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ABSTRACT

MODEL PREDICTIVE CONTROL
FOR BUILDING TEMPERATURE REGULATION

by
Weiqiang Dong

With the rapid increase in human consumption of energy, global warming is becoming an urgent problem. According to the past research, a significant part of greenhouse gas release and electricity consumption has been connected to buildings. Heating systems are the major contributor to high energy consumption of buildings. Better methods for decreasing energy consumption of buildings should be proposed and applied.

Model predictive control is widely used in building temperature regulation. Each model predictive control has its advantage. Through the comparison of several methods, this thesis discusses their respective features. The combination of a Matlab-based modeling system CVX and model predictive control makes the optimization easy.

A new idea for a model predictive control method about temperature regulation is proposed. The simulation of the disturbance will be implemented as an input. The demonstration of its advantage is shown. This thesis uses temperature index and energy index to help people evaluate the result. An index tuner is put forward to simplify the optimization.
MODEL PREDICTIVE CONTROL
FOR BUILDING TEMPERATURE REGULATION

by
Weiqiang Dong

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CHAPTER 1

INTRODUCTION

1.1 Temperature Regulation

With the rapid increase in human consumption of energy, high-tech products and natural resources, global warming is becoming an urgent problem. Better methods for decreasing the emission of greenhouse gases should be proposed and applied. It is well recognized that the major reason of the increase of green gases is the growing energy consumption.

For the requirement of sustainable development, energy saving is an urgent issue that people should pay much attention to. People have to reduce local or global environmental damage but also the usage of non-renewable fossil fuels. Organizations all over the world have their policies for energy saving. Importantly, better approaches for energy saving should be proposed. They allow people to save more money, have more jobs, increase industrial productivity, and foster economic growth per unit of energy than other alternatives.

According to the past research, 40% CO₂ release and 71% of electricity consumption are connected to buildings [1]. The temperature regulation system including the cooling and heating functions is the major reason for high energy consumption in a building. Its heating systems consume more than 50%, which means about 23% of total energy consumption [2].

The research on building temperature regulation is not only because of energy saving. The comfort level is another reason. Since the present methods for building temperature regulation have much delay and inaccuracy, people cannot always stay with comfortable temperature in a building.
1.2 Introduction to MPC

During the optimization of building temperature regulation, a proper control strategy should be implemented. Because of the particular situation of building temperature regulation, model predictive control is among the best method.

Occupancy is the most important factor for building temperature regulation. People do not care about the temperature of a zone if no one is in it. If energy would be spent for temperature regulation of an empty zone that would totally waste the energy. Although the future state of each zone cannot be confirmed in a building, people usually have some types of future schedule in using a zone, which can help one avoid the waste of any energy in an empty zone. Some methods for detecting the occupancy were proposed in the past research. An occupancy model can be built to predict the future occupancy of people [5]. In the area of real-time detection, a single camera can be used to track the behaviors of people in the building [6]. The CO2 sensor is a cheap and simple solution for the detection of occupancy [7].

The traditional control method cannot take full advantage of an occupancy schedule. However, model predictive control is a control method based on a future schedule essentially. Given a future plan, it will generate the optimal control trajectory.

In addition, traditional control methods are usually rule-based, which means that they always need to set a region for the variable during the control. If there is no limit in traditional methods, the result will be unsatisfactory. Suppose that there is a half hour meeting in one zone, the controller does nothing until people enter the zone. However, there is always delay in temperature regulation. It may take ten minutes to achieve the setting temperature. After the meeting is over, the room is empty but with comfortable temperature in next ten minutes. In addition, to reach the target
temperature quickly, the input will increase suddenly which is hard to realize and harmful for system operations. Nevertheless, in model predictive control, the input is generated by a future schedule. It means that the temperature can reach the target smoothly since meeting start time is known.

In the use of model predictive control, the importance of a future schedule should be understood. If it is unknown, a good prediction cannot be made. Then model predictive control has no difference in comparison with traditional control methods. Some discussion on prediction or identification of future events will be made in the later part of this thesis.

There are some researches discuss the realization of model predictive control in some special cases [2, 3].

There are several kinds of model predictive control, such as decentralized MPC, centralized MPC and distributed MPC [4].

The decentralized is the most widely used MPC in building temperature regulation. This thesis will discuss the characteristic of decentralized MPC in the later section. Some improvement of decentralized MPC will be generated.

The necessary characteristic of Centralized MPC is that there is one and only one, controller in the system. Thus the first step is to rebuild the system model. Combining all the zones in the building, the system model of the whole building can be generated. Through solving this model, the control signal of the central controller can be determined.

The centralized MPC aims to find the optimal input through the model of the whole building. Its result is excellent. However, because of the huge calculation required by the centralized MPC, this is not a preferred approach.

The distributed MPC can gain an excellent result as centralized MPC does [9].
Unlike centralized MPC, its computational requirement of its controller can be satisfied easily. Meanwhile, the change of the zones will not be harmful to the model.

Because of the prediction of each zone requires its neighbor zones’ information, including real time state and future prediction. An information exchange network is necessary. Although the control result of distributed MPC is as excellent as centralized MPC. It has two disadvantages. The initial one is the requirement of an information exchange network. The damage of such network will increase the error rate in the optimization. The other one is that in the distributed MPC, all the previous inputs need to be considered. If the sample volume is too large, there will be much computation.
CHAPTER 2
MODEL OF BUILDING TEMPERATURE REGULATION

2.1 Introduction

There are several kinds of models for building temperature regulation. A model with air handling unit (AHU) and variable air volume (VAV) boxes will be considered in this thesis [3].

In this model, AHU mixes the return air from zones and outside air. The mixed air is sent back to zones by a supply fan. After passing a cooling coil, the mixed air arrives at zones. Each zone has a VAV box that contains heating coils to reheat the air when it is necessary and a zone damper to control the volume of input air.

This work makes some modification to the model. The temperature of the mixed air after passing the cooling coil cannot be controlled in real time. Since many buildings may share one AHU, it is not suitable to make the air temperature of AHU convertible. Actually, it is always invariant in one month or maybe even a half year.

If temperature regulation is well performed, there is no need to use the heating coils for reheating the air during the real-time control operations, because the reheat of cooling air is a waste of energy absolutely. But the heating coil of a VAV box is still necessary for robustness. Since if there is something wrong with the cooling system and the building is too cold, the heat coils can help people raise the temperature.

The energy consumption of the system is caused by the supply fan, cooling coils and heating coils. Since the air temperature of AHU is regarded as an invariant, the energy consumption of cooling coils during the control will not be considered. Instead, the supply fan is used to control the total input. In the VAV box, the output is controlled by the damper if the heating coils are not used. It is known that the indoor
air temperature is influenced by the volume and temperature of input air. A balance of air mass and temperature should be found for the optimum result.

2.2 System Constraints

There are some constraints on the above system.

1. The temperature in the zones $T \in [T_{\text{min}}, T_{\text{max}}]$ where $T_{\text{min}}$ and $T_{\text{max}}$ are the minimum and maximum temperature that people can accept, respectively.

2. The volume of input air $V \in [V_{\text{min}}, V_{\text{max}}]$. $V$ is the volume of air which is sent to the zone. Since people always want fresh air, there is a limit for the minimum air. If the input air is too much, people may feel uncomfortable.

3. The difference of indoor temperature and outdoor temperature $\Delta T_{io} \in [\Delta T_{io\text{min}}, \Delta T_{io\text{max}}]$. People will also feel uncomfortable if the temperature difference is too large.
A cost function is an important part of a control problem. It reflects what people concern in the control process. In the building temperature regulation, there are two primary issues to be considered. The first one is the consumption of energy. Energy means money, the less energy consumption, the less expense and the less environmental impact. That is why people try to find out the best way to make the most effective temperature regulation. The second one is the effect of the control result. In another word, how comfortable a people in a building feel. Actually, temperature in a building is not the only thing that influences how people feel. This thesis focuses on the temperature only. In other words, this work assumes that the other factors make negligible impact on the human comfort level.

At the beginning, a single zone model will be considered. It gives the base-line situation. Normally, the cost function for one moment is shown as follows:

\[
J = (R_s - Y)^T (R_s - Y) + U^T \bar{R} U
\]

In the formula, \( J \) is the cost, \( Y \) is the system output, \( R_s \) is the set-point signal, \( \bar{R} \) is a tuner, and \( U \) is the system input.

A control trajectory is always connected to the predictive horizon and the control horizon. The cost at each individual moment will not be calculated.

The set-point signal means the temperature that people set for the zone in temperature regulation. The system output means the exact temperature in the zone or the measurement. A system input usually reflects how much energy people consume for the temperature regulation. Sometimes people may weigh more about the energy
consumption when the energy is more expensive. Sometimes people want more accurate temperature regulation to achieve a high comfort level. In those situations, the tuner $\overline{R}$ will be changed to meet a practical need. Hence, the cost function will be divided into two parts.

The first term is related with the objective to minimize the error between output and set-point signal.

The second term aims at reflecting the size of input, also known as, the amount of energy people use.

Thus, the concept of temperature index and energy index should be introduced. Usually, the temperature index is the absolute value of the difference between target temperature and the measured temperature. The energy index is total energy people consume.

In practice, the moments in a prediction horizon will be considered together in the cost function.

$$J = \sum_{j=1}^{N_p} \delta^k (j) \overline{R} [R_s(k + j) - Y(k + j)]^T [R_s(k + j) - Y(k + j)] + \sum_{j=2}^{N_c} U^T (k + j) \overline{R} U(k + j)$$

$N_p$ is called the prediction horizon, which indicates that the future state variables are predicted for the $N_p$ samples.

$N_c$ is called the control horizon representing the number of parameters used to capture the future control path.

The cost function above is also widely used as an objective function in model predictive control. Index $k$ in the cost function means the $k^{th}$ moment. $N_p$ represents the length of the prediction horizon, while $N_c$ means the length of the control horizon. Therefore, the target is to generate the input signal for the minimum result of the cost function in the region of given predictive horizon and control horizon.
There is $\delta(j)$ in the cost function, which is defined as follows [4]:

$$
\delta^k(j) = \begin{cases} 
1, & k + j \in \Omega \\
0, & k + j \notin \Omega 
\end{cases}
$$

$\Omega$ represents the zones in use. $\delta^k(j)$ contains the information about whether a zone is absent in the control strategy. $\overline{R^1}$ is another tuner. The combination of $\delta^k(j)$, $\overline{R^1}$ and $\overline{R}$ is important for the energy saving. If there is no person in that zone, there is no need to calculate the temperature index for that zone. After someone enters the zone, the temperature index should be considered. Two tuners $\overline{R^1}$ and $\overline{R}$ can only reflect the attitude of the balance between the temperature index and energy consumption one, but also judge the balance among zones. For example, people can pay less attention on a storage room than a room with expensive instrument. In the research about multiple zones, the cost function needs to be modified. But essentially, temperature and energy consumption are the most important issues that should be considered as the priority.
CHAPTER 4

OCCUPANCY IDENTIFICATION

The occupancy of zones should be identified. However, it is not suitable to use the word “identified”. This is simply because people could never identify the future. What people can do is to predict the future and measure the occupancy as accurately as they can.

At the beginning, it is important to set up a correct model to reflect the thermal characteristics of a building. The model should not only focus on the system state equation, but also the coefficient of thermal transmission. In the previous section, when there is no heat transmission, a perfect optimization result can be obtained. When there is a heat transmission coefficient, the system can be out of control. Many factors can affect the system model, such as material of the building, the size of the rooms in the building, and the amount of furniture and facility in the building.

Moreover, a building should have a wide and precise communication network to collect the schedule of occupancy, such as the meeting and class schedules. It is helpful if the number of attendees is known. For example, suppose that there is a 20 people meeting in one room at night on Monday. Comparing to other empty rooms, people will pay more attention to the temperature regulation for that room. On the other hand, if the vacancy of a room is known, the temperature regulation for that room can be loosed. The temperature can vary in an acceptable region and would not be harmful for the quality of the control result. Also, the weight of the temperature index can be increased in the objective function when there are many people in the zone to show its increased importance in the objective function. The information of future schedules is the main idea of model predictive control. The cost function in the
thesis intends to reflect whether the zone is occupant. The influence between the number of people and thermal effect can be tested and proved by experiments.

Last but not the least, no matter how the development of a communication network and a perfect schedule people have, it is impossible to know completely what is happening in the future. The measurement of the occupancy will influence the exact effect of the control result due to some unexpected and random events. Suppose that there is a meeting at night on Monday but the lecturer cancels it because of illness and then the zone becomes empty. Another example is that there is a fifty people class but only fifteen people attend. Consequently, it will be a waste to pay too much attention to that zone.

There are many approaches for detecting the real-time occupancy in a zone. For example, in real-time counting, the usage of a door can be counted, or use a CO₂ detector to detect the number of people in the zone. Similarly, what will be useful is a heat sensor. Finally, video cameras are the most widely used approach for counting the people in the zone. There are some effective counting methods for passing through a door [5, 6].

However, the usage of the door may not be helpful individually, because the number of pedestrians in every use of the door cannot be confirmed. Sometime the door is always open for some reasons. The CO₂ detector is low-cost and useful if there is no need for an accurate control, but there is always about 20 minutes delay of the detection [7]. Thus it may be applied when people do not pay much attention to the effect of temperature regulation. What is more, the delay may lead to a waste of energy. A heat sensor is faster and more accurate, which is similar to a CO₂ detector. On the other hand, only a rough result can be obtained.

The video cameras are used as an effective means for counting people in a
building. However, since video cameras just capture the change of the number of people passing the door, information about the distribution of people in the building should be known initially. With the development of digital image processing and facial recognition, a single camera is enough for the detection [6].

If people do not have any budget in real time detection of occupants in a building, an occupancy model for predicting the number of people in a building can be built [8]. It is low-cost but may need large data and many experiments. Since it is not a real time method, the optimal result cannot be guaranteed.

In summary, it may not be the best to use any individual method alone only. Their combination should lead to a better result. For saving money, an occupancy model should be built. But some real-time methods also need to be introduced. The video camera and digital image technologies are a good choice. Through some experiment, based on a particular situation, a proper method for building should be determined. If the energy consumption is acceptable, the occupancy model can be used individually to determine the control signal. Note that an occupancy model is especially helpful when something goes wrong with real time detection. Video cameras can help make the temperature regulation precise. For example, if there is not many people in the building, more attention can be paid to energy saving. If there is none in some zones, the temperature control device for those zones can be turned off to save more energy. These considerations will be reflected in the objective function in this thesis.

Since it is not easy to detect the real-time occupancy, in practice, the unknown occupancy of the zone is always regarded as a disturbance. In this thesis, an objective function will be used to reflect the target for temperature regulation.
CHAPTER 5

OPTIMIZATION WITH CVX

The optimal problem should be solved at every sampling time instant. Although the derivative method is direct and concise, the size of a temperature regulation problem grows rapidly when there are a realistic number of zones in a building. It is improper to use the condition of the derivative due to its huge computational task. A better method to deal with this problem should be proposed.

CVX is a Matlab-based modeling system for convex optimization, which is helpful for people to solve the calculation problem at each time instant. This thesis intends to use the CVX software developed by Information Systems Laboratory, Department of Electrical Engineering, Stanford University [10, 11].

5.1 Single Zone Method

By using the previous system model, this section aims to minimize the cost function. The only difference is that the optimization part in the receding horizon control is now performed via the CVX method. This section continues to use the temperature index $I_c$ and the energy consumption index $I_w$ to represent the goal of temperature regulation.

$$J = (R_s - Y)^T (R_s - Y) + \Delta U^T \bar{R} \Delta U$$

Here is the simulation part of the CVX method of a single zone. The target temperature remains 20°C while the state horizon and control horizon are the same. The results shown in Figures 5.1 and 5.2 are obtained by a high emphasis of input ($r_w=100$) and a low emphasis of input ($r_w=1$).
Figure 5.1 Simulation with $r_w=100$.

Figure 5.2 Simulation with $r_w=1$. 
Table 5.1 Comparison of CVX Method With Different Index Tuner

<table>
<thead>
<tr>
<th>Index tuner ((r_w))</th>
<th>Energy index</th>
<th>Temperature index</th>
<th>(U)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>69</td>
<td>597</td>
<td>4.4</td>
</tr>
<tr>
<td>1</td>
<td>413</td>
<td>393</td>
<td>106</td>
</tr>
</tbody>
</table>

The results in Figure 5.1 and 5.2 lead to the conclusion that CVX works well with the system model. The index tuner \((r_w)\) still plays an effective role in the cost function. When \(r_w= 100\), meaning that people are emphasizing to reduce the energy, the total amount of power \((U)\) people use is relatively small.

5.2 Comparison

In order to test how effective the CVX approach is, here is a comparison between the result of the CVX approach and the result of a derivative approach. In this situation, the tuner \((r_w)\) is set as 1. The other condition can be set in the same way.

The cost function used in derivative approach is shown as follows.

\[
J = (R_s - Y)^T (R_s - Y) + \Delta U^T \bar{R} \Delta U
\]

The necessary condition of the minimum \(J\) is obtained as:

\[
\frac{\partial J}{\partial \Delta U} = 0
\]

The optimal input of derivative approach can be generated by solving the equation.
Figure 5.3 Result of CVX method and derivative approach.

Figure 5.3 shows the comparison between a derivative method and CVX method. It looks like only one curve in each figure. Actually, since the value of the output and control signal of two methods are almost the same, their trajectories are overlapped. In conclusion, the CVX method can definitely be implemented to this model of temperature regulation.
The prior discussion focused on the methods implemented for a single zone. Since it is hard to calculate the optimized input directly in the derivative method when the number of zones in the building grows, the derivative method is no longer effective. This section intends to discuss about the CVX method, which will generate the answer quickly. There are several MPC methods introduced in [4], the CVX method will be applied into centralized MPC and distributed MPC, respectively.

The heat transmission plays an important role in multiple zone temperature regulation. It seems easy to solve the system if people treat multiple zones as several single zones. But, in fact, neglecting the thermal influence among zones can lead to energy waste. There is a proof of this conclusion in the later section.

A simple model can be built as follows:

\[
\begin{array}{cc}
R1 & R2 \\
R3 & R4 \\
\end{array}
\]

Figure 6.1 4-zones model.

\(R_1-R_4\) represent 4 zones in a building. Each zone has a VAV box.

The heat transmission ratio between \(R_i\) to \(R_j\) is \(B_{ij}\) (i, j = 1, 2, 3, 4). If two zones are not adjacent, \(B_{ij} = 0\). Actually, people can hardly identify the exact data of the transmission ratio. It needs many experiments and a good deal of calculation. In practice, the heat transmission cannot be neglected.
6.1 Decentralized Model Predictive Control with CVX

The decentralized model predictive control is the most used building thermal control structure with prediction nowadays. Each zone has its own controller that will not be influenced by other zones. One controller is responsible for one zone only. If the temperature is different from those of the neighbor zones, there should be a heat transfer between two zones. Since the decentralized MPC neglects the thermal transfer among the zones, it is not the optimum control method for temperature regulation.

Material of walls, amount of furniture, and the size of room will all influence the heat transmission coefficient. Although the accurate result of the coefficient is hard to calculate, its approximate value can be obtained by multiple experiments. Supposing that the coefficient of the heat transfer among zones is known, the temperature of neighbor zones can be considered as a disturbance. In other words, people will not consider the heat transfer during the optimization. Instead, the result of thermal transfer will be reflected in the process of temperature measurement.

6.2 The Discussion about Heat Transmission Coefficient

For comparison, here is the result without heat transfer first. Since each controller just responds to any temperature changes in their own zone, each zone has its own cost function. Figures 6.2 to 6.5 give the results for the 4-zone model.
Figure 6.2 Simulation without heat transfer of zone 1.

Figure 6.3 Simulation without heat transfer of zone 2.
Figure 6.4  Simulation without heat transfer of zone 3.

Figure 6.5  Simulation without heat transfer of zone 4.
Table 6.1 Temperature Index and Energy Index without Heat Transfer

<table>
<thead>
<tr>
<th>Zone</th>
<th>Temperature index</th>
<th>Energy index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>309.65</td>
<td>131.04</td>
</tr>
<tr>
<td>Zone 2</td>
<td>96.60</td>
<td>165.32</td>
</tr>
<tr>
<td>Zone 3</td>
<td>198.96</td>
<td>168.70</td>
</tr>
<tr>
<td>Zone 4</td>
<td>149.82</td>
<td>59.43</td>
</tr>
<tr>
<td>sum</td>
<td>755.03</td>
<td>524.17</td>
</tr>
</tbody>
</table>

Suppose that the coefficient of heat transmission for the four zones model is given: $B_{12}=0.1$, $B_{13}=0.15$, $B_{24}=0.15$ and $B_{34}=0.2$. There is no heat transmission if two zones are not adjacent.

The result of the situation with heat transmission is shown in Figures 6.6- 6.9:

Figure 6.6 Simulation with heat transfer of zone 1.
Figure 6.7 Simulation with heat transfer of zone 2.

Figure 6.8 Simulation with heat transfer of zone 3.
The results show that the cost of control arises significantly.

**Table 6.2** Temperature Index and Energy Index with Heat Transfer

<table>
<thead>
<tr>
<th>Zone</th>
<th>Temperature index</th>
<th>Energy index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>414.18</td>
<td>139.14</td>
</tr>
<tr>
<td>Zone 2</td>
<td>90.57</td>
<td>158.23</td>
</tr>
<tr>
<td>Zone 3</td>
<td>318.56</td>
<td>170.31</td>
</tr>
<tr>
<td>Zone 4</td>
<td>114.30</td>
<td>63.76</td>
</tr>
<tr>
<td>sum</td>
<td>937.62</td>
<td>531.44</td>
</tr>
</tbody>
</table>

Because of the high coefficient, both temperature and energy consumption indices grow. Actually there is only a little increase of the coefficient. However, the
result shows that the trajectory becomes rough. If the coefficient becomes higher, the system can no longer maintain the temperature within the ideal region.

The comparison illustrates that the thermal transfer among the zones cannot be ignored. The influence by neighbor zones should be treated as an input or disturbance if it is unknown.

From the above discussion, the real temperature can follow the set point trajectory closely in the decentralized model predictive control. However, since it does not consider the heat transfer in the optimization, a better approach to reduce the energy consumption should be established.
A traditional control method for builds is a rule-based one. For example, if the goal is to maintain the temperature of the zone in a region, the output temperature must be examined every moment to confirm that it will not exceed the limit.

In model predictive control, temperature and energy indices can be used to evaluate the control quality of the result. They could be used for realizing the limit. The high temperature index means that the temperature may exceed the limit.

The tuner $\overline{R}$ is used for finding a balance between energy consumption and temperature regulation. In practice, usually there is no much occupancy at night. The constraints can be loosened at night. The constraint of temperature is usually like the lines as shown in Figure 7.1.

Figure 7.1 Relation between tuner values and constraints.

In Figure 7.1, the constraint needs to be strictly enforced from the 7th period to 11th period since there is a meeting during that time period in the schedule. In this situation, the tuner value is set as 0.4. From the 4th to 6th periods and 12th to 14th
There is no schedule for that zone. There is a possibility that someone may come in. The tuner value is set as 0.8. The other periods belong to the night time, and the tuner can be set as 2 or more which means that people care more about the energy consumption but less about the comfort level.

The simulation of a single zone is shown next.

A) The expected region is [18.5, 21.5].

When the target temperature is 20, the value of tuner in this situation should be 1.

![Figure 7.2 Tuner value is 1.](image)

**Table 7.1** Temperature Index and Energy Index (tuner value is 1)

<table>
<thead>
<tr>
<th>Temperature index</th>
<th>Energy index</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.95</td>
<td>153.37</td>
</tr>
</tbody>
</table>
B) The expected region is [19.5, 20.5].

When the target temperature is 20, the value of tuner in this situation should be 0.4.

![Figure 7.3 Tuner value is 0.4.](image)

**Table 7.2** Temperature Index and Energy Index (tuner value is 0.4)

<table>
<thead>
<tr>
<th>Temperature index</th>
<th>Energy index</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.77</td>
<td>164.47</td>
</tr>
</tbody>
</table>

From the comparison, when the value of the tuner increases, the precision of temperature regulation is improved. Meanwhile, people do not need to set the limit anymore. Instead, people can modify the value of the tuner to make the output approach the desired set point.
CHAPTER 8
AN IDEA OF MODEL PREDICTIVE CONTROL

The decentralized model predictive control is not effective enough. The distributed model predictive control or the centralized model predictive control is complex to realize. The combination between the simplification of decentralized model predictive control and the efficiency of distributed model predictive control can be used to generate a better method.

In the decentralized model predictive control, people make no use of the heat transmission coefficient. Suppose the heat transmission coefficient is known, the system output after thermal transfer effect can be simulated.

In the temperature regulation, to optimize the system is also to overcome the effect of thermal transfer effect. The difference between the simulated temperature outputs with and without heat transmission can be assumed as another system input.

8.1 Comparison with Decentralized Model Predictive Control

This section shows the comparison between the result of decentralized predictive control and the new method by a two zones model. In this example, the heat transmission coefficient is 0.2, and the value of tuner is 1. The target temperature is 20.

The result of decentralized MPC is shown in Figures 8.1-8.2.
Figure 8.1 Result of decentralized model predictive control of zone 1.

Figure 8.2 Result of decentralized model predictive control of zone 2.
Table 8.1 Temperature Index and Energy Index of Decentralized MPC

<table>
<thead>
<tr>
<th>Zone</th>
<th>Temperature index</th>
<th>Energy index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>360.91</td>
<td>129.92</td>
</tr>
<tr>
<td>Zone 2</td>
<td>100.34</td>
<td>158.63</td>
</tr>
</tbody>
</table>

The result of the new method is given in Figures 8.3-8.4.

**Figure 8.3** Result of new method of zone 1.

**Figure 8.4** Result of new method of zone 2.
Table 8.2 Temperature Index and Energy Index of New Method

<table>
<thead>
<tr>
<th>Zone</th>
<th>Temperature index</th>
<th>Energy index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>313.87</td>
<td>127.01</td>
</tr>
<tr>
<td>Zone 2</td>
<td>99.17</td>
<td>161.18</td>
</tr>
</tbody>
</table>

8.2 Disturbance

This is a 4 zones example. The disturbance of the output is random following a uniform distribution over [-1.5, 1.5]. The set point temperature is {20, 22, 24, 18}. The coefficients of heat transmission are $B_{12}=0.1$, $B_{13}=B_{24}=0.15$, $B_{34}=0.2$. The lines in the Figure 8.5-8.8 show the limits of temperature. From the result of the previous section, the method uses the tuner instead of the constraint of temperature. The tuner value is 0.4 when there is a strict limit while the tuner value is set as 1 otherwise.

The constraint of temperature for each zone is shown by the red line in Figures 8.5-8.8.

![Figure 8.5 Constraint of temperature of zone 1.](image)

Figure 8.5 Constraint of temperature of zone 1.
Figure 8.6 Constraint of temperature of zone 2.

Figure 8.7 Constraint of temperature of zone 3.
Figure 8.8 Constraint of temperature of zone 4.

The result from decentralized model predictive is shown in Figures 8.9-8.12:

Figure 8.9 Result from decentralized MPC of zone 1.
Figure 8.10 Result from decentralized MPC of zone 2.

Figure 8.11 Result from decentralized MPC of zone 3.
The result illustrates that the decentralized model predictive control can no longer satisfy the need when there is a 7% disturbance. The result of the new method is shown in Figures 8.13-8.16.
Figure 8.14 Result from the new method of zone 2.

Figure 8.15 Result from the new method of zone 3.
From these figures, it is clear that the output temperature always stays between the constraint lines. This is a clear demonstration that the result from the new method can still meet the requirement when there is a disturbance. The result shows the new method is better than the decentralized model predictive control.
CHAPTER 9
CONCLUSIONS

The selection of a model predictive control method for temperature regulation depends on the practical situation. Usually the major disturbance is the heat transmission. If people know the future schedule and the coefficient of heat transmission, people can decrease the influence of thermal transfer. The result of control can be influenced by many factors. To improve control quality will often raise the complexity of a control strategy.

The method proposed in this thesis is more complex than decentralized MPC. For that reason the cost of temperature regulation may be raised. The result of the method is not as good as distributed MPC. Thus this method can be implemented to find the balance between simplification and precision.

The identification of occupancy is important for building regulation. The future direction will be the identification of occupancy. An occupancy model will be implemented in future research.
REFERENCES


