ABSTRACT

CONSTRUCTIVE SOLUTION METHODOLOGIES TO THE CAPACITATED NEWSVENDOR PROBLEM AND SURROGATE EXTENSION

by
Pinyuan Shan

The newsvendor problem is a single-period stochastic model used to determine the order quantity of perishable product that maximizes/minimizes the profit/cost of the vendor under uncertain demand. The goal is to find an initial order quantity that can offset the impact of backlog or shortage caused by mismatch between the procurement amount and uncertain demand. If there are multiple products and substitution between them is feasible, overstocking and understocking can be further reduced and hence, the vendor's overall profit is improved compared to the standard problem. When there are one or more resource constraints, such as budget, volume or weight, it becomes a constrained newsvendor problem.

In the past few decades, many researchers have proposed solution methods to solve the newsvendor problem. The literature is first reviewed where the performance of each of existing model is examined and its contribution is reported. To add to these works, it is complemented through developing constructive solution methods and extending the existing published works by introducing the product substitution models which so far has not received sufficient attention despite its importance to supply chain management decisions. To illustrate, this dissertation provides an easy-to-use approach that utilizes the known network flow problem or knapsack problem. Then, a polynomial in fashion

algorithm is developed to solve it. Extensive numerical experiments are conducted to compare the performance of the proposed method and some existing ones. Results show that the proposed approach though approximates, yet, it simplifies the solution steps without sacrificing accuracy. Further, this dissertation addresses the important arena of product substitute models. These models deal with two perishable products, a primary product and a surrogate one. The primary product yields higher profit than the surrogate. If the demand of the primary exceeds the available quantity and there is excess amount of the surrogate, this excess quantity can be utilized to fulfill the shortage. The objective is to find the optimal lot sizes of both products, that minimize the total cost (alternatively, maximize the profit). Simulation is utilized to validate the developed model. Since the analytical solutions are difficult to obtain, Mathematical software is employed to find the optimal results. Numerical experiments are also conducted to analyze the behavior of the optimal results versus the governing parameters. The results show the contribution of surrogate approach to the overall performance of the policy.

From a practical perspective, this dissertation introduces the applications of the proposed models and methods in different industries such as inventory management, grocery retailing, fashion sector and hotel reservation.

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my father 单鸿彪

my mother 毛安

for their unconditional and endless love.

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LIST OF SYMBOLS

B	Available budget
B_{opt}	Budget required to order the unconstrained optimal quantity of all products, $B_{opt} = \sum_{i=1}^{n} c_i x_i^*$
c_i	Purchase cost
D_i	Demand variable
$E(x_i)$	Expected total cost of Product i
$E^T(x_i)$	Taylor's expansion of the total cost function of Product i
$\Delta E\%$	Percentage of difference of optimal total cost
E_{nv}	Expected total cost of the newsvendor without surrogate product
E_{nvs}	Expected total cost of the newsvendor with surrogate product
ε	Error generated by Taylor's expansion
$f(.)$	Probability density function (PDF)
$F(.)$	Cumulative distribution function (CDF)
h_i	Holding cost
I	Interest rate used in determining carrying charge per period
k	The parameter of scale
L_{i01}	The amount of flow in ith arc from node 0 to node 1 on the left-hand side of the network
L_{12}	The amount of flow in the unique arc from node 1 to node 2 on the right-hand side of the network

La	Lagrangian relaxation of the total cost function
λ	Lagrangian multiplier
m_i	Mean of normal distribution
μ_i	Mean of exponential distribution
P_s	The probability of being in the context of the newsvendor surrogate
$P_{x_b - D_b > D_a - x_a}$	The probability of entire excess demand of Product *a* is satisfied by the left overs of Product *b*
$P_{x_b - D_b < D_a - x_a}$	The probability of all the left overs of Product *b* are used to satisfy the excess demand of Product *a*
R	Available resource
SL_i	Service level
π_i	Loss of goodwill cost for each unsatisfied demand
s_i	Salvage value
σ_i	Standard deviation of normal distribution
$U_i,\ L_i$	Upper and lower bounds of uniform distribution
v_i	Price
Var_i	Variance of demand
x_i	Order quantity of Product *i*
x_i^*	Optimal order quantity of Product *i* for uncapacitated newsvendor
x_i^{**}	Optimal order quantity of Product *i* for capacitated newsvendor
x_s	Expected value of surrogate quantity

x_{sa}	Expected quantity of Product *a* satisfied by the left overs of Product *b*
x_{sb}	Expected quantity of Product *b* used to satisfy the shortage of Product *a*
X_i	The threshold of indeterminate order quantity
$\phi()$	Standard normal distribution

LIST OF DEFINITIONS

GIM	Generic Iterative Method
BSM	Binary Search Method
LAM	Linear Approximation Method
NFM	Network Flow Method
UNI	Uniform Distribution
EXP	Exponential Distribution
NOR	Normal Distribution

CHAPTER 1

INTRODUCTION

1.1 Background

The pioneer framework to the newsvendor model can be traced back to 1888 when Edgeworth (1888) investigated the bank note reservation problem with random demand from depositors. In 1951, Morse and Kimball (1951) first mentioned the term of "newsboy" in their book and proposed an example of a newsboy who need to stock an amount of newspapers to satisfy random customers' needs while maximizing his profit. The arrival of the customers in the example was expected to follow the Poisson law. They imagined the newsboy ordering the newspapers one by one and the optimal order quantity is determined when the difference between the expected profits of buying one more newspaper and stocking current quantity is negative. An established and most familiar version of the newsvendor model was developed by Arrow and Harris (1974), where the "critical fractile" solution approach was originally derived. Since then, the newsvendor model, as a profit maximization/cost minimization framework, for inventory management problems has spawned a rich and wide-ranging body of contributions to the field.

Overall, the newsvendor problem is utilized to make a single period inventory decision with the presence of random demand which leads to understocking and overstocking cost and the goal is to find the optimal order quantity to maximize the expected profit or minimize the expected cost of the vendor. Moreover, it is one of the most popular models in analytical, behavioral and empirical research of inventory and supply chain management due to its versatility, flexibility and elegant structure. From the last century to the present, research related to the newsvendor problem has been constantly emerging, and in the future, the research in this filed will continue to develop.

1.1 Newsvendor Model

In the 1970s – 1980s, variations of the newsvendor model appeared in different areas. In 1979, Atkinson (1979) studied an incentive design problem in a newsvendor-type framework in which product price and probability distribution of retail sales are assumed to be fixed. In the case of a uniformly distributed demand, he made concrete calculations and found that a risk-averse manager will order a smaller quantity compared with the risk-neutral manager. In order to improve this situation, a delegation scheme was proposed. Moreover, when profit share is introduced as a compensation, standards-based contracts provide incentives to the manager to incorporate private information in the order size decision.

In 1980, Lau (1980) proposed two new objectives to the classical newsvendor problem which maximizes the expected profit by determining the optimal order quantity. The first alternative objective is "maximizing expected utility" and the second one is "maximizing the probability of achieving a budgeted profit". By adopting the first objective, partial moments appear in solution procedures. In order to handle this, Lau developed a set of general-purpose formulas to solve the problem and these formulas may also be applied to many other management models involving partial moments. The second objective is widely adopted by managers in real life, but it is not very common in academic literature at the time. Additionally, this paper also illustrated some of the advantages of using Schmeiser-Deutsch (1977) distribution in stochastic modelling.

In 1985, Pasternack (1985) investigated a two-echelon supply chain system in a newsvendor context. In this system, the manufacturer is assumed to have control of the channel and retailers can only choose to procure the goods or not. The decisions made by the manufacturer such as price charged to retailers and return credit of the unsold goods on the retail side will not only affect the procurement decisions of retailers but also affect the customer's demand and therefore led to lost sales and disposition. The results unveil that, in a multi-retailer environment, a manufacturer offers retailers a partial credit for all unsold goods can ensure channel coordination and achieve optimum for both manufacturer's and average retailer's profit although retailers are not affected equally.

In 1988, Lau and Lau (1988) extended the classical newsvendor problem by considering a stochastic price-demand relationship and formulated a versatile model which can handle various levels of complexities. In the solution, the order quantity and price are optimized jointly. The analytical solution can be obtained only for the simplest price-demand relationship. Furthermore, they developed efficient numerical solution procedures to deal with more complicated cases. The numerical experiments illustrate the effects of price sensitivity and demand uncertainty, and demonstrate that the performance of the classical newsvendor model is improved significantly by incorporating a price variable.

1.2 Solution to the Newsvendor Problem

In the past few decades, as the newsvendor problem has been extensively studied, the research on its solution has also increased. In this section, some established works on the solution to the newsvendor problem are introduced. The involved notations are given as follows:

Symbol	*Description*
B	Available budget
c_i	Purchase cost
$E(x_i)$	Expected total cost of Product *i*
h_i	Holding cost
I	Interest rate used in determining carrying charge per period
k	The parameter of scale
La	Lagrangian relaxation of the total cost function
λ	Lagrangian multiplier
R	Available resource
SL_i	Service level of Product *i*
π_i	Loss of goodwill cost for each unsatisfied demand
s_i	Salvage value
v_i	Price
x_i	Order quantity

X_i	The threshold of indeterminate order quantity
$\phi()$	Standard normal distribution
m_i	Mean of normal distribution
σ_i	Standard deviation of normal distribution

In 1964, Hadley and Whitin (1964) presented an example of houseware procurement in their book. In this problem, a buyer needs to purchase three items in his annual trip to Europe. The three items in the purchasing list are: 1) copper skillets, 2) salt and pepper sets, and 3) coffee makers. Based on the experience of the buyer, the demand of copper skillets and coffee makers are normally distributed and the demand of salt and pepper sets is uniformly distributed. The objective is to determine the order quantity of each item to maximize the expected profit within a budget. The Lagrangian relaxation of the problem is show as in Equation (1.1):

$$La = \sum_{i=1}^{3} E_i(x_i) + \lambda \left[\sum_{i=1}^{3} c_i x_i - B \right] \tag{1.1}$$

And the necessary condition for each item is derived as in Equation (1.2):

$$\frac{\partial La}{\partial x_i} = \frac{\partial E_i(x_i)}{\partial x_i} + \lambda c_i = 0 \tag{1.2}$$

In this case, they didn't prove that it takes the entire budget to achieve the optimum. They just pointed out that the value of Lagrangian multiplier must be negative, otherwise infeasible results are obtained (infinite order quantity of items). In order to find the optimal value of Lagrangian multiplier, they proposed solution procedures by using dynamic programming to solve the problem. However, their method has several disadvantages as stated below:

1) There is no specific procedure to avoid negative value of the optimal order quantity for each item;

2) The computation increases significantly as the number of items increases;

3) This method works only for a limited class of demand distributions.

In 1984, Nahmias and Schmidt (1984) analyzed the solution procedures proposed by Hadley and Whitin (1964) and gave some shortcomings of this method:

1) The steps guessing the optimal value of Lagrangian multiplier complicate solution procedures and hence, numerical difficulties arise;

2) Development of a software programming is required when a realistically sized problem needs to be solved and the computing time is substantial in this case.

The motivation of their work is to resolve above issues and develop efficient heuristics to solve the multi-product newsvendor problem subject to a single constraint. Consequently, four heuristics were developed and their principles are summarized as follows:

1) In the first heuristic, the values of the optimal order quantities of the unconstrained problem are scaled down to fit the available resource constraint, that is, $x_i^{**} = kx_i^*$ where $k\sum_{i=1}^{n} r_i\, x_i^* = R$;

2) In the second heuristic, the fractile point of each demand distribution is scaled to fit the available resource constraint. If it is applied to normal distribution, that is to find k to solve $\sum_{i=1}^{n}(m_i + k\sigma_i)^{+} r_i = R$ and let $x_i^{**} = m_i + k\sigma_i$ as long as this value is nonnegative;

3) In the third heuristic, they defined:

$$a_i = F(x_i^*) = \frac{v_i + \pi_i - (1 + 0.5I)c_i}{v_i + \pi_i + 0.5Ic_i - s_i} \tag{1.3}$$

$$b_i = r_i/(v_i + \pi_i + 0.5Ic_i - s_i) \tag{1.4}$$

$$t_i(\lambda) = \phi^{-1}(a_i - b_i\lambda) \tag{1.5}$$

$\phi()$ is the standard normal distribution function. A first-order Taylor's expansion is applied to $t_i(\lambda)$ at the point of $\lambda = 0$. Then the problem becomes to solve $\sum_{i=1}^{n} r_i[m_i + \widetilde{t_i}(\lambda)\sigma_i]^+ = R$ and $\widetilde{t_i}(\lambda)$ is approximation of $t_i(\lambda)$;

4) In the fourth heuristic, a second-order Taylor's expansion is applied to $t_i(\lambda)$ at the point of $\lambda = a_i/b_i - 1/2b_i$ and get:

$$t_i(\lambda) = (a_i - 0.5 - \lambda b_i)\sqrt{2\pi} \tag{1.6}$$

By solving $\sum_{i=1}^{n} r_i[m_i + \widetilde{t_i}(\lambda)\sigma_i]^+ = R$, the expression of λ can be obtained:

$$\lambda = \frac{\sum_{i=1}^{n} m_i r_i + \sqrt{2\pi}\sum_{i=1}^{n} r_i(a_i - 0.5)\sigma_i - R}{\sqrt{2\pi}\sum_{i=1}^{n} b_i\,\sigma_i r_i} \tag{1.7}$$

It is a simple algebraic function of λ.

By conducting numerical experiments, they compared above four heuristics and the Lagrangian approach. Several insights were drawn from the results:

1) The performance of each heuristic declines as the available recourse decreases;
2) When the available resource is 80% of the unconstrained solution, the cost error is under 5% and when the available resource is 50% or 25% of the unconstrained solution, the cost error in about 90% of the cases is under 5%;

3) The performance of heuristic 4 does not degrade as the number of products increases;

4) Since it is a direct algebraic expression of the problem parameters, the computing time of heuristic 4 is as much as 1000 times less than that required to find the optimum by Lagrangian approach.

In 1996, Lau and Lau (1996) analyzed Hadley and Whitin's (1964) method in detail and extended their work by presenting a solution to handle general demand distributions. In a numerical example with three products with exponential demand distribution, they observed that:

1) Unlike the uncapacitated case, the optimal order quantity of each product is affected by the characteristics of the other products when the capacity is restricted;

2) When the capacity constraint is not tight, the optimal order quantity of each product depends largely on the magnitude of the mean of demand distribution;

3) When the capacity constraint is tight, the optimal order quantity of each product depends largely on the profit contribution per unit resource consumed and this is because the service level is much lower in this case.

In an example of demand distributions with strictly positive lower bounds (uniform distribution), they observed that when resource constraint is sufficiently small, there is no unique value of the optimal order quantity for the product with zero service level. Similarly, in an example of cumulated density function with a long-left tail (normal distribution), the the convergency to the optimum is also hindered by indeterminate order quantity when Hadley and Whitin's (1964) method is applied. In order to resolve above issues, they developed a separable procedure to determine the optimal order quantity for the products with extremely small or zero service level. They defined $X_i = F_i^{-1}(SL_i^*)$ for each product and SL_i^* is a value of service level below which X_i becomes indeterminate. For instance,

$X_i = L_i$ and $X_i = \max(0,\ m_i - 4.75\sigma_i)$ in the case of uniform distribution and normal distribution respectively. Then they proposed two ways to allocate order quantity to the products with infinitesimal or zero service level after all products with sufficient large service level have been ordered with their optimal quantity and there is still a remaining budget:

1) Allocate order quantity in decreasing order of the unit profit of these products up to their X_i until the available budget is fully utilized;
2) Allocate a percentage of remaining budget to these products proportional to their X_i.

To solve the multi-product newsvendor problem with multiple constraints, they employed an adaptation of the 'active set methods' described in Luenberger (1985) and implemented a 'Ranking Heuristic' to make the process of reaching the optimum faster.

For more details of the newsvendor model and its solutions, we refer readers to the comprehensive review by Qin et al. (2011), and the handbook by Choi (2012). In Chapter 2, more recent studies of the newsvendor model and its solutions are reviewed.

1.3 Methodology Overview and Research Significance

In their seminal work, Hadley and Whitin (1964) proposed a Lagrangian dynamic programming method to solve the capacitated newsvendor problem. The procedure of this method is iterative and when the size of problem becomes large, the problem-solving process becomes intractable. Subsequently, many scholars have devoted themselves to seek more efficient solution methods. However, for decades, the solution to the newsvendor problem has not been widely disseminated. The main reason is that the existing solution methods require advanced mathematical knowledge and the steps are cumbersome. Therefore, one motivation of this dissertation is to provide an easy-to-use method and reduce the learning cost. In the development of this method, the conceptions and algorithms are referred from standard textbooks of operations research, management science, and operations management as well as others. The solution method is greatly simplified under the premise of retaining the key information of the model. Finally, a relatively straight forward and error-controllable method is obtained.

The second problem studied in this dissertation is the two-product newsvendor with substitution. More specifically, it addresses the situation where the prime product can be substituted by a surrogate one when shortages occur and the excess demand of the prime product is partially or fully fulfilled by the substitute product if overstocking of the latter exists after satisfying its particular stochastic demand. The objective basically is to find the lot sizes of these two products that minimize/maximize total cost/profit of a vendor who needs to manage inventory of perishable commodities, products with a short shelf life, or items that become technologically obsolete fast. By analyzing the context of substitution, a solid formulation of the model is developed. Subsequently, analytical solutions to the cases of the most common demand distributions such as uniform, exponential and normal distributions are investigated to show the difficulty of obtaining closed form expression of the optimal order quantity for the two products. However, thanks to the explicit formulation of the model and assistance of Mathematica software, the optimal results can be achieved in a few seconds and therefore, a series of numerical experiments are conducted to analyze the behavior of the optimal results versus the governing parameters and draw some managerial insights.

1.4 Organization of the Work

The taxonomy in this dissertation is as follows: Chapter 1 introduces the background, conceptions and early development of the newsvendor problem, as well as the purpose of writing this dissertation. Chapter 2 reviews the relevant literature. The seminal work and development of the solution to the newsvendor problem are explored. As for the extension, literatures on the newsvendor with substitution are reviewed and research involving behavior analysis are also covered to keep up with the trend in this stream. Chapter 3 presents the model formulation of the capacitated newsvendor and provides rationale and proof of the proposed method. The numerical comparisons between the proposed method and some existing ones are conducted to verify the proposed method and show its advantages. Chapter 4 derives a model formulation of the two-product newsvendor with substitution. To validate the proposed model, simulation is utilized and ranges of parameters are tested. Further, numerical experiments are conducted to show the behavior of governing parameters versus the optimum total cost as well as the optimal order quantities of the newsvendor. Chapter 5 introduces the applications of the models and solution methods developed in previous two chapters and they cover industries related to people’s daily lives. Finally, Chapter 6 summarizes the dissertation and provides some directions for future research.

CHAPTER 2

LITERATURE REVIEW

2.1 The Solution Method to the Newsvendor Problem

In the past few decades, many scholars have proposed a lot of solution methods to solve the constrained newsvendor problem. In Chapter 1, some related studies before 2000 are presented. As a continuation, more recent research in this area is reviewed in this chapter. Moon and Silver (2000) considered the fixed ordering cost in the capacitated newsvendor problem. A dynamic program was developed to solve the problem. However, when the number of products under consideration grows substantially, the required computational effort increases significantly. Consequently, the dynamic program becomes unfriendly to practitioners. Two heuristic algorithms were proposed to overcome this shortcoming. The tests showed that the heuristics can handle the distribution free case very well. Abdel-Malek and Montanari (2004, 2005a, 2005b) defined two thresholds to avoid infeasible order quantities and presented a Generic Iterative Method (GIM) to solve the capacitated newsvendor problem. Furthermore, they proposed a solution procedure for the multi-product newsvendor problem with two constraints. Abdel-Malek and Areeratchakul (2007) applied a triangle approximation which facilitates expressing the objective function in quadratic terms. They developed a solution method for the multi-product newsvendor with side constraints. Niederhoff (2007) adopted a Linear Approximation Method (LAM) to solve the newsstand problem. Then, she employed a convex separable programming to find a close approximation of the optimal solution. The convergence of the cost values is affected by the number of breakpoints used to interpolate the cost curve. This approach is iterative in nature and requires insights to determine the number and location of interpolations. Abdel-Malek et al. (2008) extended their previous work by considering random yield in newsvendor problem. Lagrangian approach, Leibniz's rule and Newton's method were applied to develop a solution method to solve the extended model. Simulated experiments were utilized to validate the proposed algorithm. Zhang et al. (2009) developed a Binary Solution Method (BSM) to solve the capacitated newsvendor problem which is applicable to general types of demand distribution functions, discrete as well as continuous. In their method, a bisection algorithm is applied to search the optimum and the judgment conditions are not complicated. However, the it usually requires more iterations to achieve the optimum. Zhang (2010) extended the constrained newsvendor problem by incorporating quantity discounts offered by suppliers. The goal is to maximize the expected profit by taking advantage of the price discounts under budget constraints. A Lagrangian relaxation approach was presented to solve this kind of problem. Also, a case of multi-constraint model and the performance of an adjusted algorithm was investigated. Chen and Chen (2010) explored the multi-product newsvendor problem with a reservation policy. In their model, the optimal order quantity and optimal discount rate are determined simultaneously and the extra-demand caused by discount rate is also considered. Numerical examples show advantages of proposed model and approach over the standard constrained newsvendor model. Moreover, the impact of willingness function on the expected profit was inspected. Abdel-Malek and Otegbeye (2013) extended the constrained multi-product newsvendor problem by considering the random yield. Separable programming and a duality approach were developed to solve the problem. It is worth mentioning that the developed method is portable in the classroom. Zhou et al. (2015) analyzed the multi-product newsvendor problem with budget and loss constraints. They divided the solution space into four cases and developed a loss-based marginal utility deleting method to avoid infeasible results. The reason for introducing loss constraint is to ensure global optimization and improve the speed of convergence. Furthermore, linearization was utilized to deal with the nonlinear loss constraint. Dash and Sahoo (2015) took into account the subjectivity in demand forecasting. They defined the demand as a fuzzy random variable and the purchasing cost as a fuzzy number. Buckley's concept of minimization and fuzzy programming was applied to obtain the optimal results. Wang et al. (2015) incorporated subjectivity, fuzziness and uncertainty in the definition of demands. In order to transfer the uncertain random model to a deterministic one, they pursued the expected cost and referred accumulated information of chance distributions. Wang and Qin (2016) considered fuzziness, randomness and their combination of the demand in the multi-product newsvendor problem. The profit maximization model was converted to a deterministic one when the chance distributions are given. They designed a hybrid simulation to estimate the chance distribution and adopted a genetic algorithm to solve the problem. Moon et al. (2016) extended the multi-product newsvendor problem by integrating multiple discounts and upgrades in the model. The comparisons of traditional and distribution-free approaches were conducted. Their study shows that when discounts and upgrades occur simultaneously, it may profit the seller and the choice of discounts, upgrades or combination of them depends on their impact on profit. Sahoo and Dash (2016) defined both the demand and storage space a fuzzy random variable. The difference between this study and other similar ones is that the storage space is also defined as a fuzzy random variable and it follows either exponential or normal distribution. In their method, the initial single-period inventory fuzzy probabilistic model is converted to a deterministic nonlinear programming problem by a ranking function method. The optimum is obtained by using LINGO software. In Table 2.1, the above studies are summarized and arranged in chronological order.

2.2 The Newsvendor Problem with Substitution and Other Extensions

In this section, the literatures on the newsvendor with substitution are reviewed. Other extensions such as human preference analysis and multi-location newsvendor problem are also included to keep up with trend in this area. In order to organize the content in this section well, the research on the newsvendor with substitution are classified by the number of vendors and products involved in the problem.

Table 2.1 Summary of Literatures on the Solution to the Newsvendor Problem

Year	Authors	Model	Number of Constraints	Solution	Optimality
1963	Hadley and Whitin	The Capacitated Newsvendor Model	Single	Dynamic Programming	Optimum
1984	Nahmias and Schmidt	The Capacitated Newsvendor Model	Single	Lagrangian Heuristic	Optimum/Near Optimum
1996	Lau and Lau	The Capacitated Newsvendor Model	Single/Multiple	Ranking Heuristic	Optimum
2000	Moon and Silver	Newsvendor Model with Fixed Ordering Cost	Single	Dynamic Programming, Marginal Allocation Heuristic and Two Stages Heuristic	Optimum/Near Optimum
2004 2005	Abdel-Malek et al.	The Capacitated Newsvendor Model	Single/Two	Generic Iterative Method (GIM)	Optimum/Near Optimum
2006 2007	Abdel-Malek and Areeratchakul	The Capacitated Newsvendor Model	Single/Multiple	Triangle Approximation and Quadratic Programming	Optimum/Near Optimum
2007	Niederhoff	The Constrained Newsvendor Model	Multiple	Linear Approximation Method (LAM)	Optimum
2008	Abdel-Malek et al.	Newsvendor Model with Random Yield	Single	Lagrangian Heuristic	Near Optimum
2009	Zhang et al.	The Capacitated Newsvendor Model	Single	Binary Solution Method (BSM)	Optimum/Near Optimum
2010	Guoqing Zhang	Newsvendor Model with Supplier Quantity Discount	Single/Multiple	Lagrangian Heuristic	Optimum
2010	Chen and Chen	Newsvendor Model with Reservation	Single	Lagrangian Heuristic	Optimum
2013	Abdel-Malek and Otegbeye	Newsvendor Model with Random Yield	Single/Multiple	Duality Approach	Optimum/Near Optimum
2015	Zhou et al.	Newsvendor Model with Loss Constraint	Two	Loss-based Marginal Utility Deleting Method and Linearization of the Loss Constraint	Optimum/Near Optimum
2015	Dash and Sahoo	Fuzzy Newsvendor Model	Single	Buckley's Concept, Fuzzy Programming Method and LINGO	Optimum
2015	Wang et al.	Uncertain Random Newsvendor Model	Single	MATLAB	Optimum
2016	Wang and Qin	Uncertain Random Newsvendor Model	Single	Hybrid Simulation and Genetic Algorithm	Optimum
2016	Moon et al.	Newsvendor Model with Discounts and Upgrades	Single	Distribution-free Approach and Binary Solution Method	Optimum
2016	Sahoo and Dash	Fuzzy Newsvendor Model	Single	Weighted Sum Method, Ranking Function Method, Chance Constraint and LINGO	Optimum/Near Optimum

The most common scenario of the newsvendor with substitution is one vendor with two products. Stavrulaki (2011) studied an inventory policy for two substitutable products with inventory dependent demand. The proposed model accounts for both the demand stimulation and substitution effects. They developed two heuristic solutions to handle these two effects. The conditions under which the heuristics can be applied effectively were also investigated by a numerical study. Kim and Bell (2011) evaluated the impact of price-driven substitution on a firm's pricing and production capacity decisions for a single period. The cases of symmetrical and asymmetrical demand substitution are explored. Zhou and Sun (2013) studied an inventory management problem with two products and an updated relation. They considered component substitution and product subsititution simultaneously. The situations of firm-driven substitution and customer-driven subsitituion were also investigated by setting different values of system parameters. They applied a two-stage dynamic programming to pursue the optimal assembled quantities for different types of prodcuts. Surti et al. (2014) indicated that the products with different degrees of substitution have different impact on supplier and retailer. Retailers prefer product with high substitution while suppliers do the opposite. In the inventory management, retailer's market power plays a key role in retailer-supplier negotiations. They also found that partial vertical integration and appropriate contracting can reduce the agency costs associated with supply chain. Krommyda et al. (2015) formulated a profit maximizing model to study the inventory management problem of two products with substitution and stock-dependent demand. In some cases, inventory shown to consumers can boost sales, so non-zero stock level at the end of the cycle is desirable to meet the increased demand. The numerical results show that retailer's profit can be improved significantly when both substitution and stock-dependent demand are considered. However, when the substitution rate is small, the profit is high and the holding costs are low, zero ending inventory policy is more preferable. Chen et al. (2015) developed a newsvendor model for two products with downward, supplier-driven substitution. The problem setting with or without customer service level objectives for both products are considered. The numerical study reveals how service commitments affects the optimal inventory levels and how substitution affects profitability. Mukhopadhyay and Goswami (2017) investigated a two-product deterministic inventory system with one-way substitution when there is a shortage of imperfect quanlity items. They presented an EOQ approach to observe the behavior of the optimal order quantities and conrresponding optimal cost when expected defective fraction is more in major item and otherwise. Fu et al. (2017) studied the trade-off between profitability generated by a flexible product and the risk of cannibalizing in a two-product newsvendor system. Their research unveils that the benefit of offering a flexible product is more salient when two products have same price, narrow profit margin or high overage risk and the demands of them are more negatively correlated, more volatile and more symmetric. When the pricing decision of the flexible product is considered, they explored the cases of linear and nonlinear relationship between price difference between the specific and flexible products and the fraction of customers switching to the flexible product. Transchel (2017) analyzed a two-product inventory management problem with price-based and stockout-based substitution. In their model, demand is derived from a linear consumer utility function and endogenous substitution rates are expressed as functions of price. In order to handle the complexity of the model, two price approximations were applied by ignoring stockout-based substitution or considering gross-margin optimization. The performance of these two approximations were analyzed under different distributions of customer valuation and different levels of market uncertainty. Yu et al. (2017) studied a stochastic production-inventory system with firm-driven product substitution and a dynamic pricing policy. The problem was formulated as a Markov decision process (MDP) and the demand of two prodcuts are price-dependent. Production, pricing and substitution decisions are optimized jointly in their model. Li and Fu (2017) investigated a problem with stock-out based substitution when the information of the joint demand distribution is limited. They formulated a two-stage robust optimization model and developed a heuristic to solve it for cases of no substitution and perfect substitution. Zhang et al. (2018) compared the efficiency of a probabilistic selling strategy and a inventory subsitution strategy. They pointed out that the profitability rendered by these two strategies is affected by similarity of products, demand uncertainty and price sensitivity of customers. Different from heuristic approach proposed by other researchers, Zhang et al. (2020) developed an analytical approach to pursue joint optimization of pricing and order quantities for two substitutable products when a newsvendor has a budget constraint. They proved the uniqueness of optimum in a general demand case and investigated the effect of budget and products' parameters on optimal order quantities. The advantage of the analytical approach is that the solutions are straightforward to calculate and have managerial appeal. Zhang et al. (2020) introduced a budget constraint in a newsvendor problem with one-way substitution. In their model, retailers can sell high-end product at a discount price when low-end products are out of stock. The numerical studies were conducted by MATLAB and the results show that when the budget increases, the optimal order quantities of two products increase, and the optimal discount decreases. Otherwise, when the budget constraint is fixed, the variation trend of the optimal order quantity of two products about governing parameters is opposite. From a practical perspective, this research has a certain guiding significance for retailers who have budget limits.

The second case of the newsvendor with substitution is one retailer with multiple substitutable products or a product line. Honhon et al. (2010) considered a newsvendor setting with a set of substitutable products and solved a joint assortment-planning and inventory management problem. In their study, customers are classified by a sequence of products arranged in decreasing order of customer's preferences. They proposed a dynamic programming heuristic and compared it with a Sample Path Gradient Algorithm (SPGA) and an Assortment-Based Substitution (ABS) heuristic. Nowadays, many stores have intelligent system that can coordinate inventory and pricing, thereby reducing the cost of the supply chain. Beyond the current retail practice, Maddah et al. (2011) proposed an integrative approach that can optimize inventory, pricing and variety jointly for a retailer’s product line with substitutable items. The growing interest in this approach comes from the development of the sophisticated demand models and the wide spread of information systems. Tan and Karabati (2013) studied an inventory management problem with stock-out based and customer-driven substitution. The substitution costs and service level requirements were considered in the problem. They proposed an approximated method that can work directly with the point-of-sales data to determine the optimal order-up-to levels. Zhang et al. (2020) studied a multi-product newsvendor problem with customer-driven substitution for both the cases of deterministic and stochastic demand. When demand is deterministic, they proved that the problem is NP-hard and a binary quadratic program can be applied to solve it. When demand is stochastic, they developed a two-stage mixed-integer programming to solve a small or medium sized problem and employed approximation algorithms to handle a large sized problem. Numerical experiments show that substitution among multiple products can reduce the risks from demand uncertainty significantly as well as increase the expected profit. The sensitivity analysis suggests retailers to order more products with high substitution rates and demand variation.

The third case of the newsvendor with substitution is a scenario of multiple retailers. Huang et al. (2011) studied a multi-product competitive newsvendor problem with shortage penalty cost and partial product substitution. An iterative algorithm was developed to solve it. They illustrated the impact of product substitution, demand correlation and demand variation on the optimal order quantities and the corresponding profits in a two-product symmetric example. In a multi-store newsvendor setting, a group of independent retailers need to deal with stochastic demand in a decentralized inventory management system. If the retailers have profit sharing agreements, they could also coordinate inventory transshipment to satisfy shortage with overage. In order to study this realistic problem, Summerfield and Dror (2012) proposed a graphic taxonomy tree that contains a family of inventory problems to explore various scenarios that may happen in the aforementioned context. Moreover, they developed a stochastic programming that can handle optimization problems which have two or more decision stages and time-dependent stochastic variables. The aim of their work is to present an explicit and well understood methodology that balances complexity of recourse options and computational tractability. In Liu et al.'s (2013) study, there are two newsvendors with loss averse preferences and product substitution. They utilized Game Theory to find the optimum and investigate the impact of loss aversion and substitution rate in this competitive environment. Unlike other studies, they demonstrated that there exists a unique Nash equilibrium under certain conditions and there is a threshold of loss aversion coefficient above which the loss averse effect dominates the competition and leads to lower total inventory in a decentralized supply chain. Ye (2014) focused on the inventory issues with both horizontal (inter-brand) and vertical (intra-brand) substitution. They analyzed centralized and decentralized inventory management systems and derived the optimal order quantities for both cases. Duong et al. (2015) formulated a two-echelon inventory management model for perishable and substitutable products. They considered three extensions and treated them separately, they are: multi-period lifetime, positive lead time and customer service level. They adopted a multi-metric approach and simulation to evaluate the performance of this inventory management model. Shou et al.

(2018) investigated a newsvendor game with two loss-averse retailers and the purpose of the research is to find optimal order quantity of product sold by each retailer under multiple mental accounts. The game theory was utilized to find the optimum and the numerical experiments show that the order quantities are decreasing in the loss aversion coefficient and substitution rate and they are increasing in shortage cost and salvage value. The numerical results also reveal that, under a centralized inventory management system, there exists a threshold of loss aversion coefficient above which the effect of loss aversion dominates the effect of competition.

As a supplement, the newsvendor problem with human preference analysis and other extensions are also reviewed. In Schweitzer and Cachon's (2000) experiment, a decision maker cannot be absolutely rational in a single-store setting. Therefore, the bias in order quantities due to decision maker's irrational factors will hinder the realization of the risk pooling benefit. Further, there is a reason to doubt the theoretical predictions in a multi-store case. In response to this, Ho et al. (2010) formally quantified the extent of the psychological biases in multi-location inventory systems. It is based on a fact that people's preferences are reference-dependent. The experiments show that their model can explain the pull-to-center biases in order quantities for both centralized and decentralized structures of inventory management system regardless of whether the profit is high or low. Bansai and Moritz (2015) conducted a series of behavioral experiments to investigate how decision-makers perform when estimating the value of substitution. The results show that subjects often overestimate the value of substitution and it is driven by subjects' fundamental and systematic behavioral biases. After identifying the major factors leading to this, they developed a simple decomposition-based approach to mitigate the overestimation of the value of substitution. This is significant because when sophisticated tool and necessary information are not available, decision-makers frequently use managerial estimates to evaluate the benefit and cost incurred by product substitution. Nagare et al. (2016) investigated the impact of bidirectional changes of demand in a single model. They presented near closed form expressions of decision variables when constraints are incorporated and the optimal order quantity and weight factor are drawn based on the revised forecasts. As mentioned before, Schweitzer and Cachon (2000) were the first to use human subjects to make newsvendor decisions in laboratory experiments. Since then, many other researchers also observed anchoring and insufficient adjustment bias in the experiments. However, few studies have revealed the relationship between human behavioral tendencies and components of solution approach to the newsvendor problem. In order to further explore the phenomena observed by Schweitzer and Cachon, Gavirneni and Robinson (2017) extended the newsvendor problem by incorporating risk aversion into the model and developed an efficient procedure to solve the problem. They unveiled that risk aversion coupled with shortage cost can comprehensively characterize the anchoring and insufficient adjustment bias. In commercial applications, the forecasting experts assist retailers to draw up an ordering plan far ahead of selling seasons. However, in some cases, the unforeseen events may significantly impact the initial demand estimation. Therefore, a revised demand forecast is needed to determine a final ordering plan. Furthermore, a weight factor is utilized to hedge against the potential impact of unforeseen events. Hrabec et al. (2017) studied the impact of advertising on operational problems such as the inventory control with marketing strategies. They introduced a concave and a S-shaped response function to demonstrate the relation between the advertising expenditure and the demand. The demand was formulated in multiplicative or additive form. In their solution, the optimal order quantity and the optimal advertising expenditure are determined simultaneously to achieve the coordination of the marketing and the production. The numerical results show that when the demand is formulated in multiplicative form, the optimal advertising expenditure never exceeds the one in the equivalent deterministic model while, it is always equal to the one in the case of additive form of demand. In Table 2.2, the research reviewed in this section are summarized and arranged in chronological order.

2.3 Knapsack Problem

In Chapter 3, an approximated approach is developed to solve the capacitated newsvendor problem by mapping the model into a network flow maximization problem or a knapsack problem. Knapsack problem is an efficient formulation of the allocation of given resource among competitive alternatives. In this section, the literatures on knapsack problem are reviewed as a supplement to the theoretical base of the method developed in Chapter 3. In 1981, Bitran and Hax (1981) proposed a recursive procedure to solve resource allocation problems by using convex knapsack problem. It determines the optimal value of at least one variable at each iteration which makes it different from other optimization algorithms. Billionnet and Calmels (1996) developed a method by using linear programming to solve the 0-1 quadratic knapsack problem. It obtains better upper bounds of solutions than other known methods. An algorithm was developed based on the heuristics proposed by Chaillou, Hansen and Mahieu (1989) and Gallo, Hammer and Simeone (1980) to obtain a good feasible solution. They also proposed a branch-and-bound algorithm to solve the problem. Marchand and Wolsey (1999) studied a mixed 0-1 knapsack problem which contains a continuous variable in constraint. By investigating the polyhedral structure of the model, they derived a separation heuristic and pointed out that the order of lifting, particularly for the continuous variable, plays an important role. This separation heuristic can be used to derive cuts for more general mixed 0-1 constraints. Bretthauer and Shetty (2002) surveyed the applications of knapsack problem such as production planning, financial modeling, stratified sampling, and capacity planning in manufacturing, health care, and computer networks. Based to the structure of the problem, it can be classified as continuous and integer problems, convex and nonconvex problems, separable and non-separable problems, and problems with additional specially structured constraints. The efficient solution methods were developed according to the characteristics of the problem. In Kogan's (2003) model, time and controllable operation rates are incorporated in the knapsack problem. Each item is put into a knapsack at a controllable production rate and the demand is unknown until the end of the production horizon. Therefore, the continuous-time problem is converted to a discrete-time problem. They proposed a polynomial-time algorithm to obtain the approximate optimum with any desirable precision. By analyzing existing algorithms to solving the continuous quadratic knapsack problem, Kiwiel (2007) discovered some cycling and wrong-convergence examples. In response to this, they developed an algorithm which is linear in time and gives encouraging computational results for large-scale problems. Zhang and Hua (2008) solved a class of convex separable nonlinear knapsack problems by taking advantage of positive marginal cost and increasing marginal loss-cost ratio. In contrast to other solution methods in the literature, their method is a unified one that can be utilized to solve the problem with equality or inequality constraints and it has linear computation complexity. Quadri et al. (2009) employed pre-procedure techniques to reduce the problem size of a particular integer quadratic multi-knapsack problem with a convex set of linear constraints and bounded integer variables. They developed a branch-and-bound algorithm to search the optimum and it outperforms other solution methods in the literature in large scaled instances. Wang et al. (2012) compared the quadratic knapsack problem and its linear representation by testing

Table 2.2 Summary of Literatures on the Newsvendor with Substitution and Other Extensions

The Newsvendor with Substitution

Year	Authors	Retailer	Products	Demand	Substitution	Solution
2010	D. Honhon, V. Gaur, S. Seshadri	One	Multiple	Assortment-dependent	Stockout based	Sample Path Gradient Algorithm/Assortment-Based Substitution Heuristic
2011	E. Stavrulaki	One	Two	Stock-dependent	Stockout based	First Order Condition/Two Heuristic Solutions
2011	D. Huang, H. Zhou, Q. Zhao	Multiple	Multiple	Independent PDF	Stockout based	Static Service Rate and Iterative Algorithm
2011	S. Kim, P. Bell	One	Two	Deterministic/Stochastic	Price-based	First Order Condition
2013	Y. Zhou, J. Sun	One	Two	Independent PDF	Stockout based	Decreased Feasible Domain and First Order Condition
2013	W. Liu, S. Song, C. Wu	Two	Two	Joint PDF	Stockout based	Game Theory
2013	B. Tan, S. Karabati	One	Multiple	Poisson	Stockout based	Genetic Algorithm
2014	C. Surti, P. Ritchie, J. Rowcroft	One	Two	Price-dependent	Stockout based	Game Theory
2014	T. Ye	One/Multiple	Multiple	Independent PDF	Stockout based	First Order Condition
2015	X. Chen, Y. Feng, M. Keblis et al.	One	Two	Joint PDF	Stockout based	Global Optimization
2015	I. Krommyda, K. Skouri, I. Konstantaras	One	Two	Stock-dependent	Stockout based	First Order Condition
2015	L. Duong, L. Wood, W. Wang	Multiple	Multiple	Discrete-Event	Stockout based	Multi-Metric Approach and Simulation Method
2017	A. Mukhopadhyay, A. Goswami	One	Two	Deterministic	Stockout based	EOQ
2017	Q. Fu, Q. Wang, X. Xu et al.	One	Two	Price-dependent	Price/Stockout based	MATLAB
2017	S. Transchel	One	Two	Discrete Choice Model	Price/Stockout based	Price Approximation
2017	Y. Yu, B. Shou, Y, Ni et al.	One	Two	Price-dependent Poisson	State-dependent	Markov Decision Process
2017	Z. Li, Q, Fu	One	Two	Joint PDF	Stockout based	Max-Min Approach/ Heuristic
2018	J. Zhang, W. Xie, S. Sarin	One	Multiple	Substitution-dependent	Stockout based	Double Greedy Algorithm/Lagrangian Relaxation Approach
2018	H. Shou, B. Gu, D. Zhang	Two	Two	Joint PDF	Stockout based	Game Theory
2018	Y. Zhang, G. Hua, S. Wang et al.	One	Two	Joint PDF	Stockout based	First Order Condition
2020	L. Zhang, G. Zhang, Z. Yao	One	Two	Price-dependent	Competition based	KKT Theorem
2020	L. Zhang, Y. Yang, J. Cai	One	Two	Price-dependent	Stockout based	KKT Theorem /MATLAB

Other Extensions

Year	Authors	Problem/Contribution
2011	T. Ho, N. Lim, T. Cui	Multi-Store Newsvendor with Human Preference/Quantification of the Extent of the Psychological Biases
2012	N. Summerfield, M. Dror	Multi-Store Newsvendor with Transshipment/ Graphic Taxonomy Tree/Multi-Stage Stochastic Programming
2015	S. Bansai, B. Moritz	Overestimation for the Value of Substitution /Decomposition-Based Approach
2016	M. Nagare, P. Dutta, N. Cheikhrouhou	Simultaneous Consideration of Bidirectional Changes in Demand/Near Closed Form of Expressions of Decision Variables
2017	D. Hrabec, K. Haugen, P. Popela	Impact of Advertising/Response Function/Joint Optimization of Order Quantity and Advertising
2017	S. Gavirneni, L. Robinson	Newsvendor wish Risk Aversion and Shortage Cost/Explanation of Anchoring and Insufficient Adjustment Bias

instances with multiple constraints and the number of variables is up to eight hundred. Standard branch-and-cut optimizers in CPLEX were utilized to conduct the comparisons and the results show that the linear models perform well on small size instances and the quadratic model is more efficient for solving medium or large size instances. Beheshti (2014) studied a class of bilevel knapsack problem which has a nonlinear integer upper-level objective and a linear binary lower-level objective. They proposed an exact solution approach by using an equivalent single-level reformulation and the linear characteristic of the lower-level objective. The efficiency of the solution approach is assured as long as good upper and lower bounds can be generated in the generic branch-and-backtrack algorithm. Fampa et al. (2020) relaxed the quadratic knapsack problem by perturbing the objective function. In this perturbation, partial quadratic information is maintained and nonconcave part is linearized. To solve the problem, they presented a cutting plane algorithm. In this approach, a primal-dual interior point procedure is employed to update the perturbation at each iteration to reduce the upper bound rendered by the relaxation. New classes of cuts are defined by lifted variables and they are derived from cover inequalities. Additionally, they noted that this method is effective for binary quadratic problems and separation problems.

2.4 Summary of Literature and Contribution of the Dissertation

In this chapter, literatures on the evolution, solution and extensions of the newsvendor problem are reviewed. In addition, some literatures on the knapsack problem are also reviewed and the purpose is to provide an alternative theoretical explanation to the solution method proposed in Chapter 3. The classical newsvendor problem seeks optimal order quantity of the perishable product under stochastic demand. One strategy adopted by many scholars is Lagrangian relaxation. A specific algorithm is developed to find the optimal Lagrangian multiplier such as generic algorithm and binary search algorithm. Another direction of the solution method is the approximation of the objective function and it is also employed in this dissertation. Then quadratic programming or separable programming is utilized to achieve the optimum. However, the solution method proposed in this dissertation differs from the existing literatures in that it adopts a network flow algorithm to solve the problem after the linearization of the objective function. As for the extensions of the newsvendor problem, one of them is the newsvendor with substitution. The problems under this category can be classified as:

1) one retailer with two products
2) one retailer with multiple products or a product line
3) multiple retailers with multiple substitutable products

The substitution can be implemented by suppliers, retailers or customers and the factors leading to substitution include stock-out, pricing, advertising, inventory visible to customers and so on. Moreover, some research further investigates the impact of human preferences on the optimal decision under the context of newsvendor with substitution.

As can be seen from Table 2.1, the solution to the constrained newsvendor problem proposed by predecessors seeks to be as close as possible to the optimum. The marginal utility of pursuing the accuracy of results is decreasing, despite the emergence of various solution methods. That is to say, although researchers spend more time and energy developing “better” approaches, the solution procedures become more complicated and harder to manipulate. More computational efforts are also required to achieve the optimum. On the contrary, the increase in the accuracy is getting smaller. Hence, one of the contributions of this dissertation is to present a simple and straight forward method that is easier to master to solve the capacitated newsvendor problem while the accuracy of the results is sufficient to meet the needs of practical applications. The significance of this dissertation is more focused on the popularization of the solution method rather than the further improvement of accuracy.

According to the literatures shown in Table 2.2, one broad stream of the extensions to the newsvendor problem is the newsvendor with substitution and the cornerstone and most common case in this vein is one retailer with two products. In order to analyze this type of problem, in Chapter 4, an explicit and easy-to-understand model of the two-product newsvendor with one-way substitution is established and simulation is utilized to validate this model. With the aid of computer, the problem can be formulated by Mathematica software intuitively and therefore a series of numerical experiments are conducted to study the relationship between selected governing parameters and optimal results. Some managerial insights are drawn based on the tables and figures used to present and visualize the results of numerical studies. In Chapter 5, the applications in different industries show the adoptability of the proposed method and the optimal results of these cases also confirm the conclusions derived in Chapter 4.

CHAPTER 3

THE CAPACITATED NEWSVENDOR PROBLEM

3.1 Introduction

When there is only one kind of newspaper, a newsvendor can decide a near optimal order quantity based on his experience and judgement. However, when the newsvendor needs to determine the optimal order quantities for various newspapers to maximize the total profit or minimize the total cost under a recourse constraint, the model of the capacitated multi-product newsvendor is needed to assist the decision making. Because newspapers are time-sensitive and the newsvendor have to make a stock before the customer's demand are realized, this model can also be applied to other similar situations in which perishable products and stochastic demand are involved. When customers' demand for a newspaper is higher than the inventory, they may abandon the purchase or go to another newsvendor. Conversely, when customers' demand for a newspaper is lower than the inventory, the unsold newspaper will be disposed. The mismatch of supply and demand generate lost sale and disposal costs. In addition, when there is a resource limitation, such as budget, weight or space, the multi-product newsvendor problem will become the capacitated newsvendor problem and the decision maker needs to pursue the optimum under this constraint.

This chapter is divided into four subsections. In the first section, the formulation of the capacitated multi-product newsvendor is presented and each term of the objective function is explained. In the second section, rational and proof of the proposed solution method to the capacitated newsvendor problem is presented. In the third section, the performance of the proposed solution method and some existing solution approaches developed by predecessors are compared through a series of numerical experiments.

3.2 The Model

Equations (3.1-3.3) show a common formulation of the capacitated multi-product newsvendor problem. As defined in the model: x_i is the order quantity of product i. The unit purchase cost, unit overstocking cost and unit understocking cost of product i are represented by c_i, h_i and v_i respectively. The stochastic demand is characterized by the random variable D_i, probability density function f(.) and the cumulative density function F(.). Equation (3.1) is the objective function. Its components are explained as follows:

- On the left-hand side, $\sum_{i=1}^{n} E(x_i)$ represents the expected total cost of the newsvendor.
- On the right-hand side, the first term, $c_i x_i$, expresses the purchase cost of product i which is the multiplication of the unit purchase cost and the order quantity of product i.
- The second term evaluates the overstocking cost of the products where, h_i is the overstocking cost per unit, and the integral $\int_0^{x_i}(x_i - D_i) f_i(D_i) dD_i$ is the expected number of overstocked items of Product i.
- Similarly, the last term shows the understocking cost, where v_i is understocking cost per unit, and the integral $\int_{x_i}^{\infty}(D_i - x_i)\, f_i(D_i) dD_i$ is the expected number of understocked items of Product i.
- Equation (3.2) is the budget constraint. It should be noted without loss of generality, the budget constraint can be replaced by space, weight, etc.

- Equation (3.3) ensures the non-negativity of the decision variable, that is the order quantity of products.

Symbols used in this section are shown as follows:

Symbol	*Description*
B	Available budget
c_i	Purchase cost of Product *i*
D_i	Demand of Product *i*
$E(x_i)$	Expected total cost of Product *i*
$f_i(D_i)$	Probability density function (PDF) of demand of Product *i*
$F_i(D_i)$	Cumulative distribution function (CDF) of demand of Product *i*
h_i	Holding cost of Product *i*
v_i	Price of Product *i*
x_i	Order quantity of Product *i*

Minimize:

$$\sum_{i=1}^{n} E(x_i) = \sum_{i=1}^{n} [c_i x_i + h_i \int_0^{x_i} (x_i - D_i) f_i(D_i) dD_i \ + v_i \int_{x_i}^{\infty} (D_i - x_i) f_i(D_i) dD_i] \quad (3.1)$$

Subject to

$$\sum_{i=1}^{n} c_i x_i \leq B \quad (3.2)$$

$$x_i \geq 0 \quad i = 1,2 \dots n \tag{3.3}$$

3.3 Rationale and Proof

For most common probability density functions, the cost function of the newsvendor is known to be convex. As shown in Figure 3.1, the more Product i are ordered, the closer it gets to the unconstrained optimum, x_i^*. When the budget is sufficient, the unconstrained optimal amount of each product can be ordered. Nevertheless, when the budget is less than B_{opt}, the optimal lot size of one or more products need to be less than its unconstrained optimum (x_i^*) to satisfy the capacity constraint. Further, when the budget is even tighter, one or more products need to be removed from the purchase list and hence the corresponding optimal order quantity is zero. In summary, one has to decide the priority of ordering the products and which of them should be discarded if it is necessary in the decision-making process. In order to simplify this procedure, cost function is linearized by Taylor's expansion at the midpoint of the unconstrained optimal order quantity ($x_i^*/2$). In Figure 3.1, the slope of the dash line represents the cost reduction rate and it is a key factor in determining ordering priority. In validation of this approximation, two probability density functions are considered: the uniform and the exponential (the behavior of the normal distribution is known to lie in between, that is, its error lies in between these two, see Table 3.1). Equations (3.4-3.14) exhibit the derivation of difference between the optimal total cost obtained by the proposed approximation and existing approaches. The notations utilized in subsequent analysis are as follows:

Symbol	*Description*
B_{opt}	Minimum budget required to order the unconstrained optimal quantity of all products, $B_{opt} = \sum_{i=1}^{n} c_i x_i^*$
$E_u(x_i)$	Expected total cost of Product *i* for uniform demand distribution
$E_e(x_i)$	Expected total cost of Product *i* for exponential demand distribution
$E^T(x_i)$	Taylor's expansion of the total cost function
$\Delta E\%$	Percentage difference between the optimal total costs
ε	Error generated by Taylor's expansion
L_{i01}	The amount of flow in *i*th arc from node 0 to node 1
L_{12}	The amount of flow in the unique arc from node 1 to node 2
μ_i	Mean of exponential distribution
U_i, L_i	Upper and lower bounds of uniform distribution
m_i	Mean of normal distribution
σ_i	Standard deviation of normal distribution
x_i^*	Optimal order quantity of Product *i* for uncapacitated newsvendor
x_i^{**}	Optimal order quantity of Product *i* for capacitated newsvendor
EXP	Exponential distribution
UNI	Uniform distribution
GIM	Generic iterative method
BSM	Binary solution method
LAM	Linear approximation method

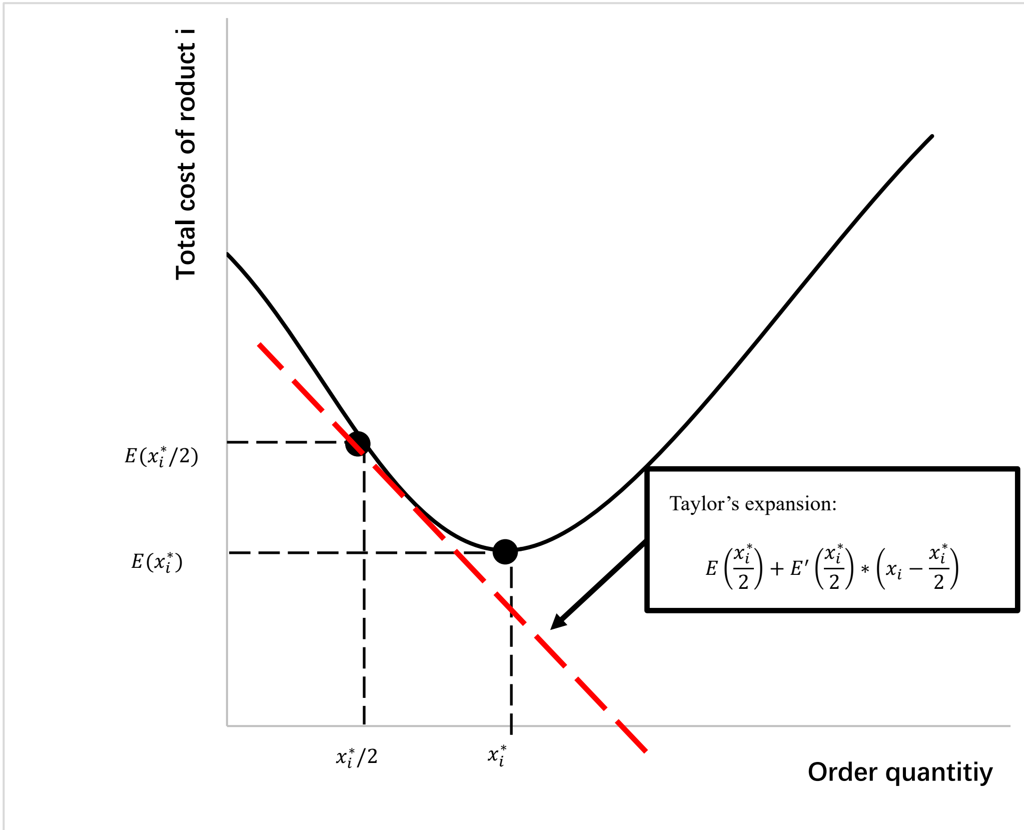


Figure 3.1 Total cost of newsvendor.

The cost function for uniform and exponential distributions are expressed in Equation (3.4) and Equation (3.5):

$$\sum_{i=1}^{n} E_u(x_i) = \sum_{i=1}^{n} (c_i - v_i)\, x_i + \frac{(v_i + h_i)}{2(b_i - a_i)} x_i^2 + v_i(\frac{b_i - a_i}{2}) \tag{3.4}$$

$$\sum_{i=1}^{n} E_e(x_i) = \sum_{i=1}^{n} (c_i - v_i)\, x_i + (v_i + h_i)(x_i + \mu_i e^{-\frac{x_i}{\mu_i}}) + v_i \mu_i \tag{3.5}$$

The slope of the cost function at $x_i = x_i^*/2$ are presented in Equation (3.6) and Equation

(3.7):

$$\sum_{i=1}^{n} E_u{}'(x_i) = \sum_{i=1}^{n} (c_i - v_i) + \frac{(v_i + h_i)}{(b_i - a_i)} * (x_i^*/2) \tag{3.6}$$

$$\sum_{i=1}^{n} E_e{}'(x_i) = \sum_{i=1}^{n} (c_i - v_i) \ + (v_i + h_i)(1 - e^{-\frac{x_i^*}{2\mu_i}}) \tag{3.7}$$

Therefore, applying Taylor's expansion to Equation (3.4) and Equation (3.5) at $x_i = x_i^*/2$ yields:

$$\begin{aligned} \sum_{i=1}^{n} E_u^T(x_i) = \sum_{i=1}^{n} (c_i - v_i)(x_i^*/2) + \frac{(v_i + h_i)}{2(b_i - a_i)} (x_i^*/2)^2 \\ + v_i \left(\frac{b_i - a_i}{2}\right) + [(c_i - v_i) + \frac{(v_i + h_i)}{(b_i - a_i)} (x_i^*/2)] * (x_i - x_i^*/2) + \varepsilon \end{aligned} \tag{3.8}$$

$$\begin{aligned} \sum_{i=1}^{n} E_e^T(x_i) = \sum_{i=1}^{n} (c_i - v_i)(x_i^*/2) + (v_i + h_i)(x_i^*/2 + \mu_i e^{-\frac{x_i^*}{2\mu_i}}) + v_i \mu_i \\ +[(c_i - v_i) \ + (v_i + h_i)(1 - e^{-\frac{x_i^*}{2\mu_i}})] * (x_i - x_i^*/2) + \varepsilon \end{aligned} \tag{3.9}$$

In Equation (3.8) and Equation (3.9), it can be seen the linear approximation dominates the total cost function (ε is negligible). To show the percentage difference of the optimal total cost ($\Delta E\%$) as expressed in Equation (3.10), numerical experiments are conducted by considering different profit ratios (v_i/c_i) and several budget tightness (B/B_{opt}). Note that v_i/c_i is assumed to be same for all products, this represents the worst-case scenario for the difference. Table 3.1 presents the results and one can see that the percentage differences are small in most of cases.

$$\Delta E\% = \frac{\sum_{i=1}^{n} E(x_i^{**}) - \sum_{i=1}^{n} E^T(x_i^{**})}{\sum_{i=1}^{n} E(x_i^{**})} \tag{3.10}$$

Table 3.1 Differences between the Optimal Total Costs Obtained by Introduced Method and Existing Approaches

$\Delta E\%$	$B/B_{opt} = 0.3$		$B/B_{opt} = 0.5$		$B/B_{opt} = 0.7$		$B/B_{opt} = 0.9$	
	UNI	EXP	UNI	EXP	UNI	EXP	UNI	EXP
$\frac{v_i}{c_i} = 1.5$	2.11%	1.18%	2.45%	1.35%	1.93%	0.97%	0.93%	0.38%
$\frac{v_i}{c_i} = 1.75$	3.66%	2.18%	4.33%	2.48%	3.48%	1.77%	1.68%	0.89%
$\frac{v_i}{c_i} = 2$	5.22%	3.27%	6.28%	3.73%	5.16%	2.65%	2.49%	1.02%

UNI = Uniform Distribution; EXP = Exponential Distribution

By referring to the maximum flow minimum cost algorithm proposed in Orlin's work (1994), the capacitated newsvendor problem can be mapped into a minimum cost flow problem as expressed in Equations (3.11-3.14). Figure 3.2 depicts this mapping and visualizes the mechanism of the proposed solution method. As exhibited in Figure 3.2, a set of arcs on the left-hand side of the network represents the purchase cost assigned to each product with flow capacity between (0, $c_i x_i^*$). On the right-hand side, the unique arc's upper bound is the available budget, B. As proved by Zhang et al. (2009), the available budget must to be fully utilized to yield the optimum which is equivalent to maximizing the flow in the network. L_{i01} is amount of flow in the ith arc from node 0 to node 1 and L_{12} is amount of flow in the unique arc from node 1 to node 2. Since there are only three nodes in the network, when the aforementioned minimum cost flow algorithm is applied, the arcs on the left-hand side are filled from top to bottom until the arc on the right-hand side is full. In order to assign the arcs on the left-hand side to each product, the profit ratio (v_i/c_i) is an indicator to determine the priority of assignment. This means the more profitable the product is, the upper arc it is assigned.

Maximize:

$$Z = \sum_{i=1}^{n} L_{i01} \tag{3.11}$$

Subject to

$$\sum_{i=1}^{n} L_{i01} = L_{12} \tag{3.12}$$

$$0 \leq L_{i01} \leq c_i x_i^* \tag{3.13}$$

$$0 \leq L_{12} \leq B \tag{3.14}$$

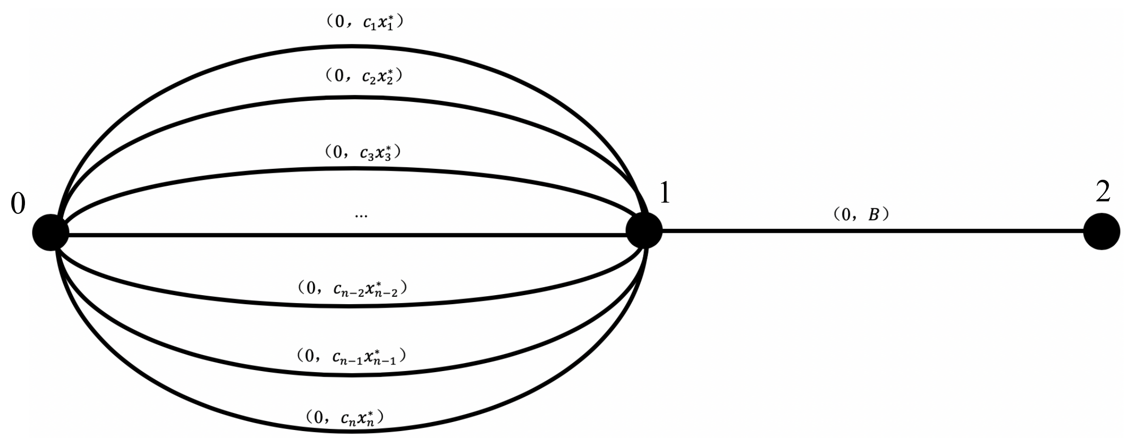


Figure 3.2 Network presentation of the capacitated newsvendor.

Alternatively, in Equations (3.15-3.17) and Figure 3.3, the capacitated newsvendor problem can also be mapped as a knapsack problem. The solution methods of the general knapsack problem are based on branch and bound and/or dynamic programming techniques. Both of them are known to be usually NP. They require taken into consideration the earlier decisions in the search of the optimality. However, in this case, one does not need to go backward, the assignments are made in deceasing order of products' profitability (v_i/c_i). Hence, an increase of the number of products is linear in nature of complexity (polynomial in time). Therefore, in the solution steps, the products are sorted in deceasing order of ratio (v_i/c_i) as priority to put them in the sack. In summary, the polynomial nature of the introduced method is shown. Also, as mentioned before, one must note that a necessary optimality condition for the capacitated newsvendor problem is the full use of its available resource which is equivalent to fill up the capacity of the knapsack as much as possible.

Maximize:

$$Z = \sum_{i=1}^{n} -E^T(x_i) \tag{3.15}$$

Subject to

$$\sum_{i=1}^{n} c_i x_i \leq B \tag{3.16}$$

$$0 \leq x_i \leq x_i^* \tag{3.17}$$

To avoid duplication and confusion, the network flow approach is utilized to illustrate steps and a flowchart (see Appendix A) of the proposed method:

Step 1: Find the optimum amount for each product independently by $F(x_i^*) = \frac{v_i - c_i}{v_i + h_i}$;

Step 2: Substitute the resulting amounts in the budget constraint, $\sum_{i=1}^{n} c_i x_i \leq B$, if satisfied, let $x_i^{**} = x_i^*$ and goto *Step 5*, Otherwise;

Step 3: Rearrange the products in the list in descending order of v_i/c_i and map the model as a network (see Figure 3.2);

Step 4: Fill each arc on the left-hand side of the network in Figure 3.2 to its maximum capacity $c_i x_i^*$, $i = 1,2, \ldots n$, till the arc on the right-hand side reaches its upper bound (note that the last arc to be filled on the left-hand side may not reach its maximum capacity);

Step 5: Use $x_i^{**} = L_{i01}/c_i$ to calculate x_i^{**}, $i = 1, 2, \ldots n$ (L_{i01} is the amount of flow in the *i*th arc on the left-hand side of the network);

Step 6: Use x_i^{**}, $i = 1,2,..n$ and Equation (3.1) to calculate the optimal total cost.

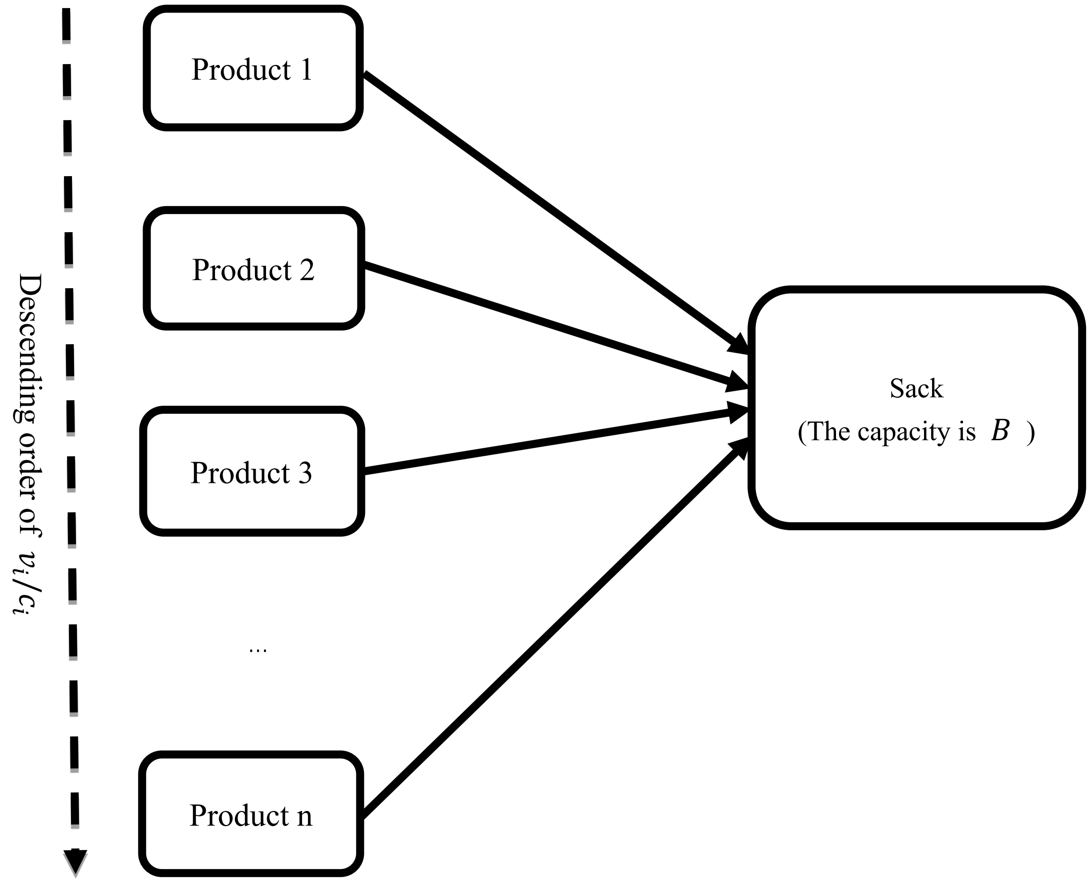


Figure 3.3 Knapsack representation of the capacitated newsvendor.

3.4 Numerical Comparisons

In this section, numerical experiments are conducted to compare the performance between the developed method and some existing ones, such as GIM (Generic Iterative Method), BSM (Binary Solution Method) and LAM (Linear Approximation Method). These three methods are known to yield exact optima or close to it depending on the probability distribution function of the demand. While exhaustive number of numerical experiments conducted to ascertain the accuracy of the developed method, the comparison reported here is based on the example which is extracted from Lau and Lau (1995) where ten products are considered (their parameters are shown in Table 3.2) and in each instance, the products have the same type of demand distribution. To show the adaptability of the proposed method in a scenario of a combination of different types of demand distributions, an instance of nine products with different demand distributions are constructed and their parameters are listed in Table 3.3. Additionally, in order to cover a reasonable range of budgets, the comparisons are conducted when the available budget is 50%, 70% and 90% of the minimum one (B_{opt}) required to order the optimal quantity of all products. Table 3.4 exhibits the optimal costs and narrow differences among them when they are obtained by the proposed method, GIM, BSM and LAM. One can see from Table 3.4 that the differences are most of time negligible (0.49% - 2.18%). Therefore, one can apply the proposed method with confidence. It should be noted that the results support the observation which is made in Section 3.3 regarding the leading effect of the profit ratio, v_i/c_i on the procurement strategy. Based on the numerical comparisons conducted in this section, the following managerial insights can be drawn:

a) As the available budget becomes closer to that needed to order the optimal number of items of each product in the list, the differences between optimal total cost obtained by the developed methodology, BSM, GIM and LAM narrows.

b) The ratio v_i/c_i dominates on deciding the priority of product purchases when the tightness of budget constraint is in a reasonable range.

c) The optimal total cost is not very sensitive to the order quantities of each product.

Table 3.2 Parameters of Products with the Same Demand Distributions

Product	Parameters			Uniform		Exponential	Normal	
	v_i	h_i	c_i	L_i	U_i	μ_i	m_i	σ_i
1	7	1	4	0	255	200	255	85
2	12	2	8	0	127	225	127	42
3	30	4	19	0	215	113	215	72
4	30	4	17	0	166	100	166	55
5	40	2	23	0	108	75	108	36
6	45	5	15	0	258	30	258	86
7	16	1	10	0	172	235	172	57
8	21	2	10	0	207	91	207	69
9	42	3	30	0	210	139	210	70
10	34	5	20	0	155	130	155	52

Table 3.3 Parameters of Products with Different Demand Distributions

Product	Parameters of product			Parameters of demand	
	v_i	h_i	c_i	L_i	U_i
1	7	1	4	0	255
2	12	2	8	0	127
3	30	4	19	0	215
				μ_i	
4	45	5	15	30	
5	16	1	10	235	
6	21	2	10	91	
				m_i	σ_i
7	20	3	10	229	76
8	15	5	7	182	61
9	10	3	4	104	35

Table 3.4 Numerical Comparisons between Proposed Method and Existing Approaches

Uniform Distribution												
Budget	5400				7600				9700			
Method	NFM	BSM	GIM	LAM	NFM	BSM	GIM	LAM	NFM	BSM	GIM	LAM
Total cost	22188	21740	21740	21805	21507	21111	21111	21213	20913	20812	20812	20941
$\Delta\boldsymbol{E}$%		2.06%	2.06%	1.76%		1.87%	1.87%	1.38%		0.49%	0.49%	-0.13%
Exponential Distribution												
Budget	4000				5600				7200			
Method	NFM	BSM	GIM	LAM	NFM	BSM	GIM	LAM	NFM	BSM	GIM	LAM
Total cost	29034	28662	28662	28685	28587	28309	28309	28325	28211	28140	28140	28160
$\Delta\boldsymbol{E}$%		1.30%	1.30%	1.22%		0.98%	0.98%	0.92%		0.25%	0.25%	0.18%
Normal Distribution												
Budget	12700				17800				23000			
Method	NFM	BSM	GIM	LAM	NFM	BSM	GIM	LAM	NFM	BSM	GIM	LAM
Total cost	40415	39551	39551	39718	37936	37285	37285	37411	36076	35722	35722	35829
$\Delta\boldsymbol{E}$%		2.18%	2.18%	1.75%		1.75%	1.75%	1.40%		0.85%	0.85%	0.69%
Combination of Distributions												
Budget	3900				5400				7000			
Method	NFM	BSM	GIM	LAM	NFM	BSM	GIM	LAM	NFM	BSM	GIM	LAM
Total cost	16935	16667	16667	16695	16292	16052	16052	16065	15812	15729	15729	15745
$\Delta\boldsymbol{E}$%		1.61%	1.61%	1.44%		1.49%	1.49%	1.41%		0.53%	0.53%	0.43%

CHAPTER 4

THE NEWSVENDOR WITH SUBSTITUTION

4.1 Introduction

The rapid advances in technology coupled with the evolution in the complexities of today's supply chains, where disruptions happen more frequently, have generated renewed interest in the newsvendor problem and its extensions as well as various areas of its applications. As evident of this, in addition to the plethora of recent publications in this vein, is the edited book by Choi (2012) where he presents a compendium of articles that include literature review, solution methods, extensions, and applications of the newsvendor problem in different sectors. Additionally, the review articles in the foresaid book cites areas where more research is needed. One of these areas is the newsvendor with surrogate product. As reported by Turken et al. (2012), the product substitution approach offers retailers an avenue to reduce customer loss. Further, they describe the approach as perfect, partial, or downward. One of the practical applications that are cited in their review paper is that in the semiconductor industry, where a lower capacity chip can be replaced by a higher capacity one (despite the fact that could reduce the profitability of the company).

In this chapter, the focus is on the newsvendor with surrogate product; one-way substitution. Situations where this occur, in addition to the one mentioned before are many. For example, in retail, when a customer does not find a fresh product and resort to a frozen one (both are perishable commodities), or s/he seeks artisan bread and settles for a lesser quality type (in both scenarios the margin of profit is less for the surrogate, however, the retailer reduces the probability of overstocking of a more expensive perishable items while hedging against the complete loss of the customer). Other situations include examples of products with high rate of technological obsolescence where a superior product can be substituted by one with a lower profit margin. Also, in fashion, shortages in a name brand designer product can be fulfilled by a generic one. As far as service industries are concerned, hotel suites can be substituted by prime rooms or luxury car can be replaced by compact ones.

In brief, this chapter contributes in the vein of two product one-way substitution models in a newsvendor environment. The objective is to develop a model to find the lot sizes of each of these two products that optimize total cost (profit) of a retailer or a supplier of perishable commodities and/or those who deal in environments of short product shelf life, as well as where items become technologically obsolete fast.

The taxonomy of this chapter is as follows: after this introduction, section 4.2 analyzes the surrogate context and formulates the model. Section 4.3 verifies the developed model by using a simulation in Excel. In section 4.4, numerical experiments are conducted to compare the newsvendor with or without surrogate and analyze the behavior of the governing parameters versus the optimal total cost as well as the optimal order quantities of the two products.

4.2 The Model

The model developed here addresses the following scenario. We consider a retailer who deals in two perishable commodities, primary product which is designated as *a* and a surrogate one defined as *b*. The demand on both products are random variables with known distribution. As shown in Equation (4.1), Product *a* is superior than Product *b* as it yields more profit. If the retailer runs out of Product *a* and the demand on that of Product *b* has been fulfilled, yet there remains left-over, then the retailer can satisfy the shortage or portion of it from the excess (note, not the other way around). The objective is to estimate the optimal order quantity of each product that minimizes the retailer's cost (or alternatively the profit). But, before presenting the mathematical model, the notations used in its development are given as follows:

Symbol	*Description*
D_a	Demand of Product a
D_b	Demand of Product b
E_{nv}	Expected total cost of the newsvendor without surrogate product
E_{nvs}	Expected total cost of the newsvendor with surrogate product
$f(.)$	Probability density function
$F(.)$	Cumulative distribution function
P_s	The probability of being in the context of the newsvendor surrogate
$P_{x_b-D_b>D_a-x_a}$	The probability of entire extra demand of Product *a* is satisfied by the left overs of Product *b*

$P_{x_b - D_b < D_a - x_a}$	The probability of all the left overs of Product *b* are used to satisfy the extra demand of Product *a*
x_i	Order quantity of Product *i*
x_s	Expected value of surrogate quantity
x_{sa}	Expected quantity of Product *a* satisfied by the left overs of Product *b*
x_{sb}	Expected quantity of Product *b* used to satisfy the shortage of Product *a*
$U_i,\ L_i$	Upper and lower bounds of uniform distribution
μ_i	Mean of exponential distribution
m_i	Mean of normal distribution
σ_i	Standard deviation of normal distribution
Var_i	Variance of demand

The upper left corner of Figure 4.1 depicts the context when the newsvendor with surrogate is applied. As stated before, the demand of Product *b* should be fulfilled before trying to satisfy the shortage in Product *a*. From the figure and in order to express the consolidated formula of the cost function, the following bullet points show the approach.

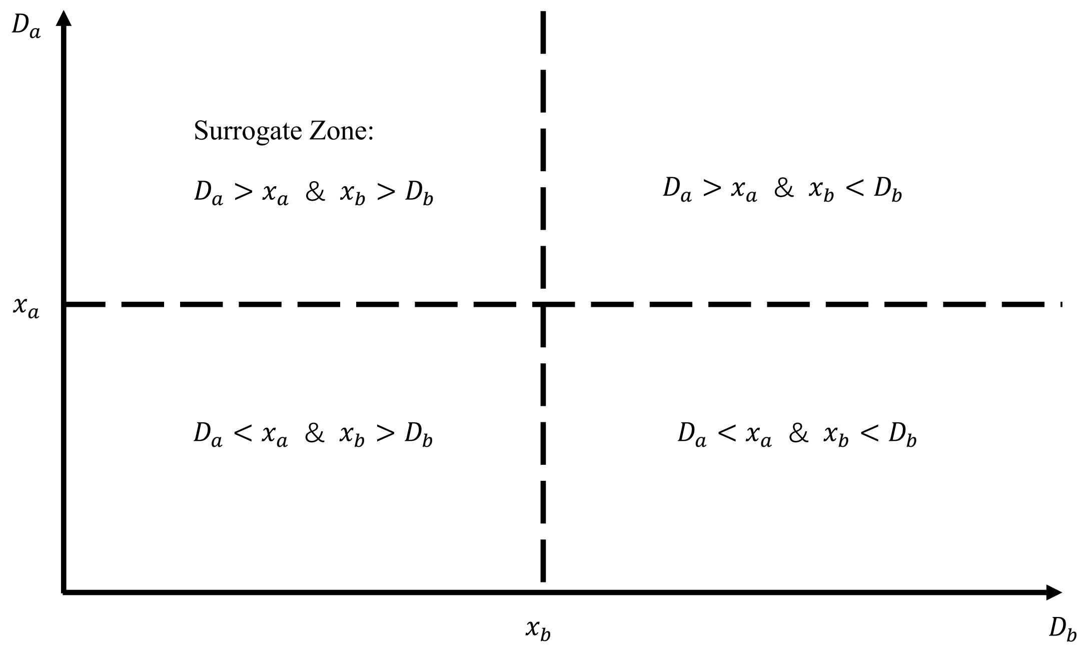


Figure 4.1 The context of newsvendor with surrogate.

Equation (4.1) shows a necessary condition of the model which is, the marginal utility of Product *a* is larger than that of Product *b*.

$$v_a - c_a > v_b - c_b \tag{4.1}$$

The probability of the activating surrogate zone is determined in Equation (4.2) by multiplying the probabilities of shortage in Product *a* and excess in Product *b*.

$$P_s = \int_{x_a}^{\infty} f_a(D_a)dD_a * \int_{0}^{x_b} f_b(D_b)dD_b \tag{4.2}$$

Consequently, two probabilities emerge, either the shortage in Product *a* can be fully fulfilled by excess in Product *b*:

$$P_{x_b-D_b>D_a-x_a} = \int_{x_a}^{x_a+x_b} f_a(D_a)\left(\int_0^{x_b-(D_a-x_a)} f_b(D_b)dD_b\right)dD_a \tag{4.3}$$

Or not fully:

$$P_{x_b-D_b<D_a-x_a} = \int_0^{x_b} f_b(D_b)\left(\int_{x_a+(x_b-D_b)}^{\infty} f_a(D_a)dD_a\right)dD_b \tag{4.4}$$

Afterwards, P_s can be rearranged as shown in Equation (4.5):

$$\begin{aligned} P_s &= \int_{x_a}^{\infty} f_a(D_a)dD_a * \int_0^{x_b} f_b(D_b)dD_b \\ &= P_{x_b-D_b>D_a-x_a} + P_{x_b-D_b<D_a-x_a} \\ &= \int_{x_a}^{x_a+x_b} f_a(D_a)\left(\int_0^{x_b-(D_a-x_a)} f_b(D_b)dD_b\right)dD_a \\ &+ \int_0^{x_b} f_b(D_b)\left(\int_{x_a+(x_b-D_b)}^{\infty} f_a(D_a)dD_a\right) \end{aligned} \tag{4.5}$$

The expected surrogate quantity can be obtained by multiplying the expected values of shortage in Product *a* by that of excess in Product *b* as in Equation (4.6):

$$\begin{aligned} x_s = &\int_{x_a}^{x_a+x_b} (D_a - x_a)f_a(D_a)\left(\int_0^{x_b-(D_a-x_a)} f_b(D_b)dD_b\right)dD_a \\ &+ \int_0^{x_b} (x_b - D_b)f_b(D_b)\left(\int_{x_a+(x_b-D_b)}^{\infty} f_a(D_a)dD_a\right)dD_b \end{aligned} \tag{4.6}$$

Equations (4.7-4.10) show the total cost function and its rearrangement:

$$
\begin{aligned}
E_{nvs} = c_a x_a + h_a \int_0^{x_a} (x_a - D_a) f_a(D_a) dD_a \; + v_a \int_{x_a}^{\infty} (D_a - x_a) f_a(D_a) dD_a \\
+ c_b x_b \\
+ h_b \left[\int_0^{x_b} (x_b - D_b) f_b(D_b) dD_b - x_s \right] \\
+ v_b \left[\int_{x_b}^{\infty} (D_b - x_b) f_b(D_b) dD_b - x_s \right]
\end{aligned} \tag{4.7}
$$

From Equation (4.7), one can see that there is a reduction in the holding cost of Product b, $-h_b x_s$, and an additional revenue of Product b, $-v_b x_s$. Thus, the total contribution of the surrogate product is $-(v_b + h_b) x_s$, and Equation (4.7) can be rearranged as in Equation (4.8):

$$
\begin{aligned}
E_{nvs} = c_a x_a + h_a \int_0^{x_a} (x_a - D_a) f_a(D_a) dD_a \; + v_a \int_{x_a}^{\infty} (D_a - x_a) f_a(D_a) dD_a \\
+ c_b x_b + h_b \int_0^{x_b} (x_b - D_b) f_b(D_b) dD_b \; + v_b \int_{x_b}^{\infty} (D_b - x_b) f_b(D_b) dD_b \\
-(v_b + h_b) x_s
\end{aligned} \tag{4.8}
$$

Equation (4.9) is derived by integrating Equation (4.8):

$$
\begin{aligned}
E_{nvs} = (c_a - v_a) x_a + v_a E(D_a) + (h_a + v_a) \int_0^{x_a} F_a(D_a)\, dD_a \\
+ (c_b - v_b) x_b + v_b E(D_b) + (h_b + v_b) \int_0^{x_b} F_b(D_b)\, dD_b \\
-(v_b + h_b) x_s
\end{aligned} \tag{4.9}
$$

Equation (4.10) further simplifies Eq. (4.9) by using E_{nv} to represent the terms of the standard newsvendor and that of the contribution of the surrogate product, respectively:

$$E_{nvs} = E_{nv} - (v_b + h_b)x_s \tag{4.10}$$

Appendix C shows the optimality conditions for the order quantities of the two products. As can be seen, it is difficult to obtain analytical solutions of their order quantities (an exception is Product a when the demand is exponentially distributed). Therefore, Mathematica software is applied to yield the optimum. In section 4.4, numerical experiments are conducted to analyze the relationship between governing parameters and optimal results. However, before that, in section 4.3, a simulated model is introduced to verify the theoretical formulation of the model as in Equations (4.8-4.10).

4.3 Verification of the Model

As can be seen from the previous section, the theoretical analysis could be prone to errors. To validate the results obtained, in this section, several simulation experiments are conducted. Table 4.1 exhibits the ranges of parameters for the two products and their demand. It covers the cases of uniform, exponential and normal demand distributions. Reproducing Equations (4.2-4.7) in Mathematica software, one can obtain the theoretical values of the optimal order quantities for the two products (it utilizes combination of Differential Evolution, Random Search, and Simulated Annealing to solve the problem). In the same manner, the theoretical values of Equations (4.2-4.7) can also be obtained. Then the theoretical values of the optimal order quantities for the two products are entered to the simulated model and make it run for 21000 days. Table 4.2 shows the average of theoretical values of Equations (4.2-4.7) and those obtained by the simulated model. One can see that the differences between them are negligible which verifies the introduced model.

Table 4.1 Ranges of Parameters in Simulation

Parameter	Product *a*	Product *b*
v_i	35-65	20-50
c_i	20-50	10-40
h_i	5-35	1-31
Uniform Distribution		
U_i	400-700	300-600
L_i	300-600	150-450
Exponential Distribution		
μ_i	500-800	300-600
Normal Distribution		
m_i	600-900	300-600
σ_i	200-300	100-200

Table 4.2 Comparisons of Theoretical and Simulated Results

Demand	Uniform Distribution		Exponential Distribution		Normal Distribution	
Model	Theoretical	Simulated	Theoretical	Simulated	Theoretical	Simulated
P_s	44.59%	44.87%	52.80%	48.18%	49.01%	59.01%
$P_{x_b-D_b>D_a-x_a}$	21.93%	22.05%	19.18%	19.21%	16.48%	16.62%
x_{sa}	7	7	29	30	19	19
$P_{x_b-D_b<D_a-x_a}$	22.66%	22.81%	33.62%	33.50%	42.73%	42.39%
x_{sb}	6	6	80	81	57	58
x_s	13	13	109	111	76	77
E_{nvs}	32238.46	32106.57	49399.40	48048.72	47814.10	46699.66

Figure 4.2 exhibits a flowchart of the simulated model (see Appendix D), its work flow and principle are explained as follows:

In Excel, RAND () generates a random decimal number between 0 and 1. Then, the random demand of uniform, exponential and normal distributions can be generated by:

1) $\mathrm{RAND}() * (U_i - L_i) + L_i$;

2) $-\mu_i LN(1 - \mathrm{RAND}())$;

3) $\mathrm{NORM.INV}(\mathrm{RAND}(), m_i, \sigma_i)$.

To figure out the probability of being in the newsvendor surrogate region as shown in Figure 4.1, two outcomes are considered for each day:

1) $D_a - x_a < 0$ or $x_b - D_b < 0$ (standard newsvendor);

2) $D_a - x_a > 0$ and $x_b - D_b > 0$ (newsvendor with surrogate).

If a condition is true, assign it a value of '1'. Otherwise, assign it a value of '0'. If substitution occurs on a certain day, the multiplication of the values of these two conditions should be '1' (Both conditions are true). Consequently, the probability of being in the newsvendor surrogate region is simply the sum of days the specific outcome appears divided by the total number of days of simulation (21000). Similarly, $P_{x_b - D_b > D_a - x_a}$ and $P_{x_b - D_b < D_a - x_a}$ can also be obtained.

If conditions $P_{x_b - D_b > D_a - x_a}$ and $D_a - x_a > 0$ are true in a certain day, then assign the quantity of $(D_a - x_a)$ for this day. To obtain the value of x_{sa} (expected quantity of Product *a* satisfied by left overs of Product *b*), the average of the quantities of $(D_a - x_a)$ in 21000 days is calculated. In the same manner, x_{sb} (expected quantity of Product *b* used to satisfy the shortage of Product *a*) can also be obtained and x_s is sum of x_{sa} and x_{sb}. Finally, the optimal total cost of the simulated model is obtained by utilizing aforementioned values.

4.4 Solutions and Numerical Analysis

From Appendix C, one can see that the close form expressions of the optimal order quantities are difficult to obtain. Therefore, Mathematica software is utilized to solve the problem. Thanks to the explicit and concise structure of the developed model, it takes only a few seconds to yield the optimal results in most of cases.

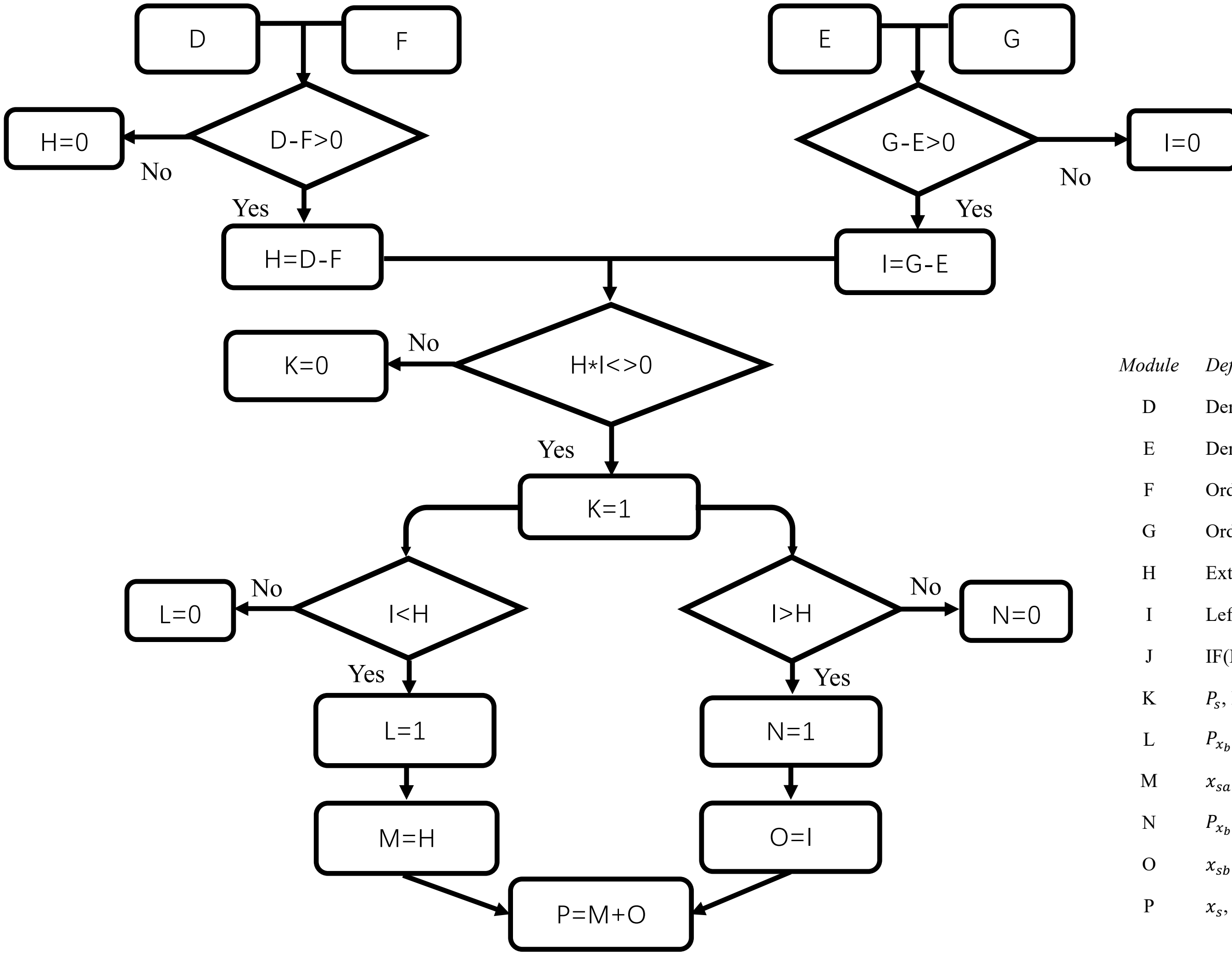
To investigate the impact of the different governing parameters such as profit margins, demand distributions and its variance, we extract from Table 4.2 the values used in the analysis. Tables (4.3-4.5) exhibit the comparisons between the optimal total costs and optimal order quantities of the two products with and without surrogate for various demand distribution. Figures (4.3-4.12) visualize the relationship between the selected governing parameters and the optimal results. Based on aforementioned tables and figures, one can make these observations:

a) As the variance of the demand of the primary product increases, the reduction in the total expected cost increases due to the utilization of surrogate product (up to 12.5%).

b) When the profit of the primary product increases its optimal order quantity also increases while that of the surrogate decreases.

c) When the variance of the demand of the primary product is large, the optimal total cost and the optimal order quantity of the surrogate increases.

d) When the profit margins of the primary product and the secondary one are close, the optimal order quantity of the primary product approaches zero.



Module	*Definition*
D	Demand of Product *a*
E	Demand of Product *b*
F	Order quantity of Product *a*
G	Order quantity of Product *b*
H	Extra demand of Product *a*, IF(D-F>0, D-F, 0)
I	Left overs of Product *b*, IF(G-E>0, G-E, 0)
J	IF(H*I<>0, “NBs”, “NB”)
K	P_s, IF(J=”NB”, 0, 1)
L	$P_{x_b-D_b>D_a-x_a}$, IF(AND(K14=1, I>H), 1, 0)
M	x_{sa}, IF(L<>0, H, 0)
N	$P_{x_b-D_b<D_a-x_a}$, IF(AND(K14=1, I<H), 1, 0)
O	x_{sb}, IF(AND(K=1, H>I), I, 0)
P	x_s, M+O

Figure 4.2 Flowchart of simulation.

Table 4.3 Optimal Results under Different Profit Ratios of the Two Products

$\frac{v_a - c_a}{v_b - c_b}$	1	1.5	2	2.5	3
Uniform Distribution					
Traditional Newsvendor					
x_a^*	329	338	344	350	355
x_b^*	221	221	221	221	221
E_{nv}^*	10000	10112	10198	10268	10325
Newsvendor with Surrogate					
x_a^*	290	320	332	340	347
x_b^*	282	254	245	239	236
E_{nvs}^*	9701	9901	10038	10141	10221
Contribution of Substitution					
$(E_{nv}^* - E_{nvs}^*)/E_{nv}^*$	2.99%	2.09%	1.57%	1.24%	1.01%
Exponential Distribution					
Traditional Newsvendor					
x_a^*	168	235	294	347	394
x_b^*	194	194	194	194	194
E_{nv}^*	19340	21009	22481	23798	24990
Newsvendor with Surrogate					
x_a^*	0	124	214	285	345
x_b^*	632	517	448	402	370
E_{nvs}^*	17419	19615	21393	22908	24238
Contribution of Substitution					
$(E_{nv}^* - E_{nvs}^*)/E_{nv}^*$	9.93%	6.63%	4.84%	3.74%	3.01%
Normal Distribution					
Traditional Newsvendor					
x_a^*	487	536	572	600	623
x_b^*	294	294	294	294	294
E_{nv}^*	18212	18865	19387	19821	20191
Newsvendor with Surrogate					
x_a^*	36	460	527	568	598
x_b^*	851	436	388	366	352
E_{nvs}^*	16886	18131	18840	19380	19821
Contribution of Substitution					
$(E_{nv}^* - E_{nvs}^*)/E_{nv}^*$	7.28%	3.89%	2.82%	2.22%	1.84%

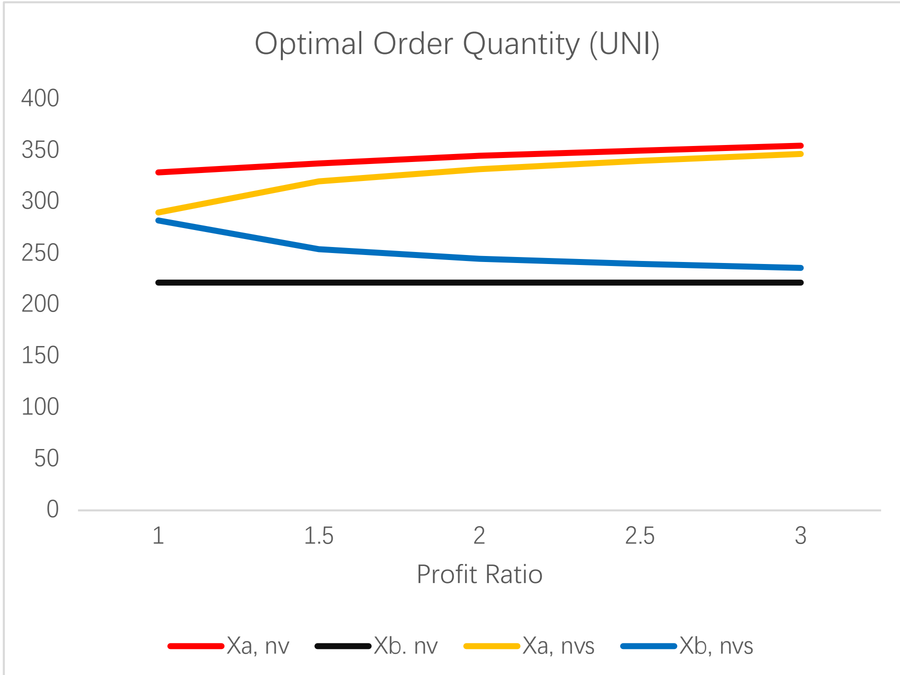


Figure 4.3 Optimal order quantity under different profit ratios of the two products with uniform distribution.

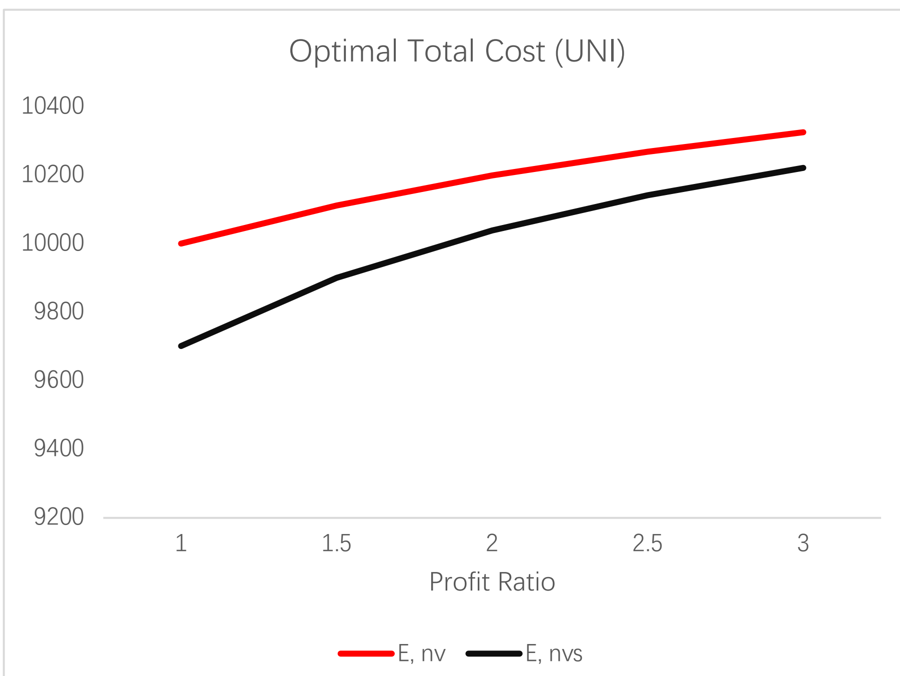


Figure 4.4 Optimal total cost under different profit ratios of the two products with uniform distribution.

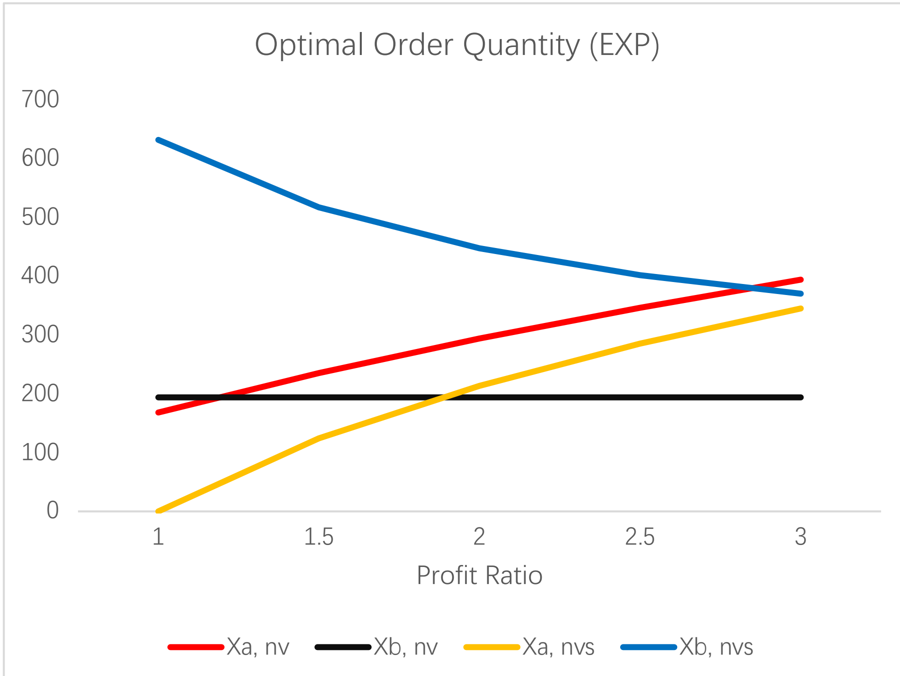


Figure 4.5 Optimal order quantity under different profit ratios of the two products with exponential distribution.

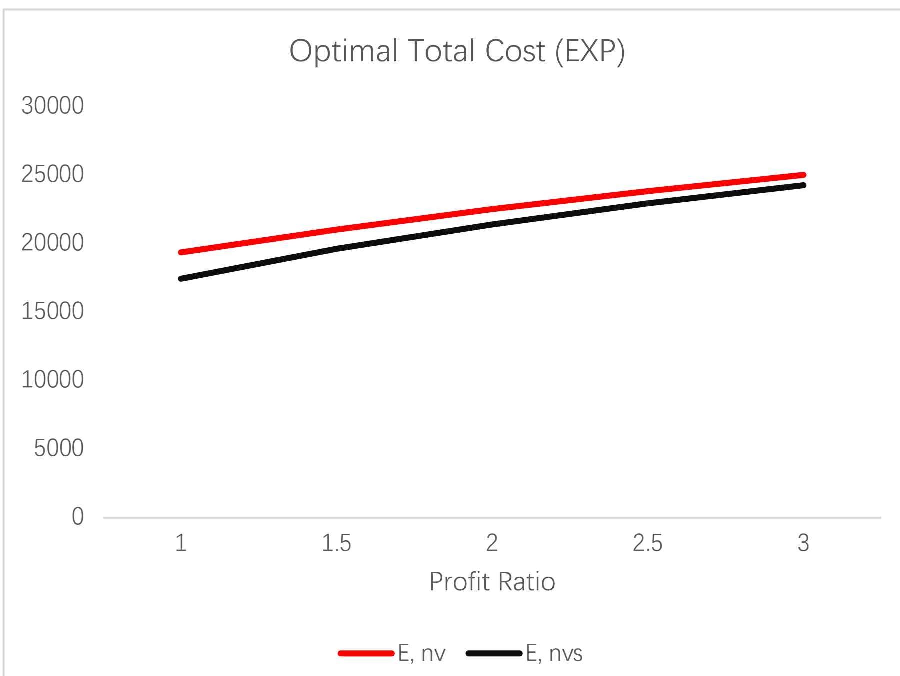


Figure 4.6 Optimal total cost under different profit ratios of the two products with exponential distribution.

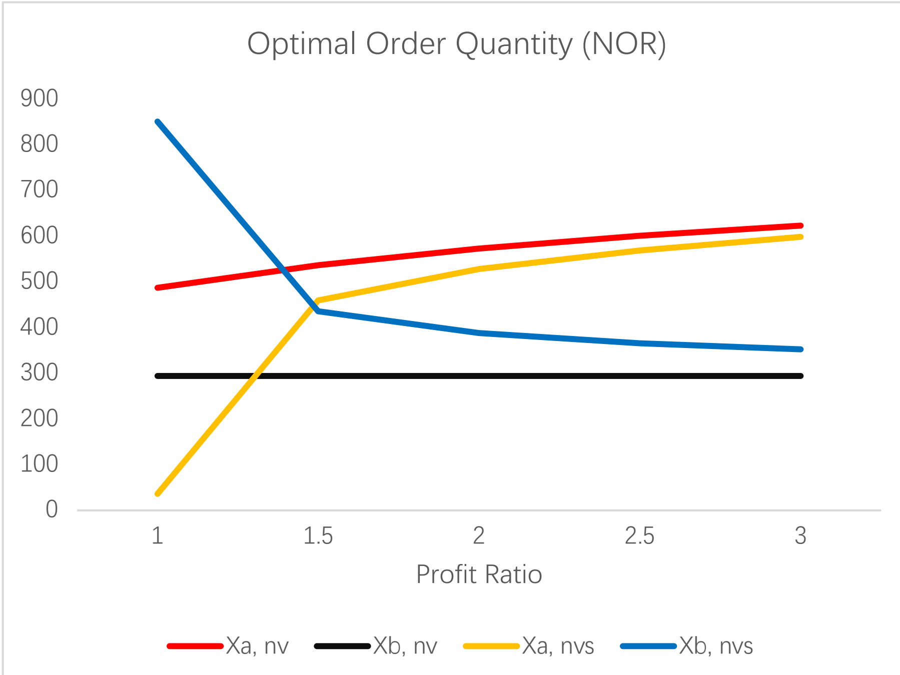


Figure 4.7 Optimal order quantity under different profit ratios of the two products with normal distribution.

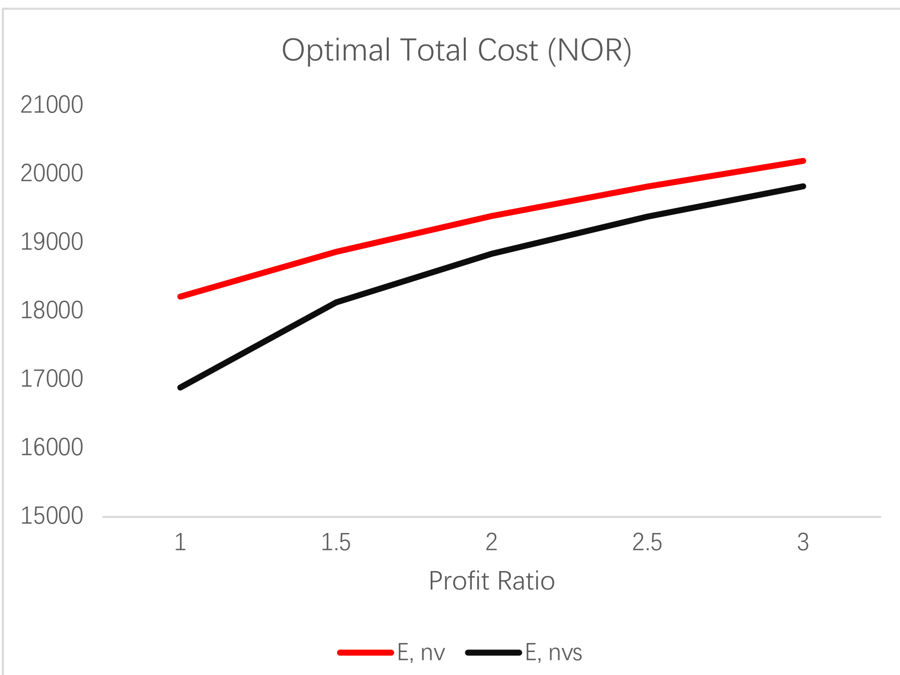


Figure 4.8 Optimal total cost under different profit ratios of the two products with normal distribution.

Table 4.4 Optimal Results under Different Variances of Demand of Product a for Uniform Distribution

Uniform Distribution								
U_a	600	800	900	950	975	995	999	1000
L_a	400	200	100	50	25	5	1	0
Var_a	3333	30000	53333	67500	75208	81675	83000	83333
U_b	300	300	300	300	300	300	300	300
L_b	250	250	250	250	250	250	250	250
Var_b	208	208	208	208	208	208	208	208
$(v_a - c_a)/Var_a * 1000$	3.000	0.333	0.188	0.148	0.133	0.122	0.120	0.120
Traditional Newsvendor								
x_a^*	475	425	400	388	381	376	375	375
x_b^*	274	274	274	274	274	274	274	274
E_{nv}^*	13819	15694	16631	17100	17334	17522	17559	17569
Newsvendor with Surrogate								
x_a^*	400	200	100	50	25	5	1.07	0.07
x_b^*	370	561	656	704	727	746	750	751
E_{nvs}^*	13285	14325	14848	15110	15241	15346	15367	15372
Contribution of Substitution								
$(E_{nv}^* - E_{nvs}^*)/E_{nv}^*$	3.86%	8.72%	10.72%	11.64%	12.08%	12.42%	12.49%	12.50%

Table 4.5 Optimal Results under Different Variances of Demand of Product a for Normal Distribution

Normal Distribution								
m_a	600	600	600	600	600	600	600	600
σ_a	150	180	200	210	215	218	218.8	219
Var_a	22500	32400	40000	44100	46225	47524	47873	47961
m_b	300	300	300	300	300	300	300	300
σ_b	100	100	100	100	100	100	100	100
Var_b	10000	10000	10000	10000	10000	10000	10000	10000
$(v_a - c_a)/Var_a * 1000$	0.444	0.309	0.250	0.227	0.216	0.210	0.209	0.209
Traditional Newsvendor								
x_a^*	515	498	487	481	479	477	476.58	476
x_b^*	294	294	294	294	294	294	294	294
E_{nv}^*	17620	17976	18212	18329	18387	18422	18428	18433
Newsvendor with Surrogate								
x_a^*	168	86	38	17	8	2	1.10	0.39
x_b^*	721	802	849	869	878	883	884.517	885
E_{nvs}^*	16521	16737	16887	16963	17001	17024	17028	17031
Contribution of Substitution								
$(E_{nv}^* - E_{nvs}^*)/E_{nv}^*$	6.24%	6.89%	7.28%	7.45%	7.54%	7.59%	7.60%	7.60%

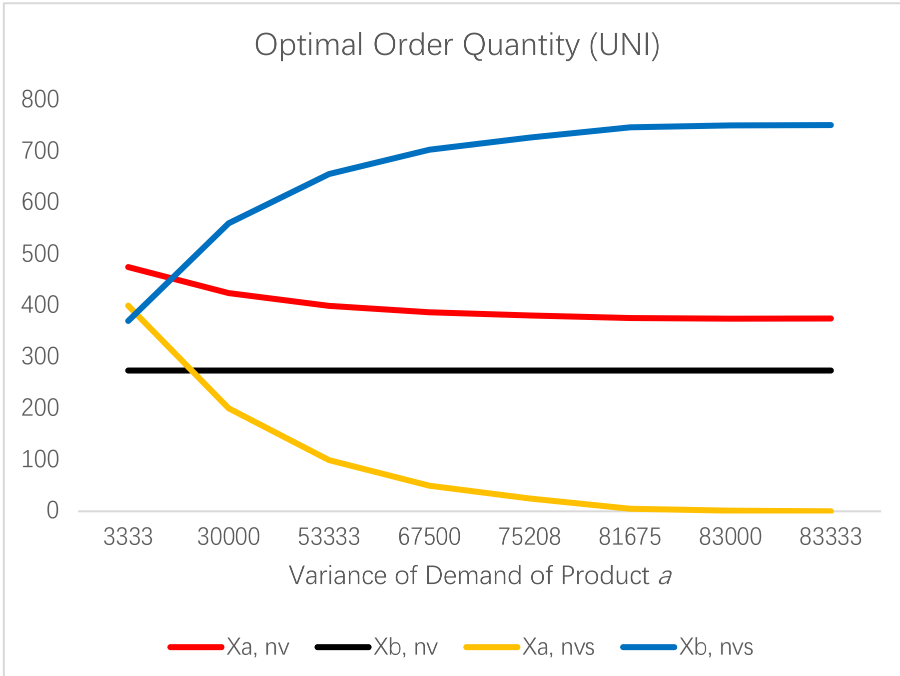


Figure 4.9 Optimal order quantity under different variances of demand of Product *a* with uniform distribution.

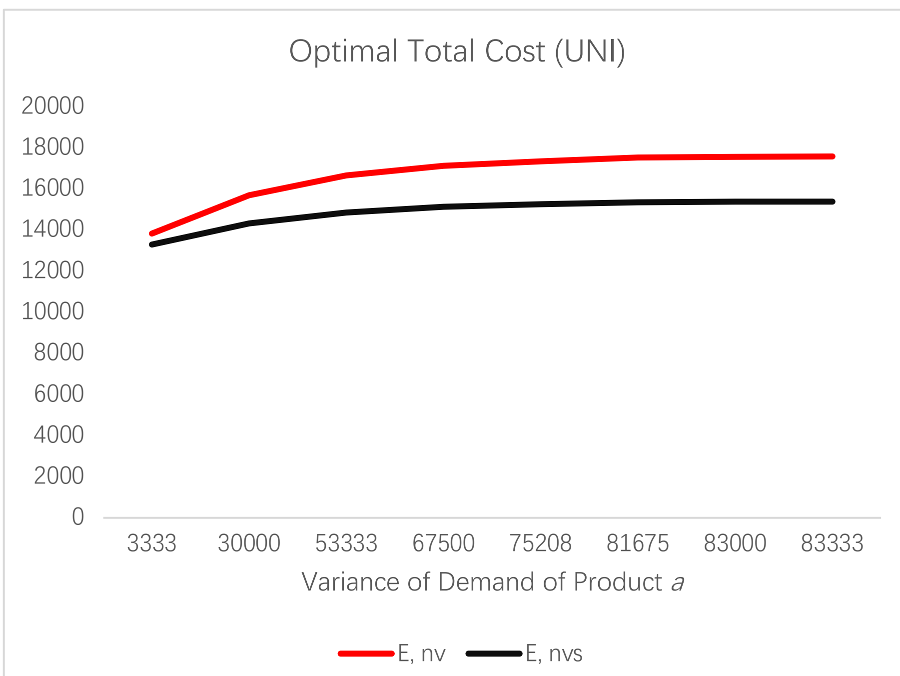


Figure 4.10 Optimal total cost under different variances of demand of Product *a* with uniform distribution.

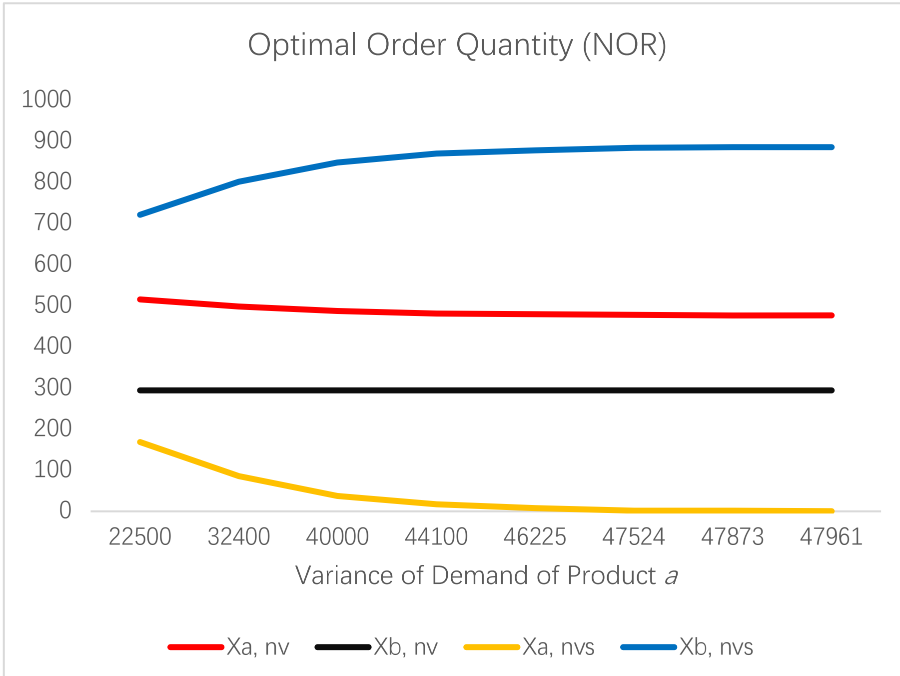


Figure 4.11 Optimal order quantity under different variances of demand of Product *a* with normal distribution.

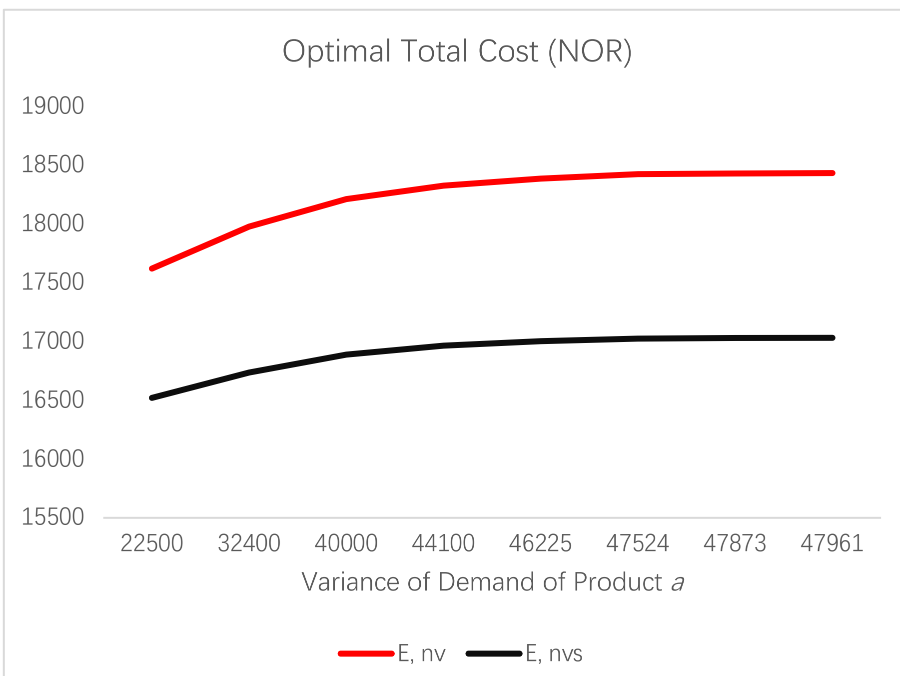


Figure 4.12 Optimal total cost under different variances of demand of Product *a* with normal distribution.

CHAPTER 5

APPLICATIONS

The application of newsvendor problem is very extensive in reality. Whether it's physical goods or virtual goods with short life cycle, as long as the model features match the newsvendor model, it can be solved by the newsvendor problem. Such universality has made research on newsvendor problem have a high degree of attention in the past few decades. In addition to solving the inventory optimization problem for perishable products, some applications of newsvendor problem in other fields are cited. In Hiller and Lieberman's book (2015), they treat the flight booking as a newsvendor problem. Due to the timeliness of the flight's reservation, the occupancy rate of flight is affected by customers' uncertain demand. Thus, airline can use the newsvendor model to determine the reservations for each flight to maximize the overall revenue of all flights. Aloi et al. (2012) consider the radio transmission service as a perishable product. By balancing service quality and facility costs, the vendor of the radio transmission service can use the newsvendor model to improve the overall benefits. As the price of assets changes, Jianqing and Yikang (2010) treat a portfolio optimization problem as a newsvendor problem. They develop an iterative process to find the optimal profit margin value within a boundary. Then the optimal quantity for each candidate entity is determined by the optimal profit margin value to maximize the return on investment. As can been from above cases, in addition to retail industry, the newsvendor model can also be utilized in airline business, communication industry and financial sector.

In this chapter, four applications of the newsvendor problem are presented: one is based on the capacitated newsvendor and the other three are related to the newsvendor with substitution. They include retailing, fashion industry and hotel industry.

5.1 The Capacitated Newsvendor: Inventory Management

To clarify the application of the network flow/knapsack method proposed in Chapter 3, in this subsection, an example of ten products extracted from Table 3.2 with exponentially distributed demand is presented. The available budget is set to \$4500. Figure 5.1 and Figure 5.2 show the mapping of the capacitated newsvendor problem to a maximum flow minimum cost problem and a knapsack problem, respectively. The optimal results rendered by the network flow/knapsack method and other existing solution methods are exhibited in Table 5.1. It should be noted that the optimal total cost obtained by the introduced method is very close to those yielded by GIM, BSM and LAM (the difference is 1.26% and 1.16%).

The following are the steps of the proposed method:

Step 1: Find the optimal amount for each product independently by $F(x_i^*) = \frac{v_i - c_i}{v_i + h_i}$;

Step 2: Then substitute the resulting amounts in the budget constraint and obtain B_{opt} = \$8008. Because B= \$4500 and $B < B_{opt}$, so the budget constraint is binding;

Step 3: Rearrange the products in Table 3.2 in descending order of corresponding v_i/c_i as shown in Table 5.1 and map it as a network as in Figure 5.1;

Step 4: Fill each arc on the left-hand side of the network in Figure 5.1 to its maximum capacity $c_i x_i^*$, till the unique arc on the right side reaches its maximum B;

Step 5: Then, use $x_i^{**} = L_{i01}/c_i$ to calculate x_i^{**}, $i = 1,2, \dots \mathrm{n}$. The results are presented in Table 5.1, x_6^{**}, x_8^{**} = 59, x_4^{**} = 48..., x_2^{**} = 0, x_9^{**} = 0;

Step 6: Use x_i^{**}, $i = 1,2, \dots \mathrm{n}$ and Equation (3.1) to calculate the optimal total cost and it is \$28890 as shown in Table 5.1.

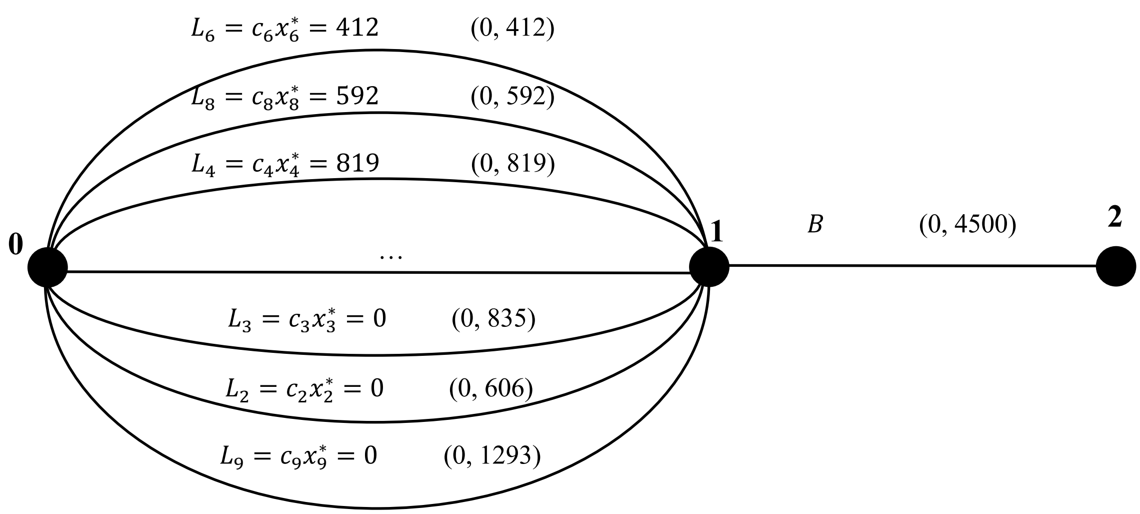


Figure 5.1 Network flow of the application of the capacitated newsvendor.

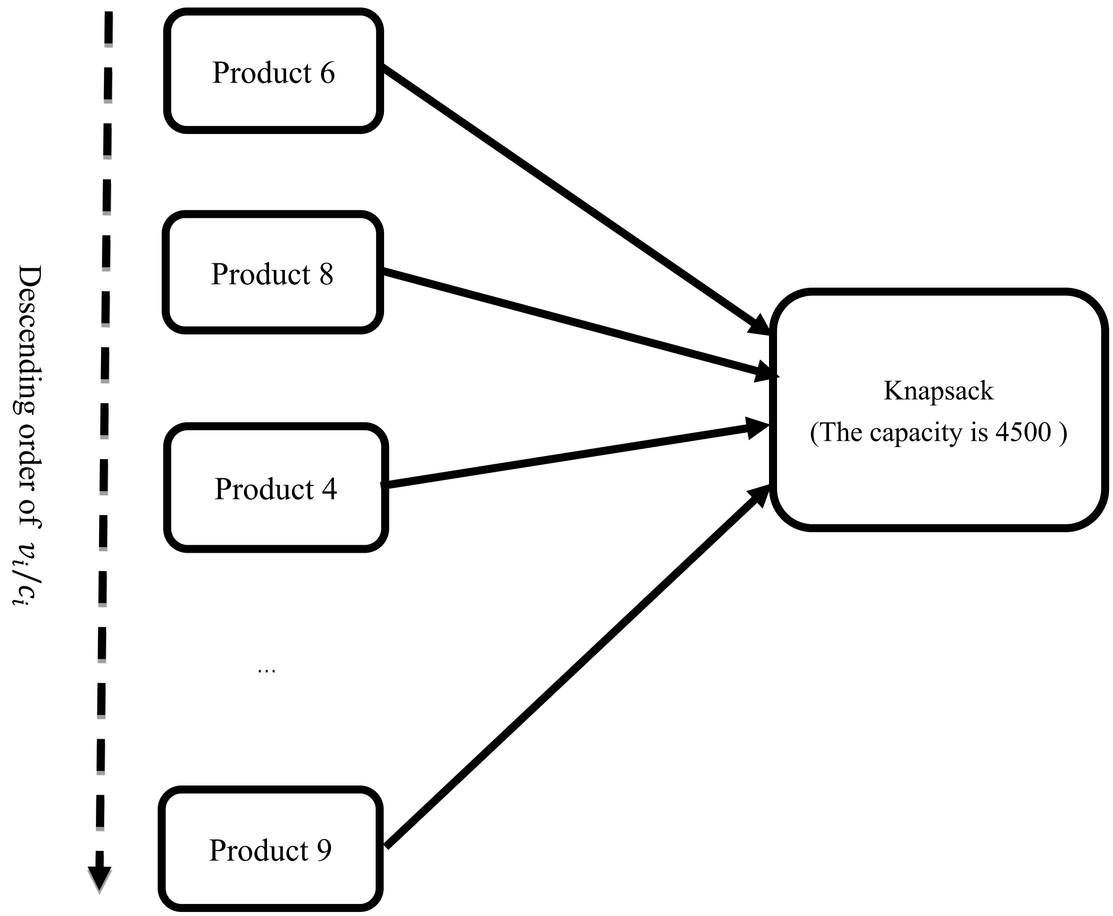


Figure 5.2 Knapsack representation of the application of the capacitated newsvendor.

Table 5.1 Parameters and Results of the Application of the Capacitated Newsvendor

Product	v_i	h_i	c_i	x_i^*	$c_i x_i^*$	μ_i	v_i/c_i	$x_{i,NFM}^{**}$	$E(x_{i,NFM}^{**})$	$x_{i,BSM}^{**}$	$E(x_{i,BSM}^{**})$	$x_{i,GIM}^{**}$	$E(x_{i,GIM}^{**})$	$x_{i,LAM}^{**}$	$E(x_{i,LAM}^{**})$
6	45	5	15	27	412	30	3.0	27	1000	59	1287	59	1287	68	1279
8	21	2	10	59	592	91	2.1	59	1620	36	2595	36	2595	36	2594
4	30	4	17	48	819	100	1.8	48	2712	23	3195	23	3195	21	3207
1	7	1	4	94	376	200	1.8	94	1270	30	2748	30	2748	35	2731
5	40	2	23	39	895	75	1.7	39	2698	24	2738	24	2738	28	2718
10	34	5	20	58	1156	130	1.7	58	4045	22	1009	22	1009	27	1000
7	16	1	10	102	1023	235	1.6	25	3632	55	3530	55	3530	48	3549
3	30	4	19	44	835	113	1.6	0	3375	42	1640	42	1640	43	1640
2	12	2	8	76	606	225	1.5	0	2700	15	5690	15	5690	10	5732
9	42	3	30	43	1293	139	1.4	0	5838	35	4100	35	4100	33	4106
SUM									28890		28531		28531		28555
										$\Delta E\%$ = 1.26%		$\Delta E\%$ = 1.26%		$\Delta E\%$ = 1.16%	

5.2 The Two-Product Newsvendor with One-Way Substitution

In this section, three applications of the two-product newsvendor with one-way substitution are presented. They include grocery retailing, fashion sector and hotel reservation. The parameters selected in the applications cover different types of demand distributions, variance of demand and profit margins of product to show the adoptability and versatility of the model proposed in Chapter 4.

Grocery Retailing: Fresh and Frozen Foods

In this case, two products with different profit margin but similar variance of demand are considered. Suppose a retailer needs to coordinate inventory of one food in two conditions: fresh and frozen. Due to the timeliness and frequency of transportation, the profit margin of fresh food is higher than that of frozen food. However, the customer's demand for food is rigid. Although fresh food is the first choice of customers, when it is sold out, the customer will buy frozen one as a substitute most of the time. Therefore, the variances of demand of fresh and frozen foods are assumed to be very close. In addition, we assume the demand of fresh and frozen foods is uniformly distributed to cover this most common distribution. In Table 5.2, the parameters and the optimal results are presented. One can see that the optimal order quantity of fresh food is greater than its mean of demand and the optimal order quantity of frozen food is less than its mean of demand. Therefore, it can be concluded that when the profit margin of the primary product is significantly higher than that of the secondary product and the variance of demand of these two products are very close, the profitability of product dominates the procurement strategy which conforms to the managerial insights drawn in Chapter 4.

Table 5.2 Parameters and Optimal Results of Application of Grocery Retailing

Parameter	Fresh Food	Frozen Food
v_i	40	15
c_i	15	10
h_i	2	5
U_i	300	200
L_i	200	100
Optimum		
x_i^*	256.787	133.903
E_{nvs}^*	5916.27	

Fashion Industry: Inter Brand Substitution

In this case, two products with similar profit margins but different variances of demand are considered. Suppose a fashion company releases Product *a* that leads a trend in a season, with the increasing sales of Product *a*, its raw materials are in short supply. In response to this, the company releases Product *b* with similar design but different fabrics as a substitute. In order to protect the brand value, the company makes the profit margins of these two products similar. The demand of the two products is assumed to follow normal distribution and exponential distribution due to the diffusion of fashion trend and people's herd mentality. Additionally, the variance of demand of Product *a* is relatively small attributed its popularity and customers' brand loyalty. Table 5.3 shows the parameters and optimal results. As can be seen from the table, the optimal order quantity of Product *a* is less than its mean of demand and the optimal order quantity of Product *b* is greater than its mean of demand. This means, in this case, the variance of demand dominates the decision making.

Table 5.3 Parameters and Optimal Results of Application of Fashion Industry

Parameter	Product *a*	Product *b*
v_i	800	700
c_i	300	250
h_i	80	50
m_i/μ_i	600	250
σ_i	200	
Optimum		
x_i^*	432.657	460.601
E_{nvs}^*	346465	

Hotel Reservation: Deluxe Suite and Standard Suite

In this case, two products with different profit margins and different variances of demand are considered. In order to increase the occupancy rate, a hotel usually provides different types of rooms to meet customers' needs. For ease of analysis, suppose there are two types of rooms: Deluxe Suite and Standard Suite. Deluxe Suite is an option for customers with more budget and it is more profitable and limited than standard one. When Deluxe Suite is fully booked, some customers may choose Standard Suite as a substitution. If a city has some well-known tourist attractions or when a hotel holds events, conferences, and exhibitions, the scale of people participating in these activities will not change much every year. Therefore, it can be assumed that customers' demand for hotel rooms follows normal distribution. In some cases, external factors like economic crisis and epidemic make people reduce non-essential expenditures. Thus, the ratio σ_i/m_i of Deluxe Suite is assumed to be relatively greater than that of the Standard Suite. In Table 5.4, the parameters and optimal reservation policy are presented. As can be seen, despite the profit margin of Deluxe Suite is significantly higher than that of the Standard Suite, the optimal reservation quantity of the Standard Suite is slightly greater than its mean of demand and as an opposite, the optimal reservation quantity of the Deluxe Suite is less than its mean of demand. Hence, it can be concluded that the ratio σ_i/m_i rather than profit margin dominates the reservation policy in this case.

Table 5.4 Parameters and Optimal Results of Application of Hotel Reservation

Parameter	Deluxe Suite	Standard Suite
v_i	600	200
c_i	300	100
h_i	50	30
m_i	300	1000
σ_i	150	250
Optimum		
x_i^*	256.415	1036.9
E_{nvs}^*	245044	

CHAPTER 6

SUMMARY AND FUTURE RESEARCH

6.1 Summary

This dissertation addresses extensions and developments in the newsvendor problem arena. Specifically, in introducing easy-to-use constructive methods for solving its capacitated problem. Also, it develops extension in the area of substitute product, where despite its importance, there has been insufficient attention given to it. To complement previous works, two polynomial in time solution methods for solving the capacitated newsvendor problem are developed. Among their salient features is the fact that they utilize solution techniques that exist in the introductory operations research, operation management, management science as well as other related fields' textbooks. These techniques are the classical newsvendor problem, network flow algorithm and knapsack problem. The introduced method should enable the dissemination of the capacitated newsvendor solution approach to wider range of interested reader. Extensive numerical experiments are conducted to compare some existing methods that are known to yield exact optima or near optimum, depending on the type of demand distributions, with the one developed here. Over a wide range of the governing parameters as well as the most common types of the demand distributions, the developed method, in addition to its ease of applying, yields narrow differences (0.49% - 2.18%) between its optimal results and those rendered by the foresaid methods. It should be noted further that one can easily alternate among different capacity constraints and decide on which of them is more restricting. More clearly, if there are constraints other than budget, such as space or weight, each can be considered alternately and the results can be assessed and compared. One should note that in addition to its ease of implementation, the developed methods can serve as a first cut approach for decision makers to ascertain whether they should choose to pursue one of the more involved iterative techniques that are available in the open literature. Additionally, it should be noted that the developed method is polynomial (P) in time. In contrary, the existing solution methods which are either non-polynomial (NP) or pseudo polynomial. The results of numerical experiments reveal three important managerial insights, they are:

1) The dominance of v_i/c_i for each product on the acquisition decision (the higher the profit of the product, the higher the priority of ordering);
2) The optimal total cost is diminutively sensitive to the optimum order quantity of each product;
3) The difference between the optimal total cost obtained by the developed approach and existing ones becomes negligible (less than 2.18%) when the budget is 50% or more than that required to order the optimum of items for each product, which is reasonable from a practical point of view.

Moreover, this dissertation models the two-product newsvendor with surrogate product. The objective is to find the optimal order quantities of each of the considered products that minimize the expected cost. Simulation experiments to validate the developed model as well as numerical analysis are provided. Since explicit solutions for the optimum order quantities are difficult to obtain, Mathematica software is utilized to yield optimal results. The developed approach helps retailers and decision makers in managing their inventory policies when one of the products can be substituted by the other. The numerical results show that the surrogate policy can significantly contribute to the reduction of the total cost (in the order of 12%). The applications of the proposed model are presented to show its adoptability in industries related to people's daily lives such as grocery retailing, fashion sector and hotel reservations. Further, the analysis supports several managerial insights:

1) It shows the effect of the increased variance of the primary product on its reduced order quantity and the increase in the surrogate amount.
2) It reveals the intuitive correlation between the profit margins of both products on their order quantities:
    - The larger the primary product profit margin leads to the lesser the amount of the surrogate quantity ordered;
    - When the profit margins of both products become very close to each other, the order quantity of the primary product approaches zero.

6.1 Future Research

Future research directions in the newsvendor arena and its extensions are immense. As technology evolves, its application becomes more pronounced. Analytics provides a significant fulcrum to the meaningful extensions and results of its numerous areas of applications.

One research direction to the capacitated newsvendor problem is to find the threshold of the problem scale that the difference between the optimal cost (profit) obtained by the proposed constructive method and existing methods becomes nonnegligible.

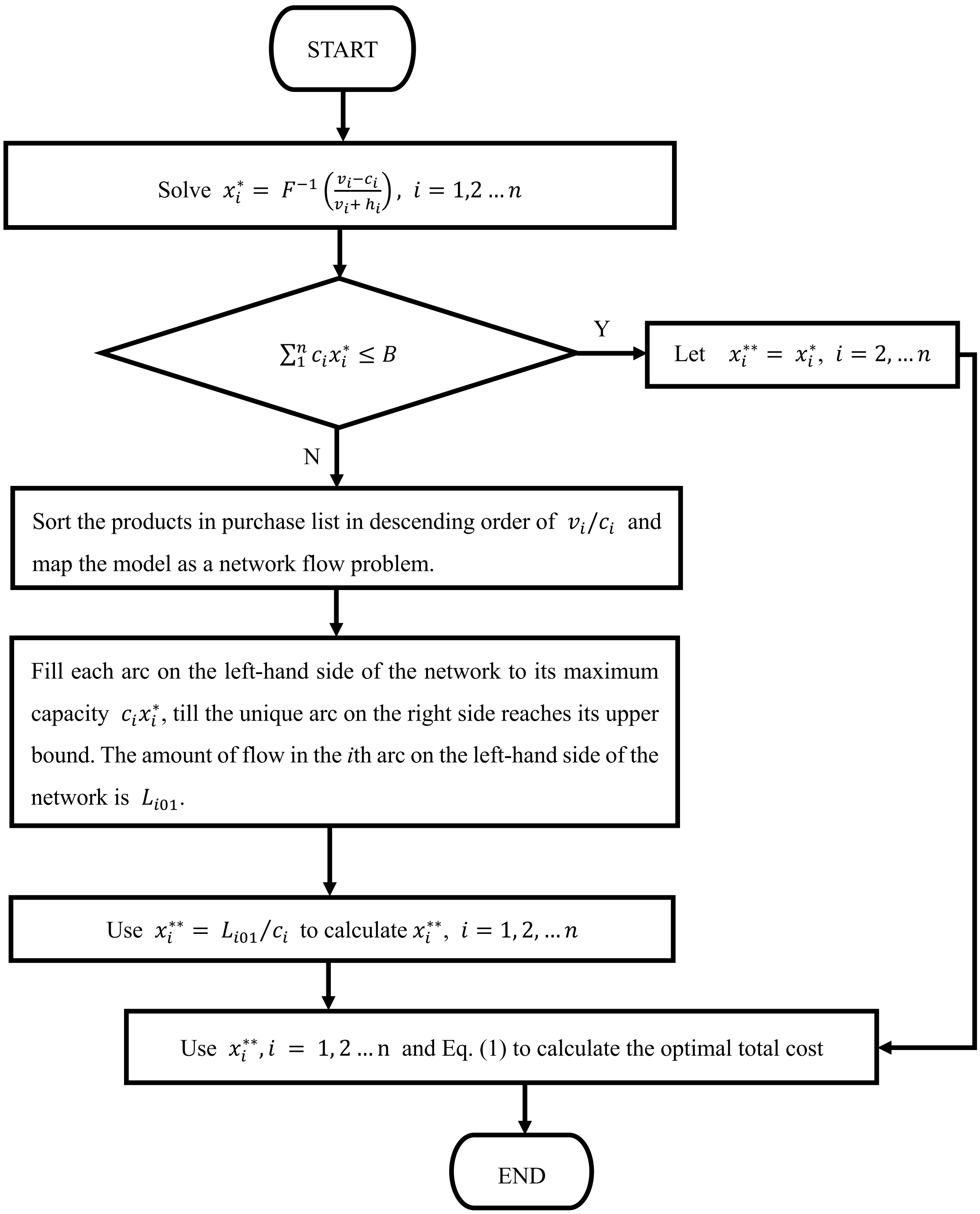
Another future research direction in this stream is to develop an approach to solve the multi-constraint newsvendor problem while ensuring its simplicity and applicability.

For the two-product newsvendor with substitution, the parameters used in the numerical experiments and applications are estimated based on experience. Therefore, the accumulation and refinement of the real data is a direction for future research. When realistic data is available, there is a potential to discover more patterns and draw more managerial insights. In addition, expanding the current model to the one with mutual substitution is another direction. The application of this extended model covers the scenarios in which customers do not have preference and their decision of substitution is not only driven by product shortages, but also by other factors such as pricing, advertising, and defect.

APPENDIX A

FLOWCHART EXHIBITING THE STEPS OF NETWORK FLOW APPROACH

This appendix is dedicated to show the flowchart of the proposed method in Chapter 3.



APPENDIX B

THE TRANSFORMATION OF OBJECTIVE FUNCTION OF NEWSVENDOR

This appendix is dedicated to show the transformation of the objective function of newsvendor problem.

$$
\begin{aligned}
E_{nv} &= c_i x_i + h_i \int_0^{x_i} (x_i - D_i) f_i(D_i)\, dD_i + v_i \int_{x_i}^{\infty} (D_i - x_i) f_i(D_i)\, dD_i \\
&= c_i x_i + h_i x_i \int_0^{x_i} f_i(D_i)\, dD_i - h_i \int_0^{x_i} D_i f_i(D_i)\, dD_i + v_i \int_{x_i}^{\infty} D_i f_i(D_i)\, dD_i \\
&\quad -v_i x_i \int_{x_i}^{\infty} f_i(D_i)\, dD_i \\
&= c_i x_i + h_i x_i F_i(x_i) - h_i \int_0^{x_i} D_i f_i(D_i)\, dD_i + v_i \left(E(D_i) - \int_0^{x_i} D_i f_i(D_i)\, dD_i \right) \\
&\quad -v_i x_i \big(1 - F_i(x_i)\big) \\
&= c_i x_i + (h_i + v_i) x_i F_i(x_i) + v_i (E(D_i) - x_i) - (h_i + v_i) \int_0^{x_i} D_i f_i(D_i)\, dD_i
\end{aligned} \tag{B.1}
$$

Implement integral by parts to $\int_0^{x_i} D_i f_i(D_i)\, dD_i$:

$$
\int_0^{x_i} D_i f_i(D_i)\, dD_i = D_i F_i(D_i)\big|_0^{x_i} - \int_0^{x_i} F_i(D_i)\, dD_i = x_i F_i(x_i) - \int_0^{x_i} F_i(D_i)\, dD_i \tag{B.2}
$$

Substitute Eq. (B.2) back into Eq. (B.1):

$$
\begin{aligned}
E_{nv} &= c_i x_i + (h_i + v_i) x_i F_i(x_i) + v_i (E(D_i) - x_i) \\
&\quad -(h_i + v_i) \left(x_i F_i(x_i) - \int_0^{x_i} F_i(D_i) dD_i \right) \\
&= (c_i - v_i) x_i + v_i E(D_i) + (h_i + v_i) \int_0^{x_i} F_i(D_i)\, dD_i
\end{aligned} \tag{B.3}
$$

APPENDIX C

OPTIMALITY CONDITIONS OF THE NEWSVENDOR WITH SUBSTITUTION

This appendix presents the necessary conditions of optimality for the uniform, exponential and normal distributions. It is obtained by the derivatives of the total cost functions, with respect to the decision variables, x_a and x_b. Thus;

a) For the uniform distribution, there are two cases:

Case I: $\boldsymbol{x_a + x_b > U_a - L_a}$

$$\begin{aligned} x_s &= \int_{x_a}^{x_a+x_b} (D_a - x_a)\frac{1}{U_a - L_a}\left(\int_0^{x_b-(D_a-x_a)} \frac{1}{U_b - L_b} dD_b\right) dD_a \\ &+ \int_0^{x_b} (x_b - D_b)\frac{1}{U_b - L_b}\left(\int_{x_a+(x_b-D_b)}^{U_a-L_a} \frac{1}{U_a - L_a} dD_a\right) dD_b \\ &= \frac{(x_a - U_a + L_a)^2}{3(U_a - L_a)(U_b - L_b)}\left(x_a + \frac{3}{2}x_b - U_a + L_a\right) + \frac{(U_a - L_a - x_a)^3}{6(U_a - L_a)(U_b - L_b)} \\ &= \frac{(U_a - L_a - x_a)^2}{6(U_a - L_a)(U_b - L_b)}(x_a + 3x_b - U_a + L_a) \end{aligned} \qquad (\mathrm{C.1})$$

$$
\begin{aligned}
E_{nvs} &= (c_a - v_a)x_a + v_a \frac{U_a + L_a}{2} + (v_a + h_a)\int_0^{x_a} \frac{D_a - L_a}{U_a - L_a} dD_a \\
&+(c_b - v_b)x_b + v_b \frac{U_b + L_b}{2} + (v_b + h_b)\int_0^{x_b} \frac{D_b - L_b}{U_b - L_b} dD_b - (v_b + h_b)x_s \\
&= c_a x_a + h_a \frac{x_a^2}{2(U_a - L_a)} \\
&+v_a\left[\frac{(U_a - L_a - x_a)^2}{2(U_a - L_a)} - \frac{(U_a - L_a - x_a)^2}{6(U_a - L_a)(U_b - L_b)}(x_a + 3x_b - U_a + L_a)\right] \\
&+ c_b x_b + h_b\left[\frac{x_b^2}{2(U_b - L_b)} - \frac{(U_a - L_a - x_a)^2}{6(U_a - L_a)(U_b - L_b)}(x_a + 3x_b - U_a + L_a)\right] \\
&+v_b\left[\frac{(U_b - L_b - x_b)^2}{2(U_b - L_b)} - \frac{(U_a - L_a - x_a)^2}{6(U_a - L_a)(U_b - L_b)}(x_a + 3x_b - U_a + L_a)\right]
\end{aligned}
\tag{C.2}
$$

Case II: $\boldsymbol{x_a + x_b < U_a - L_a}$

$$
\begin{aligned}
x_s &= \int_{x_a}^{x_a+x_b} \frac{1}{U_a - L_a}(D_a - x_a)\left(\int_0^{x_b-(D_a-x_a)} \frac{1}{U_b - L_b} dD_b\right) dD_a \\
&+ \int_0^{x_b} (x_b - D_b)\frac{1}{U_b - L_b}\left(\int_{x_a+(x_b-D_b)}^{U_a-L_a} \frac{1}{U_a - L_a} dD_a\right) dD_b \\
&= \frac{x_b^3}{6(U_a - L_a)(U_b - L_b)} + \frac{x_b^2}{6(U_a - L_a)(U_b - L_b)}[3(U_a - L_a - x_a) - 2x_b] \\
&= \frac{x_b^2}{6(U_a - L_a)(U_b - L_b)}[3(U_a - L_a - x_a) - x_b]
\end{aligned}
\tag{C.3}
$$

$$
\begin{aligned}
E_{nvs} &= (c_a - v_a)x_a + v_a \frac{U_a + L_a}{2} + (v_a + h_a)\int_0^{x_a} \frac{D_a - L_a}{U_a - L_a} dD_a \\
&+(c_b - v_b)x_b + v_b \frac{U_b + L_b}{2} + (v_b + h_b)\int_0^{x_b} \frac{D_b - L_b}{U_b - L_b} dD_b \\
&-(v_b + h_b)x_s
\end{aligned}
\tag{C.4}
$$

$$E_{nvs} = c_a x_a + c_b x_b + h_a \frac{x_a^2}{2(U_a - L_a)} + v_a \left[\frac{(U_a - L_a - x_a)^2}{2(U_a - L_a)} - \frac{x_b^2}{6(U_a - L_a)(U_b - L_b)} [3(U_a - L_a - x_a) - x_b] \right] + h_b \left[\frac{x_b^2}{2(U_b - L_b)} - \frac{x_b^2}{6(U_a - L_a)(U_b - L_b)} [3(U_a - L_a - x_a) - x_b] \right] + v_b \left[\frac{(U_b - L_b - x_b)^2}{2(U_b - L_b)} - \frac{x_b^2}{6(U_a - L_a)(U_b - L_b)} [3(U_a - L_a - x_a) - x_b] \right] \tag{C.5}$$

b) Exponential Distribution

$$\begin{aligned} x_s &= \int_{x_a}^{x_a + x_b} (D_a - x_a) \frac{1}{\mu_a} e^{-\frac{1}{\mu_a} D_a} \left(\int_0^{x_b - (D_a - x_a)} \frac{1}{\mu_b} e^{-\frac{1}{\mu_b} D_b} dD_b \right) dD_a \\ &+ \int_0^{x_b} (x_b - D_b) \frac{1}{\mu_b} e^{-\frac{1}{\mu_b} D_b} \left(\int_{x_a + (x_b - D_b)}^{\infty} \frac{1}{\mu_a} e^{-\frac{1}{\mu_a} D_a} dD_a \right) dD_b \\ &= e^{-\frac{1}{\mu_a} x_a} \left(\frac{\frac{1}{\mu_b} e^{-\frac{1}{\mu_a} x_b}}{\frac{1}{\mu_a} \left(\frac{1}{\mu_a} - \frac{1}{\mu_b} \right)} - \frac{e^{-\frac{1}{\mu_a} x_b}}{\frac{1}{\mu_a} - \frac{1}{\mu_b}} + \mu_a \right) \end{aligned} \tag{C.6}$$

$$\begin{aligned} E_{nvs} &= (c_a - v_a) x_a + v_a \mu_a + (v_a + h_a) \int_0^{x_a} \left(1 - e^{-\frac{D_a}{\mu_a}} \right) dD_a \\ &+ (c_b - v_b) x_b + v_b \mu_b + (v_b + h_b) \int_0^{x_b} \left(1 - e^{-\frac{D_b}{\mu_b}} \right) dD_b \\ &- (v_b + h_b) \left[e^{-\frac{1}{\mu_a} x_a} \left(\frac{\frac{1}{\mu_b} e^{-\frac{1}{\mu_a} x_b}}{\frac{1}{\mu_a} \left(\frac{1}{\mu_a} - \frac{1}{\mu_b} \right)} - \frac{e^{-\frac{1}{\mu_a} x_b}}{\frac{1}{\mu_a} - \frac{1}{\mu_b}} + \mu_a \right) \right] \end{aligned} \tag{C.7}$$

$$x_a^* = -\mu_a ln\left[\frac{C_a + h_a}{(v_a + h_a) - \frac{1}{\mu_a}\left(\frac{\frac{1}{\mu_b} e^{-\frac{1}{\mu_a}x_b}}{\frac{1}{\mu_a}\left(\frac{1}{\mu_a} - \frac{1}{\mu_b}\right)} - \frac{e^{-\frac{1}{\mu_b}x_b}}{\frac{1}{\mu_a} - \frac{1}{\mu_b}} + \mu_a\right)(v_b + h_b)}\right] \qquad (C.8)$$

Eq. (C.8) shows the optimal order quantity of Product *a*.

c) Normal Distribution

$$\begin{aligned} x_s = &\int_{x_a}^{x_a+x_b} (D_a - x_a)\frac{1}{\sigma_a\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D_a-m_a}{\sigma_a}\right)^2} \left(\int_0^{x_b-(D_a-x_a)} \frac{1}{\sigma_b\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D_b-m_b}{\sigma_b}\right)^2} dD_b\right) dD_a \\ &+ \int_0^{x_b} (x_b - D_b)\frac{1}{\sigma_b\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D_b-m_b}{\sigma_b}\right)^2} \left(\int_{x_a+(x_b-D_b)}^{\infty} \frac{1}{\sigma_a\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D_a-m_a}{\sigma_a}\right)^2} dD_a\right) dD_b \end{aligned} \qquad (C.9)$$

$$\begin{aligned} E_{nvs} = &(c_a - v_a)x_a + v_a m_a \\ &+(h_a + v_a)\int_0^{x_a} \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{D_a - m_a}{\sigma_a\sqrt{2}}\right)\right] dD_a \\ &+(c_b - v_b)x_b + v_b m_b \\ &+(h_b + v_b)\int_0^{x_b} \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{D_b - m_b}{\sigma_b\sqrt{2}}\right)\right] dD_b \\ &-(v_b + h_b)x_s \end{aligned} \qquad (C.10)$$

APPENDIX D

SIMULATED MODEL OF THE NEWSVENDOR WITH SUBSTITUTION

This appendix is dedicated to present the spreadsheet of the simulated model of the two-product newsvendor with substitution.

	C	D	E	F	G	H	I	J
	Day	Demand of Product A	Demand of Product B	Order Quantity of Product A	Order Quantity of Product B	Extra Demand of Product A	Left Overs of Product B	Model
		RAND()*$(U_i - L_i)$+L_i						NBs=
		$-\mu_i^{-1}$*LN(1-RAND())				IF(D14-F14>0, D14-F14, 0)	IF(G14-E14>0, G14-E14, 0)	NB=
		NORM.INV(RAND(), m_i, s_i)						IF(H14*I14<>0, ”NBs ”, ”NB”)
14	1	325	196	320	254	5	57	NBs
15	2	377	220	320	254	57	34	NBs
16	3	330	257	320	254	11	0	NB
17	4	371	269	320	254	52	0	NB
18	5	311	234	320	254	0	20	NB
19	6	336	201	320	254	16	53	NBs
20	7	305	175	320	254	0	79	NB
	...							

	J	K	L	M	N	O	P
		Simulation					
		55.63%	34.01%	11	21.62%	6	17
	Model	P_s	$P_{x_b-D_b>D_a-x_a}$	x_{sa}	$P_{x_b-D_b<D_a-x_a}$	x_{sb}	x_s
	NBs=	11683	7142		4541		
	NB=	9317	13858		16459		
		IF(J14=”NB”, 0, 1)	IF(AND(K14=1, I14>H14), 1, 0)	IF(L14<>0, H14, 0)	IF(AND(K14=1, I14<H14), 1, 0)	IF(AND(K14=1, H14>I14), I14, 0)	IF(J14=”NBs”, IF((I14>H14), H14, I14), 0)
14	NBs	1	1	5	0	0	5
15	NBs	1	0	0	1	34	34
16	NB	0	0	0	0	0	0
17	NB	0	0	0	0	0	0
18	NB	0	0	0	0	0	0
19	NBs	1	1	16	0	0	16
20	NB	0	0	0	0	0	0
	...						

	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK
	Newsvendor with Surrogate										
	Holding Amount of Product A	Stockout of Product A	Holding Amount of Product B	Stockout of Product B	Purchase Cost of Product A	Holding Cost of Product A	Lost Revenue of Product A	Purchase Cost of Product B	Holding Cost of Product B	Lost Revenue of Product B	Surrogate Revenue
						$7,535			**$2,352**		
					6393	9	1133	2537	20	142	-346
	IF(F14>D14, F14-D14, 0)	IF(D14>F14, D14-F14, 0)	IF(G14>E14, G14-E14-P14, 0)	IF(E14>G14, E14-G14, 0)	c_a*F14	h_a*AA14	v_a*AB14	c_b*G14	h_b*AC14	v_b*AD14	-(v_b+h_b)*P14
14	0	63	0	0	6393	0	2203	2537	0	0	-54
15	0	78	0	0	6393	0	2732	2537	0	0	-78
16	0	5	0	25	6393	0	177	2537	0	506	0
17	0	45	0	0	6393	0	1580	2537	0	0	-922
18	0	12	53	0	6393	0	405	2537	53	0	-243
19	0	80	0	45	6393	0	2783	2537	0	894	0
20	0	31	0	0	6393	0	1093	2537	0	0	-507
	...										

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