ABSTRACT

ON PERFORMANCE ANALYSIS OF OPTIMAL DIVERSITY COMBINING WITH IMPERFECT CHANNEL ESTIMATION

by
Yong Peng

The optimal diversity combining technique is investigated for multipath Rayleigh and Ricean fading channel with additive white Gaussian noise where only imperfect channel knowledge is available at the receiver. The non-observable estimation error contributes as an additive source of noise which is not white. Therefore, the optimal combining weight is derived taking into consideration the imperfect channel knowledge.

The bit error rate for BPSK modulation over correlated Rayleigh and Ricean fading channel is derived for minimum mean square channel estimation using pilot symbol assisted modulation. Analytical result and Monte-Carlo simulation are presented for specific channel and estimation models to demonstrate the effect of diversity combining with imperfect channel estimation on error performance in comparison with the case when perfect channel knowledge is available at the receiver. The trade-off between the channel estimation accuracy and the effective bit SNR is also discussed. The Pilot-to-Data power ratio is studied for different Rice K factors for optimizing the bit error performance.

ON PERFORMANCE ANALYSIS OF OPTIMAL DIVERSITY COMBINING WITH IMPERFECT CHANNEL ESTIMATION

by
Yong Peng

A Thesis
Submitted to the Faculty of
New Jersey Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of
Master of Science in Electrical Engineering

Department of Electrical and Computer Engineering

May 2005

APPROVAL PAGE

ON PERFORMANCE ANALYSIS OF OPTIMAL DIVERSITY COMBINING WITH IMPERFECT CHANNEL ESTIMATION

Yong Peng

Dr. Roy You, Thesis Advisor | Date
Assistant Professor of Electrical and Computer Engineering, NJIT

Dr. Hongya Ge, Committee Member | Date
Associate Professor of Electrical and Computer Engineering, NJIT

Dr. Sirin Tekinay, Committee Member | Date
Associate Professor of Electrical and Computer Engineering, NJIT

BIOGRAPHICAL SKETCH

Author: Yong Peng

Degree: Master of Science

Date: May 2005

Undergraduate and Graduate Education:

- Master of Science in Electrical Engineering,
  New Jersey Institute of Technology, Newark, NJ, 2005
- Bachelor of Engineering in Communication Engineering,
  University of Electronic Science and Technology of China, Chengdu, P.R.China, 2003

Major: Electrical Engineering

Presentations and Publications:

Yong Peng and Roy You, "Diversity Combining over Ricean Fading Channel with Imperfect Channel Knowledge," *Proceedings of the 2005 Conference on Information Science and Systems*, Baltimore, p. 59, March 2005.

To my beloved family

ACKNOWLEDGMENT

I would like to express my appreciation to my thesis advisor, Dr. Roy You, who not only provided significant help to the thesis, but also served as my research advisor and supported me with valuable resources and insightful ideas. With our collaboration, we have a paper published on the proceedings of a conference and a submission to IEEE Communications Letters. Special thanks are given to Dr. Hongya Ge and Dr. Sirin Tekinay for actively participating in my committee and giving me very good recommendations for my future carreer.

Many of my fellow graduate students in the Center for Communications and Signal Processing Research are deserving of recognition for their support. They are really resourceful and lovely persons.

TABLE OF CONTENTS

TABLE OF CONTENTS
(Continued)

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Diversity combining has often been used to combat the deleterious effect of channel fading and to provide significant improvement in system bit error performance [1, 2]. When the channel state information (CSI) is known perfectly at the receiver, this knowledge can be used in various optimal and sub-optimal combining schemes, such as maximal ratio combining (MRC) and equal gain combining (EGC). However, in practical cases, usually only imperfect CSI is available at the receiver, in which case the channel gain must be estimated and the estimation error will degrade system bit error performance.

1.1 Background Information

In [3], the effects of imperfect channel estimation on bit error rate (BER) performance for correlated fading channels are addressed and optimal combining scheme is derived. It worths pointing out that diversity combining with imperfect channel estimation for uncorrelated diversity branches [4] or diversity combining for correlated branches but with perfect channel knowledge [5] are special cases of the framework in [3]. In this thesis, the impact of imperfect channel knowledge is studied. It is shown that the channel gain can be decomposed into channel estimation and channel estimation error, which can be treated as an additional source of noise aside from the noise of the channel itself. Therefore, the combining weighting coefficients must be adjusted appropriately for optimizing the system bit error performance. The work in [3] is also extended to investigate the BER performance of diversity combining of BPSK modulation with imperfect CSI over multiple correlated Ricean fading channel, where line of sight (LoS) paths exist between the transmitter and the receiver. Since pilot symbol assisted modulation (PSAM) scheme is used for channel estimation [6, 7], the pilot-to-data symbol power allocation strategy is also evaluated in order to optimize the bit error performance of the system when the total transmitted power budget is fixed.

1.2 Objective

In this thesis, a Ricean block fading channel model is assumed and the channel observation is assumed to be jointly Gaussian with the channel gain. BPSK modulation is adopted for simplicity and better demonstration of the impact of imperfect estimation, although other type of modulation techniques can be derived similarly. The optimal detection rule using the channel observation and decision variable is investigated first. The minimum mean square error (MMSE) is used for channel estimation and since there is a detection error, combining technique with MMSE should be used instead of MRC in order to optimize the system bit error performance. Furthermore, the BER are calculated analytically for Rayleigh and Ricean fading channels under the corresponding combining method. As examples of the theory put forth, error performance analysis of frequency and spatial diversity system models with pilot symbol estimation method are calculated to demonstrate the combined effect of channel estimation and diversity combining. With pilot symbols used for channel estimation in a block fading channel model, the pilot-to-data symbol power allocation strategy is also evaluated to balance the trade-off between channel estimation error and effective bit SNR to optimize the bit error performance of the system.

This thesis is organized as follows. Based on the system and channel model introduced in Chapter 2, analytical BER expressions for Ricean fading with both perfect and imperfect CSI are derived in Chapter 3. In Chapter 4, the optimal pilot-to-data power ratio for Rayleigh fading is studied. Chapter 5 presents the numerical results for different diversity channel models. Chapter 6 concludes the thesis.

CHAPTER 2

MODELS

2.1 System Model

A general single-user diversity system is shown in Figure 2.1.

The transmitted signal s is in general complex valued. It is also assumed that s is drawn from an i.i.d. source with equal symbol probability. The channel state $\mathbf{g} = [g_1, g_2, \ldots, g_N]^H$ is a complex Gaussian random vector with mean $\mathbf{u}$ and covariance matrix

$$C_{\mathbf{gg}} = E[\mathbf{g}\mathbf{g}^H] \tag{2.1}$$

where $\mathbf{g}^H$ denotes the Hermitian of $\mathbf{g}$. Thus the channel state can be denoted as $\mathbf{g} \sim \mathcal{N}(\mathbf{u}, C_{\mathbf{gg}})$. The channel noise $\mathbf{n} = [n_1\ n_2\ \ldots\ n_N]^H$ is assumed to be circularly symmetric complex white Gaussian with covariance matrix

$$\Lambda_{\mathbf{n}} = E[\mathbf{n}\mathbf{n}^H] = N_0 I \tag{2.2}$$

which can be expressed as $\mathbf{n} \sim \mathcal{CN}(0, N_0 I)$. At the receiver, a random vector $\mathbf{d}_s = [d_1\ d_2\ \ldots\ d_N]^H$ is received where

$$\mathbf{d}_s = \mathbf{g}s + \mathbf{n}. \tag{2.3}$$

In practical cases, the channel state is not perfectly known at the receiver and only an imperfect observation of the channel state is available. The channel observation vector is denoted as $\mathbf{h} = [h_1\ h_2\ \ldots\ h_N]^H$, which is also assumed to be complex Gaussian with mean $\mathbf{u}$ and covariance matrix of

$$C_{\mathbf{hh}} = E[\mathbf{h}\mathbf{h}^H]. \tag{2.4}$$

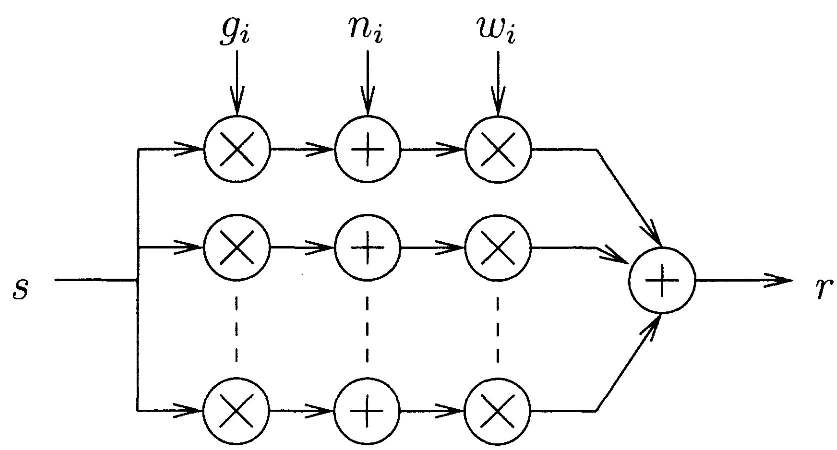

Figure 2.1 A general single user diversity combiner.

Similarly, the channel observation can be denoted as $\mathbf{h} \sim \mathcal{N}(\mathbf{u}, C_{\mathbf{hh}})$. It is also assumed that the channel observation h and channel state g are jointly Gaussian and have the cross-covariance matrix

$$C_{\mathbf{gh}} = E[\mathbf{gh}^H]. \tag{2.5}$$

Based on the channel observation $\mathbf{h}$, an estimation of the actual channel state g can be made and the combining weights can be evaluated which optimize the performance of the system. Using the combining weighting coefficients $\mathbf{w} = [w_1\ w_2\ \ldots\ w_N]^H$, the random variable r used for signal detection can be written as

$$r = \mathbf{w}^H \mathbf{d}_s = \mathbf{w}^H(\mathbf{g}s + \mathbf{n}). \tag{2.6}$$

2.2 Signal Model

Since the channel is assumed to be block fading and the channel state remains unchanged for $L = \lfloor 1/2 f_D T_s \rfloor$ symbol durations, where f_D is the Doppler frequency and T_s is the symbol duration. For each fading block, M pilot symbols are inserted for channel estimation. BPSK modulation is used with average energy E_p. Thus, the pilot symbols

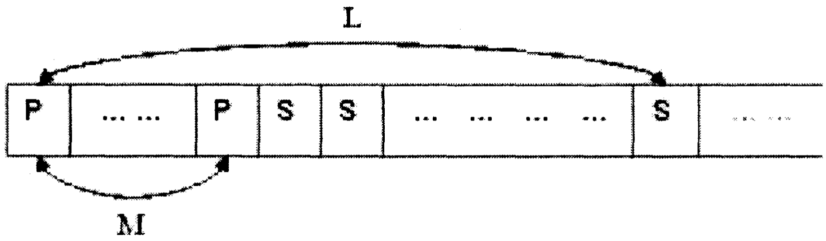

Figure 2.2 One block of the transmitted symbols.

are equally probable of $\{\sqrt{E_p}, -\sqrt{E_p}\}$. The signal symbols are with equal probability of $\{\sqrt{E_s}, -\sqrt{E_s}\}$. The SNR measures is defined as

$$\gamma_b = \frac{E_b}{N_0} \tag{2.7}$$

which is the transmit SNR per bit, where

$$E_b = \frac{(L-M)E_s + ME_p}{L-M}. \tag{2.8}$$

2.3 Channel Model

The channels which are considering here are multiple correlated Ricean fading channels. The correlation between channel gain can be over time, frequency or space or their combinations. In this thesis, block fading channels are considered, where channel gain is invariant over L symbol periods and the correlation occurs either between sub-carriers at different frequencies or between antennas at different space locations. The BER performance of the optimal diversity combining over Ricean correlated fading channel will be investigated with imperfect channel estimation under frequency diversity channel model and spatial diversity channel model.

In a frequency diversity channel model, the diversity comes from the frequency separation of the sub-carriers. The covariance matrix of the channel gain with a frequency diversity channel model is given by [8]

$$C_{\mathbf{gg}}(m,n) = \frac{\sigma_g^2}{1 + j2\pi f_d \tau_d (m-n)}. \tag{2.9}$$

τ_d is the channel delay spread, f_d is the frequency separation between two adjacent sub-carriers.

In a spatial diversity channel model, the diversity comes from the spatial separation of the adjacent antennas. It is assumed that the antennas are placed in line and spaced equally. The normalized correlation matrix for channel gain is modeled as [9]

$$C_{\mathbf{gg}}(m,n) = \sigma_g^2 J_0 \left(2\pi \frac{|m-n|\, d \cdot f_c}{C} \right), \tag{2.10}$$

where d is the distance between two adjacent antennas, f_c is the carrier frequency and C is the light speed. $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind.

2.4 Optimal Diversity Combining

To determine the optimal diversity combining weights, the pilot symbol assisted modulation (PSAM) is used for channel estimation. The received M pilot symbols are

$$\mathbf{d}_i = \mathbf{g}\sqrt{E_p} + \mathbf{n}_i, \quad i = 1, \ldots, M. \tag{2.11}$$

Then the channel observation is

$$\mathbf{h} = \frac{\sum_i \mathbf{d}_i}{M\sqrt{E_p}} = \mathbf{g} + \frac{\mathbf{n}}{\sqrt{ME_p}}. \tag{2.12}$$

Given the channel observation $\mathbf{h}$, the minimum mean square estimation (MMSE) of the channel gain $\mathbf{g}$ is given by [10]

$$\mathbf{m} = \mathbf{u} + C_{\mathbf{gh}} C_{\mathbf{hh}}^{-1} (\mathbf{h} - \mathbf{u}) \tag{2.13}$$

where $\mathbf{m} \sim \mathcal{N}(\mathbf{u}, C_{\mathbf{mm}})$ is Gaussian distributed with covariance matrix

$$C_{\mathbf{mm}} = C_{\mathbf{gh}} C_{\mathbf{hh}}^{-1} C_{\mathbf{gh}}^{H}. \tag{2.14}$$

The channel state can be expressed as the combination of the linear estimator $\mathbf{m}$ based on channel observation and a non-observable part e which corresponds to the estimation error

$$\mathbf{g} = \mathbf{m} + \mathbf{e} \tag{2.15}$$

where $\mathbf{h}$ (hence $\mathbf{m}$) and $\mathbf{e} \sim \mathcal{CN}(0, C_{\mathbf{ee}})$ are independent and

$$C_{\mathbf{ee}} = C_{\mathbf{gg}} - C_{\mathbf{gh}} C_{\mathbf{hh}}^{-1} C_{\mathbf{gh}}^{H} = C_{\mathbf{gg}} - C_{\mathbf{mm}}. \tag{2.16}$$

Diversity combining is applied by multiplying each diversity branch with the set of weighting coefficient $\mathbf{w} = [w_1, w_2, \ldots, w_N]^H$, and the decision signal is $r = \mathbf{w}^H \mathbf{g} s$. Performing the minimum distance detection, the decision rule reduces to

$$|r - \mathbf{w}^H \mathbf{m} s_0|^2 \underset{\hat{s}=s_1}{\overset{\hat{s}=s_0}{\lessgtr}} |r - \mathbf{w}^H \mathbf{m} s_1|^2. \tag{2.17}$$

2.4.1 Optimal Combining with Perfect CSI

When CSI is perfect at the receiver, i.e., $\mathbf{m} = \mathbf{h} = \mathbf{g}$, it can be shown that the conditional error probability $\Pr(e|\mathbf{g}, s)$ is

$$\Pr(e|\mathbf{g}, s) = Q\left(\sqrt{\frac{|\mathbf{w}^H \mathbf{g}|^2 |s_1 - s_0|^2}{2|\mathbf{w}^H \mathbf{w}| N_0}}\right) = Q\left(\sqrt{\frac{2|\mathbf{w}^H \mathbf{g}|^2 E_s}{|\mathbf{w}^H \mathbf{w}| N_0}}\right). \tag{2.18}$$

When $\mathbf{w} = \mathbf{g}$, the error probability is minimized and

$$\min_{\mathbf{w}} \Pr(e|\mathbf{g}, s) = Q\left(\sqrt{\frac{2|\mathbf{g}|^2 E_s}{N_0}}\right). \tag{2.19}$$

It means that when the channel gain g is perfectly known at the receiver and the noise $\mathbf{n}$ is white, maximum ratio combining (MRC) is optimal in eliminating the error probability. Then the BER is

$$P_e = \int Q(\sqrt{2\gamma_{\mathbf{b}}}\,)\, p(\mathbf{g})\, d\mathbf{g} \tag{2.20}$$

where

$$\gamma_{\mathbf{b}} = \mathbf{g}^H (N_0 I_{\mathbf{N}})^{-1} \mathbf{g} E_s. \tag{2.21}$$

2.4.2 Optimal Combining with Imperfect CSI

Applying MMSE estimation to the channel observation to obtain the channel estimate $\mathbf{m} = C_{\mathbf{gh}}C_{\mathbf{hh}}^{-1}\mathbf{h}$. As mentioned before, the channel gain can be decomposed as $\mathbf{g} = \mathbf{m}+\mathbf{e}$, where $\mathbf{e} \sim \mathcal{CN}(0, C_{\mathbf{ee}})$ is the estimation error which is independent of $\mathbf{m}$. The conditional error probability can be expressed as

$$\Pr(e|\mathbf{h}, s) = Q\left(\sqrt{\frac{2|\mathbf{w}^H\mathbf{m}|^2 E_s}{(\mathbf{w}^H(C_{\mathbf{ee}}E_s + N_0 I)\mathbf{w})}}\right). \tag{2.22}$$

The optimal weighting coefficients $\mathbf{w}$ that minimizes $\Pr(e|\mathbf{h}, s)$ is

$$\mathbf{w} = (C_{\mathbf{ee}}E_s + N_0 I)^{-1}\mathbf{m} \tag{2.23}$$

and the conditional performance error is

$$\min_{\mathbf{w}} \Pr(e|\mathbf{h}, s) \quad = \quad Q\left(\sqrt{2\mathbf{m}^H(C_{\mathbf{g}|\mathbf{h}}E_s + N_0 I)^{-1}\mathbf{m}E_s}\right). \tag{2.24}$$

The corresponding BER is

$$P_e = \int Q(\sqrt{2\gamma_{\mathrm{b}'}}\,)\, p(\mathbf{m})\, d\mathbf{m} \tag{2.25}$$

where

$$\gamma_{\mathrm{b}'} = \mathbf{m}^H(C_{\mathbf{ee}}E_s + N_0 I_{\mathrm{N}})^{-1}\mathbf{m}E_s. \tag{2.26}$$

2.5 Rice Factor

For a single diversity branch, the link quality is measured by the Rice K factor which is the ratio of the LoS power and the diffuse power in that branch. With correlated multiple diversity branches, the relation between LoS and diffuse power is still defined by the mean vector $\mathbf{u}$ and covariance matrix $C_{\mathbf{gg}}$, but there is no convenient single parameter such as the Rice K factor. In this thesis, it is assumed that Rice K factor for each of the correlated diversity branch equals each other, thus the ratio between the total LoS power and the total diffuse power over all diversity branches is also

$$K = \frac{\mathbf{u}^H \mathbf{u}}{\mathrm{tr}(C_{\mathbf{gg}})}. \tag{2.27}$$

Therefore, it is easy to see that

$$u_i^2 = (K/N) \sum_{i=1}^{N} \sigma_{\bar{g}_i}^2, \quad i = 1, \ldots, N. \tag{2.28}$$

CHAPTER 3

BER ANALYSIS

As is well known, the bit error probability P_e is evaluated by averaging the conditional error $\Pr(e|\mathbf{h}, s)$ over all possible channel observations as in equation (2.25). The traditional approach [11, 12] is to use equation (2.26) to derive the distribution of SNR $p(\gamma_{\mathrm{b}}')$ and then calculate

$$\gamma_{\mathrm{b}'} = \mathbf{m}^H (C_{\mathbf{ee}} E_s + N_0 I_{\mathrm{N}})^{-1} \mathbf{m} E_s. \tag{3.1}$$

Since the combiner output SNR γ_b is in quadratic form, the alternative expression for the $Q(\cdot)$ function can be used as described in [13]

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp(-\frac{x^2}{2\sin^2\theta}) d\theta. \tag{3.2}$$

3.1 Evaluation For Perfect Channel Knowledge

Using Eqs. (2.20), (2.21) and (3.2), the P_e for perfect CSI case can be written as

$$P_e = \frac{1}{\pi} \int_{\mathbf{g}} \int_0^{\pi/2} \exp\left[-\frac{\mathbf{g}^H (N_0 I_{\mathrm{N}})^{-1} \mathbf{g} E_s}{\sin^2\theta} \right] \frac{1}{\pi^N \det(C_{\mathbf{gg}})}$$
$$\cdot \exp\left[-(\mathbf{g} - \mathbf{u})^H C_{\mathbf{gg}}^{-1} (\mathbf{g} - \mathbf{u}) \right] d\theta \, d\mathbf{g}. \tag{3.3}$$

Applying the fact that a proper complex Gaussian (PCG) joint pdf integrates to 1, and $C_{\mathbf{gg}} = C_{\mathbf{gg}}^H$, the BER reduces to

$$P_e = \frac{1}{\pi} \int_0^{\pi/2} \left\{ \det\left[\frac{(E_s/N_0) C_{\mathbf{gg}}}{\sin^2\theta} + I_{\mathrm{N}} \right] \right\}^{-1}$$
$$\cdot \exp\{ -\mathbf{u}^H [C_{\mathbf{gg}} + (E_s/N_0)^{-1} \sin^2\theta I_{\mathrm{N}}]^{-1} \mathbf{u} \} \, d\theta. \tag{3.4}$$

In Rayleigh fading where $\mathbf{u} = \mathbf{0}$, the closed form of BER is given in [14]

$$P_e = \frac{1}{2}\sum_{n=1}^{N}\left(\prod_{\substack{i=1\\i\neq n}}^{N}\left(1-\frac{\sigma_{\tilde{g}_i}^2}{\sigma_{\tilde{g}_n}^2}\right)\right)^{-1}\left[1-\left(\sqrt{\frac{\sigma_{\tilde{g}_n}^2(E_s/N_0)}{1+\sigma_{\tilde{g}_n}^2(E_s/N_0)}}\right)\right]. \tag{3.5}$$

In Ricean fading where $\mathbf{u} \neq \mathbf{0}$, the exponential polynomial can not be eliminated inside the integration and no further simplification is possible. However, Since (3.4) is a single finite-range integral, the BER can easily be evaluated numerically for any given K.

3.2 Evaluation For Imperfect Channel knowledge

Using Eqs. (2.25), (2.26) and (3.2), the BER with imperfect channel knowledge can be written as

$$\begin{aligned} P_e = \frac{1}{\pi}\int_{\mathbf{m}}\int_0^{\pi/2} &\exp\left[-\frac{\mathbf{m}^H\Sigma^{-1}\mathbf{m}E_s}{\sin^2\theta}\right]\frac{1}{\pi^N\det\left(C_{\mathbf{mm}}\right)} \\ &\cdot\exp\left[-(\mathbf{m}-\mathbf{u})^H C_{\mathbf{mm}}^{-1}(\mathbf{m}-\mathbf{u})\right]d\theta\, d\mathbf{m}, \end{aligned} \tag{3.6}$$

where

$$\Sigma = C_{\mathbf{ee}}E_s + N_0 I_{\mathbf{N}}.$$

Integrating with respect to $\mathbf{m}$, as shown in appendix A,

$$\begin{aligned} P_e = \frac{1}{\pi}\int_0^{\pi/2} &\left\{\det\left[\frac{C_{\mathbf{mm}}\Sigma^{-1}E_s}{\sin^2\theta}+I_{\mathbf{N}}\right]\right\}^{-1} \\ &\cdot\exp\left\{-\mathbf{u}^H[C_{\mathbf{mm}}+\Sigma E_s^{-1}\sin^2\theta]^{-1}\mathbf{u}]\right\}d\theta. \end{aligned} \tag{3.7}$$

The closed form expression of BER in Rayleigh correlated fading is given in [3]

$$P_e = \frac{1}{2}\sum_{n=1}^{N}\left(\prod_{\substack{i=1\\i\neq n}}^{N}\left(1-\frac{\lambda_i}{\lambda_n}\right)\right)^{-1}\left[1-\sqrt{\frac{\lambda_n}{1+\lambda_n}}\right] \tag{3.8}$$

where

$$\lambda_n = \frac{\sigma_{\tilde{g}_n}^2 E_s}{N_0}\cdot\frac{E_p}{E_s+E_p+N_0/\sigma_{\tilde{g}_n}^2}. \tag{3.9}$$

Again, (3.7) must be evaluated by numerical integration when the channel is Ricean correlated fading with $\mathbf{u} \neq \mathbf{0}$.

CHAPTER 4

OPTIMAL PILOT-TO-DATA POWER RATIO

Since PSAM is adopted, M pilot symbols are inserted in a block of length L. As it is shown that, the channel observation is

$$\mathbf{h} = \frac{\sum_i \mathbf{d}_i}{M\sqrt{E_p}} = \mathbf{g} + \frac{\mathbf{n}}{\sqrt{ME_p}}.$$

The effective bit SNR per bit can be written as

$$\gamma_b = \frac{E_b}{N_0} \tag{4.1}$$

where

$$E_b = \frac{(L-M)E_s + ME_p}{L-M}. \tag{4.2}$$

There is a trade-off between the pilot and data symbol power. While more pilot power gives better channel estimation result, the overhead imposed by pilot symbol also reduces the effective SNR γ_b. In this chapter, the power ratio between pilot and data symbol power is studied to optimize BER performance.

The ratio between the pilot power and the total transmitted power is defined as

$$\beta = \frac{ME_p}{ME_p + (L-M)E_s}, \quad \beta \in (0,1). \tag{4.3}$$

For Rayleigh fading case, the power allocation strategy for optimal diversity combining is suggested in [15]. Substituting E_b and β into (3.9) yields

$$\begin{aligned}\lambda_n &= \frac{\sigma_{\bar{g}_n}^4 (L-M)\beta(1-\beta)\gamma_b^2}{\sigma_{\bar{g}_n}^2 \left[\beta(L-M) + (1-\beta)\right]\gamma_b + 1} \\ &\approx \frac{(L-M)\beta(1-\beta)}{(L-M)\beta + (1-\beta)} \cdot \sigma_{\bar{g}_n}^2 \cdot \gamma_b. \end{aligned} \tag{4.4}$$

The optimal β value is shown as

$$\beta = \frac{\sqrt{L-M}-1}{(L-M)-1} \tag{4.5}$$

which implies that

$$\frac{ME_p}{(L-M)E_s} \approx \frac{1}{\sqrt{L-M}}. \tag{4.6}$$

With typical $f_D T_s$ parameters, e.g. $f_D T_s = .05$, $L = 10$, the total data power approximately three times to the total pilot power should be assigned in order to approach the optimal performance.

For Rice fading case, since there is no closed form expression for P_e, the optimal β can only be obtained by numerical simulation, which is obtained in next chapter.

CHAPTER 5

EXAMPLES AND SIMULATION RESULTS

In Chapter 3, the evaluation of the bit error probability of the optimal diversity combining under both Rayleigh and Ricean correlated fading channel are investigated. Since PSAM is used for channel estimation and the optimal weighting coefficients are derived therefrom, the relationship between the channel estimation and the effective system SNR per bit are also discussed. In this chapter, examples of using pilot symbol scheme for block fading channels are given to analyze the bit error probability while taking into consideration the channel estimation accuracy and power allocation strategy for pilot symbols that optimize the system bit error performance.

It is assumed that the channel is block fading and remains unchanged for $L = \lfloor 1/2f_D T_s \rfloor$ symbol durations, where f_D is the Doppler frequency and T_s is the symbol duration. For each fading block, M pilot symbols are inserted for channel estimation and the pilot symbol has amplitude $\sqrt{E_p}$. Since the diversity branches of the channel which is considered in this thesis are correlated, to evaluate the BER performance, the covariance matrices of the system statistics should be diagonalized, such that the diversity branches can be treated as statistically independent.

It can be seen that, under the pilot symbol scheme, the covariance matrices $C_{\mathbf{gg}}$ and $C_{\mathbf{hh}}$ can be diagonalized by the same unitary matrix U as

$$C_{\mathbf{gg}} = U\Lambda_{\mathbf{gg}}U^H, \quad (5.1)$$

$$C_{\mathbf{hh}} = U\Lambda_{\mathbf{hh}}U^H = U(\Lambda_{\mathbf{gg}} + (N_o/E_p)I)U^H. \quad (5.2)$$

Define that

$$\bar{\mathbf{g}} = U^H\mathbf{g}, \quad (5.3)$$

$$\bar{\mathbf{h}} = U^H\mathbf{h}, \quad (5.4)$$

then one can show that

$$\Lambda_{\mathbf{gg}} = E[\bar{\mathbf{g}}\bar{\mathbf{g}}^H] = \text{diag}(\sigma^2_{\bar{g}_1}, \sigma^2_{\bar{g}_2}, \ldots, \sigma^2_{\bar{g}_N}), \tag{5.5}$$

$$\Lambda_{\mathbf{hh}} = E[\bar{\mathbf{h}}\bar{\mathbf{h}}^H] = \text{diag}(\sigma^2_{\bar{h}_1}, \sigma^2_{\bar{h}_2}, \ldots, \sigma^2_{\bar{h}_N}) \tag{5.6}$$

where

$$\sigma^2_{\bar{h}_i} = \sigma^2_{\bar{g}_i} + N_0/E_p \tag{5.7}$$

$$\Lambda_{\mathbf{gh}} = E[\bar{\mathbf{g}}\bar{\mathbf{h}}^H] = \text{diag}(\rho_{\bar{g}_1\bar{h}_1}, \rho_{\bar{g}_2\bar{h}_2}, \ldots, \rho_{\bar{g}_N\bar{h}_N}), \tag{5.8}$$

where

$$\rho_{\bar{g}_i\bar{h}_i} = E[\bar{g}_i\bar{h}_i^H] = \sigma^2_{\bar{g}_i} \tag{5.9}$$

then

$$\Lambda_{\mathbf{mm}} = E[\bar{\mathbf{m}}\bar{\mathbf{m}}^H] = \Lambda_{\mathbf{gh}}\Lambda^{-1}_{\mathbf{hh}}\Lambda^H_{\mathbf{gh}} \tag{5.10}$$

where

$$\bar{\mathbf{m}} = U^H\mathbf{m}. \tag{5.11}$$

5.1 Frequency Diversity

As is given in Chapter 2, the correlation matrix of the channel gain with a frequency diversity model is expressed as

$$C_{\mathbf{gg}}(m,n) = \frac{\sigma_g^2}{1 + j2\pi f_d\tau_d(m-n)}.$$

τ_d is the channel delay spread, f_d is the frequency separation between two adjacent sub-carriers.

5.1.1 Rayleigh Fading Model

Firstly, the bit error performance of the optimal diversity combining under Rayleigh Fading channel is evaluated, in which there is no LoS path between the transmitter and the receiver and the mean of the channel gain is zero. Then the performance of the optimal combining with MRC is analyzed and shown that the optimal combining outperforms the MRC in system bit error probability when the CSI is imperfectly known at the receiver. Figure

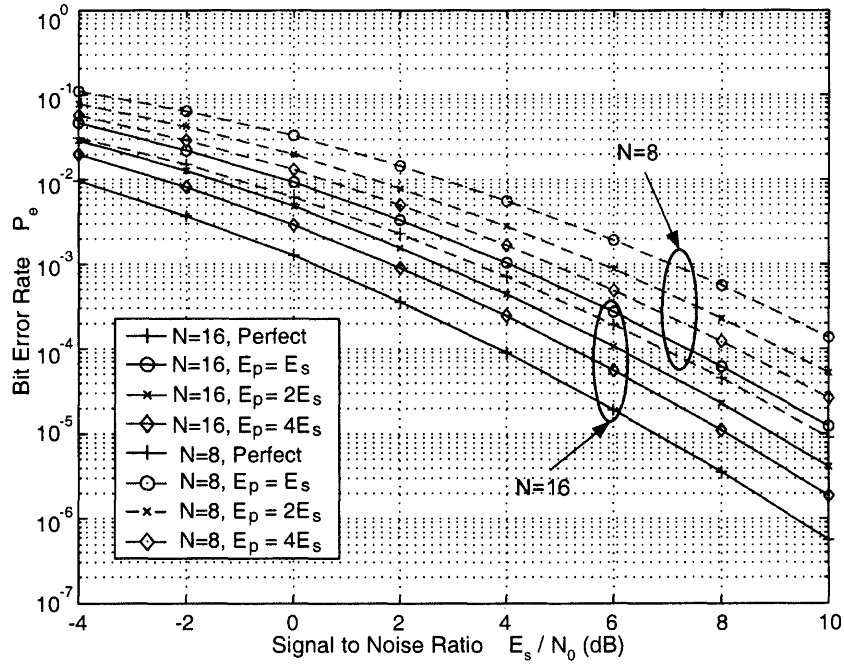

Figure 5.1 Comparison of the analytical error performance of optimal combining with perfect channel knowledge and imperfect channel knowledge in a frequency diversity model using different estimation power for $N = 8$ and $N = 16$.

5.1 cited from [3] compares the analytical BER performance of optimal combining with perfect CSI and imperfect CSI when the number of frequency diversity branches are set to $N = 8$ and $N = 16$ respectively. The channel delay spread τ_d is set to 25ns, the frequency separation f_d between two adjacent sub-carriers is 2.5MHz for $N = 8$ and 1.25MHz for $N = 16$. Therefore, the total bandwidth allocated to the channel is 20MHz. The figure shows that the BER performance can be improved by either increasing the frequency diversity branches or improving the channel observation quality, which produce less channel estimation error. Obviously, the channel observation quality can be improved by increasing the pilot-to-data power ratio. Figure 5.2 cited form [3] compares the cases

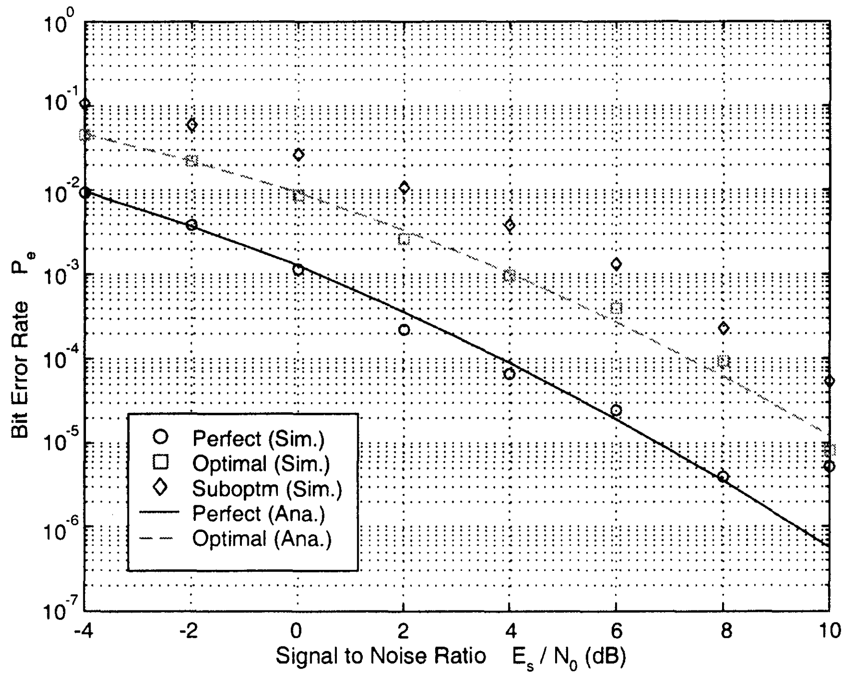

Figure 5.2 Comparison of the analytical and the Monte-Carlo simulation results for bit error performance of the perfect channel knowledge, optimal combining and sub-optimal combining with imperfect channel knowledge.

of diversity combining with perfect channel knowledge (in such case, MRC is optimal), optimal combining and MRC with imperfect channel knowledge. As shown in this figure, it is intuitive that even with optimal combining, the bit error performance will degrade when the CSI is known imperfectly in comparison with the perfect CSI, and the bit error performance is further reduced if MRC is adopted, in which the observation of the CSI h is directly used as the weighting coefficients.

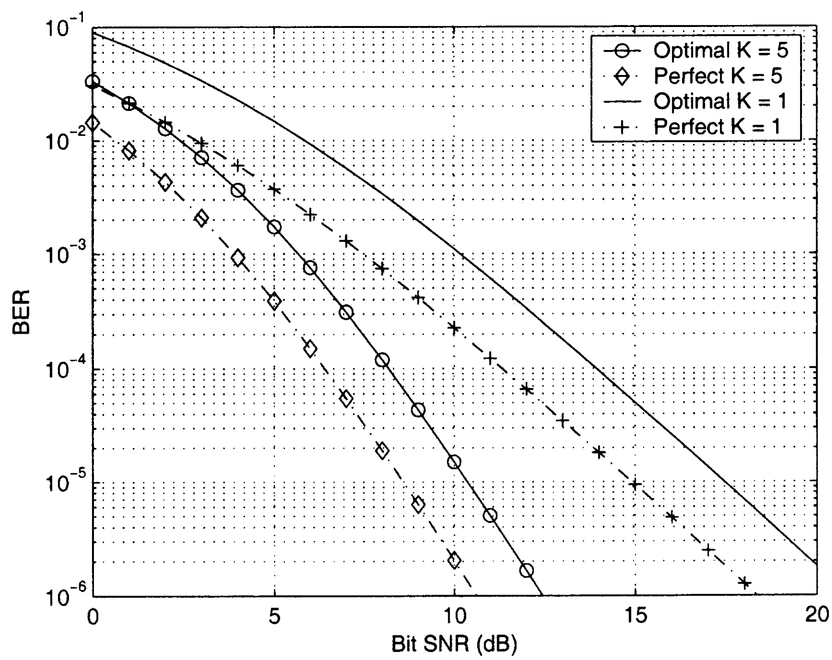

Figure 5.3 Comparison of the BER P_e vs. SNR γ_b with different Rice K factors and diversity branches using Frequency diversity model ($N = 3$, $f_d = .55$MHz, $\tau_d = 25$ns, $L = 10$, $M = 1$, $E_p = 3E_s$).

5.1.2 Ricean Fading Model

Figure 5.5 compares the BER performance of the optimal diversity combining with different link qualities and number of diversity branches. In Ricean fading channel, the channel gain has non-zero mean and the ratio between the power of the signal propagating through the LoS path and diffused paths are measured as the link quality called Rice K factor. Intuitively, with better link quality, there is less uncertainty of the channel which indicates less error probability. As it is discussed in Chapter 2, the channel is assumed to be block fading and remains unchanged for $L = [1/2f_D T_s]$ symbol durations, where f_D is the Doppler frequency and T_s is the symbol duration. The typical $f_D T_s$ parameters are used, e.g. $f_D T_s = .05, L = 10$ in this example. It is also assumed that $M = 1$, which means one pilot symbol is inserted into the block with the length of $L = 10$. The power of the pilot symbol is allocated three times as much as the data symbol. Without loss of generality, it is assumed that $\mathbf{u}^H\mathbf{u} + \mathrm{tr}(C_{\mathbf{gg}}) = 1$, where $\mathrm{tr}(\cdot)$ denotes the trace of a matrix. Obviously, with more frequency diversity branches or better link quality, i.e, larger Rice K factor, the bit error performance of the system increases in a Ricean correlated fading channel.

5.2 Spatial Diversity

As it is given in Chapter 2, the correlation matrix of the channel gain with a spatial diversity model is expressed as

$$C_{\mathbf{gg}}(m,n) = \sigma_g^2 J_0\left(2\pi\frac{|m-n|\, d\cdot f_c}{C}\right).$$

d is the distance between two adjacent antennas, f_c is the carrier frequency and C is the light speed. $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind.

5.2.1 Rayleigh Fading Model

In a spatial diversity model, the antennas are assumed to be placed in line and spaced equally. In the case of $N = 2, d = .05$m and the two antennas are uncorrelated. In the case of $N = 3, d = .025$m the three antennas are correlated. $f_c = 2.4$GHz is the carrier frequency. Similar to the frequency diversity model, Figure 5.4 from [3] also shows the trade-off between combining diversity branches and channel estimation quality. The BER performance can be improved by either increasing the number of antennas or appropriately adjusting the power allocated for channel estimation.

5.2.2 Ricean Fading Model

Figure 5.5 shows both the analytical and the Monte-Carlo simulation results of the BER performance for the optimal diversity combining using PSAM and the maximal ratio combining with perfect channel knowledge for $K = 1$ and $K = 5$. It can be seen that the analytical results agree with the Monte-Carlo simulations quite well. As expected, the BER decreases with larger Rice K factor since there is stronger LoS component and less channel uncertainty.

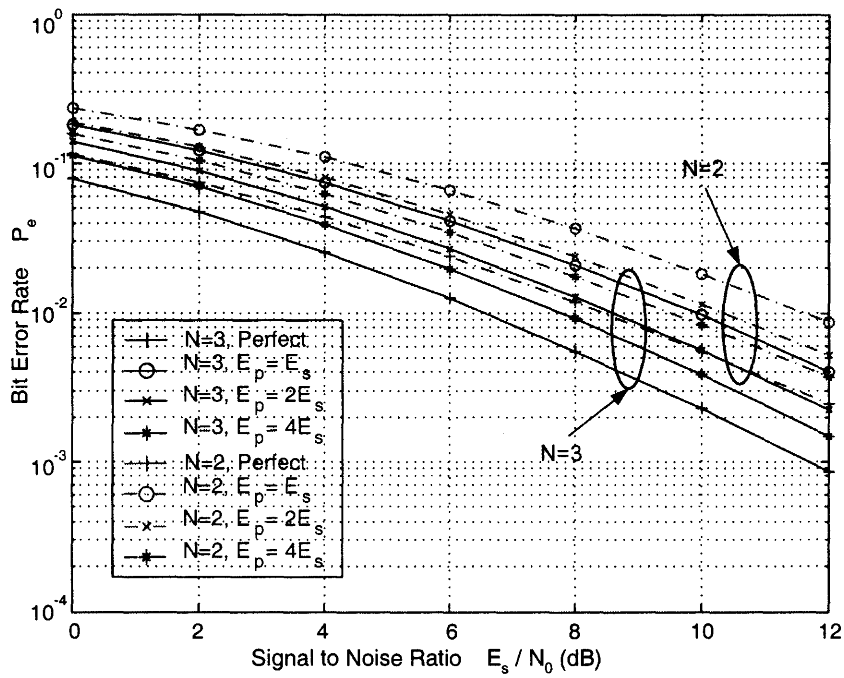

Figure 5.4 Comparison of the analytical error performance of optimal combining with perfect channel knowledge and imperfect channel knowledge in a spatial diversity model using different estimation power for $N = 2$ and $N = 3$.

5.3 Power Allocation Strategy

In order to illustrate the effect of the pilot symbol power allocation, the extreme case that $L = 2$ is discussed first, i.e., one pilot and one data symbol are transmitted within each block. For convenience, the energy ratio between the pilot and data symbol is defined as $\kappa = E_p/E_s$. Figure 5.6 compares the BER performance with different SNR and κ for both Rayleigh fading ($K = 0$) and Ricean fading ($K = 4$).

From the contour of BER when $K = 0$ on the horizontal plane, it can be seen that when κ is very small ($\kappa \ll 1$) or large, much more SNR should be spend to achieve the same BER, and the optimal κ in Ricean fading and Rayleigh fading is not the same. Since there is no closed form expression for the BER with optimal combining when the Rice factor $K \neq 0$, P_e in terms of SNR in equation (3.7) includes integration, absolute value and exponentiation, the optimal κ can not be derived analytically. Figure 5.7 shows the

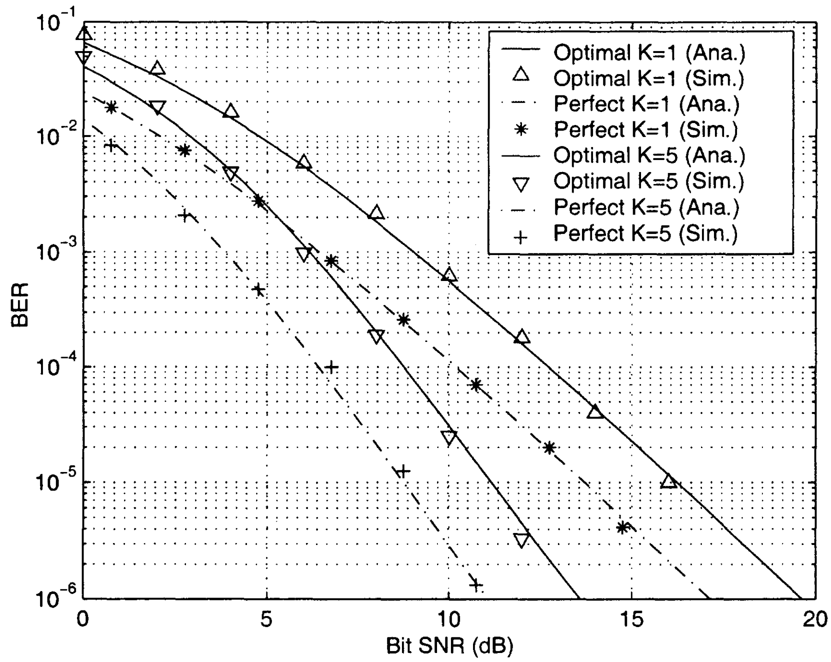

Figure 5.5 Comparison of the analytical and simulation results of BER P_e vs. SNR γ_b with different Rice K factors using spatial diversity model ($N = 3$, $d = .15$m, $f_D T_s = .05$, $L = 10$, $M = 1$, $E_p = 3E_s$).

numerical result of optimal κ. When $K \neq 0$, the optimal κ varies with SNR. This result is intuitive, since when γ is very small, the accuracy of pilot symbol estimation is highly degraded, spending more power on the pilots can not significantly improve the estimation. When γ is very large, the BER is hardly due to the estimation error. When γ is around 10 dB, more power should be assigned on pilots to achieve the best BER. Comparing the optimal κ with respect to different diversity Rice factor, when $K \neq 0$, a lot of energy can be saved on the pilot symbols to achieve the best BER in comparison with Rayleigh fading channels. Since in Ricean fading with the appearance of LoS power, the link quality is improved and the signal is more certain at the receiver, which increase the estimation accuracy, it is not needed to spend as much energy on pilot symbols to achieve the best BER when transmitted SNR is fixed.

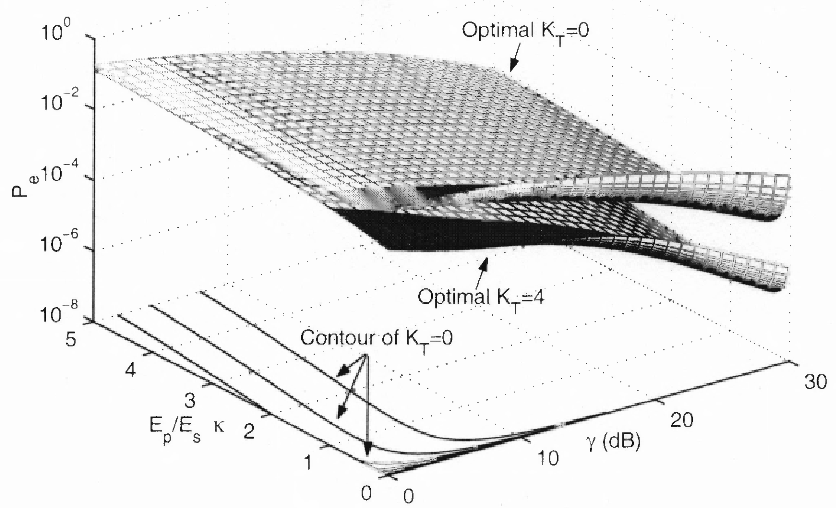

Figure 5.6 Comparison of BER vs. input SNR with different κ in frequency diversity model ($N = 5,\ \tau_d = 25$ns) for Rice factor $K = 0$ and $K = 4$.

It is interesting to see that, using optimal combining scheme for $L = 2$ with Rayleigh fading case, the BER is optimized when $\kappa = 1$, i.e., when the energy are set as $E_s = E_p$, the BER can be minimized. Investigating the closed form expression of P_e in (3.8), this result is proved theoretically in Appendix B.

5.3.1 Rayleigh Fading Model

Noting the effects of power allocation strategy of the pilot symbols, the pilot-to-data symbol power ratio optimization for the typical cases will be discussed through numerical examples. As it is discussed in Chapter 4, our optimization goal is to find the optimal ratio $\beta = \frac{ME_p}{ME_p+(L-M)E_s}$. Considering the spatial diversity channel model, a fixed length of .15m distance separation is assigned to three ($N = 3$) antennas. The antennas are assumed to be placed in line and spaced equally. As an example, $f_D T_s = .05$ is evaluated.

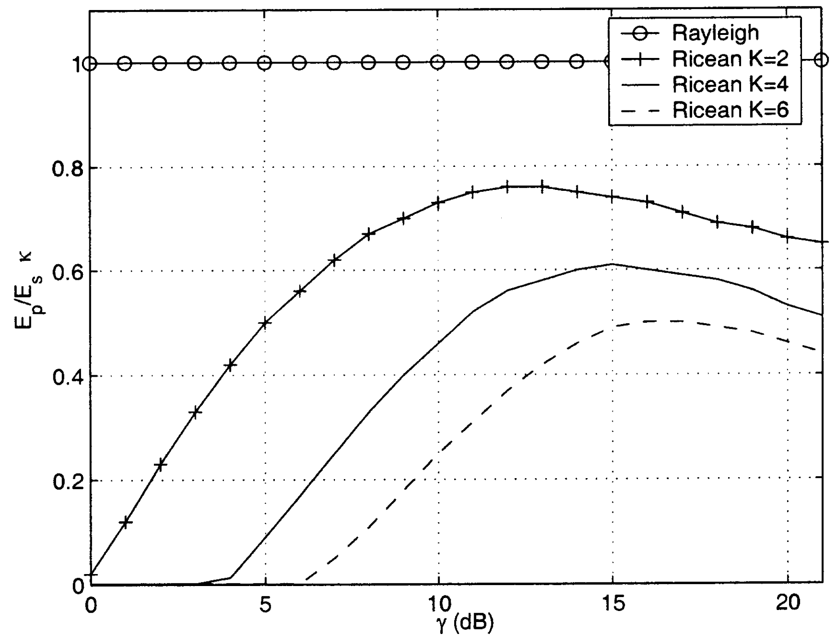

Figure 5.7 Comparison of the optimal κ in frequency diversity model ($N = 5,\ \tau_d = 25\text{ns}$) for Rice factor $K = 0$, 2, 4 and 6.

When inserting one pilot symbol to each block, the number of pilot symbol in each frame is fixed to $M = 1$ and the aim is to optimize parameter β in (4.3). With the help of (2.25) and (4.4), BER vs. β is shown in Figure 5.8 from [15].

In addition, the approximation of optimal β in (4.5) is shown as solid vertical line and compared to numerically optimal β (shown in '+') in Figure 5.8.

If $E_p = E_s$ is set as a constraint, namely, the energy of pilot and data per symbol are equal, the parameter M which optimize the system performance will be discussed. In this case, $\beta = \frac{M}{L}$ in (4.3) and with the help of (2.25), Figure 5.9 can be derived.

(4.6) can be used to get the approximates of optimal M. However, since M is an integer, the optimum *integer* number of pilot symbol can not be calculated by rounding the solution of (4.6) toward the nearest integer. Instead, both nearest integers greater and less than the solution have to be checked using (2.25).

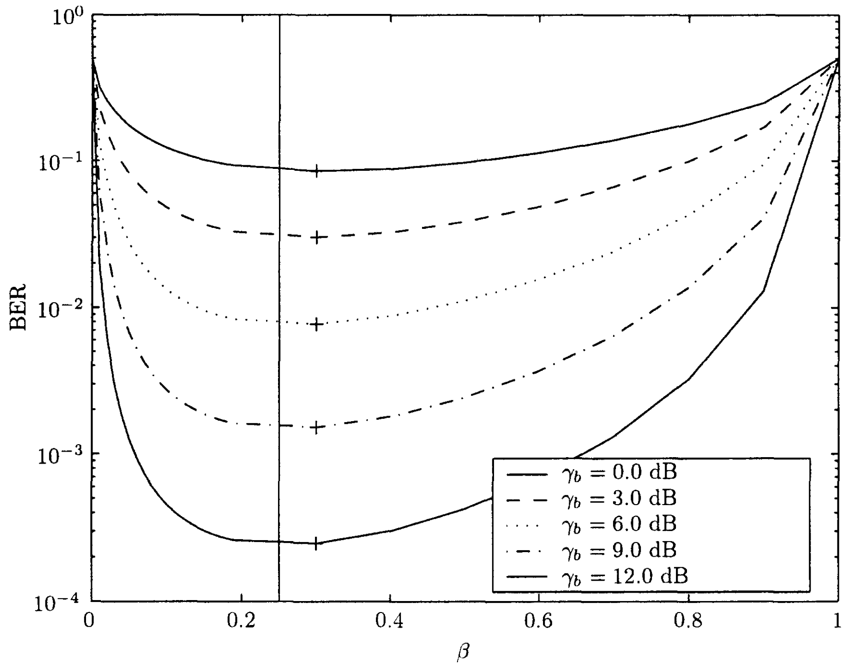

Figure 5.8 Bit Error Rate vs. β for $f_D T_b = .05$

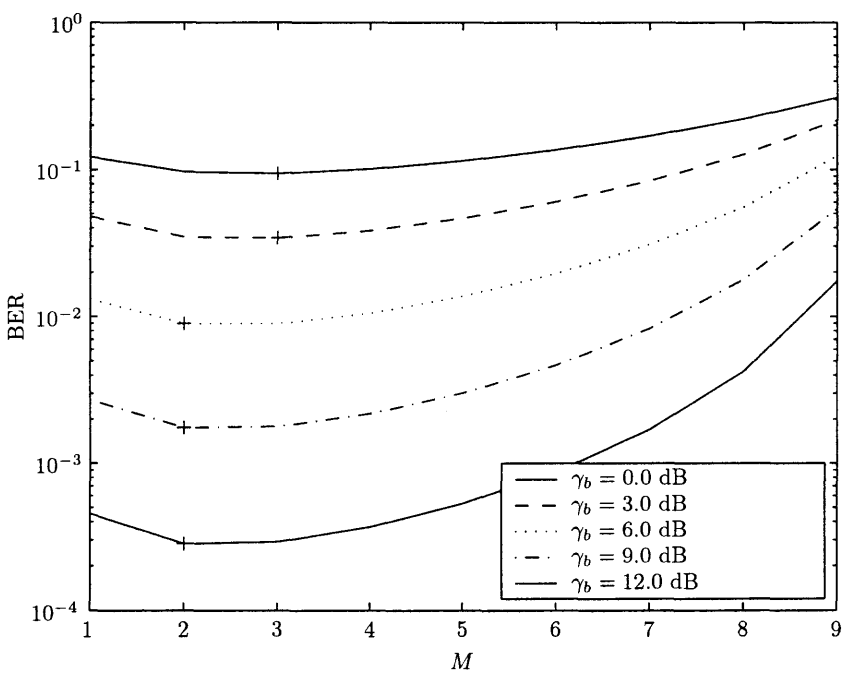

Figure 5.9 Bit Error Rate vs. M for $f_D T_b = .05$

Actually, since it is assumed that the channel is block fading and the CSI is unchanged for L symbol period, the two cases have the same implication that when assigning the total data power three times to the pilot power, the BER performance is optimal. Since there is more flexibility in assigning the pilot power arbitrarily within one symbol, thus only the $M = 1$ case is considered in the following discussion.

5.3.2 Ricean Fading Model

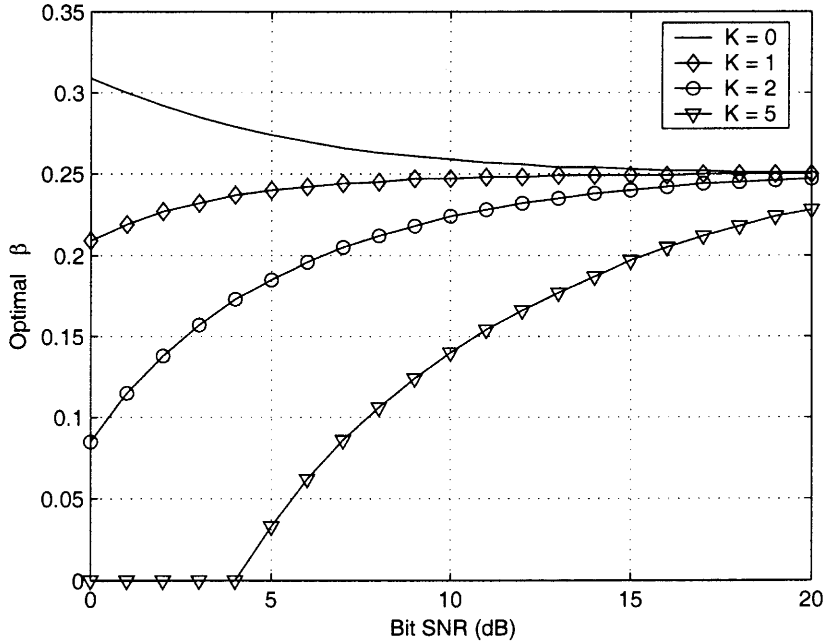

Figure 5.10 The optimum value of β vs. SNR γ_b with Rice factors $K = 0, 1, 2, 5$ ($N = 3$, $d = .15$m, $f_D T_s = .05$, $L = 10$, $M = 1$).

The trade-off between pilot symbol power and data symbol power is investigated numerically with respect to Rice K factor and effective SNR γ_b. The result is plotted in Figure 5.10. With large Rice K factor, since there is less channel uncertainty, less pilot symbol energy is needed for channel estimation. The optimal β value is also dependent on the effective SNR γ_b. At high SNR, the channel estimation error dominates. Therefore, the optimal proportion should be used to achieve the balance between estimation overhead and accuracy. At low SNR, the BER performance is dominated by channel noise and the channel estimation can not be significantly improved even if more power is allocated. Therefore, if a reliable LoS component exists, less power is needed on channel estimation.

CHAPTER 6

CONCLUSION

In this thesis, it is shown that the channel knowledge at the receiver is not perfect, channel estimation can be performed to improve the system bit error performance. The non-observable estimation error can be treated as an additional source of noise and is usually not white. Therefore, diversity combining taking consideration of MMSE channel estimation should be used for optimal combining. It is shown that the bit error rate of optimal combining with imperfect channel knowledge is worse than with perfect channel knowledge but is better than maximal ratio combining. It is also shown that the diversity combining performance over Ricean correlated fading channel is dependent on the Rice K factor as well as the power ratio between the pilot and data symbols. When more signal propagate through the LoS path between transmitter and receiver, there is less channel uncertainty and less resource is needed for channel estimation.

APPENDIX A

INTEGRATION FROM EQ. (3.6) TO (3.7)

Combining the two exponential factors in (3.7) reduces

$$P_e = \frac{1}{\pi}\int_{\mathbf{m}}\int_0^{\pi/2} \frac{1}{\pi^N \det(C_{\mathbf{mm}})} \exp(\Gamma)\, d\theta\, d\mathbf{m} \tag{A.1}$$

where

$$\Gamma = -\frac{\mathbf{m}^H (C_{\mathbf{ee}}E_s + N_0 I_{\mathbf{N}})^{-1}\mathbf{m}E_s}{\sin^2\theta} - (\mathbf{m}-\mathbf{u})^H C_{\mathbf{mm}}^{-1}(\mathbf{m}-\mathbf{u}). \tag{A.2}$$

Rearranging the the above equation reduces

$$\Gamma = -\mathbf{m}^H[\frac{(C_{\mathbf{ee}}E_s + N_0 I_{\mathbf{N}})^{-1}E_s}{\sin^2\theta} + C_{\mathbf{mm}}^{-1}]\mathbf{m} + \mathbf{m}^H C_{\mathbf{mm}}^{-1}\mathbf{u} + \mathbf{u}^H C_{\mathbf{mm}}^{-1}\mathbf{m} - \mathbf{u}^H C_{\mathbf{mm}}^{-1}\mathbf{u}.$$

Define

$$\Lambda = \frac{(C_{\mathbf{ee}}E_s + N_0 I_{\mathbf{N}})^{-1}E_s}{\sin^2\theta} + C_{\mathbf{mm}}^{-1}. \tag{A.3}$$

Patch a polynomial inside to completing squares and using the property of *Hermitian*, the above equation can be rewritten as

$$\begin{aligned}\Gamma =& -\mathbf{m}^H\Lambda\mathbf{m} + \mathbf{m}^H C_{\mathbf{mm}}^{-1}\mathbf{u} + \mathbf{u}^H C_{\mathbf{mm}}^{-1}\mathbf{m} - \mathbf{u}^H (C_{\mathbf{mm}}^{-1})^H \Lambda^{-1} C_{\mathbf{mm}}^{-1}\mathbf{u} \\ &+ \mathbf{u}^H (C_{\mathbf{mm}}^{-1})^H \Lambda^{-1} C_{\mathbf{mm}}^{-1}\mathbf{u} - \mathbf{u}^H C_{\mathbf{mm}}^{-1}\mathbf{u} \\ =& -[\mathbf{m}^H - \mathbf{u}^H (C_{\mathbf{mm}}^{-1})^H (\Lambda^{-1})^H]\Lambda(\mathbf{m} - \Lambda^{-1}C_{\mathbf{mm}}^{-1}\mathbf{u}) + \mathbf{u}^H C_{\mathbf{mm}}^{-1}\Lambda^{-1}C_{\mathbf{mm}}^{-1}\mathbf{u} - \mathbf{u}^H C_{\mathbf{mm}}^{-1}\mathbf{u} \\ =& -(\mathbf{m} - \Lambda^{-1}C_{\mathbf{mm}}^{-1}\mathbf{u})^H \Lambda (\mathbf{m} - \Lambda^{-1}C_{\mathbf{mm}}^{-1}\mathbf{u}) + [-\mathbf{u}^H (C_{\mathbf{mm}}^{-1} - C_{\mathbf{mm}}^{-1}\Lambda^{-1}C_{\mathbf{mm}}^{-1})\mathbf{u}]\end{aligned} \tag{A.4}$$

Replacing (3.7) by (A.4), P_e is given by

$$P_e = \frac{1}{\pi}\int_0^{\pi/2} \frac{1}{\det(\Lambda\, C_{\mathbf{mm}})} \cdot \exp\left[-\mathbf{u}^H (C_{\mathbf{mm}}^{-1} - C_{\mathbf{mm}}^{-1}\,\Lambda^{-1}C_{\mathbf{mm}}^{-1})\mathbf{u}\right] d\theta \tag{A.5}$$

Replacing Λ in (A.5) by (A.3) and applying the matrix inversion lemma, yield equation (3.7).

APPENDIX B

EVALUATION OF κ WITH $K = 0$

When $L = 2$, let $\gamma = \frac{(\beta-1)E_s+E_p}{(\beta-1)N_0}$, $E_p = \frac{\kappa\gamma N_0}{\kappa+1}$ and $E_s = \frac{\gamma N_0}{\kappa+1}$. Then Eqs. (3.8) and (3.9) can be rewritten as

$$\lambda_n = \frac{\kappa\sigma_{\bar{g}_n}^4\gamma^2}{(\kappa+1)^2(\sigma_{\bar{g}_n}^2\gamma+1)}. \tag{B.1}$$

and

$$P_e = \frac{1}{2}\sum_{n=1}^{N}\prod_{\substack{i=1\\i\neq n}}^{N}\left(1-\frac{\sigma_{\bar{g}_i}^4(\sigma_{\bar{g}_n}^2\gamma+1)}{\sigma_{\bar{g}_n}^4(\sigma_{\bar{g}_i}^2\gamma+1)}\right)^{-1} \cdot\left(1-\sqrt{\frac{\kappa\sigma_{\bar{g}_n}^4\gamma^2}{(\kappa+1)^2(\sigma_{\bar{g}_n}^2+1)+\kappa\sigma_{\bar{g}_n}^4\gamma^2}}\right) \tag{B.2}$$

Taking derivative with respect to κ reduces,

$$\sigma_{\bar{g}_n}^2 + 1 - \kappa^2\sigma_{\bar{g}_n}^2 - \kappa^2 = 0 \implies \kappa^2 = 1 \implies \kappa = \pm 1, \tag{B.3}$$

the polynomial is maximized. Since κ can not be negative, it is only needed to verify that whether $\kappa = 1$ can maximize the polynomial. Taking second order derivation, the following equation can be derived,

$$(-2\kappa\sigma_{\bar{g}_n}^2 - 2\kappa)|_{\kappa=1} = -2\sigma_{\bar{g}_n}^2 - 2 < 0 \tag{B.4}$$

Therefore, when $\kappa = 1$, the polynomial is maximized, i.e., the optimal BER with optimal combining scheme in the case of $K = 0$ is achieved by choosing $E_s = E_p$.

REFERENCES

[1] W. C. Jakes, *Microwave Mobile Communications.* John Wiley & Sons, Inc., 1974.

[2] J. G. Proakis, *Digital Communications*, 4th ed. McGraw-Hill, 2000.

[3] R. R. You, H. Li and Y. Bar-Ness, "Diversity combining with imperfect channel knowledge," *submitted to IEEE Trans. Commun.*, 2004.

[4] P. Schramm and R. R. Müller, "Pilot symbol assisted BPSK on Rayleigh fading channels with diversity: Performance analysis and parameter optimization," vol. 46, pp. 1560–1563, Dec. 1998.

[5] V. V. Veeravalli, "On performance analysis of signaling on correlated fading channels," *IEEE Trans. Comm.*, vol. 49, no. 11, pp. 1879–1883, November 2001.

[6] C. Tepedelenlioglu, A. Abdi and G. B. Giannakis, "Average-rate optimal PSAM transmissions over time-selective fading channels," *IEEE Wireless Commun.*, vol. 2, pp. 799–810, July 2003.

[7] S. Ohno. and G. B. Giannakis, "Average-rate optimal PSAM transmissions over time-selective fading channels," *IEEE WCNC2002*, vol. 1, pp. 17–21, March 2002.

[8] M. D. Yacoub, *Foundations of Mobile Radio Engineering.* CRC, 1993.

[9] G. Stuber, *Principles of Mobile Communication.* Norwell, MA: Kluwer Academic, 1996.

[10] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory.* Prentice Hall PTR, 1993.

[11] M. Gans, "The effect of gaussian error in maximal ratio combiners," *IEEE Trans. on Comm.*, pp. 492–500, August 1971.

[12] B. R. Tomiuk, N. C. Beaulieu, and A. A. Abu-Dayya, "General forms for maximal ratio diversity with weighting errors," *IEEE Trans. on Comm.*, pp. 488 –492, April 1999.

[13] M. K. Simon and D. Divsalar, "Some new twists to problems involving the gaussian integral," *IEEE Transactions on Communications*, vol. 46, pp. 200–210, Feb. 1998.

[14] F. A. Dietrich and W. Utschick, "Maximal ratio combining of correlated Rayleigh fading channels with imperfect channel knowledge," *IEEE Communications Letters*, vol. 7, pp. 419–421, Sept. 2003.

[15] Shengshan, Cui and Roy, You, "Pilot-to-data power ratio for diversity combining with imperfect channel estimation," *Proc. of Conference on Information Sciences and Systems 2005*, p. 227, March 2005.