ABSTRACT

KINEMATIC SYNTHESIS OF ADJUSTABLE SPATIAL FOUR AND FIVE-BAR MECHANISMS FOR FINITE AND MULTIPLY SEPARATED POSITIONS

by
Kevin Russell

Although spatial mechanisms are more general in structure than planar mechanisms, their applications are few due to the limited number of practical design tools and the complexity of those available. It is in fact the task of the future to develop effective, but practical design tools for spatial mechanisms.

This research presents several new methods for synthesizing adjustable spatial mechanisms. The first method involves the kinematic synthesis of spatial mechanisms for multi-phase motion generation. Using this method, spatial four and five-bar mechanisms can be synthesized to achieve different phases of prescribed rigid body positions. The theory of this approach has also been extended to incorporate rigid body tolerance problems. Using the tolerance problem method, spatial four-bar mechanisms can be synthesized to achieve the prescribed precise rigid body positions and also satisfy the rigid body positions within the prescribed tolerances. Both approaches use the R-R, S-S, R-S and C-S dyad displacement equations.

The second method involves the kinematic synthesis of spatial mechanisms for multi-phase multiply separated positions. Using this method, spatial four and five-bar mechanisms can be synthesized to achieve different phases of prescribed rigid body positions, velocities and accelerations. The theory of this approach has also been extended to incorporate instantaneous screw axis (ISA)

parameters. Using ISA parameters, spatial four-bar mechanisms can also be synthesized to achieve different phases of prescribed rigid body positions, velocities and accelerations. Both approaches use the R-R, S-S, R-S and C-S dyad displacement, velocity and acceleration equations.

For each method, the maximum number of prescribed rigid positions is determined for each mechanism for two and three phase problems. The spatial four and five-bar mechanisms considered in this research are the RRSS, RRSC, RSSR-SS and RSSR-SC.

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by
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KINEMATIC SYNTHESIS
OF ADJUSTABLE SPATIAL FOUR AND FIVE-BAR MECHANISMS
FOR FINITE AND MULTIPLY SEPARATED POSITIONS

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To the underdog

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CHAPTER 1

INTRODUCTION

This chapter introduces the fundamental concepts of multi-phase motion generation and multi-phase multiply separated position synthesis. The underlying principles of tolerance problems as well as the use of instant screw axis parameters in spatial mechanism synthesis are also introduced. This is followed by a review of literature related to spatial mechanism synthesis processes. Finally, the research objectives in this study are outlined.

1.1 Multi-Phase Motion Generation

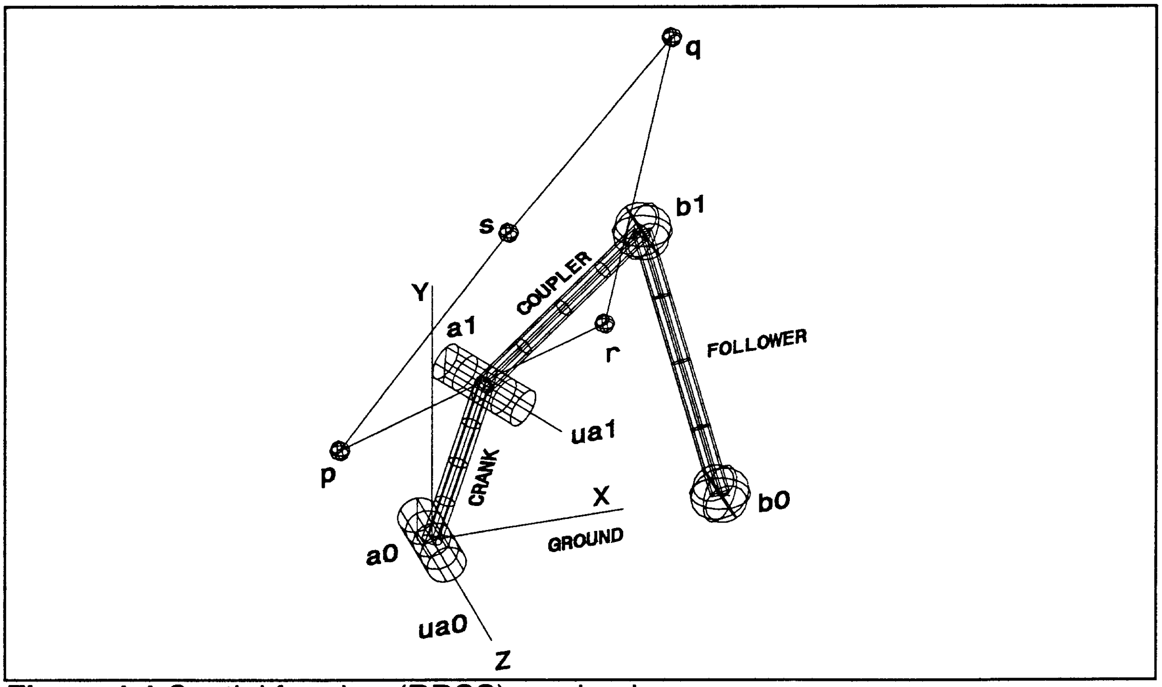

Figure 1.1 Spatial four-bar (RRSS) mechanism

When the crank of the RRSS mechanism in figure 1.1 rotates about joint axis $\mathbf{ua}_0$, the coupler and follower move accordingly. This research focuses on the motion of rigid bodies affixed to the coupler in spatial four and five-bar mechanisms. The motion of this link is generally the focus in motion generation synthesis. Points **p**, **q**, **r** and **s** in figure 1.1 denote one particular rigid body orientation. In figure 1.2 the rigid body moves through positions 1, 2 and 3 in the first phase and positions 4, 5 and 6 in the second phase. Therefore each rigid body point (**p**, **q**, **r** and **s**) for each of the six prescribed rigid body positions is illustrated in figure 1.2.

In the case of closed-loop, four and five-bar spatial mechanisms, the coupler link is capable of moving bodies in three-dimensional space. Unlike open-loop mechanisms, the closed-loop design is structurally rigid-making mechanisms of this type more effective for motion generation applications involving heavy loads and heavy fluctuating loads.

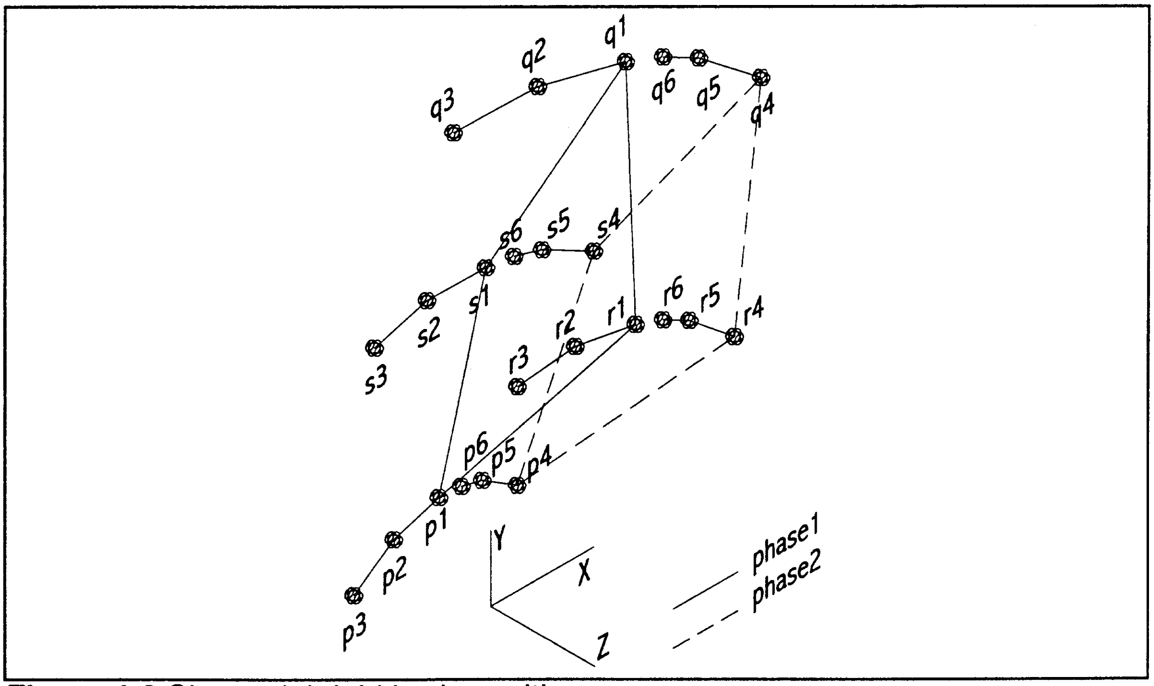

Figure 1.2 Six spatial rigid body positions

Due to the location and orientation of the six spatial rigid body positions in figure 1.2, it may not be possible for a single four-bar mechanism to achieve them all. One alternative is to separate the rigid body positions into groups or phases and synthesize, for each phase, a four-bar mechanism to achieve the rigid body positions in that phase. Another alternative is to synthesize a single four-bar mechanism with circle points, center points, crank and follower lengths that can be adjusted in order to achieve the rigid body positions in all phases. The latter alternative is better because it offers a single mechanism a greater range of flexibility.

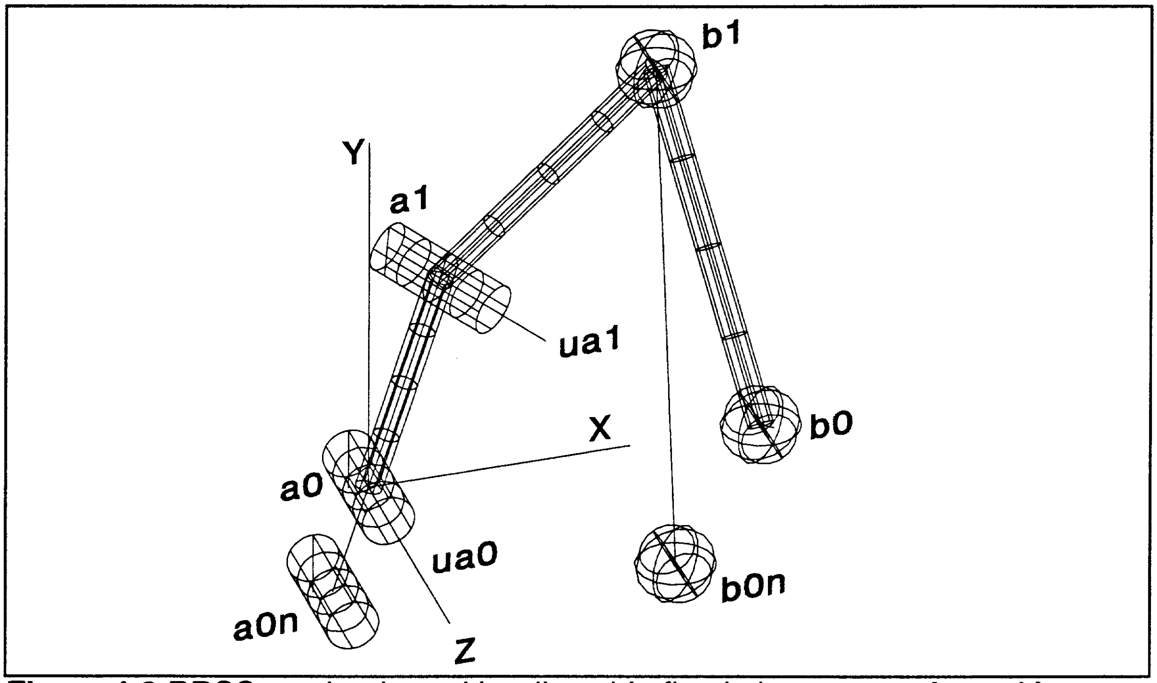

Figure 1.3 RRSS mechanism with adjustable fixed pivots $\mathbf{a}_0$ $\mathbf{a}_{0n}$, $\mathbf{b}_0$ and $\mathbf{b}_{0n}$

When the four-bar mechanism in figure 1.3 has the fixed pivots $\mathbf{a}_0$ and $\mathbf{b}_0$, the rigid body positions in phase 1 can be achieved. When $\mathbf{a}_{0n}$ and $\mathbf{b}_{0n}$ are the fixed pivots, the rigid body positions in phase 2 are achievable. Although the mechanism solution for phase 1 is not dimensionally equivalent to the mechanism solution for phase 2, both solutions can incorporate the same hardware.

As previously mentioned, rigid body positions 1-2-3 and 3-4-5 are grouped into phases (phase 1 = 1-2-3 and phase 2 = 3-4-5). A phase represents a set of rigid body parameters obtainable by a particular mechanism configuration. Figures 1.1, 1.2 and 1.3 illustrate a two-phase motion generation problem.

Figure 1.1 also illustrates a two-phase motion generation problem with no shared rigid body positions. There aren't any common (or shared) positions in phases one and two. Multi-phase motion generation problems could involve one, several or no shared positions.

Therefore, in summary, multi-phase motion generation involves calculating the mechanism adjustments necessary to achieve phases of rigid body positions. This research presents new motion generation synthesis methods for adjustable spatial four and five-bar mechanisms.

1.2 Tolerance Problem

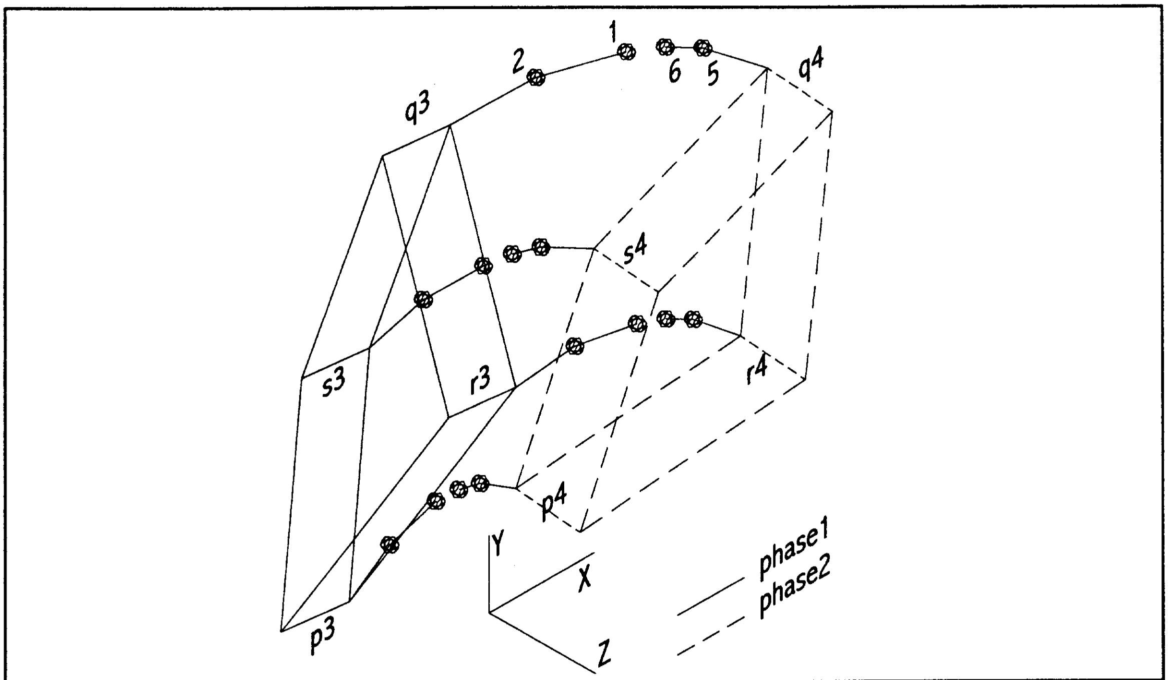

Figure 1.4 Spatial rigid body positions and position tolerance regions

Unlike figure 1.1, figure 1.4 shows a problem in which two rigid body positions (positions 3 and 4) need only lie within a particular region. These spatial regions represent tolerances placed on both rigid body positions. Positions 1, 2, 5 and 6 are not under any tolerances (they are exact rigid body positions). Just as in multi-phase motion generation synthesis, the ideal solution in this case lies in a single adjustable mechanism. This adjustable mechanism must satisfy the precise rigid body positions while remaining within the tolerance limits of the rigid body positions with tolerances.

1.3 Multi-Phase Multiply Separated Positions

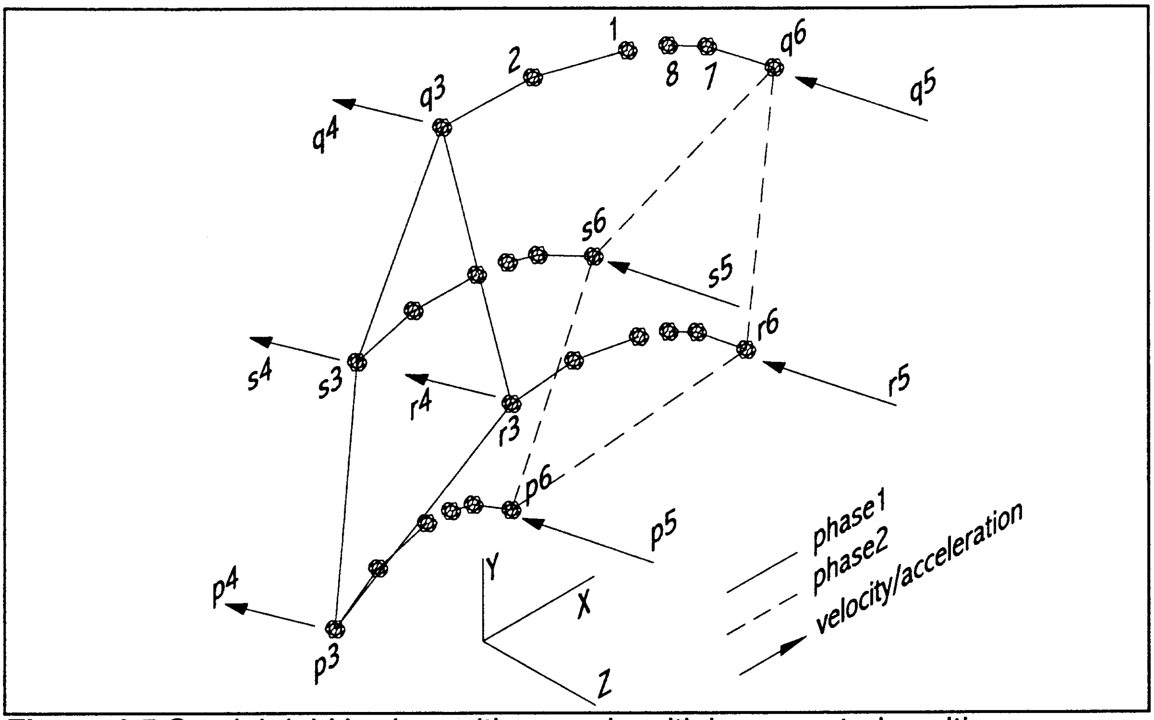

Figure 1.5 Spatial rigid body positions and multiply separated positions

Figure 1.5 illustrates a combination of finite and multiply separated positions. As mentioned previously, points **p**, **q**, **r** and **s** represent distinct rigid body locations and are finitely separated. The remaining positions (represented by arrows) are called multiply separated positions or MSPs. Multiply separated positions can represent rigid velocities and/or accelerations. All of the finite and multiply separated positions in figure 1.5 may not be achievable by a single mechanism. Again, this creates the need for a mechanism that can achieve all finite and multiply separated positions by adjusting its fixed and moving pivot locations and crank and follower lengths. Such adjustments are possible when incorporating adjustable mechanisms. Figure 1.5 illustrates a two-phase finite and multiply separated position problem with no shared positions.

Therefore, in summary, multi-phase multiply separated position synthesis involves calculating the mechanism adjustments necessary to achieve phases of rigid body positions velocities and/or accelerations. This research presents new multiply separated position synthesis methods for adjustable spatial four and five-bar mechanisms.

1.4 Instant Screw Axis Consideration

Unlike the coupler of a mechanism in planar motion, the coupler of a spatial mechanism generally undergoes screw motions. This type of motion is a combination of rotations about and translations along axes in space. In an instant in time, the coupler of a spatial mechanism would also rotate about and translate along a spatial axis. Knowing this, what is known as an instant center of rotation for a rigid body of a planar mechanism would in fact become an instant axis of screw motion for a rigid body of a spatial mechanism. This axis is called an instant screw axis or ISA. A locus of ISAs is called an axode (just as a locus of instant centers is called a centrode).

There is a close link between the axodes and both a motion on one hand and a mechanism on the other. For this reason, in synthesis, the mechanism for a prescribed motion can be found using the relations between the axodes and the motion, and the axodes and the mechanism [41]. This quote by Skreiner is illustrated in figure 1.6. As the figure shows, ISAs can also be used to calculate the rigid body multiply separated positions that are necessary for spatial mechanism synthesis.

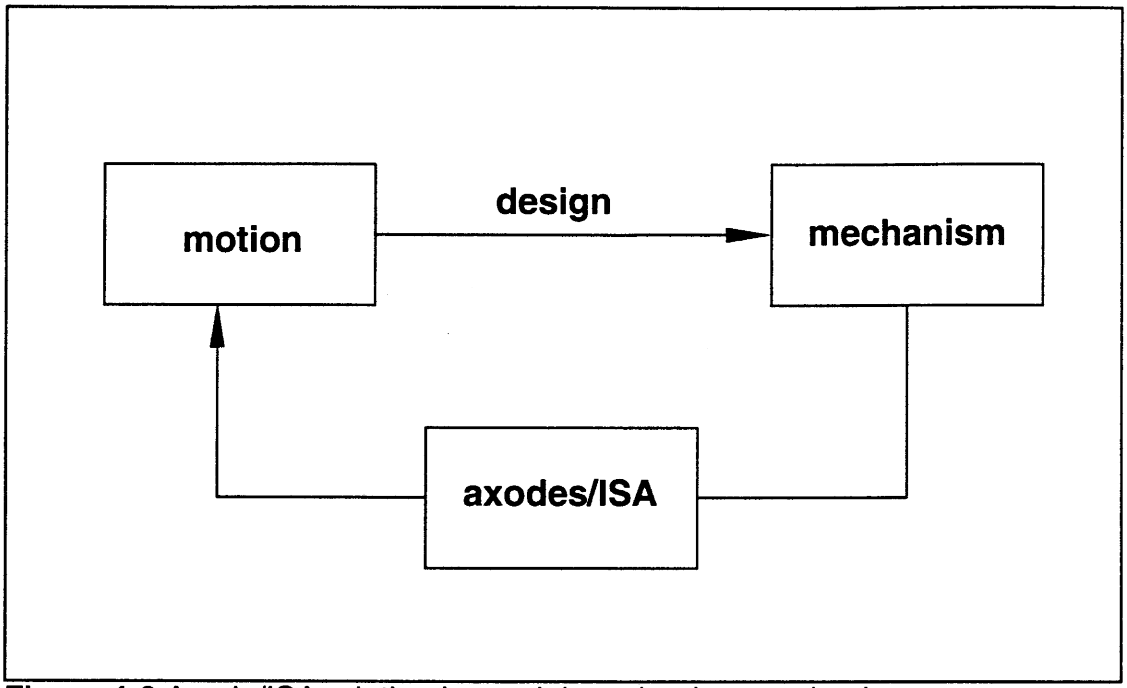

Figure 1.6 Axode/ISA relation in spatial mechanism synthesis

1.5 Review of Spatial Mechanism Synthesis

Several authors have made significant contributions in the area of spatial and adjustable spatial mechanism synthesis. Shoup [9] presented a technique for the design of an adjustable slider crank mechanism to be used as a variable displacement pump or compressor. The design technique considers velocity fluctuation, force transmission effectiveness and mechanism geometric proportions.

Sandor, Kohli, Reinholtz and Ghosal [14] presented a technique for the closed-form analytic synthesis of a five-link spatial motion generator. The motion generator mechanism consists of two grounded R-S links, one grounded C-S link, a ternary S-S-S coupler and the R-R-C fixed frame. The resulting system can be solved for unknown vectors defining the dyad in its starting position, in closed form for up to three precision positions.

Sandor, Kohli and Zhuang [15] presented a technique for the synthesizing RSSR-SRR spatial motion generators. Motion of the coupler is to be prescribed for three or four finitely separated positions and to be correlated to the prescribed input rotations of the crank and he grounded R-R link of the RRS dyad.

Yao, Xu and Fan [21] presented a method for kinematic synthesis of an RS-SRR-SS adjustable spatial motion generator for three alternate tasks. Three separate systems of synthesis equations to exactly generate the first and last positions of each task are obtained for the R-S link by co-plane, distance equations and inversion theory, and for the S-S link by a constant distance equation.

Sandor, Yang, Xu and De [33] presented a technique for synthesizing adjustable spatial motion generators by analytical methods with two exact prescribed positions and orientations for each of two different motion tasks. This mechanism is also synthesized by numerical methods to solve a nonlinear system of equations and by optimization techniques to minimize the motion errors at additional, approximately prescribed positions.

Lebedev and Marder [35] presented a vector loop method for position analysis of a spatial bimobile $RRS_FS_FS_FSS$ mechanism. This method is developed for the analytic determination of link positions of a bimobile two-loop spatial mechanism in the form of vector functions of the turn angles of the driving links and the design parameters of the mechanism.

Funbashi, Iwatsuki and Yokoyama [39] have presented a paper in which crank-length adjusting mechanisms are proposed in order to change the input-output relationships of arbitrary planar, spherical and spatial crank mechanisms and also to stop output motions of crank-rocker mechanisms during rotations of their crank shafts.

Alvarez, Cardenal and Cuadrado [10] presented a paper that outlined a simple and efficient method for optimum synthesis of multi-body systems. The proposed formulation was based on the use of a set of fully Cartesian coordinates. Using the coordinates, the system was described by a set of geometric constraints and the design requirements were introduced by a set of functional constraints.

Tavkhelidze [13] developed methods that considered the following:

1. The layout of the position of links in a spatial four-bar mechanism with one sliding pair
2. The condition of the existence of one or two cranks having rotational motion in the same mechanism

His method for solving the first problem is based on two theorems. For the application of these, a simple method was devised, when the position of the driving link is known. For the solution of the second problem, the relationship between the displacement of points in the four-link spatial mechanism with one sliding pair was found analytically and the configuration examined in which the connecting rod reaches its maximum and minimum positions. The conditions for a member to be either a crank or a rocker arm are then expressed by inequalities.

Ananthasuresh and Kramer [53] developed a closed form solution to the analysis of the RSCR mechanism. Using analysis modules developed from the geometric characteristics of the mechanism, the mechanism can be optimally synthesized for function, path and motion generation problems-satisfying conditions within prescribed accuracy limits.

Hanchak and Murray [63] developed a method for designing mechanisms composed of Revolute-Binary state Prismatic-Revolute (RBR) chains for rigid body guidance. By requiring the arrangement of the three RBR chains to share specific fixed and moving pivots, called an N-type arrangement, four positions are reachable.

1.6 Research Objectives

In response to the need for effective but practical synthesis tools for spatial mechanisms, a new technique will be presented to synthesize adjustable spatial four and five-bar mechanisms for multi-phase motion generation. Using this method, spatial mechanisms will be synthesized to achieve different phases of prescribed finitely separated rigid body positions.

As an extension of the multi-phase motion generation method, a new technique will also be presented to synthesize adjustable spatial four-bar mechanisms for multi-phase motion generation with tolerances. Using this method, spatial mechanisms will be synthesized to achieve the phases of prescribed precise rigid body positions and also satisfy the rigid body positions with prescribed tolerances.

A new technique will also be presented to synthesize adjustable spatial four and five-bar mechanisms for multi-phase motion generation and multiply separated positions. Using this method, spatial mechanisms will be synthesized to achieve different phases of prescribed rigid body positions, velocities and/or accelerations.

As an extension of the multi-phase motion generation method, a new technique will also be presented to synthesize adjustable spatial four-bar mechanisms for multi-phase motion generation and multiply separated positions using instant screw axis parameters. Using this alternative method, spatial four-bar mechanisms will be synthesized to achieve phases of prescribed rigid body positions, velocities and/or accelerations.

The maximum number of prescribed rigid body positions will also be determined for all synthesis techniques for two and three phase problems. Knowing this limit of these new techniques can better enable one to establish their usefulness and effectiveness in satisfying mechanical design applications.

CHAPTER 2

SPATIAL MECHANISMS AND DISPLACEMENT EQUATIONS

2.1 Four and Five-Bar Spatial Mechanisms

The spatial four and five-bar mechanisms considered in this research are the RRSS, RRSC, RSSR-SS and RSSR-SC. Due to their simple designs and spatial kinematics the RRSS and the RSSR-SS mechanisms are two of the most practical four and five-bar mechanism designs for spatial motion generation applications. These mechanisms have no prismatic or cylindrical joints. Such joints require rails, pins or slots to travel along. These mechanisms require only links with revolute and spherical joints. This feature makes them fairly easy to design and construct. The coupler links of the RRSS and RSSR-SS mechanisms have no passive degrees of freedom (unlike the RSSR mechanism). This feature gives the two mechanisms the capacity for motion generation applications (involving the coupler link). Unlike Bennett's linkage, the RRSS and RSSR-SS mechanisms have no rigid joint axis and link length requirements. This feature allows more freedom in the design of the two mechanisms. After considering all of these features of the RRSS and RSSR-SS mechanisms, they became two of the mechanisms of choice for this research in spatial motion generation and multiply separated position synthesis.

The RRSC and RSSR-SC mechanisms are variations of the RRSS and the RSSR-SS mechanisms. The former mechanisms incorporate only one cylindrical joint between their connections to ground. This additional degree of freedom of the C-S link makes the spatial kinematics of these mechanisms different from the RRSS and RSSR-SS. All of the qualities mentioned in the previous paragraph (minus those qualities due to the absence of cylindrical joints) can also be attributed to the RRSC and RSSR-SC mechanisms. Although these two mechanisms are more complicated in design than the RRSS and RSSR-SS, the synthesis techniques to be presented in this research can easily accommodate all four mechanisms.

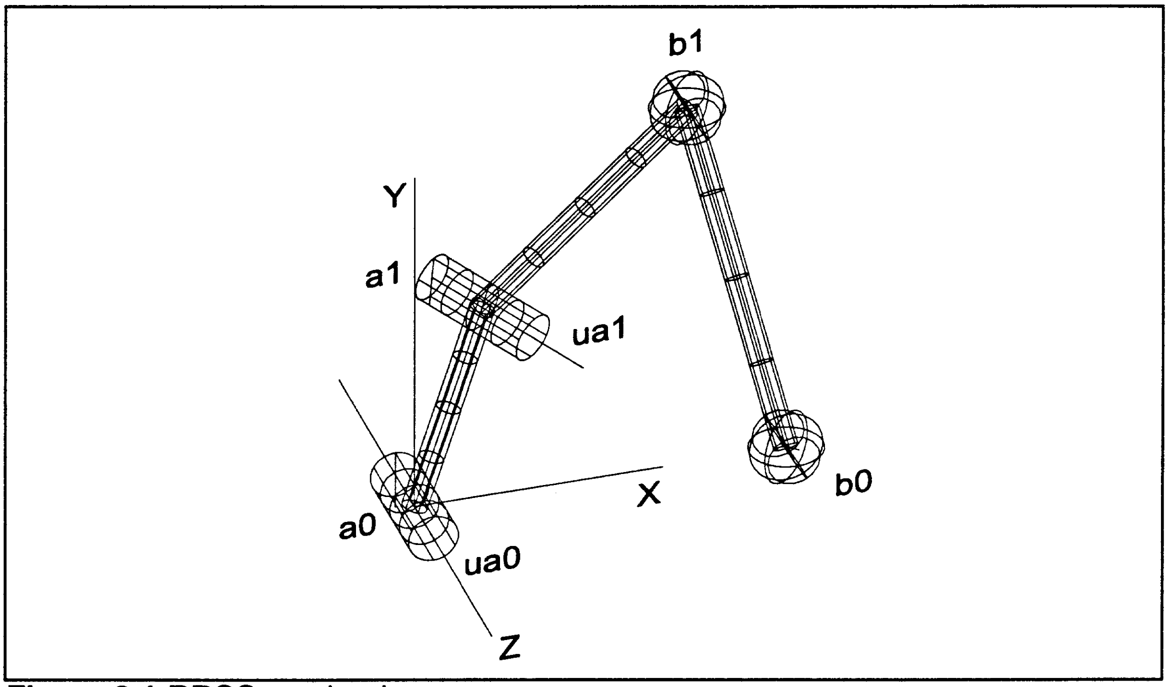

Figure 2.1 RRSS mechanism

In this research, link $\mathbf{a}_0$-$\mathbf{a}_1$ of the RRSS mechanism rotates in the X-Y plane. Joint axis $\mathbf{ua}_1$ is normal to link $\mathbf{a}_0$-$\mathbf{a}_1$.

Link $\mathbf{a}_0$-$\mathbf{a}_1$ of the RRSC mechanism (figure 2.2) rotates in the X-Y plane and link $\mathbf{b}_0$-$\mathbf{b}_1$ rotates in the x*-y* plane. Joint axis $\mathbf{ua}_1$ is normal to the link $\mathbf{a}_0$-$\mathbf{a}_1$. The origins of the X-Y-Z frame and the x*-y*-z* frame are offset by a distance **d** along the X-axis.

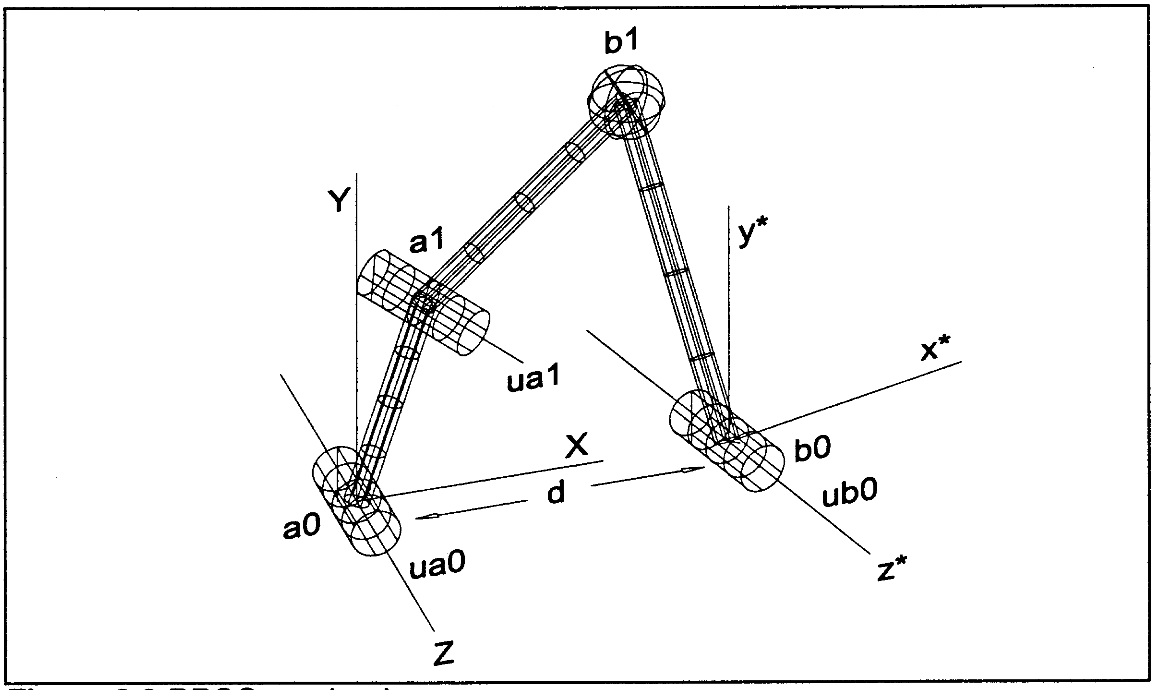

Figure 2.2 RRSC mechanism

Link $\mathbf{a}_0$-$\mathbf{a}_1$ of the RSSR-SS mechanism (figure 2.3) rotates in the X-Y plane and link $\mathbf{b}_0$-$\mathbf{b}_1$ rotates in the x*-y* plane. The origins of frame X-Y-Z and frame x*-y*-z* are offset by a distance **d** along the X-axis. Link $\mathbf{c}_0$-$\mathbf{c}_1$ is measured in the X-Y-Z frame.

Link $\mathbf{a}_0$-$\mathbf{a}_1$ of the RSSR-SC mechanism (figure 2.4) rotates in the X-Y plane, link $\mathbf{b}_0$-$\mathbf{b}_1$ rotates in the x*-y* plane and link $\mathbf{c}_0$-$\mathbf{c}_1$ in the x**-y** plane. The origins of the X-Y-Z frame and the x*-y*-z* frame are offset by a distance **d1** along the X-axis. The origins of the X-Y-Z frame and the x**-y**-z** frame are offset by a distance **d2** (measured from the X-Y-Z frame).

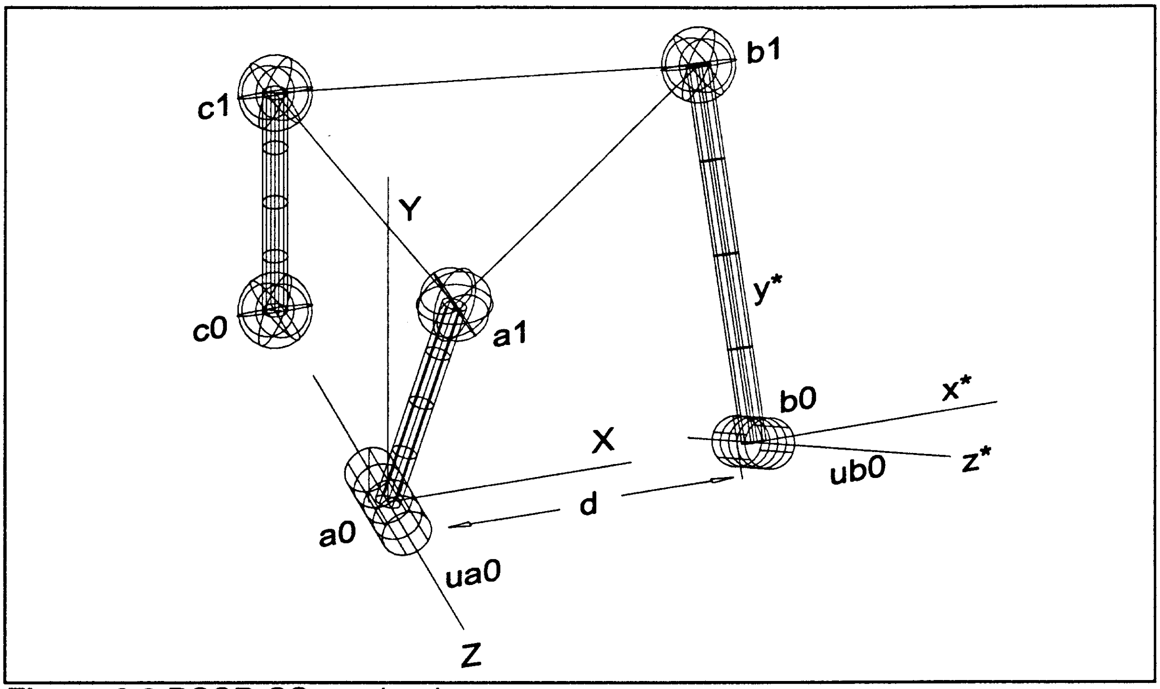

Figure 2.3 RSSR-SS mechanism

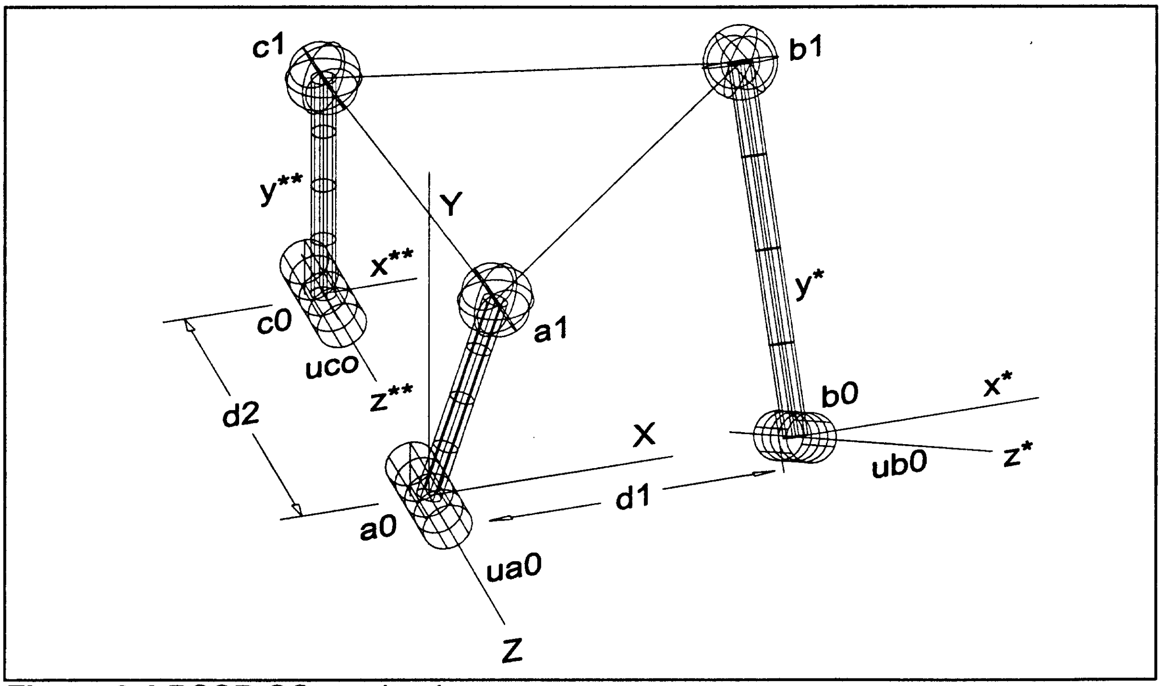

Figure 2.4 RSSR-SC mechanism

2.2 Sphere-Sphere (S-S) Link

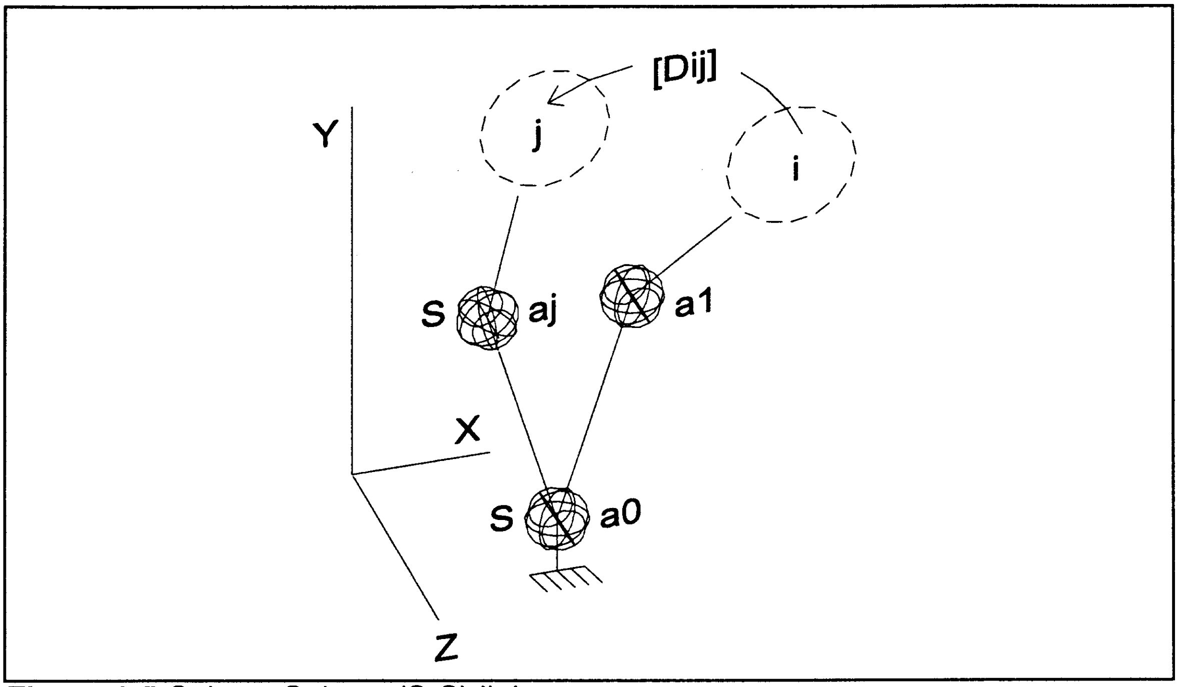

Figure 2.5 Sphere-Sphere (S-S) link

The S-S link must satisfy the constant length condition only. Given a fixed pivot $\mathbf{a}_0$ and a moving pivot $\mathbf{a}_1$, the following displacement constraint equation must be satisfied:

$$(\mathbf{a}_j - \mathbf{a}_0)^T(\mathbf{a}_j - \mathbf{a}_0) = (\mathbf{a}_j - \mathbf{a}_0)^T(\mathbf{a}_j - \mathbf{a}_0) \qquad j = 2, 3, \ldots n \tag{2.1}$$

where

$$\mathbf{a}_0 = (a_{0x}, a_{0y}, a_{0z}) \qquad \mathbf{a}_1 = (a_{1x}, a_{1y}, a_{1z}) \qquad \mathbf{a}_j = [D_{ij}]\mathbf{a}_1$$

and

$$[D_{ij}] = \begin{bmatrix} p_{jx} & q_{jx} & r_{jx} & s_{jx} \\ p_{jy} & q_{jy} & r_{jy} & s_{jy} \\ p_{jz} & q_{jz} & r_{jz} & s_{jz} \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{ix} & q_{ix} & r_{ix} & s_{ix} \\ p_{iy} & q_{iy} & r_{iy} & s_{iy} \\ p_{iz} & q_{iz} & r_{iz} & s_{iz} \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \tag{2.2}$$

Points **p**, **q**, **r** and **s** are used to mark the position of the rigid body in three-dimensional space. These four points must not all travel in the same plane in each rigid body position. This precaution is taken in order to prevent the rows in the S-S link displacement matrix (equation 2.2) from becoming proportional. With proportional rows, the matrix in equation 2.2 to be inverted cannot be inverted.

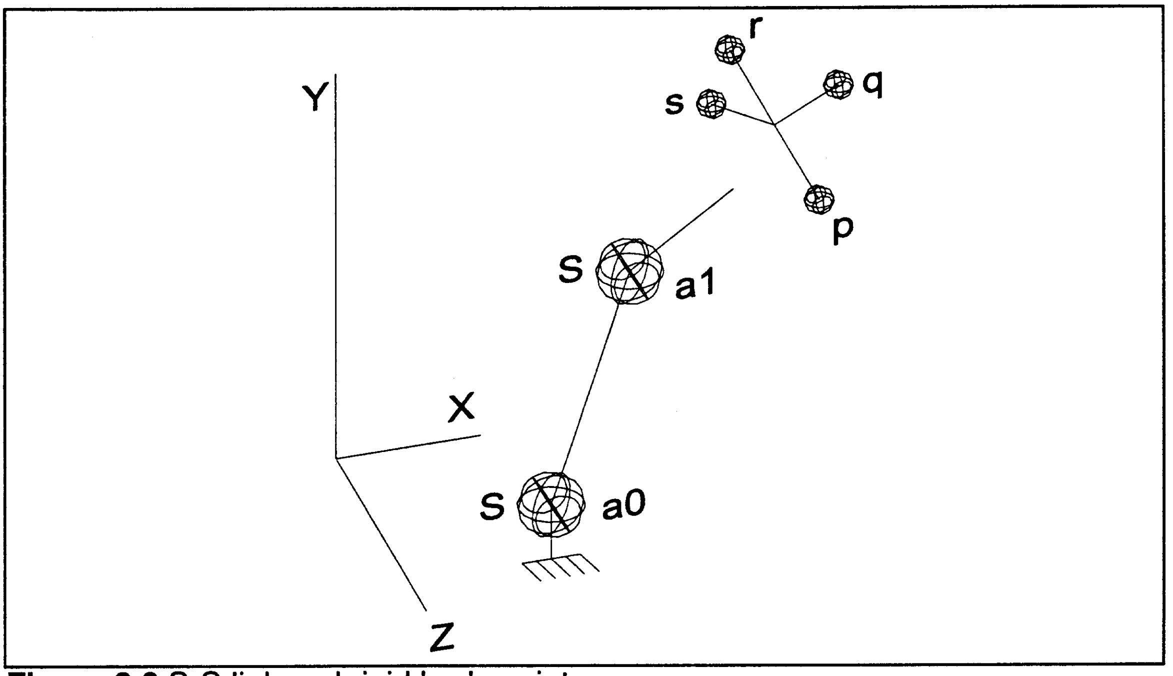

Figure 2.6 S-S link and rigid body points

Since there are six variables (a_{0x}, a_{0y}, a_{0z}, a_{1x}, a_{1y} and a_{1z}), a maximum of seven positions of a rigid body can be specified, with no arbitrary choice of parameter.

2.3 Revolute-Sphere (R-S) Link

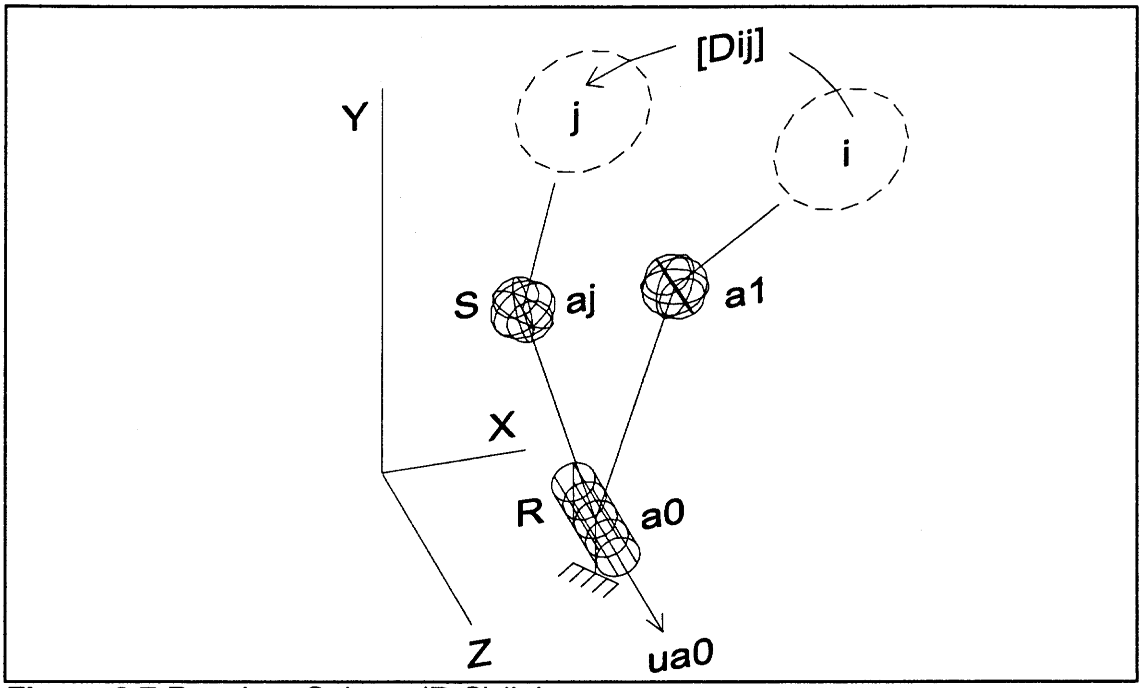

Figure 2.7 Revolute-Sphere (R-S) link

The R-S link must satisfy all S-S link constraints with one additional constraint imposed such that the spherical joint is restricted to rotation in a plane that is perpendicular to axis $\mathbf{ua}_0$ of the revolute joint. The R-S link displacement constraint equations become

$$(\mathbf{a}_j - \mathbf{a}_0)^T(\mathbf{a}_j - \mathbf{a}_0) = (\mathbf{a}_j - \mathbf{a}_0)^T(\mathbf{a}_j - \mathbf{a}_0) \quad j = 2, 3, \ldots n \tag{2.3}$$

$$(\mathbf{ua}_0)^T(\mathbf{a}_j - \mathbf{a}_0) = 0 \quad j = 1, 2, 3, \ldots n \tag{2.4}$$

$$(\mathbf{ua}_0)^T(\mathbf{ua}_0) = 1 \tag{2.5}$$

where

$\mathbf{a}_0 = (a_{0x}, a_{0y}, a_{0z})$ $\quad \mathbf{a}_1 = (a_{1x}, a_{1y}, a_{1z})$ $\quad \mathbf{ua}_0 = (ua_{0x}, ua_{0y}, ua_{0z})$ $\quad \mathbf{a}_j = [D_{ij}]\mathbf{a}_1$

These equations form a set of 8 design equations with 9 unknown scalar components of $\mathbf{ua}_0$, $\mathbf{a}_0$ and $\mathbf{a}_1$. Therefore, the number of rigid body positions that can be specified for an R-S link to be used for rigid body guidance is four, with an arbitrary choice of one of the nine unknowns.

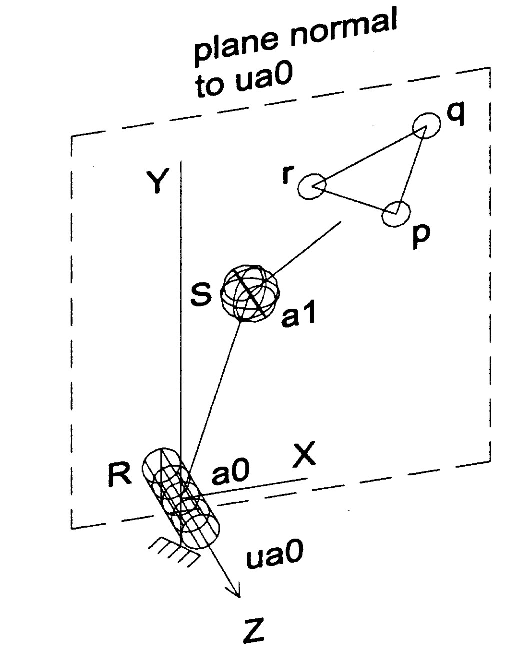

Figure 2.8 R-S link and rigid body points

When the rigid body points travel in a plane that is normal to joint axis $\mathbf{ua}_0$ (therefore prescribing the joint axis) only equation 2.3 must be satisfied. When this is the case,

$$\mathbf{a}_0 = (a_{0x}, a_{0y}) \qquad \mathbf{a}_1 = (a_{1x}, a_{1y}) \qquad \mathbf{ua}_0 = (0, 0, 1) \qquad \mathbf{a}_j = [D_{ij}]\mathbf{a}_1$$

where

$$[D_{ij}] = \begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{ix} & q_{ix} & r_{ix} \\ p_{iy} & q_{iy} & r_{iy} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \qquad (2.6)$$

Here, points **p**, **q** and **r** are used to mark the position of the rigid body in two-dimensional space. These three points must not all travel in the same line in each rigid body position. This precaution is taken in order to prevent the rows in the R-S link displacement matrix (equation 2.6) from becoming proportional. With proportional rows, the matrix in equation 2.6 to be inverted cannot be inverted.

2.4 Cylindrical-Sphere (C-S) Link

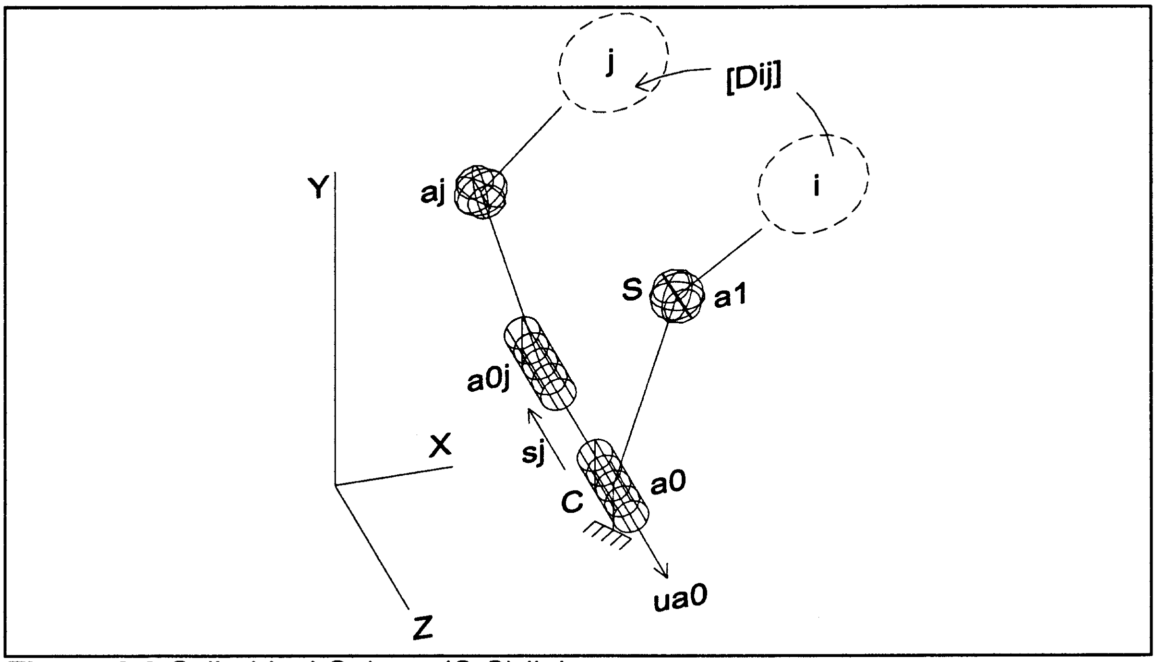

Figure 2.9 Cylindrical-Sphere (C-S) link

The C-S link must satisfy all of the constraints imposed by the R-S link plus an additional constraint equation, which accounts for the translational degree of freedom of the cylindrical joint. In this case, equations 2.3 and 2.4 are written with the coordinates of the intermediate point $\mathbf{a}_{0j}$ replacing $\mathbf{a}_0$ where

$$\mathbf{a}_{0j} = \mathbf{a}_0 + S_j\mathbf{ua}_0 \qquad (2.7)$$

The synthesis of the C-S link for three specified rigid body positions leads to a set of 12 design equations. The two added translations S_2 and S_3 give a total of

14 unknowns. Therefore, the maximum number of rigid body positions that can be specified is three with an arbitrary choice of any two scalar parameters.

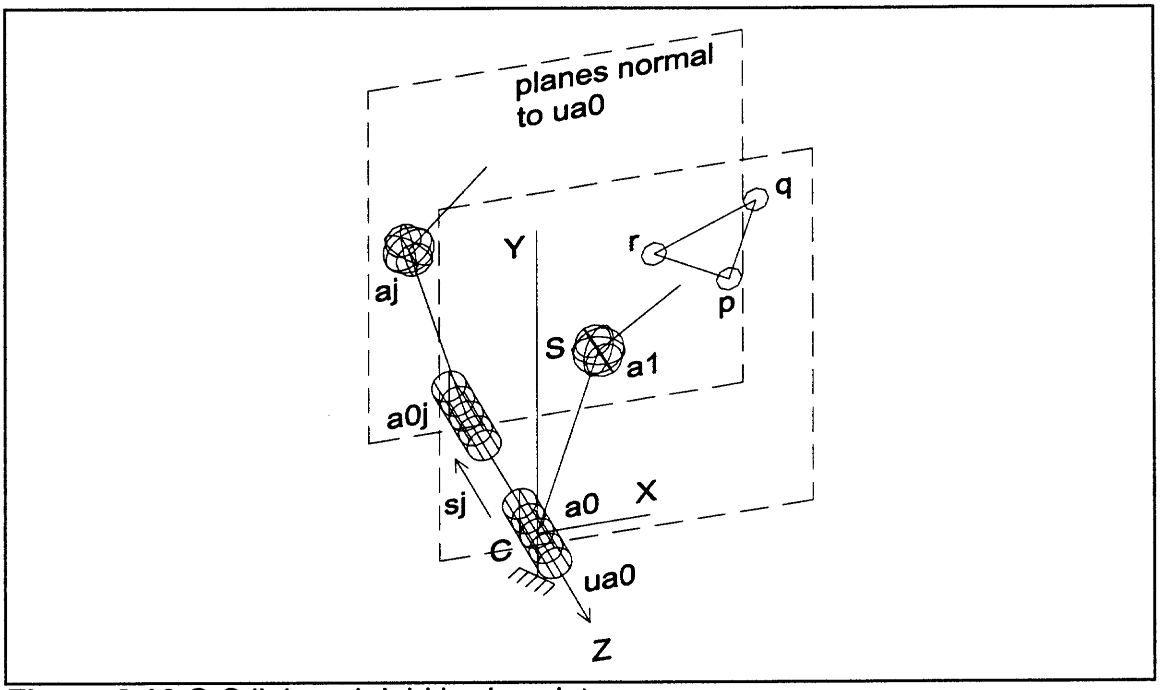

Figure 2.10 C-S link and rigid body points

When the rigid body points travel in a plane that is normal to joint axis $\mathbf{ua}_0$ (therefore prescribing the joint axis) only equation 2.3 needs satisfying. This measure also eliminates the need for translation terms (S_j = 0) in the design equations. Therefore

$$\mathbf{a}_0 = (a_{0x}, a_{0y}) \qquad \mathbf{a}_1 = (a_{1x}, a_{1y}) \qquad \mathbf{ua}_0 = (0, 0, 1) \qquad \mathbf{a}_j = [D_{ij}]\mathbf{a}_1$$

where

$$[D_{ij}] = \begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{ix} & q_{ix} & r_{ix} \\ p_{iy} & q_{iy} & r_{iy} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \tag{2.8}$$

Here, points **p**, **q** and **r** are used to mark the position of the rigid body in two-dimensional space. These three points must not all travel in the same line in each rigid body position. This precaution is taken in order to prevent the rows in the C-S link displacement matrix (equation 2.8) from becoming proportional. With proportional rows, the matrix in equation 2.8 to be inverted cannot be inverted.

Although all translation terms are now eliminated from the design equations, an initial translation term is required when assembling the synthesized mechanism. This can be accomplished by trial and error. The optimum translation term would be that which causes the assembled mechanism to produce the smallest errors between the specified rigid body points and the rigid body points produced by the synthesized mechanism..

2.5 Revolute-Revolute (R-R) Link

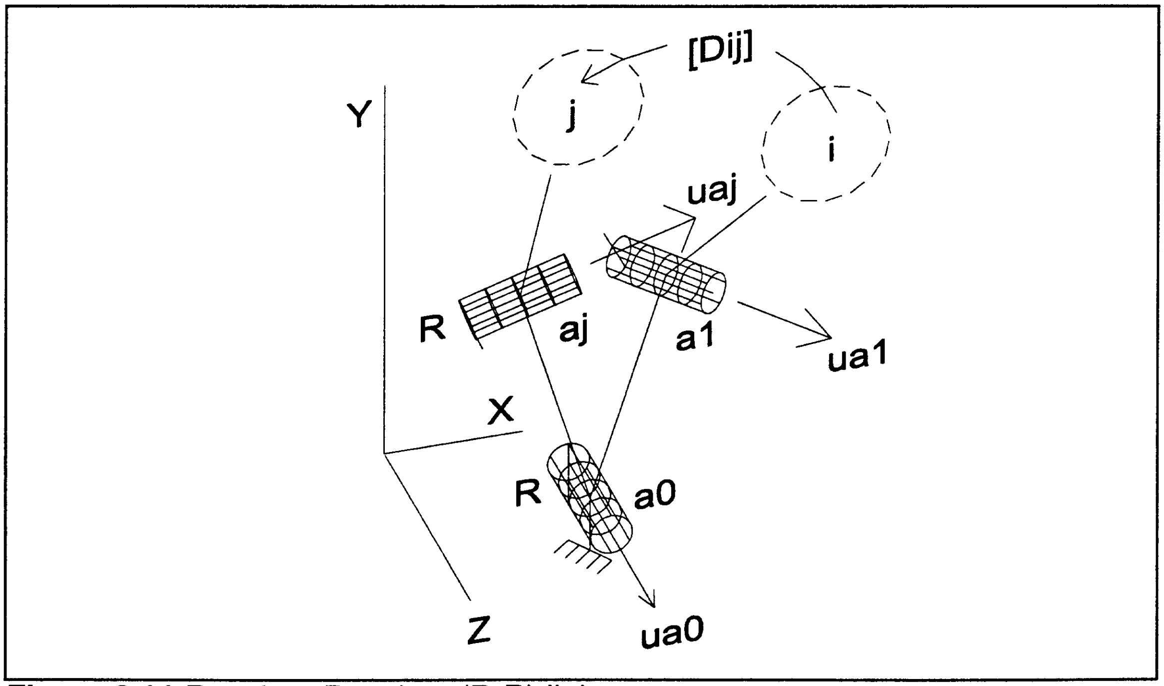

Figure 2.11 Revolute-Revolute (R-R) link

The R-R link must satisfy several conditions (in addition to the constant length condition). Given a fixed pivot $\mathbf{a}_0$ and a moving pivot $\mathbf{a}_1$ (in addition to the fixed and moving pivot joint axes $\mathbf{ua}_0$ and $\mathbf{ua}_1$), the following constraint equations are used:

$$(\mathbf{ua}_0)^T(\mathbf{a}_j - \mathbf{a}_0) = 0, \quad (\mathbf{ua}_j)^T(\mathbf{a}_j - \mathbf{a}_0) \qquad j = 1, 2, 3 \tag{2.9}$$

$$(\mathbf{ua}_0)^T(\mathbf{ua}_0) = 1, \quad (\mathbf{ua}_1)^T(\mathbf{ua}_1) = 1 \tag{2.10}$$

$$(\mathbf{a}_j - \mathbf{a}_0)^T(\mathbf{a}_j - \mathbf{a}_0) = (\mathbf{a}_j - \mathbf{a}_0)^T(\mathbf{a}_j - \mathbf{a}_0) \qquad j = 2, 3 \tag{2.11}$$

$$[(\mathbf{a}_j + \mathbf{ua}_j) - (\mathbf{a}_0 + \mathbf{ua}_0)]^T[(\mathbf{a}_j + \mathbf{ua}_j) - (\mathbf{a}_0 + \mathbf{ua}_0)] =$$
$$[(\mathbf{a}_1 + \mathbf{ua}_1) - (\mathbf{a}_0 + \mathbf{ua}_0)]^T[(\mathbf{a}_1 + \mathbf{ua}_1) - (\mathbf{a}_0 + \mathbf{ua}_0)] \qquad j = 2, 3 \tag{2.12}$$

where

$$\mathbf{a}_0 = (a_{0x}, a_{0y}, a_{0z}) \qquad \mathbf{a}_1 = (a_{1x}, a_{1y}, a_{1z}) \qquad \mathbf{a}_j = [D_{ij}]\mathbf{a}_1$$

$$\mathbf{ua}_0 = (ua_{0x}, ua_{0y}, ua_{0z}) \qquad \mathbf{ua}_1 = (ua_{1x}, ua_{1y}, ua_{1z}) \qquad \mathbf{ua}_j = [R_{ij}]\mathbf{ua}_1$$

and

$$[R_{ij}] = \begin{bmatrix} ua_{0x}{}^2V\varphi_{ij} + C\varphi_{ij} & ua_{0x}{}^2ua_{0y}{}^2V\varphi_{ij} - ua_{0z}S\varphi_{ij} & ua_{0x}{}^2ua_{0z}{}^2V\varphi_{ij} + ua_{0y}S\varphi_{ij} \\ ua_{0x}{}^2ua_{0y}{}^2V\varphi_{ij} + ua_{0z}S\varphi_{ij} & ua_{0y}{}^2V\varphi_{ij} + C\varphi_{ij} & ua_{0y}{}^2ua_{0z}{}^2V\varphi_{ij} - ua_{0x}S\varphi_{ij} \\ ua_{0x}{}^2ua_{0z}{}^2V\varphi_{ij} - ua_{0y}S\varphi_{ij} & ua_{0y}{}^2ua_{0z}{}^2V\varphi_{ij} + ua_{0x}S\varphi_{ij} & ua_{0z}{}^2V\varphi_{ij} + C\varphi_{ij} \end{bmatrix}$$

(2.12)

$$V\varphi_{ij} = 1 - \cos(\varphi_j - \varphi_i), \quad S\varphi_{ij} = \sin(\varphi_j - \varphi_i), \quad C\varphi_{ij} = \cos(\varphi_j - \varphi_i) \qquad (2.13 - 2.15)$$

These equations form a set of 12 design equations with 12 unknowns scalar components for $\mathbf{ua}_0$, $\mathbf{ua}_1$, $\mathbf{a}_0$ and $\mathbf{a}_1$. Therefore, the maximum number of rigid body positions that can be specified for an R-R link to be used for rigid body guidance is three, with no arbitrary choice of parameter.

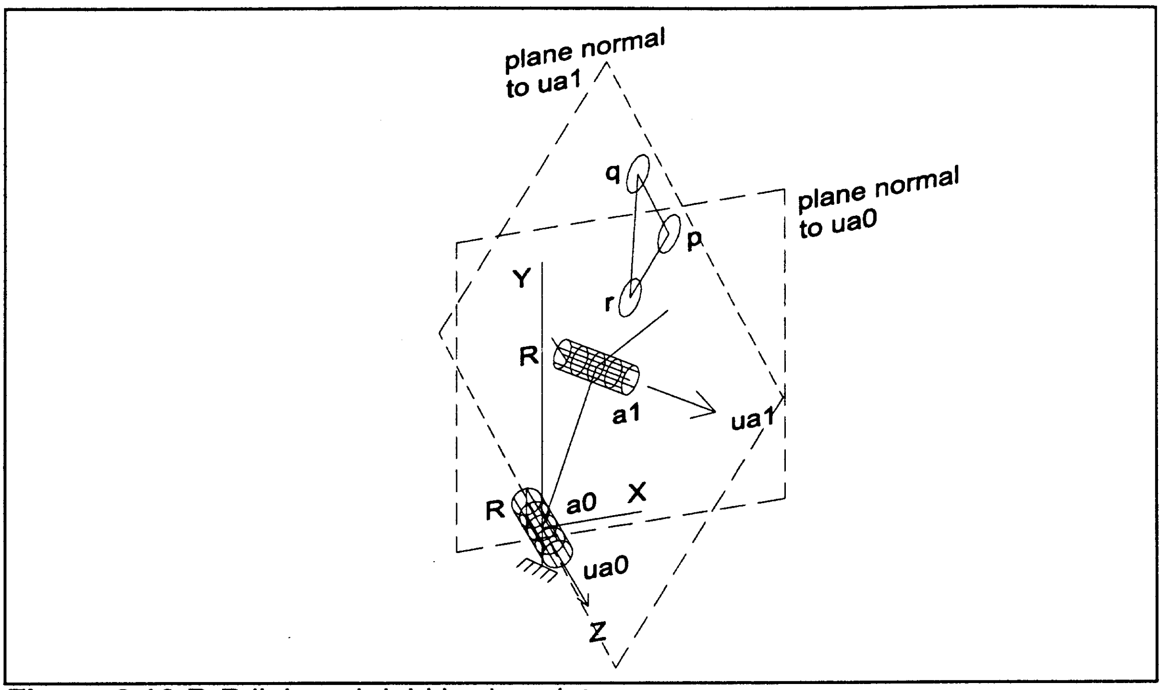

Figure 2.12 R-R link and rigid body points

When the rigid body points travel in planes that are normal to joint axis $\mathbf{ua}_j$ and are projected on plane that is normal to joint axis $\mathbf{ua}_0$ (therefore prescribing both joint axes) only equation 2.11 must be satisfied (with j = 2, 3, . . . n). Therefore

$\mathbf{a}_0 = (a_{0x}, a_{0y})$ $\mathbf{a}_1 = (a_{1x}, a_{1y})$ $\mathbf{ua}_0 = (0, 0, 1)$ $\mathbf{ua}_1 = (ua_{1x}, 0, ua_{1z})$ $\mathbf{a}_j = [D_{ij}]\mathbf{a}_1$

and

$$[D_{ij}] = \begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{ix} & q_{ix} & r_{ix} \\ p_{iy} & q_{iy} & r_{iy} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \qquad (2.16)$$

Here, points **p**, **q** and **r** are used to mark the position of the moving body in two-dimensional space. These three points must not all travel in the same line in each rigid body position. This precaution is taken in order to prevent the rows in the R-S link displacement matrix (equation 2.16) from becoming proportional.

With proportional rows, the matrix in equation 2.16 to be inverted cannot be inverted. In this research, the value of $\mathbf{ua}_1$ corresponds to an R-R link when along the positive Y-axis.

2.6 Rigid Body Point Selection Methods

The S-S link has a hemispherical workspace. This means that the S-S link can travel virtually anywhere in three-dimensional space. Due to this quality of the S-S link, the rigid body points required to synthesize this link can be used just as they appear in space. No rigid body point projections are required. The user can place the rigid body points where desired and move them anywhere is three-dimensional space

The R-S and C-S links travel in planes that are normal to their joint axes. The specified rigid body points used to synthesize these links must be projected on planes that are normal to these joint axes. This means that the joint axes of the R-S and C-S links can be specified before the actual joint link variables are calculated. By specifying a plane in space, the user establishes the value of the joint axis (since the joint axis is normal to the specified plane). The user can then place the rigid body position points where desired and move them anywhere in three-dimensional space. However, only the rigid body point coordinates obtained after projecting the points on the specified plane are used to synthesize the R-S and C-S links

A rigid body connected to an R-R link travels in a plane that is normal to $\mathbf{ua}_1$ and rotates about $\mathbf{ua}_0$. If the prescribed rigid body points travel in the same manner, both joint axes can be specified before the R-R link joint variables are calculated. By specifying a plane in space, the user establishes the initial value of $\mathbf{ua}_1$ (since $\mathbf{ua}_1$ is normal to the specified plane). The user can then place the rigid body position points where desired. The specified plane and rigid body points can be rotated (about joint axis $\mathbf{ua}_0$) and the rigid body points moved (parallel to the specified plane) to obtain additional rigid body positions. By selecting rigid body positions in this manner, the user can know the joint axes of the R-R link before synthesizing the mechanism. This "foreknowledge" of the link joint axes is one of the strengths of this synthesis method since it makes the constant length condition (eq. 2.11) the only constraint for the R-R link.

Since the four mechanisms used in this research are made up of a combination of R-R, S-S, R-S and C-S links, the rigid body point requirements for each link must satisfied in each mechanism. In the RRSS mechanism, four rigid body points are required. Four points are used to synthesize the S-S link and three of the four points are used to synthesize the R-R link. The four points must travel in a plane that is normal to $\mathbf{ua}_1$ and rotates about $\mathbf{ua}_0$ in each rigid body position. Although the X, Y and Z-coordinates of the four rigid body points are used to synthesize the S-S link, only the X and Y-coordinates of three points are used to synthesize the R-R link. The points used to synthesize the S-S link must be non-planar and the points used to synthesize the R-R link must be non-linear.

In the RRSC mechanism, three rigid body points are required. Three points are used to synthesize the C-S link and three are used to synthesize the R-R link. The three points must travel in a plane that is normal to $\mathbf{ua}_1$ and rotates about $\mathbf{ua}_0$ in each position. Although the X and Y-coordinates of the three rigid body points are used to synthesize the R-R link, only the x* and y*-coordinates of three points are used to synthesize the C-S link. The points used to synthesize the C-S link and the R-R link must be non-linear.

In the RSSR-SS mechanism, four rigid body points are required. Four points are used to synthesize the S-S link and three of the four points are used to synthesize the R-S links. Although the X, Y and Z-coordinates of the four rigid body points are used to synthesize the S-S link, only the X and Y-coordinates and the x* and y*-coordinates of three points are used to synthesize the R-S links. The points used to synthesize the S-S link must be non-planar and the points used to synthesize the R-S links must be non-linear.

In the RSSR-SC mechanism, three rigid body points are required. Three points are used to synthesize the C-S and R-S links. Although the x** and y**-coordinates of the three rigid body points are used to synthesize the C-S link, only the X and Y-coordinates and the x* and y*-coordinates of three points are used to synthesize the R-S links. The points used to synthesize the C-S link and the R-S link must be non-linear.

To judiciously perform the plane selection and rotations and rigid body point selection, rotations and displacements, a CAD package that allows one to create and manipulate models in three dimensions is required. Using AutoCAD 2000 software, all of the rigid body points and link variables given in the example problems in chapter 5 were selected. This CAD software allows one to specify a point at a dimensional accuracy of up to eight significant figures. All rigid body points and mechanism link parameters presented in this research are dimensionless. Throughout the example problems in this work, the dimensionless unit “[units]” was affixed to the displacement error magnitudes calculated.

CHAPTER 3

EQUATIONS FOR TOLERANCES, MULTIPLY SEPARATED POSITIONS AND INSTANT SCREW AXIS PARAMETERS

3.1 Introduction

It was shown in chapter 2 that the points on a moving body can be prescribed using rigid body point selection schemes that specify the joint axes of the R-S, C-S and R-R links. The result of this is that the constant length condition becomes the only constraint these links must satisfy. The S-S link need only satisfy this condition as well.

Due to these rigid body point selection schemes, the constant length condition is now limited to "n" prescribed moving body positions (theoretically). This theoretical prescribed rigid body position limit makes the constant length condition ideal for multi-phase mechanism synthesis since each additional phase corresponds to an additional number of prescribed rigid body positions. This equation is also ideal since it is now the only constraint all of the links must satisfy.

3.2 Tolerance Equations

The adjustable spatial mechanism can produce ideal solutions to problems involving rigid body point tolerances. The rigid-body displacement matrix $[D_{ij}]$ must be modified however in order to incorporate rigid body point tolerances. The modified displacement matrices become

$$[D_{ij}]=\begin{bmatrix} p_{jx}\pm\delta p_{jx} & q_{jx}\pm\delta q_{jx} & r_{jx}\pm\delta r_{jx} \\ p_{jy}\pm\delta p_{jy} & q_{jy}\pm\delta q_{jy} & r_{jy}\pm\delta r_{jy} \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} p_{ix}\pm\delta p_{ix} & q_{ix}\pm\delta q_{ix} & r_{ix}\pm\delta r_{ix} \\ p_{iy}\pm\delta p_{iy} & q_{iy}\pm\delta q_{iy} & r_{iy}\pm\delta r_{iy} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \quad (3.1)$$

for the R-S, C-S and R-R links and

$$[D_{ij}]=\begin{bmatrix} p_{jx}\pm\delta p_{jx} & q_{jx}\pm\delta q_{jx} & r_{jx}\pm\delta r_{jx} & s_{jx}\pm\delta s_{jx} \\ p_{jy}\pm\delta p_{jy} & q_{jy}\pm\delta q_{jy} & r_{jy}\pm\delta r_{jy} & s_{jy}\pm\delta s_{jy} \\ p_{jz}\pm\delta p_{jz} & q_{jz}\pm\delta q_{jz} & r_{jz}\pm\delta r_{jz} & s_{jz}\pm\delta s_{jz} \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} p_{ix}\pm\delta p_{ix} & q_{ix}\pm\delta q_{ix} & r_{ix}\pm\delta r_{ix} & s_{ix}\pm\delta s_{ix} \\ p_{iy}\pm\delta p_{iy} & q_{iy}\pm\delta q_{iy} & r_{iy}\pm\delta r_{iy} & s_{iy}\pm\delta s_{iy} \\ p_{iz}\pm\delta p_{iz} & q_{iz}\pm\delta q_{iz} & r_{iz}\pm\delta r_{iz} & s_{iz}\pm\delta s_{iz} \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1}$$

(3.2)

for the S-S link. Here $\delta\mathbf{p}$, $\delta\mathbf{q}$, $\delta\mathbf{r}$ and $\delta\mathbf{s}$ are the upper and lower rigid body point tolerances.

When the non-tolerance rigid body displacement matrices are used, circle point curves and center point curves can be generated for multi-phase finitely separated position problems. An infinite number of mechanism solutions can be selected from these curves which allow the synthesized mechanism to satisfy the precise rigid body position requirements.

When the modified rigid body displacement matrices are used, circle point regions and center point regions can be generated for multi-phase finitely separated position problems. These regions are bounded by the curves generated by using the rigid bodies with the specified upper tolerance limit and lower tolerance limits. An infinite number of mechanism solutions can be selected from these regions which allow the synthesized mechanism to satisfy both the precise rigid body position requirements and the rigid body position tolerance requirements.

3.3 Velocity and Acceleration Equations

By differentiating the constant length equation, the velocity equation can be calculated for a rigid body. Given a fixed pivot $\mathbf{a}_0$ and a moving pivot $\mathbf{a}_1$, the general rigid body velocity equation becomes

$$(\mathbf{a'}_j)^T(\mathbf{a}_j - \mathbf{a}_0) = 0 \tag{3.3}$$

where

$$\mathbf{a'}_j = [V_{ij}]\mathbf{a}_1 \qquad V_{ij} = [V][D_{ij}]$$

and

$$[V] = \begin{bmatrix} [W] & (\mathbf{p'} - [W]\mathbf{p}) \\ 0 & 0 \end{bmatrix} \tag{3.4}$$

and

$$[W] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} = \omega \begin{bmatrix} 0 & -up_z & up_y \\ up_z & 0 & -up_x \\ -up_y & up_x & 0 \end{bmatrix} \tag{3.5}$$

Matrix [V] is the spatial velocity matrix. Matrix [W] is the spatial angular velocity matrix. Variables **p** and **up** represent the reference point and the joint axis unit vector at the reference point. In this research, **p** is equivalent to the fixed pivot, and **up** is equivalent to the joint axis unit vector at the fixed pivot (therefore $\mathbf{p} = \mathbf{a}_0$ and $\mathbf{up} = \mathbf{ua}_0$).

By differentiating the velocity equation, the acceleration equation can be calculated for a rigid body. Given a fixed pivot $\mathbf{a}_0$ and a moving pivot $\mathbf{a}_1$, the general rigid body acceleration equation becomes

$$(\mathbf{a''}_j)^T(\mathbf{a}_j - \mathbf{a}_0) + (\mathbf{a'}_j)^T(\mathbf{a'}_j) = 0 \tag{3.6}$$

where

$$\mathbf{a}''_j = [A_{ij}]\mathbf{a}_1 \qquad A_{ij} = [A][D_{ij}]$$

and

$$[A] = \begin{bmatrix} [W'] & (\mathbf{p}''-[W']\mathbf{p}) \\ 0 & 0 \end{bmatrix} \tag{3.7}$$

and

$$[W'] = \begin{bmatrix} (up_x^{\,2}-1)\omega^2 & (up_x up_y \omega^2 - \dot{up}_z\omega - up_z\alpha) & (up_x up_z \omega^2 + \dot{up}_y\omega + up_y\alpha) \\ (up_x up_y \omega^2 + \dot{up}_z\omega + up_z\alpha) & (up_y^{\,2}-1)\omega^2 & (up_y up_z \omega^2 - \dot{up}_x\omega - up_x\alpha) \\ (up_x up_z \omega^2 - \dot{up}_y\omega - up_y\alpha) & (up_y up_z \omega^2 - \dot{up}_x\omega + up_x\alpha) & (up_z^{\,2}-1)\omega^2 \end{bmatrix}$$

(3.8)

Matrix [A] is the spatial acceleration matrix and matrix [W'] is the spatial angular acceleration matrix.

3.4 Instant Screw Axis Parameters

The general spatial velocity matrix in terms of instant screw axis parameters is given by

$$[V_s] = \begin{bmatrix} [W_{\mathbf{up}_0}] & (s'\mathbf{up}_0 - [W_{\mathbf{up}_0}]\mathbf{p}_0) \\ 0 & 0 \end{bmatrix} \tag{3.9}$$

where variables $\mathbf{p}_0$, $\mathbf{up}_0$, and s' correspond to a point on the ISA, the unit vector of the ISA and the linear velocity norm along the ISA. Matrix $[W_{up0}]$ is the spatial angular velocity matrix in terms of unit vector $\mathbf{up}_0$ and the angular velocity about the ISA (ω_s).

The general spatial acceleration matrix in terms of instant screw axis parameters is given by

$$[A_s] = \begin{bmatrix} [W'_{up_0}] & s''\mathbf{up}_0 + s'\mathbf{up}_0' - [W'_{up_0}]\mathbf{p}_0 - [W_{up_0}]\mathbf{p}_0' \\ 0 & 0 \end{bmatrix} \tag{3.10}$$

Matrix $[W'_{up0}]$ is the spatial angular acceleration matrix in terms of the ISA unit vector $\mathbf{up}_0$ and the angular velocity and acceleration about the ISA (ω_s and α_s).

Equating the fourth columns of [V] and $[V_s]$, and [A] and $[A_s]$, the following relations are obtained:

$$s'\mathbf{up}_0 - [W_s]\mathbf{p}_0 = \mathbf{p}' - [W]\mathbf{p} \tag{3.11}$$

$$s''\mathbf{up}_0 + s'\mathbf{up}_0' - [W_s']\mathbf{p}_0 - [W_s]\mathbf{p}_0' = \mathbf{p}'' - [W']\mathbf{p} \tag{3.12}$$

When the relation $(\mathbf{up}_0)^T(\mathbf{up}_0) = 1$ is included in equation 3.11, there are four equations for eight unknowns (up_{0x}, up_{0y}, up_{0z}, p_{0x}, p_{0y}, p_{0z}, s' and ω_s) given **p**, **p'**, **up** and ω. When the relation $(\mathbf{up}_0)^T(\mathbf{up}_0) = 1$ is included in equation 3.12, there are four equations for 16 unknowns ($\mathbf{p}_0$, $\mathbf{p}_0'$, $\mathbf{up}_0$, $\mathbf{up}_0'$, s', s'', ω_s and α_s) given **p**, **p''**, **up**, **up'**, ω and α. When equations 3.11, 3.12 and the relation $(\mathbf{up}_0)^T(\mathbf{up}_0) = 1$ are used together, there are 7 equations for 16 unknowns.

CHAPTER 4

2 AND 3 PHASE PROBLEMS FOR MULTI-PHASE MOTION GENERTAION AND MULTIPLY SEPARATED POSITIONS

4.1 Tables of Prescribed Positions and Adjustment Phases

Table 4.1 Rigid body and phase variations for R-R and S-S links

		R-R Link		S-S Link	
number of phases	number of rigid body positions	number of unknowns	number of free choices	number of unknowns	number of free choices
1	5	4	0	6	2
2	8	6	0	9	3
3	11	8	0	12	4
4	14	10	0	15	5
n	5+3(n-1)	2+2n	0	3+3n	1+n

Table 4.2 Rigid body and phase variations for R-S and C-S links

		R-S Link		C-S Link	
number of phases	number of rigid body positions	number of unknowns	number of free choices	number of unknowns	number of free choices
1	5	4	0	4	0
2	8	6	0	6	0
3	11	8	0	8	0
4	14	10	0	10	0
n	5+3(n-1)	2+2n	0	2+2n	0

The rigid body position values given in the second columns of tables 4.1 and 4.2 are the theoretical maximum prescribed position values. They are based on the number of link variables added with each additional phase. Although the R-R, R-S and C-S links require no prescribed link variables (free choices), the S-S link does. This is because, unlike the other links, the S-S link is not constrained to planar motion. As stated in chapter 1.6, part of this research involves determining the actual maximum number of prescribed rigid body positions for two and three phase problems. As the shaded rows in tables 4.1 and 4.2 indicate, this research focuses on two and three-phase problems involving the R-R, S-S R-S and C-S links.

4.2 Link Adjustment Possibilities

The following adjustments are possible for the R-R, R-S, C-S and S-S links:

Table 4.3 Mechanism link adjustment possibilities

1. adjust the moving pivots while maintaining fixed crank and follower lengths
2. adjust the fixed pivots while maintaining fixed crank and follower lengths
3. adjust the moving pivots and crank and follower lengths between phases
4. adjust the fixed pivots and crank and follower lengths between phases

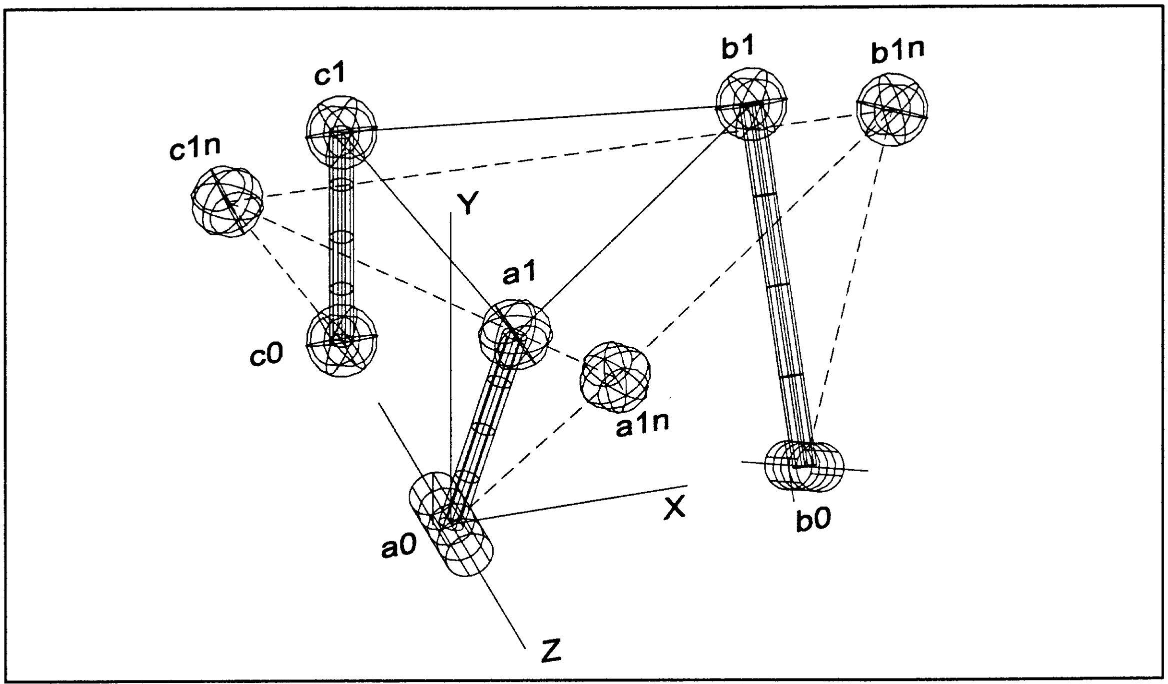

Figure 4.1 Adjustment possibility #1 for the RSSR-SS mechanism

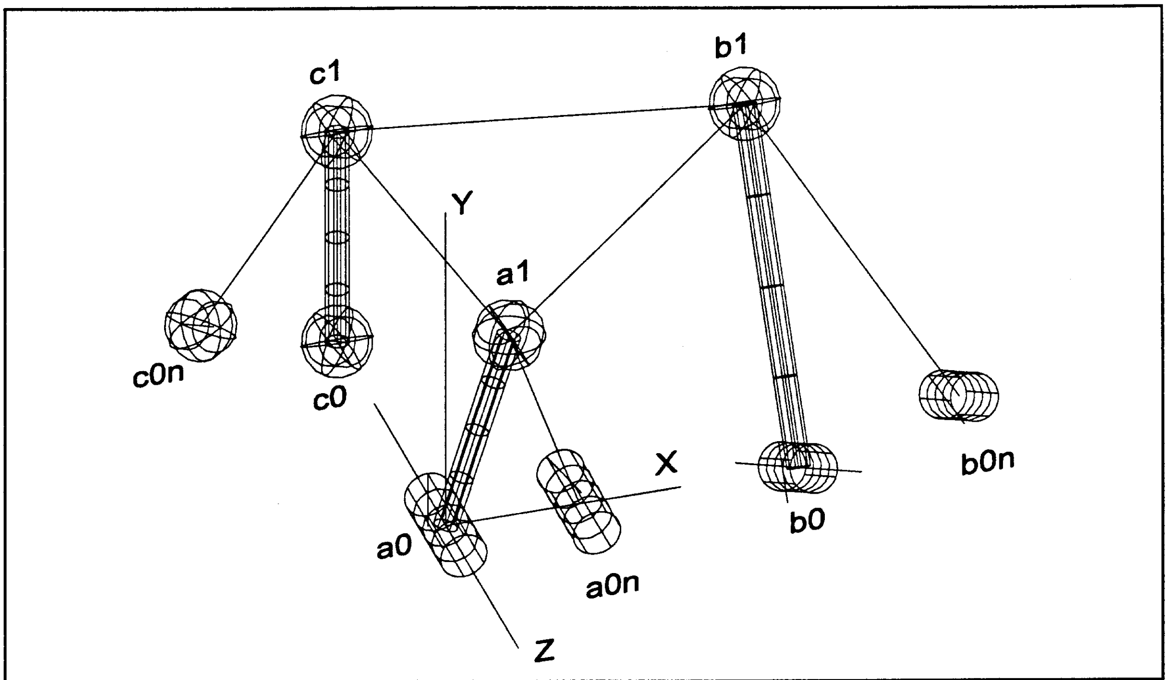

Figure 4.2 Adjustment possibility #2 for the RSSR-SS mechanism

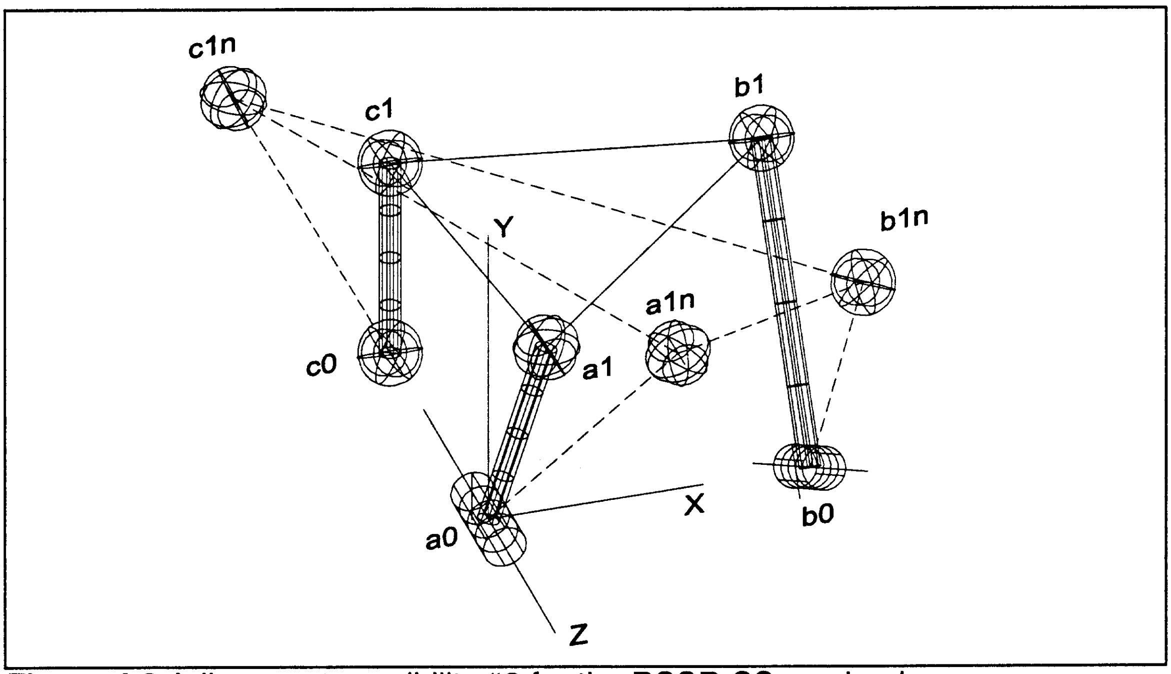

Figure 4.3 Adjustment possibility #3 for the RSSR-SS mechanism

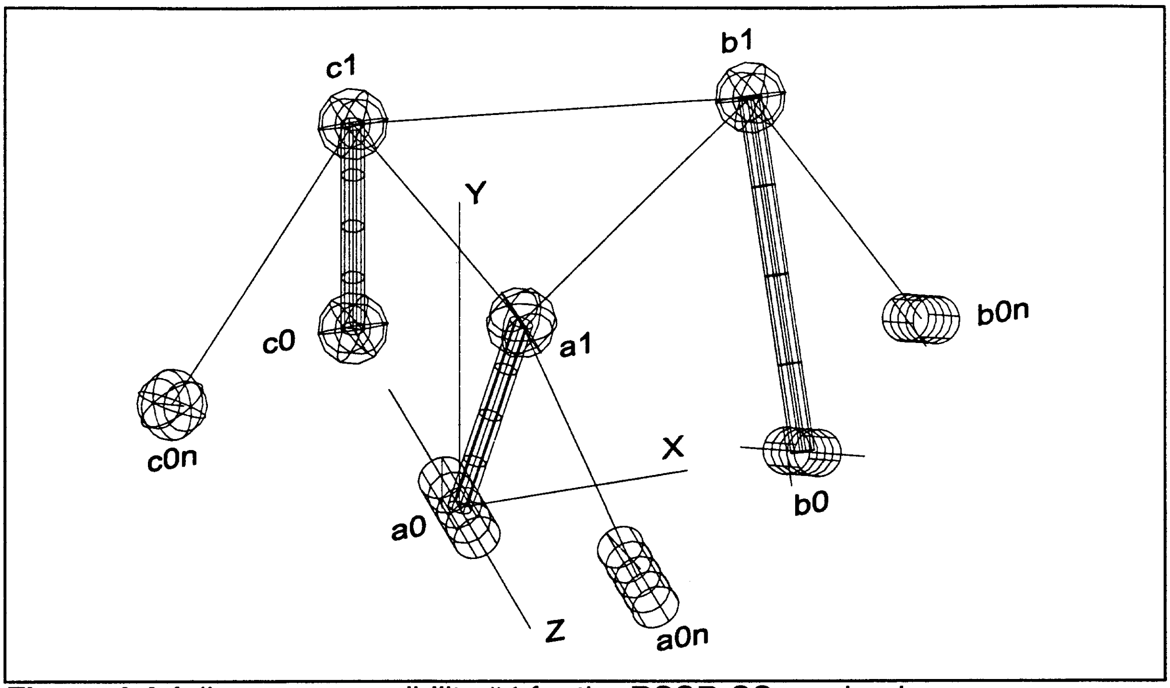

Figure 4.4 Adjustment possibility #4 for the RSSR-SS mechanism

The first and third adjustment possibilities can be applied to the R-R, R-S, C-S and S-S links in this research. The second adjustment possibility however can only be applied to the R-S and S-S links. The fourth adjustment possibility can only be applied to the R-R, R-S and S-S links in this research. For the R-R ink, it is only possible when the new fixed pivot lies along the length of the link calculated for the first phase.

The adjustments made to each link must remain within their corresponding workspaces. The S-S link has a hemispherical workspace with the link length equal to the radius of the sphere. The R-R, R-S and C-S links have planar circular workspaces (in planes normal to the fixed pivot joint axes) with the link lengths equal to the radii of the circles.

4.3 Adjustable Moving Pivot Problems

In the 2-phase moving pivot problem with fixed crank and follower lengths, the fixed pivot $\mathbf{a}_0$ and moving pivots $\mathbf{a}_1$ and $\mathbf{a}_{1n}$ are to be calculated. There is a maximum of 8 prescribed rigid body positions resulting in a set of 6 equations. This set of equations can be a combination of position, velocity and/or acceleration equations. The following are the 6 displacement equations:

Phase 1 $F_i = ([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0, \quad i = 1, 2, 3$ (4.1)

Phase 2 $F_j = ([D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n} - \mathbf{a}_0)^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0,$
$j = 4, 5, 6$ (4.2)

When expressed as velocity equations they become

Phase 1 $F_i = ([V][D_{1,i+1}]\mathbf{a}_1)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) = 0$ (4.3)

Phase 2 $F_j = ([V][D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n})^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n} - \mathbf{a}_0) = 0$ (4.4)

When expressed as acceleration equations they become

Phase 1 $F_i = ([A][D_{1,i+1}]\mathbf{a}_1)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)+([V][D_{1,i+1}]\mathbf{a}_1)^T([V][D_{1,i+1}]\mathbf{a}_1) = 0$ (4.5)

Phase 2 $F_j = ([A][D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n})^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n} - \mathbf{a}_0) +$
$([V][D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n})^T([V][D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n}) = 0$ (4.6)

In the 3-phase moving pivot problem with fixed crank and follower lengths, the fixed pivot $\mathbf{a}_0$ and moving pivots $\mathbf{a}_1$, $\mathbf{a}_{1n}$ and $\mathbf{a}_{2n}$ are to be calculated. There is a maximum of 11 prescribed rigid body positions resulting in a set of 8 equations. The following are the 8 displacement equations:

Phase 1 $F_i = ([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0, \quad i = 1, 2, 3$ (4.7)

Phase 2 $F_j = ([D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n} - \mathbf{a}_0)^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0,$
$j = 4, 5, 6$ (4.8)

Phase 3 $F_k = ([D_{9,k+3}][D_{1,9}]\mathbf{a}_{2n} - \mathbf{a}_0)^T([D_{9,k+3}][D_{1,9}]\mathbf{a}_{2n} - \mathbf{a}_0) - R_1^2 = 0,$

$k = 7, 8$ (4.9)

When expressed as velocity equations they become

Phase 1 $F_i = ([V][D_{1,i+1}]\mathbf{a}_1)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) = 0$ (4.10)

Phase 2 $F_j = ([V][D_{6,j+2}][D_{1,6}]\mathbf{a}_{1n})^T([D_{6,j+2}][D_{1,6}]\mathbf{a}_{1n} - \mathbf{a}_0) = 0$ (4.11)

Phase 3 $F_k = ([V][D_{9,k+3}][D_{1,9}]\mathbf{a}_{2n})^T([D_{9,k+3}][D_{1,9}]\mathbf{a}_{2n} - \mathbf{a}_0) = 0$ (4.12)

When expressed as acceleration equations they become

Phase 1 $F_i = ([A][D_{1,i+1}]\mathbf{a}_1)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)+([V][D_{1,i+1}]\mathbf{a}_1)^T([V][D_{1,i+1}]\mathbf{a}_1) = 0$ (4.13)

Phase 2 $F_j = ([A][D_{6,j+2}][D_{1,6}]\mathbf{a}_{1n})^T([D_{6,j+2}][D_{1,6}]\mathbf{a}_{1n} - \mathbf{a}_0) +$

$([V][D_{6,j+2}][D_{1,6}]\mathbf{a}_{1n})^T([V][D_{6,j+2}][D_{1,6}]\mathbf{a}_{1n}) = 0$ (4.14)

Phase 3 $F_k = ([A][D_{9,k+3}][D_{1,9}]\mathbf{a}_{2n})^T([D_{9,k+3}][D_{1,9}]\mathbf{a}_{2n} - \mathbf{a}_0) +$

$([V][D_{9,k+3}][D_{1,9}]\mathbf{a}_{2n})^T([V][D_{9,k+3}][D_{1,9}]\mathbf{a}_{2n}) = 0$ (4.15)

4.4 Adjustable Moving Pivot and Crank/Follower Length Problems

In the 2-phase moving pivot problem with crank and follower lengths that are adjusted between phases, the fixed pivot $\mathbf{a}_0$ and moving pivots $\mathbf{a}_1$ and $\mathbf{a}_{1n}$ are to be calculated. There is a maximum of 8 prescribed rigid body positions resulting in a set of 6 equations. This set of equations can be a combination of position, velocity and/or acceleration equations. The following are the 6 displacement equations:

Phase 1 $F_i = ([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad i = 1, 2, 3$ (4.16)

Phase 2 $F_j = ([D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n} - \mathbf{a}_0)^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n} - \mathbf{a}_0) - R_2^2 = 0$

$j = 4, 5, 6$ (4.17)

Equations 4.3 through 4.6 remain the same in two phase moving pivot problems with adjustable crank and follower lengths.

In the 3-phase moving pivot problem with crank and follower lengths that are adjusted between phases, the fixed pivot $\mathbf{a}_0$ and moving pivots $\mathbf{a}_1$, $\mathbf{a}_{1n}$ and $\mathbf{a}_{2n}$ are to be calculated. There is a maximum of 11 prescribed rigid body positions resulting in a set of 8 equations. The following are the 8 displacement equations:

Phase 1 $F_i = ([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \qquad i = 1, 2, 3 \qquad (4.18)$

Phase 2 $F_j = ([D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n} - \mathbf{a}_0)^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_{1n} - \mathbf{a}_0) - R_2^2 = 0$
$j = 4, 5, 6 \qquad (4.19)$

Phase 3 $F_k = ([D_{9,k+3}][D_{1,9}]\mathbf{a}_{2n} - \mathbf{a}_0)^T([D_{9,k+3}][D_{1,9}]\mathbf{a}_{2n} - \mathbf{a}_0) - R_3^2 = 0$
$k = 7, 8 \qquad (4.20)$

Equations 4.10 through 4.15 remain the same in three phase moving pivot problems with adjustable crank and follower lengths.

4.5 Adjustable Fixed Pivot Problems

In the 2-phase fixed pivot problem with fixed crank and follower lengths, the moving pivot $\mathbf{a}_1$ and fixed pivots $\mathbf{a}_0$ and $\mathbf{a}_{0n}$ are to be calculated. There is a maximum of 8 prescribed rigid body positions resulting in a set of 6 equations. This set of equations can be a combination of position, velocity and/or acceleration equations. The following are a set of the 6 displacement equations:

Phase 1 $F_i = ([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \qquad i = 1, 2, 3 \qquad (4.21)$

Phase 2 $F_j = ([D_{5,j+2}][D_{1,5}]\mathbf{a}_1 - \mathbf{a}_{0n})^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0$
$j = 4, 5, 6 \qquad (4.22)$

When expressed as velocity equations they become

Phase 1 $F_i = ([V][D_{1,i+1}]\mathbf{a}_1)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) = 0$ (4.23)

Phase 2 $F_j = ([V][D_{5,j+2}][D_{1,5}]\mathbf{a}_1)^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_1 - \mathbf{a}_{0n}) = 0$ (4.24)

When expressed as acceleration equations they become

Phase 1 $F_i = ([A][D_{1,i+1}]\mathbf{a}_1)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)+([V][D_{1,i+1}]\mathbf{a}_1)^T([V][D_{1,i+1}]\mathbf{a}_1) = 0$ (4.25)

Phase 2 $F_j = ([A][D_{5,j+2}][D_{1,5}]\mathbf{a}_1)^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_1 - \mathbf{a}_{0n}) + ([V][D_{5,j+2}][D_{1,5}]\mathbf{a}_1)^T([V][D_{5,j+2}][D_{1,5}]\mathbf{a}_1)= 0$ (4.26)

In the 3-phase fixed pivot problem with fixed crank and follower lengths, the moving pivot $\mathbf{a}_1$ and fixed pivots $\mathbf{a}_0$, $\mathbf{a}_{0n}$ and $\mathbf{a}_{20n}$ are to be calculated. There is a maximum of 11 prescribed rigid body positions resulting in a set of 8 equations. The following are a set of the 8 displacement equations:

Phase 1 $F_i = ([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad i = 1, 2, 3$ (4.27)

Phase 2 $F_j = ([D_{5,j+2}][D_{1,5}]\mathbf{a}_1 - \mathbf{a}_{0n})^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \quad j = 4, 5, 6$ (4.28)

Phase 3 $F_k = ([D_{9,k+3}][D_{1,9}]\mathbf{a}_1 - \mathbf{a}_{20n})^T([D_{9,k+3}][D_{1,9}]\mathbf{a}_1 - \mathbf{a}_{20n}) - R_1^2 = 0 \quad k = 7, 8$ (4.29)

When expressed as velocity equations they become

Phase 1 $F_l = ([V][D_{1,l+1}]\mathbf{a}_1)^T([D_{1,l+1}]\mathbf{a}_1 - \mathbf{a}_0) = 0$ (4.30)

Phase 2 $F_j = ([V][D_{6,j+2}][D_{1,6}]\mathbf{a}_1)^T([D_{6,j+2}][D_{1,6}]\mathbf{a}_1 - \mathbf{a}_{0n}) = 0$ (4.31)

Phase 3 $F_k = ([V][D_{9,k+3}][D_{1,9}]\mathbf{a}_1)^T([D_{9,k+3}][D_{1,9}]\mathbf{a}_1 - \mathbf{a}_{20n}) = 0$ (4.32)

When expressed as acceleration equations they become

Phase 1 $F_i = ([A][D_{1,i+1}]\mathbf{a}_1)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) + ([V][D_{1,i+1}]\mathbf{a}_1)^T([V][D_{1,i+1}]\mathbf{a}_1) = 0$ (4.33)

Phase 2 $F_j = ([A][D_{6,j+2}][D_{1,6}]\mathbf{a}_1)^T([D_{6,j+2}][D_{1,6}]\mathbf{a}_1 - \mathbf{a}_{0n}) +$
$$([V][D_{6,j+2}][D_{1,6}]\mathbf{a}_1)^T([V][D_{6,j+2}][D_{1,6}]\mathbf{a}_1) = 0 \quad (4.34)$$

Phase 3 $F_k = ([A][D_{9,k+3}][D_{1,9}]\mathbf{a}_1)^T([D_{9,k+3}][D_{1,9}]\mathbf{a}_1 - \mathbf{a}_{20n}) +$
$$([V][D_{9,k+3}][D_{1,9}]\mathbf{a}_1)^T([V][D_{9,k+3}][D_{1,9}]\mathbf{a}_1) = 0 \quad (4.35)$$

4.6 Adjustable Fixed Pivot and Crank/Follower Length Problems

In the 2-phase fixed pivot problem with crank and follower lengths that are adjusted between phases, the moving pivot $\mathbf{a}_1$ and fixed pivots $\mathbf{a}_0$ and $\mathbf{a}_{0n}$ are to be calculated. There is a maximum of 8 prescribed rigid body positions resulting in a set of 6 equations. This set of equations can be a combination of position, velocity and/or acceleration equations. The following are the 6 displacement equations:

Phase 1 $F_i = ([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad i = 1, 2, 3$ (4.36)

Phase 2 $F_j = ([D_{5,j+2}][D_{1,5}]\mathbf{a}_1 - \mathbf{a}_{0n})^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_1 - \mathbf{a}_{0n}) - R_2^2 = 0$
$j = 4, 5, 6$ (4.37)

Equations 4.23 through 4.26 remain the same in two phase fixed pivot problems with adjustable crank and follower lengths.

In the 3-phase fixed pivot problem with crank and follower lengths that are adjusted between phases, the moving pivot $\mathbf{a}_1$ and fixed pivots $\mathbf{a}_0$, $\mathbf{a}_{0n}$ and $\mathbf{a}_{20n}$ are to be calculated. There is a maximum of 11 prescribed rigid body positions resulting in a set of 8 equations. The following are a set of the 8 displacement equations:

Phase 1 $F_i = ([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,i+1}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad i = 1, 2, 3$ (4.38)

Phase 2 $F_j = ([D_{5,j+2}][D_{1,5}]\mathbf{a}_1 - \mathbf{a}_{0n})^T([D_{5,j+2}][D_{1,5}]\mathbf{a}_1 - \mathbf{a}_{0n}) - R_2^2 = 0$

$$j = 4, 5, 6 \qquad (4.39)$$

Phase 3 $F_k = ([D_{9,k+3}][D_{1,9}]\mathbf{a}_1 - \mathbf{a}_{20n})^T([D_{9,k+3}][D_{1,9}]\mathbf{a}_1 - \mathbf{a}_{20n}) - R_3^2 = 0$

$$k = 7, 8 \qquad (4.40)$$

Equations 4.30 through 4.35 remain the same in three phase fixed pivot problems with adjustable crank and follower lengths.

The terms R_1, R_2 and R_3 represent the link lengths. In synthesis problems with fixed link lengths, only one link length term is required. In synthesis problems with adjustable link lengths, two or all terms are required. The number of link length terms required depends on the number of rigid body phases given and the number of link length adjustments desired.

CHAPTER 5

EXAMPLE PROBLEMS

5.1 Two-Phase Example Problems

5.1.1 RRSS Mechanism for Finitely Separated Positions

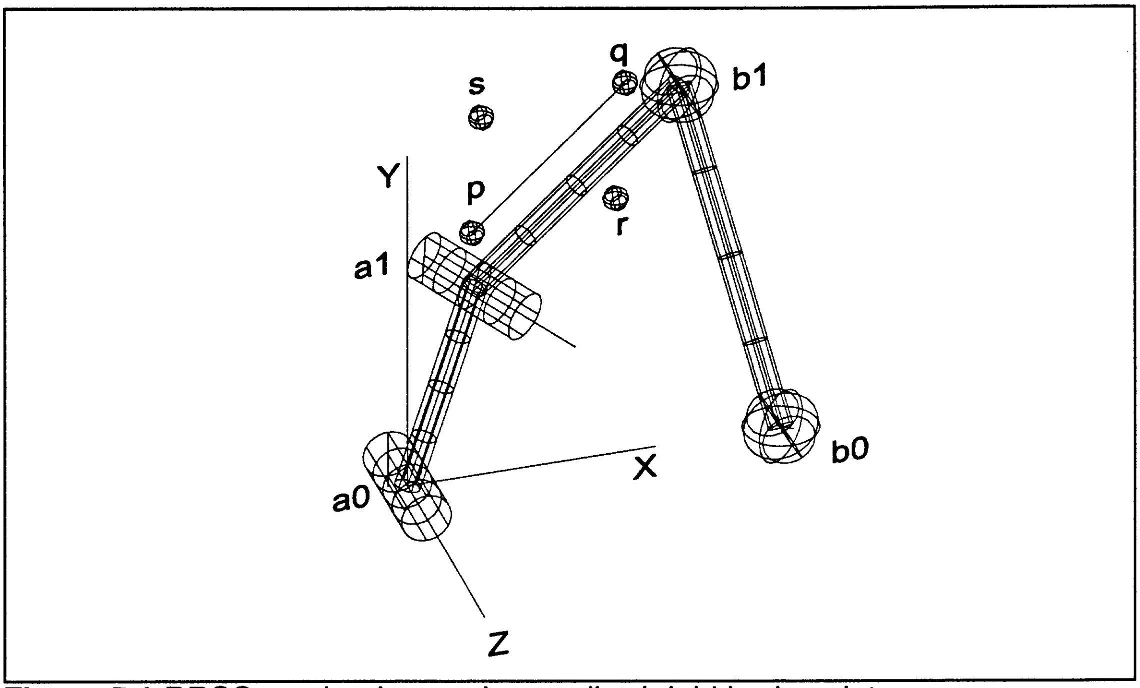

Figure 5.1 RRSS mechanism and prescribed rigid body points

The S-S link (link $\mathbf{b}_0$-$\mathbf{b}_1$) was the first link synthesized in this two-phase adjustable fixed pivot problem. The length of the S-S link remained fixed throughout each phase.

The values for 8 prescribed rigid body positions are given in table 5.1. These positions are represented by points **p**, **q**, **r** and **s**. To satisfy the design equations of the S-S link, these points do not all lie on the same plane in each rigid body position.

Table 5.1 Prescribed X-Y-Z frame rigid body positions for 2-phase RRSS fixed pivot problem with fixed crank and follower lengths

	p	q	r	s
Phase 1				
pos. 1	-0.5096.1.0541,0.1040	1.3326,2.9344,-0.2720	0.7928,1.7071,0.3485	0.5928, 1.7071,-0.6313
pos. 2	-0.6860,1.0111,0.1014	1.0406,2.9872,-0.3285	0.5737,1.7488,0.3282	0.3776,1.7070,-0.6515
pos. 3	-0.8558,0.9360,0.0972	0.7449,3.0055,-0.3779	0.3523,1.7589,0.3110	0.1676,1.6769,-0.6683
pos. 4	-1.0136,0.8301,0.0914	0.4504,2.9902,-0.4200	0.1336,1.7380,0.2971	-0.0324,1.6187,-0.6818
Phase 2				
pos. 5	-0.3353,1.0440,0.1045	1.5427,2.9038,-0.1869	0.9779,1.6620,0.3802	0.7819,1.7064,-0.5994
pos. 6	-0.1676,0.9989,0.1012	1.7294,2.8525,-0.0890	1.1475,1.5927,0.4180	0.9633,1.6808,-0.5610
pos. 7	-0.0117,0.9194,0.0934	1.8820,2.7847,0.0214	1.2946,1.5026,0.4618	1.1297,1.6323,-0.5159
pos. 8	0.1276,0.8063,0.0801	1.9867,2.7064,0.1434	1.4113,1.3965,0.5118	1.2726,1.5647,-0.4642

Note: In this problem, no rigid body positions are shared

All rigid body points in this example problem were taken using $\mathbf{ua}_1$=[sin 15°, 0, cos 15°] when the R-R link lies along the positive Y-axis.

The required S-S link variables are $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_{0n}$. Variable $\mathbf{b}_1$ represents the moving pivot of the S-S link. Variables $\mathbf{b}_0$ and $\mathbf{b}_{0n}$ represent the fixed pivots in phase 1 and phase 2 of the S-S link. Since each of these variables has three scalar components, there are a total of nine required unknowns.

$$\mathbf{b}_0=(b_{0x}, b_{0y}, b_{0z}) \qquad \mathbf{b}_1=(b_{1x}, b_{1y}, b_{1z}) \qquad \mathbf{b}_{0n}=(b_{0nx}, b_{0ny}, b_{0nz})$$

The eight prescribed rigid body positions result in six design equations. Therefore, three of the nine required unknowns were specified. Using AutoCAD 2000 software, the value of $\mathbf{b}_1$ was specified to $\mathbf{b}_1$=[1.7792, 1.7792, -0.4767]. The following six design equations were used to calculate $\mathbf{b}_0$ and $\mathbf{b}_{0n}$:

$$([D_{1,2}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,2}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \tag{5.1}$$

$$([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \tag{5.2}$$

$$([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.3)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \quad (5.4)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \quad (5.5)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \quad (5.6)$$

The term R_1 represents the length of the S-S link. Since this example problem involved fixed pivot adjustments with fixed crank and follower lengths, the value of R_1 remained constant in equations 5.1 through 5.6. The specified value for R_1 is 2. Given the following initial guesses:

$\mathbf{b}_0$=(1.2, 0.1, 0.1) $\mathbf{b}_{0n}$=(1.4, -0.1, 0)

The solution to equations 5.1 through 5.6 converged to the following using Newton's Method:

$\mathbf{b}_0$=(1.0003, 0.0260, 0.0888) $\mathbf{b}_{0n}$=(1.4862, -0.1403, 0.0019)

The R-R link (link $\mathbf{a}_0$-$\mathbf{a}_1$) was the next link synthesized in this two-phase adjustable fixed pivot problem. Like the S-S link, the length of the R-R link remained constant throughout each phase. The X and Y-coordinates of points **p**, **q** and **r** in table 5.1 were used to synthesize the R-R link. Points **p**, **q**, **r** and **s** travel in a plane that is normal to $\mathbf{ua}_j$ and rotates about $\mathbf{ua}_0$ in each coupler position. The points are also non-linear.

The required R-R link variables are $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{0n}$. Variable $\mathbf{a}_1$ represents the moving pivot of the R-R link. Variables $\mathbf{a}_0$ and $\mathbf{a}_{0n}$ represent the fixed pivots in phase 1 and phase 2 of the R-R link. Since each of these variables has two scalar components, there are a total of six required unknowns.

$\mathbf{a}_0=(a_{0x}, a_{0y})$ $\mathbf{a}_1=(a_{1x}, a_{1y})$ $\mathbf{a}_{0n}=(a_{0nx}, a_{0ny})$

The eight prescribed rigid body positions resulted in six design equations. The following set of design equations were used to calculate $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{0n}$:

$$([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.7)$$

$$([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.8)$$

$$([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.9)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \quad (5.10)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \quad (5.11)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \quad (5.12)$$

The term R_1 represents the length of the R-R link. Since this example problem involved fixed pivot adjustments with fixed crank and follower lengths, the value of R_1 remained constant in equations 5.7 through 5.12. The specified value for R_1 is 1. Given the following initial guesses:

$\mathbf{a}_0$=(0.1, 0.1) $\mathbf{a}_1$=(0.1, 0.85) $\mathbf{a}_{0n}$=(0.1, 0.1)

The solution to equations 5.7 through 5.12 converged to the following using Newton's Method:

$\mathbf{a}_0$=(-0.0361, -0.0061) $\mathbf{a}_1$=(0.0723, 0.9879) $\mathbf{a}_{0n}$=(-0.0531, -0.0034)

By using the initial rigid body position points in each phase as the starting points for the synthesized adjustable RRSS mechanism and rotating the R-R link by certain angles, the remaining positions in table 5.1 were approximated. The R-R link rotation angles for the first four rigid body positions are 90°, 100°, 110° and 120°. The R-R link rotation angles for the last four rigid body positions are 90°, 80°, 70° and 60°. These angles are measured with respect to the X-axis.

Table 5.2 Rigid body positions for synthesized mechanism for 2-phase RRSS fixed pivot problem with fixed crank and follower lengths

Phase 1

	p	q	r	s
pos. 1	-0.5096.1.0541,0.1040	1.3326,2.9344,-0.2720	0.7928,1.7071,0.3485	0.5928, 1.7071,-0.6313
pos. 2	-0.6868,1.0315,0.1006	1.0589,2.9897,-0.3336	0.5798,1.7576,0.3260	0.3837,1.7152,-0.6537
pos. 3	-0.8613,0.9754,0.0950	0.7773,3.0134,-0.3868	0.3611,1.7773,0.3071	0.1766,1.6942,-0.6722
pos. 4	-1.0271,0.8866,0.0875	0.4933,3.0055,-0.4316	0.1422,1.7661,0.2918	-0.0236,1.6452,-0.6869
		Phase 2		
pos. 5	-0.3353,1.0440,0.1045	1.5427,2.9038,-0.1869	0.9779,1.6620,0.3802	0.7819,1.7064,-0.5994
pos. 6	-0.1652,1.0045,0.1054	1.7232,2.8658,-0.0945	1.1499,1.6026,0.4137	0.9581,1.6922,-0.5636
pos. 7	-0.0056,0.9307,0.1025	1.8704,2.8131,0.0145	1.3008,1.5232,0.4555	1.1205,1.6573,-0.5190
pos. 8	0.1386,0.8233,0.0952	1.9698,2.7508,0.1403	1.4222,1.4283,0.5059	1.2600,1.6051,-0.4649

The average error magnitude between the specified rigid body positions (table 5.1) and the rigid body positions of the synthesized mechanism for positions 2, 3 and 4 is 0.0129 units. The maximum error magnitude between positions 2, 3 and 4 is 0.0564 units. It occurs at rigid body point p_y in position 4.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for position 6, 7 and 8 is 0.0118 units. The maximum error magnitude between positions 6, 7 and 8 is 0.0444 units. It occurs at rigid body point q_y in position 8.

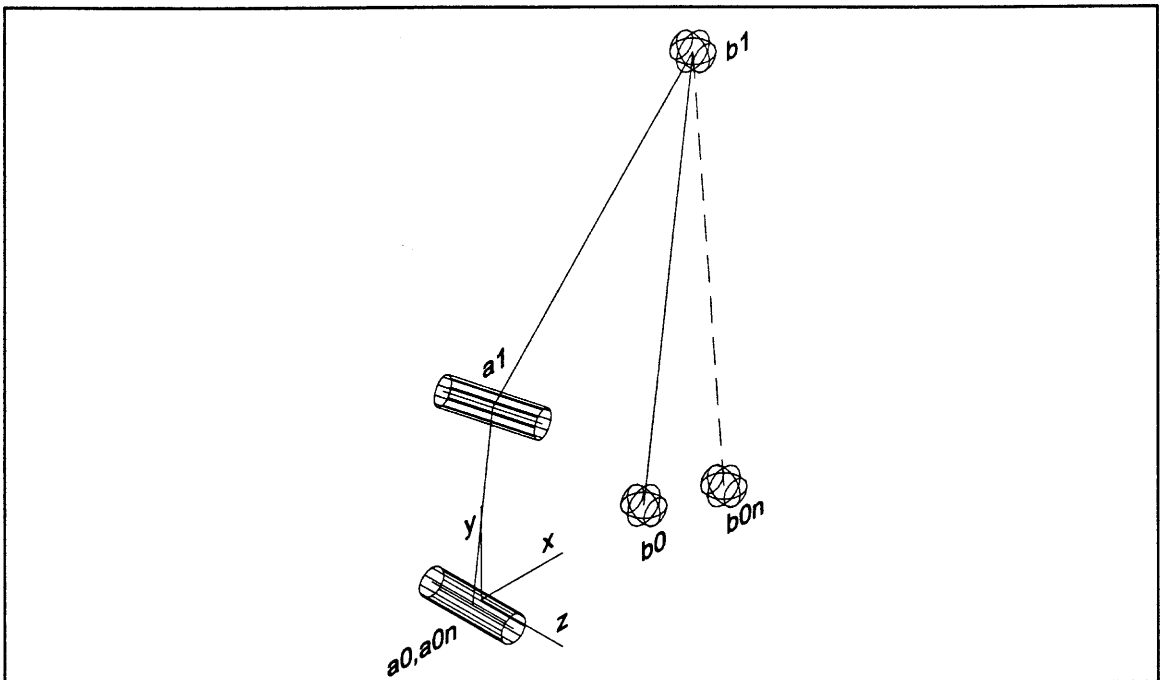

Figure 5.2 Solution to 2-phase RRSS fixed pivot problem with fixed crank and follower lengths

5.1.2 RRSS Mechanism for Finitely Separated Positions with Tolerances

In this example problem, rigid body point tolerances were incorporated to synthesize the adjustable R-R link of the RRSS mechanism in section 5.1.1. In section 5.1.1, the following design equations were used to synthesize the R-R link:

$$([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{5.13}$$

$$([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{5.14}$$

$$([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{5.15}$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \tag{5.16}$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \tag{5.17}$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \tag{5.18}$$

Equations 5.13 through 5.15 were used to calculate the R-R link parameters for phase 1 and the remaining three equations were used to calculate the R-R link parameters for phase 2. If each set of equations corresponding to each phase is solved using a range of prescribed values for one of the unknowns, a range of R-R link parameters can be calculated. These parameter ranges are equivalent to the Burmester curves used in planar four-bar mechanism synthesis. Burmester curves are the loci of circle points and center points that satisfy a particular set of rigid body positions.

Using the following initial guesses and prescribed variable ranges:

$a_{0y} = 0.1$ $\quad$ $\mathbf{a}_1 = (0.1, 0.85)$ $\quad$ $a_{0ny} = 0.1$ $\quad$ $a_{0x}=a_{0nx} = -0.9, -0.85, \ldots 0.5$

figure 5.3 illustrates the circle and center point curves calculated for each phase.

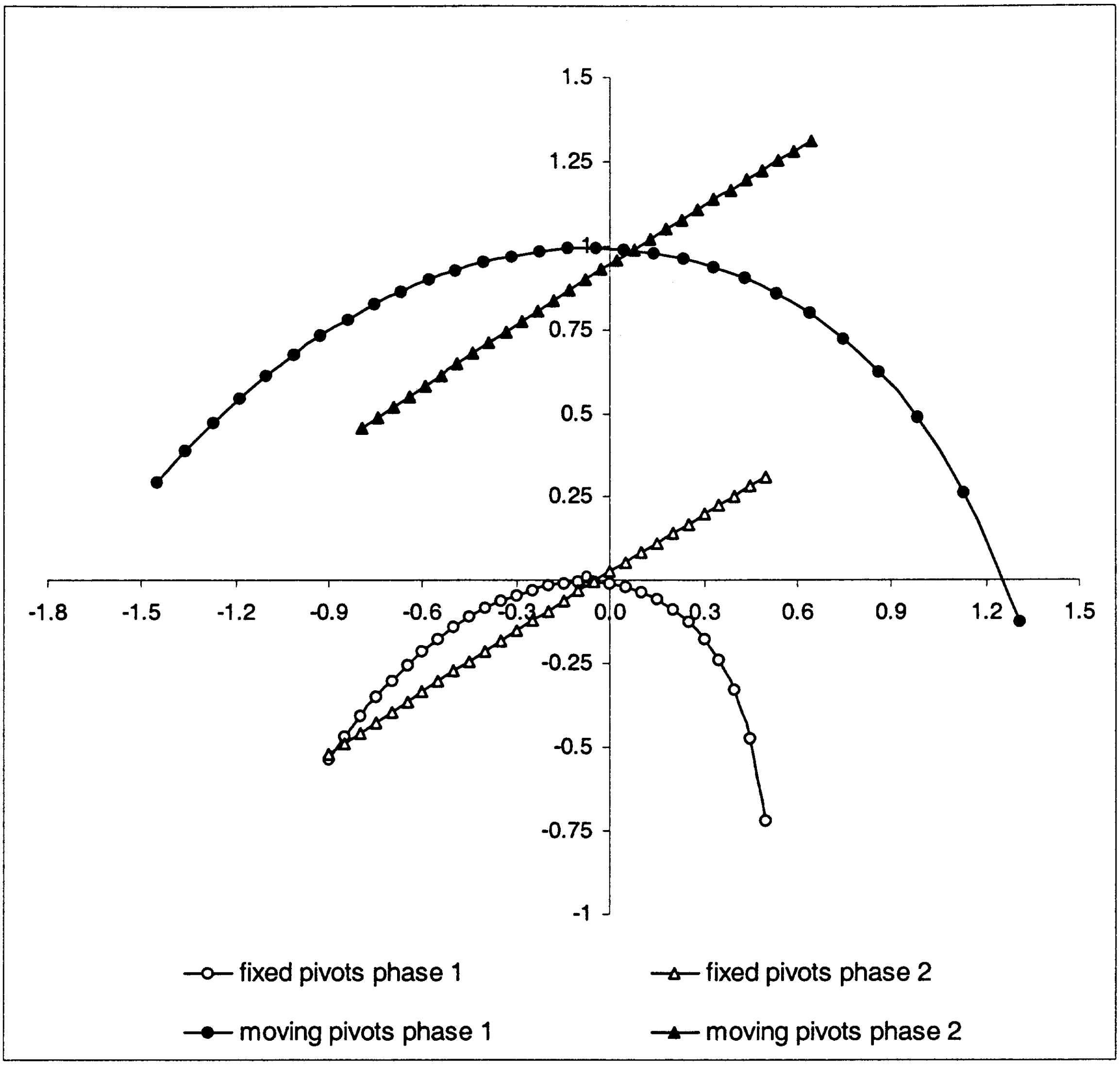

Figure 5.3 Graphical solution for R-R link in 2-phase RRSS fixed pivot problem with fixed crank and follower lengths

In figure 5.3, portions of the circle and center point curves are plotted for the rigid body positions in phases 1 and 2. The points of intersection of the circle point curves and center point curves represent the R-R link parameters that are needed to achieve the rigid body positions in both phases. In tolerance problems, these intersection points would become regions in which the R-R link parameters must remain within in order to achieve the precise rigid body positions while remaining within the limits of the rigid body positions with tolerances.

Table 5.3 Prescribed X-Y plane rigid body positions and tolerances for 2-phase RRSS fixed pivot problem with fixed crank and follower lengths

	p	q	r
	Phase 1		
pos. 1	-0.5096 $\pm\ \delta_x$. 1.0541	1.3326 $\pm\ \delta_x$, 2.9344	0.7928 $\pm\ \delta_x$, 1.7071
pos. 2	-0.6860, 1.0111	1.0406, 2.9872	0.5737, 1.7488
pos. 3	-0.8558, 0.9360	0.7449, 3.0055	0.3523, 1.7589
pos. 4	-1.0136, 0.8301	0.4504, 2.9902	0.1336, 1.7380
	Phase 2		
pos. 5	-0.3353 $\pm\ \delta_x$, 1.0440	1.5427 $\pm\ \delta_x$, 2.9038	0.9779 $\pm\ \delta_x$, 1.6620
pos. 6	-0.1676, 0.9989	1.7294, 2.8525	1.1475, 1.5927
pos. 7	-0.0117, 0.9194	1.8820, 2.7847	1.2946, 1.5026
pos. 8	0.1276, 0.8063	1.9867, 2.7064	1.4113, 1.3965

Note: In this problem δ_x = 0.1 units

It was shown in section 5.1.1 that the calculated R-R link parameters using the rigid body positions in table 5.3 (without tolerances) and equations 5.13 through 5.18 were the following:

$\mathbf{a}_0$=(-0.0361, -0.0061) $\mathbf{a}_1$=(0.0723, 0.9879) $\mathbf{a}_{0n}$=(-0.0531, -0.0034)

When using the same equations but incorporating the +δ tolerance value in table 5.3, the R-R link parameters become

$\mathbf{a}_0$=(-0.0361, -0.0061) $\mathbf{a}_1$=(0.1723, 0.9879) $\mathbf{a}_{0n}$=(-0.0531, -0.0034)

When using the same equations but incorporating the -δ tolerance value in table 5.3, the R-R link parameters become

$\mathbf{a}_0$=(-0.0361, -0.0061) $\mathbf{a}_1$=(-0.0277, 0.9879) $\mathbf{a}_{0n}$=(-0.0531, -0.0034)

All of these R-R link parameters were obtained using the same R-R link initial guesses given in section 5.1.1. These parameters are illustrated in figure 5.4. Since $\mathbf{a}_0$ and $\mathbf{a}_{0n}$ nearly overlap when expressed graphically, they will be represented by a single point ($\mathbf{a}_0$).

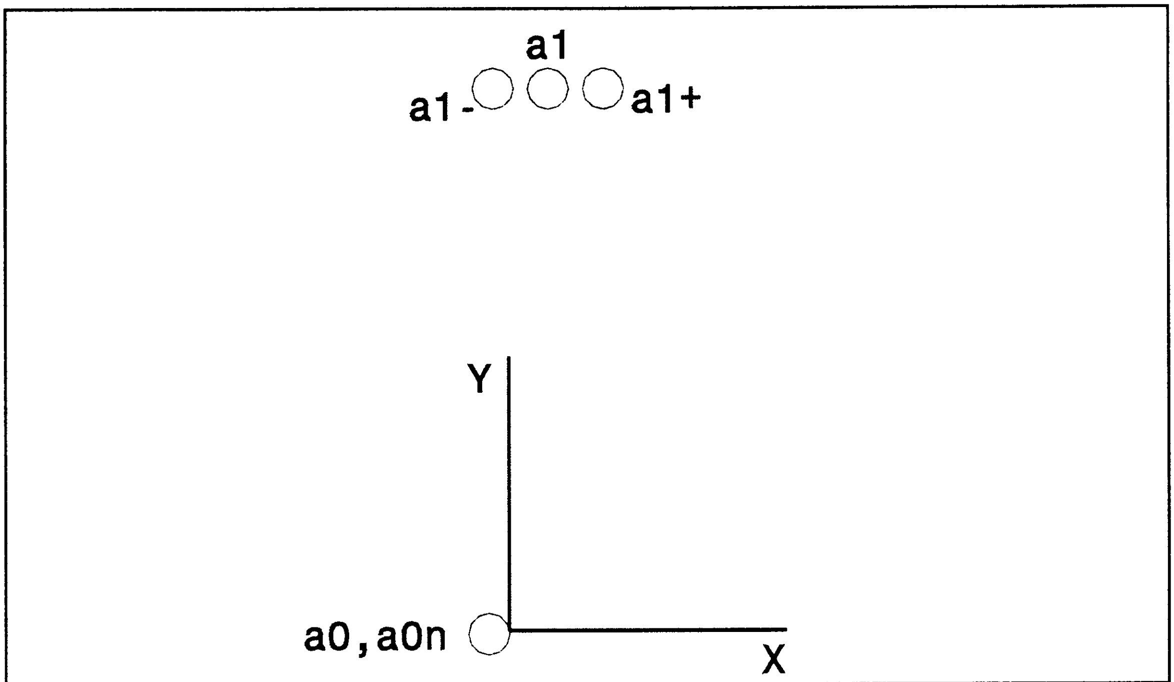

Figure 5.4 R-R link parameters with and without tolerances for 2-phase RRSS fixed pivot problem with fixed crank and follower lengths

Although at least three R-R link solutions exist in figure 5.4, to calculate the regions necessary to produce additional solutions for the R-R link, a part of the $\mathbf{a}_0$, $\mathbf{a}_1$+ and $\mathbf{a}_1$- loci must be calculated. By specifying a_{0x} and using equations 5.13 through 5.18, the following R-R link parameters were calculated:

Table 5.4 Additional R-R link parameters for 2-phase RRSS fixed pivot problem with fixed crank and follower lengths and tolerances

$\mathbf{a}_0$, $\mathbf{a}_{0n}$	$\mathbf{a}_1$	$\mathbf{a}_1$ (using $+\delta$)	$\mathbf{a}_1$ (using $-\delta$)
-0.0561*, -0.0311	0.0309, 0.9635	0.1309, 0.9635	-0.0697, 0.9635
-0.0661*, 0.0190	0.1134, 1.0120	0.2134, 1.0120	0.0134, 1.0120

*specified R-R link parameter

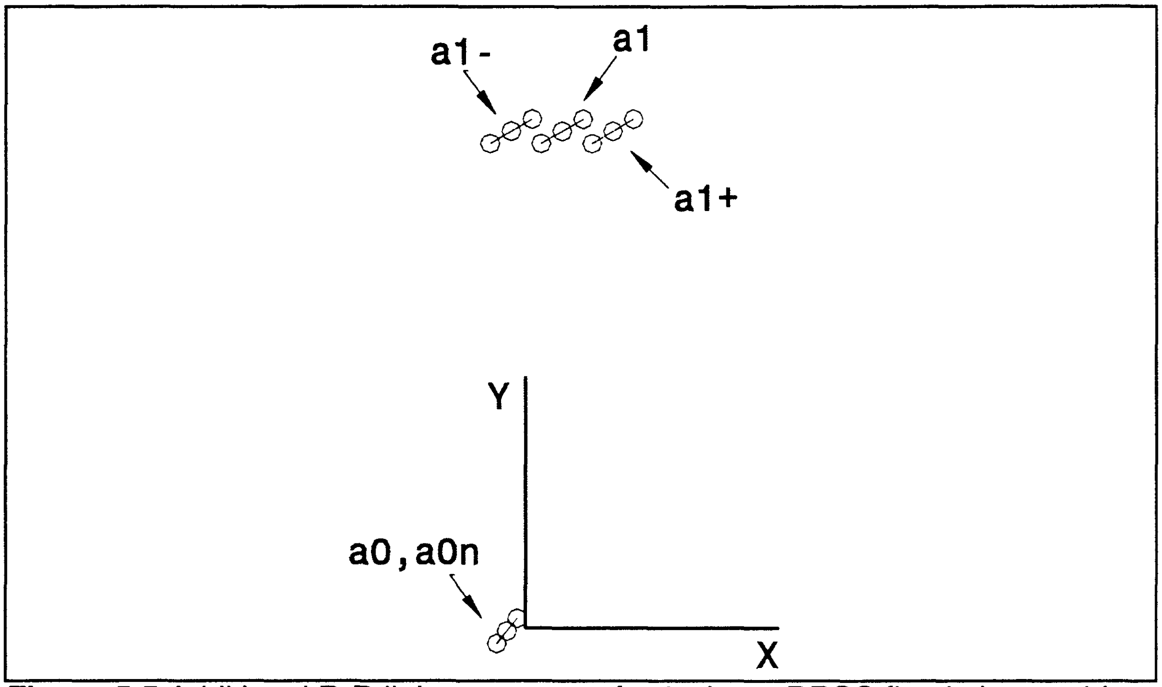

Figure 5.5 Additional R-R link parameters for 2-phase RRSS fixed pivot problem with fixed crank and follower lengths and tolerances

By connecting the peripheral points for $\mathbf{a}_1$, a region is formed. A curve is formed when the points for $\mathbf{a}_0$ are connected.

To find acceptable R-R link parameters, a circle representing the R-R link must be properly positioned. The center of this circle represents the fixed pivot of the link and the radius of the circle is the link length. Whenever the center of the circle intersects a fixed pivot region or line and the circle intersects a moving pivot line or region, an acceptable solution is found.

In figures 5.5 and 5.6 it can be seen that the fixed pivot solutions form a curve and the moving pivot solutions form a region. Using a circle with a radius of 1 (therefore $R_1=1$), the R-R link parameters were obtained.

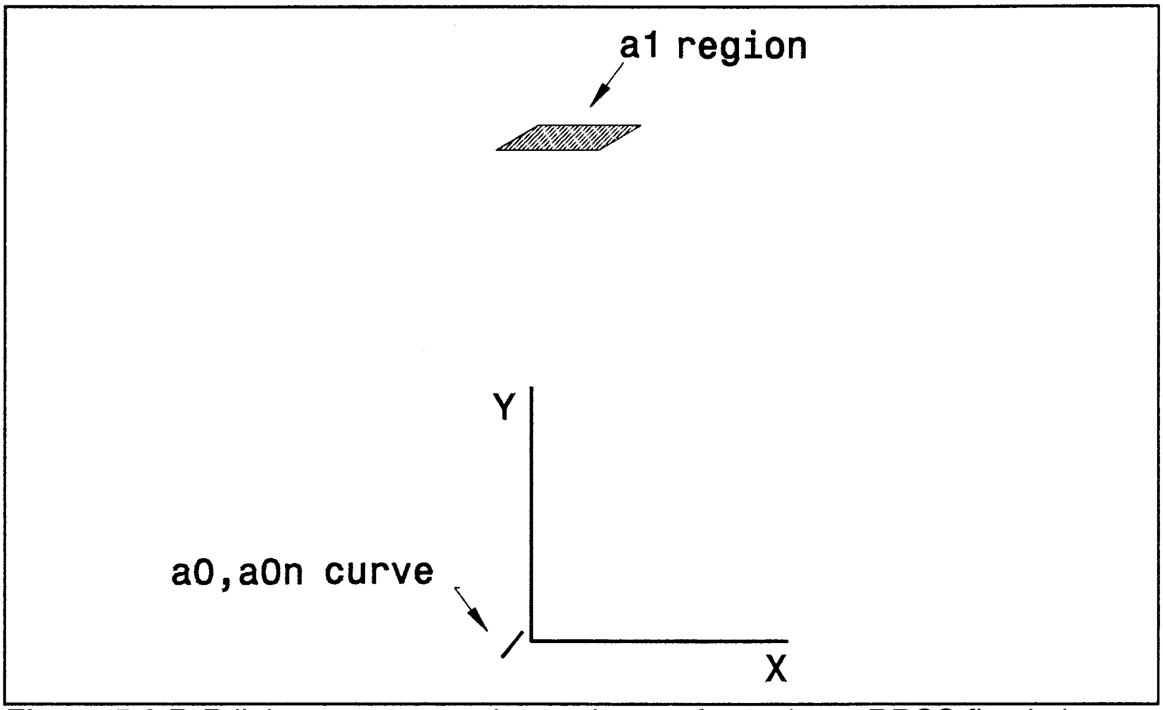

Figure 5.6 R-R link parameter region and curve for 2-phase RRSS fixed pivot problem with fixed crank and follower lengths and tolerances

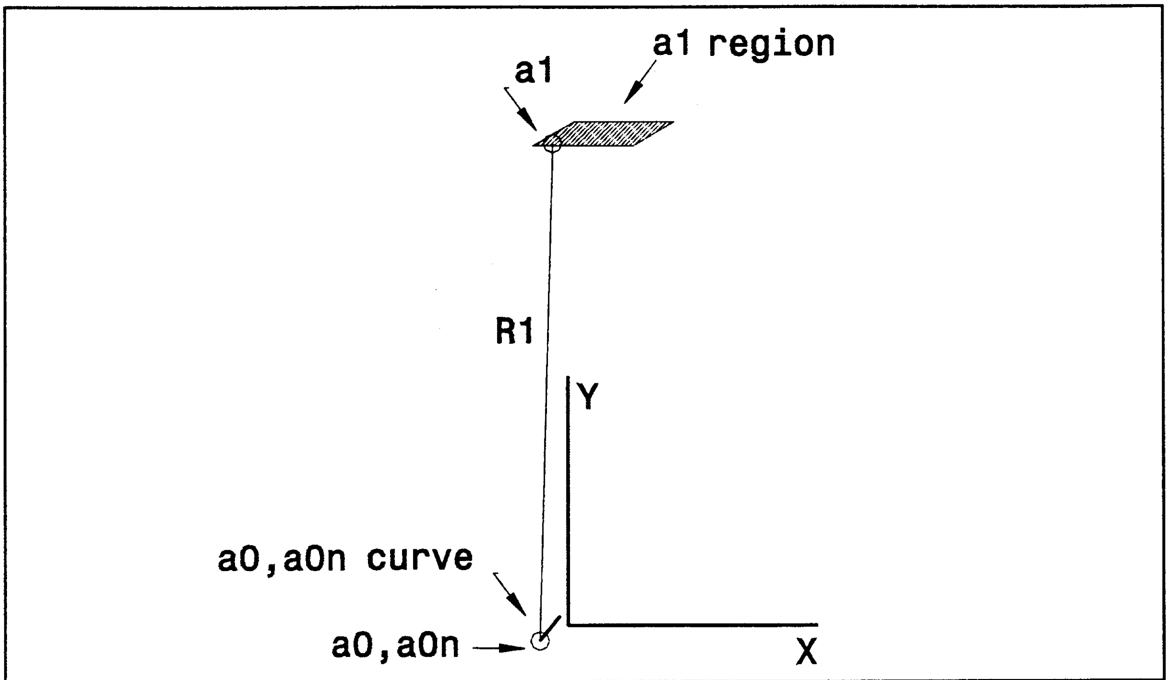

Figure 5.7 R-R link parameter selections for 2-phase RRSS fixed pivot problem with fixed crank and follower lengths and tolerances

The following R-R link parameters were selected and are shown in figure 5.7:

$\mathbf{a}_0$=(-0.0561, -0.0311) $\mathbf{a}_1$=(-0.0299, 0.9686) $\mathbf{a}_{0n}$=(-0.0561, -0.0311)

The S-S link parameters are the same as those calculated in chapter 5.1.2.

$\mathbf{b}_1$=(1.7792, 1.7792, -0.4767) $\mathbf{b}_0$=(1.0003, 0.0260, 0.0888)

$\mathbf{b}_{0n}$=(1.4862, -0.1403, 0.0019)

By using the final rigid body position points in each phase as the starting points for the synthesized adjustable RRSS mechanism and rotating the R-R link by certain angles, the remaining positions in table 5.3 were approximated. The R-R link rotation angles for the first four rigid body positions are 90°, 100°, 110° and 120°. The R-R link rotation angles for the last four rigid body positions are 90°, 80°, 70° and 60°. These angles are measured with respect to the X-axis.

Using the R-R and S-S link parameters, in addition to **ua**$_1$=[sin 15°, 0, cos 15°] when the R-R link lies along the positive Y-axis, the following rigid body positions were obtained:

Table 5.5 Rigid body positions for synthesized mechanism for 2-phase RRSS fixed pivot problem with fixed crank and follower lengths and tolerances

	p	q	r
Phase 1			
pos. 1	-0.5040, 1.0289	1.3134, 2.9296	0.7915, 1.6950
pos. 2	-0.6811, 0.9945	1.0317, 2.9832	0.5736, 1.7405
pos. 3	-0.8528, 0.9278	0.7407, 3.0032	0.3526, 1.7547
pos. 4	-1.0136, 0.8301	0.4504, 2.9902	0.1336, 1.7380
Phase 2			
pos. 5	-0.3275, 1.0616	1.5742, 2.8959	0.9937, 1.6635
pos. 6	-0.1610, 1.0103	1.7531, 2.8457	1.1599, 1.5926
pos. 7	-0.0077, 0.9250	1.8956, 2.7803	1.3020, 1.5018
pos. 8	0.1276, 0.8063	1.9867, 2.7064	1.4113, 1.3965

The average error magnitude between the specified rigid body positions without tolerances in table 5.3 and the corresponding rigid body positions of the synthesized RRSS mechanism for positions 2, 3 and 4 is 0.0074 units. The maximum error magnitude in position 1 is 0.0153 units. It occurs at rigid body point q_x in position 1.

The average error magnitude between the specified rigid body positions without tolerances in table 5.3 and the corresponding rigid body positions of the synthesized RRSS mechanism for positions 6, 7 and 8 is 0.0083 units. The maximum error magnitude between positions 5 is 0.0315 units. It occurs at rigid body point q_x in position 5.

5.1.3 RRSC Mechanism for Finite and Multiply Separated Positions with ISA parameters

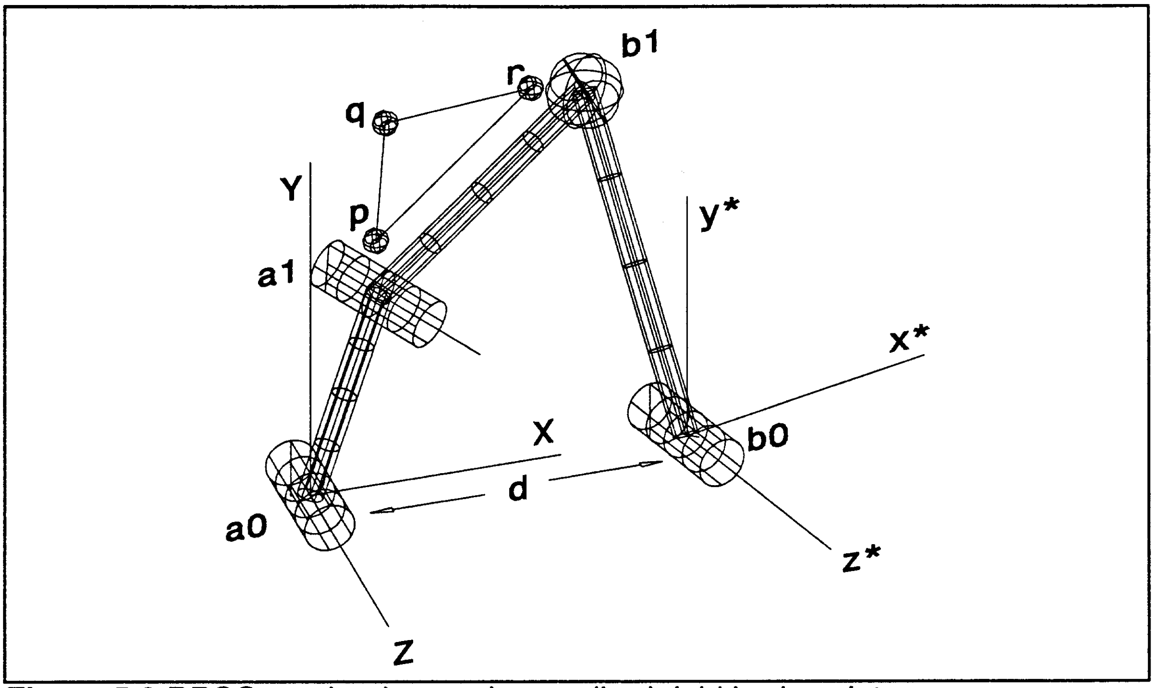

Figure 5.8 RRSC mechanism and prescribed rigid body points

The instant screw axis and R-R link parameters were calculated first in this two-phase adjustable moving pivot problem.

The values for 8 prescribed rigid body positions projected on the X-Y plane are given in table 5.6. These positions are represented by points $\mathbf{p}_{X\text{-}Y}$, $\mathbf{q}_{X\text{-}Y}$ and $\mathbf{r}_{X\text{-}Y}$. To satisfy the design equations of the R-R link, points **p**, **q** and **r** travel in a plane that is normal to $\mathbf{ua}_j$ and rotates about $\mathbf{ua}_0$ in each rigid body position. The points are also non-linear.

In addition to the 8 prescribed rigid body positions, several instant screw axis parameters are also given for the first rigid body position. The parameter $\mathbf{p}_0$ represents a point on the instant screw axis and ω_s is the angular velocity about the instant screw axis.

Table 5.6 Prescribed X-Y plane rigid body positions and ISA parameters for 2-phase RRSC moving pivot problem with fixed crank and follower lengths

Phase 1

	$p_{X\text{-}Y}$	$q_{X\text{-}Y}$	$r_{X\text{-}Y}$
pos. 1	0.1959, 1.2647	1.2089, 1.5993	0.7024, 1.8320
pos. 2	p_0=(0,-5.0499, -0.6241)	ω_s=1.5	
pos. 3	0.0126, 1.2555	1.0132, 1.6224	0.4984, 1.8386
pos. 4	-0.1668, 1.2169	0.8190, 1.6205	0.2952, 1.8171
Phase 2			
pos. 5	0.1959, 1.2647	1.2089, 1.5993	0.7024, 1.8320
pos. 6	0.0123, 1.2329	1.0031, 1.6284	0.4821, 1.8298
pos. 7	-0.1607, 1.1731	0.8037, 1.6301	0.2698, 1.7980
pos. 8	-0.3191, 1.0880	0.6138, 1.6079	0.0687, 1.7396

Note: In this problem, rigid body positions 1 and 5 are shared

These rigid body position points were calculated using **d** =1, $\mathbf{ua_1}$=[sin 10°, 0, cos 10°] (when the R-R link lies along the positive Y-axis) and $\mathbf{ub_0}$=[sin -15°, 0, cos -15°]. These joint axes were measured with respect to the X-Y-Z coordinate frame.

Although the ISA parameters $\mathbf{p_0}$ and ω_s are known, the instant screw axis unit vector ($\mathbf{up_0}$) is unknown. To calculate $\mathbf{up_0}$, the following equations were used:

$$s'\mathbf{up_0} - [W_s]\mathbf{p_0} = \mathbf{p'} - [W]\mathbf{p} \qquad (5.19)$$

$$(\mathbf{up_0})^T(\mathbf{up_0}) - 1 = 0 \qquad (5.20)$$

When the fourth columns of equations 3.4 and 3.9 are equated, the result is equation 5.19.

As mentioned in section 3.4, the variable **p** is equivalent to the fixed pivot of the link to be synthesized (in this case $\mathbf{a_0}$ of the R-R link). Since $\mathbf{a_0}$ is fixed, its derivatives are zero (therefore $\mathbf{p}'=0$). In addition, the variable **p** will also be set to zero. Although the present goal here was to determine the ISA unit vector, by making **p** equal to zero, the fixed pivot to be calculated ($\mathbf{a}_0$) will also approximate zero since **p** is equivalent to $\mathbf{a}_0$.

When expanded and simplified, equations 5.19 and 5.20 become

$$s'up_{0x} + \omega_s p_{0y} up_{0z} - \omega_s p_{0z} up_{0y} = 0 \tag{5.21}$$

$$s'up_{0y} - \omega_s p_{0x} up_{0z} + \omega_s p_{0z} up_{0x} = 0 \tag{5.22}$$

$$s'up_{0x} + \omega_s p_{0x} up_{0y} - \omega_s p_{0y} up_{0x} = 0 \tag{5.23}$$

$$up_{0x}^2 + up_{0y}^2 + up_{0z}^2 - 1 = 0 \tag{5.24}$$

Using the initial guesses $\mathbf{up}_0 = [0.5, 0.5, 0.5]$ and s' = 0.5, the solution converged to $\mathbf{up}_0 = [0, 0.99245, -0.12265]$ and s' = 0 using Newton's Method.

After the ISA unit vector has been calculated, the R-R link joint locations were calculated next. The length of this link remained fixed throughout each phase. The required variables for the R-R link are $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{1n}$. Variable $\mathbf{a}_0$ represents the fixed pivot of the R-R link. Variables $\mathbf{a}_1$ and $\mathbf{a}_{1n}$ represent the moving pivots in phases 1 and 2 of the R-R link. Since each of these variables has two scalar components, there are a total of six required variables.

$$\mathbf{a}_0=(a_{0x}, a_{0y},) \qquad \mathbf{a}_1=(a_{1x}, a_{1y}) \qquad \mathbf{a}_{1n}=(a_{1nx}, a_{1ny})$$

The seven prescribed rigid body positions and ISA parameters result in six design equations. The following set of design equations were used to calculate $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{1n}$:

$$([V_s][D_{1,1}]\mathbf{a}_1)^T([D_{1,1}]\mathbf{a}_1 - \mathbf{a}_0) = 0 \tag{5.25}$$

$$([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{5.26}$$

$$([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{14,}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{5.27}$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \tag{5.28}$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \tag{5.29}$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \tag{5.30}$$

The term R_1 represents the length of the R-R link. Since this example problem involved moving pivot adjustments with fixed crank and follower lengths, the value of R_1 remained constant in equations 5.25 through 5.30. The specified value for R_1 is 1. Given the following initial guesses:

$\mathbf{a}_0$=(0.1, 0.1) $\mathbf{a}_1$=(0.1, 0.85) $\mathbf{a}_{1n}$=(0.1, 0.85)

the solution to equations 5.25 through 5.30 converged to the following using Newton's Method:

$\mathbf{a}_0$=(0.0001, -0.0040) $\mathbf{a}_1$=(0.0014, 1.0045) $\mathbf{a}_{1n}$=(-0.2497, 0.9726)

Using ISA parameters, rigid body velocities and accelerations can be calculated. The velocities and accelerations of the rigid body are governed by the angular velocity and acceleration of the driving link. Since the C-S link can also function as a driving link in the RRSC mechanism, it will be synthesized using the calculated ISA parameters for rigid body position #1 (as they appear in the x*-y*-z* frame).

Table 5.7 Prescribed x*-y* plane rigid body positions and ISA parameters for 2-phase RRSC moving pivot problem with fixed crank and follower lengths

	$\mathbf{p}_{x^*\text{-}y^*}$	$\mathbf{q}_{x^*\text{-}y^*}$	$\mathbf{r}_{x^*\text{-}y^*}$
Phase 1			
pos. 1	-0.7857, 1.2647	0.1466, 1.5993	-0.3195, 1.8320
pos. 2	$\mathbf{up}_0$=(-0.0317,0.9924, -0.1185)		ω_s=1.5
pos. 3	-0.9643, 1.2555	-0.0456, 1.6224	-0.5215, 1.8386
pos. 4	-1.1389, 1.2169	-0.2352, 1.6205	-0.7218, 1.8171
Phase 2			
pos. 5	-0.7857, 1.2647	0.1466, 1.5993	-0.3195, 1.8320
pos. 6	-0.9637, 1.2329	-0.0529, 1.6284	-0.5349, 1.8298
pos. 7	-1.1313, 1.1731	-0.2454, 1.6301	-0.7420, 1.7980
pos. 8	-1.2843, 1.0880	-0.4278, 1.6079	-0.9374, 1.7396

Note: In this problem, rigid body positions 1 and 5 are shared

Points $\mathbf{p}_{x^*\text{-}y^*}$, $\mathbf{q}_{x^*\text{-}y^*}$ and $\mathbf{r}_{x^*\text{-}y^*}$ are the values obtained by projecting the rigid body points on the x*-y* plane. The value of the unit vector $\mathbf{up}_0$ here is the ISA as it appears in the x*-y*-z* coordinate frame. In this two-phase adjustable fixed pivot problem, the required variables are $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_{1n}$. Variable $\mathbf{b}_0$ represents the fixed pivot of the C-S link. Variables $\mathbf{b}_1$ and $\mathbf{b}_{1n}$ represent the moving pivots in phases 1 and 2 of the C-S link. Since each of these variables has two scalar components, there are a total of six required variables.

$$\mathbf{b}_0=(b_{0x}, b_{0y}) \qquad \mathbf{b}_1=(b_{1x}, b_{1y}) \qquad \mathbf{b}_{1n}=(b_{1nx}, b_{1ny})$$

The eight prescribed rigid body positions result in six design equations. The following set of design equations were used to calculate $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_{1n}$:

$$([V_s][D_{1,1}]\mathbf{b}_1)^T([D_{1,1}]\mathbf{b}_1 - \mathbf{b}_0) = 0 \qquad (5.31)$$

$$([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \qquad (5.32)$$

$$([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \qquad (5.33)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \quad (5.34)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \quad (5.35)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \quad (5.36)$$

The term R_1 represents the lengths of the C-S link. Since this example problem involved moving pivot adjustments with fixed crank and follower lengths, the value of R_1 remained constant in equations 5.31 through 5.36. The specified value for R_1 is 1.5. Given the following initial guesses:

$\mathbf{b}_0$=(0.1, 0.1) $\mathbf{b}_1$=(0.5, 1.5) $\mathbf{b}_{1n}$=(0.5, 1.5)

the solution to equations 5.31 through 5.36 converged to the following using Newton's Method:

$\mathbf{b}_0$=(-0.0001, -0.0052) $\mathbf{b}_1$=(0.3257, 1.4589) $\mathbf{b}_{1n}$=(0.4975, 1.4093)

By using the initial rigid body points in each phase as the starting points for the synthesized adjustable RRSC mechanism and rotating the R-R link by certain angles, the remaining positions in table 5.8 were approximated. The R-R link rotation angles for the first three rigid body positions are 90°, 100° and 110°. The R-R link rotation angles for the next four rigid body positions are 105°, 125°, 135° and 145°. These angles are measured with respect to the X-axis.

Table 5.8 Prescribed X-Y-Z frame rigid body positions for 2-phase RRSC moving pivot problem with fixed crank and follower lengths and ISA parameters

	p	q	r
Phase 1			
pos. 1	0.1959, 1.2647,-0.0345	1.2089, 1.5993,-0.2132	0.7024, 1.8320,-0.1238
pos. 3	0.0126, 1.2555,-0.0406	1.0132, 1.6224,-0.2256	0.4984, 1.8386,-0.1428
pos. 4	-0.1668, 1.2169,-0.0457	0.8190, 1.6205,-0.2334	0.2952, 1.8171,-0.1585
Phase 2			
pos. 5	0.1959, 1.2647,-0.0345	1.2089, 1.5993,-0.2132	0.7024, 1.8320,-0.1238
pos. 6	0.0123, 1.2329,-0.0373	1.0031, 1.6284,-0.2159	0.4821, 1.8298,-0.1340
pos. 7	-0.1607, 1.1731,-0.0389	0.8037, 1.6301,-0.2152	0.2698, 1.7980,-0.1414
pos. 8	-0.3191, 1.0880,-0.0393	0.6138, 1.6079,-0.2113	0.0687, 1.7396,-0.1461

Table 5.9 Rigid body positions for synthesized RRSC mechanism for 2-phase RRSC moving pivot problem with fixed crank and follower lengths and ISA parameters

	p	q	r
Phase 1			
pos. 1	0.1959, 1.2647,-0.0345	1.2089, 1.5993,-0.2132	0.7024, 1.8320,-0.1238
pos. 3	0.0128,1.2557,-0.0405	1.0136,1.6220,-0.2255	0.4988,1.8385,-0.1428
pos. 4	-0.1665,1.2172,-0.0456	0.8197,1.6199,-0.2333	0.2960,1.8169,-0.1584
Phase 2			
pos. 5	0.1959, 1.2647,-0.0345	1.2089, 1.5993,-0.2132	0.7024, 1.8320,-0.1238
pos. 6	0.0124,1.2335,-0.0372	1.0036,1.6282,-0.2159	0.4828,1.8300,-0.1340
pos. 7	-0.1607,1.1744,-0.0388	0.8044,1.6300,-0.2151	0.2707,1.7986,-0.1414
pos. 8	-0.3193,1.0899,-0.0392	0.6146,1.6078,-0.2110	0.0698,1.7406,-0.1460

The average error magnitude between the specified rigid body positions (table 5.8) and the rigid body positions of the synthesized mechanism for positions 3 and 4 is 0.0003 units. The maximum error magnitude between positions 3 and 4 is 0.0008 units. It occurs at rigid body point r_x in position 4.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for positions 6, 7 and 8 is 0.0004 units. The maximum error magnitude between positions 6, 7 and 8 is 0.0019 units. It occurs at rigid body point p_y in position 8.

The two initial translation magnitudes for the C-S link that were used in the RRSC mechanism to calculate the rigid body positions in table 5.9 are

S_1 =0.3440 units for phase 1 and S_5 = 0.4854 units for phase 2. They were determined by trial and error. Both displacement magnitudes lie along $\mathbf{ub_0}$ in the negative z*-axis direction.

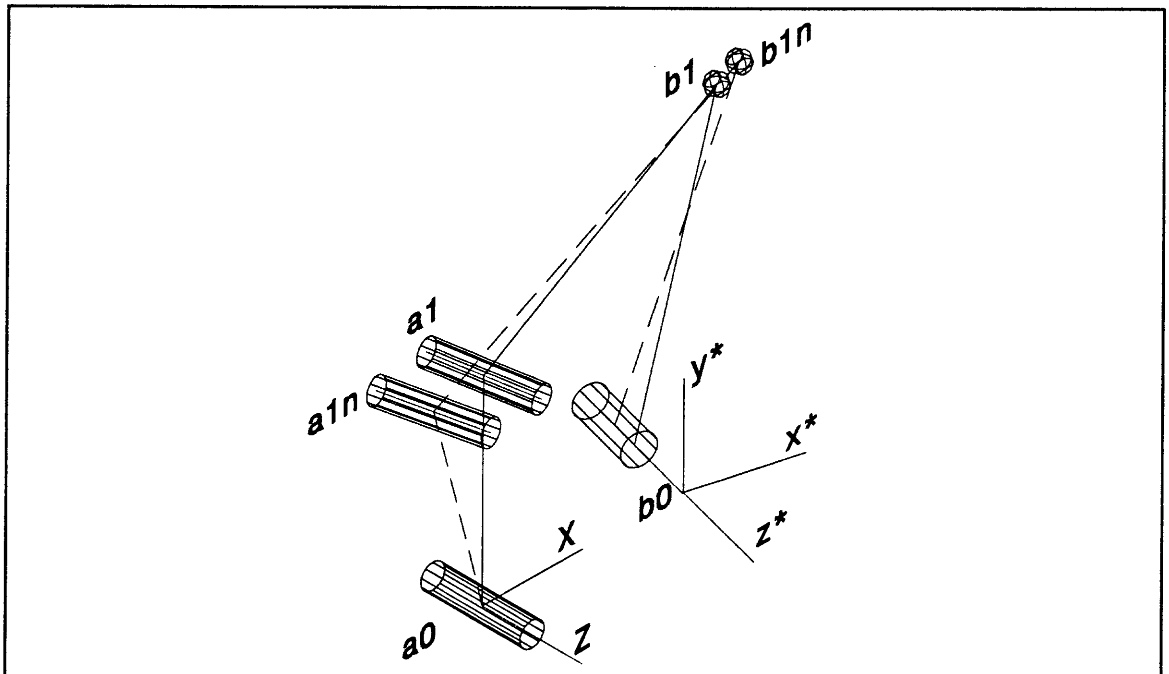

Figure 5.9 Solution to RRSC 2-phase moving pivot problem with fixed crank and follower lengths and ISA parameters.

The specified ISA parameters in table 5.6 are

$$\mathbf{p_0}=(0,-5.0499, -0.6241) \qquad \omega_s=1.5$$

The calculated ISA parameters are $\mathbf{up}_0$ = [0, 0.99245, -0.12265] and s' = 0 for the R-R link and $\mathbf{up}_0$=[-0.0317,0.9924, -0.1185] for the C-S link. The ISA unit vectors are different because the R-R and C-S links lie in different coordinate frames. These parameters correspond to the R-R and C-S links for rigid body position #1. Since this position is in phase 1, the R-R link parameters for this position are $\mathbf{a}_0$ and $\mathbf{a}_1$ and the C-S link parameters are $\mathbf{b}_0$ and $\mathbf{b}_1$. The values calculated for these parameters are the following:

$\mathbf{a}_0$=(0.0001, -0.0040) $\mathbf{a}_1$=(0.0014, 1.0045)

$\mathbf{b}_0$=(-0.0001, -0.0052) $\mathbf{b}_1$=(0.3257, 1.4589)

The velocity of $\mathbf{a}_1$ and $\mathbf{b}_1$ about the instant screw axis were calculated using the **a**' terms in equation 3.3 (where i=j=1). Equation 3.4 was replaced with equations 3.9 to incorporate the ISA parameters.

The values calculated for the velocity $\mathbf{a}_1$ and $\mathbf{b}_1$ are

$\mathbf{a}_1$'=(0.1848, -0.0003, -0.0021) $\mathbf{b}_1$'=(0.2593, -0.0579, -0.5542)

The angular velocities of the R-R and C-S links were calculated using the **a**' term equation in equation 3.3 (where i=j=1). Equation 3.4 was not replaced in this case since ISA parameters were not used. Since both links rotate in planes, only the ω_z term was used in equation 3.5 (therefore ω_x=ω_y=0). The fixed pivot velocity term was also eliminated in equation 3.4 since the fixed pivots are "fixed." When expanded the **a**' term equation becomes

$$\left[\begin{bmatrix} 0 & -\omega_z & \omega_z a_{0y} \\ \omega_z & 0 & -\omega_z a_{0x} \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]\begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} = \begin{pmatrix} a_{1x}' \\ a_{1y}' \\ 0 \end{pmatrix} \tag{5.37}$$

When the moving pivot position and velocity parameters for the R-R link were incorporated, the angular velocity values calculated were ω_z=0.1840 rad/sec for the first row in equation 5.37 and ω_z=0.2143 rad/sec for the second row. When the moving pivot position and velocity parameters for the C-S link were incorporated, the angular velocity values calculated were ω_z=0.1777 rad/sec for the first row in equation 5.37 and ω_z=0.1778 rad/sec for the second row. Since the calculated position and velocity parameters of the moving pivots of the R-R and C-S links were truncated (to four significant figures), the angular velocity values for both links are not exact matches.

5.1.4 RSSR-SS Mechanism for Finite and Multiply Separated Positions

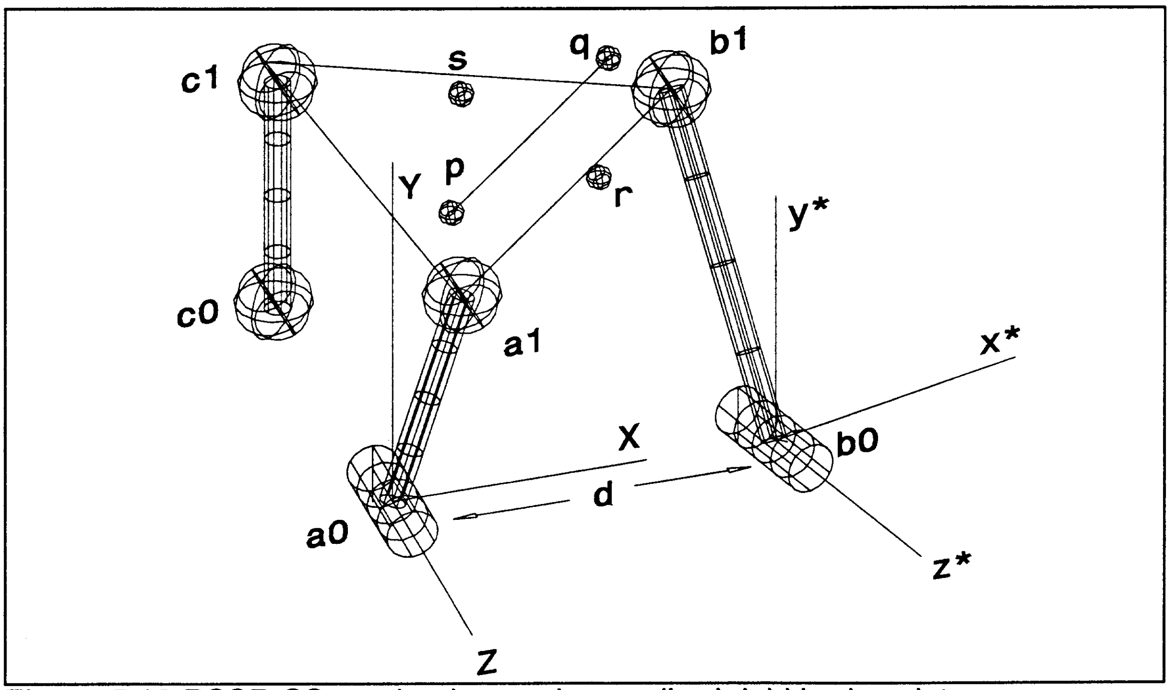

Figure 5.10 RSSR-SS mechanism and prescribed rigid body points

The R-S link ($\mathbf{a}_0$-$\mathbf{a}_1$) was the first link synthesized in this two-phase adjustable fixed pivot problem. The length of this R-S link remained fixed throughout each phase.

The values for 8 prescribed rigid body positions projected on the X-Y plane are given in table 5.10. These positions are represented by points $\mathbf{p}_{X\text{-}Y}$, $\mathbf{q}_{X\text{-}Y}$ and $\mathbf{r}_{X\text{-}Y}$. To satisfy the design equations of the R-S link, points **p**, **q** and **r** are projected in the X-Y plane in each rigid body position. The points are also non-linear.

In addition to the 8 prescribed rigid body positions, several multiply separated positions are also given. They represent the velocity of the rigid body from position 1 to position 2.

Table 5.10 Prescribed X-Y plane rigid body positions MSPs for 2-phase RSSR-SS fixed pivot problem with fixed crank and follower lengths

	$\mathbf{p}_{X\text{-}Y}$	$\mathbf{q}_{X\text{-}Y}$	$\mathbf{r}_{X\text{-}Y}$
Phase 1			
pos. 1	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
vel. 1-2	-0.4085, 10.0016	-9.9912, -0.4088	0.2867, -2.6548
pos. 2	0.6282, 2.8817	1.1455, 2.6134	0.1435, 2.3507
pos. 3	0.2508, 2.8328	0.7791, 2.5874	-0.2104, 2.2814
pos. 4	-0.1134, 2.7271	0.4264, 2.5073	-0.5467, 2.1554
Phase 2			
pos. 5	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
pos. 6	0.6316, 2.8239	1.1511, 2.5627	0.1521, 2.2887
pos. 7	0.2659, 2.7149	0.7992, 2.4876	-0.1829, 2.1584
pos. 8	-0.0762, 2.5463	0.4694, 2.3612	-0.4903, 1.9737

Note: In this problem, rigid body positions 1 and 5 are shared

All of the rigid body points in this example problem were obtained using **d**=1.5 and $\mathbf{ub}_0$=[sin 10°, 0, cos 10°] with respect to the X-Y-Z frame.

The required R-S link variables here are $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{0n}$. Variable $\mathbf{a}_1$ represents the moving pivot of the R-S link. Variables $\mathbf{a}_0$ and $\mathbf{a}_{0n}$ represent the fixed pivots in phase 1 and phase 2 of the R-S link. Since each of these variables has two scalar components, there are a total of six required unknowns.

$$\mathbf{a}_0=(a_{0x}, a_{0y}) \qquad \mathbf{a}_1=(a_{1x}, a_{1y}) \qquad \mathbf{a}_{0n}=(a_{0nx}, a_{0ny})$$

The eight prescribed rigid body positions and multiply separated positions result in six design equations. The following set of design equations were used to calculate $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{0n}$:

$$([V][D_{1,2}]\mathbf{a}_1)^T([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0) = 0 \tag{5.38}$$

$$([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{5.39}$$

$$([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{5.40}$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \tag{5.41}$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \tag{5.42}$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \tag{5.43}$$

The term R_1 represents the length of the R-S link. Since this example problem involved fixed pivot adjustments with fixed crank and follower lengths, the value of R_1 remained constant in equations 5.38 through 5.43. The specified value for R_1 is 2. Given the following initial guesses:

$$\mathbf{a}_0=(0.1, 0.1) \qquad \mathbf{a}_1=(0.1, 1.85) \qquad \mathbf{a}_{0n}=(0.3, 0.1)$$

the solution to equations 5.38 through 5.43 converged to the following using Newton's Method:

$$\mathbf{a}_0=(0.0025, -0.0142) \qquad \mathbf{a}_1=(-0.0004, 1.9851) \qquad \mathbf{a}_{0n}=(0.3913, 0.0243)$$

The other R-S link ($\mathbf{b}_0$-$\mathbf{b}_1$) was the next link synthesized in this two-phase adjustable fixed pivot problem. The multiply separated positions of the rigid body are governed by the angular velocity and acceleration of the driving link. Since link b_0-b_1 can function as a driving link, it was also synthesized using MSPs. The length of this R-S link remained fixed throughout each phase.

The values for 8 prescribed rigid body positions projected on the x*-y* plane are given in table 5.11. These positions are represented by points $p_{x^*-y^*}$, $q_{x^*-y^*}$ and $r_{x^*-y^*}$. To satisfy the design equations of the R-S link, points **p**, **q** and **r** are projected in the x*-y* plane in each rigid body position. The points are also non-linear.

In addition to the 8 prescribed rigid body positions, several multiply separated positions are also given. They represent the velocity of the rigid body from position 1 to position 2.

Table 5.11 Prescribed x*-y* plane rigid body positions and MSPs for 2-phase RSSR-SS fixed pivot problem with fixed crank and follower lengths

	$p_{x^*-y^*}$	$q_{x^*-y^*}$	$r_{x^*-y^*}$
	Phase 1		
pos. 1	-0.4737, 2.8716	0.0281, 2.5824	-0.9754, 2.3608
vel. 1-2	-0.4115, 9.9459	-9.9913, -0.4050	-0.3209, 2.6906
pos. 2	-0.8565, 2.8817	-0.3457, 2.6134	-1.3348, 2.3507
pos. 3	-1.2346, 2.8328	-0.7150, 2.5874	-1.6864, 2.2814
pos. 4	-1.5988, 2.7271	-1.0706, 2.5073	-2.0204, 2.1554
	Phase 2		
pos. 5	-0.4737, 2.8716	0.0281, 2.5824	-0.9754, 2.3608
pos. 6	-0.8543, 2.8239	-0.3405, 2.5627	-1.3267, 2.2887
pos. 7	-1.2206, 2.7149	-0.6954, 2.4876	-1.6597, 2.1584
pos. 8	-1.5644, 2.5463	1.0283, 2.3612	-1.9656, 1.9737

Note: In this problem, rigid body positions 1 and 5 are shared

The required R-S link variables here are $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_{0n}$. Variable $\mathbf{b}_1$ represents the moving pivot of the R-S link. Variables $\mathbf{b}_0$ and $\mathbf{b}_{0n}$ represent the fixed pivots in phase 1 and phase 2 of the R-S link. Since each of these variables has two scalar components, there are a total of six required unknowns.

$$\mathbf{b}_0=(b_{0x}, b_{0y}) \qquad \mathbf{b}_1=(b_{1x}, b_{1y}) \qquad \mathbf{b}_{0n}=(b_{0nx}, b_{0ny})$$

The eight prescribed rigid body positions and multiply separated positions result in six design equations. The following set of design equations were used to calculate $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_{0n}$:

$$([V][D_{1,2}]\mathbf{b}_1)^T([D_{1,2}]\mathbf{b}_1 - \mathbf{b}_0) = 0 \qquad (5.44)$$

$$([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \qquad (5.45)$$

$$([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \qquad (5.46)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \qquad (5.47)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \qquad (5.48)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \qquad (5.49)$$

The term R_1 represents the length of this R-S link. Since this example problem involved fixed pivot adjustments with fixed crank and follower lengths, the value of R_1 remained constant in equations 5.44 through 5.49. The specified value for R_1 is 2.5. Given the following initial guesses:

$$\mathbf{b}_0=(0.1, 0.1) \qquad \mathbf{b}_1=(0.5, 2.5) \qquad \mathbf{b}_{0n}=(0.1, -0.1)$$

the solution to equations 5.44 through 5.49 converged to the following using Newton's Method:

$$\mathbf{b}_0=(0.0010, 0.0153) \qquad \mathbf{b}_1=(0.5344, 2.4584) \qquad \mathbf{b}_{0n}=(0.2965, -0.0306)$$

The S-S link ($\mathbf{c}_0$-$\mathbf{c}_1$) was the last link synthesized in this two-phase adjustable fixed pivot problem. Since this S-S link cannot function as a driving link, it was not synthesized using MSPs. The length of this S-S link will remain fixed throughout each phase.

The values for 8 prescribed rigid body positions in the X-Y-Z frame are given in table 5.12. These positions are represented by points **p**, **q**, **r** and **s**. To satisfy the design equations of the S-S link, these points do not all lie in the same plane in each rigid body position.

Table 5.12 Prescribed X-Y-Z frame rigid body positions for 2-phase RSSR-SS fixed pivot problem with fixed crank and follower lengths and MSPs

	p	q	r	s
Phase 1				
pos. 1	1.0109,2.8716,-0.0460	1.5163,2.5824,-0.0690	0.5054,2.3608,-0.0230	1.0109,2.4716,0.2540
pos. 2	0.6282,2.8817,-0.0095	1.1455,2.6134,-0.0190	0.1435,2.3507,-0.0052	0.6355,2.4797,0.2877
pos. 3	0.2506,2.8328,0.0350	0.7791,2.5874,0.0336	-0.2104,2.2814,0.0155	0.2678,2.4252,0.3240
pos. 4	-0.1134,2.7271,0.0858	0.4264,2.5073,0.0875	-0.5467,2.1554,0.0387	-0.0820,2.3112,0.3616
Phase 2				
pos. 5	1.0109,2.8716,-0.0460	1.5163,2.5824,-0.0690	0.5054,2.3608,-0.0230	1.0109,2.4716,0.2540
pos. 6	0.6316,2.8239,0.0369	1.1511,2.5627,-0.0019	0.1521,2.2887,0.0125	0.6486,2.4019,0.3044
pos. 7	0.2659,2.7149,0.1271	0.7992,2.4876,0.0669	-0.1829,2.1584,0.0508	0.3065,2.2713,0.3544
pos. 8	-0.0762,2.5463,0.2237	0.4694,2.3612,0.1362	-0.4903,1.9737,0.0912	-0.0049,2.0847,0.4020

Note: In this problem, rigid body positions 1 and 5 are shared

The required S-S link variables are $\mathbf{c}_0$, $\mathbf{c}_1$ and $\mathbf{c}_{0n}$. Variable $\mathbf{c}_1$ represents the moving pivot of the S-S link. Variables $\mathbf{c}_0$ and $\mathbf{c}_{0n}$ represent the fixed pivots in phase 1 and phase 2 of the S-S link. Since each of these variables has three scalar components, there are a total of nine required unknowns.

$$\mathbf{c}_0=(c_{0x}, c_{0y}, c_{0z}) \qquad \mathbf{c}_1=(c_{1x}, c_{1y}, c_{1z}) \qquad \mathbf{c}_{0n}=(c_{0nx}, c_{0ny}, c_{0nz})$$

The eight prescribed rigid body positions result in six design equations. Therefore, three of the nine required unknowns were specified. Using AutoCAD

2000 software, the value of $\mathbf{c}_1$ was specified to $\mathbf{c}_1$=[0, 2.5, -1]. The following set of design equations were used to calculate $\mathbf{c}_0$ and $\mathbf{c}_{0n}$:

$$([D_{1,2}]\mathbf{c}_1 - \mathbf{c}_0)^T([D_{1,2}]\mathbf{c}_1 - \mathbf{c}_0) - R_1^2 = 0 \quad (5.50)$$

$$([D_{1,3}]\mathbf{c}_1 - \mathbf{c}_0)^T([D_{1,3}]\mathbf{c}_1 - \mathbf{c}_0) - R_1^2 = 0 \quad (5.51)$$

$$([D_{1,4}]\mathbf{c}_1 - \mathbf{c}_0)^T([D_{1,4}]\mathbf{c}_1 - \mathbf{c}_0) - R_1^2 = 0 \quad (5.52)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n}) - R_1^2 = 0 \quad (5.53)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n}) - R_1^2 = 0 \quad (5.54)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n}) - R_1^2 = 0 \quad (5.55)$$

The term R_1 represents the length of the S-S link. Since this example problem involved fixed pivot adjustments with fixed crank and follower lengths, the value of R_1 remained constant in equations 5.50 through 5.55. The specified value for R_1 is 2.5. Given the following initial guesses:

$\mathbf{c}_0$=(0, 0.1, -1) $\quad$ $\mathbf{c}_{0n}$=(0.1, 0.1, -0.5)

the solution to equations 5.50 through 5.55 converged to the following using Newton's Method:

$\mathbf{c}_0$=(-0.0009, -0.0003, -0.9780) $\quad$ $\mathbf{c}_{0n}$=(0.0004, 0.0180, -0.7001)

By using the initial rigid body points in each phase as the starting points for the synthesized adjustable RSSR-SS mechanism and rotating link $\mathbf{a}_0$-$\mathbf{a}_1$ by certain angles, the remaining positions in table 5.12 were approximated. The R-S link rotation angles for the first four rigid body positions are 90°, 100°, 110° and 120°. The R-S link rotation angles for the next four rigid body positions are 101°, 121°, 131° and 141°. These angles are measured with respect to the X-axis.

Table 5.13 Rigid body positions for synthesized mechanism for 2-phase RSSR-SS fixed pivot problem with fixed crank and follower lengths and MSPs

Phase 1

	p	q	r	s
pos. 1	1.0109,2.8716,-0.0460	1.5163,2.5824,-0.0690	0.5054,2.3608,-0.0230	1.0109,2.4716,0.2540
pos. 2	0.6273,2.8818,-0.0093	1.1447,2.6137,-0.0190	0.1429,2.3505,-0.0051	0.6348,2.4797,0.2878
pos. 3	0.2489,2.8327,0.0354	0.7777,2.5877,0.0337	-0.2116,2.2808,0.0159	0.2666,2.4249,0.3242
pos. 4	-0.1156,2.7266,0.0863	0.4243,2.5074,0.0874	-0.5484,2.1544,0.0393	-0.0836,2.3107,0.3619
Phase 2				
pos. 5	1.0109,2.8716,-0.0460	1.5163,2.5824,-0.0690	0.5054,2.3608,-0.0230	1.0109,2.4716,0.2540
pos. 6	0.6307,2.8237,0.0373	1.1503,2.5627,-0.0022	0.1515,2.2883,0.0133	0.6483,2.4015,0.3046
pos. 7	0.2644,2.7141,0.1280	0.7977,2.4874,0.0662	-0.1841,2.1574,0.0524	0.3060,2.2705,0.3548
pos. 8	-0.0780,2.5449,0.2247	0.4675,2.3605,0.1348	-0.4918,1.9718,0.0937	-0.0053,2.0833,0.4025

The average error magnitude between the specified rigid body positions (table 5.12) and the rigid body positions of the synthesized mechanism for positions 2, 3 and 4 is 0.0006 units. The maximum error magnitude between positions 2, 3 and 4 is 0.0022 units. It occurs at rigid body point p_x in position 4.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for position 6, 7 and 8 is 0.0007 units. The maximum error magnitude between positions 6, 7 and 8 is 0.0025 units. It occurs at rigid body point r_z in position 8.

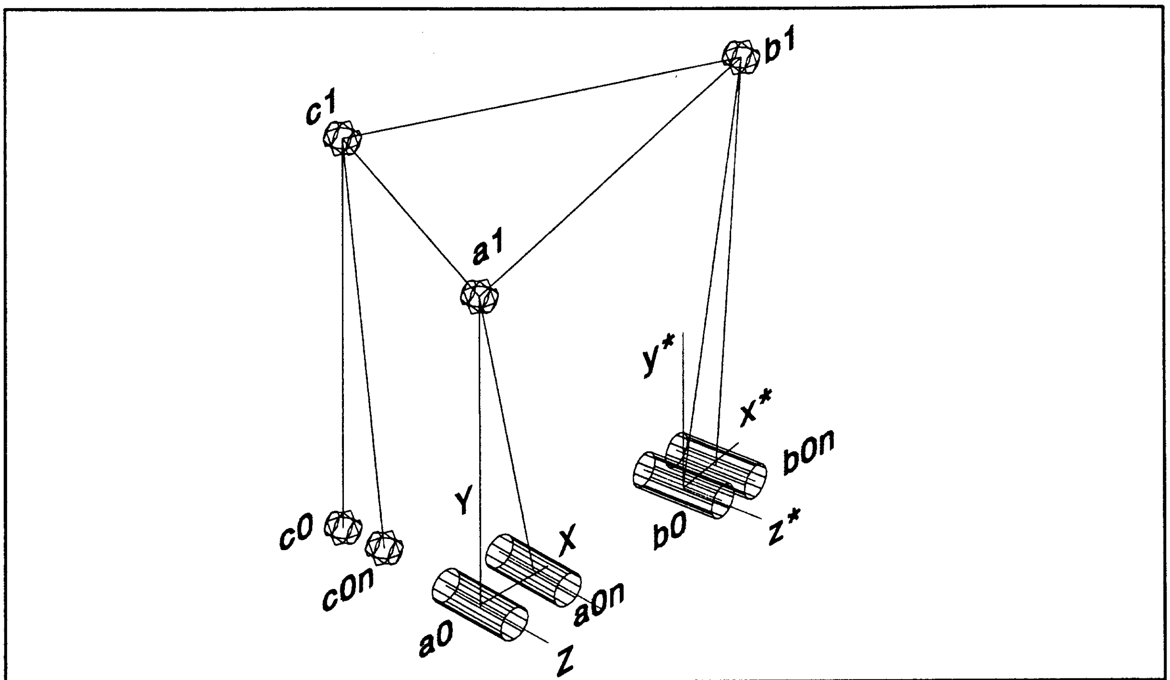

Figure 5.11 Solution to 2-phase RSSR-SS fixed pivot problem with fixed crank and follower lengths and MSPs

The specified MSP parameters in table 5.10 are

$\mathbf{Vp}_{1,2}$=(-0.4085, 10.0016) $\mathbf{Vq}_{1,2}$=(-9.9912, -0.4088) $\mathbf{Vr}_{1,2}$=(0.2867, -2.6548)

These parameters correspond to link ($\mathbf{a}_0$-$\mathbf{a}_1$) for rigid body positions 1 and 2. Since these positions are in phase 1, the R-S link parameters for this position are $\mathbf{a}_0$ and $\mathbf{a}_1$. The values calculated for these parameters are the following:

$\mathbf{a}_0$=(0.0025, -0.0142) $\mathbf{a}_1$=(-0.0004, 1.9851)

The velocity of $\mathbf{a}_1$ was calculated using the **a**' terms in equation 3.3 (where $[V_{12}]=[\mathbf{Vp}_{1,2}|\mathbf{Vq}_{1,2}|\mathbf{Vr}_{1,2}]$ with a third row of zeros). The value calculated for the velocity $\mathbf{a}_1$' is

$\mathbf{a}_1$'=(-19.8334, -0.8155)

The specified MSP parameters in table 5.11 are

$\mathbf{Vp}_{1,2}$=(-0.4115, 9.9459) $\mathbf{Vq}_{1,2}$=(-9.9913, -0.4050) $\mathbf{Vr}_{1,2}$=(-0.3209, 2.6906)

These parameters correspond to link (**b_0-b_1**) for rigid body positions 1 and 2. Since these positions are in phase 1, the R-S link parameters for this position are **b_0** and **b_1**. The values calculated for these parameters are the following:

$$\mathbf{b}_0=(-0.0010, 0.0153) \qquad \mathbf{b}_1=(0.5344, 2.4584)$$

The velocity of **b_1** was calculated using the **b'** terms in equation 3.3 (where $[V_{12}]=[\mathbf{Vp}_{1,2}|\mathbf{Vq}_{1,2}|\mathbf{Vr}_{1,2}]$ with a third row of zeros). The value calculated for the velocity **b_1'** is

$$\mathbf{b}_1'=(-24.7825, 4.3194)$$

The angular velocities of both R-S links were calculated using the **a'** term equation in equation 3.3. Since both links rotate in planes, only the ω_z term was used in equation 3.5 (therefore $\omega_x=\omega_y=0$). The fixed pivot velocity term was also eliminated in equation 3.4 since the fixed pivots are "fixed." When expanded the **a'** term equation becomes

$$\left[\begin{bmatrix} 0 & -\omega_z & \omega_z a_{0y} \\ \omega_z & 0 & -\omega_z a_{0x} \\ 0 & 0 & 0 \end{bmatrix}\left[\begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} p_{ix} & q_{ix} & r_{ix} \\ p_{iy} & q_{iy} & r_{iy} \\ 1 & 1 & 1 \end{bmatrix}^{-1}\right]\right]\begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} = \begin{pmatrix} a_{1x}' \\ a_{1y}' \\ 0 \end{pmatrix} \qquad (5.56)$$

When the rigid body position parameters and the moving pivot position and velocity parameters for link **a_0-a_1** were incorporated, the angular velocity values calculated were ω_z=10.0012 rad/sec for the first row in equation 5.56 and ω_z=9.9694 rad/sec for the second row. When the rigid body position parameters and the moving pivot position and velocity parameters for link **b_0-b_1** were incorporated, the angular velocity values calculated were ω_z=10.0018 rad/sec for the first row in equation 5.56 and ω_z=10.0241 rad/sec for the second row. Since the calculated position and velocity parameters of the moving pivots of the R-S links were truncated (to four significant figures), the angular velocity values for both links are not exact matches.

5.1.5 RSSR-SC Mechanism for Finite and Multiply Separated Positions

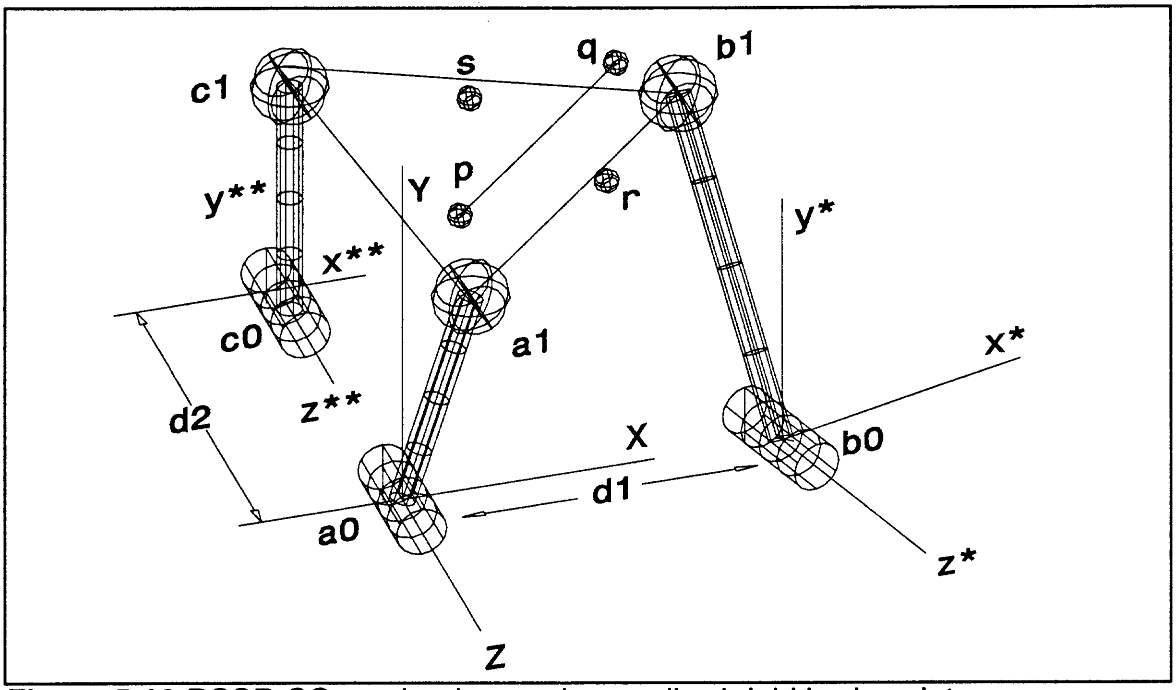

Figure 5.12 RSSR-SC mechanism and prescribed rigid body points

The R-S link ($\mathbf{a}_0$-$\mathbf{a}_1$) was the first link synthesized in this two-phase adjustable moving pivot problem. The length of this R-S link remained fixed throughout each phase.

The values for 8 prescribed rigid body positions projected on the X-Y plane are given in table 5.14. These positions are represented by points $\mathbf{p}_{X\text{-}Y}$, $\mathbf{q}_{X\text{-}Y}$ and $\mathbf{r}_{X\text{-}Y}$. To satisfy the design equations of the R-S link, points **p**, **q** and **r** are projected in the X-Y plane in each rigid body position. The points are also non-linear.

In addition to the 8 prescribed rigid body positions, several multiply separated positions are also given. They represent the velocity and acceleration of the rigid body from position 1 to position 2.

Table 5.14 Prescribed X-Y plane rigid body positions and MSPs for 2-phase RSSR-SC moving pivot problem with fixed crank and follower lengths

	$p_{X\text{-}Y}$	$q_{X\text{-}Y}$	$r_{X\text{-}Y}$
	Phase 1		
pos. 1	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
vel. 1-2	-0.2004, 10.0036	-9.9975, -0.2013	0.0703, -1.3397
accel. 1-2	-100.0556, -1.0034	1.0128, -99.9949	13.4039, 0.5691
pos. 2	0.8195, 2.8841	1.3309, 2.6051	0.3241, 2.3633
pos. 3	0.6282, 2.8817	1.1455, 2.6134	0.1435, 2.3507
pos. 4	0.4382, 2.8646	0.9612, 2.6074	-0.0350, 2.3234
	Phase 2		
pos. 5	0.8178, 2.9076	1.3302, 2.6299	0.3239, 2.3861
pos. 6	0.6239, 2.9273	1.1424, 2.6617	0.1414, 2.3957
pos. 7	0.4304, 2.9310	0.9541, 2.6782	-0.0409, 2.3898
pos. 8	0.2382, 2.9191	0.7664, 2.6795	-0.2217, 2.6385

Note: In this problem, rigid body positions are shared

All of the rigid body points in this example problem were obtained using **d1**=1.5, **d2**=[0, 0, -1], $\mathbf{ub}_0$=[sin 10°, 0, cos 10°] and $\mathbf{uc}_0$=[0, 0, 1] with respect to the X-Y-Z frame.

The required R-S link variables here are $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{1n}$. Variable $\mathbf{a}_0$ represents the fixed pivot of the R-S link. Variables $\mathbf{a}_1$ and $\mathbf{a}_{1n}$ represent the moving pivots in phase 1 and phase 2 of the R-S link. Since each of these variables has two scalar components, there are a total of six required unknowns.

$$\mathbf{a}_0=(a_{0x}, a_{0y}) \quad \mathbf{a}_1=(a_{1x}, a_{1y}) \quad \mathbf{a}_{1n}=(a_{1nx}, a_{1ny})$$

The eight prescribed rigid body positions and multiply separated positions result in six design equations. The following set of design equations were used to calculate $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{1n}$:

$$([A_{1,2}]\mathbf{a}_1)^T([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0) + ([V_{1,2}]\mathbf{a}_1)^T([V_{1,2}]\mathbf{a}_1) = 0 \quad (5.57)$$

$$([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.58)$$

$$([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.59)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \quad (5.60)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \quad (5.61)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \quad (5.62)$$

The term R_1 represents the length of the R-S link. Since this example problem involved moving pivot adjustments with fixed crank and follower lengths, the value of R_1 remained constant in equations 5.57 through 5.62. The specified value for R_1 is 2. Given the following initial guesses:

$\mathbf{a}_0$=(0.1, 0.1) $\mathbf{a}_1$=(0.1, 1.85) $\mathbf{a}_{1n}$=(0.1, 1.85)

the solution to equations 5.57 through 5.62 converged to the following using Newton's Method:

$\mathbf{a}_0$=(-0.0006, -0.0000) $\mathbf{a}_1$=(0.0000, 2.0001) $\mathbf{a}_{1n}$=(0.3456, 1.9697)

The other R-S link ($\mathbf{b}_0$-$\mathbf{b}_1$) was the next link synthesized in this two-phase adjustable moving pivot problem. The multiply separated positions of the rigid body are governed by the angular velocity and acceleration of the driving link. Since link $\mathbf{b}_0$-$\mathbf{b}_1$ can function as a driving link, it was also synthesized using MSPs. The length of this R-S link remained fixed throughout each phase.

The values for 8 prescribed rigid body positions projected on the x*-y* plane are given in table 5.15. These positions are represented by points $\mathbf{p}_{x^*-y^*}$, $\mathbf{q}_{x^*-y^*}$

and $\mathbf{r}_{x^*\text{-}y^*}$. To satisfy the design equations of the R-S link, points **p**, **q** and **r** are projected in the x*-y* plane in each rigid body position. The points are also non-linear.

In addition to the 8 prescribed rigid body positions, several multiply separated positions are also given. They represent the velocity and acceleration of the rigid body from position 1 to position 2.

Table 5.15 Prescribed x*-y* plane rigid body positions and MSPs for 2-phase RSSR-SC moving pivot problem with fixed crank and follower lengths

	$\mathbf{p}_{x^*\text{-}y^*}$	$\mathbf{q}_{x^*\text{-}y^*}$	$\mathbf{r}_{x^*\text{-}y^*}$
Phase 1			
pos. 1	-0.4737, 2.8716	0.0281, 2.5824	-0.9754, 2.3608
vel. 1-2	-0.2018, 9.9771	-9.9975, -0.1988	-0.2277, -1.3541
accel. 1-2	-99.7914, -1.0207	0.9878, -99.9951	13.5182, -2.4126
pos. 2	-0.6651, 2.8841	-0.1587, 2.6051	-1.1555, 2.3633
pos. 3	-0.8565, 2.8817	-0.3457, 2.6134	-1.3348, 2.3507
pos. 4	-1.0467, 2.8646	-0.5315, 2.6074	-1.5122, 2.3234
Phase 2			
pos. 5	-0.6636, 2.9076	-0.1571, 2.6299	-1.1538, 2.3861
pos. 6	-0.8555, 2.9273	-0.3447, 2.6614	-1.3338, 2.3957
pos. 7	-1.0485, 2.9310	-0.5334, 2.6782	-1.5140, 2.3898
pos. 8	-1.2411, 2.9191	-0.7220, 2.6795	-1.6934, 2.3685

Note: In this problem, no rigid body positions are shared

The required R-S link variables here are $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_{1n}$. Variable $\mathbf{b}_0$ represents the fixed pivot of the R-S link. Variables $\mathbf{b}_1$ and $\mathbf{b}_{1n}$ represent the moving pivots in phase 1 and phase 2 of the R-S link. Since each of these variables has two scalar components, there are a total of six required unknowns.

$$\mathbf{b}_0=(b_{0x}, b_{0y}) \qquad \mathbf{b}_1=(b_{1x}, b_{1y}) \qquad \mathbf{b}_{1n}=(b_{1nx}, b_{1ny})$$

The eight prescribed rigid body positions and multiply separated positions result in six design equations. The following set of design equations were used to calculate $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_{1n}$:

$$([A_{1,2}]\mathbf{b}_1)^T([D_{1,2}]\mathbf{b}_1 - \mathbf{b}_0) + ([V_{1,2}]\mathbf{b}_1)^T([V_{1,2}]\mathbf{b}_1) = 0 \quad (5.63)$$

$$([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.64)$$

$$([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.65)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \quad (5.66)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \quad (5.67)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \quad (5.68)$$

The term R_1 represents the length of the R-S link. Since this example problem involved moving pivot adjustments with fixed crank and follower lengths, the value of R_1 remained constant in equations 5.63 through 5.68. The specified value for R_1 is 2.5. Given the following initial guesses:

$\mathbf{b}_0$=(0.1, 0.1) $\mathbf{b}_1$=(0.5, 2.5) $\mathbf{b}_{1n}$=(1, 2.5)

the solution to equations 5.63 through 5.68 converged to the following using Newton's Method:

$\mathbf{b}_0$=(0.0716, 0.0077) $\mathbf{b}_1$=(0.4286, 2.4568) $\mathbf{b}_{1n}$=(0.8494, 2.3317)

The C-S link ($\mathbf{c}_0$-$\mathbf{c}_1$) was the last link synthesized in this two-phase adjustable moving pivot problem. The multiply separated positions of the rigid body are governed by the angular velocity and acceleration of the driving link. Since the C-S link can function as a driving link, it was also synthesized using MSPs. The length of this link remained fixed throughout each phase.

The values for 8 prescribed rigid body positions projected on the x**-y** plane are given in table 5.16. These positions are represented by points $\mathbf{p}_{x^{**}-y^{**}}$,

$q_{x^{**}\text{-}y^{**}}$ and $r_{x^{**}\text{-}y^{**}}$. To satisfy the design equations of the C-S link, points **p**, **q** and **r** are projected in the x**-y** plane in each rigid body position. The points are also non-linear.

In addition to the 8 prescribed rigid body positions, several multiply separated positions are also given. They represent the velocity of the rigid body from positions 1 to 2 and from positions 5 to 6.

Table 5.16 Prescribed x**-y** plane rigid body positions and MSPs for 2-phase RSSR-SC moving pivot problem with fixed crank and follower lengths

	$p_{x^{**}\text{-}y^{**}}$	$q_{x^{**}\text{-}y^{**}}$	$r_{x^{**}\text{-}y^{**}}$
Phase 1			
pos. 1	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
vel. 1-2	-0.2004, 10.0036	-9.9975, -0.2013	0.0703, -1.3397
pos. 2	0.8195, 2.8841	1.3309, 2.6051	0.3241, 2.3633
pos. 3	0.6282, 2.8817	1.1455, 2.6134	0.1435, 2.3507
pos. 4	0.4382, 2.8645	0.9612, 2.6074	-0.0350, 2.3233
Phase 2			
pos. 5	0.8178, 2.9076	1.3302, 2.6299	0.3239, 2.3861
vel. 5-6	-0.2234, 5.0005	-4.9825, -0.2256	-0.1029, -1.2876
pos. 6	0.6239, 2.9273	1.1424, 2.6617	0.1414, 2.3957
pos. 7	0.4304, 2.9310	0.9541, 2.6782	-0.0409, 2.3898
pos. 8	0.2382, 2.9191	0.7664, 2.6795	-0.2217, 2.3685

Note: In this problem, no rigid body positions are shared

The required C-S link variables are $\mathbf{c}_0$, $\mathbf{c}_1$ and $\mathbf{c}_{1n}$. Variable $\mathbf{c}_0$ represents the fixed pivot of the C-S link. Variables $\mathbf{c}_1$ and $\mathbf{c}_{1n}$ represent the moving pivots in phase 1 and phase 2 of the C-S link. Since each of these variables has two scalar components, there are a total of six required unknowns.

$$\mathbf{c}_0=(c_{0x}, c_{0y}) \qquad \mathbf{c}_1=(c_{1x}, c_{1y}) \qquad \mathbf{c}_{1n}=(c_{1nx}, c_{1ny})$$

The eight prescribed rigid body positions and multiply separated positions result in six design equations. The following set of design equations were used to calculate $\mathbf{c}_0$, $\mathbf{c}_1$ and $\mathbf{c}_{1n}$:

$$([V_{1,2}]\mathbf{c}_1)^T([D_{1,2}]\mathbf{c}_1 - \mathbf{c}_0) = 0 \quad (5.69)$$

$$([D_{1,3}]\mathbf{c}_1 - \mathbf{c}_0)^T([D_{1,3}]\mathbf{c}_1 - \mathbf{c}_0) - R_1^2 = 0 \quad (5.70)$$

$$([D_{1,4}]\mathbf{c}_1 - \mathbf{c}_0)^T([D_{1,4}]\mathbf{c}_1 - \mathbf{c}_0) - R_1^2 = 0 \quad (5.71)$$

$$(\{[V_{5,6}][D_{1,5}]\}\mathbf{c}_{1n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{c}_{1n} - \mathbf{c}_0) = 0 \quad (5.72)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{c}_{1n} - \mathbf{c}_0)^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{c}_{1n} - \mathbf{c}_0) - R_1^2 = 0 \quad (5.73)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{c}_{1n} - \mathbf{c}_0)^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{c}_{1n} - \mathbf{c}_0) - R_1^2 = 0 \quad (5.74)$$

The term R_1 represents the length of the C-S link. Since this example problem involved moving pivot adjustments with fixed crank and follower lengths, the value of R_1 remained constant in equations 5.69 through 5.74. The specified value for R_1 is 2.5. Given the following initial guesses:

$$\mathbf{c}_0=(0.1, 0.1) \qquad \mathbf{c}_1=(0.1, 2.4) \qquad \mathbf{c}_{1n}=(-0.5, 2.5)$$

the solution to equations 5.69 through 5.74 converges to the following using Newton's Method:

$$\mathbf{c}_0=(0.0000, -0.0000) \qquad \mathbf{c}_1=(-0.0850, 2.4931) \qquad \mathbf{c}_{1n}=(0.2762, 2.4720)$$

By using the initial rigid body points in each phase as the starting points for the synthesized adjustable RSSR-SC mechanism and rotating link $\mathbf{a}_0$-$\mathbf{a}_1$ by certain angles, the remaining positions in table 5.17 were approximated. The crank rotation angles for the first four rigid body positions are 90°, 95°, 100° and 105°. The crank rotation angles for the next four rigid body positions are 85°, 90°, 95° and 100°. These angles are measured with respect to the X-axis.

Table 5.17 Prescribed X-Y-Z frame rigid body positions for 2-phase RSSR-SC moving pivot problem with fixed crank and follower lengths and MSPs

	p	q	r
	Phase 1		
pos. 1	1.0109, 2.8716, -0.0460	1.5163, 2.5824, -0.0690	0.5054, 2.3608, -0.0230
pos. 2	0.8195, 2.8841, -0.0288	1.3309, 2.6051, -0.0445	0.3241, 2.3633, -0.0145
pos. 3	0.6282, 2.8817, -0.0095	1.1455, 2.6134, -0.0190	0.1435, 2.3507, -0.0052
pos. 4	0.4382, 2.8646, 0.0119	0.9612, 2.6074, 0.0070	-0.0350, 2.3234, 0.0048
	Phase 2		
pos. 5	0.8178, 2.9076, -0.0663	1.3302, 2.6299, -0.0674	0.3239, 2.3861, -0.0345
pos. 6	0.6239, 2.9273, -0.0832	1.1424, 2.6617, -0.0639	0.1414, 2.3957, -0.0446
pos. 7	0.4304, 2.9310, -0.0971	0.9541, 2.6782, -0.0587	-0.0409, 2.3898, -0.0532
pos. 8	0.2382, 2.9191, -0.1081	0.7664, 2.6795, -0.0520	-0.2217, 2.6385, -0.0605

Table 5.18 Rigid body positions for synthesized mechanism for 2-phase RSSR-SC moving pivot problem with fixed crank and follower lengths and MSPs

	p	q	r
	Phase 1		
pos. 1	1.0109, 2.8716, -0.0460	1.5163, 2.5824, -0.0690	0.5054, 2.3608, -0.0230
pos. 2	0.8201, 2.8841, -0.0281	1.3315, 2.6050, -0.0441	0.3246, 2.3634, -0.0143
pos. 3	0.6293, 2.8818, -0.0114	1.1465, 2.6132, -0.0198	0.1444, 2.3510, -0.0060
pos. 4	0.4397, 2.8648, 0.0074	0.9626, 2.6073, 0.0054	-0.0338, 2.3238, 0.0030
	Phase 2		
pos. 5	0.8178, 2.9076, -0.0663	1.3302, 2.6299, -0.0674	0.3239, 2.3861, -0.0345
pos. 6	0.6252, 2.9285, -0.0604	1.1434, 2.6618, -0.0519	0.1421, 2.3967, -0.0339
pos. 7	0.4320, 2.9342, -0.0516	0.9556, 2.6788, -0.0347	-0.0399, 2.3921, -0.0321
pos. 8	0.2391, 2.9248, -0.0403	0.7680, 2.6810, -0.0161	-0.2209, 2.3724, -0.0292

The average error magnitude between the specified rigid body positions (table 5.17) and the rigid body positions of the synthesized mechanism for positions 2, 3 and 4 is 0.0009 units. The maximum error magnitude between positions 2, 3 and 4 is 0.0045 units. It occurs at rigid body point p_z in position 4.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for position 6, 7 and 8 is 0.0106 units. The maximum error magnitude between positions 6, 7 and 8 is 0.0678 units. It occurs at rigid body point p_z in position 8.

The two initial translation magnitudes for the C-S link that were used in the RSSR-SC mechanism to calculate the rigid body positions in table 5.18 are S=0 units for phase 1 and S = 0 for phase 2. They were determined by trial and error.

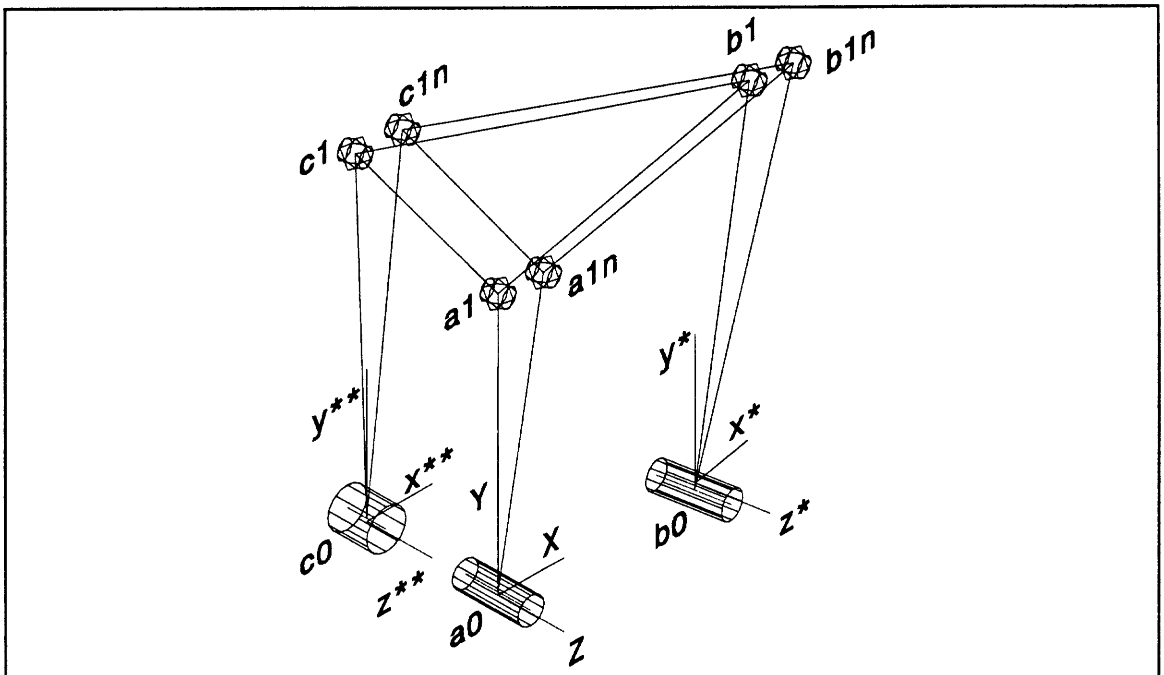

Figure 5.13 Solution to 2-phase RSSR-SC moving pivot problem with fixed crank and follower lengths and MSPs

The specified MSP parameters in table 5.14 are

Vp$_{1,2}$=(-0.2004, 10.0036) **Vq**$_{1,2}$=(-9.9975, -0.2013) **Vr**$_{1,2}$=(0.0703, -1.3397)

Ap$_{1,2}$=(-100.0556, -1.0034) **Aq**$_{1,2}$=(1.0128, -99.9949) **Ar**$_{1,2}$=(13.4039, 0.5691)

These parameters correspond to link $\mathbf{a}_0$-$\mathbf{a}_1$ for rigid body positions 1 and 2. Since these positions are in phase 1, the R-S link parameters for this position are $\mathbf{a}_0$ and $\mathbf{a}_1$. The values calculated for these parameters are the following:

$\mathbf{a}_0$=(-0.0006, -0.0000) $\mathbf{a}_1$=(-0.0000, 2.0001)

The velocity and acceleration of $\mathbf{a}_1$ were calculated using the **a'** and **a"** terms in equations 3.3 and 3.6 (where $[V_{12}]=[\mathbf{Vp}_{1,2}|\mathbf{Vq}_{1,2}|\mathbf{Vr}_{1,2}]$ and $[A_{12}]=[\mathbf{Ap}_{1,2}|\mathbf{Aq}_{1,2}|\mathbf{Ar}_{1,2}]$ with a third row of zeros). The values calculated for the velocity $\mathbf{a}_1$' and acceleration $\mathbf{a}_1$" are

$\mathbf{a}_1$'=(-19.9960, -0.4026) $\mathbf{a}_1$"=(2.0257, -199.9998)

The specified MSP parameters in table 5.15 are

$\mathbf{Vp}_{1,2}$=(-0.2018, 9.9771) $\mathbf{Vq}_{1,2}$=(-9.9975, -0.1988) $\mathbf{Vr}_{1,2}$=(-0.2277, -1.3541)

$\mathbf{Ap}_{1,2}$=(-99.7914, -1.0207) $\mathbf{Aq}_{1,2}$=(0.9878, -99.9951) $\mathbf{Ar}_{1,2}$=(13.5182, -2.4126)

These parameters correspond to link $\mathbf{b}_0$-$\mathbf{b}_1$ for rigid body positions 1 and 2. Since these positions are in phase 1, the R-S link parameters for this position are $\mathbf{b}_0$ and $\mathbf{b}_1$. The values calculated for these parameters are the following:

$\mathbf{b}_0$=(0.0716, 0.0077) $\mathbf{b}_1$=(0.4286, 2.4568)

The velocity and acceleration of $\mathbf{b}_1$ was calculated using the **a'** and **a"** terms in equations 3.3 and 3.6 (where $[V_{12}]=[\mathbf{Vp}_{1,2}|\mathbf{Vq}_{1,2}|\mathbf{Vr}_{1,2}]$ and $[A_{12}]=[\mathbf{Ap}_{1,2}|\mathbf{Aq}_{1,2}|\mathbf{Ar}_{1,2}]$ with a third row of zeros). The values calculated for the velocity $\mathbf{b}_1$' and acceleration $\mathbf{b}_1$" are

$\mathbf{b}_1$'=(-24.6483, 3.7878) $\mathbf{b}_1$"=(-40.3438, -246.0154)

The specified MSP parameters in table 5.16 are

$\mathbf{Vp}_{1,2}$=(-0.2004, 10.0036) $\mathbf{Vq}_{1,2}$=(-9.9975, -0.2013) $\mathbf{Vr}_{1,2}$=(0.0703, -1.3397)

$\mathbf{Vp}_{5,6}$=(-0.2234, 5.0005) $\mathbf{Vq}_{5,6}$=(-4.9825, -0.2256) $\mathbf{Vr}_{5,6}$=(-0.1029, -1.2876)

These parameters correspond to links $\mathbf{c}_0$-$\mathbf{c}_1$ and $\mathbf{c}_0$-$\mathbf{c}_{1n}$ for rigid body from positions 1 to 2 and from positions 5 to 6. Since these positions are in phases 1, and 2 the C-S link parameters for these position are $\mathbf{c}_0$, $\mathbf{c}_1$ and $\mathbf{c}_{1n}$. The values calculated for these parameters are the following:

$\mathbf{c}_0$=(0.0000, -0.0000) $\mathbf{c}_1$=(-0.0850, 2.4931) $\mathbf{c}_{1n}$=(0.2762, 2.4720)

The velocity and acceleration of $\mathbf{c}_1$ and $\mathbf{c}_{1n}$ were calculated using the **a**' term in equation 3.3 (where $[V_{12}]=[\mathbf{Vp}_{1,2}|\mathbf{Vq}_{1,2}|\mathbf{Vr}_{1,2}]$ and $[V_{56}]=[\mathbf{Vp}_{5,6}|\mathbf{Vq}_{5,6}|\mathbf{Vr}_{5,6}]$ with a third row of zeros). The values calculated for the velocities $\mathbf{c}_1$' and $\mathbf{c}_{1n}$' are

$\mathbf{c}_1$'=(-24.9077, -1.3522) $\mathbf{c}_{1n}$'=(-12.3670, 1.1091)

The angular acceleration of both R-S links were calculated using the **a**" term equation in equation 3.6. Since both links rotate in planes, only the up_z unit vector term was used in equation 3.8 (therefore up_x=up_y=0). The fixed pivot acceleration term was also eliminated in equation 3.7 since the fixed pivots are "fixed" (therefore $\dot{\mathbf{up}}=0$ also). When expanded the **a**" term equation becomes

$$\left[\begin{bmatrix} -\omega^2 & -\alpha & \omega^2 a_{0x} + \alpha a_{0y} \\ \alpha & \omega^2 & -\alpha a_{0x} + \omega^2 a_{0y} \\ 0 & 0 & 0 \end{bmatrix} \left[\begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} p_{ix} & q_{ix} & r_{ix} \\ p_{iy} & q_{iy} & r_{iy} \\ 1 & 1 & 1 \end{bmatrix}^{-1}\right]\right]\begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} = \begin{pmatrix} a_{1x}'' \\ a_{1y}'' \\ 0 \end{pmatrix} \quad (5.75)$$

When the rigid body position parameters and the moving pivot position and acceleration parameters for link $\mathbf{a}_0$-$\mathbf{a}_1$ were incorporated, the angular velocity and acceleration values calculated using the simultaneous equations were ω=10.0007 rad/sec and α=0.9874 rad/sec^2. When the rigid body position parameters and the moving pivot position and acceleration parameters for link

b$_0$-**b**$_1$ were incorporated, the angular velocity and acceleration values calculated using the simultaneous equations were ω=9.9972 rad/sec and α=1.0162 rad/sec^2.

The angular velocities of both R-S links and the C-S link were calculated using the **a**' term equation in equation 3.3. Since these links rotates in planes, only the ω_z term was used in equation 3.5 (therefore ω_x=ω_y=0). The fixed pivot velocity term was also eliminated in equation 3.4 since the fixed pivots are "fixed."

When the rigid body position parameters and the moving pivot position and velocity parameters for link **a**$_0$-**a**$_1$ were incorporated, the angular velocity values calculated were ω_z=10.0005 rad/sec for the first row in equation 5.56 and ω_z=10.0149 rad/sec for the second row. When the rigid body position parameters and the moving pivot position and velocity parameters for link **b**$_0$-**b**$_1$ were incorporated, the angular velocity values calculated were ω_z=9.9993 rad/sec for the first row in equation 5.56 and ω_z=10.0021 rad/sec for the second row. Since the calculated position and velocity parameters of the moving pivots of the R-S links were truncated (to four significant figures), the angular velocity values for the R-S links were not exact matches.

When the rigid body position parameters and the moving pivot position and velocity parameters for link **c**$_0$-**c**$_1$ were incorporated, the angular velocity values calculated were ω_z=9.9975 rad/sec for the first row in equation 5.56 and ω_z=10.0237 rad/sec for the second row. When the rigid body position parameters and the moving pivot position and velocity parameters for link $\mathbf{c}_0$-$\mathbf{c}_{1n}$ were incorporated, the angular velocity values calculated were ω_z=5 rad/sec for the first row in equation 5.56 and ω_z=5.0004 rad/sec for the second row. Since the calculated position and velocity parameters of the moving pivot of the C-S link was truncated (to four significant figures), the angular velocity values for the C-S link are not exact matches.

5.2 Three-Phase Example Problems

5.2.1 RRSS Mechanism for Finite and Multiply Separated Positions with ISA Parameters

The instant screw axis and R-R link parameters were calculated first in this three-phase adjustable moving pivot problem.

The values for 11 prescribed rigid body positions are given in table 5.19. These positions are represented by points **p**, **q**, **r** and **s**. To satisfy the design equations of the R-R link, points **p**, **q**, **r** and **s** travel in planes that are normal to $\mathbf{ua}_j$ and rotate about $\mathbf{ua}_0$ in each rigid body position. The points are also non-linear.

In addition to the 11 prescribed rigid body positions, several instant screw axis parameters are also given for the first rigid body position. The parameter $\mathbf{p}_0$ represents a point on the instant screw axis and ω_s and α_s are the angular velocity and acceleration about the instant screw axis.

Table 5.19 Prescribed X-Y-Z frame rigid body positions and ISA parameters for 3-phase RRSS moving pivot problem with adjustable crank and follower lengths

Phase 1

	p	q	r	s
pos. 1	-0.5096.1.0541,0.1040	1.3326,2.9344,-0.2720	0.7928,1.7071,0.3485	0.5928, 1.7071,-0.6313
	p_0=(0,-2.2834, 0.6118) ω_s=2.0 α_s=1.0 up_0' =p_0'=(0, 0, 0)			
pos. 2	-0.6860,1.0111,0.1014	1.0406,2.9872,-0.3285	0.5737,1.7488,0.3282	0.3776,1.7070,-0.6515
pos. 3	-0.8558,0.9360,0.0972	0.7449,3.0055,-0.3779	0.3523,1.7589,0.3110	0.1676,1.6769,-0.6683
pos. 4	-1.0136,0.8301,0.0914	0.4504,2.9902,-0.4200	0.1336,1.7380,0.2971	-0.0324,1.6187,-0.6818
pos. 5	-1.1540,0.6957,0.0845	0.1614,2.9428,-0.4549	-0.0777,1.6873,0.2863	-0.2185,1.5349,-0.6920

Phase 2

	p	q	r	s
pos. 6	-0.5096.1.0541,0.1040	1.3326,2.9344,-0.2720	0.7928,1.7071,0.3485	0.5928, 1.7071,-0.6313
pos. 7	-0.7137,1.0066,0.1090	1.0079,2.9872,-0.3199	0.5442,1.7473,0.3360	0.3481,1.7056,-0.6437
pos. 8	-0.9090,0.9206,0.1120	0.6806,2.9991,-0.3611	0.2948,1.7496,0.3263	0.1101,1.6679,-0.6530

Phase 3

	p	q	r	s
pos. 9	-0.5096.1.0541,0.1040	1.3326,2.9344,-0.2720	0.7928,1.7071,0.3485	0.5928, 1.7071,-0.6313
pos. 10	-0.6567,1.0161,0.0935	1.0764,2.9861,-0.3379	0.6053,1.7499,0.3198	0.4092,1.7079,-0.6599
pos. 11	-0.7995, 0.9527,0.0815	0.8160,3.0100,-0.3962	0.4142,1.7675,0.2947	0.2295,1.6850,-0.6846

Note: In this problem, rigid body positions 1, 6 and 9 are shared

All of the rigid body points in this example problem were obtained using $\mathbf{ua}_1$=[sin 15°, 0, cos 15°] when the R-R link lies along the positive Y-axis.

Although ISA parameters $\mathbf{p}_0$, ω_s, α_s, $\mathbf{up}_0'$ and $\mathbf{p}_0'$ are known, the instant screw axis unit vector ($\mathbf{up}_0$) is unknown. To calculate $\mathbf{up}_0$, the following equations were used:

$$s''\mathbf{up}_0 + s'\mathbf{up}_0' - [W_s']\mathbf{p}_0 - [W_s]\mathbf{p}_0' = \mathbf{p}'' - [W']\mathbf{p} \quad (5.76)$$

$$(\mathbf{up}_0)^T(\mathbf{up}_0) - 1 = 0 \quad (5.77)$$

When the fourth columns of equations 3.7 and 3.10 are equated, the result is equation 5.76.

As mentioned in section 3.4, the variable **p** is equivalent to the fixed pivot of the link synthesized (in this case $\mathbf{a}_0$ of the R-R link). Since $\mathbf{a}_0$ is fixed, its derivatives are zero (therefore **p''**=0). In addition, **p** itself will also be set to zero since it can be prescribed. Although the present goal here is to determine the ISA unit vector, by making **p** equal to zero, the fixed pivot that will be calculated later will also approximate zero since **p** is equivalent to $\mathbf{a}_0$.

When expanded and simplified, equations 5.76 and 5.77 become

$$s''up_{0x} - (((up_x^2-1)\omega_s^2)p_{0x} + (up_x up_y \omega_s^2 - up_z \alpha_s)p_{0y} + (up_x up_z \omega_s^2 + up_y \alpha_s)p_{0z}) = 0 \quad (5.78)$$

$$s''up_{0x} - (((up_y^2-1)\omega_s^2)p_{0y} + (up_x up_y \omega_s^2 + up_z \alpha_s)p_{0x} + (up_y up_z \omega_s^2 - up_x \alpha_s)p_{0z}) = 0 \quad (5.79)$$

$$s''up_{0x} - (((up_z^2-1)\omega_s^2)p_{0z} + (up_x up_z \omega_s^2 - up_y \alpha_s)p_{0x} + (up_x up_z \omega_s^2 + up_x \alpha_s)p_{0y}) = 0 \quad (5.80)$$

$$up_{0x}^2 + up_{0y}^2 + up_{0z}^2 - 1 = 0 \quad (5.81)$$

Using the initial guesses $\mathbf{up}_0$=[0.5, 0.5, 0.5] and s''=0.25, the solution converged to $\mathbf{up}_0$=[0, 0.96593, 0.25881] and s''=0 using Newton's Method.

After the ISA unit vector was calculated, the R-R link joint locations were calculated next. The required R-R link variables are $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{a}_{1n}$ and $\mathbf{a}_{2n}$. The length of this link was adjusted throughout each phase. Variable $\mathbf{a}_0$ represents the fixed pivot of the R-R link. Variables $\mathbf{a}_1$, $\mathbf{a}_{1n}$ and $\mathbf{a}_{2n}$ represent the moving pivots in phases 1, 2 and 3 of the R-R link. Since each of these variables has two scalar components, there are a total of eight required unknowns.

$\mathbf{a}_0=(a_{0x}, a_{0y})$ $\quad$ $\mathbf{a}_1=(a_{1x}, a_{1y})$ $\quad$ $\mathbf{a}_{1n}=(a_{1nx}, a_{1ny})$ $\quad$ $\mathbf{a}_{2n}=(a_{2nx}, a_{2ny})$

The eleven prescribed rigid body positions result in eight design equations. For the R-R link, the rigid body position #5 was not used. Instead, the ISA parameters for rigid body position #1 were incorporated. The ISA parameters account for a multiply separated rigid body position. Therefore, the data in the section labeled "position 1" in table 5.19 actually account for position 1 and position 2. As a result, the data labeled "position 2, 3 and 4" in table 5.19 can be called position 3, 4, and 5. Since the data labeled "position 5" in table 5.19 is now an extra position, it was not used. The following set of equations were used to calculate $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{a}_{1n}$ and $\mathbf{a}_{2n}$:

$$([A_s][D_{1,1}]\mathbf{a}_1)^T([D_{1,1}]\mathbf{a}_1 - \mathbf{a}_0) + ([V_s][D_{1,1}]\mathbf{a}_1)^T([V_s][D_{1,1}]\mathbf{a}_1) = 0 \quad (5.82)$$

$$([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.83)$$

$$([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.84)$$

$$([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.85)$$

$$(\{[D_{6,7}][D_{1,6}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{6,7}][D_{1,6}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_2^2 = 0 \quad (5.86)$$

$$(\{[D_{6,8}][D_{1,6}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{6,8}][D_{1,6}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_2^2 = 0 \quad (5.87)$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{a}_{2n} - \mathbf{a}_0)^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{a}_{2n} - \mathbf{a}_0) - R_3^2 = 0 \quad (5.88)$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{a}_{2n} - \mathbf{a}_0)^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{a}_{2n} - \mathbf{a}_0) - R_3^2 = 0 \quad (5.89)$$

The terms R_1, R_2 and R_3 represent the lengths of the R-R link. Since this example problem involved moving pivot adjustments with adjustable crank and follower lengths, the values of R_1, R_2 and R_3 were not identical in equations 5.82 through 5.89. The specified value for R_1, R_2 and R_3 are 1, 1.25 and 0.75. Given the following initial guesses:

$\mathbf{a}_0$=(0.1, 0.1) $\mathbf{a}_1$=(0.1, 1.2) $\mathbf{a}_{1n}$=(0.1, 1) $\mathbf{a}_{2n}$=(0.1, 0.9)

the solution to equations 5.82 through 5.89 converged to the following using Newton's Method:

$\mathbf{a}_0$=(-0.0070, 0.0077) $\mathbf{a}_1$=(0.1302, 0.9994) $\mathbf{a}_{1n}$=(0.0644, 1.2557)

$\mathbf{a}_{2n}$=(0.1867, 0.7327)

The S-S link (link $\mathbf{b}_0$-$\mathbf{b}_1$) was synthesized next in this three-phase adjustable moving pivot problem. The length of the S-S link was adjusted throughout each phase. Using ISA parameters, rigid body velocities and accelerations can be calculated. The velocities and accelerations of the rigid body are governed by the angular velocity and acceleration of the driving link. Since the S-S link cannot function as a driving link, it will not be synthesized using ISA parameters. Therefore, all of the rigid body positions in table 5.19 were used to synthesize the S-S link.

To satisfy the design equations of the S-S link, points **p**, **q**, **r** and **s** do not all lie on the same plane in each rigid body position. In this three-phase adjustable moving pivot problem, the required S-S link variables are $\mathbf{b}_0$, $\mathbf{b}_1$, $\mathbf{b}_{1n}$ and $\mathbf{b}_{2n}$. Variable $\mathbf{b}_0$ represents the fixed pivot of the S-S link. Variables $\mathbf{b}_1$, $\mathbf{b}_{1n}$ and $\mathbf{b}_{2n}$ represent the moving pivots in phases 1, 2 and 3 of the S-S link. Since each of these variables has three scalar components, there are a total of twelve required unknowns.

$\mathbf{b}_0=(b_{0x}, b_{0y}, b_{0z})$ $\mathbf{b}_1=(b_{1x}, b_{1y}, b_{1z})$ $\mathbf{b}_{1n}=(b_{1nx}, b_{1ny}, b_{1nz})$ $\mathbf{b}_{2n}=(b_{2nx}, b_{2ny}, b_{2nz})$

The eleven prescribed rigid body positions result in eight design equations. Therefore, the values of $\mathbf{b}_0$ and b_{1nz} were prescribed. Using AutoCAD 2000 software, the value of $\mathbf{b}_0$ was specified to $\mathbf{b}_o=[1, 0, 0]$ and the value of b_{1nz} was specified to $b_{1nz}=-0.4730$. The following set of equations were used to calculate b_{1nx}, b_{1ny}, $\mathbf{b}_1$ and $\mathbf{b}_{2n}$:

$$([D_{1,2}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,2}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.90)$$

$$([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.91)$$

$$([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.92)$$

$$([D_{1,5}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,5}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.93)$$

$$(\{[D_{6,7}][D_{1,6}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{6,7}][D_{1,6}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_2^2 = 0 \quad (5.94)$$

$$(\{[D_{6,8}][D_{1,6}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{6,8}][D_{1,6}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_2^2 = 0 \quad (5.95)$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{b}_{2n} - \mathbf{b}_0)^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{b}_{2n} - \mathbf{b}_0) - R_3^2 = 0 \quad (5.96)$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{b}_{2n} - \mathbf{b}_0)^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{b}_{2n} - \mathbf{b}_0) - R_3^2 = 0 \quad (5.97)$$

The terms R_1, R_2 and R_3 represent the lengths of the S-S link. Since this example problem involved moving pivot adjustments with adjustable crank and follower lengths, the values of R_1, R_2 and R_3 were not identical in equations 5.90 through 5.97. The specified value for R_1, R_2 and R_3 are 2, 2.25 and 1.75. Given the following initial guesses:

b_{1nx}=1.5 b_{1ny}=1.5 $\mathbf{b}_1$=(1.5, 1.5, -0.4) $\mathbf{b}_{2n}$=(1.5, 1.5. -0.4)

the solution to equations 5.90 through 5.97 converged to the following using Newton's Method:

b_{1nx}=1.7651 b_{1ny}=2.0623 $\mathbf{b}_1$=(1.7801, 1.7793, -0.4752)

$\mathbf{b}_{2n}$=(1.7664, 1.4739. –0.5490)

By using the initial rigid body points in each phase as the starting points for the synthesized adjustable RRSS mechanism and rotating the R-R link by certain angles, the remaining positions in table 5.20 can be approximated. The R-R link rotation angles for the first five rigid body positions are 90°, 100°, 110°, 120° and 130°. The R-R link rotation angles for the next three rigid body positions are 90°, 100° and 110°. The R-R link rotation angles for the last three rigid body positions are 90°, 100° and 110°. These angles are measured with respect to the X-axis.

Table 5.20 Rigid body positions for synthesized mechanism for 3-phase RRSS moving pivot problem with adjustable crank and follower lengths and ISA parameters

	p	q	r	s
		Phase 1		
pos. 1	-0.5096.1.0541,0.1040	1.3326,2.9344,-0.2720	0.7928,1.7071,0.3485	0.5928, 1.7071,-0.6313
pos. 2	-0.6862,1.0346,0.1008	1.0625,2.9900,-0.3340	0.5815,1.7589,0.3260	0.3854,1.7164,-0.6536
pos. 3	-0.8607,0.9810,0.0951	0.7832,3.0145,-0.3876	0.3637,1.7800,0.3071	0.1792,1.6967,-0.6722
pos. 4	-1.0273,0.8941,0.0873	0.5002,3.0075,-0.4327	0.1448,1.7698,0.2915	-0.0209,1.6488,-0.6872
pos. 5	-1.1802,0.7755,0.0779	0.2188,2.9695,-0.4695	-0.0700,1.7291,0.2792	-0.2102,1.5745,-0.6987
		Phase 2		
pos. 6	-0.5096.1.0541,0.1040	1.3326,2.9344,-0.2720	0.7928,1.7071,0.3485	0.5928, 1.7071,-0.6313
pos. 7	-0.7152,1.0182,0.1092	1.0172,2.9888,-0.3220	0.5466,1.7523,0.3355	0.3505,1.7103,-0.6441
pos. 8	-0.9142,0.9427,0.1120	0.6968,3.0038,-0.3648	0.2978,1.7600,0.3255	0.1132,1.6777,-0.6539
		Phase 3		
pos. 9	-0.5096.1.0541,0.1040	1.3326,2.9344,-0.2720	0.7928,1.7071,0.3485	0.5928, 1.7071,-0.6313
pos. 10	-0.6508,1.0530,0.0906	1.1187,2.9886,-0.3488	0.6244,1.7645,0.3144	0.4284,1.7213,-0.6652
pos. 11	-0.7951,1.0234,0.0747	0.8898,3.0214,-0.4150	0.4446,1.7989,0.2850	0.2602,1.7143,-0.6943

The average error magnitude between the specified rigid body positions (table 5.19) and the rigid body positions of the synthesized mechanism for positions 2, 3, 4 and 5 is 0.0178 units. The maximum error magnitude between positions 2, 3, 4 and 5 is 0.0798 units. It occurs at rigid point p_y in position 5.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for positions 7 and 8 is 0.0051 units. The maximum error magnitude between positions 7 and 8 is 0.0221 units. It occurs at rigid point p_y in position 8.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for positions 10 and 11 is

0.0210 units. The maximum error magnitude between positions 10 and 11 is 0.0738 units. It occurs at rigid body point q_x in position 11.

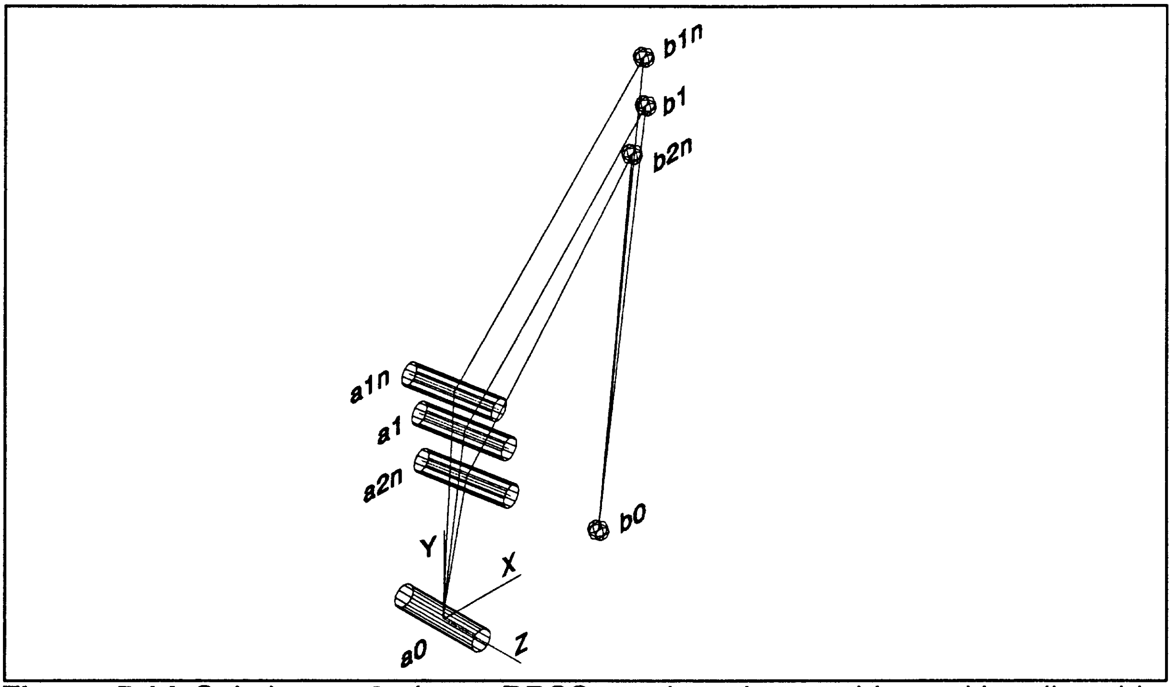

Figure 5.14 Solution to 3-phase RRSS moving pivot problem with adjustable crank and follower lengths and ISA parameters

The specified ISA parameters in table 5.19 are

$$\mathbf{p}_0=(0,-2.2834, 0.6118) \quad \omega_s=2 \quad \alpha_s=1 \quad \mathbf{up}_0' = \mathbf{p}_0'=(0, 0, 0)$$

and the calculated ISA parameters are $\mathbf{up}_0=[0, 0.96593, 0.25881]$ and s"=0. These parameters correspond to the R-R link for rigid body position #1. Since this position is in phase 1, the R-R link parameters for this position are $\mathbf{a}_0$ and $\mathbf{a}_1$. The values calculated for these parameters are the following:

$$\mathbf{a}_0=(-0.0070, 0.0077) \qquad \mathbf{a}_1=(0.1302, 0.9994)$$

The velocity and acceleration of $\mathbf{a}_1$ about the instant screw axis were calculated using the **a**' and **a**" terms in equations 3.3 and 3.6 (where i=j=1).

Equations 3.4 and 3.7 were replaced with equations 3.9 and 3.10 to incorporate the ISA parameters.

The values calculated for the velocity and acceleration of $\mathbf{a}_1$ are

$$\mathbf{a}_1'=(-0.5173, 0.0674, -0.2515) \qquad \mathbf{a}_1''=(-0.7794, -0.2341, 0.8736)$$

The angular velocities of the R-R link was calculated using the **a**' term equation in equation 3.3 (where i=j=1). Equation 3.4 was not replaced in this case since ISA parameters were not used. Since this links rotate in a plane, only the ω_z term was used in equation 3.5 (therefore $\omega_x=\omega_y=0$). The fixed pivot velocity term was also eliminated in equation 3.4 since the fixed pivot is "fixed."

When the moving pivot position and velocity parameters for the R-R link were incorporated, the angular velocity values calculated were $\omega_z=0.5176$ rad/sec for the first row in equation 5.37 and $\omega_z=0.5177$ rad/sec for the second row. Since the calculated position and velocity parameters of the moving pivots of the R-R link was truncated (to four significant figures), the angular velocity values for this links are not exact matches.

The angular acceleration of the R-R link were calculated using the **a**" term equation in equation 3.6 (where i=j=1). Equation 3.7 was not replaced in this case since ISA parameters were not used. Since the R-R link rotates in a plane, only the up_z term was used in equation 3.8 (therefore $up_x=up_y=0$). The fixed pivot acceleration term was also eliminated in equation 3.7 since the fixed pivots are "fixed" (therefore $\dot{\mathbf{up}}=0$ also). When expanded the **a**" term equation becomes

$$\left[\begin{bmatrix} -\omega^2 & -\alpha & \omega^2 a_{0x} + \alpha a_{0y} \\ \alpha & \omega^2 & -\alpha a_{0x} + \omega^2 a_{0y} \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]\begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} = \begin{pmatrix} a_{1x}'' \\ a_{1y}'' \\ 0 \end{pmatrix} \tag{5.98}$$

When the moving pivot position and acceleration parameters for link $\mathbf{a}_0$-$\mathbf{a}_1$ were incorporated, the angular velocity and acceleration values calculated using the simultaneous equations were ω=0.5746 rad/sec and α=0.7368 rad/sec^2.

5.2.2 RRSC Mechanism for Finitely Separated Positions

The R-R link ($\mathbf{a}_0$-$\mathbf{a}_1$) was the first link synthesized in this three-phase adjustable fixed pivot problem. The length of the R-R link was adjusted throughout each phase.

The values for 11 prescribed rigid body positions projected on the X-Y plane are given in table 5.21. These positions are represented by points $\mathbf{p}_{X\text{-}Y}$, $\mathbf{q}_{X\text{-}Y}$ and $\mathbf{r}_{X\text{-}Y}$. To satisfy the design equations of the R-R link, points **p**, **q** and **r** travel in a plane that is normal to $\mathbf{ua}_j$ and rotates about $\mathbf{ua}_0$ in each rigid body position. The points are also non-linear.

Table 5.21 Prescribed X-Y plane rigid body positions for 3-phase fixed pivot RRSC problem with adjustable crank and fixed follower lengths

	$p_{X\text{-}Y}$	$q_{X\text{-}Y}$	$r_{X\text{-}Y}$
	Phase 1		
pos. 1	0.1959, 1.2647	1.2089, 1.5993	0.7024. 1.8320
pos. 2	0.0126, 1.2555	1.0132, 1.6224	0.4984, 1.8386
pos. 3	-0.1668, 1.2169	0.8190, 1.6205	0.2952, 1.8171
pos. 4	-0.3375, 1.1502	0.6302, 1.5957	0.0971, 1.7689
	Phase 2		
pos. 5	0.2417, 1.2625	1.2574, 1.5896	0.7528, 1.8260
pos. 6	-0.0322, 1.2527	0.9665, 1.6248	0.4506, 1.8383
pos. 7	-0.2544, 1.2031	0.7285, 1.6140	0.2033, 1.8067
pos. 8	-0.4645, 1.1178	0.5001, 1.5702	-0.0343, 1.7397
	Phase 3		
pos. 9	0.0576, 1.2582	1.0605, 1.6186	0.5470, 1.8381
pos. 10	-0.0784, 1.2300	0.9119, 1.6227	0.3901, 1.8250
pos. 11	-0.2084, 1.1812	0.7660, 1.6123	0.2353, 1.7933

Note: In this problem, no rigid body positions are shared

All of the rigid body points in this example problem were taken using **d** =1, $\mathbf{ua}_1$=[sin 10°, 0, cos 10°] (when the R-R link lies along the positive Y-axis) and $\mathbf{ua}_0$=[sin -15°, 0, cos 15°]. These joint axes were measured with respect to the X-Y-Z coordinate frame.

The required R-R link variables are $\mathbf{a}_0$, $\mathbf{a}_1$ $\mathbf{a}_{0n}$ and $\mathbf{a}_{20n}$. Variable $\mathbf{a}_1$ represents the moving pivot of the R-R link. Variables $\mathbf{a}_0$, $\mathbf{a}_{0n}$ and $\mathbf{a}_{20n}$ represent the fixed pivots in phases 1, 2 and 3 of the R-R link. Since each of these variables has two scalar components, there are a total of eight required unknowns.

$$\mathbf{a}_0=(a_{0x}, a_{0y}) \qquad \mathbf{a}_1=(a_{1x}, a_{1y}) \qquad \mathbf{a}_{0n}=(a_{0nx}, a_{0ny}) \qquad \mathbf{a}_{20n}=(a_{20nx}, a_{20ny})$$

The eleven prescribed rigid body positions result in eight design equations. The following set of design equations were used to calculate $\mathbf{a}_0$, $\mathbf{a}_1$ $\mathbf{a}_{0n}$ and $\mathbf{a}_{20n}$:

$$([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.95)$$

$$([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.96)$$

$$([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.97)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_2^2 = 0 \quad (5.98)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_2^2 = 0 \quad (5.99)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_2^2 = 0 \quad (5.100)$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n})^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n}) - R_3^2 = 0 \quad (5.101)$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n})^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n}) - R_3^2 = 0 \quad (5.102)$$

The terms R_1, R_2 and R_3 represent the lengths of the R-R link. Since this example problem involved moving pivot adjustments with adjustable crank and fixed follower lengths, the values of R_1, R_2 and R_3 were not identical in equations 5.95 through 5.102. The specified values for R_1, R_2 and R_3 are 1, 1.5 and 0.75. Given the following initial guesses:

$\mathbf{a}_0$=(0.1, 0.1) $\mathbf{a}_1$=(0.1, 0.85) $\mathbf{a}_{0n}$=(0.1, -0.3) $\mathbf{a}_{20n}$=(0.1, 0.3)

The solution to equations 5.95 through 5.102 converged to the following using Newton's Method:

$\mathbf{a}_0$=(-0.0426, 0.0028) $\mathbf{a}_1$=(-0.0527, 1.0027) $\mathbf{a}_{0n}$=(-0.0419, -0.2473)

$\mathbf{a}_{20n}$=(-0.0436, 0.2531)

The C-S link (link $\mathbf{b}_0$-$\mathbf{b}_1$) was the next link synthesized in this three-phase adjustable fixed pivot problem. The length of the C-S link remained constant throughout each phase.

The values for 11 prescribed rigid body positions projected on the x*-y* plane are given in table 5.22. These positions are represented by points $\mathbf{p}_{x^*\text{-}y^*}$, $\mathbf{q}_{x^*\text{-}y^*}$ and $\mathbf{r}_{x^*y^*}$. To satisfy the design equations of the C-S link, points **p**, **q** and **r** are projected in the x*-y* plane in each rigid body position. The points are also non-linear.

Table 5.22 Prescribed x*-y* plane rigid body positions for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths

	$\mathbf{p}_{x^*\text{-}y^*}$	$\mathbf{q}_{x^*\text{-}y^*}$	$\mathbf{r}_{x^*\text{-}y^*}$
Phase 1			
pos. 1	-0.7857, 1.2647	0.1466, 1.5993	-0.3195, 1.8320
pos. 2	-0.9643, 1.2555	-0.0456, 1.6224	-0.5215, 1.8386
pos. 3	-1.1389, 1.2169	-0.2352, 1.6205	-0.7218, 1.8171
pos. 4	-1.3049, 1.1502	-0.4185, 1.5957	-0.9164, 1.7689
Phase 2			
pos. 5	-0.7411, 1.2625	0.1942, 1.5896	-0.2698, 1.8260
pos. 6	-1.0075, 1.2527	-0.0906, 1.6248	-0.5675, 1.8383
pos. 7	-1.2234, 1.2031	-0.3226, 1.6140	-0.8104, 1.8067
pos. 8	-1.4275, 1.1178	-0.5442, 1.5702	-1.0431, 1.7397
Phase 3			
pos. 9	-0.9208, 1.2582	0, 1.6186	-0.4747, 1.8381
pos. 10	-1.0536, 1.2300	-0.1456, 1.6227	-0.6304, 1.8250
pos. 11	-1.1802, 1.1812	-0.2874, 1.6123	-0.7832, 1.7933

Note: In this problem, no rigid body positions are shared

The required C-S link variables are $\mathbf{b}_0$, $\mathbf{b}_1$ $\mathbf{b}_{0n}$ and $\mathbf{b}_{20n}$. Variable $\mathbf{b}_1$ represents the moving pivot of the C-S link. Variables $\mathbf{b}_0$, $\mathbf{b}_{0n}$ and $\mathbf{b}_{20n}$ represent the fixed pivots in phases 1, 2 and 3 of the C-S link. Since each of these variables has two scalar components, there are a total of eight required unknowns.

$$\mathbf{b}_0=(b_{0x}, b_{0y}) \qquad \mathbf{b}_1=(b_{1x}, b_{1y}) \qquad \mathbf{b}_{0n}=(b_{0nx}, b_{0ny}) \qquad \mathbf{b}_{20n}=(b_{20nx}, b_{20ny})$$

The eleven prescribed rigid body positions result in eight design equations. The following set of design equations were used to calculate $\mathbf{b}_0$, $\mathbf{b}_1$ $\mathbf{b}_{0n}$ and $\mathbf{b}_{20n}$:

$$([D_{1,2}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,2}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.103)$$

$$([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.104)$$

$$([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.105)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \quad (5.106)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \quad (5.107)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \quad (5.108)$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{b}_1 - \mathbf{b}_{20n})^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{b}_1 - \mathbf{b}_{20n}) - R_1^2 = 0 \quad (5.109)$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{b}_1 - \mathbf{b}_{20n})^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{b}_1 - \mathbf{b}_{20n}) - R_1^2 = 0 \quad (5.110)$$

The term R_1 represents the length of the C-S link. Since this example problem involved fixed pivot adjustments with adjustable crank and fixed follower lengths, the value of R_1 remained constant in equations 5.103 through 5.110. The specified values for R_1 is 1.5. Given the following initial guesses:

$\mathbf{b}_0$=(0.1, 0.1) $\quad$ $\mathbf{b}_1$=(0.4, 1.5) $\quad$ $\mathbf{b}_{0n}$=(0.1, 0.1) $\quad$ $\mathbf{b}_{20n}$=(0.1, 0.1)

The solution to equations 5.103 through 5.110 converged to the following using Newton's Method:

$\mathbf{b}_0$=(-0.0280, 0.0096) $\quad$ $\mathbf{b}_1$=(0.2840, 1.4764) $\quad$ $\mathbf{b}_{0n}$=(-0.0310, 0.0099)

$\mathbf{b}_{20n}$=(-0.0243, 0.0092)

By using the initial rigid body points in each phase as the starting points for the synthesized adjustable RRSC mechanism and rotating the R-R link by certain angles, the remaining positions in table 5.24 can be approximated. The R-R link rotation angles for the first four rigid body positions are 90°, 100°, 110° and 120°. The R-R link rotation angles for the next four rigid body positions are 88°, 100°,

110° and 120°. The R-R link rotation angles for the last three rigid body positions are 100°, 110° and 120°. These angles are measured with respect to the X-axis.

Table 5.23 Prescribed X-Y-Z frame rigid body positions for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths

	p	q	r
	Phase 1		
pos. 1	0.1959, 1.2647, -0.0345	1.2089, 1.5993, -0.2132	0.7024. 1.8320, -0.1238
pos. 2	0.0126, 1.2555, -0.0406	1.0132, 1.6224, -0.2256	0.4984, 1.8386, -0.1428
pos. 3	-0.1668, 1.2169, -0.0457	0.8190, 1.6205, -0.2334	0.2952, 1.8171, -0.1585
pos. 4	-0.3375, 1.1502, -0.0499	0.6302, 1.5957, -0.2369	0.0971, 1.7689, -0.1708
	Phase 2		
pos. 5	0.2417, 1.2625, -0.0333	1.2574, 1.5895, -0.2103	0.7528, 1.8260, -0.1199
pos. 6	-0.0322, 1.2527, -0.0404	0.9665, 1.6248, -0.2252	0.4506, 1.8383, -0.1422
pos. 7	-0.2544, 1.2031, -0.0455	0.7285, 1.6140, -0.2331	0.2033, 1.8067, -0.1577
pos. 8	-0.4645, 1.1178, -0.0497	0.5001, 1.5702, -0.2368	-0.0343, 1.7397, -0.1702
	Phase 3		
pos. 9	0.0576, 1.2582, -0.0409	1.0605, 1.6187, -0.2261	0.5470, 1.8381, -0.1436
pos. 10	-0.0784, 1.2300, -0.0461	0.9119, 1.6227, -0.2339	0.3901, 1.8250, -0.1596
pos. 11	-0.2084, 1.1812, -0.0503	0.7660, 1.6122, -0.2371	0.2353, 1.7933, -0.1720

Table 5.24 Rigid body positions for synthesized mechanism for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths

	p	q	r
Phase 1			
pos. 1	0.1959, 1.2647, -0.0345	1.2089, 1.5993, -0.2132	0.7024. 1.8320, -0.1238
pos. 2	0.0128, 1.2557, -0.0405	1.0134, 1.6226, -0.2255	0.4985, 1.8388, -0.1427
pos. 3	-0.1661, 1.2175, -0.0453	0.8198, 1.6210, -0.2330	0.2959, 1.8176, -0.1580
pos. 4	-0.3361, 1.1516, -0.0490	0.6319, 1.5968, -0.2360	0.0988, 1.7702, -0.1699
Phase 2			
pos. 5	0.2417, 1.2625, -0.0333	1.2574, 1.5895, -0.2103	0.7528, 1.8260, -0.1199
pos. 6	-0.0320, 1.2547, -0.0404	0.9669, 1.6260, -0.2255	0.4511, 1.8399, -0.1423
pos. 7	-0.2540, 1.2068, -0.0453	0.7293, 1.6163, -0.2334	0.2043, 1.8098, -0.1578
pos. 8	-0.4640, 1.1233, -0.0492	0.5012, 1.5738, -0.2370	-0.0328, 1.7443, -0.1702
Phase 3			
pos. 9	0.0576, 1.2582, -0.0409	1.0605, 1.6187, -0.2261	0.5470, 1.8381, -0.1436
pos. 10	-0.0782, 1.2315, -0.0457	0.9124, 1.6237, -0.2330	0.3907, 1.8262, -0.1590
pos. 11	-0.2080, 1.1845, -0.0493	0.7670, 1.6145, -0.2352	0.2365, 1.7961, -0.1705

The average error magnitude between the specified rigid body positions (table 5.23) and the rigid body positions of the synthesized mechanism for positions 2, 3 and 4 is 0.0007 units. The maximum error magnitude between positions 3 and 4 is 0.0017 units. It occurs at rigid body point q_x in position 4.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for positions 6, 7 and 8 is 0.0013 units. The maximum error magnitude between positions 6, 7 and 8 is 0.0055 units. It occurs at rigid body point p_y in position 8.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for positions 10 and 11 is

0.0012 units. The maximum error magnitude between positions 10 and 11 is 0.0033 units. It occurs at rigid body point p_y in position 11.

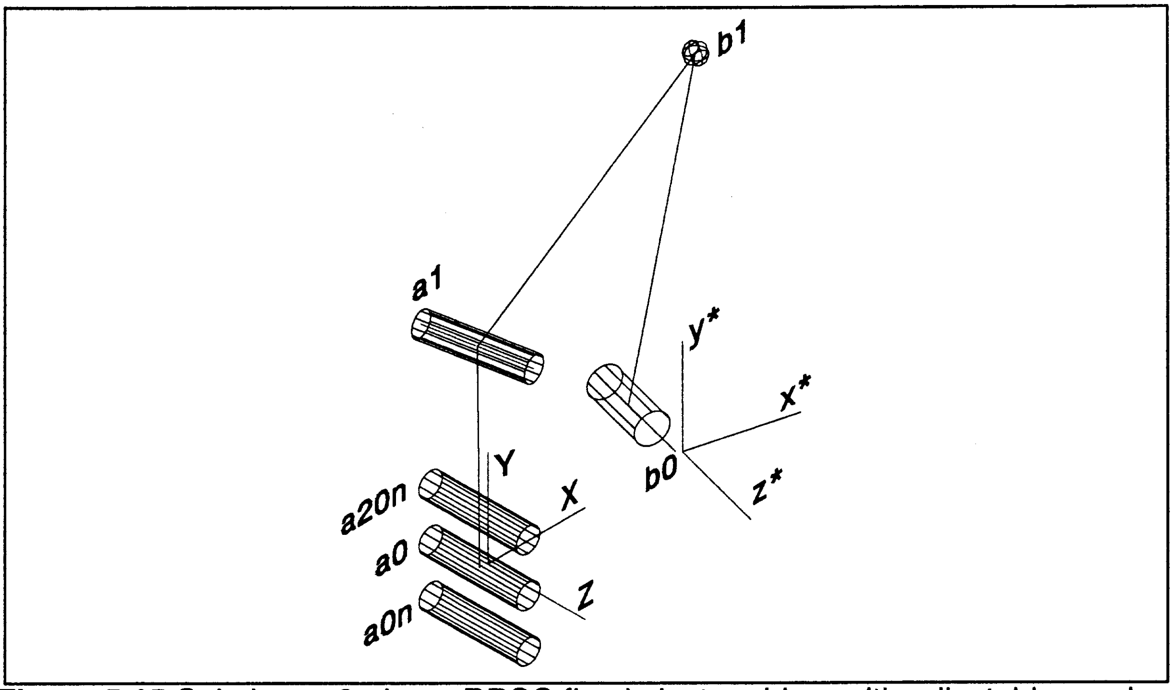

Figure 5.15 Solution to 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths ($\mathbf{b}_0$=$\mathbf{b}_{0n}$=$\mathbf{b}_{20n}$).

The initial translation magnitude for the C-S link that was used in the RRSC mechanism to calculate the rigid body positions in table 5.24 is S = 0.3440 units for phases 1, 2 and 3. It was determined by trial and error. This translation magnitude lies along $\mathbf{ub}_0$ in the negative z*-axis direction.

5.2.3 RRSC Mechanism for Finitely Separated Positions with Tolerances

In this example problem, rigid body point tolerances were incorporated to synthesize the R-R link of the RRSC mechanism in section 5.2.2. In section 5.2.2, the following design equations were used to synthesize the R-R link:

$$([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.115)$$

$$([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.116)$$

$$([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.117)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_2^2 = 0 \quad (5.118)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_2^2 = 0 \quad (5.119)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_2^2 = 0 \quad (5.120)$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n})^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n}) - R_3^2 = 0 \quad (5.121)$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n})^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n}) - R_3^2 = 0 \quad (5.122)$$

Equations 5.115 through 5.117 are used to calculate the R-R link parameters for phase 1 and equations 5.118 through 5.120 are used to calculate the R-R link parameters for phase 2. The remaining two equations are used to calculate the R-R link parameters for phase 3. If each set of equations corresponding to each phase is solved using a range of prescribed values for one of the unknowns, a range of R-R link parameters can be calculated. These solution ranges are equivalent to the Burmester curves used in planar four-bar mechanism synthesis. Burmester curves are the loci of circle points and center points that satisfy a particular set of rigid body positions.

Using the following initial guesses and prescribed variable ranges:

a_{0y}=0.1, $\mathbf{a}_1$=(0.1, 0.85), $\mathbf{a}_{0n}$=(0.1, -0.3), $\mathbf{a}_{20n}$=(0.1, 0.3), a_{0x} = -0.5, -0.45, ...0.5 and the following constant length equation included to the set of equations for phase three:

$$(\mathbf{a}_1 - \mathbf{a}_{20n})^T(\mathbf{a}_1 - \mathbf{a}_{20n}) - R_1^2 = 0 \quad (5.123)$$

figure 5.16 illustrates the circle and center point curves are calculated for each phase.

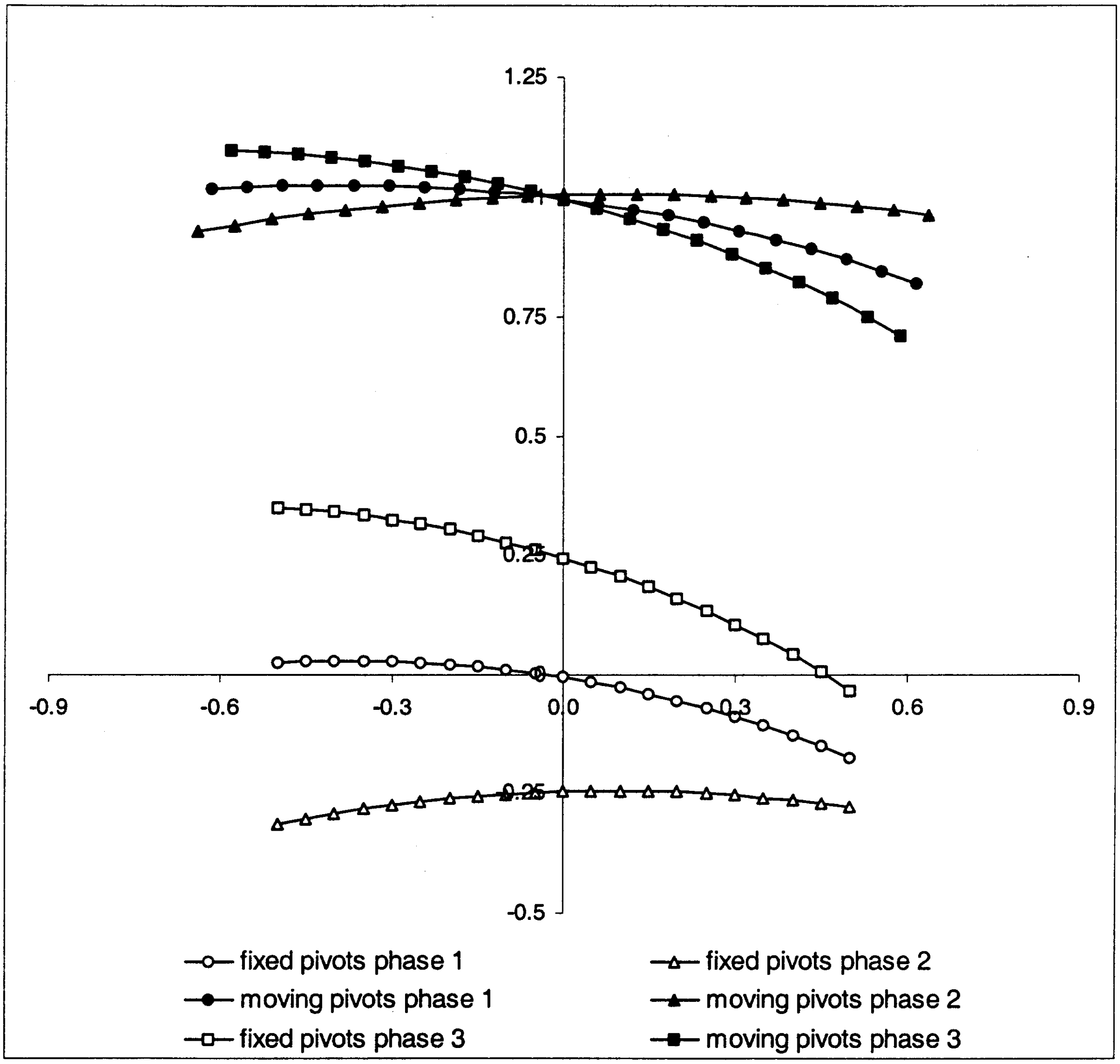

Figure 5.16 Graphical solution for R-R link for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths

In figure 5.16, portions of the circle and center point curves are given for the rigid body positions in phases 1, 2 and 3. The point of intersection of the circle point curves and the corresponding center points represent the R-R link parameters that are needed to achieve the rigid body positions in all three phases. In tolerance problems, this intersection point and corresponding center points become regions in which the R-R link parameters must remain within in order to achieve the precise rigid body positions while remaining within the limits of the rigid body positions with tolerances.

Table 5.25 Prescribed X-Y plane rigid body positions and tolerances for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths

	$\mathbf{p}_{X\text{-}Y}$	$\mathbf{q}_{X\text{-}Y}$	$\mathbf{r}_{X\text{-}Y}$
	Phase 1		
pos. 1	0.1959, 1.2647 $\pm\ \delta_y$	1.2089, 1.5993 $\pm\ \delta_y$	0.7024. 1.8320 $\pm\ \delta_y$
pos. 2	0.0126, 1.2555	1.0132, 1.6224	0.4984, 1.8386
pos. 3	-0.1668, 1.2169	0.8190, 1.6205	0.2952, 1.8171
pos. 4	-0.3375, 1.1502	0.6302, 1.5957	0.0971, 1.7689
	Phase 2		
pos. 5	0.2417, 1.2625 $\pm\ \delta_y$	1.2574, 1.5896 $\pm\ \delta_y$	0.7528, 1.8260 $\pm\ \delta_y$
pos. 6	-0.0322, 1.2527	0.9665, 1.6248	0.4506, 1.8383
pos. 7	-0.2544, 1.2031	0.7285, 1.6140	0.2033, 1.8067
pos. 8	-0.4645, 1.1178	0.5001, 1.5702	-0.0343, 1.7397
	Phase 3		
pos. 9	0.0576, 1.2582 $\pm\ \delta_y$	1.0605, 1.6186 $\pm\ \delta_y$	0.5470, 1.8381 $\pm\ \delta_y$
pos. 10	-0.0784, 1.2300	0.9119, 1.6227	0.3901, 1.8250
pos. 11	-0.2084, 1.1812	0.7660, 1.6123	0.2353, 1.7933

Note: In this problem δ_y = 0.1 units

It was shown in section 5.2.2 that the R-R link solution using the rigid body positions in table 5.25 (without tolerances) and equations 5.115 through 5.122 were the following:

$\mathbf{a}_0$=(-0.0426, 0.0028) $\mathbf{a}_1$=(-0.0527, 1.0027) $\mathbf{a}_{0n}$=(-0.0419, -0.2473)

$\mathbf{a}_{20n}$=(-0.0436, 0.2531)

When using the same equations but incorporating the $+\delta$ tolerance value in table 5.25, the R-R link parameters become

$\mathbf{a}_0$=(-0.0426, 0.0028) $\mathbf{a}_1$=(-0.0527, 1.1027) $\mathbf{a}_{0n}$=(-0.0419, -0.2473)

$\mathbf{a}_{20n}$=(-0.0436, 0.2531)

When using the same equations but incorporating the $-\delta$ tolerance value in table 5.25, the R-R link parameters become

$\mathbf{a}_0$=(-0.0426, 0.0028) $\mathbf{a}_1$=(-0.0527, 0.9027) $\mathbf{a}_{0n}$=(-0.0419, -0.2473)

$\mathbf{a}_{20n}$=(-0.0436, 0.2531)

All of these R-R link parameters were obtained using the R-R link initial guesses in section 5.2.2 and are illustrated in figure 5.17.

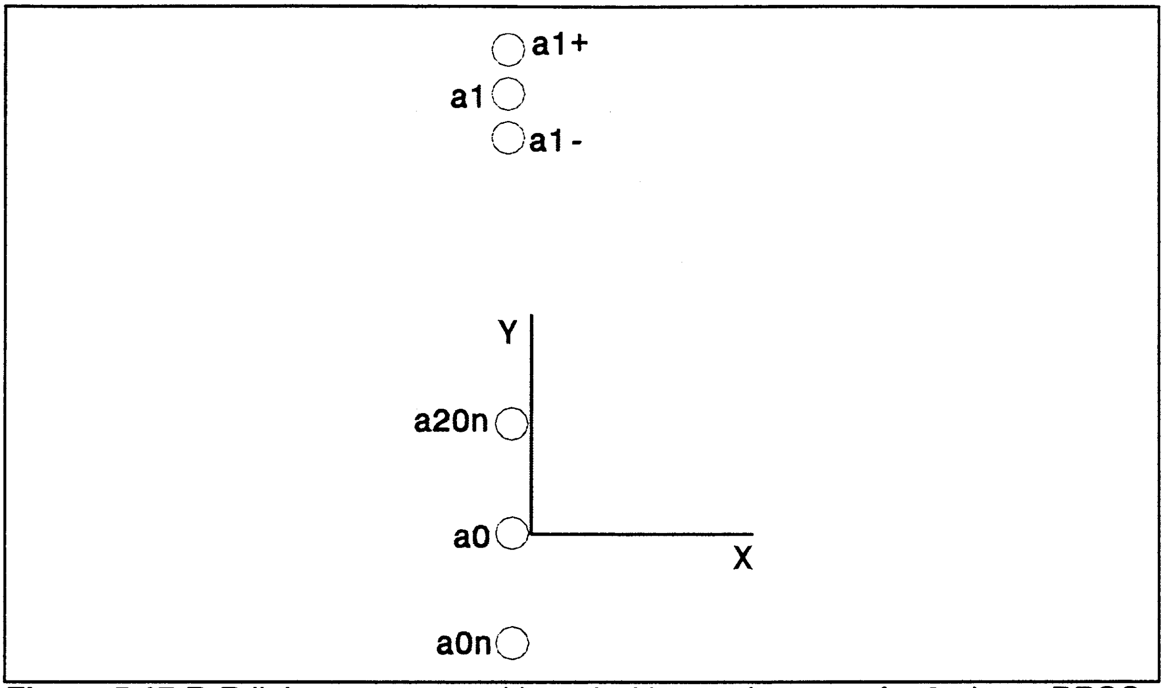

Figure 5.17 R-R link parameters with and without tolerances for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths

Although at least three R-R link solutions exist in figure 5.17, to calculate the regions necessary to produce additional solutions for the R-R link, a part of the $\mathbf{a}_0$, $\mathbf{a}_{0n}$, $\mathbf{a}_{20n}$, $\mathbf{a}_1$+ and $\mathbf{a}_1$- loci must be calculated. By specifying a_{0x} and using equations 5.115 through 5.121, the following R-R link parameters were calculated:

Table 5.26 Additional R-R link parameters for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths and tolerances

$\mathbf{a}_0$	$\mathbf{a}_1$	$\mathbf{a}_{0n}$	$\mathbf{a}_{20n}$	$\mathbf{a}_1$ (with $+\delta$)	$\mathbf{a}_1$ (with $-\delta$)
-0.0626*, 0.0019	-0.0782, 1.0015	-0.0619, - 0.2484	-0.0642, 0.2522	-0.0782, 1.1015	-0.0782, 0.9015
-0.0226*, 0.0036	-0.0272, 1.0037	-0.0219, -0.2463	-0.0230, 0.2537	-0.0271, 1.1037	-0.0271, 0.9037

* specified R-R link parameter

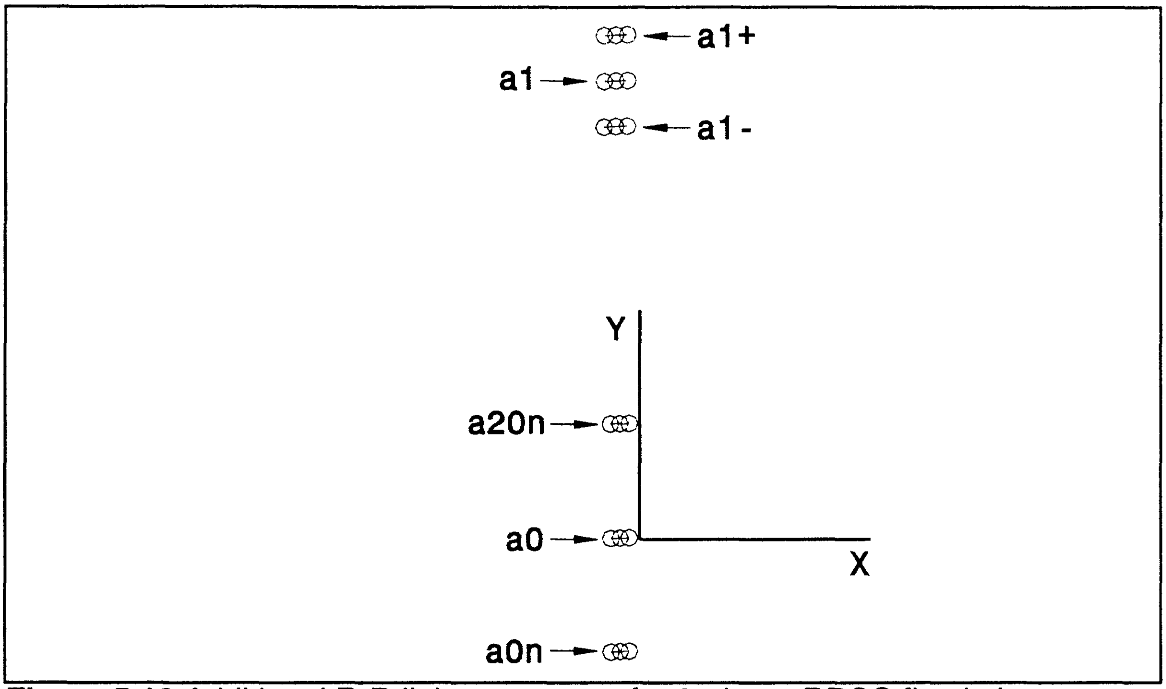

Figure 5.18 Additional R-R link parameters for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths and tolerances

By connecting the peripheral points for $\mathbf{a}_1$, a region is formed and curves are formed when the points for $\mathbf{a}_0$, $\mathbf{a}_{0n}$ and $\mathbf{a}_{20n}$ are connected. The calculated circle point regions and center point curves are illustrated in figure 5.19

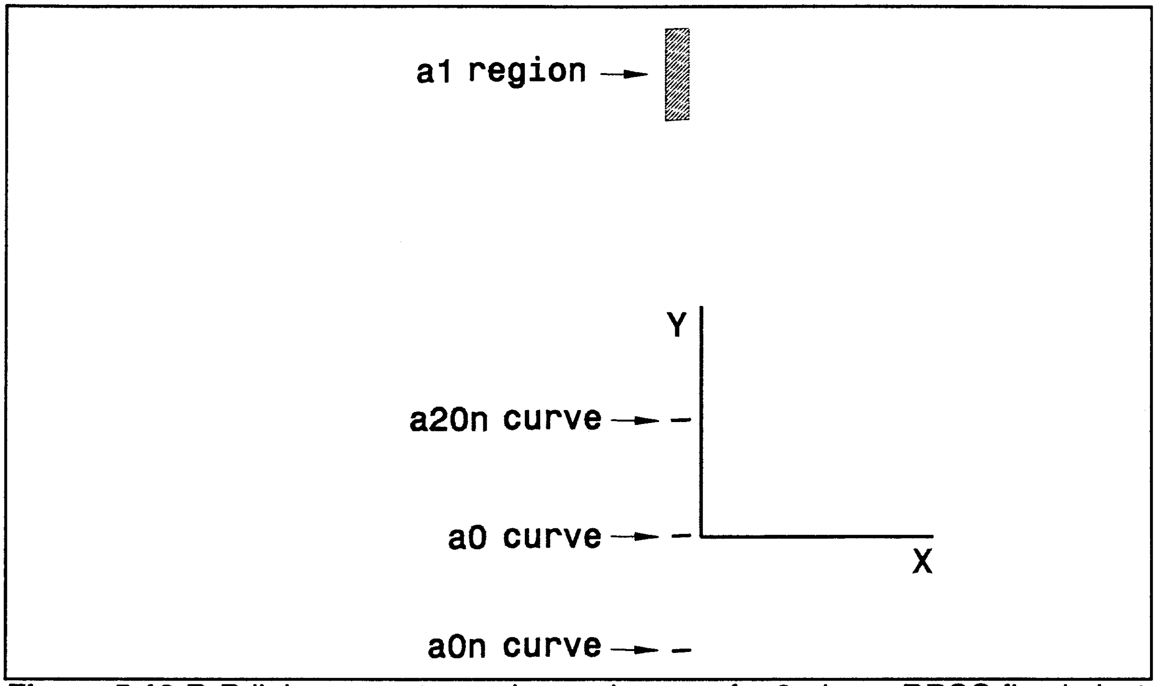

Figure 5.19 R-R link parameter region and curves for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths and tolerances

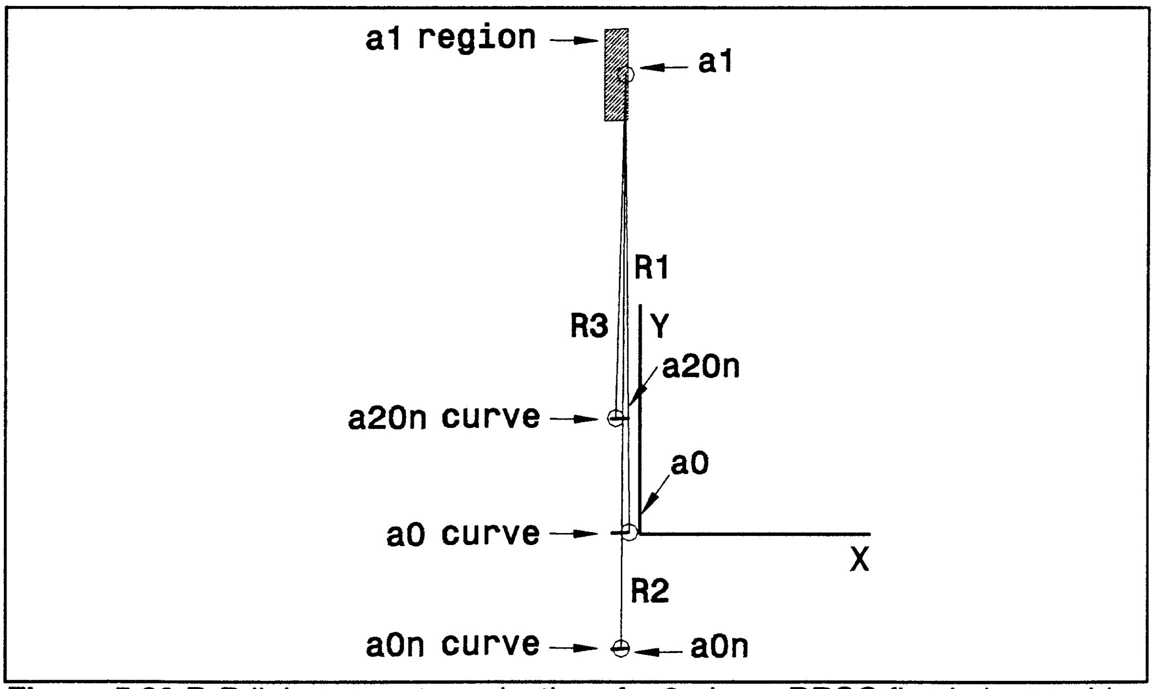

Figure 5.20 R-R link parameter selections for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths and tolerances

The following R-R link parameters were selected and are illustrated in figure 5.20:

$\mathbf{a}_0$=(-0.0226, 0.0036) $\mathbf{a}_1$=(-0.0323, 1.0035) $\mathbf{a}_{0n}$=(-0.0402, -0.2464)

$\mathbf{a}_{20n}$=(-0.0525, 0.2538)

The C-S link parameters are the same as those calculated in chapter 5.2.2.

$\mathbf{b}_0$=(-0.0280, 0.0096) $\mathbf{b}_1$=(0.2840, 1.4764) $\mathbf{b}_{0n}$=(-0.0310, 0.0099)

$\mathbf{b}_{20n}$=(-0.0243, 0.0092)

By using the final rigid body position points in each phase as the starting points for the synthesized adjustable RRSC mechanism and rotating the R-R link by certain angles, the remaining positions in table 5.25 were approximated. The R-R link rotation angles for the first four rigid body positions are 90°, 100°, 110° and 120°. The R-R link rotation angles for the next four rigid body positions are 88°, 100°, 110° and 120°. The R-R link rotation angles for the next four rigid body positions are 100°, 110° and 120°. These angles are measured with respect to the X-axis.

The rigid body positions in table 5.27 were obtained using the R-R and C-S link parameters, in addition to **d** =1, $\mathbf{ua}_1$=[sin 10°, 0, cos 10°] and $\mathbf{ua}_0$=[sin -15°, 0, cos -15°]. The value given for the joint axis $\mathbf{ua}_1$ occurs when the R-R link lies along the positive Y-axis, These joint axes were measured with respect to the X-Y-Z coordinate frame.

Table 5.27 Rgid body positions for synthesized mechanism for 3-phase RRSC fixed pivot problem with adjustable crank and fixed follower lengths and tolerances

	$p_{X\text{-}Y}$	$q_{X\text{-}Y}$	$r_{X\text{-}Y}$
		Phase 1	
pos. 1	0.1947, 1.2655	1.2080, 1.5991	0.7018, 1.8323
pos. 2	0.0116, 1.2558	1.0125, 1.6220	0.4977, 1.8385
pos. 3	-0.1675, 1.2169	0.8185, 1.6202	0.2947, 1.8169
pos. 4	-0.3375, 1.1502	0.6302, 1.5957	0.0971, 1.7689
		Phase 2	
pos. 5	0.2397, 1.2553	1.2553, 1.5891	0.7472, 1.8222
pos. 6	-0.0331, 1.2481	0.9641, 1.6240	0.4474, 1.8355
pos. 7	-0.2547, 1.2007	0.7274, 1.6135	0.2019, 1.8052
pos. 8	-0.4645, 1.1178	0.5001, 1.5702	-0.0343, 1.7397
		Phase 3	
pos. 9	0.0573, 1.2501	1.0580, 1.6166	0.5433, 1.8330
pos. 10	-0.0783, 1.2259	0.9108, 1.6215	0.3885, 1.8223
pos. 11	-0.2084, 1.1812	0.7660, 1.6123	0.2353, 1.7933

The average error magnitude between the specified rigid body positions without tolerances (in table 5.25) and the corresponding rigid body positions of the synthesized RRSC mechanism for positions 2, 3 and 4 is 0.0005 units. The maximum error magnitude in position 1 is 0.0008 units. It occurs at rigid body point p_y in position 1.

The average error magnitude between the specified rigid body positions without tolerances and the corresponding rigid body positions of the synthesized RRSC mechanism for positions 6, 7 and 8 is 0.0029 units. The maximum error magnitude in position 5 is 0.0072 units. It occurs at rigid body point p_y in position 5.

The average error magnitude between the specified rigid body positions without tolerances and the corresponding rigid body positions of the synthesized RRSC mechanism for positions 10 and 11 is 0.0036 units. The maximum displacement error magnitude in position 9 is 0.0081 units. It occurs at rigid body point p_y in position 9.

5.2.4 RSSR-SS Mechanism for Finite and Multiply Separated Positions

The R-S link ($\mathbf{a}_0$-$\mathbf{a}_1$) was the first link synthesized in this three-phase adjustable fixed pivot problem. The length of this R-S link was adjusted in the last phase.

The values for 11 prescribed rigid body positions projected on the X-Y plane are given in table 5.28. These positions are represented by points $\mathbf{p}_{X\text{-}Y}$, $\mathbf{q}_{X\text{-}Y}$ and $\mathbf{r}_{X\text{-}Y}$. To satisfy the design equations of the R-S link, points **p**, **q** and **r** are projected in the X-Y plane in each rigid body position. The points are also non-linear.

In addition to the 11 prescribed rigid body positions, several multiply separated positions are also given. They represent the velocity of the rigid body from position 1 to position 2.

Table 5.28 Prescribed X-Y plane rigid body positions and MSPs for 3-phase RSSR-SS fixed pivot problem with adjustable crank and follower lengths

Phase 1

	$\mathbf{p}_{X\text{-}Y}$	$\mathbf{q}_{X\text{-}Y}$	$\mathbf{r}_{X\text{-}Y}$
pos. 1	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
vel. 1-2	-0.4493, 11.0017	-10.9903, -0.4496	0.3153, -2.9203
pos. 2	0.6282, 2.8817	1.1455, 2.6134	0.1435, 2.3507
pos. 3	0.2506, 2.8328	0.7791, 2.5874	-0.2104, 2.2814
pos. 4	-0.1134, 2.7271	0.4264, 2.5073	-0.5467, 2.1554
		Phase 2	
pos. 5	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
pos. 6	0.6316, 2.8239	1.1511, 2.5627	0.1521, 2.2887
pos. 7	0.2659, 2.7419	0.7992, 2.4876	-0.1829, 2.1584
pos. 8	-0.0762, 2.5463	0.4694, 2.3612	-0.4903, 1.9737
		Phase 3	
pos. 9	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
pos. 10	0.6790, 2.8306	1.1969, 2.5658	0.1963, 2.2880
pos. 11	0.3588, 2.7381	0.8892, 2.5024	-0.0969, 2.1853

Note: In this problem, rigid body positions 1, 5 and 9 are shared

All of the rigid body point in this example problem were obtained using **d**=1.5 and $\mathbf{ub}_0$=[sin 10°, 0, cos 10°] with respect to the X-Y-Z frame.

The required R-S link variables here are $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{a}_{0n}$ and $\mathbf{a}_{20n}$. Variable $\mathbf{a}_1$ represents the moving pivot of the R-S link. Variables $\mathbf{a}_0$, $\mathbf{a}_{0n}$ and $\mathbf{a}_{20n}$ represent the fixed pivots in phases 1, 2 and 3 of the R-S link. Since each of these variables has two scalar components, there are a total of eight required unknowns.

$\mathbf{a}_0=(a_{0x}, a_{0y})$ $\mathbf{a}_1=(a_{1x}, a_{1y})$ $\mathbf{a}_{0n}=(a_{0nx}, a_{0ny})$ $\mathbf{a}_{20n}=(a_{20nx}, a_{20ny})$

The 11 prescribed rigid body positions and multiply separated positions result in eight design equations. The following set of design equations were used to calculate $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{a}_{0n}$ and $\mathbf{a}_{20n}$:

$$([V_{1,2}]\mathbf{a}_1)^T([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0) = 0 \quad (5.124)$$

$$([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.125)$$

$$([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.126)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \quad (5.127)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \quad (5.128)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_1 - \mathbf{a}_{0n}) - R_1^2 = 0 \quad (5.129)$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n})^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n}) - R_2^2 = 0 \quad (5.130)$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n})^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{a}_1 - \mathbf{a}_{20n}) - R_2^2 = 0 \quad (5.131)$$

The term R_1 represents the length of the R-S link in phases 1 and 2. The term R_2 represents the length of the R-S link in phase 3. Since this example problem involved fixed pivot adjustments with adjustable crank and follower lengths, the values of R_1 and R_2 were not identical in equations 5.124 through 5.131. The specified values for R_1 and R_2 are 2 and 1.75. Given the following initial guesses:

$\mathbf{a}_0$=(0.1, 0.1) $\mathbf{a}_1$=(0.1, 1.85) $\mathbf{a}_{0n}$=(0.3, 0.1) $\mathbf{a}_{20n}$=(0.3, 0.3)

the solution to equations 5.124 through 5.131 converged to the following using Newton's Method:

$\mathbf{a}_0$=(0.0025, -0.0142) $\mathbf{a}_1$=(-0.0004, 1.9851) $\mathbf{a}_{0n}$=(0.3913, 0.0243)

$\mathbf{a}_{20n}$=(0.3423, 0.2694)

The other R-S link ($\mathbf{b}_0$-$\mathbf{b}_1$) was the next link synthesized in this three-phase adjustable fixed pivot problem. The multiply separated positions of the rigid body are governed by the angular velocity and acceleration of the driving link. Since link $\mathbf{b}_0$-$\mathbf{b}_1$ can function as a driving link, it was also synthesized using MSPs. The length of this R-S link was adjusted in the last phase.

The values for 11 prescribed rigid body positions projected on the x*-y* plane are given in table 5.29. These positions are represented by points $\mathbf{p}_{x^*-y^*}$, $\mathbf{q}_{x^*-y^*}$ and $\mathbf{r}_{x^*-y^*}$. To satisfy the design equations of the R-S link, points **p**, **q** and **r** are projected in the x*-y* plane in each rigid body position. The points are also non-linear.

In addition to the 11 prescribed rigid body positions, several multiply separated positions are also given. They represent to the velocity of the rigid body from position 1 to position 2.

Table 5.29 Prescribed x*-y* plane rigid body positions and MSPs for 3-phase RSSR-SS fixed pivot problem with adjustable crank and follower lengths

	$\mathbf{p}_{x^*\text{-}y^*}$	$\mathbf{q}_{x^*\text{-}y^*}$	$\mathbf{r}_{x^*\text{-}y^*}$
Phase 1			
pos. 1	-0.4737, 2.8716	0.0281, 2.5824	-0.9754, 2.3608
vel. 1-2	-0.4526, 10.9405	-10.9904, -0.4455	-0.3530, 2.9597
pos. 2	-0.8565, 2.8817	-0.3457, 2.6134	-1.3348, 2.3507
pos. 3	-1.2346, 2.8328	-0.7150, 2.5874	-1.6864, 2.2814
pos. 4	-1.5988, 2.7271	-1.0706, 2.5073	-2.0204, 2.1554
Phase 2			
pos. 5	-0.4737, 2.8716	0.0281, 2.5824	-0.9754, 2.3608
pos. 6	-0.8543, 2.8239	-0.3405, 2.5627	-1.3267, 2.2887
pos. 7	-1.2206, 2.7149	-0.6954, 2.4876	-1.6597, 2.1584
pos. 8	-1.5644, 2.5463	1.0283, 2.3612	-1.9656, 1.9737
Phase 3			
pos. 9	-0.4737, 2.8716	0.0281, 2.5824	-0.9754, 2.3608
pos. 10	-0.8067, 2.8306	-0.2943, 2.5658	-1.2828, 2.2980
pos. 11	-1.1273, 2.7381	-0.6044, 2.5024	-1.5742, 2.1853

Note: In this problem, rigid body positions 1, 5 and 9 are shared

The required R-S variables here are $\mathbf{b}_0$, $\mathbf{b}_1$, $\mathbf{b}_{0n}$ and $\mathbf{b}_{20n}$. Variable $\mathbf{b}_1$ represents the moving pivot of the R-S link. Variables $\mathbf{b}_0$, $\mathbf{b}_{0n}$ and $\mathbf{b}_{20n}$ represent the fixed pivots in phases 1, 2 and 3 of the R-S link. Since each of these variables has two scalar components, there are a total of eight required unknowns.

$$\mathbf{b}_0=(b_{0x},\ b_{0y}) \qquad \mathbf{b}_1=(b_{1x},\ b_{1y}) \qquad \mathbf{b}_{0n}=(b_{0nx},\ b_{0ny})$$

The 11 prescribed rigid body positions and multiply separated positions result in eight design equations. The following set of design equations were used to calculate $\mathbf{b}_0$, $\mathbf{b}_1$, $\mathbf{b}_{0n}$ and $\mathbf{b}_{20n}$:

$$([V_{1,2}]\mathbf{b}_1)^T([D_{1,2}]\mathbf{b}_1 - \mathbf{b}_0) = 0 \quad (5.132)$$

$$([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.133)$$

$$([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.134)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \quad (5.135)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \quad (5.136)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_1 - \mathbf{b}_{0n}) - R_1^2 = 0 \quad (5.137)$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{b}_1 - \mathbf{b}_{20n})^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{b}_1 - \mathbf{b}_{20n}) - R_2^2 = 0 \quad (5.138)$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{b}_1 - \mathbf{b}_{20n})^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{b}_1 - \mathbf{b}_{20n}) - R_2^2 = 0 \quad (5.139)$$

The term R_1 represents the length of the R-S link in phases 1 and 2. The term R_2 represents the length of the R-S link in phase 3. Since this example problem involved fixed pivot adjustments with adjustable crank and follower lengths, the values of R_1 and R_2 were not identical in equations 5.132 through 5.139. The specified values for R_1 and R_2 are 2.5 and 2.25. Given the following initial guesses:

$\mathbf{b}_0$=(0.1, 0.1) $\mathbf{b}_1$=(0.5, 2.5) $\mathbf{b}_{0n}$=(0.1, -0.1) $\mathbf{b}_{20n}$=(0.5, 0.5)

the solution to equations 5.132 through 5.139 converged to the following using Newton's Method:

$\mathbf{b}_0$=(0.0010, 0.0153) $\mathbf{b}_1$=(0.5344, 2.4584) $\mathbf{b}_{0n}$=(0.2965, -0.0306)

$\mathbf{b}_{20n}$=(0.3202, 0.2183)

The S-S link ($\mathbf{c}_0$-$\mathbf{c}_1$) was the last link synthesized in this three-phase adjustable fixed pivot problem. Since this S-S link cannot function as a driving link, it was not synthesized using MSPs. The length of this S-S link was adjusted in the last phase.

The values for 11 prescribed rigid body positions in the X-Y-Z frame are given in table 5.30. These positions are represented by points **p**, **q**, **r** and **s**. To satisfy the design equations of the S-S link, these points do not all lie in the same plane in each rigid body position.

Table 5.30 Prescribed X-Y-Z frame rigid body positions for 3-phase RSSR-SS fixed pivot problem with adjustable crank and follower lengths and MSPs

	p	q	r	s
		Phase 1		
pos. 1	1.0109,2.8716,-0.0460	1.5163,2.5824,-0.0690	0.5054,2.3608,-0.0230	1.0109,2.4716,0.2540
pos. 2	0.6282,2.8817,-0.0095	1.1455,2.6134,-0.0190	0.1435,2.3507,-0.0052	0.6355,2.4797,0.2877
pos. 3	0.2506,2.8328,0.0350	0.7791,2.5874,0.0336	-0.2104,2.2814,0.0155	0.2678,2.4252,0.3240
pos. 4	-0.1134,2.7271,0.0858	0.4264,2.5073,0.0875	-0.5467,2.1554,0.0387	-0.0820,2.3112,0.3616
		Phase 2		
pos. 5	1.0109,2.8716,-0.0460	1.5163,2.5824,-0.0690	0.5054,2.3608,-0.0230	1.0109,2.4716,0.2540
pos. 6	0.6316,2.8239,0.0369	1.1511,2.5627,-0.0019	0.1521,2.2887,0.0125	0.6486,2.4019,0.3044
pos. 7	0.2659,2.7149,0.1271	0.7992,2.4876,0.0669	-0.1829,2.1584,0.0508	0.3065,2.2713,0.3544
pos. 8	-0.0762,2.5463,0.2237	0.4694,2.3612,0.1362	-0.4903,1.9737,0.0912	-0.0049,2.0847,0.4020
		Phase 3		
pos. 9	1.0109,2.8716,-0.0460	1.5163,2.5824,-0.0690	0.5054,2.3608,-0.0230	1.0109,2.4716,0.2540
pos. 10	0.6790,2.8306,0.0249	1.1969,2.5658,-0.0109	0.1963,2.2980,0.0075	0.6937,2.4118,0.2976
pos. 11	0.3588,2.7381,0.0990	0.8892,2.5024,0.0474	-0.0969,2.1853,0.0390	0.3933,2.3015,0.3402

Note: In this problem, rigid body positions 1, 5 and 9 are shared

The required S-S link variables are $\mathbf{c}_0$, $\mathbf{c}_1$, $\mathbf{c}_{0n}$ and $\mathbf{c}_{20n}$. Variable $\mathbf{c}_1$ represents the moving pivot of the S-S link. Variables $\mathbf{c}_0$, $\mathbf{c}_{0n}$ and $\mathbf{c}_{20n}$ represent the fixed pivots in phases 1, 2 and 3 of the S-S link. Since each of these variables has three scalar components, there are a total of 12 required unknowns.

$\mathbf{c}_0=(c_{0x}, c_{0y}, c_{0z})$ $\mathbf{c}_1=(c_{1x}, c_{1y}, c_{1z})$ $\mathbf{c}_{0n}=(c_{0nx}, c_{0ny}, c_{0nz})$ $\mathbf{c}_{20n}=(c_{20nx}, c_{20ny}, c_{20nz})$

The 11 prescribed rigid body positions and multiply separated positions result in eight design equations. Therefore, four of the 11 required unknowns were specified. Using AutoCAD 2000 software, the values of $\mathbf{c}_1$ and c_{0z} were specified to $\mathbf{c}_1$=[0, 2.5, -1] and c_{0z}=-1. The following set of design equations were used to calculate c_{0x}, c_{0y}, $\mathbf{c}_{0n}$, and $\mathbf{c}_{20n}$:

$$([D_{1,2}]\mathbf{c}_1 - \mathbf{c}_0)^T([D_{1,2}]\mathbf{c}_1 - \mathbf{c}_0) - R_1^2 = 0 \quad (5.140)$$

$$([D_{1,3}]\mathbf{c}_1 - \mathbf{c}_0)^T([D_{1,3}]\mathbf{c}_1 - \mathbf{c}_0) - R_1^2 = 0 \quad (5.141)$$

$$([D_{1,4}]\mathbf{c}_1 - \mathbf{c}_0)^T([D_{1,4}]\mathbf{c}_1 - \mathbf{c}_0) - R_1^2 = 0 \quad (5.142)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n}) - R_1^2 = 0 \quad (5.143)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n})^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n}) - R_1^2 = 0 \quad (5.144)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n})^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{c}_1 - \mathbf{c}_{0n}) - R_1^2 = 0 \quad (5.145)$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{c}_1 - \mathbf{c}_{20n})^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{c}_1 - \mathbf{c}_{20n}) - R_2^2 = 0 \quad (5.146)$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{c}_1 - \mathbf{c}_{20n})^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{c}_1 - \mathbf{c}_{20n}) - R_2^2 = 0 \quad (5.147)$$

The terms R_1 represents the length of the S-S link in phases 1 and 2. The term R_2 represents the length of the S-S link in phase 3. Since this example problem involved fixed pivot adjustments with adjustable crank and follower lengths, the values of R_1 and R_2 were not identical in equations 5.140 through 5.147. The specified values for R_1 and R_2 are 2.5 and 2.0. Given the following initial guesses:

$\mathbf{c}_{0n}$=(0.1, 0.1, -0.5) $\mathbf{c}_{20n}$=(0.1, 0.5, -0.5) c_{0x}=0 c_{0y}=0.1

the solution to equations 5.140 through 5.147 converges to the following using Newton's Method:

$\mathbf{c}_{0n}$=(0.0004, 0.0180, -0.7001) $\mathbf{c}_{20n}$=(0.0049, 0.5201, -0.7159)

c_{0x}=-0.0009 c_{0y}=-0.0003

By using the initial rigid body points in each phase as the starting points for the synthesized adjustable RSSR-SS mechanism and rotating the R-S link $\mathbf{a}_0$-$\mathbf{a}_1$ by certain angles, the remaining positions in table 5.31 can be approximated. The R-S link rotation angles for the first four rigid body positions are 90°, 100°, 110° and 120°. The R-S link rotation angles for the next four rigid body positions are 101.30°, 121.30°, 131.30° and 141.30°. The R-S link rotation angles for the last three rigid body positions are 101.30°, 121.30° and 131.30°. These angles are measured with respect to the X-axis.

Table 5.31 Rigid body positions for synthesized mechanism for 3-phase RSSR-SS fixed pivot problem with adjustable crank and follower lengths and MSPs

	p	q	r	s
Phase 1				
pos. 1	1.0109,2.8716,-0.0460	1.5163,2.5824,-0.0690	0.5054,2.3608,-0.0230	1.0109,2.4716,0.2540
pos. 2	0.6273,2.8818,-0.0093	1.1447,2.6137,-0.0190	0.1429,2.3505,-0.0051	0.6348,2.4797,0.2878
pos. 3	0.2489,2.8327,0.0354	0.7777,2.5877,0.0337	-0.2116,2.2808,0.0159	0.2666,2.4249,0.3242
pos. 4	-0.1156,2.7266,0.0863	0.4243,2.5074,0.0874	-0.5484,2.1544,0.0393	-0.0836,2.3107,0.3619
Phase 2				
pos. 5	1.0109,2.8716,-0.0460	1.5163,2.5824,-0.0690	0.5054,2.3608,-0.0230	1.0109,2.4716,0.2540
pos. 6	0.6307,2.8237,0.0373	1.1503,2.5627,-0.0022	0.1515,2.2883,0.0133	0.6483,2.4015,0.3046
pos. 7	0.2644,2.7141,0.1280	0.7977,2.4874,0.0662	-0.1841,2.1574,0.0524	0.3060,2.2705,0.3548
pos. 8	-0.0780,2.5449,0.2247	0.4675,2.3605,0.1348	-0.4918,1.9718,0.0937	-0.0053,2.0833,0.4025
Phase 3				
pos. 9	1.0109,2.8716,-0.0460	1.5163,2.5824,-0.0690	0.5054,2.3608,-0.0230	1.0109,2.4716,0.2540
pos. 10	0.6751, 2.8337, 0.0245	1.1939, 2.5708, -0.0120	0.1944, 2.2993, 0.0078	0.6917, 2.4150, 0.2972
pos. 11	0.3574, 2.7376, 0.0996	0.8880, 2.5022, 0.0467	-0.0980, 2.1844, 0.0404	0.3927, 2.3009, 0.3405

The average error magnitude between the specified rigid body positions (table 5.30) and the rigid body positions of the synthesized mechanism for positions 2, 3 and 4 is 0.0006 units. The maximum error magnitude between positions 2, 3 and 4 is 0.0022 units. It occurs at rigid body point p_x in position 4.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for position 6, 7 and 8 is 0.0007 units. The maximum error magnitude between positions 6, 7 and 8 is 0.0025 units. It occurs at rigid body point r_z in position 8.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for positions 10 and 11 is 0.0015 units. The maximum error magnitude between positions 10 and 11 is 0.0050 units. It occurs at rigid body point q_y in position 10.

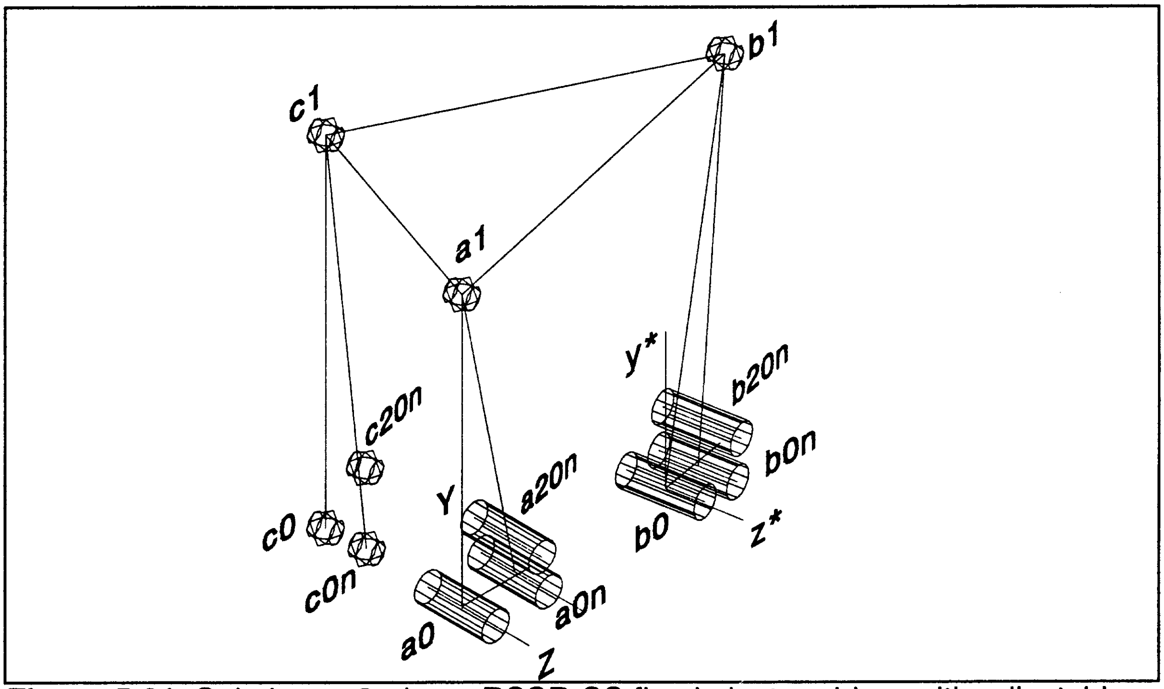

Figure 5.21 Solution to 3-phase RSSR-SS fixed pivot problem with adjustable crank and follower lengths and MSPs

The specified MSP parameters in table 5.28 are

$Vp_{1,2}$=(-0.4493, 11.0017) **$Vq_{1,2}$**=(-10.9903, -0.4496) **$Vr_{1,2}$**=(0.3153, -2.9203)

These parameters correspond to link ($\mathbf{a}_0$-$\mathbf{a}_1$) for rigid body positions 1 and 2. Since these positions are in phase 1, the R-S link parameters for this position are $\mathbf{a}_0$ and $\mathbf{a}_1$. The values calculated for these parameters are the following:

$$\mathbf{a}_0=(0.0025, -0.0142) \quad \mathbf{a}_1=(-0.0004, 1.9851)$$

The velocity of $\mathbf{a}_1$ was calculated using the **a**' terms in equation 3.3 (where $[V_{12}]=[\mathbf{Vp}_{1,2}|\mathbf{Vq}_{1,2}|\mathbf{Vr}_{1,2}]$ with a third row of zeros). The value calculated for the velocity $\mathbf{a}_1$' is

$$\mathbf{a}_1'=(-21.8169, -0.8888)$$

The specified MSP parameters in table 5.29 are

$$\mathbf{Vp}_{1,2}=(-0.4526, 10.9405) \quad \mathbf{Vq}_{1,2}=(-10.9404, -0.4455) \quad \mathbf{Vr}_{1,2}=(-0.3530, -2.9597)$$

These parameters correspond to link ($\mathbf{b}_0$-$\mathbf{b}_1$) for rigid body positions 1 and 2. Since these positions are in phase 1, the R-S link parameters for this position are $\mathbf{b}_0$ and $\mathbf{b}_1$. The values calculated for these parameters are the following:

$$\mathbf{b}_0=(-0.0010, 0.0153) \quad \mathbf{b}_1=(0.5344, 2.4584)$$

The velocity of $\mathbf{b}_1$ was calculated using the **b**' terms in equation 3.3 (where $[V_{12}]=[\mathbf{Vp}_{1,2}|\mathbf{Vq}_{1,2}|\mathbf{Vr}_{1,2}]$ with a third row of zeros). The value calculated for the velocity $\mathbf{b}_1$' is

$$\mathbf{b}_1'=(-27.2608, 4.7514)$$

The angular velocities of both R-S links were calculated using the **a**' term equation in equation 3.3. Since both links rotate in planes, only the ω_z term was used in equation 3.5 (therefore $\omega_x=\omega_y=0$). The fixed pivot velocity term was also eliminated in equation 3.4 since the fixed pivots are "fixed."

When the rigid body position parameters and the moving pivot position and velocity parameters for link $\mathbf{a}_0$-$\mathbf{a}_1$ were incorporated, the angular velocity values calculated were ω_z=11.0014 rad/sec for the first row in equation 5.56 and ω_z=10.8655 rad/sec for the second row. When the rigid body position parameters and the moving pivot position and velocity parameters for link $\mathbf{b}_0$-$\mathbf{b}_1$ were incorporated, the angular velocity values calculated were ω_z=11.0020 rad/sec for the first row in equation 5.56 and ω_z=11.0267 rad/sec for the second row. Since the calculated position and velocity parameters of the moving pivots of the R-S links were truncated (to four significant figures), the angular velocity values for both links are not exact matches.

5.2.5 RSSR-SC Mechanism for Finite and Multiply Separated Positions

The R-S link $\mathbf{a}_0$-$\mathbf{a}_1$ was the first link synthesized in this three-phase adjustable moving pivot problem. The length of this R-S link was adjusted in the last phase.

The values for 11 prescribed rigid body positions projected on the X-Y plane are given in table 5.32. These positions are represented by points $\mathbf{p}_{X\text{-}Y}$, $\mathbf{q}_{X\text{-}Y}$ and $\mathbf{r}_{X\text{-}Y}$. To satisfy the design equations of the R-S link, points **p**, **q** and **r** are projected in the X-Y plane in each rigid body position. The points are also non-linear.

In addition to the 11 prescribed rigid body positions, several multiply separated positions are also given for the first rigid body position. They represent the velocity and acceleration of the rigid body from position 1 to position 2.

Table 5.32 Prescribed X-Y plane rigid body positions and MSPs for 3-phase RSSR-SC moving pivot problem with adjustable crank and follower lengths

	$\mathbf{p}_{X\text{-}Y}$	$\mathbf{q}_{X\text{-}Y}$	$\mathbf{r}_{X\text{-}Y}$
Phase 1			
pos. 1	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
vel. 1-2	-0.1802, 9.0032	-8.9977, -0.1812	0.0633, -1.2057
accel. 1-2	-81.0692, 0.3775	-0.3690, -81.0200	10.8656, 0.3015
pos. 2	0.8195, 2.8841	1.3309, 2.6051	0.3241, 2.3633
pos. 3	0.6282, 2.8817	1.1455, 2.6134	0.1435, 2.3507
pos. 4	0.4382, 2.8646	0.9612, 2.6074	-0.0350, 2.3234
Phase 2			
pos. 5	0.8178, 2.9076	1.3302, 2.6299	0.3239, 2.3861
pos. 6	0.6239, 2.9273	1.1424, 2.6617	0.1414, 2.3957
pos. 7	0.4304, 2.9310	0.9541, 2.6782	-0.0409, 2.3898
pos. 8	0.2382, 2.9191	0.7664, 2.6795	-0.2217, 2.6385
Phase 3			
pos. 9	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
pos. 10	0.8385, 2.8830	1.3494, 2.6030	0.3421, 2.3632
pos. 11	0.6661, 2.8814	1.1823, 2.6109	0.1793, 2.3523

Note: In this problem, rigid body positions 1 and 9 are shared

All of the rigid body points in this example problem were obtained using **d1**=1.5, **d2**=[0, 0, -1], $\mathbf{ub}_0$=[sin 10°, 0, cos 10°] and $\mathbf{uc}_0$=[0, 0, 1] with respect to the X-Y-Z frame.

The required R-S link variables here are $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{a}_{1n}$ and $\mathbf{a}_{2n}$. Variable $\mathbf{a}_0$ represents the fixed pivot of the R-S link. Variables $\mathbf{a}_1$ $\mathbf{a}_{1n}$ and $\mathbf{a}_{2n}$ represent the moving pivots in phases 1, 2 and 3 of the R-S link. Since each of these variables has two scalar components, there are a total of eight required unknowns.

$\mathbf{a}_0=(a_{0x}, a_{0y},)$ $\mathbf{a}_1=(a_{1x}, a_{1y})$ $\mathbf{a}_{1n}=(a_{1nx}, a_{1ny})$ $\mathbf{a}_{2n}=(a_{2nx}, a_{2ny})$

The 11 prescribed rigid body positions and multiply separated positions result in eight design equations. The following set of design equations were used to calculate $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{a}_{1n}$ and $\mathbf{a}_{2n}$:

$$([A_{1,2}]\mathbf{a}_1)^T([D_{1,2}]\mathbf{a}_1 - \mathbf{a}_0) + ([V_{1,2}]\mathbf{a}_1)^T([V_{1,2}]\mathbf{a}_1) = 0 \quad (5.148)$$

$$([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,3}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.149)$$

$$([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{1,4}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \quad (5.150)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \quad (5.151)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \quad (5.152)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0)^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \quad (5.153)$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{a}_{2n} - \mathbf{a}_0)^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{a}_{2n} - \mathbf{a}_0) - R_2^2 = 0 \quad (5.154)$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{a}_{2n} - \mathbf{a}_0)^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{a}_{2n} - \mathbf{a}_0) - R_2^2 = 0 \quad (5.155)$$

The term R_1 represents the length of the R-S link in phases 1 and 2. The term R_2 represents the length of the R-S link in phase 3. Since this example problem involved moving pivot adjustments with adjustable crank and follower lengths, the values of R_1 and R_2 were not identical in equations 5.148 through 5.155. The specified values for R_1 and R_2 are 2 and 1.75. Given the following initial guesses:

$\mathbf{a}_0$=(0.1, 0.1) $\mathbf{a}_1$=(0.1, 1.85) $\mathbf{a}_{1n}$=(0.1, 1.85) $\mathbf{a}_{2n}$=(0.5, 1.85)

the solution to equations 5.148 through 5.155 converged to the following using Newton's Method:

$\mathbf{a}_0$=(-0.0006, -0.0000) $\mathbf{a}_1$=(0.0000, 2.0001) $\mathbf{a}_{1n}$=(0.3456, 1.9697)

$\mathbf{a}_{2n}$=(-0.0046, 1.7496)

The other R-S link ($\mathbf{b}_0$-$\mathbf{b}_1$) was the next link synthesized in this three-phase adjustable moving pivot problem. The multiply separated positions of the rigid body are governed by the angular velocity and acceleration of the driving link. Since link $\mathbf{b}_0$-$\mathbf{b}_1$ can function as a driving link, it was also be synthesized using MSPs. The length of this R-S link was adjusted in the last phase.

The values for 11 prescribed rigid body positions projected on the x*-y* plane are given in table 5.33. These positions are represented by points $\mathbf{p}_{x^*-y^*}$, $\mathbf{q}_{x^*-y^*}$ and $\mathbf{r}_{x^*-y^*}$. To satisfy the design equations of the R-S link, points **p**, **q** and **r** are projected in the x*-y* plane in each rigid body position. The points are also non-linear.

In addition to the 11 prescribed rigid body positions, several multiply separated positions are also given for the first rigid body position. They represent the velocity and acceleration of the rigid body from position 1 to position 2.

Table 5.33 Prescribed x*-y* plane rigid body positions and MSPs for 3-phase RSSR-SC moving pivot problem with adjustable crank and follower lengths

	$\mathbf{p}_{x^*\text{-}y^*}$	$\mathbf{q}_{x^*\text{-}y^*}$	$\mathbf{r}_{x^*\text{-}y^*}$
Phase 1			
pos. 1	-0.4737, 2.8716	0.0281, 2.5824	-0.9754, 2.3608
vel. 1-2	-0.1816, 8.9794	-8.9977, -0.1789	-0.2049, -1.2187
accel. 1-2	-80.8448, -0.1380	0.1107, -81.0096	10.9341, -2.0475
pos. 2	-0.6651, 2.8841	-0.1587, 2.6051	-1.1555, 2.3633
pos. 3	-0.8565, 2.8817	-0.3457, 2.6134	-1.3348, 2.3507
pos. 4	-1.0467, 2.8646	-0.5315, 2.6074	-1.5122, 2.3234
Phase 2			
pos. 5	-0.6636, 2.9076	-0.1571, 2.6299	-1.1538, 2.3861
pos. 6	-0.8555, 2.9273	-0.3447, 2.6614	-1.3338, 2.3957
pos. 7	-1.0485, 2.9310	-0.5334, 2.6782	-1.5140, 2.3898
pos. 8	-1.2411, 2.9191	-0.7220, 2.6795	-1.6934, 2.3685
Phase 3			
pos. 9	-0.4737, 2.8716	0.0281, 2.5824	0.0281, 2.5824
pos. 10	-0.6460, 2.8830	-0.1402, 2.6030	-1.1376, 2.3632
pos. 11	-0.8185, 2.8814	-0.3086, 2.6109	-1.2993, 2.3523

Note: In this problem, rigid body positions 1 and 9 are shared

The required R-S link variables here are $\mathbf{b}_0$, $\mathbf{b}_1$, $\mathbf{b}_{1n}$ and $\mathbf{b}_{2n}$. Variable $\mathbf{b}_0$ represents the fixed pivot of the R-S link. Variables $\mathbf{b}_1$ $\mathbf{b}_{1n}$ and $\mathbf{b}_{2n}$ represent the moving pivots in phases 1, 2 and 3 of the R-S link. Since each of these variables has two scalar components, there are a total of eight required unknowns.

$$\mathbf{b}_0=(b_{0x}, b_{0y}) \qquad \mathbf{b}_1=(b_{1x}, b_{1y}) \qquad \mathbf{b}_{1n}=(b_{1nx}, b_{1ny}) \qquad \mathbf{b}_{2n}=(b_{2nx}, b_{2ny})$$

The 11 prescribed rigid body positions and multiply separated positions result in eight design equations. The following set of design equations were used to calculate $\mathbf{b}_0$, $\mathbf{b}_1$, $\mathbf{b}_{1n}$ and $\mathbf{b}_{2n}$:

$$([A][D_{1,2}]\mathbf{b}_1)^T([D_{12}]\mathbf{b}_1 - \mathbf{b}_0) + ([V][D_{1,2}]\mathbf{b}_1)^T([V][D_{1,2}]\mathbf{b}_1) = 0 \quad (5.156)$$

$$([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,3}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.157)$$

$$([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0)^T([D_{1,4}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \quad (5.158)$$

$$(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \quad (5.159)$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \quad (5.160)$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0)^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \quad (5.161)$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{b}_{2n} - \mathbf{b}_0)^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{b}_{2n} - \mathbf{b}_0) - R_2^2 = 0 \quad (5.162)$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{b}_{2n} - \mathbf{b}_0)^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{b}_{2n} - \mathbf{b}_0) - R_2^2 = 0 \quad (5.163)$$

The term R_1 represents the length of the R-S link in phases 1 and 2. The term R_2 represents the length of the R-S link in phase 3. Since this example problem involved moving pivot adjustments with adjustable crank and follower lengths, the values of R_1 and R_2 were not identical in equations 5.156 through 5.163. The specified values for R_1 and R_2 are 2.5 and 2.15. Given the following initial guesses:

$\mathbf{b}_0$=(0.1, 0.1) $\mathbf{b}_1$=(0.5, 2.5) $\mathbf{b}_{1n}$=(1, 2.5) $\mathbf{b}_{2n}$=(0.5, 2)

the solution to equations 5.156 through 5.163 converged to the following using Newton's Method:

$\mathbf{b}_0$=(0.0716, 0.0077) $\mathbf{b}_1$=(0.4286, 2.4568) $\mathbf{b}_{1n}$=(0.8494, 2.3317)

$\mathbf{b}_{2n}$=(0.3592, 2.1140)

The C-S link ($\mathbf{c}_0$-$\mathbf{c}_1$) was the last link synthesized in this three-phase adjustable moving pivot problem. The multiply separated positions of the rigid body are governed by the angular velocity and acceleration of the driving link. Since this C-S link can function as a driving link, it was also synthesized using MSPs. The length of this C-S link was adjusted in the last phase.

The values for 11 prescribed rigid body positions projected on the x**-y** plane are given in table 5.34. These positions are represented by points $\mathbf{p}_{x^{**}\text{-}y^{**}}$, $\mathbf{q}_{x^{**}\text{-}y^{**}}$ and $\mathbf{r}_{x^{**}\text{-}y^{**}}$. To satisfy the design equations of the C-S link, points **p**, **q** and **r** are projected in the x**-y** plane in each rigid body position. The points are also non-linear.

In addition to the 8 prescribed rigid positions, several multiply separated positions are also given. They represent the velocity of the rigid body from positions 1 to 2 and from positions 5 to 6.

Table 5.34 Prescribed x**-y** plane rigid body positions and MSPs for 3-phase RSSR-SC moving pivot problem with adjustable crank and follower lengths

	$\mathbf{p}_{x^{**}\text{-}y^{**}}$	$\mathbf{q}_{x^{**}\text{-}y^{**}}$	$\mathbf{r}_{x^{**}\text{-}y^{**}}$
	Phase 1		
pos. 1	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
vel. 1	-0.1102, 5.5020	-5.4986, -0.1107	0.0387
pos. 2	0.8195, 2.8841	1.3309, 2.6051	0.3241, 2.3633
pos. 3	0.6282, 2.8817	1.1455, 2.6134	0.1435, 2.3507
pos. 4	0.4382, 2.8645	0.9612, 2.6074	-0.0350, 2.3233
	Phase 2		
pos. 5	0.8178, 2.9076	1.3302, 2.6299	0.3239, 2.3861
vel. 5	-0.1240, 5.5002	-5.4891, -0.1205	-0.0387, -0.7165
pos. 6	0.6239, 2.9273	1.1424, 2.6617	0.1414, 2.3957
pos. 7	0.4304, 2.9310	0.9541, 2.6782	-0.0409, 2.3898
pos. 8	0.2382, 2.9191	0.7664, 2.6795	-0.2217, 2.3685
	Phase 3		
pos. 9	1.0109, 2.8716	1.5163, 2.5824	0.5054, 2.3608
pos. 10	0.8385, 2.8830	1.3494, 2.6030	0.3421, 2.3632
pos. 11	0.6661, 2.8814	1.1823, 2.6109	0.1793, 2.3523

Note: In this problem, rigid body positions 1 and 9 are shared

These points were taken using **d1**=1.5, **d2**=[0, 0, -1], $\mathbf{ua}_0$=[sin 10°, 0, cos 10°] and $\mathbf{uc}_0$=[0, 0, 1] with respect to the X-Y-Z frame.

The required C-S link variables here are $\mathbf{c}_0$, $\mathbf{c}_1$, $\mathbf{c}_{1n}$ and $\mathbf{c}_{2n}$. Variable $\mathbf{c}_0$ represents the fixed pivot of the C-S link. Variables $\mathbf{c}_1$ $\mathbf{c}_{1n}$ and $\mathbf{c}_{2n}$ represent the moving pivots in phases 1, 2 and 3 of the C-S link. Since each of these variables has two scalar components, there are a total of eight required unknowns.

$$\mathbf{c}_0=(c_{0x}, c_{0y}) \qquad \mathbf{c}_1=(c_{1x}, c_{1y}) \qquad \mathbf{c}_{1n}=(c_{1nx}, c_{1ny}) \qquad \mathbf{c}_{2n}=(c_{2nx}, c_{2ny})$$

The 11 prescribed rigid body positions and multiply separated positions result in eight design equations. The following set of design equations were used to calculate $\mathbf{c}_0$, $\mathbf{c}_1$, $\mathbf{c}_{1n}$ and $\mathbf{c}_{2n}$:

$$([V][D_{1,2}]\mathbf{c}_1)^T([D_{1,2}]\mathbf{c}_1 - \mathbf{c}_0) = 0 \tag{5.164}$$

$$([D_{1,3}]\mathbf{c}_1 - \mathbf{c}_0)^T([D_{1,3}]\mathbf{c}_1 - \mathbf{c}_0) - R_1^2 = 0 \tag{5.165}$$

$$([D_{1,4}]\mathbf{c}_1 - \mathbf{c}_0)^T([D_{1,4}]\mathbf{c}_1 - \mathbf{c}_0) - R_1^2 = 0 \tag{5.166}$$

$$(\{[V][D_{5,6}][D_{1,5}]\}\mathbf{c}_{1n})^T(\{[D_{5,6}][D_{1,5}]\}\mathbf{c}_{1n} - \mathbf{c}_0) = 0 \tag{5.167}$$

$$(\{[D_{5,7}][D_{1,5}]\}\mathbf{c}_{1n} - \mathbf{c}_0)^T(\{[D_{5,7}][D_{1,5}]\}\mathbf{c}_{1n} - \mathbf{c}_0) - R_1^2 = 0 \tag{5.168}$$

$$(\{[D_{5,8}][D_{1,5}]\}\mathbf{c}_{1n} - \mathbf{c}_0)^T(\{[D_{5,8}][D_{1,5}]\}\mathbf{c}_{1n} - \mathbf{c}_0) - R_1^2 = 0 \tag{5.169}$$

$$(\{[D_{9,10}][D_{1,9}]\}\mathbf{c}_{2n} - \mathbf{c}_0)^T(\{[D_{9,10}][D_{1,9}]\}\mathbf{c}_{2n} - \mathbf{c}_0) - R_2^2 = 0 \tag{5.170}$$

$$(\{[D_{9,11}][D_{1,9}]\}\mathbf{c}_{2n} - \mathbf{c}_0)^T(\{[D_{9,11}][D_{1,9}]\}\mathbf{c}_{2n} - \mathbf{c}_0) - R_2^2 = 0 \tag{5.171}$$

The term R_1 represents the length of the C-S link in phases 1 and 2. The term R_2 represents the length of the C-S link in phase 3. Since this example problem involved moving pivot adjustments with adjustable crank and follower lengths, the values of R_1 and R_2 were not identical in equations 5.164 through 5.171. The specified values for R_1 and R_2 are 2.5 and 2.25. Given the following initial guesses:

$\mathbf{c}_0$=(0.1, 0.1) $\mathbf{c}_1$=(0.1, 2.4) $\mathbf{c}_{1n}$=(-0.5, 2.5) $\mathbf{c}_{2n}$=(0.1, 2.5)

the solution to equations 5.164 through 5.171converged to the following using Newton's Method:

$\mathbf{c}_0$=(0.0000, -0.0000) $\mathbf{c}_1$=(-0.0850, 2.4931) $\mathbf{c}_{1n}$=(0.2762, 2.4720)

$\mathbf{c}_{2n}$=(-0.0539, 2.2473)

By using the initial rigid body points in each phase as the starting points for the synthesized adjustable RSSR-SC mechanism and rotating the R-S link $\mathbf{a}_0$-$\mathbf{a}_1$ by certain angles, the remaining positions in table 5.37 can be approximated. The R-S link rotation angles for the first four rigid body positions are 90°, 95°, 100° and 105°. The R-S link rotation angles for the next four rigid body positions are 85°, 90°, 95° and 100°. The R-S link rotation angles for the last three rigid body positions are 90°, 95° and 100°. These angles are measured with respect to the X-axis.

Table 5.35 Prescribed X-Y-Z frame rigid body positions for 3-phase RSSR-SC moving pivot problem with adjustable crank and follower lengths and MSPs

	p	q	r
Phase 1			
pos. 1	1.0109, 2.8716, -0.0460	1.5163, 2.5824, -0.0690	0.5054, 2.3608, -0.0230
pos. 2	0.8195, 2.8841, -0.0288	1.3309, 2.6051, -0.0445	0.3241, 2.3633, -0.0145
pos. 3	0.6282, 2.8817, -0.0095	1.1455, 2.6134, -0.0190	0.1435, 2.3507, -0.0052
pos. 4	0.4382, 2.8646, 0.0119	0.9612, 2.6074, 0.0070	-0.0350, 2.3234, 0.0048
Phase 2			
pos. 5	0.8178, 2.9076, -0.0663	1.3302, 2.6299, -0.0674	0.3239, 2.3861, -0.0345
pos. 6	0.6239, 2.9273, -0.0832	1.1424, 2.6617, -0.0639	0.1414, 2.3957, -0.0446
pos. 7	0.4304, 2.9310, -0.0971	0.9541, 2.6782, -0.0587	-0.0409, 2.3898, -0.0532
pos. 8	0.2382, 2.9191, -0.1081	0.7664, 2.6795, -0.0520	-0.2217, 2.6385, -0.0605
Phase 3			
pos. 9	1.0109, 2.8716, -0.0460	1.5163, 2.5824, -0.0690	0.5054, 2.3608, -0.0230
pos. 10	0.8385, 2.8830, -0.0300	1.3494, 2.6030, -0.0464	0.3421, 2.3632, -0.0149
pos. 11	0.6661, 2.8814, -0.0109	1.1823, 2.6109, -0.0219	0.1793, 2.3523, -0.0051

Table 5.36 Rigid body positions for synthesized mechanism for 3-phase RSSR-SC moving pivot problem with adjustable crank and follower lengths and MSPs

	p	q	r
	Phase 1		
pos. 1	1.0109, 2.8716, -0.0460	1.5163, 2.5824, -0.0690	0.5054, 2.3608, -0.0230
pos. 2	0.8201, 2.8841, -0.0281	1.3315, 2.6050, -0.0441	0.3246, 2.3634, -0.0143
pos. 3	0.6293, 2.8818, -0.0114	1.1465, 2.6132, -0.0198	0.1444, 2.3510, -0.0060
pos. 4	0.4397, 2.8648, 0.0074	0.9626, 2.6073, 0.0054	-0.0338, 2.3238, 0.0030
	Phase 2		
pos. 5	0.8178, 2.9076, -0.0663	1.3302, 2.6299, -0.0674	0.3239, 2.3861, -0.0345
pos. 6	0.6252, 2.9285, -0.0604	1.1434, 2.6618, -0.0519	0.1421, 2.3967, -0.0339
pos. 7	0.4320, 2.9342, -0.0516	0.9556, 2.6788, -0.0347	-0.0399, 2.3921, -0.0321
pos. 8	0.2391, 2.9248, -0.0403	0.7680, 2.6810, -0.0161	-0.2209, 2.3724, -0.0292
	Phase 3		
pos. 9	1.0109, 2.8716, -0.0460	1.5163, 2.5824, -0.0690	0.5054, 2.3608, -0.0230
pos. 10	0.8390, 2.8828, -0.0303	1.3498, 2.6028, -0.0466	0.3425, 2.3630, -0.0151
pos. 11	0.6670, 2.8810, -0.0134	1.1831, 2.6106, -0.0234	0.1801, 2.3520, -0.0065

The average error magnitude between the specified rigid body positions (table 5.35) and the rigid body positions of the synthesized mechanism for positions 2, 3 and 4 is 0.0009 units. The maximum error magnitude between positions 2, 3 and 4 is 0.0045 units. It occurs at rigid body point p_z in position 4.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for position 6, 7 and 8 is 0.0106 units. The maximum error magnitude between positions 6, 7 and 8 is 0.0678 units. It occurs at rigid body point p_z in position 8.

The average error magnitude between the specified rigid body positions and the rigid body positions of the synthesized mechanism for position 10 and 11 is

0.0006 units. The maximum error magnitude between positions 10 and 11 is 0.0258 units. It occurs at rigid body point p_z in position 11.

The three initial translation magnitudes for the C-S link that were used in the RSSR-SC mechanism to calculate the rigid body positions in table 5.36 are S=0 units for phases 1, 2 and 3. They were determined by trial and error.

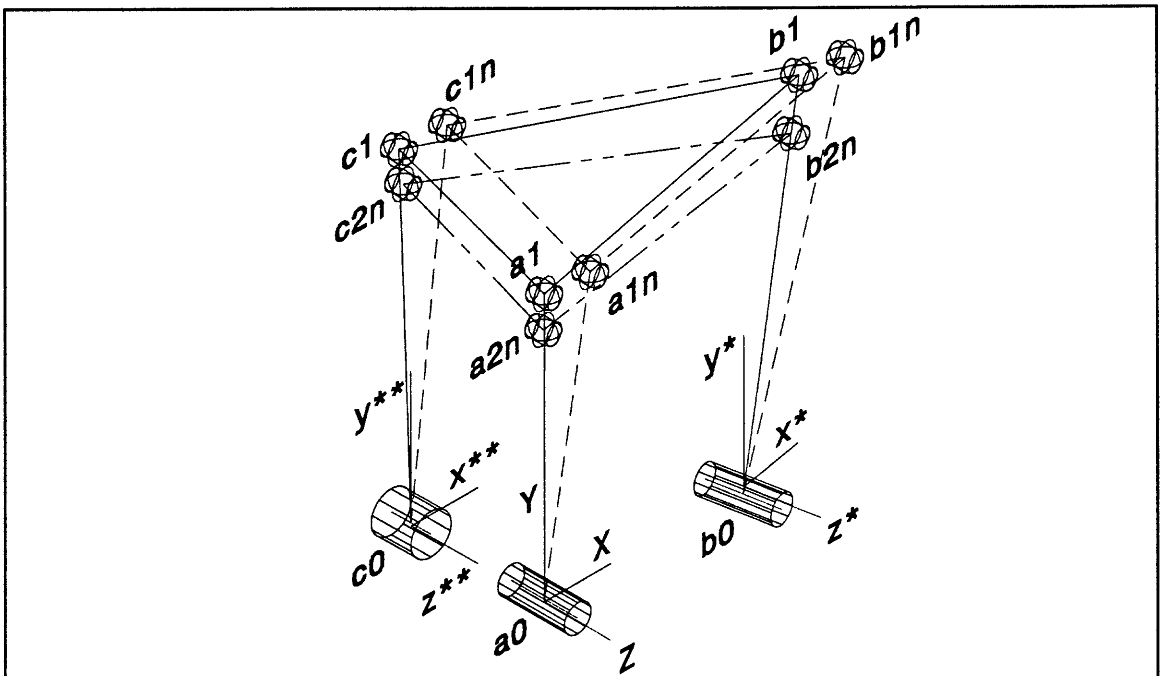

Figure 5.22 Solution to 3-phase RSSR-SC moving pivot problem with adjustable crank and follower lengths and MSPs

The specified MSP parameters in table 5.32 are

$\mathbf{Vp}_{1,2}$=(-0.1802, 9.0032) $\mathbf{Vq}_{1,2}$=(-8.9977, -0.1812) $\mathbf{Vr}_{1,2}$=(0.0633, -1.2057)

$\mathbf{Ap}_{1,2}$=(-81.0692, 0.3775) $\mathbf{Aq}_{1,2}$=(-0.3690, -81.0200) $\mathbf{Ar}_{1,2}$=(10.8656, 0.3015)

These parameters correspond to link $\mathbf{a}_0$-$\mathbf{a}_1$ for rigid body positions 1 and 2. Since these positions are in phase 1, the R-S link parameters for this position are $\mathbf{a}_0$ and $\mathbf{a}_1$. The values calculated for these parameters are the following:

$\mathbf{a}_0$=(-0.0006, -0.0000) $\mathbf{a}_1$=(-0.0000, 2.0001)

The velocity and acceleration of $\mathbf{a}_1$ were calculated using the $\mathbf{a}'$ and $\mathbf{a}''$ terms in equations 3.3 and 3.6 (where $[V_{12}]=[\mathbf{Vp}_{1,2}|\mathbf{Vq}_{1,2}|\mathbf{Vr}_{1,2}]$ and $[A_{12}]=[\mathbf{Ap}_{1,2}|\mathbf{Aq}_{1,2}|\mathbf{Ar}_{1,2}]$ with a third row of zeros). The values calculated for the velocity $\mathbf{a}_1'$ and acceleration $\mathbf{a}_1''$ are

$$\mathbf{a}_1'=(-17.9963, -0.3624) \qquad \mathbf{a}_1''=(-0.7380, -162.0481)$$

The specified MSP parameters in table 5.33 are

$$\mathbf{Vp}_{1,2}=(-0.1816, 8.9794) \quad \mathbf{Vq}_{1,2}=(-8.9977, -0.1789) \quad \mathbf{Vr}_{1,2}=(-0.2049, -1.2187)$$

$$\mathbf{Ap}_{1,2}=(-80.8448, -0.1380) \quad \mathbf{Aq}_{1,2}=(0.1107, -81.0096) \quad \mathbf{Ar}_{1,2}=(10.9341, -2.0475)$$

These parameters correspond to link $\mathbf{b}_0$-$\mathbf{b}_1$ for rigid body positions 1 and 2. Since these positions are in phase 1, the R-S link parameters for this position are $\mathbf{b}_0$ and $\mathbf{b}_1$. The values calculated for these parameters are the following:

$$\mathbf{b}_0=(0.0716, 0.0077) \qquad \mathbf{b}_1=(0.4286, 2.4568)$$

The velocity and acceleration of $\mathbf{b}_1$ was calculated using the $\mathbf{a}'$ and $\mathbf{a}''$ terms in equations 3.3 and 3.6 (where $[V_{12}]=[\mathbf{Vp}_{1,2}|\mathbf{Vq}_{1,2}|\mathbf{Vr}_{1,2}]$ and $[A_{12}]=[\mathbf{Ap}_{1,2}|\mathbf{Aq}_{1,2}|\mathbf{Ar}_{1,2}]$ with a third row of zeros). The values calculated for the velocity $\mathbf{b}_1'$ and acceleration $\mathbf{b}_1''$ are

$$\mathbf{b}_1'=(-22.1834, 3.4090) \qquad \mathbf{b}_1''=(-34.3781, -199.0835)$$

The specified MSP parameters in table 5.34 are

$$\mathbf{Vp}_{1,2}=(-0.1102, 5.5020) \quad \mathbf{Vq}_{1,2}=(-5.4986, -0.1107) \quad \mathbf{Vr}_{1,2}=(0.0387, -0.7368)$$

$$\mathbf{Vp}_{5,6}=(-0.1240, 5.5002) \quad \mathbf{Vq}_{5,6}=(-5.4891, -0.1205) \quad \mathbf{Vr}_{5,6}=(-0.0387, -0.7165)$$

These parameters correspond to links $\mathbf{c}_0$-$\mathbf{c}_1$ and $\mathbf{c}_0$-$\mathbf{c}_{1n}$ for rigid body from positions 1 to 2 and from positions 5 to 6. Since these positions are in phases 1, and 2 the C-S link parameters for these position are $\mathbf{c}_0$, $\mathbf{c}_1$ and $\mathbf{c}_{1n}$. The values calculated for these parameters are the following:

$\mathbf{c}_0$=(0.0000, -0.0000) $\mathbf{c}_1$=(-0.0850, 2.4931) $\mathbf{c}_{1n}$=(0.2762, 2.4720)

The velocity and acceleration of $\mathbf{c}_1$ and $\mathbf{c}_{1n}$ were calculated using the **a**' term in equation 3.3 (where $[V_{12}]=[\mathbf{Vp}_{1,2}|\mathbf{Vq}_{1,2}|\mathbf{Vr}_{1,2}]$ and $[V_{56}]=[\mathbf{Vp}_{5,6}|\mathbf{Vq}_{5,6}|\mathbf{Vr}_{5,6}]$ with a third row of zeros). The values calculated for the velocities $\mathbf{c}_1$' and $\mathbf{c}_{1n}$' are

$\mathbf{c}_1$'=(-13.6992, -0.7437) $\mathbf{c}_{1n}$'=(-13.6033, 1.2213)

The angular acceleration of both R-S links were calculated using the **a**" term equation in equation 3.6. Since both links rotate in planes, only the up_z unit vector term was used in equation 3.8 (therefore $up_x=up_y=0$). The fixed pivot acceleration term was also eliminated in equation 3.7 since the fixed pivots are "fixed" (therefore $\dot{\mathbf{up}}=0$ also).

When the rigid body position parameters and the moving pivot position and acceleration parameters for link $\mathbf{a}_0$-$\mathbf{a}_1$ were incorporated, the angular velocity and acceleration values calculated using the simultaneous equations were ω=9.0000 rad/sec and α=2.0015 rad/sec^2. When the rigid body position parameters and the moving pivot position and acceleration parameters for link $\mathbf{b}_0$-$\mathbf{b}_1$ were incorporated, the angular velocity and acceleration values calculated using the simultaneous equations were ω=9.0018 rad/sec and α=1.4941 rad/sec^2.

The angular velocities of both R-S links and the C-S link were calculated using the **a**' term equation in equation 3.3. Since these links rotate in planes, only the ω_z term was used in equation 3.5 (therefore $\omega_x=\omega_y=0$). The fixed pivot velocity term was also eliminated in equation 3.4 since the fixed pivots are "fixed."

When the rigid body position parameters and the moving pivot position and velocity parameters for link $\mathbf{a}_0$-$\mathbf{a}_1$ were incorporated, the angular velocity values calculated were ω_z=9.0004 rad/sec for the first row in equation 5.56 and ω_z=9.0149 rad/sec for the second row. When the rigid body position parameters and the moving pivot position and velocity parameters for link $\mathbf{b}_0$-$\mathbf{b}_1$ were incorporated, the angular velocity values calculated were ω_z=8.9994 rad/sec for the first row in equation 5.56 and ω_z=9.0018 rad/sec for the second row. Since the calculated position and velocity parameters of the moving pivots of the R-S links were truncated (to four significant figures), the angular velocity values for the R-S links were not exact matches.

When the rigid body position parameters and the moving pivot position and velocity parameters for link $\mathbf{c}_0$-$\mathbf{c}_1$ were incorporated, the angular velocity values calculated were ω_z=5.4999 rad/sec for the first row in equation 5.56 and ω_z=5.5007 rad/sec for the second row. When the rigid body position parameters and the moving pivot position and velocity parameters for link $\mathbf{c}_0$-$\mathbf{c}_{1n}$ were incorporated, the angular velocity values calculated were ω_z=5.5000 rad/sec for the first row in equation 5.56 and ω_z=5.4989 rad/sec for the second row. Since the calculated position and velocity parameters of the moving pivot of the C-S link was truncated (to four significant figures), the angular velocity values for the C-S link are not exact matches.

CHAPTER 6

CONCLUSIONS

The objectives of this work were to present several new methods for synthesizing adjustable spatial four and five-bar mechanisms for multi-phase finite and multiply separated positions. The spatial mechanisms of choice in this research were the RRSS, RRSC, RSSR-SS and the RSSR-SC and the link adjustments this work considered were the following:

- adjust the moving pivots while maintaining fixed crank and follower lengths
- adjust the fixed pivots while maintaining fixed crank and follower lengths
- adjust the moving pivots and crank and follower lengths between phases
- adjust the fixed pivots and crank and follower lengths between phases

One method presented involved the synthesis of spatial four and five-bar mechanisms for multi-phase motion generation. Using this technique, spatial mechanisms were synthesized to achieve 2 and 3 phases of finitely separated rigid body positions.

Another method presented involved the synthesis of spatial four-bar mechanisms for multi-phase motion generation with tolerances. Using this technique, spatial mechanisms were synthesized to achieve 2 and 3 phases of finitely separated rigid body positions-including positions with tolerances.

Another method presented involved the synthesis of spatial four and five-bar mechanisms for multi-phase motion generation and multiply separated positions. Using this technique spatial mechanisms were synthesized to achieve 2 and 3 phases of rigid body positions-including velocities and/or accelerations.

The final new method presented involved the synthesis of spatial four-bar mechanisms for multi-phase motion generation and multiply separated positions using instant screw axis parameters. Using this technique, spatial mechanisms were synthesized to achieve 2 and 3 phases of finitely separated rigid body positions-including velocities and/or accelerations based on instant screw axis parameters.

Using specific computer-aided rigid body point motion and selection approaches, the general displacement equations for the R-S, C-S and R-R dyads were reduced to the constant length constraint only. Using these schemes, joint axes need not be synthesized for R-S and R-R links and translation terms are not required for C-S link synthesis.

When under the constant length constraint alone, the number of prescribed rigid body positions is theoretically unlimited. This constraint is ideal since each additional phase in a multi-phase synthesis problem results in additional rigid body parameters.

In two and three-phase problems, the theoretical maximum numbers of prescribed rigid body positions are 8 and 11. Part of this research focused on determining the actual maximum prescribed rigid body position values. Every two-phase example problem in this research was solved using 8 prescribed rigid body positions and every three-phase problem was solved using 11 prescribed rigid body positions. Based on the results of the example problems in this work, the theoretical maximum rigid body position values for two and three-phase problems are achievable.

APPENDIX A

EXPANDED DISPLACEMENT, VELOCITY AND ACCELERATION EQUATIONS FOR R-R, R-S AND C-S LINKS

This appendix contains the simplified and expanded R-R, R-S and C-S link displacement, velocity and acceleration equations used in this research. The partial derivatives of these equations are also included with respect to the given components of variables $\mathbf{a}_1$ and $\mathbf{a}_0$.

Displacement Equation:

$$F1_{ij} = \left(\begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ 1 \end{pmatrix} - \begin{pmatrix} a_{0x} \\ a_{0y} \\ 1 \end{pmatrix} \right)^T \left(\begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ 1 \end{pmatrix} - \begin{pmatrix} a_{0x} \\ a_{0y} \\ 1 \end{pmatrix} \right) - R_1^2 = 0$$

(A.1)

Expanded Displacement Equation:

$$F1_{ij} = 1 + a_{0x}^2 + a_{0y}^2 - R_1^2 - 2a_{0x}a_{1x}g1_{xij} + a_{1x}^2 g1_{xij}^2 - 2a_{0y}a_{1x}g1_{yij} + a_{1x}^2 g1_{yij}^2 - 2a_{1x}g1_{zij} + a_{1x}^2 g1_{zij}^2 - 2a_{0x}a_{1y}g2_{xij} + 2a_{1x}a_{1y}g1_{xij}g2_{xij} + a1_y^2 g2_{xij}^2 - 2a_{0y}a_{1y}g2_{yij} + 2a_{1x}a_{1y}g1_{yij}g2_{yij} + a1_y^2 g2_{yij}^2 - 2a_{1y}g2_{zij} + 2a_{1x}a_{1y}g1_{zij}g2_{zij} + a_{1y}^2 g2_{zij}^2 - 2a_{0x}g3_{xij} + 2a_{1x}g1_{xij}g3_{xij} + 2a_{1y}g2_{xij}g3_{xij} + g3_{xij}^2 - 2a_{0y}g3_{yij} + 2a_{1x}g1_{yij}g3_{yij} + 2a_{1y}g2_{yij}g3_{yij} + g3_{yij}^2 - 2g3_{zij} + 2a_{1x}g1_{zij}g3_{zij} + 2a_{1y}g2_{zij}g3_{zij} + g3_{zij}^2 = 0$$

(A.2)

Partial Derivatives of Expanded Displacement Equation:

$$\frac{\partial F1_{ij}}{\partial a_{0x}} = 2a_{0x} - 2a_{1x}g1_{xij} - 2a_{1y}g2_{xij} - 2g3_{xij} \qquad \text{(A.3)}$$

$$\frac{\partial F1_{ij}}{\partial a_{0y}} = 2a_{0y} - 2a_{1x}g1_{yij} - 2a_{1y}g2_{yij} - 2g3_{yij} \qquad \text{(A.4)}$$

$$\frac{\partial F1_{ij}}{\partial a_{1x}} = -2a_{0x}g1_{xij} + 2a_{1x}g1_{xij}^2 - 2a_{0y}g1_{yij} + 2a_{1x}g1_{yij}^2 - 2g1_{zij} + 2a_{1x}g1_{zij}^2 + 2a_{1y}g1_{xij}g2_{xij} + 2a_{1y}g1_{yij}g2_{yij} + 2a_{1y}g1_{zij}g2_{zij} + 2g1_{xij}g3_{xij} + 2g1_{yij}g3_{yij} + 2g1_{zij}g3_{zij}$$

(A.5)

$$\frac{\partial F1_{ij}}{\partial a_{1y}} = -2a_{0x}g2_{xij} + 2a_{1x}g1_{xij}g2_{xij} - 2a_{0y}g2_{yij} + 2a_{1y}g2_{xij}^2 - 2g2_{zij} + 2a_{1y}g2_{yij}^2 + 2a_{1x}g1_{yij}g2_{yij} + 2a_{1x}g1_{zij}g2_{zij} + 2a_{1y}g2_{zij}^2 + 2g2_{xij}g3_{xij} + 2g2_{yij}g3_{yij} + 2g2_{zij}g3_{zij}$$

(A.6)

Velocity Equation:

$$F2_{ij} = \left(\left(\begin{pmatrix} 0 & -\omega & v_{a0x}+\omega a_{0y} \\ \omega & 0 & v_{a0y}-\omega a_{0x} \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix}\begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix}\right)^T\left(\begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix}\begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} - \begin{pmatrix} a_{0x} \\ a_{0y} \\ 0 \end{pmatrix}\right)\right) = 0$$

(A.7)

Expanded Velocity Equation:

$$\begin{aligned} F2_{ij} = &-\omega a_{0y}a_{1x}g1_{xij} + \omega a_{0x}a_{1x}g1_{yij} - a_{0x}a_{1x}v_{a0x}g1_{zij} - a_{0y}a_{1x}v_{a0y}g1_{zij} \\ &+ \omega a_{0y}a_{1x}^2g1_{xij}g1_{zij} + a_{1x}^2v_{a0x}g1_{xij}g1_{zij} - \omega a_{0x}a_{1x}^2g1_{yij}g1_{zij} + a_{1x}^2v_{a0y}g1_{yij}g1_{zij} \\ &- \omega a_{0y}a_{1y}g2_{xij} + \omega a_{0y}a_{1x}a_{1y}g1_{zij}g2_{xij} + a_{1x}a_{1y}v_{a0x}g1_{zij}g2_{xij} + \omega a_{0x}a_{1y}g2_{yij} - \\ &\omega a_{0x}a_{1x}a_{1y}g1_{zij}g2_{yij} + a_{1x}a_{1y}v_{a0y}g1_{zij}g2_{yij} - a_{0x}a_{1y}v_{a0x}g2_{zij} - a_{0y}a_{1y}v_{a0y}g2_{zij} \\ &+ \omega a_{0y}a_{1x}a_{1y}g1_{xij}g2_{zij} + a_{1x}a_{1y}v_{a0x}g1_{xij}g2_{zij} - \omega a_{0x}a_{1x}a_{1y}g1_{yij}g2_{zij} + \\ &a_{1x}a_{1y}v_{a0y}g1_{yij}g2_{zij} + \omega a_{0y}a_{1y}^2g2_{xij}g2_{zij} + a_{1y}^2v_{a0x}g2_{xij}g2_{zij} - \omega a_{0x}a_{1y}^2g2_{yij}g2_{zij} \\ &+ a_{1y}^2v_{a0y}g2_{yij}g2_{zij} = 0 \end{aligned}$$

(A.8)

Partial Derivatives of Expanded Velocity Equation:

$$\frac{\partial F2_{ij}}{\partial a_{0x}} = \omega a_{1x} g1_{yij} - a_{1x} v_{a0x} g1_{zij} - \omega a_{1x}{}^{2} g1_{yij} g1_{zij} + \omega a_{1y} g2_{yij} - \omega a_{1x} a_{1y} g1_{zij} g2_{yij} - a_{1y} v_{a0x} g2_{zij} - \omega a_{1x} a_{1y} g1_{yij} g2_{zij} - \omega a_{1y}{}^{2} g2_{yij} g2_{zij}$$

(A.9)

$$\frac{\partial F2_{ij}}{\partial a_{0y}} = -\omega a_{1x} g1_{xij} - a_{1x} v_{a0y} g1_{zij} + \omega a_{1x}{}^{2} g1_{xij} g1_{zij} - \omega a_{1y} g2_{xij} + \omega a_{1x} a_{1y} g1_{zij} g2_{xij} - a_{1y} v_{a0y} g2_{zij} + \omega a_{1x} a_{1y} g1_{xij} g2_{zij} + \omega a_{1y}{}^{2} g2_{xij} g2_{zij}$$

(A.10)

$$\frac{\partial F2_{ij}}{\partial a_{1x}} = -\omega a_{0y} g1_{xij} + \omega a_{0x} g1_{yij} - a_{0x} v_{a0x} g1_{zij} - a_{0y} v_{a0y} g1_{zij} + 2\omega a_{0y} a_{1x} g1_{xij} g1_{zij} + 2a_{1x} v_{a0x} g1_{xij} g1_{zij} - 2\omega a_{0x} a_{1x} g1_{yij} g1_{zij} + 2a_{1x} v_{a0y} g1_{yij} g1_{zij} + \omega a_{0y} a_{1y} g1_{zij} g2_{xij} + a_{1y} v_{a0x} g1_{zij} g2_{xij} - \omega a_{0x} a_{1y} g1_{zij} g2_{yij} + a_{1y} v_{a0y} g1_{zij} g2_{yij} + \omega a_{0y} a_{1y} g1_{xij} g2_{zij} + a_{1y} v_{a0x} g1_{xij} g2_{zij} - \omega a_{0x} a_{1y} g1_{yij} g2_{zij} + a_{1y} v_{a0y} g1_{yij} g2_{zij}$$

(A.11)

$$\frac{\partial F2_{ij}}{\partial a_{1y}} = -\omega a_{0y} g2_{xij} + \omega a_{0y} a_{1x} g1_{zij} g2_{xij} + a_{1x} v_{a0x} g1_{zij} g2_{xij} + \omega a_{0x} g2_{yij} - \omega a_{0x} a_{1x} g1_{zij} g2_{yij} + a_{1x} v_{a0y} g1_{zij} g2_{yij} - a_{0x} v_{a0x} g2_{zij} - a_{0y} v_{a0y} g2_{zij} + \omega a_{0y} a_{1x} g1_{xij} g2_{zij} + a_{1x} v_{a0x} g1_{xij} g2_{zij} - \omega a_{0x} a_{1x} g1_{yij} g2_{zij} + a_{1x} v_{a0y} g1_{yij} g2_{zij} + 2\omega a_{0y} a_{1y} g2_{xij} g2_{zij} + 2a_{1y} v_{a0x} g2_{xij} g2_{zij} - 2\omega a_{0x} a_{1y} g2_{yij} g2_{zij} + 2a_{1y} v_{a0y} g2_{yij} g2_{zij}$$

(A.12)

Acceleration Equation:

$$
F3_{ij} = \left(\left(\left(\begin{pmatrix} -\omega^2 & -\alpha & a_{a0x} + \omega^2 a_{0x} + \alpha a_{0y} \\ \alpha & -\omega^2 & a_{a0y} - \alpha a_{0x} + \omega^2 a_{0y} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \right) \begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} \right)^T \right.
$$

$$
\left(\begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} - \begin{pmatrix} a_{0x} \\ a_{0y} \\ 0 \end{pmatrix} \right) \Bigg) + \left(\left(\left(\begin{pmatrix} 0 & -\omega & v_{a0x} + \omega a_{0y} \\ \omega & 0 & v_{a0y} - \omega a_{0x} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \right) \begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} \right)^T \right.
$$

$$
\left. \left(\left(\begin{pmatrix} 0 & -\omega & v_{a0x} + \omega a_{0y} \\ \omega & 0 & v_{a0y} - \omega a_{0x} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \right) \begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} \right) \right) = 0
$$

(A.13)

Expanded Acceleration Equation:

$$
\begin{aligned}
& F3_{ij} = \omega^2 a_{0x} a_{1x} g1_{xij} - \alpha a_{0y} a_{1x} g1_{xij} + \alpha a_{0x} a_{1x} g1_{yij} + \omega^2 a_{0y} a_{1x} g1_{yij} - \omega^2 a_{0x}{}^2 a_{1x} g1_{zij} - \\
& \omega^2 a_{0y}{}^2 a_{1x} g1_{zij} - a_{0x} a_{1x} a_{a0x} g1_{zij} - a_{0y} a_{1x} a_{a0y} g1_{zij} - \omega^2 a_{0x} a_{1x}{}^2 g1_{xij} g1_{zij} + \alpha a_{0y} a_{1x}{}^2 g1_{xij} g1_{zij} \\
& + a_{1x}{}^2 a_{a0x} g1_{xij} g1_{zij} + 2\omega a_{1x}{}^2 v_{a0y} g1_{xij} g1_{zij} - \alpha a_{0x} a_{1x}{}^2 g1_{yij} g1_{zij} - \omega^2 a_{0y} a_{1x}{}^2 g1_{yij} g1_{zij} + \\
& a_{1x}{}^2 a_{a0y} g1_{yij} g1_{zij} - 2\omega a_{1x}{}^2 v_{a0x} g1_{yij} g1_{zij} + \omega^2 a_{0x}{}^2 a_{1x}{}^2 g1_{zij}{}^2 + \omega^2 a_{0y}{}^2 a_{1x}{}^2 g1_{zij}{}^2 + \\
& 2\omega a_{0y} a_{1x}{}^2 v_{a0x} g1_{zij}{}^2 + a_{1x}{}^2 v_{a0x}{}^2 g1_{zij}{}^2 - 2\omega a_{0x} a_{1x}{}^2 v_{a0y} g1_{zij}{}^2 + a_{1x}{}^2 v_{a0y}{}^2 g1_{zij}{}^2 + \\
& \omega^2 a_{0x} a_{1y} g2_{xij} - \alpha a_{0y} a_{1y} g2_{xij} - \omega^2 a_{0x} a_{1x} a_{1y} g1_{zij} g2_{xij} + \alpha a_{0y} a_{1x} a_{1y} g1_{zij} g2_{xij} + \\
& a_{1x} a_{1y} a_{a0x} g1_{zij} g2_{xij} + 2\omega a_{1x} a_{1y} v_{a0y} g1_{zij} g2_{xij} + \alpha a_{0x} a_{1y} g2_{yij} + \omega^2 a_{0y} a_{1y} g2_{yij} - \\
& \alpha a_{0x} a_{1x} a_{1y} g1_{zij} g2_{yij} - \omega^2 a_{0y} a_{1x} a_{1y} g1_{zij} g2_{yij} + a_{1x} a_{1y} a_{a0y} g1_{zij} g2_{yij} - 2\omega a_{1x} a_{1y} v_{a0x} g1_{zij} g2_{yij} \\
& - \omega^2 a_{0x}{}^2 a_{1y} g2_{zij} - \omega^2 a_{0y}{}^2 a_{1y} g2_{zij} - a_{0x} a_{1y} a_{a0x} g2_{zij} - a_{0y} a_{1y} a_{a0y} g2_{zij} - \\
& \omega^2 a_{0x} a_{1x} a_{1y} g1_{xij} g2_{zij} + \alpha a_{0y} a_{1x} a_{1y} g1_{xij} g2_{zij} + a_{1x} a_{1y} a_{a0x} g1_{xij} g2_{zij} + 2\omega a_{1x} a_{1y} v_{a0y} g1_{xij} g2_{zij} \\
& - \alpha a_{0x} a_{1x} a_{1y} g1_{yij} g2_{zij} - \omega^2 a_{0y} a_{1x} a_{1y} g1_{yij} g2_{zij} + a_{1x} a_{1y} a_{a0y} g1_{yij} g2_{zij} - 2\omega a_{1x} a_{1y} v_{a0x} g1_{yij} g2_{zij} \\
& + 2\omega^2 a_{0x}{}^2 a_{1x} a_{1y} g1_{zij} g2_{zij} + 2\omega^2 a_{0y}{}^2 a_{1x} a_{1y} g1_{zij} g2_{zij} + 4\omega a_{0y} a_{1x} a_{1y} v_{a0x} g1_{zij} g2_{zij} + \\
& 2 a_{1x} a_{1y} v_{a0x}{}^2 g1_{zij} g2_{zij} - 4\omega a_{0x} a_{1x} a_{1y} v_{a0y} g1_{zij} g2_{zij} + 2 a_{1x} a_{1y} v_{a0y}{}^2 g1_{zij} g2_{zij} - \omega^2 a_{0x} a_{1y}{}^2 g2_{xij} g2_{zij} \\
& + \alpha a_{0y} a_{1y}{}^2 g2_{xij} g2_{zij} + a_{1y}{}^2 a_{a0x} g2_{xij} g2_{zij} + 2\omega a_{1y}{}^2 v_{a0y} g2_{xij} g2_{zij} - \alpha a_{0x} a_{1y}{}^2 g2_{yij} g2_{zij} - \\
& \omega^2 a_{0y} a_{1y}{}^2 g2_{yij} g2_{zij} + a_{1y}{}^2 a_{a0y} g2_{yij} g2_{zij} - 2\omega a_{1y}{}^2 v_{a0x} g2_{yij} g2_{zij} + \omega^2 a_{0x}{}^2 a_{1y}{}^2 g2_{zij}{}^2 +
\end{aligned}
$$

$$\omega^2 a_{0y}{}^2 a_{1y}{}^2 g2_{zij}{}^2 + 2\omega a_{0y} a_{1y}{}^2 v_{a0x} g2_{zij}{}^2 + a_{1y}{}^2 v_{a0x}{}^2 g2_{zij}{}^2 - 2\omega a_{0x} a_{1y}{}^2 v_{a0y} g2_{zij}{}^2 + a_{1y}{}^2 v_{a0y}{}^2 g2_{zij}{}^2 = 0$$

(A.14)

Partial Derivatives of Expanded Acceleration Equation:

$$\frac{\partial F3_{ij}}{\partial a_{0x}} = -\omega^2 a_{1x} g1_{xij} + \alpha a_{1x} g1_{yij} - 2\omega^2 a_{0x} a_{1x} g1_{zij} - a_{1x} a_{a0x} g1_{zij} - \omega^2 a_{1x}{}^2 g1_{xij} g1_{zij} - \alpha a_{1x}{}^2 g1_{yij} g1_{zij} + 2\omega^2 a_{0x} a_{1x}{}^2 g1_{zij}{}^2 - 2\omega a_{1x}{}^2 v_{a0y} g1_{zij}{}^2 + \omega^2 a_{1y} g2_{xij} - \omega^2 a_{1x} a_{1y} g1_{zij} g2_{xij} + \alpha a_{1y} g2_{yij} - \alpha a_{1x} a_{1y} g1_{zij} g2_{yij} - 2\omega^2 a_{0x} a_{1y} g2_{zij} - a_{1y} a_{a0x} g2_{zij} - \omega^2 a_{1x} a_{1y} g1_{xij} g2_{zij} - \alpha a_{1x} a_{1y} g1_{yij} g2_{zij} + 4\omega^2 a_{0x} a_{1x} a_{1y} g1_{zij} g2_{zij} - 4\omega a_{1x} a_{1y} v_{a0y} g1_{zij} g2_{zij} - \omega^2 a_{1y}{}^2 g2_{xij} g2_{zij} - \alpha a_{1y}{}^2 g2_{yij} g2_{zij} + 2\omega^2 a_{0x} a_{1y}{}^2 g2_{zij}{}^2 - 2\omega a_{1y}{}^2 v_{a0y} g2_{zij}{}^2$$

(A.15)

$$\frac{\partial F3_{ij}}{\partial a_{0y}} = \omega^2 a_{1x} g1_{yij} - \alpha a_{1x} g1_{xij} - 2\omega^2 a_{0y} a_{1x} g1_{zij} - a_{1x} a_{a0y} g1_{zij} - \omega^2 a_{1x}{}^2 g1_{yij} g1_{zij} + \alpha a_{1x}{}^2 g1_{xij} g1_{zij} + 2\omega^2 a_{0y} a_{1x}{}^2 g1_{zij}{}^2 + 2\omega a_{1x}{}^2 v_{a0x} g1_{zij}{}^2 + \omega^2 a_{1y} g2_{yij} - \omega^2 a_{1x} a_{1y} g1_{zij} g2_{yij} - \alpha a_{1y} g2_{xij} + \alpha a_{1x} a_{1y} g1_{zij} g2_{xij} - 2\omega^2 a_{0y} a_{1y} g2_{zij} - a_{1y} a_{a0y} g2_{zij} - \omega^2 a_{1x} a_{1y} g1_{yij} g2_{zij} + \alpha a_{1x} a_{1y} g1_{xij} g2_{zij} + 4\omega^2 a_{0y} a_{1x} a_{1y} g1_{zij} g2_{zij} + 4\omega a_{1x} a_{1y} v_{a0x} g1_{zij} g2_{zij} - \omega^2 a_{1y}{}^2 g2_{yij} g2_{zij} + \alpha a_{1y}{}^2 g2_{xij} g2_{zij} + 2\omega^2 a_{0y} a_{1y}{}^2 g2_{zij}{}^2 + 2\omega a_{1y}{}^2 v_{a0x} g2_{zij}{}^2$$

(A.16)

$$\frac{\partial F3_{ij}}{\partial a_{1x}} = \omega^2 a_{0x} g1_{xij} - \alpha a_{0y} g1_{xij} + \alpha a_{0x} g1_{yij} + \omega^2 a_{0y} g1_{yij} - \omega^2 a_{0x}{}^2 g1_{zij} - \omega^2 a_{0y}{}^2 g1_{zij} - a_{0x} a_{a0x} g1_{zij} - a_{0y} a_{a0y} g1_{zij} - 2\omega^2 a_{0x} a_{1x} g1_{xij} g1_{zij} + 2\alpha a_{0y} a_{1x} g1_{xij} g1_{zij} + 2a_{1x} a_{a0x} g1_{xij} g1_{zij} + 4\omega a_{1x} v_{a0y} g1_{xij} g1_{zij} - 2\alpha a_{0x} a_{1x} g1_{yij} g1_{zij} - 2\omega^2 a_{0y} a_{1x} g1_{yij} g1_{zij} + 2a_{1x} a_{a0y} g1_{yij} g1_{zij} - 4\omega a_{1x} v_{a0x} g1_{yij} g1_{zij} + 2\omega^2 a_{0x}{}^2 a_{1x} g1_{zij}{}^2 + 2\omega^2 a_{0y}{}^2 a_{1x} g1_{zij}{}^2 + 4\omega a_{0y} a_{1x} v_{a0x} g1_{zij}{}^2 + 2a_{1x} v_{a0x}{}^2 g1_{zij}{}^2 - 4\omega a_{0x} a_{1x} v_{a0y} g1_{zij}{}^2 + 2a_{1x} v_{a0y}{}^2 g1_{zij}{}^2 - \omega^2 a_{0x} a_{1y} g1_{zij} g2_{xij} + \alpha a_{0y} a_{1y} g1_{zij} g2_{xij} + a_{1y} a_{a0x} g1_{zij} g2_{xij} + 2\omega a_{1y} v_{a0y} g1_{zij} g2_{xij} - \alpha a_{0x} a_{1y} g1_{zij} g2_{yij} - \omega^2 a_{0y} a_{1y} g1_{zij} g2_{yij} + a_{1y} a_{a0y} g1_{zij} g2_{yij} - 2\omega a_{1y} v_{a0x} g1_{zij} g2_{yij} - \omega^2 a_{0x} a_{1y} g1_{xij} g2_{zij} + \alpha a_{0y} a_{1y} g1_{xij} g2_{zij} + a_{1y} a_{a0x} g1_{xij} g2_{zij} + 2\omega a_{1y} v_{a0y} g1_{xij} g2_{zij} -$$

$$\alpha a_{0x}a_{1y}g1_{yij}g2_{zij} - \omega^2 a_{0y}a_{1y}g1_{yij}g2_{zij} + a_{1y}a_{a0y}g1_{yij}g2_{zij} - 2\omega a_{1y}v_{a0x}g1_{yij}g2_{zij} +$$
$$2\omega^2 a_{0x}{}^2 a_{1y}g1_{zij}g2_{zij} + 2\omega^2 a_{0y}{}^2 a_{1y}g1_{zij}g2_{zij} + 4\omega a_{0y}a_{1y}v_{a0x}g1_{zij}g2_{zij} +$$
$$2a_{1y}v_{a0x}{}^2 g1_{zij}g2_{zij} - 4\omega a_{0x}a_{1y}v_{a0y}g1_{zij}g2_{zij} + 2a_{1y}v_{a0y}{}^2 g1_{zij}g2_{zij}$$

(A.17)

$$\frac{\partial F3_{ij}}{\partial a_{1y}} = \omega^2 a_{0x}g2_{xij} - \alpha a_{0y}g2_{xij} - \omega^2 a_{0x}a_{1x}g1_{zij}g2_{xij} + \alpha a_{0y}a_{1x}g1_{zij}g2_{xij} +$$
$$a_{1x}a_{a0x}g1_{zij}g2_{xij} + 2\omega a_{1x}v_{a0y}g1_{zij}g2_{xij} + \alpha a_{0x}g2_{yij} + \omega^2 a_{0y}g2_{yij} - \alpha a_{0x}a_{1x}g1_{zij}g2_{yij} -$$
$$\omega^2 a_{0y}a_{1x}g1_{zij}g2_{yij} + a_{1x}a_{a0y}g1_{zij}g2_{yij} - 2\omega a_{1x}v_{a0x}g1_{zij}g2_{yij} - \omega^2 a_{0x}{}^2 g2_{zij} -$$
$$\omega^2 a_{0y}{}^2 g2_{zij} - a_{0x}a_{a0x}g2_{zij} - a_{0y}a_{a0y}g2_{zij} - \omega^2 a_{0x}a_{1x}g1_{xij}g2_{zij} + \alpha a_{0y}a_{1x}g1_{xij}g2_{zij} +$$
$$a_{1x}a_{a0x}g1_{xij}g2_{zij} + 2\omega a_{1x}v_{a0y}g1_{xij}g2_{zij} - \alpha a_{0x}a_{1x}g1_{yij}g2_{zij} - \omega^2 a_{0y}a_{1x}g1_{yij}g2_{zij} +$$
$$a_{1x}a_{a0y}g1_{yij}g2_{zij} - 2\omega a_{1x}v_{a0x}g1_{yij}g2_{zij} + 2\omega^2 a_{0x}{}^2 a_{1x}g1_{zij}g2_{zij} + 2\omega^2 a_{0y}{}^2 a_{1x}g1_{zij}g2_{zij} +$$
$$4\omega a_{0y}a_{1x}v_{a0x}g1_{zij}g2_{zij} + 2a_{1x}v_{a0x}{}^2 g1_{zij}g2_{zij} - 4\omega a_{0x}a_{1x}v_{a0y}g1_{zij}g2_{zij} +$$
$$2a_{1x}v_{a0y}{}^2 g1_{zij}g2_{zij} - 2\omega^2 a_{0x}a_{1y}g2_{xij}g2_{zij} + 2\alpha a_{0y}a_{1y}g2_{xij}g2_{zij} + 2a_{1y}a_{a0x}g2_{xij}g2_{zij} +$$
$$4\omega a_{1y}v_{a0y}g2_{xij}g2_{zij} - 2\alpha a_{0x}a_{1y}g2_{yij}g2_{zij} - 2\omega^2 a_{0y}a_{1y}g2_{yij}g2_{zij} + 2a_{1y}a_{a0y}g2_{yij}g2_{zij}$$
$$- 4\omega a_{1y}v_{a0x}g2_{yij}g2_{zij} + 2\omega^2 a_{0x}{}^2 a_{1y}g2_{zij}{}^2 + 2\omega^2 a_{0y}{}^2 a_{1y}g2_{zij}{}^2 + 4\omega a_{0y}a_{1y}v_{a0x}g2_{zij}{}^2 +$$
$$2a_{1y}v_{a0x}{}^2 g2_{zij}{}^2 - 4\omega a_{0x}a_{1y}v_{a0y}g2_{zij}{}^2 + 2a_{1y}v_{a0y}{}^2 g2_{zij}{}^2$$

(A.18)

where

$$\begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} = \begin{pmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_{ix} & q_{ix} & r_{ix} \\ p_{iy} & q_{iy} & r_{iy} \\ 1 & 1 & 1 \end{pmatrix}^{-1} \quad (A.19)$$

APPENDIX B

EXPANDED INSTANT SCREW AXIS VELOCITY AND ACCELERATION EQUATIONS FOR R-R, R-S AND C-S LINKS

This appendix contains the simplified and expanded R-R, R-S and C-S link displacement, velocity and acceleration equations used in this research. The velocity and acceleration equations incorporate ISA parameters. The partial derivatives of these equations are also included with respect to the given components of variables $\mathbf{a}_1$ and $\mathbf{a}_0$.

Velocity Equation:

$$F1_{ij} = \left(\begin{pmatrix} 0 & -\omega_s up_{0z} & \omega_s up_{0y} \\ \omega_s up_{0z} & 0 & -\omega_s up_{0x} \\ -\omega_s up_{0y} & \omega_s up_{0x} & 0 \end{pmatrix} \begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} \right)^T \left(\begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} - \begin{pmatrix} a_{0x} \\ a_{0y} \\ 0 \end{pmatrix} \right) = 0$$

(B.1)

Expanded Velocity Equation:

$$F1_{ij} = a_{0y}a_{1x}g1_{zij}up_{0x}\omega_s + a_{0y}a_{1y}g2_{zij}up_{0x}\omega_s - a_{0x}a_{1x}g1_{zij}up_{0y}\omega_s - a_{0x}a_{1y}g2_{zij}up_{0y}\omega_s - a_{0y}a_{1x}g1_{xij}up_{0z}\omega_s + a_{0x}a_{1x}g1_{yij}up_{0z}\omega_s - a_{0y}a_{1y}g2_{xij}up_{0z}\omega_s + a_{0x}a_{1y}g2_{yij}up_{0z}\omega_s = 0$$

(B.2)

Partial Derivatives of Expanded Velocity Equation:

$$\frac{\partial F1_{ij}}{\partial a_{0x}} = -a_{1x}g1_{zij}up_{0y}\omega_s - a_{1y}g2_{zij}up_{0y}\omega_s + a_{1x}g1_{yij}up_{0z}\omega_s + a_{1y}g2_{yij}up_{0z}\omega_s \quad \text{(B.3)}$$

$$\frac{\partial F1_{ij}}{\partial a_{0y}} = a_{1x}g1_{zij}up_{0x}\omega_s + a_{1y}g2_{zij}up_{0x}\omega_s - a_{1x}g1_{xij}up_{0z}\omega_s - a_{1y}g2_{xij}up_{0z}\omega_s \quad \text{(B.4)}$$

$$\frac{\partial F1_{ij}}{\partial a_{1x}} = a_{0y}g1_{zij}up_{0x}\omega_s - a_{0x}g1_{zij}up_{0y}\omega_s - a_{0y}g1_{xij}up_{0z}\omega_s + a_{0x}g1_{yij}up_{0z}\omega_s \quad \text{(B.5)}$$

$$\frac{\partial F1_{ij}}{\partial a_{1y}} = a_{0y}g2_{zij}up_{0x}\omega_s - a_{0x}g2_{zij}up_{0y}\omega_s - a_{0y}g2_{xij}up_{0z}\omega_s + a_{0x}g2_{yij}up_{0z}\omega_s \tag{B.6}$$

Acceleration Equation:

$$F2_{ij} = \left(\begin{pmatrix} \omega_S^2 up_{0x}^2 - \omega_S^2 & up_{0x}up_{0y}\omega_S^2 - v_{up0z}\omega_S - up_{0z}\alpha_S & up_{0x}up_{0z}\omega_S^2 + v_{up0y}\omega_S + up_{0y}\alpha_S \\ up_{0x}up_{0y}\omega_S^2 + v_{up0z}\omega_S + up_{0z}\alpha_S & \omega_S^2 up_{0y}^2 - \omega_S^2 & up_{0y}up_{0z}\omega_S^2 - v_{up0x}\omega_S - up_{0x}\alpha_S \\ up_{0x}up_{0z}\omega_S^2 - v_{up0y}\omega_S - up_{0y}\alpha_S & up_{0y}up_{0z}\omega_S^2 - v_{up0x}\omega_S + up_{0x}\alpha_S & \omega_S^2 up_{0z}^2 - \omega_S^2 \end{pmatrix} \begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} \right)^T$$

$$\left(\begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} - \begin{pmatrix} a_{0x} \\ a_{0y} \\ 0 \end{pmatrix} \right) + \left(\begin{pmatrix} 0 & -\omega_S up_{0z} & \omega_S up_{0y} \\ \omega_S up_{0z} & 0 & -\omega_S up_{0x} \\ -\omega_S up_{0y} & \omega_S up_{0x} & 0 \end{pmatrix} \begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} \right)^T$$

$$\left(\begin{pmatrix} 0 & -\omega_S up_{0z} & \omega_S up_{0y} \\ \omega_S up_{0z} & 0 & -\omega_S up_{0x} \\ -\omega_S up_{0y} & \omega_S up_{0x} & 0 \end{pmatrix} \begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ 0 \end{pmatrix} \right) = 0$$

(B.7)

Expanded Acceleration Equation:

$F2_{ij} = a_{0y}a_{1x}g1_{zij}up_{0x}\alpha_S + a_{0y}a_{1y}g2_{zij}up_{0x}\alpha_S - a_{0x}a_{1x}g1_{zij}up_{0y}\alpha_S - a_{0x}a_{1y}g2_{zij}up_{0y}\alpha_S -$

$a_{0y}a_{1x}g1_{xij}up_{0z}\alpha_S + a_{0x}a_{1x}g1_{yij}up_{0z}\alpha_S - a_{0y}a_{1y}g2_{xij}up_{0z}\alpha_S + a_{0x}a_{1y}g2_{yij}up_{0z}\alpha_S +$

$a_{0x}a_{1x}g1_{xij}\omega_S - a_{1x}^2g1_{xij}^2\omega_S + a_{0y}a_{1x}g1_{yij}\omega_S - a_{1x}^2g1_{yij}^2\omega_S - a_{1x}^2g1_{zij}^2\omega_S + a_{0x}a_{1y}g2_{xij}\omega_S -$

$2a_{1x}a_{1y}g1_{xij}g2_{xij}\omega_S - a_{1y}^2g2_{xij}^2\omega_S + a_{0y}a_{1y}g2_{yij}\omega_S - 2a_{1x}a_{1y}g1_{yij}g2_{yij}\omega_S - a_{1y}^2g2_{yij}^2\omega_S -$

$2a_{1x}a_{1y}g1_{zij}g2_{zij}\omega_S - a_{1y}^2g2_{zij}^2\omega_S - a_{0x}a_{1x}g1_{xij}up_{0x}^2\omega_S + a_{1x}^2g1_{xij}^2up_{0x}^2\omega_S -$

$a_{0x}a_{1y}g2_{xij}up_{0x}^2\omega_S + 2a_{1x}a_{1y}g1_{xij}g2_{xij}up_{0x}^2\omega_S + a_{1y}^2g2_{xij}^2up_{0x}^2\omega_S - a_{0y}a_{1x}g1_{yij}up_{0y}^2\omega_S +$

$a_{1x}^2g1_{yij}^2up_{0y}^2\omega_S - a_{0y}a_{1y}g2_{yij}up_{0y}^2\omega_S + 2a_{1x}a_{1y}g1_{yij}g2_{yij}up_{0y}^2\omega_S + a_{1y}^2g2_{yij}^2up_{0y}^2\omega_S +$

$a_{1x}^2g1_{zij}^2up_{0z}^2\omega_S + 2a_{1x}a_{1y}g1_{zij}g2_{zij}up_{0z}^2\omega_S + a_{1y}^2g2_{zij}^2up_{0z}^2\omega_S + a_{1x}^2g1_{yij}^2up_{0x}^2\omega_S^2 +$

$a_{1x}^2g1_{zij}^2up_{0x}^2\omega_S^2 + 2a_{1x}a_{1y}g1_{yij}g2_{yij}up_{0x}^2\omega_S^2 + a_{1y}^2g2_{yij}^2up_{0x}^2\omega_S^2 +$

$2a_{1x}a_{1y}g1_{zij}g2_{zij}up_{0x}^2\omega_S^2 + a_{1y}^2g2_{zij}^2up_{0x}^2\omega_S^2 - a_{0y}a_{1x}g1_{xij}up_{0x}up_{0y}\omega_S^2 -$

$a_{0x}a_{1x}g1_{yij}up_{0x}up_{0y}\omega_S^2 - a_{0y}a_{1y}g2_{xij}up_{0x}up_{0y}\omega_S^2 - a_{0x}a_{1y}g2_{yij}up_{0x}up_{0y}\omega_S^2 + a_{1x}^2g1_{xij}^2up_{0y}^2\omega_S^2$

$$+a_{1x}^2 g1_{zij}^2 up_{0y}^2 \omega_S^2 + 2a_{1x}a_{1y}g1_{xij}g2_{xij}up_{0y}^2\omega_S^2 + a_{1y}^2 g2_{xij}^2 up_{0y}^2 \omega_S^2$$
$$+2a_{1x}a_{1y}g1_{zij}g2_{zij}up_{0y}^2\omega_S^2 + a_{1y}^2 g2_{zij}^2 up_{0y}^2\omega_S^2 - a_{0x}a_{1x}g1_{zij}up_{0x}up_{0z}\omega_S^2 -$$
$$a_{0x}a_{1y}g2_{zij}up_{0x}up_{0z}\omega_S^2 - a_{0y}a_{1x}g1_{zij}up_{0y}up_{0z}\omega_S^2 - a_{0y}a_{1y}g2_{zij}up_{0y}up_{0z}\omega_S^2 +$$
$$a_{1x}^2 g1_{xij}^2 up_{0z}^2\omega_S^2 + a_{1x}^2 g1_{yij}^2 up_{0z}^2\omega_S^2 + 2a_{1x}a_{1y}g1_{xij}g2_{xij}up_{0z}^2\omega_S^2 + a_{1y}^2 g2_{xij}^2 up_{0z}^2\omega_S^2 +$$
$$2a_{1x}a_{1y}g1_{yij}g2_{yij}up_{0z}^2\omega_S^2 + a_{1y}^2 g2_{yij}^2 up_{0z}^2\omega_S^2 + a_{0y}a_{1x}g1_{zij}\omega_S v_{up0x} - 2a_{1x}^2 g1_{yij}g1_{zij}\omega_S v_{up0x} -$$
$$2a_{1x}a_{1y}g1_{zij}g2_{yij}\omega_S v_{up0x} + a_{0y}a_{1y}g2_{zij}\omega_S v_{up0x} - 2a_{1x}a_{1y}g1_{yij}g2_{zij}\omega_S v_{up0x} - 2a_{1y}^2 g2_{yij}g2_{zij}\omega_S v_{up0x}$$
$$-a_{0x}a_{1x}g1_{zij}\omega_S v_{up0y} - a_{0x}a_{1y}g2_{zij}\omega_S v_{up0y} - a_{0y}a_{1x}g1_{xij}\omega_S v_{up0z} + a_{0x}a_{1x}g1_{yij}\omega_S v_{up0z} -$$
$$a_{0y}a_{1y}g2_{xij}\omega_S v_{up0z} + a_{0x}a_{1y}g2_{yij}\omega_S v_{up0z} = 0$$

(B.8)

Partial Derivatives of Expanded Acceleration Equation:

$$\frac{\partial F2_{ij}}{\partial a_{0x}} = -a_{1x}g1_{zij}up_{0y}\alpha_s - a_{1y}g2_{zij}up_{0y}\alpha_s + a_{1x}g1_{yij}up_{0z}\alpha_s + a_{1y}g2_{yij}up_{0z}\alpha_s + a_{1x}g1_{xij}\omega_s$$
$$+a_{1y}g2_{xij}\omega_s - a_{1x}g1_{xij}up_{0x}^2\omega_s - a_{1y}g2_{xij}up_{0x}^2\omega_s - a_{1x}g1_{yij}up_{0x}up_{0y}\omega_s^2 -$$
$$a_{1y}g2_{yij}up_{0x}up_{0y}\omega_s^2 - a_{1x}g1_{zij}up_{0x}up_{0z}\omega_s^2 - a_{1y}g2_{zij}up_{0x}up_{0z}\omega_s^2 - a_{1x}g1_{zij}\omega_s v_{up0y} -$$
$$a_{1y}g2_{zij}\omega_s v_{up0y} + a_{1x}g1_{yij}\omega_s v_{up0z} + a_{1y}g2_{yij}\omega_s v_{up0z}$$

(B.9)

$$\frac{\partial F2_{ij}}{\partial a_{0y}} = a_{1x}g1_{zij}up_{0x}\alpha_s + a_{1y}g2_{zij}up_{0x}\alpha_s - a_{1x}g1_{xij}up_{0z}\alpha_s - a_{1y}g2_{xij}up_{0z}\alpha_s + a_{1x}g1_{yij}\omega_s$$
$$+a_{1y}g2_{yij}\omega_s - a_{1x}g1_{yij}up_{0y}^2\omega_s - a_{1y}g2_{yij}up_{0y}^2\omega_s - a_{1x}g1_{xij}up_{0x}up_{0y}\omega_s^2 -$$
$$a_{1y}g2_{xij}up_{0x}up_{0y}\omega_s^2 - a_{1x}g1_{zij}up_{0y}up_{0z}\omega_s^2 - a_{1y}g2_{zij}up_{0y}up_{0z}\omega_s^2 + a_{1x}g1_{zij}\omega_s v_{up0x} +$$
$$a_{1y}g2_{zij}\omega_s v_{up0x} - a_{1x}g1_{xij}\omega_s v_{up0z} - a_{1y}g2_{xij}\omega_s v_{up0z}$$

(B.10)

$$\frac{\partial F2_{ij}}{\partial a_{1x}} = a_{0y}g1_{zij}up_{0x}\alpha_s - a_{0x}g1_{zij}up_{0y}\alpha_s - a_{0y}g1_{xij}up_{0z}\alpha_s + a_{0x}g1_{yij}up_{0z}\alpha_s + a_{0x}g1_{xij}\omega_s$$
$$-2a_{1x}g1_{xij}^2\omega + a_{0y}g1_{yij}\omega_s - 2a_{1x}g1_{yij}^2\omega_s - 2a_{1x}g1_{zij}^2\omega_s - 2a_{1y}g1_{xij}g2_{xij}\omega_s -$$
$$2a_{1y}g1_{yij}g2_{yij}\omega_s - 2a_{1y}g1_{zij}g2_{zij}\omega_s - a_{0x}g1_{xij}up_{0x}^2\omega_s + 2a_{1x}g1_{xij}^2 up_{0x}^2\omega_s +$$

$$2a_{1y}g1_{xij}g2_{xij}up_{0x}{}^2\omega_s - a_{0y}g1_{yij}up_{0y}{}^2\omega_s + 2a_{1x}g1_{yij}{}^2up_{0y}{}^2\omega_s + 2a_{1y}g1_{yij}g2_{yij}up_{0y}{}^2\omega_s +$$
$$2a_{1x}g1_{zij}{}^2up_{0z}{}^2\omega_s + 2a_{1y}g1_{zij}g2_{zij}up_{0z}{}^2\omega_s + 2a_{1x}g1_{yij}{}^2up_{0x}{}^2\omega_s{}^2 + 2a_{1x}g1_{zij}{}^2up_{0x}{}^2\omega_s{}^2 +$$
$$2a_{1y}g1_{yij}g2_{yij}up_{0x}{}^2\omega_s{}^2 + 2a_{1y}g1_{zij}g2_{zij}up_{0x}{}^2\omega_s{}^2 - a_{0y}g1_{xij}up_{0x}up_{0y}\omega_s{}^2 -$$
$$a_{0x}g1_{yij}up_{0x}up_{0y}\omega_s{}^2 + 2a_{1x}g1_{xij}{}^2up_{0y}{}^2\omega_s{}^2 + 2a_{1x}g1_{zij}{}^2up_{0y}{}^2\omega_s{}^2 + 2a_{1y}g1_{xij}g2_{xij}up_{0y}{}^2\omega_s{}^2 +$$
$$2a_{1y}g1_{zij}g2_{zij}up_{0y}{}^2\omega_s{}^2 - a_{0x}g1_{zij}up_{0x}up_{0z}\omega_s{}^2 - a_{0y}g1_{zij}up_{0y}up_{0z}\omega_s{}^2 + 2a_{1x}g1_{xij}{}^2up_{0z}{}^2\omega_s{}^2$$
$$+ 2a_{1x}g1_{yij}{}^2up_{0z}{}^2\omega_s{}^2 + 2a_{1y}g1_{xij}g2_{xij}up_{0z}{}^2\omega_s{}^2 + 2a_{1y}g1_{yij}g2_{yij}up_{0z}{}^2\omega_s{}^2 + a_{0y}g1_{zij}\omega_s v_{up0x}$$
$$- 4a_{1x}g1_{yij}g1_{zij}\omega_s v_{up0x} - 2a_{1y}g1_{zij}g2_{yij}\omega_s v_{up0x} - 2a_{1y}g1_{yij}g2_{zij}\omega_s v_{up0x} - a_{0x}g1_{zij}\omega_s v_{up0y}$$
$$- a_{0y}g1_{xij}\omega_s v_{up0z} + a_{0x}g1_{yij}\omega_s v_{up0z}$$

(B.11)

$$\frac{\partial F2_{ij}}{\partial a_{1y}} = a_{0y}g2_{zij}up_{0x}\alpha_s - a_{0x}g2_{zij}up_{0y}\alpha_s - a_{0y}g2_{xij}up_{0z}\alpha_s + a_{0x}g2_{yij}up_{0z}\alpha_s + a_{0x}g2_{xij}\omega_s$$
$$- 2a_{1x}g1_{xij}g2_{xij}\omega - 2a_{1y}g2_{xij}{}^2\omega_s + a_{0y}g2_{yij}\omega_s - 2a_{1y}g2_{yij}{}^2\omega_s - 2a_{1x}g1_{yij}g2_{yij}\omega_s -$$
$$2a_{1x}g1_{zij}g2_{zij}\omega_s - 2a_{1y}g2_{zij}{}^2\omega_s - a_{0x}g2_{xij}up_{0x}{}^2\omega_s + 2a_{1x}g1_{xij}g2_{xij}up_{0x}{}^2\omega_s +$$
$$2a_{1y}g2_{xij}{}^2up_{0x}{}^2\omega_s - a_{0y}g2_{yij}up_{0y}{}^2\omega_s + 2a_{1x}g1_{yij}g2_{yij}up_{0y}{}^2\omega_s + 2a_{1y}g2_{yij}{}^2up_{0y}{}^2\omega_s +$$
$$2a_{1x}g1_{zij}g2_{zij}up_{0z}{}^2\omega_s + 2a_{1y}g2_{zij}{}^2up_{0z}{}^2\omega_s + 2a_{1x}g1_{yij}g2_{yij}up_{0x}{}^2\omega_s{}^2 + 2a_{1y}g2_{yij}{}^2up_{0x}{}^2\omega_s{}^2 +$$
$$2a_{1x}g1_{zij}g2_{zij}up_{0x}{}^2\omega_s{}^2 + 2a_{1y}g2_{zij}{}^2up_{0x}{}^2\omega_s{}^2 - a_{0y}g2_{xij}up_{0x}up_{0y}\omega_s{}^2 -$$
$$a_{0x}g2_{yij}up_{0x}up_{0y}\omega_s{}^2 + 2a_{1x}g1_{xij}g2_{xij}up_{0y}{}^2\omega_s{}^2 + 2a_{1y}g2_{xij}{}^2up_{0y}{}^2\omega_s{}^2 + 2a_{1x}g1_{zij}g2_{zij}up_{0y}{}^2\omega_s{}^2 +$$
$$2a_{1y}g2_{zij}{}^2up_{0y}{}^2\omega_s{}^2 - a_{0x}g2_{zij}up_{0x}up_{0z}\omega_s{}^2 - a_{0y}g2_{zij}up_{0y}up_{0z}\omega_s{}^2 + 2a_{1x}g1_{xij}g2_{xij}up_{0z}{}^2\omega_s{}^2$$
$$+ 2a_{1y}g2_{xij}{}^2up_{0z}{}^2\omega_s{}^2 + 2a_{1x}g1_{yij}g2_{yij}up_{0z}{}^2\omega_s{}^2 + 2a_{1y}g2_{yij}{}^2up_{0z}{}^2\omega_s{}^2 + a_{0y}g2_{zij}\omega_s v_{up0x}$$
$$- 4a_{1y}g2_{yij}g2_{zij}\omega_s v_{up0x} - 2a_{1x}g1_{zij}g2_{yij}\omega_s v_{up0x} - 2a_{1x}g1_{yij}g2_{zij}\omega_s v_{up0x} - a_{0x}g2_{zij}\omega_s v_{up0y}$$
$$- a_{0y}g2_{xij}\omega_s v_{up0z} + a_{0x}g2_{yij}\omega_s v_{up0z}$$

(B.12)

where

$$\begin{pmatrix} g1_{xij} & g2_{xij} & g3_{xij} \\ g1_{yij} & g2_{yij} & g3_{yij} \\ g1_{zij} & g2_{zij} & g3_{zij} \end{pmatrix} = \begin{pmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_{ix} & q_{ix} & r_{ix} \\ p_{iy} & q_{iy} & r_{iy} \\ 1 & 1 & 1 \end{pmatrix}^{-1} \qquad \text{(B.13)}$$

APPENDIX C

EXPANDED S-S LINK DISPLACEMENT EQUATION

This appendix contains the simplified and expanded S-S link displacement equations used in this research. The partial derivatives of the S-S link displacement equation are also included with respect to the given components of variables $\mathbf{a}_1$ and $\mathbf{a}_0$.

Displacement Equation:

$$F1_{ij} = \left(\begin{pmatrix} f1_{ij} & f5_{ij} & g1_{ij} & g5_{ij} \\ f2_{ij} & f6_{ij} & g2_{ij} & g6_{ij} \\ f3_{ij} & f7_{ij} & g3_{ij} & g7_{ij} \\ f4_{ij} & f8_{ij} & g4_{ij} & g8_{ij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ a_{1z} \\ 1 \end{pmatrix} - \begin{pmatrix} a_{0x} \\ a_{0y} \\ a_{0z} \\ 1 \end{pmatrix} \right)^T \left(\begin{pmatrix} f1_{ij} & f5_{ij} & g1_{ij} & g5_{ij} \\ f2_{ij} & f6_{ij} & g2_{ij} & g6_{ij} \\ f3_{ij} & f7_{ij} & g3_{ij} & g7_{ij} \\ f4_{ij} & f8_{ij} & g4_{ij} & g8_{ij} \end{pmatrix} \begin{pmatrix} a_{1x} \\ a_{1y} \\ a_{1z} \\ 1 \end{pmatrix} - \begin{pmatrix} a_{0x} \\ a_{0y} \\ a_{0z} \\ 1 \end{pmatrix} \right) - R_1^2 = 0$$

(C.1)

Expanded Displacement Equation:

$$\begin{aligned} F1_{ij} = {} & 1 + a_{0x}^2 + a_{0y}^2 + a_{0z}^2 - 2a_{0x}a_{1x}f1_{ij} + a_{1x}^2 f1_{ij}^2 - 2a_{0y}a_{1x}f2_{ij} + a_{1x}^2 f2_{ij}^2 - 2a_{0z}a_{1x}f3_{ij} + \\ & a_{1x}^2 f3_{ij}^2 - 2a_{1x}f4_{ij} + a_{1x}^2 f4_{ij}^2 - 2a_{1y}f8_{ij} - 2a_{0x}a_{1y}f8_{ij} - 2a_{0y}a_{1y}f8_{ij} - 2a_{0z}a_{1y}f8_{ij} + \\ & 2a_{1x}a_{1y}f1_{ij}f8_{ij} + 2a_{1x}a_{1y}f2_{ij}f8_{ij} + 2a_{1x}a_{1y}f3_{ij}f8_{ij} + 2a_{1x}a_{1y}f4_{ij}f8_{ij} + a_{1y}^2 f8_{ij}^2 - 2a_{0x}a_{1z}g1_{ij} \\ & + 2a_{1x}a_{1z}f1_{ij}g1_{ij} + 2a_{1y}a_{1z}f5_{ij}g1_{ij} + a_{1z}^2 g1_{ij}^2 - 2a_{0y}a_{1z}g2_{ij} + 2a_{1x}a_{1z}f2_{ij}g2_{ij} + 2a_{1y}a_{1z}f6_{ij}g2_{ij} \\ & + a_{1z}^2 g2_{ij}^2 - 2a_{0z}a_{1z}g3_{ij} + 2a_{1x}a_{1z}f3_{ij}g3_{ij} + 2a_{1y}a_{1z}f7_{ij}g3_{ij} + a_{1z}^2 g3_{ij}^2 - 2a_{1z}g4_{ij} + 2a_{1x}a_{1z}f4_{ij}g4_{ij} \\ & + 2a_{1y}a_{1z}f8_{ij}g4_{ij} + a_{1z}^2 g4_{ij}^2 - 2a_{0x}g5_{ij} + 2a_{1x}f1_{ij}g5_{ij} + 2a_{1y}f5_{ij}g5_{ij} + 2a_{1z}g1_{ij}g5_{ij} + g5_{ij}^2 - \\ & 2a_{0y}g6_{ij} + 2a_{1x}f2_{ij}g6_{ij} + 2a_{1y}f6_{ij}g6_{ij} + 2a_{1z}g2_{ij}g6_{ij} + g6_{ij}^2 - 2a_{0z}g7_{ij} + 2a_{1x}f3_{ij}g7_{ij} + \\ & 2a_{1y}f7_{ij}g7_{ij} + 2a_{1z}g3_{ij}g7_{ij} + g7_{ij}^2 - 2g8_{ij} + 2a_{1x}f4_{ij}g8_{ij} + 2a_{1y}f8_{ij}g8_{ij} + 2a_{1z}g4_{ij}g8_{ij} + \\ & g8_{ij}^2 - R_1^2 = 0 \end{aligned}$$

(C.2)

Partial Derivatives of Expanded Displacement Equation:

$$\frac{\partial F1_{ij}}{\partial a_{0x}} = 2a_{0x} - 2a_{1x}f1_{ij} - 2a_{1y}f5_{ij} - 2a_{1z}g1_{ij} - 2g5_{ij} \quad \text{(C.3)}$$

$$\frac{\partial F1_{ij}}{\partial a_{0y}} = 2a_{0y} - 2a_{1x}f2_{ij} - 2a_{1y}f6_{ij} - 2a_{1z}g2_{ij} - 2g6_{ij} \quad \text{(C.4)}$$

$$\frac{\partial F1_{ij}}{\partial a_{0z}} = 2a_{0z} - 2a_{1x}f3_{ij} - 2a_{1y}f7_{ij} - 2a_{1z}g3_{ij} - 2g7_{ij} \quad \text{(C.5)}$$

$$\frac{\partial F1_{ij}}{\partial a_{1x}} = -2a_{0x}f1_{ij} + 2a_{1x}f1_{ij}^2 - 2a_{0y}f2_{ij} + 2a_{1x}f2_{ij}^2 - 2a_{0z}f3_{ij} + 2a_{1x}f3_{ij}^2 - 2f4_{ij} +$$
$$2a_{1x}f4_{ij}^2 + 2a_{1y}f1_{ij}f5_{ij} + 2a_{1y}f2_{ij}f6_{ij} + 2a_{1y}f3_{ij}f7_{ij} + 2a_{1y}f4_{ij}f8_{ij} + 2a_{1z}f1_{ij}g1_{ij} + \quad \text{(C.6)}$$
$$2a_{1z}f2_{ij}g2_{ij} + 2a_{1z}f3_{ij}g3_{ij} + 2a_{1z}f4_{ij}g4_{ij} + 2f1_{ij}g5_{ij} + 2f2_{ij}g6_{ij} + 2f3_{ij}g7_{ij} +$$
$$2f4_{ij}g8_{ij}$$

$$\frac{\partial F1_{ij}}{\partial a_{1y}} = -2a_{0x}f5_{ij} + 2a_{1x}f1_{ij}f5_{ij} + 2a_{1y}f5_{ij}^2 - 2a_{0y}f6_{ij} + 2a_{1x}f2_{ij}f6_{ij} + 2a_{1y}f6_{ij}^2$$
$$-2a_{0z}f7_{ij} + 2a_{1x}f3_{ij}f7_{ij} + 2a_{1y}f7_{ij}^2 - 2f8_{ij} + 2a_{1x}f4_{ij}f8_{ij} + 2a_{1y}f8_{ij}^2 + 2a_{1z}f5_{ij}g1_{ij} \quad \text{(C.7)}$$
$$+ 2a_{1z}f6_{ij}g2_{ij} + 2a_{1z}f7_{ij}g3_{ij} + 2a_{1z}f8_{ij}g4_{ij} + 2f5_{ij}g5_{ij} + 2f6_{ij}g6_{ij} + 2f7_{ij}g7_{ij} +$$
$$2f8_{ij}g8_{ij}$$

$$\frac{\partial F1_{ij}}{\partial a_{1z}} = -2a_{0x}g1_{ij} + 2a_{1x}f1_{ij}g1_{ij} + 2a_{1y}f5_{ij}g1_{ij} + 2a_{1z}g1_{ij}^2 - 2a_{0y}g2_{ij} +$$
$$2a_{1x}f2_{ij}g2_{ij} + 2a_{1y}f6_{ij}g2_{ij} + 2a_{1z}g2_{ij}^2 - 2a_{0z}g3_{ij} + 2a_{1x}f3_{ij}g3_{ij} + 2a_{1y}f7_{ij}g3_{ij} \quad \text{(C.8)}$$
$$+ 2a_{1z}g3_{ij}^2 - 2g4_{ij} + 2a_{1x}f4_{ij}g4_{ij} + 2a_{1y}f8_{ij}g4_{ij} + 2a_{1z}g4_{ij}^2 + 2g1_{ij}g5_{ij} +$$
$$2g2_{ij}g6_{ij} + 2g3_{ij}g7_{ij} + 2g4_{ij}g8_{ij}$$

where

$$\begin{pmatrix} f1_{ij} & f5_{ij} & g1_{ij} & g5_{ij} \\ f2_{ij} & f6_{ij} & g2_{ij} & g6_{ij} \\ f3_{ij} & f7_{ij} & g3_{ij} & g7_{ij} \\ f4_{ij} & f8_{ij} & g4_{ij} & g8_{ij} \end{pmatrix} = \begin{pmatrix} p_{jx} & q_{jx} & r_{jx} & s_{jx} \\ p_{jy} & q_{jy} & r_{jy} & s_{jy} \\ p_{jz} & q_{jz} & r_{jz} & s_{jz} \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_{ix} & q_{ix} & r_{ix} & s_{ix} \\ p_{iy} & q_{iy} & r_{iy} & s_{iy} \\ p_{iz} & q_{iz} & r_{iz} & s_{iz} \\ 1 & 1 & 1 & 1 \end{pmatrix}^{-1} \quad \text{(C.9)}$$

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