# ABSTRACT

## PAPR REDUCTION IN OFDM COMMUNICATIONS WITH GENERALIZED DISCRETE FOURIER TRANSFORM

**by**
**Sertac Sayin**

The main advantage of Generalized Discrete Fourier Transform (GDFT) is its ability to design a wide selection of constant modulus orthogonal code sets, based on the desired performance metrics mimicking the engineering specs of interest. One of the main drawbacks of Orthogonal Frequency Division Multiplexing (OFDM) systems is the high Peak to Average Power Ratio (PAPR) value which is directly related to power consumption of the system. Discrete Fourier Transform (DFT) spread OFDM technology, also known as Single Carrier Frequency Division Multiple Access (SCFDMA), which has a lower PAPR value, is used for uplink channel.

In this thesis, the PAPR of DFT spread OFDM was further decreased by using a GDFT concept. The performance improvements of GDFT based PAPR reduction for various SCFDMA communications scenarios were evaluated by simulations. Performance simulation results showed that PAPR efficiency of SCFDMA systems for Binary Phase Shift Keying (BPSK), Quadrature Phase Shift Keying (QPSK) and 16 Quadrature Amplitude Modulation (16-QAM), digital modulation techniques are increased.

# PAPR REDUCTION IN OFDM COMMUNICATIONS WITH GENERALIZED DISCRETE FOURIER TRANSFORM

by
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# APPROVAL PAGE

## PAPR REDUCTION IN OFDM COMMUNICATIONS WITH GDFT

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**Sevgili annem**
**Sabite Sayın,**
**sevgili babam**
**Senai Sayın,**
**ve**
**sevgili ablam**
**Esen Sayın'a**

**To my beloved mother,**
**Sabite Sayın**
**my beloved father,**
**Senai Sayın**
**and**
**my beloved sister,**
**Esen Sayın**

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# CHAPTER 1

# INTRODUCTION

## 1.1 Objective

Objective of this thesis is to introduce a method to reduce peak to average power (PAPR) for Single Carrier Frequency Division Multiple Access (SCFDMA) with the Localized Frequency Division Multiple Access (LFDMA) subcarrier mapping for different digital modulation types like; Binary Phase Shift Keying (BPSK), Quadrature Phase Shift Keying (QPSK) and 16 Quadrature Amplitude Modulation (QAM).

As background information, Long Term Evolution (LTE), a beyond third generation mobile broadband standard, successor to UMTS and CDMA2000 (which are 3G cellular technologies), is reviewed as two parts downlink, Orthogonal Frequency Division Multiple Access (OFDMA) and uplink SCFDMA. And Generalized Discrete Fourier Transform (GDFT) is introduced.

Finally, the difference between conventional SCFDMA system and the proposed SCFDMA system is compared in terms of PAPR with different sizes of frequency band and different number of users.

## 1.2 Background Information

Since the beginning of humanity, communication is one of the basic needs. At first, they started to communicate through fire, smoke...etc and then, the telegraph was invented. Wired phone invention followed telegraph and today, with the technological improvements, people can communicate wirelessly between each other through cell phones in their pockets.

Engineers are always looking for faster, reliable and safer solutions to problems. In the wireless communication field, the technology that is used as 3rd Generation (3G) in the cell phones which is Code division Multiple Access (CDMA) based, has achieved its limits. So for further increase in throughput, the technology is changed to LTE. And LTE is based on OFDM in the downlink channel (link from station to mobile) and Discrete Fourier Transform (DFT) spread OFDM or in other words SCFDMA in the uplink channel (link from mobile to station).

The reason to use two different standards for uplink and downlink is PAPR. PAPR is directly related to battery life of the mobile. So PAPR is not a big issue for downlink because station is stationary and does not require battery. However, mobiles are using battery therefore battery life is one of the biggest issues for them. SCFDMA has similar throughput performance and essentially the same overall complexity as OFDMA. A principal advantage of SC-FDMA is the peak-to-average power ratio (PAPR), which is lower than that of OFDMA. SCFDMA is currently a strong candidate for the uplink multiple access scheme in the Long Term Evolution of cellular systems under consideration by the Third Generation Partnership Project (3GPP).

# CHAPTER 2

# LTE OVERVIEW AS OFDMA AND SCFDMA

## 2.1 OFDMA

OFDMA is used on the downlink of LTE as mentioned previously. The key concept of OFDMA is that the sub carriers can overlap with each other in a way that crosstalk is avoided. This is achieved by making the sub carriers orthogonal to each other. As a result, we can send data from different users overlapping each other in parallel sub carriers, which save bandwidth as shown in figure (2.1) below. Each trapezoid represents a subcarrier, a channel. We can see that with OFDMA we can save bandwidth.

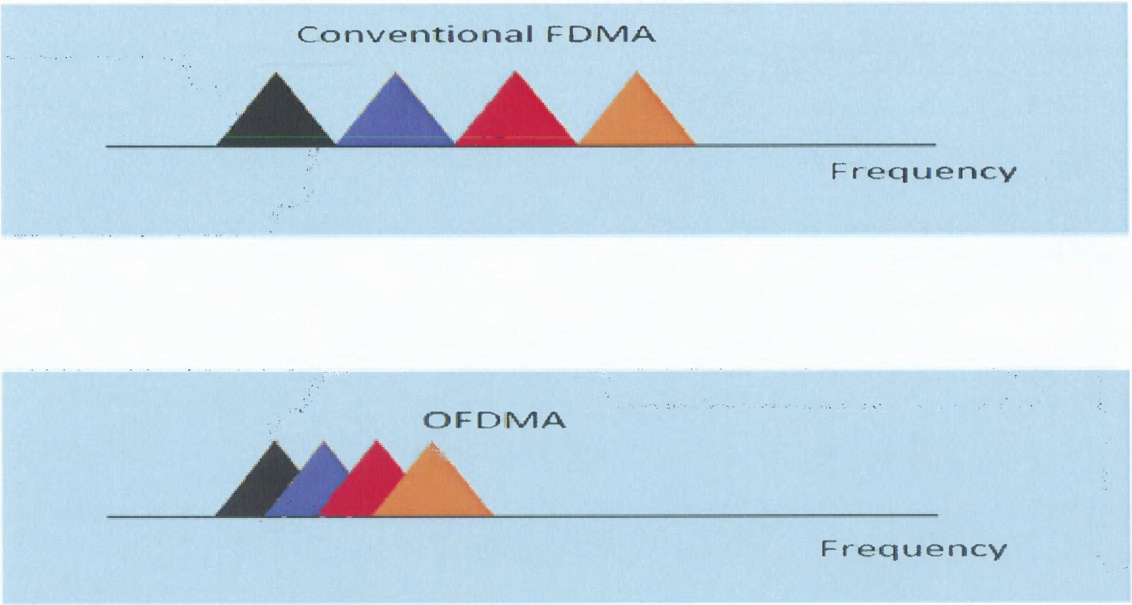


**Figure 2.1** Conventional FDMA (the upper one) and Orthogonal FDMA (the lower one).

Orthogonality not only saves the bandwidth but also removes interference between multiple sub carriers. In frequency domain the overlapping carriers are represented by SINC functions and if each sub carrier is detected at the center frequency of that specific sub carrier, due to orthogonality, the only signal that will be detected will correspond to that specific sub carrier. For this to happen, the sub carriers in the time domain have to be integer multiples of the base sinusoid with complete cycles per symbol time T as seen in figure (2.2). This ensures that the dot product of any different sub carrier frequencies will return a zero, which means the sub carriers are orthogonal (i.e. No Interference). (Figure 2.2).

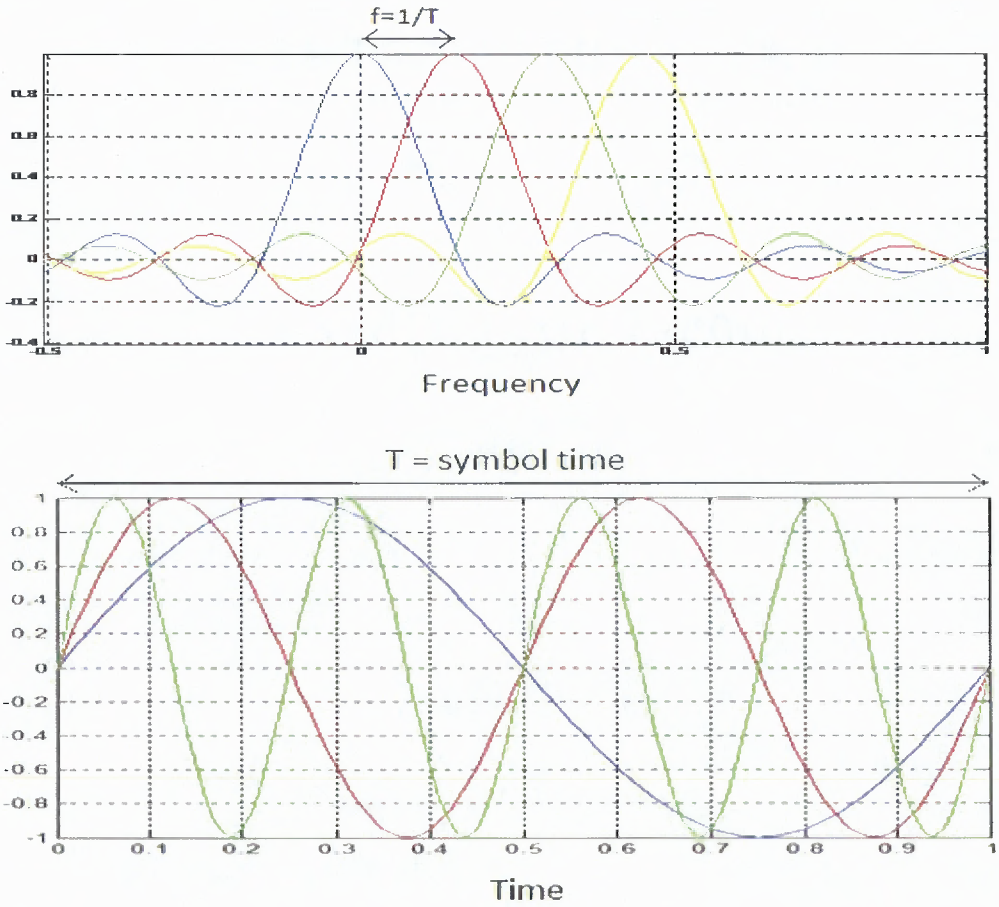


**Figure 2.2** The carrier signals in frequency domain and in time domain.

## 2.2 OFDMA Architecture

OFDMA transmission works the following way, as multiple user data enter the system; an individual sub carrier frequency is assigned to them. Depending on the channel condition adaptive coding is used for every symbol of each sub carrier. Additionally, interleaving is applied, which is used for forward error correction. Lastly, depending on the channel condition again, different modulation schemes are applied to the different sub carriers (BPSK, QPSK, 16 QAM or 64 QAM). If a channel's condition is good, 64 QAM is applied, which results in higher data rates; on the other hand, if the channel condition is not so great 16 QAM or QPSK is applied. After this is done, each user's symbols are converted from serial to parallel, and an Inverse Fast Fourier Transform (IFFT) is applied to them, which combines the multiple sub carriers and converts them from frequency domain to time domain. Once this is done, they get converted from parallel to serial. On the serial stream, a cyclic prefix and a guard band are added to every symbol, which prevents multi-path fading, intra and inter OFDM symbol interference respectively. The serial stream is then converted from digital to analog. It further gets RF modulated and amplified for wireless transmission. Since many sinusoids are added up for transmission, an abnormal waveform is created. This abnormal waveform often has many high peaks and to transmit this wave effectively a very expensive linear amplifier, which also requires high energy, has to be used, which can uniformly amplify all the peaks of the abnormal waveform. The exact opposite process is happening on the receiver side for effective signal detection in the frequency domain. This complete process is depicted in figure 2.3.

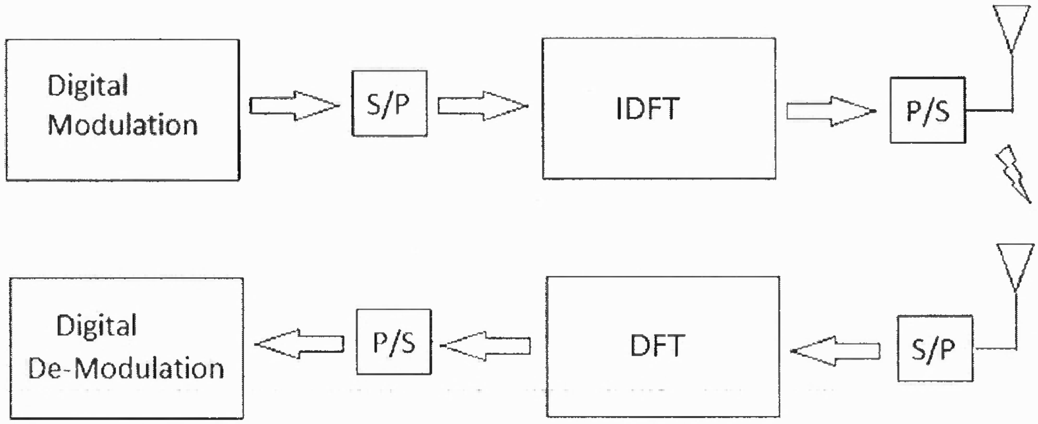


**Figure 2.3** OFDMA system block diagram (transmitter and receiver ).

## 2.3 OFDMA Frame Structure

The OFDMA radio frame structure is as follows. A radio frame consists of 10 sub frames, 1ms each. Each sub frame is made up of 2 slots, 0.5ms each and each slot has 7 symbols along with their own cyclic prefix.

## 2.4 SCFDMA

OFDMA has small frequency channels, each of which is assigned to a specific symbol. These symbols are transmitted simultaneously as in figure 2.4. As it was mentioned before, prior to transmission over the air all the multiple frequency channels are added together which creates an uncontrollable signal with high peaks. To handle this uncontrollable signal we have to use more power. Using more power is not a problem for downlink however it is one of the main issues in uplink since it increases mobile costs and decreases battery life. Because OFDMA transmits many symbols at a time, we need more power for effective transmission. So as a solution, SC-FDMA decrease the number of symbols transmitted per time, which brings the uncontrollable signal to a manageable levels. Use of wider bandwidth reduces symbol transmission time. For more clarity this can be seen in figure 2.4.

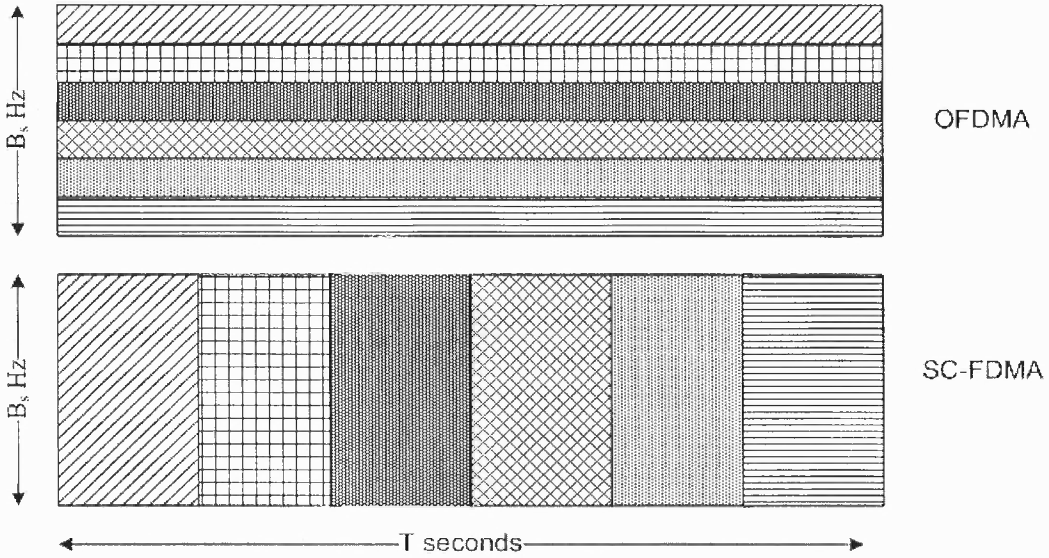


**Figure 2.4** Difference between channel representations between OFDMA and SCFDMA.

## 2.5 SCFDMA Architecture

Figure 2.5 shows SC-FDMA structure which is very similar to that of an OFDMA structure aside from the purple blocks. These blocks represent the "N-point DFT" for SC-FDMA transmitter and "N-point IDFT" for receiver. The "N" stands for number of sub carriers used by one user in the system and the "M" stands for total number of sub carriers in the system.

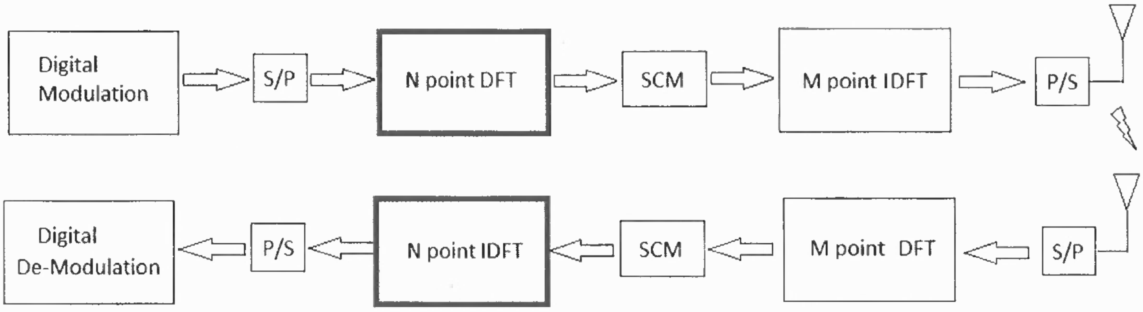


**Figure 2.5** SCFDMA system block diagram for transmitter and receiver.

As every digital communication systems, SC-FDMA also uses modulation. In SC-FDMA generally BPSK, QPSK or 16-QAM modulation scheme is used. The DFT block in the transmitter on the other hand is unique to SC-FDMA and serves specific functions in SC-FDMA. This is where the user's parallel symbol stream is converted to the frequency domain from the time domain. User symbols are then spread over N subcarriers. At this point the DFT forwards the new parallel stream to the subcarrier mapping block.

Each user has N subcarriers from the DFT block however, in the system there are total of M available subcarriers. Thus each user has to select N specific subcarriers out of M available ones. This is done in subcarrier mapping block. There are two kinds of subcarrier mapping available in SC-FDMA, localized and distributed. In the Localized SC-FDMA (LFDMA) mapping each user uses a set of adjacent subcarriers to transmit its symbols. Distributed SC-FDMA (DFDMA) mapping on the other hand, spreads the user subcarriers over the entire available signal band. Both of these mapping schemes are depicted in figure 2.7 for clarity. Each color represents different channel.

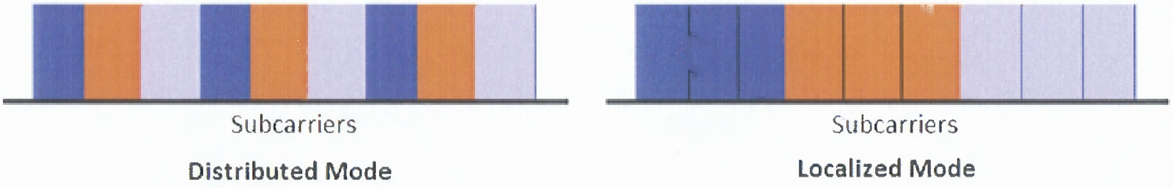


**Figure 2.6** SCFDMA sub-carrier mappings Distributed and Localized modes.

To show the difference between these subcarrier mapping methods, they have been simulated in a Matlab program. The results of which can be seen in figure 2.7. However prior to examining these results, peak to average power ratio (PAPR) of a signal has to be introduced. PAPR basically quantifies the controllability of the transmitted signal. If the PAPR of a signal is high it means that the signal has multiple high peaks. High PAPR makes the signal harder to control and more transmission power has to be used. In this simulation, we have compared three subcarrier mapping methods localized, distributed and interleaved (special type of distributed mapping) FDMA. In terms of PAPR performance figure 2.7 shows that both distributed and localized mapping's performances are the same. However, interleaved FDMA mapping has higher performance over the other methods (Approximately 8db advantage over the localized mapping). IFDMA yields an ideal PAPR performance of 0dB. However ideal PAPR doesn't guarantee higher throughput.

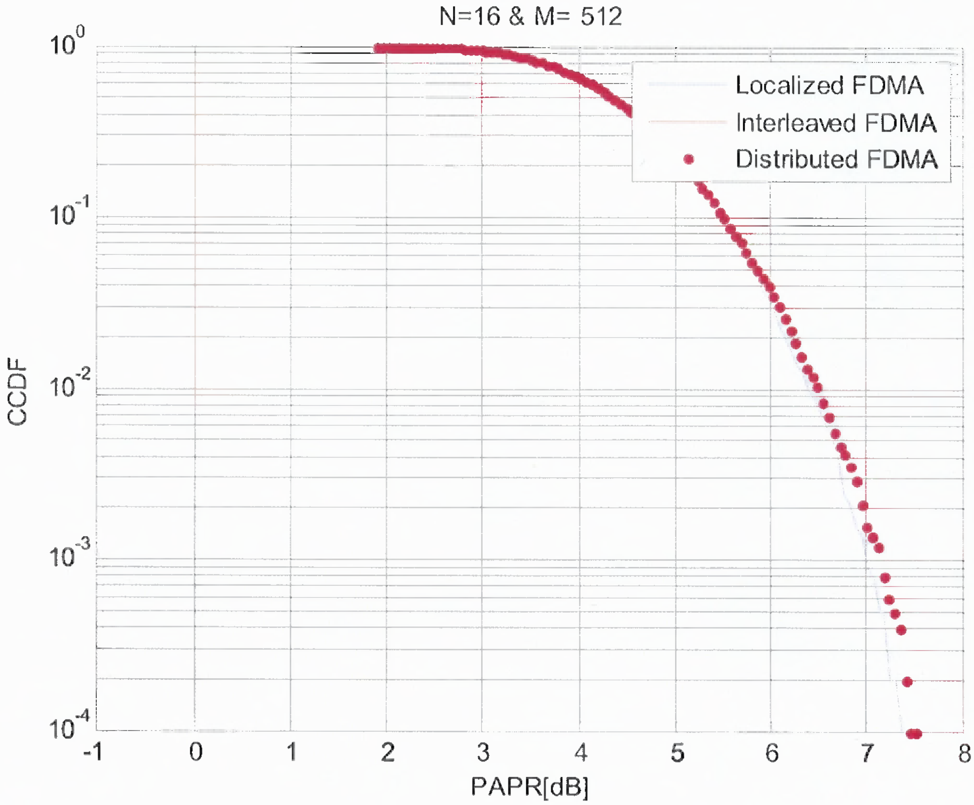


**Figure 2.7** Comparison of PAPR for different subcarrier mapping types of SCFDMA with DFT size 16 and IDFT size 512.

Once the switch was made from circuit switching to packet switching the measure for system capabilities switched from capacity to throughput. Thus, even though IFDMA has ideal PAPR compared to LFDMA, LFDMA is used in SC-FDMA because of higher throughput performance with Channel Dependent Scheduling (CDS). The comparison of different subcarrier mapping methods in terms of throughput can be seen in figure 2.8.

IDFT block in SC-FDMA is much like the IDFT block of the OFDMA. IDFT in SC-FDMA also combines the multiple subcarriers and converts the frequency domain signal to time domain prior to transmission. However the key difference is that in OFDMA all M subcarriers were occupied but in SC-FDMA only N subcarriers per user are occupied which means less number of subcarriers will be added in the IDFT. This is the main logic behind SC-FDMA.

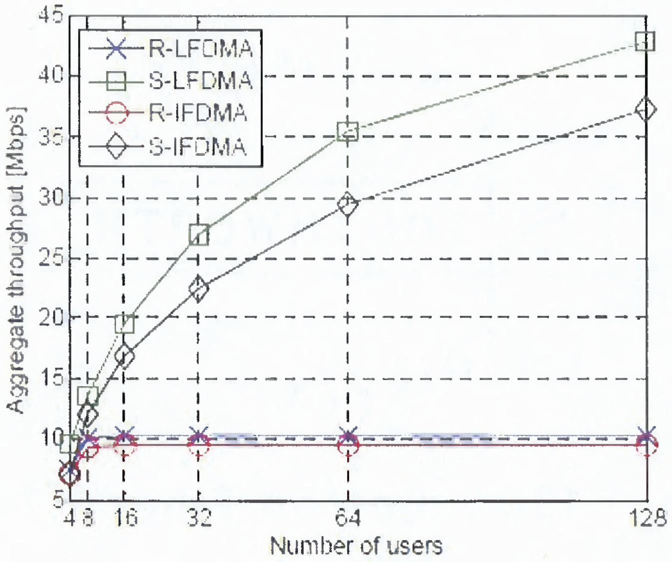


**Figure 2.8** Comparison of CCDF of PAPR for IFDMA, DFDMA, LFDMA and OFDMA with total number of subcarriers M=512, number of input symbols N=128, IFDMA spreading factor 4, DFDMA spreading factor 2, QPSK, and a (roll-off factor)=0.22.

Source: Hyung G. Myung, Junsung Lim, and David J. Goodman, "Single Carrier FDMA for Uplink Wireless Transmission", IEEE Vehicular Technology Magazine, vol. 1, no. 3, Sep. 2006, pp. 30-38

After IDFT stage the time domain signal is converted from parallel to serial stream. Cyclic Prefix is added to reduce the effects of multipath fading. Finally the signal is converted from digital to analog and transmitted over the air, same as in OFDMA. Inverse process occurs at the receiver for effective detection in the time domain.

## 2.6 SCFDMA Frame Structure

SC-FDMA frame structure is much like that of the OFDMA with minor differences. Both OFDMA and SC-FDMA have ten-millisecond frame duration. Each frame is divided into ten sub frames with duration of one millisecond each, as in OFDMA. Each sub frame is further divided into two half-millisecond slots also as in OFDMA. The slots in both carry seven symbols per slot. However, the difference between OFDMA and SC-FDMA lies within the content of these symbols. The data for OFDMA is packed within all seven symbols of the slot; such is not the case in SC-FDMA. Per slot, SC-FDMA only has six data symbols. It uses the seventh symbol as demodulation reference for channel estimation. There are multiple ways to implement this reference symbol, for example hopping pattern. However it is out of scope for this paper. Key take away here is that per slot SC-FDMA sends less data compared to OFDMA.

The resource block (RB) for both OFDMA and SC-FDMA is defined by half millisecond timeslot and 180 KHz bandwidth. Each RB is divided by 12 subcarriers of 15 KHz frequency bandwidth. Therefore the seven symbols per slot can be spread over the 12 subcarriers in the RB, as seen in the figure 2.9. A resource element in both SC-FDMA and OFDMA is defined by a subcarrier and a symbol time. Ultimately for a 20 MHz system bandwidth it is possible to have 100 usable RBs that can support 1200 subcarriers. Multiple user data is stored in resource elements, which are spread across multiple RBs. This method of data allocation makes detection scheduling very critical in both SC-FDMA and OFDMA. Thus, detection scheduling will be one of the challenge areas in LTE as it matures.

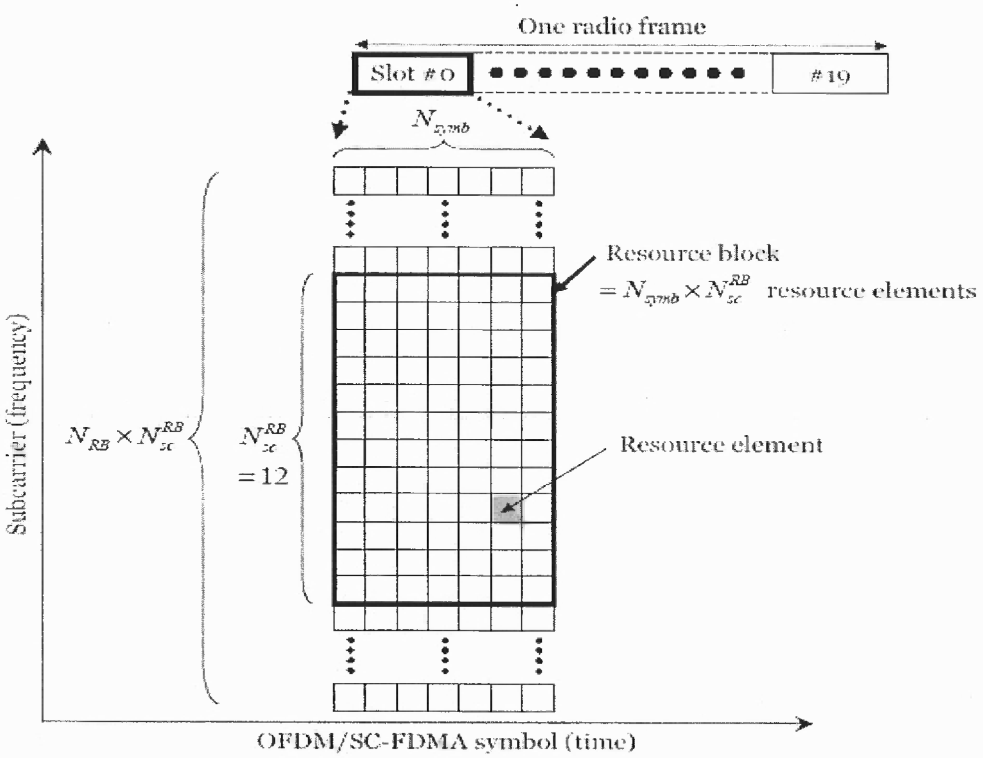


**Figure 2.9** Resource block for LTE system.

# CHAPTER 3

# GENERALIZED DFT

## 3.1 Theory of DFT and GDFT

Discrete Fourier Transform (DFT) is nothing else but special solution of Generalized Discrete Fourier Transform (GDFT). DFT is the solution of GDFT with linear phase. If z holds on equation (3.1), z is called N<sup>th</sup> root of unity.

$$z^N = 1,\ N = 1,2... \tag{3.1}$$

And if for any m, which is smaller than N, m<sup>th</sup> power of z is not equal to one, z is called primitive N<sup>th</sup> root of unity. For example, $e^{i\frac{2\pi}{5}}$ is a primitive N<sup>th</sup> root of unity for N=5. All primitive N<sup>th</sup> roots of unity satisfy the following equation;

$$\sum_{n=0}^{N-1} (z_p)^n = \frac{(z_p)^N - 1}{z_p - 1} = \begin{Bmatrix} 1,\ N = 1 \\ 0,\ N > 1 \end{Bmatrix} \ \forall p \tag{3.2}$$

Now, define a complex sequence like,

$$\mathrm{e}_r(\mathrm{n}) = (\mathrm{z}_p^r)^n = e^{i(2\pi r/N)n} \quad \begin{Bmatrix} r = 0,1,2,...,N-1 \\ n = 0,1,2,..,N-1 \end{Bmatrix} \tag{3.3}$$

If we want to write this complex number as a geometric series, with the help of (3.2)

$$\frac{1}{N}\sum_{n=0}^{N-1} e_r(n) = \frac{1}{N}\sum_{n=0}^{N-1} (z_p^r)^n =$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} e^{i(2\pi r/N)n} = \begin{Bmatrix} 1, & r = mN \\ 0, & r \neq mN \end{Bmatrix} \tag{3.4}$$

And (3.4) can be used for the orthonormality condition;

$$< e_x(n), e_y^*(n) > = \frac{1}{N}\sum_{n=0}^{N-1} e_x(n), e_y^*(n) =$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} e^{i(2\pi/N)(x-y)n} = \begin{Bmatrix} 1, & x - y = r = mN \\ 0, & x - y \neq r = mN \end{Bmatrix} \tag{3.5}$$

(*) is used to mention conjugate operation. Equation (3.4) can also be generalized by replacing r with phase.

$$\frac{1}{N}\sum_{n=0}^{N-1} e^{i(2\pi/N)rn} = \frac{1}{N}\sum_{n=0}^{N-1} e^{i(2\pi/N)\varphi(n)n} =$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} e^{i(2\pi/N)(\varphi_x(n)-\varphi_y(n))n} = \begin{Bmatrix} 1, & \varphi_x(n) - \varphi_y(n) = r = mN \\ 0, & \varphi_x(n) - \varphi_y(n) \neq r = mN \end{Bmatrix} \tag{3.6}$$

And the basis functions are;

$$e_k(n) = e^{i(2\pi/N)\varphi_k(n)n} \; k,n = 0,1,..,N-1 \tag{3.7}$$

This set is called the Generalized Discrete Fourier Transform with Non-linear Phase. DFT is a special case of GDFT with $\varphi_k(n) = 0$ in equation (3.7). Because DFT has a constant value for its phase, it has linear phase property.

Infinitely many GDFT sets with same power can be found. So, if designer of any system is not concerning about linear phase, designer will have infinitely many options up to desired optimization.

### 3.2 GDFT Design

DFT matrix is,

$$A_{DFT} = \left[ e^{i(2\pi/N)kn} \right] \; k,n = 0,1,2,...,N-1 \tag{3.8}$$

As it is said, DFT is just a special solution of GDFT. So from DFT matrix, GDFT matrix can be reconstructed.

$$A_{GDFT} = A_{DFT} * G \tag{3.9}$$

G matrix is an orthogonal, complex valued matrix.

There are several types of G matrix. These are;

- Full G Matrix
- Diagonal G Matrix Family
    - Constant Valued Diagonal Elements
    - Non-Constant Valued Diagonal Elements
    - Two Matrices with Non-constant Valued Diagonal Elements

In this thesis research, G matrix from diagonal G matrix family with non-constant valued diagonal elements is used.

**Non-Constant Valued Diagonal Elements:** With non-zero, non-constant and constant modulus diagonal elements, G matrix family of this kind is defined as;

$$G(k,n)=\begin{Bmatrix} e^{i\theta_{kk}}, & k=n \\ 0, & k\neq n \end{Bmatrix} \quad k,n=0,1,2,\ldots,N \qquad (3.10)$$

It can be seen that when DFT matrix is multiplied with the G matrix, each row of DFT matrix will be independently shifted in the phase.

# CHAPTER 4

# PEAK TO AVERAGE POWER RATIO REDUCTION IN SCFDMA WITH GDFT

## 4.1 Theory

Although OFDM technology is known for many years, because of the technological capability limits, OFDM was not an applicable system until today. With the improvements on DSP, OFDM is proposed for 4G communication systems due to its spectral efficiency, ease of implementation with Fast Fourier Transform (FFT) operation, and strong resistance to multipath resistance. As mentioned before in this thesis, the main drawback of an OFDM is its high Peak to Average Power Ratio (PAPR). This drawback is not a big problem for downlink channel (channel from station to mobile), because the station has a steady power source, whereas this drawback is a very essential issue for uplink channel (channel from mobile to station) because a mobile device has limited amount of power source. For the uplink channel a new technology, which is called SCFDMA, is used. SCFDMA, as mentioned before, has lower PAPR value when compared to OFDM technology.

The work done utilizes GDFT approach to SCFDMA instead of DFT to reduce PAPR. The work is about to find the most efficient G matrix. In the work done, first DFT is replaced by GDFT with different G matrixes up to different N (DFT size, number of subcarriers used by a single user) and M (IDFT size, total number of subcarriers in the system) numbers. This chapter is organized as follows; SCFDMA system 's described, PAPR calculations are introduced. In the following section, the proposed GDFT based, PAPR reduction technique is described and explained with graphical results and the required GDFT matrix coefficients are introduced.

## 4.2 SCFDMA ANALYSIS

Incoming data bit stream is $\underline{x} = \left(x_k ; k = 1,2,3,....,N-1\right)$,

**Figure 4.1** SCFDMA block diagram.

In the figure, N is the number of subcarriers used by single user. SCM stands for subcarrier mapping. M is the total number of subcarriers in the system.

$$\underline{\theta} = A_{DFT}\underline{x} \tag{4.1}$$

where $A_{DFT}$ is N size DFT matrix. We can describe each element of $\underline{\theta}$ vector as,

$$\theta_k = \sum_{n=0}^{N-1} x_k e^{\frac{-2\pi i}{N}kn} \quad k = 1,2,...,N-1 \tag{4.2}$$

after DFT operation subcarriers are allocated to available locations in Subcarrier Mapping block (SCM). There are different types of subcarrier mapping;

- Localized Mapping
- Distributed Mapping
    - Interleaved Mapping

For the simulations, localized mapping is used. There are several reasons for that. First, even though interleaved mapping has 0dB PAPR, interleaved mapping has lower throughput, especially with channel dependent scheduling. And one of the most beneficial sides of OFDMA/SCFDMA systems is being flexible in terms of user number and the number of subcarriers assigned to each user. However, interleaved mapping restricts this flexibility. For example, user A is making a video conference through his phone however user B is just using GPS ability of his phone. It is obvious that user A will require more band width than user B. If the allocation type is localized mapping, it is very easy and straight forward to assign a sufficient number of subcarriers to each user. However, if the allocation type is interleaved mapping, each user has to have the same number of subcarriers. So either user A will not get sufficient subcarriers or user B will allocate more subcarriers than he needs, which will lower the throughput of other users in the same cell. Because of all of the reasons above, during this research, localized mapping was used.

In Subcarrier mapping (SCM) block;

$$\tilde{\underline{\theta}}\ (k) = \left\{ \begin{array}{cc} \underline{\theta}\ (k) & k = 1, 2, ...., N \\ 0 & otherwise \end{array} \right\} \tag{4.3}$$

and after IDFT block;

$$X\ (m) = C_1 \sum_{k=0}^{M-1} \tilde{\underline{\theta}}^{i}(k) e^{i\frac{2\pi}{M}km} \qquad 0 \le m \le M-1 \tag{4.4}$$

M is the total number of available subcarriers in the system (size of the IDFT). $C_1$ is a constant. And the concern of this thesis is PAPR and PAPR for this case can be expressed as;

$$PAPR = \frac{\max|X(n)|^2}{E\left\{|X(n)|^2\right\}} \tag{4.5}$$

$$PAPR_{dB} = 10\log_{10}\left(\frac{\max\left\{|X(n)|^2\right\}}{E\left\{|X(n)|^2\right\}}\right) \tag{4.6}$$

## 4.3 Design and Performance of GDFT Based SCFDMA

From figure (4.1) and equation (4.1);

$$\underline{\tilde{\theta}} = T^{MxN}\underline{\theta} = T^{MxN}A_{DFT}^{NxN}\underline{x} \tag{4.7}$$

$$\underline{X} = A_{IDFT}^{MxM}\underline{\tilde{\theta}} = A_{IDFT}^{MxM}T^{MxN}A_{DFT}^{NxN}\underline{x} \tag{4.8}$$

T is the subcarrier mapping matrix.

On the above SCFDMA with DFT is introduced. In this thesis, we have focused on a G matrix which will change $A_{DFT}^{NxN}$ matrix into $A_{GDFT}^{NxN}$ matrix. So equation (4.8) will be;

$$\underline{X} = A_{IDFT}^{MxM}\underline{\tilde{\theta}} = A_{IDFT}^{MxM}T^{MxN}G\underbrace{A_{DFT}^{NxN}\underline{x}}_{\underline{\theta}} \tag{4.9}$$

Using generalized DFT makes the user to arrange the elements of $\underline{\theta}$ to get the lowest possible PAPR. By using G matrix, we can change the position on the complex plane of the each element of $\underline{\theta}$ vector. In this thesis, main goal is to reduce to PAPR so the arrangement should be done to minimize the PAPR.

In the example below, the number of subcarriers for each user is assumed as 4 and total number of subcarriers in the system is assumed as 16. As digital modulation type, BPSK is used. The input;

$$\underline{x}' = [1\ 1\ \text{-}1\ \ 1]$$

After DFT operation,

$$\underline{\theta}' = [2\ 2\ \text{-}2\ 2]$$

after zero padding;

$$\underline{\tilde{\theta}}'^{16x1} = [2\ 2\ \text{-}2\ 2\ 0\ 0\ \ldots\ldots 0]$$

after IDFT operation

$$\underline{X} = [\begin{array}{ll} 0.25 & +i0 \\ 0.199932022961229 & +i0.0749320229612286 \\ 0.125 & +i0.0517766952966369 \\ 0.145738835130044 & -i0.0207388351300438 \\ 0.25 & +i0 \\ 0.281037860166593 & +i0.15603786016659 \\ 0.125 & +i0.301776695296637 \\ -0.1267087182578 & +i0.25170871825 \\ -0.25 & +0i \\ -0.126708718257 & -i0.251708718257 \\ 0.1250 & -i0.30177669 \\ 0.2810378 & -i0.156037 \\ 0.2500 & +i0 \\ 0.1457388 & +i0.02073 \\ 0.12500 & -i0.05177 \\ 0.1999320 & -i0.07493 \end{array} ]$$

PAPR value of that vector is 2.32.

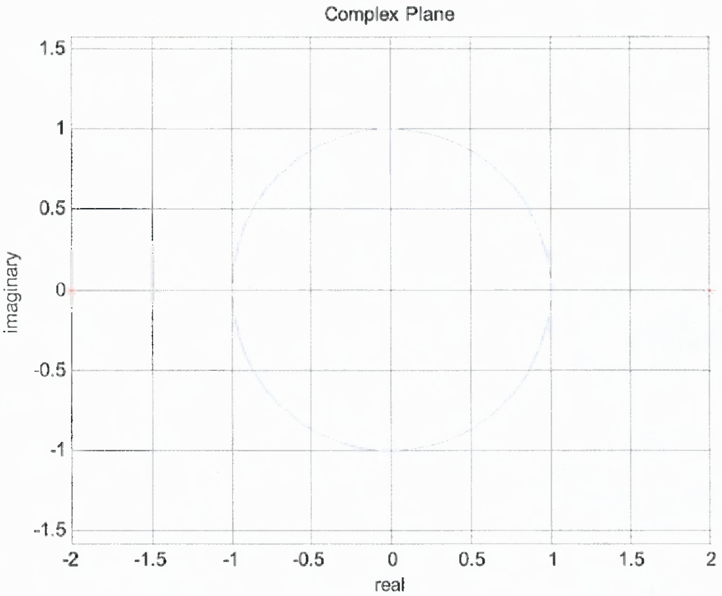
With G matrix, elements of $\underline{\theta}$ matrix have been rearranged to get the minimum lowest PAPR value at the end. On figure (4.2), how each element's phase has been changed on complex plane, can be seen. It is obvious that each particular element has been spread over complex plane to balance each other

$$\underline{x}' = [1\ 1\ {-1}\ 1]$$

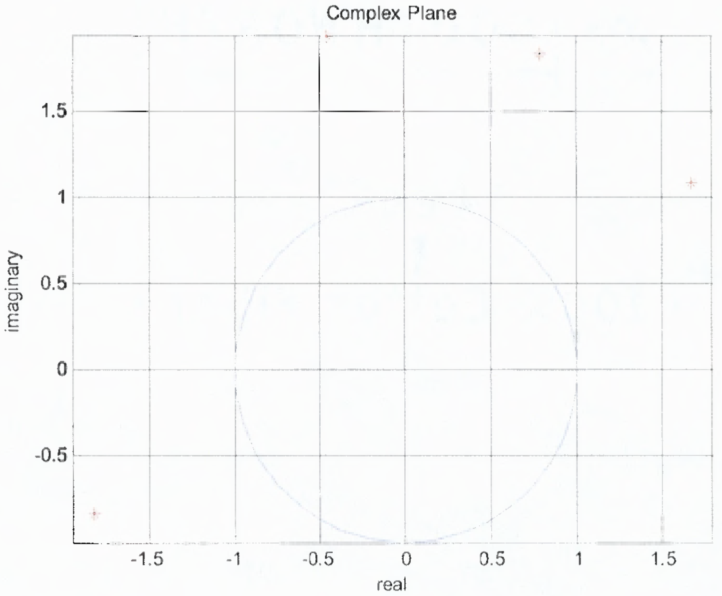
$$\underline{\theta}' = [2\ 2\ {-2}\ 2]$$

$$G\underline{\theta} = [-1.8 - i0.8 \quad 0.7 + i1.8 \quad 1.6 + i1.09 \quad -0.4 + i1.9]$$

$$\underline{\tilde{\theta}}' = [-1.8 - i0.8 \quad 0.7 + i1.8 \quad 1.6 + i1.09 \quad -0.4 + i1.9 \quad \underbrace{0\ 0\ 0 \ldots 0}_{\text{12 Zeros}}]$$



(a)



(b)

**Figure 4.2** Position of elements of $\underline{\theta}$ (a) before multiplying (b) after multiplying with G matrix.

where,

$$G = diag\{1.1\ 0.37\ 1.1\ 0.57\}$$

with that G matrix PAPR for that particular input can be reduced from 2.32 to 1.6. However, that G matrix is valid for that particular input which is [1 1 -1 1]. Even though, that G matrix works for that input, it will not work for another input. In the example below, another input was fed to the system with same G matrix.

$$\underline{x} = [1\ 1\ \text{-}1\ \text{-}1]$$

$$\underline{\theta} = [0\ 2\text{-}i2\ 0\ 2\text{+}i2]$$

$$\underline{\tilde{\theta}}' = [0\ 2\text{-}i2\ 0\ 2\text{+}i2\ \underbrace{0\ 0\ 0.....\ 0}_{\text{12 Zeros}}]$$

without G matrix PAPR value for that input is 3.01.

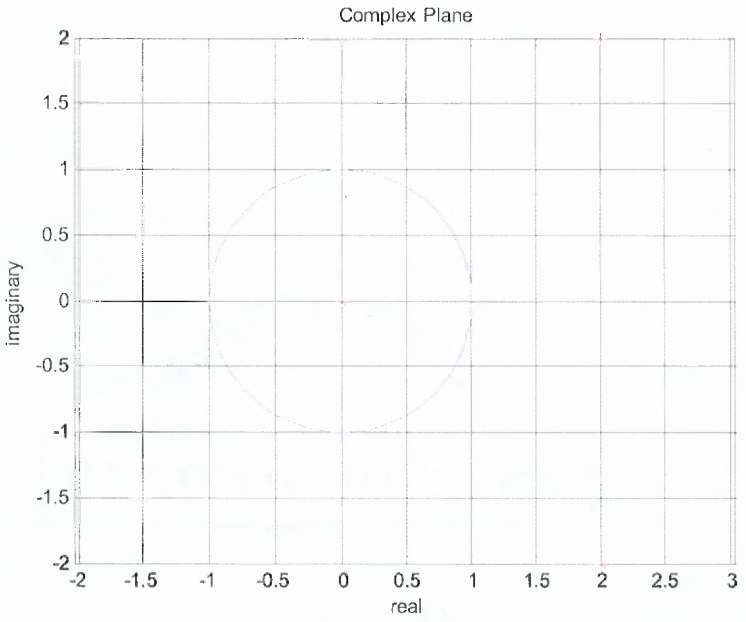
$$\underline{x}' = [1\ 1\ \text{-}1\ \text{-}1]$$

$$\underline{\theta}' = [0\ 2\text{-}i2\ 0\ 2\text{+}i2]$$

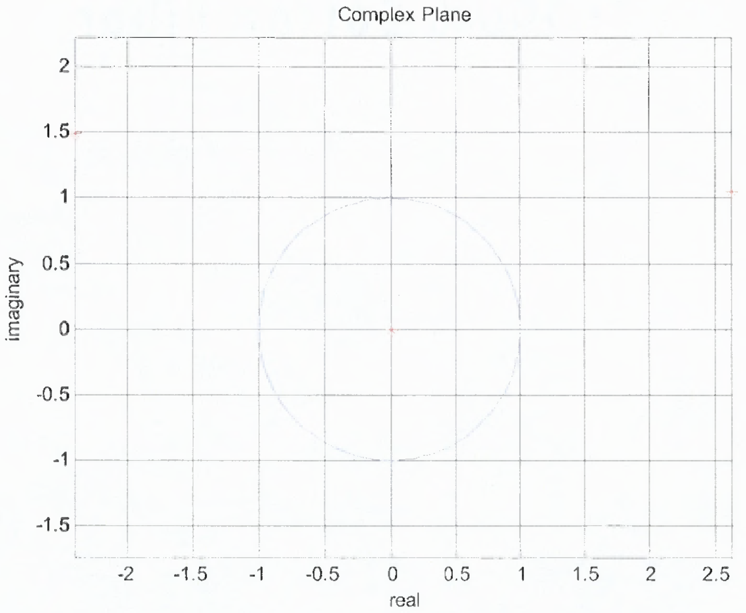
$$G\underline{\theta} = [0 \qquad 2.6 + i1.05 \qquad 0 \qquad \text{-}2.4 + i1.48]$$

$$\underline{\tilde{\theta}} = [0\quad 2.6 + i1.05 \qquad 0 \qquad \text{-}2.4 + i1.48 \qquad \underbrace{0\ 0\ 0.....0}_{\text{12 Zeros}}]$$

with G matrix, PAPR value of that input is 3.98. On figure (4.3), how each element's phase has been changed on complex plane, can be seen.



(a)



(b)

**Figure 4.3** Position of elements of $\underline{\theta}$ (a) before multiplying (b) after multiplying with G matrix.

From the example above, it can be understood that a G matrix can optimize one particular input's PAPR value, but at the same time, it can increase PAPR value of another possible input. So, a proper G matrix has to be found such that it will optimize PAPR values for all possible inputs.

To have the minimum PAPR value, after the DFT operation in the receiver side, a G matrix can be used to change the phase of each output of DFT operation and spread them on the complex plane so that after the M-size IDFT operation the signal that will be transmitted to channel has the minimum PAPR. The proposed systems block diagram can be seen on figure (4.3).

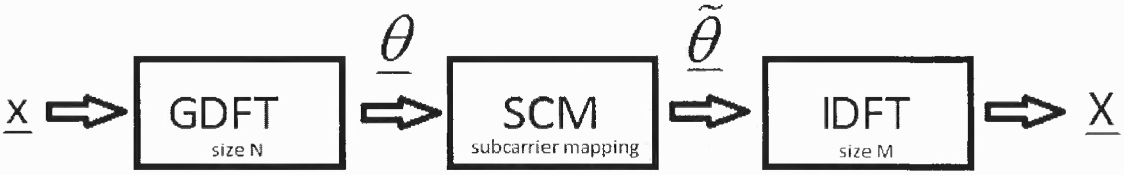


**Figure 4.4** Proposed SCFDMA system block diagram.

After this point, this master thesis has couple of graphs to compare conventional DFT spread OFDMA PAPR value and proposed GDFT spread OFDMA PAPR value. In the simulations, firstly Binary Phase Shift-Keying (BPSK), secondly Quadrature Phase Shift-Keying (QPSK) and thirdly Quadrature Amplitude Modulation (QAM) are used. For each modulation type different N and M sizes are used. As explained above, for different inputs, different G matrix should be used so for each different N&M pair, a different G matrix will be introduced. Again, because of the reasons explained before, Localized Mapping (LFDMA) is preferred for the Subcarrier Mapping (SCM) type. To get the proper results the graphs below is obtained after more than 1 million runs.

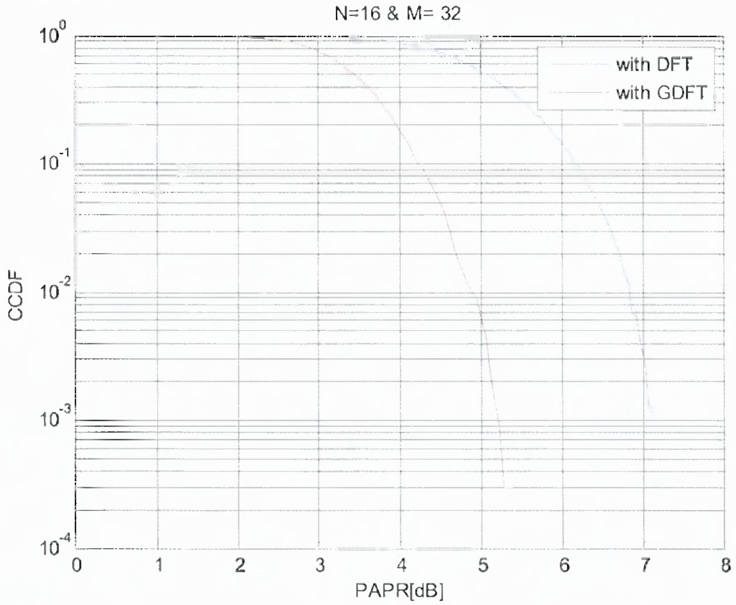


**Figure 4.5** CCDF of PAPR for SCFDMA with DFT or GDFT size (N) =16 and IDFT size=32 with BPSK digital modulation.

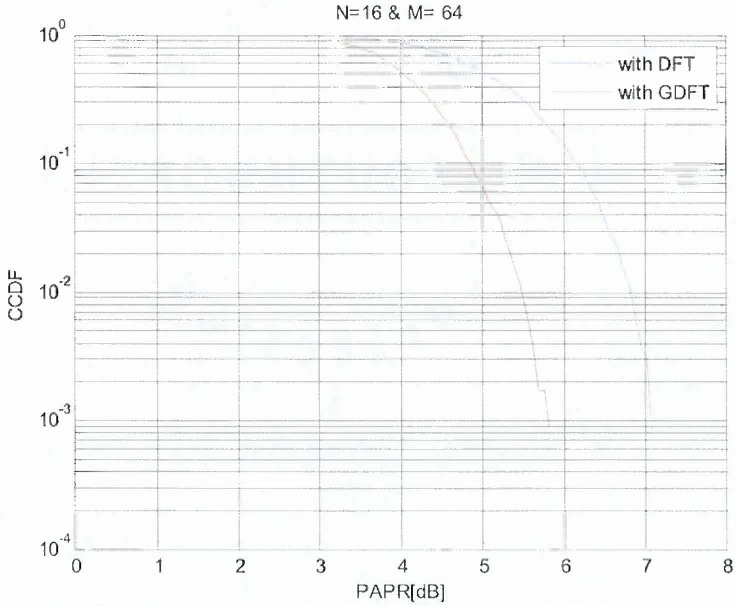


**Figure 4.6** CCDF of PAPR for SCFDMA with DFT or GDFT size (N) =16 and IDFT size=64 with BPSK digital modulation.

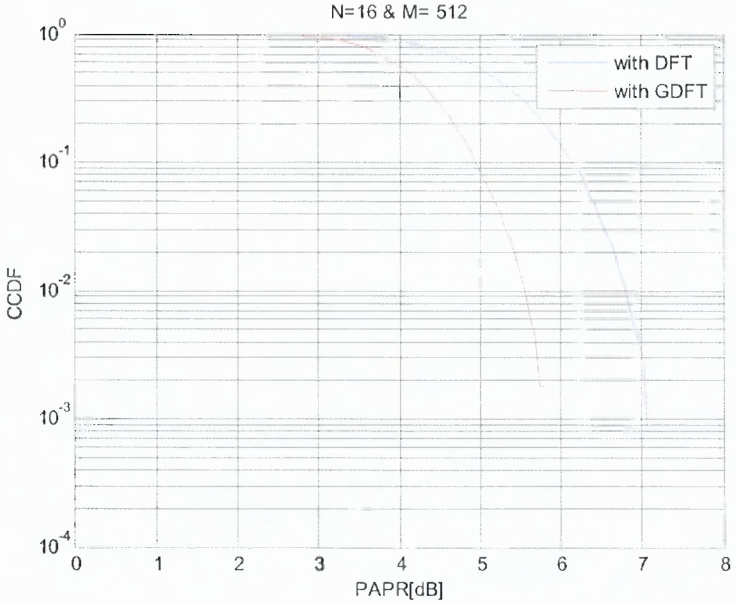


**Figure 4.7** CCDF of PAPR for SCFDMA with DFT or GDFT size (N) =16 and IDFT size=512 with BPSK digital modulation.

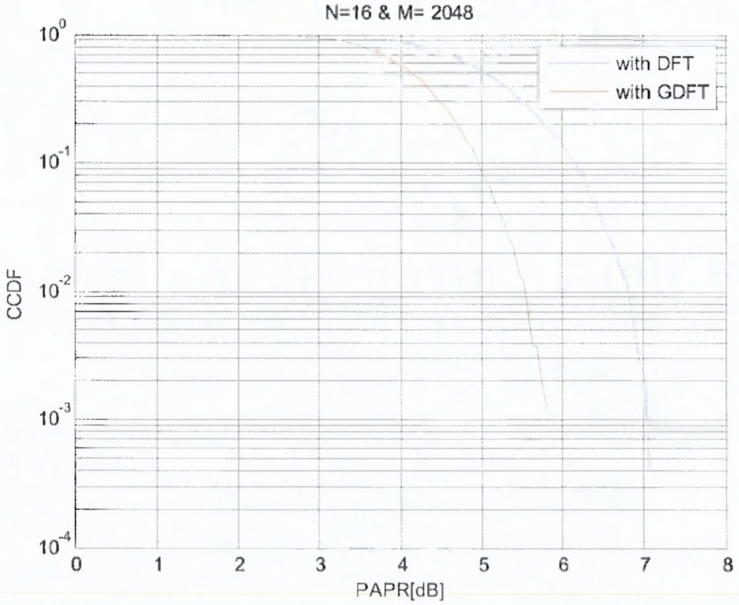


**Figure 4.8** CCDF of PAPR for SCFDMA with DFT or GDFT size (N) =16 and IDFT size=2048 with BPSK digital modulation.

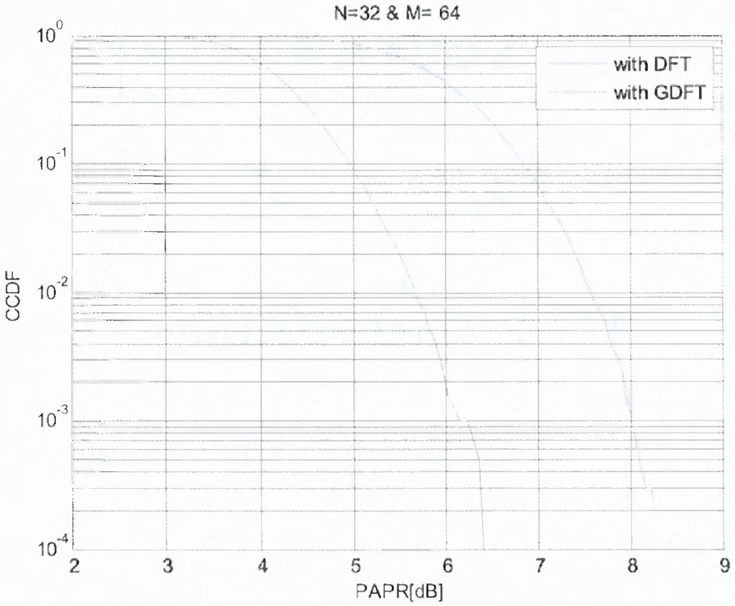


**Figure 4.9** CCDF of PAPR for SCFDMA with DFT or GDFT size (N) =32 and IDFT size=64 with BPSK digital modulation.

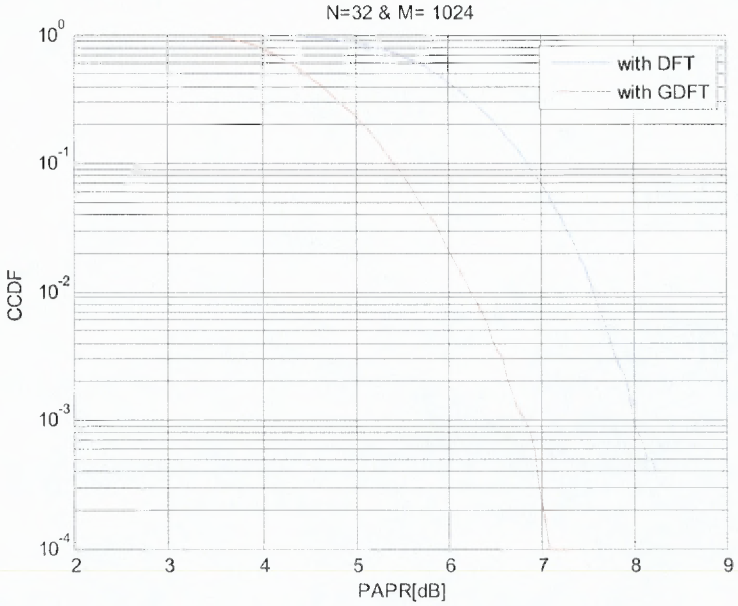


**Figure 4.10** CCDF of PAPR for DFT or GDFT size (N) =16 and IDFT size=128, SCFDMA with BPSK digital modulation.

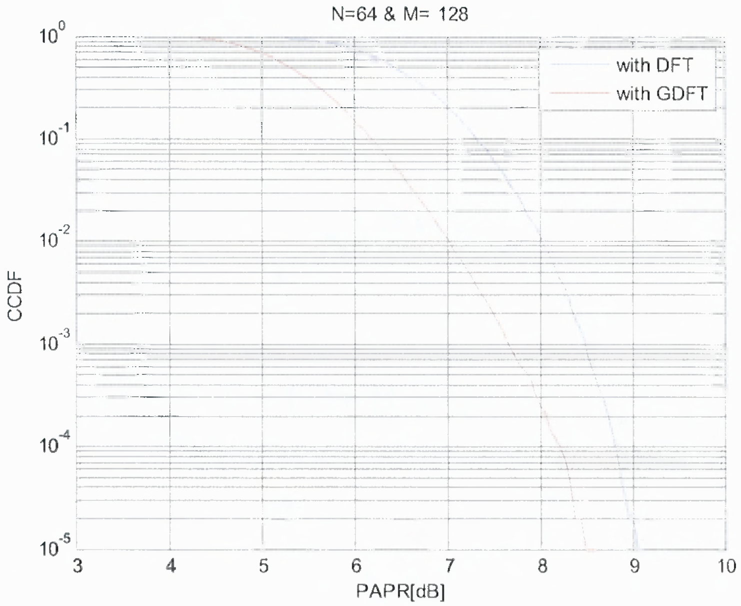


**Figure 4.11** CCDF of PAPR for DFT or GDFT size (N) =16 and IDFT size=128, SCFDMA with BPSK digital modulation.

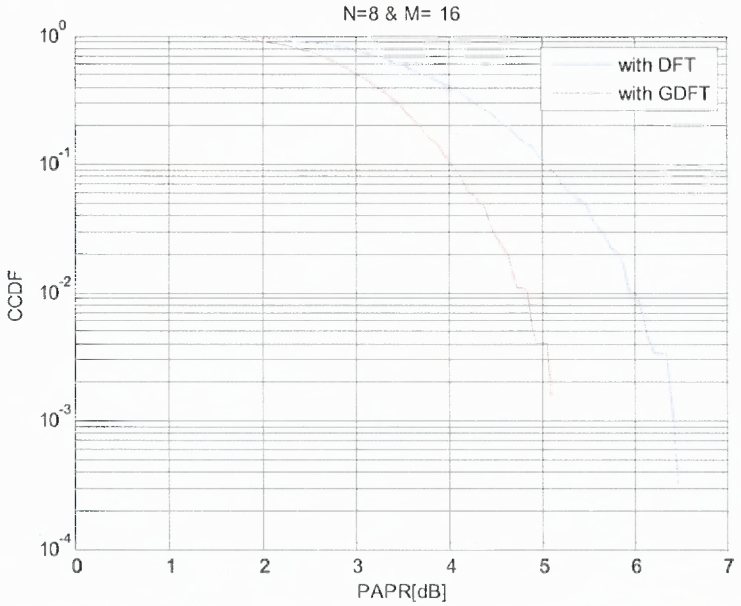


**Figure 4.12** CCDF of PAPR for DFT or GDFT size (N) =8 and IDFT size=16, SCFDMA with QPSK digital modulation.

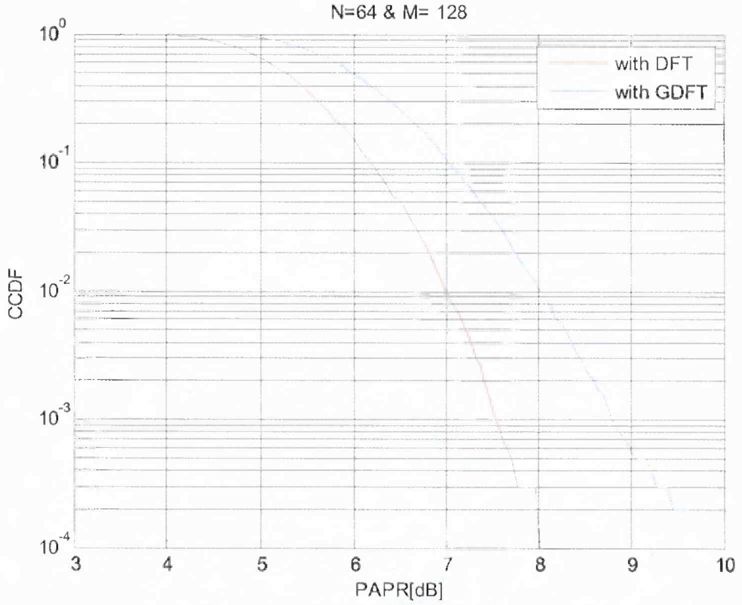


**Figure 4.13** CCDF of PAPR for DFT or GDFT size (N) =64 and IDFT size=128, SCFDMA with QPSK digital modulation.

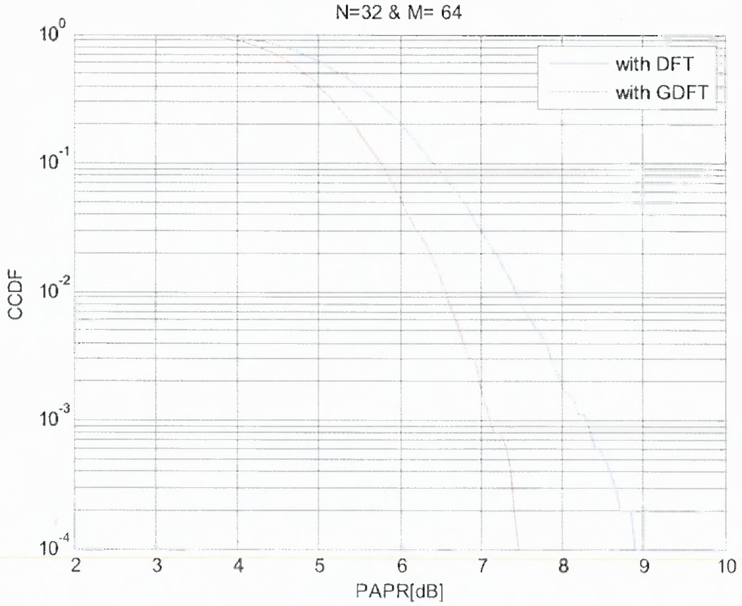


**Figure 4.14** CCDF of PAPR for DFT or GDFT size (N) =32 and IDFT size=64, SCFDMA with QAM digital modulation.

| Figure No | Diagonal entries of the G matrix that is used for simulation | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **4.4** | 0.0043 | 0.2510 | 0.2802 | 0.2380 | 0.1669 | 0.0453 | 0.0408 | |
| | 0.0112 | 0.1862 | 0.0904 | 0.1627 | 0.2282 | 0.4063 | 0.5242 | |
| **4.5** | 0.2137 | 0.4540 | 0.3715 | 0.3033 | 0.1952 | 0.0818 | 0.0601 | |
| | 0.1098 | 0.1137 | 0.2030 | 0.2779 | 0.4003 | 0.5419 | 0.6734 | |
| **4.6** | 0.5204 | 0.5862 | 0.5704 | 0.5395 | 0.4358 | 0.3046 | 0.2518 | 0.2317 |
| | 0.2404 | 0.3096 | 0.3893 | 0.5200 | 0.7036 | 0.8396 | 0.9219 | 1.0170 |
| **4.7** | 0.0727 | 0.0501 | 0.0970 | 0.2238 | 0.3561 | 0.5588 | 0.6593 | 0.7449 |
| | 0.7869 | 0.8016 | 0.7867 | 0.7361 | 0.6017 | 0.5440 | 0.4800 | 0.4968 |
| **4.8** | 0.7522 | 0.5329 | 0.5900 | 0.9694 | 0.9457 | 0.8744 | 0.8085 | 0.7170 |
| | 0.5269 | 0.5700 | 0.4703 | 0.6392 | 0.6428 | 0.5984 | 0.4424 | 0.5227 |
| | 0.5191 | 0.4902 | 0.4452 | 0.5154 | 0.5575 | 0.5190 | 0.3586 | 0.4038 |
| | 0.3336 | 0.4707 | 0.5151 | 0.6482 | 0.6388 | 0.6441 | 0.0839 | -0.0630 |
| **4.9** | 0.7653 | 0.4217 | 0.4236 | 0.9679 | 0.9331 | 0.8781 | 0.7853 | 0.6332 |
| | 0.6418 | 0.6421 | 0.6246 | 0.6354 | 0.5999 | 0.5707 | 0.5512 | 0.5459 |
| | 0.5178 | 0.5069 | 0.4833 | 0.4675 | 0.4609 | 0.4708 | 0.4352 | 0.4314 |
| | 0.4100 | 0.3747 | 0.5248 | 0.6335 | 0.7103 | 0.8014 | 0.0457 | -0.0537 |
| **4.10** | 0.0818 | 0.1014 | 0.1076 | 0.0052 | 0.8540 | 0.8260 | 0.7824 | 0.7414 |
| | 0.3345 | 0.3465 | 0.3680 | 0.7414 | 0.7037 | 0.6506 | 0.4741 | 0.4336 |
| | 0.4032 | 0.4706 | 0.5408 | 0.5029 | 0.5529 | 0.5428 | 0.3178 | 0.4943 |
| | 0.5930 | 0.5816 | 0.4112 | 0.3785 | 0.4434 | 0.4945 | 0.4716 | 0.5159 |
| | 0.5885 | 0.5251 | 0.4840 | 0.4655 | 0.3743 | 0.2968 | 0.0393 | 0.5351 |
| | 0.5234 | 0.3341 | 2.3365 | 0.3874 | 0.4168 | 0.4896 | 0.4230 | 0.4271 |
| | 0.3532 | 0.1259 | 0.1228 | 0.5370 | 0.5641 | 0.6296 | 0.2198 | 0.2678 |
| | 0.2635 | 0.6839 | 0.7362 | 0.6798 | 0.8108 | 0.0575 | 0.0600 | 0.1229 |
| **4.11** | 0.5320 | 0.4695 | 0.4070 | 0.3445 | 0.2820 | 0.2195 | 0.1570 | 0.0945 |
| **4.12** | 0.1328 | 0.1668 | 0.4314 | -0.136 | 0.6383 | 0.4789 | 0.4979 | 0.6471 |
| | 0.5707 | 0.4473 | 0.6296 | 0.6453 | 0.1427 | 0.1855 | 0.4114 | 0.4517 |
| | 0.4589 | 0.5170 | 0.3044 | 0.5166 | 0.5581 | 0.5119 | 0.3647 | 0.3319 |
| | 0.4651 | 0.4519 | 0.1955 | 0.1503 | 0.5459 | 0.6725 | 0.4692 | 0.1708 |
| | 0.3506 | 0.1477 | 0.2647 | 0.4253 | 0.5656 | 0.4258 | 0.1131 | 0.2543 |
| | 0.4215 | 0.4602 | 0.3783 | 0.1270 | 0.3868 | 0.4152 | 0.3059 | 0.4736 |
| | 0.0225 | 0.3605 | 0.3335 | 0.3509 | 0.6091 | 0.1007 | 0.2013 | 0.0329 |
| | 0.0574 | 0.2986 | 0.9456 | 0.5137 | 0.1031 | 0.2914 | 0.0992 | 0.0984 |
| **4.13** | 0.1657 | 0.1777 | 0.1974 | 0.2063 | 0.2260 | 0.2445 | 0.2575 | 0.2702 |
| | 0.2879 | 0.3027 | 0.3290 | 0.3413 | 0.3588 | 0.3827 | 0.3788 | 0.4089 |
| | 0.4208 | 0.4280 | 0.4507 | 0.4575 | 0.4754 | 0.4967 | 0.5139 | 0.5247 |
| | 0.5415 | 0.5504 | 0.5603 | 0.5873 | 0.5991 | 0.6178 | 0.6308 | 0.6573 |

# CHAPTER 5

# CONCLUSION

Wireless communication concept started with Frequency-division multiplexing (FDM) based systems. And Time-Division Multiplexing (TDM) based systems followed that. After a while, with the improvements on Digital Signal Processing technologies, Code Division Multiple Access (CDMA) based systems started to be used for wireless communication. The next step after CDMA based systems is Orthogonal Frequency-Division Multiplexing (OFDM) based systems.

One of the main drawbacks of OFDM system is its high Peak to Average Power Ratio (PAPR) value which is directly related to power consumption of the system. Even though power consumption is not a problem for downlink channel, it is one of the main issues for uplink channel because the available power for the mobile user is limited. To overcome this issue, for the uplink channel, Discrete Fourier Transform (DFT) spread OFDM or Single Carrier-FDM (SC-FDM) system is used. In this thesis PAPR value of DFT spread OFDM channel is further decreased by using Generalized DFT concept.

In this thesis, a solution is proposed to the PAPR problem of SCFDMA or in other words DFT spread OFDM by using Generalized DFT. Instead of conventional DFT, using GDFT makes the system flexible in terms of phase of each element. By using this flexibility, these elements can be reorganized on the complex plane to get the lowest PAPR value.

solution is G matrix is not input dependent for fixed N and M numbers for all inputs, same G matrix can be used. So, user does not have to use side information bits. This gives an advantage over other solutions which requires side information bits.

# APPENDIX

## MATLAB CODE USED FOR SIMULATIONS

The matlab codes that are used to find the G matrix for the lowest possible PAPR, are given within the boxes below.

### SCFDMA PAPR Simulation Matlab Code For BPSK Modulation

```
function papr(input)
totalSubcarriers = 256; % Number of total subcarriers.
numSymbols = 128; % Data block size.
numRuns = 1e6;
papr=zeros(numRuns,1);
table=ones(4000,128);
input=zeros(numSymbols,1);
color=['b'];

k=1;
for n = 1:numRuns,
    % Generate random data
    tmp = round(rand(numSymbols,1));
    tmp = tmp*2 - 1;
    data=tmp;


    X = fft(data);
    Y = zeros(totalSubcarriers,1);
    Y(1:numSymbols) = X;
    y = ifft(Y);
    papr(n) =10*log10(max(abs(y).^2) / mean(abs(y).^2));

    %-------TO CREATE TABLE -----------------
        table(k,:)=data;
        k=k+1;
    %-----------------------------------------------

end
% % %
save table table

[N,X] = hist(papr, 100);
semilogy(X,1-cumsum(N)/max(cumsum(N)),color)
title(['N=' num2str(numSymbols) ' & M= ' num2str(totalSubcarriers)]);
ylabel('CCDF'); xlabel('PAPR[dB]')
legend('with DFT')
grid on
hold on
```

## SCFDMA PAPR Simulation Matlab Code For QPSK Modulation

```
function papr(input)

totalSubcarriers = 256; % Number of total subcarriers.
numSymbols = 64; % Data block size.


numRuns = 1e4;
papr=zeros(numRuns,1);

k=1;
for n = 1:numRuns,
   % Generate random data
   tmp = round(rand(numSymbols,2));
   tmp = tmp*2 - 1;
   data = (tmp(:,1) + 1i*tmp(:,2))/sqrt(2);
   X = fft(data);
   X=X.*exp(1i*pi*input);
   Y = zeros(totalSubcarriers,1);
   Y(1:numSymbols) = X;
   y = ifft(Y);
   papr(n) =10*log10(max(abs(y).^2) / mean(abs(y).^2));

   %-------TO CREATE TABLE -----------------

   table(k,:)=data;
   k=k+1;



   %---------------------------------------------

end
save table table
[N,X] = hist(papr, 100);
semilogy(X,1-cumsum(N)/max(cumsum(N)),color)
title(['N=' num2str(numSymbols) ' & M= ' num2str(totalSubcarriers)]);
ylabel('CCDF'); xlabel('PAPR[dB]')
legend('with DFT')
grid on
hold on
```

## SCFDMA PAPR Simulation Matlab Code For QAM Modulation

```
function papr(input)

totalSubcarriers = 128; % Number of total subcarriers.
numSymbols = 16; % Data block size.
numRuns = 1e6;
papr=zeros(numRuns,1);
table=ones(numRuns,numSymbols);
% ---- to see the original performance----
input=zeros(numSymbols,1);
color=['b'];
%-----------------------------------------
sertac=1;

for n = 1:numRuns,
    % Generate random data.a
    data=ones(1,numSymbols);
    dataSet = [-3+3i -1+3i 1+3i 3+3i ...
        -3+1i -1+1i 1+1i 3+1i ...
        -3-1i -1-1i 1-1i 3-1i ...
        -3-3i -1-3i 1-3i 3-3i];
    dataSet = dataSet / sqrt(mean(abs(dataSet).^2));
    tmp = ceil(rand(numSymbols,1)*16);
    for k = 1:numSymbols,
        if tmp(k) == 0
            tmp(k) = 1;
        end
        data(1,k) = dataSet(1,tmp(k));
    end
    data = data.';

    X = fft(data);

    X=X.*exp(1i*pi*input);

    Y = zeros(totalSubcarriers,1);
    Y(1:numSymbols) = X;
    y = ifft(Y);
    papr(n) =10*log10(max(abs(y).^2) / mean(abs(y).^2));

    %-------TO CREATE TABLE -----------------
    table(sertac,:)=data;
    sertac=sertac+1;
    %-------------------------------------------


end
save table table
[N,X] = hist(papr, 100);
semilogy(X,1-cumsum(N)/max(cumsum(N)),color)
title(['N=' num2str(numSymbols) ' & M= ' num2str(totalSubcarriers)]);
ylabel('CCDF'); xlabel('PAPR[dB]')
legend('with DFT’)
```

### Matlab Code to Find Optimum G matrix Diagonal Elements

```
function papr= opt_papr(input)

totalSubcarriers =256 ;
numSymbols = 128; % Data block size.
load table
D=dftmtx(totalSubcarriers);
matrix=table';
A=dftmtx(numSymbols);
G=diag(exp(1i*pi*input));
Matrix=G*A*matrix;
Mat=[Matrix; zeros((totalSubcarriers-numSymbols),length(table))];
mat=abs(D\Mat);
papr = sum(10*log10(max(mat.^2) ./ (sum(mat.^2)/totalSubcarriers)));
```

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