number of phases). The voltage is the phase voltage to neutral. Inspection of the wave diagram indicates that the intersection of adjacent voltage waves occurs at

$$\lambda = \pi/2 - \pi/n \qquad (19)$$

and the next intersection at

$$\beta = 2\pi/n \qquad (n \neq 1) \qquad (20)$$

except for the half-wave case. The instantaneous steady-state current is given by equation 18, and the transient component now will be

$$i = I_m \sin(\omega t + \lambda - \theta) + K\varepsilon^{-\omega t\cot\theta} \qquad (21)$$

When $t = 0$, $i = I_o$

so $\quad K = I_o - I_m \sin(\lambda - \theta) \qquad (22)$

and the current during the conducting period from 0 to β is

$$i = I_m \left\{ \sin(\omega t + \lambda - \theta) + \left[\frac{I_o}{I_m} - \sin(\lambda - \theta)\right]\varepsilon^{-\omega t\cot\theta} \right\} \qquad (23)$$

The value of I_o may be determined as before, since it must be the same at each commutating point, and the equation (similar to equation 16) is

$$\frac{I_o}{I_m} = \frac{\sin(\beta + \lambda - \theta) - \sin(\lambda - \theta)}{1 - \varepsilon^{-\beta\cot\theta}} + \sin(\lambda - \theta) \qquad (n \neq 1) \qquad (24)$$

$$= 0 \qquad (n = 1) \qquad (25)$$

The peak current will occur at some angle $\omega t = \delta$, and the minimum current at some angle η (where $\eta < \delta$), both of which are found from the derivative of equation 23 set equal to zero:

$$\cos(\omega t + \lambda - \theta) - \cot\theta \left[\frac{I_o}{I_m} - \sin(\lambda - \theta) \right] \varepsilon^{-\omega t \cot\theta} = 0 \qquad (26)$$

The above equation yields two solutions for ωt, the smaller angle being the value of η, and the larger the value of δ. These angles are plotted in *Figures 7* and *5*. Substitution of the angles for ωt back in equation 23 gives the ratios of the maximum current, I_δ, and the minimum current, I_η, to the circuit parameter, I_m:

$$\frac{I_\delta}{I_m} = \sin(\delta + \lambda - \theta) + \left[\frac{I_o}{I_m} - \sin(\lambda - \theta) \right] \varepsilon^{-\delta \cot\theta} \qquad (27)$$

$$\frac{I_\eta}{I_m} = \sin(\eta + \lambda + \theta) + \left[\frac{I_o}{I_m} - \sin(\lambda - \theta) \right] \varepsilon^{-\eta \cot\theta} \qquad (n \neq 1) \qquad (28)$$

$$= 0 \qquad (n = 1) \qquad (29)$$

In these equations there appears the ratio involving the current at commutation, I_o, given in equation 24. These three current ratios are graphed in *Figures 4*, *6*, and *8*.

From these data the characteristics of any particular rectifier circuit may be computed.

Resulting Characteristics

In solving for the equations of the rectifier's characteristics, the first things to find are I_m and θ by equations 4 and 5, which are the ordinary steady-state solutions for the series inductance-resistance circuit. The resistance, R, of course, includes the resistance of the inductive filter. Next we find

$$\lambda = \pi/2 - \pi/n \qquad (n \neq 1) \qquad (19)$$

$$\beta = 2\pi/n \qquad (n \neq 1) \qquad (20)$$

or, for the half-wave circuit

$$\lambda = 0 \qquad (n = 1)$$

$$\sin(\beta - \theta) + \varepsilon^{-\beta \cot\theta} \sin\theta = 0 \qquad (n = 1) \qquad (9)$$

It is noteworthy that β in both formulas represents physically the same quantity, namely, the angle at which conduction ceases. Mathematically, however, the two β's are entirely different quantities; the former a function of n, the latter a function of θ. In circuits where at no time is the voltage zero, the end of conduction is determined solely by the start of conduction by the next tube; hence it is a function of n. But in the half-wave circuit, the current comes to zero at a point determined by the angle at which the steady-state current would come to zero, and is, therefore, a function of θ. This latter equation is best solved by use of the curve of *Figure 3*.

Each succeeding wave of current is dependent on the value which the current had at the instant conduction by the previous anode ceased, and for this reason the commutation current, I_o, is prominent in the analysis. In order to use a pure number, the ratio given by equations 24 and 25 is used. These formulas are plotted in *Figure 4*. Their general shape is easily verified as follows: If the load phase angle is zero, the load is a pure resistance so the current and voltage will be in phase. Therefore, when conduction ceases at the intersection of two adjacent voltage waves, the ratio plotted in the figure will be the same as the ratio of voltages, that is, the ratio of the voltage at the intersection, to the peak voltage. Also, consider the curve for the case of an infinite number of phases. This assumption would result in a pure d-c load voltage E_m, regardless of filtering. At zero load phase angle the impedance is R, and a current $I_m = E_m/R$ would flow at every instant, so $I_o = I_m$. At ninety degrees phase angle the d-c voltage would be applied to a pure reactance; hence an infinite current flows throughout the entire cycle. Between these extremes the commutation current varies with the amount of resistance in the circuit, thus:

$$\frac{I_o}{I_m} = \sec\theta \qquad (n = \infty) \qquad (30)$$

Also of interest are the values of the maximum and the minimum currents and the points of the cycle at which they occur. The latter are found from equation 26, which gives both solutions. Curves of δ are plotted in *Figure 5*, which shows at what angle the maximum current will exist. When the phase angle is zero, there is a pure resistance load but no transient, so that the maximum current occurs at the same point as the maximum voltage, or

$$\delta = \pi/2 - \lambda \qquad (\theta = 0) \qquad (31)$$

As the load approaches a pure inductance, for the single-phase case since λ occurs at zero degrees, the maximum current will occur at an angle approaching 180 degrees behind the point of zero current. For other numbers of phases it is obvious that the peak current must occur before the point of commutation, for if there were no interval between the peak and commutating points there would be either a discontinuity in the wave shape at commutation or the current would increase indefinitely. As is obvious, for a large number of phases there is a smaller possible range for the angle δ to vary.

The curves for the angle of minimum current are drawn in *Figure 7*. The minima must appear prior to the maxima, and will occur soon after conduction starts if there is inductance in the circuit. The fact that there is a minimum current after the voltage has begun to increase is explainable, because when the transient current begins with an initial value greater than zero it tends to drop off toward its steady-state value. Due to inductance this change occurs slowly, so that a loop is obtained after which the curve courses upward.

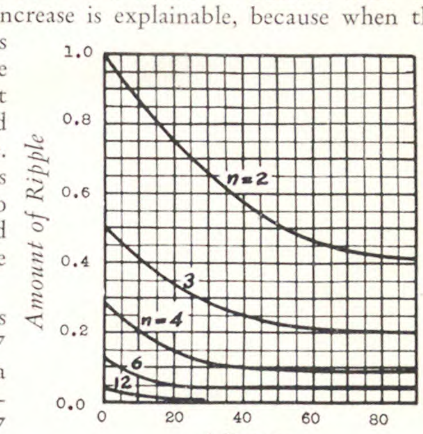

Figure 9
Amount of Ripple

With the values given in *Figures 6* and *7* the maxima and minima currents may be calculated by equations 27 and 28, and these are plotted in *Figures 6* and *8*. For an infinite number of phases the curves are identical, and for fewer phases and for smaller values of load phase angle the difference between the maximum and minimum currents increases. This difference represents the magnitude of the ripple, shown in *Figure 9*.

Amount of Ripple

The wave-shape of the d-c voltage across the resistance load has the same contour as the current wave, and varies from a maximum value of RI_δ to a minimum value of RI_η. Per cent ripple is usually defined as the rms value of the pulsating component of the load voltage, divided by the average d-c load voltage. The calculation of this expression does not seem worth while, for the actual effects of the ripple do not depend so much on this figure as on the magnitudes of the individual harmonics present. Probably just as useful a measure of the ripple magnitude is to express the maximum amplitude of the ripple from trough to peak, divided by the maximum value of the load voltage. Then we have:

$$\frac{RI_{ripple}}{RI_m} = P = \frac{I_\delta}{I_m} - \frac{I_\eta}{I_m} \qquad (32)$$

where P is the "relative amount of ripple."

At zero phase angle the amount of ripple is obviously determined from the point of intersection of the voltage (or current) waves. Hence, it is unity for the full-wave rectifier, and one-half for the three-phase rectifier. Regardless of phase angle, the amount of ripple will be unity for the half-wave circuit and zero for the rectifier with an infinite number of phases. The curves of *Figure 9* indicate the variation in ripple with number of phases and load phase angle.

Peak Values

We have computed the peak load and tube current, I_δ. The inverse peak voltage for parallel rectifiers shown in *Figure 3* is:

$$E_\gamma = 2E_m \text{ occurs at } \gamma = \pi/2 - \lambda \text{ where } n \text{ is even} \qquad (33a)$$

$$E_\gamma = E_m \text{ occurs at } \gamma = \pi/2 \text{ for } n = 1 \qquad (33b)$$

$$E_\gamma = \sqrt{3}\, E_m \text{ occurs at } \gamma = \pi/2 \text{ for } n = 3 \qquad (33c)$$

the last being the only practical case of odd values of n. For bridge circuits the inverse peak voltage will be one-half the value given in equations 33.

Average and Rms Values

From the formula for the instantaneous load current, equation 23, most of the equations of characteristics can be obtained by integration with respect to ωt from 0 to β. The average rectified current:

$$I_{avg} = \frac{1}{\beta}\int_0^\beta (\text{equation } 23)\ d(\omega t) \qquad (34)$$

yielding the lengthy expression

$$\frac{I_{avg}}{I_m} = \frac{1}{\beta}\left\{\cos(\lambda - \theta) - \cos(\beta + \lambda - \theta) + \tan\theta\left[\frac{I_o}{I_m} - \sin(\lambda - \theta)\right]\left[1 - \varepsilon^{-\beta\cot\theta}\right]\right\} \qquad (35)$$

which is nevertheless easy to solve.

However, certain approximations were arrived at which greatly simplify this calculation without detracting greatly from the accuracy for large values of n and θ. If the ripple is small it may be assumed to be of sinusoidal shape, since the error involved with respect to the total current is then very small. If this assumption is made, the average of the ripple wave about its axis will be zero, and its axis will lie half way between I_η and I_δ. Then

$$I_{avg} = I_\eta + \tfrac{1}{2}(I_\delta - I_\eta) \quad \text{(Approx.)} \qquad (36)$$

so

$$\frac{I_{avg}}{I_m} = \frac{1}{2}\left(\frac{I_\eta}{I_m} + \frac{I_\delta}{I_m}\right) \quad \text{(Approx.)} \qquad (37)$$

The equations for I_{avg} given above are the values of the rectified load current. The average value of the current in each rectifier will be I_{avg}/n.

By integration of the squared wave of equation 23, the rms values of the rectified current and the supply current per phase can be obtained. There results a readily-evaluated but unwieldy expression which will be omitted here.

To get a simpler expression which is approximate but sufficiently accurate for large n and θ, consider that the instantaneous value of the total current will be the average value plus the assumed sinusoidal ripple, or

$$i = \tfrac{1}{2}(I_\delta + I_\eta) + \tfrac{1}{2}(I_\delta - I_\eta)\sin\omega t \qquad (38)$$

The rms value of the above equation gives

$$I_{rms} = \frac{1}{I_\delta}\sqrt{(I_\delta + I_\eta)^2 + (I_\delta - I_\eta)^2} \quad \text{(Approx.)} \qquad (39)$$

which is easy to calculate.

That equation for I_{rms} is the value of the rectified load current. The rms value of the current supplied per phase will be $\frac{I_{rms}}{\sqrt{n}}$.

Curves of the average and rms values of rectified current are given in *Figures 10* and *12*. It will be noted that except for very small values of n and θ, the corresponding curves are almost identical, so that I_{avg}, which is easy to calculate, will be sufficiently accurate when used as I_{rms}.

From this effective current value the value of the voltage across the resistance may be determined:

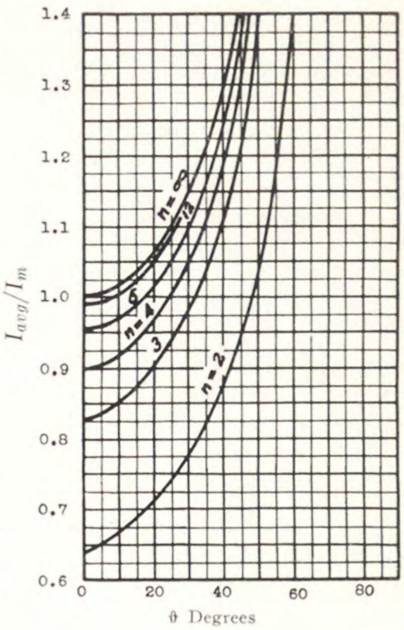

Figure 10
Relative Average Rectified Current

$$\frac{E_{rms}R}{E_m} = \frac{I_{rms}}{I_m}\cos\theta \qquad (40)$$

The instantaneous voltage

$$e = E_m\sin(\omega t + \lambda) \qquad (41)$$

and at no-load the rectified voltage

$$\frac{E_{avg}}{E_m} = \frac{n}{2\pi}\left[\cos\lambda - \cos(\beta + \lambda)\right] \quad (I_{avg} = 0) \qquad (42)$$

Under load, the voltage across the resistance is

$$E_{avg} = RI_{avg} \qquad (43)$$

proportional to I_{avg}. When considered with respect to the load phase angle, the shape of the voltage curve will not be the same as the shape of the current wave, because R is a variable with respect to θ. For values of n greater than three, the average voltage does not vary greatly with the phase angle, but rises slowly as the phase angle increases.

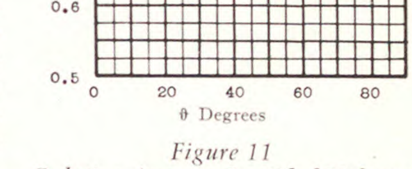

Figure 11
Relative Average Rectified Voltage

Equation 42 minus equation 43 gives the value of the voltage regulation:

$$\frac{nE_m}{2\pi}\left[\cos\lambda - \cos(\beta - \lambda)\right] - RI_{avg} \qquad (44)$$

Power Equations

Difficulties arise when consideration is given to the power relations of this type of circuit, because of the wave form, which is neither sinusoidal nor constant direct current. The power supplied is usually paid for on the basis of the readings of an ordinary a-c wattmeter, voltmeter, and ammeter, which are dynamometer instruments. The total voltamperes supplied will then be

$$VA = 0.707 nE_m I_{rms} \qquad (45)$$

The dynamometer wattmeter (or watthour meter) reads the power (or energy) due to the heating value of the current in the resistance load (and resistance of the inductive filter choke):

$$\frac{P_R}{n} = I_{rms}^2 R \qquad (46)$$

the average load power contributed by each phase of the supply.

In circuits such as those of rectifiers, where the voltage and current have different wave-shapes, the symbol *P.F.* represents only a "power factor" in the literal sense, and cannot be interpreted as the cosine of the phase angle between voltage and current, but only by the definition

$$P.F. = P_R/VA \qquad (47)$$

This cosine may be said to be that of a fictitious angle by which the rms voltage of any phase leads its rms equivalent sinusoidal current. This equivalent current has the same rms value as the actual current, and might be written as

$$i = \sqrt{2}\, I_{rms} \sin(\omega t - \arccos P.F.) \qquad (48)$$

Bridge Rectifiers

The equations for the ordinary bridge circuit, if tube drop is neglected, will be identical with those for the full-wave rectifier, except for the inverse voltage. In bridge circuits the phase reversal is accomplished by the extra pair of tubes instead of a switching operation from one supply to another 180 degrees out of phase. When one tube of a pair conducts, the other must conduct also, so that the currents in the two will be identical at all times, and will also be the same as the current through the tube of the full-wave rectifier. In the bridge circuit the same overall inverse voltage exists, but it is divided equally between the two tubes of the non-conducting pair.

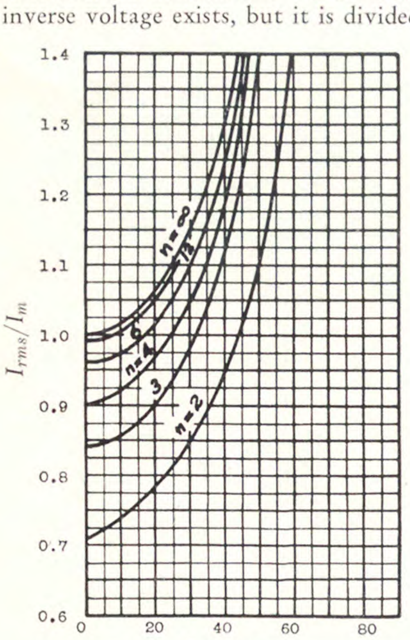

Figure 12
Relative RMS Rectified Current

Bridge circuits of any number of phases may be used, and the analogy with the corresponding parallel rectifier will be the same as that described above. Such circuits might better be referred to as series-parallel rectifiers, since the bridge configuration no longer exists.

It will be seen that such circuits are better suited to high-current requirements, since, at present, high - current tubes have inherently low inverse-voltage ratings. The circuit also permits of cheaper transformer construction for a given voltage.

Supply Impedance and Tube Drop

When the impedance of the supply circuit, the equivalent tube resistance (if it is a vacuum rectifier), and the voltage drop across the tube (if it is a vapor rectifier) are considered, the equations derived are considerably more complicated. The principal cause of additional difficulty is the existence of the phenomenon known as "overlap" at the commutating point. If there is any resistance or inductance in the source of supply, the current cannot change at an infinite rate at the instant when the two tubes have equal voltages. Then a finite time will elapse during which the current is transferred from one rectifier to the next. During this period both tubes conduct simultaneously; hence, it is known as the angle of overlap.* Another equation must be written for the conditions which obtain during this period of short-circuit, and it is this additional equation which, when solved simultaneously with the other pair of formulas, complicates the solution. Space prohibits the inclusion of such an analysis here.

The commutating period or "angle of overlap" increases with the load current and the supply impedance, and causes poor voltage regulation, so that in practice the supply impedance is kept as small as practicable.

With the method of computation given in this paper, an improvement in accuracy may be obtained if the supply impedance per phase is included in the load impedance and the tube drop subtracted from the maximum phase voltage. Then the only remaining source of error is in ignoring the angle of overlap. In a typical experimental measurement of a 0.100-amp. 3-phase circuit, a supply resistance of 10 per cent of the load resistance produced an angle of overlap of 4 degrees. By the method of computation given above there resulted approximately a 10 per cent deviation from the results obtained by the accurate method or by experimental measurement. Part of this error must be allotted to the value of inductance used, since the choke was iron-clad and highly saturated.

Design Procedure

To design a rectifier from the last four sets of curves is a simple matter. Assume that the supply frequency, the tolerable amount of ripple, the load resistance, and the desired d-c load voltage are known.

First, from *Figure 9* and economic considerations, the number of phases can be chosen. Then the curve selected will show the value of load phase angle required to produce the amount of ripple decided upon. The tangent of this angle is the ratio of filter (plus supply) reactance to load (plus filter and supply) resistance. From this ratio and the value of the supply frequency, the requisite filter inductance can be computed.

From *Figure 11* the ratio of the rectified average voltage to the maximum a-c phase voltage can be determined. From the value of the latter the rms supply voltage per phase (or, if a given supply voltage is to be utilized, the required transformer ratio) may be computed. (To E_m the rectifier voltage drop may first be added if vapor rectifiers are to be used.)

If the circuit is treated as a steady-state single-phase a-c circuit with no rectifier, I_m can be calculated, since E_m and the impedance are known. Then, from *Figures 10* and *12*, the currents of the rectifier on the d-c side may be determined. The rms current per phase on the a-c side will be the rectified value divided by $\sqrt{n}$.

Peak tube current (*Figure 6*), peak inverse tube voltage (equation 33), and average tube current (the value of average rectified current divided by n) will determine the ratings of the rectifier tubes to be employed. Voltage regulation for the rectifier circuit may be determined from *Figure 11*. The over-all

*Jolley, "A-C Rectification"

(Continued on page 14)

VIBRATION ISOLATION

By Harry F. Ritterbusch

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The need for vibration control has become a recognized fact with the advent of high-speed machinery, the desirability for quiet operation, and the necessity for preventing the transmission of vibrations to precision machinery. Furthermore, the increased knowledge of the theory underlying vibration isolation, together with the means for obtaining accurate measurements of vibrations, has resulted in the manufacture of relatively simple devices that are inexpensive and easy to install. Many cases have been recorded where excessive vibration has resulted in serious physical damage and loss of life. For instance, the failure of the Tacoma suspension bridge was attributed to dynamic instability to resist vibration, although it was difficult to predict in advance the nature of the disturbance. Another case shows that lack of vibration isolation for a refrigeration machine in an ice rink was responsible for damage to household articles in homes near the rink. Failure of machine parts due to repeated stress is a common manifestation of an uncontrolled vibration. In the aircraft industry in particular is the need for reducing vibrations of prime importance. Today, every plane manufactured in this country has incorporated within it a number of devices which serve to reduce the severity of vibrations. A representative group of applications which show the wide scope of the problem of isolation is listed below.

Internal combustion engines
Air compressors
Punch presses
Electric refrigerators
Electric motors
Fans and blowers
Turbo-generators
Business machines
Instruments and instrument panels
Printing presses
Railroad equipment
Marine equipment.

The question of physical damage as the result of vibration is not the only factor to be considered. Although many cases of vibration may not be harmful from the standpoint of mechanical design, one must also make allowance for the reaction of the human being. Noise and vibrations of the body are factors that deserve attention in the design of any automotive vehicle. The reader is aware, no doubt, of how distinctly uncomfortable engine noise in an automobile may become on a long drive. This problem of riding comfort is difficult to analyze because of the varied reactions of different people, but many concrete results have been obtained by experiment and are reported in a paper, "Human Reactions to Vibrations," by H. M. Jacklin, *Transactions S.A.E.*, 1936, Volume 31, pp. 401-407.

Before any attempt is made to discuss the methods of correcting vibrations, it is well to establish a few principles involved in the theory of vibration isolation, and define a number of the more common terms which are used in discussing the subject.

Vibration is a periodic motion which reverses itself twice during every complete cycle. Note that an important requisite of a vibration is the fact that the motion must be of a periodic nature.

Frequency is the number of complete cycles for a given unit of time. It is usually stated as so many cycles per second.

Period is the time required for one complete cycle, and is the reciprocal of frequency.

Amplitude is the maximum displacement of the body as measured from its point of static equilibrium. One also speaks of the velocity and acceleration amplitude in the motion of a vibrating body, but in this article amplitude will be confined to displacement.

Natural frequency is that frequency at which a body will vibrate after removal of the external driving force. It should be remembered that the natural frequency is an inherent characteristic of the system, and is not affected by the magnitude of the disturbance or displacement.

Forced frequency, as the name implies, is the frequency of the external driving force. In rotating machinery, the forced frequency is generally the revolutions per minute of the unit.

Resonance is a result of having a forced frequency of the same value as the natural frequency of the system.

Damping is produced by the application of an internal or external force which continually opposes a vibration. The internal or external force in mechanical vibrations is a frictional force, and, hence, any damping represents a dissipation of energy which obviously must come from the system.

Because the simplest form of vibration is a simple harmonic motion, further discussion will be based on assuming the vibration to be of this character. For more complex conditions, the vibration can be broken down into a series of harmonics of various orders. In addition, only systems of one degree of freedom will be treated in order to confine the material to its more understandable form.

The natural frequency of a body may be calculated by the following formula:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{kg}{W}}$$

in which

f_n = natural frequency
k = elastic constant of the system, force per unit deflection
g = acceleration due to gravity
W = weight of the vibrating body

If the frequency is to be in cycles per second, then g should be in inches per second per second, k in pounds per inch, and W in pounds. The above relation may be simplified by realizing that W/k represents the static deflection of the body, and, hence, the expression is reduced to:

$$f_n = 3.13\sqrt{\frac{1}{\triangle}} \text{ cycles per second,}$$

in which $\triangle$ = static deflection in inches.

It is apparent that the natural frequency is an internal characteristic of the system and can be altered in two ways, namely, by changing the elastic constant or by changing the weight of the vibrating body. A plot of the above equation is shown in figure 1.

Another equation which is of importance in the study of vibration isolation is that which gives the "transmissibility" factor for the vibration. The equation is:

$$e = \frac{1}{1 - \left(\frac{f}{f_n}\right)^2}$$

in which

e = transmissibility
f = forced frequency
Damping, zero or negligible.

Transmissibility may be defined as the ratio of the force

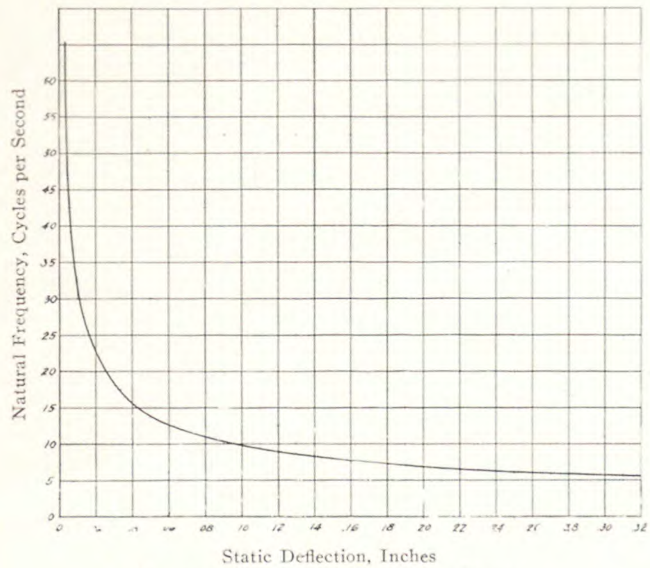

Figure 1. Natural Frequency Curve

actually transmitted to the impressed force, and indicates the effectiveness of the method which is employed to overcome excessive vibration. It is to be noted that for the case in which the natural frequency is equal to the forced frequency the transmitted force becomes infinite, and thus a resonant condition is established. Of course, no actual case occurs in which damping is zero and, therefore, the transmitted force or the amplitude of the actual vibration can never attain an infinite value. Nevertheless, the vibration can become of such magnitude that serious damage will occur within or outside the system. It can be observed from the equation that the impressed force does not have to be large in order to produce abnormally large transmitted forces, inasmuch as the transmissibility is a direct function of the frequencies of the system. In figure 2, the transmissibility is plotted as a function of the ratio f/f_n, and illustrates a very important consideration in vibration analysis. For all values of f/f_n less than $\sqrt{2}$, any attempt at isolation is useless because the transmissibility is always greater than 1, and it would, therefore, be much better to have a rigid connection, in which event the transmitted force would be equal to the impressed force. Hence, it is not often advisable to use isolation material for low-speed machinery unless the natural frequency of the machine can be reduced to a very small value as compared to the forced frequency.

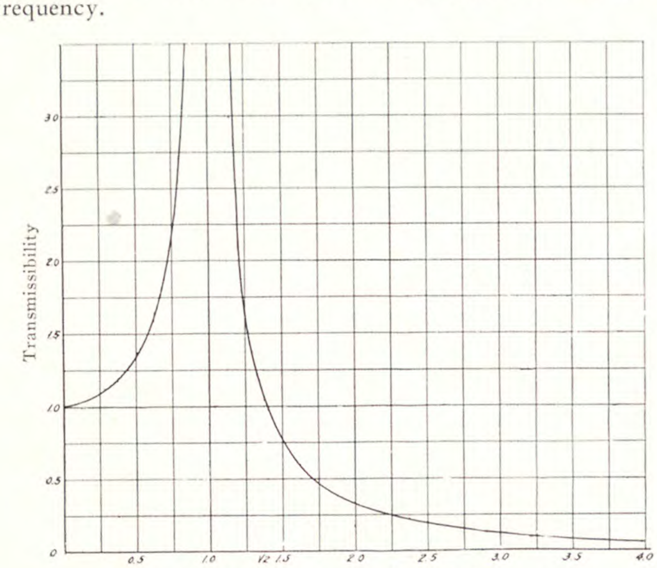

Figure 2. Transmissibility for Zero Damping

Where appreciable damping prevails, the transmissibility factor is represented by the equation:

$$e = \sqrt{\frac{1 + \left[2\frac{c}{c_c}\frac{f}{f_n}\right]^2}{\left[1 - \frac{f^2}{f_n^2}\right]^2 + \left[2\frac{c}{c_c}\frac{f}{f_n}\right]^2}}$$

in which the new terms "c" and "c_c" are the damping constant and the critical damping, respectively. The damping constant of the system in this case is assumed to be proportional to the velocity of the vibrating body, and usually can be given as pounds per inch per second, although absolute values of damping are difficult to obtain. This form of damping is termed "viscous," and represents to some degree the conditions that would obtain in an oil dashpot. Incidentally, it might be mentioned that the assumption of viscous damping makes for an easier mathematical solution of the equation of motion for the vibration. Critical damping, on the other hand, is that value of damping which distinguishes the transition point between a non-periodic disturbance and a vibration. Thus, values of c/c_c greater than one indicate that the motion is not a vibration but a gradual return of the body to its position of equilibrium. The value of c_c for a system may be obtained from the following relation:

$$c_c = 2\sqrt{\frac{Wk}{g}}$$

Note that, like natural frequency, critical damping is an inherent characteristic of the system, as it should be. In figure 3,

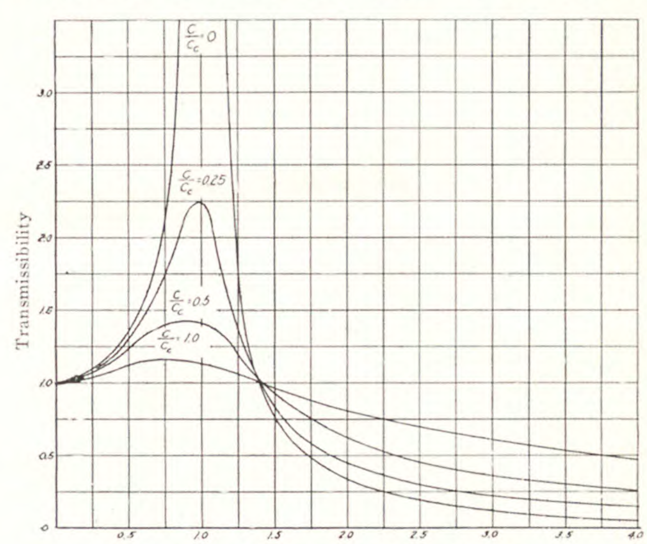

Figure 3. Transmissibility for Different Damping Factors

the transmissibility is plotted for different values of the ratio c/c_c. From an inspection of the curves it is seen that damping reduces the transmissibility for values of $f/f_n < \sqrt{2}$, which is the region where isolation is detrimental, but increases the transmissibility for values of $f/f_n > \sqrt{2}$. It would seem, therefore, that damping is of no value where the natural frequency is small compared to the disturbing frequency; yet one should remember that quite often it is necessary for the unit to pass through the resonance stage, or perhaps even operate within the region. Under such conditions, damping is a decided asset in reducing abnormal amplitudes.

The correction of excessive vibrations may be accomplished in a number of ways, but the more common methods are:

1. Elimination of the source of the vibration
2. Damping
3. Use of a "dynamic absorber"
4. Elastic suspension.

Naturally, each of these methods has a distinct application, and is somewhat limited by economic considerations.

The first method, elimination of the source of the vibration, is particularly useful in connection with rigid rotors where static and dynamic balance, in many cases, can be accomplished with little effort. By dynamic balance, the forces and couples that are induced by rotation of a member are balanced out, thereby eliminating the exciting forces which cause the vibration. To detect unbalance, there are a number of excellent balancing machines which are adapted to production work, but the cost of the machines may be prohibitive. In the case of large turbo-generators, balancing is a highly specialized field of study and should be done only by qualified experts.

Damping, as mentioned before, resorts to dissipating the energy of vibration through the action of friction before it can be transmitted outside the system. It should be realized that most applications of damping involve the use of dampers whose value is greater than the critical damping, and, therefore, the problem comes under the treatment of a non-periodic disturbance. A common illustration of such a problem is represented by shock absorbers on automotive vehicles, in which the nature of the impressed force is such a variable quantity. However, one particularly useful method for a damper is in counteracting a torsional vibration, but such a study is beyond the scope of this article.

It is of interest to mention the "dynamic absorber" although it is not, strictly speaking, considered as an isolator in the field of vibration isolation. Briefly, the function of the dynamic absorber is to purposely set up a vibration in the absorber which continually produces a force in opposition to the impressed force of the main body. The principle of the absorber can be demonstrated by the schematic arrangement shown in figure 4. Originally, the system consisted of the weight W attached to the supporting spring. Now, if an absorber be included as shown, which consists of a spring and weight whose *natural frequency is the same as the frequency of the disturbing force*, then it will be found that the smaller weight will vibrate in such a manner as to completely nullify the oscillation of the weight W. Of course, the application of this principle is limited to cases in which the frequency of the disturbance is constant.

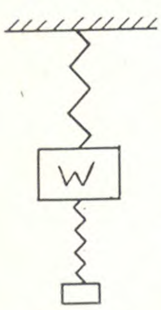

Figure 4 Simplified Dynamic Absorber

One of the best means of combating vibration is the use of an elastic suspension. However, the popular conception that any form of elastic mounting reduces vibration is entirely erroneous, and in many cases merely aggravates the condition. Furthermore, the installation of patented devices without a careful preliminary analysis may also produce poor results. It, therefore, becomes necessary to enumerate certain points that should be observed in selecting an elastic suspension.

1. Determine the lowest exciting frequency of the unit to be isolated, which in rotating machinery is generally the revolutions per minute. Use this frequency as a basis for the selection of the isolator. Any higher-frequency vibrations will be more effectively reduced if the isolator is selected for the lowest disturbance, inasmuch as the f/f_n ratio becomes larger, and consequently the transmissibility, lower.

2. Obtain the weight of the loaded machine and also the distribution of this weight. It is important to locate the correct isolator in the place where it will function as desired. For instance, where tall, top-heavy machines are involved, it is wise to place the isolators as far apart as possible to get increased stability.

3. Select the isolator which meets the requirements noted above. It should be remembered that the objective is to reduce the natural frequency of the assembly so that f/f_n is sufficiently large to give low transmissibility. A ratio of 3:1 will give good results although a greater ratio will further improve the mounting. In this case, the factor which is responsible for the change in natural frequency is the elastic constant, but another means of modifying the natural frequency would be to change the weight of the vibrating body. A practical illustration of combined weight and elastic modification is shown later on in this article.

The foregoing suggestions are more or less general, and it is important to realize that each installation requires individual treatment. There is no "cure-all" type of mounting.

Of the materials used in isolators, the most common are cork, rubber, steel, or some compounded form of these materials. Each has its particular advantages, and should satisfy the following questions:

1. Is the stiffness appropriate for the installation?

2. Is the damping coefficient of the material too large or too small?

3. Will the isolating power be reduced in a relatively short time?

4. Is the material resistant to any corrosive action to which it may be subjected?

5. Is the cost, including that of installation, justifiable?

Cork pads are often used on which to place machinery, but it should be noted that cork is particularly useful for isolating high-frequency disturbances in the sound range. The one disadvantage of cork is that it may lose its isolating power too rapidly when subjected to repeated loads. Nevertheless, cork pads have been used in many installations with good results for a number of years.

An example of one of the compounded products which are used in pad form is worth description. The pad consists of tightly twisted, closely woven, light weight cotton duck, each layer being thoroughly impregnated with a special rubber compound. These layers are built up to the required thickness and vulcanized together. The material has rather limited resilience with high damping, and under such conditions it is useful for the isolation of high-frequency vibrations. It has received widespread use at points of support in railroad trucks where predominately heavy loads are encountered.

The use of rubber in vibration isolation has found noteworthy application, especially because of the success in bonding rubber to metal. Of the three types of loads to which the mounting may be subjected (tension, compression, or shear), that of shear gives the best results. The reason for this is simply the fact that rubber in shear will give a greater deflection than in tension or compression for the same load. In other words, rubber in shear mounting offers a soft suspension and yet possesses a certain stability to any transverse loads which may be applied. Two typical forms of rubber mountings are shown in figures 5 and 6. Note that in both cases complete separation is insured between engine frame and foundation with consequent elimination of "bridging." There are numerous types of isolators for specific applications, and it is wise to secure the manufacturer's recommendation

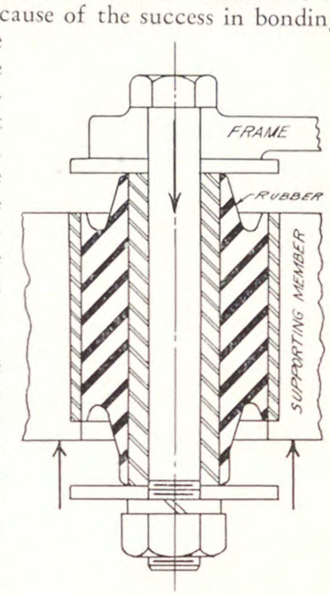

Figure 5. Rubber Tube Form Mounting
(Courtesy Lord Mfg. Co., Erie, Pa.)

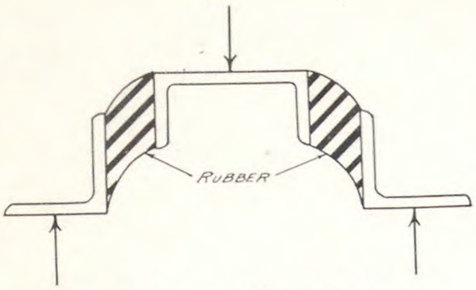

Figure 6. Rubber Channel Mounting

(Courtesy B. F. Goodrich Co., Akron, O.)

before attempting to install the units. Although rubber mountings possess some degree of damping, it is not of sufficient magnitude to decrease from the softness of the suspension, and a moderate amount is highly desirable for a number of cases, as mentioned previously. Rubber units should not be subjected to temperatures above 150° F., or even less, and proper shields should be installed if the mountings are liable to come in contact with oil.

Steel helical springs furnish an excellent mounting and are particularly adapted to heavy machines, especially large Diesel engines. One factor which favors the use of steel is its durability and elasticity, although the fact that it possesses virtually no damping necessitates careful design to guard against resonance from some overlooked source. It is true that where leaf springs are employed, the friction between the leaves offers appreciable damping. For extremely heavy machinery, where springs may not be adequate, the flexibility of steel beams furnishes an elastic form of mounting. Generally, isolation units which employ steel make use of a number of individual springs, and have some method of adjustment to provide for proper spring loading. A sample form of spring mounting is pictured in figure 7.

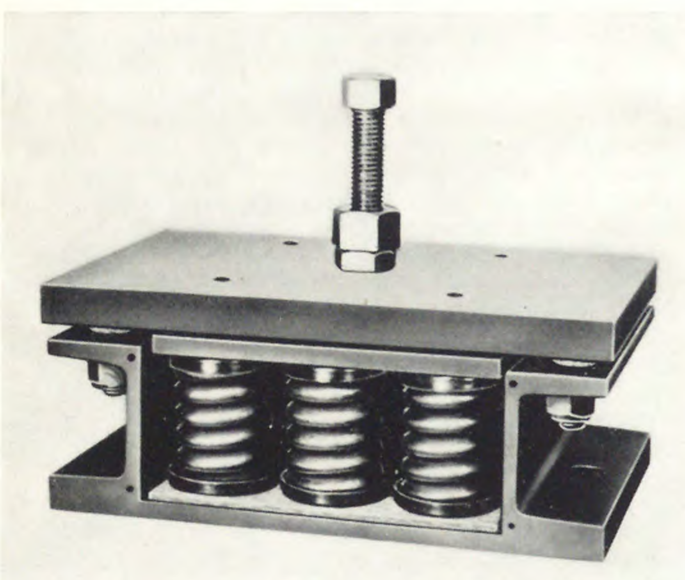

Figure 7. Spring Type Vibration Isolator

(Courtesy The Korfund Company, Long Island City, N. Y.)

There are several ways of installing spring units. One method is to place the engine frame directly on the units, but one should ascertain that the engine possesses sufficient mass to insure a low natural frequency as compared to the forced frequency. In cases where two machines are coupled together, a structural steel frame support is necessary to provide proper alignment, and, therefore, it becomes necessary to have the steel frame resting on the isolator units. One of the most interesting forms of mounting occurs when it is found that additional mass is required to provide a further decrease in the natural frequency of the system. Under such a condition, the engine is mounted on a steel frame which is encased in a concrete foundation, and the entire assembly is mounted on the spring isolators. Of course, adequate provision is made for accessibility to the units for adjustment or possible replacement. In addition, it may be necessary to investigate the soil condition, as this may influence the installation. Figure 8 illustrates the latter form of mounting

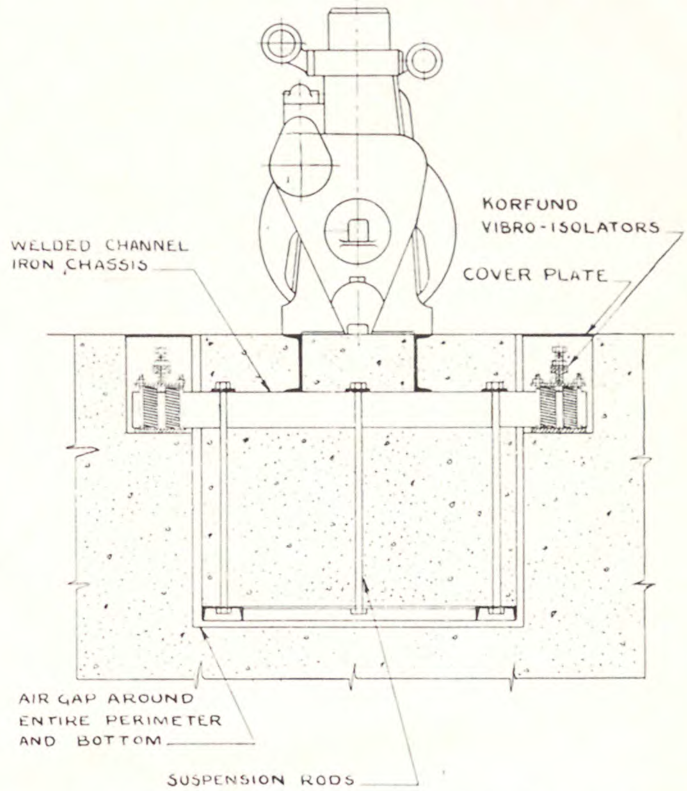

Figure 8. End Elevation of Diesel-Engine Mounting

(Courtesy The Korfund Company, Long Island City, N. Y.)

where it was necessary to place the vibration isolators outside of the foundation proper. Units which are used for such an installation can be obtained in capacities up to 30,000 pounds. It is evident that projects of the character mentioned require a detailed analysis of operating conditions, weight distribution, adaptability of isolators, etc., in order to secure best results.

One rather important consideration too often neglected is to avoid elastic suspension of equipment which must rest on flexible flooring. Such a case is difficult to analyze, and generally necessitates a cut-and-try procedure which may prove costly and altogether inadequate. To conclude, elastic suspensions for vibration isolation should be used only where there exists a definite need for them, inasmuch as some machines will function in a satisfactory manner when rigid connections are employed.

(Continued from page 10)

regulation will be this value plus the regulation of the supply.

The results thus obtained will be approximate in that the angle of overlap has been neglected. If the regulation of the supply is poor, this overlap becomes an important factor.

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THE BEAM INTEGRALS

Numerals giving the change in statical behavior of beams with variable moment of inertia as compared to beams with constant section

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Synopsis

Often beams with a variable moment of inertia are designed as beams with constant moment of inertia. This is particularly true with haunched concrete beams. It is known that this type of beams, stiffened by haunches at the supports, will have the tendency to absorb more moment at the supports, with the result that less moment is left to be taken by the actual beam between the supports. This means that the negative moments are increased, while the positive moments are decreased.

It is to be shown here how by the introduction of the "Beam Integrals" the cantilever design method may be used in order to find the moment coefficients in continuous beams with variable moment of inertia. Though the method is general in application, this article will refer only to beams with symmetric straight haunches, and subject to uniformly distributed loading.

History

The author first considered using the cantilever deflections instead of the elastic lines to determine deflections and moments when he was a freshman in Chalmers Institute of Technology, some 24 years ago. Professor Per Gullander found the idea new and assisted in developing it. However, at about that time Ostenfelt's slope deflection theory became known and, because this was a modification of the cantilever deflection idea, Prof. Gullander advised dropping the matter. Recently the old notes were recalled and, since the author had not seen the method published, they were rewritten. It was found that this method, hidden away so many years, was easily applicable to almost any condition met in the design of continuous beams and frames. Thus, the following articles were published: "Determining Beam Deflections without Integrations," *Engineering News-Record*, December 10, 1936; "Stresses in Continuous Beams for Deflecting Supports," *Civil Engineering*, June, 1938; "The Deflection of a Beam with the Use of Cantilever Deflections," *American Swedish Engineer*, October, 1936; "Moment Coefficients for Haunched Beams," *Engineering News-Record*, April 13, 1939. In all these articles the cantilever deflection method was used.

Principle Theory

The cantilever design method is based upon the various deflections of the free end of a member considered as a cantilever beam with all loads applied and the influences of other members considered as loads.

Thus a direct load will tend to bend the free end down. An imaginary load of the reaction (if upward) at the free end will

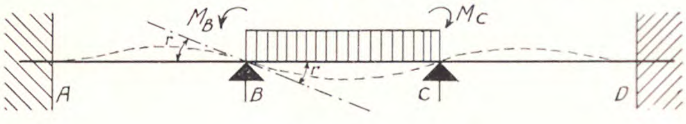

Figure 1

tend to bend this end up. A moment at the free end will either bend the free end up or down in accordance with its direction.

Each member is cut out of the system and treated separately as a cantilever beam with first one end restrained, and afterwards the other end restrained, the degree of restraining expressed by the deflection angle (r) and the moment (M) at this end.

For instance, if beam B-C in figure 1 is considered, it is cut out, and all the loads and reactions are applied as shown by figure 2. Then the beam is first assumed to be fixed in support B with the slope at this support equal to the deflection indicated by angle (r), and the movements of end (C) are figured for all the loads.

Attention is called to the fact that the angle (r) in (B) is equivalent to a movement down in (C) of (rL). It will also be noted that this angle is the same as the angle (r) at support (B) for beam A-B, and therefore is equivalent to an upward movement of A equal to (rL).

Hence the influence of the beams to the left of support B is expressed by the angle (r), and therefore will cause a deflection down of end C of beam B-C. Further, it is evident that the

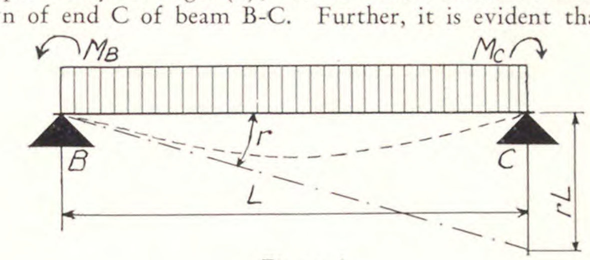

Figure 2

direct loading and the outside moment in (C) also will cause this end to move down, while the reaction in (C) will cause this end to move up.

Now, the sum of all these movements must equal the total movement in (C). If this support is fixed, the sum of these movements must equal zero.

Thereupon, the beam is considered fixed in (C), and the movements of the free end in (B) are considered the very same way.

Hence, it is evident that the design of a continuous beam may be done by simple calculus, if the movements of the free end of a cantilever beam with a similar cross section for various loadings are known.

General Assumption

The following derivations of the deflections of the free end of a fully restrained cantilever beam with variable moment of inertia are based upon the discovery by Mohr that the total deflection of the free end of a cantilever beam equals the statical moment of the M/EI_x diagram with reference to the free end.

The Deflections of the Free End of a Cantilever Beam

We have a cantilever beam of variable moment of inertia fully restrained at support A. It is assumed that the moment of inertia is less at the middle than at the end of the beam. Consider the following loading conditions:

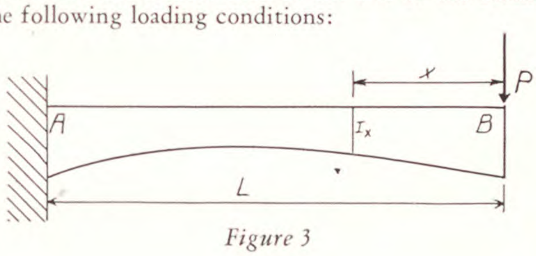

Figure 3

Case 1. Concentrated load (P) at (B)

The moment at a distance (x) from (B) will be (see figure 3)

$$M_x = P\,x \qquad (1)$$

and with a strip (dx) wide, the statical moment of this moment strip considered as a load with reference to (B) will be P x dx x, and this value divided by the variable mmoent of inertia I_x times the modulus of elasticity E, extended all over the beam from B, with x = 0, to A, with x = L, will give the total deflection at the free end B. Hence

$$d = \int_0^L \frac{Px^2\,dx}{E\,I_x} = \frac{P\,L^3}{3\,EI} \times \frac{3\,I}{L^3}\int_0^L \frac{x^2\,dx}{I_x} \qquad (2)$$

or

$$d = \frac{P\,L^3}{3\,EI}\,i_1 \qquad (3)$$

where the first "Beam Integral" is indicated as i_1. It can easily be seen that

$$i_1 = \frac{3\,I}{L^3}\int_0^L \frac{x^2\,dx}{I_x} \qquad (4)$$

Case 2. A moment (M) applied at (B)

The moment at a distance (x) from (B) will be (see figure 4)

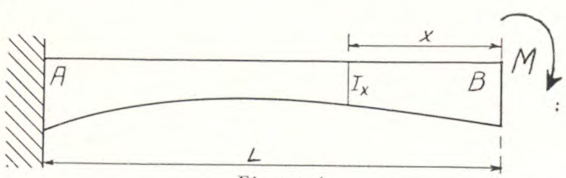

Figure 4

$$M_x = M \qquad (5)$$

and, if one assume a strip (dx) wide, the statical moment of this moment strip with reference to (B) will be M dx x, and this value divided by (E I_x) extended all over the beam from (B) with x = 0 to (A) with x = L, will give the deflection of the free end (B). Hence,

$$d = \int_0^L \frac{M\,x\,dx}{E\,I_x} = \frac{M\,L^2}{2\,EI}\,\frac{2\,I}{L^2}\int_0^L \frac{x\,dx}{I_x} \qquad (6)$$

or

$$d = \frac{M\,L^2}{2\,EI}\,i_2 \qquad (7)$$

where the second "Beam Integral" is indicated as i_2. It shows that

$$i_2 = \frac{2\,I}{L^2}\int_0^L \frac{x\,dx}{I_x} \qquad (8)$$

Case 3. Uniformly distributed load

The moment at a distance (x) from (B) will be (see figuse 5)

$$M_x = \frac{q\,x^2}{2} \qquad (9)$$

and by considering a strip (dx) wide, the statical moment of this moment strip with reference to (B) will be $\frac{q\,x^2}{2}$ dx x, and this value divided by EI_x, extended all over the beam from (B) with x = 0 to (A), with x = L, will give the deflection of the free end (B). Hence,

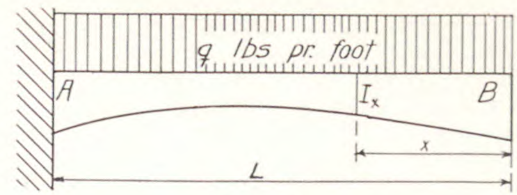

Figure 5

$$d = \int_0^L \frac{q\,x^3\,dx}{2\,EI_x} = \frac{q\,L^4}{8\,EI}\,\frac{4\,I}{L^4}\int_0^L \frac{x^3\,dx}{I_x} \qquad (10)$$

or

$$d = \frac{q\,L^4}{8\,EI}\,i_3 \qquad (11)$$

where the third "Beam Integral" is indicated as i_3. Then

$$i_3 = \frac{4\,I}{L^4}\int_0^L \frac{x^3\,dx}{I_x} \qquad (12)$$

It will be noticed that the derived formulas for the deflections are similar to formulas for the deflections for a beam with constant moment of inertia and that the former are identical with the latter in the case the i-values equal 1.

The Beam Integrals

The "Beam Integrals" are very important constants in the design of frames of any kind as soon as the members have variable moments of inertia. They depend only upon the shape of the beam. For a beam with constant moment of inertia they all equal one, and for beams with variable moments of inertia they always are less than one and positive.

They can be figured by mechanical integration or by pure mathematical integration. However, the latter method can be applied only for such changes in the moment of inertia as can be expressed by an equation.

Though the mathematical integrations have been shown only for a beam with straight haunches, similar integrations can be used for other changes in the section of a beam.

It is rather difficult to solve these integrals, but when solved they can be used for various spans, as they depend only upon the shape of the beam.

Beam with Straight Haunches

If, for instance, a beam with straight haunches as shown in figure 6 is considered, then three separate integrations have to be made, one (s_1) for the right haunch from x = 0 to x = aL,

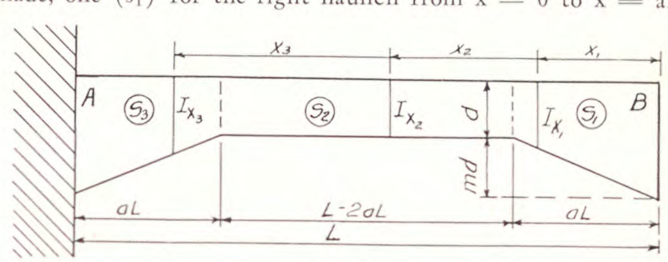

Figure 6

one (s_2) for the straight middle portion, from x = aL to x = L — aL, and one (s_3) for the left haunch from x = L — aL to x = L.

For simplicity it is assumed that the beam is symmetrical and that the moment of inertia for the weakest part of the section is called plain I.

The variable expression I_x/I, in which I_x represents the moment of inertia at the distance x from the right end, can be expressed for the right haunch (s_1)

$$\frac{I_x}{I} = \left(1 + m - \frac{mx}{aL}\right)^3 = \left(c - \frac{mx}{aL}\right)^3 \quad \text{(13)}$$

middle part (s_2) $\frac{I_x}{I} = 1$ (14)

left haunch (s_3) $\frac{I_x}{I} =$

$$\left(1 + m - \frac{m}{a} + \frac{mx}{aL}\right)^3 = \left(d + \frac{mx}{aL}\right)^3 \quad \text{(15)}$$

Beam Integral i_1

Using formula (4) we get

$$i_1 = \frac{3}{L^3}\int_0^L \frac{I}{I_x} x^2\,dx = s_1 + s_2 + s_3 \quad \text{(16)}$$

where the s-integrals have to be solved separately. Thus we get:

$$s_1 = \frac{3}{L^3}\int_0^{aL} \frac{I}{I_x} x^2\,dx \quad \text{(17)}$$

$$\text{Make } c - \frac{mx}{aL} = z; \quad \text{Hence } x = \frac{aL(c-z)}{m} \quad \text{(18, 19)}$$

$$\text{and } dx = -\frac{aL}{m}\,dz \quad \text{(20)}$$

New limits: for $x = 0$, use $z = (1 + m)$ (21)

and for $x = aL$, use $z = 1$ (22)

$$s_1 = -\frac{3a^3}{m^3}\int_{1+m}^{1}\left(\frac{c^2\,dz}{z^3} - \frac{2c\,dz}{z^2} + \frac{dz}{z}\right)$$

$$= -\frac{3a^3}{m^3}\left[-\frac{c^2}{2z^2} + \frac{2c}{z} + {}^e\!\log z\right]_{1+m}^{1} \quad \text{(23)}$$

$$\text{Hence} \quad s_1 = \frac{3a^3}{m^3}\left[{}^e\!\log(1+m) - \frac{m(2-m)}{2}\right] \quad \text{(24)}$$

$$s_2 = \frac{3}{L^3}\int_{aL}^{L-aL} x^2 dx = \frac{3}{L^3}\left[\frac{x^3}{3}\right]_{aL}^{L-aL}$$

$$= \frac{1}{L^3}\left[(L-aL)^3 - aL^3\right] = (1-a)^3 - a^3 \quad \text{(25)}$$

$$s_3 = \frac{3}{L^3}\int_{L-aL}^{L} \frac{I}{I_x} x^2\,dx \quad \text{(26)}$$

$$\text{Make } d + \frac{mx}{aL} = z; \quad \text{Hence } x = \frac{aL(z-d)}{m} \quad \text{(27, 28)}$$

$$\text{and } dx = \frac{aL}{m}\,dz \quad \text{(29)}$$

New limits: for $x = L - aL$, use $z = 1$, (30)

and for $x = L$, use $z = (1 + m)$ (31)

$$s_3 = \frac{3a^3}{m^3}\int_{1}^{1+m}\left(\frac{d^2dz}{z^3} - \frac{2d\,dz}{z^2} + \frac{dz}{z}\right)$$

$$= \frac{3a^3}{m^3}\left[\frac{d^2}{2z^2} + \frac{2d}{z} + {}^e\!\log z\right]_{1}^{1+m} \quad \text{(32)}$$

Hence $s_3 =$

$$\frac{3a^3}{m^3}\left[{}^e\!\log(1+m) - \frac{dm}{1+m}\left(2 - \frac{d(m+2)}{2(1+m)}\right)\right] \quad \text{(33)}$$

where (a) refers to the length of the haunch, (m) to the depth of the haunch, and $d = 1 + m - m/a$. (See figure 6.)

Beam Integral i_2

Using formula (8) we get

$$i_2 = \frac{2I}{L^2}\int_0^L \frac{I}{I_x} x\,dx = s_1 + s_2 + s_3 \quad \text{(34)}$$

where the s-integrals have to be solved separately. We have:

$$s_1 = \frac{2}{L^2}\int_0^{aL} \frac{I}{I_x} x\,dx \quad \text{(35)}$$

This integral is solved in the same manner that the i_1 integral was solved—by the use of (18-22). We get

$$s_1 = -\frac{2a^2}{m^2}\int_{1+m}^{1}\left(\frac{c\,dz}{z^3} - \frac{dz}{z^2}\right)$$

$$= -\frac{2a^2}{m^2}\left[-\frac{c}{2z^2} + \frac{1}{z}\right]_{1+m}^{1} \quad \text{(36)}$$

$$\text{Hence} \quad s_1 = \frac{a^2}{1+m} \quad \text{(37)}$$

$$s_2 = \frac{2}{L^2}\int_{aL}^{L-aL} x\,dx = \frac{2}{L^2}\left[\frac{x^2}{2}\right]_{aL}^{L-aL}$$

$$= \frac{1}{L^2}\left[(L-aL)^2 - aL^2\right] = 1 - 2a \quad \text{(38)}$$

$$s_3 = \frac{2}{L^2}\int_{L-aL}^{L} \frac{I}{I_x} x\,dx \quad \text{(39)}$$

This integral is solved in the same manner that the i_1-integral was solved, by the use of (27-31). We get

$$s_3 = \frac{2a^2}{m^2}\int_{1}^{1+m}\left(\frac{dz}{z^2} - \frac{d\,dz}{z^3}\right)$$

$$= \frac{2a^2}{m^2}\left[-\frac{1}{z} + \frac{d}{2z^2}\right]_{1}^{1+m} \quad \text{(40)}$$

$$\text{Hence} \quad s_3 = \frac{a^2}{m(1+m)}\left[2 - \frac{d(2+m)}{1+m}\right] \quad \text{(41)}$$

where (a) refers to the length of the haunch, (m) to the depth of the haunch, and $d = 1 + m - m/a$. (See figure 6.)

Beam Integral i_3

Using formula (12), we get

$$i_3 = \frac{4}{L^4}\int_0^L \frac{I}{I_x} x^3\,dx = s_1 + s_2 + s_3 \quad \text{(42)}$$

where the s-integrals have to be solved separately. We have

$$s_1 = \frac{4}{L^4}\int_0^{aL} \frac{I}{I_x} x^3\,dx \quad \text{(43)}$$

This integral is solved in the same manner that the i_1-integral was solved, by the use of (18-22). We get then

$s_1 =$

$$-\frac{4a^4}{m^4}\int_{1+m}^{1}\left(-\frac{c^3dz}{z^3} - \frac{3c^2dz}{z^2} + \frac{3c\,dz}{z} - dz\right) \quad \text{(44)}$$

Hence $s_1 =$

$$\frac{2a^4}{m^4}\left[6(1+m)\,{}^e\log(1+m) - m(6+3m-m^2)\right] \quad (45)$$

$$s_2 = \frac{4}{L^4}\int_{aL}^{L-aL} x^3\,dx = \frac{4}{L^4}\left[\frac{x^4}{4}\right]_{aL}^{L-aL}$$

$$= \frac{1}{L^4}\left[(L-aL)^4 - aL^4\right] = (1-a)^4 - a^4 \quad (46)$$

$$s_3 = \frac{4}{L^4}\int_{L-aL}^{L} \frac{I}{I_x}\,x^3\,dx \quad (47)$$

This integral is solved in the same manner that the i_1-integral was solved, by the use of (27-31). We here get

$s_3 =$

$$\frac{4a^4}{m^4}\int_1^{1+m}\left[-\frac{d^3\,dz}{z^3} + \frac{3d^2\,dz}{z^2} - \frac{3d\,dz}{z} + dz\right] \quad (48)$$

Hence $s_3 =$

$$\frac{2a^4}{m^4}\left[2m - 6d\,{}^e\log(1+m) + \frac{d^2 m}{1+m}\left(6 - \frac{d(2+m)}{1+m}\right)\right] \quad (49)$$

where (a) refers to the length of the haunch, (m) to the depth of the haunch, and $d = 1 + m - m/a$. (See figure 6).

Special Case: Beams with One Haunch Only

It will be noticed that these derived formulas for the beam integrals were obtained under the assumption that the beam is symmetrical. However, the same formulas can also be applied to beams not symmetrical. In this case (s_2) and (s_3) remain the same, if the values of (a) and (m) are indexed (a_1, a_3) and (m_1, m_3). Then the middle portion will have the limits (a_1L and $L - a_3L$).

The beam shown in figure 7 is assumed to have one straight haunch at A only. In this case the integrals are extended from

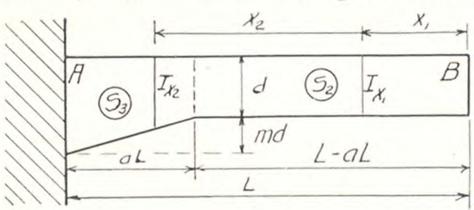

Figure 7

(B) to (A) again, and it is obvious that the computations must be divided into only two parts, so

$$i = s_2 + s_3 \quad (50)$$

where (s_2) is the integral for the right straight portion of the beam from $x = 0$ to $x = L - aL$, and (s_3) is the integral for the haunch, from $x = L - aL$ to $x = L$.

Beam Integral i_1

$$s_2 = \frac{3}{L^3}\int_0^{L-aL} x^2\,dx = (1-a)^3 \quad (51)$$

s_3 same as in (33)

Beam Integral i_2

$$s_2 = \frac{2}{L^2}\int_0^{L-aL} x\,dx = (1-a)^2 \quad (52)$$

s_3 same as in (41)

Beam Integral i_3

$$s_2 = \frac{4}{L^4}\int_0^{L-aL} x^3\,dx = (1-a)^4 \quad (53)$$

s_3 same as in (49)

Special Case: The Beam Is Haunched from End to End

This type of beam is represented in figure 8. For such a condition, formulas (33), (41) and (49) will give the integrals directly for $a = 1$.

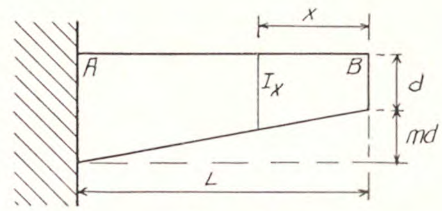

Figure 8

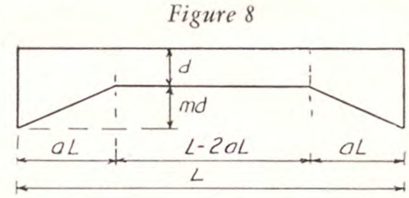

Figure 9

Table 1. The Beam Integrals for a beam with symmetrical and straight haunches.

In the following table the Beam Integrals are given for various values of (m) and (a) for a symmetrically haunched beam. For this condition only we have $i_3 = 2i_1 - i_2$.

		Value of m									
		0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
	i_1	0.850	0.755	0.692	0.642	0.612	0.586	0.565	0.548	0.535	0.523
a = 1/4	i_2	0.882	0.806	0.754	0.716	0.688	0.665	0.648	0.633	0.621	0.611
	i_3	0.818	0.705	0.629	0.576	0.536	0.506	0.483	0.464	0.448	0.435
	i_1	0.876	0.798	0.745	0.707	0.678	0.656	0.639	0.625	0.614	0.604
a = 1/5	i_2	0.906	0.844	0.803	0.773	0.750	0.732	0.718	0.707	0.697	0.689
	i_3	0.847	0.751	0.687	0.641	0.607	0.581	0.560	0.544	0.530	0.519
	i_1	0.895	0.828	0.782	0.750	0.725	0.707	0.692	0.679	0.669	0.661
a = 1/6	i_2	0.921	0.871	0.836	0.811	0.792	0.777	0.765	0.755	0.747	0.741
	i_3	0.868	0.785	0.729	0.689	0.659	0.636	0.618	0.603	0.592	0.582
	i_1	0.919	0.867	0.832	0.806	0.787	0.773	0.761	0.752	0.744	0.737
a = 1/8	i_2	0.941	0.903	0.877	0.858	0.844	0.833	0.824	0.817	0.811	0.806
	i_3	0.897	0.831	0.787	0.755	0.731	0.713	0.698	0.687	0.677	0.669

Important Notice. It will be noted that (i_1) corresponds to (φ_α), and $(3l_2 - 2i_1)$ to (φ_β) in Strassner's book *Neuere Methoden*, Volume I, Third Edition, as given on page 85, that (i_2) corresponds to (c_1) and $(3i_2 - 2i_1)$ to (c_2) in E. Suter's book *Die Methode der Festpunkte*, Second Edition, Page 727. The values (i_1) and $(3i_2 - 2i_1)$ correspond also to (α) and (β) in Taylor, Thompson & Smulski, *Concrete*, Volume II, Page 138, Fourth Edition.

The Mathematical Derivation of the Numerical Values of the Beam Integrals

Example 1. A haunched beam as shown in figure 10 is given, where $a = 1/4$, and $m = 1$. It is required to find the Beam Integrals with the use of the previously derived formulas.

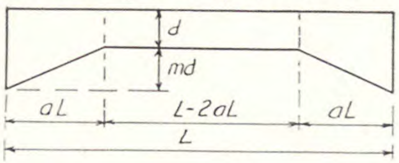

Figure 10

Figure first the value (d) from (15). We get $d = 1 + 1 - 4 = -2$.

Beam Integral i_1. From (16) we know that $i_1 = s_1 + s_2 + s_3$.

Use (24) $s_1 = \frac{3}{64}\left[{}^e\log 2 - \frac{1 + (2-1)}{2}\right] = 0.00905$

Use (25) $s_2 = \left(1 - \frac{1}{4}\right)^3 - \frac{1}{64} = 0.40625$

Use (33) $s_3 =$

$$\frac{3}{64}\left[{}^e\log 2 + \frac{2}{2}\left(2 + \frac{2(1+2)}{2(1+1)}\right)\right] = 0.19655$$

Hence, the first Beam Integral i_1 $= 0.61185$

Beam Integral i_2. From (34) we know that $i_2 = s_1 + s_2 + s_3$.

Use (37) $s_1 = \frac{1}{16(1+1)} = 0.03125$

Use (38) $s_2 = 1 - 2/4 = 0.5$

Use (41) $s_3 = \frac{1}{16(1+1)}\left[2 + \frac{2(2+1)}{1+1}\right] = 0.15625$

Hence, the second Beam Integral i_2 $= 0.68750$

Beam Integral i_3. From (42) we know that $i_3 = s_1 + s_2 + s_3$.

Use (45) $s_1 =$

$$\frac{2}{256}\left[6(1+1)\,{}^e\log(1+1) - (6+3-1)\right] = 0.00248$$

Use (46) $s_2 = \left(1 - \frac{1}{4}\right)^4 - \frac{1}{256} = 0.31250$

Use (49) $s_3 =$

$$\frac{2}{256}\left[2 + 6\,{}^e\log(1+1) + \frac{4}{1+1}\left(6 + \frac{2(2+1)}{1+1}\right)\right] = 0.22123$$

Hence the third Beam Integral i_3 $= 0.53621$

All these values can be found in table 1 for $m = 1$ and $a = 1/4$.

The Mechanical Derivation of the Numerical Values of the Beam Integrals

Example 2. Determine by mechanical integration the Beam Integrals for the same beam as shown in figure 10. We assume the beam to be 20 feet long, and that each strip is 1 foot wide, (or $dx = 1$).

Distance from B to C. of G. of strip x	Depth of strip $1+m$	I_x $(1+m)^3$	$\int \frac{x\,dx}{I_x}$ $\frac{L^2}{2} i_2$	$\int \frac{x^2\,dx}{I_x}$ $\frac{L^3}{3} i_1$	$\int \frac{x^3\,dx}{2\,I_x}$ $\frac{L^4}{8} i_3$
0.5	1.9	6.859	0.0728	0.036	0.009
1.5	1.7	4.913	0.3053	0.458	0.340
2.5	1.5	3.375	0.7408	1.852	2.314
3.5	1.3	2.197	1.5931	5.576	9.760
4.5	1.1	1.331	3.3809	15.220	34.200
(Values for s_1)			6.0929	23.142	46.623
5.5	1.0	1.000	5.5	30.25	83.19
6.5	1.0	1.000	6.5	42.25	137.31
7.5	1.0	1.000	7.5	56.25	210.94
8.5	1.0	1.000	8.5	72.25	307.06
9.5	1.0	1.000	9.5	90.25	428.69
10.5	1.0	1.000	10.5	110.25	578.81
11.5	1.0	1.000	11.5	132.25	760.44
12.5	1.0	1.000	12.5	156.25	976.56
13.5	1.0	1.000	13.5	182.25	1230.19
14.5	1.0	1.000	14.5	210.25	1524.31
(Values for s_2)			100.0	1082.50	6237.50
15.5	1.1	1.331	11.6453	180.50	1398
16.5	1.3	2.197	7.5103	123.90	1022
17.5	1.5	3.375	5.1853	90.80	794
18.5	1.7	4.913	3.7655	69.70	644
19.5	1.9	6.859	2.8429	55.45	540
(Values for s_3)			30.9493	520.35	4398
$(s_1 + s_2 + s_3)$			137.0422	1625.99	10682

Beam Integral i_2 is now obtained by dividing the sum by $400/2 = 200$.

Hence $i_2 = 0.685$

Beam Integral i_1 is now obtained by dividing the sum by $8000/3 = 2666.7$

Hence $i_1 = 0.610$

Beam Integral i_3 is now obtained by dividing the sum by $160000/8 = 20000$.

Hence $i_3 = 0.534$

The method of solution is clearly indicated by the table. Take, for instance, the third strip that extends from $x = 2$ to $x = 3$, with the distance to the centre of gravity assumed to be $x = 2.5$. Then the moment of inertia is $I_x = 3.375$, and $dx = 1$, as each strip is one foot wide.

Hence

$$\frac{x\,dx}{I_x} = \frac{2.5}{3.375} = 0.7408$$

$$\frac{x^2\,dx}{I_x} = 0.7408 \times 2.5 = 1.852$$

$$\frac{x^3\,dx}{2\,I_x} = 1.852 \times 2.5 \times 1/2 = 2.314$$

It will be noted that the difference in the found values of the integrals is very slight as compared with the mathematical values given in Table 1. The thinner the strips are, the more correct the values will be.

As a comparison, take s_2 for i_1. The correct value can very easily be figured, because the moment of inertia is constant for this part. We get

$$s_2 = \int_5^{15} x^2\,dx = \Big|_5^{15} x^3/3 = \frac{3375 - 125}{3} = 1083.3,$$

as compared to the value of 1082.50, obtained in the mechanical integration.

The Application of the Beam Integrals to an Actual Example

Example 3. A continuous beam with equal spans on four supports is considered. Each individual beam has the same

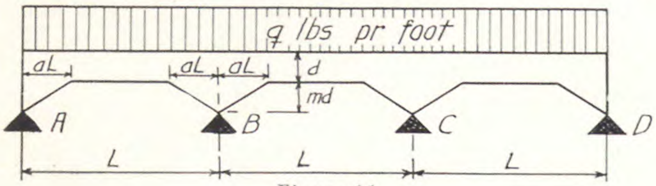

Figure 11

haunched shape. Find the moment at support B for a uniformly distributed load.

Each beam has the same shape (see figure 11), with $m = 1$ and $a = 1/4$. Therefore, the Beam Integrals will be found from Table 1 to be

$i_1 = 0.612, \quad i_2 = 0.688, \quad \text{and } i_3 = 0.536$

Use the cantilever deflection theory. Therefore, cut out each individual beam, and figure out the deflections as per formula given. The total of these deflections must equal zero, as the supports are assumed to be fixed.

Beam A-B. We consider the beam restrained in (B) and free to move at support A (see figure 12). The uniformly dis-

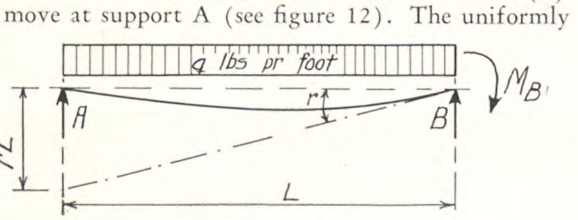

Figure 12

tributed load will tend to deflect this end as per (11), while the reaction in (A) will tend to push this end up as per (3). The angle (r) at support B corresponds to a deflection of (rL) at support A. The sum of these deflections must equal zero. Therefore, if the formulas are used to indicate these deflections, we get

$$\frac{qL^4}{8\,EI}\,i_3 - \frac{AL^3}{3\,EI} + rL = 0 \qquad (54)$$

and with a moment equation round (B) we get

$$A = \frac{qL}{2} - \frac{M_B}{L} \qquad (55)$$

so the value of (r) becomes

$$r = \frac{qL^3}{24\,EI}(4i_1 - 3i_3) - \frac{M_B L}{3\,EI} \qquad (56)$$

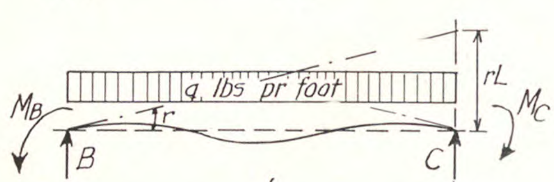

Figure 13

Beam B-C (see figure 13). The deflection in support (C) is figured. It will be noted that the moment acting in this support will tend to bend this end down. Thus, we get the total effect of the deflections to be

$$\frac{qL^4}{8\,EI}\,i_3 + \frac{M_C L^2}{2\,EI}\,i_2 - rL - \frac{CL^3}{3\,EI}\,i_1 = 0 \qquad (57)$$

and, as the beam is symmetrical, it is obvious that $M_C = M_B$, and that

$$C = gL/2 \qquad (58)$$

These values substituted in (57) will give

$$r = \frac{M_B L}{2\,EI}\,i_2 = \frac{qL^3}{24\,EI}(4i_1 - 3i_3) \qquad (59)$$

Then (56) and (59) combined will give the moment at support B to be

$$M_B = \frac{q\,L^2}{2}\;\frac{(4i_1 - 3i_3)}{3i_2 + 2i_1} \qquad (60)$$

and if the values of i_1, i_2, and i_3 are used, we get the actual moment

$$M_B = \frac{q\,L^2}{7.8} \qquad (61)$$

Should the beam be of constant moment of inertia, then all the Beam Integrals become equal to one, so (60) becomes

$$M_B = \frac{q\,L^2}{10} \qquad (62)$$

which is a known expression.

It will be noticed that the haunched beam will take more moment at the supports, and will relieve the middle part of the beam of some positive moment.

Comments by Professor H. N. Cummings, Vice-President and Head of the Civil Engineering Department, Newark College of Engineering:

Although the basic equations used in Professor Albert's method of obtaining the moments in continuous beams of varying moment of inertia are not new, the "Beam Integrals" are interesting and cleverly worked out variations for the special—and probably most frequently occurring—case of symmetric straight haunched beams. It requires only the ordinary undergraduate course in theory of structures to enable one to follow the mathematical steps by which the "i" and the "s" formulas are derived. The table of values of i_1, i_2, i_3 for the symmetrical straight haunched beam provided the designer with the means of quickly determining bending moments for about as wide a variety of designs as he will meet in ordinary practice.

Of course in the limited space available to Professor Albert, only a special case could be investigated. It will be interesting later to see how he will attack the more general problem of unsymmetrical and not-straight haunched beams. I should very much like to see him develop the graphical—or as he calls it—mechanical method in connection with this more general problem and put it in such form as to show its great power and availability not only to this but to the whole field of engineering mathematics.

THE COMBUSTION OF CARBON

By JOSEPH JOFFE, A.B., B.S., M.A., Ph.D.

Professor in Chemical Engineering, Newark College of Engineering

In spite of the great industrial importance of the combustion of carbon, our knowledge of the mechanism of this process is very incomplete. Experimental investigations are hampered by the difficulties of measurement and control which arise when an attempt is made to study the extremely rapid reactions of combustion at the elevated temperatures existing inside of a furnace.

When carbon burns in dry air or oxygen, carbon monoxide and carbon dioxide are found as constituents in the gas resulting from combustion. Carbon monoxide, however, is capable of reacting if any excess oxygen is present, and carbon dioxide may be reduced on contact with carbon. We are thus faced with a problem of simultaneous and consecutive reactions, some being primary, others secondary. We may be dealing with several heterogeneous reactions of various orders taking place on the carbon surface and with the oxidation of carbon monoxide partly on the carbon surface and partly in the gas phase. The kinetics of these reactions are further complicated by the fact that the physical process of material transfer to and from the carbon surface by diffusion and convection is often the slowest and therefore the rate-determining step. In addition, there is evidence that impurities such as water vapor in ordinary air and ash constituents in coal and coke may influence and even change the mechanism of the reactions[1, 2,*]. It has also been suggested that the external (visible) surface of the carbon is responsible for only a portion of the total reaction and that the internal (pore) surface is also involved[3].

Because of the many existing uncertainties, it is impossible to give a single unambiguous picture of what occurs at or near the carbon surface when carbon burns in air. Let us first examine the problem in terms of the generally accepted view [4, 5] that when carbon burns in the presence of excess oxygen, as in the oxidizing zone of a fuel bed, the only reaction of any importance is

$$C + O_2 = CO_2$$

Imagine a piece of carbon at furnace temperature (say, 1200° C.) suddenly brought in contact with a stream of preheated air at the same temperature. Oxygen in the immediate neighborhood of the carbon surface will react with the carbon with a speed so great that no method has as yet been devised for directly measuring it. As the oxygen supply in the immediate neighborhood of the carbon surface is depleted, additional oxygen can reach the surface only by diffusion from the main gas stream through the relatively stagnant gas film which clings to the carbon surface. Carbon dioxide generated at the carbon surface diffuses out through the gas film and mixes with the air in the main gas stream. The situation is represented by the diagram of figure 1, where the partial pressures of the gases are plotted as ordinates against distance from the carbon surface as abscissa.

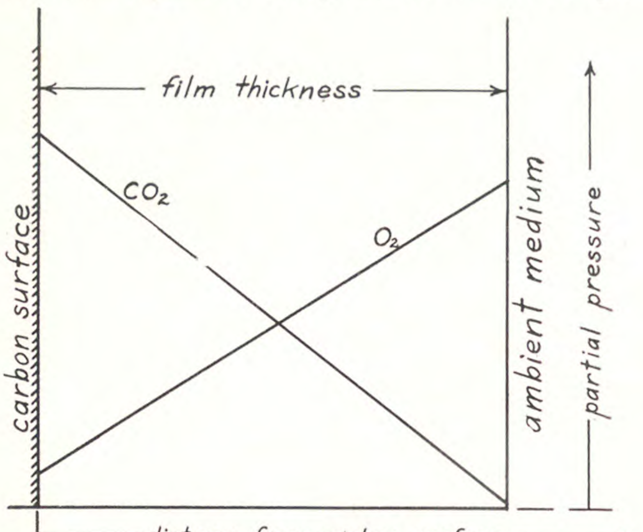

Figure 1

*See references at end of article.

In order not to complicate the discussion, a plane carbon surface has been selected for consideration in figure 1 and throughout the rest of this paper. Furthermore, although for the sake of simplicity straight lines are used in figure 1, actually the curves representing the variation of partial pressures of the gases with distance from the carbon surface are not linear[5]. One reason for this is the fact that the coefficients of diffusion vary with temperature, which is not constant throughout the film. Another reason is the fact that in that part of the film which is further away from the carbon surface some of the material transfer is by convection rather than by diffusion.

Because the chemical reaction, which was assumed to take place at the carbon surface, is strongly exothermic, the temperature of the carbon surface will rise above that of the ambient medium, until a steady state is established in which the heat liberated at the carbon surface by the chemical reaction is balanced by the heat leaving the surface in various ways. The chief heat loss from the carbon surface is by radiation, the rate of which depends on the fourth power of the absolute temperature of the carbon surface. Since the fourth power of the absolute temperature is involved, small changes in the temperature of the carbon surface correspond to large changes in the rate of combustion. Heat is also lost by conduction through the stagnant gas film which covers the carbon surface, and some heat is carried away as sensible heat by the carbon dioxide leaving the carbon surface[6]. It may be pointed out parenthetically that the effective film thickness corresponding to the process of heat transfer from the carbon surface by conduction is not necessarily identical with the effective film thickness which must be assumed for the process of material transfer of reactants and products to and from the carbon surface by diffusion[7].

That the picture presented above for the process of combustion of carbon cannot be entirely correct, is evident from the following considerations: even if the only product of combustion of carbon by oxygen is carbon dioxide, some carbon monoxide must be formed by the secondary reaction $C + CO_2 = 2CO$. That the rate of the reaction $C + CO_2$ is by no means negligible in comparison with that of the reaction $C + O_2 = CO_2$ was shown by the calculations of C. C. Furnas[8] and of M. A. Mayers[9]. Moreover, there is experimental evidence for the presence of a carbon monoxide flame in the immediate neighborhood of a burning carbon surface. Thus, S. P. Burke[10] as well as Z. Chukhanov[11] report observing these blue flames in the combustion of charcoal, while H. Davis and H. C. Hottel[12] obtained both visual and photographic evidence of blue coronas around spheres of brush carbon burning in air or in oxygen.

Based on the assumption that appreciable amounts of carbon monoxide are formed at the carbon surface, S. P. Burke and T. Schumann[13] have proposed the following mechanism for the combustion of carbon: Carbon monoxide, formed at the carbon surface, diffuses outward through the stagnant gaseous film which covers the carbon surface. Oxygen from the ambient air diffuses into the gaseous film. Somewhere in the film a zone exists where the oxygen and carbon monoxide meet and combustion of carbon monoxide takes place. Carbon dioxide is generated in this zone of combustion and diffuses both outward into the main gas stream and inward toward the carbon surface. Because of

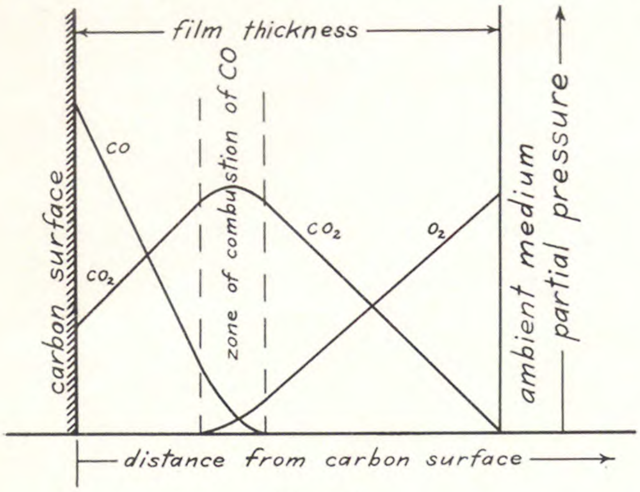

Figure 2

the carbon monoxide flame no oxygen ever reaches the carbon surface, and combustion of carbon takes place entirely by the reaction $C + CO_2 = 2CO$. The situation may be represented by the diagram of figure 2.

According to this picture, heat is being absorbed at the carbon surface by the endothermic reaction $C + CO_2 = 2CO$. At the same time the carbon surface loses heat by radiation to the furnace walls. Heat energy is supplied to the carbon surface by conduction through the gaseous film from the zone of combustion of carbon monoxide.

Burke and Schumann have shown[13] that the mechanism postulated by them gives approximate agreement with the experimental results of Smith and Gudmundsen on the combustion

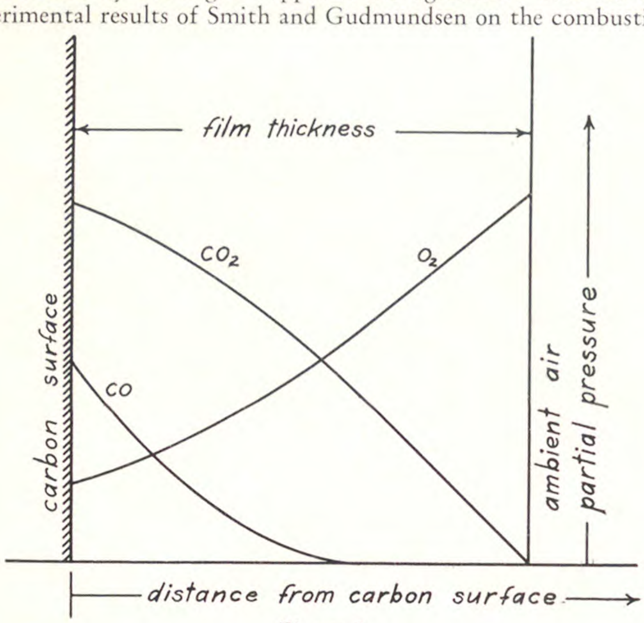

Figure 3

of small electrode-carbon spheres in dry and moist air[1]. Their theory, however, has been challenged on the ground that the rate of the reaction $C + CO_2 = 2CO$ is not sufficiently fast to account for observed combustion rates of carbon in air[14]. Moreover, gas samples obtained by Parker and Hottel by a microsampling technique from the immediate neighborhood of a burning carbon surface show the presence of oxygen[5].

It has been suggested by K. Fischbeck[15] that the reactions of carbon and oxygen and carbon and carbon dioxide proceed simultaneously at the carbon surface. The carbon monoxide formed by these reactions is consumed within the gaseous film by reaction with the oxygen which diffuses toward the carbon surface from the main gas stream. The situation is represented by the diagram of figure 3.

In drawing the curves of figure 3, representing partial pressures, the variation of temperature throughout the gas film has been disregarded. It was also assumed in drawing the carbon dioxide curve that carbon dioxide is being formed more rapidly at the carbon surface by the reaction of carbon and oxygen than it is being used up by the reaction $C + CO_2 = 2CO$. Hence, the carbon dioxide curve starts from the carbon surface with a negative slope. The carbon dioxide curve is concave down in that region of the gaseous film where oxidation of carbon monoxide to carbon dioxide takes place. If, however, carbon dioxide is consumed faster at the carbon surface by the reaction $C + CO_2 = 2CO$ than it is produced by the reaction of carbon and oxygen, the carbon dioxide curve will start from the carbon surface with a positive slope and will pass through a maximum. This will correspond to a carbon dioxide stream diffusing in two

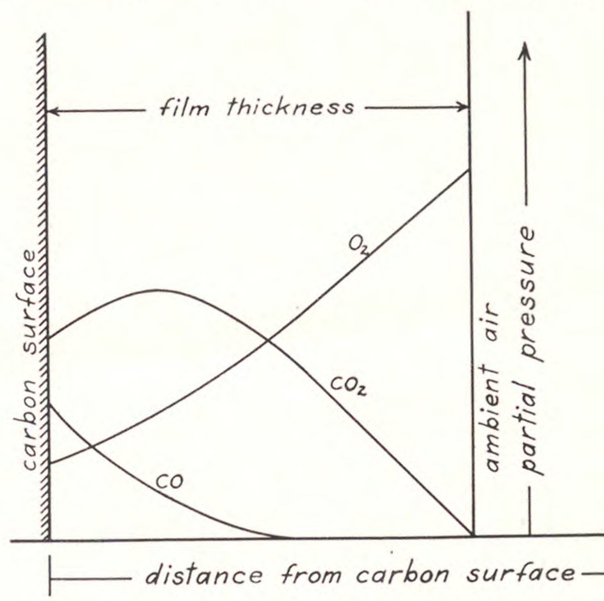

Figure 4

opposite directions from the region of maximum partial pressure, somewhat in the manner postulated by Burke and Schumann[13]. This picture has been postulated by Fischbeck[15] and is shown in figure 4.

Consideration of the factors influencing reaction and diffusion rates shows that the distribution of the gases at the carbon surface and throughout the gas film will be affected by surface temperature, furnace temperature, gas velocity, and composition of the ambient medium. Hence, no single diagram can be expected to represent accurately the combustion of carbon under all conditions. However, diagrams such as those presented in this paper are helpful in visualizing the process of combustion and show the importance of diffusional resistance as a factor determining combustion rates.

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(Continued on page 27)

THE ENGINEER AND PSYCHOLOGY

By LILLIAN M. GILBRETH, Ph.D., Sc.D., D.Eng.

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It is about thirty years since pioneer attempts were made to convince engineers that they needed a knowledge of psychology. Even the Management group, who had most contact with human problems, to whom the suggestion was made, received it politely, but very coolly. This was, perhaps, not surprising as few engineers had any training in understanding the human being beyond grammar and high school courses in hygiene and physical education. In most technical schools and universities engineers had such full schedules in science and techniques that there was little time for anything else. Such time as was available was usually given to History, Literature, or the Languages, which were too often taught by factual rather than by interpretative methods. It was more surprising that psychologists were equally cool toward engineers studying psychology and also showed a lack of interest in themselves studying the industrial situations in which the engineer worked.

"Psychology is a pure science," was frequently the answer given to any engineer wishing for psychological assistance on his industrial problems. Apparently, a "pure" science could develop only in an academic class room or laboratory!

The attitude of the psychologist has changed tremendously with the years. Although in the United States the psychologists have not pre-empted investigations of the human element in industrial and business areas, as was the case in several countries abroad, they have shown an increasing tendency to work in these areas. The reason why European psychologists pre-empted what we might call engineering areas was partially, at least, because engineers in those countries either refused to concern themselves with human element problems or did little. As a result, a more aggressive psychological group took over not only tests, guidance, and similar fields, but also advertising and selling and, less justifiably, investigations of fatigue, methods, and skill, obviously within the scope of the work of the Industrial Engineer. In this country Scientific Management, considering as it did not only problems of machines and materials, but of men and methods—early discovered that technical progress and human progress must be considered together. It was in spite of this that acceptance of psychology as necessary for the engineer was so lukewarm. The feeling seemed to be that handling of people was an "art," and that proficiency in an art is a gift which is accepted and used, and in no respect a skill which is learned.

The wholehearted devotion of the engineer to the disciplines of his science, to the effectiveness of his techniques, and to the school where he got his training—whether this was the school of hard knocks and experience, or some academic institution—made him sure not only that what he had was valuable, but that what he did not have was of questionable value.

The few hardy pioneers explored fields which might be called personnel areas, and interested themselves in what have developed into significant and useful personnel procedures. These inevitably realized the value of psychology more rapidly than did other engineers, and came increasingly to accept and ask for psychological findings. They gradually came to suggest problems needing investigation and to urge research along these lines. In the meantime, the psychologists themselves had recognized the fertile field which industry offered and had started investigations of their own in this field.

With the coming of the World War, the psychologist developed his techniques on selection and training enormously. These penetrated industry as well as the Services, and have continued to develop, from that day on. Where the psychologist became an effective personnel man or the personnel man utilized the best psychology had to offer, the results have met with great success. In fact, they have been so profitable that many people do not realize the contribution that psychology, itself, made originally and is making continuously. The patient research men and women in this field supply invaluable material and seldom get the appreciation their work justifies.

The engineer, and especially the Industrial Engineer, has come, through the years, to work more smoothly and profitably with the personnel man. He has increasingly added as a part of his training, courses in Personnel and Industrial Relations. These are, unfortunately, too often offered as electives which he may set aside for some other elective like Statistics, Political Economy, or Cost Accounting. It is a pity that the value of all these is not sufficiently realized so that they are included in all engineers' training; through more adequate secondary school preparation; integration into the four-year course; or the "fifth year" which is so valuable and so difficult for most engineers to arrange for. In spite of all of these things, many more industrial engineers preparing to enter industry are getting Personnel Relations work with each passing year. Up to the present time so few hours are available for this that only a hasty, survey, presentation can be attempted. The number of people who can succeed in the tremendous task of sensitizing the undergraduate to the point where he will carry on both in his work, and through study after graduation, is small. Yet, in spite of this, the results are encouraging and a pioneer required four year Personnel Relations course, in one college is arousing great interest everywhere.

What the engineer would seem increasingly to need is an appreciation that psychology has much to offer him if he is willing to go directly to it, as well as to utilize its findings as they penetrate into Personnel and Industrial Relations texts, courses and meetings. Psychology is a science and should offer not only findings, but a strong attraction, to engineers. It, like engineering, believes in the most accurate measurement possible and interests itself in discussing units, methods and devices of measurement. It has not, as yet, attained the results that engineering, which goes into measurement of materials and of machines, can show—but this is because the human element is far more difficult to measure. But considering the difficulties of its problems, psychology need not be ashamed of its results. Some of its difficulties are caused by the fact that it is dependent upon physiology —and the physiologist has as yet not sufficiently interested himself in those areas which are of primary concern to the engineer who comes to the psychologist for help. Some of its difficulties are caused by friction between psychologists and psychiatrists, on background, training, field, etc. Some of its difficulties are caused by the fact that psychologists have organized themselves into various "schools." Some of the members of these seem more interested in disputes, in destructive criticism, and aggrandizement than in division of responsibility and coördination of results. In attacking certain problems, such as the learning process, and the transference of skill, psychology has made splendid advancement and, fortunately for the engineer, these are problems of special interest to him.

What has the engineer who has taken a keen interest in effective handling of the human element, an open mind as to what psychology can offer him, and a willingness to devote time and energy to finding out, to expect today? First, a more general appreciation of his problems by psychologists. Few psychologists have become engineers, but an increasing number are gaining understanding of engineering problems, especially in such industrial areas as have the human element as an important factor of their problems. This is indicated by text books, some of which give excellent accounts of investigations in the industrial area, though, unfortunately, some of the best of these have been allowed to get out of date. The psychologist has also done more writing recently in a vocabulary easily mastered by the engineering mind. The engineer is in no position to criticize a technical vocabulary and by and large is not inclined to do this—but he does rightly expect technical terms to be generally accepted by the group who use them—with definitions available and understandable, and adequate illustrative material to make the meanings clear. The engineer will also find that personnel books contain in the body of the text and in footnotes very copious and adequate psychological references. He will find a welcome at psychological meetings and more and more psychological papers presented at his own technical meetings. There is, as yet, some tendency on the part of psychologists to talk up to or down to engineers, or to blame the engineer for conditions which he neither caused nor can control, but this tendency is becoming increasingly less, and psychologists spend more time and do more work in industry itself.

There is really no reason why any engineer with sufficient interest should not master at least enough of what psychology has to offer to be "at home" in the literature, at the meetings, or when talking or working with psychologists. But he has greater opportunity and even responsibility than this, both for what he can get and what he can give. He should become something of a psychologist himself, and there are certain pioneers who have done just this. They have not only studied psychology with the idea of utilizing successful results, but of evaluating methods as well as results, and of attempting to suggest possible solutions of unsolved problems. These may come from methods and devices which the engineer already uses in engineering work, or from methods and devices which engineers could supply to psychology through the utilization of engineering disciplines. The micromotion film—and the cyclegraph are two typical engineering techniques which have already proved their value as techniques of psychology. Undoubtedly many more now exist, and even more will be added, as the engineer works in this field.

It is too much to ask that any but the few gifted, or persevering, or both, go into this highly demanding and specialized field, but it is surely not too much to ask that the engineer consider what he has already done to use and assist psychology, and what remains to be done. The profession has accepted the responsibility of the effective handling not only of materials and machines, but of men. The profession has accepted, almost as an ethical code, the obligation to further technological progress as far as human progress will allow—but always to make human progress the first consideration. This demands understanding of human beings. Psychology is the science of human behavior. The engineer is pledged to accept the findings of science. Would it not seem that he must accept the findings of the science of psychology and that he would feel it his privilege to contribute to these findings—by his viewpoint—by his interest—and ultimately by his participation, not only in their use, but in their derivation?

AIRCRAFT PRODUCTION AND ITS PROBLEMS

By Jarvis C. Buxton, B.S.

Chairman, Standard Tooling Committee, Vega Aircraft Corporation, Glendale, California

The military airplane is far more complex because of structural, aerodynamic and weight requirements than is apparent in its smooth camoflaged surface. Military maneuvers, rough air, take off, landing and functional vibrations impose severe loads far in excess of those experienced in land and water travel. The airplane, to meet military requirements satisfactorily, must perform efficiently and compete in speed and maneuverability with other planes of its type. Drag, the greatest non-functional factor in aircraft design, must be reduced to a minimum and necessarily limits the surface finish and contour of the craft. The limitation of engine power requires the greatest of care in designing to the ultimate strength of the materials employed resulting in compulsory high strength weight ratios.

From the engineer's conception of the plane through the three view stage, wind tunnel tests, preliminary and final design, conservation of weight and ease and speed of fabrication must be the predominant factors governing the design. A suitable production breakdown must be decided upon so that a minimum of fabrication interference is experienced in assembly work. Each sub-assembly should be selected for its size or nature so that a few men are required to work in or on it at the same time. Assembly joints must be simple and so designed that high interchangeability is affected.

The number of ships to be built governs the extent of tooling and breakdown advisable. The present average order is 500 to 1000 planes and superficially appears to warrant semi-high production methods, but from an economical and practical standpoint a close analysis of the number of parts and assemblies in the plane shows the inadvisability of high production methods.

To many people a production rate of 50,000 airplanes a year is just another figure to meet for national defense. However, it is a challenge of such proportions that it must surpass the comparable efforts and development of 20 years in the automobile industry and 40 years in the steel industry. It means mobilization of huge quantities of men, machines and material in unheard of quantities.

The automobile industry may without materially increasing the cost of their product spend large sums on elaborate tooling and moving production lines as the cost of such tooling and methods applied to each car is small. On the other hand it is necessary to apply the cost of expensive and yet not elaborate tooling to 500 or 1000 planes.

Even though a blanket order were placed for as many ships as could be produced in a specified time other factors govern the extent of semi or high production methods to be employed. The entire industry as well as many others must be supplied with machines in quantities far in excess of those required in the past. Priority establishment has made many companies carry on under severe handicaps. There are very few plants which are not waiting for machinery to carry on certain phases of their projects. The shortage of machines handicaps the companies furnishing engines, propellers and other equipment as well as the plane fabricating plants. Dural, steel and other material shortages limits production to job lot manufacture. Large quantities of material in a semi-fabricated state require large storage spaces and represent considerable sums of money and amounts of materials tied up in a nonproductive state. It is impossible with available machinery to assign a machine to one specific part or operation. Every machine and tool must be uesd in several ways eliminating the possibility of large or complete order runs because of the delays incurred on other parts.

Supply is unable to cope with demand for skilled labor. One average aircraft plant today employs more men than was required by the entire industry ten years ago. This large spread of experienced men indicates the serious educational problems facing the industry. For every fifteen unskilled men there is but one skilled mechanic to educate and supervise various types of the work. One in ten of the experienced skilled mechanics can be classified as being capable of supervisory work. Government and educational agencies realizing the difficulties have arranged courses in every phase of the work and at all hours of the day so that both night and day employees may share in their benefits.

An experiment using women employees is being carried out in the southern California area. They are being educated to specific routine jobs and operations. Women have not been used in aircraft work previously but first reports indicate that they are easily trained and highly efficient. The release of experienced men from routine lines of work to advanced or more varied and skilled work by the use of women will automatically relieve some of the labor problems.

Skilled technical men have been borrowed from other industries and advanced manufacture and production ideas have been introduced by them. Riveting, the highest developed production operation in aircraft, is being replaced in many parts and structures by the newly developed resistance welding as it is more economical and faster. Advancements in forming dural sheets in the hardened state have eliminated many heat treatment operations. As yet magnesium machining and forming offers too many problems to be practical but a considerable amount of experimental work is being conducted because of the weight advantages gained by using the lighter metal. Steel in forming, shearing, and blanking dies has been supplanted by softer and easier worked materials. Part quantities are such that runs are completed before appreciable wear is experienced. Shear plates operated in conjunction with the Guerin forming process have decreased the number of routing or form blank cutting operations. Material shortage and time necessity have been responsible for many of the above developments.

After World War I there was a huge slump in aircraft manufacture because of the abundance of military craft readily converted for commercial use. Many people have predicted the same condition after World War II. The majority of planes, because of their military design, cannot be easily rebuilt for commercial use thus eliminating the possibility of a flooded market. This is one of the bright spots in the aircraft future. Regardless of the present "all out" effort to produce military planes the managements are looking ahead to the time when stability will again be reached. The huge capacities of the new and enlarged companies must be diverted into other channels. New aircraft fields are being found and others being investigated. We may look with confidence to a new era of improved aircraft uses in transportation after the War is over.

Comments by Dr. Frank D. Carvin, Professor and in charge of Mechanical Engineering Department, Newark College of Engineering:

The paper entitled "Aircraft Production and Its Problems" by Jarvis C. Buxton is a brief synopsis of the rather detailed thesis presented by Mr. Buxton for the degree of Mechanical Engineer at this College. Mr. Buxton graduated from Newark College of Engineering in June, 1939, with a degree of Bachelor of Science in Mechanical Engineering. He was employed by the Vega Aircraft Corporation of Glendale, California.

In 1941 he was selected as one of eight representatives of the Tooling Division of the Vega Aircraft Corporation to work out the problems involved in the subcontract of the Boeing Flying Fortress to the Douglas and Vega Corporations.

In November of 1941 he was made Chairman of the Standard Tooling Committee. This committee was charged with the responsibilities of coördinating Boeing and Vega standard operational tooling, tools standards and arranging the procurement of the necessary tooling material for the Vega-built Boeing Flying Fortress.

His paper briefly outlines some of the problems encountered in tooling up for this kind of work.

I have found the paper an exceedingly interesting synopsis of the more detailed report presented in his thesis.

WAR EMERGENCY PROGRAM

Dr. Allan R. Cullimore, President, Newark College of Engineering, has announced the following program for the College:

1. Freshmen entering in September, 1942 (for pre-freshman term see No. 2 below), will have their course scheduled so that it will be possible to graduate them in 1945. The exact time of graduation in 1945 will be determined as the emergency develops.

2. The program necessary to provide graduation in 1945 makes advisable the offering of an optional pre-freshman course to orient students, acquaint them with college methods of instruction and strengthen any weaknesses in Mathematics or Science. This course, "Introduction to Engineering," will start July 6th, if enrollment warrants, and will be given three half-days each week for ten weeks. Tuition for the ten-week course, $30.00.

3. The class of 1943 will complete a summer term in accordance with the calendar below and will graduate in January, 1943.

4. Announcements as to vacations, holidays, examination periods, etc., will be made as information regarding emergency needs becomes available.

COLLEGE CALENDAR[1]

April, 1942, to September, 1942

Good Friday	April 3
Spring Recess (Freshmen, Sophomores and Juniors)	April 13-18
Re-examinations	April 13-18
End of Second Semester (Seniors only)	May 8
End of Second Semester (Freshmen, Sophomores and Juniors)	May 30
Final Examinations (Freshmen and Sophomores only)	June 1-6
Registration (Seniors, Class of 1943, only)	June 3-5
First Semester Begins (Seniors only)	June 8
Labor Day	September 7
Re-examinations	September 8-12
Entrance Examinations	September 8-12
End of First Semester (Seniors)	September 11
Registration (Freshmen)	September 14 at 9:00 A.M. to September 16 at noon
Registration (Sophomores, Juniors and Seniors)	September 14 at 9:00 A.M. to September 19 at noon
Second Semester Begins (Seniors)	September 21
First Semester Begins (Freshmen, Sophomores and Juniors)	September 21
Registration of Freshmen Entering in February, 1943	See note below[2]

[1] Subject to emergency revision.

[2] Students desiring to enter the College in February, 1943, may apply to the Registrar for information concerning the dates of the registration period and the beginning of the spring session.

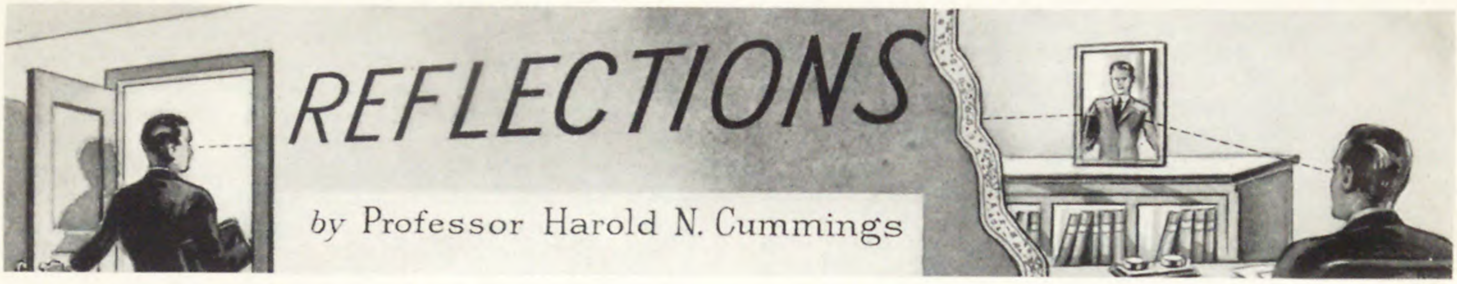
SCHOLARSHIP TO OLSEN

Reprinted from *The Percolator* of Chemist's Club, N. Y., of January, 1942

The Hoffman Scholarship of The Chemists' Club has been awarded for the school year 1941-42 to Robert T. Olsen, a candidate for the Ph.D. degree in the Department of Chemistry at Massachusetts Institute of Technology. This scholarship, founded by the late Dr. William F. Hoffman, is available in alternate years; the stipend is $800, payable in semi-annual installments of $400.

Mr. Olsen did his undergraduate work in the Newark College of Engineering, receiving the B.S. degree in chemical engineering in 1936. In 1937 Columbia University awarded him the M.S. degree in chemical engineering, after which he worked for two years with the Eastman Kodak Company, Rochester, New York. In 1939 he entered Massachusetts Institute of Technology as a candidate for the Ph.D. degree in chemistry. He is completing this work under the direction of Professor Ernest H. Huntress, and will be eligible for this degree in June, 1942.

Mr. Olsen chose as his thesis subject for the B.S. degree "Design of a Plant for the Manufacture of Propanol-2," and "Etherification in Hydrotropic Solution" for his M.S. degree. His Ph.D. dissertation involves study in the field of syntheses of coumarones.



Well, it was pleasant in the good old days to look into my "busybody"—my office mirror—and imagine that I could see the shore of the lake down east where so many long summer vacations have been spent resting up after a "full academic year" of work on a "normal" schedule of teaching. The resting usually consisted of much hard labor—clearing out brush, cutting firewood, or paddling a canoe against a headwind and rough water for hours for the sake of a little fishing—or perhaps struggling with a recalcitrant outboard motor while slowly coming to the realization that it was lucky we put a pair of oars in the boat. The long summer vacations are out, for the duration at least. Instead of dreams of vacation scenes, I see now a steady procession of young people coming in to inquire about training courses for war work—courses that will run all spring and all summer. And I see members of the staff making arrangements for their share of summer teaching of college classes or of special classes. And I do not hear any grumbling or any talk about the right to "time and a half" for working at unusual times. Sometimes I wonder if the reason our people pitch in with so little grumbling may not be that they feel sure of their privilege of grumbling and wish to retain that privilege.

I wonder if you noticed that in the preceding sentence I said the *privilege* of grumbling. I deliberately avoided using the word "right." I've done a lot of reflecting about "rights" and find myself more and more reluctant to speak of my "right" to do anything, or your "right," or that of anyone else. We use the word too carelessly, it seems to me. And that carelessness leads us to take for granted the continuation forever and forever of what we call our rights. Let's suppose, for the moment, that we think we have the "right" to grumble about the extra hours of work needed because of the war. We claim that "right," I suppose, because of the "Bill of Rights." Can that Bill of Rights be repealed? It certainly can—in actual fact by the simple process of a constitutional amendment, or in effect by an interpretation of the Constitution by the Supreme Court. Can a few of us who desire passionately to be free to grumble ad lib prevent such a repeal? We can *not*, unless we can persuade a majority of those sufficiently interested to vote at the necessary elections to go along with us and vote down a proposed repeal. In other words, we have to get the *permission* of the voting majority to continue grumbling under the guise of free speech. It seems, then, that we don't have any "right"—it's only a *privilege* and something we may lose at the will or whim of the voting majority. Does this startle you? I hope so. Does it shock you? It ought to. I've had entire classes of Seniors rise up in indignation at my suggestion that they didn't have any rights. I'm very sure I did not succeed in making clear to some of them that whatever freedom they had was theirs by permission of "we the people," and could—and might be—reduced or taken away at any time.

H. N. Cummings

March 2, 1942

ELECTRICITY AND MAGNETISM — Norman E. Gilbert, Ph.D.—The MacMillan Company, 1941.

This is primarily a survey course in electricity and magnetism for non-engineering students. As it presupposes calculus preparation, the course would usually be given in the junior year of the arts or science course. As a survey course, it covers the subject very broadly and in the most painstaking manner. Since electricity has such a wide application, everyone needs a knowledge of the subject such as this book provides.

Advanced references are included in the body of the text wherever it is felt the treatment of the topic as included in the text is too brief. It is necessary to do this, otherwise the book would be encyclopedic in size.

The first seven chapters cover some fundamentals in electricity. The next six chapters deal primarily with circuits, and then there are three chapters on current and magnetism. These are followed by electrical machinery, and after that, a chapter on conduction of gases leading up to thermionic electron tubes and the general subject of electrical communication.

At first glance one is impressed by the small, clear diagrams used for illustrations. There is no distraction from the main discussion of the text by these simple illustrations. The large number of problems at the end of each chapter in the early part of the book makes for ease in teaching the subject.

"The proof of the pudding is in the eating" might be paraphrased, "The proof of the textbook is in the teaching." As the writer has not had the opportunity to apply this acid test, his comments must be of a very general nature. The fact, however, that this is the second edition of a book published nearly nine years ago is evidence that the book found a real field of usefulness.

The writer was unable to compare the present revision with the original text. This revision naturally includes the correction of any errors that have been discovered in the use of the text for class work. The book also includes a new Chapter V which provides some fundamental theories of dielectrics and of electric induction. This is really a valuable addition to the subject.

The broadening of the discussion in Chapter XIX concerning the various systems of units is a particularly good one. The paragraph on the M.K.O.S. system of units at least establishes a basis on which to argue for the use of the ohm as the fourth unit in the electrical system of measurements.

The chapter on thermionic electron tubes has been rewritten and brought up to date in a very acceptable manner. Also the addition in Chapter XVIII of some material on electrical communications is very timely, since we find that the subject of electrical communications is receiving greater emphasis all over the country.

Finally, the author has chosen well, from the material available in Electricity and Magnetism, the topics most essential in a survey course. He has treated this material in a most logical manner and has produced a most excellent and up-to-date text for a general course in the subject.

PROFESSOR PAUL L. CAMBRELENG DRAFTED

By Robert W. Van Houten

Associate Professor in Civil Engineering, Newark College of Engineering

The long arm of the United States Army reached into the College on Friday, February 13th, to take Professor Paul L. Cambreleng into its service for the duration. Professor Cambreleng has played an important part in the guidance and personnel program of the College and his absence will leave a void which it will be difficult to fill until he returns.

Professor Cambreleng began his college education at the Newark College of Engineering, later entering Newark University from which he was graduated in 1935 with the degree of Bachelor of Arts in Economics. Recently he has been doing graduate work in the field of guidance and personnel at Teachers College, Columbia University.

After graduating from Newark University, Professor Cambreleng spent one year and one-half in the Audit Division of the Prudential Insurance Company. On February 1, 1937, he became associated with the Newark College of Engineering as Registrar. More recently as Chairman of the Committee on Entrance Credentials. He has been in charge of admissions to the College. For several years he has served on the administrative staff of the Newark Technical School and since September, 1941, has held the title of Registrar of that institution. His contributions to the development of the College's selection, admission, and registration procedures have been many and valuable. In addition to his administrative duties Professor Cambreleng also, up to last September, held the title of Instructor in Industrial Relations and since then the title of Assistant Professor in Personnel Relations. His academic activities have included the supervision of the freshman course in Principles of Engineering which is an orientation course for freshmen and the guidance of discussion groups in the junior and senior courses in Staff Control.

Professor Cambreleng has also been extremely active in the community having been a member of the Board of Directors of Youth Consultation Service, Church Mission of Help, Newark Diocese; Regional Vice-President of the Guidance and Personnel Association of New Jersey; a member of the National Vocational Guidance Association; the American Association of Collegiate Registrars; the Middle States Association of Collegiate Registrars; and Sigma Pi. He has made a conspicuous success in the guidance and personnel field and it is to be hoped that the loss which the College and the community will suffer during his leave of absence may be more than compensated for by the contributions which he is able to make in the field of personnel in the Army. All of his associates wish him success and a speedy return.

(Continued from page 22)

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Book Review on opposite page reported by Professor James C. Peet, in Charge of Electrical Engineering Department, Newark College of Engineering.

Professor Paul L. Cambreleng

READING FOR ENGINEERS

From *Engineering News-Record*, January 29, 1942, page 71

THROUGH ENGINEERING EYES—By Allan R. Cullimore. 166 pp., Pitman Publishing Co., New York and Chicago. Price, $1.50.

This book of pocket size contains extracts from the writings of 27 famous men translated into English by various authorities and in most cases bearing directly or indirectly upon the development of scientific and engineering thought. The period from 1000 B. C. to the present century is represented, but most of the space is devoted to the period preceding the birth of Christ. The longest article consists of interesting quotations from the Greek traveler and historian, Herodotus, who lived in the fifth century B. C. and from whose writings are obtained the most definite information available upon many matters of ancient customs; but Homer, Archimedes, Caesar, Roger and Francis Bacon, Leonardo, Cellini and Ben Franklin are also allotted considerable space.

Each extract is preceded by a brief statement concerning its author and these statements are among the more valuable features of the book. It is to be regretted, however, that the introductory paragraph, in the case of Leonardo da Vinci fails to do justice to his achievements as an artist. In the writer's opinion it should have included mention of his famous paintings, Mona Lisa and The Last Supper.

The compiler states that the selections have been taken from books which he has read and enjoyed, and a perusal of the volume furnishes convincing evidence of his own high taste. The book should prove of interest to all readers and be of especial value to scientific and engineering students, as well as to older and wider-read members of these professions.—Reviewed by C. M. Spofford, consulting engineer, Boston, Mass.

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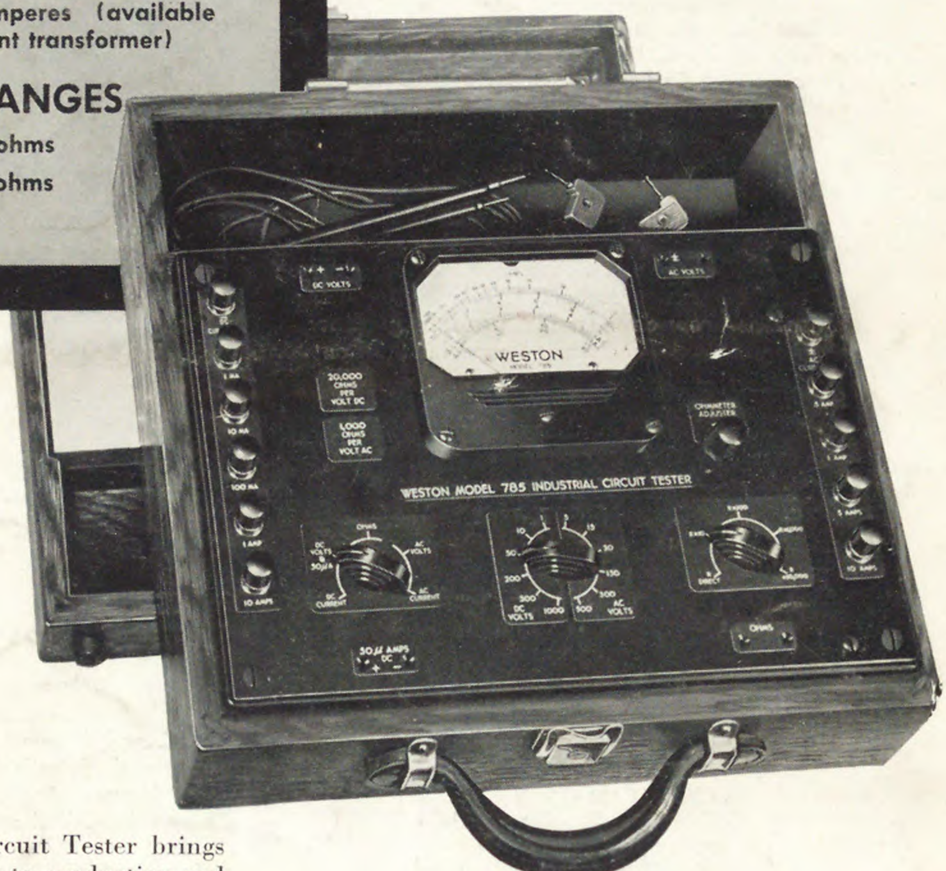

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