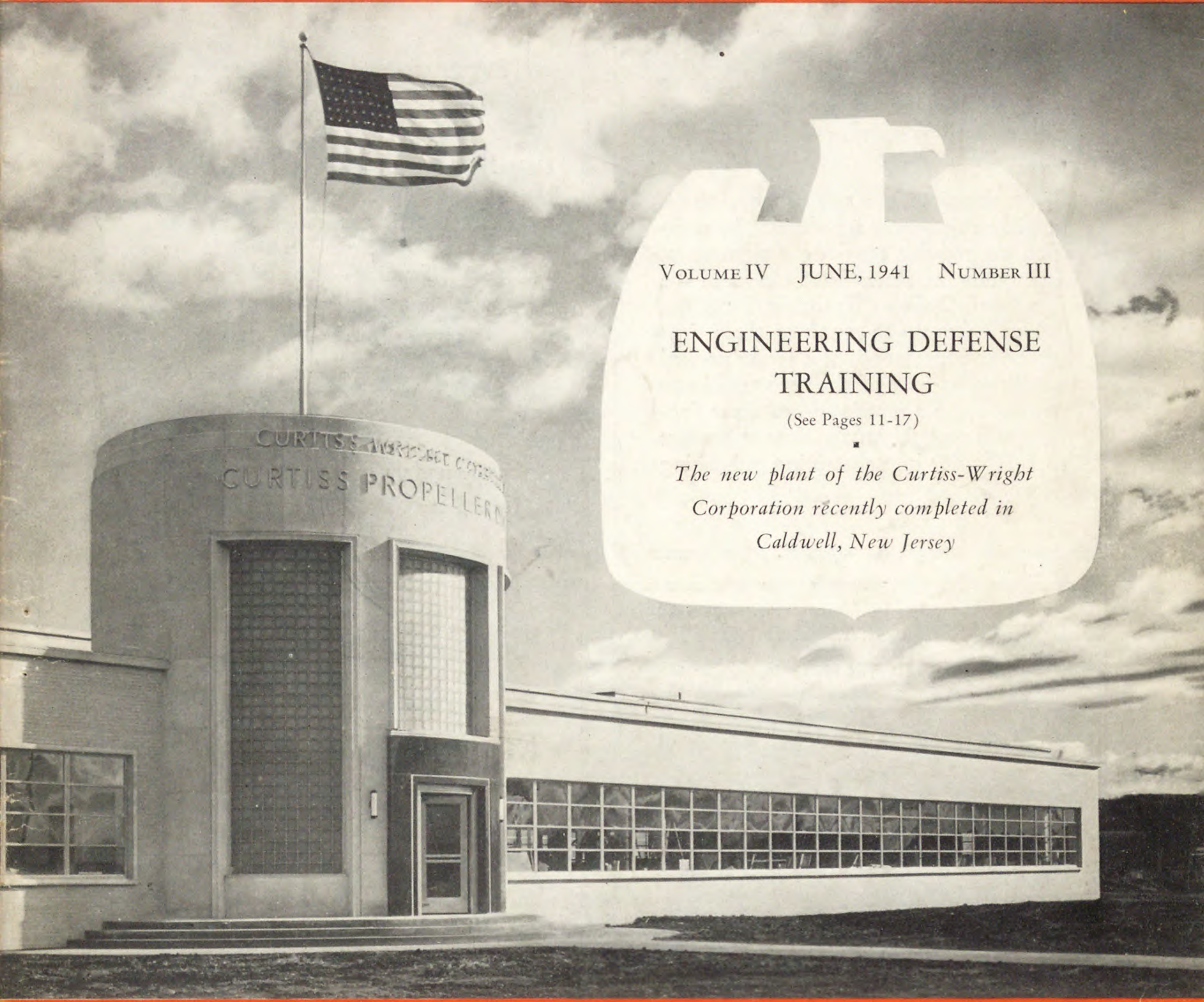
VOLUME IV JUNE, 1941 NUMBER III

ENGINEERING DEFENSE TRAINING

(See Pages 11-17)



The new plant of the Curtiss-Wright Corporation recently completed in Caldwell, New Jersey

Newark ★ ENGINEERING NOTES

Readers may be aware of the fact that this issue of the NEWARK ENGINEERING NOTES has not been published as early as planned. All the material which goes to make up the contents of the magazine is contributed by members of our faculty, alumni, and friends.

At this time, our staff is working on greatly augmented schedules. In addition to regular teaching assignments, nearly all these men have classes in the National Defense Training Program, which takes up so much of their time that our editorial board is sorely pressed to obtain articles of the type we like to publish. An appreciation of these factors will probably make clear to readers the difficulty in maintaining publishing dates.

NEWARK ENGINEERING NOTES

A Technical Magazine

for

Chemical, Civil, Electrical, and Mechanical Engineers

▪ ▪

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CONTENTS

DO YOU WANT TO BE ON OUR MAILING LIST?

From time to time you will receive a copy of the NEWARK ENGINEERING NOTES. In case you wish to receive this magazine regularly, the only thing you have to do is to write us and say so. Address your letter to the Editor, Newark Engineering Notes, Newark College of Engineering, Newark, N. J.

THE PRESIDENT'S DIARY

March 15th

I suppose that one of the very definite responsibilities of any sort of leadership is to advise, and one of the hardest problems that faces the head of any institution, in dealing with younger people, is to give sound advice under difficult and changing conditions. At this particular time it would seem extremely difficult to advise young men who are in college and who are graduating from college as to their course of procedure with respect to this whole question of National Defense.

This is particularly true when none of us is any too sure, when no values seem to be stable, and where the whole situation is in a state of change which moves ever more rapidly. Perhaps the only excuse that we can have or the only thing that can comfort us is that our lines of contact are more numerous, the points where we touch the situation are greater in number, that is, our general horizon is a bit wider than that of these young people whom we attempt to advise; and while the advice must be given, I think we all feel progressively more and more that no one knows, no one can tell, what the future holds.

There are a few basic things, however, which seem to be true, things which are even more important than the immediate emergency, things which should be characteristic of our treatment of youth in any day, any age, under any conditions, no matter how immediate a necessity seems.

In the first place, I think that with respect to joining the armed forces, the question of deferment of the draft, the question of defense industry against armed forces, should be determined primarily, perhaps, on the basis of a man's own conscience —what he feels he must do. I am advising our young men to get all the information they can from all sources, and giving all that I have, but resting the ultimate decision upon the man's conscience, his will and his desire as to the general line in which he shall help; and I think the question, "What do you feel you should do?" is the most pertinent question which can be asked these young people—this to be followed by the facts as we see them.

This approach has the advantage of what we in educational work call "motivation" and at least ensures to the man the privilege of serving in the place that he can serve best.

April 15th

In reading over this last entry, it has occurred to me that it might be wise to indicate some of the factors in the situation as I see them. The main idea that seems to spring out of this whole defense effort of ours is that what we need are primarily trained minds and trained hands to think and to work, and we do not need, as we did in the last war, simply living, moving, breathing animals to be shot at.

This has all changed, and the war will be won, I think, in the Army, in the Navy, and in our defense industries by highly skilled, trained technicians in all fields; and by technicians I would include those very highly skilled intellectual technicians whose work is entirely that of designing and of planning, for in the broad sense they are technicians as well. The cry in this war is for the machines of war—not for the men. We need ships, airplanes, tanks, trucks, tractors, and thousands of other pieces of machinery to do the job, and men as men may even be a drug on the military market; but brains, planning ability, and technical ability with the head or the hand are at a very high premium.

Personally, it seems to be entirely beside the question as to whether a young man desires to develop this thing through the Army, the Navy, or through the industrial channel, and I am of the opinion that he should do it in the place that he wants to. It should be remembered that the Army can get all of us, young or old, if it wants us and needs us. Every one of us stands ready to answer any sort of call for service which we can discharge to the United States Army. The same is perfectly true of the Navy and, in addition, the same is perfectly true with respect to those vital defense industries which need technical ability, and I hope my students will go where they want to go and give all they have to the proposition of defense.

There is one and only one limitation that I see that any young man has in days of peace, and in these times more than in days of peace. That is, any young man is very foolish to let anything interfere with his technical education or prolong it, for interference in that education or the deferring of the time when he can get into the picture as a technical man is definitely against the best interests of the nation as I see those interests. I think it is very necessary for all young men who can enter technical training to enter it immediately and to follow it definitely until they finish that particular phase of technical training for which they are fitted.

All that I see leads me to believe that we need technically trained men in all fields as we need nothing else. Moreover, we need them now and will need them increasingly as time goes on.

ALLAN R. CULLIMORE

May 2, 1941

AN ELECTRONIC FREQUENCY-METER

By RUDOLPH DEHN
Senior, Newark College of Engineering

Simple, direct-reading frequency meters which will indicate frequency with practical accuracy at least to 15,000 cycles per second have been devised in the last decade. These meters are based on the fundamental definition of capacitance, and their readings are largely independent of the waveshape of the periodic wave under investigation. The circuit to be described is a modification of an existing circuit with the intent to increase the stability and sensitivity without sacrificing independence of waveshape.

Square waves, triangular waves, and other geometric waveshapes are finding increased use in modern circuits. For instance, in video circuits, the synchronizing pulses are square waves and the sweep voltages are triangular in shape. The modern method for checking the frequency response of audio amplifiers by means of square waves is another illustration. These applications make it desirable to have an instrument for measuring the frequency of these waves.

At present there are available many instruments for accurately measuring the frequency of sine waves, both in the audio and radio frequency portions of the spectrum. However, when the frequency of odd waveshapes is to be measured, it is necessary to compare these waves with sine waves of known frequency. This comparison is made by oscillographic means and is necessarily a cumbersome procedure. Therefore, it is desirable to have an instrument for measuring directly the frequency of all these waveshapes and to do so with at least sufficient accuracy for commercial tests. The purpose of this paper is to describe a device with which frequency measurements of many waveshapes may be made in the audio range from 20 to 29,000 cycles per second.

This type of frequency meter is based on the fundamental definition of capacitance, $Q = CE$. If a condenser of good quality is charged from a constant-voltage source, a definite quantity of energy will be held on the plates of the condenser. This is the theoretical relationship which is approached in practice. Actually, the fundamental circuit of this type of frequency meter is as in figure 1.

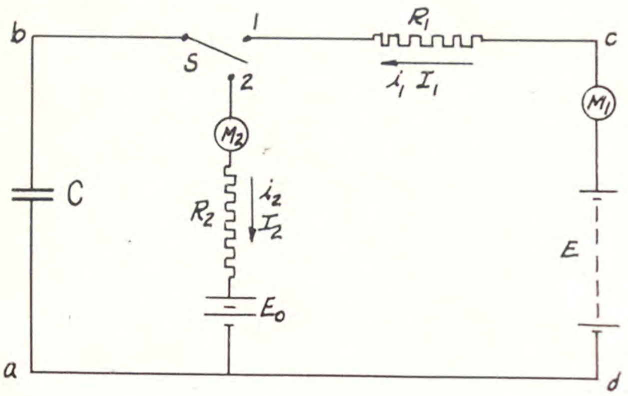


Figure 1

During one-half of the cycle of the test voltage, switch S connects the condenser to a voltage source, E, through a resistance, R_1, and the D'Arsonval instrument, M_1. The condenser then charges during this half-cycle and M_1 indicates the average charging current. During the second half of the cycle, the switch assumes the alternate position and the condenser discharges to a lower voltage, E_0, through the meter M_2, and the Resistance R_2. Here M_2 reads the average discharging current.

In appendix A the expressions for the voltage of this condenser are derived. When the condenser is charging, its instantaneous voltage is

$$e_{c1} = E - (E - E_0)\varepsilon^{-\frac{1}{2}fR_1C}$$

and the average current read on meter M_1 is

$$I_1 = fC(E - E_0)(1 - \varepsilon^{-\frac{1}{2}fR_1C})$$

The time for one half-cycle of the unknown frequency has been substituted for the time element of the equations. The similar equations for the second half of the cycle, during which the condenser discharges, are

$$e_{c2} = E_0 - (E_0 - E)\varepsilon^{-\frac{1}{2}fR_2C}$$

$$I_2 = fC(E_0 - E)(1 - \varepsilon^{-\frac{1}{2}fR_2C})$$

The significant feature of the equations for average current is that this current varies exponentially with frequency. In order to prevent a crowded instrument scale at the upper end of the frequency range, the exponential terms in the equations must be made negligible in magnitude. This is accomplished by maintaining a low RC product in the exponents. Then the indicator current will be directly proportional to frequency for a given capacitance and a given set of voltage limits.

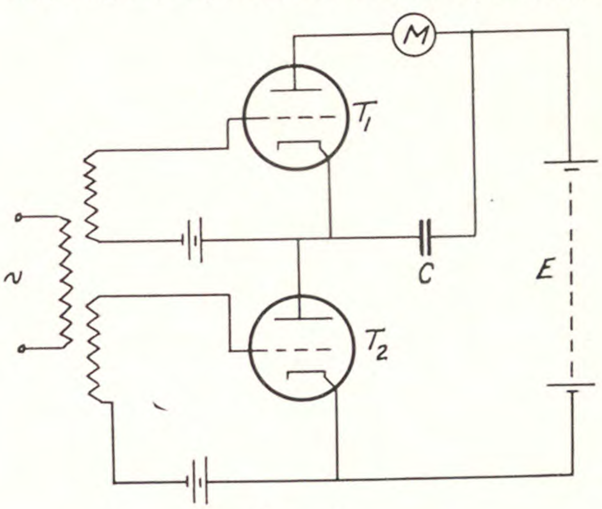


Figure 2

The problem is to design a circuit which will perform the necessary switching and make it possible to read the condenser current. One of the earliest circuits employing the principle which has been described is that presented by Guarnaschelli and Vechiacci in the *Proceedings of the I. R. E.*, p. 659, 1931. The basic circuit which they used is shown in figure 2. The grid voltage of tubes T_1 and T_2 is adjusted so that plate current will be just cut off when the constant voltage, E, is applied. The transformer is so connected that a signal voltage, e, induces two voltages 180° out-of-phase in the secondary windings. The device operates as follows: Assume that condenser C has no charge, and that a signal voltage of unknown frequency is applied. During one-half of the cycle the grid voltage of tube T_1 becomes more negative, while that of T_2 becomes more positive. This grid swing causes T_2 to become conductive, and condenser C charges up to the supply voltage, E, through the plate-to-cathode impedance of T_2. During the second half-cycle the grid voltage of T_2 becomes more negative, while the grid voltage of T_1 becomes more positive. Thus the supply is cut off and the condenser C discharges to zero voltage through T_1 and the meter. The current indicated by M will be practically

$$I = Q/t = fCE$$

In view of the exact equation for the average current, this equation is an approximation. In practice, the error incurred here is negligible. In fact, this is the favorable feature of this circuit. That is, when triodes of high plate conductance are used, the resistance represented by the tubes, although variable is low, and the exponential term in the current equation can easily be made negligible. This can be accomplished without reducing the size of the condenser unduly, hence the current indication per cycle will be high.

However, two poor features of the instrument limit its use. Both of these result from the use of the input transformer. First, as the frequency increases, the core losses in the transformer increase and the secondary voltage decreases. With specially designed transformers having low-loss cores, the maximum readable frequency is approximately 10,000 cps. The second disadvantage of the transformer is its operation when odd-shaped waves such as square waves are applied. These waves cause anomalous voltages to be developed in the secondary. If the amplitude of the signal voltage is too low, or if the waveshape is such that the conductance of each tube is too low to perform the charging and discharging function in the time available, the readings will be in error.

In connection with the signal voltage specifications, if the condenser is large enough to prevent charging essentially to the supply voltage, the actual voltage to which the condenser charges will vary with different waveshapes of the same frequency. Therefore, it is desirable to charge the condenser practically to the supply voltage and discharge it practically to zero voltage, not only to produce a linear scale on the meter, but to attain greater freedom from waveshape. Since the principle of this type of frequency meter is predicated on a constant-voltage source, some means must be provided to keep this voltage constant. Various methods may be used. The most common device makes use of the constant voltage drop across a gaseous diode, this voltage being independent of reasonable variations in current.

Probably the most recent frequency meter based on the principle we are discussing is that devised by E. L. E. Wheatcroft and G. Haley and presented in the April, 1937, issue of *Journal of Scientific Instruments.* The fundamental diagram for their frequency meter is shown in figure 3. In this circuit the tube T_1

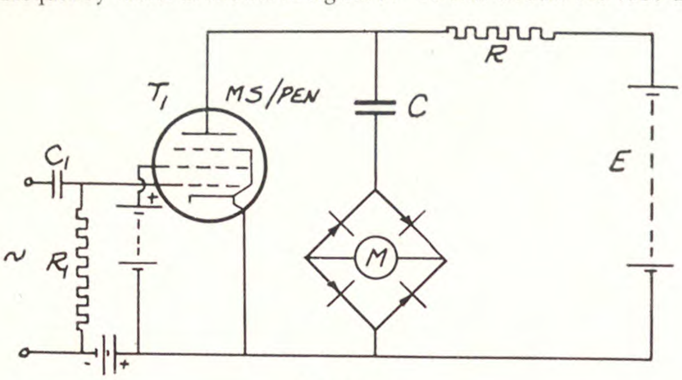


Figure 3

is a high-transconductance pentode which is operated at cutoff grid voltage. When there is no signal voltage applied, condenser C charges up to the supply potential, E, through R. When an a-c voltage is applied to the grid of the pentode and this voltage becomes more positive, the condenser discharges through the tube, approaching the steady-state anode-to-cathode voltage. When the grid voltage becomes more negative the condenser again charges to the voltage E. Both the charging and discharging currents are read on the d-c microammeter, M, which is used in conjunction with a bridge-connected copper-oxide meter rectifier.

The most important feature of this circuit is that the input transformer has been eliminated. Thus, the frequency and waveshape limitations from this source are no longer present. The authors point out that a triode could be used in place of the pentode, provided the triode may be operated with positive grid voltage. This is necessary to secure a high enough conductance between anode and cathode to (1) discharge the condenser almost to its steady-state value in the time of a half cycle, and (2) make the steady-state voltage of discharge very low. A pentode was used in this circuit because of the low anode-cathode impedance possible without a large grid voltage. Thus the input impedance of the instrument remains high. This last statement points out a disadvantage of operating the grids of the tubes at positive potentials, other than consideration of the heat developed in the tube. That is, when grid current starts to flow, the grid-cathode impedance has decreased, and the meter burden increases on the circuit under test.

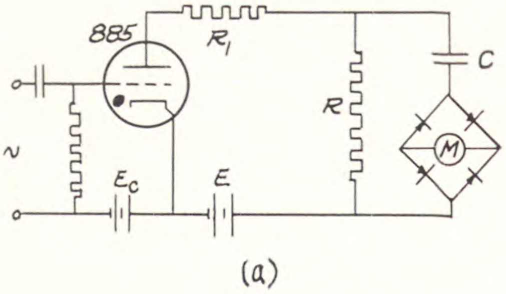
Referring to figure 3, if the anode-cathode impedance of the tube during conduction is of the same order of values as resistance R, the lower limit of the condenser voltage will be high, and the calibration will be affected by variations in the characteristics of the tube. To overcome this condition, Wheatcroft and Haley found it necessary to make R approximately 25 times greater than the conductance of the tube at small positive values of grid voltage. This fixed R at 50,000 ohms. A disadvantage here, due to the use of this high resistance, is that the condenser must be small in order to keep RC small. That is, if the condenser is to charge to practically the supply voltage, (say 99%), RC must be such that $\varepsilon^{-t/RC} \leq .01$. This means that for a given frequency, the average current will be less than that obtained with a larger condenser. A 0-500 microampere meter is used in this circuit. With three condensers the three ranges 0-150, 0-1500, and 0-15,000 cps are obtained. The capacitances of these three condensers are .00966, .00097, and .000095 ufd. respectively. Here another undesirable feature is seen. That is, for the 0-15,000 cps range in particular, leakage currents and changes in atmospheric conditions around the instrument are likely to have a relatively large effect on condensers of small capacitance. As a result, the stability and accuracy of the instrument will be impaired.

The Wheatcroft-Haley circuit seemed to be the simplest, most desirable circuit for this type of frequency meter. Hence it was used as a basis for a modified design to secure additional features. The objectives which were set up for the improvement of this circuit were:

1. Maintain simplicity
2. Extend the frequency range
3. Increase freedom from waveshape
4. Increase sensitivity

Of these objectives, the first and fourth have been realized. The second objective definitely seems to be realizable, although it has not been verified experimentally. The waveshape tolerance is the same as that of the Wheatcroft-Haley circuit.

The first circuit developed to this end is shown in figure 4(a). The tube T_1 is a type-885 gaseous triode with the grid voltage, E_c, set slightly more negative than the value required for firing at an anode voltage, E. The intended operation was as follows: With no charge on the condenser the anode of the tube is at supply voltage. When a signal is applied to the grid, the tube will fire, causing current to flow through R and R_1. The condenser voltage then approaches the IR drop in the resistance R. By the same principle upon which saw-tooth oscillator circuits function, the tube discharge becomes unstable when the condenser voltage approaches its steady-state value and conduction ceases. This periodic instability causes the condenser to discharge through R to a lower limit of zero voltage. The condenser current is read on the copper-oxide rectifier type meter as in the Wheatcroft-Haley circuit. R_1 is a resistance for limit-



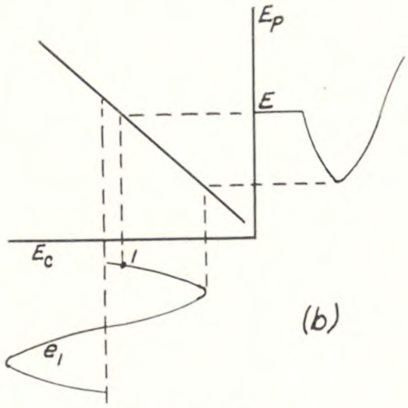


Figure 4

ing the peak current at the instant of firing and R_2 limits the grid current. A gaseous triode was chosen because of its very low impedance between anode and cathode during conduction and because the grid voltage has negligible effect on the discharge, once conduction has started.

The desired operation described above is not obtainable in this circuit. The reason is best illustrated by reference to the firing characteristic curve for the type-885 tube as shown in figure 4(b). When the sine wave voltage, e_1, is applied to the grid of the tube, the potential at which the tube will fire varies according to the projected portion of a sine wave shown opposite the anode voltage axis of figure 4(b). Of course, the tube fires as soon as the signal voltage wave reaches point 1, since the anode voltage prior to this instant was E. However, the first cycle of charge and discharge is completed before a half-cycle has elapsed when the signal frequency is less than the maximum frequency for which the charge and discharge is essentially complete. When this occurs, the condenser will immediately charge at a rate determined by the time constant, RC, and the tube will fire at some new voltage, dependent on the locus of firing voltages shown in figure 4(b). The condenser charge and discharge is not directly controlled by the signal voltage, and the current indicated by M will be too high. If the amplitude of the signal voltage is increased, the locus of anode firing voltages of figure 4(b) is such that the firing voltage changes rapidly from E to zero and from zero to E at the ends of the half-cycles. This change of the firing voltage will prevent the spontaneous saw-tooth pulses, and the meter will read properly. However, in this way the operation of the device is made susceptible to variations in the waveshape of the signal voltage, and consequently there is no advantage over the Wheatcroft-Haley circuit.

The saw-tooth oscillations can also be eliminated if the firing characteristic curve is a vertical straight line. That is, at a critical grid voltage, the tube will fire at any anode voltage. The only tube of this type available is the type OA4G cold-cathode triode. This tube was substituted for the type-885 in figure 4(a), with the starter-anode voltage, E_c, operated close to its positive critical value. When a signal was applied, the first positive swing of the a-c voltage increased the starter-anode voltage to the critical value and the tube fired. However, when the signal voltage varied in the negative direction, the OA4G would not deionize, even at low frequencies. In order to make the gaseous discharge less stable, R was increased to a maximum of 100,000 ohms. However, the tube still did not deionize during the negative half-cycle. R was not increased above 100,000 ohms, since it would be difficult to obtain a low RC product under these conditions, and no advantage over the Wheatcroft-Haley circuit would be realized.

Since the type-OA4G would not function successfully, a method was sought for modifying the circuit of figure 4(a) in order to eliminate the saw-tooth pulses. If the supply voltage could be cut off at the appropriate time, there would be no power available for the saw-tooth oscillations. The simplest method for doing this is to replace R in figure 4(a) by the anode-cathode impedance of a high-vacuum tube and applying the signal voltage to this tube. This circuit, rearranged, is shown in figure 5.

In effect, the resistance, R, of the Wheatcroft-Haley circuit has been replaced by R_1 and the type-885 gaseous triode, T_2. The gaseous triode permits a low RC time constant for charging the condenser, in addition to the low time constant for discharging already available in the Wheatcroft-Haley circuit. The operation of the device is as follows: Tube T_1 is a high-transconduction triode (type 2A3) with a bias E_{g1} set for plate current cutoff at the plate voltage, E. The grid voltage E_{g2} has been set to cause firing at a voltage slightly less than E. When the condenser, C, is discharged and the signal voltage swings the grid of T_1 in the negative direction, the anode-cathode potential of T_2 is equal to the supply voltage, and the tube fires. The condenser then charges through R_1 and T_2 to a voltage approaching the supply

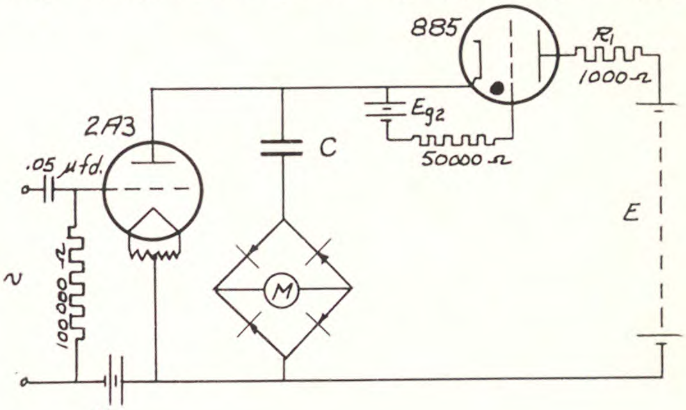


Figure 5

potential minus the constant voltage drop across T_2. When the condenser voltage reaches this value, T_2 ceases to conduct because of the decrease in anode-cathode potential below the maintaining voltage of the arc. When the signal voltage is in its positive half-cycle, T_1 starts to conduct, causing the condenser to discharge. The condenser voltage continues to drop until the supply voltage minus the condenser voltage is equal to the critical anode voltage of T_2. At this point T_2 fires again and the condenser recharges. If T_1 is still conducting when T_2 fires, the condenser charges up to the anode-cathode voltage drop of T_1. As soon as T_1 ceases to conduct, the condenser voltage approaches the maximum value it reached when it first became charged. Saw-tooth oscillations cannot occur because the condenser will not discharge until the signal swings the grid of T_1 in the positive direction. The sum of the average charging and discharging currents is read on the copper-oxide rectifier type meter, M. The function of R_1 is to limit the peak anode current of T_2 at the instant of firing.

The circuit parameters specified in figure 5 are those used in an experimental device constructed in the laboratory. A Western Electric Model 13A oscillator with a calibrated range of 30 to 10,000 cps supplied a variable frequency sine wave voltage. The operation of the circuit was easily checked with the aid of a cathode-ray oscilloscope. The rms value of the signal voltage was measured with a rectifier type meter having a constant impedance of 20,000 ohms. Various calibration curves corresponding to several values of capacitance were taken. All of the calibration curves for this circuit are linear down to approximately 100 microamperes. Below this value there is some curvature, apparently indicating that a minimum frequency is required before any reading is obtained. This curvature is probably due to the presence of the copper oxide rectifier in the circuit. That is, a copper oxide rectifier does not offer infinite impedance to the flow of inverse current. The effect of this anomalous condition is to make the meter reading of current different from the computed value, although the meter reading is still proportional to frequency. The true condenser current, including the leakage current in the rectifier probably conforms to the theoretical calculations. By the use of several capacitances, the frequency ranges of the instrument may be so proportioned that it will be unnecessary to use the scale below 100 microamperes.

Wheatcroft and Haley pointed out that they used a pentode instead of a triode for discharging the condenser because of the high transconductance of the pentode they used. In the experimental circuit a type-2A3 tube was used because this tube seems to have the highest transconductance of any of the commercially available tubes. The conductance of a type-2A3 is of the same order of magnitudes as that of the MS/PEN used by Wheatcroft and Haley, namely, 5,000 micromhos. However, in spite of this large conductance, it was not possible to discharge the condenser completely with the signal voltage available unless the supply voltage was reduced to approximately 80 volts. For a given supply voltage, the minimum signal voltage requirement varies with the capacitance used. That is, when the capacitance is large, a larger conductance for discharging is required. This necessitates a large signal voltage. In the experimental circuit of figure 5, C had a value of .005 microfarads and a minimum of 11.0 volts rms was required for sine waves. The requirement will probably be somewhat lower for square waves and higher for sharply peaked waves, such as saw-tooth waves. With the aid of an amplifier, the signal voltage may be increased in amplitude to the point where reliable operation is obtained. If the signal voltage amplitude is increased sufficiently, higher supply voltages may be used, and consequently more output current will be obtained. Any signal voltage reasonably in excess of the minimum will cause no error, since the limit to which the condenser voltage can discharge is independent of the characteristics of T_1. If the signal voltage is so large as to make the grid voltage appreciably positive, grid current will flow and may cause heating in the tube, in addition to reducing the input impedance.

A comparison between the results of the experimental investigation and the objectives which were set up at the start will serve to measure the progress attained. First, the simplicity of the Wheatcroft-Haley circuit has been retained. The gaseous triode requires a battery for grid voltage, but this does not make the instrument too cumbersome. As shown in appendix B, the probable maximum frequency measurable with this instrument is 29,000 cps if a current of one milliampere is desired. This is predicated on sufficient amplitude of the signal voltage. With ordinary amplifiers the desired amplification may not be obtained at frequencies above 20,000 cps. Although the range has been verified up to ten kc, the probable range of the instrument is from 50 to 20,000 cps, which covers the major portion of the audio frequency range. Therefore, the second objective is realizable.

The signal voltage is applied to the circuit in the same manner as in the Wheatcroft-Haley circuit, with the result that there is no greater freedom from waveshape than in the original circuit. However, the tolerance with respect to waveshape is still very broad.

The main advantage of the modified circuit of figure 5 lies in the improved stability and the greater sensitivity made possible by the use of relatively large capacitances. The maximum frequency read by the Wheatcroft-Haley circuit is 15,000 cps. This has been extended to 20,000 cps, and with sufficient signal voltage may be extended to approximately 29,000 cps. By balancing-out the steady component of current in the meter with the current from an external battery, a suitably calibrated sensitive meter can be inserted to measure minute deviations from a given frequency.

The accuracy which can be expected from the readings depends on the accuracy with which the capacitance and the voltage source remain constant. High quality condensers hold their values within approximately .2% over long periods of time. The average load on the power source is low; hence, the supply voltage may be expected to remain within one or two per cent of its nominal value if gaseous diodes are used as voltage regulators. Indicating instruments can easily be obtained with accuracies of two per cent or better. The maximum error in practice is then probably two per cent.

A circuit diagram for a complete form of the instrument of figure 5 is shown in figure 6. The constant plate voltage and the grid voltage for the 2A3 tube are obtained by means of the voltage-regulator gaseous diodes shown. The four capacitances, selected by means of the multi-contact switch, provide convenient ranges of 0-200, 0-2,000, 0-6,000, and 0-20,000 cps. The fifth contact on the selector switch short circuits the type-885 tube and enables the instrument to be used as a slide-back type vacuum tube voltmeter. The potentiometer, R_1, can have a calibrated dial marked in volts. Then the peak value of a voltage applied to the grid of the 2A3 can be approximately determined. The purpose of this connection is not to measure signal voltages; instead it is used to enable easy adjustment of the d-c bias voltage of the 2A3 for cutoff. The signal amplifier follows the specifications for voltage amplifiers in the RCA Manufacturing Co. technical bulletin RC-14 and will amplify satisfactorily from 50 to 20,000 cps. The voltage gain of the amplifier is fairly constant at 82 over most of the range. At 50 cycles and 20,000 cycles the voltage gain decreases to approximately 60.

Several unique applications of this type of meter have been made. A few of these will be described. One investigator used the instrument to measure speed. The usual electrical tachometer has an instrument which measures the voltage of a magneto, this voltage being proportional to the speed. A change in the state of the magnetic circuit of the magneto causes error in the readings. If a frequency meter is used, a small alternator can be used in place of the magneto, and the speed can be read independently of the voltage output of the small alternator. High frequencies can be determined by measuring the beat frequency between the signal voltage and the voltage from a calibrated high-frequency oscillator. Guarnaschelli and Vechiacci used their frequency meter to measure the minute capacitances between the electrodes of vacuum tubes. Their method consisted of noting the change in the frequency of an oscillator due to the insertion of the unknown capacitance in parallel with the capacitance in the circuit.

A review of the characteristics of the frequency meter which has been described reveals that the device will measure

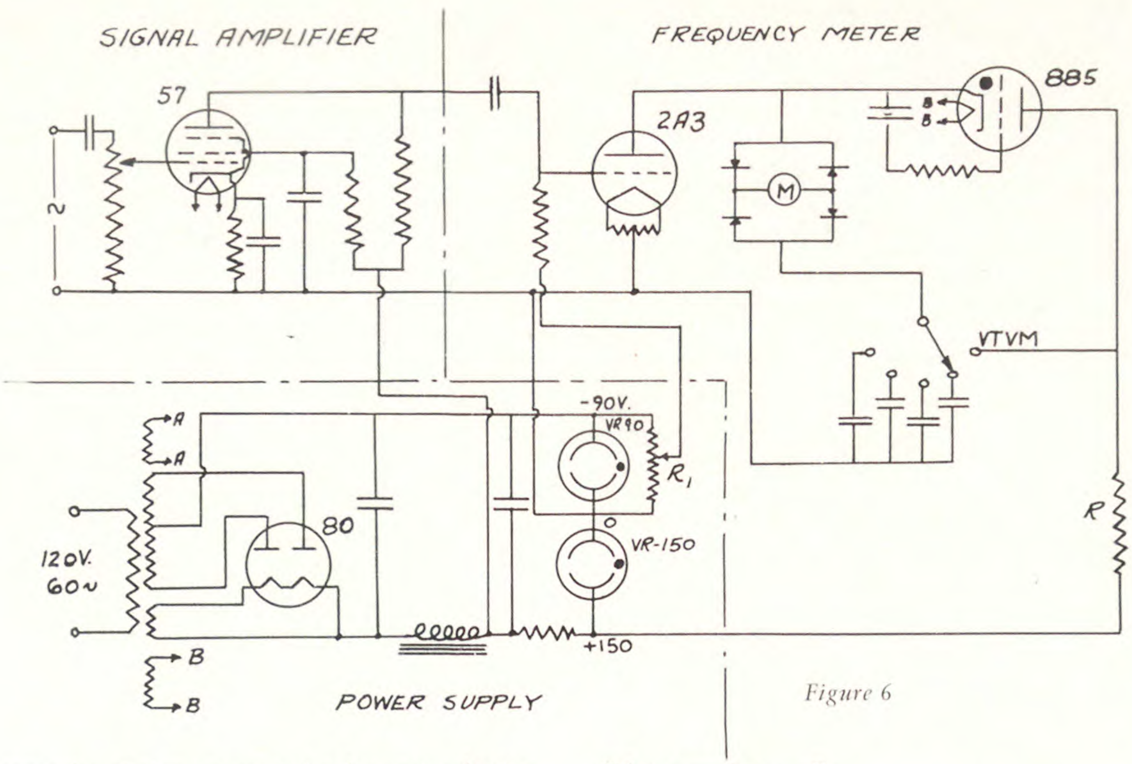


Figure 6

directly the frequencies of sine waves, and other waves having more or less harmonic content. Included are square waves and triangular waves which have a relatively high harmonic content. Also, the indication has sufficient accuracy for industrial measurements. If precision capacitances and a laboratory-type meter are used in conjunction with a well regulated power supply, the precision might well be increased to laboratory standards. An additional feature of the instrument is that the complete unit represented by figure 6 is rugged, compact, and weighs only 12 pounds.

Appendix A

The exact equations for the condenser voltage and current in the type of frequency meter discussed may be derived as follows: Referring to figure 1, the switch changes its position every half-cycle of the signal voltage. The ultimate limits of the condenser voltage will be E and E_0. Assume that the condenser voltage is E_0 and S takes the position marked 1.

In the circuit a-b-c-d-a, by Kirchhoff's law,

$$E - R_1 i_1 - e_c = 0 \qquad (1)$$

but

$$i_1 = Cpe_c \qquad (p = d/dt) \qquad (2)$$

$$E - R_1 Cpe_c - e_c = 0$$

$$(1 + R_1 Cp) e_c = E \qquad (3)$$

$$e_c = e_{cs} + e_{ct} \qquad (4)$$

where

e_{cs} = The steady-state condenser voltage = E.

e_{ct} = The transient condenser voltage.

From (3)

$$(1 + R_1 Cp) e_{ct} = 0$$

The determinental equation is

$$1 + R_1 Cm = 0$$

$$m = -1/R_1 C$$

and

$$e_{ct} = K\varepsilon^{-t/R_1C}$$

Combining according to equation (4),

$$e_c = E + K\varepsilon^{-t/R_1C} \qquad (5)$$

When $t = 0$, $e_c = E_0$

Substituting in (5),

$$E_0 = E + K$$

$$K = E_0 - E$$

and

$$e_{c1} = E - (E - E_0)\varepsilon^{-t/R_1C} \qquad (6)$$

The condenser current is

$$i_1 = Cpe_c$$

$$= C\left[\frac{1}{R_1 C}(E - E_0)\varepsilon^{-t/R_1C}\right]$$

$$= \frac{E - E_0}{R_1}\varepsilon^{-t/R_1C} \qquad (7)$$

If the condenser charges during one-half of an a-c cycle, the average charging current will be

$$I_1 = Q_1/\triangle t = \frac{\int_0^t i\,dt}{\triangle t}$$

$$= \frac{\int_0^{\frac{1}{2f}} \frac{E - E_0}{R_1}\varepsilon^{-t/R_1C}\,dt}{1/f}$$

$$I_1 = -fR_1C\left[\frac{E - E_0}{R_1}\varepsilon^{-t/R_1C}\right]_0^{½f}$$

$$= fC(E - E_0)(1 - \varepsilon^{-½fR_1C}) \qquad (8)$$

The time of flow of instantaneous current is only one-half cycle, but the current is averaged over the entire current cycle, which is the time of a complete cycle of the signal voltage. It has been assumed here that the two half-waves of the signal require equal times for their completion. In actual cases this may not be true. Where the two half-waves are unequal it merely requires a change in the time quantity appearing in the exponent.

When the condenser discharges from a voltage, E, to a lower voltage, E_0, the derivation is the same as above except for an interchange of limits and the use of R_2 in place of R_1. Hence for the discharge of the condenser,

$$e_{c2} = E_0 - (E_0 - E)\,\varepsilon^{-\frac{1}{2}fR_2C} \qquad (9)$$

$$i_2 = \frac{E_0 - E}{R_2}\,\varepsilon^{-\frac{1}{2}fR_2C}$$

$$I_2 = fC(E_0 - E)\,(1 - \varepsilon^{-\frac{1}{2}fR_2C}) \qquad (10)$$

Appendix B

The maximum frequency which may be measured is governed by the deionization time of the gaseous triode. That is, assuming sufficient signal voltage is available as a result of amplification, the condenser must charge and the gas tube must deionize in the time of a half-cycle. To this end, a type-885 gaseous triode was used instead of a gaseous diode because the 885 has a lower deionization time than that of the diodes available.

In commercial saw-tooth oscillator circuits, a frequency as high as 30,000 cycles per second is obtained. In these circuits the deionizing time of the gas tube is the limiting factor in extending the frequency range. At the maximum frequency the deionizing time is just slightly less than the time of one cycle, since the tube deionizes before the saw-tooth voltage reaches the firing voltage of the tube. Taking the deionizing time as

$$t_1 = 1/30000 = .0000333 \text{ seconds},$$

the maximum frequency which will be indicated with a current of one milliampere will be found. In the equation for the average current, since both charge and discharge currents are read in the completed instrument,

$$I = fC(E - E_0)\,(1 - \varepsilon^{-\frac{\frac{1}{2}f + t_1}{R_1C}}) - fC(E_0 - E)\,(1 - \varepsilon^{-\frac{1}{2}fR_2C})$$

This equation contains the two unknowns, f and C. A solution by successive approximations shows f to be greater than 29,000 cps but necessarily less than 30,000 cps with a capacitance of .000128 ufd. The difference in magnitudes between E and E_0 has been taken as 135 volts, and R_2 has been taken as approximately 1500 ohms from the plate characteristics of a type 2A3 tube. The exponential term is less than .01 when the indicator current is one milliampere.

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Comments by Solomon Fishman, Assistant Professor in Electrical Engineering, Newark College of Engineering

Third prize was awarded to Mr. Dehn for his presentation of this paper at the fifteenth annual convention of the student branches in the New York district of the American Institute of Electrical Engineers. This convention was held this year on the campus of Rutgers University, New Brunswick, N. J., on April 25, 1941. Ten engineering colleges in the metropolitan district were represented.

Mr. Dehn deserves particular commendation for personally initiating and completing this development. The project was undertaken primarily to fulfill the requirement of the electrical department of the Newark College of Engineering that each senior submit, previous to graduation, a paper on a topic in which he has a special interest. These papers are presented before meetings of the Newark student branch of the American Institute of Electrical Engineers and the outstanding paper is chosen for presentation before the annual convention of the student branches in the Metropolitan district.

The apparatus used in developing this frequency meter is the standard equipment available in the electronics and communication laboratory of the college. Students are encouraged to use this equipment for investigating the application of their ideas and designs which serve as the topics of their senior papers. Our laboratory is now well equipped with vacuum tubes, power supplies, cathode-ray oscillographs, variable frequency oscillators, measuring instruments, and other apparatus for the development and application of electronic devices. A large percentage of the seniors choose subjects from the electronic field as the topics for their papers.

CULLIMORE SAYS DEFENSE NEEDS MANY MORE GRADUATE ENGINEERS

(Reprinted from *Newark Evening News* of May 13, 1941)

The defense industry could use 10 times the number of men who will be graduated from engineering schools this June, Dr. Allan R. Cullimore, president of Newark College of Engineering, told a group of high school vocational guidance officers at a conference this morning at Upsala College in East Orange.

Cullimore, who is adviser for Region 5 of the National Engineering Defense Training Program, told of work being done to provide trained men for defense. He said there were about 2,000 in New Jersey schools and 20,000 being trained in industry. Upgrading courses are being given at night by Princeton, Rutgers, Stevens and Newark College of Engineering, and an intensive 12-weeks summer course for high school graduates will be given this year in the same institutions, with all expenses paid by the government.

In Three Parts

The program consists of three parts, Cullimore explained, regular vocational schools under direction of State Commissioner of Education Elliott, training within industry itself, which is being directed by the Office of Production Management, and the Engineering Defense Training Program, which comes under the United States Office of Education.

He said the purpose of the integrated program was fourfold: to find the points of resistance in defense production, to build a reserve of managerial and engineering personnel, to establish a training organization geared to almost any possible demands and bring to light all available manpower.

In the latter connection, Cullimore said the big surprise to leaders of the program has been availability of material. He said men of all ages and all degrees of previous training are offering themselves for government courses.

Colonel Robert E. Woodward of the United States Army described civilian skills of use to the army. He said there were shortages of X-ray technicians, signal corps men, armorers, electrical workers, welders, mechanics and stenographers.

Benefits in Future

Colonel Woodward also outlined benefits of army training in future civilian life. While the schooling provided by the army, he said, was primarily for needs of the service, it is bound to provide new experiences and valuable training.

After the defense forum, a panel discussion was held on "What the Employer Looks for in a Prospective Employee." Speakers included representatives of several local businesses and industries.

The afternoon session was divided into group discussions on vocations. Professor Maurice S. Trotta, head of the economics department at Upsala, was general chairman.

ENGINEERING DEFENSE TRAINING IN NEW JERSEY

Glenn Gardiner, District Director of Training-Within-Industry Division of the Office of Production Management, says:

A picture of the defense training job may be had from the realization that thirty-eight per cent defense labor must be skilled labor. To meet these training requirements the Office of Production Management has set up a separate division known as the Training-Within-Industry Division, the function of which is to assist defense contracting companies in setting up and carrying out training programs within the plants.

For this purpose the country has been divided into twenty-two districts of which the State of New Jersey has been constituted as one district. The headquarters of the Training-Within-

Glenn Gardiner

Industry Division of the Office of Production Management is located in Newark. Its personnel consists of a District Director, and Assistant District Director, together with headquarters office personnel. The District Director has four advisors, two representing industrial management and two representing labor, one from the C.I.O. and one from the A. F. of L.

In addition there is a panel of twenty-one training experts. These twenty-one men have been chosen as the outstanding industrial training executives of the State of New Jersey. They are loaned to the Federal Government on a part-time basis by their companies and are available to go into the defense companies of the state to assist them in setting up their training programs. In addition to this they call the attention of the company management to the existing educational facilities within the state which are available to help in the training program—such agencies as our State Vocational Education organization, the Voca-

(Please turn to page 27)

Dr. Allan R. Cullimore, Regional Adviser for Engineering Defense Training in Northern New Jersey, says:

Mr. Gardiner has covered in a very helpful way the defense picture as it relates to training for defense production. Particularly significant is his point that at least the final steps in any training must be done in industry.

It is undoubtedly true that no matter on what intellectual level college training is given, it cannot be made immediately useable by industry without some period of adjustment in industry itself. The Engineering Defense Training Program is particularly sensitive to this point of view, and for this reason our feeling is that coöperation with the Office of Production Man-

Dr. Allan R. Cullimore

agement is absolutely necessary to the success of our part of this tremendous endeavor.

Four million workers in industry are a great many, and of this number perhaps two hundred thousand will have to have some training and some practice in techniques above the work usually given in high school. This particular phase of the problem is not so great in point of numbers but is fundamentally necessary if production is to ultimately be obtained.

The lower management and middle management groups are vital in the defense program. The development, the engineering and the design groups are fundamental in production. Back of all this lies the research group, which is certainly as vital as any other group in the picture. In a word, this whole question of defense training is a very intimately organized, a very closely interlocking job, and its success depends primarily upon the appreciation of the relative values of various factors even while

(Please turn to page 27)

Editorials on Engineering Defense Training Administered by Newark College of Engineering

A Real Start on a Big Problem

In response to a very definite indication of a shortage of men with technical training to meet the expansion of defense industries in this area, Newark College of Engineering started a group of courses during the week of January 6, 1941. The need for short intensive courses had been determined at Washington and the United States Office of Education was authorized to direct the effort.

The start of the work here was preceded by several months of intensive study and survey as to the existing and anticipated need, and it was recognized early that while short intensive courses would probably relieve an immediate emergency situation, the expedient could not be expected to take care of a long range program. The same point of view still holds, and there seems to be no alternative to a thorough training program covering several years rather than several months, if we are to prepare enough thoroughly trained men to do the job ahead, both so far as the defense program is concerned and so far as the period after we have gotten ready in a military way. The descriptions of defense work to date which follows is offered with this strong fact in the background.

It was difficult to determine the needs exactly, particularly in view of the fact that our local industries had not yet received all of their defense contracts nor determined which materials should have precedence in manufacturing. There was available information from a survey and from an analysis of the calls for men that came to us daily. It seemed reasonable to suppose that the first major activity would be in designing and planning, that this phase would be followed by intensive production activity and by inspection and acceptance or rejection of the products. And with this preliminary projection and the recognition that a very considerable flexibility was needed to meet the situation as it developed, we got under way.

In response to an initial offering of courses in Engineering Drawing, Machine Design, Materials Inspection and Testing, and Production Supervision, about 2,000 applications came to our hands. Some 300 of the best prepared were selected from this group and training started in evening classes, taking as high as 16 hours per week.

Our normal programs of evening school, evening college, and graduate school had been under way for a semester, and a very considerable portion of our plant facilities in use.

Our first effort had not taken care of even a reasonable portion of the qualified applicants or of the need expressed by the calls coming to us. Between the opening date of January 6 and April 7 and through the coöperation of Newark University, Newark State Teachers' College, Newark Junior College, and Essex Junior College in making their plant facilities available to us, we added about 250 more trainees.

Information came to us about that time of a need in junior tool designers. The American Society of Tool Engineers provided an advisory committee; a course was designed and put into operation May 5, with 60 men enrolled. At the same time additional sections in Materials Inspection and Testing and Production Supervision involving about 190 students came into the program, and Newark Academy offered the facilities to take care of some of this load.

The first class in Materials Inspection and Testing breaking a wood beam and observing deflections and load.

Shop Layout work. Part of the training for engineering assistants, carried on in the Mechanical Laboratory.

Advanced Drafting class. Instruction in this class is planned to prepare the students for advancement in their present line of employment.

Photographs taken at Newark College of Engineering

The first courses offered made a considerable demand on time and effort of the students, in some instances perhaps too heavy a demand and as our experience developed, added courses were shortened as to the weekly load and as to the total hour content, and the original content was progressively revised.

Some very interesting information came forth as the courses went along. The older men who were actually engaged in defense industries, in many instances supporting families, and in some instances doing overtime work in industry, felt the need of training most keenly and were able to take the punishment involved in good shape. To the younger men, the seriousness of the situation and the obligation involved did not appear to make so great an impression, and the class shrinkage was considerably greater with this group. Many of them are making very commendable progress, but the application of the older group was particularly noticeable.

So far as the college is concerned, probably the most valuable part of the experience was in the learning to estimate the needs in industry, in selecting students and instructors, and enfolding the operation of an emergency program into the operation of a normal program. We have gone through pilot processes, as it were, to determine where difficulties and points of weakness were to be found, and through very much the same procedures as a business or industrial concern goes in expanding and adding facilities, and we are as least partially tooled up for production in our field.

Most of the men are already employed in defense industries and as a whole, ours has been an upgrading effort, although a considerable number were not working at the time they were enrolled and have been absorbed. We have been in touch also with a considerable number of men with non-technical background who have been able to get some hold in a short time and are available as a reserve force for further expansion and increase in employment.

We have every indication that the original courses, refined in the light of the operating experience, have met and are meeting a real need. Primary defense industries have encouraged their men to take the work and in many cases have so arranged scheduled overtime so as to make attendance at classes possible. We have in our hands a large number of letters containing such assuring statements as—"There can be no doubt as to the value to our company of the training these men secured as a result of the courses."

In a region carrying a large share of the defense contracts of the Nation it is satisfying to feel that we have made some contribution in an emergency situation. An expression of appreciation for coöperation in recommending instructors and students and for other very direct help in the program is due to such industries as the American Can Company, Crocker-Wheeler Electric Manufacturing Company, Thomas A. Edison, Inc., Foster Wheeler Corporation, R. C. A. Manufacturing Company, Inc., Westinghouse Electric & Manufacturing Company, Westinghouse Electric Elevator Company, Wright Aeronautical Corporation and innumerable other firms in the area.

Some Objectives That Have Been Attained

There are several interesting conclusions regarding what has been accomplished since its inception by the program of Engineering Training for National Defense as sponsored by the U. S. Office of Education. Any one who has been associated in any degree with the program can readily appreciate the progress made.

In its broadest phases the program of technical development as laid out for engineering colleges and affiliated institutions may be said to have fulfilled several important missions. A general analysis of the whole program as it has been devised and administered shows certain significant developments. What some of these developments are and what objectives have been attained thus far in the life of the program may be considered briefly here.

First, we might consider that a definite mission has been accomplished in the help given to industry by the furnishing of men to take care of industry's present needs. This was the first and foremost consideration of the entire plan, and this objective has certainly been met. Specific, definite, specialized needs of various plants concentrating on defense products have been met by the training of men since classes started about the first of the present year. In any consideration of the success of the plan up to this time certainly we must consider this factor.

Second, the program has succeeded in building up a reserve of men who may be utilized to meet the ever increasing demands of industry for specially trained individuals. We are gaining impetus by the hour, and a reserve to take care of future needs is one of the most important accomplishments of the work done thus far. This phase will become more apparent as the general program of National Defense reaches its maximum efficiency.

Third, it is evident that the plan to recruit for industrial technical needs the highest type of personnel is succeeding. Admission to the classes has been based on a sound preparation. Requirements have limited admission to those men who possess one, two, or three years of technical college training. While many reasons may be obvious for this procedure, if we but consider the short time required to provide supplementary training for such men, we will appreciate more readily the value of this rather restrictive condition for admission.

Fourth, and a very important point, tehre has been devised a skeleton organization of administrative and instructional groups whose greatest strength lies in their ability to expand. These groups, on the basis of their newly acquired experiences with procedures that differ in some respects with traditional educational and technical methods, are now prepared to function in a way that can take care of a much larger group of men than has been possible up to now. So integrated is such an organization with the Vocational Training Program and the Training Within Industry Division of the Office of Production Management that

Personnel Relations class at Essex County Junior College. Discussion of human relationships in industry is an important part of the course in Production Supervision.

Motion Study project at Newark Junior College. There is a maximum interest in this work as applied industrial psychology.

Class in Industrial Costing at Newark Academy. This subject is included in some of the longer courses in Production Supervision.

it will be able to deliver in ever increasing quantities the type of specially trained engineering personnel needed for various important jobs.

Opportunities Created for Individual Development

The training program is reaching men in all walks of life—all the way from this year's crop of High School Graduates to seasoned industrial executives. The training is two-fold in purpose—one of which is to provide pre-employment training to high school, junior college and senior college graduates. Each year either through graduation or for other reasons, many students discontinue college forever. A number of these have a background which makes them immediately acceptable to defense industries. Many more, however, namely students of the Arts, as well as many of the pure Sciences, have not as yet developed skills which are immediately useful in the production and manufacturing fields. To these, courses in electrical and mechanical drafting, and production supervision are offered with the idea that the intensive training provided in such studies coupled with the background possessed by every individual who comes within this classification will insure a wider range of opportunities to the individual as well as a greater acceptance by industry.

The second purpose of the training is that of up-grading individuals who are already at work in certain fields. The service which we offer here is also associated with drafting and production activities along with associated training in materials inspection, tool designing, machine designing, etc. By far the greater number of applicants who have been accepted to date have come within this classification. Most of these men desire to improve the service which they are able to offer in their chosen line of work. An appreciable number, however, wish to expand either the number of skills or the quality of skills which they can offer and thus increase their opportunities within the industrial field.

At the time of printing and since January 1, 1941, more than 1400 have registered for Engineering Defense Courses in this College alone. A number of these have expressed an appreciation for the unusual privileges which the exigencies of the hour have provided. Everyone has appreciated the opportunities which such training offers to the individual and in many cases, a determination to see the thing through has been voluntarily stated. Industry, which has the most to gain in the venture, has actively coöperated and in many instances, has made special arrangements so that certain of their employees may meet the rigid appointments of time and effort which the courses require. Punctuality and attendance at class has been excellent especially in the higher brackets. This experience combined with the reception of a steady stream of applications for the various courses which are being offered indicates a demand for such training; it indicates moreover that the program being offered by the United States Office of Education and associated schools and colleges throughout this area is providing industry with talent which it so sorely needs. When the supply is compared with the demand, however, our efforts seem insignificant or even unproductive and many years of training at the present level seem necessary if the demand ever is to be met.

The Office of Production Management is in excellent position to ascertain industry's needs for man-power and it is already rendering excellent service in this direction. The world situation today is not as much a conflict of men as it is a battle of Engineering skills. We have, for many years, placed great faith in our ability to produce and this capacity has served us well in the past. Our problem now, however, is not alone one of production but one of competition and it will remain so for many years to come. Today we must not only produce more aeroplanes; we must produce better ones. The flagship of our air force of yesterday is a training ship today and will be scrapped for its metal tomorrow. The same can be said of many other agents of destruction being used by this war-mad world today. We cannot today dwell on the futility of it all—we must meet the challenge with more and more production of better and still better commodities. We cannot go far in this direction unless our engineering skill goes apace—in fact, the development of trained personnel must precede any expansion in productive capacity. Industry must therefore produce trained men as well as things, and it is in this training program that we earnestly wish to help.

Engineering Drawing Instruction underway at Newark State Teachers' College. Younger men were attracted to this group of courses.

Examination of metal specimens by microscope and Rockwell Hardness Tester in the Metallurgical Laboratory, Newark College of Engineering, in Materials Inspection and Testing Course.

Production Supervision class at Newark University. This class in Motion Economy includes many industrial executives and supervisors.

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James A. Bradley, Dean and Associate Professor in Chemistry, has been elected counselor of the North Jersey Section of the American Chemical Society for the year 1941-42. Dean Bradley also attended the Regional Conference of the Eastern Association of Deans and Advisers of Men which was held at Fordham University, New York City, on April 26.

NATIONAL DEFENSE AND THE ENGINEERING COLLEGE

By JAMES A. BRADLEY, A.B., A.M.

Dean and Associate Professor in Chemistry, Newark College of Engineering

In these anxious days one is hardly inclined to look too far into the future. This is a time, if ever there was one, for living and working in the present. So much can happen in the next few months or weeks or days, and conditions change rapidly and unpredictably. One wonders whether even those who move nations about like chessmen can do any but the most obvious long range planning, complete and complicated though this may be. But, indistinct as the future is, we can be sure of some things. The workings of nature will not be changed, and human beings will still continue to behave like human beings, changing the least bit in their outlook century by century, but remaining fundamentally the same. They cannot be transfigured overnight by systems of thought which also have blossomed overnight and which are at present called by the high sounding name of "ideologies." In spite of minor differences, men will still have, as Shylock said, the same "organs, dimensions, senses, affections, passions." And there will be no Utopias because there are so few Utopians.

If we cannot look far ahead, what can we then do in the way of planning; for ourselves as individuals, for ourselves as citizens of a country? All but a very few of us will have the plans made for us for some time to come. To every one now there is only one course, and that is the way of coöperation. In a crisis authority is always necessarily vested in a few. Dissent becomes too dangerous even to be considered. This has been shown clearly enough in recent months in Europe. It is not necessary to go into the matter here. The shops are filled with books by writers who are pointing accusing fingers in all directions and who now see clearly the causes of failure—after the failure. But the combined wisdom and resources of great nations seemed not enough to foresee and prevent the failure.

How long the universal disease will last, and what will come out of it, we need not ask now. There are prophets enough abroad in the world, but prophecy is notably the least exact of the sciences. For lack of past experience it cannot tell us what would happen in the improbable event that men and nations should abate a little their naturally selfish instincts, or that they should put into practice the doctrine of brotherly love, or the Golden Rule, or other much admired ways of life. No one knows what would happen if these were tried universally, or even for a time by a great many. As it is, they receive chiefly mouth honor, especially on Sundays and certain winter holidays. But, to come down to earth again, it is wiser to look forward to a long hard pull and hope for the best, even though we may pray, as the President said at the annual dinner of the White House Correspondents' Association, that the end will come "sooner than any of us now dare hope."

Since our tasks will be set for all of us, it might be interesting to ask what the immediate and middle foreground of time holds for most of those who read these pages, graduate engineers and student engineers. To a certain degree this can be guessed by considering the recent impact of national defense on engineering education.

First, it might be said that the war is largely a matter of the assembly line, of the farm, of the machine, of transportation and communication. It is a war of blockade and a war in which armies attack not only armies, but "the plain people." Without a constant flow of food and materials for replacement, the side which we have elected to help cannot win. As the President has said, our country is going to be the "arsenal of democracy." In the working of the various departments of this arsenal the engineer and the technical man will be needed and needed badly. That the government is aware of this is shown by the nine million dollar fund appropriated by Congress in October for the training of men in special courses in engineering colleges all through the country. If one reads the temper of the times aright, it may be assumed that this is only a beginning. Not only now, not only during the war, but afterwards, when the greater task of rebuilding a broken world is before us, then, to quote Mr. Roosevelt again, "our country must continue to play its great part in the period of world reconstruction."

In the last war the importance of the trained man in industry was not always recognized. There was then a more general movement toward enlistment without regard to the special talents or the usefulness of the individual in non-military activities. But then there had not been the planning for possible action as in the last few years. They were quite different times and it was a quite different war. Yet even then, toward the end, technical men were being withdrawn from regular military and naval service and assigned to duties that made them more useful.

In the present war the former mistake is being avoided. It has appeared that among the warring nations technical preparedness was an index of military success. In Germany every employable person is at work or in training. Technical and vocational schools are said to be running at capacity in the captive countries. From them skilled workers have been shipped into Germany. In England and France there was an attempt to carry on defense without total organization. But later the picture changed. At the fifty-fifth annual meeting of the Engineering Institute of Canada it was reported that an English university was carrying on engineering training with no relaxation of standards, length of courses, or examinations, despite the fact that all the windows of the engineering buildings had been blown out by bomb explosions.

With its appropriation passed in October, Congress directed the U. S. Office of Education to set up short, intensive courses, of college grade in engineering schools everywhere in the country for the purpose of meeting "the shortage of engineers for service as designers, inspectors, and supervisors with the industries and Government agencies engaged in the National Defense Program," and charged the U. S. Commissioner of Education with the administration of the act (The First Supplemental Functions Appropriations Act) under the direction and supervision of the Federal Security Administration.

As soon as the act was passed, the heads of engineering schools in the metropolitan area of New York City organized themselves into a committee to survey the needs in engineering training in that district. Their report, dated November 18, 1940, was made public in abstract by the U. S. Office of Education on December 31, 1940. But before this later date the plans for training were well under way. By December 11, 1940, it was reported from Washington that 250 short intensive training courses were planned. Such basic courses as production engineering, testing and inspection methods, and engineering drawing were each to be offered by more than 50 colleges. By the first of the year the U. S. Office of Education was able to announce that the program had expanded and that 444 Engineering Defense Training Courses in 91 engineering schools were ready and that these were spread through 44 states, the District of Columbia, and Puerto Rico. These courses were to vary in length from eight weeks to six months. Some were to be given during the day, others at night. In many places highly specialized courses were given in addition to the more fundamental engineering courses mentioned above. The following list, a very small selection of those available throughout the country, will show the great diversity of the Defense Training Program: Theory of Elasticity, Concrete Inspection and Testing, Design of Bomb-proof Shelters, Diesel Engines, Explosives, Plastics, Food Inspection, Fuel Technology.

Although training courses of college grade have been favored by the Defense Program, a later report, issued March 3, 1941, from the U. S. Office of Education, gave some consideration to courses which were more on the high school and vocational school level. One might read there of courses in metal work, welding, machine shop practice, and even of courses to train clerical workers and cooks. And yet the program, in all its phases, may be considered as no more than started.

The Newark College of Engineering has been in the movement from the first, working to its full capacity. President Cullimore has been one of the leaders in this part of the country. He was a member of the New York Committee on Engineering Training for National Defense, which made a survey of the engineering training needs of New York metropolitan region, the report of which was published by the U. S. Office of Education on December 31, 1940. At the February meeting of the Engineering Institute of Canada in Hamilton, Ontario, he presented the program of special technical training for defense purposes which is being followed in the United States. He is also Regional Adviser of Region No. 5 for Engineering Defense Training. Others among the faculty of the Newark College of Engineering who are engaged very actively in the Defense Program are Professor Robert Widdop, who is in charge of Defense Training at the Newark College of Engineering, Professor H. N. Cummings, and Professor Frank A. Busse. A number of other members of the faculty are giving defense courses in a schedule which has necessarily expanded beyond the facilities of our own buildings into other institutions and colleges in the surrounding districts.

Perhaps this sketch in outline of the work being done in what may be termed mere preparation for National Defense can suggest to the reader the task which has been set for industry. It may also emphasize to the engineer and the student of engineering his importance in the country's organization, and remind him that he has a particular duty to see that, as far as he is able, he will use his special talents in the way that will best serve the country.

As the U. S. Commissioner of Education has said, no one has a blueprint of the total needs of industry. But every one must be put to work. If we are going to put all men and many women to work, and this is now being attempted in England, a large proportion of them must first be trained. And the training must be begun months before the factories are ready to take them. Else, when the factories are ready industry will have to mark time for the want of skilled hands. If, as has been stated again and again, America's chief function will be that of the production of material, the student who is planning to be an engineer must go all-out for training or not at all. In doing so, he is preparing not only "for the duration" but for the future. When the war is over, if the world is to be rebuilt, industry will change its objectives, but hardly its tempo.

We hope when that time comes that there will be combined wisdom enough in the world to set up again some stable form of civilization; that there will be a leadership for peace as resourceful and aggressive as leadership for war always seems to be; that the peace will be not merely another armistice; that the common man who fights the wars and pays the bills will not be forgotten; and that Americans will insist, and continue to insist, on their rights to seats at the council tables after a war which they did not make but which they will have to fight and finance. Until that time we shall have to be united in thought and word and action, for there is no other way.

FROM DEFENSE TRAINING TO ENGINEERING TRAINING —A LOGICAL STEP

By Robert W. Van Houten, B.S., C.E.

Acting Supervisor, Newark Technical School

Since January 1st of this year, 79,000 young men throughout this country have returned to school to study technical and engineering subjects. In so doing they have shown themselves willing to contribute their time and energy to the successful promotion of national defense. The courses which they are taking are part of the United States Office of Education's program to train men for national defense industries. These courses are of an intensive nature and are given to provide upgrading training as well as the pre-employment training.

Typical of the courses which are being given are Machine Design, Materials Inspection and Testing, Tool Design, Production Supervision and Engineering Drawing. Each of these courses has been designed to give the essentials in its own particular field and each is meeting a very real need in the present emergency. There is unquestionably a demand for men with the specific training which these and other similar courses provide. Each one has been offered only after a careful survey has been made of the personnel requirements of industry. The work which is being done is excellent and without it our industrial expansion would be considerably retarded. It should be kept in mind, however, that these courses are emergency courses and meet a specific need only. They do not in any sense provide a complete engineering education.

It would be well for the men who have returned to school and have gotten back into the swing of studying to give some thought now to the period which will follow the present national emergency. We all hope and expect that in the not too distant future the war will end and life will resume its more or less normal pace. When it does, industry will be forced to reduce the expanded staffs which it now has, and one important factor which will be considered in determining a man's worth to his company will be his education. Experience is undeniably of importance, but when the emergency program draws to a close and the company is forced to select its personnel for the normal output of the organization, it will retain those men who can in the future assume the responsibility for the management and continued development of the organization. It goes without saying that the man who has had an education in at least the essentials of engineering is in a much more advantageous position than the man who has not. It would seem, therefore, that the wise thing for the man to do who has been taking defense courses and has satisfied his immediate need would be to continue his education along broader technical lines. Men who have such foresight unquestionably will be the industrial leaders of tomorrow.

During the period since January 1st, 800 men have been taking United States Engineering Defense Training courses given by the Newark College of Engineering. Beginning September 22nd, an opportunity will be available for those men who have finished their defense training courses by that date to continue their education in the Newark Technical School. The courses which these men have been taking have been subsidized by the Federal Government. While the courses offered by the Newark Technical School are not tuition free, they are subsidized in part by both the State of New Jersey and the City of Newark.

For the past fifty-five years the Newark Technical School, which is affiliated with the Newark College of Engineering, has given technical courses in the evening. These courses furnish the essentials of a technical education, giving the fundamentals in four branches of engineering. They are designed to cover the work given in the first two years of most four-year curricula. The work included in the various programs is, therefore, equivalent to that of a Junior College in Engineering.

Students who, for economic or other reasons, wish to terminate their organized training program short of a bachelor's degree will find in these courses the subjects in Mathematics, Science, Drawing, and English that furnish the best basis for technical work. Many of the men who have been graduated from these courses hold responsible positions in production control, design work, and as assistants to research leaders.

For those men who wish to continue their education beyond the Junior College level, an opportunity is provided to proceed further and to complete the required courses for the Bachelor of Science degree.

The office of the Newark Technical School will be open evenings, Monday to Thursday, inclusive, until July 3rd and every evening, Monday to Friday, beginning immediately after Labor Day. The men who are taking defense courses are invited to call at the office to discuss the opportunities for continuing their education in the engineering field. Requests for copies of the Newark Technical School Bulletin, which we shall be glad to supply, should be addressed to the Registrar.

THE SOCIETY OF THE TRUNNION

The Society of the Trunnion was organized in March, 1929, by a group of senior students under the direction and supervision of Professor James C. Peet and Professor Albert A. Nims, for the purpose of conferring distinction upon those students and alumni of the Newark College of Engineering who have served their alma mater in a most creditable manner; to encourage academic attainment among undergraduates and professional achievement among alumni; to encourage extra curricular activities; and to develop character, leadership, and personality.

Academic attainment is inspired by admitting to the Society only those who have obtained grades which are sufficiently high to place them in the upper eighth of the junior class or the upper quarter of the senior class. Thus, the members are held in high esteem. Professional achievement among the alumni of the college is promoted by the eligibility for membership of those who have distinguished themselves in the field of their profession. Extra curricular activities are made essential as scholastic eligibility alone is not sufficient for admission to the Society. Further, all candidates are rated on the qualities of leadership, character, and personality by the faculty; this rating carries considerable weight in the competitive election.

The ultimate goal of the Society, as adopted in a resolution at its first meeting, is affiliation with the national honor Association of Tau Beta Pi. The Association, without question, is the outstanding engineering honorary organization in the country and has granted charters to seventy-two chapters in the leading colleges and universities, comprising a membership of some 30,000 engineers.

In striving toward this goal and in keeping with the ideals set forth in its purpose, the Trunnion has sponsored many academic and social activities. As pioneers in the field of Orientation, the Society in close coöperation with the Administration sponsored a series of lectures for high school seniors who were contemplating engineering as a profession. Four lectures, given by men prominent in the field of engineering they represented, presented the case for each of the four major branches of engineering. The fifth lecture, given by President Cullimore, aided the prospective engineering student in determining whether or not he was fitted for engineering, and if so, in what particular branch he was most likely to succeed. This phase of orientation has been taken over by the counseling activity at the high school and personal interviews by the department of Industrial Relations at the college. Student Aid, in the form of tutoring for delinquent freshmen is carried on by the undergraduate members of the Society. A medallion and cash award are presented each year to the senior having the highest academic honors. Socially, the Society holds several banquets and dinner meetings each year and sponsors the Joint Spring Smoker.

During the fall term the officers of the Trunnion approached their advisory council, consisting of Professors A. A. Nims, R. W. Van Houten, and W. Hazell, concerning the advisability of petitioning for a chapter in Tau Beta Pi this year. Professor Hazell contacted President C. H. Spencer and Secretary-Treasurer R. C. Matthews of the Association and arranged for their inspection of the college. These gentlemen were sufficiently impressed to suggest that the Society of the Trunnion prepare and submit a formal petition for a charter in Tau Beta Pi.

Material for the petition was gathered by the undergraduates of the society under the supervision of Mr. Oliver J. Sizelove, who was appointed alumni chairman of the Committee on Petition. The petition includes histories of the Trunnion and the College; requirements for admission and graduation; system of grading; organizations and societies; enrollment data; a list of the staff, their degrees and graduate study; teaching schedules; a list of prominent alumni; and letters of endorsement. The majority of the latter were obtained by President Cullimore from the following persons who are prominent in the profession: Dean Edward L. Moreland, of Massachusetts Institute of Technology; Dean Frank C. Stockwell of Stevens Institute of Technology; President H. S. Rogers of the Polytechnic Institute of Brooklyn; Dean A. A. Potter of Purdue University; Dean-Emeritus Dexter S. Kimball of Cornell University; Former Governor A. Harry Moore; Mr. Charles H. Jung of the New Jersey State Board of Regents; Mr. Edward F. Weston of Weston Electrical Instrument Company; Mr. Fred L. Eberhardt of Gould and Eberhardt; and Dr. Lillian M. Gilbreth of Purdue University. With unanimity these letters indicate the college has earned a well deserved place among the prominent engineering schools of the country.

The petition has received official approval of the Executive Council of Tau Beta Pi and copies have been submitted to all of the chapters for their consideration. They will voice their decision at the October convention in Philadelphia.

(Reported by Oliver J. Sizelove, Jr., Instructor in Industrial Engineering, Newark College of Engineering.)

CONTROL SURVEYS FOR ENGINEERING CONSTRUCTION

By WILLIAM S. LA LONDE, JR., S.B., M.S.
Professor in Civil Engineering, Newark College of Engineering

In much of our building construction work our control surveys are very simple. The land surveyor locates and marks the boundaries of the lot to be built upon to the degree of accuracy required as determined by the value of the property. Then the chief concern is to see that the building stays within the limits of the lot as surveyed. Sometimes, but not too often, a licensed surveyor or engineer gives the necessary lines and grades. More commonly, the contractor or his foreman sees to the establishment of them by means of a chalk-line and tape. Perhaps he is aided by an inexpensive combination of wye level and transit, but so long as the building conforms, within reason, to the dimensions called for on the plans and stays within the property lines, further accuracy is not needed, for each part is especially adjusted or built to fit as the work progresses. However, when the complexity of the construction work increases, greater care must be exercised in establishing control surveys for lines and grades, and the services of engineers and surveyors trained in the highest order of survey work are required.

These men must be thoroughly familiar with the procedures of base line measurements. The tapes used must either be individually standardized by the National Bureau of Standards or compared with a tape that has been standardized. It is always best to keep one standardized tape on hand on any large project to be used only for purposes of checking the field tapes at intervals not exceeding one month. Each field tape should have its own identification number stamped on or attached to the tape. Even the tape thermometers and spring balances should be compared with a standard thermometer and weights at intervals. To be effective, the tape thermometer should be attached to the tape with the bulb in contact with the tape. A board-backed thermometer hanging on the tripod of a transit and facing the sun with its back to the wind will give a much different reading from that of a tape thermometer properly attached to the tape.

It is customary in base line and traverse work to set hubs or monuments at the ends of the lines and to tape between these hubs, reducing the taped lengths by applying corrections for standardization, slope, sag, tension, and temperature. Most often the work is that of laying out a distance to be of a given length at a given temperature. This requires the corrections to be applied in the opposite manner to that used when measuring between two points; for that reason it behooves the man in charge of the survey to know his tape corrections and how to apply them correctly.

Surveyors in New Jersey are using coördinates more and more to definitely locate parcels of land and are appreciating the merits of the State System of Coördinates. On large scale industrial and chemical construction projects the use of a system of coördinates is absolutely vital to the proper tying together of the many working surveys needed to orient properly the multiplicity of machines and structures that go to make up a modern industrial plant. When it is remembered that it may take six months to a year to build a modern industrial plant, thereby requiring layout work to be done in winter and in summer; that machines or tanks in one building may be connected to other machines and tanks in perhaps the same or another building; that in some cases the pipe lines are to carry gases under three thousand pounds per square inch pressure; that the heavy flanges for each section of pipe must be machined to insure that the end faces be in planes at right angles to the axis of the pipe; that gaskets are of tissue paper thickness or about five-thousandths of an inch thick, then one appreciates why each and every unit must be accurately located in reference to the master plan. And this can best be done by assigning coördinates to every centerline of building column, machine, and pipe line.

To accomplish the task of accurately laying out the work, all measurements must be reduced to a given length at a given temperature, say seventy degrees Fahrenheit. If the plant is to cover a large area, of perhaps a mile square, it might be advisable to divide the property into two or four sections. In one plant, built a decade ago, the so-called east-west or main base line divided the gas plant from the by-products plant. The main north-south base line or meridian divided the two plants in such a way that everything in the low pressure gas plant had south and east coördinates, all the high pressure plant had south and west coördinates, the by-products plant had north and west coördinates, and the storage and warehousing facilities for the by-products plant had north and east coördinates. It was found expedient to have one or two extra base lines parallel and north and south of the main base line as well as to have one or two extra meridians east and west of the principal meridian. These base lines and meridians were monumented every five hundred feet, and in no case was it necessary to go farther than fifteen hundred feet from a coördinate monument to any construction work.

This well-conceived coördinate control was all but ruined by having the field work handled at the start by a man inexperienced in precise survey work. He had been on a number of small bridge construction projects on which the taping had been done by stretching the tapes with as much pull (by feel) as compensated for sag in a full tape length, by keeping the tapes approximately horizontal, and by plumbing from both ends of the tape at the same time. No thought was given to temperature, and the monuments were set in August under heat conditions that ran above one hundred degrees in the shade. The following January, when the thermometer registered in the twenties, a work survey was made by starting from one monument and closing upon another monument one thousand feet distant. It was apparent that something was wrong when the survey failed to close by eight and one-half inches. On analyzing the situation it was plain that tape corrections would have to be used thereafter. A one-hundred degree difference in temperature on a line one thousand feet long would make very close to an eighth-inch difference in the two surveys.

Just the use of tape corrections alone did not give an answer to this problem. Some of the steps taken to remedy the difficulty are of interest. First, all tapes were compared with one that had been standardized. Each field party had its own numbered tape and was issued a tape clamp and a spring balance with a built-in level bubble. Then the distance between monuments was measured, and corrections were applied for standardization, slope, sag, and tension. No temperature correction was applied, but the temperature at which the new measurements were made was recorded. After all monuments had been thus resurveyed, all existing construction work was tied with like precision to one or more monuments. The rest was a matter of judgment and expedience. First, a temperature was adopted for a base; that is, all work was considered to be the right length at sixty, seventy, or eighty degrees Fahrenheit. By choosing the one most satisfactory base certain local discrepancies were eliminated. Then chainage equations were used liberally, which threw all the remaining errors into definite places, where changes in the general layout would cause the least trouble. The overtime cost for these additional surveys, plus the cost for changes in many pieces of high pressure pipe that had already been made to order, amounted to several thousand dollars.

Perhaps one may wonder why it took five months to discover this error. The answer is that one of the first rules of the land surveyor had not been observed in carrying on the many small work surveys—to close the traverse, and that, if possible, upon a point other than the starting point. Had this check been applied when the location survey was made for the very first building, much of the difficulty later would have been eliminated, although the monuments would never have been five hundred feet apart.

The angle work was accurate in all but one case, and that one error cost several thousand dollars. Like most errors, it was made by not applying satisfactory checks. It was necessary to establish an east-and-west working line one hundred and fifty feet south of the main base line. A point was accurately set one hundred and fifty feet south of one of the base line monuments. Then, instead of setting a second point the same distance south of another monument of that base line, it was thought that time would be saved by locating the second point by measuring from a building column which was believed to be one hundred and sixty feet south of the base line, when in reality it was an offset column four inches farther south. Thus a number of buildings in the high compression group fronted on one line, and the remainder of the group fronted on another line with the angle occurring between the two buildings having a considerable interchange of high-pressure piping. Each pipe line that crossed that angle required a special section of pipe, all of them different one from another. The coördinate control for the one group of buildings had to be rotated to conform to the angle of error described. This entire error might easily have been detected if one or two extra measurements had been made or if an extra angle had been turned with the transit.

Sometimes through deliberate design part of the plant is at an angle with another part of the plant, and in these cases more than one set of coördinates controls the layout work from the start. This plan should cause no difficulty to the surveyor or engineer in charge, but may greatly simplify the actual work in the field. There is a difference between a planned layout and a makeshift one.

In another case, in order to set accurately a considerable number of points in a confined area, where, because of construction difficulties and obstructions, it was necessary to use plumb-bobs and many short tape lengths, two special base lines at right angles to each other were laid out at a convenient distance back of the particular area and at a point where it was a simple matter to tape accurately. Permanent hubs were placed on each base line in such positions that two transits when set up over a selected pair of hubs, one in each base line, could, by sighting at right angles to these base lines, have their lines of sight (direct or prolonged) intersect at the required point. Thus many points were set accurately and with a minimum of effort and size of survey party.

For large machine foundations that ran perhaps from six to ten feet below ground to eighteen to twenty feet above ground and in plan were approximately thirty-five feet wide by sixty-five feet long, it was the practice to form the outside walls of these foundations completely before the pouring of the concrete, which was accomplished in six or seven steps. To get transit lines into or out of such a box or to rely upon any points on the forms to stay in place to within two or three thousandths of a foot, between successive pours, was out of the question. Therefore, before any excavation or forming was done a very careful layout was made on the ground of the centerlines of shafts and cylinders. These lines were then thrown to the upper steel work of the building and targets inscribed thereon. After that, a transit set inside the excavation or forms could jig onto line in very short order. The transits were kept in excellent adjustment for convenience, but it was the order of the day to always reverse all instruments and to go through each operation twice. When one visualizes a building having two rows of six such foundations, carrying ammonia compressors weighing five hundred tons apiece, with gas pressures up to three thousand pounds per square inch, and all high pressure piping being ordered in advance, it is essential that a high degree of accuracy be maintained in the layout work.

In setting some machinery it is necessary to take into account the curvature of the earth. In the case of a boring mill for the large navy and coast defense guns the track on the several sections of bed frames must definitely be on a tangent to the earth's surface, and during the first world war a precise level was used to set some of these frames to the required accuracy. In the case of the United States Navy's new model testing basin, which is over one thousand feet long, the rails had to follow the curvature of the earth in order that the towing cable remain at a constant angle with the water's surface. The ends of the track would be approximately three-eighths of an inch below a line tangent to the rail at the mid-point. In placing the bed frames for the ammonia compressors referred to before, an engineer's level was used to bring the several frames to within one or two thousandths of a foot of the same elevation and then specially prepared straight-edges eight feet long were used by the machinery company. These straight-edges carried level bubble tubes ground to a very flat arc. The bubble was approximately twelve inches long, and a cigarette paper placed under one end of the straight-edge caused the bubble to move one division. Piano wires, used for setting the centerlines of cylinders and shafts to exact alignment, were established to line and grade by surveyors to within one or two thousandths of a foot. Then the machinists, with specially designed beams carrying a micrometer on one end, were able to set two wires twenty-two feet apart parallel with each other to within one ten-thousandth of an inch. The cylinder walls and other parts were set with the same accuracy.

On some construction projects it is necessary to sink shafts, and when the plant is at a sea-level location and elevations refer to sea level the surveyor has to contend with positive and negative elevations. In many cases it is a convenience to adopt a datum that is one, two, three, or four hundred feet below the sea level datum and thereby to keep all elevations of borings or of construction work positive. Passing through zero in a vertical direction seems to cause more trouble than when passing through zero horizontally as when changing from north to south coördinates.

Many industrial plants that may be reached by boat require wharfs and docks. These facilities often need to be built out to the limit of the pier head lines as established by the Federal government. As a rule there are penalties for going beyond these lines. The best control that can be had for these structures is to tie into an existing triangulation network along the shore, or if that is lacking, to establish a triangulation system expressly for the purpose to control the survey. No matter which is the procedure, a knowledge of the essentials for the establishment of a triangulation system would be of value. The control for one construction project consisted of a simple quadrilateral. A base line was carefully measured on each bank of the river and triangulation stations were established at both ends of these two base lines. No trouble was experienced the first year, but in the spring of the second year the points set by triangulation were noticed to differ considerably when the third or fourth stations were used. The answer was that one of the triangulation points had been established in a comparatively newly made shore fill

(Continued on page 27)

FUNDAMENTALS OF OPERATIONAL CALCULUS

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The contents of this paper were discussed by the writer at a colloquium sponsored by the Mathematics Department of the Newark College of Engineering on March 28, 1941. The purpose of publishing this material is to furnish in concise form a simple yet fairly comprehensive introduction to the subject of Operational Calculus. It is assumed that the reader will be familiar with the interpretation of physical systems in terms of differential equations, and that he will use this paper as a guide to learning the powerful operational methods of their solution. Throughout the following pages, therefore, the derivations of the various differential equations will not be discussed. Instead, emphasis will be on methods of solution, and many references will be given to published descriptions in more detail.

The Operational Calculus was invented by Oliver Heaviside (1850-1925)[1] and published by him in three volumes entitled *Electromagnetic Theory*. The general problem which Heaviside attacked by this method was: Given a group of linear differential equations with constant coefficients representing a physical system initially in equilibrium, find the response of the system to a suddenly introduced change in its nature.[2]

Heaviside's method was simple. He would translate the problem into the mathematical language with the aid of symbols called "operators," and would then "algebrize" the equations, or manipulate them by algebra until he obtained some recognizable form. This process is somewhat similar to ordinary integration where we convert the given integral into one of a series of standard forms listed in a table of integrals.

The manipulation of the operational equations can generally be carried out by one of three methods: (1) by expansion into an infinite series, (2) by the "Shifting Process", (3) by the "Expansion Theorem". These methods will be illustrated in the following pages. There are more elaborate operations which can be performed by the advanced mathematician, but anyone who can handle these three can use Operational Calculus.

In the interest of clarity, little attention will be paid to mathematical rigor during the first part of the paper. Heaviside himself thought most rigorous proofs silly[3] saying, "The best of all proofs is to set out the fact descriptively, so that it can be seen as a fact." At the conclusion will be given a rigorous method of verifying the validity of all results.

Let us now consider a simple illustration of Heaviside's general problem. We are to find the electric current which flows in a series circuit of inductance and resistance after a constant voltage is applied across it. We shall assume the circuit to be in equilibrium before the switch is closed. In conventional mathematics we would write the differential equation $E = Ri + L\frac{di}{dt}$ where E is the applied voltage, R the resistance, L the inductance, and i the current. We would also be obliged to determine and state the boundary conditions, namely that the current is zero when the voltage is applied at $t = 0$.

Heaviside began his simplification by stating the physical conditions of the problem in mathematical symbols. He wrote $E1 = Ri + L\frac{di}{dt}$ where 1 is a function (called the "unit function") which is zero before $t = 0$, and equal to one after $t = 0$. Thus his equation tells us that there was no voltage across the circuit before $t = 0$, and that a constant voltage E remained across it after $t = 0$. Having included in the equation all the physical conditions of the problem, it is now unnecessary to state that the initial value of the current is zero, because this fact depends on the physical conditions which have already been included. The next simplifying step was to use the letter p for d/dt. Thus $E1 = Ri + Lpi$. From here on, Heaviside "algebrized" the equation, or manipulated it by algebra until he obtained something which he could recognize. Let us follow this process.

$$E1 = (R + Lp)i$$

$$i = \frac{E1}{R + Lp} = \frac{E}{L}\frac{1}{p + a}1 \quad \text{where } a = R/L.$$

By ordinary long division,

$$i = \frac{E}{L}\left(\frac{1}{p} - \frac{a}{p^2} + \frac{a^2}{p^3} - \frac{a^3}{p^4} + \cdots\right)1$$

Now if p means differentiate with respect to t, then 1/p means integrate with respect to t. Hence $\frac{1}{p}1 = t1$. (The constant of integration is zero because the function is zero before $t = 0$.) Likewise, $\frac{1}{p^2}1$ means integrate $\frac{1}{p}1$ or $\frac{1}{p^2}1 = \frac{1}{p}\left(\frac{1}{p}1\right) = \frac{1}{p}(t1) = \frac{t^2}{2}1$.

Also $\frac{1}{p^3}1 = \frac{1}{p}\left(\frac{t^2}{2}1\right) = \frac{t^3}{2(3)}1$ and thus in general $\frac{1}{p^n}1 = \frac{t^n}{n!}1$.

Therefore,

$$i = \frac{E}{L}\left[\frac{t}{1} - \frac{at^2}{2!} + \frac{a^2t^3}{3!} - \frac{a^3t^4}{4!} + \cdots\right]1$$

$$= \frac{E}{aL}\left[\frac{at}{1} - \frac{a^2t^2}{2!} + \frac{a^3t^3}{3!} - \cdots\right]1$$

$$= \frac{E}{aL}\left[1 - \left(1 - \frac{at}{1} + \frac{a^2t^2}{2!} - \frac{a^3t^3}{3!} + \cdots\right)\right]1$$

$$= \frac{E}{aL}\left[1 - \varepsilon^{-at}\right]1$$

which is the familiar solution $i = \frac{E}{R}\left(1 - \varepsilon^{-\frac{Rt}{L}}\right)1$

From this result one of the standard operational formulas can be obtained, for

$$\frac{E}{L}\frac{1}{p + a}1 = \frac{E}{aL}\left[1 - \varepsilon^{-at}\right]1$$

and hence, $\frac{1}{p + a}1 = \frac{1}{a}\left[1 - \varepsilon^{-at}\right]1$

By a similar process of expansion it is easy to show that $\frac{p}{p + a}1 = \varepsilon^{-at}1$. This latter formula may be used to solve another common electrical circuit problem. Let us find the current which flows in a series circuit of resistance and capacitance when a constant voltage is suddenly placed across it. In conventional form we would write $E = Ri + \frac{1}{C}\int i\,dt$, and we would have to know something about the initial conditions in order to solve the problem. In the Operational Calculus the equation is written $E\,1 = Ri + \frac{i}{Cp}$. Solving for the current,

$$i = \frac{E\,1}{R + \frac{1}{Cp}} = \frac{E}{R}\,\frac{p}{p + \frac{1}{CR}}\,1$$

$$= \frac{E}{R}\,\varepsilon^{-\frac{t}{CR}}\,1$$

Note how rapidly this result was obtained, and observe that it was unnecessary to state initial conditions. (It was understood, of course, that the circuit was originally in equilibrium.)

Many operational formulas can be obtained by working backward from a series expansion of a function of time. For example,

$$\sin wt\,1 = \left[\frac{wt}{1\,!} - \frac{w^3t^3}{3\,!} + \frac{w^5t^5}{5\,!} - \cdots\right]1$$

$$= \left[\frac{w}{p} - \frac{w^3}{p^3} + \frac{w^5}{p^5} - \cdots\right]1$$

$$= \frac{wp}{p^2 + w^2}\,1$$

Heaviside had an uncanny faculty of recognizing series such as this.

Trigonometric functions can be put into operational form also by first converting to the exponential. Thus,

$$\cos wt\,1 = \frac{\varepsilon^{jwt} + \varepsilon^{-jwt}}{2}\,1 = \frac{1}{2}\left[\frac{p}{p-jw} + \frac{p}{p+jw}\right]1$$

$$= \frac{1}{2}\left[\frac{p^2 + jwp + p^2 - jwp}{p^2 + w^2}\right]1 = \frac{p^2}{p^2 + w^2}\,1$$

From the formulas for sine and cosine we get $\sin(wt \pm \varphi)\,1 = (\sin wt \cos\varphi \pm \cos wt \sin\varphi)\,1 = \frac{pw\cos\varphi \pm p^2\sin\varphi}{p^2 + w^2}\,1$, etc.

An important function in engineering is the damped sine or $\varepsilon^{-Bt}\sin(wt \pm \varphi)$. An equivalent operator for this can be derived by making use of Heaviside's "shifting".[4, 5, 6]

If a function of p operates on a function of time $f(t)$ and an exponential function of time ε^{at} as $f(p)\,[\varepsilon^{at}f(t)]$, the exponential may be shifted out of the influence of the operator by writing $f(p)\,[\varepsilon^{at}f(t)] = \varepsilon^{at}f(p + a)f(t)$. The validity of this equation can be illustrated by first letting $f(p) = p$. Then $p[\varepsilon^{at}f(t)] = \varepsilon^{at}pf(t) + f(t)a\varepsilon^{at} = \varepsilon^{at}[pf(t) + af(t)] = \varepsilon^{at}[p + a]f(t) = \varepsilon^{at}f(p + a)f(t)$. In similar manner, $p^n[\varepsilon^{at}f(t)] = \varepsilon^{at}(p + a)^nf(t)$. Thus the relation holds for $f(p) = p^n$, and will therefore hold for any function of p which can be expanded in a power series. The shifting process obviously works both ways. Thus, $\varepsilon^{at}f(p)f(t) = f(p - a)[\varepsilon^{at}f(t)]$.

Now consider $\varepsilon^{-Bt}\sin(wt \pm \varphi)\,1$

$$= \varepsilon^{-Bt}\left(\frac{pw\cos\varphi \pm p^2\sin\varphi}{p^2 + w^2}\right)1$$

Here $a = -B$, $f(p) = \frac{pw\cos\varphi \pm p^2\sin\varphi}{p^2 + w^2}$, $f(t) = 1$

Shifting the exponential to the right,

$$\varepsilon^{-Bt}\sin(wt \pm \varphi)\,1 = \frac{(p + B)\,w\cos\varphi \pm (p + B)^2\sin\varphi}{(p + B)^2 + w^2}\,\varepsilon^{-Bt}\,1$$

Replacing the exponential by its operator equivalent,

$$\varepsilon^{-Bt}\sin(wt \pm \varphi)\,1 = \frac{(p + B)\,w\cos\varphi \pm (p + B)^2\sin\varphi}{(p + B)^2 + w^2}\,\frac{p}{p + B}\,1$$

$$= \frac{pw\cos\varphi \pm p(p + B)\sin\varphi}{(p + B)^2 + w^2}\,1$$

These and other formulas can also be derived by the "Expansion Theorem".[7, 8, 9, 10] Let the operator $f(p) = \frac{Y(p)}{Z(p)}$ where $Y(p)$ is a polynomial in p of lower degree than the polynomial $Z(p)$. Let the roots of $Z(p)$ be p_1, p_2, p_3, etc. Then, by factoring the denominator and expanding by partial fractions,

$$\frac{Y(p)}{Z(p)} = \frac{Y(p)}{(p - p_1)(p - p_2)(p - p_3)\cdots}$$

$$= \frac{A}{p - p_1} + \frac{B}{p - p_2} + \frac{C}{p - p_3} + \cdots$$

It can be shown[11] that $A = \frac{Y(p_1)}{Z'(p_1)}$, $B = \frac{Y(p_2)}{Z'(p_2)}$ etc., where $Y(p_1)$ is the value of $Y(p)$ when $p = p_1$, and $Z'(p_1)$ is the value of the derivation of $Z(p)$ with respect to p when $p = p_1$. If this operator is applied to the unit function,

$$\frac{Y(p)}{Z(p)}\,1 = A\left(\frac{1}{p - p_1}\right)1 + B\left(\frac{1}{p - p_2}\right)1$$

$$+ C\left(\frac{1}{p - p_3}\right)1 + \cdots$$

$$= \frac{A}{-p_1}(1 - \varepsilon^{p_1t})\,1 + \frac{B}{-p_2}(1 - \varepsilon^{p_2t})\,1 + \cdots$$

$$= -\left(\frac{A}{p_1} + \frac{B}{p_2} + \cdots\right)1$$

$$+ \left(\frac{A}{p_1}\varepsilon^{p_1t} + \frac{B}{p_2}\varepsilon^{p_2t} + \cdots\right)1$$

The first term is $\frac{Y(0)}{Z(0)}\,1$ as can be seen by letting $p = 0$ in the original expression. Thus, after substituting for A, B, etc.,

$$\frac{Y(p)}{Z(p)}\,1 = \frac{Y(0)}{Z(0)}\,1 + \left[\frac{Y(p_1)}{p_1Z'(p_1)}\varepsilon^{p_1t} + \frac{Y(p_2)}{p_2Z'(p_2)}\varepsilon^{p_2t} + \cdots\right]1$$

$$= \left[\frac{Y(0)}{Z(0)} + \Sigma\,\frac{Y(pi)}{piZ'(pi)}\,\varepsilon^{p_it}\right]1$$

which is the "Expansion Theorem". Here we have assumed that the roots of $Z(p)$ were unequal, and that none were equal to zero.

The Expansion Theorem can be used to evaluate the important, frequently occurring operator $\frac{1}{p^2 + 2ap + B^2}\,1$

Here $Y(p) = 1$, $Z(p) = p^2 + 2ap + B^2$, $Z'(p) = 2p + 2a$. Let $-m$, $-n$ be the roots of $Z(p)$. Then by the Expansion Theorem,

$$\frac{1}{p^2 + 2ap + B^2}\,1 = \left[\frac{1}{B^2} + \frac{1}{-m(-2m + 2a)}\varepsilon^{-mt} + \frac{1}{-n(-2m + 2a)}\varepsilon^{-nt}\right]1$$

Now, $m = a - \sqrt{a^2 - B^2}$, $n = a + \sqrt{a^2 - B^2}$.

Thus $m + n = 2a$ and $\frac{1}{p^2 + 2ap + B^2}\,1$

$$= \left[\frac{1}{B^2} + \frac{1}{-m(-m + n)}\varepsilon^{-mt} + \frac{1}{-n(-n + m)}\varepsilon^{-nt}\right]1$$

$$= \left[\frac{1}{B^2} - \frac{1}{n-m}\left(\frac{\varepsilon^{-mt}}{m} - \frac{\varepsilon^{-nt}}{n}\right)\right] 1$$

In this derivation we have assumed that a^2 is larger than B^2 so that m and n are real.

Now suppose that a^2 is less than B^2. Then m and n are complex numbers involving a real and an imaginary part. When the complex values of m and n are inserted in the above formula the exponentials have complex exponents. They can, therefore, be broken up into a real exponential times a trigonometric function, so that

$$\frac{1}{p^2+2ap+B^2}\, 1 = \left[\frac{1}{B^2} - \frac{\varepsilon^{-at}}{wB}\sin(wt+\varphi)\right] 1$$

where $w = \sqrt{B^2 - a^2}$ and $\tan\varphi = \frac{w}{a}$. This substitution is an interesting algebraic exercise, but contains nothing essential to an understanding of the operational method.

Consider, however, the case where a^2 equals B^2. Here the Expansion Theorem fails because when deriving it we assumed unequal roots. We may write

$$\frac{1}{p^2+2ap+B^2}\, 1 = \frac{1}{(p+a)^2}\, 1$$

Now note that $\frac{d}{da}\left(\frac{1}{p+a}\right) = -\frac{1}{(p+a)^2} = -\frac{1}{p^2+2ap+B^2}$

Thus, $\frac{1}{p^2+2ap+B^2}\, 1 = -\frac{d}{da}\frac{1}{p+a}\, 1$

$$= -\frac{d}{da}\frac{1}{a}\left(1 - \varepsilon^{-at}\right) 1$$

$$= -\left[\frac{1}{a}\left(t\varepsilon^{-at}\right) - \frac{1}{a^2}\left(1 - \varepsilon^{-at}\right)\right] 1$$

$$= \frac{1}{a^2}\left[1 - \varepsilon^{-at}(1+at)\right] 1$$

Let us now consider a mechanical problem; to find the effect of suddenly applying a load to a spring whose movement is retarded by friction, say in the form of a shock absorber. Assume the system to be originally in equilibrium. Let the frictional force be proportional to the velocity of the load as $k\frac{dx}{dt}$. Let the spring constant be k_1, and the mass of the spring be negligible in comparison with the mass of the load. Then in conventional form, $mg = m\frac{d^2x}{dt^2} + k\frac{dx}{dt} + k_1x$ where m is the mass of the load, g the acceleration of gravity, and x the elongation. In operational form, $mg1 = mp^2x + kpx + k_1x$. Solving for x,

$$x = \frac{mg1}{mp^2+kp+k_1} = \frac{g}{p^2 + \frac{k}{m}p + \frac{k_1}{m}}\, 1$$

Thus the elongation of the spring is described by the operator which we have just evaluated by the Expansion Theorem. The solution will take one of the three forms described above depending on whether $\left(\frac{k}{2m}\right)^2$ is greater than, equal to, or less than $\frac{k_1}{m}$

The examples given so far involve the sudden application of a *constant* force. The application of a variable force is easily handled by converting the force function of time into a function of p and proceeding as before. As an illustration, let us find the current set up in a series circuit of inductance and resistance when an alternating electromotive force E sin wt is applied at $t = 0$. Using the operational notation, $E\sin wt1 = Ri + Lpi$.

Solving for i, $i = \frac{E\sin wt}{R+Lp}\, 1$

Substituting the previously derived operator for sin wt 1,

$$i = \frac{E}{R+Lp}\frac{wp}{p^2+w^2}\, 1 = \frac{Ew}{L}\frac{p}{(p^2+w^2)\left(p+\frac{R}{L}\right)}\, 1$$

This involves a standard operator[12] which gives the solution

$$i = \frac{Ew}{L}\frac{1}{w\sqrt{\frac{R^2}{L^2}+w^2}}\left[\sin(wt-B) + \varepsilon^{-\frac{Rt}{L}}\sin B\right] 1$$

$$= \frac{E}{\sqrt{R^2+w^2L^2}}\left[\sin(wt-B) + \varepsilon^{-\frac{Rt}{L}}\sin B\right] 1$$

where $\sin B = \frac{w}{\sqrt{\frac{R^2}{L^2}+w^2}}$

It is interesting to note that in order to obtain a solution to any specific problem of the type we have discussed, it is necessary to know only the one operational formula $\frac{1}{p^n}\, 1 = \frac{t^n}{n!}\, 1$

For instance, the last example might have been solved by expanding the operator by ordinary division. Thus,

$$i = \frac{Ew}{L}\frac{p}{(p^2+w^2)\left(p+\frac{R}{L}\right)}\, 1$$

$$= \frac{Ew}{L}\left[\frac{1}{p^2} - \frac{a}{p^3} + \frac{a^2-w^2}{p^4} - \frac{a(a^2-w^2)}{p^5} + \frac{w^4-a^2w^2+a^2}{p^6} - \cdots\right] 1$$

$$= \frac{Ew}{L}\left[\frac{t^2}{2!} - \frac{t^3a}{3!} + \frac{t^4(a^2-w^2)}{4!} - \frac{t^5a(a^2-w^2)}{5!} + \frac{t^6(w^4-a^2w^2+a^2)}{6!} - \cdots\right] 1$$

From this convergent series, numerical values for the current could be obtained for any specific circuit.

All the problems that we have worked so far have involved lumped parameters, that is, the mass, resistance, inductance, etc., have been concentrated at various points in the system. The solution of problems with distributed parameters (for example, the transmission line where the inductance, capacitance, etc., are distributed along the line) is more difficult because of the appearance of fractional powers of p in the operators. Even these problems can be handled, however, by the simple method of expansion into a series.

Suppose we wish to determine the current at the sending end of an infinitely long "ideal" cable to which an alternating voltage E sin wt is applied at $t = 0$. The operational expression for the current is[13] $i = Ew\sqrt{\frac{C}{R}}\left[\frac{p^{\frac{3}{2}}}{p^2+w^2}\right] 1$

which can be expanded by division to

$$i = Ew\sqrt{\frac{C}{R}}\left[p^{-\frac{1}{2}} - w^2p^{-\frac{5}{2}} + w^4p^{-\frac{9}{2}} - \cdots\right] 1$$

Now by some very bold assumptions[14], which we shall have to skip for lack of space, Heaviside concluded that $p^{-\frac{1}{2}}\, 1 =$

$\frac{2\sqrt{t}}{\sqrt{\pi}}\,1$. (The correctness of this relation will be proven later.)

From this expression other fractional powers of p can be evaluated by integration. Thus,

$$p^{-\frac{3}{2}}\,1 = \frac{1}{p}\left(p^{-\frac{1}{2}}\,1\right) = \frac{1}{p}\left(\frac{2\sqrt{t}}{\sqrt{\pi}}\right)1$$

$$= \frac{2^2 t^{\frac{3}{2}}}{3\sqrt{\pi}}\,1$$

$$p^{-\frac{5}{2}}\,1 = \frac{1}{p}\left(p^{-\frac{3}{2}}\,1\right) = \frac{2^3 t^{\frac{5}{2}}}{3(5)\sqrt{\pi}}\,1$$

$$p^{-\frac{7}{2}}\,1 = \frac{2^4 t^{\frac{7}{2}}}{3(5)(7)\sqrt{\pi}}\,1$$

$$p^{-\frac{9}{2}}\,1 = \frac{2^5 t^{\frac{9}{2}}}{3(5)(7)(9)\sqrt{\pi}}\,1$$

Using these operators, we can write the current equation as

$$i = 2Ew\sqrt{\frac{Ct}{R\pi}}\left[1 - \frac{(2wt)^2}{1(3)(5)} + \frac{(2wt)^4}{1(3)(5)(7)(9)} - \cdots\right]1$$

It is sometimes possible to express a series such as this in terms of functions which have been tabulated; e.g.

$$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots = \cosh x,$$ the hyperbolic cosine; or,

$$\frac{x^n}{2^n\,n!}\left[1 - \frac{x^2}{2(2n+2)} + \frac{x^4}{2(4)(2n+2)(2n+4)} - \cdots\right]$$

$= J_n(x)$, the n^{th}-order Bessel function. If this can be done, the arithmetical labor involved in obtaining a solution is much reduced. It is assumed, of course, that any series from which a solution is to be obtained will not be divergent. The series should preferably be convergent[15], but we may be asymptotic[16, 17, 18].

Now note that we have based our solutions with lumped or distributed parameters on the two formulas for $\frac{1}{p^n}\,1$ where n is an integer, and $p^m 1$ where m is not an integer. If we can evaluate these operators rigorously we shall have established the validity of all the results obtained thus far. Let us now turn to a more rigorous discussion of Heaviside's general problem.[19, 20]

Consider a set of simultaneous differential equations

$$(b_{11}+c_{11}p+d_{11}p^2+\cdots)x_1+\cdots+(b_{1n}+c_{1n}p+d_{1n}p^2+\cdots)x_n = E_1$$
$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$
$$(b_{n1}+c_{n1}p+d_{n1}p^2+\cdots)x_1+\cdots+(b_{nn}+c_{nn}p+d_{nn}p^2+\cdots)x_n = E_n$$

where the b's, c's, d's, $\cdots$ are constants. Note that the equations are linear in x and E.

Let us assume that $E_1 = F_1\varepsilon^{at}$, and $E_2, \cdots E_n = 0$. This latter restriction involves no loss of generality because of the linearity of the equations. Now assume a particular solution $x'_j = k_j\varepsilon^{at}$, $(j = 1, 2, \cdots n)$, where k_j is a constant. Then, $px'_j = ak_j\varepsilon^{at}$, $p^2x'_j = a^2k_j\varepsilon^{at}$, etc. Thus,

$$(b_{11}+c_{11}a+d_{11}a^2+\cdots)\varepsilon^{at}k_1+\cdots+$$
$$(b_{1n}+c_{1n}a+d_{1n}a^2+\cdots)\varepsilon^{at}k_n = F_1\varepsilon^{at}$$

etc. Thus in each equation the exponential can be factored out, leaving a set of equations which do not involve time t. To simplify the writing, let $(b_{jk}+c_{jk}a+d_{jk}a+\cdots) = H_{jk}(a)$. Then the differential equations can be written

$$H_{11}(a)k_1+H_{12}(a)k_2+\cdots+H_{1n}(a)k_n = F_1$$
$$H_{21}(a)k_1+H_{22}(a)k_2+\cdots+H_{2n}(a)k_n = 0$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$H_{n1}(a)k_1+H_{n2}(a)k_2+\cdots+H_{nn}(a)k_n = 0$$

Solving, $k_j = \frac{M_{j1}}{D}F_1$ where D is the determinant of the coefficients, and M_{j1} is the cofactor of the j^{th} column and first row.

Thus, $x'_j = \frac{M_{j1}}{D}F_1\varepsilon^{at} = \frac{F_1}{Z_{j1}}\varepsilon^{at}$ where $Z_{j1} = \frac{D}{M_{j1}}$

We have now obtained only the "particular solution" of this system of equations. Consider next the "complementary solution". Assume $x''_j = L_j\varepsilon^{bt}$. Then if x'_j is a solution, $x'_j + x''_j$ is also a solution. Now since $px''_j = bL_j\varepsilon^{bt}$, $p^2x''_j = b^2L_j\varepsilon^{bt}$, etc., the exponential can be factored out, so that

$$H_{11}(b)L_1+H_{12}(b)L_2+\cdots+H_{1n}(b)L_n = 0$$
$$H_{21}(b)L_1+H_{22}(b)L_2+\cdots+H_{2n}(b)L_n = 0$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$H_{n1}(b)L_1+H_{n2}(b)L_2+\cdots+H_{nn}(b)L_n = 0$$

The condition that a finite solution exist is that the determinant of the coefficients must vanish,[21] that is

$$D(b) = \begin{vmatrix} H_{11}(b) & \cdots & H_{1n}(b) \\ \vdots & & \vdots \\ H_{n1}(b) & \cdots & H_{nn}(b) \end{vmatrix} = 0$$

Thus b must be a root of $D(b) = 0$. Let the roots be $b_1, b_2 \cdots b_m$. Then it can be shown[21] that $x''_j = C_1\varepsilon^{b_1t} + C_2\varepsilon^{b_2t} + \cdots + C_m\varepsilon^{b_mt}$. Here the C's are arbitrary constants which must be determined by boundary conditions, while $b_1 \cdots b_m$ depend on the values of the coefficients in $D(b)$. Thus the complete solution is $x_j = x'_j + x''_j = \frac{F_1}{Z_{j1}}\varepsilon^{at} + C_1\varepsilon^{b_1t} + C_2\varepsilon^{b_2t} + \cdots + C_m\varepsilon^{b_mt}$ or, $x_j = \frac{F_1}{Z_{j1}}\varepsilon^{at} + f(t)$ where $f(t)$ depends on the boundary conditions and the coefficients of $D(b)$, but not on E_1.

Note that in obtaining this solution we have assumed only that the original equations were linear in x and E, initially in equilibrium, and that they permitted an exponential solution. The solution, then, involves a "driven" term which contains the same exponential as that applied at E_1, and a complementary term which is independent of E_1. For all dissipative systems, the complementary term will vanish with increasing time.

Assume now that E_1 should take the form of a function which is zero before $t = 0$ and which has a constant value of one for all positive values of t. (The unit function.) Let the value of x_j corresponding to this value of E_1 be $A_{j1}(t)$, and assume that $A_{j1}(t)$ is zero for all negative values of t. In general let $x = A(t)$.

Now if $E_1 = E_k$ where E_k is constant, $x = E_kA(t)$ because of the linearity of the equations. Also, if a function of constant magnitude E_k is applied when $t = k$, $x = E_kA(t - k)$ where $A(t - k)$ is zero as long as $(t - k)$ is negative. Suppose, then, that E_1 is made to vary in steps so that $E_1 = 0$ for negative values of t: $E_1 = E(0)$ from $t = 0$ to $t = \triangle t$: $E_1 = E(0) + \triangle_1 E$ from $t = \triangle t$ to $t = 2\triangle t$: $E_1 = E(0) + \triangle_1 E + \triangle_2 E$ from $t = 2\triangle t$ to $t = 3\triangle t$: etc. In other words, it has an increment $\triangle_m E$ at time $t = m\triangle t$. Then x will vary in corresponding steps, and for any particular value of t, say b,

$$x(b) = E(0)A(b) + \triangle_1 EA(b - \triangle t) + \triangle_2 EA(b - 2\triangle t) + \cdots + \triangle_n EA(b - n\triangle t)$$

$$= E(0)A(b) + \sum_{i=1}^{n} A(b - i\triangle t)\,\triangle_i E.$$

Next, let E_1 be any arbitrary function $E(t)$. This can be approximated by the above step function, and the approximation can be made to improve by decreasing the length of the steps. Thus for any value of t, say b,

$$x(b) = E(0)A(b) + \lim_{n \to \infty} \sum_{i=1}^{n} A(b - i\triangle t)\,\triangle_i E.$$

Now $i\triangle t = t$, and when $i = 1$, $t = \triangle t$; and when $i = n$, $t = b$. Hence,

$$x(b) = E(0)A(b) + \lim_{n \to \infty} \sum_{i=1}^{n} A(b - t)\frac{\triangle_i E}{\triangle t}\triangle t$$

$$= E(0)A(b) + \int_{t=0}^{t=b} A(b-t)\,\frac{dE}{dt}\,dt$$

This can be put into more convenient form by a change of variable.

Let $t = b - y$. Then

$$x(b) = E(0)A(b) + \int_{b-y=0}^{b-y=b} A(y)\frac{dE}{d(b-y)}\,d(b-y)$$

$$= E(0)A(b) + \int_{y=b}^{y=0} A(y)\frac{\partial E}{\partial y}\,dy$$

$$= E(0)A(b) - \int_0^b A(y)E'(b-y)\,dy$$

where $E'(b-y) = \frac{\partial E}{\partial y}$

Now note that $\frac{\partial}{\partial b}E(b-y) = -\frac{\partial}{\partial y}E(b-y) = -E'(b-y)$. Thus,

$$x(b) = E(0)A(b) + \int_0^b \frac{\partial}{\partial b}A(y)E(b-y)\,dy$$

$$= \frac{d}{db}\int_0^b A(y)E(b-y)\,dy.$^{[22]}$$

Now let $E(t) = \varepsilon^{at}$ where a is either a positive real number or a complex number with positive real part. Then $E(b-y) = \varepsilon^{ab}\varepsilon^{-ay}$, and

$$x(b) = \frac{d}{db}\int_0^b A(y)\varepsilon^{ab}\varepsilon^{-ay}dy$$

$$= \frac{d}{db}\left\{\int_0^{\infty} A(y)\varepsilon^{ab}\varepsilon^{-ay}dy - \int_b^{\infty} A(y)\varepsilon^{ab}\varepsilon^{-ay}dy\right\}$$

$$= a\varepsilon^{ab}\int_0^{\infty} A(y)\varepsilon^{-ay}dy - a\varepsilon^{ab}\int_b^{\infty} A(y)\varepsilon^{-ay}dy + A(b).$$

But $x(t) = \frac{\varepsilon^{at}}{Z} + f(t)$. Hence,

$$\frac{\varepsilon^{ab}}{Z} + f(b) =$$

$$a\varepsilon^{ab}\int_0^{\infty} A(y)\varepsilon^{-ay}dy - a\varepsilon^{ab}\int_b^{\infty} A(y)\varepsilon^{-ay}dy + A(b).$$

Transposing the $f(b)$, and dividing through by ε^{ab},

$$\frac{1}{Z} = a\int_0^{\infty} A(y)\varepsilon^{-ay}dy - a\int_b^{\infty} A(y)\varepsilon^{-ay}dy + \frac{A(b) - f(b)}{\varepsilon^{ab}}$$

This equation holds for all values of b, hence for $b = \infty$,

$$\frac{1}{Z} = a\int_0^{\infty} A(y)\varepsilon^{-ay}dy$$

or, $$\frac{1}{aZ(a)} = \int_b^{\infty} A(y)\varepsilon^{-ay}dy$$

This is an expression of the "Infinite Integral Theorem". Observe that it contains $Z(a)$ which is obtained from the coefficients of the original equations, and it also contains $A(y)$ a function which represents the response of the system to the application of the unit function at E_1. The term $\frac{1}{aZ(a)}$ is called the Laplacian transform[23] of $A(y)$. If one is familiar with Laplacian transforms, he can find A, given Z, by the Infinite Integral Theorem. However, for the most part this theorem is used to check the value of A found by some simpler method or by some questionable method.

It might be noted in passing that the Infinite Integral Theorem can be modified by a Fourier transform[24] so that

$$A(t) = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{1}{aZ(a)}\varepsilon^{at}\,da.$$

However, the solution of this integral is generally not easily obtained by the average engineer (or mathematician).

Now consider the original determinant. It is a function of p. Assume that for a certain case $Z(p) = p^{\frac{1}{2}}$. Then the Infinite Integral Theorem gives

$$\frac{1}{pZ(p)} = p^{-\frac{3}{2}} = \int_0^{\infty} A(y)\varepsilon^{-py}dy.$$

This equation is satisfied by $A(y) = \frac{2\sqrt{y}}{\sqrt{\pi}}$ as can be seen by integrating by parts. Thus

$$\frac{2}{\sqrt{\pi}}\int_0^{\infty} y^{\frac{1}{2}}\varepsilon^{-py}dy = p^{-\frac{3}{2}}$$

But A is the response of the system to the unit function applied at E_1. Hence $A(t)\,1 = \frac{1}{Z(p)}\,1 = p^{-\frac{1}{2}}\,1$. Thus $p^{-\frac{1}{2}}\,1 = \frac{2\sqrt{t}}{\sqrt{\pi}}\,1$, and we have checked the value that we accepted previously without proof.

In a similar manner we can show that $p^m\,1 = \frac{t^{-m}}{\Gamma(1-m)}$ except for positive integral values of m. (The expression $\Gamma(1-m)$ is called the "gamma" function" of $(1-m)$.) Using the Infinite Integral Theorem, $\frac{1}{Z(p)} = p^m$,

$$\frac{1}{pZ(p)} = \frac{1}{p^{1-m}} = \int_0^{\infty} A(y)\varepsilon^{-py}dy.$$ This integral equation is satisfied by $A(y) = \frac{y^{-m}}{\Gamma(1-m)}$ for,

$$\int_0^{\infty} \frac{y^{-m}}{\Gamma(1-m)}\varepsilon^{-py}dy = \frac{1}{\Gamma(1-m)}\int_0^{\infty} y^{-m}\varepsilon^{-py}dy = \frac{1}{\Gamma(1-m)}\left[\frac{\Gamma(1-m)}{p^{1-m}}\right] = \frac{1}{p^{1-m}}$$

The values of these definite integrals can be found in any good set of tables.[25]

It is also obvious that $\frac{1}{p^n}\,1 = \frac{t^n}{n!}\,1$, for if $\frac{1}{Z(p)} =$

$$\frac{1}{p^n}, \quad \frac{1}{pZ(p)} = \frac{1}{p^{n+1}} = \int_0^\infty A(y)\,\varepsilon^{-py}dy \text{ which}$$

is satisfied by $A(y) = \frac{y^n}{n!}$

$$\int_0^\infty \frac{y^n}{n!}\varepsilon^{-py}dy = \frac{1}{n!}\int_0^\infty y^n\varepsilon^{-py}dy =$$

$$\frac{1}{n!}\left[\frac{n!}{p^{n+1}}\right] = \frac{1}{p^{n+1}}$$

We have now laid a rigorous foundation for the fundamental processes of the Operational Calculus. Once the operational equations are written, they may be manipulated by algebraic means until a recognizable operator is encountered. The Infinite Integral Theorem serves as a convenient check on the solution. Problems with lumped parameters can generally be solved in terms of standard tabulated functions. Problems involving distributed parameters must frequently be solved in terms of infinite series.

Operational Calculus at present finds its greatest use among electrical engineers, but it is also useful in many other fields, e.g., for the study of impulsive forces, stability of aircraft, effect of earthquakes on tall buildings, transmission of heat, etc.; in short, for any physical system which can be described by a group of linear differential equations with constant coefficients, initially in equilibrium, and into which a change is suddenly introduced. If the system is not initially in equilibrium, the new solution is simply superposed on the old.[26]

Listed below are several books for general reference. Those by Berg, Bush, and Carson are best suited to the reader with limited mathematical background. The texts by Doherty and Keller, and McLachlan approach the subject from the theory of the function of the complex variable.

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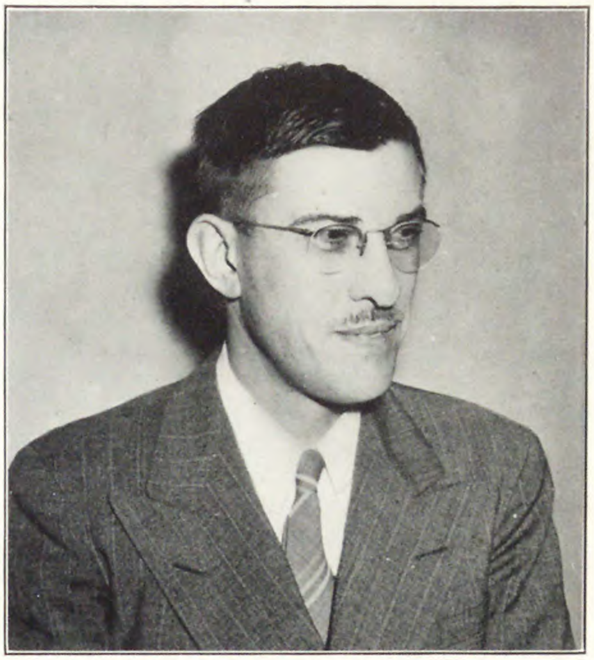
25. Pierce, B. O., *A Short Table of Integrals*, Ginn 1914
26. Bush, p297.

LOCAL TEACHER GETS NAVY POST

Appoint Professor La Londe Lieutenant in Civil Engineer Corps

Reprinted from NEWARK SUNDAY CALL of June 8, 1941



William S. La Londe, Jr.

Professor William S. La Londe, Jr., of the Civil Engineering Department of the Newark College of Engineering will step into a new title and job tomorrow as a lieutenant in the civil engineering corps of the U. S. Naval Reserve.

Professor La Londe's new post will be at the Brooklyn Navy Yard, probably in an executive capacity in the public works office, which is the drafting and designing division of the corps. He plans to commute daily from his home at 22 Bailey Road, Maplewood.

The public works office of the navy, Professor La Londe explained yesterday, has nothing directly to do with the design of seagoing vessels. It includes, however, all land construction undertaken by the navy, such as drydock facilities, railroads and piers for naval use, etc.

Active in Profession

His new assignment, Professor La Londe hopes, will not interfere with his participation in the affairs of engineering societies in which he has been active. He is a director of the metropolitan section of the American Society of Civil Engineers and chairman of committees in charge of junior activities for that group and New Jersey Society of Professional Engineers. For several years he has been faculty adviser for the Newark College of Engineering student chapter of the American Institute of Civil Engineers. During this period the local chapter three times received national commendation for its activities.

A native of Chicago, Ill., Professor La Londe attended high school at Evanston, Ill., where he joined a high school ROTC unit. When he entered Northwestern University on an academic scholarship he again enrolled for ROTC training.

After two years at Northwestern he transferred to Massachusetts Institute of Technology, from which he graduated in 1923 with a bachelor of science degree. Later he took his master's degree in civil engineering at University of Michigan.

Following his graduation from MIT Professor La Londe worked for three years in railroad and municipal engineering in California. Then came six months' experience at sea as an officer in the U. S. Coast and Geodetic Survey, making a study of water depths off the coast of Oregon. He acquired experience as a field engineer on two large projects—the construction of foundations for the Arlington Memorial Bridge in Washington and erection of a $50,000,000 industrial plant for the atmospheric nitrogen plant at Hopewell, Va.

In 1929 he came to the Newark institution as an assistant professor in civil engineering. He became associate professor in 1930 and professor in 1940, specializing in the field of construction.

Professor La Londe has held a lieutenancy in the naval reserve for five years. He is married and has two children.

PRESIDENT CULLIMORE RECEIVES HONORARY DOCTOR OF SCIENCE

Reprinted from NEWARK STAR-LEDGER of June 13, 1941

Dr. George H. Black (left), president of the University of Newark, shown last night conferring honorary degrees upon (left to right) Governor Charles Edison, Allan R. Cullimore, president of Newark College of Engineering, and Percy S. Young, Public Service Corporation executive. Degrees were awarded at seventh annual commencement exercises at the Mosque Theater last night.

The citation by Dr. George H. Black, President of University of Newark, to Allan Reginald Cullimore upon the award of the honorary degree of Doctor of Science June 12, 1941, was as follows:

Native of Illinois, that State of rugged pioneer spirit and action; trained in youth in a military academy; graduated at the Massachusetts Institute of Technology; engineer; college teacher; Dean of the Schools of Engineering of Delaware College and of the University of Toledo; Major in the United States Army in the Office of the Surgeon General; Dean and President of the Newark College of Engineering and Regional Adviser of the Northern New Jersey Defense Program under Federal Jurisdiction; distinguished scientist and civic leader:

I now, in recognition of these achievements, and acting upon the recommendation of the Committee on Higher Degrees of the University of Newark, confer upon you the honorary degree of Doctor of Science, with all the rights, privileges, and immunities thereunto appertaining.

THE MATHEMATICS COLLOQUIA

Professor Odd Albert read a paper on "The Beam Integrals" at the third of a series of mathematics colloquia, which was held at the Newark College of Engineering on Friday, May 2, at 3 P. M. Professor Albert discussed numerals which give the change in statical behavior of beams with variable moment of inertia as compared with beams of constant cross section. This form of analysis, originated by Professor Albert, has been described in part in various technical journals. The paper which was read on May 2, however, was the first complete presentation of his method.

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The last of the Mathematics Colloquia for this year was held on Friday, May 23, when Professor James H. Fithian, Head of the Mathematics Department of the Newark College of Engineering spoke on "The Theory of Relativity and its Application to Electromagnetisms." In a two-hour long lecture, which was intently followed by an interested audience, Professor Fithian showed how the phenomenon of magnetism may be explained as resulting from the forces between electric charges because of the modification of these forces when the charges are in motion.

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This series of colloquia grew out of a suggestion made at a meeting of the Mathematics Department. The suggestion found immediate favor with all members of the Department and titles were submitted for papers to be read at four meetings—one to be held during each of the remaining months of the college year. Previous to the papers by Professor Albert and Professor Fithian were those presented by Dr. John E. Freehafer on "The Application of Complex Variables to Aerofoil Theory" and by Professor Elmer C. Easton on "Operational Calculus." Because of the interest aroused by these meetings, the Department plans to conduct regular monthly discussions of topics on advanced analysis during the coming college year, 1941-1942.

WHO'S WHO IN ENGINEERING

Who's Who in Engineering, 1941, the standard biographical dictionary of the engineering profession, has just been published. The book contains the professional records of about 15,000 leading engineers in the U. S. A.

The following professors and instructors at Newark College of Engineering are listed in Who's Who in Engineering:

Dr. Allan R. Cullimore, President, of South Orange; J. Ansel Brooks, Professor Emeritus, of Montclair; Professor James C. Peet, of East Orange; Professor Harold N. Cummings, of Belleville; Professor and Dr. V. J. Stewart, of Montclair; Professor Frank N. Entwisle, of Maplewood; Professor Albert A. Nims, of Bloomfield; Dr. and Professor Frank D. Carvin, of Summit; Dr. and Professor Paul M. Giesy, of Bloomfield; Professor William S. La Londe, of Maplewood; Professor James A. Bradley, of Union; Professor Henry H. Metzenheim, of Hillside; Professor Paul C. Shedd, of Glen Ridge; Dr. and Professor T. Smith Taylor, of Caldwell; Professor James M. Robbins, of Maplewood; Professor Solomon Fishman, of Maplewood; Professor Robert W. Van Houten, of Maplewood; Professor Arthur S. Kohler, of Rutherford; Professor Odd P. Albert, of East Orange; Professor Elmer C. Easton, of Newark; Professor Frank A. Busse, of Maplewood; Mr. Clarence H. Stephans, of Nutley; Mr. Daniel C. Frost, of Montclair.

DR. FRANK D. CARVIN

Dr. Frank D. Carvin, Professor in Mechanical Engineering and in charge of the Department at Newark College of Engineering, has been elected Chairman of the Metropolitan Section of the American Society of Mechanical Engineers for the coming year.

CONTROL SURVEYS FOR ENGINEERING . . .

(Continued from page 19)

and frost action had thrown the station out of position. It was necessary to remeasure the base line and all angles in the quadrilateral and to readjust the triangulation before the control system could function correctly. From this incident it is apparent that it is not enough for a surveyor just to be able to occupy a triangulation station, but rather he must be sufficiently well trained so that he may appreciate the troubles he encounters and correct them himself if necessary.

Because of the interest in national defense, the examples cited in this paper have all been tied to an industrial plant, but that does not limit the surveyor or engineer from using these same methods on other types of engineering construction work requiring control surveys. Knowing how to measure base lines and conduct triangulation, or set up machinery is only part of the problem. To get the degree of precision needed for any particular construction project requires much continuous and conscientious effort, the exercise of ingenuity, and the sufferance by all parties concerned of long hours of work, but when everything dovetails nicely there is a degree of satisfaction in accomplishment that is gratifying.

Comment by Professor James M. Robbins

In this paper Professor LaLonde clearly points out the necessity for thorough reconnaissance, careful planning, skillful execution, and the application of all possible checks in carrying out surveys for important construction.

Referring particularly to a single important project, he quotes chapter and verse from the record to show the excessive cost, delays and difficulties unnecessarily incurred when these principles are disregarded. Successful and economic construction depends in no small measure on a layout which has been intelligently planned and executed with precision. The methods of the geodetic surveyor are being increasingly applied to heavy construction.

This paper was presented before the Third Annual Conference on Land Surveying, sponsored by the Conference Committee of the New Jersey Society of Professional Engineers, held at the Newark College of Engineering on January 25, 1941.

GLENN GARDINER SAYS:

(Continued from page 11)

tional Schools of the state and facilities available through the colleges and engineering schools of the state.

Most of the defense training that will have to be done must necessarily be done right on the job in the plant where the men work. Schools and colleges can supplement this "on the job training" and can do a certain amount of pre-employment training, all of which is important and necessary.

Therefore a great bulk of responsibility for actually doing the training job falls upon industry itself and whether we meet the challenge of the world to American industry or not is going to depend upon the success with which the hundreds and thousands of junior executives, foremen, supervisors and job instructors right out in American industrial plants do their job.

Although the task ahead of us is enormous, no one actually connected with the American defense program doubts but that American industry will come through and do the job.

DR. ALLAN R. CULLIMORE SAYS:

(Continued from page 11)

addressing ourselves to the matter directly at hand and along the line which we can make our own personal contribution.

Fortunately, in the relations we have had with the Office of Production Management in this district, this condition holds. Mr. Gardiner, while intimately interested in his own work in the Training-Within-Industry program, has been very sensitive to and appreciative of the work not only of the Engineering Defense Training program but also of the training of the State Department of Education, and it is my hope that in my own position as Regional Adviser we can see not only the importance of our own work but that we can appreciate fully the importance of the vocational program and the program of Training-Within-Industry. "Coöperation consists of diversity of function but the recognition of a common objective."