ABSTRACT

THE FLOATING CONTRACT BETWEEN RISK-AVERSE SUPPLY CHAIN PARTNERS IN A VOLATILE COMMODITY PRICE ENVIRONMENT

by
Mojisola Kike Otegbeye

In this dissertation, two separate but closely related decision making problems in environments of volatile commodity prices are addressed. In the first problem, a risk-averse commodity user's purchasing policy and his risk-neutral supplier's pricing decision, where the user can purchase his needs through contract with his supplier as well as directly from the spot market, are analyzed. The commodity user is assumed to be the supplier's sole client, and the supplier can always expand capacity, at a cost to the user, to accommodate the user's demand in excess of initially reserved capacity.

In the more generalized second problem, both parties (commodity user and supplier) are assumed to be risk averse, and both can directly access the spot market. In addition to making pricing decisions, the supplier is also faced with the challenge of establishing the right combination of in-house production and spot market engagements to manage her risk of exposure to spot price volatility under the contract. While the supplier has a frictionless buy and sell access to the spot market, the user can only access this market for buying purposes and incurs an access fee that is linearly increasing in the purchased volume.

In both problems, by adopting the mean-variance criterion to reflect aversion to risk, the decisions of both parties are explicitly characterized. Based on analytical results and numerical studies, managerial insights as to how changes in the model's parameters would affect each party's decisions are offered at length, and the implications of these results to the manager are discussed. A focal point for the dissertation is the consideration of a floating contract, the landing price of

which is contingent on the realization of the commodity's spot market price at the time of delivery. It was found that if properly designed, not only can this dynamic pricing arrangement strategically position a long-term supplier against spot market competition, but it also has the added benefit of leading to improved supply chain expected profits compared to a locked-in contract price setting. Another key finding is that when making her pricing decisions, the supplier runs the risk of over-estimating the commodity user's vulnerability at higher levels of the user's aversion to risk as well as at higher volatility of spot prices.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

Of recent, the global commodity markets have been experiencing increasing volatility of commodity prices, catalyzed by such factors as instability in global production volumes, constantly changing exchange rates, unstable political relations, and the rapid emergence of new technologies. In September of 2006, Purchasing magazine (Stundza, 2006), reported that the periodic reviews of metal price forecasts pushed up the world copper consensus average to $3.05/lb$, up from about 2.60 at midyear, and compared with just under $1.70/lb$ in 2005 while the consensus forecast for 2007 was a cloudy average of 2.60. In a related article by the same source, fluctuations in iron ore prices have led to substantial price increases from steel makers in recent years, peaking at 71.5 percent in 2005 and 18 percent in 2006.

The oil markets have also had their share of wild runs in recent times. As at July 2008, a barrel of crude oil sold for $145, and experts began to predict that it will hit $200 per barrel by December of the same year. However, as the global economy faltered, oil fell to $33 per barrel by that December, while as at July 2009, oil went for $70 per barrel, a 55 percent jump from its December 2008 price (Mouawad, 2009). In 2008, Southwest Airlines, a company well known for insuring itself against volatile prices by buying long-term oil contracts, reported two consecutive quarters of losses, as prices spiked and collapsed - all within a few months. According to a representative of the firm, "Prices were falling faster than we could de-hedge," (Mouawad, 2009).

Therefore, with increasing pressures on bottom lines, it becomes obvious that to stay competitive in today's era of escalating commodity prices, manufacturing firms must incorporate into the integrated acquisition/production planning process, the influence of commodity price volatility on decisions concerning commodity procurement strategies.

Arrow et al. (1951), and Dvoretzky et al. (1952) are among the pioneer works that form the foundation of modern stochastic inventory concepts. Girlich and Chikan (2001), gives a comprehensive study on the rise of inventory modeling under uncertainty particularly as a fallout of these two, while providing historical insight into the interrelation of mathematics and inventory modeling in searching for the best ordering rule, spanning the fields of statistics, probability theory and stochastic processes, game theory, and dynamic programming.

A review of literature reveals that for most documented works on stochastic inventory control policies, emphases have been placed on demand uncertainties, especially by the ones based on the celebrated newsvendor paradigm from the seminal work of Whitin (1955). This trend is hardly surprising given the dominant age long challenge of making inventory decisions in the face of uncertainties in demand, especially with the traditional make to stock manufacturing framework. The advent of just - in time inventory philosophies of the early 1970s (Vuppalapati (1995)) which advocates a lean approach to production, brought to the limelight, the critical role supplier reliability and/or relationship plays in facilitating manufacturing efficiency and this has largely stimulated the significantly rich literature on the subject of stochastic supply inventory control modeling. While Karlin (1958) was the first to consider the implications of yield uncertainty on inventory stocking decisions for an agricultural problem, it was the work of Silver (1976) in which the author incorporated the effect of yield uncertainty in the Economic Order Quantity (EOQ) framework that stimulated interests in researching the effects of random yields, particularly for the continuous review inventory models. An extensive survey of works in this arena of random supply is presented by Yano and Lee (1995).

The 1960s and 1970s witnessed a fundamental transition from a world dominated by Keynesian forms of national economic management to a world system dominated by the anarchy of global competition (Cypher, 1984). In particular, with the rise of OPEC in the 1970s came the oil supply shock that saw surging oil prices and the attendant inflationary pressures. This largely explains why the effects of raw material procurement price volatility on inventory policy began to gain attention in academic circles around this era. Some notable early works are those of Fabian et al. (1959), Friend (1960), Hurter and Kaminsky (1968), and Naddor (1966). While Fabian et al. (1959) pioneered research on fluctuating commodity procurement costs where they consider the case of deterministic inventories for which the price of the raw material varies from period to period, it was Kalymon (1971) who attracted scholarly interests on multiple purchase price levels by showing the optimality of a price-dependent (s, S) policy where the purchase prices of future periods assume a Markovian stochastic process and the distribution of demand in each period depends on the current purchase price. Buzacott's (1975) incorporation of inflationary effects on optimal ordering decisions stimulated the advent of research interests on how continuous changes in purchase price affect inventory policies with most of these works assuming deterministic future price with constant rate of change (Berling (2008)).

With increasing uncertainty of supply networks, globalization of businesses, product proliferation, and shortening of product life cycles, organizations are increasingly forced to look beyond their four walls to collaborate with supply chain partners (Sahay, and Mohan, 2003). Supply chain management integrates supply and demand management within and across channel partners and the coordination of all the chain's activities. Coordination is particularly important because supply chain partners have conflicting objectives, so that for the optimal supply chain performance to be realized, various incentive re-alignments must be made among channel partners.

To address this issue of supply chain coordination, several authors have offered different contractual frameworks to provide incentives for each channel partner to align it's objective with that of the supply chain. In these research efforts, a large amount of attention has been directed towards designing contracts that incorporate such features as quantity discounts (Starr and Miller (1962), Monahan (1984), and Viswanthan and Wang (2003)), returns or buy-backs (Pasternack (1985) and Lariviere and Porteus (2001)), revenue sharing (Pasternack (2001), Wang et al. (2004), Giannoccaro and Pontradolfo (2004), and Cachon and Lariviere (2005)), minimum quantity commitments (Bassok and Anupindi (1997)), and quantity flexibility (Tsay (1999)). For a detailed review of such supply chain contracts, the reader is referred to Tsay et al. (1999) and Cachon (2003).

While the supply chain literature on channel coordinating contracts is very extensive, major emphasis has been placed on where the need for coordination is driven by demand uncertainties. In contrast, scant attention has been given to the situation where supply chain partners are faced with the dilemma of making decisions in fluctuating input commodity price environments that are characteristic of most of today's commodity markets. Even among the few works that take commodity price volatility into account, most adopt the expected-value optimization approach in addressing the decision maker's problem thereby making the assumption that the decision maker is risk neutral.

In reality, the decision maker may be willing to sacrifice higher profits for lower but more stable earnings. In their experimental study, Schweitzer and Cachon (2000) found their subjects to exhibit risk-aversion towards high-value products as they systematically ordered amounts lower than that which maximizes the expected profit. Eeckhoudt et al. (1995) also showed that the risk-averse newsboy will systematically place a smaller order than that which maximizes expected profit. In a 2007 McKinsey global survey on funds allocation, more than 40 percent of the respondents described their companies as risk averse. Therefore, the need for models that capture the risk averse behavior of the decision maker (DM) as a key input in establishing the DM's optimal choices cannot be overemphasized. In my dissertation, addressing a commodity user's sourcing allocation strategy between a long-term supplier (also called the commodity producer) and an alternative spot market, I incorporate the effect of risk aversion on the user's and supplier's decision making.

In practice, it has been found that while some firms have preference for fixed contract pricing, others prefer time-varying contract price settings. For example, an executive at BHP, the world's third largest iron ore miner noted in a recent interview that while some of its clients prefer floating contract prices, the big steel makers prefer stable long-term contract prices, and in response to this realization, the company is shifting towards a mixed pricing mechanism (International Business Times, 2009). However, for virtually all the works documented in literature, there is either the inherent assumption that the buyer will settle for a fixed price contract, or the assumption that the buyer's preference is strictly a non-stationary price contract. Hence, the design of a mixed pricing contract that allows the capture of the buyer's pricing preference will no doubt constitute an important contribution to the existing supply chain contracting literature. In this dissertation, I propose a flexible contract structure that accommodates both pricing preferences. Specifically, under the proposed contract, the buyer (referred to as the commodity user) has the flexibility of requesting a purely fixed price contract or specifying the level of exposure to a future spot price of the commodity that he seeks under the contract.

In the supply chain contracting literature that addresses a buyer's optimal purchase allocation between his long-term supplier(s) and the spot market given the volatility of the commodity's spot price, there are those works that assume an unconstrained capacity for the supplier and those that take the on approach that due to the significance of installation lead time, the supplier needs to invest in capacity well ahead of the receipt of a firm order from the buyer. Very popular among the latter category are the capacity reservation contracts (e.g. Wu et al., (2002), Sethi and Feng, (2008)), and one thing these works share in common is that any demand in excess of the planned capacity will result in lost sales to the supplier. In practice however, a capacity constrained producer can temporarily expand capacity through such means as running overtime production, leasing workstations, and subcontracting, while charging its client a premium for making such temporary arrangements to accommodate the excess demand. Such flexible capacity management initiatives have been quite extensively studied in the context of stochastic demand as the underlying source of uncertainty (Mincsovics et al. (2009)).

In the first problem addressed in this dissertation, termed the purchasing, and pricing problem, I adopt the approach that while a commodity producer (supplier) initially invests in an agreed upon capacity for the commodity user, she can temporarily expand capacity to satisfy a contract order that exceeds the dedicated capacity. I make the assumption that the commodity user knows the demand for the refined commodity ahead of its realization, and what drives the uncertainty of the order he eventually places with contract is the unstable spot price of the commodity, as he is torn between purchasing from the supplier and purchasing directly from the spot market.

While seeking to maximize her share of her client's commodity needs, the supplier makes her pricing decisions in the same uncertain spot market conditions that the commodity user faces. It can thus be reasonably expected that in practice, at the time of contract negotiation, the supplier's offered price would be reflective of the spot price behavior as well her anticipation of the commodity user's response. However, with the exception of a few like Kleindorfer and Wu (2003) and Wu et al. (2002), almost all of the existing works do not reflect if and how the supplier sets the contract price based on these considerations, but rather, the focus is for the most part, on the buyer's transaction choice, given already set contract parameters. In this dissertation, I consider the interplay of the supplier's and commodity user's optimal choices.

Lastly, virtually all works in the relevant literature that take into account the buyer's and supplier's spot market accessibility ignore the possibility of the existence of a disparity in both parties' spot market transaction efficiencies. Most assume for example, that for either party, the total expense involved in using the spot market at any given point in time is the prevailing spot price. This is quite contrary to what is obtained in practice whereby if we factor in the differences in administrative costs for instance, the actual cost of spot market engagements can be unique to each market participant. In the second problem addressed in this dissertation, termed the purchasing, pricing, and production problem, I assume that the supplier has a frictionless access to the spot market, and the only expense she incurs in purchasing from this market is the spot price of the commodity. For the commodity user however, aside the spot price of the commodity, I assume that he also incurs a fee to access the spot market, and this fee is linearly increasing in the purchased volume. On the premise that the supplier is a major player in the industry of concern, the spot market price disparity assumption closely reflects what would be obtained in practice where a major supplier can leverage economies of scale superior knowledge on how to efficiently navigate the market in realizing negligible market transaction costs (exclusive of the spot price) in contrast to a smaller purchasing firm (the commodity user in this case).

One can imagine the spot access fee to be a cost the user incurs in finding the commodity producers with surplus in the spot market. The fee could also be viewed as the cost incurred by the user as a result of his last minute ordering from the spot market jeopardizing his timely delivery to the end customer (there is typically a short time lag between the time the user places his order with a supplier in the spot market and when he takes delivery of the commodity). The idea of a buyer incurring the spot price plus some additional transaction cost for each unit purchased from the spot market is well established in literature (see Kaminsky et al. (2009), and Pei (2008)). According to Pei (2008), the cost of purchasing units from the spot market is typically higher than the quoted spot price due to hidden transaction, adaptation and compatibility costs, stemming from issues like asset specificity, time specificity, product compatibility, and transactional complexity. It is worth noting that in this dissertation, the underlying cause of the disparity in the user's and supplier's spot market expense is the differences in their spot market transaction efficiencies. Neither party is assumed to have any perceptible influence on the spot price.

1.2 Objectives

This dissertation studies a purchasing firm's sourcing strategies and his supplier's choices in environments of volatile commodity prices. The dissertation is a two-part problem addressing two closely related problems. A more restrictive version of the second problem and termed the purchasing and pricing problem, the first problem seeks to derive the optimal allocation of a risk-averse commodity user's (also called user for brevity) purchasing needs between a long-term risk-neutral commodity producer (supplier) and an alternative spot market, as well as the optimal pricing decision of the supplier. In contrast to the common approach in the related literature that constrain the contract price to be either strictly fixed or strictly time-varying, I propose a contract structure that captures both cases thus providing the user the flexibility of selecting his pricing preference. In addition, while it is common in the related literature to assume that a supplier cannot satisfy demand in excess of the capacity she initial reserved for the user, I make allowance for temporary capacity expansion to satisfy excess demand.

The more elaborate second problem, termed the purchasing, pricing, and production problem, adopts the same contract pricing arrangement as the first, but relaxes the assumptions that the supplier is risk-neutral and does not have access to an alternative market (the spot market in this case), and that the user's access to the spot market is always without friction. In addition to the decisions addressed by the first problem, the latter problem also tackles the production (supply strategy) decisions of the supplier given her need to manage the risks associated with the proposed dynamic contract.

1.3 Contributions to the Literature

From the two problems to be studied, my dissertation aims at contributing to the supply chain contracting literature in the following distinct ways:

For the purchasing and pricing problem:

1. I propose a flexible contract pricing arrangement that offers the risk averse commodity user the flexibility to opt for a purely fixed price setting or a dynamic contract price that is tied to the realization of the spot market price at the time of delivery of the sourced commodity.

2. I make provisions for temporary capacity expansion so that the supplier can accommodate demand in excess of the user's reserved capacity to more closely reflect what happens in practice.

3. For both parties (commodity user and supplier), I obtain closed form optimal solutions that are reflective of the supplier's anticipation of the user's reaction to the offered contract price. In great detail, through analytical results and numerical studies, I provide managerial insights as to how changes in the parameters of the model will affect the optimal decisions of both parties. I also discuss the managerial implications of some of the more interesting findings.

The results from the analysis of the purchasing, and pricing problem reveal that when the random price component that the commodity user experiences under the contract is "low enough", not only can the proposed dynamic contract pricing arrangement strategically position a long-term supplier against spot market competition, but it also has the added benefit of leading to improved supply chain expected profits compared to a fixed contract price setting.

For the more elaborate purchasing, pricing, and production problem:

1. I again consider the same contract pricing arrangement proposed in the previous problem.

However, in this problem, both the commodity user and supplier are now considered to be risk-averse.

2. I consider the possibility of the risk-averse supplier using spot market purchases to actively manage the risk of the realized contract price falling below her production cost owing to unanticipated decrease in spot prices. It is not unusual for the spot price of a commodity to fall below its cost of production. For example, by the end of 1998, West Texas Intermediate Crude Oil (WTI) spot prices fell to near $10 per barrel , while the average production cost for twenty four of the larger United States oil companies was $8.60 per barrel, with many oil fields incurring as much as four times this average (Williams (1999)). The question of the right combination of in-house production and spot market purchases to meet the user's demand is addressed for the supplier.

3. To better reflect what is usually obtained in practice, I factor in the user's possible non-frictionless spot market access. This is in contrast to existing works in the related literature that simply assume that the only expense to the buyer in making spot market purchases is the spot market price..

4. Closed form solutions are derived and in great detail, I provide managerial insights as to how changes in the parameters of the model will affect the decisions of both parties.

The results from the analysis of the purchasing, pricing, and production problem reveal that there exists a certain threshold on the user's spot market access fee, above which his equilibrium strategy is always to solely purchase his commodity needs from contract with the supplier. Below this threshold, the user's equilibrium strategy is to use a combination of contract and spot market purchases to satisfy his needs. Where the user's equilibrium strategy is to solely source from contract, conditions were found under which the floating contract improves the user's equilibrium expected profits, while such pricing structure was discovered to be always detrimental to the supplier's and overall supply chain's expected profits. On the other hand, where the user's equilibrium strategy is to use a combination of contract and spot market, and incurs a non-zero fee to access the spot market, the floating contract always results in win-win outcomes for both the user and supplier to the overall benefit of the supply chain. However, when the user's only spot market expense is the spot price, the floating contract yields the same equilibrium expected profits and equilibrium variances of profit that would be obtained with a fixed contract. Another crucial finding is that the supplier's equilibrium variance of profit is independent of the contract's exposure to the spot price, so that in terms of gaining greater control over variable profits, the supplier is no better off with a locked in contract price.

1.4 Dissertation Structure

The rest of the dissertation is organized as follows. Following this introduction, chapter 2 provides a detailed review of the relevant literature. In this chapter, I categorize the related literature that address decision making in a volatile commodity price framework into three groups - Expected Value Optimization models, Expected Utility Optimization models, and Mean-Variance Optimization models. I also discuss the rationale behind the focus of this dissertation on the use of forward contracts to hedge commodity price increases as against the use of derivatives.

Chapter 3 addresses the purchasing, and pricing problem. The problem description for the commodity user as well as the commodity producer (henceforth called the supplier) is furnished. The models to address both parties' problems are presented and closed-form equilibrium solutions are derived. Using comparative statics analysis, the influences of model parameters on the decisions of both parties are studied, and the performance of the proposed contract price structure is compared to that of a fixed price setting. Important managerial implications of some relevant results are discussed.

In chapter 4, the more intricate purchasing, pricing, and production problem is studied. The commodity user's and supplier's problems are defined, and the relevant models are presented and analyzed. Closed-form equilibrium solutions are derived and comparative statics analysis conducted. Managerial insights are drawn from analytical results and important managerial implications are highlighted.

Chapter 5 presents the concluding remarks, summarizing the findings in this dissertation, while Chapter 6 discusses future research directions.

CHAPTER 2

LITERATURE REVIEW

2.1 Purchasing Models in a Volatile Commodity Price Framework

With the rapid growth of online spot markets for a broad range of supplies like memory chips, chemicals, energy, telecommunication bandwidth and so forth, companies are increasingly adopting spot market procurement alongside the traditional long-term contracting for their input needs (Seifert et al. (2004)). While the spot market is without doubt riskier than term contracts particularly given the variability of spot prices, its appeal over term contracts is the flexibility it offers market participants to take advantage of price dips, better adjust to fluctuating demand, and its negligible delivery lead time. For example, the spot market is estimated to represent about 20% of total iron-ore trade (Kinch (2008)) and 20% DRAM chips market (Crane (2007)). In a rather dramatic turn of events, while in 2008, spot market sales accounted for about 30% of iron-ore trades in China, by 2009, the country's iron-ore spot sales had jumped to 60% (Scott and Gordon (2009)).

The rising significance of the spot market as a viable commodity-procurement mechanism has caught the attention of the academia and a good number of researchers have either contrasted long-term to spot contracting or analyzed the optimal balance between the two. Others have simply focussed on designing the optimal procurement policy for a term contract or spot market trade. What these works all have in common is that purchasing decision is made in the face of spot price uncertainty. This chapter discusses some of these. From my extensive review of literature, I categorize the related works into three major groups based on the assumption made on the risk preference of the decision maker and the optimization approach adopted to establish the optimal choices. The first group of documented works assume risk neutrality for the decision maker (DM) and model the DM's objective as an expected-value optimization problem. The second group of documented literature assume that the decision maker is risk averse and adopt the expected utility optimization concept in modeling the DM's problem. The third group of documented works also assume that the decision maker is risk averse, but adopt the mean-variance optimization maxim in establishing the optimal decisions.

2.1.1 Expected-Value Optimization Models

Given two alternatives with the same expected value, a risk-neutral decision maker equally prefers the alternative with a risk-free outcome and that with a risky outcome. This is because the risk neutral decision maker is willing to play the long-run odds when making decisions, and evaluates alternatives according to their expected values. Such approach to risk may be justified when decisions are made over a large number of the realizations of the random event so that the Law of Large Numbers can be invoked. Suppose the decision maker's payoff at some future time, when viewed at the current decision time is a random $X(v)$ subject to his control v in some range V. Then, the risk neutral decision maker seeks to solve the expected-value optimization problem:

$$w^* = \max_{v \in V} E[X(v)], \tag{2.1}$$

where $E[X(v)]$ is the expected payoff. This optimization model can be modified accordingly to accommodate the case where the preference is to minimize a given performance measure like expected cost.

Given that the validity of risk-neutral decision making is closely tied to the applicability of the Law of Large Numbers, the expected-value optimization approach to decision making under risk becomes less suitable as the assumption that the decision maker seeks to optimize over some long-run performance measure is relaxed. This modeling technique appears however to be the most common approach that has been adopted by works in the literature that address the decision making of a buyer and sometimes of his supplier(s) given that the price of the sourced commodity is random. I now discuss some of these.

Li and Kouvelis (1999) studied impacts of time and quantity flexibility, as well as risk sharing on sourcing contracts when the purchase price of the sourced component is uncertain and its demand is known. With the objective of establishing the purchasing strategy that minimizes the buyer's expected discounted total cost, their model did not take into account the risk from the uncertainty of the material's unit price. Under the time-flexible contract, the firm observes the price movement and dynamically decides when to buy. For the risk sharing contract, a factor is incorporated to represent how much the supplier is going to share the risk with the firm and vice versa when the unit price moves against either parties. Their results reveal that contractual flexibility can effectively reduce sourcing costs in environments of price uncertainty. Closely related to the work of Li and Kouvelis is that of Fotopoulos et al. (2008) in which the authors provide a framework for identifying the expected optimum purchasing time.

Swinney and Netessine (2009) consider a risk neutral buyer with two identical suppliers to contract from. The suppliers are faced with uncertain production costs and the contract price is partially tied to the stochastic production cost component that is common to both suppliers. The authors find that where there is the possibility of supplier failure, for a long term contractual relationship, such dynamic contracts facilitate the realization of system optimal profits.

Although the practice of making trade-offs between the price stability offered by long term contracts and the speculative and flexibility benefits offered by short term contracts has been in place among practitioners for some time, especially among hi-tech companies (a classic example is the procurement risk management (PRM) initiative developed by Hewlett-Packard in mid 2000 (Nagali et al., (2008))), the stream of supply chain publications addressing this issue is quite recent. One work that appears to have led this pack is that of Cohen and Agrawal (1999) where the authors examine the contract selection problem between a long-term contract and a flexible short-term contract, and their results suggest that long-term contracts do not always dominate short-term contracts. In a very recent but related work, Li et al. (2009) provide conditions in which the long-term supplier is preferred.

Araman et al. (2000) consider a make-to-order risk neutral buyer that can purchase via a combination of a long-term contract and the spot market and the underlying source of uncertainty is the random spot price. They show that this combined use is beneficial to the buyer.

Motivated by Hewlett-Packard's procurement risk management (PRM) framework, Martinez-de-Albeniz and Simchi-Levi (2005) propose a portfolio contract that maximizes a buyer's expected profit where the buyer can select from long term contract, option contracts, and direct spot market purchase. They show that this strategy potentially drives down profit uncertainty while increasing expected profits for the buyer.

Wu et al. (2002) considered contracting arrangements between a Stackelberg producing seller and one or more buyer(s) for a non-storable good, where the sole source of uncertainty is the spot market price for that good. Both the seller and buyer(s) can either enter into long-term contracts for the supply of a fixed quantity of output or can take recourse in the spot market. The seller selects her profit maximizing contract prices (reservation cost and execution cost per unit of capacity) anticipating how the buyer will react. The risk neutral buyer in turn determines his optimal reservation level, and at some future time, upon observing the spot price, determines the optimal allotment of his needs between the contract and the spot market. Their results revealed that the seller's optimal strategy is to set her execution cost as low as possible (i.e. reveal her production cost), but extract her margin from the buyer(s) using the reservation charge. Extensions to this work are presented in Wu and Kleindorfer (2005) and Wu et al. (2001).

2.1.2 Expected Utility Models

The risk averse decision maker is reluctant to accept an alternative with an uncertain outcome and requires an additional marginal reward (risk premium) to agree to take on the additional risk. One approach commonly used in economics to model the risk averse behavior of such a decision maker is to maximize his expected utility. Initiated by Daniel Bernoulli in 1738 as a resolution of the St. Petersburg paradox (involving infinite expected values), the first important use of the expected utility theory was that of Neumann and Morgenstern (1944) where the authors, using the assumption of expected utility maximization in their ground breaking formulation of game theory, formally proved expected utility maximization to be a rational decision criteria (Schoemaker, (1982)). The expected utility theory posits that each rational decision maker has a non-decreasing utility function u(.) of the random outcome, and the decision maker seeks to optimize the expected value of this utility function. Suppose this decision maker's payoff at some future time, when viewed at the current decision time is a random $X(v)$ subject to his control v in some range V. Then, the expected utility optimization model that addresses the DM's problem is given by:

$$w^* = \max_{v \in V} E[u(X(v))], \tag{2.2}$$

where $E[u(X(v))]$ is the expected value of the nondecreasing, concave utility function of the DM's payoff. The exponential utility functions are the most widely used utility functions as they can conveniently model a broad range of risk attitudes (Corner and Corner, 1995).

As observed by Schoemaker, (1982), as a fallout of the work of Neumann and Morgenstern, the expected utility theory has almost become a major paradigm in economics theory and the author gives an extensive discussion on existing expected utility theory models, prior to and subsequent to Neumann and Morgenstern, as well as discusses the setbacks of the expected utility theory. Expected utility theory is particularly criticized for the difficulties associated with obtaining the exact utility function for each decision maker. Further, as noted by Seifert et al. (2004), while expected utility maximization offers a great avenue for analyzing the direction of change induced by a hypothetical shift in the model parameters, the approach does not however, readily lend itself to the realization of closed form solutions, so it becomes less suitable where the objective is to explicitly characterize the optimal solution. I now proceed to discuss some documented works that have adopted expected utility optimization frameworks to address risk averse decision making in a volatile commodity price environment.

Addressing the buyer-seller relationship in B2B markets, Kleindorfer and Wu (2003) present a modeling framework for a three period time line in which contracting is done through a combination of options and forwards, and the suppliers are capacitated. The buyer's objective is to maximize expected utility subject to the available contracts, while the seller seeks to maximize expected profit, jointly obtained from sales in both the contract market and the spot market, subject to the available capacity. The authors provide an excellent review of economic and managerial frameworks that have been proposed in literature to explain the structure of contracting in B2B markets in capital intensive industries that are prone to quickly fluctuating prices, like the power sector, and they pay particular interest to those works that adopted real options theory and financial engineering in modeling buyer-seller contractual relationships.

Brusset (2005) propose conditions in which a capacity constrained supplier of an input service and a buyer of such service can choose from among three different transaction forms: spot procurement, minimum purchase commitment, and quantity flexibility contracts. Both the spot price of the input service and end-market demand are exogenous stochastic processes, and while knowledge of the characteristics of the demand distribution is exclusive to the buyer, spot price distribution is common knowledge. The author derived the utility for each transaction type and subsequently proposed conditions in which the buyer will choose minimum purchase commitment (MPC) contract over spot, quantity flexibility contract (QFC) over spot, MPC over QFC and vice versa. One of the limitations of this work is that it does not provide avenue for the buyer to have a mixed purchase strategy, and the spot market only serves as a recourse in the event of supplier capacity shortage.

2.1.3 Mean-Variance Models

Originally proposed in the Nobel-Prize-winning work of Markowitz (1952) to explain and guide investment behavior, the mean-variance optimization concept is now widely used in portfolio theory to model an investor's decision under risk . Specifically, Markowitz introduced the idea to achieve a portfolio that: (1) provides the minimum variance for a given expected return or more, and (2) provides the maximum expected return for a given variance or less. Suppose the commodity user's payoff at some future time, when viewed at the current decision time, is a random $X(v)$ subject to his control v in some range V. Then, under the mean-variance maxim, the user will try to solve the problem

$$w^* = \max_{v \in V}(E[X(v)] - \beta \cdot V[X(v)]), \tag{2.3}$$

where $E[X(v)]$ is the expected payoff and $V[X(v)]$ is the variance of the payoff. Also, β is a positive constant that reflects the user's degree of risk averseness, and is considered to be the trade-off constant between the expected outcome and the variance of the outcome.

Sharpe's 1963 and 1971 simplified models of the Markowitz portfolio analysis technique could be credited for paving the way for the profound acceptance of the mean-variance approach to risk averse decision making in academic corridors and among practitioners. In particular, Sharpe's 1971 linear programming approximation of the mean-variance model has led to several attempts at linearizing the portfolio optimization problem through computationally attractive alternative risk measures (e.g. Young, (1998), Mansini and Speranza, (1999), Ogryczak, (2000), and Ruszczynski and Vanderbei (2003)). The popularity of the mean-variance approach to risk averse decision making stems from the many advantages it offers. It reduces the decision problem to a parametric quadratic programming problem thus facilitating the explicit characterization of the optimal solutions. It provides an efficient platform for trade-off analysis between the expected value and the risk of the outcome.

The mean-variance concept is however not without its criticism, most of these stemming from its sensitivity to statistical errors (Grootveld and Hallerbach (1999), Goldfarb and Iyengar (2003)). Indeed, Markowitz (1952) himself emphasized that it is important to combine both statistical techniques and the judgement of experts in establishing reasonable probability beliefs for the security in consideration, as crucial to the success of the mean-variance approach is the accuracy of the probability beliefs arrived at for such security.

Critics further point to the imperfection of variance as the risk measure owing to its symmetric property, resulting in the equal treatment of over - performance and under - performance. This has motivated several researchers to seek asymmetric risk measures in place of variance for a more general mean-risk approach to decision making (e.g. Bawa and Lindenberg (1977), Harlow and Rao (1989), Konno, (1990), Konno and Yamazaki (19991), Markowitz et al. (1993), Ogryczak and Ruszczynski (1999), Uryasev, (2000), and Rockafellar and Uryasev (2000)).

The aforementioned shortcomings not withstanding, mean-variance approach to decision making enjoys a large following in inventory modeling as the mean-variance theory is found to be more general than the expected utility theory. As stated by Bar-Shira and Finkelshtain (1999): "...The increased generality stems from the fact that the class of decision-makers whose risk preferences are representable by a mean standard deviation utility function is broader than the corresponding class of individuals whose risk preferences are representable by an expected utility function. This is because the mean standard deviation decision theory is capable of accommodating various nonlinearities in the probabilities, including the rank-dependent expected utility theory". I now proceed to discuss those works that have utilized the mean-variance rule in addressing decision making under commodity price uncertainty. Tsiang (1999) also provides justification for the mean-variance approach.

Seifert et al. (2004) analyzed optimal procurement strategies for a buyer faced with both demand and purchase price uncertainties for a given commodity, and averse to some variance of profit. This buyer can satisfy his commodity needs through a contract agreement with an uncapacitated supplier and come the day demand is to be realized, if realized demand turns out to be in excess of on-hand inventory, the excess demand is purchased from spot, otherwise, unmet demand is lost. On the other hand, if realized demand is less than on-hand inventory, the excess inventory is salvaged in the spot market at the ongoing spot price, otherwise, the excess is salvaged at some non-negative unit price less than the contract's fixed unit price.

Martinez-de Albeniz and Simchi-Levi (2006) considered a single-period inventory setting for a manufacturer faced with both demand and spot price uncertainty for the input material. The manufacturer can choose from a portfolio of option contracts and has access to the spot market to satisfy his needs. They show that there exists an efficient frontier bounded by the maximum expectation portfolio (selected by a risk-neutral buyer), and the minimum variance portfolio (selected by the buyer with infinite risk aversion).

Dong and Liu (2007) considered the equilibrium forward contract of a non-storable commodity and both the supplier and the manufacturer can trade in a spot market for the commodity. The final product demand, final product sale price, and the component spot price are all possibly correlated random variables and the two risk averse players have mean-variance preferences over their risky profits. The authors argued that the risk hedging benefit from a forward contract is what justifies its prevalence despite the availability of liquid spot markets.

2.2 Why Forward Contracts to Hedge Commodity Price Risk?

The reader might be apt to ask that given the popularity of derivative instruments such as futures, options, swaps, and commodity-linked notes in hedging price risks, what informs our stance that research on price volatility driven contracts is crucial to today's supply chain management efforts? Indeed it is true that derivative usage is an age long concept and there are tons of research papers, particularly in the finance literature, on how derivatives could be used to manage price uncertainty. However, the one thing that is clear is that while financial derivative tools have been found to enjoy high levels of appeal; even with the increasing liberalization and globalization of the commodity market, participation in the commodity derivative market is growing at a rather slow pace, particularly in the developing countries (Varangis and Larson, (1996)). In a study on derivative usage by non-financial firms in the US and Germany for instance, Bodnar and Gebhardt (1998), found that commodity derivatives came a distant third to foreign currency and interest rate derivatives in both countries.

As explained by Varangis and Larson, (1996), the hesitation by firms to participate in commodity derivatives market is largely informed by the challenges involved with overcoming certain barriers to using these markets. These include legal and regulatory barriers, policy barriers and government intervention, market know-how and awareness, basis risk, creditworthiness, and liquidity issues due to restrictions on the length of time and transaction volume. In addition, there is the need for the firm to have on hand, the cash for the payment of a premium for the purchase of options and the deposit of margins for the use of futures, so cash flow becomes an issue as well. Indeed, the criticality of liquidity and cash flow to commodity derivative market participation is evidenced in a study by Haushalter (2000) on hedging policies of oil and gas producers between 1992 and 1994, where it was found that firms with greater financial leverage used derivative markets more extensively than the smaller firms. With these barriers to the commodity derivative markets, it comes at no surprise therefore that managing price uncertainty by way of supply contracts remains the common practice among commodity users today and the following is a discussion on some of the most relevant works documented in this area, and these works also serve as excellent review sources of such supply contracts literature.

CHAPTER 3

THE PURCHASING, AND PRICING PROBLEM

3.1 Problem Description

Let the current time be t_0. I consider a risk-averse commodity user faced with a known end market demand δ at a future time t_1. As an example, the commodity user could be a major oil refiner with relatively stable aggregate demand for its wholesale customers. A spot market exists for the commodity. Suppose the current spot price is π_0 and the spot price at time t_1 is a random Π_1. The commodity user realizes ρ dollars in revenue per unit of the commodity. I suppose that the commodity user has no long-term storage facility, and the time needed for the acquired commodity to be processed at the user is negligible. The practicality of non-storability models for commodities, particularly the energy commodities, has been discussed at great lengths by Wu et al. (2002), and Kleindorfer and Wu (2003). The commodity user has two procurement options to consider, namely, either to purchase his commodity needs via contract with a supplier, who will deliver the contracted quantity at time t_1, or to purchase the balance of his needs via the spot market at time t_1. I do not prohibit the commodity user from buying more than his input commodity needs through contract, and selling off the excess to the spot market.

Let us denote the commodity user's purchase from the supplier by x. If $x \leq \delta$, the user must purchase the entire balance $\delta - x$ from the spot market. If $x > \delta$, he must sell the excess $x - \delta$ to the spot market. In my modeling approach, I view the spot market transaction volume for the commodity user as taking on a positive value in the event that he buys from spot, and a negative value in the event that he sells to spot. I denote this spot market transaction volume by y, where y is a free variable. The no storage condition implies that at all times, we should have $x + y = \delta$.

But, to avoid solving a constrained optimization problem, I assume that the commodity user incurs a huge cost ζ per squared unit of deviation of $(x + y)$ from δ. Further, the commodity user should not be able to place a negative order through his contract. That is, we should have $x \geq 0$. However, I start by relaxing this non-negativity constraint on x, and study the conditions, if any, under which it would be violated. The question the commodity user seeks to address is that of how much he should purchase through contract and how much he should purchase from or sell to the spot market.

In a bid to mitigate his exposure to the spot market price volatility, the commodity user hedges his risk by entering into a contract with his supplier. There are two sides to this contract:

(1) At the initiation of their business relationship, the supplier agrees to invest in a dedicated capacity ω, available to be called upon by the commodity user in each purchasing period throughout the life of the contract. Under this arrangement, if the commodity user requests exactly ω in a typical period of length $(t_1 - t_0)$, then the supplier's production cost is simply $\kappa\omega$, where κ is the per unit cost of production, and the supplier absorbs the entire cost of production. If however, the commodity user requests a quantity $x \neq \omega$, then the supplier's production cost takes on the quadratic form $\kappa x + \gamma \cdot (x - \omega)^2$, where γ is the additional cost incurred per squared unit of deviation from the dedicated capacity, ω. The $\gamma \cdot (x - \omega)^2$ portion is incurred as a result of the supplier having to temporarily expand capacity at the last minute or not being able to reach the full economy of scale corresponding to her capacity. It is agreed that this $\gamma \cdot (x - \omega)^2$ cost component will be completely absorbed by the commodity user, and could therefore be viewed as the penalty the commodity user incurs for upsetting the planned run of the supplier's production.

The idea of penalizing the buyer for under-used capacity in a pure forward contracting framework (i.e. no capacity reservation fee) where the underlying source of uncertainty is the random spot price of the input part has been adopted by Araman et al. (2000). The authors considered where in the bid to account for any losses she may incur due to capacity under-utilization, the supplier charges the buyer a higher unit price in the event that the buyer eventually orders less than the reserved capacity. In contrast to our work, capacity-overusage is not an option in the aforementioned.

(2) The actual purchase contract through which the commodity user places an order with the supplier: under this arrangement, at the decision time t_0, the commodity user enters into a one-time agreement with the supplier for delivery of a given quantity of the commodity at t_1. Here, the contract price is structured as an affine function of the spot price π_1 realized at t_1:

$$p(\pi_1) = \alpha\pi_1 + z. \tag{3.1}$$

I suppose that there is an already established agreement on the non-negative α, reflecting the degree of exposure to spot market conditions desired by the commodity user under the contract. I suppose further that the supplier requires that α never exceed 0.5 (the reason for this will become obvious when we derive the optimal solution to the supplier's problem). In other words, $\alpha \in [0, 0.5]$. The corresponding fixed-component term z is determined by the supplier based on her calculation of the random spot price Π_1. Thus at time t_0, while there is a known component z to the contract price, there also exists a random component to the price which floats with the spot market price. Under this "floating" contract, I assume that payment is settled at the time t_1 when delivery of the goods is made and the spot price has been realized as some π_1. I adopt the mean-variance approach described in chapter 2 to model the commodity user's aversion to risk.

3.2 Problem Formulation

Suppose the commodity user decides to contract an x quantity from the supplier and purchase a y quantity from the spot market, the realized price of the contract offered by the supplier is π, and the realized spot market price at t_1 is π_1. Then, the commodity user's payoff $f^{u0}(x, y, \pi, \pi_1)$ will be as follows:

$$f^{u0}(x, y, \pi, \pi_1) = \rho \cdot (x + y) - \zeta \cdot (x + y - \delta)^2 - \pi x - \pi_1 y - \gamma \cdot (x - \omega)^2. \tag{3.2}$$

Here, $\rho \cdot (x + y)$ is the revenue the user can earn, $\zeta \cdot (x + y - \delta)^2$ is the huge penalty incurred to the user when $(x + y)$ is different from the user's demand δ, πx is the amount the user has to pay to the supplier according to the contract, $\pi_1 y$ is the amount the user has to pay to the spot market, and $\gamma \cdot (x - \omega)^2$ is the amount the user has to pay to the supplier when the contracted quantity is different from the supplier's designed capacity ω.

Suppose the supplier has decided the z in the contract form (3.1), and the commodity user has made decisions on x and y, then, when the realized spot market price is π_1, the user earn $f^u(x, y, z, \pi_1)$, where

$$\begin{aligned} f^u(x, y, z, \pi_1) &= f^{u0}(x, y, \alpha\pi_1 + z, \pi_1) \\ &= \rho \cdot (x + y) - \zeta \cdot (x + y - \delta)^2 - x \cdot (\alpha\pi_1 + z) - \\ &\pi_1 y - \gamma \cdot (x - \omega)^2. \end{aligned} \tag{3.3}$$

After some algebra, we may work out that

$$f^u(x, y, z, \pi_1) = f_0^u(x, y, z) + f_1^u(x, y) \cdot \pi_1, \tag{3.4}$$

where

$$\begin{cases} f_0^u(x,y,z) & = \rho \cdot (x+y) - \zeta \cdot (x+y-\delta)^2 - xz - \gamma \cdot (x-\omega)^2, \\ f_1^u(x,y) & = -(\alpha x + y). \end{cases} \tag{3.5}$$

The risk-averse commodity user's objective function $f^u(x,y,z)$ is therefore determined by

$$f^u(x,y,z) = E[f^u(x,y,z,\Pi_1)] - \beta \cdot V[f^u(x,y,z,\Pi_1)]. \tag{3.6}$$

Plugging (3.4) into (3.6), we can, through algebra (see equation (6.58) in Appendix B), obtain that

$$f^u(x,y,z) = f_0^u(x,y,z) + \mu \cdot f_1^u(x,y) - \beta\sigma^2 \cdot (f_1^u(x,y))^2, \tag{3.7}$$

where I have used μ to denote the mean $E[\Pi_1]$ of Π_1 and σ^2 the variance $V[\Pi_1]$ of Π_1. When (3.5) is further plugged into (3.7), we may get

$$\begin{aligned} f^u(x,y,z) \quad & = -(\beta\sigma^2\alpha^2 + \gamma + \zeta) \cdot x^2 - (\zeta + \beta\sigma^2) \cdot y^2 - \\ & 2(\zeta + \beta\sigma^2\alpha) \cdot xy - xz + (\rho + 2\zeta\delta + 2\gamma\omega - \alpha\mu) \cdot x + \\ & (\rho + 2\zeta\delta - \mu) \cdot y - (\zeta\delta^2 + \gamma\omega^2). \end{aligned} \tag{3.8}$$

Given the contract coefficient z, the commodity user will solve $\max_{x,y} f^u(x,y,z)$ to obtain the optimal contract quantity $x^u(z)$ and optimal spot market purchase quantity $y^u(z)$.

When the commodity user's ordering quantity is x while the realized contract price is π, the supplier's profit realized at t_1 is $f^s(x,\pi)$, where

$$f^s(x,\pi) = (\pi - \kappa) \cdot x. \tag{3.9}$$

Recall that κ is the per unit production cost charged to the supplier. At time t_0, the risk-neutral supplier seeks to maximize her expected profit by choosing a proper value for the contract coefficient z, given her anticipation of the commodity user's response $x^u(z)$. Hence, the supplier's objective function is $F^s(z)$, with

$$F^s(z) = E[f^s(x^u(z), \alpha\Pi_1 + z)] = (\alpha\mu - \kappa + z) \cdot x^u(z). \tag{3.10}$$

The supplier will solve $\max_z F^s(z)$ to obtain the equilibrium contract coefficient z^s. Therefore, the commodity user's equilibrium contract quantity will be $x^u = x^u(z^s)$ and his equilibrium spot purchase quantity $y^u = y^u(z^s)$.

3.3 Analysis

In solving the commodity user's problem, I adopt a two-step approach. In the first step, we fix x and then proceed to solve for the $y^u(x,z)$ that maximizes $f^u(x,y,z)$ defined in (3.8). It is easy to see that $f^u(x,y,z)$ is quadratic in y with a negative second order coefficient. The optimal spot market purchase, $y^u(x,z)$, can therefore be obtained by setting the first order derivative of $f^u(x,y,z)$ with respect to y to zero, i.e.,

$$\partial f^u(x,y,z)/\partial y \mid_{y=y^u(x,z)} = 0. \tag{3.11}$$

From the above, we obtain that $y^u(x,z)$ is independent of z, and hence can be written as $y^u(x)$:

$$\begin{aligned} y^u(x) &= (-2(\zeta - \beta\sigma^2\alpha) \cdot x + \rho - \mu + 2\zeta\delta)/(2 \cdot (\beta\sigma^2 + \zeta)) \\ &= (\zeta/(\zeta + \beta\sigma^2)) \cdot (\delta - x) + (\rho - \mu - 2\beta\sigma^2 x\alpha)/(2 \cdot (\zeta + \beta\sigma^2)). \end{aligned} \tag{3.12}$$

It is obvious from (3.12) that, when the off-demand penalty ζ approaches $+\infty$, the spot purchase quantity $y^u(x)$ will tend to $\delta - x$. In the second step, we substitute $y^u(x)$ of (3.12) into (3.8). The commodity user's objective function will become

$$\begin{aligned}G^u(x,z) &= f^u(x,y^u(x),z)\\&= -[\gamma+\zeta\cdot(1-\alpha)^2-(\zeta^2/(\beta\sigma^2+\zeta))\cdot(1-\alpha)^2]\cdot x^2-[z-\rho\\&-2\gamma\omega-2\zeta\delta+\rho\alpha+(\zeta\cdot(1-\alpha)\cdot(\rho-\mu+2\zeta\delta))/(\zeta+\beta\sigma^2)+\\&2\zeta\delta\alpha]\cdot x+((\rho-\mu+2\zeta\delta)^2)/(4\cdot(\zeta+\beta\sigma^2))-\gamma\omega^2-\zeta\delta^2\end{aligned} \tag{3.13}$$

Again, we observe that $G^u(x,z)$ is quadratic in x with a negative second order coefficient. We can therefore establish $x^u(z)$ by setting

$$\partial G^u(x,z)/\partial x\mid_{x=x^u(z)}=0, \tag{3.14}$$

from which we obtain

$$\begin{aligned}x^u(z) &= [-(\beta\sigma^2+\zeta)\cdot z+(1-\alpha)\cdot(\beta\sigma^2\rho+\zeta\mu+2\beta\sigma^2\zeta\delta)\\&+2\gamma\omega\cdot(\beta\sigma^2+\zeta)]/[2\beta\sigma^2\zeta\cdot(1-\alpha)^2+2\gamma\cdot(\beta\sigma^2+\zeta)].\end{aligned} \tag{3.15}$$

Substituting (3.15) into the supplier's objective function (3.10), we obtain

$$\begin{aligned}F^s(z) &= [-(\zeta+\beta\sigma^2)/(2\beta\sigma^2\zeta\cdot(1-\alpha)^2+2\gamma\cdot(\zeta+\beta\sigma^2))]\cdot z^2\\&+[(\beta\sigma^2\rho\cdot(1-\alpha)+(\kappa-\alpha\mu)\cdot(\zeta+\beta\sigma^2)+\zeta\mu\cdot(1-\alpha)\\&+2\gamma\omega\cdot(\zeta+\beta\sigma^2)+2\beta\sigma^2\zeta\delta\cdot(1-\alpha))\\&/(2\beta\sigma^2\zeta\cdot(1-\alpha)^2+2\gamma\cdot(\zeta+\beta\sigma^2))]\cdot z\\&-[(\kappa-\alpha\mu)\cdot(\beta\sigma^2\rho\cdot(1-\alpha)+\zeta\mu\cdot(1-\alpha)\\&+2\gamma\omega\cdot(\zeta+\beta\sigma^2)+2\beta\sigma^2\zeta\delta\cdot(1-\alpha)]\\&/[2\beta\sigma^2\zeta\cdot(1-\alpha)^2+2\gamma\cdot(\zeta+\beta\sigma^2)].\end{aligned} \tag{3.16}$$

It is clear from (3.16) that $F^s(z)$ is quadratic in z with a negative second order coefficient. We can therefore set

$$dF^s(z)/dz\,|_{z=z^s}= 0, \tag{3.17}$$

from which we may obtain

$$\begin{aligned} z^s = &[\kappa \cdot (\beta\sigma^2 + \zeta) + \beta\sigma^2\rho \cdot (1-\alpha) + \zeta\mu \cdot (1-2\alpha) \\ &+ 2\gamma\omega \cdot (\beta\sigma^2+\zeta) + 2\beta\sigma^2\zeta\delta \cdot (1-\alpha) - \beta\sigma^2\mu\alpha]/[2\cdot(\beta\sigma^2+\zeta)]. \end{aligned} \tag{3.18}$$

Plugging (3.18) into (3.15), we obtain

$$\begin{aligned} x^u = x^u(z^s) = &[\beta\sigma^2\rho \cdot (1-\alpha) - \kappa \cdot (\beta\sigma^2+\zeta) + \zeta\mu + 2\gamma\omega \cdot (\beta\sigma^2+\zeta) \\ &+ 2\beta\sigma^2\zeta\delta \cdot (1-\alpha) + \beta\sigma^2\alpha\mu] \\ &/[4\cdot(\beta\sigma^2\zeta \cdot (1-\alpha)^2 + \gamma \cdot (\beta\sigma^2+\zeta))]. \end{aligned} \tag{3.19}$$

Plugging the above into (3.12), we obtain

$$\begin{aligned} y^u &= y^u(x^u(z^s)) \\ &= (\rho - \mu + 2\zeta\delta)/(2\cdot(\beta\sigma^2+\zeta)) - 2\cdot(\zeta+\beta\sigma^2\alpha)\cdot[\beta\sigma^2\rho\cdot(1-\alpha)- \\ &\kappa\cdot(\beta\sigma^2+\zeta) + \zeta\mu + 2\gamma\omega\cdot(\beta\sigma^2+\zeta) + 2\beta\sigma^2\zeta\delta\cdot(1-\alpha) + \beta\sigma^2\alpha\mu]/ \\ &[8\cdot(\beta\sigma^2+\zeta)\cdot(\beta\sigma^2\zeta\cdot(1-\alpha)^2 + \gamma\cdot(\beta\sigma^2+\zeta))] \end{aligned} \tag{3.20}$$

If we let ζ go to its intended value $+\infty$, we will have

$$\begin{cases} x^u &= [\mu - \kappa + 2\beta\sigma^2\delta \cdot (1-\alpha) + 2\gamma\omega]/[4\beta\sigma^2 \cdot (1-\alpha)^2 + 4\gamma], \\ y^u &= \delta - x^u = \delta - [\mu - \kappa + 2\beta\sigma^2\delta \cdot (1-\alpha) + 2\gamma\omega]/ \\ & \quad [4\beta\sigma^2 \cdot (1-\alpha)^2 + 4\gamma], \\ z^s &= [\kappa + \mu \cdot (1-2\alpha) + 2\gamma\omega + 2\beta\sigma^2\delta \cdot (1-\alpha)]/2. \end{cases} \tag{3.21}$$

When ζ takes its limit value, we see that $x^u + y^u = \delta$ is always maintained. Also, it is reasonable to assume that $\kappa < \mu$, i.e., it costs the supplier less to produce a unit item than for the item to be purchased from the spot market. If so, we may observe from (3.21) that x^u will never assume negative values. If we substitute z^s of (3.21) in the supplier's contract price $p(\pi_1)$ of (3.1), we arrive at the following:

$$p(\pi_1) = (\kappa + \mu)/2 \; + \gamma\omega + \beta\sigma^2\delta \cdot (1-\alpha). \tag{3.22}$$

From (3.22), it becomes obvious that $p(\pi_1)$ is decreasing in α, so that the supplier's opportunity to charge the commodity user a premium over the expected spot price deteriorates with α. Thus, as α gets very large, the more the supplier runs the risk of realizing a contract price that hardly compensates for her unit cost of production, κ. For these reasons, we assume that as a matter of policy, the supplier is not willing to entertain an $\alpha > 0.5$.

Let E^u be the commodity user's expected profit under equilibrium, V^u be the variance of his profit under equilibrium, E^s be the supplier's expected profit under equilibrium, and E^{sc} be the total supply chain expected profit under equilibrium. From (3.4), we may find that

$$E^u = E[f^u(x^u, y^u, z^s, \Pi_1)] = f_0^u(x^u, y^u, z^s) + \mu \cdot f_1^u(x^u, y^u). \tag{3.23}$$

If we plug (3.21) into (3.23), we can get a closed form expression for E^u. We opt not to present the result here due to its excessive length. Using (3.5), (3.6), and (3.7), we may derive that

$$V^u = \sigma^2 \cdot (f_1^u(x^u, y^u))^2 = \sigma^2 \cdot (\alpha x^u + y^u)^2, \tag{3.24}$$

which, after substitutions for x^u and y^u from (3.21), becomes

$$V^u = \sigma^2 \cdot [\delta/2 + (2\gamma\delta - (1-\alpha) \cdot (\mu - \kappa + 2\gamma\omega))/(4\gamma + 4\beta\sigma^2 \cdot (\alpha - 1)^2)]^2. \tag{3.25}$$

Substituting x^u of (3.21) in the supplier's expected profit of (3.10), we have

$$E^s = F^s(z^s) = [\mu - \kappa + 2\beta\sigma^2\delta \cdot (1-\alpha) + 2\gamma\omega]^2/[8 \cdot (\beta\sigma^2 \cdot (1-\alpha)^2 + \gamma)]. \tag{3.26}$$

From (3.5), (3.10), and y^u of (3.21), we may obtain

$$E^{sc} = E^u + E^s = \rho\delta - \gamma \cdot (x^u - \omega)^2 - \mu \cdot (\delta - x^u) - \kappa x^u. \tag{3.27}$$

3.4 A Comparative Statics Study

I now study how x^u, y^u, and z^s are influenced by the problem's parameters. According to (3.21) through (3.26), the following is a list of relevant parameters:

α–the degree of exposure to spot price volatility sought by the commodity user under the contract;

β–the commodity user's degree of risk averseness;

γ–the severity of the off-capacity penalty that is passed from the supplier to the commodity user;

ω–the capacity that the supplier dedicates to the commodity user; and,

μ–the expected value of the spot price; and,

σ^2–the variance of the spot price.

Besides, the following two parameters are subject to scaling:

δ–the fixed demand level; and,

κ–the supplier's unit production cost.

3.4.1 Effects of α, the Contract's Exposure to Spot Price

My findings concerning α can be summarized as follows.

Proposition 1 *(a) Define α_0 so that*

$$\alpha_0 = 1 - \frac{\sqrt{(\mu - \kappa + 2\gamma\omega)^2 + 4\beta\sigma^2\delta^2\gamma} - (\mu - \kappa + 2\gamma\omega)}{2\beta\sigma^2\delta}.$$

When the user's exposure to spot price under the contract $\alpha < \alpha_0$, his equilibrium contract ordering level x^u will be increasing in α; otherwise, x^u will be decreasing in α.

(b) Define ω_0 so that

$$\omega_0 = \frac{\beta\sigma^2\delta \cdot (1 - \alpha)}{\gamma + 2\beta\sigma^2 \cdot (1 - \alpha)^2} - \frac{\mu - \kappa}{2\gamma}.$$

When $\alpha - \alpha_0$ and $\omega - \omega_0$ are of opposite signs, the total supply chain equilibrium expected profit E^{sc} will be increasing in α, and decreasing in α otherwise.

(c) The supplier's equilibrium fixed charge z^s is always decreasing in α.

(d) Define α_1 so that

$$\alpha_1 = 1 - \frac{2\gamma\delta}{\mu - \kappa + 2\gamma\omega}.$$

The supplier's equilibrium expected profit E^s is increasing in α when $\alpha < \alpha_1$, and decreasing in

α *otherwise.*

Obviously, the more exposure to spot price that the commodity user has under the contract, the closer the price variability associated with the contract gets to that of spot, and as such, the less the opportunity for the supplier to leverage contract price stability in charging higher prices. In addition, the user seeking a high degree of exposure to spot price under the contract, reflects a low level of hesitation on his part to engage in direct spot purchase. This explains the observed decrease in his equilibrium contract ordering level x^u for high values of α (i.e. $\alpha > \alpha_0$). Where the sought exposure is low (i.e. $\alpha < \alpha_0$), so that the user's hesitation to spot exposure is high, the accompanying reduction in the supplier's contract charge as $\alpha < \alpha_0$ increases serves as an added incentive for the user to increase his contract order. The observed increase in E^s as $\alpha < \alpha_1$ increases implies that for the supplier, the growth in the equilibrium volume of order the user routes her way is guaranteed to more than compensate for her reduced equilibrium contract charge.

Given its lengthy expression, analyzing the effect of the problem parameters on the user's equilibrium expected profit E^u is a cumbersome exercise. Hence, I have opted for a numerical approach in conducting such an analysis. Also, while the effects of all other parameters on the user's equilibrium variance of profit, V^u are analytically tractable, studying the effect of α on V^u is analytically demanding. The first numerical example (results presented in Figure 3.1) is used to illustrate the effect of α on V^u and E^u. Varying α, we set $\beta = 0.0005$, $\gamma = 0.3$, $\delta = \omega = 100$, $\mu = 58$, $\sigma^2 = 315$, $\kappa = 30$, and $\rho = 100$. Both V^u and E^u are found to increase in α. Obviously, the higher the value of α, the less price stability the contract offers, thus the observed increase in V^u. Also, since an increased α reduces the premium placed on contract price stability, the supplier is forced to reduce her contract charge to better compete with the spot market. This explains the observed increase in E^u.

Let us now focus on the effect of α on the total supply chain equilibrium expected profit E^{sc}.

It is easy to show that ω_0 is very small so that $\omega > \omega_0$ is a condition that holds most often. Now,

$$\omega_0 = \frac{\beta\sigma^2\delta \cdot (1-\alpha)}{\gamma + 2\beta\sigma^2 \cdot (1-\alpha)^2} - \frac{\mu - \kappa}{2\gamma}. \tag{3.28}$$

We see from (3.28) that as γ tends to zero, ω_0 tends to $-\infty$. As γ becomes very large compared to $2\beta\sigma^2 \cdot (1-\alpha)^2$,

$$\omega_0 \approx \delta \cdot (\frac{\beta\sigma^2 \cdot (1-\alpha)}{\gamma}) - \frac{\mu - \kappa}{2\gamma}. \tag{3.29}$$

Since $\alpha \in [0, 0.5]$, if γ is very large compared to $2\beta\sigma^2 \cdot (1-\alpha)^2$, then γ will be very large compared to $\beta\sigma^2 \cdot (1-\alpha)$, so that ω_0 approaches very small values. Therefore, for practical parameter values, $\omega > \omega_0$, so that realistically, as $\alpha < \alpha_0$ increases, E^{sc} increases. I now proceed to numerically compare the $\alpha > 0$ contract to a fixed price contract ($\alpha = 0$) on the basis of the total supply chain equilibrium expected profit E^{sc} of (3.27). In this study, I present three different scenarios of parameter settings. In the first scenario, all parameters of the problem are the same as in Figure 3.1. In scenario 2, $\beta = 0.001$, $\gamma = 0.7$, $\omega = 50$, and all other parameters are kept the same as in scenario 1, while in the third scenario, all the parameter values of scenario 2 remain the same with the exception of σ^2, ω, and γ which are respectively set to 472, 30, and 0.2.

In all instances, the total supply chain equilibrium expected profit E^{sc} was found to be increasing in α when $\alpha < \alpha_0$, and decreasing in α otherwise (see Figure 3.2). These results suggest that improvements in the total supply chain profits can be expected when higher values of α result in higher order quantities that the user routes to the supplier. Further, it can be verified from the plot that for each of the 3 scenarios, the percentage improvement an $\alpha = \alpha_0$ contract has over a fixed price contract ($\alpha = 0$) on the total supply chain equilibrium expected profit, E^{sc}, is respectively 9.7%, 1.6%, and 5.9%. It can also be observed that for scenarios 1, and 3, $\alpha_0 > 0.5$. Hence, while there are instances where $\alpha_0 > 0.5$ might lead to the maximum total supply chain profit, the supplier's unwillingness to be exposed to the risk of dramatic reductions in profit inhibits the realization of such maximum supply chain profit.

Thus, most importantly, for small enough α, i.e. $\alpha < \alpha_0$, the analytical and numerical results reveal that the floating contract price setting, $\alpha\pi_1 + z$, outperforms a fixed price contract when viewed in the context of the total supply chain equilibrium expected profits. The analytical results further show that the floating contract price is beneficial to the supplier as well when $\alpha < \alpha_1$. Indeed, by offering the commodity user reasonable measure of exposure to spot market conditions under the contract, the supplier is able to capture the user's desire to take advantage of the occurrence of a downward swing in spot price. By so doing, the supplier is better positioned to curtail the attractiveness of spot market purchase to the user.

3.4.2 Effects of β, the Commodity User's Degree of Risk Averseness

Concerning β, my findings are summarized below.

Proposition 2 *(a) Define γ_0 so that*

$$\gamma_0 = \frac{(\mu - \kappa) \cdot (1 - \alpha)}{2 \cdot (\delta - \omega \cdot (1 - \alpha))}.$$

When the severity of off-capacity penalty $\gamma < \gamma_0$, the user's equilibrium contract ordering level x^u is decreasing in his level of risk aversion β while his equilibrium variance of profit V^u is increasing in β. When $\gamma > \gamma_0$, the trends are reversed.

(b) The supplier's equilibrium fixed charge z^s is increasing in β.

(c) Define β_0 so that

$$\beta_0 = \frac{(\mu - \kappa) \cdot (1 - \alpha) - 2\gamma \cdot (2\delta - \omega \cdot (1 - \alpha))}{2\sigma^2 \delta \cdot (1 - \alpha)^2}.$$

The supplier's equilibrium expected profit E^s is decreasing in β when $\beta < \beta_0$, and increasing in β otherwise.

A corollary of this result is that although the commodity user will naturally get increasingly concerned about reducing variance of profits as his degree of risk aversion β increases, when $\gamma < \gamma_0$ however, his urgency at gaining expected profits outweighs his urgency to reduce variance of profits. To explain this result, let us first derive the incremental cost Δ that the commodity user incurs to purchase an additional unit from contract. The total expected cost to the commodity user to place a contract order is given by

$$C^u = (\alpha\mu + z^s) \cdot x^u \ + \ \gamma \cdot (x^u - \omega)^2. \tag{3.30}$$

Therefore,

$$\Delta = \partial C^u / \partial x^u = \alpha\mu + z^s \ + \ 2\gamma \cdot (x^u - \omega). \tag{3.31}$$

Substituting (3.21) for x^u and z^s, Δ reduces to

$$\begin{aligned} \Delta = (\mu + \kappa)/2 - \gamma\omega + (2\gamma \cdot (\mu - \kappa + 2\gamma\omega - 2\beta\sigma^2\delta \cdot (\alpha - 1))) \\ /(4\gamma + 4\beta\sigma^2 \cdot (\alpha - 1)^2) - \beta\sigma^2\delta \cdot (\alpha - 1). \end{aligned} \tag{3.32}$$

Since

$$\begin{aligned} \partial\Delta/\partial\gamma = (\beta\sigma^2 \cdot (\alpha - 1)^2 \cdot (\mu - \kappa + 2\beta\sigma^2\delta(1 - \alpha) - 2\beta\sigma^2\omega \cdot (1 - \alpha)^2)/ \\ (2 \cdot (\beta\sigma^2 \cdot (1 - \alpha)^2 + \gamma)^2) > 0, \end{aligned} \tag{3.33}$$

it is clear that Δ increases in γ. Now, let us consider the case where $\gamma = \gamma_0$. It is easy to verify that (3.32) reduces to

$$\Delta_{(\gamma=\gamma_0)} = \mu + \beta\sigma^2\delta \cdot (1 - \alpha). \tag{3.34}$$

Since Δ increases in γ, it is thus obvious that for $\gamma > \gamma_0$, the incremental contract cost, $\Delta_{(\gamma>\gamma_0)}$, is always greater than μ, the expected unit spot price of the commodity. Under this circumstance, a risk neutral commodity user ($\beta = 0$), would consider the contract deal to be less attractive than the spot market and would therefore prefer to order his commodity needs through the latter. However, as his risk aversion grows, the more of an issue price volatility becomes to him, and the more order he places with contract.

On the other hand, $\gamma < \gamma_0$, leads to $\Delta_{(\gamma<\gamma_0)} < \mu + \beta\sigma^2\delta \cdot (1 - \alpha)$. This implies a $\Delta_{(\gamma<\gamma_0)} < \mu$ for the risk neutral commodity user. This risk neutral user will therefore prefer to place his needs through contract. Notice that Δ grows with β. Thus as the commodity user's degree of risk aversion increases, the deteriorating offer he receives from the supplier increases the possibility of the contract price turning out higher than the spot price. Since the low $\gamma < \gamma_0$ penalty affords the user a higher flexibility to place an off-capacity contract order, the consequence is an increased inclination of the user to consider spot purchase in response to the higher contract charge that accompanies an increased aversion to risk.

We see also from Proposition 2 that while the supplier will always seek to take advantage of her client's spot market shy behavior by charging higher prices for higher degrees of risk aversion, she however does so to her loss when the commodity user's aversion to risk, β, is low (i.e. when $\beta < \beta_0$). Therefore, the supplier's opportunistic response to her client's aversion to risk could unduly drive the user into the spot market with the effect of a reduced bottom line profit for the supplier.

I use the following numerical example (results presented in Figures 3.3 and 3.4) to illustrate the effect of β on E^u. Varying β and setting $\alpha = 0.4$ while keeping all other parameters the same as in Figure 3.1, we first set $\gamma = 0.309 > \gamma_0 = 0.209$ and then set $\gamma = 0.159 < \gamma_0$. It can be observed from Figures 3.3 and 3.4 that in both instances, E^u decreases with β. This trend can be directly traced to the deteriorating contract deal as well as the increased hesitation of the user to spot purchase as he gets more risk averse.

3.4.3 Effects of γ, the Severity of the Off-capacity Penalty

Concerning γ, we have the following result.

Proposition 3 *(a) The user's equilibrium contract ordering level x^u is decreasing in γ, the severity of the off-capacity penalty.*

(b) Define γ_1 so that

$$\gamma_1 = \frac{(\mu - \kappa) \cdot (1 - \alpha) - 2\delta\beta\sigma^2 \cdot (1 - \alpha)^2}{2 \cdot (2\delta - \omega \cdot (1 - \alpha))}.$$

$\gamma = \gamma_1$ results in a purchasing portfolio with zero standard deviation for the user, and his equilibrium variance of profit V^u is decreasing in γ when $\gamma < \gamma_1$, and increasing in γ otherwise.

(c) The supplier's equilibrium fixed charge z^s is increasing in γ.

(d) Define γ_2 so that

$$\gamma_2 = \frac{\mu - \kappa + 2\beta\sigma^2 \cdot (1 - \alpha) \cdot (\delta - 2\omega \cdot (1 - \alpha))}{2\omega}.$$

The supplier's equilibrium expected profit E^s is decreasing in γ when $\gamma < \gamma_2$, and increasing in γ otherwise.

The commodity user's increased inclination to consider spot market purchase for higher values of γ is triggered by the supplier's opportunistic response to the reduced contract flexibility that a higher γ affords the user. This increased spot market exposure for higher values of γ explains the observed increase in the user's equilibrium variance of profit V^u. When $\gamma = \gamma_1$, it can be verified that $x^u = \delta/(1-\alpha)$. Since it's been established that x^u decreases in γ; $\gamma < \gamma_1$ implies $x^u > \delta/(1-\alpha)$, so that at the very least (when $\alpha = 0$), the user will purchase his entire needs δ from contract while for a nonzero α, $x^u > \delta$. As this $\gamma < \gamma_1$ penalty increases, the less incentive the user has to purchase beyond his actual needs δ from contract with the hope of selling the excess at a spot price premium in the future time t_1, so he reduces the excess contract purchase. The user's reduced sell-to-spot market transaction volume explains the observed decrease in V^u as $\gamma < \gamma_1$ increases.

It can easily be verified that when $\gamma = \gamma_2$, the equilibrium strategy for the commodity user is to purchase the entire capacity ω that the supplier has dedicated to him. This implies that $x^u < \omega$ when $\gamma > \gamma_2$. It must mean that for the supplier, a $\gamma > \gamma_2$ guarantees that the drop in x^u will always be more than compensated by the gain in z^s as γ increases, so that her equilibrium expected profit E^s is always increasing in $\gamma > \gamma_2$ as our results show.

On the other hand, a low γ (i.e. $\gamma < \gamma_2$), implies $x^u > \omega$. The implication of this is that when $\gamma < \gamma_2$, the commodity user will always seek to purchase beyond the dedicated capacity from contract, but as this γ increases, the less incentive he has to do so, resulting in loss of revenue for the supplier. This explains why E^s is decreasing in $\gamma < \gamma_2$.

I now present a numerical example to illustrate the effect of γ on the commodity user's equilibrium expected profit, E^u. Keeping all other parameters of Figure 3.4 the same, it can be verified that γ_1=0.019. Varying the value of γ from $0.014 < \gamma_1$ upwards, it is observed (see Figure 3.5) that E^u is decreasing in γ and this can be readily attributed to the supplier's opportunistic reaction to higher values of γ. Therefore from Figure 3.5, we see that a reduced penalty γ improves the commodity user's efficient frontier by facilitating the realization of higher equilibrium expected profits at lower equilibrium variances of profits.

Lastly, from equation (3.27), it can be easily observed that when $x^u < \omega$ (which implies $\gamma > \gamma_2$), the supply chain equilibrium expected profit E^{sc} is strictly decreasing in γ. Thus, while $\gamma > \gamma_2$ penalties lead to improved expected profits for the supplier, their deteriorating effects on the user's expected profits however are such that the overall supply chain expected profits will always take a plunge.

3.4.4 Effects of ω, the Capacity Reserved at the Supplier

The following is what we can firmly say concerning ω.

Proposition 4 *(a) The user's equilibrium contract ordering level x^u increases in ω, the capacity he reserves with the supplier.*

(b) Define ω_1 so that

$$\omega_1 = \frac{4\delta\gamma - (1-\alpha)\cdot(\mu - \kappa) + 2\delta\beta\sigma^2\cdot(1-\alpha)^2}{2\gamma\cdot(1-\alpha)}.$$

$\omega = \omega_1$ yields a zero standard deviation purchasing portfolio for the user. The user's equilibrium variance of profit V^u is decreasing in ω when $\omega < \omega_1$, and increasing in ω otherwise.

(c) The supplier's equilibrium fixed charge z^s and her equilibrium expected profit E^s are increasing in ω.

It can be expected that a higher reserved capacity would compel the user to raise his contract order since among other considerations, the user seeks to minimize the penalty he incurs for capacity under-usage. Thus by investing in a higher capacity for the user, the supplier is better able to curtail spot market competition and increase her earnings potential.

Recall that when we studied the effect of the off-capacity penalty γ on the user's equilibrium strategy, $\gamma \geq \gamma_2$ leads to $x^u \leq \omega$. In studying the effect of ω on the user's equilibrium expected profit E^u, of particular interest is where the user's equilibrium response is to order no more than the reserved capacity from contract. I keep all parameters the same as in Figure 3.5, but set $\gamma = 0.4 > \gamma_2 = 0.354$, and vary ω. We would expect that since higher values of ω would serve to reduce spot market competition for the supplier, the user's expected profit should deteriorate as ω increases. The results of this numerical study (presented in Figure 3.6) are consistent with this expectation.

3.4.5 Effects of μ, the Expected Spot Price of the Commodity

Concerning μ, the following is true.

Proposition 5 *(a) The user's equilibrium contract ordering level x^u increases in μ, the expected spot price of the commodity.*

(b) Define μ_0 so that

$$\mu_0 = \frac{4\delta\gamma - (1-\alpha)\cdot(2\gamma\omega - \kappa) + 2\delta\beta\sigma^2\cdot(1-\alpha)^2}{1-\alpha}.$$

When $\mu = \mu_0$, a zero standard deviation purchasing portfolio is realized for the user. The user's equilibrium variance of profit V^u is decreasing in μ when $\mu < \mu_0$ and increasing in μ otherwise. Further, when $\mu > \mu_0$, the user's equilibrium strategy is always to place a contract order, $x^u > \delta$, with the hope of selling the excess units at a spot price premium in the future time t_1.

(c) The supplier's equilibrium fixed charge z^s and her equilibrium expected profit E^s are increasing in μ.

That the user's equilibrium contract ordering level x^u increases in μ comes as no surprise since for the same spot price variance, a higher expected spot price increases the likelihood of the realized spot price turning out to be greater than the contract price, and the commodity user's natural response would be to increase his contract purchase. For the supplier, the reduced spot market threat owing to an increased spot price premium gives her the flexibility to increase her contract charge with less worry of losing her share of the user's needs to the spot market.

To study the effect of μ on the commodity user's equilibrium expected profit E^u, I vary μ, set $\gamma = 0.4$, and keep all other parameters the same as in Figure 3.5. The results (Figure 3.7) reveal that E^u initially decreases as the expected spot price μ increases and this is as expected since an increased μ not only places a premium on the spot price, but also, on the fixed contract charge as well. When $\mu > \mu_0 = 235.5$ however, E^u begins to increase in μ, and we recall from proposition 5 that when $\mu > \mu_0$, the user is sure to speculate through the contract.

3.4.6 Effects of σ^2, the Spot Price Volatility

We may conclude the following concerning σ^2.

Proposition 6 *Recall that, in Proposition 2, γ_0 is define through*

$$\gamma_0 = \frac{(\mu - \kappa) \cdot (1 - \alpha)}{2 \cdot (\delta - \omega \cdot (1 - \alpha))}.$$

(a) When the severity of off-capacity penalty $\gamma < \gamma_0$, the user's equilibrium contract ordering level x^u is decreasing in the spot price volatility σ^2, and increasing in σ^2 otherwise.

(b) The user's equilibrium variance of profit V^u is always increasing in σ^2.

(c) The supplier's equilibrium fixed charge z^s is increasing in σ^2.

(d) Define σ_0^2 so that

$$\sigma_0^2 = \frac{(\mu - \kappa) \cdot (1 - \alpha) - 2\gamma \cdot (2\delta - \omega \cdot (1 - \alpha))}{2\beta\delta \cdot (1 - \alpha)^2}.$$

The supplier's equilibrium expected profit E^s is decreasing in σ^2 when $\sigma^2 < \sigma_0^2$, and increasing in σ^2 otherwise.

We see that the opportunistic response of the supplier to an increased spot price volatility by way of an increased contract charge could unduly drive the commodity user into the spot market, and unless the spot price volatility is high enough (i.e. $\sigma^2 > \sigma_0^2$), such opportunistic tendency would only serve to lower bottom line E^s for the supplier. Where the spot price volatility is high, the user's hesitation in engaging in spot market transactions is heightened so that the loss of sales volume becomes less of a worry for the supplier.

The result of V^u increasing everywhere in σ^2 is intuitive since the more volatile the spot price gets, the higher the price volatility the user is faced with both under the contract and in the spot market. Setting $\gamma = 0.309$, $\mu = 58$, and varying σ^2 while keeping all other parameters of Figure 3.7 the same, we find that a reduced spot price volatility allows for a higher equilibrium expected profit for the commodity user (results presented in Figure 3.8).

3.5 Managerial Implications

The findings that there are instances in which the risk averse commodity user could become less hesitant in considering purchasing from the spot market with increased levels of risk aversion β, spot price volatility σ^2, or off-capacity order penalty γ, provide some very interesting insights. We would ordinarily expect that higher levels of β and σ^2 should cause the user to be further drawn to the price stability offered by the contract, while higher values of γ would be expected to curtail the user's flexibility to order less than the reserved capacity ω from contract. We however find that the supplier's exploitation of the commodity user's vulnerability at elevated values of these parameters by charging higher unit prices, could sometimes trigger, quite the opposite reactions from the user and with sometimes detrimental effects on the supplier by way of reduced bottom line expected profits. Thus, while it is reasonable to expect that the supplier's bargaining edge would increase with higher values of β, σ^2, or γ, our results seem to indicate that the supplier runs the risk of overestimating the increased leverage afforded her by higher values of these parameters. This underscores the need for the supplier to have a sound understanding of the value that her client places on the price stability offering of her contract.

By allowing the contract price to be tied to the realization of the spot market price, the supplier runs the risk of the realized contract price turning out to be less than what she would have obtained from a fixed price contract if the spot price plunged. However, we have found that when the level of the contract's exposure to the spot price is low enough, the supplier gains the advantage of better competing with the spot market for the commodity user's business, resulting in higher expected profits compared to her guaranteed profits under a fixed price setting. Therefore, aside the commonplace argument that a floating contract price arrangement affords the supplier the opportunity to reap the benefits of upward movements in spot prices, we find that if properly designed, this pricing strategy has the strategic benefit of reducing the threat that spot market competition poses to the supplier.

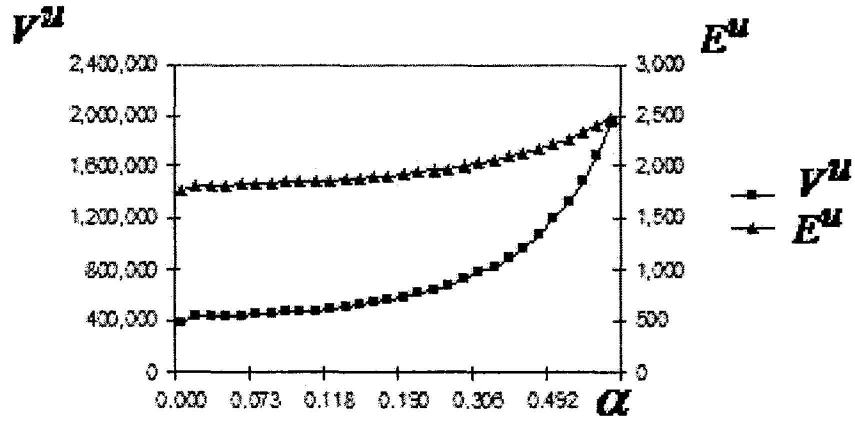

Figure 3.1 Effect of α on E^u and V^u

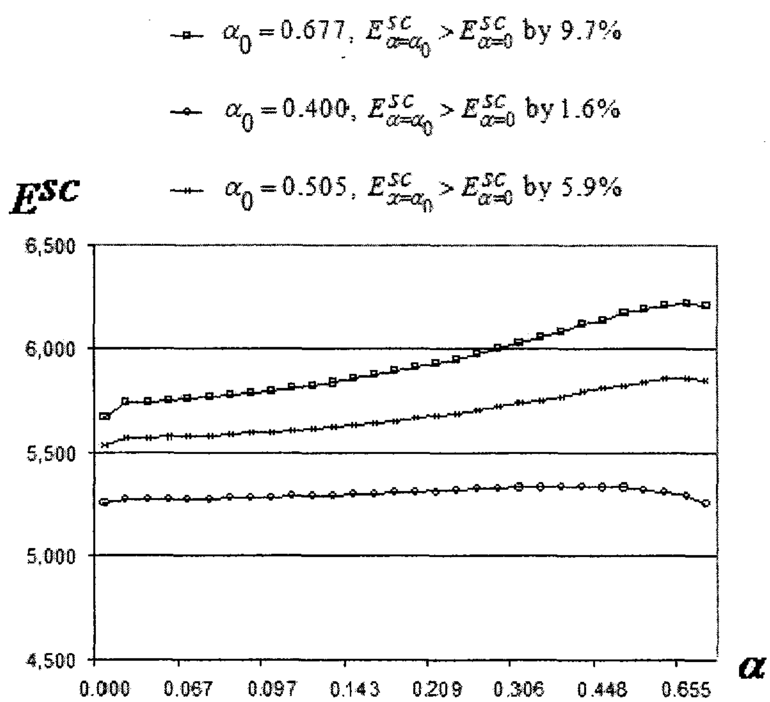

Figure 3.2 Effect of α on E^{sc}

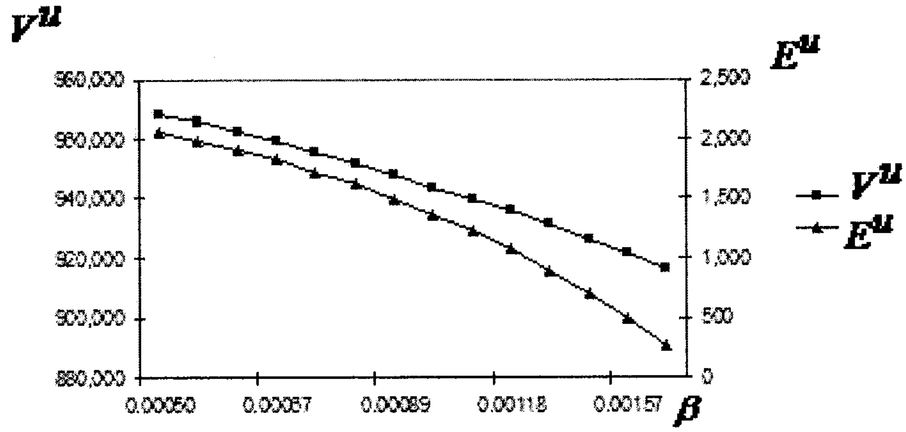

Figure 3.3 Effect of β on E^u and V^u for $\gamma > \gamma_0$

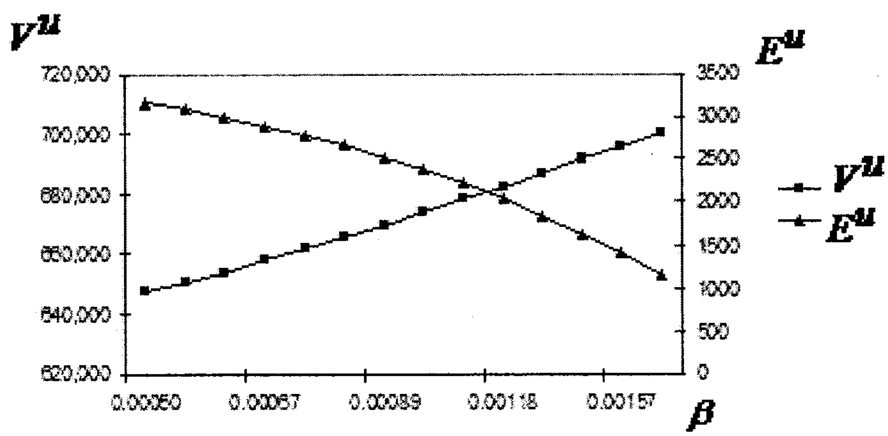

Figure 3.4 Effect of β on E^u and V^u for $\gamma < \gamma_0$

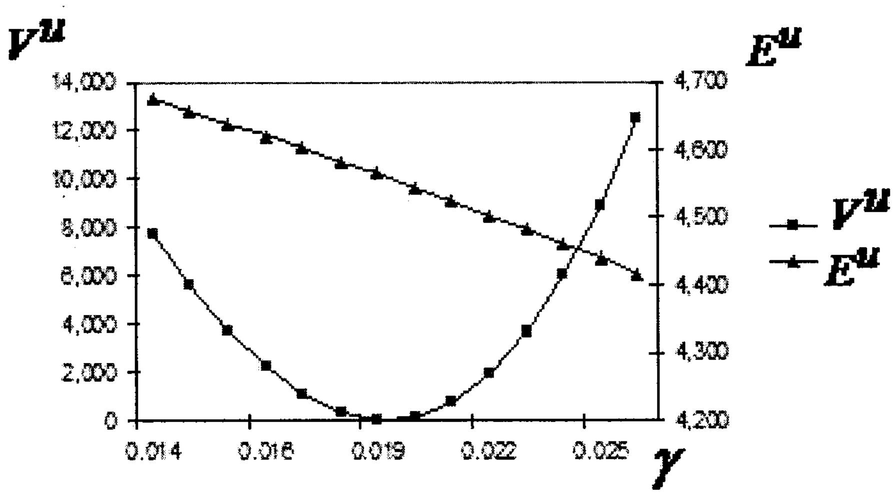

Figure 3.5 Effect of γ on E^u and V^u

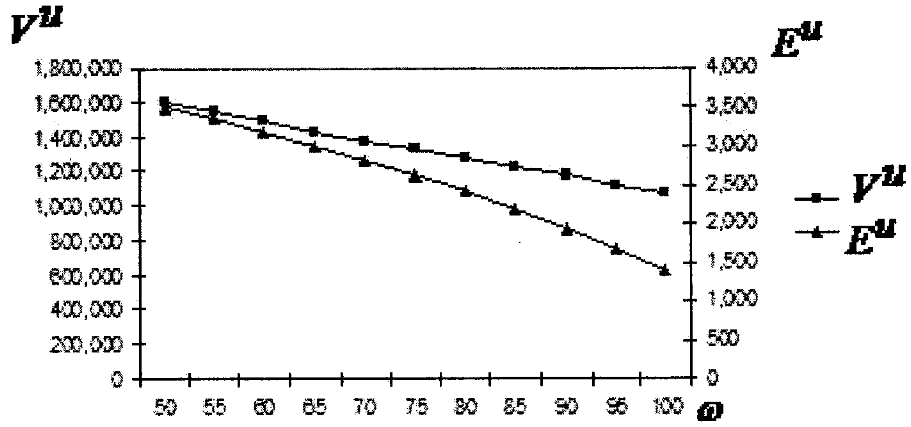

Figure 3.6 Effect of ω on E^u and V^u

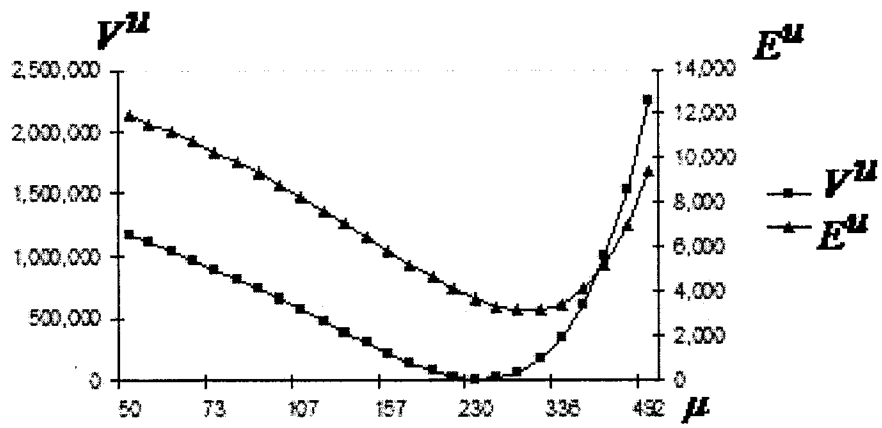

Figure 3.7 Effect of μ on E^u and V^u

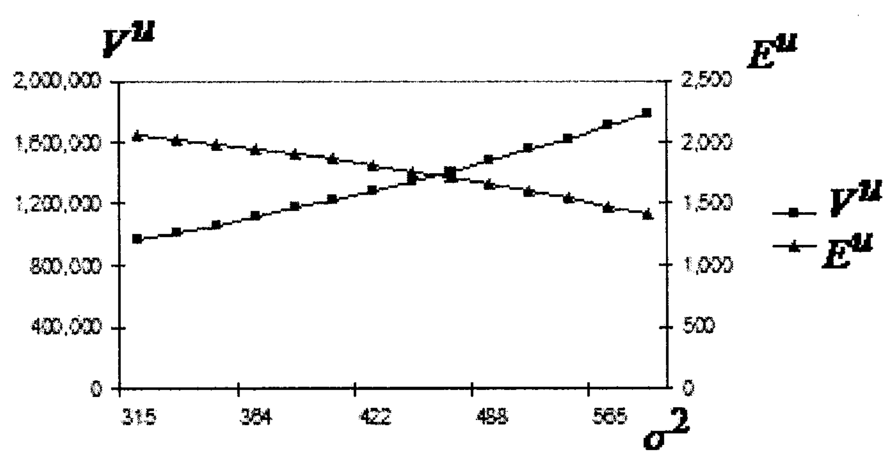

Figure 3.8 Effect of σ^2 on E^u and V^u

CHAPTER 4

THE PURCHASING, PRICING, AND PRODUCTION PROBLEM

4.1 Problem Description

Let the current time be t_0. I again consider a risk-averse commodity user faced with a known end market demand δ at a future time t_1. A spot market exists for the commodity. Suppose the spot price at time t_1 is a random Π. The commodity user realizes ρ dollars in revenue per unit of the commodity. As before, I suppose that storage is prohibitive for the commodity user and the time needed for the acquired commodity to be processed at the user is negligible. The commodity user has two procurement options to consider, namely, contract with a risk-averse supplier, who will deliver the contracted quantity at time t_1, or to purchase the balance of his needs from the spot market at time t_1.

Let us denote the commodity user's purchase from the supplier by x. If $x < \delta$, the user must purchase the entire balance $\delta - x$ from the spot market. The commodity user should not be able to place a negative order through his contract. That is, we should have $x \geq 0$. However, I start by relaxing this non-negativity constraint on x, and study the conditions, if any, under which it would be violated. The question the commodity user seeks to address is that of how much he should purchase through contract and how much he should purchase from the spot market.

Under the contractual arrangement with the supplier, at the decision time t_0, the commodity user enters into a one-time agreement with the supplier for delivery of a given quantity x of the commodity at t_1. Again, the contract price is structured as an affine function of the spot price π realized at t_1:

$$p(\pi) = \alpha\pi + y. \tag{4.1}$$

I suppose that there is an already established agreement on the non-negative α, reflecting the degree of exposure to spot price desired by the commodity user under the contract. The corresponding fixed-component term y is determined by the supplier based on her calculation of the random spot price Π. Like before, I assume that the contract payment is settled at the time t_1 when delivery of the goods is made and the spot price has been realized as some π. Unlike the purchasing and pricing problem that excluded the supplier's spot market participation, it is assumed here that the supplier can easily sell off her excess stock to the spot market or buy from this market to satisfy the user's demand in excess of her available stock. Thus, in addition to establishing the contract's fixed charge y, the supplier also faces the question of what production quantity z to commit to.

For each unit of production, I suppose the supplier incurs some κ amount. While any fixed cost that might be incurred by the supplier in engaging in spot market transactions is assumed to be negligible, as discussed in the introduction, it is assumed that the commodity user incurs some γ spot market access fee per unit purchased from this market. While the user is allowed to buy from the spot market, he is not allowed to sell to this market unlike the previous problem that gives the user both buy and sell access to the spot market. Allowing only spot market buying access for the user is more realistic since given that the user's demand is known, there are little to no practical incentives for the user to engage in speculative selling activities.

The mean-variance approach is adopted in modeling the commodity user's and supplier's risk aversion. I denote the measure of the commodity user's risk aversion as β^u, while β^s denotes the measure of the supplier's aversion to risk.

4.2 Problem Formulation

Suppose the commodity user decides to contract an x quantity from the supplier, the realized price of the contract offered by the supplier is ϕ, the realized spot market price at t_1 is π, and his per unit spot access fee is γ. Then, the commodity user's payoff $f^{u0}(x, \phi, \pi)$ will be as follows:

$$f^{u0}(x, \phi, \pi) = \rho\delta - \phi x - \pi \cdot (\delta - x) - \gamma \cdot (\delta - x). \tag{4.2}$$

Here, $\rho\delta$ is the revenue the user can earn, ϕx is the amount the user has to pay to the supplier according to the contract, $\pi \cdot (\delta - x)$ is the cost of purchasing from the spot market (not including the spot market access fee), and $\gamma \cdot (\delta - x)$ is the total fee the user incurs in accessing the spot market.

Suppose the supplier has decided the y in the contract form (4.1), and the commodity user has made his decision on x, then, when the realized spot market price is π, the user will earn $\psi^u(x, y, \pi)$, where

$$\psi^u(x, y, \pi) = f^{u0}(x, \alpha\pi + y, \pi). \tag{4.3}$$

Combining (4.2) and (4.3), we may work out that

$$\psi^u(x, y, \pi) = f_0^u(x, y) + f_1^u(x) \cdot \pi, \tag{4.4}$$

where

$$\begin{cases} f_0^u(x, y) & = \rho\delta - xy - \gamma \cdot (\delta - x), \\ f_1^u(x) & = (1 - \alpha) \cdot x - \delta. \end{cases} \tag{4.5}$$

The risk-averse commodity user's objective function $f^u(x, y)$ is therefore determined by

$$f^u(x, y) = E[\psi^u(x, y, \Pi)] - \beta^u \cdot V[\psi^u(x, y, \Pi)]. \tag{4.6}$$

Plugging (4.4) into (4.6), we can obtain that

$$f^u(x,y) = f_0^u(x,y) + \mu \cdot f_1^u(x) - \beta^u \sigma^2 \cdot (f_1^u(x))^2, \tag{4.7}$$

where I have used μ to denote the mean $E[\Pi]$ of Π and σ^2 the variance $V[\Pi]$ of Π. Therefore, the commodity user's problem can be formulated as follows:

$$\begin{aligned} \max \quad & f^u(x,y) \\ \text{s.t.} \quad & x \le \delta. \end{aligned} \tag{4.8}$$

Here as well as in the supplier's formulation, I ignore nonnegativity constraint, as they all turn out to be always satisfied. Given the contract coefficient y, the commodity user will solve (4.8) to obtain the equilibrium contract quantity $x^u(y)$ and consequently, the equilibrium spot market purchase quantity $\delta - x^u(y)$.

Suppose the commodity user responds to the supplier's offer with an ordering quantity x, the realized contract price is ϕ, the supplier has decided a production quantity z, and she can buy from or sell to the spot market at t_1 at the prevailing spot price π. Then, the supplier's profit realized at t_1 is $f^{s0}(x,\phi,z,\pi)$, where

$$\begin{aligned} f^{s0}(x,\phi,z,\pi) &= \phi x - \kappa z + \pi \cdot (z-x)^+ - \pi \cdot (z-x)^- \\ &= \phi x - \kappa z + \pi \cdot (z-x). \end{aligned} \tag{4.9}$$

Recall that κ is the per unit production cost charged to the supplier, and z is the quantity the supplier produces. When the contract price follows the form in (4.1) and the commodity user's equilibrium ordering quantity is $x^u(y)$, the supplier's payoff will be

$$\psi^s(y, z, \pi) = f^{s0}(x^u(y), \alpha\pi + y, z, \pi). \tag{4.10}$$

Combining (4.9) and (4.10), we may work out that

$$\psi^s(y, z, \pi) = f_0^s(y, z) + f_1^s(y, z) \cdot \pi, \tag{4.11}$$

where

$$\begin{cases} f_0^s(y, z) & = y \cdot x^u(y) - \kappa \cdot z, \\ f_1^s(y, z) & = z - (1 - \alpha) \cdot x^u(y). \end{cases} \tag{4.12}$$

The risk-averse supplier's objective function $f^s(y, z, \lambda)$ is therefore determined by

$$f^s(y, z) = E[\psi^s(y, z, \Pi)] - \beta^s \cdot V[\psi^s(y, z, \Pi)]. \tag{4.13}$$

Plugging (4.11) into (4.13), we can obtain that

$$f^s(y, z) = f_0^s(y, z) + \mu \cdot f_1^s(y, z) - \beta^s \sigma^2 \cdot (f_1^s(y, z))^2, \tag{4.14}$$

The supplier's problem can be formulated as

$$\begin{array}{ll} \max & f^s(y, z) \\ \text{s.t.} & y, z \text{ are free.} \end{array} \tag{4.15}$$

The supplier will solve (4.15) to obtain the equilibrium contract coefficient y^s and equilibrium production quantity z^s. The commodity user's equilibrium contract quantity will be $x^u = x^u(y^s)$.

I make the following reasonable assumption:

(a1) $\kappa < \mu$, i.e., it is expected to cost the supplier less to produce a unit item than for the item to be purchased from the spot market.

4.3 Analysis

When (4.5) is plugged into (4.7) and then further into (4.8), we may get

$$\begin{aligned} \max \quad & f^u(x,y) = \rho\delta - xy - \gamma\cdot(\delta - x) - \mu\cdot(\delta - x\cdot(1-\alpha)) - \\ & \qquad\qquad \beta^u\sigma^2\cdot(\delta - x\cdot(1-\alpha))^2 \\ \text{s.t.} \quad & x \le \delta. \end{aligned} \tag{4.16}$$

The Lagrangian for (4.16), after regrouping, is given by:

$$\begin{aligned} L^u(x,y,\lambda) = & -\beta^u\sigma^2\cdot(1-\alpha)^2\cdot x^2 - (\lambda - \gamma + y - (1-\alpha)\cdot(\mu + 2\beta^u\sigma^2\delta))\cdot x \\ & + \delta\cdot(\rho - \mu - \gamma - \beta^u\sigma^2\delta + \lambda), \end{aligned} \tag{4.17}$$

where λ is the multiplier corresponding to the constraint $x \le \delta$. Clearly, $L^u(x,y,\lambda)$ is quadratic in x with a negative second order coefficient. The Karush-Kuhn-Tucker (KKT) conditions, which are sufficient and necessary for optimality of the user's problem, are as follows:

$$\frac{\partial L^u(x,y,\lambda)}{\partial x} = 0, \qquad \text{Lagrangian Maximizer,} \tag{4.18}$$

$$x \le \delta, \qquad \text{Explicit Feasibility,} \tag{4.19}$$

$$\lambda\cdot(x-\delta) = 0, \qquad \text{Complementary Slackness,} \tag{4.20}$$

$$\lambda \ge 0, \qquad \text{Positive Shadow Price.} \tag{4.21}$$

For the time being, we may pretend that λ is a given parameter. Now, let us associate all entities defined earlier with a λ field. We may solve (4.18) to obtain $x^u(y, \lambda)$:

$$x^u(y, \lambda) = \frac{(\mu + 2\beta^u \sigma^2 \delta) \cdot (1 - \alpha) - y + \gamma - \lambda}{2\beta^u \sigma^2 \cdot (1 - \alpha)^2}. \tag{4.22}$$

Plugging (4.22) into (4.12) and then further into the supplier's objective function (4.14), we obtain

$$\begin{aligned} f^s(y, z, \lambda) &= -\beta^s \sigma^2 z^2 + (\mu - \kappa - \beta^s \sigma^2 \vartheta / (\beta^u \sigma^2 \cdot (1 - \alpha))) \cdot z \\ &\quad -\beta^s \sigma^2 \vartheta^2 / (2\beta^u \sigma^2 \cdot (1 - \alpha))^2 \\ &\quad +\mu \vartheta / (2\beta^u \sigma^2 \cdot (1 - \alpha)) - y\vartheta / (2\beta^u \sigma^2 \cdot (1 - \alpha)^2), \end{aligned} \tag{4.23}$$

where I have let

$$\vartheta = \lambda - \gamma + y - (1 - \alpha) \cdot (\mu + 2\beta^u \sigma^2 \delta). \tag{4.24}$$

I adopt a two-step approach in solving the supplier's problem. We first fix y and then proceed to solve for the $z^s(y, \lambda)$ that maximizes $f^s(y, z, \lambda)$. It is easy to see that $f^s(y, z, \lambda)$ is quadratic in z with a negative second order coefficient. The equilibrium production quantity $z^s(y, \lambda)$ can therefore be obtained by setting the first order derivative of $f^s(y, z, \lambda)$ with respect to z to zero, i.e.,

$$\frac{\partial f^s(y, z, \lambda)}{\partial z} \Big|_{z = z^s(y, \lambda)} = 0. \tag{4.25}$$

From this we may obtain

$$z^s(y, \lambda) = \frac{\mu - \kappa}{2\beta^s \sigma^2} - \frac{\lambda - \gamma + y - (1 - \alpha) \cdot (\mu + 2\beta^u \sigma^2 \delta)}{2\beta^u \sigma^2 \cdot (1 - \alpha)}. \tag{4.26}$$

In the second step, we substitute (4.26) into (4.23). The supplier's objective function will become

$$
\begin{aligned}
F^s(y,\lambda) \;&= f^s(y, z^s(y,\lambda)) = -y^2/(2\beta^u\sigma^2\cdot(1-\alpha)^2) \\
&-(\eta/(2\beta^u\sigma^2\cdot(1-\alpha)^2) - \kappa/(2\beta^u\sigma^2\cdot(1-\alpha)))\cdot y \\
&+((\mu-\kappa)/(2\beta^s\sigma^2) - \eta/(2\beta^u\sigma^2\cdot(1-\alpha)))\cdot(\mu-\kappa-\beta^s\sigma^2) \\
&-\eta^2/(2\beta^u\sigma^2\cdot(1-\alpha))^2 + \eta/(2\beta^u\sigma^2\cdot(1-\alpha)),
\end{aligned}
\tag{4.27}
$$

where I have let

$$
\eta = \lambda - \gamma - (1-\alpha)\cdot(\mu + 2\beta^u\sigma^2\delta). \tag{4.28}
$$

We observe that (4.27) is quadratic in y with a negative second order coefficient. We can therefore establish $y^s(\lambda)$ by setting

$$
\frac{\partial F^s(y,\lambda)}{\partial y}\,|_{y=y^s(\lambda)} = 0, \tag{4.29}
$$

from which we obtain

$$
y^s(\lambda) = (\frac{\mu+\kappa}{2} + \beta^u\sigma^2\delta)\cdot(1-\alpha) + \frac{\gamma-\lambda}{2}. \tag{4.30}
$$

Plugging (4.30) into (4.22) and (4.26), we obtain

$$
x^u(\lambda) = \frac{(\mu-\kappa+2\beta^u\sigma^2\delta)\cdot(1-\alpha)+\gamma-\lambda}{4\beta^u\sigma^2\cdot(1-\alpha)^2}, \tag{4.31}
$$

$$
z^s(\lambda) = \frac{\mu-\kappa}{4\beta^u\sigma^2} + \frac{\mu-\kappa}{2\beta^s\sigma^2} + \frac{\delta}{2} + \frac{\gamma-\lambda}{4\beta^u\sigma^2\cdot(1-\alpha)}. \tag{4.32}
$$

Now, let us try to fill the value for λ. From the user's optimality conditions (4.19) through (4.21), two optimal solutions are possible namely:

$$\begin{aligned} &\text{Case 1:} && \lambda > 0 \text{ and } x^u(\lambda) = \delta, \\ &\text{Case 2:} && \lambda = 0 \text{ and } x^u(\lambda) \leq \delta. \end{aligned} \tag{4.33}$$

Let us concentrate on case 1 first. From (4.31), setting $x^u(\lambda) - \delta = 0$, we obtain λ to be

$$\lambda = (\mu - \kappa) \cdot (1 - \alpha) + \gamma - 2\beta^u \sigma^2 \delta \cdot (1 - \alpha) \cdot (1 - 2\alpha). \tag{4.34}$$

From (4.34), it becomes clear that $\lambda > 0$ if and only if

$$\gamma > (1 - \alpha) \cdot (2\beta^u \sigma^2 \delta \cdot (1 - 2\alpha) - \mu + \kappa). \tag{4.35}$$

Therefore, when the fee to access the spot market is sufficiently high, the user will be compelled to purchase all of his commodity needs from contract. Substituting (4.34) for λ in (4.30) and (4.32), we obtain

$$\begin{cases} y^s = \kappa \cdot (1 - \alpha) + 2\beta^u \sigma^2 \delta \cdot (1 - \alpha)^2, \\ z^s = \delta \cdot (1 - \alpha) + (\mu - \kappa)/(2\beta^s \sigma^2). \end{cases} \tag{4.36}$$

By assumption (a1), we may observe from (4.36) that neither y^s nor z^s will ever assume any negative value.

Let us then focus on case 2. Substituting zero for λ in (4.30), (4.31), and (4.32), we obtain

$$\begin{cases} x^u = ((\mu - \kappa + 2\beta^u \sigma^2 \delta) \cdot (1 - \alpha) + \gamma)/(4\beta^u \sigma^2 \cdot (1 - \alpha)^2), \\ y^s = ((\mu + \kappa + 2\beta^u \sigma^2 \delta) \cdot (1 - \alpha) + \gamma)/2, \\ z^s = (\mu - \kappa)/(4\beta^u \sigma^2) + (\mu - \kappa)/(2\beta^s \sigma^2) + \delta/2 + \gamma/(4\beta^u \sigma^2 \cdot (1 - \alpha)). \end{cases} \tag{4.37}$$

Again by assumption (a1), we may observe from (4.37) that none of x^u, y^s, and z^s will ever assume any negative value. The above x^u will remain below δ when the opposite to (4.35) is true. From our results thus far, we can state the following.

Proposition 7 *When the user's spot market access fee is so high as to satisfy (4.35), the user's equilibrium strategy will be to purchase all of his commodity needs through contract, i.e.,* $x^u = \delta$*; also, the supplier's equilibrium strategy will be given by (4.36). When the opposite is true, the user's equilibrium strategy will be to satisfy his commodity needs through a combination of contract and spot market purchases; more particularly, equilibrium strategies of the user and the supplier will be given by (4.37).*

At the signing of the contract, the commodity user's payoff is the random Ψ^u, determined through

$$\Psi^u = \psi^u(x^u, y^s, \pi), \tag{4.38}$$

where $\psi^u(\cdot)$ is defined in (4.3); at the same time, the supplier's payoff is the random Ψ^s, determined through

$$\Psi^s = \psi^s(y^s, z^s, \pi), \tag{4.39}$$

where $\psi^s(\cdot)$ is defined in (4.10). Let us now embark on a study of $E[\Psi^u]$, $V[\Psi^u]$, $E[\Psi^s]$, $V[\Psi^s]$, $E[\Psi^u] + E[\Psi^s]$, and $E[p(\pi)]$ where $p(\pi)$ is defined in (4.1).

For the commodity user, using (4.5), (4.6), and (4.7), we may derive that

$$\begin{cases} E[\Psi^u] & = f_0^u(x^u, y^s) + \mu \cdot f_1^u(x^u) \\ & = \rho\delta - z^s x^u - \gamma \cdot (\delta - x^u) - \mu \cdot (\delta - x^u \cdot (1-\alpha)), \\ V[\Psi^u] & = \sigma^2 \cdot (f_1^u(x^u))^2 = \sigma^2 \cdot (\delta - x^u \cdot (1-\alpha))^2. \end{cases} \tag{4.40}$$

For the supplier, using (4.1), (4.11), (4.12), and (4.13), we may derive that

$$\begin{cases} E[p(\pi)] & = \alpha\mu + y^s, \\ E[\Psi^s] & = f_0^s(y^s, z^s) + \mu \cdot f_1^s(x^u, z^s) \\ & = y^s \cdot x^u - \kappa z^s + \mu \cdot (z^s - (1-\alpha) \cdot x^u), \\ V[\Psi^s] & = \sigma^2 \cdot (f_1^s(x^u, z^s))^2 = \sigma^2 \cdot (z^s - (1-\alpha) \cdot x^u)^2. \end{cases} \tag{4.41}$$

For case 1 with $x^u = \delta$, substituting (4.36) in (4.40), (4.41), and $\alpha\mu + z^s$, we obtain

$$\begin{cases} E[\Psi^u] & = \rho\delta - \alpha\mu\delta - \delta \cdot (1-\alpha) \cdot (\kappa + 2\beta^u\sigma^2\delta \cdot (1-\alpha)), \\ V[\Psi^u] & = \alpha^2\sigma^2\delta^2, \\ E[\Psi^s] & = 2\beta^u\sigma^2\delta^2 \cdot (1-\alpha)^2 + (\mu-\kappa)^2/(2\beta^s\sigma^2), \\ V[\Psi^s] & = \sigma^2 \cdot (\mu-\kappa)^2/(2\beta^s\sigma^2)^2, \\ E[\Psi^u] + E[\Psi^s] & = \rho\delta - \kappa\delta + (\mu-\kappa)^2/(2\beta^s\sigma^2) - \alpha\delta \cdot (\mu-\kappa), \\ E[p(\pi)] & = \alpha \cdot (\mu-\kappa) + \kappa + 2\beta^u\sigma^2\delta \cdot (1-\alpha)^2. \end{cases} \tag{4.42}$$

For case 2, substituting (4.37) in (4.40), (4.41), and $\alpha\mu + z^s$, we obtain

$$\begin{cases} E[\Psi^u] & = (2\gamma \cdot (\mu - \kappa) \cdot (1-\alpha) + \gamma^2)/(8\beta^u\sigma^2 \cdot (1-\alpha)^2) \\ & \quad + (\mu - \kappa)^2/(8\beta^u\sigma^2) - \delta \cdot ((\beta^u\sigma^2\delta/2) + \mu - \rho + \gamma), \\ V[\Psi^u] & = \sigma^2 \cdot (\delta + ((\mu - \kappa + 2\beta^u\sigma^2\delta) \cdot (1-\alpha) + \gamma) \\ & \quad /(4\beta^u\sigma^2 \cdot (1-\alpha)^2))^2, \\ E[\Psi^s] & = (\beta^u\sigma^2\delta^2/2) + (\mu-\kappa)^2/(2\beta^s\sigma^2) + (\delta \cdot (\mu - \kappa)/2) \\ & \quad + (\mu-\kappa)^2/(8\beta^u\sigma^2) + (\gamma^2 + 2\gamma \cdot (\mu - \kappa + 2\beta^u\sigma^2\delta) \cdot \\ & \quad (1-\alpha))/(8\beta^u\sigma^2 \cdot (1-\alpha)^2), \\ V[\Psi^s] & = \sigma^2 \cdot (\mu-\kappa)^2/(2\beta^s\sigma^2)^2, \\ E[\Psi^u] + E[\Psi^s] & = (\mu-\kappa)^2/(2\beta^s\sigma^2) + (\mu-\kappa)^2/(4\beta^u\sigma^2) \\ & \quad - \delta \cdot (((\mu+\kappa)/2) - \rho + \gamma) + (\gamma^2 + \\ & \quad 2\gamma \cdot (\mu - \kappa + \beta^u\sigma^2\delta) \cdot (1-\alpha))/(4\beta^u\sigma^2 \cdot (1-\alpha)^2), \\ E[p(\pi)] & = ((\mu + \kappa + \gamma)/2) + \beta^u\sigma^2\delta + (\alpha/2) \cdot (\mu - \kappa - 2\beta^u\sigma^2\delta). \end{cases} \tag{4.43}$$

4.4 A Comparative Statics Study

I now study how x^u, y^s, z^s, $E[\Psi^u]$, $V[\Psi^u]$, $E[\Psi^s]$, $V[\Psi^s]$, $E[\Psi^u] + E[\Psi^s]$, and $E[p(\pi)]$ are influenced by the problem's parameters. According to (4.36) through (4.43), the following is a list of relevant parameters:

(I) α–the degree of the contract's exposure to spot price volatility;

(II) β^u–the commodity user's degree of risk aversion;

(III) β^s–the supplier's degree of risk aversion;

(IV) γ–the user's spot market access fee;

(V) μ–the expected value of the spot price; and,

(VI) σ^2–the variance of the spot price.

Besides, the following two parameters are subject to scaling:

(VII) δ–the fixed demand level; and,

(VIII) κ–the supplier's unit production cost.

4.4.1 Effects of α, the Contract's Exposure to Spot Price

My findings concerning α can be summarized as follows.

Proposition 8 *When (4.35) holds so that the user's equilibrium strategy is to solely source his needs through contract, the following will happen:*

(a) The user's equilibrium variance of profit $V[\Psi^u]$ is increasing in the contract's spot price exposure α.

Define α_0 so that

$$\alpha_0 = 1 - \frac{\mu - \kappa}{4\beta^u \sigma^2 \delta}.$$

The user's equilibrium expected profit $E[\Psi^u]$ is increasing in α when $\alpha < \alpha_0$, and decreasing in α otherwise.

(b) The supplier's equilibrium fixed charge y^s, equilibrium production volume z^s, and equilibrium expected profit $E[\Psi^s]$, are all decreasing in α. When $\alpha > \alpha_0$, the supplier's equilibrium expected contract price $E[p(\pi)]$, is increasing in α, and decreasing in α otherwise. Her equilibrium variance of profit $V[\Psi^s]$ is not impacted by α.

(c) The total supply chain equilibrium expected profit $E[\Psi^u] + E[\Psi^s]$ is decreasing in α. When the opposite to (4.35) is true so that the user's equilibrium strategy is to use a combination of contract

and spot market to source his needs, the following will happen:

(d) The user's equilibrium contract quantity x^u, and equilibrium expected profit $E[\Psi^u]$, are increasing in α. His $V[\Psi^u]$ is decreasing in α.

(e) For the supplier, y^s, and $E[p(\pi)]$ are decreasing in α, while z^s is increasing in α. Her equilibrium expected profit $E[\Psi^s]$ is increasing in α, while her $V[\Psi^s]$ is again not impacted by α.

(f) For the supply chain, $E[\Psi^u] + E[\Psi^s]$ is increasing in α.

Discussion of Propositions 8(a-c)

From proposition 8(b), we see that while the supplier will always to respond to an increased contract spot exposure α with a reduced fixed charge y, she is however more hesitant to reduce y when α is already very high (i.e. $\alpha > \alpha_0$) compared to when α is already low enough (i.e. $\alpha > \alpha_0$). This is what we would expect in practice since a high level of α puts the supplier at great risk of exposure to spot price volatility and the supplier would naturally try to mitigate this exposure with higher expected contract prices. Therefore, we see that while the user benefits from higher levels of $\alpha < \alpha_0$ with his equilibrium expected profit $E[\Psi^u]$ peaking at $\alpha = \alpha_0$, his $E[\Psi^u]$ goes downhill with the deteriorating pricing deals from the supplier for higher levels of $\alpha > \alpha_0$.

We observe from proposition 8(b) that the supplier's equilibrium production volume z^s is decreasing in α. There are two possible explanations for this observation. Firstly, when $\alpha < (\mu - \kappa)/(2\beta^s\delta)$ so that $z^s > \delta$, the supplier's equilibrium strategy is to produce more than the user's demand δ and sell the excess to the spot market. In this case, for the profit variance minimizing supplier, the more the spot price exposure she already has through the contract, the more she would want to cut back on her direct exposure to the spot market by reducing her spot market selling volume. This also explains why the supplier's equilibrium variance of profit $V[\Psi^s]$ is not impacted by α. Secondly, when $\alpha > (\mu - \kappa)/(2\beta^s\delta)$ $z^s < \delta$, the supplier's equilibrium strategy involves satisfying the user's needs through a combination of her production activity and spot market buying engagements.

Indeed, in practice, instances can be found where although it might be expected to be cheaper to produce an item than to source the item through the spot market, the item's extreme spot price volatility might make the latter option more attractive to the supplier. In this case, the supplier is more concerned with the possibility of the spot price dropping below her production cost κ. Hence, as α increases, the more she would seek to satisfy the user's demand at a cost closer to the realized spot price due to the fear that the realized contract price may not compensate for her production cost. This again explains why α has no effect on $V[\Psi^s]$. From these two explanations, it is straightforward to see why notwithstanding the fact that the supplier gets to charge higher contract prices as $\alpha > \alpha_0$ increases, her equilibrium expected profit $E[\Psi^s]$ is nonetheless strictly decreasing in α. In the first case where $z^s > \delta$, the reduced spot market selling volume reduces her expected revenue generating potential. For $z^s < \delta$, by assumption (a1), her increased reliance on the spot market to satisfy the user's demand increases her expected cost.

From proposition 8(c), we can conclude that when the user's equilibrium strategy is to purchase solely through the contract, even when the dynamic contract pricing arrangement is beneficial to the user, its negative effect on the supplier is always significant enough to reduce the overall supply chain equilibrium expected profit. It is safe to predict that under this sole sourcing strategy, it would take a user with superior bargaining strength to negotiate such dynamic pricing with the supplier.

Discussion of Proposition 8(d-f)

From propositions 8(d-f), we see that when the user's equilibrium strategy is to use a combination of contract and spot market to source his commodity needs, dynamic contract pricing has a win-win effect on the user-supplier relationship to the overall benefit of the supply chain. As a higher level of exposure to spot price is sought under the contract, the supplier responds with a reduced contract price to lure the user to the contract. This is because a higher α would naturally indicate that the user is less spot market shy, thereby raising the threat the spot market poses to the supplier. The user in turn responds to the improved contract offer by way of an increased contract quantity that more than compensates the supplier for her reduced price. The observation that z^s is increasing in α is directly attributable to the increased x^u.

Thus, perhaps, the most important finding in this section is that when the supplier is in competition with the spot market for the user's business, by offering the user a dynamic contract price, the supplier can gain a strategic advantage over the spot market. When there is no such spot market threat, dynamic contract pricing is never expected to work to the supplier's benefit.

To illustrate the effect of α on equilibrium expected profits, we present two numerical examples. In the first example (Figure 4.1), varying α from 0 to 1, we fix the other parameters as follows: $\rho = 100, \kappa = 40, \delta = 300, \gamma = 3, \beta^u = 0.0002, \beta^s = 0.0001, \mu = 60, \sigma^2 = 150$, so that the user's equilibrium strategy is always to purchase his total needs from contract. In the second example (Figure 4.2), $\rho = 100, \kappa = 20, \delta = 1000, \gamma = 2, \beta^u = 0.0003, \beta^s = 0.0002, \mu = 40, \sigma^2 = 150$, while α is varied from 0 to 0.37, so that the user's equilibrium strategy is to use a combination of contract and spot market. We see that the supply chain's equilibrium expected profit is deteriorating in α in Figure 4.1, while in Figure 4.2, an improvement of 1.13% of the supply chain equilibrium expected profit is realized for $\alpha = 0.37$ over $\alpha = 0$ (fixed price contract). It should be noted that our selected values of γ are in line with what would be expected in practice, as typically, the spot market unit transaction cost of a commodity can be roughly a tenth of its spot price (see Secomandi and Kekre (2009)).

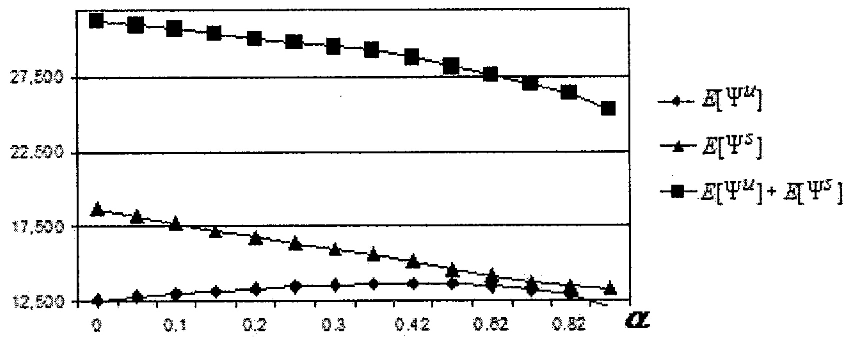

Figure 4.1 Effect of α on $E[\Psi^u]$, $E[\Psi^s]$, and $E[\Psi^u] + E[\Psi^s]$ for $x^u = \delta$.

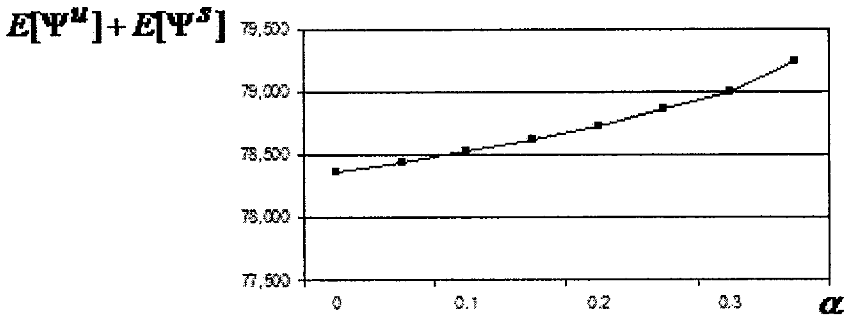

Figure 4.2 Effect of α on $E[\Psi^u] + E[\Psi^s]$ for $x^u \leq \delta$.

4.4.2 Effects of Risk Aversion Factors β^u and β^s.

Concerning β^u, and β^s, my findings are summarized below.

Proposition 9 *When (4.35) holds, the following will happen:*

(a) The user's equilibrium expected profit $E[\Psi^u]$ is decreasing in β^u, while β^s has no effect on $E[\Psi^u]$; also, β^u and β^s have no impact on $V[\Psi^u]$.

(b) For the supplier, y^s and $E[\Psi^s]$ are increasing in β^u, while β^s has no effect on these. Both z^s and $V[\Psi^s]$ are decreasing in β^s, and not impacted by β^u.

When the opposite of (4.35) is true, the following will happen:

(c) The user's equilibrium contract quantity x^u is decreasing in β^u. His $E[\Psi^u]$ is decreasing in β^u and $V[\Psi^u]$ is increasing in β^u. Also, β^s has no effect on the user's equilibrium results.

(d) For the supplier, y^s and $E[\Psi^s]$ are increasing in β^u, while β^s has no effect on these. Both z^s and $V[\Psi^s]$ are decreasing in β^s, and while z^s is also decreasing in β^u, $V[\Psi^s]$ is not impacted by this parameter.

(e) Both the user's $E[\Psi^u]$ and supplier's $E[\Psi^s]$ increasing rates in α are decreasing in β^u, while both are unaffected by β^s.

Discussion of Proposition 9(a-f)

We see from Propositions 9(b & d) that while increased levels of the supplier's aversion to risk do not affect her pricing leverage, she always exploits increased levels of the user's risk aversion β^u by charging higher contract prices. Even when the user is increasingly forced into the spot market as her β^u increases due to the supplier's deteriorating deals, the supplier's gain in per unit contract price more than compensates for the lost sales volume as indicated in Propositions 9(c & d). We can therefore conclude that for the risk-averse user-supplier relationship, the supplier has the bargaining edge. The observation that the supplier's z^s is always decreasing in β^s is intuitive as we would expect the supplier to cut down on her spot market selling engagements as she gets more risk-averse. Proposition 9(d)'s observation that z^s is decreasing in the user's risk aversion β^u is a direct result of x^u's decline with β^u.

Overall, while we would ordinarily expect the user to find the relative stability offered by the contract more attractive as he gets increasingly risk-averse and order more from contract, we find that the supplier's opportunistic response to increased β^u triggers quite the opposite reaction from the user. We however find that this opportunistic behavior on the part of the supplier could result in the lowering of her profit earning potential as we see from Proposition 9(e) that the gains in the supplier's equilibrium expected profits with increasing α occur at slower rates for higher levels of the user's risk aversion.

4.4.3 Effects of γ, the User's Spot Market Access Fee

My findings concerning γ are given as follows.

Proposition 10 *When (4.35) is true, the following will happen:*

(a) γ has no impact on the equilibrium results.

When the opposite of (4.35) holds, the following will happen:

(b) The user's equilibrium contract quantity x^u is increasing in γ. His $E[\Psi^u]$ is decreasing in γ, while $V[\Psi^u]$ is increasing in γ.

(c) For the supplier, y^s (and consequently $E[p(\pi)]$), z^s, and $E[\Psi^s]$ are all increasing in γ. γ has no effect on $V[\Psi^s]$.

(d) Both the user's $E[\Psi^u]$ and supplier's $E[\Psi^s]$ increasing rates in α are increasing in γ.

Discussion of Proposition 10(a-c)

The effects of γ on equilibrium results are intuitive. Obviously, when the user's equilibrium strategy is to use a combination of contract and spot market, the more it costs the user to access the spot market, the more the user would want to shy away from this market. The reduced spot market threat in turn provides the supplier a higher pricing leverage to capitalize on. It can be seen from (4.43), that for very low values of γ, the improvements in supply chain coordination that can be realized from the dynamic contract can be very marginal. In fact, for $\gamma = 0$, the equilibrium expected profits $E[\Psi^u]$ and $E[\Psi^s]$, and equilibrium variances of profit $V[\Psi^u]$ and $V[\Psi^s]$, are unaffected by α. Therefore, when the user's spot market access fee γ is negligible and the user's equilibrium strategy is to use a combination of contract and spot market purchases, the dynamic $\alpha > 0$ contract yields the same performance as a locked-in $\alpha = 0$ contract. In this case, the only benefit of the dynamic contract from the user's perspective is that it allows the user to take advantage of plunges in spot prices if such do occur. On the part of the supplier, the dynamic contract serves to increase her share of the user's business and allows her to gain from spot price hikes. The observation that higher equilibrium expected profits are realized from the floating contract with higher values of γ can be explained as follows. Since the user's hesitation to access the spot market increases in γ, the higher the γ therefore, the more eager is the user to take advantage of the improved contract offer that accompanies an increase in α. The effect on the supplier is that her share of the user's business increases in α at a faster pace with higher values of γ.

Figure 4.3 illustrates how the expected profit improvements realized from an $\alpha > 0$ contract becomes less significant with negligible spot access fee γ. Keeping all other parameters of Figure 4.2 the same, we plot $E[\Psi^u] + E[\Psi^s]$ against α for $\gamma = 5, 3, 1, 0.5$. It can be seen from Figure 4.3 that while for $\gamma = 5$, an improvement of 3% in supply chain equilibrium expected profit is realized for an $\alpha = 0.37$ contract over an $\alpha = 0$ contract, only a 0.27% improvement is realized for $\gamma = 0.5$.

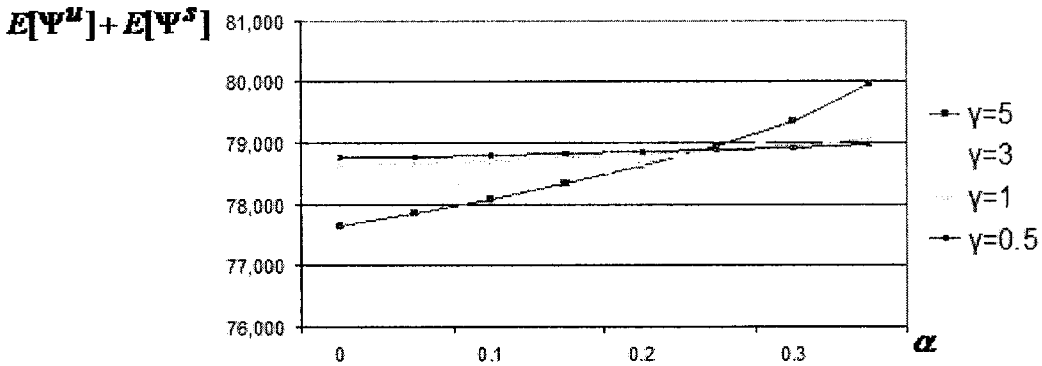

Figure 4.3 Significance of γ on Supply Chain Improvements with $\alpha > 0$ Contracts.

4.4.4 Effects of μ, the Expected Spot Price of the Commodity

Concerning μ, the following is true.

Proposition 11 *When (4.35) holds, the following will happen:*

(a) For the user, $E[\Psi^u]$ *is decreasing in* μ*, while* μ *has no effect on* $V[\Psi^u]$.

(b) For the supplier, $E[p(\pi)]$, z^s, $E[\Psi^s]$, *and* $V[\Psi^s]$ *are all increasing in* μ. μ *has no impact on* y^s.

When the opposite of (4.35) is true, the following will happen:

(c) For the user, x^u *is increasing in* μ*, while* $E[\Psi^u]$ *is decreasing in* μ *and* $V[\Psi^u]$ *is increasing in* μ.

(d) All the supplier's equilibrium results are increasing in μ.

Discussion of Proposition 11(a-d)

It is interesting to find that when the spot market poses no threat to the supplier (i.e. $x^u = \delta$), the supplier's equilibrium fixed charge y^s is independent of the expected spot market price. This brings to bear, the importance of spot market competition in putting the supplier in check in her pricing decisions as indicated by the dependence of y^s on μ when $x^u < \delta$. As we would also expect, a higher μ raises the chances of realizing a high spot price, and the user becomes more cautious in considering spot market engagements. This explains why x^u is increasing in μ as indicated in proposition 11(c). The supplier in turn, not only capitalizes on the reduced spot market threat by charging higher values of the contract coefficient y, but also gains from the increased spot price premium. These result in higher expected profits for the supplier and lower expected profits for the user. Overall, we can conclude that higher expected spot prices always work to the supplier's benefit, and never the user's.

4.4.5 Effects of σ^2, the Spot Price Volatility

The following can be concluded concerning σ^2.

Proposition 12 *When (4.35) is true, the following will happen:*

(a) For the user, $E[\Psi^u]$ is decreasing in σ^2 and $V[\Psi^u]$ is increasing in σ^2.

(b) For the supplier, y^s (and consequently $E[p(\pi)]$) is increasing in σ^2, while z^s and $V[\Psi^s]$ are decreasing in σ^2.

Define σ_0^2 so that

$$\sigma_0^2 = \frac{\mu - \kappa}{2\delta \cdot (1-\alpha) \cdot (\beta^s \beta^u)^{0.5}}.$$

The supplier's $E[\Psi^s]$ is increasing in σ^2 when $\sigma^2 > \sigma_0^2$ and decreasing in σ^2 otherwise.

When the opposite of (4.35) is true, the following will happen:

(c) For the user, x^u, and $E[\Psi^u]$ are decreasing in σ^2, while $V[\Psi^u]$ is increasing in σ^2.

(d) For the supplier, y^s (and consequently $E[p(\pi)]$) is increasing in σ^2, while z^s, and $V[\Psi^s]$ are decreasing in σ^2.

Define σ_1^2 so that

$$\sigma_1^2 = \left(\frac{\beta^s \cdot (\gamma + (\mu - \kappa) \cdot (1-\alpha))^2 + 4\beta^u \cdot (\mu-\kappa)^2 \cdot (1-\alpha)^2}{4\beta^s \cdot (\beta^u \delta \cdot (1-\alpha))^2}\right)^{0.5}.$$

The supplier's $E[\Psi^s]$ is increasing in σ^2 when $\sigma^2 > \sigma_1^2$ and decreasing in σ^2 otherwise.

Discussion of Proposition 12(a-d)

The observation that y^s is increasing in σ^2 comes at no surprise as we would expect the supplier to mitigate the increased risk by charging higher prices. The result that the supplier's z^s is decreasing in σ^2 can be attributed to the following. First is her need to reduce exposure to the increasingly volatile spot market when her equilibrium strategy involves selling excess units to the spot market.

Second is her need to get the cost of satisfying the user's demand as close as possible to the realized spot price when her equilibrium strategy is to use a combination of her production and spot market purchase volumes.

We would expect that when the user's equilibrium strategy is to use a combination of contract and spot market, the more volatile the spot price gets, the more the risk-averse user would want stay away from the spot market. It is therefore interesting to see from proposition 12(c) that x^u is decreasing in σ^2. This contradiction is attributed to the supplier's opportunistic response to higher values of σ^2 by way of higher contract charges as indicated in 12(d). We however observe that except σ^2 is high enough (i.e. $\sigma^2 > \sigma_1^2$) so that the user is extremely cautious in trading in the spot market and responds less quickly to exorbitant charges by the supplier, the supplier's opportunistic tendency has the undesired effect of reducing her bottom line equilibrium expected profits. Proposition 12(b)'s finding that when the user's equilibrium strategy is to solely use contract, the supplier's equilibrium expected profit is increasing in $\sigma^2 > \sigma_0^2$ and decreasing in σ^2 otherwise, can only lead us to conclude that for the supplier, the drop in z^s occurs at a slower pace relative to the pace of her gains in y^s when $\sigma^2 > \sigma_0^2$, while the reverse is the case when $\sigma^2 < \sigma_0^2$.

4.5 Managerial Implications

Just as we saw in the previous problem, we find yet again instances in which the risk-averse commodity user could become less hesitant in considering purchasing from the spot market with increased levels of risk aversion β^u, and spot price volatility σ^2. We would expect that higher levels of β^u and σ^2 should cause the user to be further drawn to the contract's price stability offering. We however find that when the user's equilibrium strategy is to use a combination of contract and spot market to source his needs, the supplier's exploitation of the commodity user's vulnerability at elevated values of these parameters by charging higher contract prices, always triggers, quite the opposite reactions from the user. In particular, it was interesting to find that the user's reaction to the supplier's opportunistic response to increased levels of σ^2 could sometimes have detrimental effects on the supplier by way of reduced bottom line expected profits. Thus, while it is reasonable to expect that the supplier's bargaining edge would increase with higher values of β^u, and σ^2, we see yet again that the supplier runs the risk of overestimating the increased pricing leverage afforded her by higher values of these parameters. It is thus imperative that supplier understands the value that her client places on price stability.

By offering a floating contract that is tied to the realization of the spot market price, the supplier is clearly exposed to the risk of realizing a contract price that hardly compensates her for her cost of production. However, we have found that when the user's equilibrium strategy is to split his purchasing needs between contract and spot market, with the floating contract, the supplier gains the advantage of competing better against the spot market for the commodity user's business. When the user's spot market access fee is not negligible, the supplier's competitive advantage with the dynamic contract results in higher expected profits, compared to her guaranteed profits under a fixed price setting. Therefore, just as we saw with the previous problem, this pricing strategy clearly has the strategic benefit of reducing spot market threat for the supplier. Furthermore, it was discovered that where the level of spot exposure sought under the contract is high, by using the right combination of in-house production and spot market purchases to satisfy her client's needs, the supplier is able to mitigate her exposure to spot price volatility under such dynamic contracts.

Moreover, with the supplier's equilibrium variance of profit independent of the contract's exposure to the spot price, in terms of gaining greater control over variable profits, the supplier is no better off with a fixed contract price. Therefore, while the concept of a procurement portfolio for the risk mitigating buying firm has been widely studied (e.g. the Hewlett-Packard's PRM), I have demonstrated in this dissertation that a supplier offering a floating price contract to edge out spot market competition, can successfully manage the associated risks by adopting a portfolio of supply strategies. Overall, my findings from both problems studied support the contention that even with today's proliferation of online spot markets for a broad range of commodities (and near-commodities), well structured long-term contractual relationships are sustainable.

CHAPTER 5
CONCLUSION

The purchasing, and pricing problem addressed in the first part of this dissertation explored the effects of risk aversion, spot price behavior, dynamic contract pricing and capacity reservation on the purchasing decision of a risk-averse commodity user and the pricing decision of his risk-neutral supplier. The demand faced by the commodity user is assumed to be known and it is assumed that the supplier can temporarily expand capacity to accommodate a contract order above the reserved capacity. Further, the supplier faces competition from the spot market for her client's business.

Using the mean-variance rule to model the commodity user's aversion to risk, closed-form equilibrium solutions were derived for the commodity user and supplier, and managerial insights were drawn from analytical results and numerical studies. While it was found that higher dedicated capacities and higher expected spot prices would always result in higher equilibrium expected profits for the supplier, conditions were found in which the supplier's opportunistic response to her client's increased aversion to risk and increased spot price volatility would result in the lowering of her earnings potential. It was also discovered that low penalties for off-capacity contract orders and high levels of exposure to spot price sought by the commodity user under the contract can potentially raise the threat that the spot market poses to the supplier.

It was shown through analytical results and numerical studies that when the level of spot price exposure the commodity user seeks under the contract is not too high, the proposed dynamic contract is superior to a fixed price contract arrangement by way of improved equilibrium expected profits for the supply chain. Analytical results also showed that the dynamic contract can strategically position the supplier against spot market competition when the contract's spot exposure is low enough, while it was found through numerical studies that an increased contract spot exposure always leads to an improved bottom line expected profit for the commodity user.

The more elaborate purchasing, pricing, and production problem addressed in the second part of this dissertation explored the effects of dynamic contract pricing, risk aversion, spot price behavior, and non-frictionless spot market access on the purchasing decision of a risk-averse commodity user and the pricing and production decisions of his risk-averse supplier. Again, the demand faced by the commodity user is assumed to be known and both the user and supplier have direct access to the spot market. While the supplier can buy from or sell to the spot market at the prevailing spot price, the user can only buy from the spot market at the spot price plus some access fee that is linearly increasing in the spot market purchase volume. As is the case with the first problem, the contract pricing arrangement is such that the supplier's offered price is tied to the realized spot price of the commodity at the time of delivery.

Modeling both parties' aversion to risk using the mean-variance maxim, closed-form equilibrium solutions were derived for the commodity user and supplier, and managerial insights were drawn from analytical results. A threshold on the user's spot market access fee was derived, above which, the user's equilibrium strategy is always to purchase all his commodity needs from contract. Below this threshold, his equilibrium strategy involves using a combination of contract and spot market to satisfy his needs. Where the user's equilibrium strategy is to solely source from contract, it was found that though the proposed dynamic pricing arrangement may benefit the user, such pricing structure never works to the supplier's and overall supply chain's benefits. When the user's spot market access fee is non-zero and his equilibrium strategy is to use a portfolio of contract and spot market, it was discovered that the dynamic contract pricing arrangement always leads to better supply chain coordination with win-win outcomes for both parties (user and supplier). Conditions on the problem parameters were derived, under which, the supplier's equilibrium strategy entails using a portfolio of supply strategies (in-house production and spot market purchases) to mitigate her risk of exposure to spot price volatility under the contract.

Where the user uses a portfolio of contract and spot market, the supplier's opportunistic response to higher levels of the user's aversion to risk (β^u), and spot price volatility (σ^s), by charging higher prices, has the unexpected effect of driving the user further into the spot market. While notwithstanding the lost business from the user, her exploitation of higher values of β^u was found to always result in increased equilibrium expected profits for the supplier, it was however discovered that such opportunistic behavior diminishes the rate of increase in her equilibrium expected profits with higher values of α, the contract exposure to spot price. Furthermore, when the spot price is not so volatile, the supplier's exploitation of higher values of σ^2 only serves to hurt her bottom line expected profits. While the supplier gains pricing advantage with increased expected spot price μ as well as increased user's spot access fee γ, the user's equilibrium expected profit dwindles as either of these parameters increases. Lastly, it was discovered that while the supplier's aversion to risk has no effect on the commodity user's decision making process, the supplier's equilibrium strategy is to produce less units of the commodity as she gets more risk averse.

CHAPTER 6
FUTURE RESEARCH DIRECTIONS

This chapter proposes some possible extensions to the work conducted in this dissertation and discusses an emerging and closely related research direction.

6.1 Effects of Sport Market Yield

In this work, I have assumed that the commodity user will always get the exact amount of the order he places with the spot market, so spot market yield was not considered an issue. In practice however, it is not always guaranteed that a buyer will get the exact units of the item sought from the spot market at the given point in time. For example, at the time it is needed, the item might be hard to find in the spot market due to shortage in supply. Even where the availability of the item is not an issue, there may be quality issues associated with the purchased item, given that most spot market transactions are one-off trading engagements between strangers. Therefore, while from past experience with his long-term supplier's quality, a buyer may know for instance, that 1 unit of an input material bought from the supplier will yield 1 unit of the end product, he may not be able to accurately make the same prediction regarding the quality of output that can be experienced from spot purchase.

To this end, it would be interesting to study the effects of spot market reliability on both the commodity user and the supplier, especially since in this case, the supplier will be able to leverage the supply reliability offering in addition to the higher price stability that the contract enjoys.

6.2 Effects of Disparity in Spot Market Price Distributions

In this dissertation, while I have modeled some imperfections in the spot market in terms of how the buyer can access it in the purchasing, pricing, and production problem, I have nonetheless adopted the same widespread assumption in literature that the commodity user and supplier both face the same spot market price distribution. As suggested by Wu et al. (2002), a reasonable generalization that one could consider is the case in which both parties face different spot price distributions.

6.3 Relational Contracts

In this dissertation, I have treated the commodity producer's (supplier's) cost of production as fixed. In practice however, it is not uncommon for a commodity producer to experience cost uncertainty in her production operations. For example, while the cost of finding the iron ore may be fixed for an iron ore producer, she may be exposed to the fluctuations in the prices of the energy commodities she relies on for her operations.

With growing concerns about the effects of rising costs on suppliers' bottom lines, of recent, there's been a gradual emergence of works in the supply chain contracting literature addressing the issue of contracting under supplier production cost uncertainty. These works have been mainly inspired by the success of the buyer-supplier alliances observed in the Japanese automotive industry. These alliances are characterized by, among others, long-term relationships and risk-sharing contracting practices wherein the buyer assumes some of the risk associated with for instance, the supplier's production cost by having the contract price tied to the supplier's realized cost of production. This risk-sharing pricing strategy serves to alleviate the suppliers' exposure to cost increases particularly those derived from exogenous factors like increased commodity prices. Such cooperative buyer-supplier relationships largely account for the productive and financial edge the

Japanese auto suppliers hold over their U.S counterparts with far less or even non-existing buyer-supplier cooperative alliances (see McMillan (1990), Dyer and Ouchi (1993), Dyer (1996), Dyer et al. (1998) for a contrast of the Japanese auto supply networks, also referred to as Keiretsu, and their U.S counterparts).

Under the Japanese-style risk-sharing contract pricing strategy, the contract price is set dynamically, usually starting from a mark-up of the supplier's costs, and adjusted periodically to reflect changes in the supplier's production costs (Camuffo et al. (2005), McMillan (1990)). A key basis for such risk-sharing contracts is the supplier's anticipation of future opportunities to transact with the buyer over an extended period of time so that the theory of repeated games and relational contracts come into play (Camuffo et al. (2005)). Advanced by the Nobel-prize-winning work of Aumann (1959), the theory of repeated games captures the idea that faced with the prospect of recurrent interactions over a long period of time, a rational player would take into consideration, the impact of his current decision on the future decisions of other players, so that reputation effects and fear of retribution induces cooperative behavior. The seminal three-part paper of Harsanyi (1967, 1968) which allowed for a coherent formulation of games of incomplete information, paved the way for the beginning of a theory of repeated games with incomplete information wherein players have an incentive to conceal or reveal private information (Aumann et al. (1995)).

Relational contract theory focusses on the relationship between contracting parties in a repeated interaction framework and posits that this leads to cooperation and to implicit obligations being self-enforcing (Hviid, (1999)). According to Goetz and Scott (1981), "A contract is relational to the extent that the parties are incapable of reducing important terms of the arrangement to well-defined obligations. Such definitive obligations may be impractical because of the inability to identify uncertain future conditions or because of inability to characterize complex adaptations adequately even when the contingencies themselves can be identified in advance". Therefore, it is the value of future relationship that serves to support the enforcement of relational contracts as their incomplete characterization makes direct legal enforcements impossible (see Baker et al. (2002) and Hviid, (1999) for an extensive review of relational contracts).

A key construct underlying the level of success that can be achieved with self-enforcing contracts is the degree to which the contracting parties trust the other to not place personal gains over the fulfilment of promised obligations. Closely related to trust is the notion of reputation - an expectation about an individual's behavior given the observation of his past behavior. Generally, the higher the reputation of an individual, the more trustworthy that individual is considered to be, and the brighter is the prospect of future dealings. It can thus be expected that how much a trading partner values his reputation is closely linked to the value he places on the future relationship.

The concept of reputation, particularly how it can be elicited from self-seeking economic agents and its effect on future trading decisions has been widely studied in the information systems literature (Bolton et al. (2005), Dellarocas (2005, 2006), Pavlou and Dimoka (2006), Resnick et al. (2006), Ba and Pavlou (2002), and Lucking-Reiley et al. (2000)). This can be largely attributed to the advent of electronic markets in which anonymous traders transact with each other usually on a one-shot basis thus making the realization of trust in such virtual trading environments a very complex issue compared to the conventional brick-and-mortar business communities.

The role of reputation in government contracting has also received significant amount of attention (Lewis (1986), Arvan and Leite (1990), and Perez-Castrillo and Riedinger (2004)). This can be largely attributed to the fact that with most long-term government sponsored projects, at the time of contract negotiation, not all information concerning the project are available thereby making it difficult to credibly commit to a firm course of action over the life of the project.

Building long-term supplier relationship is gaining increasing recognition as an important aspect of supply chain management. However, most of the supply chain contracting literature have focused on one-shot, legally binding contracts (as is the case with this dissertation), thereby eliminating the value of future relationships and constraining the involved parties to ignore reputation (Ren et al. (2006)). It is only of recent that serious academic consideration has been given to the viability of relational contracts in sustaining long-term supply chain relationships, with most of these works addressing the optimal choices for the buyer. Few works have considered relational contracts in which the buyer takes on the risk of the supplier's production cost increases, and of particular note are those of Swinney and Netessine (2009), and Babich (2008). However, in deriving the buyer's optimal choices, the supplier's willingness to act in good faith in correctly revealing and bringing down costs for the sake of future business opportunities is assumed to be guaranteed. These works fail to take into account the impact of the value of the future business on the supplier's readiness to forgo short-term gains in serving the interest of the buyer. Filling this gap will no doubt constitute an important contribution to the supply chain contracting literature.

APPENDIX A PROOF OF PROPOSITIONS

Proof of Proposition 1: I start by analyzing the effect of α on the user's decision. Taking the first order derivative of x^u with respect to α, we may derive that

$$\frac{\partial x^u}{\partial \alpha} = \frac{\beta\sigma^2 \cdot ((\mu - \kappa) \cdot (1-\alpha) + 2\gamma\omega \cdot (1-\alpha) + \beta\sigma^2\delta \cdot (1-\alpha)^2 - \gamma\delta)}{2 \cdot (\beta\sigma^2 \cdot (1-\alpha)^2 + \gamma)^2}. \tag{6.1}$$

Hence, $\partial x^u / \partial \alpha > 0$ if and only if

$$\beta\sigma^2\delta \cdot (1-\alpha)^2 + (\mu - \kappa + 2\gamma\omega) \cdot (1-\alpha) - \gamma\delta > 0. \tag{6.2}$$

The solution to

$$\beta\sigma^2\delta \cdot (1-\alpha)^2 + (\mu - \kappa + 2\gamma\omega) \cdot (1-\alpha) - \gamma\delta = 0 \tag{6.3}$$

is $\alpha = \alpha_0$, where α_0 is defined in the statement of the proposition. Thus, we have $\partial x^u/\partial\alpha > 0$ if and only if $\alpha < \alpha_0$.

Now I show that E^{sc} has a similar trend. To this end, by plugging (3.21) into (3.27) and taking derivative, we may obtain

$$\begin{aligned} \partial E^{sc}/\partial\alpha = {} & [(\beta\sigma^2 \cdot ((\mu - \kappa) \cdot (1-\alpha) + 2\gamma\omega \cdot (1-\alpha) + \beta\sigma^2\delta \cdot (1-\alpha)^2 - \\ & \gamma\delta)) \times (\gamma \cdot (\mu - \kappa) + 2\gamma^2\omega + 2\beta\sigma^2 \cdot (1-\alpha)^2 \cdot (\mu - \kappa) + \\ & 4\gamma\beta\sigma^2\omega \cdot (1-\alpha)^2 - 2\gamma\beta\sigma^2\delta \cdot (1-\alpha))]/[4 \cdot (\gamma + \beta\sigma^2 \cdot (1-\alpha)^2)^3]. \end{aligned} \tag{6.4}$$

It can be verified that

$$(\gamma \cdot (\mu - \kappa) + 2\gamma^2\omega + 2\beta\sigma^2 \cdot (1-\alpha)^2 \cdot (\mu - \kappa) + 4\gamma\beta\sigma^2\omega \cdot (1-\alpha)^2 - 2\gamma\beta\sigma^2\delta \cdot (1-\alpha)) > 0, \tag{6.5}$$

if and only if

$$\omega > (\beta\sigma^2\delta \cdot (1-\alpha))/(\gamma + 2\beta\sigma^2 \cdot (1-\alpha)^2) \;-\; (\mu - \kappa)/2\gamma. \tag{6.6}$$

Thus, from (6.4) and (6.6), E^{sc} will be increasing in α, if and only if

$$\begin{aligned} &(\mu-\kappa)\cdot(1-\alpha) + 2\gamma\omega\cdot(1-\alpha) + \beta\sigma^2\delta\cdot(1-\alpha)^2 - \gamma\delta > 0, \text{ and} \\ &\omega > (\beta\sigma^2\delta\cdot(1-\alpha))/(\gamma + 2\beta\sigma^2\cdot(1-\alpha)^2) - (\mu-\kappa)/2\gamma, \end{aligned} \tag{6.7}$$

or both left-hand sides are negative. We recall that the first inequality of (6.7) is the same condition for which x^u is increasing in α. Hence, E^{sc} is increasing in α if $\alpha - \alpha_0$ and $\omega - \omega_0$ are of opposite signs, and decreasing in α otherwise, where α_0 and ω_0 are defined in the statement of the proposition.

It can be seen right away from (3.21) that the supplier's z^s is strictly decreasing in α. On the other hand, taking the first order derivative of E^s with respect to α, we may derive

$$\begin{aligned} \partial E^s/\partial\alpha = {} & \beta\sigma^2\cdot(\mu - \kappa + 2\gamma\omega + 2\delta\beta\sigma^2\cdot(1-\alpha))\times \\ & ((\mu-\kappa)\cdot(1-\alpha) - 2\gamma\delta + 2\gamma\omega\cdot(1-\alpha))/(4\cdot(\beta\sigma^2\cdot(1-\alpha)^2+\gamma)^2). \end{aligned} \tag{6.8}$$

It is clear that $\partial E^s/\partial\alpha > 0$ if and only if

$$(\mu-\kappa)\cdot(1-\alpha) - 2\gamma\delta + 2\gamma\omega\cdot(1-\alpha) > 0. \tag{6.9}$$

Therefore, we have $\partial E^s/\partial\alpha > 0$ if and only if $\alpha < \alpha_1$, where α_1 is defined in the statement of the proposition. ∎

Proof of Proposition 2: Let us start by analyzing the effect of β on the commodity user's decision. Taking the first order derivative of x^u of (3.21) with respect to β, we have

$$\begin{aligned} \partial x^u/\partial\beta = -\sigma^2 \cdot (1-\alpha) \cdot ((\mu-\kappa) \cdot (1-\alpha) - 2\gamma \cdot (\delta - \omega \cdot (1-\alpha)))/ \\ 4 \cdot (\beta\sigma^2 \cdot (1-\alpha)^2 + \gamma)^2. \end{aligned} \tag{6.10}$$

Hence, $\partial x^u/\partial\beta > 0$ if an only if

$$(\mu-\kappa) \cdot (1-\alpha) - 2\gamma \cdot (\delta - \omega \cdot (1-\alpha)) < 0. \tag{6.11}$$

Therefore, we have $\partial x^u/\partial\beta > 0$ if and only if $\gamma > \gamma_0$, where γ_0 is defined in the statement of the proposition. On the other hand, it is easily observed from (3.25) that when $\gamma > \gamma_0$, V^u is decreasing in β. Let us now study the effect of β on V^u for $\gamma < \gamma_0$. When $\gamma < \gamma_0$, V^u is increasing in β if and only if

$$\delta/2 > (2\gamma\delta - (1-\alpha) \cdot (\mu - \kappa + 2\gamma\omega))/(4\gamma + 4\beta\sigma^2 \cdot (\alpha-1)^2)). \tag{6.12}$$

Now,

$$\begin{aligned} &(2\gamma\delta - (1-\alpha) \cdot (\mu - \kappa + 2\gamma\omega))/(4\gamma + 4\beta\sigma^2 \cdot (\alpha-1)^2)) = \\ &\quad \gamma \cdot (\delta - \omega \cdot (1-\alpha))/(2 \cdot (\gamma + \beta\sigma^2 \cdot (1-\alpha))) \; - \; ((1-\alpha) \cdot (\mu - \kappa)) \\ &\quad /(4 \cdot (\gamma + \beta\sigma^2 \cdot (\alpha-1)^2)). \end{aligned} \tag{6.13}$$

It is obvious that

$$\gamma \cdot (\delta - \omega \cdot (1-\alpha))/(2 \cdot (\gamma + \beta\sigma^2 \cdot (1-\alpha))) < \gamma\delta/2\gamma = \delta/2. \tag{6.14}$$

Thus clearly,

$$\delta/2 > (2\gamma\delta - (1-\alpha)\cdot(\mu - \kappa + 2\gamma\omega))/(4\gamma + 4\beta\sigma^2\cdot(\alpha-1)^2)). \tag{6.15}$$

It is straightforward to see from equation (3.21) that as β increases, z^s increases everywhere. From (3.26), we may derive that

$$\begin{aligned}\partial E^s/\partial\beta = [&\sigma^2\cdot(1-\alpha)\cdot(\mu-\kappa+2\gamma\omega+2\beta\sigma^2\delta\cdot(1-\alpha))\times \\ &((\kappa-\mu)\cdot(1-\alpha)+4\gamma\delta-2\gamma\omega\cdot(1-\alpha)+2\beta\sigma^2\delta\cdot(1-\alpha)^2)] \\ &/(8\cdot(\beta\sigma^2(1-\alpha)^2+\gamma)^2).\end{aligned} \tag{6.16}$$

Hence, $\partial E^s/\partial\beta > 0$ if and only if

$$((\kappa-\mu)\cdot(1-\alpha)+4\gamma\delta-2\gamma\omega\cdot(1-\alpha)+2\beta\sigma^2\delta\cdot(1-\alpha)^2) > 0. \tag{6.17}$$

Therefore, we have $\partial E^s/\partial\beta > 0$ if and only if $\beta > \beta_0$, where β_0 is defined in the statement of the proposition. ■

Proof of Proposition 3: I start by analyzing the effect of γ on the user's decision. Taking the first order derivative of x^u of (3.21) with respect to γ, we have:

$$\begin{aligned}\partial x^u/\partial\gamma = -&(\mu-\kappa+2\delta\beta\sigma^2\cdot(1-\alpha)-2\omega\beta\sigma^2\cdot(1-\alpha)^2) \\ &/(4\cdot(\gamma+\beta\sigma^2\cdot(1-\alpha)^2)^2) < 0.\end{aligned} \tag{6.18}$$

Taking the first order derivative of V^u with respect to γ, we derive

$$\begin{aligned}\partial V^u/\partial\gamma = {} & [\sigma^2 \cdot (1-\alpha) \cdot (\mu - \kappa + 2\delta\beta\sigma^2 \cdot (1-\alpha) - 2\omega\beta\sigma^2 \cdot (1-\alpha)^2) \times \\ & ((\kappa-\mu) \cdot (1-\alpha) + 4\gamma\delta - 2\gamma\omega \cdot (1-\alpha) + 2\delta\beta\sigma^2 \cdot (1-\alpha)^2)] \\ & /(8 \cdot (\beta\sigma^2 \cdot (1-\alpha)^2)^3).\end{aligned} \tag{6.19}$$

Hence, $\partial V^u/\partial\gamma > 0$ if and only if

$$(\kappa-\mu) \cdot (1-\alpha) + 4\gamma\delta - 2\gamma\omega \cdot (1-\alpha) + 2\delta\beta\sigma^2 \cdot (1-\alpha)^2 > 0. \tag{6.20}$$

Therefore, $\partial V^u/\partial\gamma > 0$ if and only if $\gamma > \gamma_1$, where γ_1 is defined in the statement of the proposition.

Further, from (3.5) and (3.7), we have $V^u = \sigma^2 \cdot (\alpha x^u + y^u)^2$. When $\gamma = \gamma_1$, it can be verified that

$$x^u = \frac{\delta}{1-\alpha}; \text{ and } y^u = -\frac{\alpha\delta}{1-\alpha}, \tag{6.21}$$

so that V^u reduces to zero.

It is straightforward to see from (3.21) that as γ increases, z^s increases everywhere. For the effect of γ on the supplier's E^s, taking the first order derivative of E^s with respect to γ, we may derive that

$$\begin{aligned}\partial E^s/\partial\gamma = {} & [(2\omega\beta\sigma^2 \cdot (1-\alpha)^2 + 2\omega\gamma)^2 - (\kappa - \mu - 2\delta\beta\sigma^2 \cdot (1-\alpha) + \\ & 2\omega\beta\sigma^2 \cdot (1-\alpha)^2)^2]/(8 \cdot (\beta\sigma^2 \cdot (1-\alpha)^2 + \gamma)^2).\end{aligned} \tag{6.22}$$

Hence, $\partial E^s/\partial\gamma > 0$ if and only if

$$(2\omega\beta\sigma^2 \cdot (1-\alpha)^2 + 2\omega\gamma)^2 > (\kappa - \mu - 2\delta\beta\sigma^2 \cdot (1-\alpha) + 2\omega\beta\sigma^2 \cdot (1-\alpha)^2)^2. \tag{6.23}$$

Therefore, $\partial E^s/\partial\gamma > 0$ if and only if $\gamma > \gamma_2$, where γ_2 is defined in the statement of the proposition. ∎

Proof of Proposition 4: It is readily observed from (3.21), that the user's x^u is increasing in ω. Likewise, it is straightforward to see that the supplier's z^s (from (3.21)) and her equilibrium expected profit E^s (from (3.26)) are increasing in ω.

Taking the first order derivative of the user's equilibrium variance of profit with respect to ω, we derive

$$\begin{aligned}\partial V^u/\partial\omega = [&-(1-\alpha)\cdot\gamma\sigma^2\cdot(\delta - ((1-\alpha)\cdot(\mu-\kappa+2\gamma\omega+2\delta\beta\sigma^2\cdot \\ &(1-\alpha)))/(4\cdot(\gamma+\beta\sigma^2\cdot(\alpha-1)^2))]/(\gamma+\beta\sigma^2\cdot(\alpha-1)^2).\end{aligned} \tag{6.24}$$

Hence, $\partial V^u/\partial\omega > 0$ if and only if

$$\begin{aligned}&\delta - ((1-\alpha)\cdot(\mu-\kappa+2\gamma\omega+2\delta\beta\sigma^2\cdot(1-\alpha)))/ \\ &(4\cdot(\gamma+\beta\sigma^2\cdot(\alpha-1)^2) < 0,\end{aligned} \tag{6.25}$$

which requires $\omega > \omega_1$, where ω_1 is defined in the statement of the proposition. It can be verified that when $\omega = \omega_1$, $x^u = \delta/(1-\alpha)$ so that as was shown in the proof of proposition 3, V^u reduces to zero. ∎

Proof of Proposition 5: From (3.21), it is straightforward to see that the commodity user's x^u

and the supplier's z^s are increasing in μ. Further, for the supplier, her E^s of (3.26) is increasing in μ.

Taking the first order derivative of the user's equilibrium variance of profit V^u with respect to μ, we may derive

$$\begin{aligned}\partial V^u/\partial\mu = [-(1-\alpha)\cdot 2\sigma^2\cdot(\delta-((1-\alpha)\cdot(\mu-\kappa+2\gamma\omega+2\delta\beta\sigma^2\cdot \\ (1-\alpha)))/(4\cdot(\gamma+\beta\sigma^2\cdot(\alpha-1)^2))]/(4\cdot(\gamma+\beta\sigma^2\cdot(\alpha-1)^2)).\end{aligned} \tag{6.26}$$

Hence, $\partial V^u/\partial\mu > 0$ if and only if

$$\begin{aligned}&\delta-((1-\alpha)\cdot(\mu-\kappa+2\gamma\omega+2\delta\beta\sigma^2\cdot(1-\alpha)))/ \\ &(4\cdot(\gamma+\beta\sigma^2\cdot(\alpha-1)^2)<0,\end{aligned} \tag{6.27}$$

and this requires $\mu > \mu_0$, where μ_0 is defined in the statement of the proposition. It can be verified that when $\mu = \mu_0$, $x^u = \delta/(1-\alpha)$ so that as earlier shown in the proof of proposition 3, V^u reduces to zero. Further, since x^u increases in μ, it implies that when $\mu > \mu_0$, $x^u > \delta/(1-\alpha)$, so that $x^u > \delta$. ■

Proof of Proposition 6: I start by analyzing the effect of σ^2 on the user's decision. Taking the first order derivative of x^u with respect to σ^2, we have

$$\frac{\partial x^u}{\partial\sigma^2} = \frac{\beta\cdot(1-\alpha)\cdot[(\kappa-\mu)\cdot(1-\alpha)+2\gamma\delta-2\gamma\omega\cdot(1-\alpha)]}{4\cdot(\beta\sigma^2\cdot(1-\alpha)^2+\gamma)^2}. \tag{6.28}$$

It is clear that $\delta x^u/\delta\sigma^2 > 0$ if and only if

$$(\kappa-\mu)\cdot(1-\alpha)+2\gamma\delta-2\gamma\omega\cdot(1-\alpha) > 0. \tag{6.29}$$

Therefore, $\delta x^u/\delta\sigma^2 > 0$ if and only if $\gamma > \gamma_0$, where γ_0 is defined in the statement of proposition 2.

When $2\gamma\delta < (1-\alpha)\cdot(\mu-\kappa+2\gamma\omega)$ (i.e. $\gamma < \gamma_0$), it is straightforward to see that as σ^2 increases, $(2\gamma\delta-(1-\alpha)\cdot(\mu-\kappa+2\gamma\omega))/(4\gamma+4\beta\sigma^2\cdot(\alpha-1)^2)$ increases everywhere. Hence, when $\gamma < \gamma_0$, V^u of (3.25) strictly increases in σ^2.

Now, when $\gamma > \gamma_0$, it is clear that $2\gamma\delta-(1-\alpha)\cdot(\mu-\kappa+2\gamma\omega)/(4\gamma+4\beta\sigma^2\cdot(\alpha-1)^2)$ decreases in σ^2. We note that the expression for V^u can be re-written as

$$\begin{aligned} V^u = (\sigma\delta/2 \;+\; \sigma\cdot(2\gamma\delta-(1-\alpha)\cdot(\mu-\kappa+2\gamma\omega))/ \\ (4\gamma+4\beta\sigma^2\cdot(\alpha-1)^2))^2. \end{aligned} \tag{6.30}$$

Since we have already established from the proof of Proposition 2 that $\delta/2 > (2\gamma\delta-(1-\alpha)\cdot(\mu-\kappa+2\gamma\omega))/(4\gamma+4\beta\sigma^2\cdot(\alpha-1)^2))$, it means that an increase in $\sigma\delta/2$ as σ^2 increases will always be greater than whatever decrease $\sigma\cdot(2\gamma\delta-(1-\alpha)\cdot(\mu-\kappa+2\gamma\omega))/(4\gamma+4\beta\sigma^2\cdot(\alpha-1)^2)$ may experience from the increased σ^2. The implication of this is that V^u will always increase in σ^2.

It is straightforward to see from (3.21) that as σ^2 increases, z^s increases everywhere. Taking the first order derivative of E^s of (3.26) with respect to σ^2, we may derive that

$$\begin{aligned} \partial E^s/\partial\sigma^2 = {} & [\beta\cdot(1-\alpha)\cdot(\mu-\kappa+2\gamma\omega+2\beta\sigma^2\delta\cdot(1-\alpha) \\ & \times((\kappa-\mu)\cdot(1-\alpha)+4\gamma\delta-2\gamma\omega\cdot(1-\alpha)+2\beta\sigma^2\delta\cdot(1-\alpha)^2)] \\ & /(8\cdot(\beta\sigma^2\cdot(1-\alpha)^2+\gamma)). \end{aligned} \tag{6.31}$$

$\partial E^s/\partial\sigma^2 > 0$ if and only if

$$(\kappa-\mu)\cdot(1-\alpha)+4\gamma\delta-2\gamma\omega\cdot(1-\alpha)+2\beta\sigma^2\delta\cdot(1-\alpha)^2 > 0. \tag{6.32}$$

Therefore, $\partial E^s/\partial\sigma^2 > 0$ if and only if $\sigma^2 > \sigma_0^2$, where σ_0^2 is defined in the statement of the proposition. ∎

Proof of Proposition 8: (a)Taking the first order derivative of $E[\Psi^u]$ of (4.42) with respect to α, we have

$$\frac{\partial E[\Psi^u]}{\partial\alpha} = \delta\cdot(\kappa + 2\beta^u\sigma^2\delta\cdot(1-\alpha)) - \mu\delta + 2\beta^u\sigma^2\delta^2\cdot(1-\alpha). \tag{6.33}$$

It is clear from (6.33) that $(\partial E[\Psi^u]/\partial\alpha) > 0$ if and only if

$$\alpha < \alpha_0, \tag{6.34}$$

where α_0 is as defined in the statement of the proposition. The observation that $V[\Psi^u]$ is increasing in α can be directly made from (4.42).

(b) The effects of α on y^s and z^s can be directly observed from (4.36), and its effect on $E[\Psi^s]$ can be deduced from (4.42). Taking the first order derivative of $E[p(\pi)]$ of (4.42) with respect to α, we have

$$\frac{\partial E[p(\pi)]}{\partial\alpha} = \mu - \kappa - 4\beta^u\sigma^2\delta\cdot(1-\alpha), \tag{6.35}$$

so that $(\partial E[p(\pi)]/\partial\alpha) > 0$ if and only if

$$\alpha > \alpha_0, \tag{6.36}$$

where α_0 is as defined in the statement of the proposition.

(c) The effect of α on $E[\Psi^u] + E[\Psi^s]$ can be directly inferred from (4.42).

Where the opposite of (4.35) holds:

(d)The effects of α on x^u and $E[\Psi^u]$ can be respectively directly observed from (4.37) and (4.43).

Taking the first order derivative of $V[\Psi^u]$ of (4.43) with respect to α, we obtain

$$\frac{\partial V[\Psi^u]}{\partial \alpha} = \frac{\gamma\sigma^2 \cdot (\mu - \kappa - 2\beta^u\sigma^2\delta) \cdot (1-\alpha)}{8(\beta^u\sigma^2)^2 \cdot (1-\alpha)^3}. \tag{6.37}$$

From (6.37), we see that $(\partial V[\Psi^u]/\partial\alpha) > 0$ if and only if

$$(\mu - \kappa) > 2\beta^u\sigma^2\delta. \tag{6.38}$$

Now, the opposite of (4.35) is given by:

$$\gamma \leq (1-\alpha) \cdot (2\beta^u\sigma^2\delta \cdot (1-2\alpha) - \mu + \kappa). \tag{6.39}$$

But at the minimum, $\gamma = 0$. Hence, (6.39) implies that

$$\begin{aligned} &(1-\alpha) \cdot (2\beta^u\sigma^2\delta \cdot (1-2\alpha) - \mu + \kappa) \geq 0, \\ \text{or} \qquad & 2\beta^u\sigma^2\delta \cdot (1-2\alpha) \geq \mu - \kappa. \end{aligned} \tag{6.40}$$

If (6.40) holds, then clearly,

$$2\beta^u\sigma^2\delta \geq \mu - \kappa. \tag{6.41}$$

(6.41) implies that (6.38) is not feasible. Therefore, $(\partial V[\Psi^u]/\partial\alpha) \leq 0$ so that $V[\Psi^u]$ is decreasing in α.

(e) The effects of α on y^s and z^s are obvious from (4.37), while its effect on $E[\Psi^s]$ can be deduced from (4.43). From (4.43), we observe that $E[p(\pi)]$ is increasing in α if and only if

$$(\mu - \kappa) > 2\beta^u\sigma^2\delta, \tag{6.42}$$

a condition we have shown earlier to be infeasible. Hence, $E[p(\pi)]$ is decreasing in α.

(f) The effect of α on $E[\Psi^u] + E[\Psi^s]$ is obvious from (4.43). ∎

Proof of Proposition 9: The results proposed in (a) and (b) can be deduced directly from (4.42), and (4.36).

(c) Taking the first order derivative of x^u of (4.37) with respect to β^u, we have

$$\frac{\partial x^u}{\partial \beta^u} = -\frac{(\mu - \kappa) \cdot (1 - \alpha) + \gamma}{4(\beta^u)^2 \sigma^2 \cdot (1 - \alpha)^2} < 0. \tag{6.43}$$

The first order derivative of $V[\Psi^u]$ of (4.43) with respect to β^u is given by

$$\frac{\partial V[\Psi^u]}{\partial \beta^u} = -\frac{(\mu - \kappa - 2\beta^u \sigma^2 \delta) \cdot (1 - \alpha) \cdot ((\mu - \kappa) \cdot (1 - \alpha) + \gamma)}{8(\beta^u)^3 \sigma^2 \cdot (1 - \alpha)^2}. \tag{6.44}$$

From (6.44), it becomes clear that $(\partial V[\Psi^u]/\partial \beta^u) > 0$ if and only if

$$\mu - \kappa - 2\beta^u \sigma^2 < 0. \tag{6.45}$$

From (6.41), we know that (6.45) always holds when the opposite of (4.35) is true. Hence, $(\partial V[\Psi^u]/\partial \beta^u) > 0$. The effect of β^u on $E[\Psi^u]$ can be directly observed from (4.43), while it is obvious from (4.37) and (4.43) that β^s has no effect on the user's equilibrium results.

The supplier's results proposed in (d) can be deduced directly from (4.37), and (4.43).

(e) It is obvious from (4.43), that:

$$\frac{\partial^2 E[\Psi^u]}{\partial \alpha \partial \beta^u} = \frac{\partial^2 E[\Psi^s]}{\partial \alpha \partial \beta^u} = -\frac{\gamma \cdot (\gamma + (\mu - \kappa) \cdot (1 - \alpha))}{4(\beta^u)^2 \sigma^2 \cdot (1 - \alpha)^3}. \tag{6.46}$$

■

Proof of Proposition 10: (a) Can be directly observed from (4.36), and (4.42).

(b) The effects of γ on x^u and $V[\Psi^u]$ are respectively obvious from (4.37), and (4.43). Taking the first order derivative of $E[\Psi^u]$ of (4.43) with respect to γ, we obtain

$$\frac{\partial E[\Psi^u]}{\partial \gamma} = \frac{(\mu - \kappa) \cdot (1 - \alpha) + \gamma}{4\beta^u \sigma^2 \cdot (1 - \alpha)^2} - \delta. \tag{6.47}$$

From (6.47), it can be verified that $(\partial E[\Psi^u]/\partial \gamma) > 0$ if and only if

$$\gamma > (1 - \alpha) \cdot (4\beta^u \sigma^2 \delta \cdot (1 - \alpha) - \mu + \kappa). \tag{6.48}$$

But proposition 10(b) is derived under condition (6.39), so that (6.48) cannot hold. Hence, $(\partial E[\Psi^u]/\partial \gamma) \leq 0$ under condition (6.39).

(c) The effects of γ on the supplier's equilibrium results can be directly observed from (4.37), and (4.43).

(d) It can be easily deduced from (4.43) that $\partial^2 E[\Psi^u]/\partial \alpha \partial \gamma$ and $\partial^2 E[\Psi^s]/\partial \alpha \partial \gamma$ are positive. ■

Proof of Proposition 11: The results proposed in (a) and (b) can be deduced directly from (4.42), and (4.36).

(c)From (4.37), and (4.43) we respectively observe that x^u and $V[\Psi^u]$ are increasing in μ. Taking the first order derivative of $E[\Psi^u]$ with respect to μ, we have

$$\frac{\partial E[\Psi^u]}{\partial \mu} = \frac{\mu - \kappa}{4\beta^u \sigma^2} - \delta + \frac{\gamma}{4\beta^u \sigma^2 \cdot (1 - \alpha)}. \tag{6.49}$$

From (6.49), $(\partial E[\Psi^u]/\partial\mu) > 0$ if and only if

$$\gamma > (1-\alpha)\cdot(4\beta^u\sigma^2\delta - \mu + \kappa). \tag{6.50}$$

Again, proposition 11(c) is derived under condition (6.39), so that (6.50) cannot hold. Hence, $(\partial E[\Psi^u]/\partial\mu) \leq 0$ under condition (6.39).

For the supplier, proposition 11(d) can be directly deduced from (4.37), and (4.43).

■

Proof of Proposition 12: (a) Can be deduced directly from (4.42).

(b) The impacts of σ^2 on the supplier's y^s, z^s, $E[p(\pi)]$, and $V[\Psi^s]$ can be directly observed from (4.36) and (4.42). Taking the first order derivative of $E[\Psi^s]$ with respect to σ^2, we obtain

$$\frac{\partial E[\Psi^s]}{\partial\sigma^2} = 2\beta^u\delta^2\cdot(1-\alpha)^2 - \frac{(\mu-\kappa)^2}{2\beta^s\sigma^4}. \tag{6.51}$$

It can be verified that $(\partial E[\Psi^s]/\partial\sigma^2) > 0$ if and only if

$$\sigma^2 > \sigma_0^2, \tag{6.52}$$

where σ_0^2 is as defined in the statement of the proposition.

(c) The effect of σ^2 on $E[\Psi^u]$ can be directly inferred from (4.43). The first order derivative of x^u with respect to σ^2 is given by

$$\frac{\partial x^u}{\partial\sigma^2} = -\frac{(\mu-\kappa)\cdot(1-\alpha)+\gamma}{4\beta^u\sigma^4\cdot(1-\alpha)^2}. \tag{6.53}$$

From (6.53), it is obvious that $(\partial x^u/\partial\sigma^2) < 0$ everywhere. Taking the first order derivative of $V[\Psi^u]$ with respect to σ^2, we obtain

$$\frac{\partial V[\Psi^u]}{\partial\sigma^2} = \frac{\delta^2}{4} - \frac{((\mu-\kappa)\cdot(1-\alpha)+\gamma)^2}{(4\beta^u\sigma^2\cdot(1-\alpha))^2}, \tag{6.54}$$

It can be verified that $(\partial V[\Psi^u]/\partial\sigma^2) > 0$ if and only if

$$\gamma < (1-\alpha)\cdot(2\beta^u\sigma^2\delta - \mu + \kappa). \tag{6.55}$$

Since proposition 12 is derived under condition (6.39), we see right away that 6.55 holds anywhere condition (6.39) is applicable so that $(\partial V[\Psi^u]/\partial\sigma^2) > 0$ everywhere.

(d) The effects of σ^2 on y^s, z^s, and $V[\Psi^s]$ are obvious from (4.37), and (4.43). Taking the first order derivative of $E[\Psi^s]$ with respect to σ^2, we have

$$\frac{\partial E[\Psi^s]}{\partial\sigma^2} = \frac{-2\gamma\cdot(\mu-\kappa)\cdot(1-\alpha)-\gamma^2}{8\beta^u\sigma^4\cdot(1-\alpha)^2} - \frac{(\mu-\kappa)^2\cdot(4\beta^u+\beta^s)}{8\beta^u\beta^s\sigma^4} + \frac{\beta^u\delta^2}{2}. \tag{6.56}$$

From (6.56), it can be verified that $(\partial E[\Psi^s]/\partial\sigma^2) > 0$ if and only if

$$\sigma^2 > \sigma_1^2, \tag{6.57}$$

where σ_1^2 is as defined in the statement of the proposition. ∎

APPENDIX B TRANSFORMATION OF EQUATION (3.6) INTO EQUATION (3.7)

$$\begin{aligned}
F^u(x,y,z) &= E[f^u(x,y,z,\Pi_1)] - \beta \cdot E[(f^u(x,y,z,\Pi_1))^2] + \\
&\quad \beta \cdot (E[f^u(x,y,z,\Pi_1)])^2 \\
&= E[f_0^u(x,y,z) + f_1^u(x,y,z) \cdot \Pi_1] - \beta \cdot E[(f_0^u(x,y,z))^2 + \\
&\quad 2f_0^u(x,y,z) f_1^u(x,y,z) \cdot \Pi_1 + (f_1^u(x,y,z))^2 \cdot (\Pi_1)^2] + \\
&\quad \beta \cdot (E[f_0^u(x,y,z) + f_1^u(x,y,z) \cdot \Pi_1])^2 \\
&= f_0^u(x,y,z) + f_1^u(x,y,z) \cdot E[\Pi_1] - \beta \cdot (f_0^u(x,y,z))^2 - \\
&\quad 2\beta \cdot f_0^u(x,y,z) \cdot f_1^u(x,y,z) \cdot E[\Pi_1] - \\
&\quad \beta \cdot (f_1^u(x,y,z))^2 \cdot E[\Pi_1^2] + \beta \cdot (f_0^u(x,y,z))^2 + \\
&\quad 2\beta \cdot f_0^u(x,y,z) \cdot f_1^u(x,y,z) \cdot E[\Pi_1] + \beta \cdot (f_1^u(x,y,z))^2 \cdot (E[\Pi_1])^2 \\
&= f_0^u(x,y,z) + f_1^u(x,y,z) \cdot E[\Pi_1] - \\
&\quad \beta \cdot (f_1^u(x,y,z))^2 \cdot (E[\Pi_1^2] - (E[\Pi_1])^2) \\
&= f_0^u(x,y,z) + f_1^u(x,y,z) \cdot E[\Pi_1] - \beta \cdot (f_1^u(x,y,z))^2 \cdot Var[\Pi_1].
\end{aligned} \tag{6.58}$$

■

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