ABSTRACT

ON RATE CAPACITY AND SIGNATURE SEQUENCE ADAPTATION IN DOWNLINK OF MC-CDMA SYSTEM

by
Jianming Zhu

This dissertation addresses two topics in the MC-CDMA system: rate capacity and adaptation of users' signature sequences. Both of them are studied for the downlink communication scenario with multi-code scheme.

The purpose of studying rate capacity is to understand the potential of applying MC-CDMA technique for high speed wireless data communications. It is shown that, to maintain high speed data transmission with multi-code scheme, each mobile should cooperatively decode its desired user's encoded data symbols which are spread with different signature sequences simultaneously. Higher data rate can be achieved by implementing dirty paper coding (DPC) to cooperatively encode all users' data symbols at the base station. However, the complexity of realizing DPC is prohibitively high. Moreover, it is found that the resource allocation policy has profound impact on the rate capacity that can be maintained in the system. Nevertheless, the widely adopted proportional resource allocation policy is only suitable for the communication scenario in which the disparity of users' channel qualities is small. When the difference between users' channel qualities is large, one must resort to non-proportional assignment of power and signature sequences.

Both centralized and distributed schemes are proposed to adapt users' signature sequences in the downlink of MC-CDMA system. With the former, the base station collects complete channel state information and iteratively adapts all users' signature sequences to optimize an overall system performance objective function, e.g. the weighted total mean square error (WTMSE). Since the proposed centralized scheme is designed such that each iteration of signature sequence adaptation decreases the

WTMSE which is lower bounded, the convergence of the proposed centralized scheme is guaranteed.

With the distributed signature sequence adaptation, each user's signature sequences are *independently* adapted to optimize the associated user's individual performance objective function with no regard to the performance of other users in the system. Two distributed adaptation schemes are developed. In one scheme, each user adapts its signature sequences under a pre-assigned power constraint which remains unchanged during the process of adaptation. In the other scheme, pricing methodology is applied so that the transmission power at the base station is properly distributed among users when users' signature sequences are adapted. The stability issue of these distributed adaptation schemes is analyzed using game theory frame work. It is proven that there always exists a set of signature sequences at which no user can unilaterally adapt its signature sequences to further improve its individual performance, given the signature sequences chosen by other users in the system.

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CHAPTER 1

INTRODUCTION

The wireless communications industry has been experiencing an increasing demand for its services, not only in volume, but also in diversity. There is growing desire to integrate various types of applications, such as voice, data, text, image and video, within the same wireless system. The trend of providing multimedia services requires the future wireless communication systems to be *data* orientated and capable of supporting high speed data transmission.

However, it is always a challenge to achieve high speed data transmission over a wireless link. There are two main difficulties associated with this problem. One originates from the the fading nature of the wireless channels. The other is the multiple access problem which lies in the fact that a common medium, i.e. air, need to be shared by multiple users.

In general, a typical wireless channel consists of multiple number of signal paths, called multipath. Their constructive or destructive reception results in fading property, which is characterized by randomness. Furthermore, due to the short wave-length of the carrier signal, a wireless channel varies quickly as a function of the terminal or nearby environment's movement. Both effects will significantly distort the transmitted signal, and result in poor system performance.

Furthermore, a wireless system does not have a dedicated medium which connects a transmitter and its corresponding receiver. Instead, a common communication medium, i.e. air, needs to be shared by all transmitters and receivers in the system. As multiple transmitters and receivers share the same communication medium, they cause mutual interference against each other. Such mutual interference, known as multiple access interference (MAI), is detrimental to the system performance.

In an effort to solve the problems of channel fading and multiple access, Multi-Carrier Code Division Multiple Access (MC-CDMA) was proposed in [1] by combining the well known Orthogonal Frequency Division Multiplexing (OFDM) and Direct Sequence Code Division Multiple Access (DS-CDMA). With this technique, the whole channel bandwidth is divided in frequency domain into a set of parallel orthogonal sub-channels. Each data symbol, after being modified separately by a different chip of a signature sequence (an operation known as *spreading in frequency domain*), is transmitted simultaneously on all those sub-channels. Data symbols designated to different users are transmitted simultaneously over the same set of sub-channels but with distinct signature sequences.

Since each data symbol is transmitted over all available sub-channels, MC-CDMA is ready to exploit the frequency diversity of the underlying wireless link, which makes MC-CDMA robust to channel fading. Furthermore, with MC-CDMA, multiple access problem is resolved by spreading data symbols designated to different users with signature sequences that have low cross-correlation. Due to its robustness against channel fading and its flexibility in handling multiple access problem, MC-CDMA has drawn considerable attention in both the industrial and the academic communities [1–5]. Preliminary study has shown great potential of this technique for future wireless communication systems.

1.1 Overview

In this dissertation, two topics in MC-CDMA system are addressed in details: one is the data rate that can be achieved; the other is how users' signature sequences can be adapted to improve the performance of a MC-CDMA based communication system.

In general, there exist in a wireless communication system two communication links: the downlink, for communication from the base station to mobiles; and the uplink, for communication from the mobiles to the base station. The growth of wireless Internet access resulted in requiring much higher data rate in the downlink than in the uplink, making the downlink become the bottleneck that dominantly restricts the system performance. Therefore, this dissertation concentrates on the downlink scenario.

To obtain high data rate, multi-code scheme is assumed, such that each user can transmit several data symbols simultaneously, where each data symbol is spread by a distinct signature sequence. Note that, with multi-code scheme, the MC-CDMA system considered in this dissertation is more general than the traditional MC-CDMA system which assigns single signature sequence to each user, the purpose of which is to provide more insights into MC-CDMA technique. Particularly as was shown in [4], multi-code scheme is capable of supporting multi-rate, a critical feature required by future wireless communication systems to provide multimedia services.

1.1.1 Rate Capacity

The purpose of studying rate capacity is to understand the potential of applying MC-CDMA technique in wireless data communications.[1] Since the downlink of MC-CDMA is a point-to-multipoint communication system, its rate capacity is characterized by a rate vector, whose elements represent the rates that users in the system can simultaneously maintain under the same channel condition.

In general, the downlink communication scenario fits the model of broadcast channel in information theory. Specifically, with the identical information being transmitted over all available sub-channels, the downlink of MC-CDMA can be modeled as a vector broadcast channel. Even though there is extensive research

[1]More detailed background introduction on rate capacity see Chapter 3.

for the broadcast channel, the capacity of vector broadcast channel is still an open question in the literature [6, 7].

In this dissertation, the rate capacity that can be achieved in the downlink of MC-CDMA is studied under various combinations of coding strategies and resource allocation policies.

It is shown that, to maintain high speed data transmission under multi-code scheme, each mobile's should cooperatively decode its desired user's encoded data symbols which are spread with distinct signature sequences (multi-code). Further improvement in rate capacity can be achieved by implementing dirty paper coding (DPC) to cooperatively encode all users' data symbols at the base station. However, the complexity of realizing DPC is prohibitively high.

It is found that the system capacity region heavily depends on the resource allocation policy employed in the system. When the disparity of users' channel qualities is small, one can proportionally assign power and signature sequences among users without suffering significant penalty in the resultant system capacity region. However, when the difference of users' channel qualities is large, a much larger system capacity region can be obtained with non-proportional assignment of power and signature sequences.

1.1.2 Signature Sequence Adaptation

Recently, there was extensive research on the signature sequence adaptation for the CDMA based communication systems, most of which focused on the uplink scenario. The study of signature sequence adaptation in the downlink scenario is scarce and limited, even though, for future wireless communications, most transmission is expected to take place in that direction.[2]

[2] More detailed background introduction on adaptation of users' signature sequences see Chapter 4.

This dissertation addresses the problem of signature sequence adaptation in the downlink of MC-CDMA under frequency selective responses. Both *centralized* and *distributed* adaptation of users' signature sequences are considered.

With the centralized signature sequence adaptation, all users' signature sequences are *jointly* adapted for a global system performance based upon the channel state information of the system (which is defined as the collection of all users' channel station information). Particularly, a centralized adaptation scheme, which is developed in this dissertation, aims at minimizing the weighted total mean square error (WTMSE) of the system. With the proposed centralized adaptation scheme, the base station collects the complete channel state in the system and iteratively adapts all users' signature sequences so that each iteration of signature sequence adaptation decreases the WTMSE which is lower bounded. Hence, the convergence of the proposed centralized adaptation scheme is guaranteed.

With the distributed signature sequence adaptation, each user's signature sequences are *independently* adapted to optimize the associated user's individual performance objective function with no regard to the performance of other users in the system. This implies that the adaptation of one user's signature sequences is based upon that user's channel state information only. Two distributed adaptation schemes are developed in this dissertation. In one scheme, each user adapts its signature sequences under a pre-assigned power constraint which remains unchanged during the process of adaptation. In the other scheme, pricing methodology is applied such that the transmission power at the base station is properly distributed among users when users' signature sequences are adapted. The stability issue of these distributed adaptation schemes is analyzed with game theory frame work. It is proven that there always exists a set of signature sequences at which no user can unilaterally adapt its signature sequences to further improve its individual performance, given the signature sequences chosen by other users in the system. Note that, the existence of such signature sequence set does not depend on the system channel condition, or the number of users in the system, or the number of signature sequences assigned to each user.

Simulation results are presented to demonstrate and compare the performance improvement of these adaptation schemes. It is shown that, the centralized adaptation scheme is more efficient in improving system performance, while the distributed adaptation schemes are much easier to be implemented in practical communication scenarios. It is also found that pricing is an adequate mechanism to increase the efficiency of the distributed signature sequence adaptation.

1.2 Notations

The notation used in this dissertation is as follows. Unbold lower case letters, bold lower case letters and bold upper case letters are used to denote scalars, vectors and matrices, respectively. $(\cdot)^T$ denotes the transpose operation, $(\cdot)^H$ denotes the Hermitian transpose operation, $\det(\cdot)$ denotes the determinant operation, and $tr(\cdot)$ denotes the trace operation. An N-dimensional identity matrix is denoted as either $\mathbf{I}_N$ or $\mathbf{I}$.

1.3 Organization

The dissertation is composed of seven chapters including the present Introduction chapter. Chapter 2 provides a concise matrix formulation to model MC-CDMA system in discrete frequency domain. The rate capacity of a MC-CDMA system is studied in Chapter 3. Chapter 4 focuses on the centralized signature sequence adaptation. Chapter 5 concerns with the distributed signature sequence adaptation, where arbitrary power assignment among users is assumed. In Chapter 6, the distributed signature sequence with pricing is investigated. Finally, the conclusions presented in Chapter 7.

CHAPTER 2

MATRIX FORMULATION OF MC-CDMA SYSTEM

This chapter is to provide a matrix formulation for MC-CDMA system in discrete frequency domain. Since both the modeling and the analysis of MC-CDMA system require an accurate characterization of the effects of the wireless channel, a brief review of dispersive channels is also presented in this chapter.

As shown in [1], MC-CDMA is an OFDM based multiple access technique; its modulation is implemented with the help of an OFDM modem; and its multiple access is achieved by spreading data symbols in frequency domain with distinct signature sequences. Therefore, the transmitted MC-CDMA signal is indeed an OFDM signal. A systematic framework is provided in this chapter to analyze the effects of the underlying channel on the transmitted OFDM signal. With this framework, a concise matrix formulation for MC-CDMA system is presented. The need to apply multi-code scheme for MC-CDMA system for achieving high speed data transmission is also justified.

The rest of the chapter is organized as follows. Section 2.1 briefly reviews the characteristics of the dispersive channel. A systematic framework is presented in Section 2.2 to analyze the channel effect on the transmitted OFDM signals. Section 2.3 presents the model of MC-CDMA system in matrix-vector format, which will be used throughout the rest of the dissertation.

2.1 Dispersive Channel

The wireless channel is a very challenging environment. The transmission path between the transmitter and the receiver can vary from a simple line-of-sight to one that is severely obstructed by buildings, mountains, and foliage. The modeling of a radio channel has historically been one of the most difficult parts of mobile radio system design. Typically, it is done in a statical fashion based on measurements made specifically for an intended communication system. In this section, a channel description that fits the needs of this dissertation is provided. More detailed treatment can be found in [8–10].

Generally speaking, the effects of radio channels can be classified into two main categories: the large scale path loss which describes the average received signal strength at a give distance from the transmitter, and the small scale fading which characterizes the variability of the signal strength in close spatial proximity to a particular location.

2.1.1 Large Scale Path Loss

Two factors affect the large scale path loss, one is propagation path loss, the other is shadowing.

Propagation path loss describes the signal attenuation from the transmitter to the receiver as a function of the distance between them. Many models have been developed from empirical measurements, where the effects of antenna hight, carrier frequency and terrain type are taken into account. It is found, under most circumstances, the received signal power decreases logarithmically with distance. Therefore, for simplicity, the propagation path loss model is always given as [9]

$$G \propto d^{-n} \tag{2.1}$$

where d is the distance between the transmitter and the receiver; n is the path loss exponent. Typically, n ranges from 2 to 4, depending on the specific application considered.

However, the model of propagation path loss given in (2.1) does not consider the fact that the surrounding environmental clutter may be vastly different at two different locations having the same distance from the transmitter. The measured received signal strength always fluctuates around the *average* value predicted by the propagation path loss model. This phenomenon is known as shadowing and its characteristics is usually done statistically. The shadowing effect, in addition to the propagation path loss, is usually modeled as

$$G = 10^{\xi/10} \tag{2.2}$$

where ξ is a Gaussian random variable with zero mean and standard deviation chosen from 6 to 8 dB.

The large scale path loss is usually constant over the frequency band of a band-pass signal, such as the signal studied in this dissertation. However, as the receiver moves, the large scale path loss will become time variant. Nevertheless, such time variation is much slower compared with the fast signal strength fluctuations characterized by the small scale fading.

2.1.2 Small Scale Fading

Small scale fading, or *fading*, is a phenomenon which describes the rapid fluctuation of the amplitude of a received radio signal over a short period of time or travel distance. It is caused by *multiple propagation.* As multiple reflections of the transmitted signal produce different propagation paths to the receiver, a number of replicas of the transmitted signal arrive at the receiver with different delays, phases and amplitudes. The constructive or destructive combination of these multiple signals results in fluctuation in received signal strength.

The small scale variations of a radio channel can directly be related to the impulse response of the underlying channel. Due to its time variant nature, the impulse response is usually given in the form $c(t, \tau)$, which represents the response of the channel at time instant t due to an impulse applied at time instant $t - \tau$. Then, $r(t)$, the output of channel with time variant impulse response $c(t, \tau)$, can be expressed as

$$r(t) = \int x(t-\tau)c(t,\tau)d\tau = \int x(\tau)c(t,t-\tau)d\tau \tag{2.3}$$

where $x(t)$ is the channel input.

In practise, it is useful to discretize the multipath delay axis τ of the impulse response into equal delay segments called *excess delay bins*, where each bin has a time delay width equal to $\tau_{i+1} - \tau_i$, where τ_0 is equal to 0 and represents the first arriving signal at the receiver. Let $i = 0$, then $\tau_1 - \tau_0$ is equal to the time delay bin width given by $\Delta\tau$. For convenience, $\tau_0 = 0$, $\tau_1 = \Delta\tau$ and $\tau_i = i\Delta\tau$ for $i = 0, \cdots, L$, where $L + 1$ represents the total number of possible equally spaced multipath components. Any number of multipath signals received within the ith bin are represented by a single resolvable multipath component having delay τ_i. This technique of quantizing the delay bins determines the time delay resolution of the channel model.

In this way, the time variant impulse response can be expressed as

$$c(t,\tau) = \sum_{i=0}^{L} \underbrace{\alpha_i(t)\exp(j\phi_i(t))}_{g_i(t)} \delta(t-\tau_i) \tag{2.4}$$

where, for the ith multipath component, $\alpha_i(t)$, $\phi_i(t)$ and τ_i is the amplitude attenuation, phase shift and excess delay, respectively.

In a rich scattering environment, where there are a large number of paths arriving within the ith excess delay bin, the central limit theorem can be applied, such that $g_i(t) \triangleq \alpha_i(t)\exp(j\phi_i(t))$ may be modeled as a complex-valued Gaussian random process. Then, the impulse response $c(t,\tau)$ can be fully characterized by its auto-correlation function, which is defined as

$$R(t_1,t_2;\tau_1,\tau_2) = \frac{1}{2}E\left[c(t_1,\tau_1)c^*(t_2,\tau_2)\right] \tag{2.5}$$

In general, for the wireless communications, the radio channel satisfies what is called *wide-sense stationary uncorrelated scattering* (WSSUS). Wide-sense stationary

(WSS) means that the auto-correlation of the impulse response $R(t_1, t_2; \tau_1, \tau_2)$ is a function of time difference $\Delta t \triangleq t_2 - t_1$. Uncorrelated scattering (US) means that the correlation between multipath components with different delays is uncorrelated.

2.2 Analysis of OFDM System

Figure 2.1 illustrates the implementation of a typical OFDM system. As shown there, at the transmitter, a stream of data symbols $x(n)$ is serial-to-parallel converted, such that the serial stream is grouped into consecutive blocks of size N. To be specific, the ith block of data symbols is denoted as a vector[3] $\mathbf{x}[i] \triangleq [x_i(0), \cdots, x_i(N-1)]^T$ with $x_i(n) \triangleq x(iN+n)$.

The N-point IDFT (inverse discrete Fourier transform) of $\mathbf{x}[i]$ can then be expressed as

$$\bar{\mathbf{x}}[i] = \mathcal{F}^H \mathbf{x}[i] \tag{2.6}$$

where $\mathcal{F}$ is a $N \times N$ DFT (discrete Fourier transform) matrix whose (m,n)th entry is given as $\mathcal{F}(m,n) = \frac{1}{\sqrt{N}} \exp\left(-j\frac{2\pi}{N}mn\right)$.

After the parallel-to-serial conversion of $\bar{\mathbf{x}}[i]$, the cyclic prefix with N_g samples are inserted, such that the resultant $\bar{N} = N + N_g$ samples is expressed as

$$\tilde{\mathbf{x}}[i] = \mathbf{T}_{\text{cp}} \mathcal{F}^H \mathbf{x}[i] \tag{2.7}$$

where $\mathbf{T}_{\text{cp}} \triangleq \left[\mathbf{I}_{\text{cp}}^T, \mathbf{I}_N^T\right]^T$ is a concatenation of the last N_g row of a $N \times N$ identity matrix $\mathbf{I}_N$ (that is denoted as $\mathbf{I}_{\text{cp}}$) and the identity matrix $\mathbf{I}_N$ itself.

These $\bar{N}$ samples are send into channel sequentially at rate $1/T_s$ via a digital to analog converter. Hence, for the ith block, the sequence of information-bearing

[3]Note that, in this chapter, the index of N dimensional vectors and matrices takes value from 0 to $N-1$, such that, the first element of vector $\mathbf{y}$ is denoted as $y(0)$; and the first column of matrix $\mathbf{Y}$ is denoted by $\mathbf{Y}(n,0)$ with $n \in [0, N-1]$.

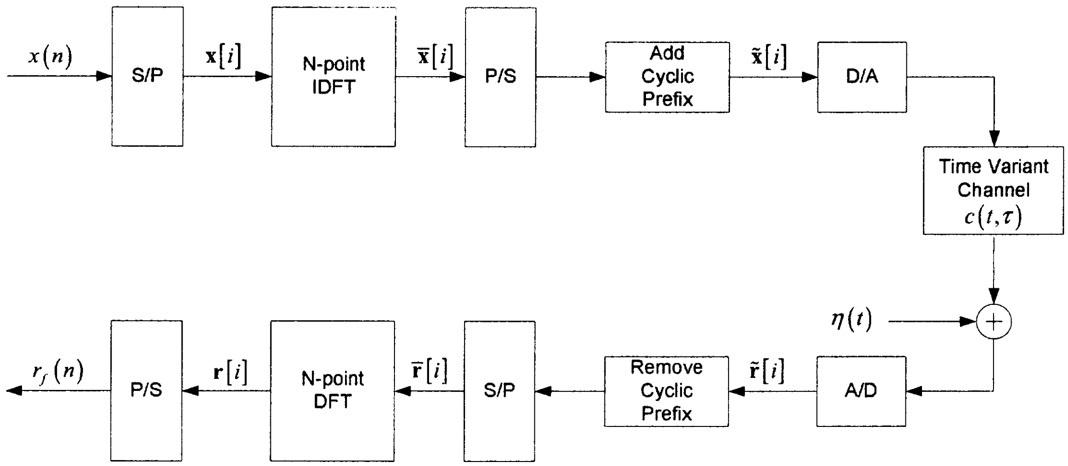


Figure 2.1 OFDM system diagram.

samples before the digital-to-analog converter can be expressed as

$$\tilde{x}_i(m) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(iN+n) \exp\left(j\frac{2\pi n(m-N_g)}{N}\right) \quad m = 0, \cdots, \bar{N}-1 \tag{2.8}$$

which is indeed the sampled version of the complex envelop of ith OFDM symbol in continuous time domain

$$\tilde{x}_i(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(iN+n) \exp\left(j\frac{2\pi n(t-N_gT_s)}{NT_s}\right) \quad t \in [iT, (i+1)T) \tag{2.9}$$

where $T = \bar{N}T_s$ is the duration of one OFDM symbol including cyclic prefix. From (2.9), it is clear that the vector $\mathbf{x}[i]$ represents the signals transmitted on all sub-channels during one OFDM symbol duration, where the separation between two adjacent sub-channel is $1/NT_s$.

In general, the sequence of samples sent into the digital-to-analog converter can be expressed as

$$\tilde{x}(t) = \sum_{l=-\infty}^{\infty} \tilde{x}_{\lfloor \frac{l}{\bar{N}} \rfloor} \left(l \bmod \bar{N}\right) \delta(t - lT_s) \tag{2.10}$$

where $\lfloor \cdot \rfloor$ is the floor operation, l is the sample index. Suppose the digital-to-analog converter at the transmitter is modeled as a spectral shaping filter $\Psi_{\mathrm{Tx}}(t)$, and the analog-to-digital convertor at the receiver is modeled by a receiver filter $\Psi_{\mathrm{Rx}}(t)$, the overall impulse response of the cascade of transmitter filter, continuous-channel and receiver filter can then be denoted as $h(t,\tau) \triangleq \Psi_{\mathrm{Tx}}(t) \star c(t,\tau) \star \Psi_{\mathrm{Rx}}(t)$, where $c(t,\tau)$ represents the continuous time dispersive channel and $\star$ is the linear convolution. With $h(t,\tau)$, the received baseband signal can be written as

$$\begin{aligned}
\tilde{r}(t) &= \int \tilde{x}(t-\tau)h(t,\tau)d\tau + \eta(t) \star \Psi_{\mathrm{Rx}}(t) \\
&= \sum_l \tilde{x}_{\lfloor \frac{l}{\bar{N}} \rfloor}\left(l \bmod \bar{N}\right) \int \delta(t-\tau-lT_s)h(t,\tau)d\tau + \eta(t) \star \Psi_{\mathrm{Rx}}(t) \\
&= \sum_l \tilde{x}_{\lfloor \frac{l}{\bar{N}} \rfloor}\left(l \bmod \bar{N}\right) h(t, t-lT_s) + \eta(t) \star \Psi_{\mathrm{Rx}}(t) \qquad (2.11)
\end{aligned}$$

where $\eta(t)$ is the additive white Gaussian noise (AWGN) with zero mean and variance of σ^2.

By sampling the received signal $\tilde{r}(t)$ at the rate $1/T_s$, the discrete-time received sequence, defined as $\tilde{r}(n) \triangleq \tilde{r}(t)|_{t=nT_s}$, is given as

$$\tilde{r}(n) = \sum_l \tilde{x}_{\lfloor \frac{l}{\bar{N}} \rfloor}\left(l \bmod \bar{N}\right) h_n(n-l) + \tilde{\eta}(n) \qquad (2.12)$$

where, $h_n(l) \triangleq h(t,\tau)|_{t=nT_s,\tau=lT_s}$ is the sampled version of time variant channel response, and $\tilde{\eta}(n) \triangleq [\eta(t) \star \Psi_{\mathrm{Rx}}(t)]\,|_{t=nT_s}$ is the sampled AWGN.

It is clear that $h_n(l)$ represents the response of channel at time instant n due to an impulse applied at time instant $n-l$. In the following, $h_n(l)$ is assumed to be a finite impulse response (FIR) whose order is no greater than L, such that $h_n(l) = 0\ \forall l > L$. From the above equation, it is easy to show that $\tilde{\eta}(n)$ is a Gaussian random variable with zero mean and variance of σ^2.

Then, in discrete time domain, the collection of $\bar{N}$ samples of received signal, which corresponds to ith transmitted OFDM symbol, can be expressed as

$$\tilde{\mathbf{r}}[i] = \mathbf{H}_{i,0}\tilde{\mathbf{x}}[i] + \mathbf{H}_{i,1}\tilde{\mathbf{x}}[i-1] + \tilde{\eta}[i] \qquad (2.13)$$

where $\tilde{\eta}[i]$ is a vector of AWGN samples with covariance matrix $\sigma^2 \mathbf{I}_{\bar{N}}$, and for $l = 0, 1$ the matrices $\mathbf{H}_{i,l}$ are defined as

$$\mathbf{H}_{i,0}(m,n) = \begin{cases} h_{i\bar{N}+m}(m-n) & 0 \le m-n \le L \\ 0 & \text{elsewhere} \end{cases} \tag{2.14}$$

i.e.

$$\mathbf{H}_{i,0} = \begin{bmatrix} h_{i\bar{N}}(0) & 0 & 0 & \cdots & 0 \\ \vdots & h_{i\bar{N}+1}(0) & 0 & \cdots & 0 \\ h_{i\bar{N}+L}(L) & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & h_{i\bar{N}+\bar{N}-1}(L) & \cdots & h_{i\bar{N}+\bar{N}-1}(0) \end{bmatrix} \tag{2.15}$$

and

$$\mathbf{H}_{i,1}(m,n) = \begin{cases} h_{i\bar{N}+m}\left(\bar{N}-n+m\right) & \bar{N}-L \le n-m < \bar{N} \\ 0 & \text{elsewhere} \end{cases} \tag{2.16}$$

i.e.

$$\mathbf{H}_{i,1} = \begin{bmatrix} 0 & \cdots & h_{i\bar{N}}(L) & \cdots & h_{i\bar{N}}(1) \\ \vdots & \ddots & 0 & \ddots & \vdots \\ 0 & \cdots & \ddots & \cdots & h_{i\bar{N}+L-1}(L) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \tag{2.17}$$

As shown in Figure 2.1, the cyclic prefix are dropped at the receiver side. Such an operation of dropping cyclic prefix can be expressed as

$$\bar{\mathbf{r}}[i] = \mathbf{R}_{\text{cp}}\tilde{\mathbf{r}}[i] = \mathbf{R}_{\text{cp}}\mathbf{H}_{i,0}\tilde{\mathbf{x}}[i] + \mathbf{R}_{\text{cp}}\mathbf{H}_{i,1}\tilde{\mathbf{x}}[i-1] + \mathbf{R}_{\text{cp}}\tilde{\eta}[i] \tag{2.18}$$

where $\mathbf{R}_{\text{cp}} \triangleq \left[0_{N \times N_g}, \mathbf{I}_N\right]$.

As long as $N_g \geq L$, $\mathbf{R}_{\text{cp}}\mathbf{H}_{i,1} = 0_{N\times\bar{N}}$, and $\bar{\mathbf{r}}[i]$ can be simplified to

$$\bar{\mathbf{r}}\left[i\right] = \mathbf{R}_{\text{cp}}\mathbf{H}_{i,0}\tilde{\mathbf{x}}\left[i\right] + \bar{\eta}\left[i\right] \tag{2.19}$$

where $\bar{\eta}\left[i\right] \triangleq \mathbf{R}_{\text{cp}}\tilde{\eta}\left[i\right]$. Hence, if the cyclic prefix is chosen to be longer than the maximum delay spread of the underlying channel, there exists no inter (OFDM) symbol interference between adjacent OFDM symbols.

Therefore, the index i in Equation (2.19) can be dropped; and in the sequel, the received OFDM symbol in discrete time domain is expressed as

$$\bar{\mathbf{r}} = \mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}\mathcal{F}^H\mathbf{x} + \bar{\eta} \tag{2.20}$$

where $\mathbf{H}_0$ is a $\bar{N} \times \bar{N}$ matrix, whose (m,n)th entry is given as

$$\mathbf{H}_0\left(m,n\right) = \begin{cases} h_m\left(m-n\right) & 0 \leq m-n \leq L \\ 0 & \text{elsewhere} \end{cases} \tag{2.21}$$

After N-point DFT, the received OFDM symbol in discrete frequency domain is obtained as

$$\mathbf{r} = \mathcal{F}\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}\mathcal{F}^H\mathbf{x} + \eta \tag{2.22}$$

where $\eta \triangleq \mathcal{F}\bar{\eta}$ follows Gaussian distribution $\mathcal{CN}(0, \sigma^2\mathbf{I}_N)$.

By defining $\bar{\mathbf{H}} \triangleq \mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}$ and $\mathbf{H} \triangleq \mathcal{F}\bar{\mathbf{H}}\mathcal{F}^H$, Equation (2.22) can be simplified:

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \eta \tag{2.23}$$

Note that, Equation (2.23) describes the input-output relation of a OFDM system in discrete frequency domain.

From Equation (2.23), the nth element in vector $\mathbf{r}$, which represents the received signal on the nth sub-channel, can be expressed as[4]

$$r_f(n) = \mathbf{H}(n,n)x(n) + \sum_{m \neq n} \mathbf{H}(n,m)x(m) + \eta(n) \tag{2.24}$$

where $\mathbf{H}(n,m)$ is the (n,m)th entry of matrix $\mathbf{H}$.

After some calculation, it shows that the (m,n)th entry of matrix $\mathbf{H}$ can be expressed as

$$\mathbf{H}(m,n) = \mathbf{H}_f(n, (m-n)\,\mathrm{mod}N) \tag{2.25}$$

where matrix $\mathbf{H}_f$ is the 2-dimensional N-point DFT of matrix $\mathbf{H}_t$, whose (l,k)th entry is defined as

$$\mathbf{H}_t(l,k) = \begin{cases} h_{k+N_g}(l) & 0 \leq l \leq L \\ 0 & \text{elsewhere} \end{cases} \tag{2.26}$$

More specifically, the relation between $\mathbf{H}_f$ and $\mathbf{H}_t$ can be expressed as

$$\mathbf{H}_f(m,n) = \frac{1}{N} \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} \mathbf{H}_t(l,k) e^{-j\frac{2\pi}{N}(ml+nk)} \tag{2.27}$$

Then, Equation (2.24) can be rewritten as

$$r_f(n) = \mathbf{H}_f(n,0)\,x(n) + \sum_{m \neq n} \mathbf{H}_f(m, (n-m)\,\mathrm{mod}N)\,x(m) + \eta(n) \tag{2.28}$$

From the definition of matrix $\mathbf{H}_t$ given in (2.26), it is clear that matrix $\mathbf{H}_t$ describes the channel response within one OFDM symbol duration: each column of matrix $\mathbf{H}_t$ represents the instantaneous channel impulse response at a given time instant; and each row of matrix $\mathbf{H}_t$ shows how a particular multipath component varies with time.

---

[4]Subscribe f stands for frequency domain, while subscribe t for time domain.

When channel response varies within one OFDM symbol duration, columns in matrix $\mathbf{H}_t$ are different from each other. Then, from Equation (2.26) and Equation (2.27), the first of column of matrix $\mathbf{H}_f$ can be expressed as

$$\mathbf{H}_f(n,0) = \frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{l=0}^{L} h_{k+N_g}(l)e^{-j\frac{2\pi}{N}nl}\right] \quad \text{for } n = 0, \cdots, N-1 \tag{2.29}$$

which represents the average frequency response of the underlying time-variant channel within one OFDM symbol duration, while the remaining $N-1$ columns in matrix $\mathbf{H}_f$ are non-zero. Hence, it can be deduced from (2.28) that the data symbol transmitted on the nth sub-channel will be interfered by the data symbols transmitted on sub-channels other than the nth sub-channel. Such interference is known as inter-carrier interference (ICI) and is detrimental to the system performance. Clearly, ICI is generated due to the time variation of the underlying channel.

In the special case when channel is time invariant within one OFDM symbol duration, matrix $\mathbf{H}_t$ contains identical columns, i.e., $h_{k_1+N_g}(l) = h_{k_2+N_g}(l) = h(l)$ for $k_1 \neq k_2$. Then, by (2.27), matrix $\mathbf{H}_f$ only contains non-zero elements in its first column. That means matrix $\mathbf{H}$ is diagonal. Specifically, the elements in the first column of matrix $\mathbf{H}_f$ (or the diagonal elements of matrix $\mathbf{H}$) can be expressed as

$$\mathbf{H}(n) \triangleq \mathbf{H}_f(n,0) = \sum_{l=0}^{L} h\,(l)\exp\left(-j\frac{2\pi}{N}nl\right) \quad \text{for } n = 0, \cdots, N-1 \tag{2.30}$$

which reflects the channel response in frequency domain. Furthermore, Equation (2.28) is simplified as

$$r_f\,(n) = \mathbf{H}\,(n)\,x\,(n) + \eta\,(n) \quad \text{for } n = 0, \cdots, N-1 \tag{2.31}$$

which means there exists no ICI in the system, and the whole channel, which is usually frequency selective, is divided into a set of parallel orthogonal sub-channels with flat frequency response.

As implied by Equation (2.31), for a OFDM system without inter-symbol interference and inter-carrier interference, equalization of each data with coherent detection simply amounts to a normalization by a complex scalar. More important, each symbol can be detected separately, an approach that can be extremely successful in preventing errors caused by strong channel attenuation or noise in specific sub-channels.

From above discussion, it is obvious that OFDM is attractive for practical applications only if there is no inter-symbol interference and inter-carrier interference. These conditions can be met by carefully choosing the OFDM symbol duration. First of all, in order to prevent inter-symbol interference, OFDM symbol duration should be chosen long enough to include the cyclic prefix which is longer than the maximum delay spread of the underlying channel. Secondly, to prevent inter-carrier interference, the chosen OFDM symbol duration should be much less than the coherence time of the underlying channel, such that the channel response within one OFDM symbol duration can be well approximated as time invariant.

2.3 Modeling of MC-CDMA

The term multi-carrier CDMA (MC-CDMA) was used in [1] to identify an orthogonal multi-carrier modulation in which transmitted data symbols are copied over all available sub-channels (an operation known as spreading in the frequency domain), and encoded by different signature sequences in order to be distinguished at the receiver side.

Figure 2.2 shows the MC-CDMA transmitter for user k. The input information data symbol x_k, which has duration of T, is first copied N times. Each copy is then multiplied by a single chip of a signature sequence $\mathbf{c}_k$ of length N, denoted as $\mathbf{c}_k = [c_{n,0}, \cdots, c_{n,N-1}]^T$. Then, those N products are simultaneously modulated on a set of sub-carriers, where the separation between two adjacent sub-carriers is set

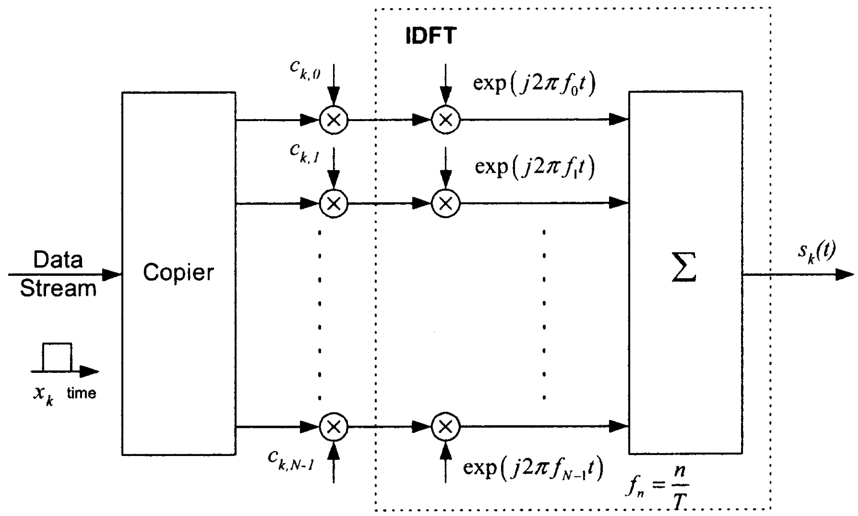


Figure 2.2 Diagram of MC-CDMA transmitter.

to $1/T$. Such modulation can be implemented by IDFT as in OFDM. The main difference is that MC-CDMA scheme transmits the same data symbol in parallel through the set of sub-carriers whereas the OFDM scheme transmits different data symbols. The complex equivalent low-pass transmitted MC-CDMA signal for user k is given by

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} c_{k,n} x_k \exp\left(j2\pi \frac{nt}{T}\right) \tag{2.32}$$

where N is also known as the *processing gain* of the MC-CDMA system.

In discrete frequency domain, the equivalent transmitted signal for user k can be characterized by a vector

$$\mathbf{s}_k = \mathbf{c}_k x_k \tag{2.33}$$

where $\mathbf{s}_k = [s_{k,0}, \cdots, s_{k,N-1}]^T$ is a N dimensional vector whose elements represent the signal to be transmitted on the sub-channels.

In general, cyclic prefix, which is not shown in Figure 2.2 for simplicity, is inserted between adjacent MC-CDMA symbols to prevent inter-symbol interference caused by multipath fading [2,3,11]. However, the insertion of cyclic prefix reduces the symbol rate at which each signature sequence can convey. This is because, as argued in the last section, the data symbol duration (the reciprocal of data symbol rate) should be chosen long enough to include the cyclic prefix which must be larger than the maximum delay spread of the underlying channel. Therefore, to achieve high speed data transmission, multi-code scheme is suggested, wherein each user transmits several data symbols in parallel, with each data symbol being spread by a distinct signature sequence.

With multi-code scheme, the transmitted MC-CDMA signal for user k in discrete frequency domain is denoted by

$$\mathbf{s}_k = \mathbf{C}_k \mathbf{x}_k \tag{2.34}$$

where $\mathbf{x}_k \triangleq [x_{k,1}, \cdots, x_{k,M_k}]$ is a M_k dimensional vector representing the data symbols transmitted in parallel; M_k is the number of parallel transmitted data symbols; $\mathbf{C}_k \triangleq [\mathbf{c}_{k,1}, \cdots, \mathbf{c}_{k,M_k}]^T$ is a $N \times M_k$ matrix whose columns are distinct signature sequences.

Without loss of generality, it is assumed that data symbols transmitted in parallel are energy normalized and uncorrelated, such that $E\left[\mathbf{x}_k \mathbf{x}_k^H\right] = \mathbf{I}_{M_k}$. Then, the transmitted power for user k can be given as $tr\left(\mathbf{C}_k \mathbf{C}_k^H\right)$.

2.3.1 Downlink Scenario

For multi-user case, all active users are communicating with a common base station. For the downlink communication scenario, which is shown in Figure 2.3, the transmitted signal designated to different users are synchronously summed together at base station. Hence, the transmitted signal at the base station can be characterized as

$$\mathbf{s} = \sum_{k=1}^{K} \mathbf{s}_k = \sum_{k=1}^{K} \mathbf{C}_k \mathbf{x}_k \tag{2.35}$$

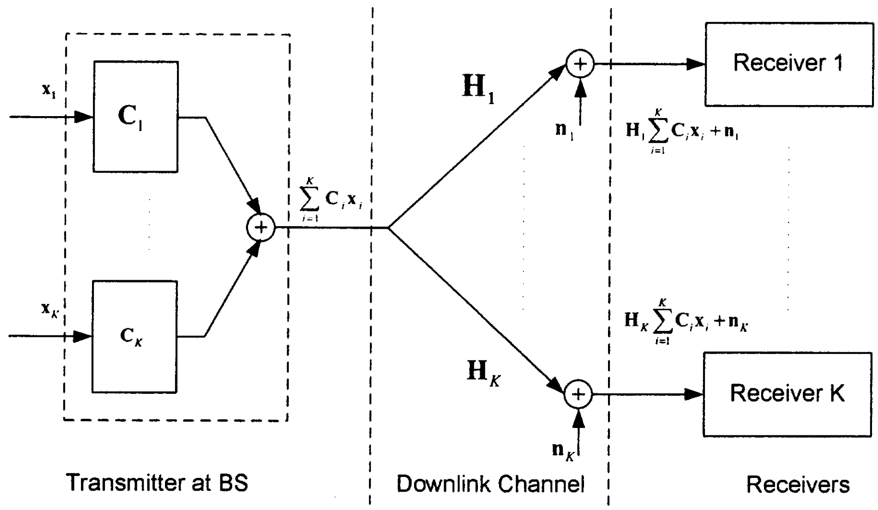


Figure 2.3 Downlink scenario of MC-CDMA system.

where K is the number of active users in the system. In this way, $M = \sum_{k=1}^{K} M_k$ is the total number of signature sequences used by the system.

Extending (2.23), the received signal at a particular user's mobile, say user k's mobile, can be expressed as

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{C}_k \mathbf{x}_k + \mathbf{H}_k \sum_{l \neq k} \mathbf{C}_k \mathbf{x}_k + \mathbf{n}_k \tag{2.36}$$

where $\mathbf{H}_k$ reflects the channel effect of the wireless channel which links the base station and mobile k, and $\mathbf{n}_k$ represents the AWGN observed at mobile k.

In the rest of dissertation, it is assumed that data symbol duration has been carefully selected such that the channel response can be approximated as time invariant within one data symbol duration. Then, matrix $\mathbf{H}_k$ in (2.36) can be modeled as a diagonal matrix, whose diagonal elements are characterized by a N dimensional vector

$$\mathbf{h}_k = \sqrt{g_k}\left[h_{k,0}, \cdots, h_{k,n}, \cdots, h_{k,N-1}\right]^T \tag{2.37}$$

where g_k is a real scalar representing the path loss due to propagation and shadowing; and $h_{k,n}$ is a Gaussian random variable with zero mean and unit variance, which represents the instantaneous channel fading realization.

Because the channel response is always determined by the surrounding environment, $\mathbf{h}_k$ is assumed to be independent among users. However, due to the proximity and the partial overlap of different sub-channels, fading is assumed to be correlated across sub-channels. Such fading correlation can be determined by the channel multipath delay profile. Following the model in [8] where the delay profile is assumed to be exponential distributed, the fading correlation between the mth sub-channel and the nth sub-channel is easily found to be

$$\rho_k(m,n) \triangleq E\left[h_{k,m}h_{k,n}^*\right] = \frac{1 - j2\pi\tau_d(m-n)\Delta f}{1 + (2\pi\tau_d(m-n)\Delta f)^2} \tag{2.38}$$

where τ_d is the r.m.s (root mean square) delay spread and Δf is the separation between two adjacent sub-channels.

Since path loss g_k's have been explicitly incorporated, the variances of AWGN at all mobiles are assumed to be same; i.e. $E\left[\mathbf{n}_k\mathbf{n}_k^H\right] = \sigma^2\mathbf{I}_N$ for all k.

2.3.2 Uplink Scenario

For completeness of describing MC-CDMA system, the uplink scenario is briefly discussed in the sequel.

For the uplink communication scenario, transmission from mobiles to the base station is shown in Figure 2.4, wherein all mobiles transmitting asynchronously. In this subsection, for simplicity, only quasi-synchronous transmission is considered, in which all mobiles in the system are coordinated so that signals from all mobile arrive at the base station within a pre-assigned receipt window. By choosing the cyclic prefix longer than the receipt window, the received signal at the base station can be

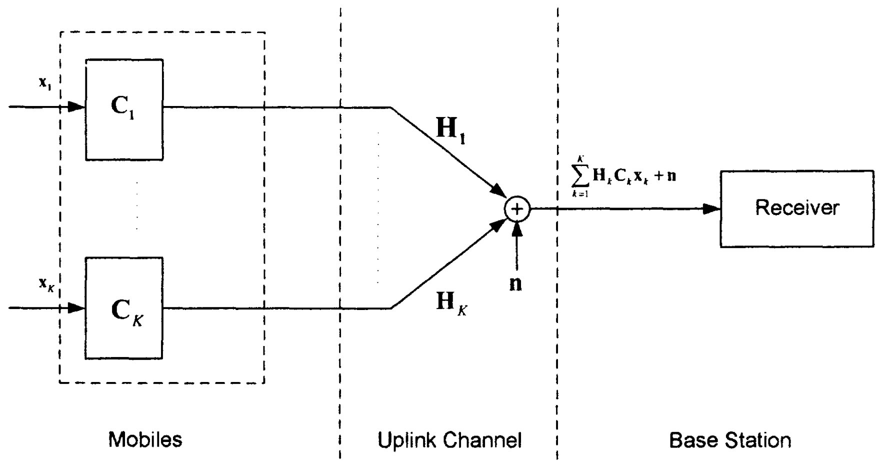


Figure 2.4 Uplink scenario of MC-CDMA system.

modeled by

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{C}_k \mathbf{x}_k + \mathbf{n} \tag{2.39}$$

where $\mathbf{H}_k$ reflects the effect of the wireless channel which links mobile k and the base station, and $\mathbf{n}$ represents the AWGN observed at the base station with zero mean and covariance matrix $E\left[\mathbf{n}\mathbf{n}^H\right] = \sigma^2 \mathbf{I}_N$.

2.4 Chapter Summary

In this chapter, a brief review of dispersive channel is presented. By carefully analyzing the effect of underlying channel on the transmitted OFDM signal, a matrix formulation of MC-CDMA system in discrete frequency domain is established, which will be used for the rest of dissertation.

CHAPTER 3

RATE CAPACITY IN MC-CDMA SYSTEM

There is always much interest in providing high speed data transmission over a wireless link. This is partially driven by the desire to integrate various data centric services, such as text, packet data, image and video, into a single wireless communication system. In order to provide multimedia services, the wireless communication system is migrating from *voice* orientated to *data* orientated. As the result, the system capacity of interest is shifting from the user capacity to the rate capacity.

This chapter studies the data rate that can be achieved in the downlink of MC-CDMA system. The purpose is to understand the potential of applying MC-CDMA technique for wireless data communication systems.

3.1 Introduction

In general, the downlink communication scenario, in which a single transmitter sends independent information to multiple uncoordinated receivers, fits the model of the broadcast channel in the information theory. As a multi-user communication system, the rate capacity is defined with a rate vector, whose elements represent the rates that all users in the system can simultaneously maintain on same channel condition [6,12].

The rate capacity for broadcast channel was first studied by Cover in [13] and Bergmans in [14], respectively. It is shown that, for Gaussian degraded broadcast channel, the capacity is achieved by applying superposition coding at the common transmitter and successive decoding at the receivers [6, 12–14]. Later, Hughes-Hartogs [15] and Tse [16, 17] independently characterized the rate capacity for parallel Gaussian broadcast channel. Most recently, extending these results, Li and Goldsmith derived the ergodic [18] and outage capacity [19] for frequency flat fading broadcast channel. Note that, a fading channel can always be modeled as a set of parallel channels with each of parallel channels corresponding to a fading state.

However, for MC-CDMA technique, identical information is transmitted over all available sub-channels simultaneously. Therefore, the downlink of MC-CDMA, which is formulated by (2.36), fits the model of a *vector* broadcast channel, rather than a degraded broadcast channel. Even though, there is extensive research on broadcast channel, the rate capacity for vector broadcast channel is still an open question in the literature.

Reference [20] studied the outage rate capacity that can be achieved in the downlink of MC-CDMA system under frequency selective fading channels. Multi-code scheme was assumed in that work. However, the result presented there was based on two implicit assumptions:

1. the data symbols spread by different signature sequences are independently encoded and decoded at the base station and mobiles, respectively;

2. for each user, the assigned power is proportional to the number of its assigned signature sequences.

In order to fully understand the rate capacity in the downlink of MC-CDMA system, various combinations of coding strategies and resource allocation policies are considered in this chapter. Totally, three coding strategies, which differ in the way to encode and decode the data symbols spread by different signature sequences; and two resource allocation policies, which differ in the way to assign power and signature sequences among users, are considered. This work seeks to understand, through a simulation study, the potential gains from introducing cooperation to encode and decode the data symbols spread with distinct signature sequences. The influence of resource allocation policy on the rate capacity, and its dependence on users' channel qualities, are also considered.

The rest of chapter is organized as follows. Section 3.2 presents the MMSE receiver used at each mobile. In Section 3.3, three coding strategies are presented and their corresponding rate capacities under given channel responses are obtained. The ergodic and the outage rate capacities for fading channels are then given in Section 3.4. Via simulation, the effect of coding strategies and resource allocation policies on the resultant rate capacity is presented in Section 3.5, where frequency selective fading channels are assumed. Finally, Section 3.6 summaries the chapter.

3.2 Linear MMSE Receiver

At each mobile, a linear MMSE receiver is used to demodulate its desired user's transmitted data symbols. Specifically, the MMSE receiver at mobile k, which is composed of M_k linear MMSE receiver filters, can be expressed as:

$$\mathbf{G}_k = [\mathbf{g}_{k,1}, \cdots, \mathbf{g}_{k,M_k}] = \mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{C}_k \tag{3.1}$$

where $\mathbf{g}_{k,i}$ is the linear MMSE receiver filter which reconstructs the data symbol spread by user k's ith signature sequence $\mathbf{c}_{k,i}$,

$$\mathbf{R}_k = \mathbf{H}_k \left(\sum_{l=1}^{K} \mathbf{C}_l \mathbf{C}_l^H \right) \mathbf{H}_k + \sigma^2 \mathbf{I}_N \tag{3.2}$$

is the covariance matrix of the received signal.

Then, the residual error at the output of those MMSE receiver filters can be expressed as:

$$\mathbf{e}_k \triangleq [e_{k,1}, \cdots, e_{k,M_k}]^T = \mathbf{x}_k - \mathbf{G}_k^H \mathbf{r}_k \tag{3.3}$$

And the covariance matrix of the residual error, which is also known as user k's individual mean square error matrix, can be shown to equal

$$\mathbf{MSE}_k \triangleq E\left[\mathbf{e}_k \mathbf{e}_k^H\right] = \left[\mathbf{I}_{M_k} + \mathbf{C}_k^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{C}_k\right]^{-1} \tag{3.4}$$

where

$$\mathbf{Z}_k = \mathbf{H}_k \left(\sum_{l \neq k} \mathbf{C}_l \mathbf{C}_l^H \right) \mathbf{H}_k + \sigma^2 \mathbf{I}_N \tag{3.5}$$

is the covariance of the MAI observed at mobile k.

Furthermore, the output of ith MMSE receiver filter at mobile k can be explicitly expressed as

$$\hat{x}_{k,i} = \mathbf{g}_{k,i}^H \mathbf{r}_k = \frac{\mathrm{SINR}_{k,i}}{1 + \mathrm{SINR}_{k,i}} x_{k,i} + z'_{k,i} \tag{3.6}$$

where

$$\mathrm{SINR}_{k,i} = \mathbf{c}_{k,i}^H \mathbf{H}_k^H \left(\mathbf{Z}_k + \mathbf{W}_{k,i} \right)^{-1} \mathbf{H}_k \mathbf{c}_{k,i} \tag{3.7}$$

with $\mathbf{W}_{k,i} = \mathbf{H}_k \sum_{j=1, j\neq i}^{M_k} \left(\mathbf{c}_{k,j} \mathbf{c}_{k,j}^H \right) \mathbf{H}_k^H$, and

$$z'_{k,i} = \mathbf{g}_{k,i}^H \left[\mathbf{H}_k \left(\sum_{j \neq i} \mathbf{c}_{k,j} x_{k,j} + \sum_{l \neq k} \mathbf{C}_l \mathbf{x}_l \right) + \mathbf{n}_k \right] \tag{3.8}$$

Since $z'_{k,i}$, the residual interference plus noise at the output, has zero mean and variance of $\frac{\mathrm{SINR}_{k,i}}{\left(1+\mathrm{SINR}_{k,i}\right)^2}$, the $\mathrm{SINR}_{k,i}$ given by (3.7) turns out to be the output signal to interference plus noise ratio for the ith MMSE receiver filter at mobile k.

Indeed, $\mathrm{SINR}_{k,i}$ has close relationship with the matrix $\mathbf{MSE}_k$:

$$\mathrm{SINR}_{k,i} = \frac{1}{\left[\mathbf{MSE}_k\right]_{i,i}} - 1 \tag{3.9}$$

where $\left[\mathbf{MSE}_k\right]_{i,i}$ is the ith diagonal element of matrix $\mathbf{MSE}_k$.

It is well known that the linear MMSE receiver is the optimum linear receiver. It maximizes the output signal to interference plus noise ratio for each data symbol which is spread with a distinct signature sequences. For the multi-code scheme, it also minimizes the covariance *matrix* of the residual error at each mobile.

The linear MMSE receiver is very attractive for practical applications because it admits adaptive [21–23] and blind-adaptive [24,25] implementations. Moreover, since those adaptive and blind-adaptive implementations require as little side information as a conventional matched filter based receiver, the linear MMSE receiver is well suited for downlink communication scenario.

3.3 Instantaneous Rate Capacity

This section studies the instantaneous rate capacity that can be achieved under given channel responses. Totally, three coding strategies are considered, which differ in the way to encode and decode the data symbols spread by distinct signature sequences.

3.3.1 Independent Coding Across Signature Sequences

First, consider the coding strategy through which, for each user, the data symbols spread by different signature sequences are *independently encoded and decoded* at the base station and the mobile, respectively.

As shown in the upper part of Figure 3.1, at the transmitter side, the data stream of user k is first serial to parallel converted into a set of M_k sub-streams; each sub-stream is then independently encoded before being spread for transmission. At the receiver side, the outputs of MMSE receiver filters, which correspond to the sub-streams spread with different signature sequences, are independently decoded, and then parallel to serial converted.

By viewing the spreading at the base station and the filtering at the receiver as part of the *equivalent channel*, the link from the base station to the kth mobile can be decomposed into a set of M_k sub-channels. As indicated by the lower part of Figure 3.1, each sub-channel is associated with a signature sequence and its corresponding MMSE receiver filter. In this way, the corresponding rate capacity for user k is given as the summation of the mutual information $I\left(x_{k,i};\hat{x}_{k,i}\right)$ of all those sub-channels, such that

$$\mathcal{R}_k^{\mathrm{I}} = \sum_{i=1}^{M_k} I\left(x_{k,i};\hat{x}_{k,i}\right) \tag{3.10}$$

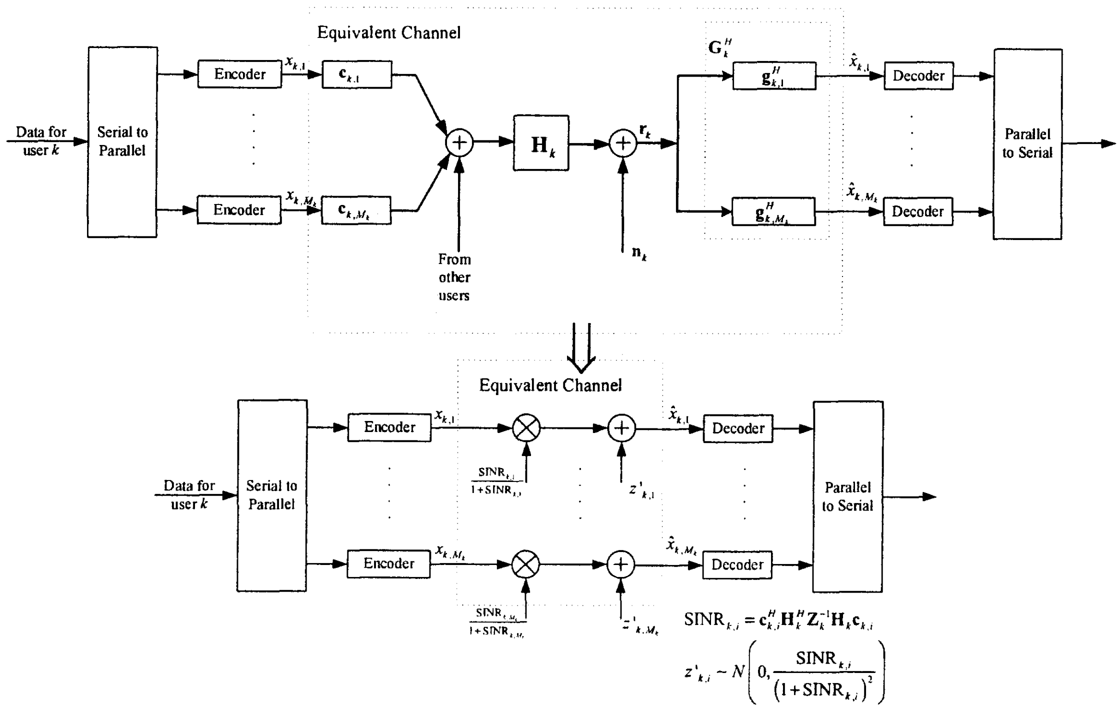


Figure 3.1 Independent coding across signature sequences.

Specifically, the ith sub-channel of user k (with $1 \le i \le M_k$), which is associated with user k's ith signature sequence and its corresponding MMSE receiver filter, can be modeled by Equation (3.6). That is, the encoded data symbol $x_{k,i}$ is properly amplified $\frac{\mathrm{SINR}_{k,i}}{1+\mathrm{SINR}_{k,i}}$ times, and then corrupted by residual interference plus noise $z'_{k,i}$, which is characterized as a random variable with zero mean and covariance of $\frac{\mathrm{SINR}_{k,i}}{\left(1+\mathrm{SINR}_{k,i}\right)^2}$. Due to the fact that $z'_{k,i}$ can be well approximated by a Gaussian random variable [23], the maximum data rate for user k is achieved by using Gaussian signalling. The resultant user k's rate capacity is then given as

$$\begin{aligned} \mathcal{R}_k^{\mathrm{I}} &= \sum_{i=1}^{M_k} \frac{1}{2} \log_2\left(1 + \mathrm{SINR}_{k,i}\right) \qquad (3.11) \\ &= -\sum_{i=1}^{M_k} \frac{1}{2} \log_2\left(\left[\mathbf{MSE}_k\right]_{i,i}\right) \\ &= \sum_{i=1}^{M_k} \frac{1}{2} \log_2\left(1 + \mathbf{c}_{k,i}^H \mathbf{H}_k^H \left(\mathbf{Z}_k + \mathbf{W}_{k,i}\right)^{-1} \mathbf{H}_k \mathbf{c}_{k,i}\right) \end{aligned}$$

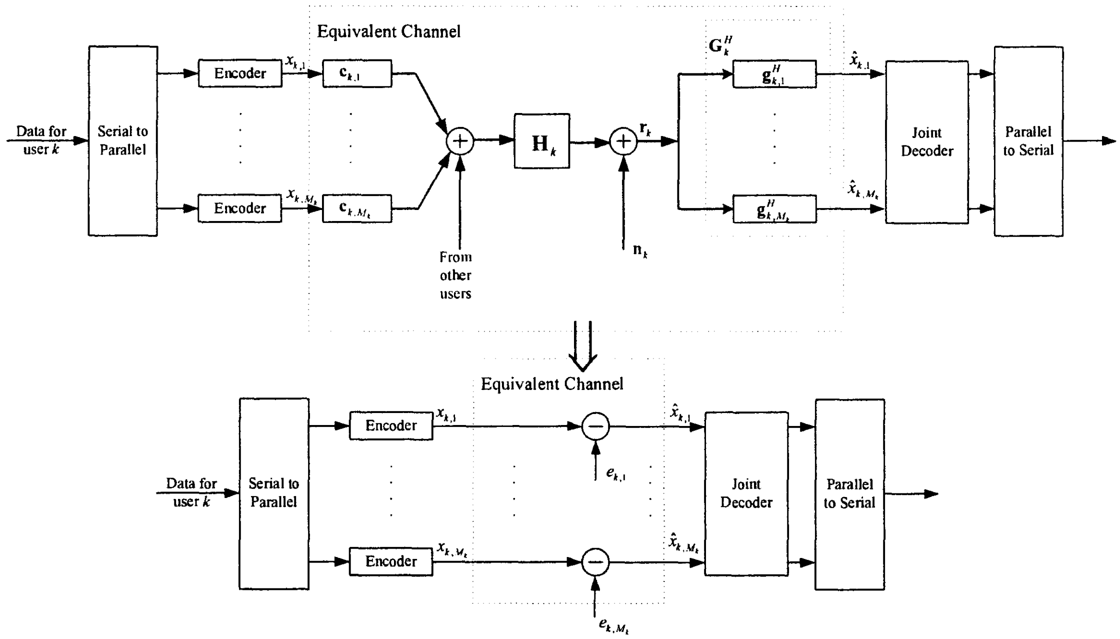


Figure 3.2 Cooperative decoding across signature sequences.

3.3.2 Cooperative Decoding Across Signature Sequences

Next, consider the coding strategy with which, at each mobile, the desired user's encoded data symbols are *cooperatively decoded*, even though those data symbols are conveyed by different signature sequences.

In general, the covariance matrix of the residual error at the output of each user's MMSE receiver filters is non-diagonal. This is because, as indicated by Equation (3.8), the MMSE receiver filter does not totally eliminate the interference which arises from the parallel transmission of the multiple symbols spread with distinct signature sequences. As a result, the outputs of each user's MMSE receiver filters are correlated. It is then conceivable that higher data rate can be achieved for each user by exploiting such correlation.

As shown in Figure 3.2, a joint decoder is applied at mobile to exploit the correlation among the outputs of MMSE receiver filters. By considering the spreading at the transmitter and the filtering at the receiver as part of the *equivalent channel*, the link from the base station to the kth user can be modeled as

$$\hat{\mathbf{x}}_k = \left[\hat{x}_{k,1}, \cdots, \hat{x}_{k,M_k}\right]^T = \mathbf{x}_k - \mathbf{e}_k \tag{3.12}$$

whose rate capacity is given as

$$\mathcal{R}_k^{\mathrm{II}} = I\left(\mathbf{x}_k; \hat{\mathbf{x}}_k\right) \tag{3.13}$$

With Gaussian signalling adopted by each user, both the received signal and the MAI at mobile k follow Gaussian distribution. Hence, from linear estimation theory [26], the residual error $\mathbf{e}_k$ is Gaussian distributed and independent of $\hat{\mathbf{x}}_k$. Therefore, from (3.13), user k's rate capacity can be derived as

$$\begin{aligned} \mathcal{R}_k^{\mathrm{II}} &= h\left(\mathbf{x}_k\right) - h\left(\mathbf{x}_k|\hat{\mathbf{x}}_k\right) \\ &= h\left(\mathbf{x}_k\right) - h\left(\mathbf{e}_k\right) \\ &= -\frac{1}{2}\log_2 \det\left(\mathbf{MSE}_k\right) \\ &= \frac{1}{2}\log_2 \det\left(\mathbf{I}_{M_k} + \mathbf{C}_k^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{C}_k\right) \end{aligned} \tag{3.14}$$

Due to the Hadamard inequality [27] that: the determinant of a positive definite matrix is less than or equal to the products of its diagonal elements with equality if and only if the matrix itself is diagonal, it is easy to show:

$$\mathcal{R}_k^{\mathrm{II}} = -\frac{1}{2}\log\det\left(\mathbf{MSE}_k\right) \geq -\sum_{i=1}^{M_k}\frac{1}{2}\log\left(\left[\mathbf{MSE}\right]_{i,i}\right) = \mathcal{R}_k^{\mathrm{I}} \tag{3.15}$$

That means the joint decoding of outputs of each user's MMSE receiver filters will definitely increase the corresponding user's rate capacity.

It is clear that, in the presence of the MAI and the additive noise, the rate capacity that can be achieved by user k is less than or equal to the the mutual information between the transmitted data symbol $\mathbf{x}_k$ and the received signal $\mathbf{r}_k$. With Gaussian signalling, the mutual information between the transmitted data symbol $\mathbf{x}_k$

and the received signal $\mathbf{r}_k$ can be derived as

$$\begin{aligned} I(\mathbf{r}_k;\mathbf{x}_k) &= h(\mathbf{r}_k) - h(\mathbf{r}_k|\mathbf{x}_k) \\ &= \frac{1}{2}\log\frac{\det\left(\mathbf{Z}_k + \mathbf{H}_k\mathbf{C}_k\mathbf{C}_k^H\mathbf{H}_k^H\right)}{\det(\mathbf{Z}_k)} \\ &= \frac{1}{2}\log\det\left(\mathbf{I}_N + \mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k\mathbf{C}_k^H\mathbf{H}_k^H\right) \\ &= \frac{1}{2}\log\det\left(\mathbf{I}_{M_k} + \mathbf{C}_k^H\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k\right) \end{aligned} \tag{3.16}$$

where the determinant identity $\det(\mathbf{I}+\mathbf{AB}) = \det(\mathbf{I}+\mathbf{BA})$ is used in the last step. It is obvious that $I(\mathbf{r}_k;\mathbf{x}_k) = \mathcal{R}_k^{\mathrm{II}}$. That means, for each user, the joint decoding of the outputs of its MMSE filters is capacity lossless.

Even though the joint decoding of outputs of MMSE filters is capacity lossless, the cost of its implementation is prohibitively high when the number of parallel transmitted sub-streams is large. Therefore, in the following, another decoding structure, known as the successive interference cancelation (SIC) decoding, is considered.

By using the matrix inverse lemma

$$(\mathbf{A}+\mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left(\mathbf{C}^{-1}+\mathbf{DA}^{-1}\mathbf{B}\right)^{-1}\mathbf{DA}^{-1} \tag{3.17}$$

the MMSE filters at mobile k, which was once given by (3.1), can be rewritten as

$$\begin{aligned} \mathbf{G}_k &= \left(\mathbf{Z}_k + \mathbf{H}_k\mathbf{C}_k\mathbf{C}_k^H\mathbf{H}_k^H\right)^{-1}\mathbf{H}_k\mathbf{C}_k \\ &= \mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k\left(\mathbf{C}_k^H\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k + \mathbf{I}_{M_k}\right)^{-1} \\ &= \mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k\,\mathbf{MSE}_k \end{aligned} \tag{3.18}$$

where the last equality is obtained by using (3.4).

Equation (3.18) indicates that user k's linear MMSE filters can be modeled as a cascaded of a set of matched filters $\mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k$ and a set of estimation filters $\mathbf{MSE}_k$, as shown in Figure 3.3. More specifically, with $\mathbf{r}_k$ as the received signal which is given

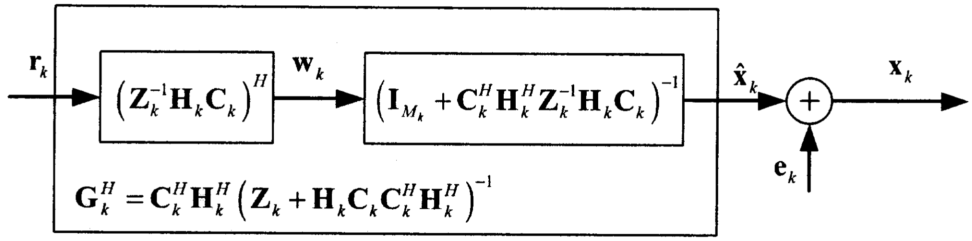


Figure 3.3 Cascade model of linear MMSE receiver for mobile k.

by (2.36), the outputs of the matched filters can be expressed as

$$\mathbf{w}_k = \left(\mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k\right)^H \mathbf{r}_k = \mathbf{C}_k^H\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k\mathbf{x}_k + \mathbf{C}_k^H\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{z}_k \tag{3.19}$$

where $\mathbf{z}_k \triangleq \mathbf{H}_k \sum_{l\neq k} \mathbf{C}_l\mathbf{x}_l + \mathbf{n}_k$ is the MAI plus additive noise observed at the input of mobile k. And, the outputs of the estimation filters can be expressed as

$$\hat{\mathbf{x}}_k = \mathbf{MSE}_k\mathbf{w}_k = \mathbf{x}_k - \mathbf{e}_k \tag{3.20}$$

where $\mathbf{e}_k$ is the residual error which, as presented in Section 3.2, has zero mean and covariance matrix of $\mathbf{MSE}_k$.

With Cholesky factorization [27,28], the covariance matrix of the residual error $\mathbf{e}_k$ can be uniquely decomposed as

$$\mathbf{MSE}_k = \left(\mathbf{C}_k^H\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k + \mathbf{I}_{M_k}\right)^{-1} = \mathbf{B}_k^{-1}\mathbf{\Delta}_k^{-1}\mathbf{B}_k^{-H} \tag{3.21}$$

where matrix $\mathbf{B}_k$ is upper triangular and monic (having all ones along the diagonal), and matrix $\mathbf{\Delta}_k$ is real and diagonal.

Define $\mathbf{e}'_k = \mathbf{B}_k\mathbf{e}_k$. It is easy to verify that the covariance matrix of $\mathbf{e}'_k$ preserves the determinant of the covariance matrix of $\mathbf{e}_k$:

$$\begin{aligned}
\det(E[\mathbf{e}'_k\mathbf{e}'^H_k]) &= \det\left(\mathbf{B}_k E\left[\mathbf{e}_k\mathbf{e}_k^H\right]\mathbf{B}_k^H\right) \\
&= \det(\mathbf{B}_k)\det\left(E\left[\mathbf{e}_k\mathbf{e}_k^H\right]\right)\det(\mathbf{B}_k^H) \\
&= \det\left(\mathbf{MSE}_k\right)
\end{aligned} \tag{3.22}$$

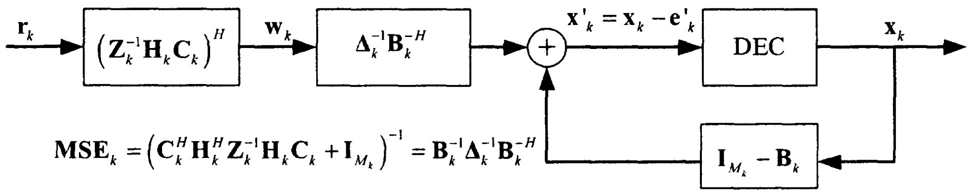


Figure 3.4 Successive interference cancelation structure at mobile k.

Moreover, the components of $\mathbf{e}'_k$ are uncorrelated, because

$$E\left[\mathbf{e}'_k\mathbf{e}'^H_k\right] = E\left[\mathbf{B}_k\mathbf{e}_k\mathbf{e}_k^H\mathbf{B}_k^H\right] = \mathbf{B}_kE\left[\mathbf{e}_k\mathbf{e}_k^H\right]\mathbf{B}_k^H = \mathbf{\Delta}_k^{-1} \tag{3.23}$$

From (3.20) and (3.21), the estimation filters can be reorganized into a feedback configuration:

$$\mathbf{x}_k = \left(\mathbf{C}_k^H\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k + \mathbf{I}_{M_k}\right)^{-1}\mathbf{w}_k + \mathbf{e}_k \tag{3.24}$$

$$\mathbf{x}_k = \mathbf{B}_k^{-1}\mathbf{\Delta}_k^{-1}\mathbf{B}_k^{-H}\mathbf{w}_k + \mathbf{e}_k \tag{3.25}$$

$$\mathbf{B}_k\mathbf{x}_k = \mathbf{\Delta}_k^{-1}\mathbf{B}_k^{-H}\mathbf{w}_k + \mathbf{e}'_k \tag{3.26}$$

$$\mathbf{x}_k = \underbrace{\mathbf{\Delta}_k^{-1}\mathbf{B}_k^{-H}\mathbf{w}_k + (\mathbf{I}_{M_k} - \mathbf{B}_k)\,\mathbf{x}_k}_{\mathbf{x}'_k} + \mathbf{e}'_k \tag{3.27}$$

as presented by Figure 3.4.

Indeed, the feedback structure given in Figure 3.4 is a so called "*generalized decision feedback equalizer*" (GDFE) [29], in which $\mathbf{x}_k$ is recovered from $\mathbf{x}'_k$ in a component-wise fashion. That is, $x_{k,i}$, the ith component of $\mathbf{x}_k$, is decoded based upon the observation of $x'_{k,i}$, the ith component of $\mathbf{x}'_k$, for $1 \leq i \leq M_k$.

Because matrix $\mathbf{B}_k$ is upper triangle and monic, the decoding of $\mathbf{x}_k$ can be implemented by using back substitute. Specifically, as indicated by (3.27), the last component of $\mathbf{x}_k$, x_{k,M_k}, can be decoded first from the observation of x'_{k,M_k}. Once x_{k,M_k} is successfully decoded, x'_{k,M_k-1} can be constructed by subtracting the interference caused by x_{k,M_k}, and is then used to decode x_{k,M_k-1}. Further, after x_{k,M_k-1} are decoded, x'_{k,M_k-2} can constructed by subtracting the interference caused by both x_{k,M_k} and x_{k,M_k-1}, and is then used to decode x_{k,M_k-2}. Such process continues until all components in $\mathbf{x}_k$ are decoded. It is clear that, in this way, the data symbols which are spread with different signature sequences are decoded and canceled one after another. Thus the feedback structure given in Figure 3.4 is a SIC structure.

As all users using Gaussian signalling, $\mathbf{e}'_k$ follows Gaussian distribution. Now, the components of $\mathbf{e}'_k$ are independent (as indicated by (3.23)), and the components of input $\mathbf{x}'_k$ are independent (by assumption given in Chapter 2). Then, with SIC structure, the equivalent channel connecting the base station and mobile k can be converted into a set of independent AWGN sub-channels, each with a capacity [30,31] $\frac{1}{2}\log(\mathtt{SNR}_{k,i})$ with $1 \leq i \leq M_k$, where $\mathtt{SNR}_{k,i}$ is the biased signal to noise ratio defined as [32, Eq. (31)]:

$$\mathtt{SNR}_{k,i} = \frac{E[|x_{k,i}|^2]}{E[|e'_{k,i}|^2]} = \delta_{k,i} \tag{3.28}$$

in which $\delta_{k,i}$ is the ith diagonal element of matrix $\mathbf{\Delta}_k$. Thus, the data rate that can be achieved by user k with SIC structure is derived as

$$\mathcal{R}_k^{\mathrm{SIC}} = \frac{1}{2}\log\prod_{i=1}^{M_k}\delta_{k,i} = \frac{1}{2}\log\det(\mathbf{\Delta}_k) = -\frac{1}{2}\log\det(\mathbf{MSE}_k) \tag{3.29}$$

This is precisely the capacity that user k can achieve by jointly decoding the outputs of its linear MMSE receiver filters. That means the SIC structure is also capacity lossless.

Note that, at each mobile, for either the joint decoding structure or the SIC decoding structure, cooperation has been introduced to decode the desired user's encoded data symbols which are spread with distinct signature sequences.

3.3.3 Cooperative Encoding Across Signature Sequences

From previous sub-sections, it can be seen that each user's rate capacity is restricted by the MAI at its corresponding mobile. Meanwhile, it is known that, for the downlink communication, data symbols from all users are available at the base station before transmission. Then, it is natural to ask if users' rate capacities can be improved by encoding all users' data symbol in a way such that the MAI in the system can be *pre-canceled.*

Costa [33] showed that the rate capacity of the standard scalar single-user additive white Gaussian noise channel is *unchanged* in the presence of an independent additive Gaussian interferer, provided that the interferer's signal is known non-causally to the transmitter. This result has been extended to the more general vector Gaussian channel in [7], such that:

Lemma 3.1 ([7], page 34) *Given a fixed power constraint, a Gaussian vector channel with side information* $\mathbf{y} = \mathbf{x} + \mathbf{s} + \mathbf{z}$*, where* $\mathbf{z}$ *and* $\mathbf{s}$ *are independent Gaussian random vectors, and* $\mathbf{s}$ *is known non-causally at the transmitter but not at the receiver, has the same capacity as if* $\mathbf{s}$ *does not exist. Further, the capacity-achieving* $\mathbf{x}$ *is statistically independent of* $\mathbf{s}$*.*

Lemma 3.1 effectively shows that, while encoding a particular user's signal, the transmitter can perform a pre-cancelation of the interfering signal *without* a power or rate penalty as long as the interference is known in advance at the transmitter. Such a process is termed as "writing on dirty paper" in the literature. The corresponding coding technique is nicknamed dirty paper coding (DPC), whose implementation is based upon an idea called random binning [6, Chapter 14] [34].

Figure 3.5 shows the diagram of the base station applying dirty paper coding, in which $\pi(\cdot)$ is a certain permutation representing a particular encoding order among users. More specifically, let $\pi(k)$ denote the encoding order index of user k, and $\pi^{-1}(l)$ denote the user index of the lth encoded user.

Suppose user k is the lth encoded user at the base station, i.e., $k = \pi^{-1}(l)$ with $1 < l < K$. As shown in Figure 3.5, user k's data symbols are encoded with a DPC

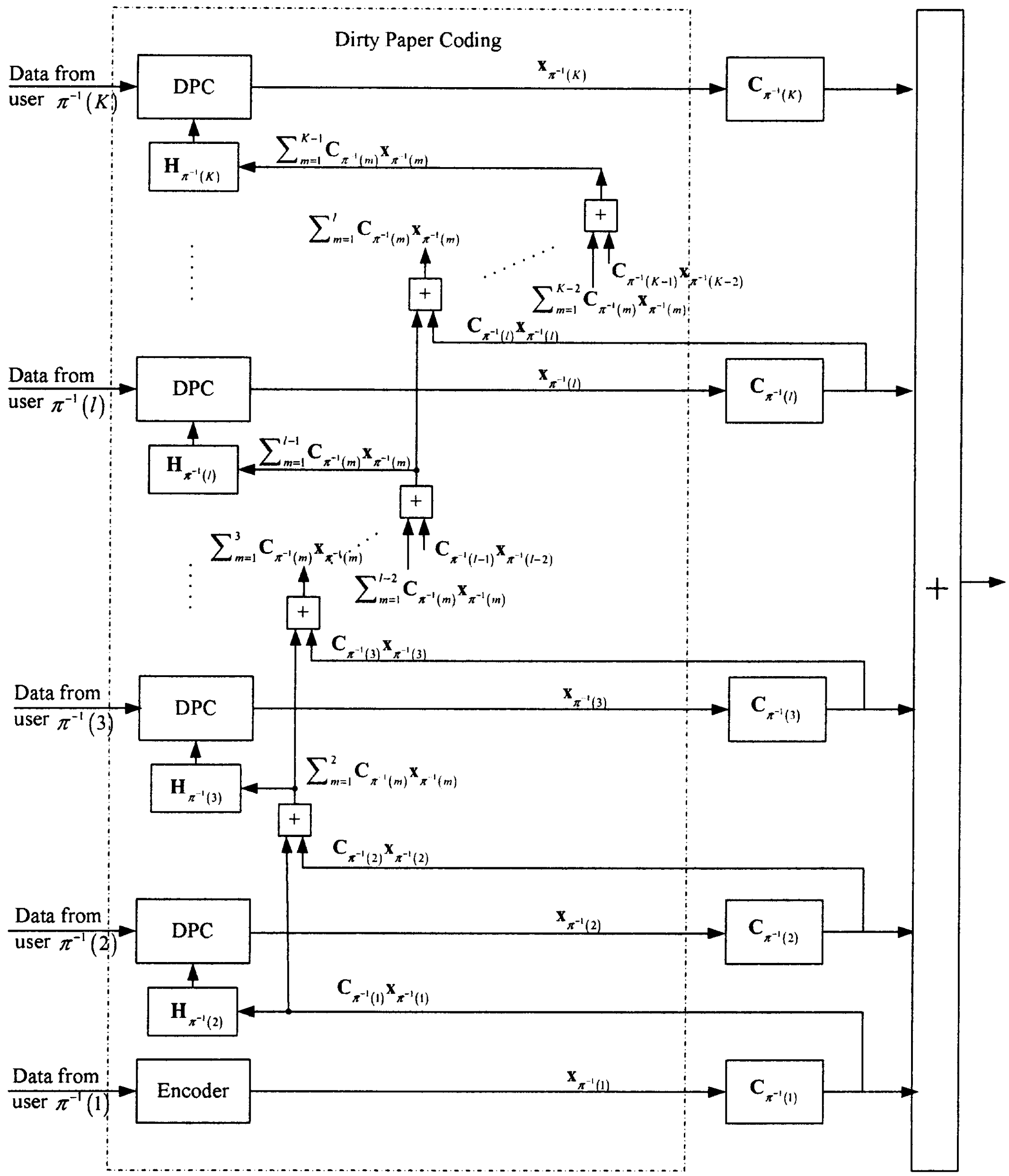


Figure 3.5 Base station with dirty paper coding.

encoder which takes consideration of interference

$$\mathbf{H}_{\pi^{-1}(l)}\left(\sum_{m}^{l-1}\mathbf{C}_{\pi^{-1}(m)}\mathbf{x}_{\pi^{-1}(m)}\right)=\mathbf{H}_k\left(\sum_{\pi(m)<\pi(k)}\mathbf{C}_m\mathbf{x}_m\right) \tag{3.30}$$

Note that, the received signal at mobile k can be expressed as

$$\mathbf{r}_k=\mathbf{H}_k\mathbf{C}_k\mathbf{x}_k+\mathbf{H}_k\sum_{\pi(m)<\pi(k)}\mathbf{C}_m\mathbf{x}_m+\mathbf{H}_k\sum_{\pi(m)>\pi(k)}\mathbf{C}_m\mathbf{x}_m+\mathbf{n}_k \tag{3.31}$$

where the second term is the MAI due to the previously encoded users, and the third term is the interference which arises from users encoded afterwards. With $\mathbf{H}_k$ perfectly known at the base station in advance, the DPC technique ensures that user k will not suffer any capacity penalty for the MAI due to those previously encoded users. However, the rate capacity for user k is still restricted by the MAI from users encoded *after* user k. In this case, the following rate capacity can be achieved at mobile k:

$$\mathcal{R}_k^{\mathrm{III}}=\frac{1}{2}\log\det\left(\mathbf{I}_{M_k}+\mathbf{C}_k^H\mathbf{H}_k^H\mathbf{W}_k^{-1}\mathbf{H}_k\mathbf{C}_k\right) \tag{3.32}$$

where

$$\mathbf{W}_k=\mathbf{H}_k\left(\sum_{\pi(k)<\pi(l)}\mathbf{C}_l\mathbf{C}_l^H\right)\mathbf{H}_k^H+\sigma^2\mathbf{I}_N \tag{3.33}$$

is the covariance of MAI from the users encoded after user k.

Since $\mathbf{Z}_k$ is the covariance of MAI plus noise observed at mobile k, it is easy to show that $\mathbf{Z}_k-\mathbf{W}_k$ is always positive definite for all k except when $k=\pi^{-1}(1)$. Therefore [27]

$$\begin{aligned}&\det\left(\mathbf{I}_{M_k}+\mathbf{C}_k^H\mathbf{H}_k^H\mathbf{W}_k^{-1}\mathbf{H}_k\mathbf{C}_k\right)\geq\det\left(\mathbf{I}_{M_k}+\mathbf{C}_k^H\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k\mathbf{C}_k\right)\\ \Longleftrightarrow\quad&\mathcal{R}_k^{\mathrm{III}}\geq\mathcal{R}_k^{\mathrm{II}}\end{aligned} \tag{3.34}$$

That means, DPC will definitely improve users' rate capacities.

As indicated by Figure 3.5, with DPC, certain level of cooperation is introduced to encode the data streams which are spread with different signature sequences. Note that, DPC improves users' rate capacities at the cost of increased complexity at the base station, since all users' channel state information has to be available there in advance. From practical point of view, realizing DPC may require prohibitively high complexity.

3.4 Ergodic and Outage Capacity

In the last section, users' rate capacities in the downlink of MC-CDMA system are obtained for specific channel realization. However, wireless channel is a fading channel, whose state changes randomly. Therefore, in this section, the ergodic and the outage rate capacities [35] are considered. For both cases, it is assumed that, for each user, the underlying channel is a block fading channel, i.e., the channel remains fixed with one channel use (at least one MC-CDMA symbol duration) and then changes to a new channel realization.

3.4.1 Ergodic Capacity

Ergodic capacity is defined for a fading channel with a long term delay constraint. The basic assumption is that the transmission time is long enough to reveal the long-term ergodic properties of the fading channel. That is, as a codeword length goes to infinity, the fading process of the underlying channel which is covered by a codeword becomes asymptotically ergodic. In other words, a codeword covers all channel states according to the channel's probability distribution.

Regarding to the three coding strategies discussed in the last section, each user's corresponding ergodic capacity is given as [35]:

$$\mathcal{R}_{k_{\text{ergodic}}}^{\text{I/II/III}} = E\left[\mathcal{R}_k^{\text{I/II/III}}\right] \tag{3.35}$$

where $\mathcal{R}_k^{\mathrm{I/II/III}}$ is the instantaneous capacity under a particular channel realization, and the expectation $E[\cdot]$ is taken over the probability distribution of that user's fading channel.

Note that, at data rates lower than the ergodic capacities given by (3.35), each user can achieve the error probability which decays exponentially with the codeword length by using the associated coding strategies. In practice, the ergodic capacity is usually used to evaluate the rate that can be achieved under fast fading channel, or for the application which does not have stringent restriction on decoding delay.

3.4.2 Outage Capacity

However, there exists scenario in which user's codeword can only span an arbitrary but fixed number of channel states while the duration of those channel states are long enough to be approximated as infinity. This situation typically occurs when stringent delay constraints are imposed, as is the case, for example, in speech transmission and video conference over wireless channels. Also, the similar situation happens for the case that the underlying wireless channel is slow fading.

In this case, the fading process of the underlying channel which is covered by a codeword is non-ergodic. Thus, there does not exist Shannon capacity. The concept of capacity versus outage [35] has to be invoked, where the capacity is viewed as a random entity depending on the instantaneous random channel realization.

For the three coding strategies discussed in the last section, the user k's corresponding outage probability evaluated at rate R is given as

$$P_{\mathrm{out}}^{(k)}(R) = Pr\left(\mathcal{R}_k^{\mathrm{I/II/III}} \le R\right) \tag{3.36}$$

where $\mathcal{R}_k^{\mathrm{I/II/III}}$ is the instantaneous capacity under a particular channel realization, and the probability $Pr(\cdot)$ is taken over the probability distribution of that user's fading channel.

Then, user k's outage capacity with outage probability of ϵ is defined as [35]

$$\mathcal{R}_{k_{\text{out}}}^{\text{I/II/III}}(\epsilon) = R : Pr\left(\mathcal{R}_{k}^{\text{I/II/III}} \leq R\right) = \epsilon \tag{3.37}$$

Specifically, the rate at which the outage probability approaches 0 is referred as zero-outage capacity and expressed as

$$\mathcal{R}_{k_{\text{delay-limit}}}^{\text{I/II/III}} = \lim_{\epsilon \to 0_{+}} \mathcal{R}_{k_{\text{out}}}^{\text{I/II/III}}(\epsilon) \tag{3.38}$$

which is also denoted as delay-limited capacity [35].

Clearly, the calculation of both ergodic and outage capacities relies on the distribution of the rate capacity under fading channel responses, which is difficult to derive analytically, especially for general frequency selective fading channels. Therefore, numerical study is used in the next section to show the ergodic and the outage capacity for the downlink of MC-CDMA system under frequency selective fading channel responses.

3.5 Simulation

In this section, the rate capacity for the downlink of MC-CDMA system is studied by means of simulation. A 2-user system is considered, in which each user employs multiple signature sequences for transmission.

In the simulation, the system setup follows the parameters in IEEE 802.11a [36] and HIPERLAN type 2 [37]. That is an OFDM-based system with total bandwidth of 20MHz. The whole channel bandwidth is divided into 64 sub-carriers, of which 52 sub-carriers are used for transmission. Among those 52 sub-carriers, 4 of them are used for pilot signal transmission, whose objective is to enable mobile receiver to implement channel estimation. The remaining 48 sub-carriers are used for informative data transmission. The separation between adjacent sub-carriers is 312.5kHz. The symbol duration of each OFDM symbol is 4μs, with cyclic prefix of length 0.8μs.

It is assumed that, throughout the simulation, the cyclic prefix is long enough to completely eliminate inter (OFDM) symbol interference.

Each user's frequency selective fading channel is assumed to be an independent Rayleigh fading channel, with its r.m.s. delay spread set as 25ns, a typical value for indoor communication. Two sets of path loss settings are considered in the following: one is that g_1 and g_2 are chosen such that $\frac{g_1}{\sigma^2} = 30$dB and $\frac{g_2}{\sigma^2} = 5$dB, which corresponds to the case that the difference in channel quality between 2 users is large; the other is that g_1 and g_2 are chosen such that $\frac{g_1}{\sigma^2} = 20$dB and $\frac{g_2}{\sigma^2} = 15$dB, which corresponds to the case that the disparity in channel quality between 2 users is small.

Walsh-Hadamard signature sequences, which have been widely adopted for the research of MC-CDMA in the literature, are assumed in the simulation. That is, total $M = 48$ Walsh-Hadamard signature sequences, with processing gain of $N = 48$, are shared between 2 users, such that $M_1 + M_2 = 48$. And the total transmission power at the base station is given as $\sum_{k=1}^{2} \left(\mathbf{C}_k^H \mathbf{C}_k\right) = 48$.

Note that, signature sequence and power are two types of resource to be shared by 2 users. It is conceivable that, the distribution of power and signature sequences between users, which is usually depends on the resource allocation policy adopted in the system, will influence each user's rate capacity. To reveal the impact of resource allocation policy on the resultant rate capacity, two resource allocation policies are considered in the following:

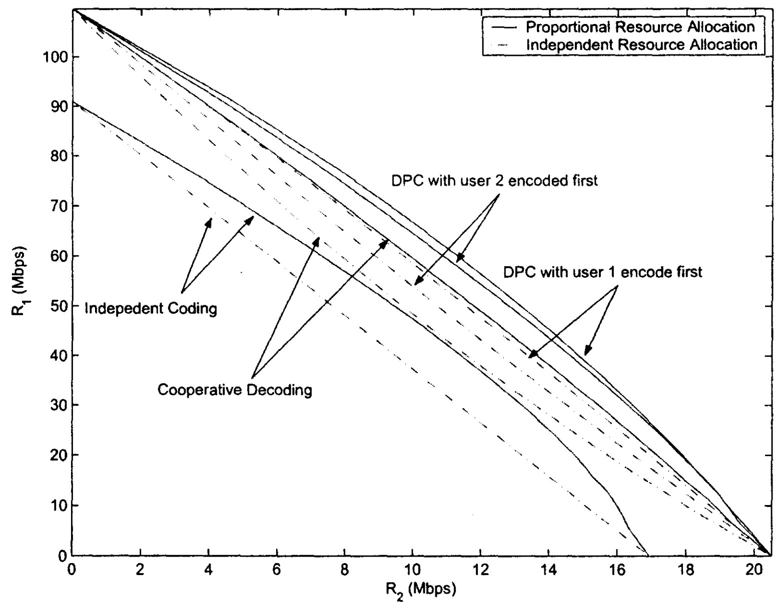
1. *Proportional Resource Allocation*: in which, each user is assigned power proportional to the number of its assigned signature sequences. Note that, this is the most widely adopted resource allocation policy considered in the literature for the study of MC-CDMA system.

2. *Independent Resource Allocation*: in which, power assignment and signature sequence assignment are regarded as two separate processions, such that, for each user, its assigned power is independent of the number of its assigned signature sequence.

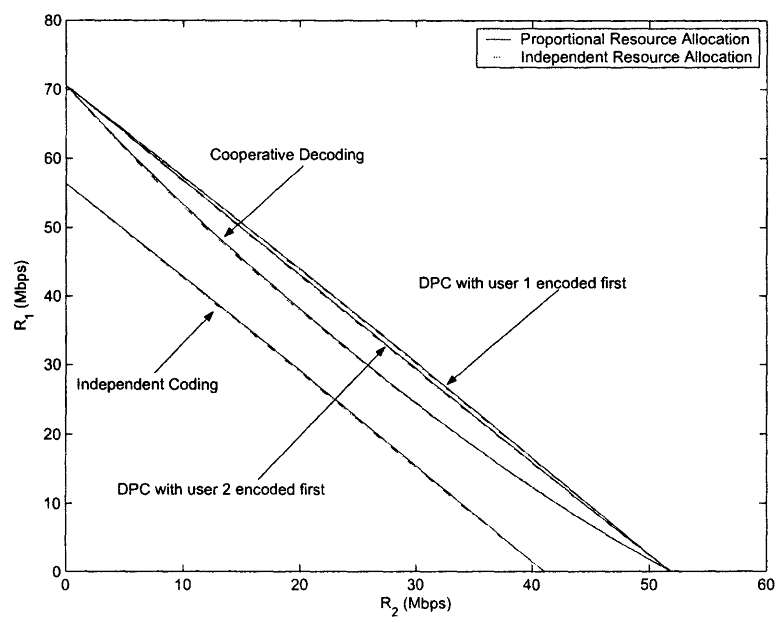
For various combination of coding strategies and resource allocation policies, the corresponding ergodic and outage capacities are presented in Figure 3.6 and Figure 3.7 respectively. In those figures, the sub-figure (a) is for the case that $\frac{g_1}{\sigma^2}$ = 30dB and $\frac{g_2}{\sigma^2}$ = 5dB, and the sub-figure (b) is for the case that $\frac{g_1}{\sigma^2}$ = 20dB and $\frac{g_2}{\sigma^2}$ = 15dB. Each line in the figures represents the boundary of capacity region achieved with a particular combination of a coding strategy and a resource allocation policy. That is, for each line, any point below represents a rate pair that both users can maintain simultaneously by applying the corresponding combination of coding strategy and resource allocation policy.

As shown in these figures, for either ergodic or outage capacity, significant rate capacity improvement can be obtained by using more sophisticated coding strategies which cooperatively encode and/or decode those data symbols spread by different signature sequences. To maintain high speed data transmission, each mobile's should cooperatively decode its desired users' data symbols which are spread with distinct signature sequences. Further improvement in rate capacity can be achieved by implementing dirty paper coding (DPC) to cooperatively encode all users' data symbols at the base station. It is interesting to notice that, for DPC, a larger capacity region is always achieved by first encoding the signal from user with better channel quality. Therefore, in practice, it only needs to consider the DPC with encoding order in which the user with better channel quality is encoded earlier.

Also shown in these figures, the resource allocation policy has an important impact on the resultant capacity region. A larger capacity region is always obtained with independent resource allocation policy. Intuitively, this is because, without the proportional relation between power and signature sequence assignment, there is more degree of freedom in assigning resource among users. However, the advantage of applying independent resource allocation policy is prominent only when the channel quality between two users is large. When the channel quality between two users is

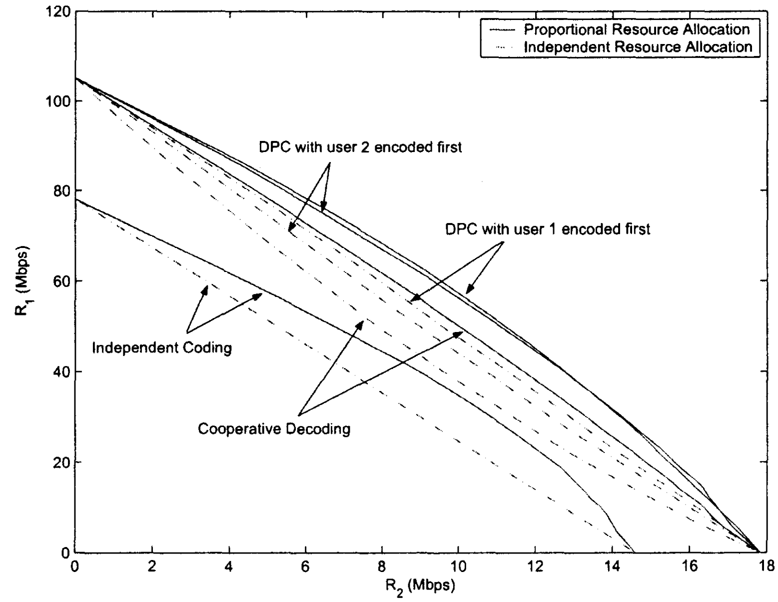


(a) $\frac{g_1}{\sigma^2} = 30\text{dB}$ and $\frac{g_2}{\sigma^2} = 5\text{dB}$

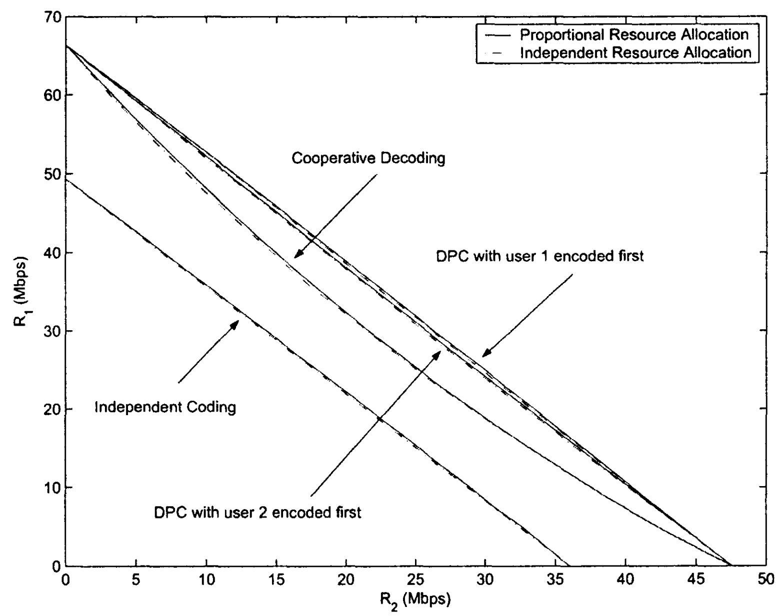


(b) $\frac{g_1}{\sigma^2} = 20\text{dB}$ and $\frac{g_2}{\sigma^2} = 15\text{dB}$

Figure 3.6 Ergodic capacity regions for Rayleigh frequency selective fading channels under various combinations of coding strategies and resource allocation policies.



(a) $\frac{g_1}{\sigma^2} = 30\text{dB}$ and $\frac{g_2}{\sigma^2} = 5\text{dB}$



(b) $\frac{g_1}{\sigma^2} = 20\text{dB}$ and $\frac{g_2}{\sigma^2} = 15\text{dB}$

Figure 3.7 Outage capacity regions for Rayleigh frequency selective channels under various combination of coding strategies and resource allocation policies, with outage probability of $\epsilon = 0.1$.

close, as shown in Figure 3.6(b) and Figure 3.7(b), the capacity region obtained with proportional resource allocation is almost as large as that achieved with independent resource allocation.

3.6 Chapter Summary

This chapter studies the rate capacity for the downlink of MC-CDMA system under different combinations of coding strategies and resource allocation policies. It is shown that, with multi-code scheme, to maintain high speed data transmission, each mobile's should cooperatively decode its desired users' encoded data symbols which are spread with distinct signature sequences. Further improvement in rate capacity can be achieved by implementing dirty paper coding (DPC) to cooperatively encode all users' data symbols at the base station. However, the complexity of realizing DPC is prohibitively high. Furthermore, system capacity region heavily depends on the resource allocation policy adopted in the system. When the disparity of users' channel qualities is small, one can proportionally assign power and signature sequences among users without suffering significant penalty in the resultant system capacity region. However, when the difference of users' channel qualities is large, a much larger system capacity region is obtained with non-proportional assignment of power and signature sequences.

CHAPTER 4

CENTRALIZED SIGNATURE SEQUENCE ADAPTATION

For MC-CDMA system, orthogonal signature sequences like Walsh-Hardamad sequences are always used to spread data symbols designated to different users. However, in practice, the orthogonality of these signature sequences is destroyed by the underlying frequency selective channels. Inevitably, there exists MAI in the system, which will severely deteriorate the system performance. One efficient method to improve the system performance is to apply multi-user detection (MUD) on the receiver side, so that the MAI encountered at receivers is explicitly taken into account in receiver design. It is conceivable that, in addition to applying MUD on the receiver side, the system performance can be further improved if users' signature sequences at the transmitter side could be properly adapted according to the channel state information in the system.

4.1 Introduction

The idea of signature sequence adaptation has attracted extensive research interests. The pioneer work on signature sequence adaptation can be traced back to [38], in which the problem of *single* user signature sequence adaptation was dealt with. In that work, the desired user, of which the signature sequence was adapted, was assumed to experience frequency flat channel response and fixed multiple access interference. By assuming the chip of the desired user's signature sequence to be any real value (as opposed to the traditional binary value $\{+1, -1\}$), that user's signature sequence was iteratively updated with its corresponding energy normalized MMSE receiver filter. It was proved that such an updating approach could monotonically decrease the desired user's mean square error (MSE). However, in that work, there was no analysis showing the impact of such an adaptation scheme on the system performance or the performance of other users.

Since then, there has been extensive research on *multi*-user signature sequence adaptation, most of which concerns with the uplink scenario. A popular example is to find users' signature sequences that maximize the sum capacity for the synchronous CDMA system under frequency flat channel response when the received powers from mobiles are fixed. In [39], the signature sequences that optimize the sum capacity were derived when the received powers from different mobiles are all equal. In [40], the result of [39] was generalized to the situation in which the received powers are not constraint to be equal to each other. Furthermore, subject to the requirement that all users in the system meet quality-of-service (QoS) thresholds, which were expressed as required signal to interference plus noise ratios, references [41] and [42] designed the signature sequences and the corresponding power control policy to minimize the required total power (i.e., the sum of all users' powers) for the synchronous CDMA system under frequency flat channel response. The difference between these two references is that, in [41] a linear MMSE receiver was assumed at the base station, while in [42] a decision-feedback receiver (SIC receiver) was used. Most recently, reference [43] studied the optimal signature sequences for the asynchronous CDMA system with frequency flat channel response. Inspired by the above cited works, iterative algorithms were developed for adapting users' signature sequences under various communication scenarios, such as single-cell with frequency flat channels in [44–47], single-cell with frequency selective channels in [48–50], multi-cell with frequency flat channels in [51,52] and multi-cell with frequency selective channels [53].

However, very little attention has been paid to signature sequence adaptation in the downlink scenario, even though, for future broadband wireless communication systems, most transmission is expected to take place in this direction. In [54], algorithms were proposed to adapt users' signature sequences to pre-compensate the channel multipath and eliminate the MAI at all mobile rec4eivers simultaneously. In that work, users' mobile receivers were assumed to be pre-assigned and fixed during the process of adaptation. In [55–57], under the assumption that each mobile adopts an MMSE receiver, Gao and Wong proposed iterative algorithms to construct the signature sequence set which could support users with a uniform target signal to interference plus noise ratio. However, the proposed adaptation algorithms were only suitable for the case of frequency flat channel.

This chapter is to address the problem of signature sequence adaptation in the downlink of MC-CDMA system, with frequency selective channel responses and multi-user linear receivers. The objective of signature sequence adaptation is to optimize an overall system performance objective function, e.g. the weighted total mean square error (WTMSE) of the system. Note that, the coefficients of multi-user linear receivers are functions of the signature sequences currently engaged in the system. When users' signature sequences are adapted at the base station, the receiver filters at mobiles must be adapted as well. Therefore, the signature sequence adaptation considered in this chapter can be modeled as an optimization problem which jointly constructs users' signature sequences and receiver filters to minimize the WTMSE of the system.

The rest of this chapter is organized as follows. Section 4.2 formulates the problem of signature sequence adaptation in the downlink of MC-CDMA system. In Section 4.3, the optimal pair of signature sequences and receiver filters which minimize the WTMSE are identified. In Section 4.4, an adaptive algorithm is proposed to iteratively construct the optimal signature sequences and the corresponding receiver filters under realistic communication conditions. Simulation results are presented in Section 4.5 to examine the performance of the proposed adaptation algorithm. Finally, summaries are provided in Section 4.6.

4.2 Problem Fomulation

Suppose the adopted linear receiver filters at mobile k are explicitly denoted as a $N \times M_k$ matrix $\mathbf{G}_k$, the mean square error incurred at mobile k can be expressed as

$$\text{MSE}_k \triangleq E\left[(\hat{\mathbf{x}}_k - \mathbf{x}_k)^H(\hat{\mathbf{x}}_k - \mathbf{x}_k)\right] \tag{4.1}$$

where $\hat{\mathbf{x}}_k = \mathbf{G}_k^H \mathbf{r}_k$ is the soft decision at the output of linear receiver filters, $\mathbf{r}_k$ is the received signal at mobile k as given by (2.36). Specifically, MSE_k can be expressed in detail as:

$$\begin{aligned}
\text{MSE}_k &= tr\left(E\left[(\hat{\mathbf{x}}_k - \mathbf{x}_k)(\hat{\mathbf{x}}_k - \mathbf{x}_k)^H\right]\right) \\
&= tr\left(E\left[(\mathbf{G}_k^H\mathbf{r}_k - \mathbf{x}_k)(\mathbf{r}_k^H\mathbf{G}_k - \mathbf{x}_k^H)\right]\right) \\
&= tr\left(\mathbf{G}_k^H E[\mathbf{r}_k\mathbf{r}_k^H]\mathbf{G}_k - \mathbf{G}_k^H E[\mathbf{r}_k\mathbf{x}_k^H] - E[\mathbf{x}_k\mathbf{r}_k^H]\mathbf{G}_k + E[\mathbf{x}_k\mathbf{x}_k^H]\right) \\
&= tr\left(\mathbf{G}_k^H\mathbf{R}_k\mathbf{G}_k - \mathbf{G}_k^H\mathbf{H}_k\mathbf{C}_k - \mathbf{C}_k^H\mathbf{H}_k^H\mathbf{G}_k + \mathbf{I}_{M_k}\right) \qquad (4.2)
\end{aligned}$$

where $E[\mathbf{r}_k\mathbf{r}_k^H] \triangleq \mathbf{R}_k$ is the covariance matrix of received signal as shown in (3.2).

Since MSE_k only reflects a particular user's individual performance, the weighted total mean square error (WTMSE) defined as

$$\text{WTMSE} = \sum_{k=1}^{K} \mu_k \text{MSE}_k \tag{4.3}$$

is chosen as the measurement which characterizes the overall system performance, where μ_k is the weighting factor assigned to user k.

Note that, with carefully chosen weighting factor μ_k's, WTMSE can be used to represent the overall system performance of various communication scenarios. In the special case with $\mu_k = 1\ \forall k$, WTMSE is simply the sum of all users' mean square error, which is referred as total mean square error (TMSE) of the system. As a matter of fact, TMSE has be extensively used as an performance objective for signature sequence adapation in the uplink, such as in [45] for frequency flat channels and in [48, 49] for frequency selective channels.

In practice, μ_k's are usually determined by higher layer protocol based upon each user's required service and priority. The selection of the weighting factor μ_k's is beyond the scope of this dissertation. In the following, the μ_k's are assumed given (and fixed) during the process of signature sequence adaptation.

By substituting (4.2) into (4.3), the WTMSE can be expressed in terms of users' signature sequences as well as the linear receiver filters:

$$\text{WTMSE} = \sum_{k=1}^{K} \mu_k tr\left(\mathbf{G}_k^H \mathbf{R}_k \mathbf{G}_k - \mathbf{G}_k^H \mathbf{H}_k \mathbf{C}_k - \mathbf{C}_k^H \mathbf{H}_k^H \mathbf{G}_k + \mathbf{I}_{M_k}\right) \tag{4.4}$$

The objective of signature sequence adaptation is to find the set of signature sequences $\{\mathbf{C}_k\}_{k=1}^K$ and receiver filters $\{\mathbf{G}_k\}_{k=1}^K$ such that WTMSE defined in (4.4) is minimized.

Note that, the higher transmission power at the base station, the better performance one may expect. It is trivial to optimize the system performance with infinite transmission power. Therefore, signature sequences are always adapted under certain transmission power constraints. In the rest of this chapter, the total transmission power at the base station is chosen as the optimization constraint, because in the downlink scenario the base station is the common transmitter for all mobiles.

With P as the total transmission power at the base station, the optimization problem of minimizing WTMSE over the signature sequences and receiver filters can be formulated as

$$\min_{\{\mathbf{C}_k\}_{k=1}^K, \{\mathbf{G}_k\}_{k=1}^K} \sum_{k=1}^{K} \mu_k tr\left(\mathbf{G}_k^H \mathbf{R}_k \mathbf{G}_k - \mathbf{G}_k^H \mathbf{H}_k \mathbf{C}_k - \mathbf{C}_k^H \mathbf{H}_k^H \mathbf{G}_k + \mathbf{I}_{M_k}\right) \tag{4.5}$$

$$\text{s.t.} \sum_{k=1}^{K} tr\left(\mathbf{C}_k \mathbf{C}_k^H\right) \leq P$$

The definition of signature sequence given in Chapter 2 implies that the norm of a signature sequence reflects the power at which a data symbol is transmitted. Then, under the total power constraint, the transmission power at the base station is automatically distributed among K active users when the signature sequences engaged in the system are updated. Moreover, since each user is assigned with multiple signature sequences, the transmission power is also optimally distributed among each user's multiple transmitted data symbols.

4.3 Optimal Signature Sequences and Receiver Filters

The optimization problem given in (4.5) can be solved with an iterative process known as alternating minimization [58]. The idea is to fix the value of all but one of the variables in the function and optimize over that variable. One then iterates between different variables by optimizing over one variable at a time.

4.3.1 Alternative Minimization

First consider the receiver filter optimization, assuming the signature sequences $\mathbf{C}_k\ \forall k$ are fixed. It is easy to prove that, with fixed signature sequences, the minimization of (4.4) with respect to the receiver filters at mobile k is equivalent to minimizing MSE_k defined in (4.2). This is a simple consequence of the fact that a mobile k's receiver filters do not affect the mean square error of any other user but itself. Thus, if the signature sequences are fixed, all mobiles need to set their receiver filters to the MMSE filters. More specifically, for mobile k, the optimal filters are given as

$$\mathbf{G}_k = \mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{C}_k$$

which is the expression of user k' MMSE receiver filters shown in (3.1).

Next, consider the signature sequence optimization, by assuming the receiver filters are fixed. Because the optimization of signature sequences is carried out under a power constraint, a Lagrangian is constructed:

$$\begin{aligned} L\left(\{\mathbf{C}_k\}_{k=1}^K, \{\mathbf{G}_k\}_{k=1}^K, \alpha\right) &= \sum_{k=1}^{K} \mu_k tr\left(\mathbf{G}_k^H \mathbf{R}_k \mathbf{G}_k - \mathbf{G}_k^H \mathbf{H}_k \mathbf{C}_k - \mathbf{C}_k^H \mathbf{H}_k^H \mathbf{G}_k + \mathbf{I}_{M_k}\right) \\ &+ \alpha\left(\sum_{k=1}^{K} tr\left(\mathbf{C}_k \mathbf{C}_k^H\right) - P\right) \end{aligned} \tag{4.6}$$

where $\alpha \geq 0$ is the Lagrangian multiplier corresponding to the total power constraint at the base station.

It is known that, for arbitrary matrices $\mathbf{E}$ and $\mathbf{F}$ with proper size, $tr(\mathbf{E}+\mathbf{F}) = tr(\mathbf{E}) + tr(\mathbf{F})$ and $tr(\mathbf{EF}) = tr(\mathbf{FE})$. Using these two equalities repeatedly, one can obtain

$$\sum_{k=1}^{K} \mu_k tr\left(\mathbf{G}_k^H \mathbf{R}_k \mathbf{G}_k\right) = \sum_{k=1}^{K} tr\left(\mathbf{C}_k^H \mathbf{A} \mathbf{C}_k\right) \tag{4.7}$$

where

$$\mathbf{A} \triangleq \sum_{l=1}^{K} \mu_l \mathbf{H}_l^H \mathbf{G}_l \mathbf{G}_l^H \mathbf{H}_l \tag{4.8}$$

After some straightforward algebraic manipulations, the Lagrangian in (4.6) can be rewritten as

$$L\left(\{\mathbf{C}_k\}_{k=1}^K, \{\mathbf{G}_k\}_{k=1}^K, \alpha\right) = \sum_{k=1}^{K} L_k\left(\mathbf{G}_k, \mathbf{C}_k, \alpha\right) + \sum_{k=1}^{K} \Xi_k \tag{4.9}$$

where

$$L_k\left(\mathbf{G}_k, \mathbf{C}_k, \alpha\right) = tr\left(\mathbf{C}_k^H\left(\mathbf{A} + \alpha \mathbf{I}_N\right)\mathbf{C}_k - \mu_k \mathbf{G}_k^H \mathbf{H}_k \mathbf{C}_k - \mu_k \mathbf{C}_k^H \mathbf{H}_k^H \mathbf{G}_k\right) \tag{4.10}$$

and

$$\Xi_k = \mu_k tr\left(\mathbf{I}_{M_k} + \sigma^2 \mathbf{G}_k^H \mathbf{G}_k\right) \tag{4.11}$$

It is clear that, with fixed receiver structure $\mathbf{G}_k$'s, the user k's optimal signature sequences which minimize the Lagrangian given in (4.6) are those minimizing $L_k\left(\mathbf{G}_k, \mathbf{C}_k, \alpha\right)$.

Lemma 4.1 *For any positive definite matrix* $\mathbf{Q}$*,*

$$\arg\min_{\mathbf{E}} tr\left(\mathbf{E}^H\mathbf{Q}\mathbf{E} - \mathbf{E}^H\mathbf{F} - \mathbf{F}^H\mathbf{E}\right) = \mathbf{Q}^{-1}\mathbf{F} \tag{4.12}$$

where $\mathbf{E}$ *and* $\mathbf{F}$ *are matrices of proper size.*

Proof: Denote the function of $\mathbf{E}$ on the left side of (4.12) by $f(\mathbf{E})$. It is easy to show that

$$f\left(\mathbf{Q}^{-1}\mathbf{F} + \mathbf{Z}\right) = f\left(\mathbf{Q}^{-1}\mathbf{F}\right) + tr\left(\mathbf{Z}^H\mathbf{Q}\mathbf{Z}\right) \tag{4.13}$$

where the last term is nonnegative due to the nonnegative definiteness of $\mathbf{Z}^H\mathbf{Q}\mathbf{Z}$ [27].

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By Lemma 4.1, user k's optimal signature sequence $\mathbf{C}_k$, which minimizes the Lagrangian for the given receiver filters $\mathbf{G}_k$'s, can be found as

$$\mathbf{C}_k = \mu_k \left(\mathbf{A} + \alpha\mathbf{I}_N\right)^{-1} \mathbf{H}_k^H \mathbf{G}_k \tag{4.14}$$

where $\alpha \geq 0$ is the Lagrangian multiplier which needs to be chosen such that the resulting signature sequences satisfy the total power constraint.

According to the *complementary slackness* condition [59]

$$\alpha\left(\sum_{k=1}^{K} tr\left(\mathbf{C}_k\mathbf{C}_k^H\right) - P\right) = 0, \tag{4.15}$$

α is either a positive value such that the chosen signature sequences satisfy the power constraint with equality, or 0 when the power of the chosen signature sequences is less than the power limit.

With the Hermitian matrix $\mathbf{A}$ given in (4.8) being eigenvalue decomposed as

$$\mathbf{A} = \mathbf{V}\mathbf{D}\mathbf{V}^H \tag{4.16}$$

where $\mathbf{D} = diag\left(d_1, \cdots, d_N\right)$; d_n are the eigenvalues and $\mathbf{V}$ contains the corresponding eigenvectors. Using (4.14), the transmission power with the selected signature sequences can be expressed as:

$$\sum_{k=1}^{K} tr\left(\mathbf{C}_k\mathbf{C}_k^H\right) = \sum_{k=1}^{K} tr\left(\mu_k^2\left(\mathbf{D}+\alpha\mathbf{I}_N\right)^{-2}\mathbf{V}^H\mathbf{H}_k^H\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_k\mathbf{V}\right) \tag{4.17}$$

By defining matrix

$$\mathbf{B}_k \triangleq \mu_k^2\mathbf{V}^H\mathbf{H}_k^H\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_k\mathbf{V} \tag{4.18}$$

which has $b_{i,j}^{(k)}$ as its (i,j)th entry, Equation (4.17) can be simplified as

$$\sum_{k=1}^{K} tr\left(\mathbf{C}_k\mathbf{C}_k^H\right) = \sum_{k=1}^{K}\sum_{n=1}^{N}\frac{b_{n,n}^{(k)}}{\left(d_n+\alpha\right)^2} \tag{4.19}$$

It is obvious that the Lagrangian α which satisfies total power constraint with equality is the non-negative real solution of

$$\sum_{k=1}^{K}\sum_{n=1}^{N}\frac{b_{n,n}^{(k)}}{\left(d_n+\alpha\right)^2} = P \tag{4.20}$$

Define

$$z\left(\alpha\right) = \sum_{k=1}^{K}\sum_{n=1}^{N}\frac{b_{n,n}^{(k)}}{\left(d_n+\alpha\right)^2} \tag{4.21}$$

Note that, both $b_{n,n}^{(k)}$'s, the diagonal entries of matrix $\mathbf{B}_k$, and d_n's, the eigenvalues of Hermitian matrix $\mathbf{A}$, are nonnegative due to the positive semidefiniteness of these two matrices. It is easy to verify that the function $z(\alpha)$ is a monotonically decreasing function of non-negative α, hence, there exists at most one non-negative solution of α that satisfies the (4.20).

Therefore, the value of the Lagrange multiplier α is the only non-negative real solution of (4.20) if it exists; otherwise, it is 0.

4.3.2 Iterative Processing

For the constrained optimization problem defined in (4.5), Equation (3.1) and Equation (4.14) are the necessary conditions for optimality. From these two necessary conditions, the optimal signature sequences and the corresponding receiver filters that minimize the WTMSE can be obtained numerically via an iterative process presented in the following.

Algorithm 4.1 *Assuming all users' channel responses* $\mathbf{H}_k$ $\forall k$ *are available, and the weighting factors* μ_k $\forall k$ *are given,*

1. *Initialization*

   (a) *For* $k = 1, \cdots, K$, *randomly generate signature sequences* $\mathbf{C}_k$ *for initialization, such that* $\sum_{k=1}^{K} tr\left(\mathbf{C}_k\mathbf{C}_k^H\right) = P$;

   (b) *For the given signature sequence set* $\{\mathbf{C}_k\}_{k=1}^{K}$, *find the receiver filters* $\mathbf{G}_k$ *for every* k *according to (3.1);*

   (c) *Calculate WTMSE following (4.4);*

2. *Updating*

   (a) *Update user* k*'s signature sequences* $\mathbf{C}_k$ *for* $k = 1, \cdots, K$ *following (4.14), in which* α *is a non-negative real value such that the updated signature sequences satisfy the total power constraint;*

   (b) *Update user* k*'s receiver filters* $\mathbf{G}_k$ *for* $k = 1, \cdots, K$ *according to (3.1);*

   (c) *Calculate* new *WTMSE following (4.4).*

3. *If the difference between the new and old WTMSE is larger than the desired accuracy threshold, go back to step 2. Otherwise terminate the process.*

Clearly, in Algorithm 4.1, the signature sequences and receiver filters are updated alternatively. Each step of updating, either the update of signature sequence or the update receiver filters, lowers the WTMSE. Due to the fact that the WTMSE is lower bounded, the proposed iterative process will converge in finite number of updating.

4.3.3 Fixed-Point Property

After convergence, the resulting optimal signature sequences, which minimize the WTMSE in the system, satisfy following fixed point property:

$$\mathbf{C}_k = \mu_k \left(\sum_{l=1}^{K} \mu_l \mathbf{T}_l \mathbf{C}_l \mathbf{C}_l^H \mathbf{T}_l + \alpha \mathbf{I}_N \right)^{-1} \mathbf{T}_k \mathbf{C}_k \;\; \forall k \tag{4.22}$$

where

$$\mathbf{T}_k = \mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k \tag{4.23}$$

Note that, such fixed-point property is obtained by substituting (3.1) into (4.14).

Equation (4.23) can also be written as

$$\mathbf{T}_k = \bar{\mathbf{H}}_k^H \left(\bar{\mathbf{H}}_k \left(\sum_{l=1}^{K} \mathbf{C}_l \mathbf{C}_l^H \right) \bar{\mathbf{H}}_k + \mathbf{I}_N \right)^{-1} \bar{\mathbf{H}}_k \tag{4.24}$$

with $\bar{\mathbf{H}}_k = \mathbf{H}_k/\sigma$. This expression will be used in the next section to adapt users' signature sequences in realistic communication scenario.

4.4 Centralized Signature Sequence Adaptation

Though the iterative process outlined in the last section can numerically search the optimal signature sequence set and the corresponding receiver filters under arbitrary given channel responses, such a process may not be feasible for practical implementation.

Note that, in the iterative process presented in Algorithm 4.1, the updating of signature sequences and receiver filters are interdependent. In realistic communication scenarios, the base station (transmitter) and mobiles (receivers) are separated geographically. Hence, it is impractical to exchange large amount of overhead information between them during the process of communication.

Therefore, in the following, a joint adaptation scheme is developed, in which the signature sequences at the base station and the receiver filters at the mobiles

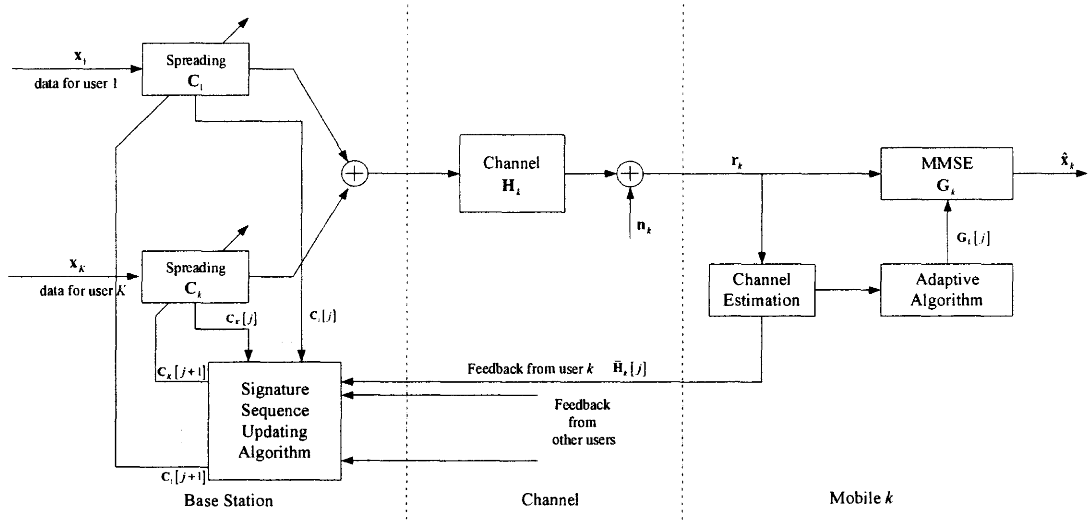


Figure 4.1 System diagram for centralized signature sequence sdaptation in the downlink of MC-CDMA system.

are adapted simultaneously. Such a joint adaptation scheme is illustrated in Figure 4.1, where for simplicity, only mobile k is presented. In the figure, $\mathbf{C}_k[j]$ and $\mathbf{G}_k[j]$ represent user k' signature sequences used at the base station and its corresponding receiver filters at the mobile, respectively.

With this structure, the base station transmits data symbols in blocks with the current signature sequences. The receiver filters at mobile side are adapted independently based on the MMSE criterion. Each mobile also estimates its channel condition and sends it back to the base station. The base station uses the feedback information to update all users' signature sequences; and then employs the updated signature sequences to transmit the data of the next block.

Specifically, the proposed adaptation algorithm can be summarized as follows,

Algorithm 4.2 *Suppose the data symbols designated to different users are transmitted from the base station in blocks,*

1. *The jth block data for user k is transmitted using current signature sequences $\mathbf{C}_k[j]$. Pilot signals and training sequences are inserted. (The initial signature sequence set $\mathbf{C}_k[0]$ could be generated arbitrarily).*

2. *Each mobile adaptively adjusts its receive filters to converge to the corresponding MMSE filters [25] with the help of training sequences. Each mobile also estimates its current channel response via inserted pilot signals. Then, for* $1 \leq k \leq K$, $\bar{\mathbf{H}}_k[j]$ *defined as*

$$\bar{\mathbf{H}}_k[j] = \frac{\mathbf{H}_k[j]}{\sigma} \tag{4.25}$$

is fed back to the base station, where $\mathbf{H}_k[j]$ *is the estimated channel response at mobile* k.

3. *With the knowledge of current signature sequence set* $\mathbf{C}_k[j]$ *and channel responses* $\bar{\mathbf{H}}_k[j]$, $\mathbf{C}_k[j+1]$*'s are updated for all* k *at the base station via*

$$\mathbf{C}_k[j+1] = \mu_k \left(\sum_{l=1}^{K} \left(\mu_l \mathbf{T}_l^H[j] \mathbf{C}_l[j] \mathbf{C}_l^H[j] \mathbf{T}_l[j] \right) + \alpha \mathbf{I} \right)^{-1} \mathbf{T}_k^H[j] \mathbf{C}_k[j] \tag{4.26}$$

where $\mathbf{T}_k[j]$ *is constructed as*

$$\mathbf{T}_k[j] = \bar{\mathbf{H}}_k^H[j] \left(\bar{\mathbf{H}}_k[j] \mathbf{S}[j] \bar{\mathbf{H}}_k^H[j] + \mathbf{I} \right)^{-1} \bar{\mathbf{H}}_k[j] \tag{4.27}$$

$$\mathbf{S}[j] = \sum_{l=1}^{K} \mathbf{C}_l[j] \mathbf{C}_l^H[j] \tag{4.28}$$

and α *is chosen to make updated signature sequences* $\mathbf{C}_k[j+1]$ *with* $k = 1, \cdots, K$ *satisfy the total power power constraint;*

4. *The updated signature sequences* $\mathbf{C}_k[j+1]$ *is employed to transmit* $j+1$*th block of data to mobile* k.

Note that, for the proposed adaptation scheme, the base station is only one decision maker in the system, which determines all users' signature sequences. However, each user's signature sequences are adapted by using other users' channel state information adaptation. Therefore, the proposed adaptation scheme is termed as *centralized signature sequence adaptation scheme.*

4.5 Simulation Result

In this section, both the convergence property of the proposed adaptation scheme and the system performance achieved with the proposed adaptation scheme are presented.

4.5.1 Convergence Property

First, to study the convergence property for the proposed centralized signature sequence adaptation scheme, a simulation is carried out in a system setup with processing gain $N = 6$ (sub-channels) and $K = 10$ active users. Each user is assigned with $M_k = 1$ signature sequence, therefore, the total number of signature sequences engaged in the system is $M = 10$. The total power constraint at the base station is set to be $P = 10$. Each user's weighting factor is set to $\mu_k = 1$, such that all users' signature sequences are jointly adapted to minimize the TMSE in the system. Because the number of signature sequences M is larger than the processing gain N, the resultant optimal signature sequences are nontrivial.

AWGN channel is assumed in the simulation, such that user k's channel response is characterized by $\mathbf{h}_k = g_k\mathbf{1} = g\mathbf{1}$ for all $k = 1, \cdots, K$, where $\mathbf{1}$ is a vector whose elements are all ones. In this circumstance, it can be proven analytically [40, 41, 45] that the optimal signature sequence are *generalized Welch-bound-equality (WBE) sequences*, and the optimal TMSE is given as

$$M - N + \frac{1}{1 + \frac{P}{N}\frac{g}{\sigma^2}}. \tag{4.29}$$

The convergence property of the proposed centralized signature sequence adaptation scheme is presented in Figure 4.2 and Figure 4.3, respectively. In Figure 4.2, path loss g is set such that $\frac{g}{\sigma^2} = 10$dB; while in Figure 4.3, path loss g is set such that $\frac{g}{\sigma^2} = 25$dB. As expected, the proposed centralized scheme lowers the TMSE at each iteration and converges to the optimal TMSE value. It is shown that the proposed adaptation scheme converges faster with lower g. For example,

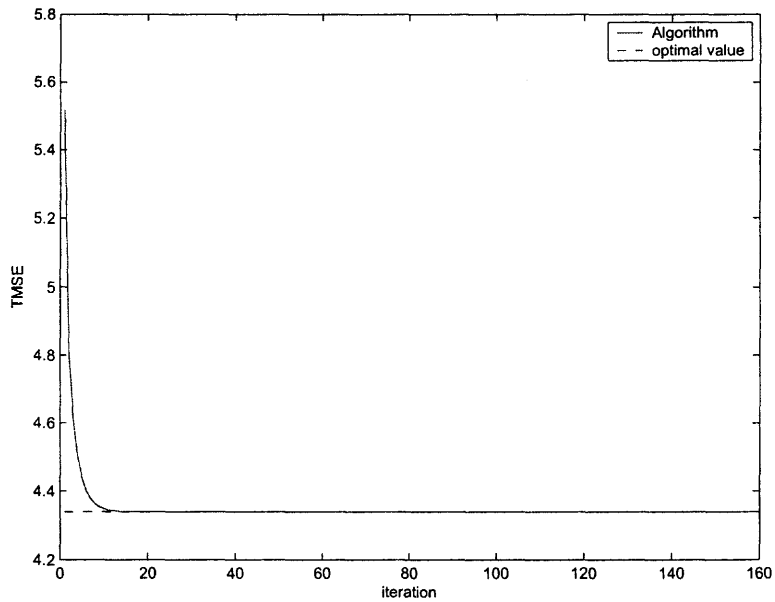


Figure 4.2 Convergence property of the proposed centralized adaptation scheme in AWGN channel with $N = 6$, $K = 10$ and $\frac{g_k}{\sigma^2} = 10$dB for $k = 1, \cdots, 10$.

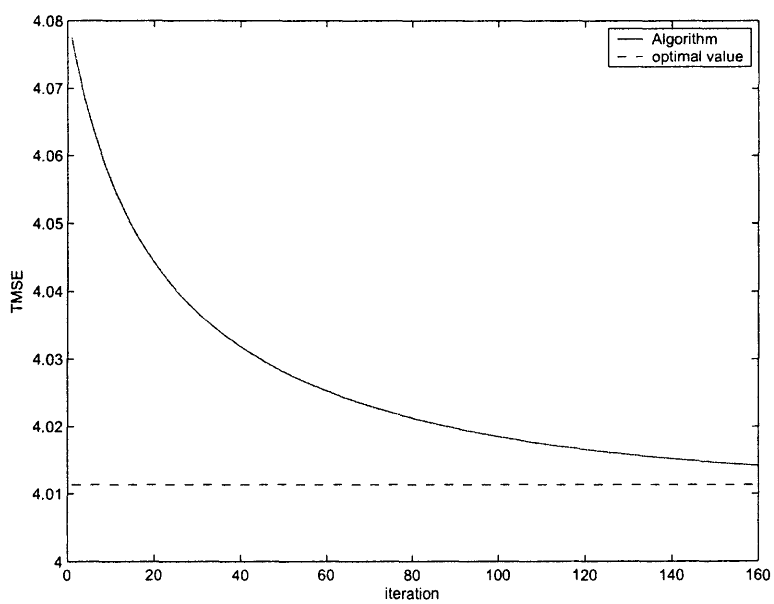


Figure 4.3 Convergence property of the proposed centralized adaptation scheme in AWGN channel with $N = 6$, $K = 10$ and $\frac{g_k}{\sigma^2} = 25$dB for $k = 1, \cdots, 10$.

with $\frac{g}{\sigma^2} = 10$dB, the convergence occurs within 20 iterations, while with $\frac{g}{\sigma^2} = 25$dB, the proposed adaptation scheme takes more than 160 iterations to converge.

4.5.2 System Performance

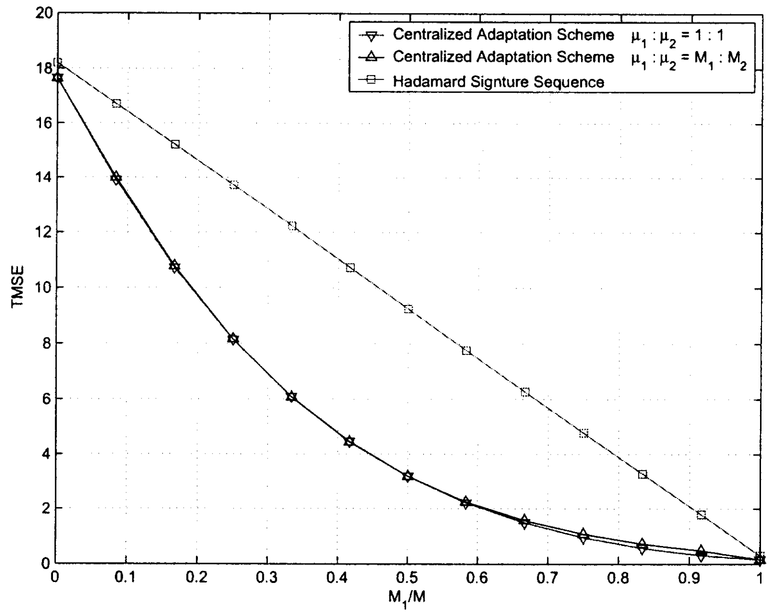
Next, consider the system performance achieved with the proposed adaptation scheme under frequency selective fading channels. For simplicity, only a 2-user system is considered, in which each user is assigned with a number of signature sequences for data transmission.

The system setup follows parameters of wireless LAN specified in IEEE 802.11a and HIPERLAN type 2 standards. Totally, $M = 48$ signature sequences are employed in the system.

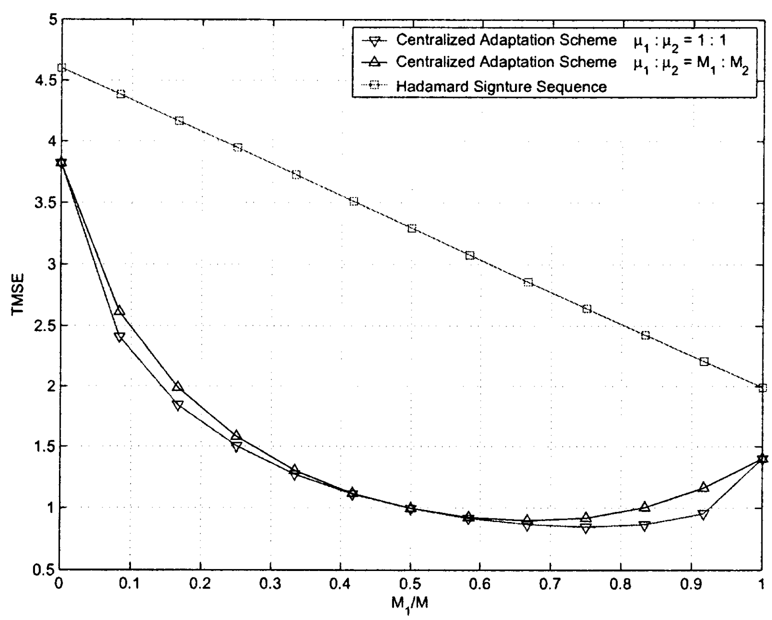
Independent Rayleigh frequency selective channel is assumed to each user, which has exponential multipath delay profile with r.m.s. delay spread set to 25ns. Two sets of path losses are considered in the simulation. In one case, g_1 and g_2 are set such that $\frac{g_1}{\sigma^2} = 30$dB and $\frac{g_2}{\sigma^2} = 5$dB, which corresponds to the scenario in which one user's channel quality is much better than the other. In the other case, the path losses g_1 and g_2 are chosen such that $\frac{g_1}{\sigma^2} = 20$dB and $\frac{g_2}{\sigma^2} = 15$dB, which corresponds to the scenario in which two users' channel qualities are close.

Two weighting factor settings are considered in the simulation. With $\mu_1 : \mu_2 = 1 : 1$, users' signature sequence are adapted to minimize the TMSE in the system. In the other case, weighting factors μ_1 and μ_2 are chosen such that $\mu_1 : \mu_2 = M_1 : M_2$.

Figure 4.4 depicts the total mean square error (TMSE) that can be achieved under different signature sequence assignment, wherein each signature sequence assignment refers to a pair of M_1 and M_2 which characterizes how the total $M = 48$ signature sequences are distributed between two users. Figure 4.4(a) is for the case that $\frac{g_1}{\sigma^2} = 30$dB and $\frac{g_2}{\sigma^2} = 5$dB, and Figure 4.4(b) is for the case that $\frac{g_1}{\sigma^2} = 20$dB and $\frac{g_2}{\sigma^2} = 15$dB. For comparison purpose, the TMSE achieved with traditional



(a) $\frac{g_1}{\sigma^2} = 30\text{dB}$ and $\frac{g_2}{\sigma^2} = 5\text{dB}$



(b) $\frac{g_1}{\sigma^2} = 20\text{dB}$ and $\frac{g_2}{\sigma^2} = 15\text{dB}$

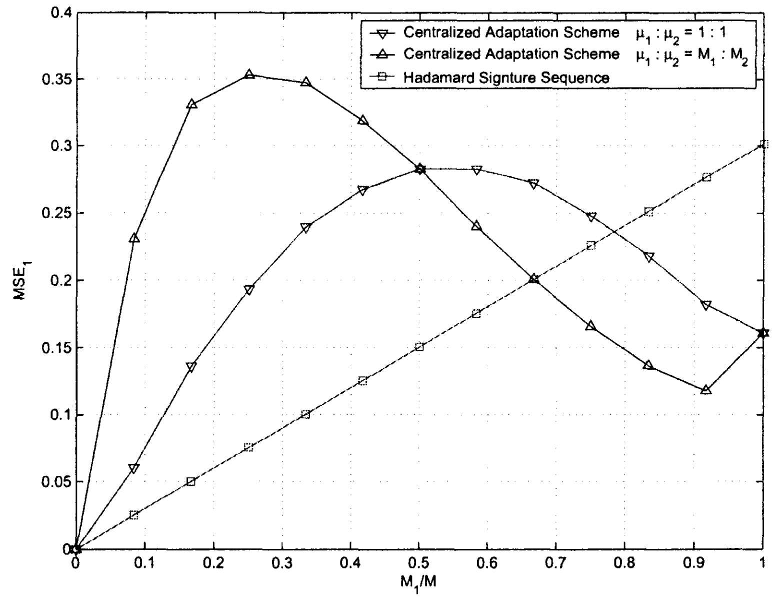
Figure 4.4 TMSE achieved with centralized adaptation scheme under frequency selective Rayleigh fading channels with different weighting factor settings, channel quality settings.

Walsh-Hadamard signature sequences are also add. It is observed that the proposed adaptation scheme generates users' signature sequences which achieve much better performance than the traditional Walsh-Hardmada signature sequences. It is shown that the TMSE is not sensitive to the weighting factors chosen in the system. For either path loss setting, the TMSE achieved under different weight factor settings are almost identical.

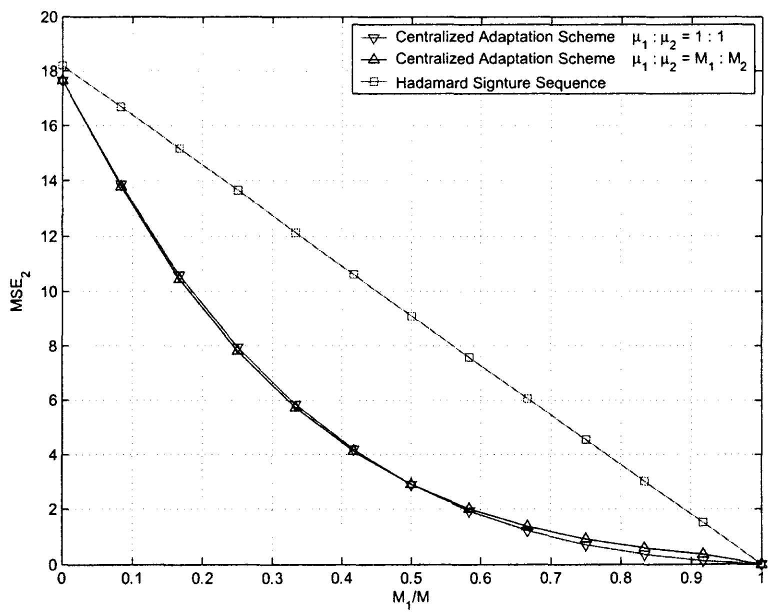
Figure 4.5 and Figure 4.6 present individual user's MSE performance, where the first corresponds to the case that $\frac{g_1}{\sigma^2}$ = 30dB and $\frac{g_2}{\sigma^2}$ = 5dB; and the second corresponds to the case that $\frac{g_1}{\sigma^2}$ = 20dB and $\frac{g_2}{\sigma^2}$ = 15dB. Sub-figure (a) and (b) of those figures present the individual MSE performance of user 1 and 2, respectively.

It can be observed that the selection of weighting factors μ_1 and μ_2 have strong effect on the individual user's performance, especially when the channel quality disparity between two users is large. As presented in Figure 4.5 and Figure 4.6, the individual MSE performance of user 1, the user with much better channel quality, changes significantly for different weighting factor settings; while the individual performance of user 2, the user with worse channel quality, remains almost unchanged for different weighting factor settings.

Since the proposed adaptation scheme is designed to improve the performance from a system viewpoint, some users' individual performances may be scarified under certain circumstance. Usually, those users with better channel qualities will be scarified. For example, as shown in the sub-figure (a)s of Figure 4.5 and Figure 4.6, in many cases, the proposed centralized adaptation scheme achieves worse individual MSE performance for user 1 than the traditional Walsh-Hadamard sequences. However, as indicated in the sub-figure (b)s of Figure 4.5 and Figure 4.6, the proposed signature sequence adaptation scheme can always significantly improve the individual performance of user 2; the one with worse channel quality in the system.

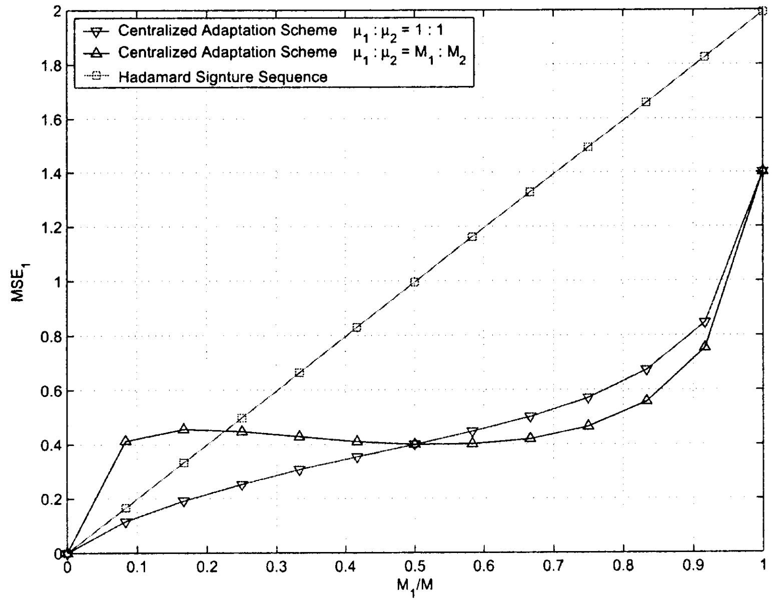


(a) User 1's individual MSE

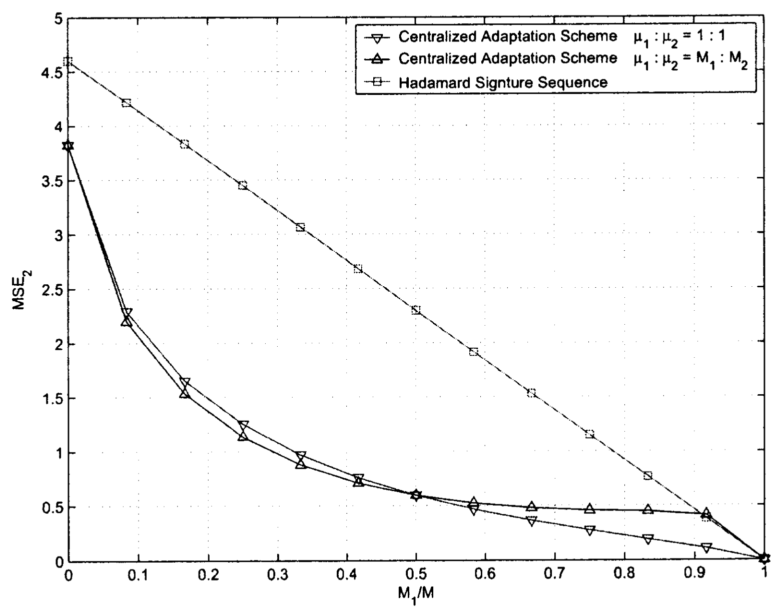


(b) User 2's individual MSE

Figure 4.5 Users' individual MSE achieved with centralized adaptation scheme when $\frac{g_1}{\sigma^2} = 30\text{dB}$ and $\frac{g_2}{\sigma^2} = 5\text{dB}$.



(a) User 1's individual MSE



(b) User 2's individual MSE

Figure 4.6 Users' individual MSE achieved with centralized adaptation scheme when $\frac{g_1}{\sigma^2} = 20\text{dB}$ and $\frac{g_2}{\sigma^2} = 15\text{dB}$.

4.6 Chapter Summary

This chapter focuses on finding the optimal signature sequences (as well as the corresponding receiver filters) which minimize the weighted total mean square error in the downlink scenario. The characteristics of such optimal signature sequences have been identified. A centralized signature sequence adaptation scheme is then developed to construct such optimal signature sequences under arbitrarily given channel response. Following the proposed scheme, the base station collects channel state information from mobiles and jointly adapts all users' signature sequence for a common global system performance. The convergence of the proposed adaptation algorithm is proven, and its convergence property is shown by the simulation. For frequency selective fading channels, simulation result indicates that the proposed signature sequence can dramatically improve the system performance.

CHAPTER 5

DISTRIBUTED SIGNATURE SEQUENCE ADAPTATION

The signature sequence adaptation scheme considered in the last chapter is centralized in nature. That is, the base station — the only decision maker in the system — collects the complete channel state information of the system and use it to *jointly* determine all users' signature sequence for a system wide performance objective, namely the weighted total mean square error. However, from the point view of practical implementation, such centralized scheme may not be suitable for certain applications.

First of all, because the centralized scheme requires the complete system channel station information, each mobile in the system has to feed its observed channel response back to the base station. When a system is to work in an unlicensed band, mobiles will probably receive background interference generated from other communication systems operating on the same frequency band, whose characteristics may be different from mobile to mobile and even time variant. In this case, to fulfill the centralized adaptation scheme, each mobile has to feed back the statistics of its background interference in addition to its channel response, which will make the amount of feedback information prohibitively large.

Moreover, for an existing wireless communication system which migrats to support signature sequence adaptation, there are probably many "obsolete" mobiles which may not have the capability to feed back its channel state information. Without complete knowledge of the system wide channel state information, the base station can not properly adapt the signature sequences for those "advanced" users who are ready for signature sequence adaptation.

Furthermore, in future generation wireless communication systems, users may simultaneously require different services which have different performance requirements. It will be quite difficult to construct a tractable comprehensive objective function which can efficiently reflects all users' diverse performance requirements.

To overcome these obstacles, a distributed adaptation scheme is developed in this chapter for signature sequence adaptation. With the proposed scheme, each user's signature sequences, on one hand, are decided based on the corresponding user's channel state information, and on the other hand, are optimized for that particular user's own performance objective.

The rest of the chapter is organized as follows. Section 5.1 presents the proposed signature sequence adaptation scheme. Section 5.2 focuses on how each user's signature sequences are adapted to fulfil the corresponding user's performance objective based upon the local channel state information. Section 5.3 adopts game theory framework to study the stability of the proposed scheme from a system viewpoint. Simulation results about the proposed distributed scheme is then presented in Section 5.4. Finally Section 5.5 summarizes this chapter.

5.1 Distributed Signature Sequence Adaptation Scheme

Figure 5.1 illustrates the system diagram of the downlink of a MC-CDMA system for distributed signature sequence adaptation.

As shown in the figure, each mobile adopts a linear MMSE receiver to demodulate its desired user's data symbols which are spread with signature sequences currently engaged in the system. At the same time, each mobile updates its corresponding user's signature sequences based upon its local channel state information, and then feeds them back to the base station for future data transmission.

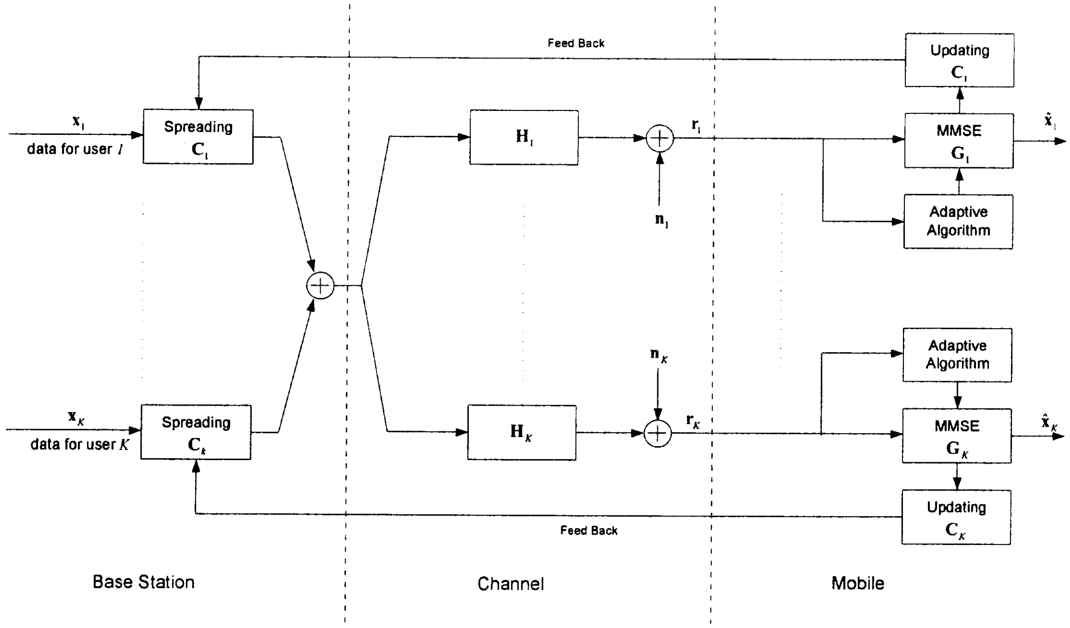


Figure 5.1 Block diagram of downlink MC-CDMA system with distributed signature sequence adaptation.

Note that, for each mobile, it can only extract the channel state information of the wireless link which connects the base station and itself. In this way, each mobile's knowledge about the system channel state information is incomplete. Therefore, with limited system channel state information, each user are designed to adapt its signature sequence to optimize its own performance, with no consideration to the performance of other users in the system. For simplicity, it is assumed in this chapter that, during the adaptation process, the power for each user is pre-assigned and fixed.

As indicated in Figure 5.1, the base station just follows the instructions from the mobiles and uses the feedback signature sequences for data transmission. Since the base station is not involved in the decision of any user's signature sequence, all users' signature sequences are adapted independently and non-cooperatively. In other words, the proposed adaptation scenario follows what in termed *distributed* fashion.

5.2 Adaptation of Each User's Signature Sequences

This section focuses on how each user's signature sequences are independently adapted to optimize the corresponding user's individual performance. The interaction among users during the process of adaptation will be dealt with in the next section.

With MMSE receiver filters, user k's mean square error matrix $\mathbf{MSE}_k$ can be expressed as

$$\mathbf{MSE}_k = \left[\mathbf{I}_{M_k} + \mathbf{C}_k^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{C}_k\right]^{-1}$$

which was once given in (3.4) and (3.5). In the following, three performance objectives are considered for adapting user k's signature sequences $\mathbf{C}_k$. They are:

- the mean square error that may incur at mobile k;
- the rate capacity that can be achieved by user k;
- the maximum diagonal element of matrix $\mathbf{MSE}_k$.

As will be shown later, for these optimization objectives, the adaptation of $\mathbf{C}_k$ is based upon the knowledge of matrix $\mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k$, where $\mathbf{H}_k$ is the channel response of the wireless link from the base station to mobile k, and $\mathbf{Z}_k$ is the covariance of MAI observed at mobile k. Even though (3.5) indicates that $\mathbf{Z}_k$ is a function of the signature sequences of other users in the system, the matrix $\mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k$ can be estimated from the input of user k's mobile by replacing $\mathbf{H}_k$ with the corresponding channel estimation $\hat{\mathbf{H}}_k$ and estimating $\mathbf{Z}_k$ via

$$\hat{\mathbf{Z}}_k = \frac{1}{J}\left(\sum_{j=1}^{J} \mathbf{r}_k(j)\,\mathbf{r}_k(j)^H\right) - \hat{\mathbf{H}}_k \mathbf{C}_k \left(\hat{\mathbf{H}}_k \mathbf{C}_k\right)^H \tag{5.1}$$

where $\mathbf{r}_k(j)$ is the input of mobile k at instant j and J is the number of samples used to estimate $\mathbf{Z}_k$. Therefore, the adaptation of user k's signature sequences can be implemented by using the received signal at mobile k without any further information.

5.2.1 Minimize Mean Square Error

First, consider signature sequences adaptation which minimizes user's individual mean square error.

Note that, user k's individual mean square error MSE_k is the trace of the mean square error matrix $\mathbf{MSE}_k$. Therefore, to find the signature sequences $\mathbf{C}_k$ which minimizes user k's mean square error can be formulated as

$$\mathbf{C}_k^{\mathrm{tr}} = \arg\min_{\mathbf{C}_k} tr\left(\mathbf{MSE}_k\right) \quad \text{s.t. } tr\left(\mathbf{C}_k\mathbf{C}_k^H\right) \leq P_k \tag{5.2}$$

where P_k is user k's pre-assigned power limit.

To solve this optimization problem, matrix $\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k$ is eigenvalue decomposed as follows,

$$\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k = \bar{\mathbf{U}}_k\mathbf{\Lambda}_k\bar{\mathbf{U}}_k^H \tag{5.3}$$

where $\mathbf{\Lambda}_k = diag(\lambda_{k_1}, \cdots, \lambda_{k_N})$ is a diagonal matrix composed with eigenvalues; and $\bar{\mathbf{U}}_k$ is a unitary matrix composed with the corresponding eigenvectors. Without loss of generality, suppose those eigenvalues are arranged in descending order such that $\lambda_{k_1} \geq \cdots \geq \lambda_{k_N}$.

For notational brevity, define $\mathbf{B}_k \triangleq \bar{\mathbf{U}}_k^H\mathbf{C}_k$, from which $\mathbf{C}_k = \bar{\mathbf{U}}_k\mathbf{B}_k$ can be uniquely determined by matrix $\mathbf{B}_k$. Using the matrix inverse lemma, after certain linear algebra calculation, user k's mean square error can be rewritten in term of matrix $\mathbf{B}_k$:

$$\mathrm{MSE}_k = M_k - N + tr\left(\left[\mathbf{I}_N + \mathbf{\Lambda}_k^{\frac{1}{2}}\mathbf{B}_k\mathbf{B}_k^H\mathbf{\Lambda}_k^{\frac{1}{2}}\right]^{-1}\right) \tag{5.4}$$

Since matrix $\bar{\mathbf{U}}_k$ is unitary, it is easy to see

$$tr\left(\mathbf{B}_k\mathbf{B}_k^H\right) = tr\left(\bar{\mathbf{U}}_k^H\mathbf{C}_k\mathbf{C}_k^H\bar{\mathbf{U}}_k\right) = tr\left(\mathbf{C}_k\mathbf{C}_k^H\right) \tag{5.5}$$

In this way, the problem of finding $\mathbf{C}_k$ which minimizes $tr\left(\mathbf{MSE}_k\right)$ under the power constraint P_k is equivalent to finding $\mathbf{B}_k$ which minimizes

$$\mathcal{E}_k^{\mathrm{tr}}\left(\mathbf{B}_k\right) \triangleq tr\left(\left[\mathbf{I}_N + \mathbf{\Lambda}_k^{\frac{1}{2}}\mathbf{B}_k\mathbf{B}_k^H\mathbf{\Lambda}_k^{\frac{1}{2}}\right]^{-1}\right) \tag{5.6}$$

with subject to

$$tr\left(\mathbf{B}_k\mathbf{B}_k^H\right) \leq P_k \tag{5.7}$$

Lemma 5.1 *For an $N \times N$ positive definite matrix $\mathbf{A}$, whose (m,n)th entry is denoted as $[\mathbf{A}]_{m,n} = a_{m,n}$, it holds true that $tr(\mathbf{A}^{-1}) \geq \sum_{i=1}^{N} 1/a_{i,i}$, where the equality holds if and only if $\mathbf{A}$ is diagonal.*

Proof: Note that function $g(x) = \frac{1}{x}$ for $x > 0$ is a strictly convex function, since $\frac{d^2}{dx^2}g(x) = 2x^{-3} > 0$. Then, following Lemma A.2 given in Appendix, function $\phi(\mathbf{x})$ defined as $\phi(x_1, \cdots, x_n) \triangleq \sum_{i=1}^{N} 1/x_i$ is strictly Schur convex.

Consider an arbitrary $N \times N$ positive definite matrix $\mathbf{A}$ with eigenvalues $(\lambda_1, \cdots, \lambda_N)$ and diagonal elements $(a_{1,1}, \cdots, a_{N,N})$. By Lemma A.3 presented in Appendix, it follows that $(\lambda_1, \cdots, \lambda_N)$ majorizes $(a_{1,1}, \cdots, a_{N,N})$.

Hence, following the definition of Schur convexity,

$$\begin{aligned} \phi(\lambda_1, \cdots, \lambda_N) &\geq \phi(a_{1,1}, \cdots, a_{N,N}) && \text{(5.8a)} \\ \sum_{i=1}^{N}\frac{1}{\lambda_i} &\geq \sum_{i=1}^{N}\frac{1}{a_{i,i}} && \text{(5.8b)} \\ tr(\mathbf{A}^{-1}) &\geq \sum_{i=1}^{N}\frac{1}{a_{i,i}} && \text{(5.8c)} \end{aligned}$$

Moreover, since function $\phi(x_1, \cdots, x_n)$ is strictly Schur convex, the equality of (5.8c) holds if and only if $(\lambda_1, \cdots, \lambda_N)$ is a permutation of $(a_{1,1}, \cdots, a_{N,N})$, which implies matrix $\mathbf{A}$ is diagonal. ∎

By Lemma 5.1, $\mathcal{E}_k^{\mathrm{tr}}\left(\mathbf{B}_k\right)$ is minimized only when the matrix $\mathbf{I}_N + \mathbf{\Lambda}_k^{\frac{1}{2}}\mathbf{B}_k\mathbf{B}_k^H\mathbf{\Lambda}_k^{\frac{1}{2}}$ is diagonal. Note that $\mathbf{I}_N + \mathbf{\Lambda}_k^{\frac{1}{2}}\mathbf{B}_k\mathbf{B}_k^H\mathbf{\Lambda}_k^{\frac{1}{2}}$ is diagonal if and only $\mathbf{B}_k\mathbf{B}_k^H$ is diagonal.

Hence the optimal $\mathbf{B}_k^{\mathrm{tr}}$ which minimizes (5.4) should satisfy

$$\mathbf{B}_k^{\mathrm{tr}}\mathbf{B}_k^{\mathrm{tr}H} = diag(\lambda_{k_1}^{\mathrm{tr}}, \cdots, \lambda_{k_N}^{\mathrm{tr}}) \tag{5.9}$$

Then, $\mathcal{E}_k^{\mathrm{tr}}(\mathbf{B}_k^{\mathrm{tr}})$ can be simplified as a function of $\left\{\lambda_{k_i}^{\mathrm{tr}}\right\}_{i=1}^{N}$:

$$\mathcal{E}_k^{\mathrm{tr}}(\mathbf{B}_k^{\mathrm{tr}}) = \sum_{i=1}^{N} \frac{1}{1+\lambda_{k_i}\lambda_{k_i}^{\mathrm{tr}}} \tag{5.10}$$

Note that, matrix $\mathbf{C}_k$ is of dimension $N \times M_k$, therefore the rank of $\mathbf{B}_k\mathbf{B}_k^H$ is no larger than M_k. In other words, within $\left\{\lambda_{k_i}^{\mathrm{tr}}\right\}_{i=1}^{N}$, there exist at most M_k non-zero elements. Since the eigenvalues $\{\lambda_{k_i}\}_{i=1}^{N}$ are arranged in descending order, it can be deduced from Equation (5.10) that $\mathcal{E}_k^{\mathrm{tr}}(\mathbf{B}_k^{\mathrm{tr}})$ is minimized if and only if the first M_k elements of $\left\{\lambda_{k_i}^{\mathrm{tr}}\right\}_{i=1}^{N}$ are non-zero, i.e. $\lambda_{k_i}^{\mathrm{tr}} \neq 0$ for $i = 1, \cdots, M_k$. As the result, $\mathbf{B}_k^{\mathrm{tr}}\mathbf{B}_k^{\mathrm{tr}H}$ can be explicitly expressed as

$$\mathbf{B}_k^{\mathrm{tr}}\mathbf{B}_k^{\mathrm{tr}H} = diag(\lambda_{k_1}^{\mathrm{tr}}, \cdots, \lambda_{k_{M_k}}^{\mathrm{tr}}, \underbrace{0, \cdots, 0}_{N-M_k}) \tag{5.11}$$

That means matrix $\mathbf{B}_k^{\mathrm{tr}}$ should take form

$$\mathbf{B}_k^{\mathrm{tr}} = \begin{bmatrix} \sqrt{\mathbf{\Lambda}_k^{\mathrm{tr}}}\mathbf{W}_k^H \\ \mathbf{0} \end{bmatrix} \tag{5.12}$$

where $\mathbf{\Lambda}_k^{\mathrm{tr}} \triangleq diag(\lambda_{k_1}^{\mathrm{tr}}, \cdots, \lambda_{k_{M_k}}^{\mathrm{tr}})$ and $\mathbf{W}_k$ is an arbitrary $M_k \times M_k$ unitary matrix. Then, the corresponding optimal matrix $\mathbf{C}_k^{\mathrm{tr}}$ can be obtained as:

$$\mathbf{C}_k^{\mathrm{tr}} = \bar{\mathbf{U}}_k\mathbf{B}_k^{\mathrm{tr}} = \mathbf{U}_k\sqrt{\mathbf{\Lambda}_k^{\mathrm{tr}}}\mathbf{W}_k^H \tag{5.13}$$

where $\mathbf{U}_k$ is composed with first M_k columns of $\bar{\mathbf{U}}_k$. Clearly, matrix $\mathbf{U}_k$ is composed of eigenvectors corresponding to the M_k largest eigenvalues of matrix $\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k$.

Using $\mathbf{C}_k^{\mathrm{tr}}$ given in (5.13), user k's mean square error becomes

$$f_k^{\mathrm{tr}}(\lambda_{k_1}^{\mathrm{tr}}, \cdots, \lambda_{k_{M_k}}^{\mathrm{tr}}) = \sum_{i=1}^{M_k} \frac{1}{1+\lambda_{k_i}\lambda_{k_i}^{\mathrm{tr}}} \tag{5.14}$$

Then the optimization problem defined in (5.2) is simplified to find the set of $\left\{\lambda_{k_i}^{\text{tr}}\right\}_{i=1}^{M_k}$ which minimizes (5.14), that is

$$\begin{aligned} \min_{\left\{\lambda_{k_i}^{\text{tr}}\right\}_{i=1}^{M_k}} \quad & \sum_{i=1}^{M_k} \frac{1}{1+\lambda_{k_i}^{\text{tr}}\lambda_{k_i}} \\ \text{s.t.} \sum_{i=1}^{M_k} \lambda_{k_i}^{\text{tr}} &\leq P_k \\ \lambda_{k_i}^{\text{tr}} &\geq 0 \text{ for } i=1,\cdots,M_k \end{aligned} \tag{5.15}$$

where $\lambda_{k_i}^{\text{tr}} \geq 0$ for $i = 1, \cdots, M_k$ are due to positive semi-definiteness of matrix $\mathbf{B}_k\mathbf{B}_k^H$.

Because $f(x) = 1/(1+ax)$ is a convex function in x for $a \geq 0$ and $x \geq 0$, it is easy to show that the objective function given in (5.15) is a convex function for $\lambda_{k_i}^{\text{tr}}$'s [59]. Further, since the power constraints are linear functions of $\lambda_{k_i}^{\text{tr}}$'s, the optimization problem defined in (5.15) is indeed a standard convex optimization problem [59].

Therefore, to solve such an optimization problem, one can establish a Lagrangian as:

$$L = \sum_{i=1}^{M_k} \frac{1}{1+\lambda_{k_i}^{\text{tr}}\lambda_{k_i}} + \delta\left(\sum_{i=1}^{M_k} \lambda_{k_i}^{\text{tr}} - P_k\right) - \sum_{i=1}^{M_k} \alpha_i \lambda_{k_i}^{\text{tr}} \tag{5.16}$$

where δ is a Lagrangian multiplier corresponding to the power limit for user k, and $\{\alpha_i\}_{i=1}^{M_k}$ are Lagrangian multipliers corresponding to the non-negative constraints for $\left\{\lambda_{k_i}^{\text{tr}}\right\}_{i=1}^{M_k}$. According to the KKT condition, both δ and $\{\alpha_i\}_{i=1}^{M_k}$ are non-negative [59].

By setting the derivative of the Lagrangian expressed in (5.16) with respect to $\lambda_{k_i}^{\text{tr}}$ to 0, one then have

$$\frac{\partial L}{\partial \lambda_{k_i}^{\text{tr}}} = -\frac{\lambda_{k_i}}{\left(1+\lambda_{k_i}^{\text{tr}}\lambda_{k_i}\right)^2} + \delta - \alpha_i = 0 \tag{5.17}$$

which is the necessary and sufficient condition that an optimal $\lambda_{k_i}^{\text{tr}}$ must satisfy.

From (5.17) and the complementary slackness condition $\alpha_i \lambda_{k_i}^{\mathrm{tr}} = 0$ for $i = 1, \cdots, M_k$, the optimal $\lambda_{k_i}^{\mathrm{tr}}$ can be derived as [59]

$$\lambda_{k_i}^{\mathrm{tr}} = \left(\sqrt{\frac{1}{\delta \lambda_{k_i}}} - \frac{1}{\lambda_{k_i}} \right)^+ \tag{5.18}$$

where $(x)^+ \triangleq \max(0, x)$ and δ is a positive value which make the resultant optimal $\{\lambda_{k_i}^{\mathrm{tr}}\}_{i=1}^{M_k}$ satisfy the power constraint with equality. Note that, for $\lambda_{k_i}^{\mathrm{tr}} > 0$, $\lambda_{k_i}^{\mathrm{tr}}$ is a monotonic decreasing function of δ, therefore, there exists a unique positive δ that makes the resultant optimal $\{\lambda_{k_i}^{\mathrm{tr}}\}_{i=1}^{M_k}$ satisfy the power constraint with equality.

As a summary, user k's optimal signature sequences which minimize user k's mean square error can be expressed as:

$$\mathbf{C}_k^{\mathrm{tr}} = \mathbf{U}_k \sqrt{\mathbf{\Lambda}_k^{\mathrm{tr}}} \mathbf{W}_k^H \tag{5.19}$$

where $\mathbf{W}_k$ is an arbitrary $M_k \times M_k$ unitary matrix; $\mathbf{\Lambda}_k^{\mathrm{tr}} = diag(\lambda_{k_1}^{\mathrm{tr}}, \cdots, \lambda_{k_{M_k}}^{\mathrm{tr}})$ with

$$\lambda_{k_i}^{\mathrm{tr}} = \left(\sqrt{\frac{1}{\delta \lambda_{k_i}}} - \frac{1}{\lambda_{k_i}} \right)^+ \quad \text{for} \quad i \in \{1, M_k\}$$

and δ is chosen so that $\sum_{i=1}^{M_k} \lambda_{k_i}^{\mathrm{tr}} = P_k$; $\{\lambda_{k_i}\}_{k=1}^{M_k}$ are the M_k largest eigenvalue of matrix $\mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k$; and $\mathbf{U}_k$ is composed of the corresponding eigenvectors.

5.2.2 Maximize Rate Capacity

This subsection consider the adaptation of user k's signature sequence which maximizes user k's rate capacity.

For general downlink communication scenario, the information designated to different users is independently encoded at the base station. In this case, the rate capacity that can be achieved for each user is closely related to the determinant of that user's mean square error matrix. As can be deduced from Chapter 3, to maximize user k's rate capacity is equivalent to minimize $\det(\mathbf{MSE}_k)$. Therefore, the adaptation of user k's signature sequence with objective to maximize user k's rate capacity can be formulated as:

$$\mathbf{C}_k^{\text{det}} = \arg\min_{\mathbf{C}_k} \det\left(\mathbf{MSE}_k\right) \quad (5.20)$$
$$\text{s.t. } tr\left(\mathbf{C}_k\mathbf{C}_k^H\right) \le P_k$$

With $\mathbf{\Lambda}_k$ and $\mathbf{B}_k$ defined as in the last subsection, the determinant of $\mathbf{MSE}_k$ can be rewritten as a function of $\mathbf{B}_k$:

$$\mathcal{E}_k^{\text{det}}\left(\mathbf{B}_k\right) = \det\left[\left(\mathbf{I}_N + \mathbf{\Lambda}_k^{\frac{1}{2}}\mathbf{B}_k\mathbf{B}_k^H\mathbf{\Lambda}_k^{\frac{1}{2}}\right)^{-1}\right] \quad (5.21)$$

By the Hadamard inequality [6, 27], it is easy to show that $\det(\mathbf{MSE}_k)$ is minimized if and only if matrix $\mathbf{B}_k\mathbf{B}_k^H$ is diagonal. In other words, the optimal $\mathbf{B}_k^{\text{det}}$ which minimizes $\det(\mathbf{MSE}_k)$ can be explicitly expressed as

$$\mathbf{B}_k^{\text{det}}\mathbf{B}_k^{\text{det}H} = diag\left(\lambda_{k_1}^{\text{det}}, \cdots, \lambda_{k_N}^{\text{det}}\right) \quad (5.22)$$

Using $\mathbf{B}_k^{\text{det}}$, the determinant of $\mathbf{MSE}_k$ can be simplified as a function of $\left\{\lambda_{k_i}^{\text{det}}\right\}_{i=1}^N$, such that

$$\mathcal{E}_k^{\text{det}}\left(\mathbf{B}_k^{\text{det}}\right) = \prod_{i=1}^{N} \frac{1}{1+\lambda_{k_i}\lambda_{k_i}^{\text{det}}} \quad (5.23)$$

Because the rank of $\mathbf{B}_k\mathbf{B}_k^H$ is no larger than M_k as argued in the last subsection, there are at most M_k non-zero elements among $\left\{\lambda_{k_i}^{\text{det}}\right\}_{i=1}^N$. With $\{\lambda_{k_i}\}_{i=1}^N$ ordered in decreasing order, it is easy to deduced that the minimization of $\mathcal{E}_k^{\text{det}}\left(\mathbf{B}_k^{\text{det}}\right)$ is achieved if and only if the first M_k elements in $\left\{\lambda_{k_i}^{\text{det}}\right\}_{i=1}^N$ are non-zero. That means,

$$\mathbf{B}_k^{\text{det}}\mathbf{B}_k^{\text{det}H} = diag(\lambda_{k_1}^{\text{det}}, \cdots, \lambda_{k_{M_k}}^{\text{det}}, \underbrace{0, \cdots, 0}_{N-M_k}) \quad (5.24)$$

Following similar argument presented in the last subsection, it can be deduced that user k's optimal signature sequences $\mathbf{C}_k^{\text{det}}$ should be expressed as

$$\mathbf{C}_k^{\text{det}} = \mathbf{U}_k \sqrt{\mathbf{\Lambda}_k^{\text{det}}} \mathbf{W}_k^H \tag{5.25}$$

where $\mathbf{U}_k$ and $\mathbf{W}_k$ are defined as before, and $\mathbf{\Lambda}_k^{\text{det}} \triangleq diag(\lambda_{k_1}^{\text{det}}, \cdots, \lambda_{k_{M_k}}^{\text{det}})$.

Now, seeking $\mathbf{C}_k^{\text{det}}$ which minimizes $\det(\mathbf{MSE}_k)$ can be simplified to looking for a set of $\left\{\lambda_{k_i}^{\text{det}}\right\}_{i=1}^{M_k}$ which minimizes

$$f_k^{\text{det}}(\lambda_{k_1}^{\text{det}}, \cdots, \lambda_{k_{M_k}}^{\text{det}}) = \prod_{i=1}^{M_k} \frac{1}{1 + \lambda_{k_i}^{\text{det}} \lambda_{k_i}} \tag{5.26}$$

Such an optimization problem can be formulated as

$$\begin{aligned} \min_{\left\{\lambda_{k_i}^{\text{det}}\right\}_{i=1}^{M_k}} & \quad \prod_{i=1}^{M_k} \frac{1}{1 + \lambda_{k_i}^{\text{det}} \lambda_{k_i}} \\ \text{s.t.} \ \sum_{i=1}^{M_k} \lambda_{k_i}^{\text{det}} & \leq P_k \\ \lambda_{k_i}^{\text{det}} & \geq 0 \text{ for } i = 1, \cdots, M_k \end{aligned} \tag{5.27}$$

where $\lambda_{k_i}^{\text{det}} \geq 0$ is due to the positive semi-definiteness of matrix $\mathbf{B}_k \mathbf{B}_k^H$.

With

$$\frac{\partial^2}{\partial(\lambda_{k_i}^{\text{det}})^2} f_k^{\text{det}}(\lambda_{k_1}^{\text{det}}, \cdots, \lambda_{k_{M_k}}^{\text{det}}) = \frac{2\left(\lambda_{k_i}^{\text{det}}\right)^2}{\left(1 + \lambda_{k_i}^{\text{det}} \lambda_{k_i}\right)^2} \prod_{l=1}^{M_k} \frac{1}{1 + \lambda_{k_l}^{\text{det}} \lambda_{k_l}} \tag{5.28}$$

for $i = 1, \cdots, M_k$, and

$$\frac{\partial^2}{\partial \lambda_{k_i}^{\text{det}} \partial \lambda_{k_j}^{\text{det}}} f_k^{\text{det}}(\lambda_{k_1}^{\text{det}}, \cdots, \lambda_{k_{M_k}}^{\text{det}}) = \frac{\lambda_{k_i}^{\text{det}} \lambda_{k_j}^{\text{det}}}{\left(1 + \lambda_{k_i}^{\text{det}} \lambda_{k_i}\right)\left(1 + \lambda_{k_j}^{\text{det}} \lambda_{k_j}\right)} \prod_{l=1}^{M_k} \frac{1}{1 + \lambda_{k_l}^{\text{det}} \lambda_{k_l}} \tag{5.29}$$

for any $i \, \text{and} \, j \in \{1, M_k\}$ with $i \neq j$, the Hessian of $f_k^{\text{det}}(\lambda_{k_1}^{\text{det}}, \cdots, \lambda_{k_{M_k}}^{\text{det}})$ can be expressed as [59]

$$\nabla^2 f_k^{\text{det}}(\lambda_{k_1}^{\text{det}}, \cdots, \lambda_{k_{M_k}}^{\text{det}}) = \prod_{l=1}^{M_k} \frac{1}{1 + \lambda_{k_l}^{\text{det}} \lambda_{k_l}} \mathbf{Q} \tag{5.30}$$

where

$$\mathbf{Q} \triangleq \begin{bmatrix} \frac{\lambda_{k_1}^{\det}}{1+\lambda_{k_1}^{\det}\lambda_{k_1}} \\ \vdots \\ \frac{\lambda_{kM_k}^{\det}}{1+\lambda_{kM_k}^{\det}\lambda_{kM_k}} \end{bmatrix} \begin{bmatrix} \frac{\lambda_{k_1}^{\det}}{1+\lambda_{k_1}^{\det}\lambda_{k_1}} \\ \vdots \\ \frac{\lambda_{kM_k}^{\det}}{1+\lambda_{kM_k}^{\det}\lambda_{kM_k}} \end{bmatrix}^T + \begin{bmatrix} \frac{(\lambda_{k_1}^{\det})^2}{(1+\lambda_{k_1}^{\det}\lambda_{k_1})^2} & & \\ & \ddots & \\ & & \frac{(\lambda_{kM_k}^{\det})^2}{(1+\lambda_{kM_k}^{\det}\lambda_{kM_k})^2} \end{bmatrix}$$

Because $\nabla^2 f_k^{\det}$ is positive definite, the objective function $f_k^{\det}(\lambda_{k_1}^{\det}, \cdots, \lambda_{kM_k}^{\det})$ is a convex function in $\lambda_{k_i}^{\det}$'s [59]. Furthermore, because the constrain functions are all linear functions in $\lambda_{k_i}^{\det}$'s, the optimization problem defined in (5.27) is a convex optimization problem.

The Lagrangian for the optimization problem defined in (5.27) can then be established as

$$L = \prod_{i=1}^{M_k} \frac{1}{1+\lambda_{k_i}^{\det}\lambda_{k_i}} + \delta\left(\sum_{i=1}^{M_k} \lambda_{k_i}^{\det} - P_k\right) - \sum_{i=1}^{M_k} \alpha_i \lambda_{k_i}^{\det} \tag{5.31}$$

where δ is a Lagrangian multiplier corresponding to the power limit for user k, and $\{\alpha_i\}_{i=1}^{M_k}$ are Lagrangian multipliers corresponding to the non-negative constraints for $\left\{\lambda_{k_i}^{\det}\right\}_{i=1}^{M_k}$. Note that, both δ and $\{\alpha_i\}_{i=1}^{M_k}$ are non-negative according to the KKT condition [59].

By setting the derivative of the Lagrangian expressed in (5.31) with regard to $\lambda_{k_i}^{\det}$ to 0, one then has

$$\frac{\partial L}{\partial \lambda_{k_i}^{\det}} = -\frac{\lambda_{k_i}}{1+\lambda_{k_i}^{\det}\lambda_{k_i}} + \delta' - \alpha_i' = 0 \tag{5.32}$$

where

$$\delta' = \delta \prod_{l=1}^{M_k} \left(1+\lambda_{k_l}^{\det}\lambda_{k_l}\right) \tag{5.33}$$

$$\alpha_i' = \alpha_i \prod_{l=1}^{M_k} \left(1+\lambda_{k_l}^{\det}\lambda_{k_l}\right) \tag{5.34}$$

Then, from Equation (5.32) and complementary slackness condition $\alpha_i \lambda_{k_i}^{\text{det}} = 0$, the optimal $\lambda_{k_i}^{\text{det}}$ can be derived as [59]

$$\lambda_{k_i}^{\text{det}} = \left(\frac{1}{\delta'} - \frac{1}{\lambda_{k_i}}\right)^+ \tag{5.35}$$

where δ' is a unique positive value which makes the resultant optimal $\{\lambda_{k_i}^{\text{det}}\}_{i=1}^{M_k}$ satisfy the power constraint with equality. Note that, (5.35) is the traditional formulation for water-falling power allocation [6, 60].

As a summary, user k's optimal signature sequences which maximizes user k's mutual information can be expressed as

$$\mathbf{C}_k^{\text{det}} = \mathbf{U}_k \sqrt{\mathbf{\Lambda}_k^{\text{det}}} \mathbf{W}_k^H \tag{5.36}$$

where $\mathbf{U}_k$ and $\mathbf{W}_k$ are as defined before, $\mathbf{\Lambda}_k^{\text{det}} = diag(\lambda_{k_1}^{\text{det}}, \cdots, \lambda_{k_{M_k}}^{\text{det}})$ with

$$\lambda_{k_i}^{\text{det}} = \left(\frac{1}{\delta} - \frac{1}{\lambda_{k_i}}\right)^+ \quad \text{for } i \in \{1, M_k\}$$

and δ is chose so that $\sum_{i=1}^{M_k} \lambda_{k_i}^{\text{det}} = P_k$

5.2.3 Minimize Maximum MSE

This subsection will consider the scenario in which user k's signature sequences are adapted to minimize the maximum diagonal element of matrix $\mathbf{MSE}_k$.

Note that, for MC-CDMA system employing multi-code scheme, each user transmits several sub-streams of data in parallel, where each sub-stream is spread with a distinct signature sequence. In general, a user's overall performance, like the average BER performance, is dominated by the sub-stream with highest mean square error. Because the mean square error associated with user k's ith sub-stream is given by the ith diagonal element of matrix $\mathbf{MSE}_k$, it makes sense for user k to adapt its signature sequences to minimize the maximum diagonal element of matrix $\mathbf{MSE}_k$ as an attempt to improve its overall performance. Formally, such adaptation of user k's signature sequence can be formulated as

$$\mathbf{C}_k^{\mathrm{MAX}} = \arg\min_{\mathbf{C}_k} \max_{i=1,\cdots,M_k} \left([\mathbf{MSE}_k]_{i,i}\right) \tag{5.37}$$
$$\text{s.t. } tr\left(\mathbf{C}_k\mathbf{C}_k^H\right) \le P_k$$

where $[\mathbf{MSE}_k]_{i,i}$ is the ith diagonal element of matrix $\mathbf{MSE}_k$.

Consider a function defined as

$$f\left(\{x_i\}_{i=1}^n\right) \triangleq \max_{i=1,\cdots,n} (x_i) \tag{5.38}$$

with $x_i \ge 0$ for $i = 1, \cdots, n$ and $n \ge 1$.

According to Definition A.1 given in Appendix, it is easy to show that $f\left(\{x_i\}_{i=1}^n\right) = \max_{i=1,\cdots,n}(x_i) = x_{[1]}$. If $(x_1, \cdots, x_n)$ majorizes $(y_1, \cdots, y_n)$, it must be true that $x_{[1]} \ge y_{[1]}$ (See Definition A.2 in Appendix), and therefore $f\left(\{x_i\}_{i=1}^n\right) \ge f\left(\{y_i\}_{i=1}^n\right)$. This means $f\left(\{x_i\}_{i=1}^n\right)$ is a Schur-convex function in $(x_1, \cdots, x_n)$, following Definition A.3 in Appendix.

Since $(x_1, \cdots, x_n)$ majorizes $\left(\sum_{i=1}^n x_i/n, \cdots, \sum_{i=1}^n x_i/n\right)$ as stated by Lemma A.1 in Appendix, then, by the definition of Schur-convex,

$$f\left(\{x_i\}_{i=1}^n\right) \ge \frac{1}{n}\sum_{i=1}^n x_i \tag{5.39}$$

It is clear that in (5.39) the equality holds if $\{x_i\}_{i=1}^n$ are identical.

From (5.39), it follows that

$$\max_{i=1,\cdots,M_k} \left([\mathbf{MSE}_k]_{i,i}\right) = f\left(\{[\mathbf{MSE}_k]_{i,i}\}_{i=1}^{M_k}\right) \ge \frac{1}{M_k}\sum_{i=1}^{M_k}[\mathbf{MSE}_k]_{i,i} = \frac{1}{M_k} tr(\mathbf{MSE}_k) \tag{5.40}$$

That means, the optimal signature sequences $\mathbf{C}_k^{\mathrm{MAX}}$ which minimize the maximum diagonal elements of $\mathbf{MSE}_k$ should, on one hand, minimize $tr(\mathbf{MSE}_k)$ — the mean square error at mobile k; and on the other hand, make the diagonal elements in the resultant matrix of $\mathbf{MSE}_k$ identical.

For notation brevity, let $\mathbf{MSE_k}(\mathbf{C}_k)$ denote the mean square error matrix at mobile k when $\mathbf{C}_k$ is used to spread user k's data symbols. Lemma A.4 in Appendix implies that, for any given non-zero $\mathbf{C}_k$, one can always find a unique M_k dimensional unitary matrix $\mathbf{W}_k$ so that matrix $\mathbf{MSE_k}(\mathbf{C}_k\mathbf{W}_k^H) = \mathbf{W}_k\mathbf{MSE_k}(\mathbf{C}_k)\mathbf{W}_k^H$ has identical diagonal elements. Such unitary matrix $\mathbf{W}_k$ can be found by using algorithm developed in [40, Sect. IV-A].

Hence, the optimal signature sequences $\mathbf{C}_k^{\mathrm{MAX}}$ which minimizes the maximum diagonal elements of $\mathbf{MSE}_k$ can be explicitly given as

$$\mathbf{C}_k^{\mathrm{MAX}} = \mathbf{U}_k\sqrt{\mathbf{\Lambda}_k^{\mathrm{tr}}}\mathbf{W}_k^H \tag{5.41}$$

where $\mathbf{U}_k$ and $\mathbf{\Lambda}_k^{\mathrm{tr}}$ are defined in subsections 5.3.1 which minimize the mean square error at mobile k, $\mathbf{W}_k$ is the unique M_k dimensional unitary matrix which make the resultant $\mathbf{MSE}_k$ have identical diagonal elements.

5.2.4 Some Remarks

Regarding to the three adaptation objectives presented in previous sub-sections, it is clear that the resultant optimal signature sequences for use k can be expressed with a unified format:

$$\mathbf{C}_k^* = \mathbf{U}_k\sqrt{\mathbf{\Lambda}_k^*}\mathbf{W}_k^H \tag{5.42}$$

In the above equation, $\mathbf{U}_k$ is composed of eigenvectors corresponding to the M_k largest eigenvalues of matrix $\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k$. $\mathbf{\Lambda}_k^*$ is either $\mathbf{\Lambda}_k^{\mathrm{tr}}$ or $\mathbf{\Lambda}_k^{\mathrm{det}}$, depending on the performance objective to be optimized. $\mathbf{W}_k$ is a M_k dimensional unitary matrix. For the adaptation which aims to minimize user k's mean square error or maximize user k's rate capacity, $\mathbf{W}_k$ could be an arbitrary unitary matrix, such that one can set $\mathbf{W}_k = \mathbf{I}_{M_k}$ for simplicity. However, for the adaptation which aims to minimize the maximum diagonal element of matrix $\mathbf{MSE}_k$, $\mathbf{W}_k$ is a unique unitary matrix which makes the resultant $\mathbf{MSE}_k$ have identical diagonal elements.

It can be seen that, no matter which performance objective is chosen, user k's optimal signature sequences $\mathbf{C}_k^*$ is based upon the matrix $\mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k$, where $\mathbf{Z}_k$ is a function of signature sequences of other users in the system. As argued before, even though user k does not access the knowledge of other users' signature sequences, the matrix $\mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k$ can always be estimated from the received signal at mobile k. Therefore, the optimal $\mathbf{C}_k^*$ can be adaptively obtained at mobile k without any further information.

As implied by Equation (5.42), the adaptation of users' signature sequences is interdependent. That means, one user's adaptation of its own signature sequences will trigger other users in the system to adapt their signature sequences, which will in turn trigger another around of signature sequence adaptation in the system, and so on. Hence, it is natural to ask if there is a stable solution for the proposed scheme at which no user has incentive to unilaterally adapts its own signature sequences given the signature sequences chosen by other users in the system? Note that, the existence of such a stable solution is critical. Without it, the proposed adaptation scheme will turn out to be a chaos in which each user endlessly adapts its signature sequence as the response to other users' adaptation of their signature sequences.

5.3 Stability of Distributed Adaptation Scheme

One feature of the considered signature sequence adaptation is that users in the system are selfish, such that each user adapts its signature sequences to optimize its own individual performance objective without considering the performance of other users in the system. Another feature is that the adaptation of users' signature sequences is interdependent. These two features indicate that users in the system are indeed competing with each other to achieve best results for themselves. Therefore, the considered signature sequence adaptation can be well formulated into a K player non-cooperative game, in which each user in the system is a player to compete against each other. In the following, game theory framework will be adopted to analyze the proposed distributed signature sequence adaptation scheme from a system wide viewpoint, and hence answer the posed question.

5.3.1 Non-Cooperative Game Modeling

In the general case, a K player non-cooperative game can be described as $G = [\mathcal{N}, \{\mathcal{S}_k\}, \{u_k(\cdot)\}]$, where $\mathcal{N}$ is the index set for players in the game, $\mathcal{S}_k$ is the strategy set, and $u_k(\cdot)$ is the performance objective that player k tries to optimize [61].

In the game of signature sequence adaptation considered, $\mathcal{N} = \{1, \cdots, K\}$ is the set of user index in the system. A choice of user k's signature sequence $\mathbf{C}_k$ is termed in game theory terminology as a "strategy". Therefore, $\mathcal{S}_k$ is the set of all possible signature sequences $\mathbf{C}_k$ that user k may choose, which is indeed the set of all $N \times M_k$ matrices satisfying user k's power constraint. Formally, it can be defined as $\mathcal{S}_k = \left\{\mathbf{C}_k : tr\left(\mathbf{C}_k\mathbf{C}_k^H\right) \leq P_k\right\}$, with P_k as the power constraint of user k. Then K -tuple $\mathbf{C} = (\mathbf{C}_1, \cdots, \mathbf{C}_K) \in \mathcal{S}$ denotes the outcome of the game in terms of each user's strategy, where $\mathcal{S} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_K$ is the strategy space of Cartesian product of $\mathcal{S}_k$ of all k. The performance objective function of user k is denoted as $u_k(\mathbf{C})$. Alternatively, $u_k(\mathbf{C}_k, \mathbf{C}_{-k})$ is used to emphasize that user k has control over its own strategy $\mathbf{C}_k$ only, where $\mathbf{C}_{-k}$ is the $K-1$-tuple $(\mathbf{C}_1, \cdots, \mathbf{C}_{k-1}, \mathbf{C}_{k+1}, \cdots, \mathbf{C}_K)$ representing the strategies selected by all users except user k. The strategy space of all users excluding user k is denoted by $\mathcal{S}_{-k} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_{k-1} \times \mathcal{S}_{k+1} \times \cdots \times \mathcal{S}_K$.

For the signature sequence adaptations considered in last section, each user's performance objective to be optimized is related to that user's mean square error matrix $\mathbf{MSE}_k$, that is $u_k(\mathbf{C}_k, \mathbf{C}_{-k})$ can be the trace, the determinant or the maximum diagonal element of matrix $\mathbf{MSE}_k$. In this way, the non-cooperative game, in which each user tries to optimize its individual performance by adapting its signature sequences in a distributed fashion, can be formally expressed as

$$\min_{\mathbf{C}_k \in \mathcal{S}_k} u_k\left(\mathbf{C}_k, \mathbf{C}_{-k}\right), \text{ for all } k \in \mathcal{N} \tag{5.43}$$

Since one user's signature sequences that optimize the corresponding user's individual performance of interest depend on the signature sequences of all the other users in the system, then, it is necessary to characterize a set of signature sequences where all users are satisfied with the performance they achieved given the choices of other users. Such a set is called an *equilibrium* operation point.

5.3.2 Nash Equilibrium

The solution that is most widely used for game theoretic problem is the *Nash equilibrium* [61].

Definition 5.1 *The outcome* $\mathbf{C} = (\mathbf{C}_1, \cdots, \mathbf{C}_K)$ *is a Nash equilibrium of game* $G = [\mathcal{N}, \{\mathcal{S}_k\}, \{u_k(\cdot)\}]$ *if, for every* $k \in \mathcal{N}$, $u_k\left(\mathbf{C}_k, \mathbf{C}_{-k}\right) \leq u_k\left(\mathbf{C}'_k, \mathbf{C}_{-k}\right)$ *for all* $\mathbf{C}'_k \in \mathcal{S}_k$.

According to this definition, it is clear that, at Nash equilibrium, no user can further improve its performance by making individual changes in its signature sequences given the signature sequences chosen by other users. Therefore, a Nash equilibrium is an outcome of strategies such that each user's strategy is a *best response* to the other users' strategies [61].

Mathematically, user k's best response, denoted as r_k, is a mapping which specifies user k's optimal strategies for each fixed combination of strategies chosen by its opponents. Therefore, for a general case, r_k is defined as

$$\begin{aligned} r_k\left(\mathbf{C}_{-k}\right) \triangleq\ & \{\mathbf{C}_k \in \mathcal{S}_k : u_k\left(\mathbf{C}_k, \mathbf{C}_{-k}\right) \\ & \leq u_k\left(\mathbf{C}'_k, \mathbf{C}_{-k}\right) \text{ for all } \mathbf{C}'_k \in \mathcal{S}_k\} \end{aligned} \tag{5.44}$$

Specifically, for the non-cooperative game of signature sequence adaptation considered in this chapter, the best response of user k can be defined by using the unified format of user k's optimal signature sequences given by (5.42), such that

$$r_k\left(\mathbf{C}_{-k}\right) \triangleq \mathbf{C}_k^* = \mathbf{U}_k\sqrt{\mathbf{\Lambda}_k^*}\mathbf{W}_k^H \tag{5.45}$$

where $\mathbf{U}_k$, $\mathbf{\Lambda}_k^*$ and $\mathbf{W}_k$ have been explained in previous sections.

From system point of view, the adaptation of all users' signature sequences can be reflected by a correspondence $r : \mathcal{S} \to \mathcal{S}$ defined as the Cartesian product of the r_k for all k, such that

$$r\left(\mathbf{C}\right) \triangleq r_1\left(\mathbf{C}_{-1}\right) \times \cdots \times r_K\left(\mathbf{C}_{-K}\right) \tag{5.46}$$

The definition of Nash equilibrium implies that the signature sequence $\mathbf{C} = \left(\mathbf{C}_1, \cdots, \mathbf{C}_K\right)$ is a Nash equilibrium of the non-cooperative game if and only if $\mathbf{C}_k \in r_k\left(\mathbf{C}_{-k}\right)$ for all $k \in \mathcal{N}$. Note that, a fixed point of r is an outcome of strategies $\mathbf{C}$ such that $\mathbf{C} \in r\left(\mathbf{C}\right)$, i.e., $\mathbf{C}_k \in r_k\left(\mathbf{C}_{-k}\right)$ for each user. Thus a fixed point of best response r is a Nash equilibrium. In other words, to show the existence of Nash equilibrium is equivalent to prove that there exists a fixed point in the correspondence $r\left(\cdot\right)$.

The proof of existence of Nash equilibrium is based on the Kakutani's fixed point theorem, which is stated as follows [62],

Theorem 5.1 *Let $\varphi : S \to S$ be an upper semi-continuous correspondence from a non-empty, compact, convex set $S \subset R^n$ into itself such that for all $x \in S$, the set $\varphi\left(x\right)$ is convex and non-empty, then $\varphi\left(\cdot\right)$ has a fixed point, i.e. there is a x^* where $x^* \in \varphi\left(x^*\right)$.*

First, it is easy to show that the strategy space of considered non-cooperative signature sequence adaptation game $\mathcal{S}$ is compact, convex and non-empty. Second, the correspondence $r : \mathcal{S} \to \mathcal{S}$ defined in (5.46) is non-empty and convex for any given $\mathbf{C}$. This is because, for an arbitrarily given signature sequence set denoted as $\mathbf{C}$, one can always get a unique $\mathbf{C}^* = \left\{\mathbf{C}_1^*, \cdots, \mathbf{C}_K^*\right\}$ by applying (5.45). Third, as implied by

(5.19), (5.36) and (5.41) that each element of $\mathbf{C}_k^*$ is a continuous function of elements of $\mathbf{C}_{-k}$, it is easy to prove that the correspondence $r(\cdot)$ has a close graph. Therefore, by Kakutani's fixed point theorem, it is sufficient to prove that correspondence r has a fixed point. In other words, there always exists a Nash equilibrium for the considered non-cooperative signature sequence adaptation game. Note that, the existence of Nash equilibrium does not depends on the number of users in the system, or the number of signature sequences each user may use, or the channel condition that each user may experience.

Following is an iterative algorithm that can be used to determine the signature sequences at a Nash Equilibrium, in which users' signature sequences are calculated in a round-robin fashion.

Algorithm 5.1 *Considering a K-user system in which the kth user is assigned with M_k signature sequences and transmit power P_k*

1. *A random set of signature sequences is generated, such that for $k = 1, \cdots, K$, $\mathbf{C}_k$ is of size $N \times M_k$ and tr $\left(\mathbf{C}_k\mathbf{C}_k^H\right) = P_k$;*

2. *for $k = 1$ to K*

   (a) *Construct matrix $\mathbf{H}_k^H\mathbf{Z}_k^{-1}\mathbf{H}_k$ using information gathered at mobile k;*

   (b) *Based upon the optimization objective of user k, calculate $\mathbf{C}_k^*$ by using (5.19), (5.36) or (5.41);*

   (c) *update user k's signature sequences with $\mathbf{C}_k \leftarrow \mathbf{C}_k^*$.*

3. *Repeating step 2) until desired accuracy is reached.*

Mathematically, it is difficult to prove that Algorithm 5.1 will definitely converge to a Nash Equilibrium. However, after numerous simulation, it was found that, under reasonable communication scenarios, Algorithm 5.1 can always converge to a set of user signature sequences at which no user in the system has intention to unilaterally adapts its signature sequences.

Algorithm 5.1 suggests that the proposed distributed signature sequence adaptation scheme is stable if users' signature sequences can be adapted in a sequential manner. To ensure the sequential adaptation of users' signature sequences in realistic communication scenarios, each mobile can initialize a timer with a randomly selected back-off interval after its most recent signature sequence adaptation, and is allowed to exsert signature sequence adaptation only after the timer is expired. Since each user's back-off interval between two consecutive adaptations is selected randomly and independently, it is almost impossible for two users to simultaneously adapt their signature sequences. That means, no central controller is required to maintain the sequential adaptation of users' signature sequences. Therefore, the whole system can work in a complete distributed manner.

5.4 Simulation Result

This section is to present the convergence property of the proposed distributed signature sequence adaptation scheme and the system performance achieved by applying such a scheme.

5.4.1 Convergence Property

First, consider the convergence properties for the distributed adaptation scheme. The simulation is carried out in a MC-CDMA system with $N = 6$ sub-channels and $K = 10$ active users. Each user is assigned with $M_k = 1 \ \forall k$ signature sequence, and each user's power constraint is set as $P_k = 1 \ \forall k$. Therefore, the total number of signature sequences engaged in the system is $M = 10$, and the total transmission power at the base station is $P = 10$.

Figure 5.2 and Figure 5.3 shows the convergence property of the proposed adaptation schemes under AWGN channel, that is kth user's channel response is characterized by $\mathbf{h}_k = g_k\mathbf{1}$ for all $k = 1, \cdots, K$, where $\mathbf{1}$ is a vector whose elements are all ones. In Figure 5.2, g_k's are set such that $\frac{g_k}{\sigma^2} = 10$dB $\forall k$; while in Figure 5.3, g_k's are set such that $\frac{g_k}{\sigma^2} = 25$dB $\forall k$. For comparison purpose, the convergence property of the centralized adaptation scheme proposed in last chapter is also presented, where the total power constraint is set to $P = 10$, such that the total transmission power at the base station is identical for the two adaptation schemes under consideration. Here, the objective of the centralized adaptation scheme is to minimize the TMSE in the system. As shown in the figures, for either path loss setting, the distributed algorithm converges to the optimal TMSE very fast. Because the simulation is carried out under assumption that all user's channel responses are identical, each iteration of the proposed distributed adaptation scheme lowers the TMSE.

In Figure 5.4 and Figure 5.5, the convergence property of the distributed adaptation scheme is presented under arbitrarily generated frequency selective channel (a snapshot of the fading channel), where the channel coefficients (elements of $\mathbf{h}_k,\ \ k = 1, \cdots, K$) are independently generated while satisfy $\frac{1}{N}||\mathbf{h}_k||^2 = g_k$. In Figure 5.4, g_k's are set such that $\frac{g_k}{\sigma^2} = 10$dB $\forall k$; while in Figure 5.5, g_k's are set such that $\frac{g_k}{\sigma^2} = 25$dB $\forall k$. Once again, the convergence property of the centralized scheme proposed in the last chapter is presented for comparison purpose, which is obtained under the total power constraint $P = 10$, and with objective to minimize the TMSE.

It is observed that both distributed and centralized adaptation schemes converge with finite number of iterations. However, these two adaptation schemes converge to different TMSEs. Better TMSE performance is achieved by using centralized adaptation scheme. Note that, the centralized adaptation scheme applied in the simulation is designed to minimize the TMSE of system, therefore the TMSE achieved by the centralized adaptation scheme is the optimal TMSE performance of the system. However, for the distributed adaptation scheme, each iteration of adaptation is based upon incomplete system channel state information; and each user adapts its signature

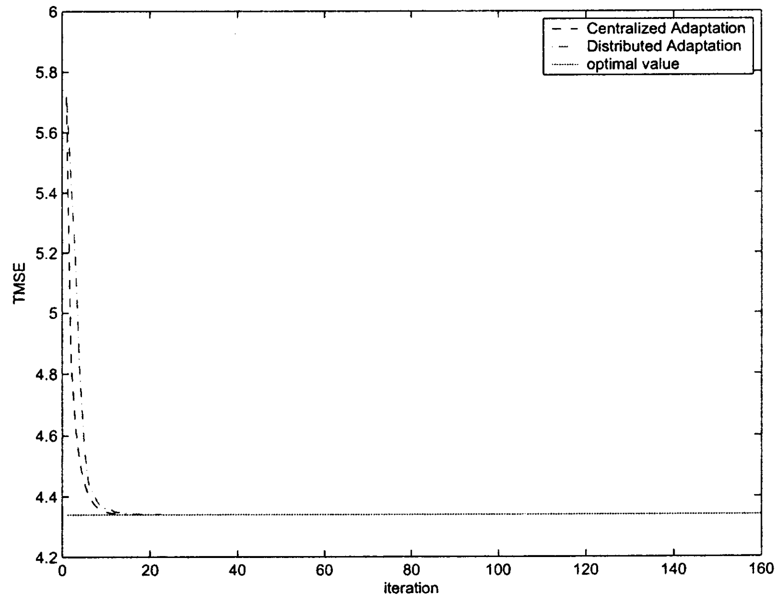


Figure 5.2 Convergence property of distributed adaptation scheme in AWGN channel with $N = 6$, $K = 10$ and $\frac{g_k}{\sigma^2} = 10\text{dB}$ for all $k = 1, \cdots, 10$.

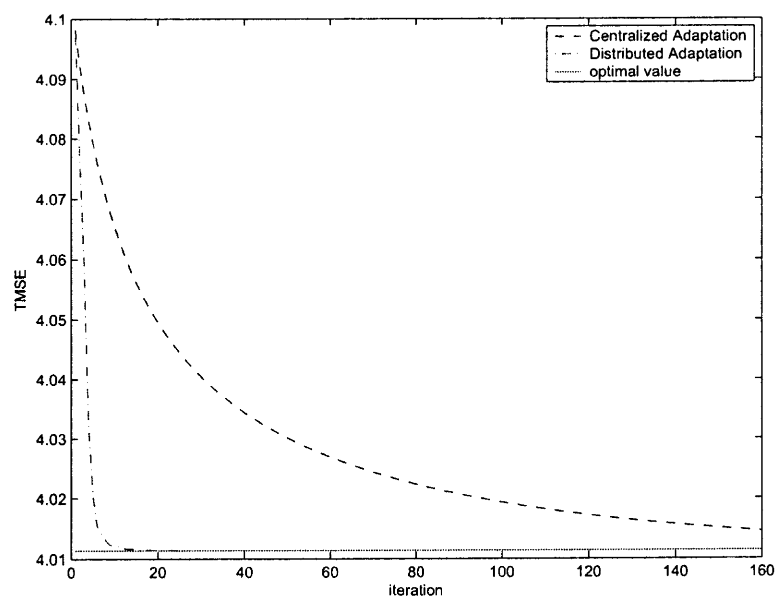


Figure 5.3 Convergence property of distributed adaptation scheme in AWGN channel with $N = 6$, $K = 10$ and $\frac{g_k}{\sigma^2} = 25\text{dB}$ for all $k = 1, \cdots, 10$.

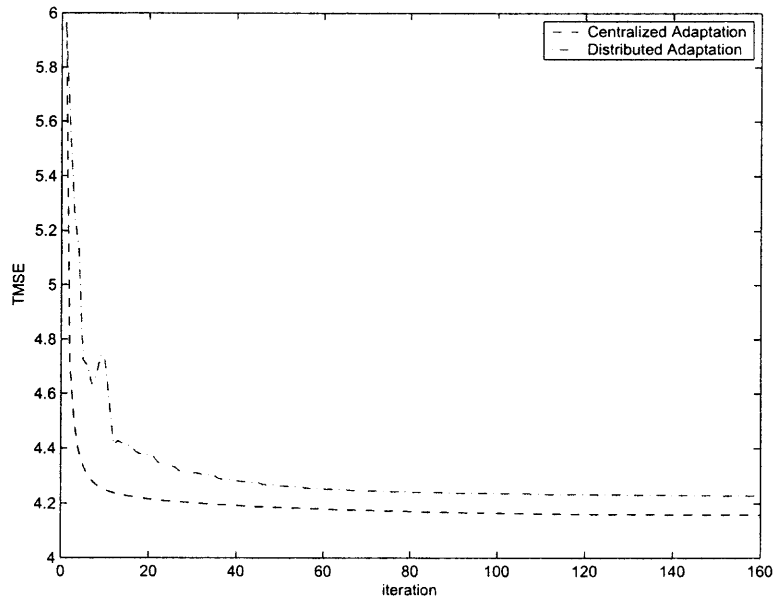


Figure 5.4 Convergence property of the distributed adaptation scheme under arbitrary frequency selective channel with $N = 6$, $K = 10$ and $\frac{g_k}{\sigma^2} = 10\text{dB}$ for all $k = 1, \cdots, 10$

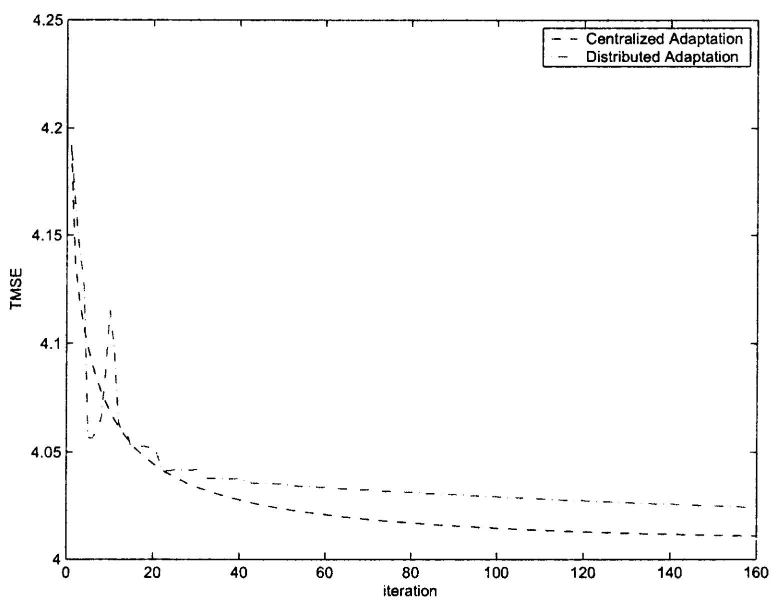


Figure 5.5 Convergence property of the distributed adaptation scheme under arbitrary frequency selective channel with $N = 6$, $K = 10$ and $\frac{g_k}{\sigma^2} = 25\text{dB}$ for all $k = 1, \cdots, 10$.

sequences without considering the TMSE performance of the system. Therefore, the distributed adaptation scheme always achieve worse TMSE performance than the centralized adaptation scheme.

Also shown in those figures, the distributed adaptation scheme does not lower the TMSE at each iteration of adaptation. This is because, at each iteration, the distributed adaptation scheme only concerns the performance of the user whose signature sequence is adapted, where no consideration is taken for the system performance, such as the TMSE performance.

5.4.2 System Performance

In the following, consider the system performance that could be achieved by applying the proposed distributed adaptation schemes under frequency selective fading channels. As before, a 2-user system is considered, which follows parameter specified in IEEE 802.11a and HIPERLAN type 2. Independent Rayleigh frequency selective channel is assumed for each user, which has exponential multipath delay profile with r.m.s. delay spread set to 25ns. Totally $M = 48$ signature sequences are engaged in the system, which are to be shared by those two users. The power assigned to user k is assumed to be proportional to the number of signature sequence assigned to user k, such that $P_k = \frac{M_k}{M}P$ for $k = 1, 2$, where $P = 48$ is the total transmission power assumed at the base station.

First, consider the performance that can be achieved under different signature sequence assignment, where a particular signature sequence assignment refers to a pair of M_1 and M_2 which characterize how $M = 48$ signature sequences are distributed between two users.

Figure 5.6 and Figure 5.7 present the system performance in term of TMSE that could be achieved by applying the proposed distributed signature sequence adaptation with different path loss settings: in Figure 5.6, the path losses are set such that

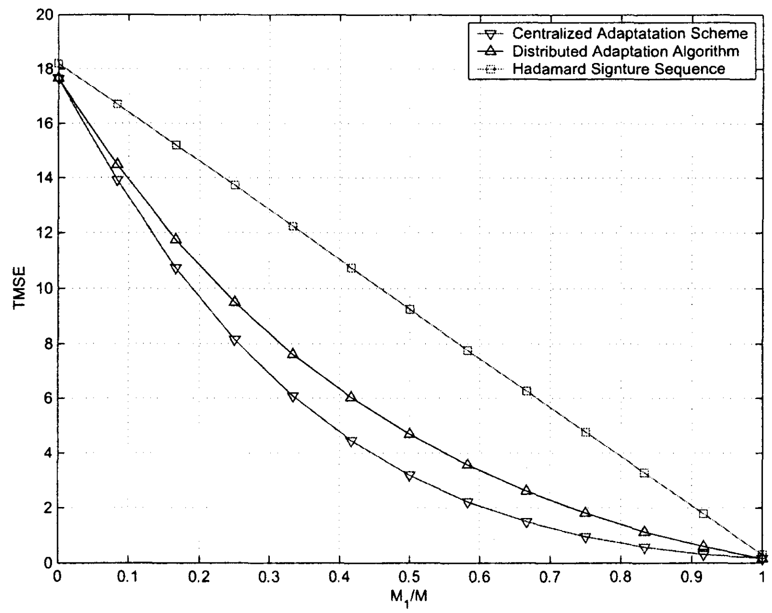


Figure 5.6 TMSE of 2-user system using distributed adaptation scheme under Rayleigh fading channel with $\frac{g_1}{\sigma^2}$ = 30dB and $\frac{g_2}{\sigma^2}$ = 5dB.

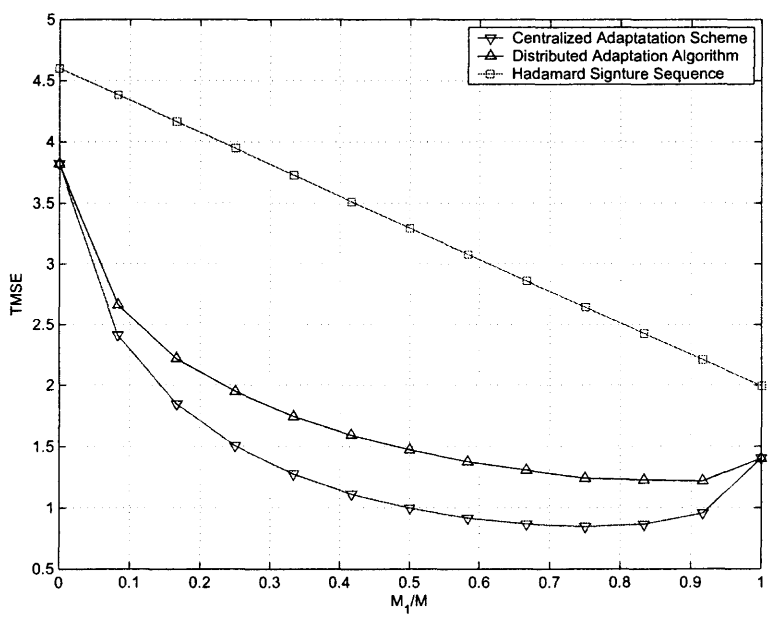
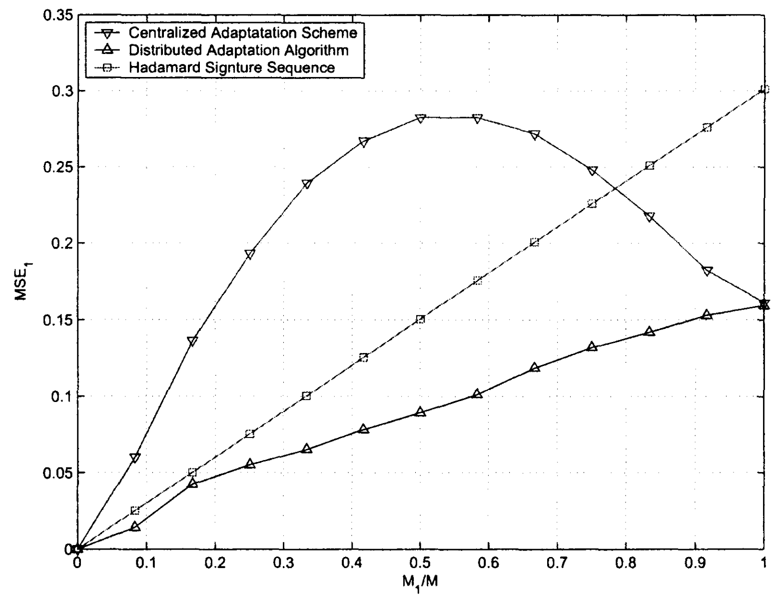
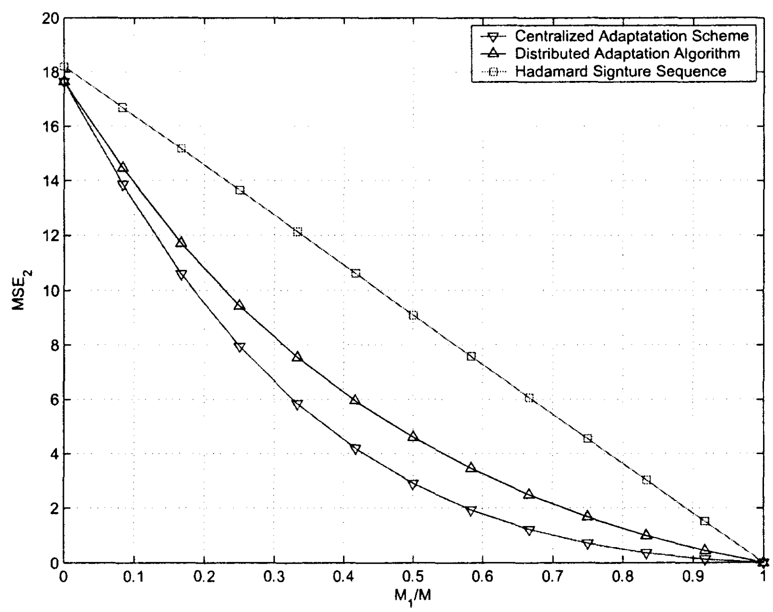


Figure 5.7 TMSE of 2-user system using distributed adaptation scheme under Rayleigh fading channel with $\frac{g_1}{\sigma^2}$ = 20dB and $\frac{g_2}{\sigma^2}$ = 15dB.

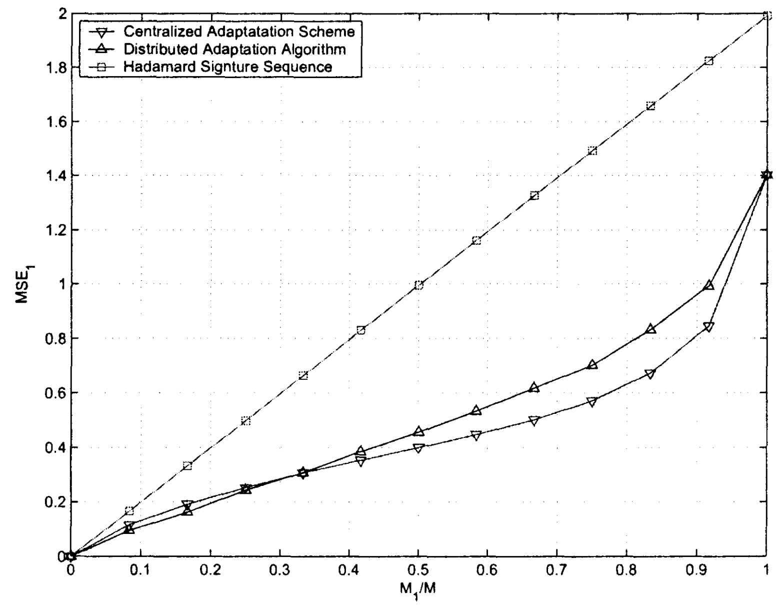


(a) User 1's individual MSE

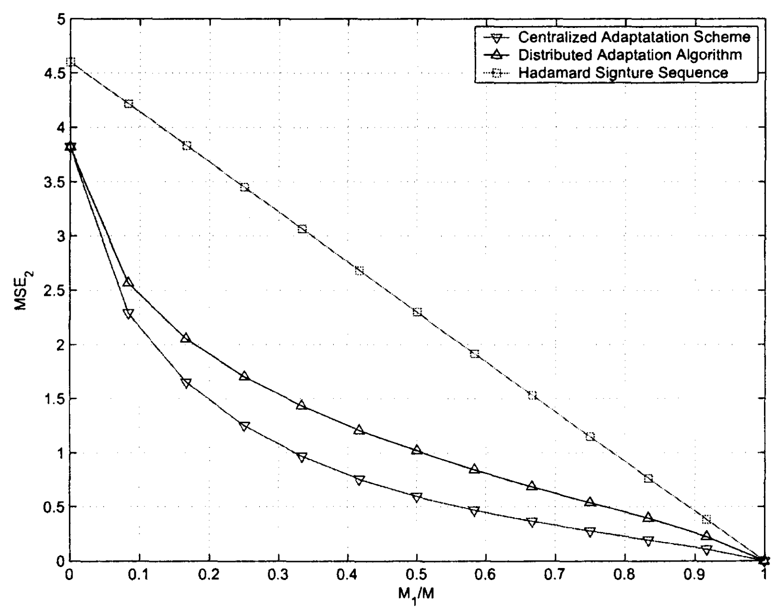


(b) User 2' individual MSE

Figure 5.8 Users' individual MSE performance in the 2-user system under Rayleigh frequency selective fading channel with $\frac{g_1}{\sigma^2}$ = 30dB and $\frac{g_2}{\sigma^2}$ = 5dB.



(a) User 1's individual MSE



(b) User 2's individual MSE

Figure 5.9 Users' individual MSE performance in the 2-user system under Rayleigh frequency selective fading channel with $\frac{g_1}{\sigma^2} = 20\text{dB}$ and $\frac{g_2}{\sigma^2} = 15\text{dB}$.

$\frac{g_1}{\sigma^2} = 30$dB and $\frac{g_2}{\sigma^2} = 5$dB; in Figure 5.7, the path losses are set such that $\frac{g_1}{\sigma^2} = 20$dB and $\frac{g_2}{\sigma^2} = 15$dB. Users' individual MSE performances under the same simulation setup are presented in Figure 5.8 and Figure 5.9, in which the subplot (a) corresponds to user 1 and the subplot (b) is associated with user 2. During the simulation, each user is assumed to adapt its signature sequences to minimize its MSE. Moreover, to clarify the property of the proposed distributed scheme, the performance that could be achieved by employing the centralized adaptation scheme proposed in the last chapter and the traditional Walsh-Hadamard signature sequences are also added to these figures, where the centralized adaptation scheme is carried out to minimize the TMSE in the system.

From Figure 5.6 and Figure 5.7, it is shown that, in terms of TMSE performance, the distributed adaptation scheme can achieve much better performance than the traditional Walsh-Hadamard signature sequences. However, the distributed scheme is not as efficient as its centralized counterpart in improving system performance. From the perspective of user's individual MSE performance, it is found (see Figure 5.8 and 5.9) that distributed adaptation scheme can always achieve better performance for both users than the traditional Walsh-Hadamard signature sequences, under either path loss setting. However, for the scenario in which users' channel quality disparities are close, the distributed adaptation scheme is not as efficient as the centralized adaptation scheme in improving users' individual performances. Since, as depicted in Figure 5.9, centralized adaptation scheme can always achieve better individual MSE performance for both users than the proposed distributed adaptation scheme, the proposed distributed adaptation scheme is not Pareto efficient.

Next, consider the performance achieved by applying the distributed signature sequence adaptation scheme under fixed signature sequence assignment with $M_1 = M_2 = 24$. The simulation is carried out in a scenario in which: a) the path loss g_2 is fixed such that $\frac{g_2}{\sigma^2} = 15$dB; b) the path loss g_1 varies such that $\frac{g_1}{\sigma^2} \in [0, 30]$dB.

Figure 5.10 presents users' average BER (bit error rate) performance. In this figure, each user adapts its signature sequences to minimize the maximum diagonal element of that user's mean square error matrix. 16-QAM is assumed during the simulation. For comparison purpose, the average BER performance that could be achieved by employing the centralized adaptation scheme proposed in the last chapter and the traditional Walsh-Hadamard signature sequences are also presented in Figure 5.10, where the centralized adaptation scheme is carried out to minimize the TMSE in the system. It is observed that, for either user, the proposed distributed scheme can achieve much better BER performance than the traditional Walsh-Hadamard signature sequences. However, as depicted in the figure, the centralized adaptation scheme is more efficient than the proposed distributed adaptation scheme in improving BER performance for either user.

Figure 5.11 shows users' individual capacity achieved by applying the proposed distributed adaptation scheme, where the objective of each user is to maximize its own rate capacity. The individual capacity performance achieved with Walsh-Hadamard signature sequences is also presented in the figure. Obviously, larger capacity can be achieved for each user by using the proposed distributed adaptation scheme than using the traditional Walsh-Hadamard signature sequences.

Finally, Figure 5.12 and Figure 5.13 present users' individual MSE and capacity performance that is achieved by applying the proposed distributed adaptation scheme when user 1 adapts its signature sequence to optimize its capacity and user 2 adapts its signature sequence to optimize its MSE. In these figures, the relevant performance obtained with Walsh-Hadamard signature sequences is presented. It can be seen that, by applying the proposed distributed adaptation scheme, larger individual capacity and lower individual MSE are obtained for user 1 and user 2, respectively.

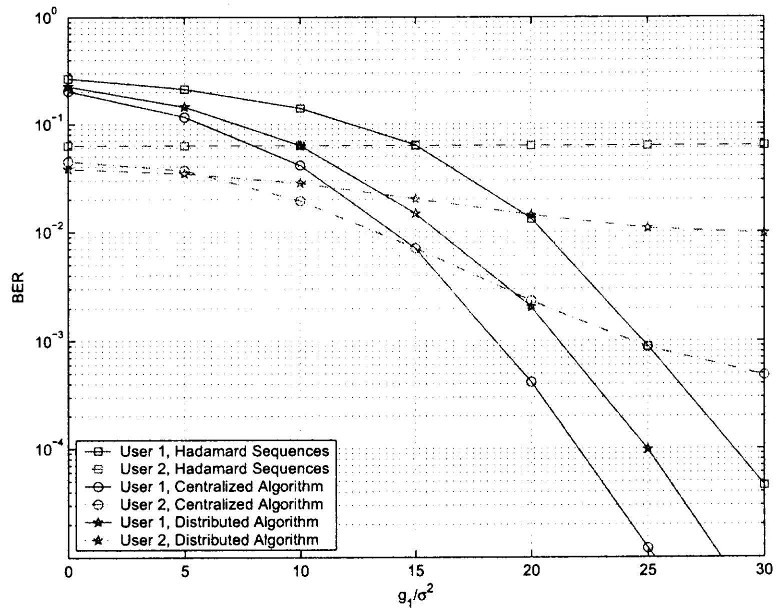


Figure 5.10 Users' average BER performance in a 2-user system when each user adapts its signature sequences to minimize the maximum diagonal element of the corresponding user's mean square error matrix.

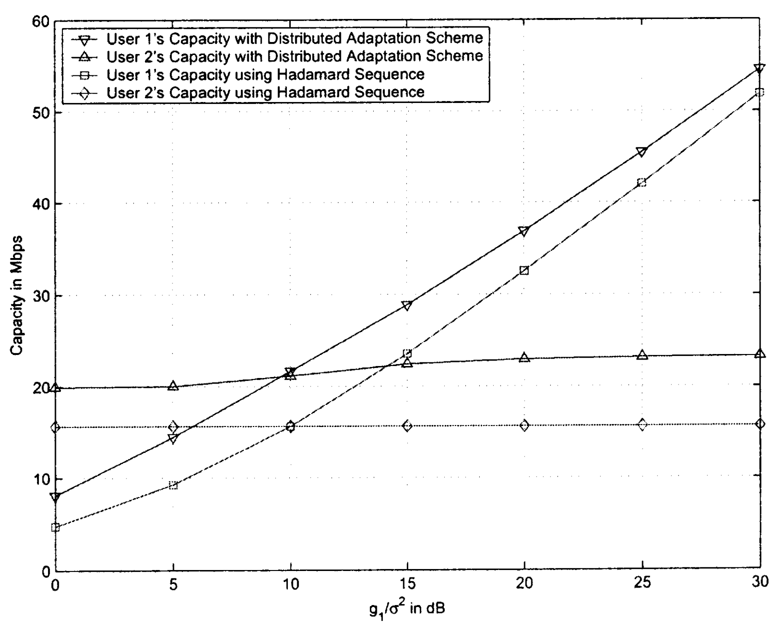


Figure 5.11 Users' individual capacity in a 2-user system when each user adapts its signature sequences to maximize its capacity.

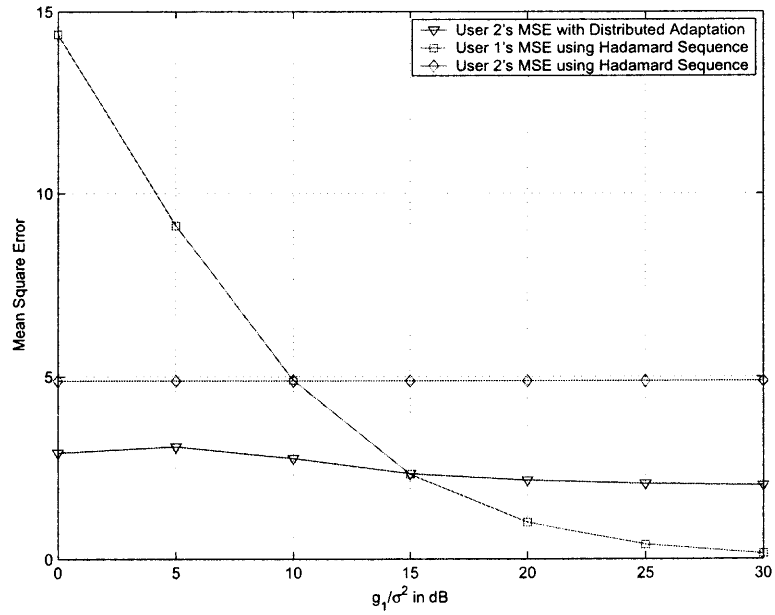


Figure 5.12 Users' individual MSE performance of 2-user system, when user 1 tries to maximize its capacity and user 2 tries to optimize its MSE.

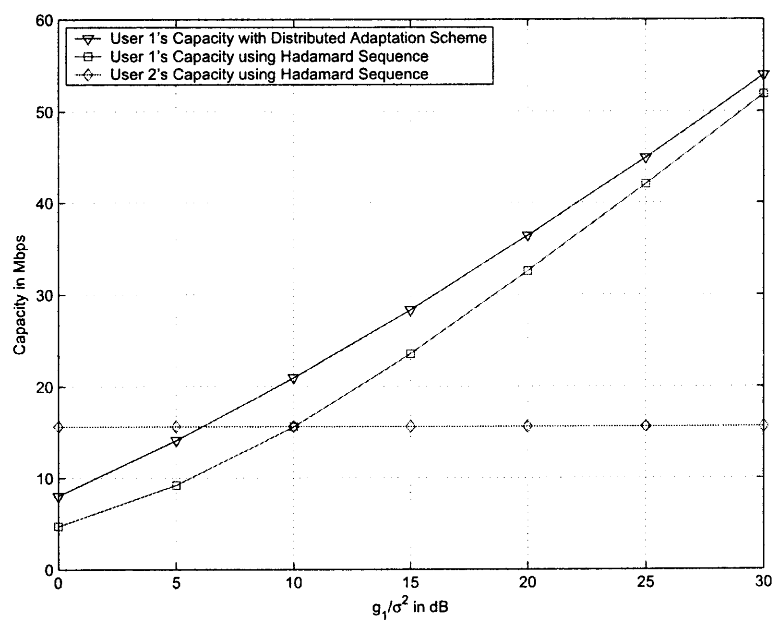


Figure 5.13 Users' individual capacity of 2-user system, when user 1 tries to maximize its capacity and user 2 tries to optimize its MSE.

5.5 Chapter Summary

This chapter considered the distributed adaptation of users' signature sequence in the downlink of MC-CDMA system. For simplicity, it is assume that the transmission power assigned to each user is fixed during the process of adaptation. Each user's signature sequences are independently adapted base upon the corresponding user's local channel state information. Totally, three performance objectives are considered for adapting users' signature sequences. Even though each user adapts its signature sequence without consider the performance of other users in the system, the considered distributed adaptation scenario is proven to have a stable solution, at which every user will be satisfied with its performance and has no incentive to further unilaterally adapts its signature sequences. Simulation result shows that the proposed adaptation schemes can achieve much better performance than the traditional Walsh-Hadamard sequence. However, it is found that, in terms of improving system performance like the TMSE performance, the proposed distributed scheme may not be as efficient as the centralized scheme considered in Chapter 4.

CHAPTER 6

DISTRIBUTED ADAPTATION OF USERS' SIGNATURE SEQUENCE WITH PRICING MECHANISM

For the distributed signature sequence adaptation scheme considered in the last chapter, each user adapts its signature sequences under an arbitrary pre-assigned power constraint which remains unchanged throughout the process of adaptation. In an effort to efficiently utilize the transmission power at the base station, a new distributed signature sequence adaptation scheme is proposed in this chapter, which has additional flexibility of power control. That is, with the distributed adaptation scheme proposed in this chapter, the base station's transmission power is properly distributed among users based upon the system channel state information during the process of adapting users' signature sequences.

In this chapter, the additional power control capability introduced to the proposed distribution signature sequence adaptation scheme is realized by applying the method of pricing. Therefore, the adaptation scheme proposed is referred *distributed adaptation scheme with pricing mechanism.*

As a matter of fact, the pricing methodology has once been applied for distributed non-cooperative power control in the uplink of DS-CDMA based wireless communication systems. For example, in [63,64], pricing methodology was adopted to improve the Pareto efficiency of the resultant power control policy. Recently, pricing mechanism was employed in [65] to restrict users from generating extra MAI so as to improve the performance from a system viewpoint.

It is worthy noting that there is a fundamental difference between this chapter and the cited work [63–65]. In this chapter, users' signature sequences are adapted under a total power constraint, while in [63–65], each user adapted its transmission power under an independent power constraint. Therefore, from the viewpoint of a non-cooperative game, users' strategy spaces for distributed signature sequence adaptation considered in this chapter are *interdependent.* However, for references [63–65], users' strategy spaces for distributed power control were *independent* against each other.

The rest of chapter is organized as follows. Section 6.1 formulates the problem of distributed adaptation of users' signature sequences with pricing mechanism. Section 6.2 focuses on how each user adapts its signature sequence under such circumstance. Section 6.3 studies the whole process of adaptation from a system viewpoint. Section 6.4 presents the performance that could be achieved by applying pricing mechanism under a typical communication scenario. Finally, the summary is provided in Section 6.5.

6.1 Problem Formulation

The distributed signature sequence adaptation with pricing mechanism is modeled as a non-cooperative game, which is denoted as[5]

$$\min_{\mathbf{C}_k \in \mathcal{Q}_k} u_k^c(\mathbf{C}_k; \mathbf{C}_{-k}) \quad \text{for } k \in [1, K] \tag{6.1}$$

where, $u_k^c(\mathbf{C}_k; \mathbf{C}_{-k})$ is the objective function that user k tries to optimize by adapting its signature sequences based upon local channel state information; $\mathcal{Q}_k$ is user k's strategy space which is defined as the set of all possible signature sequences that user k may select.

Specifically, with pricing mechanism, $u_k^c(\mathbf{C}_k; \mathbf{C}_{-k})$ is defined as

$$u_k^c(\mathbf{C}_k; \mathbf{C}_{-k}) = \mathrm{MSE}_k(\mathbf{C}_k; \mathbf{C}_{-k}) + c_k(\mathbf{C}_k; \mathbf{C}_{-k}) \tag{6.2}$$

[5]Notations $u_k^c(\mathbf{C}_k; \mathbf{C}_{-k})$ and $\mathcal{Q}_k$ are used to emphasize that, with pricing mechanism, each user's objective function and strategy space will take different format from those given in Chapter 5.

where $\text{MSE}_k(\mathbf{C}_k; \mathbf{C}_{-k})$ is the mean square error incurred at mobile k; $c_k(\mathbf{C}_k; \mathbf{C}_{-k})$ is the pricing function associated with user k.

Suppose each mobile adopts a linear MMSE receiver to demodulate its desired user's transmitted data symbols, the mean square error at mobile k is given as

$$\text{MSE}_k(\mathbf{C}_k; \mathbf{C}_{-k}) = tr\left((\mathbf{I}_{M_k} + \mathbf{C}_k^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{C}_k)^{-1}\right) \tag{6.3}$$

For simplicity, $c_k(\mathbf{C}_k; \mathbf{C}_{-k})$ is defined as a linear function of user k's transmission power, that is

$$c_k(\mathbf{C}_k, \mathbf{C}_{-k}) \triangleq c \cdot tr(\mathbf{C}_k \mathbf{C}_k^H) \tag{6.4}$$

where c (with $c > 0$) is the unit price which is identical to all users in the system.

In this way, user k's objective function defined in (6.2) can be explicitly expressed as:

$$u_k^c(\mathbf{C}_k; \mathbf{C}_{-k}) = tr\left((\mathbf{I}_{M_k} + \mathbf{C}_k^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{C}_k)^{-1}\right) + c \cdot tr(\mathbf{C}_k \mathbf{C}_k^H) \tag{6.5}$$

Here, $u_k^c(\mathbf{C}_k; \mathbf{C}_{-k})$ can be regarded as the total cost associated with transferring data from the base station to mobile k. On one hand, there is cost due to communication error, which is inevitable because of the presence of MAI and additive noise. Such cost is represented by the mean square error at mobile k. On the other hand, there is cost arising from using transmission power, which is introduced by a linear pricing function with a non-zero unit price c. It is obvious that, for any user, the mean square error can be lowered by increasing transmission power. In the case $c = 0$, user k will intend to use as much power as possible to lower its mean square error. However, the non-zero unit price c discourages this behavior. Indeed, the non-zero unit price c reflects the relative level of penalty paid for using power against the benefit obtained by lowering mean square error. Because the power required by user k to minimize $u_k^c(\mathbf{C}_k; \mathbf{C}_{-k})$ is a function of the unit price applied in the system, the unit price c can be regarded as a control signal which regulates the power assigned to each user at the base station.

To effectively assign base station's transmission power among users during the process of adaptation, all users adapt their signature sequences under a common power constraint, that is:

$$\sum_{k=1}^{K} tr(\mathbf{C}_k\mathbf{C}_k^H) \leq P \tag{6.6}$$

where P is the total transmission power available at the base station. Therefore, the joint strategy space of the considered non-cooperative game $\mathcal{Q} = \mathcal{Q}_1 \times \cdots \times \mathcal{Q}_K$ can be expressed as

$$\mathcal{Q} = \left\{ (\mathbf{C}_1, \cdots, \mathbf{C}_K) : \sum_{k=1}^{K} tr(\mathbf{C}_k\mathbf{C}_k^H) \leq P \right\} \tag{6.7}$$

And user k's strategy space can be expressed as

$$\mathcal{Q}_k = \{\mathbf{C}_k : tr(\mathbf{C}_k\mathbf{C}_k^H) \leq P_{k_R}\} \tag{6.8}$$

where $P_{k_R} = P - \sum_{l \neq k} tr(\mathbf{C}_l\mathbf{C}_l^H)$. It is clear that users' individual strategy spaces $\mathcal{Q}_k$'s are interdependent.

Note that, to apply the distributed adaptation with pricing mechanism in a realistic communication system, each user in the system should know the unit price c in advance. Meanwhile, due to the interdependency of users' strategy spaces, only one user can adapt its signature sequences at any given time. That is, users should adapt their signature sequences in a sequential manner. It is easy to notice that users' strategy spaces will vary during the process of sequential signature sequence adaption. Therefore, each user should be able to identify its strategy space before its signature sequences are being adapted.

By temporarily assuming each user has full knowledge of unit price c and its own strategy space, the next section will focuse on how an individual user adapts its signature sequences under pricing mechanism. In Section 6.3, an algorithm which is suitable for adapting users' signature sequences under pricing mechanism will be developed for realistic communication scenarios. With the proposed algorithm, each user will be able to know the unit price c and determine its strategy space during the process of adaptation.

6.2 Individual User's Optimal Signature Sequence with Pricing

With assumption the unit price c and strategy space $\mathcal{Q}_k$ are both available for user k, the adaptation of user k's signature sequence under pricing mechanism can be formulated into following optimization problem:

$$\min_{\mathbf{C}_k} tr\left((\mathbf{I}_{M_k} + \mathbf{C}_k^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{C}_k)^{-1}\right) + c \cdot tr(\mathbf{C}_k \mathbf{C}_k^H) \tag{6.9}$$
$$\text{s.t. } tr\left(\mathbf{C}_k \mathbf{C}_k^H\right) \leq P_{k_R}$$

where P_{k_R} is user k's power limit which characterizes user k's strategy space $\mathcal{Q}_k$.

Let matrix $\mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k$ be eigenvalue decomposed as in the last chapter, such that

$$\mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k = \bar{\mathbf{U}}_k \mathbf{\Lambda}_k \bar{\mathbf{U}}_k^H$$

where $\mathbf{\Lambda}_k = diag(\lambda_{k_1}, \cdots, \lambda_{k_N})$ is a diagonal matrix composed with eigenvalues arranged in descending order; and $\bar{\mathbf{U}}_k$ is a unitary matrix composed with the corresponding eigenvectors.

By defining $\mathbf{B}_k \triangleq \bar{\mathbf{U}}_k^H \mathbf{C}_k$ and applying matrix inverse lemma, the objective function of (6.9) which user k tries to optimize can be expressed as a function of matrix $\mathbf{B}_k$, such as

$$M_k - N + tr\left(\left[\mathbf{I}_N + \mathbf{\Lambda}_k^{\frac{1}{2}} \mathbf{B}_k \mathbf{B}_k^H \mathbf{\Lambda}_k^{\frac{1}{2}}\right]^{-1}\right) + c \cdot tr(\mathbf{B}_k \mathbf{B}_k^H) \tag{6.10}$$

Clearly, searching $\mathbf{C}_k$ which minimizes the objective function of (6.9) is equivalent to searching $\mathbf{B}_k$ which minimizes the following objective function

$$\mathcal{E}_k^{\text{price}}(\mathbf{B}_k) = tr\left(\left[\mathbf{I}_N + \mathbf{\Lambda}_k^{\frac{1}{2}}\mathbf{B}_k\mathbf{B}_k^H\mathbf{\Lambda}_k^{\frac{1}{2}}\right]^{-1}\right) + c \cdot tr(\mathbf{B}_k\mathbf{B}_k^H) \tag{6.11}$$

According to Lemma 5.1, it is easy to find out that $\mathcal{E}_k^{\text{price}}(\mathbf{B}_k)$ is minimized if and only if $\mathbf{B}_k\mathbf{B}_k^H$ is diagonal. Then, the optimal $\mathbf{B}_k^{\text{price}}$ should satisfy

$$\mathbf{B}_k^{\text{price}}\mathbf{B}_k^{\text{price}H} = diag(\lambda_{k_1}^{\text{price}} \cdots \lambda_{k_N}^{\text{price}}) \tag{6.12}$$

and the corresponding objective function $\mathcal{E}_k^{\text{price}}(\mathbf{B}_k^{\text{price}})$ can be simplified as a function of $\left\{\lambda_{k_i}^{\text{price}}\right\}_{i=1}^N$:

$$\mathcal{E}_k^{\text{price}}(\mathbf{B}_k^{\text{price}}) = \sum_{i=1}^{N}\left[\frac{1}{1+\lambda_{k_i}\lambda_{k_i}^{\text{price}}} + \lambda_{k_i}^{\text{price}}\right] \tag{6.13}$$

From its definition, it is clear that matrix $\mathbf{B}_k$ is of size $N \times M_k$ and has rank no larger than M_k. Hence, there exist at most M_k non-zero elements in $\left\{\lambda_{k_i}^{\text{price}}\right\}_{i=1}^N$. By (6.13), it is easy to deduced that $\mathcal{E}_k^{\text{price}}(\mathbf{B}_k^{\text{price}})$ is minimized if and only if the first M_k elements of $\left\{\lambda_{k_i}^{\text{price}}\right\}_{i=1}^N$ is non-zero. Then, the optimal $\mathbf{B}_k^{\text{price}}$ can be expressed as

$$\mathbf{B}_k^{\text{price}} = \begin{bmatrix} \sqrt{\mathbf{\Lambda}_k^{\text{price}}}\mathbf{W}_k^H \\ \mathbf{0} \end{bmatrix} \tag{6.14}$$

where $\mathbf{\Lambda}_k^{\text{price}} \triangleq diag(\lambda_{k_1}^{\text{price}}, \cdots, \lambda_{k_{M_k}}^{\text{price}})$, and $\mathbf{W}_k$ is an arbitrary $M_k \times M_k$ unitary matrix.

The next step is to find the optimal set of $\left\{\lambda_{k_i}^{\text{price}}\right\}_{i=1}^{M_k}$ which minimizes function

$$f_k^{\text{price}}(\lambda_{k_1}^{\text{price}}, \cdots, \lambda_{k_{M_k}}^{\text{price}}) = \sum_{i=1}^{M_k}\left[\frac{1}{1+\lambda_{k_i}\lambda_{k_i}^{\text{price}}} + \lambda_{k_i}^{\text{price}}\right] \tag{6.15}$$

subject to $\sum_{i=1}^{M_k} \lambda_{k_i}^{\text{price}} \leq P_{k_R}$ and $\lambda_{k_i} \geq 0$ for $i = 1, \cdots, M_k$. Such an optimization problem can be formulated as

$$\begin{aligned} \min_{\left\{\lambda_{k_i}^{\text{price}}\right\}_{i=1}^{M_k}} \quad & f_k^{\text{price}}\left(\lambda_{k_1}^{\text{price}}, \cdots, \lambda_{k_{M_k}}^{\text{price}}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{M_k} \lambda_{k_i}^{\text{price}} \leq P_{k_R} \\ & \lambda_{k_i}^{\text{price}} \geq 0 \quad \forall i \in [1, M_k] \end{aligned} \tag{6.16}$$

To solve the optimization problem given by (6.16), a Lagrangian is constructed

$$L = \sum_{i=1}^{M_k} \left[\frac{1}{1 + \lambda_{k_i} \lambda_{k_i}^{\text{price}}} + c \cdot \lambda_{k_i}^{\text{price}} \right] + \delta \left(\sum_{i=1}^{M_k} \lambda_{k_i}^{\text{price}} - P_{k_R} \right) - \sum_{i=1}^{M_k} \alpha_i \lambda_{k_i}^{\text{price}} \tag{6.17}$$

where δ is a Lagrangian multiplier corresponding to user k's power constraint; α_i with $i = 1, \cdots, M_k$ are Lagrangian multipliers corresponding to the non-negative constraints on $\left\{\lambda_{k_i}^{\text{price}}\right\}_{i=1}^{M_k}$.

The KKT optimality conditions for the considered optimization problem can then be expressed as [59]

$$\frac{\partial L}{\partial \lambda_{k_i}^{\text{price}}} = 0 \quad \text{for } i = 1, \cdots, M_k \tag{6.18a}$$

$$\sum_{i=1}^{M_k} \lambda_{k_i}^{\text{price}} - P_{k_R} \leq 0 \tag{6.18b}$$

$$\delta \geq 0 \tag{6.18c}$$

$$\delta \left(\sum_{i=1}^{M_k} \lambda_{k_i}^{\text{price}} - P_{k_R} \right) = 0 \tag{6.18d}$$

$$-\lambda_{k_i}^{\text{price}} \leq 0 \quad \text{for } i = 1, \cdots, M_k \tag{6.18e}$$

$$\alpha_i \geq 0 \quad \text{for } i = 1, \cdots, M_k \tag{6.18f}$$

$$\alpha_i \lambda_{k_i}^{\text{price}} = 0 \quad \text{for } i = 1, \cdots, M_k \tag{6.18g}$$

Note that (6.16) is a convex optimization problem, because its objective function given in (6.15) is convex for $\left\{\lambda_{k_i}^{\text{price}}\right\}_{i=1}^{M_k}$ and its constraint functions are all linear.

Thus, the KKT optimality conditions shown above are the necessary and sufficient conditions for the optimum $\left\{\lambda_{k_i}^{\text{price}}\right\}_{i=1}^{M_k}$ [59].

From (6.18a), it is clear that the optimal $\lambda_{k_i}^{\text{price}}$ (with $i \in \{1, M_k\}$) must satisfy

$$-\frac{\lambda_{k_i}}{\left(1+\lambda_{k_i}\lambda_{k_i}^{\text{price}}\right)^2}+c+\delta-\alpha_i=0 \tag{6.19}$$

According to the complementary slackness condition given by (6.18g), the Lagrangian multiplier α_i equals 0 if the optimal $\lambda_{k_i}^{\text{price}}$ takes a positive value. Hence, the optimal non-zero $\lambda_{k_i}^{\text{price}}$ can be obtained by solving

$$\frac{\lambda_{k_i}}{\left(1+\lambda_{k_i}\lambda_{k_i}^{\text{price}}\right)^2}=c+\delta \tag{6.20}$$

Based upon the complementary slackness condition (6.18d), one can find that δ equals 0 if the total transmission power consumed by user k is less than P_{k_R}. In this case, the optimal $\lambda_{k_i}^{\text{price}}$ can be obtained as

$$\lambda_{k_i}^{\text{price}}=\left(\frac{1}{\sqrt{c\lambda_{k_i}}}-\frac{1}{\lambda_{k_i}}\right)^+ \tag{6.21}$$

However, the complementary slackness condition (6.18d) also implies that, if Lagrangian multiplier δ takes a positive value, then the power consumed by user k equals to P_{k_R}. In this case, the optimal $\lambda_{k_i}^{\text{price}}$ is expressed as

$$\lambda_{k_i}^{\text{price}}=\left(\frac{1}{\sqrt{(c+\delta)\lambda_{k_i}}}-\frac{1}{\lambda_{k_i}}\right)^+ \tag{6.22}$$

where δ is a positive value such that $\sum_{i=1}^{M_k}\lambda_{k_i}^{\text{price}}=P_{k_R}$. Since the optimal $\lambda_{k_i}^{\text{price}}$ is a decreasing function of Lagrangian multiplier δ, the positive δ which satisfies $\sum_{i=1}^{M_k}\lambda_{k_i}^{\text{price}}=P_{k_R}$ is unique.

Then, the optimal $\mathbf{C}_k^{\text{price}}$ which minimizes (6.5) under unit price c and power limit P_{k_R} is obtained as

$$\mathbf{C}_k^{\text{price}}=\bar{\mathbf{U}}_k\mathbf{B}_k^{\text{price}}=\mathbf{U}_k\sqrt{\mathbf{\Lambda}_k^{\text{price}}}\mathbf{W}_k^H \tag{6.23}$$

where $\mathbf{U}_k$ is composed with eigenvectors corresponding to the M_k largest eigenvalues of matrix $\mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k$, $\mathbf{W}_k$ is an arbitrary $M_k \times M_k$ unitary matrix, and $\mathbf{\Lambda}_k^{\text{price}}$ is a M_k dimentional diagonal matrix with its ith diagonal element given as

$$\lambda_{k_i}^{\text{price}} = \left(\frac{1}{\sqrt{(c+\delta)\lambda_{k_i}}} - \frac{1}{\lambda_{k_i}} \right)^+$$

in which δ is the minimum non-negative value which statisfies $\sum_{i=1}^{M_k} \lambda_{k_i}^{\text{price}} \leq P_{k_R}$.

6.3 Adaptation Algorithm with Pricing

Mathematically, the adaptation of user k's signature sequence can be characterized by the relation [61]

$$r_k^{\text{price}}\left(\mathbf{C}_{-k}\right) = \mathbf{C}_k^{\text{price}} \tag{6.24}$$

which maps each $\mathbf{C}_{-k}$ to user k's optimal signature sequences $\mathbf{C}_k^{\text{price}}$.

Moreover, define the mapping $r^{\text{price}}(\cdot)$ as the Cartesian product of $r_k^{\text{price}}(\cdot)$ for all $k \in [1, K]$ such that

$$r^{\text{price}}\left(\mathbf{C}\right) = r_1^{\text{price}}\left(\mathbf{C}_{-1}\right) \times \cdots \times r_K^{\text{price}}\left(\mathbf{C}_{-K}\right) \tag{6.25}$$

where $\mathbf{C} = (\mathbf{C}_1, \cdots, \mathbf{C}_K)$ is a K tuple which represents the signature sequences chosen by all users in the system. It is clear that $r^{\text{price}}(\cdot)$ reflects how users' signature sequences are adapted from the system point of view.

According to its definition, the mapping $r^{\text{price}}(\cdot)$ has following properties:

1. $r^{\text{price}}(\cdot)$ is non-empty and convex for any given $\mathbf{C}$;
2. $r^{\text{price}}(\cdot)$ has a close graph;
3. the set of all possible $\mathbf{C}$ is compact, non-empty and convex.

Then by Kakutani's fixed point theorem, it can be proven that there always exist a fixed point such that, for $k = 1, \cdots, K$, $\mathbf{C}_k^* = r_k\left(\mathbf{C}_{-k}^*\right)$. That means, even though each user's signature sequences are updated independently, there always exists an equilibrium at which no user can further improve its performance objective by unilaterally updating its signature sequences, given the signature sequences selected by other users in the system.

As argued before, due to the interdependency of users' strategy spaces, only one user is allowed to adapt its signature sequences at any given time. In other words, by applying pricing mechanism, users should adapt their signature sequences in a sequential manner.

Following is a totally asynchronous adaptation algorithm which is suitable for applying pricing mechanism in real-life communication scenario.

Algorithm 6.1 *Suppose, in the system, the total transmission power at base station is P and unit price applied for each user is c.*

The base station consistently broadcasts unit price c and the remaining transmission power at base station $P_{0_R} = P - \sum_{k=1}^{K} tr\left(\mathbf{C}_k \mathbf{C}_k^H\right)$ with $\mathbf{C}_k$ as the current signature sequences of user k.

At the time when mobile k is to adapts user k's signature sequences:

1. *Calculate the transmission power constraint*

$$P_{k_R} = P_{0_R} + tr\left(\mathbf{C}_k \mathbf{C}_k^H\right) = P - \sum_{l \neq k} tr\left(\mathbf{C}_l \mathbf{C}_l^H\right)$$

2. *Apply (6.23) to obtain the optimal signature sequence $\mathbf{C}_k^{\text{price}}$;*

3. *Feed $\mathbf{C}_k^{\text{price}}$ back to the base station, and let the base station use the feedback signature sequence for spreading user k's data symbols;*

4. *Initialize a timer with a randomly chosen back-off period. Only after the timer expires, can user k adapt its signature sequences again.*

From Step 4 of Algorithm 6.1, it is clear that, for any user, the interval between its two consecutive iterations of signature sequence adaptation is randomly selected. Then, it is unlikely that two or more than two users will adapt their signature sequences simultaneously. In other words, at any given time instance, only one user can have its signature sequences adapted. Therefore users' signature sequences are adapted in a sequential manner.

6.4 Simulation Result

This section presents the system performance that can be achieved with the proposed adaptation scheme. The simulation is carried out under WLAN channel model. A 2-user system is considered, where the total number of signature sequences engaged in the system is assumed to be $M = 48$; and the total transmission power at the base station is set as $P = 48$. The channel for each user is independent Rayleigh frequency selective channel with exponential multipath delay profile, whose r.m.s. delay spread is set to 25ns. The path losses of these two users are set such that $g_1/\sigma^2 = 30$dB and $g_2/\sigma^2 = 5$dB.

Figure 6.1 presents the TMSE performance that can be achieved by the proposed distributed scheme which employs pricing mechanism. Three signature sequence assignments are considered in the simulation, which are characterized with different pairs of M_1 and M_2 as shown in Figure 6.1. For comparison purpose, Figure 6.1 also presents the TMSE performance that can be achieved by the centralized adaptation scheme proposed in Chapter 4 and the distributed adaptation scheme proposed in Chapter 5.

For the centralized adaptation scheme considered in the simulation, users' signature sequences are jointly adapted to minimize the TMSE in the system. For the distributed adaptation scheme proposed in Chapter 5, proportional power assignment is assumed in the simulation, that is the power assigned to each user is proportional

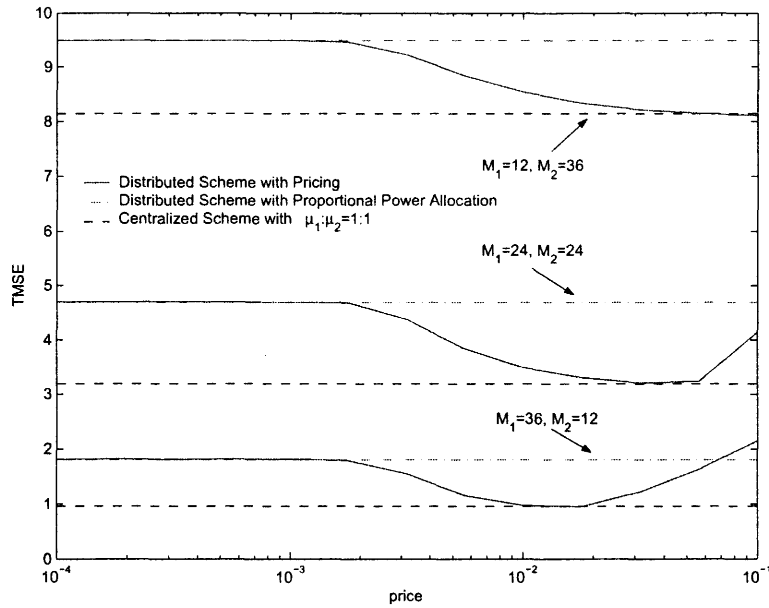


Figure 6.1 TMSE performance achieved by the distributed signature sequence adaptation scheme with pricing.

to the number of signature sequence to that user, that is, for $k = 1, 2$, $P_k = \frac{M_k}{M} P$. The same proportional signature sequence assignment is also used in the simulation as the initial power assignment for the proposed distributed adaptation scheme which employs pricing mechanism. As a remark, the TMSE performance achieved with the centralized adaptation scheme is the optimal TMSE performance in the system.

From Figure 6.1, it is obvious that pricing mechanism can effectively improve the system performance, even though only a linear pricing function was used. It is observed, with a properly chosen unit price c, the distributed adaption scheme with pricing mechanism can approach the optimal TMSE performance in the system. It is also noticed that the unit price which achieves the optimal TMSE performance varies for different signature sequence assignments. As shown in the figure, significant performance improvement can be obtained for the considered signature sequence assignments when the unit price chosen is within the range between 0.01 and 0.02.

6.5 Chapter Summary

In this chapter, a distributed scheme is proposed to adapt users' signature sequences for the downlink of MC-CDMA system. The pricing methodology has been adopted in the proposed distributed adaptation scheme, such that each user's objective contains a pricing function in addition to a performance measurement matric. The purpose of introducing pricing functions is to bring certain level of cooperation to the users, so that the transmission power at the base station can be properly distributed among users during the process of adaptation, which in turn will improve the system performance. It is shown that a linear pricing function with a properly selected unit price can effectively improve the efficiency of the distributed signature sequence adaptation scheme.

CHAPTER 7

CONCLUSIONS

This dissertation addresses two topics for MC-CDMA system. One is the rate capacity that can be achieved. The other is the adaptation of users' signature sequences. Both topics are studied for the downlink scenario.

Rate Capacity

For the rate capacity in the downlink of MC-CDMA system, the objective is to understand the potential of applying MC-CDMA technique for high speed wireless data communications. Various combinations of coding strategies and resource allocation policies were considered. The system rate capacity under typical fading channel condition is investigated through a simulation study. It is shown that, to maintain high speed data transmission under multi-code scheme, each mobile's should cooperatively decode its desired users' encoded data symbols which are spread with distinct signature sequences. Further improvement in rate capacity can be obtained by applying dirty paper coding (DPC) to cooperatively encode all users' data symbols at the base station. However, the complexity of implementing DPC is prohibitively high. It is also found that the system capacity region heavily depends on the resource allocation policy adopted in the system. When the disparity of users' channel qualities is small, one may proportionally assign power and signature sequences among users without suffering significant penalty in the resultant system capacity region. However, when the difference of users' channel qualities is large, one may resort to non-proportional assignment of power and signature sequences.

Signature Sequence Adaptation

Both centralized and distributed schemes are developed for adapting users' signature sequences in the downlink of MC-CDMA system.

With *the centralized adaption scheme*, the base station collects complete channel state information of the system and iteratively adapts all users' signature sequences to optimize the weighted total mean square error (WTMSE). During the process, each iteration of signature sequence adaptation decreases the WTMSE which is bounded from below. Therefore, the convergence of the proposed centralized adaption scheme is guaranteed.

For *the distributed signature sequence adaptation*, each user adapts its signature sequences to optimize its individual performance objective function, with no regard to the performance of other users in the system. The adaptation of one user's signature sequences is based upon that user's channel state information only. Two distributed adaptation schemes are developed. In one scheme, each user adapts its signature sequence under a pre-determined power constrain which remains unchanged during the process of adaptation. In the other scheme, pricing methodology is employed such that the transmission power at the base station can be properly distributed among users when users' signature sequences are adapted. The stability of these distributed adaptation schemes is analyzed with game theory frame work. It is shown that there always exists a set of signature sequences at which no user can unilaterally adapt its signature sequences to improve its individual performance, given the signature sequences chosen by other users in the system.

Thus, in addition to apply MUD at the receiver side, both centralized and distributed adaption of users' signature sequences can help improve the performance in the downlink of MC-CDMA system. The centralized adaptation scheme is more efficient in improving system performance, while the distributed adaptation schemes are much easier to be implemented in practical communication scenarios. It is also found that pricing is a good mechanism to increase the efficiency of the distributed signature sequence adaptation.

APPENDIX

BRIEF DESCRIPTION OF MAJORIZATION

This Appendix summarizes some basic notions and results about theory of majorization which are utilized in Chapter 5.

Definition A.1 *For any vector* $\mathbf{x} \in \mathbb{R}^n$, *let*

$$x_{[1]} \geq \cdots \geq x_{[n]}$$

denote the components of $\mathbf{x}$ *in decreasing order, or called the* order statistics *of* $\mathbf{x}$.

Definition A.2 *For* $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, *say that* $\mathbf{x}$ *is majorized by* $\mathbf{y}$ *(or* $\mathbf{y}$ *majorizes* $\mathbf{x}$*) if*

$$\begin{aligned} \sum_{i=1}^{k} x_{[i]} &\leq \sum_{i=1}^{k} y_{[i]} \quad k = 1, \cdots, n-1 \\ \sum_{i=1}^{n} x_{[i]} &= \sum_{i=1}^{n} y_{[i]} \end{aligned}$$

Definition A.3 ([66], 3.A.1) *A real valued function* $\phi : \mathbb{R}^n \to \mathbb{R}$ *is said to be* Schur convex *if* $\phi(\mathbf{x}) \leq \phi(\mathbf{y})$ *for all* $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ *such that* $\mathbf{y}$ *majorizes* $\mathbf{x}$. *If, in addition,* $\phi(\mathbf{x}) < \phi(\mathbf{y})$ *whenever* $\mathbf{y}$ *majorizes* $\mathbf{x}$ *but* $\mathbf{x}$ *is not a permutation of* $\mathbf{y}$, *then* ϕ *is said to be* strictly Schur convex.

Lemma A.1 ([66], p.7) *Let* $\mathbf{x} \in \mathbb{R}^n$ *and* $\mathbf{1} \in \mathbb{R}^n$ *denote the constant vector with* $1_j \triangleq \sum_{j=1}^{n} x_j / n$. *Then,* $\mathbf{1}$ *is majorized by* $\mathbf{x}$.

Lemma A.2 ([66], 3.C.1) *If function* $g : \mathbb{R} \to \mathbb{R}$ *is convex, then* $\phi(x_1, \cdots, x_n) = \sum_{i=1}^{n} g(x_i)$ *is Schur convex. Consequently,* $\mathbf{y}$ *majorizes* $\mathbf{x}$ *implies* $\phi(\mathbf{x}) \leq \phi(\mathbf{y})$. *If, in addition that* $g : \mathbb{R} \to \mathbb{R}$ *is strictly convex, then* ϕ *is strictly convex.*

Lemma A.3 ([66], 9.B.1) *Let matrix* $\mathbf{R}$ *be a* $n \times n$ *Hermitian matrix with diagonal elements* $(d_1, \cdots, d_n)$ *and eigenvalues* $(\lambda_1, \cdots, \lambda_n)$, *then* $(\lambda_1, \cdots, \lambda_n)$ majorizes $(d_1, \cdots, d_n)$

Lemma A.4 ([66], 9.B.2) *For any* $\mathbf{x} \in \mathbb{R}^n$, *there exists a real symmetric (and therefore Hermitian matrix) with equal diagonal elements and eigenvalues given by* $\mathbf{x}$.

REFERENCES

[1] N. Yee, J. Linnartz, and G. Fettweis, "Multi-carrier CDMA indoor wireless radio networks," in *Proc. IEEE PIMRC*, Yokohama, Japan, Sept. 1993, pp. 109–113.

[2] K. Fazel and G. Fettweis, *Multicarrier Spreading-Spectrum.* Kluwer Academic Publishers, 1997.

[3] S. Hara and R. Prasad, *Multicarrier Techniques for 4th Generation Communications.* Artech House, 2003.

[4] M. Tan, P. Zong, and Y. Bar-Ness, "Multi-rate access schemes for MC-CDMA," *Wireless Personal Communications*, vol. 27, no. 2, pp. 149–182, Nov. 2003.

[5] C. Ibars and Y. Bar-Ness, "Comparing the performance of coded multiuser OFDM and coded MC-CDMA over fading channels," in *Proc. IEEE GLOBECOM*, San Antonio, TX, Nov. 2001.

[6] T. Cover and J. Thomas, *Elements of Information Theory.* John Wiley, 1991.

[7] W. Yu, "Competition and cooperation in multi-user communication enviroments," Ph.D. dissertation, Stanford University, Department of Electrical and Computer Engineering, June 2002.

[8] W. Jacks, *Microwave Mobile Communications.* New York, Wiley, 1974.

[9] T. Rappaport, *Wireless Communications Principles and Practice.* Englewood Cliffs, NJ: Prentice Hall, 1996.

[10] P. Bello, "Characterization of randomly time-variant linear channels," *IEEE Transactions on Communications*, vol. 11, no. 4, pp. 360–393, Dec. 1963.

[11] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Communications Magazine*, vol. 35, no. 12, pp. 126–133, Dec. 1997.

[12] T. Cover, "Comments on broadcast channels," *IEEE Transactions on Information Theory*, vol. 44, no. 6, pp. 2524–2530, Oct. 1998.

[13] ——, "Broadcast channels," *IEEE Transactions on Information Theory*, vol. IT-18, no. 1, pp. 2–14, Jan. 1972.

[14] P. Bergmans, "A simple converse for broadcast channels with additive white Gaussian noise," *IEEE Transactions on Information Theory*, vol. IT-20, no. 2, pp. 279–280, Mar. 1974.

[15] D. Hughes-Hartogs, "The capacity of a degraded spectral Gaussian broadcast channel," Ph.D. dissertation, Stanford University, Department of Electrical and Computer Engineering, July 1975.

[16] D. Tse, "Optimal power allocation over parallel Gaussian broadcast channels," in *Proc. IEEE ISIT*, 1997, p. 27.

[17] ——, "Optimal power allocation over parallel Gaussian broadcast channels," http://www.eecs.berkeley.edu/~dtse/broadcast2.pdf.

[18] L. Li and A. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels – Part I: ergodic capacity," *IEEE Transactions on Information Theory*, vol. 47, no. 3, pp. 1083–1102, Mar. 2001.

[19] ——, "Capacity and optimal resource allocation for fading broadcast channels – Part II: outage capacity," *IEEE Transactions on Information Theory*, vol. 47, no. 3, pp. 1103–1127, Mar. 2001.

[20] C. Ibars and Y. Bar-Ness, "Outage capacities of a multi-carrier WLAN downlink under different resource sharing techniques," in *Proceeding of ISSSTA 2002*, Prague, Czech Republic, Sept. 2002, pp. 144–149.

[21] P. Rapajic and B. Vucetic, "Adaptive receiver structures for asynchronous CDMA system," *IEEE Journal on Selected Areas in Communications*, vol. 12, no. 4, pp. 685–697, May 1994.

[22] U. Madhow and M. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Transactions on Communications*, vol. 42, no. 12, pp. 3178–3188, Dec. 1994.

[23] H. Poor and S. Verdú, "Probability of error in MMSE multiuser detection," *IEEE Transactions on Information Theory*, vol. 43, no. 3, pp. 858–871, May 1997.

[24] M. Honig, U. Madhow, and S. Verdú, "Blind adaptive multiuser detection," *IEEE Transactions on Information Theory*, vol. 41, no. 4, pp. 944–960, July 1995.

[25] M. Honig and M. Tsatsanis, "Adaptive techinque for multiuser CDMA," *IEEE Signal Processing Mag.*, vol. 17, no. 3, pp. 49–61, May 2000.

[26] L. Scharf, *Statistical Signal Processing: Detection, Estimation and Time Seriers Analysis.* Addison-Wesley, 1990.

[27] R. Horn and C. Johnson, *Matrix Analysis.* Combridge University Press, 1990.

[28] G. Golub and C. V. Loan, *Matrix Computation.* The Johns Hopkins University Press, 1996.

[29] J. Cioffi and G. Forney, "Generalized decision-feedback equalization for packet transmission with ISI and Gaussian noise," in *Communication, Computation, Control and Signal Processing*, A. Paulraj, V. Roychowdhury, and C. Schaper, Eds. Kluwer Academic Publishers, 1997.

[30] W. Yu and J. Cioffi, "Multiuser detection in vector multiple access channels using generalized decision feedback equalization," in *5th International Conference on Signal Processing, World Computer Congress*, vol. 3, Beijing, China, Aug. 2000, pp. 1771–1777.

[31] J. Cioffi, G. Dudevoir, M. Eyuboglu, and G. Forney, "MMSE decision-feedback equalizers and coding – Part II: coding results," *IEEE Transactions on Communications*, vol. 43, no. 10, pp. 2595–2604, Oct. 1995.

[32] ——, "MMSE decision-feedback equalizers and coding – Part I: equalization results," *IEEE Transactions on Communications*, vol. 43, no. 10, pp. 2582–2594, Oct. 1995.

[33] M. Costa, "Writing on dirty paper," *IEEE Transactions on Information Theory*, vol. 29, no. 3, pp. 439–441, May 1983.

[34] D. Slepian and J. K. Wolf, "Noiseless coding of correlated information sources," *IEEE Transactions on Information Theory*, vol. 19, no. 4, pp. 471–480, July 1973.

[35] G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels," *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1468–1489, July 1999.

[36] IEEE Std 802.11a-1999, "Wireless LAN medium access control (MAC) and Physical layer (PHY) specifications: High-speed physcial layer in the 5GHz band."

[37] ETSI, "Broadband radio access networks (BRAN); HIPERLAN type 2; physical (PHY) layer," Feb. 2001.

[38] P. Rapajic and B. Vucetic, "Linear adaptive transmitter-receiver structures for asynchronous CDMA systems," *European Trans. Telecommun.*, vol. 6, no. 1, pp. 21–27, Jan.-Feb. 1995.

[39] M. Rupf and J. Massey, "Optimum sequence multisets for synchronous code-division multiple-access channels," *IEEE Transactions on Information Theory*, vol. 40, no. 4, pp. 1261–1266, July 1994.

[40] P. Viswanath and V. Ananthara, "Optimal sequences and sum capacity of synchronous CDMA systems," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 1984–1991, Sept. 1999.

[41] P. Viswanath, V. Ananthara, and D. Tse, "Optimal sequences, power control, and user capacity of synchronouse CDMA systems with linear MMSE multiuser receivers," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 1968–1983, Sept. 1999.

[42] T. Guess, "Optimal sequences for CDMA with decision-feedback receivers," *IEEE Transactions on Information Theory*, vol. 49, no. 4, pp. 886–900, Apr. 2003.

[43] S. Ulukus and R. Yates, "User capacity of asynchronous CDMA systems with matched filter receivers and optimum signature," *IEEE Transactions on Information Theory*, vol. 50, no. 5, pp. 903–909, May 2004.

[44] ——, "Iterative construction of optimum signature sequence sets in synchronous CDMA systems," *IEEE Transactions on Information Theory*, vol. 47, no. 5, pp. 1989–1998, July 2001.

[45] S. Ulukus and A. Yener, "Iterative joint optimization of CDMA signature sequences and receiver filters," in *Proc. CISS*, Princetion, NJ, Mar. 2002.

[46] T. Wong and T. Lok, "Transmitter adaptation in multicode DS-CDMA systems," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 1, pp. 69–82, Jan. 2001.

[47] C. Sung and K. Leung, "Distributed seqeunce adaptation for capacity maximization of DS-CDMA systems," in *Proc. IEEE ICC*, vol. 3, 2001, pp. 796–800.

[48] G. Rajappan and M. Honig, "Signature sequence adaptation for DS-CDMA with multipath," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 2, pp. 384–395, Feb. 2002.

[49] J. Concha and S. Ulukus, "Optimization of CDMA signature sequences in multipath channels," in *Proc. IEEE VTC*, vol. 3, Rhodes, Greece, May 2001, pp. 1978–1982.

[50] F. Horlin and L. Vandendorpe, "CA-CDMA: channel-adpated CDMA for MAI/ISI-free burst transmission," *IEEE Transactions on Communications*, vol. 51, no. 2, pp. 275–283, Feb. 2003.

[51] C. Sung, K. Shum, and K. Leung, "Multi-objective power control and signature sequence adaptation for synchronouse CDMA system – a game theoretic viewpoint," in *Proc. IEEE ISIT*, 2003, p. 335.

[52] C. Sung and K. Leung, "On the stability of distributed sequence adaptation for cellular asynchronous DS-CDMA systems," *IEEE Transactions on Information Theory*, vol. 49, no. 7, pp. 1828–1831, July 2003.

[53] O. Popescu, "Interference avoidance for wireless systems with multiple receivers," Ph.D. dissertation, Rutgers, The State University of New Jersey, Department of Electrical and Computer Engineering, May 2004.

[54] M. Brandt-Pearce and A. Dharap, "Transmitter-based multiuser interference rejection for the down-link of a wireless CDMA system in a multipath enviroment," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, pp. 407–417, Mar. 2000.

[55] L. Gao and T. Wong, "Power control and spreading sequence allocation in a CDMA forward link," *IEEE Transactions on Information Theory*, vol. 50, no. 1, pp. 105–124, Jan. 2004.

[56] L. Gao and T. Wang, "Joint transmitter-receiver adaptation over DS-CDMA forward link," in *Proc. IEEE GLOBECOM*, San Antonio, Texas, Nov. 2001, pp. 1297–1301.

[57] ——, "Iterative construction of spreading sequences in CDMA forward link," in *Proc. IEEE VTC*, Vancouver, BC, Canada, Sept. 2002, pp. 515–519.

[58] D. Bertsekas, *Nonlinear Programming.* Athena Scientific, 1999.

[59] S. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge University Press, 2004.

[60] J. G. Proakis, *Digital Communications.* McGraw-Hill, Inc., 1995.

[61] D. Fudenberg and J. Tirole, *Game Theory.* The MIT Press, 1991.

[62] G. Debreu, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium.* New York: Wiley, 1959.

[63] C. Saraydar, N. Mandayam, and D. Goodman, "Efficient power control via pricing in wireless data neworks," *IEEE Transactions on Communications*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[64] C. Saraydar, "Pricing and power control in wireless data networks," Ph.D. dissertation, Rutgers, The State University of New Jersey, Department of Electrical and Computer Engineering, Jan. 2001.

[65] C. Sung and W. Wang, "A noncooperative power control game for multirate CDMA data networks," *IEEE Transactions on Wireless Communications*, vol. 2, no. 1, pp. 186–194, Jan. 2003.

[66] A. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications.* New York: Academic, 1979.