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ABSTRACT

UNIAXIAL AND TRIAXIAL BEHAVIOR OF HIGH STRENGTH CONCRETE WITH AND WITHOUT STEEL FIBERS

^{by} Xiaobin Lu

This study first presents an extensive experimental research program on the true uniaxial and triaxial compression behavior for both high strength concrete (HSC) and steel fiber reinforced high strength concrete (SFHSC). The experimental study mainly focuses on the octahedral shear stress \sim strain relationship of those two types of concrete, which is adopted as the basis to develop a new incremental constitutive model. Emphasis is also put on the investigation of the variation of tangent Poisson's ratio under not only uniaxial but also triaxial stress conditions. The effect of cyclic loading on this parameter is also addressed.

According to this research, under triaxial compression, there is no apparent advantage of steel fiber reinforced high strength concrete (SFHSC) over high strength concrete (HSC) in terms of triaxial strength, ductility and stress ~ strain behavior. The compressive meridians and the peak octahedral shear stress (τ_{octp}) versus peak octahedral shear strain (γ_{octp}) relationships for the two types of concrete can be virtually expressed by a single expression respectively.

Unlike most of the previous incremental constitutive models, the proposed new model utilizes the experimentally acquired octahedral shear stress $(\tau_{oct}) \sim$ octahedral shear strain (γ_{oct}) relationship instead of the fictitious concept of "equivalent uniaxial

strain" to locate the peak point of the triaxial stress \sim strain curve, which ensures its capability of simulating the whole load \sim deformation process for both HSC and SFHSC, including the descending branch in the stress \sim strain curve. The results from the model analysis comply with the experimental data fairly well under moderate confining pressures.

UNIAXIAL AND TRIAXIAL BEHAVIOR OF HIGH STRENGTH CONCRETE WITH AND WITHOUT STEEL FIBERS

by Xiaobin Lu

A Dissertation Submitted to the Faculty of New Jersey Institute of Technology in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Civil Engineering

Department of Civil and Environmental Engineering

January 2005

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Dedicated to my beloved wife Zhe and little Leo (Qiqi)

and

my Mom

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LIST OF SYMBOLS

- f_c' : Uniaxial compressive strength of concrete.
- f_t : Uniaxial tensile strength of concrete.
- σ_1 : Stress in the first principal direction (axial stress in triaxial compression test).
- σ_2 : Stress in the second principal direction.
- σ_3 : Stress in the third principal direction. (in triaxial compression test, $\sigma_2 = \sigma_3$ =confining pressure).
- σ_{1c} : Peak stress in the first principal direction.
- σ_{2c} : Peak stress in the second principal direction.
- σ_{3c} : Peak stress in the third principal direction.
- σ_{oct} : Octahedral normal stress.
- σ_m : Mean normal stress.
- τ_{oct} : Octahedral shear stress.
- τ_{octp} : Peak octahedral shear stress.
- τ_m : Mean shear stress.
- β : Stress ratio σ_i / σ_{ic} (i=1, 2 and 3).
- ε_1 : Strain in the first principal direction (axial strain in triaxial compression test).
- ε_2 : Strain in the second principal direction.
- ε_3 : Strain in the third principal direction. (in triaxial compression test, $\varepsilon_2 = \varepsilon_3$ =circumferential strain).
- ε_{1c} : Peak strain in the first principal direction.

LIST OF SYMBOLS (Continued)

- ε_{2c} : Peak strain in the second principal direction.
- ε_{3c} : Peak strain in the third principal direction.
- ε_{oct} : Octahedral normal strain.
- γ_{oct} : Engineering octahedral shear strain.
- γ_{octp} : Peak engineering octahedral shear strain.
- ε_{cu} : Peak axial strain in uniaxial compression.
- ε_{iu} : Equivalent uniaxial strain in "i" direction (i=1, 2 and 3).
- ε_{iuc} : Peak equivalent uniaxial strain in "i" direction (i=1, 2 and 3).
- γ_m^* : Peak engineering octahedral shear strain in terms of equivalent uniaxial strain.
- ε_m^* : Peak octahedral normal strain in terms of equivalent uniaxial strain.
- v_0 : Initial tangent Poisson's ratio.
- v_{tf} : Tangent Poisson's ratio at peak stress.
- v_{tp} : Tangent Poisson's ratio after peak stress.
- v_s : Secant Poisson's ratio.
- v_t : Tangent Poisson's ratio.
- E_i : Tangent elastic modulus in "i" direction (i=1, 2 and 3).
- E_0 : Initial elastic modulus.

LIST OF SYMBOLS (Continued)

- G_t : Tangent shear modulus.
- G_0 : Initial shear modulus.
- K_t : Tangent bulk modulus.
- K_0 : Initial bulk modulus.

CHAPTER 1

INTRODUCTION

1.1 General

Nowadays, with the wide use of the high strength concrete (HSC) and the steel fiber reinforced high strength concrete (SFHSC) in heavy duty structures like high rise buildings and nuclear power plants, their properties under multiaxial stress conditions have become a great concern for the researchers around the world. However, most of the previous studies mainly focused on the multiaxial strength properties (failure criteria) and impractical triaxial stress ~ strain curves which could not be incorporated easily into the constitutive model. And also most of the analysis methods just assumed the Poisson's ratio and sometimes even the elastic modulus to be a constant throughout the whole loading process, which by no mean fitted with the nonlinear characteristics of the concrete. Although some models have considered the variation of elastic modulus and Poisson's ratio with stress condition, they used the fictitious concept of the "equivalent uniaxial strain" as the analysis tool.

Based on an extensive experimental program, this study intends to introduce a simple but practical incremental constitutive model using the octahedral shear stress versus octahedral shear strain relationship with readily-defined input parameters (initial elastic modulus E₀, initial Poisson's ratio v_0 , and uniaxial compressive strength f_c') to simulate the whole nonlinear stress ~ strain relationship (ascending and descending branches) for high strength concrete (HSC) and steel fiber reinforced high strength concrete (SFHSC) under proportional loading in triaxial compression. In this model, the

non-linear variation for both the elastic modulus E and Poisson's ratio v with stress condition will be taken into consideration.

To simplify the nomenclature, hereinafter the high strength concrete and the steel fiber reinforce high strength concrete will be referred to as HSC and SFHSC, respectively.

1.2 Research Objectives

This research mainly consists of two parts — experimental program and model analysis. The experimental program of this study is concentrated on investigating the behavior of both HSC and SFHSC in the following aspects:

- To develop a new way to perform the true uniaxial compression test by providing the uniform lateral expansion for the whole cylinder;
- 2) To evaluate the applicability of different failure criteria, including the Mohr-Coulomb failure criterion and the Willam-Warnke failure criterion under triaxial compression. Comparisons with the results of numerous previous studies on normal strength concrete, high strength concrete and steel fiber reinforce concrete are also to be made;
- 3) To establish the triaxial stress ~ strain relationships, including axial stress (σ_1) versus axial strain (ε_1) and lateral strain (ε_3), octahedral normal stress (σ_{oct}) versus volume change ($3\varepsilon_{oct}$), and octahedral shear stress (τ_{oct}) versus engineering octahedral shear strain (γ_{oct}) under triaxial compression;

- 4) To establish the relationship between the peak octahedral shear stress τ_{octp} and peak engineering octahedral shear strain γ_{octp} under triaxial compression, which will be adopted in the proposed simple incremental constitutive model for proportional loading;
- 5) To examine the properties of concrete including ultimate strength and triaxial stresss ~ strain relationships under different load paths in triaxial compression;
- To study the Poisson's ratio variation under triaxial compression for both HSC and SFHSC;
- To evaluate the effect of cyclic loading on the stress ~ strain behavior and Poisson's ratio variation of HSC and SFHSC under triaxial compression.

Based on the present experimental results, the model analysis portion is aimed first at evaluating the existing incremental constitutive models for concrete under multiaxial stress conditions, and emphasis is put on examining the validity of the concept of "equivalent uniaxial strain" that has been developed to simulate the stress ~ strain behavior in all three orthotropic axes. And then, a simple incremental constitutive model applicable to both HSC and SFHSC in triaxial compression is proposed. Based on the octahedral shear stress (τ_{oct}) versus engineering octahedral shear strain (γ_{oct}) relationship, this model is capable of simulating the whole stress ~ strain behavior (both ascending and descending branch) of HSC and SFHSC under proportional loading in the triaxial compression. Finally the model prediction has been compared with the experimental data.

1.3 Research Originalities

The experimental and theoretical originalities of this study are listed as follows:

- Designed a simple but effective lubricated loading platen to ensure a true uniaxial compression for concrete cylinders, which is more reliable in determining the variation of uniaxial properties of concrete, especially Poisson's ratio v;
- Studied the effect of h/d ratio of concrete cylinder under the lubricated loading platen and found that the h/d ratio of 1.5 is more likely to give uniform lateral expansion of concrete cylinders under uniaxial compression;
- Introduced a strong and flexible insulation sleeve to the triaxial test specimens which is both easy for the axial and lateral strain measurements and capable of maintaining a high successful rate of tests even under confining pressures as high as 70MPa (10ksi);
- Revealed the similarity between HSC and SFHSC in terms of the Mohr-Coulomb failure criterion in triaxial compression;
- Demonstrated the possibility of a uniform expression for the compressive meridian of the Willam-Warnke failure criterion for both HSC and SFHSC in triaxial compression;
- 6) Studied the triaxial stress ~ strain relationships (ascending and descending branches) for both HSC and SFHSC under different confining pressures and load paths in triaxial compression, including the axial stress ~ axial strain and lateral strain, octahedral normal stress ~ volume change, octahedral shear stress ~ octahedral shear strain and equivalent uniaxial stress ~ strain curves, and also evaluated the validity of using the Saenz equation to make the approximation;

- Developed an innovative relationship between the peak octahedral shear stress and peak engineering octahedral shear strain for both HSC and SFHSC under different load paths and confining pressures;
- Studied the Poisson's ratio variation for both HSC and SFHSC under triaxial compression;
- Proposed an equation for the tangent Poisson's ratio variation for both HSC and SFHSC based on the uniaxial and triaxial compression tests;
- 10) Studied experimentally the effect of cyclic loading in triaxial compression to the behavior of HSC and SFHSC, including stiffness degradation and Poisson's ratio variation;
- 11) Evaluated the validity of the "equivalent uniaxial strain" in the incremental constitutive modeling analysis;
- 12) Proposed a new incremental constitutive model based on the octahedral shear stress and engineering octahedral shear strain relationship for both HSC and SFHSC under proportional loading in triaxial compression, which is capable of simulating the whole load ~ deformation process (ascending and descending branches).

CHAPTER 2

UNIAXIAL AND TRIAXIAL COMPRESSION TESTS OF HSC AND SFHSC — LITERATURE REVIEW

2.1 Uniaxial Compression Test

2.1.1 End Restraint of Specimen

Under the standard concrete cylinder compressive test prescribed in ASTM, a lateral frictional restraint exists between the loading platen and the end of the specimen, and this restraint increases proportionately as the axially applied monotonic compressive load goes up. This frictional restraint will act as a lateral confinement at the specimen end, thus unavoidably enhancing the "compressive strength" of the target specimen. Mindess, Young and Darwin (2002) has stated that the end confining pressure, exhibiting in a form of shear stress, is greatest right at the specimen end and gradually dies out at a distance from each end approximately $(\sqrt{3}/2)d$, where d is the diameter of the specimen. Due to the existence of such an end friction, the typical compressive failure of the standard 4×8 in. (h/d=2) specimen exhibits an obvious cone failure characteristic, and only a small central portion of the cylinder is in true uniaxial compression, the remainder being in a state of triaxial stress (see Figure 2.1). Consequently, the standard specimen test tends to overestimate the "pure" compressive strength of the concrete cylinder.

The apparent compressive strength of concrete specimen will increase accordingly as the volume of concrete subjected to lateral restraint increases (see Figure 2.2), which accounts for the relatively higher compressive strengths of cylinders with h/d ratios less than 2~2.5 under the compressive test method of ASTM standard. It has been shown (Mindess, Young and Darwin 2002) that, as a general rule, for specimens

subjected to end restraint, an h/d ratio of 3.0 is high enough to provide true uniaxial compression in the central part of the concrete (see Figure 2.2).



Figure 2.1 Typical failure patterns for concrete cylinders in compression: (a) confinement at both ends; (b) confinement at one end and splitting failure at the other; (c) splitting failure.



Figure 2.2 Slenderness effect for the end restraint.

2.1.2 Compressive Failure Pattern in Perfect Uniaxial Compression

Mindess, Young and Darwin (2002) have stated that "on a more fundamental level, to speak at all of a compression failure of concrete (or most other materials) is incorrect". Compression tends to squeeze the atoms and molecules closer together, so it is hard to see how a "pure" compressive failure occurs in the concrete.

Unlike ductile materials, the concrete's tensile capacity is far less than its compressive counterpart. The tensile capacity mainly includes two parts, namely tensile strength and tensile strain, which are only about 1/10 to 1/15 of their corresponding compressive counterparts of the normal strength concrete. For HSC, the concrete becomes more brittle and that ratio gets even lower.

When the concrete is subjected to the pure compression without end restraint, due to the Poisson's effect, the specimen will undergo lateral expansion simultaneously with the axial deformation. It has been suggested that the tensile strain capacity of concrete varies from 0.0001 to 0.0002. Given an approximate Poisson's Ratio of 0.2, such lateral tensile strains will occur at fairly low compressive stress, leading to a pattern of splitting failure parallel to the longitudinal direction (see Figure 2.1(c)). Theoretically, this may be the natural failure mode of concrete in pure compression.

2.1.3 End Friction-Reducing Measures

In practice, the pure compression status of the concrete specimen can hardly be achieved. So actually, the true compressive strength is inevitably overestiomated. In order to get as close as possible to the pure compression status, thus to achieve the true uniaxial properties of the concrete, end friction-reducing measures must be taken. Conclusively, all the existing measures fall into two categories. **2.1.3.1 Brush Platen** The brush platen may be the best testing apparatus to achieve the true uniaxial compression. It is capable of providing relatively better and consistent testing results which have been widely quoted by some researchers (Guo 1997, and Van Mier et. al 1997). The platen consists of filaments about $5\times3mm$ in cross section, with gaps of about 0.2mm between them. This platen allows the concrete to expand laterally with little restraint (see Figure 2.3).

However, this kind of loading platen has a very complicated structure and high cost, and meanwhile, large deviations still exist in the testing results. Therefore, the brush platen is not a widely used approach.



Figure 2.3 Brush platen.

2.1.3.2 Lubricated Loading Platen This testing setup is aimed at decreasing the friction between the loading platen and the end of the concrete cylinder. Normally one or
more friction-reducing sheet(s) are put between the loading platen and the ground end of the cylinder. Lubricant, in most cases bearing grease, is sprayed uniformly and thinly between the loading platen and friction-reducing sheet (see Figure 2.4).

Putting lubricant between the friction-reducing sheet and the cylinder end is usually avoided since there is a concern that a reversal of boundary restraint may occur when excessive grease is applied (Van Mier et. al 1997, see Figure 2.5).



Figure 2.4 Schematic drawing of the lubricated loading platen.



Figure 2.5 Reversal of boundary restraint under excessive grease.

The idealistic situation is that no friction exists between the rigid loading platen and the friction-reducing sheet and that the friction reducing material should satisfy the following condition:

$$\left(\frac{\nu}{E}\right)$$
 of friction reducing material = $\left(\frac{\nu}{E}\right)$ of concrete (2-1)

where v is the Poisson's ratio and E is the elastic modulus.

Under this condition, the friction reducing material and the concrete specimen will have the identical lateral strain ($\varepsilon' = v\varepsilon = \left(\frac{v}{E}\right)\sigma_c$), which will guarantee that there

will be no friction between the friction reducing sheet and the concrete cylinder. But unfortunately, it is hardly possible to find such an appropriate material. Therefore, instead, much emphasis has been put on decreasing the friction between the frictionreducing sheet and the specimen end.

Teflon sheet is widely accepted as an ideal option for reducing the end friction (Guo 1997, and Van Mier et. al 1997). However, there seems no consensus on how thick the teflon sheet should be. In the joint research program coordinated by RILEM (Van Mier et. al 1997), a variety of thickness from 0.05mm to 1.0mm was used in the laboratories worldwide, and no definitive conclusion concerning this issue has been made. Only a minor effort was undertaken, and it was found that there was no apparent difference between the teflon sheets with the thickness of 0.127mm and 0.254mm. Researchers in Tsinghua University of China conducted extensive program (Guo 1997) to search for the most appropriate end friction-reducing material, and it came out to be that the 2mm thick teflon sheet seemed to be quite efficient.

But, the disadvantage of the teflon sheet is its much lower elastic modulus and notably higher Poisson's Ratio. Much like the rubber, the Poisson's Ratio of teflon can reach 0.46 while the elastic modulus is as low as $0.3\sim0.8$ GPa(43~115 ksi). Given a Poisson's Ratio for the concrete ($f_c'=8$ ksi) of 0.18 and the elastic modulus of 35GPa(5000 ksi), we can have the following large discrepancy:

$$\left(\frac{\nu}{E}\right)_{\text{teflon}} = \frac{0.46}{0.3\text{GPa}} = 0.15/\text{GPa} \gg \left(\frac{\nu}{E}\right)_{\text{concrete}} = \frac{0.18}{35\text{GPa}} = 0.005/\text{GPa}$$

It can be found clearly that the $\left(\frac{\nu}{E}\right)_{\text{teflon}}$ is almost 30 times as large as $\left(\frac{\nu}{E}\right)_{\text{concrete}}$, which

raises a doubt that the excessive lateral expansion of the teflon will exert a reversal of the boundary restraint as shown in Figure 2.5, although teflon itself has a very low friction coefficient.

2.1.3.3 Combined End-Friction Reducing Measure Based on all the comments and analysis made above, it may be a feasibly better way to solve this dilemma by adopting a Sandwich-like setup combining teflon sheet, fairly thin aluminum foil and bearing grease (see Figure 2.6).

For the ordinary aluminum, the elastic modulus is around 73GPa(10600ksi) and

the Poisson's Ratio is about 0.33, so $\left(\frac{v}{E}\right)_{\text{aluminum}} = \frac{0.33}{73\text{GPa}} = 0.0045/\text{GPa}$, which is

almost equal to $\left(\frac{\nu}{E}\right)_{\text{concrete}}$ ($f_c' = 8$ ksi). The bearing grease between the teflon sheet and the elements of the symplectic structure that the symplectic structure that the symplectic structure that the symplectic structure that the symplectic structure structure that the symplectic structure struc

the aluminum foil will further reduce the friction between them. The combined effect will probably make the concrete specimen comparatively closer to the "true" compression.

The thickness of the teflon sheet and aluminum foil should be fairly thin in order to further decrease the restraint force. It is postulated that the thickness of 0.1mm~0.2mm will be appropriate in the future uniaxial and triaxial experiments.



Figure 2.6 Combined End-Friction Reducing Measure.

2.1.4 Determination of Poisson's Ratio

Probably the most classic description of the Poisson's ratio variation with stress condition is from Ottosen's constitutive model (Ottosen 1979). A stress index is predefined as $\beta = \sigma_1 / \sigma_{1f}$ such that with σ_2 and σ_3 being constant, concrete failure will occur when σ_1 reaches σ_{1f} . Then the secant Poisson's Ratio v_s can be expressed as following:

$$\beta < 0.8, v_s = v_i = \text{constant}$$
 (2-2a)

$$0.8 < \beta < 1.0, \quad v_{s} = v_{f} - (v_{f} - v_{i}) \sqrt{1 - \left(\frac{\beta - 0.8}{0.2}\right)^{2}}$$
 (2-2b)

where: for normal strength concrete, $v_i = 0.16 \sim 0.20$, $v_f = 0.36$

There is a dearth of data available to illustrate the Poisson's ratio of the HSC since it is hard to measure. Normally the ASTM standard testing method is adopted to

measure the Poisson's Ratio at 40% f_c , and in most cases the Poisson's Ratio is assumed to be constant.

Candappa et. al (2001) studied the Poisson's ratio variation for HSC. Using 40MPa, 70MPa and 100MPa concrete, he found out that the trends of the Poisson's ratio variation for all those concretes were essentially the same with that stipulated by Ottosen's model, but with a smaller β value of 0.7 for 40MPa concrete. He also found out that the peak secant Poisson's ratios for all the three concretes, around 0.5, were much larger than 0.36 stipulated in the Ottosen's model. Meanwhile, he observed that after the peak, the Poisson's ratio continued to increase, and shortly after the stress dropped a little bit, the Poisson's ratio could even reach 1.0.



Figure 2.7 Poisson's Ratio variation in Ottosen's model.

However, a critical issue is raised on how the Poisson's ratio is measured. Most researchers paid much attention to improve the accuracy of the testing devices. They employed clip gage (or circumferential extensometer) and axial extensometer to replace the traditional electrical strain gages, but they all neglected a possible defect for the standard setup— end restraint. It has been shown in the previous section that the friction between the loading platen and specimen will act as a lateral confinement at the specimen end, thus increasing the strength of the specimen. Due to the same reason, the internal cracks developed in the middle part of the concrete under certain compressive stress are restrained by the concrete portion at both ends. So it is quite possible that because of the existence of those cracks, the lateral expansion of the middle part of concrete will develop much faster than the axial contraction, leading to a rapid increase of the Poisson's ratio near failure.

Another issue which needs to be considered is whether it is meaningful to measure the Poisson's ratio in the post peak region. Poisson's ratio represents the ratio of lateral strain to axial strain under a uniaxial normal stress. However, after the concrete passes the peak stress point, numerous visible and invisible cracks have been formed. At this time, the concrete is not the "pure concrete". If the Poisson's ratio is still considered, the widening of the cracks is inevitably counted as a part, maybe an important part of the lateral strain. So far, this issue has not been addressed clearly by any researcher.

Therefore it is of importance to measure the Poisson's ratio of concrete in a "pure" compression status, or at least in a practically "true" uniaxial compressive condition, thus to provide accurate description about the actual variation of Poison's ratio with the stress conditions.

2.2 Triaxial Tests

2.2.1 Axial Stress~Strain Response under Lateral Confining Pressures

Up until now, most of the axial stress~strain curves under lateral confinement are constructed using the triaxial compression tests in which longitudinal stress is applied as

the confining stress is held constant (Xie et. al 1995, Imaran and Pantazopoulou 1996, and Candappa et. al 2001). In this case, the lateral confining pressure is always applied to the target value before the application of axial load, and then the axial load is gradually applied. The following graph (Figure 2.8) from Xie et. al (1995) has been occasionally quoted by many researchers as the typical curves for the HSC under confining pressure.



Figure 2.8 Typical axial stress ~ strain curve for HSC under different confining pressures and load path used.

The characteristics of this type of stress ~ strain curve can be summarized as follows:

- With the increase of the confining pressure, the axial strength of the HSC increases noticeably. From 0 to 29.3 MPa, the increase of confining pressure will cause more than 3 times of axial strength gain.
- 2) With the increase of the confining pressure, the axial peak strain value also increases enormously. It jumps from about 0.0025 under uniaxial compression to roughly 0.025 under a confining pressure of 29.3 MPa.
- Under comparatively higher lateral confining pressures, the HSC behaves much more like a ductile material.
- 4) Due to the lateral constraint of the confining pressure, it will take more axial load to reach a certain longitudinal deformation. Therefore the initial ascending part of the curve under higher confining pressure is fairly steeper than the one under lower confining pressure.

Some researchers (Ansari and Li 1998, and Chern 1992) tended to construct the axial stress-strain curve under confining pressure using the proportional load path in which the axial and lateral confining pressures were simultaneously applied to the specimen until the confining pressure reached the targeted value, and then the axial stress was further increased to failure (see figure 2.9). Some discrepancies exist in those axial stress-strain curves under this load path (see Figure 2.9 (Ansari and Li 1998) and Figure 2.10 (Chern 1992)). In Figure 2.10, the slopes of the initial ascending part of the curve under some confining pressures tend to be smaller than that of the uniaxial compression, whereas the opposite situation happens in Figure 2.11.



Figure 2.9 Load path of proportional loading.

2.2.2 Axial Stress~Strain Curves for Steel Fiber Reinforced Concrete under Lateral Confining Pressures

Chern (1992) reported early in 1992 that the increase of the volume ratio of steel fiber in the normal strength concrete does not influence the axial strength and deformation capacity notably under a confining pressure less than 30 MPa (see Figure 2.11). However, when this lateral confining pressure reaches 70MPa ($\sigma_{2/3} / f_c' = 350\%$), the strengthening effect of the steel fiber begins to manifest itself with 1% of steel fiber leading to roughly 10% of axial strength gain.



Figure 2.10 Axial stress~strain curve under proportional loading path (Ansari 1998).



Figure 2.11 Axial stress ~ strain curve under proportional loading path (Chern 1992).



Figure 2.12Axial stress ~ strain response of SFRC with different volume ratio of fiber.(a) uniaxial(b) confining pressure 10 MPa

2.2.3 Stress ~ Strain Curve in Multiaxial Stress Condition for Nonlinear Analysis

It can be found out from the literature search that the previous researches have focused mainly on the qualitative aspects of the triaxial stress \sim strain curves, for instance, the general trend of the axial stress versus the longitudinal strain under certain lateral

confining pressures. However, this kind of information is not enough to provide a guideline for the nonlinear analysis of concrete structures subjected to multiaxial loading condition, since it has not established an applicable constitutive model for concrete under various stress conditions.

In the nonlinear analysis, the triaxial stress~strain curve usually adopts the basic shape of the uniaxial stress ~ strain curve. For instance, in the Ottosen's constitutive model (Fig. 2-13), a nonlinear index β is defined as σ_{1c}/σ_1 such that with σ_2 and σ_3 remaining constant, the concrete fails when the axial stress σ_1 reaches σ_{1c} . Sargin's uniaxial compressive stress ~ strain curve is used (Guo 1997):

$$-\frac{\sigma}{f_c} = \frac{-A\frac{\varepsilon}{\varepsilon_c} + (D-1)\left(\frac{\varepsilon}{\varepsilon_c}\right)^2}{1 - (A-2)\frac{\varepsilon}{\varepsilon_c} + D\left(\frac{\varepsilon}{\varepsilon_c}\right)^2}$$
(2-3)

and all the parameters in the equation are replaced by corresponding triaxial ones:

$$-\frac{\sigma}{f_c} = \frac{\sigma}{\sigma_{1f}}, \quad A = \frac{E_0}{E_p} = \frac{E_i}{E_f}, \quad \frac{\varepsilon}{\varepsilon_c} = \frac{\varepsilon}{\varepsilon_f} = \frac{\sigma/E_s}{\sigma_{1f}/E_f}$$
(2-4)

where

 E_0 : initial tangent elastic modulus in uniaxial stress ~ strain curve; E_p : peak secant elastic modulus in uniaxial stress ~ strain curve;

- E_i : initial tangent elastic modulus in triaxial stress ~ strain curve;
- E_f : peak secant elastic modulus in triaxial stress ~ strain curve;

and E_s : peak secant elastic modulus in triaxial stress ~ strain curve.



Figure 2.13 Uniaxial and triaxial stress ~ strain curves in Ottosen's model.

This kind of approximation does not reflect the fact that the uniaxial compressive stress~strain curve is different from that under multiaxial stress conditions (see Figure 2.7). And, as it will be shown later, it is very difficult to accurately define those triaxial parameters like E_f and ε_f .

Another way to develop a equivalent triaxial stress~strain curve, as adopted in the ANSYS, is to use Von Mises equivalent stress (σ_e^t) or strain (ε_e^t) :

$$\varepsilon_{e}^{t} = \frac{1}{\sqrt{2}(1+\nu)} \left[\left(\varepsilon_{x} - \varepsilon_{y} \right)^{2} + \left(\varepsilon_{y} - \varepsilon_{z} \right)^{2} + \left(\varepsilon_{z} - \varepsilon_{x} \right)^{2} + \frac{3}{2} \left(\varepsilon_{xy}^{2} + \varepsilon_{yz}^{2} + \varepsilon_{zx}^{2} \right) \right]^{1/2}$$

$$= \frac{1}{\sqrt{2}(1+\nu)} \left[\left(\varepsilon_{1} - \varepsilon_{2} \right)^{2} + \left(\varepsilon_{2} - \varepsilon_{3} \right)^{2} + \left(\varepsilon_{3} - \varepsilon_{1} \right)^{2} \right]^{1/2}$$

$$\sigma_{e}^{t} = \frac{1}{\sqrt{2}} \left[\left(\sigma_{1} - \sigma_{2} \right)^{2} + \left(\sigma_{2} - \sigma_{3} \right)^{2} + \left(\sigma_{3} - \sigma_{1} \right)^{2} \right]^{1/2}$$
(2-5)
$$(2-5)$$

This equivalent triaxial stress \sim strain curve can be degraded into uniaxial compression or tension curve under respective situations. However, this equivalent triaxial stress \sim strain curve needs further study since it combines all the stresses, strains,

and Poisson's ratio in the multiaxial stress condition together, and it does not consider the variation of Poisson's ratio with respect to the stress status. Furthermore, there is a dearth of information available to prove the similarity between this type of curve and the actual triaxial stress ~ strain behavior of concrete.

2.2.4 Strength of Concrete Under Triaxial Stress Conditions

2.2.4.1 Failure Criteria of Concrete In formulating the general failure criterion, a clear concept of "failure" should be defined. Chen (1982) defined the failure of the concrete as "the ultimate loading capacity of a test specimen". Failure of the concrete elements usually occurs under complex loading conditions, therefore, the state of stress at failure is generally complicated. The study on concrete behavior under multiaxial stress conditions is essential to develop the general failure criterion.

A failure criterion of isotropic materials based on the multiaxial stress conditions must be an invariant function of stress state independent of the choice of the coordinate system by which stress is defined:

$$f(\sigma_1, \sigma_2, \sigma_3) = 0 \tag{2-7}$$

where:

where:

 $\sigma_1, \sigma_2, \sigma_3$ are three principal stresses at failure.

However, those three principal stresses can be expressed in term of the combinations of three principal stress invariants I_1 , J_2 and J_3 , and for the brittle material like concrete, the failure criterion is affected by the hydrostatic pressure ($I_1/3$). Therefore the failure criterion can be further developed as:

$$f(I_1, J_2, J_3) = 0$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$
(2-8)

$$J_{2} = \frac{1}{6} \left[(\sigma_{1} - \sigma_{2})^{2} + (\sigma_{2} - \sigma_{3})^{2} + (\sigma_{3} - \sigma_{1})^{2} \right]$$
$$J_{3} = (\sigma_{1} - \sigma_{m})(\sigma_{2} - \sigma_{m})(\sigma_{3} - \sigma_{m})$$

It has been found (Guo 1997 and Chen 1982) that the failure surface of concrete in the three coordinate axes of principal stress has a nearly triangular cross section for the small stresses and becomes increasingly bulged (more circular) at high hydrostatic stress conditions. Numerous types of concrete failure criteria have been developed aimed at defining the shape of the failure surface, and according to the number of variables, the failure criteria can be categorized into one-parameter, two-parameter, three-parameter, four-parameter and five parameter models. The heydays of the failure criteria study came around 1970s, when the famous Druker-Prager criterion, Ottosen criterion, Willam-Warnke criterion and Hsieh-Ting-Chen criterion were all published. Of them, the 5parameter Willam-Warnke criterion was considered to be the most appropriate one to describe the triaxial behavior of the concrete, so that Chen (1982) stated in early 1980 that "In view of fluctuations of experimental results, there is little need for further refinements of present failure-surface model".

Right now the 5-parameter Willam-Warnke criterion has been widely used in the multiaxial FE analysis of concrete elements, including the most acknowledged commercial computing software – ANSYS. However, the 2-parameter Mohr-Coulomb criterion has also been successfully applied to concrete due to its simplicity and fairly reasonable accuracy in some specific cases.

2.2.4.2 Mohr-Coulomb Failure Criterion

2.14) is the classic failure criterion used in many applications, and assumes only two different failure modes: the sliding (shear) failure and the separation (tensile) failure.

The Mohr-Coulomb Criterion takes the form as follows:

$$|\tau| = c + \sigma \tan \phi \tag{2-9}$$

The Mohr-Coulomb Criterion (Figure



Figure 2.14 Mohr-Coulomb failure criterion.

where: τ : the maximum shear stress of failure plane

c: cohesion

 ϕ : internal friction angle

and σ : normal stress acting on the failure plane (compression positive)

The equation above can be rewritten by using only the principal stresses as:

$$\frac{\sigma_1}{f_c} = 1 + k \frac{\sigma_3}{f_c} \qquad k = \frac{1 + \sin\phi}{1 - \sin\phi}$$
(2-10)

where σ_1 and σ_3 are the major and minor principal stresses (compression is positive) respectively, and $f_c^{'}$ is the uniaxial compressive strength.

The main disadvantages of Mohr-Coulomb criterion can be summarized as follows:

- 1) The intermediate stress σ_2 is not considered for the failure. This will easily lead to an obviously erroneous conclusion that the uniaxial and biaxial compressive strengths of concrete are the same, which is contradictory to the test results (Chen 1982).
- 2) The tensile and compressive meridians are all straight lines, which become far deviant from the actual failure surface when the hydrostatic pressure is high. And the failure surface is not a smooth one, also contradictory to the common consensus.
- 3) If tension is considered in this criterion, it will inevitably lead to a relationship between the uniaxial compressive (f_c) and uniaxial tensile stress (f_t) — $\frac{f_c}{f_t} = \frac{1 + \sin \phi}{1 - \sin \phi} = k$. Normally, the ratio $\frac{f_c}{f_t}$ of normal strength concrete is around 10~15. However, according to the early test result of Richart et. al (Chen 1982), the k is roughly 4.1, which is far less than the value derived by the criterion.

Although the Mohr-Coulomb Criterion has so many apparent weaknesses, due to extreme simplicity and fair approximate estimation in manual calculations, up until now a lot of researches have still been undertaken especially on the normal HSC (Xie et. al 1995, Imaran and Pantazopoulou 1996, Cadappa et. al 2001, Ansari and Li 1998, and Lan 1997) and steel fiber reinforced concrete (Nielson 1998 and Pantazopoulou 2001) (Table 2.1).

There seems be an obvious disagreement among those researchers on the value of k, which varies from the lowest 2.6 to a much higher 6.7. Table 2.1 lists the testing results of k value from some researchers around the world.

Some researchers gave detailed descriptions about the failure mode of the concrete specimen subjected to lateral confining pressure, which is totally different from that of the uniaxial compression. Chern (1992) found that the cylinders, with or without steel fiber, all failed in a shear mode under confining pressures from 10MPa to 70MPa $(\sigma_3 / f_c = 50\% \sim 350\%)$; Ansari and Li (1998) stated that the majority of the HSC cylinders failed in shear as indicated by 45 degree diagonal crack in the specimens under lateral confining pressure ratios σ_3 / f_c ranging from 20% to 90%; Guo (1997) used the cubic concrete specimens and found out that all the specimens were compressed along the axial direction while stretched in the other two, and there were obvious inclined cracks on lateral surfaces, which hinted a inclined shearing failure mode; Sfer and Carol (2002) studied the cylinder failure modes under $\sigma_3 / f_c^{'}$ from 25% to 180%, and they found that under relatively lower confinement, the cylinders were likely to fail in a inclined failure mode, much the same as described by the other people, but when the confining pressure became lager, a failure of the combination of split and shear seemed to dominate (see Figure 2.15). It should be noted that in their experiments, Sfer and Carol inserted a 0.1mm thick teflon sheet between the loading platen and the cylinder end to minimize the end friction.

Table 2.1 Summarization of k in $\frac{\sigma_1}{f_c} = 1 + k \frac{\sigma_3}{f_c}$

				1	
Ref.	Concrete Type	Specimen	Load Path	σ_3/f_c	k
Richart (Chen 1982)	Normal strength	Cylinder			4.1
	HSC 60MPa	Cylinder d55 ×		50%	4.85
Xie et. al (1995)	HSC 92MPa	h110 mm	Figure 2.8		5.09
	HSC 119MPa				4.47
	HSC 73MPa	Cylinder		70%	3.42
Imran	HSC 47MPa	d54 × h108 mm	Figure 2.8	91%	3.75
(1996)	Normal strength, 29MPa			73%	4.19
Candappa (2001)	HSC 42,61,73,103 MPa	Cylinder d100 × h200 mm	Figure 2.8	30%	5.30
Ansari and	HSC 48MPa	Cylinder		88%	3.0
Li (1998)	HSC 71, 107 MPa	d100 × h200 mm	Figure 2.9	93%	2.6
Nielsen	SFHSC 165 MPa, fiber	Cylinder d100 ×	Figure 2.9	40%	5.07
(1998)	volume ratio 4%	h200 mm	_	85%	4.05
Guo (1997)	Normal strength 24MPa	Cubic 70.7 mm	Figure 2.9	90%	5.26
Sfer and	Normal strength	Cylinder	Figure 2.9	30%	3.76
(2002)	<i>52, 5000 a</i>	h300 mm	1 Iguit 2.9	183%	2.80
Chern	Normal strength 20.5MPa SFC 22.8MPa yealume	Cylinder	Figure 2.0	195% 342% 175%	3.65 2.98 3.83
Chern (1992)	ratio 1%	d54 ×	1 iguit 2.7	307%	3.25
	SFC	h108 mm		163%	3.94
	ratio 2%			286%	3.43



Figure 2.15 Schematic failure mode under different σ_3 by Sfer and Carol (2002).

It can be seen clearly from all the testing results in Table 2.1 that although there is a great disagreement on the ultimate strength (k value) of concrete for the Mohr-Coulomb criterion, the general failure modes are essentially the same—shear failure. If one applies the Mohr-Coulomb failure criterion with a tension cutoff (maximum strain or stress), this combined failure criterion will describe the failure of concrete as either tensile or shear types. The tensile type of splitting fracture is governed by the maximum stress or strain, while the shear type of slip fracture is controlled by the maximum-shear-stress criterion of the Mohr-Coulomb criterion.

There is a trend that under lower confining pressures the k value seems to be higher, and when the confining pressure increases, the slope of the $\sigma_1 / f_c \sim \sigma_3 / f_c$ relationship becomes much smoother. It is reasonable since it has been found out that the meridians of the concrete failure surface are bulged curves rather than straight lines. But given the extreme simplicity and that most of the applicable cases are under a fairly low level of confinement, the Mohr-Coulomb criterion can still be employed as a first estimation of concrete strength under multiaxial compressive status.

However, from the previous research results, one can not find a consensus on the k value, neither can he draw a conclusion on the influence of concrete strength (f_c) ,

confinement level (σ_3 / f_c) , steel fiber introduction and cylinder type to this k value. The testing results are just scattered, and it is difficult to judge which one is more reliable and appropriate. Therefore it is of importance to enrich the existing database to provide more information in determining the k, especially for the HSC and SFHSC under normal confinement level.

2.2.4.3 Five-Parameter Willam Warnke Criterion The tensile meridian

and compressive meridian on the failure surface of the concrete can be expressed as two parabolas in terms of $\frac{\tau_m}{f_c}$ and $\frac{\sigma_m}{f_c}$ to fit the actual curved meridians obtained from the

multiaxial experiments:

$$\frac{r_t}{\sqrt{5}f_c} = \frac{\tau_{mt}}{f_c} = a_0 + a_1 \frac{\sigma_m}{f_c} + a_2 \left(\frac{\sigma_m}{f_c}\right)^2 \qquad \left(\eta = 0^\circ\right) \tag{2-11a}$$

$$\frac{r_c}{\sqrt{5}f_c} = \frac{\tau_{mc}}{f_c} = b_0 + b_1 \frac{\sigma_m}{f_c} + b_2 \left(\frac{\sigma_m}{f_c}\right)^2 \qquad (\eta = 60^\circ)$$
(2-11b)

where: rt and rc: the distance from the tensile meridian and compressive meridian to the

hydrostatic axis respectively (Fig. 2-16)

$$\sigma_m = I_1 / 3 = (\sigma_1 + \sigma_2 + \sigma_3) / 3$$

$$\tau_m = \frac{1}{\sqrt{15}} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2}$$



Figure 2.16 Tensile (η =0°) and compressive meridians (η =60°).

On the deviatoric plane, three identical parts of an ellipse are used to connect those points on the meridians. So eventually, the failure surface on the deviatoric plane (Fig.2-17) can be expressed as:

$$r(\sigma_m,\eta) = \frac{2r_c(r_c^2 - r_i^2)\cos\eta + r_c(2r_i - r_c)\left[4(r_c^2 - r_i^2)\cos^2\eta + 5r_i^2 - 4r_ir_c\right]^{\frac{1}{2}}}{4(r_c^2 - r_i^2)\cos^2\eta + (r_c - 2r_i)^2} \quad (2-12)$$

where:

$$\cos \eta = \frac{2\sigma_1 - \sigma_2 - \sigma_3}{\sqrt{2} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{\frac{1}{2}}}$$

$$r_{c}$$
, $r_{i} = \frac{1}{\sqrt{3}}\sqrt{(\sigma_{1} - \sigma_{2})^{2} + (\sigma_{2} - \sigma_{3})^{2} + (\sigma_{3} - \sigma_{1})^{2}} = \sqrt{3}\tau_{oct}$

$$\sigma_m = I_1 / 3 = (\sigma_1 + \sigma_2 + \sigma_3) / 3$$



Figure 2.17 Failure surface and deviatoric plane.

Xie et. al (1995) conducted an extensive study on the compressive meridian of HSC. In his research, three different HSCs with $f_c^{'}$ of 60.2MPa, 92.2MPa and 119MPa were tested under triaxial stress condition. The lateral confinement level $\sigma_3 / f_c^{'}$ was approximately 50%, and proportional loading path (Figure 2.8) was adopted. The compressive meridians of those three concretes are shown in Figure 2.18.



Figure 2.18 Compressive meridians of HSC by Xie et. al (1995).

It can be seen from Figure 2.18 that the compressive meridians of HSC with different compressive strength are essentially the same, and the curves fit the Willam-Warnke's meridian expression very well ($R^2=0.9923\sim0.9993$). There seems to be a trend that the compressive meridian of concrete with higher strength tends to be lower than those with lower compressive strength.

Imaran and Pantazopoulou (1996) studied the triaxial behavior for both the normal strength concrete (f_c = 28.6MPa) and HSC (f_c = 47.4MPa and 73.4MPa). The confinement level he used is approximately 70%~90% f_c (Figure 2.19)



Figure 2.19 Compressive meridians of HSC by Imaran (1996).

Ansari an Li (1998) also studied three HSCs with f_c of 48MPa, 72MPa and 109MPa respectively. The confining pressure used in this research is around 90% f_c (see Figure 2.20).



Figure 2.20 Compressive meridians of HSC by Ansari and Li (1998).

From Figure 2.20 it shows that the curved meridians of higher strength concrete seem to be more deviant from that of concrete with relatively lower compressive strength than in the two previous cases.

If all the triaxial data of the HSC are put together in one graph (see Figure 2.21), large discrepancies will be observed. It is hard to say which one is more appropriate for HSC since there are not enough additional testing results available.



Figure 2.21 Compressive meridians for HSC from different researchers.

Chern (1992) conducted an extensive program on the triaxial behavior of the steel fiber reinforced normal strength concrete. He changed the volume ratio of steel fiber in the concrete from 0%, 1% to 2%, and the concrete compressive strength $f_c^{'}$ was only around 22MPa. In his tests, the confining pressure level was up to 340% $f_c^{'}$. The compressive and tensile meridians of those concretes are shown in Figure 2.22.

Ishikawa et. al (2000) also studied the tensile and compressive meridians for the HSC with 1% volume ratio steel fiber and a $f_c^{'}$ of 80MPa. The results are also listed in Figure 2.22 to compare with those of Chern's normal strength concrete experiments.



Figure 2.22 Tensile and compressive meridians of steel fiber reinforced concrete.

From Chern's results, it can be seen that under relatively lower confining pressures, all the compressive meridians for concrete with 0%, 1% and 2% steel fiber are essentially the same. However, when high confining pressure (around 100% $f_c^{\,i}$) occurs, the concrete with higher volume ratio of steel fiber tends to show stronger triaxial strength behavior. But Ishikawa's equation for HSC is remarkably deviant from Chern's, even under low confining pressures. One cannot attribute this to HSC since no other sources of data is available.

However, the tensile meridians for all the concrete, regardless of high strength or normal strength and with steel fiber or without, follow the same trend.

CHAPTER 3

UNIAXIAL COMPRESSION OF STEEL FIBER REINFORCED HIGH STRENGTH CONCRETE (SFHSC)

3.1 Lubricated Loading Platen

Generally, as discussed in Chapter 2, there are two kinds of end friction-reducing measures — brush platen and lubricated loading platen. The brush platen was deemed as the best testing apparatus to achieve the true uniaxial compression (Mindess et. al 2002 and Guo 1997). It is capable of providing relatively better and consistent testing results which have been widely quoted by some researchers (Rashid et. al 2002, Kupfer et. al 1969, Van Mier et. al 1997, Gerstle 1981, and Kupfer and Gerstle 1973). However, this kind of loading platen has a very complicated structure and high cost, and meanwhile, large deviations still exist in the testing results.

The more economic alternative is to use the lubricated loading platen which is aimed at decreasing the friction between the loading platen and the end of the concrete cylinder. Teflon sheet is widely accepted as a practical option for reducing end friction (Mindess et. al 2002, Guo 1997, Kupfer et. al 1969, Van Mier et. al 1997, Gerstle 1981, and Kupfer and Gerstle 1973). However, there seems to be no consensus on the effect of its thickness. In the joint research program coordinated by RILEM (Van Mier et. al 1997), a variety of thickness from 0.05 mm to 1.0 mm were used in the laboratories worldwide, while no definitive conclusion concerning thickness has been made. It has only been found that there is no apparent difference between the teflon sheets with the thickness of 0.125mm and of 0.250mm. Researchers in Tsinghua University of China (Guo 1997) recommended that the teflon sheet with a 2mm thickness should be used.



Figure 3.1 Designed lubricated loading platen.

A specially designed lubricated loading platen as shown in Figure 3.1 is used in this study. Two 0.125mm (0.005 in.) thick teflon sheets are used to exploit the much lower friction between them. The thin 0.015 mm (0.0006 in.) thick aluminum foil has twofold functions. First, bearing grease can be thinly applied between this foil and the teflon sheet without inducing the possible reversal of boundary restraint (Van Mier et. al 1997) as has been discussed in Chapter 2. And secondly, since the v/E of the aluminum is quite close to that of the HSC, it will lead to a almost identical lateral expansion for both the concrete specimen and the aluminum foil under uniaxial compression.

3.2 Lateral Strain Measurement

In this study, circumferential extensometers were used to detect the lateral strain of the concrete cylinder. The circumferential extensometer is a specially designed "clip gauge" for measuring the average lateral expansion along the whole circumference of the cylinder, and it is advantageous over the normally used electrical strain gauge which can only catch the length change of a small portion of the total cylinder circumference. Given the remarkable heterogeneity of the concrete and highly localized deformation, the

circumferential extensometer is apparently much more reliable to provide accurate and consistent average circumferential deformations, which can then be converted easily into the lateral strains.

A solid stainless steel rod with a diameter of 4"(100mm) and a height of 8"(200mm) was used to verify the accuracy of the circumferential extensometer, which was installed right at the mid-height of the rod together with an electrical strain gage along the circumferential direction. A 900 kN compressive load was applied to the rod and the lateral strains detected by the circumferential extensometer and strain gage are shown in Figure 3.2.



Figure 3.2 Relationship between the lateral strains from the circumferential extensioneter and strain gage.

It can be clearly seen from Figure 3.2 that the lateral strains from circumferential extensometer and strain gage exhibit a terrific linear relationship. The slope is slightly less than 1.0 just because those two measuring instruments could not be perfectly positioned symmetrically opposite to each other along the height of the specimen.

3.3 Experimental Program

The concrete mix proportions are listed in table 3.1. Lafarge Type I cement, 3/8" (10mm) crushed gravel satisfying ASTM C33 gradation requirement, and ASTM No. 2 grade river sand were employed for all the mixes. Hook-ended, 30 mm long steel fiber with a diameter of 0.5 mm was used. To achieve good workability, highly effective superplasticizer was introduced to all the mixes. Right after mixing, the fresh concrete was cast into 4"×8" (10mm×20mm) standard cylinder molds, and then they were immediately moved into the curing room in which 100% humidity and a nearly constant temperature of 20~25°C were maintained. After 2 days of curing inside, the specimens were demolded. At 21 days, the specimens were taken out, sawed to the required lengths, carefully ground for both ends and taken back to the curing room until the testing age.

Table 3.1	Mix Proport	ions of HSC	and SFHSC
Table 3.1	Mix Proport	ions of HSC	and SFHSC

			· · · · · · · · · · · · · · · · · · ·
	HSC	SF-0.5%	SF-1.0%
$f_c^{'}$ at 28 days (MPa)	77.5	80.1	79.0
Initial elastic modulus E ₀ (MPa)	46500	45900	48800
Volume ratio of steel fiber	0	0.5%	1.0%
Cement (kg/m ³)	520	520	520
Sand (kg/m ³)	760	760	760
Coarse aggregate (kg/m ³)	1000	1000	1000
Water (kg/m ³)	170	170	170
Water-cement ratio	0.33	0.33	0.33
Superplasticizer (L/m ³)	4.3	4.3	4.3
Slump (mm)	225	190	85

All the uniaxial compression tests were performed in a very stiff frame MTS servo-hydraulic closed loop testing machine which has a frame stiffness of 11.0×10^9 N/m

 $(6.3 \times 10^7 \text{ lb/in})$. Displacement control mode with a constant displacement rate of 0.005 mm/s (0.0002 in/s) was employed in all tests.

3.4 Experimental Results

3.4.1 Uniaxial Compressive Strength

The compressive strengths of the SFHSC with different h/d ratios under different loading platens are listed in Table 3.2. All the results are averages of 3 specimens. It can be seen from the table that under rigid loading platen, the compressive strength tends to increase when h/d ratio is decreased from 2.0 to 1.0. At h/d =1.0, the compressive strength is $7\%\sim10\%$ higher than that of h/d=2.0, indicating the influence of h/d ratio on the end restraint effect. But under the designed lubricated loading platen, the compressive strengths of different h/d ratios are almost exactly the same. For a standard cylinder (h/d=2.0), the strength ratio between lubricated and rigid loading platen is slightly below 93%.

Loading platen	1% stee	l fiber volu	me ratio	0.5% steel fiber volume ratio			
	h/d =1.0	h/d =1.5	h/d =2.0	h/d =1.0	h/d =1.5	h/d =2.0	
Rigid (MPa)	87.5	81.8	79.0	85.6	80.5	80.1	
Lubricated (MPa)	73.9	73.9	73.3	73.9	73.7	73.9	
Lubricated/ Rigid	84.5%	90.3%	92.8%	86.3%	91.6%	92.4%	

T	ab	le 3	3.2	U	Jni	axia	al (Con	ipre	essiv	ve S	Stre	engt	h o	f S	SFF	HS (С
													<u> </u>					

3.4.2 Deformations

Two circumferential extensometers were employed to measure the lateral strains of nearly all the specimens, with one being installed right at the mid-height and the other approximately 1/6h from one end. The purpose is to demonstrate whether the specimen has a tendency of a uniform lateral expansion under the uniaxial compression. Axial extensometers were used to detect the axial strains, with 4 inch gage length for h/d = 2.0and 2 inch gage length for h/d = 1.0 and 1.5. The axial and lateral strains at peak load for specimens with different h/d ratios under two types of loading platens are shown in Table 3.3, Table 3.4 and Figure 3.3, respectively.

	Axial str	ain at pe	ak (με)	Lateral strain at peak (με)						
Loading	h/d	h/d	h/d	h/d=	1.0	h/d=	1.5	h/d=2	2.0	
platen	=1.0	=1.5	=2.0	middle	end	middle	end	middle	end	
Rigid	2735	2673	2632	1966	-	1561	1042	1328	971	
Lubricated	2511	2576	2424	1279	1091	1084	1062	751	941	

 Table 3.3
 Deformation of SFHSC (1% Fiber Volume Ratio) at Peak

 Table 3.4
 Deformation of SFHSC (0.5% Fiber Volume Ratio) at Peak

Loading platen	Axial strain at peak (με)				Lateral strain at peak (με)						
	h/d	h/d	h/d	h/d=	1.0	h/d=1.5		h/d=2.0			
	=1.0 =1.5 =2	=2.0	middle	end	middle	end	middle	end			
Rigid	3180	2836	3042	1778	-	1667	1266	1671	1013		
Lubricated	2456	2673	2572	1125	836	1120	1024	812	884		

Under rigid loading platen, for h/d=1.5 and 2.0, the peak lateral strains at the midheight of the specimen are $37\%\sim65\%$ larger than those correspondingly near the end. These notable differences clearly demonstrate the considerable end restraint imposed to the specimen by the rigid loading platen. But once the designed lubricated loading platen was employed, those strain differences were greatly diminished (see Figure 3.3). mid-height zend





Under the lubricated loading platen, there is a general trend that at h/d=2.0, the peak lateral strain at the mid-height is less than that near the end, and at h/d=1.0 a reverse situation occurs. However, the h/d of 1.5 seems to be the transition point where the lateral strains at the mid-height and end are essentially the same. Thus in terms of uniform lateral expansion, h/d=1.5 tends to be more appropriate in providing the true uniaxial compression test.

Figure 3.4 shows the stress ~ strain curves of 1% fiber volume ratio SFHSC cylinders with h/d = 1.5 under both rigid and lubricated loading platens. It can be clearly seen that under lubricated loading platen, the lateral strains at both mid-height and end comply with each other pretty well not only before but also at and after peak. However, those lateral strains under rigid loading platen start to diverge from each other around 90% of the peak load, and after peak, the difference increase tremendously. The 0.5% fiber volume ratio SFHSC also behaves in the same manner.



Figure 3.4 Uniaxial stress ~ strain curve of 1% fiber volume ratio SFHSC (h/d=1.5).

The lateral strains measured at the mid-height of the specimens, as have been done by most previous researches in studying the Poisson's ratio and bulk modulus, are remarkably different under two different loading platens. For instance, the average midheight peak lateral strain of the 1% fiber volume ratio SFHSC with h/d=1.5 was apparently reduced from 1561 $\mu\epsilon$ under rigid loading platen to 1084 $\mu\epsilon$ under the lubricated one — a considerable 31% decrease, but the corresponding axial strains only experienced comparatively a much lower 4% reduction.

3.4.3 Volume Change and Bulk Modulus K

Bulk modulus K represents the volumetric stress and strain relationship of concrete and is defined as:

$$K = \frac{d\sigma_{oct}}{3d\varepsilon_{oct}}$$
(3-1)

where

re:
$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$
 and $\varepsilon_{oct} = \frac{1}{3}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

Testing data under lubricated loading platen and a h/d ratio of 1.5 were adopted to achieve the variation of bulk modulus K under different stress conditions, since they would be comparatively consistent with the uniform lateral expansion under uniaxial compression, as can be seen from Figure 3.3. The results are shown in Figure 3.5 and Figure 3.6 respectively.

The K variations of those two types of steel fiber reinforced SFHSC follow the same trend. Initially, a linear elastic response between the octahedral stress σ_{oct} and volume change $(3\varepsilon_{oct})$ develops up to approximately 95% of the ultimate strength, and then the concrete starts to dilate. After peak, σ_{oct} tends to decrease linearly with the volume expansion of the concrete, with a K about 5%~6.6% of that in the initial elastic volumetric contraction stage (K₀). Therefore, it may be fairly reasonable to take only two constant bulk moduli K for before and after peak respectively in the analysis.



Figure 3.5 Bulk modulus of 1% fiber volume ratio SFHSC before and after peak $(\varepsilon_1 / \varepsilon_{cu} \le 2)$.



Figure 3.6 Bulk modulus of 0.5% fiber volume ratio SFHSC before and after peak $(\varepsilon_1 / \varepsilon_{cu} \le 2)$.
However, it has to be noted that the deformation of the concrete becomes quite volatile when the cracks inside the concrete fully develops under uniaxial compression, which will greatly influence the measurement of the axial strain since the length of the gage can not catch the localized damage of the cylinder. It has been observed from all the tests that when the axial strain is less than two times of the peak axial strain, all the axial stress ~ strain curves are smooth and free from shape irregularities, which will provide more reliable information for the volumetric change. That's why a limit of $\varepsilon_1 / \varepsilon_{cu} \leq 2$ has been stipulated.

3.4.4 Poisson's Ratio

The Poisson's ratio of concrete is defined as the ratio of lateral strain to axial strain under the uniaxial compression or tension. Most of the published data (Persson 1999, Rashid 2002, and Gao et. al 1997) were attained by employing the ASTM C 469, in which the direct strain measurement was performed in the standard uniaxial compression test under a compressive stress of 40% $f_c^{'}$. Normally, a constant Poisson's ratio for concrete is assumed, which is reasonable under certain low stress levels when the concrete exhibits essentially linear elasticity. However, Mindess, Young and Darwin (2002) stated that, at approximately 70% of the $f_c^{'}$, the microcracking inside the concrete parallel to the direction of the stress starts to develop much faster up until the failure, thus inducing a highly inelastic feature within that stage, which may greatly affect the apparent Poisson's ratio .

Depending on the computer analysis model, either secant or tangent Poisson's ratio is employed. For the normal strength concrete, the classic description of the secant

Poisson's Ratio variation with stress condition is from Ottosen's constitutive model (Ottosen 1979) (Fig. 3-7), in which stress index is predefined as $\beta = \sigma_1 / \sigma_{1f}$ such that with σ_2 and σ_3 being constant, concrete failure will occur when σ_1 reaches σ_{1f} . The secant Poisson's Ratio ν_s can be expressed as following:

$$\beta < 0.8, v_s = v_i = \text{constant}$$
 (3-2a)

$$0.8 < \beta < 1.0, \quad v_s = v_f - \left(v_f - v_i\right) \sqrt{1 - \left(\frac{\beta - 0.8}{0.2}\right)^2}$$
 (3-2b)

where for normal strength concrete: $v_i = 0.16 \sim 0.20$, $v_f = 0.36$



Figure 3.7 Poisson's Ratio (secant) variation in Ottosen's model.

A cubic polynomial expression of tangent Poisson's ratio under uniaxial compression with respect to the ratio of the axial strain (ε_c) to the peak axial strain (ε_{cu}) was proposed by Kupfer, Hilsdorf, and Rusch (1969):

$$v = v_0 \left[1 + 1.3763 \left(\frac{\varepsilon_c}{\varepsilon_{cu}} \right) - 5.36 \left(\frac{\varepsilon_c}{\varepsilon_{cu}} \right)^2 + 8.586 \left(\frac{\varepsilon_c}{\varepsilon_{cu}} \right)^3 \right]$$
(3-3)

where: v_0 is the initial tangent Poisson's ratio

From this equation, tangent Poisson's ratio will start to rise dramatically after $\varepsilon_c / \varepsilon_{cu} = 50\%$. Under $v_0 = 0.20$, the tangent Poisson's ratio at peak will reach 1.12, demonstrating a great volatility of Poisson's ratio near the peak.

However, for the SFHSC, there is a dearth of information available concerning the Poisson's ratio variation under different stress conditions. And in most of the previous research programs, especially those using cylinders, end restraints were not taken into consideration and concrete specimens were just assumed to be subjected to uniform lateral expansion. This is untrue since it has been demonstrated by many researchers (Mindess et. al 2002, Guo 1997, and Van Mier et. al 1997) that there exists a considerable friction between the rigid loading platen and specimen end, which acts as a lateral confinement at the specimen end, thus increasing the strength of the specimen. Mindess, Young and Darwin (2002) revealed that for a standard cylinder with h/d = 2.0, only the small central portion of concrete is in the true uniaxial compression, while the remainder virtually is in triaxial stress status (see Figure 2-1 (a)). Basically, the concrete does not possess a "compression failure" since the tensile strain capacity of concrete is much less than its compressive counterpart. Under a true uniaxial compression status, due to the Poisson's effect, the concrete should theoretically have a splitting tensile failure mode as shown in Figure 2-1(b).

Figure 2-1(a) is the typical cone failure of a standard cylinder (h/d=2.0) compression test from which most of the existing data concerning Poisson's ratio variation are achieved. It can be clearly seen that only the small central portion of concrete is in true uniaxial compression. Because the strain measurement instruments are

mostly installed on the surface of the specimen, the tested axial and lateral strains are actually the responses of the concrete portion which is in a complicated stress condition, especially near the failure. Therefore, to attain an accountable Poisson's ratio, it is of great importance to reduce the end restraint for the concrete to reach a "true" uniaxial compression status.

As shown in previous sections of this chapter, concrete specimen with h/d=1.5 under designed lubricated loading platen is more likely to provide the uniform lateral expansion under uniaxial compression, therefore this kind of specimens were employed to study the Poisson's ratio variation with stress condition. Figures 3.8 and 3-9 show these variations for both 1% and 0.5% fiber volume ratio SFHSC under different loading platens until peak load. It can be clearly seen that the secant Poisson's ratios below 70% of peak stress and the tangent Poisson's ratios below 50% of peak stress for both of the two concretes under different end conditions are essentially the same, to which a constant of 0.20~0.22 can be applied. Beyond those ranges, the Poisson's ratios under lubricated and rigid loading platens experience increasing divergence. For instance, right at the peak, secant Poisson's ratio ranges from 0.36 to 0.42 under lubricated platen compared with 0.63 to 0.71 under rigid platen, and the difference between the two tangent Poisson's ratios becomes even wider, with 0.85 to 1.25 under lubricated platen versus 3.21 to 4.02 under rigid platen.



Rigid loading platen

Figure 3.8 Poisson's ratio variation until peak for SFHSC (1% fiber volume ratio) with h/d=1.5.



Rigid loading platen

Figure 3.9 Poisson's ratio variation until peak for SFHSC (0.5% fiber volume ratio) with h/d=1.5.

Another issue deserving discussion is the place where a tangent Poisson's ratio of 0.5, the threshold indicating the end of concrete contraction, occurs. Under the rigid loading platen, it occurs around 83%~86% of the peak stress compared with approximately 95% under lubricated platen, which coincides with the point of zero incremental volume change in the graph of octahedral normal stress versus volume change (Figures 3.5 and 3-6) and represents the start of dilatation. Gerstle (1981) also reported that in the case of biaxial compression, concrete dilatation occurred immediately preceding the failure, but he acknowledged there existed a lot of contradictory results which indicated that this dilatation occurred at stress levels of 70% to 85% of failure. He attributed this discrepancy to the possible influence of the load application and strain measurements.

As discussed above, when the stress level is below 70% of the failure, the secant Poisson's ratio of SFHSC can be taken as a constant of 0.2. The secant Poisson's ratio at peak falls in a range from 0.36 to 0.42, which is quite close to that stipulated in the Ottosen's constitutive model for ordinary concrete. Eventually, the variation of secant Poisson's ratio of SFHSC under uniaxial compression can be expressed as following:

$$\frac{\sigma_1}{f_c'} \le 70\%, \qquad v_s = 0.20$$

$$70\% \le \frac{\sigma_1}{f_c'} \le 100\%, \qquad v_s = 0.69 \left(\frac{\varepsilon_1}{\varepsilon_{cu}}\right)^2 - 0.67 \left(\frac{\varepsilon_1}{\varepsilon_{cu}}\right) + 0.39$$

$$1.0 \le \left(\frac{\varepsilon_1}{\varepsilon_{cu}}\right) \le 2.0, \qquad v_s = -0.28 \left(\frac{\varepsilon_1}{\varepsilon_{cu}}\right)^2 + 1.30 \left(\frac{\varepsilon_1}{\varepsilon_{cu}}\right) - 0.62$$

$$(3-4)$$

Figure 3.10 shows the tangent Poisson's ratio variation for SFHSC before peak under true uniaxial compression (lubricated loading platen) as compared with the equation proposed by Kupfer et. al (1969) for ordinary concrete under biaxial loading. It is quite apparent that Kupfer's equation is applicable to SFHSC when the initial tangent Poisson's ratio ν_0 is taken as 0.2. Elwi and Murray (1979) proposed that a limit value of 0.5 be placed on the tangent Poisson's ratio and insisted that this yield realistic estimates of stress.



Figure 3.10 Tangent Poisson's ratio variation for SFHSC before peak compared with Kupfer's equation (1969).

Figure 3.11 shows the variation of tangent Poisson's ratio of SFHSC after failure under true uniaxial compression (lubricated loading platen). It seems fairly reasonable to adopt a constant of 1.35 for v_t after failure ($\varepsilon_t / \varepsilon_{cu} < 2.0$).



Figure 3.11 Tangent Poisson's ratio for SFHSC after peak ($\varepsilon_1 / \varepsilon_{cu} < 2.0$).

CHAPTER 4

TRIAXIAL COMPRESSION TEST SETUP

4.1 Testing Apparatus

All the triaxial compression tests in this study for both HSC and SFHSC were performed by the MTS 810 material testing system. It is a very stiff frame MTS servo-hydraulic closed loop testing machine with a frame stiffness rated at 11.0×10^9 N/m. It has a 4600KN (1,000,000 lbs) load capacity with the triaxial cell being able to apply up to 84MPa (12ksi) confining pressure. The test setup is shown in Figure 4.1.

Before the triaxial compression test starts, the pressure chamber in Figure 4.1 is filled with pressure oil by the pressure intensifier of the MTS system, and then the intensifier will press the oil to create the target confining pressure. In this study, the axial compressive stress will be always designated as σ_1 while the confining pressure as σ_2 and σ_3 , so it can be clearly seen that actually this testing setup can only provide uniform confining pressures ($\sigma_2 = \sigma_3$). However, the advantage of this machine is that it can be purposely programmed to perform various kinds of combinations of axial stress σ_1 and confining pressure σ_3 on the concrete specimen, including cyclic loading.

In order to get a successful triaxial compression test done, three aspects need to be meticulously taken care of, namely the specimen, extensometers (axial and circumferential) and the isolation plastic sleeve.



Figure 4.1 Triaxial test setup.

4.2 Specimen Treatment

The surface condition of the concrete specimen is vital to a successful triaxial compression test, since under high oil pressures, even a normal air void hidden right below the surface of the specimen which is innocuous under uniaxial compression will be easily penetrated through by the pressurized oil. This may cause a punching failure of the

isolation sleeve, which will lead to a direct contact between the specimen side surface and the confining oil. The oil can then penetrate through any possible cracks occurring under compression and acts as a splitting force inside the specimen.

To avoid this negative scenario, two measures were adopted in this study. First, a highly effective superplasticizer Sika2000 was employed instead of the previous Sika86 in the triaxial compression concrete mixture. Sika2000 is capable of producing flowable concrete mixture with high workability and consistency under low water to cement ratio, thus greatly decreasing the density of relatively large air voids beneath the side surface. However, even this is not enough to totally eliminate those voids, especially when the slump of the fresh concrete was severely reduced with the introduction of steel fiber at 1% volume ratio. In this case, the second measure is a must, although tedious and harrowing. The side surface of the specimen was scratched thoroughly using a steel blade until all the hidden voids could be clearly seen, and then all the relatively large voids (with a diameter roughly larger than 0.5mm) were filled up with cement paste. After this work was done, the specimen was taken back to the curing room where it would stay until the test.

To achieve a good contact between the specimen and the loading platen, both ends of the specimen were carefully ground. To make the $100mm \times 150mm$ (4×6 in.) specimen, a $100mm \times 200mm$ (4×8 in.) was cut at the top to eliminate the potential weak layer due to vibration during casting, and then it was ground to the specified length.

Chapter 3 shows that 100mm×150mm (4×6 in.) specimen with ground ends under designed lubricated loading platen was likely to produce a uniform lateral expansion even after peak load under uniaxial compression. Thus the same setup was also used in the

triaxial compression test, and ordinary 100mm×200mm (4×8 in.) specimens were also employed to make the comparisons of the experimental results between these two cylinders.

4.3 Axial and Circumferential Extensometers

Axial and circumferential extensometers have a remarkable advantage over traditional strain gage in measuring large deformations, which is extremely important under triaxial compression where the axial and lateral strains can both be large compared with their uniaxial counterparts. However, the measurement accuracy of these extensometers needs to be addressed before the tests in order to maintain the credibility of the experimental results.

The manufacturer (MTS company 1994) clearly stipulates that if the difference between the shunt calibration voltage value and the reference one of the extensometer is less than 15 mv, the measurements from the tests can be satisfactorily accurate. So in this study, shunt calibration was made before each triaxial test, and it was found that the error of the circumferential extensometer was consistently within that range but that of the axial extensometer was slightly away with a maximum difference of 30 mv. However, given that the total reference voltage value is 9.33 v, this slightly larger difference will be likely not to cause any appreciable experimental errors.

Another checkup with the extension test is to use a solid steel rod to perform a standard compression test, like the one described in Chapter 3. Hereby another set of steel rod uniaxial compression test results is provided in Figure 4.2 and Figure 4.3, respectively.



Figure 4.2 Axial stress ~ axial strain of steel rod under uniaxial compression.

It can be found from those two figures that the elastic modulus and the Poisson's ratio of the steel rod are 205,000 MPa and 0.2667 respectively, which fall into the reasonable range for this material. The slight difference from the reference value may be attributed to the installation positions of the extensometer on the specimen. Again, almost perfect linear relationships were achieved for those two graphs.



Figure 4.3 Lateral strain ~ axial strain of steel rod under uniaxial compression.

As a conclusion, using the axial and circumferential extensometers to measure the axial and lateral strains can give much confidence to the measurement accuracy of the tests.

4.4 Plastic Isolation Sleeve

4.4.1 Selection and Mechanical Properties of the Sleeve

Probably the most important task of setting up a triaxial compression test is to select the appropriate isolation sleeve to protect the highly pressurized oil from a direct contact with the side surface of the specimen.

Although the side surface of the specimen has been treated carefully as is mentioned above, there may still be some air voids unrevealed during the surface treatment, leaving a high probability of oil penetration under high confining pressure. Therefore, firstly the selected isolation sleeve should have a high tensile strength together with a good ductility. Secondly, the selected isolation sleeve should not be so thick as the rubber sleeves commonly used by many researchers in conducting the triaxial compression tests, which will be hard for the accurate measurements of strains. And finally, this isolation sleeve should be in tight contact with the side surface of the specimen after installation for the sake of (axial and circumferential) strain measurement and oil infiltration prevention. After a time-consuming search and comparison, the PVC transparent heat-shrink tubing with a original inner diameter of 100mm (4 in.) and a wall thickness of 1.85 mm (0.073 in.) was adopted in this study.

The tensile stress ~ strain curve of this PVC material is shown in Figure 4.4.



Figure 4.4 Tensile stress ~ strain curve of the PVC material.

The tensile capacity of this PVC is larger than 50% (the strain measurement stopped at the maximum capacity of the extensometer) and the ultimate tensile strength is around 50 MPa. Ansari and Li (1998) used another kind of heat-shrink tubing which has a maximum tensile capacity of 30 MPa, much lower than that of this study.

Installation of this tubing is quite easy. The surface-treated specimen is placed into this sleeve and two 100mm-diameter loading platens with end friction reducing measures (see Chapter 3) are also inserted into the sleeve with one at each end of the specimen. Then the assembly is heated inside an electrical oven for about 5 min. at a sustained temperature of 140~150 °C. After heating, the specimen is left at room temperature to cool down. The final procedure is to wrap the electrical tape around the joint of the isolation sleeve and the loading platen to provide additional sealing control. When it is all done the specimen is ready to go for a triaxial compression test.

Due to all of the careful specimen treatment considerations, only 2 out of more than 30 triaxial compression specimens failed before the experiment was completed, for a successful rate well above 90%.

4.4.2 Elimination of Deformation of the Sleeve from the Total Lateral Strain

It is obvious that the existence of the PVC sleeve affects the measurement of the total lateral strain, especially under high confining pressures when the wall of the sleeve is squeezed in the radial direction to cause a thickness decrease.

This effect is evaluated using two triaxial compression tests of the steel rod, one with the isolation sleeve and one without. During the test, the ratio of the axial stress to confining pressure is kept as 4.342 all the way up to the confining pressure of 70 MPa, which is exactly the maximum confining pressure the true concrete specimen will undergo. Figure 4.5 shows the lateral (circumferential) strains in those two different situations and the final radial contractions of the sleeve under different confining pressures are also calculated.

The contribution of the contraction of the PVC sleeve under different confining pressure σ_3 to the total lateral strain can be expressed as follows:

$$\varepsilon_{\rm PVC} = 5.263\sigma_3 \quad (\times 10^{-6})$$
 (4-1)

where ε_{PVC} is the lateral strain (in contraction) of the PVC sleeve under confining pressure σ_3 .



Figure 4.5 Lateral contractions of the sleeve under different confining pressures.

The other possible source of radial contraction of the sleeve may come from the Poisson's effect of the circumferential elongation of the sleeve due to the lateral expansion of the concrete specimen. But this only accounts for a negligible part of the whole lateral strain, and thus it can be ignored without causing significant errors. Consequently, in the data analysis of a true triaxial compression test of a concrete specimen, this ε_{PVC} should be subtracted from the total lateral strain directly read from the experiment to get the actual lateral strain of the concrete specimen. Also, it has been shown from the same kind of verification tests that the sleeve has little effect on the measurement of axial strain under triaxial compression.

CHAPTER 5

ANALYSIS OF TRIAXIAL COMPRESSION TEST RESULTS OF HIGH STRENGTH CONCRETE (HSC)

5.1 Experimental Program

The mix proportion of the HSC for triaxial compression test is the same with that of the uniaxial compression test (Chapter 3). However, since high-quality washed aggregates (sand and crushed gravel) had been used up in the previous tests, relatively low-quality sand and crushed gravels had to be adopted for the triaxial specimens instead, which attributed to the decrease of the standard compressive strength f_c' (down from 77.5 MPa to 68.0 MPa) and the initial elastic modulus E₀ (down from 46500 MPa to 40700MPa).

Table 5.1 Mix Proportion of HSC

Water- cement ratio	Water (kg/m ³)	Cement (kg/m ³)	Sand (kg/m ³)	Coarse aggregate (kg/m ³)	Sika2000 (L/m ³)	Slump (mm)	<i>f</i> ['] _c (MPa)
0.33	170	520	760	1000	4.3	235	68.0

The specimen curing, treatment and test setup have already been discussed in Chapter 4. In this triaxial compression test, $100 \times 200 \text{ mm} (4" \times 6")$ concrete cylinders with both ends ground were tested under the designed lubricated loading platen mainly through two different load paths: T-1 and T-2. The load application procedures of those two load paths are shown Figure 5.1.



Figure 5.1 Triaxial load path for T-1 and T-2.

The difference between T-1 and T-2 lies in the different σ_3 / σ_1 ratios before the target confining pressure σ_3 is reached. The ratio of σ_3 / σ_1 is 1/2 for T-1 and 1/6 for T-2, respectively. In both load paths, load control was employed before the target confining pressure with the increasing rate of σ_3 being 7 MPa/min for T-1 and 3-4 MPa/min for T-2 respectively. After the target σ_3 , both paths adopted displacement control all the way until the failure (including strain softening) of the specimen. The displacement rate for both cases fell in a range from 0.005 mm/s (0.0002 in/s) to 0.006mm/s (0.00025 in/s). The displacement control was applied in order to get the descending branch of the triaxial stress ~ strain relationship.

5.2 Failure Criterion

5.2.1 Mohr-Coulomb Criterion

According Mohr's criterion, material's failure will occur for all stress states at which the largest of Mohr circles is just tangent to the envelop defined by $|\tau| = f(\sigma)$. If this

envelop is set to be a straight line as shown in Figure 5.2, it will become the Coulomb's equation:

$$|\tau| = c + \sigma \tan \phi \tag{5-1}$$

where c is the cohesion and ϕ is the internal-friction angle of the material. The intermediate principal stress σ_2 will have no influence on the failure.



Figure 5.2 Mohr-Coulomb failure criterion.

If the uniaxial compressive strength f_c is considered and simple geometric relationship is exploited from Figure 5.2, Equation (5-1) can be transferred into the following form:

$$\frac{\sigma_1}{f_c'} = 1 + k \frac{\sigma_3}{f_c'} \qquad k = \frac{1 + \sin\phi}{1 - \sin\phi}$$
(5-2)

The sign convention used hereafter is that the compression is positive and $\sigma_1 \ge \sigma_2 \ge \sigma_3$. In this part of experiment, a total of 3 uniaxial compression tests and 14 triaxial compression tests under various confining pressures were performed. The details of the experiment, including the specimen size, end treatment, load path, and testing

control mode have been already discussed earlier in this chapter. The test results for Mohr-Coulomb Criterion are listed Table 5.2 and Figure 5.3.

Load path	Confining pressure	Axial strength	σ_3 / f_c	σ_1 / f_c
	σ_3 (MPa)	σ_1 (MPa)		
Uniaxial	0	67.0	0.000	1.000
T-1	3.5	84.9	0.052	1.267
T-1	7	99.0	0.104	1.478
T-1	14	130.7	0.209	1.951
T-2	14	132.7	0.209	1.981
T-2	14	134.9	0.209	2.013
T-2	14	135.5	0.209	2.022
T-2	21	157.1	0.313	2.345
T-3	21	154.0	0.313	2.299
T-2	21	161.2	0.313	2.406
T-1	28	180.2	0.418	2.690
T-2	28	179.9	0.418	2.685
T-1	42	229.1	0.627	3.419
T-1	56	276.0	0.836	4.119

Table 5.2 Triaxial Strength of HSC for Mohr-Coulomb Failure Criterion



Figure 5.3 Mohr-Coulomb failure criterion for HSC.

It can be seen clearly form Table 5.2 that due to the painstakingly careful preexperiment quality control work such as the raw material preparation, the concrete mixing, the specimen making and end grinding, the successful isolation membrane selection and its proper installation, the efficient end friction reducing measure, and the meticulous exploitation of the advanced MTS system, the triaxial compression testing results turn out to be highly consistent. For instance, under the same confining pressure of 14MPa and the identical load path, the triaxial strengths of 3 specimens are 132.7MPa, 134.9MPa and 135.5MPa respectively, resulting in a deviant error from the mean value less than 2%.

In this experiment, the maximum confining pressure is 56MPa (8 ksi), roughly 85% of the uniaxial compressive strength f_c of the HSC. This confinement level is well above most of the values used for HSC with the same strength grade in previous researches discussed in Chapter 2 (Table 2.1). From this study, k is 4.0 and

$$\frac{\sigma_1}{f_c'} = 1 + 4.0 \frac{\sigma_3}{f_c'}$$
(5-3)

From Figure 5.2 it can also been shown that $\frac{f_c}{f_t} = \frac{1 + \sin \phi}{1 - \sin \phi} = k$, indicating the ratio of compressive strength to tensile strength of the HSC should be equal to 4.0. This value is far lower than the well-accepted f_c'/f_t ratio of 10~15, thus the Mohr-Coulomb criterion is not applicable when tensile failure is involved. Also, it is found from Equation (5-3) that the internal friction angle ϕ of the HSC is 36.9°.

5.2.2 Willam-Warnke Failure Criterion

As discussed in Chapter 3, Willam-Warnke failure criterion is probably the most widely used criterion in concrete. It has been adopted in the popular finite element analysis software ANSYS. The meridian equation takes the form of follows:

$$\frac{\tau_m}{f_c'} = a_0 + a_1 \frac{\sigma_m}{f_c'} + a_2 \left(\frac{\sigma_m}{f_c'}\right)^2$$
(5-4)

where:

$$\sigma_m = I_1 / 3 = (\sigma_1 + \sigma_2 + \sigma_3) / 3$$

$$\tau_{m} = \frac{1}{\sqrt{15}} \left[(\sigma_{1} - \sigma_{2})^{2} + (\sigma_{2} - \sigma_{3})^{2} + (\sigma_{3} - \sigma_{1})^{2} \right]^{1/2}$$

The test results for Willam-Warnke Criterion are listed Table 5.3.

Load path	Confining pressure σ_3 (MPa)	Axial strength σ_1 (MPa)	σ_m / f_c	$ au_m / f_c'$
Uniaxial	0	67	0.333	0.365
T-1	3.5	84.9	0.457	0.444
T-1	7	99	0.562	0.501
T-1	14	130.7	0.790	0.636
T-2	14	132.7	0.800	0.647
T-2	14	134.9	0.810	0.659
T-2	14	135.5	0.813	0.662
T-2	21	157.1	0.991	0.742
T-3	21	154	0.975	0.725
T-2	21	161.2	1.011	0.764
T-1	28	180.2	1.175	0.829
T-2	28	179.9	1.174	0.828
T-1	42	229.1	1.558	1.020
T-1	56	276	1.930	1.199

Table 5.3 Triaxial Strength of HSC for Willam-Warnke Failure Criterion

And for the HSC, this study shows that the compressive meridian of Willam-Warnke failure criterion takes the form of:

$$\frac{\tau_m}{f_c'} = 0.165 + 0.638 \frac{\sigma_m}{f_c'} - 0.055 \left(\frac{\sigma_m}{f_c'}\right)^2$$
(5-5)

Figure 5.4 shows the comparison of the compressive meridians of HSC in terms

of $\frac{\tau_{oct}}{f_c'}$ and $\frac{\sigma_{oct}}{f_c'}$ between this study ($f_c' = 67.0$ MPa) and some previous researches from

Xie (1995, $f_c' = 60.2$ MPa), Pantazopoulou (2001, $f_c' = 73.4$ MPa) and Ansari (1998, $f_c' = 72.2$ MPa).



Figure 5.4 Compressive meridian of Willam-Warnke failure criterion for HSC.



Figure 5.5 Comparison of compressive meridians of HSC.

From Figure 5.5, it can be seen that under approximately the same compressive strength grade, the compressive meridians from Xie, Pantazopoulou and this study

comply with one another very well while that of Ansari deviates from the mainstream, especially at the higher end of the octahedral stresses.

In this study, due to the upper limit of the testing machine to apply the confining pressure and the relatively high compressive strength of concrete, the ratio of σ_{oct} to f_c' is lower than 2.0. Using the low strength concrete ($f_c'=20.5$ MPa), Chern (1992) performed a comparatively higher confining pressure (340% f_c') and reached a σ_{oct} to f_c' ratio of 5.6. The comparison between those two meridians is shown in Figure 5.6.





Figure 5.6 indicates that even though their strengths are quite different from each other, the compressive meridians of the high and normal strength concrete fairly follow the same trend, especially at low σ_{oct} to f_c' ratios. For the normal strength concrete, when the σ_{oct} to f_c' ratio reaches 4.0, the compressive meridian starts to become more curved. It is still unclear whether HSC will exhibit the same kind of feature since no experimental data is available around that extremely high confining pressure region.

5.3 Triaxial Stress ~ Strain Relationship

5.3.1 Strains at Peak Stress under Triaxial Compression

Under different confining pressures from 0 to 56 MPa (8ksi), the relationship between the axial strain and lateral strain at peak is shown in Table 5.4 and Figure 5.7.

Table 5.4 shows that with the increasing of the lateral confining pressure σ_3 , the axial and lateral strains at peak both increase dramatically. The relationship between the

 $\frac{\varepsilon_{\rm lc}}{\varepsilon_{\rm cu}}$ and $\frac{\sigma_{\rm 3}}{f_{\rm c}^{'}}$ can be expressed as:

$$\frac{\varepsilon_{1c}}{\varepsilon_{cu}} = 1 + 19.21 \left(\frac{\sigma_3}{f_c'} \right)$$
(5-6a)

where ε_{cu} represents the peak axial strain in uniaxial compression.

Load path	Confining pressure σ_3 (MPa)	Axial strength σ_1 (MPa)	Peak lateral strain ε_{3c} (×10 ⁻⁶)	Peak axial strain ε_{1c} (×10 ⁻⁶)
Uniaxial	0	67	-1000	2514
T-1	3.5	84.9	-2168	4660
T-1	7	99	-3609	7759
T-1	14	130.7	-5209	12373
T-2	14	132.7	-4649	12515
T-2	14	134.9	-5846	13454
T-2	14	135.5	-5440	13728
T-2	21	157.1	-6547	18257
T-3	21	154	-5739	16609
T-2	21	161.2	-7782	19429
T-1	28	180.2	-8679	25007
T-2	28	179.9	-8865	24089
T-1	42	229.1	-11084	32130
T-1	56	276	-12376	40582

Table 5.4 Axial and Lateral Strains at Peak under Different Confining Pressures



Figure 5.7 Peak strain ratio versus confining pressure level for HSC.

There is also a close relationship between the lateral peak strain ε_{3c} and axial peak strain ε_{1c} when the confining pressure increases (as shown in Figure 5.8), which can be expressed as follows:

$$\frac{\varepsilon_{3c}}{\varepsilon_{cu}} = -0.009 \, l \left(\frac{\varepsilon_{lc}}{\varepsilon_{cu}}\right)^2 + 0.4522 \frac{\varepsilon_{lc}}{\varepsilon_{cu}}$$
(5-6b)



Figure 5.8 Relationship between axial and lateral strains at peak for HSC.

From Table 5.4, it seems that different load paths do not have an appreciable influence on the axial strength, the peak axial strain and the peak lateral strain, which agrees with Guo's findings (1997).

5.3.2 Axial Stress versus Strains under Various Confining Pressures

Under load path T-1 and different confining pressures from 0 to 56 MPa, the axial stress \sim strain curves of HSC under triaxial compression are shown on Figure 5.9.



Figure 5.9 Axial stress versus axial and lateral strains of HSC in triaxial compression under different confining pressures (load path T-1).

Under load path T-1, the confining pressure σ_3 and axial pressure σ_1 were gradually applied to the specimen simultaneously with a constant σ_3 to σ_1 ratio of 1/2 until the target confinement was reached. Therefore, due to the Poisson's effect, the axial strain development was contained to some extend by the confining pressure σ_3 , which resulted in steeper initial slopes of the axial stress \sim strain curves under triaxial compression than that of the uniaxial counterpart, as can be seen on Figure 5.9.

However, the load path T-1 is by no means close to the proportional load path since right at the target confinement σ_3 , axial pressure σ_1 is far away from its peak value σ_{1c} . For instance, when the target confining pressure σ_3 (28 MPa) is reached, σ_1 is only 56 MPa, only 31% of the peak value of 180.2 MPa. In this case, load path T-2 will be more appropriate in simulating proportional loading in which a constant σ_3 to σ_1 ratio of 1/6 is maintained until the target confinement. At target σ_3 of 21 MPa and 28 MPa, σ_1 is 126MPa (80% σ_{1c}) and 168MPa (93.4% σ_{1c}) respectively. The comparison of the triaxial stress ~ strain curves between those two load paths under σ_3 =28 MPa is shown in Figure 5.10.



Figure 5.10 Comparison of triaxial stress ~ strain curves under different load paths.

From Figure 5.10, those two sets of stress ~ strain curves under load path T-1 and T-2 are essentially the same in terms of the peak axial stress σ_{1c} , peak axial strain ε_{1c} and

peak lateral strain ε_{3c} . The slight difference between those curves may be attributed to the different Poisson's effect from the lateral confinement which is larger under T-1.

5.3.3 Octahedral Stress ~ Strain Relationship

The triaxial stress \sim strain curves discussed above are more qualitative than quantitative, and they can only reflect the general trend of the stress \sim strain variation under some specific load conditions. In order to be applicable in the more complicated practical cases, other types of strain \sim strain curves need to be developed. Octahedral stress \sim strain relationship is among them.

There are two types of octahedral stress ~ strain relationships—octahedral normal stress (σ_{oct}) versus volume change ($3 \varepsilon_{oct}$) and octahedral shear stress (τ_{oct}) versus engineering octahedral shear strain (γ_{oct}), where

$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) = \frac{I_1}{3}$$
(5-7a)

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sqrt{\frac{2J_2}{3}}$$
(5-7b)

$$\varepsilon_{oct} = \frac{1}{3} (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$$
 (5-7c)

$$\gamma_{oct} = \frac{2}{3}\sqrt{\left(\varepsilon_1 - \varepsilon_2\right)^2 + \left(\varepsilon_2 - \varepsilon_3\right)^2 + \left(\varepsilon_3 - \varepsilon_1\right)^2}$$
(5-7d)

Under the stress and strain increments, the tangent bulk modulus K_t and tangent shear modulus G_t can be expressed as follows:

$$K_{t} = \frac{d\sigma_{oct}}{3d\varepsilon_{oct}} = \frac{E_{t}}{3(1-2v_{t})}$$
(5-8a)

$$G_{t} = \frac{d\tau_{oct}}{d\gamma_{oct}} = \frac{E_{t}}{2(1+v_{t})}$$
(5-8b)

where E_t and v_t are the tangent elastic modulus and tangent Poisson's ratio respectively.

Gerstle (1981) developed a simple formulation using those equations above for the biaxial compression. The representation decoupled the volume change and shapechange portions of the stress ~ strain relationship, assuming the bulk modulus K is only a function of the volumetric and the shear modulus G is only a function of the deviatoric stress or strain level.

5.3.3.1 Octahedral Normal Stress versus Volume Change The relationship between octahedral normal stress (σ_{oct}) and volume change (3 ε_{oct}) under load path T-1 and different confining pressures is shown in Figure 5.11.



Figure 5.11 σ_{oct} versus volume change (3 ε_{oct}) under load path T-1 for HSC.

The major similarity of all those curves in Figure 5.11 is that the concrete starts to dilate ($d\varepsilon_{oct} = 0$) right before the peak axial stress σ_{1c} (at a ratio to σ_{1c} of roughly 95%).

At the descending branch, the concrete volume expands remarkably just with a small drop in axial stress, demonstrating a violent development of crack growth inside the concrete.

Under relatively low confining pressures (less than 7 MPa), the relationship between octahedral normal stress and volume change before the start of dilation can be treated as linear, leading to a constant bulk modulus K_t . However, this linear relationship no longer exists under higher confining pressures.

Figure 5.12 shows the difference between σ_{oct} versus volume change curves under different load paths at the confining pressure of 28MPa. It seems that the proportional loading tends to produce a smoother slope change before the concrete begins to dilate.



Figure 5.12 σ_{act} versus volume change under different load path.

5.3.3.2 Octahedral Shear Stress versus Engineering Octahedral Shear Strain

The relationship between octahedral shear stress (τ_{oct}) and engineering octahedral shear strain (γ_{oct}) under load path T-1 and different confining pressures is shown on Figure 5.13.

Those curves in Figure 5.13 essentially follow the same trend, and if they are normalized by their respective peak octahedral shear stresses and peak engineering octahedral shear strains, all of them except for the uniaxial compression will almost coincide with one another, as can be seen from Figure 5.14.



Figure 5.13 $\tau_{oct} \sim \gamma_{oct}$ curves under load path T-1 for HSC.

The comparison of $\tau_{oct} \sim \gamma_{oct}$ curves under different load path (T-1 and T-2) but at the same confining pressure (28MPa) is shown in Figure 5.15. Although there is an appreciable difference between the σ_{oct} versus volume change curves under different load path T-1 and T-2 (Figure 5.12), the two $\tau_{oct} \sim \gamma_{oct}$ curves closely approximate to each other, indicating that the load path influences the tangent bulk modulus much more than the tangent shear modulus.



Figure 5.14 Normalized $\tau_{oct} \sim \gamma_{oct}$ curves under different confining pressures under load path T-1 for HSC.



Figure 5.15 $\tau_{oct} \sim \gamma_{oct}$ curves under different load paths at confining pressure 28 MPa for HSC.

5.3.3.3 Formulation of Octahedral Shear Stress versus Engineering Octahedral

Shear Strain Sargin's equation is widely used in simulating the stress ~ strain relationship of concrete, and it has been adopted in the Ottosen's constitutive model (Ottosen 1979). The equation can be expressed as follows:

$$\frac{\sigma}{f_c'} = \frac{\frac{E_0}{E_s} \frac{\varepsilon}{\varepsilon_c} + (D-1) \left(\frac{\varepsilon}{\varepsilon_c}\right)^2}{1 + \left(\frac{E_0}{E_s} - 2\right) \frac{\varepsilon}{\varepsilon_c} + D \left(\frac{\varepsilon}{\varepsilon_c}\right)^2}$$
(5-9a)

The parameters in Equation (5-9a) are defined in Figure 5.16. D is a constant to define the shape of descending branch of the curve.



Figure 5.16 Parameters in Equation (5-9a).

If D is taken as 1.0, then Equation (5-9a) will be changed to Saenz equation adopted by Darwin and Pecknold (1977) in their constitutive model based on biaxial compression tests, which will be discussed later in this study.

$$\sigma = \frac{E_0 \varepsilon}{1 + \left(\frac{E_0}{E_s} - 2\right) \frac{\varepsilon}{\varepsilon_c} + \left(\frac{\varepsilon}{\varepsilon_c}\right)^2}$$
(5-9b)
Rewrite Equation (5-9b) in terms of octahedral shear stress and engineering octahedral shear strain will yield:

$$\tau_{oct} = \frac{G_0 \gamma_{oct}}{1 + \left(\frac{G_0}{G_s} - 2\right) \frac{\gamma_{oct}}{\gamma_{octp}} + \left(\frac{\gamma_{oct}}{\gamma_{octp}}\right)^2}$$
(5-10a)

The results using Equation (5-10a) to approximate the $\tau_{oct} \sim \gamma_{oct}$ curves under relatively low confining pressures ($\sigma_3 \leq 28$ MPa) are shown in Figure 5.17.



Figure 5.17 Approximation of $\tau_{oct} \sim \gamma_{oct}$ curves using Saenz equation under low confining pressures for HSC.

However, Saenz Equation will no longer be applicable under much higher confining pressures ($\sigma_3 > 28MPa$), when it will deviate notably from the experimental data especially at the ascending branch (see Figure 5.18). In this case, Saenz equation is modified to address this deviation:

$$\tau_{oct} = \frac{0.75G_0\gamma_{oct}}{1 + \left(0.75\frac{G_0}{G_s} - 2\right)\frac{\gamma_{oct}}{\gamma_{octp}} + \left(\frac{\gamma_{oct}}{\gamma_{octp}}\right)^2}$$
(5-10b)

Due to the low occurrence probability of very high confining pressures ($\sigma_3 > 60\% f_c$) in practical cases, the Saenz Equation (5-10a) is more appropriate to approximate the true $\tau_{oct} \sim \gamma_{oct}$ relationship of HSC under triaxial compression.



Figure 5.18 Approximation of $\tau_{oct} \sim \gamma_{oct}$ curves using modified Saenz equation under high confining pressures for HSC.

5.3.3.4 Relationship between Peak Octahedral Shear Stress τ_{octp} and Peak Engineering Octahedral Shear Strain γ_{octp} In order to employ Saenz Equation (5-10a), the peak secant shear moduli G_s of HSC under different stress combinations in triaxial compression need to be determined. This is to be done by establishing a relationship between τ_{octp} and γ_{octp} , as is shown in Table 5.5 and Figure 5.19. Figure 5.19 shows that there is a fairly linear relationship between τ_{octp} and γ_{octp}

which takes a form of:

$$\tau_{octp} = 1.458 \times 10^{-3} \gamma_{octp} + 29.30 \tag{5-11}$$

 τ_{octp} can be determined by using the failure criterion discussed earlier in this chapter, and then the γ_{octp} can be calculated from Equation (5-11).

Load path	Confining pressure	Axial strength	Peak lateral	Peak axial	Peak γ_{octp}	Peak
	σ_3 (MPa)	σ_{lc} (MPa)	$(\times 10^{-6})$	$(\times 10^{-6})$	(×10°)	(MPa)
Uniaxial	0	67	-1000	2514	3313	31.6
T-1	3.5	84.9	-2168	4660	6438	38.4
T-1	7	99	-3609	7759	10718	43.4
T-1	14	130.7	-5209	12373	16576	55.0
T-2	14	132.7	-4649	12515	16182	56.0
T-2	14	134.9	-5846	13454	18196	57.0
T-2	14	135.5	-5440	13728	18072	57.3
T-2	21	157.1	-6547	18257	23385	64.2
T-3	21	154	-5739	16609	21070	62.7
T-2	21	161.2	-7782	19429	25655	66.1
T-1	28	180.2	-8679	25007	31759	71.7
T-2	28	179.9	-8865	24089	31069	71.6
T-1	42	229.1	-11084	32130	40743	88.2
T-1	56	276	-12376	40582	49929	103.7

Table 5.5 τ_{octp} and γ_{octp} of HSC under Different Load Conditions



Figure 5.19 Relationship between τ_{octp} and γ_{octp} for HSC.

Until now the only thing which needs to be defined in Equation (5-10a) to finalize the shape of $\tau_{oct} \sim \gamma_{oct}$ curve of HSC in triaxial compression is the initial shear modulus G₀. From the triaxial experiments, the G₀ and K₀ of HSC under different load paths are shown in Table 5.6.

Load path	Confining	Axial	G ₀	Average G ₀	K ₀	Average K ₀
	pressure	strength	(×10 ⁶	(×10 ⁶	(×10 ⁶	(×10 ⁶
	σ_3 (MPa)	σ_{lc} (MPa)	MPa)	MPa)	MPa)	MPa)
Uniaxial	0	67	0.0173	0.0173	0.0218	0.0210 *
	7	99	0.0181		0.0170	
	14	130.7	0.0167		0.0186	
T-1	28	180.2	0.0162	0.0170	0.0175	0.0176
	42	229.1	0.0168		0.0170	
	56	276	0.0169		0.0181	
	14	132.7	0.0162		0.0150	
	14	134.9	0.0183		0.0182	
	14	135.5	0.0193	0.0181	0.0161	0.0160
T-2	21	157.1	0.0181		0.0156	
	21	161.2	0.0196		0.0155	
	28	179.9	0.0173		0.0156	

Table 5.6G₀ and K₀ of HSC under Different Load Paths

From table 5.6 it can be seen that although there is a small difference between the initial shear moduli G_0 under different load paths, this variation is trivial compared with that of the initial bulk moduli K_0 . Therefore it is quite reasonable to take an initial shear modulus G_0 of 0.0175×10^6 MPa for the HSC under such loading cases.

5.4 Equivalent Uniaxial Stress ~ Strain Relationship

The concept of equivalent uniaxial strss ~ strain relationship was originally put forward by Darwin and Pecknold (1977) in an attempt to simulate the variation of tangent elastic modulus E_i with the stress or strain change along the specific axis "i" of the orthotropic material in biaxial compression. Elwi and Murray (1979) extended this concept to triaxial compression. The detailed description of the equivalent uniaxial strain ε_{iu} and its corresponding peak value ε_{ic} will be provided later in Chapter 7.

The definitions of the incremental and total equivalent uniaxial strain $d\varepsilon_{iu}$ and ε_{iu} are as follows:

$$d\varepsilon_{iu} = \frac{d\sigma_i}{E_i}$$
 and $\varepsilon_{iu} = \int \frac{d\sigma_i}{E_i}$ (5-12)

where Ei is the tangent elastic modulus in "i" direction under the current stress condition.

It is obvious from Equation (5-12) that $d\varepsilon_{iu}$ represents the increment of strain in direction "i" that concrete would exhibit if subjected to a uniaxial stress increment $d\sigma_i$ with other stress increments being equal to be zero. Therefore, if the relationship between stress σ_i and equivalent uniaxial strain ε_{iu} is defined, the variation of the tangent elastic modulus E_i can be determined. The load path T-1 is especially useful for deriving the equivalent uniaxial stress ~ strain curves under triaxial compression, which can be illustrated by Figure 5.20 and Table 5.7. In Figure 5.20, σ_1 and σ_3 are first increased proportionately to σ_1 and σ_3 under a constant σ_3/σ_1 ratio of 1/2, and then with σ_3 unchanged, σ_1 is increased until the failure strength σ_{1c} is reached. However, if the stress ratio was kept the same with the initial value of 1/2 until failure, σ_1 and σ_3 would follow the path of the dash line to σ_{1c} and σ_{3c} respectively. From Equation (5-5), σ_{1c} is more than 700 MPa.



Figure 5.20 Load path T-1.

Table 5.7 σ_1 / σ_{1c} fo	r Different σ_3
--	------------------------

Load path	Confining pressure σ_3 (MPa)	σ_{1c} (MPa)	σ'_1 at target σ_2 (MPa)	$\sigma'_{1} / \sigma'_{1c}$ (%)
	3.5		7	<1.0
	7		14	<2.0
T-1	14	>700	28	<4.0
	28		56	<8.0
	42		84	<12.0
	56		112	<16.0

From Equation (5-12), the equivalent uniaxial strain in "1" direction can be rewritten as:

$$\varepsilon_{1u} = \int \frac{d\sigma_1}{E_1} = \int_{(1)} \frac{d\sigma_1}{E_1} + \int_{(2)} \frac{d\sigma_1}{E_1}$$

For the second integration part $\int_{(2)} \frac{d\sigma_1}{E_1}$, since after the target the confining pressure σ_3 is

kept unchanged, so at sub-path (2) $d\sigma_2 = d\sigma_3 = 0$. Thus this part of equivalent uniaxial strain can be directly obtained from the experimental strain data in "1" direction. For the first integration part $\int_{(1)} \frac{d\sigma_1}{E_1}$, at the end of sub-path (1), σ'_1 / σ'_{1c} is less than 20%, which

leads to a fairly good approximation that E_1 can be treated as the initial values E_0 . In this case $\int_{(1)} \frac{d\sigma_1}{E_1}$ can be easily rewritten as $\frac{\sigma_1}{E_0}$. For a HSC, from the uniaxial compression

test, E_0 can be taken as 0.040×10^6 MPa. So finally, the total equivalent uniaxial strain is obtained by simply combining the two parts together.

Likewise, the equivalent uniaxial strain in "2" or "3" direction can be expressed as:

$$\varepsilon_{3u} = \int \frac{d\sigma_3}{E_3} = \int_{(1)} \frac{d\sigma_3}{E_3} + \int_{(2)} \frac{d\sigma_3}{E_3}$$

Since at sub-path (2), the confining pressure is kept constant, so ε_{3u} only depends on the first integration part, which can be simplified as $\frac{\sigma_3}{E_0}$.

As will be discussed later in Chapter 7 that Saenz equation can be rewritten in terms of the equivalent uniaxial strain as follows:

$$\sigma_{i} = \frac{\varepsilon_{iu}E_{0}}{1 + (\frac{E_{0}}{E_{s}} - 2)\frac{\varepsilon_{iu}}{\varepsilon_{iuc}} + \left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^{2}}$$
(5-13)

where E_0 : initial tangent elastic modulus

 ε_{iu} : equivalent uniaxial strain in "i" direction

 E_s : = σ_{ic} / ε_{iuc} , the secant elastic modulus at the peak point

 σ_{ic} peak stress in the "i" direction

 ε_{iuc} peak equivalent uniaxial strain in "i" direction

The equivalent uniaxial stress \sim strain curves of HSC under load path T-1 together with their corresponding Saenz approximations (Equation (5-13)) are shown in Figure 5.21.



Figure 5.21 Equivalent uniaxial stress ~ strain curves together with corresponding Saenz approximations under different confining pressures for HSC.

From Equation (5-13), the peak equivalent uniaxial strain ε_{ic} needs to be determined before Sanez equation can be used to approximate the equivalent uniaxial

stress ~ strain relationship. Elwi and Murray (1979) assumed that the ultimate (failure) surface of the equivalent uniaxial strain is analogous to that of the σ_{ic} and the Willam-Warnke failure criterion is still applicable by simply replacing $f_c^{'}$, τ_m , σ_m , σ_1 , σ_2 and σ_3 in Equation (5-4) with ε_{cu} , γ_m^* , ε_m^* , ε_{1uc} , ε_{2uc} and ε_{3uc} , respectively, where the * in the notation represents the equivalent uniaxial quantities. Table 5.8 and Figure 5.22 show the compressive meridian of the equivalent uniaxial strain ε_{iuc} of HSC from this study.

Load	σ_{3}	E _{2uc} , E _{3uc}	E _{luc}	<i>E</i> [*] _m	γ_m^*	$\varepsilon_m^*/\varepsilon_{cu}$	$\gamma_m^* / \varepsilon_{cu}$
path	(MPa)	(×10 ⁻⁶)	(×10 ⁻⁶)	(×10 ⁻⁶)	(×10 ⁻⁶)		
	uniaxial	0	2514	838	918	0.333	0.365
	3.5	87.5	4712	1629	1689	0.648	0.672
	7	175	7830	2727	2795	1.085	1.112
T-1	14	350	12474	4391	4427	1.747	1.761
	28	700	24952	8784	8856	3.494	3.523
	42	1050	31786	11295	11223	4.493	4.464
	56	1400	39680	14160	13978	5.632	5.560

Table 5.8 Compressive Meridian of the Equivalent Uniaxial Strain ε_{iuc}

From Figure 5.22, the compressive meridian of the equivalent uniaxial strain ε_{ic} takes a form of

$$\frac{\gamma_m^*}{\varepsilon_{cu}} = 0.9836 \frac{\varepsilon_m^*}{\varepsilon_{cu}} + 0.0443$$
(5-14)



Figure 5.22 Compressive meridian of the equivalent uniaxial strain for HSC.

5.5 Poisson's Ratio in Triaxial Compression

Some researchers (Chen 1982, Guo 1997 and Zhu 1998) have lamented the dearth of data on the Poisson's ratio under multiaxial stress conditions. Up until now, almost all of the available empirical equations about the variation of Poisson's ratio with stress status are derived from the uniaxial compression tests. Among them, Kuper's equation (Kupfer et. al 1969) is represented in terms of the ratio of current strain ε_c to the peak value ε_{cu} :

$$v = v_0 \left[1 + 1.3763 \left(\frac{\varepsilon_c}{\varepsilon_{cu}} \right) - 5.36 \left(\frac{\varepsilon_c}{\varepsilon_{cu}} \right)^2 + 8.586 \left(\frac{\varepsilon_c}{\varepsilon_{cu}} \right)^3 \right]$$
(5-15)

Given the initial v_0 to be 0.20, the tangent Poisson's ratio at peak will be 1.12.

Based on Ottosen's description, Guo (1997) proposed an equation in terms of the ratio of current stress σ_i to the peak value σ_{ic} , β :

$$v_{t} = \begin{cases} v_{0} & 0 < \beta \le 0.8 \\ v_{tf} - (v_{tf} - v_{0})\sqrt{1 - 25(\beta - 0.8)^{2}} & 0.8 < \beta \le 1.0 \end{cases}$$
(5-16)
$$v_{0} = 0.2$$

where:
$$v_{tf} = \begin{cases} 0.15 & \sigma_{i} < 0 \text{ tension} \\ 1.08 & \sigma_{i} > 0 \text{ compression} \end{cases}$$

Both of those two equations are only applicable at the ascending branch of the stress \sim strain curve.

Load path T-1 is expedient to derive the tangent Poisson's ratio variation in triaxial compression status after the target confining pressure is reached, when $d\sigma_3$ is 0 and consequently v_i is just equal to $d\varepsilon_3 / d\varepsilon_1$. This variation with the axial stress ratio σ_1 / σ_{1c} under different confining pressures is shown in Figure 5.23.

It can be seen from Figure 5.23 that the Poisson's ratio variation of HSC with stress ratio β under different confining pressures are highly changeable. The lower the confining pressure, the higher the initial tangent Poisson's ratio v_0 . When confining pressure is 0 (uniaxial compression), v_0 is 0.20 and v_{if} is 1.12, which well fits with the prediction of those two empirical equations. However, when confining pressure is as high as 42 or 56 MPa, v_0 is just around 0.10 and v_{if} is about 0.6. It seems there is no accurate way to describe the changing rule of Poisson's ratio of the HSC, which may be attributed to the erratic non-linear behavior of this material.

As will be discussed later in Chapter 7, in order to maintain a positive definite stiffness matrix of the constitutive equation, some certain limits were imposed to the Poisson's ratio by some researchers. Elwi and Murray (1979) put a 0.5 limit, but they only considered the ascending part of the stress ~ strain curve. Also it can be seen from Figure 5.23 this limit of 0.5 is by no means close to the actual scenario that the tangent Poisson's ratios of HSC at peak under different confining pressures (0 ~ 42 MPa) are all around 1.0 or larger than that. For the descending branch, it can be seen from Figure 5.23 that Poisson's ratio is highly unpredictable with a wide range from about 1.0 (high confining pressures) to 2.0 (low confining pressures), but there seems to be a general trend that when the confining pressure is higher than 14 MPa, the Poisson's ratio after peak lingers around 1.0.



Figure 5.23 Poisson's ratio variation of HSC with stress ratio β .

In view of all those aspects, an expression for Poisson's ratio of HSC under triaxial compression in terms of stress ratio $\beta(\sigma_i / \sigma_{ic})$ is given as follows:

$$v_{t} = \begin{cases} v_{0} & 0 < \beta \le 0.5 \\ v_{tf} - (v_{tf} - v_{0}) \sqrt{1 - 8(\beta - 0.5)^{3}} & 0.5 < \beta \le 1.0 \\ v_{tp} & \varepsilon_{iu}/\varepsilon_{ic} > 1.0 \end{cases}$$
where $v_{0} = 0.15$, $v_{tf} = 1.0$ and $v_{tp} = 1.0$ (5-17)



Figure 5.24 shows the approximation of the proposed expression to the experimental results.

Figure 5.24 Proposed Poisson's ratio of HSC under triaxial compression.

The initial tangential Poisson's ratio v_0 is taken as 0.15, which is appropriate under moderate confining pressures (3.5MPa ~ 14 MPa), and this value is smaller than that of the uniaxial compression ($v_0 = 0.2$). Up to 50% percent of stress ratio σ_i / σ_{ic} , the tangent Poisson's ratio is deemed as a constant, and 1.0 is adopted for v_t at the peak. It is very hard to predict the tangent Poisson's ratio of HSC after peak, which is highly dependent on the confining pressure. At this stage it is presumably taken as a constant 1.0, which is found to be more appropriate to illustrate the experimental results of most tests, especially under relatively large confining pressures.

5.6 Cyclic Loading in Triaxial Compression

Cyclic loading was applied after the target confining pressures had been reached for two confining pressures (14 MPa and 21 MPa). The unloading took the load control mode

with the decrease rate of axial stress being 40 MPa/min and the confining pressure being kept unchanged. After axial stress was reduced to the target value, the control mode was switched back to the original displacement control again all the way up to the next load cycle. The testing results are shown in Figure 5.25 and Figure 5.26.

From Figure 25 and Figure 26 it can be clearly seen that a more obvious cyclic loop in this stress ~ strain curve will be formed if the unloading goes back to a lower axial stress. Although it does not seem to be very apparent, the slope of each unloading and reloading portion actually does undergo a notable degradation, as can be found in Figure 25(a) and 25(c). For instance, under confining pressure of 14 MPa, just after 4 cycles which were separated almost equally by the displacement control, the shear modulus G lost about 64% (0.0181×10^6 MPa to 0.0066×10^6 MPa) of its initial value, and right before the peak, G lost about 40% (0.0181×10^6 MPa to 0.0105×10^6 MPa).



Figure 5.25(a) Axial stress ~ strain curve in cyclic loading under confining pressure σ_3 =14MPa for HSC.



Figure 5.25(b) Octahedral normal stress $\sigma_{oct} \sim$ volume change in cyclic loading under confining pressure σ_3 =14MPa for HSC.



Figure 5.25(c) $\tau_{oct} \sim \gamma_{oct}$ in cyclic loading under confining pressure σ_3 =14MPa for HSC.



Figure 5.26(a) Axial stress ~ strain curve in cyclic loading under confining pressure σ_3 =21MPa for HSC.



Figure 5.26(b) Octahedral normal stress $\sigma_{oct} \sim$ volume change in cyclic loading under confining pressure σ_3 =21MPa for HSC.



Figure 5.26(c) $\tau_{oct} \sim \gamma_{oct}$ in cyclic loading under confining pressure σ_3 =21MPa for HSC.

After the comparison between the triaxial stress \sim strain curves of monotonic loading and cyclic loading under the same confining pressures, it is found that the envelops of the cyclic loading curves maintain the same shape with those of the monotonic loading.

5.7 Strains during Cyclic Loading

The relationship between the lateral strain ε_3 and axial strain ε_1 for HSC during cyclic loading is shown in Figure 5.27.

It seems that the strain development during cyclic loading does not deviate much from the mainstream of that under monotonic loading. Since during cyclic loading the confining pressure maintains the same, it is quite reasonable to assume that the Poisson's ratio of HSC during cyclic loading takes the same tangent value at the point where the unloading occurs.



Figure 5.27 Lateral strain $\varepsilon_3 \sim axial strain \varepsilon_1$ during cyclic loading under confining pressure $\sigma_3=14MPa$ for HSC.

CHAPTER 6

TRIAXIAL COMPRESSION TEST RESULTS OF STEEL FIBER REINFORCED HIGH STRENGTH CONCRETE (SFHSC)

6.1 Experimental Program

The raw materials and mix proportion of the SFHSC are essentially the same with those of the HSC listed in Table 5-1 except for the additional introduction of 1% volume ratio of hook ended 30mm long steel fiber with a diameter of 0.5 mm. Compared with the SFHSC in Chapter 3, due to the change of aggregates, the standard compressive strength f_c is down from 79.0 MPa to 70.3 MPa and initial elastic modulus E₀ is down from 48800 MPa to 41400 MPa.

Table 6.1Mix Proportion of SFHSC

Water- cement ratio	Water (kg/m ³)	Cement (kg/m ³)	Sand (kg/m ³)	Crushed stone (kg/m ³)	Sika2000 (L/m ³)	Slump (mm)	Steel fiber (kg/m ³)
0.33	170	520	760	1000	4.3	100	78

100 mm×150mm (4×6 in.) concrete cylinders with both ends ground under designed lubricated loading platen were mainly used in this part. However, to make comparison, several tests using regular 100 mm×200mm (4×8 in.) cylinders under standard rigid loading platen were also performed to study the difference between those two situations in terms of triaxial strength and triaxial stress ~ strain behavior.

In this part, three different load paths were adopted, namely T-2, T-3 and T-4. There is also another load path T-2' which is just a derivative of load path T-2 and aimed at coping with high confining pressures which cannot be reached simply by the application of T-2. Those three load paths are shown in Figure 6.1.



Figure 6.1 Different load paths used in triaxial compression of SFHSC. $(\sigma_1$ -axial stress, σ_3 -confining pressure)

Load control was employed before the target confining pressure σ_3 was reached with the increasing rate of σ_3 being 7 MPa/min. After the target σ_3 , displacement control was adopted all the way up to the failure (including strain softening) of the specimen. The displacement rate was controlled within a range from 0.005 mm/s (0.0002 in/s) to 0.010mm/s (0.0004 in/s).

6.2 Failure Criterion

6.2.1 Mohr-Coulomb Failure Criterion

The triaxial compression test results of the SFHSC concerning the Mohr-Coulomb failure criterion are listed in Table 6.2 and Figure 6.2.

Load path	σ_3 (MPa)	$\sigma_{\rm l}$ (MPa)	$\sigma_{3}/f_{c}^{'}$	σ_1 / f_c
uniaxial	0	69.0	0	1
T-2	7	105.2	0.101	1.525
T-2	14	136.8	0.203	1.983
T-2 (100×200 mm cylinder)	14	139.0	0.203	2.014
T-2	21	164.1	0.304	2.378
T-2 (100×200 mm cylinder)	21	162.3	0.304	2.352
T-2	28	186.5	0.406	2.703
T-2'	28	189.6	0.406	2.748
T-3	28	191.6	0.406	2.777
T-4	28	191.8	0.406	2.780
T-2 (100×200 mm cylinder)	28	189.6	0.406	2.748
T-2'	42	239.0	0.609	3.464
T-2'	56	282.2	0.812	4.090
T-2'	63	308.2	0.913	4.467
T-2'	70	324.1	1.014	4.697

 Table 6.2
 Triaxial Strength of SFHSC for Mohr-Coulomb Failure Criterion



Figure 6.2 Mohr-Coulomb failure criterion of SFHSC.

From Figure 6.2 it is shown that the Mohr-Coulomb failure criterion of SFHSC can be expressed as:

$$\frac{\sigma_1}{f_c'} = 1 + 3.95 \frac{\sigma_3}{f_c'}$$
(6-1)

Compared with that of the HSC (Equation (5-3)) in which k is equal to 4.0, those two equation are essentially the same. From Equation (5-2), it is indicated that the introduction of the steel fibers does not influence the internal friction angle ϕ of the HSC.

6.2.2 Willam-Warnke Failure Criterion

The experimental results for establishing the Willam-Warnke failure criterion are listed in Table 6.3, and the compressive meridian is shown in Figure 6.3.

Load path	σ_3 (MPa)	σ_{i} (MPa)	σ_m / f_c'	$ au_m / f_c'$
uniaxial	0	69.0	0.333	0.365
T-2	7	105.2	0.576	0.520
T-2	14	136.8	0.796	0.650
T-2 (100×200 mm cylinder)	14	139.0	0.807	0.662
T-2	21	164.1	0.996	0.757
T-2 (100×200 mm cylinder)	21	162.3	0.987	0.748
T-2	28	186.5	1.171	0.839
T-2'	28	189.6	1.186	0.855
T-3	28	191.6	1.196	0.866
T-4	28	191.8	1.197	0.867
T-2 (100×200 mm cylinder)	28	189.6	1.186	0.855
T-2'	42	239.0	1.560	1.043
T-2'	56	282.2	1.904	1.197
T-2'	63	308.2	2.098	1.298
T-2'	70	324.1	2.242	1.345

 Table 6.3
 Triaxial Strength of SFHSC for Willam-Warnke Failure Criterion

Figure 6.3 indicates that regardless of the different load paths and specimen sizes, there is a good correlation between $\sigma_m / f_c^{'}$ and $\tau_m / f_c^{'}$. The compressive meridian of the SFHSC can be expressed as

$$\frac{\tau_m}{f_c'} = 0.160 + 0.653 \frac{\sigma_m}{f_c'} - 0.055 \left(\frac{\sigma_m}{f_c'}\right)^2$$
(6-2)

The compressive meridian for the HSC is defined in Chapter 5 as



Figure 6.3 Compressive meridian of Willam - Warnke failure criterion for SFHSC.

Figure 6.4 shows the data points of the compressive meridians for both HSC and SFHSC. It can be found that the difference between them is really trivial, so a uniform compressive meridian can be adopted for both of the concretes.

(5-5)



Figure 6.4 Comparison of compressive meridians of HSC and SFHSC.

In the previous section, it has been found that the introduction of steel fiber does not affect the Mohr-Coulomb failure criterion (k and internal friction angle ϕ) in triaxial compression. Combined with what has been found in this section, it seems that the introduction of steel fiber makes no difference for HSC under triaxial compression in terms of ultimate surface of stress.

Figure 6.5 shows the comparison of the compressive meridians of the SFHSC from this study and some previous researches (Chern 1992 and Ishikawa et al 2000). The testing results of Chern (1992) were from the steel fiber reinforced normal strength concrete ($f'_c \approx 25$ MPa), while those of Ishikawa et al (2000) were obtained from the fiber reinforced concrete which had the same strength grade with this study. All of the experimental data essentially follow the same trend except for that the Ishikawa's data have relatively lower upper bound of σ_m / f'_c .



Figure 6.5 Comparison of compressive meridians of SFHSC for this study and results from Ishikawa et al (2000) and Chern (1992).

6.3 Triaxial Stress ~ Strain Relationship

6.3.1 Strains at Peak Stress under Triaxial Compression

Table 6.4 shows the peak lateral strain ε_{3c} and peak axial strain ε_{1c} of SFHSC under different load paths and confining pressures.

From Table 6.4 it seems that different load paths do not have a substantial influence on the peak lateral strain ε_{3c} and the peak axial strain ε_{1c} . Under the confining pressure of 28 MPa, the cylinders under load paths T-2, T-3 and T-4 almost have the same ε_{3c} and ε_{1c} . The only exception occurs under load path T-2', where the peak lateral strain ε_{3c} is quite abnormal and even larger than the value under a confining pressure of 42 MPa. This was later found out to be the fallout of a highly non-uniform lateral expansion of the cylinder under that load path.

Load path	Confining pressure σ_3 (MPa)	Axial strength σ_1 (MPa)	Peak lateral strain ε_{3c} (×10 ⁻⁶)	Peak axial strain ε_{lc} (×10 ⁻⁶)
uniaxial	0	69.0	-1361	2777
T-2	7	105.2	-3472	7082
T-2	14	136.8	-5440	12711
T-2 (100×200 mm cylinder)	14	139.0	-4625	12090
T-2	21	164.1	-6750	16973
T-2 (100×200 mm cylinder)	21	162.3	-8011	20195
T-2	28	186.5	-9165	25722
T-2'	28	189.6	-13145 *	27439
T-3	28	191.6	-9858	24379
T-4	28	191.8	-9866	24053
T-2 (100×200 mm cylinder)	28	189.6	-9748	25230
T-2'	42	239.0	-12313	33259
T-2'	56	282.2	-13867	42168
T-2'	63	308.2	-15497	45089
T-2'	70	324.1	17338	48203

 Table 6.4
 Axial and Lateral Strains at Peak under Different Confining Pressures

* abnormal

Table 6.5 compares the peak lateral strain ε_{3c} and peak axial strain ε_{1c} of SFHSC with those of HSC under certain confining pressures. It indicates that under triaxial compression, the introduction of the steel fibers into the HSC does not provide the obvious advantage in terms of the peak lateral strain ε_{3c} and peak axial strain ε_{1c} , which means the deformation capacity of SFHSC before failure does not have a noticeable improvement compared with HSC under triaxial compression.

Confining pressure	ε_{3c} (>	<10 ⁻⁶)	ε_{lc} (×10 ⁻⁶)		
σ_3 (MPa)	HSC	SFHSC	HSC	SFHSC	
0	-1000	-1361	2514	2777	
7	-3609	-3472	7759	7082	
14	-5286	-5033	13027	12400	
21	-6689	-7381	18098	18584	
28	-8772	-9659	24548	24846	
42	-11084	-12313	32130	33259	
56	-12376	-13867	40582	42168	

Table 6.5 Comparison of ε_{3c} and ε_{1c} between HSC and SFHSC

As shown in Figure 6.6, for the SFHSC, the relationship between $\frac{\varepsilon_{lc}}{\varepsilon_{cu}}$ and $\frac{\sigma_3}{f_c}$ (where ε_{cu} is the peak axial strain in uniaxial compression and f_c is the uniaxial compressive strength) can be expressed as:



$$\frac{\varepsilon_{1c}}{\varepsilon_{cu}} = 1 + 17.33 \left(\frac{\sigma_3}{f_c}\right)$$
(6-3a)

Figure 6.6 $\frac{\varepsilon_{lc}}{\varepsilon_{cu}}$ versus $\frac{\sigma_3}{f_c}$ for SFHSC under triaxial compression.

Compared with Equation (5-6a), this expression is a little bit different in that the constant "17.33" is a little lower. However, it should be noted that this difference is caused by the additional two more points (σ_3 =63 MPa and 70 MPa). Were those points ignored, Equation (6-3a) would be almost same with Equation (5-6a).

Like HSC, a good relationship between the lateral peak strain ε_{3c} and axial peak strain ε_{1c} under different confining pressures is shown in Figure 6.7, which can be expressed as follows:

$$\frac{\varepsilon_{3c}}{\varepsilon_{cu}} = -0.0055 \left(\frac{\varepsilon_{1c}}{\varepsilon_{cu}}\right)^2 + 0.4367 \frac{\varepsilon_{1c}}{\varepsilon_{cu}}$$
(6-3b)



Figure 6.7 Relationship between axial and lateral strains at peak for SFHSC under triaxial compression.

6.3.2 Axial Stress versus Strains under Various Confining Pressures

Under load path T-2 (T-2' for confining pressure 42MPa, 56MPa and 70MPa), the axial stress ~ strain (axial and lateral) curves of SFHSC are shown in Figure 6.8. The load path T-2' for confining pressure 28MPa, 42MPa, 56MPa and 70MPa is defined in Figure 6.9. It should be noted that T-2' was just modified from T-2 in order to reach high confining pressure before the failure of the specimens.

The load paths T-2 and T-2' are designed to maintain a lateral expansion of the specimen under triaxial compression since the increment of confining pressure σ_3 is much smaller than that of the axial stress σ_1 , which is good for a more accurate lateral strain measurement (it has been found that the circumferential extensometer is more sensitive in detecting expansion than contraction since it is only driven by two springs). Thus in Figure 6.8, the lateral strains ε_3 are all in expansion (negative side).

Compared with the triaxial stress \sim strain curves of HSC (load path T-1) as shown in Figure 5-9, the shape of the triaxial stress \sim strain curve of SFHSC undergoes some variation under high confining pressures (56MPa, 70MPa). In those two cases, before the target confining pressure is reached, the curves can be approximated by two straight lines (see Figure 6.8).



Figure 6.8 Axial stress versus strain curves of SFHSC at different confining pressures (Load path T-2).



Figure 6.9 Load path T-2' under different confining pressures σ_3 .

The triaxial stress ~ strain curves of SFHSC under different load paths for confining pressure $\sigma_3 = 28$ MPa are shown in Figure 6.10. For load path T-4 (Figure 6.1), the specimen was first applied to an axial stress of 42 MPa (60% f_c).



Figure 6.10 Axial stress versus strain curves of SFHSC under different load paths at $\sigma_3 = 28$ MPa.

It can be seen from Figure 6.10 that the four axial stress \sim axial strain curves follow the same trend except that T-3 and T-4 are somehow steeper than T-2 and T-2' under high axial stresses. This is understandable since around that region the confining pressures for T-3 and T-4 is relatively higher than those of T-2 and T-2'.

6.3.3 Octahedral Stress~Strain Relationship

6.3.3.1 Octahedral Normal Stress versus Volume Change The Octahedral normal stress versus volume change of SFHSC under load path T-2 at different confining pressures (0 ~ 70 MPa) is shown in Figure 6.11. Like HSC (Figure 5.11), the initial slopes, which are the initial tangent bulk moduli K_t ($K_t = \frac{\sigma_{oct}}{3\varepsilon_{oct}}$), are essentially the same

for all the confining pressures under the same load path T-2. However, with the increase

of the confining pressure, the tangent bulk moduli tend to become larger. As can be seen from Figure 6.11, the curves under higher confining pressures become steeper with the increase of σ_{act} .



Figure 6.11 σ_{oct} versus volume change of SFHSC at different confining pressures (load path T-2).

Figure 6.11 also shows that SFHSC starts to reverse from contraction to dilation just right before the peak stress is reached, which complies with Gerstle's description (Gerstle 1981) for normal strength concrete. At that turnaround point, the stress ratio $\sigma_{oct} / \sigma_{octp}$ is around 95%. Similar to the uniaxial test result, the bulk modulus, which is

equal to
$$\frac{d\sigma_{oct}}{3d\varepsilon_{oct}}$$
, can be approximately adopted as a constant after the peak

The effect of the load path on the volume change of SFHSC under the same confining pressure is shown in Figure 6.12. Like HSC (Figure 5-12), there are some differences between the maximum contractions of SFHSC under triaxial compression, but the variation trends are almost the same.



Figure 6.12 σ_{oct} versus volume change of SFHSC under different load paths at $\sigma_3 = 28$ MPa

6.3.3.2 Octahedral Shear Stress τ_{oct} ~ Engineering Octahedral Shear Strain γ_{oct}

As will be discussed later in Chapter 7, $\tau_{oct} \sim \gamma_{oct}$ curve under triaxial compression is of great importance to establish a constitutive model for both HSC and SFHSC and is the key to this study.

Figure 6.13 depicts the $\tau_{oct} \sim \gamma_{oct}$ curves of SFHSC for different confining pressures under load path T-2 and T-2'. Under high confining pressures (56MPa and 70MPa), like the axial stress ~ strain relationship (Figure 6.8), the $\tau_{oct} \sim \gamma_{oct}$ curves also deviate from the typical parabolic shapes and manifest some kind of bilinear characteristic before the target confining pressures.

Figure 6.14 illustrates the normalized version of Figure 6.13. It can be found that upon normalized, there is clearly a general trend for all $\tau_{oct} \sim \gamma_{oct}$ curves except that the

slope of the uniaxial one before peak is steeper than the triaxial ones. The post peak deviation from the mainstream for $\tau_{oct} \sim \gamma_{oct}$ curve under the confining pressure of 14MPa may be attributed to experimental errors.



Figure 6.13 $\tau_{oct} \sim \gamma_{oct}$ curves of SFHSC under different confining pressures (load path T-2 and T-2').



Figure 6.14 Normalized $\tau_{oct} \sim \gamma_{oct}$ curves at different confining pressures (0,7,14,21,28,42,56,70 MPa) under load path T-2 and T-2'.

Figure 6.15 shows the $\tau_{oct} \sim \gamma_{oct}$ curves of SFHSC under different load paths for the confining pressure of 28MPa. Since they are essentially following the same trend, it should be appropriate to use a uniform expression to approximate all of them.



Figure 6.15 $\tau_{oct} \sim \gamma_{oct}$ curves of SFHSC under different load paths for $\sigma_3 = 28$ MPa.

Just like HSC, Saenz equation is used to make the approximation. It can be expressed in terms of $\tau_{oct} \sim \gamma_{oct}$ as follows:

$$\tau_{oct} = \frac{G_0 \gamma_{oct}}{1 + \left(\frac{G_0}{G_s} - 2\right) \frac{\gamma_{oct}}{\gamma_{octp}} + \left(\frac{\gamma_{oct}}{\gamma_{octp}}\right)^2}$$
(6-4)

where G_0 : initial tangent shear modulus

 ${\rm G_s}$: = $\tau_{\it octp}$ / $\gamma_{\it octp}$, the secant shear modulus at the peak

 $\tau_{\it octp}$ peak octahedral shear stress

 γ_{oct} : octahedral shear strain

Figure 6.16 demonstrates that the experimental $\tau_{oct} \sim \gamma_{oct}$ curves comply with the Saenz equation fairly well under moderate confining pressures ($\sigma_3 \leq 28$ MPa). However, under high σ_3 (42MPa, 56MPa, and 70MPa), the experimental $\tau_{oct} \sim \gamma_{oct}$ curves are far below the Saenz prediction. In this case, a modified Saenz equation like Equation (5-10b) can be adopted (see Chapter 5 and Chapter 7).



Figure 6.16 Experimental $\tau_{oct} \sim \gamma_{oct}$ curves and Saenz approximation of SFHSC at different confining pressures (load path T-2).

6.3.3.3 Relationship between Peak Octahedral Shear Stress τ_{octp} and Peak Engineering Octahedral Shear Strain γ_{octp} Like HSC, the relationship between
Load path	Confining	Axial	Peak lateral	Peak axial	Peak	Peak
	pressure	strength	strain ε_{3c}	strain ε_{1c}	γ_{octp}	$ au_{octp}$
	σ_3 (MPa)	$\sigma_{\rm lc}$ (MPa)	(×10 ⁻⁶)	(×10 ⁻⁶)	(×10 ⁻⁶)	(MPa)
uniaxial	0	69.0	-1361	2777	3901	32.5
T-2	7	105.2	-3472	7082	9950	46.3
T-2	14	136.8	-5440	12711	17113	57.9
T-2	21	164.1	-6750	16973	22366	67.5
T-2	28	186.5	-9165	25722	32892	74.7
T-2'	28	189.6	-13145*	27439*	*	*
T-3	28	191.6	-9858	24379	32279	77.1
T-4	28	191.8	-9866	24053	31979	77.2
T-2						
100x200mm	28	189.6	-9748	25230	32978	76.2
T-2'	42	239.0	-12313	33259	42966	92.9
T-2'	56	282.2	-13867	42168	52830	106.6
T-2'	63	308.2	-15497	45089	57121	115.6
T-2'	70	324.1	-17338	48203	61793	119.8

Table 6.6 $\tau_{octp} \sim \gamma_{octp}$ of SFHSC under Different Load Conditions

* abnormal



Figure 6.17 $\tau_{octp} \sim \gamma_{octp}$ of SFHSC under triaxial compression.

From Table 6.6 and Figure 6.17, the relationship between τ_{octp} and γ_{octp} of SFHSC can be written as:

$$\tau_{octp} = 1.456 \times 10^{-3} \gamma_{octp} + 30.4 \tag{6-5}$$

It is quite close to the same equation of HSC, which is expressed as:

$$\tau_{octp} = 1.458 \times 10^{-3} \gamma_{octp} + 29.30 \tag{5-11}$$

If those two groups of data (HSC and SFHSC) are plotted on the same graph (Figure 6.18), it is quite clear that all the data can be expressed by a single expression, either Equation (5-11) or Equation (6-5). There is really not much difference between SFHSC and HSC in terms of the peak octahedral shear stress and strain.



Figure 6.18 $\tau_{octp} \sim \gamma_{octp}$ of SFHSC and HSC.

6.4 Equivalent Uniaxial Stress ~ Strain Relationship

The detailed description on deriving the equivalent uniaxial strain has been given in section 5.4 of Chapter 5, and is hereby abridged. The calculation was based on load path T-2 (σ_3 =7MPa and 14MPa) and T-3 (σ_3 =28MPa). Results for SFHSC are shown in Table 6.7, Table 6.8, Figure 6.19 and Figure 6.20, respectively.

Table 6.7	σ'_{1c}	for Different	σ_{2}
			- 3

Load path	Confining pressure σ_3 (MPa)	$\sigma_{1c}^{'}$ (MPa)	σ'_1 at target σ_3 (MPa)	$ \begin{array}{c} \sigma_1'/\sigma_{1c} \\ (\%) \end{array} $
T-2	7		42	20.5
	14	205	84	41.0
T-3	28	+∞	28	0

Table 6.8 Compressive Meridian of the Equivalent Uniaxial Strain ε_{iuc}

Load path	σ_3	$\mathcal{E}_{2uc}, \mathcal{E}_{3uc}$	E _{luc}	\mathcal{E}_m^*	γ_m^*	$\varepsilon_m^*/\varepsilon_{cu}$	$\gamma_m^*/\varepsilon_{cu}$
	(MPa)	(×10 ⁻⁶)	(×10 ⁻⁶)	(×10 ⁻⁶)	(×10 ⁻⁶)		
uniaxial	0	0	2777.5	926	1014	0.33	0.37
T-2	7	175	7243.936	2531	2581	0.91	0.93
	14	350	12582.81	4428	4467	1.59	1.61
T-3	28	700	24718.89	8706	8770	3.13	3.16



Figure 6.19 Compressive meridian of equivalent uniaxial strain ε_{iuc} of SFHSC and HSC under triaxial compression.

All the data from both SFHSC and HSC for the compressive meridian of the equivalent uniaxial strain are shown in Figure 6.19, which can be summarized into a regression equation of which R^2 is equal 1.0:

$$\frac{\gamma_m^*}{\varepsilon_{cu}} = -0.0061 \left(\frac{\varepsilon_m^*}{\varepsilon_{cu}}\right)^2 + 1.0192 \frac{\varepsilon_m^*}{\varepsilon_{cu}} + 0.0162$$
(6-6)



Figure 6.20 Equivalent uniaxial stress~strain curves and Saenz approximation of SFHSC under different confining pressures.

6.5 Comparison of Stress ~ Strain behavior of Different Specimen Sizes under Triaxial Compression

In this research, the 100mm×150mm (4"×6") cylinders together with designed lubricated loading platen are mainly used to study the triaxial compression behavior of SFHSC. It is necessary to make a comparison with the commonly used testing method in which standard 100mm×200mm (4"×8") cylinders and rigid loading platens are employed. Under the same load path T-2 in triaxial compression, it has been found from Table 6.2 and 6-3 that those two methods make no noticeable difference in terms of triaxial strength and failure criterion. Also Table 6.4 shows that the peak strain values (ε_{1c} and ε_{3c}) for both two cases are close to each other.

Figures 6.21, 6-22 and 6-23 show that at the ascending branch, the triaxial stress ~ strain behavior (axial stress ~ strain, σ_{oct} ~ volume change and $\tau_{oct} \sim \gamma_{oct}$) of those two different specimens together with their respective loading platens comply with each other very well. However, at the descending branch, 100mm×200mm (4"×8") cylinders with rigid loading platens seem to provide better ductility than the 100mm×150mm (4"×6") cylinders with the designed lubricated loading platen. One possible explanation for this phenomenon is that the extra confining effect imposed by the end effect of the standard cylinder will manifest itself much stronger at the descending branch than the ascending branch when this end effect is cancelled out by the true confining pressures inside the triaxial chamber.



Figure 6.21 Axial stress~strain curves of the two specimen sizes under triaxial compression (load path T-2).



Figure 6.22 σ_{oct} ~ volume change of the two specimen sizes under triaxial compression (load path T-2).



Figure 6.23 $\tau_{oct} \sim \gamma_{oct}$ of the two specimen sizes under triaxial compression (load path T-2).

6.6 Poisson's Ratio in Triaxial Compression

The tangent Poisson's ratio of SFHSC under triaxial compression is shown in Figure 6.24. In this part of experiment only two cases, namely confining pressure 28MPa under load path T-3 and confining pressure 7MPa under load path T-2 are suitable for evaluating this variation. It is found that the tangent Poisson's ratio after peak v_{ip} takes a constant value from 0.9 to 1.2 for these two cases.



Figure 6.24 Variation of tangent Poisson's ratio of SFHSC with stress ratio under triaxial compression.

It can be seen from Figure 6.24 that the variation of v_t with stress condition for HSC (Equation 5-17) can also be applied to SFHSC under triaxial compression. Thus, a uniform expression can be developed for both of them.

6.7 Triaxial Stress ~ Strain Behavior under Cyclic Loading in Triaxial Compression Like HSC, the load control mode was used in the unloading path with the decrease rate of axial stress being 40 MPa/min and the confining pressure being kept unchanged. After axial stress was released to the target value, the control mode was switched back to the original displacement control again all the way up to the next load cycle.

The typical cyclic loading behavior is shown in Figure 6.25 (a) (b) and (c) for the load path T-3 and the confining pressure of 28 MPa. The other cases are shown in the appendix A. Like HSC, SFHSC also experienced appreciable degradation in terms of the elastic modulus E (Figure 6-25(a)) and shear modulus G (Figure 6-25(c)) under cyclic loading, and there seems no good way to simulate this degradation trend. However, the cyclic loading does not change the general envelop of the triaxial stress ~ strain curves of SFHSC.





Figure 6.25 Triaxial stress ~ strain curves of SFHSC in cyclic loading under confining pressure $\sigma_3=28$ MPa (load path T-3). (a) $\sigma_1 \sim \varepsilon_1$ and ε_3 (b) $\sigma_{oct} \sim 3\varepsilon_{oct}$ (c) $\tau_{oct} \sim \gamma_{oct}$

6.8 Strains During Cyclic Loading

The typical relationship of lateral strain ε_3 versus axial strain ε_1 of SFHSC during cyclic loading is shown in Figure 6.26. The confining pressure is 28 MPa under load path T-3. For the other cases, the results are shown in Appendix B.

It can be seen from Figure 6.26 that, like HSC, the variation of those two strains under cyclic loading does not deviates too much from the mainstream relationship, and it may be applicable to take a constant tangent Poisson's ratio for a single unloading and reloading process, which is equal to the value right at the unloading point.



Figure 6-26 Axial strain $\varepsilon_1 \sim$ lateral strain ε_3 of SFHSC during cyclic loading under confining pressure σ_3 =28MPa (load path T-3).

CHAPTER 7

AN INCREMENTAL CONSTITUTIVE MODEL FOR HSC AND SFHSC UNDER TRIAXIAL COMPRESSION

7.1 Introduction of Orthotropic Incremental Constitutive Models

7.1.1 Darwin-Pecknold Constitutive Model – 2D

Darwin and Pecknold (1977) proposed a constitutive stress ~ strain relationship to model the general biaxial loading histories (including cyclic loading). This model is supposed to be used in conjunction with the finite element analysis, so an incremental (tangent) rather than total (secant) form has been adopted. In this model, the concrete is treated as an incrementally linear elastic material, and at the end of each stress or strain increment, the material properties, essentially E (elastic modulus) and v (Poisson's ratio), are adjusted to reflect the latest changes in deflection (strain) or load (stress).

If the shear stress is neglected and only the principal stress axes are considered, the incremental stress ~ strain relationship can be described as the following:

$$\begin{cases} d\sigma_1 \\ d\sigma_2 \end{cases} = \frac{1}{1 - \nu_1 \nu_2} \begin{bmatrix} E_1 & \nu_2 E_1 \\ \nu_1 E_2 & E_2 \end{bmatrix} \begin{cases} d\varepsilon_1 \\ d\varepsilon_2 \end{cases}$$
(7-1)

where E_i : tangent elastic modulus in the "i" direction

 v_i : tangent Poisson's ratio induced by the stress in "i" direction

If symmetry is assumed in the stiffness matrix, then $v_2 E_1 = v_1 E_2$. Let $v^2 = v_1 v_2$, Equation (7-1) can be regritten into another form:

$$\begin{cases} d\sigma_1 \\ d\sigma_2 \end{cases} = \frac{1}{1 - \nu^2} \begin{bmatrix} E_1 & \nu \sqrt{E_1 E_2} \\ \nu \sqrt{E_1 E_2} & E_2 \end{bmatrix} \begin{cases} d\varepsilon_1 \\ d\varepsilon_2 \end{cases}$$
(7-2a)

The constitutive matrix in Equation (7-2a) will be totally defined once E_1 , E_2 and ν are given. Those three variables will be determined by using an initiative concept – equivalent uniaxial strain, ε_{iu} .

The concept of "equivalent uniaxial strain" was originally developed by Darwin and Pecknold to duplicate the actual biaxial stress \sim strain curves directly from the uniaxial counterpart and then use it to simulate the variation of elastic modulus and Poisson's ratio under different stress conditions. This concept can be better illustrated if one reshuffles the Equation (7-2a) to Equation (7-2b):

$$\begin{cases} d\sigma_1 \\ d\sigma_2 \end{cases} = \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix} \begin{cases} d\varepsilon_{1u} \\ d\varepsilon_{2u} \end{cases}$$
(7-2b)

where $d\varepsilon_{iu}$ is the "equivalent uniaxial strain" increment. Comparing it with the original equation, one can easily achieve

$$d\varepsilon_{1u} = \frac{d\varepsilon_1}{1 - v^2} + \frac{v \cdot d\varepsilon_2}{1 - v^2} \sqrt{\frac{E_2}{E_1}} \quad \text{and} \quad d\varepsilon_{2u} = \frac{d\varepsilon_2}{1 - v^2} + \frac{v \cdot d\varepsilon_1}{1 - v^2} \sqrt{\frac{E_1}{E_2}}$$

The incremental equivalent uniaxial strain $d\varepsilon_{iu}$, which can be either positive or negative, may be defined more explicitly from Equation (7-2b) as

$$d\varepsilon_{iu} = \frac{d\sigma_i}{E_i}$$

The physical meaning of $d\varepsilon_{iu}$ is clearly demonstrated from the last equation as the increment of strain in the "i" direction that concrete would experience if subjected to a uniaxial stress increment $d\sigma_i$ with other stress increment equal to zero. For a nonlinear

material like concrete, equivalent uniaxial strain ε_{iu} represents the portion of strain in the "i" direction eliminating Poisson's effect that controls the material behavior such as softening and failure. It should be noted that the equivalent uniaxial strain is a fictitious material index which is invented to simplify the procedures to attain the other true parameters such as elastic modulus E_i and Poisson's ratio v.

Uniaxial stress ~ strain curve proposed by Saenz (shown in Equation (7-3)) was adopted in this constitutive model by replacing the uniaxial strain ε_i with corresponding equivalent uniaxial strain ε_{iu} :

$$\sigma_{i} = \frac{\varepsilon_{iu}E_{0}}{1 + (\frac{E_{0}}{E_{s}} - 2)\frac{\varepsilon_{iu}}{\varepsilon_{iuc}} + \left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^{2}}$$
(7-3)

where E_0 : initial tangent elastic modulus

 E_{s} : = $\sigma_{\scriptscriptstyle ic}$ / $\varepsilon_{\scriptscriptstyle iuc}$, the secant modulus at the peak point

 σ_{ic} : peak stress in the "i" direction

 ε_{iuc} : peak equivalent uniaxial strain in the "i" direction

Differentiating both sides of Equation (7-3) with respect to equivalent uniaxial strain ε_{iu} will yield the tangent elastic modulus E_i :

$$E_{i} = \frac{d\sigma_{i}}{d\varepsilon_{iu}} = E_{0} \frac{1 - \left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^{2}}{\left(1 + \left(\frac{E_{0}}{E_{s}} - 2\right)\frac{\varepsilon_{iu}}{\varepsilon_{iuc}} + \left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^{2}\right)^{2}}$$
(7-4)

In Equations (7-3) and (7-4), if σ_{ic} and ε_{iuc} can be determined, then the total equivalent stress ~ strain relationship will be well defined. The biaxial compressive strength σ_{ic} of

normal strength concrete under proportional loading has been thoroughly studied by many researchers. Kupfer and Gerstle (1973) have suggested an analytical maximum strength envelop for biaxial loading as shown in Figure 7.1.



Figure 7.1 Biaxial strength envelop by Kupfer and Gerstle (1973).

Complying with this envelop, Kupfer and Gerstle also proposed simplified expressions of biaxial strength for different stress combinations. For compression-compression, the σ_{ic} is approximated by

$$\left(\frac{\sigma_{1c} + \sigma_{2c}}{f_{c}'}\right)^{2} - \frac{\sigma_{2c}}{f_{c}'} - 3.65 \frac{\sigma_{1c}}{f_{c}'} = 0 \quad (\sigma_{1c} \ge \sigma_{2c})$$
(7-5)

$$\sigma_{1t} = \left(1 - 0.8 \frac{\sigma_{2c}}{f_c}\right) f_t \tag{7-6}$$

$$\sigma_{2c} = \frac{1+3.28\alpha}{\left(1+\alpha\right)^2} f_c' \quad \alpha = \frac{\sigma_{1t}}{\sigma_{2c}}$$
(7-7)

For tension-tension, a constant biaxial tensile strength equal to the uniaxial tensile strength f_t was recommended.

Compared with σ_{ic} , determining ε_{iuc} is highly empirical and hypothetical. The following equation is used:

$$\varepsilon_{iuc} = \varepsilon_{cu} \left[\frac{\sigma_{ic}}{f'_c} R - (R - 1) \right]$$
(7-8)

in which $\varepsilon_{\scriptscriptstyle cu}$ is the peak strain for the uniaxial compression and

$$R = \frac{\frac{\varepsilon_{iuc}(\alpha = 1)}{\varepsilon_{cu}} - 1}{\frac{\sigma_{ic}(\alpha = 1)}{f_c^{'}} - 1} \qquad \alpha = \frac{\sigma_1}{\sigma_2}$$
(7-9)

 ε_{iuc} ($\alpha = 1$) can be calculated from the actual peak strain assuming a constant Poisson's ratio equal to 0.2. It is indicated that R is approximately 3 from the available data.

However, Equation (7-9) does not fit well for σ_{ic} which is less than f_c . In this case ε_{iuc} is taken as:

$$\varepsilon_{iuc} = \varepsilon_{iu} \left[-1.6 \left(\frac{\sigma_{ic}}{f_c'} \right)^3 + 2.25 \left(\frac{\sigma_{ic}}{f_c'} \right)^2 + 0.35 \left(\frac{\sigma_{ic}}{f_c'} \right) \right]$$
(7-10)

Based on the uniaxial and biaxial experimental results from Kupfer, Hilsdorf, and Rusch (1973), Darwin and Pecknold (1977) proposed that the so-called "effective" Poisson's ratio v, which is equal to $\sqrt{v_1v_2}$, would be defined correspondingly in different stress combination domains: for tension-tension and compression-compression v = 0.2; for uniaxial compression and tension-compression,

$$\nu = 0.2 + 0.6 \left(\frac{\sigma_2}{f_c}\right)^4 + 0.4 \left(\frac{\sigma_1}{f_t}\right)^4$$
(7-11)

where σ_1 and σ_2 represent compression and tension components, and f_c and f_t stand for the uniaxial compressive and tensile strength of the concrete, respectively. For uniaxial tension, concrete is assumed to be a linear elastic material with $\nu = 0.2$.

Right now all the unknown parameters in Equation (7-2a) have been determined. As the load is applied, the equivalent uniaxial strain ε_{iu} is calculated, accumulated and then entered into Equation (7-4) to determine the current tangent moduli E₁ and E₂.

This model is relatively simple and the parameters can be calibrated from the uniaxial stress \sim strain curve and numerous reported biaxial experimental studies. However, it is only applicable to the planar problems and cannot account for the 3-D situations. Moreover, during the construction of the equivalent uniaxial stress \sim strain curve, it assumes a constant Poisson's ratio value of 0.2, which is contradictory to the fact that this ratio will deviate progressively after about 60% of the peak stress. And what's more important, due to the lack of experimental data at the descending branch of the stress \sim strain curve, this model can only simulate the behavior of concrete at the ascending part under biaxial loading.

7.1.2 3D Incremental Concrete Constitutive Relationship

Elwi and Murray (1979) extended the usage of the orthotropic incremental constitutive model to the 3D domain. This model is arranged to be expressed in an explicit formulation which can be readily incorporated into the finite element analysis, applicable in the cyclic loading and determined completely by the conventional parameters such as uniaxial compressive strength f'_c , uniaxial tensile strength f_r , uniaxial crushing strain ε_{cu} and initial elastic modulus E₀. Like the Darwin-Pecknold 2D model, the concept of equivalent uniaxial strain is also introduced in this model and becomes the basis upon which the incremental elastic moduli and Poisson's ratios are derived in terms of strain parameters.

7.1.2.1 Form of Incremental Constitutive Relation This 3D model initially is intended for use in the axisymmetrical structures. If orthotropy is assumed, the incremental stress~strain constitutive matrix can be written as a 4×4 matix when referred to the orthotropic principal axes:

$$\begin{cases} d\varepsilon_{1} \\ d\varepsilon_{2} \\ d\varepsilon_{3} \\ d\gamma_{12} \end{cases} = \begin{bmatrix} E_{1}^{-1} & -v_{12}E_{2}^{-1} & -v_{13}E_{3}^{-1} & 0 \\ -v_{21}E_{1}^{-1} & E_{2}^{-1} & -v_{23}E_{3}^{-1} & 0 \\ -v_{31}E_{1}^{-1} & -v_{32}E_{2}^{-1} & E_{3}^{-1} & 0 \\ 0 & 0 & 0 & G_{12}^{-1} \end{bmatrix} \begin{bmatrix} d\sigma_{1} \\ d\sigma_{2} \\ d\sigma_{3} \\ d\tau_{12} \end{bmatrix}$$
(7-12)

where E_i represents the elastic modulus in the axis of orthotropy "i" and v_{ij} represents the possion's effect of strain occurring in the "i" direction induced by the stress in the "j" direction. G is the shear modulus.

If symmetry is assumed in the constitutive matrix of Equation (7-12), then:

$$v_{12}E_1 = v_{21}E_2$$
 $v_{13}E_1 = v_{31}E_3$ $v_{23}E_2 = v_{32}E_3$ (7-12a)

Substituting those constraints into Equation (7-12) and rewriting it yield

$$\begin{cases} d\varepsilon_{1} \\ d\varepsilon_{2} \\ d\varepsilon_{3} \\ d\gamma_{12} \end{cases} = \begin{bmatrix} \frac{1}{E_{1}} & \frac{-\mu_{12}}{\sqrt{E_{1}E_{2}}} & \frac{-\mu_{13}}{\sqrt{E_{1}E_{3}}} & 0 \\ & \frac{1}{E_{2}} & \frac{-\mu_{23}}{\sqrt{E_{2}E_{3}}} & 0 \\ symmetrical & \frac{1}{E_{3}} & 0 \\ & & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} d\sigma_{1} \\ d\sigma_{2} \\ d\sigma_{3} \\ d\tau_{12} \end{bmatrix}$$
(7-13)

which, upon inversion, becomes

$$\{d\sigma\} = [C]\{d\varepsilon\}$$
(7-14)

where incremental tangent stiffness matrix [C] is given by

$$[C] = \frac{1}{\phi} \begin{bmatrix} E_1(1-\mu_{32}^2) & \sqrt{E_1E_2}(\mu_{13}\mu_{32}+\mu_{12}) & \sqrt{E_1E_3}(\mu_{12}\mu_{32}+\mu_{13}) & 0 \\ E_2(1-\mu_{13}^2) & \sqrt{E_2E_3}(\mu_{12}\mu_{13}+\mu_{32}) & 0 \\ symmetrical & E_3(1-\mu_{12}^2) & 0 \\ & \phi G_{12} \end{bmatrix}$$
(7-15)

where

$$\mu_{12}^{2} = v_{12}v_{21} \quad \mu_{23}^{2} = v_{23}v_{32} \quad \mu_{13}^{2} = v_{13}v_{31}$$

$$\phi = 1 - \mu_{12}^{2} - \mu_{23}^{2} - \mu_{13}^{2} - 2\mu_{12}\mu_{23}\mu_{13}$$
 (7-15a)

If it is assumed that no particular direction will be favored with respect to the shear modulus, which leads to an invariant G under the transformation of current orthotropic principal axes to a nonorthotropic set of axes, the result will be

$$G_{12} = \frac{1}{4\phi} \left[E_1 + E_2 - 2\mu_{12}\sqrt{E_1E_2} - (\sqrt{E_1}\mu_{23} + \sqrt{E_2}\mu_{31})^2 \right]$$
(7-15b)

Although Equations (7-14) and (7-15) define the incremental constitutive relationship, the six incremental hypoelastic moduli, E_1 , E_2 , E_3 , μ_{12} , μ_{23} and μ_{31} are

required to be determined. Herein the concept of equivalent uniaxial strain is introduced. The technique is described as follows.

Arbitrarily rewrite Equation (7-14) as

$$\begin{cases} d\sigma_{1} \\ d\sigma_{2} \\ d\sigma_{3} \\ d\tau_{12} \end{cases} = \begin{bmatrix} E_{1}B_{11} & E_{1}B_{12} & E_{1}B_{13} & 0 \\ E_{2}B_{21} & E_{2}B_{22} & E_{2}B_{23} & 0 \\ E_{3}B_{31} & E_{3}B_{32} & E_{3}B_{33} & 0 \\ 0 & 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} d\varepsilon_{1} \\ d\varepsilon_{2} \\ d\varepsilon_{3} \\ d\gamma_{12} \end{bmatrix}$$
(7-16)

and let

$$d\sigma_{1} = E_{1}(B_{11}d\varepsilon_{1} + B_{12}d\varepsilon_{2} + B_{13}d\varepsilon_{3})$$

$$d\sigma_{2} = E_{2}(B_{21}d\varepsilon_{1} + B_{22}d\varepsilon_{2} + B_{23}d\varepsilon_{3})$$

$$d\sigma_{3} = E_{3}(B_{31}d\varepsilon_{1} + B_{32}d\varepsilon_{2} + B_{33}d\varepsilon_{3})$$

$$d\tau_{12} = G_{12}d\gamma_{12}$$

and then Equation (7-16) can be rewritten as

$$\begin{cases} d\sigma_{1} \\ d\sigma_{2} \\ d\sigma_{3} \\ d\tau_{12} \end{cases} = \begin{bmatrix} E_{1} & 0 & 0 & 0 \\ 0 & E_{2} & 0 & 0 \\ 0 & 0 & E_{3} & 0 \\ 0 & 0 & 0 & G_{12} \end{bmatrix} \begin{cases} d\varepsilon_{1u} \\ d\varepsilon_{2u} \\ d\varepsilon_{3u} \\ d\gamma_{12} \end{cases}$$
(7-17)

Then the incremental and total equivalent uniaxial strain $d\varepsilon_{iu}$ and ε_{iu} can be evaluated from

$$d\varepsilon_{iu} = \frac{d\sigma_i}{E_i}$$
 and $\varepsilon_{iu} = \int \frac{d\sigma_i}{E_i}$ (7-17a)

Again, $d\varepsilon_{iu}$ represents the increment of strain in direction "i" that concrete would exhibit if subjected to a uniaxial stress increment $d\sigma_i$ with other stress increments equal to be zero. It depends on the current stress condition and belongs to a fictitious parameter that is only significant as a base to evaluate the variation of other material indices, like elastic modulus and Poisson's ratio. 7.1.2.2 Equivalent Uniaxial Stress ~ Strain Curve In this model, a modified Saenz uniaxial compressive stress ~ strain relationship which incorporates the description of the descending branch and can be used to simulate both the tensile and compressive responses of concrete is adopted. The equation is written in terms of equivalent uniaxial strain as

$$\sigma_{i} = \frac{\varepsilon_{iu}E_{0}}{1 + (R + \frac{E_{0}}{E_{s}} - 2)\frac{\varepsilon_{iu}}{\varepsilon_{iuc}} + (1 - 2R)\left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^{2} + R\left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^{3}}$$
(7-18)

where:

$$E_s = \frac{\sigma_{ic}}{\varepsilon_{iuc}} \tag{7-18a}$$

$$R = \frac{E_0}{E_s} \frac{\sigma_{ic}}{\left(\frac{\varepsilon_{if}}{\varepsilon_{iuc}} - 1\right)^2} - \frac{1}{\frac{\varepsilon_{if}}{\varepsilon_{iuc}}}$$
(7-18b)

In Equation (7-18), E₀ represents the initial tangent elastic modulus; σ_{ic} is the maximum stress in the "i" direction that occurs for the current particular principal stress ratio, and ε_{iuc} represents the corresponding equivalent uniaxial strain; $(\sigma_{ij}, \varepsilon_{ij'})$ is a selected point on the descending part of the equivalent uniaxial stress ~ strain curve. Due to the high volatility of the descending branch of the uniaxial stress ~ strain curve, Elwi and Murray recommended an alternative from the flexural behavior of concrete, which gives

$$\varepsilon_{if} = 4\varepsilon_{ic}$$
 $\sigma_{if} = \frac{\sigma_{ic}}{4}$ (7-18c)

Now Equation (7-18) is to be used to derive the incremental elastic moduli E_i in Equation

(7-15) by taking
$$E_i = \frac{d\sigma_i}{d\varepsilon_{iu}}$$
:

$$E_i = E_0 \frac{1 + (2R - 1)\left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^2 - 2R\left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^3}{\left(1 + (R + \frac{E_0}{E_s} - 2)\frac{\varepsilon_{iu}}{\varepsilon_{iuc}} - (2R - 1)\left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^2 + R\left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^3\right)^2}$$
(7-19)

7.1.2.3 Poisson's Ratio In this model, Poisson's ratio is determined by the uniaxial compression data of Kupfer, Hilsdorf and Rusch (1969), which is expressed as

$$v = v_0 \left[1.0 + 1.3763 \frac{\varepsilon}{\varepsilon_{cu}} - 5.3600 \left(\frac{\varepsilon}{\varepsilon_{cu}} \right)^2 + 8.586 \left(\frac{\varepsilon}{\varepsilon_{cu}} \right)^3 \right]$$
or
$$v = v_0 f \left(\frac{\varepsilon}{\varepsilon_{cu}} \right)$$
(7-20)

where ε is the uniaxial strain, ε_{cu} is the uniaxial peak strain and v_0 is the initial Poisson's ratio.

In this model, it is assumed that the Poisson's ratio v_{ij} in Equation (7-15) is only the function of the strain or stress in the "j" direction, which will yield only independent Poisson's ratios:

$$v_{21} = v_{31} = v_1, \qquad v_{12} = v_{32} = v_2, \qquad v_{13} = v_{23} = v_3$$

By replacing those two uniaxial values ε and ε_{cu} with corresponding equivalent uniaxial strains, Elwi and Murray assumed that the three independent Poisson's ratio v_i (i=1,2,3) can also be postulated as follows:

$$v_{i} = v_{0} \left[1.0 + 1.3763 \frac{\varepsilon_{iu}}{\varepsilon_{ic}} - 5.3600 \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^{2} + 8.586 \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^{3} \right]$$

$$or \quad v_{i} = v_{0} f \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)$$
(7-21)

and the equivalent Poisson's ratio μ_{ii} in Equation (7-15) can be written as

$$\mu_{12}^{2} = v_{1}v_{2}, \qquad \mu_{23}^{2} = v_{2}v_{3} \qquad \mu_{31}^{2} = v_{3}v_{1}$$
 (7-21a)

In this model, the variable ϕ in Equation (7-15) is set to be a non-negative value. Therefore a limiting value of 0.5, which corresponds to a limit of zero incremental volume change, has been placed on the Poisson's ratio v_i determined by Equation (7-21). Elwi and Murray insisted that placing this limit would yield realistic estimates of stress. However, it has been shown in the octahedral normal stress (σ_{oct}) versus volume change ($3\varepsilon_{oct}$) curves of the uniaxial and triaxial compression in the former chapters of this thesis that this zero incremental volume change occurs only at approximately 93%~95% of the maximum stress. For an instance at the peak stress of the uniaxial compression, the tangent incremental Poisson's ratio can be as large as 1.0, and after that point it has a tendency even to keep increasing a little bit on the descending branch of the stress ~ strain curve. Therefore, the variable ϕ may not always be a non-negative number as it is postulated to be.

7.1.2.4 Ultimate Surface The surface in the 3D stress space that defines the ultimate strengths σ_{ic} for any combination of stress conditions is called the ultimate surface. In this model, the Willam-Warnke 5-parameter ultimate surface is adopted to determine the maximum σ_{ic} in Equation (7-18) and (7-19). As discussed in the previous sections, the

tensile meridian and compressive meridian stipulated in this failure criterion can be expressed as follows:

$$\frac{r_{t}}{\sqrt{5}f_{c}} = \frac{\tau_{mt}}{f_{c}} = a_{0} + a_{1}\frac{\sigma_{m}}{f_{c}} + a_{2}\left(\frac{\sigma_{m}}{f_{c}}\right)^{2} \qquad (\eta = 0^{\circ})$$
(7-22a)

$$\frac{r_c}{\sqrt{5}f_c} = \frac{\tau_{mc}}{f_c} = b_0 + b_1 \frac{\sigma_m}{f_c} + b_2 \left(\frac{\sigma_m}{f_c}\right)^2 \qquad (\eta = 60^\circ)$$
(7-22b)

On the deviatoric plane, three identical parts of an ellipse are used to connect those points on the meridians:

$$r(\sigma_m,\eta) = \frac{2r_c(r_c^2 - r_i^2)\cos\eta + r_c(2r_i - r_c)\left[4(r_c^2 - r_i^2)\cos^2\eta + 5r_i^2 - 4r_ir_c\right]^{\frac{1}{2}}}{4(r_c^2 - r_i^2)\cos^2\eta + (r_c - 2r_i)^2}$$
(7-22c)

where:

$$\cos \eta = \frac{2\sigma_1 - \sigma_2 - \sigma_3}{\sqrt{2} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{\frac{1}{2}}}$$

$$r_c \,, r_t = \frac{1}{\sqrt{3}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sqrt{3}\tau_{oct}$$

$$\sigma_m = I_1 / 3 = (\sigma_1 + \sigma_2 + \sigma_3) / 3$$

$$\tau_m = \frac{1}{\sqrt{15}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

The determination of the coefficient a_0 , a_1 , a_2 , b_0 , b_1 and b_2 has been already fully covered previously.

However, Equations (7-18) and (7-19) also require the evaluation of peak equivalent uniaxial strains ε_{iuc} . So in this model, it is assumed that the ultimate surface of the equivalent uniaxial strain is analogous to that of the σ_{ic} and the Willam-Warnke

failure criterion is still applicable by just replacing f'_{c} , τ_{m} , σ_{n} , σ_{1} , σ_{2} , and σ_{3} in Equation (7-22) with ε_{cu} , ε_{m}^{*} , γ_{m}^{*} , ε_{luc} , ε_{2uc} and ε_{3uc} respectively, where the * in the notation represents the equivalent uniaxial quantities. As can be clearly seen from its definition, equivalent uniaxial strains ε_{iu} are fictitious values that cannot be directly observed except for those corresponding to the uniaxial tension and compression, which means at least three different fictitious points are needed in order to determine the final shape of the ultimate surface of the equivalent uniaxial strain. Elwi and Murray did not provide a detailed procedure on how to get those new points, but instead they just put forward a very vague description like "These points, have been determined, herein, by trial and error until reasonable strain correspondence was obtained".

7.1.2.5 Implementation of the Constitutive Model The pivotal part of the model implementation is to determine the elastic modulus using Equation (7-19). The procedures can be summarized as follows:

1) Determine the shape of the ultimate surface of peak stresses σ_{ic} in the stress space and that of the peak equivalent uniaxial strains ε_{iuc} in the fictitious equivalent uniaxial strain space;

2) Input the initial elastic modulus $E_i = E_0$ and initial Poisson's ratio $v_i = v_0$;

3) Input the initial strain increment $d\varepsilon_i$, use Equation (7-17a) to calculate the equivalent uniaxial strain $d\varepsilon_{iu}$ and ε_{iu} , use Equation (7-14) to calculate $d\sigma_i$ and σ_i ;

4) Find σ_{ic} by extending a line from the origin point of the stress space through the current stress point σ_i until it penetrates the ultimate surface of peak stresses;

5) Find ε_{ic} by extending a line from the origin point of the equivalent uniaxial strain space through the current strain point ε_{iu} until it penetrates the ultimate surface of peak equivalent uniaxial strain;

6) Construct the constitutive matrix [C] of Equation (7-15) again by evaluating tangent elastic modulus E_i from Equation (19) and Poisson's ratio v_i from Equation (7-20);

7) Go back to procedure 3 and continue to input new strain increment $d\varepsilon_i$.

It can be clearly seen from the above-mentioned procedures that the model implementation depends on the initial stiffness of each stress increment. Also the proportional loading case is assumed.

7.2 Evaluation on Previous Incremental Models

7.2.1 Theoretical Limitation of Elwi's Model on Load Path and Poisson's Ratio

In Equation (7-12), it is assumed that the matrix is symmetrical, which will inevitably lead to:

$$v_{12}E_1 = v_{21}E_2$$
 $v_{13}E_1 = v_{31}E_3$ $v_{23}E_2 = v_{32}E_3$ (7-12a)

This argument is probably true under two circumstances: First, when the stress status is lower than a certain level under which the concrete behaves like a linear elastic material in all principal directions so that the elastic moduli and Poisson's ratios in all directions are approximately equal to E_0 and v_0 ; the other case is in the proportional loading where all principal stresses increase proportionately with respect to one another until failure:

$$\frac{\sigma_1}{\sigma_{1c}} = \frac{\sigma_2}{\sigma_{2c}} = \frac{\sigma_3}{\sigma_{3c}} = \beta$$
(7-23)

Although the elastic modulus and Poisson's ratio are mostly expressed in terms of strain

ratios (
$$\frac{\varepsilon_{iu}}{\varepsilon_{ic}}$$
 in Equation (7-19) and (7-21)), clearly they are also functions of stress ratios

 $\frac{\sigma_i}{\sigma_{ic}}$. Therefore, under a proportional loading, the elastic moduli and Poisson's ratios in

all principal directions are kept the same all the time, which will guarantee the symmetry of the stiffness matrix.

However, that assumption may not be right under non-proportional loading cases. For instance, just like the triaxial compression loading path shown in Figure 7.2, if the slope α_2 is larger than α_1 , then near the failure point $\frac{\sigma_1}{\sigma_{1c}} < \frac{\sigma_3}{\sigma_{3c}}$. Since the tangent elastic modulus decreases while Poisson's ratio increases with the increasing of the stress ratio $\frac{\sigma_i}{\sigma_{ic}}$, it will lead to $E_1 > E_3$ and $v_{13} > v_{31}$, so obviously the equilibriums in Equation

(7-12a) will not be satisfied any more.



Figure 7.2 A non-proportional loading case.

However, as Guo (1997) stated, statistically, proportional loading is the most probable loading case in the real situations, so this assumption does have a wide applicability.

The ϕ value in Equation (7-15) is set to be non-negative, which will inevitably put an upper limit of 0.5 to all the Poisson's ratios v_i . This treatment probably intends to maintain the stiffness matrix [C] in Equation (7-15) to be positive definite for the sake of the stability of finite element solutions. But this limit is clearly at odds with the experimental observations from this research and other studies which show that the tangent Poisson's ratio of concrete near the peak is well around 1.0.

Bathe (1979) also addressed the issue of stiffness matrix indefiniteness. In his model, a constant Poisson's ratio was adopted throughout the whole loading process, including ascending and descending portions of the stress ~ strain curve, whereas the tangent elastic moduli E_i can be negative on the descending branch. Therefore, if the negative elastic moduli are introduced into the stiffness matrix, the stiffness matrix may become indefinite and the element assemblage stiffness matrix may not be positive definite, which will result in relatively large errors to the solution of finite element equations. To circumvent this dilemma, a zero value (actually a small positive value) for E_i is employed for stiffness calculations, however the real negative E_i is used in stress calculations. This strategy, as stated by Bathe, is a direct generalization of the common incremental procedures employed to analyze perfectly-plastic conditions and conditions of complete stress release.

7.2.2 Some Discussion on Positive Definiteness of Stiffness Matrix [C]

Consider the stiffness matrix in Equation (7-15)

$$[C] = \frac{1}{\phi} \begin{bmatrix} E_1(1-\mu_{32}^2) & \sqrt{E_1E_2}(\mu_{13}\mu_{32}+\mu_{12}) & \sqrt{E_1E_3}(\mu_{12}\mu_{32}+\mu_{13}) & 0 \\ & E_2(1-\mu_{13}^2) & \sqrt{E_2E_3}(\mu_{12}\mu_{13}+\mu_{32}) & 0 \\ symmetrical & E_3(1-\mu_{12}^2) & 0 \\ & & & & & & & \\ \end{bmatrix}$$
(7-15)

$$\mu_{12}^{2} = v_{12}v_{21} \quad \mu_{23}^{2} = v_{23}v_{32} \quad \mu_{13}^{2} = v_{13}v_{31}$$

$$\phi = 1 - \mu_{12}^{2} - \mu_{23}^{2} - \mu_{13}^{2} - 2\mu_{12}\mu_{23}\mu_{13}$$
 (7-15a)

$$G_{12} = \frac{1}{4\phi} \Big[E_1 + E_2 - 2\mu_{12}\sqrt{E_1E_2} - (\sqrt{E_1}\mu_{23} + \sqrt{E_2}\mu_{31})^2 \Big]$$

$$= \frac{1}{4\phi} \Big[E_1 + E_2 - 2\mu_{12}\sqrt{E_1E_2} - (\mu_{23}^2E_1 + \mu_{31}^2E_2 + 2\mu_{23}\mu_{31}\sqrt{E_1E_2}) \Big]$$
(7-15b)
$$= \frac{1}{4\phi} \Big[E_1(1 - \mu_{23}^2) + E_2(1 - \mu_{31}^2) - 2(\mu_{12} + \mu_{23}\mu_{31})\sqrt{E_1E_2}) \Big]$$

The necessary and sufficient condition for [C] to be a positive definite matrix is that the determinants associated with all its upper-left submatrices, $det(C^{[k]})$, are all positive.

$$\det(C^{[1]}) = \frac{E_1(1-\mu_{32}^2)}{\phi} > 0$$
(7-24a)

$$\det(C^{[2]}) = E_1 E_2 > 0 \tag{7-24b}$$

$$\det(C^{[3]}) = \phi E_1 E_2 E_3 > 0 \tag{7-24c}$$

$$\det(C^{[4]}) = \phi E_1 E_2 E_3 G_{12} > 0 \tag{7-24d}$$

For a proportional loading case $\frac{\sigma_1}{\sigma_{1c}} = \frac{\sigma_2}{\sigma_{2c}} = \frac{\sigma_3}{\sigma_{3c}} = \beta$, if Equations (7-24 a-d) are

to be satisfied, the following restrictions should be met:

1) The tangent Poisson's ratio v_i should be less than 0.5 before σ_{ic} , but after this peak on the descending branch of multiaxial stress ~ strain curve, $0.5 < v_i < 1.0$; 2) The tangent elastic modulus E_i is positive before multiaxial peak stress σ_{ic} , and after that point it can take a negative value, representing the descending branch of multiaxial stress~strain curve. However, $\sqrt{E_i E_j}$ (i≠j and i, j=1,2,3) should be always a positive value, which indicates that σ_i reaches its respective σ_{ic} simultaneously for all directions (i=1,2,3), an apparent outcome of the proportional loading;

3) According to Equations (7-15b) and (7-24d), G₁₂ should always be a positive value.

The extension of the technique to the general three-dimensional situation is simple, since this only requires the definition of the additional incremental shear moduli G_{23} and G_{31} that can be determined in the same manner as G_{12} (Equation 7-15b).

7.2.3 Evaluation on the Equivalent Uniaxial Strain

The key to the previous incremental constitutive models is to obtain the variation of tangent elastic modulus E_i through the whole load ~ deformation process. Equation (7-4) was used in Darwin and Pecknold's model while Elwi's model adopted Equation (7-19) which is just a modification of Equation (7-4).

$$E_{i} = \frac{d\sigma_{i}}{d\varepsilon_{iu}} = E_{0} \frac{1 - \left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^{2}}{\left(1 + \left(\frac{E_{0}}{\sigma_{ic} / \varepsilon_{iuc}} - 2\right)\frac{\varepsilon_{iu}}{\varepsilon_{iuc}} + \left(\frac{\varepsilon_{iu}}{\varepsilon_{iuc}}\right)^{2}\right)^{2}}$$
(7-4)

It can be seen from Equation (7-4) that in order to have the variation of E_i , the equivalent uniaxial strain ε_{iu} and the peak equivalent uniaxial strain ε_{iuc} must be input. Although ε_{iu} has been defined by Equation (7-17a), ε_{iuc} still remains a problem. As discussed before, Elwi (1979) just assumed that the ultimate surface of the equivalent uniaxial strain took the same format and shape as those of the ultimate stress surface by replacing the stress parameters in Equation (7-22) with the corresponding equivalent uniaxial strain ones. He did not describe clearly on how to derive this ultimate surface, but he did list the results for the normal strength concrete from Kupfer and Schickert-Winkler. Those two data together with the experimental results from this study (HSC and SFHSC) are shown in Figure 7.3.



Figure 7.3 Comparison of the compressive meridians of equivalent uniaxial strain from Kupfer, Schickert-Winkler and this study.

Remarkable difference can be found from Figure 7.3. From the computer model analysis following the model implementation procedures listed above, it has been found that the ultimate surface of the equivalent uniaxial strain is highly unreliable to be used to predict the stress \sim strain behavior of HSC under triaxial compression. In this study, this ultimate surface was obtained by adopting load path T-1, and if it is used to check the load path T-2, there is a large difference between the prediction and the actual stress \sim

strain behavior. Sometimes the computational results do not make any sense. Therefore, unlike the ultimate surface of the stress which is independent of the load path, the ultimate surface of the equivalent uniaxial strain seems to have no fixed expression and the load path seems to have a substantial effect on its shape.

From the model analysis results, another important job that can not be implemented by the ultimate surface of the equivalent uniaxial strain is the synchronization of the stress σ_i and equivalent uniaxial strain ε_{iu} to their corresponding peak values σ_{ic} and ε_{iuc} . From Equation (7-4) it can be easily understood that if the triaxial stress ~ strain behavior around peak is to be well predicted, σ_i and ε_{iu} should get to their respective ultimate surface (σ_{ic} and ε_{iuc}) simultaneously in order to get a zero tangent elastic modulus right at the peak. Since this assumed ultimate surface of the equivalent uniaxial strain does not truly reflect the deformation behavior of the HSC, there is no wonder why the equivalent uniaxial strain always precedes or lag behind stress development greatly, which in turn results in the considerable difference between the prediction and experimental results.

7.3 A Simple 3D Incremental Model for Proportional Loading under Triaxial Compression for HSC and SFHSC

7.3.1 Tangent Elastic Modulus Ei

As discussed above, Equation (7-4) is not practical to evaluate the variation of the tangent elastic modulus E_i since it is based on the fictitious parameters ε_{iuc} which is hard to determine and also dependent on the load path. Thus in this model, the concept of the equivalent uniaxial strain is disregarded. Consequently, a more direct and accurate way to simulate the variation of the E_i needs to be developed.

This new approach comes from the similarity between the equivalent uniaxial stress ~ strain curve and the octahedral shear stress τ_{oct} ~ octahedral shear strain γ_{oct} curve under triaxial compression.

It can be seen from Chapter 5 and Chapter 6 that, under triaxial compression for both HSC and SFHSC, the octahedral shear stress $\tau_{oct} \sim$ octahedral shear strain γ_{oct} curves also take a form of Saenz equation. For the confining pressures no more than 28 MPa, they can be expressed as

$$\tau_{oct} = \frac{G_0 \gamma_{oct}}{1 + \left(\frac{G_0}{G_s} - 2\right) \frac{\gamma_{oct}}{\gamma_{octp}} + \left(\frac{\gamma_{oct}}{\gamma_{octp}}\right)^2}$$
(7-25a)

while for higher confining pressures

$$\tau_{oct} = \frac{0.75G_0\gamma_{oct}}{1 + \left(0.75\frac{G_0}{G_s} - 2\right)\frac{\gamma_{oct}}{\gamma_{octp}} + \left(\frac{\gamma_{oct}}{\gamma_{octp}}\right)^2}$$
(7-25b)

where G_0 : initial tangent shear modulus

 G_s : = $\tau_{octp} / \gamma_{octp}$, the secant shear modulus at the peak

 τ_{octp} peak octahedral shear stress

 γ_{octp} peak octahedral shear strain

 γ_{oct} : octahedral shear strain

$$G_{t} = \frac{d\tau_{oct}}{d\gamma_{oct}} = G_{0} \frac{1 - \left(\frac{\gamma_{oct}}{\gamma_{octp}}\right)^{2}}{\left(1 + \left(\frac{G_{0}}{G_{s}} - 2\right) + \left(\frac{\gamma_{oct}}{\gamma_{octp}}\right)^{2}\right)^{2}}$$
(7-26)

To determine γ_{octp} is much easier, since from Chapters 5 and 6 we know there are well-developed relationships between the τ_{octp} and γ_{octp} . For HSC,

$$\tau_{octp} = 1.458 \times 10^{-3} \gamma_{octp} + 29.30 \tag{7-27a}$$

and for SFHSC,

$$\tau_{octp} = 1.456 \times 10^{-3} \gamma_{octp} + 30.4 \tag{7-27b}$$

Different load paths do not influence Equations (7-27a) and (7-27b). By increasing the load proportionately to the ultimate stress surface, one can get the peak value σ_{1C} , σ_{2C} and σ_{3C} , and thus τ_{octo} can be obtained by

$$\tau_{octp} = \frac{1}{3} \sqrt{(\sigma_{1c} - \sigma_{2c})^2 + (\sigma_{2c} - \sigma_{3c})^2 + (\sigma_{3c} - \sigma_{1c})^2}$$
(7-28)

Now γ_{octp} is available from Equations (7-27a) and (7-27b) for both HSC and SFHSC.

Since the equivalent uniaxial stress ~ strain curve and the octahedral shear stress τ_{oct} ~ octahedral shear strain γ_{oct} curve under triaxial compression follow the same trend (Saenz equation), and $G_t = \frac{E_t}{2(1+v_t)}$, so the relationship between the tangent elastic

modulus E_t and tangent shear modulus G_t can be written as follows:

$$E_{t} = E_{0} \frac{G_{t}}{G_{0}} \frac{1 + v_{t}}{1 + v_{0}}$$
(7-29)

where E_0 and v_0 are constant inputs, $G_0 = \frac{E_0}{2(1+v_0)}$, G_t can be achieved from Equation

(7-26), and v_t can be obtained from Equation (7-31) which will be discussed below.

Since there will be one tangent elastic modulus in all three directions, this model is only good at proportional loading when the elastic moduli are essentially the same for all three directions since σ_i / σ_{ic} (i=1,2,3) are identical, but it can be used to simulate the post peak behavior as will be discussed later in this chapter.

7.3.2 Tangent Poisson's Ratio v_i

Although the tangent Poisson's ratio v_i can be expressed as functions of both stress ratio and strain ratio, in this model, it is assumed that v_i is only the function of the stress ratio:

$$v_i = f\left(\frac{\sigma_i}{\sigma_{ic}}\right) \tag{7-30}$$

Under proportional loading in triaxial compression when $\frac{\sigma_i}{\sigma_{ic}}$ (i=1,2,3) are the same, v_i

will take a uniform value for all three directions.

From the data analysis of triaxial compression experiments made in the previous sections, the tangent Poisson's ratios for both HSC and SFHSC under triaxial compression can be expressed in terms of stress ratios as follows:

Ascending branch $(\frac{\gamma_{oct}}{\gamma_{octp}} \le 1)$

$$v_{i} = v_{0} \qquad \beta = \frac{\sigma_{i}}{\sigma_{ic}} \le 50\%$$

$$v_{i} = v_{if} - (v_{if} - v_{0})\sqrt{1 - 8(\beta - 0.5)^{3}} \qquad 50\% < \beta = \frac{\sigma_{i}}{\sigma_{ic}} \le 100\%$$
(7-31a)

Descending branch $\left(\frac{\gamma_{oct}}{\gamma_{octp}} \ge 1\right)$

$$v_i = v_{ip} \tag{7-31b}$$

Where v_0 represents the initial Poisson's ratio, v_{if} represents the tangent Poisson's ratio at the peak stress σ_{ic} , and v_{ip} signifies the tangent Poisson's ratio at the descending branch of the stress ~ strain curve.

In the uniaxial compression test, the initial Poisson's ratio v_0 of both HSC and SFHSC is well around 0.20, while under triaxial compression, this value falls short of 0.15. Taking into account the possible influence of the lateral confinement on the Poisson's effect, this model adopts 0.15 for v_0 . If the positive definiteness of the stiffness matrix in constitutive Equation is not considered, v_{if} is taken as 1.0 to comply with the experimental results. And for the post peak branch, based on the experimental finding from this study, v_{ip} is just assumed to be a constant and given a value of 1.0.

7.3.3 Incremental Constitutive Relationship

If only the principal stresses and principal strains are considered, the incremental constitutive relationship can be written as:

$$\begin{cases} d\sigma_{1} \\ d\sigma_{2} \\ d\sigma_{3} \end{cases} = \frac{1}{\phi} \begin{bmatrix} E_{1}(1-\mu_{32}^{2}) & \sqrt{E_{1}E_{2}}(\mu_{13}\mu_{32}+\mu_{12}) & \sqrt{E_{1}E_{3}}(\mu_{12}\mu_{32}+\mu_{13}) \\ E_{2}(1-\mu_{13}^{2}) & \sqrt{E_{2}E_{3}}(\mu_{12}\mu_{13}+\mu_{32}) \\ symmetrical & E_{3}(1-\mu_{12}^{2}) \end{bmatrix} \begin{bmatrix} d\varepsilon_{1} \\ d\varepsilon_{2} \\ d\varepsilon_{3} \end{bmatrix}$$
(7-32a)
$$\mu_{12}^{2} = v_{1}v_{2} \quad \mu_{23}^{2} = v_{2}v_{3} \quad \mu_{13}^{2} = v_{1}v_{3}$$

$$\phi = 1 - \mu_{12}^{2} - \mu_{23}^{2} - \mu_{13}^{2} - 2\mu_{12}\mu_{23}\mu_{13}$$
 (7-32b)

As Guo (1997) stated the proportional loading is the most probable loading path in practical situations. If that is assumed, according to the discussions in the previous sections, the incremental constitutive Equation (7-32) can be simplified as

$$\begin{cases}
d\sigma_1 \\
d\sigma_2 \\
d\sigma_3
\end{cases} = \frac{E_t}{(1+\nu_t)(1-2\nu_t)} \begin{bmatrix}
1-\nu_t & \nu_t & \nu_t \\
\nu_t & 1-\nu_t & \nu_t \\
\nu_t & \nu_t & 1-\nu_t
\end{bmatrix} \begin{cases}
d\varepsilon_1 \\
d\varepsilon_2 \\
d\varepsilon_3
\end{cases}$$
(7-33)

If the positive definiteness of the stiffness matrix is not considered, the variation of the tangent Poisson's ratio with stress condition can adopt Equation (7-31).



7.3.5 Model Verification by Experimental Results

7.3.5.1 SFHSC The comparisons between the model expectation and experimental results under load path T-2, which is close to the proportional loading, for moderate confining pressure σ_3 =7MPa, 14MPa and 21MPa are shown in Figures 7.4, 7-5 and 7-6.

The input parameters for the model are as follows:

 $E_0 = 0.041 \times 10^6 MPa$, $G_0 = \frac{E_0}{2(1+v_0)}$, v_0, v_{tf} , and v_{tp} are from Equation (7-31), and

 $f_c^{'}$ =69.0MPa.

The compressive meridian for SFHSC: $\frac{\tau_m}{f_c'} = 0.160 + 0.653 \frac{\sigma_m}{f_c'} - 0.055 \left(\frac{\sigma_m}{f_c'}\right)^2$



Figure 7.4 Comparison of experiment data and model prediction for SFHSC under confining pressure $\sigma_3 = 7$ MPa (load path T-2).



Figure 7.5 Comparison of experiment data and model prediction for SFHSC under confining pressure $\sigma_3 = 14$ MPa (load path T-2).



Figure 7.6 Comparison of experiment data and model prediction for SFHSC under confining pressure $\sigma_3 = 21$ MPa (load path T-2).



Figure 7.7 Comparison of experiment data and model prediction for SFHSC under confining pressure $\sigma_3 = 28$ MPa (load path T-2).

From Figures 7.4, 7-5 and 7-6, it can be seen that the proposed simple model complies well with the experimental data under relatively moderate confining pressures, especially at the ascending branches. It can also possibly be used to predict the post peak concrete behavior.

However, when the confining pressure reaches 28MPa, as is shown in Figure 7.7, the model prediction will experience some appreciable deviation from the experimental data, especially for the lateral strain ε_3 just before the peak stress. This may be attributed to the deviation of the triaxial stress ~ strain relationship from the Saenz equation (see Figures 6-13 and 6-16). As the confining pressure continues to increase, the discrepancy becomes even more discernible. In this case, the modified Saenz equation (Equation (7-25b)) may be employed to replace Equation (7-25a) to address the problem.

7.3.5.2 HSC The input parameters for the model are as follows:

 $E_0 = 0.040 \times 10^6 MPa$, $G_0 = \frac{E_0}{2(1+v_0)}$, v_0, v_{if} , and v_{ip} are from Equation (7-31), and $f_c = 67.0$ MPa.

The compressive meridian for HSC: $\frac{\tau_m}{f_c'} = 0.165 + 0.638 \frac{\sigma_m}{f_c'} - 0.055 \left(\frac{\sigma_m}{f_c'}\right)^2$



Figure 7.8 Comparison of experiment data and model prediction for HSC under confining pressure $\sigma_3 = 21$ MPa (load path T-2).



Figure 7.9 Comparison of experiment data and model prediction for HSC under confining pressure $\sigma_3 = 14$ MPa (load path T-2).

CHAPTER 8

CONCLUSIONS

8.1 On Uniaxial Compression of HSC and SFHSC

1) The lateral strain at the end of the cylinder is much less than that right at the midheight of the cylinder under the rigid loading platen, indicating a substantial confinement at both ends imposed by the platen. While the designed flexible loading platen tends to diminish this confinement, especially for cylinders with an h/d ratio of 1.5 which essentially has the same lateral strains at ends and mid-height, a clear sign of comparatively better uniform expansion under fairly "true" uniaxial compression.

2) Under the rigid loading platen, the uniaxial compressive strength of SFHSC increases as the h/d ratio decreases, but for the designed lubricated loading platen, the same strength for all h/d ratios (1.0, 1.5 and 2.0) is achieved.

3) Under the lubricated loading platen, the concrete undergoes volume contraction all the way to around 95% of the ultimate strength when it starts to dilate. The variation of the bulk modulus (K) of steel fiber reinforced HSC within a certain deformation range $(\varepsilon / \varepsilon_u < 2.0)$ can be simulated by two straight lines, representing ascending and descending part respectively. The slope of the descending part is about 5%~7% of that of the initial ascending part.

4) The secant Poisson's ratio below 70% of the peak stress and the tangential Poisson's ratio below 50% of the peak stress for SFHSC under different loading platens are essentially the same, to which a constant of 0.20~0.22 can be applied. Beyond those ranges, the Poisson's ratios under lubricated and rigid loading platens experience increasing divergence from each other.

5) Kupfer's equation, originally used for the ordinary concrete, can be adopted to express the variation of the tangential Poisson's ratio of SFHSC before failure:

$$v = v_0 \left[1 + 1.3763 \left(\frac{\varepsilon_c}{\varepsilon_{cu}} \right) - 5.36 \left(\frac{\varepsilon_c}{\varepsilon_{cu}} \right)^2 + 8.586 \left(\frac{\varepsilon_c}{\varepsilon_{cu}} \right)^3 \right]$$

where: $v_0 = 0.20$

6) Within a certain range after failure ($\varepsilon / \varepsilon_u < 2.0$), the tangential Poisson's ratio of SFHSC can be reasonably taken as a constant, 1.35.

7) The secant Poisson's ratio of SFHSC can be expressed using the following three segmented equations:

$$\frac{\sigma_1}{f_c'} \le 70\%, \qquad v_s = 0.20$$

$$70\% \le \frac{\sigma_1}{f_c'} \le 100\%, \qquad v_s = 0.69 \left(\frac{\varepsilon_1}{\varepsilon_{cu}}\right)^2 - 0.67 \left(\frac{\varepsilon_1}{\varepsilon_{cu}}\right) + 0.39$$

$$1.0 \le \left(\frac{\varepsilon_1}{\varepsilon_{cu}}\right) \le 2.0, \qquad v_s = -0.28 \left(\frac{\varepsilon_1}{\varepsilon_{cu}}\right)^2 + 1.30 \left(\frac{\varepsilon_1}{\varepsilon_{cu}}\right) - 0.62$$

8.2 On Triaxial Compression Tests of HSC and SFHSC

1) Under triaxial compression, Mohr-Coulomb failure criterion can be applied to both HSC and SFHSC:

$$\frac{\sigma_1}{f_c'} = 1 + k \frac{\sigma_3}{f_c'}$$

where σ_1 is the ultimate axial strength and σ_3 is the ultimate confining pressure up to 70 MPa. The parameter k can be adopted as 4.0 for both HSC and SFHSC, which is about the mean value of the previous researches on normal strength concrete and HSC found in the literature search. And the introduction of steel fibers does not affect the internal friction angle ϕ in the Mohr-Coulomb failure criterion.

2) Under triaxial compression, the compressive meridians of Willam-Warnke failure criterion can be written as

For HSC:
$$\frac{\tau_m}{f_c'} = 0.165 + 0.638 \frac{\sigma_m}{f_c'} - 0.055 \left(\frac{\sigma_m}{f_c'}\right)^2$$

For SFHSC: $\frac{\tau_m}{f_c'} = 0.160 + 0.653 \frac{\sigma_m}{f_c'} - 0.055 \left(\frac{\sigma_m}{f_c'}\right)^2$

Upon comparison between those two, it is found that they are essentially the same on the ultimate surface.

The compressive meridian for HSC obtained in this study complies well with the results from Xie (1995) and Pantazopoulou (2001), but deviates from Ansari's findings (1998). Also it follows the same trend with that from Chern's research (1992) using the normal strength concrete.

With the shortage of such information concerning SFHSC from the previous studies, the compressive meridian of SFHSC obtained in this study is quite close to Ishikawa's result (2000). And it also complies well with Chern's result (1992) for steel fiber reinforced normal strength concrete.

Therefore, it seems that the concrete strength grade and the introduction of the steel fiber do not have a substantial influence on the compressive meridian on the ultimate surface of Willam-Warnke failure criterion.

3) For both HSC and SFHSC in triaxial compression, different load paths considered in this study (T-1, T-2, T-2', T-3 and T-4) seem to have insignificant effect on the ultimate strength σ_{1c} and the shape of axial stress ~ axial and lateral strain curves and $\tau_{oct} \sim \gamma_{oct}$ curve.

4) For both HSC and SFHSC, the relationship between the octahedral shear stress τ_{oct} and the engineering octahedral shear strain γ_{oct} under moderate confining pressure (less than 28MPa) in triaxial compression can be described by Saenz equation:

$$\tau_{oct} = \frac{G_0 \gamma_{oct}}{1 + \left(\frac{G_0}{G_s} - 2\right) \frac{\gamma_{oct}}{\gamma_{octp}} + \left(\frac{\gamma_{oct}}{\gamma_{octp}}\right)^2}$$

For relatively higher confining pressures, the following modified Saenz equation applies much better:

$$\tau_{oct} = \frac{0.75G_0\gamma_{oct}}{1 + \left(0.75\frac{G_0}{G_s} - 2\right)\frac{\gamma_{oct}}{\gamma_{octp}} + \left(\frac{\gamma_{oct}}{\gamma_{octp}}\right)^2}$$

5) Good relationships between the peak octahedral shear stress τ_{octp} and the peak engineering octahedral shear strain γ_{octp} exist for both HSC and SFHSC under triaxial compression. They can be expressed as $\tau_{octp} = 1.458 \times 10^{-3} \gamma_{octp} + 29.30$ for HSC and $\tau_{octp} = 1.456 \times 10^{-3} \gamma_{octp} + 30.4$ for SFHSC. Those two equations are almost the same when plotted on the same graph, and therefore can be represented by a single equation.

As discussed in Chapter 7, this $\tau_{octp} \sim \gamma_{octp}$ relationship is of great importance in establishing the simple constitutive model for both HSC and SFHSC under triaxial compression in that the peak octahedral shear strain γ_{octp} can be predicted through τ_{octp} , which can be easily obtained from the ultimate stress surface. And then γ_{octp} and τ_{octp} are used to describe the variation of Elastic modulus and Poisson's ratio with stress status.

6) The initial tangent Poisson's ratio v_0 (0.10~0.15) for both HSC and SFHSC under triaxial compression is lower than their uniaxial counterparts (around 0.20). The variation of v_1 with stress status for HSC and SFHSC under triaxial compression can be expressed as follows:

$$v_{t} = \begin{cases} v_{0} & 0 < \beta \le 0.5 \\ v_{tf} - (v_{tf} - v_{0}) \sqrt{1 - 8(\beta - 0.5)^{3}} & 0.5 < \beta \le 1.0 \\ v_{tp} & \varepsilon_{i} / \varepsilon_{ic} > 1.0 \text{ (descending branch)} \end{cases}$$

where $v_{0} = 0.15$, $v_{tf} = 1.0$ and $v_{tp} = 1.0$
 $\beta = \sigma_{i} / \sigma_{ic}$

7) The cyclic loading does not change the overall envelop shapes of the axial stress (σ_1) ~ strain (ε_1 and ε_3), octahedral normal stress (σ_{oct}) ~ volume change ($3\varepsilon_{oct}$), and

octahedral shear stress (τ_{oct}) ~ engineering octahedral shear strain (γ_{oct}) curves of HSC and SFHSC under triaxial compression. However, slope degradation on the cyclic branches does occur and becomes worse with the increase of the cycle numbers. But there seems to be no apparent changing rule for this degradation.

8) The axial strain $(\varepsilon_1) \sim$ lateral strain (ε_3) relationship at the cyclic branches does not deviate much from the mainstream $\varepsilon_1 \sim \varepsilon_3$ curve for both HSC and SFHSC under triaxial compression. This phenomenon indicates a fair assumption can be made that the tangent Poisson's ratio along the cyclic branches is equal to that value at the unloading point of the stress ~ strain curve.

9) From this study, it is found that the introduction of steel fiber into the HSC does not seem to provide any remarkable advantages in terms of strength and ductility over the normal HSC under triaxial compression.

8.3 On Incremental Constitutive Model for HSC and SFHSC

1) In order to establish the incremental constitutive model to simulate the whole load \sim deformation process of concrete, the stress \sim strain relationship needs to be correctly defined. The most commonly used method is to simulate the concrete behavior under multiaxial stress conditions by copying the uniaxial stress \sim strain relationship. Thus the concept of equivalent uniaxial strain was developed to determine the variation of tangent elastic moduli E_t and Poisson's ratios v_t in all directions.

However, this equivalent uniaxial strain is fictitious and based on unverified assumptions, and its ultimate surface is hard to get and highly dependent on each specific case. Therefore, when applied to the model analysis, it will lead to a big discrepancy that the peak stress and peak equivalent uniaxial strain do not occur simultaneous, which makes the analysis results highly unreliable.

2) The experimental program of this study has shown good relationships between the peak octahedral shear stress τ_{octp} and the peak engineering octahedral shear strain γ_{octp} for both HSC and SFHSC under triaxial compression. So under proportional loading, the peak strain value γ_{octp} can be clearly determined through the peak stress σ_{ic} by the ultimate surface of stress. Therefore, the $\tau_{oct} \sim \gamma_{oct}$ curve (ascending and descending branches) can be defined by using the Saenz equation. And then the relationship between the elastic modulus E_i , shear modulus G_i and Poisson's ratio v_i is employed to achieve the variation of E_i which is to be adopted in the constitutive equation.

This proposed model based on the octahedral shear stress ~ strain relationship is physically clear and every parameter needed is clearly defined rather than assumed. The advantage is that it can achieve a occurrence simultaneity of peak stress and peak strain, which is of great importance in simulating the whole deformation process of both SFHSC and HSC, including the descending branch.

3) The model prediction complies well with the experimental results of HSC and SFHSC in triaxial compression under moderate confining pressures.

APPENDIX A

TRIAXIAL STRESS ~ STRAIN CURVES OF SFHSC IN CYCLIC LOADING UNDER DIFFERENT CONFINING PRESSURES AND LOAD PATHS

This appendix contains the triaxial stress ~ strain relationship curves for the SFHSC in cyclic loading under different confining pressures ($\sigma_3 = 7 \sim 70$ MPa) and load paths (T-2, T-2', T-3 and T-4). They are arranged in groups by the specific load condition, and the graphs from the first to the third of each group are the axial stress (σ_1) versus the axial strain (ε_1) and lateral strain (ε_3) curves, the octahedral normal stress (σ_{oct}) versus the volume change ($3\varepsilon_{oct}$) curve and the octahedral shear stress (τ_{oct}) versus the engineering octahedral shear strain (γ_{oct}) curve, respectively.



Figure A.1 Triaxial stress ~ strain curves in cyclic loading under confining pressure $\sigma_3=7MPa$ (load path T-2).



Figure A.2 Triaxial stress ~ strain curves in cyclic loading under confining pressure σ_3 =14MPa (load path T-2).



Figure A.3 Triaxial stress ~ strain curves in cyclic loading under confining pressure σ_3 =21MPa (load path T-2).



Figure A.4 Triaxial stress ~ strain curves in cyclic loading under confining pressure σ_3 =28MPa (load path T-2).



Figure A.5 Triaxial stress ~ strain curves in cyclic loading under confining pressure $\sigma_3=28$ MPa (load path T-2').



Figure A.6 Triaxial stress ~ strain curves in cyclic loading under confining pressure σ_3 =28MPa (load path T-3).



Figure A.7 Triaxial stress ~ strain curves in cyclic loading under confining pressure σ_3 =28MPa (load path T-4).



Figure A.8 Triaxial stress ~ strain curves in cyclic loading under confining pressure σ_3 =42MPa (load path T-2').



Figure A.9 Triaxial stress ~ strain curves in cyclic loading under confining pressure σ_3 =56MPa (load path T-2').



Figure A.10 Triaxial stress ~ strain curves in cyclic loading under confining pressure σ_3 =70MPa (load path T-2').

APPENDIX B

LATERAL STRAIN ~ AXIAL STRAIN RELATIONSHIP DURING CYCLIC LOADING FOR SFHSC

This appendix presents the axial strain $\varepsilon_1 \sim$ lateral strain ε_3 relationship of SFHSC during cyclic loading under different confining pressures σ_3 (7MPa to 56MPa) and load paths (T-2, T-2' and T-4).



Figure B.1 Lateral strain $\varepsilon_3 \sim \text{axial strain } \varepsilon_1$ during cyclic loading under confining pressure σ_3 =7MPa (load path T-2).



Figure B.2 Lateral strain $\varepsilon_3 \sim \text{axial strain } \varepsilon_1$ during cyclic loading under confining pressure σ_3 =14MPa (load path T-2).



Figure B.3 Lateral strain $\varepsilon_3 \sim \text{axial strain } \varepsilon_1$ during cyclic loading under confining pressure σ_3 =21MPa (load path T-2).



Figure B.4 Lateral strain $\varepsilon_3 \sim axial$ strain ε_1 during cyclic loading under confining pressure σ_3 =28MPa (load path T-2).



Figure B.5 Lateral strain $\varepsilon_3 \sim \text{axial strain } \varepsilon_1$ during cyclic loading under confining pressure σ_3 =42MPa (load path T-2').



Figure B.6 Lateral strain $\varepsilon_3 \sim axial$ strain ε_1 during cyclic loading under confining pressure σ_3 =28MPa (load path T-4).



Figure B.7 Lateral strain $\varepsilon_3 \sim axial strain \varepsilon_1$ during cyclic loading under confining pressure σ_3 =56MPa (load path T-2').



Figure B.8 Lateral strain $\varepsilon_3 \sim axial strain \varepsilon_1$ during cyclic loading under confining pressure σ_3 =28MPa (load path T-2').

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