ABSTRACT

OPTIMIZING INCENTIVES FOR SYSTEMS WITH HETEROGENEOUS AGENTS

by
Chen Chen

This dissertation explores new models and applications based on the game theory of incentives. This exploration starts with controlling an invasive insect problem to address one of the most significant challenges facing our forests, the invasion of the Emerald ash borer (EAB), a non-native, wood-boring insect that threatens to kill most ash trees in North America, through designing two new cost-sharing programs between the landowners and local governments. Ash trees are one of North America's most widely distributed tree genera and a vital part of the green infrastructure of cities, where they provide residents with numerous social, economic, and ecological benefits.

Current strategies to slow ash mortality due to the EAB infestation include surveillance of ash tree health coupled with insecticide treatment and/or removal of infested trees. Most ash trees grow on private land, and the growing spread of EAB infestation is largely due to the lack of a private-public partnership in its control. Local governments need programs to induce landowners to undertake actions to slow ash mortality.

A principal-agent modeling framework is presented to design two new programs in which a local government offers reimbursements to landowners to cover a portion of their management costs. Two mathematical models are designed for each program: one in which the reimbursement is based on the number of infested trees and another in which reimbursement is based on the number of treated trees. The numerical analysis shows that neither the optimal treatment decision nor the reimbursement in both programs is in general monotonic concerning the initial infestation level; rather,

they depend on treatment effectiveness and the likelihood of the new infestations. Compared to the infestation-based reimbursement program, the treatment-based reimbursement program induces the landowner to treat more trees through a higher reimbursement and provides a higher overall benefit. The approach shown in this dissertation is expected to inspire other private-public partnerships to solve various environmental and societal spatio-temporal problems through better resource sharing, such as the management of water, land, and wildfire.

Given the required reimbursement assigned to private lands, the government needs to address the problem of budget allocation among public and private sites. An integrated game theory-mixed integer framework is designed to allocate resources to the management decisions on both public and private sites over space and time to maximize the profit of the government. The attack rate of EAB of this integrated model is validated by predicting the real attack rate based on the real infestation EAB data.

The dissertation then focuses on studying the implications of emissions policies in a Carbon Capture and Storage (CCS) system. The excessive emission of CO_2 is supercharging the natural greenhouse effect, which causes the rise of temperature, further affects climate and sea levels, and even increases extreme weather and natural disasters. In order to induce emitters to capture as much as possible CO_2, the principal-agent framework is designed between the CCS operator and emitters.

Specifically, the principal (CCS operator) offers a menu of contracts to agents (emitters) whose demand may follow different distributions, and the government may or may not introduce the cap-and-trade policy (free allocated allowances given to emitters) into the market. Two scenarios are examined: 1) the cap-and-trade policy is lunched, and 2) the cap-and-trade policy is not lunched. The principal-agent framework is presented to design optimal contracts for emitters by the CCS operator. The principal prefers to offer efficient quantities to the agents regardless of the demand levels when there is only one type of agent with no carbon allowance assigned by the government; however, emitters are induced to capture all their emissions when the cap-and-trade policy is launched. When there are two different types of agents, mostly the emitters are induced to capture all their emissions with the allowance assigned by the government; however, they always do not have enough incentive to capture all emissions when the cap-and-trade policy is not implemented on the market.

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CHAPTER 1

INTRODUCTION

This dissertation mainly focuses on two aspects of environmental sustainability. One is the non-native invasive species controlling and management problem. Invasive species are a major threat to the economy, the environment, health, and thus human well-being. Infestations by non-native insects, in particular, can severely affect valuable trees and have cascading effects on other species in the environment. Preventing the growing spread of invasive alien species has been one of the United Nations (U.N.) Sustainable Development Goals due to the threats to biodiversity from invasive species [29, 102].

Another focus of my dissertation is on emissions management, specially the Carbon Capture and Storage (CCS) system. Carbon emissions play a critical role in global warming and climate change, which has significant and costly effects on our communities, our health, and the climate. In particular, global warming is disrupting national economies of many countries and affecting lives due to rising sea levels and extreme weather events [103].

In this chapter, we provide the research motivation and objectives of each topic as well as the organization of the rest of the dissertation.

1.1 Motivation

The larvae of EAB are the destructive stage of the pest. EAB adults are present from late May through August. They are roughly a half-inch long and dark metallic green. They feed on ash leaves throughout the 3-6 week lifespan. Even though their feeding does not damage or affect the tree, their eggs are laid on the surface of the bark, in bark cracks and crevices, or just under the outer bark of ash trees. With 2-3 weeks in the hatch, the larvae from the eggs immediately begins chewing through the outer bark to the phloem. Larvae begin feeding in late July, but most feeding and growth occur from August to October. Larvae feed in S-shaped tunnels, called galleries, in the phloem. As the larva feeds and grows, the galleries get larger. The galleries disrupt the transport of nutrients and water within the trees. Most larvae overwinter as prepupae in cells found about a half-inch into the sapwood or outer bark. So, at the beginning of infection, the infected trees are very hard to be distinguished by observing.

The emerald ash borer (EAB) is a wood-boring insect in Asia. Those invasive species were found first in southeastern Michigan in 2002. Since those wood-boring insect spread rapidly after their introduction and has been found in most states (35 states) of the eastern U.S., south to Louisiana and Georgia, and west as far as Colorado. So far, the EAB has already killed uncountable millions of ash trees in the USA as well as Canada, and biological control has become a major focus of the Emerald Ash Borer Program of USDA and its cooperators.

Private landowners have a great number of ash trees in cities. For example, 75% of the ash trees belong to private property owners in the city of Aberdeen (City of Aberdeen Parks, Recreation and Forestry Department, 2018). For dealing with infected ash trees by EAB, different private owners have a different attitude. Some private owners intend to treat their own trees and there exist some landowners who are unwilling to treat infected trees, which may impact ash trees in neighboring areas. One main challenge in addressing the EAB problem is that current management options lack collaboration between the city and private landowners (City of Aberdeen Parks, Recreation and Forestry Department, 2018).

Most previous studies on forest invasive species have considered the case of a central planner, typically the government, which is relevant for public lands in the western U.S (e.g. Horie et al. [69], Epanchin-Niell et al. [54], Hauser and McCarthy

[62]) while ignoring the fact that more than half of U.S. forests are owned or managed by private parties [8].

Siriwardena et al. [117] is the only exception. They consider the effects of the private landowners and examine the scope for bargaining between adjacent governments to control and slow the spread of an invasive species across the boundaries of jurisdictions with mixed public and private landowners. However, their focus was on studying cooperative bargaining across a mix of public and private land within municipalities to control EAB spread. Furthermore, former studies provided models and information to evaluate the costs and benefits of ash search, treatment and removal [85, 95], yet there is no principal-agent game-based model of contract design in the literature that addresses the government-private landowner dynamics and helps evaluate the costs and benefits of private landowners' potential contribution to the EAB intervention planning.

Global warming is another issue that needs our attention, which has significant and costly effects on our communities, our health, and the climate. For example, global warming causes the rise of sea level and the melting of ice which will give rise to sea level as well as the increasing of coastal flooding. Wildfires are increasing and wildfire season is getting longer in the Western U.S., and more destructive hurricanes occur as temperatures rise. What is more, the rise of the global average temperature leads to extreme climates. Further, a changing climate affects the range of plants and animals, changing their behavior and causing disruptions up and down the food chain. The range of some warm-weather species will expand, while those that depend on cooler environments will face shrinking habitats and potential extinction. Global warming does have a bad effect on the whole world.

According to NASA, the global average surface temperature rose 0.6 to 0.9 degrees Celsius from 1906 to 2005. Moreover, as Figure 1.1 shows, despite ups and downs from year to year, the global average surface temperature is rising. And the temperature is certain to go up further.

Unless we take immediate action to reduce global warming emissions, these impacts will proceed to intensify, grow ever more catastrophic and damaging, and progressively affect the entire planet like rising seas, increased coastal flooding, more extreme weather events and so on.

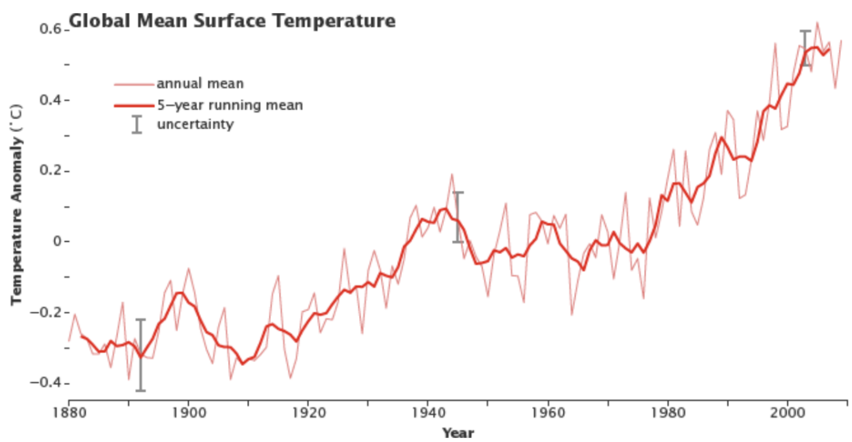


Figure 1.1 The changing of the global mean surface temperature.

Carbon dioxide plays a critical role in global warming. The 2017 annual report of the United States Environmental Protection Agency (EPA) stated that carbon dioxide contributed 81% of greenhouse gas emission. Figure 1.2 displays the increase of CO_2 and Since the Industrial Revolution began in about 1750, carbon dioxide levels have increased by nearly 38 percent as of 2009 [60]. Further, the average of global atmospheric carbon dioxide was 405.0 $\pm$ 0.1 ppm, a new record high, and there was a 2.2 $\pm$ 0.1 ppm increase between 2016 and 2017, according to the State of the Climate in 2017 [19]. However, Compared to the pre-industrial era, the global average temperature growth must be limited to 2℃. Or it will cause many disasters around the world. The world will be a lot drier, impacting economies, agriculture, infrastructure, and weather patterns. To achieve this, we need to produce a scale of effort to carbon dioxide reduction.

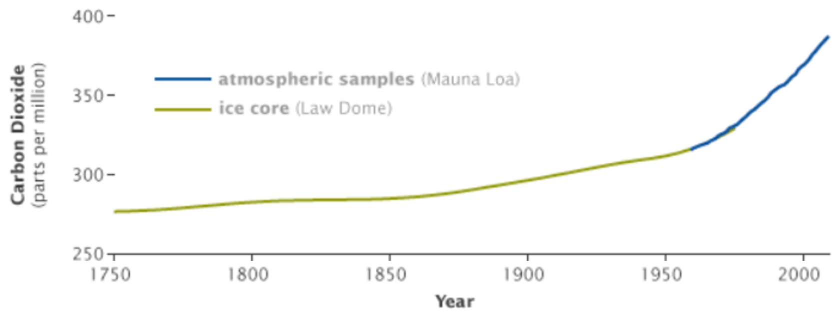


Figure 1.2 The changing of the global annual mean of carbon dioxide.

Table 1.1 Technology Area Contribution to Global Cumulative CO_2 Reductions

Technology	**Percentage**
Efficiency	34%
CCS	32%
Fuel switching	18%
Renewables	15%
Nuclear	1%
Total	100%

Table 1.1 displays the main technologies contributing in percentage to carbon dioxide mitigation [2]. Among those technologies, carbon capture and storage technology is ranked as the second most effective technology in reducing carbon dioxide emissions.

Carbon capture and storage, or CCS, is a family of technologies and techniques that enable the capture of carbon dioxide (CO_2) from fuel combustion or industrial processes, the transport of CO_2 via ships or pipelines, and its storage underground, in depleted oil and gas fields and deep saline formations preventing the carbon dioxide from entering the atmosphere. Currently, there are 21 large-scale CCS projects in operation that capture over 37 million tones of CO_2 per annum (Mtpa) globally. Some recent projects include:

- Petra Nova Carbon Capture, operational since January 2017, is the world's largest post-combustion CO_2 capture system presently in operation. Production unit 8 of the W. A. Parish power plant near Houston, Texas, was retrofitted with a 1.4 Mtpa post-combustion CO_2 capture facility. The captured CO_2 is transported via pipeline to an oil field near Houston for enhanced oil recovery.

- Illinois Industrial CCS. This project integrate newly built compression and dehydration facilities with an existing corn-to-ethanol plant in Decatur, Illinois. The captured CO2 is transported to a nearby injection well for dedicated geological storage, and injection operations commenced in April 2017. The new facilities, when combined with existing facilities constructed under the (now completed) Illinois Basin Decatur Project, can achieve a total CO2 injection capacity of approximately 1 Mtpa.

- Ji Lin Oil field CO2 EOR. It injects CO2 for enhanced oil recovery (EOR) in low permeability reservoirs of the Jilin oil field in northeast China. The CO2 is captured from a nearby natural gas processing plant at the Chang Ling gas field and transported by pipeline. The cumulative CO2 injection was 1.12 million tonnes for pilot and demonstration-scale operation. After 12 years of pilot and demonstration tests, the commercial operation began in 2018. The injection capacity reaches 600,000 tonnes of CO2 per annum.

To meet the Paris 2℃ target, we need 2500 CCS facilities with an average capacity of 1.5 Mtpa by 2040. Clearly, the adoption of CCS technology needs to increase drastically. The wide adoption of CCS technology, however, faces many challenges: (1) technological development is required to reduce the costs of CCS technologies; (2) a marked increase in government commitment to deploying CCS; (3) the strategic need for CO_2 transport and storage infrastructure; and (4) policies that support CCS deployment and CCS pricing mechanism in terms of practical economics.

We mainly focus on designing a principal-agent (PA) framework between CO_2 emitters and a CCS operator who is willing to transport and store captured CO_2 as a service. When emissions are free, as they currently are in the U.S. and other countries, emitters are unwilling to adopt the CCS technology as it is costly. Therefore, we must assume that there is either a quantity-based penalty (or tax) for emissions or there is a cap on the overall allowed emissions.

We initially take the perspective of the CCS operator and design mechanisms to induce emitters to participate in a CCS system run by the CCS operator. We adopt the principal-agent framework in designing the mechanisms, where the principal is the CCS operator and the agent is an emitter. However, the information structure differs from the classical principal-agent model in the following: (1) the agent's demand is uncertain and thus they are modeled as a demand distribution with discrete demand levels; (2) the agent privately observe his demand distribution (or ”type”) when contracting, but the actual demand is realized after committing to a contract; (3) the principal does not observe the agent's demand distribution, but s/he forms a prior belief of the likelihood over some demand distributions.

1.2 Summary of Research Objectives

In Chapter 2, we develop a principal-agent contracting model to incentivize the participation of private landowners with heterogeneous types, based on the number of ash trees, infection level, and treatment. We consider the infestation-based and treatment-based reimbursement models to identify optimal treatment decisions under various conditions.

In Chapter 3, we integrate the principal-agent contracting model with the resource allocation model for EAB surveillance and control in order to determine the optimal budget allocation among public trees and private trees. To avoid the non-linearity and solve our problem, we consider different linearization methods to address the non-linearity.

In Chapter 4, we design a principal-agent mechanism framework with heterogeneous demands of the agents (emitters) who are incentivized to participate in all periods once they have selected their contracts. We construct a cap-and-trade model to optimally allocate a set of emissions permits among the CCS operator and the emitters, and compare the cap-and-trade model with the quantity-based emission penalty model in terms of the total quantity sequestered, the aggregated utility of the agents and the profitability of the principal.

1.3 Organization of the Dissertation

The remainder of this dissertation is organized as follows. Chapter 2 builds a principal-agent framework to study the collaboration between the government and private landowners to control the impact of an invasive insect (Emerald Ash borer). Chapter 3 builds a data-driven integrated game theory-optimization framework to allocate resources to different management decisions in both public and private lands. Chapter 4 presents an incentive mechanism to induce the emitters to capture more CO_2 with or without a cap-and-trade policy, and the conclusion and future directions are discussed in Chapter 5.

CHAPTER 2

A GAME-THEORETIC APPROACH TO INCENTIVE LANDOWNERS TO MITIGATE AN EMERALD ASH BORER OUTBREAK

2.1 Introduction

Invasive species are alien plants, animals, or pests, which cause significant economic and environmental damage by harming biodiversity and degrading the environment [87, 125]. The cost of invasive species to the United States' economy is estimated to be more than 120 billion dollars each year, and the associated costs continue to increase [110]. The national and international communities, such as the United Nations' Global Invasive Species Program (GISP) and National Invasive Species Council (NISC) have called for rapid management and control of invaders to minimize their harmful impacts on sustainability and human well-being [29].

One prime example of an invasive species is the emerald ash borer (EAB), a non-native forest insect, which causes close to 100% mortality of native ash trees. They are not only one of North America's most widely distributed tree genera, but also invaluable commodities with an expected life span of 120 to 260 periods [68]. Municipalities endeavor to maximize the number of live ash trees because trees continue to provide numerous social, economic, and ecological benefits, such as providing a habitat for many animal and insect species, producing oxygen, improving soil quality, and reducing pollution [124]. Unfortunately, EAB has spread to 35 states in the U.S. and five Canadian provinces [52] and killed millions of ash trees, costing homeowners and cities millions of dollars. EAB is likely to extirpate ash trees, resulting in devastating economic and ecological impacts [84, 63]. In cities and communities, EAB is projected to cost homeowners and local governments billions of dollars for treatment or removal and replacement of landscape ash trees over the next decade [84]. In response, cities and communities develop EAB management plans, including the application of systemic insecticides to kill EAB adults or larvae and the preemptive removal of infested trees before larvae can complete development [63]. Some of the limited intervention budget should be allocated to discovering the location of infestations in the early stages by surveying ash trees for the existence of EAB larva before any application of treatment or removal [63].

Typically, through Parks and Recreation's Forestry Division, the City Forester is responsible for management and intervention planning for EAB invasion in community forests involving public trees [43]. As stated by [118], tree surveys suggest that each city has approximately 250,000 green and white ash (formally known as Fraxinus Pennsylvanica and Fraxinus Americana), 30% of which are on public property [85, 120]. A significant number of ash trees in cities is privately owned. For example, 75% of the ash trees belong to private property owners in the city of Aberdeen. When addressing private trees, removing trees as they die is currently the only option the city has. Property owners are responsible for treating privately owned trees. However, every landowner makes their decisions based on their own goals and desires and does not consider the impact of their management action or no-action on the neighbor's location.

One main challenge to address the EAB problem is that current management options lack collaboration between the city and private landowners [43]. The problem is exacerbated by the fragmented private forest landowner-ships, which can act as drivers of biological invasions at the landscape level [8].

Most previous studies on forest invasive species have considered the case of a local government, who manages public lands [69], while ignoring the fact that the majority of U.S. trees is owned by private parties [8]. The study of [118] is the only exception; however, it focuses on studying cooperative bargaining across a mix of public and private land within municipalities to control EAB spread. Furthermore, former studies provided models and information to evaluate the costs and benefits of ash search, treatment, and removal [85, 95, 79], yet there is no work in the mechanism design literature that addresses the public-private dynamics and helps evaluate the costs and benefits of the landowners' potential contribution to the EAB intervention.

We use a principal-agent framework where a local government (i.e., a principal) incentivizes a landowner (i.e., an agent) to mitigate the negative impact of the EAB on ash trees through a cost-sharing program. The local government offers a reimbursement to cover a portion of the landowner's costs of inspection, treatment, and removal. We analyze two models, where the reimbursement is based on the infestation level (or the number of infested trees) in one and on the treatment decision (or the number of treated trees) in the other.

Each model considers a two-period planning horizon to capture the evolution of infestation status of the private land based on the landowner's decisions of whether or not to participate in the cost-sharing program, how many ash trees to treat based on the initial infestation level, and the costs associated with the anticipated outcome. If the landowner does not participate in the program, he is assumed not to inspect or treat any ash trees. Because infested trees that are not treated will die with certainty, and dead trees are considered hazardous, the landowner will bear the cost of removal. If participating, the landowner will first identify the infested ash trees through surveillance. Next, the landowner will decide whether to mitigate or not in the first period, with the understanding that his treatment decision affects the infestation level in both periods. Specifically, the infested trees that are successfully treated in the first period would not be infested in the second period since healthy ash trees that are treated would develop EAB resistance which lasts two periods [96]. Unsuccessfully-treated ash trees, on the other hand, would die and incur removal costs. Further, if all of the infested ash trees are treated in the first period, the remaining healthy ash trees would be infested at a lower rate in the second period. If, however, none or only some of the infested trees were treated in period one, the remaining healthy trees would be infested at a higher rate in period two [79, 31].

The local government's utility is the valuation of the expected number of surviving trees at the end of the second period minus the reimbursement required to incentivize the landowner's participation. Because the utility function of an agent who participates in the cost-sharing program is not monotonic in the number of infested trees, both the optimal treatment decision and the reimbursement vary with the input parameters, such as the treatment effectiveness and the second-period attack rates.

In the infestation-based reimbursement, the local government offers a payment based on the landowner's reported infestation level. After receiving the payment, the landowner decides on how many trees to treat. This is a moral hazard problem and thus the local government must design the award based on the infestation level such that if the landowner treats the desirable number of trees post-reimbursement, his utility will be maximized. We find three possible treatment decisions that can be optimal: treating all (infested and healthy) trees, treating only the infested trees, and treating none of them. We provide the analytical solution for optimal reimbursement and characterize the conditions under which each of the treatment decisions can be optimal. Additionally, we show through numerical analysis that neither the optimal treatment decision nor the reimbursement is in general monotonic with respect to the infestation level.

The local government announces a reimbursement schedule, which is non-decreasing in the number of treated trees in the treatment-based reimbursement model. Upon finding out the infestation level, the landowner decides on how many trees to treat based on the reimbursement schedule. After presenting a service receipt to the local government, the landowner receives the reimbursement. Since the infestation level is not reported by the landowner, the local government deals with an adverse selection problem in which the reimbursement schedule must be designed to induce the landowner to make the desirable treatment decision prior to receiving the reimbursement. We find eight optimal reimbursement schedules and identify the conditions under which each can be optimal.

We use scenario analysis to compare the efficacy of the two cost-sharing models and conclude that the treatment-based reimbursement program is superior. Despite the higher reimbursement required, the treatment-based reimbursement program induces the landowner to treat more trees, and as a result, the local government can achieve a higher expected objective function value.

2.2 Literature Review

2.2.1 Management of Invasive Species and the EAB

Several optimization models have been presented to study the surveillance [97, 20, 106, 22, 13, 69, 54, 79] and control [4, 26, 65, 70, 77, 85] of invasive species. For a detailed background in invasive species management and a review of optimization models to detect and control biological invasions, see, e.g., Büyüktahtakın and Haight [29].

Previous work has presented optimization studies on the surveillance and control of the EAB and other similar forest invasive pests under a limited management budget [85, 97, 69, 133, 79, 22, 21]. Among those, Kovacs et al. [85] develop a spatio-temporal optimization model, in which a local government determines the quantities of public and private trees to treat and remove over time, to maximize the benefits of surviving trees minus the net costs of management, subject to constraints on municipal budgets and access to private trees. Kıbış et al. [79] address the cost-effective allocation of resources to jointly optimize the surveillance and control decisions to maximize the benefits of healthy ash trees by saving as many trees as possible over multiple periods. Later, Bushaj et al. [21] extend the formulations of [79] and [22] to a risk-averse multi-stage stochastic program and solve the complex formulation using a scenario-dominance cuts algorithm and insights into risk-averse management.

There are multiple stakeholders in the management of forest pest species (i.e., government, cities, forester, landowners). Because the infestation of an invasive forest pest is dynamic and spatial, an outbreak may impact all stakeholders involved. Therefore, the effective management of invasive species, including non-native forest insects, requires increasing collaboration and coordination among stakeholders. Most studies in invasive species management have considered a local government, which is often the government while ignoring other parties impacted by the government's decisions [69]. Furthermore, the government is responsible only for the public lands, while private properties are typically managed by the landowners.

There is an increasing trend in the game-theoretical research that addresses the dynamics between multiple stakeholders in invasive species management in recent periods [25, 91, 17, 9, 44]. For example, Liu and Sims [91] also address spatially-connected decision-makers (i.e., individual landowners, state and federal agencies, etc.), and provide a corrective mechanism in which individuals compensate invaded individuals for control actions that preserve uninvaded areas. Specifically, the authors develop an economic model to identify the timing and sequence of side-payments in which uninvaded and fully invaded individuals compensate individuals currently engaged in control for actions. Bhat and Huffaker [17] develop a two-person differential game model to characterize a dynamic contract that allows renegotiation and variable transfer payments between owners of two independently-harvested, ecologically-dependent mammal populations.
Atallah et al. [9] use non-cooperative and cooperative games to determine aggregate payoffs between two managers whose independent production processes are spatially connected through a network. Siriwardena et al. [118] develop a dynamic model of cooperative Nash bargaining to examine how the mix of land ownership within each

municipality affects the path of a negotiated transfer payment from the uninfested to the infested jurisdiction. Cobourn et al. [44] employ a Nash bargaining framework to examine the scope for bargaining between two municipalities, one of which is infested with an invasive species and the other is not.

There is a growing interest in using game theory for designing transfer or subsidy payments in invasive species management between the stakeholders. However, to the best of our knowledge, the partnership of government and private landowners in controlling and managing an invasive pest has not been addressed in the mechanism design literature.

2.2.2 Incentive Design using the Principal-Agent Framework

The Principal-Agent framework was developed to better align the interest of two parties who have conflicting interests and achieve a better outcome via incentive structuring when there is information asymmetry. It has been widely adopted in many studies, mostly in the coordination of decentralized players in supply chain management [123, 130, 131, 81, 40] and in production [71, 42].

However, there is a growing body of literature on public-private partnerships or authority-led initiatives, such as funding and auditing of non-profit organizations [112], information sharing among farmers [128], emissions mitigation through carbon-capturing [36], infrastructure development [108], container-inspection policy [12], and performance-based contracts in health service [58, 72, 5, 7].

In the context of invasive species management, the private information is how many trees are already infested, which is learned only after the private landowner (an agent) pays for surveillance by a professional. The local government (the principal) may require the landowner to submit such information prior to distributing the reimbursement. In this case, the local government faces a moral hazard problem because the landowner decides on how many trees to treat after receiving the reimbursement. Alternatively, the local government may ask the landowner to submit the receipt from the professional tree care company that performed the treatment. The local government thus faces an adverse selection problem. Traditionally, adverse selection occurs when one party has more information than the other and therefore has an advantage in entering a contract. Here, the landowner knows the infestation level but the local government does not. Therefore, the reimbursement schedule, which is a function of the number of trees treated, must induce the landowner to treat the desirable number of trees prior to receiving the reimbursement.

2.2.3 Key Contributions

To the best of our knowledge, this work is among the first that adopts the principal-agent framework in the management of invasive species to induce the collaboration between a local government and a private landowner. We hypothesize that government subsidy can be used to motivate the participation of landowners and as a result, improve the outcome of reducing the harmful impacts of an invasive forest beetle. Our game-theoretic models could also be adapted to a variety of other public-private relationships that require joint resource contributions over space and time, such as the management of water, land, and wildfire.

Next, our models differ from an assumption in the classic principal-agent framework in that the agent's utility function does not satisfy the single-crossing property. Specifically, when the infestation level is high in the first period, it may be desirable to not treat any trees, especially when the treatment is ineffective, which leads to fewer trees being successfully treated. On the other hand, when the infestation level is low initially, it may be desirable not to treat only infested trees (and not more) because the treatment is not very effective. It may also be advantageous to treat all trees in order to prevent any new infestations. Consequently, the optimal treatment decision in our models differs from the seminal result of [94].

Recall that in the optimal solution of their work, the quantity offered to the agent is non-decreasing in his type. Further, the highest type receives the efficient amount, which is computed without the presence of information asymmetry. Lower types, however, receive less than efficient amounts. In certain cases, the lowest types may not be served. In the context of invasive species management, this would mean that a landowner with a higher infestation level will be induced to treat more trees. However, we show that the optimal number of trees to be treated is, in general, non-monotonic at the infestation level in either the infestation-based reimbursement model or the treatment-based reimbursement model.

The rest of the paper is organized as follows. We introduce the infestation-based reimbursement model where the infestation level is verifiable by the local government in Section 2.3. Here, both analytical solutions and insights from numerical results are provided. In Section 2.4, we present the treatment-based reimbursement where the number of trees treated is verifiable instead. Several sets of optimal treatment decisions are identified. In Section 2.5, we compare the efficacy of the two models against the case when no cost-sharing programs are offered. Section 2.6 concludes this chapter.

2.3 The Infestation-based Reimbursement Model

In the infestation-based reimbursement (IBR) model, we assume that the local government (she henceforth) can verify the infestation level, i.e., the number of infested trees, of the private land. This can be achieved by requiring the landowner to submit a surveillance report issued by a professional tree service. Based on the reported infestation level, she selects a payment (or reimbursement) to maximize her expected utility. Because the landowner (he henceforth) receives a payment before treating any trees, the local government must consider what he would do after receiving the payment. The landowner is assumed to be rational; thus, his optimal decision depends on the reimbursement, the evolution of the number of infested trees in the two periods, and the consequences of each possible treatment decision.

Characterization of the infestation level of private property. Let n denote the number of ash trees a landowner has on his private land. This information is considered common knowledge because it can be counted along the curb and without accessing the landowner's property. The likelihood of an ash tree being infested by the EAB is π, which is assumed to be an input parameter. We also refer π as the attack rate, which can be computed based on the proportion of infested tree in nearby lands, to the landowner's property if such information is available. The local government, such as [104], may have databases and websites that keep track of the infested trees. The infestation level, denoted by i, is only identifiable through surveillance, such as visual inspection or lab testing, by professional tree services. Here, we assume thorough surveillance that involves lab testing. Let α denote the surveillance cost per tree. Its value depends on the surveillance method, with lab testing being more costly than visual inspection. We assume that i follows a binomial distribution with the rate π. We consider the marginal cost of surveillance, treatment, and removal as constants, and public and private sites share the same marginal cost in the IBR and TBR models. Because each private site is likely to have small number of ash trees, a professional tree company would likely to quote per unit costs. Though it is possible for public sites to incur lower costs due to economy of scales, assuming these costs as a function of number of infested ash trees would create non-linearity that would complicate the models. In future work, we may want to explore non-linear functions. See Section 5.2 for more discussions. Table 2.1 summarizes the notation used in both models.

The landowner's available options. Left untreated, an infested tree will die with certainty within four years. Further, a dead tree is hazardous and thus, must be removed at the cost of c. Treatments, such as soil injections or trunk injections, can be applied to target the EAB larvae residing in the tree and stop them from killing the ash trees. The cost of treatment is β per tree. Unfortunately, treatment is not effective when a tree is in a late stage of infestation or after two years of the initial infestation. Therefore, the treatment's success rate is assumed to be ρ. Further, treated trees for which the treatment is unsuccessful will die and must be removed at the landowner's expense.

Evolution of the infestation level. Let $q(i)$ denote the number of treated trees in the first period. If no trees are infested ($i = 0$) or if all of the infested trees are treated ($q(i) = n \geq i$) in the first period, the chance of a healthy, untreated tree becoming newly infested in the next period is assumed to decrease from π to π^l. On the other hand, if any of the infested trees are left untreated ($q(i) < i$), the attack rate in the second period increases from π to π^h. Further, any healthy tree that is treated in period one becomes EAB resistant for the next two years; that is, it will not be infested in period two. If all of the trees are already infested ($i = n$), there will be no new infestation in the second period.

The landowner's expected utility if he does *not* participate in the cost-sharing program. In this case, he is assumed to neither inspect nor treat any trees in either period. Otherwise, he should participate to offset the cost by getting reimbursement from the local government.

Let θ denote the landowner's marginal value of a healthy ash tree. His value of having w surviving trees at the end of the planning horizon, denoted by $V(w)$, is thus $\theta \cdot w$. Let a_0 denote the decision of not participating in the cost-sharing program, then his expected utility given the infestation level (i) is as follows:

Table 2.1 Notation

Input parameters:	
n	The number of ash trees a landowner has.
π	The first-period attack rate, i.e., the probability that an ash tree is infested with the EAB. $\bar{\pi} = 1 - \pi$.
i	The infestation level in period one follows a binomial distribution with rate π: $i \sim B(n, \pi)$.
c	Cost of removing a dead tree due to the EAB infestation.
α	Cost of inspecting an ash tree.
β	Cost of treating an ash tree.
ρ	Treatment success rate. $\bar{\rho} = 1 - \rho$.
θ	Marginal value of a landowner having a surviving ash tree.
s	Marginal value the local government has for a healthy ash tree.
π^h	The high (resp. low) second-period attack rate, i.e., the probability that a healthy ash (resp. π^l) tree will become infested in period two if the number of treated trees in period one is less than (resp. greater than or equal to) the number of infested trees. $\pi^l < \pi < \pi^h$; $\bar{\pi}^h = 1 - \pi^h$; $\bar{\pi}^l = 1 - \pi^l$.

Value function:	
$v(w, d)$	The value function of the local government of having w surviving ash trees while losing d trees either untreated or unsuccessfully-treated at the end of the second period: $v(w, d) = s \cdot w - \beta \cdot d$.
$V(w)$	The value function of the landowner having w surviving ash trees at the end of the second period: $V(w) = \theta \cdot w$.

Decision variables only applicable to the infestation-based reimbursement model:	
$q(i)$	The number of trees a landowner will treat given i out of n ash trees are infested. $q(i)$ can be greater than, equal to, or less than i.
$r(i)$	Reimbursement offered by the local government for having i infested ash trees.

Decision variables only applicable to the treatment-based reimbursement model:	
$\mathbf{q}$	$\mathbf{q} = [q(0), q(1), \cdots, q(n)]$ is a $(n+1)$-tuple that prescribes the number of trees to be treated based on infestation level i, where $q(i) \in [0, n]$.
$\mathbf{r}$	$\mathbf{r} = [r(0), r(1), \cdots, r(n)]$ is a $(n+1)$-tuple that prescribes the reimbursement scheme based on the treatment decision $q(i)$.

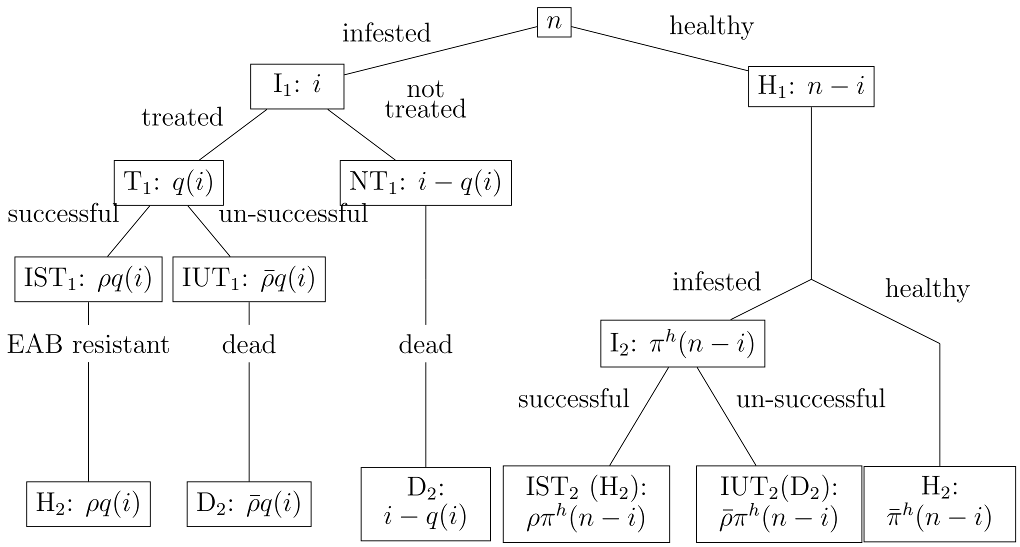
$$\phi(a_0|n,i) = \begin{cases} \theta\bar{\pi}^l n - c\pi^l n & \text{if } i = 0 \\ \theta\bar{\pi}^h(n-i) - c(i + \pi^h(n-i)) & \text{if } 0 < i < n \\ -cn & \text{if } i = n, \end{cases} \tag{2.1}$$

where $\bar{\pi}^l = 1 - \pi^l$ and $\bar{\pi}^h = 1 - \pi^h$.

First, if there are no infested trees ($i = 0$) in the first period, the attack rate in the second period is π^l, and therefore, $\pi^l n$ trees are expected to be infested and die without treatment. The term $\theta\bar{\pi}^l n$ in Equation (2.1) is the landowner's value of having $\bar{\pi}^l n$ surviving trees while $c\pi^l n$ is the cost of removing $\pi^l n$ dead trees. Second, if some trees are infested ($0 < i < n$) in period one, inaction will lead to an increased attack rate (π^h), the expected trees to be infested in period two is $\pi^h(n-i)$. A total of $i + \pi^h(n-i)$ trees will die, and the removal cost associated is $c \cdot (i + \pi^h(n-i))$. The landowner's expected utility is thus the difference between the value of healthy trees, $\theta \cdot \bar{\pi}^h(n-i)$, and the removal cost. Last, if all trees are already infested in the first period, a lack of treatment would result in the death of all. The landowner would incur a removal cost of cn.

The landowner's expected utility if he participates in the cost-sharing program. In this case, he will first pay for an inspection to identify the infested trees. The inspection cost of all trees is αn, where α is the unit cost. Upon learning the infestation level i, the landowner decides on how many trees to treat, denoted by $q(i)$. He considers the evolution of the infestation, which depends on his first-period treatment decision. Figures 2.1 and 2.2 depict the two possible progressions of infestation.

Figure 2.1 illustrates the consequence of not treating all infested trees in the first period, i.e., $q(i) < i$. When $q(i)$ out of the i infested trees are treated, the $i - q(i)$ untreated ones will die and must be removed. Further, the $\bar{\rho}q(i)$ unsuccessfully-



I_t denotes the number of newly infested trees in period t, where $t = 1, 2$. H_t denotes the number of healthy trees in period t, T_t the number of treated trees in period t, IST_t the number of infested trees that are successfully-treated in period t, IUT_t the number of infested trees that are un-successfully-treated in period t, NT_t the number of trees that are not treated in period t, and D_t the number of dead trees in period t.

Figure 2.1 Progression of infestation in two consecutive periods when some infested trees are *not* treated in the first period ($q(i) < i$).

treated trees will also not survive, whereas the $\rho q(i)$ successfully-treated ones will recover and stay healthy in the second period. Because not all infested trees are treated, the attack rate among the $n-i$ healthy trees increases to π^h in the second period. As a result, $\pi^h(n-i)$ trees may become infested in period two while $\bar{\pi}^h(n-i)$ ones remain healthy. For simplicity, we assume that all newly infested trees will be treated, then $\rho\pi^h(n-i)$ trees will become healthy, but $\bar{\rho}\pi^h(n-i)$ trees may die because the treatment is unsuccessful.

To summarize, the overall number of treated trees is $t = q(i)+\pi^h(n-i)$, the total number of trees survived by the end of period two is $w = \rho q(i)+\bar{\pi}^h(n-i)+\rho\pi^h(n-i)$, and the aggregated number of dead trees is $k = i - q(i) + \bar{\rho} q(i) + \bar{\rho}\pi^h(n-i)$. The landowner's expected utility when $q(i) < i$ is

$$
\begin{aligned}
\phi(q(i), r(i)|n, i, q(i) < i) = \quad & \theta \cdot w + r(i) - \alpha \cdot n - \beta \cdot t - c \cdot k \\
= \quad & \theta \cdot [\rho q(i) + \bar{\pi}^h(n - i) + \rho\pi^h(n - i)] + r(i) - \alpha \cdot n \\
& -\beta \cdot [q(i) + \pi^h(n - i)] - c \cdot [i - \rho q(i) + \bar{\rho}\pi^h(n - i)].
\end{aligned} \tag{2.2}
$$

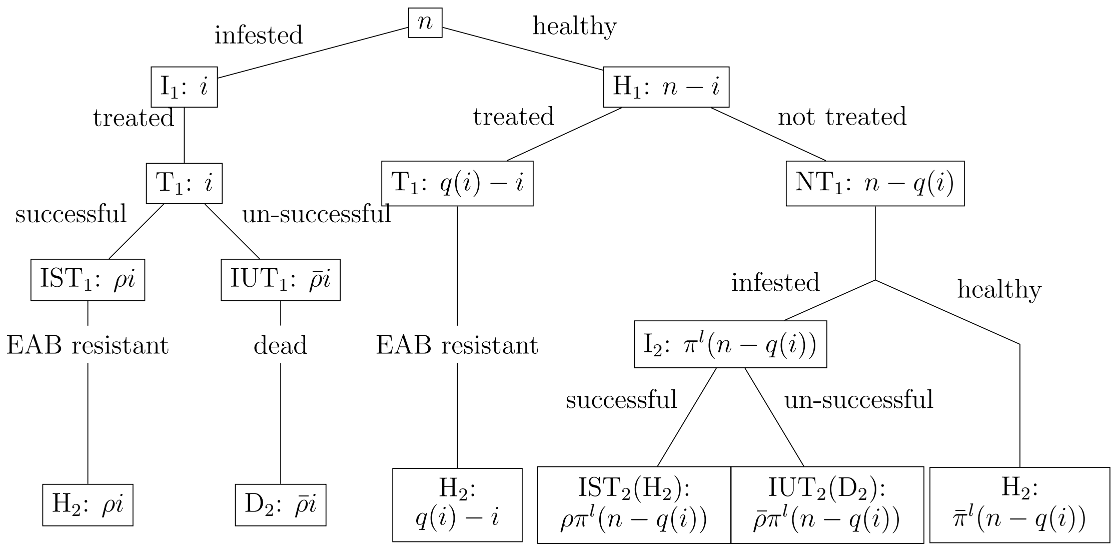
The first two terms in Equation (2.2) are the landowner's value of having w surviving trees and the reimbursement received from the local government. The next three terms represent the inspection cost, the treatment cost, and the removal cost, respectively.

Similarly, Figure 2.2 shows the outcome of treating not only infested trees but some healthy trees in the first period, i.e., $q(i) \geq i$. Because all infested trees (i) are treated, the successfully-treated trees (ρi) will survive and stay healthy in the second period, while the unsuccessfully-treated ones ($\bar{\rho} i$) would die. The healthy trees that are treated ($q(i) - i$) in the first period become resistant to EAB. Further, the attack rate among the healthy, untreated trees ($n - q(i)$) decreases to π^l in the second period. Therefore, $\pi^l(n - q(i))$ trees are expected to be infested in period two, while $\bar{\pi}^l(n - q(i))$ trees are expected to remain healthy. Since all newly infested trees in the second period are assumed to be treated, $\rho\pi^l(n - q(i))$ of them are expected to be successfully-treated and survive, while the rest are expected to die.

The landowner's expected utility when $q(i) \geq i$ is

$$
\begin{aligned}
\phi(q(i), r(i)|n, i, q(i) \geq i) = \quad & \theta \cdot w + r(i) - \alpha \cdot n - \beta \cdot t - c \cdot k \\
= \quad & \theta \cdot [q(i) - \bar{\rho} i + \bar{\pi}^l(n - q(i)) + \rho\pi^l(n - q(i))] + r(i) \\
& -\alpha \cdot n - \beta \cdot [q(i) + \pi^l(n - q(i))] - c \cdot [\bar{\rho} i + \bar{\rho}\pi^l(n - q(i))].
\end{aligned} \tag{2.3}
$$

The terms in Equation (2.3) are similar to those in Equation (2.2), except that the total number of treated trees is $t = i + (q(i) - i) + \pi^l(n - q(i))$, the aggregated number of trees survived by the end of the second period is $w = (q(i) - i) + \rho i + \bar{\pi}^l(n - q(i)) + \rho\pi^l(n - q(i))$, and the overall number of dead trees is $d = \bar{\rho} i + \bar{\rho}\pi^l(n - q(i))$.



I_t denotes the number of newly infested trees in period t, where $t = 1, 2$. H_t denotes the number of healthy trees in period t, T_t the number of treated trees in period t, IST_t the number of infested trees that are successfully treated in period t, IUT_t the number of infested trees that are un-successfully-treated in period t, NT_t the number of trees that are not treated in period t, and D_t the number of dead trees in period t.

Figure 2.2 Progression of infestation in two consecutive periods when *all* of the infested trees are treated in the first period ($q(i) \geq i$).

Let μ_i be an indicator such that $\mu_i = 1$ when $q(i) < i$ and $\mu_i = 0$ otherwise. $\bar{\mu}_i = 1 - \mu_i$. The landowner's expected utility from participating in the cost-sharing program is

$$\phi(q(i), r(i)|n, i) = \mu_i \cdot \phi(q(i), r(i)|n, i, q(i) < i) + \bar{\mu}_i \cdot \phi(q(i), r(i)|n, i, q(i) \geq i). \tag{2.4}$$

Sequence of events. Figure 2.3 illustrates the interactions between the local government and the landowner over two consecutive periods if he participates in the cost-sharing program. At the beginning of the first period, both parties have the same knowledge about the ash trees on the landowner property: there are n ash trees, and the attack rate is π. The landowner pays for a tree care professional to inspect these ash trees and finds out the number of trees (i) that are already infested. He reports the information to the local government, who then offers a financial award ($r(i)$) that depends on the infestation level (i). Upon receiving the reimbursement $r(i)$, the landowner then decides how many trees ($q(i)$) to treat. If he does not treat all of the infested trees ($q(i) < i$) in the first period, then the outcome of his decision over the next two periods is depicted in Figure 2.1. On the other hand, if he treats not only all of the infested trees but also some healthy trees, the outcome follows Figure 2.2. It is worth noting that the local government neither verifies how many trees were treated at the end of the first period nor offers an additional financial award in the second period.

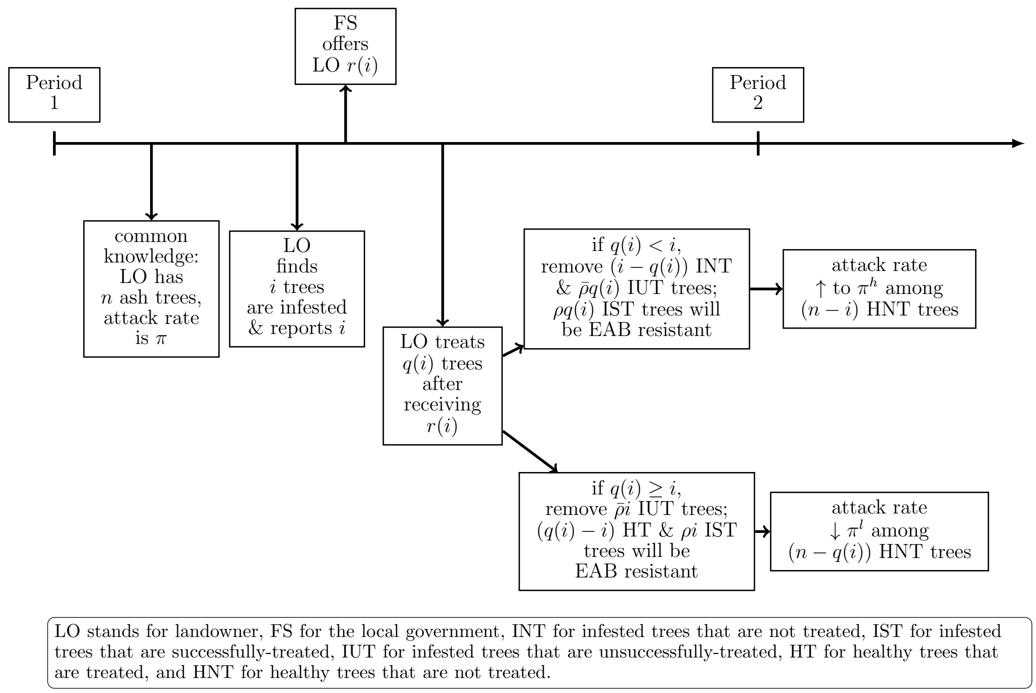


Figure 2.3 Sequence of events for the infestation-based reimbursement model.

The forester's optimization problem. We assume that both the forester and the landowner are rational and make decisions to maximize their utilities over a two-period planning horizon to maximize their utilities. Let $v(w, d) = s \cdot w - \beta \cdot k$ denotes the local government's value function. The first term is the local government's value of having w surviving trees, where s is the marginal value of a tree. The second term, $\beta \cdot k$, is the local government's penalty of having d infested trees either untreated or unsuccessfully-treated. This penalty is included because if an infested tree on the landowner's parcel is not treated or not successfully treated, then ash trees in nearby parcels would be more likely to be infested in the following period. The local government's goal is to maximize her utility function, which is the difference between her value function and the reimbursement. In the case where $q(i) < i$, as shown in Figure 2.1, the number of surviving trees at the end of the period is $w = \rho q(i) + \bar{\pi}^h(n-i) + \rho\pi^h(n-i)$, while the number of untreated or unsuccessfully-treated tree is $k = i - \rho q(i) + \bar{\rho}\pi^h(n-i)$. Similarly, in the case where $q(i) \geq i$, $w = q(i) - \bar{\rho}i + \bar{\pi}^l(n - q(i)) + \rho\pi^l(n - q(i))$ and $k = \bar{\rho}i + \bar{\rho}\pi^l(n - q(i))$, as illustrated in Figure 2.2.

The local government's problem is as follows:

$$
\begin{aligned}
\max_{q(i),r(i)} \Psi(q(i), r(i)|n, i) = \quad & \mu_i \cdot \Big(s \cdot \big[\rho q(i) + \bar{\pi}^h(n-i) + \rho\pi^h(n-i)\big] - \beta \cdot \big[i - \rho q(i) + \bar{\rho}\pi^h(n-i)\big] \Big) \\
& + \bar{\mu}_i \cdot \left(\begin{array}{l} s \cdot \big[q(i) - \bar{\rho}i + \bar{\pi}^l(n - q(i)) + \rho\pi^l(n - q(i))\big] \\ -\beta \cdot \big[\bar{\rho}i + \bar{\rho}\pi^l(n - q(i))\big] \end{array} \right) - r(i) \\
\text{s.t.} \quad & \phi(q(i), r(i)|n, i) \geq \phi(a_0|n, i) \qquad\qquad\qquad\qquad\qquad\qquad \text{(IR)} \\
& \phi(q(i), r(i)|n, i) \geq \phi(j, r(i)|n, i) \qquad \forall\, 0 \leq j \leq n \qquad\qquad (\text{IC}_{ij}) \\
\text{and} \quad & q(i), r(i) \geq 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad (\text{NN}_i)
\end{aligned}
\tag{2.5}
$$

The objective function in Equation (2.5) is the net expected utility of the local government, considering two scenarios: (1) when $q(i) < i$ (or equivalently, $\mu_i = 1$) and (2) when $q(i) \geq i$ (or $\mu_i = 0$). The first line of the objective function is the value of having $w = \rho q(i) + \bar{\pi}^h(n-i) + \rho\pi^h(n-i)$ surviving trees at the end of the second period minus the penalty from loosing $d = i - \rho q(i) + \bar{\rho}\pi^h(n-i)$ trees when $q(i) < i$. Similarly, the second line of the objective function is the difference between the value of having $w = q(i) - \bar{\rho}i + \bar{\pi}^l(n - q(i)) + \rho\pi^l(n - q(i))$ surviving trees and the penalty from loosing $d = \bar{\rho}i + \bar{\rho}\pi^l(n - q(i))$ trees when $q(i) \geq i$. For simplicity, we assume that the penalty associated with a dead tree is equal to the cost of its treatment. The last term is the cost of providing the financial award to the landowner.

The individual rationality (IR) constraint ensures the landowner is incentivized to participate in the cost-sharing program. The incentive compatibility (IC_j) constraints ensure that the landowner will prefer to treat the number of trees desired by the local government $q(i)$ based on the infestation level (i), rather than another treatment decision (j). The non-negativity constraints (NN_i) ensure that both $q(i)$ and $r(i)$ are greater than or equal to zero.

2.3.1 Analytical Solutions

We define the following parameters and conditions in order to characterize the optimal solutions analytically.

First, let $a_1 := \beta - \rho(\theta + c)$ and $a_2 := \beta - \rho(s + \beta + \theta + c)$. It is apparent that $a_1 > a_2$. Further, let $\dot{\rho} := \frac{\beta}{s+\beta+\theta+c}$ and $\ddot{\rho} := \frac{\beta}{\theta+c}$, then $0 < \dot{\rho} < \ddot{\rho}$. Because the cost of treatment is generally much lower than the sum of the valuation of a live ash tree and the removal cost, $\ddot{\rho}$ is likely to be small. We can categorize the treatment effectiveness (ρ) into three levels.

Definition 1. *As summarized in Table 2.2, the treatment is considered to be* less effective *if* $0 \leq \rho < \dot{\rho}$*,* somewhat effective *if* $\dot{\rho} \leq \rho < \ddot{\rho}$*, and* very effective *if* $\ddot{\rho} \leq \rho \leq 1$.

Table 2.2 Categorization of Treatment Effectiveness (ρ)

Condition	ρ
$a_1 > a_2 > 0$ or $0 \leq \rho < \dot{\rho}$	Less effective
$a_1 > 0 \geq a_2$ or $\dot{\rho} \leq \rho < \ddot{\rho}$	Somewhat effective
$0 \geq a_1 > a_2$ or $\ddot{\rho} \leq \rho \leq 1$	Very effective

Table 2.3 Classification of the Low Second-period Attack Rates (π^l)

Condition	π^l
$b_1 > b_2 > 0$ or $0 \leq \pi^l < \dot{\pi}^l$	low
$b_1 > 0 \geq b_2$ or $\dot{\pi}^l \leq \pi^l < \ddot{\pi}^l$	medium
$0 \geq b_1 > b_2$ or $\ddot{\pi}^l \leq \pi^l \leq 1$	high

Second, let $b_1 := \beta - \pi^l[\bar{\rho}(\theta + c) + \beta]$, $b_2 := \beta - \pi^l[\bar{\rho}(s + \beta + \theta + c) + \beta]$, and $b_1 > b_2$. Further, let $\dot{\pi}^l := \frac{\beta}{\bar{\rho}(s+\beta+\theta+c)+\beta}$ and $\ddot{\pi}^l := \frac{\beta}{\bar{\rho}(\theta+c)+\beta}$, which are used to classify the different levels of the low second-period attack rate (π^l). Since $s+\beta+\theta+c > \theta+c$, $0 < \dot{\pi}^l < \ddot{\pi}^l < 1$.

Definition 2. *As shown in Table 2.3, the low second-period attack rate* (π^l) *is considered* low *if* $0 \leq \pi^l < \dot{\pi}^l$, medium *if* $\dot{\pi}^l \leq \pi^l < \ddot{\pi}^l$, *and* high *if* $\ddot{\pi}^l \leq \pi^l \leq 1$.

Third, let $\dot{\pi}^h(i) := \max\{0, \min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}\}$ and $\ddot{\pi}^h(i) := \max\{0, \min\{1, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$ when π^l when π^l is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$). Further, let $\dot{\pi}^h(i) := \max\{0, \min\{1, \frac{\beta}{\bar{\rho}(\theta+c)+\beta} + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}\}$ and $\ddot{\pi}^h(i) := \max\{0, \min\{1, \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$ when π^l is high ($\ddot{\pi}^l \leq \pi^l \leq 1$). Both $\dot{\pi}^h(i)$ and $\ddot{\pi}^h(i)$ are valid for $0 < i < n$, and they describe the different levels of the high second-period attack rate (π^h). As shown in Appendix A.1.1, $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$ when the treatment is either less effective or somewhat effective ($0 \leq \rho < \ddot{\rho}$). $\dot{\pi}^h(i)$ increases (decreases) in i when a_1 is positive (negative), while $\ddot{\pi}^h(i)$ increases (decreases) in i when $\alpha + a_1$ is positive (negative).

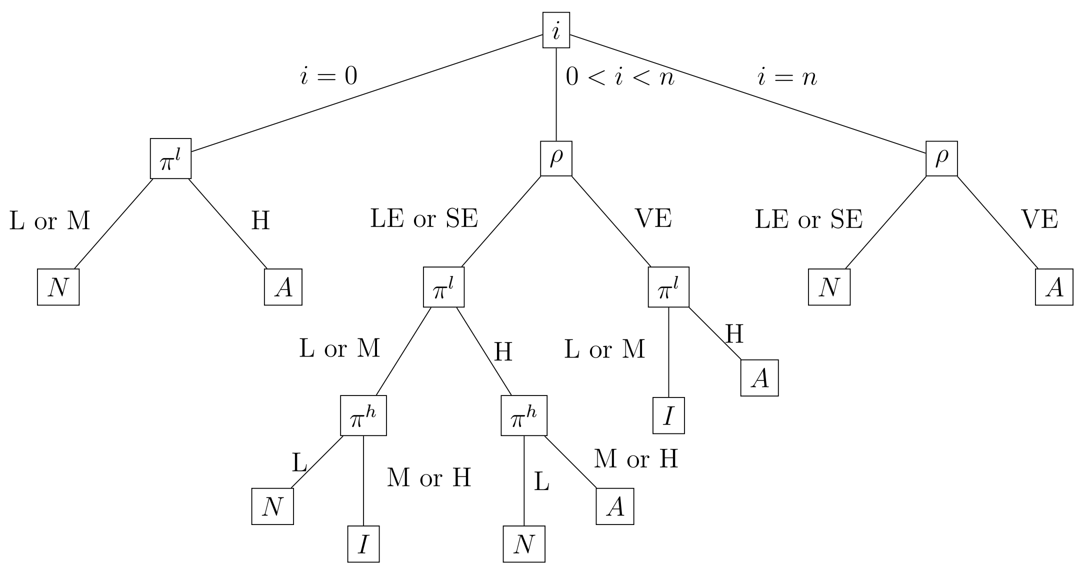
Definition 3. *We classify the high second-period attack rate* (π^h) *under two scenarios. In the first scenario, the treatment is less effective or somewhat effective* ($0 \leq \pi^l < \ddot{\pi}^l$). π^h *is considered* low *if* $0 \leq \pi^h < \dot{\pi}^h(i)$, medium *if* $\dot{\pi}^h(i) \leq \pi^h < \ddot{\pi}^h(i)$, *and* high *if* $\ddot{\pi}^h(i) \leq \pi^h \leq 1$. *In the second scenario, the treatment is very effective.* π^h *is considered* low *when* $0 \leq \pi^h < \ddot{\pi}^h(i)$ *and* high *if* $\ddot{\pi}^h(i) \leq \pi^h \leq 1$. *Table 2.4 summarizes the conditions used to define these levels.*

Table 2.4 Classification of the High Second-period Attack Rate (π^h)

Levels of π^h when $0 \leq \rho < \ddot{\rho}$	
Condition	π^h
$0 \leq \pi^h < \dot{\pi}^h(i)$	low
$\dot{\pi}^h(i) \leq \pi^h < \ddot{\pi}^h(i)$	medium
$\ddot{\pi}^h(i) \leq \pi^h \leq 1$	high

Levels of π^h when $\ddot{\rho} \leq \rho \leq 1$	
Condition	π^h
$0 \leq \pi^h < \ddot{\pi}^h(i)$	low
$\ddot{\pi}^h(i) \leq \pi^h \leq 1$	high

2.3.1.1 General characteristics of optimal solutions. We find three types of optimal treatment decisions in the first period: treating no trees (N as an abbreviated notation), treating only infested trees (I), and treating all trees (A). As shown in Figure 2.4, the optimal treatment decision depends on the infestation level (i), the treatment effectiveness (ρ), and the two second-period attack rates (π^l and π^h). At the top level, the tree diagram is split into three nodes based on how many trees are infested in the first period. The treatment decision is simple when there are no infested trees ($i = 0$). In this case, the value of the low second-period attack rate (π^l) governs whether to treat all trees or to treat none. Recall that a healthy tree, if not treated in the first period, may become infested in the second period with probability π^l. If π^l is high, treating all trees in the first period will prevent them from becoming infested in the second period. On the other hand, if π^l is low or medium, healthy trees are unlikely to be infested in the second period, and thus, there is no need for the landowner to treat them in the first period. Similarly, when all trees are infested ($i = n$), the treatment decision is solely driven by whether or not the treatment is very effective. If so, it is worth the landowner's effort to treat the infested trees. Otherwise, the landowner is better off removing all.



L stands for low, M stands for medium, H stands for high, LE stands for less effective, SE stands for somewhat effective, and VE stands for very effective.

Figure 2.4 The optimal treatment decision vs. key parameters.

When some (but not all) trees are infested, the treatment decision is jointly determined by ρ, π^l, and π^h. Therefore, the middle branch of the tree is further split into multiple levels. Rather than discussing each branch separately, we summarize the results based on the optimal treatment decision. First, it is optimal for the landowner not to treat any trees (N) if the treatment is not very effective and π^h is low. As illustrated in Figure 2.3, not treating all of the infested trees will lead to a higher attack rate in the second period. However, because π^h is low, the chance of a healthy tree becoming infested is still small. As a result, the infestation level will not increase significantly in the second period. On the other hand, trees that are infested have a slim chance of surviving after treatment when the treatment is not very effective. Therefore, the benefit of treating infested trees does not outweigh the cost of treatment, and the landowner is better to remove the infested trees.

Second, treating only infested trees (I) is optimal for the landowner if π^l is low or medium, and either the treatment is very effective or π^h is medium or high. Because π^l is low or medium, healthy trees in the first period are less likely to get infested in the second period, and thus, they do not require preventative treatment. In the first case where the treatment is very effective, an infested tree that is treated has a high chance of surviving at the end of the case. In the second case, where π^h is medium or high, I is beneficial even when treatment is not very effective because it prevents the infestation level from increasing significantly in the second period.

Third, the landowner is induced to treat all trees (A) otherwise. First, when the treatment is very effective and π^l is high, treating all trees will not only enable the infested trees to survive but also allow the healthy trees to become EAB resistant. Second, when the treatment is not very effective, π^l is high, and π^h is medium or high, the landowner still treats all trees to ensure that none of the healthy trees will become infested in the second period.

In the next sections, we discuss the optimal solutions in detail under three settings: when no trees are infested ($i = 0$), when all trees are infested ($i = n$), and when some trees are infested ($0 < i < n$).

2.3.1.2 Optimal solution characteristics when no trees are infested.

Because there are no infested trees ($i = 0$) in the first period, the second-period attack rate is π^l no matter what the landowner chooses to do in the previous period. Let $q^*(i)$ and $r^*(i)$ denote the optimal treatment option selected by the landowner and the reimbursement from the local government, respectively. The characteristics of the optimal solution when no trees are infested are summarized in Proposition 1. See Appendix A.1.2.1 for the proof of Proposition 1.

Proposition 1. *When no trees are infested in the first period, the optimal treatment decision and the optimal reimbursement depend on the low second-period attack rate* (π^l)*, as shown in Table 2.5.*

If π^l is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$), the landowner is induced to not treat any trees (N) in the first period. The reimbursement decreases in the treatment effectiveness (ρ). Moreover, it is less than or equal to the inspection cost (αn) if the treatment is very effective ($\ddot{\rho} \leq \rho \leq 1$) and greater than the inspection cost otherwise.

Table 2.5 The Optimal Solution When $i = 0$

Conditions	Optimal Treatment	Optimal Reimbursement
$0 \leq \pi^l < \ddot{\pi}^l$	$q^*(0) = 0$ (N)	$r^*(0) = \max\left\{0, \alpha n + a_1 \pi^l n\right\}$
$\ddot{\pi}^l \leq \pi^l \leq 1$	$q^*(0) = n$ (A)	$r^*(0) = \max\left\{0, \alpha n + \left(\beta - (\theta + c)\pi^l\right) n\right\}$

If π^l is high ($\ddot{\pi}^l \leq \pi^l \leq 1$), the local government encourages the landowner to treat all trees (A). The reimbursement decreases in π^l. Further, it is less than the inspection cost when π^l is very high ($\pi^l > \max\{\frac{\beta}{\theta+c}, \ddot{\pi}^l\}$). Otherwise, the reimbursement is greater than the inspection cost.

Insights from Proposition 1. First, even though the local government does not want the landowner to treat any trees (N) in the first period when π^l is low or medium, the reimbursement must include the future cost or benefit from treating newly infested trees in the second period. If the treatment is not very effective ($0 \leq \rho < \ddot{\rho}$), the net loss for the landowner is $a_1 = \beta - \rho(\theta + c)$ per infested tree.

Anticipating $\pi^l n$ trees to be infested in the second period, the local government needs to reimburse the net expected loss from unsuccessful treatment, $a_1\pi^l n$, to induce the landowner to participate in the cost-sharing program. Therefore, the reimbursement is greater than the inspection cost, i.e., $r^*(0) > \alpha n$. As the treatment effectiveness (ρ) increases, a_1 decreases. When ρ becomes very effective, a_1 turns negative and represents the landowner's net gain per infested tree. As a result, the reimbursement becomes lower than or equal to the inspection cost ($r^*(0) \leq \alpha n$) and can even be reduced to zero. Therefore, the operator $\max\{0, \alpha n + a_1\pi^l n\}$ is used to prevent the reimbursement from being negative.

Second, the local government's strategy changes drastically when the possibility of a tree becoming infested in the second period is high ($\ddot{\pi}^l < \pi^l \leq 1$). The local government prefers the landowner to treat all trees (A) in the first period. This is because not only do all of them become resistant to EAB in the second period but also the reimbursement is likely to be smaller than the inspection cost since $\beta - (\theta + c)\pi^l$ can also be negative when π^l is greater than $\frac{\beta}{\theta+c}$.

2.3.1.3 Optimal solution characteristics when all trees are infested.

In the case where all trees are already infested in the first period, any untreated trees will die, while the successfully-treated ones will be EAB resistant in the next period. We summarize the results in Proposition 2. The proof of Proposition 2 is in Appendix A.1.2.2.

Proposition 2. *When all trees are infested in the first period, the optimal treatment decision and the optimal reimbursement are driven by the effectiveness of the treatment (ρ), as presented in Table 2.6. If the treatment is less or somewhat effective, the local government only reimburses the landowner's inspection cost, and consequently, the landowner will not treat any trees (N).*

Table 2.6 The Optimal Solution When $i = n$

Condition	Optimal Treatment	Optimal Reimbursement
$0 \leq \rho < \ddot{\rho}$	$q^*(n) = 0$ (N)	$r^*(n) = \alpha n$
$\ddot{\rho} \leq \rho \leq 1$	$q^*(n) = n$ (A)	$r^*(n) = \max\{0, \alpha n + a_1 n\} < \alpha n$

If, on the other hand, the treatment is very effective, the local government can induce the landowner to treat all trees (A) with a reimbursement less than the inspection cost.

Insights from Proposition 2. The most notable characteristic of the optimal solution is that the reimbursement is always less than or equal to the inspection cost. If the treatment is not very effective, the landowner would not treat any trees, and consequently, all trees would die. Since the landowner is ultimately responsible for removing the dead ash trees, the local government only needs to compensate the landowner with the inspection cost.

The landowner would treat all trees if the treatment is very effective. Therefore, the cost of the inspection is offset by the landowner's utility from successfully-treated trees and consequently the local government only covers a portion of the inspection cost to induce the landowner to participate. The reimbursement monotonically decreases as ρ increases, and it can be reduced to zero when ρ is really high, or equivalently, when $\rho \geq \frac{\alpha+\beta}{\theta+c}$.

2.3.1.4 Optimal solution characteristics when some trees are infested. We divide our discussion into two scenarios based on the treatment effectiveness. First, we discuss the scenario where the treatment is very effective. Proposition 3 summarizes the key results. See Appendix A.1.2.3 for proof of Proposition 3.

Proposition 3. *If the treatment is very effective, the optimal treatment decision is driven by the low second-period attack rate (π^l) while the reimbursement depends on the high second-period attack rate (π^h), as shown in Table 2.7.*

When π^l is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$), the landowner is induced to treat all infested trees (I). The local government offers a positive reimbursement when π^h is low or medium. Otherwise, the reimbursement is reduced to zero.

Table 2.7 The Optimal Solution When $0 < i < n$, and the Treatment is Very Effective ($\rho \geq \ddot{\rho}$)

π^l is low or medium: $0 \leq \pi^l < \ddot{\pi}^l$		
Condition	Optimal Treatment	Optimal Reimbursement
$0 \leq \pi^h < \ddot{\pi}^h(i)$	$q^*(i) = i\ (I)$	$r^*(i) = \alpha n + a_1 n - [a_1 \bar{\pi}^l + (\theta + c)(\pi^h - \pi^l)](n-i) > 0$
$\ddot{\pi}^h(i) \leq \pi^h \leq 1$	$q^*(i) = i\ (I)$	$r^*(i) = 0$

π^l is high: $\ddot{\pi}^l \leq \pi^l \leq 1$		
Condition	Optimal Treatment	Optimal Reimbursement
$0 \leq \pi^h < \ddot{\pi}^h(i)$	$q^*(i) = n\ (A)$	$r^*(i) = \alpha n + a_1 n - [a_1 - \beta + (\theta + c)\pi^h](n-i) > 0$
$\ddot{\pi}^h(i) \leq \pi^h \leq 1$	$q^*(i) = n\ (A)$	$r^*(i) = 0$

When π^l is high ($\ddot{\pi}^l \leq \pi^l \leq 1$), then the landowner chooses to treat all trees (A). The local government may offer a positive reimbursement when the value of π^h is low or medium.

It is worth noting that because the landowner will at least treat the infested trees in the first period, π^h would not be realized in the next period. However, the reimbursement decreases as π^h increases. When π^h is high, the reimbursement is zero. This is because as the risk of healthy trees getting infested in the second period increases, the landowner would have to treat the trees anyhow. Therefore, the local government needs to offer a small award to induce the landowner to participate.

Next, we present the optimal solution when the treatment is not very effective. Proposition 4 highlights the key results. The proof of Proposition 4 is in Appendix A.1.2.4.

Proposition 4. *If the treatment is not very effective, the optimal solution depends on both of the second-period attack rates (π^l and π^h), as illustrated in Table 2.8.*

When π^h is low ($0 \leq \pi^h < \dot{\pi}^h(i)$), the landowner is induced not to treat any trees (N) regardless of the value of π^l. The local government offers a reimbursement that is higher than the inspection cost, that is, $r^*(i) = \alpha n + a_1\pi^h(n-i) > \alpha n$.

Table 2.8 Optimal Solution When $0 < i < n$, and the Treatment is Not Very Effective ($\rho < \ddot{\rho}$)

π^l is low or medium: $0 \leq \pi^l < \ddot{\pi}^l$		
Condition	Optimal Treatment	Optimal Reimbursement
$0 \leq \pi^h < \dot{\pi}^h(i)$	$q^*(i) = 0$ (N)	$r^*(i) = \alpha n + a_1\pi^h(n-i) > \alpha n$
$\dot{\pi}^h(i) \leq \pi^h < \ddot{\pi}^h(i)$	$q^*(i) = i$ (I)	$r^*(i) = \alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta + c)(\pi^h - \pi^l)](n-i) > 0$
$\ddot{\pi}^h(i) \leq \pi^h \leq 1$	$q^*(i) = i$ (I)	$r^*(i) = 0$

π^l is high: $\ddot{\pi}^l \leq \pi^l \leq 1$		
Condition	Optimal Treatment	Optimal Reimbursement
$0 \leq \pi^h < \dot{\pi}^h(i)$	$q^*(i) = 0$ (N)	$r^*(i) = \alpha n + a_1\pi^h(n-i) > \alpha n$
$\dot{\pi}^h(i) \leq \pi^h < \ddot{\pi}^h(i)$	$q^*(i) = n$ (A)	$r^*(i) = \alpha n + a_1 n - [a_1 - \beta + (\theta + c)\pi^h](n-i) > 0$
$\ddot{\pi}^h(i) \leq \pi^h \leq 1$	$q^*(i) = n$ (A)	$r^*(i) = 0$

When π^h is medium ($\dot{\pi}^h(i) \leq \pi^h \leq \ddot{\pi}^h(i)$), then the value of π^l determines the optimal solution. When π^l is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$), the landowner is induced to treat all infested trees (I). The reimbursement is positive. Further, r^* increases in i if $a_1\bar{\pi}^l + (\theta + c)(\pi^h - \pi^l) < 0$ and decreases in i otherwise. When π^l is high ($\ddot{\pi}^l \leq \pi^l \leq 1$), the local government prefers the landowner to treat all trees (A), which includes both infested and healthy trees.

When π^h is high ($\ddot{\pi}^h(i) \leq \pi^h \leq 1$), the reimbursement is zero. The landowner will treat all trees when π^l is high and only the infested trees (I) otherwise.

Unlike the results from when the treatment is very effective, π^h can be realized in the second period when it is low ($0 \leq \pi^h < \dot{\pi}^h(i)$). In such a scenario, the local government prefers the landowner not to treat any trees in the first period because the effectiveness of the treatment is not sufficiently high. Therefore, inducing the landowner to treat infested trees is too costly for the local government, and it outweighs the benefit. In the next period, the remaining healthy trees would be infested with π^h. Therefore, the local government offers a reimbursement that covers not only the inspection cost but also part of the treatment cost in the second period.

2.3.2 Numerical Insights

We illustrate the impact of key input parameters, such as the second-period attack rates (π^h and π^l), and the treatment effectiveness (ρ) on the optimal solution. Four scenarios are created by varying (1) the low second-period attack rate (π^l) between high and low or medium and (2) the treatment effectiveness (ρ) between very effective and not very effective (less or somewhat effective). Additionally, in each scenario, we pick one or two values of π^h to show the relationship between the optimal solution and the key input parameters.

2.3.2.1 Scenario 1: The low second-period attack rate is low or medium, and the treatment is not very effective.

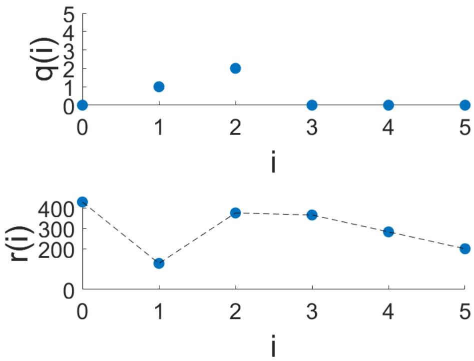
We present two examples under the scenario where the low second-period attack rate is low or medium ($\pi^l < \ddot{\pi}^l$), and the treatment is not very effective ($\rho < \ddot{\rho}$) by varying the value of the high second-period attack rate (π^h). The other values used in the numerical examples are summarized in Table 2.9.

Table 2.9 Parameters Used under Scenario 1 ($\pi^l < \ddot{\pi}^l$ and $\rho < \ddot{\rho}$)

n	α	β	c	θ	s	ρ	π	π^l	$\ddot{\pi}^l$	$\ddot{\rho}$	a_1	b_1
5	40	294	500	50	100	0.20	0.29	0.25	0.40	0.53	184	111

Example 1: The high second-period attack rate (π^h) is low for some infestation levels and medium for others. Figure 2.5 illustrates the optimal solution when $\pi^h = 0.45$. It is optimal to treat the infested trees (I) only when the infestation level is low ($1 \leq i \leq 2$) and not to treat any trees (N) otherwise. We use the notation of $N_0^0 I_1^2 N_3^5$ to represent the optimal treatment decisions (OTDs), where the subscript after each action (N or I) is the starting infestation level and the superscript is the ending infestation level. N_3^5 means not treating any trees when the infestation level is between 3 and 5.

To better understand the optimal solution, we examine the values provided in the table in Figure 2.5. First, when no trees are infested ($i = 0$), the optimal decision



i	$\dot{\pi}^h(i)$	$\ddot{\pi}^h(i)$	π^h Level	$q^*(i)$	$r^*(i)$
0	-	-	-	0	\$430
1	0.31	0.51	medium	1	\$128
2	0.42	0.68	medium	2	\$376
3	0.63	1	low	0	\$366
4	1	1	low	0	\$283
5	-	-	-	0	\$200

Figure 2.5 The optimal solution under Scenario 1 when $\pi^h = 0.45$. Plot (left): The optimal treatment decisions vs. infestation level (top) and the reimbursement vs. infestation level (bottom). Table (right): Categorization of π^h, the optimal treatment decision, and the reimbursement for each infestation level.

is not to treat any trees in the first period (N). Because all trees are healthy and the low second-period attack rate (π^l) is either low or medium, it is unnecessary for the landowner to treat any trees. However, the local government offers a reimbursement of $r^*(0) = \alpha n + a_1\pi^l n = \$430$, as the analytical solution shown in Table 2.5. The reimbursement is much higher than the inspection cost ($\alpha n = \$200$). Because the treatment is not very effective ($\rho < \ddot{\rho}$ or $a_1 > 0$), the local government needs to offer a high enough incentive for the landowner to treat newly infested trees in the second period.

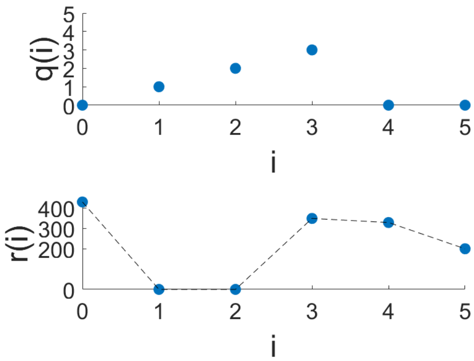
Next, we examine the solution when some (but not all) trees are infested ($0 < i < n$). As appeared in Figure 2.5, $\dot{\pi}^h(1) = 0.31 < \pi^h = 0.45 < \ddot{\pi}^h(1) = 0.51$ and $\dot{\pi}^h(2) = 0.42 < \pi^h = 0.45 < \ddot{\pi}^h(2) = 0.68$. Therefore, π^h is medium when i is 1 or 2. Table 2.8 shows that it is optimal to treat only the infested trees ($q^*(i) = i$) and the reimbursement is $r^*(i) = \alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta + c)(\pi^h - \pi^l)](n - i)$, which increases in i. On the other hand, π^h is low when i is 3 or 4 because $\pi^h = 0.45 < \dot{\pi}^h(3) = 0.63 < \dot{\pi}^h(4) = 1$. As per Table 2.8, not treating any trees ($q^*(i) = 0$) is optimal and the reimbursement, $r^*(i) = \alpha n + a_1\pi^h(n - i)$, decreases in i. As a result, neither the treatment decision nor the reimbursement is monotonic in the infestation level (i).

Last, when all trees are already infested ($i = n = 5$), the local government only reimburses the landowner the inspection cost. Even though the landowner is expected to remove all trees because treatment is not very effective, none of the removal cost is substituted by the local government.

In summary, a higher reimbursement is required when the local government wants to induce the landowner to deviate from his best action when no reimbursement is offered. In this example, the landowner receives the highest reimbursement when there are no trees infested in the first period because he is encouraged to treat all newly infested trees in the second period even though the treatment is not effective. This decision would not be optimal for him if the reimbursement is not so high. On the other hand, when there is only one tree infested in the first period, the landowner receives very little financial award because his best action would be to treat the infested tree to prevent a much higher chance ($\pi^h = 0.45$) of having newly infested trees in the second period.

Example 2: The high second-period attack rate (π^h) can be low, medium, or high. As illustrated by Figure 2.6, the optimal treatment decision when $\pi^h = 0.70$ is very similar to that in the previous example. That is, it is optimal to treat only the infested trees (I) when the infestation level (i) is low (between 1 and 3) while not treating any trees (N) otherwise (at zero or above 3). $N_0^0 I_1^3 N_4^5$ thus representing the OTDs. Differently from the previous example, the landowner will treat the infested trees without any reimbursement from the local government when the infestation level is sufficiently low ($i = 1$ or 2).

We focus our discussion on the reimbursement when some trees are infested ($0 < i < n$) because the optimal solution when none (or all) of the trees are infested is the same as the previous example. Because $\ddot{\pi}^h(1) = 0.51 < \ddot{\pi}^h(2) = 0.68 < \pi^h = 0.70$, π^h is high when the infestation level is low ($i = 1, 2$). As the analytical solution shown in Table 2.8, the optimal treatment decision is to treat the infested trees ($q^*(i) = i$),



i	$\dot{\pi}^h(i)$	$\ddot{\pi}^h(i)$	π^h Level	$q^*(i)$	$r^*(i)$
0	-	-	-	0	\$430
1	0.31	0.51	high	1	\$0
2	0.42	0.68	high	2	\$0
3	0.63	1	medium	3	\$349
4	1	1	low	0	\$329
5	-	-	-	0	\$200

Figure 2.6 The optimal solution under Scenario 1 when $\pi^h = 0.70$. Plot (left): The optimal treatment decisions vs. infestation level (top) and the reimbursement vs. infestation level (bottom). Table (right): Categorization of π^h, the optimal treatment decision, and the reimbursement for each infestation level.

and the optimal reimbursement is zero ($r^*(i) = 0$). Given that the majority of the trees are healthy ($i \leq 2$) in the first period and not treating the infested trees will result in a high chance (70%) of any healthy tree becoming infested in the second period, the landowner must treat the infested trees. As a result, no financial incentives are needed from the local government.

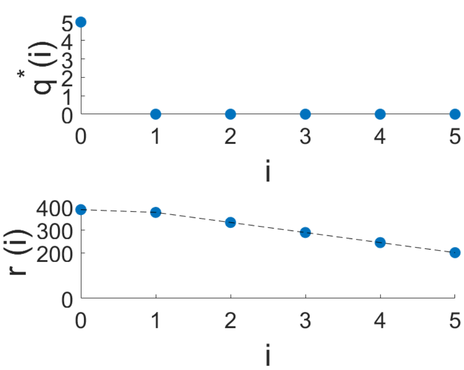
The optimal solution changes when the majority of the trees are already infested in the first period ($i \geq 3$). When the infestation level reaches 3, π^h is medium (since $\dot{\pi}^h(3) = 0.63 < \pi^h = 0.70 < \ddot{\pi}^h(3) = 1$), the local government will need to provide a substantial reimbursement, higher than inspection cost, to induce the landowner to treat the infested trees. When the infestation level is at or above 4, the landowner would not treat any trees due to the treatment being not very effective. However, the local government still needs to offer a reimbursement higher than or equal to the inspection cost to induce the landowner to participate.

2.3.2.2 Scenario 2: The low second-period attack rate is high, and the treatment is not very effective. Regarding the scenario where the low second-period attack rate π^l is high ($\pi^l \geq \ddot{\pi}^l$), and the treatment is not very effective ($\rho < \ddot{\rho}$), we present the optimal solution under two values of π^h. Table 2.10 provides the values used in this scenario. It is worth mentioning that even though π^l is high, it is still lower than the first-period attack rate ($\pi^l < \pi$).

Table 2.10 Parameters Used under Scenario 2 ($\pi^l \geq \ddot{\pi}^l$ and $\rho < \ddot{\rho}$)

n	α	β	c	θ	s	ρ	π	π^l	$\ddot{\pi}^l$	$\ddot{\rho}$	a_1	b_1
5	40	294	900	50	100	0.15	0.28	0.27	0.27	0.31	152	-14

Example 1: The high second-period attack rate (π^h) is low for all levels of infestation. The optimal solution when $\pi^h = 0.29$ is shown in Figure 2.7. Treating all trees (A) is optimal only when no trees are infested ($i = 0$). Otherwise, it is best not to treat any trees (N). The OTDs are thus $A_0^0 N_1^5$. Given that π^h is low and π^l is high, they are not sufficiently different. Therefore, whether or not to treat the infested trees in the first period has very little effect on the possibility of healthy trees becoming infested in the second period. Further, because the treatment is not very effective, the treatment cost (and eventual removal cost when it fails) outweighs the potential benefit of treating the infested trees. The only exception would be when no trees are infested, as treating all trees allow them to be EAB resistant in the second period.



i	$\dot{\pi}^h(i)$	$\ddot{\pi}^h(i)$	π^h Level	$q^*(i)$	$r^*(i)$
0	-	-	-	5	\$388
1	0.30	0.40	low	0	\$376
2	0.36	0.49	low	0	\$332
3	0.47	0.65	low	0	\$288
4	0.82	1.00	low	0	\$244
5	-	-	-	0	\$200

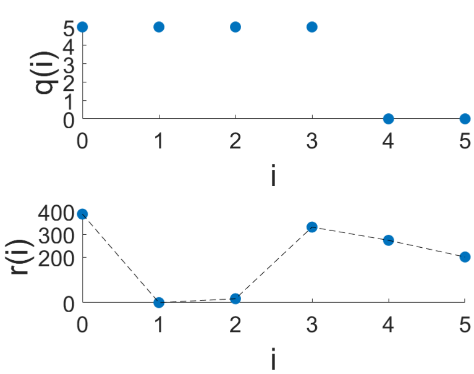
Figure 2.7 The optimal solution under Scenario 2 when $\pi^h = 0.29$. Plot (left): The optimal treatment decisions vs. infestation level (top) and the reimbursement vs. infestation level (bottom). Table (right): Categorization of π^h, the optimal treatment decision, and the reimbursement for each infestation level.

As the analytical solution presented in Table 2.5, the optimal treatment decision and the reimbursement depend only on π^l. In the case of $\pi^l \geq \ddot{\pi}^l$, the optimal solution is $q^*(0) = n = 5$ and $r^*(0) = \alpha n + [\beta - \pi^l(\theta + c)]n = \$388$. When some trees are infested ($0 < i < n$), the reimbursement, as per Table 2.8, is $r^*(i) = \alpha n + a_1 \pi^h (n - i)$. Because a_1 is positive, the reimbursement decreases in i, as illustrated in Figure 2.7.

Example 2: The high second-period attack rate (π^h) can be low, medium, or high. As shown in Figure 2.8, the OTDs are $A_0^3 N_4^5$ when $\pi^h = 0.48$. That is, treat all trees (A) so long as the number of infested trees is not large while not treating any trees (N) otherwise. In this example, π^l is high and very close to π. Further, π^h is much higher than π^l.

Treating all trees means some infested trees may survive while all healthy trees would become EAB resistant in the second period. The potential benefit from treating all trees far exceeds the sum of treatment cost and the expected removal cost when the number of infested trees is small. Therefore, the reimbursement is zero ($r^*(1) = r^*(2) = 0$). When $i = 3$, however, a positive reimbursement is needed from the local government to balance the benefit from treating all trees and the expected costs. As per the analytical solution in Table 2.8, $r^*(i) = \alpha n + a_1 n - [a_1 - \beta + (\theta + c)\pi^h](n - i) > 0$.

When the number of infested trees is high ($i \geq 4$), treating them is no longer beneficial because the treatment is not very effective, and there are almost no healthy trees to protect.

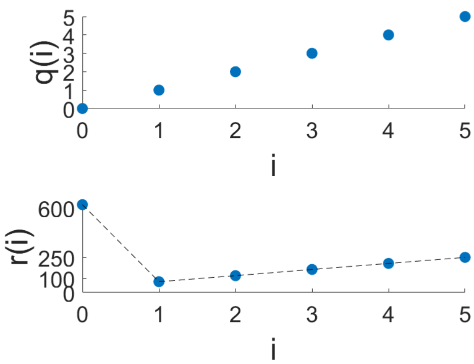


i	$\dot{\pi}^h(i)$	$\ddot{\pi}^h(i)$	π^h Level	$q^*(i)$	$r^*(i)$
0	-	-	-	5	\$388
1	0.30	0.40	high	5	\$0
2	0.36	0.49	medium	5	\$17
3	0.47	0.65	medium	5	\$331
4	0.82	1.00	low	0	\$273
5	-	-	-	0	\$200

Figure 2.8 The optimal solution under Scenario 3 when $\pi^h = 0.48$. Plot (left): The optimal treatment decisions vs. infestation level (top) and the reimbursement vs. infestation level (bottom). Table (right): Categorization of π^h, the optimal treatment decision, and the reimbursement for each infestation level.

2.3.2.3 Scenario 3: The low second-period attack rate is low or medium, and the treatment is very effective.

Figure 2.9 presents the optimal solution for the numerical example using the values listed in Table 2.11. The optimal treatment is to treat only infested trees (I) regardless of the infestation level (i). The OTDs are thus $N_0^0 I_1^4 A_5^5$. This result is primarily due to two factors. First, because the treatment is very effective, the infested trees that are treated have a high rate of survival. Therefore treating the infested trees is a viable and economic strategy. Second, since π^l is much lower than $\ddot{\pi}^l$ (the cutoff value for the low second-period attack rate to be considered high), the likelihood of a healthy tree becoming infested in the second period is considered to be low or medium. Consequently, there is no need to treat any of the healthy trees in the first period.



i	$\ddot{\pi}^h(i)$	π^h Level	$q^*(i)$	$r^*(i)$
0	-	-	0	\$625
1	0.42	low	1	\$77
2	0.45	low	2	\$120
3	0.50	low	3	\$164
4	0.66	low	4	\$207
5	-	-	5	\$250

Figure 2.9 The optimal solution under Scenario 3 when $\pi^h = 0.40$. Plot (left): The optimal treatment decisions vs. infestation level (top) and the reimbursement vs. infestation level (bottom). Table (right): Categorization of π^h, the optimal treatment decision, and the reimbursement for each infestation level.

Because the treatment is very effective ($\rho \geq \ddot{\rho}$), $a_1 < 0$. The reimbursement when no (resp. all) trees are infested, as per Table 2.5 (resp. Table 2.6), is $r^*(0) = \max\{0, \alpha n + a_1 \pi^l n\} = \$625$ (resp. $r^*(n) = \max\{0, \alpha n + a_1 n\} = \$250$) and less than the inspection cost ($\alpha n = \$750$). This means that the local government only needs to cover a portion of the inspection cost in order for the landowner to participate.

Further, since $\pi^h = 0.40 < \ddot{\pi}^h(1) = 0.42 < \cdots < \ddot{\pi}^h(4) = 0.66$, π^h is low for all infestation levels. As the analytical solution shown in Table 2.7, the reimbursement

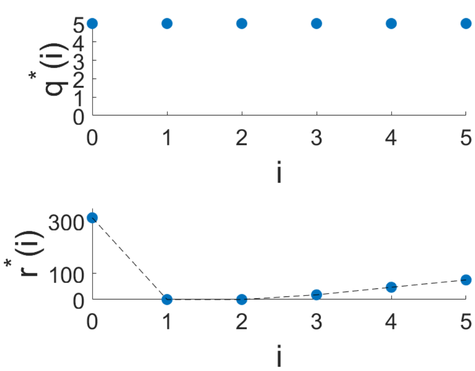
Table 2.11 Parameters Used under Scenario 3 ($\pi^l < \ddot{\pi}^l$ and $\rho \geq \ddot{\rho}$).

n	α	β	c	θ	s	ρ	π	π^l	π^h	$\ddot{\pi}^l$	$\ddot{\rho}$	a_1	b_1
5	150	294	738	50	100	0.50	0.30	0.25	0.40	0.43	0.37	-100	122

when some trees are infested is $r^*(i) = \alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta + c)(\pi^h - \pi^l)](n - i)$, which is also less than the inspection cost as $a_1 < 0$. As the number of infested trees increases, however, $r^*(i)$ increases, and thus, the local government needs to offer a slightly higher reimbursement to induce the landowner to participate. An increasing pattern in reimbursement is thus observed in Figure 2.9.

2.3.2.4 Scenario 4: The low second-period attack rate is high, and the treatment is very effective.

We present the optimal solution using the numerical values summarized in Table 2.12. As shown in Figure 2.10, it is optimal to treat all trees (A) regardless of the infestation level, making A_0^5 the OTDs. In this example, $\pi^l > \ddot{\pi}^l$ (the cutoff value above which the low second-period is considered high), and thus, the landowner reduces the possibility of any healthy trees from getting infested in the second period to zero by treating all of them in the first period.



i	$\ddot{\pi}^h(i)$	π^h Level	$q^*(i)$	$r^*(i)$
0	-	-	5	\$313
1	0.32	high	5	\$0
2	0.33	high	5	\$0
3	0.34	low	5	\$18
4	0.38	low	5	\$47
5	-	-	5	\$75

Figure 2.10 The optimal solution under Scenario 4 when $\pi^h = 0.33$. Plot (left): The optimal treatment decisions vs. infestation level (top) and the reimbursement vs. infestation level (bottom). Table (right): Categorization of π^h, the optimal treatment decision, and the reimbursement for each infestation level.

As per the analytical solution presented in Tables 2.5 and 2.6, the reimbursement when no trees are infested and when all trees are infested are $r^*(0) = \max\{0, \alpha n +$

$(\beta - (\theta + c)\pi^l)n\} = \$313$ and $r^*(n) = \max\{0, \alpha n + a_1 n\} = \$75$, respectively. Both of them are less than the inspection cost, $\alpha n = \$500$.

Table 2.12 Parameters Used under Scenario 4 ($\pi^l \geq \ddot{\pi}^l$ and $\rho \geq \ddot{\rho}$)

n	α	β	c	θ	s	ρ	π	π^l	π^h	$\ddot{\pi}^l$	$\ddot{\rho}$	a_1	b_1
5	100	200	900	50	100	0.30	0.30	0.25	0.33	0.23	0.21	-85	-16

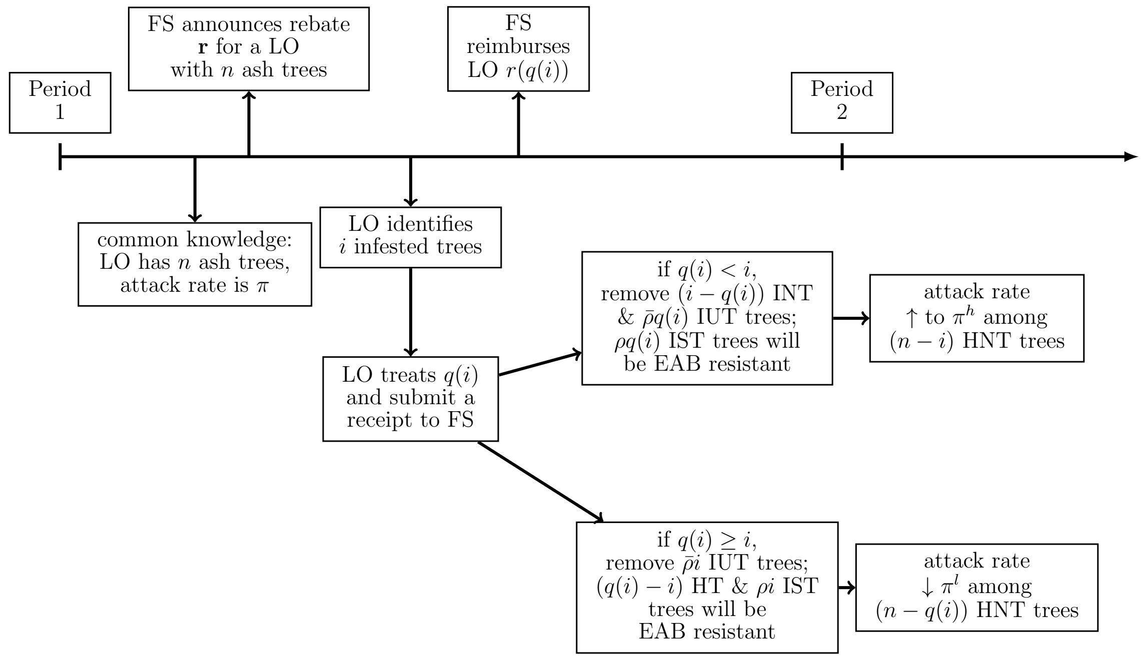
When $i = 1$ or 2, π^h is high since $\pi^h = 0.33 \geq \ddot{\pi}^h(2) = 0.33 > \ddot{\pi}^h(1) = 0.32$. Therefore, the reimbursement is $r^*(1) = r^*(2) = 0$ as per Table 2.7. Because the treatment is very effective, treating the infested trees reduces the chance of treated infested trees from dying. Further, for the healthy trees that are treated, they become EAB resistant in the second period. The benefit from surviving trees greatly exceeds the cost of treatment, and thus the local government does not provide any financial award to the landowner. However, when the infestation level is higher ($i = 3, 4$), π^h is low as $\pi^h = 0.33 < \ddot{\pi}^h(3) = 0.34 < \ddot{\pi}^h(4) = 0.38$, the reimbursement is $r^*(i) = \alpha n + a_1 i + \beta(n-i) - (\theta + c)\pi^h(n-i)$. This implies that the local government will need to provide a positive, albeit very small, reimbursement to induce the landowner to treat all trees.

2.4 The Treatment-based Reimbursement Model

In this section, we discuss the treatment-based reimbursement (TBR) model, where the number of treatment trees (q) is verifiable. This can be achieved if the local government requires the submission of a service receipt issued by a professional tree care service who provided the EAB treatment. Differently from the previous model, the infestation level (i) is assumed to be not verifiable. Moreover, the reimbursement is given to the landowner after treatment, not prior to.

Sequence of events. The interactions between the local government and the landowner over two consecutive periods if a landowner participates in the cost-sharing program are illustrated in Figure 2.11. At the start of the first period, both parties have the same knowledge about the ash trees: the landowner has n ash trees, and the attack rate is π. The local government announces a reimbursement schedule, $\mathbf{r} = [r(0), r(1), \cdots, r(n)]$, that prescribes the reimbursement corresponds to the number of treated trees (q). As an example, $r(0)$ is the reimbursement if no trees are treated ($q = 0$). After inspection, the landowner finds out the infestation level (i); however, he does not report this information to the local government. Instead, he decides the number of trees to be treated ($q(i)$) based on the infestation level (i) and the reimbursement schedule ($\mathbf{r}$). After treatment, the landowner submits the service receipt that shows the number of treated trees to the local government. He then receives the reimbursement according to the prescribed schedule. If some infested trees are not treated in the first period ($q(i) < i$), the consequence of his decision over the next two periods is depicted in Figure 2.1. Otherwise, it follows Figure 2.2. Similar to the assumptions made in the previous model, the reimbursement is only available in the first period. Therefore, a landowner who wishes to participate must sign-up before the infestation level is revealed. Further, for simplicity, we assume that the landowner will treat all newly infested trees in the second period if he participates in the program.

Landowner's expected utility when he does *not* participate in the program. Because the first-period attack rate is π, each tree is assumed to be infested equally likely. If no trees are infested ($i = 0$) in the first period, the attack rate in the second period decreases to π^l. The expected number of trees infested in period two is $\pi^l n$. Consequently, these trees would die, and the landowner would incur a removal cost of $c\pi^l n$. The expected number of surviving trees at the end of the second period is therefore $\bar{\pi}^l n$. The landowner's utility is the difference between the value of surviving trees and the cost of removal. Since $\bar{\pi}^n$ is the probability of zero trees being infested in period one, the weighted expected utility if no trees are infested is $\bar{\pi}^n \cdot (\theta\bar{\pi}^l n - c\pi^l n)$. On the other hand, if all trees are infested ($i = n$), the landowner incurs a removal cost of cn when all trees are infested. π^n is the probability



LO stands for landowner, FS for local government, INT for infested trees that are not treated, IST for infested trees that are successfully-treated, IUT for infested trees that are unsuccessfully-treated, HT for healthy trees that are treated, and HNT for healthy trees that are not treated.

Figure 2.11 Sequence of events for the treatment-based reimbursement model.

that all trees are infested, and therefore the landowner's weighted expected utility is $\pi^n \cdot (-cn)$.

If the number of infested trees is i $(0 < i < n)$ in period one, the attack rate is increased to π^h in the absence of any treatment. The death toll is the sum of the infested trees (i) in the first period and the expected number of trees that will be infested $(\pi^h(n-i))$ in the second period. The number of surviving trees at the end of period two is $\bar{\pi}^h(n-i)$. Because the probability of i trees being infested in the first period is $\binom{n}{i}\pi^i\bar{\pi}^{n-i}$, the landowner's weighted expected utility is therefore $\binom{n}{i}\pi^i\bar{\pi}^{n-i} \cdot \Big(\theta\bar{\pi}^h(n-i) - c(\pi^h(n-i)+i)\Big)$.

The landowner's expected utility, $\Phi(a_0|n)$, can be computed as follows:

$$\begin{aligned}\Phi(a_0|n) = & \sum_0^n \tbinom{n}{i}\pi^i\bar{\pi}^{n-i} \cdot \phi(a_0|n,i) \\ = & \bar{\pi}^n \cdot \Big(\theta\bar{\pi}^l n - c\pi^l n\Big) + \sum_{i=1}^{n-1} \tbinom{n}{i}\pi^i\bar{\pi}^{n-i} \cdot \Big(\theta\bar{\pi}^h(n-i) - c(\pi^h(n-i)+i)\Big) + \pi^n \cdot \Big(-cn\Big).\end{aligned} \tag{2.6}$$

Landowner's expected utility when he participates in the program. If the landowner does not treat all infested trees ($q(i) < i$), his expected utility would be

$$\begin{aligned}\phi\Big(q(i), r(q(i))|n, i, q(i) < i\Big) = \;& \theta \cdot [\rho q(i) + \bar{\pi}^h(n-i) + \rho\pi^h(n-i)] + r(q(i)) \\ & -\alpha \cdot n - \beta \cdot [q(i) + \pi^h(n-i)] - c \cdot [i - \rho q(i) + \bar{\rho}\pi^h(n-i)].\end{aligned} \tag{2.7}$$

The terms in Equation (2.7) are similar to those in Equation (2.2) except that $r(i)$ has been replaced by $r(q(i))$. Similarly, the landowner's expected utility is

$$\begin{aligned}\phi\Big(q(i), r(q(i))|n, i, q(i) \geq i\Big) = \;& \theta \cdot [q(i) - \bar{\rho}i + \bar{\pi}^l(n - q(i)) + \rho\pi^l(n - q(i))] + r(q(i)) \\ & -\alpha \cdot n - \beta \cdot [q(i) + \pi^l(n - q(i))] - c \cdot [\bar{\rho}i + \bar{\rho}\pi^l(n - q(i))].\end{aligned} \tag{2.8}$$

if he treats not only all infested trees but also some healthy trees ($q(i) \geq i$). Recall that μ_i is defined as an indicator such that $\mu_i = 1$ when $q(i) < i$ and $\mu_i = 0$ otherwise. $\bar{\mu}_i = 1 - \mu_i$. The landowner's expected utility from participating in the cost-sharing program is thus

$$\Phi(\mathbf{q}, \mathbf{r}|n) = \sum_0^n \binom{n}{i} \pi^i \bar{\pi}^{n-i} \cdot \phi\Big(q(i), r(q(i))|n, i\Big), \tag{2.9}$$

where

$$\phi\Big(q(i), r(q(i))|n, i\Big) = \mu_i \cdot \phi\Big(q(i), r(q(i))|n, i, q(i) < i\Big) + \bar{\mu}_i \cdot \phi\Big(q(i), r(q(i))|n, i, q(i) \geq i\Big). \tag{2.10}$$

Government's optimization problem. The local government's objective is to maximize her net expected utility, which is the difference between her value from the surviving trees and the reimbursement provided to the landowner. We can write the problem of the local government as follows:

$$
\max_{\mathbf{q},\mathbf{r}} \Psi(\mathbf{q},\mathbf{r}|n) = \sum_{i=0}^{n} \binom{n}{i} \pi^i \bar{\pi}^{n-i} \cdot
\begin{pmatrix}
\mu_i \cdot \Big(s \cdot [\rho q(i) + (\bar{\pi}^h + \rho\pi^h)(n-i)] - \beta \cdot [i - \rho q(i) + \bar{\rho}\pi^h(n-i)] \Big) \\
+\bar{\mu}_i \cdot \Big(s \cdot [q(i) - \bar{\rho} i + (\bar{\pi}^l + \rho\pi^l)(n - q(i))] - \beta \cdot [\bar{\rho} i + \bar{\rho}\pi^l(n - q(i))] \Big) \\
-r(q(i))
\end{pmatrix}
$$

$$
\text{s.t.} \quad
\begin{array}{lll}
\Phi(\mathbf{q},\mathbf{r}|n) \geq \Phi(a_0|n) & & \text{(IR)} \\
\phi(q(i)|n,i) \geq \phi(j|n,i) & \forall\, 0 \leq i \leq n, \quad 0 \leq j \leq n & (\text{IC}_{ij}) \\
r(q(i)) \geq r(q(i)-1) & \forall\, 1 \leq q(i) \leq n & (\text{MON}_i) \\
q(i), r(q(i)) \geq 0 & \forall\, 0 \leq i \leq n & (\text{NN}_i)
\end{array}
\tag{2.11}
$$

The objective function in Equation (2.11) is the local government's net expected utility given a set of treatment decisions ($\mathbf{q}$) and a reimbursement schedule ($\mathbf{r}$). The individual rationality (IR) constraint encourages the landowner to participate in the cost-sharing program by ensuring the landowner's expected utility is higher if he participates. The incentive compatibility (IC_{ij}) constraints induce the landowner to pick the treatment decision ($q(i)$) desired by the local government. The monotonic (MON_i) constraints ensure the reimbursement is non-decreasing in the number of treated trees. Finally, both the number of treated trees and the reimbursement are restricted to be non-negative (NN_i).

2.4.1 Optimal Solutions

Some parameters that were defined in the previous model, such as $\dot{\rho}, \ddot{\rho}, \dot{\pi}^l$, and $\ddot{\pi}^l$, are also used in this model. We introduce a few new parameters to characterize the optimal solutions in addition to the former notation. All of them are used to characterize the optimal solutions. First, let $\hat{\pi}^h := \max\{0, \min\{1, \pi^l + \frac{a_1 - b_1}{\bar{\rho}(\theta+c)+\beta} \cdot (n-1)\}\}$, which represents a cutoff value that classifies π^h.

Definition 4. *As shown in Table 2.13, the high second-period attack rate (π^h) is considered* low *if* $0 \leq \pi^h < \hat{\pi}^h$ *and* high *if* $\hat{\pi}^h \leq \pi^h \leq 1$.

Table 2.13 Categorization of the High Second-period Attack Rate (π^h)

Condition	π^h
$0 \leq \pi^h < \hat{\pi}^h$	low
$\hat{\pi}^h \leq \pi^h \leq 1$	high

Next, let $\hat{\pi}^l(\rho) := \max\{\dot{\pi}^l, \frac{\rho(\theta+c)}{\bar{\rho}(\theta+c)+\beta}\}$ when the low second-period attack rate (π^l) is medium ($\dot{\pi}^l \leq \pi^l < \ddot{\pi}^l$), and the treatment ($\rho$) is somewhat effective ($\dot{\rho} \leq \rho < \ddot{\rho}$). $\hat{\pi}^l(\rho)$ is less than $\ddot{\pi}^l$ because $\ddot{\pi}^l - \frac{\rho(\theta+c)}{\bar{\rho}(\theta+c)+\beta} = \frac{\beta-\rho(\theta+c)}{\bar{\rho}(\theta+c)+\beta} > 0$ and $\ddot{\pi}^l > \dot{\pi}^l$. We use $\hat{\pi}^l(\rho)$ to further classify π^l.

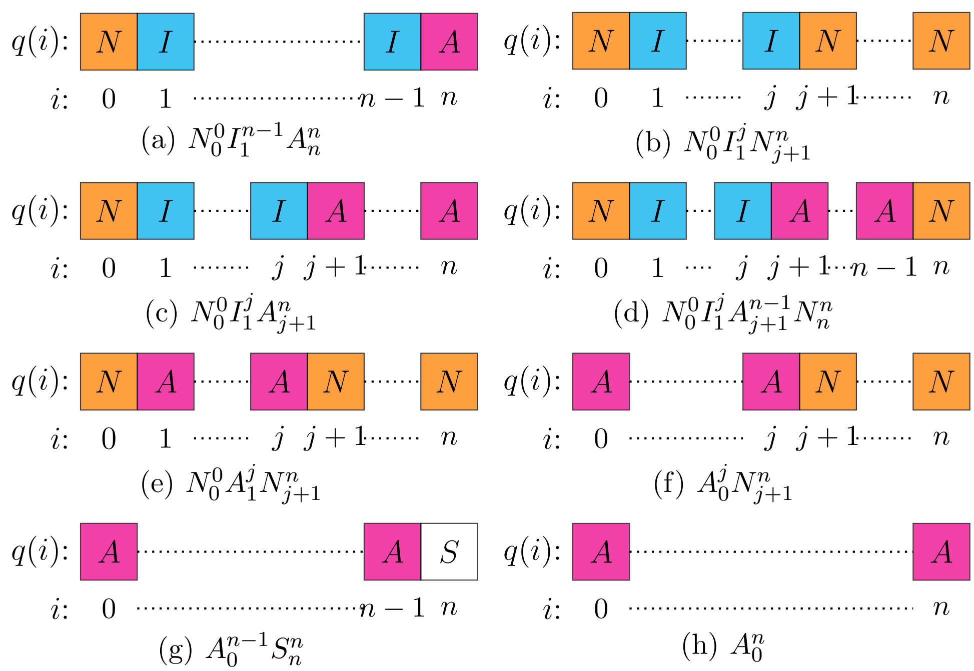
Definition 5. *The low second-period attack rate (π^l) is considered as* medium-low *if* $\dot{\pi}^l \leq \pi^l < \hat{\pi}^l(\rho)$ *and* medium-high *if* $\hat{\pi}^l(\rho) \leq \pi^l < \ddot{\pi}^l$, *as illustrated by Table 2.14.*

Table 2.14 Categorization of the Low Second-period Attack Rate (π^l)

Condition	π^l
$\dot{\pi}^l \leq \pi^l \leq \hat{\pi}^l(\rho)$	medium-low
$\hat{\pi}^l(\rho) < \pi^l < \ddot{\pi}^l$	medium-high

2.4.1.1 Optimal treatment decisions. As illustrated in Figure 2.12, we identify eight sets of optimal treatment decisions (OTD). First, let $N_0^0 I_1^{n-1} A_n^n$ denote not treating any trees (N) when no trees are infested, treating only infested trees (I) when the infestation level (i) in the first period is between 1 and $n-1$, and treating all trees (A) when all are infested. Second, $N_0^0 I_1^j N_{j+1}^n$ represents not treating any trees when none are infested, treating only infested trees (I) when the infestation level (i) is between 1 and j, and not treating any trees (N) when the infestation level is between $j+1$ and n. Similarly, $N_0^0 I_1^j A_{j+1}^n$ depicts not treating any trees when none are infested, treating all infested trees (I) when the infestation level is at or below j and treating all trees (A) when more than j trees are infested. $N_0^0 I_1^j A_{j+1}^{n-1} N_n^n$ represent the most complex set of decisions where only infested trees are treated (I)

if the number of trees being infested is between 1 and j, all trees are treated (A) if the infestation level is higher than j, but none is treated if either no trees or all trees are infested. $N_0^0 A_1^j N_{j+1}^n$ denotes treating all trees (A) when the infestation level is between 1 and j while not treating any trees otherwise ($i = 0$ or $j + 1 \leq i \leq n$). $A_0^j N_{j+1}^n$ deviates slightly from $N_0^0 A_1^j N_{j+1}^n$ in that all trees are treated even when zero trees are infested. $A_0^{n-1} S_n^n$ denotes treating all trees (A) so long as not of them are already infested ($i < n$), but treating some trees (S) when all of them are already infested. Last, A_0^n represents treating all trees regardless of the infestation level.

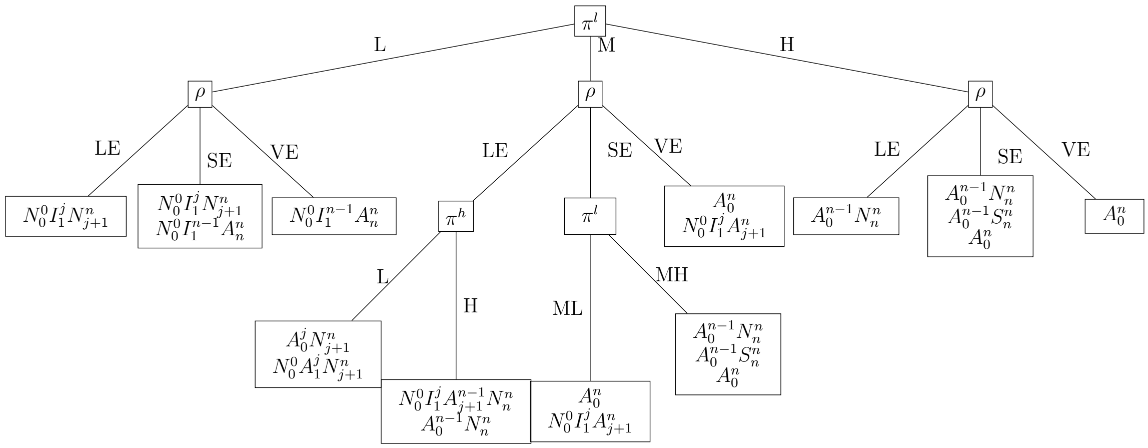


(a) $N_0^0 I_1^{n-1} A_n^n$ (b) $N_0^0 I_1^j N_{j+1}^n$ (c) $N_0^0 I_1^j A_{j+1}^n$ (d) $N_0^0 I_1^j A_{j+1}^{n-1} N_n^n$ (e) $N_0^0 A_1^j N_{j+1}^n$ (f) $A_0^j N_{j+1}^n$ (g) $A_0^{n-1} S_n^n$ (h) A_0^n

Figure 2.12 Eight sets of optimal treatment decisions (OTD).

Figure 2.13 shows when each set of OTDs can be optimal. At the top level, the tree is divided into three branches based on the value of π^l, the low second-period attack rate.

First, let us discuss the left branch of the tree diagram as the results are fairly intuitive. When π^l is low, either $N_0^0 I_1^{n-1} A_n^n$ or $N_0^0 I_1^j N_{j+1}^n$ can be optimal. A commonality between the two sets is to treat only the infested trees (I) when the infestation level is at or below a cutoff value (j) in the first period. This is primarily



L, M and H stand for π^l is low, medium and high, respectively. ML and MH stand for π^l is medium-low and medium-high, respectively. LE, SE and VE stand for the treatment is less effective, somewhat effective, and very effective, respectively.

Figure 2.13 Optimal Treatment Decisions vs. Key Parameters.

because π^l is low, and thus the chance of a healthy tree, not treated in the first period, becoming infested in the second period is low. The difference between the two sets of treatment decisions is what to do when the infestation level is above j. If the treatment (ρ) is very effective, $N_0^0 I_1^{n-1} A_n^n$ is optimal. The landowner treats only infested trees in the first period regardless of the infestation level. On the other hand, if the treatment is less effective, very few infested and treated trees will survive. Therefore, the landowner would not treat any trees in the first period, and thus, $N_0^0 I_1^j N_{j+1}^n$ is optimal. Unfortunately, the remaining healthy tree(s) will face a higher attack rate (π^h) in the second period. When the treatment is somewhat effective, either $N_0^0 I_1^{n-1} A_n^n$ or $N_0^0 I_1^j N_{j+1}^n$ can be optimal. Therefore, a numerical enumeration is needed in order to find the OTDs.

Next, we examine the OTDs when π^l is high. As shown by the right branch of the tree diagram in Figure 2.13, either one of the following sets can be optimal: A_0^n, $A_0^{n-1} S_n^n$, $A_0^{n-1} N_n^n$. All three sets of decisions agree to treat all trees in the first period, except when all trees are infested, to avoid the realization of a high π^l in the second period. When all trees are already infested in the first period, however, the decision depends on the treatment effectiveness. If the treatment is very effective, then treating all trees is optimal because the chance of survival is high. On the contrary, if the treatment is not effective, then all trees should be removed. Interestingly, when the treatment is somewhat effective, it can be optimal to treat some (but not all) trees.

The OTDs when π^l is medium, as shown in the middle branch of the tree diagram in Figure 2.13, are driven by the treatment effectiveness. If the treatment is very effective, either A_0^n or $N_0^0 I_1^j A_{j+1}^n$ can be optimal. Both sets of decisions agree to treat all trees if the infestation level is beyond some threshold j. Below that value, the landowner may either treat only infested trees or treat all. If the treatment is somewhat effective and π^l is medium-low, then either A_0^n or $N_0^0 I_1^j A_{j+1}^n$ is optimal, which is the same as that when the treatment is very effective. On the other hand, if the treatment is somewhat effective and π^l is medium-high, then the set of optimal decisions is the same as when π^l is high and the treatment is somewhat effective, which is to treat all trees except when all trees are already infested. If the treatment is less effective, then the value of the high second-period attack rate (π^h) determines the optimal treatment decisions. Specifically, when π^h is low, either $N_0^0 A_1^j N_{j+1}^n$ or $A_0^j N_{j+1}^n$ is optimal. The landowner would treat all trees if the infestation level is low, while not treating any trees if the infestation is high. This result is driven by the fact that the treatment is less effective and therefore should be used mainly for the prevention of trees becoming infested in the next period. When π^h is high, either $A_0^{n-1} N_n^n$ or $N_0^0 I_1^j A_{j+1}^{n-1} N_n^n$ is optimal. Both OTDs agree on treating all trees if the infestation level surpasses a cutoff value (j), so long as not all of them are already infested. Again, the main purpose of treating all trees is to prevent healthy trees in the first period from becoming infested in the second.

2.4.2 Numerical Insights

In this section, we further examine the impact of the second-period attack rates (π^h and π^l) and the treatment effectiveness (ρ) on the optimal solution. Three scenarios are created by varying π^l from low to high to medium.

2.4.2.1 Scenario 1: The low second-period attack rate is low. We present two examples when the low second-period attack rate is low ($\pi^l < \dot{\pi}^l$) by varying the treatment effectiveness (ρ). The treatment is less effective in the first example while it is more effective in the second. The other input parameters and calculated cutoff values used in the two examples are listed in Table 2.15.

Table 2.15 Input Parameters Used and Calculated Cutoffs When π^l is Low

Input Parameters									Calculated Cutoffs		
n	α	β	c	θ	s	π	π^l	π^h	$\dot{\rho}$	$\ddot{\rho}$	$\dot{\pi}^l$
5	40	294	738	50	100	0.30	0.20	0.4	0.25	0.37	0.24

Example 1: The treatment is less effective ($\rho = 0.20 < \dot{\rho} = 0.25$). The optimal solution is shown in Figure 2.14. The table on the left presents the menu of reimbursement based on the number of treated trees (q). The local government announces this menu at the beginning of period one. The bottom plot on the right illustrates the OTDs of the landowner are based on the infestation level (i) while the top plot shows the corresponding reimbursement he would receive. The landowner would only treat the infested trees (I) when the infestation level is at or below 3 while not treating any trees (N) when the infestation level is at 0 or above 3. Since π^l is low, treating infested trees when the infestation level is low is beneficial to the landowner because the attack rate in the second period is decreased to 0.20. However, there is little benefit in treating the infested trees when most trees are already infested, and the treatment is less effective. Therefore, the OTDs in this example are $N_0^0 I_1^3 N_4^5$.

We observe that the reimbursement is a step function of the number of the treated trees (q) in the first period: the landowner receives \$64 for treating less than

q	$r^*(q)$
0	\$64
1	\$64
2	\$64
3	\$103
4	\$103
5	\$103

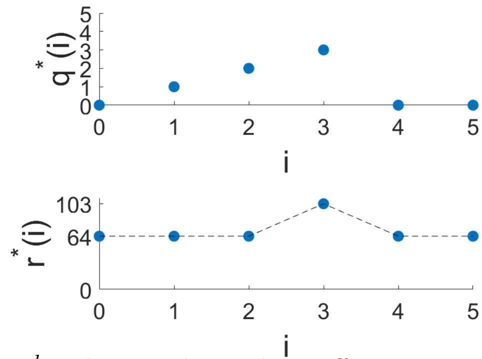


Figure 2.14 The optimal solution when π^l is low and ρ is less effective. Table (left): The optimal reimbursement schedule. Plot (right): The OTDs (top) and the corresponding reimbursement (bottom).

three trees while receiving \$103 for treatment more than that. Both reimbursement amounts are much lower than the inspection cost (αn = \$200). Further, the reimbursement the landowner would receive is non-monotonic in the infestation level (i). In this example, the landowner only gets the higher reimbursement when the infestation level is three. He would receive a smaller reimbursement otherwise.

Example 2: The treatment is very effective ($\rho = 0.40 > \ddot{\rho} = 0.37$). Figure 2.15 illustrates the optimal solution. The landowner would treat all infested trees regardless of the infestation level. This result is quite intuitive because the benefit from treating the infested trees sufficiently outweighs the cost. Moreover, since π^l is low, there is no need to treat healthy trees in the first period to prevent more trees from becoming infested in the second period. The OTDs are thus $N_0^0 I_1^5 A_5^5$. Interestingly, the reimbursement is zero for any number of treated trees, which suggests that treating infested trees is a superior option for the landowner, and thus, the local government does not need to provide any financial award.

2.4.2.2 Scenario 2: The low second-period attack rate is high. When π^l is high, the optimal treatment decision is to treat all trees (A) so long as not all of them are infested in the first period. When all trees are infested, however, the optimal treatment decision depends on the treatment effectiveness. To illustrate

q	$r^*(q)$
0	\$0
1	\$0
2	\$0
3	\$0
4	\$0
5	\$0

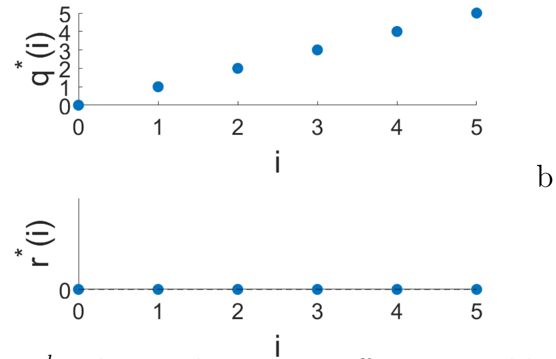


Figure 2.15 The optimal solution when π^l is low and ρ is very effective. Table (left): The optimal reimbursement schedule. Plot (right): The OTDs (top) and the corresponding reimbursement (bottom).

this relationship, we provide two examples where the treatment is somewhat effective ($\rho = 0.30$) in one and very effective ($\rho = 0.38$) in another. The other input parameters used and calculated cutoff values in these numerical examples are listed in Table 2.16.

Table 2.16 Input Parameters Used and Cutoff Values When π^l is High and ρ is Somewhat Effective

Input Parameters									Calculated Cutoffs	
n	α	β	c	θ	s	π	π^l	π^h	$\dot{\rho}$	$\ddot{\rho}$
5	40	294	738	50	100	0.40	0.38	0.50	0.25	0.37

Example 1: The treatment is somewhat effective ($\dot{\rho} = 0.25 < \rho = 0.30 < \ddot{\rho} = 0.37$). The optimal solution is presented in Figure 2.16. The OTDs are $A_0^4 S_5^5$, where $S = 1$. When all trees are infested, the landowner would only treat one tree. The reason for this decision is largely due to the reimbursement schedule, which is presented in the left table of Figure 2.16. In order to induce the landowner to treat all trees when not all of them are infested in the first period, the local government sets the reimbursement to a lower value (\$58) for treating any number of trees between 1 and 4 while offering a much higher compensation (\$102) for treating 5 trees. Given that the treatment is only somewhat effective, the landowner would not want to treat all. Therefore, he would choose to treat the minimum number of trees ($q^*(5) = 1$) to get the lower reimbursement.

q	$r^*(q)$
0	\$0
1	\$58
2	\$58
3	\$58
4	\$58
5	\$102

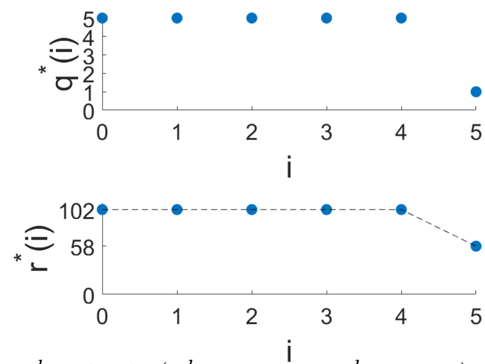


Figure 2.16 The optimal solution when π^l is high ($\pi^l = 0.38 > \ddot{\pi}^l = 0.35$) and ρ is somewhat effective. Table (left): The optimal reimbursement schedule. Plot (right): The OTDs (top) and the corresponding reimbursement (bottom).

Example 2: The treatment is very effective ($\ddot{\rho} = 0.37 < \rho = 0.38$). As shown in Figure 2.17, the OTDs are A_0^5. Because the treatment is very effective, treating all trees even when all of them are already infested in the first period is beneficial to the landowner. Further, the local government does not need to provide any reimbursement.

q	$r^*(q)$
0	\$0
1	\$0
2	\$0
3	\$0
4	\$0
5	\$0

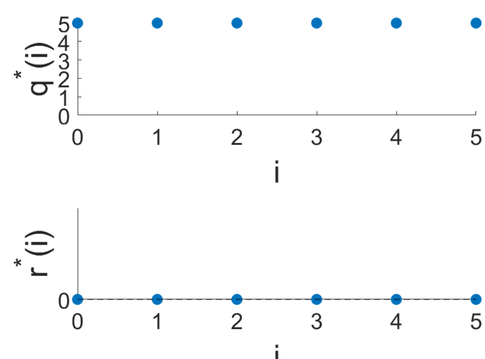


Figure 2.17 The optimal solution when π^l is high ($\pi^l = 0.38 \geq \ddot{\pi}^l = 0.38$) and ρ is very effective. Table (left): The optimal reimbursement schedule. Plot (right): The OTDs (top) and the corresponding reimbursement (bottom).

2.4.2.3 Scenario 3: The low second-period attack rate is medium.

As discussed in Section 2.4.1.1, when π^l is medium ($\dot{\pi}^l \leq \pi^l < \ddot{\pi}^l$), there are two key factors that determine the optimal treatment decisions. First, when the treatment is less effective, the value of the high second-period attack rate (π^h) determines which set of treatment decisions among A_0^j, $N_0^0 A_1^j N_{j+1}^n$, and $N_0^0 I_1^j A_{j+1}^{n-1} N_n^n$ is optimal. Second, when the treatment is somewhat effective, the relationship between π^l and ρ decides what would the set of optimal treatment decisions look like. An example of each aforementioned scenario is presented next.

2.4.2.3.1 *The effect of π^h on the OTDs when the treatment is less effective.*

We present three examples to illustrate the impact of π^h on the optimal solution by varying its value among 0.35, 0.50 and 0.75. Table 2.17 lists the other parameters and calculated cutoffs used in all three examples.

Table 2.17 Parameters Used When π^l is Medium and ρ is Less Effective

Input Parameters									Calculated Cutoffs				
n	α	β	c	θ	s	ρ	π	π^l	$\dot{\rho}$	$\ddot{\rho}$	$\dot{\pi}^l$	$\ddot{\pi}^l$	$\hat{\pi}^h$
5	40	294	738	50	100	0.20	0.30	0.25	0.25	0.38	0.24	0.32	0.57

Example 1: Figure 2.18 shows the optimal solution when π^h is 0.35, which is considered low since $\pi^h < \hat{\pi}^h$. The OTDs are $A_0^3 N_4^5$. Because π^l is medium, the landowner would prefer to avoid the chance of healthy trees getting infested in the second period. Therefore, he would choose to treat all trees if the infestation level in the first period is at or below 3. However, when the infestation level is above 3, he would not treat any trees because the treatment is less effective. To encourage the landowner to treat all trees, the local government sets a generous reimbursement of \$534 for treating all trees. The reimbursement for any other treatment decision, however, is only a token amount of \$18.

Example 2: As shown in Figure 2.19, the set of optimal treatment decisions, $N_0^0 A_1^3 N_4^5$, when $\pi^h = 0.50$ varies slightly from that when $\pi^h = 0.35$. The landowner

q	$r^*(q)$
0	\$18
1	\$18
2	\$18
3	\$18
4	\$18
5	\$534

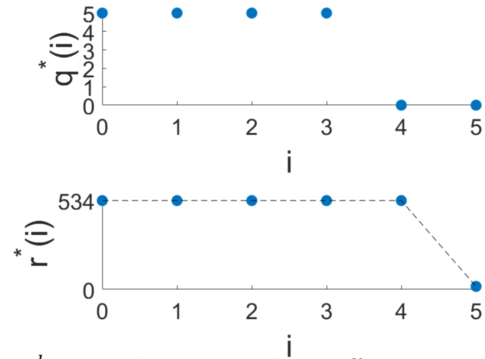


Figure 2.18 The optimal solution when π^l is medium, ρ is less effective, and $\pi^h = 0.35$. The set of optimal treatment decisions is $A_0^3 N_4^5$.Table (left): The optimal reimbursement schedule. Plot (right): The OTDs (top) and the corresponding reimbursement (bottom).

would not treat any trees when no trees have been infested in the first period. This is because the local government provides a much lower financial award (\$252) for treating all trees. His other treatment decisions when there is at least one infested tree, however, are the same as those when $\pi^h = 0.35$ since not treating all trees would lead to a much higher attack rate in the second period.

q	$r^*(q)$
0	\$0
1	\$0
2	\$0
3	\$0
4	\$0
5	\$252

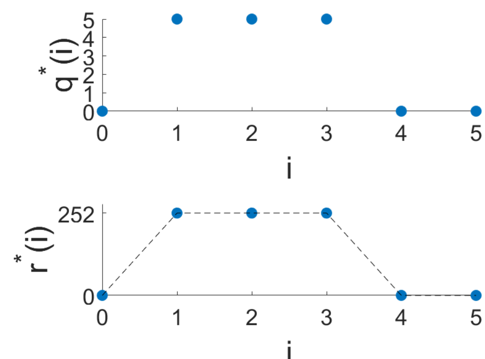


Figure 2.19 The optimal solution when π^l is medium, ρ is less effective, and $\pi^h = 0.50$. The set of optimal treatment decisions is $N_0^0 A_1^3 N_4^5$. Table (left): The optimal reimbursement schedule. Plot (right): The OTDs (top) and the corresponding reimbursement (bottom).

Example 3: When π^h is 0.75, it is considered high because $\pi^h \geq \hat{\pi}^h$. As per Figure 2.20, Even though the reimbursement scheme is quite similar to that when $\pi^h = 0.50$, the OTDs are $I_0^1 A_2^4 N_5^5$ instead. Because the reimbursement for treating all trees is only \$189, the landowner would not treat all trees when the infestation level is 0 or 1. Instead, he would treat only infested trees and take the risk of newly infested trees with a medium attack rate ($\pi^l = 0.25$). However, when the infestation level is between 2 and 4, treating all trees can prevent healthy trees from becoming infested in the second period. When all trees are infested, treating them is not optimal since the treatment is less effective.

q	$r^*(q)$
0	\$0
1	\$0
2	\$0
3	\$0
4	\$0
5	\$189

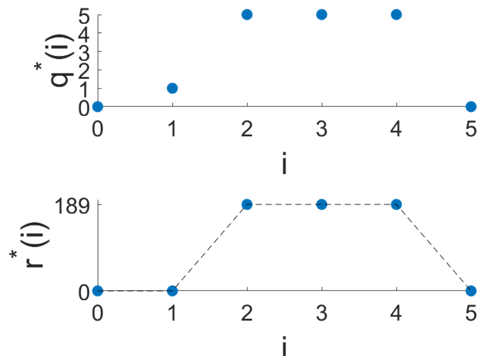


Figure 2.20 The optimal solution when π^l is medium, ρ is less effective, and $\pi^h = 0.75$. Table (left): The optimal reimbursement schedule. Plot (right): The OTDs (top) and the corresponding reimbursement (bottom).

2.4.2.3.2 ***The effect of the relationship between π^l and ρ on the OTDs when the treatment is somewhat effective.*** Recall that π^l is considered medium-low if it is less than $\hat{\pi}^l(\rho)$ and medium-high otherwise (see Definition 4). We present a numerical example for each case. The other parameters and calculated cutoffs are shown in Table 2.18.

Table 2.18 Input Parameters Used and Calculated Cutoffs When π^l is Medium and ρ is Somewhat Effective

Input Parameters									Calculated Cutoffs				
n	α	β	c	θ	s	ρ	π	π^h	$\dot{\rho}$	$\ddot{\rho}$	$\dot{\pi}^l$	$\ddot{\pi}^l$	$\hat{\pi}^l(\rho)$
5	40	200	900	50	100	0.20	0.30	0.60	0.16	0.21	0.17	0.21	0.20

Example 1: The OTDs when π^l is 0.17, as shown in Figure 2.21, are $N_0^0 I_1^2 A_3^5$. The local government does not provide any financial award unless all trees are treated. This result is intuitive since π^l is medium-low, and the chance of healthy trees becoming newly infested in the second period is fairly low. Therefore, only infested trees need to be treated in the first period so long as the infestation level is sufficiently

low (below 3). However, if the infestation level is already high (at or above 3), the landowner is better off treating all of them as the treatment is somewhat effective, and this is the only option for the landowner to receive partial reimbursement.

q	$r^*(q)$
0	\$0
1	\$0
2	\$0
3	\$0
4	\$0
5	\$74

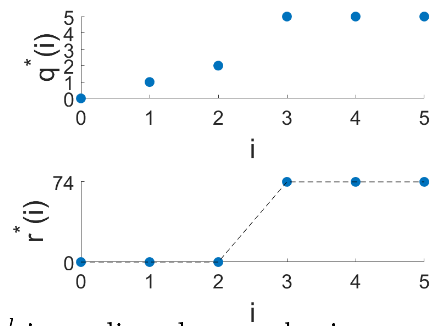


Figure 2.21 The optimal solution when π^l is medium-low, and ρ is somewhat effective. Table (left): The optimal reimbursement schedule. Plot (right): The OTDs (top) and the corresponding reimbursement (bottom).

Example 2: Figure 2.22 shows that when π^l is 0.20, the OTDs are $A_0^4 N_5^5$. The local government only sets a positive reimbursement when all trees are treated. As a result, the landowner treats all trees so long as not all of them are already infested in the first period. This allows the healthy trees to be EAB resistant in the second period. However, if all trees are already infested, given that the treatment is somewhat effective, the landowner would not treat any trees.

q	$r^*(q)$
0	\$0
1	\$0
2	\$0
3	\$0
4	\$0
5	\$40

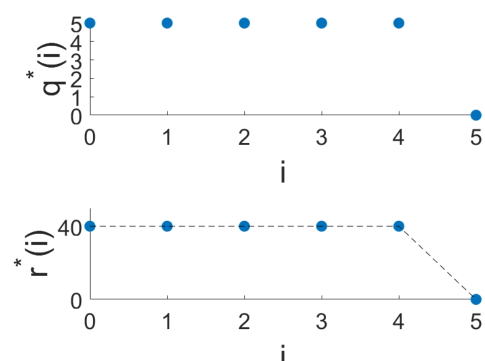


Figure 2.22 The optimal solution when π^l is medium-high and ρ is somewhat effective. Table (left): The optimal reimbursement schedule. Plot (right): The OTDs (top) and the corresponding reimbursement (bottom).

2.5 Model Comparisons and Managerial Insights

In this section, we first compare the efficacy of the two cost-sharing models (IBR in Section 2.3 and TBR in Section 2.4) in several metrics. We then summarize the managerial insights provided from the analytical solutions and the numerical results of the two models, as well as from the model comparisons.

2.5.1 Model Comparisons

In addition to the IBR and TBR models, we also consider a third model in which the local government does not offer a cost-sharing (or NCS) program. We analyze four scenarios created by varying the treatment effectiveness between very effective and not very effective and the low second-period attack rate between low, medium and high.

2.5.1.1 Scenario 1: The low second-period attack rate is low or medium, and the treatment is not very effective. Table 2.19 presents the expected number of surviving trees (E_{trees}), the expected reimbursement (E_r), the expected objective function value (E_{obj}) of the local government, and the OTDs of the three models. Under NCS, the local government does not offer any financial award and therefore, E_r is zero. The measure E_{trees} is computed as the expected number of surviving trees, assuming that the landowner would not take any actions in mitigating the negative impact of EAB. That is, the landowner's OTDs are N_0^5. The E_{obj} is computed as the expected value of $\Psi(q(i), r(i)|n, i)$, the objective function value in Equation (2.5) where both $q(i)$ and $r(i)$ are set to zero, over i. Similarly, E_r in IBR is the expected value of $r^*(i)$ over i while E_{obj} is the expected value of $\Psi(q^*(i), r^*(i)|n, i)$ over i. In TBR, E_r is the expected value of $r^*(q(i))$ over i and E_{obj} is $\Psi(\mathbf{q}^*, \mathbf{r}^*|n)$, the objective function value of Equation (2.11).

In this scenario, π^l is medium-low ($\dot{\pi}^l = 0.24 = \hat{\pi}^l(\rho) = 0.24 < \pi^l = 0.25 < \ddot{\pi}^l = 0.32$), ρ is less effective ($\rho = 0.20 < \dot{\rho} = 0.25$), and π^h varies from low ($\pi^h = 0.35$ or

Table 2.19 Comparisons under Scenario 1 ($\pi^l < \ddot{\pi}^l$ and $\rho < \ddot{\rho}$)

	$\pi^h = 0.35$				$\pi^h = 0.50$				$\pi^h = 0.70$			
	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD
IBR	2.97	270	-622	$N_0^0 I_1^2 N_3^5$	3.07	108	-420	$N_0^0 I_1^3 N_4^5$	3.07	71	-386	$N_0^0 I_1^3 N_4^5$
TBR	3.77	518	-504	$A_0^3 N_4^5$	3.60	202	-329	$N_0^0 A_1^3 N_4^5$	3.34	90	-244	$N_0^0 I_1^1 A_2^4 N_5^5$
NCS	2.36	0	-411	N_0^5	1.96	0	-537	N_0^5	1.43	0	-705	N_0^5

Note: Other parameters used: $n = 5$, $\alpha = 40$, $\beta = 294$, $c = 738$, $\theta = 50$, $s = 100$, $\rho = 0.20$, $\pi = 0.30$, $\pi^l = 0.25$. Calculated cutoffs: $\dot{\rho} = 0.25$, $\ddot{\rho} = 0.38$, $\dot{\pi}^l = 0.24$, $\ddot{\pi}^l = 0.32$, $\hat{\pi}^l(\rho) = 0.24$, $\hat{\pi}^h = 0.57$.

$0.50 < \hat{\pi}^h = 0.57$) to high ($\pi^h = 0.70 > \hat{\pi}^h = 0.57$). The OTDs are $N_0^0 I_1^j N_{j+1}^n$ under IBR. As π^h increases, the index j increases. This suggests that the infested trees will be treated up to a higher infestation level. While the landowner's OTD vary based on the value of π^h, a commonality among them is that the landowner will treat all (healthy and infested) trees for some infestation levels. As a result, the expected number of surviving trees at the end of the planning horizon is the highest under TBR. Further, despite the higher reimbursement required under TBR, its expected objective function value is higher than that of IBR.

Interestingly, not offering a cost-sharing program may be desirable to the local government even though it always leads to the lowest number of surviving trees. Case in point: When π^h is 0.35, the expected objective function value is the highest under NCS.

2.5.1.2 Scenario 2: The low second-period attack rate is high, and the treatment is not very effective.

We present the numerical results in Table 2.20 when π^l is high ($\pi^l = 0.40 > \ddot{\pi}^l = 0.32$), ρ is somewhat effective ($\dot{\rho} = 0.25 < \rho = 0.30 < \ddot{\rho} = 0.37$), and π^h is low ($\pi^h = 0.42$ or 0.55 or $0.70 < \hat{\pi}^h = 0.78$).

Under IBR, the landowner will treat all trees when the infestation level is below a certain cutoff (j) and not treat any trees beyond the cutoff ($A_0^j N_{j+1}^n$). As π^h increases, the landowner would treat all trees up to a higher infestation level to prevent the high risk of getting newly infested trees in the next period–consequently, j increases.

Table 2.20 Comparisons under Scenario 2 ($\pi^l \geq \ddot{\pi}^l$ and $\rho < \ddot{\rho}$)

	$\pi^h = 0.42$				$\pi^h = 0.55$				$\pi^h = 0.70$			
	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD
IBR	3.13	189	-447	$A_0^2N_3^5$	3.47	55	-164	$A_0^3N_4^5$	3.58	29	-86	$A_0^4N_5^5$
TBR	3.59	215	-269	$A_0^4S_5^5$ (S=3)	3.59	59	-116	$A_0^4S_5^5$ (S=1)	3.58	0	-58	$A_0^4N_5^5$
NCS	1.76	0	-607	N_0^5	1.42	0	-701	N_0^5	1.02	0	-809	N_0^5

Note: Other parameters used: $n = 5$, $\alpha = 40$, $\beta = 294$, $c = 738$, $\theta = 50$, $s = 100$, $\rho = 0.30$, $\pi = 0.40$, $\pi^l = 0.38$. Calculated cutoffs: $\dot{\rho} = 0.25$, $\ddot{\rho} = 0.37$, $\dot{\pi}^l = 0.26$, $\ddot{\pi}^l = 0.35$, $\hat{\pi}^l(\rho) = 0.28$, $\hat{\pi}^h = 0.78$.

The landowner takes a more aggressive approach in TBR. He treats all trees except when all of them are already infested in the first period. As the reimbursement from the local government decreases in π^h, he treats fewer trees when $i = 5$. Both the expected number of surviving trees and the expected objective function value are highest under TBR. This shows that offering reimbursement based on the number of treated trees is superior under this scenario.

2.5.1.3 Scenario 3: The low second-period attack rate is low or medium, and the treatment is very effective. Table 2.21 summarizes the numerical results when π^l is medium ($\pi^l = 0.29 \leq \dot{\pi}^l = 0.29$), the treatment is very effective ($\rho = 0.38 > \ddot{\rho} = 0.37$), and π^h is high ($\pi^h = 0.73$ or 0.53 or $0.33 > \hat{\pi}^h = 0$).

Table 2.21 Comparisons under Scenario 3 ($\pi^l < \ddot{\pi}^l$ and $\rho \geq \ddot{\rho}$)

	$\pi^h = 0.33$				$\pi^h = 0.53$				$\pi^h = 0.73$			
	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD
IBR	3.44	102	-330	$N_0^0I_1^4A_5^5$	3.44	33	-147	$N_0^0I_1^4A_5^5$	3.44	33	-147	$N_0^0I_1^4A_5^5$
TBR	4.07	337	-204	A_0^5	3.44	0	-114	$N_0^0I_1^4A_5^5$	3.44	0	-114	$N_0^0I_1^4A_5^5$
NCS	2.38	0	-329	N_0^5	1.85	0	-459	N_0^5	1.31	0	-589	N_0^5

Note: Other parameters used: $n = 5$, $\alpha = 40$, $\beta = 294$, $c = 738$, $\theta = 50$, $s = 100$, $\rho = 0.38$, $\pi = 0.30$, $\pi^l = 0.29$. Calculated cutoffs: $\dot{\rho} = 0.25$, $\ddot{\rho} = 0.37$, $\dot{\pi}^l = 0.29$, $\ddot{\pi}^l = 0.38$, $\hat{\pi}^l(\rho) = 0.38$, $\hat{\pi}^h = 0$.

When π^h is 0.33, the OTDs are $N_0^0I_1^4A_5^5$ under IBR and A_0^5 under TBR. Even though the local government offers a higher expected reimbursement under TBR, the expected objective function value is higher under TBR as a result of treating all trees and having a high success rate. As π^h gets higher (0.53 or 0.73), the landowner only treats the infested trees ($N_0^0 I_1^4 A_5^5$) under both IBR and TBR. Because the expected reimbursement required under TBR is zero, the objective function value is higher.

2.5.1.4 Scenario 4: The low second-period attack rate is high, and the treatment is very effective.

We present the numerical results in Table 2.22 when π^l is high ($\pi^l = 0.25 > \ddot{\pi}^l = 0.23$), the treatment is very effective ($\rho = 0.30 > \ddot{\rho} = 0.21$), and π^h is high ($\pi^h = 0.70$ or 0.50 or $0.33 > \hat{\pi}^h = 0$).

Table 2.22 Comparisons under Scenario 4 ($\pi^l \geq \ddot{\pi}^l$ and $\rho \geq \ddot{\rho}$)

	$\pi^h = 0.33$				$\pi^h = 0.50$				$\pi^h = 0.70$			
	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD
IBR	3.95	56	129	A_0^5	3.95	53	132	A_0^5	3.95	53	132	A_0^5
TBR	3.95	39	146	A_0^5	3.95	0	185	A_0^5	3.95	0	185	A_0^5
NCS	2.41	0	-178	N_0^5	1.96	0	-273	N_0^5	1.43	0	-385	N_0^5

Note: Other parameters used: $n = 5$, $\alpha = 40$, $\beta = 200$, $c = 900$, $\theta = 50$, $s = 100$, $\rho = 0.30$, $\pi = 0.30$, $\pi^l = 0.25$. Calculated cutoffs: $\dot{\rho} = 0.16$, $\ddot{\rho} = 0.21$, $\dot{\pi}^l = 0.19$, $\ddot{\pi}^l = 0.23$, $\hat{\pi}^l(\rho) = 0.33$, $\hat{\pi}^h = 0$.

The OTD under both IBR and TBR are A_0^5 regardless of the value of π^h. Similar to the results in the previous scenario, the expected number of trees and the expected objective function value are highest under TBR, which attests to the superiority of providing reimbursement based on the number of treated trees over the infestation level.

2.5.2 Managerial Insights

For the treatment-based reimbursement model, we find three optimal decisions based on the treatment effectiveness and the second-period attack rates: treat none of the trees, treat all (healthy and infested) trees, or treat only the infested trees. We provide a complete characterization of the optimal treatment decision and the reimbursement analytically for each infestation level.

First, not treating any of the trees is optimal if one of the following conditions holds: (1) none of the trees have been infested in the first period, and the low second-period attack rate is either low or medium, i.e., the attack rate in the second period would be low or medium if treatment was applied in the first period; (2) all trees have been infested in the first period, and the treatment is not very effective; or (3) some of the trees are infested and the high second-period attack rate is low, i.e., the attack rate in the second period would be low even if no treatments were applied in the first period. Treating only the infested trees is advantageous only when some (but not all) trees are infested, and either of the following is true: (i) the treatment is very effective, and the low second-period attack rate is low or medium, (ii) the treatment is not very effective, the low second-period attack rate is low, and the high second-period attack rate is medium or high. In all other cases, treating all trees is optimal.

Next, the reimbursement is only higher than the inspection cost when some trees are infested in the first period, the treatment is not very effective, and the high second-period attack rate is low. In all other scenarios, the reimbursement is no greater than the inspection cost. When some (but not all) trees are infested in the first period and the high second-period attack rate is high, the local government does not need to provide any reimbursement. The landowner will treat all of the infested trees and may even treat all of the healthy trees. Last, neither the treatment decision nor the reimbursement is monotonic at the infestation level.

For the treatment-based reimbursement model, the optimal treatment decisions vary with key parameters, such as the treatment effectiveness and the second-period attack rates, and a few takeaways are worth mentioning. First, when the infestation level is low (excluding the case where none of the trees are infested), the local government always encourages the landowner to treat, at a minimum, all of the infested trees. Further, the higher the low second-period attack rate is, the more likely he will be incentivized to treat all (healthy and infested) trees. Two, when the treatment is very effective, treating all trees can be optimal regardless of the infestation level. In this case, the reimbursement required to induce the landowner to treat all trees is very small, if not zero. Third, when the treatment is not very effective, and the infestation level in the first period is already high, not treating any trees may be optimal.

When compared to the infestation-based reimbursement model, the treatment-based reimbursement model induces the landowner to treat more trees. Even though the expected reimbursement is higher, the local government achieves a greater objective function value because more ash trees would survive at the end of the planning horizon. Under the scenario where the low second-period attack rate is very low, and the treatment is not very effective, the local government may consider not offering any cost-sharing program because the high expected reimbursement required in the treatment-based reimbursement model leads to a lower objective function value.

2.6 Conclusions

In this work, we develop two cost-sharing models for a local government to induce the participation of a private landowner in mitigating the negative impact of emerald ash borer on ash trees. Both types of information asymmetry are considered. In the infestation-based reimbursement model, the local government encounters a moral hazard problem because the reimbursement, distributed to the landowner prior to treatment, must incentivize the landowner to treat the desired number of trees based on the infestation level post-reimbursement. The local government, on the other hand, deals with an adverse selection problem in the treatment-based reimbursement model. She designs the reimbursement schedule so that the landowner would not treat more than the optimal number of trees in order to claim a higher reimbursement.

For both models, we identify possible optimal treatment decision(s) and the conditions under which each treatment decision can be optimal. We analytically characterize the optimal treatment decision, and the reimbursement in the infestation-based reimbursement model. In the treatment-based reimbursement model, however, numerical results are used to analyze the reimbursement schedule. Additionally, we investigate the efficacy of the cost-sharing programs by comparing the two cost-sharing models to the one where the local government does not offer any cost-sharing program. We conclude that in all scenarios but one, the treatment-based reimbursement model achieves the highest number of surviving trees and the highest objective function value of the local government, even though the reimbursement required to run the program is the highest. The only exception is when the treatment is not effective and both of the second-period attack rates are low.

Our future work would focus on two tasks. First, develop a heuristic to find the optimal solution for the treatment-based reimbursement model. Currently, finding the optimal treatment decisions and the reimbursement schedule requires a complete search with all possible combinations of treatment decisions. Because we have been able to narrow down the structures of the optimal treatment decisions, they can be used to reduce the search time. Second, integrate the game-theoretic models with an optimization model to maximize the number of surviving ash trees in both public and private lands under a limited budget through a public-private partnership. This integrated framework will provide optimal budget allocations for managing public trees and running the cost-sharing programs with landowners that are heterogeneous in the number of ash trees and attack rates.

CHAPTER 3

DATA-DRIVEN INTEGRATED GAME THEORY-OPTIMIZATION FRAMEWORK TO PUBLIC-PRIVATE PARTNERSHIPS FOR EAB MANAGEMENT

3.1 Introduction

Invasive species are plants, animals, or pathogens which are non-native to the ecosystem under consideration, the introduction of which causes or is likely to cause a large economic or environmental harm. For example, Xu et al. [129] show the total economic losses caused by invasive alien species to China were \$14.45 billion. Invading alien species in the United States causes major environmental damages and losses, adding up to almost \$120 billion per year [110]. Kettunen et al. [76] present that the total cost of invasive species in Europe, based on documented costs, is estimated to be at least \$13.1 billion per year and probably over \$21 billion. Emerald ash borer (EAB) is one of the harmful invasive species, first discovered in southeastern Michigan in 2002, is a wood-boring insect native to Asia. EAB larvae feed on the inner bark of ash trees and thus disrupt the tree's ability to transport water and nutrients. Emerald Ash Borer was discovered in New Jersey in May 2014 in Somerset County. As October of 2018, infestations of EAB are found throughout 35 states in the United States and the Canadian provinces of Ontario, Quebec, New Brunswick, Nova Scotia, and Manitoba [52], which has killed tens of millions of ash trees since 2002 in North America and caused hundreds of millions of dollars economic losses.

Due to the pressing need to control the spread of EAB, researchers have developed optimization models to search, treat and remove ash trees [134, 135, 79, 22, 21] or explore the collaboration between the Forest Service and the private landowners as shown in Chapter 2. Unlike former work, this chapter explores the government's budget allocation strategies to optimize surveillance, treatment, and removal decisions on both public and private sites at each period of a multi-period horizon EAB management planning problem.

Most studies on public-private partnerships that use game theory combined with optimization focus on energy sectors. To our knowledge, this chapter is among the first to present an integrated game-optimization framework in public-private partnerships with an application to tackle general natural resources conservation problems, such as managing the harmful impacts of a forest insect infestation on tree genera. Specifically, we present a data-driven integrated game-theory-optimization model to allocate the limited government budget to survey, treat and remove ash trees among public trees and optimally reimburse private landowners for EAB management. We build a multiple regression model to predict the next period attack rate based on the real EAB infestation data collected by [57, 83] to consider the uncertainty of the attack rate. We apply our integrated mathematical model to the case of New Jersey EAB infestation with various public-private tree distribution scenarios on a landscape including 5×5 sites, where each site has a size of 0.1 acres. Our findings provide practical insights into budget allocation among different management options between public and private locations under different budgets and treatment effectiveness rates.

3.2 Literature Review and Key Contributions

This section presents a review of the articles that study game-theoretical models in public service and public-private partnerships, integrated game theory and optimization approaches, and optimization methods for EAB management, and then gives this chaper's key contributions and managerial insights.

3.2.1 Principal-Agent Models in Public Services

The principal-agent (PA) framework has been applied to many fields in public services, such as real options, carbon emission reduction, health-care delivery, and infrastructure design and operation [115, 58, 36, 10, 108]. Chapter 2 builds a PA framework to study the collaboration between the government and private landowners to control the impact of an invasive insect (Emerald Ash borer).

Silaghi and Sarkar [115] present a principal-agent model for public-private partnership investment. They study concession contracts between a private firm and a government in the presence of moral hazard within a real-options framework. The authors address the limitation in inducing the firm to exert the effort of previous work and illustrate the importance of considering the cost of a bailout option. The PA framework also plays a significant role in reducing carbon emissions, as shown in the study of [36]. In their paper, Cai and Singham [36] design the optimal carbon capture and storage system contracts to induce the emitters whose demand is heterogeneous to participate in the carbon capture and storage system. Fuloria and Zenios [58] study a principal-agent framework to address the health-care delivery system with two non-cooperative parties, including a purchaser of medical services and a specialized provider. The authors propose a dynamic principal-agent model to consider the best payment system for the purchaser among the fee-for-service system, the capitation system, and outcomes-adjusted payment system. They find that the outcomes-adjusted payment system can significantly improve patient life expectancy. Auriol and Picard [10] focus on the trade-off between allocative efficiency and the cost of public funds in build-operate-and-transfer concession. When concession candidates do not have any better information, the government always avoids build-operate-and-transfer concessions. However, when information becomes asymmetric, the results depend on the shadow costs of public funds. Paez-Perez and Sanchez-Silva [108] propose a dynamic principal-agent framework for the cooperation between the public entity and the private entity in infrastructure. The authors consider the impact of the infrastructure's natural deterioration on the actions of entities.

3.2.2 Game Theory and Optimization Models

Game theory and optimization methods have been combined to tackle various industry and energy-related problems. Dai and Qiao [50] propose a model to maximize the total profits of wind and conventional power producers from both the energy market and a new bilateral reserve market by using stochastic programming and game theory to generate the optimal bidding strategies. A Nash Equilibrium is obtained by [50] to settle the reserve price between wind and conventional power producers.

Das and Tripathi [51] introduce an adaptive and intelligent energy-efficient routing technique based on the fusion of non-cooperative two-person zero-sum game theory and linear programming. Here, the linear programming technique is used to solve the mixed strategy games of a larger dimension payoff matrix. The comparison between this combined method and the previous methods shows that this new method not only reduces energy consumption but also prolongs the lifetime of the network.

To deal with the transboundary water conflicts between two cities, Zeng et al. [139] integrate cooperative game theory and mathematical programming. The results from Rubinstein bargaining are chosen to allocate the initial water and pollutant discharge rights for all players. Based on the solutions of Rubinstein bargaining, the cooperative game theory model can reallocate the water and pollutant discharge rights between two cities. Further, the numerical solutions show that the reallocation can bring the maximum net benefit.

A consumption scheduling mechanism is proposed by [143] for home and neighborhood area load demand management in the smart grid, based on mixed-integer linear programming and the coordination game. There are two scenarios shown in this mechanism, and the effectiveness of both scenarios are verified by using a simulation. In those two scenarios, the distributed one is solved by using the results of the game theory problem.

Zamarripa et al. [138] present a multi-objective mixed-integer linear programming problem assisted with a game theory (the nonzero-sum game) optimization framework to improve supply chain planning decision-making. The multi-objective problem is solved by the $\epsilon-$constraint method and the game theory framework is used to support the decision-making to handle the uncertainty of the cooperative and competitive scenarios.

3.2.3 Resource Allocation Optimization and EAB Management

Resource allocation is a core problem in many combinatorial optimization problems [126, 109, 18]. The applications of the budget-constrained resource allocation vary from agriculture and energy [46, 45, 73, 47], capital asset management and production planning [30, 34, 90, 61, 32, 33, 23], and healthcare [78, 49, 24, 137, 136]. In invasive species management, spatio-temporal resource allocation problems have often been considered due to the dynamic nature of the problem that occurs over a geographical location [67, 66, 28, 27]. We refer the reader to the review of [29], which discusses the complexities of the spatio-temporal optimization and resource allocation challenges as well as mathematical programming approaches in invasive species management.

Various resource optimization methods have been proposed for EAB management. For example, Yemshanov et al. [134] explore the optimal survey resource allocation to maximize the expected number of transmission pathways covered by survey locations and the expected number of survey locations that have at least one pest introduction. Yemshanov et al. [135] study a multi-day surveillance approach to minimize the expected number of sites with undetected infested trees or minimize the expected number of undetected infested trees. The authors find that the managers prioritize surveying the sites with high host densities and the high risk of infestation to minimize the expected number of undetected infested trees. However, the managers tend to survey a larger area at lower sampling densities to minimize the expected number of sites with undetected infested trees. Kıbış et al. [79] propose a multi-stage stochastic mixed-integer programming model to explore the optimal surveillance, treatment, and removal decisions to control the spread of EAB in Bonneville, Minnesota, USA. The authors show that delaying surveillance is never optimal in almost all infestation and budget scenarios. The presented model in [79] is extended by [22] by considering the uncertainty of infestation levels and the impact of distance on the infestation to study the EAB management in Winnipeg, MB, Canada. Bushaj et al. [21] consider managers' risk aversion, and they find that the managers prefer to remove ash trees at a higher cost instead of treating them as they become more risk-averse. Those papers mentioned above explore the optimal operations and management planning for EAB infestation in public areas. Those papers we mentioned above explore the optimal operations planning of EAB infestation in the public area. Chapter 2 presents a principal-agent framework to design cost-sharing programs for the government to induce a private landowner to act on private lands. There are two cost-sharing programs where the reimbursement is either based on the infestation level or the number of treated trees shown in their paper, and the optimal treatment decisions are among treating none of the trees, treating all trees, and treating only infested trees according to the treatment effectiveness and the second-period attack rate.

3.2.4 Key Contributions and Insights

Despite the recent progress in integrating game theory and mathematical programming with mainly energy applications, studies that use game theory in combination with optimization are limited, especially in public services. Furthermore, we are not aware of any other integrated game-optimization framework in public-private partnerships with an application in general natural resources conservation problems, such as tackling a forest insect infestation.

(a) To our knowledge, this study is among the first to integrate a principal-agent framework with a mixed-integer programming (MIP) model to motivate public-private partnerships and guide the government in its budget allocation for social good.

(b) Game theory and mathematical programming are typically two distinct fields. Unprecedentedly, we have synchronized both MIP and game-theoretical models' assumptions and blended them to tackle the resource allocation problem surrounding the EAB management in public and private sites.

(c) We integrate the solution of the principal-agent model based on Chapter 2 into an MIP to optimize the resource allocation among the public and private sites to survey, treat, and remove ash trees with a limited government budget. The model optimizes the treatment budget for public sites, while deciding on the reimbursement level for tree treatments in private sites. As opposed to Chapter 2's constant attack rate, the attack rate is dynamically updated in our integrated game theory-MIP framework. While the MIP model on the EAB management in public sites is based on those presented in [79] and [22], different than those studies, we formulate non-linearities as a function of the attack rate and former treatment decisions. We linearize all non-linearities that arise due to the changing attack rates with respect to the treatment level by replacing them with equivalent constraints and additional binary variables, resulting in a linear optimization formulation. The treatment and removal cannot exceed the number of infested trees in the mathematical models of [79] and [22]. However, we consider a more general model so that treatment can be applied to healthy trees to make the susceptible trees become EAB resistant.

(d) We consider three types of ash trees in our model: healthy, infested and dead ash trees, compared to four infestation levels of ash trees in [22] to synchronize both game theory and MIP models and incorporate the game theory model's solutions into the optimization formulation. We assume that the surveillance occurs in each period in the public sites, and the infestation realization is computed by infestation dynamics equations in the model. The infestation dynamics equations in our MIP model compute the infestation rate as a function of treatment instead of using high or low survey outcomes with different probabilities. Furthermore, we introduce the success rate of treatment to account for the effectiveness of treatment and surveillance jointly.

(e) Bushaj et al. [22] apply a distance-dependent approach to estimate the constant spread probabilities at four 1-km distances from the central site, and Chapter 2 uses a constant attack rate in the next period to compute the newly infested trees. However, different than both studies, we incorporate the uncertainty in the next period's attack rate in each site in each period by building a multiple regression model. This regression model is integrated into the MIP model to dynamically predict the next period's attack rate depending on the current attack rate and the surrounding infestation based on a real EAB infestation data set. We also use this regression model to predict the high attack rate in the next period in the game theory model instead of using the constant next period's attack rate to get optimal treatment, removal, and reimbursement decisions in private sites. Those decisions are treated as the input in the modified optimization model to compute the optimal decisions for surveying, treating, and removing ash trees from the public sites, and the reimbursement amount for the private sites.

3.3 Integrated Game Theory-Optimization Mathematical Formulation

This section presents notation used throughout the chapter (Section 3.3.1), essential features and assumptions made regarding the mathematical models (Section 3.3.2), data analysis to calibrate the attack rates (Section 3.3.3), and the integration of the game-theoretic and optimization models (Section 3.3.4). Then Section 3.3.5 presents the mathematical formulation for the integrated game theory and optimization model that synergizes all elements described in this section.

3.3.1 Model Notation

This section summarizes all notations used in our mathematical model (Table 3.1).

Table 3.1 Notation

Sets and Indices:	
Γ_u	Set of all public sites.
Γ_r	Set of all private sites.
Γ	Set of all sites; $\Gamma = \Gamma_u \cup \Gamma_r$.
Θ_i	Set of all neighboring sites of site i.
T	Set of time periods, $T = \{1, 2, 3\}$.
i	Index for all sites, $i \in \Gamma$.
j	Index for all neighboring sites of the site i, $j \in \Theta_i$
t	Index for time periods, $t \in T$.
Input Parameters:	
π_i^0	Initial attack rate, the probability of an ash tree is infested with the EAB at the beginning in site $i \in \Gamma$.
N_i^0	Initial ash trees population in site $i \in \Gamma$.
I_i^0	Initial number of infested ash trees in site $i \in \Gamma$.
D_i^0	Initial number of dead ash trees in site $i \in \Gamma$.
δ^t	Discount factor at time t which is equal to $\frac{1}{(1+\tau)^t}$, where τ is a discount rate.
θ_D	Penalty value of each dead tree.
α	Marginal value of a healthy tree.
η	Rate of reduction of the attack rate in the previous period, $\eta \in [0, 1)$.
c_1	Cost of surveying a tree.
c_2	Cost of treating a tree.
c_3	Cost of removing a tree.
ψ	Total budget available throughout the planning horizon.
r_i	Reimbursement assigned to the private site $i \in \Gamma_r$.
ρ	Success rate of treating an EAB infested trees, $\bar{\rho} = 1 - \rho$.
$q_{S,i}$	Treated healthy trees in private site $i \in \Gamma_r$.
$q_{I,i}$	Treated infested trees in private site $i \in \Gamma_r$.
ω_0	Regression weight for the previous attack rate on the current high attack rate.
ω_I	Regression weight for the ratio of neighbors' infested trees to all trees in previous period on the current high attack rate.
n_i	Number of neighboring sites of site i.
Decision variables:	
N_i^t	Total number of ash trees at site $i \in \Gamma$, at period t.
S_i^t	Number of susceptible ash trees at site $i \in \Gamma$, at period t.
I_i^t	Number of infested trees at site $i \in \Gamma$, at period t.
$\dot{D}_i^t$	Number of cumulative dead trees at site $i \in \Gamma$, at period t.
D_i^t	Number of newly dead trees at site $i \in \Gamma$, at period t.
$\tilde{H}_i^t$	Number of treated healthy trees at period t in site $i \in \Gamma$.
$\tilde{I}_i^t$	Number of treated infested trees at period t in site $i \in \Gamma$.

$\hat{I}_i^t$	Number of removed infested trees at period t in site $i \in \Gamma$.
$\hat{D}_i^t$	Number of removed dead trees at period t in site $i \in \Gamma$.
V_i^t	Number of surveyed trees at site $i \in \Gamma$, at period t.
W_i^t	Number of awarded ash trees at site $i \in \Gamma$, at period t. $W_i^t = H_i^t + \rho \cdot \tilde{I}_i^t$.
Binary Decision variables:	
μ_i^t	An indicator parameter such that $\mu_i^t = 1$ when the number of treated and removed infested trees is less than the number of infested trees in site $i \in \Gamma$ and period t; otherwise, $\mu_i^t = 0$ in site $i \in \Gamma$ and period t. $\bar{\mu}_i^t = 1 - \mu_i^t$.
Linearization-related variables:	
x_{i1}^t	Binary variable, $\pi_{h,i}^{t+1} = 1$ if $x_{i1}^t = 0$ and $\pi_{h,i}^t = \omega_0 \cdot \pi_i^t + \omega_I \cdot \frac{\sum_{j \in \Theta_i} (I_j^t - \tilde{I}_j^t - \hat{I}_j^t)}{n_i}$ otherwise.
x_{i2}^t	Binary variable, all susceptible trees in period t are infested in the period $t+1$ if $x_{i2}^t = 1$ and not all susceptible trees in period t are infested in the period $t+1$ otherwise.
A_i^{t+1}	Substitute variable of $\mu_i^t \cdot \pi_{h,i}^{t+1}$
B_i^{t+1}	Substitute variable of $\bar{\mu}_i^t \cdot \pi_{l,i}^{t+1}$
$y_{i,1}^{t+1}$	Substitute variable of $\frac{1}{2}(\pi_i^{t+1} + S_i^t)$
$y_{i,2}^{t+1}$	Substitute variable of $\frac{1}{2}(\pi_i^{t+1} - S_i^t)$
$y_{i,1,k}^{t+1}$	Breakpoints of $(y_{i,1}^{t+1})^2$; $k = 0, 1, 2, ..., m$.
$y_{i,2,k}^{t+1}$	Breakpoints of $(y_{i,2}^{t+1})^2$; $k = 0, 1, 2, ..., m$.
λ_{ink}^{t+1}	Weight of the kth breakpoint (y_{ink}^{t+1}) in the linearization function; $n = 1, 2$ and $k = 0, 1, 2, ..., m$.
$\omega_{i,n,k}^{t+1}$	Binary variable to make at most two adjacent λ's are greater than zero.

3.3.2 Model Features and Assumptions

The transmission of EAB is affected by many factors, such as the uncertain attack rates and the behavior and decisions of the government and private landowners to treat and remove ash trees on their specific lands. Therefore, we incorporate some important features and make assumptions about the possible behavior of the decision-makers in the integrated model formulation.

(a) If the total number of treated and removed infested trees is less than the number of the infested trees in site i in period t, the chance of an untreated healthy tree getting infested in the next period $t+1$ increases from π_i^t to $\pi_{h,i}^{t+1}$ and decreases to $\pi_{l,i}^{t+1}$ otherwise. So the attack rate site i in the period $t+1$, ${\pi_i}^{t+1} := \mu_i^t \pi_{h,i}^{t+1} + \bar{\mu}_i^t \pi_{l,i}^{t+1}$, where μ_i^t is an indicator variable such that $\mu_i^t = 1$

when the total number of treated and removed infested trees is less than the infested trees; otherwise, $\mu_i^t = 0$ in site i in the period t. $\bar{\mu}_i^t = 1 - \mu_i^t$.

(b) High attack rate $\pi_{h,i}^{t+1}$ in period $t+1$ is estimated by using real EAB infestation data, as described in Section 3.3.3. The data was collected by [83] and [57] in Toledo, Ohio, from 2005 to 2011, from 10 different sites, and each site includes 3 to 6 plots, a 400 m^2 circular landscape. We assume the attack rate $\pi_{l,i}^{t+1}$ in the period $t+1$ is represented by a percent of the attack rate in the previous period t, i.e., $\pi_{l,i}^{t+1} = \eta {\pi_i}^t$ and $\eta \in [0, 1)$.

(c) The untreated or unsuccessfully-treated infested trees will be dead in the next period for the infested trees, and the treated infested trees will not spread infestation even though the infested trees may not be treated successfully. On the other hand, the successfully-treated infested trees and treated healthy trees will become EAB resistant in the next period.

(d) The government is willing to treat healthy and infested trees to control the spread of EAB in public sites, and surveillance is applied in public sites in each period. The private landowner also treats healthy and infested trees in their private sites to save more ash trees and reduce associated costs with treating infested and removing dead trees.

(e) The government budget is only used to survey ash trees and treat and remove infested ash trees in public sites, and reimburse the landowner for their surveillance, treatment, and removal in private sites. The total budget is not used for removing the dead trees in public sites because the city has a separate budget for removing dead public trees. However, the private landowner only removes dead trees and does not remove infested trees.

(f) In public sites, the treatment can apply to healthy and infested trees, and removal can only apply to infested trees, as discussed above. We assume that the private landowners can treat healthy and infested trees on their own sites. Furthermore, landowners must remove all dead trees in their own sites.

3.3.3 Data Analysis to Estimate Attack Rates

The attack rate is estimated by using the real EAB infestation data, which was collected from 10 different sites in Maumee Bay State Park in Toledo, Ohio, from 2005 to 2011 [57, 83]. Each site is a forested area with homogeneous tree species and includes three plots where each plot represents a 0.1 acres circular landscape. Data consists of the number of individual ash trees that were tracked in these sites and were not treated or removed. Based on their ash canopy conditions, the ash trees are clustered into three classes (healthy, infested, and dead trees). The latitude and longitude of each plot of each site are also shown in this data [57, 83].

The attack rate is defined as the ratio of the newly infested trees in the current period to the number of susceptible trees of the previous period. To improve the accuracy of the EAB infestation data, we remove the observations that involve a decreasing attack rate because no treatment and removal are applied to those sites. As a result of the data cleansing exercise, a total of 120 observations were extracted and used out of a total of 294 observations. After several computational experiments using different factors, we find the previous average attack rate in each site and the average number of infested trees from the neighboring sites play a significant role in the current period's attack rate. As shown in Table 3.2, the correlation coefficient between attack rate and previous attack rate is 0.8, and the correlation coefficient between attack rate and the average neighboring sites infested trees is 0.6. These results imply a strong linear relationship between attack rate and previous attack rate, and a strong linear relationship between the attack rate and the average neighboring infested trees. What is more, the correlation coefficient between the previous attack rate and the average neighboring infested trees is 0.5, which means the relationship between those two factors is not strong enough, so we ignore the interaction impact between the previous attack rate and the average neighboring infested trees on the attack rate. Here, the neighboring sites refer to areas whose distance from the current site is less than 1 mile.

Table 3.2 Correlation Coefficients between Any Two Factors

	Attack rate	Previous attack rate	Average neighboring infested trees
Attack rate	1.0	0.8	0.6
Previous attack rate	0.8	1.0	0.5
Average neighboring infested trees	0.6	0.5	1.0

A multiple regression model is developed to analyze the relationship between the next period's attack rate and two factors which are mentioned above. Our regression analysis that is run under the 95% confidence level with no constant in Excel has resulted in the following equation:

$$\pi_i^{t+1} = \omega_0 \cdot \pi_i^t + \omega_I \cdot \frac{\sum\limits_{j \in \Theta_i} I_j^t}{n_i}, \tag{3.1}$$

where n_i is the number of neighboring sites of site i. The coefficients of the previous attack rate and the average number of infested trees in the neighboring sites are shown in Table 3.3.

Table 3.3 Coefficient and p-value of Each Variable Used in the Regression Analysis

Regression Input	Coefficients	p-value
Previous attack rate (π_i^t)	$\omega_0 = 1.28$	0.00
Average neighboring infested trees ($\frac{\sum_{j \in \Theta_i} I_j^t}{n_i}$)	$\omega_I = 0.06$	0.00

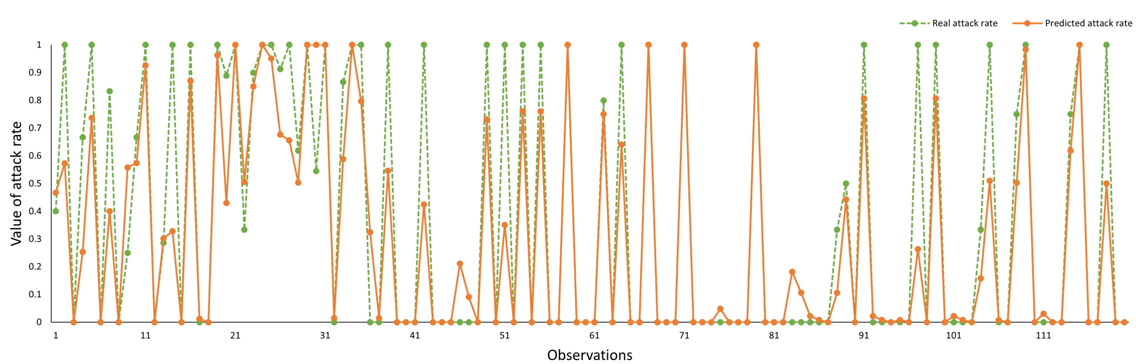


Figure 3.1 Comparison between the real attack rate and the predicted attack rate for 120 observations.

Table 3.3 shows a statistical analysis to compare the predicted and real observations. Here, the p-value of all variables is less than 0.05, and the statistical measure R^2 is 0.854, implying that the performance of the regression model is good. Figure 3.1 displays the value of the real attack rate and the predicated attack rate from the regression model (3.1). As shown in Figure 3.1, the predicted attack rate also visually fits well with the real attack rate in all observations.

3.3.4 Game Theory-Optimization-Data Analysis Integration Schema

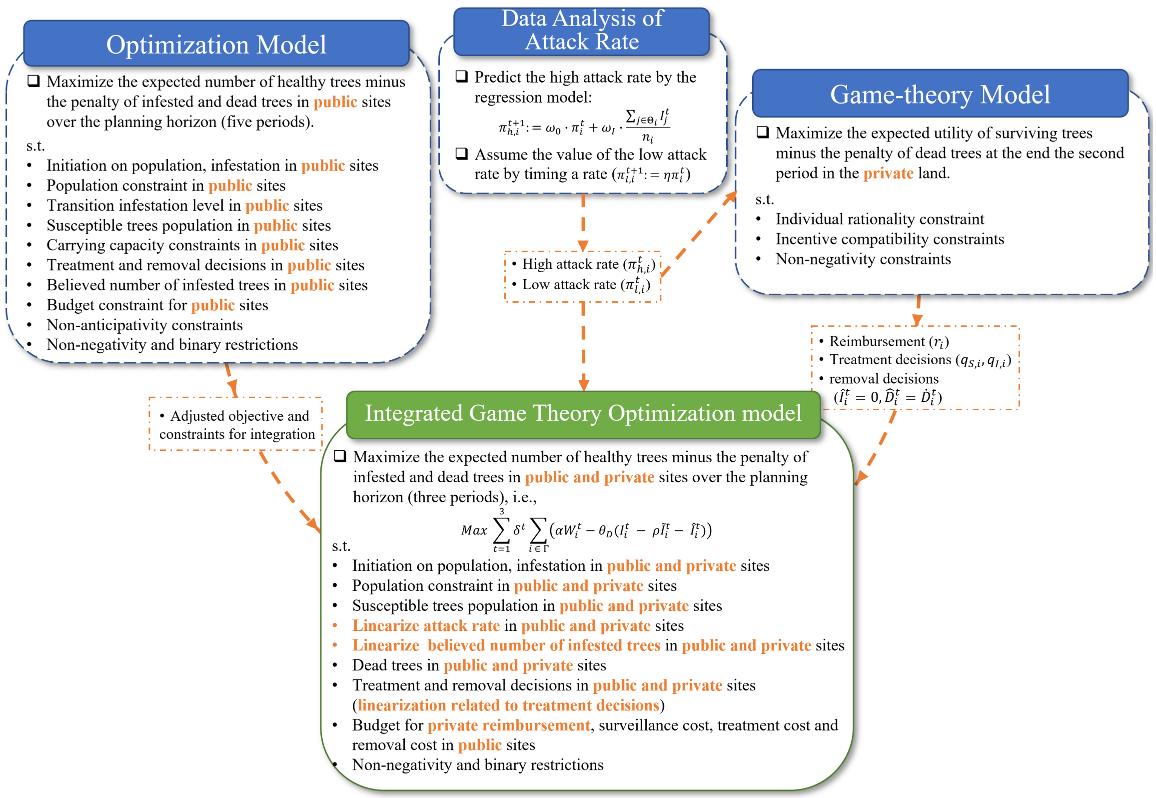


Figure 3.2 Game Theory, Optimization, and Data Analysis Integration Schema.

The optimization model shown on the left side of Figure 3.2 displays a multi-stage stochastic mixed-integer programming optimization model [22] that allocates resources to apply surveillance, treatment, and removal of ash trees over time in public sites. However, the game-theory model (Section A.2.1), shown on the right side of Figure 3.2, presents a principal-agent framework to calculate the optimal reimbursement assigned by the government to induce private landowners to treat and remove ash trees on their lands to prevent the spread of EAB. Our model integrates two models, i.e., the optimization model and the game-theory model, to allocate the resources to both public and private sites.

Bushaj et al. [22] have explored a five-period time horizon, and three classes of infestation asymptomatic, symptomatic, and dead trees. Surveillance can be implemented in each period. However, in our model, all ash trees are divided into three classes, i.e., healthy trees, infested trees, and dead trees, and surveillance is applied in each period over the three-period time horizon. Furthermore, the optimization model calculates the newly expected infested trees by using the impact rate of each infested tree, which is defined as the number of new infestations per infested tree. But the game-theory model defines and uses the attack rate as a given probability of a healthy tree being infested to compute the newly infested trees. To integrate those two models, we need a unified method to count the newly infested trees. After performing data analysis using the actual EAB infestation data mentioned in Section 3.3.3, we estimate the high attack rate by a multiple regression model and the low attack rate by timing the previous attack rate with a rate of reduction η, as shown under the "Data Analysis of Attack Rate" in Figure 3.2. Further, the integration model gets the inputs of the treatment, removal, and reimbursement from the game-theory model with the attack rate calculated by the regression model or timing a rate η.

Based on the results of the game theory model, when there are no infested trees on the private land, both treatment and reimbursement depend on the low attack rate of the next period ($\pi_{l,i}^{t+1}$). If $\pi_{l,i}^{t+1}$ is not large, the landowner has the incentive not to treat any trees ($q_{S,i} = q_{I,i} = 0$). The reimbursement (r_i) decreases the treatment effectiveness (ρ). Further, the reimbursement is no greater than the surveillance cost ($c_1 N_i^0$) if the treatment is very effective; otherwise, it is greater than the surveillance cost. If $\pi_{l,i}^{t+1}$ is large, the private landowner is induced to treat all trees ($q_{S,i} = N_i^0 - I_i^0, q_{I,i} = I_i^0$). The reimbursement ($r_i$) decreases in $\pi_{l,i}^{t+1}$. Furthermore, it is less than the surveillance cost when π^l is very high and greater than the surveillance cost otherwise.

When all trees are already infested in the private land, the treatment effectiveness plays a significant role in both treatment and reimbursement. If the treatment is not very effective, the private landowner is just assigned the surveillance cost as the reimbursement, and the landowner does not treat any trees ($q_{S,i} = q_{I,i} = 0$). On the contrary, if the treatment is very effective, the private landowner is induced to treat all trees ($q_{S,i} = N_i^0 - I_i^0, q_{I,i} = I_i^0$) even if the reimbursement is less than the surveillance cost.

When there are some trees (not all) infested in the private land and the treatment is very effective, the government encourages the private landowner to only treat infested trees ($q_{S,i} = 0, q_{I,i} = I_i^0$) if the next period's low attack rate is not very high and the private landowner is induced to treat all trees ($q_{S,i} = N_i^0 - I_i^0, q_{I,i} = I_i^0$)otherwise. Further, the private landowner can get the reimbursement when the next period's high attack rate is not very high, and no reimbursement otherwise.

When there are some trees (not all) infested in the private land and the treatment is not very effective, the private landowner does not have enough incentive to treat any trees even with a reimbursement which is higher than the surveillance cost when the next period's high attack rate is low. When the next period's high attack rate is not low, the landowner is induced to treat all infested trees ($q_{S,i} = 0, q_{I,i} = I_i^0$) if the next period's low attack rate is not very high; otherwise, the landowner prefers to treat all trees ($q_{S,i} = N_i^0 - I_i^0, q_{I,i} = I_i^0$). The results from the game theory model will be inputted into the integrated model to mimic the surveillance, treatment, and removal decisions on the private land and guide the allocation of the total budget among public and private sites.

Since we use the attack rate to compute newly infested trees, we have two different types of non-linear terms in constraints, one is a product of a binary variable and a continuous variable, and another is a product of two continuous variables. As a result, we provide the corresponding linearization equivalent equations to replace those non-linear terms in our constraints.

3.3.5 Mathematical Formulation

The mathematical formulation for our integrated game theory and optimization model (3.2)-(3.32) is given below.

$$\max \sum_{t=1}^{3} \delta^t \sum_{i\in\Gamma} \Big(\alpha \cdot W_i^t - \theta_D \cdot (I_i^t - \rho \tilde{I}_i^t - \hat{I}_i^t)\Big) \tag{3.2}$$

Subject to:

(a) Constraints that apply to both private and public sites

- Initiation on Population, infestation, and dead trees:

$$N_i^1 = N_i^0 \quad \forall i \in \Gamma \tag{3.3}$$

$$I_i^1 = I_i^0 \quad \forall i \in \Gamma \tag{3.4}$$

$$\dot{D}_i^1 = \dot{D}_i^0 \quad \forall i \in \Gamma \tag{3.5}$$

- Population constraints:

$$N_i^{t+1} = N_i^t - \hat{I}_i^t - \hat{D}_i^t \quad \forall i \in \Gamma \quad \text{and} \quad t = 1, 2 \tag{3.6}$$

- Healthy trees:

$$H_i^t = N_i^t - \dot{D}_i^t - I_i^t \quad \forall i \in \Gamma, \quad \text{and} \quad t \in T \tag{3.7}$$

- Awarding trees:

$$W_i^t = H_i^t + \rho \cdot \tilde{I}_i^t \quad \forall i \in \Gamma, \quad \text{and} \quad t \in T \tag{3.8}$$

- Susceptible tree population can be infested in the next period after treatment and removal:

$$S_i^t = H_i^t - \tilde{H}_i^t \quad \forall i \in \Gamma, \quad \text{and} \quad t \in T \tag{3.9}$$

- Believed attack rate:

$$\pi_{h,i}^{t+1} = \min\{1, \omega_0 \cdot \pi_i^t + \omega_I \cdot \frac{\sum_{j \in \Theta_i} (I_j^t - \tilde{I}_j^t - \hat{I}_j^t)}{n_i}\} \quad \forall i \in \Gamma \quad \text{and} \quad t = 1, 2 \tag{3.10}$$

$$\pi_{l,i}^{t+1} = \eta \pi_i^t \quad \forall i \in \Gamma \quad \text{and} \quad t = 1, 2 \tag{3.11}$$

$$\pi_i{}^1 = \pi_i^0 \quad \forall i \in \Gamma \tag{3.12}$$

$$\pi_i{}^{t+1} = \mu_i^t \cdot \pi_{h,i}^{t+1} + \bar{\mu}_i^t \cdot \pi_{l,i}^{t+1} \quad \forall i \in \Gamma \quad \text{and} \quad t = 1, 2 \tag{3.13}$$

- Believed number of newly infestations before treatment and removal:

$$I_i^{t+1} = \pi_i^{t+1} S_i^t \quad \forall i \in \Gamma \quad \text{and} \quad t = 1, 2 \tag{3.14}$$

- Believed number of newly dead trees before treatment and removal:

$$D_i^{t+1} = I_i^t - \rho \cdot \tilde{I}_i^t - \hat{I}_i^t \quad \forall i \in \Gamma \quad \text{and} \quad t = 1, 2 \tag{3.15}$$

- Believed number of cumulative dead trees before treatment and removal:

$$\dot{D}_i^{t+1} = \dot{D}_i^t - \hat{D}_i^t + D_i^{t+1} \quad \forall i \in \Gamma \quad \text{and} \quad t = 1, 2 \tag{3.16}$$

(b) Treatment and removal decisions in public sites ($i \in \Gamma_u$)

$$\tilde{H}_i^t \leq N_i^t - (I_i^t + D_i^t) \quad \forall i \in \Gamma_u \quad \text{and} \quad t \in T \tag{3.17}$$

$$\tilde{I}_i^t + \hat{I}_i^t \leq I_i^t \quad \forall i \in \Gamma_u, t \in T \tag{3.18}$$

$$\tilde{I}_i^t + \hat{I}_i^t \geq \bar{\mu}_i^t \cdot I_i^t \quad \forall i \in \Gamma_u, t \in T \tag{3.19}$$

$$\hat{D}_i^t = 0 \quad \forall i \in \Gamma_u, t \in T \tag{3.20}$$

(c) Treatment and removal decisions only in private sites ($i \in \Gamma_r$)

$$\tilde{H}_i^1 = q_{S,i} \quad \forall i \in \Gamma_r \tag{3.21}$$

$$\tilde{I}_i^1 = q_{I,i} \quad \forall i \in \Gamma_r \tag{3.22}$$

$$\tilde{H}_i^t = 0 \quad \forall i \in \Gamma_r \quad \text{and} \quad t = 2, 3 \tag{3.23}$$

$$\tilde{I}_i^2 = I_i^2 \quad \forall i \in \Gamma_r \tag{3.24}$$

$$\tilde{I}_i^3 = 0 \quad \forall i \in \Gamma_r \tag{3.25}$$

$$\hat{I}_i^t = 0 \quad \forall i \in \Gamma_r, t \in T \tag{3.26}$$

$$\hat{D}_i^1 = D_i^0 \quad \forall i \in \Gamma_r, t \in T \tag{3.27}$$

$$\hat{D}_i^{t+1} = I_i^t - \rho \cdot \tilde{I}_i^t \quad \forall i \in \Gamma_r \quad \text{and} \quad t = 1, 2 \tag{3.28}$$

(d) Surveillance decisions:

$$V_i^1 = N_i^1 \quad \forall i \in \Gamma_u \tag{3.29}$$

$$V_i^{t+1} = \tilde{I}_i^t + S_i^t \quad \forall i \in \Gamma_u \quad \text{and} \quad t = 1,2 \tag{3.30}$$

(e) Budget (private reimbursement and public treatment, removal, and surveillance costs):

$$\sum_{i \in \Gamma_r} r_i + c_1 \cdot \sum_{i \in \Gamma_u} \sum_{t=1}^{3} V_i^t + c_2 \cdot \sum_{i \in \Gamma_u} \sum_{t=1}^{3} (\tilde{H}_i^t + \tilde{I}_i^t) + c_3 \cdot \sum_{i \in \Gamma_u} \sum_{t=1}^{3} (\hat{I}_i^t + \hat{D}_i^t) \leq \psi \tag{3.31}$$

(f) Non-negativity and binary restrictions:

$$\tilde{H}_i^t, \tilde{I}_i^t, \hat{I}_i^t, \hat{D}_i^t, N_i^t, I_i^t, D_i^t, S_i^t, H_i^t, V_i^t \geq 0 \quad \mu_i^t \in \{0,1\} \quad \forall \quad t \in T, i \in \Gamma \tag{3.32}$$

The objective function (3.2) represents the utility of the government over the planning horizon, which is defined by the discounted value of surviving trees minus the penalty on the infested, and dead trees. Equations (3.3), (3.4), and (3.5) present the initial values for the number of healthy, infested, and dead ash trees, respectively, in each site i at the beginning of the planning horizon. Equation (3.6) computes the remaining population of the ash trees after some of the infested and dead trees are removed from the ash tree population. Equation (3.7) represents the number of healthy trees at each period in each site, and Equation (3.8) shows the number of awarding trees at each period in each site. Equation (3.9) estimates the number of healthy trees susceptible to an infestation in the next period. We assume that the successfully treated infested trees will become healthy and EAB resistant in the next period, and treated healthy trees also will be EAB resistant in the next period. As a result, as shown in Equation (3.9), the number of susceptible trees, which may be infested in the next period, equals the number of healthy trees minus the number of treated healthy trees in period t.

Our multiple regression model described in Section 3.3.3 utilizes the real data on the EAB infestation to predict the high attack rate in the next period considering the current attack rate and the average infestation from the surrounding sites. We also must guarantee that the high attack rate should not exceed 1. Consequently, the high attack rate is calculated using the following equation (Equation (3.10)):

$$\pi_{h,i}^{t+1} = \min\{1, \omega_0 \cdot \pi_i^t + \omega_I \cdot \frac{\sum_{j \in \Theta_i} (I_j^t - \tilde{I}_j^t - \hat{I}_j^t)}{n_i}\} \quad (\forall i \in \Gamma; \quad t = 1, 2)$$

Equation (3.10) defines the high attack rate at period $t+1$ in site i ($i \in \Gamma$) when the total number of treated and removed infested trees is less than the number of the infested trees at period t in site i ($i \in \Gamma$). Equation (3.10), which is non-linear, is replaced with the following constraints to obtain its equivalent linearization for each $i \in \Gamma$ and $t = 1, 2$:

$$\pi_{h,i}^{t+1} \leq 1 \tag{3.10.1}$$

$$\pi_{h,i}^{t+1} \leq \omega_0 \cdot \pi_i^t + \omega_I \cdot \frac{\sum_{j \in \Theta_i} (I_j^t - \tilde{I}_j - \hat{I}_j^t)}{n_i} \tag{3.10.2}$$

$$1 - \pi_{h,i}^{t+1} \leq x_{i1}^t \cdot M \tag{3.10.3}$$

$$\omega_0 \cdot \pi_i^t + \omega_I \cdot \frac{\sum_{j \in \Theta_i} (I_j^t - \tilde{I}_j - \hat{I}_j^t)}{n_i} - \pi_{h,i}^{t+1} \leq (1 - x_{i1}^t) \cdot M \tag{3.10.4}$$

where M is a big number and x_{i1}^t is a binary variable, which takes a value of zero if $\omega_0 \cdot \pi_i^t + \omega_I \cdot \frac{\sum_{j \in \Theta_i} (I_j^t - \tilde{I}_j - \hat{I}_j^t)}{n_i} > 1$, implying that $\pi_{h,i}^{t+1} = 1$, and one, i.e., $\pi_{h,i}^{t+1} = \omega_0 \cdot \pi_i^t + \omega_I \cdot \frac{\sum_{j \in \Theta_i} (I_j^t - \tilde{I}_j - \hat{I}_j^t)}{n_i}$, otherwise.

Equation (3.11) defines the low attack rate when the government or the private landowner applies treatment or removal to all infested trees at period t in site i ($i \in \Gamma$), which means $\tilde{I}_i^t + \hat{I}_i^t \geq I_i^t$. If all infested trees are treated or removed, the attack rate will become $\pi_{l,i}^{t+1}$, which must be lower than the previous attack rate, and we use a rate η as a multiplier of the current attack rate to compute the low attack rate. The attack rate is initialed at the beginning (period 1) for each site by Equation (3.12). Equation (3.13) computes the attack rate at period $t+1$ in each site after treatment and removal are applied in period t as a function of the high, and low attack rates and treatment and removal level applied. Specifically, the next period's attack rate increases or decreases depending on the number of treated and removed infested trees at the current period in site i ($i \in \Gamma$). Further, the attack rate will become the high attack rate ($\pi_{h,i}^{t+1}$) if not all infested trees are treated or removed, i.e., $\mu_i^t = 1$, (See Equation (3.10)) and the low attack rate ($\pi_{l,i}^{t+1}$) otherwise, i.e., $\mu_i^t = 0$ (See Equation (3.11)).

Further, Equation (3.13) is a non-linear constraint and can be linearized by the following constraints:

$$\pi_i^{t+1} = A_i^{t+1} + B_i^{t+1} \tag{3.13.1}$$

$$LB_1 \cdot \mu_i^t \leq A_i^{t+1} \leq UB_1 \cdot \mu_i^t \tag{3.13.2}$$

$$\pi_{h,i}^{t+1} - UB_1 \cdot \bar{\mu}_i^t \leq A_i^{t+1} \leq \pi_{h,i}^{t+1} - LB_1 \cdot \bar{\mu}_i^t \tag{3.13.3}$$

$$LB_2 \cdot \bar{\mu}_i^t \leq B_i^{t+1} \leq UB_2 \cdot \bar{\mu}_i^t \tag{3.13.4}$$

$$\pi_{l,i}^{t+1} - UB_2 \cdot \mu_i^t \leq B_i^{t+1} \leq \pi_{l,i}^{t+1} - LB_2 \cdot \mu_i^t \tag{3.13.5}$$

where $LB_1 = 0$, $UB_1 = 1$ and $LB_2 = 0$, $UB_2 = \eta$, are the lower and upper bounds of $\pi_{h,i}^{t+1}$ and $\pi_{l,i}^{t+1}$, respectively. A_i^{t+1} is the substitute variable of $\mu_i^t \cdot \pi_{h,i}^{t+1}$ and B_i^{t+1} is the substitute variable of $\bar{\mu}_i^t \cdot \pi_{l,i}^{t+1}$. Further, π_i^{t+1} can be represented by $(A_i^{t+1} + B_i^{t+1})$, and A_i^{t+1} as well as B_i^{t+1} are restricted by the linear constraints from Equation (3.13.2) to (3.13.5). When some of the infested trees are left untreated or unremoved ($\mu_i^t = 1$), Equation (3.13.2) to (3.13.5) become

$$LB_1 \leq A_i^{t+1} \leq UB_1$$

$$\pi_{h,i}^{t+1} \leq A_i^{t+1} \leq \pi_{h,i}^{t+1}$$

$$0 \leq B_i^{t+1} \leq 0$$

$$\pi_{l,i}^{t+1} - UB_2 \leq B_i^{t+1} \leq \pi_{l,i}^{t+1} - LB_2$$

As a result, $A_i^{t+1} = \pi_{h,i}^{t+1}$ and $B_i^{t+1} = 0$, and thus $\pi_i^{t+1} = \pi_{h,i}^{t+1}$. When all infested trees are treated or removed ($\mu_i^t = 0$), Equation (3.13.2) to (3.13.5) become

$$0 \leq A_i^{t+1} \leq 0$$

$$\pi_{h,i}^{t+1} - UB_1 \leq A_i^{t+1} \leq \pi_{h,i}^{t+1} - LB_1$$

$$LB_2 \leq B_i^{t+1} \leq UB_2$$

$$\pi_{l,i}^{t+1} \leq B_i^{t+1} \leq \pi_{l,i}^{t+1}$$

As a result, $A_i^{t+1} = 0$ and $B_i^{t+1} = \pi_{l,i}^{t+1}$, and thus $\pi_i^{t+1} = \pi_{l,i}^{t+1}$.

As shown in Equation (3.14), a susceptible tree in period t has a π_i^{t+1} chance of being infested in the next period, so the number of newly infested trees is that the susceptible trees in period t multiplied by the period $t+1$ attack rate. Equation (3.14) is the product of two continuous variables which should be linearized. To perform linearization, first, the product of two continuous variables should be converted into a separable function. We introduce two new variables, $y_{i,1}^{t+1}$ and $y_{i,2}^{t+1}$, which are, respectively, defined as:

$$y_{i,1}^{t+1} = \frac{1}{2}(\pi_i^{t+1} + S_i^t)$$
$$y_{i,2}^{t+1} = \frac{1}{2}(\pi_i^{t+1} - S_i^t)$$

where $0 \leq y_{i,1}^{t+1} \leq \frac{1+N_i^0}{2}$ and $-\frac{N_i^0}{2} \leq y_{i,2}^{t+1} \leq \frac{1}{2}$. Now, $\pi_i^{t+1} S_i^t$ can be replaced by the separable function:

$$(y_{i,1}^{t+1})^2 - (y_{i,2}^{t+1})^2$$

Then, we use the piecewise linearization technique of [14] to present the equivalent linearization of $(y_{i,1}^{t+1})^2 - (y_{i,2}^{t+1})^2$. Let $f(x) = x^2$. $f(x)$ have x_0, x_1, x_2,..., x_m breakpoints, and $f(x_1)$, $f(x_2)$, $f(x_3)$, $f(x_4)$ be related corresponding function values. If $LB \leq x \leq UB$, we may choose the $m+1$ breakpoints: $x_k = LB + \frac{k \cdot (UB-LB)}{m}$ $(k = 0, 1, ..., m)$, respectively. Given that λ_k represents the kth non-negative weights, $f(x)$ can be replaced with its approximate function, $\tilde{f}(x)$, which can be represented by linear equations as given below:

$$\tilde{f}(x) = \sum_{k=0}^{m} \lambda_k f(x_k)$$
$$x = \sum_{k=0}^{m} \lambda_k x_k$$
$$\sum_{k=0}^{m} \lambda_k = 1$$

with added restrictions that at most two adjacent λ's are greater than zero, i.e., $\lambda_k \leq \beta_k + \beta_{k+1}$ and $k = 0, 1, ..., (m-1)$, $\lambda_m \leq \beta_m$, and $\sum_{k=0}^{m} \beta_k = 1$ (β_k is a binary variable). Using the linearization method shown above and letting $\tilde{\tilde{g}}(y_{i,1}^{t+1}, y_{i,2}^{t+1})$ denote the approximated function value of $(y_{i,1}^{t+1})^2 - (y_{i,2}^{t+1})^2$, and $\tilde{\tilde{g}}(y_{i,1}^{t+1}, y_{i,2}^{t+1})$ can be represented by the following linear constraints:

$$\tilde{\tilde{g}}(y_{i,1}^{t+1}, y_{i,2}^{t+1}) = \sum_{k=0}^{m} \lambda_{i,1,k}^{t+1} f(y_{i,1,k}^{t+1}) - \sum_{k=0}^{m} \lambda_{i,2,k}^{t+1} f(y_{i,2,k}^{t+1}) \tag{3.14.1}$$

$$y_{i,1}^{t+1} = \sum_{k=0}^{m} \lambda_{i,1,k}^{t+1} y_{i,1,k}^{t+1} \tag{3.14.2a}$$

$$y_{i,2}^{t+1} = \sum_{k=0}^{m} \lambda_{i,2,k}^{t+1} y_{i,2,k}^{t+1} \tag{3.14.2b}$$

$$\sum_{k=0}^{m} \lambda_{i,1,k}^{t+1} = 1 \tag{3.14.3a}$$

$$\sum_{k=0}^{m} \lambda_{i,2,k}^{t+1} = 1 \tag{3.14.3b}$$

$$\lambda_{i,1,k}^{t+1} \leq \beta_{i,1,k}^{t+1} + \beta_{i,1,k+1}^{t+1}, \lambda_{i,1,m}^{t+1} \leq \beta_{i,1,m}^{t+1} \qquad k = 0, 1, ..., (m-1) \tag{3.14.4a}$$

$$\lambda_{i,2,k}^{t+1} \leq \beta_{i,2,k}^{t+1} + \beta_{i,2,k+1}^{t+1}, \lambda_{i,2,m}^{t+1} \leq \beta_{i,2,m}^{t+1} \qquad k = 0, 1, ..., (m-1) \tag{3.14.4b}$$

$$\sum_{k=0}^{m} \beta_{i,1,k}^{t+1} = 1 \tag{3.14.5a}$$

$$\sum_{k=0}^{m} \beta_{i,2,k}^{t+1} = 1 \tag{3.14.5b}$$

$$\beta_{i,1,k}^{t+1} \in \{0, 1\} \qquad k = 0, 1, ..., m \tag{3.14.6}$$

Because $\tilde{\tilde{g}}(y_{i,1}^{t+1}, y_{i,2}^{t+1})$ denotes the approximated function value of $\pi_i^{t+1} S_i^t$, its value may exceed the value of S_i^t. To ensure that the number of newly infested trees, I_i^{t+1}, is not more than S_i^t, we incorporate the following function setting I_i^{t+1} to the minimum of $\tilde{\tilde{g}}(y_{i,1}^{t+1}, y_{i,2}^{t+1})$ and S_i^t, i.e.,

$$I_i^{t+1} = \min(\tilde{\tilde{g}}(y_{i,1}^{t+1}, y_{i,2}^{t+1}), S_i^t). \tag{3.14.7}$$

Here we have a new non-linear constraint which can be lineareized by the following constraints:

$$I_i^{t+1} <= \tilde{\tilde{g}}(y_{i,1}^{t+1}, y_{i,2}^{t+1}) \tag{3.14.7a}$$

$$I_i^{t+1} <= S_i^t \tag{3.14.7b}$$

$$\tilde{\tilde{g}}(y_{i,1}^{t+1}, y_{i,2}^{t+1}) - I_i^{t+1} <= x_{i2}^t \cdot M \tag{3.14.7c}$$

$$S_i^t - I_i^{t+1} <= (1 - x_{i2}^t) \cdot M \tag{3.14.7d}$$

where x_{i2}^t is a binary variable and M is a big number. the approximated expression's value is not more than S_i^t, $I_i^{t+1} = \tilde{\tilde{g}}(y_{i,1}^{t+1}, y_{i,2}^{t+1})$ and x_{i2}^t takes value zero, otherwise, $I_i^{t+1} = S_i^t$ and x_{i2}^t takes value one.

We assume that the infested trees will be dead in the next period if those trees are not treated, or treatment is unsuccessful. Based on this assumption, Equation (3.15) and Equation (3.16) give the number of newly dead trees and the number of total dead trees at period $t+1$ in site i ($i \in \Gamma$) after treatment and removal are applied at period t in that site respectively. Equation (3.17) shows that the treatment can be applied to healthy trees in public sites, and those treated healthy trees will become EAB resistant in the next period. Both treatment and removal can be applied to the infested trees in public sites in each period, as shown in Equation (3.18).

As shown in Equation (3.19), all infested trees are treated or removed in period t in site i ($i \in \Gamma$) if $\mu_i^t = 0$, and not all infested trees are treated or removed if $\mu_i^t = 1$. The right-hand side of constraint (3.19) is a non-convex term which is a product of a binary variable $\bar{\mu}_i^t$ and a continuous variable I_i^t for which $0 \leq I_i^t \leq N_i^0$ holds. We introduce a continuous variable z_i^t to represent the term $\bar{\mu}_i^t \cdot I_i^t$, and the equivalent linearization constraints to replace Equation (3.19) are presented below:

$$0 \leq z_i^t \leq N_i^0 \cdot \bar{\mu}_i^t \quad \forall i \in \Gamma_u, t \in T \tag{3.19.1}$$

$$I_i^t - N_i^0 \cdot \mu_i^t \leq z_i^t \leq I_i^t \quad \forall i \in \Gamma_u, t \in T \tag{3.19.2}$$

$$\tilde{I}_i^t + \hat{I}_i^t \geq z_i^t \quad \forall i \in \Gamma_u, t \in T \tag{3.19.3}$$

When $\mu_i^t = 0$, the above constraints become:

$$0 \leq z_i^t \leq N_i^0 \quad \forall i \in \Gamma_u, t \in T$$

$$I_i^t \leq z_i^t \leq I_i^t \quad \forall i \in \Gamma_u, t \in T$$

$$\tilde{I}_i^t + \hat{I}_i^t \geq z_i^t \quad \forall i \in \Gamma_u, t \in T$$

The above three constraints imply that $z_i^t = I_i^t$ and $\tilde{I}_i^t + \hat{I}_i^t \geq I_i^t$, i.e., the treated and removed infested trees are not less than the infested trees in public site i in period t. On the contrary, when $\mu_i^t = 1$, Equations (3.19.1) to (3.19.3) become:

$$0 \leq z_i^t \leq 0 \quad \forall i \in \Gamma_u, t \in T$$

$$I_i^t - N_i^0 \leq z_i^t \leq I_i^t \quad \forall i \in \Gamma_u, t \in T$$

$$\tilde{I}_i^t + \hat{I}_i^t \geq z_i^t \quad \forall i \in \Gamma_u, t \in T$$

The above three constraints indicate that $z_i^t = 0$ and $\tilde{I}_i^t + \hat{I}_i^t \geq 0$, the total number of treated and removed infested trees are non-negative in public site i in period t.

Equation (3.20) implies that dead trees in public sites will not be removed, nor will any budget be allocated because another specified city budget is used to remove dead public trees. Equations (3.21) and (3.22) represent the treatment decisions for the private land taken from the game-theory model as shown in Appendix A.2.1 and used as an input into the optimization model. Further, the treatment decisions on healthy trees and infested trees in private sites in period one are shown in Equations (3.21) and (3.22), respectively. The private landowner will treat only infested trees without considering the treatment of healthy trees in the second period; however, no trees will be treated in the third period, which is implied by Equations (3.23), (3.24), and (3.25), respectively. Furthermore, Equations (3.26), (3.27), and (3.28) imply that the private landowner will only remove all dead trees in each period.

Equation (3.29) illustrates that all ash trees will be surveyed at the beginning period in all public sites. According to assumptions that untreated infested trees will be dead in the next period, treated healthy trees will become EAB resistant. As a result, it is only necessary survey treated infested trees and susceptible trees in the previous period. As shown in Equation (3.30), the number of surveyed trees at period $t + 1$ is equal to the number of treated infested trees plus the number of susceptible trees at period t in each public sites.

The total budget imposed on the reimbursement allocated to the private sites and the total cost of surveillance, treatment, and removal over the planning horizon allocated to the public sites is implied by Equation (3.31). According to the budget constraint (3.31), the expenses related to the government's reimbursement given to the private landowners plus the cost of surveillance, treatment, and removal of ash trees in the public sites over all periods must be less than or equal to the total budget available throughout the planning horizon. The reimbursement is an input obtained from the solution of the game-theory model. Equation (3.32) assures that all original decision variables are non-negative, and the indicator variable μ_i^t is binary.

3.4 Numerical Solutions

In this section, we present numerical solutions under different cases by considering different budget levels, different ash tree distribution levels in public sites, and different success rate of treatment levels which all play a significant role on optimal decisions in both public and private sites. We applied our integrated principal-agent framework with mixed-integer optimization model to find the optimal treatment and removal decisions in studied 5×5 sites at each period to maximize the utility of government over the planning horizon.

3.4.1 Model Application and Data

Based on the real EAB infestation data collected from 14 different surveyed areas in Maumee Bay State Park in Toledo, Ohio, from 2005 to 2011 [57, 83], and each area includes three sites. Each site is a 0.1-acre circular landscape. As a result, the study area was divided into 0.1-acre survey units (we call sites in this chapter). We study a 5×5 girdded area, because most adults fly less than 328 feet when ash trees are near shown in [84]. We calculate the population of ash trees in all sites in each area. As shown in Figure 3.3, we found the minimum population among those sites is 1 and the maximum number of ash trees among those sites is 42, and the medium number of ash trees among all sites is 11. We treat the sites as sites with a low density of ash trees where the number of ash trees is less than or equal to 11; otherwise, we treat the sites as sites with a high density of ash trees.

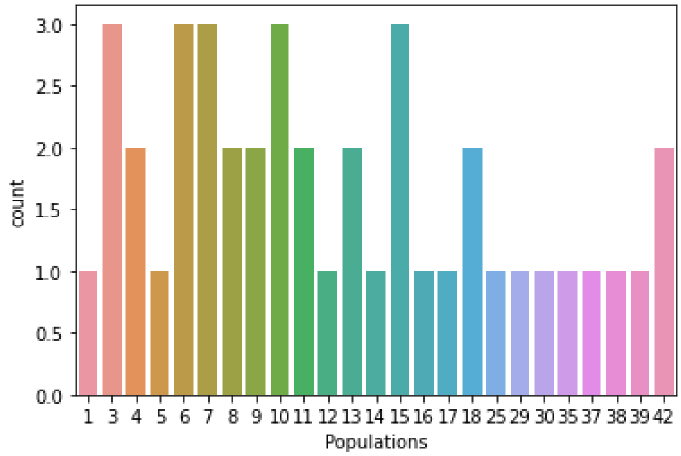


Figure 3.3 Population of ash trees in all plots.

The average number of ash trees among the low-density sites is around seven, and the average number of ash trees among the high-density sites is around 24. So we use seven as the low distribution level of ash trees and 24 as the high distribution level of ash trees in public sites. Furthermore, The mean number of ash trees in single-family house land is around one [122]. So we use three as the population of ash trees in each private site to cover the maximum population of ash trees in private lands.

The ash trees are present in specific public areas, such as streets and parks. We consider two scenarios on different levels (low and high) of ash tree distribution in public sites to show the numerical solutions. We consider the street public sites to have a low distribution level of ash trees because a certain space between two trees is required, and the public park public sites have a high distribution level of ash trees. Figure 3.4 presents the four different cases on the different ash trees distributions levels and the different number of public sites. The number of ash trees in each private site is 3, and the number of ash trees in each public site is 7 for street scenarios, and each private site has three ash trees, and each public site has 24 ash trees in both two park scenarios.

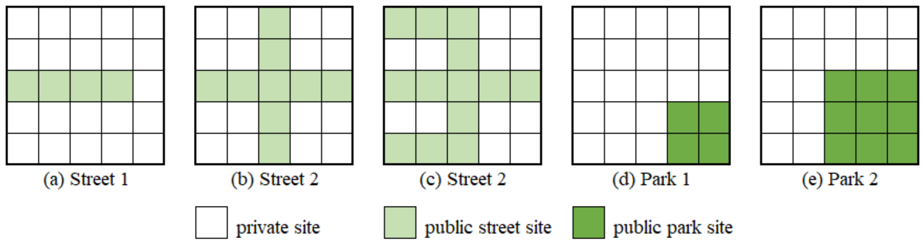


Figure 3.4 Different distribution level of ash trees in the public sites. The number of ash trees in each private site, each public street site, public park site are 3, 7, 24, respectively.

As shown in Figure 3.4, Street 1 shows the street scenario that there are 21 private sites and there are four public sites; Street 2 represents the scenario that has 16 private and nine public sites; Street 3 presents the street scenario which has 12 public sites and 13 private sites. Park 1 shows the park scenario where there are 21 private sites and four public sites, and Park 2 presents the park scenario, which has 16 private sites and nine public sites.

We assume we survey a tree by branch sampling method in the public sites, and the surveillance cost per tree is estimated \$124, and the cost of treating an infested tree is estimated at \$180 from [132]. The monetary value of an alive ash tree was estimated at \$72 from [79]. Morris County took down ash trees along county rights-of-ways in Morris Township and Long Hill in the first round of cutting in 2019. The Board of Freeholders approved a resolution to award a \$498,465 contract to Landing-based Tree King for the first round of tree removal (The contractor is responsible for removing and properly recycling the downed trees), and this initial project includes the removal of 880 trees in total in Long Hill and Morris Township [105]. So we estimate that the cost of removing an ash tree is \$300.

As shown in the cost-sharing model, the optimal treatment decision in private sites is mainly driven by the success rate of treatment (ρ) and the low second-period attack rate ($\pi^2_{l,i}$). To distinguish the success rate of treatment and the low second-period attack rate in different levels, we define the following parameters. First, let $\dot{\rho} := \frac{\beta}{s+\beta+\theta+c}$ and $\ddot{\rho} := \frac{\beta}{\theta+c}$, then $0 < \dot{\rho} < \ddot{\rho}$. Because the cost of treatment is generally much lower than the sum of the valuation of a live ash tree and the removal cost, $\ddot{\rho}$ is likely to be small. Then we can categorize the treatment effectiveness (ρ) into three levels.

Definition 6. *As summarized in Table 3.4, the treatment is considered to be* less effective *if* $\rho \in [0, \dot{\rho})$, somewhat effective *if* $\rho \in [\dot{\rho}, \ddot{\rho})$, *and* very effective *if* $\rho \in [\ddot{\rho}, 1]$.

Definition 7. *As shown in Table 3.5, the low second-period attack rate ($\pi^2_{l,i}$) in site i is considered* low *if* $\pi^2_{l,i} \in [0, \dot{\pi}^2_{l,i})$, medium *if* $\pi^2_{l,i} \in [\dot{\pi}^2_{l,i}, \ddot{\pi}^2_{l,i})$, *and* high *if* $\pi^2_{l,i} \in [\ddot{\pi}^2_{l,i}, 1]$.

Table 3.4 Categorization of Treatment Effectiveness (ρ)

Condition	ρ
$\rho \in [0, \dot{\rho})$	less effective
$\rho \in [\dot{\rho}, \ddot{\rho})$	somewhat effective
$\rho \in [\ddot{\rho}, 1]$	very effective

Table 3.5 Classification of the Low Second-period Attack Rate ($\pi^2_{l,i}$)

Condition	$\pi^2_{l,i}$
$\pi^2_{l,i} \in [0, \dot{\pi}^2_{l,i})$	low
$\pi^2_{l,i} \in [\dot{\pi}^2_{l,i}, \ddot{\pi}^2_{l,i})$	medium
$\pi^2_{l,i} \in [\ddot{\pi}^2_{l,i}, 1]$	high

In the following sections, we will explore the impact of different factors on the optimal treatment and removal decisions among public and private sites and run 30 scenarios by different initial infestation in private sites where the infestation follows binomial distribution ($I^0_i \sim B(N^0_i, \pi^0_i)$ and $i \in \Gamma$). In Section 3.4.2, we focus on the budget and the success rate of treatment's effect on the optimal treatment and removal decisions, respectively. In Section 3.4.3, we study the optimal action decisions for the different numbers of private sites with and without the cost-sharing program. In Section 3.4.4, we explore the impact of the different distribution levels of ash trees in the public sites on the budget allocations and management options on both private and public sites.

3.4.2 Different Budget Levels and Success Rate of Treatment Levels

We present nine cases by considering three different budget levels (\$15000, \$25000, \$35000) and three success rate of treatment levels ($\rho = 0.2, 0.4, 0.6$). Here, we focus on street 2 type area where there are nine public sites and 16 private sites. The input parameters and cutoff values used are listed in Table 3.6.

Table 3.6 Input Parameters Used and Calculated Cutoffs for All Budget Levels

Input Parameters								Calculated Cutoffs			
c_1	c_2	c_3	θ	α	π_i^0	$\pi_{l,i}^2$	ρ	$\dot{\pi}_{l,i}^2$	$\ddot{\pi}_{l,i}^2$	$\dot{\rho}$	$\ddot{\rho}$
124	180	300	50	72	0.4	0.32	0.2	0.27	0.39	0.30	0.51
							0.4	0.33	0.46		
							0.6	0.43	0.56		

3.4.2.1 Different budget levels. We first explore the impact of the budget on management decisions under different treatment effectiveness. Figures 3.5, 3.6 and 3.7 show the different budget allocations among reimbursement assigned to private landowners, surveillance, treatment, and removal cost applied to public sites when $\rho = 0.20, 0.4, 0.6$, respectively. As shown in those figures, we find that the government allocates the same budget on the surveillance in public sites and the reimbursement for private sites. More budget is assigned to the treatment in public sites when the total budget increases and the removal cost increases firstly and then decreases.

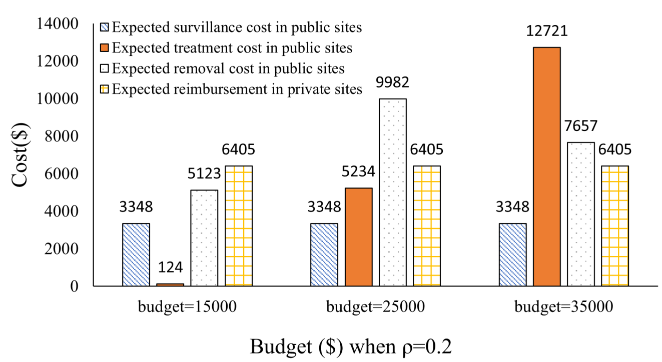


Figure 3.5 Surveillance, treatment, removal cost and reimbursement for different budget levels when $\rho = 0.2$.

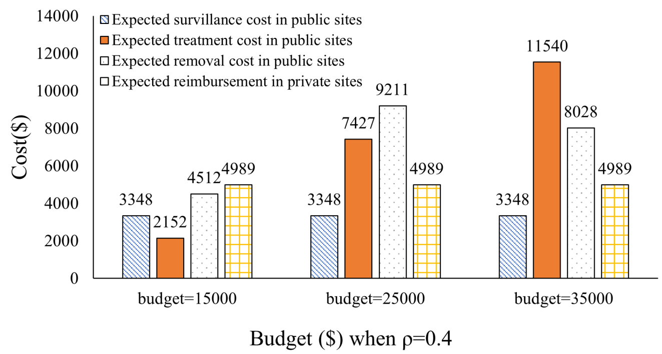


Figure 3.6 Surveillance, treatment, removal cost and reimbursement for different budget levels when $\rho = 0.4$.

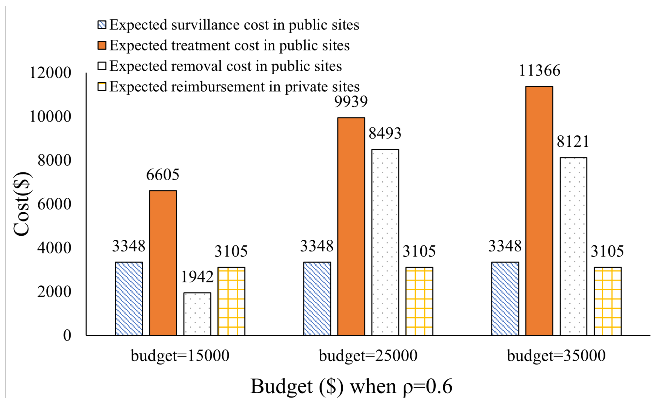


Figure 3.7 Surveillance, treatment, removal cost and reimbursement for different budget levels when $\rho = 0.6$.

When the government only assigns \$15000 budget to control the spread of EAB, the budget is very limited regardless effectiveness of treatment, and not all infested trees can be treated or removed.

When the total budget rises, the government will spend the rest budget on treating susceptible trees after taking the cost of removing all infested ash trees in the public sites in the first period. Since the success rate of treatment is very less effective, there is a high probability that the treated infested trees will also die in the next period. But the treated susceptible trees must become EAB resistant in the next period, and when the budget is enough, the treatment of the susceptible trees occurs earlier in the public sites.

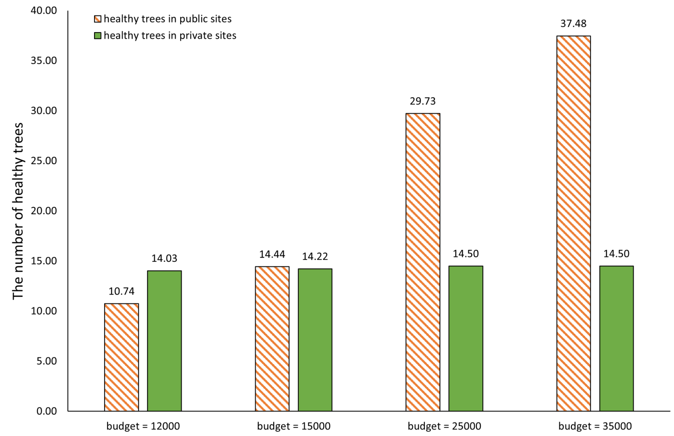


Figure 3.8 The total expected healthy trees in all public sites and private sites respectively for different budget levels. Where $c_1 = \$124$ per tree, $c_2 = \$180$ per tree, $c_3 = \$300$ per tree, $\rho = 0.2$, and the initial attack rate is 0.4.

The quantity of total healthy trees at the end of the horizon increases with the increase of the total budget. The number of healthy trees in all public sites increases largely when the government assigns more and more budget. However, the increase of healthy trees in all private sites is few.

3.4.2.2 Different success rate of treatment levels. In this section, we explore the impact of the success rate of treatment on the management decisions and the budget allocations for the treatment, removal in public sites, and the reimbursement in the private sites. We increase the success rate of treatment from 0.2 to 0.6.

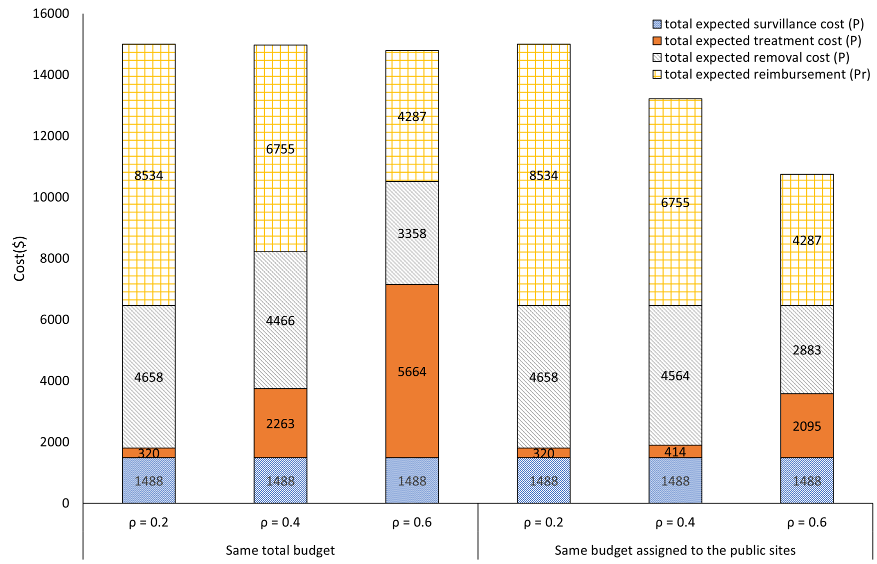


Figure 3.9 Surveillance, treatment, removal cost and reimbursement for different success rates of treatment with the same budget. Where $c_1 = \$124$ per tree, $c_2 = \$180$ per tree, $c_3 = \$300$ per tree, and the initial attack rate is 0.4.

As shown in Figure 3.9, the allocated budget for the surveillance does not change regardless of the increase in treatment success rate. However, the total reimbursement assigned to the private sites decreases with the increase in the success rate of treatment. Because private landowners have more incentives when the success rate of treatment is higher, the required reimbursement from the government is less.

The right-hand side of Figure 3.9 shows that there is more budget that can be assigned to the public sites when the success rate of treatment is increasing with the same total budget, as the reimbursement assigned to private sites is less and less. As a result, the allocated budget for the treatment and removal in the public sites is more with a higher success rate of treatment.

The left-hand side of Figure 3.9 shows the different budget allocations for the surveillance, treatment, removal, and reimbursement with the same budget on the treatment and removal decisions in different success rates of treatment. The proportion of the budget for the treatment in the public sites increases. Conversely, the proportion of the budget for the removal in public sites decreases with the increase in the success rate of treatment.

Table 3.7 The Percentage of Healthy and Infested Trees Being Taken Actions

	budget =\$15000			budget = \$25000			budget = \$35000		
	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$
Treated healthy trees percentage	0.3%	1.2%	5.2%	28.4%	38.3%	50.1%	62.3%	57.3%	57.1%
Treated infested trees percentage	0.9%	24.3%	72.2%	0.0%	0.2%	1.0%	0.0%	0.0%	0.1%
Removed infested trees percentage	35.1%	33.5%	15.2%	100.0%	99.8%	99.0%	100.0%	100.0%	99.9%

Table 3.7 shows that the newly infested trees, treated susceptible trees, treated infested trees, removed infested trees, and removed dead trees in each site for each period with different levels of the success rate of treatment and the same budget assigned to treatment and removal in all public sites. As shown in Table 3.7, the budget assigned on the public sites is almost all allocated to remove infested trees when the success rate of treatment is very low ($\rho = 0.2$), and the rest budget is applied to treat susceptible trees in the public sites. When the success rate of treatment increases to 0.4, there is more budget assigned to treatment in the public sites. Furthermore, the increased budget assigned to the treatment is allocated to treating infested trees in public sites. The proportion of treatment on the budget assigned to the public sites is largely improved when the success rate of treatment becomes 0.6. Further, a large proportion of the treatment cost is used for treating infested trees in public sites.

3.4.3 Number of Privates Sites

In this section, we present the optimal solutions under there street scenarios (Street 1, Street 2, and Street 3) with or without cost-sharing programming when the total budget is set as \$15000.

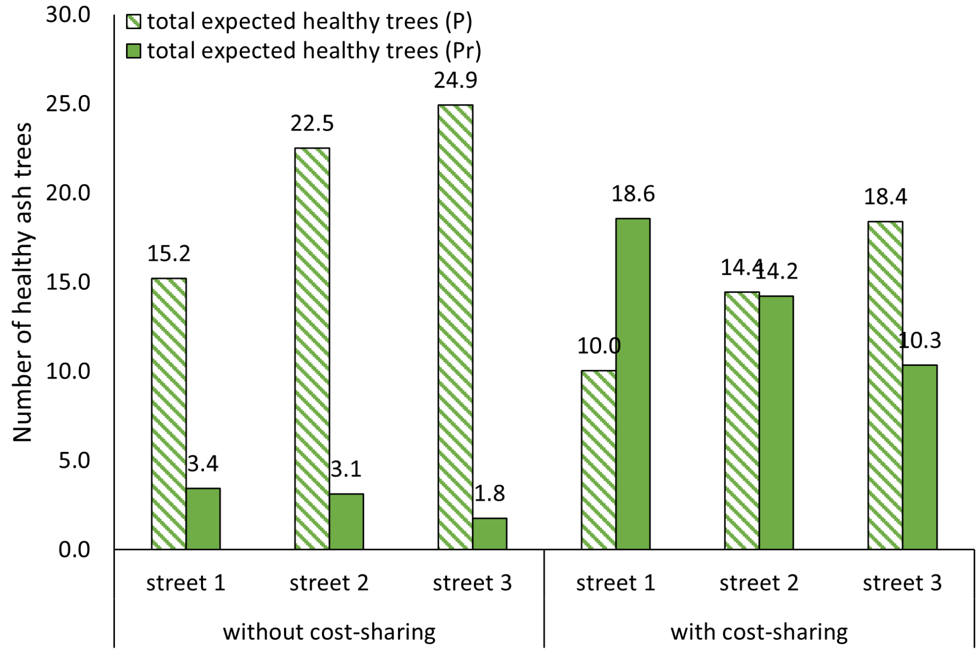


Figure 3.10 Total expected healthy ash trees in all public and private sites respectively for different street scenarios. Where $c_1 = \$124$ per tree, $c_2 = \$180$ per tree, $c_3 = \$300$ per tree, $\rho = 0.2$, and the initial attack rate is 0.4.

As shown in Figure 3.10, the number of healthy ash trees in public sites increases, and the number of healthy trees in private sites decreases when there are more and more private sites with cost-sharing or without cost-sharing programming. However, there is more budget applied to treatment and removal in public sites without cost-sharing programming compared to the case with cost-sharing programming. But accordingly, there are fewer healthy ash trees in private sites without cost-sharing programming compared to the case with cost-sharing programming. As a result, the cost-sharing program may reduce the budget allocated for treatment or removal in public sites, which further causes the reduced healthy trees. However, the reimbursement given by the government can induce private landowners to take action to save more ash trees.

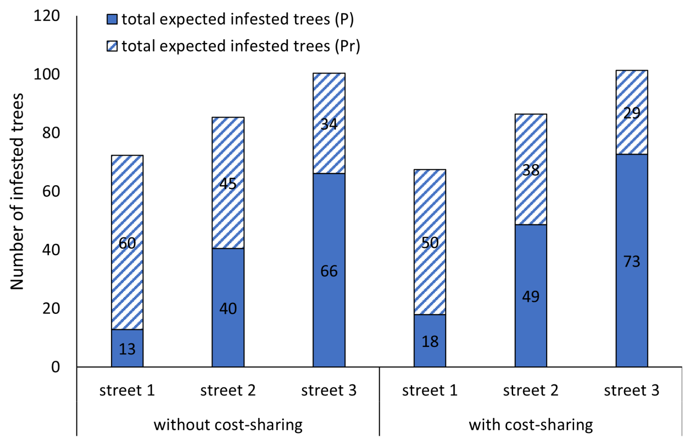


Figure 3.11 Total expected infested ash trees in all public and private sites respectively for different street scenarios. Where $c_1 = \$124$ per tree, $c_2 = \$180$ per tree, $c_3 = \$300$ per tree, $\rho = 0.2$, and the initial attack rate is 0.4.

Figure 3.11 presents the total infestation becomes worse and worse with the reduction of the number of private sites. But the number of infested ash trees in private sites is decreasing.

3.4.4 Street vs Park

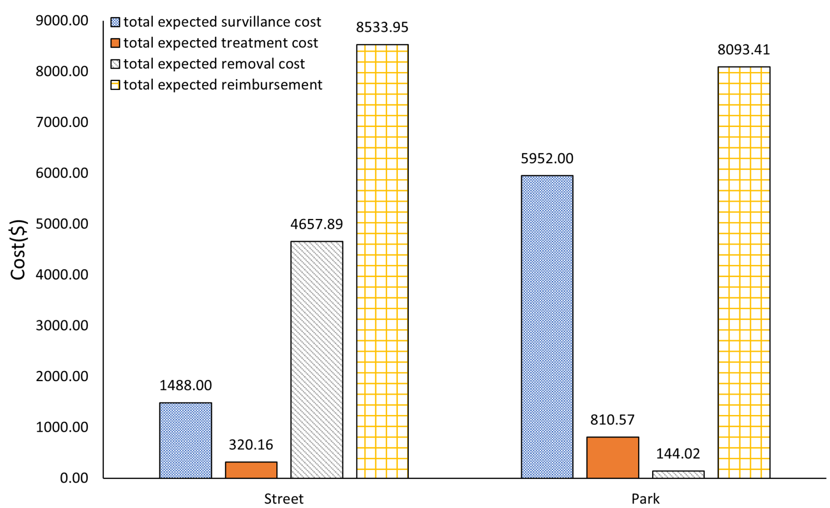


Figure 3.12 Surveillance, treatment, removal cost and reimbursement for street and park scenarios. Where $c_1 = \$124$ per tree, $c_2 = \$180$ per tree, $c_3 = \$300$ per tree, $\rho = 0.2$, and the initial attack rate is 0.4.

The government needs to allocate more budget to the surveillance of public ash trees when public sites belong to a park compared with the case when public sites belong to the streets. As a result, less budget can be allocated to treatment and removal in public sites when public sites are park areas.

CHAPTER 4

EVALUATING THE IMPLICATIONS OF EMISSIONS POLICIES ON A CARBON CAPTURE AND STORAGE SYSTEM

4.1 Introduction

Carbon Capture and Storage (CCS) is a technology that captures the produced CO_2 at the pollution sources before it is released into the atmosphere at major point sources. By compressing CO_2 to a supercritical fluid, it can then be transported and stored underground permanently. Although CCS is suitable in many heavy polluting industries, such as electricity, steel, cement, etc., almost all CCS facilities are used in the power electricity sector. There are 51 large-scale CCS facilities globally ([59]). Nineteen of them are in operation with a total capacity of 40 million metric tonnes per annum (Mtpa). However, to meet the Paris Agreement of keeping the global average temperature increase well below 2^oC, it is estimated that over 2000 CCS facilities with an average capacity of 1.5 Mtpa are needed.

Currently, coal- and natural gas-fired power plants produce 28% and 35% of the U.S. electricity sector, respectively. This sector contributes to 28% of greenhouse gas emissions in the U.S., and 82% of it is CO_2. Even though the share of electricity produced by coal is estimated to drop to 17% by 2050, natural gas is predicted to increase to 39% [53]. With such heavy reliance on fossil fuels, CCS can play a key role in mitigating emissions because it has the potential to capture 90% of generated CO_2.

The critical hurdle to widespread CCS adoption is not the technology itself but the lack of regulatory environments that assign a monetary cost to emitting CO_2 into the atmosphere. At present, several countries have created regulatory environments that encourage CCS, while other countries are moving in that direction.

The Furthering carbon capture, Utilization, Technology, Underground storage, and Reduced Emissions (FUTURE) Act, a U.S. Senate bill introduced in July 2017, proposes to award \$35 per tonne of CO_2 stored via EOR and \$50 per tonne of CO_2 used for dedicated storage, and allows a 12-year period for companies to claim the tax credit.

For CCS technology to be deployed on a commercial scale, it requires a transportation network connecting CO_2 emission sources (emitters henceforth) to underground storage sites. A third-party operator (CCS operator henceforth) may offer the transportation and disposition of CO_2 as a service. Because the storage operator owns the pipeline as well as the storage site, emitters only need to invest in the CO_2 capturing technology. Similarly, a market-based incentive structure is required to encourage users to reduce emissions by participating in CCS. Without either regulatory environments or economic incentives, emitters are unwilling to bear the additional expense associated with capturing, transporting, and storing CO_2.

Our research investigates the effectiveness of incentives on participation in CCS and the collaboration between emitters and a CCS operator. We use the Principal-Agent framework to capture the dynamics and the decision processes among the players.

We advance the literature by making two new contributions. One, we investigate the impact of allocating allowance to agents on the total quantities being captured and stored as well as the overall information rent given to the agents to incentivize them to participate. We provide analytical solutions to the special case where the allocated allowance is zero. We investigate the impact of providing allowance based on the agent's demand distribution through numerical analysis. Two, we evaluate a common cost model where the cost of emission depends on the total emission quantity instead of the individual emission quantity.

The rest of this chapter is organized as follows. Section 4.2 reviews relevant literature. In Section 4.3, we present a two-stage optimization model. In the 1st stage, we find the optimal quantities given the demand distributions and allowances. In the 2nd stage, we select the allowances that minimize the expected allowance while maximizing the overall captured quantity. We present analytical and numerical solutions where there is only one type of agent, and each has two demand levels in Section 4.4. Section 4.5 shows the analytical solutions for special cases where there are two different types of agents with two demand levels. We illustrate the numerical solutions in Section 4.6 and make a comparison between the non-cap-and-trade policy and the cap-and-trade policy in Section 4.7. Section 4.8 summarizes the managerial insights, and Section 4.9 concludes the paper.

4.2 Literature Review

There are three related bodies of literature: adopting the principal-agent framework, economic incentives for reducing carbon, CCS system deployment, and cap-and-trade mechanism.

In the traditional principal-agent framework, information asymmetry only occurs at the beginning. Once an agent announces his decision, such as selecting a contract, the agent's characterization, such as his private value for a service, becomes public. The principal is able to design a menu of contracts (multiple offers) for the agent to self-select. The contracts are incentive-compatible such that the agent reveals his characterization once he picks a contract. Maskin and Riley [94] show that, so long as the single-crossing property holds, a nonlinear price-quantity schedule can discriminate among a set of buyers with discrete valuations. In the optimal schedule, only the highest type agent is offered the efficient amount, the amount that would be offered if there is no information asymmetry between the principal and the agent. All other types are offered amounts with downward distortions. Further, the information rent increases in the agent's type. The lowest type receives zero information rent. This framework has been adopted in supply chain contracting [48, 107, 93, 130, 123, 11, 92, 131, 41, 111], as well as in the pricing of digital goods [121, 16], in service contracting [80, 3], and in resource allocation [71, 6, 72].

Similar to prior work [36, 116], we relax the assumptions on the information structure of the traditional principal-agent framework in two ways. One, the agent's characteristic is uncertain. Specifically, even though the agent knows his demand distribution, his actual demand varies from one period to the next after he enters into a binding contract. Two, the principal does not know the demand distribution of the agent but can estimate a finite number of possible distributions. To deal with the uncertainty post contracting, we introduce contracts with multiple options. The agent can select any option from the contract after his demand is realized in each period.

In recent years, efforts have been made to study carbon emissions-related issues in the production planning and design of supply chains. Song and Leng [119] obtain the optimal production quantities under three common carbon emissions policies: a strict cap on emissions, carbon tax, and cap-and-trade. Cachon [35] derives and compares the optimal retail supply chain designs under three different objectives: minimizing operating costs, minimizing carbon emissions, and minimizing both. Benjaafar et al. [15] also consider how firms' operational decisions need to be adjusted when accounting for carbon emissions under different carbon policies. Caro et al. [37] assert that double counting of emissions is needed to induce firms to make the optimal abatement efforts. Fleten et al. [56] derive optimal investment schedules for a CO_2 value chain with EOR and CO_2 permit options. An alternative real options model for choosing the optimal time to invest in capture technology under uncertain CCS costs is presented in [113].

Due to the high cost of transporting CO_2 via trucks [64], it is prudent to consider a permanent pipeline network for any long-term sequestration effort. The problem of designing a minimum-cost network is well studied [98]. However, many additional considerations arise in the design of CO_2 networks. For example, Keating et al. [74] and Middleton et al. [100] demonstrate that the geologic reservoir uncertainty has large cost implications for building a CCS infrastructure. Similarly, fluctuating network loads are caused by seasonal and daily variations in power plant output or failures in various network components, and thus the heterogeneous emissions profiles at different plants greatly impact the performance of the CCS network [99].

Different types of business models have been proposed to deal with the capture, transportation, and storage of CO_2, including "self build and operate" and "pay at the gate" [55]. The "self build and operate" model takes a vertically-integrated approach, in which emitters handle the entire chain of capture, transportation, and storage. This means that each emitter needs to obtain permits for reservoirs and build the infrastructure to operate and maintain these facilities. Emitters are intimidated not only by the high capital cost this approach requires, but also by their lack of expertise in performing such operations. Nevertheless, existing literature [98, 101] has provided the design of an optimal vertically-integrated CCS infrastructure that minimizes the total cost of capturing, transporting, and storing CO_2. Kemp and Kasim [75] and Klokk et al. [82] have incorporated Enhanced Oil Recovery (EOR) to create a value chain of CO_2, that is, selling a portion of captured CO_2 to companies with EOR operations. The "pay at the gate" business model allows a third-party operator to offer the transportation and disposition of CO_2 as a service. Because the storage operator owns the pipeline as well as the storage site, emitters only need to invest in the CO_2 capturing technology.

Since the notion of the government assigning the allowance to the emitter to emit pollution was proposed over thirty years ago, there have been many papers exploring the market-based environmental policies. Among those policies, the cap-and-trade policy is one of the most widely discussed mechanisms. Some scholars focus on the carbon allowance price. The oil shocks have a long-run asymmetric effect on the carbon allowance price and do cause the price volatility of carbon allowance from 2013 to 2020, which is shown in [142]. The carbon allowance price and emission reduction have a significant positive correlation, and industry coverage, the annual decline factor, and the free allowance rate affect the carbon allowance price, as shown in [89]. Adekoya [1] has explored a new method to predict the carbon allowance price. Some scholars paid the effort to study carbon allowance allocation policies. Zhang et al. [141] display the different allocation schemes that affect the electricity price even in other industries. It is recommended by [127] that China set a high initial free allowance and then gradually reduce it to achieve emission reduction targets while reducing economic losses. Shojaei and Mokhtar [114] present a two-step optimization mechanism to allocate the carbon allowance and found that the allocation method is more logical if including the consideration of regional heterogeneity. There are some researches exploring other topics about cap and trade. Li et al. [88] illustrate the impact of consumers' low-carbon preference level on the emission index of the government's decision. Chai et al. [38] study if cap and trade mechanism can be a benefit for re-manufacturing, and they conclude that the cap-and-trade policy is advantageous for re-manufacturing in the ordinary and green markets.

Zhang and Xu [140] propose a multi-item production planning with a cap and trade mechanism and find that emitters will get more profit under the cap and trade policy when the penalty cost is the same as the trading price. Chan and Morrow [39] evaluates the impacts of the cap and trade program, which involves nine states of the United States, in emission and damages from co-pollutants. The results show that the cap and trade program not only reduces the emission of CO_2 but also lowers the emission and associated damages of SO_2. However, this paper also addresses two factors that diminish the overall benefit of the program. But Kroes et al. [86] illustrates that carbon tradeability may not induce emitters to reduce the emission of CO_2 even with a free allowance.

4.3 Model Formulation

The principal is a CCS operator who transports and stores emission quantity q in exchange for a total payment of t. She has a cost function of $s(q) = \beta q$, and thus her profit is $t - s(q)$. An agent is an emitter who must pay the penalty for emitting. We assume that the per-unit penalty depends on the emission quantity, θ.

As a result, an agent who generates θ_{dn} emission but captures q ($q < \theta_{dn}$) emits $\theta_{dn} - q$. We refer to $\theta_{dn} - q$ as the net emission quantity. Let a_d denote the emission allowance allocated to a type-d agent. If the net emission $\theta_{dn} - q$ is over the cap, then the emitter needs to pay $\frac{\alpha}{2}(\theta_{dn} - a_d)^2$.

When the allocated allowance is greater than or equal to the net emission quantity ($a_d \geq \theta_{dn} - q$), it is free for the agent to emit $\theta_{dn} - q$. Moreover, he can collect a revenue of $\frac{\alpha}{2}(a_d - \theta_{dn} + q)^2$ by selling unused allowance. The agent, however, does incur a cost of γq for capturing q emission.

On the other hand, when the allocated allowance is less than the generated emission quantity ($a_d < \theta_{dn} - q$), the total cost of capturing q and purchasing the needed allowance is $\gamma q + \frac{\alpha}{2}(\theta_{dn} - q - a_d)^2$.

A type-d agent's value function when $q \leq \theta_{dn}$ is therefore

$$v(q, \theta_{dn}) = \begin{cases} \frac{\alpha}{2} \cdot q^2 + (\alpha(a_d - \theta_{dn}) - \gamma) \cdot q & \text{if} \quad \theta_{dn} \leq a_d \\ \frac{\alpha}{2} \cdot q^2 - (\alpha(\theta_{dn} - a_d) + \gamma) \cdot q + \alpha(\theta_{dn} - a_d)^2 & \text{if} \quad \theta_{dn} - q < a_d < \theta_{dn} \\ -\frac{\alpha}{2} \cdot q^2 + (\alpha(\theta_{dn} - a_d) - \gamma) \cdot q & \text{if} \quad a_d \leq \theta_{dn} - q \end{cases} \tag{4.1}$$

For simplify, we use $\hat{\tau}(q_{dn})$ to represent $v(q_{dn}, \theta_{dn})$ when $\theta_{dn} - q_{dn} < a_d < \theta_{dn}$ and $\bar{\tau}(q_{dn})$ to represent $v(q_{dn}, \theta_{dn})$ when $a_d < \theta_{dn} - q_{dn}$.

When the agent picks the option (q,t), that q is actually greater than θ_{dn}. The agent captures all emissions emit zero, so she doesn't need to pay the penalty but pay the capturing cost $\gamma\theta_{dn}$. In addition, she can get the revenue of $\frac{\alpha}{2}a_d^2$ by selling unused allowance.

If the agent will not participate in the cost-sharing program, the agent needs to pay the penalty $\frac{\alpha}{2}(\theta_{dn}-a_d)^2$ when the allowance a_d is less than θ_{dn}. But if $\theta_{dn} \leq a_d$, there is no penalty and the agent can get revenue of $\frac{\alpha}{2}(a_d-\theta_{dn})^2$ of selling unused allowance.

A type-d agent's value function when $q > \theta_{dn}$ is therefore:

$$v(q,\theta_{dn}) = \begin{cases} \frac{\alpha}{2}\cdot\theta_{dn}^2 + (\alpha(a_d-\theta_{dn})-\gamma)\cdot\theta_{dn} & \text{if} \quad \theta_{dn} \leq a_d \\ \frac{\alpha}{2}\cdot\theta_{dn}^2 - (\alpha(\theta_{dn}-a_d)+\gamma)\cdot\theta_{dn} + \alpha(\theta_{dn}-a_d)^2 & \text{if} \quad a_d < \theta_{dn} \end{cases} \tag{4.2}$$

The agent's emission quantity is uncertain, which comes from two sources. First, the electricity demand of a power plant may vary from one period to the next due to seasonality. Second, the generated emission depends on the operational efficiency and the capturing technology used at the power plant. As a result, the emission quantity may follow a number (D) of distributions. We thus refer to each distribution as θ_{dn}, where $d = 1, \cdots, D$. Though the principal does not know which distribution an agent has, she forms a prior probability distribution over θ_{dn} as follows:

$$\mathbf{\Theta}_d = \begin{bmatrix} \boldsymbol{\theta}_1 \\ \vdots \\ \boldsymbol{\theta}_d \\ \vdots \\ \boldsymbol{\theta}_N \end{bmatrix} = \begin{bmatrix} [\theta_{11},\cdots\theta_{1n},\cdots\theta_{1N}] \\ \vdots \\ [\theta_{d1},\cdots\theta_{dn},\cdots\theta_{dN}] \\ \vdots \\ [\theta_{D1},\cdots\theta_{Dn},\cdots\theta_{DN}] \end{bmatrix} \text{ with probabilities } \begin{bmatrix} \mu_1[\pi_{11},\cdots\pi_{1n},\cdots\pi_{1N}] \\ \vdots \\ \mu_d[\pi_{d1},\cdots\pi_{dn},\cdots\pi_{dN}] \\ \vdots \\ \mu_D[\pi_{D1},\cdots\pi_{Dn},\cdots\pi_{DN}] \end{bmatrix}$$

Figure 4.1 illustrates when an agent makes a decision and its implications subsequently. We refer to the agent whose demand follows θ_{dn} as a type-d agent henceforth. In the contracting stage, the principal does not know the agent's demand distribution. Therefore, she offers a menu of contracts $\{x_d\}_{d=1,\cdots,D}$ to the agent. Each contract contains multiple options: $x_d = [(q_{d1}, t_{d1}), \cdots, (q_{dN}, t_{dN})]$. The agent selects a contract $x_{d'}$ that maximizes his expected utility. Upon committing to the contract x_d, the agent can only pick an option within x_d during the subsequent implementation stage. As an example, in the first period ($t = 1$), the agent observes his realized demand θ_{dn} and selects an option, say $(q_{d'm}, t_{d'm})$, that maximizes his utility in that period. In the second period ($t = 2$), a different demand ($\theta_{dn'}$) may be realized. As a result, another option, $(q_{d'm'}, t_{d'm'})$, is selected instead.

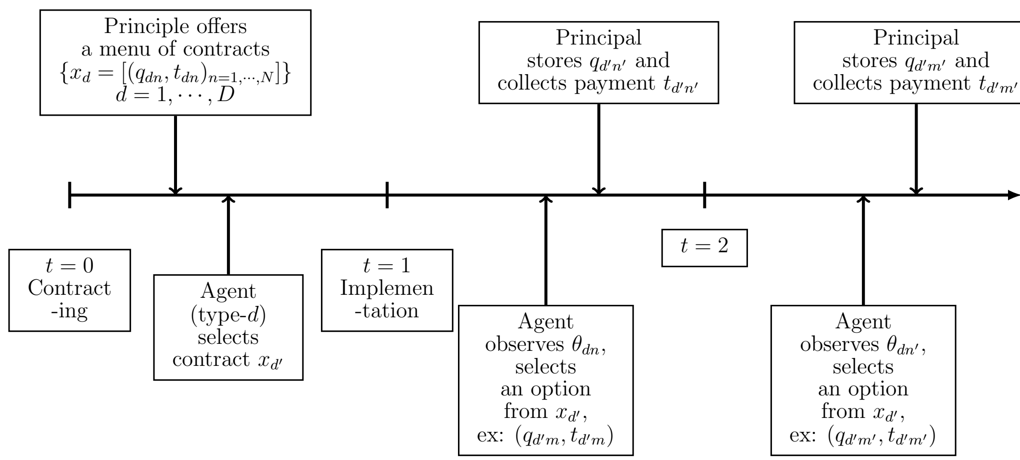


Figure 4.1 Sequence of Events.

According to the revelation principle, the principal should design a menu of incentive-compatible contracts to induce truth-telling from the agent. Therefore, contract x_d should generate the highest expected utility for a type-d agent among all contracts. Moreover, when the agent's realized demand is θ_{dn}, the option (q_{dn}, t_{dn}) should provide the agent the highest utility when compared to all the other options in contract x_d.

The purpose of the CCS operator is to maximize the profit by providing the designed contracts to the emitters. However, the government wants the emitters to capture the CO_2 as much as they can with the optimal allocated allowance(a_d). To satisfy those two objects, we treat these as a two-stage problem. We first explore the optimal profit of the CCS operator, and then we compare the expected capturing CO_2 quantity under different a_d to find the maximized expected capturing quantity.

For inducing the emitters to receive the contract designed by the CCS operator, the CCS operator should make sure that the emitter's expected value ($\pi_{dl}v(q_{dl}, \theta_{dl}) + \pi_{dh}v(q_{dh}, \theta_{dh})$) is not greater than the expected payment ($\pi_{dl}t_{dl} + \pi_{dh}t_{dh}$) offered by her/him.

We begin by illustrating the optimal solutions when a cap-and-trade market is not available. These results are used to compare those under the two types of cap-and-trade policies.

Let Φ denote the principal's expected profit and $\Delta_{dn} = v(q_{dn}, \theta_{dn}) - t_{dn}$ denote the information rent given to a type-d agent when his generated emission is θ_{dn}. The first stage is exploring the principal's optimization problem, which is shown as follows:

$$\begin{array}{llll} \max & \Phi = \sum\limits_{d} \mu_d \left[\sum\limits_{n} \pi_{dn}(t_{dn} - \beta q_{dn}) \right] & & \\ \text{s.t.} & \sum\limits_{n} \pi_{dn}\Delta_{dn} \geq 0 & d = 1, \cdots, D & (IR_d) \\ & \sum\limits_{n} \pi_{dn}\Delta_{dn} \geq \sum\limits_{n} \pi_{dn} \max\limits_{n'}\{v(q_{d'n'}, \theta_{dn}) - t_{d'n'}\} & d, d' = 1, \cdots, D, \, d' \neq d & (IC_{dd'}) \\ & \Delta_{dn} \geq v(q_{dn'}, \theta_{dn}) - t_{dn'} & n, n' = 1, \cdots, N, \, n' \neq n & (IC_{dnn'}) \\ \text{and} & q_{dn} \geq 0 & d, d' = 1, \cdots, D, \, n = 1, \cdots, N & (NN_{dn}) \end{array} \tag{4.3}$$

The CCS operator wants to maximize her/his expected profit and design the optimal contract $((q_{dl}, t_{dl}), (q_{dh}, t_{dh}))$. The objective function consists of weighted profit terms for each level. IR_d is an individual rationality constraint to incentive the emitter to participate. $IC_{dd'}$ constraints make the type d agent prefer to pick contract x_d instead of other contracts by ensuring the expected utility from choosing contract x_d is greater than or equal to the expected utility of other contracts. IC_{dhl} and IC_{dlh} are incentive compatibility constraints that make the emitter prefer the option that the CCS operator designs. UB_{dn} are the upper bond constraints to ensure capturing a quantity of each demand level is less than or equal to the actual demand quantity. NN_{dn} constraints make contract quantity and payment non-negative.

In the second stage, let Q denote the expected capturing quantities of CO_2. We want to find the optimal minimum expected allowance ($\sum_d \mu_d a_d$), which can maximize the expected capturing quantity. The government's optimization problem becomes

$$\max_{a_d} \quad Q = \sum_d \mu_d \left(\pi_{dl} q^*_{dl}(a_d) + \pi_{dh} q^*_{dh}(a_d)\right) \tag{4.4}$$

4.4 Single Distribution: $D = 1$

When there is only one type of agent, we treat it as a single distribution. In this section, we present the optimal solutions for different demand levels with and without the allowance.

4.4.1 Demand Level is Known: $N = 1$

We first present the optimal solution for the special case where the agent's demand distribution θ_{dn} is known and there is no demand uncertainty. That is, when $D = 1$ and $N = 1$.

Proposition 5. *When the agent's demand is certain, and the government does not assign any allowance to the agent i.e., $\theta_{dn} > \frac{\gamma+\beta}{\alpha}$ and $a_d = 0$, the optimal price and quantity are $(\overline{q}_{dn}, \overline{t}_{dn})$ (the proof is shown in Appendix A.3.1.1), where*

$$\overline{q}_{dn} = \theta_{dn} - \tfrac{\gamma+\beta}{\alpha}, \quad \overline{t}_{dn} = \bar{\tau}(\overline{q}_{dn}) = (\alpha\theta_{dn} - \gamma)\overline{q}_{dn} - \tfrac{\alpha}{2}\overline{q}^2_{dn}. \tag{4.5}$$

We refer to $\overline{q}_{dn}$ as the efficient quantity and $\overline{t}_{dn}$ as the full price. We use E to represent the case that capturing quantity is equal to the efficient quantity.

Proposition 6. *When the government does assign the allowance, the optimal allowance* $a_d = (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$*, if the agent's demand is known i.e.,* $\theta_{dn} > \frac{\gamma+\beta}{\alpha}$*, the optimal price and quantities are* $(\hat{q}_{dn}, \hat{t}_{dn})$ *(the proof is shown in Appendix A.3.1.2), where*

$$\hat{q}_{dn} = \theta_{dn}, \quad \hat{t}_{dn} = \hat{\tau}(\hat{q}_{dn}) = \tfrac{\alpha}{2}\theta_{dn}^2 - (\alpha(\theta_{dn} - a_d) + \gamma)\theta_{dn} + \alpha(\theta_{dn} - a_d)^2 \tag{4.6}$$

The quantity $\hat{q}_{dn}$ is referred to as the full quantity, and F represents capturing the full quantity. The price $\hat{t}_{dn}$ is referred to as the sacrificial price. Further, since the capturing cost of the agent is greater than the marginal cost of the principal $\gamma > \beta$, as a result, $\bar{t}(\bar{q}_{dn}) - \hat{t}(\hat{q}_{dn}) = \frac{\gamma^2-\beta^2}{2\alpha} + \alpha(\theta_{dn} - a_d)a_d > 0$, the full price is higher than the sacrificial price i.e., $\bar{t}(\bar{q}_{dn}) > \hat{t}(\hat{q}_{dn})$.

4.4.2 Single Bi-level Distribution: $N = 2$

Let us next show the case that the demand is uncertain. However, there is only one type of agent. And we will summarise the optimal quantity and price in two different cases.

4.4.2.1 The government does not assign any allowance to the agent.

First, we present the optimal solution for the case that the government gives no allowance.

Proposition 7. *When the agent's demand distribution is known, i.e.,* $\theta_{dn} = \boldsymbol{\theta_d}$*, the optimal quantities and prices are (the proof is shown in Appendix A.3.1.3):*

	$(q_{dl}^*, \quad t_{dl}^*)$	$(q_{dh}^*, \quad t_{dh}^*)$
d	$(\bar{q}_{dl}, \quad \bar{\tau}(\bar{q}_{dl}) - \Delta_{dl}^*)$	$(\bar{q}_{dh}, \quad \bar{\tau}(\bar{q}_{dh}) - \Delta_{dh}^*)$

where

$$\Delta_{dl}^{*} = -\pi_{dh}\alpha(\theta_{dh} - \theta_{dl})(\theta_{dl} - \tfrac{\gamma+\beta}{\alpha}),$$
$$\Delta_{dh}^{*} = \pi_{dl}\alpha(\theta_{dh} - \theta_{dl})(\theta_{dl} - \tfrac{\gamma+\beta}{\alpha}). \tag{4.7}$$

The optimal quantity at both demand levels is the efficient quantity. However, the price designed for the low demand level is higher than the full price, and the agent with the high demand level will be charged a lower price than her/his full price. Further, It's important to note the net (or expected) information rent given to the agent is

$$E\Delta_{d}^{*} = \pi_{dl}\Delta_{dl}^{*} + \pi_{dh}\Delta_{dh}^{*} = 0. \tag{4.8}$$

4.4.2.2 The government does assign the agent an allowance to emit CO_2.

In this section, we first list the optimal solution at the low and high levels of a_d, and the optimal a_d which maximizes the capturing CO_2 is shown afterward.

Proposition 8. *When the agent's demand distribution is known and the allowance is low ($0 < a_d < (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$), the optimal quantities and prices are (the proof is shown in Appendix A.3.1.4):*

	$(q_{dl}^{*},\quad t_{dl}^{*})$	$(q_{dh}^{*},\quad t_{dh}^{*})$
d	$(\overline{q}_{dl} - a_d,\quad \overline{\tau}(\overline{q}_{dl} - a_d) - \Delta_{dl}^{*})$	$(\overline{q}_{dh} - a_d,\quad \overline{\tau}(\overline{q}_{dh} - a_d) - \Delta_{dh}^{*})$

where

$$\Delta_{dl}^{*} = -\pi_{dh}\alpha(\theta_{dh} - \theta_{dl})q_{dl}^{*},$$
$$\Delta_{dh}^{*} = \pi_{dl}\alpha(\theta_{dh} - \theta_{dl})q_{dl}^{*}. \tag{4.9}$$

The optimal quantity of both demand levels is the efficient quantity ($\pi_{dl}\theta_{dl} + \pi_{dh}\theta_{dh} - \frac{\gamma+\beta}{\alpha}$) subtracting the allowance a_d, so the expected quantity of capturing CO_2 under the low allowance.

Proposition 9. *When the agent's demand distribution is known and the allowance is low ($a_d \geq (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$), the optimal quantities and prices are (the proof is shown in Appendix A.3.1.5):*

	$(q^*_{dl},\quad t^*_{dl})$	$(q^*_{dh},\quad t^*_{dh})$
d	$(\theta_{dl},\quad \hat{\tau}(\theta_{dl}) - \Delta^*_{dl})$	$(\theta_{dh},\quad \hat{\tau}(\theta_{dh}) - \Delta^*_{dh})$

where

$$\begin{aligned} \Delta^*_{dl} &= -\pi_{dh}\left[\omega \cdot v(\theta_{dh}, \theta_{dh}) + \bar{\omega} \cdot v(\theta_{dl}, \theta_{dh}) - v(\theta_{dl}, \theta_{dl})\right], \\ \Delta^*_{dh} &= \pi_{dl}\left[\omega \cdot v(\theta_{dh}, \theta_{dh}) + \bar{\omega} \cdot v(\theta_{dl}, \theta_{dh}) - v(\theta_{dl}, \theta_{dl})\right]. \\ & \textit{and} \quad \omega \in [0,1]. \end{aligned} \tag{4.10}$$

Capturing all quantities is the optimal choice for the agent with low and high demand levels. Therefore, the expected quantity is $\pi_{dl}\theta_{dl} + \pi_{dh}\theta_{dh}$.

What is more, no matter what is the value of the allowance a_d, the principal is able to induce the agent to participate and capture the efficient, even full quantities without giving any net information rent. S/he can achieve the desired behavior by charging a premium price when the generated emission is low and a reduced price when the generated emission is high.

We show the optimal solutions under the different values of a_d for the single bi-level distribution case above. Table 4.1 illustrates the expected capturing CO_2 (E_q), the agent's expected information rent ($E\Delta$), and the principle's utility (Φ). Let $\Phi_0 = \pi_{dl} \cdot \frac{\alpha}{2}\bar{q}^2_{dl} + \pi_{dh} \cdot \frac{\alpha}{2}\bar{q}^2_{dh}$ which is the principal's optimal utility when there is no allowance assigned by the government. As shown in Table 4.1, the emitter will capture the most CO_2 when the allowance is not less than $(1+\sqrt{2}) \cdot \frac{\gamma+\beta}{\alpha}$, however, the utility of the principal will reduce.

Table 4.1 Comparison of the Single Bi-level Distribution with Different Values of a_d

	E_q	$E\Delta$	Φ
$a_d = 0$	$\pi_{dl}\bar{q}_{dl} + \pi_{dh}\bar{q}_{dh}$	0	Φ_0
$0 < a_d < (1+\sqrt{2}) \cdot \frac{\gamma+\beta}{\alpha}$	$\pi_{dl}\bar{q}_{dl} + \pi_{dh}\bar{q}_{dh} - a_d$	0	$\Phi_0 + \frac{\alpha}{2}a_d^2 - \alpha(\pi_{dl}\bar{q}_{dl} + \pi_{dh}\bar{q}_{dh})a_d$
$(1+\sqrt{2}) \cdot \frac{\gamma+\beta}{\alpha} \le a_d$	$\pi_{dl}\theta_{dl} + \pi_{dh}\theta_{dh}$	0	$\Phi_0 - \frac{\alpha}{2}(\frac{\gamma+\beta}{\alpha})^2 - \alpha(\pi_{dl}\theta_{dl} + \pi_{dh}\theta_{dh})a_d$

What is more, the government actually wants to set an allowance as small as possible, and the emitter can still be induced to capture maximum CO_2. Consequently, the optimal a_d^* is $(1+\sqrt{2}) \cdot \frac{\gamma+\beta}{\alpha}$.

4.5 Dual Distributions: $D = 2$ and $N = 2$

4.5.1 The Allowance is Not Given by the Government to Any Agent

In this section, we explore the case when the distribution of the two types of agents partially overlap, $\theta_{1l} < \theta_{2l} < \theta_{1h} < \theta_{2h}$ and there is no allowance assigned by the government. We will list two types of analytical solutions in the following.

4.5.1.1 All efficient quantities contracts. We first explore the all-efficient-quantities case. If all demand levels are offered efficient quantities, and the net information rent for each type is zero. We refer to this as all efficient quantities contracts, and the solutions are shown in Table 4.2.

Table 4.2 Efficient Contracts

	$(q_{dl}^*, \quad t_{dl}^*)$	$(q_{dh}^*, \quad t_{dh}^*)$
$d = 1$	$(\bar{q}_{1l}, \quad \bar{\tau}_{1l}(\bar{q}_{1l}) - \Delta_{1l}^*)$	$(\bar{q}_{1h}, \quad \bar{\tau}_{1h}(\bar{q}_{1h}) - \Delta_{1h}^*)$
$d = 2$	$(\bar{q}_{2l}, \quad \bar{\tau}_{2l}(\bar{q}_{2l}) - \Delta_{2l}^*)$	$(\bar{q}_{2h}, \quad \bar{\tau}_{2h}(\bar{q}_{2h}) - \Delta_{2h}^*)$

the information rents are:

$$\Delta^*_{1l} = -\pi_{1h} \cdot \alpha(\theta_{1h} - \theta_{1l})q^*_{1l},$$
$$\Delta^*_{1h} = \pi_{1l} \cdot \alpha(\theta_{1h} - \theta_{1l})q^*_{1l};$$
$$\Delta^*_{2l} = -\pi_{2h} \cdot \alpha(\theta_{2h} - \theta_{2l})q^*_{2l},$$
$$\Delta^*_{2h} = \pi_{2l} \cdot \alpha(\theta_{2h} - \theta_{2l})q^*_{2l}.$$

In this particular solution, the constraints IR_1, IR_2, IC_{12} and IC_{1lh} are binding. All demand levels can be offered their related efficient quantities. However, the net information rent is zero regardless of the demand level. The principal can maximize her/his profit by charging a higher price for a low demand level and decreasing the price for a high demand level compared to his/her full price.

4.5.1.2 A single contract. When the principal only serves one type of agents, we call this solution a single contract. The constraints IR_1, IR_2, IC_{21}, IC_{1hl}, IC_{1lh}, IC_{2hl} are binding. The optimal solutions are shown in Table 4.3.

Table 4.3 A Single Contract

	$(q^*_{dl},\quad t^*_{dl})$	$(q^*_{dh},\quad t^*_{dh})$
$d=1$	$(0,\quad 0)$	$(0,\quad 0)$
$d=2$	$(\overline{q}_{2l},\quad \overline{\tau}_{2l}(\overline{q}_{2l}) - \Delta^*_{2l})$	$(\overline{q}_{2h},\quad \overline{\tau}_{2h}(\overline{q}_{2h}) - \Delta^*_{2h})$

where

$$\Delta^*_{2l} = \quad -\pi_{2h}\alpha(\theta_{2h} - \theta_{2l})q^*_{2l},$$
$$\Delta^*_{2h} = \quad \pi_{2l}\alpha(\theta_{2h} - \theta_{2l})q^*_{2l}.$$

In this solution, the principal only serves the type-2 agent, and s/he can offer the efficient quantity for both demand levels and charge a higher price than the full price when the demand level is low and a reduced price when the demand level is high to achieve her/his purpose when μ_1 is low.

4.5.2 The Allowance is Given by the Government to At Least One Type of Agent

The government may set an allowance to make the principal design a contract to induce the emitters to capture as much as possible CO_2. We found that there are two kinds of optimal solutions. The first one is capturing full quantity regardless of the type of agent and the demand level of agents. The second one is capturing efficient quantity for the type-1 agent and full quantity for the type-2 agent regardless of demand levels.

4.5.2.1 Full quantities are optimal solutions. When both types of agents choose to capture all quantities ($q_{dn} = \theta_{dn}$ $d = 1, 2$ and $n = l, h$) and the government will assign a large allowance to both types of agents. The agent's information rent with the different types and demand levels can be different. The expected information rent given to the type-1 agent is always equal to zero; however, the expected information rent of the type-2 agent can be positive or zero. We present two examples in the case when the expected information rent of the type-2 agent is zero or positive.

(1) the type-2 agent's expected information rent is zero ($\pi_{2l}\Delta_{2l} + \pi_{2h}\Delta_{2h} = 0$). We present an example of this solution in Table 4.4 which occurs when the constraints IR_1, IR_2, and IC_{12} are binding.

Table 4.4 The Expected Information Rent of the Type-2 Agent is Zero

	$(q^*_{dl},\ t^*_{dl})$	$(q^*_{dh},\ t^*_{dh})$
$d = 1$	$(\theta_{1l},\ \hat{\tau}(\theta_{1l}) - \Delta^*_{1l})$	$(\theta_{1h},\ \hat{\tau}(\theta_{1h}) - \Delta^*_{1h})$
$d = 2$	$(\theta_{2l},\ \hat{\tau}(\theta_{2l}) - \Delta^*_{2l})$	$(\theta_{2h},\ \hat{\tau}(\theta_{2h}) - \Delta^*_{2h})$

The following expands the terms in Table 4.4:

$$\begin{aligned} \Delta_{1l}^* = & \quad -\pi_{1h} \cdot (v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})) \\ \Delta_{1h}^* = & \quad \pi_{1l} \cdot (v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})) \,. \end{aligned}$$

$$\begin{aligned} \Delta_{2l}^* = & \quad v(\theta_{2l}, \theta_{2l}) - \pi_{1l} \cdot v(\theta_{2l}, \theta_{1l}) - \pi_{1h} \cdot v(\theta_{2l}, \theta_{1h}) \\ \Delta_{2h}^* = & \quad -\tfrac{\pi_{2l}}{\pi_{2h}} \cdot \Delta_{2l}^*. \end{aligned}$$

In this solution, even though the quantities offered to all types and demand levels of the agents are full quantities, the expected information rent of both two types of agents is zero ($E\Delta_d^* = 0, d = 1, 2$). The type-1 agent with the high emission demand will be charged a lower price than the sacrificial price; however, s/he will pay more price than the sacrificial price when the generated emission is low. The type-2 agent will be charged a premium price when the emission level is low and a reduced price when the emission level is high when π_{1h} is high enough.

(2) the type-2 agent's expected information rent is positive ($\pi_{2l}\Delta_{2l} + \pi_{2h}\Delta_{2h} > 0$). We present an example of this solution in Table 4.5 which occurs when the constraints IR_1, IC_{21}, IC_{1hl} and IC_{2hl} are binding.

Table 4.5 The Expected Information Rent of the Type-2 Agent is Positive

	$(q_{dl}^*,$	$t_{dl}^*)$	$(q_{dh}^*,$	$t_{dh}^*)$
$d = 1$	$(\theta_{1l},$	$\hat{\tau}(\theta_{1l}) - \Delta_{1l}^*)$	$(\theta_{1h},$	$\hat{\tau}(\theta_{1h}) - \Delta_{1h}^*)$
$d = 2$	$(\theta_{2l},$	$\hat{\tau}(\theta_{2l}) - \Delta_{2l}^*)$	$(\theta_{2h},$	$\hat{\tau}(\theta_{2h}) - \Delta_{2h}^*)$

The following expands the terms in Table 4.5:

$$\begin{aligned}
\Delta_{1l}^* = &\quad -\pi_{1h} \cdot (v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})) \\
\Delta_{1h}^* = &\quad \pi_{1l} \cdot (v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}))\,.
\end{aligned}$$

$$\begin{aligned}
\Delta_{2l}^* = &\quad \delta_1 - \pi_{2h} \cdot (v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})) \\
\Delta_{2h}^* = &\quad \delta_1 + \pi_{2l} \cdot (v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l}))\,.
\end{aligned}$$

and,

$$\begin{aligned}
\delta_1 = &\quad \pi_{2l} \cdot \max\{v(\theta_{1l}, \theta_{2l}) - v(\theta_{1l}, \theta_{1l} + \Delta_{1l}^*, v(\theta_{1h}, \theta_{2l}) - v(\theta_{1h}, \theta_{1h} + \Delta_{1h}^*\} \\
&\quad +\pi_{2h} \cdot \max\{v(\theta_{1l}, \theta_{2h}) - v(\theta_{1l}, \theta_{1l} + \Delta_{1l}^*, v(\theta_{1h}, \theta_{2h}) - v(\theta_{1h}, \theta_{1h} + \Delta_{1h}^*\} \geq 0.
\end{aligned}$$

The principal offers the full quantity for both demand levels of both two types of agents in this solution. The net information rent of the type-1 agent is zero, and the principal will charge a higher price when the generated emission is low and a reduced price when the emission demand is high. However, the net information rent of the type-2 agent is positive, and the agent with a high emission demand must have a lower price which is less than the sacrificial price.

4.5.2.2 Efficient quantities for the type-1 agent and full quantities for the type-2 agent are the optimal solutions. Capturing efficient quantity for the type-1 agent and full quantity for the type-2 agent is optimal only if the distribution for θ is partial overlapping ($\theta_{1l} < \theta_{2l} < \theta_{1h} < \theta_{2h}$) and the allowance assigned to the type-1 agent is zero, i.e., $a_1 = 0$ but the allowance assigned to the type-2 agent is large.

In this section, we show two examples in the case when the expected information rent of the type-1 agent is zero or positive.

(1) the type-1 agent's expected information rent is zero ($\pi_{1l}\Delta_{1l} + \pi_{1h}\Delta_{1h} = 0$).

We show an example of when IR_1, IR_2 and IC_{1hl} are binding in Table 4.6.

Table 4.6 The Expected Information Rent of the Type-1 Agent is Zero

	$(q^*_{dl}, \quad t^*_{dl})$	$(q^*_{dh}, \quad t^*_{dh})$
$d = 1$	$(\overline{q}_{1l}, \quad \tau(\overline{q}_{1l}) - \Delta^*_{1l})$	$(\overline{q}_{1h}, \quad \tau(\overline{q}_{1h}) - \Delta^*_{1h})$
$d = 2$	$(\theta_{2l}, \quad \hat{\tau}(\theta_{2l}) - \Delta^*_{2l})$	$(\theta_{2h}, \quad \hat{\tau}(\theta_{2h}) - \Delta^*_{2h})$

The following expands the terms in Table 4.6:

$$\begin{aligned} \Delta^*_{1l} &= \; -\pi_{1h}\alpha(\theta_{1h} - \theta_{1l})q_{1l}. \\ \Delta^*_{1h} &= \; \pi_{1l}\alpha(\theta_{1h} - \theta_{1l})q_{1l}. \end{aligned}$$

$$\begin{aligned} \Delta^*_{2l} &= \; -\pi_{2h} \cdot \delta_1. \\ \Delta^*_{2h} &= \; \pi_{2l} \cdot \delta_1. \end{aligned}$$

and,

$$\delta_1 = \; v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})$$

In this optimal contract, the principal provides the type-1 agent with the efficient quantity while charging a reduced price for the high emission level and a higher price for the low emission level. Full quantity is offered to the type-2 agent, and s/he with low emission demand will be charged a higher price; however, the lower price will be assigned to the high emission demand of the type-2 agent. What is more, the net expected information rent of both types of agents is zero.

(2) the type-1 agent's expected information rent is positive ($\pi_{1l}\Delta_{1l} + \pi_{1h}\Delta_{1h} > 0$).

We show an example of when IR_2 and IC_{12} are binding in Table 4.7.

Table 4.7 The Expected Information Rent of the Type-1 Agent is Positive

	$(q^*_{dl},$	$t^*_{dl})$	$(q^*_{dh},$	$t^*_{dh})$
$d=1$	$(\overline{q}_{1l},$	$\tau(\overline{q}_{1l}) - \Delta^*_{1l})$	$(\overline{q}_{1h},$	$\tau(\overline{q}_{1h}) - \Delta^*_{1h})$
$d=2$	$(\theta_{2l},$	$\hat{\tau}(\theta_{2l}) - \Delta^*_{2l})$	$(\theta_{2h},$	$\hat{\tau}(\theta_{2h}) - \Delta^*_{2h})$

The following expands the terms in Table 4.7:

When $\pi_{1l} = \pi_{2l}$,

$$\begin{aligned} \Delta^*_{2l} = &\ -\pi_{2h} \cdot (v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})). \\ \Delta^*_{2h} = &\ \pi_{2l} \cdot (v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})). \end{aligned}$$

and,

$$\begin{aligned} \Delta^*_{1l} &= \pi_{1l}\left(v(\theta_{2l}, \theta_{1l}) - v(\theta_{2l}, \theta_{2l}) + \Delta^*_{2l}\right) + \pi_{1h}\left(v(\theta_{2h}, \theta_{1h}) - v(\theta_{2h}, \theta_{2h}) + \Delta^*_{2h}\right) \\ &\quad -\pi_{1h} \cdot \alpha(\theta_{1h} - \theta_{1l})q_{1l}. \\ \Delta^*_{1h} &= \pi_{1l}\left(v(\theta_{2l}, \theta_{1l}) - v(\theta_{2l}, \theta_{2l}) + \Delta^*_{2l}\right) + \pi_{1h}\left(v(\theta_{2h}, \theta_{1h}) - v(\theta_{2h}, \theta_{2h}) + \Delta^*_{2h}\right) \\ &\quad +\pi_{1l} \cdot \alpha(\theta_{1h} - \theta_{1l})q_{1l}. \end{aligned}$$

We present the case when $\pi_{1l} = \pi_{2l}$ and the case when $\pi_{1l} \neq \pi_{2l}$ is shown in the Appendix A.3.2.2. In this solution, the efficient quantities are given to the type-1 agent with a positive net expected information rent.

4.6 Numerical Solutions

We introduce the different kinds of optimal solutions when capturing all quantities is optimal for both two types of agents. In this section, we explore the conditions which cause different solutions and the reason why the principal prefers not to offer all quantities to the type-1 agent in some cases.

Table 4.8 Solutions Types for Various μ_1, π_{1l} and π_{2l}

(a) When $\mu_1 = 0.25$

π_{1l}	0.25	0.50	0.75
$\pi_{2l} = 0.25$	$\triangleright$	$\diamond$	$\bigcirc$
$\pi_{2l} = 0.50$	$\square$	$\triangleright$	$\triangleright$
$\pi_{2l} = 0.75$	$\diamond$	$\diamond$	$\triangleright$

(b) When $\mu_1 = 0.50$

π_{1l}	0.25	0.50	0.75
$\pi_{2l} = 0.25$	$\triangleleft$	$\triangleright$	$\triangleright$
$\pi_{2l} = 0.50$	$\square$	$\triangleleft$	$\triangleright$
$\pi_{2l} = 0.75$	$\diamond$	$\triangleright$	$\triangleleft$

(c) When $\mu_1 = 0.75$

π_{1l}	0.25	0.50	0.75
$\pi_{2l} = 0.25$	$\triangle$	$\triangleleft$	$\triangleleft$
$\pi_{2l} = 0.50$	$\square$	$\triangle$	$\triangleleft$
$\pi_{2l} = 0.75$	$\diamond$	$\square$	$\triangleright$

Note: We use $(\theta_{1l}, \theta_{1h}, \theta_{2l}, \theta_{2h}) = (2, 4, 3, 5)$ in Mt. Pink colors correspond to the case that $E\Delta_2 = 0$ and blue colors correspond to the case that $E\Delta_2 > 0$ for F_1F_2 solution. White color represents E_1F_2 solution. Different color's degrees of pink and blue represent different solutions.

Table 4.8 displays the six various solutions when μ_1 is respectively fixed to 0.25, 0.50 and 0.75. Each color represents a unique solution. Among those colors, the solutions represented by the pink and blue colors have one thing in common, which is all demand levels in both types of agents capture their full demand of CO_2. However, the white cell with a circle pattern represents another optimal solution: the type-2 agent captures full quantity while the type-1 agent captures efficient quantity.

The patterns diamond and square in the pink cells show solutions that the expected information rent of both types of agents is zero. Among those pink cells, the light pink cells with a diamond pattern correspond to the case when IR_1, IR_2, and IC_{12} constraints are binding, and the dark pink cells with the square pattern correspond to the case when IR_1 and IR_2 bind.

Blue color cells represent the solutions when the expected information rent of the type-1 agent is zero; however, the type-2 agent's expected information rent is positive. The case when IR_1 and IC_{21} are binding is represented by the dark blue cells with a right triangle pattern. The cornflower blue cells with the left triangle pattern correspond to the case when IR_1, IC_{21}, IC_{1hl} and IC_{2hl} bind. What's more, the sky blue cells with a triangle pattern correspond to the case when IR_1, IC_{21}, IC_{12}, and IC_{1hl} bind.

As shown in Table 4.8, when π_{1l} is low and π_{2l} is high, the expected information rent of the type-2 agent intends to be zero. But when $\mu_1 = 0.25$, $\pi_{1l} = 0.75$ and $\pi_{2l} = 0.25$, the optimal solution is that the type-1 agent captures efficient quantities and the type-2 agent captures the full quantities represented by the white cell with a circle pattern. We will explore the reason why the full quantities for both types of agents are not optimal in the following.

Table 4.9 Solution Types for Various μ_1, π_{1l} and π_{2l}

(a) When $\pi_{2l} = 0.1$

π_{1l}	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
$\mu_1 = 0.05$			$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\ominus$	$\ominus$
$\mu_1 = 0.10$							$\oplus$	$\oplus$	$\ominus$	$\ominus$
$\mu_1 = 0.15$								$\ominus$	$\ominus$	$\ominus$
$\mu_1 = 0.20$								$\ominus$	$\ominus$	$\ominus$
$\mu_1 = 0.25$										$\ominus$

(b) When $\pi_{2l} = 0.3$

π_{1l}	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
$\mu_1 = 0.05$			$\otimes$	$\otimes$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\ominus$
$\mu_1 = 0.10$				$\otimes$	$\otimes$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\ominus$
$\mu_1 = 0.15$					$\otimes$	$\oplus$	$\oplus$	$\oplus$	$\ominus$	$\ominus$
$\mu_1 = 0.20$						$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\ominus$
$\mu_1 = 0.25$									$\oplus$	$\ominus$

(c) When $\pi_{2l} = 0.5$

π_{1l}	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
$\mu_1 = 0.05$					$\otimes$	$\otimes$	$\oplus$	$\oplus$	$\oplus$	$\oplus$
$\mu_1 = 0.10$						$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$
$\mu_1 = 0.15$									$\ominus$	$\oplus$
$\mu_1 = 0.20$										
$\mu_1 = 0.25$										

To find the reason why the full quantities for both two types of agents are not optimal when $\mu_1 = 0.25$, $\pi_{1l} = 0.75$, and $\pi_{2l} = 0.25$, we reduce the intervals between μ_1, π_{1l} and π_{2l} changes as shown in Table 4.9. In Table 4.9, the gray cells with no patterns represent the case that full quantities are optimal regardless of types of agents and demand levels. Other cells represent that efficient quantities are optimal for the type-1 agent and the full quantities are optimal for the type-2 agent.

Among the colorful cells (not gray), the lime color cells with a minus symbol in a circle correspond to the case when the constraints IR_1 and IR_2 are both binding. The pear green cells with a plus symbol in a circle correspond to the case when IR_1, IR_2, and IC_{1hl} are binding, and the corn yellow color cells with a times symbol in a circle correspond to the case when IR_2 , IC_{12}, and IC_{1hl} are binding.

Table 4.9 just displays the solutions when π_{2l} is 0.1, 0.3, and 0.5. Because if we continue to increase the value of π_{2l}, the optimal solutions will be full quantities for all kinds of agents, which are the same as not increasing the value of μ_1 more than 0.25. In other words, the efficient quantities designed for the type-1 agent and the full quantities designed for the type-2 agent are optimal only when the value of $\mu_2 \times \pi_{2h}$ is large. Further, the increased revenue from the type-1 agent can not reach the increased information rent to the type-2 agent; therefore, full quantities are not optimal for the type-1 agent when $\mu_2 \times \pi_{2h}$ is large.

What we also noticed is that the allowance given to the agent is large ($a_d \geq (1+\sqrt{2}) \cdot \frac{\gamma+\beta}{\alpha}$) if full quantities are designed for that agent. However, if efficient quantities are optimal for an agent, it is not necessary to assign any allowance to that agent ($a_d = 0$).

4.7 Comparison

In this section, we explore the difference between the non-cap-and-trade policy and the cap-and-trade policy. The cap-and-trade policy is that the government provides an allowance, and the emitters will pay the penalty if s/he emits the extra CO_2 out of the allowance into the air; however, the emitters can buy and sell the allowance on the market. The non-cap-and-trade policy means that the government does not assign any allowance to the emitters, and there is no cap-and-trade policy launched.

The emitters will pay the penalty once s/he emits the CO_2 into the air. To clearly distinguish different contracts offered by the principal, we use various symbols to represent different contracts shown in Table 4.10. The optimal solutions for different

Table 4.10 The Symbols of the Different Contracts

E_d	the capturing quantity of the type d agent is efficient
F_d	the type d agent captures all demand quantity
S_d	the quantities offered to low and high demand levels of the type d agent are different
P_d	the quantities offered to low and high demand levels of the type d agent are same
H_d	only serve the type d agent when the demand level is high
N_d	not serve the type d agent when the demand level is high

values of μ_1, π_{1l}, and π_{2l} when the demand levels of a type-1 agent are less than those of a type-2 agent are shown in Table 4.11, 4.12, 4.13, 4.14, 4.15, and 4.16. It provides an optimal allowance to a type-1 agent (a_1^*) and a type-2 agent (a_2^*), the optimal expected information rent obtained by a type-1 agent $E\Delta_1^*$ and by a type-2 agent $E\Delta_2^*$, the principal's expected profit (Φ^*), the aggregated storage quantity (EQ^*), the total aggregated value of the principal and agents (ES^*), and the aggregated value of the government (EG^*). It also shows the optimal storage quantities by a type-1 agent (q_{1l}^0, q_{1h}^0) and by a type-2 agent (q_{2l}^0, q_{2h}^0), the optimal expected information rent obtained by a type-1 agent $E\Delta_1^0$ and by a type-2 agent $E\Delta_2^0$, the principal's expected profit (Φ^0), the aggregated storage quantity (EQ^0), the total aggregated value of the principal and agents (ES^0), and the aggregated value of the government (EG^0) when zero allowances are given to either type of agent. The total aggregated value of the principal and agents in the cost-sharing program is computed by the sum of the profit of the principal and information rent from agents, i.e.,

$$ES = \sum_d \mu_d \left(\pi_{dl}(t_{dl} - \beta q_{dl}) + \pi_{dh}(t_{dh} - \beta q_{dh}) + \pi_{dl}\Delta_{dl} + \pi_{dh}\Delta_{dh}\right)$$

Further, the value of government is calculated by the penalty got from agents minus the cost of allowance given to agents, i.e.,

$$EG = \sum_{d} \mu_d \left(\pi_{dl} (\frac{\alpha}{2} (\theta_{dl} - q_{dl})^2 - \frac{\alpha}{2} a_d^2) + \pi_{dh} (\frac{\alpha}{2} (\theta_{dh} - q_{dh})^2 - \frac{\alpha}{2} a_d^2) \right)$$

4.7.1 Optimal Allowance for Partial Overlapping Demand Distributions: $\theta_{1l} < \theta_{2l} < \theta_{1h} < \theta_{2h}$

Table 4.11 Partial Overlapping: when $\mu_1 = 0.25$

	Cap-and-trade model									Non-cap-and-trade model								
	$\pi_{2l} = 0.25$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.6	1.5	0.00	51.48	$\$1,329$	4.25	$\$1381$	$\$-254$	H_1E_2	$(0.0, 0.8)$	$(2.8, 4.8)$	0.00	128.25	$\$2,078$	3.37	$\$2206$	$\$110$
0.50	F_1F_2	0.6	2.4	0.00	0.00	$\$1,186$	4.13	$\$1186$	$\$-631$	N_1E_2	$(0.0, 0.0)$	$(2.8, 4.8)$	0.00	0.00	$\$2,061$	3.22	$\$2061$	$\$116$
0.75	E_1F_2	0.0	2.2	0.00	0.00	$\$1,187$	4.00	$\$1187$	$\$-519$	N_1E_2	$(0.0, 0.0)$	$(2.8, 4.8)$	0.00	0.00	$\$2,061$	3.22	$\$2061$	$\$86$
	$\pi_{2l} = 0.50$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.6	1.2	0.00	0.00	$\$1,218$	3.88	$\$1218$	$\$-167$	H_1E_2	$(0.0, 2.8)$	$(2.8, 4.8)$	0.00	150.00	$\$1,863$	3.37	$\$2013$	$\$36$
0.50	F_1F_2	0.6	1.5	0.00	40.76	$\$1,015$	3.75	$\$1056$	$\$-254$	H_1E_2	$(0.0, 0.8)$	$(2.8, 4.8)$	0.00	85.50	$\$1,665$	2.95	$\$1751$	$\$1056$
0.75	F_1F_2	2.0	1.8	0.00	125.34	$\$817$	3.63	$\$942$	$\$-490$	N_1E_2	$(0.0, 0.0)$	$(2.8, 4.8)$	0.00	0.00	$\$1,654$	2.85	$\$1654$	$\$86$
	$\pi_{2l} = 0.75$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.6	0.6	0.00	0.00	$\$1,158$	3.50	$\$1158$	$\$-51$	E_1E_2	$(1.8, 3.8)$	$(2.8, 4.8)$	0.00	0.00	$\$1,662$	3.30	$\$1662$	$\$6$
0.50	F_1F_2	0.6	1.2	0.00	0.00	$\$861$	3.38	$\$861$	$\$-167$	H_1E_2	$(0.0, 3.2)$	$(2.8, 4.8)$	0.00	3.24	$\$1,495$	2.88	$\$1498$	$\$36$
0.75	F_1F_2	0.6	1.5	0.00	30.03	$\$701$	3.25	$\$731$	$\$-254$	H_1E_2	$(0.0, 0.8)$	$(2.8, 4.8)$	0.00	42.75	$\$1,252$	2.52	$\$1295$	$\$76$

Note: Optimal storage quantities, the aggregated storage quantity, and the principal's expected profit under optimal allowances vs. those under zero allowances.

Table 4.12 Partial Overlapping: when $\mu_1 = 0.50$

	Cap-and-trade model									Non-cap-and-trade model								
	$\pi_{2l} = 0.25$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	1.0	1.5	0.0	148.63	$\$1,049$	4.00	$\$1198$	$\$-232$	S_1E_2	$(0.8, 2.8)$	$(2.8, 4.8)$	0.00	328.50	$\$1,805$	3.30	$\$2133$	$\$71$
0.50	F_1F_2	1.6	1.5	0.0	275.63	$\$760$	3.75	$\$1035$	$\$-344$	S_1E_2	$(0.3, 2.3)$	$(2.8, 4.8)$	0.00	278.25	$\$1,566$	2.80	$\$1844$	$\$130$
0.75	F_1F_2	2.0	1.5	0.0	268.13	$\$744$	3.50	$\$1013$	$\$-447$	S_1E_2	$(0.1, 0.8)$	$(2.8, 4.8)$	0.00	108.83	$\$1,386$	2.30	$\$1495$	$\$207$
	$\pi_{2l} = 0.50$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.6	1.2	0.0	0.00	$\$1,198$	3.75	$\$1198$	$\$-129$	S_1E_2	$(0.8, 3.3)$	$(2.8, 4.8)$	0.00	150.22	$\$1,752$	3.24	$\$1903$	$\$37$
0.50	F_1F_2	1.4	1.2	0.0	240.24	$\$686$	3.50	$\$927$	$\$-243$	S_1E_2	$(0.8, 2.8)$	$(2.8, 4.8)$	0.00	257.00	$\$1,405$	2.80	$\$1662$	$\$71$
0.75	F_1F_2	1.6	1.5	0.0	196.98	$\$558$	3.25	$\$785$	$\$-344$	S_1E_2	$(0.5, 1.8)$	$(2.8, 4.8)$	0.00	94.83	$\$1,172$	2.30	$\$1367$	$\$130$
	$\pi_{2l} = 0.75$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.6	0.6	0.0	0.00	$\$1,158$	3.50	$\$1158$	$\$-51$	E_1E_2	$(1.8, 3.8)$	$(2.8, 4.8)$	0.00	0.00	$\$1,662$	3.30	$\$1662$	$\$6$
0.50	F_1F_2	0.8	1.2	0.0	2.05	$\$808$	3.25	$\$810$	$\$-149$	S_1E_2	$(0.8, 3.5)$	$(2.8, 4.8)$	0.00	63.80	$\$1,359$	2.72	$\$1423$	$\$41$
0.75	F_1F_2	1.2	1.2	0.0	157.30	$\$481$	3.00	$\$641$	$\$-243$	S_1E_2	$(0.8, 2.8)$	$(2.8, 4.8)$	0.00	185.50	$\$1,005$	2.30	$\$1190$	$\$71$

Note: Optimal storage quantities, the aggregated storage quantity, and the principal's expected profit under optimal allowances vs. those under zero allowances.

Tables 4.11, 4.12 and 4.13 illustrate the optimal solutions for both two models where demands follow partial overlapping distributions ($\theta_{1l} < \theta_{2l} < \theta_{1h} < \theta_{2h}$)

Mostly, the principal will design the contracts to induce both types of agents to capture all demands, i.e., the optimal contract type is F_1F_2 when there is an allowance

Table 4.13 Partial Overlapping: when $\mu_1 = 0.75$

	Cap-and-trade model									Non-cap-and-trade model									
	$\pi_{2l} = 0.25$																		
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	1.0	1.5	0.00	148.63	$1,049	4.00	$1233	$ − 98	S_1E_2	(1.5, 3.5)	(2.8, 4.8)	0.00	211.92	$1,710	3.30	$1921	$29
0.50	F_1F_2	1.6	1.5	0.00	275.63	$760	3.75	$933	$ − 193	S_1E_2	(1.3, 3.3)	(2.8, 4.8)	0.00	246.37	$1,360	2.80	$1607	$48
0.75	F_1F_2	2.0	1.5	0.00	268.13	$744	3.50	$671	$ − 235	S_1E_2	(1.2, 2.8)	(2.8, 4.8)	0.00	260.94	$1,021	2.30	$1282	$71
	$\pi_{2l} = 0.50$																		
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.6	1.2	0.00	0.00	$1,198	3.75	$1178	$ − 90	S_1E_2	(1.5, 3.6)	(2.8, 4.8)	0.00	95.48	$1,696	3.25	$1792	$20
0.50	F_1F_2	1.4	1.2	0.00	240.24	$686	3.50	$915	$ − 98	S_1E_2	(1.5, 3.5)	(2.8, 4.8)	0.00	176.17	$1,310	2.80	$1486	$29
0.75	F_1F_2	1.6	1.5	0.00	196.98	$588	3.25	$630	$ − 159	S_1E_2	(1.4, 3.1)	(2.8, 4.8)	0.00	208.64	$961	2.30	$1170	$48
	$\pi_{2l} = 0.75$																		
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.6	0.6	0.00	0.00	$1,158	3.50	$1158	$ − 51	E_1E_2	(1.8, 3.8)	(2.8, 4.8)	0.00	0.00	$1,662	3.30	$1662	$6
0.50	F_1F_2	0.8	1.2	0.00	2.05	$808	3.25	$850	$ − 90	S_1E_2	(1.5, 3.7)	(2.8, 4.8)	0.00	58.09	$1,297	2.75	$1355	$21
0.75	F_1F_2	1.4	1.2	0.00	154.44	$487	3.00	$597	$ − 98	S_1E_2	(1.5, 3.5)	(2.8, 4.8)	0.00	140.42	$910	2.30	$1050	$29

Note: Optimal storage quantities, the aggregated storage quantity, and the principal's expected profit under optimal allowances vs. those under zero allowances.

assigned by the government in the market. Further, the government often allocates a large allowance to the agent who can be induced to capture all quantities of demand. What is more, the type-1 agent can not get the expected information rent; however, the type-2 agent may get the expected information rent. As shown in Tables 4.11, 4.12, and 4.13, the left sides of those three tables display the optimal solutions for the cap-and-trade model. Except for the case when $\mu_1 = 0.25$, $\pi_{1l} = 0.75$, and $\pi_{2l} = 0.25$, the principal offers a full quantities contract (F_1F_2) in other cases with a large allowance assigned to both types of agents. When $\mu_1 = 0.25$, $\pi_{1l} = 0.75$, and $\pi_{2l} = 0.25$, the type-1 agent is offered the efficient quantities for low as well as high demand levels, and the principal designs the full quantities contract to the type-1 agent for both levels of demands, i.e., the optimal contract is E_1F_2, shown in Table 4.11.

If the cap-and-trade policy is not implemented, the type-2 agent is always offered efficient quantities by the principal, and the type-1 agent always fails to receive a positive expected information rent despite the principal offering several various types of contracts to the type-1 agents.

When $\pi_{1l} = 0.25$ and $\pi_{2l} = 0.75$, regardless of the value of μ_1, the principal designs the efficient quantities for both two types of agents without expected information rent for both agents if the government does not give the agents any allowance (See the Right sides of Tables 4.11, 4.12, and 4.13). Furthermore, we observed that the expected demand of each type of agent is equivalent, i.e, $\pi_{1l}\theta_{1l} + \pi_{1h}\theta_{1h} = \pi_{2l}\theta_{2l} + \pi_{2h}\theta_{2h}$.

As shown on the right side of Table 4.11, There are three different types of contracts, H_1E_2, N_1E_2, and E_1E_2. When the principal optimally offers H_1E_2 contract to the agents, the expected information rent given to the type-2 agent is positive. If offering E_1E_2 instead of H_1E_2 to agents at this time, the principal has to give the type-2 agent more information rent to induce her/him to pick the option designed for her/him by the principal. But once π_{1l} increases, the revenue from serving the type-1 agent can not reach the expected information rent given to the type-2 agent. As a result, the principal prefers not to serve the type-1 agent, and the expected information rent assigned to the type-2 reduces to zero. This is shown in Table 4.11, when π_{1l} rises to 0.50 from 0.25 and π_{2l} is 0.25 or when π_{1l} rises to 0.75 from 0.50, and π_{2l} is 0.50.

Tables 4.12 and 4.13 show another type of contract S_1E_2. The type-1 agent is offered the separating contract without expected information rent, and the type-2 agent is offered efficient quantities with positive expected information rent, and offering efficient quantities to the type-1 agent is more costly instead of the separating contract at that time. What is more, when there is no allowance assigned to both types of agents, the expected captured quantity of the type-1 agent is monotonically increasing when the value of μ_1 increases (from 0.25 to 0.75). Since the revenue from the type-1 agent plays more and more contributions and the principal purposefully serves more quantity to the type-1 agent with the growth of the probability of being the type-1 agent. Furthermore, the type-2 agent is offered efficient quantities without allowance assigned to both types of agents, as shown in Tables 4.11 to 4.13.

Compared with the non-cap-and-trade policy, the cap-and-trade policy always makes the principal offer more quantities to agents, which means $EQ^* > EQ^0$. As shown in Tables 4.11 to 4.16, the optimal contract offered by the principal mostly is F_1F_2, meaning agents are induced to capture all demand quantities for the cap-and-trade model. So the expected captured CO_2 quantity of the non-cap-and-trade model can not exceed the expected captured CO_2 quantity of the cap-and-trade model when capturing all demand quantities is picked by all demand levels agents respectively. There does exist another optimal contract for the cap-and-trade model, which is E_1F_2. E_1F_2 becomes optimal when μ_1 is small. Further, when μ_1 is small, the principal prefers to only serve the high demand level of the type-1 agent less than efficient quantity or even not serve the type-1 agent and the type-2 agent always is induced to capture his/her efficient quantity for the non-cap-and-trade model (H_1E_2 or N_1E_2). As a result, even though when E_1F_2 is the optimal contract for the cap-and-trade model, the total expected captured CO_2 when the cap-and-trade policy is implemented is still more than the total expected captured CO_2 when the cap-and-trade policy is not implemented. The cap-and-trade policy can make the principal design the contract to induce agents to capture more CO_2. However, the principal will sacrifice her/his profit ($\Phi^* < \Phi^0$) and the whole system value is also reduced compared to the non-cap-and-trade case ($ES^* < ES^*$), and the government also pays the cost to induce agents to capture more CO_2 ($EG^* < EG^0$) which are also shown in Tables 4.11 to 4.16.

4.7.2 Optimal Allowance for Complete Overlapping Demand Distributions: $\theta_{2l} < \theta_{1l} < \theta_{1h} < \theta_{2h}$

In section 4.7.2, we display the optimal solutions for both two models where demands follow partial overlapping distributions ($\theta_{2l} < \theta_{1l} < \theta_{1h} < \theta_{2h}$) in Tables 4.14, 4.15, and 4.16.

The left side of each table shows the optimal solutions for the cap-and-trade model, and there is only one type of optimal contract designed by the principal that is

Table 4.14 Complete Overlapping: when $\mu_1 = 0.25$

	Cap-and-trade model									Non-cap-and-trade model								
	$\pi_{2l} = 0.25$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.6	1.6	0.00	0.00	$1,344	4.25	$1344	$ − 287	P_1E_2	(2.0, 2.0)	(1.8, 4.8)	0.00	214.50	$2,087	3.54	$2302	$50
0.50	F_1F_2	0.6	2.0	0.00	0.00	$1,240	4.13	$1240	$ − 442	P_1E_2	(1.0, 1.0)	(1.8, 4.8)	0.00	168.47	$1,977	3.30	$2145	$84
0.75	F_1F_2	2.7	1.4	0.00	263.69	$1,045	4.0	$1308	$ − 471	P_1E_2	(0.05, 0.05)	(1.8, 4.8)	0.00	10.12	$1,938	3.05	$1948	$130
	$\pi_{2l} = 0.50$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.9	1.0	0.00	12.44	$1,126	3.88	$1138	$ − 136	E_1E_2	(2.8, 3.8)	(1.8, 4.8)	0.00	0.00	$1,864	3.36	$1864	$6
0.50	F_1F_2	0.6	1.2	0.00	0.00	$1,083	3.75	$1083	$ − 167	P_1E_2	(2.4, 2.4)	(1.8, 4.8)	0.00	16.82	$1,752	3.08	$1769	$25
0.75	F_1F_2	0.6	1.6	0.00	0.00	$974	3.63	$974	$ − 287	P_1E_2	(2.0, 2.0)	(1.8, 4.8)	0.00	109.01	$1,593	2.98	$1702	$30
	$\pi_{2l} = 0.75$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	1.5	0.8	0.00	0.00	$752	3.50	$752	$ − 149	E_1S_2	(2.8, 3.8)	(1.2, 4.8)	33.14	0.00	$1,275	2.49	$1337	$38
0.50	F_1F_2	1.2	0.6	0.00	0.00	$795	3.38	$795	$ − 90	E_1S_2	(2.8, 3.8)	(1.4, 4.8)	18.89	0.00	$1,240	2.49	$1278	$29
0.75	F_1F_2	0.9	0.8	0.00	0.00	$724	3.25	$724	$ − 98	E_1S_2	(2.8, 3.8)	(1.5, 4.8)	0.66	0.00	$1,206	2.49	$1218	$22

Note: Optimal storage quantities, the aggregated storage quantity, and the principal's expected profit under optimal allowances vs. those under zero allowances.

Table 4.15 Complete Overlapping: when $\mu_1 = 0.50$

	Cap-and-trade model									Non-cap-and-trade model								
	$\pi_{2l} = 0.25$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.6	1.4	0.00	2.05	$1,356	4.00	$1358	$ − 166	P_1E_2	(2.8, 2.8)	(1.8, 4.8)	0.00	216.81	$1,935	3.43	$2152	$47
0.50	F_1F_2	0.6	1.8	0.00	0.00	$1,202	3.75	$1202	$ − 257	P_1E_2	(2.4, 2.4)	(1.8, 4.8)	0.00	263.31	$1,749	3.32	$2012	$60
0.75	F_1F_2	1.8	1.6	0.00	279.47	$704	3.50	$984	$ − 415	P_1E_2	(2.0, 2.0)	(1.8, 4.8)	0.00	286.93	$1,591	3.03	$1878	$75
	$\pi_{2l} = 0.50$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	1.2	0.8	0.00	139.14	$960	3.75	$1099	$ − 149	E_1E_2	(2.8, 3.8)	(1.8, 4.8)	0.00	0.00	$1,851	3.42	$1851	$6
0.50	F_1F_2	0.6	1.2	0.00	0.00	$1,084	3.50	$1084	$ − 129	P_1E_2	(2.8, 2.8)	(1.8, 4.8)	0.00	32.73	$1,666	3.05	$1699	$28
0.75	F_1F_2	1.5	1.0	0.0	243.63	$633	3.38	$877	$ − 232	P_1E_2	(2.5, 2.5)	(1.8, 4.8)	0.0	102.22	$1,477	2.89	$1579	$34
	$\pi_{2l} = 0.75$																	
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	1.5	0.8	0.00	0.00	$786	3.50	$786	$ − 207	E_1S_2	(2.8, 3.8)	(0.9, 4.8)	0.00	0.00	$1,452	2.70	$1502	$43
0.50	F_1F_2	1.2	0.8	0.0	0.00	$756	3.25	$756	$ − 149	E_1S_2	(2.8, 3.8)	(1.1, 4.8)	0.00	0.00	$1,353	2.66	$1383	$31
0.75	F_1F_2	1.2	0.6	0.0	67.64	$652	3.00	$720	$ − 129	E_1S_2	(2.8, 3.8)	(1.5, 4.8)	0.00	0.00	$1,255	2.67	$1264	$16

Note: Optimal storage quantities, the aggregated storage quantity, and the principal's expected profit under optimal allowances vs. those under zero allowances.

F_1F_2 (See Tables 4.14, 4.15, and 4.16). All demand levels of both types of agents are offered the full quantities with a large allowance given by the government; however, the type-1 agent may acquire positive information rent instead of nothing, and the type-2 agent gets no expected information rent. On the contrary, when $\mu_1\pi_{2l}$ is large which is shown in Table 4.16.

When the government decides not to apply the cap-and-trade policy, the optimal solutions are shown in the left side of each table (See Tables 4.14, 4.15, and 4.16). When E_1E_2 is the optimal contract designed by the principal, the agents can be induced to choose the efficient quantity designed for her/him without any expected information rent given to agents. As shown in Tables 4.14, 4.15, and 4.16, when $\pi_{1l} = 0.25$ and $\pi_{2l} = 0.50$ regardless of the value of μ_1, both types of agents are

Table 4.16 Complete Overlapping: when $\mu_1 = 0.75$

	Cap-and-trade model									Non-cap-and-trade model									
	$\pi_{2l} = 0.25$																		
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.6	1.2	0.00	35.34	$1,307	3.75	$1343	$ − 90	S_1E_2	(2.8, 3.5)	(1.8, 4.8)	0.00	143.66	$1,863	3.49	$2007	$22
0.50	F_1F_2	0.9	0.8	0.00	35.34	$927	3.75	$1134	$ − 110	S_1E_2	(2.7, 3.3)	(1.8, 4.8)	0.00	182.57	$1,642	3.25	$1824	$28
0.75	F_1F_2	0.9	1.2	0.00	164.40	$776	3.00	$941	$ − 138	S_1E_2	(2.6, 2.8)	(1.8, 4.8)	0.00	201.43	$1,431	3.00	$1632	$36
	$\pi_{2l} = 0.50$																		
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	0.9	1.2	0.00	0.00	$1,085	3.63	$1085	$ − 138	E_1E_2	(2.8, 3.8)	(1.8, 4.8)	0.00	0.00	$1,839	3.48	$1839	$6
0.50	F_1F_2	0.6	1.2	0.00	0.00	$1,085	3.50	$1085	$ − 90	S_1E_2	(2.8, 3.5)	(1.8, 4.8)	0.00	39.66	$1,616	3.18	$1656	$15
0.75	F_1F_2	0.6	1.2	0.00	26.36	$914	2.88	$941	$ − 90	S_1E_2	(2.7, 3.1)	(1.8, 4.8)	0.00	76.54	$1,396	2.94	$1473	$20
	$\pi_{2l} = 0.75$																		
π_{1l}	$Type$	a_1^*	a_2^*	$E\Delta_1^*$	$E\Delta_2^*$	Φ^*	EQ^*	ES^*	EG^*	$Type$	(q_{1l}^0, q_{1h}^0)	(q_{2l}^0, q_{2h}^0)	$E\Delta_1^0$	$E\Delta_2^0$	Φ^0	EQ^0	ES^0	EG^0
0.25	F_1F_2	1.2	2.0	60.99	0.00	$831	3.50	$892	$ − 297	E_1S_2	(2.8, 3.8)	(0.9, 4.8)	0.00	0.00	$1,639	3.12	$1664	$20
0.50	F_1F_2	0.9	2.0	78.69	0.00	$791	3.13	$870	$ − 230	E_1S_2	(2.8, 3.8)	(1.1, 4.8)	0.00	0.00	$1,472	2.98	$1487	$16
0.75	F_1F_2	0.9	0.6	0.00	9.15	$747	2.75	$756	$ − 100	E_1S_2	(2.8, 3.8)	(1.5, 4.8)	0.00	0.00	$1,305	2.86	$1309	$10

Note: Optimal storage quantities, the aggregated storage quantity, and the principal's expected profit under optimal allowances vs. those under zero allowances.

offered efficient quantities, respectively, with no expected information rent, which means this case is cost-less for the principal in general.

Compared with the partial-overlapping distribution case, the principal does not always offer efficient quantities to the type-2 agent when the demand distribution is complete-overlapping. As shown in Tables 4.14, 4.15, and 4.16, the type-2 agent is offered the separating contract when $\pi_{2l} = 0.75$ regardless of the value of μ_1 and π_{1l}. Offer contract E_1S_2 instead of all efficient quantities since the principal has to pay the higher information rent to the type-1 agent to induce her/him to pick efficient quantities designed for her/him.

Further, there is another type of contract which is not mentioned before which is P_1E_2 shown in Tables 4.14 and 4.15. Except for the case when E_1E_2 is the optimal contract, all efficient quantities ($q_{dn} = \bar{q}_{dn}, d = 1, 2$ and $n = l, h$) are not incentive compatible anymore and the pooling contract designed to the type-1 agent and efficient contract designed to the type-2 agent become incentive compatible when $\pi_{2l} = 0.25, 0.50$ shown in Table 4.14 and 4.15. Those two tables also display the type-2 agent is offered efficient quantities for both demand levels with a positive expected information rent, and the type-1 agent is offered a pooling contract with no expected information rent by the government.

Similarly, when $\mu_1 = 0.75$, except for the case when $\pi_{1l} = 0.25$ and $\pi_{2l} = 0.50$ shown in Table 4.16, the type-2 agent is offered efficient quantities for both demand levels with a positive expected information rent and the type-1 agent is offered a separating contract. In this separating contract, the agent is offered a higher quantity than her/his efficient quantity if s/he is at a higher demand level and a less quantity than her/his efficient quantity otherwise. The pooling contract at this time is not an incentive as this contract is not even feasible.

When the non-cap-and-trade policy is launched, similar to the partial overlapping distribution case, the expected captured quantity of the type-1 agent is monotonically increasing when the value of μ_1 increases from 0.25 to 0.75. But the expected capturing quantity of the type-2 agent is monotonically decreasing with the rise of the value of μ_1.

4.8 Managerial Insight

When the demand distribution is known, there are two levels of demand uncertain. If the government does not launch a cap-and-trade policy, the principal offers the efficient quantity to the agent regardless of the demand level, and the agent can not get the expected information rent. However, when the cap-and-trade policy is implemented on the market and the optimal allowance is allocated to the agent, the agent has an incentive to capture all demand even without expected information rent. Although the cap-and-trade policy can make the principal design a contract to induce the agent to capture all quantities, the principal can set a higher price for the agent when there is no allowance assigned to the agent. As a result, the profit of the profit will decrease compared with the profit when the government does not give any allowance to the agent.

When the distributions of two types of agents partially overlap, and the non-cap-and-trade policy is launched, the type-2 agent is always offered an efficient quantities contract (E_2), and the type-1 agent can not acquire any expected information rent regardless of the type of his contract. Further, we summarize the following insights under this condition: (1) When the expected demand of the type-1 agent is the same as the expected demand of the type-2 agent, the principal often designs all efficient quantities contract, i.e., E_1E_2, to the agents with no cost overall. (2) If the chance of being the type-1 agent with the high demand level is low, either offering a non-participating to θ_{1l} or not serving the type-1 agent is the optimal contract. Since the revenue from offering more quantity to the type-1 agent can not increase as much as the rise of the type-2 agent's information rent. (3) When the probability of being the type-1 agent with the high demand level increases, only serving the high demand level of the type-1 agent and not serving the type-1 agent contracts becomes costly. As a result, the principal offers the separating contract instead, and more expected quantities are offered to the type-1 agent when the probability of being the type-1 agent increases.

When the demand distributions become complete-overlapping, and the chance of having a high demand level of the type-2 agent is small without an allowance given by the government, the principal offers efficient quantities to the type-1 agent instead of the type-2 agent and offers the separating contract to the type-2 agent with no expected information rent.

When the cap-and-trade policy is launched by the government, what we conclude: (1) mostly, the policy will make the principal offer the full quantities to all demand levels of the agents with a large allowance assigned by the government (F_1F_2). (2) However, when the chance of being the type-2 agent with a high demand level becomes large, there does exist a case that full quantities contracts are not incentive compatible for the type-1 agent even if the government sets a large allowance for her/him. What is more, CO_2 that the agent captures is even less than the efficient quantity with an allowance. As a result, the government is not willing to assign any allowance to that agent who can not be motivated to capture the full demand of CO_2 no matter how much allowance is given to her/him, and the principal will design the efficient quantities contract instead with no given allowance, and full quantities contract is designed to another type of agent with a large allowance.

The cap-and-trade policy can make the principal design the contracts to induce agents to capture as much as CO_2 they emit, and mostly the agents are induced to capture all demand, and the captured CO_2 is more compared with the non-cap-and-trade policy is implemented. However, the principal has to sacrifice her/his benefit compared with the non-cap-and-trade case as he can not set a price as much as in the non-cap-and-trade case. Further, the government also pays the cost as she will get the penalty from agents and needs to pay the cost of setting the allowance to agents.

4.9 Conclusions

In this chapter, we build a principal-agent framework to explore the implications of uncertainty in agents' heterogeneous demands on the principal's contracts when a cap-and-trade policy is applied or not. We consider two types of hidden information under the cap-and-trade case and non-cap-and-trade case separately. The first one is the agent's type is known by the principal and agent, but the demand level is only known by the agent. Another one is that the principal even does not know the type of agent; however, the agents know their types and demand levels. The principal designs the menu of contracts for all agents, and each agent will pick the option in the chosen contract after s/he realizes her/his demand level.

We display the analytical solutions under both types of hidden information when the government assigns the allowance in the market, i.e., whether a cap-and-trade policy is launched or not. We present the special types of analytical solutions when there are two distributions for both cap-and-trade and non-cap-and-trade models. The cap-and-trade policy and the types of demand distributions play a significant role in the types of optimal solutions. When a cap-and-trade policy is implemented in the market, mostly, full quantities contracts become optimal regardless of the demand distribution. However, efficient quantities contract to the type-1 agent, and full quantities contract to the type-2 agent become optimal when the probability of being the high demand level of the type-2 agent is large in partial-overlapping distribution. When a cap-and-trade policy is implemented in the market, we just list the analytical solutions of the all-efficient-quantities contract, and the single contract, other types of contracts (pooling, separating, only serving high demand level) are shown in the numerical solutions (Section 4.7).

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

In this chapter, we present the conclusions of the dissertation and discuss the future work directions opened by this research.

5.1 Summary of Contributions

This dissertation made novel research contributions to controlling the spread of Emerald ash borer and reducing CO_2 emissions. The high-level goal of this dissertation is to design the principal-agent framework to optimize the collaboration between the principal and agents.

We are among the first to apply the principal-agent framework to induce the collaboration between the government and private landowners on managing invasive species which is shown in chapter 2. We address a non-classical principal-agent problem which does not meet the single-crossing property by building two cost-sharing programs, where the reimbursement is based on either the infestation level or the number of treated trees. We explore the impact of the government on incentives of private landowners. Our model can also be applied to the public-private relationships by exploring the joint resource, which can strengthen the collaborations between public and private roles.

This is the first trying to integrate a principal-agent framework with a mixed-integer programming to address the budget allocation and make the collaboration between the government and private landowners in Chapter 3. To reduce the complexity of the problem, we linearize all non-linearities. We also make use of the machine learning method to dynamically predict the next attack rate in each site to consider the uncertainty of the attack rate.

Chapter 4 presents a principal-agent model to induce the emitters to capture as much as possible CO_2 through the Carbon and Storage System with or without the cap-and-trade policy. We investigate the impact of allocating carbon allowance to agents on the total quantities being captured and stored as well as the overall information rent given to the agents to incentivize them to participate. We provide analytical solutions to the special case where the allocated allowance is zero or positive. Through numerical analysis, we investigate the impact of providing allowance based on the agent's demand distribution, and we compare the optimal solutions when either the carbon allowance is assigned or not assigned by the government.

5.2 Future Work

This research took the first steps toward studying and addressing the application of the principal-agent framework. There are several interesting directions that we plan to investigate in future work.

In Chapter 2, we study the cost-sharing programs between the government and private landowners and the optimal reimbursement to induce the participation of private landowners. Our future work would focus on two tasks. The first one is to develop a heuristic to find the optimal solution for the treatment-based reimbursement model. Finding the optimal treatment decisions and the reimbursement schedule requires a complete search with all possible combinations of treatment decisions. Because we have been able to narrow down the structures of the optimal treatment decisions, they can be used to reduce the search time. The second one is to provide analytical solutions for the optimal treatment decision and the reimbursement under each infestation level. Further, we plan to validate both models by checking the results using data and input parameters from New Jersey against those from Minnesota.

In Chapter 3, we develop a data-driven integrated game theory-mixed integer programming framework to allocate the resources to management decisions in both public and private areas over space and time to maximize the government's profit and save more ash trees. For future work, first, the presented MIP model could be implemented on a larger scale beyond a five-by-five gridded landscape with a larger number of trees in each grid cell. In this case, the cost function regarding treatment could be defined as a non-linear function of the number of treated trees. We also can build different cost functions among public and private sites since the government can award a contract with a tree service company to reduce each cost. These modifications in cost would significantly complicate the model, thus, necessitating the development of non-linear MIP algorithms. Furthermore, we will perform a sensitivity analysis to explore how the change in input parameters affects optimal management decisions. Then we will explore solution algorithms to tackle the complexity of the MIP formulation resulting from the linearization of the non-linear inequalities and the addition of fixed costs for treatment and removal to reduce the computation time.

In Chapter 4, we allow the expected allowance set by the government to be un-restricted. In future work, we would like to set an upper bound for the expected allowance. According to results obtained from the cap-and-trade model, full quantities contracts are most likely to be optimal for both types of agents, meaning agents will capture all demand and sell all allowance to the market. However, the selling price of the allowance should decrease if massive emission allowances are in the trading market. Therefore, we can build a dynamic pricing mechanism where the selling price depends on the quantity of the total allowance on the market. What is more, in addition to the penalty of CO2 emissions and the cost of offering the allowances, the government may want to account for the benefit from the improvement of the environment. In future work, we plan to consider the improvement as an additional component of the objective function.

APPENDIX A

MORE EXPLANATIONS FOR EACH CHAPTER

A.1 More Notes of Chapter 2

A.1.1 Relationship between the Two Cutoff Values of the High Second-period Attack Rate

We show that $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$ when $\rho < \ddot{\rho}$. The proof is separated into two cases: when π^l is low or medium and when π^h is high.

Case 1: When the low second-period attack rate is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$). As discussed earlier, $\dot{\pi}^h$ and $\ddot{\pi}^h$ are defined as follows: $\dot{\pi}^h(i) := \max\{0, \min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}\}$ and $\ddot{\pi}^h(i) := \max\{0, \min\{1, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$. Because $\ddot{\pi}^h$ is in between 0 and 1, we divide our discussion into two sub-cases.

Sub-case 1: If $\frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} \geq 1$, then $\ddot{\pi}^h(i) = 1$. Because $\dot{\pi}^h$ is also between 0 and 1, $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$.

Sub-case 2: If $\frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} < 1$, then $\ddot{\pi}^h(i) = \max\{0, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}$. $\dot{\pi}^h(i)$ is either zero or $\min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}$. If $\dot{\pi}^h(i) = 0$, $\ddot{\pi}^h(i) = \max\{0, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\} \geq 0 = \dot{\pi}^h(i)$, i.e., $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$. Otherwise, $\dot{\pi}^h(i) = \min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}$. Equivalently, $-\dot{\pi}^h(i) = -\min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\} \geq -\left(\pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\right)$. Since $\ddot{\pi}^h(i) = \max\{0, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}$ $\geq \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}$, the difference of the two cutoffs is thus

$$\begin{aligned}
\ddot{\pi}^h(i) - \dot{\pi}^h(i) &\geq \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} - \left(\pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\right) \\
&= \frac{\alpha+\pi^l a_1}{\theta+c} + \frac{\alpha(\bar{\rho}(\theta+c)+\beta)+a_1^2}{(\theta+c)(\bar{\rho}(\theta+c)+\beta)} \cdot \frac{i}{n-i}.
\end{aligned}$$

Because $\frac{\alpha(\bar{\rho}(\theta+c)+\beta)+a_1^2}{(\theta+c)(\bar{\rho}(\theta+c)+\beta)} \cdot \frac{i}{n-i} \geq 0$ for $0 \leq i < n$, $\ddot{\pi}^h(i) - \dot{\pi}^h(i) \geq \frac{\alpha+\pi^l a_1}{\theta+c}$. When $0 \leq \rho < \ddot{\rho}$, $a_1 > 0$, and thus, $\ddot{\pi}^h(i) - \dot{\pi}^h(i) \geq \frac{\alpha+\pi^l a_1}{\theta+c} > 0$.

Case 2: When the low second-period attack rate is high ($\ddot{\pi}^l \leq \pi^l \leq 1$). Recall $\dot{\pi}^h$ and $\ddot{\pi}^h$ are defined as follows: $\dot{\pi}^h(i) := \max\{0, \min\{1, \frac{\beta}{\bar{\rho}(\theta+c)+\beta} + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}\}$ and $\ddot{\pi}^h(i) := \max\{0, \min\{1, \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$. Similar to the previous case, our discussion is separated into two sub-cases.

Sub-case 1: If $\frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} \geq 1$, then $\ddot{\pi}^h(i) = 1$. Because $\dot{\pi}^h$ is also between 0 and 1, $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$.

Sub-case 2: If $\frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} < 1$, $\ddot{\pi}^h(i) = \max\{0, \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}$. $\dot{\pi}^h(i)$ is either zero or $\min\{1, \frac{\beta}{\bar{\rho}(\theta+c)+\beta} + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}$. If $\dot{\pi}^h(i) = 0$, $\ddot{\pi}^h(i) = \max\{0, \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\} \geq 0 = \dot{\pi}^h(i)$. Therefore, $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$. Otherwise,

$$\begin{aligned}
\ddot{\pi}^h(i) - \dot{\pi}^h(i) &= \max\{0, \tfrac{\beta+\alpha}{\theta+c} + \tfrac{\alpha+a_1}{\theta+c} \cdot \tfrac{i}{n-i}\} - \min\{1, \tfrac{\beta}{\bar{\rho}(\theta+c)+\beta} + \tfrac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \tfrac{i}{n-i}\} \\
&\geq \tfrac{\beta+\alpha}{\theta+c} + \tfrac{\alpha+a_1}{\theta+c} \cdot \tfrac{i}{n-i} - \left(\tfrac{\beta}{\bar{\rho}(\theta+c)+\beta} + \tfrac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \tfrac{i}{n-i}\right) \\
&= \tfrac{\beta a_1+\alpha(\bar{\rho}(\theta+c)+\beta)}{\theta+c} + \tfrac{\alpha(\bar{\rho}(\theta+c)+\beta)+a_1^2}{(\theta+c)(\bar{\rho}(\theta+c)+\beta)} \cdot \tfrac{i}{n-i}.
\end{aligned}$$

Since $\frac{\alpha(\bar{\rho}(\theta+c)+\beta)+a_1^2}{(\theta+c)(\bar{\rho}(\theta+c)+\beta)} \cdot \frac{i}{n-i} \geq 0$ for $0 \leq i < n$, $\ddot{\pi}^h(i) - \dot{\pi}^h(i) \geq \frac{\beta a_1+\alpha(\bar{\rho}(\theta+c)+\beta)}{\theta+c}$. When $0 \leq \rho < \ddot{\rho}$, $a_1 > 0$, and thus, $\ddot{\pi}^h(i) - \dot{\pi}^h(i) \geq \frac{\beta a_1+\alpha(\bar{\rho}(\theta+c)+\beta)}{\theta+c} > 0$.

We can thus conclude that $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$ when $0 \leq \rho < \ddot{\rho}$. □

A.1.2 Proof of Propositions

A.1.2.1 Proof of Proposition 1. When no trees are infested, the objective function value in Eq. (2.5) is $\Psi(q(0), r(0)|n, 0) = (s\bar{\rho} + \beta)\pi^l q(0) + (s - (s\bar{\rho} + \beta)\pi^l)n - r(0)$. Because $(s\bar{\rho} + \beta)\pi^l > 0$, Ψ increases in $q(0)$ and decreases in $r(0)$. From the IR and NN constraints, we get the lower bound of the reimbursement as $\max\{0, b_1 q(0) + \alpha n + a_1 \pi^l n\}$, where $a_1 := \beta - \rho(\theta + c)$ and $b_1 := \beta - \pi^l[\bar{\rho}(\theta + c) + \beta]$.

Further, the IC constraints require $b_1(q(0)-j) \leq 0$ for all j. The remainder of the proof is divided into two cases: when π^l is low/medium and when π^l is high.

Case 1: When the low second-period attack rate is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$).

In this case, $b_1 > 0$. Since $b_1(q(0)-j) \leq 0$ must hold for all j, $q(0)$ must be less than j $\forall$ $0 \leq j \leq n$. $q(0) = 0$ is hence the only feasible and consequently the optimal treatment decision. The optimal reimbursement is thus $r^*(0) = \max\{0, \alpha n + a_1 \pi^l n\}$. If the treatment is very effective ($\ddot{\rho} \leq \rho \leq 1$), $a_1 \leq 0$ and hence $r^*(0) \leq \alpha n$. Otherwise, $r^*(0) = \alpha n + a_1 \pi^l n > \alpha n$.

Case 2: When the low second-period attack rate is high ($\ddot{\pi}^l \leq \pi^l < 1$). Here, $b_1 \leq 0$.

If $b_1 < 0$, $q(0)$ must equal to n to satisfy $q(0) - j \geq 0$ for all j or equivalently, $b_1(q(0)-j) \leq 0$. Therefore, $q(0) = n$ is the only feasible solution. If $b_1 = 0$, all IC constraints are satisfied because $b_1(q(0)-j) = 0 \leq 0$ for all j. Because Ψ increases in $q(0)$ and IC constraints are all satisfied, it is optimal to set $q^*(0) = n$ and $r^*(0) = \max\{0, \alpha n + (\beta - (\theta + c)\pi^l)n\}$. Further, when π^l is greater than both $\frac{\alpha+\beta}{\theta+c}$ and $\ddot{\pi}^l$, $\alpha n + (\beta - (\theta + c)\pi^l)n < 0$, and thus, $r^*(0) = 0$. □

A.1.2.2 Proof of Proposition 2. When all trees are infested, the objective function value in Eq. (2.5) is $\Psi(q(n), r(n)|n, n) = \rho(s + \beta)q(n) - \beta n - r(n)$. Because $\rho(s + \beta) > 0$, Ψ increases in $q(n)$. We proceed with the proof by discussing the optimal solution for each category of treatment effectiveness.

Case 1: When the treatment is very effective ($\ddot{\rho} \leq \rho \leq 1$). In this case, $a_1 \leq 0$.

If we set $q(n) = n$, then $\mu_n = 0$ by definition. The IC constraints require $a_1(n-j) \leq 0$ to hold for all j. Because $a_1 \leq 0$, $n - j$ must be non-negative, and thus, all IC constraints are satisfied, making $q(n) = n$ a feasible solution. The lower bound of the reimbursement obtained from the IR and NN constraints is $\max\{0, \alpha n + a_1 n\}$. Since Ψ decreases in $r(n)$, setting it to the lower bound maximizes Ψ.

If $q(n)$ is less than n, then $\mu_n = 1$ by definition. If $a_1 < 0$, the IC constraint $-a_1(n - q(n)) \leq 0$ is violated because $n - q(n) > 0$. Therefore, a feasible solution exists only when $a_1 = 0$. Since Ψ increases in $q(n)$, the best solution when $q(n) < n$ is $q(n) = n - 1$. and $r(n) = \alpha n$. The lower bound of $r(n)$ is $\max\{0, a_1 q(n) + \alpha n\}$ instead. However, when compared to $\Psi(n, \max\{0, \alpha n + a_1 n\}|n, n)$, $\Psi(n-1, \alpha n|n, n)$ is lower when $a_1 = 0$: $\Psi(n-1, \alpha n|n, n) - \Psi(n, \max\{0, \alpha n + a_1 n\}|n, n) = (\rho(s+\beta)(n-1) - \beta n - \alpha n) - (\rho(s+\beta)n - \beta n - \alpha n) = -\rho(s+\beta) < 0$. Therefore, $q(n) = n-1$ and $r(n) = \alpha n$ cannot be an optimal solution. In conclusion, the optimal solution is $q^*(n) = n$ and $r^*(n) = \max\{0, \alpha n + a_1 n\}$ when the treatment ρ is very effective.

Case 2: When the treatment is not very effective ($0 \leq \rho < \ddot{\rho}$). Here, $a_1 > 0$.

If we assume $q(n) = n$, then $\mu_n = 0$ by definition. The lower bound of the reimbursement is $\max\{0, \alpha n + a_1 n\}$. IC constraints are satisfied if $a_1(n - j) \leq 0$ for all j. Because $a_1 > 0$ and $n - j \geq 0$ for all j, IC constraints are violated. Therefore, $q(n)$ can not be n.

If, on the other hand, $q(n) < n$, $\mu_n = 1$. The lower bound of $r(n)$ is $\max\{0, a_1 q(n) + \alpha n\}$. To satisfy IC constraints, i.e., $a_1(n - j) \leq 0$ for all j, $q(n) - j$ must be non-positive for all j. Consequently, $q(n) = 0$ is the only feasible solution, and thus, optimal. Since Ψ decreases in $r(n)$, $r^*(n) = \alpha n$. $\square$

A.1.2.3 Proof of Proposition 3. Recall that Proposition 3 pertains to the case when some trees are infested, and the treatment is very effective. In this case, $a_1 \leq 0$. First, we argue that the number of treated trees must be no less than the number of infested trees ($q(i) \geq i$) by showing that IC constraints are violated if $q(i) < i$. The IC constraints can be re-written as $(\bar{\rho}(\theta+c)+\beta)(\pi^h - \pi^l)(n-i) - a_1(i - q(i)) - b_1(j-i) \leq$

0. Equivalently, $LHS = \pi^l + \frac{a_1(i-q(i))}{\left(\bar{\rho}(\theta+c)+\beta\right)(n-i)} + \frac{b_1(j-i)}{\left(\bar{\rho}(\theta+c)+\beta\right)(n-i)} \geq \pi^h$ must hold for all $j \in [i,n]$. If π^l is low or medium, $b_1 > 0$. At least one of the IC constraints is violated. Specifically, when $j = i$, the third term in LHS is zero. The second term is non-positive because $a_1 \leq 0$ and $q(i) < i$. Hence, $LHS \leq \pi^l$, which is less than π^h by assumption. Therefore, the IC constraint is violated. Similarly, if π^l is high, $b_1 \leq 0$. Here, all IC constraints are violated since $LHS \leq \pi^l < \pi^h$. As a result, $q(i) \geq i$.

The objective function value in Eq. (2.5) is thus $\Psi = (s+\beta)\bar{\rho}\pi^l q(i) - \bar{\rho}(s+\beta)i + (s-(s+\beta)\bar{\rho}\pi^l)n - r(i)$. Because $(s+\beta)\bar{\rho}\pi^l > 0$, the objective value increases in $q(i)$ and decreases in $r(i)$. A lower bound of $r(i)$ obtained from the IR and NN constraints is $\max\{0, b_1 q(i) - (\theta - \alpha - (\bar{\rho}(\theta+c)+\beta)\pi^l)n + \bar{\rho}(\theta+c)i + \phi(a_0|i)\}$. Further, to satisfy the IC constraints, the following inequalities must hold simultaneously: $(\bar{\rho}(\theta+c)+\beta)(\pi^l(n-q(i)) - \pi^h(n-i)) + a_1(i-j) + \beta(q(i)-i) \leq 0$ for all $j \in [0,i)$ and $b_1(q(i)-j) \leq 0$ for all $j \in [i,n]$. Next, we divide the proof into two cases: when π^l is low/medium and when it is high.

Case 1: When the low second-period attack rate is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$). Here, $b_1 > 0$.

In order to satisfy the IC constraints or $b_1(q(i)-j) \leq 0$ for all $j \in [i,n]$, $q(i)-j$ needs to be non-positive. We can get $q(i) = i$ is the only feasible solution, and thus, optimal. To minimize the objective function value, $r^*(i) = \max\{0, \alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h - \pi^l)](n-i)\}$. $r^*(i)$ decreases in π^h. As per its definition, $\ddot{\pi}^h(i) = \max\{0, \min\{1, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$, which is $\leq \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}$. Since $\alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} - \pi^l)](n-i) = 0$, $\alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h - \pi^l)](n-i) > 0$ for all $\pi^h > \ddot{\pi}^h(i)$. Thus, $r^*(i) = \alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h - \pi^l)](n-i) > 0$. Similarly, when $\pi^h \leq \ddot{\pi}^h(i)$, $\alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h - \pi^l)](n-i) \leq 0$. Therefore, $r^*(i) = 0$.

Case 2: When the low second-period attack rate is high ($\ddot{\pi}^l \leq \pi^l \leq 1$). In this case, $b_1 \leq 0$. If $b_1 = 0$, the IC constraints are satisfied automatically. Since

Ψ increases in $q(i)$, setting $q(i) = n$ is optimal. If, on the other hand, $b_1 < 0$, $q(i) - j$ needs to be non-negative for all $j \in [i, n]$ to satisfy the second sets of IC constraints. Therefore, $q(i) = n$ is the only feasible solution, and thus, optimal. To minimize the objective function value, $r(i)$ should take its lower bound, which is simplified to $r^*(i) = \max\{0, \alpha n + a_1 n - (a_1 - \beta + (\theta + c)\pi^h)(n - i)\}$. $r^*(i)$ decreases in π^h.

Recall that $\ddot{\pi}^h(i)$ is defined as $\ddot{\pi}^h(i) = \max\{0, \min\{1, \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$, which is $\leq \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}$. Because $\alpha n + a_1 n - (a_1 - \beta + (\theta + c)(\frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}))(n - i) = 0$, $\alpha n + a_1 n - (a_1 - \beta + (\theta + c)\pi^h)(n - i) > 0$ when $\pi^h > \ddot{\pi}^h$. Hence, $r^*(i) = \alpha n + a_1 n - (a_1 - \beta + (\theta + c)\pi^h)(n - i) > 0$. On the other hand, when $\pi^h \leq \ddot{\pi}^h$, $\alpha n + a_1 n - (a_1 - \beta + (\theta + c)\pi^h)(n - i) \leq 0$, and thus, $r^*(i) = 0$. □

A.1.2.4 Proof of Proposition 4. Proposition 4 discusses the optimal solution when some trees are infested, and the treatment is not very effective. a_1 is positive in this case. The proof is separated into two cases: when π^l is low/medium and when it is high. Each case is then further divided into two sub-cases: when π^h is low and when it is medium/high.

Case 1: When the low second-period attack rate is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$). Here, $b_1 > 0$.

Sub-case 1: When the high second-period attack rate is low ($0 \leq \pi^h < \dot{\pi}^h$).

First, we show that no feasible solutions exist when $q(i) \geq i$. The IC constraints require $(\bar{\rho}(\theta + c) + \beta)\ [\pi^l(n - q(i)) - \pi^h(n - i)] + a_1(i - j) + \beta(q(i) - i) \leq 0$ for all $j \in [0, i)$ and $b_1(q(i) - j) \leq 0$ for all $j \in [i, n]$ to hold jointly. Because $b_1 > 0$, to satisfy the second set of IC constraints, $q(i) = i$ is the only solution that can be feasible. The first set of IC constraints are simplified to $(\bar{\rho}(\theta + c) + \beta)(\pi^l - \pi^h)(n - i) + a_1(i - j) \leq 0$ for all $j \in [0, i)$. Recall that $\dot{\pi}^h$ is defined as $\dot{\pi}^h(i) = \max\{0, \min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}\}$. Since $a_1 > 0$, $\pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i} > 0$, and thus, $\dot{\pi}^h(i) \leq \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}$. Moreover, $(\bar{\rho}(\theta + c) + \beta)[\pi^l - (\pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i})](n - i) + a_1(i - j) = 0$ when $j = 0$.

Because the expression $(\bar{\rho}(\theta+c)+\beta)(\pi^l-\pi^h)(n-i)+a_1(i-j)$ decreases in π^h, $(\bar{\rho}(\theta+c)+\beta)(\pi^l-\pi^h)(n-i)+a_1(i-j)>0$ when $j=0$ for any $\pi^h<\dot{\pi}^h$. $q(i)=i$ is not a feasible solution since at least one of the IC constraints is violated.

Next, we examine whether a feasible solution exist when $q(i)<i$. In this case, $\mu_i=1$. The objective function value in Eq. (2.5) is $\Psi=\rho(s+\beta)q(i)+(\bar{\pi}^h+\rho\pi^h)s(n-i)-\bar{\rho}\beta i-\bar{\rho}\pi^l\beta n-r(i)$, which increases in $q(i)$ and decreases in $r(i)$. To satisfy the IC constraints, both $a_1(q(i)-j)\leq 0$ for all $j\in[0,i)$ and $(\bar{\rho}(\theta+c)+\beta)(\pi^h(n-i)-\pi^l(n-j))-a_1(i-q(i))-\beta(j-i)\leq 0$ for all $j\in[i,n]$ must hold. Because $a_1>0$, $q(i)$ must be zero to satisfy the first set of the IC constraints and $q(i)<i$. These constraints are simplified to $(\bar{\rho}(\theta+c)+\beta)(\pi^h(n-i)-\pi^l(n-j))-a_1 i-\beta(j-i)\leq 0$ for all $j\in[i,n]$. As shown earlier, $\dot{\pi}^h(i)\leq\pi^l+\frac{a_1}{\bar{\rho}(\theta+c)+\beta}\cdot\frac{i}{n-i}$. Therefore, for any $\pi^h<\ddot{\pi}^h$, $(\bar{\rho}(\theta+c)+\beta)(\pi^h(n-i)-\pi^l(n-j))-a_1(i-q(i))-\beta(j-i)=(\bar{\rho}(\theta+c)+\beta)(\pi^h-\pi^l)(n-i)-a_1 i-b_1(j-i)$ $<(\bar{\rho}(\theta+c)+\beta)(\pi^l+\frac{a_1}{\bar{\rho}(\theta+c)+\beta}\cdot\frac{i}{n-i}-\pi^l)(n-i)-a_1 i-b_1(j-i)=-b_1(j-i)\leq 0$ since $b_1>0$ and $j\in[i,n]$. As a result, the second set of IC constraints are satisfied.

The lower bound of $r(i)$ obtained from the IR and NN constraints is $\max\{0,a_1q(i)-(\theta-\alpha-(\bar{\rho}(\theta+c)+\beta)\pi^h)n+(\theta+c-(\bar{\rho}(\theta+c)+\beta)\pi^h)i+\phi(a_0|i)\}$. This lower bound when $q^*(i)=0$ is simplified to $r^*(i)=\alpha n+a_1\pi^h(n-i)$, which is greater than αn since $a_1>0$.

Sub-case 2: When the high second-period attack rate is medium or high. Here $\dot{\pi}^h\leq\pi^h\leq 1$ and similar to the previous sub-case, we argue that $q(i)\geq i$ when $\pi^h\geq\dot{\pi}^h$. Here, $\mu_i=0$ and the objective function value is $\Psi=(s+\beta)\bar{\rho}\pi^l q(i)-\bar{\rho}(s+\beta)i+(s-(s+\beta)\bar{\rho}\pi^l)n-r(i)$, which increases in $q(i)$ and decreases in $r(i)$. The IC constraints require $(\bar{\rho}(\theta+c)+\beta)(\pi^l(n-q(i))-\pi^h(n-i))+a_1(i-j)+\beta(q(i)-i)\leq 0$ for all $j\in[0,i)$ and $b_1(q(i)-j)\leq 0$ for all $j \in [i, n]$. Since $b_1 > 0$ and $q(i) \geq i$, $q(i)$ must be i to satisfy the second set of IC constraints. Similar to the previous sub-case, we can show that the first set of constraints become $(\bar{\rho}(\theta + c) + \beta)(\pi^l - \pi^h)(n - i) + a_1(i - j) \leq 0$ for all $j \in [0, i)$, which are satisfied when $\pi^h \geq \dot{\pi}^h$.

The lower bound of $r(i)$ obtained from the IR and NN constraints is $\max\{0, b_1 q(i) - (\theta - \alpha - (\bar{\rho}(\theta + c) + \beta)\pi^l)n + \bar{\rho}(\theta + c)i + \phi(a_0|i)\}$. Since Ψ decreases in $r(i)$, the lower bound maximizes Ψ. Further, the lower bound can be simplified to $\max\{0, \alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta + c)(\pi^h - \pi^l)](n - i)\}$, which decreases in π^h. Recall that $\pi^h := \max\{0, \min\{1, \frac{\alpha+\pi^l a_1}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$. Because $a_1 > 0$, $\pi^h \leq \frac{\alpha+\pi^l a_1}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}$. For any $\pi^h < \ddot{\pi}^h$, $\alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta + c)(\pi^h - \pi^l)](n - i) > \alpha n + a_1 n - \left[a_1\bar{\pi}^l + (\theta + c)\left(\frac{\alpha+\pi^l a_1}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\right)\right](n - i) = 0$. On the other hand, when $\pi^h \geq \ddot{\pi}^h$, $\alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta + c)(\pi^h - \pi^l)](n - i) \leq 0$. Therefore, $r^*(i) = 0$.

Case 2: When the low second-period attack rate is high ($\ddot{\pi}^l \leq \pi^l < 1$). In this case, $b_1 \leq 0$.

Sub-case 1: When the high second-period attack rate is low. Here, $0 \leq \pi^h < \dot{\pi}^h$. We first show that no feasible solutions exists when $q(i) \geq i$. The IC constraints are simplified as follows: $(\bar{\rho}(\theta + c) + \beta)(\pi^l(n - q(i)) - \pi^h(n - i)) + a_1(i - j) + \beta(q(i) - i) \leq 0$ for all $j \in [0, i)$ and $b_1(q(i) - j) \leq 0$ for all $j \in [i, n]$. Because $b_1 \leq 0$, $q(i) = n$. At least one of the first set of the IC constraints is violated when $\pi^h < \dot{\pi}^h$. Therefore, $q(i) = n$ is not a feasible solution. Next, using the same logic as Sub-case 1 of case 1, we can conclude that it is optimal to set $q^*(i) = 0$ and consequently $r^*(i) = \alpha n + a_1\pi^h(n - i)$.

Sub-case 2: When the high second-period attack rate is medium or high. Here $\dot{\pi}^h \leq \pi^h \leq 1$. As established earlier, when $\pi^h > \dot{\pi}^h$, $q(i) \geq i$. Since $\mu_i = 0$, the objective function value is $\Psi = (s + \beta)\bar{\rho}\pi^l q(i) - \bar{\rho}(s + \beta)i + (s - (s + \beta)\bar{\rho}\pi^l)n - r(i)$, which increases in $q(i)$ and decreases in

$r(i)$. To satisfy the IC constraints, two sets of conditions need to hold: $(\bar{\rho}(\theta + c) + \beta)(\pi^l(n - q(i)) - \pi^h(n - i)) + a_1(i - j) + \beta(q(i) - i) \leq 0$ and $b_1(q(i) - j) \leq 0$ for all $j \in [i, n)$. Because $b_1 \leq 0$, $q(i) = n$ is the only feasible solution that meets the second set of the IC constraints.

A lower bound obtained from the IR and NN constraints when $q^*(i) = n$ is simplified to $\max\{0, \alpha n + a_1 i + \beta(n - i) - (\theta + c)\pi^h(n - i)\}$, which decreases in π^h. For any value any $\pi^h < \ddot{\pi}^h$, $\alpha n + a_1 i + \beta(n - i) - (\theta + c)\pi^h(n - i) > \alpha n + a_1 i + \beta(n - i) - (\theta + c)\ddot{\pi}^h(n - i) \geq \alpha n + \beta n - \rho(\theta + c)i - (\theta + c)\left(\frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\right)(n - i) = 0$. On the other hand, when $\pi^h \geq \ddot{\pi}^h$, $\alpha n + a_1 i + \beta(n - i) - (\theta + c)\pi^h(n - i) \leq 0$, and thus, $r^*(i) = 0$. $\square$

A.2 More Notes of Chapter 3

A.2.1 Game-theory Model

A.2.1.1 Model Notion.

Table A.1 Notations

Input parameters:	
N_i^1	The number of ash trees a landowner has in site i at the beginning.
π_i^1	The first-period attack rate in site i, i.e., the probability that an ash tree is infested with the EAB. $\bar{\pi} = 1 - \pi$.
I_i^1	The infestation level (number of infested trees) in period one in site i follows a binomial distribution with rate π_i^1: $I_i^1 \sim B(N_i^1, \pi_i^1)$.
c_1	Cost of inspecting an ash tree.
c_2	Cost of treating an ash tree.
c_3	Cost of removing a dead tree.
ρ	Treatment success rate. $\bar{\rho} = 1 - \rho$.
θ	Marginal value of a landowner having a surviving ash tree.
α	Marginal value the city forester has for a healthy ash tree.
$\pi_{h,i}^2$ (resp. $\pi_{l,i}^2$)	The high (resp. low) second-period attack rate, i.e., the probability that a healthy ash tree will become infested in period 2 in site i if the number of treated trees in period one is less than (resp. greater than or equal to) the number of infested trees. $\pi_{l,i}^2 < \pi_i^1 < \pi_{h,i}^2$; $\bar{\pi}_{h,i}^2 = 1 - \pi_{h,i}^2$; $\bar{\pi}_{l,i}^2 = 1 - \pi_{l,i}^2$.
Value function:	
$v(p, d)$	The value function of the city forester of having p surviving ash trees while losing d trees either untreated or unsuccessfully-treated at the end of the second period: $v(p, d) = s \cdot p - \beta \cdot d$.
$V(p)$	The value function of the landowner having p surviving ash trees at the end of the second period: $V(p) = \theta \cdot p$.
Decision variables only applicable to the infestation-based reimbursement model:	
$q_{S,i}$	The number of healthy trees a landowner will treat given I_i^1 out of N_i^1 ash trees, and the treated healthy trees will become EAB resistant.
$q_{I,i}$	The number of infested trees a landowner will treat given I_i^1 out of N_i^1 ash trees.
q_i	The number of trees a landowner will treat given I_i^1 out of N_i^1 ash trees are infested. q_i can be greater than, equal to, or less than I_i^1. $q_i = q_{S,i} + q_{I,i}$. If $q_i < I_i^1$, $q_{S,i} = 0$, otherwise, $q_{I,i} = I_i^1$.
r_i	Reimbursement offered to the landowner for having I_i^1 infested ash trees.

A.2.1.2 Formulation. This section simply shows the game theory model applied to the private site i and mentioned in schema 3.3.4. Let θ denote the landowner's marginal value of a healthy ash tree, and a_0 represents not participating in the cost-sharing program, then his expected utility given the infestation level (I_i^1) in site i is as follows:

$$\phi(a_0|N_i^1, I_i^1) = \begin{cases} \theta\bar{\pi}_{l,i}^2 N_i^1 - c_3\pi_{l,i}^2 N_i^1 & \text{if } I_i^1 = 0 \\ \theta\bar{\pi}_{h,i}^2(N_i^1 - I_i^1) - c_3(I_i^1 + \pi_{h,i}^2(N_i^1 - I_i^1)) & \text{if } 0 < I_i^1 < N_i^1 \\ -c_3 N_i^1 & \text{if } I_i^1 = N_i^1, \end{cases} \tag{A.1}$$

where $\bar{\pi}_{l,i}^2 = 1 - \pi_{l,i}^2$ and $\bar{\pi}_{h,i}^2 = 1 - \pi_{h,i}^2$.

First, if there are no infested trees $(I_i^1 = 0)$ in the first period, the attack rate in the second period is $\pi_{l,i}^2$, and therefore, $\pi_{l,i}^2 N_i^1$ trees are expected to be infested and die without treatment. The term $\bar{\pi}_{l,i}^2 N_i^1$ in Equation (A.1) is the landowner's value of having $\bar{\pi}^l n$ surviving trees while $c_3\pi_{l,i}^2 N_i^1$ is the cost of removing $\pi_{l,i}^2 N_i^1$ dead trees. Second, if some trees are infested $(0 < I_i^1 < N_i^1)$ in period one, inaction will lead to an increased attack rate $(\pi_{h,i}^2)$, the expected trees to be infested in period two is $\pi_{h,i}^2(N_i^1 - I_i^1)$. A total of $I_i^1 + \pi_{h,i}^2(N_i^1 - I_i^1)$ trees will die, and the removal cost associated is $c_3 \cdot (I_i^1 + \pi_{h,i}^2(N_i^1 - I_i^1))$. The landowner's expected utility is thus the difference between the value of healthy trees, $\theta \cdot \bar{\pi}_{h,i}^2(N_i^1 - I_i^1)$, and the removal cost. Last, if all trees are already infested in the first period, a lack of treatment would result in the death of all. The landowner would incur a removal cost of $c_3 N_i^1$.

If the private landowner does participate in the cost-sharing program, the landowner's expected utility considering two scenarios: (1) when $q_i < I_i^1$ in site i in period 1, where $q_i = q_{S,i} + q_{I,i}$ and $q_{S,i} = 0$ as $q_i < I_i^1$.

$$\phi(q_i, r_i|N_i^1, I_i^1, q_i < I_i^1) = \theta \cdot p_1 + r_i - c_1 \cdot N_i^1 - c_2 \cdot t_1 - c_3 \cdot d_1. \tag{A.2}$$

The first two terms in Equation (A.2) are the landowner's value of having p_1 surviving trees, $p_1 = \rho q_i + \bar{\pi}^2_{h,i}(N^1_i - I^1_i) + \rho\pi^2_{h,i}(N^1_i - I^1_i)$, and the reimbursement received from the city forester. The left three terms represent the surveillance cost, the treatment cost where $t_1 = q_i + \pi^2_{h,i}(N^1_i - I^1_i)$, and the removal cost where $d_1 = I^1_i - \rho q_i + \bar{\rho}_i \pi^2_{h,i}(N^1_i - I^1_i)$, respectively.

(2) when $q_i \geq I^1_i$ in site i in period 1

$$\phi(q_i, r_i | N^1_i, I^1_i, q_i \geq I^1_i) = \quad \theta \cdot p_0 + r_1 - c_1 \cdot N^1_i - c_2 \cdot t_0 - c_3 \cdot d_0. \tag{A.3}$$

The first two terms in Equation (A.3) are the landowner's value of having p_0 surviving trees, $p_0 = q_i - \bar{\rho} I^1_i + \bar{\pi}^2_{l,i}(N^1_i - q_i) + \rho\pi^2_{l,i}(N^1_i - q_i)$ and the reimbursement received from the city forester. The left three terms represent the surveillance cost, the treatment cost where $t_0 = I^1_i + (q_i - I^1_i) + \pi^2_{l,i}(N^1_i - q_i)$, and the removal cost where $d_0 = \bar{\rho}_i I^1_i + \bar{\rho}_i \pi^2_{l,i}(N^1_i - q_i)$, respectively.

The game-theory model is listed in the following:

$$\max_{q_i, r_i} \Psi(q_i, r_i | N^1_i, I^1_i) = \mu^1_i \cdot \Big(\alpha \cdot p_1 - c_2 \cdot d_1\Big) + \bar{\mu}^1_i \Big(\alpha \cdot p_0 - c_2 \cdot d_0\Big) - r_i \tag{A.4}$$

Subject to:

- IR

$$\phi(q_i, r_i | N^1_i, I^1_i) \geq \phi(a_0 | N^1_i, I^1_i) \quad i \in \Gamma_r \tag{A.5}$$

- $\mathrm{IC}_{I^1_i g}$

$$\phi(q_i, r_i | N^1_i, I^1_i) \geq \phi(g, r_i | N^1_i, I^1_i) \quad \forall 0 \leq g \leq N^1_i, i \in \Gamma_r \tag{A.6}$$

- NN

$$q_i, r_i \geq 0 \quad i \in \Gamma_r \tag{A.7}$$

The objective function shown in Equation (A.4) is the net expected utility of the city forester, which has two scenarios: (1) when $q_i < I_i^1$ (or equivalently, $\mu_i^t = 1$) and (2) when $q_i \geq I_i^1$ (or $\mu_i^1 = 0$). The first term of the objective function is the value of having p_1 surviving trees at the end of the second period minus the penalty for losing d_1 trees when $q_i < I_i^1$. Similarly, the second term of the objective function is the difference between the value of having p_0 surviving trees and the penalty for losing d_0 trees when $q_i \geq I_i^1$. For simplicity, we assume that the penalty associated with a dead tree is equal to the cost of its treatment. The last term is the cost of providing the financial award to the landowner.

The individual rationality (IR) constraint ensures the landowner has enough incentive to participate in the cost-sharing program. The incentive compatibility (IC$_j$) constraints induce the landowner to treat the number of trees designed by the city forester q_i based on the infestation level (I_i^1), rather than another treatment decision (g) in site i in period 1. The non-negativity constraints (NN) ensure that both q_i and r_i are non-negative.

A.3 More Notes of Chapter 4

A.3.1 The Proof of Propositions

A.3.1.1 The Proof of Proposition 5.

Proof. When $D = 1$ and $N = 1$ and the allowance $a_d = 0$, the problem becomes

$$\begin{aligned} \max \quad & \Phi = -\tfrac{\alpha}{2}q_{dn}^2 + \alpha(\theta_{dn} - \tfrac{\gamma+\beta}{\alpha})q_{dn} - \Delta_{dn} & \\ \text{s.t.} \quad & \Delta_{dn} \geq 0 & (IR_d) \\ \text{and} \quad & q_{dn} \geq 0 & (NN_{dn}) \end{aligned}$$

There is only one type of agent with one level demand, so the problem is under complete information and the principal can get the maximum profit when the information rent $\Delta_{dn}^* = 0$. What is more, the principal's profit is convex function with respect to q_{dn} and $\frac{\partial \Phi}{\partial q_{dn}} = -\alpha q_{dn} + \alpha(\theta_{dn} - \frac{\gamma+\beta}{\alpha})$, so the optimal contract is shown as follows:

$$q_{dn}^* = \theta_{dn} - \tfrac{\gamma+\beta}{\alpha} = \overline{q}_{dn}, \quad t_{dn}^* = v(q_{dn}^*, \theta_{dn}) - \Delta_{dn}^* = \overline{\tau}(q_{dn}^*) = \overline{\tau}(\overline{q}_{dn}) = \overline{t}_{dn}. \tag{A.8}$$

□

A.3.1.2 The Proof of Proposition 6.

Proof. When $D = 1$ and $N = 1$ and the allowance $a_d > 0$, the problem actually is

$$\begin{aligned} \max \quad & \Phi = v(q_{dn}, \theta_{dn}) - \Delta_{dn} - \beta q_{dn} & \\ \text{s.t.} \quad & \Delta_{dn} \geq 0 & (IR_d) \\ \text{and} \quad & q_{dn} \geq 0 & (NN_{dn}) \end{aligned}$$

Similar as the Proof A.3.1.1, the principal can get the maximum profit when the information rent $\Delta^*_{dn} = 0$ and the problem becomes the maximize problem i.e. $\Phi = v(q_{dn}, \theta_{dn}) - \beta q_{dn}$ when $q_{dn} \geq 0$. Figure A.1 and A.2 show the plots between Φ and q_{dn} when $a_d < (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$ and $a_d \geq (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$ respectively. The points in those two figures are listed in Table A.2. As shown in Table A.2, $\Phi|_A = \Phi|_C > \Phi|_B$. When $0 < a_d < (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$, $(\sqrt{2}-1)a_d - \frac{\gamma+\beta}{\alpha} < 0$ and $(1+\sqrt{2})a_d + \frac{\gamma+\beta}{\alpha} > 0$, so $\frac{\alpha}{2}((\sqrt{2}-1)a_d - \frac{\gamma+\beta}{\alpha})((1+\sqrt{2})a_d + \frac{\gamma+\beta}{\alpha})$ is negative when $a_d < (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$ and non-negative otherwise. Therefore, $\Phi|_A = \Phi|_C > \Phi|_D$ if the allowance is less than $(1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$ and $\Phi|_A = \Phi|_C < \Phi|_D$ if the allowance is more than or equal to $(1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$.

Table A.2 The Points in Figures A.1 and A.2

	q_{dn}	Φ
Point A	$\theta_{dn} - a_d - \frac{\gamma+\beta}{\alpha}$	$\frac{\alpha}{2}(\theta_{dn} - a_d - \frac{\gamma+\beta}{\alpha})^2$
Point B	$\theta_{dn} - a_d$	$\frac{\alpha}{2}(\theta_{dn} - a_d - \frac{\gamma+\beta}{\alpha})^2 - \frac{\alpha}{2}(\frac{\gamma+\beta}{\alpha})^2$
Point C	$\theta_{dn} - a_d + (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$	$\frac{\alpha}{2}(\theta_{dn} - a_d - \frac{\gamma+\beta}{\alpha})^2$
Point D	θ_{dn}	$\frac{\alpha}{2}(\theta_{dn} - a_d - \frac{\gamma+\beta}{\alpha})^2 + \frac{\alpha}{2}((\sqrt{2}-1)a_d - \frac{\gamma+\beta}{\alpha})((\sqrt{2}+1)a_d + \frac{\gamma+\beta}{\alpha})$

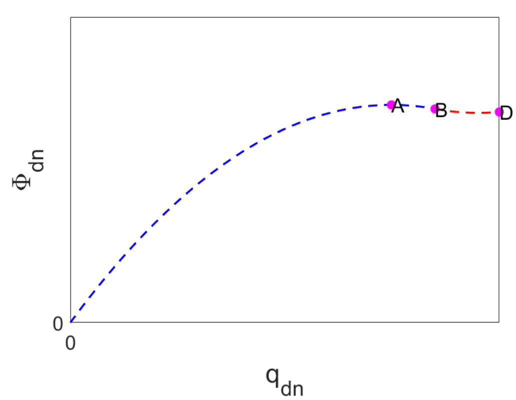
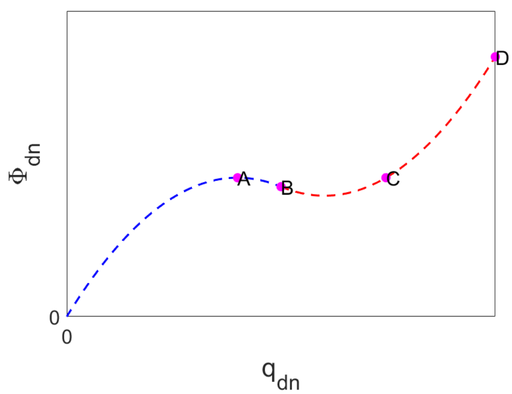


Figure A.1 Φ_{dn} when $0 < a_d < (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$.

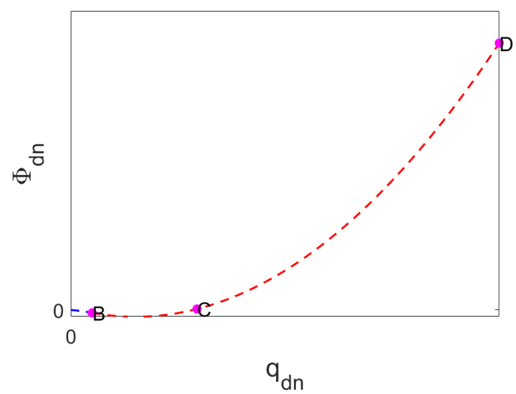
The blue dotted curve line shows the case when $0 \leq q_{dn} \leq \theta_{dn} - a_d$ and the red dotted line shows the case when $\theta_{dn} - a_d < q_{dn} \leq \theta_{dn}$. The point A is the top point of the blue dotted curve. The point B is the intersection point of these two curves. And the point D is the upper boundary point. As shown in Figure A.1, Φ reaches the maximum value when $q_{dn} = \theta_{dn} - a_d - \frac{\gamma+\beta}{\alpha}$ i.e. at the point A. So when

$0 < a_d < (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$, the optimal quantity and price are

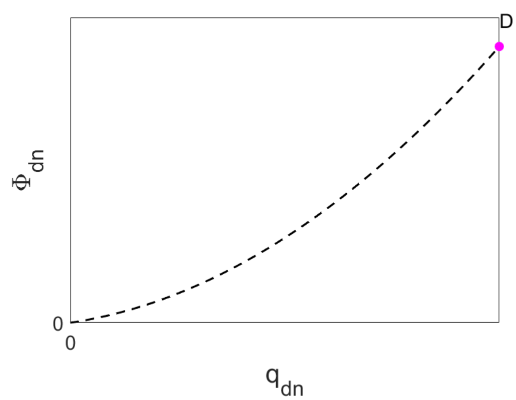
$$q_{dn} = \theta_{dn} - a_d - \frac{\gamma+\beta}{\alpha} = \overline{q}_{dn} - a_d, \quad t_{dn} = \overline{\tau}(q_{dn}) \tag{A.9}$$



(a) Φ_{dn} when $a_d < \theta_{dn} - \frac{\gamma+\beta}{\alpha}$



(b) Φ_{dn} when $\theta_{dn} - \frac{\gamma+\beta}{\alpha} \le a_d < \theta_{dn}$



(c) Φ_{dn} when $a_d \ge \theta_{dn}$

Figure A.2 Φ_{dn} when $a_d \ge (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$.

Figure A.2 shows the different trends of Φ when vary q_{dn} at the various a_d when $a_d \ge (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$. There are four different highlighted points in the figure.

The plot of Φ when $a_d < \theta_{dn} - \frac{\gamma+\beta}{\alpha}$ is shown in Figure A.2(a). In this case, according to Eq. 4.1, Φ is equal to $\hat{\tau}(q_{dn}) - \beta q_{dn}$ when $q_{dn} \le \theta_{dn} - a_d$ and $\overline{\tau}(q_{dn}) - \beta q_{dn}$ otherwise. Φ is non-negative for all q_{dn}. The maximum value of Φ is at the point D.

Figure A.2(b) illustrates the case that the principal earns a non-positive utility when $0 \leq q_{dn} \leq \theta_{dn} - a_d$. $\Phi|_D - \Phi|_B = \frac{\alpha}{2}((\sqrt{2}-1)a_d - \frac{\gamma+\beta}{\alpha})((\sqrt{2}+1)a_d + \frac{\gamma+\beta}{\alpha})$. Because $a_d \geq (\sqrt{2}+1)\frac{\gamma+\beta}{\alpha}$, $(\sqrt{2}-1)a_d - \frac{\gamma+\beta}{\alpha} \geq 0$. We can get $\Phi|_D \geq \Phi|_B$. As a result, the point D is the optimal point of the plot in Figure A.2(b).

When $a_d > \theta_{dn}$, Φ increases in q_{dn}. The plot of Φ is shown in Figure A.2(c). The principal gets the optimal utility when the agent captures all emission. In conclusion, when $a_d \geq (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$, the optimal quantity and price are

$$q_{dn} = \theta_{dn}, \quad t_{dn} = v(q_{dn}, \theta_{dn}) \tag{A.10}$$

In summary, the maximum capturing CO_2 quantity with the minimum allowance are shown as follows:

$$a_d^* = (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}, \quad q_{dn}^* = \theta_{dn}, \quad t_{dn}^* = \hat{\tau}(\theta_{dn}). \tag{A.11}$$

□

A.3.1.3 The Proof of Proposition 7.

Proof. When $D = 1$ and $N = 2$ and the allowance $a_d = 0$, the problem becomes

$$
\begin{aligned}
\max \quad & \Phi = \pi_{dl}(-\tfrac{\alpha}{2}q_{dl}^2 + \alpha(\theta_{dl} - \tfrac{\gamma+\beta}{\alpha})q_{dl} - \Delta_{dl}) + \pi_{dh}(-\tfrac{\alpha}{2}q_{dh}^2 + \alpha(\theta_{dh} - \tfrac{\gamma+\beta}{\alpha})q_{dh} - \Delta_{dh}) \\
\text{s.t.} \quad & \pi_{dl}\Delta_{dl} + \pi_{dh}\Delta_{dh} \geq 0 \qquad (IR_d) \\
& \Delta_{dh} - \Delta_{dl} \geq \alpha(\theta_{dh} - \theta_{dl})q_{dl} \qquad (IC_{dhl}) \\
& \Delta_{dl} - \Delta_{dh} \geq v(q_{dh}, \theta_{dl}) - v(q_{dh}, \theta_{dh}) \qquad (IC_{dlh}) \\
\text{and} \quad & q_{dn} \geq 0 \qquad (NN_{dn})
\end{aligned}
$$

The principal can get the maximum profit when the expected information rent $\pi_{dl}\Delta^*_{dl} + \pi_{dh}\Delta^*_{dh} = 0$. What is more, the principal's profit is convex function with respect to q_{dl}, q_{dh} and $\frac{\partial\Phi}{\partial q_{dn}} = -\alpha q_{dn} + \alpha(\theta_{dn} - \frac{\gamma+\beta}{\alpha})$ $(n = l, h)$, so the optimal capturing quantity for different demand level is shown as follows:

$$
\begin{aligned}
q^*_{dn} = \theta_{dn} - \tfrac{\gamma+\beta}{\alpha} = \overline{q}_{dn}, \\
\text{and} \qquad n = l, h
\end{aligned}
\tag{A.12}
$$

and,

$$
\begin{aligned}
\pi_{dl}\Delta^*_{dl} + \pi_{dh}\Delta^*_{dh} = 0, \\
\Delta^*_{dh} - \Delta^*_{dl} \geq \alpha(\theta_{dh} - \theta_{dl})\overline{q}_{dl}, \\
\Delta^*_{dl} - \Delta^*_{dh} \geq v(\overline{q}_{dh}, \theta_{dl}) - v(\overline{q}_{dh}, \theta_{dh})
\end{aligned}
\tag{A.13}
$$

So we can have,

$$
\begin{aligned}
\Delta^*_{dl} = -\pi_{dh}\left(\omega(v(\overline{q}_{dh}, \theta_{dl}) - v(\overline{q}_{dh}, \theta_{dh})) + \bar{\omega}\alpha(\theta_{dh} - \theta_{dl})\overline{q}_{dl}\right) < 0, \\
\Delta^*_{dh} = \pi_{dl}\left(\omega(v(\overline{q}_{dh}, \theta_{dl}) - v(\overline{q}_{dh}, \theta_{dh})) + \bar{\omega}\alpha(\theta_{dh} - \theta_{dl})\overline{q}_{dl}\right) > 0, \\
\text{and} \qquad \omega \in [0, 1]
\end{aligned}
\tag{A.14}
$$

From above, the optimal price offered by the principal is shown in the following:

$$
\begin{aligned}
t^*_{dn} = v(q^*_{dn}, \theta_{dn}) - \Delta^*_{dn} = v(\overline{q}_{dn}, \theta_{dn}) - \Delta^*_{dn} = \overline{t}(\overline{q}_{dn}) - \Delta^*_{dn}, \\
\text{and} \qquad n = l, h
\end{aligned}
\tag{A.15}
$$

□

A.3.1.4 The Proof of Proposition 8.

Proof. When $D = 1$ and $N = 2$ and the allowance $a_d > 0$, the problem actually becomes

$$\begin{aligned}
\max \quad & \Phi = \pi_{dl}(v(q_{dl}, \theta_{dl}) - \Delta_{dl} - \beta q_{dl}) + \pi_{dh}(v(q_{dh}, \theta_{dh}) - \Delta_{dh} - \beta q_{dh}) & \\
\text{s.t.} \quad & \pi_{dl}\Delta_{dl} + \pi_{dh}\Delta_{dh} \geq 0 & (IR_d) \\
& \Delta_{dh} - \Delta_{dl} \geq v(q_{dl}, \theta_{dh}) - v(q_{dl}, \theta_{dl}) & (IC_{dhl}) \\
& \Delta_{dl} - \Delta_{dh} \geq v(q_{dh}, \theta_{dl}) - v(q_{dh}, \theta_{dh}) & (IC_{dlh}) \\
\text{and} \quad & q_{dn} \geq 0 & (NN_{dn})
\end{aligned}$$

The principal can get the maximum profit when the expected information rent $\pi_{dl}\Delta^*_{dl} + \pi_{dh}\Delta^*_{dh} = 0$ and the problem i.e.

$$\begin{aligned}
&\max \quad \Phi = \sum_{q_{dl}, q_{dh}} \pi_{dn}(v(q_{dn}, \theta_{dn}) - \beta q_{dn}), \\
&\Delta_{dh} - \Delta_{dl} = \omega(v(q_{dh}, \theta_{dh}) - v(q_{dh}, \theta_{dl})) + \bar{\omega}(v(q_{dl}, \theta_{dh}) - v(q_{dl}, \theta_{dl})). \\
&\text{and} \qquad \omega \in [0, 1].
\end{aligned} \tag{A.16}$$

First, we just consider the non-constraint problem $\max \quad \Phi = \sum_{q_{dl}, q_{dh}} \pi_{dn}(v(q_{dn}, \theta_{dn}) - \beta q_{dn})$. The principal gets the maximum profit when $v(q_{dl}, \theta_{dl}) - \beta q_{dl}$ and $v(q_{dh}, \theta_{dh}) - \beta q_{dh}$ get the maximum value respectively. According to A.3.1.2, when $0 < a_d < (1 + \sqrt{2})\frac{\gamma+\beta}{\alpha}$, the optimal quantity is,

$$q_{dn} = \theta_{dn} - a_d - \tfrac{\gamma+\beta}{\alpha} = \bar{q}_{dn} - a_d \qquad (n = l, h) \tag{A.17}$$

Then we check the solutions we get if make IC_{1hl} and IC_{1lh} satisfied i.e. the inequality equation $v(q_{dh}, \theta_{dh}) - v(q_{dh}, \theta_{dl}) \geq v(q_{dl}, \theta_{dh}) - v(q_{dl}, \theta_{dl})$ satisfied.

When $\theta_{dh} - \theta_{dl} \leq \frac{\gamma+\beta}{\alpha}$

$$\begin{aligned} & v(q_{dh}, \theta_{dh}) - v(q_{dh}, \theta_{dl}) - (v(q_{dl}, \theta_{dh}) - v(q_{dl}, \theta_{dl})) \\ & = \alpha(\theta_{dh} - \theta_{dl})(q_{dh} - q_{dl}) \\ & = \alpha(\theta_{dh} - \theta_{dl})^2 > 0 \end{aligned} \tag{A.18}$$

When $\theta_{dh} - \theta_{dl} > \frac{\gamma+\beta}{\alpha}$ and $\theta_{dh} - a_d - \frac{\gamma+\beta}{\alpha} < \theta_{dl}$

$$\begin{aligned} & v(q_{dh}, \theta_{dh}) - v(q_{dh}, \theta_{dl}) - (v(q_{dl}, \theta_{dh}) - v(q_{dl}, \theta_{dl})) \\ & = -\frac{\alpha}{2}q_{dh}^2 + (\alpha(\theta_{dh} - a_d) - \gamma)q_{dh} - (\frac{\alpha}{2}q_{dh}^2 - (\alpha(\theta_{dl} - a_d) + \gamma)q_{dh} + \alpha(\theta_{dl} - a_d)^2) \\ & \quad -\alpha(\theta_{dh} - \theta_{dl})q_{dl} \\ & = -\alpha q_{dh}^2 + \alpha(\theta_{dh} - a_d + \theta_{dl} - a_d)q_{dh} - \alpha(\theta_{dl} - a_d)^2 - \alpha(\theta_{dh} - \theta_{dl})q_{dl} \\ & = -\alpha q_{dh}^2 + \alpha(q_{dh} + q_{dl} + 2\frac{\gamma+\beta}{\alpha})q_{dh} - \alpha(q_{dl} + \frac{\gamma+\beta}{\alpha})^2 - \alpha(q_{dh} - q_{dl})q_{dl} \\ & = 2\alpha\frac{\gamma+\beta}{\alpha}q_{dh} - \alpha(\frac{\gamma+\beta}{\alpha})^2 - 2\alpha\frac{\gamma+\beta}{\alpha}q_{dl} \\ & = 2\alpha\frac{\gamma+\beta}{\alpha}(\theta_{dh} - \theta_{dl} - \frac{1}{2}\frac{\gamma+\beta}{\alpha}) > 0 \end{aligned} \tag{A.19}$$

When $\theta_{dh} - \theta_{dl} > \frac{\gamma+\beta}{\alpha}$ and $\theta_{dh} - a_d - \frac{\gamma+\beta}{\alpha} \geq \theta_{dl}$

$$\begin{aligned} & v(q_{dh}, \theta_{dh}) - v(q_{dh}, \theta_{dl}) - (v(q_{dl}, \theta_{dh}) - v(q_{dl}, \theta_{dl})) \\ & = -\frac{\alpha}{2}q_{dh}^2 + (\alpha(\theta_{dh} - a_d) - \gamma)q_{dh} - (\frac{\alpha}{2}\theta_{dl}^2 - (\alpha(\theta_{dl} - a_d) + \gamma)\theta_{dl} + \alpha(\theta_{dl} - a_d)^2) \\ & -\alpha(\theta_{dh} - \theta_{dl})q_{dl} \\ & \geq -\frac{\alpha}{2}q_{dh}^2 + (\alpha(\theta_{dh} - a_d) - \gamma)q_{dh} - (\frac{\alpha}{2}q_{dh}^2 - (\alpha(\theta_{dl} - a_d) + \gamma)q_{dh} + \alpha(\theta_{dl} - a_d)^2) \\ & \quad -\alpha(\theta_{dh} - \theta_{dl})q_{dl} > 0 \end{aligned} \tag{A.20}$$

In summary, the solutions we get make IC_{1hl} and IC_{1lh} satisfied. So when $0 < a_d < (1 + \sqrt{2})\frac{\gamma+\beta}{\alpha}$, the optimal quantities and price are

	$(q_{dl}^*,\quad t_{dl}^*)$	$(q_{dh}^*,\quad t_{dh}^*)$
d	$(\overline{q}_{dl} - a_d,\quad \overline{\tau}(\overline{q}_{dl} - a_d) - \Delta_{dl}^*)$	$(\overline{q}_{dh} - a_d,\quad \overline{\tau}(\overline{q}_{dh} - a_d) - \Delta_{dh}^*)$

where

$$\begin{aligned}\Delta_{dl}^* &= -\pi_{dh}\left[\omega \cdot (v(q_{dh}^*, \theta_{dh}) - v(q_{dh}^*, \theta_{dl})) + \bar{\omega} \cdot (v(q_{dl}^*, \theta_{dh}) - v(q_{dl}^*, \theta_{dl}))\right], \\ \Delta_{dh}^* &= \pi_{dl}\left[\omega \cdot (v(q_{dh}^*, \theta_{dh}) - v(q_{dh}^*, \theta_{dl})) + \bar{\omega} \cdot (v(q_{dl}^*, \theta_{dh}) - v(q_{dl}^*, \theta_{dl}))\right].\end{aligned} \tag{A.21}$$

$$\text{and} \quad \omega \in [0, 1].$$

□

A.3.1.5 The Proof of Proposition 9.

Proof. Similar as Proof A.3.1.4, when $a_d \geq (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$, the optimal quantity is,

$$q_{dn} = \theta_{dn} \qquad (n = l, h) \tag{A.22}$$

Then we check the solutions we get if make IC_{1hl} and IC_{1lh} satisfied i.e. the inequality equation $v(q_{dh}, \theta_{dh}) - v(q_{dh}, \theta_{dl}) \geq v(q_{dl}, \theta_{dh}) - v(q_{dl}, \theta_{dl})$ satisfied.

$$\begin{aligned}&v(q_{dh}, \theta_{dh}) - v(q_{dh}, \theta_{dl}) - (v(q_{dl}, \theta_{dh}) - v(q_{dl}, \theta_{dl})) \\ &= v(\theta_{dh}, \theta_{dh}) - v(\theta_{dl}, \theta_{dh}) > 0\end{aligned} \tag{A.23}$$

In summary, the solutions we get make IC_{1hl} and IC_{1lh} satisfied. So when $a_d \geq (1+\sqrt{2})\frac{\gamma+\beta}{\alpha}$, the optimal quantities and price are

	$(q_{dl}^*,\quad t_{dl}^*)$	$(q_{dh}^*,\quad t_{dh}^*)$
d	$(\theta_{dl},\quad \hat{\tau}(\theta_{dl}) - \Delta_{dl}^*)$	$(\theta_{dh},\quad \hat{\tau}(\theta_{dh}) - \Delta_{dh}^*)$

where

$$\Delta^*_{dl} = -\pi_{dh}\left[\omega \cdot v(\theta_{dh}, \theta_{dh}) + \bar{\omega} \cdot v(\theta_{dl}, \theta_{dh}) - v(\theta_{dl}, \theta_{dl})\right],$$
$$\Delta^*_{dh} = \pi_{dl}\left[\omega \cdot v(\theta_{dh}, \theta_{dh}) + \bar{\omega} \cdot v(\theta_{dl}, \theta_{dh}) - v(\theta_{dl}, \theta_{dl})\right]. \tag{A.24}$$
$$\text{and} \quad \omega \in [0,1].$$

□

A.3.2 Analytical Solutions for None Overlapping and Complete Overlapping Distributions

A.3.2.1 Non-overlapping Distribution Case. When the distribution is not overlapping, the optimal quantities and prices are The quantities are always full

	$(q^*_{dl}, \quad t^*_{dl})$	$(q^*_{dh}, \quad t^*_{dh})$
$d = 1, 2$	$(\theta_{dl}, \quad \hat{\tau}(\theta_{dl}) - \Delta^*_{dl})$	$(\theta_{dh}, \quad \hat{\tau}(\theta_{dh}) - \Delta^*_{dh})$

quantities, however, the prices are different depending on the different binding constraints. The value of each information rent is listed in the following when the different constraints are binding.

(1) IR_1, IR_2, IC_{1hl} are binding.

$$\Delta^*_{ll} = \ -\pi_{1h}\left(v(q_{1l}, \theta_{1h}) - v(q_{1l}, \theta_{1l})\right).$$

$$\Delta^*_{lh} = \ \pi_{1l}\left(v(q_{1l}, \theta_{1h}) - v(q_{1l}, \theta_{1l})\right).$$

$$\Delta^*_{2l} = \ -\pi_{2h}\left(\omega \cdot (v(q_{2h}, \theta_{2h}) - v(q_{2l}, \theta_{2l})) + (1-\omega) \cdot (v(q_{2l}, \theta_{2h}) - v(q_{2l}, \theta_{2l}))\right).$$

$$\Delta_{2h}^* = \pi_{2l}\left(\omega \cdot (v(q_{2h}, \theta_{2h}) - v(q_{2l}, \theta_{2l})) + (1-\omega) \cdot (v(q_{2l}, \theta_{2h}) - v(q_{2l}, \theta_{2l}))\right).$$

$\omega \in [0, 1]$

(2) IR_1, IC_{21}, IC_{2hl} are binding.

When $a_1 \leq 2$, Let,

$$\tau_1 = v(q_{1l}, \theta_{2l}) - v(q_{1h}, \theta_{2l}) + v(q_{1h}, \theta_{1h}) - v(q_{1l}, \theta_{1l}).$$

$$\Delta_{1l}^* = -\pi_{1h}\tau_1.$$

$$\Delta_{1h}^* = \pi_{1l}\tau_1.$$

$$\tau_2 = \pi_{2l}(v(q_{1h}, \theta_{2l}) - v(q_{1h}, \theta_{1h})) + \pi_{2h}(v(q_{1h}, \theta_{2h}) - v(q_{1h}, \theta_{1h})) + \Delta_{1h}.$$

$$\Delta_{2l}^* = \tau_1 - \pi_{2h}\tau_2.$$

$$\Delta_{1h}^* = \tau_1 + \pi_{2l}\tau_2.$$

When $a_1 > 2$,

Let,

$$\begin{aligned}\delta_1 = \ & \omega_1 \cdot \min\{v(\theta_{1l}, \theta_{2h}) - v(\theta_{1h}, \theta_{2h}) + v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}), v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})\} \\ & + (1-\omega_1) \cdot \max\{v(\theta_{1l}, \theta_{2l}) - v(\theta_{1h}, \theta_{2l}) + v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}), v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})\}\end{aligned}$$

$$
\begin{aligned}
\Delta_{1l}^{*} = & \quad -\pi_{1h}\delta_1 \\
\Delta_{1h}^{*} = & \quad \pi_{1l}\delta_1
\end{aligned}
$$

Let,

$$
\begin{aligned}
\delta_2 = & \quad \pi_{2l} \cdot (v(\theta_{1h}, \theta_{2l}) - v(\theta_{1h}, \theta_{1h}) + \Delta_{1h}^{*}) \\
& \quad \pi_{2h} \cdot (v(\theta_{1l}, \theta_{2h}) - v(\theta_{1l}, \theta_{1l}) + \Delta_{1l}^{*}) \,.
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{2l}^{*} = & \quad \delta_2 - \pi_{2h} \cdot (v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})) \\
\Delta_{2h}^{*} = & \quad \delta_2 + \pi_{2l} \cdot (v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})) \,.
\end{aligned}
$$

(3) IR_1, IC_{21}, IC_{1hl}, IC_{2hl} are binding.

$$
\begin{aligned}
\Delta_{1l}^{*} = & \quad -\pi_{1h} \cdot (v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})) \\
\Delta_{1h}^{*} = & \quad \pi_{1l} \cdot (v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})) \,.
\end{aligned}
$$

Let,

$$
\begin{aligned}
\delta_2 = & \quad \pi_{2l} \cdot (v(\theta_{1l}, \theta_{2l}) - v(\theta_{1l}, \theta_{1l}) + \Delta_{1l}^{*}) \\
& \quad \pi_{2h} \cdot (v(\theta_{1h}, \theta_{2h}) - v(\theta_{1h}, \theta_{1h}) + \Delta_{1h}^{*})
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{2l}^{*} = & \quad \delta_2 - \pi_{2h} \cdot (v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})) \\
\Delta_{2h}^{*} = & \quad \delta_2 + \pi_{2l} \cdot (v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})) \,.
\end{aligned}
$$

(4) IR_1, IC_{21} are binding.

Let,

$$\begin{aligned}\delta_1 = \;& \omega_1 \cdot \min\{v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}), v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\} \\ & (1-\omega_1) \cdot \max\{v(\theta_{1l},\theta_{2l}) - v(\theta_{1h},\theta_{2l}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}), v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}.\end{aligned}$$

$$\begin{aligned}\Delta_{1l}^* = \;& -\pi_{1h} \cdot \delta_1 \\ \Delta_{1h}^* = \;& \pi_{1l} \cdot \delta_1.\end{aligned}$$

Let,

$$\begin{aligned}\delta_2 = \;& \omega_2 \cdot \min\{v(\theta_{2l},\theta_{1l}) - v(\theta_{2h},\theta_{1l}) + v(\theta_{2h},\theta_{2h}) - v(\theta_{2l},\theta_{2l}), \\ & v(\theta_{2l},\theta_{1h}) - v(\theta_{2h},\theta_{1h}) + v(\theta_{2h},\theta_{2h}) - v(\theta_{2l},\theta_{2l}), v(\theta_{2h},\theta_{2h}) - v(\theta_{2l},\theta_{2l})\} \\ & +(1-\omega_2) \cdot \left(v(\theta_{2l},\theta_{2h}) - v(\theta_{2l},\theta_{2l})\right).\end{aligned}$$

$$\begin{aligned}\Delta_{2l}^* = \;& \pi_{2l} \cdot (v(\theta_{1h},\theta_{2l}) - v(\theta_{1h},\theta_{1h}) + \Delta_{1h}^*) + \pi_{2h} \cdot (v(\theta_{1l},\theta_{2h}) - v(\theta_{1l},\theta_{1l}) + \Delta_{1l}^*) - \pi_{2h} \cdot \delta_2 \\ \Delta_{2h}^* = \;& \pi_{2l} \cdot (v(\theta_{1h},\theta_{2l}) - v(\theta_{1h},\theta_{1h}) + \Delta_{1h}^*) + \pi_{2h} \cdot (v(\theta_{1l},\theta_{2h}) - v(\theta_{1l},\theta_{1l}) + \Delta_{1l}^*) + \pi_{2l} \cdot \delta_2.\end{aligned}$$

and $\omega_1, \omega_2 \in [0,1]$.

A.3.2.2 Partial overlapping distribution case. When the distribution is partial overlapping ($\theta_{1l} < \theta_{2l} < \theta_{1h} < \theta_{2h}$), there are two kinds of optimal quantities offered to the type-1 agent: efficient quantities and full quantities.

First, we list the analytical solutions when the efficient quantities are offered to the type-1 agent. The optimal quantities and prices are shown in the following: The

	$(q^*_{dl},$	$t^*_{dl})$	$(q^*_{dh},$	$t^*_{dh})$
$d=1$	$(\overline{q}_{1l},$	$\tau(\overline{q}_{1l})-\Delta^*_{1l})$	$(\overline{q}_{1h},$	$\tau(\overline{q}_{1h})-\Delta^*_{1h})$
$d=2$	$(\theta_{2l},$	$\hat{\tau}(\theta_{2l})-\Delta^*_{2l})$	$(\theta_{2h},$	$\hat{\tau}(\theta_{2h})-\Delta^*_{2h})$

value of information rent depends on the different binding constraints.

(1) IR_2 and IC_{12} are binding.

When $\pi_{1l} > \pi_{2l}$,

$$\begin{aligned}
\Delta^*_{2l} = \quad & -\pi_{2h}\cdot\min\{v(\theta_{2h},\theta_{2h})-v(\theta_{2l},\theta_{2l}),\\
& \tfrac{1}{\pi_{1l}-\pi_{2l}}\left(\pi_{1l}(v(\theta_{2l},\theta_{1l})-v(\theta_{2l},\theta_{2l}))+\pi_{1h}(v(\theta_{2h},\theta_{1h})-v(\theta_{2h},\theta_{2h}))\right)\}\\
\Delta^*_{2h} = \quad & \pi_{2l}\cdot\min\{v(\theta_{2h},\theta_{2h})-v(\theta_{2l},\theta_{2l}),\\
& \tfrac{1}{\pi_{1l}-\pi_{2l}}\left(\pi_{1l}(v(\theta_{2l},\theta_{1l})-v(\theta_{2l},\theta_{2l}))+\pi_{1h}(v(\theta_{2h},\theta_{1h})-v(\theta_{2h},\theta_{2h}))\right)\}.
\end{aligned}$$

When $\pi_{1l} < \pi_{2l}$,

$$\begin{aligned}
\Delta^*_{2l} = \quad & -\pi_{2h}\cdot\max\{v(\theta_{2h},\theta_{2l})-v(\theta_{2l},\theta_{2l}),\\
& \tfrac{1}{\pi_{1l}-\pi_{2l}}\left(\pi_{1l}(v(\theta_{2l},\theta_{1l})-v(\theta_{2l},\theta_{2l}))+\pi_{1h}(v(\theta_{2h},\theta_{1h})-v(\theta_{2h},\theta_{2h}))\right)\\
& v(\theta_{2l},\theta_{1h})-v(\theta_{2h},\theta_{1h})+v(\theta_{2h},\theta_{2h})-v(\theta_{2l},\theta_{2l})\}\\
\Delta^*_{2h} = \quad & \pi_{2l}\cdot\max\{v(\theta_{2h},\theta_{2l})-v(\theta_{2l},\theta_{2l}),\\
& \tfrac{1}{\pi_{1l}-\pi_{2l}}\left(\pi_{1l}(v(\theta_{2l},\theta_{1l})-v(\theta_{2l},\theta_{2l}))+\pi_{1h}(v(\theta_{2h},\theta_{1h})-v(\theta_{2h},\theta_{2h}))\right)\\
& v(\theta_{2l},\theta_{1h})-v(\theta_{2h},\theta_{1h})+v(\theta_{2h},\theta_{2h})-v(\theta_{2l},\theta_{2l})\}.
\end{aligned}$$

When $\pi_{1l} = \pi_{2l}$ and $\omega_2 \in [0,1]$,

$$\begin{aligned}
\Delta_{2l}^* = \quad & -\pi_{2h} \cdot [\omega_2 \cdot (v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})) \\
& +(1-\omega_2) \cdot \max\{v(\theta_{2h}, \theta_{2l}) - v(\theta_{2l}, \theta_{2l}), v(\theta_{2l}, \theta_{1h}) - v(\theta_{2h}, \theta_{1h}) \\
& +v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})\}] \\
\Delta_{2h}^* = \quad & \pi_{2l} \cdot [\omega_2 \cdot (v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})) \\
& +(1-\omega_2) \cdot \max\{v(\theta_{2h}, \theta_{2l}) - v(\theta_{2l}, \theta_{2l}), v(\theta_{2l}, \theta_{1h}) - v(\theta_{2h}, \theta_{1h}) \\
& +v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})\}].
\end{aligned}$$

and,

$$\begin{aligned}
\Delta_{1l}^* \quad & = \pi_{1l} \left(v(\theta_{2l}, \theta_{1l}) - v(\theta_{2l}, \theta_{2l}) + \Delta_{2l}^* \right) + \pi_{1h} \left(v(\theta_{2h}, \theta_{1h}) - v(\theta_{2h}, \theta_{2h}) + \Delta_{2h}^* \right) \\
& -\pi_{1h} \alpha (\theta_{1h} - \theta_{1l}) q_{1l} \\
\Delta_{1h}^* \quad & = \pi_{1l} \left(v(\theta_{2l}, \theta_{1l}) - v(\theta_{2l}, \theta_{2l}) + \Delta_{2l}^* \right) + \pi_{1h} \left(v(\theta_{2h}, \theta_{1h}) - v(\theta_{2h}, \theta_{2h}) + \Delta_{2h}^* \right) \\
& +\pi_{1l} (\theta_{1h} - \theta_{1l}) q_{1l}.
\end{aligned}$$

(2) IR_1, IR_2 and IC_{1hl} are binding.

$$\begin{aligned}
\Delta_{1l}^* = \quad & -\pi_{1h} \left(v(q_{1l}, \theta_{1h}) - v(q_{1l}, \theta_{1l}) \right). \\
\Delta_{1h}^* = \quad & \pi_{1l} \left(v(q_{1l}, \theta_{1h}) - v(q_{1l}, \theta_{1l}) \right).
\end{aligned}$$

Let,

$$\begin{aligned}
\zeta = \ & \omega \cdot \min\{v(\theta_{2l}, \theta_{1h}) - v(\theta_{2h}, \theta_{1h}) + v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l}), \\
& v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})\} \\
& +(1-\omega) \cdot \max\{\tfrac{1}{\pi_{2h}} \cdot (\pi_{1l} v(\theta_{2l}, \theta_{1l}) + \pi_{1h} v(\theta_{2l}, \theta_{1h}) - v(\theta_{2l}, \theta_{2l}))\,, \\
& v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})\}
\end{aligned}$$

$$\begin{aligned}
\Delta^*_{2l} = \ & -\pi_{2h}\zeta. \\
\Delta^*_{2h} = \ & \pi_{2l}\zeta.
\end{aligned}$$

(3) IR_1 and IR_2 are binding.

Let,

$$\begin{aligned}
\zeta_1 = \ & \omega_1 \cdot \min\{v(q_{1l}, \theta_{2l}) - v(q_{1h}, \theta_{2l}) + v(q_{1h}, \theta_{1h}) - v(q_{1l}, \theta_{1l}), \\
& v(q_{1l}, \theta_{2h}) - v(q_{1h}, \theta_{2h}) + v(q_{1h}, \theta_{1h}) - v(q_{1l}, \theta_{1l}), \\
& v(q_1 h, \theta_{1h}) - v(q_{1h}, \theta_{1l})\} \\
& +(1-\omega_1) \cdot \max\{\tfrac{1}{\pi_{1h}} \cdot (\pi_{2l} v(q_{1l}, \theta_{2l}) + \pi_{1h} v(q_{1l}, \theta_{2h}) - v(q_{1l}, \theta_{1l}))\,, \\
& v(q_{1l}, \theta_{1h}) - v(q_{1l}, \theta_{1l})\}
\end{aligned}$$

$$\begin{aligned}
\zeta_2 = \ & \omega_2 \cdot min\{v(\theta_{2l}, \theta_{1l}) - v(\theta_{2h}, \theta_{1l}) + v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l}), \\
& v(\theta_{2l}, \theta_{1l}) - v(\theta_{2h}, \theta_{1l}) + v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l}), \\
& v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})\} \\
& +(1-\omega_2) \cdot \max\{\tfrac{1}{\pi_{2h}} \cdot (\pi_{1l} v(\theta_{2l}, \theta_{1l}) + \pi_{1h} v(\theta_{2l}, \theta_{1h}) - v(\theta_{2l}, \theta_{2l}))\,, \\
& v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})\}
\end{aligned}$$

$$
\begin{aligned}
\Delta_{1l}^{*} = & \quad -\pi_{1h} \cdot \zeta_1 \\
\Delta_{1h}^{*} = & \quad \pi_{1l} \cdot \zeta_1 \\
\Delta_{2l}^{*} = & \quad -\pi_{2h} \cdot \zeta_2 \\
\Delta_{2h}^{*} = & \quad \pi_{2l} \cdot \zeta_2.
\end{aligned}
$$

Then we show the analytical solutions when the full quantities are offered to the type-1 agent.The optimal quantities and prices are The quantities are always

	$(q_{dl}^{*}, \quad t_{dl}^{*})$	$(q_{dh}^{*}, \quad t_{dh}^{*})$
$d = 1, 2$	$(\theta_{dl}, \quad \hat{\tau}(\theta_{dl}) - \Delta_{dl}^{*})$	$(\theta_{dh}, \quad \hat{\tau}(\theta_{dh}) - \Delta_{dh}^{*})$

full quantities, however, the prices are different depends on the different binding constraints. The value of each information rent is listed in the following when the different constraints are binding.

(1) IR_1, IR_2, IC_{12} are binding.

$$
\begin{aligned}
\Delta_{2l}^{*} = & \quad v(\theta_{2l}, \theta_{2l}) - \pi_{1l} \cdot v(\theta_{2l}, \theta_{1l}) - \pi_{1h} \cdot v(\theta_{2l}, \theta_{1h}) \\
\Delta_{2h}^{*} = & \quad -\tfrac{\pi_{2l}}{\pi_{2h}} \cdot \Delta_{2l}^{*}.
\end{aligned}
$$

Let,

$$
\begin{aligned}
\delta_1 = & \quad \omega_1 \cdot (v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})) \\
& +(1 - \omega_1) \cdot \max\{v(\theta_{1l}, \theta_{2l}) - v(\theta_{1h}, \theta_{2l} + v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}), \\
& v(\theta_{1l}, \theta_{2l}) - v(\theta_{1h}, \theta_{2l} + v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}), v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})\}
\end{aligned}
$$

$$\begin{aligned}\Delta_{1l}^* &= \pi_{1h} \cdot \delta_1 \\ \Delta_{1h}^* &= -\pi_{1l} \cdot \delta_1.\end{aligned}$$

(2) IR_1, IC_{21}, IC_{1hl}, IC_{2hl} are binding.

$$\begin{aligned}\Delta_{1l}^* &= -\pi_{1h} \cdot (v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})) \\ \Delta_{1h}^* &= \pi_{1l} \cdot (v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}))\,.\end{aligned}$$

Let,

$$\begin{aligned}\delta_2 = \;& \pi_{2l} \cdot \max\{v(\theta_{1l}, \theta_{2l}) - v(\theta_{1l}, \theta_{1l} + \Delta_{1l}^*, v(\theta_{1h}, \theta_{2l}) - v(\theta_{1h}, \theta_{1h} + \Delta_{1h}^*\} \\ & +\pi_{2h} \cdot \max\{v(\theta_{1l}, \theta_{2h}) - v(\theta_{1l}, \theta_{1l} + \Delta_{1l}^*, v(\theta_{1h}, \theta_{2h}) - v(\theta_{1h}, \theta_{1h} + \Delta_{1h}^*\}.\end{aligned}$$

$$\begin{aligned}\Delta_{2l}^* &= \delta_2 - \pi_{2h} \cdot (v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})) \\ \Delta_{1h}^* &= \delta_2 + \pi_{2l} \cdot (v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l}))\,.\end{aligned}$$

(3) IR_1, IC_{21}.

$$\begin{aligned}\Delta_{1l}^* &= -\pi_{1h} \cdot (v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})) \\ \Delta_{1h}^* &= \pi_{1l} \cdot (v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}))\,.\end{aligned}$$

Let,

$\delta_1 = v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})$ if $v(\theta_{2l}, \theta_{1h}) - v(\theta_{2h}, \theta_{1h}) + v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l}) < v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})$,

$\delta_1 = v(\theta_{2l}, \theta_{1h}) - v(\theta_{2h}, \theta_{1h}) + v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})$ if $v(\theta_{2l}, \theta_{1h}) - v(\theta_{2h}, \theta_{1h}) + v(\theta_{2h}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l}) \geq v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})$

$$\delta_1' = v(\theta_{2l}, \theta_{2h}) - v(\theta_{2l}, \theta_{2l})$$

$$\begin{aligned} \delta_2 = \quad & \pi_{2l} \cdot \max\{v(\theta_{1l}, \theta_{2l}) - v(\theta_{1l}, \theta_{1l} + \Delta_{1l}^*, v(\theta_{1h}, \theta_{2l}) - v(\theta_{1h}, \theta_{1h} + \Delta_{1h}^*\} \\ & + \pi_{2h} \cdot \max\{v(\theta_{1l}, \theta_{2h}) - v(\theta_{1l}, \theta_{1l} + \Delta_{1l}^*, v(\theta_{1h}, \theta_{2h}) - v(\theta_{1h}, \theta_{1h} + \Delta_{1h}^*\}. \end{aligned}$$

$$\begin{aligned} \Delta_{2l}^* &= \delta_2 - \pi_{2h} \cdot (\omega \cdot \delta_1 + (1 - \omega) \cdot \delta_1') \\ \Delta_{2hl}^* &= \delta_2 + \pi_{2l} \cdot (\omega \cdot \delta_1 + (1 - \omega) \cdot \delta_1') \end{aligned}$$

(4) IR_1, IC_{12}, IC_{21}, IC_{1hl} are binding.

$$\begin{aligned} \Delta_{1l}^* = \quad & -\pi_{1h} \cdot (v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})) \\ \Delta_{1h}^* = \quad & \pi_{1l} \cdot (v(\theta_{1l}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})). \end{aligned}$$

Let,

$$\Delta_2 = \quad \pi_{2l} \cdot (v(\theta_{1l}, \theta_{2l}) - v(\theta_{1l}, \theta_{1l}) + \Delta_{1l}^*) + \pi_{2h} \cdot (v(\theta_{1h}, \theta_{2h}) - v(\theta_{1h}, \theta_{1h}) + \Delta_{1h}^*)$$

$$\Delta_{2l}^* = v(\theta_{2l}, \theta_{2l}) - \pi_{1l} \cdot v(\theta_{2l}, \theta_{1l}) - \pi_{1h} \cdot v(\theta_{2l}, \theta_{1h})$$
$$\Delta_{2h}^* = \frac{1}{\pi_{2h}} \cdot (\delta_2 - \pi_{2l} \cdot \Delta_{2l}^*) .$$

A.3.2.3 Complete overlapping distribution case. When the distribution is complete overlapping, the optimal quantities and prices are The quantities are always

	$(q_{dl}^*, \quad t_{dl}^*)$	$(q_{dh}^*, \quad t_{dh}^*)$
$d = 1, 2$	$(\theta_{dl}, \quad \hat{\tau}(\theta_{dl}) - \Delta_{dl}^*)$	$(\theta_{dh}, \quad \hat{\tau}(\theta_{dh}) - \Delta_{dh}^*)$

full quantities, however, the prices are different depends on the different binding constraints. The value of each information rent is listed in the following when the different constraints are binding.

(1) IR_1, IC_{12}, IC_{21} are binding.

When $\pi_{1l} \geq \pi_{2l}$,

$$\begin{aligned}
\Delta_{1l}^* = & -\pi_{1h} \cdot \max\{v(\theta_{1l}, \theta_{2h}) - v(\theta_{1h}, \theta_{2h}) + v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}), \\
& v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})\} \\
\Delta_{1h}^* = & \pi_{1l} \cdot \max\{v(\theta_{1l}, \theta_{2h}) - v(\theta_{1h}, \theta_{2h}) + v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}), \\
& v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l})\}
\end{aligned}$$

When $\pi_{1l} < \pi_{2l}$,

$$\Delta_{1l}^* = -\pi_{1h} \cdot (v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}))$$
$$\Delta_{1h}^* = \pi_{1l} \cdot (v(\theta_{1h}, \theta_{1h}) - v(\theta_{1l}, \theta_{1l}))$$

and,

$$\begin{aligned}
\Delta_{2l}^* = &\quad v(\theta_{2l},\theta_{2l}) - \pi_{1l}v(\theta_{2l},\theta_{1l}) - \pi_{1h}v(\theta_{2l},\theta_{1h}) \\
\Delta_{2h}^* = &\quad \tfrac{1}{\pi_{2h}} \cdot (\pi_{2l}(v(\theta_{1l},\theta_{2l}) - v(\theta_{1l},\theta_{1l})) + \pi_{2h}(v(\theta_{1h},\theta_{2h}) - v(\theta_{1h},\theta_{1h})) \\
&\quad +(\pi_{1l} - \pi_{2l}) \cdot (\Delta_{1h}^* - \Delta_{1l}^*) - \pi_{2l}\Delta_{2l}^*)
\end{aligned}$$

(2) IR_1, IC_{21} are binding.

$$\begin{aligned}
\Delta_{1l}^* = &\quad -\pi_{1h}\left(\omega \cdot (v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\right. \\
&\quad \left. +(1-\omega) \cdot \max\{0, v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\right) \\
\Delta_{1h}^* = &\quad \pi_{1l}\left(\omega \cdot (v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}))\right. \\
&\quad \left. +(1-\omega \cdot \max\{0, v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\right)
\end{aligned}$$

$\omega \in [0,1]$ Let,

$$\delta = \pi_{2l}\left(v(\theta_{1l},\theta_{2l}) - v(\theta_{1l},\theta_{1l}) + \Delta_{1l}^*\right) + \pi_{2h}\left(v(\theta_{1h},\theta_{2h}) - v(\theta_{1h},\theta_{1h}) + \Delta_{1h}^*\right)$$

$$\begin{aligned}
\Delta_{2l}^* = &\quad \delta - \pi_{2h}\left(\omega \cdot (v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\right. \\
&\quad \left. +(1-\omega) \cdot \max\{0, v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\right) \\
\Delta_{2h}^* = &\quad \pi_{1l}\left(\omega \cdot (v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}))\right. \\
&\quad \left. +(1-\omega) \cdot \max\{0, v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\right)
\end{aligned}$$

(3) IR_2, IC_{12} are binding.

When $v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}) \geq v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l})$,

$$\begin{aligned}\delta = \Delta^*_{1h} - \Delta^*_{1l} = \quad & \omega \cdot \max\{v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}),\\ & v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\\ & +(1-\omega)\cdot(v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l}))\end{aligned}$$

When $v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}) < v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l})$,

$$\delta = \Delta^*_{1h} - \Delta^*_{1l} = \quad \omega\cdot(v(\theta_{1h},\theta_{1h}) - c(\theta_{1l},\theta_{1l})) + (1-\omega)\cdot(v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l}))$$

And $\omega \in [0,1]$.

$$\begin{aligned}\Delta^*_{2l} = \quad & -\pi_{2h}\left(v(\theta_{2l},\theta_{1h}) - v(\theta_{2h},\theta_{1h}) + v(\theta_{2h},\theta_{2h}) - v(\theta_{2l},\theta_{2l})\right)\\ \Delta^*_{2h} = \quad & \pi_{2l}\left(v(\theta_{2l},\theta_{1h}) - v(\theta_{2h},\theta_{1h}) + v(\theta_{2h},\theta_{2h}) - v(\theta_{2l},\theta_{2l})\right)\end{aligned}$$

$$\pi_{1l}\Delta^*_{1l} + \pi_{1h}\Delta^*_{1h} = \quad \pi_{1l}v(\theta_{2l},\theta_{1l}) + \pi_{1h}v(\theta_{2l},\theta_{1h}) - v(\theta_{2l},\theta_{2l}) + \Delta^*_{2l} = \delta'$$

$$\begin{aligned}\Delta^*_{1l} = \quad & \delta' - \pi_{1h}\delta\\ \Delta^*_{1h} = \quad & \delta' + \pi_{1l}\delta\end{aligned}$$

(4) IR_1, IR_2, IC_{12} are binding.

Let,

$$\begin{aligned} \delta = \quad & \tfrac{1}{\pi_{1l}-\pi_{2l}} \cdot (\pi_{2l}(v(\theta_{1l},\theta_{1l}) - v(\theta_{1l},\theta_{2l})) + \pi_{2h}(v(\theta_{1h},\theta_{1h}) - v(\theta_{1h},\theta_{2h}))) \\ & \pi_{1l} \neq \pi_{2l} \end{aligned}$$

When $\pi_{1l} > \pi_{2l}$,

$$\begin{aligned} \Delta^*_{1l} = \quad & -\pi_{2h}\left(\omega \cdot \min\{v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}), \delta\}\right. \\ & +(1-\omega) \cdot \max\{v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l}), \\ & \left. v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\right) \\ \Delta^*_{1h} = \quad & \pi_{2l}\left(\omega \cdot \min\{v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}), \delta\}\right. \\ & +(1-\omega) \cdot \max\{v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l}), \\ & \left. v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\right) \end{aligned}$$

When $\pi_{1l} < \pi_{2l}$,

$$\begin{aligned} \Delta^*_{1l} = \quad & -\pi_{2h}\left(\omega \cdot (v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}))\right. \\ & +(1-\omega) \cdot \max\{v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l}), \\ & \left. v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\right) \\ \Delta^*_{1h} = \quad & \pi_{2l}\left(\omega \cdot (v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}))\right. \\ & +(1-\omega) \cdot \max\{v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l}), \\ & \left. v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\right) \end{aligned}$$

When $\pi_{1l} = \pi_{2l}$ and $\omega \in [0,1]$,

$$\begin{aligned}
\Delta_{1l}^* = \quad & -\pi_{2h}\left(\omega \cdot (v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}))\right. \\
& +(1-\omega)\cdot \max\{v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l}), \delta, \\
& \left. v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\right) \\
\Delta_{1h}^* = \quad & \pi_{2l}\left(\omega \cdot (v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l}))\right. \\
& +(1-\omega)\cdot \max\{v(\theta_{1l},\theta_{1h}) - v(\theta_{1l},\theta_{1l}), \delta, \\
& \left. v(\theta_{1l},\theta_{2h}) - v(\theta_{1h},\theta_{2h}) + v(\theta_{1h},\theta_{1h}) - v(\theta_{1l},\theta_{1l})\}\right)
\end{aligned}$$

and,

$$\begin{aligned}
\Delta_{2l}^* = \quad & v(\theta_{2l},\theta_{2l}) - \pi_{1l}v(\theta_{2l},\theta_{1l}) - \pi_{1h}v(\theta_{2l},\theta_{1h}) \\
\Delta_{2h}^* = \quad & -\tfrac{\pi_{1l}}{\pi_{2h}}\left(v(\theta_{2l},\theta_{2l}) - \pi_{1l}v(\theta_{2l},\theta_{1l}) - \pi_{1h}v(\theta_{2l},\theta_{1h})\right)
\end{aligned}$$

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