ABSTRACT

OPTIMIZING INTEGRATED SERVICE FOR A TRANSIT ROUTE WITH HETEROGENEOUS DEMAND

by
Yavuz Yusuf Ulusoy

The methodology developed in this dissertation attempts to optimize integrated service that minimizes the total cost, including user and supplier costs, of a transit route with heterogeneous demand. While minimizing total cost, a set of practical constraints, such as capacity, operable fleet size and frequency conservation, are considered.

The research problem is presented in three scenarios, consisting of various service patterns (e.g., all-stop, short-turn and express) under heterogeneous demand. A logit-based model was used to estimate passenger transfer demand. An exhaustive search method was developed to find the optimal solutions for a simplified transit route with six stops, and a Genetic Algorithm (GA) was developed to find the optimal solution for a real-world, large scale transit route. The optimized variables include the combination of service patterns, the associated service frequencies, and stops skipped by the express service.

A six-stop transit route was designed and analyzed via a proof-of-concept demonstration to ensure that the developed models are capable of finding the optimal solutions. A sensitivity analysis was conducted, which enables transit planners to quantify the impact of various model parameters (e.g., user value of time, vehicle capacity, operating cost, etc.) to the decision variables and the objective function. Finally, the developed models and solution algorithm were applied to optimize integrated service for a real world bus route in New Jersey.

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by
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To
My beloved wife and family

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CHAPTER 1

INTRODUCTION

1.1 Background

Improving public transportation system efficiency and level of service attempts to stimulate transit demand at the least operating cost, which has been recognized as a potential way of mitigating traffic congestion, increasing mobility and reducing environmental impacts. The development of efficient and convenient transit service has been frequently discussed in past decades (Vuchic and Newell, 1968; Byrne, 1976; Wirashinghe, 1980; Tsao and Schonfeld, 1984; Kuah and Perl, 1988; Spasovic and Schonfeld, 1993; Spasovic at al, 1994; Chien and Spasovic, 2002; Chien, 2005; Zhao and Zeng, 2006 and 2008) to achieve sustainable public transportation systems.

Daily travel demand is increasing in the United States. From 1995 to 2005, daily travel on major US roadways increased 34% from 2.79 to 3.73 billion vehicle miles of travel. This increase worsened the congestion problem in regions of all sizes, especially in highly populated metropolitan areas. Congestion caused urban Americans to travel 4.16 billion hours more, which resulted in purchasing and burning an extra 2.81 billion gallons of fuel for a congestion cost of $87.2 billion in 2007. Congestion mostly impacts the people who typically travel during the peak hours. The annual delay for the peak-period traveler was 38 hours and the same traveler wasted 26 gallons of fuel (Schrank and Lomax, 2007 and 2009).

Public transportation modes play an important role in providing travel options, specifically to people in congested regions. Without public transportation services, the urban areas would have suffered an additional 646 million person-hours of delay and consumed 398 million more gallons of fuel. This brings an additional $13.7 billion congestion cost, which is a 16 percent increase over 2007 levels in urban areas.

The growth of transportation problems, including congestion, increasing gasoline prices and vehicle emissions, is forcing transportation agencies at all levels to consider better public transportation initiatives to reduce the impact to our environment. Thus, sustainable transportation considerations started to shift the emphasis in public spending and actions on increasing capacity to improving service efficiency, which encourages high value land use, increases productivity, reduces transportation and infra-structure costs, and offers cost effective services (GDRC, 2008).

As a major player in the urban transportation system, public transit has been widely recognized as a potential mode that may increase productivity, provide more job opportunities, promote retail sales, and rationalize urban development patterns, in addition to reducing air pollution, lowering energy consumption, improving mobility, and mitigating traffic congestion. Moreover, providing mobility for people with low incomes, disabled or unable to drive, elderly, children, or those who do not own a car, public transit offers meaningful travel alternatives.

There were approximately 51 billion passenger-miles of travel on public transportation systems in the urban areas in 2005 (NTD, 2007). The annual travel ranges from an average of 18 million passenger miles per year in small urban areas to about 2.7 billion miles in metropolitan areas. With suburban sprawl and dispersion of employment, automobile use is challenging public transportation systems. An operationally and economically efficient transit system can help to meet these requirements while potentially reducing congestion and energy consumption.

An effective and efficient transportation mode can provide competitive service under practical constraints, such as limited operating budget and excessive demand, to maximize the users' benefits at the lowest supplier cost. Toward this aim, this study focuses on developing optimal service strategies which minimize total cost, including user and supplier costs, for a generalized transit route with heterogeneous demand.

1.2 Problem Statement

In metropolitan regions, the demand on a transit route is generally distributed heterogeneously along the route, and setting a conventional (local) service may not be an efficient and cost effective way to satisfy the route demand. In designing services for transit routes, one of the main problems faced by transit planners is developing optimized service patterns and associated service frequencies to minimize total (including user and supplier) cost while satisfying the route demand.

In traditional service strategies, the ratio of maximum passenger flow to vehicle capacity (based on desired occupancy) is deemed as the minimum required frequency to meet passenger demand. In such a case, operating vehicles from one end of the route to the other may be inefficient due to low occupancy of segments where demand is lighter. Implementing integrated transit service patterns (SPs) such as the all-stop, short-turn, and express (stop-skipping) services, may be beneficial. Most studies (Furth, 1987 and 1988; Ceder 1988 and 1989, Delle Site et al, 1998) have focused on maximizing the utilization of the vehicles and/or minimizing the supplier cost, whereas few ones involved in the integration of SPs considering potential transfer demand.

The development of an analytical model to optimize integrated (e.g., all-stop, short-turn, express) SPs and associated service frequencies to minimize the total system cost is desirable for not only optimizing the problem but also exploring the relationship among decision variables and model parameters.

1.3 Objective and Work Scope

The objective of this study is to develop an analytical model to optimize transit SPs and associated service frequencies for a generalized transit route with heterogeneous demand. The proposed model is aimed to search for the most beneficial service patterns (SPs) and associated frequencies for efficient and effective operation. Heterogeneous demand conditions and potential transfer demand are considered within the optimization process.

This dissertation is intended to optimize transit SPs especially for underutilized operations because of demand heterogeneity and quantify the benefit. The sensitivity of critical model parameters (e.g., spatial demand distribution, value of user's time, etc.), affecting the objective total cost is analyzed, while the optimal relationship between the model parameters and decision variables is investigated.

There are several alternatives to combine various transit SPs to optimize the transit operation. In this dissertation three SPs (see Table 1.1, in which "A", "S", and "E" represent all-stop, short-turn, and express SPs, respectively) are used to create two different models to analyze the integration of SPs shown in Figure 1.1.

Table 1.1 Studied Service Patterns (SPs)

Symbols	**Service Patterns (SPs)**
A	All-Stop SP
S	Short-Turn SP
E	Express (Stop-skipping) SP

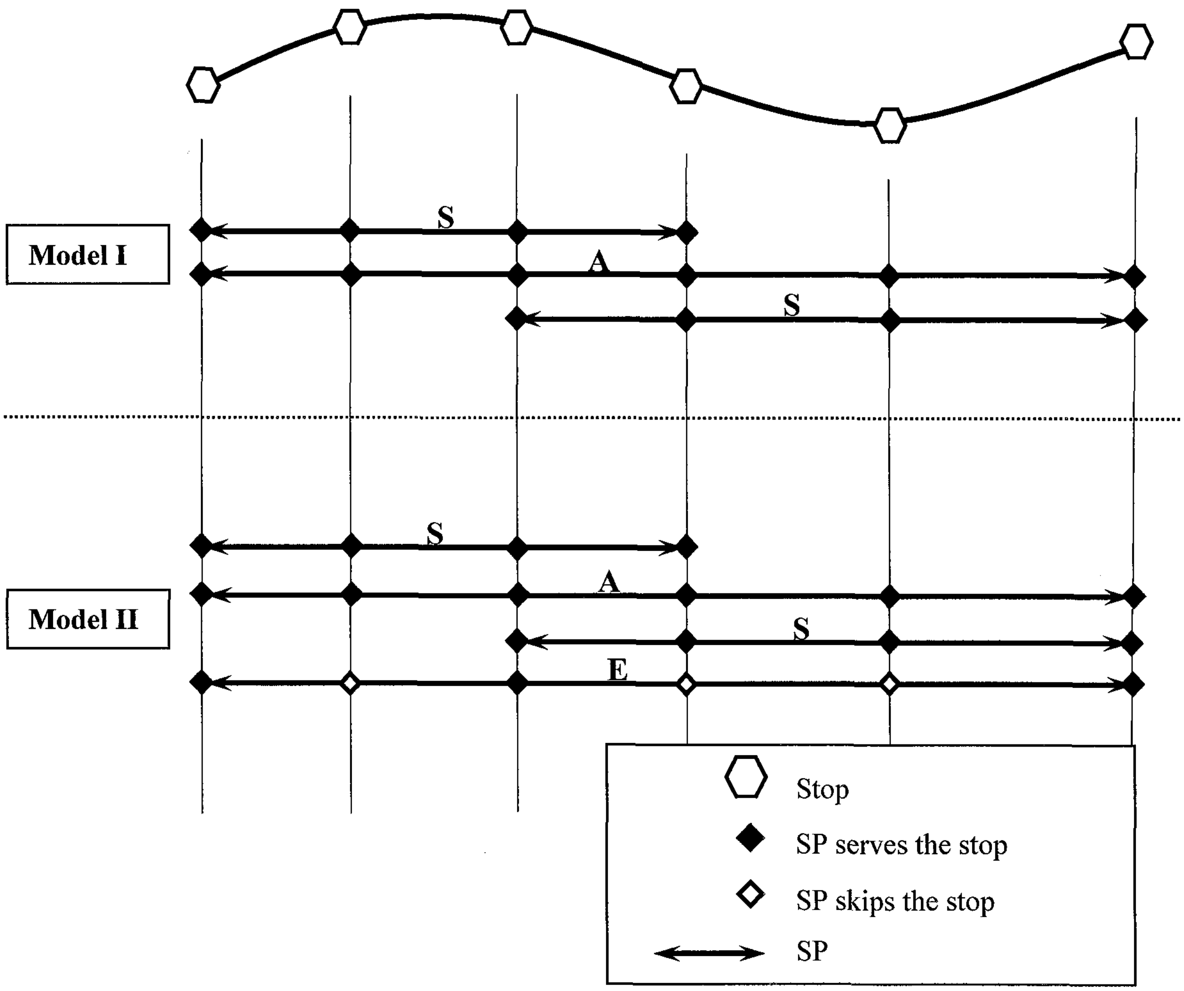

Figure 1.1 Transit service patterns considered in the proposed models.

Two models are proposed in Table 1.2 to deal with different integrated services. Model I is developed for optimal integrated all-stop and short-turn SPs to minimize the total cost. An all-stop SP serves every stop between the end terminals of the studied route, while a short-turn SP_{ij} is a smaller scale all-stop service, which serves every stop between short-turn points i and j.

Table 1.2 Proposed Models for Integrated Services

Models	Integrated Services
I	A + S
II	A + S + E

Model II, enhanced from Model I, is developed to analyze the integrated SPs including all-stop, short-turn and express (stop-skipping) services, in which a number of stops are skipped by the express SP. Similar to Model I, Model II also minimizes the total cost that is yielded by the optimized SPs, associated service frequencies, and the stops skipped by the express service. A logit-based model is used to estimate transfer demand.

1.4 Organization of the Dissertation

This dissertation is organized into six chapters. Chapter 1 introduces the research problem and presents the objective and work scope. Chapter 2 discusses the results of literature reviews on various models and techniques employed in previous optimization strategies related to transit operations. Chapter 3 describes the methodology for the developed models to optimize the SPs and associated service frequencies. A mathematical model is developed and analytically optimized by considering two different models defined in the work scope. Chapter 4 introduces an Exhaustive Search Algorithm and a Genetic Algorithm (GA) approaches to solve the optimization problems defined in three scenarios. Chapter 5 presents two case studies. First, a six-stop transit route is designed and analyzed via a proof-of-concept demonstration to ensure that the developed models are capable of finding the optimal solutions. Then, a case study of a real world transit route (New Jersey Transit Springfield Avenue Line) is presented, in which the developed model is used to search for the optimal solution with GA. Finally, findings and recommendations are presented, and potential extensions of this study are summarized in Chapter 6.

CHAPTER 2

LITERATURE REVIEW

Previous studies related to this research are summarized in this chapter, and are discussed in five sections. The state-of-the-art operating strategies for improving transit services are discussed in Section 2.1. In Section 2.2, general transit network optimization problems are investigated and summarized. A number of transit passenger assignment studies related to this study are reviewed in Section 2.3. To solve the research problem, solution algorithms and heuristics for non-linear optimization problems are presented in Section 2.4. Finally, the literature review is summarized in Section 2.5.

2.1 Transit Service Strategies

In public transportation systems, adding and/or removing transit facilities (e.g., stop locations, railway tracks, etc.) to improve system performance is expensive and sometimes not applicable because of topographic limitations as well as institutional regulations. It is relatively easier to improve performance by adjusting operating strategies of transit services. Furth et al. (1984) investigated various transit operating strategies focusing on downtown-oriented bus routes, in which the advantages and disadvantages of each strategy were discussed.

The minimum service frequency during peak period on the route level used to be determined by the demand of the maximum load segment divided by vehicle capacity. However, poor utilization, in terms of occupancy, may be expected on other route segments where demand is low. To improve space utilization, Furth et al. (1984)

introduced the short-turn service to serve high demand segments of a conventional all-stop service.

The short-turn service is particularly suitable for corridors where the demand peaks at a specific zone/segment and tapers off substantially outside of it. A number of studies conducted by Ceder (1984, 1987a, and 1987b) focused on designing a transit frequency and timetable which minimized the fleet size considering short-turn services subject to a capacity constraint.

Transit planners/schedulers usually considered the short-turn service as a viable way to reduce supplier cost. However, the passenger wait time may be increased because of reduced frequency of all-stop trips due to limited operable fleet size. While applying the short-turn service to high demand segments, Furth (1987) found that the fleet size and the user wait time significantly reduced, especially during the peak period.

The critical design issue in short-turn service is to determine the turn-back points, schedule vehicles to balance passenger loads among the services, and minimize fleet size and passenger wait time. Ceder (1989) identified the turn-back points to design short-turn services based on passenger load profile. The total number of short-turn services is optimized to find the minimum fleet size. Delle Site and Filippi (1998) optimized vehicle size, service frequencies and fare for the short-turn service by considering temporal demand conditions. A method is proposed to optimize bus operations by explaining the effects on service patterns and the trade-off between users' and operator's costs.

Another operating strategy for transit services is controlling the stop-skipping for transit vehicles. In a stop-skipping strategy, also called vehicle expressing, vehicles are instructed to skip a set of stops along a route. Vuchic (1973) described and evaluated the stop-skipping strategy and concluded that the primary operational characteristics of stop-skipping were: (1) scheduled speed was increased; (2) the frequency of stopping was reduced; (3) headways at stations that were not served by all vehicles were increased; (4) there was no direct connection between those stations served by different vehicles; and (5) services became more complicated.

The stop-skipping service can be used to balance vehicle loads, minimize fleet size, and improve schedule adherence or vehicle headway regulation which reduces passenger wait time. Ercolano (1984) evaluated the stop-skipping service for bus operations by comparing some performance characteristics (e.g., vehicle operating speed, passenger travel time, etc.) of stop-skipping and local services. It was found that a stop-skipping service may reduce the total travel time of passengers, increase average operating speed, attract more passengers, and reduce the supplier cost. Eberlein (1995) formulated a non-linear model to identify skipped stops/stations for express service. Since the dwell time and stop delay are eliminated, the average travel time per passenger may be reduced.

Suh et al (2002) introduced an express subway system in Seoul, Korea, in which a stop-skipping service was considered. The total travel time savings were estimated subject to given origin-destination (OD) demand, distance between stations, headway, and operating speed. It was found that the total passenger travel time decreased up to 7.8 percent by operating a stop-skipping service.

Wilson et al (1992) explored various operational controls applied to a light rail line in Boston, Massachusetts, in which stop-skipping was regarded as an important option for the transit supplier to improve system performance, albeit it may cause the loss of some portion of demand because of inconvenience to some passengers.

The stop-skipping strategy was investigated in several studies (Li et al., 1991; Fu and Liu, 2003; Sun and Hickman, 2005) mainly focusing on dynamic and real-time scheduling problems. Li et al. (1991) formulated a stochastic programming model for a real-time scheduling problem using stop-skipping services. The objective was to minimize schedule deviation on the route and maximize service coverage for passenger demand. The skipped stops were determined before a vehicle was dispatched from the terminal. A numerical example was given to demonstrate the results recommended by the developed stochastic model to improve the transit operation.

Fu and Liu (2003) developed a dynamic scheduling strategy that was aimed to optimally balance the user and supplier costs, similar to that discussed by Li et al. (1991), a nonlinear programming approach was used. An exhaustive search algorithm was developed to find the optimal solution; however, it was only suitable for small scale transit networks. Sensitivity analysis was conducted based on data collected from a real world bus route, including passenger demand, the variation of bus travel times, and the headway at the dispatching terminal. Results indicated that a stop-skipping service may be effective in the context of high passenger demand with short service headway within a range of travel time variation.

Sun and Hickman (2005) developed policies for implementing the stop-skipping strategy in real-time operations. Two policies (basic and alternative) were formulated with a nonlinear integer programming approach. Under the basic policy, the skipped segment was defined by a beginning stop and an end stop, within which all stops will be completely skipped. Unlike the basic policy, vehicles can drop-off and pick-up passengers at stops in the skipped segments under the alternative policy. A simulation approach was used to examine the performance of these two policies, considering passenger distribution, beginning and end stops for skipped segments and skipped segment lengths as well as vehicle travel time variability. Possibilities of the improved stop-skipping policies for real-time operations were investigated.

Chien et al. (2009) optimized integrated service patterns (SPs) (e.g., all-stop and short-turn SPs) and the associated service frequencies for a transit route with heterogeneous demand that minimized the total cost, subject to operable fleet size and capacity constraints. The percentage of transfer demand was determined by a logit-based model. Later, Ulusoy et al. (2010) introduced an express service into the all-stop and short-turn services. A model was developed and the objective function was the total cost, consisting of supplier and user costs. The decision variables included integrated SPs, the associated service frequencies, and the number of stations skipped by the express service. The impact of model parameters (e.g., value of time, vehicle operating cost, skipped stations, transfer stations etc.) affecting the optimal service decisions were evaluated. By considering aggregate effects of various service patterns and station-based demand distributions, the developed method offered to quantify the savings and improved system performance. Compared to a traditional all-stop only service, operating short-turn services reduced operating costs and vehicle miles traveled, however the waiting cost of passengers increased because of the reduced service frequency in low demand segments. Integrating the express service into all-stop services (with short-turn service) significantly reduced the in-vehicle time of passengers, which compensated for the increased wait and transfer times.

2.2 Transit Network Optimization

Transit network optimization has been studied to find the optimal network and route structure with associated service frequencies that minimize the operator and user costs (Hurdle, 1973; Newell, 1979; Chien and Schonfeld, 1997), maximize service coverage (Spasovic and Schonfeld, 1993; Spasovic et al., 1994), maximize user benefits (Hasselstrom, 1981; Kocur and Hendrickson, 1982), minimize user travel time (Silman, 1974; Dubois et al., 1979; Mandl, 1980; Ceder and Wilson, 1986; Israeli and Ceder, 1991), and maximize the number of direct trips (Van Nes et al., 1988; Zhoa and Ubaka, 2004). An overview of transit network studies is given in Table 2.1.

Hasselstrom (1981) optimized a set of bus routes and associated service frequencies simultaneously. Optimal bus routes and their frequencies were determined for maximum user benefits in the context of a variable demand formulation. Kocur and Hendrickson (1982) considered elastic demand while optimizing transit networks using a linear approximation of logit mode share model. The revenue, cost and user benefits considering optimal route spacing and bus frequency were analyzed.

Table 2.1 Studies Related to Transit Network Optimization and Design

Year	Authors	Objectives	Decision Variables	Constraints
1979	Dubois et al	Minimize travel time	Route design, Frequency settings	Operating budget
1979	Newell	Minimize total cost	Route design	Operating budget
1981	Hasselstrom	Minimize transfer, maximize service coverage	Route design, Frequency settings	Operating budget
1986	Ceder & Wilson	Minimize travel, transfer and waiting time	Route design, Frequency settings	Minimum frequency
1988	LeBlanc	Minimize operator cost, maximize transit usage	Frequency settings	Fleet size
1988	Van Nes et al	Maximize number of direct trips	Route design, Frequency settings	Fleet size, Operating budget
1991	Israeli & Ceder	Minimize travel time, and fleet size	Route design, Frequency settings	-
1991	Baaj & Mahmassani	Route network AI-based representation	Transit network design	Multi-constraints
1992	Bookbinder & Desilets	Disutility function to optimize transfer	Timetable, Headway	Passenger Demand, Fleet Size
1995	Shih & Mahmassani	Minimize travel time and fleet Size	Route Design, Frequency settings	Multi-constraints
1995	Baaj & Mahmassani	Multi-object approach	Route design, Frequency settings	Multi-constraints
1995	Constantin & Floran	Minimize the total travel and waiting time	Frequency Settings	Fleet size
1997	Chien & Schonfeld	Minimize total cost	Route and station spacing, Headway	Capacity
1998	Pattnaik et al	Minimize operator costs and passenger travel time	Route design	Frequency, Load factor
2001	Chien et al	Minimize total cost	Route Location, Headway	Capacity, Budget
2001	Ngamchai & Lovell	Minimize total cost	Route design, Frequency settings	Fleet Size, Capacity
2002	Chien & Spasovic	Maximize operator profit and social welfare	Headway, Route spacing, Fare	Capacity
2002	Chakroborty & Dwivedi	Minimize waiting time	Fleet Size, Route Schedules	Headways, Transfer time
2004	Agrawal & Mathew	Minimize total cost	Route Design and Frequency settings	Load Factor, Capacity, Minimum frequency
2005	Lee & Vuchic	Minimize passenger's travel time	Route design, Frequency settings	Fleet Size, Capacity
2005	Chien	Minimize total cost	Headway, Vehicle size, Route Choice	Capacity, Budget, Vehicle schedule
2006	Fan & Machemehl	Minimize user costs	Route Design and Frequency settings	Fleet size, Headways, Load factor
2006	Zhoa & Zeng	Minimize transfers, maximize service coverage	Route Design and Frequency settings	Passenger demand, Budget, Level of service
2008	Zhoa & Zeng	Minimize user costs	Route Design, Frequency and Timetable settings	Fleet size, Load factor, Capacity

Several studies (Silman, 1974; Dubois et al, 1979; Mandl, 1980; Ceder and Wilson, 1986; Israeli and Ceder, 1991; Zhoa and Ubaka, 2004) were conducted to minimize passenger travel time by altering the route structure. Silman et al. (1974) developed a model for planning urban bus systems, which minimizes the total travel time through a recursive route addition and deletion process. The service frequencies associated with the generated routes were optimized by a gradient search method subject to a limited number of buses.

Van Nes et al. (1988) proposed a method to design an optimal transit network, which maximizes the number of direct trips subject to a given fleet size and operating budget. The relationship between the supply and demand sides for different transit services was examined. Route design, service frequency settings, and passenger assignment were evaluated using a modal split model considering variable demand.

Silman et al. (1974) developed a method to generate bus routes between zones which yield the minimized total vehicle travel time. Considering rectangular grid networks, several studies (Fawaz and Newell, 1976; Wirasinghe, 1980; Chien and Schonfeld, 1997; Chien and Spasovic, 2002) analyzed transit service areas with heterogeneous land use and demand characteristics. Chien and Schonfeld (1997) divided a service region into small rectangular zones to analyze the impact of land use, service area, demand pattern, and travel speed on user and operator costs. As an extension of that study, Chien and Spasovic (2002) optimized a transit network considering a heterogeneous urban environment and demand elasticity.

Mandl (1980) expressed a transit network structure by “nodes”, “links” and “routes”. A node represented a specific point for loading or unloading and/or transfer in the network, a link connected a pair of nodes and represented a particular mode of transportation between nodes, and a route was a sequence of nodes. With these definitions, Fan and Machemehl (2006) developed a route generation procedure to find feasible routes and the associated service frequencies. A heuristic algorithm was used to optimize the set of routes from a very large solution space.

Several models (Lampkin et al, 1967; Last, 1976; Scheele, 1977, 1980) have been developed for simultaneously optimizing choice of routes and frequencies. Frequency settings for transit operations were devoted to the satisfaction of maximum passenger demand. Scheele (1980) developed a non-linear model to determine the routes and service frequencies to minimize the total travel time of passengers for a given fleet size and demand conditions.

Minimization of passenger waiting time is an important objective in setting the frequency problem. Salzborn (1972) determined frequencies, given passengers' arrival rate, to minimize passenger waiting time. Furth and Wilson (1981) formulated the problem of determining route frequencies and they developed a method that aimed to maximize the social benefits, including wait time savings. Han and Wilson (1982) presented a model which aimed to minimize the passenger waiting time. An algorithm which consisted of the base allocation to minimize fleet size, and the surplus allocation to minimize the maximum crowding level was developed to design the frequencies.

Constantin and Floran (1995) presented a model and solution method for the frequency setting problem to minimize the passengers' total expected travel and waiting time under fleet size constraints. A non-linear non-convex mixed integer programming model was formulated and a projected sub-gradient algorithm was used to find optimal line frequencies considering the passengers route choices.

Gao et al. (2003) proposed a bi-level programming technique to deal with the frequency setting problem. In the upper-level problem the objective was to minimize the total deterrence of the transit system (consisting of in-vehicle and waiting time) and the cost caused by frequency setting. The lower-level model was a transit equilibrium assignment model to describe the route alternatives to transit users. A heuristic algorithm was designed to help transit planners to adjust an existing transit network to evolutions in the demand and in various other parameters.

2.3 Passenger Assignment in Transit Networks

The allocation of vehicles and their frequencies are highly dependent on the number of passengers assigned to the services on a transit route (Shih et al., 1998). Several studies investigated the problem of selecting transit service for passengers, which was considered either as a separate problem (Dial, 1967; Rapp and Gehner, 1976), or as a sub-problem of more complex models, such as transit network design problem (Lampkin and Saalmans, 1967; Scheele, 1977; Mandl, 1979; Hasselstrom, 1981).

Transit service choice models, also called transit assignment models (see Table 2.2), aim to present the decisions made by passengers for selecting transit services. Lampkin and Saalmans (1967) assumed that the passenger at the stop ignored transit services that were obviously bad and chose the first vehicle to arrive from among other routes.

Table 2.2 Transit Passenger Assignment Models

Year	Author (Year)	Methodology	Objective	Decision Variables	Constraints
1975	Chriqui and Robillard	Common Lines	Minimize total travel (wait+ in-vehicle) time of passengers	Link Frequency	Common Line Set
1982	Han and Wilson	Lexicographic	Minimize number of transfers	Service Frequency	Fleet Size, Vehicle Capacity
1988	Nguyen and Pallottino	Hyperpath	Minimize total travel (wait+ in-vehicle) time of passengers	Link Frequency	Link Volume
1989	Speiss and Floran (1989)	Optimal Assignment / Frequency-Based	Minimize total travel (wait+ in-vehicle) time of passengers	Link Frequency	Link Volume
1990	Baaj and Mahmassani	Frequency-Based + Lexicographic	Minimize total travel (wait+ in-vehicle+ transfer penalty) time of passengers	Route Frequency	Policy headways, Load Factor.
1993	De Cea and Fernandez	User Equilibrium (UE)	Minimize total travel (wait+ in-vehicle) time of passengers	Line Frequency	Transit Operation Condition (Congestion)
1999	Lam et al	Stochastic User Equilibrium (SUE)	Minimize total travel (wait +in-vehicle+ passenger overload delay) time of passengers	Line Frequency	Link Capacity, Vehicle Capacity

A passenger at a stop frequently has a choice between a number of lines (services), referred to as common lines, which will get him/her directly or indirectly to his/her destination. Chriqui and Robillard (1975) presented a framework for the common lines problem. In that study, the passenger at a stop selected the sub-set of lines which minimized passenger's expected travel time. They introduced a heuristic algorithm to find the optimal choice set.

In their passenger assignment study, Han and Wilson (1982) proposed a lexicographic methodology, in which passengers aimed to minimize the number of transfers during their trips. Shih et al (1998) presented a trip assignment model for timed-transfer transit systems using a lexicographic methodology. They found that while more trips are assigned to the routes with high frequency in uncoordinated transit networks, in fully coordinated network with a common route frequency, demand is completely assigned to the competing path with the minimum travel cost. Baaj and Mahmassani (1990 and 1995) adapted a lexicographic strategy and frequency-based approach with modifications to account for the undesirability of paths with excessive travel times.

Nguyen and Pallottino (1988) demonstrated the optimal strategies by using a graph theoretic approach. They introduced the concept of hyper-paths and proposed an equilibrium model, in which passenger waiting times were considered constant and independent of passenger volumes. A hyper-path connecting an origin to a destination included all the paths that could be used by a passenger. This methodology was able to solve the transit assignment problem, especially in congested conditions.

In the frequency-based approach, the simulation of path choice for high-frequency transit systems has been commonly performed on the basis of the concept of optimal assignment strategies presented by Spiess and Florian (1989). In the optimal assignment strategy, passengers were allowed to reach their destination at the minimum expected cost. They formulated waiting time and service route selection probabilities using service frequencies.

De Cea and Fernandez (1993) presented a user equilibrium assignment model for the transit assignment problem on congested systems, in which "effective frequency" was introduced to determine passenger waiting times at stations. A lower value of frequency was formulated and used to calculate waiting time in the model, because it will become harder for a passenger to get on a bus belonging to a congested line.

Lam et al. (1999) expanded the De Cea and Fernandez's (1993) study by developing a stochastic user equilibrium (SUE) transit assignment model. The logit-based model was used to evaluate the passenger route choice probability. Later, Lam et al. (2002) expanded that study and developed a capacity restraint transit assignment model with elastic line frequency. Line frequencies were affected by passenger flows. They considered congestion for strategic planning with the assumption of fixed OD demand. They defined that the line frequency should be dependent on the vehicle dwelling time at each station. A numerical example was illustrated for the solution algorithms and to demonstrate the applicability of the model.

2.4 Optimization Algorithms and Heuristics

The transit network design problem is a combinatorial problem in nature and many parameters (e.g., route spacing, route length, stop spacing, vehicle size, vehicle headway or frequency, and passenger waiting times) need to be determined to solve the realistic size networks. A number of previous studies (Lampkin and Saalmans, 1967; Byrne and Vuchic, 1972; Silman et al, 1974; Byrne, 1976; Rapp and Gehner, 1976; Dubois et al., 1979; Mandl, 1979; Hasselstrom, 1981; Oldfield and Bly, 1988; LeBlanc, 1988; Bookbinder and Désilets, 1992; Kuah and Perl, 1988; Chang, 1990; and Ceder and Israeli, 1998) in transit network design involved various mathematical optimization techniques. They also introduced heuristic algorithms or certain simplification assumptions to limit the solution search space or to reduce the optimization objectives to a particular network structure. Complex problems were converted to a solvable problem using heuristic algorithms.

The advantage of applying heuristic approaches is to achieve relatively faster acceptable solution to problems of any size. It is relatively easy to incorporate various constraints into the solution procedures, since heuristic approaches usually select solutions from a possible solution space that already meets most of the design constraints (Zhao and Gan, 2003). On the other hand, the solution from a heuristic approach may not be always globaly optimal, which is the main disadvantage of heuristic approaches.

Heuristic approaches are usually a combination of applications of guidelines and procedures for the route selection and bus frequency/headway determination, based on criteria established from past experience, ridership and demand data, cost and feasibility constraints, intuition of the transit planners, as well as some policies out of certain social and/or political considerations (Chua and Silcock, 1982; Axhausen and Smith, 1984; Baaj, 1990). The route network structures obtained from heuristic approaches tend to be of certain types that are intuitive and conceptually easy to understand or accept by planners. They are usually shaped by historical reasons or affected by existing systems that have evolved gradually with demographic changes in the urban areas they serve. Pearl (1984) developed heuristics methods as "criteria, methods, or principles for determining which among several alternative courses of action promises to be the most effective in order to achieve some goals." Heuristic methods are usually problem dependent since their search criteria, principles, and guidelines are domain or problem specific.

The last three decades have witnessed an increasing interest in meta-heuristic approaches (such as Tabu Search, Genetic Algorithm, etc.) for solving optimization problems. The main differences between the heuristic methods and meta-heuristic algorithms are that traditional heuristic approaches are strong problem-specific and depend heavily on personal knowledge and experiences, while meta-heuristic algorithms are relatively less problem-specific since these methods usually apply to a variety of applications in different fields with little or no modifications in their basic search criteria and principles (Zhao and Zeng, 2006).

The meta-heuristic approaches tend to follow one of two different outlines. The first set generates a large set of possible routes and then iteratively examines different subsets of the routes in an attempt to find a near-optimal solution. In the second approach, a potential route layout is generated, and then one or more of the routes in the solution are changed in an attempt to find better solutions. Many techniques have been developed to approximate the optimal solutions for the transit network problems. These include Tabu Search (Fan and Machemehl, 2004; Lei and Yan, 2007), Simulated Annealing (Fan and Machemehl, 2004, 2006b), Hill Climbing (Zhao and Ubaka, 2004), Ant Colony (Yu et al., 2005), and Genetic Algorithm (GA) (Pattnaik et al, 1998; Chien et al, 2001; Bielli et al, 2002; Ngamchai and Lovell, 2003; Fan and Machemehl, 2006a).

GAs are search and optimization methods based on the principle of natural selection (Holland, 1975). The basic idea behind GAs is that individuals and their off-springs that best fit or adapt to the surrounding environment have the best chance to survive. In a typical GA, a population of individuals (usually potential solutions) undergoes a sequence of transformations through the application of "genetic operators" and a selection process. Those individuals that best fit the surrounding environments (usually defined by a problem's objective functions and constraints) will have a better chance to survive the selection process, and their off-springs may have a better chance to survive the transformation and selection processes of the next generation. After some number of generations, the solutions converge, and the individual with the best fitness score represents the optimum solution of the system.

Unlike the mathematical solution search schemes (Haupt and Haupt, 1998), GA formulations do not require the calculation of the gradient matrix and any other higher order derivative matrices, or their approximation, of the objective function with respect to all the unknowns. The calculation of the gradient matrix or its approximation is a major computational burden in traditional mathematical optimization approaches. A GA-based method directly carries out its search on a population of individuals (i.e., potential solutions) and the objective functions themselves, not their derivatives. Therefore, there is no need to formulate a system of governing equations that represent or simulate the relationship between various parameters and unknowns mathematically. This is particularly attractive for practical applications where it is difficult to establish a mathematical formulation to accurately and effectively simulate complex situations. Transit network design problems are good examples of such cases. Constraints are relatively easy to incorporate into GA. By imposing large penalties on potential solutions that violate certain constraints reduces their survival possibility in the selection process. This may be especially suitable to problems where constraints are complicated and unable to be properly defined. Finally, GA has been an active research field for the past several decades and results have been widely used in various application fields. There are many existing algorithms and computer codes.

Integrating different service strategies and setting their frequencies is a large combinatorial problem, especially for a long distance route with a large number of stops.

The solution space increases exponentially as the number of stops increases. GA is very efficient and effective in solving combinatorial optimization problems (Gen and Cheng, 1997). Chakroborty et al.(1995) used GA in bus route network scheduling problems and indicated that GA is an efficient tool for solving transit network optimization problems. Caramia et al (2001) presented an iterative scheme based on GA to evaluate and improve the performance of existing bus networks by reducing the average travel time and management cost. Numerical results for a real world problem indicated that GA might be more effective in terms of computational time compared with classical assignment methods.

GA can solve almost any type of objective function (e.g. linear, nonlinear, integer, mix-integer, logical or discontinuous) subject to a set of constraints (Dandy and Engelhardt, 2001). Chien et al.(2001) developed a GA to search for the optimal bus route including route generator and genetic operators. The developed GA started with an initial population size and street pattern of a service area, which consists of three genetic operators (e.g. reproduction, crossover, and mutation). The function of the crossover operator was to generate new routes based on existing routes. Since certain segments of the different routes would be desirable for optimal operation, by combining these segments, a better route can be obtained.

Chakroberty (2003) demonstrated the effectiveness of GAs in solving the transit network design problem by using a Swiss transit network, which was first used by Mandl (1979) and subsequently used by other researches such as Baaj and Mahmassani (1991) and Kidwai (1998). He presented a GA-based procedure to effectively handle the transit network problems which cannot be solved using traditional optimization methods.

2.5 Summary

Transit network design studies were mainly focused on finding the optimal network and route structure with associated service frequencies that minimize the operator and user costs; maximize service coverage, user benefits and number of direct trips; minimize user travel time; and maximize the number of direct trips.

Local (all-stop) service is considered for every optimized route and none of the previous studies considered different SPs (e.g., all-stop, short-turn, and express services) in transit network optimization. Service frequencies on the route level were calculated using the maximum demand in a specific time period and location (or route segment). Minimum frequencies were obtained dividing the maximum demand by the vehicle capacity considering a desired occupancy. However, on other route segments where demand is low, the transit operation may not be effective because of insufficient seat utilization. Integrated (e.g., all-stop, short-turn and express) services can be a solution for underutilized operations.

Integrating different service patterns and setting associated service frequencies can be a large combinatorial problem especially for long corridors with a large number of stops. The solution space increases exponentially as the number of stops increases. Genetic Algorithms (GAs) were found very efficient and effective in terms of computational time in solving combinatorial optimization problems.

In the following chapter, two models are developed to address the planning, and designing of various SPs which operate in the same transit route in different segments and with different frequencies. An exhaustive search method and a genetic algorithm approach are implemented in Chapter 4 for the solution of the developed models.

CHAPTER 3

METHODOLOGY

The two models proposed to optimize various transit services presented in Chapter 1 are discussed in this chapter. The development of Model I is discussed in Section 3.1, which minimizes the total cost for integrated all-stop and short-turn service for a generalized transit route with heterogeneous demand, subject to three constraints to ensure frequency conservation, sufficient capacity and operable fleet size. Model II enhances Model I by considering an express service in the modeling and optimization processes, and is presented in Section 3.2. Model II minimizes the total cost, subject to the same constraints considered in Model I. Various performance measures discussed in Section 3.3 can be used to analyze the effectiveness of the optimized solutions to the system operation. Finally, a brief summary of the methodology is given in Section 3.4.

3.1 Model I - All-Stop and Short-Turn Service Patterns

Model I attempts to minimize the total cost of integrated all-stop and short-turn transit services generated by the optimized service frequencies. Note that the service pattern (SP) defined in this study is equivalent to overlapped bus routes which share some stops along the route. An all-stop SP serves all stops from the beginning stop to the end stop of the studied route. A short-turn $SP_{i,j}$ is considered as an all-stop service, which serves all intermediate stops between stops i and j.

3.1.1 System Assumptions

The following assumptions are made for formulating the objective total cost function and constraints:

1. The studied route shown in Figure 3.1 has *n* stops. The origin-destination (OD) demand and the spacing between stops are given, which may be obtained from a demand analysis that considers demand elasticity and the relative attractiveness of alternative modes. The definitions of model parameters are summarized in Appendix A.

2. Passenger arrivals at stops are uniform within a given time period, and vehicle arrivals are deterministic. Thus, the average passenger wait/transfer time may be assumed to be half the headway.

3. A SP originating from stop *i* and terminating at stop *j*, denoted as $SP_{i,j}$, serves every stop in between. The inbound and outbound frequencies for a SP are even ($f_{i,j} = f_{j,i}$), and the sizes of vehicles running for all SPs are identical in units of spaces/vehicle.

4. The number of passengers who would make transfer between SPs are negligible.

5. When required, eligible turn-back stops, denoted as *g*, can be pre-determined by the supplier, depending on the layout of stop location.

3.1.2 Model Formulation

3.1.2.1 Total Cost (TC)

The objective function considered here is the total cost, denoted as TC, incurred by the users and the service provider as shown in Figure 3.2. Thus, TC is defined as the sum of user (C_U) and supplier (C_O) costs. The model parameters include the distances between stops, user's value of time, vehicle operating speed, vehicle capacity and vehicle operating cost. The decision variables to be optimized include SPs and the associated frequencies which yield the minimum cost operation. Note that all cost components considered in this study are on an hourly basis and are discussed next.

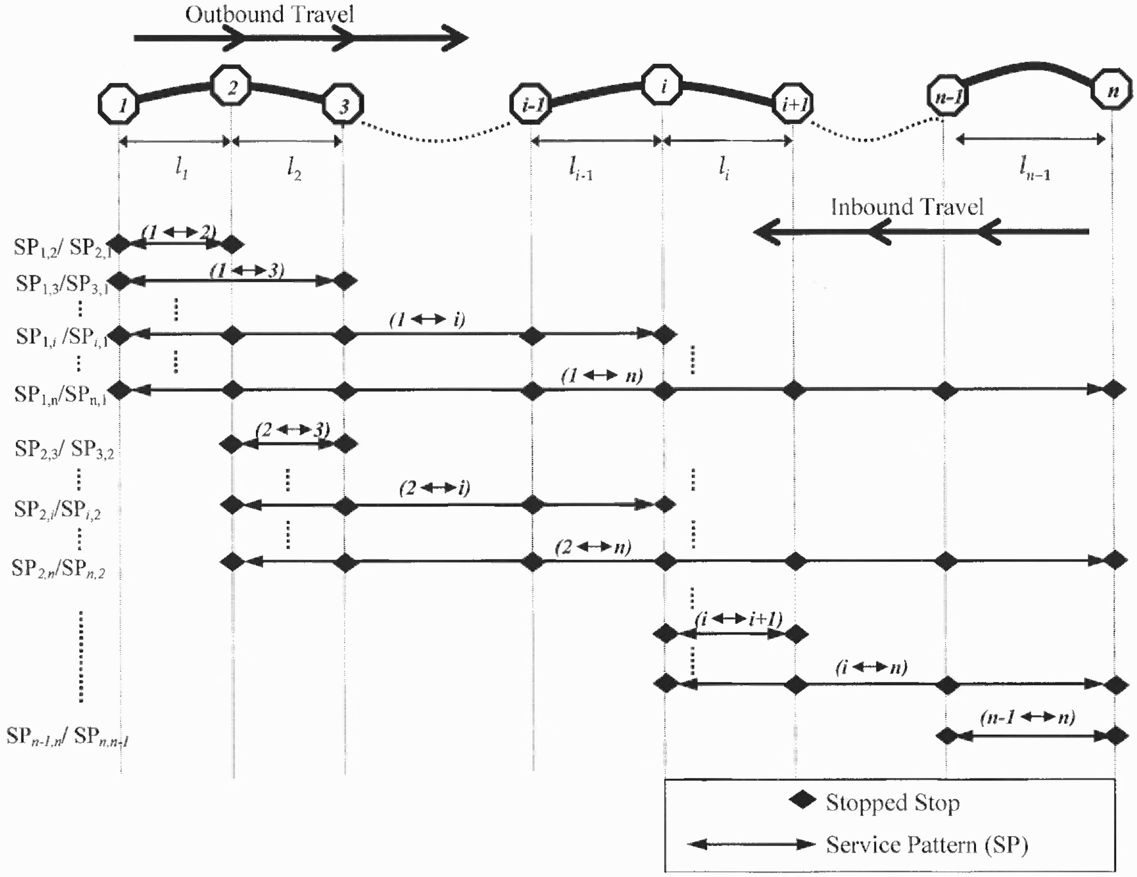

Figure 3.1 Service patterns in Model I.

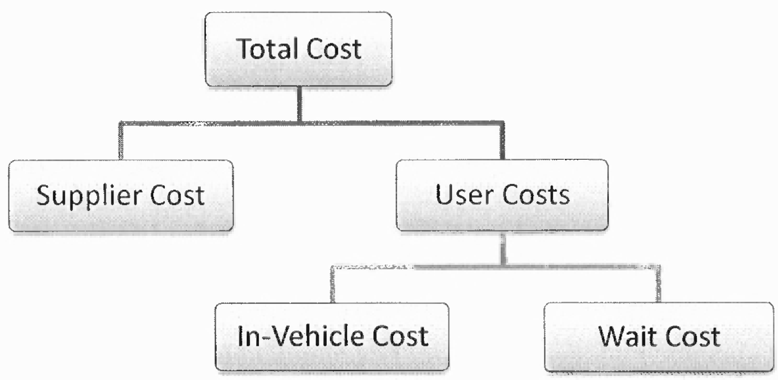

Figure 3.2 Cost structure for Model I.

3.1.2.2 User Cost (C_U)

The user cost, denoted as C_U, is defined as the sum of passenger access, wait, in-vehicle and transfer costs. The access cost to stop is constant in this study because the length of the route and the locations of the stops are known and as defined above in assumption 1, the number of passengers who would transfer between SPs are negligible. Thus, access and transfer costs are therefore excluded from C_U and not considered in the optimization processes. Thus,

$$C_U = C_W + C_I \tag{3.1}$$

where C_W, and C_I represent wait, and in-vehicle costs, respectively.

Wait Cost (C_W)

The wait cost is the product of hourly demand, average wait time, and the value of user's time. The average wait time for passengers traveling from stop i to j, denoted as $t_{W_{i,j}}$, is formulated in Equation 3.2. Generally, $t_{W_{i,j}}$ is a fraction of headway denoted as α_w. Note that the average headway is the inverse of the service frequency of all SPs. Thus,

$$t_{W_{i,j}} = \begin{cases} \dfrac{\alpha_w}{\sum_{s=1}^{i}\sum_{t=j}^{n} f_{s,t}} & for\ \ i<j \\ \dfrac{\alpha_w}{\sum_{s=i}^{n}\sum_{t=1}^{j} f_{s,t}} & for\ \ j<i \end{cases} \tag{3.2}$$

where n is the number of stops, and $f_{s,t}$ represents the frequency of $SP_{s,t}$ from stop s to t. The right hand side of Eq. 3.2 is the inverse of the total frequency (also the average headway) multiplied by α_w for approximating the average wait time. For outbound traffic ($i < j$) the index of the origin stop s varies from 1 to i, and the index of the destination stop t varies from j to n. Eq. 3.2 ensures that the average wait time is based on passengers from stop i to j using a service connecting stops i and j. The total wait cost C_W is the sum of wait costs incurred by passengers of all origin-destination (OD) pairs multiplied by the corresponding wait times and the value of time μ. Thus,

$$C_W = \sum_{i=1}^{n}\sum_{j=1}^{n} q_{i,j} t_{W_{i,j}} \mu \quad (3.3)$$

In-vehicle Cost (C_I)

The in-vehicle cost is defined as the product of demand, travel time and the value of user's time of all OD pairs. Thus,

$$C_I = \sum_{i=1}^{n}\sum_{j=1}^{n} q_{i,j} t_{I_{i,j}} \mu \quad (3.4)$$

where $t_{I_{i,j}}$, the average in-vehicle time from stop i to j, is the average travel distance divided by average speed. Thus,

$$t_{I_{i,j}} = \begin{cases} \sum_{d=i}^{j-1}\left(\frac{l_d}{v_d} + w_{d+1}\right) & \text{for } i < j \\ \sum_{d=j}^{i-1}\left(\frac{l_d}{v_d} + w_{d+1}\right) & \text{for } i > j \end{cases} \quad (3.5)$$

where v_d is the average vehicle operating speed from stop d to d+1; l_d represents the spacing between stops d and d+1; and w_{d+1} is the average dwell time at stop $d+1$.

3.1.2.3 Supplier Cost (C_O)

The supplier cost, denoted as C_O, is incurred by vehicles operating for all SPs. In general, for a SP serving from stop i to j, the supplier cost is the product of vehicle travel time from stop i to j, denoted as $T_{i,j}$, vehicle frequency, denoted as $f_{i,j}$, and hourly vehicle operating cost, denoted as b. Thus, the total supplier cost is

$$C_O = \sum_{i=1}^{n}\sum_{j=1}^{n} f_{i,j} T_{i,j} b \tag{3.6}$$

$T_{i,j}$ is defined as the vehicle travel time from stop i to j plus the layover time, denoted as t_o. Thus,

$$T_{i,j} = \begin{cases} \sum_{d=i}^{j\text{-}1}\left(\frac{l_d}{v_d} + w_{d+1}\right) + t_o & for\ i < j \\ \sum_{d=j}^{i\text{-}1}\left(\frac{l_d}{v_d} + w_{d+1}\right) + t_o & for\ i > j \end{cases} \tag{3.7}$$

Finally, the total cost (TC), is the sum of C_U and C_O, derived as

$$\text{TC} = \sum_{i=1}^{n}\sum_{j=1}^{n} q_{i,j}\left(t_{W_{i,j}} + t_{I_{i,j}}\right)\mu + \sum_{i=1}^{n}\sum_{j=1}^{n} f_{i,j} T_{i,j} b \tag{3.8}$$

3.1.3 Constraints

In this section, considering realistic limitations in planning transit services, three constraints, including frequency conservation, capacity, and fleet size constraints are formulated and discussed.

3.1.3.1 The Frequency Conservation Constraint

The frequency conservation constraint is formulated to ensure that the hourly vehicle inbound and outbound service frequencies at any SP are even. Thus,

$$f_{i,j} = f_{j,i} \quad \forall i, j \tag{3.9}$$

where $f_{i,j}$ represent the frequency of SP from stop i to j. The frequency conservation avoids the number of vehicles exceeding the garage capacity at end stops in all time periods (i.e. peak or off-peak).

3.1.3.2 The Capacity Constraint

The capacity constraint formulated in Eq. 3.10 is to ensure that there are always sufficient spaces available to satisfy the demand. The average headway on link *m*, denoted as h_m, must be less than or equal to H_m to guarantee that the service capacity is greater than the demand. Thus,

$$h_m \le H_m \quad \forall m \tag{3.10}$$

Note that the headways for both directions (h_m) are identical and equal to the inverse of total frequency of all SPs serving through link *m*. Thus,

$$h_m = \frac{1}{\sum_{s=1}^{m} \sum_{t=m+1}^{n} f_{s,t}} \quad \forall m \tag{3.11}$$

The maximum headway on link *m*, denoted as H_m, is determined by the maximum demand on link *m* for both inbound and outbound directions, which is equal to the vehicle capacity, denoted as *C*, divided by the maximum demand. Thus,

$$H_m = \frac{C}{Max(O_m, I_m) \quad \forall m} \tag{3.12}$$

where O_m and I_m represent the outbound and inbound demand on link m and can be obtained by Eqs. 3.13 and 3.14, respectively.

$$O_m = \sum_{s=1}^{m} \sum_{t=m+1}^{n} q_{s,t} \quad \forall m \tag{3.13}$$

$$I_m = \sum_{s=m+1}^{n} \sum_{t=1}^{m} q_{s,t} \quad \forall m \tag{3.14}$$

3.1.3.3 The Fleet Size Constraint

The fleet size constraint as formulated in Eq. 3.15 ensures that the optimal service frequency does not exceed the maximum service frequency due to the operable fleet size. Given that the operable fleet size is denoted as F, it must be greater than or equal to the sum of required fleet sizes for all SPs. The round trip vehicle travel time divided by headway (inverse of frequency) is the fleet size. Thus,

$$F \geq \sum_{i=1}^{n} \sum_{j=1}^{n} f_{i,j} T_{i,j} \tag{3.15}$$

where $T_{i,j}$ represents the vehicle round trip travel time for $SP_{i,j}$ and has been discussed while formulating Eq. 3.7.

3.2 Model II - Integrated All-Stop, Short-Turn, and Express SPs

Model I would be beneficial for conditions especially where demand is concentrated on certain links of the route. For the majority of demand involving local travel, all-stop and short-turn services discussed in Section 3.1 would be sufficient. However, for demand concentrating at certain OD pairs (e.g., intercity travel pattern), introducing an express service (e.g., stop-skipping) strategy may be beneficial.

The purpose of this part is to develop a model which optimizes all-stop, short-turn, and express transit SPs and associated service frequencies that yield the minimum cost operation considering heterogeneous demand. Model II is developed by considering the integration of an express SP into the all-stop and short-turn SPs. Transfering of passengers is also considered from express to all-stop (including short-turn) service and vice versa. Note, that express service serves only to the designated stops along the route, which is optimized to yield the minimum total cost. For a transit line with given locations of stops and OD demand, Model II consists of an objective total cost function and three constraints to ensure frequency conservation and sufficient capacity subject to operable fleet size.

3.2.1 System Assumptions

The assumptions discussed below are made for formulating the objective total cost function and associated constraints for Model II:

1. There are n stops allocated on the studied route as shown in Figure 3.3. The stop locations and OD based demand are given, which may be obtained from a demand analysis that considers demand elasticity and the relative attractiveness of alternative modes. The ridership of all SPs is estimated by a logit-based model, considering travel times including wait, transfer and in-vehicle times. The average passenger wait/transfer time is approximately half the headway.

2. Three types of SPs (e.g., all-stop, short-turn and express SPs) are considered in Model II. All-stop and short-turn SPs are considered as local SPs. An all-stop SP serves all stops from the beginning stop to the end stop of the studied route. A short-turn $SP_{i,j}$ is considered as a smaller scale all-stop service, which serves all intermediate stops between stops i and j. An express SP, denoted as SP_E, serves both end stops while a number of intermediate stops may be skipped.

3. Fares are identical for all SPs, and there is no transfer charge. Passengers may transfer from an express service to a local (an all-stop or a short-turn) SP or vice versa. No transfer is available between local SPs. The demand with more than one transfer per trip is negligible. The choice of transfer stop used by passengers is determined by the shortest travel time among the eligible transfer stops.

4. All intermediate stops can be used as turn-back points to configure short-turn services. However, if necessary, eligible turn-back stops may be pre-determined by the supplier, dependent on the layout of the stops over the studied route.

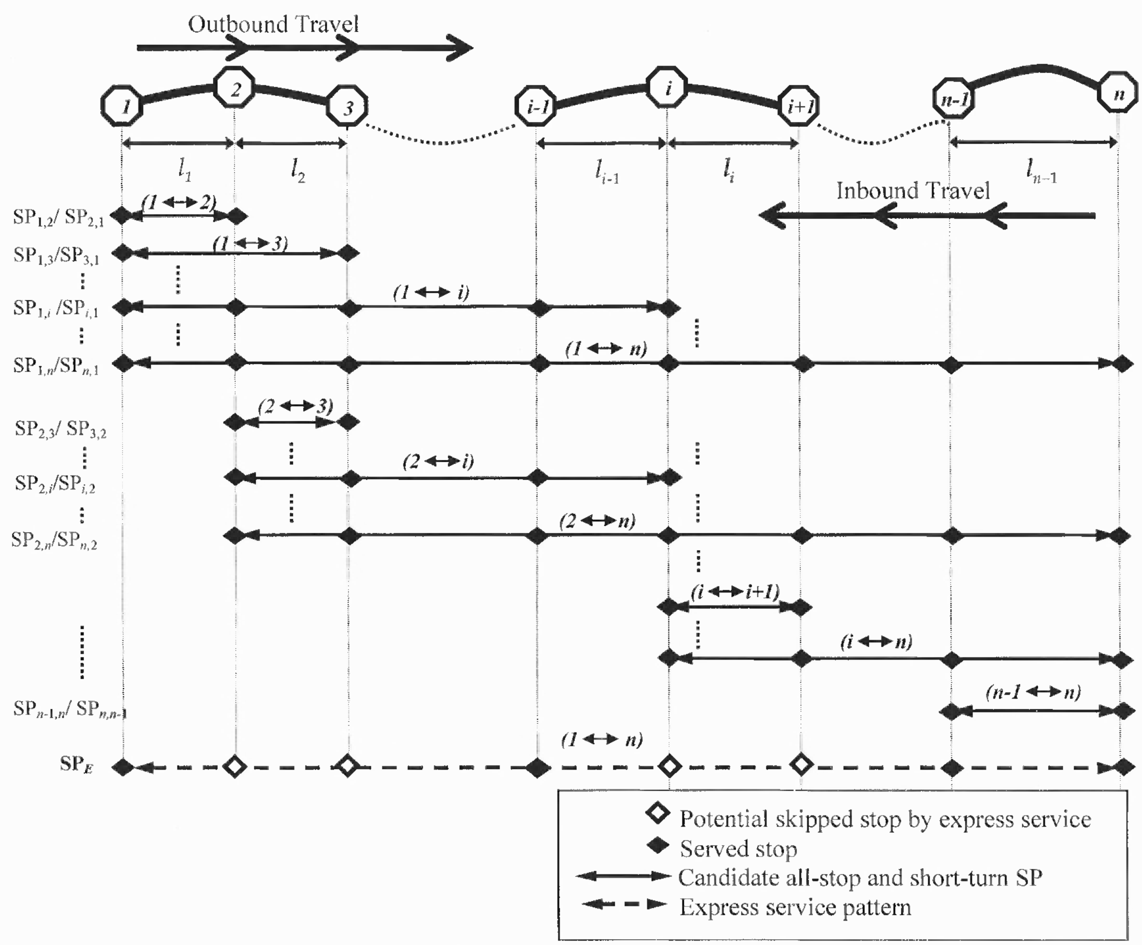

Figure 3.3 Service patterns in Model II.

3.2.2 Demand Estimation

Every passenger has two choices to begin their journey: Local (e.g, all-stop or short-turn) SP and express SP (SP_E) as shown in Figure 3.4. Since the fares are identical for all services, the travel time per passenger trip, defined as the sum of wait, transfer and in-vehicle times, is used to determine the demand of each service. For instance, the demand, denoted as $D_{i,j,k}$, from station *i* to *j* can be classified into *k* categories, where

$k=1$: Begin with SP_E and reach the destination without transfer

$k=2$: Begin with SP_E and reach the destination with transfer

$k=3$: Begin with local (i.e., all-stop or short-turn) SP and reach the destination without transfer

$k=4$: Begin with local SP and reach the destination with transfer

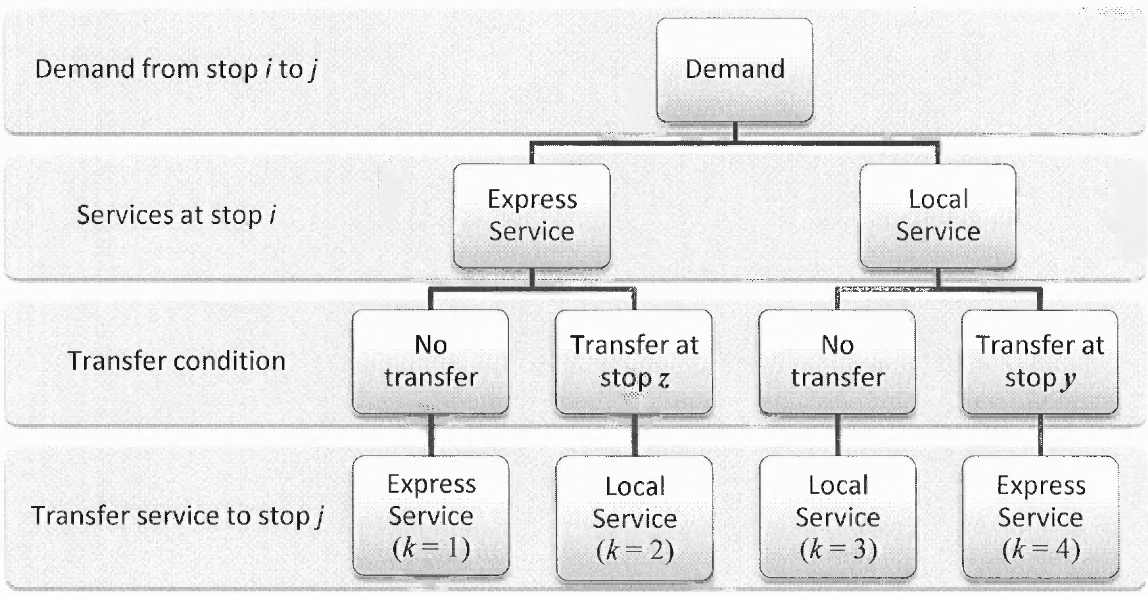

Figure 3.4 Classification of demand.

The percentage of demand selecting SP_E from stop *i* to *j*, denoted as $\psi_{i,j}$, can be estimated by a logit-based model as

$$\psi_{i,j} = \frac{\sum_{k=1}^{2} e^{-(\theta_W t_{W_{i,j,k}} + \theta_R t_{R_{i,j,k}} + \theta_I t_{I_{i,j,k}})}}{\sum_{k=1}^{4} e^{-(\theta_W t_{W_{i,j,k}} + \theta_R t_{R_{i,j,k}} + \theta_I t_{I_{i,j,k}})}} \lambda_i \quad \forall i, j;\ k = 1,2,3,4 \tag{3.16}$$

where θ_W, θ_R and θ_I are positive parameters, indicating the sensitivity of demand to the length of wait time ($t_{W_{i,j,k}}$), transfer time ($t_{R_{i,j,k}}$), and in-vehicle time ($t_{I_{i,j,k}}$). As defined in Equation 3.17, the variable indicating whether stop i is skipped by the SP_E, is denoted as λ_i. Thus,

$$\lambda_i = \begin{cases} 1; \text{ Stop } i \text{ is served by } SP_E \\ 0; \text{ Stop } i \text{ is skipped by } SP_E \end{cases} \forall i \tag{3.17}$$

In Eq. 3.17, $\lambda_i = 0$ represents that station i is skipped by SP_E, which ensures that no passenger will board on SP_E at stop i. Considering the demand from stop i to j, denoted as $q_{i,j}$, the ones using SP_E is the product of $\psi_{i,j}$ and $q_{i,j}$. The demand of SP_E from stop i to j without transfer (i.e., $k = 1$) is denoted as $D_{i,j,1}$. Thus,

$$D_{i,j,1} = q_{i,j}\psi_{i,j}\lambda_i\lambda_j \ \forall i,j \tag{3.18}$$

Passengers who board on SP_E at stop i and need to transfer (e.g., $k = 2$ and $\lambda_j = 0$) to reach stop j, denoted as $D_{i,j,2}$, can be formulated as

$$D_{i,j,2} = q_{i,j}\psi_{i,j}\lambda_i\left(1-\lambda_j\right) \ \forall i,j \tag{3.19}$$

Passengers who start the journey from stop i with the local SP to reach destination stop j do not transfer (i.e., $k = 3$) if the origin station is already served by SP_E (e.g., $\lambda_i = 1$, $\lambda_j = 0$ or 1). However, some passengers transfer to SP_E to reach destination stop j (i.e., $k = 4$) if $\lambda_i = 0$ and $\lambda_j = 1$. In this regard, the percentage of demand $q_{i,j}$ which starts with the local SP at origin stop i, transfers to the SP_E at stop y, and then reach a destination stop j, denoted as $\varphi_{i,j}$, is

$$\varphi_{i,j} = \frac{e^{-\left(\theta_R t_{R_{i,j,4}} + \theta_I t_{I_{y,j,1}}\right)}}{e^{-\left(\theta_R t_{R_{i,j,4}} + \theta_I t_{I_{y,j,1}}\right)} + e^{-\theta_I t_{I_{y,j,3}}}} \lambda_j \quad for\ i < y < j\ or\ i > y > j \tag{3.20}$$

where y is the index of transfer stop and passenger transfer station choices based on the shortest travel time [e.g., $\min\left(t_{I_{i,y,3}} + t_{R_{i,j,4}} + t_{I_{y,j,1}}\right)$]. In Equation 3.20, the transfer time from local vehicles to SP$_E$ vehicles is $t_{R_{i,j,4}}$ and in-vehicle time with SP$_E$ is $t_{I_{y,j,1}}$, while $t_{I_{y,j,3}}$ represents the in-vehicle time of the local SP from stop y to stop j. Considering the demand from stop i to j (i.e., $q_{i,j}$), and the percentage of demand using local SP ($1-\psi_{i,j}$), the demand of local SPs from stop i to stop j without transfer, denoted as $D_{i,j,3}$ is

$$D_{i,j,3} = q_{i,j}\left(1-\psi_{i,j}\right)\lambda_i + q_{i,j}\left(1-\psi_{i,j}\right)\left(1-\lambda_i\right)\left(1-\lambda_j\right) + q_{i,j}\left(1-\psi_{i,j}\right)\left(1-\lambda_i\right)\lambda_j\left(1-\varphi_{i,j}\right) \ \forall i,j \tag{3.21}$$

In Equation 3.21, the first term on the right hand side determines the local SP demand that will not transfer to SP$_E$ as the origin stop is already served by SP$_E$ (e.g., $\lambda_i = 1$, $\lambda_j = 0$ or 1); the second term ensures that the demand of local SP cannot transfer to SP$_E$ as the origin and destination stops are not served (e.g., $\lambda_i = 0$, $\lambda_j = 0$); and the third term calculates the demand of local SPs who have the option for, but are not willing to transfer to SP$_E$.

Passengers who start their journey with the local SP may transfer to SP$_E$, if SP$_E$ serves destination stop (e.g., $\lambda_j = 1$). Thus, $D_{i,j,4}$, is formulated as

$$D_{i,j,4} = q_{i,j}\left(1-\psi_{i,j}\right)\left(1-\lambda_i\right)\lambda_j\varphi_{i,j} \ \forall i,j \tag{3.22}$$

3.2.3 Model Formulation

3.2.3.1 Total Cost (TC)

The objective function considered in Model II is Total Cost (TC), which includes user (C_U) and supplier (C_O) costs as shown in Figure 3.5. The decision variables to be optimized are the service frequencies of integrated SPs that minimize the total cost. Note that, as defined in Model I, all cost components are formulated on an hourly basis and are discussed next.

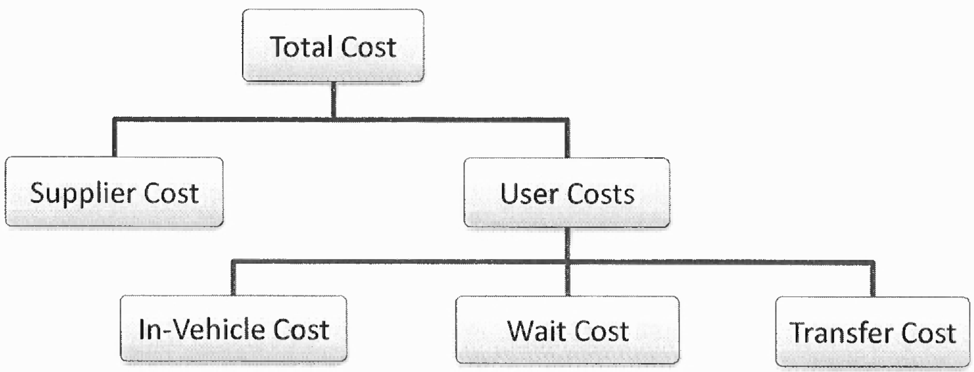

Figure 3.5 Cost structure for Model II.

3.2.3.2 User Cost (C_U)

The user cost, denoted as C_U, is defined as the sum of wait, transfer, and in-vehicle costs. Thus,

$$C_U = C_W + C_R + C_I \tag{3.23}$$

where C_W, C_R and C_I represent wait, transfer and in-vehicle costs, respectively.

Wait Cost (C_W)

The wait cost is the product of hourly demand, average wait time, and the user's value of time. In general, the average wait time is a fraction of headway, denoted as α_w, and the headway is the inverse of service frequency. The average wait time, denoted as $t_{W_{i,j,k}}$, for passengers who use SP_E from stop i to stop j is

$$t_{W_{i,j,k}} = \frac{\alpha_w}{f_E} \quad \forall\ i, j;\ k = 1,2;\ \lambda_i = 1 \tag{3.24}$$

Note that SP_E may skip certain stops. Similarly, the average wait time for passengers who use the local SP at stop i is

$$t_{W_{i,j,k}} = \begin{cases} \dfrac{\alpha_w}{\sum_{s=1}^{i}\sum_{t=j}^{n} f_{s,t}} & for \quad i < j \\ \dfrac{\alpha_w}{\sum_{s=i}^{n}\sum_{t=1}^{j} f_{s,t}} & for \quad i > j \end{cases} \qquad k = 3,4 \tag{3.25}$$

The total wait cost C_W is the sum of wait costs incurred by passengers using local and express SPs multiplied by the corresponding wait time and the value of time μ. Thus,

$$C_W = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{4} D_{i,j,k} t_{W_{i,j,k}} \mu \tag{3.26}$$

Transfer Cost (C_R)

For passengers who use SP_E from stop i and transfer to the local (all-stop or short-turn) SP via stop z to destination stop j, the transfer time denoted as $t_{R_{i,j,2}}$ is the inverse of the total frequency (=average headway) multiplied by α_R, the ratio of average transfer time to headway. Thus,

$$t_{R_{i,j,2}} = \begin{cases} \dfrac{\alpha_R}{\sum_{s=1}^{z}\sum_{t=j}^{n} f_{s,t}} & for\ i < z < j \\ \dfrac{\alpha_R}{\sum_{s=z}^{n}\sum_{t=1}^{j} f_{s,t}} & for\ i > z > j \end{cases} \quad \forall i,j \tag{3.27}$$

where n is the number of stops, z is the index of the transfer station, and $f_{s,t}$ represents the frequency of SP from stop s to t. For outbound traffic (i.e. ($i < z < j$) the index of origin stop s varies from 1 to i, and the index of destination stop t varies from z to n. Note that z is determined by the shortest travel time of a passenger starting with SP_E, transferring to the local SP via stop z, and getting off at destination stop j. Similarly, the transfer time for passengers from stop i beginning with the local SP, transferring to SP_E via stop y, denoted as $t_{R_{i,j,4}}$, can be formulated as

$$t_{R_{i,j,4}} = \frac{\alpha_R}{f_E} \quad \forall i,j \tag{3.28}$$

where $t_{R_{i,j,4}}$ is a fraction (α_R) of SP_E's headway (the inverse of frequency denoted as f_E). The transfer cost C_R is incurred by all transfer passengers, which is equal to the product of transfer demand, average transfer time and value of user's time. Thus,

$$C_R = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{4} D_{i,j,k} t_{R_{i,j,k}} \mu \tag{3.29}$$

In-vehicle Cost (C_I)

The in-vehicle cost is defined as the product of hourly demand of all OD pairs, the corresponding in-vehicle time and the value of user's time. The in-vehicle time, denoted as $t_{I_{i,j,k}}$, is a weighted average of the demand using the local SPs and SP_E from the origin stop i to the destination stop j. The in-vehicle time of passenger with SP_E is shorter than that with the local SP due to reduced stop delays (i.e., dwell time and acceleration/deceleration delay) from skipped stops.

For passengers who start the journey with SP_E from stop i to destination stop j without transfer, the in-vehicle time denoted as $t_{I_{i,j,1}}$ can be formulated as

$$t_{I_{i,j,1}} = \begin{cases} \sum_{d=i}^{j-1} \left(\frac{l_d}{v_d} + w_{d+1}\lambda_{d+1} \right) & for\ i < j \\ \sum_{d=j}^{i-1} \left(\frac{l_d}{v_d} + w_{d+1}\lambda_{d+1} \right) & for\ i > j \end{cases} \tag{3.30}$$

where v_d is the vehicle speed from stop d to d+1; l_d represents the spacing between stops d and d+1; and w_{d+1} is the average delay per stop. Similarly, in-vehicle time for passengers with local SP from stop i to destination stop j, without transfer, denoted as $t_{I_{i,j,3}}$, is

$$t_{I_{i,j,3}} = \begin{cases} \sum_{d=i}^{j-1} \left(\frac{l_d}{v_d} + w_{d+1} \right) & for\ i < z < j \\ \sum_{d=j}^{i-1} \left(\frac{l_d}{v_d} + w_{d+1} \right) & for\ i > z > j \end{cases} \quad \forall i, j \tag{3.31}$$

However, a transfer at station z to the local SP is needed for passengers (i.e., $k = 2$) who use SP_E from stop i to stop j which is not served by SP_E. In this case, the in-vehicle time can be formulated as

$$t_{I_{i,j,2}} = t_{I_{i,z,1}} + t_{I_{z,j,3}} \quad \forall i, j \tag{3.32}$$

where $t_{I_{i,z,1}}$ can be obtained using Equation 3.30 and $t_{I_{z,j,3}}$, the in-vehicle time of the local SP from stop *z* to destination stop *j*, can be obtained from Equation 3.31. Passengers (i.e., $k = 4$) who use the local SP from stop *i*, may transfer to SP_E via stop *y* to expedite their trips to destination stop *j*. Thus,

$$t_{I_{i,j,4}} = t_{I_{i,y,3}} + t_{I_{y,j,1}} \quad \forall i, j \tag{3.33}$$

where $t_{I_{y,j,1}}$, is formulated as Eq. 3.30.

Finally, C_I is the sum of in-vehicle costs incurred by passengers using SP_E and the local (i.e., all-stop and short-turn) SP multiplied by the corresponding in-vehicle times and the value of user's time μ. Thus,

$$C_I = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{4} D_{i,j,k} t_{I_{i,j,k}} \mu \tag{3.34}$$

3.2.3.3 Supplier Cost (C_O)

The supplier cost, denoted as C_O, is incurred by vehicles operating for all SPs. In general, for $SP_{i,j}$, the supplier cost is the product of vehicle travel time from stop *i* to *j*, denoted as $T_{i,j}$, vehicle frequency denoted as $f_{i,j}$, and hourly vehicle operating cost, denoted as b. Similarly, for SP_E, the supplier cost is the product of vehicle travel time denoted as T_E, frequency denoted as f_E, and hourly vehicle operating cost *b*. Thus, the total supplier cost is

$$C_O = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} f_{i,j} T_{i,j} + f_E T_E \right) b \tag{3.35}$$

$T_{i,j}$ and T_E are defined as the vehicle travel time for local and express SP, respectively, and the layover time at the end stop of each SP is denoted as t_o. The vehicle travel time is the sum of moving time along the route and dwell time at stops. Thus,

$$T_{i,j} = \begin{cases} \sum_{d=i}^{j-1}\left(\frac{l_d}{v_d} + w_{d+1}\right) + t_o & for\ i < j \\ \sum_{d=j}^{i-1}\left(\frac{l_d}{v_d} + w_{d+1}\right) + t_o & for\ i > j \end{cases} \tag{3.36}$$

$$T_E = \begin{cases} \sum_{d=i}^{j-1}\left(\frac{l_d}{v_d} + w_{d+1}\lambda_{d+1}\right) + t_o & for\ i < j \\ \sum_{d=j}^{i-1}\left(\frac{l_d}{v_d} + w_{d+1}\lambda_{d+1}\right) + t_o & for\ i > j \end{cases} \tag{3.37}$$

Finally, the total cost (TC), is defined as the sum of C_U and C_O, and is derived as

$$\text{TC} = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{4} D_{i,j,k}\left(t_{W_{i,j,k}} + t_{R_{i,j,k}} + t_{I_{i,j,k}}\right)\mu + \sum_{i=1}^{n}\sum_{j=1}^{n}\left(f_{i,j}T_{i,j} + f_E T_E\right)b \tag{3.38}$$

The decision variables in the objective total cost function formulated above include the service frequencies of all service patterns, $f_{i,j}$ and f_E for all pairs of stops *i* and *j*. The minimum total cost is yielded by the optimal service frequencies. An SP whose frequency is equal to zero indicates that the SP is not part of the optimized integrated service.

3.2.4 Constraints

Considering realistic limitations of vehicle operations, three constraints, including frequency conservation, capacity, and fleet size constraints, are formulated and discussed next.

3.2.4.1 Frequency Conservation Constraint

The frequency conservation constraint is designed to ensure that the number of vehicles will be even at the end stops. Thus,

$$f_{i,j} = f_{j,i} \quad \forall i, j \tag{3.39}$$

where $f_{i,j}$ represents the frequency of SP from stop i to j.

3.2.4.2 Capacity Constraints

The capacity constraints formulated in Eqs. 3.40 and 3.41 are designed to ensure that the service capacities from local (all-stop and short-turn) and express SPs are sufficient for the demand. For local SPs, the average headway of link l, denoted as $h_{L,l}$, must be less than or equal to the maximum headway denoted as $H_{L,l}$ of the link, so that the service capacity is capable of satisfying the demand. Thus,

$$h_{L,l} \leq H_{L,l} \quad \forall l \tag{3.40}$$

where link l connects stops l and $l+1$. Similarly, Eq. 3.41 is formulated for the SP_E. Thus,

$$h_{E,l} \leq H_{E,l} \quad \forall l \tag{3.41}$$

where $h_{E,l}$ represents the headway of SP_E of link l which must be less than or equal to the maximum headway of the SP_E, denoted as $H_{E,l}$.

Note that $h_{L,l}$ ($h_{E,l}$) is equal to the inverse of the total frequency of local SPs (SP_E) serving in link l. The headways for both directions in $h_{L,l}$ and $h_{E,l}$ are identical, which are

$$h_{L,l} = \frac{1}{\sum_{s=1}^{l} \sum_{t=l+1}^{n} f_{s,t}} \quad \forall l \tag{3.42}$$

$$h_{E,l} = \frac{1}{f_E} \quad \forall l \tag{3.43}$$

It is worth noting that $H_{L,l}$ ($H_{E,l}$) is determined by the maximum demand using local SPs (SP_E) traveling on link l, which is equal to the vehicle capacity, denoted as C, divided by the demand on the maximum load link. Thus,

$$H_{L,l} = \frac{C}{Max\left(\mathrm{O}_{L,l}, \mathrm{I}_{L,l}\right) \quad \forall l} \tag{3.44}$$

where $\mathrm{O}_{L,l}$ and $\mathrm{I}_{L,l}$ represent the outbound and inbound demand for local SPs in link l and can be obtained by Eqs. 3.45 and 3.46, respectively.

$$\mathrm{O}_{L,l} = \sum_k \mathrm{O}_{L,l,k} \quad k = 2,3,4 \tag{3.45}$$

$$\mathrm{I}_{L,l} = \sum_k \mathrm{I}_{L,l,k} \quad k = 2,3,4 \tag{3.46}$$

where k is the index for the demand categories which was described in section 3.2.2. $\mathrm{O}_{L,l,k}$ and $\mathrm{I}_{L,l,k}$ are the outbound and inbound demand for local SP in demand category k and formulated in Eqs. 3.47-49 and Eqs. 3.50-52 for $k = 2$, 3 and 4, respectively.

Eq. 3.47 is formulated to calculate the demand transferring from SP_E to local SP ($z \leq l$) at stop z, through link l to destination stop t. Demand in category 2 (k = 2) travelling from stop s to t is denoted as $D_{s,t,2}$ and formulated using Eq. 3.19. Demand for category 3 (k = 3) in link l using local SP without any transfer is formulated in Eq. 3.48. Demand starting with a local SP and using link l before transferring at stop y ($l < y$) to SP_E is given in Eq. 3.49.

$$\mathrm{O}_{L,l,2} = \sum_{s=1}^{l} \sum_{t=l+1}^{n} D_{s,t,2} \quad \mathrm{s} \leq z \leq l \leq t \tag{3.47}$$

$$O_{L,l,3} = \sum_{s=1}^{l} \sum_{t=l+1}^{n} D_{s,t,3} \ \ for\ s < t; \forall l \tag{3.48}$$

$$O_{L,l,4} = \sum_{s=1}^{l} \sum_{t=l+1}^{n} D_{s,t,4} \quad s \le l < y \le t \tag{3.49}$$

where s and t represent the indices of origin and destination stops, respectively. $D_{s,t,3}$ and $D_{s,t,4}$ represent the demand from category 3 (k = 3) and 4 (k = 4) travel from stop s to t, respectively. Note that demand for k = 2, 3, and 4 in Eq. 3.46 is formulated in Eqs. 3.50, 3.51 and 3.52, respectively.

$$I_{L,l,2} = \sum_{s=l+1}^{n} \sum_{t=1}^{l} D_{s,t,2} \quad s \ge z > l \ge t \tag{3.50}$$

$$I_{L,l,3} = \sum_{s=l+1}^{n} \sum_{t=1}^{l} D_{s,t,3} \quad for\ s > t; \forall l \tag{3.51}$$

$$I_{Ll4} = \sum_{s=l+1}^{n} \sum_{t=1}^{l} D_{st4} \quad s \ge l \ge y \ge t \tag{3.52}$$

Similar to Eq. 3.44, the maximum headway for SP_E, denoted as $H_{E,l}$, is

$$H_{E,l} = \frac{C}{Max\left(O_{E,l}, I_{E,l}\right) \ \ \forall l} \tag{3.53}$$

where $O_{E,l}$ and $I_{E,l}$ represent the outbound and inbound demand for SP_E in link l and can be formulated similar to $O_{L,l}$ and $I_{L,l}$ discussed above. Thus

$$O_{E,l} = \sum_{k} O_{E,l,k} \quad k = 1,2,4 \tag{3.54}$$

$$I_{E,l} = \sum_{k} I_{E,l,k} \quad k = 1,2,4 \tag{3.55}$$

Eqs. 3.56 to 3.58 are formulated to determine the outbound ($O_{E,l}$) and Eqs. 3.59 to 3.61 are formulated to calculate the inbound ($I_{E,l}$) demand for SP_E.

$$\mathrm{O}_{E,l,1} = \sum_{s=1}^{l} \sum_{t=l+1}^{n} D_{s,t,1} \quad for\ s < t; \forall l \tag{3.56}$$

$$\mathrm{O}_{E,l,2} = \sum_{s=1}^{l} \sum_{t=l+1}^{n} D_{s,t,2} \quad \mathrm{s} \leq l < z \leq t \tag{3.57}$$

$$\mathrm{O}_{E,l,4} = \sum_{s=1}^{l} \sum_{t=l+1}^{n} D_{s,t,4} \quad \mathrm{s} \leq y \leq l \leq t \tag{3.58}$$

$$\mathrm{I}_{E,l,1} = \sum_{s=l+1}^{n} \sum_{t=1}^{l} D_{s,t,1} \quad for\ s > t; \forall l \tag{3.59}$$

$$\mathrm{I}_{E,l,2} = \sum_{s=l+1}^{n} \sum_{t=1}^{l} D_{s,t,2} \quad s \geq l \geq z \geq t \tag{3.60}$$

$$\mathrm{I}_{E,l,4} = \sum_{s=l+1}^{n} \sum_{t=1}^{l} D_{s,t,4} \quad s \geq y > l \geq t \tag{3.61}$$

3.2.4.3 Fleet Size Constraint

The fleet size constraint formulated as Equation 3.62 ensures that the optimized service frequency does not exceed the maximum service frequency due to limited operable fleet size, denoted as F. Given that F must be greater than or equal to the total fleet size for operating all SP*s*, the fleet size can be obtained from the product of vehicle round trip travel time and its service frequency. Thus,

$$F \geq \sum_{i=1}^{n} \sum_{j=1}^{n} f_{i,j} T_{i,j} + f_E T_E \tag{3.62}$$

where $T_{i,j}$ and T_E represent the vehicle round trip travel times for the local SPs and the SP_E, respectively, and have been discussed while formulating Eqs. 3.36 and 3.37.

3.3 Performance Measures

Various performance measures (Vuchic, 2007) have been used in this study to analyze the effectiveness of the optimized solution to the system operation. In this section, a number of equations are formulated for vehicle miles traveled, passenger miles traveled, average load factors, average costs, average vehicle miles traveled per vehicle, average number of passenger per bus, average number of passenger per vehicle mile, etc.

3.3.1 Vehicle Miles of Travel

Vehicle miles of travel denoted as VMT, is the sum of distances travelled by all vehicles of a transit fleet during an hour. Thus,

$$\text{VMT} = \sum_{i=1}^{n}\sum_{j=1}^{n} f_{i,j} l_{i,j} + 2 f_E l_{1,n} \tag{3.63}$$

3.3.2 Passenger Miles of Travel

Passenger miles of travel denoted as PMT, is the product of the number of passengers carried and the average trip length. Thus,

$$\text{PMT} = \sum_{i=1}^{n}\sum_{j=1}^{n} q_{i,j} l_{i,j} \tag{3.64}$$

3.3.3 Load Factor

One of the important indicators of system productivity is the utility level of transit service or load factor (LF), which can be defined as the total passenger-miles traveled divided by the total space-miles provided. Thus,

$$\mathrm{LF} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} q_{i,j} l_{i,j}}{\sum_{i=1}^{n}\sum_{j=1}^{n} f_{i,j} l_{i,j} c + 2 f_E l_{1,n} c} \tag{3.65}$$

Note that the total passenger-miles traveled (the nominator) in Eq. 3.65 considers both inflow and outflow demand, and the denominator reflects total space-miles provided for both inbound and outbound trips.

3.3.4 Average Vehicle Miles of Travel

The average vehicle miles of travel denoted as VMT_a is defined as the ratio of VMT to fleet size. Thus,

$$VMT_a = \frac{\mathrm{VMT}}{F} \tag{3.66}$$

3.3.5 Average Passengers per Vehicle Mile of Travel

The average passengers per VMT (P_{vm}) is defined as the ratio of the total number of passengers to the vehicle miles of travel. Thus,

$$P_{vm} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} q_{i,j}}{\mathrm{VMT}} \tag{3.67}$$

3.3.6 Average Passengers per Vehicle

The average passengers per vehicle also known as the system efficiency, denoted as P_v, is defined as the ratio of the total number of passengers to fleet size, which indicates how efficiently vehicles are used in terms of the number of trips per vehicle. Thus,

$$P_v = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} q_{i,j}}{F} \tag{3.68}$$

3.3.7 Average Cost per Vehicle Mile of Travel

One of the common consumption indicators is the average cost per vehicle mile of travel, denoted as C_{vm}, which is defined as the supplier cost divided by the VMT. It indicates the cost per unit of transit service offered. Thus,

$$C_{vm} = \frac{C_O}{\text{VMT}} \tag{3.69}$$

3.3.8 Average Cost per Passenger

The average cost per passenger, denoted as C_p, is the ratio of supplier cost to the total number of passengers served. Thus,

$$C_p = \frac{C_O}{\sum_{i=1}^{n}\sum_{j=1}^{n} q_{i,j}} \tag{3.70}$$

3.3.9 Average Cost per Passenger Mile of Travel

The average cost in dollars per passenger mile of travel, denoted as C_{pm}, is defined as the supplier cost divided by the total passenger miles traveled. Thus,

$$C_{pm} = \frac{C_O}{PMT} \tag{3.71}$$

3.3.10 Average Passenger Travel Time

The average passenger travel time, denoted as T_a, is defined as the ratio of the sum of the total passenger wait, transfer and in-vehicle time to the OD demand. Thus,

$$T_a = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{4} D_{i,j,k}\left(t_{I_{i,j,k}} + t_{R_{i,j,k}} + t_{W_{i,j,k}}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n} q_{i,j}} \tag{3.72}$$

where $D_{i,j,k}$ is the demand from stop i to j in category k and formulated in the demand estimation section. The $t_{W_{i,j,k}}$, $t_{R_{i,j,k}}$ and $t_{I_{i,j,k}}$ represent wait, transfer and in-vehicle time of passengers, respectively.

3.4 Summary

In this chapter, the objective total cost functions and sets of constraints (e.g., capacity, fleet size, frequency conservation) for Models I and II were formulated. Model I was developed to minimize the total cost of integrated all stop and short-turn SPs yielded by the optimized service frequencies. Model I would be beneficial for conditions especially where demand is concentrated on certain segments of the route. However, for demand concentrating at certain OD pairs, such as intercity travel pattern, an express service strategy would be beneficial, and it was introduced in Model II. Model II was developed by considering the integration of an express SP into the all-stop and short-turn SPs. Transfering of passengers is also considered in Model II. Various performance measures (e.g., vehicle miles travel, load factor, average passenger per vehicle, etc.) were also formulated in this Chapter to analyze the effectiveness of the optimized solutions.

The combination and the interdependent relations among the decision variables (e.g., SPs, service frequencies and stops skipped by SP_E) form a combinatorial optimization problem that is difficult to optimize analytically. Therefore, solution algorithms are developed in the following chapter to search for the optimal solution.

CHAPTER 4

SOLUTION ALGORITHMS

As discussed previously, the objective of this dissertation is to develop models which optimize transit service patterns and associated service frequencies to minimize the total cost functions. The decision variables of the developed models include service patterns (SPs), associated frequencies and stops skipped by an express service (Model II only). The combination and the interdependent relations among these decision variables form a combinatorial optimization problem that is difficult to optimize analytically. Therefore, solution algorithms are developed in this chapter to search for the optimal solution.

In Section 4.1, an Exhaustive Search Algorithm (ESA) is presented to optimize the local (all-stop and short-turn) SPs and associated service frequencies (Model I) as well as the express service pattern (SP_E), its service frequency and stops skipped by SP_E (Model II). Due to increasing computation time for a large number of stops, ESA may not be the best approach to optimize the objective function for routes with a large number of stops. Therefore, a Genetic Algorithm (GA) based on an integer string representation is developed in Section 4.2 to search for the optimal decision variables, simultaneously. A brief summary of the solution algorithms is given in Section 4.3.

4.1 Exhaustive Search Algorithm (ESA)

ESA is programmed to find frequencies for the local (i.e., all-stop and short-turn) SPs for Model I, and local, express SPs as well as the stops skipped by the SP_E for Model II. ESA is described below and shown in Figure 4.1. It is used to search for optimized local service frequencies, which minimize total cost for Model I:

Step 1: Input all baseline values (e.g., vehicle capacity, operating cost, user value, number of stops, stop spacing's, vehicle operating cost, etc.) and set upper and lower boundaries of service frequencies. Input O/D demand and compute maximum headway (H_m) using Eq. 3.12.

Step 2: Select a set of initial solutions for all service frequencies ($f_{i,j}$) and Calculate the average headway (h_m) using Eq. 3.11.

Step 3: Check capacity constraint using Eq. 3.10. If it satisfies condition $h_m \leq H_m$ continue to next step, otherwise set new combination of $f_{i,j}$ values and repeat Step 3.

Step 4: Calculate wait ($t_{W_{i,j}}$) and in-vehicle ($t_{I_{i,j}}$) times.

Step 5: Calculate all cost components (i.e., C_W, C_I, C_O and TC)

Step 6: Check all combinations of service frequencies. If all combinations were considered continue to next step, otherwise select new frequencies and go to Step 3.

Step 7: Determine the minimum total cost and Stop.

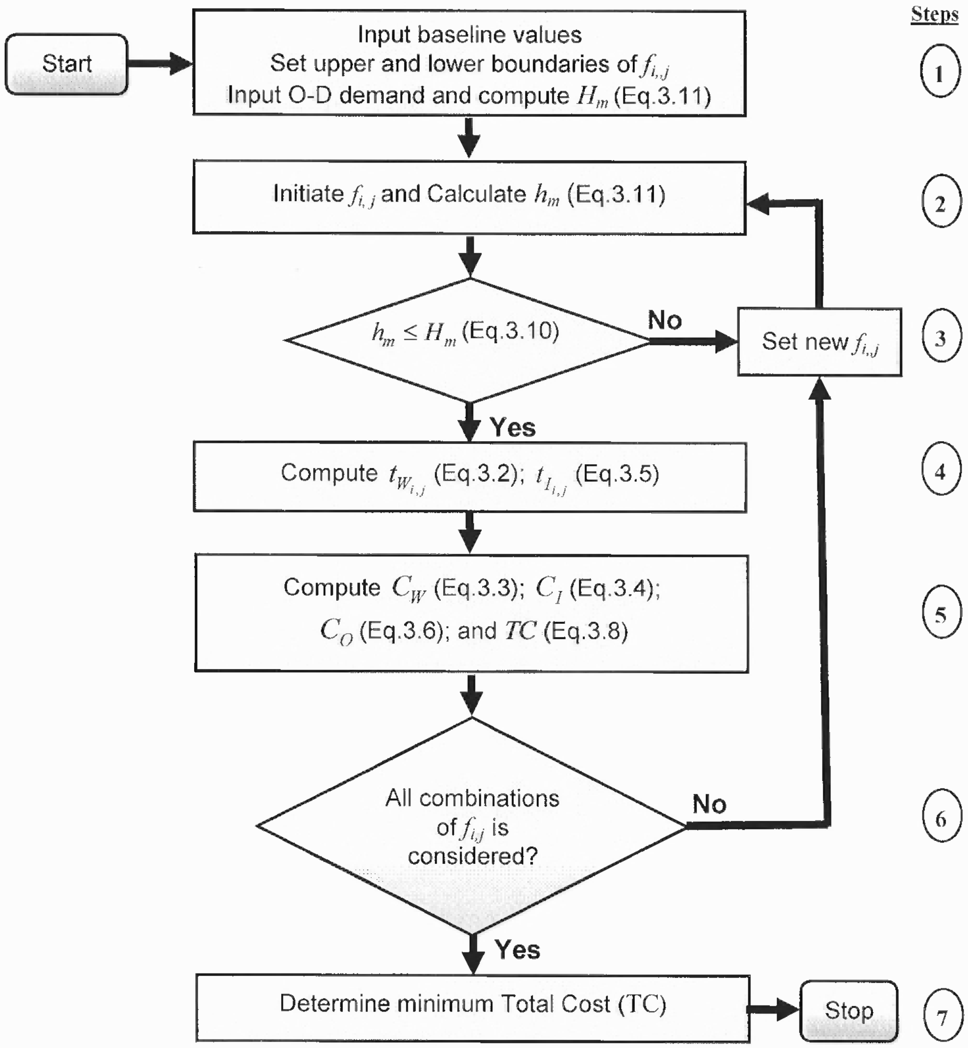

Figure 4.1 Flow chart of the exhaustive search algorithm in Model I.

Similar to the Model I, ESA is used to search for optimized service frequencies for minimized total cost in Model II, which is explained below and shown in Figure 4.2.

Step 1: Input all baseline values (e.g., capacity, operating cost, user value, number of stops, stop spacing's, vehicle operating cost, etc.) and set upper and lower boundaries of service frequencies. Input origin-destination (OD) demand and compute maximum headways ($H_{L,l}$ and $H_{E,l}$) using Eqs. 3.44 and 3.53. Set initial configuration of skipped stops (λ_i).

Step 2: Select a set of initial solutions for all decision variables (e.g., $f_{i,j}$ and f_E, skipped stops by SP_E, etc.)

Step 3: Calculate the average $h_{L,l}$ and $h_{E,l}$ using Eqs. 3.42 and 3.43, respectively.

Step 4: Check capacity constraints. If conditions $h_{L,l} \le H_{L,l}$ and $h_{E,l} \le H_{E,l}$ are satisfied continue to next step otherwise set new $f_{i,j}$ and f_E values and repeat step 3.

Step 5: Calculate wait ($t_{W_{i,j,k}}$), transfer ($t_{R_{i,j,k}}$), and in-vehicle ($t_{I_{i,j,k}}$) times. Re-determine transfer stops for transfer passengers.

Step 6: Check transfer stops. If transfer stops remain the same continue to next step, otherwise go to Step 5.

Step 7: Determine $\psi_{i,j}$, $\varphi_{i,j}$ and $D_{i,j,k}$ and calculate all cost components.

Step 8: Check all possible service frequencies. If all combinations are considered continue to next step, otherwise select new frequencies and go to Step 3.

Step 9: Check all possible skipped stop configurations. If all combinations are not considered set new configuration of skipped stops and go to Step 4, otherwise report the minimum cost and optimal solution and Stop.

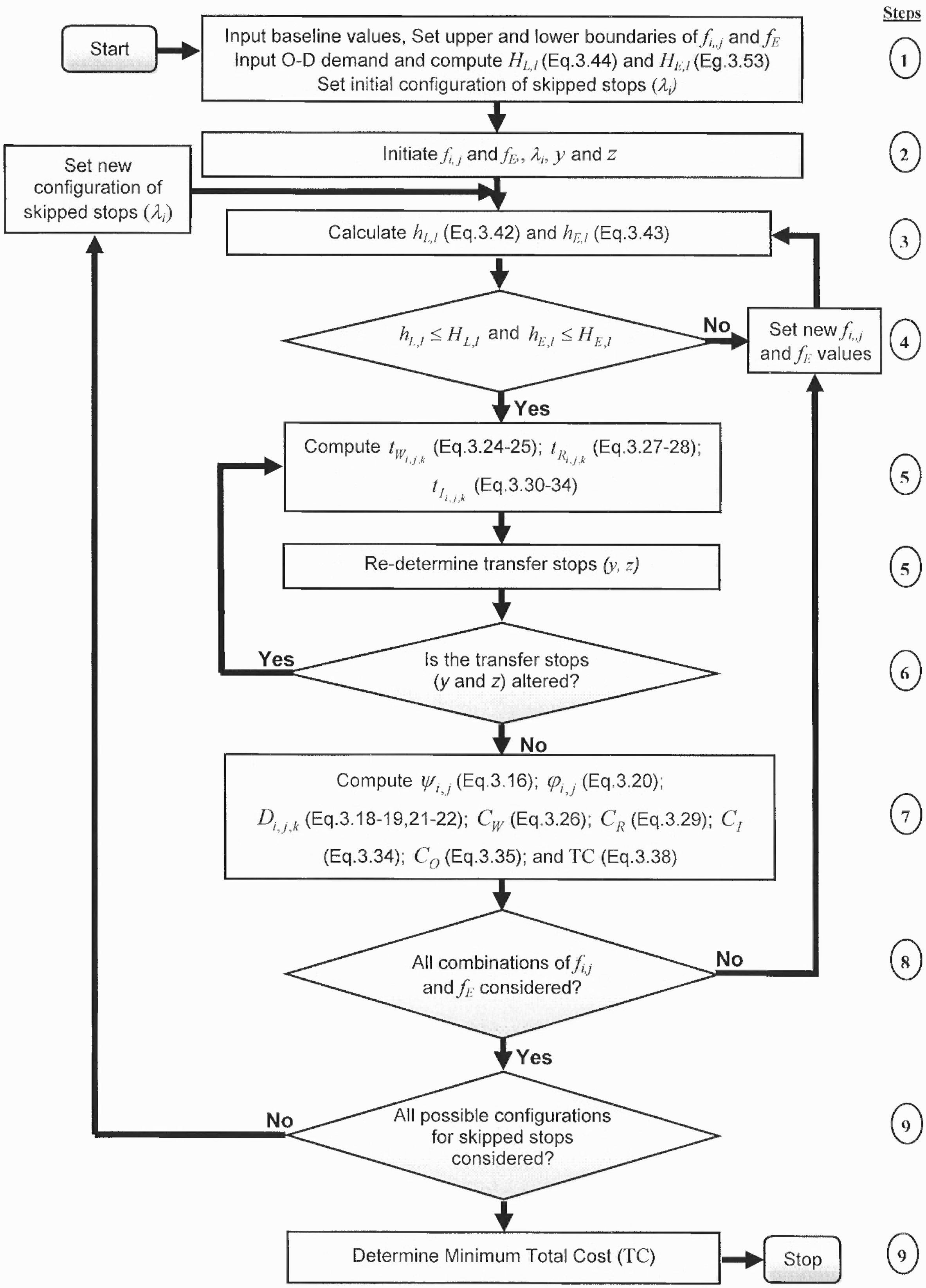

Figure 4.2 Flow chart of the exhaustive search algorithm in Model II.

Integrating three different SPs and setting associated service frequencies are a large combinatorial problem especially for long corridors with large number of stops. The solution space increases exponentially as the number of stops increases. A Genetic Algorithm (GA) is implemented in the following section to solve Model II for routes with a large number of stops.

4.2 Genetic Algorithm (GA)

GA is a stochastic algorithm which mimics the natural phenomena of genetic inheritance and Darwinian strife for survival (Michalewicz, 1999) to search for the optimal solution. A GA includes five major components:

- A criterion for evaluating the performance of a solution. The objective total cost function developed in Equation 3.38 in Chapter 3 is the criterion.
- A genetic representation for encoding feasible solutions. An efficient genetic representation needs to accommodate all decision variables and reduce the difficulties of encoding and decoding a solution, which is a key component of a GA. An efficient data structure can also facilitate the process of generating new valid solutions and reducing computation time. In this study a string representation is developed to transform the optimization problem into a GA.
- Reproduction processes to produce offspring solutions. Crossover and Mutation operators are developed in the integer string genetic representation to generate new solutions in the potential solution space, and are discussed in Section 4.2.2.
- A selection mechanism for promoting the evolution of good solutions. The elitist selection method is utilized for developed genetic representation and is discussed in Section 4.2.4.
- A constraint handling method is used to search the feasible solution space. A penalty value is added for solutions which violate constraints as defined in Section 4.2.5, which is applicable to the generic representation.

The step procedure to implement the developed GA is summarized below and depicted in Figure 4.3.

Step 1: Generate the initial group of random feasible solutions. The GA starts from the initial group of solutions as first generation, called population pool.

Step 2: Translate binary codes into real numbers for each corresponding chromosome. Calculate service frequencies and determine skipped stops by express service.

Step 3: Apply the capacity and fleet size constraints to verify that each solution satisfies the constraints.

Step 4: Calculate the objective value (i.e., total cost) for each chromosome based upon the service frequencies of all SPs and skipped stops by express service.

Step 5: Elect the solutions with good performance to produce new solutions (i.e., offspring) in accordance with the elitist selection method discussed in Section 4.2.4.

Step 6: Obtain the new generations by re-combining the preceding chromosomes (i.e., the solutions selected in Step 5) using crossover and mutation. Thus, a new population pool is formed for the next generation.

Step 7: Use the constraint handling method discussed in Section 4.2.5 to verify that each new solution satisfies the capacity and fleet size constraints.

Step 8: The new solution will replace their parent solutions in the population pool. A new population pool is formed for the next generation.

Step 9: Terminate the GA process and output the optimized solutions, if the predefined stop criteria (i.e., maximum iterations or minimum total cost) is satisfied. Otherwise go to Step 3.

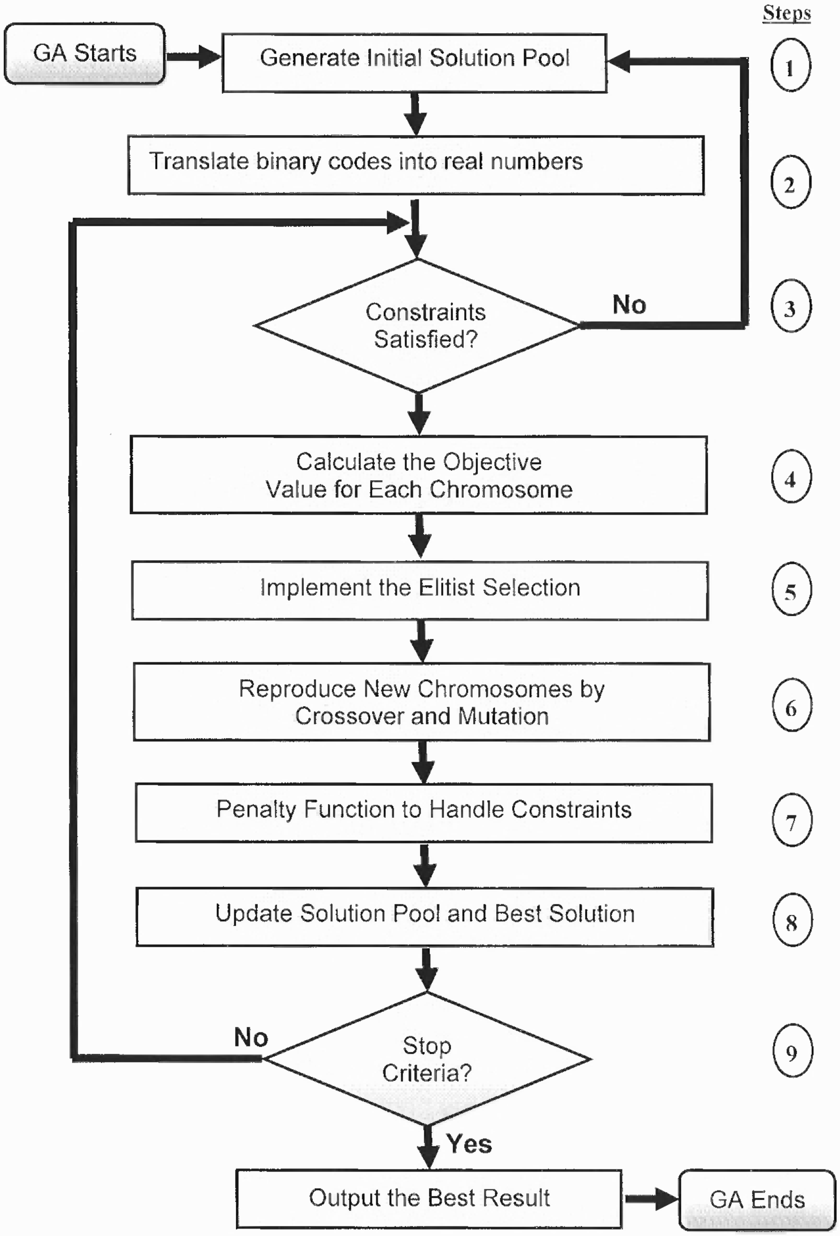

Figure 4.3 Flow chart of the genetic algorithm.

4.2.1 Encoding and Decoding Schemes

This section introduces the encoding and decoding scheme of the genetic representation. Moreover, the procedures of reproduction, (i.e., crossover and mutation) are discussed in Sections 4.2.2. In order to apply the GA to the developed model, a chromosome, denoted as *G*, consisting of two parts of genes is encoded as shown in Figure 4.4. Every one cell represents one gene. An integer string consisting of a series of cells is designed to represent various service frequencies denoted as Part 1, the information of whether stops are skipped by SP_E, denoted as Part 2.

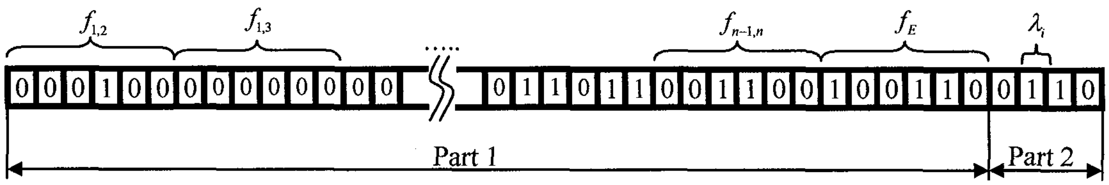

Figure 4.4 Encoding scheme of a chromosome representation.

The genes decoded in Part 1 correspond to the value of service frequencies including all-stop, short-turn and express services. The service frequency is a positive integer number, and in this study maximum value of the service frequency is considered to be no more than 60 bus/hr. Six cells are required for coding the service frequency in binary mode. As shown in Figure 4.3, every six cells represent one service frequency. The total number of local SPs for all-stop and short-turn services is $\binom{n}{2}$. Considering the frequency conservation constraint (Eq. 3.39) there will be $\frac{1}{2}\text{x}\binom{n}{2}$ different service frequencies for all-stop and short-turn SPs. The last six genes represent the express service frequency. The total number of genes in Part 1, denoted as $p1$, can be calculated as

$$p1 = 6\left[\frac{\binom{n}{2}}{2} + 1\right] \tag{4.1}$$

For a six-stop transit route, there are 31 SPs with 16 different service frequencies for all-stop and short-turn services and one for express service. Therefore, a total number of 16 genes are designed to search for optimal service frequencies with GA. The service frequency encoded in Part 1 of the chromosome shown in Figure 4.4 can be decoded based on binary, six-digit genes illustrated in the first row of Table 4.1, in which the value of the third row is the product of the encoding factor, denoted as 2^{r-1} and the value of the rth binary gene, denoted as $g_{r,i,j}$. An example given in Table 4.1 represents two sets of genes, $f_{1,2}(101000)$ and $f_{1,3}(000000)$, which can be translated into decimal numbers using Eq. 4.2. This equation can be used for all 16 services, including the express service.

$$f_{i,j} = \sum_{r=1}^{6} g_{r,i,j} 2^{r-1} \tag{4.2}$$

where $g_{r,i,j}$ is the value of the rth gene of $SP_{i,j}$ where r = 1,2,…,6.

As discussed in the model formulation in Chapter 3, SP_E may skip some of the stops along the route. The variable indicating whether stop i is skipped by the SP_E, denoted as λ_i, is encoded in Part 2 of the chromosome as shown in Figure 4.4. In Part 2, an integer attribute is assigned to define the genes corresponding to each station, while "1" indicates that SP_E serves the stop and "0" is for a skipped stop. Considering the 1st and nth stops are always served by the express service, there will be a total of (n-2) genes required in Part 2.

Table 4.1 Representation of the Genes in Part 1

$i=1,\ j=2$								
1	Genes ($g_{r,i,j}$)	1	0	1	0	0	0	Total
2	Encoding Factor (2^{r-1})	2^0	2^1	2^2	2^3	2^4	2^5	63
3	Value ($g_{r,i,j}$)(2^{r-1})	1	0	4	0	0	0	5
$i=1,\ j=3$								
1	Genes ($g_{r,i,j}$)	0	0	0	0	0	0	Total
2	Encoding Factor (2^{r-1})	2^0	2^1	2^2	2^3	2^4	2^5	63
3	Value ($g_{r,i,j}$)(2^{r-1})	0	0	0	0	0	0	0

4.2.2 Reproduction: Crossover and Mutation

During the processes of reproduction, the classic genetic operators (i.e., crossover and mutation) are adopted to produce new solutions by altering their parent solutions (i.e., solution strings in a previous population pool). Since GA is a stochastic algorithm, the probabilities of performing the crossover and mutation operations are defined as crossover and mutation ratios, denoted as r_X and r_M respectively, and are pre-determined model parameters. The procedures of crossover and mutation are illustrated in Tables 4.2 and 4.3, respectively, for the twelve-digit genes of a chromosome.

The crossover operation generates chromosomes by exchanging genes from their parents. It is used to gestate better offspring by inheriting good genes (i.e., lower total cost in the fitness evaluation) from their parents. The often-used crossovers are one-point, two-point, and multipoint crossovers. The criteria of selecting a suitable crossover depend on the length and structure of chromosomes. In this study, two-point crossovers were adopted by selecting crossover points from each part of the chromosome.

Table 4.2 One-Point and Two-Point Crossovers

	One-Point	Point 1		**Two-Point**	Point 1	Point 2	
Before Crossover	Chromosome A	101100	010101	Chromosome A	1011	0001	0101
	Chromosome B	001000	111001	Chromosome B	0010	0011	1001
After Crossover	Chromosome A	101100	111001	Chromosome A	1011	0011	0101
	Chromosome B	001000	010101	Chromosome B	0010	0001	1001

The mutation operation randomly selects a chromosome from the population and change the r^{th} bit. It is used to generate new chromosomes. The mutation is usually performed with a probability p ($0<p<1$), meaning that only a portion of the genes in a chromosome will be selected to be mutated. The two-point mutation is adopted in this study for each part of the chromosomes.

Table 4.3 One-Point and Two-Point Mutations

	One-Point		Two-Point	
Before Crossover	Chromosome A	101100010101 ↑	Chromosome A	101100010101 ↑ ↑
After Crossover	Chromosome A	101000010101 ↑	Chromosome A	101000010001 ↑ ↑

4.2.3 Evaluation Criterion

The performance of each solution is evaluated by a fitness function (i.e., minimum total cost in this study). For total cost minimization problems, the solution with a minimum cost is identified as a better solution with greater probability to be selected to reproduce new solutions in the next generation. Better solutions have higher probabilities to evolve into the next generation of the population pool by implementing the elitist selection as discussed in the next section.

4.2.4 Elitist Selection

The elitist selection, developed by Michalewicz (1999), is utilized to guarantee that the current generation solutions with good performance can always evolve into the next generation.

Figure 4.5 illustrates the process of selection of better solutions, where the population size and the ratio of selection are denoted as P and r_S, respectively, and are two pre-determined GA parameters. Prior to the selection process, the solutions in the current generation (e.g., t^{th} generation) are sorted in an ascending order based on their objective values. The first solution in the generation represents the best optimized SPs, associated service frequencies and optimized skipped stops by SP_E in terms of the lowest total cost. As shown in Figure 4.5, the top $(P)(r_S)$ parent solutions in the t^{th} generation are chosen to reproduce identical $(P)(r_S)$ new offspring in the $(t+1)^{th}$ generation, and the worst $(P)(r_S)$ solutions with higher costs in the t^{th} generation are discarded. Then, the original top $(P)(r_S)$ and the remaining $(P)(1\text{-}2r_S)$ solutions are replicated to the population pool of the $(t+1)^{th}$ generation to maintain a constant population size P. The elitist selection is used in each generation until the proposed GA reaches the criteria of terminating the search process.

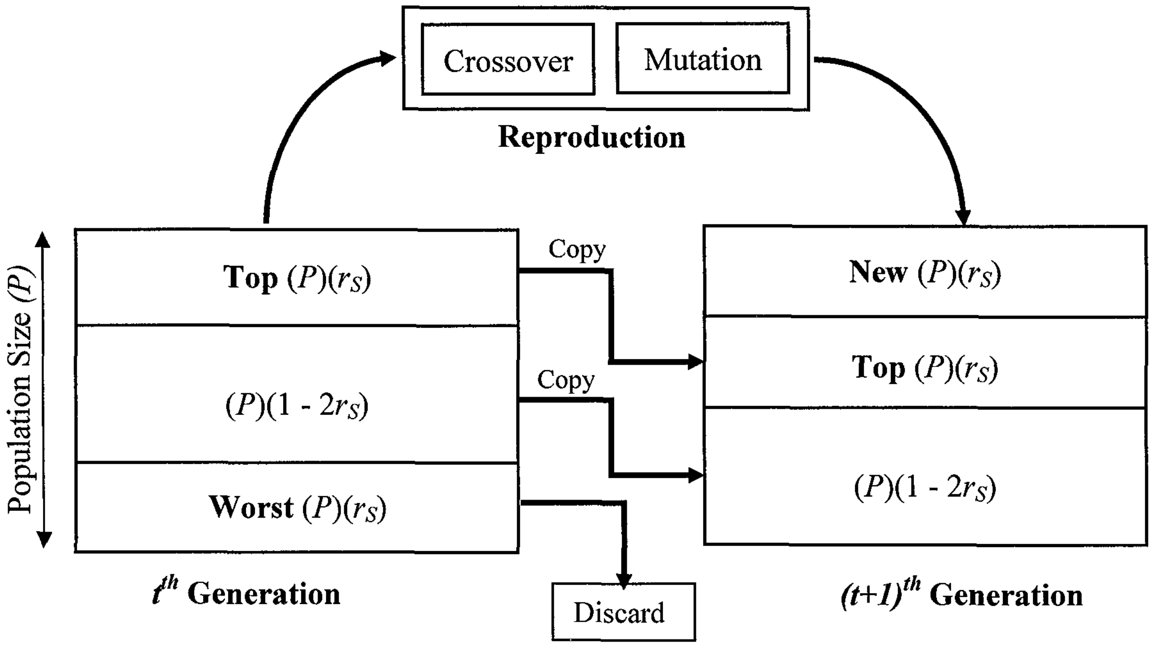

Figure 4.5 The elitist selection mechanism.

4.2.5 Constraint Handling

The objective total cost function formulated in Eq. 3.38 is optimized subject to capacity, fleet size and frequency conservation constraints. A penalty value is applied to handle the constraints. A large penalty value is added to the objective value of the solutions violating the constraint(s). Thus, infeasible solutions may be discarded during the evaluation process. Using this approach the constrained problem is transformed into a non-constraint one. The total cost function is transformed as follows:

$$\mathrm{TC} = \begin{cases} \mathrm{TC}(x) & x \in \text{feasible region} \\ \mathrm{TC}(x) + \text{Penalty Value} & x \notin \text{feasible region} \end{cases} \tag{4.3}$$

4.3 Summary

In this chapter, two solution algorithms were developed to solve the optimization problems discussed in Chapter 3. First, an exhaustive search method is used to optimize the developed models. The exhaustive search algorithm is capable and efficient to deal with smaller scale transit routes with limited number of stops. On the other hand, the genetic algorithm is developed to search for optimal solutions in reasonable time periods for transit routes with many stops. In the following chapter, two case studies will demonstrate the applicability of the solutions algorithms developed here.

CHAPTER 5

CASE STUDY

This chapter demonstrates the applicability of the models developed in Chapter 3 and the solution algorithms developed in Chapter 4 to optimize the transit service pattern problem. Two transit routes, including a hypothetical six-stop and a real world bus routes, are discussed. In Section 5.1, a six-stop bus route is used to demonstrate the applicability and effectiveness of the developed models for a small scale bus route and investigate the relationships among the model parameters and decision variables. Three scenarios are evaluated and sensitivity analyses are conducted and the results of these scenarios are compared and discussed. In Section 5.2, a real world bus route (New Jersey Transit Springfield Avenue Line) is used to test the applicability of the developed models and algorithm for a large scale transit corridor with many stops. The results of three scenarios for the real world bus route are also compared and discussed in this section. Finally, the summary of this chapter is presented in Section 5.3.

5.1 Case I - A Six-Stop Bus Route

The purpose of this section is to demonstrate the applicability and effectiveness of the developed models to a small scale bus route and investigate the relationships among the objective function, model parameters and decision variables. A hypothetical six-stop bus transit route is utilized, while the peak OD demand matrix and feasible SPs are illustrated in Table 5.1 and Figure 5.1, respectively.

Table 5.1 OD Demand Matrix of a Six-Stop Bus Route (pass/hr)

From(*i*) \ To(*j*)	1	2	3	4	5	6
1	0	10	97	21	92	202
2	10	0	12	14	17	38
3	85	10	0	42	91	211
4	15	9	66	0	78	93
5	21	11	61	63	0	72
6	199	21	214	10	21	0

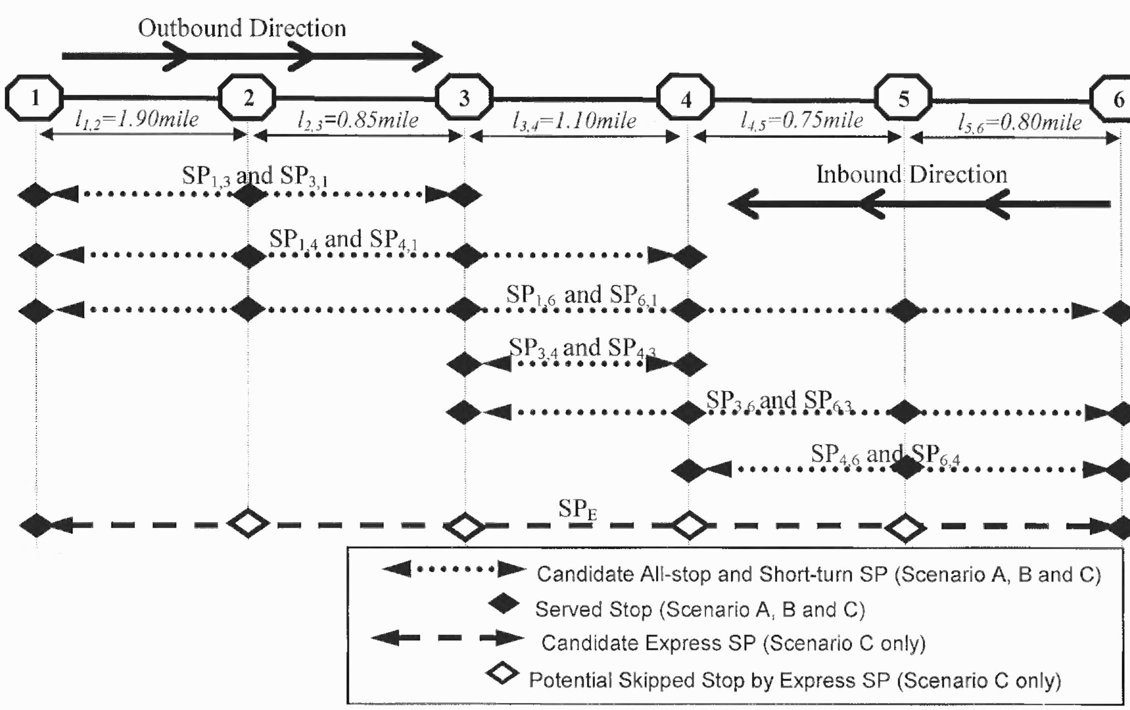

Figure 5.1 Configuration of a six-stop bus route and candidate service patterns.

The study bus route has a length of 5.4 miles, on which passengers are able to access the service only at designated stops, of which Stops 3 and 4 are potential turn-back points for short-turn services. The average bus travel speed is assumed to be 25 mph (miles/hour), and the vehicle capacity is 50 pass/bus with hourly operating cost of 120 \$/bus-hr. The weight factors of headway used to approximate wait and transfer times, denoted as α_W and α_R, respectively, are identical and equal to 0.5. The user's value of time is assumed to be 10 \$/pass-hr. The operable fleet size is assumed to be 21 buses, and it acts as the upper bound of the fleet size during the optimization processes. All baseline values of the model parameters are summarized in Table 5.2.

Table 5.2 Baseline Values of the Model Parameters

Parameters	Descriptions	Baseline Values
b	Bus operating cost	120 \$/bus-hr
C	Bus capacity	50 pass/bus
n	Total number of stops	6 stops
F	Maximum operable fleet size	21 buses
$f_{i,j}$	Frequency of all-stop and short turn SP from stop i to j	To be determined
f_E	Frequency of express SP	To be determined
l_d	Distance from stop d to $d+1$	See Figure 5.1
g	Eligible stops for turn-back	Stops 3 and 4
$q_{i,j}$	Demand from stop i to j	See Table 5.1
t_o	Layover time at the end stop	0.1 hr
v_d	Average vehicle speed from stop d to $d+1$	25 miles/hr
θ_I	Sensitivity parameter of demand to the length of in-vehicle time	1.33
θ_R	Sensitivity parameter of demand to the length of transfer time	2.00
θ_W	Sensitivity parameter of demand to the length of wait time	2.00
w_d	Average stop delay and dwelling time at stop d	0.06 hr
α_R	Ratio of the average transfer time to headway	0.5
α_w	Ratio of the average wait time to headway	0.5
μ	User's value of time	10 \$/pass-hr

The exhaustive search algorithm developed in Chapter 4 is used to minimize the objective total cost function (Eq. 3.13), considering frequency conservation, service capacity, and fleet size constraints. Three scenarios are defined as illustrated in Table 5.3, through which the applicability of the developed model and the solution algorithm are examined. For Scenario A, an all-stop SP serving every stop from the beginning to the end of the route is considered. In addition to the all-stop SP, the effectiveness of the integrated all-stop and short-turn SPs are investigated in Scenario B. Finally, the integrated operation of all-stop, short-turn and express SPs are analyzed in Scenario C.

Table 5.3 Scenarios of Study Service Patterns

Scenarios	Service Patterns
A	All-stop
B	All-stop + Short-turn
C	All-stop + Short-turn + Express

5.1.1 All-Stop SP – Scenario A

The all-stop SP denoted as $SP_{1,6}$ and $SP_{6,1}$ shown in Figure 5.2 is considered under Scenario A. Based on the OD demand of the study route illustrated in Table 5.1, the optimized service frequency is 17 bus/hr that achieves the minimum cost of 8,582 $/hr The associated performance measures (e.g., fleet size, vehicle miles travel, average load factors, etc.) and all cost components are calculated and shown in Table 5.4.

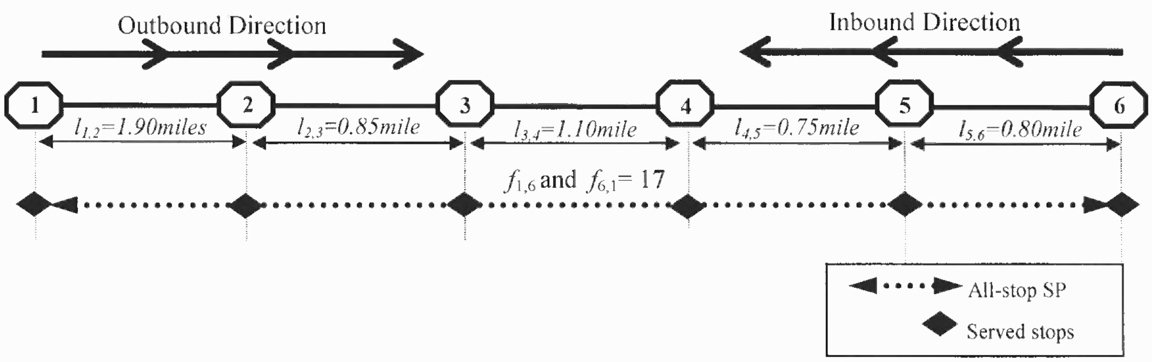

Figure 5.2 Optimized frequency of the all-stop SP in Scenario A.

Table 5.4 Optimal Results of Scenario A

Variables	**Units**	**Optimal Values**
$f_{1,6}(f_{6,1})$	buses/hr	17
Outbound LF	-	0.68
Inbound LF	-	0.53
Average LF	-	0.61
Fleet Size	buses	21
VMT	bus-mile/hr	184
PMT	pass-mile/hr	5574
V_a	mph	8.77
VMT_a	miles/bus	8.74
P_{vm}	pass/bus-mile	10.38
P_v	pass/bus	90.76
C_p	$/pass	1.32
C_{pm}	$/pass-mile	0.45
C_{vm}	$/bus-mile	13.69
T_a	hr/pass	0.32
C_W	$/hr	561
C_R	$/hr	0
C_I	$/hr	5,508
C_O	$/hr	2,513
Total Cost	**$/hr**	**8,582**

The performance measures defined and formulated in Chapter 3 are used here to evaluate the service under Scenario A. The passenger miles traveled denoted as PMT, the product of the number of passengers carried and average trip length, is 5574 pass-mile/hr. The vehicle miles traveled, denoted as VMT, the sum of distances traversed by buses per hour, is 184 bus-miles/hr. The average bus speed, denoted as V_a, is 8.77 mph, which yields an average passenger travel time (T_a) of 0.32 hr/pass.

The efficiency of bus usage, denoted as VMT_a, is 8.74 miles/bus, which is defined as VMT divided by the fleet size. Considering the intensity of bus service usage, the ratio of hourly demand to VMT, denoted as P_{vm}, is 10.38 pass/bus-mile. A higher P_{vm} represents a more efficient service. The average cost per passenger, per PMT, and per VMT, denoted as C_p, C_{pm} and C_{vm} are 1.32 \$/pass, 0.45 \$/pass-mile and 13.69 \$/bus-mile, respectively.

The service from Stop 1 to 6 is defined as the outbound direction and vice versa for the inbound direction. As shown in Figure 5.3, the link and average load factor (LF) for both outbound and inbound services are 0.68 and 0.53, respectively. It was found that the highest occupancy rate is at the outbound direction between Stops 4 and 5 with a LF of 0.97, while the LF between Stops 1 and 3 are less than the average LF for each service direction. The performance measures of Scenario A will be utilized to make comparisons with those derived under Scenarios B and C.

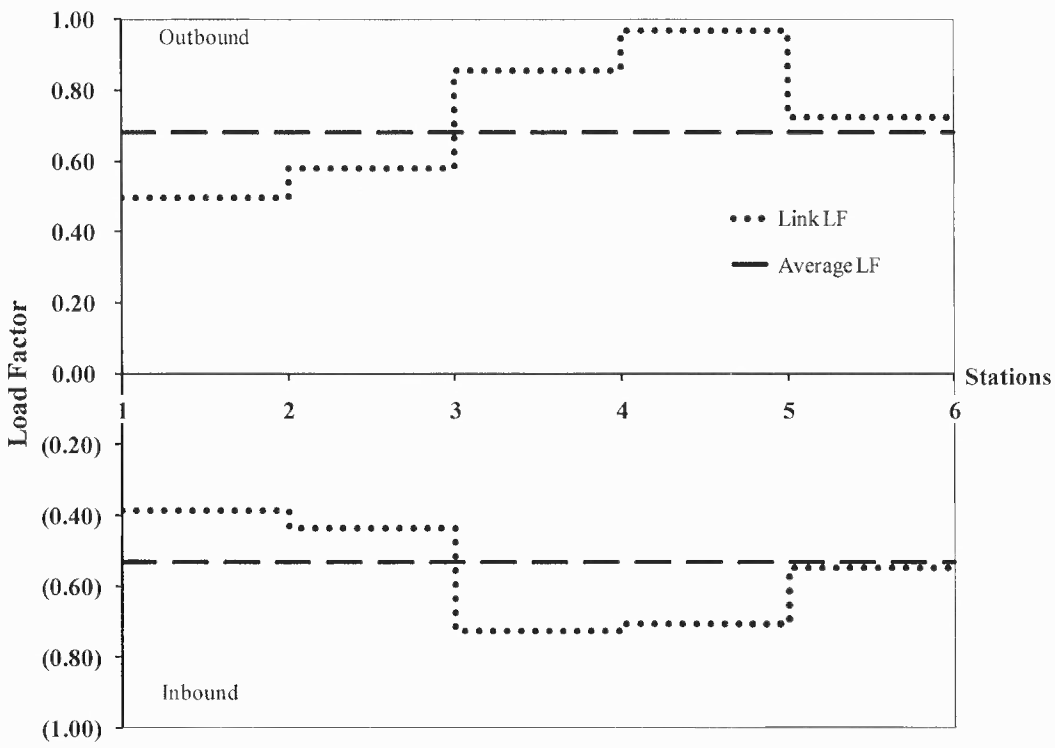

Figure 5.3 Load factor under optimal operation in Scenario A.

5.1.2 Sensitivity Analysis – Scenario A

In this section, the relationship between decision variables (e.g., SP, service frequency) and model parameters (e.g. bus operating cost, user value of time, bus capacity, bus speed, etc.) are investigated. For instance, costs (i.e., total, user and supplier costs) versus service frequency are illustrated in Figure 5.4, in which the minimum total cost of 7,882 $/hr is achieved at the optimized service frequency of 8 buses/hr. However, this solution violates the capacity constraint. The dotted lines in Figure 5.4 indicate infeasible solutions due to insufficient service capacity. Thus, the optimized service frequency is 17 buses/hr, which yields the minimum total cost of 8,582 $/hr.

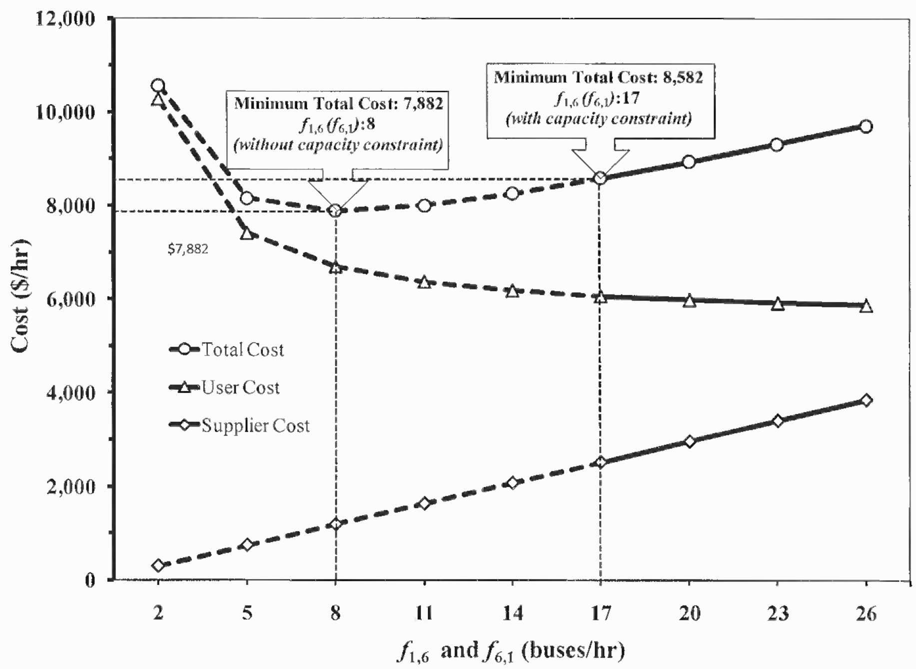

Figure 5.4 Costs vs. service frequency in Scenario A.

The optimized frequency versus the bus operating cost with and without capacity constraint are illustrated in Figure 5.5. As the bus operating cost increases from 20 to 200 $/bus-hr under constraint optimization, the optimized service frequency first decreases from 20 to 17 buses/hr and stays on 17 buses/hr when the bus operating cost is over 28 $/bus-hr, while it decreases from 20 to 6 buses/hr as the capacity constraint is removed and a smaller number of passengers is served.

The dotted line in Figure 5.6 indicates that the optimized service frequency is significantly affected by the user's value of time, which increases as the user's value of time increases to reduce the increased user's cost. As discussed earlier, the service frequency must be equal to or greater than 17 buses/hr to satisfy the capacity constraint. Under constrained optimization, the optimized frequency remains constant as the user's value of time stays below 43 $/hr.

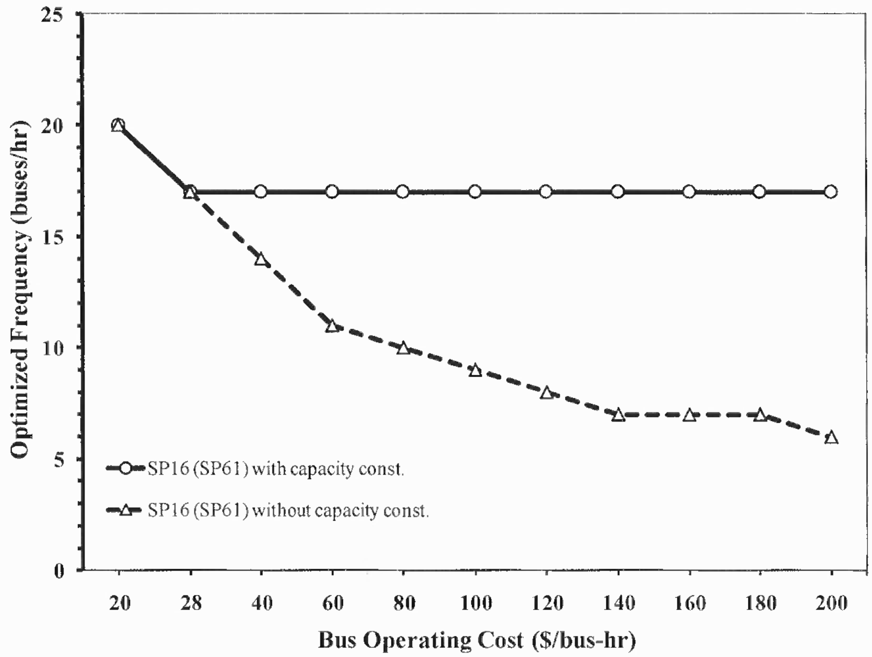

Figure 5.5 Optimized service frequency vs. bus operating cost in Scenario A.

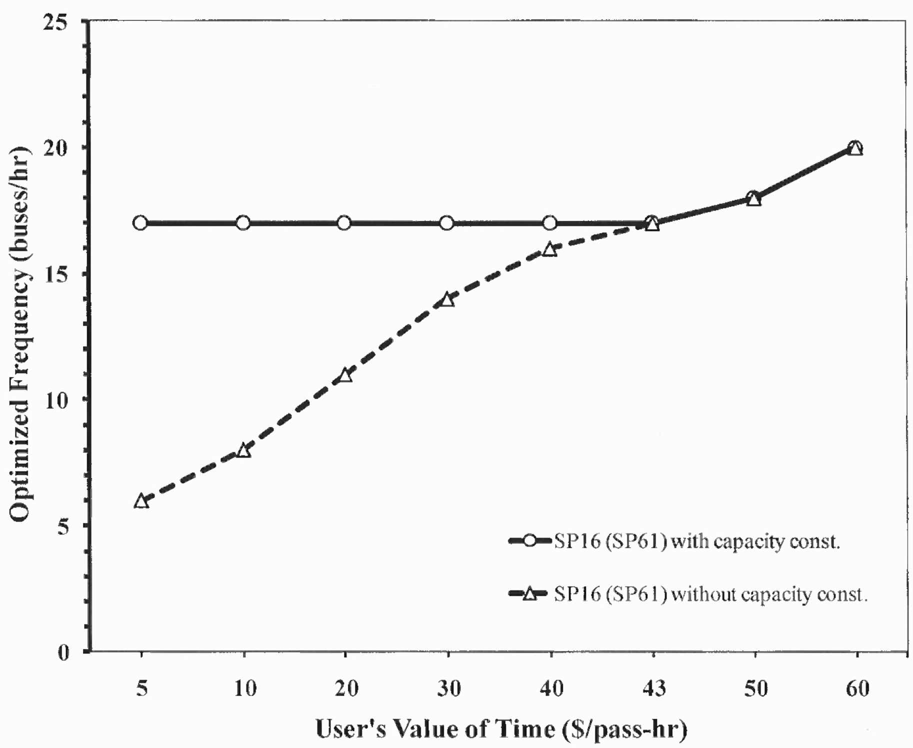

Figure 5.6 Optimized service frequency vs. user's value of time in Scenario A.

To investigate the relationship between optimized frequency and different demand, Figure 5.7 is developed by varying the demand from 20% to 140%. A solid line shows the optimized service frequency with bus capacity of 50 bus/hr while in the dotted line bus capacity increased to a large positive number (e.g., > 500 pass/bus) to optimize service frequency without the capacity constraint. Both optimized frequencies with and without considering the capacity constraint increase proportionally as the route demand increases.

Bus capacity is an important parameter which influences the optimized service frequency. While fixing the bus operating cost, the optimal bus capacity and optimized service frequency by varying the demand are illustrated in Figure 5.8. As discussed earlier, a bus capacity of 50 pass/bus requires a service frequency of 17 buses/hr which satisfies the capacity constraint and yields a total cost of 8,582 $/hr. However, using a bus capacity of 103 (instead of 50) pass/bus, decreases the optimized service frequency from 17 to 8 buses/hr which satisfies the capacity constraint and yields a minimum total cost of 7,882 $/hr for the study route in Scenario A.

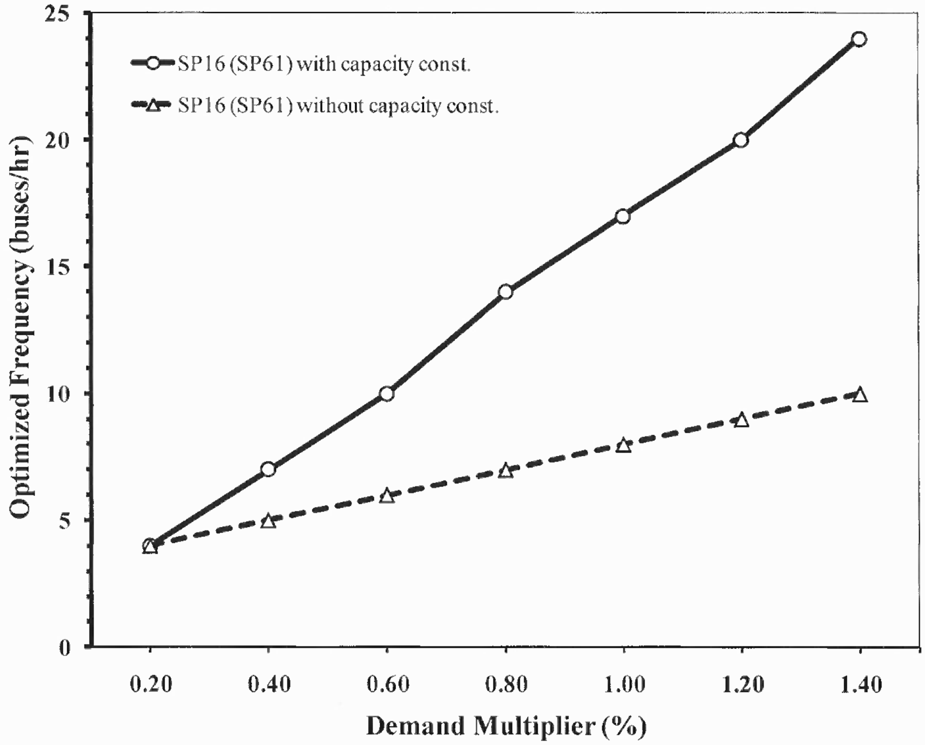

Figure 5.7 Optimized service frequency vs. demand multiplier in Scenario A.

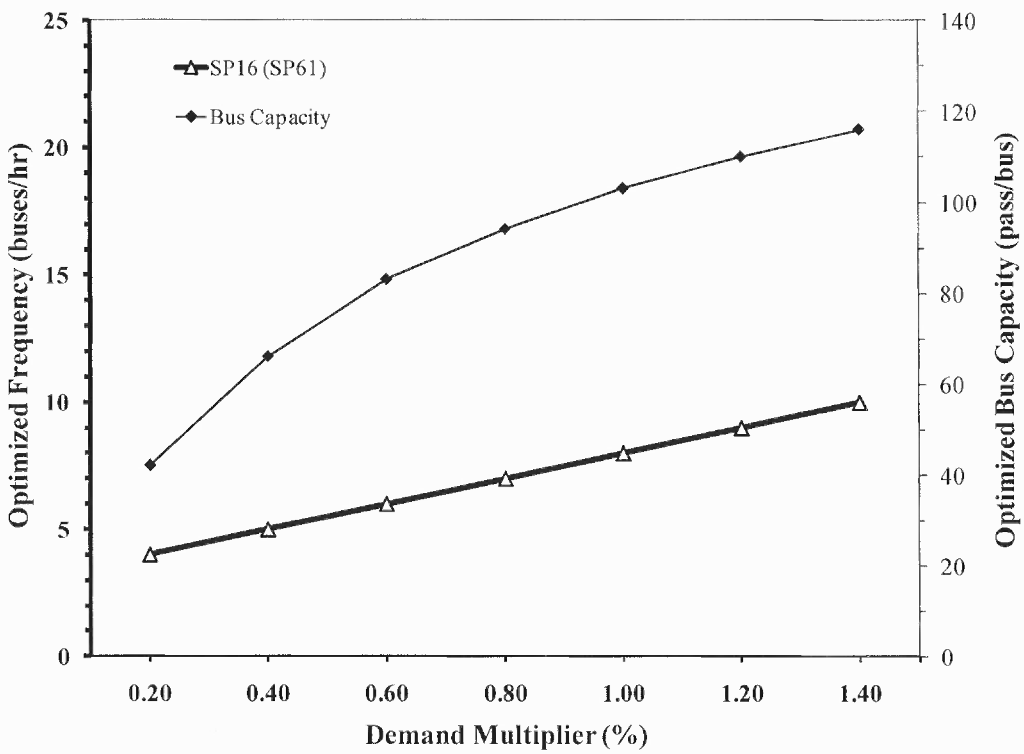

Figure 5.8 Optimized service frequency and bus capacity vs. demand multiplier in Scenario A.

As the bus capacity increases in Figure 5.9, the optimized $f_{1,6}$ and $f_{6,1}$ decrease and a lower supplier cost as well as total cost may be expected. However, increasing bus capacity may reduce the optimized service frequency (increase the optimized headway), which increases passenger wait time. As discussed earlier, Figure 5.9 also shows that bus capacities over 103 pass/bus yield the same minimum total cost of 7,882 $/hr.

Figure 5.10 shows the impact of bus capacity on optimal fleet size and average load factor. Decreasing bus capacity increases the optimal fleet size because more frequent service is needed to satisfy the capacity constraint. It is found that the average LFs vary between 0.59 to 0.63 for buses with a capacity of 60 and less while the variation range of average LF increases (e.g., 0.55~0.62) as the bus capacity increases. A bus capacity of 103 pass/hr achieves the lowest fleet size of 10 buses and yields the highest average LF of 0.63.

The effect of bus capacity on average passenger travel time denoted as T_a is indicated in Figure 5.11. More frequent service is needed when bus capacity decreases, which may decrease the passengers' travel time because of reduced wait time.

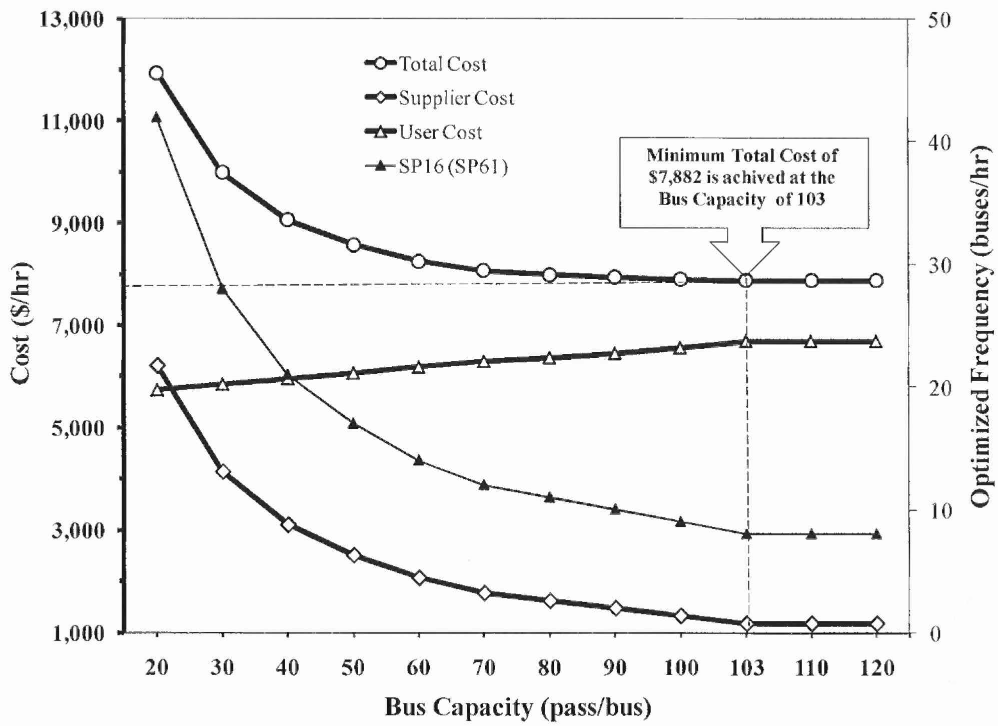

Figure 5.9 Costs and optimized service frequency vs. bus capacity in Scenario A.

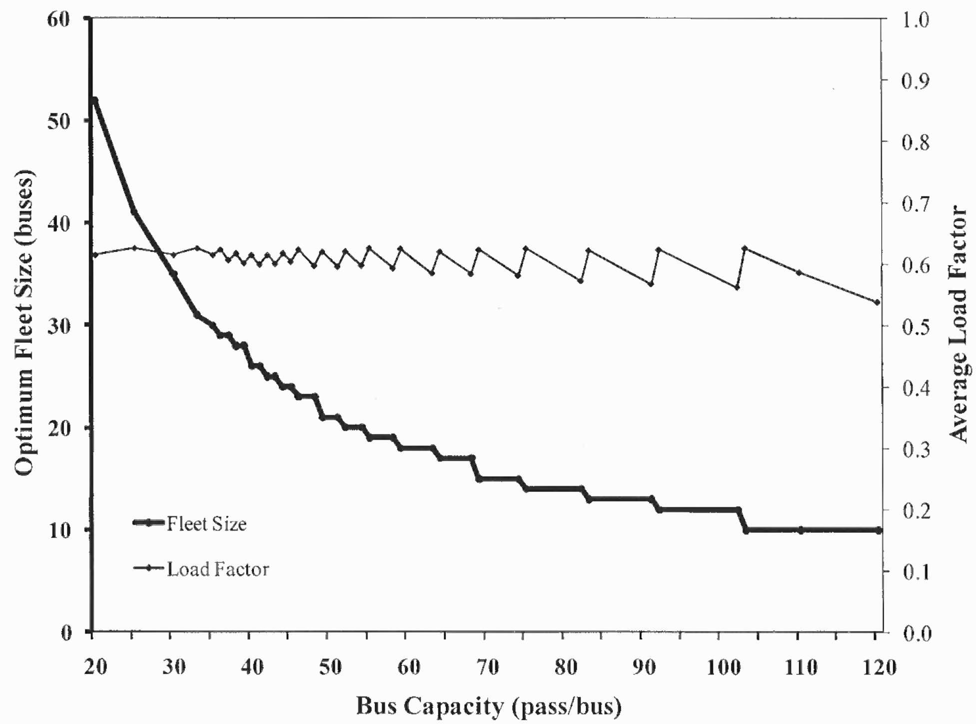

Figure 5.10 Optimal fleet size and average load factor vs. bus capacity in Scenario A.

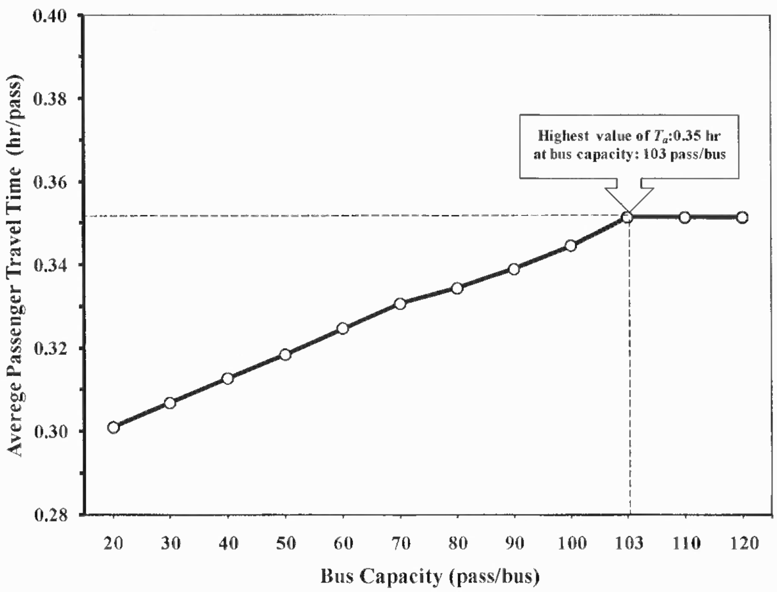

Figure 5.11 Average passenger travel time vs. bus capacity in Scenario A.

The impact of bus capacity on the average cost per VMT (C_{vm}) and average cost per PMT (C_{pm}) in Scenario A is illustrated in Figure 5.12. Bus capacity is increased without increasing the bus operating cost. It is found that C_{pm} decreases as the bus capacity increases because of the reduced supplier cost as shown in Figure 5.9. However, the bus capacity variation does not impact the C_{vm} due to the proportional change in supplier cost and VMT.

The impacts of average bus speed on cost components (i.e., user and supplier cost) and optimized frequency; and impacts on optimized fleet size and average passenger travel time are demonstrated in Figures 5.13 and 5.14, respectively. In this case, increasing the bus speed has no impact on optimized service frequency due to the capacity constraint. However, the optimal fleet size is reduced due to the reduction in bus round trip travel time as the speed increases. Therefore, reduction in bus travel time decreases the supplier cost. In addition, the buses in higher speeds decrease the average travel time of passengers and lower user costs.

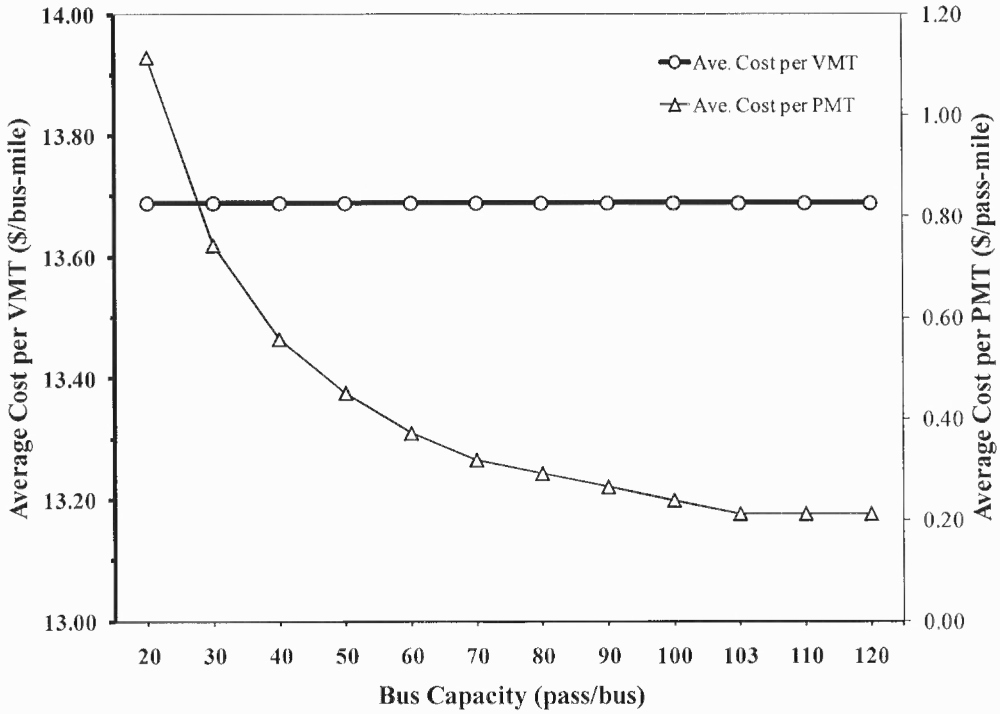

Figure 5.12 Average cost per vehicle miles travel (C_{vm}) and per passenger miles travel (C_{pm}) vs. bus capacity in Scenario A.

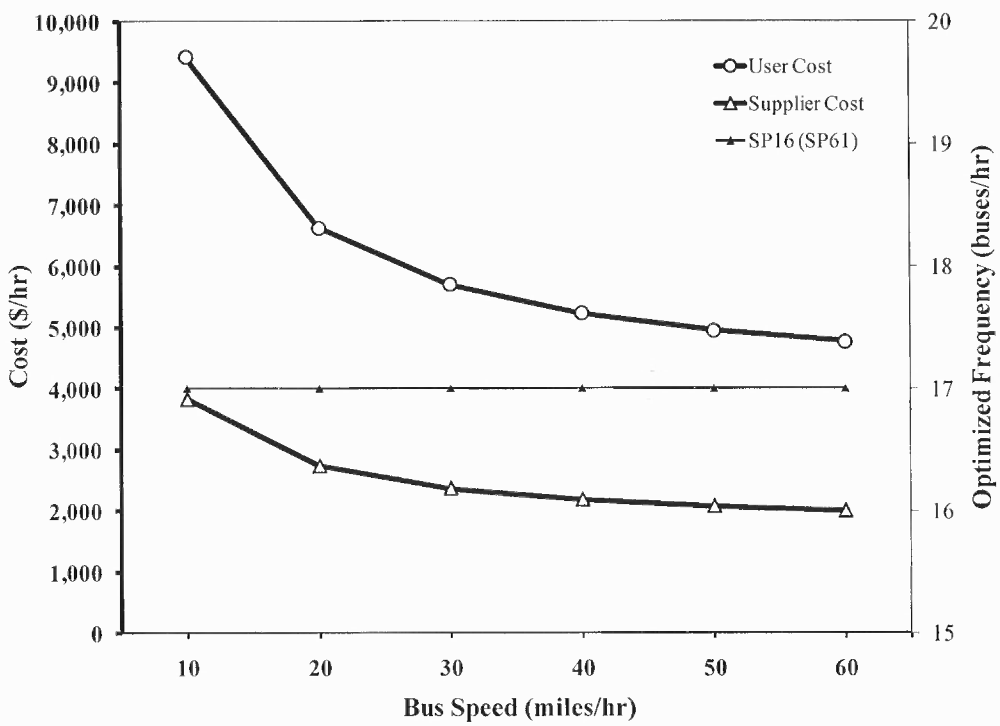

Figure 5.13 Costs and optimized service frequency vs. bus speed in Scenario A.

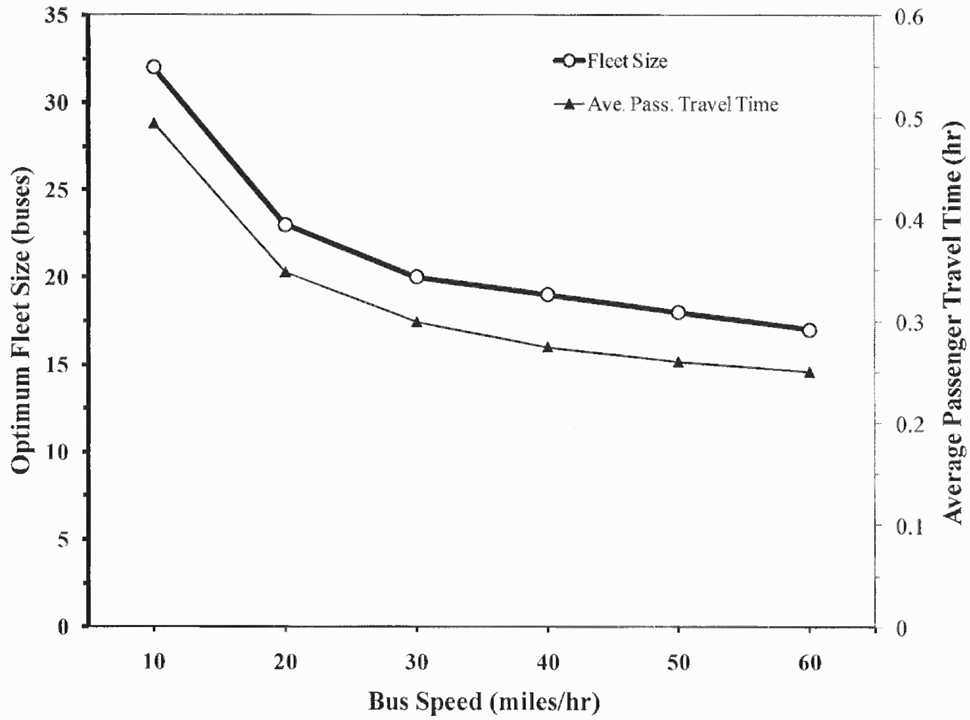

Figure 5.14 Optimal fleet size and average passenger travel time vs. bus speed in Scenario A.

5.1.3 All-Stop and Short-Turn SPs – Scenario B

In addition to the all-stop SP described in Scenario A, short-turn SPs are introduced in this section as shown in Figure 5.15. As described earlier, Stops 3 and 4 are eligible as the turn-back stops of the short-turn services. Therefore, 5 different short-turn SPs are available (e.g., $SP_{1,3}$ ($SP_{3,1}$), $SP_{1,4}$ ($SP_{4,1}$), $SP_{3,4}$ ($SP_{4,3}$), $SP_{3,6}$ ($SP_{6,3}$) and $SP_{4,6}$ ($SP_{6,4}$)) for the six-stop bus route described in Section 5.1. Based on the OD demand illustrated in Table 5.1, the optimized service frequencies of $SP_{1,6}$ ($SP_{6,1}$), $SP_{3,6}$ ($SP_{6,3}$) and $SP_{4,6}$ ($SP_{6,4}$), are 10, 5 and 2 buses/hr respectively, and 0 buses/hr for $SP_{1,3}$ ($SP_{3,1}$), $SP_{1,4}$ ($SP_{4,1}$), $SP_{3,4}$ ($SP_{4,3}$). This operation has a minimum total cost of 8,354 \$/hr. The associated measures (e.g., fleet size, VMT, average LFs, etc.) and all cost components derived from the optimized frequencies are calculated and shown in Table 5.5. Note that, the model is able to optimize the turn-back stops, while the determination of the eligible stops may be based on the bus supplier's discretion due to the stop layout.

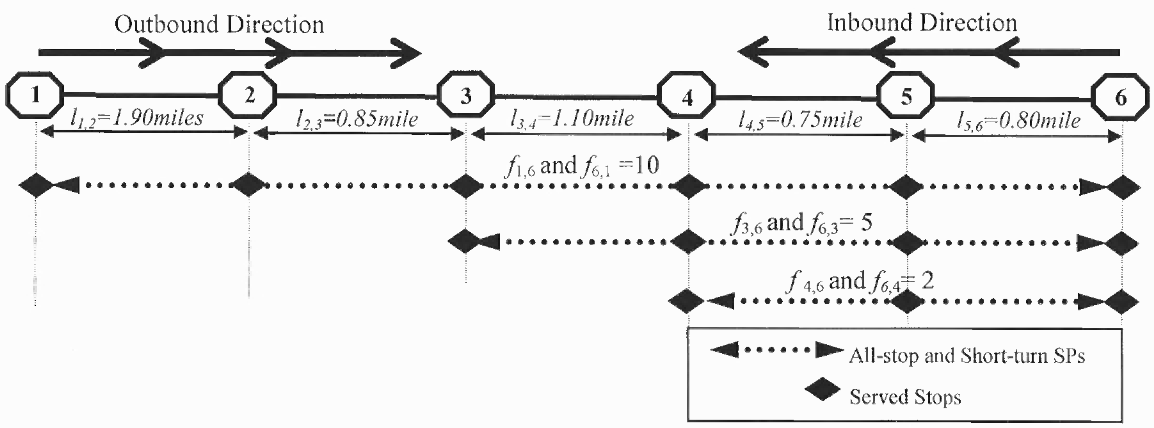

Figure 5.15 Optimized service patterns and service frequencies in Scenario B.

The performance measures are used to evaluate the system performance under Scenario B. The passenger miles traveled (PMT) and vehicle miles traveled (VMT) are 5574 pass-mile/hr and 141 bus-miles/hr, respectively. The average route speed (V_a) is 8.26 mph, which yields an average passenger travel time (T_a) of 0.33 hr/pass. The efficiency of bus usage (VMT_a) and intensity of bus service usage (P_{vm}) are 7.82 miles/bus and 13.55 pass/bus-mile, respectively. The average cost per passenger (C_p), per PMT (C_{pm}) and per VMT (C_{vm}), are 1.09 \$/pass, 0.37 \$/pass-miles and 14.76 \$/bus-miles, respectively.

Table 5.5 Optimal Results of Scenario B

Variables	**Units**	**Optimal Values**
$f_{1,3}(f_{3,1})$	buses/hr	0
$f_{1,4}(f_{4,1})$	buses/hr	0
$f_{1,6}(f_{6,1})$	buses/hr	10
$f_{3,4}(f_{4,3})$	buses/hr	0
$f_{3,6}(f_{6,3})$	buses/hr	5
$f_{4,6}(f_{6,4})$	buses/hr	2
Outbound LF	-	0.89
Inbound LF	-	0.70
Average LF	-	0.79
Fleet Size	buses	18
VMT	bus-mile/hr	141
PMT	pass-mile/hr	5574
V_a	mph	8.26
VMT_a	miles/bus	7.82
P_{vm}	pass/bus-mile	13.55
P_v	pass/bus	105.89
C_p	$/pass	1.09
C_{pm}	$/pass-mile	0.37
C_{vm}	$/bus-mile	14.76
T_a	hr/pass	0.33
C_W	$/hr	769
C_R	$/hr	0
C_I	$/hr	5508
C_O	$/hr	2077
Total Cost	**$/hr**	**8,354**

Figure 5.16 shows the link and average LF for both outbound and inbound services are illustrated. The average LF's of outbound and inbound services are 0.89 and 0.70, respectively. It was found that the highest occupancy rate is at the outbound

direction between Stops 2 to 5 with LF of 0.99 between Stop 2 and 3 and decreasing to 0.97 after Stop 3, while the LF between Stops 1 to 2 and 5 to 6 are less than the average LF of each service direction.

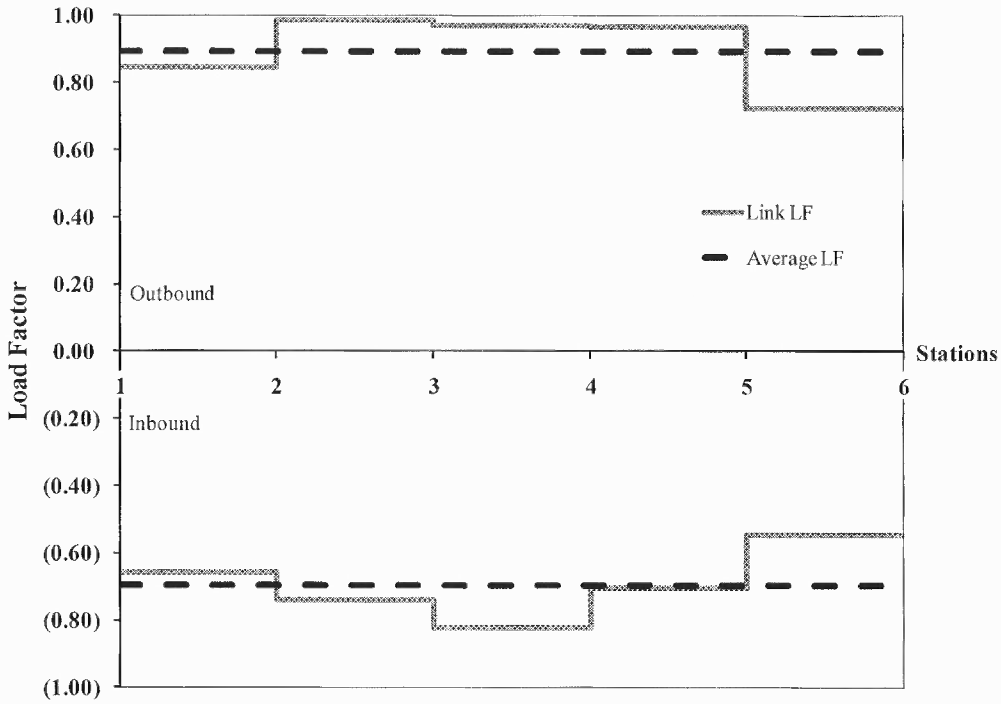

Figure 5.16 Load factor under optimal operation in Scenarios B.

5.1.4 Sensitivity Analysis – Scenario B

In this section, the relationships among decision variables (e.g., SPs, service frequencies), performance measures (e.g., fleet size, VMT, LF, T_a, etc.) and model parameters (e.g. bus operating cost, user value of time, bus capacity, bus speed etc.) are investigated. For instance, optimized short-turn service frequencies and costs (i.e., total, user and supplier costs) versus the all-stop service frequency ($f_{1,6}$ and $f_{6,1}$) are illustrated in Figure 5.17 and 5.18, respectively. When the $f_{1,6}$ and $f_{6,1}$ is more than 17 buses/hr, all demand can be satisfied by an all-stop service and all optimized short-turn service frequencies become 0 buses/hr. The optimized value of $f_{1,6}$ (and $f_{6,1}$) is 10 buses/hr and increasing $f_{1,6}$ and $f_{6,1}$ above 10 buses/hr, increases the supplier cost. However, increasing $f_{1,6}$ (and $f_{6,1}$) decreases the user cost due to the passenger waiting time decrease. Therefore, the optimized service frequencies of $f_{1,6}$ ($f_{1,6}$), $f_{3,6}$ ($f_{6,3}$), and $f_{4,6}$ ($f_{6,4}$) are achieved as 10, 5, and 2 buses/hr, respectively, which yields a minimum total cost of 8,354 \$/hr.

The sensitivity of optimized service frequencies versus the bus operating cost is illustrated in Figure 5.19. As the bus operating cost increases from 40 to 100 \$/bus-hr, the optimized service frequency of all-stop SPs (i.e., $SP_{1,6}$ and $SP_{6,1}$) decreases from 17 to 10 buses/hr, while the optimized service frequencies of short-turn SPs (e.g., $SP_{3,6}$ ($SP_{6,3}$) and $SP_{4,6}$ ($SP_{6,4}$)) increase to balance the increased supplier cost. However, all service frequencies remain the same when the bus operating cost is over 100 \$/bus-hr due to the capacity constraint.

Figure 5.20 shows the impact on optimal VMT and average LF by increasing the bus operating cost from 40 to 200 \$/bus-hr and the user's value of time and other baseline variables are fixed. VMT decreases as the operating cost increases and reaches the minimum value of 141 bus-miles/hr when the bus operating cost is 100 \$/bus-hr which yields the maximum LF of 0.79.

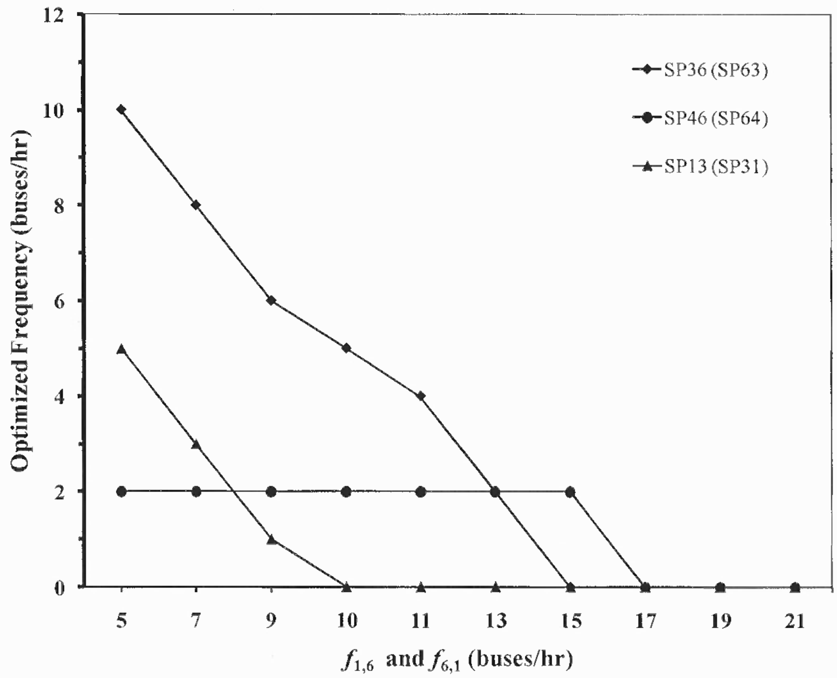

Figure 5.17 Optimized short-turn service frequencies vs. service frequency of $SP_{1,6}$ ($SP_{6,1}$) in Scenario B.

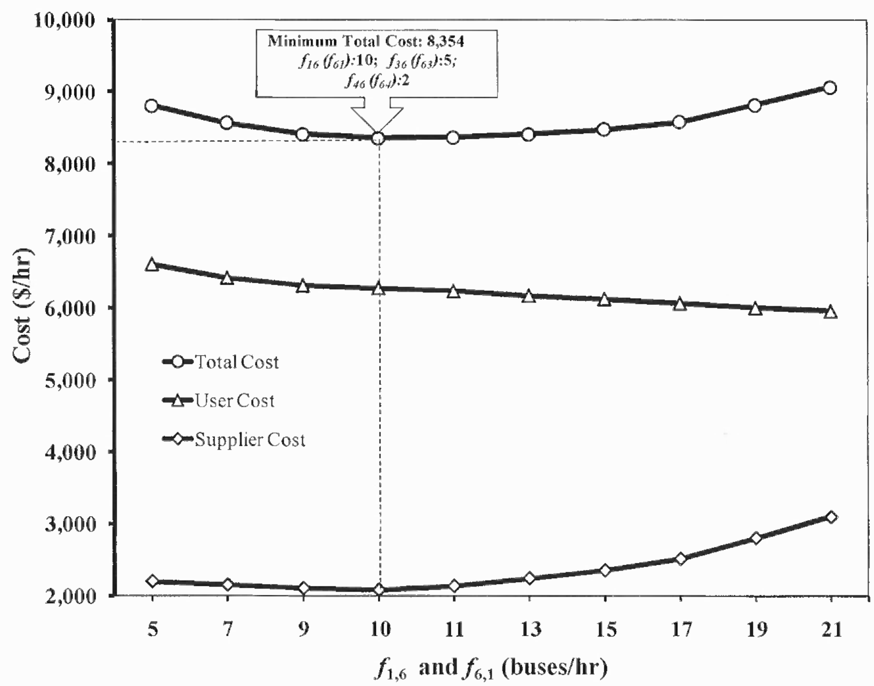

Figure 5.18 Costs vs. service frequency of $SP_{1,6}$ ($SP_{6,1}$) in Scenario B.

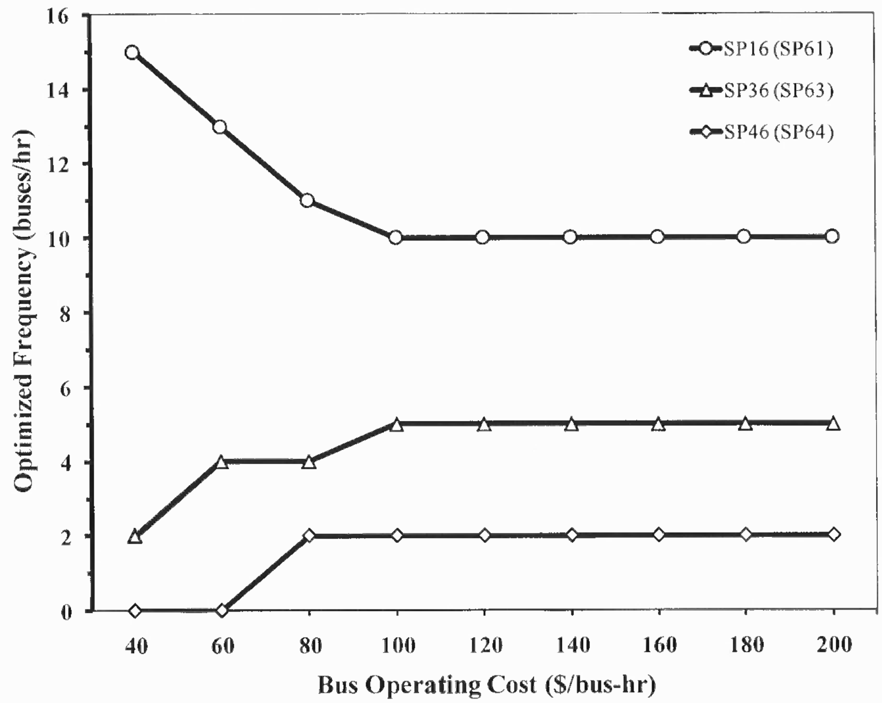

Figure 5.19 Optimized service frequency vs. bus operating cost in Scenario B.

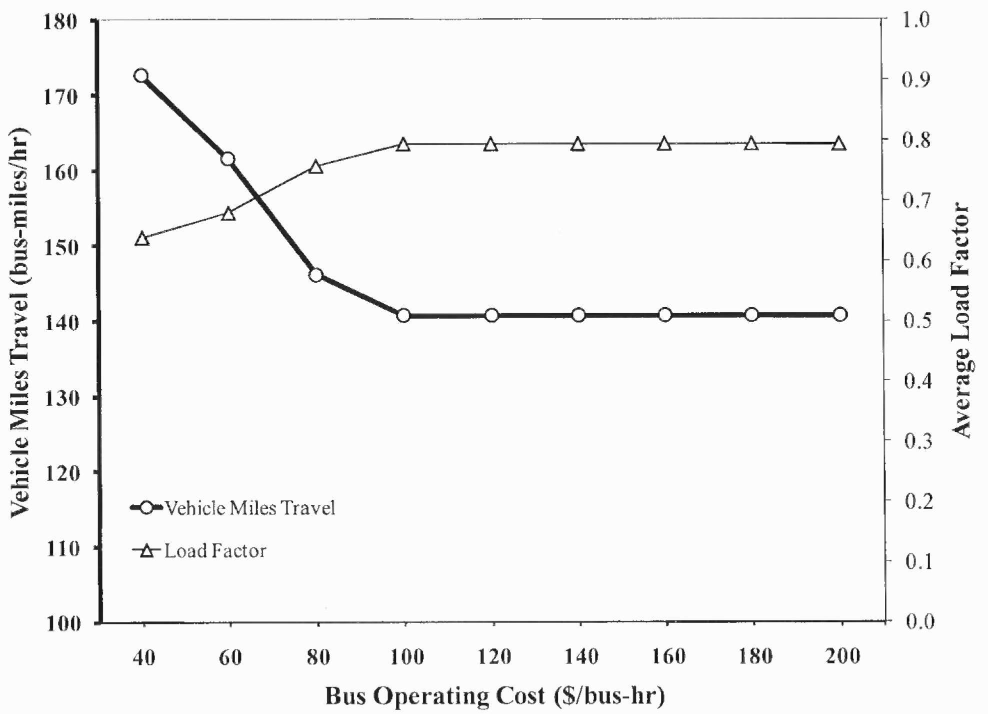

Figure 5.20 Optimal vehicle miles travel and average load factor vs. bus operating cost in Scenario B.

Figure 5.21 shows the impact on optimized service frequencies by varying the user's value of time from 5 to 50 \$/pass-hr. The all-stop service frequency increases and the short-turn service frequencies decrease as the user's value of time increases to balance the increasing user costs. In other words, increasing the user's value of time increases the need of more buses, which yields the supplier cost increase.

Figure 5.22 shows the impact of user's value of time on the optimal fleet size and average load factor. Increasing the user's value of time increases the optimal fleet size due to the replacement of the short-turn services by the all-stop service. Using the full route service instead of short-turn services also decreases the average LF and increases the supplier cost because more buses are needed to satisfy the increasing vehicle miles traveled.

In Figure 5.23, the impact of user's value of time on C_{vm} and C_{pm} is demonstrated. C_{vm} decreases as the bus capacity increases because of VMT increases proportionally higher than supplier cost. However, increasing the user's value of time increases the average cost per PMT (C_{pm}) because of the supplier cost increase while the PMT remains the same.

Figure 5.24 shows that the average passenger per bus (P_v) and per VMT (P_{vm}) decrease as the user's value of time increases, due to the increase of the optimal fleet size as discussed earlier in Figure 5.22.

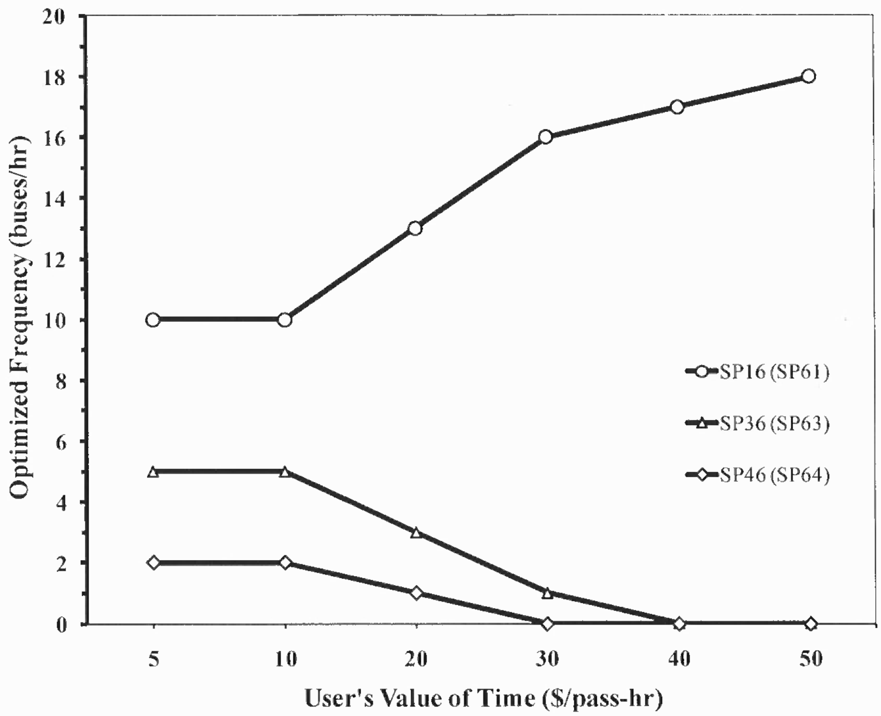

Figure 5.21 Optimized service frequencies vs. user's value of time in Scenario B.

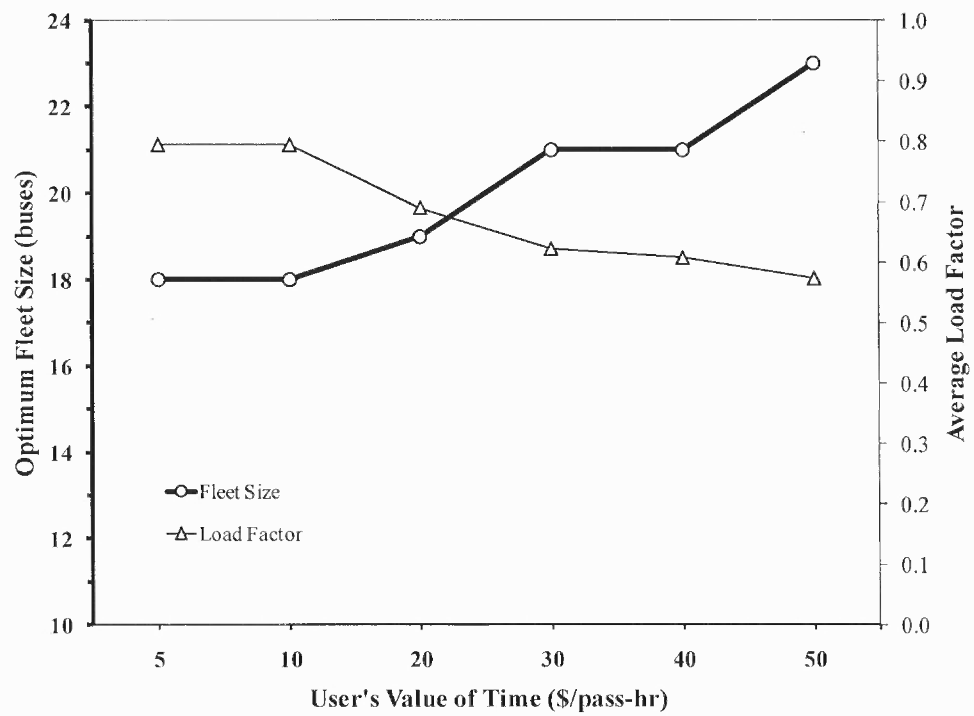

Figure 5.22 Optimal fleet size and average load factor vs. user's value of time in Scenario B.

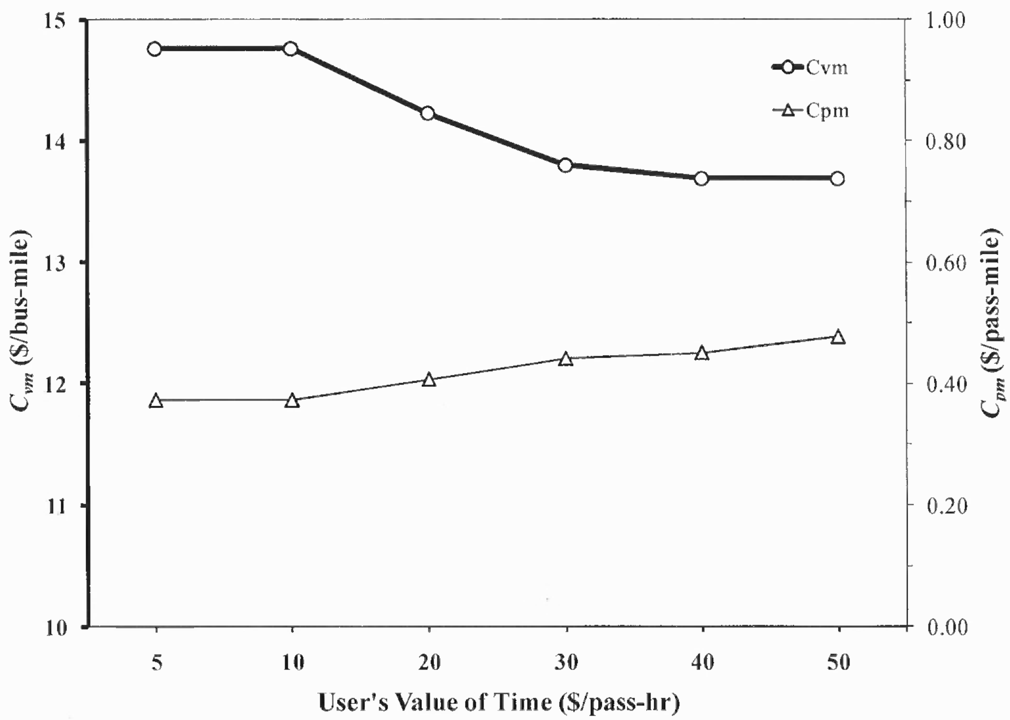

Figure 5.23 Average cost per VMT (C_{vm}) and average cost per PMT (C_{pm}) vs. bus operating cost in Scenario B.

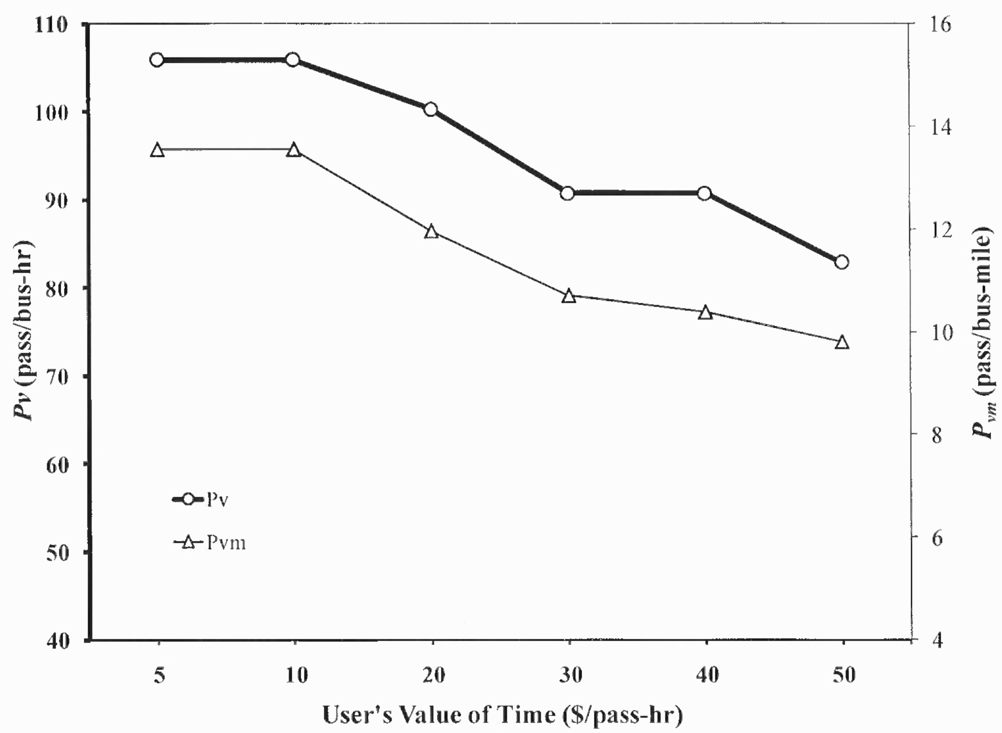

Figure 5.24 Average number of passengers per bus (P_v) and average number of passengers per VMT (P_{vm}) vs. users value of time in Scenario B.

To investigate the relationship between optimized all stop and short-turn service frequencies and different demand, Figure 5.25 is developed, in which the demand multiplier varies from 20% to 140%. All the optimized service frequencies increase to satisfy the capacity constraint.

The optimal bus capacity and associated optimized service frequency are illustrated in Figure 5.26. As discussed before, the fixed bus capacity of 50 pass/bus requires that $f_{1,6}$ ($f_{6,1}$), $f_{3,6}$ ($f_{6,3}$) and $f_{4,6}$ ($f_{6,4}$), are 10, 5 and 2 buses/hr, respectively. As illustrated in Figure 5.26, using a bus capacity of 103 pass/bus for the given route demand (i.e., demand multiplier is 100%), eliminates all short-turn SPs and decreases the $f_{1,6}$ ($f_{6,1}$) from 10 to 8 buses/hr which satisfies the given OD demand and yields a minimum total cost of 7,882 \$/hr. It is worth to note that, only $f_{1,6}$ and $f_{6,1}$ is used as the optimized frequency to achieve the minimum total cost when vehicle capacity is grater than the optimal bus capacity.

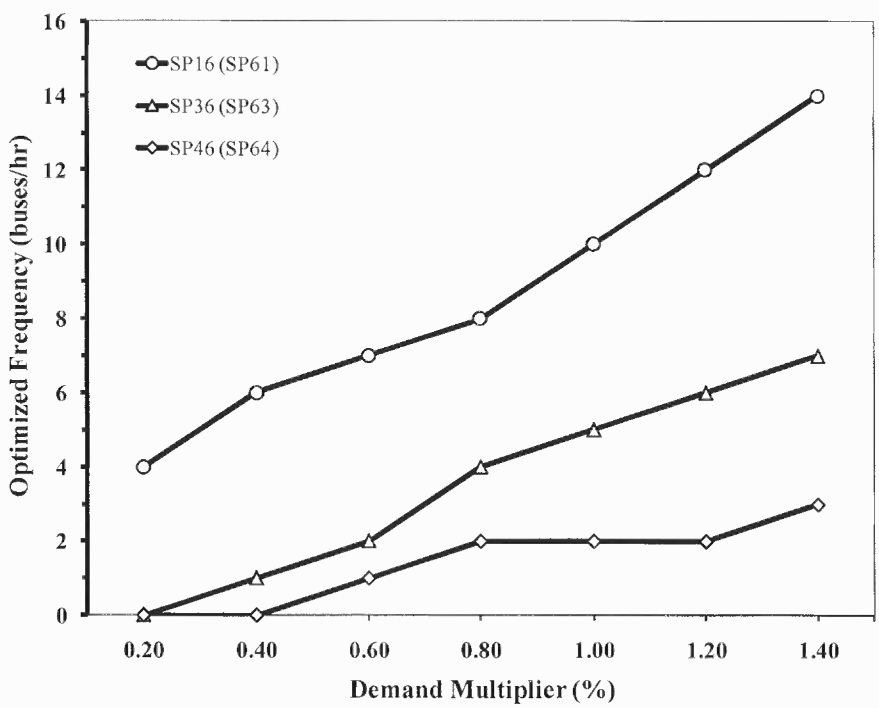

Figure 5.25 Optimized frequency vs. demand multiplier in Scenario B.

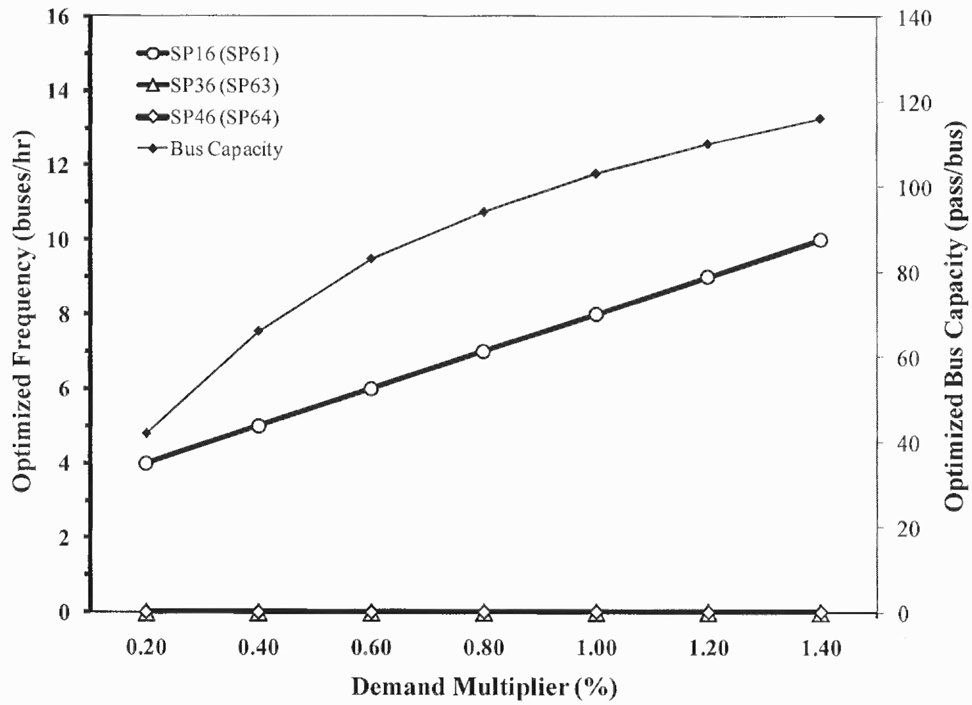

Figure 5.26 Optimal bus capacity and relevant optimized service frequency vs. demand multiplier in Scenario B.

As shown in Figure 5.27, as the bus capacity increases, the optimized service frequencies decrease. Higher bus capacity also lowers supplier as well as total cost as shown in Figure 5.28. However, increased bus capacity may reduce service frequency because of increased headway, which increases passenger wait time. Figure 5.28 also shows that bus capacities over 103 pass/hr satisfy the capacity constraint and yield the lowest minimum total cost of 7,882 $/hr.

Figure 5.29 shows the impact on C_{vm} and C_{pm} by varying the bus capacity from 20 to 200 pass/bus. The average cost per VMT decreases because of the need of less frequent service which yields lower fleet size and VMT. Average cost per PMT significantly decreases because of the constant PMT as supplier cost decreases.

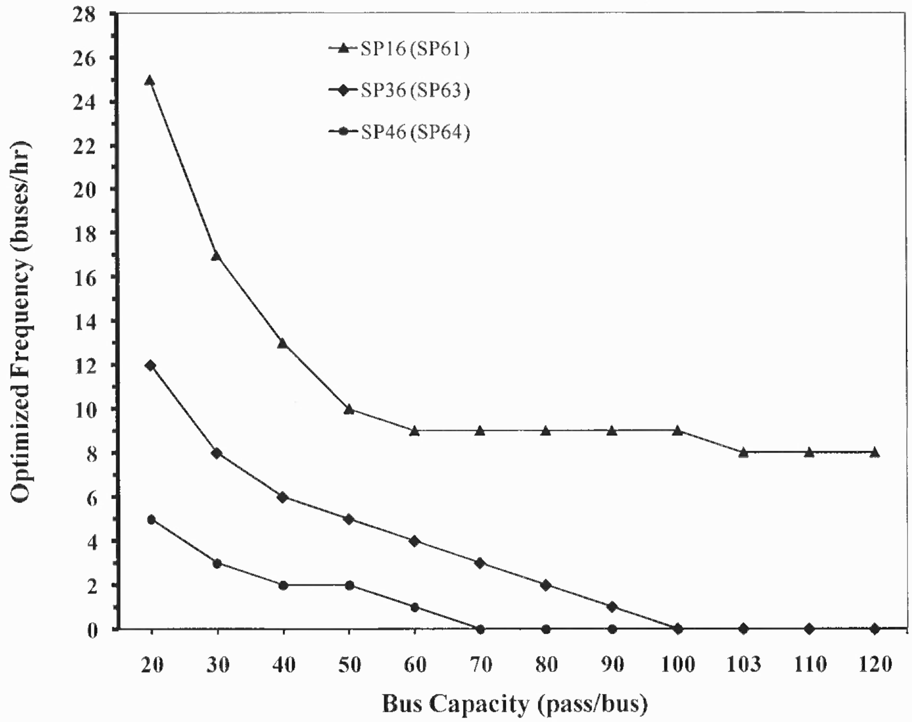

Figure 5.27 Optimized service frequencies vs. bus capacity in Scenario B.

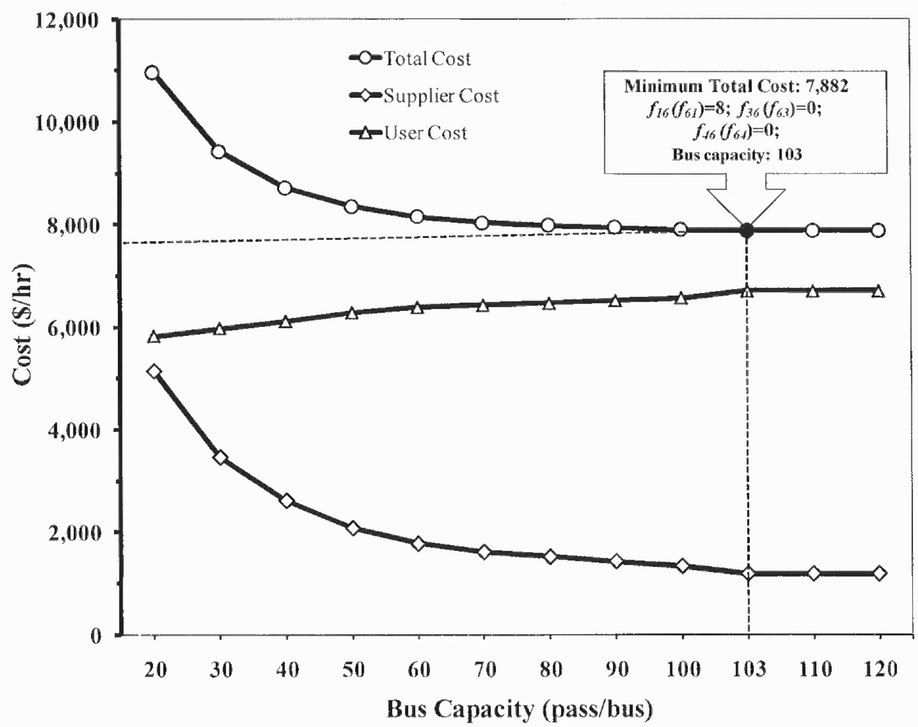

Figure 5.28 Costs vs. bus capacity in Scenario B.

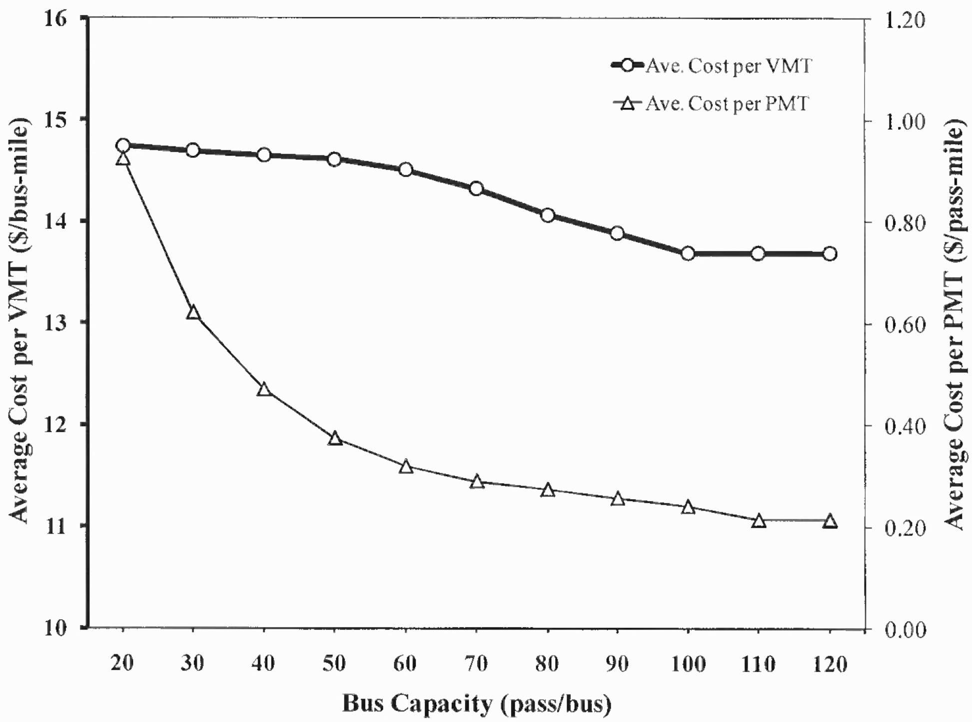

Figure 5.29 Average cost per VMT (C_{vm}) and per PMT (C_{pm}) vs. bus capacity in Scenario B.

The impacts of average bus speed on cost components (i.e., user and supplier cost) and optimized frequencies; and impacts on optimal fleet size and average passenger travel time are demonstrated in Figures 5.30 and 5.31, respectively. In Scenario B, increasing the bus speed up to the 50 miles/hr has no impact on optimized service frequency. However, optimal fleet size is reduced due to the reduction in bus round trip travel time as the speed increases. Therefore, a reduction in bus travel time decreases the supplier cost. In addition, the buses in higher speeds decrease the average travel time of passengers, which lowers user costs.

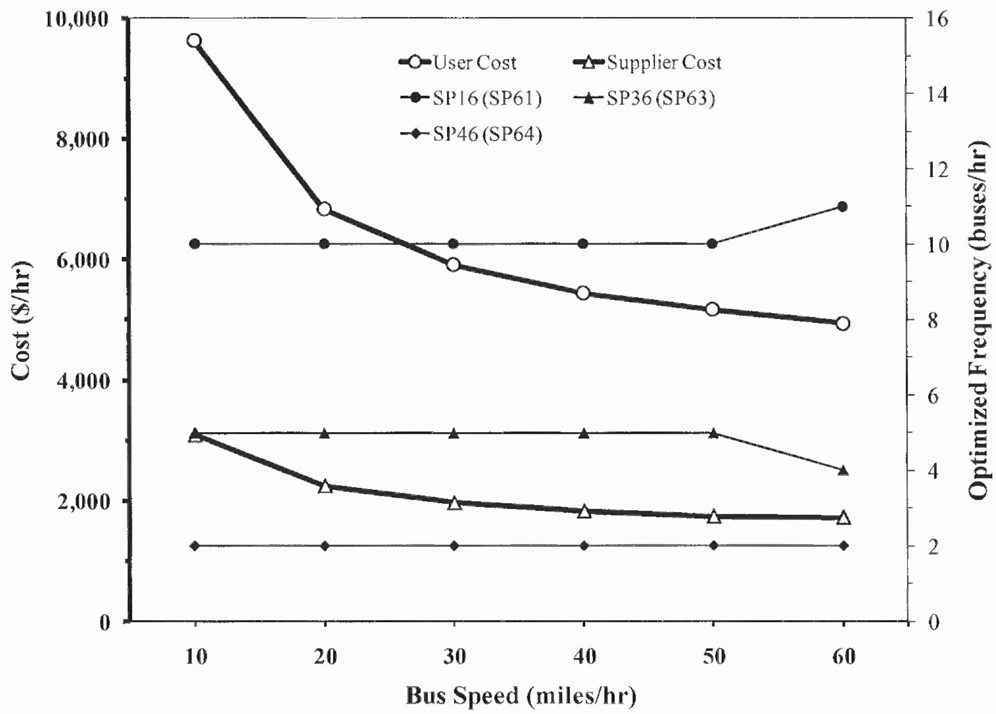

Figure 5.30 User and supplier costs and optimized frequencies vs. bus speed in Scenario B.

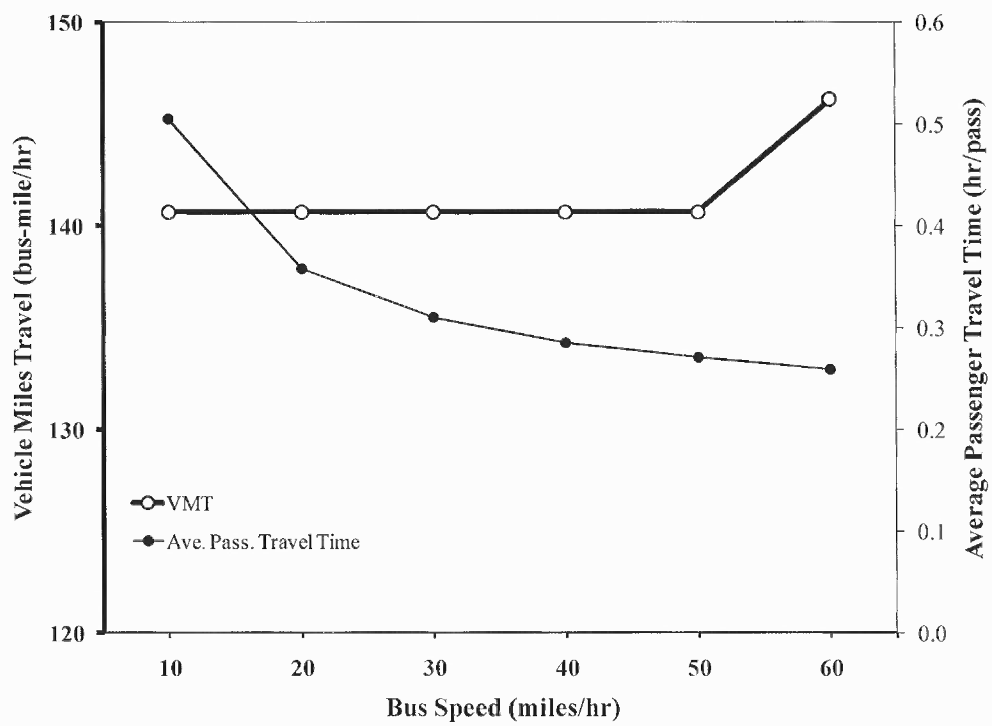

Figure 5.31 VMT and average passenger travel time (T_a) vs. bus speed in Scenario B.

5.1.5 Integrated All-Stop, Short-Turn and Express SPs – Scenario C

In addition to all-stop and short-turn SPs described in Scenario B, an express SP denoted as SP_E is introduced in this section as shown in Figure 5.32. SP_E may skip some of the stops which are determined during the optimization process. Considering the OD demand illustrated in Table 5.1 and baseline values given in Table 5.2, it was found that Stops 2, 4 and 5 are skipped by express services and the optimized service frequencies of $SP_{1,6}$ ($SP_{6,1}$), $SP_{3,6}$ ($SP_{6,3}$), $SP_{4,6}$ ($SP_{6,4}$) and SP_E are 4, 2, 2 and 9 buses/hr, respectively. This operation achieves the minimum total cost of 7,317 \$/hr. The associated measures (e.g., fleet size, VMT, VMT_a P_v, P_{vm}, T_a, and LF etc.) and all cost components derived from the optimized frequencies are calculated and shown in Table 5.6.

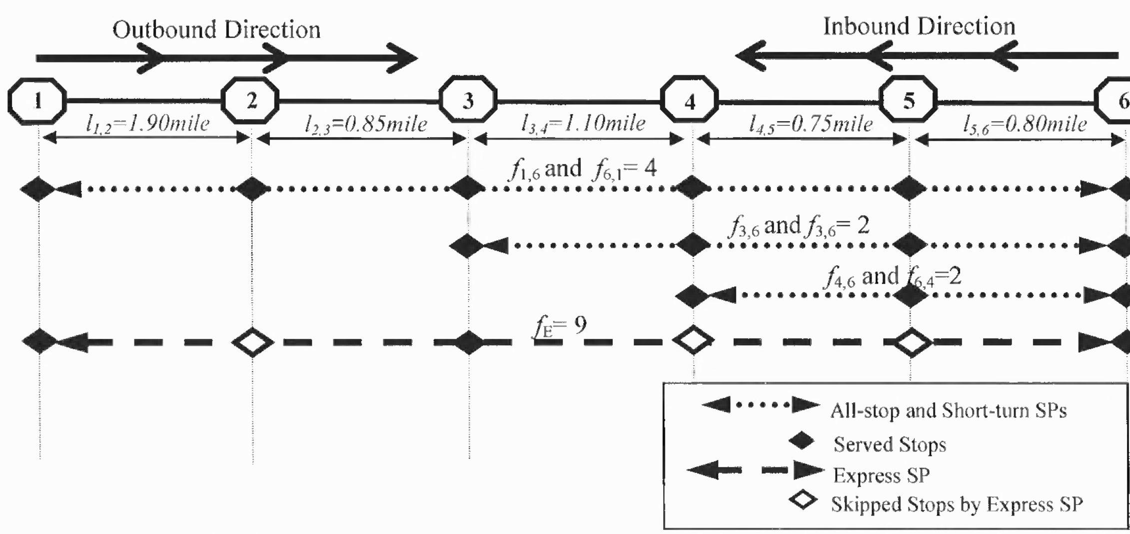

Figure 5.32 Optimized SPs in Scenario C.

In scenario C, PMT and VMT are calculated as 5574 pass-miles/hr and 141 bus-miles/hr, respectively. The average route speed (V_a) is 10.75 miles/hr which yields an average passenger travel time (T_a) of 0.29 hr. VMT_a and P_{vm} is 9.83 miles/bus and 13.55 pass/bus-mile, respectively. Average hourly supplier costs per passenger (C_p), per PMT

(C_{pm}) and per VMT (C_{vm}), are 1.09 \$/pass/hr, 0.37 \$/pass-miles/hr and 14.76 \$/bus-miles/hr, respectively.

Table 5.6 Optimal Results of Scenario C

Variable	**Units**	**Optimal Values**
$f_{1,3}(f_{3,1})$	buses/hr	0
$f_{1,4}(f_{4,1})$	buses/hr	0
$f_{1,6}(f_{6,1})$	buses/hr	4
$f_{3,4}(f_{4,3})$	buses/hr	0
$f_{3,6}(f_{6,3})$	buses/hr	2
$f_{4,6}(f_{6,4})$	buses/hr	2
f_E	buses/hr	9
Outbound LF	-	0.79
Inbound LF	-	0.62
Average LF	-	0.71
Fleet Size	buses	16
VMT	bus-mile/hr	157
PMT	pass-mile/hr	5574
V_a	mph	10.75
VMT_a	miles/bus	9.83
P_{vm}	pass/bus-mile	12.12
P_v	pass/bus	119.13
C_p	\$/pass	0.97
C_{pm}	\$/pass-mile	0.33
C_{vm}	\$/bus-mile	11.79
T_a	hr/pass	0.29
C_W	\$/hr	1,275
C_R	\$/hr	143
C_I	\$/hr	4,045
C_O	\$/hr	1,854
Total Cost	**\$/hr**	**7,317**

The link and average LFs for both outbound and inbound services are illustrated in Figure 5.33. The average LF's of outbound and inbound services are 0.79 and 0.62, respectively. It was found that the highest occupancy rate is at the outbound direction between Stops 3 and 4 with LF of 0.97, while the LF between Stops 1 to 3 and 5 to 6 are less than the average LF of each service direction.

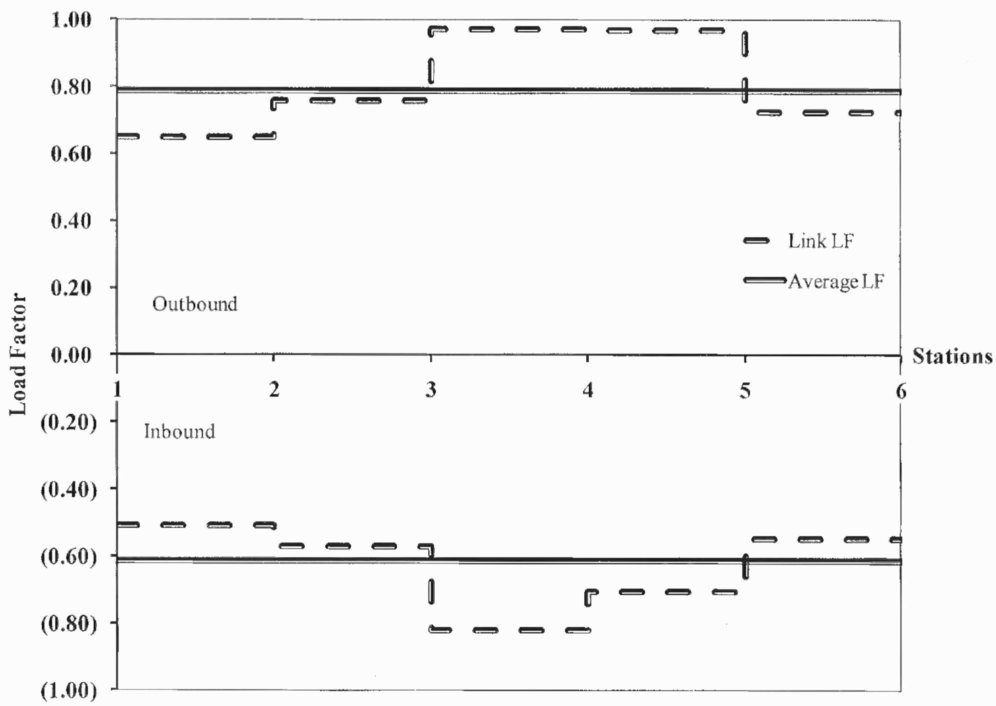

Figure 5.33 Load factor under optimal operation in Scenario C.

5.1.6 Sensitivity Analysis – Scenario C

The previous section discussed the minimized total cost and optimized service frequencies considering three SPs (i.e., all-stop, short-turn and express) while utilizing the baseline values of the input parameters shown in Table 5.2. In this section, the relationship between decision variables (e.g., SPs, service frequencies, skipped stops by express service, etc.) and model parameters (e.g. bus operating cost, user's value of time, bus capacity, bus speed, etc.) are investigated.

The optimized service frequencies (i.e., all-stop, short-turn and express) versus bus operating cost are illustrated in Figure 5.34. Increased bus operating costs encourage the short-turn SP (e.g. $SP_{3,6}$) but discourage the full route SPs (e.g., $SP_{1,6}$ and SP_E) to reduce the supplier cost.

Figure 5.35 shows that the VMT decreases as the bus operating cost increases due to the increase of short-turn services and the decrease of full route service frequencies to balance the increasing supplier cost.

Figure 5.36 shows how average passengers per bus (P_v) are affected by bus operating cost. P_v increases as the bus operating cost increases due to the decrease of VMT as well as fleet size. It is also found that average cost per VMT increases proportionally as the bus operating cost increases.

The impact on optimized service frequencies by varying the user's value of time from 5 to 50 $/bus-hr is illustrated in Figure 5.37. The full route service (i.e., all-stop and express) frequencies increase and short-turn service frequencies decrease as the user's value of time increases to minimize the passenger's wait, transfer and in-vehicle times.

Figure 5.38 shows how the VMT and LF are affected by the user's value of time. VMT increases as the user's value of time increases due to the decreased short-turn service frequencies and increased all-stop and express service frequencies. The increased VMT and fleet size reduce the average LF.

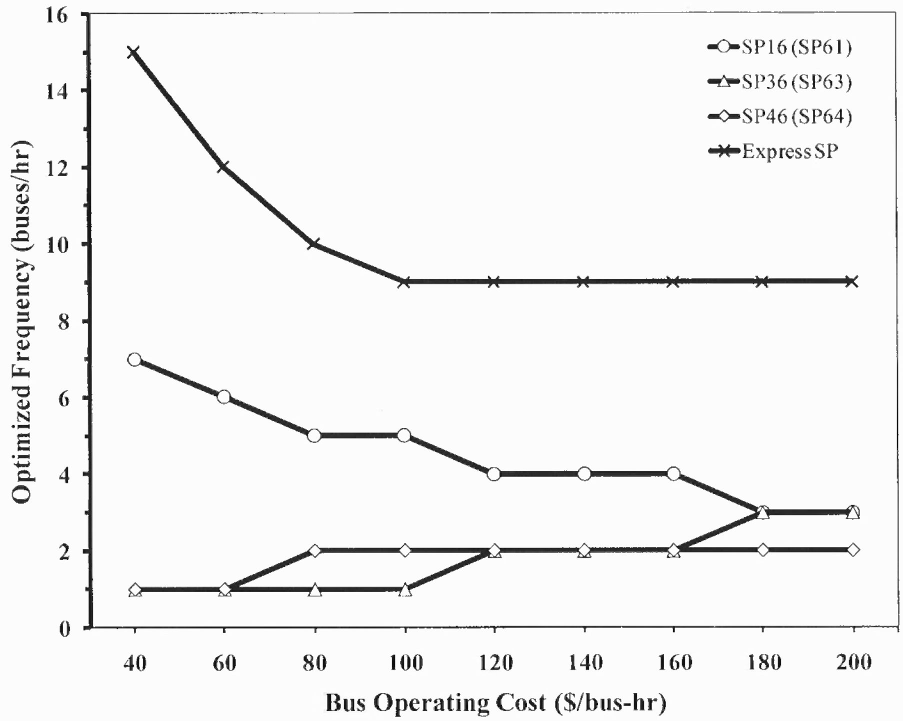

Figure 5.34 Optimized service frequency vs. bus operating cost in Scenario C.

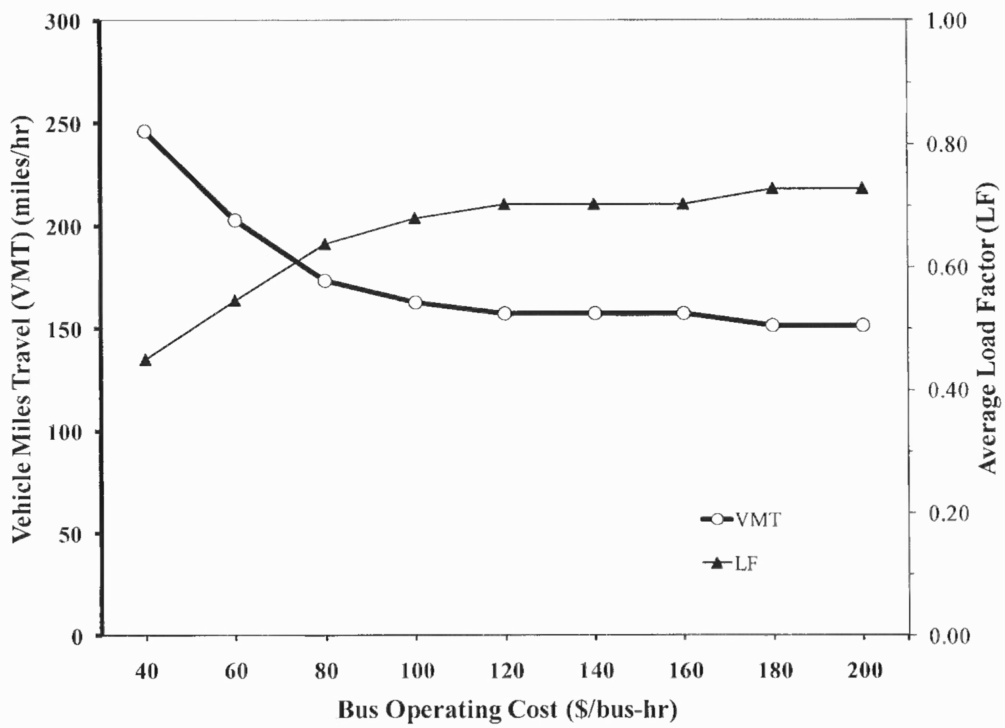

Figure 5.35 Vehicle miles travel and average load factor vs. bus operating cost in Scenario C.

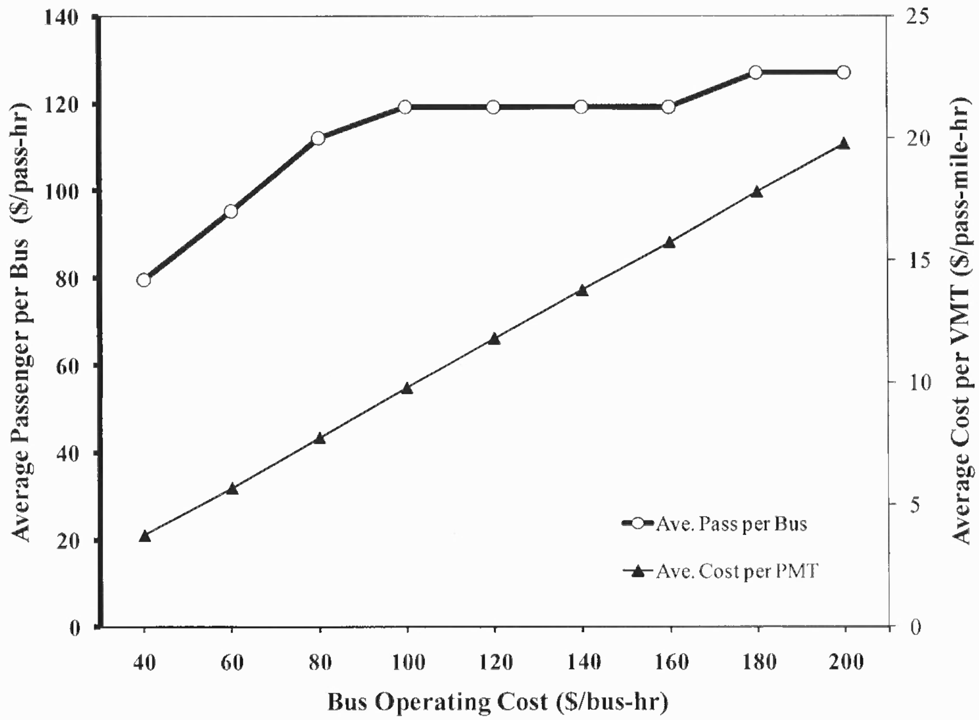

Figure 5.36 Average number of passenger per bus and average cost per VMT vs. bus operating cost in Scenario C.

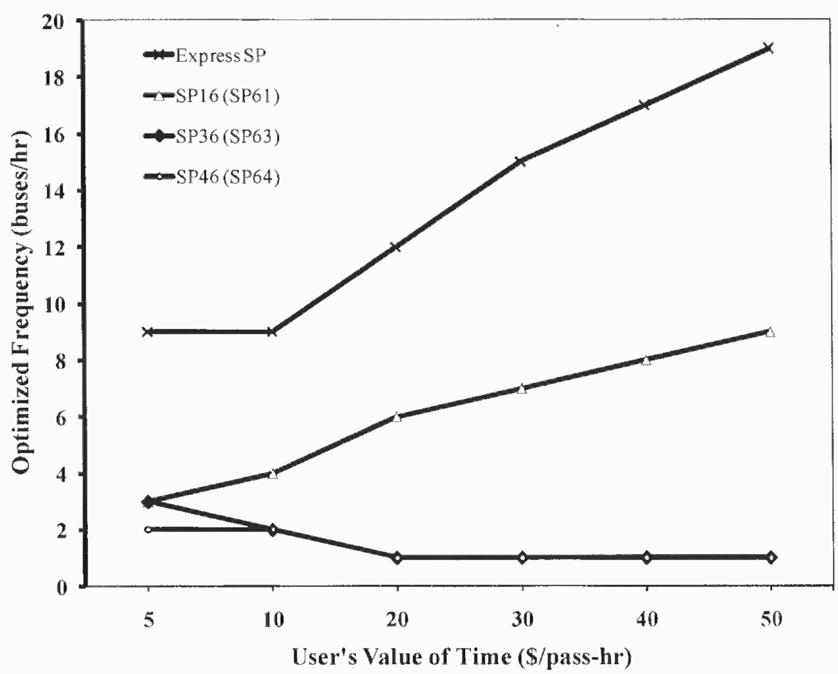

Figure 5.37 Optimized service frequency vs. user's value of time in Scenario C.

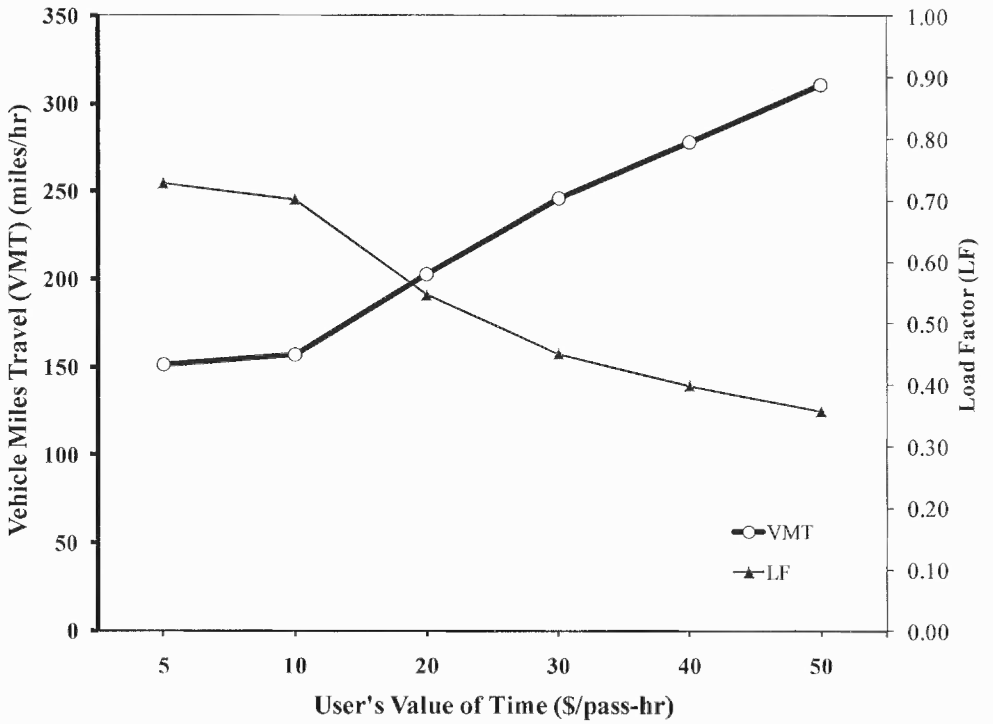

Figure 5.38 Vehicle miles travel and average load factor vs. user's value of time in Scenario C.

Figure 5.39 shows the calculated total wait and transfer time of passengers under various user's value of time. Increasing the user's value of time from 10 $/hr to 50 $/hr decreases both the total wait and transfer times by almost 32%.

Figure 5.40 and 5.41 illustrate the impact on P_v, P_{vm} and C_{vm}, C_{pm}, respectively by varying the user's value of time from 5 to 50 $/bus-hr. The average number of passengers per bus (P_v), and per vehicle miles traveled (P_{vm}) decrease as the user's value of time increase due to the increased VMT as previously showed in Figure 5.38. Figure 5.41 shows that C_{vm}, decreases as the user's value of time increases because VMT increases more than supplier cost. However C_{pm} increases as the user's value of time increases due to the fact that PMT remains fixed as the supplier cost increases.

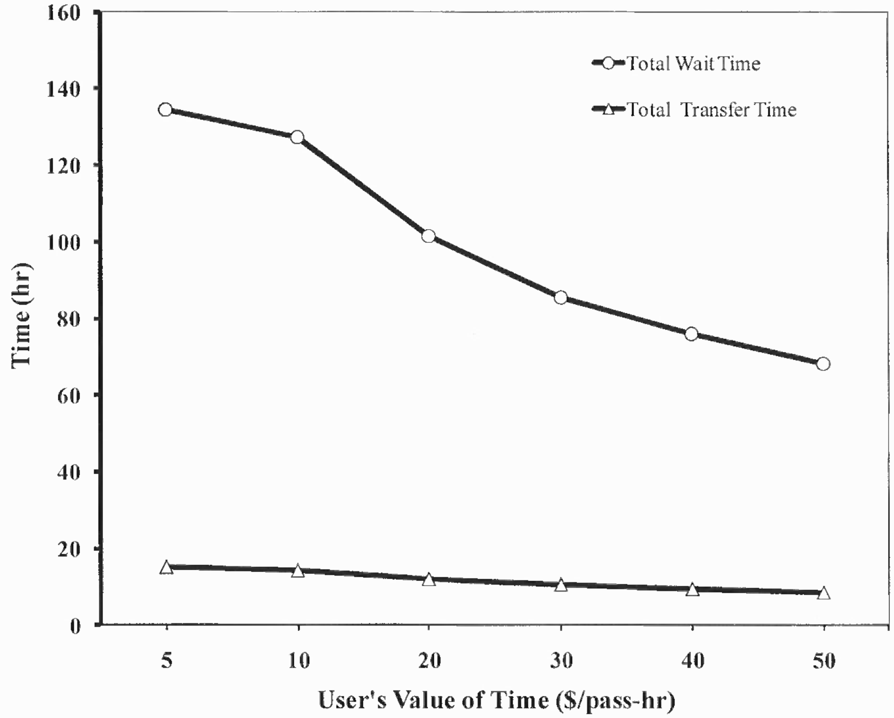

Figure 5.39 Total wait and transfer time of passengers vs. user's value of time in Scenario C.

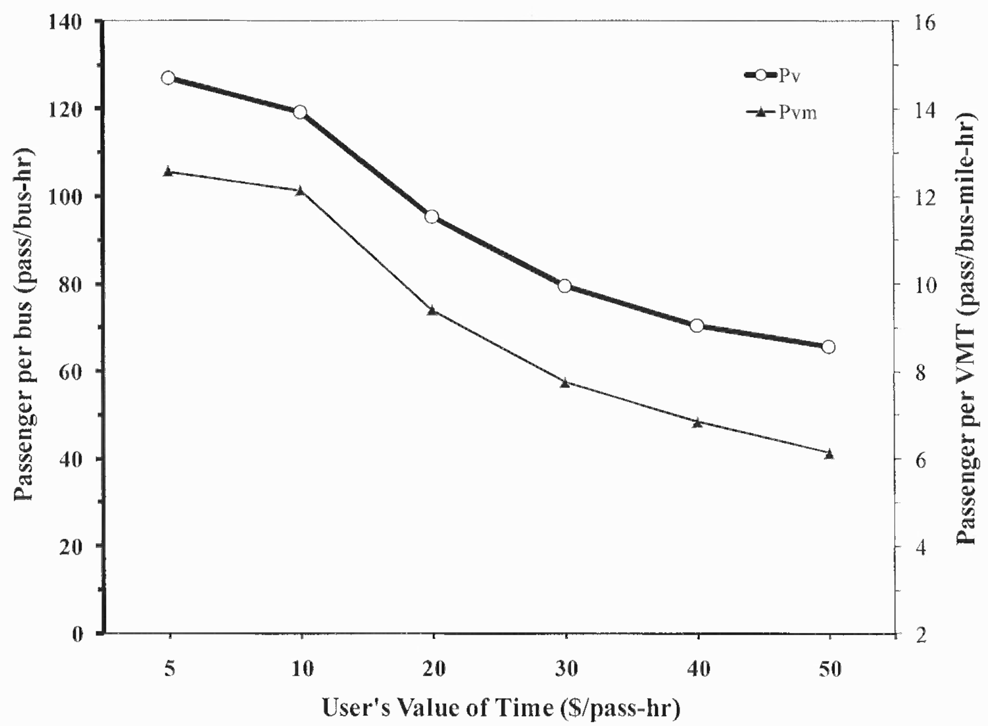

Figure 5.40 Average number of passengers per bus (P_v) and number of passengers per vehicle miles travel (P_{vm}) vs. user's value of time in Scenario C.

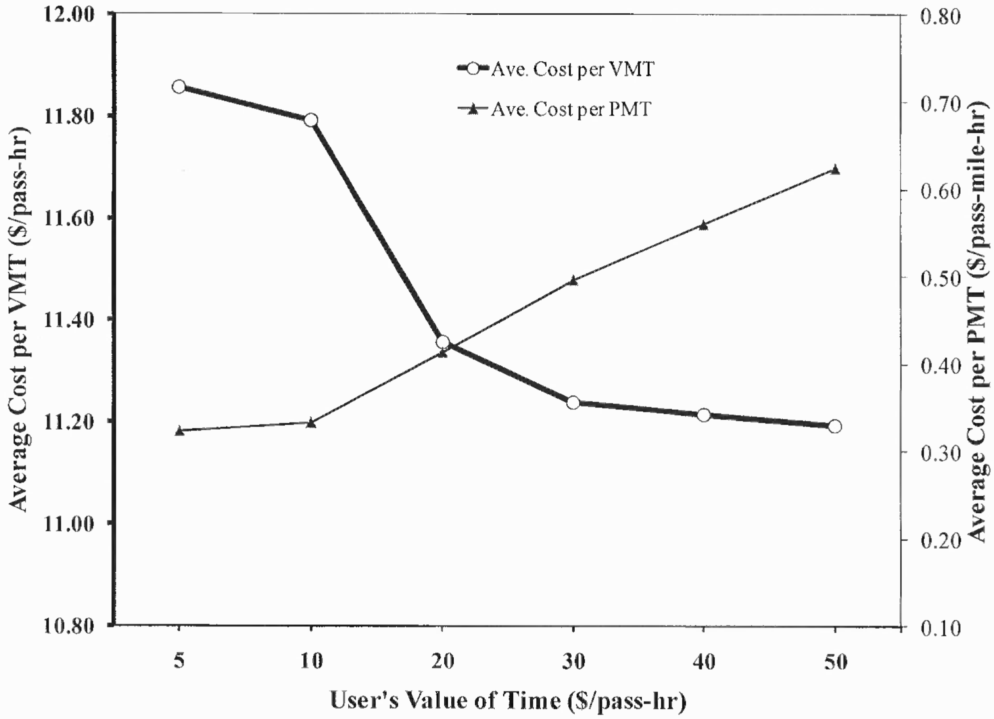

Figure 5.41 Average supplier cost per passenger and average supplier cost per passenger miles traveled vs. user's value of time in Scenario C.

To investigate the relationship between optimized all-stop, short-turn and express service frequencies and different demand, Figure 5.42 is developed, in which the demand multiplier varies from 20% to 140%. In general, all service frequencies increase to satisfy the capacity constraint. However, the express service frequency increases faster compared to the local (i.e., all-stop and short-turn) service frequencies.

The optimal bus capacity and associated service frequencies are illustrated in Figure 5.43. The minimum total costs are achieved at the optimal bus capacities while satisfying the given demand. It is found that the optimal bus capacity for the OD demand of the studied route is 76 pass/bus which yields a $f_{1,6}$ ($f_{6,1}$), $f_{3,6}$ ($f_{6,3}$) and f_E, of 4, 1 and 8 buses/hr, respectively with a minimum total cost of 7,249 $/hr.

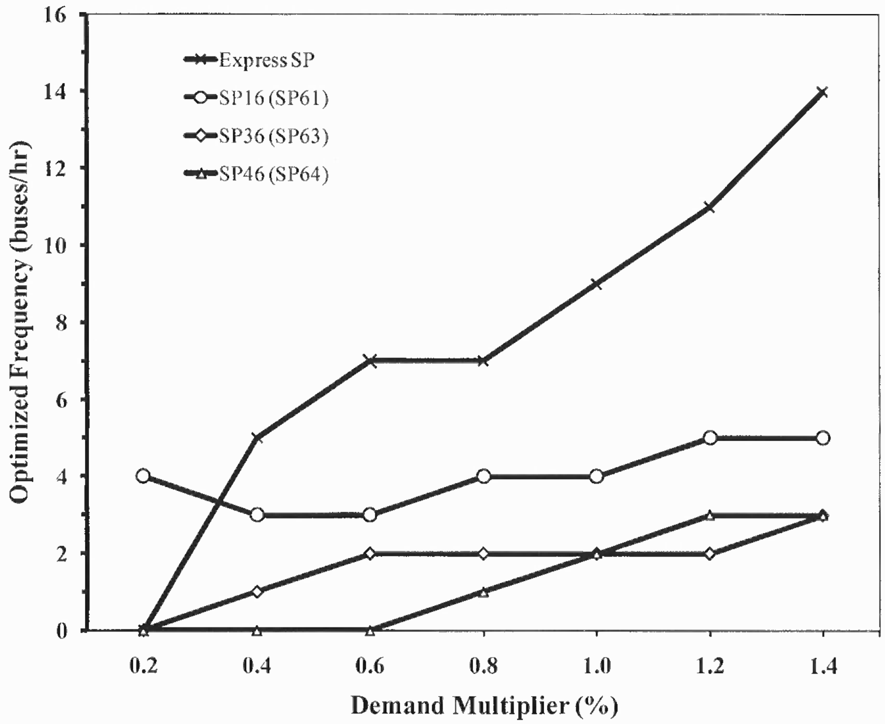

Figure 5.42 Optimized frequencies vs. demand multiplier in Scenario C.

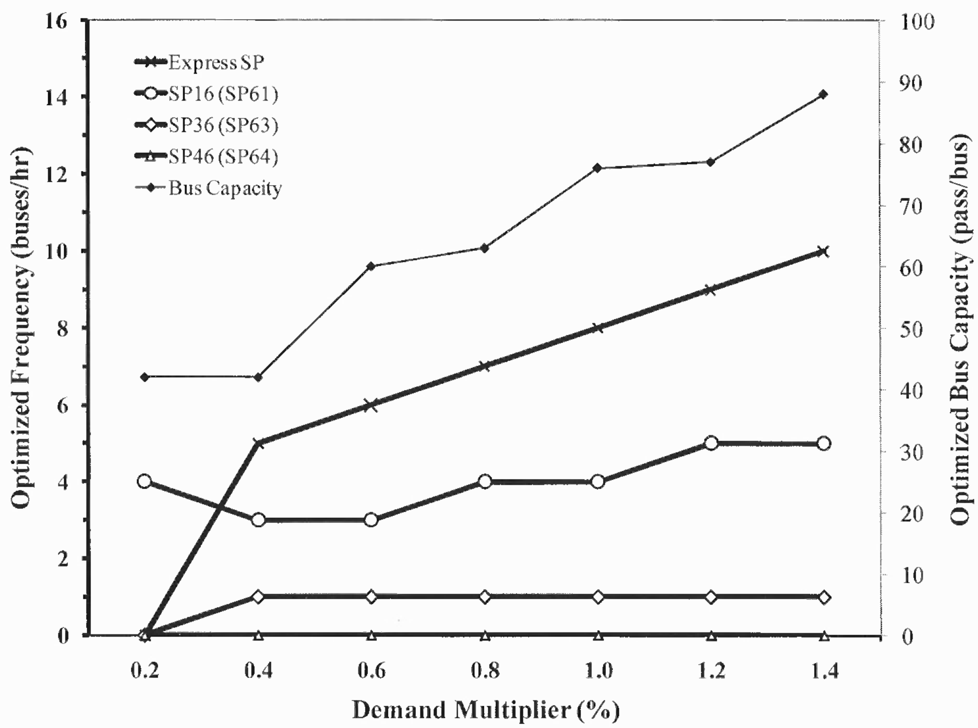

Figure 5.43 Optimized bus capacity and relevant optimized service frequencies vs. demand multiplier in Scenario C.

As shown in Figures 5.44 and 5.45, as bus capacity increases, optimized service frequencies of all-stop, short-turn and express SPs decrease and a lower supplier cost as well as total cost may be expected. However increasing bus capacity may increase the headway (or reduce the optimized service frequency), which increases passenger wait time. It is also found that bus capacities over 76 pass/bus yield the same minimum total cost of 7,249 $/hr.

Figure 5.46 shows the impact of bus capacity on the optimal VMT and average LF. Decreasing bus capacity increases the optimal VMT because more frequent service is needed to satisfy the route demand. It is found that the highest average LFs are obtained at the smaller bus capacities and increasing bus capacity decreases the LF.

Figure 5.47 shows the impact of bus capacity on average costs. Average costs decrease as bus capacity increases. Minimum average cost per VMT and per PMT is achieved as 11.28 $/bus-mile and 0.27 $/pass-mile, respectively if the bus capacity is over 76 pass/bus. Note that for the analysis of bus capacity illustrated in Figures 4.43 through 4.47, the unit bus operating cost is assumed constant regardless of vehicle capacity.

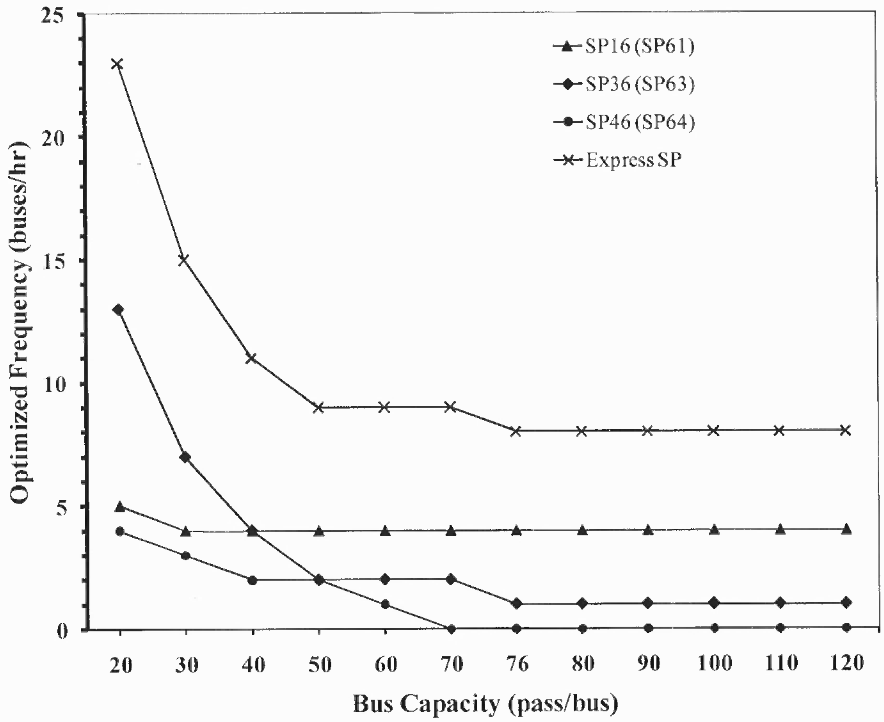

Figure 5.44 Optimized service frequency vs. bus capacity in Scenario C.

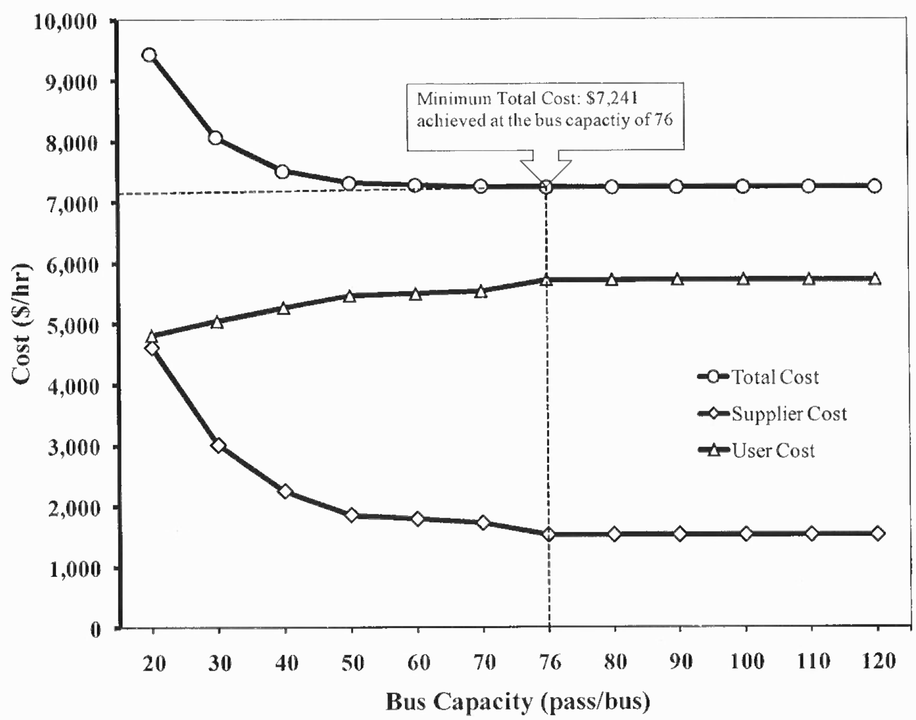

Figure 5.45 Costs vs. bus capacity in Scenario C.

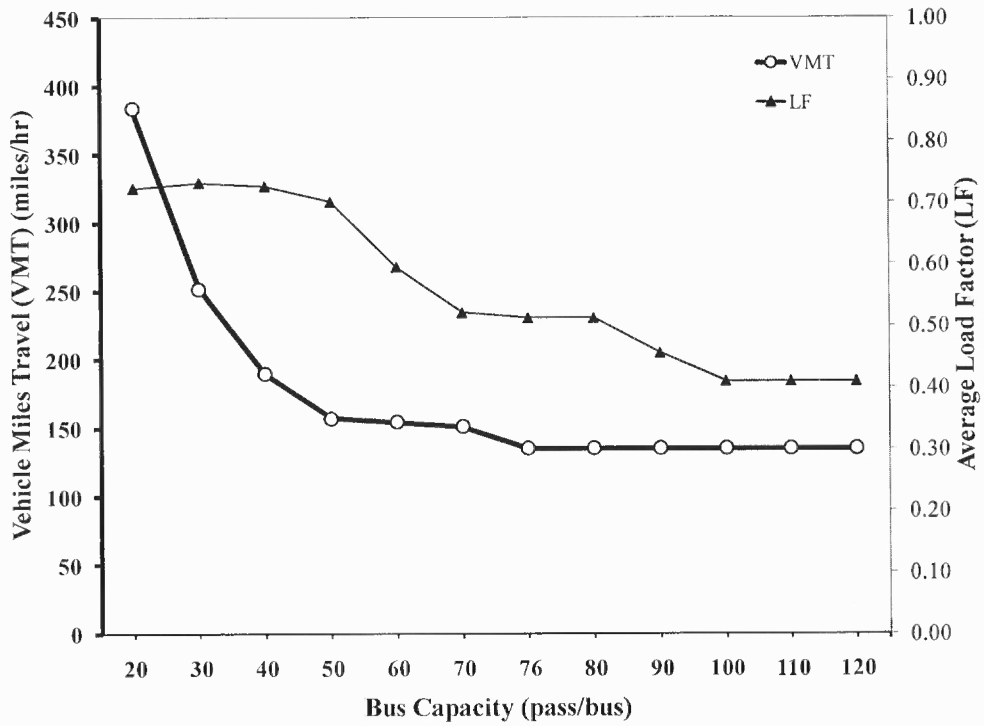

Figure 5.46 Vehicle miles of travel and load factor vs. bus capacity in Scenario C.

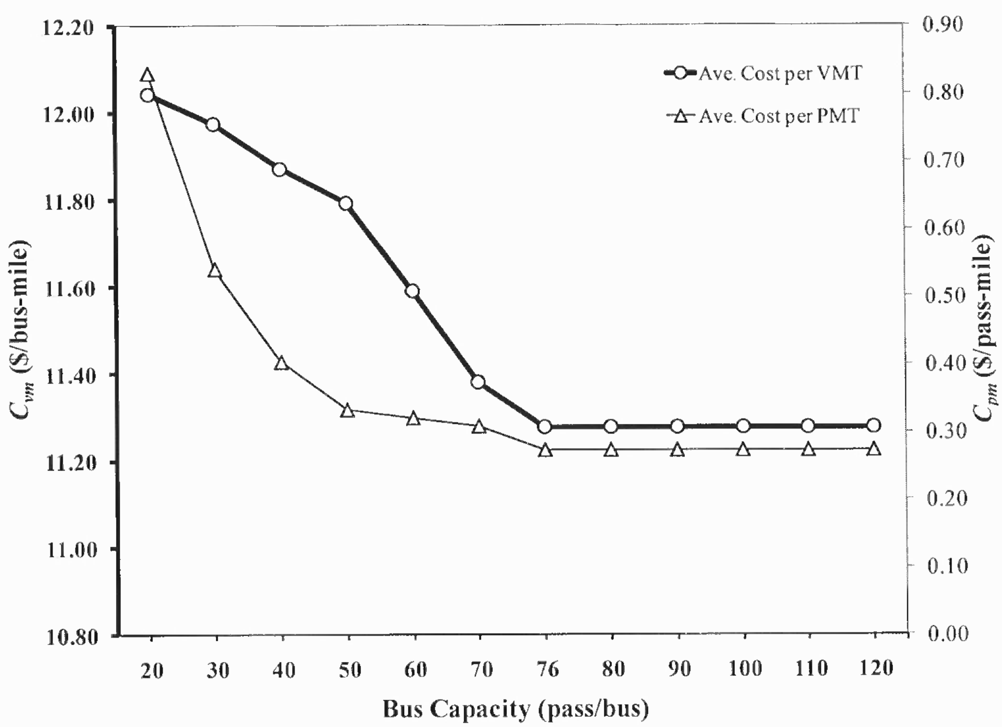

Figure 5.47 Average cost per VMT and per PMT vs. bus capacity in Scenario C.

The impacts of average bus speed on optimized frequencies, optimal VMT and average passenger travel time, and on cost components (i.e., user and supplier cost), are demonstrated in Figures 5.48, 5.49 and 5.50, respectively. In Scenario C, increasing the bus speed encourages the full length SPs which yields increased VMT. As shown in Figure 5.49, faster buses decrease the average travel time of passengers which yields a lower user costs. The bus round trip travel time is reduced because of the increasing bus operating speed, which yields a lower supplier cost.

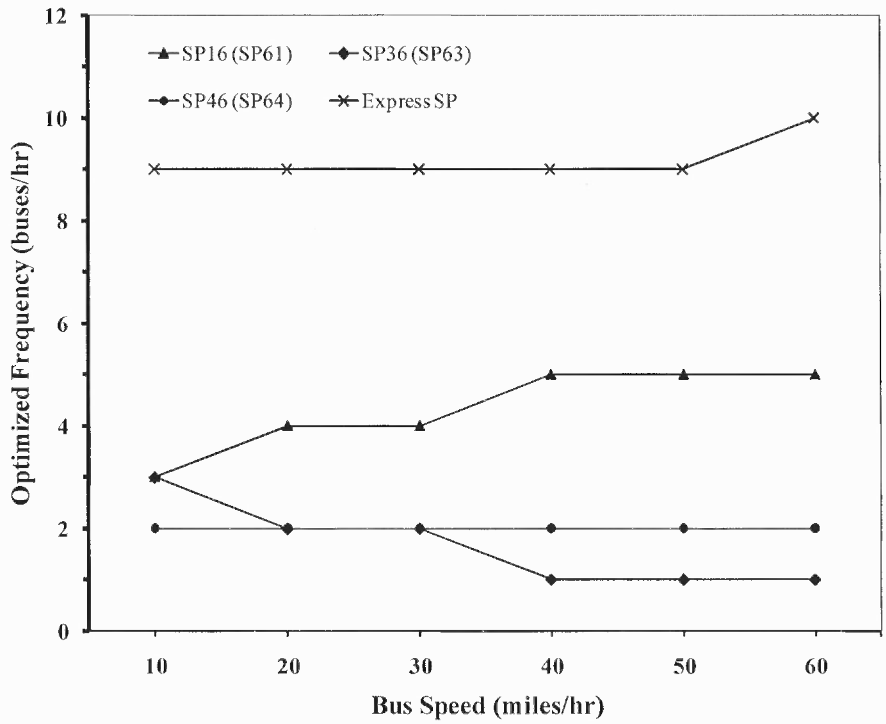

Figure 5.48 Optimized service frequencies vs. bus speed in Scenario C.

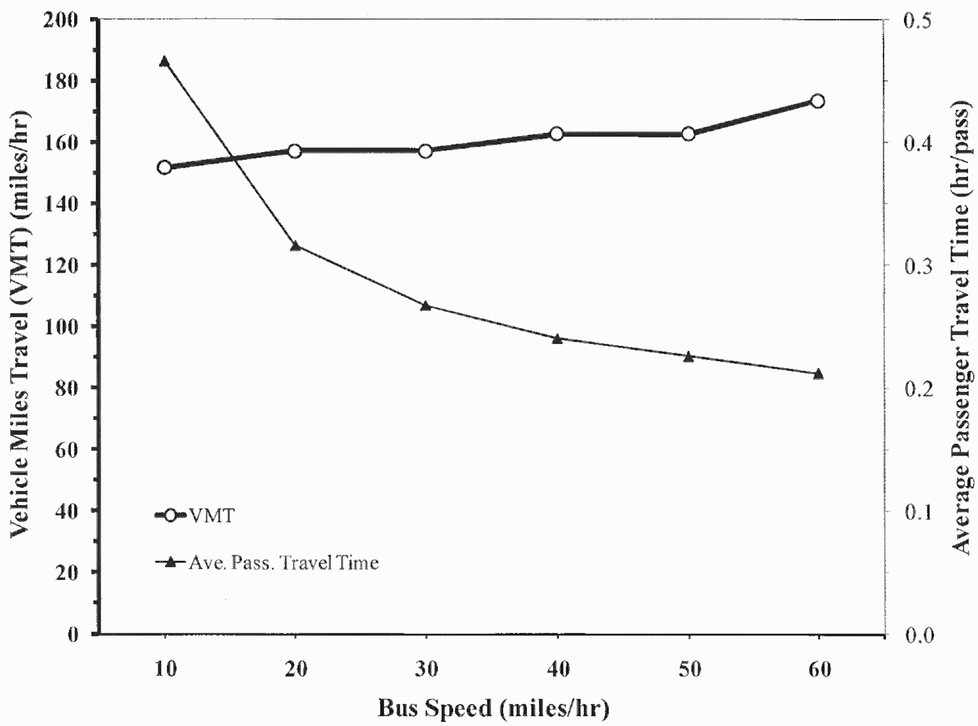

Figure 5.49 Vehicle miles travel and average passenger travel time vs. bus speed in Scenario C.

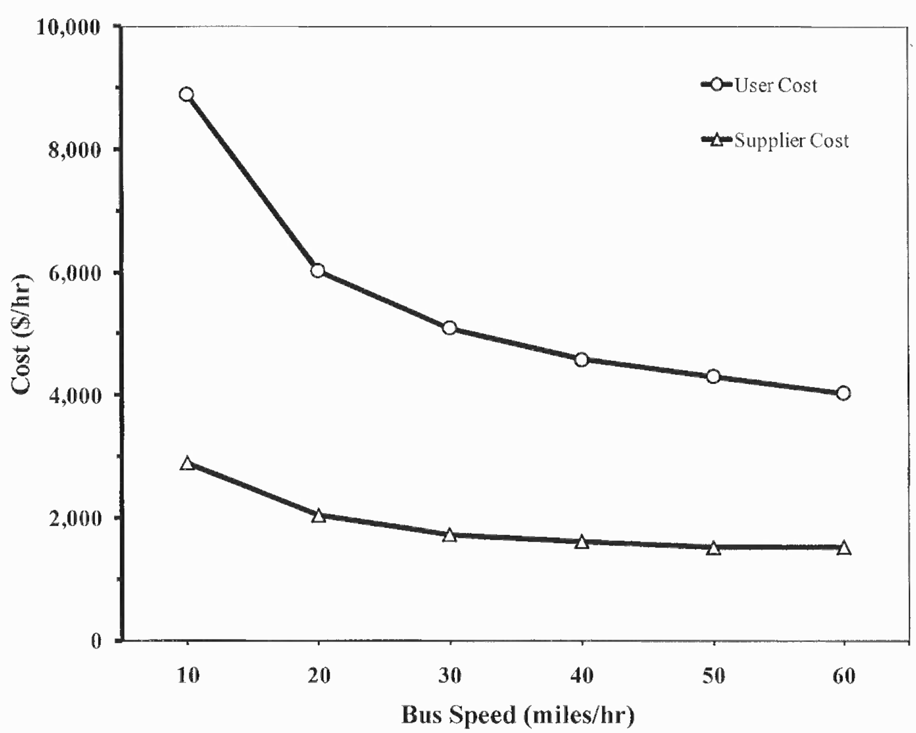

Figure 5.50 User and supplier costs vs. bus speed in Scenario C.

One of the decision variables in Scenario C is the skipped stops by the SP_E. Table 5.7 shows the impact on all costs and the optimized service frequencies by varying the stops skipped by SP_E. By comparing total and in-vehicle costs of a local (1^{st} row) and a SP_E configuration serving Stops 1 and 6 (2^{nd} row), total cost and in-vehicle cost are about 409 \$/hr and 930 \$/hr higher than those without a SP_E configuration. However, wait and supplier costs are 275 and 246 \$/hr less because of reduced frequencies in inner stops and the increased fleet size needed for the express service.

Similarly, different configurations of served stops by express service (e.g., Stops 1, 3, 6; Stops 1, 4, 6; Stops 1, 3, 4, 6; etc.) are evaluated, and optimized results are given in Table 5.7. Among all alternatives, it was found that the optimal integrated service (3^{rd} row) achieves the lowest total cost because the lowest values of in-vehicle and supplier costs are obtained in this configuration.

Table 5.7 Optimized Services with Alternative Skipped Stops

Stops Served by Express Service (stops)	Transfer Stops (stops)	Optimized SPs (Frequency) (buses/hr)	Wait Cost ($/hr)	Transfer Cost ($/hr)	In-Vehicle Cost ($/hr)	Supplier Cost ($/hr)	Total Cost ($/hr)
NA	-	$SP_{1,6}$-$SP_{6,1}$ (10) $SP_{3,6}$-$SP_{6,3}$ (5) $SP_{4,6}$–$SP_{6,4}$ (2)	769	0	5,508	2,077	8,354
1, 6	-	$SP_{1,6}$-$SP_{6,1}$ (6) $SP_{3,6}$-$SP_{6,3}$ (6) $SP_{4,6}$-$SP_{6,4}$ (1) SP_E (9)	1,044	0	4,578	2,323	7,945
1, 3 and 6	**3**	**$SP_{1,6}$-$SP_{6,1}$ (4)** **$SP_{3,6}$-$SP_{6,3}$ (2)** **$SP_{4,6}$-$SP_{6,4}$ (2)** **SP_E (9)**	**1,275**	**143**	**4,045**	**1,854**	**7,317**
1, 4 and 6	4	$SP_{1,6}$-$SP_{6,1}$ (7) $SP_{3,6}$-$SP_{6,3}$ (6) SP_E (6)	1,121	88	4,597	2219	8,025
1, 2, 3 and 6	2, 3	$SP_{1,6}$-$SP_{6,1}$ (4) $SP_{3,6}$-$SP_{6,3}$ (2) $SP_{4,6}$-$SP_{6,4}$ (2) SP_E (9)	1,198	167	4,464	1,983	7,812
1, 3, 4 and 6	3, 4	$SP_{1,6}$-$SP_{6,1}$ (4) $SP_{3,6}$-$SP_{6,3}$ (2) SP_E (12)	1,037	201	4,496	2,205	7,939
1, 3, 5 and 6	3, 5	$SP_{1,6}$-$SP_{6,1}$ (4) $SP_{3,6}$-$SP_{6,3}$ (1) SP_E (13)	1060	78	4,406	2232	7,776

5.1.7 Results Comparison

The optimal results of decision variables (e.g., SPs, frequencies, etc.), minimized costs (e.g., wait, transfer, in-vehicle, supplier costs, etc.), and associated performance measures (e.g., fleet size, load factor, P_v, P_{vm}, V_a, VMT_a, C_p, C_{pm}, T_a, etc.) obtained from previous sections in Scenarios A, B and C are compared and discussed in this section. The optimal results for service frequencies and minimized costs are shown in Table 5.8.

Table 5.8 Optimized Solutions and Minimized Costs of Scenarios A, B and C

Parameters	Units	Scenario A	Scenario B	Scenario C
$f_{1,3}(f_{3,1})$	buses/hr	N/A	0	0
$f_{1,4}(f_{4,1})$	buses/hr	N/A	0	0
$f_{1,6}(f_{6,1})$	buses/hr	17	10	4
$f_{3,4}(f_{4,3})$	buses/hr	N/A	0	0
$f_{3,6}(f_{6,3})$	buses/hr	N/A	5	2
$f_{4,6}(f_{6,4})$	buses/hr	N/A	2	2
f_{E}	buses/hr	N/A	N/A	9
C_W	\$/hr	561	769	1,275
C_R	\$/hr	0	0	143
C_I	\$/hr	5,508	5,508	4,045
C_O	\$/hr	2,513	2,077	1,854
Total Cost	**\$/hr**	**8,582**	**8,354**	**7,317**

N/A: Not applicable.

In Scenario A, the optimized service frequency of 17 buses/hr yields a total cost of 8,532 \$/hr and comparing Scenario B to Scenario A, two short-turn SPs (e.g., SP_{36} (SP_{63}) and SP_{46} (SP_{64}) reduces the total cost about 2.7% (228 \$/hr). The major benefit of introducing the short-turn SP is the reduction of the supplier cost by almost 8% (436 \$/hr). However, the wait cost of passenger is increased from 561 to 769 \$/hr due to the decreased service frequencies in some segments of the route.

Comparing the optimal results of Scenarios B and C, introducing the SP_E to the study route reduces the in-vehicle cost of passengers from 5,508 to 4,045 \$/hr and the supplier cost from 2,077 to 1,854 \$/hr. However some passengers may have the option to transfer, which generates a transfer cost of 143 \$/hr. Also the wait cost of passengers is raised to 1,275 \$/hr because the express service skips some of the stops and it may not be available to all passengers.

The performance measurers under Scenarios A, B and C are presented in Table 5.9. The minimum total cost of 7,317 \$/hr is achieved in Scenario C. It indicates that the optimal integrated all-stop, short-turn, and express service is recommended to achieve a minimum cost operation. Scenario C yields the lowest average costs per passenger (0.97 \$/pass), per passenger miles traveled (0.33 \$/pass-mile), and per vehicle miles traveled (11.79 \$/bus-mile)

Comparing the results of Scenario B to that of Scenario A, VMT is reduced from 184 to 141 miles/hr due to the encouragement of short-turn services instead of an all-stop service. However, introducing the express service in Scenario C increases the VMT to 157 miles/hr because of the replacement of some short-turn services with express service which serves the full length of the route.

The express service also increases the average route speed (V_a) in Scenario C because the stop delays are eliminated by the express buses at the skipped stops (e.g., 2, 4 and 5). Thus, V_a of Scenario C is 1.98 and 2.49 mph higher than V_a of Scenario A and Scenario B, respectively.

Due to the decrease of VMT of Scenario B in comparison with Scenario A, P_{vm} increases from 10.38 to 13.55 pass/bus-mile. As discussed earlier, VMT is increased in Scenario C (comparing to Scenario B) due to the integration of express service which decreases P_{vm} to 12.12 pass/bus-mile. However, the maximum value of 119 passengers per bus (P_v) occurs in Scenario C due to the lowest optimum fleet size of 16 buses.

Introducing the short-turn service into the all-stop only service may slightly increase the average passenger travel time (T_a) from 0.32 to 0.33 hr/pass because of the increased wait time of passengers in some parts of the route where the short-turn SPs are not available. However, integrating the express service to the study route may significantly reduce the T_a (0.29 hr/pass) due to the eliminated stop delays at skipped stops.

Table 5.9 Performance Measures under Optimal Operation of Scenarios A, B and C

Parameters	Units	Scenario A	Scenario B	Scenario C
Outbound LF	-	0.68	0.89	0.79
Inbound LF	-	0.53	0.70	0.62
Average LF	-	0.61	0.79	0.71
Fleet Size	buses	21	18	16
VMT	bus-mile/hr	184	141	157
PMT	pass-mile/hr	5574	5574	5574
V_a	mph	8.77	8.26	10.75
VMT_a	miles/bus	8.74	7.82	9.83
P_{vm}	pass/bus-mile	10.38	13.55	12.12
P_v	pass/bus	91	106	119
C_p	$/pass	1.32	1.09	0.97
C_{pm}	$/pass-mile	0.45	0.37	0.33
C_{vm}	$/bus-mile	13.69	14.76	11.79
T_a	hr/pass	0.32	0.33	0.29

The LFs at each link for both outbound and inbound services between Stops 1 and 6 in Scenarios A, B and C are illustrated in Figure 5.51. Comparing to Scenario A, the average LFs in Scenario B increased 30% for the outbound and 32% for the inbound directions because short-turn SPs serve the high demand segments of the route and some all-stop services are eliminated which decrease the number of buses serving the full length of the route.

Compared to Scenario B, the average LFs in Scenario C decreased by 13% at both outbound and inbound directions because integrating the express service increases the number of buses serving the full length of the route.

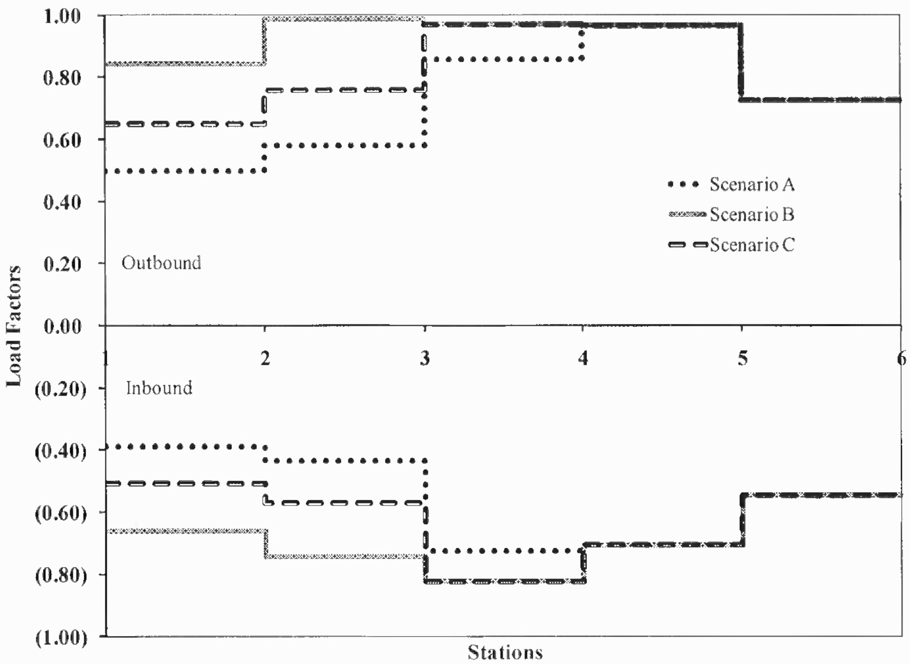

Figure 5.51 Load factors under optimal operation in Scenarios A, B and C.

It was found that the average LFs in Scenario A are less than those in Scenario B between Stops 1 and 4 because the available service in Scenario A requires all buses to serve the full length of the route and short-turn SPs ($SP_{3,6}$ and $SP_{4,6}$) in Scenario B balance the loads by serving the high demand segments.

The average LFs between Stops 1 and 3 in Scenario C are less than those in Scenario B because the total number of service frequencies between these stops is almost 30% more in Scenario C compared to Scenario B.

5.1.8 Number of Decision Variables and Solution Algorithms

The number of decision variables in Scenarios B and C can be estimated based on the equations summarized in Table 5.10, which increase exponentially in Scenario C as the number of stops increases. Note that *n* and *g* represent the total number of stops and the total number of eligible turn-back stops, respectively.

Table 5.10 Number of Decision Variables in Three Scenarios

	Scenario A	**Scenario B**	**Scenario C**
Total Number of Decision Variables	1	$\binom{g}{2}$	$\left(2^{(n-2)}-1\right)\binom{g}{2}$

As shown in Figure 5.52, it can be found that more than 10,000 decision variables, consisting of the combinations of SPs, service frequencies and configurations of stops skipped by express service need to be generated when the number of stops is 10. It is worth to note that, increasing the stops from 10 to 15 (i.e., 50% increase), increases the decision variables from 10,000 to 1,000,000 (i.e., 10,000% increase).

All intermediate stops on the studied route may not be eligible as a turn-back point for short-turn services due to the location and demand limitations of the stop. Eligible turn-back stops may be pre-determined by the transit supplier as initial input parameters in the model and the developed model may optimize the turn back stops among these stops. This process may significantly reduce the total number of decision variables as demonstrated in Figure 5.53.

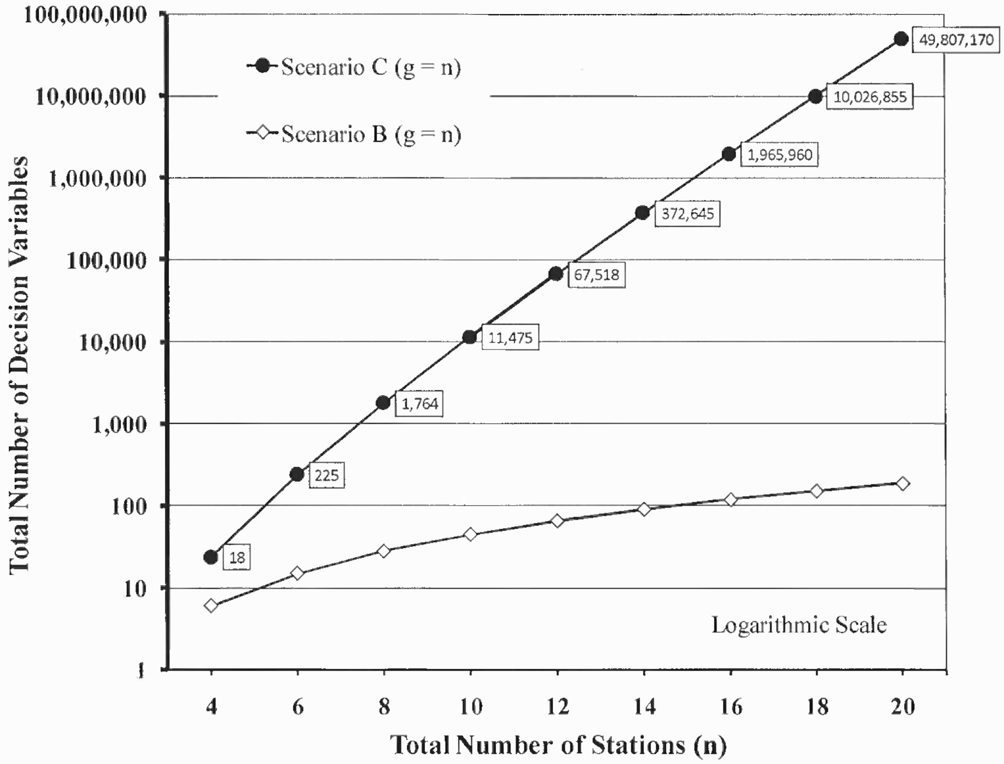

Figure 5.52 Number of decision variables vs. number of stations.

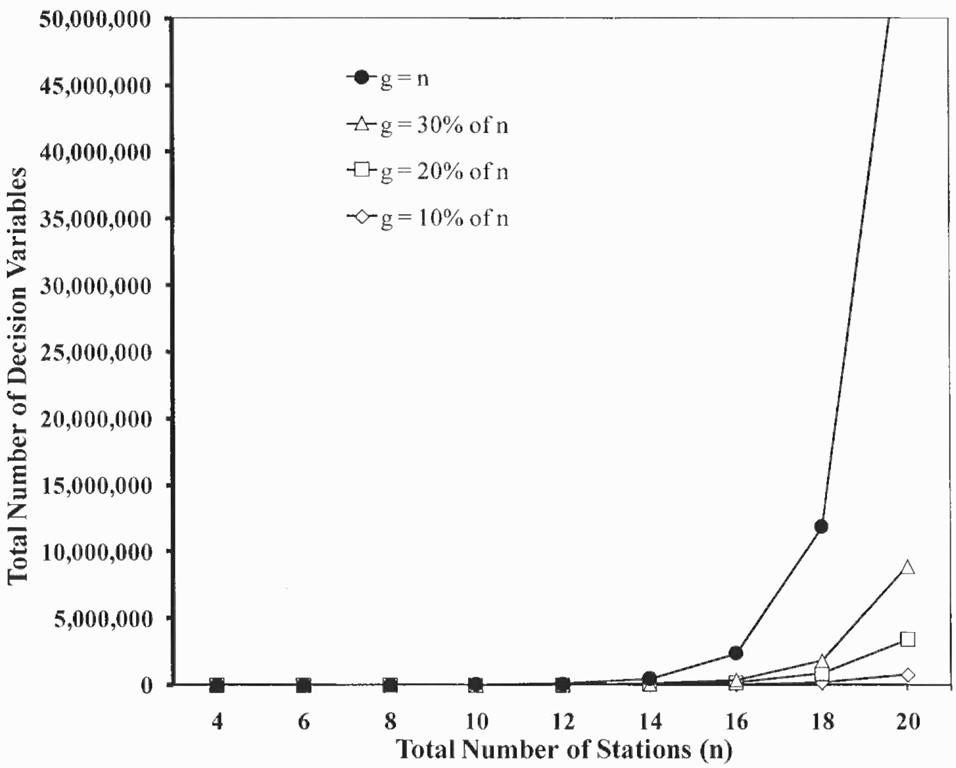

Figure 5.53 Number of decision variables vs. number of eligible turn-back stops.

The total number of decision variables is 15 for Case I (6-stop bus route with two intermediate turn-back stops) in Scenario C. The developed exhaustive search algorithm computes the optimum solution in about 14 minutes with an Intel Core 2 Duo 2.4 MHz processor and 4GB Ram. The total number of decision variables is 11,475 for a 10-stop bus route (see Figure 5.52) considering all intermediate stops are eligible as turn back points which yields a computation time of more than 7 days and 11 hours to find the optimum solution. Due to the exponential increase of the decision variables the calculation time becomes unreasonable as the number of stops increases. Therefore, a heuristic approach is required to handle Case II (69-stop bus route) which is optimized using a Genetic Algorithm (GA). The developed GA optimizes the 69-stop route (number of decision variable is about 4.13E+21) in 14 hours.

To verify the reliability of the developed GA, Model II is optimized with 30 different runs for Scenario C using Case I. The minimized total costs achieved by the GA range from 10,124 to 10,952 $/hr as illustrated in Figure 5.54. The dotted line in Figure 5.54 represents the global optimum solution which is obtained from the exhaustive search algorithm. All 30 samples are in the 8% range of the global optimum solution, and most of the samples (23 out of 30) are within a 3% range of the global optimum solution.

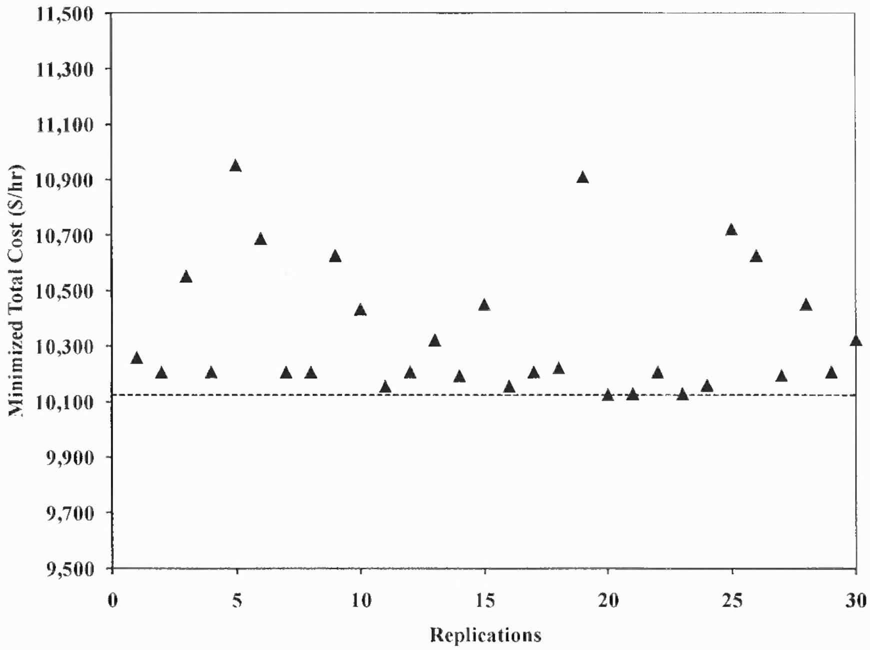

Figure 5.54 Minimized total costs using GA over 30 separate runs for Case I.

5.2 Case II – New Jersey Transit Springfield Avenue Line

The purpose of this section intends to demonstrate the applicability and effectiveness of the developed Model II and GA for a large scale bus route with many stops.

The New Jersey Transit Springfield Avenue Line is approximately 10 miles long serving 71 stops as shown in Figure 5.55. It connects Maplewood, Irvington, and Newark along Springfield Avenue. Detailed stop information such as stop numbers and stop spacing is given in Appendix B. It is worth to note that the bus line branches out at three different locations (Essex County Correction Center, NJ Transit Bus complex and Firmenich Way) at the Newark side of the route as shown in Figure 5.55 and these three locations are assumed to be one stop. Thus, the Springfield Avenue Line is adopted into the developed model as a 69-stop bus route. The peak OD demand matrix for the bus line is estimated using on-off NJ Transit on-site survey data and is presented in Appendix C.

Eight intermediate stops (i.e., stops 12, 20, 34, 38, 42, 46, 50 and 53) are selected as eligible turn-back points for short-turn service, if there is a need. The average bus travel speed is 25 miles/hr with capacity of 50 pass/bus and hourly operating cost of 120 \$/bus-hr. The factors used to approximate wait and transfer times, denoted as α_w and α_R, are identical and equal to 0.5, while the user's value of time is assumed to be 10 \$/pass-hr. The operable fleet size for the studied route is 48 buses, which will act as the upper bound of the optimal fleet size for minimum cost operation. All baseline values of the model parameters are summarized in Table 5.11.

5.2.1 All-Stop SP – Scenario A

Scenario A is considered in Case II, in which a single SP serving every stop from the beginning to the end of the study bus route, denoted as $SP_{1,69}(SP_{69,1})$. Based on the OD demand given in Appendix C, the optimized service frequency is 15 bus/hr that achieves the minimum cost of 14,621 \$/hr, and the associated measures (e.g., fleet size, VMT, LF, C_p, C_{pm}, C_{vm} and cost components, etc.) are calculated and shown in Table 5.12.

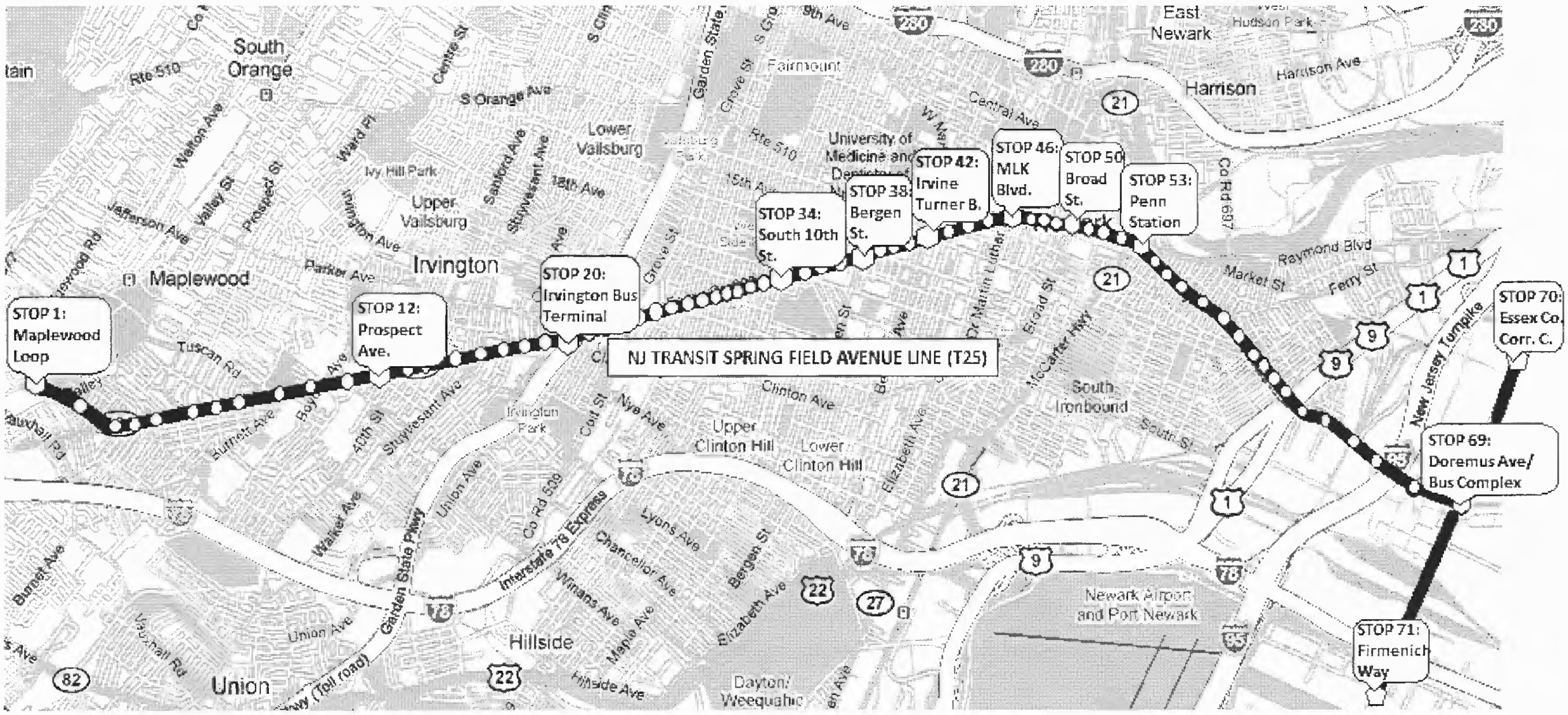

Figure 5.55 Configuration of the NJ Transit Springfield Avenue Line.

Table 5.11 Baseline Values of Model Parameters in Case II

Parameters	Descriptions	Baseline Values
b	Bus operating cost	120 \$/bus-hr
C	Bus capacity	50 pass./bus
n	Total number of stops	69 stops
F	Maximum operable fleet size	48 bus
$f_{i,j}$	Frequency of all-stop and short turn SP from stop *i* to *j*	To be determined
f_E	Frequency of express SP	To be determined
l_d	Distance from stop *d* to *d+1*	See Appendix B
g	Eligible stops for turn-back	1,12,20,34,38, 42,46,50,53,69
$q_{i,j}$	Demand from stop *i* to *j*	See Appendix C
t_o	Layover time at the end stop	0.2 hr
v_d	Average vehicle speed from stop *d* to *d+1*	25 miles/hr
θ_I	Demand parameter associated with in-vehicle time	1.33
θ_R	Demand parameter associated with transfer time	2.00
θ_W	Demand parameter associated with wait time	2.00
w_d	Average stop delay including dwell time at stop *d*	0.015 hr
α_R	Ratio of the average transfer time to headway	0.5
α_w	Ratio of the average wait time to headway	0.5
μ	User's value of time	10 \$/pass-hr
P	Population size	250
r_S	Selection ratio in GA	0.3
r_X	Crossover ratio in GA	0.3
r_M	Mutation ratio in GA	0.3

The performance measures defined and formulated in Chapter 3 are used here to evaluate the system performance under Scenario A. PMT and VMT are 4860 pass-miles/hr and 274 bus-miles/hr, respectively. The average speed of the route (V_a) is 5.75 mph, which yields an average passenger travel time (T_a) of 0.5 hr/pass. The VMT_a and P_{vm} are 5.70 miles/bus and 6.52 pass/bus-mile, respectively. The average costs per passenger, per PMT and per VMT are 3.20 \$/pass, 1.17 \$/pass-mile and 20.85 \$/bus-mile, respectively. The average LFs of outbound and inbound services are 0.44 and 0.27, respectively, which yield an average route LF of 0.36.

Table 5.12 Optimal Results of Scenario A in Case II

Variable	**Units**	**Optimal Values**
$f_{1,69}$ ($f_{69,1}$)	bus/hr	15
Outbound LF	-	0.44
Inbound LF	-	0.27
Average LF	-	0.36
Fleet Size	buses	48
VMT	bus-mile/hr	274
PMT	pass-mile/hr	4860
V_a	mph	5.75
VMT_a	miles/bus	5.70
P_{vm}	pass/bus-mile	6.52
P_v	pass/bus	37
C_p	$/pass	3.20
C_{pm}	$/pass-mile	1.17
C_{vm}	$/bus-mile	20.85
T_a	hr/pass	0.50
C_W	$/hr	595
C_R	$/hr	0
C_I	$/hr	8,321
C_O	$/hr	5,705
Total Cost	**$/hr**	**14,621**

5.2.2 All-Stop and Short-Turn SPs – Scenario B

In addition to the all-stop SP, short-turn SPs are introduced in this section. Eight intermediate stops (i.e., Stops 12, 20, 34, 38, 42, 46, 50 and 53) are eligible as the turn-back stops of the short-turn services. Therefore, 44 different short-turn SPs are available for Case II. Based on the OD demand illustrated in Appendix C, the optimized service frequencies of $SP_{1,69}$ ($SP_{69,1}$), $SP_{12,53}$ ($SP_{53,12}$), $SP_{12,69}$ ($SP_{69,12}$), $SP_{20,38}$ ($SP_{38,20}$) and $SP_{20,46}$ ($SP_{46,20}$), are 4, 5, 2, 1 and 3 buses/hr respectively, and 0 buses/hr for the rest of the SPs. This operation achieves the minimum total cost of 13,525 \$/hr. The associated measures (e.g., fleet size, VMT, average LFs, etc.) and all cost components derived from the optimized frequencies are calculated and shown in Table 5.13.

The performance measures are used to evaluate the system performance under Scenario B. The passenger miles traveled (PMT) and vehicle miles traveled (VMT) are 4,860 pass-mile/hr and 168 bus-miles/hr, respectively. The average route speed (V_a) is 5.14 mph, which yields an average passenger travel time (T_a) of 0.54 hr/pass.

The efficiency of bus usage (VMT_a) and intensity of bus service usage (P_{vm}) are 4.94 miles/bus and 10.63 pass/bus-mile, respectively. The average cost per passenger (C_p), per PMT (C_{pm}) and per VMT (C_{vm}), are 2.24 \$/pass, 0.82 \$/pass-miles and 23.82 \$/bus-mile, respectively. The average LF's of outbound and inbound services are 0.64 and 0.44, respectively, with an average LF of 0.54 for Case II.

Table 5.13 Optimal Results of Scenario B in Case II

Variables	**Units**	**Optimal Values**
$f_{1,69}$ ($f_{69,1}$)	buses/hr	4
$f_{12,53}$ ($f_{53,12}$)	buses/hr	5
$f_{12,69}$ ($f_{69,12}$)	buses/hr	2
$f_{20,38}$ ($f_{38,20}$)	buses/hr	1
$f_{20,46}$ ($f_{46,20}$)	buses/hr	3
Outbound LF	-	0.64
Inbound LF	-	0.44
Average LF	-	0.54
Fleet Size	buses	34
VMT	bus-mile/hr	168
PMT	pass-mile/hr	4860
V_a	mph	5.14
VMT_a	miles/bus	4.94
P_{vm}	pass/bus-mile	10.63
P_v	pass/bus	52
C_p	\$/pass	2.24
C_{pm}	\$/pass-mile	0.82
C_{vm}	\$/bus-mile	23.82
T_a	hr/pass	0.54
C_W	\$/hr	1,204
C_R	\$/hr	0
C_I	\$/hr	8,321
C_O	\$/hr	3,999
Total Cost	\$/hr	**13,525**

5.2.3 Integrated All-Stop, Short-Turn and Express SPs – Scenario C

In addition to all-stop and short-turn SPs described in Scenario B, an express SP denoted as SP_E is introduced in this Section. SP_E may skip some of the stops which are determined during the optimization process. Considering the OD demand given in Appendix C and the baseline values given in Table 5.11, it was found that 62 stops are skipped and only stops 1, 12, 20, 38, 50, 53 and 69 are served by express service with a service frequency of 5 buses/hr. The optimized all-stop and short-turn service frequencies of $SP_{1,53}$ ($SP_{53,1}$), $SP_{1,69}$ ($SP_{69,1}$), $SP_{12,46}$ ($SP_{46,12}$) $SP_{12,53}$ ($SP_{53,12}$), $SP_{12,69}$ ($SP_{69,12}$) and $SP_{20,42}$ ($SP_{42,20}$) are 1, 3, 2, 2, 3 and 3 buses/hr, respectively. This operation achieves the minimum total cost of 13,030 $/hr. The associated measures (e.g., fleet size, VMT, VMT_a P_v, P_{vm}, T_a, and LF, etc.) and all cost components derived from the optimized frequencies are calculated and shown in Table 5.14.

In scenario C, PMT and VMT are calculated as 4860 pass-miles/hr and 235 bus-miles/hr, respectively. The average route speed (V_a) is 8.55 miles/hr which yields an average passenger travel time (T_a) of 0.49 hr. VMT_a and P_{vm} is 6.53 miles/bus and 7.59 pass/bus-mile, respectively. Average hourly supplier costs per passenger (C_p), per PMT (C_{pm}) and per VMT (C_{vm}), are 2.36 $/pass/hr, 0.87 $/pass-miles/hr and 17.92 $/bus-miles/hr, respectively. The average LF of outbound and inbound services are 0.46 and 0.29, respectively, with an average LF of 0.38 for Case II.

Table 5.14 Optimal Results of Scenario C in Case II

Variable	**Units**	**Optimal Values**
$f_{1,53}(f_{53,1})$	buses/hr	1
$f_{1,69}(f_{69,1})$	buses/hr	3
$f_{12,46}(f_{46,12})$	buses/hr	2
$f_{12,53}(f_{53,12})$	buses/hr	2
$f_{12,69}(f_{69,12})$	buses/hr	3
$f_{20,42}(f_{42,20})$	buses/hr	3
f_{E}	buses/hr	5
Outbound LF	-	0.46
Inbound LF	-	0.29
Average LF	-	0.38
Fleet Size	buses	36
VMT	bus-mile/hr	235
PMT	pass-mile/hr	4860
V_a	mph	8.55
VMT_a	miles/bus	6.53
P_{vm}	pass/bus-mile	7.59
P_v	pass/bus	50
C_p	\$/pass	2.36
C_{pm}	\$/pass-mile	0.87
C_{vm}	\$/bus-mile	17.92
T_a	hr/pass	0.49
C_W	\$/hr	1,492
C_R	\$/hr	352
C_I	\$/hr	6,976
C_O	\$/hr	4,209
Total Cost	**\$/hr**	**13,030**

5.2.4 Results Comparison

The optimal results of decision variables (e.g., SPs, service frequencies etc.), minimized costs (e.g., wait, transfer, in-vehicle, supplier costs, etc.) and associated performance measures obtained from previous sections in Scenarios A, B and C are compared and discussed in this section. The optimal results for service frequencies and minimized costs are shown in Table 5.15.

In Scenario A, the optimized service frequency of 15 buses/hr yields the total cost of 14,621 \$/hr and comparing Scenario B to A, 4 short-turn SPs (e.g., $SP_{12,53}$ ($SP_{53,12}$), $SP_{12,69}$ ($SP_{69,12}$), $SP_{20,38}$ ($SP_{38,20}$) and $SP_{20,46}$ ($SP_{46,20}$) reduce the total cost about 7.5% (1,096 \$/hr). The major benefit of introducing the short-turn SP is reducing the supplier cost by almost 30% (1,706 \$/hr). However, the wait cost for passengers is increased from 595 to 1,204 \$/hr due to the decreased service frequencies in some segments of the route.

Comparing the optimal results of Scenarios B and C, by introducing the express SP to the study route reduces the in-vehicle cost of passengers from 8,321 to 6,976 \$/hr. However some passengers may have the option to transfer, which generates a transfer cost of 352 \$/hr. Also the wait cost of passengers is increased to 1,492 \$/hr because the express service skips some of the stops and it may not be available all of the passengers.

The performance measurers under Scenarios A, B and C are presented in Table 5.16. The minimum total cost of 13,030 \$/hr is achieved in Scenario C which indicates that the optimal integrated all-stop, short-turn, and express service is preferable for minimum cost operation. Scenario C only yields the lowest average cost per vehicle mile traveled (17.92). The lowest average costs per passenger (2.24 \$/pass), and per passenger mile traveled (0.82 \$/pass-mile) are generated by Scenario B.

Table 5.15 Optimized Frequencies and Minimized Costs in Scenarios A, B and C

Parameters	**Units**	**Scenario A**	**Scenario B**	**Scenario C**
$f_{1,53}(f_{53,1})$	buses/hr	NA	0	1
$f_{1,69}(f_{69,1})$	buses/hr	15	4	3
$f_{12,46}(f_{46,12})$	buses/hr	NA	0	2
$f_{12,53}(f_{53,12})$	buses/hr	NA	5	2
$f_{12,69}(f_{69,12})$	buses/hr	NA	4	3
$f_{20,38}(f_{38,20})$	buses/hr	NA	1	0
$f_{20,42}(f_{42,20})$	buses/hr	NA	0	3
$f_{20,46}(f_{46,20})$	buses/hr	NA	3	0
f_E	buses/hr	NA	NA	5
C_W	$/hr	595	1,204	1,492
C_R	$/hr	0	0	352
C_I	$/hr	8,321	8,321	6,976
C_O	$/hr	5,705	3,999	4,209
TC	**$/hr**	**14,621**	**13,525**	**13,030**

Comparing the results of Scenario B to that of Scenario A, VMT is reduced from 274 to 168 miles/hr due to the encouragement of short-turn services instead of an all-stop service. However, introducing the express service in Scenario C increases the VMT to 235 miles/hr because of the replacement of some short-turn services with express service, which serves the full length of the route.

The express service also increases the average route speed (V_a) in Scenario C because the stop delays are eliminated by the express buses at the skipped stops. Thus, V_a of Scenario C is 2.80 and 3.41 mph higher than V_a of Scenario A and B, respectively.

Due to the decrease of VMT in Scenario B comparing it to Scenario A, P_{vm} increases from 6.52 to 10.63 pass/bus-mile. As discussed earlier, VMT is increased in

Scenario C (compared to Scenario B) due to the integrated operation of express service which yields a lower P_{vm} (12.12 pass/bus-mile). However, the maximum value of 52 passengers per bus (P_v) occurs in Scenario B due to the lowest optimum fleet size of 34 buses.

Introducing the short-turn service into the all-stop only service may increase the average passenger travel time (T_a) from 0.50 to 0.54 hr/pass because of the increased wait time of passengers in some parts of the route where the short-turn SPs are not available. However, integrating the express service to the studied route may significantly reduce the T_a (0.49 hr/pass) due to the eliminated stop delays at skipped stops.

Table 5.16 Performance Measures in Scenarios A, B and C

Parameters	Units	Scenario A	Scenario B	Scenario C
Outbound LF	-	0.44	0.64	0.46
Inbound LF	-	0.27	0.44	0.29
Average LF	-	0.36	0.54	0.38
Fleet Size	buses	48	34	36
VMT	bus-mile/hr	274	168	235
PMT	pass-mile/hr	4860	4860	4860
V_a	mph	5.75	5.14	8.55
VMT_a	miles/bus	5.70	4.94	6.53
P_{vm}	pass/bus-mile	6.52	10.63	7.59
P_v	pass/bus	37	52	50
C_p	$/pass	3.20	2.24	2.36
C_{pm}	$/pass-mile	1.17	0.82	0.87
C_{vm}	$/bus-mile	20.85	23.82	17.92
T_a	hr/pass	0.50	0.54	0.49

5.3 Summary

In this chapter two case studies were used to demonstrate the applicability of the developed models, in which three scenarios are evaluated and optimal results are compared. The relationships among the objective function, model parameters and decision variables were investigated in a small scale (e.g., six-stop bus route) transit route. Also the applicability of Model II to a large scale bus route was demonstrated using a Genetic Algorithm in a real world example.

Compared to a traditional all-stop only service, adding short-turn services reduced operating costs and vehicle miles travelled. However, the wait cost of passengers increased because of the reduced service frequency on low demand segments. Integrating SP_E into local (e.g., all-stop and short-turn) SPs significantly reduced the in-vehicle time of passengers and supplier cost. This benefit was sufficient to compensate for the increases in wait and transfer cost. The findings and conclusions from this chapter and future expansions of this research are presented next.

CHAPTER 6

CONCLUSION AND FUTURE RESEARCH

While many studies (Furth, 1987 and 1988; Ceder 1988 and 1989, Delle Site et al, 1998) concentrated on transit service optimization by maximizing the utilization of vehicles and/or minimizing supplier cost, only few involved the optimizing of integrated service patterns (SPs) considering a heterogeneous demand. In this research, an integrated service (e.g., all-stop, short-turn and express) and associated frequencies have been defined, formulated, optimized and analyzed. A logit-based model was used to calculate transfer passengers from one SP to another. The solutions that minimize total cost (i.e., the sum of user and supplier cost) of a real world transit route were found by algorithms (i.e., an Exhaustive Search Algorithm and a Genetic Algorithm).

A hypothetical six-stop bus route was introduced in Case I, which demonstrated the applicability of the developed models for a small scale bus route. Three scenarios were evaluated and sensitivity analyses were conducted, in which the optimized results and performance measures were compared and discussed. The relationship among the objective function, model parameters, and decision variables was explored. In Case II, a real world transit route, the New Jersey Transit Springfield Avenue Line, was introduced to demonstrate the applicability and the efficiency of the developed models in a large scale bus route.

6.1 Findings

The purpose of this section is to discuss the results and the relationship among decision variables and model parameters (e.g., value of time, vehicle operating cost, skipped stations, transfer stations, etc.) for the case studies introduced in Chapter 5. The major findings are summarized as follows:

(1) Case I:

- By comparing the optimal results among all scenarios, the minimum total cost of 7,317 \$/hr was achieved in Scenario C. It indicates that the optimal integrated all-stop, short-turn, and express service is capable of minimizing the total cost subject to a set of constraints and given an OD demand distribution.

- $SP_{1,6}$ ($SP_{6,1}$), $SP_{3,6}$ ($SP_{6,3}$), $SP_{4,6}$ ($SP_{6,4}$) and SP_E achieved the optimal solution in Scenario C with the service frequencies of 4, 2, 2 and 9 buses/hr, respectively. It was also found that, SP_E skipped stops 2, 4 and 5, and Stop 3 was used as a transfer location.

- The optimal solution under Scenario C also yielded the lowest average cost per passenger (0.97 \$/pass), average cost per passenger-mile traveled (0.33 \$/pass-mile), and average cost per vehicle-mile traveled (11.79 \$/bus-mile).

- The lowest vehicle-miles traveled (VMT) were achieved by the optimal solution under Scenario B, which also yielded the highest average load factor of 0.79 and average passenger travel time (T_a) of 0.33 hr/pass.

- Integrating the express service into all-stop and short-turn services significantly increased the average travel speed (V_a). It was found that V_a of Scenario C is 23% and 30% faster than those of Scenarios A and B, respectively.

- As bus operating cost increased, the optimized service frequencies decreased to reduce the increase of supplier cost, and increased user wait and transfer time.

- As user's value of time increased, the optimized service frequencies increased, which decreased wait and transfer time, but increased the supplier cost.

- As bus capacity increased, the optimized service frequencies of all SPs decreased, reducing the supplier cost as well as total cost. However increased bus capacity reduced the optimized service frequency, which increased the passenger wait time and user cost.

- The computation time needed for the exhaustive search algorithm (ESA) implemented in Case I to search for the optimal results was 14 minutes with an Intel Core 2 Duo 2.4 MHz processor and 4GB Ram. However, the complexity of the problem, in terms of the number of decision variables, was exponentially increasing (see Figure 5.52), ESA needed more than 7 days to solve a similar problem with 10 stops.

- The global optimal result obtained via ESA was guaranteed because it was found from the results of all feasible solutions. This result was applied to compare the optimal result obtained via GA. It was found that the solutions from both algorithms were fairly close (see Figure 5.54), while the computation time needed for the GA in Case I to search for the optimal results was less than 2 minutes with the same computer specifications. Therefore, the GA was used to search for the optimal solutions in a large scale bus route discussed in Case II.

(2) Case II:

- By comparing the optimal results among all Scenarios, the minimum total cost of 13,030 $/hr was achieved in Scenario C. It indicates that the optimal integrated all-stop, short-turn, and express service is preferred for a minimum cost operation subject to a set of constraints and given an OD demand distribution.

- $SP_{1,53}$ ($SP_{53,1}$), $SP_{1,69}$ ($SP_{69,1}$), $SP_{12,46}$ ($SP_{46,12}$), $SP_{12,53}$ ($SP_{53,12}$), $SP_{12,69}$ ($SP_{69,12}$), $SP_{20,42}$ ($SP_{42,20}$) and SP_E achieved the optimal solution in Scenario C with the frequencies of 1, 3, 2, 3, 2, 3 and 5 buses/hr, respectively. It was also found that, SP_E served only Stops 1, 12, 20, 38, 50, 53 and 69 along the route.

- The optimal solution under Scenario C also yielded the lowest average cost per vehicle-mile traveled (17.92 $/bus-mile). However the average lowest costs per passenger (2.24 $/pass) and per passenger-mile traveled (0.82 $/pass-mile) is obtained by the optimal solution under Scenario B.

- The lowest VMT was achieved by the optimal solution under Scenario B, which also yielded the highest average load factor of 0.54 and average passenger travel time (T_a) of 0.54 hr/pass.

- Integrating the express service into local (e.g., all-stop and short-turn) services significantly increased the average travel speed (V_a). It was found that V_a of Scenario C is 49% and 66% faster than those of Scenario A and B, respectively.

- The computation time needed for the GA implemented in Case II to search for the optimal result was 14 hours with an Intel Core 2 Duo 2.4 MHz processor and 4GB Ram.

6.2 Conclusions

The conclusions and recommendations for this study based on the findings from the case studies and sensitivity analyses are summarized as follows:

- This study presented an approach to optimize the integrated transit SPs and associated service frequencies for a transit route with heterogeneous demand, which minimized the total cost subject to capacity, fleet size and frequency conservation constraints. The developed method intends to fill a gap in the area of transit network modeling by considering integrated SPs into the optimization processes.

- By considering aggregate effects (e.g., alternative travel options, transfer conditions, etc.) of various SPs and stop/station-based OD demand distributions, the developed method offered a practical and efficient approach to quantify the costs savings and improved system performance.

- The modeling approach suggested in this study is very flexible, and can be utilized to optimize a generalized transit route as soon as the OD demand distribution and the route/stop locations are available. Transit agencies may easily adopt the developed model and solution algorithms with minor modification to estimate costs (e.g., user and supplier) and evaluate system performance.

- Passenger transfer is an important concept for integrated transit services. In both Cases I and II, the user cost of the route was improved by transfer demand due to the reduced travel time of transfer passengers to/from an express service when available. Therefore, increasing transfer options (such as timed transfers) may improve the passenger's travel time as well as reduce the supplier costs.

- Compared to a traditional all-stop only service with heterogeneous demand, operating short-turn service may reduce the supplier cost. However, the wait cost of passengers increased because of the reduced service frequency on the segments with light demand. Integrating an express service may significantly reduce the in-vehicle time of passengers, albeit the wait and transfer cost may increase. This saved in-vehicle cost seemed sufficient to compensate for the increased wait and transfer costs. In the cases given in this research, it was found that integrated service should be encouraged to reduce supplier cost with the least impact to user benefits.

- The studied transit service optimization problem is non-linear, mixed-integer, and combinatorial, which was difficult to solve by using any classical optimization methods due to the interdependent relationships among the decision variables and the non-differentiable objective function. Thus, special solution algorithms (e.g., exhaustive search and genetic algorithms) were developed to search for the optimal solutions.

- The total number of decision variables increased exponentially as the number of stops increased. Therefore, exhaustive search algorithms would not be a practical approach for routes with large numbers of stops. A heuristic algorithm (e.g. genetic algorithm) was introduced to improve the search speed of the optimal solution in Case II.

6.2 Future Research

In this section, several possible extensions are suggested to enhance the models developed and analyses conducted in this research. It should be noted that some extensions may complicate the processes to enhance the developed models. Future research for the transit service design and optimization problem can be extended but not be limited to the following aspects:

- An immediate extension of this study will be integrating express short-turn SPs in which the express services can make short-turn trips. Throughout this extension the new model may have more options to demonstrate the advantages of integrated SPs.

- The developed models in this study were designed especially for bus operation, and may easily be extended to rail operations considering some specific conditions, such as track alignment, and for other service providers, such as parcel service and airline operations.

- An assumption of constant unit vehicle operating cost was made for developing the model in this research, regardless of the vehicle capacity. This assumption can be relaxed to improve the accuracy of supplier cost estimation.

- By considering the number of passengers boarding at stops and average vehicle operating speed along the route, dwell time and stop delays may be calculated. Therefore, estimation of the in-vehicle cost of passengers as well as the supplier cost may be improved.

- Developed GA adds a penalty value to eliminate the results which violate the capacity and fleet size constraints. The GA can be enhanced by generating a new repair strategy for these solutions to improve the search for the optimal solution.

- The developed model may be enhanced by considering an elastic demand in which passengers may be attracted to/from other routes and/or modes. This extension may increase the functionality of the model for practical use.

- Optimizing integrated service for a transit network with several transit routes share a bus terminal with known stop/route locations and OD demand distribution will be a challenging extension of this study, which also involves the issues of scheduling and routing transit vehicles. In addition, vehicle capacity, the composition of vehicle fleet, and timed transfer may be considered in the context of the problem, so the optimized network-wide system performance may be assured.

- Another extension of this study may focus on integrating intermodal transit services such as rail and bus services for the same corridor. Moreover, optimization of timed transfer may also be considered which yields increased route performance, in terms of user and supplier costs.

APPENDIX A

DEFINITIONS OF MODEL PARAMETERS

Parameters	Descriptions	Units
b	Vehicle operating cost	\$/veh-hr
C	Vehicle capacity	spc/veh
C_I	User in-vehicle cost	\$/hr
C_O	Supplier cost	\$/hr
C_p	Average cost per passenger	\$/pass
C_R	User transfer cost	\$/hr
C_U	Total user cost	\$/hr
C_{vm}	Average cost per vehicle mile of travel	\$/veh-mile
C_W	User wait cost	\$/hr
$D_{i,j,k}$	Demand from i to j in category k	pass/hr
i	Index of origin stop	-
j	Index of destination stop	-
k	Index of demand categories	-
I_m	Inbound demand on link m	pass/hr
$\mathrm{I}_{E,l}$	Inbound demand for express services in link l	pass/hr
$\mathrm{I}_{E,l,k}$	Inbound demand for express services in link l using category k	pass/hr
$\mathrm{I}_{L,l}$	Inbound demand for local services in link l	pass/hr
$\mathrm{I}_{L,l,k}$	Inbound demand for local services in link l using category k	pass/hr
F	Maximum operable fleet size	vehicle
$f_{i,j}$	Frequency of $SP_{i,j}$	veh/hr
f_E	Frequency of express service pattern	veh/hr
$H_{E,l,}$	Maximum headway on link l for express services	hr/veh
$h_{E,l}$	Average headway of link l for express services	hr/veh
$H_{L,l}$	Maximum headway on link l for local services	hr/veh
$h_{L,l}$	Average headway of link l for local services	hr/veh
H_m	Maximum headway on link m	hr/veh
h_m	Average headway on link m	hr/veh
l	Link between stops l and l+1	-
LF	Load factor	-
l_d	Distance between stop d and d+1	mile
n	Number of stops	-
g	Eligible stops for turn-back	stop
O_m	Outbound demand on link m	pass/hr
$\mathrm{O}_{E,l}$	Outbound demand for express services in link l	pass/hr
$\mathrm{O}_{E,l,k}$	Outbound demand for express services in link l using category k	pass/hr

Parameters	Descriptions	Units
$O_{L,l}$	Outbound demand for local services in link l	pass/hr
$O_{L,l,k}$	Outbound demand for local services in link l using category k	pass/hr
$q_{i,j}$	Demand from stop i to j	pass/hr
$SP_{s,t}$	Service pattern serving from stop s to t	-
P	Population size in genetic algorithm	-
PMT	Passenger miles of travel	pass-mile/hr
P_v	Average passengers per vehicle	pass/bus
P_{vm}	Average passengers per vehicle mile of travel	pass/bus-mile
r_S	Selection ratio in GA	-
r_X	Crossover ratio in GA	-
r_M	Mutation ratio in GA	-
$t_{R_{i,j,k}}$	Transfer time for demand from stop i to j using category k	hr
t_o	Layover time at the end terminal	hr
$t_{W_{i,j}}$	Wait time for demand from stop i to j	hr
$t_{W_{i,j,k}}$	Wait time for demand from stop i to j using category k	hr
$t_{I_{i,j}}$	In-vehicle time for demand from stop i to j	hr
$t_{I_{i,j,k}}$	In-vehicle time for demand from stop i to j using category k	hr
T_a	Average passenger travel time	hr
T_E	Vehicle travel time for express services	hr
$T_{i,j}$	Vehicle travel time for local services from stop i to j	hr
λ_i	Variable indicating whether stop i is served by express service	-
α_R	Ratio of the average transfer time to headway	-
$T_{i,j}$	Vehicle travel time from stop i to j	hr
λ_i	Variable indicating whether stop i is served by express service	-
α_R	Ratio of the average transfer time to headway	-
VMT	Vehicle miles of travel	veh-mile/hr
VMT_a	Average vehicle miles of travel	miles/veh
V_a	Average speed of the route	mph
v_d	Average vehicle speed from stop d to $d+1$	mile/hr
w_d	Average stop delay and dwelling time at stop d	hr
y	Index of transfer stops from all-stop to express service	-
z	Index of transfer stops from express to all-stop service	-
α_w	Ratio of the average wait time to headway	-
θ_I	Sensitivity parameter of demand to the length of in vehicle time	-
θ_R	Sensitivity parameter of demand to the length of transfer time	-
θ_W	Sensitivity parameter of demand to the length of wait time	-
$\varphi_{i,j}$	Percentage of demand using transfer from all-stop to express service	%
$\psi_{i,j}$	Percentage of demand selecting express service first	%
μ	User's value of time	\$/pass-hr

APPENDIX B

BUS STOP INFORMATION FOR CASE II

Stop ID	Stop Location	Distance from previous stop (mile)	Eligible for turn-back
1	Maplewood Loop	0.000	Yes
2	Raymond Terrace	0.199	No
3	Mildred Terrace	0.095	No
4	Springfield Ave.	0.189	No
5	Laurel Ave.	0.057	No
6	Indiana St.	0.189	No
7	Princeton St.	0.227	No
8	Tuscan St.	0.341	No
9	Burnett Ave.	0.199	No
10	Boyden Ave.	0.189	No
11	Chancellor Ave.	0.133	No
12	Prospect Ave.	0.170	Yes
13	38th St.	0.246	No
14	Sanford Ave.	0.152	No
15	Lyons Ave.	0.123	No
16	Stuyvesant Ave.	0.142	No
17	Lincoln Pl.	0.180	No
18	New St.	0.161	No
19	Union Ave.	0.095	No
20	Irvington Bus Terminal	0.038	Yes
21	Bruen Ave.	0.265	No
22	Maple Ave.	0.076	No
23	Grove St.	0.076	No
24	Harrison Pl.	0.095	No
25	Ellis Ave.	0.095	No
26	Avon Ave.	0.076	No
27	South 20th St.	0.095	No
28	South 18th St.	0.114	No
29	South 17th St.	0.057	No
30	South 15th St.	0.114	No
31	South 14th St.	0.057	No
32	Pierce St.	0.104	No
33	South 11th St.	0.076	No
34	South 10th St.	0.066	Yes
35	18th Ave.	0.038	No
36	Muhammad Ali Blvd.	0.170	No
37	Fairmount Ave.	0.142	No
38	Bergen St.	0.114	Yes
39	West Kinney St.	0.095	No
40	Sayre St.	0.123	No
41	Boyd St.	0.057	No
42	Irvine Turner Blvd.	0.114	Yes
43	Prince St.	0.133	No

Stop ID	Stop Location	Distance from previous stop (mile)	Eligible for turn-back
44	Broome St.	0.066	No
45	West St.	0.085	No
46	Martin Luther King Blvd.	0.189	Yes
47	Branford Pl.	0.038	No
48	Washington St.	0.265	No
49	Halsey Street	0.076	No
50	Broad St.	0.095	Yes
51	Mulberry St.	0.152	No
52	McCarter Hwy.	0.152	No
53	Penn Station - Alling St	0.057	Yes
54	Edison Pl.	0.152	No
55	Union St.	0.038	No
56	Congress St.	0.114	No
57	Madison St.	0.104	No
58	Adams St.	0.095	No
59	Van Buren St.	0.095	No
60	Polk St.	0.047	No
61	Patterson St.	0.170	No
62	Ann St.	0.142	No
63	Garrison - Barbara St.	0.142	No
64	Jabez St.	0.114	No
65	Avenue K	0.170	No
66	Avenue L	0.152	No
67	Hyatt Ave.	0.322	No
68	Avenue P	0.341	No
69	Doremus Ave./ NJT Bus Complex	0.275	Yes
70	Firmenich Way /	0.833	No
71	Essex Co. Correction Center	0.252*	No

* Distance from Stop #69

APPENDIX C

OD DEMAND MATRIX FOR CASE II

(pass/hr)

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23
S1	0	0	0	0	0	0	0	0	1	0	0	2	0	1	1	0	1	0	0	1	0	0	0
S2	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	3	0	0	0
S3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0
S4	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0
S5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S6	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	2	0	1	0	0	1
S7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	1	0	2	0	0	0
S10	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0
S11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S12	5	4	1	2	2	0	0	0	2	0	0	0	0	0	0	1	0	1	0	2	0	0	0
S13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	1	0	0
S14	3	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
S15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
S16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	1	0
S17	2	0	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
S18	3	2	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
S19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S20	4	1	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S21	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S22	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S23	2	2	2	0	0	0	2	0	2	0	0	2	0	2	0	0	0	0	0	0	0	0	0
S24	1	0	1	0	1	0	0	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0
S25	0	0	0	0	0	0	0	0	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0
S26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S27	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
S28	1	0	0	1	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
S29	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
S30	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
S31	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
S32	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S34	2	0	0	1	0	0	0	1	0	0	1	0	2	0	1	1	0	0	0	0	0	0	0
S35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	S24	S25	S26	S27	S28	S29	S30	S31	S32	S33	S34	S35	S36	S37	S38	S39	S40	S41	S42	S43	S44	S45	S46
S1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	0	2
S2	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
S3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0
S5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1
S6	0	0	0	0	0	0	0	0	0	0	2	0	1	2	2	0	1	1	1	1	0	0	0
S7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S8	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
S9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
S10	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
S12	0	0	0	1	0	0	0	0	0	0	1	0	0	0	2	0	0	0	2	0	0	0	3
S13	0	0	0	1	1	0	1	0	0	0	1	0	0	0	2	0	1	1	4	0	0	0	5
S14	0	0	1	0	1	1	2	2	0	0	1	0	0	1	4	0	1	0	5	1	1	0	6
S15	0	0	0	0	0	0	1	1	0	0	1	0	0	0	1	1	0	0	2	0	0	0	3
S16	1	0	1	1	1	1	2	1	1	1	3	0	1	1	3	0	1	1	5	1	0	0	8
S17	0	0	0	1	0	0	1	1	0	0	2	1	1	0	0	0	0	2	4	0	0	1	5
S18	0	0	0	1	0	0	1	0	0	1	2	2	1	0	0	0	0	0	6	3	0	0	2
S19	0	0	0	0	0	0	2	0	0	0	0	3	2	0	4	2	1	0	4	2	3	1	8
S20	0	1	0	1	0	1	4	3	1	2	6	2	1	2	6	2	3	4	5	1	2	2	9
S21	0	0	0	0	0	0	0	1	0	0	1	1	0	0	2	1	0	0	0	1	0	0	2
S22	0	0	0	0	1	0	1	1	0	1	2	1	2	1	2	0	1	3	3	1	0	0	4
S23	0	0	0	0	0	0	0	0	0	0	1	1	0	0	2	1	1	0	0	0	0	0	3
S24	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	1	4	1	0	0	2
S25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S26	0	0	0	0	0	0	0	0	0	0	2	2	0	0	2	3	2	2	5	0	0	0	0
S27	0	0	0	0	0	0	0	0	0	0	0	1	1	0	6	2	2	2	5	0	2	1	5
S28	0	0	0	0	0	0	0	0	0	0	2	0	0	0	1	0	0	0	0	0	0	0	2
S29	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
S30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0	1	4	2	2	0	7
S31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	2	1	2
S32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	2
S33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2
S35	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	S47	S48	S49	S50	S51	S52	S53	S54	S55	S56	S57	S58	S59	S60	S61	S62	S63	S64	S65	S66	S67	S68	S69
S1	0	0	1	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S4	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S7	0	0	0	4	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S8	0	2	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S9	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S10	0	0	2	2	0	0	2	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	1
S11	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S12	0	0	0	5	0	0	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S13	0	1	1	5	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S14	0	0	0	9	0	0	4	0	0	0	0	0	0	3	0	0	1	0	0	0	0	0	3
S15	1	1	0	4	0	1	2	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	1
S16	1	1	0	9	0	1	3	0	0	0	0	0	0	5	1	1	1	1	0	1	1	1	3
S17	1	1	2	6	0	0	4	2	0	0	0	0	0	1	1	0	0	0	0	0	1	0	2
S18	0	0	1	8	1	0	3	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	3
S19	1	2	4	12	1	1	9	1	0	0	0	0	0	1	2	1	0	1	0	2	1	2	6
S20	1	3	1	19	1	1	14	0	1	0	0	0	2	4	1	0	1	0	0	1	0	2	8
S21	1	0	0	3	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S22	0	0	0	14	0	0	15	2	0	2	0	0	2	0	2	0	0	0	0	1	0	0	1
S23	1	0	0	3	0	0	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S24	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S25	0	0	0	2	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	3
S27	1	4	0	10	0	0	7	1	0	3	0	0	2	0	0	1	0	0	1	0	0	0	3
S28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S29	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1
S30	1	1	0	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S31	1	0	0	3	0	0	4	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1
S32	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S33	0	0	0	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S34	0	0	0	1	0	0	2	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	2
S35	0	0	0	4	0	0	3	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23
S36	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
S37	1	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0
S38	1	1	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
S39	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
S40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
S41	2	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	2	1	1	0	2	2	3
S42	2	2	0	0	0	0	0	4	0	5	0	4	0	3	2	3	1	2	1	2	2	0	0
S43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
S44	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0
S45	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0
S46	1	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0
S47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S50	3	0	0	0	0	0	0	1	1	0	3	0	0	0	3	0	0	3	0	2	0	0	2
S51	2	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	0	0	3
S52	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	2
S53	3	0	0	0	0	0	0	0	7	5	8	11	9	6	5	8	0	7	0	10	6	0	4
S54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	2
S61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
S62	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	2	0	0	0	1	0	0	0
S63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S64	0	0	0	0	0	2	0	0	0	1	2	0	0	0	0	0	0	0	0	0	0	0	1
S65	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
S66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S67	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
S68	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S69	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	S24	S25	S26	S27	S28	S29	S30	S31	S32	S33	S34	S35	S36	S37	S38	S39	S40	S41	S42	S43	S44	S45	S46
S36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S39	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
S40	1	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S41	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S42	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S43	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S44	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S45	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S46	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S49	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0
S50	0	3	0	0	3	2	5	3	3	3	4	3	2	3	11	3	0	3	0	0	0	0	0
S51	0	3	0	0	0	2	2	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S52	0	1	1	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S53	0	0	5	0	0	0	4	0	7	3	3	0	0	0	4	0	0	0	4	0	0	0	3
S54	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	1	0	1	0	1	0	0
S55	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
S56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1
S57	0	1	0	0	0	1	0	1	0	0	1	0	0	1	1	0	1	0	1	0	0	1	0
S58	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
S59	0	1	0	1	0	0	1	0	1	0	0	1	0	1	0	1	0	0	1	0	1	0	1
S60	1	0	2	2	2	2	1	0	1	2	2	0	0	0	1	0	0	2	1	0	0	0	2
S61	0	0	2	0	0	0	2	0	0	2	0	0	0	2	0	0	0	2	1	0	1	2	2
S62	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	2	2	0	0	0	2
S63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S64	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
S65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0
S66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S67	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
S68	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S69	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0

	S47	S48	S49	S50	S51	S52	S53	S54	S55	S56	S57	S58	S59	S60	S61	S62	S63	S64	S65	S66	S67	S68	S69
S36	0	0	0	2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S37	0	0	0	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S38	0	0	0	0	0	0	7	1	3	2	0	0	3	0	0	0	3	0	0	0	0	0	5
S39	0	0	0	3	0	0	2	0	1	1	1	0	0	0	1	1	1	1	0	0	0	0	2
S40	0	0	0	2	0	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S41	0	0	0	0	0	0	2	0	0	0	0	0	0	0	2	1	0	0	2	0	0	0	3
S42	0	0	0	7	1	1	5	0	0	0	1	0	0	1	1	1	1	0	1	0	1	0	4
S43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S44	0	0	0	3	1	1	2	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
S45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S48	0	0	0	0	0	0	3	0	2	1	0	1	1	0	2	0	0	0	0	0	0	0	5
S49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S50	0	0	0	0	0	0	12	0	0	0	1	1	0	0	2	0	0	0	0	3	3	4	35
S51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S53	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	2	0	3	3	3	25
S54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S56	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S57	1	0	1	0	1	0	3	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	3
S58	1	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S59	0	0	0	3	1	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S60	0	0	0	4	1	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	4
S61	0	0	0	4	2	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S62	0	0	0	5	0	2	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S63	0	0	0	3	3	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S64	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S65	0	0	0	1	0	0	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S66	0	0	0	4	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S67	0	0	0	1	0	0	7	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
S68	0	0	0	2	0	0	6	0	0	2	1	0	0	1	0	0	0	0	0	0	0	0	0
S69	0	0	0	3	0	0	6	0	0	2	1	0	0	2	0	0	0	0	0	0	0	0	0

APPENDIX D

MATLAB CODE FOR CASE II

```
%       Binary Genetic Algorithm
%       Case II: 69-stop example
%       Minimize the objective function designated in TotalCost.m
%       Set the parameters in parts A and B
%_____________________________________________________________
%       Display start time
day=clock;
disp(datestr(datenum(day(1),day(2),day(3),day(4),day(5),day(6)),0))
clear
%_____________________________________________________________
%       A.Setup the GA
ff='TotalCost';              % objective function
npar=46;                     % number of SPs to be optimized
npar2=67;                    % number of stops to be skipped/stopped initial and final stops excluded
maxit=500;                   % max number of iterations
popsize=500;                 % set population size
mincost=-9999999;            % set minimum cost to stop GA
mutrate=0.3;                 % set mutation rate
selection=0.3;               % fraction of population kept
nbits=6;                     % number of bits in each parameter
Nt=nbits*npar+npar2;         % total number of bits in a chromosome
%_____________________________________________________________
%       B.Input Data
q = xlsread('data.xls', 'OD');          % demand;
len = xlsread('data.xls', 'LEN');       % stop spacing
dwl =  xlsread('data.xls', 'DWL');      % stop delay+ dwell time
S= xlsread('data.xls', 'S');            % eligible turn back stops
vol =  xlsread('data.xls', 'VOL');      % vehicle speed
n =69;                                  % total number of stops
c=50;                                   % capacity
alfaW=0.5;
alfaR=0.5;
uservalue=10;
operatorcost=120;
layover= 0.3;
tetW=2;
tetR=2;
tetV=1.333;
%_____________________________________________________________
%       Create the initial population
iga=0;                                  % generation counter initialized
pop=round(rand(popsize,Nt));            % random population of 1s and 0s
par=gadecode4(pop,nbits,npar2);         % convert binary to integer values
par2=pop(:,Nt-npar2+1:Nt);              % stations skipped condition
cost=TotalCost(par,popsize,par2,q,len,dwl,S,vol,n,alfaW,alfaR,uservalue,operatorcost,layover,c,tetW,tetR,tetV);
 [cost,ind]=sort(cost);                 % min cost in element 1
par=par(ind,:);    pop=pop(ind,:); par2=par2(ind,:);   % sorts population with lowest cost first
minc(1)=min(cost);                      % minc contains min of population
%_____________________________________________________________
```

```
%         Iterate through generations
while iga<maxit
  iga=iga+1;                                          % increments generation counter
  keep=floor(selection*popsize);                      % #population members that survive
  rest=popsize-keep;
%_______________________________________________________
%        Pair and mate
  M=ceil(keep/2);                                     % number of matings
  prob=flipud([1:keep]'/sum([1:keep]));               % weights chrom. based upon position in list
  odds=[0 cumsum(prob(1:keep))'];                     % probability distribution function
  pick1=rand(1,M);                                    % mate #1
  pick2=rand(1,M);                                    % mate #2
%       ma and pa contain the indices of the chromosomes that will mate
  ic=1;
  while ic<=M
    for id=2:keep+1
      if pick1(ic)<=odds(id) & pick1(ic)>odds(id-1)
        ma(ic)=id-1;
      end % if
      if pick2(ic)<=odds(id) & pick2(ic)>odds(id-1)
        pa(ic)=id-1;
      end % if
    end % id
    ic=ic+1;
  end % while
  %_______________________________________________________
  %       Performs mating using double point crossover
  ix=1:2:keep;                                                    % index of mate #1
  xp1=ceil(rand(1,M)*((Nt/2)-1));                                 % crossover point 1
  xp2=ceil(Nt/2)+ceil(rand(1,M)*((Nt/2)-1));                      % crossover point 2
    pop(rest+ix,:)=[pop(ma,1:xp1) pop(pa,xp1+1:xp2) pop(ma,xp2+1:Nt)];     % first offspring
    pop(rest+ix+1,:)=[pop(pa,1:xp1) pop(ma,xp1+1:xp2) pop(pa,xp2+1:Nt)];   % second offspring
 %_______________________________________________________
 %        Mutate the population
  nmut=ceil((popsize-1)*Nt*mutrate);                          % total number of mutations
  mrow=ceil(rand(1,nmut)*keep);                               % row to mutate
  mcol=ceil(rand(1,nmut)*Nt);                                 % column to mutate
  for ii=1:nmut
    pop(rest+mrow(ii),:)=pop(mrow(ii),:);
    pop(rest+mrow(ii), mcol(ii))=abs(pop(rest+mrow(ii),mcol(ii))-1);     % toggles bits
  end % ii
 %_______________________________________________________
%         The population is re-evaluated for cost
  par(2:popsize,:)=gadecode4(pop(2:popsize,:),nbits,npar2); % decode
  par2(2:popsize,:)=pop(2:popsize,Nt-npar2+1:Nt);
  cost(2:popsize)=TotalCost(par(2:popsize,:),popsize-
1,par2(2:popsize,:),q,len,dwl,S,vol,n,alfaW,alfaR,uservalue,operatorcost,layover,c,tetW,tetR,tetV);
%_______________________________________________________
%        Sort the costs and associated parameters
  [cost,ind]=sort(cost);
  par=par(ind,:);  pop=pop(ind,:);par2=par2(ind,:);
%_______________________________________________________
%        Do statistics for a single nonaveraging run
 minc(iga+1)=min(cost);
%_______________________________________________________
```

```
%        Stopping criteria
   if iga>maxit | cost(1)<mincost
     break
   end
[iga cost(1)]
 disp([num2str(par(1,:))])
disp([num2str(par2(1,:))])
end %iga
%_______________________________________________________________
%        Displays the output and end time
day=clock;
disp(datestr(datenum(day(1),day(2),day(3),day(4),day(5),day(6)),0))
disp(['optimized function is ' ff])
format short g
disp(['popsize = ' num2str(popsize) ' mutrate = ' num2str(mutrate) ' # par = ' num2str(npar)])
disp(['#generations=' num2str(iga) ' best cost=' num2str(cost(1))])
disp(['best solution'])
disp([num2str(par(1,:))])
disp([num2str(par2(1,:))])
disp('binary genetic algorithm')
disp(['each parameter represented by ' num2str(nbits) ' bits'])
%        End of Binary GA
%_______________________________________________________________
%_______________________________________________________________
%        gadecode4.m
%        Decode binary to integer values
%        chrom: population
%        bits: number of bits/parameter
%
function f=gadecode4(chrom,bits,npar2)
 [M,N]=size(chrom);
npar=(N-npar2)/bits;                              % number of variables
quant=(2.^([1:bits]-1)');
ct=reshape(chrom(:,1:N-npar2)',bits,npar*M)';     % each column contains one variable
pr=(ct*quant);                                    % DA conversion and un-normalize variables
parcal=(pr>=1);
par=parcal.*pr;
f=reshape(par,npar,M)';                           % reassemble population
%        End of gadecode4.m
%_______________________________________________________________
%_______________________________________________________________
%        TotalCost.m
%         GA Fitness Function
%         x: vehicle frequency
%        par2: Stopped/Skipped stops
function f=TotalCost(x,popsize,par2,q,len,dwl,S,vol,n,alfaW,alfaR,uservalue,
                     operatorcost,layover,c,tetW,tetR,tetV)
freq=zeros(n,n);
freqexp=zeros(n,n);
lamda= zeros(1,n);
z= zeros(n,n);
y= zeros(n,n);
for p=1:popsize
    lamda(1)=1;               lamda(2)= par2(p,1);      lamda(3)= par2(p,2);
    lamda(4)= par2(p,3);      lamda(5)= par2(p,4);      lamda(6)= par2(p,5);
    lamda(7)= par2(p,6);      lamda(8)= par2(p,7);      lamda(9)= par2(p,8);
```

```
    lamda(10)= par2(p,9);    lamda(11)= par2(p,10); lamda(12)= par2(p,11);  lamda(13)=par2(p,12);
    lamda(14)=par2(p,13);   lamda(15)=par2(p,14);   lamda(16)=par2(p,15);   lamda(17)=par2(p,16);
    lamda(18)=par2(p,17);   lamda(19)=par2(p,18);   lamda(20)=par2(p,19);   lamda(21)=par2(p,20);
    lamda(22)=par2(p,21);   lamda(23)=par2(p,22);   lamda(24)=par2(p,23);   lamda(25)=par2(p,24);
    lamda(26)=par2(p,25);   lamda(27)=par2(p,26);   lamda(28)=par2(p,27);   lamda(29)=par2(p,28);
    lamda(30)=par2(p,29);   lamda(31)=par2(p,30);   lamda(32)=par2(p,31);   lamda(33)=par2(p,32);
    lamda(34)=par2(p,33);   lamda(35)=par2(p,34);   lamda(36)=par2(p,35);   lamda(37)=par2(p,36);
    lamda(38)=par2(p,37);   lamda(39)=par2(p,38);   lamda(40)=par2(p,39);   lamda(41)=par2(p,40);
    lamda(42)=par2(p,41);   lamda(43)=par2(p,42);   lamda(44)=par2(p,43);   lamda(45)=par2(p,44);
    lamda(46)=par2(p,45);   lamda(47)=par2(p,46);   lamda(48)=par2(p,47);   lamda(49)=par2(p,48);
    lamda(50)=par2(p,49);   lamda(51)=par2(p,50);   lamda(52)=par2(p,51);   lamda(53)= par2(p,52);
    lamda(54)=par2(p,53);   lamda(55)=par2(p,54);   lamda(56)=par2(p,55);   lamda(57)=par2(p,56);
    lamda(58)=par2(p,57);   lamda(59)=par2(p,58);   lamda(60)=par2(p,59);   lamda(61)=par2(p,60);
    lamda(62)=par2(p,61);   lamda(63)=par2(p,62);   lamda(64)=par2(p,63);   lamda(65)=par2(p,64);
    lamda(66)=par2(p,65);   lamda(67)=par2(p,66);   lamda(68)=par2(p,67);   lamda(69)=1;
   freq(1,12)=x(p,1);   freq(1,20)=x(p,2);   freq(1,34)=x(p,3);   freq(1,38)=x(p,4);
   freq(1,42)=x(p,5);   freq(1,46)=x(p,6);   freq(1,50)=x(p,7);   freq(1,53)=x(p,8);
   freq(1,69)=x(p,9);   freq(12,20)=x(p,10);   freq(12,34)=x(p,11);   freq(12,38)=x(p,12);
   freq(12,42)=x(p,13);   freq(12,46)=x(p,14);   freq(12,50)=x(p,15);   freq(12,53)=x(p,16);
   freq(12,69)=x(p,17);   freq(20,34)=x(p,18);   freq(20,38)=x(p,19);   freq(20,42)=x(p,20);
   freq(20,46)=x(p,21);   freq(20,50)=x(p,22);   freq(20,53)=x(p,23);   freq(20,69)=x(p,24);
   freq(34,38)=x(p,25);   freq(34,42)=x(p,26);   freq(34,46)=x(p,27);   freq(34,50)=x(p,28);
   freq(34,53)=x(p,29);   freq(34,69)=x(p,30);   freq(38,42)=x(p,31);   freq(38,46)=x(p,32);
   freq(38,50)=x(p,33);   freq(38,53)=x(p,34);   freq(38,69)=x(p,35);   freq(42,46)=x(p,36);
   freq(42,50)=x(p,37);   freq(42,53)=x(p,38);   freq(42,69)=x(p,39);   freq(46,50)=x(p,40);
   freq(46,53)=x(p,41);   freq(46,69)=x(p,42);   freq(50,53)=x(p,43);   freq(50,69)=x(p,44);
   freq(53,69)=x(p,45);   freqexp(1,69)=x(p,46);
   freq(12:69,1)=freq(1,12:69);   freq(20:69,12)=freq(12,20:69);
   freq(34:69,20)=freq(20,34:69);   freq(38:69,34)=freq(34,38:69);
   freq(42:69,38)=freq(38,42:69);   freq(46:69,42)=freq(42,46:69);
   freq(50:69,46)=freq(46,50:69);   freq(53:69,50)=freq(50,53:69);
   freq(69,53)=freq(53,69);   freqexp(69,1)=freqexp(1,69);
   [tW]=waittime(n,alfaW,freqexp,lamda,freq);
   [Tloc,Texp]=vehtraveltime(n,len, vol, dwl, lamda,layover);
%_____________________________________________________________
%        Determine transfer stops (y) and (z)
 [z,y]=mintravel(n, alfaR, freq, freqexp, len, vol, dwl, lamda,z,y,S);
[tR]=transfertime(n, alfaR, freq, freqexp,z,y);
[tV]=invehicletime(n, len, vol, dwl, lamda,z,y);
[dem]=demand(n,q, lamda, alfaR, len,vol,dwl, alfaW,freqexp,freq,z,y,tetW,tetR,tetV);
[z1,y1]=mintravel(n, alfaR, freq, freqexp, len, vol, dwl, lamda,z,y,S);
while (z1 ~= z) & (y1~=y)
    [z,y]=mintravel(n, alfaR, freq, freqexp, len, vol, dwl, lamda,z,y,S);
    [tR]=transfertime(n, alfaR, freq, freqexp,z,y);
    [tV]=invehicletime(n, len, vol, dwl, lamda,z,y);
    [dem]=demand(n,q, lamda, alfaR, len,vol,dwl, alfaW,freqexp,freq,z,y,tetW,tetR,tetV);
    [z1,y1]=mintravel(n, alfaR, freq, freqexp, len, vol, dwl, lamda,z,y,S);
end %while
[z,y]=mintravel(n, alfaR, freq, freqexp, len, vol, dwl, lamda,z,y,S);
[tR]=transfertime(n, alfaR, freq, freqexp,z,y);
[tV]=invehicletime(n, len, vol, dwl, lamda,z,y);
[dem]=demand(n,q, lamda, alfaR, len,vol,dwl, alfaW,freqexp,freq,z,y,tetW,tetR,tetV);
[hdwL,hdwE,HmaxL,HmaxE]=constraints(n,c,freqexp,freq,dem,z,y);
if hdwL<=HmaxL & hdwE<=HmaxE                          % check demand Constraints
   i=1:n;
   j=1:n;
   k=1:4;
```

```
    wt=dem(i,j,k).*tW(i,j,k);
    tt=dem(i,j,k).*tR(i,j,k);
    ivt=dem(i,j,k).*tV(i,j,k);
    sw=freq(i,j).*Tloc(i,j)+freqexp(i,j).*Texp(i,j);
    WaitCost=sum(sum(sum(wt))).*uservalue;
    TransferCost=sum(sum(sum(tt))).*uservalue;
    InVehicleCost=sum(sum(sum(ivt))).*uservalue;
    SupplierCost=sum(sum(sw)).*operatorcost;
   f(p,1)=WaitCost+TransferCost+InVehicleCost+SupplierCost; % total cost/capacity const. satisfied
else
   f(p,1)=999999999999999;                              % total cost /with Penalty Value
end
end
end
%       End of TotalCost.m
%_______________________________________________________________
%_______________________________________________________________
%       demand.m
%       Demand distribution among four categories k=1 to 4
function [dem]=demand(n,q, lamda, alfaR, len,vol,dwl, alfaW,freqexp,freq,z,y,tetW,tetR,tetV)
[psi] = perexp(n,alfaR,len,vol,dwl,alfaW,freqexp,lamda,freq,z,y,tetW, tetR,tetV);
[phi]=perlocexp(n,len, vol, dwl, lamda, alfaR, freq, freqexp,z,y, tetR, tetV);
dem=zeros(n,n,4);
for i=1:n
   for j=1:n
      dem(i,j,1)=((q(i,j).*psi(i,j)).*lamda(i)).*lamda(j);
      if i<j && i<z(i,j) && z(i,j) <j
         dem(i,j,2)=((q(i,j).*psi(i,j)).*lamda(i)).*(1-lamda(j));
      elseif i>j && i>z(i,j) && z(i,j)>j
         dem(i,j,2)=((q(i,j).*psi(i,j)).*lamda(i)).*(1-lamda(j));
      else
         dem(i,j,2)=0;
      end
      if i<j && i<y(i,j) && y(i,j) <j
         dem(i,j,3)=(q(i,j).*(1-psi(i,j))).*lamda(i)+(((q(i,j).*(1-psi(i,j))).*(1-lamda(i))).*(1-lamda(j)))+((q(i,j).*((1-
psi(i,j))).*(1-lamda(i))).*lamda(j)).*(1-phi(i,j));
         dem(i,j,4)=(q(i,j).*(1-psi(i,j)).*(1-lamda(i)).*lamda(j)).*phi(i,j);
      elseif i>j && i>y(i,j) && y(i,j)>j
         dem(i,j,3)=(q(i,j).*(1-psi(i,j))).*lamda(i)+(((q(i,j).*(1-psi(i,j))).*(1-lamda(i))).*(1-lamda(j)))+((q(i,j).*((1-
psi(i,j))).*(1-lamda(i))).*lamda(j)).*(1-phi(i,j));
         dem(i,j,4)=(q(i,j).*(1-psi(i,j)).*(1-lamda(i)).*lamda(j)).*phi(i,j);
      else
         dem(i,j,3)=q(i,j).*(1-((psi(i,j).*lamda(i)).*lamda(j)));
         dem(i,j,4)=0;
      end
   end
end
end
%       End of demand.m
%_______________________________________________________________
%_______________________________________________________________
%       waittime.m
%       Waiting time calculation
%        tW: Wait time for passengers
function[tW]=waittime(n,alfaW,freqexp,lamda,freq)
tW=zeros(n,n,4);
for i=1:n
```

```
    for j=1:n
      if sum(sum(freqexp))~=0 && lamda(i)==1
        if i<j
          s=1:i;
          t=j:n;
          tW(i,j,1)=alfaW./sum(sum(freqexp(s,t)));
          tW(i,j,2)=(alfaW./sum(sum(freqexp(s,t))));
        end
        if i>j
          s= i:n;
          t= 1:j;
          tW(i,j,1)=alfaW./sum(sum(freqexp(s,t)));
          tW(i,j,2)=(alfaW./sum(sum(freqexp(s,t))));
        end
      end
      if i<j
        s=1:i;
        t=j:n;
        tW(i,j,3)=alfaW./sum(sum(freq(s,t)));
        tW(i,j,4)=alfaW./sum(sum(freq(s,t)));
      end
      if i>j
        s= i:n;
        t= 1:j;
        tW(i,j,3)=alfaW./sum(sum(freq(s,t)));
        tW(i,j,4)=alfaW./sum(sum(freq(s,t)));
      end
    end
  end
  end
  %      End of waittime.m
  %_____________________________________________________________
  %_____________________________________________________________
  %      transfertime.m
  %      Transfer time calculation
  %      tR: transfer time for passengers
  function[tR]=transfertime(n, alfaR, freq, freqexp,z,y)
  tR=zeros(n,n,4);
  for i=1:n
    for j=1:n
      if i<j && i<z(i,j) && z(i,j)<j
        s=1:z(i,j);
        t=j:n;
        tR(i,j,2)=alfaR./sum(sum(freq(s,t)));
      end
      if i>j && i>z(i,j) && z(i,j)>j
        s=z(i,j):n;
        t=1:j;
        tR(i,j,2)=alfaR./sum(sum(freq(s,t)));
      end
      if i<j && i<y(i,j) && y(i,j)<j
        s=1:y(i,j);
        t=j:n;
        tR(i,j,4)=alfaR./sum(sum(freqexp(s,t)));
      end
      if i>j && i>y(i,j) && y(i,j)>j
        s=y(i,j):n;
```

```
      t=1:j;
      tR(i,j,4)=alfaR./sum(sum(freqexp(s,t)));
    end
  end
end
end
%       end of transfertime.m
%______________________________________________________________
%______________________________________________________________
%       invehicletime.m
%       in-vehicle time calculation
%       tV: in-vehicle time for passengers
function[tV]=invehicletime(n, len, vol, dwl, lamda,z,y)
tV=zeros(n,n,4);
for i=1:n
  for j=1:n
    if i<j
      d=i:j-1;
      tV(i,j,1)=sum((len(d)./vol(d))+(dwl(d+1).*lamda(d+1)));
      tV(i,j,3)=sum((len(d)./vol(d))+dwl(d+1));
    end
    if i>j
      d=j:i-1;
      tV(i,j,1)=sum((len(d)./vol(d))+(dwl(d+1).*lamda(d+1)));
      tV(i,j,3)=sum((len(d)./vol(d))+dwl(d+1));
    end
  end
end
for i=1:n
  for j=1:n
    if i<j
      if i<z(i,j)  && z(i,j)<j
        tV(i,j,2)=tV(i,z(i,j),1)+tV(z(i,j),j,3);
      end
      if i<y(i,j) && y(i,j)<j
        tV(i,j,4)=tV(i,y(i,j),3)+tV(y(i,j),j,1);
      end
    end
    if i>j
      if i>z(i,j) && z(i,j)>j
        tV(i,j,2)=tV(i,z(i,j),1)+tV(z(i,j),j,3);
      end
      if i>y(i,j) && y(i,j)>j
        tV(i,j,4)=tV(i,y(i,j),3)+tV(y(i,j),j,1);
      end
    end
  end
end
end
%       End of invehicletime.m
%______________________________________________________________
%______________________________________________________________
%       mintravel.m
%       Determines the transfer location (z) and (y) for Case II
%       z: transfer stop from express to local service
%       y: transfer stop from local to express service
%
```

```
function[z,y]=mintravel(n, alfaR, freq, freqexp, len, vol, dwl, lamda,z,y,S)
 [tR]=transfertime(n, alfaR, freq, freqexp,z,y);
[tV]=invehicletime(n, len, vol, dwl, lamda,z,y);
trvexploc=99999999999.*ones(n,n,n);
trvlocexp=99999999999.*ones(n,n,n);
for i=1:n
   for j=1:n
      if i<j
         for zx=i+1:j-1
            if  S(zx)==1 && lamda(zx)==1 && i<zx && zx<j
               trvexploc(zx,i,j) = tV(i,zx,1)+tR(i,j,2)+tV(zx,j,3);
            end
         end
         for yx=i+1:j-1
            if S(yx)==1 && lamda(yx)==1 && i<yx && yx<j
               trvlocexp(yx,i,j)= tV(i,yx,3)+tR(i,j,4)+tV(yx,j,1);
            end
         end
      end
      if i>j
         for zx=j+1:i-1
            if S(zx)==1 && lamda(zx)==1 && i>zx && zx>j
               trvexploc(zx,i,j) = tV(i,zx,1)+tR(i,j,2)+tV(zx,j,3);
            end
         end
         for yx= j+1:i-1
            if  S(yx)==1 && lamda(yx)==1 && i>yx && yx>j
               trvlocexp(yx,i,j)= tV(i,yx,3)+tR(i,j,4)+tV(yx,j,1);
            end
         end
      end
   end
end
[trvexp, zb]=min(trvexploc);
[trvloc, yb]= min(trvlocexp);
zxt=[zb(:,:,1);zb(:,:,2);zb(:,:,3);zb(:,:,4);zb(:,:,5);zb(:,:,6);zb(:,:,7);zb(:,:,8);zb(:,:,9);zb(:,:,10);zb(:,:,11);zb(:,:,12);zb(:,:,1
3);zb(:,:,14);zb(:,:,15);zb(:,:,16);zb(:,:,17);zb(:,:,18);zb(:,:,19);zb(:,:,20);zb(:,:,21);zb(:,:,22);zb(:,:,23);zb(:,:,24);zb(:,:,25
);zb(:,:,26);zb(:,:,27);zb(:,:,28);zb(:,:,29);zb(:,:,30);zb(:,:,31);zb(:,:,32);zb(:,:,33);zb(:,:,34);zb(:,:,35);zb(:,:,36);zb(:,:,37);
zb(:,:,38);zb(:,:,39);zb(:,:,40);zb(:,:,41);zb(:,:,42);zb(:,:,43);zb(:,:,44);zb(:,:,45);zb(:,:,46);zb(:,:,47);zb(:,:,48);zb(:,:,49);z
b(:,:,50);zb(:,:,51);zb(:,:,52);zb(:,:,53);zb(:,:,54);zb(:,:,55);zb(:,:,56);zb(:,:,57);zb(:,:,58);zb(:,:,59);zb(:,:,60);zb(:,:,61);zb
(:,:,62);zb(:,:,63);zb(:,:,64);zb(:,:,65);zb(:,:,66);zb(:,:,67);zb(:,:,68);zb(:,:,69)];
yxt=[yb(:,:,1);yb(:,:,2);yb(:,:,3);yb(:,:,4);yb(:,:,5);yb(:,:,6);yb(:,:,7);yb(:,:,8);yb(:,:,9);yb(:,:,10);yb(:,:,11);yb(:,:,12);yb(:,:,1
3);yb(:,:,14);yb(:,:,15);yb(:,:,16);yb(:,:,17);yb(:,:,18);yb(:,:,19);yb(:,:,20);yb(:,:,21);yb(:,:,22);yb(:,:,23);yb(:,:,24);yb(:,:,25
);yb(:,:,26);yb(:,:,27);yb(:,:,28);yb(:,:,29);yb(:,:,30);yb(:,:,31);yb(:,:,32);yb(:,:,33);yb(:,:,34);yb(:,:,35);yb(:,:,36);yb(:,:,37);
yb(:,:,38);yb(:,:,39);yb(:,:,40);yb(:,:,41);yb(:,:,42);yb(:,:,43);yb(:,:,44);yb(:,:,45);yb(:,:,46);yb(:,:,47);yb(:,:,48);yb(:,:,49);y
b(:,:,50);yb(:,:,51);yb(:,:,52);yb(:,:,53);yb(:,:,54);yb(:,:,55);yb(:,:,56);yb(:,:,57);yb(:,:,58);yb(:,:,59);yb(:,:,60);yb(:,:,61);yb
(:,:,62);yb(:,:,63);yb(:,:,64);yb(:,:,65);yb(:,:,66);yb(:,:,67);yb(:,:,68);yb(:,:,69)];
zl=(zxt>=2);
yl=(yxt>=2);
z=zl.*zxt;
y=yl.*yxt;
end
%        End of mintravel.m
%_____________________________________________________________
%_____________________________________________________________
%        perexp.m
%        Percentage of demand selecting express service
```

```
function [psi] = perexp(n,alfaR,len,vol,dwl,alfaW,freqexp,lamda,freq,z,y,tetW, tetR,tetV)
psi=zeros(n,n);
ab=zeros(n,n);
bc=zeros(n,n);
[tW]=waittime(n,alfaW,freqexp,lamda,freq);
[tR]=transfertime(n, alfaR, freq, freqexp,z,y);
[tV]=invehicletime(n, len, vol, dwl, lamda,z,y);
for i=1:n
   for j=1:n
      if i~=j && sum(sum(freqexp))~=0
         k=1:2;
         ab(i,j)=sum(sum(exp(-60*(tetW.*tW(i,j,k)+tetR.*tR(i,j,k)+tetV.*tV(i,j,k)))));
         k=1:4;
         bc(i,j)=sum(sum(exp(-60*(tetW.*tW(i,j,k)+tetR.*tR(i,j,k)+tetV.*tV(i,j,k)))));
         psi(i,j)=(ab(i,j)/bc(i,j)).*lamda(i);
      end
   end
end
end
%        end of perexp.m
%_____________________________________________________________
%_____________________________________________________________
%        perlocexp.m
%        Percentage of local to express demand
function [phi]=perlocexp(n,len, vol, dwl, lamda, alfaR, freq, freqexp,z,y, tetR, tetV)
 [tR]=transfertime(n, alfaR, freq, freqexp,z,y);
[tV]=invehicletime(n, len, vol, dwl, lamda,z,y);
b=zeros(n,n);
c=zeros(n,n);
a=zeros(n,n);
phi=zeros(n,n);
for i=1:n
   for j=1:n
      if (i<y(i,j) && y(i,j)<j) || (i>y(i,j) && y(i,j)>j)
         a(i,j)=60*tetR.*tR(i,j,4);
         b(i,j)=60*tetV.*tV(y(i,j),j,1);
         c(i,j)=60*tetV.*tV(y(i,j),j,3);
         phi(i,j)=((exp(-(a(i,j)+b(i,j)))))./(exp(-(a(i,j)+b(i,j)))+exp(-c(i,j)))).*lamda(j);
      else
         phi(i,j)=0;
      end
   end
end
%        End of perlocexp.m
%_____________________________________________________________
%_____________________________________________________________
%        vehtraveltime.m
%        Calculate  vehicle travel time
%        Tloc: local vehicles travel time
%        Texp: express vehicles travel time
function[Tloc,Texp]=vehtraveltime(n,len, vol, dwl, lamda,layover)
Tloc=zeros(n,n);
Texp=zeros(n,n);
for i=1:n
   for j=1:n
      if i<j
         d=i:j-1;
```

```
        Tloc(i,j)=sum((len(d)./vol(d))+dwl(d+1))+layover;
        Texp(i,j)=sum((len(d)./vol(d))+dwl(d+1).*lamda(d+1))+layover;
      end
      if i>j
        d=j:i-1;
        Tloc(i,j)=sum((len(d)./vol(d))+dwl(d+1))+layover;
        Texp(i,j)=sum((len(d)./vol(d))+dwl(d+1).*lamda(d+1))+layover;
      end
    end
end
end
%       End of vehtraveltime.m
%______________________________________________________________
%______________________________________________________________
%       constraints.m
%       Capacity constraints in Model II
%       hdwL: local service headway
%       hdwE: express service headway
%       HmaxL: maximum local service headway
%       HmaxE: maximum express service headway
function [hdwL,hdwE,HmaxL,HmaxE]=constraints(n,c,freqexp,freq,dem,z,y)
hdwL=zeros(1,n-1);
hdwE=zeros(1,n-1);
HmaxL=zeros(1,n-1);
HmaxE=zeros(1,n-1);
OutL=zeros(n-1,4);
inL=zeros(n-1,4);
OutE=zeros(n-1,4);
inE=zeros(n-1,4);
for l=1:n-1
    OutDdL=zeros(n,n,4);
    inDdL=zeros(n,n,4);
    OutDdE=zeros(n,n,4);
    inDdE=zeros(n,n,4);
    OutDemL=zeros(n-1,4);
    inDemL=zeros(n-1,4);
    OutDemE=zeros(n-1,4);
    inDemE=zeros(n-1,4);
    for s=1:l
      for t=l+1:n
        OutDdL(s,t,3)=dem(s,t,3);
        if  s<=z(s,t) && z(s,t)<=l
          OutDdL(s,t,2)=dem(s,t,2);
        end
        if  l<y(s,t) && y(s,t)<=t
          OutDdL(s,t,4)=dem(s,t,4);
        end
        OutDdE(s,t,1)=dem(s,t,1);
        if  l<z(s,t) && z(s,t)<=t
          OutDdE(s,t,2)=dem(s,t,2);
        end
        if  s<=y(s,t) && y(s,t)<=l
          OutDdE(s,t,4)=dem(s,t,4);
        end
      end
    end
    for s=l+1:n
```

```
    for t=1:l
      inDdL(s,t,3)=dem(s,t,3);
      inDdE(s,t,1)=dem(s,t,1);
      if  s>=z(s,t) && z(s,t)>l
        inDdL(s,t,2)=dem(s,t,2);
      end
      if  l>=y(s,t) && y(s,t)>=t
        inDdL(s,t,4)=dem(s,t,4);
      end
      if  l>=z(s,t) && z(s,t)>=t
        inDdE(s,t,2)=dem(s,t,2);
      end
      if  s>=y(s,t) && y(s,t)>l
        inDdE(l,4)=dem(s,t,4);
      end
    end
  end
  for k=1:4
    s=1:l;
    t=l+1:n;
    OutDemL(l,k)=sum(sum(OutDdL(s,t,k)));
    OutDemE(l,k)=sum(sum(OutDdE(s,t,k)));
  end
  for k=1:4;
    s=l+1:n;
    t=1:l;
    inDemL(l,k)=sum(sum(inDdL(s,t,k)));
    inDemE(l,k)=sum(sum(inDdE(s,t,k)));
  end
  k= [2 3 4];
  OutL(l)=sum(OutDemL(l,k));
  inL(l)=sum(inDemL(l,k));
  HmaxL(l) =c./max(OutL(l),inL(l));
  k= [1 2 4];
  OutE(l)=sum(OutDemE(l,k));
  inE(l)=sum(inDemE(l,k));
  HmaxE(l) =c./max(OutE(l),inE(l));
  s=1:l;
  t=l+1:n;
  hdwL(l)=1/sum(sum(freq(s,t)));
  hdwE(l)=1/sum(sum(freqexp(s,t)));
end
end
%       End of constraints.m
%______________________________________________________________
```

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