ABSTRACT

OPTIMIZING REPLENISHMENT ORDER QUANTITIES IN UNCOORDINATED SUPPLY CHAINS

by
Atipol Kanchanapiboon

Many modern supply chains can be described as a series of uncoordinated suppliers. That is each supplier establishes their individual inventory and production policies on both the input and output sides. In these supply links there is minimal coordination between suppliers, and typically only prices and delivery guarantees are contracted. As a consequence, the inventory behavior and associated costs do not exhibit standard patterns. This makes it difficult to model and optimize these chains using classical inventory models. The common approach, therefore, for evaluating uncoordinated supply chains is to use Supply Chain Analytics software. These retrieve operational data from Enterprise Resource Planning (ERP) systems and then characterize the historical inventory performance behavior.

Nearier (2008) developed a joint production inventory model for estimating inventory costs in uncoordinated chains as an alternative to supply chain analytics. They proposed a $(Q, R, \delta)^2$ relationship between each pair of sequential suppliers, where Q is the order quantity, R is the reorder level, and δ is the production or consumption rate. In this arrangement each part has two inventory locations: (i) on the output side of the seller, and (ii) on the input side of the buyer. In this dissertation, the $(Q, R, \delta)^2$ model was extended. Three specific research tasks were accomplished in this regard.

First, the inventory estimation accuracy of the original $(Q, R, \delta)^2$ model was improved. This was accomplished by deriving a more reliable estimate of the residual

inventory at the end of each supply cycle. Further, a more accurate model of the inventory behavior in supply cycles where the seller has no production was developed. A discrete inventory simulation was used to demonstrate a significant improvement in the estimation accuracy, from a 10-30 % error range to within 5% error on average.

Second, a prescriptive model for deriving the optimal Q when reducing inventory costs in a $(Q, R, \delta)^2$ supply relationship was developed. From simulation studies, it was found that due to differences in production batch sizes, production rates, and replenishment order quantities, the inventory cost function exhibits a non-differentiable step-wise convex behavior. Further, the steps are observed to occur at integer ratios of Q and the buyer's production batch. This behavior makes it difficult to analytically derive the optimal Q, which could occur at one of the step points or any intermediate point. A golden section based search heuristic for efficiently deriving the optimal Q was developed.

Third, the robustness of Q to demand shifts was studied. A demand shift occurs wherever the mean demand jumps to a higher or lower level, similar to a moving average forecast. The demand shift range beyond, which there is significant deterioration in inventory costs and a change in the supply policy Q is justified, was determined. Two supply policies were studied: (i) fixed delivery batch and (ii) fixed production period. For each stochastic demand shift behavior, a delivery batch size or production period that minimizes the total cost of both suppliers is selected.

OPTIMIZING REPLENISHMENT ORDER QUANTITIES IN UNCOORDINATED SUPPLY CHAINS

by
Atipol Kanchanapiboon

A Dissertation
Submitted to the Faculty of
New Jersey Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of
Doctor of Philosophy in Industrial Engineering

Department of Mechanical and Industrial Engineering

May 2009

APPROVAL PAGE

OPTIMIZING REPLENISHMENT ORDER QUANTITIES IN UNCOORDINATED SUPPLY CHAINS

Atipol Kanchanapiboon

03/10/09

Dr. Sanchoy Das, Dissertation Advisor — Date
Professor of Mechanical and Industrial Engineering, NJIT

3/10/09

Dr. Reggie J. Caudill, Committee Member — Date
Professor of Mechanical and Industrial Engineering, NJIT

3/10/09

Dr. Athanassios Bladikas, Committee Member — Date
Associate Professor of Mechanical and Industrial Engineering, NJIT

3/10/09

Dr. George Abdou, Committee Member — Date
Associate Professor of Mechanical and Industrial Engineering, NJIT

3/10/09

Dr. Cheickna Sylla, Committee Member — Date
Associate Professor of School of Management, NJIT

BIOGRAPHICAL SKETCH

Author: Atipol Kanchanapiboon

Degree: Doctor of Philosophy

Date: May 2009

Undergraduate and Graduate Education:

- Doctor of Philosophy in Industrial Engineering,
  New Jersey Institute of Technology, Newark, NJ, 2009
- Master of Science in Industrial Engineering,
  New Mexico State University, Las Cruces, NM, 2003
- Bachelor of Engineering in Chemical Engineering,
  Chulalongkorn University, Bangkok, Thailand, 1999

Major: Industrial Engineering

Presentations and Publications:

Kanchanapiboon, A. and Das, S, "Robustness of a Joint Supply Policy between Two Manufacturers under Shifting Demand", Proceedings of the 20th Annual Conference of the Production and Operations Management Society, Orlando, Florida, May 1 - 4, 2009.

Cecil, J., Kanchanapiboon, A. (2007) "Virtual Engineering Approaches in Product and Process Design", International Journal of Advanced Manufacturing Technology, 31(9-10), pp. 846-856.

Nearier, R., Kanchanapiboon, A. and Das, S., "Evaluating the Supply Chain Responsiveness to Changes in Market Demand", Proceedings of the 17th Annual Conference of the Production and Operations Management Society, Boston, Massachusetts, April 28 - May 1, 2006.

Nearier, R., Kanchanapiboon, A., Das, S., "Profit and Cost Distribution Analysis Across the Supply Chain Using the NEXUS Model", Proceedings of 2005 IEEE International Conference on Service Operations and Logistics, and Informatics, Beijing, China, August 10-12, 2005.

Kanchanapiboon, A., Palanivel, K., Rajanyagam V. and Brahmankar, S., "Optimizing Service Processes: The Case of Streamlining Mortgage Loan Portfolio Activities for Financial Institutions", Proceedings of Advanced Simulation Technologies Conference 2003 (ASTC'03), Orlando, Florida, March 30-April 4, 2003.

Cecil, J. A., Kanchanapiboon, A., Kanda, P. and Muthaiyan, A., "A Virtual Prototyping Test Bed for Electronics Assembly", Proceedings of the 27th Annual IEEE International Electronics Manufacturing Technology (IEMT) Symposium, San Jose, July 17-18, 2002, pp.130-136.

Cecil, J. A. and Kanchanapiboon, A., "A Virtual Prototyping Framework for Electronics Manufacturing", Proceedings of the 2002 International Mechanical Engineering Congress, New Orleans, Nov. 17-22, 2002.

To my father, Jumpol, my mother, Atinuch, and my little sister, Benjapa.
For all of your love and support.

ACKNOWLEDGMENT

I pleased to take this opportunity to extend my sincere appreciation to my advisor, Prof. Sanchoy Das for providing me with valuable advice and motivation throughout my course of study. His research skills, patience, and understanding were crucial to my research completion. I would like to thank my committee members: Prof. Reggie J. Caudill, Dr. Athanassios Bladikas and Dr. George Abdou for their great suggestions and guidance, including the feedback on my dissertation writing. I would also like to thank Dr. Cheickna Sylla for his time and helpful advice in my research ideas.

I give my deepest appreciation to Dr. Ronald S. Kane, the Dean of Graduate Studies, and the Department of Mechanical and Industrial Engineering for allocating funds for my Ph.D. program study. Special thanks to the Graduate Student Association at NJIT for providing me travel awards to present my research in academic conferences.

Finally, thanks to my Ph.D. research fellow, Dr. Robert C Nearier, for his encouragement and support in developing and validating models. I am greatly indebted to my parents and sister for naturing and cherishing my academic endeavors. Without their help and support, I would never made it this far.

TABLE OF CONTENTS

TABLE OF CONTENTS
(Continued)

TABLE OF CONTENTS
(Continued)

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES
(Continued)

CHAPTER 1

INTRODUCTION

1.1 Introduction to the Problem

A supply chain can be considered as a network of production and distribution facilities, with multiple point of sale terminations. Supply chains as physical networks are not new, but since the late 1980s, there was a considerable evolution in the practice of supply chain management (SCM). SCM can be described as a total systems approach to efficiently managing the entire flow of information, materials, and services from raw material suppliers through factories and warehouses to the end customer so as to minimize cost and maximize service. In particular SCM uses advanced information technology, to coordinate the activities of (i) procurement, (ii) production, (iii) warehousing, (iv) distribution logistics, and (v) retail sales so as to meet the supply chain objectives and optimize the performance metrics. In this dissertation our focus is exclusively on the materials flow component of SCM.

The two key materials flow related SCM performance metrics are (i) *Inventory Turnover Ratios* – Annual cost of material goods sold divided by average daily inventory value, and (ii) *Customer Service Rate* – Customer orders that are filled within the target time. Clearly, one of the most important functions in SCM is inventory control and the SCM research literature is rich with papers focused on this topic. A key decision of the inventory control function is specifying the inventory quantities at each location (or supplier) in the chain. Inventory is a major cost driver in the performance of the supply chains, and there is considerable research on this subject. From a literature review, it is found that the majority of the existing supply chain inventory research topics assume the supply relationships are coordinated or synchronized, and there are a large number of papers on the subject. Typically, these papers assume production occurs in a lot-for-lot mode or in integer multiples.

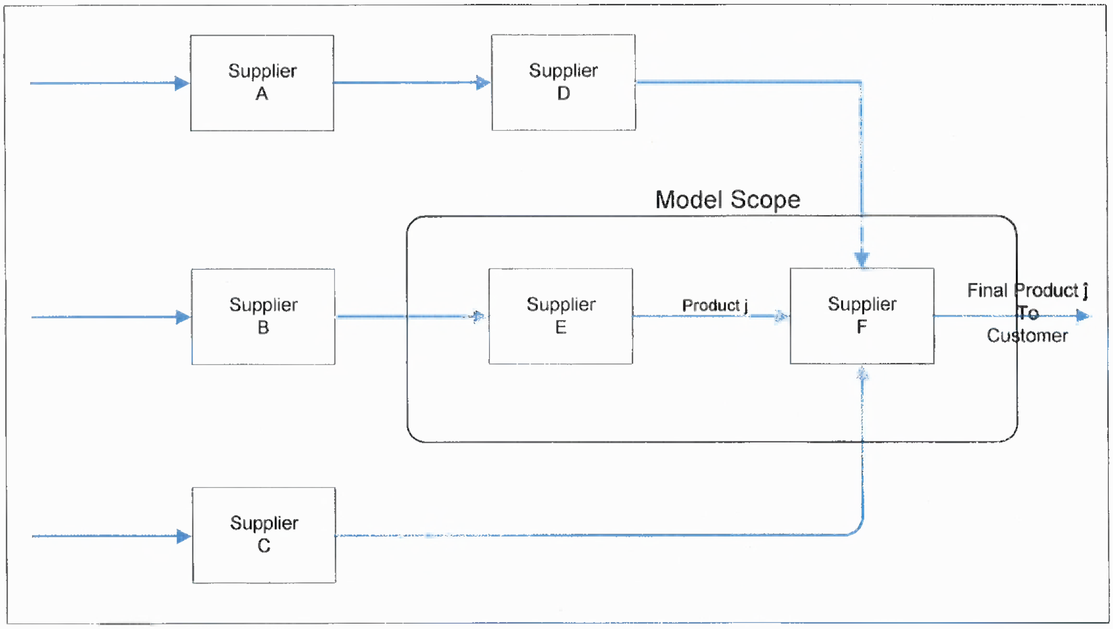


Figure 1.1 Scope of $(Q, R, \delta)^2$ supply inventory model in the supply chain.

Consider Figure 1.1 above, which illustrates a supply chain with six suppliers. Supplier A, B, and C receive materials from outside the supply chain and these are then processed through the network. There are then six manageable inventory relationships in this chain: A→D, D→F, B→E, E→F, C→F, and F→Customer. In a coordinated chain these relationships are centrally managed and shipments occur in a synchronized manner. Figure 1.2 shows the inventory behavior of a fully coordinated supply chain

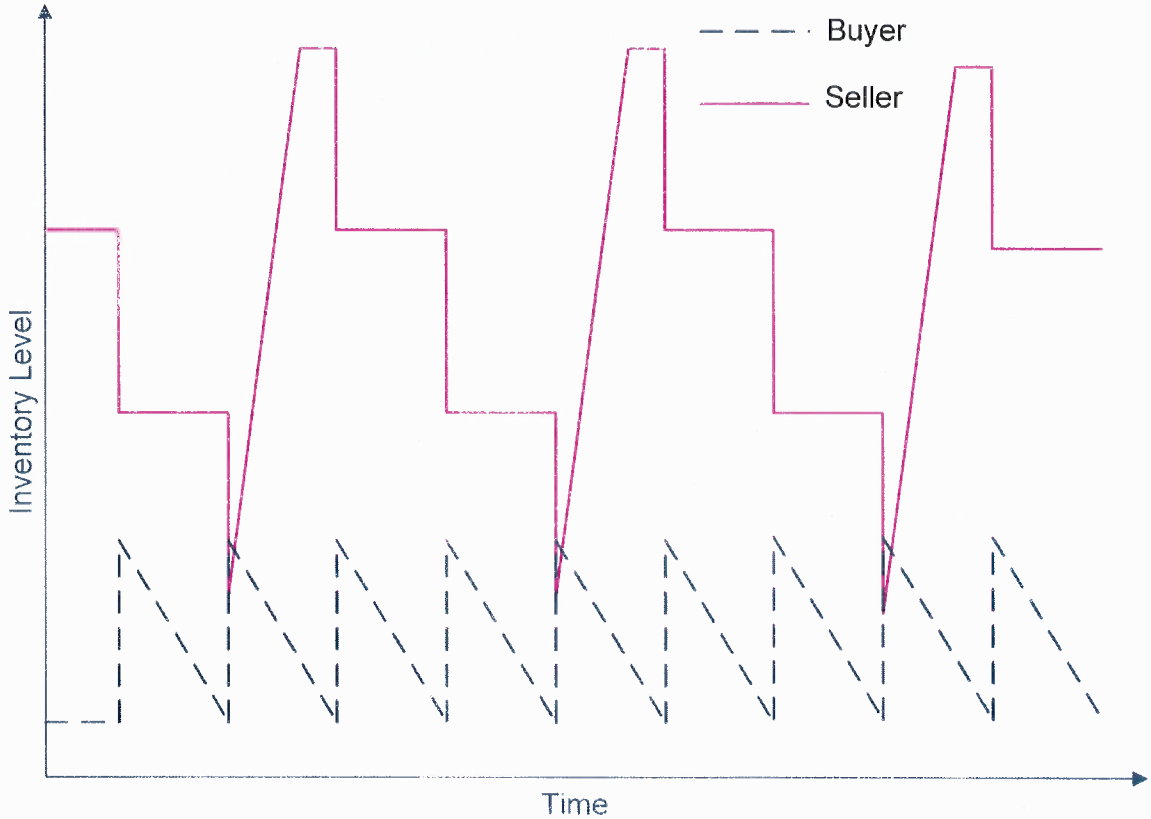


Figure 1.2 The inventory behavior in fully coordinated supply relationship.

However, in many modern supply chains the coordination assumption does not hold. These supply chains can be described as a series of uncoordinated supplier relationships. That is each supplier establishes their individual inventory and production policies on both the input and output sides. In these supply links there is minimal coordination between suppliers, and typically only supply prices and delivery guarantees are contracted. As a consequence, the inventory behavior and associated costs do not exhibit standard patterns. Figure 1.3 illustrates the typical inventory behavior in an uncoordinated supply relationship. The y-axis shows the inventory level of the buyer and seller over the time in x-axis. It is clear that the lack of a clear inventory pattern makes it difficult to estimate the inventory costs and consequently effectively manage the supply policy. This makes it difficult to model and optimize these chains using classical inventory models. The common approach, therefore, for evaluating uncoordinated supply chains is to use Supply Chain Analytics software. These retrieve operational data from ERP systems and then provide inventory performance metrics. Going back to Figure 1.1, the analytics would provide estimates of the inventory range of each of the six inventory relationship pairs.

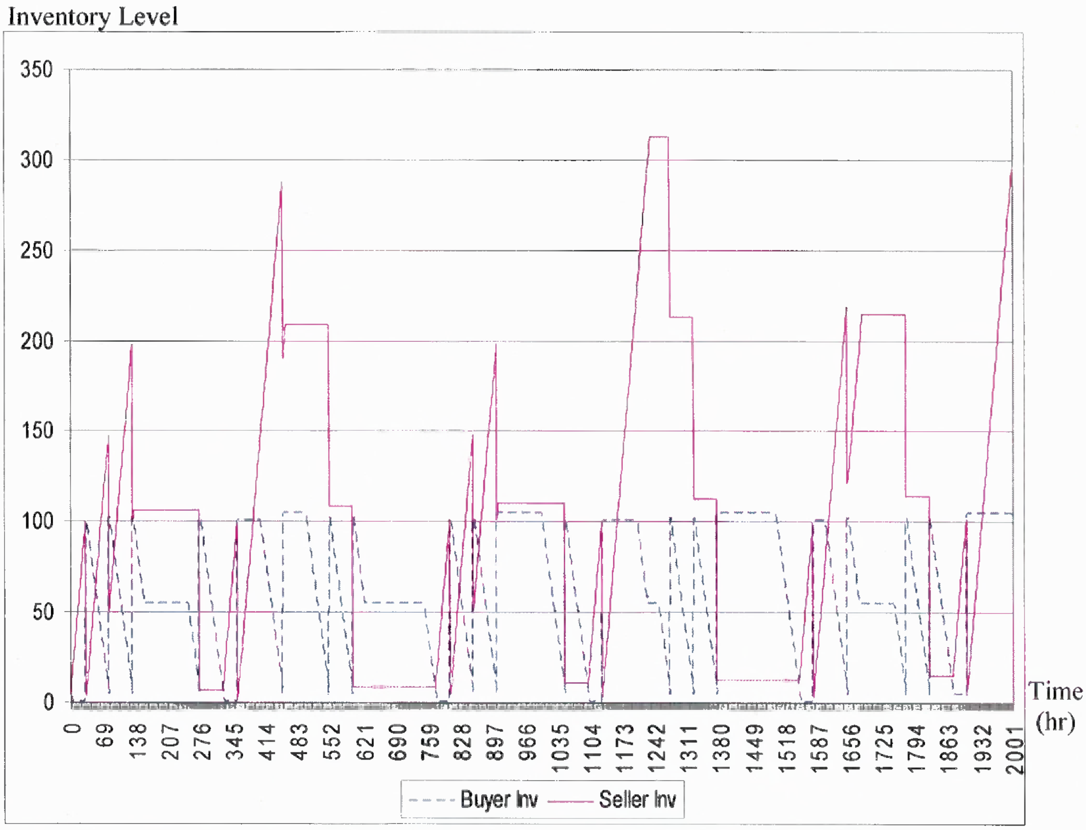


Figure 1.3 The typical inventory behavior in an uncoordinated supply relationship.

Nearier (2008) developed a joint production inventory model for estimating the inventory costs in uncoordinated chains, as an alternative to the use of Supply Chain Analytics software. Nearier proposed a $(Q, R, \delta)^2$ *relationship* between each pair of sequential suppliers, where Q is the supply order quantity, R the reorder level, and δ the production rate. The scope of their model is a single supply pair as shown in Figure 1.1.

But their model is applicable to any pair in the network, and hence can be used to estimate inventory costs across the network.

The $(Q, R, \delta)^2$ model focuses on a pair of two sequential suppliers. The first supplier (seller E) produces a product that is shipped as an input material to the second supplier (buyer F) to be used to make a final product for end customers. In this arrangement, each part has two inventory locations: (i) on the output side of the seller, and (ii) on the input side of the buyer. Nearier (2008) found that since the inventory policies are not coordinated, the inventory behavior is not easily characterized and tends to exhibit long irregular cycles primarily due to the difference in production rates (δ), production batch sizes, and the selection of supply order quantities (Q) for logistics convenience. Nearier's proposed $(Q, R, \delta)^2$ model is an approximation since it approximates the average inventory behavior across a range of supply cycles. He considered several supply relationships by capturing the inventory behavior for each supplier in that case. From several case studies, he concluded that the joint inventory cost for a supply pair tends to be a stepped convex function. In developing their descriptive model, they found that the residual inventory at the end of each supply cycle was a key determinant of the inventory costs.

1.2 Problem Statement

There continues to be a need for models which provide more reliable insights to the inventory behavior of uncoordinated supply chains. In this context, it proposed that by either optimal or sub-optimal coordination of (i) the supply quantity (Q) or (ii) the seller's production batch size (B) considerable inventory cost benefits can be achieved. Note that not a full coordination was proposing, but rather a partial coordination where feasible.

This research extends Nearier's (2008) descriptive model to the prescriptive level. It is assumed that demand is deterministic and constant, but could shift periodically. Since the objective function is known to have a step-wise convex behavior, search based optimization procedures need to be developed. There is also a need to develop the model to determine the supply policy in uncertain demand conditions.

1.3 Research Objective

The overall objective of this research was to extend the $(Q, R, \delta)^2$ inventory model and make it more readily applicable in the management and operation of uncoordinated supply chains. The research was organized into the following three research objectives each of which was accomplished:

Research Objective #1: The inventory estimation accuracy of the original $(Q, R, \delta)^2$ model was improved. This was accomplished by deriving a more reliable estimate of the residual inventory at the end of each supply cycle. Furthermore, a more accurate model of the inventory behavior in supply cycles where the seller has no production was developed. A discrete inventory simulation was used to demonstrate through hypothesis testing a significant improvement in the estimation accuracy, from a 10-30 % error range to within 5% error on average.

Research Objective #2: A prescriptive model for deriving the optimal Q when reducing inventory costs in a $(Q, R, \delta)^2$ supply relationship was developed. From simulation studies, it was found that due to differences in production batch sizes, production rates, and replenishment order quantities, the inventory cost function exhibits a non-differentiable step-wise convex behavior. In addition, the steps were observed to occur at integer ratios of Q and the buyer's production batch. This behavior makes it difficult to analytically derive the optimal Q, which could occur at one of the step points or any intermediate point. A golden Section based search heuristic for efficiently deriving the optimal Q was developed.

Research Objective #3: The robustness of Q to demand shifts was studied. A demand shift occurs wherever the mean demand jumps to a higher or lower level, similar to a moving average forecast. The demand shift range, beyond which there is significant deterioration in inventory costs and a change in the supply policy Q is justified, was determined. Two supply policies were studied: (i) fixed delivery batch and (ii) fixed production period. For each stochastic demand shift behavior, a delivery batch size or production period that minimizes the total cost of both suppliers is selected.

1.4 Research Significance

The improved $(Q, R, \delta)^2$ model provides a more reliable approach for estimating inventory costs in uncoordinated supply relationships. This research develops a prescriptive model for deriving optimal order quantities and production batch sizes in $(Q, R, \delta)^2$ settings. These models will enable supply chain managers to suggest partial coordination strategies which improve the network inventory costs, without significant operational changes at each supplier. Moreover, this prescriptive model will provide suppliers with a decision support tool for negotiating and setting supply parameters so as to minimize their joint inventory costs. This research also quantifies the effect of changing demand conditions on inventory costs given a fixed $(Q, R, \delta)^2$ policy. This, in turn, will let supplier know when to renegotiate or reset the supply parameters and what should be the optimal supply policy in the uncertain demand conditions.

1.5 Organization of Dissertation

In Chapter 2, literature review in the area of joint inventory replenishment policy and supply chain inventory policy are presented. Chapter 3 provides the modification in $(Q, R, \delta)^2$ inventory model for more reliable inventory approximation, including the model validation with inventory simulation. Chapter 4 includes the algorithm to find the optimal order quantity using modified golden Section search. Chapter 5 analyzes the robustness of supply chains in shifting demand conditions and presented the selection of the optimal supply policies. The conclusion of the research and future research suggestion are listed in Chapter 6.

CHAPTER 2

LITERATURE SURVEY

2.1 Jointed Economic Lot Size Model

In supply chain models, one of the classical problems is optimization of the logistic cost in the chain. Traditional models focus on finding the optimal order quantity from the basic EOQ model which considers the inventory and order cost of only the buyer in the chain. This model yields high inventory costs to the vendor. In the real chain, it is unlikely that the vendor will agree with the EOQ model from the buyer. The cost is not optimal for the buyer.

In order to find the order quantity that is optimal to both vendor and buyer in the chain, Baneejee (1986) proposed a joint economic-lot-size model (JELS) for both buyer and vendor. The vendor produced each lot for each buyer order (lot-for-lot basis). The model assumes deterministic conditions of the demand rate. The assumption of lot-for-lot restriction is relaxed in Goyal (1988). It is assumed that the vendor production quantity is an integral multiple of the buyer order quantity. The shipments of equal sizes assume to occur after the production is completed. By relaxing the lot-for-lot assumption, the total cost of both parties is equal to or less than the cost in the original JELS model. Later, Goyal (1995) relaxed the assumption that the shipment size increase from previous shipment based on the production and demand rate. Hill (1997) later added that the successive shipment should increase by a factor between one and the ratio or production and demand rate. The time to consume current shipment is expected to equal the time to produce next shipment. The procedure to modify shipment size in each subsequent delivery is in Goyal (2000).

Considering the procurement policy, Goyal and Deshmukh (1992) determined the optimal lot size of jointed consideration of raw material and finished products. This integrated procurement-production (IPP) system optimization gives a lower cost than separated optimization. Later, Lee (2005) developed IPP model with a single product, one manufacturer and one buyer. Six cost components are included in the objective function: (a) manufacturer's raw material order cost (b) manufacturer's raw material holding cost (c) manufacturer's production setup cost (d) manufacturer's finished product order cost (e) buyer's finished product order cost and (f) buyer's inventory holding cost. The objective is to minimize the average inventory and production costs. The jointed consideration has a lower average inventory and cost than separated optimization of manufacturer and buyer.

More study on JELS is base on different assumption and parameters. Visawanathan and Piplani (2001) proposed an inventory model of one-vendor, multi-buyer supply chain. A common replenishment epochs or time period is defined as a coordinating policy between vendor and buyers. The model yields a higher inventory on the buyer side which can be compensated by price discount from vendor. Zhao (2004) developed an optimal order quantities model for a supplier-retailer supply chain. The transportation and uses of vehicles for delivery are considered. A modified EOQ model is proposed. Braglia and Zavanella (2003) discussed 'Consignment Stock' (CS) in automotive manufacturing domain. An analytical model of CS in single-vendor single-buyer was proposed. The condition to get a success when implementing the CS is also analyzed. Pujawan and Kingsman (2002) derived a model for JELS of one-vendor and one-buyer. There are multiple shipments for each buyer order and the production can be divided in more than one batch. Two replenishment policies are analyzed: (a) the delivery occurs only after the whole production batch is completed and (b) the delivery can occur as soon has there is sufficient inventory for delivery. The model is used for studying the effect of information sharing and synchronization by comparing individual decision and joint decision. It is concluded that the coordination of vendor and buyer yields a better performance for the whole supply chain. Wu and Ouyang (2003) extended the vendor-buyer system to a shortage is allowed case. It is found that the combined total cost is lower than no-shortage case.

The just in time concept and production scheduling are also mention in recent research. Zimmer (2001) analyzed a single-period order and delivery planning model within a just-in-time setting. The model focuses on both the overall performance and the allocation of the cost to each party in the chain. Chan and Kingsman (2007) suggested the coordination of single-vendor multi-buyer supply chain by synchronizing production and delivery cycles. The scheduled delivery days from buyer will be shared with seller to plan the production cycle. The buyers are allow choosing their delivery lot sizes and order cycles. It is proven that this policy is better than independent optimization or restricted the buyer to a predetermined common order cycle. Boute et al. (2007) analyzed two echelon supply chain with single retailer and single manufacturer. The production is make-to-order basis. It is found that smooth order pattern results in shorter and less variable production and replenishment lead times, which compensate the effect of higher retailer safety stock. By including the impact of order decision on lead times, the order pattern can be smoothed without increasing in stock levels. This policy benefits both parties.

In parameter synchronization, Sucky (2003) discussed the most assumption for the JELS which is the supplier has the complete information of buyer cost structure. The assumption is not realistic because buyer does not have incentive to do that. This research provided a bargaining model about buyer cost structure. It is assumed that buyer has power to use EOQ if the negotiation does not success. The algorithm was also implemented as software application. Park (2006) developed a JELS supply chain model with multiple-manufacturer and single-retailer. The demand is deterministic and constant. The order quantity, which is allocated to multiple manufacturers, placed for each period is based on the EOQ policy. The deterministic variables are cycle length, the frequency of shipment, and the production allocation for each manufacture. The problem is a concave minimization problem; therefore, the closed form solution of cycle length and the production allocation in terms of production allocation ratios is proposed. The near-optimal heuristic algorithm is provided.

Information sharing and negotiation between vendor and buyer was considered in some models. Kelle et al. (2003) analyzed two cases of buyer-supplier relationship and negotiation: (a) supplier's dominance with larger production and delivery lot size (b) Buyer's dominance with smaller delivery lot size and more frequent shipment. The analysis assume that the buyer's order is filled by a number of identical delivery lot size while the production lot size is an integer multiple of the shipment size. On the other hand, Sucky (2003) discussed that the most assumption for the JELS which is the supplier has the complete information of buyer cost structure. The assumption is not realistic because buyer does not have incentive to share that information. A bargaining model about buyer cost structure is provided. It is assumed that buyer has power to use

EOQ if the negotiation does not success. The algorithm was also implemented as software application.

Only few multi-echelon JELS are found. Chiu and Huang (2003) proposed a JIT multi-echelon model of suppliers in series. The delivery lead times are random. The member exchanges information in order to make production or replenishment. The model uses a time buffer and emergency borrowing policies in order to deal with the uncertain delivery lead times to avoid stock outs. The problem is found to be a complex mixed nonlinear integer programming problem. Combination of A proposed search methodology and a simulated annealing algorithm is used to find a near-optimal solution.

2.2 Supply Chain Inventory Policy

The efficiency of different inventory policy is studied in Hoberg et al. (2007). The linear control theory is used to compare three inventory policies in the supply chain for the effect to order and inventory variability. With inventory-on-hand-policy, order quantities are determined from the current inventory at that echelon. Pending orders from previous periods are ignored. On the other hand, based stock policy usually used in the chain with low set-up cost. There are two based stock policies: (a) installation stock, which based on local information and (b) echelon-stock policy, which based on system-wide information in the chain. From the analytical model, it is concluded that the inventory-on-hand policy is unstable, while the installation-stock and echelon-stock policies are stable and less fluctuated.

CHAPTER 3

IMPROVED $(Q, R, \delta)^2$ INVENTORY MODEL

The $(Q, R, \delta)^2$ model, originally proposed by Nearier (2008), is a parametric supply chain analytic model that provides an estimate of the inventory costs between a pair of suppliers within a supply chain network. The model considers both the production and replenishment activities of each supplier. The original $(Q, R, \delta)^2$ model was derived from the jointed economic lot size (JELS) model first proposed by Bannerjee (1986).

Referring to Figure 1.1, the $(Q, R, \delta)^2$ model consists of a set of UPMs (Unit Process Model) which quantify the inventory relationship for each supply pair. Since there are six manageable inventory relationships in the Figure 1.1 supply chain (A→D, D→F, B→E, E→F, C→F, and F→Customer), there are then six interrelated UPMs. Each UPM estimates the inventory costs between a buyer and a seller for a single procured product. The UPMs themselves are linked by the demand explosion which cascades through the network. For example, the E→F UPM is driven by demand for the final product, while the B→E UPM is driven by demand for the intermediate product j. But the demand for j is exploded from the final product demand. In this chapter; therefore, an improved $(Q, R, \delta)^2$ inventory estimation was developed.

3.1 Introduction to Original $(Q, R, \delta)^2$ Inventory Model

In this Section, a brief introduction to the assumptions, key parameters, and cost components of the $(Q, R, \delta)^2$ model are presented.

3.1.1 Assumptions

In the formulation of the $(Q, R, \delta)^2$, the following assumptions are made:

- The final product demand is deterministic.
- The consumption rate of product by a downstream buyer is not greater than the production rate of product by an upstream seller.
- The final demand from the market is never greater than the production rate of the last supplier in the supply chain.
- The replenishment from buyer to seller is instantaneous, that is the transport time is considered to be zero.
- The output from the production is available for shipment immediately to the buyer or final customer.
- Stockouts or backorders are not allowed.
- The supply prices and transport cost is fixed regardless of the size of replenishment order quantity. That is there are no quantity discounts.

3.1.2 Key Parameters

The key parameters which describe the buyer-seller supply relationship in an uncoordinated supply chain were introduced below. At the this point only a pair of suppliers was considered, the seller (1) and the buyer (2) as shown in Figure 3.1.

Q Replenishment order quantity shipped from the seller to buyer. Once Q is decided it is fixed for the planning horizon.

R_1 Production reorder level of the seller. That is, when the output side inventory level reaches R_1 then a production order is triggered.

R_2 Replenishment reorder level of the buyer. That is, when the input side inventory level reaches R_2 then a replenishment order is triggered.

δ_1 Output production rate of the seller, units/day

δ_2 Output production rate of the buyer, units/day

Z Bill of materials explosion between buyer and seller. That is, Z units of the seller output are required to make one unit of the buyer output. It is assumed that Z is integer.

B_1 Production batch size of the seller

B_2 Production batch size of the buyer

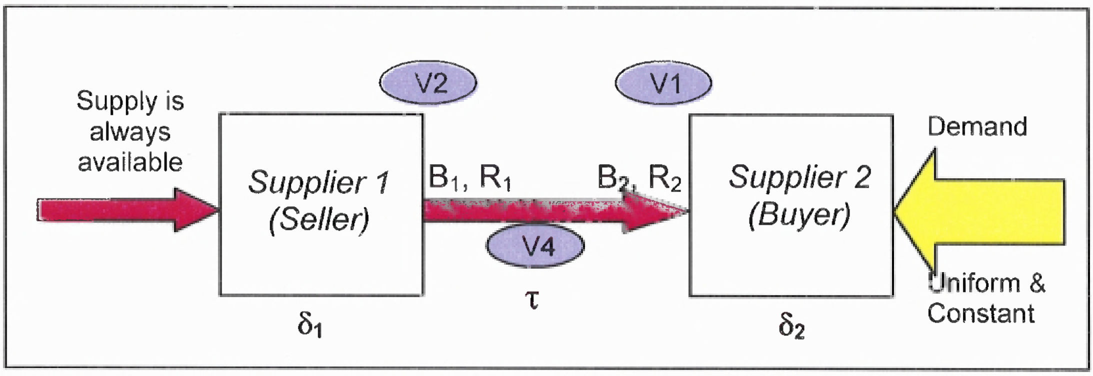


Figure 3.1 $(Q, R, \delta)^2$ inventory model for a buyer and a seller.

According to Figure 3.1, the right supplier (buyer) produces the final product at the rate of δ_2 with the production batch size of B_2. The left supplier (seller) produces product j which is used to make final product with a BOM quantity of Z. Therefore, B_2Z units of product j are required to make one product batch of the final product, and the consumption rate is δ_2Z. When the buyer input-side inventory for product j drops to replenishment reorder level R_2, then the order quantity Q will be shipped from the left supplier (seller). Until the output-side inventory of seller reaches production reorder level

R_l, the production order of NB_l is released (N is the number of production batches needed to satisfy Q from the buyer). The production occurs at the rate of δ_l until the quantity NB_l has completed and the production stops. The cycle repeats again.

3.1.3 Cost components

To develop the inventory cost estimation Nearier (2008) identified the following cost components between a supplier pair, these are denoted in Figure 3.1:

V1 Unit input side inventory cost of the buyer

V2 Unit output side inventory cost of the seller

V3 Unit order cost for the buyer

V4 Unit in-transit inventory cost from seller to buyer

The approximation of inventory pattern proposed by Nearier et al (2005) for buyer and seller is shown in Figure 3.2. The supply cycle is defined as the length of the time that it take for the customer to consume a production batch B_2 of the buyer. The supply cycle time length can be calculated by T:

$$T = B_2/D \tag{3.1}$$

The following notations are introduced:

V Total unit inventory cost in the supply chain

Ch Annual unit inventory cost of item between seller and buyer

One of the key questions in setting up the $(Q, R, \delta)^2$ model was, *what unit should the inventory costs be anchored to ?* The conclusion from our research was that the anchor should be a unit of final product. This allows the model to normalize the differences in cycle times and BOM explosion across the supply chain. The above costs are unitized in this scale. As an example the V2 cost is not per unit of product j, but rather per unit of final product, even though the V2 cost pertains to product j.

A second key question was, for each supply pair *what should be the cycle time over which we study the inventory behavior?* The assumption here is that the behavior repeats every cycle. Now it is know that in uncoordinated supply chains the repetition cycle is very long, and in many instances could be longer than a year. But for modeling purposes it was found that a shorter cycle time can give us a relatively accurate estimate of the inventory behavior. Therefore, for a supply pair the supply cycle was defined as the length of the time that it takes for the customer to consume a production batch B_2 of the buyer, which is the same as the time between two subsequent production starts at the buyer. If D is the annual demand for the final product then the supply cycle time length is denoted by T in equation (3.1) and is given by:

$$T = B_2/D$$

The first step in developing the $(Q, R, \delta)^2$ model was to try and capture the approximate inventory behavior between the supplier pair during the supply cycle. This is shown in Figure 3.2, note that this represents the nominal or average behavior for a specific parametric case. In reality each supply cycle repetition will demonstrate a variation of Figure 3.2 inventory behavior. Furthermore, when the parameters change, then even the nominal behavior will change.

The following graph in Figure 3.2 shows the inventory on the input side of the buyer, while the lower graphs shows that for the outside side of the seller. Each graph is divided into two phases, a production plus replenishment phase, and a no production phase during which the residual inventory level remains unchanged. Next, in Figure 3.2, inventory behavior for buyer and supplier was described in detail. This includes an abbreviated derivation of the $(Q, R, \delta)^2$ inventory estimation model. Note that V1 and V2 cost will initially be derived as a dollar cost per supply cycle. The unit cost is then calculated by taking the supply cycle cost and dividing this with the final product demand per supply cycle.

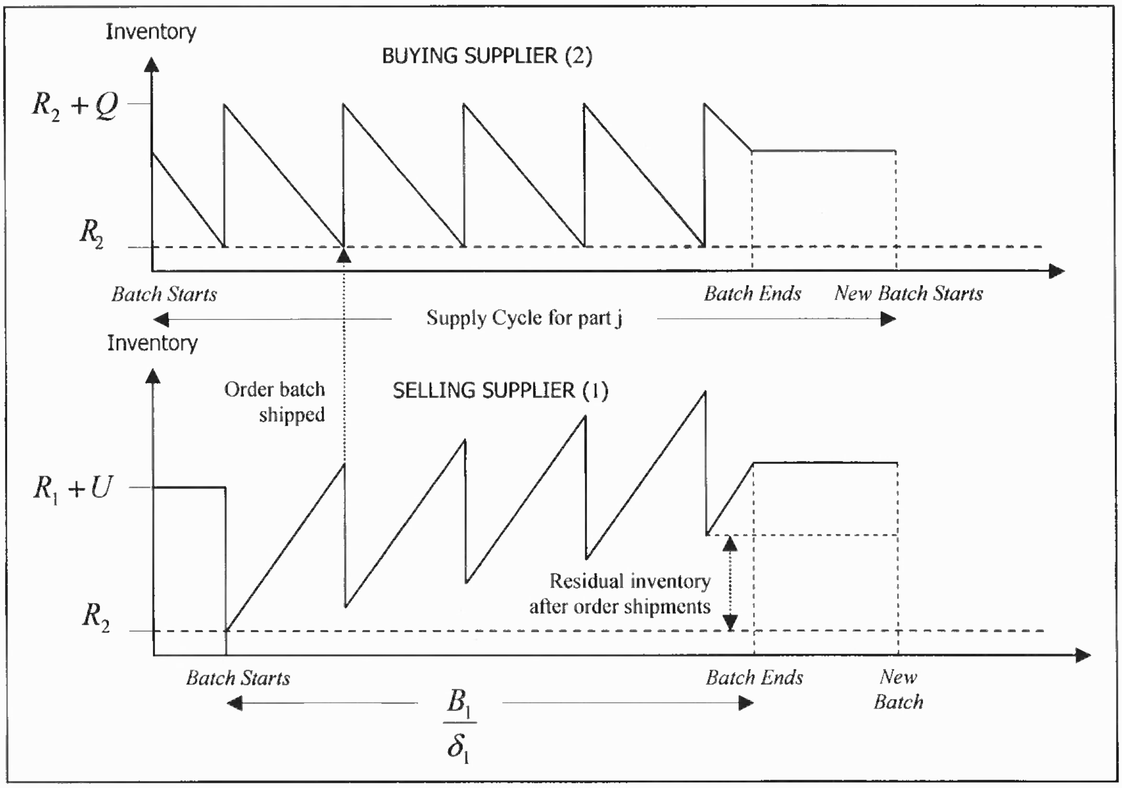


Figure 3.2 Nominal inventory behavior between buyer and seller.

a. Input Side Buyer Inventory Cost

Referring to the upper graph in Figure 3.2, the supply cycle starts when the production of a new batch of the final product is initiated. The inventory behavior is described by the following events:

- Production of a new final product batch starts.

- The input side inventory declines until it reaches the replenishment reorder level R_2.
- The buyer then places an order for Q, and the order is immediately shipped from the seller.
- On receipt of seller shipment the input side inventory jumps to R_2+Q.
- For the nominal case, it is assumed that the order quantity Q is smaller than the buyer consumption batch size ZB_2. The replenishment behavior therefore repeats several times.
- At some point the production batch for the final product is completed and the input side inventory is frozen at the residual level.
- In a coordinated chain the residual level will typically be the same every cycle and in the ideal case will be equal to R_2, in an uncoordinated chain the residual inventory level changes from cycle to cycle.
- The inventory level is frozen till a new production batch for the final product is initiated. Depending on the production rates and demand rates, the frozen cycle could be quite long.

The frequency of replenishments orders in the supply cycle depends on the size of Q. Therefore, the inventory cost of the buyer can be divided into two cases by the size of order quantity. The above was labeled as Case-1, in which the buyer needs to deliver more than one order during each supply cycle. Alternatively, Case-2 is when the order quantity Q is larger than the buyer consumption batch size ZB_2. In this later case, each order replenished to the buyer will be used in more than one supply cycle. The buyer does not need to place an order in every supply cycle. The inventory cost derivations for each case follows.

Case 1- For small Q, the replenishment cycle are divided into two sections, full replenishment cycle and partial replenishment cycle. In order to calculate the inventory cost for input parts, the following integer functions are defined:

$$\text{The number of full replenishment cycle: } X1 = Int^{+}\left\{\frac{ZB_2}{Q}\right\} \tag{3.2}$$

$$\text{The partial replenishment cycle: } Y1 = Int^{-}\left\{\frac{ZB_2}{Q}\right\} \tag{3.3}$$

Function Int^{+} and Int^{-} separate the value into integer and fractional part respectively. Using the definition of the above defined integer functions; the V1 cost can be identified as:

$$V1 = \frac{Ch}{T}\left\{\frac{R_2B_2}{D} + (X1+Y1)\left(\frac{Q}{2}\right)\left(\frac{Q}{\delta_2 Z}\right) + QB_2(1-Y1)\left(\frac{1}{D}-\frac{1}{\delta_2}\right)\right\} \quad \text{when } Q < ZB_2 \tag{3.4}$$

Case 2- For large Q, each replenishment for the buyer consists of more than one supply cycle length. The cost per supply cycle length is the average inventory cost of both cycle with replenishment and cycles without replenishment. Two integer functions are defined:

Number of consecutive supply cycle without replenishment: $X2 = Int^{+}\left\{\frac{Q}{ZB_2}\right\} - 1$ (3.5)

The partial supply cycle: $Y2 = Int^{-}\left\{\frac{Q}{ZB_2}\right\}$ (3.6)

From the above integer functions, The V1 cost can be defined as:

$$V1 = \frac{Ch}{T}\left\{\frac{R_2 B_2}{D}\right\} + \frac{Ch}{T}\left\{\frac{Y2 \cdot B_2^2 Z^2}{2Q}\right\} + \frac{Ch}{T}\left(\frac{B_2^3 Z^2}{2Q}\right)\left\{\frac{1}{D}\left(\frac{X2^2}{D} - \frac{X2}{2} + \frac{1}{2}\right) + \frac{1}{\delta_2}\left(X2 + \frac{5}{4}\right)\right\} \quad \text{when } Q > ZB_2 \tag{3.7}$$

b. Output Side Seller Inventory Cost

Referring to the lower graph in Figure 3.2, the seller supply cycle start coincides with that of the buyer, which is the production of a new batch of the final product at the buyer. The seller inventory behavior is then described by the following events:

- Production of a new final product batch starts at the buyer. The seller inventory level remains frozen.
- A replenishment order for quantity Q is received from the buyer and is immediately shipped, the seller inventory drops by Q.
- When the seller inventory reaches the reorder level R_1, the seller begins production of a new batch of product j.
- The inventory level then increases at the rate δ_1. It is assumed that the seller production rate δ_1 is greater than the buyer consumption rate is $Z\delta_2$.

- At some point the seller receives another replenishment order, which is shipped immediately and the inventory drops again by Q. Since $\delta_l > Z\delta_2$ each subsequent drop point is higher than the previous drop point.
- For the nominal case, it is assumed that the seller will continue producing after the last replenishment order is shipped.
- The seller completes production of the batch and the residual inventory level is frozen for the remainder of the supply cycle.

The frequency of seller production orders in the supply cycle depends on the size of B_l. Therefore, the inventory cost of the buyer can be divided into 2 cases by the size of B_l. The above is labeled as Case-1, in which the seller produces one or more production batches during each supply cycle. Alternatively, in Case-2 B_l is much larger than is $Z\delta_2$, consequently a production supply cycle is followed by one or more supply cycles in which there is no seller production. The following parameters are introduced to describe the seller inventory costs:

N = Number of production batches produced in a supply cycle

$$N = Int^{Next}\left\{\frac{B_2 Z}{B_1}\right\} \tag{3.8}$$

X3 = Number of replenishments shipped occurring during seller production

$$X3 = \min\left[Int^{+}\left\{\left(\frac{NB_1}{\delta_1}\right)\left(\frac{\delta_2 Z}{Q}\right)\right\}, X1\right] \tag{3.9}$$

U = The residual inventory at the beginning of no-activity period

$$U = NB_1 - X1.Q \tag{3.10}$$

X4 = Number of seller production cycle in 10 supply cycle

$$X4 = Int^{+}\left\{\frac{10B_2Z}{NB_1}\right\}+1 \tag{3.11}$$

Int^{Next} Round-up function of the value to the next integer

The cost V2 for a supply cycle can be derived from this formula:

V2 = average inventory cost of supply cycle with and without production

$$V2 = \frac{X4}{10}(cycle\_with\_production)+\left(1-\frac{X4}{10}\right)(cycle)\_no\_production) \tag{3.12}$$

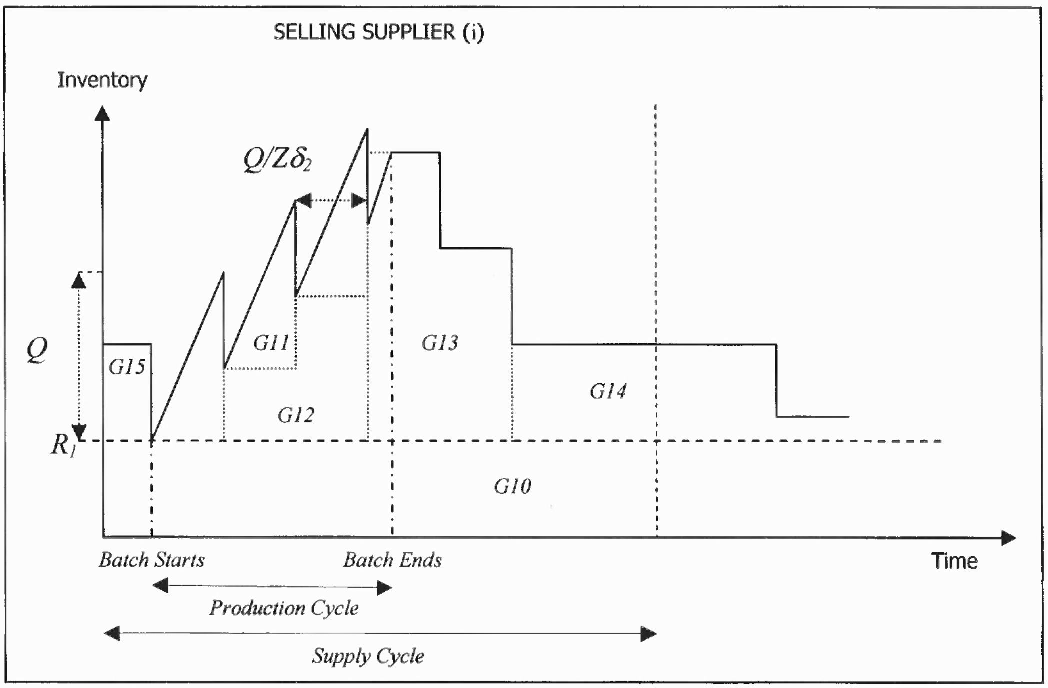


Figure 3.3 Output side seller inventory behavior (case-1).

Case-1: Figure 3.3 shows the detailed inventory behavior for case-1. To derive the total inventory cost during the seller's output inventory cycle, Figure 3.3 was broke up into the following areas: G10- the reorder or safety stock inventory; G11- the full cycle triangular inventory when the production and replenishment cycles overlap; G12- the full cycle step inventory when the production and replenishment cycles overlap; G13 – the partial production cycle step inventory plus the step inventory during the remainder of the replenishment cycle; and G14 – the no activity inventory.

$$V2_{prod} = G10 + G11 + G12 + G13 + G14 + G15 + G16 \tag{3.13}$$

When $$G10 = R_1\left(\frac{B_2}{D}\right) \tag{3.14}$$

$$G11 = X3\left(\frac{\delta_2}{2}\right)\left(\frac{Q}{\delta_2 Z}\right)^2 \tag{3.15}$$

$$G12 = X3(X3-1)\left(\frac{Q^2}{\delta_2 Z}\right)\left(\frac{\delta_1}{\delta_2 Z}-1\right) \tag{3.16}$$

$$G13 = \frac{(X1-X3)(X1-X3+1)}{2}\cdot Q\left(\frac{Q}{\delta_2 Z}\right)-\frac{\delta_1}{2}\left(\frac{NB_1}{\delta_1}-\frac{X3\cdot Q}{\delta_2}\right)^2 \tag{3.17}$$

$$G14 = U\left\{\frac{B_2}{D}-X3\left(\frac{Q}{\delta_2 Z}\right)\right\} \tag{3.18}$$

$$G15 = \frac{Q^2}{2\delta_2 Z} \tag{3.19}$$

$$G16 = (NB_1 - B_2 Z)\left(\frac{10}{X4}-1\right) \tag{3.20}$$

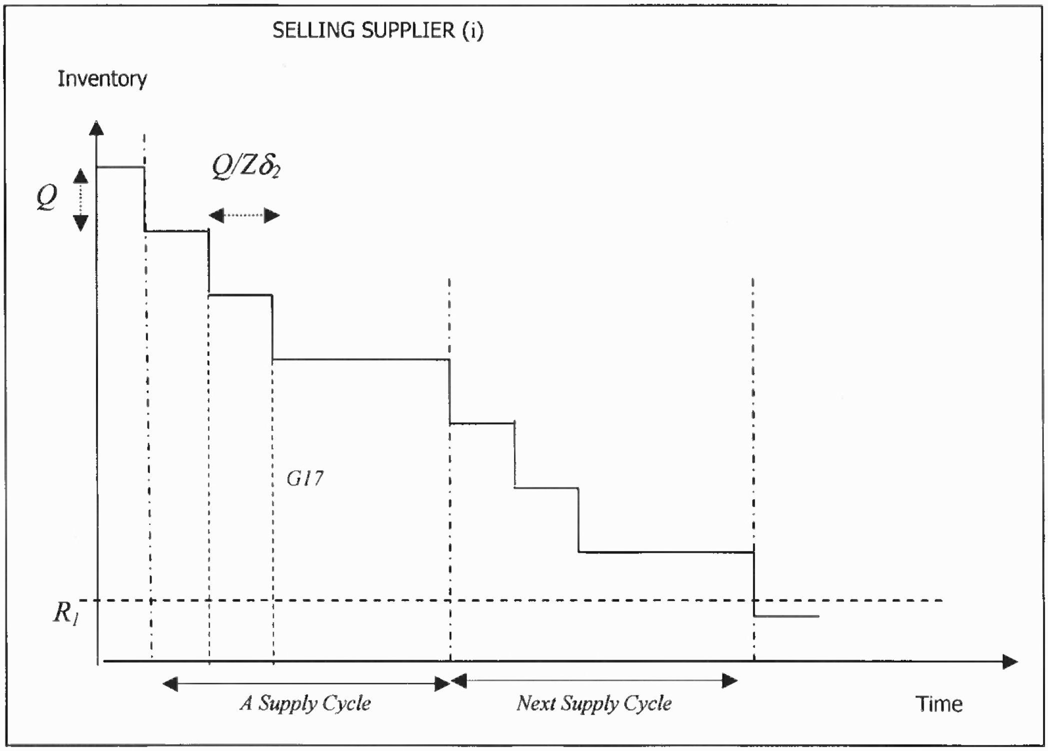


Figure 3.4 Output side seller inventory behavior (case-2).

Case-2: Figure 3.4 shows the details of the inventory behavior when there are supply cycles in which no production occurs. The approach is to consider the no-production cycles separately, which is derived as follows:

$$V2_{\text{no-prod}} = G17 \tag{3.21}$$

$$\text{where, } G17 = X1(X1+1)\left(\frac{Q}{2}\right)\left(\frac{Q}{\delta_2 Z}\right) \tag{3.22}$$

c. Order cost to the buyer

The order cost per supply cycle is rather straight forward and is simply a function of the number of replenishments per supply cycle, and is derived as follows:

$$V3 = Co\left\{\frac{B_2 Z}{Q}\right\} \tag{3.23}$$

3.1.4 Strategies for Improving the Cost Accuracy

The original $(Q, R, \delta)^2$ inventory cost model derived in Section 3.1.3 provided the following V1 and V2 cost approximations:

$$V1 = \frac{Ch}{T}\left\{\frac{R_2 B_2}{D} + (X1 + Y1)\left(\frac{Q}{2}\right)\left(\frac{Q}{\delta_2 Z}\right) + QB_2(1 - Y1)\left(\frac{1}{D} - \frac{1}{\delta_2}\right)\right\} \quad \text{when } Q < ZB_2 \tag{3.24}$$

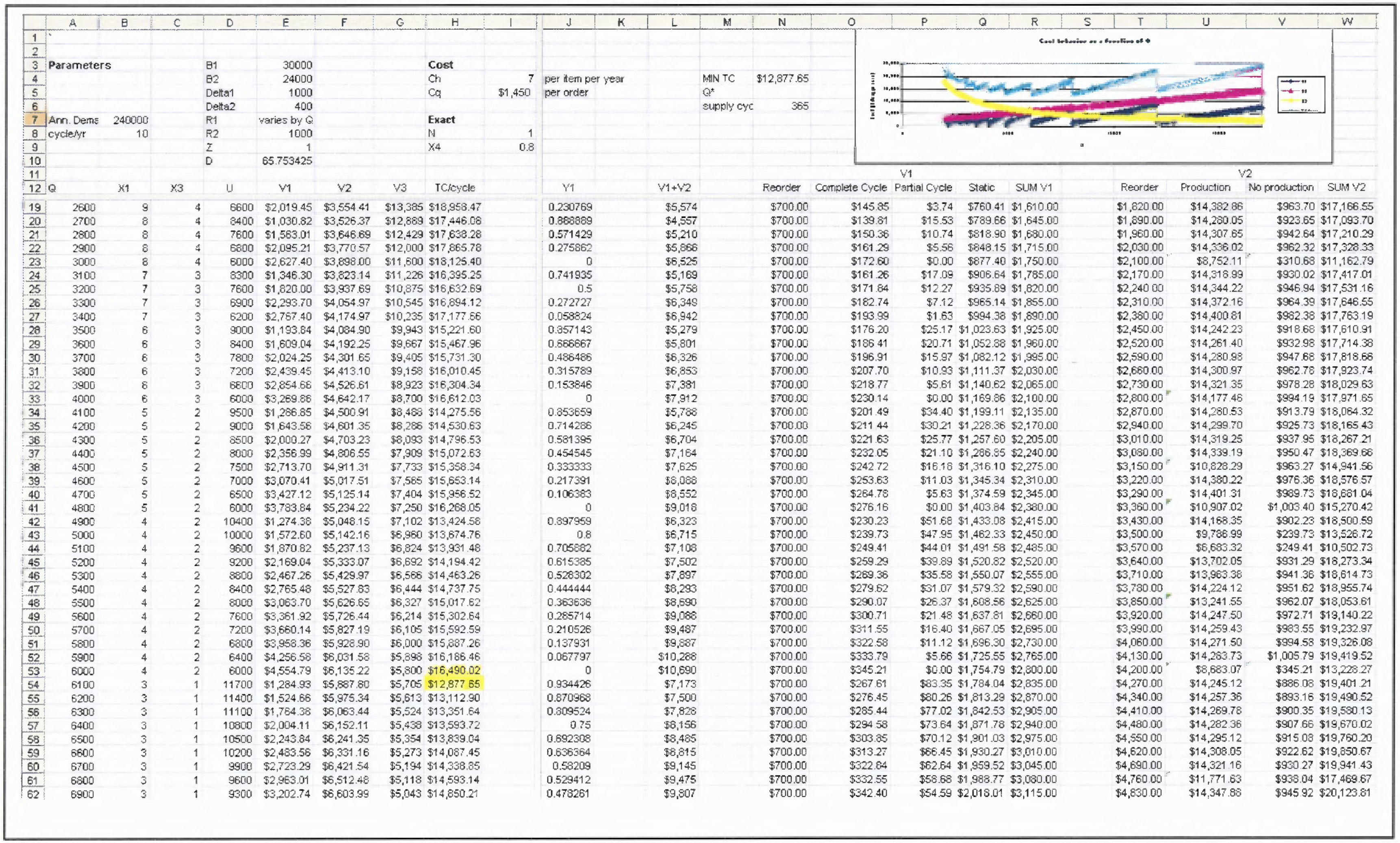
$$\begin{aligned} V1 = {} & \frac{Ch}{T}\left\{\frac{R_2 B_2}{D}\right\} + \frac{Ch}{T}\left\{\frac{Y2 \cdot B_2^2 Z^2}{2Q}\right\} \\ & + \frac{Ch}{T}\left(\frac{B_2^3 Z^2}{2Q}\right)\left\{\frac{1}{D}\left(\frac{X2^2}{D} - \frac{X2}{2} + \frac{1}{2}\right) + \frac{1}{\delta_2}\left(X2 + \frac{5}{4}\right)\right\} \quad \text{when } Q > ZB_2 \end{aligned} \tag{3.25}$$

$$\begin{aligned} V2 = {} & \frac{Ch}{T}\left\{\frac{R_1 B_2}{D} + \frac{Q^2}{\delta_2 Z}\left[0.5X1^2 + 0.5X1\right] + \frac{X4}{10}\left[\frac{Q^2}{\delta_2 Z}\cdot\left(0.5 \cdot X3^2 \frac{\delta_1}{\delta_2 Z} - X1 \cdot X3 + .5\right)\right]\right. \\ & \left. - \frac{(NB_1 - X3 \cdot Q)^2}{2\delta_1} + U\left(\frac{B_2}{D} - [X3 + 0.5]\left(\frac{Q}{\delta_2 Z}\right)\right) + (NB_1 - B_2 Z)\left[\frac{10}{X4} - 1\right]\right\}; when \frac{B_1}{B_2 Z} < 2 \end{aligned} \tag{3.26}$$

Simulation tests showed that this model provided an estimation error in the 15-20% range. The research strategy to improve this accuracy was to study a range of parametric cases and identify where the estimation error was occurring. This was followed by an analytical investigation to identify the cause of the error. Finally the estimation equations were modified to reduce the error rates. The above research strategy was executed in the following steps:

- A discrete inventory simulation was created to evaluate the accuracy error of the original model.
- The simulation model was developed as a spreadsheet in Microsoft Excel. All of the above equations were programmed, and simulator generates not only the V1 and V2 cost components but their sub-components as well.
- The model takes input operating parameters, including B_1, B2, Q, R_1, R_2, δ_1, δ_2 and the cost parameters Ch1, Ch2, Co into the calculation. It was decided to use the time scale of 'hour' because the smallest time shown in the $(Q, R, \delta)^2$ is in hour.
- The model uses "If/Else" function to determine the action of the suppliers in the next hour. The action can be (a) the production starts, (b) the production stops, (c) the order is shipped, and (d) the new order is placed. This action affects the inventory levels and the subsequent actions of the other supplier. The demand that the seller will have to fulfill is lumpy because of the ordering behavior of the buyer.
- Because of the non-repeated behavior of this joint-inventory model, the longest simulation time period as possible was chosen. The simulation is limited to maximum number of rows in Microsoft Excel. Therefore, it was decided to run the simulation for 65,000 hours or about 7.5 years. The average inventory levels on buyer input side (V1) and seller output side (V2) were multiplied by the cost to calculate the inventory cost.

- The supply cycle length in (Q, R, $\delta)^2$ inventory model are varied depends on the parameters used. Therefore, both the cost from the simulation model and the cost from the (Q, R, $\delta)^2$ model were converted to the annual cost for comparison.
- Seven benchmark tests with different levels of coordination in the supply chain contract were developed. This includes fully-coordinated, partial coordinated, and uncoordinated.
- The error size, which is the difference of annual inventory cost between the original inventory estimation model and the simulation model, was calculated. The buyer costs V1 from the estimation model are differentiated into four components: (a) reorder inventory, (b) full replenishment cycle inventory, (c) partial replenishment inventory, and (d) static inventory at the end of supply cycle. The seller costs V2 from the estimation model are differentiated into three components: (a) reorder inventory, (b) production period inventory, and (c) no production period inventory.
- Finally, the variance in the estimation model and simulation model, the inventory component that causes the variation is identified. The additional experiment is performed by increasing in the resolution of the simulation model to study the behavior of the inventory cost behavior. As a result, this original (Q, R, $\delta)^2$ replenishment model is found to be inaccurate while there was no replenishment during the production (shown in Section 3.2) and incorrect estimation of estimation of residual inventory at the end of supply cycle (shown in Section 3.3). From the findings, a revised estimation model was created to improve the accuracy. The new model (shown in Section 3.4) is validated in Section 3.5.



Parameters			Cost			
	B1	30000	Ch	7	per item per year	MIN TC	$12,877.65
	B2	24000	Cq	$1,450	per order	Q*	
	Delta1	1000				supply cyc	365
	Delta2	400	Exact				
Ann. Dema 240000	R1	varies by Q	N	1			
cycle/yr 10	R2	1000	X4	0.8			
	Z	1					
	D	65.753425					

Row	Q	X1	X3	U	V1	V2	V3	TC/cycle	Y1	V1+V2	V1 Reorder	V1 Complete Cycle	V1 Partial Cycle	V1 Static	SUM V1	V2 Reorder	V2 Production	V2 No production	SUM V2
19	2600	9	4	6600	$2,019.45	$3,554.41	$13,385	$18,958.47	0.230769	$5,574	$700.00	$145.85	$3.74	$760.41	$1,610.00	$1,820.00	$14,382.86	$963.70	$17,166.55
20	2700	8	4	8400	$1,030.82	$3,526.37	$12,889	$17,446.08	0.888889	$4,557	$700.00	$139.81	$15.53	$789.66	$1,645.00	$1,890.00	$14,280.05	$923.65	$17,093.70
21	2800	8	4	7600	$1,563.01	$3,646.69	$12,429	$17,638.28	0.571429	$5,210	$700.00	$150.36	$10.74	$818.90	$1,680.00	$1,960.00	$14,307.65	$942.64	$17,210.29
22	2900	8	4	6800	$2,095.21	$3,770.57	$12,000	$17,865.78	0.275862	$5,866	$700.00	$161.29	$5.56	$848.15	$1,715.00	$2,030.00	$14,336.02	$962.32	$17,328.33
23	3000	8	4	6000	$2,627.40	$3,898.00	$11,600	$18,125.40	0	$6,525	$700.00	$172.60	$0.00	$877.40	$1,750.00	$2,100.00	$8,752.11	$310.68	$11,162.79
24	3100	7	3	8300	$1,346.30	$3,823.14	$11,226	$16,395.25	0.741935	$5,169	$700.00	$161.26	$17.09	$906.64	$1,785.00	$2,170.00	$14,316.99	$930.02	$17,417.01
25	3200	7	3	7600	$1,820.00	$3,937.69	$10,875	$16,632.69	0.5	$5,758	$700.00	$171.84	$12.27	$935.89	$1,820.00	$2,240.00	$14,344.22	$946.94	$17,531.16
26	3300	7	3	6900	$2,293.70	$4,054.97	$10,545	$16,894.12	0.272727	$6,349	$700.00	$182.74	$7.12	$965.14	$1,855.00	$2,310.00	$14,372.16	$964.39	$17,646.55
27	3400	7	3	6200	$2,767.40	$4,174.97	$10,235	$17,177.66	0.058824	$6,942	$700.00	$193.99	$1.63	$994.38	$1,890.00	$2,380.00	$14,400.81	$982.38	$17,763.19
28	3500	6	3	9000	$1,193.84	$4,084.90	$9,943	$15,221.60	0.857143	$5,279	$700.00	$176.20	$25.17	$1,023.63	$1,925.00	$2,450.00	$14,242.23	$918.68	$17,610.91
29	3600	6	3	8400	$1,609.04	$4,192.25	$9,667	$15,467.96	0.666667	$5,801	$700.00	$186.41	$20.71	$1,052.88	$1,960.00	$2,520.00	$14,261.40	$932.98	$17,714.38
30	3700	6	3	7800	$2,024.25	$4,301.65	$9,405	$15,731.30	0.486486	$6,326	$700.00	$196.91	$15.97	$1,082.12	$1,995.00	$2,590.00	$14,280.98	$947.68	$17,818.66
31	3800	6	3	7200	$2,439.45	$4,413.10	$9,158	$16,010.45	0.315789	$6,853	$700.00	$207.70	$10.93	$1,111.37	$2,030.00	$2,660.00	$14,300.97	$962.78	$17,923.74
32	3900	6	3	6600	$2,854.66	$4,526.61	$8,923	$16,304.34	0.153846	$7,381	$700.00	$218.77	$5.61	$1,140.62	$2,065.00	$2,730.00	$14,321.35	$978.28	$18,029.63
33	4000	6	3	6000	$3,269.86	$4,642.17	$8,700	$16,612.03	0	$7,912	$700.00	$230.14	$0.00	$1,169.86	$2,100.00	$2,800.00	$14,177.46	$994.19	$17,971.65
34	4100	5	2	9500	$1,286.85	$4,500.91	$8,488	$14,275.56	0.853659	$5,788	$700.00	$201.49	$34.40	$1,199.11	$2,135.00	$2,870.00	$14,280.53	$913.79	$18,064.32
35	4200	5	2	9000	$1,643.56	$4,601.35	$8,286	$14,530.63	0.714286	$6,245	$700.00	$211.44	$30.21	$1,228.36	$2,170.00	$2,940.00	$14,299.70	$925.73	$18,165.43
36	4300	5	2	8500	$2,000.27	$4,703.23	$8,093	$14,796.53	0.581395	$6,704	$700.00	$221.63	$25.77	$1,257.60	$2,205.00	$3,010.00	$14,319.25	$937.95	$18,267.21
37	4400	5	2	8000	$2,356.99	$4,806.55	$7,909	$15,072.63	0.454545	$7,164	$700.00	$232.05	$21.10	$1,286.85	$2,240.00	$3,080.00	$14,339.19	$950.47	$18,369.66
38	4500	5	2	7500	$2,713.70	$4,911.31	$7,733	$15,358.34	0.333333	$7,625	$700.00	$242.72	$16.18	$1,316.10	$2,275.00	$3,150.00	$10,828.29	$963.27	$14,941.56
39	4600	5	2	7000	$3,070.41	$5,017.51	$7,565	$15,653.14	0.217391	$8,088	$700.00	$253.63	$11.03	$1,345.34	$2,310.00	$3,220.00	$14,380.22	$976.36	$18,576.57
40	4700	5	2	6500	$3,427.12	$5,125.14	$7,404	$15,956.52	0.106383	$8,552	$700.00	$264.78	$5.63	$1,374.59	$2,345.00	$3,290.00	$14,401.31	$989.73	$18,681.04
41	4800	5	2	6000	$3,783.84	$5,234.22	$7,250	$16,268.05	0	$9,018	$700.00	$276.16	$0.00	$1,403.84	$2,380.00	$3,360.00	$10,907.02	$1,003.40	$15,270.42
42	4900	4	2	10400	$1,274.38	$5,048.15	$7,102	$13,424.58	0.897959	$6,323	$700.00	$230.23	$51.68	$1,433.08	$2,415.00	$3,430.00	$14,168.35	$902.23	$18,500.59
43	5000	4	2	10000	$1,572.60	$5,142.16	$6,960	$13,674.76	0.8	$6,715	$700.00	$239.73	$47.95	$1,462.33	$2,450.00	$3,500.00	$9,786.99	$239.73	$13,526.72
44	5100	4	2	9600	$1,870.82	$5,237.13	$6,824	$13,931.48	0.705882	$7,108	$700.00	$249.41	$44.01	$1,491.58	$2,485.00	$3,570.00	$6,683.32	$249.41	$10,502.73
45	5200	4	2	9200	$2,169.04	$5,333.07	$6,692	$14,194.42	0.615385	$7,502	$700.00	$259.29	$39.89	$1,520.82	$2,520.00	$3,640.00	$13,702.05	$931.29	$18,273.34
46	5300	4	2	8800	$2,467.26	$5,429.97	$6,566	$14,463.26	0.528302	$7,897	$700.00	$269.36	$35.58	$1,550.07	$2,555.00	$3,710.00	$13,963.38	$941.36	$18,614.73
47	5400	4	2	8400	$2,765.48	$5,527.83	$6,444	$14,737.75	0.444444	$8,293	$700.00	$279.62	$31.07	$1,579.32	$2,590.00	$3,780.00	$14,224.12	$951.62	$18,955.74
48	5500	4	2	8000	$3,063.70	$5,626.65	$6,327	$15,017.62	0.363636	$8,690	$700.00	$290.07	$26.37	$1,608.56	$2,625.00	$3,850.00	$13,241.55	$962.07	$18,053.61
49	5600	4	2	7600	$3,361.92	$5,726.44	$6,214	$15,302.64	0.285714	$9,088	$700.00	$300.71	$21.48	$1,637.81	$2,660.00	$3,920.00	$14,247.50	$972.71	$19,140.22
50	5700	4	2	7200	$3,660.14	$5,827.19	$6,105	$15,592.59	0.210526	$9,487	$700.00	$311.55	$16.40	$1,667.05	$2,695.00	$3,990.00	$14,259.43	$983.55	$19,232.97
51	5800	4	2	6800	$3,958.36	$5,928.90	$6,000	$15,887.26	0.137931	$9,887	$700.00	$322.58	$11.12	$1,696.30	$2,730.00	$4,060.00	$14,271.50	$994.58	$19,326.08
52	5900	4	2	6400	$4,256.58	$6,031.58	$5,898	$16,186.46	0.067797	$10,288	$700.00	$333.79	$5.66	$1,725.55	$2,765.00	$4,130.00	$14,283.73	$1,005.79	$19,419.52
53	6000	4	2	6000	$4,554.79	$6,135.22	$5,800	$16,490.02	0	$10,690	$700.00	$345.21	$0.00	$1,754.79	$2,800.00	$4,200.00	$8,683.07	$345.21	$13,228.27
54	6100	3	1	11700	$1,284.93	$5,887.80	$5,705	$12,877.65	0.934426	$7,173	$700.00	$267.61	$83.35	$1,784.04	$2,835.00	$4,270.00	$14,245.12	$886.08	$19,401.21
55	6200	3	1	11400	$1,524.66	$5,975.34	$5,613	$13,112.90	0.870968	$7,500	$700.00	$276.45	$80.26	$1,813.29	$2,870.00	$4,340.00	$14,257.36	$893.16	$19,490.52
56	6300	3	1	11100	$1,764.38	$6,063.44	$5,524	$13,351.64	0.809524	$7,828	$700.00	$285.44	$77.02	$1,842.53	$2,905.00	$4,410.00	$14,269.78	$900.35	$19,580.13
57	6400	3	1	10800	$2,004.11	$6,152.11	$5,438	$13,593.72	0.75	$8,156	$700.00	$294.58	$73.64	$1,871.78	$2,940.00	$4,480.00	$14,282.36	$907.66	$19,670.02
58	6500	3	1	10500	$2,243.84	$6,241.35	$5,354	$13,839.04	0.692308	$8,485	$700.00	$303.85	$70.12	$1,901.03	$2,975.00	$4,550.00	$14,295.12	$915.08	$19,760.20
59	6600	3	1	10200	$2,483.56	$6,331.16	$5,273	$14,087.45	0.636364	$8,815	$700.00	$313.27	$66.45	$1,930.27	$3,010.00	$4,620.00	$14,308.05	$922.62	$19,850.67
60	6700	3	1	9900	$2,723.29	$6,421.54	$5,194	$14,338.85	0.58209	$9,145	$700.00	$322.84	$62.64	$1,959.52	$3,045.00	$4,690.00	$14,321.16	$930.27	$19,941.43
61	6800	3	1	9600	$2,963.01	$6,512.48	$5,118	$14,593.14	0.529412	$9,475	$700.00	$332.55	$58.68	$1,988.77	$3,080.00	$4,760.00	$11,771.63	$938.04	$17,469.67
62	6900	3	1	9300	$3,202.74	$6,603.99	$5,043	$14,850.21	0.478261	$9,807	$700.00	$342.40	$54.59	$2,018.01	$3,115.00	$4,830.00	$14,347.88	$945.92	$20,123.81

Figure 3.5 The (Q, R, δ)^2 model screenshot.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Parameters												
2	Seller		Other Parameters		Buyer		Final customer						Assumption
3	B1	24000	Q	5100	B2	15000							
4	R1	5100	h	$7	R2	1000							
5	Prod rate	3	b(per event)	$5,000	Cons rate	2	D	1					
6			Order Cost	$1,000									
7													
8	Ini Inv	5100			Ini Inv	1000	Ini Out	0		P(SO)			
9					Last echelon					0.000%			
10		Seller Output Side			Buyer Input Side					Buyer Output Side			
11		Avg Inv	#stockout	P(SO)	Avg Inv	#stockout	P(SO)			Avg Inventory			
12		1861.116	0	0.0000%	4857.884	0	0.0000%	Transaction		619	Stockout Events		
13	Time	Inv	IsProducing	Accum	Input Inv	IsConsuming	Accum	Order placed	Order [illegible]	Output Inv	SellerOutput	BuyerInput	BuyerOutput
14	0	5100	0	0	1000	1	0	1	1	0	0	0	0
15	1	3	1	3	6098	1	2	0	0	1	0	0	0
16	2	6	1	6	6096	1	4	0	0	2	0	0	0
17	3	9	1	9	6094	1	6	0	0	3	0	0	0
18	4	12	1	12	6092	1	8	0	0	4	0	0	0
19	5	15	1	15	6090	1	10	0	0	5	0	0	0
20	6	18	1	18	6088	1	12	0	0	6	0	0	0
21	7	21	1	21	6086	1	14	0	0	7	0	0	0
22	8	24	1	24	6084	1	16	0	0	8	0	0	0
23	9	27	1	27	6082	1	18	0	0	9	0	0	0
24	10	30	1	30	6080	1	20	0	0	10	0	0	0
25	11	33	1	33	6078	1	22	0	0	11	0	0	0
26	12	36	1	36	6076	1	24	0	0	12	0	0	0
27	13	39	1	39	6074	1	26	0	0	13	0	0	0
28	14	42	1	42	6072	1	28	0	0	14	0	0	0
29	15	45	1	45	6070	1	30	0	0	15	0	0	0
30	16	48	1	48	6068	1	32	0	0	16	0	0	0
31	17	51	1	51	6066	1	34	0	0	17	0	0	0
32	18	54	1	54	6064	1	36	0	0	18	0	0	0
33	19	57	1	57	6062	1	38	0	0	19	0	0	0
34	20	60	1	60	6060	1	40	0	0	20	0	0	0
35	21	63	1	63	6058	1	42	0	0	21	0	0	0
36	22	66	1	66	6056	1	44	0	0	22	0	0	0
37	23	69	1	69	6054	1	46	0	0	23	0	0	0
38	24	72	1	72	6052	1	48	0	0	24	0	0	0
39	25	75	1	75	6050	1	50	0	0	25	0	0	0
40	26	78	1	78	6048	1	52	0	0	26	0	0	0
41	27	81	1	81	6046	1	54	0	0	27	0	0	0
42	28	84	1	84	6044	1	56	0	0	28	0	0	0
43	29	87	1	87	6042	1	58	0	0	29	0	0	0
44	30	90	1	90	6040	1	60	0	0	30	0	0	0
45	31	93	1	93	6038	1	62	0	0	31	0	0	0
46	32	96	1	96	6036	1	64	0	0	32	0	0	0
47	33	99	1	99	6034	1	66	0	0	33	0	0	0
48	34	102	1	102	6032	1	68	0	0	34	0	0	0

Sheet2 / Sheet3 /

Figure 3.6 The inventory simulation model.

The (Q, R, δ)^2 model in Figure 3.5 is compared with the inventory simulation model screen shot in Figure 3.6 to identify and improve the model.

In (Q, R, δ)^2 model in Figure 3.5, the upper left Section shows input parameters to the model. The lower left Section shows the V1, V2, and V3 costs at different replenishment batch size. The detail V1 and V2 cost differentiated by the component is in lower right Section. The inventory behavior graph by each cost component and the total cost graph is shown in the upper Section. All cost shown in the Table is the cost per supply cycle. The cost need to be converted to the annual cost by multiplied them by the number of supply cycle per year.

The simulation model in Figure 3.6 takes the input parameters from the top section of the spreadsheet. Each row in the Table shows the inventory snapshot at each time period (hour). The flags is used to determine the action of buyer or seller in the next period. 'IsProducing' flag identifies whether seller is producing an output. 'IsConsumping' flag identifies whether the buyer is consuming input materials shipped from seller. The size of seller production batch and buyer consumption batch are tracked by 'Accum' value in the Table. The 'Order Placed' and 'Order Shipped' flags determined the activities of the buyer order placement process and seller order fulfilling process. The output side inventory level is also showed in the next column of the spreadsheet. The average inventory level at this location at the top of seller and buyer inventory column is used to calculated annual inventory cost. The Microsoft Excel screenshot and formulas are shown in Appendix A.2 and B.2 respectively.

3.2 Seller Inventory in Case of No Replenishment During the Production

The original $(Q, R, \delta)^2$ inventory model assumes that there is at least one replenishment during the production period of supply cycle ($X3 \geq 1$). Consider the X3 function, it is found that there is a case when all replenishment occurs after the production completes. The production has been completed before the first replenishment occurs. The approximation of inventory pattern of the seller output side V2 when there was no replenishment to the buyer during the production ($X3 = 0$) follows a different pattern as shown in Figure 3.7

The cost V2 for a supply cycle for this special case can be derived from equation (3.12):

V2 = average inventory cost of supply cycle with and without production

$$\mathrm{V2_{prod}} = \frac{X4}{10}(cycle\_with\_production) + \left(1 - \frac{X4}{10}\right)(cycle)\_no\_production)$$

The inventory cost for the cycle with production was calculated from

$$\mathrm{V2_{prod}} = \mathrm{G10 + G11 + G12 + G13 + G14} \tag{3.27}$$

In this case, the G10 and G16 remain unchanged while G11 to G15 was revised as:

The G11 represents inventory from when the seller production starts, production completes, the inventory becomes constant, until the first replenishment occurs. G11 is derived as:

$$G11 = \frac{1}{2}\left(\frac{2Q}{\delta_2 Z} - \frac{B_1}{\delta_1}\right) \cdot NB_1 \tag{3.28}$$

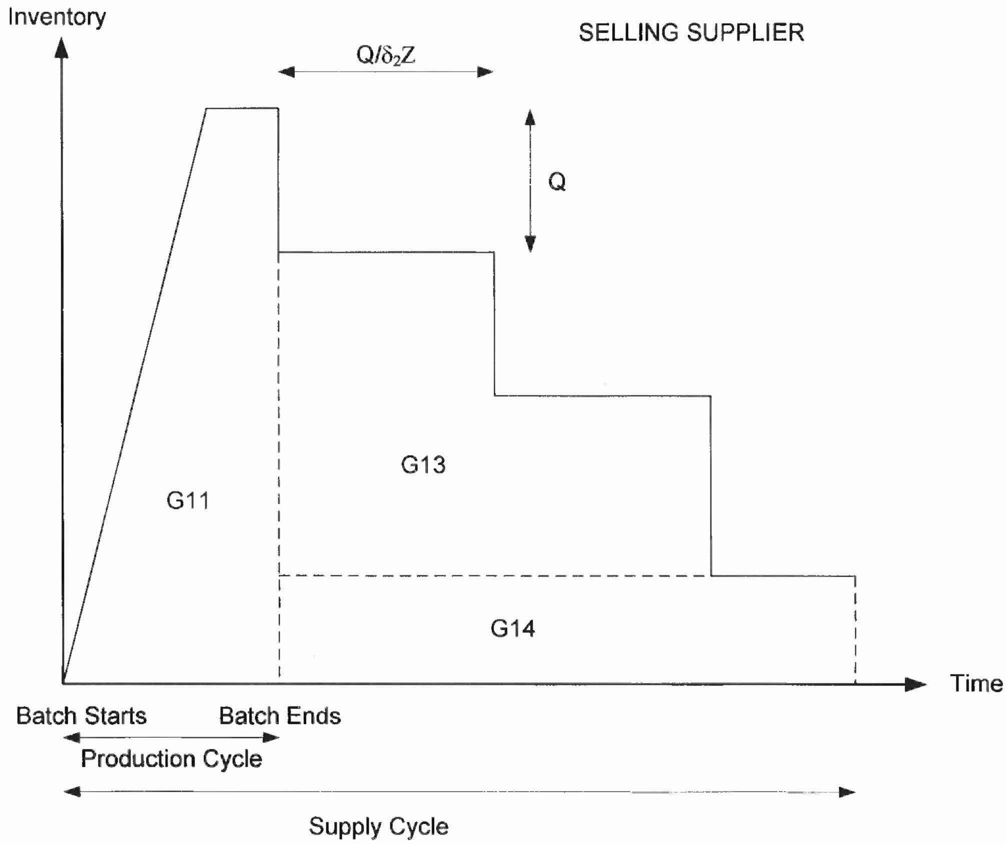


Figure 3.7 The inventory in supply cycle with production of the seller when X3 = 0.

The G12 inventory which shows the step pattern in the original model is not apply to this special case.

$$G12 = 0 \tag{3.29}$$

The G13 inventory shows the inventory in the dynamic part during the first replenishment until the last replenishment. G14 is derived as:

$$G13 = \frac{X1(X1+1)}{2} \cdot Q \cdot \left(\frac{Q}{\delta_2 Z} \right) \tag{3.30}$$

The G14 inventory shows the base inventory (static inventory) from the first to the last replenishment. G14 is derived as:

$$G14 = U\left(\frac{B_2}{D} - \frac{Q}{\delta_2 Z} \right) \tag{3.31}$$

The G15 and G16 is eliminated by the replacement of more accurate residual inventory approximation described in Section 3.3

3.3 Estimation of Supply Cycle Residual Inventory

This Section presents the modification of residual inventory at the end of seller and buyer supply cycle from the original $(Q, R, \delta)^2$ model. This modification enhances the accuracy of the V1 and V2 cost. In Section 3.1, the residual inventory of seller is presented. The residual inventory of buyer is in Section 3.2

3.3.1 Extra Residual at the End of Seller Inventory Cycle

The accumulation of seller's residual inventory from cycle with production results in cycle without production. The estimation of supply cycle residual inventory in the original model (G15 and G16) does not include extra residual inventories carried from preceding cycle to succeeding cycle. This inaccuracy causes the inventory cost to be significantly lower than the actual inventory cost. This can be seen from an example in Table 3.1.

Table 3.1 Example of Residual Inventory in Seller's Supply Cycle

Supply Cycle	1	2	3	4	5	6	7	8	9	10
Production Quantity	1.6	1.6	0	1.6	1.6	0	1.6	0	1.6	1.6
Residual Inventory	0.6	1.2	0.2	0.8	1.4	0.4	1	0	0.6	1.2

Let θ equals to the ratio of total production amount for each seller production cycle to the buyer consumption batch size.

$$\theta = \frac{NB_1}{B_2 Z} \tag{3.32}$$

Note the θ always greater than or equal to 1 to allow the seller to have enough supply for a consumption batch of buyer.

The inventory pattern will repeat itself after a number of cycles. For example in Table 3.1, after eight supply cycles, the inventory in the ninth cycle will be identical to the first period. Therefore, it is possible to estimate the average extra residual inventory for each cycle. This extra residual inventory for cycle with production and cycle without production follows different patterns. The steps to calculate inventory pattern is shown in Section 3.3.1 and 3.3.2.

a) Estimation of residual inventory in cycle with production

From the observation, the number of production cycles from the first production cycle till the production cycle it take before the extra residual inventory repeats the first production cycle again can be calculate from an inverse of greatest common divisor of θ and θ-1. It is also found that after sorting entire extra inventory in ascending order, they follow arithmetic sequence with

d = greatest common divisor of θ and θ-1

$a_1 = \theta$-1

n = 1/d

The average of this sequence is:

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_n = \frac{n(2a_1 + (n-1)d)}{2}$$

$$S_n = \frac{n[2(\theta - 1) + (n-1)d]}{2} \tag{3.33}$$

Therefore, an average of all sequence is

$$\bar{a}_n = \frac{S_n}{n} = 2(\theta - 1) + (n-1)d \tag{3.34}$$

Substitute n = 1/d:

$$\bar{a}_n = \frac{2\theta - 1 - d}{2} \tag{3.35}$$

Therefore, G15 inventory in the original model can be replaced by

$$G15 = \frac{(1-d)}{2}\theta \cdot ZB_2 \text{ when } \theta = \frac{NB_1}{B_2 Z}, \tag{3.36}$$

d = greatest common divisor of θ and θ-1

b) Estimation of residual inventory in cycle without production

It was found that the only feasible case for seller residual inventory is when an average of one seller production quantity (NB_1) is used in greater than one buyer consumption batch size (B_2Z). It was assumed that the extra residual inventory in cycle with no production follows uniform distribution with minimum of zero.

According to equation (3.21), Assume the supply cycle start from period 1, 2, 3, …, (t-1), t, …, d before the next supply cycle repeat the first cycle again when t is the period with the maximum extra residual inventory in cycle with no production. The extra residual inventory are I_1, I_2, I_3, …, I_{t-1}, I_t,…, I_d respectively. It is found that

$$I_{t-1} = \underset{n=1->\infty}{Max}\left[Int^{-}\{n\theta\}\right]$$

And $I_t = I_2 + I_{t-1}$

$$= (0 + \theta - 1 - 1) + \underset{n=1->\infty}{Max}\left[Int^{-}\{n\theta\}\right]$$

$$I_t = (\theta - 2) + Max\left[\underset{n=1\rightarrow\infty}{Int}^{-}\{n\theta\}\right] \quad (3.37)$$

Therefore, G16 inventory in the original model can be replaced by

$$G15 = \frac{I_t}{2}\theta B_2 \text{ when } \theta = \frac{NB_1}{B_2 Z}, \quad (3.38)$$

I_t = The maximum extra residual inventory in no production cycle

3.3.2 Extra Residual at the End of Buyer Inventory Cycle

On the buyer side, every single inventory cycle consists of consumption activity. The original model assumes that the residual inventory after the consumption completes is always the same for every cycle and equals to the first cycle. A more accurate estimation was proposed by analyzing the behavior of the actual residual inventory in each cycle and find the average of them to calculate new buyer input-side inventory cost. An example of buyer's inventory behavior can be seen in Table 3.2.

Table 3.2 Example of Residual Inventory in Buyer's Supply Cycle

Supply Cycle	1	2	3	4	5	6	7	8	9	10
# of Input Order	4	4	4	4	3	4	4	4	4	3
Residual Inventory	0.2	0.4	0.6	0.8	0.0	0.2	0.4	0.6	0.8	0.0

In this Table, let the consumption amount for each order to be 3.8 (X1=3 and Y1 = 0.8). From the observation, the number of consumption cycles repeat in five cycles. The residual inventory is independent from X1. Similar to the case for seller, after sorting entire extra inventory in ascending order, they follow an arithmetic sequence with

d = greatest common divisor of Y1 and 1-Y1

$a_1 = 0$, $n = 1/d$

According to $\overline{a}_n = \frac{(1-d)}{2}$ in Section 3.1, the average of this sequence is

$$\overline{a}_n = \frac{(1-d)}{2}$$

Therefore, the G4 inventory in the original model can be replaced by

$$G4 = \frac{(1-d)}{2} Q \cdot (B_2)\left(\frac{1}{D} - \frac{1}{\delta_2}\right) \tag{3.39}$$

When d = greatest common divisor of Y1 and 1-Y1

3.4 Revised (Q, R, $\delta)^2$ Inventory Model

From the revised (Q, R, $\delta)^2$ model, the cost function for each cost component was as follows:

3.4.1 Buyer Input Side Inventory Cost

- When production batch size is smaller than order quantity ($Q < ZB_2$)

$$V1 = \frac{Ch}{T}\left\{\frac{R_2 B_2}{D} + (X1+Y1)\left(\frac{Q}{2}\right)\left(\frac{Q}{\delta_2 Z}\right) + QB_2(1-Y1)\left(\frac{1}{D} - \frac{1}{\delta_2}\right)\right\} \tag{3.40}$$

- When production batch size is larger than order quantity ($Q > ZB_2$)

$$V1 = \frac{Ch}{T}\left\{\frac{R_2 B_2}{D}\right\} + \frac{Ch}{T}\left\{\frac{Y2 \cdot B_2^2 Z^2}{2Q}\right\} + \frac{Ch}{T}\left(\frac{B_2^3 Z^2}{2Q}\right)\left\{\frac{1}{D}\left(\frac{X2^2}{D} - \frac{X2}{2} + \frac{1}{2}\right) + \frac{1}{\delta_2}\left(X2 + \frac{5}{4}\right)\right\} \tag{3.41}$$

3.4.2 Seller Output Side Inventory Cost

$$V2 = \frac{Ch}{T}\left\{\frac{R_1 B_2}{D} + \frac{Q^2}{\delta_2 Z}\left[0.5X1^2 + 0.5X1\right] + \frac{X4}{10}\left[\frac{Q^2}{\delta_2 Z} \cdot \left(0.5 \cdot X3^2 \frac{\delta_1}{\delta_2 Z} - X1 \cdot X3 + .5\right)\right]\right. \tag{3.42}$$

$$\left. - \frac{(NB_1 - X3 \cdot Q)^2}{2\delta_1} + U\left(\frac{B_2}{D} - [X3 + 0.5]\left(\frac{Q}{\delta_2 Z}\right)\right) + (NB_1 - B_2 Z)\left[\frac{10}{X4} - 1\right]\right\}; when \frac{B_1}{B_2 Z} < 2$$

3.4.3 Order Cost

$$V3 = Co\left\{\frac{B_2 Z}{Q}\right\} \tag{3.43}$$

3.5 Model Validation

In order to confirm the improved accuracy of the revised $(Q, R, \delta)^2$ inventory model in Section 3.4, there is a need to develop the validation procedure that will compare the accuracy of the original model by Nearier (2008) to this revised model.

3.5.1 Experimental Details

The simulation model in Section 3.1.4 is used for testing the accuracy of the model. It was decided to study the accuracy of revised $(Q, R, \delta)^2$ model from 50 experiments in Table 3.3 and 3.4.

Table 3.3 Experimental Parameters (scenario #1 - #30)

	Parameters					
Exp#	B1	B2	Delta1	Delta2	D (/yr)	Q
1	24000	15000	1000	400	240000	500
2	24000	15000	1000	400	240000	1000
3	24000	15000	1000	400	240000	2000
4	24000	15000	1000	400	240000	3000
5	24000	15000	1000	400	240000	4000
6	24000	15000	1000	400	240000	4500
7	24000	15000	1000	400	240000	4800
8	24000	15000	1000	400	240000	5000
9	24000	15000	1000	400	240000	5100
10	24000	15000	1000	400	240000	5200
11	24000	15000	1000	400	240000	5250
12	24000	15000	1000	400	240000	5500
13	24000	15000	1000	400	240000	6000
14	24000	15000	1000	400	240000	6800
15	24000	15000	1000	400	240000	7000
16	24000	15000	1000	400	240000	8000
17	24000	15000	1000	400	240000	9000
18	24000	15000	1000	400	240000	10000
19	24000	15000	1000	400	240000	12500
20	24000	15000	1000	400	240000	15000
21	24000	15000	1000	400	50000	5100
22	24000	15000	1000	400	100000	5100
23	24000	15000	1000	400	120000	5100
24	24000	15000	1000	400	180000	5100
25	24000	15000	1000	400	360000	5100
26	24000	15000	1000	400	480000	5100
27	24000	15000	1000	400	600000	5100
28	24000	15000	1000	400	800000	5100
29	24000	15000	1000	400	1000000	5100
30	24000	15000	1000	400	1200000	5100

Table 3.4 Experimental Parameters (scenario #31 - #50)

	Parameters					
Exp#	B1	B2	Delta1	Delta2	D (/yr)	Q
31	30000	15000	1000	400	240000	5100
32	20000	15000	1000	400	240000	5100
33	25000	15000	1000	400	240000	5100
34	15000	15000	1000	400	240000	5100
35	50000	24000	1000	400	240000	5100
36	30000	24000	1000	400	240000	5100
37	15000	15000	1000	400	240000	3000
38	15000	15000	1000	400	240000	5000
39	15000	15000	1000	400	240000	6000
40	15000	15000	1000	400	240000	10000
41	30000	24000	1000	400	240000	3000
42	30000	24000	1000	400	240000	5000
43	30000	24000	1000	400	240000	6000
44	30000	24000	1000	400	240000	10000
45	15000	15000	1000	400	120000	5100
46	15000	15000	1000	400	480000	5100
47	15000	15000	1000	400	1000000	5100
48	30000	24000	1000	400	120000	5100
49	30000	24000	1000	400	480000	5100
50	30000	24000	1000	400	1000000	5100

The base case is in experiment #9 with the following parameters:

- Holding cost of the product (Ch) = \$7 per year across the chain
- Seller production batch size (B1) = 24000
- Buyer production batch size (B2) = 15000
- Seller production rate ($\delta 1$) = 1000 per hour
- Buyer production rate ($\delta 2$) = 400 per hour

- Seller production reorder level (R1) = 3000
- Buyer production reorder level (R2) = Q (to prevent the stockout)
- Order Quantity (Q) = 5100
- Annual Consumer demand rate for final product (D) = 240,000

At least one of the parameters from the base case was adjusted to study the accuracy of the model. In experiment #1 to #20, the order quantity (Q) is varied from 500 to 15000. The annual demand from the customer (D) is varied from 50,000 to 1,200,000 in experiment #21 to #30. The ratio of seller to buyer batch size is varied from 2.083 to 1 in experiment #31 to #36. Experiment #37 to #40 studies the effect of varying to varying the order quantity when the batches are synchronized. Experiment #41 to #50 varied 2 parameters at a time.

The experiment calculated V1 and V2 annual costs from the simulation and the estimated the revised $(Q, R, \delta)^2$ inventory model.

3.5.2 Experimental Results

The experimental result from 50 experiments in Section 3.5.1 is shown in Table 3.5 and 3.6. The error is measured as the percentage of difference in an annual cost from the $(Q, R, \delta)^2$ model and the annual cost from simulation. The size of V1 error is smaller than the size of V2 error.

Table 3.5 Experimental Results (Experiment #1 - #25)

	Buyer's Annual Cost							Seller's Annual Cost					
Exp#	V1S	#1	#2	#3	#4	V1A	% Err.	V2S	#1	#2	#3	V2A	% Err.
1	$8,633	$7,000	$288	$0	$1,462	$8,750	1.4%	$81,881	$3,500	$65,919	$11,219	$80,638	-1.5%
2	$10,741	$7,000	$575	$0	$2,925	$10,500	-2.2%	$83,273	$7,000	$65,601	$11,327	$83,928	0.8%
3	$14,935	$7,000	$1,074	$77	$5,849	$14,000	-6.3%	$86,079	$14,000	$68,756	$11,097	$93,853	9.0%
4	$17,760	$7,000	$1,726	$0	$8,774	$17,500	-1.5%	$90,254	$21,000	$65,025	$11,759	$97,784	8.3%
5	$23,056	$7,000	$1,841	$460	$11,699	$21,000	-8.9%	$91,958	$28,000	$67,606	$10,637	$106,242	15.5%
6	$22,671	$7,000	$2,330	$259	$13,161	$22,750	0.3%	$104,349	$31,500	$52,224	$11,370	$95,094	-8.9%
7	$25,908	$7,000	$2,651	$110	$14,038	$23,800	-8.1%	$94,706	$33,600	$52,574	$11,852	$98,025	3.5%
8	$24,970	$7,000	$2,877	$0	$14,623	$24,500	-1.9%	$110,207	$35,000	$64,995	$12,190	$112,185	1.8%
9	$24,731	$7,000	$1,995	$939	$14,916	$24,850	0.5%	$113,207	$35,700	$67,610	$10,120	$113,430	0.2%
10	$25,989	$7,000	$2,074	$917	$15,208	$25,200	-3.0%	$112,633	$36,400	$65,185	$10,209	$111,794	-0.7%
11	$24,976	$7,000	$2,114	$906	$15,354	$25,375	1.6%	$110,610	$36,750	$65,830	$10,254	$112,834	2.0%
12	$26,147	$7,000	$2,321	$844	$16,086	$26,250	0.4%	$115,515	$38,500	$62,880	$10,486	$111,865	-3.2%
13	$28,932	$7,000	$2,762	$690	$17,548	$28,000	-3.2%	$100,082	$42,000	$50,906	$10,982	$103,888	3.8%
14	$31,704	$7,000	$3,547	$365	$19,888	$30,800	-2.9%	$124,985	$47,600	$56,354	$11,866	$115,820	-7.3%
15	$31,805	$7,000	$3,759	$268	$20,473	$31,500	-1.0%	$123,513	$49,000	$51,645	$12,104	$112,749	-8.7%
16	$36,971	$7,000	$2,455	$2,148	$23,397	$35,000	-5.3%	$106,043	$56,000	$61,563	$9,716	$127,279	20.0%
17	$38,752	$7,000	$3,107	$2,071	$26,322	$38,500	-0.7%	$131,067	$63,000	$39,452	$10,205	$112,658	-14.0%
18	$43,110	$7,000	$3,836	$1,918	$29,247	$42,000	-2.6%	$141,420	$70,000	$44,116	$10,752	$124,868	-11.7%
19	$50,601	$7,000	$5,993	$1,199	$36,558	$50,750	0.3%	$164,587	$87,500	$26,030	$12,370	$125,899	-23.5%
20	$61,623	$7,000	$8,630	$0	$43,870	$59,500	-3.4%	$173,067	$105,000	$70,889	$14,348	$190,237	9.9%
21	$22,996	$7,000	$416	$196	$17,239	$24,850	8.1%	$119,015	$35,700	$70,195	$8,343	$114,237	-4.0%
22	$23,586	$7,000	$831	$391	$16,627	$24,850	5.4%	$115,532	$35,700	$69,515	$8,810	$114,025	-1.3%
23	$23,905	$7,000	$998	$469	$16,383	$24,850	4.0%	$114,404	$35,700	$69,242	$8,997	$113,940	-0.4%
24	$24,492	$7,000	$1,496	$704	$15,649	$24,850	1.5%	$114,608	$35,700	$68,426	$9,559	$113,685	-0.8%
25	$24,169	$7,000	$2,993	$1,408	$13,449	$24,850	2.8%	$110,474	$35,700	$65,977	$11,242	$112,919	2.2%

Table 3.6 Experimental Results (Experiment #26 - #30)

	Buyer's Annual Cost							Seller's Annual Cost					
Exp#	V1S	#1	#2	#3	#4	V1A	% Err.	V2S	#1	#2	#3	V2A	% Err.
26	$24,212	$7,000	$3,991	$1,878	$11,982	$24,850	2.6%	$107,227	$35,700	$64,345	$12,364	$112,409	4.8%
27	$23,906	$7,000	$4,988	$2,347	$10,514	$24,850	3.9%	$104,373	$35,700	$62,712	$13,487	$111,899	7.2%
28	$23,566	$7,000	$6,651	$3,130	$8,069	$24,850	5.4%	$96,690	$35,700	$59,991	$15,357	$111,049	14.9%
29	$23,272	$7,000	$8,314	$3,912	$5,624	$24,850	6.8%	$94,712	$35,700	$57,270	$17,228	$110,198	16.4%
30	$22,870	$7,000	$9,976	$4,695	$3,179	$24,850	8.7%	$89,961	$35,700	$54,549	$19,098	$109,348	21.6%
31	$24,066	$7,000	$1,995	$939	$14,916	$24,850	3.3%	$130,950	$35,700	$72,850	$2,993	$111,543	-14.8%
32	$24,066	$7,000	$1,995	$939	$14,916	$24,850	3.3%	$100,228	$35,700	$62,868	$1,496	$100,064	-0.2%
33	$24,066	$7,000	$1,995	$939	$14,916	$24,850	3.3%	$116,544	$35,700	$69,325	$9,396	$114,421	-1.8%
34	$24,066	$7,000	$1,995	$939	$14,916	$24,850	3.3%	$82,230	$35,700	$50,561	$0	$86,261	4.9%
35	$25,228	$7,000	$2,494	$440	$14,916	$24,850	-1.5%	$199,189	$35,700	$118,339	$41,429	$195,467	-1.9%
36	$25,228	$7,000	$2,494	$440	$14,916	$24,850	-1.5%	$134,750	$35,700	$97,523	$2,494	$135,717	0.7%
37	$17,760	$7,000	$1,726	$0	$8,774	$17,500	-1.5%	$61,155	$21,000	$6,904	$0	$27,904	-54.4%
38	$24,970	$7,000	$2,877	$0	$14,623	$24,500	-1.9%	$67,944	$35,000	$6,616	$0	$41,616	-38.7%
39	$28,932	$7,000	$2,762	$690	$17,548	$28,000	-3.2%	$80,287	$42,000	$43,454	$0	$85,454	6.4%
40	$43,110	$7,000	$3,836	$1,918	$29,247	$42,000	-2.6%	$101,185	$70,000	$19,969	$0	$89,969	-11.1%
41	$25,918	$7,000	$1,726	$0	$8,774	$17,500	-32.5%	$116,737	$21,000	$87,521	$3,107	$111,628	-4.4%
42	$27,000	$7,000	$2,494	$440	$14,916	$24,850	-8.0%	$129,655	$35,000	$97,870	$2,397	$135,267	4.3%
43	$45,298	$7,000	$3,452	$0	$17,548	$28,000	-38.2%	$118,356	$42,000	$86,831	$3,452	$132,283	11.8%
44	$47,504	$7,000	$4,795	$959	$29,247	$42,000	-11.6%	$144,150	$70,000	$30,658	$2,877	$103,535	-28.2%
45	$23,905	$7,000	$998	$469	$16,383	$24,850	4.0%	$83,654	$35,700	$50,449	$0	$86,149	3.0%
46	$24,212	$7,000	$3,991	$1,878	$11,982	$24,850	2.6%	$79,909	$35,700	$50,597	$0	$86,297	8.0%
47	$23,272	$7,000	$8,314	$3,912	$5,624	$24,850	6.8%	$71,868	$35,700	$50,810	$0	$86,510	20.4%
48	$25,538	$7,000	$1,247	$220	$16,383	$24,850	-2.7%	$138,405	$35,700	$97,480	$1,247	$134,427	-2.9%
49	$24,740	$7,000	$4,988	$880	$11,982	$24,850	0.4%	$127,239	$35,700	$97,599	$4,988	$138,287	8.7%
50	$23,496	$7,000	$10,392	$1,834	$5,624	$24,850	5.8%	$110,928	$35,700	$97,771	$10,392	$143,863	29.7%

From Table 3.7, the average size of V1 error is 3.5% while the average size of V2 error is generally within 8%.

Table 3.7 V1 Error of the (Q, R, δ)^2 Model from the Experiments

Exp_no	Simulation	Model	Difference (%)
1	$8,633	$8,750	1.36
2	$10,741	$10,500	2.24
3	$14,935	$14,000	6.26
4	$17,760	$17,500	1.46
5	$23,056	$21,000	8.92
6	$22,671	$22,750	0.35
7	$25,908	$23,800	8.14
8	$24,970	$24,500	1.88
9	$24,731	$24,850	0.48
10	$25,989	$25,200	3.04
11	$24,976	$25,375	1.60
12	$26,147	$26,250	0.39
13	$28,932	$28,000	3.22
14	$31,704	$30,800	2.85
15	$31,805	$31,500	0.96
16	$36,971	$35,000	5.33
17	$38,752	$38,500	0.65
18	$43,110	$42,000	2.57
19	$50,601	$50,750	0.30
20	$61,623	$59,500	3.45
21	$22,996	$24,850	8.06
22	$23,586	$24,850	5.36
23	$23,905	$24,850	3.95
24	$24,492	$24,850	1.46
25	$24,169	$24,850	2.82

Exp_no	Simulation	Model	Difference (%)
26	$24,212	$24,850	2.64
27	$23,906	$24,850	3.95
28	$23,566	$24,850	5.45
29	$23,272	$24,850	6.78
30	$22,870	$24,850	8.66
31	$24,066	$24,850	3.26
32	$24,066	$24,850	3.26
33	$24,066	$24,850	3.26
34	$24,066	$24,850	3.26
35	$25,228	$24,850	1.50
36	$25,228	$24,850	1.50
37	$17,760	$17,500	1.46
38	$24,970	$24,500	1.88
39	$28,932	$28,000	3.22
40	$43,110	$42,000	2.57
41	$25,918	$17,500	32.48
42	$27,000	$24,850	7.96
43	$45,298	$28,000	38.19
44	$47,504	$42,000	11.59
45	$23,905	$24,850	3.95
46	$24,212	$24,850	2.64
47	$23,272	$24,850	6.78
48	$25,538	$24,850	2.69
49	$24,740	$24,850	0.45
50	$23,496	$24,850	5.76

The V1 cost from (Q, R, δ)^2 inventory model is shown in four components:

- #1: Replenishment reorder inventory
- #2: Complete replenishment cycle inventory
- #3: Partial replenishment cycle inventory
- #4: Static Inventory after the consumption stops

The V2 cost from (Q, R, δ)^2 inventory model is shown in 4 components:

- #1: Production reorder inventory

- #2: Production and replenishment cycle inventory
- #3: Replenishment cycle inventory

Table 3.8 V2 Error of the (Q, R, δ)^2 Model from the Experiments

Exp_no	Simulation	Model	Difference (%)
1	$81,881	$80,638	1.52
2	$83,273	$83,928	0.79
3	$86,079	$93,853	9.03
4	$90,254	$97,784	8.34
5	$98,083	$106,242	8.32
6	$104,349	$95,094	8.87
7	$94,706	$98,025	3.50
8	$110,207	$112,185	1.79
9	$113,207	$113,430	0.20
10	$112,633	$111,794	0.75
11	$110,610	$112,834	2.01
12	$115,515	$111,865	3.16
13	$100,082	$103,888	3.80
14	$124,985	$115,820	7.33
15	$123,513	$112,749	8.71
16	$134,982	$127,279	5.71
17	$131,067	$112,658	14.05
18	$141,420	$124,868	11.70
19	$164,587	$125,899	23.51
20	$173,067	$190,237	9.92
21	$119,015	$114,237	4.01
22	$115,532	$114,025	1.30
23	$114,404	$113,940	0.41
24	$114,608	$113,685	0.81
25	$110,474	$112,919	2.21

Exp_no	Simulation	Model	Difference (%)
26	$107,227	$112,409	4.83
27	$104,373	$111,899	7.21
28	$122,640	$111,049	9.45
29	$114,789	$110,198	4.00
30	$106,812	$109,348	2.37
31	$130,950	$111,543	14.82
32	$100,228	$100,064	0.16
33	$116,544	$114,421	1.82
34	$82,230	$86,261	4.90
35	$199,189	$195,467	1.87
36	$134,750	$135,717	0.72
37	$24,200	$27,904	15.31
38	$37,199	$41,616	11.88
39	$80,287	$85,454	6.44
40	$101,185	$89,969	11.08
41	$116,737	$111,628	4.38
42	$129,655	$135,267	4.33
43	$118,356	$132,283	11.77
44	$144,150	$103,535	28.18
45	$83,654	$86,149	2.98
46	$92,302	$86,297	6.51
47	$82,720	$86,510	4.58
48	$138,405	$134,427	2.87
49	$127,239	$138,287	8.68
50	$110,928	$143,863	29.69

From Figure 3.8, it is found that the error in V1 from the (Q, R, δ)^2 model is mostly within 5% with only three scenarios when the V1 error is more than 10%. While, only about a half of the V2 error is within 5%. The total inventory cost, which is a combination of V1 and V2 cost, was compared to the total annual cost from the simulation in Figure 3.9. The error in total inventory cost is approximately between V1 error and V2 error size.

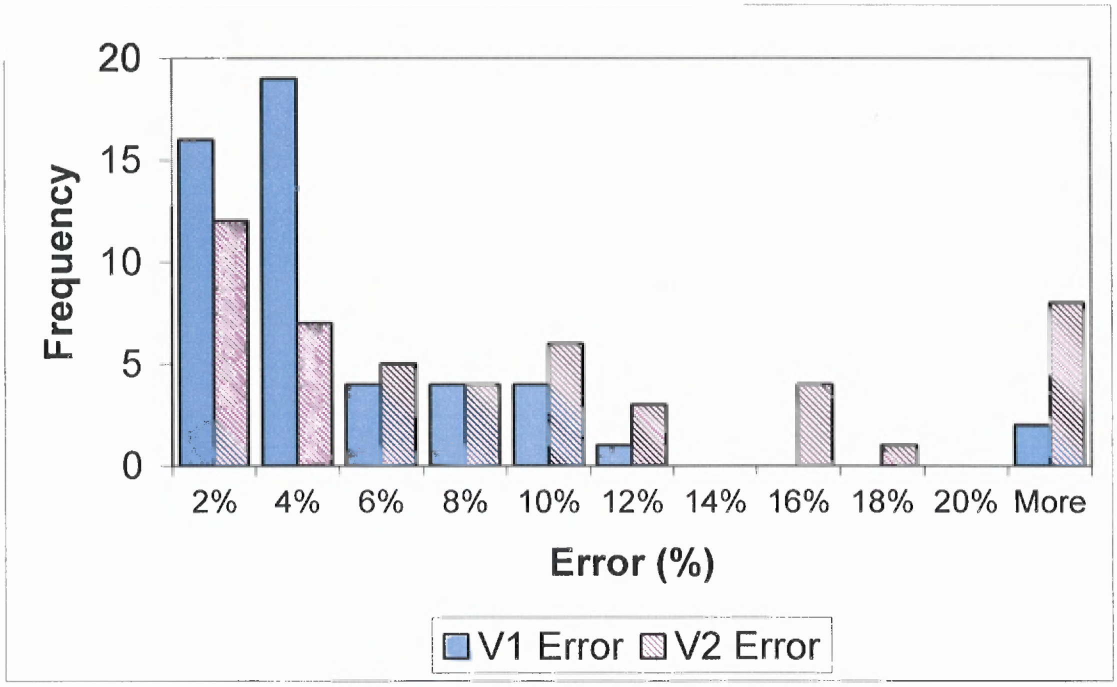


Figure 3.8 Histogram of V1 and V2 error distribution.

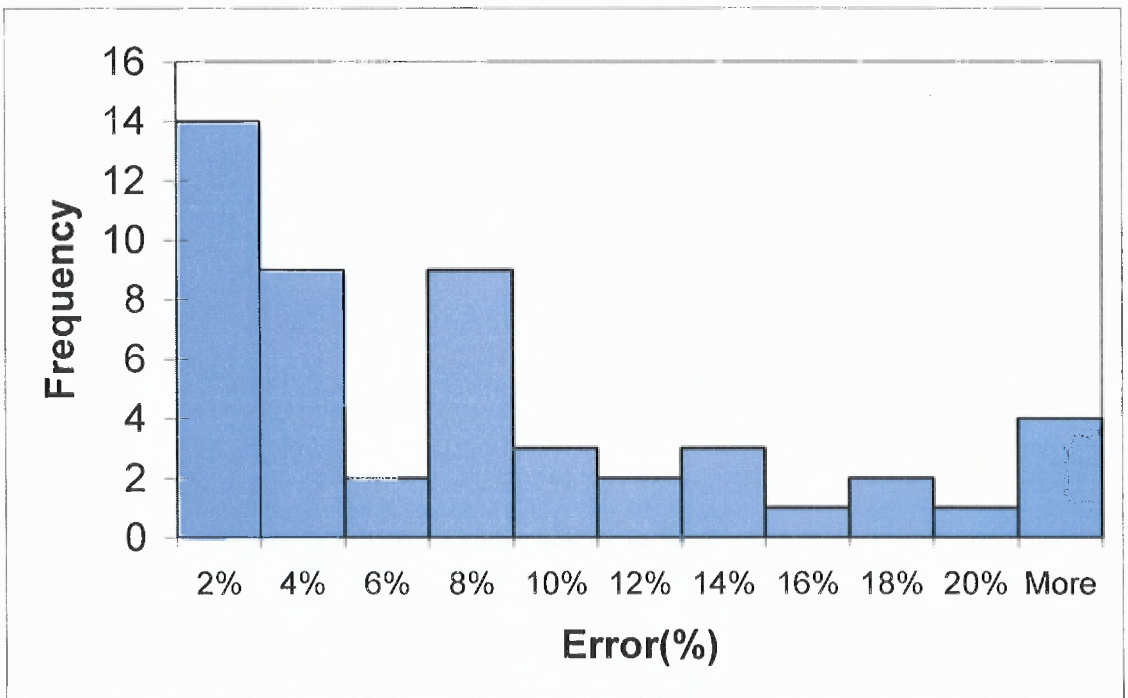


Figure 3.9 Histogram of total annual cost error distribution.

The new model improves the accuracy of inventory estimation, comparing to the original model by Nearier (2008), which has an error size at 10 – 30%. These results were statistically validated in Section 3.5.3.

3.5.3 Hypothesis Testing Results

In order to validate the model, the hypothesis testing method was used. The test was performed using SAS software. The following hypothesis was used to test V1 accuracy:

Null Hypothesis (Ho): $\left|\frac{V1_M - V1_S}{V1_M}\right| \geq 5\%$

Alternate Hypothesis (Ha): $\left|\frac{V1_M - V1_S}{V1_M}\right| < 5\%$

When $V1_M$ is the annual cost from the $(Q, R, \delta)^2$ model and

$V1_S$ is the annual cost from the inventory simulation model

The SAS test report is shown in Figure 3.10, 3.11, and 3.12.

From SAS result, the P-value from t-test was less than .05, the Ho was rejected and the Ha was accepted. It was concluded that the error of V1 cost approximation in $(Q, R, \delta)^2$ model is less than 5% at 95% confidence. V1 error size was expected to be in between the 95 percent confidence interval of 2.71% and 4.22%.

```
                         V1 Model Validation
                    One Sample t-test for a Mean

Sample Statistics for Difference

      N           Mean         Std. Dev.        Std. Error
   -------------------------------------------------------
     50           3.47           2.66             0.38

Hypothesis Test

   Null hypothesis:   Mean of Difference => 5
   Alternative:       Mean of Difference <  5

          t Statistic       Df       Prob > t
          ---------------------------------
            -4.075          49        <.0001

95 % Confidence Interval for the Mean

        Lower Limit:              2.71
        Upper Limit:              4.22
```

Figure 3.10 SAS test report for V1.

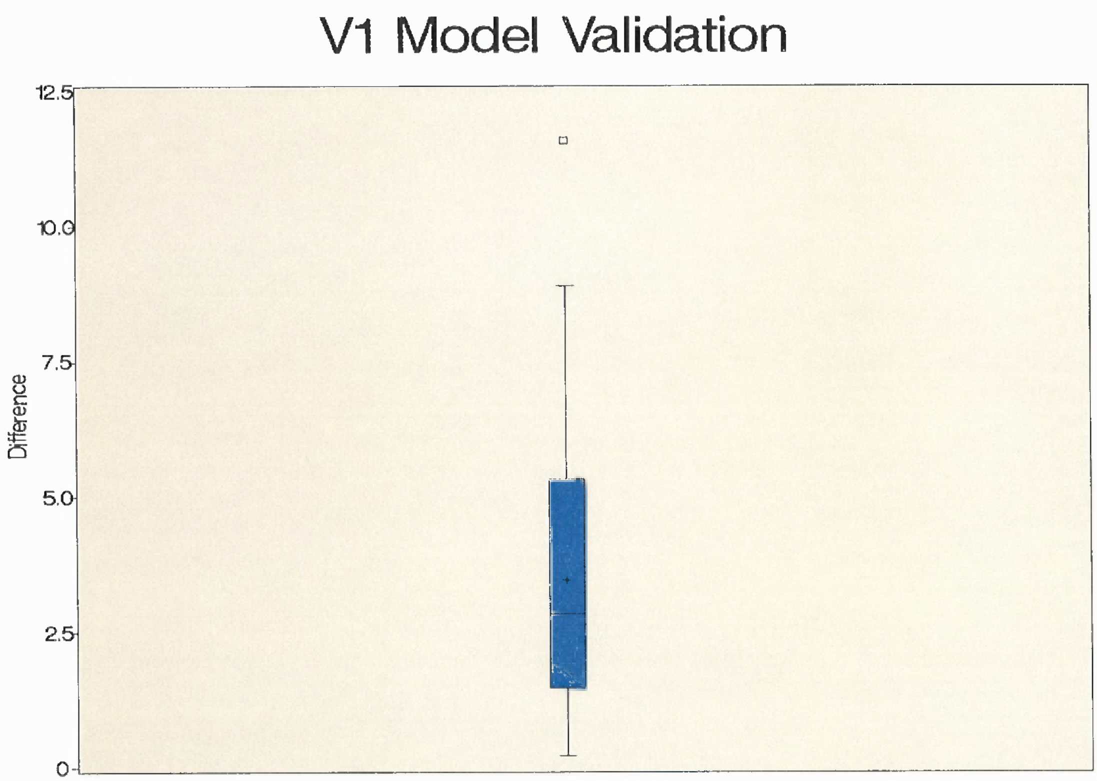


Figure 3.11 SAS box plot of V1 error (percentage).

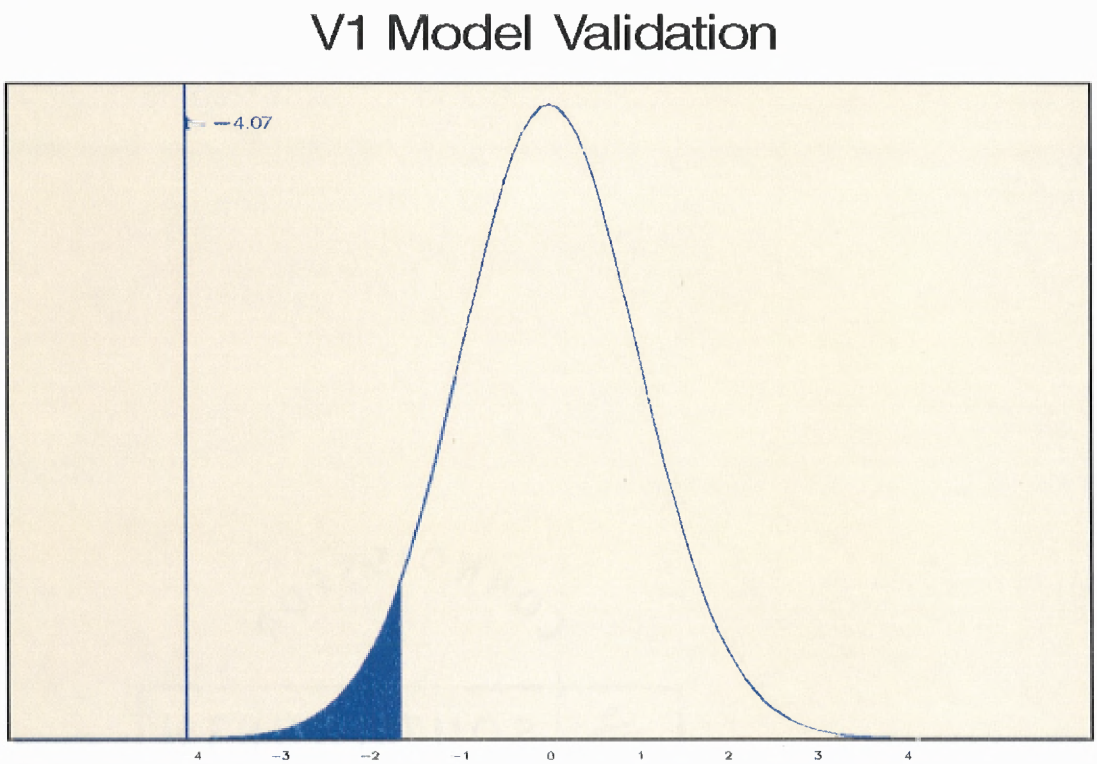


Figure 3.12 SAS t-distribution plot of V1 test.

Using the same testing procedure, the hypothesis used to test V2 accuracy was defined as:

Null Hypothesis (Ho): $\left|\dfrac{V2_M - V2_S}{V2_M}\right| \geq 9\%$

Alternate Hypothesis (Ha): $\left|\dfrac{V2_M - V2_S}{V2_M}\right| < 9\%$

When $V2_M$ is the annual cost from the $(Q, R, \delta)^2$ model and

$V2_S$ is the annual cost from the inventory simulation model

The initial test at 5% failed. It is found that the test passed at 9% error threshold. The SAS test report is shown in Figure 3.13, 3.14 and 3.15.

```
                         V2 Model Validation

                     One Sample t-test for a Mean

Sample Statistics for Difference

      N          Mean        Std. Dev.       Std. Error
   ----------------------------------------------------
     50          6.85          6.63             0.94

Hypothesis Test

   Null hypothesis:   Mean of Difference => 9
   Alternative:       Mean of Difference <  9

          t Statistic       Df       Prob > t
          ---------------------------------
            -2.291          49        0.0132

95 % Confidence Interval for the Mean

        Lower Limit:              4.97
        Upper Limit:              8.74
```

Figure 3.13 SAS test report for V2.

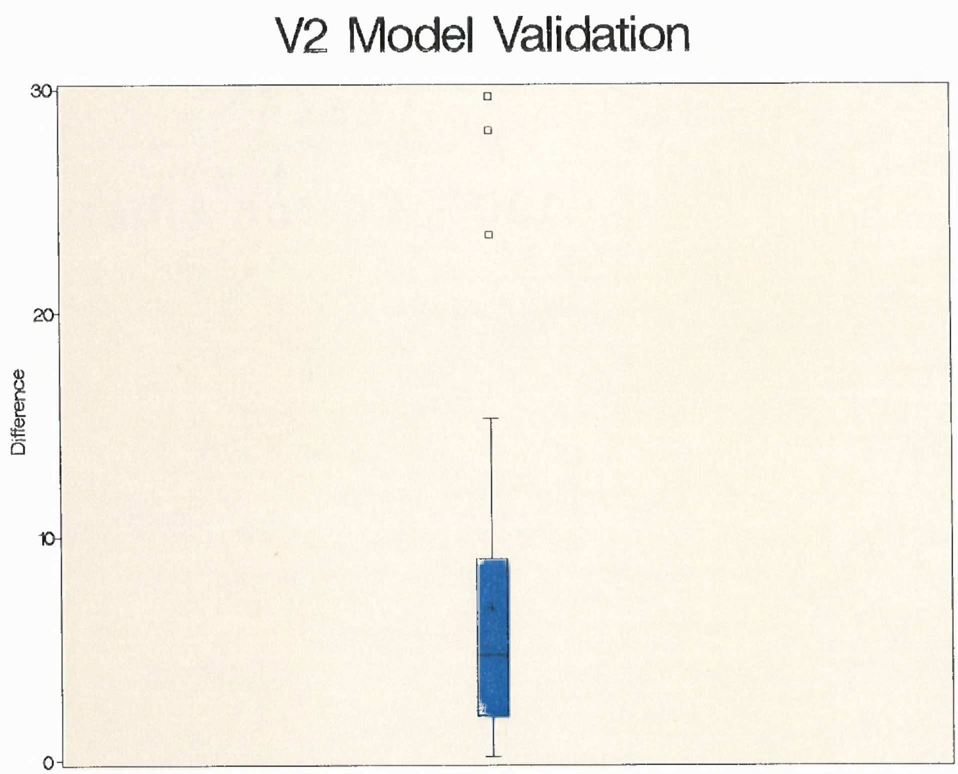


Figure 3.14 SAS Box plot of V2 error (percentage).

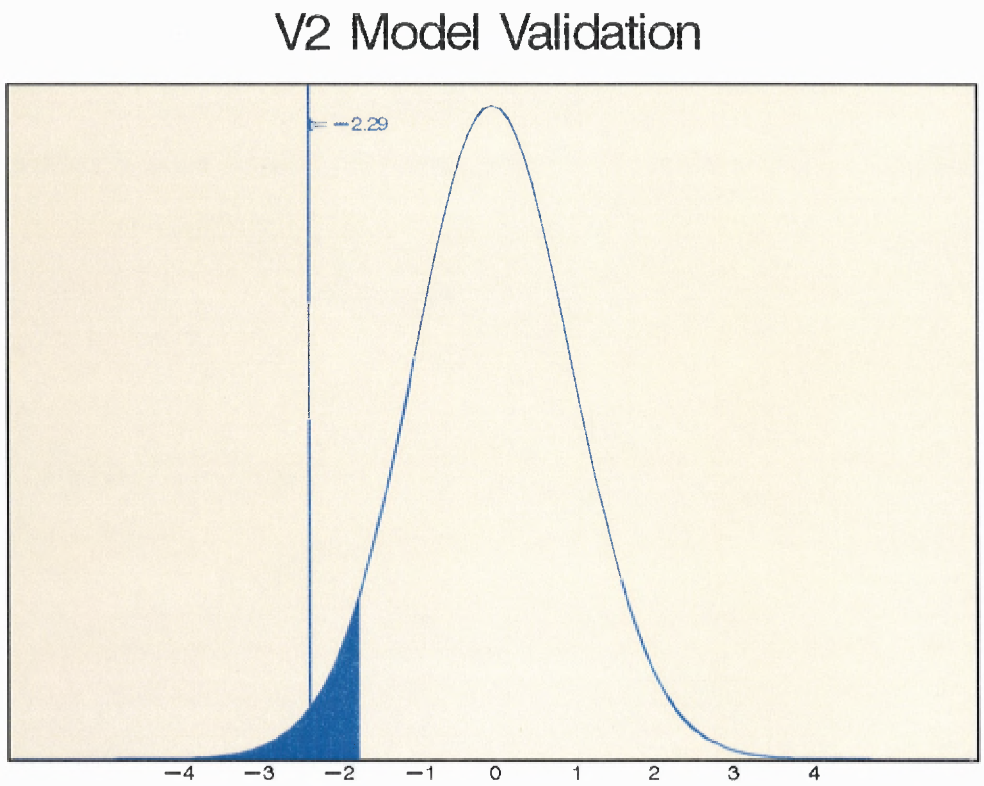


Figure 3.15 SAS t-distribution plot of V2.

From SAS result, the P-value from t-test is less than .05, the Ho was rejected and the Ha was accepted. It was concluded that the error of V2 cost approximation in $(Q, R, \delta)^2$ model is less than 9% at 95% confidence. V2 error size was expected to be in between the 95 percent confidence interval of 4.97% and 8.74%.

From the hypothesis testing result, it was found that the improved $(Q, R, \delta)^2$ improved the accuracy of the model from the original model by Nearier (2008). The model is more accurate in predicting the V1 cost because of the inventory behavior is less complex and depend on a constant demand from the final customer. The V2 cost approximation has larger error, but still within 9% range. This is because the lumpy demand behavior from the order by buyer. From the validation test results, It was concluded that this $(Q, R, \delta)^2$ inventory gives a good approximation of inventory cost in joint-production inventory model.

3.6 Effect of Disruption on Supply Chain Performance

The $(Q, R, \delta)^2$ model assumes that the production batch sizes are fixed, the production rates are constant and based on the pre-determined parameters. In addition, the stock outs are not allowed. The only cost incurred at each supplier is an inventory holding cost.

In the actual production suppliers, the production batch sizes and rates may not be at the full rate (or full capacity). This is known as 'production/supply disruption in supply chains.' The disruption is defined as the difference in the production rate or the production batch size from the designed capacity in which caused the shortage supplying the product to the buyers or customers. There are several causes of the disruption. In this section, the $(Q, R, \delta)^2$ model in figure 3.1 with two suppliers, single product, single input part, with constant demand was used. Two types of disruption are included in the model:

Type-1 disruption This disruption is from the reduced production rate, which can occurred from the partial breakdown of the machine, the fatigue of the labors, or the defects of the input parts.

Type-2 disruption The disruption is from uncompleted batch size can occurred from the complete break down of the machine that the batch need to be stopped and the new set up need to be perform.

From the above type-1 and type-2 disruption, three costs associated with disruption were defined:

a) Shortage cost at seller output for not be able to fulfill order from buyer (W1). This cost is the cost of the seller (left supplier) for not being able to fulfill the incoming order from the buyer. The unit of this cost is dollar/item/year. The larger the shortage in shipping out the order is, the higher is cost is. Let Cs be the unit shortage cost at the seller. The cost in each period is calculated from this formula:

$$W1 = (\text{Order received} - \text{Output parts available}) * Cs \tag{3.44}$$

(when the order received is greater than the parts available)

b) Buyer Production disruption cost for suspending the batch because of input material shortage (W2). This cost occurs when the new batch that should be start cannot start or the batch that already started need to be suspended because the shortage of input material from the seller. This cost is independent from the size of shortage. The disruption in (a) triggers this cost in the nest period. The unit of this cost is dollar/year. Let Cb be the buyer production disruption cost. The cost in each period is calculated from this formula:

$$W2 = Cb \text{ (when the production batch is suspended)} \tag{3.45}$$

c) Customer Back order cost at the final product (W3). The consumption on the final customer side (output product of the right supplier) is assumed to be constant and continuous. This costs triggered when this final product inventory going below zero at buyer output. The unit of this cost is dollar/item/year. It is assumed that all demand from the final customers is always fulfilled later. The more negative the inventory level on buyer output side is, the higher this cost is. Let Cf be the unit back order cost at the buyer output. The cost in each period is calculated from this formula:

W3 = number of units the final customer inventory shortage * Cf (3.46)

(when the inventory level on seller output side is negative)

3.6.1 Disruption Simulation Model

The simulation model was created to study the effect of disruption to supply chain performance. The model was developed from the discrete inventory cost simulation in Section 3.1.4. Microsoft Excel was a software tool used to model this behavior. From the original model in Section 3.1.4, a few parameters and columns are added. This includes three disruption/shortage cost parameters, the disruption cost in each time period. The formulas that simulate the behavior of the buyer and seller behavior were updated.

In the revised simulation model, the following assumptions were changed:

- The back order is allowed at final customer level (output side of the buyer supplier).
- The production rate restriction of the buyer and seller were removed. The buyer production rate may greater than the seller production rate. However, the final customers demand restriction ($D < \delta_1$, $D < \delta_2$) still remains.
- The buyer production batch may be delayed or suspended if there is not sufficient inventory in the next time period. The production may start or resume after the replenishment input part order was received.

- Only full replenishment order was allowed. In case the output inventory of the seller is not enough for a replenishment Q, the shipment will be delayed until the inventory increased beyond Q.

3.6.2 Disruption Simulation Results

The model in Section 3.6.2 was used in this section. The experiment assumed the disruption at the seller production rate (type 1 disruption). The following condition was used in the model.

- The seller production rates were as follows:
  - 60% probability that the production rates were at the full capacity.
  - 40% probability that the production rates were smaller than the specification. In this experiment, the actual rate followed the uniform distribution from 75-100% of the full capacity.
- The production rates (both full and reduced case) were constant in the same production batch
- There was no disruption at the buyer production batch size, or production rates.
- The production rates were varied from 400 – 2000 parts/hour.
- All other parameters are same with the base case experiment in Section 3.5.1.
- The length of the simulation was 65000 hours.
- The shortage costs includes shortage cost at seller output (W1), buyer production disruption cost (W2) and customer back order cost at the final customer (W3).
- The inventory costs include inventory cost at the buyer input product (V1) and inventory cost at the buyer output product (V2).

The three shortage costs parameters used were Cs = $15 /item/yr, Cb = $1,000,000 per year, and Cf = $25 /item/year. The holding cost is fixed at $7 /item/yr at all locations in the chain.

Table 3.9 The Annual Inventory and Backorder Cost in Disruption Simulation

	Parameters						Max	Annual Cost		
Exp#	B1	B2	δ1	δ2	D (/yr)	Q	Disrup	Inv. ($)	Backorder ($)	Total ($)
1	24000	15000	1000	400	240000	5100	75%	$131,729	$14	$131,743
2	24000	15000	1000	500	240000	5100	75%	$131,868	$13	$131,881
3	24000	15000	1000	600	240000	5100	75%	$133,302	$13	$133,315
4	24000	15000	1000	700	240000	5100	75%	$132,085	$10,640	$142,725
5	24000	15000	1000	800	240000	5100	75%	$132,426	$15,430	$147,855
6	24000	15000	1000	900	240000	5100	75%	$147,147	$17,322	$164,469
7	24000	15000	1000	1000	240000	5100	75%	$133,294	$21,999	$155,293
8	24000	15000	1000	1200	240000	5100	75%	$134,046	$25,396	$159,442
9	24000	15000	1000	1500	240000	5100	75%	$137,739	$25,595	$163,334
10	24000	15000	1000	2000	240000	5100	75%	$126,092	$31,185	$157,277

Table 3.9 shows the experimental results. The buyer production rate was 40% chance likely to have a disruption to a reduced rate of 750 to 1000. When the buyer production rates were varied from 400 to 1200 parts / hour, there was no significant change in inventory costs. The back order cost was minimal at the buyer production rate of 600 parts/ hour or less. The only source of this cost at low production rate is from W3 (final customer back order). There was a significant jump in the cost when the buyer production rate closes to lower threshold of the seller production rate (750 parts /hour). From the experiments, this jump is when the buyer production rate changed to 700 pats/hour. This was when the supply chain start experiencing the W1 (seller shortage

cost) and W2 (buyer production disruption cost). At higher buyer production rate (more than 1500 parts/hour), the inventory cost started decreasing.

CHAPTER 4

OPTIMIZATION OF REPLENISHMENT ORDER QUANTITY

4.1 Order Quantity and Inventory Cost Relationship

From (Q, R, $\delta)^2$ jointed production inventory model in Chapter 3, the study the behavior of replenishment order quantity Q was chosen. An assumption that the order quantity will be constant for each shipment in a supply cycle was made. In this chapter, the behavior of the order quantity to the total inventory cost was studied. This is because order quantity is a common parameter that can be easily adjusted by suppliers in the chain.

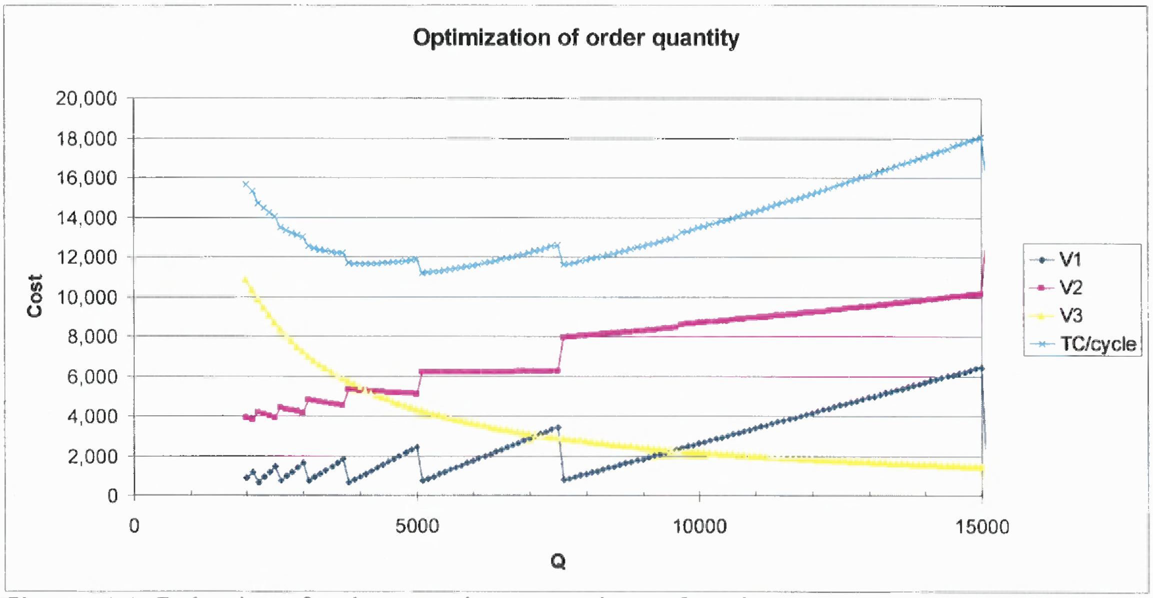


Figure 4.1 Behavior of order quantity vs. total cost function.

From Figure 4.1, which shows the behavior of the buyer incoming inventory cost (V1), seller outgoing inventory cost (V2), and order cost (V3) when order quantity is less than consumption batch size (ZB_2), it is found that

V1 is a step function with a non-continuous location at the integer fractional of consumption batch size of the buyer, such as ZB_2, $ZB_2/2$, $ZB_2/3$, $ZB_2/4$, $ZB_2/5$, ... For each step of the V1, the function increased almost linearly. The lowest V1 cost was at one of the integer fractional point (step point) of the ZB_2.

V2 is also a step function with a non-continuous location at the integer fractional of consumption batch size of the buyer (similar to V1). At lower Q, V2 decreased as the Q increases. However, at the higher Q, V2 increased with increasing Q. The lowest V2 cost may or may not be at the integer fractional point (step point of the ZB_2).

V3 follows a traditional unit order cost behavior. As the Q increase, the V3 exponentially decrease.

Total cost function TC will generally decrease at a smaller Q and increase at a larger Q. The analysis of total cost function is included in Section 4.2

4.2 Optimal Order Quantity Search Methodology

4.2.1 Evaluating the Convexity of the Integer-relaxed Model

In order to prove the convexity of the total cost function with the Q, the first derivative of total cost function was used. However, because of step-wised behavior of TC, it is not possible to differentiate the exact function TC. This step behavior is from the integer variable in V1 and V2. Therefore, an approximation method to eliminate the step behavior of TC was used. The following approximation of the integer function is used:

Case 1 For small Q ($Q < ZB_2$) it is assumed that the buyer consumption batch is exactly an integer multiple of the shipment ($ZB_2 = nQ$). The Int^+ function in X1 can be removed and the partial cycle term Y1 will be zero; therefore:

The number of full supply cycle: $X1 = Int^{+}\left\{\frac{ZB_2}{Q}\right\} \sim \frac{ZB_2}{Q}$ (4.1)

The partial supply cycle: $Y1 = Int^{-}\left\{\frac{ZB_2}{Q}\right\} \sim 0$ (4.2)

From the above approximated X1 and Y1, V1 was estimated as follows:

$$V1 = \frac{Ch}{T}\left\{\frac{R_2B_2}{D} + (X1)\left(\frac{Q}{2}\right)\left(\frac{Q}{\delta_2 Z}\right) + QB_2\left(\frac{1}{D} - \frac{1}{\delta_2}\right)\right\} \tag{4.3}$$

Case 2 For large Q ($Q > ZB_2$), it is assumed that the shipment is exactly an integer multiple of the buyer consumption batch size. The shipment will be used exactly for an integer of buyer supply cycle with no residual at the end ($nZB_2 = Q$). Furthermore, the seller production batch size B_1 is not larger than the buyer consumption batch size ZB_2. The buyer consumption batch size is also a multiple of the seller production batch size. From these assumptions, the following integer variables can be approximated:

N = Number of production batches produced in a supply cycle

$$N = Int^{Next}\left\{\frac{B_2Z}{B_1}\right\} \sim \frac{B_2Z}{B_1} \tag{4.4}$$

X3 = Number of replenishments occurring during seller production

$$X3 = \min\left[Int^{+}\left\{\left(\frac{NB_1}{\delta_1}\right)\left(\frac{\delta_2 Z}{Q}\right)\right\}, X1\right] \tag{4.5}$$

$$X3 \sim \min\left[Int^{+}\left\{\left(\frac{\frac{B_2 Z}{B_1}B_1}{\delta_1}\right)\left(\frac{\delta_2 Z}{Q}\right)\right\}, X1\right] \tag{4.6}$$

$$X3 \sim \left\{\left(\frac{B_2 Z}{\delta_1}\right)\left(\frac{\delta_2 Z}{Q}\right)\right\}$$

For residual inventory U, the estimated X1 and N is substituted:

U = The residual inventory at the beginning of no-activity period

$= NB_1 - X1.Q$

$$U \sim B_2Z - B_2Z = 0 \text{ (No residual inventory U)} \tag{4.7}$$

X4 = Number of seller production cycle in 10 supply cycle

$$X4 = Int^{+}\left\{\frac{10B_2Z}{NB_1}\right\}+1 \sim \frac{10B_2Z}{NB_1}+1 \tag{4.8}$$

After substitute estimated X1, Y1, N, X3, and U into V2, substitute V1, V2, and V3, into function TC = V1 + V2 + V3. it is found that there is always a root to the equation:

$$\frac{dTC}{dQ} = 0 \ ; Q > 0 \tag{4.9}$$

Therefore, there is always an optimal order quantity for a given set of parameters in our $(Q, R, \delta)^2$ model. Is was also found that

$$\frac{d^2TC}{dQ^2} > 0 \ ; Q > 0 \tag{4.10}$$

This confirms that the optimal order quantity is a minimal point. Therefore, there is an optimal order quantity Q* that minimizes the total cost.

According to the behavior of function TC, two Lemmas were proposed:

Lemma 1: For a given total cost function TC(Q), the optimal order quantity Q* of TC(Q) in many cases will be an integer fraction (B_2/i ; i = 1, 2, 3, ...) of B_2.

Lemma 2: if Lemma 1 does not hold, the Q* will be in the preceding or succeeding section ($B_2/(i-1)$, B_2/i] or (B_2/i, $B_2/(i+1)$] when TC(B_2/i) is the minimum cost for all i = 1, 2, 3, ...

These 2 lemmas will be used in selecting and developing the search methodology. The detail of is shown in Section 4.2.2 and 4.2.3.

4.2.2 Need for Golden Section Search

From Section 4.1 and 4.2.1, the global trend of total cost function TC follows a step-wised convex behavior. Because of the non-continuous point in function TC, using the first derivative to find the optimal replenishment order quantity Q* is not feasible. Therefore, there is a need to develop a search technique to find the optimal order quantity. For this research, Golden Section search technique was chosen to be applied to locate optimal answer for TC because it does not require the function to be differentiable and it is able to find the optimal quickly.

The Golden Section search, introduced by Kiefer (1953), is a search technique for finding the minimum or maximum of a function by successively narrowing the range of value inside the range of the values, which the minimum or maximum is known to be exist. The name 'Golden Section' derives from the distance between three points, which forms a 'Golden ratio.' The example of Golden Section search is shown in Figure 4.2. According to the figure, the Golden Section search is performed in a range of x_1 to x_3 to find the minimum of f(x). Next, the position of x_2 is chosen (mostly, by using Golden Section ratio). The search starts from finding the value of $f(x_1)$, $f(x_2)$, and $f(x_3)$. If $f(x_2)$ is less than $f(x_1)$ and $f(x_3)$, the optimal is within x_1 and x_3. If $f(x_2)$ is not less than $f(x_1)$ and $f(x_3)$, the optimal is not within x_1 and x_3 and the search range may have to be expanded until that the optimal is found within the search range.

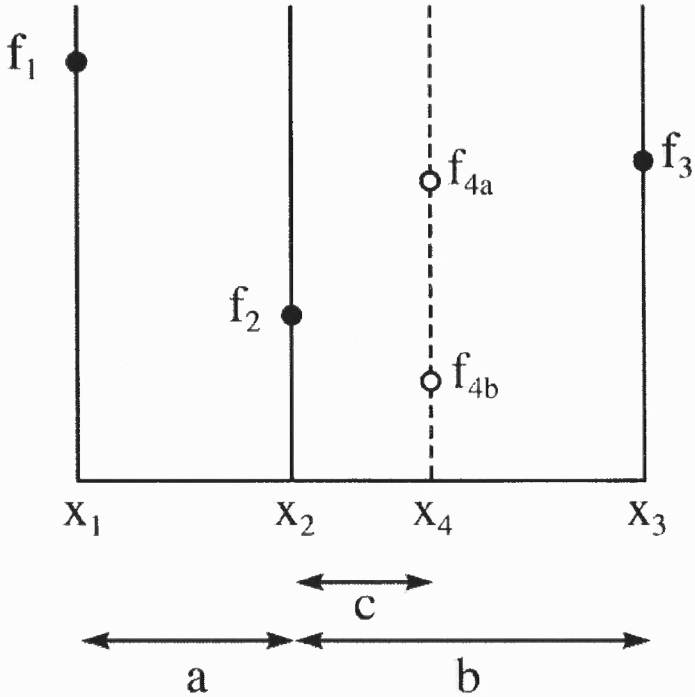


Figure 4.2 Golden section search by Kiefer (1953).

After that, another mid-point in the larger section of the search using Golden ratio (in this example is x_4 between x_2 and x_3) is chosen. There will be 2 possibilities: (a) If $f(x_4)$ is greater than $f(x_2)$ (for example, f_{4a}), then a minimum is within x_1 and x_3. Therefore, the new search points are x_1, x_2, and x_4. ,or (b) If $f(x_4)$ is less than $f(x_2)$ (for example, f_{4b}), then a minimum is within x_2 and x_3. Therefore, the new search points are x_2, x_4, and x_3. Using this criteria and repeat these steps until 2 consecutive f(x) are within specified value, it is concluded that the last value of x is an optimal answer that minimizes the cost.

However, because of the step behavior of the function, additional steps is needed to find the starting ranges, and verify the final value from the search.

4.2.3 Modified Golden Section Search Steps

In this section, the detail of the search methodology that used in finding optimal order quantity is shown. The modified Golden section search steps are:

Step1: let the total annual cost function f(x)= TC(Q), and Q* = optimal order quantity. Calculate all total cost at each integer fractional of buyer consumption batch size ($TC(ZB_2/i)$ when i = 1, 2, 3, ...).

Step 2: Find the $\hat{Q}^*$, which is the estimated optimal order quantity, by selecting the $\hat{Q} = ZB_2/\hat{i}$ that gives the lowest cost TC.

Step 3: Perform a golden section search within a range $Q = (B_2/\hat{i} - 1 , B_2/\hat{i} + 1]$ by using the steps in section 4.2.2. Using Golden ratio $\frac{1+\sqrt{5}}{2} \approx 1.61803$, the length of each section in Figure 4.2 as:

$a = 0.8\ \hat{Q}$

and $b = 1.32\ \hat{Q}$

Step 4: From range Q, a, and b in step 3, find the starting left point, right point, and middle point $f(a^{(0)})$, $f(b^{(0)})$, and $f(x^{(0)})$, respectively.

Step 5: From 3 points in step 4, calculate $f(a^{(0)})$, $f(b^{(0)})$, $f(x^{(0)})$ from the TC formula in chapter 3.

Step 6: If $f(x^{(0)}) < f(a^{(0)})$ and $f(x^{(0)}) < f(b^{(0)})$, the optimal is within $a^{(0)}$ and $b^{(0)}$, continue the search. Otherwise, extend the search range to the next preceding or succeeding i and repeat step 3-6 again.

Step 7: Find $x^{(1)} = (1/2.6)(b - x^{(0)}) + x^{(0)}$ and calculate $f(x^{(1)})$

Step 8: If f($x^{(1)}$) > $x^{(0)}$) then the optimal will be in Q = (a, $x^{(1)}$) and the point a, $x^{(0)}$, and $x^{(1)}$ will be used in the next search iteration.

Otherwise, then the optimal will be in Q = ($x^{(1)}$, b) and the point $x^{(0)}$, $x^{(1)}$, and b will be used in the next search iteration.

Step 9: repeat step 7-8 by calculating the mid-point of the next iteration of the search from $x^{(i+1)} = (1/2.6)(x^{M(i)} - x^{R(i)}) + x^{M(i)}$ when M(i) and R(i) is the middle point and right point for iteration i. The search will be performed until the difference of 2 consecutive Q is within the range of stopping condition $\varepsilon = |x^{(i)} - x^{(i-1)}|$

Step 10: Validate the final answer by perform a check in $Q \pm n\phi$ when $\phi = ZB_2/10$ and n = 1, 2, 3, 4, 5 to make sure that the optimal answer is correct.

4.2.4 Using Microsoft Excel to Perform a Modified Golden Section Search

In order to perform the search in Section 4.2.3, Microsoft Excel with Visual Basic Application to perform was used the search. The work sheet requires following parameters as input:

- Seller production batch size (B_1)
- Buyer production batch size (B_2)
- Seller production batch size (δ_1)
- Buyer production batch size (δ_2)
- Seller production reorder level (R_1)
- Buyer replenishment reorder level (R_2)
- Buyer bill or material Quantity (Z)
- Final customer demand rate (D)

- Annual part holding cost ($/year)
- Order cost ($/order)

At the beginning of the search, the macro determines the potential order quantity (integer fractional of buyer consumption batch size) that will be use as a candidate for determining the starting search range. The range is from the preceding potential Q to succeeding potential Q that gives the lowest TC(Q).

The macro calculates the mid-point and pastes it in the proper cell to calculate the total cost using the linked formulas and input parameters. Next, it compares all three points and identifies the next search range in the next iteration. The result is passed to the next row. The range is narrowed down for each iteration until the answer for the preceding search is in the closed range with the succeeding iteration. The last row will show the optimal order quantity and the minimal total cost (objective value).

From the optimal quantity, the final answer have to be validated by perform a check in $Q \pm n\phi$ when $\phi = ZB_2/10$ and n = 1, 2, 3, 4, 5 to make sure that the optimal answer is correct.

4.3 Numerical Example

The proposed modified Golden section search in Section 4.2 was tested with the following numerical example:

There are two suppliers: seller and buyer. The first supplier (seller) produces a single output that is shipped to the second supplier (buyer) to produce a final product for customers. The demand is deterministic and constant. Both of the suppliers follow the $(Q, R, \delta)^2$ inventory policy. The following numerical parameters were used:

- Holding cost of the product (Ch) = $7 per year across the chain
- Seller production batch size (B1) = 24000
- Buyer production batch size (B2) = 15000
- Seller production rate (δ1) = 1000 per hour
- Buyer production rate (δ2) = 400 per hour
- Seller production reorder level (R1) = 3000
- Buyer production reorder level (R2) = 1000
- Annual Consumer demand rate for final product (D) = 320,000
- Order Cost (Co)= $1450 per order

The graph showing the behavior of the inventory costs, order costs, and total cost is shown in Figure 4.3.

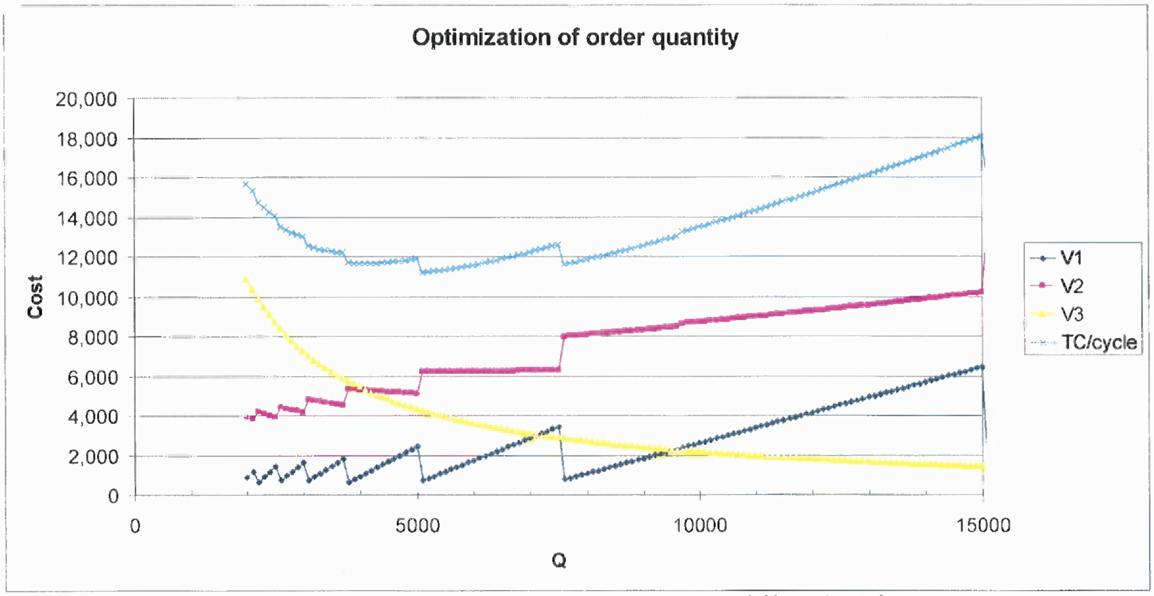


Figure 4.3 Graph for optimization of the order quantity while Ch = \$7.

According to graph, is found that from all candidates of the potential order quantity ZB_2, $ZB_2/2$, $ZB_2/3$, $ZB_2/4$, … The $\hat{Q} = ZB_2/3$ gives the lowest cost. Therefore, a search is performed between $Q = ZB_2/4$ to $Q = ZB_2/2$. After executing the search macro in Microsoft Excel within the search range, the optimal order quantity Q* is still being at the same point $\hat{Q} = ZB_2/3 = 5000$ with the minimal total cost at \$11,216.16. This follows the lemma 1 in section 4.2.1.

On the other hand, when the cost parameter value changed, it was found that the behavior of the optimal value changes. In Figure 4.4, the annual holding cost was modified from \$7 to \$2.

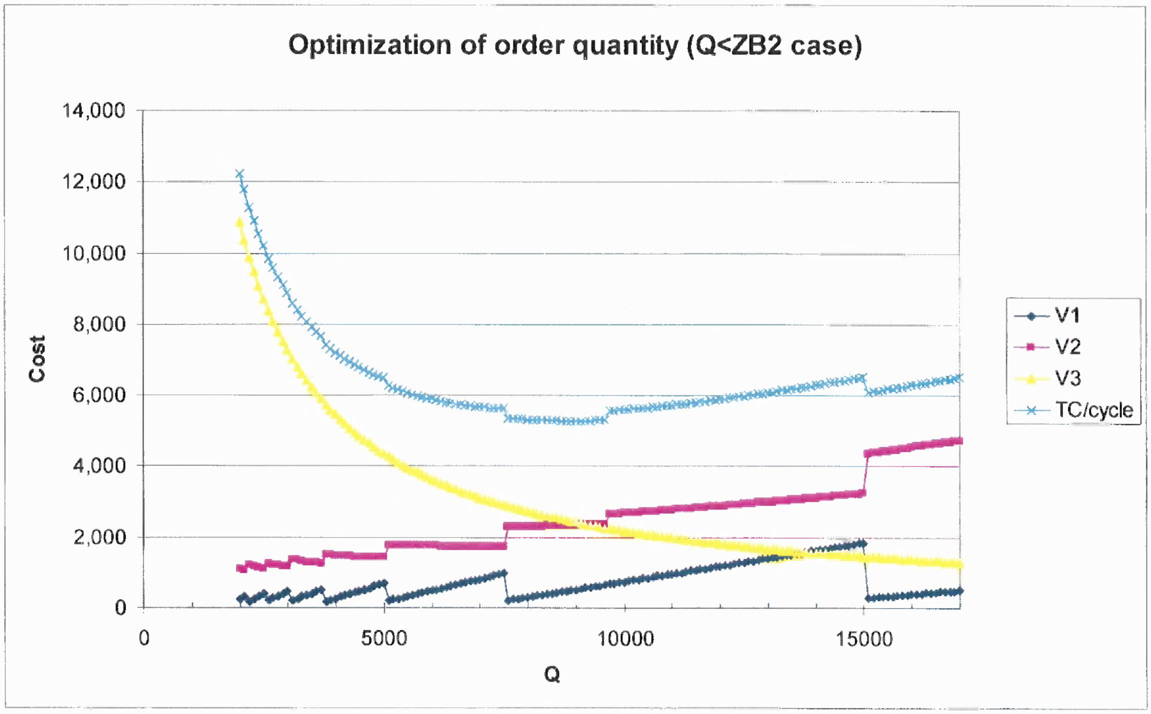


Figure 4.4 Graph for optimization of the order quantity while Ch = \$2.

According to the search, it was found that from all candidates of the potential order quantity, the $\hat{Q}$ = $ZB_2/3$ = 7500 gives the lowest cost. Therefore, a search was performed between Q = $ZB_2/3$ to Q = ZB_2. After executing the search macro in Microsoft Excel within the search range, the optimal order quantity Q* is no longer at the $\hat{Q}$ = 7500. Instead, the Q* is at Q = 9200 with the minimal total cost at \$5378.29. This optimal behavior follows the lemma 2 in section 4.2.1.

4.4 Conclusion of the Order Quantity Optimization

From the graphical and mathematical analysis, it is concluded that the total cost function for the order quantity follows the step-wised convex behavior with a step point at the integer fraction of buyer consumption batch size. This behavior is from the residual inventory at the end of supply cycle which results in integer function in $(Q, R, \delta)^2$ inventory model. From the analysis, the function has the optimal with minimal cost. The optimal may be at order quantity equals to the integer fraction of buyer consumption batch size that yields the lowest cost. However, there is also a chance that it will be around that point.

Because of step-wised behavior, the function is not differentiable. Therefore, it is infeasible to find the optimal using traditional differentiation method. In this chapter, the Golden section search is chose as a search methodology to find the optimal answer. The extra steps are included with original Golden section search to find the starting range for the search and validate the optimal from the search. The numerical example shows two cases while the optimal is at the step point and around the step point.

CHAPTER 5

ROBUSTNESS OF THE ORDER POLICY UNDER CHANGING DEMAND LEVELS

5.1 Uncertainty in a Supply Chain

In the Chapter 4, the optimal replenishment quantity Q in $(Q, R, \delta)^2$ was developed. An assumption for this model was the annual demand level of the final product was fairly constant at the same level over the year. However, this may not be the case in the actual supply chain, which the product demand level can be changed over the time. This is called 'demand shift.' The demand shift is defined as the event whenever the mean demand jumps to a higher or lower level, similar to moving average forecast.

Supply chain may keep the same supply policy when the demand shift occurs at smaller level because the change in inventory costs is not significant. Therefore, it is not necessary for the chain to make a change in supply policy. However, when the size of the demand shift becomes larger, there will be a significant deterioration in inventory costs and a change in supply policy is justified. The limits of all demand levels was defined as the demand shift range.

In this chapter, the case when the demand shifts occur out of the demand shift range was analyzed. This shift makes the current optimal supply policy no longer justified. There was a need to find a new supply policy for this uncertain demand scenario. A new matrix 'Supply policy robustness' was set to measure the ability of a supply policy to handle an uncertain demand behavior. Extra costs incurred from keeping the existing supply policy Q from the previous demand level, comparing with adjusting the policy to the new optimal Q at the new demand level shows the inefficiency of this supply policy. From this research, the supply policy robustness was calculated as an expected dollar amount of this extra cost over the demand shift range. The lower this cost is, the more robustness of supply chain policy.

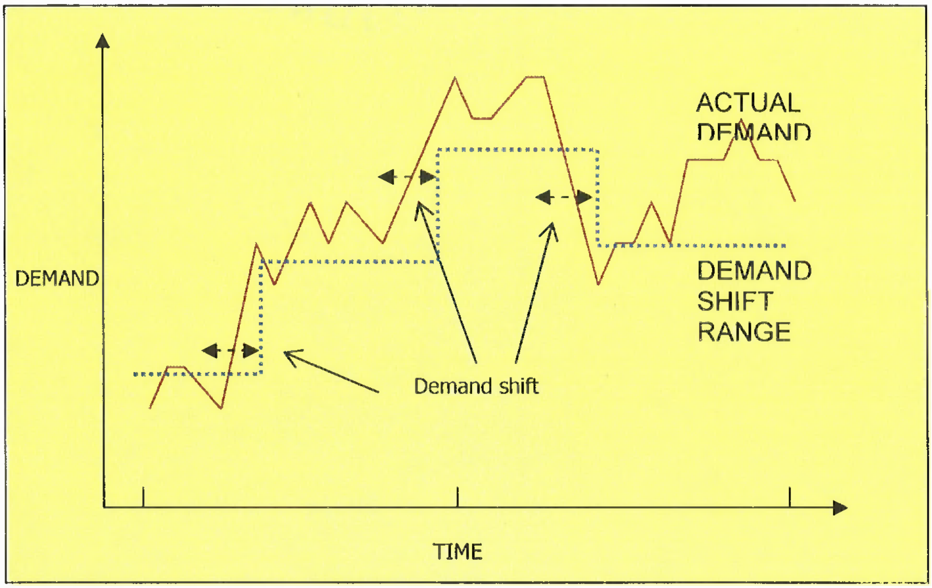


Figure 5.1 The demand shifts over the time.

Figure 5.1 shows the historical demand level of a final product from a supply chain. The actual demand level was approximated into three demand shift occurrences. The demand level starts at the lower demand range. After the first upper shift, the demand increases to the average demand level. The upper demand range is reached after the second upper shift. The demand level falls back to the average level again after the lower shift. In this chapter, the demand shift was represented by a uniform distribution with the maximum and minimum value of demand shift range as parameters.

Because of unconventional inventory behavior of supply chain shown in Chapter 3, the optimal supply policy is unlikely to be at the optimal supply policy at the average demand level. There was a need for the supply chain to develop alternative supply policy to minimize the total expected inefficient cost (maximize the supply chain robustness). three strategies have been developed in this research:

(a) Fixed delivery batch size strategy

(b) Fixed production period strategy

(c) Combined strategy

5.2 Fixed Delivery Batch Size Strategy

As described in Section 5.1, the delivery batch size of product j from seller to buyer depends on the annual demand level. Theoretically, whenever there is a demand shift, it is necessary for the chain to adjust the supply contract policy. This could be accomplished using the modified golden section search in Section 4.2.

In the actual supply chain, the demand shifts (smaller or larger sizes) occurs every period (daily, weekly, or bi-weekly). It is infeasible to continuously adjust the replenishment order quantity to the new demand levels after every demand shift event. This is because for each change in the supply policy incurred the extra operating cost from the modification in resource and labor. In addition, the delivery schedule needs to be modified because of the new replenishment order size.

The numerical example for this fixed delivery batch size strategy was completed using the same scenario from Section 4.3.

As a consequence, a fixed delivery batch size Q** that minimized the total expected cost over the demand shift range was selected. For simplicity, it was assumed that the demand level follows a uniform distribution [D_1, D_2] when D_1 is the lower limit of the demand shift range and D_2 is the upper limit of the demand shift range. The Q** is the Q that minimizes this supply chain robustness from this integral term:

$$\int_{D_1}^{D_2} \{TC(Q^{**}) - TC(Q^{*})\}\, dD$$ When Q* is the actual optimal Q at each demand level

In this policy, the seller production batch size B_1 is fixed at the optimal value B_1*, which is calculated at the average demand level.

A numerical example showed how the cost changes while adjusting to the replenishment order quantity. The following parameters are used:

- Holding cost of the product (Ch) = $7 per year across the chain
- Seller production batch size (B1) = 24000
- Buyer production batch size (B2) = 15000
- Seller production rate (δ1) = 1000 per hour
- Buyer production rate (δ2) = 400 per hour
- Seller production reorder level (R1) = replenishment order quantity (to prevent stock out event)
- Buyer production reorder level (R2) = 1000
- Annual Consumer demand range for final product (D) = U[150000, 350000]
- Order Cost (Co)= $1450 per order

Using the modified golden section search in the previous experiment at the demand level of 240,000 gives a Q* at 5376. In Table 5.1, the supply chain robustness over the above demand shift range at this replenishment order quantity is calculated.

Table 5.1 The Supply Chain Robustness When Q = 5376

Demand	At Q=5376	TC At Q* (c)	Q*	DTC (c)	DTC (%)	Area
150000	117.68	117.33	5001	0.3495	0.0030	
160000	112.06	111.79	5001	0.2730	0.0024	480524586
170000	107.10	106.90	5001	0.2054	0.0019	392977786
180000	102.70	102.55	5001	0.1454	0.0014	305430985
190000	98.75	98.70	5120	0.0499	0.0005	178275216
200000	95.20	95.17	5120	0.0345	0.0004	81916343
210000	91.99	91.97	5120	0.0205	0.0002	55963335
220000	89.08	89.07	5120	0.0151	0.0002	38087121
230000	86.40	86.40	5280	0.0058	0.0001	23229267
240000	83.92	83.92	5376	0.0000	0.0000	6635562
250000	81.72	81.63	5440	0.0896	0.0011	111972451
260000	79.64	79.50	5600	0.1490	0.0019	305637137
270000	77.72	77.25	7501	0.4702	0.0061	828412525
280000	75.94	75.15	7501	0.7931	0.0106	1745088190
290000	74.28	73.19	7501	1.0937	0.0149	2696273216
300000	72.73	71.35	7501	1.3744	0.0193	3647458241
310000	71.28	69.64	7501	1.6369	0.0235	4598643266
320000	69.92	68.04	7501	1.8829	0.0277	5549828291
330000	68.64	66.53	7501	2.1141	0.0318	6501013316
340000	67.44	65.11	7501	2.3317	0.0358	7452198341
350000	66.31	63.77	7501	2.5369	0.0398	8403383366
					SUM	43402948538

The first column shows the annual demand level over the demand shift range. The second column is the unit inventory cost (in cents) when the order quantity is fixed at the Q* of demand level of 240000. The third column shows the unit inventory cost (in cents) when the inventory order quantities are continuously adjusted to Q* at each demand level after every demand shift event. From the table, this Q* are ranged from 5001 to 7501. The last three columns calculate the extra cost incurred when the order quantity was fixed at 5376. From the calculation, it is found that the expected extra cost incurred is $1736.12 per year. Figure 5.2 shows the unit cost comparison with this scenario:

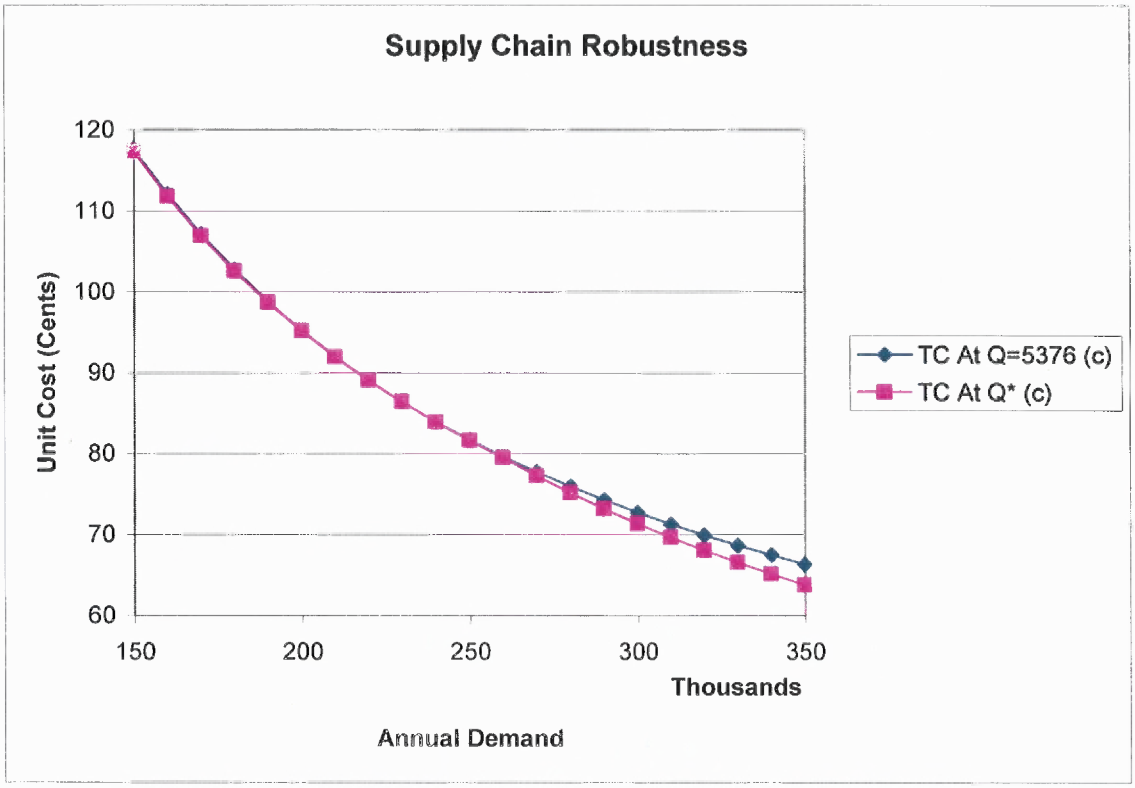


Figure 5.2 Unit cost comparison when Q** = 5376.

It is found that, when the Q is fixed at 240000, the extra cost at high demand level is much higher. In Table 5.2, the Q** was moved to the Q* at demand level of 250000. The second column in the table shows the unit inventory cost (in cents) when the order quantity is fixed at the Q** = 5440 of demand level of 250000. The last two columns calculate the extra cost incurred when the order quantity was fixed at 5440. From the calculation, the expected extra cost incurred is $1561.40. Therefore, changing the Q** = 5376 (from D = 240k) to Q** = 5440 (from D = 250k) yielded an annual saving of 10.06%. On the other hand, the fixed replenishment order quantity policy at the D = 250k is more robust that policy at the D = 240k.

Table 5.2 The supply chain robustness when Q = 5440

Demand	At Q =5440	TC At Q* (c)	Q*	DTC (c)	Area
150000	118.33	117.33	5001	0.9992	
160000	112.59	111.79	5001	0.8071	1395038747
170000	107.53	106.90	5001	0.6376	1187654150
180000	103.04	102.55	5001	0.4870	980269554
190000	99.01	98.70	5120	0.3105	733275988
200000	95.39	95.17	5120	0.2221	517079320
210000	92.11	91.97	5120	0.1421	371288515
220000	89.14	89.07	5120	0.0694	225497711
230000	86.42	86.40	5280	0.0202	99586215
240000	83.92	83.92	5376	0.0023	26049755
250000	81.63	81.63	5440	0.0000	2764695
260000	79.51	79.50	5600	0.0167	21773337
270000	77.55	77.25	7501	0.2985	424710929
280000	75.73	75.15	7501	0.5847	1221548799
290000	74.04	73.19	7501	0.8512	2052896028
300000	72.45	71.35	7501	1.1000	2884243257
310000	70.97	69.64	7501	1.3327	3715590486
320000	69.59	68.04	7501	1.5508	4546937715
330000	68.28	66.53	7501	1.7557	5378284944
340000	67.06	65.11	7501	1.9486	6209632173
350000	65.90	63.77	7501	2.1305	7040979402
				SUM	39035101720

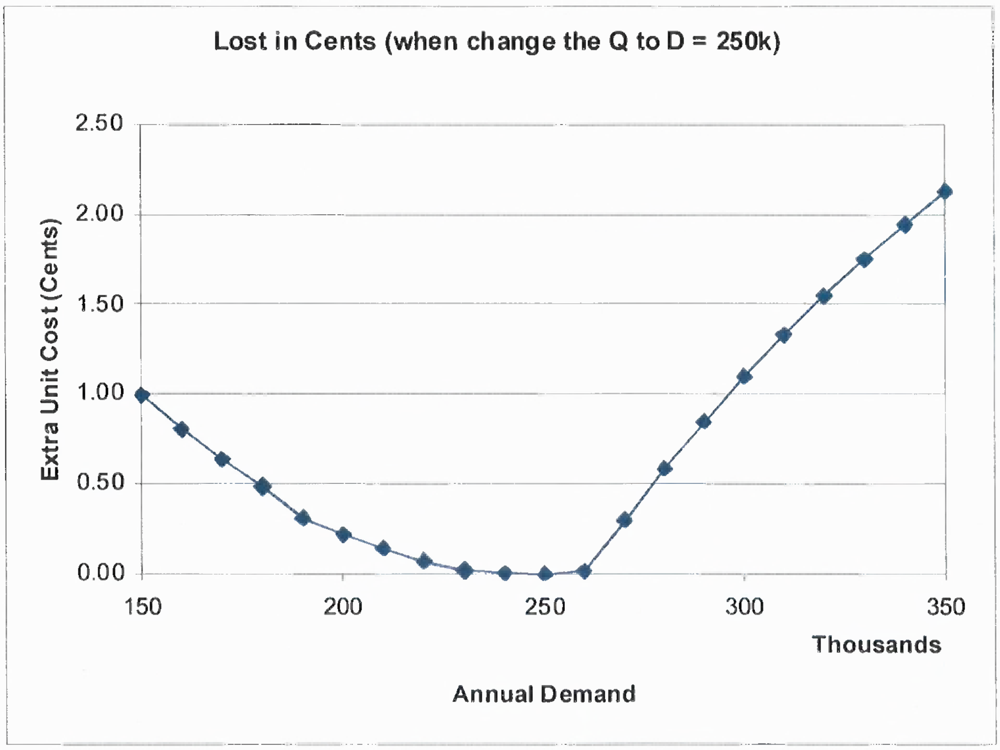


Figure 5.3 Extra cost from using a fixed Q at D = 250k.

Figure 5.3 shows the extra unit inventory cost from setting the order quantity at 250000. The cost went up from zero to about 1 cent at the lower limit of the demand shift range (D = 150k) and 2.2 cents at the upper limit (D = 350k).

5.3 Fixed Production Period Strategy

The other strategy that the supply chain contract may use to maximize the robustness in the supply chain with demand uncertainty is to determine the seller production period frequency. This is the length of time from the starting of the new batch to the time the subsequent batch starts. The production period may be every 5 days, every week, every 2 weeks, or every month, etc. The sizes of the batch are varied by the demand level D and can be determined from:

$$B_1 = \frac{D}{T} \text{ ; When T is the production period} \tag{5.1}$$

Fixed production period strategy simplifies the labor and machine resource scheduling tasks. The manager of the seller production facility knows exactly how many batches they will be running in a year and when they should be started.

The process to determine this task is to select a fixed production period T** that minimized the total expected cost over the demand shift range. For simplicity, it was assumed that the demand level follows a uniform distribution [D_1, D_2] when D_1 is the lower limit of the demand shift range and D_2 is the lower limit of the demand shift range. The T* is the T that minimizes this supply chain robustness integral term:

$$\int_{D_1}^{D_2} \left\{ TC(T^{**}) - TC(T^{*}) \right\} dD$$ When T* is the actual optimal T at each demand level.

The delivery batch size Q was fixed and set to the optimal value Q*, which was calculated at the average demand level.

A numerical example showed how the cost changes while the production period was adjusted. The similar parameters with Section 5.2 are used, except the order quantity was fixed at the Q = 5376, which was the optimal order quantity Q* at the average demand level.

Using the modified golden section search for optimal production batch size at each demand level, the extra cost incurred was calculated. The cost measured the robustness of this supply policy.

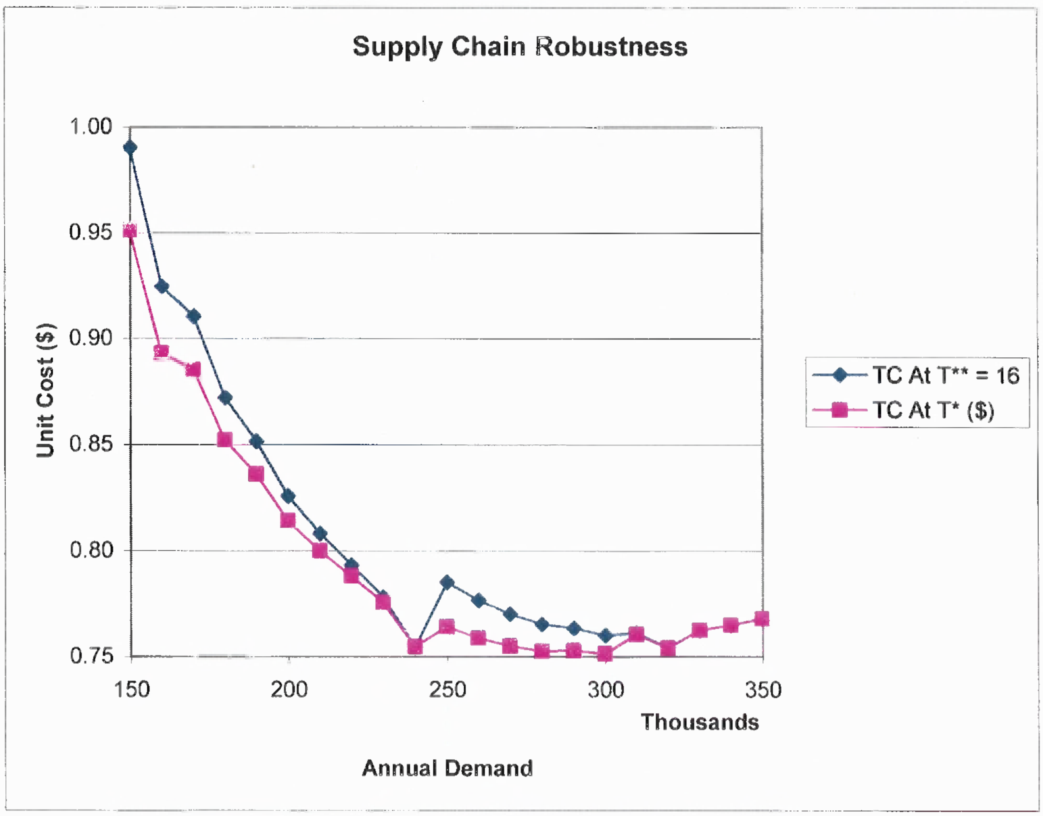


Figure 5.4 The unit cost comparison when T** = 16.

Figure 5.4 shows the cost comparison of using a fixed supply policy with 16 production periods per year. This means the new batch starts every 3.25 weeks. From the graph, this policy works well at the higher demand level. The extra cost when the demand level is greater than 300,000 was very small. The extra cost beyond the demand of 240k jumps up and decreases steadily as the demand increase. The extra cost below the demand level of 240k, decrease steadily as the demand decrease. This extra cost is highest at the demand level of 150k. This extra cost is shown in Figure 5.5.

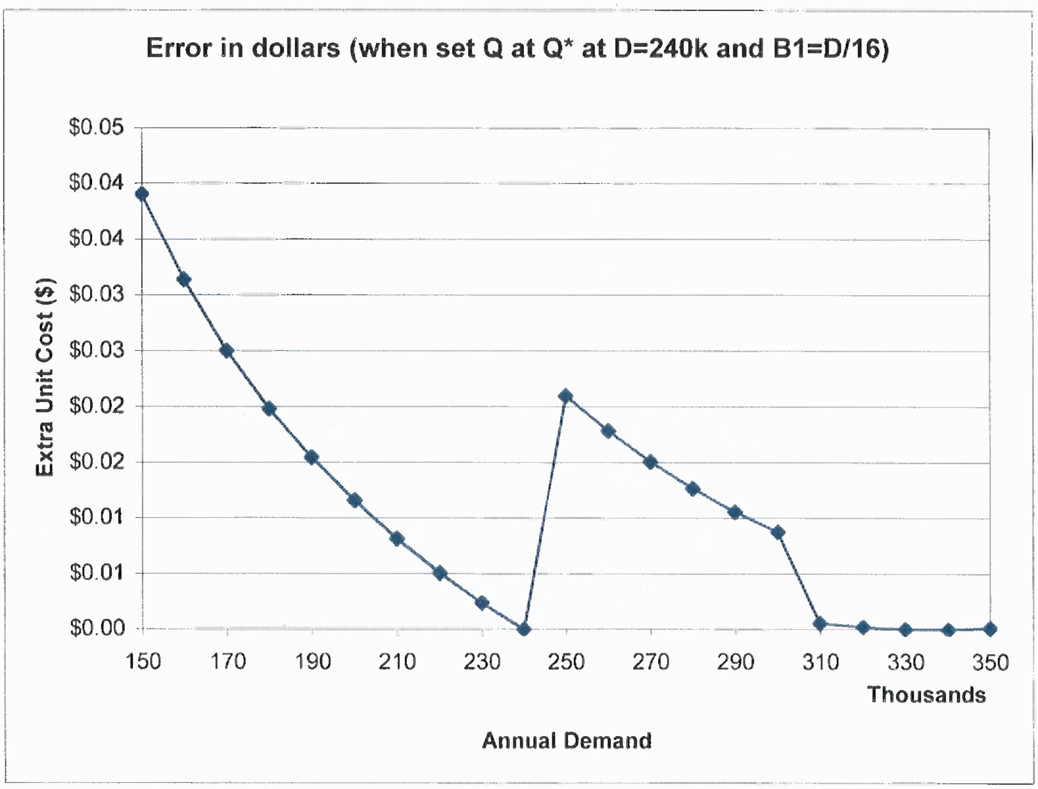


Figure 5.5 Extra cost from using a fixed T at 16 per year.

5.4 Combined Strategy

The combined strategy used both fixed replenishment delivery batch size and fixed production period strategy to achieve the highest robustness in the chain. The process start from finding the Q** that maximizes the total expected cost over the demand shift range when the seller production batch size is fixed at B_l*, which is calculated from the average demand level (similar to Section 5.2). The Q** is the Q that maximizes the supply chain robustness from this integral:

$$\int_{D_1}^{D_2} \{TC(Q^{**}) - TC(Q^{*})\}\, dD$$ When Q* is the actual optimal Q at each demand level.

After getting the Q**, the same process with fixed production period in Section 5.3 was performed. However, instead of using the optimal value Q* is calculated at the average demand level, the Q** from the above calculation step as an order quantity that will is used in finding the T** from the robustness equation. The T** is the T that maximizes the supply chain robustness from this integral:

$$\int_{D_1}^{D_2} \{TC(T^{**}, Q^{**}) - TC(T^{*}, Q^{**})\}\, dD$$

When Q** is the optimal Q at that maximize the robustness (minimize the total expected extra inventory cost) over the demand shift range and T* is the actual optimal T at each demand level.

Using combined policy, the supply policy contract will be set at the replenishment order quantity of Q** and production period T**. This policy is most efficient in maximizes the robustness of the chain by minimizing the robustness of the extra cost incurred from using fixed policy instead of continuous supply policy adjustment.

CHAPTER 6

CONCLUSION

This dissertation focused on modeling the inventory behavior between supplier pairs in an uncoordinated supply chain. The majority of the existing supply chain research assumes that supply relationships are coordinated or synchronized typically with a central SCM manager. In reality, many supply chains have minimal coordination even when they are managed through an ERP system. The inventory behavior of uncoordinated supply chains are known to be complex and difficult to estimate. This research presented models to reliably estimate the inventory costs, and demonstrated the use of these models to achieve at least partial coordination.

6.1 Significant Results

The $(Q, R, \delta)^2$ developed by Nearier (2008) has been revised to improve the accuracy in inventory approximation of residual inventory for buyer and seller in Chapter 3. The revised model added the new seller inventory in case of no replenishment during the production. The new model has been validated with the inventory simulation experiment. From the statistical test with 50 experiments, the new model reduced the errors of the original model from 10-30% to within 5% on the buyer side and 7% on the seller side. This revised model was used in later part of the research.

In Chapter 4, the revised $(Q, R, \delta)^2$ was proven to be a non-differentiable step-wised convex function. From numerical examples, it is verified that the optimal replenishment lot size may not be at the step point. This dissertation derived the heuristic to approximate the optimal replenishment lot size. This approximate lot size was used as a starting point in a modified golden section search for an actual optimal lot size. The process has been automated using Macro modules in Microsoft Excel.

Finally, the robustness of $(Q, R, \delta)^2$ supply policy derived is studied in Chapter 5. The study assumed that the demand can be shifted to new levels over the time. The robustness is defined as the extra cost incurred while keeping the fixed policy to the new demand levels. The model proposed three strategies to maximize the robustness (minimize the extra cost incurred) for a given demand pattern. The proposed strategies are (a) fixed batch strategy, (b) fixed period strategy, and (c) combined strategy. The fixed batch strategy calculates the optimal replenishment batch size. The fixed period strategy calculates the frequency of starting a new production batch size. It is found that combined fixed batch and fixed period strategy gave best result for a given scenario.

6.2 Future Research Suggestions

Several future research tasks and objectives based on this work are possible, including:

- Further evolution of the $(Q, R, \delta)^2$ models with objective of improving the estimation accuracy in the 2-3% range. Especially, in approximate the residual inventory in production and no-production cycle.
- Some assumption may be removed from the model. The transportation lead time should reflect the inventory received at the buyer incoming inventory side. The production lead time may also delay the output from the seller. There may be an uncertainty in the lead time.
- The backorder may be allowed. The seller may choose to hold the buyer order on backorder for a certain time before shipping it out.

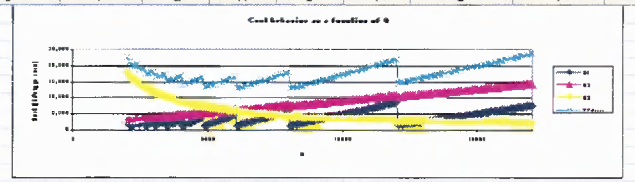
- The different demand distribution may be studied, e.g., normal, triangular, log-normal, gamma, exponential, etc.
- Multi-echelons in the chain may be developed from this research.

APPENDIX A

MICROSOFT EXCEL SCREENSHOTS

Microsoft Excel screenshots used in the models are shown in Figure A.1 to A.3. The related formulas are shown in Appendix B.

Row	A	B	D	E	H	I	J	M	N
1	`								
2									
3	Parameters		B1	30000	Cost				
4			B2	24000	Ch	7	per item per year	MIN TC	$12,877.65
5			Delta1	1000	Cq	$1,450	per order	Q*	
6			Delta2	400				supply cyc	365
7	Ann. Dema	240000	R1	varies by Q	Exact				
8	cycle/yr	10	R2	1000	N	1			
9			Z	1	X4	0.8			
10			D	65.753425					



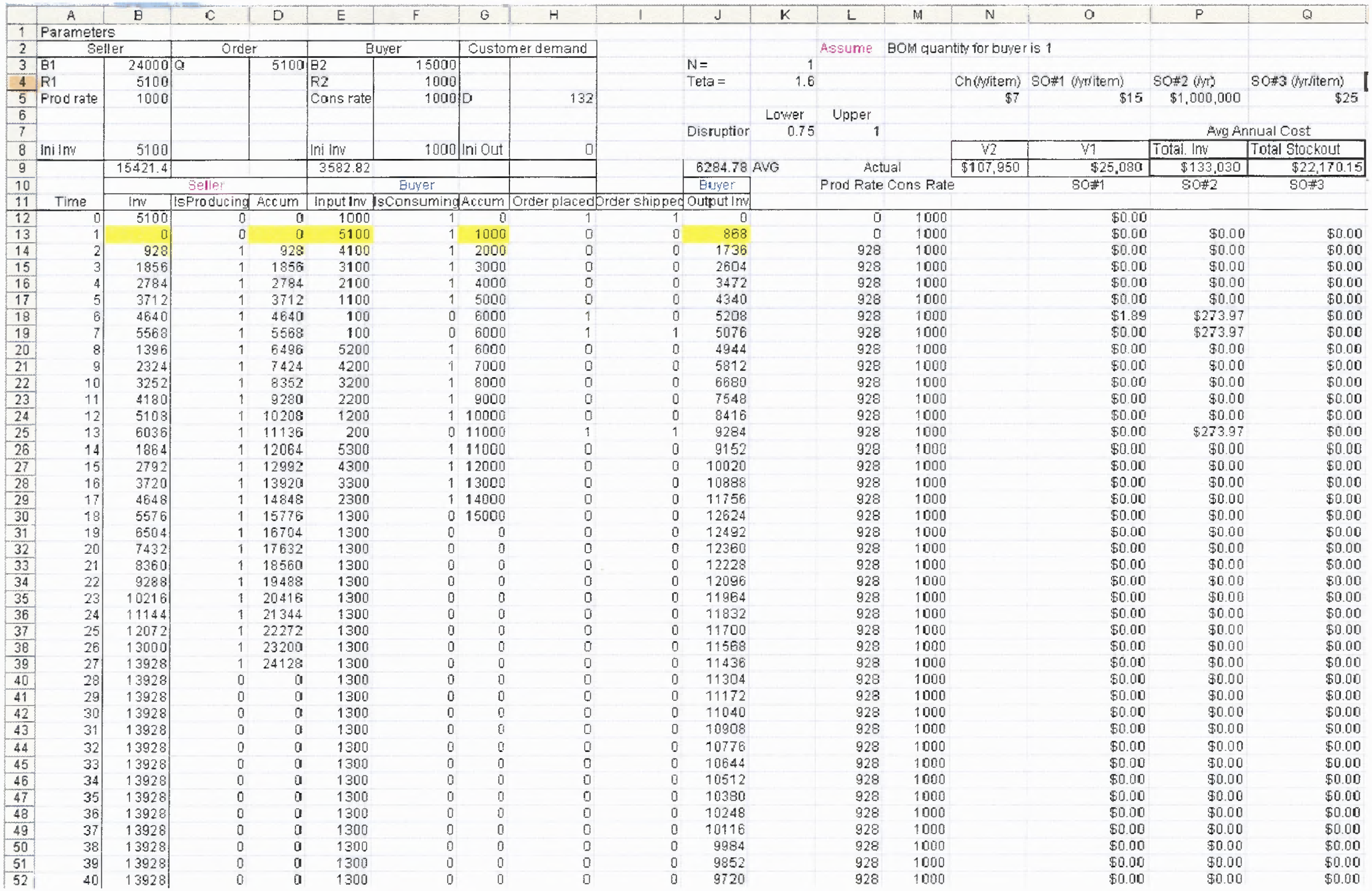
Row	Q	X1	X3	U	V1	V2	V3	TC/cycle	Y1	V1+V2
19	2600	9	4	6600	$2,019.45	$3,554.41	$13,385	$18,958.47	0.230769	$5,574
20	2700	8	4	8400	$1,030.82	$3,526.37	$12,889	$17,446.08	0.888889	$4,557
21	2800	8	4	7600	$1,563.01	$3,646.69	$12,429	$17,638.28	0.571429	$5,210
22	2900	8	4	6800	$2,095.21	$3,770.57	$12,000	$17,865.78	0.275862	$5,866
23	3000	8	4	6000	$2,627.40	$3,898.00	$11,600	$18,125.40	0	$6,525
24	3100	7	3	8300	$1,346.30	$3,823.14	$11,226	$16,395.25	0.741935	$5,169
25	3200	7	3	7600	$1,820.00	$3,937.69	$10,875	$16,632.69	0.5	$5,758
26	3300	7	3	6900	$2,293.70	$4,054.97	$10,545	$16,894.12	0.272727	$6,349
27	3400	7	3	6200	$2,767.40	$4,174.97	$10,235	$17,177.66	0.058824	$6,942
28	3500	6	3	9000	$1,193.84	$4,084.90	$9,943	$15,221.60	0.857143	$5,279
29	3600	6	3	8400	$1,609.04	$4,192.25	$9,667	$15,467.96	0.666667	$5,801
30	3700	6	3	7800	$2,024.25	$4,301.65	$9,405	$15,731.30	0.486486	$6,326
31	3800	6	3	7200	$2,439.45	$4,413.10	$9,158	$16,010.45	0.315789	$6,853
32	3900	6	3	6600	$2,854.66	$4,526.61	$8,923	$16,304.34	0.153846	$7,381
33	4000	6	3	6000	$3,269.86	$4,642.17	$8,700	$16,612.03	0	$7,912
34	4100	5	2	9500	$1,286.85	$4,500.91	$8,488	$14,275.56	0.853659	$5,788
35	4200	5	2	9000	$1,643.56	$4,601.35	$8,286	$14,530.63	0.714286	$6,245
36	4300	5	2	8500	$2,000.27	$4,703.23	$8,093	$14,796.53	0.581395	$6,704
37	4400	5	2	8000	$2,356.99	$4,806.55	$7,909	$15,072.63	0.454545	$7,164
38	4500	5	2	7500	$2,713.70	$4,911.31	$7,733	$15,358.34	0.333333	$7,625
39	4600	5	2	7000	$3,070.41	$5,017.51	$7,565	$15,653.14	0.217391	$8,088
40	4700	5	2	6500	$3,427.12	$5,125.14	$7,404	$15,956.52	0.106383	$8,552
41	4800	5	2	6000	$3,783.84	$5,234.22	$7,250	$16,268.05	0	$9,018
42	4900	4	2	10400	$1,274.38	$5,048.15	$7,102	$13,424.58	0.897959	$6,323
43	5000	4	2	10000	$1,572.60	$5,142.16	$6,960	$13,674.76	0.8	$6,715
44	5100	4	2	9600	$1,870.82	$5,237.13	$6,824	$13,931.48	0.705882	$7,108
45	5200	4	2	9200	$2,169.04	$5,333.07	$6,692	$14,194.42	0.615385	$7,502
46	5300	4	2	8800	$2,467.26	$5,429.97	$6,566	$14,463.26	0.528302	$7,897
47	5400	4	2	8400	$2,765.48	$5,527.83	$6,444	$14,737.75	0.444444	$8,293
48	5500	4	2	8000	$3,063.70	$5,626.65	$6,327	$15,017.62	0.363636	$8,690
49	5600	4	2	7600	$3,361.92	$5,726.44	$6,214	$15,302.64	0.285714	$9,088
50	5700	4	2	7200	$3,660.14	$5,827.19	$6,105	$15,592.59	0.210526	$9,487
51	5800	4	2	6800	$3,958.36	$5,928.90	$6,000	$15,887.26	0.137931	$9,887
52	5900	4	2	6400	$4,256.58	$6,031.58	$5,898	$16,186.46	0.067797	$10,288
53	6000	4	2	6000	$4,554.79	$6,135.22	$5,800	$16,490.02	0	$10,690
54	6100	3	1	11700	$1,284.93	$5,887.80	$5,705	$12,877.65	0.934426	$7,173
55	6200	3	1	11400	$1,524.66	$5,975.34	$5,613	$13,112.90	0.870968	$7,500
56	6300	3	1	11100	$1,764.36	$6,063.44	$5,524	$13,351.64	0.809524	$7,828
57	6400	3	1	10800	$2,004.11	$6,152.11	$5,438	$13,593.72	0.75	$8,156
58	6500	3	1	10500	$2,243.84	$6,241.35	$5,354	$13,839.04	0.692308	$8,485
59	6600	3	1	10200	$2,483.56	$6,331.16	$5,273	$14,087.45	0.636364	$8,815
60	6700	3	1	9900	$2,723.29	$6,421.54	$5,194	$14,338.85	0.58209	$9,145
61	6800	3	1	9600	$2,963.01	$6,512.48	$5,118	$14,593.14	0.529412	$9,475
62	6900	3	1	9300	$3,202.74	$6,603.99	$5,043	$14,850.21	0.478261	$9,807

		V1					V2			
Row	Q	Reorder	Complete Cycle	Partial Cycle	Static	SUM V1	Reorder	Production	No production	SUM V2
19	2600	$700.00	$145.85	$3.74	$760.41	$1,610.00	$1,820.00	$14,382.86	$963.70	$17,166.55
20	2700	$700.00	$139.81	$15.53	$789.66	$1,645.00	$1,890.00	$14,280.05	$923.65	$17,093.70
21	2800	$700.00	$150.36	$10.74	$818.90	$1,680.00	$1,960.00	$14,307.65	$942.64	$17,210.29
22	2900	$700.00	$161.29	$5.56	$848.15	$1,715.00	$2,030.00	$14,336.02	$962.32	$17,328.33
23	3000	$700.00	$172.60	$0.00	$877.40	$1,750.00	$2,100.00	$8,752.11	$310.68	$11,162.79
24	3100	$700.00	$161.26	$17.09	$906.64	$1,785.00	$2,170.00	$14,316.99	$930.02	$17,417.01
25	3200	$700.00	$171.84	$12.27	$935.89	$1,820.00	$2,240.00	$14,344.22	$946.94	$17,531.16
26	3300	$700.00	$182.74	$7.12	$965.14	$1,855.00	$2,310.00	$14,372.16	$964.39	$17,646.55
27	3400	$700.00	$193.99	$1.63	$994.38	$1,890.00	$2,380.00	$14,400.81	$982.38	$17,763.19
28	3500	$700.00	$176.20	$25.17	$1,023.63	$1,925.00	$2,450.00	$14,242.23	$918.68	$17,610.91
29	3600	$700.00	$186.41	$20.71	$1,052.88	$1,960.00	$2,520.00	$14,261.40	$932.98	$17,714.38
30	3700	$700.00	$196.91	$15.97	$1,082.12	$1,995.00	$2,590.00	$14,280.98	$947.68	$17,818.66
31	3800	$700.00	$207.70	$10.93	$1,111.37	$2,030.00	$2,660.00	$14,300.97	$962.78	$17,923.74
32	3900	$700.00	$218.77	$5.61	$1,140.62	$2,065.00	$2,730.00	$14,321.35	$978.28	$18,029.63
33	4000	$700.00	$230.14	$0.00	$1,169.86	$2,100.00	$2,800.00	$14,177.46	$994.19	$17,971.65
34	4100	$700.00	$201.49	$34.40	$1,199.11	$2,135.00	$2,870.00	$14,280.53	$913.79	$18,064.32
35	4200	$700.00	$211.44	$30.21	$1,228.36	$2,170.00	$2,940.00	$14,299.70	$925.73	$18,165.43
36	4300	$700.00	$221.63	$25.77	$1,257.60	$2,205.00	$3,010.00	$14,319.25	$937.95	$18,267.21
37	4400	$700.00	$232.05	$21.10	$1,286.85	$2,240.00	$3,080.00	$14,339.19	$950.47	$18,369.66
38	4500	$700.00	$242.72	$16.18	$1,316.10	$2,275.00	$3,150.00	$10,828.29	$963.27	$14,941.56
39	4600	$700.00	$253.63	$11.03	$1,345.34	$2,310.00	$3,220.00	$14,380.22	$976.36	$18,576.57
40	4700	$700.00	$264.78	$5.63	$1,374.59	$2,345.00	$3,290.00	$14,401.31	$989.73	$18,681.04
41	4800	$700.00	$276.16	$0.00	$1,403.84	$2,380.00	$3,360.00	$10,907.02	$1,003.40	$15,270.42
42	4900	$700.00	$230.23	$51.68	$1,433.08	$2,415.00	$3,430.00	$14,168.35	$902.23	$18,500.59
43	5000	$700.00	$239.73	$47.95	$1,462.33	$2,450.00	$3,500.00	$9,786.99	$239.73	$13,526.72
44	5100	$700.00	$249.41	$44.01	$1,491.58	$2,485.00	$3,570.00	$6,683.32	$249.41	$10,502.73
45	5200	$700.00	$259.29	$39.89	$1,520.82	$2,520.00	$3,640.00	$13,702.05	$931.29	$18,273.34
46	5300	$700.00	$269.36	$35.58	$1,550.07	$2,555.00	$3,710.00	$13,963.38	$941.36	$18,614.73
47	5400	$700.00	$279.62	$31.07	$1,579.32	$2,590.00	$3,780.00	$14,224.12	$951.62	$18,955.74
48	5500	$700.00	$290.07	$26.37	$1,608.56	$2,625.00	$3,850.00	$13,241.55	$962.07	$18,053.61
49	5600	$700.00	$300.71	$21.48	$1,637.81	$2,660.00	$3,920.00	$14,247.50	$972.71	$19,140.22
50	5700	$700.00	$311.55	$16.40	$1,667.05	$2,695.00	$3,990.00	$14,259.43	$983.55	$19,232.97
51	5800	$700.00	$322.58	$11.12	$1,696.30	$2,730.00	$4,060.00	$14,271.50	$994.58	$19,326.08
52	5900	$700.00	$333.79	$5.66	$1,725.55	$2,765.00	$4,130.00	$14,283.73	$1,005.79	$19,419.52
53	6000	$700.00	$345.21	$0.00	$1,754.79	$2,800.00	$4,200.00	$8,683.07	$345.21	$13,228.27
54	6100	$700.00	$267.61	$83.35	$1,784.04	$2,835.00	$4,270.00	$14,245.12	$886.08	$19,401.21
55	6200	$700.00	$276.45	$80.26	$1,813.29	$2,870.00	$4,340.00	$14,257.36	$893.16	$19,490.52
56	6300	$700.00	$285.44	$77.02	$1,842.53	$2,905.00	$4,410.00	$14,269.78	$900.35	$19,580.13
57	6400	$700.00	$294.58	$73.64	$1,871.78	$2,940.00	$4,480.00	$14,282.36	$907.66	$19,670.02
58	6500	$700.00	$303.85	$70.12	$1,901.03	$2,975.00	$4,550.00	$14,295.12	$915.08	$19,760.20
59	6600	$700.00	$313.27	$66.45	$1,930.27	$3,010.00	$4,620.00	$14,308.05	$922.62	$19,850.67
60	6700	$700.00	$322.84	$62.64	$1,959.52	$3,045.00	$4,690.00	$14,321.16	$930.27	$19,941.43
61	6800	$700.00	$332.55	$58.66	$1,988.77	$3,080.00	$4,760.00	$11,771.63	$938.04	$17,469.67
62	6900	$700.00	$342.40	$54.59	$2,018.01	$3,115.00	$4,830.00	$14,347.88	$945.92	$20,123.81

Figure A.1 Microsoft Excel screenshot of $(Q, R, \delta)^2$ order quantity optimization experiments.

	A	B	C	D	E	F	G	H	I	J	K
1	Parameters										
2	Seller		Order		Buyer		Customer demand				
3	B1	24000	Q	5100	B2	15000				N =	1
4	R1	5100			R2	1000				Teta =	1.6
5	Prod rate	1000			Cons rate	400	D	33		Ubar=	0.2
6											
7											
8	Ini Inv	5100			Ini Inv	1000	Ini Out	0			
9		16343.48			3414.946					6957.694	AVG
10			Seller			Buyer				Buyer	
11	Time	Inv	IsProducing	Accum	Input Inv	IsConsuming	Accum	Order placed	Order shipped	Output Inv	
12	0	5100	0	0	1000	1	0	1	1	0	
13	1	0	0	0	5700	1	400	0	0	367	
14	2	1000	1	1000	5300	1	800	0	0	734	
15	3	2000	1	2000	4900	1	1200	0	0	1101	
16	4	3000	1	3000	4500	1	1600	0	0	1468	
17	5	4000	1	4000	4100	1	2000	0	0	1835	
18	6	5000	1	5000	3700	1	2400	0	0	2202	
19	7	6000	1	6000	3300	1	2800	0	0	2569	
20	8	7000	1	7000	2900	1	3200	0	0	2936	
21	9	8000	1	8000	2500	1	3600	0	0	3303	
22	10	9000	1	9000	2100	1	4000	0	0	3670	
23	11	10000	1	10000	1700	1	4400	0	0	4037	
24	12	11000	1	11000	1300	1	4800	0	0	4404	
25	13	12000	1	12000	900	1	5200	1	1	4771	
26	14	7900	1	13000	5600	1	5600	0	0	5138	
27	15	8900	1	14000	5200	1	6000	0	0	5505	
28	16	9900	1	15000	4800	1	6400	0	0	5872	
29	17	10900	1	16000	4400	1	6800	0	0	6239	
30	18	11900	1	17000	4000	1	7200	0	0	6606	

Figure A.2 Microsoft Excel screenshot for inventory simulation model without stockouts.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Parameters																
2	Seller		Order		Buyer		Customer demand					Assume	BOM quantity for buyer is 1				
3	B1	24000	Q	5100	B2	15000				N =	1						
4	R1	5100			R2	1000				Teta =	1.6			Ch($/yr/item)	SO#1 ($/yr/item)	SO#2 ($/yr)	SO#3 ($/yr/item)
5	Prod rate	1000			Cons rate	1000	D	132						$7	$15	$1,000,000	$25
6											Lower	Upper					
7										Disruptior	0.75	1				Avg Annual Cost	
8	Ini Inv	5100			Ini Inv	1000	Ini Out	0						V2	V1	Total. Inv	Total Stockout
9		15421.4			3582.82					6284.78	AVG	Actual		$107,950	$25,080	$133,030	$22,170.15
10			Seller			Buyer				Buyer		Prod Rate	Cons Rate		SO#1	SO#2	SO#3
11	Time	Inv	IsProducing	Accum	Input Inv	IsConsuming	Accum	Order placed	Order shipped	Output Inv							
12	0	5100	0	0	1000	1	0	1	1	0		0	1000		$0.00		
13	1	0	0	0	5100	1	1000	0	0	868		0	1000		$0.00	$0.00	$0.00
14	2	928	1	928	4100	1	2000	0	0	1736		928	1000		$0.00	$0.00	$0.00
15	3	1856	1	1856	3100	1	3000	0	0	2604		928	1000		$0.00	$0.00	$0.00
16	4	2784	1	2784	2100	1	4000	0	0	3472		928	1000		$0.00	$0.00	$0.00
17	5	3712	1	3712	1100	1	5000	0	0	4340		928	1000		$0.00	$0.00	$0.00
18	6	4640	1	4640	100	0	6000	1	0	5208		928	1000		$1.89	$273.97	$0.00
19	7	5568	1	5568	100	0	6000	1	1	5076		928	1000		$0.00	$273.97	$0.00
20	8	1396	1	6496	5200	1	6000	0	0	4944		928	1000		$0.00	$0.00	$0.00
21	9	2324	1	7424	4200	1	7000	0	0	5812		928	1000		$0.00	$0.00	$0.00
22	10	3252	1	8352	3200	1	8000	0	0	6680		928	1000		$0.00	$0.00	$0.00
23	11	4180	1	9280	2200	1	9000	0	0	7548		928	1000		$0.00	$0.00	$0.00
24	12	5108	1	10208	1200	1	10000	0	0	8416		928	1000		$0.00	$0.00	$0.00
25	13	6036	1	11136	200	0	11000	1	1	9284		928	1000		$0.00	$273.97	$0.00
26	14	1864	1	12064	5300	1	11000	0	0	9152		928	1000		$0.00	$0.00	$0.00
27	15	2792	1	12992	4300	1	12000	0	0	10020		928	1000		$0.00	$0.00	$0.00
28	16	3720	1	13920	3300	1	13000	0	0	10888		928	1000		$0.00	$0.00	$0.00
29	17	4648	1	14848	2300	1	14000	0	0	11756		928	1000		$0.00	$0.00	$0.00
30	18	5576	1	15776	1300	0	15000	0	0	12624		928	1000		$0.00	$0.00	$0.00
31	19	6504	1	16704	1300	0	0	0	0	12492		928	1000		$0.00	$0.00	$0.00
32	20	7432	1	17632	1300	0	0	0	0	12360		928	1000		$0.00	$0.00	$0.00
33	21	8360	1	18560	1300	0	0	0	0	12228		928	1000		$0.00	$0.00	$0.00
34	22	9288	1	19488	1300	0	0	0	0	12096		928	1000		$0.00	$0.00	$0.00
35	23	10216	1	20416	1300	0	0	0	0	11964		928	1000		$0.00	$0.00	$0.00
36	24	11144	1	21344	1300	0	0	0	0	11832		928	1000		$0.00	$0.00	$0.00
37	25	12072	1	22272	1300	0	0	0	0	11700		928	1000		$0.00	$0.00	$0.00
38	26	13000	1	23200	1300	0	0	0	0	11568		928	1000		$0.00	$0.00	$0.00
39	27	13928	1	24128	1300	0	0	0	0	11436		928	1000		$0.00	$0.00	$0.00
40	28	13928	0	0	1300	0	0	0	0	11304		928	1000		$0.00	$0.00	$0.00
41	29	13928	0	0	1300	0	0	0	0	11172		928	1000		$0.00	$0.00	$0.00
42	30	13928	0	0	1300	0	0	0	0	11040		928	1000		$0.00	$0.00	$0.00
43	31	13928	0	0	1300	0	0	0	0	10908		928	1000		$0.00	$0.00	$0.00
44	32	13928	0	0	1300	0	0	0	0	10776		928	1000		$0.00	$0.00	$0.00
45	33	13928	0	0	1300	0	0	0	0	10644		928	1000		$0.00	$0.00	$0.00
46	34	13928	0	0	1300	0	0	0	0	10512		928	1000		$0.00	$0.00	$0.00
47	35	13928	0	0	1300	0	0	0	0	10380		928	1000		$0.00	$0.00	$0.00
48	36	13928	0	0	1300	0	0	0	0	10248		928	1000		$0.00	$0.00	$0.00
49	37	13928	0	0	1300	0	0	0	0	10116		928	1000		$0.00	$0.00	$0.00
50	38	13928	0	0	1300	0	0	0	0	9984		928	1000		$0.00	$0.00	$0.00
51	39	13928	0	0	1300	0	0	0	0	9852		928	1000		$0.00	$0.00	$0.00
52	40	13928	0	0	1300	0	0	0	0	9720		928	1000		$0.00	$0.00	$0.00

Figure A.3 Microsoft Excel screenshot for inventory simulation model with disruption/stockouts.

APPENDIX B

MICROSOFT EXCEL FORMULAS FOR EXPERIMENTS

Microsoft Excel formulas used in the Appendix A's models are shown in Table B.1 to B.3.

Table B.1 Microsoft Excel Formulas for $(Q, R, \delta)^2$ Model

Parameters

B1 = $E$3

B2 = $E$4

Delta1 = $E$5

Delta2 = $E$6

R1 = $E$7 (varies by Q)

R2 = $E$8

Z = $E$9

Annual Demand = $B$7

Cq = $I$5

N = $I$8 = ROUNDUP($E$4*$E$9/$E$3,0)

X4 = $I$9 = $E$4*$E$9/($I$8*$E$3)

D = $E$10 = $B$7/3650

Cycle/yr = B7/E4

Sample Calculation of the Cost at Each Size of Order Quantity (each row in MS Excel)

Q = $A$19

X1 = $B$19 = INT($E$4*$E$9/A19)

X3 = $C$19 =MIN(INT($I$8*$E$3*$E$9*$E$6/(A19*$E$5)),B19)

U = $D$19 = ($I$8*$E$3-B19*A19)

V1 = $E$19

=$I$4*(($E$8*$E$4/$E$10)+(B19+J19)*(A19/2)*(A19/($E$6*$E$9))+(A19*$E$4*(1-J19)*(1/$E$10-1/$E$6)))/3650

V2 = $F$19

=(($I$4/3650)*A19*$E$4/$E$10)+(($I$4/3650)*(A19^2/($E$6*$E$9))*(0.5*B19^2+0.5*B19))+((($I$4/3650)*$I$9/10)*((A19^2/($E$6*$E$9))*((C19^2-2.5*C19+1.5)*($E$5/($E$6*$E$9))+(2.5*C19-0.5*C19^2-B19*C19-1.5))-(($I$8*$E$3-(C19-1)*A19)^2/(2*$E$5))+D19*($E$4/$E$10-(C19-0.5)*(A19/($E$6*$E$9)))+($I$8*$E$3-$E$4*$E$9)*(10/$I$9-1)))

V3 = $G$19 =$E$4*$E$9*$I$5/A19

TC/cycle = $H$19 = =SUM(E19:G19)

Y1 = $J$19 =$E$4*$E$9/A19-INT($E$4*$E$9/A19)

Sample Calculation of the individual V1 Cost Component at Each Size of Order Quantity (each row in MS Excel)

Reorder = $N$19 = ($I$4/3650)*$E$8*$E$4/$E$10

Complete Cycle = $O$19 =($I$4/3650)*(B13)*(A13/2)*(A13/($E$6*$E$9))

Partial Cycle = $P$19 =($I$4/3650)*(J13)*(A13/2)*(A13/($E$6*$E$9))

Static = $Q$19 =($I$4/3650)*A13*$E$4*(0.5)*(1/$E$10-1/$E$6)

SUM V1 = $R$19 =SUM(N13:Q13)

Sample Calculation of the individual V1 Cost Component at Each Size of Order Quantity (each row in MS Excel)

Reorder = $T$19 =($I$4/3650)*A13*$E$4/$E$10

Production = $U$19

=($I$4/3650)*($I$9)*((C13)*($E$6/2)*(A13/($E$6*$E$9))^2+0+(C13)*(C13-1)*(A13^2/($E$6*$E$9))*($E$5/($E$6*$E$9)-1)+0+(B13-C13)*(B13+1-C13)*(A13/2)*(A13/($E$6*$E$9))-($E$5*($I$8*$E$3/$E$5-C13*A13/$E$6)^2)/2+0+(1-0*(IF(J13=0,0,(1-J13)/2)))*$E$4*($E$4/$E$10-C13*(A13/($E$6*$E$9)))+0.4*$E$4*C13*A13/($E$4*$E$9))

No production = $V$19

=($I$4/3650)*(1-$I$9)*(B13*(B13+1)*(A13/2)*(A13/($E$6*$E$9))+0.2*$E$4*$E$4/$E$10)

SUM V2 = =SUM(T13:V13)

Table B.2 Microsoft Excel Formulas for Discrete Inventory Simulation Model without Stockout

Seller Parameters

B1 = $B$3

R1 = $B$4

Prod Rate = $B$5

Initial Inventory = $B$8

Buyer Parameters

B2 = $F$3

R2 = $F$4

Cons rate = $F$5

Initial Input Inventory = $F$8

Initial Output Inventory = $H$8

Other Parameters

Q = $D$3

D = $H$5

Ch = $7 /item/year

Sample Calculation of each time hour time period (each row in MS Excel)

Time = $A$13

Seller Output Inv = $B$13 =IF(C13=0,B12-I12*$D$3,B12+$B$5-I12*$D$3)

Seller IsProducing flag = $C$13

=IF(OR(B12<$B$4,AND(C12=1,D12<$B$3),AND(H12=1,I12=0)),1,0)

Seller Batch Accum. = $D$13 =IF(C13=1,D12+$B$5,0)

Buyer Input Inv = $E$13

=IF(I12=0,IF(E12-$F$5*F12>=0,E12-$F$5*F12,E12),E12-$F$5*F12+$D$3)

Buyer IsConsuming flag = $F$13

=IF(AND(F12=1,G12+$F$5<$F$3),IF(E13-$F$5>=0,1,0),IF(J12-$H$5<0,IF(E13-$F$5>=0,1,0),0))

Buyer Batch Accum = $G$13 = =IF(G12<$F$3,G12+F12*$F$5,0)

Order placed flag = $H$13 =IF(E13>$F$4,0,1)

Order shipped flag = $I$13 =IF(AND(H13=1,B13>=$D$3),1,0)

Buyer Output Inv = $J$13 =J12+F12*$F$5-$H$5

Inventory Cost Calculation

Avg seller outgoing Inv = $B$9 =AVERAGE(B12:B65012)

Avg buyer incoming Inv = $E$9 =AVERAGE(E12:E65012)

V1 = $O$9 =E9*7

V2 = $N$9 =B9*7

Table B.3 Microsoft Excel Formulas for Discrete Inventory Simulation Model with Disruption

Seller Parameters

B1 = $B$3

R1 = $B$4

Prod Rate = $B$5

Initial Inventory = $B$8

Buyer Parameters

B2 = $F$3

R2 = $F$4

Cons rate = $F$5

Initial Input Inventory = $F$8

Initial Output Inventory = $H$8

Other Parameters

Q = $D$3

D = $H$5

Lower limit seller production rate disruption = $K$7

Upper limit seller production rate disruption = $L$7

Ch = $N$4

Cs = $O$4

Cb = $P$4

Cf = $Q$4

Sample Calculation of each time hour time period (each row in MS Excel)

Time = $A$13

Seller Output Inv = $B$13 =IF(C13=0,B12-I12*$D$3,B12+L13-I12*$D$3)

Seller IsProducing flag = $C$13

=IF(OR(B12<$B$4,AND(C12=1,D12<$B$3),AND(H12=1,I12=0)),1,0)

Seller Batch Accum. = $D$13 =IF(C13=1,D12+L13,0)

Buyer Input Inv = $E$13

=IF(I12=0,IF(E12-M13*F12>=0,E12-M13*F12,E12),E12-M13*F12+$D$3)

Buyer IsConsuming flag = $F$13

=IF(F12=1,IF(G12+M13>=$F$3,0,IF(E13-

M13>=0,1,0)),IF(OR(G12=0,G12>=$F$3),IF(J12-$H$5<=0,1,0),IF(E13-M13>=0,1,0)))

Buyer Batch Accum = $G$13 =IF(G12<$F$3,G12+F12*M13,0)

Order placed flag = $H$13 =IF(OR(E13<=$F$4, AND(F13=0,G13<$F$3, G13>0)),1,0)

Order shipped flag = $I$13 =IF(AND(H13=1,B13>=$D$3),1,0)

Buyer Output Inv = $J$13 =J12+F12*M13-$H$5

Actual production rate = $L$13

=IF(AND(C14=1,C13=0),IF(RAND()<0.6,$B$5,ROUND($B$5*($K$7+RAND()*($L$7 -$K$7)),0)),L13)

Actual Consumption rate = $M$13 =$F$5

Inventory Cost Calculation

Avg seller outgoing Inv = $B$9 =AVERAGE(B12:B65012)

Avg buyer incoming Inv = $E$9 =AVERAGE(E12:E65012)

Stock Out Cost Calculation

SO#1 cost = $O$13 = IF(AND(H13=1,I13=0),($D$3-B13)*$O$5/3650,0)

SO#2 cost = $P$13 =IF(AND(F13=0,G13<$F$3, G13>0),$P$5/3650,0)

SO#3 cost =$Q$13 =IF(J13<0,J13*(-1)*$Q$5/3650,0)

Total SO cost = $R$13 =O13+P13+Q13

Total Cost calculation

Total Inventory Cost = $P$9 =N9+O9

Total Stockout Cost =SUM(R12:R65012)*3650/65000

REFERENCES

1. Banerjee, A. (1986) A Joint Economic-Lot-Size Model for Purchase and Vendor. *Decision Sciences,* 17, 292-311.

2. Boute, R.N., Disney S.M., Lambrecht, M.R. and Houdt, B.V. (2007) An integrated production and inventory model to dampen upstream demand variability in the supply chain. *European Journal of Operational Research,* 178, 121-142.

3. Braglia, M. and Zavanella, L. (2003) Modeling an industrial strategy for inventory management in supply chains: the 'Consignment Stock' case. *International Journal of Productional Research,* 16, 3793-3808.

4. Chan, C.K. and Kingsman, B.G. (2007) Coordination in a single-vendor multi-buyer supply chain by synchronizing delivery and production cycles. *Transportation Research Part E,* 43, 90-111.

5. Chiu, H.N. and Huang, H.L. (2003) A multi-echelon integrated JIT inventory model using the time buffer and emergency borrowing policies to deal with random delivery lead times. *International Journal of Production Research,* 41, 2911-2931.

6. Goyal, S.K. (1988) A Joint Economic-Lot-Size Model for Purchase and Vendor: A comment. *Decision Sciences,* 19, 236-241.

7. Goyal, S.K. (1995) A one-vendor multi-buyer integrated inventory model: A comment. *European Journal of Operational Research,* 125, 429-430.

8. Goyal, S.K. (2000) On improving the single-vendor single buyer integrated production inventory model with a generalized policy. *European Journal of Operational Research,* 125, 429-430.

9. Goyal, S.K. and Deshmukn, S.G. (1992) Integrated procument-production systems: A review. *European Journal of Operational Research,* 62, 1-10.

10. Hill, R.M. (1997) The single-vendor single-buyer integrated production-inventory model with a generalized policy. *European Journal of Operational Research,* 107, 493-499.

11. Hoberg K., Bradley J.R, and Thonemann, U.W. (2007) Analyzing the effect of the inventory policy on order and inventory variability with linear control theory. *European Journal of Operational Research,* 176, 1620-1642.

12. Kiefer, J. (1953) Sequential minimax search for a maximum. *Proceedings of the American Mathematical Society*, 4, 502–506.

13. Kelle, P., Al-khateeb, F. and Miller, P.A. (2003) Partnership and negotiation support by joint optimal ordering/setup policies for JIT. *International Journal of Production Economics,* 81-82, 431-441.

14. Lee, W. (2005) A joint economic lot size model for raw material ordering, manufacturing setup, and finished goods delivering. *Omega,* 33, 163-174.

15. Nearier, R.C. (2008) Modeling Inventory and Responsiveness Costs in a Supply Chain. *PhD Dissertation, New Jersey Institute of Technology*, Newark, New Jersey.

16. Pujawan, I.N. and Kingsman, B.G. (2002) Joint optimization of timing synchronization in a buyer supplier inventory system. *International Journal of Operational and Quantitative Management,* 8, 93-110.

17. Sucky, E.S. (2003) A single buyer-single supplier bargaining problem with asymmetric information – theoretical approach and software implementation, in *Procedings of the 36th Hawaii International Conference on System Sciences (HICSS'03).*

18. Visawanathan, S. and Piplani, R. (2001) Coordinating supply chain inventories through common replenishment epochs. *European Journal of Operational Research,* 129, 277-286.

19. Wu, K. and Ouyang, L. (2003) An integrated single-vendor single-buyer inventory system with shortage derived algebraically. *Production Planning and Control,* 14, 555-561.

20. Zimmer, K. (2001) Supply chain coordination with uncertain just-in-time delivery. *International Journal of Production Economics,* 77, 1-15.