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ABSTRACT

A GAIT GENERATION MECHANISM FOR LEG REHABILITATION THERAPY

**by
Yazan Ahmad Manna**

While most take their ability to walk for granted, some are unable to walk secondary to any number of pathologies, such as traumatic brain injury (TBI), spinal cord injury (SCI), cerebrovascular accidents (CVA), cerebral palsy (CP), multiple sclerosis (MS), Parkinson's disease, as well as various orthopedic conditions. Decreased activity has been shown to be associated with rapidly deconditioning and other co-morbidities. Rehabilitation techniques that afford patients the ability to begin reconditioning through walking sooner may ultimately enhance their return to a better quality of life.

The overall goal of this study is to design a gait generation mechanism for rehabilitation of paralyzed legs. This mechanism should provide an appropriate afferent input to the spinal cord by moving the legs in a physiological way.

This thesis focuses on the dimensional synthesis of a four-bar linkage to reproduce the desired ankle trajectory in the sagittal plane.

The data of sagittal flexion/extension at the hip and knee joints from healthy people are used to define the desired ankle trajectory during normal gait cycle. A path generation four-bar mechanism is then synthesized to obtain proper link and coupler dimensions. The resulted coupler curve matches well with the desired ankle trajectory.

**A GAIT GENERATION MECHANISM FOR LEG
REHABILITATION THERAPY**

by
Yazan Ahmad Manna

**A Thesis
Submitted to the Faculty of
New Jersey Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of
Master of Science in Mechanical Engineering**

Department of Mechanical Engineering

August 2005

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APPROVAL PAGE

**A GAIT GENERATION MECHANISM FOR LEG
REHABILITATION THERAPY**

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To my beloved Mother, my dear Father, my Sister and my little Brother

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LIST OF SYMBOLS

hfe	Hip Flexion and Extension angles.
kfe	Knee Flexion and Extension angles.
apd	Ankle Plantar Flexion and DorsiFlexion angles.
X_0	X-axis of frame {0}.
Y_0	Y-axis of frame {0}.
X_1	X-axis of frame {1}.
Y_1	Y-axis of frame {1}.
X_2	X-axis of frame {2}.
Y_2	Y-axis of frame {2}.
X_3	X-axis of frame {3}.
Y_3	Y-axis of frame {3}.
α_{i-1}	Twist angle of link i-1.
a_{i-1}	Length of link i-1.
d_i	Offset of link i.
θ_i	Angle of joint i.
${}^{i-1}T_i$	Transformation matrix between frame “i-1” and frame “i”.
$C\theta$	Cos θ .
$S\theta$	Sin θ .
0R_3	The orientation matrix that defines the cosine directions of frame {3} and frame {0}.

LIST OF SYMBOLS
(Continued)

${}^{base}R_0$	The orientation matrix that defines the cosine directions of frame {0} and the base frame.
${}^{S3}X_{base}$	The distance from the origin of frame {S3} and the base frame in the X- direction of frame {S3}.
${}^{S3}Y_{base}$	The distance from the origin of frame {S3} and the base frame in the Y- direction of frame {S3}.
H	Height of a person
r1	Frame length.
Q1	Frame angle.
r2	Crank length.
r3	Coupler length.
r4	Rocker length.
Cr1	Coupler point length or radius.
Beta1	Angle from coupler line to coupler point.

CHAPTER 1

PROBLEM STATEMENT

Recent studies have confirmed that regular treadmill training can improve walking capabilities in incomplete spinal cord-injured subjects [1]. To assist the functional recovery of people with spinal cord injury, an appropriate afferent input to the spinal cord will help in the therapy of the patient [1]. The main goal of treadmill training is to "teach" the patient to walk again [1]. One type of therapy is accomplished with the help of physiotherapists. At the beginning of this therapy, the leg movements of the patients have to be assisted by physiotherapists during gait on the moving treadmill [1]. The physical capabilities and the individual experience of the therapists usually limit this training [1]. Another method of therapy can be achieved using rehabilitation robotics as shown in Figure 1.1. In this therapy, actuators move joints of the patient in a physiological way by imposing joint movements known from recordings in healthy subjects [1]. This method has shown its effectiveness.

Several gait trainers were designed, such as Driven Gait Orthosis (DGO) [1], Supported Treadmill Ambulation Training (STAT) [2], and Mechanized Gait Trainer [3].



Figure 1.1 Rehabilitation Robotics used in therapy of spinal-cord injuries.

In the DGO, a gait trainer has been developed that can move the legs of a patient in a physiological way on the moving treadmill. The orthosis is adjustable in size so different patients can use it. Actuators at the knee and hip joints are controlled by a position controller [1]. In the STAT, the program of therapy involves simultaneously supporting a portion of the patient's weight while gait training on a treadmill [2]. The Mechanized Gait Trainer simulates the phases of gait, supports the subjects according to their abilities, and controls the center of mass in the vertical and horizontal directions.

However, due to the high cost of these trainers, many disabled people do not get the therapy they require. Two main advantages of the robotic training or gait trainers in general are identified. First, the reproducibility of the movement since it will be possible to test the effects of different gait parameters (speed, step length) [1]. Therapists usually have to practice for a longer time until they are able to perform optimal training [1]. Nevertheless, therapists are still needed to ensure the effectiveness of the training by monitoring the progress in locomotion and to supervise the training session. Secondly, the locomotor training sessions can be prolonged and the walking speed can be adjusted.

The overall goal of this study is to design a gait generation mechanism for rehabilitation of paralyzed legs. This mechanism should move the legs in a physiological way. To accomplish this specified task, this thesis will focus on the dimensional synthesis of a four-bar linkage to reproduce the desired ankle trajectory in the sagittal plane. The data of sagittal flexion/extension at the hip and knee joints from healthy people are used to define the desired ankle trajectory during normal gait cycle. A path generation four-bar mechanism is then synthesized to obtain proper link and coupler dimensions. A typical elliptical machine, which is also based on a four-bar mechanism, found in fitness centers

is shown in Figure 1.2. The path of the foot generated by this four-bar mechanism is shown in Figure 1.3. As will be seen in chapter two, this path or profile of the foot does not resemble the path generated by healthy people. A profile, or a trajectory, of healthy people's foot, which represents a physiological gait pattern will be constructed in chapter two. The path of the elliptical machine is generated using a MATLAB m-file called "fourbar_analysis" which will be discussed in chapter three.

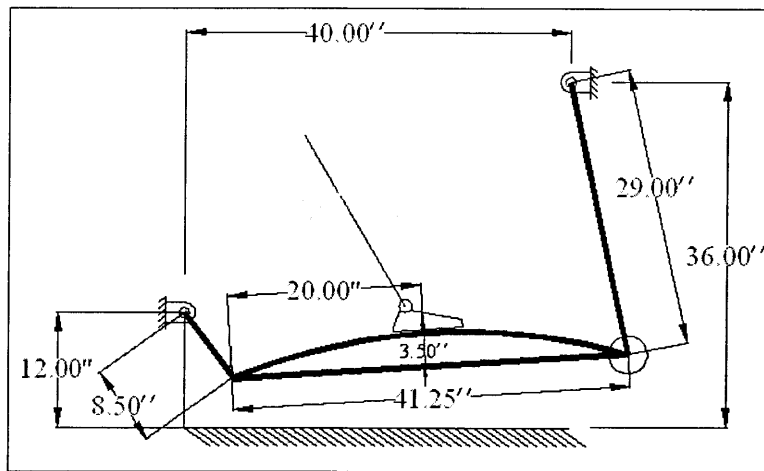


Figure 1.2 A typical elliptical machine found in a fitness center.

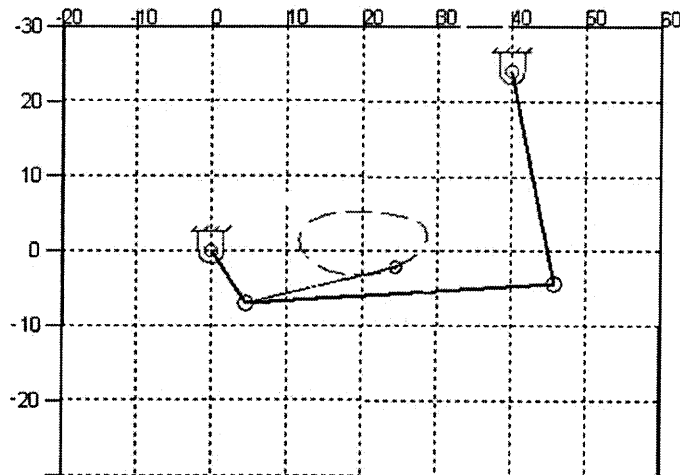


Figure 1.3 Path of the foot generated by the elliptical machine in Figure 1.2. The units are in inches.

CHAPTER 2

GENERATING THE MOTION PROFILE OF A PHYSIOLOGICAL GAIT PATTERN

2.1 Introduction

A profile, or a trajectory, of healthy people's foot, which represents a physiological gait pattern, is to be generated by a mechanism so that the patient's legs will be trained accordingly. The focus of this study will be on the gait in the sagittal plane. In this chapter, the trajectory of the healthy people's foot is studied for the design of the mechanism. Section 2.2 presents the gait data used in this chapter. Section 2.3 talks about the forward kinematics. Section 2.4 talks about plotting the trajectory of human's foot.

2.2 Gait Data

To generate the physiological gait pattern in the sagittal plane, Hip Flexion and Extension (hfe) angles, Knee Flexion and Extension (kfe) angles, and Ankle Plantar Flexion and DorsiFlexion (apd) angles are required. The definition and sign convention of these angles are shown in Figure 2.1.

The data, which were measured for 10 young people, are taken from Clinical Gait Analysis web site [4]. The joint angles are shown in Figures 2.2 through 2.4, where the gait cycle is divided into 50 equal intervals. Thus, there are 51 data points and each interval represents 2% of the whole cycle.

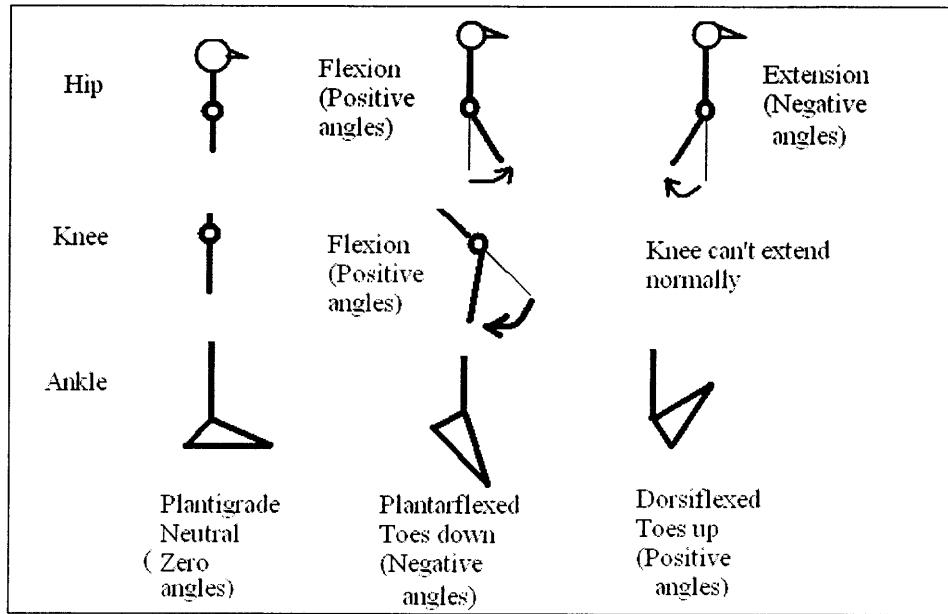


Figure 2.1 Definition and sign convention of Hip Flexion and Extension angles, Knee Flexion and Extension angles, and Ankle Plantar Flexion and DorsiFlexion angles.

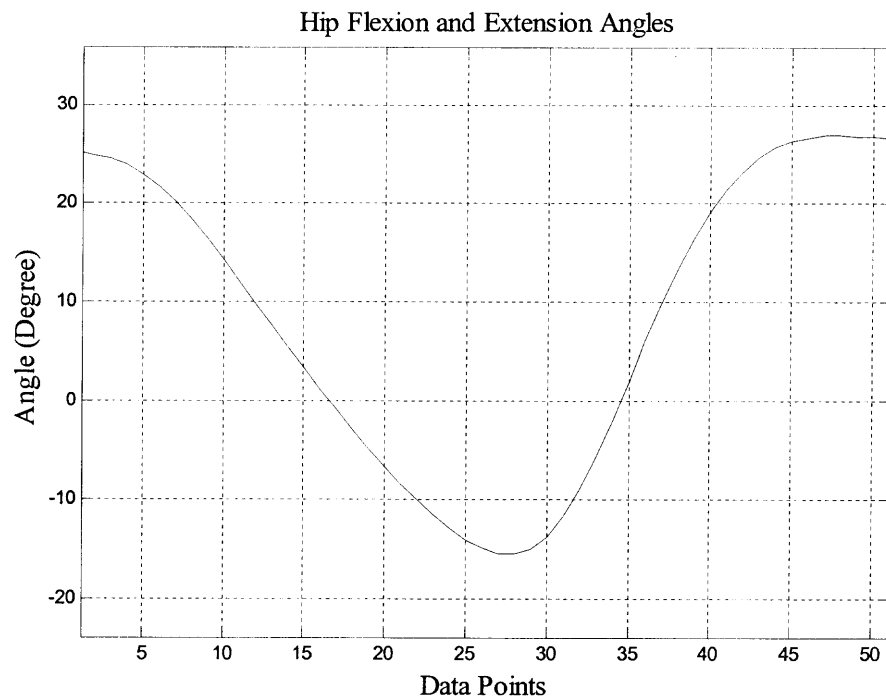


Figure 2.2 Hip Flexion and Extension Angles versus the 51 Data Points.

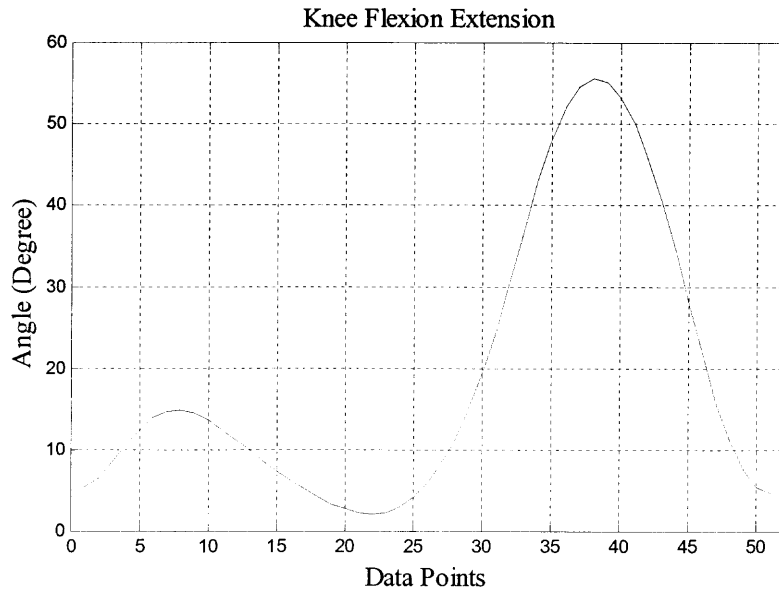


Figure 2.3 Knee Flexion and Extension Angles versus the 51 Data Points.

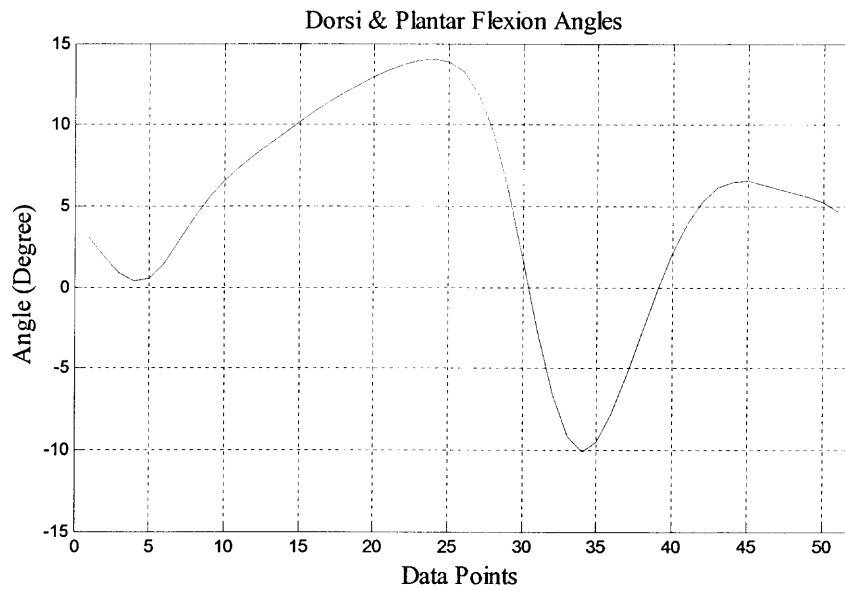


Figure 2.4 Dorsi and Plantar Flexion Angles versus the 51 Data Points.

It is noted from the above Figures that the angle data do not end at the same angle that the data started. This will cause a small discontinuity in the foot trajectory that is to be plotted. The numerical values of these angles and their standard deviations are shown in Tables A.1, A.2 and A.3 of appendix A.

2.3 Forward Kinematics

Kinematics is the science of motion without regard to the forces which cause it. A very basic problem in the study of a mechanical robot manipulation is the forward kinematics. This is the static geometrical problem of computing the position of the robot if a rotation of joints occurred. This can be done using a matrix called "the transformation matrix" [5]. In the context of this research, one can treat the upper and lower legs as a two-link planar robot manipulator connected to torso via the hip and knee joint. The foot can be treated as an end-effector. The upper leg or the thigh will be called link one, and the lower leg will be called link two. When the hip, knee, and ankle joints rotate, the ankle will travel through a trajectory. This trajectory is to be found out using the forward kinematics. Firstly, a universe coordinate system to which our descriptions will refer need to be defined. This coordinate system, shown in Figure 2.5, is called "base frame", which can be identified by X_{base} and Y_{base} axes in the Figure. The hip, knee, and ankle joints are also shown in this Figure. The frames, which are shown in Figure 2.5, are attached to each joint according to the convention used in reference [5].

Frame $\{0\}$, which is identified by X_0 and Y_0 on the Figure, is attached to the hip joint and does not rotate with this joint. Its coordinates, relative to the base frame, are at point (X_{0b}, Y_{0b}) . Frame $\{1\}$ is attached to the hip joint and rotates with this joint. Frame $\{1\}$ aligns with frame $\{0\}$ when the hip joint or "hfe" is zero. Frame $\{2\}$ is attached to the knee joint and rotates with this joint. Frame $\{2\}$ aligns with frame $\{1\}$ when the knee joint or "kfe" is zero. Frame $\{3\}$ is attached to the ankle joint and rotates with this joint. Frame $\{3\}$ aligns with frame $\{2\}$ when the ankle joint or "apd" is zero.

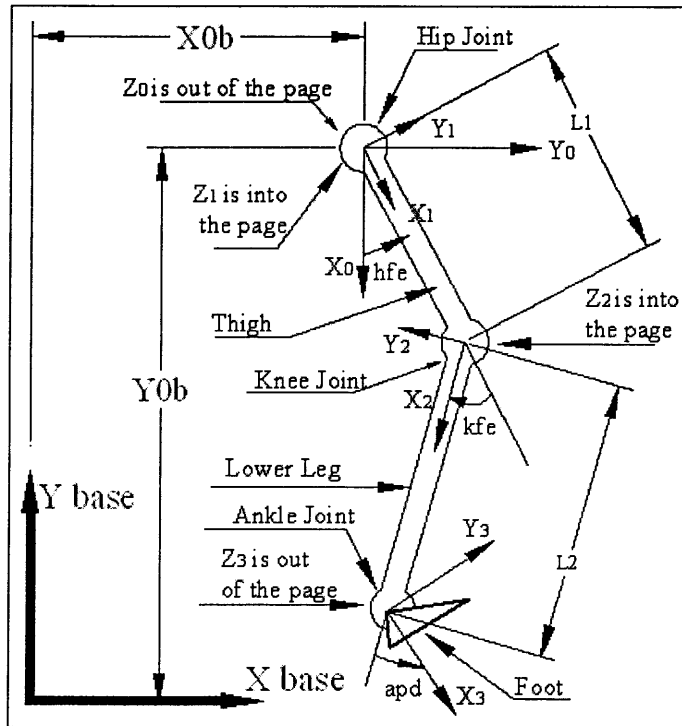


Figure 2.5 Link frame assignments

Some parameters that describe each link and the way that the two links are connected will be presented next. These parameters are called “The Denavit-Hartenberg” (D-H) parameters. These are shown in Table 2.1.

Table 2.1 Denavit-Hartenberg parameters for the two links in Figure 2.5

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	hfe
2	180	L1	0	kfe
3	180	L2	0	apd

where: α_{i-1} : Twist angle of link $i-1$, (degree).

a_{i-1} : Length of link $i-1$, (meter).

d_i : Offset of link i, (meter).

θ_i : Angle of joint i, (degree). [5]

To find the transformation matrix between frame “i-1” and frame “i”, ${}^{i-1}T_i$, the corresponding Denavit-Hartenberg parameters are substituted in the general formula for the transformation matrix which is given by equation 2.1 below.

$${}^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -d_i^* S\alpha_{i-1} \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & d_i^* C\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.1)$$

where $C\theta$ is shorthand for $\cos \theta$, $S\theta$ for $\sin \theta$ and so on.

Substituting the D-H parameters in 2.1 yields the following transformation matrices:

$${}^0T_1 = \begin{bmatrix} C_{hfe} & -S_{hfe} & 0 & 0 \\ S_{hfe} & C_{hfe} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.2)$$

$${}^1T_2 = \begin{bmatrix} C_{kfe} & -S_{kfe} & 0 & L_1 \\ -S_{kfe} & -C_{kfe} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.3)$$

$${}^2T_3 = \begin{bmatrix} C_{apd} & -S_{apd} & 0 & L_2 \\ -S_{apd} & -C_{apd} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.4)$$

Carrying out the matrix multiplication of 0T_1 and 1T_2 gives:

$${}^0T_2 = \begin{bmatrix} C_{(hfe-kf\theta)} & S_{(hfe-kf\theta)} & 0 & L_1 C_{hfe} \\ S_{(hfe-kf\theta)} & -C_{(hfe-kf\theta)} & 0 & L_1 S_{hfe} \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.5)$$

Carrying out the matrix multiplication of 0T_2 and 2T_3 gives:

$${}^0T_3 = \begin{bmatrix} C_{(hfe-kfe+apd)} & -S_{(hfe-kfe+apd)} & 0 & L_2 C_{(hfe-kfe)} + L_1 C_{hfe} \\ S_{(hfe-kfe+apd)} & C_{(hfe-kfe+apd)} & 0 & L_2 S_{(hfe-kfe)} + L_1 S_{hfe} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.6)$$

The transformation matrix 0T_3 describes the position and the orientation of frame {3} (or the foot) relative to frame {0} when the hip, knee and ankle joint rotate. For example, if the hip joint rotates by 90 degrees, the knee joint by 90 degrees and the ankle joint by 45 degrees, we will get the configuration shown in Figure 2.6.

Substituting these angles in equation 2.6 will give:

$${}^0T_3 = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 & L_2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & L_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.7)$$

The first three rows and three columns of 0T_3 represent a three by three matrix called “orientation matrix”. This orientation matrix is given by:

$${}^0R_3 = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.8)$$

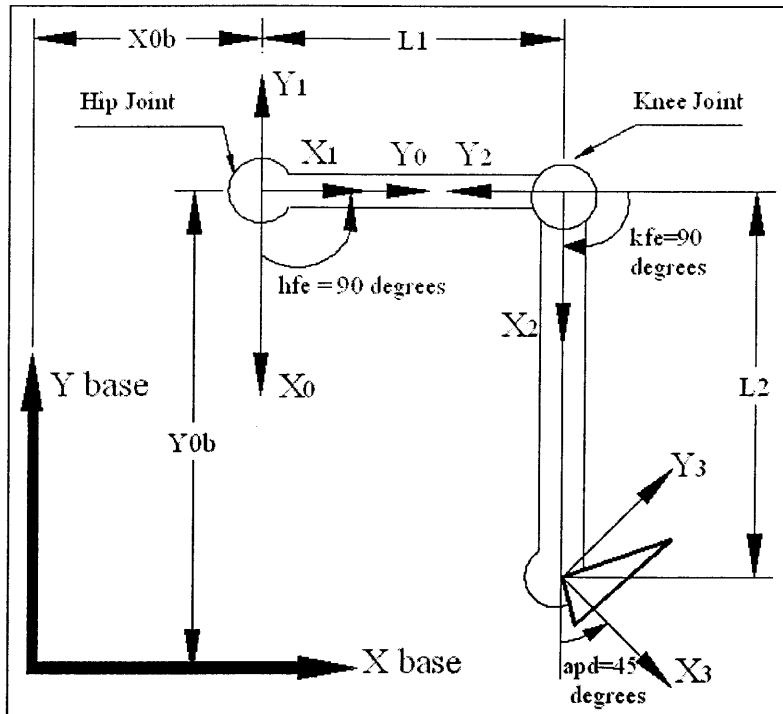


Figure 2.6 Configuration of the two links when $hfe = kfe = 90$ degrees and when $apd = 45$ degrees.

The elements of this orientation matrix are often referred to as direction cosines [5]. These elements describe the orientation of the axes of frame {3} relative to frame {0}. For example, element (2, 1) is $1/\sqrt{2}$. This suggests that the cosine of the angle between Y_0 and X_3 is $1/\sqrt{2}$. Thus, the angle between Y_0 and X_3 is 45 degrees. This can be verified by looking into Figure 2.6. Also, element (1, 2) has a value of $-1/\sqrt{2}$.

This indicates that the angle between X0 and Y3 is 135 degrees as can be verified from Figure 2.6.

The first three elements of column four in equation 2.7 represent the position of the origin of frame {3} relative to frame {0}. The X, Y and Z coordinates of frame {3} relative to frame {0} are: L2, L1, and 0 respectively. Visual inspection into Figure 2.6 can verify these values. Thus, a description, which includes position and orientation, of frame {3} relative to frame {0} was obtained.

We now need a transformation matrix that describes the position and orientation of frame {0} relative to the base frame. To derive this matrix, the “X-Y-Z fixed angles” method is used, [5]. It is assumed originally that frame {0} coincides with the base frame. To reach to the same orientation that frame {0} has in Figure 2.5, frame {0}, which coincides with the base frame, is rotated about the Z-axis of the base frame by an angle of 90 degrees clockwise. This angle is considered as a negative rotation since the positive rotation should match the right hand rule in rotating about the Z-axis. Thus, we substitute a negative 90 degrees in the orientation matrix given below by equation 2.9. This orientation matrix defines the cosine directions of a frame when it rotates about the Z-axis of another frame through an angle of θ degrees.

$$R_z(\theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.9)$$

When $\theta = -90$ degrees, the orientation matrix becomes:

$${}^{base}R_0 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.10)$$

where ${}^{base}R_0$ is the orientation matrix that defines the cosine directions of frame {0} relative to the base frame when frame {0} rotates about the Z-axis of the base frame through angle of 90 degrees clock-wise.

To construct the transformation matrix ${}^{base}T_0$, we still need the position of the origin of frame {0} relative to the base frame. It can be easily seen that the origin of frame {0} is located at $(X_{0b}, Y_{0b}, 0)$. Thus, ${}^{base}T_0$ becomes:

$${}^{base}T_0 = \begin{bmatrix} 0 & 1 & 0 & X_{0b} \\ -1 & 0 & 0 & Y_{0b} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.11)$$

We need the transformation matrix relating frame {3} and the base frame. Thus, we multiply equation 2.11, ${}^{base}T_0$, by equation 2.6, 0T_3 , to get ${}^{base}T_3$ as shown by equation 2.12:

$${}^{base}T_3 = \begin{bmatrix} S(hfe-kfe+apd) & C(hfe-kfe+apd) & 0 & X_{0b} + L_1 S_{hfe} + L_2 S(hfe-kfe) \\ -C(hfe-kfe+apd) & S(hfe-kfe+apd) & 0 & Y_{0b} - (L_1 C_{hfe} + L_2 C(hfe-kfe)) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.12)$$

Taking the same angles rotated by the joints in Figure 2.6, and substituting them in 2.12 gives the following matrix:

$${}^{base}T_3 = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & X_{0b} + L_1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & Y_{0b} - L_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.13)$$

The orientation matrix part and the position part of this transformation matrix are given by equation 2.14 and 2.15, respectively.

$${}^{base}R_3 = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.14)$$

$${}^{base}P_3 = \begin{bmatrix} X_{0b} + L_1 \\ Y_{0b} - L_2 \\ 0 \end{bmatrix} \quad (2.15)$$

From equation 2.14, it can be seen that the cosine of the angle between Y-base and X3 is $-1/\sqrt{2}$, which means that the angle between the two axes is 135 degrees as can be verified from Figure 2.6. The same procedure can be used to check the other axes.

From equation 2.15, the origin of frame {3} is located, relative to the base frame, at $X = X_{0b} + L_1$ and $Y = Y_{0b} - L_2$, as can be verified also from Figure 2.6.

2.4 Plotting the Ankle Trajectory of a Human Foot

As mentioned in section 2.2, when the hip, knee, and ankle joints rotate, the foot will travel through a trajectory. This trajectory is to be plotted using equation 2.12 which was derived in section 2.3.

A MATLAB m-file, which can be found in Appendix B, is used to generate this trajectory. This m-file is called “for_kinm” which stands for forward kinematics. It uses the angle data “hfe” and “kfe” in Appendix A as its input. It will generate three trajectories corresponding to three different heights which are: 1.37, 1.75 and 2.13 meters. These three trajectories are shown in Figure 2.7. As shown, the size of each trajectory differs depending on the height used. The origin of the Figure represents the origin of the base frame. One can simply choose another value of the height and find the corresponding trajectory. A relation between the body height and the body segment lengths is shown in Figure 2.8.

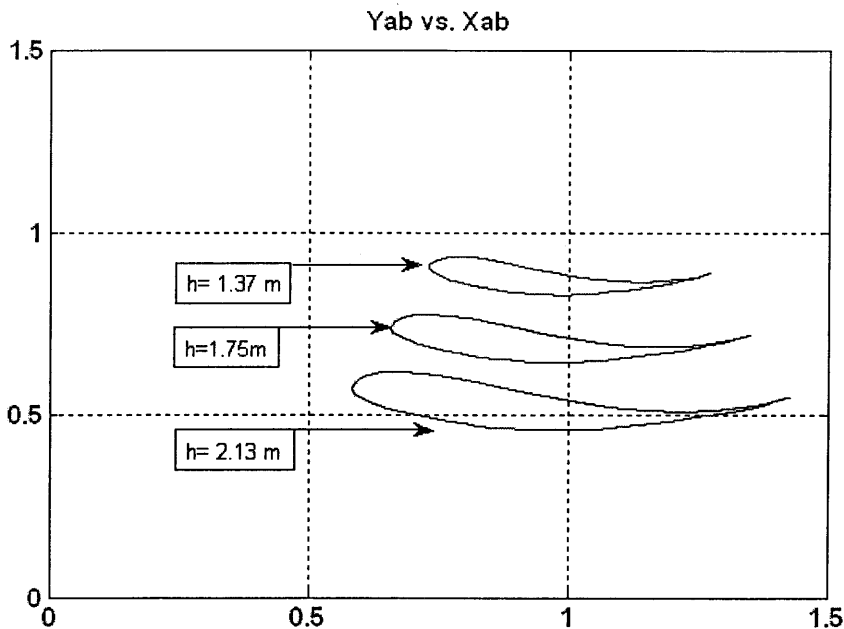


Figure 2.7 Three trajectories of the foot corresponding to three heights: 1.37, 1.75 and 2.13 meters.

From this Figure, the upper leg’s length or the thigh’s length is equal to 0.245 of the body height H . And the lower leg’s length is 0.246 of H .

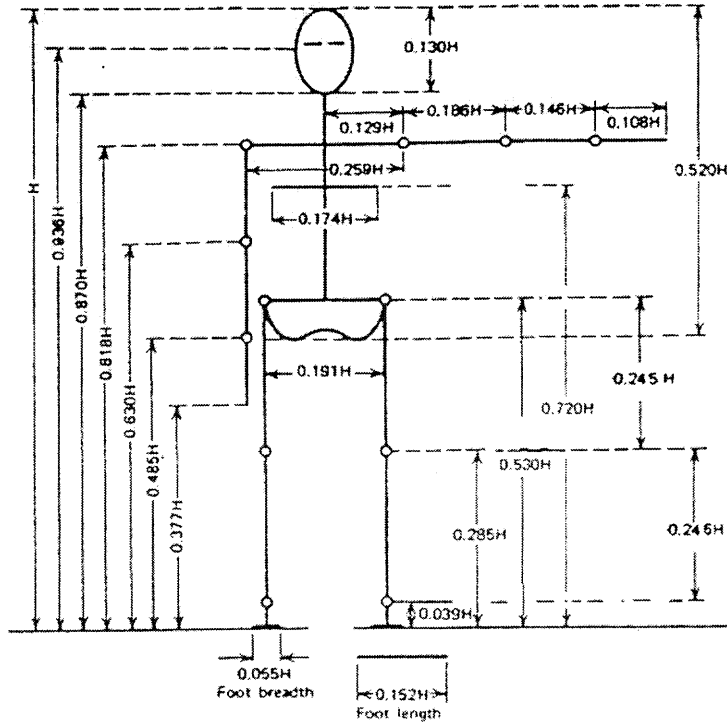


Figure 2.8 Body segment lengths expressed as a fraction of body height H .

These two ratios are used in the `for_kinm` m-file. The position part of the transformation matrix in equation 2.12 contains the X , Y , and Z coordinates of the ankle joint (frame {3}) relative to the base frame. This motion is planar, and so the Z coordinate is zero. The “ x_{ab} ” and “ y_{ab} ” symbols given in the program represent the X and Y coordinates of the ankle joint relative to the base frame. The program will calculate the “ x_{ab} ” and “ y_{ab} ” values and plot them as shown in Figure 2.7. Thus, an ankle trajectory relative to the hip, which is generated when a person walks, was obtained. In the next chapter, a four-bar mechanism is designed to generate this desired trajectory.

CHAPTER 3

DESIGNING A FOUR-BAR MECHANISM

3.1 Introduction

In this chapter, suitable ratios of link lengths of a four-bar mechanism will be found to generate the desired trajectory discussed in chapter 2. Section 3.2 talks about displacement trajectory charts. Section 3.3 talks about designing the four-bar mechanism.

3.2 Displacement Trajectory Charts

One of the methods to design a four-bar mechanism which will give the desired trajectory, is trying to match this desired trajectory with an atlas of coupler curves in which a displacement trajectory of a network of points on coupler of a four-bar linkage are accurately reproduced for a wide variety of link ratios [6]. In this atlas, the charts are reproduced with the driving crank length link in all cases the same. Changing the actual link lengths but maintaining the same link length ratios does not alter the motion characteristics of the linkage. It merely introduces a scale change [6]. Thus if a linkage is designed twice the size of that shown the trajectory of a given point will be twice as large but unchanged in shape [6]. This feature means that once a trajectory is found by choosing a suitable links ratio, the scale can be changed according to the height of the person to produce the trajectories shown in Figure 2.7 in chapter two.

All the charts in this atlas are from Crank-Rocker type linkage. In this type of linkage, the input link (crank) is capable of rotation through a complete revolution while the output link (rocker) will oscillate between two positions. An example of a trajectory chart is shown in Figure 3.1. This is a schematic drawing that shows four motion trajectories, shown in dashed line, for four coupler points on the coupler link, for a certain link ratios.

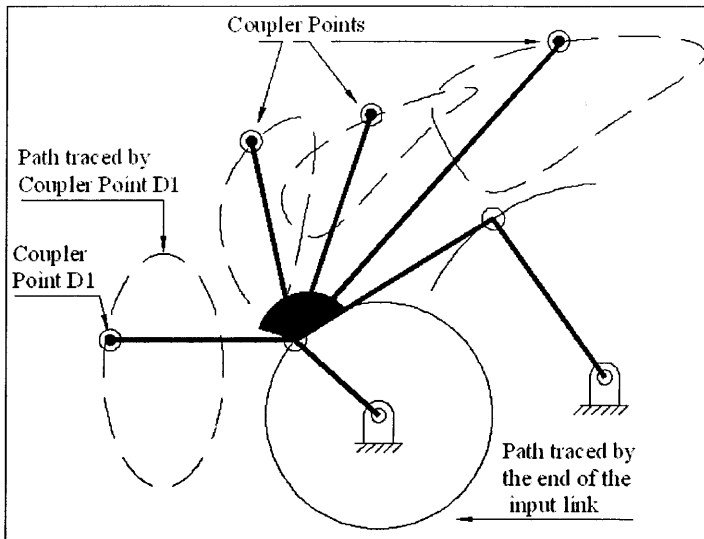


Figure 3.1 Schematic drawing that shows four motion trajectories, shown in dashed line, for four coupler points.

As shown in Figure 3.1, each coupler point will produce a different motion trajectory in shape and size. The atlas has many charts with different link ratios and different locations of the coupler points. This gives a wide range of motion trajectories. Choosing another set of link ratios, for Figure 3.1, will give different motion trajectories for the same coupler points.

3.3 Designing the Four-Bar Mechanism

As mentioned in section 3.2, there is a wide range of motion trajectories for different coupler points with different link ratios. Visual inspection to the charts can help in selecting different trajectories which are close in shape to the desired trajectory. After that, we need to plot these selected trajectories and the desired trajectory in the same figure and compare between them and choose the suitable trajectory, and hence the suitable link ratios. Fine tuning of the link ratios can then be performed to produce better match in the shape of the trajectory.

To do this comparison, we need first to draw the coupler curve (motion trajectory) of a given coupler point in a given four-bar mechanism. There are six parameters needed to draw the displacement trajectory or coupler curve of a coupler point. These are: frame length, frame angle, coupler length, rocker length, coupler point radius (length), and the angle from the coupler line to the coupler point. A four-bar mechanism with these parameters is shown in Figure 3.2.

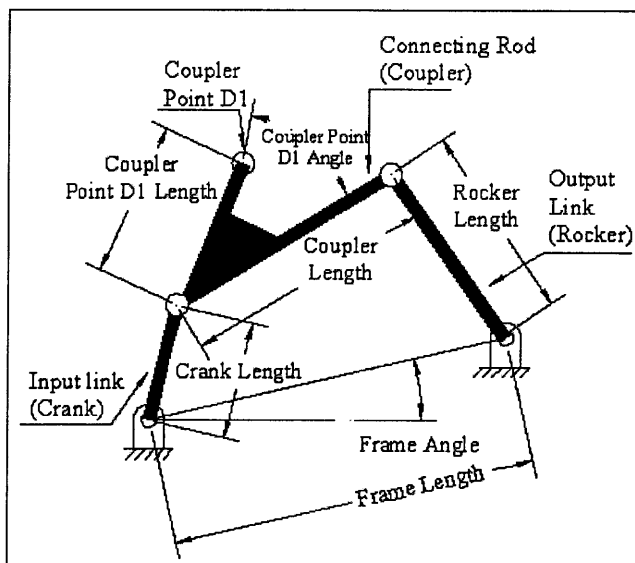


Figure 3.2 A four-bar mechanism

A MATLAB m-file taken from reference [7] is used to plot a coupler curve. This m-file is called “fourbar_analysis”. Originally, this m-file is used to analyze a crank-rocker mechanism for position, velocity, and torque, and coupler curves [7]. Since our interest is only in the position of the crank-rocker mechanism and the coupler curve at the moment, the command lines of the program dealing with velocity and torques of the mechanism are deleted and only those command lines relevant to the position and the coupler curve are kept. The command lines in the “for_kinm” m-file are plugged into the modified “fourbar_analysis” m-file with a few command lines added to them.

The nomenclature used in the “fourbar_analysis” m-file is shown in Figure 3.3, where r_1 is the frame length, Q_1 is the frame angle, r_2 is the crank length, r_3 is the coupler length, r_4 is the rocker length, cr_1 is the coupler point length or radius, and $Beta_1$ is the angle from coupler line to coupler point.

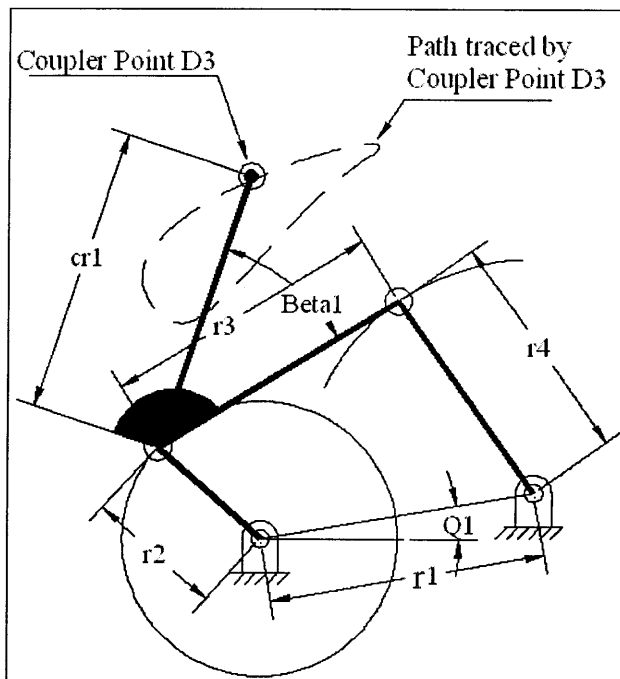


Figure 3.3 Nomenclature used in the “fourbar_analysis” m-file.

Now, once a coupler point trajectory of a certain links ratios in the atlas is to be plotted, the five parameters, mentioned at the beginning of this section, must be measured from the atlas and plugged in this m-file. Many trajectory curves, in the atlas, are close in shape to the trajectories shown in Figure 2.7. However, many of these curves are different in their size from the size of the desired trajectory and different in their orientations. To handle the size issue, we need to scale the link ratios and thus the size of the trajectory as was mentioned in the beginning of section 3.2. So, a parameter is plugged into the “fourbar_analysis” m-file to change the scale of the link ratios. Another issue needs to be resolved in the orientation of the trajectory. Figure 3.4 shows how different mechanisms having different link ratios produce different orientations of coupler trajectories.

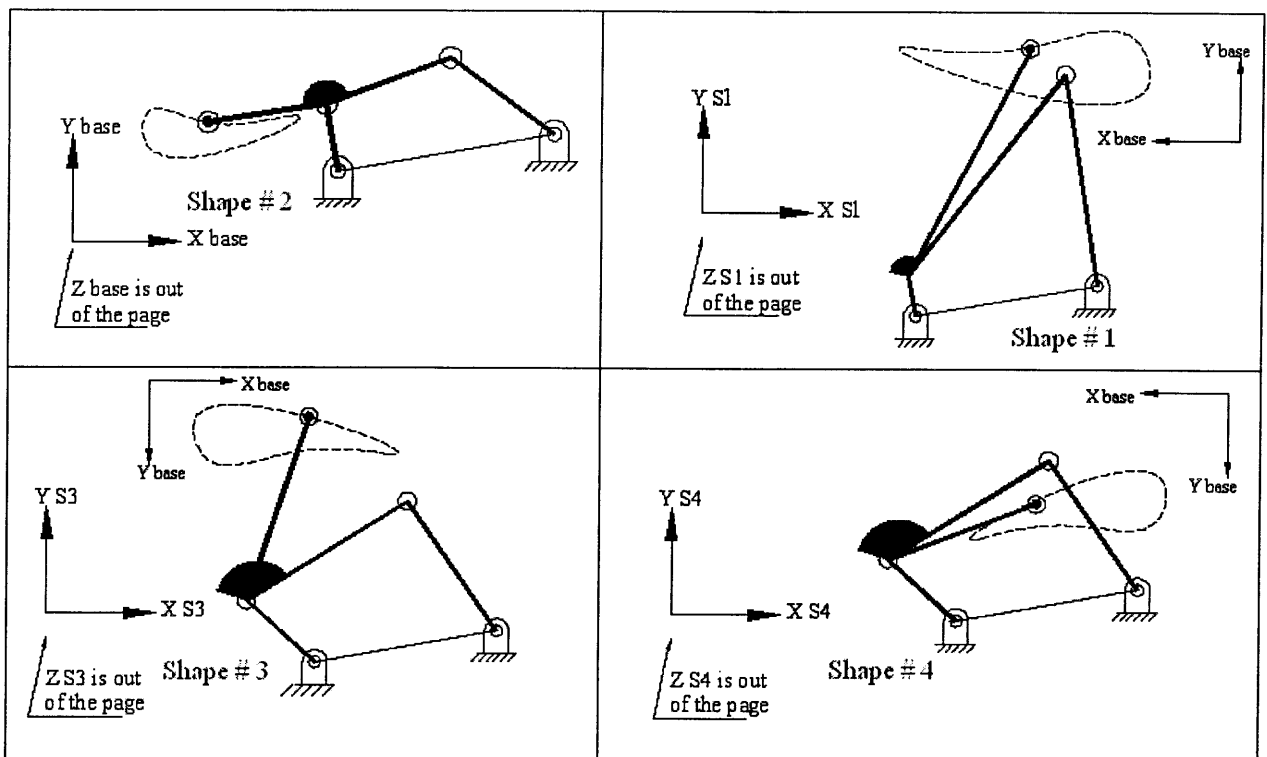


Figure 3.4 Different orientations of coupler trajectories for different link ratios.

The trajectories in Figure 2.7 look like shape # 2 in Figure 3.4, i.e., the tail of the shape lies to the right of the trajectory and points towards up direction, and so the orientation is the same. If any trajectory in the atlas is found to have an orientation like any of the other three shapes in Figure 3.4, then we need to map the trajectory in Figure 2.7 to the same orientation of the trajectory selected from the atlas. This can be done using transformation matrices, as was done in chapter two.

First, the orientation matrix part is derived then the whole transformation matrix is used. If we have shape # 3, then we can reach this shape by rotating shape # 2 about X S3 by 180 degrees. Substituting this angle into the following orientation matrix [5]:

$$R_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix} \quad (3.1)$$

Yields:

$$R_{X S3}(180) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (3.2)$$

The transformation matrix becomes:

$${}^{S3}T_{base} = \begin{bmatrix} 1 & 0 & 0 & {}^{S3}X_{base} \\ 0 & -1 & 0 & {}^{S3}Y_{base} \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.3)$$

Carrying out the multiplication of equation 3.3 and 2.12 will give ${}^{S3}T_3$.

$${}^{S3}T_3 = \begin{bmatrix} S(hfe-kfe+apd) & C(hfe-kfe+apd) & 0 & {}^{S3}X_{base}+X_{0b}+L_1S_{hfe}+L_2S(hfe-kfe) \\ C(hfe-kfe+apd) & -S(hfe-kfe+apd) & 0 & {}^{S3}Y_{base}-1*(Y_{0b}-(L_1C_{hfe}+L_2C(hfe-kfe))) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.4)$$

The position part of this transformation matrix will give the desired trajectory in an orientation similar to that of shape # 3. The values of ${}^{S3}X_{base}$ and ${}^{S3}Y_{base}$ can be chosen to adjust the position of the desired trajectory so that it coincides with the coupler curve resulting from a certain four-bar mechanism. As noticed, only the position part of 3.4 is used in plotting the desired trajectory and the generated trajectory from a four-bar mechanism. The orientation part is not important to us in this stage.

Similarly, if we have shape # 4, then we can reach this shape by rotating shape # 2 about Z S4 by 180 degrees. If we have shape # 1, then we can reach this shape by rotating shape # 2 about Y S1 by 180 degrees. The orientation matrices needed for these two rotations are:

$$R_Z(\theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.5)$$

$$R_Y(\theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \quad (3.6)$$

Using the same procedure used above and taking only the position parts, we get equations 3.7, 3.8, 3.9, and 3.10. Equations 3.7 and 3.8 are used if we have shape # 4 and 3.9 and 3.10 are used when we have shape # 1.

$$S^4 X_3 = S^4 X_{base-1} * (X_{0b} + L_1 S_{hfe} + L_2 S_{(hfe - kfe)}) \quad (3.7)$$

$$S^4 Y_3 = S^4 Y_{base-1} * (Y_{0b} - (L_1 C_{hfe} + L_2 C_{(hfe - kfe)})) \quad (3.8)$$

$$S^1 X_3 = S^1 X_{base-1} * (X_{0b} + L_1 S_{hfe} + L_2 S_{(hfe - kfe)}) \quad (3.9)$$

$$S^1 Y_3 = S^1 Y_{base+1} * (Y_{0b} - (L_1 C_{hfe} + L_2 C_{(hfe - kfe)})) \quad (3.10)$$

The position part of 3.4 and equations 3.7 and 3.8 are plugged into the “fourbar_analysis” m-file. Once a shape is identified, the number of the shape is entered into the program and the program will plot the desired trajectory in the desired orientation. Many four-bar mechanisms with different link ratios, form the atlas, were tested using the “fourbar_analysis” m-file. The best trajectory found is shown in Figure 3.5. The dashed line represents the trajectory generated by the four-bar mechanism, while the continuous line represents the desired trajectory. Figure 3.6 is a zoom figure of Figure 3.5. The desired trajectory is plotted for a height of 1.75 meters. And so, this four-bar mechanism is suitable for this height. The link ratios, coupler point angle and length are shown below. The “x” represents a scale factor to scale the link ratios and thus the trajectory generated to be suitable with a given trajectory of a certain height. For a height of 1.75 meters, “x” is 2.83.

$$r1 = 3/x \quad (\text{frame length})$$

$$Q1 = -30.25 \text{ degrees} \quad (\text{frame angle})$$

$$r2 = 1/x \quad (\text{crank length})$$

$$r3 = 2.5/x \quad (\text{coupler length})$$

$$r4 = 2.0/x \quad (\text{rocker length})$$

$cr1 = 18 / (6.3 * x)$ (coupler point radius)

$Beta1 = 11$ degrees (coupler point angle)

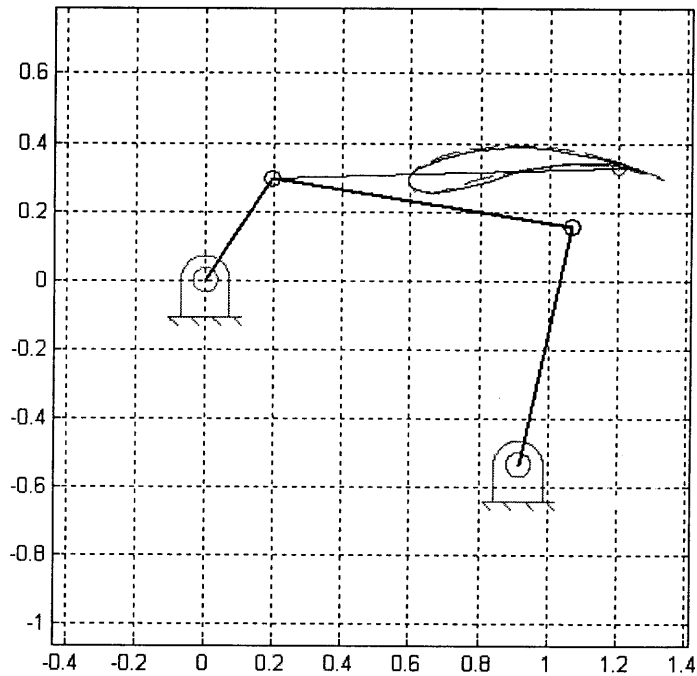


Figure 3.5 The desired trajectory compared with the trajectory generated by a four-bar mechanism for a person's height of $H = 1.75$ meters.

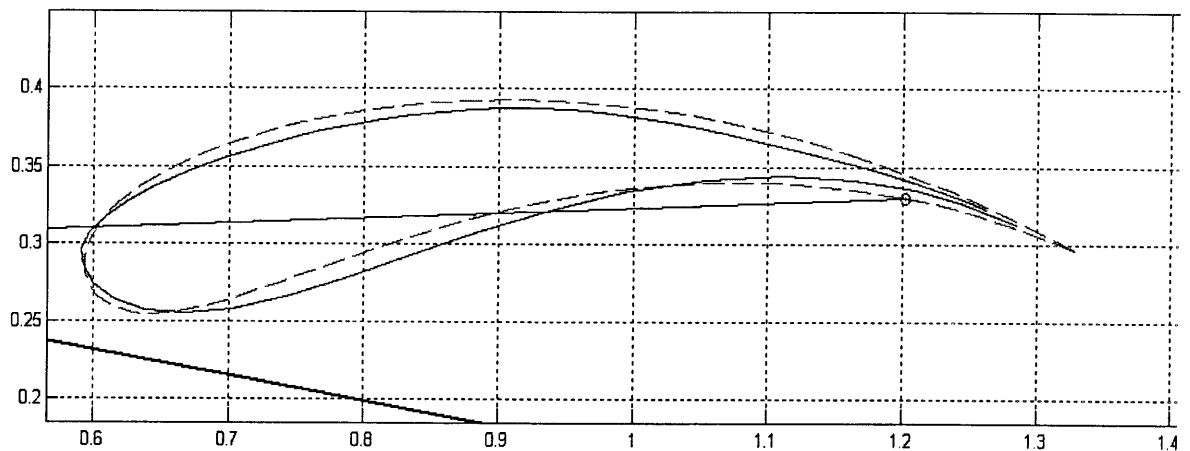


Figure 3.6 A zoom figure of Figure 3.5 for a person's height of $H = 1.75$ meters.

For heights of 2 and 1.37 meters, the desired trajectories and the generated trajectories are shown in Figures 3.7 and 3.8, respectively.

It is noted that the size of the trajectories differs depending on the height of the person used. However, these trajectories have the same shape. A scale factor that scales the profile according to the height was found and embedded into the “fourbar_analysis” m-file to give the required link dimensions along with the profile once a height is given.

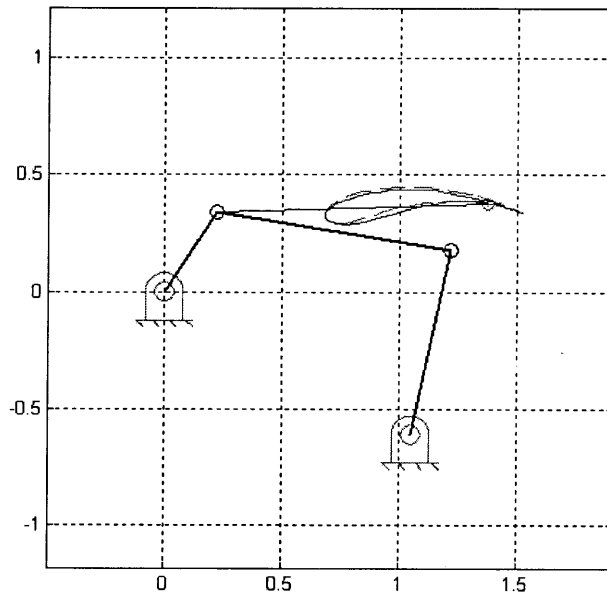


Figure 3.7 The desired trajectory compared with the trajectory generated by a four-bar mechanism for a person's height of $H=2$ meters.

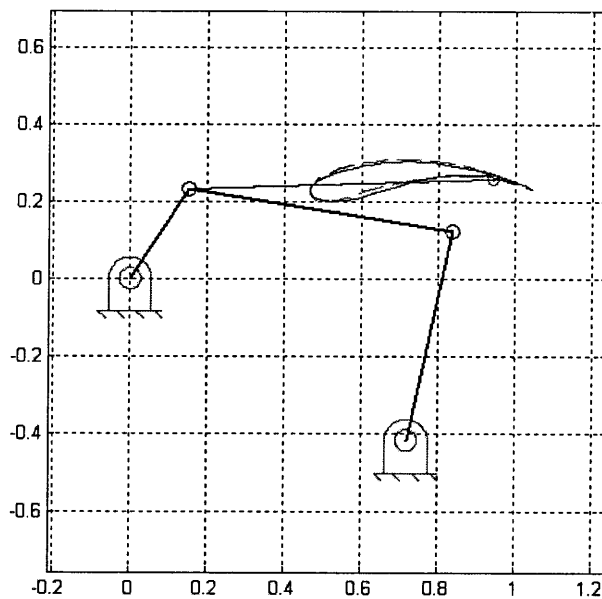


Figure 3.8 The desired trajectory compared with the trajectory generated by a four-bar mechanism for a person's height of $H=1.37$ meters.

CHAPTER 4

CONCLUSION

4.1 Results

A path generation four-bar mechanism was synthesized to obtain proper link and coupler dimensions. The resulted coupler curve matches well with the desired ankle trajectory. This gait generation mechanism can be used for rehabilitation of paralyzed legs. It should provide an appropriate afferent input to the spinal cord by moving the legs in a physiological way. Figure 4.1 shows a typical four-bar mechanism to be used for rehabilitation therapy with a profile generated for a person's height of 1.75 meters.

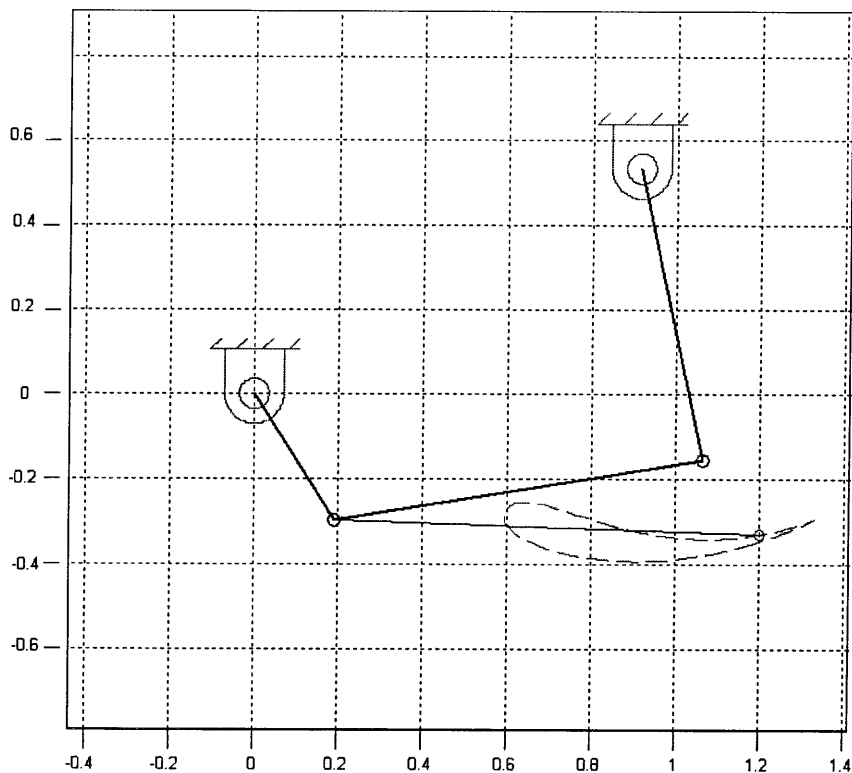


Figure 4.1 A typical four-bar mechanism with a profile generated for a person's height of 1.75 meters.

4.2 Future Research

Future research of this Gait Generation Mechanism includes:

- Using the inverse kinematics to find the flexion/extension at the hip and knee joints resulting from applying this mechanism and compare it to the original data.
- Finding the orientation of the ankle joint and adding a degree of freedom to obtain this orientation during a normal gait cycle.
- Building a gait trainer that along with this mechanism will form a complete gait trainer that can be used in rehabilitation therapy.

APPENDIX A
ANGLES DATA

The young people's data for Hip Flexion and Extension (hfe) angles, Knee Flexion and Extension (kfe) angles, and Ankle Plantar Flexion and DorsiFlexion (apd) angles are shown, along with their standard deviations, in Tables A.1, A.2, and A.3, respectively.

Table A.1 Data for Hip Flexion and Extension (hfe) angles.

Data #	hfe (degrees)	Standard Deviation	Data #	hfe	Standard Deviation
1	25.264	7.674	27	-15.417	9.081
2	24.899	7.687	28	-15.485	8.762
3	24.459	7.746	29	-14.962	8.418
4	23.849	7.853	30	-13.722	8.091
5	22.942	7.997	31	-11.715	7.836
6	21.706	8.177	32	-8.99	7.673
7	20.165	8.359	33	-5.68	7.573
8	18.362	8.514	34	-1.961	7.489
9	16.367	8.63	35	1.98	7.399
10	14.282	8.715	36	5.946	7.304
11	12.146	8.789	37	9.751	7.215
12	9.974	8.876	38	13.257	7.135
13	7.776	8.993	39	16.371	7.088
14	5.569	9.158	40	19.063	7.061
15	3.374	9.359	41	21.333	7.063
16	1.221	9.57	42	23.193	7.105
17	-0.874	9.759	43	24.645	7.182
18	-2.894	9.899	44	25.682	7.299
19	-4.833	9.977	45	26.343	7.443
20	-6.677	9.997	46	26.709	7.599
21	-8.424	9.968	47	26.868	7.734
22	-10.063	9.9	48	26.899	7.844
23	-11.567	9.806	49	26.854	7.918
24	-12.901	9.684	50	26.773	7.944
25	-14.029	9.536	51	26.632	7.94
26	-14.894	9.344			

Table A.2 Data for Knee Flexion and Extension (kfe) angles.

Data #	kfe (degrees)	Standard Deviation	Data #	kfe	Standard Deviation
1	5.375	4.867	27	8.167	4.777
2	6.416	4.777	28	11.149	5.104
3	8.289	4.729	29	14.844	5.611
4	10.476	4.659	30	19.335	6.2
5	12.443	4.596	31	24.625	6.78
6	13.899	4.615	32	30.556	7.234
7	14.735	4.685	33	36.788	7.437
8	14.88	4.757	34	42.803	7.305
9	14.431	4.837	35	48.029	6.835
10	13.578	4.934	36	52.015	6.174
11	12.443	5.002	37	54.51	5.604
12	11.163	5.042	38	55.473	5.384
13	9.846	5.083	39	54.961	5.566
14	8.557	5.151	40	53.069	6.065
15	7.333	5.25	41	49.894	6.728
16	6.181	5.378	42	45.561	7.393
17	5.11	5.509	43	40.286	7.913
18	4.139	5.598	44	34.294	8.226
19	3.301	5.624	45	27.951	8.321
20	2.633	5.578	46	21.679	8.18
21	2.186	5.47	47	15.907	7.76
22	2.032	5.31	48	11.053	7.074
23	2.252	5.112	49	7.467	6.225
24	2.929	4.902	50	5.38	5.5
25	4.113	4.736	51	4.722	5.076
26	5.839	4.668			

Table A.3 Data for Ankle Plantar Flexion and DorsiFlexion (apd) angles.

Data #	apd (degrees)	Standard Deviation	Data #	apd	Standard Deviation
1	3.029	3.914	27	11.882	5.52
2	1.896	3.886	28	9.473	6.46
3	0.886	3.767	29	5.947	7.583
4	0.353	3.582	30	1.612	8.537
5	0.54	3.396	31	-2.853	8.947
6	1.487	3.288	32	-6.689	8.729
7	2.905	3.252	33	-9.202	8.09
8	4.326	3.215	34	-10.11	7.353
9	5.546	3.154	35	-9.512	6.673
10	6.562	3.104	36	-7.789	6.037
11	7.408	3.057	37	-5.408	5.44
12	8.15	3.003	38	-2.77	4.918
13	8.843	2.969	39	-0.169	4.509
14	9.519	2.947	40	2.151	4.18
15	10.188	2.94	41	4.021	3.915
16	10.83	2.939	42	5.354	3.732
17	11.428	2.959	43	6.166	3.629
18	11.97	3.02	44	6.521	3.577
19	12.467	3.132	45	6.54	3.554
20	12.923	3.286	46	6.355	3.553
21	13.336	3.476	47	6.094	3.57
22	13.692	3.701	48	5.841	3.612
23	13.955	3.942	49	5.583	3.693
24	14.065	4.193	50	5.241	3.81
25	13.902	4.482	51	4.669	3.971
26	13.266	4.886			

APPENDIX B
FOR_KINM M-FILE

A MATLAB m-file is used to generate the trajectory of a human's foot or the ankle joint. This m-file is called "for_kinm" which stands for forward kinematics. The program is shown below by italic.

```
% This function computes the position of the Ankle joint or relative to the base frame  
hfe= [ 25.2640; 24.8990; 24.4590; 23.8490; 22.9420; 21.7060; 20.1650;  
18.3620; 16.3670; 14.2820; 12.1460; 9.9740; 7.7760; 5.5690; 3.3740; 1.2210 ;  
-0.8740; -2.8940; -4.8330; -6.6770; -8.4240; -10.0630; -11.5670; -12.9010;  
-14.0290; -14.8940; -15.4170; -15.4850; -14.9620; -13.7220; -11.7150; -8.9900;  
-5.6800; -1.9610; 1.9800; 5.9460; 9.7510; 13.2570; 16.3710; 19.0630; 21.3330;  
23.1930; 24.6450; 25.6820; 26.3430; 26.7090; 26.8680; 26.8990 ; 26.8540; 26.7730;  
26.6320];  
  
kfe= [ 5.3750; 6.4160; 8.2890; 10.4760; 12.4430; 13.8990; 14.7350;  
14.8800; 14.4310; 13.5780; 12.4430; 11.1630; 9.8460; 8.5570; 7.3330;  
6.1810; 5.1100; 4.1390; 3.3010; 2.6330; 2.1860; 2.0320; 2.2520;  
2.9290; 4.1130; 5.8390; 8.1670; 11.1490; 14.8440; 19.3350; 24.6250;  
30.5560; 36.7880; 42.8030; 48.0290; 52.0150; 54.5100; 55.4730; 54.9610;  
53.0690; 49.8940; 45.5610; 40.2860; 34.2940; 27.9510; 21.6790; 15.9070;  
11.0530; 7.4670; 5.3800; 4.7220];  
  
h=1.37; % Length of a person (meter)
```

```

Figure, hold on, axis([0 1.5 0 1])

for j=1:1:3    % for loop is used to generate different trajectories corresponding to
               % different heights.

l1=0.245*h; % Length of upper leg (meter)

l2=0.246*h; % Length of lower leg (meter)

i=1:1:51;    % Number of data samples

% The data angles are in degrees and are converted to radians in the x and y
% relations. A plot for the position of the ankle joint will be plotted relative to the
% base frame. This is the absolute frame that the four-bar mechanism will be
% working according to its coordinates.

x0b=1;       % x0b and y0b locate the origin of frame zero relative to the base
y0b=1.5;     % frame, these are arbitrary and can be changed

xab= x0b+ l1*sin (hfe*pi/180) +l2*sin ((hfe-kfe)*pi/180);
yab= y0b+ -1*(l1*cos (hfe*pi/180) +l2* cos ((hfe-kfe)*pi/180));

% xab and yab relations locate the position of the Ankle Joint relative to the base
% frame

plot(xab,yab),grid, title('Yab vs. Xab')

h=h+.38;     % Increment of h. The profile will be plotted at three values of h
               % which are: 1.37, 1.75 and 2.13 meters

end          % End of for loop

```


REFERENCES

- [1] Gery Colombo, Matthias Joerg, Reinhard Schreier, Volker Dietz, Treadmill Training of Paraplegic Patients Using a Robotic Orthosis. *Journal of Rehabilitation Research and Development* Vol. 37 No. 6, November/December 2000, p. 693-700.
- [2] Michael S. Wilson, Huma Qureshy, Elizabeth J. Protas, S. Ann Holmes, Thomas A. Krouskop, Arthur M. Sherwood. Equipment Specifications for Supported Treadmill Ambulation Training: A Technical Note. *Journal of Rehabilitation Research and Development* Vol. 37 No. 4, July/August 2000
- [3] Stefan Hesse and Dietmar Uhlenbrock, A Mechanized Gait Trainer for Restoration of Gait, *Journal of Rehabilitation Research and Development* Vol. 37 No. 6, November/December 2000.
- [4] Clinical Gait Analysis Normative Gait Database. Retrieved **April 1, 2005** from the World Wide Web: <http://guardian.curtin.edu.au/cga/data/index.html> .
- [5] Craig, John J., (1989). *Introduction to Robotics, Mechanics and Control*. Third Edition, Pearson Education, Inc.
- [6] Hrones, John A. and Nelson, George L., (1951). *Analysis of The Four Bar Linkage*. Published jointly by The Technology Press of the Massachusetts Institute of Technology and John Wiley & Sons, Inc., New York.
- [7] Kenneth J. Waldron, Gray L. Kinzel, (1999). *Kinematics, Dynamics, and Design of Machinery*. John Wiley & Sons, Inc.