ABSTRACT

METHOD FOR THEORETICALLY DETERMINING THE LOCUS AND LOCATION OF THE TRANSMISSION ZEROS IN MICROWAVE FILTER NETWORKS

By
Keehong Um

This dissertation presents a theoretical investigation of a practical method to determine *quantitatively* the locations and loci of complex *transmission zeros* (TZ's) of positively and negatively cross-coupled RF or microwave bandpass filter networks.

Bandpass filters can be effectively designed by adjusting the locations of TZ's in the complex s-domain. To locate TZ's, this practical method uses chain matrices for subsystems (discrete parts of the network) of the filter network, and can be extended to other types of filters with cross-coupled sections.

An important result is that a complex doublet, triplet and/or quadruplet, (one-, two-, or four-pairs) of TZ's are shown to result solely from the cross-coupled portion of the circuit.

The several closed-forms of expressions called the *TZ characteristic equation* (TZCE) are obtained in terms of element values of the filter network. The locations and loci of TZ's are obtained by solving the relevant equations. These TZCE's are derived by taking advantage of the bridged-T structure for the cross-coupled part.

The reason for this dissertation is to locate TZ's without *having* to evaluate the entire transfer function, with all the infinite and DC TZ's as well as the *transmission poles* (TP's).

In the first chapter, definitions of voltage transfer function and *chain* (ABCD) matrix are discussed to investigate terminated two-port system. The relation between cascaded chain matrices and voltage transfer function is shown.

In the second chapter, a practical bandpass filter network with cross-coupled element is discussed in great detail. The derivations of TZ characteristic equations, the solutions of the equations, and the locations and loci of the TZ's are discussed so that this approach can be extended to generalized networks, including those consisting of combinations of lumped and distributed elements. The transfer function results from a concatenation of chain matrices, and it is expressed as a ratio of rational polynomials, with PR and Hurwitz properties. The reduction of the transfer function into factored polynomials allows for location and identification of TZ's.

In the third and fourth chapters, the application of the theory is discussed. The denominator *characteristic equation* (CE) is solved to locate *reflection zeros* (RZ's), referred to here in as *transmission poles* (TP's). Note that this identity $(TP's \equiv RZ's)$ pertains only to the lossless cases. Further examination of lossy networks is part of the work planned in the future.

Several examples of networks are introduced to find out location and locus of the transmission zeros, by directly considering the cancellation of the common terms in the numerator and denominator polynomials to obtain the canonical expressions of characteristic equations.

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APPROVAL PAGE

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E. Niver, Keehong Um, Alex A. Zakidov, Ray Baughman and Anvarar A. Zakhidov, *"Tunable Split-Ring Resonator Array for Left-Handed Electromagnetic Meta-Materials"*, Proceedings of PIERS 2002, July 1-5, 2002 in Cambridge, Massachusetts, USA.

R.V. Snyder, Edip Niver, Keehong Um, Sang-Hoon Shin, *"Suspended Resonators For Filters-Reduced λ_g Excitation of Evanescent Cavities Using High Dielectric Constant Feedlines"*, IEEE Transactions on Microwave Theory and Techniques, Vol. 50, No. 12, December 2002. This is an extended version of the previous one.

O. H. Gokce, P. Kenny, J. R.Markham, K. Um, E. Niver, J. G. Flemming, S. Y. Lin, Z.Li, K.-M. Ho, *"Temperature- dependent Radiative Properties of a Three-dimensional Photonic Crystals in the infrared Region"*, Photonic and Electromagnetic Crystal Structures (PECS) – IV. October 28-31, 2002, Los Angeles, California, USA.

USA Patent:

R.V. Snyder, Edip Niver, Keehong Um, Sang-Hoon Shin, *"Evanescent Waveguide"*, USA patent filed.

To my deceased father, Maldong Um

To my mother, Soonjee Park

To my wife, Eunyoung Lee

To my only son, Kangil Um

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CHAPTER 1

TERMINATED TWO-PORT SYSTEMS FOR THE ANALYSIS OF CROSS-COUPLED FILTER NETWORKS

In this chapter several fundamental concepts on microwave filter networks are introduced. For the cascaded systems the chain matrices are most conveniently used to derive the voltage transfer function with cascaded two-port subsystems. The concepts of voltage transfer function of the two-port system are introduced. The convenient relations of transfer function and chain matrix are used to find the transmission zeros.

NOMENCLATURE

Rational polynomial function: A polynomial quotient of two polynomials.

$H(s)$: Transfer function. The ratio of output to input quantities of a linear time-invariant system in Laplace domain.

$N(s)$: Numerator polynomial of $H(s)$.

$D(s)$: Denominator polynomial of $H(s)$.

Canonic: The simplest possible.

Canonic transfer function: Transfer function with all common terms cancelled out between numerator and denominator polynomials.

Canonic numerator: Numerator of a canonic transfer function.

Canonic denominator: Denominator of a canonic transfer function.

Transmission zeros: The roots of numerator polynomial of a canonical transfer function.

Stationary (Static) zeros: The stationary (static) zeros are the zeros that do not change location in spite of the change of the element values comprising the system. The stationary zeros are located at the origin of the complex s-plane.

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Dynamic zeros: The dynamic zeros are the zeros that do change the locations as a function of the element values comprising the system. It is located in finite plane or infinite plane. The dynamic zeros are of the 2 types.

Zero-σ dynamic zeros: The dynamic zeros that move only along the $j\omega$-axis.

Nonzero-σ dynamic zeros: The dynamic zeros that can move onto any other locations in the $j\omega$-axis of the complex s-plane.

Chain (ABCD) matrix: A matrix that relates output voltage and current to input voltages and current.

Two-port system: A system that has one input and one output.

1.1   Voltage Transfer Function of a Linear System

The one-sided Laplace transform, as the primary analysis tool for time-invariant systems, is a mathematical operation indicated symbolically by $\mathcal{L}[f(t)]$, and defined for a transformable function $f(t)$ that is zero for $t < 0$ as [1]

$$\mathcal{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}\,dt \,. \qquad (1.1)$$

In Equation (1.1), the variable s is a complex frequency variable.

Given a linear system, it is conventional, although not universal, to define transfer function as the s-domain ratio of the Laplace transform of the output signal (response) to the Laplace transform of the input signal (source).

To define the transfer function, the linear system is assumed to be a circuit where all initial conditions are zero.

If a system has multiple independent sources, the transfer function for each source

can be found, and the principle of superposition is used to find the response to all sources.

As one of the possible forms of transfer function, that relates input quantities to output quantities, a voltage transfer function is defined.

To define the voltage transfer function, consider a linear system with an input and an output signals, shown in Figure 1.1.
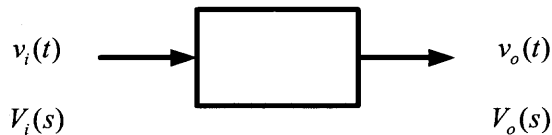
Figure 1.1 Input/output of a linear system.

In Figure 1.1, $v_i(t)$ and $v_o(t)$ are the time domain input and out signals, and the corresponding Laplace transform pairs are $V_i(s)$ and $V_o(s)$, respectively.

The voltage transfer function of the linear system of the figure above is defined as the ratio of output to input [2]

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{b_0 s^m + \cdots + b_m}{a_0 s^n + \cdots + a_n} . \qquad (1.2)$$

In Equation (1.2), $H(s)$ is a rational function of complex variable s. The transfer function $H(s)$ is the frequency-domain description of a linear time-invariant system and is a necessary function for analysis and synthesis in this domain [1].

A method for determining the transfer function of systems (filters) composed of lumped constants (those described by ordinary constant-coefficient differential equations) is investigated.

1.2 Methods to Find Transfer Function

To analyze a network, several methods are used. Using the following methods, the voltages and/or currents, to be used in Equation (1.2) can be found.

Simplifying the circuit: Combine and simplify the elements from the load to the source until there are one source and one equivalent load impedance. Employ Kirchoff's voltage law (KVL), Kirchoff's current law (KCL), Ohm's law, and/or current division to calculate all currents and voltages in the network current division from the source side to the load side until all branch currents are found. Find the ratio of output voltages (currents) to input voltages (currents).

Loop analysis on each mesh: Use Kirchoff's voltage law (KVL) to determine current in the network. Once the currents are known, Ohm's law can be used to calculate voltages. If the network contains N independent loops, then N linearly independent simultaneous equations are required to obtain Equation (1.2).

Nodal analysis on each node: Use Kirchoff's current law (KCL) to find node voltages with one node selected as the reference node. Assign branch currents for non-reference nodes. If the network contains N independent nodes, then N-1 linearly independent simultaneous equations are required to characterize the network. Set up linearly independent simultaneous equations. Solve for the unknown node voltages to obtain Equation (1.2).

Beside these, an impulse response method [2], eigen function method [3], and Mason's rule [4] can be applied to derive a transfer function.

A simple method to obtain the transfer function will depend upon the relationships that exist between the branch currents and node voltages of the ladder. It is the use of *chain* (ABCD) matrix [5].

1.3 Transmission Zeros

Given a voltage transfer function with the form of Equation (1.2), it can be expressed as

$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{N(s)}{D(s)} \; . \tag{1.3}$$

In Equation (1.3), *H(s)* is a rational polynomial function expressed as a polynomial quotient of two polynomials *N(s)*, the numerator polynomial, and *D(s)*, the denominator polynomial [6].

After the common term cancellation, *N(s)* and *D(s)* do not have any common terms. Then *H(s)*, *N(s)*, and *D(s)* are called "of the canonical form".

Transmission zeros (TZ's) are defined as the roots of canonical forms of the numerator polynomial of the transfer function. Reflection zeros or transmission poles are defined as the roots of canonical forms of the denominator polynomial.

Equating *N(s)* to zero, the equation,

$$N(s) = 0 \tag{1.4}$$

is obtained. This equation is defined as the *TZ characteristic equation* (or TZCE). The roots of Equation (1.4) are the *transmission zeros* (TZ's) of the system. *Transmission poles* (TP's) are defined as the roots of canonical forms of denominator polynomial of the transfer function. Equating *D(s)* to zero, the equation,

$$D(s) = 0 \tag{1.5}$$

is obtained. This equation is defined as the TP characteristic equation. The roots of Equation (1.5) are the *transmission poles* (or *reflection zeros*) of the system.

1.4 Two Types of Transmission Zeros

According to the possible locations of the TZ's in the complex s-domain, TZ's can be classified as two different types.

Stationary (Static) zeros: The stationary zeros are the zeros that do not change location in spite of the change of the element values comprising the system. The stationary zeros are located at the origin of the complex s-plane.

Dynamic zeros: The dynamic zeros are the zeros that do change the locations as a function of the element values comprising the system. These are located in finite plane or infinite plane. The dynamic zeros are of the 2 types:

(i) Zero-σ dynamic zeros: The dynamic zeros that move only along the $j\omega$-axis.

(ii) Nonzero-σ dynamic zeros: The dynamic zeros that can move onto any other

locations in the $j\omega$-axis of the complex s-plane.

1.5 Definition of Chain Matrix

In analyzing some electrical systems, the locations of terminal pairs where signals are either fed in or extracted are referred as to the ports of the system. A two-port system is a system that has one input and one output. Since the two-port is the most fundamental form for electrical networks and systems, it has been studied extensively. In order to characterize the behavior of a two-port network, *measured data* (currents and voltage) at both ends of the network must be obtained.

Synthesizing a large and complex linear system may be simplified by first designing subsections of the system. By first designing these less complex models and then connecting them, the whole system is completed. If the subsections are modeled by a two-port system, synthesis involves the analysis of the interconnected two-port system.

One of the ways to interconnect two-port system is the cascaded connection. The cascaded connection is important because it occurs frequently in the modeling of large systems. In using the parameters of the individual two-port systems to obtain the parameters of the interconnected systems, the *chain parameters* (*ABCD parameters*) are best suited for describing the cascaded connections [7].

Figure 1.2 represents the basic two-port building block to define chain matrix. This system should be a linear system with the following restrictions.

1. There can be no energy stored within the system.

2. There can be no independent sources within the system (dependent sources are permitted).

3. All external connections must be made to either input port or the output port, i.e., no such connections are allowed between ports.
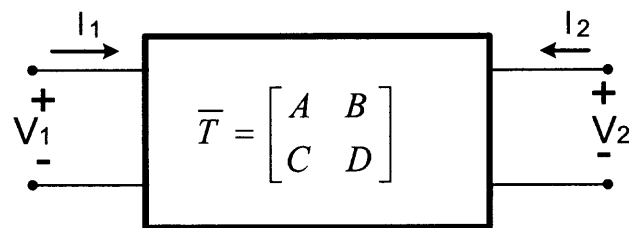
Figure 1.2 System to define chain matrix.

Two input variables and two output variables are assigned on the input and output terminals, in terms of s-domain variables, V_1, I_1, V_2, and I_2. The two input variables are V_1 and I_1. The two output variables are V_2 and I_2.

The chain parameters are used to relate the voltage and current at one port to voltage and current at the other port. In explicit form,

$$V_1 = AV_2 - BI_2 \qquad\qquad (1.6.a)$$

$$I_1 = CV_2 - DI_2 \qquad\qquad (1.6.b)$$

where A, B, C, and D are the chain parameters. In matrix form, Equation (1.6) is written by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}. \qquad\qquad (1.7)$$

For convenience, the chain matrix in Equation (1.7) is written as

$$\overline{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}. \qquad\qquad (1.8)$$

From Equations (1.6.a) and (1.8), the entry A of Equation (1.8) is given by

$$A = \overline{T}(1,1) = \left.\frac{V_1}{V_2}\right|_{I_2=0}. \qquad\qquad (1.9)$$

Equation (1.9) means that entry A is the entry (1, 1) of chain matrix $\overline{T}$, obtained by open-circuiting port #2. From Equations (1.2) and (1.6.a), the voltage transfer function can be expressed as

$$H(s) \equiv \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{1}{A} = \frac{1}{\overline{T}(1,1)}.$$

(1.10)

Equation (1.10) tells that if the entry (1.1) of the chain matrix is known the transfer function can be obtained.

1.6 Chain Matrix of Cascaded Two-port Networks

The cascade connection of a pair of two-port networks is considered as in Figure 1.3 [8].
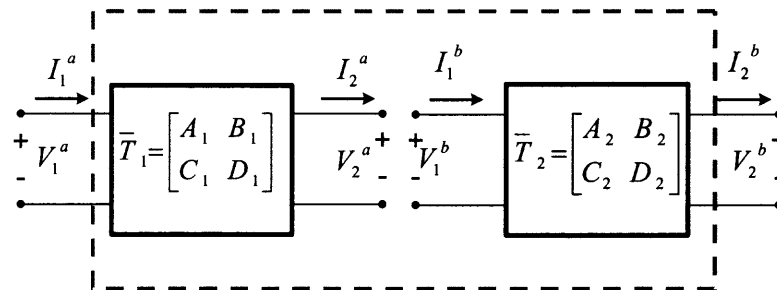
Figure 1.3 Cascade connection of a pair of two-port networks.

At microwave frequencies (300 MHz -300 GHz) of operation, chain parameters are very difficult (if not impossible) to *measure*, because the short and open circuits to AC signals are difficult to implement. Therefore, a new parameter called the *scattering parameter (or s-parameter)*, which can be obtained from chain matrices, is defined in terms of traveling waves [5].

Each two-port system in the figure above is expressed in terms of chain matrix as

$$\begin{bmatrix} V_1^{\,a} \\ I_1^{\,a} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2^{\,a} \\ I_2^{\,a} \end{bmatrix} \qquad (1.11.a)$$

$$\begin{bmatrix} V_1^{\,b} \\ I_1^{\,b} \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2^{\,b} \\ I_2^{\,b} \end{bmatrix} \qquad (1.11.b)$$

The final system is constructed when the connection is made, by combining the two, with

$$V_2^{\,a} = V_1^{\,b} \quad \text{and} \quad I_2^{\,a} = I_1^{\,b}. \qquad (1.12)$$

Substituting Equation (1.11.b) into (1.11.a) with (1.12), the following expression is obtained;

$$\begin{bmatrix} V_1^{\,a} \\ I_1^{\,a} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} V_2^{\,a} \\ I_2^{\,a} \end{bmatrix}. \qquad (1.13.a)$$

Multiplying the two *chain (ABCD)* matrices in Equation (1.13.a), the simplified relation

$$\begin{bmatrix} V_1^{\,a} \\ I_1^{\,a} \end{bmatrix} = \begin{bmatrix} A_1 A_2 + B_1 C_2 & A_1 B_2 + B_1 D_2 \\ C_1 A_2 + D_1 C_2 & C_1 B_2 + D_1 D_2 \end{bmatrix} \cdot \begin{bmatrix} V_2^{\,a} \\ I_2^{\,a} \end{bmatrix}. \qquad (1.13.b)$$

is obtained. Equation (1.13.b) shows that for a cascaded system, the input variables are related to output variables by the products of chain matrices of individual two-port subsystems. It should be noted that this result could be extended to the case of any number of cascaded two-port systems. With the form of Equation (1.8), the chain matrix

of n cascaded networks can be represented as the product of each of the chain matrix by

$$\bar{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{i=1}^{n} \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \prod_{i=1}^{n} \bar{T}_i. \qquad (1.14)$$

In Equation (1.14), Π is the symbol for product of n chain matrices.

Bandpass filters can be effectively designed by adjusting the locations of *transmission zeros* (TZ's) and *transmission poles* (TP's) in the complex s-domain. Given a filter network, determining the TZ locations as a function of element values includes deriving the transfer function.

Here, a practical method for determination of the complex TZ locations of the cross-coupled bandpass filter is discussed. This technique uses chain matrices for subsystems (discrete parts of the network), and can be extended to other types of filters with cross-coupled sections.

An important result is that a complex doublet and/or quadruplet (one-, two-, or four-pairs) of TZ's are shown to result solely from the cross-coupled portion of the circuit. Modifications to the cross coupled portion have only a small effect on the TP's (otherwise known as reflection zeros).

The method for determining the locus and location of TZ's for both positively and negatively cross-coupled bandpass filters will be considered below.

The several closed-forms of expressions in terms of elements are obtained, and TZ's are located by solving what is called the TZ characteristic equation. This is derived by taking advantage of the bridged-T structure for the cross-coupled part.

CHAPTER 2

BRIDGED-T CROSS-COUPLED FILTER NETWORKS

2.0 Introduction

A specific filter network with a cross-coupled element added between two shunt-connected resonators is considered. Since a filter network is a two-port system, it can be described by two-port parameters. When a large and complex filter network is to be constructed by cascading the unit subsystems, the *chain* (*ABCD*) parameters are mostly conveniently used to describe it.

Snyder and Bozarth [9] discussed the analysis and design of an active resonator using the hybrid configuration transistor circuit by sectioning the whole system to introduce the bridged-T structure. The structure was used to derive the computed input impedance suitable for the studies of resonators under various biases and load conditions.

The transfer function of the isolated passive networks composed of R's and C's with a cross-coupled section was derived and the one pair of complex zeros and a number of real zeros were discussed [10]-[11].

Levy [12]-[13] discussed the realization of transmission zero (TZ) locations in the complex $(\sigma + j\omega)$ plane by positively or negatively cross-coupling a pair of nonadjacent elements in the microwave filter, and Wenzel [14] discussed the TZ movement in cross-coupled (CC) filters, based on *qualitative* rules. No *quantitative* information was provided, and in this dissertation, such will be provided.

A new technique will be introduced in this chapter. A new technique to determine TZ's from the cross-coupled filter network obtained from the initially synthesized ladder network is presented. By adding a cross-coupled bridge on the ladder

network, TZ's are produced in the complex *s*-plane, in a doublet, a quadruplet, or a sextuplet (one-, two-, or three-pairs) of locations. Production of the finite-frequency complex pairs of TZ's is shown to result solely from the cross-coupled portion of the circuit. The network is described as a connection of cascaded two-port networks.

As is always the case, multiplication of chain matrices enables computation of the total transfer function of the filter system.

In this dissertation, the location and motion of the TZ's will be *quantitatively* examined.

The location and locus of complex zeros in the left half-plane (LHP) and right half-plane (RHP), as a result of perturbing the element values of *L* and/or *C* are determined from the numerator polynomial of the transfer function.

It is known that the chain matrix of *n* cascaded networks can be represented as the product of each chain matrix given by

$$\overline{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{i=1}^{n} \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \prod_{i=1}^{n} \overline{T}_i \ . \tag{2.1}$$

In Equation (2.1), $\overline{T}_i$ is the chain matrix of the *i-th* system. Since the size of each system is 2×2, the resultant matrix is also 2×2. Voltage transfer function *H(s)* and the entry (1, 1) of the resultant matrix $\overline{T}$ have the relationship,

$$H(s) = \frac{1}{A} = \frac{1}{\overline{T}(1,1)} = \frac{N(s)}{D(s)} . \tag{2.2}$$

In Equation (2.2), *H(s)* is a polynomial quotient of two polynomials *N(s)* and *D(s)*, and known as a *rational polynomial function*. Where, *N(s)* and *D(s)* are the numerator and denominator polynomials of *H(s)*, respectively.

They may or may not have common terms to be cancelled out. After the common terms, if any, are cancelled out, the canonical form of transfer function is obtained.

The numerator polynomial of the canonical polynomial is the transmission zero characteristic polynomial.

NOMENCLATURE

Locus: The path of motion for dynamic TZ's or TP's as functions of cross-coupling.

Doublet: Two transmission zeros in complex conjugate pairs, with real part zero.

Quadruplet: Four transmission zeros, with two TZ's are in complex conjugate pairs, respectively.

Hurwitz polynomial $f(s)$: Polynomial whose roots of $f(s) = 0$ is in LHP.

$\overline{T}(i, j)$: The entry located at the *i*-th row and *j*-th column of 2×2 chain matrix $\overline{T}$.

Ladder network: A network composed of series-connected and parallel-connected elements, such that every element is alternately in series-connected and shunt-connected as a signal travels from the source to the load.

Cross-coupling: An additional connection of element between two nodes in the network.

Chebyshev response: A filter response, with ripples in the passband and/or stopband.

$S_i (i = 1-5)$:   The subsystem built at the *i*-th location of the cascaded network, with $i = 1$ for the 1st subsystem numbered from the source side.

$\overline{T}_i (i = 1-5)$: The chain matrix of $S_i (i = 1-5)$.

Zm, Z_m , $\overline{Z}m$, or $\overline{Z}_m$:The Laplace impedance of the *m*-th subsystem with only one element.

Zmn, Z_{mn}, $\overline{Z}mn$, or $\overline{Z}_{mn}$: The Laplace impedance of the *n*-th element of the *m*-th subsystem, with more than one element.

Cm (or Cm): Capacitor of *m*-th subsystem with only one capacitor.

Lm (or Lm):   Inductor of *m*-th subsystem with   only one inductor.

Lmn (or Lmn): Inductor as the *n*-th element of *m*-th subsystem.

Cmn (or Cmn): Capacitor as the *n*-th element of *m*-th subsystem.

Bridged-T: A T-network with a cross-coupling element between two series elements.

NEm = The numerator of the matrix entry E in the *m*-th (*m*=1-5) subsystem.

DEm = The denominator of the matrix entry *E* in the *m*-th (*m*=1-5) subsystem.
    The 2nd variable E must be one the followings:
        A = the entry (1, 1) of chain matrix.
        B = the entry (1, 2) of chain matrix.
        C = the entry (2, 1) of chain matrix.
        D = the entry (2, 2) of chain matrix
    Am = the entry (1, 1) of chain matrix of *m*-th subsystem.
    Bm = the entry (1, 2) of chain matrix of *m*-th subsystem.
    Cm = the entry (2, 1) of chain matrix of *m*-th subsystem.
    Dm = the entry (2, 2) of chain matrix of *m*-th subsystem.

a_{mn} = Polynomial coefficient of s^n of *m*-th subsystem.

Polynomial equation: $f(s) = a_m s^m + a_{m-1} s^{m-1} + ... + a_o = 0$. The highest degree *m* is
    greater than 1 in the *m*-th degree polynomial.

Monomial equation: $f(s) = s = 0$.

"0_+" : The very small positive value almost equal to zero.

"∞_-": The very big positive value almost equal to (very close to) infinity.

2.1 The Ladder Network

A frequently used ladder network is composed of series-connected and parallel-connected

elements as shown in Figure 2.1. The pattern is that every other element is alternatively

in series-connected and shunt-connected as a signal travels from the source to the load.

So the subsystem S_i ($i = 1 - 4$) makes a ladder network, where the subscript i is used to indicate the system is the i-th subsystem. Subsystem S5 is an external load connected to the ladder network. The network is an initially synthesized ladder networks without any cross-coupling. It is a four-pole (four resonators) band pass filter. Four shunt-connected *LC* resonators have impedances Z2, Z4, Z6 and Z8, due to the parallel *LC* components composed of (L2, C2), (L4, C4), (L6, C6), and (L8, C8), respectively.

The impedances Z3, Z5, and Z7 are due to the series-connected elements, and could be inductors and/or capacitors, respectively. The impedances Z1 and Z9 represent the source and load impedances of 50 Ohms, respectively.
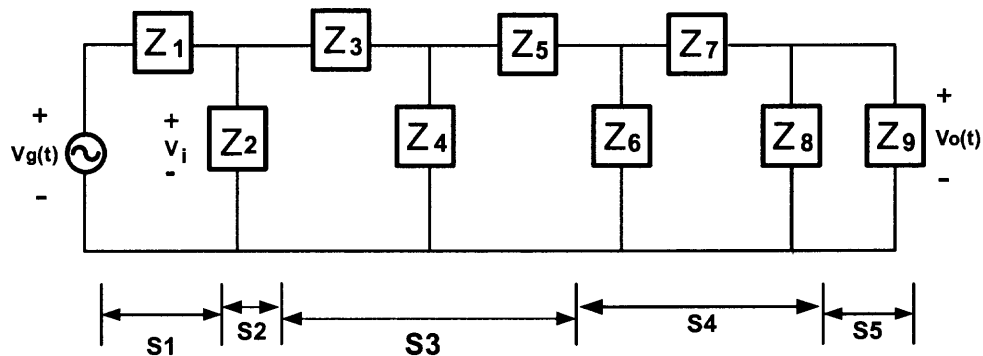
Figure 2.1 Ladder network without cross-coupling.

In the figure above, Zm, ($m = 1-9$), is the Laplace impedance of each element, where subscript m means the m-th element. Since the impedance is a complex number, it should be expressed as $\overline{Zm}$. However, it is understood that Zm implies $\overline{Zm}$.

Signal $v_g(t)$ is the input signal and signal $v_o(t)$ is the output signal in time domain, respectively. In the analysis of this filter network, the Laplace transform is used.

In frequency domain, the generic response of the ladder network, for example,

without cross-coupled element added, is shown in Figure 2.2. In the figure, $m = -2$ is used to indicate the slope of the attenuation of the response is -2, and fc is used to indicate the center frequency of the filter [15].
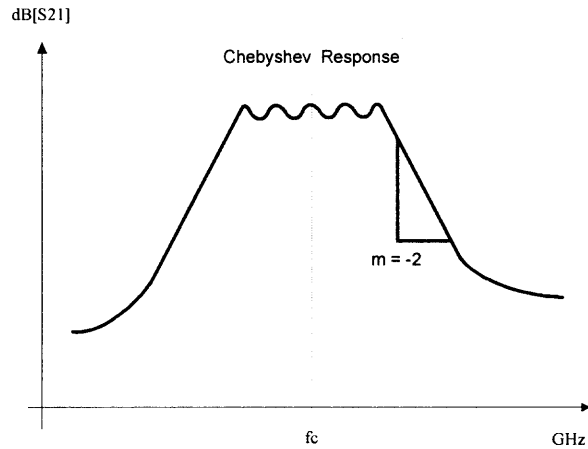
Figure 2.2 Insertion loss of a ladder network, without cross-coupling.

The generic response of the filter, for example, with cross-coupled element added, is shown in Figure 2.3.
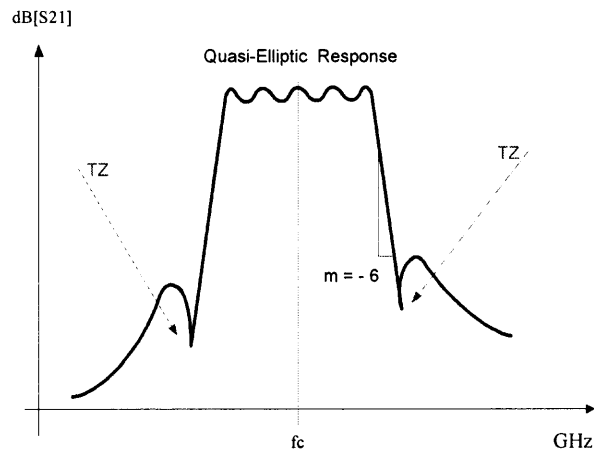
Figure 2.3 Improved insertion loss of a cross-coupled filter.

In the figure, $m = -6$ is used to indicate the slope of the attenuation of the response is -6, and f_c is used to indicate the center frequency. Two TZ's are located at the both sides of passband.

The transition slope of Figure 2.3 is steeper than that of Figure 2.2. This occurs due to the addition of a cross-coupling element between the two resonators.

There are several possibilities to add cross-coupled elements for the filter network. A few examples, to be considered, are as follows:

1) Without skipping any resonators (adjacent resonators),

2) Skipping one resonator,

3) Skipping two resonators.

When more than three resonators are skipped, they can be simplified to no. 2 or no. 3 above. Then the analysis follows the same procedure. Therefore, in this chapter, the 2nd case above will be considered. Using these results, the 1st and 3rd cases will be investigated in Chapter 3.

In each case of filter configurations, coupling can be achieved in two different types: one is *negative cross-coupling* (NCC); the other is *positive cross-coupling* (PCC). Negative cross coupling means that the sign of cross coupling opposes the sign of the main line coupling (i.e. capacitive cross coupling in an inductively coupled circuit, or inductive cross coupling in a capacitive coupled main line).

In a negatively cross-coupled implementation, the series-connected elements are all inductors (or capacitors) and the cross-coupled element is a capacitor (or inductor).

These two filters have the same locations for the finite frequency TZ's (but not for infinite frequency or DC TZ's, and not necessarily the same TP's.

In a positively cross-coupled implementation, the series-connected elements are all inductors (or capacitors) and the cross-coupled element is an inductor (or a capacitor). These two filters have the same TZ locations.

A cross-coupled filter network skipping one resonator is first analyzed, for both negative cross coupling and positive cross coupling.

2.2 Cross-coupled (CC) Filter Configuration

In Figure 2.1, connecting the two resonators Z_2 and Z_6, skipping one resonator Z_4, can add a cross-coupling element. Likewise, the two resonators Z_4 and Z_8 can be connected, skipping one resonator Z_6. These two networks have the same TZ locations and locus. Locus is defined as the path of motion for dynamic TZ's as functions of cross-coupling.

A cross-coupled filter of Figure 2.4 is considered. The cascaded chain matrices of five subsystems sectioned is used to conveniently represent the system. For the cross-coupled subsystem an equivalent system in the form of bridged-T network can be used to determine chain matrices. The analysis on the cross-coupled microwave filters also will show the sectioning the whole filter system into several subsystems. The chain-parameters for each subsystem are derived. Since the cross-coupled circuit is the bridged-T structure, the chain parameters of the structure are first found. With all the chain parameters, the transfer function is found.

From the transfer function, the locations of TZ's are found from the canonical form of the numerator polynomial of the transfer function. The whole filter network is considered to be composed of five subsystems cascaded.

Since the cross-coupled subsystem S3 is the bridged-T structure, the chain parameters of this structure are first to be determined. With all the chain parameters determined for the five subsystems, the transfer function is found. As stated above, from the transfer function, the locations of TZ's are found from the canonical form of the numerator polynomial of the transfer function.

The overall filter network is sectioned into five subsystems (Si, i =1-5) as shown in Figure 2.4. Each system is characterized by its own chain matrix of size 2×2.
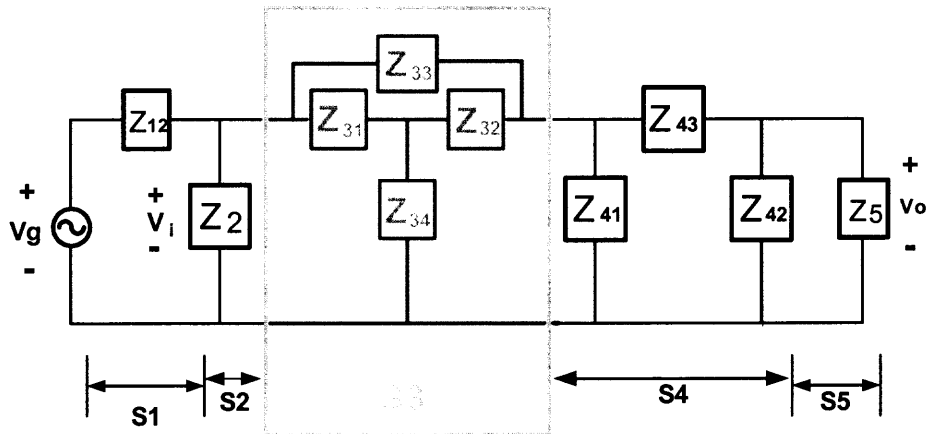
Figure 2.4 A block diagram of cross-coupled filter network.

In the figure above, Zm and Zmn as used herein are defined by

Z_m : The Laplace impedance of the m-th subsystem with only one element.

Z_{mn} : The Laplace impedance of the n-th element of the m-th subsystem, with more than one element.

For example, Z_2 means the Laplace impedance of the element of the 2nd subsystem, and Z_{32} means the Laplace impedance of the 2nd element of 3rd subsystem. Following the definitions above, the Z_2, Z_{34}, Z_{41} and Z_{42} represent the impedances due to the shunt-

connected tank circuits composed of (L2, C2), (L34, C34), (L41, C41), and (L42, C42), respectively.

In the figure above, all impedances are consisted of inductors (capacitors) and all shunt impedances are consisted of parallel *LC*'s.

Impedances Z31, Z32, and Z43 are due to series-connected inductors L31, L32, and L43, or capacitors C31, C32, and C43, respectively. For a negatively cross-coupled network, impedance Z33 is due to a single cross-coupled capacitor (*or* inductor) C33 (*or* L33), while for a positively cross-coupled network, impedance Z33 is due to a single cross-coupled inductor (*or* capacitor) L33 (*or* C33), respectively. The impedances Z1 and Z5 represent source and load impedances of 50 Ohms.

2.3 Negatively Cross-coupled (NCC) Filter Network

In Figure 2.5, the series-connected elements are all inductors. A negatively cross-coupled filter network is obtained by using capacitor impedance for Z33 connected between the 1st and the 3rd resonators, as shown in Figure. If the series-connected elements are all capacitors, the cross-coupled (CC) elements should be an inductor to result in the same locations for the TZ's. Here is the first case to be considered.

A cross-coupled circuit, or a bridge-T circuit, is installed from the 1st resonator (Z2) and the 3rd resonator (Z41). The whole system is considered to be composed of five subsystems (S1, S2, S3, S4, and S5) connected in cascade. Therefore, the *chain* (ABCD) matrix of the whole system is expressed by

$$\overline{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \overline{T}_1 \cdot \overline{T}_2 \cdot \overline{T}_3 \cdot \overline{T}_4 \cdot \overline{T}_5 . \tag{2.3}$$

In Equation (2.3), each entry of five chain matrices must be expressed in terms of Laplace impedance shown in the Figure 2.5.
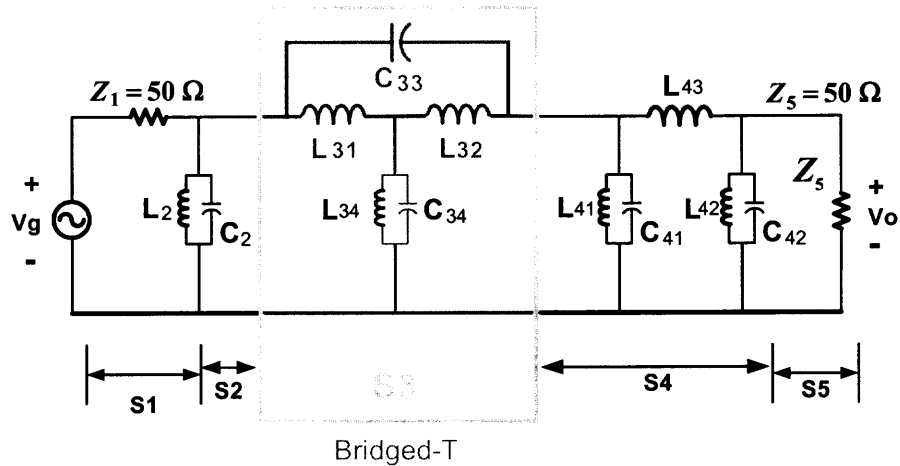
Figure 2.5 A negatively cross-coupled filter network.

In the figure above, the impedances (i.e. Laplace impedances) of the elements are expressed as:

$$\overline{Z}_1 = 50;$$

$$\overline{Z}_2 = \frac{s L_2}{L_2 C_2 s^2 + 1};$$

$$\overline{Z}_{31} = sL_{31}, \ \overline{Z}_{32} = sL_{32}, \ \overline{Z}_{33} = 1/sC_{33}, \overline{Z}_{34} = \frac{sL_{34}}{L_{34}C_{34}s^2 + 1}; \qquad (2.4)$$

$$\overline{Z}_{41} = \frac{sL_{41}}{L_{41}C_{41}s^2 + 1}, \ \overline{Z}_{42} = \frac{sL_{42}}{L_{42}C_{42}s^2 + 1}, \ \overline{Z}_{43} = sL_{43} \ ;$$

$$\overline{Z}_5 = 50.$$

The chain matrices of the network of Equation (2.3) are given by

$$\overline{T}_1 = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix} \tag{2.5.a}$$

$$\overline{T}_i = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}, \quad i = 2,3,\&4. \tag{2.5.b}$$

$$\overline{T}_5 = \begin{bmatrix} 1 & 0 \\ 1/50 & 1 \end{bmatrix} \tag{2.5.c}$$

Equation (2.5) shows the chain matrices of each subsystem, i.e. $\overline{T}_1, \overline{T}_2, \overline{T}_3, \overline{T}_4$, and $\overline{T}_5$. These matrices are due to the series source impedance R, shunt resonator #1, bridged-T subsystem, π-network, and the load impedance, respectively.

In Equation (2.3), matrix entry $\overline{T}(1,1)$ is dependent on each of the cascaded five networks. In Equation (2.5.b), all of the 12 entries of three matrices should be expressed in terms of Laplace impedances given in Equation (2.4). From Equation (2.2), the voltage transfer function $H(s)$ has the numerator polynomial $N(s)$ and denominator polynomial and $D(s)$, respectively. Using a MATLAB program, the chain matrices in Equation (2.3) are obtained based on the following detailed procedures.

Rational polynomial expressions of matrix entries

In Equation (2.3), to ensure that the conditions of the realizations of Hurwitz polynomial and /or polynomial of even degree for the complex conjugate roots is imposed in the numerator and denominator of a rational polynomial function, the rational expressions of any matrix entries are defined in this dissertation.

The i-th chain matrix $\overline{T}_i$ of the i-th subsystem of a filter network is a 2×2 matrix with four entry A_i, B_i, C_i, and D_i, since these are defined from the two-port

systems. Any matrix obtained by mathematically manipulating any numbers of 2×2 matrices is also 2×2 matrix. Let the entry Xi of the chain matrix $\overline{T}_i$ represent any of the matrix entry Ai, Bi, Ci, or Di. Four of these entries are meant by

$$Ai \equiv Entry \ (1,1) \ of \ the \ \overline{T}_i, \quad Bi \equiv Entry \ (1,2) \ of \ the \ \overline{T}_i,$$
$$Ci \equiv Entry \ (2,1) \ of \ the \ \overline{T}_i, \quad Di \equiv Entry(2,2) \ of \ the \ \overline{T}_i.$$

Each of the entry Xi of matrix $\overline{T}_i$ has a numerator polynomial $f_i(s)$ and a denominator polynomial $g_i(s)$. Therefore, entry Xi can be expressed in terms of two quantities as

$$Xi = \frac{f_i(s)}{g_i(s)} .$$

The numerator function $f_i(s)$ has its own numerator $n(f_i(s))$ and denominator $d(f_i(s))$. The denominator function $g_i(s)$ has its own numerator $n(g_i(s))$ and denominator $d(g_i(s))$. Therefore, Xi can be expressed in terms of the four quantities as

$$Xi = \frac{f_i(s)}{g_i(s)} = \frac{\dfrac{n(f_i(s))}{d(f_i(s))}}{\dfrac{n(g_i(s))}{d(g_i(s))}} .$$

To get a rational polynomial function for the entry Xi, the following expression is used.

$$Xi = \frac{f_i(s)}{g_i(s)} = \frac{\dfrac{n(f_i(s))}{d(f_i(s))}}{\dfrac{n(g_i(s))}{d(g_i(s))}} = \frac{n(f_i(s)) \cdot d(g_i(s))}{n(g_i(s)) \cdot d(f_i(s))} .$$

The resultant numerator is a polynomial, and the resultant denominator is also a polynomial. Two notations NXi and DXi are introduced as

$$NXi \equiv n\left(f_i(s)\right) \cdot d\left(g_i(s)\right)$$

and

$$DXi \equiv n\left(g_i(s)\right) \cdot d\left(f_i(s)\right).$$

Matrix entry Xi is given by a rational polynomial function as

$$Xi \equiv \frac{NXi}{DXi}.$$

This expression is used to represent a rational polynomial. The numerator and denominator may or may not have common terms.

By a subsystem approach, microwave or RF filter networks are quantitatively investigated in this dissertation.

2.3.1 Chain Matrices of Each Subsystem

The filter network is composed of five subsystems, S1, S2, S3, S4, and S5. Each subsystem is considered in terms of its chain matrix.

- **System S1**

The 1st subnetwork is composed of source impedance $\overline{Z}_1 = \overline{Z}_g = 50\ \Omega$ and the ground line. The *chain (ABCD)* matrix, $\overline{T}_1$, of the series-connected impedance is given by

$$\overline{T}_1 = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}. \qquad\qquad (2.6)$$

All entries of matrix Equation (2.6) are constant, so $\overline{T}_1$ is not a function of s. Therefore, the 1st system does not have zeros nor poles in the s-plane. The value 50 of entry (2, 1) affects the magnitude of the transfer function for the whole system.

- **System S2**

The 2nd filter network is composed of impedance Z_2, shunt-connected to the ground line. Since Z_2 is a parallel connection of L_2 and C_2, it is expressed as

$$Z_2 = \frac{s\,L_2}{L_2\,C_2\,s^2 + 1} \ .$$ (2.7)

The chain ($ABCD$) matrix, $\overline{T}_2$, is given by

$$\overline{T}_2 = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix} .$$ (2.8)

In Equation (2.8), the subscript 2 means the subsystem S2.

From Equation (2.8), the entry (2, 1) of the matrix is expressed as

$$C_2 = 1/Z_2 = \frac{NC2}{DC2} = \frac{L_2 C_2\,s^2 + 1}{L_2\,s} \ .$$ (2.9)

For an efficient mathematical calculations, the symbols as used herein in the whole dissertation are defined as follows;

NEm = the *numerator* of the matrix entry E in the m-th (m =1-5) subsystem.

DEm = the *denominator* of the matrix entry E in the m-th (m=1-5) subsystem.

The 2nd variable E must be one the followings:

A = the entry (1, 1) of chain matrix.
B = the entry (1, 2) of chain matrix.
C = the entry (2, 1) of chain matrix.
D = the entry (2, 2) of chain matrix.

In Equation (2.9),

$NC2$ is the *numerator* polynomial of the *entry* (2, 1) of the subsystem $S2$.

$DC2$ is the *denominator* polynomial of the *entry* (2, 1) of the subsystem $S2$.

From Equation (2.9) the following expressions are obtained, respectively.

$$NC2 = L_2 C_2 s^2 + 1,$$
(2.10.a)

$$DC2 = L_2 s .$$
(2.10.b)

- **System S3**

The 3rd network is the cross-coupled subsystem, which is considered as a *bridged-T network*. The *chain* (*ABCD*) matrix $\overline{T}_3$ of the subsystem is expressed by

$$\overline{T}_3 = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} .$$
(2.11)

In Equation (2.11), the four entries of the matrix are expressed as follows:

$$A_3 = \frac{Z_{31}(Z_{32} + Z_{33}) + (Z_{31} + Z_{32} + Z_{33})Z_{34}}{Z_{31}Z_{32} + (Z_{31} + Z_{32} + Z_{33})Z_{34}} \, , \qquad (2.12.a)$$

$$B_3 = \frac{Z_{33}(Z_{31}Z_{32} + Z_{31}Z_{34} + Z_{32}Z_{34})}{Z_{31}Z_{32} + (Z_{31} + Z_{32} + Z_{33})Z_{34}} \, , \qquad (2.12.b)$$

$$C_3 = \frac{Z_{31} + Z_{32} + Z_{33}}{Z_{31}Z_{32} + (Z_{31} + Z_{32} + Z_{33})Z_{34}} \, , \qquad (2.12.c)$$

$$D_3 = 1 + \frac{Z_{32}Z_{33}}{Z_{31}Z_{32} + (Z_{31} + Z_{32} + Z_{33})Z_{34}} \, . \qquad (2.12.d)$$

a) $\overline{T}_3(1,1)$ of System S3

The entry $\overline{T}_3(1,1)$, or A_3 , is a *rational polynomial* (a ratio of two polynomials),

$$A_3 = \frac{NA3}{DA3} \, . \qquad (2.13)$$

In Equation (2.13), $NA3$ is the numerator polynomial of A_3, and $DA3$ is the

denominator polynomial of A_3. These are expressed as follows, respectively:

$$
\begin{aligned}
NA3 \;=\; & (L31L32C33L34C34)\,s^4 \\
& + (\, L31\ L32C33 + L31L34\ C34 + L34L31C33 + L34\ L32\ C33)\,s^2 \\
& \qquad\qquad\qquad + L34 + L31 \qquad\qquad\qquad\qquad (2.14)\\
\equiv\; & a_{34}\ s^4 + a_{32}\ s^2 + a_{30} \, ,
\end{aligned}
$$

where

$$a_{34} = L31L32C33L34C34,$$
$$a_{32} = L31\ L32C33 + L31L34\ C34 + L34L31C33 + L34\ L32\ C33$$
$$\quad = (L31\ L32 + L34L31 + L34\ L32\ )\,C33 + L31L34\ C34, \quad \text{and}$$
$$a_{30} = L34 + L31 \, .$$

$$DA3 = (L_{31}\ L_{32}\ C_{33}L_{34}\ C_{34})\ s^4 + (\ L_{31}\ L_{32} + L_{34}\ L_{31} + L_{34}L_{32})\ C_{33}\ s^2 + L_{34}$$
$$\equiv a_{34}\ s^4 + a_{32}s^2 + a_{30}, \qquad\qquad (2.15)$$

where
$$a_{34} = L_{31}L_{32}C_{33}L_{34}C_{34},$$
$$a_{32} = (\ L_{31}\ L_{32} + L_{34}\ L_{31} + L_{34}L_{32})\ C_{33}, \quad \text{and}$$
$$a_{30} = L_{34}.$$

In Equation (2.14), notations Lmn (or L_{mn}), Cmn (or C_{mn}), and a_{mn} are defined by

Lmn = Inductor as the n-th element of the m-th subsystem,

Cmn = Capacitor as the n-th element of the m-th subsystem,

a_{mn} = Coefficient of s^n in a polynomial of the m-th subsystem.

These definitions are valid in the remainder of the dissertation.

b) $\overline{T}_3(1,2)$ of System S3

The entry $\overline{T}_3(1,2)$, or B_3, is a *rational polynomial* (a ratio of two polynomials),

$$B_3 = \frac{NB3}{DB3}. \qquad\qquad (2.16)$$

In Equation (2.16), $NB3$ is the numerator polynomial of B_3, and $DB3$ is the denominator polynomial of B_3, which is expressed as follows in terms of element values, respectively.

$$NB3 = (\ L_{31}L_{32}\ L_{34}\ C_{34})s^3 + (\ L_{31}L_{32} + L_{34}\ L_{32} + L_{34}L_{31})\ s \qquad (2.17)$$
$$\equiv s \cdot (a_{32}s^2 + a_{30}),$$
where

$$a_{32} = L31\ L32\ L34\ C34\ ,\quad \text{and}$$

$$a_{30} = L31L32 + L34\ L32 + L34L31\ .$$

$$DB3 = (L31\ L32\ C33L34\ C34)\ s^4 + (\ L31\ L32 + L34\ L31 + L34L32)C33\ s^2\ +\ L34$$
$$\equiv a_{34}\,s^4 + a_{32}\,s^2 + a_{30}\,, \tag{2.18}$$

where
$$a_{34} = L31L32\ C33L34C34\ ,$$
$$a_{32} = (L31\ L32 + L34\ C31 + L34\ L32)C33\ ,\quad \text{and}$$
$$a_{30} = L34\ .$$

c) $\overline{T}_3(2,1)$ of System S3

The entry $\overline{T}_3(2,1)$, or C_3 is a *rational polynomial* (a ratio of two polynomials),

$$C_3 = \frac{NC3}{DC3}\ . \tag{2.19}$$

In Equation (2.19), $NC3$ is the numerator polynomial of C_3 , and $DC3$ is the

denominator polynomial of C_3, which is expressed as follows, respectively.

$$NC3 = (L31L34\ C34 + L32\ L34\ C34\ )C33\ s^4 + (\ L31C33 + L32C33 + L34\ C34)s^2\ +\ 1 \tag{2.20}$$

$$DC3 = (L31\ L32C33\ L34\ C34)\,s^5 + (L31\ L32 + L34\ L31 + L34\ L32)\ C33\ s^3\ +\ L34\ s$$
$$\equiv s\cdot(a_{34}s^4 + a_{32}s^2 + a_{30})\,, \tag{2.21}$$

where
$$a_{34} = L31L32\ C33\ L34C34\ ,$$
$$a_{32} = (L31\ L32 + L34\ L31 + L34\ L32)\ C33,\quad \text{and}$$
$$a_{30} = L34\ .$$

d) $\overline{T}_3(2,2)$ **of System S3**

The entry $\overline{T}_3(2,1)$, or D_3, is a *rational polynomial function*,

$$D_3 = \frac{ND3}{DD3}.$$ (2.22)

In Equation (2.22), $ND3$ is the numerator polynomial of rational function D_3, and

$DD3$ is the denominator polynomial of D_3. These are respectively given by

$$ND3 = (\text{L31L32C33L34C34})\, s^4 +$$
$$(\text{L31L32C33+L34L32C33+L32L34C34+L34L31C33})\, s^2 + (\text{L34 + L32})\,,$$ (2.23)

$$DD3 = (\text{L31 L32 C33 L34 C34})\, s^4 + (\text{L31 L32 + L34 L31 + L34 L32})\,\text{C33}\; s^2 + \text{L34}$$
$$\equiv a_{34}s^4 + a_{32}s^2 + a_{30}\,,$$ (2.24)

where
$$a_{34} = \text{L31 L32 C33 L34 C34},$$
$$a_{32} = \text{L31 L32 + L34 L31 + L34 L32})\,\text{C33},\quad \text{and}$$
$$a_{30} = \text{L34}.$$

- **System S4**

    The 4$^{\text{th}}$ network is composed of Z_{41}, Z_{42}, and Z_{43}, which is a π- network.

    Z_{41} is a network of parallel connection of L_{41} and C_{41}, and shunt-connected.

    Z_{42} is a network of parallel connection of L_{42} and C_{42}, and shunt-connected.

    Z_{43} is just an impedance of single inductor, L_{43}.

The chain matrix, $\overline{T}_4$, is given by

$$\overline{T}_4 = \begin{bmatrix} A_4 & B_4 \\ C_4 & D_4 \end{bmatrix}. \tag{2.25}$$

The four entries of Equation (2.25) are expressed as

$$A_4 = 1 + \frac{Z_{43}}{Z_{42}} \qquad\qquad B_4 = Z_{43}$$

$$C_4 = \frac{1}{Z_{41}} + \frac{1}{Z_{42}} + \frac{Z_{43}}{Z_{41} Z_{42}} \qquad D_4 = 1 + \frac{Z_{43}}{Z_{41}} \;. \tag{2.26}$$

In Equation (2.26), each impedance of the matrix entries is expressed in terms of Laplace impedances as [16]

$$Z_{41} = \frac{sL_{41}}{s^2 L_{41} C_{41} + 1},$$

$$Z_{42} = \frac{sL_{41}}{s^2 L_{41} C_{42} + 1}, \tag{2.27}$$

$$Z_{43} = s L_{43}.$$

The impedances in Equation (2.27) are used in (2.26) to obtain matrix entries. Each entry is calculated as the following procedures.

a) $\overline{T}_4(1,1)$ **of System S4**

The entry $\overline{T}_4(1,1)$, or A_4, is a rational *polynomial* function (a ratio of two polynomials)

$$A_4 = \frac{NA4}{DA4} = \frac{(L_{43}L_{42}C_{42})s^2 + (L_{42} + L_{43})}{L_{42}} . \tag{2.28}$$

In Equation (2.28), $NA4$ is the numerator polynomial of A_4, and $DA4$ is the

denominator polynomial of A_4, which is expressed as

$$NA4 \quad = \quad (L_{43}\,L_{42}\,C_{42})\,s^2 \,+\, L_{43} + L_{42}, \tag{2.29.a}$$

$$DA4 \quad = \quad L_{42}. \tag{2.29.b}$$

b) $\overline{T}_4(1,2)$ **of System S4**

The entry $(1, 2)$ of $\overline{T}_4$ is given by

$$B_4 = \frac{NB4}{DB4} = L_{43} \cdot s . \tag{2.30}$$

In Equation (2.30), the quantity $NB4$ represents the numerator polynomial of B_4, and

$DB4$ represents the denominator polynomial of B_4, which is expressed as, respectively.

$$NB4 = L_{43} \cdot s, \tag{2.31.a}$$

$$DA4 = 1. \tag{2.31.b}$$

c) $\overline{T}_4(2,1)$ of System S4

The entry $\overline{T}_4(2,1)$, or C_4, is a *rational polynomial* function (a ratio of two polynomials),

$$C_4 = \frac{NC4}{DC4}.$$

(2.32)

In Equation (2.32), $NC4$ is the numerator polynomial of C_4, and $DC4$ is the denominator polynomial of C_4, which is expressed as follows, respectively.

$$
\begin{aligned}
NC4 = & (L_{43}\,L_{41}\,C_{41}\,L_{42}\,C_{42})\ s^4 \\
& + (L_{42}\,L_{41}\,C_{41} + L_{41}\,L_{42}\,C_{42} + L_{43}\,L_{41}\,C_{41} + L_{43}\,L_{42}\,C_{42})\ s^2 \\
& + L_{42}\ + L_{41} + L_{43} \\
\equiv & \ a_{44}\ s^4 + a_{42}s^2 + a_{40}, \\
& \text{where} \\
& a_{44} = L_{43}\,L_{41}\,C_{41}\,L_{42}\,C_{42}, \\
& a_{42} = L_{42}\,L_{41}\,C_{41} + L_{41}\,L_{42}\,C_{42} + L_{43}\,L_{41}\,C_{41} + L_{43}\,L_{42}\,C_{42}, \text{ and} \\
& a_{40} = L_{42}\ + L_{41} + L_{43}.
\end{aligned}
$$

(2.33)

$$DC4 \ = \ s\ L_{41}\,L_{42}.$$

(2.34)

d) $\overline{T}_4(2,2)$ of System S4

The entry $\overline{T}_4(2,2)$, or C_4, is a *rational polynomial* (a ratio of two polynomials),

$$D_4 \ = \ \frac{ND4}{DD4} \ = \ \frac{\left(L_{41}C_{41}L_{43}\right) s^2 + \left(L_{41}+L_{43}\right)}{L_{41}}.$$

(2.35)

In Equation (2.35), $ND4$ is the numerator polynomial of D_4 in chain matrix, and $DD4$ is the denominator polynomial of D_4, which is expressed as follows, respectively.

$$ND4 \quad = \quad (L_{41}C_{41} \; L_{43}) \; s^2 + \; (L_{41} + L_{43}), \qquad (2.36.a)$$

$$DD4 \quad = \quad L_{41}. \qquad (2.36.b)$$

• **System S5**

The 5th subnetwork is composed of load impedance $Z_L = 50\,\Omega$ shunt-connected to the ground line. The *chain matrix* is given by

$$\overline{T}_L = \begin{bmatrix} 1 & 0 \\ 1/50 & 1 \end{bmatrix}. \qquad (2.37)$$

All entries of matrix Equation (2.37) are constant. Therefore the 5th system does not have zeros nor poles in any s-plane.

2.3.2 Transfer Function of the Filter Network

• **General Form of Transfer Function**

Equations (2.6)-(2.37) show all the necessary *chain* (ABCD) matrices of subsystems. Using this information, the transfer function of Figure 2.5 is obtained. From the relation given in Equation (2.2), the transfer function of the whole system is written as

$$H(s) = \frac{N(s)}{D(s)}. \qquad (2.38)$$

In Equation (2.38), $N(s)$ is the numerator of polynomial of $H(s)$, and $D(s)$ is the denominator polynomial of $H(s)$, and have the following expressions, respectively.

$$N(s) = 50 \cdot DC2 \cdot (DA3 \cdot DB3 \cdot DC3 \cdot DD3) \cdot (DA4 \cdot DC4 \cdot DD4) \qquad (2.39)$$

$$
\begin{aligned}
D(s) = (\ &50 \ NA4 \ DB3 \ DD3 \ DC4 \ DD4 \ DC3 \ DC2 \\
&+ 2500 \ NA4 \ DB3 \ DD3 \ DC4 \ DD4 \ DC3 \ NC2 \\
&+ NB4 \ DA4 \ DB3 \ DD3 \ DC4 \ DD4 \ DC3 \ DC2 \\
&+ 50 \ NB4 \ DA4 \ DB3 \ DD3 \ DC4 \ DD4 \ DC3 \ NC2\ ) \cdot NA3 \\
&+ 2500 \ NC4 \ DA3 \ DC3 \ DA4 \ DD4 \ NB3 \ DD3 \ NC2 \\
&+ 2500 \ NC4 \ DA3 \ DC3 \ DA4 \ DD4 \ ND3 \ DC2 \ DB3 \\
&+ 2500 \ NA4 \ DB3 \ DD3 \ DC4 \ DD4 \ NC3 \ DC2 \ DA3 \\
&+ 50 \ NC4 \ DA3 \ DC3 \ DA4 \ DD4 \ NB3 \ DD3 \ DC2 \\
&+ 50 \ NB4 \ DA4 \ DB3 \ DD3 \ DC4 \ DD4 \ NC3 \ DC2 \ DA3 \\
&+ ND4 \ DA3 \ DC3 \ DA4 \ DC4 \ NB3 \ DD3 \ DC2 \\
&+ 50 \ ND4 \ DA3 \ DC3 \ DA4 \ DC4 \ NB3 \ DD3 \ NC2 \\
&+ 50 \ ND4 \ DA3 \ DC3 \ DA4 \ DC4 \ ND3 \ DC2 \ DB3
\end{aligned} \qquad (2.40)
$$

As defined before, the notations, for example, are used to mean the following;

DB3 means denominator polynomial of entry B, or (1, 2), of subsystem $S3$.

ND4 means numerator polynomial of entry D, or (2, 2), of subsystem $S4$.

Equations (2.39) and (2.40) represent the numerator and denominator polynomials of the transfer function of the whole filter system, respectively. To find out actual polynomials of complex variable s, the values of $L's$ and $C's$ of the each subsystem should be used .

Depending on the existence of common terms in the numerator polynomial and the denominator polynomial, the relevant terms will be cancelled, so that $N(s)$ and $D(s)$ should be prime polynomials to determine the locations of transmission zeros.

The expression of Equations (2.39) and (2.40) hold for any network composed of five cascaded subsystems.

a) Numerator Polynomial

Equation (2.39) of the whole system of Figure 2.5 has eight variable terms, which are given as follows:

DC2 from Equation (2.10)

$$DC2 = L_2 s.$$

(2.41)

DA3 from Equation (2.15)

$$DA3 = (L_{31} L_{32} C_{33}L_{34} C_{34})\, s^4 + (L_{31} L_{32} + L_{34} L_{31} + L_{34}L_{32})\, C_{33}\, s^2 + L_{34}$$
$$\equiv a_{34}\, s^4 + a_{32}\, s^2 + a_{30},$$
$$\text{where } a_{34} = L_{31}L_{32}C_{33}L_{34}C_{34},$$
$$a_{32} = (L_{31} L_{32} + L_{34} L_{31} + L_{34}L_{32})\, C_{33}, \text{ and}$$
$$a_{30} = L_{34}.$$

(2.42)

DB3 from Equation (2.18)

$$DB3 = (L_{31}L_{32} C_{33}L_{34} C_{34})\, s^4 + (L_{31} L_{32} + L_{34} L_{31} + L_{34}L_{32})\, C_{33}\, s^2 + L_{34}$$
$$\equiv a_{34}\, s^4 + a_{32}\, s^2 + a_{30},$$
$$\text{Where, } a_{34} = L_{31} L_{32} C_{33}L_{34} C_{34},$$
$$a_{32} = (L_{31} L_{32} + L_{34} L_{31} + L_{34}L_{32})\, C_{33},$$
$$a_{30} = L_{34}.$$

(2.43)

DC3 from Equation (2.21)

$$DC3 = (L_{31}L_{32} C_{33}L_{34} C_{34})\, s^5 + (L_{31} L_{32} + L_{34} L_{31} + L_{34} L_{32})\, C_{33}\, s^3 + L_{34}\, s$$
$$\equiv s \cdot (a_{34}\, s^4 + a_{32}\, s^2 + a_{30}),$$
$$\text{where } a_{34} = L_{31} L_{32} C_{33}L_{34} C_{34},$$
$$a_{32} = (L_{31} L_{32} + L_{34} L_{31} + L_{34}L_{32})\, C_{33},$$
$$a_{30} = L_{34}.$$

(2.44)

DD3 from Equation (2.24)

$$DD3 = (L_{31}L_{32} L_{34} C_{34})C_{33} s^4 + (L_{31} L_{32} + L_{34} L_{31} + L_{34} L_{32})C_{33} s^2 + L_{34}$$
$$= (a_{34}s^4 + a_{32}s^2 + a_{30}), \tag{2.45}$$

where $a_{34} = L_{31} L_{32} C_{33}L_{34} C_{34}$,

$a_{32} = (L_{31} L_{32} + L_{34} L_{31} + L_{34}L_{32}) C_{33}$,

$a_{30} = L_{34}$.

DA4 from Equation (2.29.b)

$$DA4 = L_{42}. \tag{2.46}$$

DC4 from Equation (2.34)

$$DC4 = s \cdot L_{41} \cdot L_{42}. \tag{2.47}$$

DD4 from Equation (2.36.b)

$$DD4 = L_{41}. \tag{2.48}$$

Substituting Equations (2.41)-(2.48) into Equation (2.39), a numerator polynomial of the following form is obtained.

$$N(s) = 50 \cdot DC2 \cdot DA3 \cdot DB3 \cdot DC3 \cdot DD3 \cdot DA4 \cdot DC4 \cdot DD4$$

$$= 50 \cdot L_2 s \cdot (a_{34} s^4 + a_{32} s^2 + a_{30}) \cdot (a_{34} s^4 + a_{32} s^2 + a_{30})$$
$$\cdot s (a_{34} s^4 + a_{32} s^2 + a_{30}) \cdot (a_{34} s^4 + a_{32} s^2 + a_{30})$$
$$\cdot L_{42} \cdot (s L_{41} L_{42}) \cdot L_{41}.$$

$$\tag{2.49}$$

Equation (2.49) is rewritten as

$$N(s) = 50 \cdot L_2 \cdot (L_{41}L_{42})^2 \cdot s^3 \cdot (a_{34} s^4 + a_{32} s^2 + a_{30})^4. \tag{2.50}$$

Polynomial $N(s)$ given in Equation (2.50) is an odd polynomial. A necessary condition for the Hurwitz polynomial requires that all coefficients of polynomial $N(s)$ are strictly positive, and without any missing terms in $N(s)$. Since all of the even-degree terms of Equation (2.50) are missing, $N(s)$ does not satisfy the necessary condition. Therefore, $N(s)$ is not a Hurwitz polynomial [17]. This means that, not all of the roots of equation $N(s)=0$ are in the *left-half plane* (LHP). Some roots may be on the $j\omega$-axis and/or some roots may be in the *right-half plane* (RHP). By Equations (2.50) itself, there exist a 3rd degree static zeros at the origin due to the term s^3, and 16th degree dynamic zeros due to the term $(a_{34}\, s^4 + a_{32}\, s^2 + a_{30})^4$.

Since the possible common term has not yet been cancelled, the expression for $N(s)$ is not in the canonical form. To obtain TZ's, the canonical form is required. Therefore, it is not reasonable to use Equation (2.50) to find transmission zeros of the filter network.

When common-term pole-zero cancellation is accomplished, the expression Equation (2.50) reduces to canonical form. To obtain the canonical form, a MATLAB program is employed [18].

This canonical expression will be shown later, with the use of the MATLAB program.

b) Denominator Polynomial

The denominator polynomial Equation (2.40) of the whole system of Figure 2.5 is expressed again. Each term is given as follows. The *transmission poles* (reflection zeros) are the roots of the denominator polynomial.

From Equation (2.40),

$$
\begin{aligned}
D(s) \ = (\ \ & 50 \ \text{NA4 DB3 DD3 DC4 DD4 DC3 DC2} \\
+ \ & 2500 \ \text{NA4 DB3 DD3 DC4 DD4 DC3 NC2} \\
+ \ & \text{NB4 DA4 DB3 DD3 DC4 DD4 DC3 DC2} \\
+ \ & 50 \ \text{NB4 DA4 DB3 DD3 DC4 DD4 DC3 NC2} \ ) \ \text{NA3} \\
+ \ & 2500 \ \text{NC4 DA3 DC3 DA4 DD4 NB3 DD3 NC2} \\
+ \ & 2500 \ \text{NC4 DA3 DC3 DA4 DD4 ND3 DC2 DB3} \\
+ \ & 2500 \ \text{NA4 DB3 DD3 DC4 DD4 NC3 DC2 DA3} \\
+ \ & 50 \ \text{NC4 DA3 DC3 DA4 DD4 NB3 DD3 DC2} \\
+ \ & 50 \ \text{NB4 DA4 DB3 DD3 DC4 DD4 NC3 DC2 DA3} \\
+ \ & \text{ND4 DA3 DC3 DA4 DC4 NB3 DD3 DC2} \\
+ \ & 50 \ \text{ND4 DA3 DC3 DA4 DC4 NB3 DD3 NC2} \\
+ \ & 50 \ \text{ND4 DA3 DC3 DA4 DC4 ND3 DC2 DB3}
\end{aligned}
$$
(2.40)

Substituting Equations (2.41)-(2.48) into Equation (2.40), a non-canonical form of denominator polynomial is obtained. Since this polynomial is not used to obtain TZ locations, it is not shown here.

Therefore, the next step is to find canonical forms of numerator and denominator polynomials.

- **Canonical Form of Transfer Function**

a) Canonical Numerator Polynomial

Numerator polynomial Equation (2.39) expressed in terms of Laplace impedances, and denominator polynomial Equation (2.40) expressed in terms of Laplace impedances should be compared to find out the possible common terms in order to accomplish the pole-zero cancellations. After the cancellation, the remaining zeros and poles will be considered. To obtain the canonical forms of numerator and denominator polynomials, the MATLAB program is used.

From Equation (2.3), the chain matrix of filter network is computed by the multiplications of five chain matrices ($\overline{T}_1, \overline{T}_2, \overline{T}_3, \overline{T}_4$, and $\overline{T}_5$.). Each chain matrix has a size of 2×2. The final chain matrix is again of the size 2×2. The entry (1, 1) is noted as $\overline{T}(1,1)$.

The inverse of the matrix is the transfer function of the whole system. The transfer function is a rational polynomial. From the prime polynomials the pole and zeros are found.

A MATLAB program to calculate the canonical form of the numerator polynomial is attached as appendix A. The polynomial is obtained as the 5th degree polynomial.

The polynomial is expressed as

$$
\begin{aligned}
N(s) = {}& 50 \cdot L_2\, L_{41} L_{42}\ s \\
& [\ L_{31} L_{32} C_{33} L_{34} C_{34}\ s^4 + (\ L_{31}\ L_{32} + L_{34}\ L_{31} + L_{34} L_{32})\ C_{33}\ s^2 + L_{34} \\
\equiv {}& k \cdot s \cdot [a_{34} s^4 + a_{32} s^2 + a_{30}],
\end{aligned}
$$

(2.51)

where,

$$k\ =\ 50\ L_2\ L_{41} L_{42},$$
$$a_{34}\ =\ L_{31} L_{32} C_{33} L_{34} C_{34},$$
$$a_{32}\ =\ (\ L_{31}\ L_{32} + L_{34}\ L_{31} + L_{34} L_{32})\ C_{33},\ \text{and}$$
$$a_{30}\ =\ L_{34}.$$

The 4th degree polynomial $[a_{34} s^4 + a_{32} s^2 + a_{30}]$ given in Equation (2.51) is an even polynomial that produces a dynamic quadruplet of complex zeros.

The quadruplet is only due to the cross-coupled subsystem. Worth of emphasizing, this will be discussed in Section 2.3.3.

Compared to the non-canonical form of Equation (2.50), the orders of static and dynamic zeros have been reduced. It is Equation (2.51), not (2.50), that should be used to locate TZ's.

A necessary condition for the Hurwitz polynomial requires that all coefficients of polynomial are strictly positive, with no missing terms. Since all of the odd-degree terms of are missing in the polynomial $[a_{34}s^4 + a_{32}s^2 + a_{30}]$, Equation (2.51) does not satisfy the necessary condition. Therefore, Equation (2.51) is not a Hurwitz polynomial.

This means that, not all of the roots of equation $N(s) = 0$ in the LHP. Some roots may be on the $j\omega$-axis and/or some roots may be in the RHP.

Solving the *transmission zero characteristic equation* (TZCE), $N(s) = 0$, there exist a single static zero at the origin due to the term s. Other than that there are four dynamic zeros in LHP, on the $j\omega$-axis and/or in the RHP, due to the 4th degree even polynomial, $[a_{34}s^4 + a_{32}s^2 + a_{30}]$.

Given a transfer function, the total number of zeros is equal to the total number of poles, if the entire s-plane domain is taken into account. If some zeros or some poles are not located in the finite region of the s-plane, they are located at infinity.

The degree of the denominator polynomial is eight. The degree of the numerator polynomial is five. Since the degree of numerator polynomial is five in the finite s-plane, there should be three zeros in the infinite locations. The five finite zeros are considered. Equation (2.51) shows that three pole-zero pairs were cancelled at the origin, leaving only one zero.

Before pole-zero cancellations, the numerator polynomial includes the term of 4th degree polynomial to the 4th power, $(a_{34} s^4 + a_{32} s^2 + a_{30})^4$.

But after cancellation, the numerator polynomial includes only the term of 4th degree polynomial to the 1th power, ($a_{34} s^4 + a_{32} s^2 + a_{30}$).

From Equation (2.51), equating $N(s) = 0$ to find roots, the expression is obtained as

$$f(s) = s \cdot [a_{34}s^4 + a_{32}s^2 + a_{30}] \qquad (2.52)$$

$$\equiv f_1(s) \cdot f_2(s) = 0.$$

The polynomial Equation (2.52) is the 5th degree polynomial, where the three coefficients (a_{34}, a_{32}, and a_{30}) are real positive numbers calculated from the $L's$ and $C's$ of the whole filter network of Figure 2.5. These coefficients are given in Equation (2.51). Each of the factored polynomials of Equation (2.52) is expressed as follows.

$$f_1(s) = s \; , \qquad (2.53.a)$$

$$f_2(s) = a_{34} s^4 + a_{32} s^2 + a_{30}. \qquad (2.53.b)$$

The Equation (2.52) has five roots. One is obtained from $f_1(s) = s$ of Equation (2.53.a), and the other four from $f_2(s) = a_{34} s^4 + a_{32} s^2 + a_{30}$ of (2.53.b). These all constitute the total five solutions of the filter system of Figure 2.5.

b) Canonical Denominator Polynomial

The MATLAB program to calculate denominator polynomial of the transfer function

is attached as appendix A. The results obtained from the program show that the polynomial is an 8th degree polynomial with eight terms,

$$D(s) = a_8 s^8 + a_7 s^7 + a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_0 \ . \qquad (2.54)$$

Enumerated coefficients of Equation (2.54) are expressed as follows:

$a_8 =$ (2500 L2 C2 L31 L32 L34 C34 L43 L41 C41 L42 C42
  + 2500 L2 L31 L32 C33 L34 C34 L43 L41 C41 L42 C42   (2.55.a)
  + 2500 L41 L2 C2 L31 L32 C33 L34 C34 L43 L42 C42)

$a_7 =$ (50 L41 L2 L31 L32 C33 L34 C34 L43 L42 C42
  + 50 L42 L2 L31 L32 C33 L34 C34 L43 L41 C41
  + 50 L43 L41 L42 L2 C2 L31 L32 C33 L34 C34   (2.55.b)
  + 50 L42 L2 C2 L31 L32 L34 C34 L43 L41 C41
  + 50 L2 L31 L32 L34 C34 L43 L41 C41 L42 C42)

$a_6 =$ (L42 L2 L31 L32 L34 C34 L43 L41 C41
  + 2500 L2 C2 L31 L32 L34 C34 L42 L41 C41
  + 2500 L2 C2 L31 L32 L43 L41 C41 L42 C42
  + 2500 L41 L2 C2 L31 L32 C33 L34 C34 L42
  + 2500 L41 L2 C2 L31 L32 C33 L34 C34 L43
  + 2500 L41 L31 L32 C33 L34 C34 L43 L42 C42
  + 2500 L41 L2 C2 L34 L31 C33 L43 L42 C42
  + 2500 L41 L2 L34 C34 L31 C33 L43 L42 C42
  + 2500 L2 L31 L32 C33 L34 C34 L41 L42 C42
  + 2500 L41 L2 L34 C34 L32 C33 L43 L42 C42
  + 2500 L31 L32 L34 C34 L43 L41 C41 L42 C42
  + L43 L41 L42 L2 L31 L32 C33 L34 C34   (2.55.c)
  + 2500 L2 C2 L34 L31 L43 L41 C41 L42 C42
  + 2500 L2 L31 L32 C33 L34 C34 L43 L41 C41
  + 2500 L2 L31 L32 C33 L34 C34 L43 L42 C42
  + 2500 L2 L31 L32 C33 L43 L41 C41 L42 C42
  + 2500 L41 L2 C2 L31 L34 C34 L43 L42 C42
  + 2500 L2 L31 L32 C33 L34 C34 L42 L41 C41
  + 2500 L41 L2 C2 L31 L32 C33 L43 L42 C42
  + 2500 L2 C2 L32 L34 L43 L41 C41 L42 C42
  + 2500 L2 L32 C33 L34 L43 L41 C41 L42 C42
  + 2500 L41 L2 C2 L32 C33 L34 L43 L42 C42
  + 2500 L2 L32 L34 C34 L43 L41 C41 L42 C42
  + 2500 L2 L34 L31 C33 L43 L41 C41 L42 C42

+ 2500 L2 C2 L31 L32 L34 C34 L41 L42 C42
+ 2500 L2 C2 L31 L32 L34 C34 L43 L41 C41
+ 2500 L2 C2 L31 L32 L34 C34 L43 L42 C42)

a_5 = (50 L42 L2 L31 L32 C33 L43 L41 C41
+ 50 L43 L41 L42 L2 C2 L34 L31 C33
+ 50 L2 L31 L32 L34 C34 L42 L41 C41
+ 50 L2 L31 L32 L34 C34 L41 L42 C42
+ 50 L41 L2 L31 L34 C34 L43 L42 C42
+ 100 L41 L2 L42 L31 L32 C33 L34 C34
+ 50 L41 L2 L34 L32 C33 L43 L42 C42
+ 50 L2 L34 L32 L43 L41 C41 L42 C42
+ 50 L43 L41 L42 L2 C2 L31 L32 C33
+ 50 L2 L34 L31 L43 L41 C41 L42 C42
+ 50 L42 L2 L31 L32 C33 L34 C34 L43
+ 50 L41 L2 L31 L32 C33 L34 C34 L43
+ 50 L43 L41 L42 L31 L32 C33 L34 C34
+ 50 L42 L2 C2 L31 L32 L43 L41 C41
+ 50 L42 L2 C2 L34 L32 L43 L41 C41
+ 50 L43 L41 L42 L2 C2 L34 L32 C33   (2.55.d)
+ 50 L41 L2 L34 L31 C33 L43 L42 C42
+ 50 L42 L2 L34 L31 C33 L43 L41 C41
+ 50 L42 L2 C2 L34 L31 L43 L41 C41
+ 50 L43 L41 L42 L2 C2 L31 L34 C34
+ 50 L42 L31 L32 L34 C34 L43 L41C41
+ 50 L42 L2 C2 L31 L32 L34 C34 L41
+ 50 L42 L2 C2 L31 L32 L34 C34 L43
+ 50 L42 L2 L34 L32 C33 L43 L41 C41
+ 50 L2 L31 L32 L43 L41 C41 L42 C42
+ 50 L42 L2 L32 L34 C34 L43 L41 C41
+ 50 L43 L41 L42 L2 L34 C34 L31 C33
+ 50 L43 L41 L42 L2 L34 C34 L32 C33
+ 50 L2 L31 L32 L34 C34 L43 L41 C41
+ 50 L2 L31 L32 L34 C34 L43 L42 C42
+ 50 L41 L2 L31 L32 C33 L43 L42 C42)

a_4 = (L42 L2 L34 L31 L43 L41 C41
+ 2500 L31 L32 L34 C34 L41 L42 C42 + 2500 L31 L32 L34 C34 L43 L41 C41
+ 2500 L2 C2 L31 L32 L43 L41 C41 + 2500 L2 C2 L34 L31 L43 L42 C42
+ 2500 L2 C2 L31 L32 L41 L42 C42 + 2500 L31 L32 L34 C34 L42 L41 C41
+ 2500 L31 L32 L34 C34 L43 L42 C42 + 2500 L41 L31 L34 C34 L43 L42 C42
+ 2500 L2 C2 L31 L32 L34 C34 L41 + 2500 L2 L31 L32 C33 L43 L41 C41
+ 2500 L2 L31 L32 C33 L43 L42 C42 + 2500 L41 L2 L34 C34 L32 C33 L42
+ 2500 L41 L2 L34 C34 L32 C33 L43 + 2500 L2 L31 L32 C33 L42 L41 C41

+ 2500 L41 L2 C2 L31 L34 C34 L43 + 2500 L41 L2 L34 C34 L31 C33 L42
+ 2500 L41 L31 L32 C33 L43 L42 C42 + 2500 L41 L2 L34 C34 L31 C33 L43
+ 2500 L2 L31 L32 C33 L41 L42 C42 + 2500 L2 L34 L32 C33 L42 L41 C41
+ 2500 L2 L34 L32 C33 L41 L42 C42 + 2500 L31 L32 L43 L41 C41 L42 C42
+ 2500 L2 L31 L32 C33 L34 C34 L42 + 2500 L2 L32 L34 C34 L41 L42 C42
+ 2500 L34 L32 L43 L41 C41 L42 C42 + 2500 L34 L31 L43 L41 C41 L42 C42
+ 2500 L2 L32 L43 L41 C41 L42 C42 + L43 L41 L42 L2 L34 L31 C33
+ L43 L41 L42 L2 L31 L32 C33 + 2500 L2 C2 L31 L32 L34 C34 L42
+ L42 L2 L31 L32 L34 C34 L41 + L42 L2 L31 L32 L34 C34 L43   (2.55.e)
+ L42 L2 L31 L32 L43 L41 C41 + 2500 L2 L34 L32 C33 L43 L41 C41
+ 2500 L2 L34 L32 C33 L43 L42 C42 + 2500 L2 C2 L31 L32 L34 C34 L43
+ 2500 L2 C2 L34 L32 L43 L41 C41 + 2500 L2 C2 L31 L32 L43 L42 C42
+ 2500 L2 C2 L34 L32 L42 L41 C41 + 2500 L2 L31 L32 C33 L34 C34 L41
+ L43 L41 L42 L2 L34 L32 C33 + L43 L41 L42 L2 L31 L34 C34
+ 2500 L2 L32 L34 C34 L43 L41 C41 + 2500 L2 L32 L34 C34 L43 L42 C42
+ L42 L2 L34 L32 L43 L41 C41 + 2500 L41 L2 L32 C33 L43 L42 C42
+ 2500 L2 C2 L34 L32 L43 L42 C42 + 2500 L2 C2 L34 L31 L42 L41 C41
+ 2500 L2 C2 L34 L31 L41 L42 C42 + 2500 L2 C2 L34 L31 L43 L41 C41
+ 2500 L41 L34 L31 C33 L43 L42 C42 + 2500 L41 L34 L32 C33 L43 L42 C42
+ 2500 L2 L34 L31 C33 L43 L41 C41 + 2500 L2 L34 L31 C33 L43 L42 C42
+ 2500 L41 L2 L34 C34 L43 L42 C42 + 2500 L41 L2 C2 L34 L31 C33 L42
+ 2500 L41 L2 C2 L34 L31 C33 L43 + 2500 L41 L2 C2 L31 L32 C33 L42
+ 2500 L41 L2 C2 L31 L32 C33 L43 + 2500 L41 L2 C2 L34 L43 L42 C42
+ 2500 L41 L2 C2 L34 L32 C33 L42 + 2500 L2 L34 L43 L41 C41 L42 C42
+ 2500 L2 L32 L34 C34 L42 L41 C41 + 2500 L2 L31 L32 C33 L34 C34 L43
+ 2500 L41 L2 L31 C33 L43 L42 C42 + 2500 L41 L2 C2 L34 L32 C33 L43
+ 2500 L41 L2 C2 L31 L34 C34 L42 + 2500 L41 L2 C2 L31 L43 L42 C42
+ 2500 L2 C2 L31 L32 L42 L41 C41 + 2500 L2 C2 L34 L32 L41 L42 C42
+ 2500 L2 L34 L31 C33 L42 L41 C41 + 2500 L2 L34 L31 C33 L41 L42 C42
+ 2500 L41 L31 L32 C33 L34 C34 L42 + 2500 L41 L31 L32 C33 L34 C34 L43)

$a_3 =$ (50 L41 L2 L34 L31 C33 L43
+ 50 L41 L2 L31 L34 C34 L42 + 100 L41 L2 L34 L32 C33 L42
+ 50 L43 L41 L42 L31 L32 C33 + 50 L42 L2 L34 L32 C33 L43
+ 50 L42 L2 L32 L34 C34 L41 + 50 L41 L2 L34 L43 L42 C42
+ 50 L2 L34 L31 L41 L42 C42 + 50 L2 L34 L31 L43 L41 C41
+ 50 L42 L31 L32 L43 L41 C41 + 50 L42 L34 L32 L43 L41 C41
+ 50 L2 L34 L31 L42 L41 C41 + 50 L42 L2 C2 L34 L31 L43
+ 50 L42 L2 L34 L31 C33 L43 + 50 L2 L34 L32 L43 L41 C41
+ 50 L2 L34 L32 L43 L42 C42 + 50 L43 L41 L42 L2 L31 C33
+ 50 L42 L34 L31 L43 L41 C41 + 50 L42 L2 L32 L34 C34 L43
+ 50 L42 L2 C2 L34 L31 L41 + 50 L2 L34 L31 L43 L42 C42
+ 50 L42 L2 L32 L43 L41 C41 + 50 L42 L2 C2 L34 L32 L41
+ 50 L43 L41 L42 L2 L34 C34 + 50 L2 L31 L32 L34 C34 L41
+ 50 L2 L31 L32 L34 C34 L42 + 50 L43 L41 L42 L2 L32 C33   (2.55.f)
+ 50 L43 L41 L42 L34 L32 C33 + 50 L42 L2 L34 L43 L41 C41

$+ 50 \ L42 \ L2 \ C2 \ L34 \ L32 \ L43 + 50 \ L43 \ L41 \ L42 \ L34 \ L31 \ C33$
$+ 50 \ L42 \ L2 \ C2 \ L31 \ L32 \ L41 + 50 \ L42 \ L2 \ C2 \ L31 \ L32 \ L43$
$+ 50 \ L2 \ L34 \ L32 \ L42 \ L41 \ C41 + 50 \ L2 \ L31 \ L32 \ L34 \ C34 \ L43$
$+ 50 \ L41 \ L2 \ L31 \ L32 \ C33 \ L43 + 50 \ L2 \ L34 \ L32 \ L41 \ L42 \ C42$
$+ 50 \ L2 \ L31 \ L32 \ L42 \ L41 \ C41 + 100 \ L41 \ L2 \ L31 \ L32 \ C33 \ L42$
$+ 50 \ L2 \ L31 \ L32 \ L41 \ L42 \ C42 + 50 \ L2 \ L31 \ L32 \ L43 \ L41 \ C41$
$+ 50 \ L2 \ L31 \ L32 \ L43 \ L42 \ C42 + 50 \ L41 \ L2 \ L31 \ L34 \ C34 \ L43$
$+ 100 \ L41 \ L2 \ L34 \ L31 \ C33 \ L42 + 50 \ L41 \ L2 \ L34 \ L32 \ C33 \ L43$
$+ 50 \ L41 \ L2 \ L31 \ L43 \ L42 \ C42 + 50 \ L42 \ L31 \ L32 \ L34 \ C34 \ L41$
$+ 50 \ L42 \ L31 \ L32 \ L34 \ C34 \ L43 + 50 \ L43 \ L41 \ L42 \ L31 \ L34 \ C34$
$+ 50 \ L43 \ L41 \ L42 \ L2 \ C2 \ L34 + 50 \ L43 \ L41 \ L42 \ L2 \ C2 \ L31$
$+ 50 \ L42 \ L2 \ L31 \ L32 \ C33 \ L43)$

$a_2 \ = (\ 2500 \ L41 \ L2 \ C2 \ L31 \ L42$
$+ 2500 \ L41 \ L31 \ L34 \ C34 \ L43 + 2500 \ L41 \ L31 \ L32 \ C33 \ L42$
$+ 2500 \ L41 \ L31 \ L32 \ C33 \ L43 + 2500 \ L41 \ L2 \ L43 \ L42 \ C42$
$+ 2500 \ L41 \ L2 \ L34 \ C34 \ L42 + 2500 \ L41 \ L2 \ C2 \ L31 \ L43$
$+ 2500 \ L41 \ L31 \ L43 \ L42 \ C42 + 2500 \ L34 \ L32 \ L42 \ L41 \ C41$
$+ 2500 \ L2 \ C2 \ L34 \ L31 \ L43 + 2500 \ L31 \ L32 \ L34 \ C34 \ L42$
$+ 2500 \ L41 \ L34 \ L31 \ C33 \ L42 + 2500 \ L41 \ L34 \ L31 \ C33 \ L43$
$+ 2500 \ L41 \ L2 \ L34 \ C34 \ L43 + 2500 \ L2 \ L34 \ L32 \ C33 \ L41$
$+ 2500 \ L2 \ L34 \ L32 \ C33 \ L43 + 2500 \ L41 \ L2 \ L32 \ C33 \ L43$
$+ 2500 \ L2 \ L32 \ L34 \ C34 \ L42 + 2500 \ L2 \ L32 \ L34 \ C34 \ L41$
$+ 2500 \ L2 \ L32 \ L34 \ C34 \ L43 + 2500 \ L2 \ L34 \ L31 \ C33 \ L42$
$+ 2500 \ L2 \ L34 \ L31 \ C33 \ L41 + 2500 \ L41 \ L34 \ L32 \ C33 \ L43$
$+ 2500 \ L2 \ C2 \ L34 \ L31 \ L41 + 2500 \ L2 \ L34 \ L32 \ C33 \ L42$
$+ 2500 \ L34 \ L32 \ L41 \ L42 \ C42 + 2500 \ L2 \ L34 \ L31 \ C33 \ L43$
$+ 2500 \ L31 \ L32 \ L42 \ L41 \ C41 + 2500 \ L31 \ L32 \ L41 \ L42 \ C42$
$+ 2500 \ L31 \ L32 \ L43 \ L41 \ C41 + L43 \ L41 \ L42 \ L2 \ L31$ $(2.55.g)$
$+ 2500 \ L34 \ L31 \ L43 \ L41 \ C41 + 2500 \ L34 \ L31 \ L43 \ L42 \ C42$
$+ 2500 \ L34 \ L32 \ L43 \ L42 \ C42 + 2500 \ L34 \ L31 \ L42 \ L41 \ C41$
$+ 2500 \ L34 \ L32 \ L43 \ L41 \ C41 + 2500 \ L34 \ L31 \ L41 \ L42 \ C42$
$+ L42 \ L2 \ L31 \ L32 \ L43 + L42 \ L2 \ L34 \ L32 \ L41 + L42 \ L2 \ L34 \ L32 \ L43$
$+ L43 \ L41 \ L42 \ L2 \ L34 + 2500 \ L2 \ L34 \ L42 \ L41 \ C41 + L42 \ L2 \ L34 \ L31 \ L41$
$+ L42 \ L2 \ L34 \ L31 \ L43 + 2500 \ L2 \ L32 \ L43 \ L41 \ C41$
$+ 2500 \ L2 \ C2 \ L31 \ L32 \ L42 + 2500 \ L2 \ C2 \ L31 \ L32 \ L41$
$+ 2500 \ L2 \ C2 \ L31 \ L32 \ L43 + 2500 \ L2 \ C2 \ L34 \ L32 \ L42$
$+ 2500 \ L31 \ L32 \ L34 \ C34 \ L43 + 2500 \ L2 \ L31 \ L32 \ C33 \ L42$
$+ 2500 \ L2 \ L31 \ L32 \ C33 \ L41 + 2500 \ L31 \ L32 \ L34 \ C34 \ L41$
$+ 2500 \ L2 \ L31 \ L32 \ C33 \ L43 + 2500 \ L2 \ C2 \ L34 \ L32 \ L41$
$+ L42 \ L2 \ L31 \ L32 \ L41 + 2500 \ L2 \ L32 \ L43 \ L42 \ C42$
$+ 2500 \ L31 \ L32 \ L43 \ L42 \ C42 + 2500 \ L41 \ L31 \ L34 \ C34 \ L42$
$+ 2500 \ L41 \ L34 \ L43 \ L42 \ C42 + 2500 \ L41 \ L34 \ L32 \ C33 \ L42$
$+ 2500 \ L41 \ L2 \ L31 \ C33 \ L42 + 2500 \ L41 \ L2 \ L31 \ C33 \ L43$
$+ 2500 \ L41 \ L2 \ L32 \ C33 \ L42 + 2500 \ L41 \ L2 \ C2 \ L34 \ L43$
$+ 2500 \ L41 \ L2 \ C2 \ L34 \ L42 + 2500 \ L2 \ L34 \ L43 \ L41 \ C41$

$$+ 2500 \ L2 \ L34 \ L43 \ L42 \ C42 + 2500 \ L2 \ L32 \ L42 \ L41 \ C41$$
$$+ 2500 \ L2 \ L32 \ L41 \ L42 \ C42 + 2500 \ L2 \ C2 \ L34 \ L32 \ L43$$
$$+ 2500 \ L2 \ C2 \ L34 \ L31 \ L42 \quad + 2500 \ L2 \ L34 \ L41 \ L42 \ C42)$$

$$
\begin{aligned}
a_1 = \ (\ & 50 \ L42 \ L34 \ L32 \ L43 + 100 \ L41 \ L2 \ L42 \ L34 + 50 \ L43 \ L41 \ L42 \ L31 \\
& + 50 \ L42 \ L31 \ L32 \ L41 + 50 \ L2 \ L31 \ L32 \ L42 + 50 \ L41 \ L2 \ L31 \ L42 \\
& + 50 \ L2 \ L34 \ L32 \ L42 + 50 \ L42 \ L34 \ L31 \ L43 + 50 \ L41 \ L2 \ L31 \ L43 \\
& + 50 \ L2 \ L34 \ L31 \ L42 + 50 \ L2 \ L31 \ L32 \ L43 + 50 \ L42 \ L34 \ L32 \ L41 \\
& + 50 \ L2 \ L31 \ L32 \ L41 + 50 \ L2 \ L34 \ L32 \ L43 + 50 \ L42 \ L2 \ L34 \ L43 \\
& + 50 \ L2 \ L34 \ L32 \ L41 + 50 \ L42 \ L31 \ L32 \ L43 + 50 \ L42 \ L2 \ L32 \ L43 \\
& + 50 \ L43 \ L41 \ L42 \ L2 + 50 \ L41 \ L2 \ L34 \ L43 + 50 \ L42 \ L2 \ L32 \ L41 \\
& + 50 \ L2 \ L34 \ L31 \ L41 + 50 \ L42 \ L34 \ L31 \ L41 + 50 \ L2 \ L34 \ L31 \ L43 \\
& + 50 \ L43 \ L41 \ L42 \ L34)
\end{aligned}
$$

(2.55.h)

$$
\begin{aligned}
a_0 = \ & 2500 \ L41 \ L34 \ L43 + 2500 \ L34 \ L31 \ L42 \ + \ 2500 \ L2 \ L34 \ L42 \\
& + 2500 \ L31 \ L32 \ L41 + 2500 \ L2 \ L34 \ L41 \ + 2500 \ L2 \ L34 \ L43 \\
& + 2500 \ L2 \ L32 \ L42 + 2500 \ L2 \ L32 \ L41 + 2500 \ L2 \ L32 \ L43 \\
& + 2500 \ L34 \ L32 \ L41 + 2500 \ L41 \ L34 \ L42 + 2500 \ L41 \ L2 \ L42 \\
& + 2500 \ L41 \ L2 \ L43 + 2500 \ L34 \ L31 \ L41 + 2500 \ L34 \ L31 \ L43 \\
& + 2500 \ L34 \ L32 \ L43 \ + 2500 \ L31 \ L32 \ L42 + 2500 \ L41 \ L31 \ L42 \\
& + 2500 \ L31 \ L32 \ L43 + 2500 \ L41 \ L31 \ L43 \ + 2500 \ L34 \ L32 \ L42
\end{aligned}
$$

(2.55.i)

In Equation (2.55.a-i), all of the coefficients ($a_8, a_7, a_6, a_5, a_4, a_3, a_2,$ and a_0) are positive numbers, since they are composed of element values of inductors and capacitors comprising the filter network.

2.3.3 Transmission Zeros of the Filter Network

To find out the complex *transmission zeros* (TZ's) of the filter network, the 5th degree polynomial equation is to be solved by using the equality, $s = \sigma + j\omega$.

In the canonical form of numerator polynomial given in Equation (2.52), each of the factored polynomials is expressed again as follows.

$$f_1(s) = s \;, \qquad\qquad (2.53.a)$$

$$f_2(s) \;=\; a_{34}\, s^4 + a_{32}\, s^2 + a_{30} \;. \qquad\qquad (2.53.b)$$

- **Monomial Equation** $f_1(s) = s = 0$

Equation (2.53.a) gives the monomial equation,

$$f_1(s) = s = 0 \;. \qquad\qquad (2.56)$$

Equation (2.56) represents a single stationary transmission zero at the origin as shown in Figure 2.6.

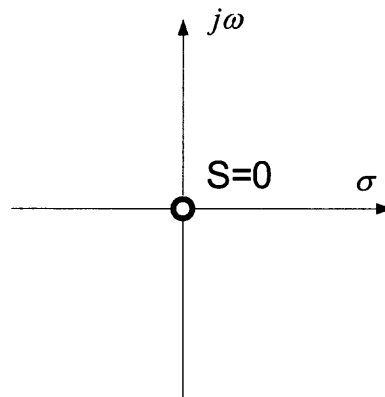**Figure 2.6** A single stationary zero at origin.

- **Polynomial Equation** $f_2(s) = a_{34}\, s^4 + a_{32}\, s^2 + a_{30}$

The polynomial $f_2(s)$ has three coefficients ($a_{34}, a_{32},$ & a_{30}). The first subscript i of coefficients a_{ij} indicates the i-th subsystem.

Therefore, the polynomial $f_2(s) = a_{34}\, s^4 + a_{32}\, s^2 + a_{30}$ comes only from the 3rd

subsystem, and describes the 3rd subsystem *S3* of the Figure 2.3. The 3rd subsystem is a *bridged-T circuit*, which gives a cross-coupling between resonator no. 1 and resonator no. 3. It will be shown that this circuit generate a *quadruple complex zeros* for the whole filter network. The 4th degree polynomial of *bridged-T circuit* has the following equation,

$$f_2(s) = a_{34} s^4 + a_{32} s^2 + a_{30} = 0.$$

(2.57)

The 4th degree polynomial of Equation (2.57) has four solutions. Depending on the values of three coefficients (a_{34}, a_{32} & a_{30}) three different *(mutually exclusive)* cases are possible. Each of different (mutually exclusive) case, there are four solutions. The three cases of coefficients restrictions are noted as;

$$(a_{32}^2 > 4 a_{34} a_{30}), \quad \text{or}$$

(2.58)

$$(a_{32}^2 = 4 a_{34} a_{30}), \quad \text{or}$$

(2.59)

$$(a_{32}^2 < 4 a_{34} a_{30}).$$

(2.60)

The three different cases given in Equations (2.58), (2.59), and (2.60) are considered.

a) Case 1: Coefficients of $f_2(s)$ with $a_{32}^2 > 4 a_{34} a_{30}$

For the condition of $a_{32}^2 > 4 a_{34} a_{30}$ given in Equation (2.58), four solutions from two quadratic equations,

$$s^2 = \frac{-a_{32} + \sqrt{a_{32}^2 - 4a_{34}a_{30}}}{2a_{34}} \tag{2.61}$$

and

$$s^2 = \frac{-a_{32} - \sqrt{a_{32}^2 - 4a_{34}a_{30}}}{2a_{34}} \tag{2.62}$$

are obtained. The Equation (2.61) produces two transmission zeros. The Equation (2.62) produces again two transmission zeros.

1. Two Solutions of Equation (2.61)

With $s = \sigma + j\omega$ into (2.61), using $s^2 = (\sigma + j\omega)^2 = \sigma^2 - \omega^2 + j2\sigma\omega$, the relation,

$$\sigma^2 - \omega^2 + j2\sigma\omega \equiv \frac{-a_{32} + \sqrt{a_{32}^2 - 4a_{34}a_{30}}}{2a_{34}} \tag{2.63}$$

is satisfied.

Solving Equation (2.63) by equating real and imaginary parts, respectively, one pair of solutions

$$\text{i)} \quad \sigma = 0 \ , \quad \omega = \sqrt{\frac{a_{32} - \sqrt{a_{32}^2 - 4a_{34}a_{30}}}{2a_{34}}} \tag{2.64.a}$$

and

$$\text{ii)} \quad \sigma = 0 \ , \quad \omega = -\sqrt{\frac{a_{32} - \sqrt{a_{32}^2 - 4a_{34}a_{30}}}{2a_{34}}} \tag{2.64.b}$$

or, the other pair of solutions,

$$\text{i) } \omega = 0 \ , \ \sigma = \sqrt{\frac{-a_{32} + \sqrt{a_{32}^2 - 4 a_{34} a_{30}}}{2 a_{34}}} \qquad (2.65.\text{a})$$

and

$$\text{ii) } \omega = 0 \ , \ \sigma = -\sqrt{\frac{-a_{32} + \sqrt{a_{32}^2 - 4 a_{34} a_{30}}}{2 a_{34}}} \qquad (2.65.\text{b})$$

are obtained. The value of σ itself should be real. So Equation (2.65) cannot be meaningful solutions. Only Equation (2.64) is a pair of solutions.

2. Two Solutions of Equation (2.62)

With $s = \sigma + j\omega$ into (2.62), using $s^2 = (\sigma + j\omega)^2 = \sigma^2 - \omega^2 + j2\sigma\omega$, the relation,

$$\sigma^2 - \omega^2 + j2\sigma\omega \equiv \frac{-a_{32} - \sqrt{a_{32}^2 - 4 a_{34} a_{30}}}{2 a_{34}} \qquad (2.66)$$

is satisfied. Solving Equation (2.66) by equating real and imaginary parts, respectively, one pair of solutions

$$\text{i) } \quad \sigma = 0 \ , \ \omega = \sqrt{\frac{+a_{32} + \sqrt{a_{32}^2 - 4 a_{34} a_{30}}}{2 a_{34}}} \qquad (2.67.\text{a})$$

and

$$\text{ii)} \quad \sigma = 0 \ , \quad \omega = -\sqrt{\dfrac{+a_{32} + \sqrt{a_{32}^{\,2} - 4\,a_{34}\,a_{30}}}{2\,a_{34}}} \tag{2.67.b}$$

or the other pair of solutions,

$$\text{i)} \quad \omega = 0 \ , \quad \sigma = \sqrt{\dfrac{-a_{32} - \sqrt{a_{32}^{\,2} - 4\,a_{34}\,a_{30}}}{2\,a_{34}}} \tag{2.68.a}$$

and

$$\text{ii)} \quad \omega = 0 \ , \quad \sigma = -\sqrt{\dfrac{-a_{32} - \sqrt{a_{32}^{\,2} - 4\,a_{34}\,a_{30}}}{2\,a_{34}}} \tag{2.68.b}$$

are obtained. But since σ cannot be negative, Equation (2.68) cannot be the meaningful

solutions. Only Equation (2.67) is a pair of solutions.

Therefore, a set of four of solutions, Equations (2.64) and (2.67) are given by

$$\sigma = 0 \ , \quad \omega = \sqrt{\dfrac{a_{32} - \sqrt{a_{32}^{\,2} - 4\,a_{34}\,a_{30}}}{2\,a_{34}}} \quad \equiv \omega_1^{\,+} \tag{2.69.a}$$

and

$$\sigma = 0 \ , \quad \omega = -\sqrt{\dfrac{a_{32} - \sqrt{a_{32}^{\,2} - 4\,a_{34}\,a_{30}}}{2\,a_{34}}} \quad \equiv \omega_1^{\,-} \tag{2.69.b}$$

and

$$\sigma = 0 \ , \quad \omega = \sqrt{\dfrac{+a_{32} + \sqrt{a_{32}^{\,2} - 4\,a_{34}\,a_{30}}}{2\,a_{34}}} \quad \equiv \omega_2^{\,+} \tag{2.69.c}$$

and

$$\sigma = 0 \ , \quad \omega = -\sqrt{\frac{+a_{32} + \sqrt{a_{32}^2 - 4a_{34}a_{30}}}{2a_{34}}} \quad \equiv \omega_2^- \ . \qquad (2.69.d)$$

In Equation (2.69), $\sigma = 0$ in the four solutions. That is, the real part of the complex frequency ($s = \sigma + j\omega$) is zero.

But the imaginary part ω is not zero, and cannot be zero at the same time for the given conditions of $a_{32}^2 > 4a_{34}a_{30}$ to be satisfied. This means that the four TZ's cannot be located at the origin, since they are all different.

The *transmission zero* (TZ) locations are determined by coefficients as follows:

1. If $a_{34} = 0$, then the two zeros (ω_2^+ and ω_2^-) are located at infinity, but the other two zeros (ω_1^+ and ω_1^-) cannot be determined.

2. If $a_{30} = 0$, then the two zeros (ω_1^+ and ω_1^-) are located at origin, but the other two zeros (ω_2^+ and ω_2^-) are located at

$$\omega_2^+ = \sqrt{\frac{a_{32}}{a_{34}}} \qquad (2.70.a)$$

and

$$\omega_2^+ = -\sqrt{\frac{a_{32}}{a_{34}}} \ . \qquad (2.70.b)$$

The two *transmission zeros* (TZ's) given by Equation (2.70) are in complex conjugate pairs on $j\omega$-axis.

The TZ locations given by Equations (2.69) and (2.70) are shown in Figure 2.7.
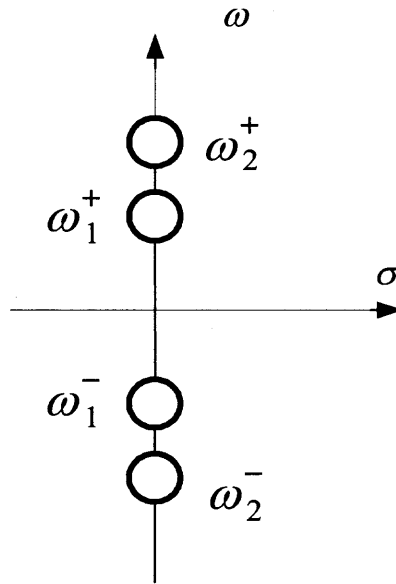
Figure 2.7 Quadruplet zero locations in complex plane; four complex zeros on $j\omega$ -axis.

b) Case 2: Coefficients of $f_2(s)$ **with** $a_{32}^{\,2} = 4\,a_{34}\,a_{30}$

Suppose the relative value of $a_{32}^{\,2}$ becomes smaller or the relative value of $4\,a_{34}\,a_{30}$

becomes bigger to have the relation $a_{32}^{\,2} = 4\,a_{34}\,a_{30}$. Equations (2.61) & (2.62) can be

reduced to the expressions,

$$s^2 = \frac{-a_{32} + \sqrt{a_{32}^{\,2} - 4\,a_{34}\,a_{30}}}{2\,a_{34}} = \frac{-a_{32}}{2a_{34}} \tag{2.71}$$

and

$$s^2 = \frac{-a_{32} - \sqrt{a_{32}^{\,2} - 4\,a_{34}\,a_{30}}}{2\,a_{34}} = \frac{-a_{32}}{2a_{34}} \tag{2.72}$$

Equations (2.71) and (2.72) have four solutions of equal magnitude.

1. Two Solutions of Equation (2.71)

With $s = \sigma + j\omega$ into the Equation (2.71), using $s^2 = (\sigma + j\omega)^2 = \sigma^2 - \omega^2 + j2\sigma\omega$,

Equation (2.71) has two sets of possible solutions,

$$\sigma = 0, \ \omega = +\sqrt{\frac{a_{32}}{2a_{34}}} \tag{2.73.a}$$

$$\sigma = 0, \ \omega = -\sqrt{\frac{a_{32}}{2a_{34}}} \tag{2.73.b}$$

or

$$\omega = 0, \ \sigma = +j\sqrt{\frac{a_{32}}{2a_{34}}} \tag{2.74.a}$$

$$\omega = 0, \ \sigma = -j\sqrt{\frac{a_{32}}{2a_{34}}} \tag{2.74.b}$$

The value of σ, as the real part of complex variable s, must be real. Therefore, solution given by Equation (2.74) is physically meaningless.

2. Two Solutions of Equation (2.72)

With $s = \sigma + j\omega$ into the Equation (2.71), using $s^2 = (\sigma + j\omega)^2 = \sigma^2 - \omega^2 + j2\sigma\omega$,

Equation (2.72) has two sets of possible solutions,

$$\sigma = 0, \ \omega = \sqrt{\frac{a_{32}}{2a_{34}}} \ , \tag{2.75.a}$$

and

$$\sigma = 0, \omega = -\sqrt{\frac{a_{32}}{2a_{34}}} \quad .$$ (2.75.b)

or

$$\omega = 0, \sigma = +j\sqrt{\frac{a_{32}}{2a_{34}}} \quad ,$$ (2.76.a)

and

$$\omega = 0, \sigma = -j\sqrt{\frac{a_{32}}{2a_{34}}} \quad .$$ (2.76.b)

The value of σ is real. Therefore, solution (2.76) is physically meaningless.

If $a_{32}{}^2 = 4\,a_{34}\,a_{30}$, then Equations (2.73) and (2.75) constitute a set of solutions:

$$\sigma = 0, \quad \omega = \sqrt{\frac{a_{32}}{2a_{34}}}$$ (2.77.a)

$$\sigma = 0, \quad \omega = -\sqrt{\frac{a_{32}}{2a_{34}}}$$ (2.77.b)

$$\sigma = 0, \quad \omega = \sqrt{\frac{a_{32}}{2a_{34}}}$$ (2.77.c)

$$\sigma = 0, \quad \omega = -\sqrt{\frac{a_{32}}{2a_{34}}}$$ (2.77.d)

The locations of four transmission zeros given by Equation (2.77) have zero real part. The magnitudes of the four TZ's are all the same.

Therefore, the TZ's are located on $j\omega$-axis. These TZ's are shown in Figure 2.8.
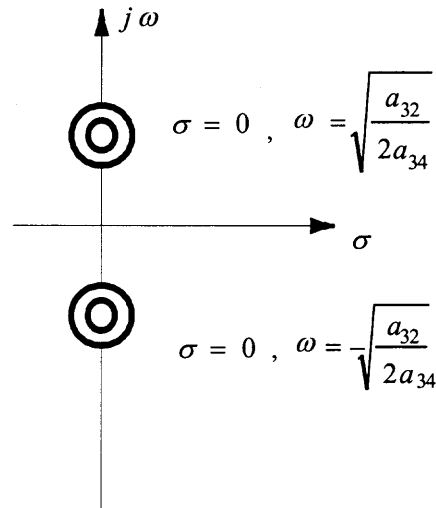
Figure 2.8 Complex quadruplet zero locations: two pairs of double zeros are on $j\omega$-axis.

Consider the relation given by the inequality $a_{32}^{2} = 4a_{34}a_{30}$ inside the square root in the Equations (2.71) and (2.72). For this equality expression, if any of the coefficient of a_{32}, a_{34}, or a_{30} is zero, then all of the other two coefficients are zeros.

Then the 4th degree polynomial equation $f_2(s) = a_{34}s^4 + a_{32}s^2 + a_{30} = 0$ given by Equation (2.57) does not exist. So, there can be no complex solutions of transmission zeros.

Therefore, none of the coefficients are zeros. Here again, the two pairs of zeros are different each other, and they cannot be positioned at the origin. The distances of the locations of all of the TZ's from the origin are all the same.

c) Case 3: Coefficients of $f_2(s)$ with $a_{32}{}^2 < 4 a_{34} a_{30}$

For the condition of $a_{32}{}^2 < 4 a_{34} a_{30}$ given in Equation (2.60), four solutions from two quadratic equations,

$$s^2 = \frac{-a_{32} + j\sqrt{4 a_{34} a_{30} - a_{32}{}^2}}{2 a_{34}} \qquad (2.78)$$

and

$$s^2 = \frac{-a_{32} - j\sqrt{4 a_{34} a_{30} - a_{32}{}^2}}{2 a_{34}} \qquad (2.79)$$

are obtained.

Equation (2.78) has two simultaneous solutions. And at the same time, Equation (2.79) has two simultaneous solutions.

1. Two Solutions of Equation (2.78)

With $s = \sigma + j\omega$ into (2.78), using $s^2 = (\sigma + j\omega)^2 = \sigma^2 - \omega^2 + j2\sigma\omega$, the relation ,

$$\sigma^2 - \omega^2 + j2\sigma\omega \equiv \frac{-a_{32} + j\sqrt{4 a_{34} a_{30} - a_{32}{}^2}}{2 a_{34}} \qquad (2.80)$$

is satisfied. Solving Equation (2.80) by equating real and imaginary parts, respectively,

$$\sigma^2 - \omega^2 = \frac{-a_{32}}{2a_{34}} \equiv k_1 \qquad (2.81)$$

and

$$2\sigma\omega = \frac{\sqrt{4a_{34}a_{30} - a_{32}{}^2}}{2a_4} \equiv k_2 \tag{2.82}$$

are obtained. In Equations (2.81) and (2.82) the new variables k_1 and k_2 are introduced for the sake of convenience. From Equation (2.82), the relation

$$\sigma = \frac{k_2}{2\omega} \tag{2.83}$$

is obtained. With Equation (2.83) into (2.81),

$$4\omega^4 + 4(k_1)\omega^2 - (k_2)^2 = 0 \tag{2.84}$$

is obtained. Solving the 4th degree Equation (2.84) in terms of ω^2, the quadratic

$$\omega^2 = \frac{-(k_1) \pm \sqrt{(k_1)^2 + (k_2)^2}}{2} \tag{2.85}$$

is obtained. In Equation (2.85), ω^2 cannot be negative. Choosing only + from $\pm$ in the numerator, the quadratic expression, with the introduction of a new variable k_3,

$$\omega^2 = \frac{-(k_1) + \sqrt{(k_1)^2 + (k_2)^2}}{2} \equiv k_3 \tag{2.86}$$

is obtained.

Solving Equation (2.86) for ω, and from Equation (2.86), the two sets of simultaneous solutions,

$$\omega = +\sqrt{k_3}, \quad \sigma = \frac{1}{2}\frac{k_2}{\sqrt{k_3}} \qquad (2.87.a)$$

and

$$\omega = -\sqrt{k_3}, \quad \sigma = -\frac{1}{2}\frac{k_2}{\sqrt{k_3}} \qquad (2.87.b)$$

are obtained.

2. Two Solutions of Equation (2.79)

With $s = \sigma + j\omega$ into the Equation (2.71), using $s^2 = (\sigma + j\omega)^2 = \sigma^2 - \omega^2 + j2\sigma\omega$, the equality relation,

$$\sigma^2 - \omega^2 + j2\sigma\omega \equiv \frac{-a_{32} - j\sqrt{4a_{34}a_{30} - a_{32}^2}}{2a_{34}} \qquad (2.88)$$

is obtained.

Solving Equation (2.88) by equating real and imaginary parts, respectively, the two relations

$$\sigma^2 - \omega^2 = \frac{-a_{32}}{2a_{34}} \equiv k_1 \qquad (2.89)$$

and

$$2\sigma\omega = \frac{-\sqrt{4a_{34}a_{30} - a_{32}^2}}{2a_4} \equiv -k_2 \qquad (2.90)$$

are obtained.

From Equation (2.90), the relation

$$\sigma = \frac{-k_2}{2\omega} \tag{2.91}$$

is obtained.

With Equation (2.91) into (2.89), the equation

$$4\omega^4 + 4(k_1)\omega^2 - (k_2)^2 = 0 \tag{2.92}$$

is derived.

Solving Equation (2.92), the relation

$$\omega^2 = \frac{-(k_1) \pm \sqrt{(k_1)^2 + (k_2)^2}}{2} \tag{2.93}$$

is obtained.

In Equation (2.93), ω^2 cannot be negative. Choosing only + from $\pm$ in the numerator, the following relation is obtained.

$$\omega^2 = \frac{-(k_1) \pm \sqrt{(k_1)^2 + (k_2)^2}}{2} \equiv k_3 \tag{2.94}$$

Solving Equation (2.94) for ω, and from Equation (2.91), two sets of possible and meaningful solutions,

$$\omega = +\sqrt{k_3}, \quad \sigma = -\frac{1}{2}\frac{k_2}{\sqrt{k_3}} \qquad (2.95.a)$$

and

$$\omega = -\sqrt{k_3}, \quad \sigma = +\frac{1}{2}\frac{k_2}{\sqrt{k_3}} \qquad (2.95.b)$$

are obtained.

Therefore, $f(s) = 0$ has a set of four meaningful solutions, Equations (2.87) and (2.95), in terms of ω and σ, which are expressed as follows again, respectively. Four notations $\omega^+, \omega^-, \sigma^+$, and σ^- are introduced for the sake of convenience.

$$\omega = \sqrt{k_3} = \omega^+, \qquad \sigma = \frac{1}{2}\frac{k_2}{\sqrt{k_3}} = \sigma^+ \qquad (2.96.a)$$

$$\omega = -\sqrt{k_3} = \omega^-, \qquad \sigma = \frac{1}{2}\frac{k_2}{\sqrt{k_3}} = \sigma^+ \qquad (2.96.b)$$

$$\omega = \sqrt{k_3} = \omega^+, \qquad \sigma = -\frac{1}{2}\frac{k_2}{\sqrt{k_3}} = \sigma^- \qquad (2.96.c)$$

$$\omega = -\sqrt{k_3} = \omega^-, \qquad \sigma = -\frac{1}{2}\frac{k_2}{\sqrt{k_3}} = \sigma^- \qquad (2.96.d)$$

The simultaneous quadruplet *transmission zero* (TZ) locations given in Equation (2.96) are shown in Figure 2.9.
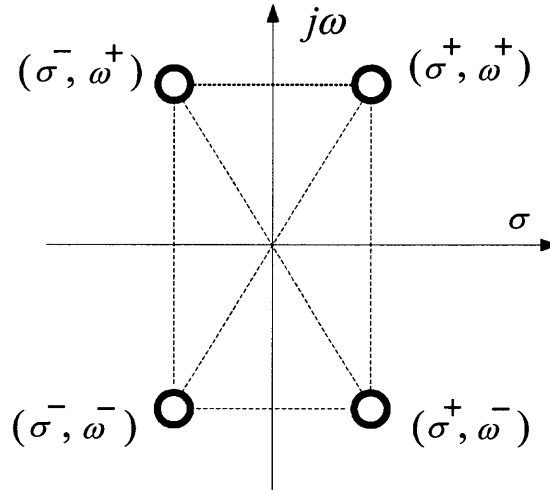
Figure 2.9 Complex quadruplet zero locations.

In Figure 2.9, the four transmission zeros are found in the mirror image locations with respect to real and imaginary axes.

The relation of $a_{32}^{2} < 4 a_{34} a_{30}$ is considered. If any of the coefficients a_{34} or a_{30} is zero, the inequality cannot be true. If a_{32} is zero (which means that C33 is zero), then a_{34} is zero. This relation is not reasonable. Therefore, a_{32} cannot be zero.

2.3.4 Denominator Polynomial

The denominator polynomial of Equation (2.54) is expressed again,

$$D(s) = a_8 s^8 + a_7 s^7 + a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 . \quad (2.54)$$

Polynomial $D(s)$ of Equation (2.54) is an 8th degree polynomial. All the coefficients

(a_8, a_7, a_6, a_5, a_4, a_3, a_2, and a_0) are positive real numbers given by Equation (2.55),

since these come from the real values of realizable $L's$ and $C's$. So, the system has

eight finite *transmission poles* (reflection zeros).

A necessary condition for the Hurwitz polynomial requires that all coefficients of

$D(s)$ are strictly positive, with no missing terms.

Since no terms of Equation (2.54) are missing, $D(s)$ satisfies the necessary

condition of a Hurwitz polynomial. In fact, since the crossed-coupled filter is realizable

with $L's$ and $C's$, the filter system is a stable linear system. Therefore, it should have

all system transmission poles in the strict left-half plane (LHP).

However, transmission zeros can be located in any place such as LHP, $j\omega$-axis,

and/or right-half plane (RHP).

2.3.5 Locus of Transmission Zeros

Representing the single stationary zero at origin and the four dynamic zeros at non-origin,

the locus of *transmission zeros* (TZ's) is shown in Figure 2.10.

In the figure below, f_0 is the center frequency of the cross-coupled bandpass

filter, and $0.7f_0$ is a break frequency. Break frequency is the frequency or point at

which two or more branches of the locus come together and then part. In other words,

break frequency is the frequency at which the incoming locus becomes the outgoing

locus.

Before the four TZ's meet at the break frequency of $0.7f_0$, the dynamic zeros are

zero - σ dynamic zeros, moving only along the $j\omega$-axis. But after the TZ's meet at the
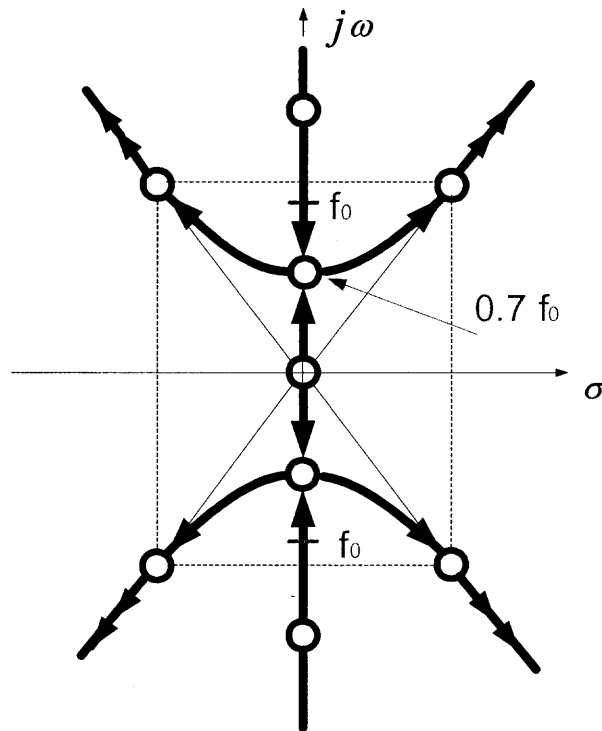
Figure 2.10 Transmission zero locus of the cross-coupled network in Figure 2.5.

Point of $0.7 f_0$, they began to separate. Therefore, after the break point, the dynamic zeros are *nonzero - σ* dynamic zeros. These *nonzero - σ* dynamic zeros are located at the four corners of a rectangle.

As the elements values are changing, the locations of transmission zeros are changing also. The rules of the locus in the figure above are as follows:

1. When there is no cross-coupling, there is a single stationary zero at the origin.

2. When there is a cross-coupling, quadruplet dynamic zeros additionally exist as a complex quadruplet. Comparing the relative value of $4a_{34}a_{30}$ to a_{32}^2, the locus of zero is summarized as follows.

1) At the first moment, when $a_{30} = 0$ (i.e. $L34=0$), two zeros (ω_1^+ and ω_1^-) are at the origin, and the other two zeros, ω_2^+ and ω_2^-, are located at $+\infty$ and $-\infty$, respectively.

2) As a_{30} increases from zero value, under the condition of $4a_{34}a_{30} < a_{32}^2$, two origin zeros (ω_1^+ and ω_1^-) start moving away from the origin while two non-origin zeros (ω_2^+ and ω_2^-) start moving toward the origin.

The four quadratic transmission zeros are moving separately(but, not independently) on the $j\omega$ -axis, as $zero-\sigma$ dynamic zeros, until two positive zeros (ω_1^+ and ω_2^+) meet at a positive break point, and at the same time two negative zeros (ω_1^- and ω_2^-) meet at a negative break points.

3) As a_{30} increases further , a_{34} increases, or a_{32}^2 decreases to meet the condition of $4a_{34}a_{30} = a_{32}^2$. At the moment the condition $4a_{34}a_{30} = a_{32}^2$ is satisfied, two non-origin positive zeros (ω_1^+ and ω_2^+) meet and overlap at a positive break point of $\omega = \sqrt{a_{32}/2a_{34}}$, while two non-origin negative transmission zeros (ω_1^- and ω_2^-) meet and overlap at a negative break point of $\omega = -\sqrt{a_{32}/2a_{34}}$.

4) As a_{30} increases further , a_{34} increases, or a_{32}^2 decreases to meet the condition of $4a_{34}a_{30} > a_{32}^2$.

At the moment the condition $4a_{34}a_{30} > a_{32}^2$ is satisfied, two non-zero positive zeros (ω_1^+ and ω_2^+) at $\omega = \sqrt{a_{32}/2a_{34}}$ start separating. One zero moves to the RHP and the other zero moved to the LHP.

At the same time, the two non-zero negative transmission zeros (ω_1^- and ω_2^-) at $\omega = -\sqrt{a_{32}/2a_{34}}$ start separating. One zero moves to the RHP and the other zero moves to the LHP. The separated complex zeros follow a hyperbolic path to move away to the locations in infinite s-plane. The asymptotes of the hyperbola are $\omega = \pm\sigma$.

The general properties of zero loci are stated as follows, including the 1st three rules [6]:

1. Symmetrical with respect to real axis and imaginary axis.

2. Dynamic zeros travel in opposite directions.

3. Continuous change of elements produces continuous loci.

4. Break points are located only on $j\omega$ -axis.

5. TZ locus does not intercept σ -axis, except the origin.

2.4 Positively Cross-coupled Filter Network

In the filter network shown in Figure 2.5, a capacitor C33 was used as the cross-coupling element of the subsystem S3. If an inductor (L33) is used in place of C33, then a *positive cross-coupled* (PCC) RF filter network can be obtained as shown in Figure 2.11.

2.4.1 Characteristic Polynomial

The procedures to derive the *transmission zero characteristic equation* (TZCE) is the same as the *negatively cross-coupled* (NCC) filter case. Without repeating the same details, the final form of TZCE is obtained by using a modified MATLAB program.
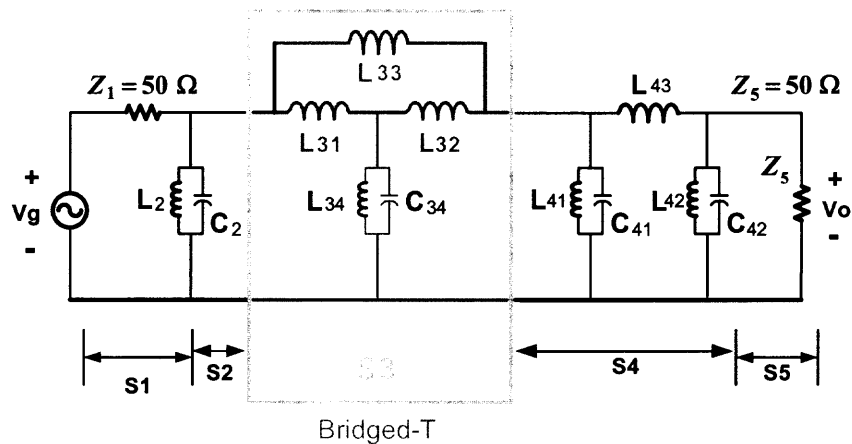
Figure 2.11 Filter network with an inductor cross-coupling.

The filter of the figure above is analyzed to have the following characteristic polynomial,

$$N(s) = 50L_2L_{41}L_{42} \ s \cdot$$
$$[L_{31}L_{32}L_{34}C_{34} \ s^2 + (L_{31}L_{32} + L_{34}L_{33} + L_{31}L_{34} + L_{32}L_{34})] \tag{2.97}$$

The polynomial given in Equation (2.97) is a 3rd degree numerator polynomial. This numerator polynomial is composed of two functions factored.

2.4.2 TZ Characteristic Equation

Equating Equation (2.97) to zero , i.e. *N(s)=0*, the *TZ characteristic equation* (TZCE) is

obtained as

$$f(s) = f_1(s) \cdot f_2(s) = 0 , \qquad (2.98)$$

where,

$$f_1(s) = s = 0 , \qquad (2.99)$$

$$f_2(s) = a_{32} s^2 + a_{30} = 0 . \qquad (2.100)$$

The coefficients in the quadratic Equation (2.100) are given by

$$a_{32} = L_{31} L_{32} L_{34} C_{34} , \qquad (2.101.a)$$

$$a_{30} = (L_{31} L_{32} + L_{34} L_{33} + L_{34} L_{31} + L_{34} L_{32}) . \qquad (2.101.b)$$

The coefficients a_{32} and a_{30} given in Equations (2.100) and (2.101) are both positive.

Monomial Equation (2.99) has a single stationary zero. Equation (2.100) has two

dynamic transmission zeros. Since a_{32} and a_{30} are both positive, the roots are pure

imaginary. Therefore, the roots are given by

$$s = \pm j \sqrt{\frac{a_{30}}{a_{32}}} \quad . \qquad (2.102)$$

The two TZ's given by Equation (2.102) are zero-σ dynamic zeros. There are two

extreme cases for the values of cross-coupled inductor: one is very small but not zero,

and the other case is the infinity, which means that there is no cross-coupling.

For the sake of convenience, the *"very small positive value almost equal to zero"* and the *"very big positive value almost equal to (very close to) infinity"* are noted as $"0_+"$ and $"\infty_-"$, respectively, as are called in Laplace transform theory.

When L33 is very small, the characteristic polynomial is expressed as follows:

$$N(s) \cong 50L_2L_{41}L_{42} \ s \cdot$$
$$[L_{31}L_{32}L_{34}C_{34} \ s^2 + (L_{31}L_{32} + \ "0_+" + L_{31}L_{34} + L_{32}L_{34}) \ ] \ . \tag{2.103}$$

Equating Equation (2.103) to zero, i.e. *N(s)=0*, the following form of TZ characteristic equation,

$$f(s) = s \cdot (a_{32}s^2 + a'_{30}) = 0$$
$$\equiv f_1(s) \cdot f_2(s) = 0 . \tag{2.104}$$

is obtained, where the coefficients are given by

$$a_{32} = L_{31}L_{32}L_{34}C_{34} , \tag{2.105.a}$$

$$a'_{30} = (L_{31}L_{32} + "0_+" + L_{34}L_{31} + L_{34}L_{32}) . \tag{2.105.b}$$

Coefficient a_{32}, in Equation (2.105.a) is the same as that of Equation (2.101.a). It is noted that the coefficients of Equation (2.105) are non-negative, since they are expressed in terms of elements values. Each of the factored polynomial of Equation (2.104) is expressed as follows:

$$f_1(s) = s = 0 , \tag{2.106}$$

$$f_2(s) = a_{32}s^2 + a'_{30} = 0 \ . \tag{2.107}$$

Equation (2.104) shows that the network in Figure 2.11 has three finite TZ's; a single zero located at the origin due to Equation (2.106) and two zeros due to Equation (2.107).

2.4.3 Transmission Zeros of System

a) Monomial Equation $\quad f_1(s) = s = 0$

Equation (2.106) produces a single stationary TZ at the origin.

b) Quadratic Polynomial $\quad f_2(s) = a_{32}s^2 + a'_{30}$

This polynomial again is only due to the cross-coupled network, since the coefficients are in the form of a_{ij}, with $i=3$, $j=2$ and a'_{ij} with $i=3, j=0$.

Where, the subscript i =3 means the 3$^{\text{rd}}$ subsystem, i.e. the bridged-T system. The characteristic equation for the bridged-T system is give by a quadratic equation,

$$a_{32}s^2 + a'_{30} = 0 \ . \tag{2.108}$$

The solutions of Equation (2.108) are given by

$$s = \pm j \sqrt{\frac{a'_{30}}{a_{32}}} \ . \tag{2.109}$$

From Equations (2.101.b) and (2.105.b), it is clear that the two coefficients of the relevant characteristic equations have the inequality relation,

$$a'_{30} \quad < \quad a_{30} \ , \tag{2.110}$$

since Equation (2.101.b) contains the term $L_{34}L_{33}$, but Equation (2.105.b) does not.

From Equation (2.110), the magnitude of s given in Equation (2.102) is bigger than that given in Equation (2.109). This means that zero location of s given in Equation (2.109) is further away from the origin.

2.4.4 Locus of Transmission Zeros

From Equations (2.102), (2.106), and (2.109), *transmission zeros* (TZ's) are located as
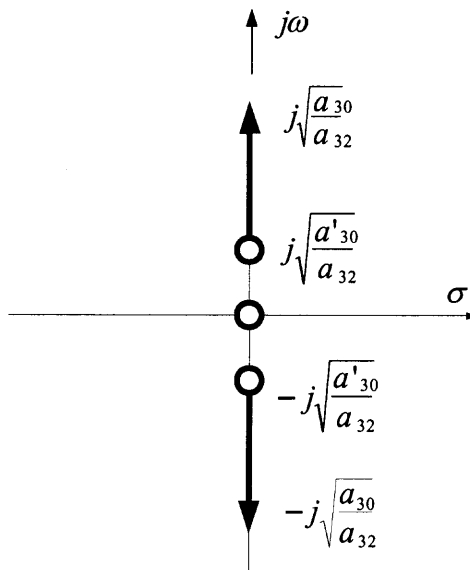
Figure 2.12 Transmission zero locus of network given in Figure 2.11.

shown in Figure 2.12. As shown in the figure above, there is a single *stationary* (static) zero at the origin, and there are two *zero - σ dynamic zeros* on the $j\omega$-axis. Since the TZ's are confined only on the $j\omega$- *axis*, there are no *nonzero - σ dynamic zeros*.

As the cross-coupled inductor L33 increases, from "0_{+}" to "∞_{-}", the transmission zero locus start from the value of Equation (2.109) and approaches (2.102).

When there is no cross-coupling, there is no L33 at all. It is open circuited. This is the case where Laplace impedance of L33 is pure infinity. At this time, there are no dynamic zeros. There is only a single stationary zero at the origin.

It is noted that dynamic zeros can be existing only for the case,

$$0 < L_{33} < \infty \ . \tag{2.111}$$

Physically, if the inductor L_{33} as a cross-coupling element does not exist in the filter network of Figure 2.11, then the Laplace impedance of L33 is infinity, and hence it is open circuited.

Mathematically, if the Laplace impedance of L33 is infinity, then the only term to be considered in the parenthesis of Equation (2.97) is L_{33}. All the other terms are relatively small. Without any loss of generality, all the other terms are neglected. Therefore, the characteristic equation is reduced to become

$$N(s) = 50 \ L_2 \underline{L_{34}} L_{41} L_{42} \ s \ \ . \tag{2.112}$$

In fact, Equation (2.112) could be directly obtained from Equation (2.97).

2.5 Chapter Summary

A *cross-coupled* (CC) filter network is formed by adding a cross-coupling bridge on the initially-synthesized ladder network. Considering the cross-coupled section as a bridged-T subsystem, and the whole network to be a cascaded connection, from input

terminal to output terminal, the rules of chain matrix were applied to derive the transfer function.

After the cancellations of the common terms in numerator and denominator polynomials, canonical forms of transfer function is calculated. From this canonical form of the transfer function, the canonical forms of numerator and denominator polynomials are obtained. By equating the canonical forms of numerator polynomial to zero, the *transmission zero characteristic equation* (TZCE) is obtained.

When a negative or positive cross-coupling element is added, skipping one resonator, a 5th order or a 3rd order TZCE is obtained.

The TZCE's are factored into a product of monomial and polynomial equations. Due to the monomial, single stationary zero is located at the origin. On the other hand, a 4th or a 2nd degree polynomial, which comes solely from the cross-coupled subsystem, gives the two pairs of (i.e. quadruplet) complex zeros, or one pair of (i.e. doublet) complex zeros.

The polynomials have positive coefficients. Depending on the perturbed element values, the coefficients are varying. Based on the varying coefficients, the TZCE gives different solutions. The continuous change of solutions produces the TZ locus on the complex plane.

CHAPTER 3

BRIDGED-T CROSS-COUPLED FILTER NETWORKS: WITHOUT SKIPPING ANY RESONATORS AND SKIPPING TWO RESONATORS

In Chapter 2, the theoretical derivation from the *cross-coupled* (CC) filter network was considered in great detail. The cross-coupling element was added between the two resonators, skipping only one resonator.

In this chapter, the following filter networks will be discussed:

(1) the filter networks with cross-coupling elements without skipping any resonators, and

(2) the filter networks with cross-coupling elements skipping two resonators.

The canonical form of *transmission zero characteristic equations* (TZCE's) are obtained by considering the common term cancellations between numerator and denominator polynomials.

TZCE will be solved to locate TZ locations and to obtain TZ locus.

NOMENCLATURE

Positively cross-coupled (PCC) network: A network where sign of the cross-coupling is the same as the sign of the main line coupling (i.e. inductive cross-coupling in an inductively coupled circuit).

Negatively cross-coupled (NCC) network: A network where sign of the cross-coupling is the opposite as the sign of the main line coupling (i.e. inductive cross-coupling in a capacitively coupled circuit).

Transmission zero characteristic equation (TZCE): The canonical numerator polynomial
     set equal to zero.

LHP: Left-half plane

RHP: Right-half plane

3.1 Cross-coupled Filter Network Without Skipping Any Resonators; i.e. Cross-coupling Adjacent Resonators

The filter in Figure 3.1 is an example of negative cross-coupled filter, where "negative"

means that the sign of the cross coupling opposes the sign of the main line coupling (i.e.

capacitive cross coupling in an inductively coupled circuit). Another case of negative

coupling is for the inductive cross coupling in a capacitively coupled main line). Both of

these negatively cross-coupled filters have the same characteristic equations.
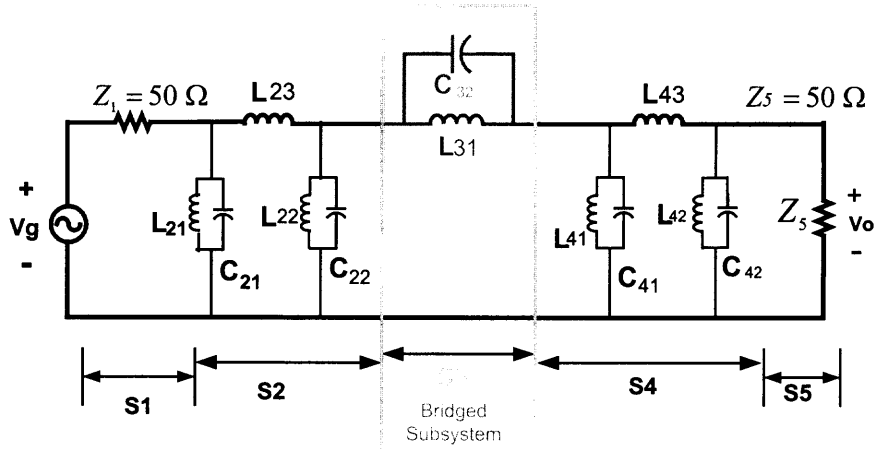

Figure 3.1 Negatively cross-coupled (NCC) network.

The overall filter network is sectioned into five subsystems $(S_i, i = 1-5)$. Each system

is characterized by its own chain matrix of size 2×2. The L31 and C32 in subsystem S3

can be represented as an equivalent impedance by considering a parallel connection of C32 and L31. The whole system is conveniently represented by the cascaded chain matrices of five subsystems sectioned. The impedances of the elements are given by the following expressions;

$$Z_1 = 50,$$

$$Z_{21} = L_{21} \,/\!/\, C_{21} = \frac{sL_{21}}{L_{21}C_{21}s^2 + 1},$$

$$Z_{22} = L_{22} \,/\!/\, C_{22} = \frac{sL_{22}}{L_{22}C_{22}s^2 + 1},$$

$$Z_{23} = s\,L_{23},$$

$$Z_3 = L_{31} \,/\!/\, C_{32} = \frac{sL_{31}}{L_{31}C_{32}s^2 + 1}, \tag{3.1}$$

$$Z_{41} = L_{41} \,/\!/\, C_{41} = \frac{sL_{41}}{L_{41}C_{41}s^2 + 1},$$

$$Z_{42} = L_{42} \,/\!/\, C_{42} = \frac{sL_{42}}{L_{42}C_{42}s^2 + 1},$$

$$Z_{43} = s\,L_{43},$$

$$Z_5 = 50.$$

In Figure 3.1, the chain matrix of the each subsystem is expressed as

$$\overline{T}_1 = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}, \tag{3.2.a}$$

$$\overline{T_i} = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}, \ i = 1,2,3,\&\,4; \tag{3.2.b}$$

$$\overline{T_5} = \begin{bmatrix} 1 & 0 \\ 1/50 & 1 \end{bmatrix}. \tag{3.2.c}$$

Equation (3.2) defines the chain matrices of each subsystem. That is, $\overline{T_1}, \overline{T_2}, \overline{T_3}, \overline{T_4}$, and $\overline{T_5}$ are the chain matrices of the series source impedance R, π-network (composed of Z21, Z22, and Z23), series-connected LC-parallel subsystem, π-network(composed of Z41,Z42,and Z43), and the load impedance of 50 Ohms, respectively.

In Equation (2.2), $\overline{T}(1,1)$ is dependant on the each of the cascaded five networks. In Equation (3.2.b), all of the 12 entries of three matrices should be expressed in terms of Laplace impedances given in Equation (3.1). As shown in Equation (2.2), the voltage transfer function $H(s)$ is represented by the numerator and denominator polynomials $N(s)$ and $D(s)$, respectively.

The canonical form of the numerator polynomial in the transfer function is obtained as the 3rd degree polynomial,

$$N(s) = 50\,L_{21}L_{22}L_{41}L_{42}\,s \cdot [L_{31}C_{32}\,s^2 + 1]. \tag{3.3}$$

Equating Equation (3.3) to zero, $N(s)=0$, the TZ characteristic equation is expressed as a product of two functions,

$$f(s) = s \cdot (a_{32}\,s^2 + 1) \tag{3.4}$$

$$\equiv f_1(s) \cdot f_2(s) = 0.$$

In Equation (3.4), there is only one coefficient in the quadratic. The coefficient is given by the product of two elements in the subsystem S3,

$$a_{32} = L_{31} C_{32} .$$

(3.5)

It is noted that the coefficient a_{32} is nonnegative in Equation (3.5), since it is expressed in terms of element values.

When there is no cross-coupling, the cross-coupling element does not exist. That means C32=0. In this case the cross-coupled network reduces to the ladder network. Then, Equation (3.5) makes $a_{32} = 0$ and there is single stationary zero at origin.

There cannot be dynamic zeros. So there will be no complex zeros (in this case, complex doublet zeros) produced. Therefore, for the *positively cross-coupled* (PCC) network to have complex doublet zeros in finite *s*-plane, the assumption is C32 is not zero.

To calculate the *transmission zeros* (TZ's) of the PCC filter network, each of the factored polynomial of Equation (3.4) is written as

$$f_1(s) = s$$

(3.6)

$$f_2(s) = a_{32} s^2 + 1 .$$

(3.7)

Equation (3.4) shows that the network in Figure 3.1 has three finite TZ's; a single zero located at the origin due to Equation (3.6), and two *zero - σ dynamic zeros* due to Equation (3.7).

By solving the simultaneous Equations (3.6) and (3.7), the finite *transmissions zeros*

(TZ's) of the filter network are determined as follows.

a) Monomial Equation $f_1(s) = s = 0$

The monomial given by Equation (3.6) produces a single stationary transmission zero

(TZ) at the origin, as shown in Figure 3.2.
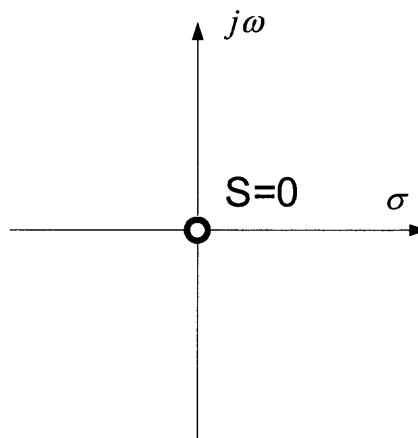
Figure 3.2 A single stationary (static) zero located at origin.

b) Quadratic Polynomial $f_2(s) = a_{32}s^2 + 1$

This polynomial is only due to the cross-coupled network, since the coefficients are in

the form of a_{i2}, $i = 3$, where the subscript $i = 3$ means the 3rd system, i.e. the cross-

coupled subsystem. The characteristic equation for the cross-coupled subsystem is

obtained by $f_2(s) = 0$, i.e.

$$f_2(s) = a_{32} s^2 + 1 = 0 .$$ (3.8)

In Equation 3.8, the coefficient a_{32} is the positive real number, so the quadratic equation has two imaginary solutions,

$$s = \pm j \sqrt{\frac{1}{a_{32}}} \, . \qquad (3.9)$$

For the infinite value of L31, the value of a_{32} in Equation (3.9) is to be investigated. When there is no cross-coupling (i.e. C32=0), a_{32}=0 in Equation (3.5).

If a_{32}=0 in Equation (3.9), then the two dynamic zeros are located in the infinite s-plane. As the values of C32 is increasing continuously from 0 (i.e. no-cross-coupling) to very big number, a_{32} is also increasing from 0 to a very big number.

It is noted that dynamic zeros can exist only when the cross-coupling element C32 has the following range of values,

$$0 \leq C32 < \infty \, . \qquad (3.10)$$

Accordingly, the zero locations move from $\pm\infty$ to a very small number close to origin (but not zero) along the $j\omega$ -axis. Perturbed element values of the filter network generate a different coefficient.

c) TZ Locus

Based on the stationary zero of Figure 3.2 and Equations (3.9-10), it is clear that the transmission zero locus is obtained as shown in Figure 3.3. The two dynamic zeros are approaching from the $\pm\infty$ locations to the origin.
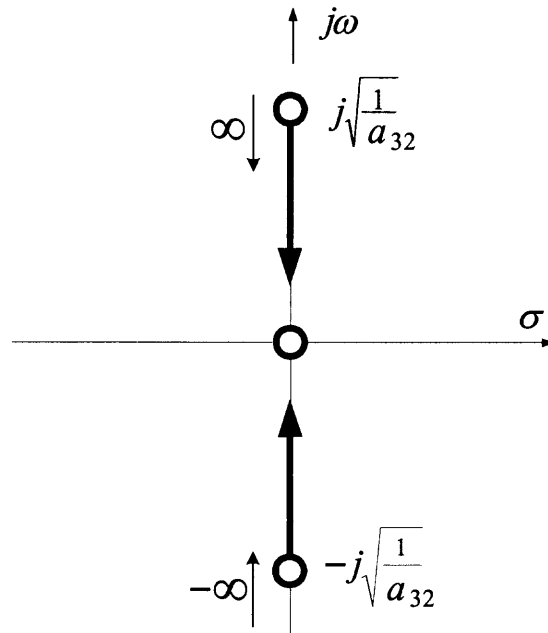
Figure 3.3 Zero locus of the filter network of Figure 3.1.

Thus, without skipping a resonator, a single TZ results from the tank circuit (S3) resonance. This is the expected result and simply helps validate the generality of the theory.

Positively cross-coupled (PCC) network

In Figure 3.1, coupling element C32 is used for the *negatively cross-coupled* (NCC) filter network. If L32 is used instead, a *positively cross-coupled* (PCC) network is obtained, where "positive(ly)" means that the sign of the cross coupling is the same as the

sign of the main line coupling (i.e. inductive cross coupling in an inductively coupled circuit). Another case of positive coupling is for the capacitive cross coupling in a capacitively coupled main line). Both of these positively cross-coupled filters have the same characteristic equations.

For the PCC filter network, the series impedance of subsystem S3 is given by

$$Z_3 = L_{31} \, // \, L_{32} = \frac{s \, L_{31} L_{32}}{L_{31} + L_{32}} \, . \tag{3.11}$$

Using Equation (3.11), the canonical form of the numerator polynomial is calculated as the 1st degree monomial,

$$N(s) = 50 \, L_{21} L_{22} \cdot (L_{31} + L_{32}) \cdot L_{41} L_{42} \cdot s \, . \tag{3.12}$$

From Equation (3.12), with $N(s) = 0$, the TZ characteristic equation is given by

$$f(s) = s = 0 \, . \tag{3.13}$$

Equations (3.12) and (3.13) show that there is a single stationary zero at the origin. There are no dynamic zeros.

Again, there is no resonance in S3 and the result is expected from conventional network theory, merely helps validate the generality of this theory.

3.2 Negatively Cross-coupled Filter Network, Skipping Two Resonators

A cross-coupled filter network, skipping two resonators, is investigated in detail. Without the loss of generality, the cross-coupled subsystem is assumed to be the 3rd subsystem, as shown in Figure 3.4.

The *transmission zeros* (TZ's) can be obtained by solving the "*transmission zero characteristic equation* (TZCE)", which is derived from the canonical transfer function of the network.

To analyze this filter network, the whole system is considered to be composed of five subsystems cascaded from the input port to the output port.
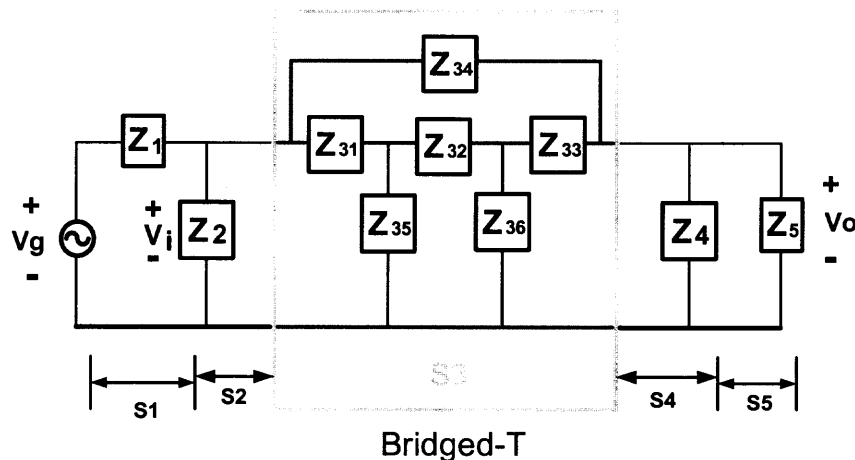
Figure 3.4 Cross-coupled filter network, skipping two resonators.

In the block diagram shown in Figure 3.4, Z_1 is the source impedance; Z_2, Z_{35}, Z_{36}, and Z_4 are shunt-connected LC-resonators; and Z_5 is the load impedance.

In Figure 3.4, the subscripts are used to indicate each of five subsystems. A pair of two-port network is equivalent if the characterizing parameters are identical. By transforming the S3 into an equivalent circuit, the figure can be simplified for analysis.

The filter network in Figure 3.5 is an equivalent network. In Figures 3.4 and 3.5, the subsystems S3's of the two networks should have the same terminal voltages and currents so that the transmission parameters are the same.

Bridged-T
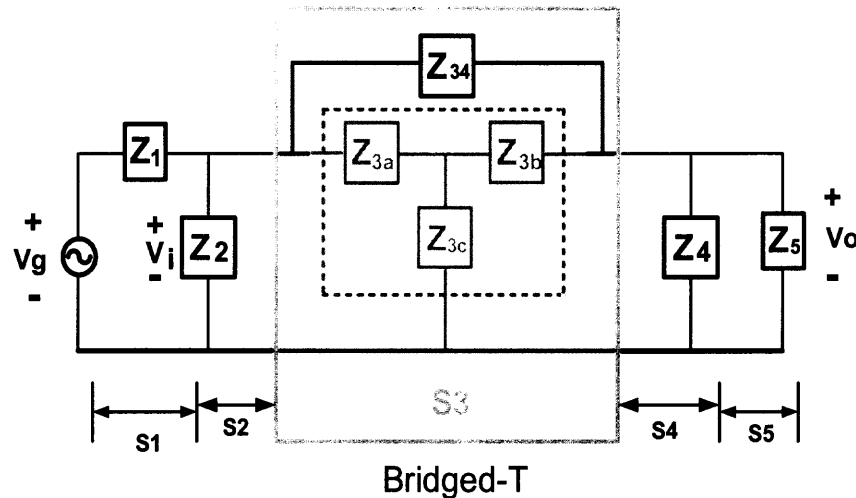
Figure 3.5 Cross-coupled network, equivalent to Figure 3.4.

The network in Figure 3.5 is the same form as the one in Chapter 2. To investigate the filter, the filter is conveniently sectioned to use the characteristics of *chain* (ABCD) matrices.

The necessary and sufficient conditions for the two-port network to be equivalent, the terminal voltages and terminal currents should be equal. For the networks given in Figures 3.4 and 3.5, the terminal characteristics of the subsystem S3 should be equivalent.

By several procedures of $\Delta \leftrightarrow Y$ transformations of the T-network in the subsystem S3, the equivalent system is derived. The impedance Z34 of the cross-coupling element keeps the same value for the two networks.

Only the T-network is transformed, with all others unaltered in the two networks.

Without showing the derivations of transformations in details, the resultant expressions

are given by

$$Z_{3a} = Z_{31} + \left(\frac{Z_{35} + Z_{32}}{Z_{35} + Z_{32} + Z_{36}} \right) \quad , \qquad (3.14.a)$$

$$Z_{3b} = Z_{33} + \left(\frac{Z_{36} + Z_{32}}{Z_{35} + Z_{32} + Z_{36}} \right) \quad , \qquad (3.14.b)$$

$$Z_{3c} = \frac{Z_{36} \cdot Z_{35}}{Z_{35} + Z_{32} + Z_{36}} \quad . \qquad (3.14.c)$$

For the cross-coupled filter network shown in Figure 3.4, each of the elements is

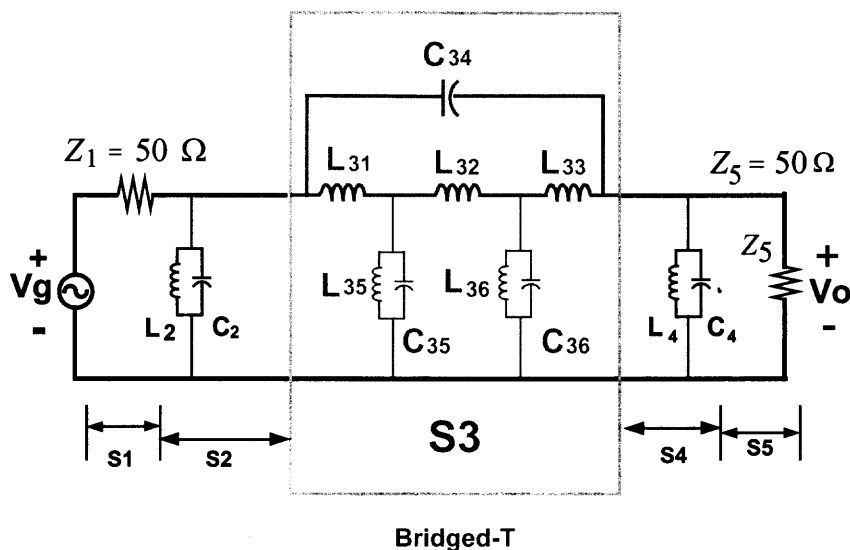specified as in Figure 3.6 for analysis.

Bridged-T

Figure 3.6 Negatively cross-coupled network, skipping two resonators.

In the figure above, all the elements values are shown in real numbers. These elements are given by Laplace impedances in Ohms;

$$Z_1 = 50;$$

$$Z_2 = \frac{s\,L_2}{L_2\,C_2\,s^2 + 1}\;;$$

$$Z_{31} = sL_{31},\;\; Z_{32} = sL_{32},\;\; Z_{33} = sL_{33},\;\; Z_{34} = 1/sC_{33}\;; \qquad (3.15)$$

$$Z_{35} = \frac{sL_{35}}{L_{35}C_{35}\,s^2 + 1},\, Z_{36} = \frac{sL_{36}}{L_{36}C_{36}\,s^2 + 1}\;;$$

$$Z_4 = \frac{s\,L_4}{L_4\,C_4\,s^2 + 1}\;;$$

$$Z_5 = 50\,.$$

In terms of filter element values given by Equation (3.15), Equation (3.14) is rewritten as

$$Z_{3a} = s\cdot L_{31} + \frac{s^2\cdot L_{35}L_{32}}{\left(L_{35}C_{35}\cdot s^2 + 1\right)\left(\dfrac{s\cdot L_{35}}{L_{35}C_{35}\cdot s^2 + 1} + s\cdot L_{32} + \dfrac{s\cdot L_{36}}{L_{36}C_{36}\cdot s^2 + 1}\right)}, \qquad (3.16.a)$$

$$Z_{3b} = \frac{s^2\cdot L_{35}L_{36}}{\left(L_{35}C_{35}\cdot s^2 + 1\right)\left(L_{36}C_{36}\cdot s^2 + 1\right)\left(\dfrac{s\cdot L_{35}}{L_{35}C_{35}\cdot s^2 + 1} + s\cdot L_{32} + \dfrac{s\cdot L_{36}}{L_{36}C_{36}\cdot s^2 + 1}\right)}, \qquad (3.16.b)$$

$$Z_{3c} = s\cdot L_{33} + \frac{s^2\cdot L_{35}L_{36}}{\left(L_{36}C_{36}\cdot s^2 + 1\right)\left(\dfrac{s\cdot L_{35}}{L_{35}C_{35}\cdot s^2 + 1} + s\cdot L_{32} + \dfrac{s\cdot L_{36}}{L_{36}C_{36}\cdot s^2 + 1}\right)}. \qquad (3.16.c)$$

Substituting Equation (3.15) into (3.16), the filter network is analyzed. For the purpose, a *chain* (ABCD) matrix of each subsystem is derived.

3.2.1 Chain Matrices of Each Subsystem

The filter network is composed of five subsystems, S1, S2, S3, S4, and S5. Starting from the system S1, all five subsystems are considered.

- **System S1**

The 1st subsystem is composed of source impedance $Z_1 = Z_g = 50\,\Omega$ and ground line. The *chain* (*ABCD*) matrix, $\overline{T}_1$, is given by

$$\overline{T}_1 = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}.$$

(3.17)

All entries of matrix Equation (3.17) are constant, so $\overline{T}_1$ is not a function of s.

- **System S2**

The 2nd network is composed of Laplace impedance Z_2, shunt-connected to ground line. Since Z_2 is due to the parallel connection of L_2 and C_2, it is expressed as

$$Z_2 = \frac{s\,L_2}{L_2\,C_2\,s^2 + 1}\ ,$$

(3.18)

The chain (*ABCD*) matrix, $\overline{T}_2$, is given by

$$\overline{T}_2 = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix}. \tag{3.19}$$

From Equation (3.19), the entry (2, 1) is written as

$$C_2 = 1/Z_2 = \frac{NC2}{DC2} \tag{3.20}$$

In Equation (3.20), NC2 is the numerator polynomial of *C2*, and DC2 is the denominator polynomial of *C2*, which is expressed as, respectively,

$$NC2 = L_2 C_2 s^2 + 1, \tag{3.21.a}$$

$$DC2 = L_2 s. \tag{3.21.b}$$

- **System S3**

The 3rd network is the *bridged-T network*. The *chain* (*ABCD*) matrix $\overline{T}_3$ is given by

$$\overline{T}_3 = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix}, \tag{3.22}$$

In Equation (3.22), each of the four entries of the matrix are defined as, in terms of Laplace impedances,

$$A_3 = \frac{Z_{3a}\,(Z_{3b}+Z_{34})+(Z_{3a}+Z_{3b}+Z_{34})\,Z_{33}}{Z_{3a}Z_{3b}+(Z_{3a}+Z_{3b}+Z_{34})\,Z_{33}}, \qquad (3.23.a)$$

$$B_3 = \frac{Z_{34}\,(Z_{3a}\,Z_{3b}+Z_{3a}\,Z_{33}+Z_{3b}\,Z_{33})}{Z_{3a}\,Z_{3b}+(Z_{3a}+Z_{3b}+Z_{33})Z_{33}}, \qquad (3.23.b)$$

$$C_3 = \frac{Z_{3a}+Z_{3b}+Z_{34}}{Z_{3a}Z_{3b}+(Z_{3a}+Z_{3b}+Z_{34})\,Z_{33}}, \qquad (3.23.c)$$

$$D_3 = 1 + \frac{Z_{3b}Z_{34}}{Z_{3a}\,Z_{3b}+(Z_{3a}+Z_{3b}+Z_{34})Z_{33}}. \qquad (3.23.d)$$

In Equation (3.23) the right hand side of equality is expressed in terms of impedances given in Equation (3.14) and Figure 3.6.

a) $\overline{T}_3(1,1)$ **of System S3**

The entry $\overline{T}_3(1,1)$, or A_3 , is a *rational polynomial* (a ratio of two polynomials).

$$A_3 = \frac{NA3}{DA3}. \qquad (3.24)$$

In Equation (3.24), $NA3$ is the numerator polynomial of A_3, and $DA3$ is the denominator polynomial of A_3. These are expressed as follows, respectively:

NA3 = L31 L35 L36 C36 L33 C34 L32 C35 S^6

$\quad$ + (L31 L32 L35 C35 L36 C3 + C35 L31 L35 L32 L36 C34
$\quad$ + C35 L31 L35 L33 C34 L36 + C35 L31 L35 L33 C34 L32
$\quad$ + L33 C34 L32 L35 L36 C36 + L36 L35 L31 L33 C34 C36
$\quad$ + L36 L31 L33 C34 C36 L32) S^4

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (3.25.a)

+ (C35 L31 L35 L36 + C35 L31 L35 L32 + L32 L35 L36 C36
+ L36 L35 C34 L32 + L31 L35 L36 C36 + L36 L35 L31 C34
+ L36 L35 L33 C34 + L36 L31 C36 L32 + L36 L32 L31 C34
+ L36 L31 L33 C34 + L35 L33 C34 L32 + L35 L31 L33 C34
+ L31 L33 C34 L32) S^2

+ L36 L35 + L31 L36 + L35 L32 + L31 L35 + L31 L32

DA3= L31 L35 L36 C36 L33 C34 L32 C35 S^6

+ (C35 L31 L35 L32 L36 C34+ C35 L31 L35 L33 C34 L36
+ C35 L31 L35 L33 C34 L32 + L33 C34 L32 L35 L36 C36
+ L36 L35 L31 L33 C34 C36 + L36 L31 L33 C34 C36 L32) S^4

(3.25.b)

+ (L36 L35 C34 L32 + L36 L35 L33 C34+ L36 L35 L31 C34
+ L36 L32 L31 C34 + L36 L31 L33 C34 + L35 L33 C34 L32
+ L35 L31 L33 C34 + L31 L33 C34 L32) S^2

+ L36 L35

b) $\overline{T}_3(1,2)$ **of System S3**

The entry $\overline{T}_3(1,2)$, or B_3, is a *rational polynomial* (a ratio of two polynomials) of

$$B_3 \ = \ \frac{NB3}{DB3} \ .$$

(3.26)

In Equation (3.26), the number "3" implies that all of these symbols are assigned to the subsystem S3. The notations $NB3$ is the numerator polynomial of B_3, and $DB3$ is the denominator polynomial of B_3, which are expressed in terms of element values of the filter network, as follows, respectively.

NB3= C35 L31 L35 L36 C36 L33 L32 S^5

$+$ (C35 L31 L35 L32 L36 + C35 L31 L35 L33 L36
$+$ C35 L31 L35 L33 L32 + L33 L32 L35 L36 C36
$+$ L31 L35 L36 C36 L33 + L36 L31 L33 C36 L32) S^3

(3.27.a)

$+$ (L35 L32 L36 + L31 L35 L36 + L36 L33 L35
$+$ L31 L32 L36 + L36 L31 L33 + L35 L32 L33
$+$ L35 L31 L33 + L31 L33 L32) S

DB3= L31 L35 L36 C36 L33 C34 L32 C35 S^6

$+$ (C35 L31 L35 L32 L36 C34 + C35 L31 L35 L33 C34 L36
$+$ C35 L31 L35 L33 C34 L32+ L33 C34 L32 L35 L36 C36
$+$ L36 L35 L31 L33 C34 C36+ L36 L31 L33 C34 C36 L32) S^4

(3.27.b)

$+$ (L36 L35 C34 L32 + L36 L35 L33 C34 + L36 L35 L31 C34
$+$ L36 L32 L31 C34 + L36 L31 L33 C34+ L35 L33 C34 L32
$+$ L35 L31 L33 C34 + L31 L33 C34 L32) S^2

$+$ L36 L35

c) $\overline{T}_3(2,1)$ **of System S3**

The entry $\overline{T}_3(2,1)$, or C_3 , is a *rational polynomial* (a ratio of two polynomials)

$$C_3 = \frac{NC3}{DC3} \quad .$$

(3.28)

In Equation (3.28), $NC3$ is the numerator polynomial of C_3 , and $DC3$ is the denominator

polynomial of C_3 .

These are expressed in terms of element values of the filter network, as follows,

respectively.

NC3= (L33 C34 L32 L35 C35 L36 C36 + L31 C34 L32 L35 C35 L36 C36) S^6

    + (L33 C34 L36 L35 C35 + L33 C34 L32 L36 C36
    + L31 C34 L32 L35 C35 + L33 C34 L32 L35 C35
    + L36 L31 L35 C36 C34 + L36 L31 C36 L32 C34
    + L36 L35 L32 C36 C34 + L32 L36 C34 L35 C35
    + L32 L35 C35 L36 C36+ L33 C34 L35 L36 C36
    + L31 C34 L36 L35 C35) S^4 (3.29.a)

    + (L35 L31 C34+ L35 L36 C36 + L36 L35 C35
    + L32 L36 C34 + L32 L35 C35 + L32 L36 C36
    + L32 L31 C34 + L33 C34 L32 + L35 C34 L32
    + L33 C34 L36 + L36 L31 C34 + L35 L33 C34) S^2

    + L36 + L35 + L32

DC3= C35 L31 L35 L36 C36 L33 C34 L32 S^7

    + (C35 L31 L35 L32 L36 C34
    + C35 L31 L35 L33 C34 L36 + C35 L31 L35 L33 C34 L32
    + L33 C34 L32 L35 L36 C36 + L36 L35 L31 L33 C34 C36
    + L36 L31 L33 C34 C36 L32) S^5 (3.29.b)

    + (L36 L35 C34 L32 + L36 L35 L33 C34
    + L36 L35 L31 C34 + L36 L32 L31 C34 + L36 L31 L33 C34
    + L35 L33 C34 L32 + L35 L31 L33 C34 + L31 L33 C34 L32) S^3

    + L35 L36 S

d) $\overline{T}_3(2,2)$ **of System S3**

The entry $\overline{T}_3(2,1)$, or D_3, is a *rational polynomial* (a ratio of two polynomials),

$$D_3 = \frac{ND3}{DD3} \quad .$$
        (3.30)

In Equation (3.30), polynomial $ND3$ is the numerator polynomial of D_3, and $DD3$ is

the denominator polynomial of D_3, as follows, respectively.

ND3= L31 L35 L36 C36 L33 C34 L32 C35 S^6

    + (L33 L32 L35 C35 L36 C36
    + C35 L31 L35 L32 L36 C34 + C35 L31 L35 L33 C34 L36
    + C35 L31 L35 L33 C34 L32 + L33 C34 L32 L35 L36 C36
    + L36 L35 L31 L33 C34 C36 + L36 L31 L33 C34 C36 L32) S^4

<div align="right">(3.31.a)</div>

    + (C35 L35 L32 L36 + C35 L36 L33 L35 + C35 L35 L32 L33
    + L36 L35 C34 L32+ L36 L35 L33 C36 + L36 L35 L33 C34
    + L36 L35 L31 C34 + L36 L32 L33 C36 + L36 L32 L31 C34
    + L36 L31 L33 C34 + L35 L33 C34 L32 + L35 L31 L33 C34
    + L31 L33 C34 L32) S^2

    + L36 L35 + L32 L36 + L33 L36 + L33 L35 + L33 L32

DD3= L31 L35 L36 C36 L33 C34 L32 C35 S^6

    + (C35 L31 L35 L32 L36 C34 + C35 L31 L35 L33 C34 L36
    + C35 L31 L35 L33 C34 L32 + L33 C34 L32 L35 L36 C36
    + L36 L35 L31 L33 C34 C36 + L36 L31 L33 C34 C36 L32) S^4

<div align="right">(3.31.b)</div>

    + (L36 L35 C34 L32 + L36 L35 L33 C34
    + L36 L35 L31 C34 + L36 L32 L31 C34
    + L36 L31 L33 C34+ L35 L33 C34 L32
    + L35 L31 L33 C34 + L31 L33 C34 L32) S^2 + L36 L35

- **System S4**

The *chain (ABCD)* matrix , $\overline{T}_4$, is given by

$$\overline{T}_4 = \begin{bmatrix} A_4 & B_4 \\ C_4 & D_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/Z_4 & 1 \end{bmatrix} .$$

<div align="right">(3.32)</div>

The 4nd network is composed of Laplace impedance Z_4, shunt-connected to ground line. Since Z_4 is due to the parallel connection of L_4 and C_4, it is expressed as

$$Z_4 = \frac{s\,L_4}{L_4\,C_4\,s^2 + 1}\,.$$

(3.33)

From Equation (3.33), the entry $(2,\,1)$ is expressed as

$$C_4 = 1/Z_4 = \frac{NC4}{DC4}\,.$$

(3.34)

In Equation (3.34), the quantity NC4 is the numerator polynomial of C4, and DC4 is the denominator polynomial of C4;

$$NC4 = L_4 C_4\,s^2 + 1,$$

(3.35.a)

$$DC4 = L_4\,s\,.$$

(3.35.b)

- **System S5**

The 5th subnetwork is composed of load impedance $Z_L = 50\,\Omega$ shunt-connected to the ground line. The chain matrix is given by

$$\overline{T}_L = \begin{bmatrix} 1 & 0 \\ 1/50 & 1 \end{bmatrix}\,.$$

(3.36)

All entries of matrix Equation (3.36) are constant. They are not a function of s.

Equations (3.24), (3.26), (3.28), (3.30), and (3.34) are expressed in term of filter elements. Replacing these with Laplace impedances given in Equation(3.1), the TZCE is obtained.

3.2.2 Canonical Numerator Polynomial

By using the MATLAB program of Appendix B, the canonical form of the numerator polynomial, that is, the transmission zero characteristic polynomial is obtained as

$$N(s) = 50L_2L_4\, s \cdot [a_{36}\, s^6 + a_{34}\, s^4 + a_{33}\, s^2 + a_{30}], \qquad (3.37)$$

where, in Equation (3.37) the coefficients are given by

$$a_{36} = 50\ L_{31}L_{32}\ L_{33}\ C_{34}L_{35}\ C_{35}\ L_{36}\ C_{36}\ ,$$

$$
\begin{aligned}
a_{34} =\ & L_{31}\ L_{32}\ C_{34}L_{35}\ C_{35}\ L_{36} + L_{31}L_{32}\ L_{33}\ C_{34}\ L_{35}\ C_{35} \\
&+ L_{32}\ L_{33}\ C_{34}\ L_{35}\ L_{36}\ C_{36} + L_{31}\ L_{32}\ L_{33}\ C_{34}\ L_{36}\ C_{36} \\
&+ L_{31}\ L_{33}\ C_{34}\ L_{35}\ C_{35}\ L_{36} + L_{31}\ L_{33}\ C_{34}L_{35}\ L_{36}\ C_{36}\ ),
\end{aligned}
$$

$$
\begin{aligned}
a_{33} =\ & (\ L_{32}\ C_{34}L_{35}\ L_{36} + L_{32}\ L_{33}\ C_{34}\ L_{35} + L_{31}\ L_{32}\ C_{34}\ L_{36} \\
&+ L_{31}\ L_{32}L_{33}\ C_{34} + L_{31}\ C_{34}L_{35}\ L_{36} + L_{33}\ C_{34}\ L_{35}\ L_{36} \\
&+ L_{31}\ L_{33}\ C_{34}\ L_{35} + L_{31}\ L_{33}\ C_{34}\ L_{36}\ ),
\end{aligned}
$$

$$a_{30} =\ L_{35}\ L_{36}\ .$$

(3.38)

3.2.3 Transmission Zeros of System

In Equation (3.37), equating *N(s)=0* gives the following form of transmission zero characteristic equation,

$$f(s) = s \cdot \left(a_{36} s^6 + a_{34} s^4 + a_{33} s^2 + a_{30} \right) = 0$$
$$\equiv f_1(s) \cdot f_2(s) = 0 \tag{3.39}$$

When there is no cross-coupling, C34=0, and therefore a_{36}, a_{34}, and a_{33} become zeros. Therefore, Equation (3.39) has only one zero at s=0, the origin of the complex plane.

Only when there is a cross-coupling, it is possible that Equation(3.39) can produce complex zeros.

Each of the factored polynomial of Equation (3.39) is expressed as follows;

$$f_1(s) = s, \tag{3.40.a}$$

$$f_2(s) = a_{36} s^6 + a_{34} s^4 + a_{33} s^2 + a_{30}. \tag{3.40.b}$$

Equation (3.39) shows that the network in Figure 3.6 has seven finite transmission zeros. A single *stationary zero* (static zero) is located at the origin due to Equation (3.40.a), and six dynamic zeros are located at non-origin due to Equation (3.40.b).

The zeros are determined as follows.

a) Monomial Equation $f_1(s) = s = 0$

From Equation (3.40.a), the equation

$$f_1(s) = s = 0 \tag{3.41}$$

is obtained. Equation (3.41) shows a single stationary zero at origin.

b) Sixth degree Polynomial Equation $f_2(s) = a_{36}\, s^6 + a_{34} s^4 + a_{33}\, s^2 + a_{30} = 0$

The 6th degree even polynomial $f_2(s)$ has four coefficients a_{ij} (i.e. a_{36}, a_{34}, a_{32}, and a_{30}). The first subscript i of coefficients a_{ij} indicates the i-th subsystem.

Therefore, this polynomial $f_2(s) = a_{36}s^6 + a_{34}s^4 + a_{33}s^2 + a_{30}$ comes only from the 3rd subsystem, and describes the 3rd subsystem $S3$ of the Figures 3.4 and 3.5. The 3rd subsystem is equivalent to a *bridged-T circuit*, which gives a cross-coupling between resonator # 1 and resonator # 4, skipping two resonators # 2 and 3 in the middle. It will be shown that this circuit can generate a quadruple of *complex zeros* in the response.

The 6th degree polynomial of *bridged-T circuit* has the following equation,

$$f_2(s) = a_{36}s^6 + a_{34}s^4 + a_{33}s^2 + a_{30} = 0. \tag{3.42}$$

The 6th degree polynomial of Equation (3.42) has six solutions. Depending on the relative values of four coefficients (a_{36}, a_{34}, a_{32}, and a_{30}) of the polynomial, there can be

three different (mutually exclusive) cases are possible. The task here is to solve the Equation (3.42). Let the change of variable $S \equiv s^2$. The Equation (3.42) can be expressed as

$$S^3 + \frac{a_{34}}{a_{36}} S^2 + \frac{a_{33}}{a_{36}} S + \frac{a_{30}}{a_{36}} = 0. \tag{3.43}$$

With

$$\frac{a_{34}}{a_{36}} \equiv a_2, \quad \frac{a_{33}}{a_{36}} \equiv a_1, \quad \text{and} \quad \frac{a_{30}}{a_{36}} \equiv a_0 \tag{3.44}$$

Equation (3.43) is written as

$$S^3 + a_2 S^2 + a_1 S + a_0 = 0. \tag{3.45}$$

With another changes of variables,

$$q = \frac{1}{3} a_1 - \frac{1}{9} a_2^2, \tag{3.46.a}$$

$$r = \frac{1}{6} \left(a_1 a_2 - 3 a_0 \right) - \frac{1}{27} a_2^3, \tag{3.46.b}$$

and, with another change of variables,

$$\alpha = \sqrt[3]{r + \sqrt{q^3 + r^2}}, \tag{3.47.a}$$

$$\beta = \sqrt[3]{r - \sqrt{q^3 + r^2}}, \tag{3.47.b}$$

the three solutions of Equation (3.45) take the following expressions:

$$S_1 \equiv s_1{}^2 = (\alpha + \beta) - \frac{a_2}{3}, \tag{3.48.a}$$

$$S_2 \equiv s_2{}^2 = -\frac{1}{2}(\alpha + \beta) - \frac{a_2}{3} + j\frac{\sqrt{3}}{2}(\alpha - \beta), \tag{3.48.b}$$

$$S_3 \equiv s_3{}^2 = -\frac{1}{2}(\alpha + \beta) - \frac{a_2}{3} - j\frac{\sqrt{3}}{2}(\alpha - \beta). \tag{3.48.c}$$

With the conditions given in Equations (3.46), (3.47), and (3.48), the three solutions of Equation (3.45) are classified as three categories as follows [19]:

i) If $q^3 + r^2 > 0$, there are one real root and a pair of complex conjugate roots,

ii) If $q^3 + r^2 = 0$, all three roots are real, at least two roots are equal,

iii) If $q^3 + r^2 < 0$, all three root are real, and unequal.

In terms of complex variable s in 6 degree polynomial, the roots are as follows.

Equation (3.42) is the 6th degree polynomial with all odd terms missing. It is not a Hurwitz polynomial. This means that not all the roots are in *left-half plane*(LHP). Therefore, it is possible that some roots are on the $j\omega$-axis and/or *right-half plane* (RHP), but a passive network cannot have RHP roots.

The mathematically possible cases of roots obtained from the equation are as follows:

    i) Six real roots and no complex roots,

    ii) Four real roots and two complex roots,

    iii) Two real roots and four complex roots,

    iv) No real roots and six complex roots.

In practical sense, $s = \sigma + j\omega$, where σ should be a positive real number. Therefore, only the last case produces the non-trivial solutions.

To find out zeros, the three cases are considered as follows.

Case 1: If $q^3 + r^2 > 0$, then there are one real root and a pair of complex conjugate roots. Let the roots are expressed in terms of polar form,

$$S_i \equiv s_i{}^2 = A_i\, e^{j\,\phi_i}, \quad i = 1, 2, \text{ and } 3. \tag{3.49}$$

In Equation (3.49),

    S_i's are the roots of the 3$^{\text{rd}}$ degree polynomial equation,

    s_i's are the roots of the 6$^{\text{th}}$ degree polynomial equation,

    A_i is the magnitude, and

    ϕ_i is the angle of the complex roots.

The three solutions S_i, $i = 1-3$, are given by

$$S_1 = A_1 \, e^{j0},$$ (3.50.a)

$$S_2 = A_2 \, e^{j\phi_2},$$ (3.50.b)

$$S_2^* = A_2 \, e^{-j\phi_2}.$$ (3.50.c)

Writing in terms of s, the roots of 6th degree polynomial equation are obtained.

$$s_1 = \sqrt{A_1} \quad , \qquad s_1^* = -\sqrt{A_1} \quad ;$$ (3.51.a)

$$s_2 = \sqrt{A_2} \, e^{j(\pi + \theta_2)} , s_2 = \sqrt{A_2} \, e^{j\theta_2} \quad ;$$ (3.51.b)

$$s_2^* = \sqrt{A_2} \, e^{-j(\pi + \theta_2)} \, , \quad s_2^* = \sqrt{A_2} \, e^{-j\theta_2} \, .$$ (3.51.c)

In Equations (3.50) and (3.51), the two phase angles, θ_2 and ϕ_2, of the transmission

zeros (TZ's) of the cross-coupled filter network have the relations $\theta_2 = \dfrac{\phi_2}{2}$, and "*"

means the complex conjugate.

With $|s_1| < |s_2|$, the transmission zero locations are shown in Figure 3.7.
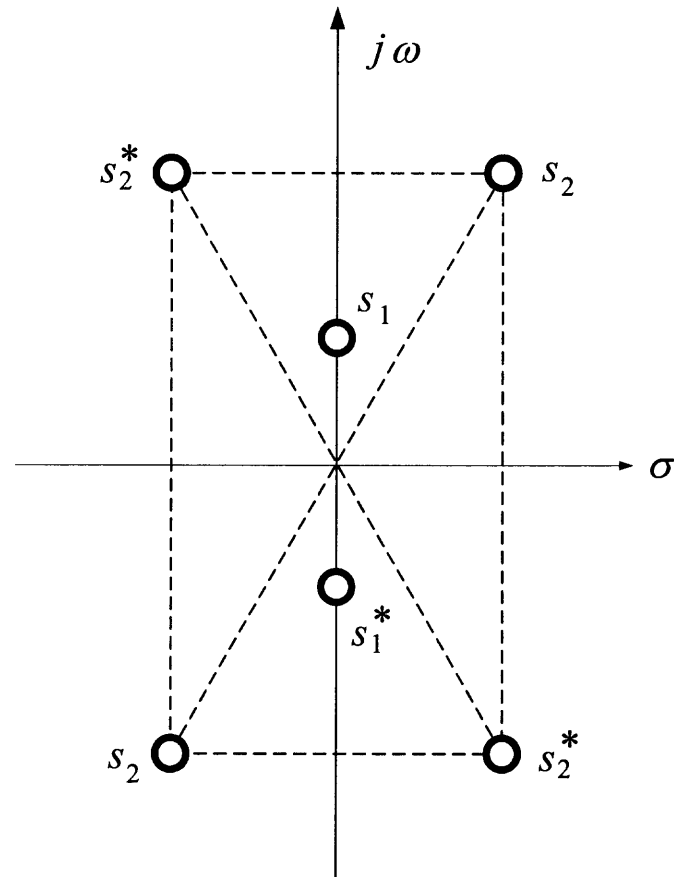
Figure 3.7 TZ locations for Case 1.

Case 2 : If $q^3 + r^2 = 0$, then there are all three roots possible, at least two are equal.

The solutions are expressed in terms of polar form, as in Equation (3.49). The three TZ

solutions $S_i, i = 1 - 3$, are given by two different cases:

$$Case\ i)\quad S_1 = A_1 e^{j0}, \quad S_2 = A_2 e^{j0}, \quad S_2 = A_2 e^{j0} \tag{3.52}$$

$$Case\ ii)\quad S_1 = A_1 e^{j0}, \quad S_1 = A_1 e^{j0}, \quad S_1 = A_1 e^{j0} \tag{3.53}$$

Writing in terms of s, the roots of 6th degree polynomial equation, the followings are obtained.

For the case of Equation (3.52),

$$s_1 = \sqrt{A_1} \, , \quad s_1^* = -\sqrt{A_1} \ ; \qquad (3.54.a)$$

$$s_2 = \sqrt{A_2} \, , \quad s_2^* = -\sqrt{A_2} \ ; \qquad (3.54.b)$$

$$s_2 = \sqrt{A_2} \, , \quad s_2^* = -\sqrt{A_2} \ . \qquad (3.54.c)$$

With $|s_1| < |s_2|$, the transmission zero locations are shown in Figure 3.8.
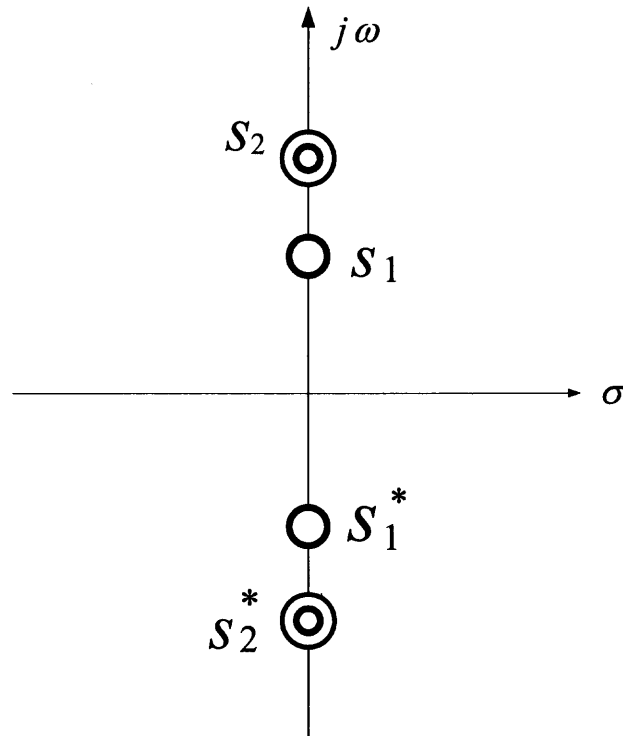
Figure 3.8 Transmission zero locations for Case 2- i .

For the case of Equation (3.53),

$$s_1 = \sqrt{A_1} \, , \qquad s_1^* = -\sqrt{A_1} \quad ; \tag{3.55.a}$$

$$s_1 = \sqrt{A_1} \, , \qquad s_1^* = -\sqrt{A_1} \quad ; \tag{3.55.b}$$

$$s_1 = \sqrt{A_1} \, , \qquad s_1^* = -\sqrt{A_1} \quad . \tag{3.55.c}$$

From Equation (3.55), the transmission zero locations are shown in Figure 3.9.
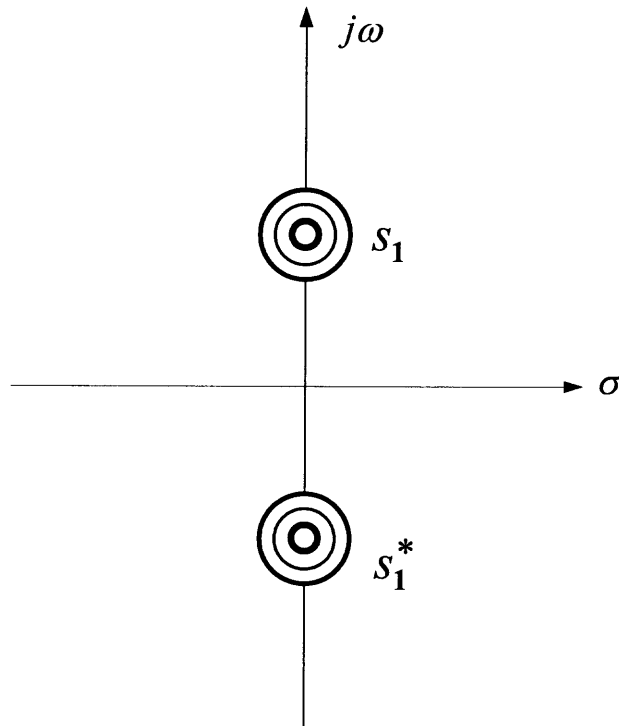

Figure 3.9 Transmission zero locations for Case 2-ii.

Case 3 : If $q^3 + r^2 < 0$, then there are all three real roots, with none of them are equal. Let the roots are expressed in terms of polar form, as in Equation (3.48),

$$S_i \equiv s_i^{\ 2} = A_i \, e^{j \, \phi_i} \, , \quad i = 1, 2, \& 3. \quad (3.49)$$

Then the three solutions S_i, $i = 1 - 3$, are given by

$$S_1 = A_1 \, e^{j\phi_1} \, , \qquad\qquad (3.56.a)$$

$$S_2 = A_2 \, e^{j\phi_2} \, , \qquad\qquad (3.56.b)$$

$$S_3 = A_3 \, e^{-j\phi_3} \, . \qquad\qquad (3.56.c)$$

Writing in terms of s, the roots of 6th degree polynomial equation, the solutions are obtained as

$$s_1 = \sqrt{A_1} \, , \quad s_1^{\ *} = -\sqrt{A_1} \; ; \qquad\qquad (3.57.a)$$

$$s_2 = \sqrt{A_2} \, , \quad s_2^{\ *} = -\sqrt{A_2} \; ; \qquad\qquad (3.57.b)$$

$$s_3 = \sqrt{A_3} \, , \quad s_1^{\ *} = -\sqrt{A_3} \; . \qquad\qquad (3.57.c)$$

Assuming $|s_1| < |s_2| < |s_3|$, the transmission zero locations are shown in Figure 3.10.
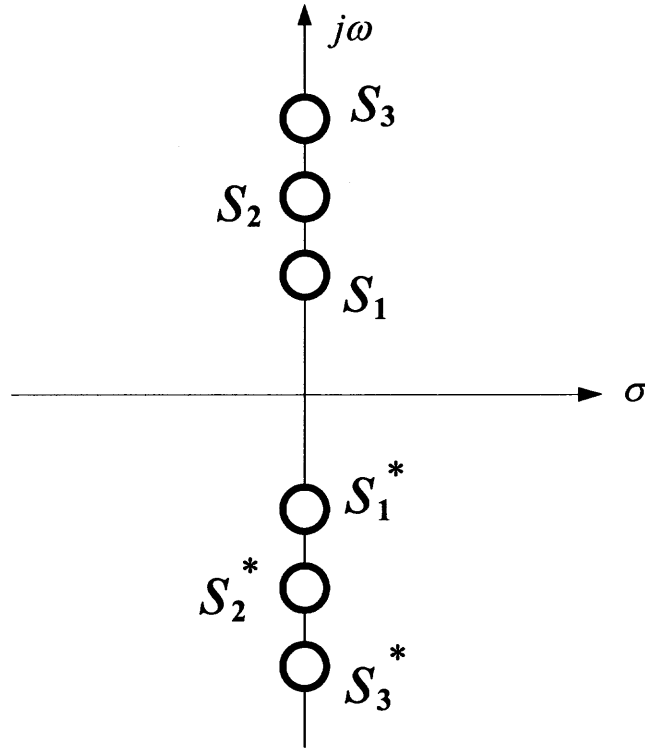
Figure 3.10 Transmission zero locations for Case 3.

3.2.4 Locus of Transmission Zeros

Based on Figures 3.7, 3.8, and 3.10, the transmission zero locus is shown in Figure 3.11.
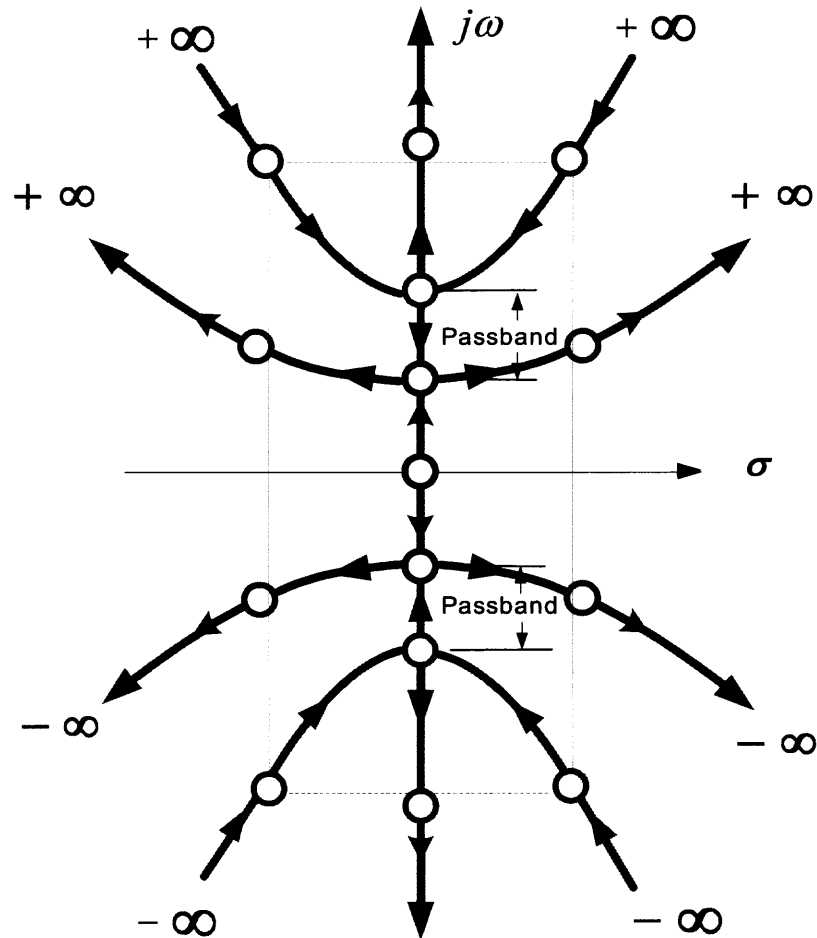
Figure 3.11 Transmission zero locus based on Figures 3.7, 3.8, and 3.10.

In Figure 3.11 transmission zeros are located at the both ends of passband. When the zeros are approaching, the width of passband is decreasing. In the extreme case, the transmission zeros are overlapping. As a result, the passband is disappeared. This case is considered in Figure 3.12.

Based on Figures 3.7, 3.9, and 3.10, the transmission zero locus is shown in Figure 3.12.
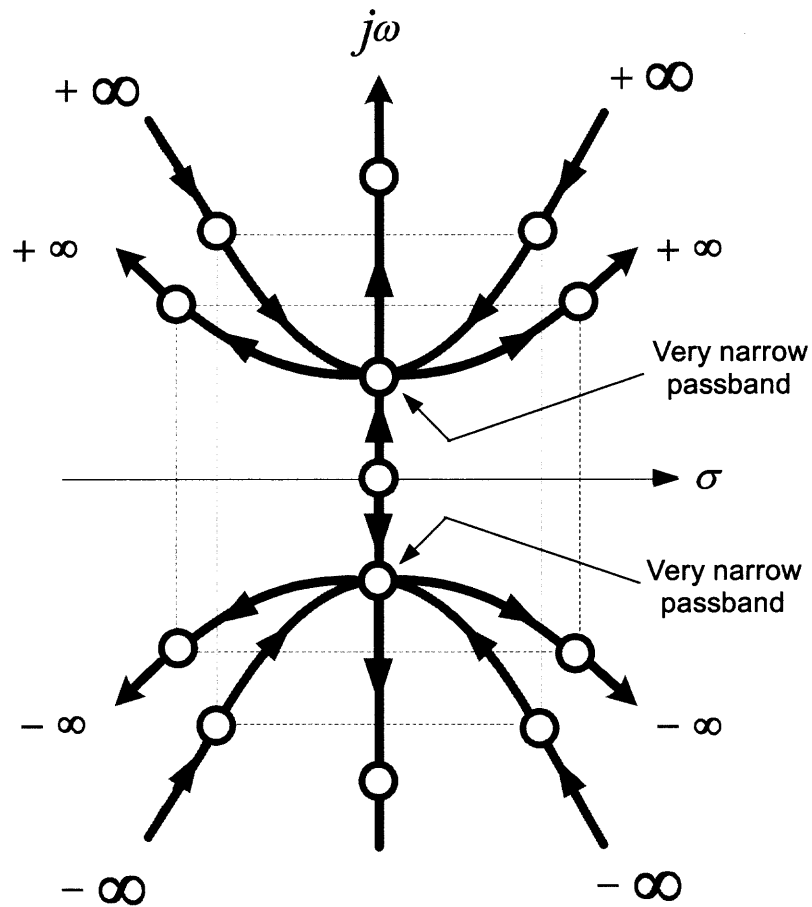
Figure 3.12 Transmission zero locus based on Figures 3.7, 3.9, and 3.10.

In Figure 3.12, two dynamic zeros are overlapping. The passband is very narrow. In the limit case, the passband is disappeared.

3.3 Positively Cross-coupled (PCC) Network

In Section 3.2, a *negatively cross-coupled* (NCC) filter was investigated. In this section, a *positively cross-coupled* (PCC) is considered. Since the procedure to derive the TZ characteristic equation is the same, the detailed discussions are avoided. Instead, as a start, TZCE is used to find out TZ locations and locus.

Figure 3.13 Positively cross-coupled (PCC) Network.

Following the same procedures as in the NCC network, The TZ characteristic equation is obtained as

$$f(s) = s \cdot [a_{34}s^4 + a_{32}s^2 + a_{30}]$$

(3.58)

$$\equiv f_1(s) \cdot f_2(s) = 0 .$$

In Equation (3.58), each of the factored terms are expressed as

$$f_1(s) = s = 0,$$
(3.59)

$$f_2(s) = a_{34}s^4 + a_{32}s^2 + a_{30} = 0.$$
(3.60)

In Equation (3.60), the coefficients are given by
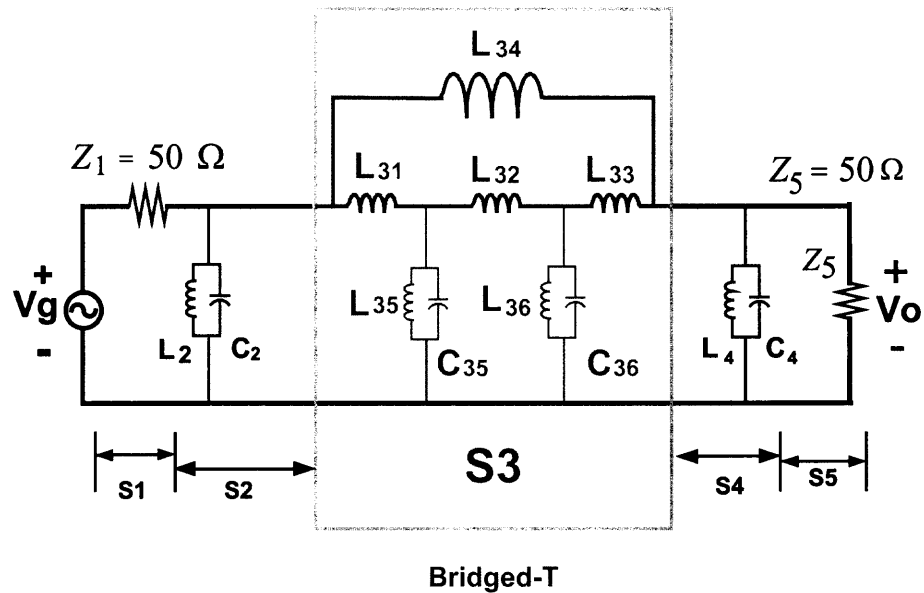
$$a_{34} = L_{31}\ L_{32}\ L_{33}\ L_{35}\ C_{35}\ L_{36}\ C_{36},$$

$$\begin{aligned} a_{32} = (\ &L_{31}\ L_{33}\ L_{35}\ C_{35}\ L_{36} + L_{31}\ L_{33}\ L_{35}\ L_{36}\ C_{36} \\ &+ L_{32}\ L_{33}L_{35}\ L_{36}\ C_{36} + L_{31}L_{32}\ L_{33}\ L_{36}\ C_{36} \\ &+ L_{31}\ L_{32}\ L_{35}\ C_{35}\ L_{36} + L_{31}\ L_{32}\ L_{33}\ L_{35}\ C_{35}\ ), \end{aligned}$$
(3.61)

$$\begin{aligned} a_{30} = (\ &L_{31}\ L_{35}\ L_{36} + L_{31}\ L_{33}\ L_{35} + L_{31}\ L_{33}\ L_{36} \\ &+ L_{32}L_{35}\ L_{36} + L_{32}L_{33}\ L_{35} + L_{31}\ L_{32}\ L_{36} \\ &+ L_{31}L_{32}\ L_{33} + L_{34}\ L_{36}\ L_{35} + L_{33}\ L_{35}\ L_{36}\ ). \end{aligned}$$

The coefficients a_{34}, a_{32}, and a_{30} given in Equation (3.61) are all positive.

Without cross-coupling, the canonical form of numerator polynomial is given by

$$N(s) = 50\,L_2 L_{35} L_{36} L_4 \cdot s$$

$$\equiv 50\,K \cdot s\ .$$
(3.62)

In Equation (3.62), the parameter K is given by

$$K = L_2 L_{35} L_{36} L_4\ .$$
(3.63)

This shows that numerator polynomial $N(s)$ is simply a monomial, where the positive coefficient K in Equation (3.63) is simply calculated from the product of all of the shunt inductors in all the resonators. This conclusion is true for all the previous example networks.

Since Equation (3.58) is the same form as Equation (2.52) in Chapter 2, the TZ locus is also the same as the Figure 2.10.

3.4 Chapter Summary

In this chapter, the microwave bandpass filter network with negative and/or positive cross-coupling element is discussed. The cross-coupling element is added between two resonators.

The filters assembled with cross-coupling element, skipping no resonators and skipping two resonators, are investigated.

As was theoretically investigated in Chapter 2, the filter network is sub-sectioned into 5 subsystems cascaded to take advantage of the properties of chain matrix. By solving the TZCE, the locations and locus of the TZ's are theoretically derived.

The 1st filter produces a TZCE, expressed by the product of a monomial and a quadratic equation. The TZ's are composed of a single stationary zero and two $zero\text{-}\sigma$ dynamic zeros. The location and locus of TZ's are plotted in the complex plane.

The 2nd filter is analyzed by introducing the equivalent filter network for the cross-coupled subsection. The TZCE is expressed by the product of a monomial and the 6th degree even polynomial. The solution of the polynomial produces a three pairs of complex dynamic zeros. The location and locus of TZ's are plotted in the complex plane.

CHAPTER 4

NUMERICAL EXAMPLE OF PRACTICAL FILTER NETWORK

In Chapters 2 and 3, the *transmission zero* (TZ) locations and locus of a cross-coupled filter was investigated. The elements L and C were used without specifying values.

With the unknown element values of the filter network, the 2nd, 4th, and 6th degree characteristic equations for the dynamic TZ's were derived with unknown coefficients. The loci of TZ's are the results of characteristic equations with unknown parameters.

By continuous change of element values, the coefficients of the *characteristic equations* (CE's) are changing. The solutions the CE's are the locus of transmission zeros in complex plane. Therefore, the *locus* is obtained. Once the coefficients are given in terms of element values, the coefficients of the TZCE are expressed in terms of real numbers. Then, the solution of the equation with real number coefficients is obtained to represent *locations of zeros,* not locus.

As was proved in Chapters 2 and 3, the cross-coupled subsystem produced the complex TZ's, depending on the relative values of the element values (and therefore, the value of coefficients of transmission zero characteristic equations), there was a possibility of complex transmission zeros. That means that the transmission zero characteristic equations are solely due to the cross-coupled subsystem.

In this chapter, a practical cross-coupled filter with real element values is considered. The closed-form expression in terms of element values is obtained, locating TZ's by solving the TZ characteristic equation. It again verifies the important result that an integer pairs of complex TZ's (such as doublet, triplet, and/or quadruplet, and TZ's) are shown to result solely from the cross-coupled portion of the circuit.

NOMENCLATURE

Q: Quality factor (Selectivity) of a network. Ratio of the center frequency to the
bandwidth, used to measure the width of the passband.

ADS: Advanced Design Systems, a circuit and EM simulator of Agilent.

S11: Reflection coefficient seen at port 1 when port 2 is terminated in matched load.

S21: Transmission coefficient from port 1 to port 2.

Insertion loss: $IL = -20\log|S21|$ dB.

VNA: Vector Network Analyzer.

4.1 Lossless Filter

4.1.1 Lossless Filter Configuration

An ideal or lossless *negatively cross-coupled* (NCC) lossless bandpass filter network,

synthesized with the numerical real values of all the elements, is considered in Figure 4.1.
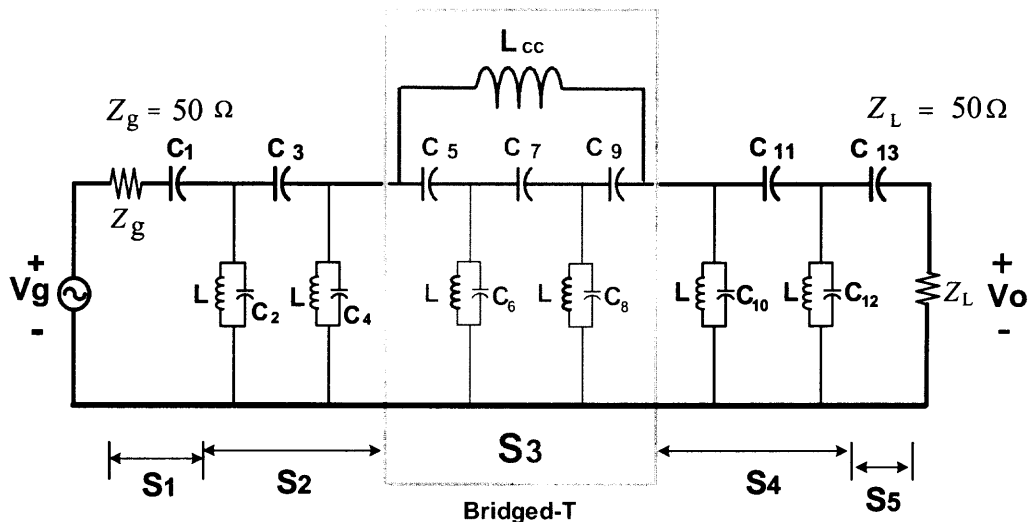

Figure 4.1 NCC filter network, with elements values specified.
(If finite quality factor is taken into account, then practical lossy filter is obtained.
This practical bandpass filter is designed and realized at RS Microwave Company Inc..
Butler, New Jersey, USA. It is discussed in Section 4.2.)

Since the *quality factor* (selectivity) is not considered in the reactive elements of the filter shown in Figure 4.1, the filter is a *lossless* (ideal) filter. The practical lossy filter obtained by considering the finite selectivity will be investigated in the next Section 4.2.

There are six *LC*-resonators, shunt-connected, with two in the crossed-coupled subsection. As was investigated in the previous chapters, this filter can be investigated by considering as a cascaded connection of five subsystems S_i ($i = 1-5$), to make use of the properties of chain matrices. Each of the element values in the figure is as follows;

1. Source impedance; Zg= 50 Ω.

2. Series- or shunt-connected capacitors have the values of Q=1000, and given by

$$
\begin{aligned}
&C1 = 16.20 \text{ pF}; \quad C2 = 35.90 \text{ pF}; \quad C3 = 3.00 \text{ pF}; \quad C4 = 49.00 \text{ pF}; \\
&C5 = 2.30 \text{ pF}; \quad C6 = 50.90 \text{ pF}; \quad C7 = 8.10 \text{ pF}; \quad C8 = 50.90 \text{ pF}; \\
&C9 = 2.30 \text{ pF}; \quad C10 = 49.10 \text{ pF}; \quad C11 = 2.70 \text{ pF}; \quad C12 = 37.00 \text{ pF}; \\
&C13 = 14.70 \text{ pF}.
\end{aligned} \tag{4.1}
$$

3. Shunt-connected inductors have the values given by

   L3 = L5 = L35 = L36 = L6 = L8 = 100 nH, and Q=180.

4. Cross-coupling (CC) inductor; Lcc=19200 nH, and Q=30 .

5. Load impedance; Z_L = 50 Ω.

4.1.2 Filter Response

For the lossless bandpass filter shown in Figure 4.1, the quality factor Q is considered to be infinity. With $Q = \infty$ and the finite element values given by Equation (4.1), the simulated result of the filter represents the response of a bandpass characteristics, as shown in Figure 4.2. The figure is obtained by a circuit simulator, Ansoft Ensemble [21].

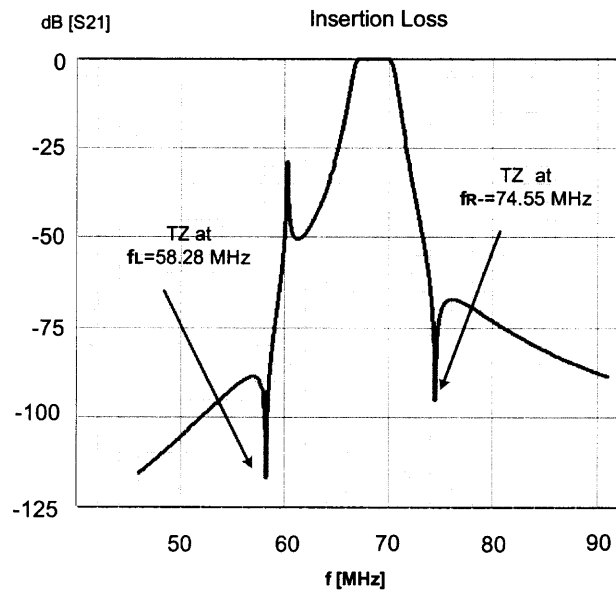**Figure 4.2** Response of the filter given in Figure 4.1

To see in more detail the locations of reflection zeros of the passband, the figure is clearly magnified for the frequency range of (67-75 MHz) and for the insertion loss of (0.0-0.4 dB). The figure shows three pairs of refection zeros at about 0.0 dB values.
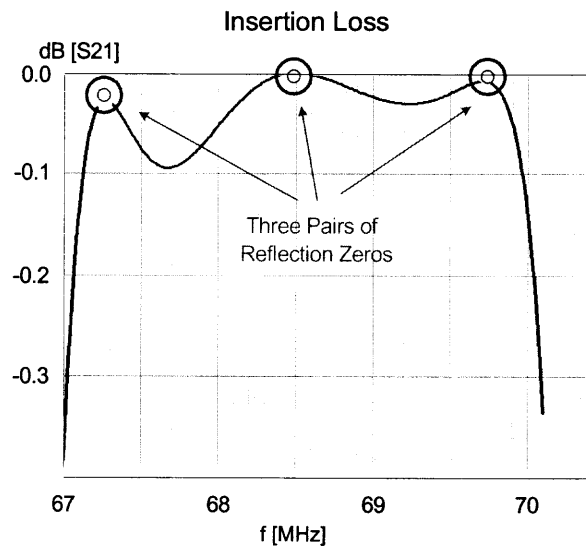

Figure 4.3 Negatively coupled filter network, with elements values specified.

There are six reflection zeros indicated in the figure. The frequency distance based on the reflection zero is;

$$(69.7\text{-}67.3) \text{ MHz} = 2.4 \text{ MHz}. \qquad (4.2)$$

Result of Equation (4.2) is also found by S21 response plots, page 3, designed and measured by RS Microwave Company Incorporated.

4.1.3 Transmission Zero Characteristic Equation

The network in Figure 4.1 is composed of five subsystems cascaded, with six resonators. The subsystem S3 has two resonators. The cross-coupled filter network is investigated by obtaining TZCE.

By using the modified MATLAB program in Appendix B, the canonical numerator and denominator polynomials of the bandpass filter are obtained. From those polynomials, TZ's and TP's are obtained

The locations and locus of transmission poles of the feedback control systems are the major concern of the control engineers. In the feedback control system, locations and locus of transmission poles are investigated in the topic of root locus. Considering the locations of poles, the feedback system should be designed to satisfy the stability criteria.

The main purpose of the control engineer is to design a feedback controller (compensator), such as proportional controller, integrators and/or differentiators (called PID controllers).   To test the stability of system, the control theory discusses the denominator polynomials of the transfer function, and the denominator polynomial must have Hurwitz characteristics.

On a safe side, the Nyquist plot and Routh-Hurwitz theory are also used in compensation for the root locus.

However, for microwave engineers, the main concern is TZ locations rather than TP locations.

In this dissertation, the TZ's are the main concern. Therefore, only the TZ locations and locus are investigated in detail. For TP's, the locations are discussed, but the locus is not.

By modifying the MATLAB program in Appendix B, with element values of Equation (4.1), the TZCE is given by

$$f(s) = k \cdot s^7 \cdot [a_6 \cdot s^6 + a_4 \cdot s^4 + a_2 \cdot s^2 + a_0] = 0. \tag{4.3}$$

In Equation (4.3), the coefficients are given by

$$
\begin{aligned}
k &= 0.2020529420 \times 10^{110}, \\
a_6 &= 0.4606599478 \times 10^{63}, \\
a_4 &= 0.2067329192 \times 10^{82}, \\
a_2 &= 0.6864817635 \times 10^{99}, \\
a_0 &= 0.5599361855 \times 10^{116}.
\end{aligned}
\tag{4.4}
$$

Equation (4.3) produces 13 TZ's, i.e. seven stationary zeros at origin and six dynamic zeros at non-origin. There is one at infinity, which will become clear in the next section.

From Equations (4.3) and (4.4), the TZ's are found to have the following values.

$$s = 0, 0, 0, 0, 0, 0, 0, \tag{4.5}$$

$$s = 10^9 \times (0 - 2.03331195711816\,j), \tag{4.6.a}$$
$$s = 10^9 \times (0 + 2.03331195711816\,j), \tag{4.6.b}$$

$$s = 10^9 \times (0 - 0.46821064295932\,j), \tag{4.7.a}$$
$$s = 10^9 \times (0 + 0.46821064295932\,j), \tag{4.7.b}$$

$$s = 10^9 \times (0 - 0.36621421575419\,j), \tag{4.8.a}$$
$$s = 10^9 \times (0 + 0.36621421575419\,j)\,. \tag{4.8.b}$$

TZ's given in Equation (4.5) show that there are seven stationary zeros at origin.

Equations (4.6.a-b), (4.7.a-b), and (4.8.a-b) show that the *zero - σ* TZ's of the filter network given in Figure 4.1 are in complex conjugate pairs, respectively. These three pairs of TZ's are dynamic complex TZ's on $j\omega$-axis.

4.1.4 Locations of Transmission Zeros

All of the 13 zeros obtained by Equations (4.5-8) are on the $j\omega$-axis as shown in the Figure 4.4. There are seven static zeros at origin. The other six dynamic zeros are at non-origin. The dynamic TZ's given in Equation (4.6) are indicated at the top and bottom locations.

The TZ's given by the Equations (4.7) and (4.8) are indicated in Figure 4.4, by the frequency relation of $\omega = \pm 2\pi f$, where $f_R = 74.55$ MHz and $f_L = 58.28$ MHz, respectively, obtained from Figure 4.2.

2.5×10^9

$j\omega$

2

$\omega = 2.0333\,[rad/s]$

1.5

TZ plot

1

$\omega = \pm 2\pi f$

$= \pm 2\pi \times 74.55\,MHz$

$= \pm 0.4682\,[rad/\sec]$

0.5

Imaginary
axis

0

σ

-0.5

$\omega = \pm 2\pi f$

-1

$= \pm 2\pi \times 58.28\,MHz$

$= \pm 0.3662\,[rad/\sec]$

-1.5

-2

$\omega = -2.0333\,[rad/s]$

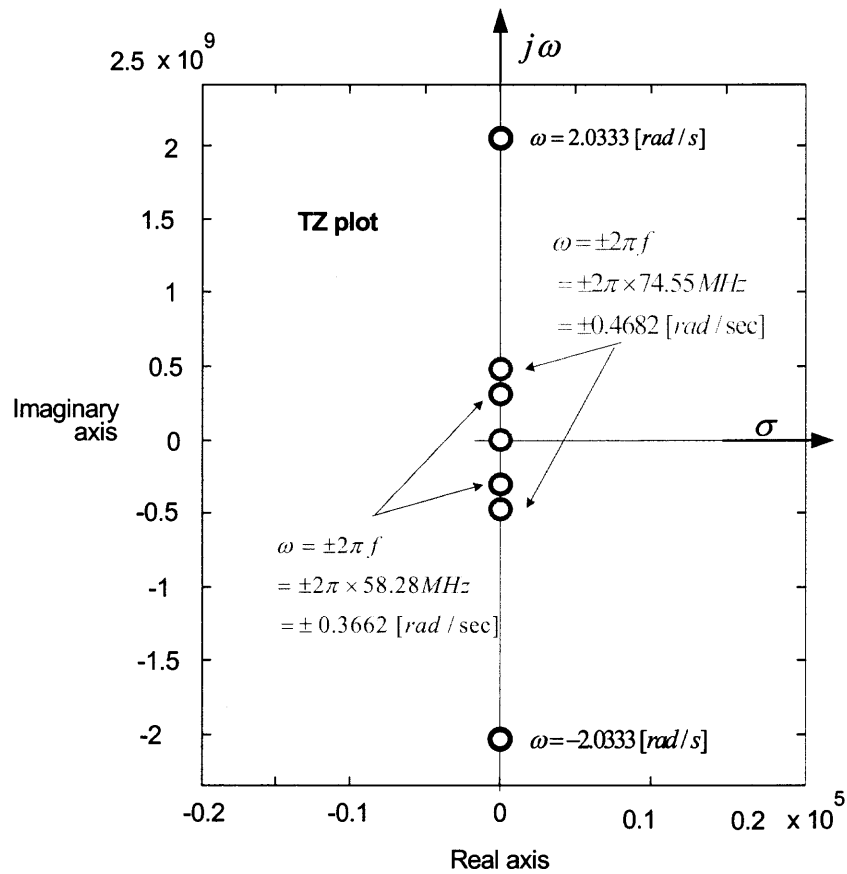-0.2   -0.1   0   0.1   0.2 $\times 10^5$

Real axis

Figure 4.4 Complex conjugates TZ locations of filter given in Figure 4.1.

It is clear that TZ locations in Figure 4.4 satisfy the TZ location and locus investigated in Chapter 3. They are in the mirror image with respect to real axis.

4.1.5 Transmission Poles of Denominator Polynomial

The denominator polynomial is given by a 14th degree polynomial;

$$
\begin{aligned}
D(s)= {}&0.216\times10^{170}\ s^{14} + 0.784\times10^{179}\ s^{13} + 0.975\times10^{188}\ s^{12} \\
&+ 0.902\times10^{197}\ s^{11} + 0.898\times10^{206}\ s^{10} + 0.429\times10^{215}\ s^{9} \\
&+ 0.378\times10^{224}\ s^{8} + 0.108\times10^{233}\ s^{7} + 0.866\times10^{241}\ s^{6} \\
&+ 0.152\times10^{250}\ s^{5} + 0.113\times10^{259}\ s^{4} + 0.114\times10^{267}\ s^{3} \\
&+ 0.785\times10^{275}\ s^{2} + 0.352\times10^{283}\ s + 0.228\times10^{292}
\end{aligned}
$$

$$(4.9)$$

Transmission pole CE, i.e. D(s) = 0, from Equation (4.9) produces the 14 poles given by

$$s = 10^{9} \times$$

$$
\begin{array}{lll}
-1.84346523620811 & & \\
-1.70885736782103 & & \\
-0.01272688553032 & + & 0.44228013639003\ j \\
-0.01240041265150 & - & 0.43493513336094\ j \\
-0.01240041265150 & + & 0.43493513336094\ j \\
-0.00802741537402 & - & 0.42758913719640\ j \\
-0.00802741537402 & + & 0.42758913719640\ j \\
-0.00357826279085 & - & 0.44152059049047\ j \\
-0.00357826279085 & + & 0.44152059049047\ j \\
-0.00280277335778 & - & 0.42010065163781\ j \\
-0.00280277335778 & + & 0.42010065163781\ j \\
-0.00000661605496 & - & 0.37868253963818\ j \\
-0.00000661605496 & + & 0.37868253963818\ j\ .
\end{array}
$$

$$(4.10)$$

Since D(s) is a strict Hurwitz polynomial, all the poles in Equation (4.10) are in the LHP.

None of the poles are on the $j\omega$-axis nor in RHP.

These 14 poles are plotted in the entire s-plane, as shown in Figure 4.5. Two real

poles are located too far away to be shown in this figure.

To see in more detail the pole locations of the filter around the origin, the pole

plot is clearly magnified in the range of $(-25)\times 10^{6}-(+5)\times 10^{6}$ for the real axis, and in

the range of $(-6)\times10^{8}-(+6)\times10^{8}$ for the imaginary axis. The figure shows six pairs of

poles , with each of the pairs in mirror image of real axis.
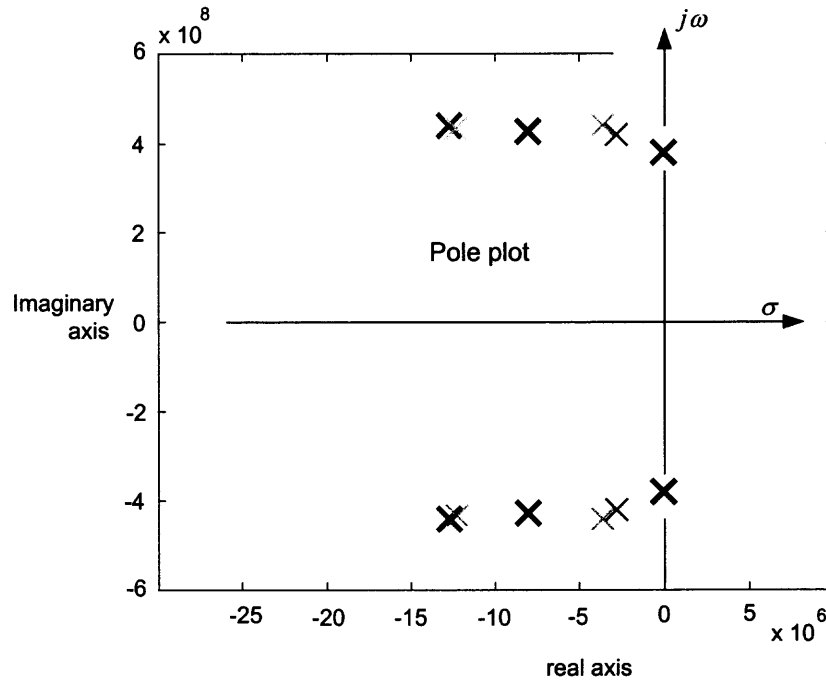
Figure 4.5 Transmission pole locations of filter given in Figure 4.1.

4.1.6 Locations of Transmission Zeros and Poles

To show all the zeros and poles in the same plot, Figure 4.6 is plotted below.
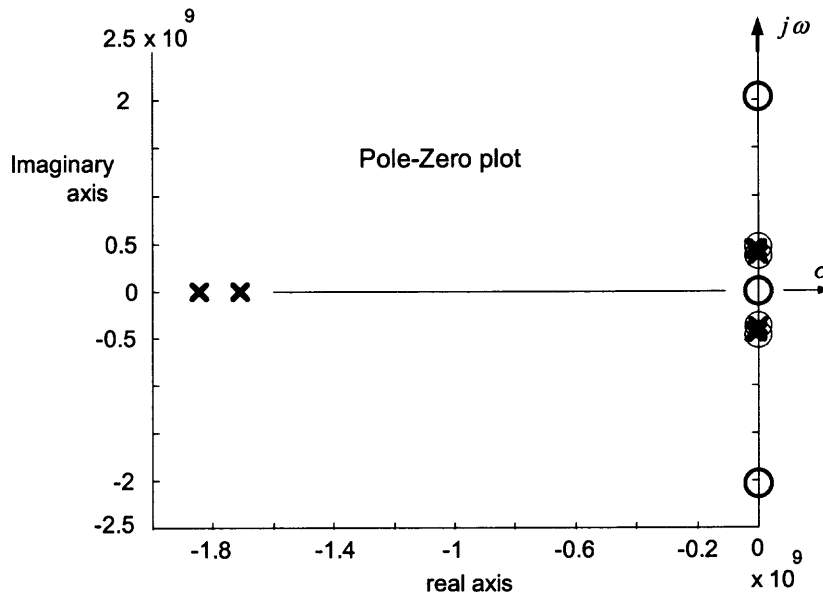
Figure 4.6 Transmission pole/zero locations of filter given in Figure 4.1.

The poles shown in Figure 4.5 are located close to $j\omega$ -axis. They are too closely located to be clearly seen. Two positive dynamic zeros are too close each other on $+j\omega$ -axis and two negative dynamic zeros are too close each other on $-j\omega$ -axis. At the origin, seven-fold static (stationary) zeros are positioned.

4.2 Lossy Cross-coupled Filter

4.2.1 Lossy Filter Configuration

By considering Q in the inductors and capacitors, the losses of the filter are considered.

The ideal inductor L is represented as the addition of series resistor,

$$R = \frac{2\pi f_c L}{Q}.$$ (4.11)

The ideal capacitor C is represented as the addition of parallel conductance,

$$G = \frac{2\pi f_c C}{Q}.$$ (4.12)

In Equations (4.11) and (4.12), f_c is the center frequency of the bandpass filter, Q is the quality factor (or selectivity), and the L and C are lossless inductors and capacitors, respectively.

The NCC bandpass filter designed and   realized at RS Microwave Inc. has the center frequency of 68.5000 MHz, and the values of inductors and capacitors are given in Equation (4.1) [21].

With all of these values considered in inductors and capacitors, the lossy circuit is obtained. By using Equations (4.11) and (4.12), the values of series resistances and parallel conductances are calculated.

The calculated resistor values are used in series with the ideal inductors, and the calculated conductance values are used in parallel with the ideal capacitors.

4.2.2 Simulation of Lossy Filter

As one of the circuit simulators, ADS (Advanced Design Systems) manufactured by the Agilent Company can take direct values of Q in the capacitors and inductors to build schematics. Or it can take the values of R and G in the additional elements. The simulated response by ADS is shown in Figure 4.7 [22].

As shown in the figure, The frequency range to be considered is from 41 MHz to 91 MHz. The center frequency of the bandpass filter is shown to be fc = 68.5 MHz.

Since the filter is a practical (non-ideal) bandpass filter, the maximum insertion loss in the passband is not 0 dB, but it is about −5 dB.
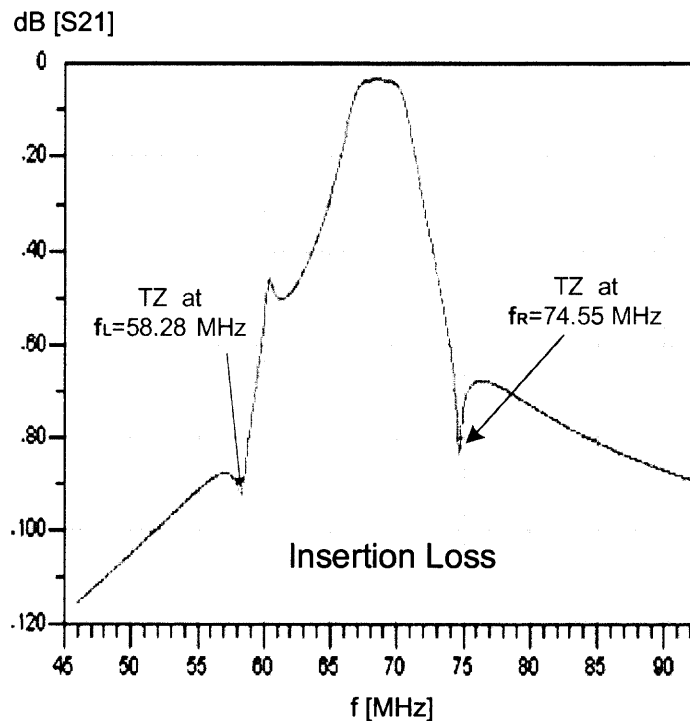
Figure 4.7 Simulated response of lossy filter, obtained by ADS.

The plot of insertion loss S21 [dB] obtained by ADS is based on the locations of zeros and poles. In ADS the zero vectors and pole vectors are considered to plot s-parameters (S11, S12, S21, and S22), phase, and group delay,…etc.. The simulator ADS does not have a function to show pole/zero locations. Without showing pole/zero location, it just shows the filter responses.

As shown in Figures 4.2 and 4.7, TZ's are positioned at the both ends of passband region. It is clear that the TZ positions obtained by simulation are exactly matched with the theoretically calculated values given by Equations (4.7) and (4.8).

Since the magnitudes of s values in the equations are computed in terms of radian frequency ω, TZ locations in the Figure 4.2 and 4.7 are verified by

$$f_L = \frac{10^9 \times (\ 0.36621421575419\ )}{2\pi} = 58.28\,MHz \qquad (4.12.a)$$

and

$$f_R = \frac{10^9 \times (\ 0.46821064295932\ )}{2\pi} = 78.55\,MHz\,. \qquad (4.12.b)$$

In Equation (4.12), f_L and f_R are the left frequency and right frequency of passband of Figures 4.2 and 4.7, respectively.

Therefore, the theory developed in the previous chapters are valid for locating the TZ's of the filter network shown in Figure 4.1.

4.2.3 Measured Response of Lossy Filter Network
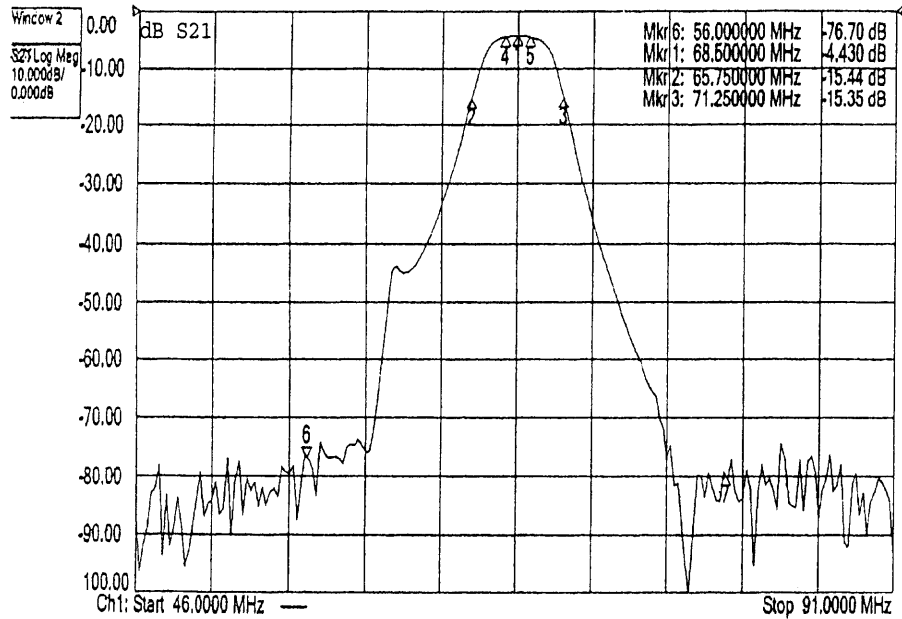
Channel 1
Center Frequency 68.5000 MHz
Span Frequency 45.0000 MHz
Start Frequency 46.0000 MHz
Stop Frequency 91.0000 MHz
Number of Points 201
Power 0.000000 dBm
IF Bandwidth 35.000000kHz
Sweep Type: Linear
Sweep Time 6.030000mSec

Trace Marker Summary

Window 2

S21

Mkr #	Ref	Frequency (MHz)	Response
1		68.5	-4.4305 dB
2		65.75	-15.443 dB
3		71.25	-15.351 dB
4		67.75	-4.6174 dB
5		69.25	-4.6685 dB
6		56	-76.704 dB

Figure 4.8 Measured response of lossy filter from VNA.

The *negatively cross-coupled* (NCC) bandpass filter network with finite quality factors given in Equation (4.1) is designed and realized at RS Microwave Company Inc.. Butler, New Jersey, USA.

The response of the realized filter measured by *vector network analyzer* (*VNA*) is shown in Figure 4.8. To have the same frequency range as that of ADS, the same frequency range (46 MHz to 91 MHz) is considered.

Comparing Figures (4.7) with (4.8), it is clear that the figures agrees well with each other.

This result again shows that the theory developed in the earlier chapters are valid for locating the TZ's of the cross-coupled filter network.

4.3 Chapter Summary

In this chapter, a lossy and lossless cross-coupled filters with real element values are considered to verify the theory developed by the author in the earlier chapters.

A lossless NCC filter network is first designed, and the response is obtained by simulation. Using the theory, a closed-form expression of *transmission zero characteristic equation* (TZCE) in terms of elements is obtained. The derived TZCE is a 13th degree polynomial which produces seven stationary zeros and six $zero - \sigma$ dynamic zeros.

By considering the finite Q in the reactive elements, a practical microwave filter is designed and realized at RS Microwave Company Inc.. It is shown that, by this practical filter network, complex TZ's are only due to the cross-coupled element.

The TZ locations quantitatively calculated from the theory developed in the author's dissertation and the simulated and measured results of TZ locations of the filter

designed at RS Microwave Company filter *do* agree with each other.

For the RS Microwave bandpass filter, the negatively cross-coupling inductor has a value of *Lcc* =19200 nH. With this value of inductance, the TZ's are located as shown in Figure 3.10 in the dissertation.

If *Lcc* is increasing, the TZ's on the $j\omega$-axis are moving to have the forms given in Figure 3.8. If *Lcc* is increasing further, the overlapped TZ's are beginning to split from the axis. Two TZ's are on the $j\omega$-axis, but the four TZ's are located as a quadruplet. It is shown in Figure 3.7.

Therefore, locations of TZ's are obtained. It again verifies the important result that cross-coupled filter produces the complex TZ's.

The *transmission poles* (TP's) derived by the theory are shown in the pole-zero plot. However, the TP's are not yet compared with the realization and/or simulation. This work will be included in the future work.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

In this dissertation, a theoretical investigation of a practical method to determine *quantitatively* the locus and location of complex *transmission zeros* (TZ's) in the cross-coupled microwave filter network was presented.

To take advantage of chain matrices applied to cascaded subsystem, the cross-coupled subsystem was considered as a bridged-T network. Since a filter network is two-port linear system, the transfer function was derived by taking advantage of the chain matrices applied to cascaded subsystem.

The subsystem was characterized by its own chain matrix. The cascaded chain matrices represent the whole filter network. The matrix entry (1, 1) was used to find transfer function.

The transfer function was expressed as a ratio of numerator polynomial and denominator polynomial. After the common terms were cancelled out in numerator and denominator, the canonical form of transfer function was obtained.

The canonical form of numerator polynomial was defined as the *transmission zero characteristic equation* (TZCE). The TZCE was expressed as a product of a monomial and an even polynomial. The even polynomial was shown to be originated only from the cross-coupled portion of the filter network.

The monomial produced a stationary zero at the origin, and the even polynomial produced a doublet, quadruplet, and sextuplet complex TZ's. A continuous perturbation of the element values (L or C) of the filter network resulted in the loci of TZ's.

The *quantitative* investigation in this dissertation is unique in that it *theoretically* proved that cross-coupled filter produces complex TZ's.

Many other types of cross-coupled filters are possible. A cross-coupling element could be a parallel and/or series connected element. The cross-coupling branch could be nested inside another cross-coupling branch, a distributed device such as a transmission line could be combined with distributed elements.

Future work will include these kinds of filter networks with various cross-coupling elements added.

APPENDIX A

NOMENCLATURE

This nomenclature is used to define or explain the terminology, notations, and symbols used in this dissertation. Some definitions are generally acknowledged and some are defined only in this dissertation.

CHAPTER 1

Rational polynomial function : A polynomial quotient of two polynomials.

H(s): Transfer function. The ratio of output to input quantities of a linear time-invariant system in Laplace domain.

N(s): Numerator polynomial of H(s).

D(s): Denominator polynomial of H(s).

Canonic: The simplest possible.

Canonic transfer function: Transfer function with all common terms cancelled out between numerator and denominator polynomials.

Canonic numerator: Numerator of a canonic transfer function.

Canonic denominator: Denominator of a canonic transfer function.

Transmission zeros: The roots of numerator polynomial of a canonical transfer function.

Stationary (static) zeros: The stationary zeros are the zeros that do not change location in spite of the change of the element values comprising the system. The stationary zeros are located at the origin of the complex s-plane.

Dynamic zeros: The dynamic zeros are the zeros that do change the locations as a function of the element values comprising the system. It is located in finite plane or infinite plane. The dynamic zeros are of the 2 types.

Zero-σ dynamic zeros: The dynamic zeros that move only along the $j\omega$-axis.

Nonzero-σ dynamic zeros: The dynamic zeros that can move onto any other locations in the $j\omega$-axis of the complex s-plane.

Two-port system: A system that has one input and one output.

Chain (ABCD) matrix: A matrix that relates output voltage and current to input voltage and current of a two-port system.

CHAPTER 2

Locus: The path of motion for dynamic TZ's or TP's as functions of cross-coupling.

Doublet: Two transmission zeros in complex conjugate pairs, with real part zero.

Quadruplet: Four transmission zeros, with two TZ's are in complex conjugate pairs, respectively.

Hurwitz polynomial $f(s)$: Polynomial whose roots of the $f(s) = 0$ is in LHP.

$\overline{T}(i,j)$: The entry located at the i-th row and j-th column of 2×2 chain matrix $\overline{T}$.

Ladder network: A network composed of series-connected and parallel-connected elements, such that every element is alternately in series-connected and shunt-connected as the signal travel from the source to the load.

Cross-coupling: An additional connection of element between two nodes in the network.

Chebyshev response: A filter response, with ripples in the passband and/or stopband.

$S_i (i = 1 - 5)$:   The subsystem built at the i-th location of the cascaded network, where, the subscript $i = 1$ means the 1st subsystem numbered from the source side.

$\overline{T}_i (i = 1 - 5)$: The chain matrix of $S_i (i = 1 - 5)$.

Zm, Z_m, $\overline{Z}m$, or $\overline{Z}_m$:The Laplace impedance of the m-th subsystem with only one element.

Zmn, Z_{mn}, $\overline{Z}mn$, or $\overline{Z}_{mn}$: The Laplace impedance of the n-th element of the m-th subsystem, with more than one element.

Cm *(or C_m)*:   Capacitor of m-th subsystem with only one capacitor.

Lm *(or L_m)*:   Inductor of m-th subsystem with only one inductor.

Lmn *(or L_{mn})*: Inductor as the n-th element of m-th subsystem.

Cmn: Capacitor as the *n*-th element of *m*-th subsystem.

Bridged-T: A T-network with a cross-coupling element between two series elements.

NEm = the numerator of the matrix entry E in the *m*-th (*m*=1-5) subsystem.

DEm = The denominator of the matrix entry *E* in the *m*-th (*m*=1-5) subsystem.
   The 2nd variable E must be one the followings:
      A = the entry (1,1) of chain matrix.
      B = the entry (1,2) of chain matrix.
      C = the entry (2,1) of chain matrix.
      D = the entry (2,2) of chain matrix
   Am = the entry (1,1) of chain matrix of *m*-th subsystem.
   Bm = the entry (1,2) of chain matrix of *m*-th subsystem.
   Cm = the entry (2,1) of chain matrix of *m*-th subsystem.
   Dm = the entry (2,2) of chain matrix of *m*-th subsystem.

a_{mn} = Polynomial coefficient of s^n of *m*-th subsystem.

Polynomial equation: $f(s) = a_m s^m + a_{m-1} s^{m-1} + ... + a_o = 0$. The highest degree *m* is
   greater than 1 in the *m*-th degree polynomial.

Monomial equation: $f(s) = s = 0$.

"0_+" : The very small positive value almost equal to zero.

"∞_-": The very big positive value almost equal to (very close to) infinity.

CHAPTER 3

Positively cross-coupled (PCC) network: A network where sign of the cross coupling is
   the same as the sign of the main line coupling (i.e. inductive cross-coupling in an
   inductively coupled circuit).

Negatively cross-coupled (NCC) network: A network where sign of the cross coupling is
   the opposite as the sign of the main line coupling (i.e. inductive cross-coupling in
   an capacitively coupled circuit).

Transmission zero characteristic equation (TZCE): The canonical numerator polynomial set equal to zero.

LHP: Left-half plane.

RHP: Right-half plane.

CHAPTER 4

Q: Quality factor (Selectivity) of a network. Ratio of the center frequency to the bandwidth, used to measure the width of the passband.

ADS: Advanced Design Systems, a circuit and EM simulator of Agilent.

S11 : Reflection coefficient seen at port 1 when port 2 is terminated in matched load.

S21: Transmission coefficient from port 1 to port 2.

Insertion loss: $IL = -20\log|S21|$ dB.

VNA: Vector Network Analyzer.

APPENDIX B

MATLAB PROGRAM FOR FIGURE 2.5

This program is used to compute the numerator and denominator polynomials of the cross-coupled filter in terms of symbolic variables of L's and C's in complex s-domain for the filer network of Figure 2.5.

```
clear

%---------------------
% 1st ckt = Source impedance
T1=[1 50;0 1];

%---------------------
% 2nd ckt =  L2//C2

syms   s L2 C2

Z2=s*L2 /((L2*C2)*s^2+1);

A2=1 ;        B2=0;
C2=1/Z2 ;     D2=1;

C2=simplify(C2);
[NC2,DC2]=numden(C2);
NC2=simplify(NC2);
DC2=simplify(DC2);
C2p=NC2/DC2;

T2=[A2 B2;C2p D2];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 3rd ckt = bridgeT
% A3=T3(1,1)
syms   s L31 L32 C33  L34 C34

Z31=s*L31;  Z32=s*L32;
Z33= 1/ (s*C33);
Z34=(s*L34) / ((L34*C34)*s^2+1);

den_Z=Z31*Z32+(Z31+Z32+Z33)*Z34;

A3=(Z31*(Z32+Z33)+(Z31+Z32+Z33)*Z34)/den_Z;
A3=simplify(A3);
[NA3,DA3]=numden(A3);
NA3=simplify(NA3);
NA3=collect(NA3,s);
DA3=simplify(DA3);
DA3=collect(DA3,s);

A3p=NA3/DA3;
```

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```
%--------------------------
% 3rd ckt = bridgeT
% B3=T3(1,2)

B3=Z33*(Z31*Z32+Z32*Z34+Z34*Z31)/den_Z;
B3=simplify(B3);
[NB3,DB3]=numden(B3);
NB3=simplify(NB3);
NB3=collect(NB3,s);
DB3=simplify(DB3);
DB3=collect(DB3,s);

B3p=NB3/DB3;

%--------------------------
% 3rd ckt = bridgeT
% C3 = T3(2,1)


C3=(Z31+Z32+Z33)/den_Z;
C3=simplify(C3);
[NC3,DC3]=numden(C3);
NC3=simplify(NC3);
NC3=collect(NC3,s);
DC3=simplify(DC3);
DC3=collect(DC3,s);

C3p=NC3/DC3;

%--------------------------
% 3rd ckt = bridgeT
% D3,T3(2,2)

D3=1+ ((Z32*Z33)/den_Z);
D3=simplify(D3);
[ND3,DD3]=numden(D3);
ND3=simplify(ND3);
ND3=collect(ND3,s);

DD3=simplify(DD3);
DD3=collect(DD3,s);
D3p=ND3/DD3;
%--------------------------
T3=[A3p B3p;C3p D3p];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 4th ckt = pi ckt
syms  s L41 C41 L42 C42 L43
Z41=(s*L41)/(s^2*L41*C41+1);
Z42=(s*L42)/(s^2*L42*C42+1);
Z43=s*L43;

A4=1+Z43/Z42;
A4=simplify(A4);
[NA4,DA4]=numden(A4);
NA4=simplify(NA4);
DA4=simplify(DA4);
```

```
A4p=NA4/DA4;

%-------------------------
B4=Z43;
B4=simplify(B4);
[NB4,DB4]=numden(B4);
NB4=simplify(NB4);
NB4=collect(NB4,s);
DB4=simplify(DB4);
DB4=collect(DB4,s);
B4p=NB4/DB4;

%-------------------------

C4=1/Z41+1/Z42+Z43/(Z41*Z42);
C4=simplify(C4);
[NC4,DC4]=numden(C4);
NC4=simplify(NC4);
NC4=collect(NC4,s);
DC4=simplify(DC4);
DC4=collect(DC4,s);
C4p=NC4/DC4;
%-------------------------
D4=1+(Z43/Z41);
D4=simplify(D4);
[ND4,DD4]=numden(D4);
ND4=simplify(ND4);
ND4=collect(ND4,s);
DD4=simplify(DD4);
DD4=collect(DD4,s);
D4p=ND4/DD4;
%-------------------------
T4=[A4p B4p;C4p D4p];

%%%%%%%%%%%%%%%%%%%%%%%%
%5th ckt = Load ZL
T5=[1 ,0 ; 1/50 , 1];

%=================

T=T1*T2*T3*T4*T5;
H=1/T(1,1)
%-------------------------
H=simplify(H);
[nH,dH]=numden(H);
%--------------

n=collect(nH,s);
d=collect(dH,s);

% The end of program
```

APPENDIX C

MATLAB PROGRAM FOR FIGURE 3.6

This program is used to compute the numerator and denominator polynomials of the cross-coupled filter in terms of symbolic variables of L's and C's in complex s-domain for the filer network of Figure 3.6.

```
clear

%---------------------------
% 1st ckt = Source impedance
T1=[1 50;0 1];

%--------------------------
% 2nd ckt =  L2//C2
syms   s L2 C2

Z2=s*L2 /((L2*C2)*s^2+1);

A2=1 ;        B2=0;
C2=1/Z2 ;     D2=1;

C2=simplify(C2);
[NC2,DC2]=numden(C2);
NC2=simplify(NC2);
DC2=simplify(DC2);
C2p=NC2/DC2;

T2=[A2 B2;C2p D2];

%-----------------------
% 3rd ckt = bridgeT
% A3=T3(1,1)
syms   s L31 L32 L33 C34  L35 C35 L36 C36


Z31=s*L31;  Z32=s*L32; Z33=s*L33;
Z34= 1/ (s*C34);
Z35=(s*L35) /((L35*C35)*s^2+1);
Z36=(s*L36) /((L36*C36)*s^2+1);

Z_delta=Z35+Z32+Z36;
Z37=Z35*Z32/Z_delta;
Z38=Z32*Z36/Z_delta;

Z3a=Z31+Z37;
Z3b=Z38+Z33;
Z39= (Z35*Z36)/Z_delta ;

den_Z=Z3a*Z3b+(Z3a+Z3b+Z34)*Z39;

A3=(Z3a*(Z3b+Z34)+(Z3a+Z3b+Z34)*Z39)/den_Z;
A3=simplify(A3);
```

```
[NA3,DA3]=numden(A3);
NA3=simplify(NA3);
NA3=collect(NA3,s);
DA3=simplify(DA3);
DA3=collect(DA3,s);

A3p=NA3/DA3;

%---------------------------
% 3rd ckt = bridgeT
% B3=T3(1,2)

syms   s L31 L32 L33 C34 L35 C35 L36 C36


Z31=s*L31;   Z32=s*L32; Z33=s*L33;
Z34= 1/(s*C34);
Z35=(s*L35)/((L35*C35)*s^2+1);
Z36=(s*L36)/((L36*C36)*s^2+1);

Z_delta=Z35+Z32+Z36;
Z37=Z35*Z32/Z_delta;
Z38=Z32*Z36/Z_delta;

Z3a=Z31+Z37;
Z3b=Z38+Z33;
Z39= (Z35*Z36)/Z_delta ;

den_Z=Z3a*Z3b+(Z3a+Z3b+Z34)*Z39;

B3= Z34*(Z3a*Z3b+Z3b*Z39+Z39*Z3a)/den_Z;

B3=simplify(B3);
[NB3,DB3]=numden(B3);
NB3=simplify(NB3);
NB3=collect(NB3,s);
DB3=simplify(DB3);
DB3=collect(DB3,s);

B3p=NB3/DB3;

%--------------------
% 3rd ckt = bridgeT
% C3 = T3(2,1)

syms   s L31 L32 L33 C34 L35 C35 L36 C36

Z31=s*L31;   Z32=s*L32; Z33=s*L33;
Z34= 1/(s*C34);
Z35=(s*L35)/((L35*C35)*s^2+1);
Z36=(s*L36)/((L36*C36)*s^2+1);

Z_delta=Z35+Z32+Z36;
Z37=Z35*Z32/Z_delta;
Z38=Z32*Z36/Z_delta;

Z3a=Z31+Z37;
```

```
Z3b=Z38+Z33;
Z39=(Z35*Z36)/Z_delta ;

den_Z=Z3a*Z3b+(Z3a+Z3b+Z34)*Z39;

C3=(Z3a+Z3b+Z34)/den_Z;
C3=simplify(C3);
[NC3,DC3]=numden(C3);
NC3=simplify(NC3);
NC3=collect(NC3,s);
DC3=simplify(DC3);
DC3=collect(DC3,s);

C3p=NC3/DC3;

%------------------------
% 3rd ckt = bridgeT
% D3,T3(2,2)

syms  s L31 L32 L33 C34 L35 C35 L36 C36

Z31=s*L31;  Z32=s*L32; Z33=s*L33;
Z34= 1/ (s*C34);
Z35=(s*L35)/((L35*C35)*s^2+1);
Z36=(s*L36)/((L36*C36)*s^2+1);

Z_delta=Z35+Z32+Z36;
Z37=Z35*Z32/Z_delta;
Z38=Z32*Z36/Z_delta;

Z3a=Z31+Z37;
Z3b=Z38+Z33;
Z39=(Z35*Z36)/Z_delta ;

den_Z=Z3a*Z3b+(Z3a+Z3b+Z34)*Z39;

D3=1+ ((Z3b*Z34)/den_Z);
D3=simplify(D3);
[ND3,DD3]=numden(D3);
ND3=simplify(ND3);
ND3=collect(ND3,s);

DD3=simplify(DD3);
DD3=collect(DD3,s);
D3p=ND3/DD3;
%------------------------
T3=[A3p B3p;C3p D3p];

%------------------------
% 4th ckt =  L4//C4

syms   s L4 C4

Z4=s*L4 /((L4*C4)*s^2+1);

A4=1 ;       B4=0;
C4=1/Z4 ;    D4=1;
```

```
C4=simplify(C4);
[NC4,DC4]=numden(C4);
NC4=simplify(NC4);
DC4=simplify(DC4);
C4p=NC4/DC4;

T4=[A4 B4;C4p D4];

%--------------------
%5th ckt = Load ZL
T5=[1 ,0 ; 1/50 , 1];

%=================

T=T1*T2*T3*T4*T5;
H=1/T(1,1);
%-------------------
H=simplify(H);
[nH,dH]=numden(H);

%---------------

n=collect(nH,s);
d=collect(dH,s);

% The end of program
```

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