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# ABSTRACT <br> EVALUATING THE IMPACT OF ADOPTING 3D PRINTING SERVICES ON THE RETAILERS 

by<br>Sharareh Rajaei Dehkordi

As additive manufacturing technology becomes more responsive to consumers' demand, one important question for the retailers is whether they should provide 3D printing services in their brick-and-mortar store in addition to the traditional off-the-shelf product? If so, what should be the retailers pricing scheme to achieve a higher profit? What should be the optimal inventory level of off-the-shelf products? What is the optimal capacity of 3D printers? In this study, stochastic models are examined to capture the joint optimal 3D product price and capacity of 3D printers to maximize retailer's expected profit while considering consumer product choices. Moreover, a stochastic model is developed to capture joint optimal pre-made inventory level and 3D product price to maximize retailer's expected profit considering 3D services are offered in the off-the-shelf stock-out situations as a one-way substitution. Utilizing the Markov Decision Process, a framework for queuing systems is developed to examine the performance of various production/inventory strategies that optimize the system's performance. Here, four strategies are developed: (i) providing only off-the-shelf products, (ii) providing only 3D printed products, (iii) substituting the shortage of the off-the-shelf products by 3D printed products, and (iv) providing consumers the options of selecting either the off-the-shelf product or the customized product produced by additive manufacturing. In essence, this approach assists decision makers in both capacity planning and inventory management. For all models, analytical results and numerical examples are given in order to demonstrate managerial insights.

## EVALUATING THE IMPACT OF ADOPTING 3D PRINTING SERVICES ON THE RETAILERS

by<br>Sharareh Rajaei Dehkordi

A Dissertation<br>Submitted to the Faculty of New Jersey Institute of Technology in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Industrial Engineering<br>Department of Mechanical and Industrial Engineering

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## APPROVAL PAGE

# EVALUATING THE IMPACT OF ADOPTING 3D PRINTING SERVICES ON THE RETAILERS 

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## CHAPTER 1

## INTRODUCTION

Nowadays, three-dimensional printing technology is recognized as the new manufacturing revolution, which has been developed rapidly. This technology opens up a new prospect to manufacturers, as well as retailers. The on-site production with a one-step manufacturing process and geometrical freedom in design allow the companies to produce the products as they visualize, without traditional manufacturing constraints. Giant retailers and service providers, such as Toys R Us, Staples, Macy's, Hershey's, Amazon, McDonald's, UPS and FedEx, are considering 3D printed products in their product portfolios. In particular, it is important for the retailers to work on the design and pricing of the product portfolios to attract more customers, gain more market share, and earn more profits. By adopting the 3D printing technology, retailers can react to customers' demands fast. 3D printing manufacturing provides a variety of features for customers, such as creating customized products, complex parts while the process is simple manufacturing process with less waste of materials. Furthermore, retailers will end up with less inventory, reduction in lead times, and transportation costs.

In this chapter, the research motivation of this study on the impact of adapting 3D printing services on retailers are discussed in Section 1.1. Background and problem statement of the research are explained in Section 1.2. This dissertation follows four research objectives which are proposed in Section 1.3 and the contributions are provided in Section 1.4.

### 1.1 Research Motivation

three-dimensional printing technology, like much other new technologies, is developing rapidly. [Columbus, 2015] conducts a survey on the impact of 3D printing on the


Rapid market growth


Figure 1.1 Siemens prediction of 3D printing market growth. Source: Siemens. (2014). Pictures of the future. The Magazine for Research and Innovation.
market share. It is reported that the global 3D printing market value has been 2.3 billion dollars in 2013 and is predicted to reach 8.6 billion dollars by 2020, attaining a Compound Annual Growth Rate (CAGR) of 20.6\%. Another prediction by Siemens anticipates that 3D printing will become $50 \%$ cheaper and up to $400 \%$ faster in the next five years. Siemens is also foreseeing that 3D Printing will gain an 8.3 billion dollars global market by 2023. Figure 1.1 shows the Siemens prediction of 3D printing market growth.

As retailers' competition becomes more severe, they try to optimize their product portfolios by adopting new technologies, such as 3D printing technology, to attract more customers and achieve more market share in order to gain more profit. One of the world's largest and best-known toy stores, Toys' R' Us, sets up 3D printing kiosks in some of their branches in the US in order to assess the increase in their profit margins and market shares. They try to increase customer satisfaction,
and eventually, the company profits by providing 3D printers to produce custom designed action figures and toys for their consumers.

The Hershey Company, which is commonly called Hershey's, is one of the largest chocolate manufacturers in North America. In 2015, the Hershey Company developed a chocolate 3D printer in collaboration with 3D Systems. The 3D printer is named the CocoJet printer. Hershey's chocolate company is now 3D printing uniquely designed candy. The products can be complicated hexagons and intricately laced patterned chocolate, as well as customized design. Furthermore, Hershey's can get a scan of their customers and provide them a 3D printed chocolate figurine of themselves.

In 2014, one of the United States largest office supply chain stores, Staples, has been piloting 3D printing services in its stores in Canada, allowing customers and small businesses to create their personalized products. In 2015, Staples has started rolling out 3D printing services in most of their 2,000 store locations and providing online 3D printing services which offer customers a user-friendly interface that allows them either to upload their 3D models or purchase 3D printed products.

Macy's is the other giant retailer that decides to provide 3D printing cubes in one of the New York City's stores, allowing customers to print jewelry and iPhone cases.

With the variety of manufacturing possibilities offered by 3D printing, even online retailers try to get on the 3D printing bandwagon. Amazon, Shapeways, Ownphones and 3Dshoes provide the options of either buying pre-made 3D products or making customized ones for their customers.

McDonald's, on the other hand, provides 3D printed toys for Happy Meals to increase customer satisfaction and revenue margins. Jeff DeGrange, the former manager of Boeing Phantom prognosticated that one day, Boeing will be building parts on demand at the point of use even when it is in space on aircraft carriers [Khajavi et al., 2014].

To obtain more market share, delivery services may adopt 3D printing technology as well. One of the giants of delivery services, UPS, starts using 3D printing centers to improve supply chains dynamics, reduce transportation cost and shorten delivery lead times. Its 3D project is run by an Atlanta startup, called CloudDDM LLC that UPS invested in during 2014. The two companies strategic plan is to expand more than nine hundred 3D printers and their mission is to open the 3 D printing services outside the U.S. [Lindsay and Laura, 2015]. As an example of UPS 3D project, 3D printing services were receiving an order of forty mounting brackets for paper towel dispensers. The order comes from a division of Georgia-Pacific LLC that makes dispensers, dixie cups, and cutlery. By using 3D printing technology, CloudDDM printed the mounts and UPS shipped them to a Georgia-Pacific by the next morning [Lindsay and Laura, 2015].

UPS is not the only delivery company exploring the printing business. In 2015, TNT Express started using 3D printing services at some locations across Germany.

On the other hand, Amazon.com Inc has filed a patent for 3D printing trucks, aimed at producing on-demand system printing goods from inside delivery vehicles [Lindsay and Laura, 2015].

One of the supply chain management challenges is to reduce the inventory and transportation costs. Moreover, in order to increase customer satisfaction, they have to provide shorter lead times from order to delivery. 3D printing is a completely different form of manufacturing which changes some of the underlying assumptions for supply chain management systems to improve the above-mentioned concerns. By using 3D printing technology, manufacturers can produce products on demand, which result in the reduction of maintaining safety inventory, such as work-in-process inventory and finished products in transport and in-stock inventory. Therefore, there is less obsolescence of existing stock. The raw materials need to be close at hand and products can be made closer to the consumer with shorter lead times from order to
delivery. This technology provides infinite customization flexibilities without any cost penalties and the production of these customized products can take place in front of customers at the push of a button.

Furthermore, as 3D printing is more of an on-demand or builds to order process, it is agile and better able to react to customer demands. In another word, instead of manufacturers producing products based on forecasted demands, 3D printing provides real-time demand for the manufacturer.

Gartner reported a worldwide survey at the end of 2014 . The survey was coordinated to determine how and why organizations are using or planning to use 3D printing technologies. Survey participants were three hundred thirty individuals employed by organizations with at least one hundred employees, that are using or planning to use 3D printing technology. The survey concluded that rapid prototyping is the most cited benefit and use of 3D printing, but the applications far exceed this sole use. The technology is used for developing products, creating new products which are impossible or difficult to produce by traditional technologies because of their high degree of complexity, reducing cost, increasing efficiency, developing customized products, improving supply chain logistics and expanding the product line [Meulen and Rivera, 2014].

### 1.2 Background and Problem Statement

### 1.2.1 Retailers' Challenges in Supply Chain Management

As business becomes ever more competitive, retailers must seek every opportunity to increase their market share and reduce their costs.

By adopting new technologies, such as 3D printing technology retailers attract more customers in order to increase their market share. This technology provides infinite product customization flexibilities for consumers without any cost penalties and the production of these customized products can take place in front of customers


Figure 1.2 The worldwide classification of the 3D printing application. Source: Meulen, R. Rivera, J. (2014). High Acquisition and Start-Up Costs Are Delaying Investment in 3D Printers. Gartner.
at the push of a button. The pie chart in Figure 1.2 shows the worldwide classification of the 3D printing application. As a result of this survey, the major applications of 3D manufacturing for businesses are providing product development, adding a new product line and using it to create personalized parts or complex shaped items. Price Waterhouse Cooper (PWC) has conducted another survey of manufacturers that adopt 3D technology and as a result, they announce the only reason those small businesses can keep pace with the larger ones is by using 3D printing technologies [Piazza and Alexander, 2015]. Of all business operations, supply chain optimization is perhaps the most critical place where increased control and visibility can reduce costs. Nowadays, the important supply chain challenges are reducing the inventory and transportation costs. In another word, in order to increase customer satisfaction, they have to provide shorter lead times from order to delivery. By adopting new technologies such as 3D printing technology, retailers can provide products on demand, which result in the reduction of maintaining safety inventory, such as work-in-process inventory and finished products in transport and in-stock inventory.

Therefore, there is less obsolescence of existing stock. The raw materials need to be close at hand and products can be made closer to the consumer with shorter lead times from order to delivery.

### 1.2.2 3D Printing Manufacturing Process

3D printing technology has sparked scientists' imagination in 1970's. At first, it was known as Rapid Prototyping and was invented with the intent of allowing engineers to create prototypes of their designs in a more time effective manner.

For the first time, in the early 1970s, a Japanese researcher, Dr. Hideo Kodama, first invented the layered approach of 3D printing technology. Later in 1986, Dr. Chuck Hull patented Stereolithography as a method of producing 3D printed products by printing successive layers upon layers of an object, starting from the bottom layer to the top layer. In 1986, Hull founded the world's first 3D printing company, 3D Systems Inc. In 2015, the market value of 3D Systems was 170.5 million dollars, which had risen 12.5 percent compared to the previous year [Tita, 2015].

The 3D printing process is a manufacturing process in the category of additive manufacturing process. There are a large number of technologies which employ additive manufacturing, some of the more widely used include i) 3D Printing (3DP), ii) Stereolithography (SL), iii) Fused deposition modeling (FDM) and iv) Selective laser sintering (SLS).

Since the development of many of these technologies has occurred simultaneously, there are various similarities, as well as distinct differences between each one [Kulkarni et al., 2000]. Many of the aforementioned technologies are limited to the rapid prototyping, as they do not allow common engineering materials to be processed with sufficient mechanical properties. However, 3D printing technology can be used to produce final products with a widening range of material, high precision and high final quality [Kruth et al., 2007].

In the 3D printing manufacturing process, products are built on a layer-by-layer additive basis through the series of cross-sectional slices, rather than subtracting material from a larger piece of material, which is called subtractive manufacturing. It can be said that 3D printers work in a manner similar to traditional inkjet printers, but instead of using multi-colored inks, the 3D printers use the liquid resin of the raw materials to build the final product.

A 3D printer is a type of industrial robot and in order to produce an object, it uses 3D computer-aided-design(CAD) programs to create digital models which can be saved and reused at the end of the product design process. 3D printers can utilize CAD data from commercial programs marketed by SolidWorks and Autodesk or free design packages, such as Blender and Google SketchUp [Berman, 2012]. The next step is the conversion of CAD information to the STL file, which is then sent to the 3D printer to manufacture the product. The 3D printer starts at the bottom of the design and builds up successive layers of liquid resin. In some cases, it uses a computer-controlled ultraviolet laser to harden each of the layers in the specified cross-section pattern until the final product is produced.

Generally, the 3D printing manufacturing process follows the following three different steps as listed:

- Creating the 3D image of the product in computer-aided-design(CAD) program.
- Converting CAD information to the STL file and sending them to the 3D printer.
- Manufacturing the product in a layer-by-layer, cross-sectional pattern.

Figure 1.3 shows the different steps of the 3D printing manufacturing process from designing the product by the customer to converting it into the CAD file, then STL file and producing with the 3 D printers.
[Gibson et al., 2010] defines below seven key steps as the 3D printing manufacturing process:


Figure 1.3 Three steps of the 3D printing manufacturing process.

- Conceptualization and CAD
- Conversion to STL
- Transfer and manipulation of STL file on the 3D printer
- Machine setup
- Building the product
- Product removal and clean up
- Post-processing of the product

Several raw materials can be used in the 3D printing manufacturing and the number of these available raw materials are increasing fast. The most common materials of the 3D printing process are classified in different categories as below.

- Plastics, such as Nylon, Polyamide, ABS, PLA
- Metals, such as Aluminium, Stainless steel, Silver, Gold, Bronze, several alloys
- Biomaterials and food, such as chocolate, pasta, and meat
- Ceramics, such as cement and sand

The cost of the 3D printing technology depends on a variety of factors. [Berger, 2013] defines different cost factors for the 3D printing manufacturing process and classifies them in below five different groups.

- Operation costs
- Machine costs
- Labor costs
- Material costs
- Energy and overhead costs

He anticipates that the 3D printing manufacturing costs will decrease in the next five years and $74 \%$ of the costs consist of the machine cost, operation cost, labor cost, energy, and overhead expenses, while $26 \%$ of these costs are direct costs for materials. [Munguía et al., 2008] define similar cost factors for the 3D printing manufacturing process.

### 1.2.3 The Application of The 3D Printing Technology

The 3D printing technology is capable of building the most durable, stable, repeatable with high accuracy parts for different industries, such as automotive, aerospace and medical industries. It can produce very simple products to very complex ones, or manufacturing mass customization products, such as toys, art, and jewelry. Moreover, the 3D printing technology is able to produce micro products, for instance, mimicking structures of biological origin, microfluidic elements such as filters or mixers on microfluidic chips, micro-lens arrays or prisms with complex surface shapes, as well as macro ones. For example, Local Motors produces 3D printed cars and Dutch-design company, MX3D, have developed 3D printers that will build a steel bridge over a canal in Amsterdam by 2018.

The application of 3 D printing is very wide. It is generally used for concept modeling, rapid prototyping, manufacturing tools, and end user parts.

Concept modeling is when the firms extend their visions by testing out more new projects and developing only the right ones before presenting them to their
superiors. Rapid prototyping is creating realistic prototypes with the look and feel of real products. 3D printer technology can manufacture quick, low-volume tools and custom fixtures as well. Producing customer products is another application of the 3D printing technology, which is the focus of this research as well. As retailer competition becomes more severe, they try to optimize their product portfolios by adopting 3D printing technology to attract more customers and achieve more market share in order to gain more profit.

In general, the application of 3D printing has expanded in different industries, such as Automotive, Aerospace, Defense, Healthcare, Education, and Research, Consumer Products, Architecture, Art, Nanotechnologies, Repair, and Tooling. The pie chart in Figure 1.2.3 displays the percentage of the 3D printing manufacturing usage in different industries. Consumer products, with 18.4 percent of the total 3D printing applications is the second major usage of the 3D printing technology, which is the concern of this dissertation as well.


Figure 1.4 The application of the 3D printing technology in different industries. Source: Meulen, R. Rivera, J. (2014). High Acquisition and Start-Up Costs Are Delaying Investment in 3D Printers. Gartner.

### 1.2.4 3D Printing Technology in Comparison to Other Technologies

There are vastly different types of manufacturing processes that one can use based on the technical and managerial requirements. The majority of manufacturing processes can be classified into four major groups, i) Subtractive Manufacturing, ii) Additive

Manufacturing, iii) Formative Manufacturing and iv) Joining Manufacturing. The subcategories of each class are outlined in Figure 1.5. Comparing 3D printing


Figure 1.5 Manufacturing process classification.
technology with other technologies shows that 3D printing production is a one-step manufacturing process, with no assembly process and a short setup time, which requires a low level of operator expertise. 3D printing manufacturing reduces the amount of human interaction needed to create a product. Furthermore, creating the part directly from the consumer design ensures that the product precisely represents the consumer's intent and thus reduces inaccuracies found in traditional manufacturing processes. The 3D printing process is inherently green. Since the material is added layer-by-layer, only the material needed for the part is used in the production and there is almost zero waste, which is in contrast to the traditional subtractive manufacturing processes, such as machining, where the desired product is carved out of the raw material. [Campbell et al., 2011].
[Holmström et al., 2010] propose the individual characteristics of the 3D printing production which leads to the following benefits:

- Possibility to quickly change the design.
- Allows economical custom products.
- Allows product to be optimized for function.
- Small production batches are feasible and economical.
- No significant tooling is needed.
- Possibility to reduce waste.
- Shorter lead times.
- Lower inventory level.
- Less transportation cost.

Table 1.1 is provided to summarize the advantages and limitations of 3D printing manufacturing compared to other technologies.

Mass customization is used to produce customized products for consumers. Mass customization as well as 3D printing technique, combine flexibility with the production process in order to satisfy specific customer needs. The 3D printing technology has significant differences compared to the mass customization which are listed in Table 1.2.

### 1.3 Research Objective

This dissertation accomplished the following four research objectives:
Research objective 1: Considering the retailers provide 3D printing services in the brick-and-mortar stores, in addition to the traditional, off-the-shelf products, we develop mathematical models to maximize retailer's expected profit and capture the optimal 3D product price and capacity of 3D printers, considering the following assumptions: (i) The retailer offers two types of products, off-the-shelf and 3D printing products. (ii) Consumer selection behavior follows either the Cut-off model

Table 1.1 3D Printing in Comparison to Other Technologies (Injection Molding, Cuttingbased Machinery)

## The Advantages of 3D Printing Technology

Precise physical replication of products
Less waste of materials
Free manufacturing complexity
Free products variety
Feasible and Economical small production batches
Design and production of customized products
No assembly needed
No need for costly tools, molds, jigs, and fixtures
Short setup time
Optimised for function products
Automated manufacturing and reduction of human interaction
Unlimited design space
Ability to easily change and share designs
Ability to outsource manufacturing

## The Limitations of 3D Printing Technology

Limited strength
Resistance to heat

Limited choice of materials and colors
High start-up costs of implementing 3D printing strategies
Higher costs for large production compared to injection molding and other technologies

Table 1.2 The Comparison of the 3D Printing Technology and Mass Customization

| Characteristic | 3D Printing Technology | Mass Customization |
| :---: | :---: | :---: |
| Supply chain management | Raw materials are provided from small number of vendors | Requires a high degree of supply chain integration as the component parts come from multiple suppliers |
| Supply chain management | Ability to produce custom products at no WIP and unsold finished inventory | Ability to produce custom products at low WIP inventory with no unsold finished inventory |
| Manufacturing process | One step production by using CAD file and additive manufacturing technologies to 3D print part | Building of partially constructed products and different combinations of pre-assembled modular parts |
| Range of products | Prototypes; Consumer's orders; Medical/Dental applications; Replacement parts | Computers; watches; Shoes; <br> Jeans |

or the Multinomial Logit model. (iii) No inventory limitation is considered for the off-the-shelf products.

Research objective 2: Investigate the effect of having stochastic consumer valuation on the retailer's expected profit. The joint optimal price of 3D products
and 3D printer capacity are captured. The same approach as described in the first research objective is used and the following two cases are made: (i) The customer valuation for the 3D products follows a stochastic distribution, while the valuation for the off-the-shelf products is deterministic. (ii) The customer valuation for both products is stochastic.

Research objective 3: Considering the retailer provides off-the-shelf products in the brick-and-mortar store, and in the stock-out situation, 3D printing services can be used as the substitution. The goal is based on the Newsvendor model, capture the joint optimal ordering quantity for the off-the-shelf product and the price of 3D product to maximize the retailer's expected profit, while considering consumer product selection behavior. The result will be compared with the traditional order up to policy when 3D products are not available. Here are the assumptions: (i) Consumers choose off-the-shelf products if they are available, otherwise, they consider 3D printed products. (ii) While the utility of off-the-shelf product depends on its price and availabilities, the utility of a 3D product depends on its price and production time.

Research objective 4: Utilizing the Markov Decision Process, a framework for queuing systems is developed to examine the performance of each of the following strategies: (i) Providing only off-the-shelf products, (ii) Providing only 3D printed products, (iii) Substituting the shortage of the off-the-shelf products by 3D printed products, and (iv) Providing consumers the options of selecting either the off-the-shelf product or the customized product produced by additive manufacturing. In essence, this research objective assists decision makers in both capacity planning and inventory management.

### 1.4 Contribution

This dissertation intends to make three major contributions to the literature as follows: (i) Develop a framework that combines consumer selection models with
queuing models to determine the joint optimal 3D product price and 3D printer capacity. (ii) Investigate the impact of heterogeneous customer valuation on the retailer's product offering decisions. (iii) Develop a framework of queuing systems by utilizing the Markov Decision Process, to examine the model of the retailer providing consumers the options of selecting either the off-the-shelf product or the customized product produced by additive manufacturing. The performance of this strategy is compared to the three strategies of providing only off-the-shelf products, providing only 3 D printed products, and substituting the shortage of the off-the-shelf products by 3D printed products.

## CHAPTER 2

## LITERATURE REVIEW

### 2.1 Additive Manufacturing and 3D Printing Technology

Additive Manufacturing (AM) is the process of joining materials to make the products layer upon layer. Additive manufacturing is divided into a large number of technologies. Some of the more widely used include Stereolithography(SL), Fused Deposition Modeling (FDM), Selective Laser Sintering (SLS) and 3D printing. Since the development of many of these technologies has occurred simultaneously, there are significant similarities, as well as differences between each one of them [Kulkarni et al., 2000].

In this section, different classifications of the literature review on additive manufacturing are provided. In the second paragraph, the studies on the additive manufacturing process and the advantages and limitations of the process compared to the other manufacturing processes are discussed. In the third paragraph, the literature on the effect of additive manufacturing on product design is reviewed. The fourth paragraph discusses the application of the additive manufacturing in different industries. The fifth paragraph provides studies on production planning of additive manufacturing and in the last paragraph, the research on the cost factor of additive manufacturing is presented.

The literature that studies additive manufacturing and the 3D printing technology is vast. The additive manufacturing process, its constraints and conveniences respect to other technologies, has received much attention by researchers and have motivated numerous theoretical and empirical validations in a range of applications. [Piazza and Alexander, 2015] deploy a complemental review paper on the additive manufacturing technology, its manufacturing process, the technical pros and cons and
the application of this technology in different industries. [Levy et al., 2003] evolve a systematic material dependent classification of layer manufacturing techniques and explain the specific characteristics of additive manufacturing. Other researchers, like [Kulkarni et al., 2000], [Reeves, 2009] and [Weller et al., 2015], study on additive manufacturing process and its production advantages and disadvantages. [Wang et al., 2013] investigate significant factors affecting the fabricating quality of additive manufacturing product's surface. [Long et al., 2017] explore the characteristics and limitation of the 3D printing technology. They investigate the current situation of Chinese manufacturing, its problems and discuss the potential impact of the 3D printing on its development. [Schniederjans, 2017] conduct a survey analysis on the impact and the main drivers of the intention-to-adopt the 3D printing technology on manufacturing systems. [Petrovic et al., 2011], [Xu et al., 2015] and [Gardan, 2016] explore the impact of the 3D printing technology on manufacturing systems. They present that adoption of the layer-by-layer additive manufacturing will result in producing customisation with complete flexibility in design, reducing time-to-market due to the high speed of the process, maximum material savings and producing lightweight structures.

The other area of operations, which has been proposed by many researchers, is product design. The product design is changed distinctly within the adoption of additive manufacturing technology. A number of articles have been written on the impact of additive manufacturing on the product design compared to the traditional design ([Hague et al., 2003] and [Hague et al., 2004]). The joining-material nature of the additive manufacturing processes consequences in removing many of the limitations in the conventional technologies, such as subtractive manufacturing or formative processes ([Hao et al., 2010]). The unique characteristics of additive manufacturing systems require new design tools and practices to be developed and implemented. [Mellor et al., 2014] study these characteristics and implement
a framework to produce high-value products and new business opportunities. [Emmelmann et al., 2011] develop additive manufacturing techniques for new designing of lightweight products in the aircraft industry. [Berger, 2013] proposes that additive manufacturing has several advantages, such as it can produce an object of virtually any shape and design, even the products which are not producible with traditional manufacturing techniques because of their freedom of shapes. In additive manufacturing, the complexity of design is free and the technology has fewer production steps compared to other technologies. The technology enables weight reduction via topological optimization as well.

Majority of the papers evolve different applications of the additive manufacturing in various industries. In the first place, additive manufacturing or 3D printing technology was used for prototyping. [Reeves, 2009] demonstrates different examples of additive manufacturing prototype applications. In recent years, the new applications of this manufacturing opened up. [Kruth et al., 1998] present that the advancements in the 3D printing are widening the material range used by the machines, improving production precision and final quality, and reducing machine acquisition cost. This improvement changes the application of the 3D printing technology from producing a prototype to the final products. [Levy et al., 2003] expand the application of the additive manufacturing in rapid prototyping, air-cooling ducts for aircraft, hearing aid and prosthesis equipment. [Emmelmann et al., 2011] study on bionic lightweight products used in the aircraft industry. [Gebhardt et al., 2010] propose the additive manufacturing technology for making dental parts. [Melican et al., 2001] discuss an application of the 3D printing technology in orthopedic implant surfaces.

Another research area of additive manufacturing operations is the production planning. Articles on the additive manufacturing production planning are still lacking, [Munguía et al., 2008] propose production planning strategies employed
at thirty-six additive manufacturing centers in Spain. The authors use personal interviews with technicians and survey analysis to identify and categorize various additive manufacturing process strategies, including part orientation strategies, layering strategies, build volume strategies, and support generation strategies.

Some studies focus on cost factor of the additive manufacturing process.[Costabile et al., 2017] analyze the existing literature on the cost models of adopting the 3D printing technology from an operations management point of view. They investigate the strengths and weaknesses of different models as well. [Hayes and Jaikumar, 1991] show that by implementing new technologies, like additive manufacturing or 3D printing, an enormous part of the labor cost shifts to essentially fixed costs of the newly developed technology. [Munguía et al., 2008] identify four key cost factors for additive manufacturing processes: operation costs, machine costs, labor costs and material costs. [Ruffo et al., 2006] develop a cost estimation model for both direct and indirect cost of additive manufacturing and laser sintering processes. [Berger, 2013] propose different cost factors in the additive manufacturing process and based on surveys a cost estimation model and a forecast cost model till 2023 are presented.

### 2.2 The Impact of Additive Manufacturing on Supply Chains

3D printing technology, with its fast advancements in the technology, material range, final product quality and affordability has the potential to fundamentally revolutionize supply chains. There are some studies on the effect of 3D printing on the logistic system in the recent years. Researchers explored some managerial opportunities of adopting 3D printing, such as downsizing the inventory level, reducing transportation cost, and shortening the lead times in supply chain management context. In the following, the related studies are investigated.

A rich class of studies has developed in the literature on the reduction of transportation cost and shortening of lead times in supply chain systems by adopting

3D printing techniques. Furthermore, 3D printing technology can potentially change the traditional inventory management of supply chain systems in different industries, which will result in cost reduction and efficiency improvement of the supply chain system.
[Walter et al., 2004] and [Piazza and Alexander, 2015] propose that as firms incline to produce products on demand with additive manufacturing, the amount of safety stock they need to keep on hand will decrease. Inventory costs are a significant portion of manufacturers' costs. Moreover, in traditional manufacturing, if a manufacturer does not have a part in inventory, they have to order the part, which will result in a delay in the production line. As mentioned before, adopting additive manufacturing technologies can reduce these issues and costs.
[Mashhadi et al., 2015] study the impact of additive manufacturing on the environmental, operational and supply chain configurations. They show that in the supply chain context, additive manufacturing results in increasing information flow, while the material flow decreases. Moreover, adopting additive manufacturing technology will result in on-demand producing, lead times shortening and expedited shipments and unnecessary international transportation removing. They use the inventory level and the lead time as two performance measures and under agent-based simulation system and system dynamics simulation show how additive manufacturing can improve the supply chain system.
[Berman, 2012] propose that 3D printing provides the option of removing unsold finished goods inventory for the firms. The paper argues that 3D printing will significantly reduce the advantages of producing small lot sizes in low-wage countries by reducing the need for factory workers.
[Thomas and Gilbert, 2014] work on the business cost-effectiveness of the 3D printing process and mention that 3D printing technology can reduce the amount of transportation cost of the supply chain system.
[Nyman and Sarlin, 2014] explore some managerial opportunities, such as the possibility of producing a variety of products, reduction of the lead time, and reducing of the wasting material, as well as some barriers of 3D printing technology in a supply chain context.
[Tuck et al., 2006] have provided some effects on supply chain methodologies and principles that will occur with the advent of 3D printing technology. Three supply chain management principles of lean, agile, and leagile supply, as well as the aspect of mass customization, are considered. It proposes that 3D printing technology will change the design, which results in the reduction in the number of components for assemblies and waste of time, cost, and material. Moreover, the ability to produce products on demand, closer to the customers would have profound effects on the lead times and reduction of inventory costs. In fact, there is no need to produce products cheaply miles away for the low volume and custom products, by adopting 3D printing technology, they can be produced faster and closer to the customers in a more convenient location.

Companies along the spare parts supply chain have crucial challenges in their costly inventory management system and ability to provide prompt repair and an appropriate transportation system. Most of the spare parts are infrequently needed but they have to be kept in stock in order to ensure fast service time in emergency situations. Aircraft spare parts demand pattern follows a 20/80 Pareto curve, which means that $80 \%$ of the demand of spare parts are needed frequently and they only cover $20 \%$ of the supply chain inventory cost. While $80 \%$ of the supply chain cost is due to the $20 \%$ amount of infrequently needed parts. Using 3D printing technology can reduce the inventory expenditure by producing on demand of barely used spare parts ([Liu et al., 2014]).

Recently, researchers like [Walter et al., 2004], [Holmström et al., 2010], [Khajavi et al., 2014], [Wullms et al., 2014], [Liu et al., 2014] and [Pour and Zanoni, 2017]
investigate the impact of the 3D printing process in the spare parts supply chain. They present that reduction in the inventory holding cost, cutting down transportation cost and producing on-demand products of spare parts are the consequences of providing 3D printing centers close to the point of use.

In particular, [Walter et al., 2004] propose the effect of additive manufacturing on the aircraft spare part industry. An analysis of spare part orders reveals that most parts are only infrequently needed and a lot of infrequently sold parts have to be stored for a very long time, which generates high inventory holding and logistics costs. Thus, the paper presents the need to reduce inventory, cut lead-time, and delivery costs. The article demonstrates that for small plastic parts, the additive manufacturing technology would compete with injection molding with lower material cost.
[Holmström et al., 2010] deploy two distinct approaches to integrate the 3D printing technology into the spare parts supply chain of the aircraft industry. In the first approach, 3D printing technology is deployed in centralized distribution centers to produce infrequently needed spare parts on demand. This approach ensures the efficient usage of the 3D printing capacity but produced parts need to be shipped to the demand points, which will result in increasing the response time. On the other hand, when the response time is critical and the demand for spare parts are high, the second approach is preferred to use, which is distributing 3D printing centers at every service location. This approach will result in the diminishing of inventory holding and transportation costs and an accelerated response time.
[Wullms et al., 2014] present different criteria where a portfolio of spare parts is selected. The spare parts can be produced by using either the 3D printing technology or traditional techniques. The chosen case study is Philips Healthcare spare part system and they deploy a mathematical model for the last time buying spare part products. The results show that additive manufacturing can be used
to replace the safety stock which results in inventory cost savings in last minute product-buying decisions. [Liu et al., 2014] investigate three supply chain scenarios of the conventional, as-is supply chain, centralized AM supply chain and distributed AM supply chain. The paper concludes that the use of additive manufacturing would bring opportunities for reducing the required inventory for aircraft spare part industries.
[Khajavi et al., 2014] propose the readiness of the additive manufacturing technology in its current state and the future evolution of technology to effect on the centralized and decentralized spare part supply chain management. The study compares the total cost of each scenario, including personnel cost, material cost, spare parts transportation cost, inventory carrying cost, machine downtime cost, inventory obsolescence cost, and initial investment in the 3D printing machines. The results demonstrate that 3D printers' purchasing cost is the major obstacle to a decentralized deployment scenario. Therefore, by utilizing the future generation of 3D printing machines, this scenario could be identified with the lower total cost than the centralized production one. As a result of this paper, the higher automation, the lower 3D printers purchasing cost and the shorter production time, will result in the change in spare parts supply chain operations.

### 2.3 Assortment Planning and Customer Choice Model

A retailer's assortment is defined by the set of products carried in his store at a period of time. The aim of assortment planning is to propose an assortment for a retailer in order to maximize his profit, which is subject to various constraints, such as limited data, limited financial resources, limited capacity for holding inventory or limited shelf space. Given all the limitations, assortment planning requires a tradeoff between three elements: defining different categories of products the retailer has to carry, SKUs in each category, and amount of the inventory for each SKU [Kök et al.,

2009]. In the following paragraphs, literature on different customer choice models, Multinomial Logit model (MNL) and Nested Logit (NL) model, is explained.

Assortment planning, with the aspect of consumer choice models, has been the focus of numerous academic and industry studies. [Kök et al., 2009] present a comprehensive review of the literature on the assortment planning and practical applications. [Mussa and Rosen, 1978] consider a product line design problem, in which the utility function is a linear function of the quality of the model, and as a result, the optimal sets of products and their prices are provided.

The Multinomial Logit model (MNL) is a discrete choice model which assumes that consumers are rational utility maximizers. The model is an intuitive, frequentlyused type of consumer choice model. The MNL model was first proposed by McFadden (1980), who was later awarded the 2000 Nobel Prize in Economics.
[Anderson et al., 1992], in their book, explain the probability of customers choosing a product under MNL model and prove that optimal pricing policy for a group of products is a specific absolute markup policy. [Hanson and Martin, 1996] propose that the profit function of a retailer selling multiple substitutable products under the MNL model is not concave in prices. They assume that the demand function is deterministic and present a procedure to find a path of the prices from the global optimal of a related, but concave profit function, to the global optimal of the non-concave profit function. They indicate that finding the optimal prices may require sophisticated search techniques.
[Hopp and Xu, 2005] and [Anderson et al., 1992] demonstrate different approaches to show the optimal retailer's profit function using the MNL model. [Cachon et al., 2005] develop three models for a retail assortment problem based on the Multinomial Logit model to demonstrate consumer choice behavior, incorporating search costs. They deploy that ignoring consumer search in demand estimation can result in lower retailer's expected profit function for an assortment with less variety
compared to the optimal solution. The study compares the result with a heuristic equilibrium as well.
[Aydin and Ryan, 2000] consider a retailer's problem of pricing a product line using the MNL choice model. The pricing of the optimal product line is provided under three different strategies for retailer's aspect. In the first strategy, a retailer has a product line with fixed prices and considers the addition of a new product. In the second one, the retailer has a pre-selected set of products and has to determine the optimal prices, and finally, a retailer can select any subset of products from a set of potential ones. For all strategies, the optimal amount and the optimal prices of the products are calculated.
[Cachon and Kök, 2007] study the assortment planning problem with two multiproduct retailers. In this paper, customers can choose between two retailers and a no-purchase option. The consumer behavior follows the MNL model. The retailers' assortment decisions with centralized and decentralized disciplines are investigated and the properties of the optimal solution are determined and compared to the decentralized solution. The decentralized assortment planning is likely to result in the lower variety, higher prices, and significantly lower profits than the optimal solution. However, a centralized optimal solution is almost not implementable in practice due to the complexity of the model.

The Nested Logit (NL) model is an expanded version of the MNL model in which customers have different hierarchy of the product selection. At the upper level, a branch, or nest, consisting of multiple similar products is chosen, then at the lower level, the product selection will be within the chosen nest. The Nested Logit model is used in the modeling competition between two or more companies with multiple products [Kök et al., 2009, Anderson et al., 1992, Cachon et al., 2006]. [Wang, 2012] investigate an optimal pricing model under the MNL and NL models. Moreover, efficient computational algorithms are developed. The article calculates the Nash
equilibrium and delivered management insights through analytical and numerical results.
[Li and Huh, 2011] propose the concavity of the retailer's revenue and profit functions respective to the market share, when a retailer is selling multiple differentiated products with demand given by either the MNL or the NL model. The optimal solution of the retailer's profit function is evolved.
[Gallego and Wang, 2014] study the firms that sell multiple differentiated substitutable products and consumers' purchase behavior follows the MNL and the NL models with product-differentiated price sensitivities and general nest coefficients. The problem is to price the products to maximize the expected total profit. It proposes that the adjusted markup, which is defined as the price minus the cost, minus the reciprocal of price sensitivity, is constant for all products within a nest at optimality which results in the reduction of the problem's dimension to a single variable per nest.

### 2.4 Joint Pricing and Inventory Control of Substitutes Products

Studies on the joint pricing and inventory management on product substitution models have progressed rapidly in the recent years. One of the pioneers of studies in this area is [Petruzzi and Dada, 1999]. They examine the single product price-dependent newsvendor problem in order to find joint optimal stocking quantity and selling price. The article also reviews and develops insights into a dynamic inventory extension of the problem.
[Bassok et al., 1999] characterize the structure of the optimal policy for a single period, multiple products inventory management problem with different demand classes and downward substitution. Moreover, they assume proportional costs and revenues with a constant marginal cost of the substitution and develop a model for the profit function. They find a greedy allocation policy for the optimal solution.
[Birge et al., 1998] assume a single period model of a firm produces two products with price-dependent demands, in which the firm can find optimal pricing or capacity for one or both of the products. The demand is assumed to be uniformly distributed and the model shows the pricing and capacity decisions policies by considering three different scenarios. In the first one, the firm is a price taker for both of the products and the goal is to find optimal capacity for each product. In the second case, the firm needs to decide on the optimal amount of the capacity for one of the products and the optimal price for the other product. While in the last case, the capacity is fixed for both of the products and the problem is to find optimal prices. Moreover, by different numerical results, they are able to show that the pricing and capacity decisions are affected significantly by the system parameters.

Studies consider modeling consumer choice behavior for substitution models using the MNL and NL models. An example of deterministic consumer choice model is [Pentico, 1976] and [Pentico, 1988]. They respectively proposed one-dimensional and two-dimensional assortment planning, considering deterministic demand and using EOQ model for inventory costs. They evolved an optimal solution with an efficient dynamic programming formulation.

One class of literature on the assortment planning and inventory management topic is focused on the assortment-based substitution models. In these studies, customers make their choice from the given assortment without knowledge of the product availability and there is no substitution option in the stock-out situation. Therefore, when one product runs out of the stock, customers who prefer that product do not switch to another product and will leave without buying anything.
[Ryzin and Mahajan, 1999] deploy the assortment planning under the MNL model and inventory management under the newsboy model to represent the retailer's inventory cost, with identical prices for the assortment-based substitution case. The
optimal solution is found and insights are provided on how various factors affect the optimal level of assortment variety.
[Aydin and Porteus, 2008] investigate optimal prices and inventory levels of multiple products in a given assortment. The article assumes a single firm in a single period price-dependent newsvendor model. The consumer's demand follows the MNL model. Moreover, they consider the stochastic demand for a product is a function of the prices of all products, unmet demands become lost sales and leftover inventory has no value and present that although the profit function is non-concave, the problem is well behaved in the sense that there is a unique vector of prices and inventory levels that satisfy the first-order conditions, which is the optimal solution of the problem.
[Maddah and Bish, 2007] consider a model of joint pricing, inventory and variety decisions to maximize retailer's expected profit function under the MNL consumer choice model and newsvendor inventory setting. They derive the structure of the optimal assortment for some special cases. They drive structural properties of the optimal prices and propose a heuristic solution procedure, which is shown to be effective through a numerical study.

The other class of literature is focused on the stock-out-based substitution models, in which customer selection choices are based on the products available in the stock at the time of their visit to the store. [Mahajan and van Ryzin, 2001] evolve and analyze a single period, stochastic inventory model in which a sequence of heterogeneous customers dynamically substitute among products in the stock-out situation. They show that, under general assumptions, total sale of each product is concave in their own inventory levels and the marginal value of an additional unit of the given product is decreasing in the inventory levels of other products. However, the expected profit function is not concave, thus the authors propose a stochastic gradient for the assortment stock-out-based substitution case to determine the optimal assortment and inventory levels to maximize expected profit function.

By comparing the algorithm to the heuristic one, it is proven that substitution effects can have a significant impact on an assortment's profits.
[Honhon et al., 2010] propose the problem of determining the optimal assortment and inventory levels to maximize the expected profit in a single-period problem considering dynamic substitution. Moreover, a heuristic algorithm for the general case is provided.
[Kök and Fisher, 2007] study a stock-out-based assortment planning and develop an algorithmic process for retailers to determine the best assortment of different stores. They present a procedure for estimating the parameters of demand and substitution behavior for products, including the products that have not been carried in stores previously, and they deploy an iterative optimization heuristic for solving the assortment planning problem.
[Chiang and Monahan, 2005] and [Takahashi et al., 2011] present a two-echelon dual-channel supply chain model in which stocks are kept in both the manufacturer warehouse as well as the retail store. Customers can place their orders through either online or traditional retail store channels. [Chiang and Monahan, 2005] propose a cost-structure model of holding cost and lost sale cost, while [Takahashi et al., 2011] add setup cost and delivery cost to the previous cost factors. The optimal inventory level is calculated using Markov analysis.
[ Xu et al., 2016] and [Yu et al., 2017] investigate the optimal inventory, pricing and substitution policies of a two-product inventory system. [ Xu et al., 2016] assume that supplier has the option of offering substitution at each price level or discount level and the customer may or may not accept the offer, with the acceptance probability decreasing in the substitution price. They consider an N-period selling season, with a one-time replenishment at the beginning of each season. The optimal dynamic substitution-pricing policy and replenishment quantities for each period are calculated, using the stochastic dynamic programming approach. While, [Yu et al.,

2017] consider a Markov decision process and characterize the structure of the optimal control policy and price. They demonstrate that the optimal base-stock level for each product depends on the inventory level of the other product and it features a monotonic property. Moreover, the optimal prices can be either decreasing or increasing in the inventory levels, depending on the forms of demand functions.

### 2.5 Queuing Models

Historically, the subject of the queuing system has been developed largely in different studies of engineering, operations research, and computer science context. In the following paragraphs, the literature of the queuing theory considering cost-benefit model and retrial queuing models are discussed.

A queuing model is considered in the systems in which arriving customers have the choice of either joining the waiting positions, considering the acquiring benefits of getting the service, or declining to join the queue. The decision of each customer is following the cost-benefit model of the customer. [Naor, 1969] presents a queuing model considering a cost-benefit model. Self-optimization of each customer, overall optimization, and the imposition of tolls on newly arriving customers are investigated and shown. The study concludes that these strategies lead to the social optimality attainment. [Adler and Naor, 1969] study a queuing model with customer revenue function. The paper presents a self-optimization and social optimization model by considering joining and balking strategy.

Recently, there have been significant contributions on retrial queuing systems. In these systems, arriving customers, who find all the servers busy and waiting lines occupied are constrained to either leave the system or join the retrial orbit to try again for the service after a random period of time. Retrial queues have been widely used in different research areas.
[Mokaddis et al., 2007] consider a single server retrial queue, where the server is subjected to starting failure and the system has a single vacation. New customers who find the server busy or down would repeat the request after a random period of time. [Choi et al., 1998] develop $\mathrm{M} / \mathrm{M} / \mathrm{c}$ retrial queues with geometric loss and feedback. They proposed the joint generating function of the number of busy servers and the queue length.
[Kumar et al., 2002] evolve a retrial queue with the Bernoulli feedback and the server is considered to have a failure rate. The retrial time is assumed to follow an arbitrary distribution and the customers have FCFS discipline. As a result of the paper, the necessary and the sufficient conditions for the stability of the system and various performance measures are calculated.
[Artalejo and Falin, 2002] describe the retrial queuing model and compared the result with standard queues with waiting for positions and queues with losses. The paper provided a survey of main results for both single server $M / G / 1$ type and multiserver $M / M / c$ type retrial queues and distinguished similarities and dissimilarities between the retrial queues and the standard counterparts.
[Choi and Chang, 1999] propose a retrial queue with two types of calls, retrial group with finite capacity and geometric loss. They resulted in the derivation of the joint distribution of two queue lengths, the waiting time distribution and the distribution of the busy period. [Falin, 2010] develops a batch arrival retrial queue. The study calculates and discusses the necessary and sufficient condition for joint distribution of the number of customers in the queue and the number of customers in the retrial orbit in the steady state. [Wang and Zhao, 2007] propose a discrete-time retrial queue, considering all the arriving customers require the first essential service, while only some of them ask for the second optional service and the server obtained a failure rate. As a result, the stationary distribution and performance measures of the system in the steady state are presented.

## CHAPTER 3

## RETAILER'S PRICING AND CAPACITY OPTIMIZATION

### 3.1 Introduction

A retailer's assortment is defined by the set of products carried in the store at a period of time in order to maximize the expected profit function respect to the various constraints, such as having the limitation in production capacity, holding inventory capacity or financial resources. Moreover, a retailer tends to modify the assortment based on different situations, for instance entering new products to the market or changing in consumer selection behavior.

As 3D printing becomes agile and better able to react to customer demands, one important question for the retailers is whether they should provide 3D printing services in their brick and mortar store in addition to the traditional off-the-shelf product? If so, what should be the retailer's pricing scheme to achieve a higher profit? Moreover, consumers often know what kind of products they want. Each product has a specific utility value to the customers. Therefore, the expected profit function of a retailer should consider the direct costs and revenues of the product, as well as the probability of customers choosing every single one of the products.

In this chapter, two different consumer selection models are proposed to anticipate customer selection behavior. The Cut-off model and the multinomial logit model (MNL) model.

In the Cut-off model, the maximum capacity of the 3D printing system or equivalently the cut-off number of consumers who are willing to serve with 3D products are calculated. The cut-off number of customers depends on the prices of the products versus the customer's value of products, considering the waiting time of producing 3D products. It's assumed that the customers who join the system are
willing to get 3D products unless the number of customers in the system exceeds the cut-off number of consumers, in which they prefer to buy off-the-shelf products.

The multinomial logit model (MNL) is a discrete choice model which assumes that consumers are rational utility maximizers. The model is intuitive, frequently used and successfully applied for any type of consumer choice model. The MNL model first proposed by [McFadden, 1980], who was later awarded the 2000 Nobel Prize in Economics. Using the model for substitutable products has been received lots of attention by researchers and it has motivated variant analytical research and numerical validations in a large range of applications [McFadden, 1980], [Anderson et al., 1992], [Aydin and Ryan, 2000], [Hopp and Xu, 2005], [Cachon and Kök, 2007], and [Gallego and Wang, 2014].

In the second model, it is assumed that customer selection behavior follows the MNL model. The retailer offers two types of products: 3D and off-the-shelf products and consumers weight the products based on prices of the products interact with the customer's value of each item and 3D production time. Thus, in order to maximize retailer's profit, pricing must be considered simultaneously with the production limitation of the products.

In this chapter, we answer the question of the retailer's pricing scheme by examining retailers' optimal joint decisions on the pricing scheme and 3D printer's capacity, while considering consumers' heterogeneous preferences for self-designed, 3D printed product versus the off-the-shelf product. We use a multi-server queue to capture customers' product selection process and its impact on the retailer's expected profit. Moreover, the 3D printer failure rate is considered, so if the customer does not satisfy with the 3D printing product, s/he can receive another 3D product immediately.

### 3.2 Consumer Purchase Behavior Follows The Cut-off Model

As 3D printing technology becomes agile and better able to react customer demands, retailers decide to provide 3D printing services in their brick and mortar stores in addition to the traditional off-the-shelf product, with the goal of attracting more market share to maximize their profits.

In this section, it is assumed consumer purchase behavior follows the Cut-off model. The arrival rate of customers to the system follows a Poisson distribution with the rate of $\lambda$. It's assumed that the customers who join the system are willing to get the 3D products unless the number of customers in the system exceeds the cut-off number of consumers. In that case, they prefer to buy off-the-shelf products and the cut-off number of customers depends on the prices of the products versus the customer's value of products, considering the waiting time of producing the 3D product.

It is assumed that customers have high value for their custom designed 3D products compare to off-the-shelf ones. On the other hand, producing 3D printing products is a timely manufacturing so the customers need to be in a queue. In this model, the system behaves like a queuing system with multi-server and limited capacity. It is assumed that 3D service time follows an Exponential distribution with the rate of $\mu$, which is independent of arrival rate. The 3D printers have a determined failure rate of $1-p$. Therefore, if the customer does not satisfy with the 3D printing product, s/he can receive another 3D product immediately.

The objective function is to calculate retailer's joint optimal decisions on the pricing scheme of the 3 D product and the total capacity of 3 D printers in order to maximize retailer's expected profit function. In Table 3.1, the model parameters and decision variables are defined. Figure 3.2 demonstrates the queue system of the model. The capacity of the system in this truncated queue model of $M / M / C / K$ is equal to the cut-off number of customers in the system (K), which is calculated in

Table 3.1 Summary of Notation
Parameters:
$p_{s}$ : Selling price of off-the-shelf product/unit
$v_{s}$ : Customer value of off-the-shelf product/unit
$c_{s}$ : Procurement cost of off-the-shelf product/unit
$v_{d}$ : Customer value of 3D product/unit
$c_{d}: 3 \mathrm{D}$ production cost/unit
$c_{w}$ : Waiting cost/unit
$c_{s e}$ : Capacity cost/3D printer
$p$ : Success probability of producing 3D product/unit
$\lambda: \quad$ Customer arrival rate/hr
$\mu: \quad 3 \mathrm{D}$ printer service rate/hr
$T$ : Number of working hours/day
Decision Variables:
$p_{d}$ : Selling price of 3D product/unit
$C$ : Number of 3D printers

Equation 3.7. The probability of being the $n_{t h}$ customer in the system is calculated as Equation 3.1. In which, $\pi_{0}$ is the probability of having zero customers in the system.


Figure $3.1 \mathrm{M} / \mathrm{M} / \mathrm{C} / \mathrm{K}$ queue system in the Cut-off model.

$$
\begin{align*}
& \pi_{0}=\left[\sum_{n=0}^{C-1} \frac{1}{n!} \cdot\left(\frac{\lambda}{\mu}\right)^{n}+\frac{\left(\frac{\lambda}{\mu}\right)^{C} \cdot\left(1-\left(\frac{\lambda}{c \mu}\right)^{K-C+1}\right)}{c!\cdot\left(1-\frac{\lambda}{c \mu}\right)}\right]^{-1} \\
& \pi_{n}= \begin{cases}\frac{1}{n!} \cdot\left(\frac{\lambda}{\mu}\right)^{n} \cdot \pi_{0} \quad \text { if } n<C \\
\frac{1}{C!\cdot C^{n-c}} \cdot\left(\frac{\lambda}{\mu}\right)^{n} \cdot \pi_{0} \quad \text { if } C \leq n \leq K\end{cases} \tag{3.1}
\end{align*}
$$

The conditional waiting time in the queuing system of $M / M / C$ is calculated as following and the proof can be found in Appendix A.

$$
W_{q}(t \mid n)= \begin{cases}\frac{1}{\mu \cdot p} & \text { if } n \leq C  \tag{3.2}\\ \frac{n}{C \cdot \mu \cdot p} & \text { if } C<n\end{cases}
$$

The customer utility in buying one unit of the off-the-shelf product is calculated as:

$$
\begin{equation*}
\Psi_{s}=v_{s}-p_{s} \tag{3.3}
\end{equation*}
$$

The customer expected utility in buying one unit of the 3 D printing product is calculated as:

$$
\Psi_{d}=\left\{\begin{array}{lc}
v_{d}-p_{d}-c_{w} \cdot \frac{1}{\mu \cdot p} & \text { if } n \leq C  \tag{3.4}\\
v_{d}-p_{d}-c_{w} \cdot \frac{n}{C \cdot \mu \cdot p} & \text { if } n>C
\end{array}\right.
$$

Retailer profit in selling one unit of the off-the-shelf product is calculated as:

$$
\begin{equation*}
\Phi_{s}=p_{s}-c_{s} \tag{3.5}
\end{equation*}
$$

Retailer expected profit in selling one unit of the 3D printing product is calculated as:

$$
\begin{equation*}
\Phi_{d}=p_{d}-\frac{c_{d}}{p}-c_{s e} \tag{3.6}
\end{equation*}
$$

It's assumed that the customers who have entered the retail store are willing to receive the 3D service unless the customer utility of the 3D product is less than customer utility of an off-the-shelf product. In that case, they prefer to buy the off-the-shelf product. Therefore, the cut-off number of customers in the system, $K$, presents if a number of customers in the system including the one who is willing to join, is less than $K$, then the customer will join the system. Otherwise, he will buy off-the-shelf item.

The cut-off number of customers depend on the prices of the products versus the customer value of products, considering the waiting time of producing the 3D item and it follows Equation 3.7.

$$
\begin{equation*}
K=\left\lfloor\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(\left(v_{d}-p_{d}\right)-\left(v_{s}-p_{s}\right)\right)\right\rfloor \tag{3.7}
\end{equation*}
$$

The consumer expected utility is calculated as the Equation 3.8. The first two terms are the customer expected utility of buying 3D products, while the last term is the expected utility of purchasing the off-the-shelf products.

$$
\begin{align*}
\Psi= & \sum_{n=0}^{C-1} \pi_{n} \cdot\left(v_{d}-p_{d}-c_{w} \cdot \frac{1}{\mu \cdot p}\right)+\sum_{n=C-1}^{K-1} \pi_{n} \cdot\left(v_{d}-p_{d}-c_{w} \cdot \frac{n}{C \cdot \mu \cdot p}\right)  \tag{3.8}\\
& +\pi_{K} \cdot\left(v_{s}-p_{s}\right)
\end{align*}
$$

The total retailer expected profit is calculated as the Equation 3.9. The first term is the expected profit of selling the 3D products. The second term is the expected profit of selling off-the-shelf products and the last term is the 3D printers capacity cost.

$$
\begin{align*}
\Phi & =\sum_{n=0}^{K-1} \lambda \cdot T \cdot \pi_{n} \cdot\left(p_{d}-\frac{c_{d}}{p}\right)+\lambda \cdot T \cdot \pi_{K} \cdot\left(p_{s}-c_{s}\right)-c_{s e} \cdot C  \tag{3.9}\\
& =\lambda \cdot T \cdot\left(1-\pi_{K}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)+\lambda \cdot T \cdot \pi_{K} \cdot\left(p_{s}-c_{s}\right)-c_{s e} \cdot C
\end{align*}
$$

In which $\pi_{n}(n=1, \ldots, K)$ is the probability of being the $n_{t h}$ customer in the retail store.

### 3.2.1 Analytical Solution

In this section, we present the general form of the optimal solution under different scenarios. We start from the simplest case where there is one 3D printing service (M/M/1/K), and then the extension for the number of $C 3 \mathrm{D}$ printers will be provided (M/M/C/K).

The optimal 3D product price that maximizes the retailer's profit function needs to satisfy $\Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}+\epsilon}$ and $\Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}-\epsilon}$. In which $\epsilon$ is the 3 D price interval that changes the cut-off number of customers in one unit. For the case that retailer provides $C$ number of 3 D printers, $\epsilon=\frac{c_{w}}{C \cdot \mu \cdot p}$, which is calculated by solving the system of equations listed in Equation 3.10.

$$
\begin{align*}
& K^{*}=\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}-v_{s}+p_{s}\right) \\
& K^{*}-1=\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}-\epsilon-v_{s}+p_{s}\right)  \tag{3.10}\\
& K^{*}+1=\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}+\epsilon-v_{s}+p_{s}\right)
\end{align*}
$$

Proposition 1. The optimal 3D product price that maximizes the retailer expected profit function, while the retailer provides just one 3D printer, satisfies:

$$
\begin{aligned}
& \frac{c_{d}}{p}-c_{s}+p_{s}+\frac{c_{w} \cdot\left(1-\rho^{\left(K^{*}+1\right)}\right) \cdot\left(1-\rho^{\left(K^{*}-1\right)}\right)}{\mu \cdot p \cdot(1-\rho)^{2} \cdot \rho^{\left(K^{*}-1\right)}}<p_{d}^{*}<\frac{c_{d}}{p}-c_{s}+p_{s}+\frac{c_{w} \cdot\left(1-\rho^{\left.\left(K^{*}+1\right)\right)^{2}}\right.}{\mu \cdot p \cdot(1-\rho)^{2} \cdot \rho^{K^{*}}} \\
& p_{d}^{*} \in\left\{v_{d}-v_{s}+p_{s}, v_{d}-v_{s}+p_{s}-\epsilon, v_{d}-v_{s}+p_{s}-2 \epsilon, v_{d}-v_{s}+p_{s}-3 \epsilon, \ldots\right\}^{+}
\end{aligned}
$$

in which $\rho=\frac{\lambda}{\mu}$ and $\epsilon=\frac{c_{w}}{\mu \cdot p}$.
Corollary 1.1. At the optimal 3D product price, the optimal cut-off number of customers satisfies:

$$
\frac{\mu p}{c_{w}}\left(v_{d}-v_{s}+c_{s}-\frac{c_{d}}{p}\right)-\frac{\left(1-\rho^{K^{*}+1}\right)^{2}}{(1-\rho)^{2} \cdot \rho^{K^{*}}}<K^{*}<\frac{\mu p}{c_{w}}\left(v_{d}-v_{s}+c_{s}-\frac{c_{d}}{p}\right)-\frac{\left(1-\rho^{K^{*}+1}\right)\left(1-\rho^{K^{*}-1}\right)}{(1-\rho)^{2} \cdot \rho^{K^{*}-1}}
$$

Corollary 1.2. For the general case, When retailer provides the number of C 3D printers, the optimal 3D product price that maximizes the retailer expected profit
function satisfies:

$$
\begin{aligned}
& \frac{c_{d}}{p}-c_{s}+p_{s}-\frac{c_{w} \cdot\left(1-\pi_{\left(K^{*}-1\right)}\right)}{C \cdot \mu \cdot p \cdot\left(\pi_{\left.K^{*}-\pi_{\left(K^{*}-1\right)}\right)}\right.}<p_{d}^{*}<\frac{c_{d}}{p}-c_{s}+p_{s}-\frac{c_{w} \cdot\left(1-\pi_{\left(K^{*}+1\right)}\right)}{C \cdot \mu \cdot p \cdot\left(\pi_{\left(K^{*}+1\right)}-\pi_{K^{*}}\right)} \\
& p_{d}^{*} \in\left\{v_{d}-v_{s}+p_{s}, v_{d}-v_{s}+p_{s}-\epsilon, v_{d}-v_{s}+p_{s}-2 \epsilon, v_{d}-v_{s}+p_{s}-3 \epsilon, \ldots\right\}^{+}
\end{aligned}
$$

in which $\epsilon=\frac{c_{w}}{C \cdot \mu \cdot p}$.
Corollary 1.3. In the general case with $C$ number of $3 D$ printers, at the optimal $3 D$ product price, the optimal cut-off number of customers satisfies:

$$
\frac{1-\pi_{K^{*}+1}}{\pi_{K^{*}+1}-\pi_{K^{*}}}+\frac{C \cdot \mu \cdot p}{c_{w}}\left(v_{d}-v_{s}+c_{s}-\frac{c_{d}}{p}\right)<K^{*}<\frac{1-\pi_{K^{*}-1}}{\pi_{K^{*}}-\pi_{K^{*}-1}}+\frac{C \cdot \mu \cdot p}{c_{w}}\left(v_{d}-v_{s}+c_{s}-\frac{c_{d}}{p}\right)
$$

The proof of the proposition and the subsequent corollaries can be found in Appendix B.

### 3.2.2 Computational Solutions

In this section, we present numerical solutions as well as sensitivity analysis of the Cut-off model on the governing parameters.

In the first place, the 3 D product price, $p_{d}$, is changed between the range of $\$ 11$ to $\$ 23$ and the optimal retailer's expected profit, optimal number of 3D printers and optimal cut-off number of customers are calculated at each price. The following parameters are used: $v_{s}=\$ 20, p_{s}=\$ 15, c_{s}=\$ 8, v_{d}=\$ 28, c_{d}=\$ 3, c_{w}=\$ 3, \mu=21$, $\lambda=20, p=0.8, T=8 h r, c_{s e}=\$ 10$.

As shown in Table 3.2 and Figure 3.2, by increasing 3D product price, clearly the retailer's profit and the optimal number of 3D printer will increase, meaning that retailer prefers to sell more of 3D printing products, so he will provide more 3D printers. On the other hand, by increasing the 3 D product price, the optimal cut-off number of customers, $K$, will decrease as fewer customers incline to buy 3D items.

The optimal 3D product price is at $p_{d}^{*}=\$ 22$, with optimal two 3D printers, $\left(C^{*}=2\right)$, and the retailer's optimal expected profit is $\Phi^{*}=\$ 2899.6$.

At $p_{d}=\$ 23$, the customer's utility margin of off-the-shelf products, $\left(v_{s}-p_{s}\right)$, becomes equal to the customer's utility margin of 3 D printing products, $\left(v_{d}-p_{d}\right)$, and for greater than this price the customer's utility margin of off-the-shelf becomes greater than that of 3 D printing products, therefore, the optimal Cut-off number of customers (K), is equal to zero and customers prefer to buy off-the-shelf products, so the optimal number of 3D printers become zero and retailer's profit decreases to $\$ 1120$.

Table 3.2 The Effect of $p_{d}$ on Retailer's Expected Profit and Optimal 3D Capacity

| $p_{d}$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Phi^{*}$ | 1149.9 | 1309.5 | 1468.8 | 1627.6 | 1785.7 | 1943 | 2100 | 2260 | 2420 | 2580 | 2740 | 2899.6 | 1120 |
| $C^{*}$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 0 |
| $K^{*}$ | 67 | 61 | 56 | 50 | 44 | 39 | 67 | 56 | 44 | 33 | 22 | 11 | 0 |



Figure 3.2 The effect of $p_{d}$ on the retailer in the Cut-off model.

Considering the same parameters, put $p_{d}$ equal to $\$ 15$ and varying $p_{s}$ from $\$ 8$ to $\$ 20$, provide the results of Table 3.3 and Figure 3.3. By increasing $p_{s}$, the optimal retailer's profit will increase. The optimal number of 3D printer decreases as retailer prefers to provide more of the off-the-shelf products for the customers. Moreover, by increasing $p_{s}$ as the customer's utility margin of 3D printing products, $\left(v_{d}-p_{d}\right)$,
becomes higher than the customer's utility margin of off-the-shelf products, $\left(v_{s}-p_{s}\right)$, the optimal cut-off number of customers increases as well. At the point of $p_{s}=\$ 14$, $K^{*}$, drops to 39 , as the optimal number of 3 D printers drops from 2 to 1 . At point $p_{s}$ from $\$ 15$ to $\$ 19$, Cut-off number of customers increases by increasing $p_{s}$, means that by increasing $p_{s}$, customers prefer to buy from 3D products.

For $p_{s}$ equal or greater than $\$ 20$, the retailer's utility margin of off-the-shelf products, $\left(p_{s}-c_{s}\right)$, becomes equal and higher than the retailer's utility margin of 3D printing products, $\left(p_{d}-c_{d}\right)$, therefore the retailer tends to provide just off-the-shelf products, and $C^{*}$ and $K^{*}$ decrease to zero. The retailer's optimal expected profit, $\Phi^{*}$, will increase as $p_{s}$ increases.

Table 3.3 The Effect of $p_{s}$ on Retailer's Expected Profit and Optimal 3D Capacity

| $p_{s}$ | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Phi^{*}$ | 1780 | 1780 | 1780 | 1784.2 | 1787.6 | 1789.7 | 1790.8 | 1791.3 | 1791.5 | 1791.5 | 1791.4 | 1920 |
| $C^{*}$ | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| $K^{*}$ | 22 | 33 | 44 | 56 | 67 | 39 | 44 | 50 | 56 | 61 | 67 | 0 |



Figure 3.3 The effect of $p_{s}$ on retailer in the Cut-off model.

Considering parameters, $v_{s}=20, p_{s}=15, c_{s}=8, v_{d}=28, p_{d}=15, c_{d}=3$, $c_{w}=3, \mu=2, p=0.8, T=8, c_{s e}=10$ and changing $\lambda$ between 10 to 25 , we observed
that optimal retailer's profit and optimal Cut-off number of customers increase. The results are shown in Table 3.4 and Figure 3.4.

Table 3.4 Sensitivity Analysis of $\lambda$ in the Cut-off Model

| $\lambda$ | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Phi^{*}$ | 838.9 | 924.6 | 1008.8 | 1094.2 | 1178.8 | 1264.4 | 1348.7 | 1434.1 | 1518.6 | 1603.9 | 1688.5 | 1773.7 |
| $C^{*}$ | 6 | 6 | 7 | 7 | 8 | 8 | 9 | 9 | 10 | 10 | 11 | 11 |
| $K^{*}$ | 25 | 25 | 29 | 29 | 34 | 34 | 38 | 38 | 42 | 42 | 46 | 46 |



Figure 3.4 Sensitivity Analysis of $\lambda$ in the Cut-off model.

### 3.2.3 Stochastic Customer Valuation for 3D Product

It is assumed that consumers heterogeneous preferences for 3D products are classified in two different groups, high preference customers,$v_{d h}$, with the probability of $\beta_{h}$ and low preference customers, $v_{d l}$, with the probability of $\beta_{l}$.

$$
\left\{\begin{array}{lll}
v_{d h}=\alpha_{h} \cdot v_{s} & \text { with probability of } & \beta_{h}  \tag{3.11}\\
v_{d l}=\alpha_{l} \cdot v_{s} & \text { with probability of } & \beta_{l}
\end{array}\right.
$$

In which $\beta_{h}+\beta_{l}=1$. As shown in the Equation 3.11, customer preference in choosing a 3 D product is a function of customer preference in choosing the off-the-shelf product.

The aim of the model is to obtain the retailer's joint optimal decisions on the 3D product price and the total number of 3 D printers.

The cut-off number of customers in the system follows the Equation 3.12. The $K_{h}$ presents the cut-off number of customers with high 3D product preference in the system, while the $K_{l}$ presents the cut-off number of customers with low 3D item preference in the system.

$$
\begin{align*}
& K_{h}=\left\lfloor\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(\left(v_{d h}-p_{d}\right)-\left(v_{s}-p_{s}\right)\right)\right\rfloor=\left\lfloor\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(\left(\alpha_{h}-1\right) \cdot v_{s}-p_{d}+p_{s}\right)\right\rfloor  \tag{3.12}\\
& K_{l}=\left\lfloor\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(\left(v_{d l}-p_{d}\right)-\left(v_{s}-p_{s}\right)\right)\right\rfloor=\left\lfloor\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(\left(\alpha_{l}-1\right) \cdot v_{s}-p_{d}+p_{s}\right)\right\rfloor
\end{align*}
$$

The retailer's expected profit function is provided in the Equation 3.13.

$$
\begin{align*}
\Phi= & \lambda \cdot T \cdot\left[\sum_{n=0}^{K_{l}-1} \pi_{n} \cdot\left(p_{d}-\frac{c_{d}}{p}\right)+\sum_{n=K_{l}}^{K_{h}-1} \pi_{n} \cdot\left(\beta_{h} \cdot\left(p_{d}-\frac{c_{d}}{p}\right)+\beta_{l} \cdot\left(p_{s}-c_{s}\right)\right)\right. \\
& \left.+\pi_{K_{h}} \cdot\left(p_{s}-c_{s}\right)\right]-c_{s e} \cdot C \tag{3.13}
\end{align*}
$$

In which $\pi_{n}, n=1, \ldots, K_{h}$, is the probability of being the $n_{t h}$ customer in the $M / M / C / K_{h}$ queue system which is shown in Figure 3.5 and Equation 3.14.


Figure 3.5 $\mathrm{M} / \mathrm{M} / \mathrm{C} / K_{h}$ queue system of the Cut-off model with stochastic 3D products customer value.

$$
\begin{align*}
& \pi_{0}=\left[\sum_{n=0}^{C} \frac{1}{n!} \cdot\left(\frac{\lambda}{\mu}\right)^{n}+\sum_{n=C+1}^{K_{l}} \frac{1}{C!\cdot C^{n-c}} \cdot\left(\frac{\lambda}{\mu}\right)^{n}+\sum_{n=K_{l}+1}^{K_{h}} \frac{\beta_{h}\left(n-K_{l}\right)}{C!\cdot C^{n-c}} \cdot\left(\frac{\lambda}{\mu}\right)^{n}\right]^{-1} \\
& \pi_{n}= \begin{cases}\frac{1}{n!} \cdot\left(\frac{\lambda}{\mu}\right)^{n} \cdot \pi_{0} \quad \text { if } n<C \\
\frac{1}{C!\cdot C^{n-c}} \cdot\left(\frac{\lambda}{\mu}\right)^{n} \cdot \pi_{0} \quad \text { if } C \leq n<K_{l} \\
\frac{\beta_{h}\left(n-K_{l}\right)}{C!\cdot C^{n-c}} \cdot\left(\frac{\lambda}{\mu}\right)^{n} \cdot \pi_{0} \quad \text { if } K_{l} \leq n<K_{h}\end{cases} \tag{3.14}
\end{align*}
$$

### 3.2.4 Computational Solutions

To generalize our findings in this section, numerical solutions of the model are presented. The 3D product price, $p_{d}$, is changed between the range of $\$ 11$ to $\$ 30$ and the optimal retailer's expected profit, the optimal number of 3D printers and the optimal cut-off number of customers are calculated. The following parameters are used: $v_{s}=\$ 20, p_{s}=\$ 15, c_{s}=\$ 8, c_{d}=\$ 3, \alpha_{h}=1.5, \alpha_{l}=1.3, \beta_{h}=0.5, \beta_{l}=0.5$, $c_{w}=\$ 3, \mu=21, \lambda=20, p=0.8, T=8 h r, c_{s e}=\$ 10$.

As shown in Table 3.5 and Figure 3.6, from $p d=\$ 11$ to $p d=\$ 20$, by increasing 3D product price the Optimal retailer's expected profit and the optimal number of 3D printers increase.

As shown in Figure 3.7, in the range of $p_{d}=11$ to 20 , both types of low preference and high preference customers are willing to have 3D products, after this point, the cut-off number of low preference customers becomes zero and they will buy just off-the-shelf products.

As retailer's profit margin of off-the-shelf products, $\left(p_{s}-c_{s}\right)$, is lower than of 3D products, the retailer's expected profit will decrease at the point of $p_{d}=\$ 21$ but after that, by increasing $p_{d}$, retailer's optimal profit increases. At the point of $p_{d}=\$ 25$, the cut-off number of both types of customers are zero and all the customers will be served with off-the-shelf products. The optimal retailer's profit is equal to $\$ 1120$.

Table 3.5 The Effect of $p_{d}$ on Retailer's Expected Profit and Optimal 3D Capacity in the MNL Model With Stochastic Customer Utility

| $p_{d}$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Phi^{*}$ | 1150 | 1309 | 1468 | 1626 | 1783 | 1940 | 2100 | 2260 | 2420 | 2580 |
| $C^{*}$ | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 |
| $K_{l}^{*}$ | 56 | 50 | 44 | 39 | 33 | 44 | 39 | 33 | 22 | 11 |
| $K_{h}^{*}$ | 78 | 72 | 67 | 61 | 56 | 100 | 89 | 78 | 67 | 56 |
| $p_{d}$ | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| $\Phi^{*}$ | 1930 | 2010 | 2090 | 2160 | 1120 | 1120 | 1120 | 1120 | 1120 | 1120 |
| $C^{*}$ | 1 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| $K_{l}^{*}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $K_{h}^{*}$ | 22 | 16 | 11 | 11 | 0 | 0 | 0 | 0 | 0 | 0 |



Figure 3.6 The effect of changing $p_{d}$ on the retailer in the Cut-off model with stochastic customer utility.

### 3.2.5 Managerial Insights

The developed model considers the impact of the flexible terms, including $\alpha_{h}, \alpha_{l}$ and $\beta_{h}$, on the retailer's expected profit, optimal 3D product price, optimal number of


Figure 3.7 The effect of $p_{d}$ on the cut-off number of customers with high and low preferences.

3D printers and market shares of high preference and low preference customers. The parameters are as defined in Section 3.2.4, For $\alpha_{l}$ sensitivity analysis, $\alpha_{h}$ is set equal to 1.7 and for $\alpha_{h}$ sensitivity analysis, $\alpha_{l}$ is set equal to 1.4. The implications of the results from our analysis are summarized as the following.

As Table 3.6 presents, the retailer's expected profit, as well as optimal 3D product price, will decrease as $\beta_{h}$ decreases. The optimal number of 3D printers remains the same. At the Cut-off point of $\beta_{h}=0.698$, there are two optimal results of $p_{d}^{*}=\$ 24$ and $p_{d}^{*}=\$ 21$ with retailer's expected profit of $\$ 2580$. Before this cut-off, low reference customers tend to get served by off-the-shelf products and for the $\beta_{h}$ greater than the cut-off value, two types of customers incline to have 3D products.

As Table 3.7 shows, by increasing $\alpha_{h}$, high preference customers are more eager to get 3D products and their Cut-off number of customers increases. The retailer's expected profit, as well as optimal 3D product price, will decrease. Till $\alpha_{h}=1.9$ both types of customers incline to buy 3D products. By increasing $\alpha_{h}$, high preference customers are more willing to have 3D product so optimal 3D product price increases as well as retailer's profit while low reference customers tend to get served by off-theshelf products.

Table 3.6 Sensitivity Analysis of $\beta_{h}$ in The Cut-off Model With Stochastic Customer Utility

| $\beta_{h}$ | $p_{d}^{*}$ | $C^{*}$ | $\Phi^{*}$ | $K_{l}^{*}$ | $K_{h}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.9 | 24 | 2 | 3008 | 0 | 11 |
| 0.7 | 24 | 2 | 2584 | 0 | 11 |
| 0.698 | 24 | 2 | 2580 | 0 | 11 |
|  | 20 | 2 | 2580 | 11 | 56 |
| 0.5 | 20 | 2 | 2580 | 11 | 56 |
| 0.3 | 20 | 2 | 2580 | 11 | 56 |
| 0.1 | 20 | 2 | 2580 | 11 | 56 |

Table 3.7 Sensitivity Analysis of $\alpha_{h}$ in the Cut-off Model With Stochastic Customer Utility

| $\alpha_{h}$ | $p_{d}^{*}$ | $C^{*}$ | $\Phi^{*}$ | $K_{l}^{*}$ | $K_{h}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.5 | 22 | 2 | 2900 | 11 | 33 |
| 1.7 | 22 | 2 | 2900 | 11 | 78 |
| 1.9 | 22 | 2 | 2900 | 11 | 123 |
| 2 | 34 | 2 | 2960 | 0 | 11 |
| 2.5 | 44 | 2 | 3760 | 0 | 11 |

As Table 3.8 provides, retailer's expected profit will decrease as $\alpha_{l}$ decreases. For $\alpha_{l}$ greater than 1.2, both types of customers incline to buy 3D products. For $\alpha_{l}$ less than 1.2, low preference customers tend to get served by off-the-shelf products.

Table 3.8 Sensitivity Analysis of $\alpha_{l}$ in the Cut-off Model With Stochastic Customer Utility

| $\alpha_{l}$ | $p_{d}^{*}$ | $C^{*}$ | $\Phi^{*}$ | $K_{l}^{*}$ | $K_{h}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.4 | 22 | 2 | 2900 | 11 | 78 |
| 1.3 | 20 | 2 | 2580 | 11 | 100 |
| 1.2 | 28 | 2 | 2480 | 0 | 11 |
| 1 | 28 | 2 | 2480 | 0 | 11 |
| 0.5 | 28 | 2 | 2480 | 0 | 11 |

### 3.3 Consumer Purchase Behavior Follows The Multinomial Logit Model

In this model, it's assumed that the retail store is capable of providing two types of products: Off-the-shelf and 3D printing products. The arrival rate of customers to the system follows a Poisson distribution with the rate of $\lambda$. Consumers who enter the retail store, have heterogeneous preferences for 3D printed versus off-the-shelf products. The probability of customer choosing each product is determined using the MNL model. This probability depends on the prices of the products versus the customer value of products, considering the waiting time of producing the 3D product. Moreover, because producing 3D printing products is a timely process, consumers join a queue and system behave like a multi-server queuing system. It is assumed that 3D service time follows an Exponential distribution with the rate of $\mu$, which is independent of arrival rate. The 3D printers have a determined failure rate of $1-p$. Therefore, if a customer does not satisfy with the 3D product, $\mathrm{s} /$ he can receive another product immediately. Assuming that the system has $C$ 3D printers and customer enter to the 3D queue with the arrival rate of $q \lambda$, the queuing system of the model is demonstrated in Figure 3.3. The customer probability of choosing 3D products, $q$, is described in Equation 3.20.


Figure $3.8 \mathrm{M} / \mathrm{M} / \mathrm{C} / \infty$ queue system in the MNL model.

$$
\begin{align*}
& \pi_{0}=\left[\sum_{n=0}^{C-1} \frac{1}{n!} \cdot\left(\frac{q \lambda}{\mu}\right)^{n}+\sum_{n=C}^{\infty} \frac{(q \lambda)^{n}}{C!\cdot C^{n-C} \cdot \mu^{n}}\right]^{-1} \\
& \pi_{n}= \begin{cases}\frac{1}{n!} \cdot\left(\frac{q \lambda}{\mu}\right)^{n} \cdot \pi_{0} & \text { if } n<C \\
\frac{1}{C!\cdot C^{n-C}} \cdot\left(\frac{q \lambda}{\mu}\right)^{n} \cdot \pi_{0} \quad \text { if } C \leq n\end{cases} \tag{3.15}
\end{align*}
$$

The objective function is to calculate retailers' optimal joint decisions on the pricing scheme of the 3 D product and the capacity of 3 D printers in order to maximize retailer's expected profit function. In Table 3.9, the parameters and decision variables of the model are defined.

The customer utility in buying one unit of the off-the-shelf product is calculated as:

$$
\begin{equation*}
\Psi_{s}=v_{s}-p_{s} \tag{3.16}
\end{equation*}
$$

The customer utility in buying one unit of 3D printing product is calculated as:

$$
\begin{equation*}
\Psi_{d}=v_{d}-p_{d}-\frac{c_{w}}{C . \mu \cdot p} \tag{3.17}
\end{equation*}
$$

The retailer profit in selling a unit of the off-the-shelf product is calculated as:

$$
\begin{equation*}
\Phi_{s}=p_{s}-c_{s} \tag{3.18}
\end{equation*}
$$

The retailer profit in selling one unit of 3D printing product is calculated as:

$$
\begin{equation*}
\Phi_{d}=p_{d}-\frac{c_{d}}{p}-c_{s e} \tag{3.19}
\end{equation*}
$$

## Table 3.9 Summary of Notation

Parameters:
$p_{s}$ : Selling price of off-the-shelf product/unit
$v_{s}$ : Customer value of off-the-shelf product/unit
$c_{s}$ : Procurement cost of off-the-shelf product/unit
$v_{d}$ : Customer value of 3D product/unit
$c_{d}: 3 \mathrm{D}$ production cost/unit
$c_{w}$ : Waiting cost/unit
$c_{s e}$ : Capacity cost/3D printer
$p$ : Success probability of producing 3D product/unit
$\lambda: \quad$ Customer arrival rate/hr
$\mu: \quad 3 \mathrm{D}$ printer service rate/hr
$T$ : Number of working hours/day
Decision Variables:
$p_{d}$ : Selling price of 3D product/unit
$C$ : Number of 3D printers

Lets $q$ defines the customer preference probability of choosing 3D products, which also known as market share of the 3D product and $\bar{q}$ defines the customer preference probability of selecting off-the-shelf products or market share of the off-the-shelf product.

$$
\begin{equation*}
q=\frac{e^{\left(v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p)}\right.}}{e^{\left(v_{d}-p_{d}-\frac{c}{c} \cdot \mu \cdot p\right)}+e^{\left(v_{s}-p_{s}\right)}} \quad \text { where, } \bar{q}=1-q \tag{3.20}
\end{equation*}
$$

It is easily verified that, the derivatives of the market shares with respect to 3 D product price and total number of 3D printers are given by:

$$
\begin{align*}
& \frac{d q}{d p_{d}}=-q \cdot \bar{q} \\
& \frac{d q}{d C}=\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q \cdot \bar{q}  \tag{3.21}\\
& \frac{d \bar{q}}{d p_{d}}=q \cdot \bar{q} \\
& \frac{d \bar{q}}{d C}=-\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q \cdot \bar{q}
\end{align*}
$$

3D product's market share is decreasing in its own price and increasing in the total number of 3D printers. On the other hand, off-the-shelf product's market share is increasing in 3D product price and decreasing in the total number of 3D printers. The total customer expected utility function is calculated as Equation 3.22.

$$
\begin{align*}
\Psi & =\lambda \cdot T \cdot \sum_{n=0}^{\infty} \pi_{n} \cdot\left(q_{d}\left(v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)+q_{s} \cdot\left(v_{s}-p_{s}\right)\right)  \tag{3.22}\\
\Psi & =\lambda \cdot T \cdot\left(q_{d}\left(v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)+q_{s} \cdot\left(v_{s}-p_{s}\right)\right)
\end{align*}
$$

The total retailer's expected profit function is calculated as Equation 3.23.

$$
\begin{align*}
& \Phi=\lambda \cdot T \cdot \sum_{n=0}^{\infty} \pi_{n} \cdot\left(q \cdot\left(p_{d}-\frac{c_{d}}{p}\right)+\bar{q} \cdot\left(p_{s}-c_{s}\right)\right)-c_{s e} \cdot C  \tag{3.23}\\
& \Phi=\lambda \cdot T \cdot\left(q \cdot\left(p_{d}-\frac{c_{d}}{p}\right)+\bar{q} \cdot\left(p_{s}-c_{s}\right)\right)-c_{s e} \cdot C
\end{align*}
$$

In which $\pi_{n}(n=0, \ldots)$ is the probability of being the $n_{t h}$ customer in the queuing system.

### 3.3.1 Analytical Solutions

As mentioned before, the objective function is to calculate retailers' optimal joint decisions on the pricing scheme of the 3 D product and the capacity of 3 D printers in order to maximize retailer's expected profit function.

The Proposition 2, demonstrates the optimal 3D price and optimal 3D printers capacity of the model to maximize retailer's expected profit.

Proposition 2. Assuming customer purchase behavior follows the MNL model, under the condition of $\frac{2}{3} \leq q \leq 1$, the retailer's expected profit function is concave and the joint optimal 3D product price and optimal 3D printer's capacity satisfy the below equations:

$$
\begin{gathered}
p_{d}^{*}=p_{s}-c_{s}+\frac{c_{d}}{p}+1+e^{v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}-v_{s}+p_{s}} \\
C^{* 2} \cdot e^{\frac{c_{w}}{\mu \cdot p \cdot c^{*}}} \cdot\left(e^{v_{d}-p_{d}-\frac{c_{w}}{C^{*} \cdot \mu \cdot p}}+e^{v_{s}-p_{s}}\right)^{2}=\frac{\lambda \cdot T \cdot c_{w}}{c_{s e} \cdot \mu} \cdot e^{v_{d}-p_{d}+v_{s}-p_{s}} \cdot\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)
\end{gathered}
$$

Corollary 2.1. Assuming consumers have the additional option of purchasing nothing, with the probability of $q_{n}$, besides the options of choosing the 3D product, with the probability of $q_{d}$ or choosing the off-the-shelf product, with the probability of $q_{s}$. Using the MNL model, the customer preference probability of each option is captured as:

$$
\begin{aligned}
& q_{d}=\frac{e^{\left(v_{d}-p_{d}-\frac{c_{w}}{c \cdot \mu \cdot p}\right)}}{1+e^{\left(v_{d}-p_{d}-\frac{c^{-}}{c \cdot \mu \cdot p}\right)}+e^{\left(v_{s}-p_{s}\right)}} \\
& q_{s}=\frac{e^{\left(v_{s}-p_{s}\right)}}{1+e^{\left(v_{d}-p_{d}-\frac{c . \mu}{c \cdot \mu \cdot p}\right)}+e^{\left(v_{s}-p_{s}\right)}} \\
& q_{n}=\frac{1}{1+e^{\left(v_{d}-p_{d}-\frac{c_{\omega}}{C \cdot \mu \cdot p)}\right.}+e^{\left(v_{s}-p_{s}\right)}}
\end{aligned}
$$

The 3D product's market share is decreasing in her own price and increasing in the total number of $3 D$ printers, on the other hand, off-the-shelf product and purchasing nothing market shares are increasing in 3D product price and decreasing in the total number of 3D printers.

The retailer's expected profit function is:

$$
\begin{align*}
& \Phi=\lambda \cdot T \cdot \sum_{n=0}^{\infty} \pi_{n} \cdot\left(q_{d} \cdot\left(v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)+q_{s} \cdot\left(v_{s}-p_{s}\right)\right)  \tag{3.24}\\
& \Phi=\lambda \cdot T \cdot\left(q_{d} \cdot\left(v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)+q_{s} \cdot\left(v_{s}-p_{s}\right)\right)
\end{align*}
$$

Which is concave under the condition of:

$$
\left(2 q_{d}^{* 2} \cdot \lambda \cdot T+C^{*} \cdot c_{s e} \cdot\left(1-2 q_{d}^{*}\right)\right) \cdot \frac{q_{d}^{*}}{C^{*} \cdot\left(1-2 q_{d}^{* 2}\right)}>\frac{c_{s e}^{3}}{\lambda \cdot T}
$$

The optimal price for $3 D$ products and the optimal capacity of $3 D$ printers satisfy the below equations.

$$
\begin{gathered}
p_{d}^{*}-\frac{e^{\left(v_{d}-\frac{c_{w}}{C^{*} \cdot \mu \cdot p}\right)}}{1+e^{\left(v_{s}-p_{s}\right)}} \cdot e^{-p_{d}^{*}}=1+\frac{c_{d}}{p}+\frac{p_{s}-c_{s}}{\left(1+e^{\left(v_{s}-p_{s}\right)}\right)} \\
q_{d}{ }^{*}=\frac{c_{s e} \cdot \mu \cdot p \cdot C^{* 2}}{\lambda \cdot T \cdot c_{w}}
\end{gathered}
$$

The proof of the Proposition and Corollary can be found in the Appendix C.

### 3.3.2 Computational Solutions

To generalize our findings, in this section we present numerical solutions of the MNL model. In the first place, 3D product price, $p_{d}$, is changed between the range of $\$ 11$ to $\$ 29$ and the optimal retailer's expected profit and the optimal number of 3D printers are calculated. The following parameters are used: $v_{s}=\$ 20, p_{s}=\$ 15, c_{s}=\$ 8$, $v_{d}=\$ 28, c_{d}=\$ 3, c_{w}=\$ 3, \mu=21, \lambda=20, p=0.8, T=8, c_{s e}=\$ 10$.

As shown in Table 3.10 and Figure 3.9, by increasing 3D product price, the retailer's expected profit increases till the optimal $p_{d}$ and will decrease after that. The optimal 3D product price is at $p_{d}^{*}=\$ 21$, with the optimal number of two 3D printers and the retailer's expected profit is $\Phi^{*}=\$ 2531.9$.

If the 3D product price reaches $\$ 29$ and more, the customer's utility for buying the 3D product will become negative, the market share of this product, $q_{d}$, becomes zero and the retailer does not need to provide any 3 D printer. Therefore, the profit will drop to $\$ 1120$, meaning that the retailer just sells off-the-shelf products.

As shown in Figure 3.10, by increasing 3D product price, customers incline to buy less from 3D products and more from off-the-shelf products, which decrease 3D market share as well. On the other hand, by increasing number of 3D printers the market share of the 3D product will increase.

Table 3.10 Effect of $p_{d}$ on Retailer's Expected Profit and Optimal Number of 3D Printers in the MNL Model

| $p_{d}$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Phi^{*}$ | 1150 | 1310 | 1470 | 1629.9 | 1789.7 | 1949.1 | 2107.1 | 2261.1 | 2402.7 | 2509.6 |
| $C^{*}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $p_{d}$ | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| $\Phi^{*}$ | 2531.9 | 2390.2 | 2046.7 | 1640.9 | 1355.1 | 1211 | 1150.6 | 1126 | 1120 | 1120 |
| $C^{*}$ | 2 | 2 | 3 | 2 | 2 | 1 | 1 | 1 | 0 | 0 |



Figure 3.9 Effect of $p_{d}$ on retailer's expected profit and optimal 3D printer capacity in the MNL model.


Figure 3.10 Effect of $p_{d}$ and $C$ on the 3D product market share.

Considering the same parameters, let's set $p_{d}$ equal to $\$ 15$, and $p_{s}$ varies between the range of $\$ 8$ to $\$ 20$. The results are computed in Table 3.11 and Figure 3.11. By increasing $p_{s}$, the optimal retailer's profit will strictly increase and the retailer prefers to offer more of the off-the-shelf products. Furthermore, the optimal number of 3D printers decreases. As shown in Figure 3.12, by increasing $p_{s}$ the market share of 3D
products will increase and distinctly the market share of off-the-shelf products will decrease.

Table 3.11 Effect of $p_{S}$ on Retailer's Expected Profit and Optimal 3D Printer Capacity in the MNL Model

| $p_{s}$ | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Phi^{*}$ | 1270.2 | 1571.9 | 1709.6 | 1762.7 | 1781.1 | 1787.1 | 1789.1 | 1789.7 | 1789.9 | 1790 | 1790 | 1790 | 1920 |
| $C^{*}$ | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |



Figure 3.11 Effect of $p_{s}$ on retailer's expected profit and optimal 3D printer capacity in the MNL model.


Figure 3.12 Effect of $p_{s}$ on 3D product market share.

### 3.3.3 Stochastic Customer Valuation for 3D Product

In this section, similar to Section 3.2 .3 of the Cut-off model, it is assumed that customer value of choosing off-the-shelf products is deterministic, while their preferences for 3D products are classified into two categories, high preference customer $\left(v_{d h}\right)$, with probability of $\beta_{h}$ and low preference customers $\left(v_{d l}\right)$, with probability of $\beta_{l}$. In which, $\beta_{h}+\beta_{l}=1$.

As shown in Equation 3.25, customer preferences to select 3D products are a function of customer preferences to choose off-the-shelf products. The aim of the model here is to obtain retailers' joint optimal decisions on 3D product price and 3D printer capacity.

$$
\begin{cases}v_{d h}=\alpha_{h} \cdot v_{s} & \text { with probability of }  \tag{3.25}\\ \beta_{h} \\ v_{d l}=\alpha_{l} \cdot v_{s} & \text { with probability of } \beta_{l}\end{cases}
$$

The customer preference probability or market share of choosing 3D printing product is defined as Equation 3.26. $1-q_{i}$, fori $=h, l$, captures the market share of the off-the-shelf product.

$$
\begin{equation*}
q_{i}=\frac{e^{\alpha_{i} \cdot v_{s}-p_{d}-\frac{c_{w}}{c \cdot \mu \cdot p}}}{1+e^{\alpha_{i} \cdot v_{s}-p_{d}-\frac{c}{c \cdot u \cdot p}}+e^{v_{s}-p_{s}}} \quad \text { for } \mathrm{i}=\mathrm{h}, \mathrm{l} \tag{3.26}
\end{equation*}
$$

The total retailer's expected profit is calculated as Equation 3.27. In which, the first and second terms present the retailer's profit of selling 3D items and off-the-shelf ones to the consumers with the high value of 3 D service and low value of 3 D service respectively. The third term denotes the 3D printers capacity cost.

$$
\begin{align*}
\Phi & =\lambda \cdot T \cdot\left[\beta_{h} \cdot\left(q_{h} \cdot\left(p_{d}-\frac{c_{d}}{p}\right)+\left(1-q_{h}\right) \cdot\left(p_{s}-c_{s}\right)\right)\right.  \tag{3.27}\\
& \left.+\beta_{l} \cdot\left(q_{l} \cdot\left(p_{d}-\frac{c_{d}}{p}\right)+\left(1-q_{l}\right) \cdot\left(p_{s}-c_{s}\right)\right)\right]-c_{s e} \cdot C
\end{align*}
$$

Proposition 3. Given consumer value to buy off-the-shelf product follows a uniform distribution, $v_{s} \sim U[a, b]$, and customer's value to buy one unit of 3D printing
product is a coefficient of off-the-shelf customer value, $v_{d}=\alpha \cdot v_{s}$. The customer preference probability of choosing 3D printing product, q, and off-the-shelf product , $\bar{q}$, are calculated as:

$$
q=\frac{1}{(b-a) \cdot(1-\alpha)} \cdot\left[\ln \left(1+e^{\left((1-\alpha) a-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)-\ln \left(1+e^{\left((1-\alpha) b-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)\right]
$$

Where $\bar{q}=1-q$.
The retailers expected profit function is concave under the condition of:

$$
\begin{aligned}
& (\alpha-1) \cdot v_{s}+p_{s}-p_{d}^{*}-\frac{c_{w}}{C^{*} \mu p} \leq 0 \\
& \frac{\left(p_{d}^{*}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)}{C^{* 2}} \cdot\left[\frac{e^{\left((\alpha-1) a+p_{s}-p_{d}^{*}-\frac{c_{w}}{C^{*} \cdot \mu \cdot p}\right)}}{\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}^{*}-\frac{c_{w}}{C^{*} \cdot \mu \cdot p}\right)}\right)^{2}}-\frac{e^{\left((\alpha-1) b+p_{s}-p_{d}^{*}-\frac{c_{w}}{C^{*} \cdot \mu \cdot p}\right)}}{\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}^{*}-\frac{c w}{C^{*} \cdot \mu \cdot p}\right)}\right)^{2}}\right] \\
& \cdot\left(-C^{*} \cdot\left(p_{d}^{*}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)-\frac{2 c_{w}}{\mu \cdot p}\right)>\frac{c_{s e} \cdot(1-\alpha) \cdot(b-a)}{\lambda \cdot T}
\end{aligned}
$$

The optimal price for 3D products and the optimal 3D printer capacity satisfy the below equations:

$$
\begin{aligned}
& \left(\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}^{*}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)^{-1}-\left(1+e^{\left((\alpha-1) b+p_{s}-p_{d}^{*}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)^{-1}\right) \cdot\left(p_{d}^{*}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)= \\
& \ln \left(1+e^{\left((1-\alpha) b-p_{s}+p_{d}^{*}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)-\ln \left(1+e^{\left((1-\alpha) a-p_{s}+p_{d}^{*}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right) \\
& \quad \frac{1}{C^{* 2}} \cdot\left(\frac{1}{1+e^{(\alpha-1) b+p_{s}-p_{d}-\frac{c}{C^{*} \cdot \mu \cdot p}}}-\frac{1}{1+e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C^{*} \cdot \mu \cdot p \cdot p}\right.}}\right)=\frac{c_{s e} \cdot(b-a) \cdot(1-\alpha) \cdot \mu \cdot p}{\lambda \cdot T \cdot c_{w} \cdot\left(p_{d}^{*}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)}
\end{aligned}
$$

The proof of Proposition can be found in Appendix D

### 3.3.4 Computational Solutions

To generalize our findings in this section, we present numerical solutions of the model. The 3D product price, $p_{d}$, is varied between the range of $\$ 11$ to $\$ 30$ and the retailer's optimal expected profit and the optimal number of 3D printers are calculated. The following parameters are used: $v_{s}=\$ 20, p_{s}=\$ 15, c_{s}=\$ 8, c_{d}=\$ 3, \alpha_{h}=1.5$, $\alpha_{l}=1.3, \beta_{h}=0.5, \beta_{l}=0.5, c_{w}=\$ 3, \mu=21, \lambda=20, p=0.8, T=8 h r, c_{s e}=\$ 10$.

As shown in Table 3.12 and Figure 3.3.4, because the market share of the 3D product is high, by increasing 3D product price from $p d=\$ 11$ to $p d=\$ 20$, the retailer's expected profit will increase. For the prices greater than $p d=\$ 20$ as 3D market share decreases, the retailer's expected profit starts to decrease. At the point of $p_{d}=\$ 30$, the market share of 3 D product for both high and low preferences customers are equal to zero, therefore retailer just sells off-the-shelf products.

As provided in Table 3.12 and Figure 3.3.4, the market share of 3D products is decreasing in 3D product price and customers are more willing to buy off-the-shelf products.

Table 3.12 Sensitivity Analysis of $p_{d}$ in the MNL Model With Stochastic Customer Utility

| $p_{d}$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Phi^{*}$ | 1150 | 1310 | 1470 | 1629.7 | 1789 | 1949.7 | 2099.5 | 2237.9 | 2339 | 2365.1 |
| $C^{*}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| $q_{h}^{*}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.99 |
| $q_{l}^{*}$ | 1 | 1 | 1 | 1 | 1 | 0.99 | 0.98 | 0.95 | 0.86 | 0.72 |
| $p_{d}$ | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| $\Phi^{*}$ | 2299.5 | 2183.9 | 2065.3 | 1906.8 | 1668.8 | 1418.9 | 1249.3 | 1168.2 | 1133.2 | 1120 |
| $C^{*}$ | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 0 |
| $q_{h}^{*}$ | 0.98 | 0.95 | 0.87 | 0.72 | 0.48 | 0.26 | 0.11 | 0.04 | 0.02 | 0 |
| $q_{l}^{*}$ | 0.48 | 0.26 | 0.11 | 0.04 | 0.02 | 0.01 | 0 | 0 | 0 | 0 |

### 3.3.5 Managerial Insights

The developed model considers the impact of the flexible terms, including $\alpha_{h}, \alpha_{l}$ and $\beta_{h}$, on the retailer's expected profit, optimal 3D product price, optimal number of 3D printers and market shares of high preference and low preference customers. The parameters are as defined as in Section 3.3.4, and for $\alpha_{l}$ sensitivity analysis, $\alpha_{h}$ is set equal to 1.7 and for $\alpha_{h}$ sensitivity analysis, $\alpha_{l}$ is set equal to 1.4. The implications of the results from our analysis are summarized in the following:

As Table 3.13 presents, the retailer's expected profit, as well as optimal 3D product price, will decrease as $\beta_{h}$ decreases. The optimal number of 3D printers


Figure 3.13 Sensitivity Analysis of $p_{d}$ in the MNL model with stochastic customer utility.


Figure 3.14 Effect of $p_{d}$ on 3D product market share.
decreases to one but as retailer's marginal profit of 3D product, $\left(p_{d}-c_{d}\right)$, is higher than the marginal profit of off-the-shelf product, $\left(p_{s}-c_{s}\right)$, retailer prefers to provide one 3D printer.

Market share of 3D product for high value and low value customers increase in $\beta_{h}$, as $p_{d}$ decreases.

At the cut-off point of $\beta_{h}=0.4735$, there are two optimal results of $p_{d}^{*}=\$ 21$ and $p_{d}^{*}=\$ 32$ with retailer's expected profit of $\$ 2633$ and the optimal number of one 3 D printer. for the $\beta_{h}$ greater than the cut-off value, low reference customers tend
to get served by off-the-shelf products and for the $\beta_{h}$ less than the cut-off value, two types of customers incline to have 3D products.

Table 3.13 Sensitivity Analysis of $\beta_{h}$ in The MNL Model With Stochastic Customer Utility

| $\beta_{h}$ | $p_{d}^{*}$ | $C^{*}$ | $\Phi^{*}$ | $q_{h}^{*}$ | $q_{l}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.9 | 33 | 2 | 4005 | 0.873 | 0 |
| 0.7 | 32 | 1 | 3361 | 0.946 | 0 |
| 0.5 | 32 | 1 | 2718 | 0.946 | 0 |
| 0.4735 | 32 | 1 | 2633 | 0.946 | 0 |
|  | 21 | 1 | 2633 | 1 | 0.865 |
| 0.3 | 21 | 1 | 2595 | 1 | 0.865 |
| 0.1 | 21 | 1 | 2553 | 1 | 0.865 |

As Table 3.14 shows, by increasing $\alpha_{h}$, high preference customers are more eager to get 3D products and the retailer's expected profit, as well as optimal 3D product price, will increase.

At the cut-off point of $\alpha_{h}=1.952$, there are two optimal results of $p_{d}^{*}=\$ 22$ with $C^{*}=2$ and $p_{d}^{*}=\$ 31$ with $C^{*}=1$ with retailer's expected profit of $\$ 2645$. Before the Cut-off value, both types of customers incline to buy 3D products. By increasing $\alpha_{h}, q_{h}^{*}$ will increase to one but as the 3D price increases the $q_{l}^{*}$ will decrease. After the cut-off value, as the 3D price increases the $q_{l}^{*}$ will decrease to zero and low reference customers tend to get served by off-the-shelf products and the $C^{*}$ will drop to one.

As Table 3.15 provides, before the Cut-off point of $\alpha_{l}=1.263$, both types of customers incline to buy 3D products with the high market share of 3D products. Moreover, retailer's expected profit and optimal 3D product price will decrease as

Table 3.14 Sensitivity Analysis of $\alpha_{h}$ in the MNL Model With Stochastic Customer Utility

| $\alpha_{h}$ | $p_{d}^{*}$ | $C^{*}$ | $\Phi^{*}$ | $q_{h}^{*}$ | $q_{l}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.5 | 21 | 1 | 2622 | 0.98 | 0.85 |
| 1.7 | 22 | 2 | 2644 | 1 | 0.72 |
| 1.9 | 22 | 2 | 2645 | 1 | 0.72 |
| 1.952 | 22 | 2 | 2645 | 1 | 0.72 |
|  | 31 | 1 | 2645 | 0.95 | 0 |
| 2 | 32 | 1 | 2718 | 0.95 | 0 |
| 2.5 | 41 | 1 | 3480 | 0.98 | 0 |

$\alpha_{l}$ decreases. After the cut-off value, the $q_{l}^{*}$ will decrease to zero and low reference customers tend to get served by off-the-shelf products.

Table 3.15 Sensitivity Analysis of $\alpha_{L}$ in The MNL Model With Stochastic Customer Utility

| $\alpha_{l}$ | $p_{d}^{*}$ | $C^{*}$ | $\Phi^{*}$ | $q_{h}^{*}$ | $q_{l}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.4 | 22 | 2 | 2644 | 1 | 0.72 |
| 1.3 | 20 | 2 | 2370 | 1 | 0.72 |
| 1.263 | 19 | 1 | 2267 | 1 | 0.75 |
|  | 26 | 1 | 2267 | 0.95 | 0 |
| 1.2 | 26 | 1 | 2265 | 0.95 | 0 |
| 1 | 26 | 1 | 2264 | 0.95 | 0 |
| 0.5 | 26 | 1 | 2264 | 0.95 | 0 |

## CHAPTER 4

## RETAILER'S PRICING AND INVENTORY MANAGEMENT

### 4.1 Introduction

With the wide variety of manufacturing possibilities offered by 3D printing technology, retailers prefer to have 3 D printed products next to their off-the-shelf ones to satisfy consumers' willing to buy different products.

From the customer aspect, most of the consumers are willing to substitute a product rather than go home empty-handed. Therefore, the 3D product can be considered as a substitute for the off-the-shelf product in the stock-out situation. Substitution policy can result in improvements of supply chains and increasing retailers' profit.

Many retailers have been applying integrated pricing and inventory management strategies to maximize their profit. As [Federgruen and Heching, 1999] have shown, a combined optimal price and inventory management policy can result in the $6.5 \%$ profit improvement for a retailer compared to the pricing policy without inventory management consideration. The primary goal of this chapter is to characterize the optimal pricing and inventory policies in a retail store with the combined off-the-shelf and 3D products in the presence of substitution policy.

### 4.2 The Model

In this section, we consider a retail store which is capable of providing two types of products: Off-the-shelf product, whose demand is satisfied by the on-hand inventory. In the stock-out situation, retailer offers 3D service as the substitution policy.

The arrival rates of both types of products follow the Poisson distribution. It is assumed that service time for producing 3D product follows the Exponential distributions with mean of $\frac{1}{\mu}$. Furthermore, the 3D printer has a predefined failure
rate of $1-p$. Therefore, if the customer does not satisfy with the 3D printing product, $\mathrm{s} /$ he can receive another one immediately.

The system costs include the purchase cost of the off-the-shelf product, the manufacturing cost of the 3 D product, the holding cost of off-the-shelf product inventories, the 3D printer capacity cost and the lost sale cost of unsatisfied demand.

The objective function of the model is to maximize the retailer's expected profit by coordinating the off-the-shelf product inventory and pricing decisions. In Table 4.1, the parameters and decision variables of the model are defined.

As mentioned before, it is assumed that customers are inclined to buy the off-the-shelf product. In the stock-out situation, the retailer offers the 3D product to the customers as the substitution product. Customers have high value for their custom designed 3D products, compared to purchase nothing. On the other hand, producing 3D printing products is a timely manufacturing so the customers need to be in a queue. In this model, the system behaves like a queuing system with multiserver, in which each server is a 3D printer with limited system capacity. The conditional waiting time in the queuing system of $\mathrm{M} / \mathrm{M} / \mathrm{C}$ is calculated as following and the proof can be found in Appendix A.

$$
W_{q}(t \mid n)= \begin{cases}\frac{1}{\mu \cdot p} & \text { if } n \leq C  \tag{4.1}\\ \frac{n}{C \cdot \mu \cdot p} & \text { if } C<n\end{cases}
$$

The customer expected utility in buying one unit of the 3 D printing product is calculated as:

$$
\Psi_{d}=\left\{\begin{array}{lc}
v_{d}-p_{d}-c_{w} \cdot \frac{1}{\mu \cdot p} & \text { if } n \leq C  \tag{4.2}\\
v_{d}-p_{d}-c_{w} \cdot \frac{n}{C \cdot \mu \cdot p} & \text { if } n>C
\end{array}\right.
$$

It's assumed that the customer purchase behavior in the case of stock-out follows the Cut-off model. So, the customers who have entered the retail store are willing to get

Table 4.1 Summary of Notation
Parameters:

$$
\begin{array}{ll}
p_{s}: & \text { Selling price of off-the-shelf product/unit } \\
v_{s}: & \text { Customer value of off-the-shelf product/unit } \\
c_{s}: & \text { Procurement cost of off-the-shelf product/unit } \\
v_{d}: & \text { Customer value of 3D product/unit } \\
c_{d}: & \text { 3D production cost/unit } \\
c_{w}: & \text { Waiting cost/unit } \\
C: & \text { Total number of 3D printers } \\
c_{s e}: & \text { Capacity cost/3D printer } \\
p: & \text { Success probability of producing 3D product/unit } \\
\lambda: & \text { Customer arrival rate/hr } \\
\mu: & 3 \mathrm{D} \text { printer service rate/hr } \\
h: & \text { Holding cost/unit } \\
l: & \text { Lost sale cost/unit }
\end{array}
$$

Decision Variables:
$p_{d}$ : Selling price of 3D product/unit
$y_{s}^{w}$ : Inventory level of off-the-shelf product with substitution option
$y_{s}^{w o}$ : Inventory level of off-the-shelf product without substitution option
the 3 D product unless the customer utility of the 3 D product is less than zero. In that case, they prefer to purchase nothing. The cut-off number of customers in the system, $K$, means the customer inclined to join the 3D service if the current number of customer in the system including him is less than $K$. Otherwise, he will leave without buying anything. The cut-off number of customers depends on the price and the customer value of the 3D product, considering the waiting time of production and
it follows Equation 4.3.

$$
\begin{equation*}
K=\left\lfloor\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}\right)\right\rfloor \tag{4.3}
\end{equation*}
$$

The model follows the truncated queue model of $M / M / C / K$, which is shown in Figure 4.2. The capacity of the queue system is equal to the cut-off number of customers in the system $(K)$, which is calculated in Equation 4.3. The probability of being $n_{t h}$ customer in the system is calculated as Equation 4.4.


Figure $4.1 \mathrm{M} / \mathrm{M} / \mathrm{C} / \mathrm{K}$ queue system in the Cut-off model.

$$
\begin{align*}
& \pi_{0}=\left[\sum_{n=0}^{C-1} \frac{1}{n!} \cdot\left(\frac{\lambda}{\mu}\right)^{n}+\frac{\left(\frac{\lambda}{\mu}\right)^{C} \cdot\left(1-\left(\frac{\lambda}{c \mu}\right)^{K-C+1}\right)}{c!\cdot\left(1-\frac{\lambda}{c \mu}\right)}\right]^{-1} \\
& \pi_{n}= \begin{cases}\frac{1}{n!} \cdot\left(\frac{\lambda}{\mu}\right)^{n} \cdot \pi_{0} \quad \text { if } n<C \\
\frac{1}{C!\cdot C^{n-c}} \cdot\left(\frac{\lambda}{\mu}\right)^{n} \cdot \pi_{0} \quad \text { if } C \leq n \leq K\end{cases} \tag{4.4}
\end{align*}
$$

Here, we consider two different scenarios: The retail store just provides off-the-shelf products without considering any substitution policy (benchmark case), and the retail store provides the 3D product, next to the off-the-shelf ones as a substitute in the stock-out situation.

In the first scenario, the retailer's expected profit is calculated as the Equation 4.5.

$$
\begin{align*}
\Phi\left(y_{s}\right) & =p_{s} \cdot \sum_{D=0}^{y_{s}} D \cdot p(D)+p_{s} \cdot \sum_{D=y_{s}+1}^{\infty} y_{s} \cdot p(D)  \tag{4.5}\\
& -c_{s} \cdot y_{s}-h \cdot \sum_{D=0}^{y_{s}}\left(y_{s}-D\right) \cdot p(D)-l \cdot \sum_{y_{s}+1}^{\infty}\left(D-y_{s}\right) \cdot p(D)
\end{align*}
$$

The first two terms in Equation 4.5, compute the retailer's expected sales revenue of off-the-shelf products. The third term denotes the procurement cost of off-the-shelf
products. The fourth and the fifth terms show the holding cost of off-the-shelf unsold inventory and the lost sales cost respectively.

In the second scenario, the retailer's expected profit is calculated as the Equation 4.6.

$$
\begin{align*}
\Phi\left(y_{s}, p_{d}\right) & =p_{s} \cdot \sum_{D=0}^{y_{s}} D \cdot p(D)+p_{s} \cdot \sum_{D=y_{s}+1}^{\infty} y_{s} \cdot p(D) \\
& -C \cdot c_{s e}-c_{s} \cdot y_{s}-h \cdot \sum_{D=0}^{y_{s}}\left(y_{s}-D\right) \cdot p(D)  \tag{4.6}\\
& +\left(\sum_{i=0}^{K-1} \pi_{i} \cdot\left(p_{d}-c_{d} / p\right)-\pi_{K} \cdot l\right) \cdot \sum_{y_{s}+1}^{\infty}\left(D-y_{s}\right) \cdot p(D)
\end{align*}
$$

The first two terms in Equation 4.6, computes the retailer's expected sales revenue of off-the-shelf products. The third term shows the 3D printers capacity cost. The fourth term denotes the procurement cost of off-the-shelf products. The fifth term is the holding cost of the off-the-shelf unsold inventory and the last term corresponds to the retailer's expected 3D product sales profit in the stock-out situation and the lost sales cost respectively.

### 4.3 Analytical Solution

In this section, we present the general form of the optimal decisions under two different scenarios. We start from the simplest case where there is one 3D printing service (M/M/1/K), and then the extension for the number of $C 3 \mathrm{D}$ printers will be provided (M/M/C/K).

The optimal inventory level of the off-the-shelf product that maximize the retailer's profit function needs to satisfy $\Phi_{y_{s}^{w}}>\Phi_{y_{s}^{w}+1}$ and $\Phi_{y_{s}^{w}}>\Phi_{y_{s}^{w}-1}$.

Proposition 4. The optimal inventory level of the off-the-shelf product that maximize the retailer expected profit function, while the retailer provides one $3 D$ printer, satisfies:

$$
F\left(y_{s}^{w}-1\right) \leq \frac{p_{s}-c_{s}-\left(1-\pi_{K}\right) \cdot\left(p_{d}-c_{d} / p\right)+\pi_{K} \cdot l}{p_{s}+h-\left(1-\pi_{K}\right) \cdot\left(p_{d}-c_{d} / p\right)+\pi_{K} \cdot l} \leq F\left(y_{s}^{w}\right)
$$

In which $\pi_{K}=\frac{\rho^{K} \cdot(1-\rho)}{1-\rho^{K+1}}$, and $F($.$) is the cumulative distribution function of demand.$

Corollary 4.1. In the benchmark case, where there is no substitution option, the optimal inventory level of the off-the-shelf product is calculated as:

$$
F\left(y_{s}^{w o}-1\right) \leq \frac{p_{s}-c_{s}+l}{p_{s}+h+l} \leq F\left(y_{s}^{w o}\right)
$$

The optimal 3D product price that maximize the retailer's profit function needs to satisfy $\Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}+\epsilon}$ and $\Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}-\epsilon}$. In which $\epsilon$ is the 3 D price interval that would change the cut-off number of customers in one unit. For the case that retailer provides $C$ number of 3 D printers, $\epsilon=\frac{c_{w}}{C \cdot \mu \cdot p}$, and is calculated by solving the system of equations in 4.7.

$$
\begin{align*}
& K^{*}=\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}\right) \\
& K^{*}-1=\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}-\epsilon\right)  \tag{4.7}\\
& K^{*}+1=\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}+\epsilon\right)
\end{align*}
$$

Proposition 5. The optimal 3D product price that maximizes the retailer's expected profit function, while the retailer provides one 3D printer, satisfies:

$$
\begin{aligned}
& \frac{c_{w} \cdot\left(1-\rho^{\left(K\left(p_{d}^{*}\right)+1\right)}\right) \cdot\left(1-\rho^{\left(K\left(p_{d}^{*}\right)-1\right)}\right)}{\mu \cdot p \cdot(1-\rho)^{2} \cdot \rho^{\left(K\left(p_{d}^{*}\right)-1\right)}}+\frac{c_{d}}{p}-l \leq p_{d}^{*} \leq \frac{c_{w} \cdot\left(1-\rho^{\left(K\left(p_{d}^{*}\right)+1\right)}\right)^{2}}{\mu \cdot p \cdot(1-\rho)^{2} \cdot \rho^{K\left(p_{d}^{*}\right)}}+\frac{c_{d}}{p}-l \\
& p_{d}^{*} \in\left\{v_{d}, v_{d}-\epsilon, v_{d}-2 \epsilon, v_{d}-3 \epsilon, \ldots\right\}^{+}
\end{aligned}
$$

in which $\rho=\frac{\lambda}{\mu}, \rho<1$ and $\epsilon=\frac{c_{w}}{\mu \cdot p}$.
Corollary 5.1. At the optimal 3D product price, the optimal cut-off number of customers satisfies:

$$
\frac{\mu \cdot p}{c_{w}} \cdot\left(v_{d}-\frac{c_{d}}{p}+l\right)+\frac{\left(1-\rho^{\left(K^{*}+1\right)}\right)^{2}}{(1-\rho)^{2} \cdot \rho^{K^{*}}} \leq K^{*} \leq \frac{\mu \cdot p}{c_{w}} \cdot\left(v_{d}-\frac{c_{d}}{p}+l\right)-\frac{\left(1-\rho^{K^{*}}\right)^{2}}{(1-\rho)^{2} \cdot \rho^{\left(K^{*}-1\right)}}
$$

Corollary 5.2. For the general case that the retailer provides the number of C 3D printers, the optimal $3 D$ product price that maximize the retailer's expected profit function satisfies:

$$
\begin{aligned}
& \frac{c_{w} \cdot\left(1-\pi_{\left(K^{*}-1\right)}\right)}{C \cdot \mu \cdot p \cdot\left(\pi_{K^{*}-1}-\pi_{\left(K^{*}\right)}\right)}+\frac{c_{d}}{p}-l \leq p_{d}^{*} \leq \frac{c_{w} \cdot\left(1-\pi_{\left(K^{*}+1\right)}\right)}{C \cdot \mu \cdot p \cdot\left(\pi_{K^{*}}-\pi_{\left(K^{*}+1\right)}\right)}+\frac{c_{d}}{p}-l \\
& p_{d}^{*} \in\left\{v_{d}, v_{d}-\epsilon, v_{d}-2 \epsilon, v_{d}-3 \epsilon, \ldots\right\}^{+}
\end{aligned}
$$

in which $\epsilon=\frac{c_{w}}{C \cdot \mu \cdot p}$, and $\pi_{n}$ is the probability of being the $n_{t h}$ customer in the queue system of $M / M / C / K$ and calculated as the Equation 4.4.

Corollary 5.3. In the general case with $C$ number of $3 D$ printers, at the optimal $3 D$ product price, the optimal cut-off number of customers satisfies:

$$
\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-\frac{c_{d}}{p}+l\right)-\frac{\left(1-\pi_{\left(K^{*}+1\right)}\right)}{\left(\pi_{K^{*}}-\pi_{\left(K^{*}+1\right)}\right)} \leq K^{*} \leq \frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-\frac{c_{d}}{p}+l\right)+\frac{\left(1-\pi_{\left(K^{*}-1\right)}\right)}{\left(\pi_{K^{*}-1}-\pi_{\left(K^{*}\right)}\right)}
$$

The proof of the Propositions and all the subsequent Corollaries can be found in the Appendix E.

### 4.4 Computational Solutions and Managerial Insights

To generalize our findings in this section, numerical solutions of the model are presented. The following parameters are used: $v_{s}=\$ 30, p_{s}=\$ 30, c_{s}=\$ 8, v_{d}=\$ 30$, $c_{d}=\$ 8, \mu=10, \lambda=25, p=0.8, C=1, c_{w}=\$ 4, h=5, l=22, T=8 h r, c_{s e}=\$ 10$. As shown in Figures and the joint optimal solution is at $\left(p d^{*}=28, y^{w}=27\right)$ with the optimal retailer's profit of $\$ 466$. Retailer's profit function is concave respect to either of the decision variables. As shown in Table 4.2, the optimal result of


Figure 4.2 Retailer's expected profit function vs. $y_{s}$.
two scenarios are compared. The optimal retailer's profit in the first scenario, in which retailer does not consider any substitution policy, is $\Phi=\$ 452$ at optimal


Figure 4.3 Retailer's expected profit function vs. $p_{d}$.
inventory level of $y_{s}=29$. While, in the second scenarios, in which retailer provides substitution policy, the optimal retailer's profit is $\Phi=\$ 466$ at optimal inventory level of $y_{s}=27$, with the optimal 3D price of $p d=\$ 28$ and capacity of four customers in the 3D queue. Therefore, by adopting 3D printing technology as substitution policy, the retailer's expected profit increases as the optimal inventory level of off-the-shelf product decreases.

Table 4.2 Comparison of Two Scenarios

| The Cut-off Model | $p_{d}^{*}$ | $K^{*}$ | $y^{w}$ | $\Phi^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
| With Substitution Policy | 28 | 4 | 27 | 466 |
| Without Substitution Policy | - | - | 29 | 452 |

By increasing either the holding cost or the lost sales cost, the difference between the retailer's expected profit with the substitution option and that without the substitution option increases. Figure 4.4 demonstrates the effect of changing the lost sales on two scenarios. As shown in Figure 4.5, by increasing the customer arrival rate, the retailer's expected profits with and without the substitution option increase, while the difference between these two decreases.


Figure 4.4 Sensitivity analysis of lost sales on two scenarios.


Figure 4.5 Sensitivity analysis of arrival rate on two scenarios.

## CHAPTER 5

## RETAILERS CAPACITY AND INVENTORY DECISIONS

### 5.1 Introduction

As the additive manufacturing technology becomes more responsive to consumers' demand, one important question arises for the retailer of a brick-and-mortar store: what should be the level of inventory to keep the considered product in storage that maximizes the retailer's profit given the availability of the 3D printers. In this chapter, we compare various production/inventory strategies that optimize the system's performance. Utilizing the Markov Decision Process, we develop a framework of queuing systems to examine the performance of each of the following strategies: (1) providing only off-the-shelf products, (2) providing only 3D printed products, (3) substituting the shortage of the off-the-shelf products by 3D printed products, and (4) providing consumers the options of selecting either the off-the-shelf product or the customized product produced by additive manufacturing. In essence, our approach assists decision makers in both capacity planning and inventory management. Additionally, we characterize the conditions under which each of the strategies aforementioned will be optimal. Analytical results and examples are given to demonstrate managerial insights for these four strategies.

### 5.2 The Model

In this section, we develop a queuing-inventory model in which the retail store is capable of providing two types of products: Off-the-shelf and 3D printing products. The system receives a stochastic demand from two customer segments: those who prefer the off-the-shelf product versus those who prefer the 3D product. The demand of the off-the-shelf product is met with the on-hand inventory. The demand of 3D printing product is fulfilled through the 3D printer. The arrival rates of both types
of products follow the Poisson distribution and each segment has an independent demand arrival rate. demand rate of the 3D printed consumers is $\lambda_{d}$, while off-theshelf consumers have the demand rate of $\lambda_{s}$.

It's assumed that the customers who have joined the system are willing to get either the 3D printing or off-the-shelf products with a certain probability. Let $\lambda_{s}=\lambda \cdot \alpha$ and $\lambda_{d}=\lambda \cdot(1-\alpha)$ be the demand rate of the off-the-shelf and 3D products respectively, while $\lambda=\lambda_{s}+\lambda_{d}$ is the total demand rate of the products.

In the case of stock-out, the off-the-shelf customers will shift to the 3D printing queue with a specific probability $\left(\beta_{s}\right)$. While, the 3D customers who reach the maximum capacity of the system will shift to off-the-shelf products with another predefined probability $\left(\beta_{d}\right)$.

It is assumed that 3D production time, service time and replenishment lead times for the off-the-shelf product follow independent exponential distributions with means of $\frac{1}{\mu_{d}}, \frac{1}{\mu_{s}}$ and $\frac{1}{\nu}$ respectively.

The flows of the products are illustrated in Figure 5.1.
The system costs include off-the-shelf procurement cost, off-the-shelf shipping cost, holding cost of the off-the-shelf product inventories, manufacturing cost of the 3D product, the 3D printer capacity cost and the lost sale cost of unsatisfied demand.

The objective of the model is to maximize the retailer's expected profit by coordinating the off-the-shelf product inventory and 3D capacity decisions.

We also analyze the performance of the three possible benchmark strategies:

- providing only off-the-shelf products.
- providing only 3D products.
- Substituting The Shortage of Off-the-shelf Products by 3D Products.

In Table 5.1, parameters, random variables and decision variables of the model are defined.


Figure 5.1 The product flows of the two-echelon supply system.

### 5.2.1 The Markov System

Let $\pi_{n, i, d}$ be the steady-state probability of having $n$ customers in the system, $i$ on-hand off-the-shelf items, and $d$ number of customers in the 3 D printer queue. The transition diagram of the model is shown in Figure 5.2.

Table 5.1 Summary of Notation
Parameters:
$p_{s}$ : Selling price of off-the-shelf product/unit
$v_{s}: \quad$ Customer value of off-the-shelf product/unit
$c_{s}$ : Procurement cost of off-the-shelf product/unit
$c_{s h}$ : Shipping cost of off-the-shelf product/order
$p_{d}: \quad$ Selling price of 3 D product/unit
$v_{d}$ : Customer value of 3 D product/unit
$c_{d}: \quad 3 \mathrm{D}$ production cost/unit
$c_{w}$ : Waiting cost/unit
$c_{s e}: \quad$ Capacity cost/3D printer
$\lambda: \quad$ Total demand rate
$\alpha: \quad$ Off-the-shelf product preference rate, $0 \leq \alpha \leq 1$
$\beta_{s}: \quad 3 \mathrm{D}$ product substitution rate, $0 \leq \beta_{s} \leq 1$
$\beta_{d}: \quad$ Off-the-shelf product substitution rate, $0 \leq \beta_{d} \leq 1$
$\mu_{d}: \quad 3 \mathrm{D}$ printer service rate
$\mu_{s}: \quad$ Service rate of off-the-shelf product
$\nu: \quad$ Off-the-shelf product's replenishment parameter
$h: \quad$ Holding cost/unit
$l: \quad$ Lost sale cost/unit
Random Variables:
$\pi_{n, i, d}$ : Steady-state probability of having $n$ customers in the system, $i$ on-hand off-the-shelf items, and $d$ customers in the 3D queue

Decision Variables:
$y_{s}$ : Inventory level of off-the-shelf product
$C: \quad$ Number of 3D printers


Figure 5.2 Transition diagram of the model, retailer offers both 3D and off-the-shelf products.

The balance equation of the transition diagram is given by Equation 5.1.

$$
\begin{align*}
& \left(A_{i, d} \cdot \lambda+B_{n, i, d} \cdot\left(\lambda_{s}+\beta_{d} \lambda_{d}\right)+C_{n, i} \cdot\left(\lambda_{d}+\beta_{s} \lambda_{s}\right)+\left(n \cdot D_{d}+C \cdot E_{n, d}\right) \cdot \mu_{d}\right. \\
& \left.+F_{n, i, d} \cdot \mu_{s}+G_{i} \cdot \nu\right) \cdot \pi_{n, i, d}=I_{i, d} \cdot \lambda_{d} \cdot \pi_{(n-1), i,(d-1)}+J_{n, i, d} \cdot \lambda_{s} \cdot \pi_{(n-1), i, d} \\
& +K_{n, i} \cdot\left(\lambda_{s}+\beta_{d} \lambda_{d}\right) \cdot \pi_{(n-1), i, C}+L_{n, i} \cdot\left(\lambda_{d}+\beta_{s} \lambda_{s}\right) \cdot \pi_{(n-1), 0,(d-1)}  \tag{5.1}\\
& +\mu_{s} \cdot \pi_{(n+1),(i+1), d}+\left(C-(C-n-1) \cdot\left(1-E_{n, d)}\right) \cdot \mu_{d} \cdot \pi_{(n+1), i,(d+1)}\right. \\
& +H_{n, i, d} \cdot \nu \cdot \pi_{n, 0, d} \\
& \forall n=0,1,2, \ldots, \forall i=0, \ldots, y_{s}, \forall d=0, \ldots, C .
\end{align*}
$$

Where,

$$
\begin{aligned}
& A_{i, d}= \begin{cases}1 & \text { if } \quad i \neq 0, d<C \\
0 & \text { o.w }\end{cases} \\
& B_{n, i, d}= \begin{cases}1 & \text { if } \quad i \neq 0, d=C, n \geq C \\
0 & \text { o.w }\end{cases} \\
& C_{n, i}= \begin{cases}1 & \text { if } \quad i=0, n<C \\
0 & \text { o.w }\end{cases} \\
& D_{d}= \begin{cases}1 & \text { if } 1 \leq d<C \\
0 & \text { o.w }\end{cases} \\
& E_{n, d}= \begin{cases}1 & \text { if } \quad d=C, n \geq C \\
0 & \text { o.w }\end{cases} \\
& F_{n, i, d}= \begin{cases}1 & \text { if } \quad i \neq 0, n>d \\
0 & \text { o.w }\end{cases} \\
& G_{i}= \begin{cases}1 & \text { if } i=0 \\
0 & \text { o.w }\end{cases} \\
& H_{n, i, d}= \begin{cases}1 & \text { if } \quad i=y_{s}, n=d \\
0 & \text { o.w }\end{cases}
\end{aligned}
$$

$$
\begin{gathered}
I_{i, d}= \begin{cases}1 & \text { if } \quad i \neq 0, d \neq 0 \\
0 & \text { o.w }\end{cases} \\
J_{n, i, d}=\left\{\begin{array}{lll}
1 & \text { if } & i \neq 0, n \neq 0, n \neq d \\
0 & \text { o.w }
\end{array}\right. \\
K_{n, i}= \begin{cases}1 & \text { if } \quad i \neq 0, n>C \\
0 & \text { o.w }\end{cases} \\
L_{i, d}= \begin{cases}1 & \text { if } \\
i=0,1 \leq n \leq C \\
0 & \text { o.w }\end{cases}
\end{gathered}
$$

The left side of Equation 5.1 demonstrates the average transition out of the state ( $n, i, d$ ), while the right side reflects the average transition into the state $(n, i, d)$.

The first three terms present the arrival rate out of the state $(n, i, d)$. The fourth term states if there is at least one customer in the 3D line, there will be a 3D service rate out of the state. The fifth term captures off-the-shelf service rate when inventory level is not zero and at least one customer waiting for off-the-shelf product. In the sixth term, the $G_{i}$ shows if the inventory is depleted after the service of a customer is completed, a replenishment order is instantaneously triggered with the replenishment lead time exponentially distributed with the parameter of $\nu$. On the right-hand side of Equation 5.1, the first four terms indicate the transitions due to satisfying demand by the 3D printer and on-hand inventory of the off-the-shelf product. The fifth and sixth term denote the service rate of off-the-shelf and 3D into
the state $(n, i, d)$ respectively. In the last term, $H_{n, i, d}$ denotes the replenishment order of off-the-shelf product just goes to states $\left(n, y_{s}, d\right)$.

The steady-state probabilities can be solved by the corresponding system of linear equations of the balance equations given in 5.1 and normalizing constraint of 5.2.

$$
\begin{equation*}
\sum_{n=0}^{\infty} \sum_{i=0}^{y_{s}} \sum_{d=0}^{C} \pi_{n, i, d}=1 \tag{5.2}
\end{equation*}
$$

### 5.2.2 The Stochastic Model

In this section, the stochastic model of the retailer's expected profit is presented. The system costs include off-the-shelf procurement cost, off-the-shelf shipping cost, the holding cost of the off-the-shelf product inventories, manufacturing cost of the 3D product, the 3D printer capacity cost and, the lost sale cost of unsatisfied demand. The objective of the model is to maximize the retailer's expected profit by capturing the joint optimal of off-the-shelf ordering quantity and 3D capacity.

Assume that the system runs for an infinite horizon, given the transition diagram of Figure 5.2 and steady-state probabilities of Equation 5.1, the average inventory holding cost of the off-the-shelf product is calculated as:

$$
\begin{equation*}
C_{h}=h \cdot \sum_{n=0}^{\infty} \sum_{i=0}^{y_{s}} \sum_{d=0}^{C} i \cdot \pi_{n, i, d} \tag{5.3}
\end{equation*}
$$

In which, $h$ is the inventory holding cost per product, and the rest of the equation is the expected inventory level of the off-the-shelf products.

In this model, In the case of stock-out, the off-the-shelf customers will shift to the 3D printing queue with a specific probability $\left(\beta_{s}\right)$. While the 3D customers who find the 3D servers busy will turn to off-the-shelf products with another predefined probability $\left(\beta_{d}\right)$. Considering the transition diagram of Figure 5.2 and steady-state
probabilities of Equation 5.1, the average inventory lost sales cost is calculated as:

$$
\begin{align*}
C_{l} & =l \cdot\left(\left(1-\beta_{d}\right) \cdot \lambda_{d} \cdot \sum_{i=1}^{y_{s}} \sum_{n=C}^{\infty} \pi_{n, i, C}+\left(1-\beta_{s}\right) \cdot \lambda_{s} \cdot \sum_{n=0}^{C-1} \pi_{n, 0, n}\right.  \tag{5.4}\\
& \left.+\lambda \cdot \sum_{n=C}^{\infty} \pi_{n, 0, C}\right)
\end{align*}
$$

In which, $l$ is the lost sales cost per item. In Equation 5.4 the first term is 3D customer lost sales cost when all the 3D servers are busy and consumers are not willing to substitute with off-the-shelf products. The second term is lost sales of off-the-shelf customers in the stock-out. While the third term denotes the lost sales cost of both type of products.

As mentioned in Table 5.1, $p_{d}$ is the selling price of the 3 D product and $c_{d}$ is allocated to the 3D printing manufacturing cost. The capacity cost of each 3D printer is denoted as $c_{s e}$ and the retailer provides totally $C 3 \mathrm{D}$ printers. Suppose that service time for producing 3D product follows an Exponential distribution with the mean of $\frac{1}{\mu_{d}}$ and the order rate of 3D customer follows the Poisson distribution with the mean of $\lambda_{d}$. Therefore, the 3D product profit function is defined as:

$$
\begin{equation*}
\phi_{d}=\left(p_{d}-c_{d}\right) \cdot\left(\lambda_{d} \cdot \sum_{n=0}^{\infty} \sum_{i=1}^{y_{s}} \sum_{d=1}^{C-1} \pi_{n, i, d}+\left(\lambda_{d}+\beta_{s} \cdot \lambda_{s}\right) \cdot \sum_{n=0}^{C-1} \pi_{n, 0, n}\right)-C \cdot c_{s e} \tag{5.5}
\end{equation*}
$$

The first term presents the difference between the 3 D price and expected 3 D production cost per part. The second term is the average 3D customers who enter to the system while the off-the-shelf inventory is still available. The third term captures the average arrival rate of 3D customers as well as the off-the-shelf customers who join the 3D line in the stock-out situation. The last term presents the total 3D printers capacity cost.

If the inventory is depleted after the service of a customer is completed, a replenishment order is instantaneously triggered with the replenishment lead time exponentially distributed with the parameter of $\nu$. The size of a replenishment order is deterministic and equal to the optimal ordering quantity. Let's denote $c_{s}$ as the
procurement cost and $c_{s h}$ as the shipping cost per order. Total procurement and shipping cost can be defined as:

$$
\begin{equation*}
C_{s}+C_{s h}=\left(c_{s} \cdot y_{s}+c_{s h}\right) \cdot \nu \cdot\left(\sum_{n=0}^{C} \pi_{n, 0, n}+\sum_{n=C+1}^{\infty} \pi_{n, 0, C}\right) \tag{5.6}
\end{equation*}
$$

The retailer expected profit function can be written as:

$$
\begin{align*}
\Phi\left(y_{s}\right) & =p_{s} \cdot\left(\lambda_{s} \cdot \sum_{n=0}^{\infty} \sum_{i=1}^{y_{s}} \sum_{d=0}^{C} \pi_{n, i, d}+\beta_{d} \cdot \lambda_{d} \cdot \sum_{n=C+1}^{\infty} \sum_{i=1}^{y_{s}} \pi_{n, i, C}\right) \\
& +\left(p_{d}-c_{d}\right) \cdot\left(\lambda_{d} \cdot \sum_{n=0}^{\infty} \sum_{i=1}^{y_{s}} \sum_{d=1}^{C-1} \pi_{n, i, d}+\left(\lambda_{d}+\beta_{s} \cdot \lambda_{s}\right) \cdot \sum_{n=0}^{C-1} \pi_{n, 0, n}\right) \\
& -h \cdot \sum_{n=0}^{\infty} \sum_{i=0}^{y_{s}} \sum_{d=0}^{C} i \cdot \pi_{n, i, d}-l \cdot\left(\left(1-\beta_{d}\right) \cdot \lambda_{d} \cdot \sum_{i=1}^{y_{s}} \sum_{n=C}^{\infty} \pi_{n, i, C}\right) \\
& -l \cdot\left(\left(1-\beta_{s}\right) \cdot \lambda_{s} \cdot \sum_{n=0}^{C-1} \pi_{n, 0, n}+\lambda \cdot \sum_{n=C}^{\infty} \pi_{n, 0, C}\right) \\
& -\left(c_{s} \cdot y_{s}+c_{s h}\right) \cdot \nu \cdot\left(\sum_{n=0}^{C} \pi_{n, 0, n}+\sum_{n=C+1}^{\infty} \pi_{n, 0, C}\right)-C \cdot c_{s e} \tag{5.7}
\end{align*}
$$

The first term in Equation 5.7, computes the retailer's expected sales revenue of off-the-shelf products. The second term denotes the retailer's expected sales profit of 3D products, while the third term corresponds to the holding cost of off-the-shelf unsold inventory. The fourth and the fifth terms show the lost sales cost. The sixth term presents the purchase cost and the shipping cost of off-the-shelf products and the last term is the 3D printers capacity cost.

### 5.3 Benchmark Strategies

In this section, we examine the performance of three possible benchmark strategies:

- providing only off-the-shelf products.
- providing only 3D products.
- Substituting the shortage of off-the-shelf products by 3D products.


### 5.3.1 Providing Only Off-the-shelf Products

In this section, we develop a stochastic model in which the retail store just provides off-the-shelf products. The customer arrival rate to the system follows the Poisson
distribution with the rate of $\lambda$. It is assumed that service time and replenishment lead time follow independent Exponential distributions with means of $\frac{1}{\mu_{s}}$ and $\frac{1}{\nu}$ respectively, which are independent of arrival rate distribution as well.

The system costs include procurement cost, shipping cost, holding cost of unsold inventories, and lost sales cost of unsatisfied demand in the stock-out situation. The objective of the model is to capture the optimal ordering quantity which maximizes the retailer's expected profit.

Here, let $\pi_{n, i}$ be the steady-state probability of having $n$ customers in the system and $i$ items of inventory level. The transition diagram of the model is shown in Figure 5.3.


Figure 5.3 Transition diagram of providing only off-the-shelf products.

The balance equation of the transition diagram is given by Equation 5.8.

$$
\begin{align*}
& \left(A_{i} \cdot \lambda+B_{n, i} \cdot \mu_{s}+\left(1-A_{i}\right) \cdot \nu\right) \cdot \pi_{n, i}=A_{i} \cdot \lambda \cdot \pi_{(n-1), i}+C_{i} \cdot \mu_{s} \cdot \pi_{(n+1),(i+1)} \\
& +\left(1-C_{i}\right) \cdot \nu \cdot \pi_{n, 0} \quad \forall n=0,1,2, \ldots, \forall i=0, \ldots, y_{s} . \tag{5.8}
\end{align*}
$$

Where,

$$
\begin{aligned}
A_{i} & = \begin{cases}0 & \text { if } \quad i=0 \\
1 & \text { o.w }\end{cases} \\
B_{n, i} & =\left\{\begin{array}{lll}
1 & \text { if } & n \neq 0, i \neq 0 \\
0 & \text { o.w }
\end{array}\right. \\
C_{i} & =\left\{\begin{array}{lll}
0 & \text { if } & i=y_{s} \\
1 & \text { o.w }
\end{array}\right.
\end{aligned}
$$

The left side of Equation (5.8) demonstrates the transition out of the state ( $n, i$ ) while, the right side reflects the average transition into the state $(n, i)$. The first term is the consumer arrival rate at the state $(n, i)$, which is zero in the stock-out situation. $B_{n, i}$ in the second term presents that there is no service rate when either the number of customer in the system or inventory level is zero. $\left(1-A_{i}\right)$ in the third term states, the product replenishment just happens when inventory level reaches zero. On the right-hand side of Equation (5.8), the first term captures the arrival rate to the state $(n, i)$, which is zero in the stock-out situation. The second term denotes the service rate to the state, which is zero when inventory level is $y_{s}$. The last term indicates the off-the-shelf replenishment order.

Given the transition diagram in Figure (5.3), and based on the balance equations of the states and normalizing constraint, the probability of being in each state is shown as Proposition 6.

Proposition 6. The Markov process of the case "Retailer only provides off-the-shelf products" is ergodic if and only if $\lambda<\mu_{s}$. The probability of being in each state is
calculated as:

$$
\begin{align*}
& \pi_{n, i}=\left(\frac{\mu_{s}}{\mu_{s}-\lambda} \cdot\left(y_{s}+\frac{\lambda}{\nu}\right)\right)^{-1} \cdot\left(\frac{\lambda}{\mu_{s}}\right)^{n} \quad \forall n=0,1,2, \ldots, \forall i=1, \ldots, y_{s}  \tag{5.9}\\
& \pi_{n, 0}=\left(\frac{\mu_{s}}{\mu_{s}-\lambda} \cdot\left(y_{s}+\frac{\lambda}{\nu}\right)\right)^{-1} \cdot\left(\frac{\lambda}{\mu_{s}}\right)^{n} \cdot \frac{\lambda}{\nu} \quad \forall n=0,1,2, \ldots
\end{align*}
$$

The proof can be found in Appendix F.
The retailer expected profit function is calculated as:

$$
\begin{align*}
\Phi\left(y_{s}\right) & =p_{s} \cdot \lambda \cdot \sum_{n=0}^{\infty} \sum_{i=1}^{y_{s}} \pi_{n, i}-h \cdot \sum_{n=0}^{\infty} \sum_{i=1}^{y_{s}} i \cdot \pi_{n, i}-l \cdot \lambda \cdot \sum_{n=0}^{\infty} \pi_{n, 0}  \tag{5.10}\\
& -c_{s} \cdot y_{s} \cdot \nu \cdot \sum_{n=0}^{\infty} \pi_{n, 0}-c_{s h} \cdot \nu \cdot \sum_{n=0}^{\infty} \pi_{n, 0}
\end{align*}
$$

The first term in Equation (5.10), computes the retailer's expected sales revenue. The second term is the holding cost of the unsold inventory. The third is the lost sales cost in the stock-out situation, the fourth term corresponds to the expected procurement cost and the last term is shipping cost per order.

We present the optimal inventory level for the retailer's profit $\left(y_{s}^{*}\right)$ in the following proposition.

Proposition 7. The optimal order quantity, while the retailer just provides off-theshelf products follows:

$$
y_{s}^{*}=\left\lfloor\frac{\sqrt{\delta}+1}{2}-\frac{\lambda}{\nu}\right\rfloor
$$

where $\delta$ is equal to $\left(\frac{2 \lambda-\nu}{\nu}\right)^{2}+\frac{8 \lambda\left(\left(p_{s}+l-c_{s}\right) \lambda+c_{s h} \nu\right)}{h \nu}$, which is non-negative all the time.
Corollary 7.1. $y_{s}^{*}$ increases in $\lambda$. However, $y_{s}^{*}$ decreases in $\nu$ when $c_{s h}<\frac{1}{2} \cdot\left(p_{s}+\right.$ $\left.l-c_{s}\right) \cdot\left(\frac{\lambda}{h} \cdot\left(p_{s}+l-c_{s}\right)-1\right)$.

See Appendix G for the proof.

### 5.3.2 Providing Only 3D Products

Here, it is assumed that retail store just provides 3D products. The customer arrival rate to the system follows the Poisson distribution with the rate of $\lambda$ and

3D production time follows the independent Exponential distribution with the mean of $\frac{1}{\mu_{d}}$, which is independent of arrival rate distribution as well. We assume because 3 D production is a time-consuming process, consumers will join a queue.

Let $\pi_{n}$ be the steady-state probability of having $n$ customers in the system. The transition diagram of the model is shown in Figure 5.4.


Figure 5.4 Transition diagram of providing only 3D products.

The state probability of being the $n_{t h}$ consumer in the queue is given by Equation 5.11.

$$
\begin{equation*}
\pi_{n}=\frac{\frac{\lambda^{n}}{n!\mu_{d}^{n}}}{\sum_{i=0}^{C} \frac{\lambda^{i}}{i!\mu_{d}^{2}}} \quad \forall n=0,1,2, \ldots, C \tag{5.11}
\end{equation*}
$$

The system costs include 3D printers capacity cost, 3D production cost, and lost sales cost of unsatisfied demand when all the servers are busy. The objective of the model is to capture the optimal number of 3 D printers which maximizes the retailer's expected profit. The retailer expected profit function is calculated as:

$$
\begin{equation*}
\Phi(C)=\left(p_{d}-c_{d}\right) \cdot \lambda \cdot\left(1-\pi_{C}\right)-l \cdot \lambda \cdot \pi_{C}-C \cdot c_{s e} \tag{5.12}
\end{equation*}
$$

The first term in Equation 5.12, computes the retailer's expected sales profit. The second term is the lost sales cost and the last term is 3 D printers capacity cost.

### 5.3.3 Substituting The Shortage of Off-the-shelf Products by 3D Products

In this benchmark model, the retail store provides off-the-shelf products, while they consider 3D products as the substitution items when off-the-shelf products are out of stock. The customer arrival rate to the system follows the Poisson distribution with the rate of $\lambda$. It is assumed that off-the-shelf service time, off-the-shelf replenishment
lead time and 3D production time follow independent Exponential distributions with means of $\frac{1}{\mu_{s}}, \frac{1}{\nu}$ and $\frac{1}{\mu_{d}}$ respectively, which are independent of arrival rate distribution as well.

When off-the-shelf products are out of stock, the products can be substituted by 3D ones and the customers will shift to the 3D printing queue with the probability of $\beta_{s}$. Furthermore, the retail store provides $C$ 3D printers in which they can serve $C$ customers in the queue. When the 3D service reaches its maximum capacity, the rest of the customers will become lost sale.

The system costs include off-the-shelf procurement cost, manufacturing cost of the 3D product, holding cost of unsold inventories, shipping cost per order, the 3D printer capacity cost and lost sales cost of unsatisfied demand in the stock-out situation. The objective of the model is to present the joint optimal ordering quantity of off-the-shelf products and 3D capacity that maximizes the retailer's expected profit.

Here, let $\pi_{n, i, d}$ be the steady-state probability of having $n$ customers in the system, $i$ items of off-the-shelf inventory level and $d$ number of customers in the 3D queue.

The transition diagram of the model with $C$ 3D printers is shown in Figure 5.5. The balance equation of the transition diagram is given in Equation 5.13.

$$
\begin{align*}
& \left(\left(A_{i}+E_{i, d} \cdot \beta_{s}\right) \cdot \lambda+B_{n, i} \cdot \mu_{s}+\left(n E_{n, i}+C \cdot F_{i, d}\right) \cdot \mu_{d}+\left(1-A_{i}\right) \cdot \nu\right) \cdot \pi_{n, i, d}= \\
& \left(A_{i}+D_{n, i} \cdot \beta_{s}\right) \cdot \lambda \cdot \pi_{(n-1), i, d}+\left((n+1) \cdot E_{i, d}+C \cdot F_{i, d}\right) \cdot \mu_{d} \cdot \pi_{(n+1), i,(d+1)}+ \\
& G_{i} \cdot \nu \cdot \pi_{n, 0, n}+\left(1-G_{i}\right) \cdot \mu_{s} \cdot \pi_{(n+1),(i+1), 0} \\
& \forall n=0,1,2, \ldots, \forall i=0, \ldots, y_{s}, \forall d=0, \ldots, C . \tag{5.13}
\end{align*}
$$

Where,

$$
A_{i}= \begin{cases}0 & \text { if } \quad i=0 \\ 1 & \text { o.w }\end{cases}
$$



Figure 5.5 Transition diagram of providing first off-the-shelf then 3D product, with $C$ 3D printers.

$$
\begin{gathered}
B_{n, i}= \begin{cases}1 & \text { if } \quad n \neq 0, i \neq 0 \\
0 & \text { o.w }\end{cases} \\
D_{n, i}=\left\{\begin{array}{lll}
1 & \text { if } & i=0,1 \leq n \leq C \\
0 & \text { o.w }
\end{array}\right. \\
E_{i, d}
\end{gathered}=\left\{\begin{array}{ll}
1 & \text { if } \quad i=0, d<C \\
0 & \text { o.w }
\end{array}\right\} \begin{array}{ll}
1 & \text { if } \quad i=0, d=C \\
0 & \text { o.w }
\end{array}
$$

$$
G_{i}= \begin{cases}1 & \text { if } i=y_{s} \\ 0 & \text { o.w }\end{cases}
$$

The left side of Equation (5.13) demonstrates the transitions out of the state $(n, i, d)$, while, the right side reflects the average transition into the state $(n, i, d)$. The first term presents arrival rate comes out of all the states except when off-the-shelf inventory level reaches zero and all $C$ 3D printers are busy. The second and third terms are the consumer rate from off-the-shelf and 3D products respectively, going out of the state $(n, i, d)$. If the inventory is depleted after the service of a customer is completed, a replenishment order is instantaneously triggered with the replenishment lead time exponentially distributed with the parameter of $\nu$. On the right-hand side of Equation (5.13), the first term captures the transitions due to the satisfying demand by the on-hand inventory of off-the-shelf products or 3D services. The second term states serving a customer with 3D printers. The next term capture the off-the-shelf replenishment order with the size of $y_{s}$. The last term shows the service rate of $\mu_{s}$ coming from the state $(n+1, i+1, d)$ to the state $(n, i, d)$.

Proposition 8. Assuming there is one $3 D$ printer with capacity of serving one customer at a time with the rate of $\mu_{d}$, the Markov process of the model is ergodic if
and only if $\lambda<\mu_{s}$ and the probability of being in each state is calculated as:

$$
\begin{align*}
& \pi_{0, i, 0}=\left(\left(y_{s}+\frac{\lambda}{\nu}\right) \cdot \frac{\mu_{s}}{\mu_{s}-\lambda}+\frac{\lambda}{\mu_{s}} \cdot \frac{\mu_{d}-\mu_{s}}{\nu+\mu_{d}+\lambda} \cdot\left(1+\frac{\lambda}{\nu}\right) \cdot\left(1+\frac{\lambda}{\mu_{d}}\right)+\frac{\lambda}{\mu_{s}} \cdot \frac{\lambda}{\mu_{d}} \cdot \frac{\mu_{s}-\mu_{d}}{\nu}\right)^{-1} \\
& \forall i=1, \ldots, y_{s}-1 \\
& \pi_{0, y_{s}, 0}=\left(1+\frac{\lambda}{\mu_{s}} \cdot \frac{\mu_{d}-\mu_{s}}{\lambda+\nu+\mu_{d}}\right) \cdot \pi_{0, i, 0} \\
& \pi_{0,0,0}=\frac{\lambda}{\nu} \cdot\left(1+\frac{\lambda}{\mu_{s}} \cdot \frac{\mu_{d}-\mu_{s}}{\lambda+\nu+\mu_{d}}\right) \cdot \pi_{0, i, 0} \\
& \pi_{1,0,1}=\frac{\lambda}{\mu_{d}} \cdot\left(\frac{\lambda}{\nu}+\frac{\lambda}{\mu_{s}} \cdot \frac{\mu_{d}-\mu_{s}}{\lambda+\nu+\mu_{d}} \cdot\left(1+\frac{\lambda}{\nu}\right)\right) \cdot \pi_{0, i, 0} \\
& \pi_{n, i, 0}=\left(\frac{\lambda}{\mu_{s}}\right)^{n} \cdot \pi_{0, i, 0} \quad \forall n=1,2, \ldots, \forall i=1, \ldots, y_{s} \\
& \pi_{n, 0,1}=\frac{\lambda}{\nu} \cdot\left(\frac{\lambda}{\mu_{s}}\right)^{n} \cdot \pi_{0, i, 0} \quad \forall n=2,3, \ldots, \forall i=1, \ldots, y_{s} \tag{5.14}
\end{align*}
$$

The retailer expected profit function is calculated as:

$$
\begin{align*}
\Phi\left(y_{s}\right) & =p_{s} \cdot \lambda \cdot \sum_{n=0}^{\infty} \sum_{i=1}^{y_{s}} \pi_{n, i, 0}+\left(p_{d}-c_{d}\right) \cdot \beta_{s} \cdot \lambda \cdot \sum_{n=0}^{C-1} \pi_{n, 0, n} \\
& -h \cdot \sum_{n=0}^{\infty} \sum_{i=1}^{y_{s}} i \cdot \pi_{n, i, 0}-c_{s} \cdot y_{s} \cdot \nu \cdot \sum_{n=0}^{\infty} \pi_{n, 0, n}-c_{s h} \cdot \nu \cdot \sum_{n=0}^{\infty} \pi_{n, 0, n} \\
& -l \cdot\left(\left(1-\beta_{s}\right) \cdot \sum_{n=0}^{C-1} \pi_{n, 0, n}+\sum_{n=C}^{\infty} \pi_{n, 0, C}\right) \cdot \lambda-C \cdot c_{s e} \tag{5.15}
\end{align*}
$$

The first term in Equation (5.15), computes the retailer's expected sales revenue of off-the-shelf products. The second term denotes the retailer's expected sales profit of 3D products. The third term corresponds to the holding cost of the off-the-shelf unsold inventory. While the fourth term capture the expected purchase cost of off-the-shelf products. The fifth term presents shipping cost of off-the-shelf products per order. The sixth term denotes the lost sales cost and the last term is the capacity cost of the 3 D printers.

The optimal ordering quantity for off-the-shelf products, $\left(y_{s}^{*}\right)$, is computed in the following proposition.

Proposition 9. The optimal ordering quantity, while the retailer only provides one 3D printer as a substitution option follows:

$$
y_{s}^{*}=\left\lfloor\sqrt{\delta}+\frac{1}{2}-\gamma\right\rfloor
$$

where,

$$
\begin{gathered}
\gamma=\frac{\lambda}{\nu}+\frac{\mu_{s}-\lambda}{\mu_{s}} \cdot\left(\frac{\lambda\left(\mu_{d}-\mu_{s}\right)}{\mu_{s}\left(\lambda+\nu+\mu_{d}\right)}\left(1+\frac{\lambda}{\nu}\right)\left(1+\frac{\lambda}{\mu_{d}}\right)+\frac{\lambda^{2}\left(\mu_{s}-\mu_{d}\right)}{\nu \mu_{s} \mu_{d}}\right) \\
\delta=\frac{1}{4}+\gamma \cdot\left(\gamma-1+\frac{2 \lambda\left(\mu_{s}-\mu_{d}\right)\left(\mu_{s}-\lambda\right)}{\mu_{s}^{2}\left(\lambda+\nu+\mu_{d}\right)}-\frac{2}{h} \cdot\left(c_{s h} \cdot \nu+l \cdot \lambda-\left(p_{s}-c_{s}\right) \cdot \lambda\right)\right) \\
+\frac{2\left(\mu_{s}-\lambda\right)}{h \mu_{s}} \cdot\left(\frac{\lambda\left(\mu_{d}-\mu_{s}\right)}{\mu_{s}\left(\lambda+\nu+\mu_{d}\right)} \cdot\left(c_{s h} \cdot \nu+l \cdot \lambda-\left(p_{s}-c_{s}\right) \cdot \lambda\right)-\frac{\lambda^{2}}{\nu}\left(1+\frac{\lambda\left(\mu_{d}-\mu_{s}\right)}{\mu_{s}\left(\lambda+\nu+\mu_{d}\right)}\right)\left(p_{d}-\frac{c_{d}}{p}+l\right)\right)
\end{gathered}
$$

### 5.4 Computational Solutions and Managerial Insights

Nowadays, 3D printing technology is adopted by footwear industries to make customfit, sustainably made shoes for every pair of feet. Shape and length of toes, foot arch and heel have impact on the footwear that works best for an individual, which motivate companies to focus on 3D printed orthopedic shoes and insoles.

Feetz Company is one of the pioneers in producing 3D shoes and insoles, which has a partnership with giant shoe retailers such as Designer Shoe Warehouse known as DSW Company. The company has two kiosks at DSW stores, the New York City and the San Fransico branches. They use SizeMe technology to get the scan of consumers feet. The consumer 3 D model will be converted to CAD file as an input to the 3 D printers and 3 D shoes will be produced using 3 D printers. The Feetz Company patented Flex Knit materials to design affordable, comfortable and long-lasting custom-designed shoes.

To derive the numerical result, we combined publicly accessible data and expert views where sufficient data were unavailable. The results are shown in Table 5.2. We choose shoe insoles as our product. The product has a similar off-the-shelf and 3 D price as $\$ 18$ per item. The main purpose of our numerical study is to gain managerial insights through a qualitative investigation of how system parameters affect the performance of the models.

Based on the information from Table 5.2 , the following parameters are used: $p_{s}=\$ 18, c_{s}=\$ 4, c_{s h}=\$ 10, p_{d}=\$ 18, c_{d}=\$ 3, \lambda=18, \mu_{s}=90, \mu_{d}=5, \nu=3$, $h=\$ 3, l=\$ 14$, and $c_{s e}=\$ 12$.

Table 5.2 Cost Breakout of the 3D Products

| 3D printer investment cost | $\$ 7,000$ |
| :--- | :--- |
| 3D printer maintenance cost (per year) | $\$ 1,072$ |
| Software purchase | $\$ 3,000$ |
| Software maintenance and updates cost (per year) | $\$ 1,200$ |
| 3D printer depreciation time | 5 years |
| 3D production time | $90-100$ minutes |
| 3D production rate (per day) | 5 |
| 3D build material | Flex Knit |
| 3D product material cost (per filament) | $\$ 45$ |
| 3D items produced per filament | 15 |

### 5.4.1 Effects of Consumer Preference Rate of Choosing Products ( $\alpha$ )

In this section, the impact of different values of off-the-shelf consumer preference rate, $\alpha$, on optimal retailer's profit, ordering quantity and 3D capacity are investigated. The value is changed between the range of 0 to 1 with the step value of 0.1 and for each value of $\alpha$, joint optimal ordering quantity and 3D capacity and the corresponding retailer's profit are computed. Figure 5.6 demonstrates the optimal retailer's expected profit for four different substitution options scenarios. As a result, when substitution rates are one, the retailer's profit increases monotonically. When 3D consumer substitution rate is less than one, by increasing $\alpha$, the retailer's expected profit will increase. However, the result is counterintuitive when off-the-shelf consumer substitution rate is less than one and 3D consumer substitution rate is one. By increasing $\alpha$, the retailer's expected profit will decrease.

The plots in Figures 5.7 and 5.8 indicate that, when substitution rates are one the optimal 3D capacity increases in off-the-shelf consumer preference rate, while the optimal ordering quantity decreases in $\alpha$. When off-the-shelf consumer substitution
rate is less than one, the 3D capacity decreases while the optimal ordering quantity increases as $\alpha$ increases. When 3D consumer substitution rate is less than one and, all of the off-the-shelf consumers use the substitution option the optimal ordering quantity is non-monotonic, while the optimal 3D capacity decreases in off-the-shelf consumer preference rate.


Figure 5.6 The effect of $\alpha$ on retailer's expected profit.

### 5.4.2 Effects of Product Substitution Rates $\left(\beta_{s}\right.$ and $\left.\beta_{d}\right)$

As mentioned before, the model specifies that, when a stock-out occurs in either product, customers shift to the other product with a known probability. We defined 3 D consumer substitution rate, $\beta_{d}$, as the proportion of the 3 D consumers who will buy the off-the-shelf products when all 3D printers are busy. $\left(1-\beta_{d}\right)$ proportion of the 3D consumers becomes lost sales. The off-the-shelf consumer substitution rate, $\beta_{s}$, is the proportion of the off-the-shelf customers who will join the 3 D queue in the case of off-the-shelf product stock-out and $\left(1-\beta_{s}\right)$ proportion of the off-the-shelf consumers will result in lost sales.


Figure 5.7 The effect of $\alpha$ on optimal ordering quantity.


Figure 5.8 The effect of $\alpha$ on optimal 3D capacity.

In this section, we investigate the impact of substitution rates on the performance of the two strategies of substituting the shortage of off-the-shelf by 3 D products and the main strategy which is providing both options. We consider three different values of the off-the-shelf consumer preference rate, $\alpha=0.25, \alpha=0.5$, and $\alpha=0.75$.

Figures 5.9, 5.10 and 5.11 show that in the main model of providing both options, by increasing $\beta_{s}$, as lost sales cost decreases, retailer's expected profit increases. Moreover, the optimal ordering quantity will decrease and the optimal 3D capacity will increase as more consumers opt to join the 3D services. For the low value of $\beta_{s}$, the highest profit comes from the case with the low off-the-shelf consumer preference rate. By increasing $\beta_{s}$, the highest profit comes from the case with the high off-the-shelf consumer preference rate.


Figure 5.9 The effect of $\beta_{s}$ on retailer's expected profit.

As Figures 5.12, 5.13 and 5.14 demonstrate, in the main model by increasing $\beta_{d}$, as lost sales cost decreases, retailer's expected profit increases. Moreover, the optimal ordering quantity will increase and the optimal 3D capacity will decrease as more consumers opt to buy off-the-shelf products when all of the 3D printers are busy. For the low value of $\beta_{d}$, the highest profit comes from the case with high off-the-shelf consumer preference rate. For a range of $\beta_{d}$ with the medium value, the highest profit comes from the case with the medium off-the-shelf consumer preference rate


Figure 5.10 The effect of $\beta_{s}$ on optimal ordering quantity.


Figure 5.11 The effect of $\beta_{s}$ on optimal 3D capacity.
and for the high $\beta_{d}$, the highest profit comes from the case with the high off-the-shelf consumer preference rate.

As Figure 5.15 presents in the retailer's strategy of providing first off-the-shelf then 3D products by increasing $\beta_{s}$, as lost sales cost decreases, retailer's expected


Figure 5.12 The effect of $\beta_{d}$ on retailer's expected profit.


Figure 5.13 The effect of $\beta_{d}$ on optimal ordering quantity.
profit increases. Moreover, the optimal ordering quantity will decrease and the optimal 3D capacity will increase as more consumers opt to join the 3D services.


Figure 5.14 The effect of $\beta_{d}$ on optimal 3D capacity.


Figure 5.15 The effect of $\beta_{s}$ on the first off-the-shelf then 3D model.

### 5.4.3 Evaluation of Model Parameters

The impact of demand rate $(\lambda)$ : Here, $\beta_{s}=\beta_{d}=1$ and $\alpha=0.5$, other parameters are as described before. As shown in Figure 5.16, by increasing $\lambda$, the retailer's expected profit will increase. When $\lambda$ is small the strategy of providing just off-the-
shelf is optimal. For the medium arrival rate to the system, the optimal strategy is the main strategy that retailer provides consumers the options of selecting either the off-the-shelf product or 3D product. Finally, for the high values of arrival rate the optimal strategy is to substitute the shortage of the off-the-shelf products by 3D printed products. As presented in Figures 5.17 and 5.18, by increasing arrival rate, the optimal 3D capacity will increase. In the same level of 3D capacity, the optimal ordering quantity will increase. However, when the optimal 3D capacity increases the optimal ordering quantity may decrease.


Figure 5.16 The effect of arrival rate of demand on retailer's expected profit.

The impact of replenishment order's parameter ( $\nu$ ): Here, $\beta_{s}=\beta_{d}=1$ and $\alpha=0.5$, other parameters are as described before. As demonstrated in Figure 5.19, by increasing $\nu$, the retailer's expected profit will increase. Figures 5.20 and 5.21 capture that by increasing the off-the-shelf replenishment order's parameter, the optimal 3D capacity will decrease. In the same level of 3D capacity, the optimal ordering quantity will decrease. However, when the optimal 3D capacity decreases the optimal ordering quantity may increase. In the case of all off-the-shelf consumers use the substitution option; for the low values of $\nu$, the strategy of substituting the


Figure 5.17 The effect of arrival rate of demand on optimal ordering quantity.


Figure 5.18 The effect of arrival rate of demand on optimal 3D capacity.
shortage of off-the-shelf products by 3D ones is the optimal strategy. When $\nu$ is high providing just off-the-shelf is the optimal strategy. Otherwise, the main model which is providing options of selecting either off-the-shelf or 3D product is the optimal strategy.


Figure 5.19 The effect of replenishment order parameter on retailer's expected profit.


Figure 5.20 The effect of replenishment order parameter on optimal ordering quantity.

The impact of off-the-shelf service rate $\left(\mu_{s}\right)$ : Here, $\alpha=0.5$, other parameters are as described before. As shown in Figures 5.22, 5.23 and 5.24, the off-the-shelf service rate does not have any impact on retailer's expected profit and optimal ordering quantity of providing just off-the-shelf products strategy.


Figure 5.21 The effect of replenishment order parameter on optimal 3D capacity.

In the retailer's strategy of substituting the shortage of off-the-shelf products by 3D ones, by increasing off-the-shelf service rate the retailer's expected profit, and optimal 3D capacity increase, while the optimal ordering quantity monotonically decreases. On the other hand, in the strategy of providing both options the retailer's expected profit and optimal ordering quantity increase in off-the-shelf service rate, while the optimal 3D capacity decreases. Moreover, the strategy of providing options of selecting either off-the-shelf or 3D product may be optimal for high values of substitution rates as it depends on consumers' substitution behavior.

The impact of 3D production rate $\left(\mu_{d}\right)$ : Here, $\beta_{s}=\beta_{d}=0.5$ and $\alpha=0.5$, other parameters are as described before. As shown in Figures 5.25, 5.26 and 5.27, the 3 D production rate does not have any impact on retailer's expected profit and 3D capacity of providing just 3D services strategy. However, in the retailer's strategy of substituting the shortage of off-the-shelf products by 3 D ones, by increasing 3 D production rate the retailer's expected profit increases and optimal ordering quantity decreases. The counterintuitive results capture in the main model of "providing options of selecting either off-the-shelf or 3D product". In this model, the retailer's


Figure 5.22 The effect of off-the-shelf service rate on retailer's expected profit.


Figure 5.23 The effect of off-the-shelf service rate on optimal ordering quantity.
expected profit increases in 3D production rate, while the model shows inconstancy in optimal ordering quantity and optimal 3D capacity.


Figure 5.24 The effect of off-the-shelf service rate on optimal 3D capacity.


Figure 5.25 The effect of 3D production rate on retailer's expected profit.

The impact of 3D capacity cost $\left(c_{s e}\right)$ : Here, $\beta_{s}=\beta_{d}=1$ and $\alpha=0.5$, other parameters are as described before. Figures 5.28, 5.29 and 5.30 illustrate the impact of 3 D capacity cost on the retailer. They capture that by increasing the 3D


Figure 5.26 The effect of 3D production rate on optimal ordering quantity.


Figure 5.27 The effect of 3D production rate on optimal 3D capacity.
capacity cost, as we expected, the retailer's expected profit will decrease in all the related strategies. Moreover, the optimal ordering quantity monotonically increases, while the optimal 3D capacity decreases.

Moreover, for the low shipping cost, the retailer's strategy of substituting the shortage of off-the-shelf products by 3D ones is optimal when 3D capacity cost is low. Providing just off-the-shelf products is the optimal strategy when 3D capacity cost is high. Otherwise, providing options of selecting either off-the-shelf or 3D product is the optimal strategy. For the high shipping cost, providing just 3D is the optimal strategy when 3D capacity cost is low, providing just off-the-shelf products is the optimal strategy when 3D capacity cost is high. Otherwise, providing options of selecting either off-the-shelf or 3D product is the optimal strategy.


Figure 5.28 The effect of 3D capacity cost on retailer's expected profit.


Figure 5.29 The effect of 3D capacity cost on optimal ordering quantity.


Figure 5.30 The effect of 3D capacity cost on optimal 3D capacity.

## CHAPTER 6

## CONCLUSION AND FUTURE EXTENSIONS

In this dissertation, we evaluate the impact of adopting 3D printing services on the expected profit of retailers by considering different research objectives. It is assumed that retailer provides 3D printing services in the brick-and-mortar stores, in addition to the traditional, off-the-shelf products.

First, we develop a stochastic model combined with a queuing system to maximize the retailer's expected profit, by capturing the optimal 3D product price and 3D capacity. In this model, consumer product selection follows either the Cut-off model or the Multinomial Logit model. Moreover, we extend our model to investigate the effect of consumers having stochastic product valuation on the retailers expected profit and joint optimal 3D product price and 3D capacity. Assuming 3D consumer valuation is higher than off-the-shelf consumer value, we conclude that the optimal retailers expected profit in the cut-off model is higher than the Multinomial Logit model. Furthermore, we conclude that the model with stochastic consumer valuation, the optimal retailers expected profit is lower than the one in the deterministic model.

We also develop a stochastic model combined with the Newsvendor framework to maximize retailer's profit by calculating joint optimal 3D product price and inventory of off-the-shelf product. In this model, it is assumed that the retailer provides off-the-shelf products in the brick-and-mortar store, while, in the case of stock-out the 3 D printing service can be the substitution option. It is assumed that consumers have product selection behavior in choosing the substitution option, which follows the Cut-off model. We present both analytical and numerical results to presents the joint optimal 3D product price and inventory of off-the-shelf product. Comparing this results which the case of no substitution option demonstrates with providing 3D
services as a substitution option, retailer gains higher profit by providing a lower level of off-the-shelf products. Furthermore, by increasing either the holding or the lost sales cost, the difference between the retailer's expected profit with the substitution option and that without the substitution option increases. By increasing the customer arrival rate, the retailer's expected profits with and without the substitution option increase, while the difference between these two decreases.

Finally, we develop a queuing-inventory model in which the system receives a stochastic demand from two customer segments: those who prefer the off-the-shelf product versus those who prefer the 3D product. The demand for the off-the-shelf product is met with the on-hand inventory, while the demand for 3D printing product is fulfilled through the 3D printer. As 3D printing process is a time consuming one, customers need to join a queue. In the case of stock-out, the off-the-shelf customers will turn to the 3D printing queue with a specific probability. While the 3D customers who reach the maximum capacity of the system will shift to off-the-shelf products with another predefined probability. We analyze this model and compare it with three different benchmark strategies: (i) the retailer just provides off-the-shelf products, (i) the retailer just provides 3D products, (iii) the retailer provides first off-the-shelf products and in the stock-out situation the 3D service is an option. In essence, these queue frameworks assist decision makers in both capacity planning and inventory management. Additionally, we characterize the conditions under which each of the strategies aforementioned will be optimal. Just off-the-shelf strategy is optimal when demand rate is low, or replenishment lead time is low, or 3D production rate is low, or 3D capacity cost is high. Just 3D is optimal when replenishment lead time is high and $\beta_{s} \neq 1$, or 3D capacity cost is low but shipping cost is high. First off-the-shelf strategy is optimal when demand rate is high and shipping cost is low, or replenishment lead time is high and $\beta_{s}=1$, or 3D capacity cost and shipping cost are low. Otherwise, both options is the optimal strategy.

This dissertation has laid the groundwork for several future research opportunities. In our models, we assume there is one type of off-the-shelf product, which can also be produced using 3D technology. This assumption can expand to multi-product problem. It is assumed that consumer arrival rate to the system follows the Poisson distribution. Moreover, the off-the-shelf service time, 3D production time and replenishment lead time follow independent Exponential distribution. A future extension is to consider other distributions for either of these system parameters such as the Normal distribution. In the last model, it is assumed that queuing system of off-the-shelf products has infinite capacity. A natural extension is to consider a finite capacity for off-the-shelf customers in the store as well. Moreover, in the last model, it is assumed that customer preference rate of choosing a product is deterministic, which can be expanded to consumer product choice models.

## APPENDIX A

## PROOF OF CONDITIONAL WAITING TIME IN THE QUEUING SYSTEM

The conditional waiting time in the queuing system of $\mathrm{M} / \mathrm{M} / 1$ is calculated as the following:

$$
\begin{aligned}
& \quad W_{q}(t \mid n)=p[\mathrm{n} \text { completion } \leq t \mid \text { arrival found } \mathrm{n} \text { in system }] \\
& \quad=\int_{0}^{t} \frac{\mu(\mu x)^{n-1} \cdot \cdot^{-\mu x}}{(n-1)!} d x \\
& =\frac{(\mu)^{n}}{(n-1)!} \int_{0}^{t} x^{n-1} e^{-\mu x} d x \\
& =\frac{\mu^{n}}{(n-1)!} \cdot \frac{e^{-\mu x}}{-\mu} \cdot \sum_{i=0}^{n} \frac{t^{n-i} \cdot(n-1)!}{\mu^{i-1}(n-i)!}+1 \\
& \quad=1-\sum_{i=0}^{n} \frac{t^{n-i} \cdot e^{-\mu t} \mu^{n-i}}{(n-i)!} \\
& E\left(w_{q}(t \mid n)\right)=\int_{0}^{\infty} t \cdot \frac{d W_{q}(t \mid n)}{d t} \\
& E\left(w_{q}(t \mid n)\right)=\int_{0}^{\infty} t \cdot \frac{d\left(1-\sum_{i=0}^{n} \frac{t^{n-i} \cdot e^{-\mu t} \mu^{n-i}}{(n-i)!}\right)}{d t} \\
& =\sum_{i=0}^{n} \frac{\mu^{n-i}}{(n-i)!} \int_{0}^{\infty} t \cdot\left(\mu e^{-\mu t} \cdot t^{n-i}-(n-i) \cdot t^{n-i-1} \cdot e^{-\mu t}\right) d t \\
& =\sum_{i=0}^{n} \int_{0}^{\infty}\left(\frac{\mu^{n-i+1}}{(n-i)!} t^{n-i+1} \cdot e^{-\mu t}-\frac{\mu^{n-i}}{(n-i-1)!} e^{-\mu t} \cdot t^{n-i}\right) d t \\
& =\sum_{i=0}^{n}\left(\frac{\mu^{n-i+1}}{(n-i)!} \int_{0}^{\infty} t^{n-i+1} \cdot e^{-\mu t} \cdot d t-\frac{\mu^{n-i}}{(n-i-1)!} \int_{0}^{\infty} e^{-\mu t} \cdot t^{n-i} \cdot d t\right) \\
& =\left.\sum_{i=0}^{n} \frac{\mu^{n-i+1}}{(n-i)!} \cdot \frac{e^{-\mu t}}{-\mu}\left(t^{n-i+1}-\frac{(n-i+1) t^{n-i}}{-\mu}+\ldots+\frac{(-1)^{n-i+1}(n-i+1)!}{-\mu^{n-i+1}}\right)\right|_{0} ^{\infty} \\
& - \\
& =\left.\frac{\mu^{n-i}}{(n-i-1)!} \cdot \frac{e^{-\mu t}}{-\mu}\left(t^{n-i}-\frac{(n-i) t^{n-i-1}}{-\mu}+\ldots+\frac{(-1)^{n-i}(n-i)!}{-\mu^{n-i}}\right)\right|_{0} ^{\infty} \\
& =\sum_{i=1}^{n}\left(\frac{n-i+1}{\mu}-\frac{n-i}{\mu}\right)=\frac{n}{\mu}
\end{aligned}
$$

With C number of 3D printers and failure rate of p for each 3 D printer, the conditional waiting time in the system is calculated as:

$$
\begin{aligned}
& W_{q}(t \mid n)=p[\mathrm{n}-\mathrm{c}+1 \text { completion } \leq t \mid \text { arrival found } \mathrm{n} \text { in system }] \\
& =\int_{0}^{t} \frac{c \mu(c \mu x)^{n-c} \cdot e^{-c \mu x}}{(n-c)!} d x \\
& =1-\int_{t}^{\infty} \frac{c \mu(c \mu x)^{n-c} \cdot e^{-c \mu x}}{(n-c)!} d x \\
& =1-\sum_{i=0}^{n-c} \frac{t^{i} \cdot e^{-c \mu t}(c \mu)^{i}}{i!}
\end{aligned}
$$

Similar to the previous calculation, the result can be computed as $E\left(w_{q}(t \mid n)\right)=\frac{n-C}{C \cdot \mu \cdot p}$.

$$
W_{q}(t \mid n)=\left\{\begin{array}{l}
\frac{1}{\mu \cdot p} \quad \text { if } n \leq C \\
\frac{1}{\mu \cdot p}+\frac{n-C}{C \cdot \mu \cdot p}=\frac{n}{C \cdot \mu \cdot p} \quad \text { if } n>C
\end{array}\right.
$$

## APPENDIX B

## PROOF OF PROPOSITION 1

When the retailer provides one 3 D printer, the optimal 3 D product price, $p_{d}^{*}$, is calculated as:

$$
\begin{aligned}
& \Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}+\epsilon} \rightarrow \\
& \lambda \cdot T \cdot\left(\left(p_{d}{ }^{*}-\frac{c_{d}}{p}\right) \cdot\left(1-\pi_{K^{*}}\right)+\pi_{K^{*}} \cdot\left(p_{s}-c_{s}\right)\right)-c_{s e}> \\
& \lambda \cdot T \cdot\left(\left(p_{d}{ }^{*}+\epsilon-\frac{c_{d}}{p}\right) \cdot\left(1-\pi_{K^{*}-1}\right)+\pi_{K^{*}-1} \cdot\left(p_{s}-c_{s}\right)\right)-c_{s e} \\
& \rightarrow p_{d}^{*}>\frac{c_{d}}{p}-c_{s}+p_{s}+\frac{c_{w} \cdot\left(1-\rho^{\left(K^{*}+1\right)}\right) \cdot\left(1-\rho^{\left(K^{*}-1\right)}\right)}{\mu \cdot p \cdot\left(1-\rho^{2}\right) \cdot \rho^{\left(K^{*}-1\right)}} \\
& \Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}-\epsilon} \rightarrow \\
& \\
& \lambda \cdot T \cdot\left(\left(p_{d}^{*}-\frac{c_{d}}{p}\right) \cdot\left(1-\pi_{K^{*}}\right)+\pi_{K^{*}} \cdot\left(p_{s}-c_{s}\right)\right)-c_{s e}> \\
& \lambda \cdot T \cdot\left(\left(p_{d}^{*}-\epsilon-\frac{c_{d}}{p}\right) \cdot\left(1-\pi_{K^{*}-1}\right)+\pi_{K^{*}-1} \cdot\left(p_{s}-c_{s}\right)\right)-c_{s e} \\
& \rightarrow p_{d}^{*}<\frac{c_{d}}{p}-c_{s}+p_{s}+\frac{c_{w} \cdot\left(1-\rho^{\left.\left(K^{*}+1\right)\right)^{2}}\right.}{\mu \cdot p \cdot\left(1-\rho^{2}\right) \cdot \rho^{K^{*}}}
\end{aligned}
$$

The optimal 3D product price, $p_{d}^{*}$, satisfies:

$$
\frac{c_{d}}{p}-c_{s}+p_{s}+\frac{c_{w} \cdot\left(1-\rho^{\left(K^{*}+1\right)}\right) \cdot\left(1-\rho^{\left(K^{*}-1\right)}\right)}{\mu \cdot p \cdot(1-\rho)^{2} \cdot \rho^{\left(K^{*}-1\right)}}<p_{d}^{*}<\frac{c_{d}}{p}-c_{s}+p_{s}+\frac{c_{w} \cdot\left(1-\rho^{\left(K^{*}+1\right)}\right)^{2}}{\mu \cdot p \cdot(1-\rho)^{2} \cdot \rho^{K^{*}}}
$$

For the general case, that retailer provides the number of C 3D printers, the optimal 3 D product price that maximizes the retailer expected profit function is calculated
as:

$$
\begin{aligned}
& \Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}+\epsilon} \rightarrow \\
& \lambda \cdot T \cdot\left(\left(p_{d}{ }^{*}-\frac{c_{d}}{p}\right) \cdot\left(1-\pi_{K^{*}}\right)+\pi_{K^{*}} \cdot\left(p_{s}-c_{s}\right)\right)-C \cdot c_{s e}> \\
& \lambda \cdot T \cdot\left(\left(p_{d}{ }^{*}+\epsilon-\frac{c_{d}}{p}\right) \cdot\left(1-\pi_{K^{*}-1}\right)+\pi_{K^{*}-1} \cdot\left(p_{s}-c_{s}\right)\right)-C \cdot c_{s e} \\
& \rightarrow p_{d}^{*}>\frac{c_{d}}{p}-c_{s}+p_{s}-\frac{c_{w} \cdot\left(1-\pi_{\left(K^{*}-1\right)}\right)}{C \cdot \mu \cdot p \cdot\left(\pi_{K^{*}}-\pi_{\left(K^{*}-1\right)}\right)} \\
& \Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}-\epsilon} \rightarrow \\
& \lambda \cdot T \cdot\left(\left(p_{d}{ }^{*}-\frac{c_{d}}{p}\right) \cdot\left(1-\pi_{K^{*}}\right)+\pi_{K^{*}} \cdot\left(p_{s}-c_{s}\right)\right)-C \cdot c_{s e}> \\
& \lambda \cdot T \cdot\left(\left(p_{d}{ }^{*}-\epsilon-\frac{c_{d}}{p}\right) \cdot\left(1-\pi_{K^{*}-1}\right)+\pi_{K^{*}-1} \cdot\left(p_{s}-c_{s}\right)\right)-C \cdot c_{s e} \\
& \rightarrow p_{d}^{*}<\frac{c_{d}}{p}-c_{s}+p_{s}-\frac{c_{w} \cdot\left(1-\pi_{\left(K^{*}+1\right)}\right)}{C \cdot \mu \cdot p \cdot\left(\pi_{\left(K^{*}+1\right)}-\pi_{K^{*}}\right)}
\end{aligned}
$$

So the optimal 3D product price, $p_{d}^{*}$, satisfies:

$$
\frac{c_{d}}{p}-c_{s}+p_{s}-\frac{c_{w} \cdot\left(1-\pi_{\left(K^{*}-1\right)}\right)}{C \cdot \mu \cdot p \cdot\left(\pi_{K^{*}}-\pi_{\left(K^{*}-1\right)}\right)}<p_{d}^{*}<\frac{c_{d}}{p}-c_{s}+p_{s}-\frac{c_{w} \cdot\left(1-\pi_{\left(K^{*}+1\right)}\right)}{C \cdot \mu \cdot p \cdot\left(\pi_{\left(K^{*}+1\right)}-\pi_{K^{*}}\right)}
$$

In the $\mathrm{M} / \mathrm{M} / \mathrm{C} / \mathrm{K}$ queuing system, $\pi_{n}$ is the probability of being the $n_{t h}$ customer in the queue, which follows the below equations:

$$
\begin{aligned}
& \pi_{K}=\frac{1}{C!\cdot C^{K-c}} \cdot\left(\frac{\lambda}{\mu}\right)^{K} \cdot\left[\sum_{n=0}^{C-1} \frac{1}{n!} \cdot\left(\frac{\lambda}{\mu}\right)^{n}+\frac{\left(\frac{\lambda}{\mu}\right)^{C} \cdot\left(1-\left(\frac{\lambda}{c \mu}\right)^{K-C+1}\right)}{c!\cdot\left(1-\frac{\lambda}{c \mu}\right)}\right]^{-1} \\
& \pi_{K-1}=\frac{1}{C!\cdot C^{K-1-c}} \cdot\left(\frac{\lambda}{\mu}\right)^{K-1} \cdot\left[\sum_{n=0}^{C-1} \frac{1}{n!} \cdot\left(\frac{\lambda}{\mu}\right)^{n}+\frac{\left(\frac{\lambda}{\mu}\right)^{C} \cdot\left(1-\left(\frac{\lambda}{c \mu}\right)^{K-C}\right)}{c!\cdot\left(1-\frac{\lambda}{c \mu}\right)}\right]^{-1} \\
& \pi_{K+1}=\frac{1}{C!\cdot C^{K+1-c}} \cdot\left(\frac{\lambda}{\mu}\right)^{K+1} \cdot\left[\sum_{n=0}^{C-1} \frac{1}{n!} \cdot\left(\frac{\lambda}{\mu}\right)^{n}+\frac{\left(\frac{\lambda}{\mu}\right)^{C} \cdot\left(1-\left(\frac{\lambda}{c \mu}\right)^{K-C+2}\right)}{c!\cdot\left(1-\frac{\lambda}{c \mu}\right)}\right]^{-1}
\end{aligned}
$$

## APPENDIX C

## PROOF OF PROPOSITION 2

The first order derivatives of the retailer's expected profit function with respect to the aforementioned decision variables are calculated as:

$$
\begin{aligned}
\frac{d \Phi}{d p_{d}}= & \lambda \cdot T \cdot q \cdot \bar{q} \cdot\left(p_{s}-c_{s}-p_{d}+\frac{c_{d}}{p}+1+e^{v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}-v_{s}+p_{s}}\right)=0 \\
& p_{d}^{*}=p_{s}-c_{s}+\frac{c_{d}}{p}+1+e^{v_{d}-p_{d}^{*}-\frac{c_{w}}{C \cdot \mu \cdot p}-v_{s}+p_{s}} \\
\frac{d \Phi}{d C}= & \lambda \cdot T \cdot\left(\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q \cdot \bar{q} \cdot\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)\right)-c_{s e}=0 \\
& C^{* 2} e^{\frac{c_{w}}{\mu \cdot p \cdot c^{*}}}\left(e^{v_{d}-p_{d}-\frac{c_{w}}{C^{*} \cdot \mu \cdot p}}+e^{v_{s}-p_{s}}\right)^{2}=\frac{\lambda \cdot T \cdot c_{w}}{c_{s e} \cdot \mu} e^{v_{d}-p_{d}+v_{s}-p_{s}}\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)
\end{aligned}
$$

The second order derivatives of the retailer's expected profit function respect to the decision variables result in the below equations:

$$
\begin{aligned}
\frac{d \Phi^{2}}{d p_{d}} & =\lambda \cdot T \cdot\left[-q \cdot \bar{q}^{2} \cdot\left(p_{s}-c_{s}-p_{d}+\frac{c_{d}}{p}+1+e^{v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}-v_{s}+p_{s}}\right)\right. \\
& +q^{2} \cdot \bar{q} \cdot\left(p_{s}-c_{s}-p_{d}+\frac{c_{d}}{p}+1+e^{v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}-v_{s}+p_{s}}\right) \\
& \left.+q \cdot \bar{q} \cdot\left(-1-e^{v_{d}-p_{d}-\frac{c_{w}}{C \cdot \cdot \cdot \cdot p}-v_{s}+p_{s}}\right)\right] \\
& =\lambda \cdot T \cdot q \cdot \bar{q}\left[(-1+2 q)\left(p_{s}-c_{s}-p_{d}+\frac{c_{d}}{p}+1+e^{v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu^{2} \cdot p}-v_{s}+p_{s}}\right)\right. \\
& \left.-1-e^{v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}-v_{s}+p_{s}}\right] \\
& =\lambda \cdot T \cdot q \cdot \bar{q}\left[-1-e^{v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}-v_{s}+p_{s}}\right] \leq 0 \\
\frac{d \Phi^{2}}{d C^{2}} \quad & \lambda \cdot T \cdot \frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q \cdot \bar{q}\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right) \cdot\left(-2 C^{-1}+\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot(1-2 q)\right) \\
& =c_{s e} \cdot C^{-2} \cdot\left(-2 C+\frac{c_{w}}{\mu \cdot p} \cdot(1-2 q)\right) \\
\frac{d \Phi^{2}}{d p_{d} d C} & =\frac{\lambda \cdot T \cdot c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q \cdot \bar{q}\left[\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right) \cdot(1-2 q)+1\right]
\end{aligned}
$$

Observe that the second order derivative of the retailer's expected profit function respect to the 3D printing products' price is nonpositive all the time, the other
negative definite conditions are calculated as below:

$$
\begin{align*}
& \frac{d \Phi^{2}}{d p_{d}{ }^{2}} \cdot \frac{d \Phi^{2}}{d C^{2}}-\left(\frac{d \Phi^{2}}{d p_{d} d C}\right)^{2}>0 \\
& \frac{-\lambda \cdot T}{\mu^{2} \cdot p^{2} \cdot C^{* 2}} \cdot\left(3 q^{3}-5 q^{2}\right)+2 q \cdot\left(\frac{c_{w} \cdot c_{s e}}{\mu \cdot p}-\frac{\lambda \cdot T}{\mu^{2} \cdot p^{2} \cdot C^{* 2}}\right)+2 C^{*} \cdot c_{s e}-\frac{c_{w} \cdot c_{s e}}{\mu \cdot p}>0  \tag{C.1}\\
& \frac{-\lambda \cdot T}{\mu^{2} \cdot p^{2} \cdot C^{* 2}} \cdot q\left(3 q^{2}-5 q+2\right)+\frac{c_{w} \cdot c_{s e}}{\mu \cdot p}(2 q-1)+2 C^{*} \cdot c_{s e}>0
\end{align*}
$$

The relaxed condition is: $\quad \frac{2}{3} \leq q \leq 1$.

So under the condition of $\frac{2}{3} \leq q \leq 1$, the retailer's profit function is concave and the optimal price for 3D printing products and the optimal number of 3D printers follow the below equations.

$$
\begin{gathered}
p_{d}^{*}=p_{s}-c_{s}+\frac{c_{d}}{p}+1+e^{v_{d}-p_{d}{ }^{*}-\frac{c_{w}}{C \cdot \mu \cdot p}-v_{s}+p_{s}} \\
C^{* 2} \cdot e^{\frac{c_{w}}{\mu \cdot p \cdot c^{*}}} \cdot\left(e^{v_{d}-p_{d}-\frac{c_{w}}{C^{*} \cdot \mu^{\prime} \cdot p}}+e^{v_{s}-p_{s}}\right)^{2}=\frac{\lambda \cdot T \cdot c_{w}}{c_{s e} \cdot \mu} \cdot e^{v_{d}-p_{d}+v_{s}-p_{s}} \cdot\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)
\end{gathered}
$$

The Multinomial Logit Model Considering No Purchase Option:
Let's $q_{d}$ defines the customer's preference probability or market share of choosing 3 D printing product, $q_{s}$ defines market share of off-the-shelf product and $q_{n}$ defines customer's probability of purchasing nothing.

$$
\begin{aligned}
& q_{d}=\frac{e^{\left(v_{d}-p_{d}-\frac{c_{w}}{c \cdot \mu \cdot p}\right)}}{1+e^{\left(v_{d}-p_{d}-\frac{c}{c} \cdot{ }^{c} \cdot p \cdot p\right)}+e^{\left(v_{s}-p_{s}\right)}} \\
& q_{s}=\frac{e^{\left(v_{s}-p_{s}\right)}}{1+e^{\left(v_{d}-p_{d}-\frac{c}{c} \cdot \mu \cdot p\right)}+e^{\left(v_{s}-p_{s}\right)}} \\
& q_{n}=\frac{1}{1+e^{\left(v_{d}-p_{d}-\frac{c_{\omega}}{C \cdot \mu \cdot p)}\right.}+e^{\left(v_{s}-p_{s}\right)}}
\end{aligned}
$$

The derivatives of the market shares with respect to the 3D product price and total number of 3D printers are calculated as below:

$$
\begin{aligned}
& \frac{d q_{d}}{d p_{d}}=-q_{d} \cdot\left(1-q_{d}\right) \\
& \frac{d q_{s}}{d p_{d}}=q_{d} \cdot q_{s} \\
& \frac{d q_{n}}{d p_{d}}=q_{d} \cdot q_{n} \\
& \frac{d q_{d}}{d C}=\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q_{d} \cdot\left(1-q_{d}\right) \\
& \frac{d q_{s}}{d C}=-\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q_{d} \cdot q_{s} \\
& \frac{d q_{n}}{d C}=-\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q_{d} \cdot q_{n}
\end{aligned}
$$

In order to find the retailers optimal joint decisions on 3D printing products' pricing and the total number of 3 D printers, the first order derivatives of the retailer's expected profit function with respect to aforementioned decision variables are taken, the results are shown in the below equations:

$$
\begin{aligned}
& \frac{d \Phi}{d p_{d}}=\lambda \cdot T \cdot q_{d} \cdot\left(-\left(1-q_{d}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)+q_{s} \cdot\left(p_{s}-c_{s}\right)+1\right)=0 \\
& \left(q_{s}+q_{n}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)=q_{s} \cdot\left(p_{s}-c_{s}\right)+1 \\
& q_{n} \cdot\left(1+e^{\left(v_{s}-p_{s}\right)}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)=q_{n} \cdot e^{\left(v_{s}-p_{s}\right)} \cdot\left(p_{s}-c_{s}\right)+1 \\
& \left(1+e^{\left(v_{s}-p_{s}\right)}\right) \cdot p_{d}=\left(1+e^{\left(v_{s}-p_{s}\right)}\right) \cdot \frac{c_{d}}{p}+e^{\left(v_{s}-p_{s}\right)} \cdot\left(p_{s}-c_{s}\right)+\left(q_{n}\right)^{-1} \\
& p_{d}\left(1+e^{\left(v_{s}-p_{s}\right)}\right)-1-e^{\left(v_{d}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}-e^{\left(v_{s}-p_{s}\right)}=\frac{c_{d}}{p}\left(1+e^{\left(v_{s}-p_{s}\right)}\right)+\left(p_{s}-c_{s}\right) e^{\left(v_{s}-p_{s}\right)} \\
& p_{d}^{*}-\frac{e^{\left(v_{d}-c_{w} \cdot c_{u}\right)}}{\left(1+e^{\left(v_{s}-p_{s}\right)}\right)} \cdot e^{-p_{d}^{*}}=1+\frac{c_{d}}{p}+\frac{p_{s}-c_{s}}{\left(1+e^{\left(v_{s}-p_{s}\right)}\right)} \\
& \quad \frac{d \Phi}{d C}=\lambda \cdot T \cdot\left(\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q_{d} \cdot\left(\left(1-q_{d}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)-q_{s} \cdot\left(p_{s}-c_{s}\right)\right)\right)-c_{s e}=0 \\
& \quad \frac{\lambda T c_{w}}{\mu p c_{s e}}\left(\left(p_{d}-\frac{c_{d}}{p}\right)\left(1+e^{v_{s}-p_{s}}\right)-\left(p_{s}-c_{s}\right) e^{v_{s}-p_{s}}\right)=\frac{C^{2}\left(1+e^{v_{d}-p_{d}-\frac{c_{w}}{C \mu p}}+e^{v_{s}-p_{s}}\right)^{2}}{e^{\left(v_{d}-p_{d}-\frac{c_{w}}{C C_{p}}\right)}} \\
& \quad C^{* 2} \cdot e^{\frac{C^{*} \mu p}{c_{w}}} \cdot\left(1+e^{\left(v_{d}-p_{d}-\frac{c_{w}}{C^{*} \mu_{p}}\right)}+e^{\left(v_{s}-p_{s}\right)}\right)^{2}= \\
& \quad \frac{\lambda \cdot T \cdot c_{w} \cdot e^{\left(v_{d}-p_{d}\right)}}{\mu \cdot p \cdot c_{s e}} \cdot\left[\left(1+e^{\left(v_{s}-p_{s}\right)}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)-e^{\left(v_{s}-p_{s}\right)} \cdot\left(p_{s}-c_{s}\right)\right]
\end{aligned}
$$

Moreover, combination of these two equations provides the following optimal result:

$$
q_{d}{ }^{*}=\frac{c_{s e} \cdot \mu \cdot p \cdot C^{* 2}}{\lambda \cdot T \cdot c_{w}}
$$

Taking the second order derivatives of retailer's expected profit function with respect to the decision variables result in the below equations:

$$
\begin{aligned}
\frac{d \Phi^{2}}{d p_{d}{ }^{2}}= & \lambda T\left[-\left(p_{d}-\frac{c_{d}}{p}\right) \cdot\left[-q_{d}\left(1-q_{d}\right)^{2}+q_{d} \cdot\left(q_{d} \cdot\left(q_{s}+q_{n}\right)\right)\right]-q_{d} \cdot\left(1-q_{d}\right)\right. \\
+ & \left.\left(p_{s}-c_{s}\right) \cdot\left[-q_{d} \cdot q_{s} \cdot\left(1-q_{d}\right)+q_{d}^{2} \cdot q_{s}\right]+1\right] \\
= & \left(2 q_{d}-1\right)\left(-q_{d} \cdot\left(1-q_{d}\right)\left(p_{d}-\frac{c_{d}}{p}\right)+q_{d} \cdot q_{s} \cdot\left(p_{s}-c_{s}\right)\right)-q_{d} \cdot\left(1-q_{d}\right) \\
= & -q_{d}^{2} \leq 0 \\
\frac{d \Phi^{2}}{d C^{2}}= & \frac{\lambda \cdot T \cdot c_{w}}{\mu \cdot p \cdot C^{2}}\left[-2 \cdot C^{-1} \cdot q_{d} \cdot\left(\left(1-q_{d}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)-q_{s} \cdot\left(p_{s}-c_{s}\right)\right)\right. \\
& +\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q_{d} \cdot\left(1-q_{d}\right) \cdot\left(\left(1-q_{d}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)-q_{s} \cdot\left(p_{s}-c_{s}\right)\right) \\
& \left.-\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q_{d}^{2} \cdot\left(\left(1-q_{d}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)-q_{s} \cdot\left(p_{s}-c_{s}\right)\right)\right] \\
= & c_{s e} \cdot q_{d}^{-1} \cdot\left(-2 C^{-1} \cdot q_{d}+\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot\left(q_{d} \cdot\left(1-q_{d}\right)-q_{d}^{2}\right)\right) \\
& =\frac{-2 \cdot \cdot_{s e}}{C}+\frac{c_{w} \cdot c_{s e} \cdot\left(1-2 q_{d}\right)}{\mu \cdot p \cdot C^{2}} \\
\frac{d \Phi^{2}}{d C d p_{d}}= & \frac{\lambda \cdot T \cdot c_{w}}{\mu \cdot p \cdot C^{2}}\left[\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q_{d} \cdot\left(1-q_{d}\right) \cdot\left(\left(1-q_{d}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)-q_{s} \cdot\left(p_{s}-c_{s}\right)\right)\right. \\
= & \left.\frac{c_{w}}{\mu \cdot p \cdot C^{2}} \cdot q_{d}^{2} \cdot\left(\left(1-q_{d}\right) \cdot\left(p_{d}-\frac{c_{d}}{p}\right)-q_{s} \cdot\left(p_{s}-c_{s}\right)\right)\right] \\
= & \frac{c_{w} \cdot c_{s e}}{\mu \cdot p \cdot C^{2} \cdot q_{d}}\left[q_{d} \cdot\left(1-q_{d}\right)-q_{d}^{2}\right] \\
= & \frac{c_{w} \cdot c_{s e}}{\mu \cdot p \cdot C^{2}} \cdot\left(1-2 q_{d}\right)
\end{aligned}
$$

Observe that the second order derivative of retailer's expected profit function with respect to the 3D printing products' price is negative or equal to zero all the time, the other negative definite condition is calculated as below:

$$
\begin{aligned}
& \frac{d \Phi^{2}}{d p_{d}} \cdot \frac{d \Phi^{2}}{d C^{2}}-\left(\frac{d \Phi^{2}}{d p_{d} d C}\right)^{2}>0 \\
& -\frac{q_{d}^{*}}{2} \cdot\left(-2 \cdot c_{s e} \cdot C^{*-1}+\frac{c_{w} \cdot c_{s e} \cdot\left(1-2 q_{d}^{*}\right)}{\mu \cdot p \cdot C^{* 2}}\right)-\left(\frac{c_{w} \cdot c_{s e} \cdot\left(1-2 q_{d}^{*}\right)}{\mu \cdot p \cdot C^{* 2}}\right)^{2}>0 \\
& -\frac{q_{d}^{*}}{2} \cdot\left(\frac{-2 \cdot c_{s e} \cdot \mu^{2} \cdot \cdot^{2} \cdot C^{* 4}}{C \cdot c_{w}^{2} \cdot c_{s e}^{2} \cdot\left(1-2 q_{d}^{*}\right)^{2}}\right)+\frac{\mu \cdot p \cdot C^{* 2}}{c_{w} \cdot c_{s e} \cdot\left(1-2 q_{d}^{*}\right)}>1 \\
& \frac{q_{d}^{*} \cdot \lambda \cdot T}{c_{s e}^{2} \cdot\left(1-2 q_{d}^{*}\right)} \cdot\left(\frac{2 q_{d}^{*} \cdot \lambda \cdot \lambda}{C^{*} \cdot c_{s e} \cdot\left(1-2 q_{d}^{*}\right)}+1\right)>1 \\
& \left(2 q_{d}^{* 2} \cdot \lambda \cdot T+C^{*} \cdot c_{s e} \cdot\left(1-2 q_{d}^{*}\right)\right) \cdot \frac{q_{d}^{*}}{C \cdot\left(1-2 q_{d}^{* 2}\right)}>\frac{c_{s e}^{3}}{\lambda \cdot T}
\end{aligned}
$$

So under the condition of $\left(2 q_{d}^{* 2} \cdot \lambda \cdot T+C^{*} \cdot c_{s e} \cdot\left(1-2 q_{d}^{*}\right)\right) \cdot \frac{q_{d}^{*}}{C \cdot\left(1-2 q_{d}^{* 2}\right)}>\frac{c_{s e}^{3}}{\lambda \cdot T}$, the retailer's profit function is concave and the optimal price for 3D printing products and the optimal number of 3D printers follow the below equations.

$$
\begin{gathered}
p_{d}^{*}-\frac{e^{\left(v_{d}-\frac{c_{w}}{C^{*} \cdot \mu \cdot p}\right)}}{\left(1+e^{\left(v_{s}-p_{s}\right)}\right)} \cdot e^{-p_{d}^{*}}=1+\frac{c_{d}}{p}+\frac{p_{s}-c_{s}}{\left(1+e^{\left(v_{s}-p_{s}\right)}\right)} \\
q_{d}{ }^{*}=\frac{c_{s e} \cdot \cdot \cdot \cdot \cdot \cdot \cdot C^{* 2}}{\lambda \cdot T \cdot c_{w}}
\end{gathered}
$$

## APPENDIX D

## PROOF OF PROPOSITIONS 3

The customer's preference probability of choosing 3D printing product, $q_{d}$, as well as the off-the-shelf product,$q_{s}$, is calculated as:

$$
\begin{aligned}
& q_{d}=\int_{a}^{b} \frac{e^{\left(v_{d}-p_{d}-\frac{c_{w}}{c \cdot \mu \cdot p}\right)}}{e^{\left(v_{d}-p_{d}-\frac{c_{0}}{C \cdot \mu \cdot \mu \cdot p}\right.}+e^{\left(v_{s}-p_{s}\right)}} \cdot \frac{1}{b-a} \cdot d v_{s} \\
& =\int_{a}^{b} \frac{1}{1+e^{\left((1-\alpha) v_{s}-p_{s}+p_{d}+\frac{c_{w}}{C, \mu \cdot p)}\right.}} \cdot \frac{1}{b-a} \cdot d v_{s} \\
& =\int_{a}^{b}\left(1-\frac{e^{\left((1-\alpha) v_{s}-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{1+e^{\left((1-\alpha) v_{s}-p_{s}+p_{d}+\frac{c . c .}{C \cdot p}\right)}}\right) \cdot \frac{1}{b-a} \cdot d v_{s} \\
& =\frac{1}{(b-a) \cdot(1-\alpha)} \cdot\left[\ln \left(1+e^{\left((1-\alpha) a-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)-\ln \left(1+e^{\left((1-\alpha) b-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)\right]
\end{aligned}
$$

The first derivatives of the market shares with respect to 3D product price and total number of 3D printers are given by the Equations D. 1 and D.1.

$$
\begin{gathered}
\frac{d q_{d}}{d p_{d}}=\frac{1}{(b-a)(1-\alpha)} \cdot\left[\frac{e^{\left((1-\alpha) a-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{1+e^{\left((1-\alpha) a-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}-\frac{e^{\left((1-\alpha) b-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{1+e^{\left((1-\alpha) b-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}\right] \\
\frac{d q_{d}}{d C}=\frac{c_{w}}{(b-a) \cdot(1-\alpha) \cdot \mu \cdot p \cdot C^{2}} \cdot\left[\frac{e^{\left((1-\alpha) b-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{1+e^{\left((1-\alpha) b-p_{s}+p_{d}+\frac{c}{C \cdot \mu \cdot p}\right)}}-\frac{e^{\left((1-\alpha) a-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{1+e^{\left((1-\alpha) a-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}\right]
\end{gathered}
$$

In order to find the retailers' optimal joint decisions on 3D printing products' pricing and the total number of 3 D printers, the first order derivatives of the retailer's expected profit function respect to aforementioned decision variables are taken, results are shown in equations D. 1 and D.1.

$$
\begin{aligned}
\frac{d \Phi}{d p_{d}} & =\lambda \cdot T\left(\frac{d q_{d}}{d p_{d}} \cdot\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)+q_{d}\right) \\
& =\frac{\lambda \cdot T}{(b-a)(1-\alpha)} \cdot\left[\left(\frac{1}{1+e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{c \cdot u \cdot p}\right)}}-\frac{1}{1+e^{\left((\alpha-1) b+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}\right)\right. \\
& \cdot\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)+\ln \left(1+e^{\left((1-\alpha) a-p_{s}+p_{d}+\frac{c_{w}}{C \cdot \mu \cdot p \cdot p}\right)}\right) \\
& \left.-\ln \left(1+e^{\left((1-\alpha) b-p_{s}+p_{d}+\frac{c_{w}}{(\cdot \cdot \mu \cdot p}\right)}\right)\right]=0
\end{aligned}
$$

$$
\begin{aligned}
\frac{d \Phi}{d C} & =\lambda \cdot T\left(\frac{d q_{d}}{d C} \cdot\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)\right)-c_{s e} \\
& =\frac{\lambda \cdot T \cdot c_{w} \cdot\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)}{(b-a) \cdot(1-\alpha) \cdot \mu \cdot p \cdot C^{2}} \cdot\left(\left(1+e^{\left((\alpha-1) b+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)^{-1}\right. \\
& \left.-\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)^{-1}\right)-c_{s e}=0
\end{aligned}
$$

In order to prove that the function is concave, the second order derivative of retailer's expected profit function with respect to the decision variables are taken, results are shown as Equations D. 1 and D. 1 and D.1.

$$
\begin{aligned}
& \frac{d \Phi^{2}}{d p_{d}{ }^{2}}= \frac{\lambda \cdot T \cdot\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)}{(b-a)(1-\alpha)} \cdot\left[\frac{e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)^{2}}-\frac{e^{\left((\alpha-1) b+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)^{2}}\right] \\
& \frac{d \Phi^{2}}{d C^{2}}= \frac{\lambda \cdot T \cdot c_{w} \cdot\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)}{(b-a) \cdot(1-\alpha) \cdot \mu \cdot p \cdot C^{2}} \cdot\left[-C^{-3} \cdot\left(\left(1+e^{\left((\alpha-1) b+p_{s}-p_{d}-\frac{c_{w}}{C^{*} \cdot \mu \cdot p}\right)}\right)^{-1}\right.\right. \\
&\left.-\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C^{*} \cdot \mu \cdot p}\right)}\right)^{-1}\right) \\
&\left.+\frac{c_{w}}{C^{4} \cdot \mu \cdot p} \cdot\left(\frac{e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)^{2}}-\frac{e^{\left((\alpha-1) b+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)^{2}}\right)\right] \\
& \frac{d \Phi^{2}}{d C d p_{d}}= \frac{\lambda \cdot T \cdot c_{w}}{(b-a) \cdot(1-\alpha) \cdot \mu \cdot p \cdot C^{2}} \cdot\left[\left[\frac{e^{\left((\alpha-1) b+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{\left.\left(1+e^{\left((\alpha-1) b+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)^{2}-\frac{e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}}{\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right)^{2}}\right]}\right.\right. \\
& \cdot\left(p_{d}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)+\left(1+e^{\left((\alpha-1) b+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)}\right) \\
&-1 \\
& \quad-\left(1+e^{\left.\left.\left((\alpha-1) a+p_{s}-p_{d}-\frac{c_{w}}{C \cdot \mu \cdot p}\right)\right)^{-1}\right]}\right.
\end{aligned}
$$

The second order derivative of the retailer's expected profit with respect to the 3D printing products' price is negative or equal to zero when $(\alpha-1) \cdot v_{s}+p_{s}-$ $p_{d}^{*}-\frac{c_{w}}{C^{*} \mu p} \leq 0$ and the negative definite condition, $\frac{d \Phi^{2}}{d p_{d}{ }^{2}} \cdot \frac{d \Phi^{2}}{d C^{2}}-\left(\frac{d \Phi^{2}}{d p_{d} d C}\right)^{2}>0$, is satisfied when $\frac{\left(p_{d}^{*}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)}{C^{* 2}} \cdot\left[\frac{e^{\left((\alpha-1) a+p_{s}-p_{d}^{*}-\frac{c^{w}}{C^{*}} \cdot \mu \cdot p\right)}}{\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}^{*}-\frac{c^{*}}{c^{*} \cdot \mu \cdot p}\right)}\right)^{2}}-\frac{e^{\left((\alpha-1) b+p_{s}-p_{d}^{*}-\frac{c_{w}}{C^{*}} \cdot \underline{\mu \cdot p}\right)}}{\left(1+e^{\left((\alpha-1) a+p_{s}-p_{d}^{*}-\frac{c}{C^{*}} \cdot \mu \cdot p \cdot p\right.}\right)^{2}}\right]$. $\left(-C^{*} \cdot\left(p_{d}^{*}-\frac{c_{d}}{p}-p_{s}+c_{s}\right)-\frac{2 c_{w}}{\mu \cdot p}\right)>\frac{c_{s e} \cdot(1-\alpha) \cdot(b-a)}{\lambda \cdot T}$.

## APPENDIX E

## PROOF OF PROPOSITIONS 4 AND 5

The optimal inventory level of the off-the-shelf products in a single 3D printing service is calculated as:

$$
\begin{aligned}
\Phi_{y_{s}^{w}}>\Phi_{y_{s}^{w}+1} \rightarrow & \\
& p_{s} \cdot \sum_{D=0}^{y_{s}^{w}} D \cdot p(D)+p_{s} \cdot \sum_{D=y_{s}^{w}+1}^{\infty} y_{s}^{w} \cdot p(D) \\
& -C \cdot c_{s e}-c_{s} \cdot y_{s}^{w}-h \cdot \sum_{D=0}^{y_{s}^{w}}\left(y_{s}^{w}-D\right) \cdot p(D) \\
& +\left(\sum_{i=0}^{K-1} \pi_{i} \cdot\left(p_{d}-c_{d} / p\right)-\pi_{K} \cdot l\right) \cdot \sum_{y_{s}^{w}+1}^{\infty}\left(D-y_{s}\right) \cdot p(D)> \\
& p_{s} \cdot \sum_{D=0}^{y_{s}^{w}+1} D \cdot p(D)+p_{s} \cdot \sum_{D=y_{s}^{w+2}}^{\infty} y_{s}^{w} \cdot p(D) \\
& -C \cdot c_{s e}-c_{s} \cdot y_{s}^{w}-h \cdot \sum_{D=0}^{y_{s}^{w+1}\left(y_{s}^{w}-D\right) \cdot p(D)} \\
& +\left(\sum_{i=0}^{K-1} \pi_{i} \cdot\left(p_{d}-c_{d} / p\right)-\pi_{K} \cdot l\right) \cdot \sum_{y_{s}^{w}+2}^{\infty}\left(D-y_{s}\right) \cdot p(D) \rightarrow \\
& c_{s}+\left(1-\pi_{K}\right) \cdot\left(p_{d}-c_{d} / p\right)-\pi_{K} \cdot l-p_{s} \\
& +F\left(y_{s}^{w}\right) \cdot\left(h+p_{s}-\left(1-\pi_{K}\right) \cdot\left(p_{d}-c_{d} / p\right)+\pi_{K} \cdot l\right) \geq 0 \rightarrow \\
& F\left(y_{s}^{w}\right) \geq \frac{p_{s}-c_{s}-\left(1-\pi_{K}\right) \cdot\left(p_{d}-c_{d} / p\right)+\pi_{K} \cdot l}{p_{s}+h-\left(1-\pi_{K}\right) \cdot\left(p_{d}-c_{d} / p\right)+\pi_{K} \cdot l} \\
\Phi_{y_{s}^{w}}>\Phi_{y_{s}^{w}-1} \rightarrow & \\
& p_{s} \cdot \sum_{D=0}^{y_{s}^{w}} D \cdot p(D)+p_{s} \cdot \sum_{D=y_{s}^{w}+1}^{\infty} y_{s}^{w} \cdot p(D) \\
& -C \cdot c_{s e}-c_{s} \cdot y_{s}^{w}-h \cdot \sum_{D=0}^{y_{s}^{w}}\left(y_{s}^{w}-D\right) \cdot p(D) \\
& +\left(\sum_{i=0}^{K-1} \pi_{i} \cdot\left(p_{d}-c_{d} / p\right)-\pi_{K} \cdot l\right) \cdot \sum_{y_{s}^{w}+1}^{\infty}\left(D-y_{s}\right) \cdot p(D)> \\
& p_{s} \cdot \sum_{D=0}^{y_{s}^{w}-1} D \cdot p(D)+p_{s} \cdot \sum_{D=y_{s}^{w}}^{\infty} y_{s}^{w} \cdot p(D) \\
& -C \cdot c_{s e}-c_{s} \cdot y_{s}^{w}-h \cdot \sum_{D=0}^{y_{s}^{w}-1}\left(y_{s}^{w}-D\right) \cdot p(D) \\
& +\left(\sum_{i=0}^{K-1} \pi_{i} \cdot\left(p_{d}-c_{d} / p\right)-\pi_{K} \cdot l\right) \cdot \sum_{y_{s}^{w}}^{\infty}\left(D-y_{s}\right) \cdot p(D) \rightarrow \\
& p_{s}-c_{s}-\left(1-\pi_{K}\right) \cdot\left(p_{d}-c_{d} / p\right)+\pi_{K} \cdot l \\
& +F\left(y_{s}^{w}-1\right) \cdot\left(h+p_{s}-\left(1-\pi_{K}\right) \cdot\left(p_{d}-c_{d} / p\right)+\pi_{K} \cdot l\right) \geq 0 \rightarrow \\
& F\left(y_{s}^{w}-1\right) \leq \frac{p_{s}-c_{s}-\left(1-\pi_{K}\right) \cdot\left(p_{d}-c_{d} / p\right)+\pi_{K} \cdot l}{p_{s}+h-\left(1-\pi_{K}\right) \cdot\left(p_{d}-c_{d} / p\right)+\pi_{K} \cdot l}
\end{aligned}
$$

The optimal price of 3D printed products in a single 3D printing service:
Assuming there is a single 3D printing service (M/M/1/K), the optimal 3D product price that maximizes retailer's profit function needs to satisfy $\Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}+\epsilon}$ and $\Phi_{p_{d}^{*}}>$ $\Phi_{p_{d}^{*}-\epsilon}$. In which $\epsilon$ can be calculated as:

$$
\begin{aligned}
& K^{*}=\frac{\mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}\right) \\
& K^{*}-1=\frac{\mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}-\epsilon\right) \\
& K^{*}+1=\frac{\mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}+\epsilon\right)
\end{aligned}
$$

which result in $\epsilon=\frac{c_{w}}{\mu \cdot p}$. Assuming $\rho<1$, the optimal 3D product price that maximize retailer's profit function satisfies:

$$
\begin{aligned}
& \Phi_{p_{d}^{*}} \geq \Phi_{p_{d}^{*}+\epsilon} \rightarrow \\
& p_{d}^{*} \geq \frac{c_{w} \cdot\left(1-\rho^{(K+1)}\right) \cdot\left(1-\rho^{(K-1)}\right)}{\mu \cdot p \cdot(1-\rho)^{2} \cdot \rho^{(K-1)}}+\frac{c_{d}}{p}-l \\
& \Phi_{p_{d}^{*}} \geq \Phi_{p_{d}^{*}-\epsilon} \rightarrow \\
& p_{d}^{*} \leq \frac{c_{w} \cdot\left(1-\rho^{(K+1)}\right)^{2}}{\mu \cdot p \cdot(1-\rho)^{2} \cdot \rho^{K}}+\frac{c_{d}}{p}-l
\end{aligned}
$$

In which $\rho=\frac{\lambda}{\mu}$.
The optimal price of 3 D printed products in a multi 3D printing service:

Assuming there are C 3D printing services (M/M/C/K), the optimal 3D product price that maximizes retailer's profit function follows needs to satisfy $\Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}+\epsilon}$ and $\Phi_{p_{d}^{*}}>\Phi_{p_{d}^{*}-\epsilon}$. In which $\epsilon$ calculated by solving the system of equations below:

$$
\begin{aligned}
& K^{*}=\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}\right) \\
& K^{*}-1=\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}-\epsilon\right) \\
& K^{*}+1=\frac{C \cdot \mu \cdot p}{c_{w}} \cdot\left(v_{d}-p_{d}^{*}+\epsilon\right)
\end{aligned}
$$

And $\epsilon$ is calculated as $\frac{c_{w}}{C \cdot \mu \cdot p}$. Therefore, the optimal 3D product price that maximizes retailer's profit function satisfies:

$$
\begin{aligned}
& \Phi_{p_{d}^{*}} \geq \Phi_{p_{d}^{*}+\epsilon} \rightarrow \\
& p_{d}^{*} \geq \frac{c_{w} \cdot\left(1-p_{\left(K^{*}-1\right)}\right)}{C \cdot \mu \cdot p \cdot\left(\pi_{K^{*}-1}-\pi_{\left(K^{*}\right)}\right)}+\frac{c_{d}}{p}-l \\
& \Phi_{p_{d}^{*}} \geq \Phi_{p_{d}^{*}-\epsilon} \rightarrow \\
& p_{d}^{*} \leq \frac{c_{w} \cdot\left(1-\pi_{\left(K^{*}+1\right)}\right)}{C \cdot \mu \cdot p \cdot\left(\pi_{K^{*}}-\pi_{\left(K^{*}+1\right)}\right)}+\frac{c_{d}}{p}-l
\end{aligned}
$$

## APPENDIX F

## PROOF OF PROPOSITIONS 6

Proof. In the transition diagram of the model, shown in the Figure 5.3:

$$
\begin{aligned}
& \text { for } \quad i=y_{s}: \\
& \left\{\begin{array}{l}
\lambda \cdot \pi_{0, y_{s}}=\nu \cdot \pi_{0,0} \quad \text { if } \quad n=0 \\
\left(\lambda+\mu_{s}\right) \pi_{n, y_{s}}=\lambda \cdot \pi_{(n-1), y_{s}}+\nu \cdot \pi_{n, 0} \quad \text { if } \quad n>0
\end{array}\right. \\
& \text { for } \quad 0<i<y_{s}: \\
& \left\{\begin{array}{l}
\lambda \cdot \pi_{0, i}=\mu_{s} \cdot \pi_{1,(i+1)} \quad \text { if } \quad n=0 \\
\left(\lambda+\mu_{s}\right) \pi_{n, i}=\lambda \cdot \pi_{(n-1), i}+\mu_{s} \cdot \pi_{n+1, i+1} \quad \text { if } \quad n>0
\end{array}\right.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
& \pi_{0, i}=\frac{\mu_{s}}{\lambda} \cdot \pi_{1,(i+1)} \\
& \pi_{1, i}=\frac{1}{\lambda+\mu_{s}} \cdot\left(\lambda \cdot \pi_{0, i}+\mu_{s} \cdot \pi_{2,(i+1)}\right)=\frac{\mu_{s}}{\lambda+\mu_{s}}\left(\pi_{1, i+1}+\pi_{2,(i+1)}\right) \\
& \pi_{2, i}=\frac{\mu_{s} \cdot \lambda}{\left(\lambda+\mu_{s}\right)^{2}}\left(\pi_{1, i+1}+\pi_{2,(i+1)}\right)+\frac{\mu_{s}}{\lambda+\mu_{s}}\left(\pi_{3,(i+1)}\right) \\
& \vdots \\
& \pi_{n, i}=\frac{\mu_{s} \cdot \lambda^{(n-1)}}{\left(\lambda+\mu_{s}\right)^{n}}\left(\pi_{1, i+1}+\pi_{2,(i+1)}\right)+\frac{\mu_{s} \cdot \lambda^{(n-2)}}{\left(\lambda+\mu_{s}\right)^{(n-1)}}\left(\pi_{3, i+1}\right)+\ldots+\frac{\mu_{s}}{\lambda+\mu_{s}} \cdot \pi_{(n+1),(i+1)} \\
& \text { for } \quad i=0: \\
& \nu \cdot \pi_{n, 0}=\mu_{s} \cdot \pi_{(n+1), 1}
\end{aligned}
$$

So sum of the state probabilities with $i$ inventory level, when $i>0$, can be calculated as:

$$
\begin{aligned}
& P_{n>0}(i)=\sum_{n>0} \pi_{n, i}=\left(\pi_{1, i+1}+\pi_{2,(i+1)}\right) \cdot \frac{\mu_{s}}{\lambda+\mu_{s}} \cdot \sum_{n=1}^{\infty}\left(\frac{\lambda}{\lambda+\mu_{s}}\right)^{(n-1)} \\
& +\pi_{3, i+1} \cdot \frac{\mu_{s}}{\lambda+\mu_{s}} \cdot \sum_{n=2}^{\infty}\left(\frac{\lambda}{\lambda+\mu_{s}}\right)^{(n-2)}+\ldots=\sum_{n>0} \pi_{n,(i+1)}=P_{n>0}(i+1) ; \forall i .
\end{aligned}
$$

In a similar way it can be proved that, $\quad P_{n>1}(i)=P_{n>1}(i+1) ; \forall i$.
Therefore, $\forall i ; \pi_{1, i}$ 's are equal.
$P_{n>2}(i)=P_{n>2}(i+1)$. So, $\forall i ; \pi_{2, i}$ 's are equal and so on.

In general, at each number of customer's level, all the $\pi_{n, i}$ 's are equal for $i=1,2, \ldots, y_{s}$.
Based on the balance equations:

$$
\begin{aligned}
& \pi_{1, i}=\frac{\lambda}{\mu_{s}} \cdot \pi_{0, i} \\
& \pi_{2, i}=\frac{\lambda+\mu_{s}}{\mu_{s}} \cdot \frac{\lambda}{\mu_{s}} \cdot \pi_{0, i}-\frac{\lambda}{\mu_{s}} \cdot \pi_{0, i}=\left(\frac{\lambda}{\mu_{s}}\right)^{2} \cdot \pi_{0, i} \\
& \pi_{3, i}=\frac{\lambda+\mu_{s}}{\mu_{s}} \cdot\left(\frac{\lambda}{\mu_{s}}\right)^{2} \cdot \pi_{0, i}-\left(\frac{\lambda}{\mu_{s}}\right)^{2} \cdot \pi_{0, i}=\left(\frac{\lambda}{\mu_{s}}\right)^{2} \cdot \pi_{0, i} \\
& \vdots \\
& \pi_{n, i}=\left(\frac{\lambda}{\mu_{s}}\right)^{n} \cdot \pi_{0, i}
\end{aligned}
$$

Also, based on the balance equations if $i=y_{s}$ :

$$
\begin{aligned}
& \pi_{0,0}=\frac{\lambda}{\nu} \cdot \pi_{0, y_{s}} \\
& \pi_{n, 0}=\frac{1}{\nu} \cdot\left(\left(\lambda+\mu_{s}\right) \cdot \pi_{n, y_{s}}-\lambda \cdot \pi_{(n-1), y_{s}}\right)=\frac{1}{\nu} \cdot\left(\left(\lambda+\mu_{s}\right)-\lambda \cdot \frac{\mu_{s}}{\lambda}\right) \cdot \pi_{n, y_{s}}=\frac{\lambda}{\nu} \cdot \pi_{n, y_{s}}
\end{aligned}
$$

All in all, the balance equations can be written as:

$$
\left\{\begin{array}{l}
\pi_{n, i} \text { 's } \quad \text { are equal for each } n \quad \text { if } \quad i \neq 0 \\
\pi_{n, i}=\frac{\lambda}{\mu_{s}} \cdot \pi_{n, i-1} \quad \text { for each } n \\
\pi_{n, 0}=\frac{\lambda}{\nu} \cdot \pi_{n, i} \quad \text { for each } n \quad \text { if } \quad i=0
\end{array}\right.
$$

Therefore, based on normalizing constraint:

$$
\begin{aligned}
& \sum_{n=0}^{\infty} \sum_{i=0}^{y_{s}} \pi_{n, i}=1 \\
& \left.\left(y_{s} \cdot \sum_{n=0}^{\infty}\left(\frac{\lambda}{\mu_{s}}\right)^{n}+\frac{\lambda}{\nu} \cdot \sum_{n=0}^{\infty}\left(\frac{\lambda}{\mu_{s}}\right)^{n}\right)\right) \pi_{0, y_{s}}=1 \rightarrow \pi_{0, y_{s}}=\left(\frac{\mu_{s}}{\mu_{s}-\lambda} \cdot\left(y_{s}+\frac{\lambda}{\nu}\right)\right)^{-1} \\
& \pi(n, i)=\left(\frac{\mu_{s}}{\mu_{s}-\lambda} \cdot\left(y_{s}+\frac{\lambda}{\nu}\right)\right)^{-1} \cdot\left(\frac{\lambda}{\mu_{s}}\right)^{n} \\
& \pi(n, 0)=\left(\frac{\mu_{s}}{\mu_{s}-\lambda} \cdot\left(y_{s}+\frac{\lambda}{\nu}\right)\right)^{-1} \cdot\left(\frac{\lambda}{\mu_{s}}\right)^{n} \cdot \frac{\lambda}{\nu}
\end{aligned}
$$

## APPENDIX G

## PROOF OF PROPOSITION 7

Proof. Based on the Equation (5.10), the retailer's expected profit function can be rewritten as:

$$
\begin{aligned}
\Phi\left(y_{s}^{*}\right) & =p_{s} \cdot \lambda \cdot\left(1-\sum_{n=0}^{\infty} \pi_{n, 0}\right)-\left(l \cdot \lambda+c_{s h} \cdot \nu+c_{s} \cdot y_{s} \cdot \nu\right) \cdot \sum_{n=0}^{\infty} \pi_{n, 0} \\
& -h \cdot \sum_{i=1}^{y_{s}^{*}} i \cdot \sum_{n=0}^{\infty} \pi_{n, i} \\
& =p_{s} \cdot \lambda-\left(\left(p_{s}+l\right) \cdot \lambda+\left(c_{s h}+c_{s} \cdot y_{s}\right) \cdot \nu\right) \cdot \sum_{n=0}^{\infty}\left(\frac{\mu}{\mu-\lambda} \cdot\left(y_{s}+\frac{\lambda}{\nu}\right)\right)^{-1} \cdot\left(\frac{\lambda}{\mu}\right)^{n} \\
& -h \cdot \frac{y_{s}^{*}\left(y_{s}^{*}+1\right)}{2} \cdot \sum_{n=0}^{\infty}\left(\frac{\mu}{\mu-\lambda} \cdot\left(y_{s}+\frac{\lambda}{\nu}\right)\right)^{-1} \cdot\left(\frac{\lambda}{\mu}\right)^{n} \\
& =p_{s} \cdot \lambda-\left(\left(p_{s}+l\right) \cdot \lambda+\left(c_{s h}+c_{s} \cdot y_{s}\right) \cdot \nu\right) \cdot \frac{\lambda}{y_{s}^{*}+\frac{\lambda}{\nu}}-h \cdot \frac{y_{s}^{*}\left(y_{s}^{*}+1\right)}{2} \cdot \frac{1}{y_{s}^{*}+\frac{\lambda}{\nu}} \\
& \Phi\left(y_{s}^{*}\right)>\Phi\left(y_{s}^{*}+1\right) \\
& \left(\left(p_{s}+l\right) \cdot \lambda+c_{s h} \cdot \nu\right) \cdot \frac{\lambda}{\nu}\left(\frac{1}{y_{s}^{*}+1+\frac{\lambda}{\nu}}-\frac{1}{y_{s}^{*}+\frac{\lambda}{\nu}}\right)+c_{s} \cdot \lambda\left(\frac{y_{s}^{*}+1}{y_{s}^{*}+1+\frac{\lambda}{\nu}}-\frac{y_{s}^{*}}{y_{s}^{*}+\frac{\lambda}{\nu}}\right) \\
& +h \cdot \frac{\left(y_{s}^{*}+1\right)}{2} \cdot\left(\frac{y_{s}^{*}+2}{y_{s}^{*}+1+\frac{\lambda}{\nu}}-\frac{y_{s}^{*}}{y_{s}^{*}+\frac{\lambda}{\nu}}\right)>0 \\
& \left(y_{s}^{*}\right)^{2}+\left(\frac{2 \lambda}{\nu}+1\right) \cdot y_{s}^{*}+2 \cdot \frac{\lambda}{\nu} \cdot\left(1-\left(\left(p_{s}+l-c_{s}\right) \cdot \lambda+c_{s h} \cdot \nu\right) \frac{1}{h}\right)>0 \\
& y_{s}^{*}>-\frac{1}{2}-\frac{\lambda}{\nu}+\frac{\sqrt{\delta}}{2}
\end{aligned}
$$

Where, $\delta$ is equal to $\left(\frac{2 \lambda-\nu}{\nu}\right)^{2}+\frac{8 \lambda\left(\left(p_{s}+l-c_{s}\right) \lambda+c_{s h} \nu\right)}{h \nu}$.

$$
\begin{aligned}
& \Phi\left(y_{s}^{*}\right)>\Phi\left(y_{s}^{*}-1\right) \\
& \left(\left(p_{s}+l\right) \cdot \lambda+c_{s h} \cdot \nu\right) \cdot \frac{\lambda}{\nu}\left(\frac{1}{y_{s}^{*}+\frac{\lambda}{\nu}}-\frac{1}{y_{s}^{*}-1+\frac{\lambda}{\nu}}\right)+c_{s} \cdot \lambda\left(\frac{y_{s}^{*}}{y_{s}^{*}+\frac{\lambda}{\nu}}-\frac{y_{s}^{*}-1}{y_{s}^{*}-1+\frac{\lambda}{\nu}}\right) \\
& +h \cdot \frac{y_{s}^{*}}{2} \cdot\left(\frac{y_{s}^{*}+1}{y_{s}^{*}+\frac{\lambda}{\nu}}-\frac{y_{s}^{*}-1}{y_{s}^{*}-1+\frac{\lambda}{\nu}}\right)<0 \\
& \left(y_{s}^{*}\right)^{2}+\left(\frac{2 \lambda}{\nu}-1\right) \cdot y_{s}^{*}-2 \cdot \frac{\lambda}{h \nu} \cdot\left(\left(p_{s}+l-c_{s}\right) \cdot \lambda+c_{s h} \cdot \nu\right)<0 \\
& y_{s}^{*}<\frac{1}{2}-\frac{\lambda}{\nu}+\frac{\sqrt{\delta}}{2}
\end{aligned}
$$

And, $\delta$ is equal to $\left(\frac{2 \lambda-\nu}{\nu}\right)^{2}+\frac{8 \lambda\left(\left(p_{s}+l-c_{s}\right) \lambda+c_{s h} \nu\right)}{h \nu}$.

## Proof of the Corollary:

Proof.

$$
\begin{aligned}
& y_{s}^{*}=\left\lfloor\frac{1}{2}-\frac{\lambda}{\nu}+\left(\left(\frac{\lambda}{\nu}\right)^{2} \cdot\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)\right)+\lambda \cdot\left(\frac{-1}{\nu}+\frac{2 c_{s h}}{h}\right)+\frac{1}{4}\right)^{\frac{1}{2}}\right\rfloor \\
& \frac{d y_{s}^{*}}{d \lambda}= \\
&-\frac{1}{\nu}+\frac{1}{2}\left(\left(\frac{\lambda}{\nu}\right)^{2} \cdot\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)\right)+\lambda \cdot\left(\frac{-1}{\nu}+\frac{2 c_{s h}}{h}\right)+\frac{1}{4}\right)^{\frac{-1}{2}} \\
& \cdot\left(\frac{2 \lambda}{\nu^{2}} \cdot\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)\right)-\frac{1}{\nu}+\frac{2 c_{s h}}{h}\right) \\
&=-\frac{1}{\nu}+\frac{\frac{2 \lambda}{\nu^{2}} \cdot\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)\right)-\frac{1}{\nu}+\frac{2 c_{s h}}{h}}{2 \cdot\left(\left(\left(\frac{\lambda}{\nu}\right)^{2} \cdot\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)\right)+\lambda \cdot\left(\frac{-1}{\nu}+\frac{2 c_{s h}}{h}\right)+\frac{1}{4}\right)^{\frac{1}{2}}\right.} \\
&= \frac{2 \lambda}{\nu^{2}}\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)\right)-\frac{1}{\nu}+\frac{2 c_{s h}}{h} \\
&-\frac{2}{\nu} \sqrt{\left(\frac{\lambda}{\nu}\right)^{2}\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)\right)+\lambda\left(\frac{-1}{\nu}+\frac{2 c_{s h}}{h}\right)+\frac{1}{4}}
\end{aligned}
$$

Which is always greater than zero and proves that $y_{s}^{*}$ is always increasing in $\lambda$.

$$
\begin{aligned}
\frac{d y_{s}^{*}}{d \nu} & =\frac{\lambda}{\nu^{2}}+\frac{1}{2}\left(\frac{-2 \lambda^{2}}{\nu^{3}} \cdot\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)\right)+\frac{2 \lambda^{2}}{h \nu^{2}}\left(p_{s}+l-c_{s}\right)+\frac{\lambda}{\nu^{2}}\right) \\
& \cdot \sqrt{\left(\frac{\lambda^{2}}{\nu^{2}} \cdot\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)\right)-\frac{\lambda}{\nu}+\frac{2 \lambda c_{s h}}{h}+\frac{1}{4}\right)} \\
& =\frac{\lambda}{\nu^{2}}+\frac{\frac{-\lambda^{3}}{\nu^{3}} \cdot\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)\right)+\frac{\lambda^{2}}{h \nu^{2}}}{\sqrt{\left(\frac{\lambda}{\nu}\right)^{2} \cdot\left(1+\frac{2 \nu}{h}\left(p_{s}+l-c_{s}\right)+\frac{\lambda}{2 \nu^{2}}\right.}}
\end{aligned}
$$

Which is less than zero if $\left(p_{s}+l-c_{s}\right) \cdot\left(\frac{\lambda}{h} \cdot\left(p_{s}+l-c_{s}\right)-1\right)-2 c_{s h}>0$. In another words, $y_{s}^{*}$ is decreasing in $\nu$, if $\left(p_{s}+l-c_{s}\right) \cdot\left(\frac{\lambda}{h} \cdot\left(p_{s}+l-c_{s}\right)-1\right)-2 c_{s h}>0$.

## BIBLIOGRAPHY

[Adler and Naor, 1969] Adler, I. and Naor, P. (1969). Social optimization versus selfoptimization in waiting lines. Technical Report TR-126, Stanford University Department of Statistics, Stanford, CA.
[Anderson et al., 1992] Anderson, S. P., De Palma, A., and Thisse, J. F. (1992). Discrete choice theory of product differentiation. MIT Press, Cambridge, MA.
[Artalejo and Falin, 2002] Artalejo, J. and Falin, G. (2002). Standard and retrial queueing systems: a comparative analysis. Revista Matematica Complutense, 15(1):101-129.
[Aydin and Porteus, 2008] Aydin, G. and Porteus, E. L. (2008). Joint inventory and pricing decisions for an assortment. Operations Research, 56(5):1247-1255.
[Aydin and Ryan, 2000] Aydin, G. and Ryan, J. K. (2000). Product line selection and pricing under the multinomial logit choice model. In Proceedings of the 2000 MSOM Conference.
[Bassok et al., 1999] Bassok, Y., Anupindi, R., and Akella, R. (1999). Single-period multiproduct inventory models with substitution. Operations Research, 47(4):632-642.
[Berger, 2013] Berger, R. (2013). Additive manufacturing: a game changer for the manufacturing industry. Roland Berger Strategy Consultants GmbH, Munich, 1(5.1).
[Berman, 2012] Berman, B. (2012). 3d printing: the new industrial revolution. Business Horizons, 55(2):155-162.
[Birge et al., 1998] Birge, J. R., Drogosz, J., and Duenyas, I. (1998). Setting single-period optimal capacity levels and prices for substitutable products. International Journal of Flexible Manufacturing Systems, 10(4):407-430.
[Cachon and Kök, 2007] Cachon, G. P. and Kök, A. G. (2007). Category management and coordination in retail assortment planning in the presence of basket shopping consumers. Management Science, 53(6):934-951.
[Cachon et al., 2005] Cachon, G. P., Terwiesch, C., and Xu, Y. (2005). Retail assortment planning in the presence of consumer search. Manufacturing $\mathcal{E}$ Service Operations Management, 7(4):330-346.
[Cachon et al., 2006] Cachon, G. P., Terwiesch, C., and Xu, Y. (2006). The impact of consumer search, firm entry and competition on assortment and pricing. Marketing Science Forthcoming.
[Campbell et al., 2011] Campbell, T., Williams, C., Ivanova, O., and Garrett, B. (2011). Could 3d printing change the world. Technologies, Potential, and Implications of Additive Manufacturing, Atlantic Council, Washington, DC.
[Chiang and Monahan, 2005] Chiang, W.-y. K. and Monahan, G. E. (2005). Managing inventories in a two-echelon dual-channel supply chain. European Journal of Operational Research, 162(2):325-341.
[Choi and Chang, 1999] Choi, B. and Chang, Y. (1999). Map 1, map 2/m/c retrial queue with the retrial group of finite capacity and geometric loss. Mathematical and Computer Modelling, 30(3):99-113.
[Choi et al., 1998] Choi, B., Kim, Y., and Lee, Y. (1998). The m/m/c retrial queue with geometric loss and feedback. Computers $8 \mathcal{B}$ Mathematics with Applications, 36(6):41-52.
[Columbus, 2015] Columbus, L. (2015). Roundup of 3d printing market forecasts and estimates. Forbes, 10:2016.
[Costabile et al., 2017] Costabile, G., Fera, M., Fruggiero, F., Lambiase, A., and Pham, D. (2017). Cost models of additive manufacturing: a literature review. International Journal of Industrial Engineering Computations, 8(2):263-283.
[Emmelmann et al., 2011] Emmelmann, C., Petersen, M., Kranz, J., and Wycisk, E. (2011). Bionic lightweight design by laser additive manufacturing (lam) for aircraft industry. In SPIE Eco-Photonics, pages 80650L-80650L. International Society for Optics and Photonics.
[Falin, 2010] Falin, G. (2010). A single-server batch arrival queue with returning customers. European Journal of Operational Research, 201(3):786-790.
[Federgruen and Heching, 1999] Federgruen, A. and Heching, A. (1999). Combined pricing and inventory control under uncertainty. Operations Research, 47(3):454-475.
[Gallego and Wang, 2014] Gallego, G. and Wang, R. (2014). Multiproduct price optimization and competition under the nested logit model with product differentiated price sensitivities. Operations Research, 62(2):450-461.
[Gardan, 2016] Gardan, J. (2016). Additive manufacturing technologies: state of the art and trends. International Journal of Production Research, 54(10):3118-3132.
[Gebhardt et al., 2010] Gebhardt, A., Schmidt, F.-M., Hötter, J.-S., Sokalla, W., and Sokalla, P. (2010). Additive manufacturing by selective laser melting the realizer desktop machine and its application for the dental industry. Physics Procedia, 5:543-549.
[Gibson et al., 2010] Gibson, I., Rosen, D. W., Stucker, B., et al. (2010). Additive manufacturing technologies. Springer, New York, NY.
[Hague et al., 2003] Hague, R., Campbell, I., and Dickens, P. (2003). Implications on design of rapid manufacturing. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 217(1):25-30.
[Hague et al., 2004] Hague, R., Mansour, S., and Saleh, N. (2004). Material and design considerations for rapid manufacturing. International Journal of Production Research, 42(22):4691-4708.
[Hanson and Martin, 1996] Hanson, W. and Martin, K. (1996). Optimizing multinomial logit profit functions. Management Science, 42(7):992-1003.
[Hao et al., 2010] Hao, L., Mellor, S., Seaman, O., Henderson, J., Sewell, N., and Sloan, M. (2010). Material characterisation and process development for chocolate additive layer manufacturing. Virtual and Physical Prototyping, 5(2):57-64.
[Hayes and Jaikumar, 1991] Hayes, R. H. and Jaikumar, R. (1991). Requirements for successful implementation of new manufacturing technologies. Journal of Engineering and Technology Management, 7(3):169-175.
[Holmström et al., 2010] Holmström, J., Partanen, J., Tuomi, J., and Walter, M. (2010). Rapid manufacturing in the spare parts supply chain: alternative approaches to capacity deployment. Journal of Manufacturing Technology Management, 21(6):687-697.
[Honhon et al., 2010] Honhon, D., Gaur, V., and Seshadri, S. (2010). Assortment planning and inventory decisions under stockout-based substitution. Operations Research, 58(5):1364-1379.
[Hopp and Xu, 2005] Hopp, W. J. and Xu, X. (2005). Product line selection and pricing with modularity in design. Manufacturing \& Service Operations Management, $7(3): 172-187$.
[Khajavi et al., 2014] Khajavi, S. H., Partanen, J., and Holmström, J. (2014). Additive manufacturing in the spare parts supply chain. Computers in Industry, 65(1):50-63.
[Kök and Fisher, 2007] Kök, A. G. and Fisher, M. L. (2007). Demand estimation and assortment optimization under substitution: Methodology and application. Operations Research, 55(6):1001-1021.
[Kök et al., 2009] Kök, A. G., Fisher, M. L., and Vaidyanathan, R. (2009). Assortment planning: review of literature and industry practice. In Retail Supply Chain Management, pages 99-153. Springer, New York, NY.
[Kruth et al., 1998] Kruth, J. P., Leu, M. C., and Nakagawa, T. (1998). Progress in additive manufacturing and rapid prototyping. CIRP Annals-Manufacturing Technology, 47(2):525-540.
[Kruth et al., 2007] Kruth, J. P., Levy, G., Klocke, F., and Childs, T. (2007). Consolidation phenomena in laser and powder-bed based layered manufacturing. CIRP Annals-Manufacturing Technology, 56(2):730-759.
[Kulkarni et al., 2000] Kulkarni, P., Marsan, A., and Dutta, D. (2000). A review of process planning techniques in layered manufacturing. Rapid Prototyping Journal, 6(1):18-35.
[Kumar et al., 2002] Kumar, B. K., Madheswari, S. P., and Vijayakumar, A. (2002). The $\mathrm{m} / \mathrm{g} / 1$ retrial queue with feedback and starting failures. Applied Mathematical Modelling, 26(11):1057-1075.
[Levy et al., 2003] Levy, G. N., Schindel, R., and Kruth, J.-P. (2003). Rapid manufacturing and rapid tooling with layer manufacturing (lm) technologies, state of the art and future perspectives. CIRP Annals-Manufacturing Technology, 52(2):589-609.
[Li and Huh, 2011] Li, H. and Huh, W. T. (2011). Pricing multiple products with the multinomial logit and nested logit models: concavity and implications. Manufacturing $\mathcal{E}$ Service Operations Management, 13(4):549-563.
[Lindsay and Laura, 2015] Lindsay, E. and Laura, S. (2015). Ups tests a 3d printing service. Technical report, Wall Street Journal, New York, NY.
[Liu et al., 2014] Liu, P., Huang, S. H., Mokasdar, A., Zhou, H., and Hou, L. (2014). The impact of additive manufacturing in the aircraft spare parts supply chain: supply chain operation reference (scor) model based analysis. Production Planning $\mathcal{E G}^{\prime}$ Control, 25(13-14):1169-1181.
[Long et al., 2017] Long, Y., Pan, J., Zhang, Q., and Hao, Y. (2017). 3d printing technology and its impact on chinese manufacturing. International Journal of Production Research, 55(5):1488-1497.
[Maddah and Bish, 2007] Maddah, B. and Bish, E. K. (2007). Joint pricing, assortment, and inventory decisions for a retailer's product line. Naval Research Logistics (NRL), 54(3):315-330.
[Mahajan and van Ryzin, 2001] Mahajan, S. and van Ryzin, G. (2001). Stocking retail assortments under dynamic consumer substitution. Operations Research, 49(3):334-351.
[Mashhadi et al., 2015] Mashhadi, A. R., Esmaeilian, B., and Behdad, S. (2015). Impact of additive manufacturing adoption on future of supply chains. In International Manufacturing Science and Engineering Conference, pages V001T02A064-V001T02A064. American Society of Mechanical Engineers.
[McFadden, 1980] McFadden, D. (1980). Econometric models for probabilistic choice among products. Journal of Business, pages S13-S29.
[Melican et al., 2001] Melican, M. C., Zimmerman, M. C., Dhillon, M. S., Ponnambalam, A. R., Curodeau, A., and Parsons, J. R. (2001). Threedimensional printing and porous metallic surfaces: A new orthopedic application. Journal of Biomedical Materials Research, 55(2):194-202.
[Mellor et al., 2014] Mellor, S., Hao, L., and Zhang, D. (2014). Additive manufacturing: A framework for implementation. International Journal of Production Economics, 149:194-201.
[Meulen and Rivera, 2014] Meulen, R. and Rivera, J. (2014). Gartner survey reveals that high acquisition and start-up costs are delaying investment in 3d printers. Technical report, Gartner, Stamford, CT.
[Mokaddis et al., 2007] Mokaddis, G., Metwally, S., and Zaki, B. (2007). A feedback retrial queuing system with starting failures and single vacation. Tamkang Journal of Science and Engineering, 10(3):183-192.
[Munguía et al., 2008] Munguía, J., de Ciurana, J., and Riba, C. (2008). Pursuing successful rapid manufacturing: a users' best-practices approach. Rapid Prototyping Journal, 14(3):173-179.
[Mussa and Rosen, 1978] Mussa, M. and Rosen, S. (1978). Monopoly and product quality. Journal of Economic Theory, 18(2):301-317.
[Naor, 1969] Naor, P. (1969). The regulation of queue size by levying tolls. Econometrica: Journal of The Econometric Society, 37:15-24.
[Nyman and Sarlin, 2014] Nyman, H. J. and Sarlin, P. (2014). From bits to atoms: 3d printing in the context of supply chain strategies. In System Sciences (HICSS), 201447 th Hawaii International Conference, pages 4190-4199. IEEE.
[Pentico, 1976] Pentico, D. W. (1976). The assortment problem with nonlinear cost functions. Operations Research, 24(6):1129-1142.
[Pentico, 1988] Pentico, D. W. (1988). The discrete two-dimensional assortment problem. Operations Research, 36(2):324-332.
[Petrovic et al., 2011] Petrovic, V., Vicente Haro Gonzalez, J., Jorda Ferrando, O., Delgado Gordillo, J., Ramon Blasco Puchades, J., and Portoles Grinan, L. (2011). Additive layered manufacturing: sectors of industrial application shown through case studies. International Journal of Production Research, 49(4):1061-1079.
[Petruzzi and Dada, 1999] Petruzzi, N. C. and Dada, M. (1999). Pricing and the newsvendor problem: a review with extensions. Operations Research, 47(2):183-194.
[Piazza and Alexander, 2015] Piazza, M. and Alexander, S. (2015). Additive manufacturing: a summary of the literature. Urban Publications, Cleveland State University, Maxine Goodman Levin College of Urban Affairs, 1319.
[Pour and Zanoni, 2017] Pour, M. A. and Zanoni, S. (2017). Impact of merging components by additive manufacturing in spare parts management. Procedia Manufacturing, 11:610-618.
[Reeves, 2009] Reeves, P. (2009). Additive manufacturing: a supply chain wide response to economic uncertainty and environmental sustainability. Econolyst Limited, The Silversmiths, Crown Yard, Wirksworth, Derbyshire, UK, DE4(4ET).
[Ruffo et al., 2006] Ruffo, M., Tuck, C., and Hague, R. (2006). Cost estimation for rapid manufacturing-laser sintering production for low to medium volumes. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 220(9):1417-1427.
[Ryzin and Mahajan, 1999] Ryzin, G. v. and Mahajan, S. (1999). On the relationship between inventory costs and variety benefits in retail assortments. Management Science, 45(11):1496-1509.
[Schniederjans, 2017] Schniederjans, D. G. (2017). Adoption of 3d-printing technologies in manufacturing: a survey analysis. International Journal of Production Economics, 183:287-298.
[Takahashi et al., 2011] Takahashi, K., Aoi, T., Hirotani, D., and Morikawa, K. (2011). Inventory control in a two-echelon dual-channel supply chain with setup of production and delivery. International Journal of Production Economics, 133(1):403-415.
[Thomas and Gilbert, 2014] Thomas, D. S. and Gilbert, S. W. (2014). Costs and cost effectiveness of additive manufacturing. NIST Special Publication, 1176:12.
[Tita, 2015] Tita, B. (2015). 3d printer firms fall flat, as buyers wait for new models. Technical report, Wall Street Journal, New York, NY.
[Tuck et al., 2006] Tuck, C., Hague, R., and Burns, N. (2006). Rapid manufacturing: impact on supply chain methodologies and practice. International Journal of Services and Operations Management, 3(1):1-22.
[Walter et al., 2004] Walter, M., Holmström, J., Tuomi, H., and Yrjölä, H. (2004). Rapid manufacturing and its impact on supply chain management. In Proceedings of The Logistics Research Network Annual Conference, pages 9-10.
[Wang et al., 2013] Wang, D., Yang, Y., Yi, Z., and Su, X. (2013). Research on the fabricating quality optimization of the overhanging surface in slm process. The International Journal of Advanced Manufacturing Technology, 65(9-12):14711484.
[Wang and Zhao, 2007] Wang, J. and Zhao, Q. (2007). A discrete-time geo/g/1 retrial queue with starting failures and second optional service. Computers $\mathcal{E}$ Mathematics with Applications, 53(1):115-127.
[Wang, 2012] Wang, R. (2012). Multiproduct pricing management and design of new service products. PhD thesis, Columbia University, New York, NY.
[Weller et al., 2015] Weller, C., Kleer, R., and Piller, F. T. (2015). Economic implications of 3d printing: market structure models in light of additive manufacturing revisited. International Journal of Production Economics, 164:43-56.
[Wullms et al., 2014] Wullms, B., Arts, J. J., Topan, E. E., van der Schoot, H., and Schindler-Chaloub, J. (2014). Additive manufacturing in the spare parts supply chain. Master's thesis, Eindhoven Univerity of Technology, Eindhoven, Netherlands.
[Xu et al., 2016] Xu, H., Yao, D. D., and Zheng, S. (2016). Optimal policies for a two-product inventory system under a flexible substitution scheme. Production and Operations Management, 25(6):1088-1105.
[Xu et al., 2015] Xu, X., Meteyer, S., Perry, N., and Zhao, Y. F. (2015). Energy consumption model of binder-jetting additive manufacturing processes. International Journal of Production Research, 53(23):7005-7015.
[Yu et al., 2017] Yu, Y., Shou, B., Ni, Y., and Chen, L. (2017). Optimal production, pricing, and substitution policies in continuous review production-inventory systems. European Journal of Operational Research, 260(2):631-649.

