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ABSTRACT

INTERMODAL TRANSIT SYSTEM COORDINATION WITH DYNAMIC VEHICLE DISPATCHING

by
Md. Shoaib Chowdhury

In most urban areas where transit demand is spread widely, passengers may be served by an intermodal transit system, consisting of a rail transit line (or a bus rapid transit route) and a numbers of feeder routes connecting at different transfer stations. In such a system, passengers may need one or more transfers to complete their journey. Therefore, scheduling vehicles operating in the system with special attention to reduce transfer time can contribute significantly service quality improvements. In this study two models, one for coordination of a general intermodal transit system and another for dynamic dispatching of vehicles on coordinated routes, are presented.

Schedule synchronization may significantly reduce transfer delays at transfer stations where various routes interconnect. Since vehicle arrivals are stochastic, slack time allowances in vehicle schedules may be desirable to reduce the probability of missed connections. An objective total cost function, including supplier and user costs, are formulated for optimizing the coordination of a general intermodal transit network. A four-stage procedure is developed for determining the optimal coordination status among routes at every transfer station. Considering stochastic feeder vehicle arrivals at transfer stations, the slack times of coordinated routes are optimized, by balancing the savings from transfer delays and additional cost from slack delays and operating costs. The model is used to optimize the coordination of an intermodal transit network under different

demand situations, while the impact of various factors (e.g., demand, standard deviation of vehicle arrival times, etc) on coordination is examined.

For dynamic vehicle dispatching control, the decision whether a coordinated vehicle should be held to wait for late vehicles can be made by minimizing the dynamic total cost objective function formulated in this study. The time-varied objective total cost function, including supplier and user costs, is developed for determining the optimal dynamic dispatching times of all coordinated vehicles at transfer stations. A numerical example is provided to demonstrate the application of the dynamic dispatching model, while vehicle holding times are optimized and dispatching costs are analyzed under different delay variations of coordinated vehicles arrival times.

**INTERMODAL TRANSIT SYSTEM COORDINATION
WITH DYNAMIC VEHICLE DISPATCHING**

by
Md. Shoaib Chowdhury

**A Dissertation
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APPROVAL PAGE

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WITH DYNAMIC VEHICLE DISPATCHING**

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This dissertation is dedicated to
my parents and teachers from whom I have had the opportunity to acquire knowledge

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CHAPTER 1

INTRODUCTION

The level of service of an intermodal transit system is dependent significantly on transfer times. In urban areas where demand is spread widely, providing a door-to-door public transportation service is very expensive. An intermodal transit system with appropriate coordination may be a cost effective alternative to serve the residents.

An efficient intermodal transit system design should provide good transfer facilities to travelers by balancing supplier and user costs. Low operating headway, which can substantially reduce transfer time, is not always cost effective due to the variations of demand among transit routes over space and time. Aside from decreasing headways (and thus operating more vehicles), timed transfers may be the best way to reduce transfer time in a transit network.

This research develops a procedure to optimize coordination for a linear intermodal transit system, called trunk line with feeders system [Gray and Hoel, 1992], by minimizing the total costs. Two cost models, the route coordination model and the dynamic dispatching model, are developed for different purposes. A coordination model is developed to optimize headways for all transit routes and slack times for all coordinated routes by minimizing the total cost including supplier and user costs. For some transit systems in a relatively uncongested environment, timed transfers may be successful without advanced technology and intensive management, especially, as the headway variation is small and longer slack time is tolerable. However, in a congested environment with available advanced transit operational information, timed transfer

systems can be significantly improved by making dynamic holding or dispatching decision for ready vehicles at transfer stations if the number of connecting passengers on the late incoming vehicles are known. The dynamic dispatching model is formulated for optimizing holding times for vehicles operating on coordinated routes.

1.1 Problem Statement

In an intermodal transit system, integrating a rail line and numbers of feeder bus routes, passengers require one or more transfers to complete their journey. Transfers among routes are used to (1) eliminate the need for direct routes connecting all origin-destination pairs and (2) concentrate passengers on major routes served by high speed equipment. However, long transfer times significantly reduce the service quality and cause ridership reductions. Effective coordination can significantly increase the attractiveness and productivity of the intermodal transit system [Hicky, 1992; Vuchic, 2000].

Because of dynamic traffic congestion, vehicle breakdown, incidents, and the variation of demand over space and time, maintaining a stable headway along a bus route is quite difficult. Pure schedule synchronization for connecting routes at transfer stations may not reduce the transfer time effectively. Holding times (slack times) added into the schedules of coordinated routes may be required to increase the probability of successful connections, which can be optimized through minimizing the total cost including user and supplier costs. However, it is not always cost effective to coordinate all connected transit routes at a transfer station. The difficulty of finding an efficient coordinated group of routes depends on the design of a intermodal network, which is outside the scope of this study, the number of routes connected to the transfer station and their associated

operating headways. When the coordination among multiple transfer stations is considered, the complexity of the problem increases further. Transfer coordination may not always be successful if large slack times are not tolerable by a transit system, whose headway variation is large. Therefore, if real time data are available, the operation of the intermodal transit system can be tremendously improved with optimal control of arrival times and dynamic dispatching of ready vehicles.

1.2 Objectives and Scope

Public transportation systems should serve people efficiently under available transit resources and reasonable costs. This research is intended to provide cost effective coordination of intermodal transit service with special attention to coordinated transfers. An intermodal transit network consisting of a rail line and a number of feeder bus routes connected at different transfer stations is considered, while the objectives of this research are to

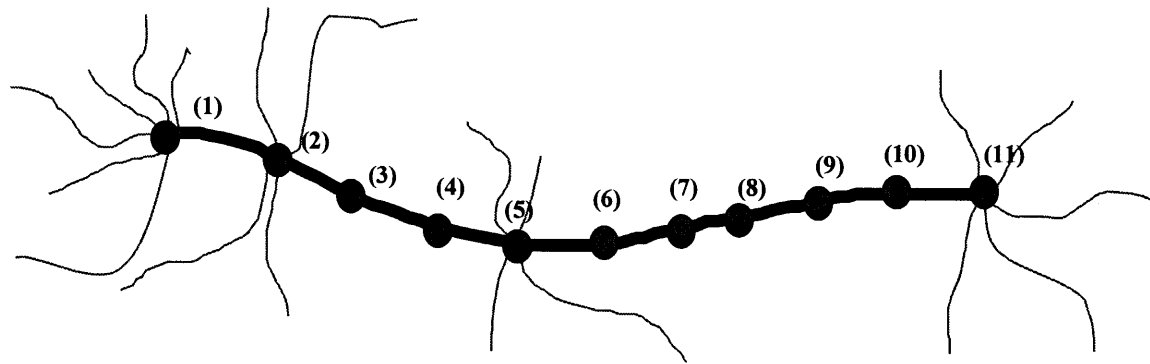
- (1) develop models for optimizing timed transfers in a given intermodal transit network with multiple transfer stations under both deterministic and stochastic conditions;
- (2) apply these models and determine the best type of coordinated transfer operations (e.g., full coordination, partial coordination and no coordination) in the intermodal transit system;
- (3) enhance the microscopic simulation model CORSIM to emulate Automatic Vehicle Location (AVL) systems for estimating bus arrival time distributions, and;
- (4) develop models to dispatch vehicles on coordinated routes at the lowest vehicle operating, transfer and in-vehicle costs.

The developed models can be used to evaluate the benefit of a timed transfer transit system with multiple transfer stations at which many feeder bus routes are connected. The transit network analyzed in this study is shown in Figure 1-1.

1.3 Motivation

Intermodal transit system planning and design has been an increasing interest with the passage of the Intermodal Surface Transportation Efficiency Act (ISTEA) of 1991. The purpose of ISTEA is *“to develop a National Intermodal Transportation System that is economically sound, provides the foundation for the nation to compete in the global economy and will move people and goods in an energy efficient manner”* [US Department of Transportation, 1991]. ISTEA’s Successor, the Transportation Equity Act for the 21st century (TEA-21) further supports and encourages the development of an intermodal system.

The continuing decline of ridership and productivity in some transit systems clearly indicates that public transportation operators are facing difficulties in providing adequate level of service at reasonable social and financial costs. The efficiency of public transportation systems can be increased through proper coordination, which minimizes passenger transfer time from one route to another. Thus, a better level of service (LOS) can be provided to passengers at a lower operating cost.

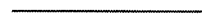


Legend:

Transfer station



Feeder bus route



Rail line



Figure 1-1 Intermodal Transit Network

1.4 Innovations

Models for scheduling coordinated transfers among a rail line and its feeder bus routes, and dynamic dispatching of vehicles on coordinated routes at transfer stations are developed. It is expected that the models developed in this study can be used to design a coordinated intermodal transit system that will significantly improve service quality, ridership, and productivity. Two major innovations in this research that help to coordinate the proposed intermodal transit system efficiently are:

1. determining the coordinated routes and optimizing common headways, and slack times, which minimize total cost.
2. dynamically optimizing holding time for coordinated vehicles at transfer stations.

1.5 Technical Approach

In this study the total cost functions are formulated for coordinating an intermodal transit system and dynamically dispatching vehicles on coordinated feeder routes, respectively. The decision variables (e.g., headways and slack times) in both functions are optimized by minimizing total cost.

A four-stage procedure is developed for optimizing the coordination model, which helps to identify various levels of coordination, such as no coordination, partial coordination and full coordination. If vehicles are operated without coordination, the total cost function is affected by the standard deviation of vehicle arrival times and headways. Under coordinated operations, the total cost function is affected by both the standard deviation and the distribution of vehicle arrival times. However, it is difficult to obtain

analytical solutions for the decision variables because of the stochastic nature of the vehicle arrival distributions. Therefore, Powell's method [Powell M. J. D., 1964; Himmelblan 1972], a multidimensional search algorithm, is used to find the optimal decision variables. Considering route coordination, the transfer cost is formulated based on the joint probability of vehicle arrival distributions. It is found intractable to calculate the joint probability of vehicle arrival times analytically using realistic vehicle arrival distributions (e.g., normal and lognormal distribution). Therefore, numerical integration (Gaussian Quadrature [Engeln-Mullges, G., and Uhlig, F., 1996]) is used to calculate the joint probability of vehicle arrivals.

In the dynamic dispatching model, the optimizable decision variable is the vehicle holding time determined at each decision time point. Due to the non-convex nature of the total cost function and the inability to calculate the probability of vehicle arrival times, analytical solutions (closed form) of decision variables (i.e., holding times) are not easy to obtain. Therefore, an incremental line search procedure is applied to find the optimal holding time, which can minimize total cost. A procedure is also developed in this study for evaluating the holding time periodically. The enhanced CORSIM model is used to generate data (e.g., vehicle departure times at various checkpoints and vehicles arrival times at transfer stations) for the dynamic vehicle dispatching model.

1. 6 Organization of the Dissertation

This dissertation consists of seven chapters. Chapter 1 covers an introduction, problem statement, objectives and scope, and research motivation. In Chapter 2, the relevant literature are reviewed, covering scheduling and coordination problem of intermodal

transit systems, vehicle holding strategies and service reliability, nonlinear optimization algorithms, transit simulation, and passenger and vehicle arrival distributions. In Chapter 3, the objective total cost function is formulated for planning coordinated transfers among rail and feeder buses at multiple transfer stations. A four-stage procedure is developed for determining different levels of coordination, which minimizes total cost. The total cost function for dynamic dispatching of vehicles on coordinated routes and the enhancement of CORSIM are discussed in Chapters 4 and 5, respectively. Numerical examples for testing the coordination and dispatching models and the sensitivity analysis between decision variables and various model parameters are presented in Chapter 6. Finally, in Chapter 7, general conclusions and recommendations for future study are presented.

CHAPETER 2

LITERATURE REVIEW

Previous studies that were found related to this research are summarized in this chapter, which is organized into seven sections. The first three sections cover the state-of-the-art review of scheduling and coordination problem of intermodal transit systems. Section 2.4 covers vehicle holding strategies. Nonlinear optimization algorithms are reviewed in section 2.5. Literature related to transit simulation and passenger/vehicle arrival distributions are discussed in Sections 2.6 and 2.7, respectively. Finally, a brief summary concludes the chapter.

2.1 Transit Scheduling Problems

In the design of optimal transit operations, a common objective is to minimize total cost, including operator and user costs [Hurdle, 1973; Wirasinghe, Hurdle and Newell, 1977; Wirasinghe and Ho, 1982; Kuah and Pert, 1988; Lee and Schonfeld, 1991; Spasovic and Schonfeld, 1993; Chien, 1995, Chien and Schonfeld, 1997]. Some of the previous studies [Hurdle, 1973; Kuah and Pert, 1988; Wirasinghe, 1980; Chien and Schonfeld, 1997] formulated mathematical models for a rail system served with a number of feeder buses. However, none of them considered coordination for a rail line and its feeder buses. Hurdle [1973] formulated a total cost function (including supplier and user costs), while the optimal locations of parallel feeder bus lines and headways were found. Wirasinghe [1980] optimized feeder bus route locations, bus headways and rail station locations by minimizing the total cost function. Later, Kuah and Pert [1988] developed an analytical

model for optimizing a feeder bus system. The decision variables, such as route spacing, operating headway and stop spacing were jointly optimized. Chien and Schonfeld [1997] conducted a similar study, but considered more decision variables, such as bus stop spacings under a heterogeneous environment. They formulated a model to jointly optimize the service provided by a rail line and its feeder bus system. Considering demand to be distributed irregularly along the service area, the rail line length, rail station spacing, rail headway, bus headways, bus route and stop spacings with and without rail-bus coordination were optimized. In that model, both rail and bus headways were assumed to be deterministic.

2.2 Analytical Approaches on Transfer Operations

Bus transfer policies and practices in the United States were described and summarized by Nelson, Brand and Mandel [1982]. Later, Bakker, Calkin, and Sylvester [1988] described how a radial bus route system of Capital Metro in Austin, Texas was modified to a multi-centered timed transfer network. The aim of the timed transfer network was to provide better service to transfer passengers by ensuring that coordinated vehicles meet at selected timed transfer stations. Note that the developed strategy was only suitable for a service area with low demand density.

Clever [1997] and Maxwell [1999] introduced a European concept of integrated fixed interval timed transfer strategy to coordinate intercity rail and other public transportation modes. Maxwell [1999] also discussed how a European timed transfer strategy could be applied in an intercity rail system in Northern California. Becker and Spielberg [1999] discussed how a multi-centered timed-transfer network for Tidewater

Regional Transit (TRT) in Virginia has established. TRT, which serves the city of Norfolk and Virginia metropolitan area, began implementing timed transfer since 1989. The whole area was served by twenty transfer centers. Results indicated that the timed transfer network improved transfer efficiency as well as reduced average travel time while ridership, revenue and productivity increased.

Reynolds and Hixson [1992] developed a computer program to present a detailed graphic display of vehicle arrivals and departures on selected routes connecting at a transfer point and calculated average transfer times. After observing vehicle arrival and departure times for a period of time on the selected routes, the program calculated the number of possible meets, good meets, and the percentage of good meets. Note that a possible meet is any appearance of the feeder vehicle at the transfer point; a good meet is one that fits the wait criteria defined by the planner. This program could be a helpful tool to identify where transfer improvement is needed between local and long-haul routes.

Whitney and Brill (1998) used a simulation approach to improve the transfer facility between Bridge-Pratt Street Station and Frankford Terminal Transportation Center in Philadelphia. The Bridge-Pratt Street Station served around 17,600 boardings per day of which 56 percent were transfer passengers from buses or trackless trolley. The Frankford Terminal served 23,000 boardings per day of which 50 percent were transfer passengers from Market-Frankford Subway Elevated (MFSE). After improving the transfer facility between the two terminals, transfer time was significantly reduced by minimizing transfer-walking time. However, the arrival synchronization issues were not discussed in that study.

Liu, Pendyala, and Polzin [1996] assessed the disutility of transit trip associated with transfers. They used discrete choice models to quantify transfer penalties and found that the value of transfer penalty could be more than twice the wait time while assuming that the wait time is one-half of the headway.

2.3 Deterministic/ Probabilistic Models for Transfer Operations

The variation of vehicle arrival times will lessen the benefit of coordination among transit routes. However, it has been neglected in many studies. For instance, Rapp and Gehner [1976] developed a deterministic model to schedule vehicle connections and used a computerized graphic process involving iterative modifications of departure times to minimize transfer delays. Salzborn [1972] developed a deterministic model to solve a bus-scheduling problem. While a passenger arrival rate was given, the optimal vehicle departure rate was determined by minimizing the fleet size and the total passenger wait time. Although the model was formulated for a single bus route, it could be extended to schedule a pair of bus routes connected at a terminal. Later, Salzborn [1980] developed guidelines for scheduling an inter-town bus system linking a string of interchanges, and applied combinatorial group theory to construct arrival and departure timetables. However, the route demand and transfer cost were not considered in that study. Ceder and Tal [1999] described the problem of maximal synchronization in generating a timetable for a given bus network. They formulated a mixed integer linear programming problem with the objective to maximize synchronization (i.e., maximal number of simultaneous bus arrivals) for deterministic bus arrivals at transfer locations, while satisfying the constraints for both maximum and minimum service headways.

The impact of probabilistic vehicle arrivals to the coordination problem has been considered in several studies [Hall, 1985; Lee and Schonfeld, 1991; Knoppers and Muller, 1995; The Bookbinder and Desilets, 1992; Chien 1995; Chien and Chowdhury, 1997; Chowdhury and Chien, 2000]. Hall [1985] developed an analytical model for scheduling vehicle arrivals at transportation terminals considering exponential vehicle arrival distributions. The main weakness of that model was the use of an exponential vehicle arrival distribution, which may not be realistic in many situations. Moreover, only wait time was considered in the objective function. Other factors, such as operator cost and in-vehicle time, which may affect vehicle scheduling, were neglected. Bookbinder and Desilets [1992] developed a model for optimizing vehicle departure times and showed how timed transfers could reduce the passenger wait time, and how layover time could be added to vehicle schedules at transfer points to ensure successful connections.

Lee and Schonfeld [1991] formulated a model to determine optimal vehicle slack times for a transfer terminal serving multiple bus routes. Stochastic vehicle arrivals were considered while formulating the objective coordinated transfer cost function. Analytical results were derived for both empirical and Gumbel distributions of vehicle arrival times. They found that the standard deviation of vehicle arrival times is an important factor affecting the duration of slack time. Later, Chien [1995] formulated a model for coordinating an intermodal transit system that consists of multiple bus routes at each transfer station and multiple transfer stations along a rail line. A method was developed to coordinate transit routes at transfer stations with different common headways and slack times, while normal bus arrival time distributions were assumed.

Knoppers and Muller [1995] investigated the possibilities and limitations of coordinated transfers in public transit systems. The optimal transfer times were obtained while considering stochastic arrivals of feeder vehicles and deterministic arrivals of vehicles operating on the major transit route. It was found that coordination was worthwhile when the arrival time standard deviation on the feeder line at the transfer point is less than 40 % of the headway on the major service network. However, only one directional transfer was considered in that study.

2.4 Vehicle Holding Strategies and Service Reliability

Many previous studies addressed controlling to the operation of transit vehicles to improving service reliability. Abkowitz and Engelstein [1984] developed a method for maintaining transit service regularity. Their objective was to find optimal control points for both headway and schedule based holding for a single route. Both strategies were evaluated on bus routes operating in Los Angeles. They concluded that headway variation decreased after applying headway based control. Later, Abkowitz, Eiger, and Engelstein [1986] examined the effect of a threshold-based holding control strategy on reducing bus headway variation at downstream stops. A simple algorithm was developed to determine the control point and threshold headway which would yield the greatest reduction in the total wait time including wait time at stops and in-vehicle delay at control stops. They found that control in reducing the headway variation is more effective at stops that are close to the control point.

Abkowitz and Tozzi [1997] examined the impact of ridership profiles on the effectiveness of headway control. Five boarding and alighting profiles were examined,

including (1) boarding at the beginning of the route and alighting at the end of the route, (2) boarding at the beginning of the route and alighting at the middle and end of the route, (3) boarding at the beginning of the route and alighting at the middle of the route, (4) boarding and alighting uniformly along the route, and (5) boarding at the middle of the route and alighting at the end of the route. They found that the implementation of headway control achieved the greatest reduction of wait time when passengers were boarding at the middle of the route and alighting at the end of the route.

Turnquist and Blume [1980] developed a method and tested the potential effectiveness of headway control strategies for a transit system, while considering headway variability, the number of delayed in-vehicle passengers and the number of passengers who will benefit from the reduced wait time. That method was effective for a system operating with shorter headway (i.e., for headway ≤ 10 minutes) where passenger arrivals at bus stops were random. Later, Turnquist [1981] analyzed several control strategies for improving transit service reliability. He indicated that schedule-based holding strategies worked better for low-frequency service (less than 10 buses per hour), while zone scheduling or signal preemption was more effective for mid-frequency service (10 to 30 buses per hour). The headway-based holding (if an appropriate control point could be found) and an exclusive lane combined with signal preemption were suggested to be applied in a system with high-frequency dispatching situations (more than 30 buses per hour). However, none of the above holding strategies considered the impact of transfer coordination.

Abkowitz, Josef, and Driscoll [1987] developed a computer simulation model programmed in FORTRAN to evaluate four timed transfer strategies: (1) unscheduled

transfers, (2) scheduled transfers (without vehicle holding), (3) scheduled transfers (holding vehicles operating on a low frequency route until vehicles operating on a higher frequency route arrive), and (4) scheduled transfers (always holding the early arriving vehicle). They simulated a network which consisted of two routes with a single transfer point and found that route characteristics including scheduled headway, percentage of transferring and through passengers at the transfer point and distance from the route origin to the transfer point play significant roles in determining a preferable transfer strategy. Simulation results showed that the double holding strategy was preferable when the headways of both routes were equal, while scheduled strategy (without vehicle holding) was preferable when the headways of both routes were unequal but multiples of one another.

Dessouky, Hall, Nowroozi, and Mourikas [1999] developed a simulation model for assessing various bus holding strategies at timed transfer stations. Several holding strategies were examined including (1) hold a vehicle until all coordinated vehicles arrive, (2) dispatch the vehicle at its schedule departure time, (3) hold the vehicle up to predefined fixed time, and (4) hold the vehicle up to predefined fixed time if at least one coordinated vehicle is predicted to arrive during the holding time with at least one transferring passenger. Simulation results showed that real time vehicle arrival information significantly reduces vehicles departure lateness without increasing the number of passengers missing their connections.

Lee and Schonfeld [1994] formulated a model for deciding whether to hold or dispatch coordinated vehicles at a transfer terminal. Holding times were optimized by minimizing the total cost, which included operator cost of holding vehicle, holding cost

of onboard passengers and missed connection cost of passengers from late connecting vehicles. In that model, missed connecting passenger wait time were assumed to be equal to scheduled headway and connection delay cost of passengers from late incoming vehicles were neglected.

Intelligent Transportation Systems (ITS) technologies can significantly improve passenger intermodal operations and services [Miller and Tsao, 2000]. Computer aided bus dispatching systems has been in the field or recently implemented in many transit properties [Federal Transit Administration, Update '98]. Tri-Met [Strathman, et. al, 1999], the transit provider in Portland, has implemented satellite-based Global Positioning Systems (GPS) to track vehicle locations. Bus dispatchers use real time bus locations and schedule deviation information to dispatch buses. The Ann Arbor (Michigan) Transportation Authority (AATA) deployed advanced public transportation system (APTS) technologies in its bus transit routes [Levine, Hong, Gug, and Rodriguez 2000]. The system called "Advanced Operating System" (AOS) enabled digital bus-to-bus communications to improve the transfer between buses. Buses among coordinated routes can locate other vehicle positions through the digital communication system and can request the holding of early vehicles to ensure the successful connection of passengers from late vehicles. The system did not optimize holding time, rather a preset maximum (up to five minutes) holding is used. It was reported that the system is capable of improving transfer efficiency.

2.5 Optimization Algorithms

Algorithms used for optimizing multivariate non-linear functions can be categorized in two groups: optimization with and without using the derivatives of the objective function. Widely applied derivative-typed algorithms, such as the Frank-Wolfe algorithm [Hillier and Lieberman, 1995], optimize non-linear functions through sequential linear approximations of the objective function. The gradient algorithms (e.g., gradient search procedure, generalized reduced gradient method) optimize multivariate unconstrained non-linear functions using the gradient at a trial solution point of each iteration. However, both types of algorithms may fail to find the optimal solution if the objective function is not differentiable.

Algorithms without using derivatives may be the choice to optimize the objective functions that are non-differentiable. Hooke and Jeeves [Himmelblan, 1972; Bazaraa, Sherali and Shetty 1993] proposed a heuristic for n-dimensional direct search called pattern search. The algorithm performs two types of search including an exploratory search and a pattern search. Exploratory search begins with function evaluation at the vectors of initial guesses of the independent variables. Then each variable is changed by incremental amounts in rotation to find the direction for minimization in the pattern search. Rosenbrock [Himmelblan, 1972] proposed a direct successive unidimensional search method, similar to the exploratory search developed by Hooke and Jeeves. To reach the optimal solution from an initial point the Rosenbrock method takes steps in orthogonal directions.

Nelder and Mead [Nelder and Mead, 1964; Himmelblan, 1972] minimized a function containing multiple independent variables using vertices of a flexible

polyhedron in an n-dimensional euclidean space. In each search, trail vectors in the vertices of the simplex are chosen to minimize the objective function. The vertex in the n-dimensional Euclidean space, which maximize the function value is projected through the the centroid of the remaining vertices. Then a better point is searched along the projected line that minimizes the objective function. This process will continue until the minimized objective function converges.

Powell [Powell M. J. D., 1964; Himmelblan, 1972; Press, et. al, 1992] developed a method which minimized quadratic functions by a successive uni-dimensional search along a set of n linear independent and mutually conjugate directions from an initial point. However, in some situations, a modified version of Powell method that does not possess the property of quadratic convergence was used because of the linear dependency among variables. In such situations, when an objective function is non-quadratic, the Hessian matrix of the function could be used to find the conjugated search direction [Avriel, 1976].

Himmelblau [1972] evaluated several unconstrained non-linear optimization algorithms, based on their execution times (seconds) to reach a given value of selected functions. He used a CDC 6600 computer to determine the execution time. Based on execution times of 11 problems weighting equally, he rated algorithms superior, good and fair classes, as shown in Table 2-1. We found that the Powell method ranks in the superior class.

Table 2-1 Evaluation of Unconstrained Optimization Algorithms from Execution Times

Classification	Algorithm
Superior	Fletcher (D) Davidon-Fletcher-Powell (D) Broyden (D) Powell (ND)
Good	Goldstein-Price (D)
Fair	Nelder-Mead (ND) Rosenbrock (ND) Fletcher-Reeves (D) Hooke-Jeeves (ND)

Source: Himmelblau M. David [1972]

*D= Derivative Type Algorithm

*ND= Non-Derivative Type Algorithm

2.6 Simulation of Transit Vehicles

With the advent of powerful computers, simulation has become a practical approach for evaluating complex transit operations. The simulation of transit operations enhances the capabilities of transit planners and operators to quickly and economically collect necessary data. (e.g., variation of bus travel times, bus arrival times at transfer stations etc.). Vandebona and Richardson [1985] developed TRAMS (Transit Route Animation and Modeling by Simulation) for simulating light rail transit operations. However, TRAMS cannot simulate high density bus operations as it is unable to simulate overtaking and merging maneuvers. CORSIM [FHWA, 1996] is able to simulate transit operations while considering car following and lane changing behavior. It can simulate traffic operations for almost any networks, including freeways and urban streets, while considering traffic control at both signalized and unsignalized intersections. Two types of

bus stops (on-line and off-line) can be handled simultaneously. However, in CORSIM, the duration of the bus dwell time is determined mainly by a mean dwell time rather than the number of boarding and alighting passengers. Thus, the stochastic nature of bus operations can not be simulated appropriately. A detailed review of other traffic simulation models can be found in [Chien, Mouskos and Chowdhury, 1999].

A major contributor to the degradation of the level of service is the bunching of buses caused by random variations in dwell times at stops as a result of the variable numbers of boarding/alighting passengers and the operation of wheel chair lifts on buses, and in-bus travel times incurred by different levels of traffic congestion. Thus, when a bus falls slightly behind schedule, it may result in a larger number of passengers waiting at the next stop. Additionally, this may lead to further delays and even more abnormal numbers of boarding and alighting passengers at further downstream stops. As a result, that bus may keep falling further behind its schedule. Conversely, the following bus, behind the late bus, encounters fewer passengers than usual and shorter dwell times, allowing it to catch up with the preceding bus. The resulting irregular headways degrade the transit system's productivity and efficiency. Although CORSIM can be viewed as one of the most powerful microscopic corridor simulators, it does not model bus operations properly, especially when processing dwell times at stops. CORSIM deals with bus dwell times by simply relying on mean dwell times specified by users and embedded statistical distributions rather than loading and unloading demand. Thus, the actual dwell time determined in CORSIM is extracted from a distribution and can be regarded as a random variable. This deficiency may generate unreasonable simulation results [Chien, Chowdhury, Mouskos, and Ding 1999].

In this study CORSIM is enhanced to generate emulated real time information, such as vehicle arrival and departure times at stops. This information can be potentially used to collect transit related data to implement a dynamic dispatching model.

2.7 Passenger and Vehicle Arrival Distributions

To enhance the CORSIM model to simulate bus operations, a thorough review of the literature on bus route service characteristics, such as passenger waiting times, passenger arrival rates, bus service and dwell times at bus stops was conducted and is summarized below.

Analytical approaches have been extensively employed in the analysis of service time [Adamski, 1992; Guenther and Sinha, 1983; Kraft, 1977; Zografos and Levinson, 1986], wait time [Jolliffe and Hutchinson, 1975; O'Flaherty and Mangan, 1970; Seddon and Day, 1974; Turnquist, 1978], operational control [Abkowitz, Eiger and Engelstein, 1986; Koffman, 1978; Osuna and Newell, 1972], and reliability of service [Guenther and Hamat, 1988b; Talley and Becker, 1987; Turnquist, 1978]. Generally, analytical approaches require fewer computations than simulation approaches and avoid the variance problems inherent in simulation, but are incapable of modeling stochastic and complex systems in a behaviorally realistic manner.

The most commonly assumed average passenger waiting time is one half of the vehicle headway. It can be sustained only if the headways are completely regular and passenger arrivals are random. However, this approximation is not always valid, especially when the transit headway is long and the passenger arrivals are not random. A

model for estimating the passenger waiting time $E(W)$ was developed by Welding in 1957, as shown in Eq. 2-1.

$$E(W) = E(H)/2 + V(H)/2E(H) \quad (2-1)$$

where $E(H)$ and $V(H)$ represent the mean and the variance of vehicle headways, respectively. However, Eq. 2-1 can be used for estimating the average passenger waiting time only when passenger arrivals are uniformly distributed within the headway H . This model was subsequently used in various studies [Osuna and Newell, 1972; Larson and Odoni, 1976; Lee and Schonfeld, 1991; Chien, 1995].

Jolliffe and Hutchinson [1975] provided a behavioral explanation of the association between observed bus and passenger arrivals at ten bus stops in suburban London. They categorized passengers into three groups: those who arrived coincidentally with the bus; those who arrived at random; and those who arrived at the optimal time (the time at which the expected waiting time is smallest). They indicated that the average passenger wait time could be 30 percent less when the bus arrival time could be predictable.

O'Flaherty and Mangan [1970] analyzed passenger wait times at bus stops for various service headways in central areas of London, Harrogate, and Leeds. They concluded that in evening peak periods the average wait time varied from approximately one-half to one-third of the headway, when the headway varied from 5 to 12 minutes. However, the average wait time appeared to be greater than the headway if the headway was less than two minutes. This phenomenon was associated with severe irregularities in bus service. In addition, as the bus frequency decreased from 50 buses to 5 buses per hour, the average wait time only increased from 2 to 3.5 minutes.

Seddon and Day [1974] estimated passenger wait times $E(W)$ by a regression model, as formulated in Eq. 2-2, based on data collected from the City of Manchester in England.

$$E(W) = 1.71 + 0.57 E(W_r) \quad (2-2)$$

where $E(W_r)$ represents the average wait time for randomly arriving passengers. Later, Turnquist [1978] proposed the following model (Eq. 2-3):

$$E(W) = \omega E(W_n) + (1 - \omega) E(W_r) \quad (2-3)$$

where ω , $E(W_n)$ and $E(W_r)$ represent the proportion of non-random arrivals, expected wait time for non-random, and random arrivals, respectively.

Bowman and Turnquist [1981] developed a passenger arrival model at transit stops. The model was validated using data from the Chicago area. The model indicated that the passenger wait time is much more sensitive to schedule reliability and much less sensitive to service frequency.

Headway variation among connecting routes is the primary cause of increased transfer time at transit transfer stations. Abkowitz, Eiger, and Engelstein [1986] developed an empirical headway variation model based on Monte Carlo simulations and concluded that headway variation at stops did not increase linearly with the stop location along a route. The model was validated using data collected from Route 44 in Los Angeles. The vehicle arrival distribution at any down stream station or transfer station may vary from route to route, location to location even time to time due to various geometric, control, and traffic conditions. Turnquist [1978] found that the distribution of bus arrival times at a stop was lognormal, and used that distribution for predicting

passenger wait times. Tally and Becker [1987] developed a model to estimate the probability of bus arrival time, while assuming the bus inter-arrival times are exponentially distributed. Later, Guenther and Hamat [1988b] found that the difference between the observed and the scheduled bus arrival times at a bus stop in Milwaukee followed a Gamma distribution. They also found that buses tended to arrive late during peak periods because of passenger demand and traffic congestion.

Adamski [1992] proposed probabilistic models of passenger service processes at bus stops. Since dwell time variation has important implications in the service regularity of bus operation, he computed the mean and standard deviation of the stop time based on the Exponential, Gamma and Erlang probability distributions of boarding/alighting times. After comparing the results with real world observations, he concluded that all three distributions can be used to predict dwell time variability depending on demand and levels of precision. Koffman [1978] used 4.3 seconds for each boarding passenger and 2 seconds for each alighting passenger in bus route simulations. These values were found by Kulash [1970] after analyzing a bus route in Cambridge. Zografos and Levinson [1986] analyzed passenger boarding times for a bus system operating at the University of Connecticut. They found that a passenger took 2 seconds to board on a no-fare and uncrowded bus. In crowded situations, the boarding time increased linearly with the number of in-vehicle passengers and the rank of a passenger in the waiting line.

Kraft and Deutchman [1977] demonstrated that the boarding times of bus systems in San Diego, Montreal, and New Brunswick, New Jersey fitted a k-Erlang function, where k was the number of bus doors. The average boarding times for passengers getting on/off a two-door and a one-door buses were approximately 2 seconds and 3.5 seconds,

respectively. With the findings identified in this study, the Highway Capacity Manual [HCM, 1997] indicated that the average boarding and alighting times depend on many factors, including the number and width of doors, the number and height of steps, the type of door actuation control, fare collection system, amount of baggage or parcels carried by passengers, procedures and time required to serve wheel chair passengers, seating configuration, the mix of alighting vs. boarding, and the condition and configuration of the pavement, curb and stop area.

Guenther and Sinha [1983] found that passenger boardings and alightings at stops on low ridership routes in Milwaukee fitted a Poisson distribution. Additionally, the bus dwell times, based on data collected in Lafayette, Indiana, were demonstrated to decrease with the natural logarithm of the number of passenger boardings and alightings at stops. They concluded that the negative binomial distribution was a good descriptor of passenger boardings and alightings over a range of ridership levels. Guenther and Hamat [1988a] studied transit dwell time under complex fare structure. Based on the data collected from three routes of the Southeastern Michigan Transportation Authority (SEMTA), they found that alighting time per passengers significantly decreases with more passengers per stop, while boarding time per passenger increase instead of decrease as the number of boardings increases at a stop. They also showed that boarding and alighting times were lognormally distributed.

2.8 Summary

The literature review gave an overview of how transit coordination design was studied in the past decades. The major limitations found in most studies were the assumptions of

deterministic vehicle arrival patterns and their inapplicability to large-scale intermodal transit networks. In addition, the interdependency among transfer stations with multiple routes connecting at multiple transfer stations was not considered. Moreover, most of the objective functions developed in previous studies were over simplified by neglecting important cost components (e.g., wait, in-vehicle and supplier costs) that are sensitive to transfer coordination. These deficiencies limit the applicability of those models to optimize transfer coordination in public transportation systems.

In Chapter 3, a procedure will be developed to evaluate various options for coordinated operations at multiple transfer stations. The total cost function for the coordinated routes will contain multiple variables (i.e., common headways of coordinated route groups, and slack times of all routes scheduled for coordination). Such a cost function could be categorized as unconstrained, multi-dimensional, non-linear function. A review of non-linear programming algorithms gives an overview on various algorithms and helps to select an algorithm that can solve such an objective function.

CHAPTER 3

INTERMODAL TRANSIT SYSTEM COORDINATION

3.1 Introduction

Synchronization of vehicle arrivals at transfer stations may be cost effective. However, due to demand variations over time and space, efficient coordination is difficult to achieve. Coordinated transit operations among various transit routes are classified into three categories in this study: full coordination, partial coordination, and no coordination. For a system with full coordination the operating headways of all connecting routes at a transfer station are all synchronized. For a system with partial coordination two or more but not all routes are synchronized. A system without coordination has all its routes operate independently. An optimization procedure is developed in this chapter to determine the best coordination strategy for a general intermodal transit system, containing a rail transit line (or a major transit route) with many feeder routes serving many transfer stations.

A given intermodal transit network as shown in Figure 1-1 is applied to formulate the objective total cost function consisting of user and operator costs. The headways for all transit routes and slack times for all coordinated routes, which minimize the total cost function, are optimized. The slack time is defined as the schedule delay time (the time difference between the mean arrival time and the schedule departure time) of vehicles and applies only to coordinated routes.

3.2 System Assumptions

To formulate the total cost function and evaluate coordinated operations, the following assumptions are made.

1. Transit demand is assumed to be independent with the quality of transit service (i.e., fixed demand). The demand pattern of feeder bus routes is many-to-one with a uniform distribution over space and at a given time period. This assumption may be relaxed when demand functions and passenger arrival distributions are known or can be derived empirically.
2. Locations of transit facilities (e.g., routes and stations), supply parameters (e.g., vehicle sizes, operating speeds and cost) and demand parameters (e.g., value of user's time, demand density and distribution) of the analyzed system are given.
3. Trains operating on the rail line have an exclusive right-of-way and their arrivals at stations are assumed to be deterministic. Bus arrivals at transfer stations are stochastic considering traffic congestion on streets and delays at intersections.
4. One common headway per transfer station is considered if coordination is desirable. To relax this assumption in the future, the multiple common headways situation discussed by Lee and Schonfeld [1991] can be used.
5. The probability that a vehicle arrival delay is longer than its headway is small enough to be negligible.

3.3 Rail, Bus and Transfer Demand

Demand of bus and rail is assumed to be fixed and distributed uniformly over time and space. Transfer demand is defined as the number of passengers transferring from one route to another at transfer stations. Transfer demand can be bus-to-bus, bus-to-rail and rail-to-bus transfer demands. The formulation of bus, rail, and transfer demand is discussed below.

Bus Demand

The feeder bus route directions leading to and coming away from transfer stations are denoted as directions 1 and 2, respectively. Passenger demand using feeder bus routes is either many-to-one (i.e., from bus stops to transfer stations) or one-to-many (i.e., from transfer stations to bus stops). If δ represents bus service direction ($\delta = 1$ or 2), $I_{ij\delta}$ denotes the hourly demand of bus route j at transfer station i , which can be obtained from Eq. 3-1:

$$I_{ij\delta} = \begin{cases} \sum_{d=1}^2 U_{ijd} + \sum_{k=1}^{m_i} U_{ijk} & \text{for } \delta = 1, 1 \leq i \leq n, 1 \leq j \leq m_i \\ \sum_{d=1}^2 U_{idj} + \sum_{k=1}^{m_i} U_{ikj} & \text{for } \delta = 2, 1 \leq i \leq n, 1 \leq j \leq m_i \end{cases} \quad (3-1)$$

where d represents the rail service direction ($d = 1$: from station 1 to n , and $d = 2$: from station n to 1). In Eq. 3-1, U_{ijd} and U_{idj} represent the transfer demand from bus route j at station i to rail direction d and vice versa, respectively. Similarly, U_{ijk} and U_{ikj} are transfer demands from bus route j to route k , and vice versa, where k varies from 1 to m_i , the number of feeder bus routes connecting at station i .

Rail Demand

Rail demand at each station for both rail directions can be also obtained from the demand functions formulated in Eq. 3-1. Note that rail stations are not necessarily served by feeder bus routes. If station i is served by several feeder bus routes, the inflow demand I_{id} of the rail direction d is the sum of demand from all feeder routes at the station to rail direction d plus demand α_{id} from other modes (walk, taxi, park-and-ride) to the station. Similarly, the outflow Q_{id} is the summation of demand transferring from rail direction d to all feeder routes plus the demand β_{id} destining at that station. Thus, if a rail station is not served by feeder bus routes, the rail inflows and outflows are simply the demand originating from and destining to rail at that station. The directional inflow and outflow of rail demand at station i is formulated in Eqs. 3-2 and 3-3, respectively.

$$I_{id} = \sum_{j=1}^{m_i} U_{ijd} + \alpha_{id} \quad \text{for } d \in \{1,2\}, 1 \leq i \leq n \quad (3-2)$$

$$Q_{id} = \sum_{j=1}^{m_i} U_{idj} + \beta_{id} \quad \text{for } d \in \{1,2\}, 1 \leq i \leq n \quad (3-3)$$

3.4 Model Formulation

To determine cost-effective coordination for an intermodal transit system, an objective total cost function, including supplier and user costs, is formulated in this section. The supplier cost is incurred by operating trains and buses in the system, while the user cost includes wait, transfer, and in-vehicle costs. Since demand is fixed and the locations of transit routes and stations are given, the user access cost is a constant and will not affect the optimization results. Thus, the access cost can be omitted in the objective function.

The cost structure of the route coordination model is shown in Figure 3-1. All variables used to formulate the coordination model are defined in Appendix A. The derivation of each cost component is discussed below.

3.4.1 Supplier Cost (C_o)

The total supplier cost defined in this study is the sum of rail C_{or} and bus C_{ob} operator costs:

$$C_o = C_{or} + C_{ob} \quad (3-4)$$

Both operator costs are formulated on the basis of the vehicle round trip travel time, in which the layover time is assumed to be constant and will not be considered in the total cost function. The operator cost is defined as the product of fleet size and vehicle operating cost (u_r for trains and u_b for buses). The vehicle fleet size can be obtained from the round trip time divided by the operating headway (H_r for trains or H_{ij} for bus route j at station i). For example, the rail operator cost is the product of rail fleet size F_r and average rail operating cost u_r , where F_r can be obtained by the rail round trip time T_r^R divided by the headway H_r . Ignoring the layover time, the rail round trip time consists of motion time T_r^M , stop delay time T_r^S and dwell time T_r^D :

$$T_r^R = T_r^M + T_r^S + T_r^D \quad (3-5)$$

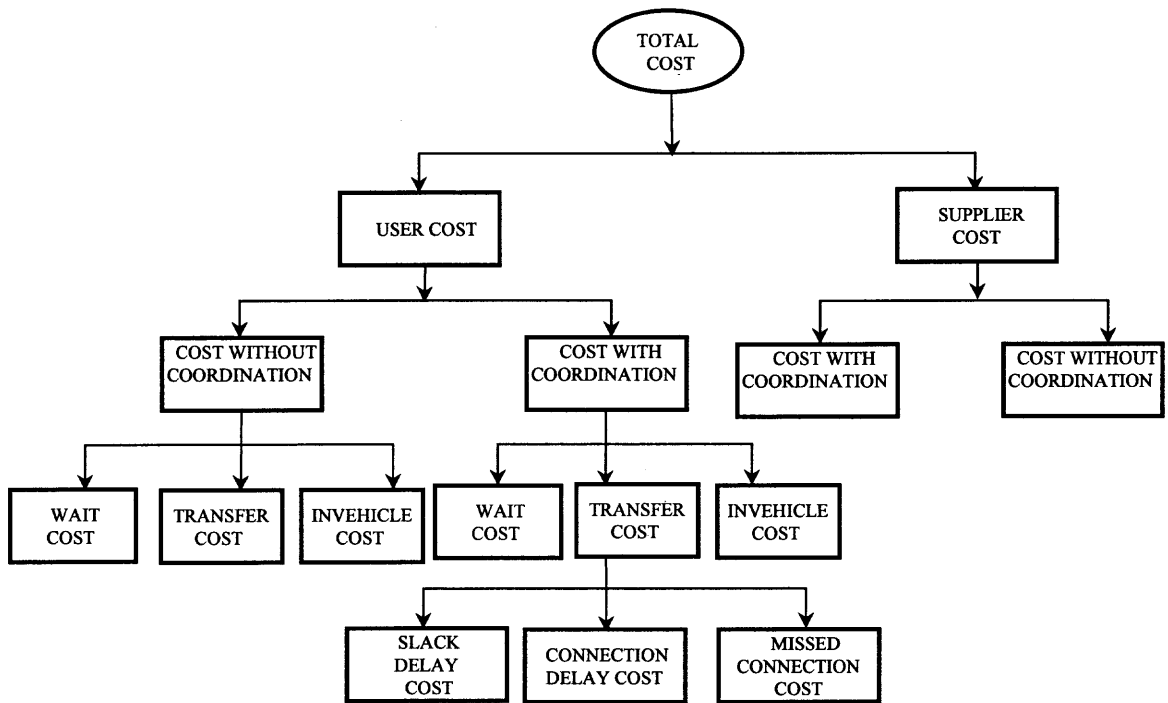


Figure 3-1 Cost Structure of Route Coordination Model

The derivations of T_r^M , T_r^S , and T_r^D are discussed in Appendix B. The rail operator cost is

$$C_{or} = \frac{T_r^R u_r}{H_r} \quad (3-6)$$

The bus operator cost of route j at station i can be similarly derived as the product of bus fleet size F_{ij} and operating cost u_b . The bus fleet size can be obtained by the bus round trip time T_{ij}^R divided by its headway H_{ij} . The bus round trip time consists of average bus motion time and dwell time, while assuming that the stop delay time incurred at bus stops and intersections is taken into consideration by using an average operating speed. Therefore, the bus motion time accounts for bus movements along the route (excluding dwell time at stops) and can be obtained by dividing bus route length L_{ij} by average bus operating speed S_{ij} . In addition, the dwell time can be obtained as the product of the number of inflow or outflow passengers of the route and the average service time q_b for one passenger boarding or alighting from a vehicle. Since one passenger generates two actions (boarding and alighting), the average round trip dwell time experienced by a bus can be obtained by multiplying demand by two. Thus, the bus round trip time can be formulated as

$$T_{ij}^R = 2 \left(\frac{L_{ij}}{S_{ij}} + \frac{\sum_{\delta=1}^2 I_{ij\delta} H_{ij}}{q_b} \right) \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq m_i \quad (3-7)$$

Considering stochastic vehicle arrivals in coordinated bus operations, it may be cost-effective to add slack time into the bus schedule to increase the probability of a successful connection with other coordinated vehicles. The slack time is part of travel

time, and the bus fleet size is the sum of bus round trip time and slack time divided by bus headway. Thus, the total bus operator cost can be derived as follows:

$$C_{ob} = \sum_{i=1}^n \sum_{j=1}^{m_i} \frac{(T_{ij}^R + K_{ij})u_b}{H_{ij}} \quad 1 \leq i \leq n, \quad 1 \leq j \leq m_i \quad (3-8)$$

where K_{ij} represents the slack time for buses operating on route j at station i .

3.4.2 User Cost (C_U)

The total user cost C_U is defined here as the sum of user waiting C_w , transferring C_t , and in-vehicle travelling C_v costs and can be formulated as

$$C_U = C_w + C_t + C_v \quad (3-9)$$

The formulation of user wait, transfer, and in-vehicle costs is discussed below.

Wait Cost

The wait cost, incurred by passengers waiting for buses or trains, is the product of average wait time, demand, and the value of user's wait time u_w . The total wait cost is the sum of the wait-bus cost C_{wb} and the wait-train cost C_{wr} :

$$C_w = C_{wb} + C_{wr} \quad (3-10)$$

Note that feeder buses serving a many-to-one demand pattern (i.e., from local bus stops to the transfer station) are moving in direction 1, and those serving a one-to-many demand pattern (i.e., the transfer station to local bus stops) are moving in direction 2. There are no passengers waiting at local bus stops in direction 2. Assuming that the average wait time is one half the headway, the wait-bus and wait-train cost can be estimated by Eqs. 3-11 and 3-12 after the coordination status of the rail or bus routes is determined.

$$C_{wb} = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^{m_i} H_{ij} I_{ij1} u_w \quad (3-11)$$

where I_{ij1} is the demand of bus route j in direction 1 at station i .

$$C_{wr} = \frac{1}{2} \sum_{i=1}^n \sum_{d=1}^2 H_r \alpha_{id} u_w \quad (3-12)$$

The wait-train cost derived in Eq. 3-12 is incurred by the passengers who can access train stations without using the feeder service. The cost incurred by passengers transferring from buses to trains is discussed next.

Transfer Cost

The transfer cost, incurred by transfer passengers from one route to another, can be obtained from the product of average transfer time, transfer demand, and the value of user wait time. The total transfer cost C_t , consisting of bus-to-bus C_{tbb} (incurred by passengers transferring from buses to buses), rail-to-bus C_{trb} (incurred by passengers transferring from rail to buses) and bus-to-rail C_{tbr} (incurred by passengers transferring from buses to rail) can be formulated as follows:

$$C_t = C_{tbb} + C_{trb} + C_{tbr} \quad (3-13)$$

where C_{tbb} , C_{trb} and C_{tbr} are formulated in Eqs. 3-14, 3-15, and 3-16, respectively.

$$C_{tbb} = \sum_{i=1}^n \sum_{j=1}^{m_i} \sum_{k=1}^{m_i} [y_{ikj} T_{ikj}^C + (1 - y_{ikj}) T_{ikj}^N] J U_{ikj} u_w \quad (3-14)$$

$$C_{trb} = \sum_{i=1}^n \sum_{j=1}^{m_i} \sum_{d=1}^2 [y_{idj} T_{idj}^C + (1 - y_{idj}) T_{idj}^N] J U_{idj} u_w \quad (3-15)$$

$$C_{tbr} = \sum_{i=1}^n \sum_{d=1}^2 \sum_{j=1}^{m_i} [y_{idj} T_{ijd}^C + (1 - y_{idj}) T_{ijd}^N] J U_{ijd} u_w \quad (3-16)$$

In Eqs. 3-14, 3-15 and 3-16, the first and the second terms inside the brackets represent the transfer times with and without coordination, respectively. The binary variable (i.e. 0 or 1) y_{idj} indicates the status of coordination between rail direction d and bus route j at station i . Similarly, y_{ikj} represents the coordination status of bus routes k and j at station i . Considering the benefit of coordination among routes, both coordinated and not coordinated transfer times for bus-to-bus (T_{ikj}^C and T_{ikj}^N), rail-to-bus (T_{idj}^C and T_{idj}^N) and bus-to-rail (T_{ijd}^C and T_{ijd}^N) transfers are formulated and discussed next.

Bus-to-bus Transfer Time

In general, the time required for passengers transferring from bus route k to j at station i depends on the coordination between the routes and the vehicle arrival distribution over time. If route j is not coordinated with route k , the average transfer time T_{ikj}^N can be estimated based on the mean and variance of headways (vehicle inter-arrival times) of route j , which was developed by Welding [1957], as shown in Eq. 3-17.

$$T_{ikj}^N = \frac{1}{2} H_{ij} \left(1 + \frac{\text{var}(H_{ij})}{H_{ij}^2} \right) \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq m_i, 1 \leq k \leq m_i \quad (3-17)$$

where H_{ij} and $\text{var}(H_{ij})$ represent the mean and the variance of headways on route j at station i , respectively.

However, if both routes are coordinated, the average transfer time T_{ikj}^C experienced by passengers transferring from route k (deliver vehicles) to j (pickup vehicles) can be obtained from the joint probability function of vehicle arrival times. The average transfer time depends on the slack time reserved in the schedule for dispatching

the pickup vehicle and its arrival distribution. Therefore, the transfer time is a function of slack time and headway, and depends on the pickup and delivery vehicle arrival times under three situations: (1) both pickup and delivery vehicles arrive on time (2) the pickup vehicle is late, and (3) the pickup vehicle left before the arrival of the delivery vehicle. Under the first two situations, a coordinated transfer is successfully made. In the third situation, all passengers from the delivery vehicle will miss the pickup vehicle and wait for the next pickup one. Therefore, the bus-to-bus transfer time consists of slack delay time, connection delay time, and missed connection time.

(1) Slack Delay Time

It was mentioned earlier that the purpose of adding slack time to vehicle schedules in coordinated routes is to increase the probability of successful transfer connections. The slack delay time incurred by passengers transferring from route k to j , called K_{jk} , is shown in Figure 3-2.

(2) Connection Delay Time

The connection delay time is the delay experienced by transfer passengers when the connection between two coordinated vehicles is successfully made, while the pickup vehicle arrives behind schedule. Two situations may exist: (1) the delivery vehicle arrives before its schedule and the pickup one arrives after its schedule, and (2) both the delivery and pickup vehicles arrive after their schedule but the delivery vehicle arrives before the pickup one.

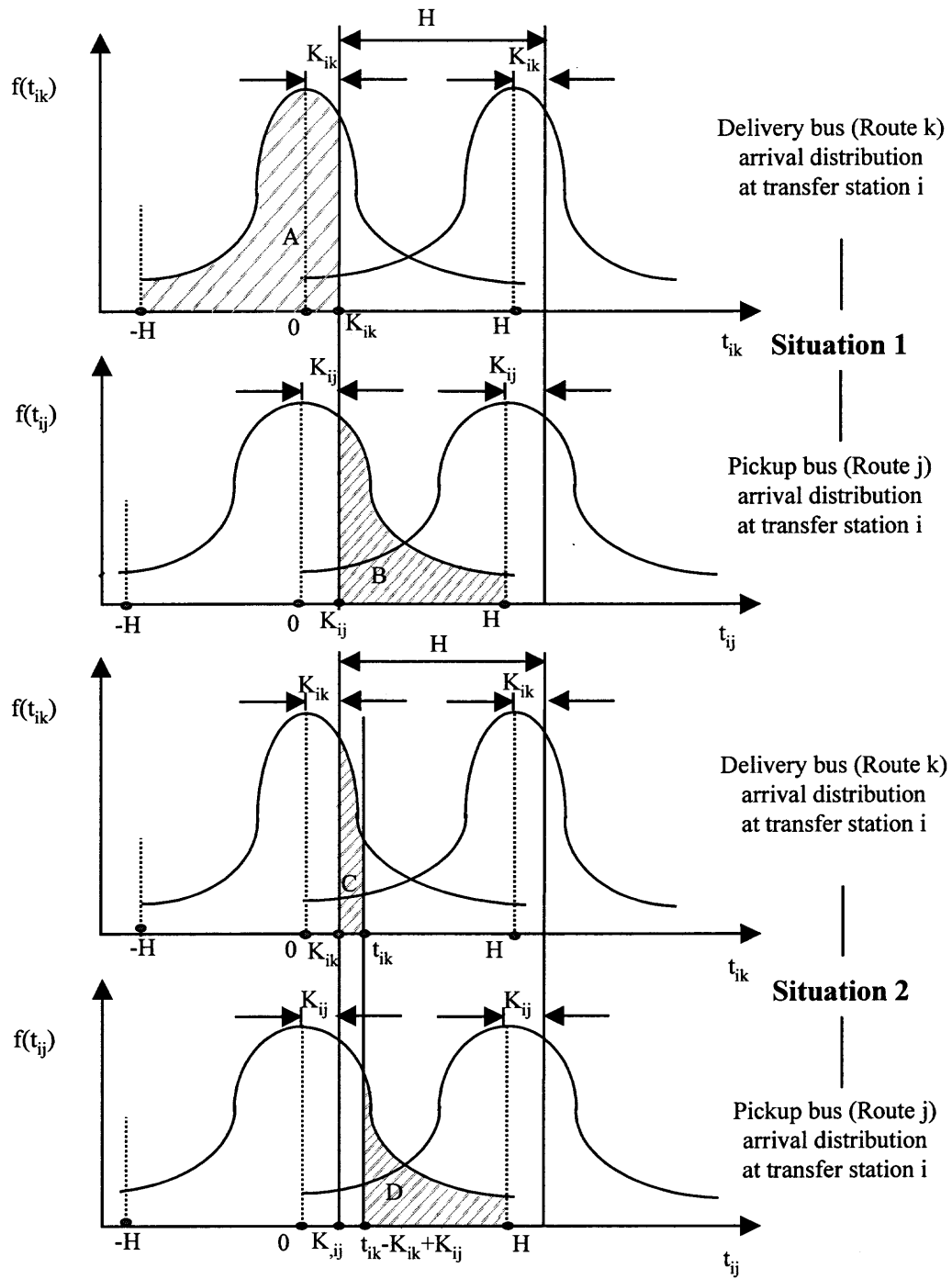


Figure 3-2 Joint Probability of Connection Delay

Since vehicle arrivals on one route are assumed to be independent from the vehicle arrival on another route, the connection delay time is a function of the joint probability of delivery and pickup vehicle arrival times. In Figure 3-2, situation 1 shows that the delivery vehicle (on route k) arrives before the scheduled time (area A) and the pickup vehicle (on route j) arrives after the scheduled time (area B). Therefore, the delivery vehicle arrival time ranges between $-H$ (representing a common headway to coordinate the vehicle arrivals) to K_{ik} , while the pickup vehicle's arrival time ranges between K_{ij} and H . In the same figure, situation 2 shows that both the delivery and pickup vehicle's arrive behind schedule but the delivery vehicle arrives (area C) before the arrival of the pickup vehicle (area D). Therefore, the pickup bus arrival time ranges between $t_{ik} - K_{ik} + K_{ij}$ and H , and the delivery bus arrival time ranges between K_{ik} and t_{ik} . Thus, the connection delay time for passengers transferring from route k to route j at station i is formulated as

$$D_{ikj} = \int_{-H}^{K_{ik}} f(t_{ik}) dt_{ik} \int_{K_{ij}}^H (t_{ij} - K_{ij}) f(t_{ij}) dt_{ij} + \int_{K_{ik}}^H f(t_{ik}) dt_{ik} \int_{t_{ik} - K_{ik} + K_{ij}}^H (t_{ij} - t_{ik} + K_{ik} - K_{ij}) f(t_{ij}) dt_{ij}$$

for $1 \leq i \leq n, 1 \leq j \leq m_i, 1 \leq k \leq m_i$ (3-18)

(3) Missed Connection Time

The missed connection delay time is the delay experienced by passengers when connections between two vehicles on coordinated routes fail due to the departure of the pickup vehicle before the arrival of the delivery vehicle. The missed connection delay time may exist under two situations: (1) the pickup vehicle arrives before schedule and the delivery vehicle arrives after its scheduled time and (2) both delivery and pickup

vehicles arrive behind schedule, but the delivery bus arrives after the departure of the pickup vehicle. Under both situations, transfer passengers will miss the pickup vehicle and wait for the next one.

In Figure 3-3, situation 1 shows that the pickup vehicle arrives early (area F), while the delivery vehicle arrives behind schedule (area E). Under this situation, the pickup vehicle arrival time ranges between $-H$ to K_{ij} , and the delivery vehicle arrival time ranges between K_{ik} to H . In situation 2, both the delivery and pickup vehicles arrive behind schedule, but the delivery vehicle arrives (Area G) after the departure of the pickup vehicle (area H). Therefore, the arrival time of the delivery vehicle ranges from t_{ik} to H , and the pickup vehicle arrival time ranges between K_{ij} to $t_{ik} - K_{ik} + K_{ij}$, where t_{ik} varies from K_{ik} to H . The average missed connection time is the sum of the product of areas E and F and the product of areas G and H, multiplied by the common headway H , as formulated below.

$$M_{ikj} = \left[\int_{K_{ik}}^H f(t_{ik}) dt_{ik} \int_{-H}^{K_{ij}} f(t_{ij}) dt_{ij} + \int_{K_{ik}}^H f(t_{ik}) dt_{ik} \int_{K_{ij}}^{t_{ik} - K_{ik} + K_{ij}} f(t_{ij}) dt_{ij} \right] H$$

$$\text{for } 1 \leq i \leq n, 1 \leq j \leq m_i, 1 \leq k \leq m_i \quad (3-19)$$

By adding up the slack delay, the connection delay and the missed connection times, the average transfer time T_{ikj}^C for passengers transferring from route k to j at station i is

$$T_{ikj}^C = K_{ik} + D_{ikj} + M_{ikj} \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq m_i, 1 \leq k \leq m_i \quad (3-20)$$

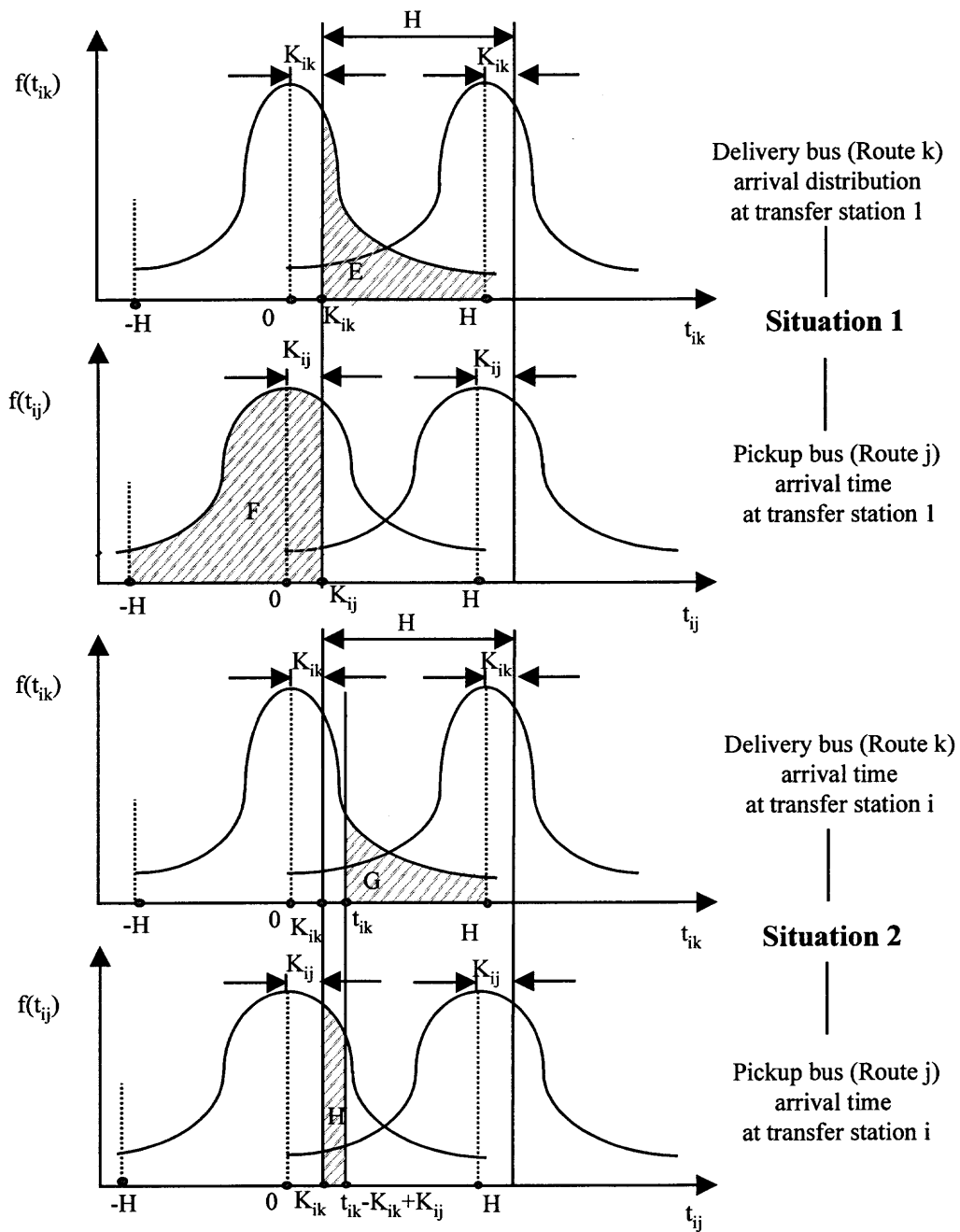


Figure 3-3 Joint Probability of Missed Connection Delay

T_{ikj}^C can be obtained by using the value of the slack delay K_{ik} , and the results obtained from Eqs. 3-18 and 3-19.

Rail-to-bus and Bus-to-rail Transfer Times

The rail-to-bus and bus-to-rail transfer times also depend on the status of coordination between rail and bus routes. If bus route j at station i is not coordinated with rail direction d , the average transfer time from rail direction d to bus route j , T_{idj}^N , and from bus route j to rail direction d , T_{ijd}^N , can be estimated by Eqs. 3-21 and 3-22, respectively.

$$T_{idj}^N = \frac{1}{2} H_{ij} \left(1 + \frac{\text{var}(H_{ij})}{H_{ij}^2} \right) \quad \text{for } 1 \leq i \leq n, d \in \{1,2\}, 1 \leq j \leq m_i \quad (3-21)$$

$$T_{ijd}^N = \frac{1}{2} H_r \quad \text{for } 1 \leq i \leq n, d \in \{1,2\}, 1 \leq j \leq m_i \quad (3-22)$$

The difference between Eqs. 3-21 and 3-22 is that the variance of train arrivals at stations in the second equation is zero because of assumed deterministic train arrivals. Under coordinated operation, the average transfer times in the categories of rail-to-bus T_{idj}^C and bus-to-rail T_{ijd}^C are determined based on the bus and train arrival distributions. Since trains arrive deterministically, the transfer time from rail direction d to bus route j only depends on the probability that buses arrive behind schedule as shown in Figure 3-4 (area X), where the range of bus arrival time varies from K_{ij} to H . On the other hand, the transfer time for passengers from route j at station i to rail direction d depends on the slack delay time K_{ij} and the probability of missed connection time M_{ijd} due to the late arrivals of buses on route j as shown in Figure 3-4 (area X), where the range of bus arrival time varies from K_{ij} to H .

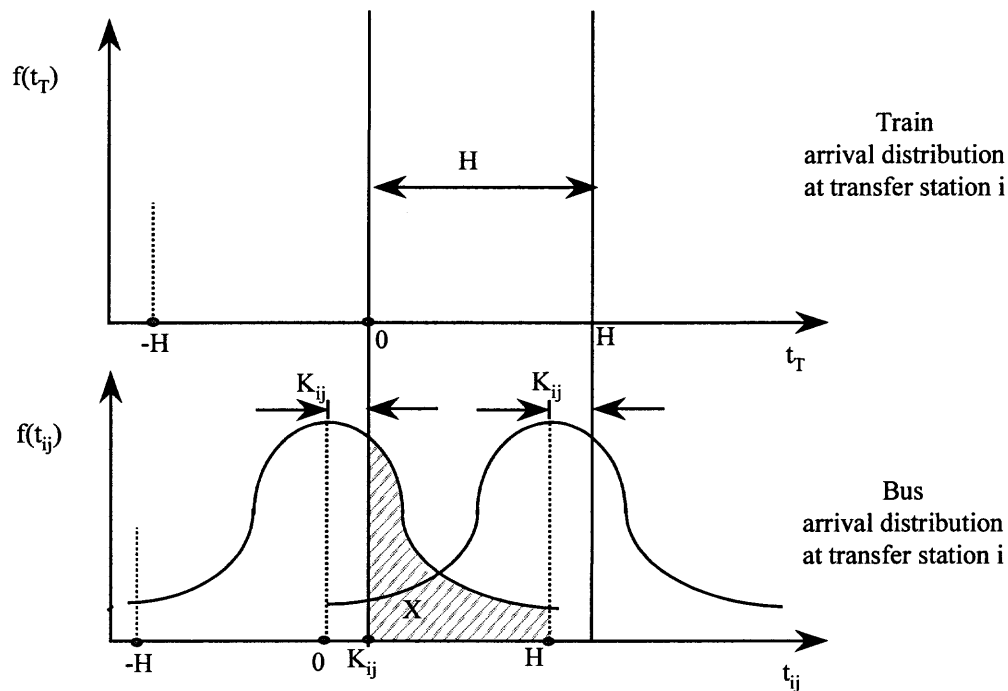


Figure 3-4 Train and Bus Arrival Distributions with Coordination

The average transfer times at station i experienced by passengers transferring from rail direction d to bus route j and from bus route j to rail direction d are formulated in Eqs. 3-23 and 3-24, respectively.

$$T_{idj}^C = \int_{K_{ij}}^H (t_{ij} - K_{ij}) f(t_{ij}) dt_{ij} \quad \text{for } 1 \leq i \leq n, d \in \{1,2\}, 1 \leq j \leq m_i \quad (3-23)$$

$$T_{ijd}^C = K_{ij} + \int_{K_{ij}}^H f(t_{ij}) dt_{ij} H \quad \text{for } 1 \leq i \leq n, d \in \{1,2\}, 1 \leq j \leq m_i \quad (3-24)$$

where t_{ij} , K_{ij} , and $f(t_{ij})$ represent the bus arrival time, slack time and bus arrival distribution, respectively. The total transfer cost could be obtained from Eq. 3-13 after

substituting the corresponding transfer times (derived from Eqs. 3-17 through 3-24) into Eqs. 3-14 through 3-16.

In-vehicle Cost (C_v)

In general, the in-vehicle cost C_v is the product of the average in-vehicle time, the corresponding demand, and the value of users' in-vehicle time u_v . The in-vehicle cost includes two components: in-bus cost C_{vb} and in-train cost C_{vr} .

$$C_v = C_{vb} + C_{vr} \quad (3-25)$$

(1) In-bus Cost

The in-bus cost is formulated on the basis of the known average bus journey time, which accounts for bus moving and dwell times. The bus moving time is the journey distance divided by the average bus operating speed (S_{ij}) as discussed previously. The average journey distance for passengers is one half of the route length ($L_{ij}/2$) because the demand is assumed to be uniformly distributed over the route. In addition, the average bus dwell time is demand divided by the passenger boarding/alighting rate. Thus, the in-bus cost is

$$C_{vb} = \sum_{i=1}^n \sum_{j=1}^{m_i} \sum_{\delta=1}^2 \left(\frac{L_{ij}}{2S_{ij}} + \frac{H_{ij} I_{ij\delta}}{2q_b} \right) I_{ij\delta} u_v \quad (3-26)$$

where $I_{ij\delta}$ represents the demand for bus route j at station i in direction δ .

(2) In-train Cost

The in-train cost, incurred by in-train passengers is generated by trains moving between stations, and dwelling at stations:

$$C_{vr} = C_{vr1} + C_{vr2} \quad (3-27)$$

where C_{vr1} and C_{vr2} represent the components of in-train costs caused by vehicle moving and dwelling times, respectively. Similar to the in-bus cost, the in-train cost is defined as the product of the rail demand, average in-train time and the value of users' in-vehicle time. A portion of in-train cost, constituted by the train moving time, is the sum of inter-station travel times (interstation spacing between station i and $i+1$ divided by average operating speed = l_i / V_i), multiplied by the corresponding travel demand between each pair of stations and the value of users' in-vehicle time. C_{vr1} is formulated as

$$C_{vr1} = \sum_{i=1}^{n-1} \sum_{d=1}^2 \frac{A_{id} l_i u_v}{V_i} \quad (3-28)$$

where A_{id} represents the rail demand between station i and $i+1$ for direction d and is formulated below

$$A_{i1} = \sum_{l=1}^i (I_{l1} - Q_{l1}) \quad \text{for } 1 \leq i \leq n-1 \quad (3-29)$$

$$A_{i2} = \sum_{m=i+1}^n (I_{m2} - Q_{m2}) \quad \text{for } 1 \leq i \leq n-1 \quad (3-30)$$

where I_{l1} , Q_{l1} , I_{m2} , and Q_{m2} represent inflow and outflow demand at station l and m in direction 1 and 2, respectively. The derivation of average interstation train operating speed V_i is discussed in Appendix B.

Another portion of in-train cost C_{vr2} is caused by trains dwelling at stations. C_{vr2} is defined as the product of in-train passengers, the dwell time, and the value of users' in-vehicle time. The duration of dwell time at stations depends on the number of boarding

and alighting passengers and the boarding/alighting rate. For a given demand, the longer the headway, the more passengers wait at stations increasing the dwell time. Therefore, the dwell time at a station is the sum of inflow and outflow demand at the station multiplied by the headway and divided by the passenger boarding/alighting rate q_r .

Thus, C_{vr2} can be formulated as

$$C_{vr2} = \sum_{i=2}^{n-1} \sum_{d=1}^2 [A_{id} - \{I_{id}(2-d) + Q_{id}(d-1)\}] (I_{id} + Q_{id}) \frac{H_r u_v}{q_r} \quad \text{for } 1 \leq i \leq n, d \in \{1,2\} \quad (3-31)$$

The total cost function C_T can be obtained by substituting the derived cost functions into Eq. 3-32. The optimal decision variables (e.g., common headway and slack times) can be obtained by minimizing Eq. 3-32 subject to capacity and non-negativity constraints formulated in Eqs. 3-33 and 3-34, respectively.

$$C_T(H_{ij}, H_r, K_{ij}) = C_O(H_{ij}, H_r, K_{ij}) + C_U(H_{ij}, H_r, K_{ij}) \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq m_i \quad (3-32)$$

$$\text{Subject to: } \begin{cases} P_{ij} \geq D_{ij} \\ P_r \geq D_r \end{cases} \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq m_i \quad (3-33)$$

$$\begin{cases} H_{ij} > 0 \\ H_r > 0 \\ K_{ij} \geq 0 \end{cases} \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq m_i \quad (3-34)$$

where P_r , P_{ij} , D_r and D_{ij} represent the service capacities of the rail line and bus route j at station i and the maximum loads of the rail line and bus routes, respectively. The derivation of P_{ij} and P_r are discussed in the Appendix C.

3.5 Optimization

In this section the optimization of decision variables (i.e., rail and bus headways, and slack times of coordinated routes) for the analyzed intermodal transit system are discussed. All the decision variables are optimized by minimizing the total cost function formulated in Eq. 3-32, while the coordinated routes at each transfer station are determined by a four-stage optimization algorithms developed in this section.

If no coordination is used in the system, headways will be optimized analytically. However, if coordination is used, headways and slack time for each coordinated route will be jointly optimized by applying a multi-dimensional non-linear optimization algorithm. Powell's method is selected in this study to optimize the problem. Capacity constraints are taken into consideration while determining the optimized headways for coordinated routes.

Stage-I: No Coordination

At this Stage, since coordination among routes is not considered, the total cost derived in Eq. 3-32 is purely a function of route headways. After taking the derivative of the total cost function with respect to each route headway and setting it equal to zero, the optimal headway of each route can be obtained from Eq. 3-35. Capacity constraints are considered in the optimization process. The uniqueness of the optimal headway depends on the convexity of the objective function. The function with the decision variables (i.e., route headways) is strictly convex only if the second derivative of the cost function with respect to headway is positive.

After taking the first derivative of the total cost function with respect to bus headway H_{ij} and setting it equal to zero, the analytical relations for the headway of bus route j at station i with various demand parameters (e.g., route length, operating speed, demand and value of user's times) is derived and shown in Eq. 3-35.

$$H_{ij} = \sqrt{\frac{\frac{2L_{ij}}{S_{ij}} u_b + 0.5 \text{var}(H_{ij}) (\sum_{k=1}^{m_i} U_{ikj} + \sum_{d=1}^2 U_{idj}) u_w}{\frac{1}{2} (\sum_{k=1}^{m_i} U_{ikj} + \sum_{d=1}^2 U_{idj}) u_w + \frac{1}{2} I_{ijl} u_w + \sum_{\delta=1}^2 \frac{(I_{ij\delta})^2}{2q_b} u_v}} \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq m_i \quad (3-35)$$

In addition, the second derivative of the total cost function with respect to H_{ij} is

$$\frac{\partial^2 TC}{\partial H_{ij}^2} = \frac{4L_{ij}}{H_{ij}^3 S_{ij}} u_b + \frac{\text{var}(H_{ij})}{H_{ij}^3} (\sum_{k=1}^{m_i} U_{ikj} + \sum_{d=1}^2 U_{idj}) u_w \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq m_i \quad (3-36)$$

In Eq. 3-36, since all parameters (e.g., round trip travel time, and average vehicle operating cost) are positive, the second derivative of the total cost function with respect to H_{ij} is positive. Therefore, the optimal H_{ij} obtained from Eq. 3-35 is unique.

Similarly, the first derivative of the total cost function with respect to rail headway gives the following analytical relations

$$H_r = \sqrt{\frac{(T_r^M + T_r^S) u_r}{\frac{1}{2} \sum_{i=1}^n \sum_{d=1}^2 (\alpha_{id} + \sum_{j=1}^{m_i} U_{ijd}) u_w + \sum_{i=2}^{n-1} \left\{ \sum_{d=1}^2 [A_{id} - \{I_{id}(2-d) + Q_{id}(d-1)\}] (I_{id} + Q_{id}) \right\} \frac{u_v}{q_r}}} \quad (3-37)$$

In addition, the second derivative of the total cost function with respect to H_r is

$$\frac{\partial^2 TC}{\partial H_r^2} = \frac{2(T_r^M + T_r^S) u_r}{H_r^3} \quad (3-38)$$

In Eq. 3-38, all parameters (e.g., motion time, stop delay time, demand, average operating cost) are positive, and the optimal H_r obtained from Eq. 3-37 is also unique.

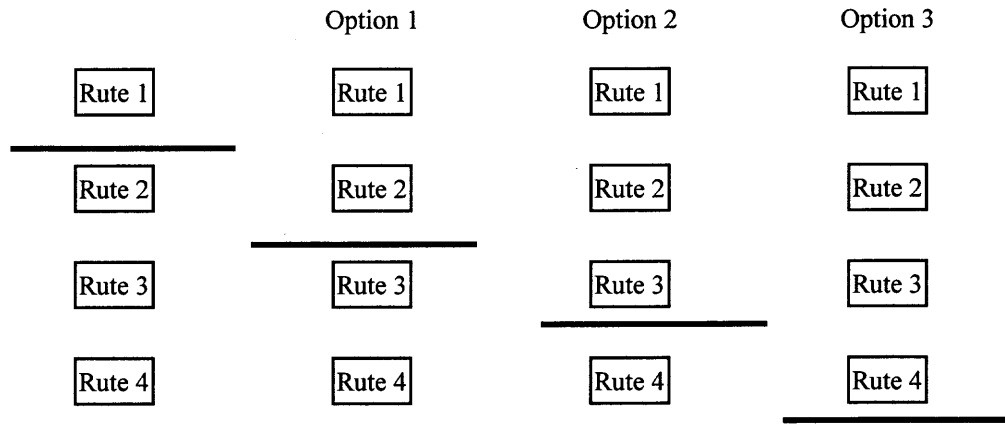
Stage II: Bus Route Coordination at Isolated Transfer Stations

The procedure for optimizing bus route coordination at isolated transfer stations is discussed in this section. First, all bus routes at each transfer station are arranged in a descending order based on the headways obtained from Eq. 3-35. Then, a boundary line is initiated between the two largest headways as shown in Figure 3-5, in which all bus routes at each transfer station operate without coordination. The total cost at this situation is computed by substituting the optimal values of the decision variables into Eq. 3-32, which is identical to that obtained at stage I.

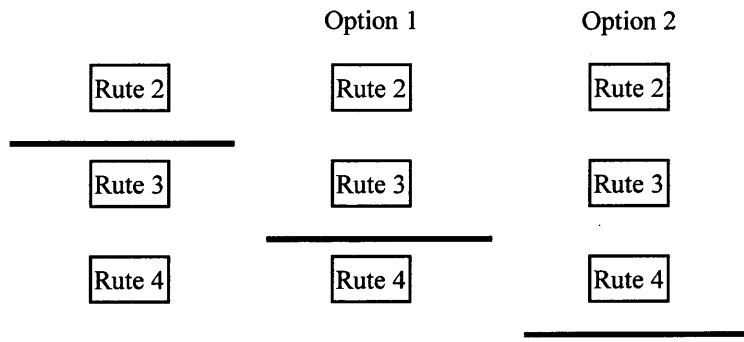
The boundary line then moves downward to include a new route at a time for considering coordination. After the first move, there are two routes above the boundary line, and both the routes are considered to be coordinated, while all other routes below the boundary line operate without coordination. The corresponding minimum total cost can be computed again using Eq. 3-32 after determining the optimal common headway and slack times by using Powell's algorithm [Powell M. J. D., 1964; Himmelblan 1972; Press, H. et. al. 1992].

The boundary line continues to move downward a step at a time until all routes are above the boundary line. Thus, the first cycle is completed. After completing the first cycle, the route with the longest headway is not considered to be coordinated with other routes. The route is thus removed from the list in the second and all subsequent cycles.

Cycle 1



Cycle 2



Cycle 3

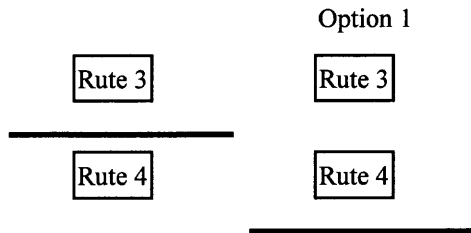


Figure 3-5 Example for Determining Bus Route Coordination

The optimal common headway and slack times yielding the minimum total cost are computed by using Powell's algorithm. The number of cycles varies with the number of bus routes connecting at the transfer station. For example, if there are m_i bus routes connecting at transfer station i , the total number of cycles required to be completed is $m_i - 1$. After searching the optimal solutions for all cycles, the minimum total cost can be found. The most cost-effective coordination within the system also emerges. Figure 3-5 shows the procedure to determine the optimal coordination for four routes connecting at a transfer station.

The step by step procedure that determines the optimal headway for each coordinated group and the optimal slack time for each route within the coordinated group is described below. Figure 3-6 shows the flow diagram of bus route coordination procedures.

- Step 1 List all routes by sorted headways (obtained from Eq. 3-35) in a descending order and set the boundary line between the top two routes in the list.
- Step 2 Calculate the total cost without coordination by substituting the optimal values of decision variables into Eq. 3-32.
- Step 3 Shift the boundary line to the next route and optimize the common headway and slack times for all routes above the boundary line by using Powell's algorithm.

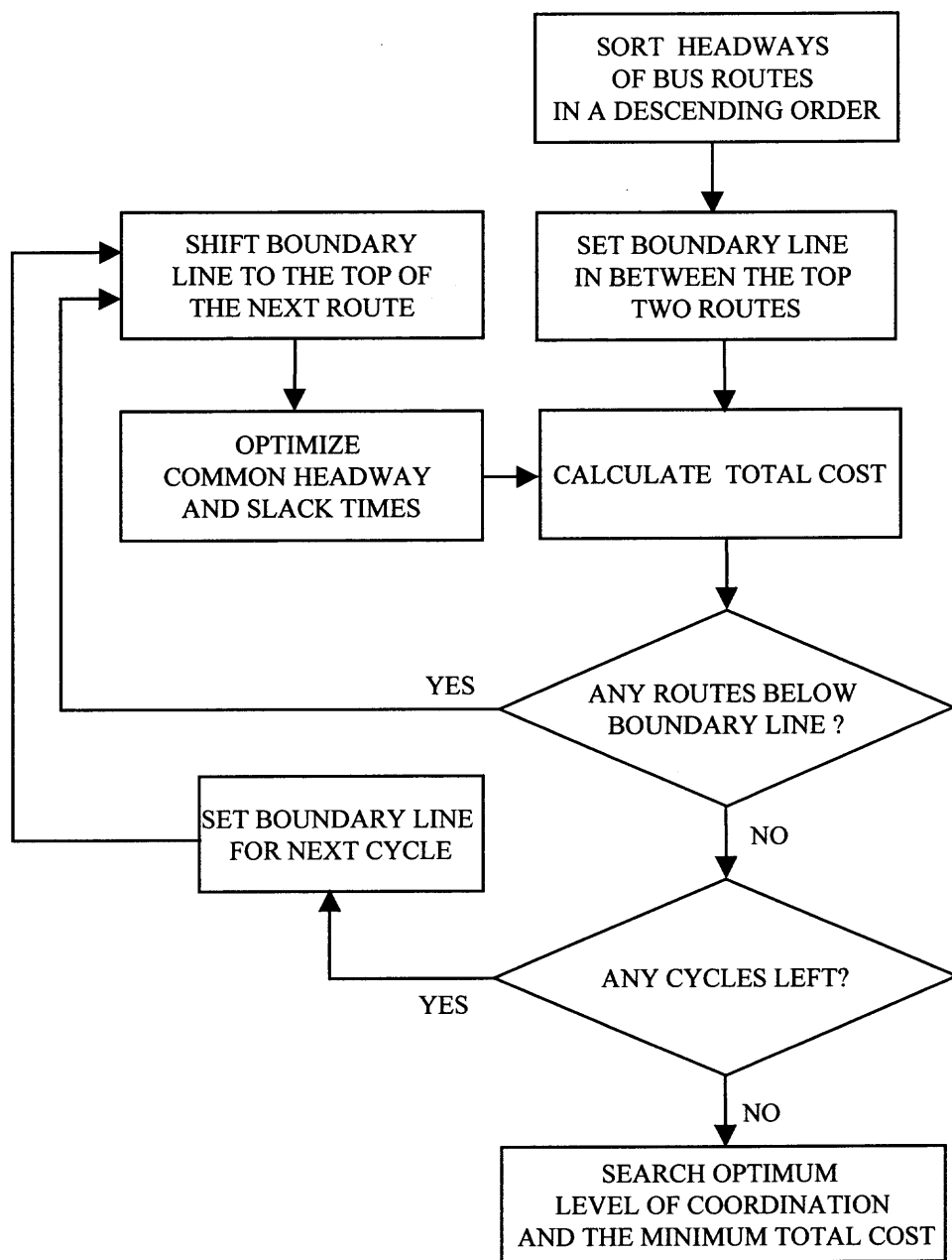


Figure 3-6 Stage-II Procedure Optimizing Bus Coordination at Isolated Transfer Station

- Step 4 Calculate the total cost using Eq. 3-32 after substituting optimal common headway and slack times of all coordinated routes (routes above the boundary line) obtained from the minimization of the total cost objective function using Powell's algorithm and the optimal headways of routes that are not coordinated (all routes below boundary line) obtained from Eqs. 3-35 and 3-37. Record the minimum total cost and the coordinated status of each route.
- Step 5 If there are routes below the boundary line in the list go to Step 3; otherwise, go to Step 6.
- Step 6 Remove the first route from the current list and consider this route operates without coordination at all subsequent iterations. If more than one route remain in the list, shift the boundary line to the next route and go to Step 3; otherwise, go to Step 7.
- Step 7 Pick among all iterations the one with the minimum total cost. The corresponding coordination plan is the optimum.

Stage-III: Rail-Bus Coordination at Isolated Transfer Station

At this stage, the coordinated bus groups identified at Stage II are considered to be coordinated with the rail line. At each transfer station, the identified coordinated group is evaluated for coordination with rail directions 1 and 2 separately. While evaluating rail-bus coordination, the headway of the bus routes at other transfer stations obtained from Stage II will not be changed. The procedure, shown in Figure 3-7, can help to determine which rail direction and bus routes should be coordinated.

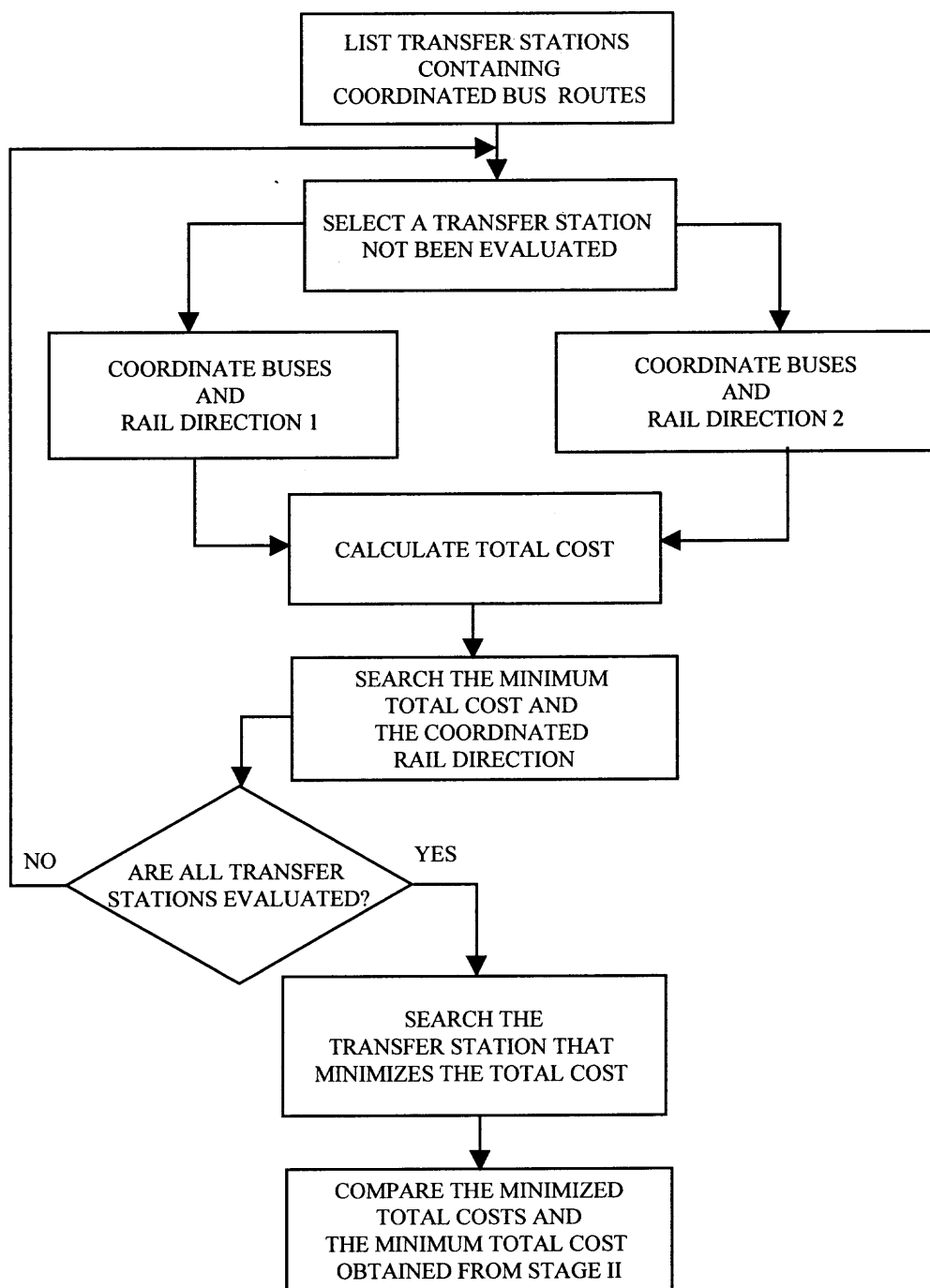


Figure 3-7 Stage-III Procedure

Optimizing Rail-Bus Coordination for Isolated Transfer Station

The step by step procedure is described below.

- Step 1 List all transfer stations that contain coordinated groups.
- Step 2 From the list select a transfer station, which has not been evaluated, and find the optimal variables for the coordination of rail directions 1 and 2 and the coordinated group. Record the coordinated total cost (Eq. 3-32), the optimal common headways, and slack times.
- Step 3 Search the minimum total cost obtained at Step 2. Record the minimum total cost and the coordinated rail direction. If all stations are evaluated go to Step 4; otherwise go to Step 2.
- Step 4 Search the minimum total cost from the recorded total costs in Step 3. Record the minimum total cost for rail-bus coordination.
- Step 5 Compare the total cost recorded in Step 4 and the minimum total cost obtained from Stage II. The minimum total cost scheme determines the optimum coordination.

Stage IV: Networkwide Coordination

The procedure developed in this Stage is to achieve rail-bus coordination for multiple transfer stations. All transfer stations containing rail-bus coordination obtained from Step 3 at Stage III are evaluated. The procedure developed at this stage is similar to that developed at Stage II (see Figure 3-8).

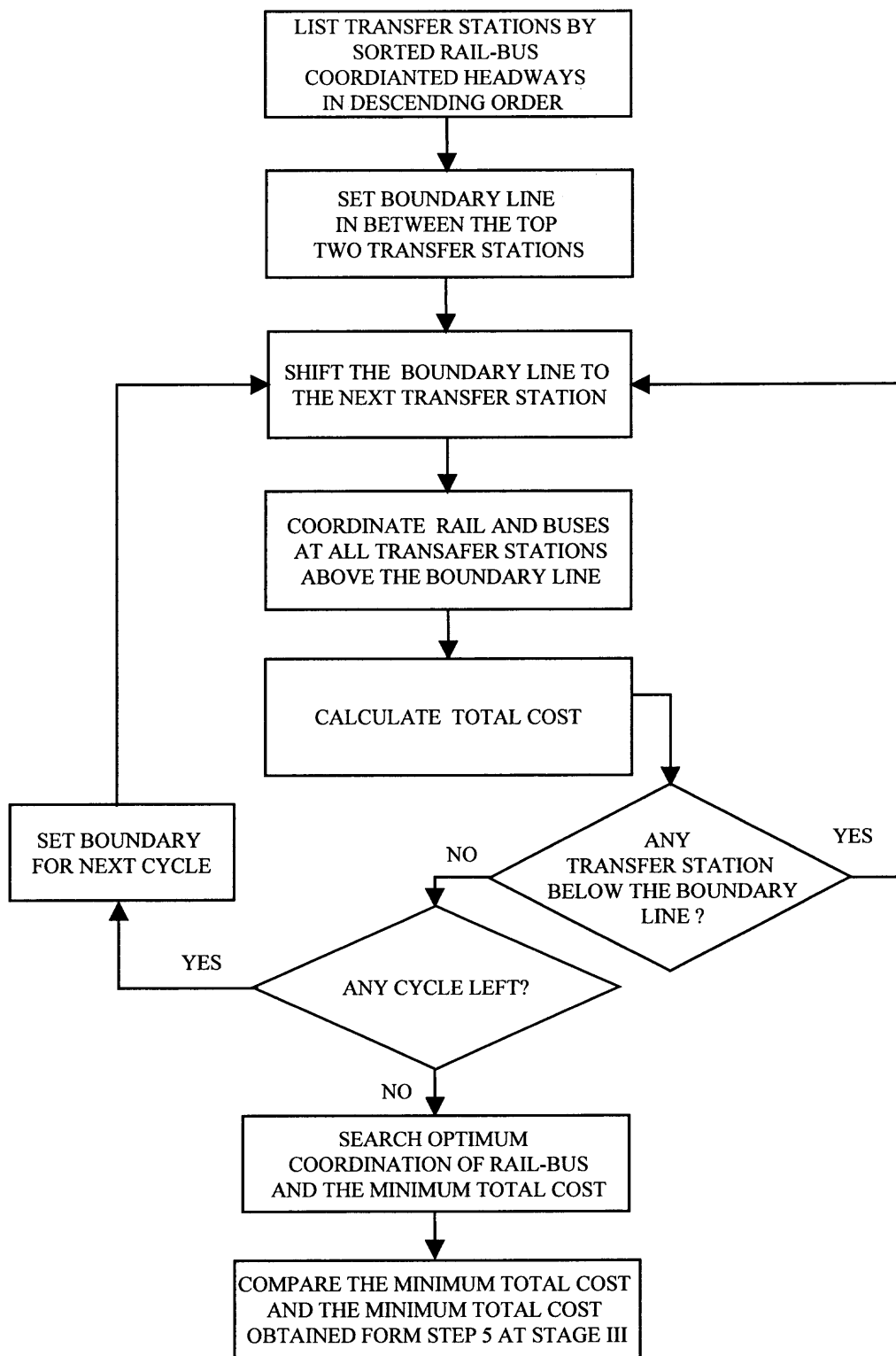


Figure 3-8 Stage-IV Procedure Optimizing Coordination for Multiple Transfer Stations

The step by step procedure is discussed below.

- Step 1 List all coordinated transfer stations identified at Stage III by sorted common headways in a descending order and set the boundary line between the top two transfer stations in the list.
- Step 2 Shift the boundary line to the next transfer station. Coordinate rail service and buses at all transfer stations above the boundary line. Optimize the common headway and slack times for all coordinated bus routes and calculate the total cost using Eq. 3-32.
- Step 3 If there is no transfer station below the boundary line, go to Step 4; otherwise, go to Step 2.
- Step 4 Remove the first transfer station from the list and consider that there is no rail-bus coordination at this transfer station at all subsequent iterations. If there are more than one transfer stations remaining in the current list, set the boundary line between the top two transfer stations and go to Step 2; otherwise go to Step 5.
- Step 5 Search the minimum total cost by comparing the total costs calculated in Step 2.
- Step 6 Compare the total costs obtained in Step 5 and the minimum total cost obtained from Step 5 at Stage III. The coordination scheme of each route in the intermodal transit system is the one that generates the minimum total cost.

3.6 Efficiency of Optimization Procedure

To determine the optimal intermodal transit system coordination, combinations of coordinated routes need to be evaluated. However, the number of combination increases significantly as the number of transfer stations and feeder routes increase. Therefore, the computation time for the whole optimization process will be huge.

In this section a comparative analysis is conducted. The numbers of iterations required to solve the intermodal coordination problem by applying the developed four-Stage procedure and searching all combinations of coordination are investigated. The numbers of iterations required at Stage II for the intermodal network containing m_i feeder routes at station i with and without applying the proposed procedure are shown in Table 3-1. The iterations required at Stages III and IV with n transfer stations are shown in Table 3-2. Both tables show that the number of iterations increase as the number of feeder routes and transfer stations increases. For example, if there are 9 feeder routes connected at transfer station i , the number of iterations required to seek a coordinated route group at Stage II by applying the proposed method is 36. However, if all combinations are evaluated, the number of evaluation will be 502. Similarly, considering 9 transfer stations in the system the total number of iterations required together at Stages III and IV with and without applying the proposed method are 54 and 19682, respectively.

Table 3-1 Total Number of Iterations Required at Stage-II

# OF ROUTES (m_i)	# OF ITERATIONS	
	ALL COMBINATIONS $\sum_{k=2}^n C_k^n$	PROPOSED PROCEDURE $\frac{n(n-1)}{2}$
2	1	1
3	4	3
4	11	6
5	15	10
6	57	15
7	120	21
8	247	28
9	502	36

C_k^n represents the number of combinations of n different things, taken k at a time, without repetitions.

Table 3-2 Total Number of Iterations Required at Stages III and IV

# OF TRANSFER STATIONS INVOLVE WITH COORDINATED BUS ROUTES (n)	# OF ITERATIONS	
	ALL COMBINATIONS $2n + \sum_{i=2}^n [(2^i)(C_i^n)]$	PROPOSED PROCEDURES $2n + \frac{n(n-1)}{2}$
1	2	2
2	8	5
3	26	9
4	80	14
5	242	20
6	728	27
7	2186	35
8	6560	44
9	19682	54

CHAPTER 4

DYNAMIC VEHICLE DISPATCHING

4.1 Introduction

The model developed in Chapter 3 can be used to determine the most cost-effective coordination for a given transit network by optimizing vehicle headways and slack times. It was assumed in Chapter 3 that the transfer demand among transit routes in the network was uniformly distributed over a specific time period. However, in reality, both the transfer demand and the vehicle arrivals at a transfer station vary from time to time. Considering the variation of vehicle arrival times at transfer stations, the fixed slack time optimized in Chapter 3 can not guarantee that coordinated transfers are cost effective in the following conditions: (1) if a coordinated delivery vehicle arrives after the reserved slack time of a pickup vehicle ready at the transfer station, but the delivery vehicle carries more transfer passengers than expected, (2) if a successful connection can be made within the reserved slack time, and (3) if the number of transfer passengers is significantly low. If condition 1 occurs, the pickup vehicle should be held longer. At conditions 2 and 3, the ready pick-up vehicle should be dispatched immediately without holding for the late delivery vehicle. Therefore, a dynamic vehicle dispatching strategy is important for improving transfer efficiency.

The holding decision for a coordinated pick-up vehicle ready at a transfer station should be evaluated by minimizing the dynamic total cost objective function. The total cost function consists of operator cost caused by holding a vehicle, and the connection delay and missed connection costs incurred by transfer passengers. All cost components

can be formulated if the holding duration is known and the arrival time of a late vehicle is predictable. The benefits of coordinated transfers will encourage transit providers to use AVL for real-time monitoring and supervising vehicle operations and thus accurately predicting vehicle arrival times. An enhanced simulation model [Chien, Chowdhury, Mouskos, and Ding, 1999] is required to emulate an AVL environment and deriving vehicle arrival distributions from various locations along bus routes to the transfer station.

Since the rail line usually provides trunk service, trains will carry higher demand than feeder buses. Moreover, in contrast to feeder bus routes, the rail line serves many transfer stations. The holding decision made for trains may significantly increase user wait cost at down-stream stations. Therefore, the expenses for holding a train may be greater than the benefit gaining from the reduction of the passenger transfer time. Hence, the decision of holding trains is not expected to be made.

In this chapter, the objective total cost function is formulated, assuming that the feeder vehicle dispatching time can be determined based on real-time information. A procedure for optimizing dynamic vehicle dispatching for coordinated vehicles is developed based on the known vehicle arrival distribution and transfer demand.

4.2 System Assumptions

In addition to the assumptions made in chapter 3, the following ones are made for formulating the objective total cost function affected by a dynamic vehicle-dispatching decision.

1. Vehicle arrival times at transfer stations are assumed to be either known or predictable. In Chapter 5, the enhanced CORSIM model will be introduced for

emulating an AVL system, and the method for deriving vehicle arrival distributions will be introduced.

2. According to assumption 1, the locations of late vehicles and their arrival distributions at transfer stations are available. To estimate late vehicle arrival times, a number of checkpoints (can be located at any bus stops) along the route are designated. The travel time variation from any checkpoint to the transfer station could be calculated based on the simulation outputs. Any decent arrival time prediction model, such as artificial neural networks (ANNs) [Chien and Ding 1999], multivariate regression models [Abdelfattah and Khan 1997, Zeng and Lin 1999] and Kalman filtering models [Wall and Dailey, 999] can be applied for this purpose.
3. Transfer demand from one vehicle to another is known or predictable. In the numerical example discussed in Chapter 6, the transfer demand will be given, while various dynamic dispatching strategies will be evaluated.

4.3 Cost Functions

The objective function for dynamic vehicle dispatching on coordinated routes is defined by the total cost, including supplier and user costs. Unlike the cost components defined in Chapter 3, the supplier cost considered in this chapter is caused by holding a vehicle, while the user cost, including the connection delay and missed connection costs, incurred by transfer passengers is affected by the holding decision for the vehicle. The cost structure of the dynamic vehicle dispatching model is shown in Figure 4-1. All variables used to formulate the total cost function are defined in Appendix A.

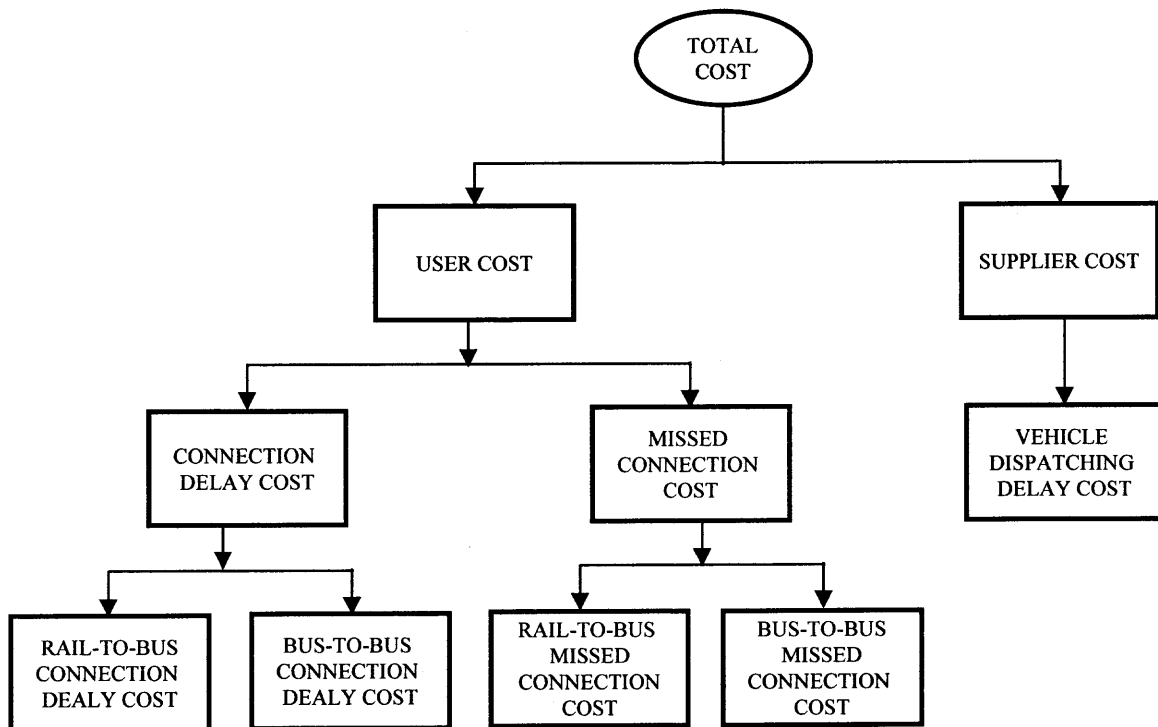


Figure 4-1 Cost Structure for Dynamic Vehicle Dispatching Model

The dispatching decision for a coordinated vehicle will be evaluated when the departure time of the vehicle is approached. If the vehicle is determined to be held for a late coordinated vehicle, its dispatching time will be re-evaluated periodically (e.g., every 30 seconds in this study). While evaluating the dispatching decision, the cost associated with holding the vehicle is independent of the dispatching decision of other coordinated vehicles that have arrived at the transfer station. Thus, the cost functions for dispatching vehicles can be determined individually. Hence, the optimal dispatching time (either with or without holding) of each coordinated vehicle can be determined by minimizing total cost.

The total cost TC_v for dispatching a vehicle v on route j at station i is

$$TC_v = C_v^O + C_v^C + C_v^M \quad (4-1)$$

where C_v^O , C_v^C , and C_v^M represent the operator cost, the connection delay, and missed connection costs caused by holding vehicle v , respectively.

Supplier Cost

To evaluate the holding decision for a ready vehicle, the delay cost for the vehicle can be formulated as

$$C_v^O = t_v^h u_b \quad (4-2)$$

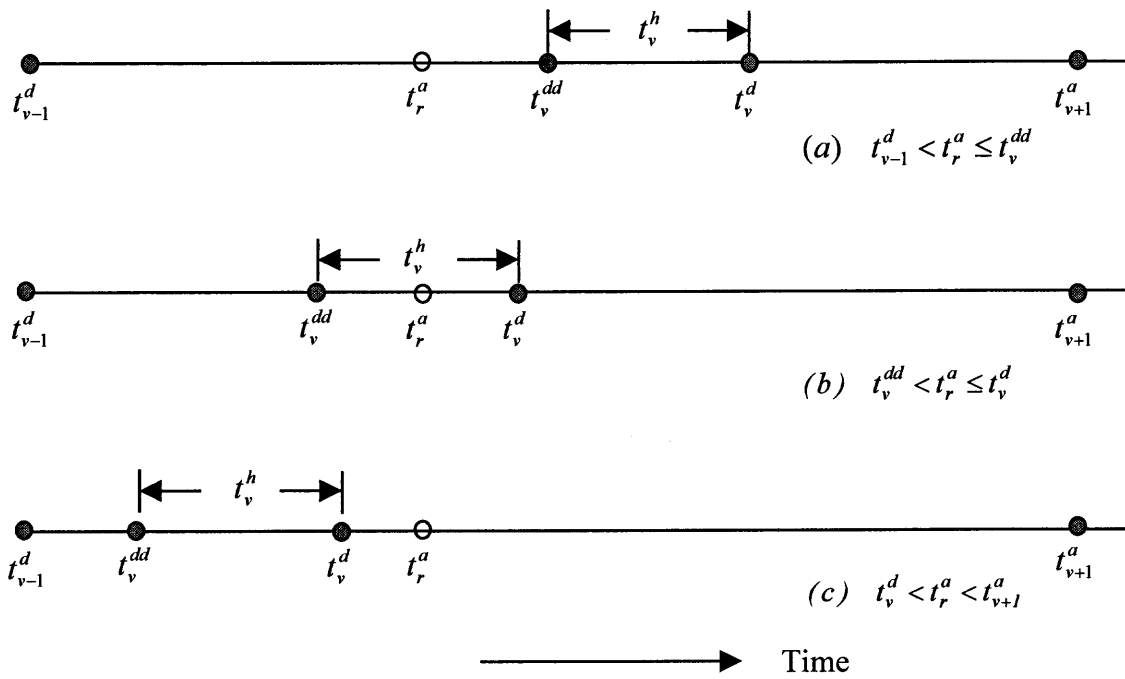
where v represents the holding vehicle, while t_v^h and u_b are holding time and vehicle operating cost, respectively.

Connection Delay Cost

The connection delay cost is incurred by passengers arriving during the interdeparture time of vehicles $v-1$ and v , while vehicle $v-1$ is the vehicle arriving at the transfer station prior to vehicle v . Since the passengers may transfer from trains or other bus routes, the connection delay cost is thus classified into bus-to-bus $C_{b,v}^C$ and rail-to-bus $C_{r,v}^C$ connection delay costs.

In this study, the train arrivals at the station are assumed to be deterministic. Therefore, $C_{r,v}^C$ is simply defined as the connection delay time multiplied by the transfer demand from rail and the value of users' wait time. The rail-to-bus connection delay time for vehicle v , calculated at each dispatching decision time, depends on whether (1) the train arrives during the previous vehicle (say $v-1$) departure time t_{v-1}^d and the dispatching decision time t_v^{dd} , and (2) the train arrives between t_v^{dd} and the departure time t_v^d of vehicle v , as shown in Figure 4-2.

For example, in Figure 4-2a, it is shown that if the train arrives at t_r^a between t_{v-1}^d and t_v^{dd} , the connection delay time is exactly equal to the vehicle holding time t_v^h . If the train arrives between t_v^{dd} and t_v^d as shown in Figure 4-2b, the connection delay time is $(t_v^{dd} + t_v^h - t_r^a)$. However, if the train arrives between t_v^d and t_{v+1}^a as shown in Figure 4-2c, the connection delay time is zero, since the train passengers will not catch vehicle v . Thus, $C_{r,v}^C$ can be formulated as



t_{v-1}^d : departure time of vehicle v-1

t_r^a : train arrival time

t_v^{dd} : dispatching decision time of vehicle v

t_v^d : departure time of vehicle v

t_{v+1}^a : arrival time of vehicle v+1

t_v^h : holding time of vehicle v

Figure 4-2 Vehicle Arrivals and Dispatching Decision Time at a Transfer Station

$$C_{r,v}^C = \begin{cases} \sum_r (t_v^{dd} + t_v^h - t_r^a) U_{r,v} u_w & \text{if } t_v^{dd} < t_r^a \leq t_v^{dd} + t_v^h \\ t_v^h \sum_r U_{r,v} u_w & \text{if } t_{v-1}^d < t_r^a \leq t_v^{dd} \\ 0 & \text{otherwise} \end{cases} \quad \forall r \quad (4-3)$$

where t_v^{dd} , t_v^h , t_r^a and $U_{r,v}$ represent the dispatching decision time and holding time of vehicle v , the arrival time of train r , and the transfer demand from train r to vehicle v , respectively. The dispatching decision time (or re-evaluation time) can be determined after knowing the arrival time t_v^a of vehicle v , the evaluation number n for holding vehicle v , and the evaluation interval Δ (e.g., 30 seconds), and formulated as

$$t_v^{dd} = t_v^a + (n-1)\Delta \quad (4-4)$$

For example, at the first dispatching decision time, n is equal to 1. Thus, the first dispatching decision time of vehicle v is t_v^a . The duration of Δ can be adjusted depending on traffic and demand conditions over time.

Similar to the derivation of $C_{r,v}^C$, $C_{b,v}^C$ is affected by the arrival distribution of the delivery vehicle at the transfer station, and is evaluated at each dispatching decision time point. $C_{b,v}^C$ is incurred by passengers transferring from all delivery vehicles arriving between t_{v-1}^a and t_v^d . Due to stochastic vehicle arrivals, the bus-to-bus connection delay cost incurred by transfer passengers from all delivery vehicles arriving between t_v^{dd} and t_{v+1}^a is formulated based on the probability distribution of the late delivery vehicle arriving during the holding time of vehicle v .

For example, if the dispatching delay time of a ready vehicle v is t_v^h (see Figure 4-3), the connection delay cost of transfer passengers from late vehicle b to vehicle v is the transfer demand $U_{b,v}$ multiplied by the probability of vehicle b arriving between t_v^{dd} and t_v^d (area A), the corresponding wait time, and the value of users' wait time u_w .

Thus, the bus-to-bus connection delay cost can be formulated as

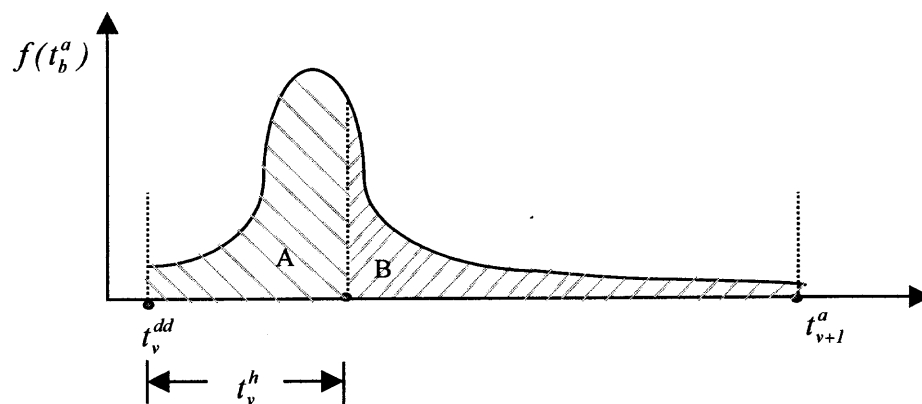
$$C_{b,v}^C = \begin{cases} t_v^h \sum_b U_{b,v} u_w & \text{if } t_{v-1}^d < t_b^a \leq t_v^{dd} \\ \sum_b \int_{t_v^{dd}}^{t_v^{dd} + t_v^h} f(t_b^a) [(t_v^{dd} + t_v^h) - t_b^a] dt_b^a U_{b,v} u_w & \text{if } t_v^{dd} < t_b^a \leq t_v^{dd} + t_v^h \end{cases} \quad \forall b \quad (4-5)$$

where $f(t_b^a)$ and $U_{b,v}$ represent the probability distribution for the arrival of vehicle b , and the transfer demand from vehicle b to v , respectively.

Missed Connection Cost

The missed connection cost is incurred by passengers transferring from coordinated vehicles, who will miss vehicle v . Similar to the formulation of the connection delay cost, the missed connection cost can also be classified into rail-to-bus $C_{r,v}^M$ and bus-to-bus $C_{b,v}^M$. The rail-to-bus missed connection cost, formulated in Eq. 4-6, is defined as the missed connection delay time multiplied by transfer demand and the value of users' wait time, where the missed connection delay time is the time difference between the arrivals of train r and vehicle $v+1$.

$$C_{r,v}^M = \begin{cases} \sum_r (t_{v+1}^a - t_r^a) U_{r,v} u_w & \text{if } t_v^d < t_r^a \leq t_{v+1}^a \\ 0 & \text{otherwise} \end{cases} \quad \forall r \quad (4-6)$$



$f(t_b^a)$: probability density function of bus b's arrival time

t_v^{dd} : dispatching decision time of vehicle v

t_v^h : holding time of vehicle v

t_{v+1}^a : arrival time of vehicle v+1

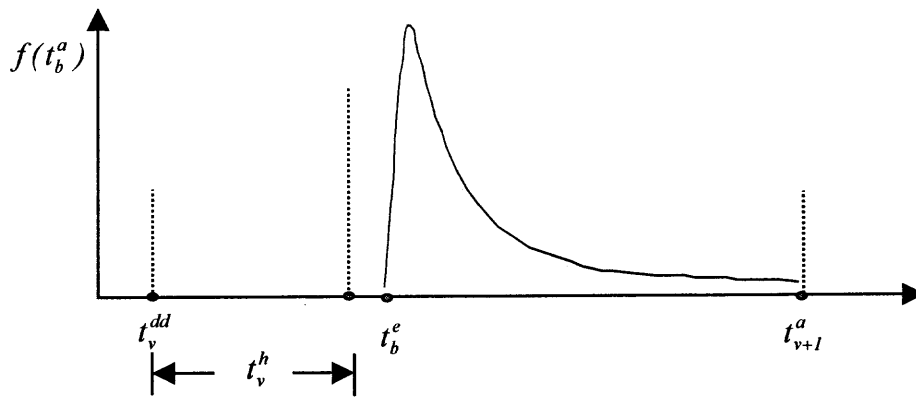
Figure 4-3 Probability Distribution of Late Vehicle Arrivals

where t_{v+1}^a is the arrival time of vehicle $v + 1$ (the follower of vehicle v). The bus-to-bus missed connection cost is formulated based on the probability of a missed connection, which can be determined from the vehicle arrival distribution. As shown in Figure 4-3, a missed connection occurs when the late vehicle b arrives between t_v^d and t_{v+1}^a (area B). At this situation, the bus-to-bus missed connection cost $C_{b,v}^M$ is defined as the transfer demand $U_{b,v}$ multiplied by the probability of a missed connection, corresponding waiting time, and the value of users' wait time:

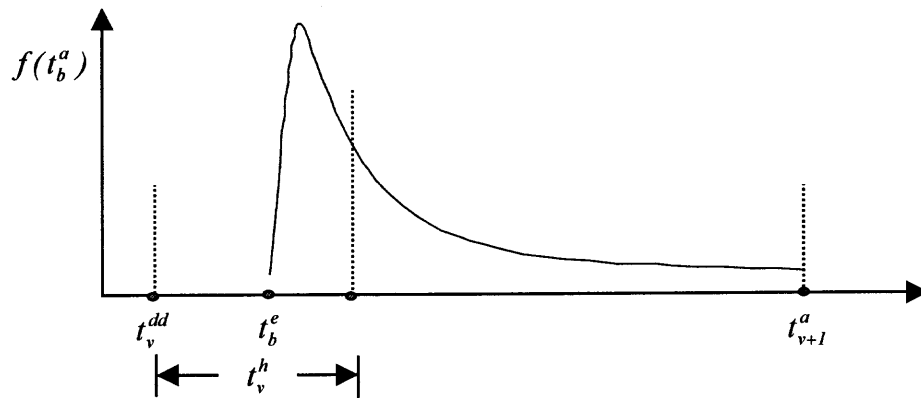
$$C_{b,v}^M = \sum_b \left[\int_{t_v^d + t_v^h}^{t_{v+1}^a} f(t_b^a) [t_{v+1}^a - t_b^a] dt_b^a U_{b,v} u_w \quad \forall b \right] \quad (4-7)$$

The cost of connection delay and missed connection incurred by passengers transferring from late vehicles are affected by the arrival distribution of the late vehicles. Therefore, connection delay and missed connection cost formulated in Eqs. 4-5 and 4-7 may need to be restructured depending on the arrival distribution of the late vehicles. Such a case would be when the vehicle arrival distribution is very skewed to the right (i.e., lognormal and exponential distribution [Hines and Montgomery, 1990]).

While formulating connection delay and missed connection costs with such distributions (i.e., lognormal or exponential) of vehicle arrival times, the relationship among dispatching decision time, earliest arrival times of the late vehicles, and the duration of holding time, has to be identified. For example, in Figure 4-4(a), it is shown-



$$(a) \quad t_{v+1}^a > t_b^e > t_v^{dd} + t_v^h$$



$$(b) \quad t_v^{dd} + t_v^h > t_b^e > t_v^{dd}$$

$f(t_b^a)$ = arrival distribution of vehicle b

t_v^{dd} = dispatching decision time of vehicle v

t_v^h = optimal holding time of vehicle v

t_b^e = earliest arrival time of vehicle b

t_{v+1}^a = schedule arrival time of vehicle v+1

Figure 4-4 Late Vehicle Arrival Distribution (Lognormal)

that if vehicle v is dispatched ($t_v^d = t_v^{dd} + t_v^h$) before the earliest arrival time t_b^e of late vehicle b , there will be no connection delay cost since the transfer passengers from late vehicle b will not catch vehicle v . However, if vehicle v is dispatched after the earliest arrival times of vehicle b as shown in Figure 4-4 (b), both connection delay and missed connection costs exist. The connection delay and missed connection costs, considering a late vehicle arrival distribution with finite earliest arrival time, are formulated in Eqs. 4-8 and 4-9, respectively.

$$C_{b,v}^C = \sum_b t_{b,v}^C U_{b,v} u_w \quad \forall b \quad (4-8)$$

$$C_{b,v}^M = \sum_b t_{b,v}^M U_{b,v} u_w \quad \forall b \quad (4-9)$$

where $t_{b,v}^C$ and $t_{b,v}^M$ represent connection delay and missed connection delay times, and are formulated in Eqs. 4-10 and 4-11, respectively.

$$t_{b,v}^C = \begin{cases} \int_{t_b^e}^{t_v^{dd} + t_v^h} f(t_b^a) [t_v^{dd} + t_v^h - (t_b^e + t_b^a)] dt_b^a & \text{if } t_v^{dd} + t_v^h > t_b^e \\ 0 & \text{otherwise} \end{cases} \quad \forall b \quad (4-10)$$

$$t_{b,v}^M = \begin{cases} \int_{t_v^a + t_v^h}^{t_{v+1}^a} f(t_b^a) [t_{v+1}^a - (t_b^e + t_b^a)] dt_b^a & \text{if } t_v^{dd} + t_v^h > t_b^e \\ \int_{t_b^e}^{t_{v+1}^a} f(t_b^a) [t_{v+1}^a - (t_b^e + t_b^a)] dt_b^a & \text{otherwise} \end{cases} \quad \forall b \quad (4-11)$$

4.4 Estimation of Vehicle Arrival Times

The development of advanced models for predicting transit vehicle arrival times at a transfer station has been discussed by Chien and Ding [1999]. In this study, the vehicle arrival times are estimated based on the vehicle arrival distribution at a transfer station

from various checkpoints along the route. CORSIM, which will be discussed in Chapter 5, is enhanced to generate data for determining the distribution of vehicle arrival times.

To collect vehicle departure times from various locations, a number of checkpoints are placed along a bus route. After simulating bus operations on the route, bus departure times from all checkpoints and arrival times at the transfer station can be obtained from the simulation output. The travel times of individual vehicles from any checkpoint to the transfer station can be determined as well.

The mean \bar{t}_p and the standard deviation S_p of travel times from each checkpoint to the transfer station are calculated by using Eqs. 4-12 and 4-13, respectively.

$$\bar{t}_p = \frac{1}{N} \sum_{b=1}^N t_{b,p} \quad (4-12)$$

$$S_p = \sqrt{\frac{1}{N-1} \sum_{b=1}^N (t_{b,p} - \bar{t}_p)^2} \quad (4-13)$$

where $t_{b,p}$ and N represent the travel time of vehicle b from checkpoint p to station i and the sample size, respectively. The simulation can generate the shortest travel time from any checkpoint to the transfer station. An example of the travel time distribution of late vehicles from various distances to the transfer station is shown in Figure 4-5.

After knowing the travel times between checkpoints and the transfer station, the travel time (shortest or mean) from various locations (other than the checkpoints) to the transfer station can be obtained by linear interpolation of travel times at adjacent (up-and down-stream) checkpoints. At the dispatching decision time, the late vehicle arrival time (mean and earliest) from its current location to the transfer station is the dispatching -

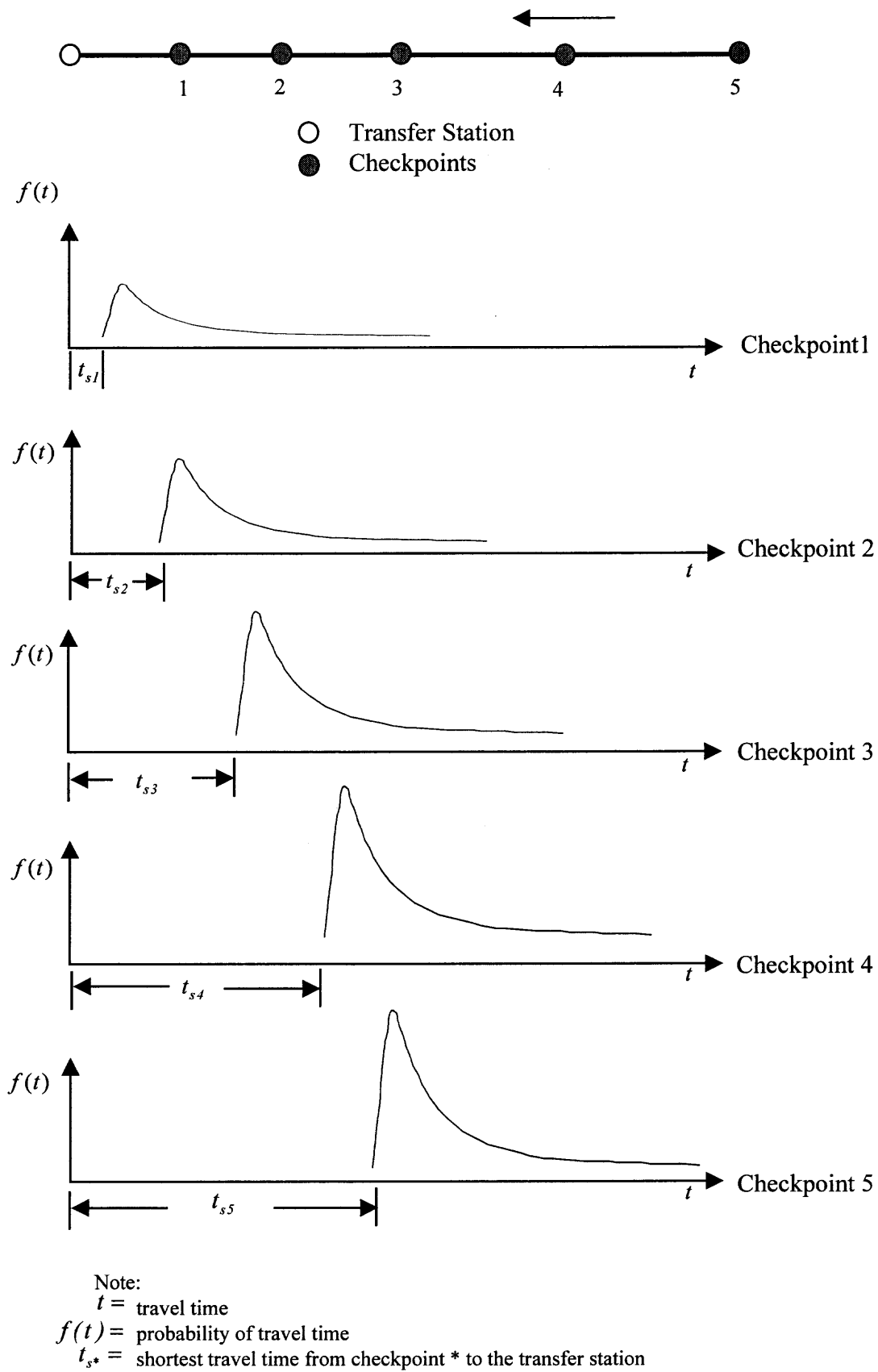


Figure 4-5 Travel Time Distributions from Various Checkpoints to a Transfer Station

decision time plus the travel time (mean and shortest). The standard deviation of vehicle arrival times from any location to the transfer station can also be obtained by linear interpolation of the standard deviation of travel times at adjacent (up-and down-stream) check-points.

4.5 Procedure for Dynamic Vehicle Dispatching

The model developed in Section 4.3 for dynamic dispatching of coordinated vehicles contains an objective total cost function, which will be minimized by optimizing the timing for dispatching coordinated vehicles. Within the holding time, the dispatching decision will be re-evaluated periodically (e.g., 30 seconds) to adjust the holding decision dynamically. At each dispatching decision (or re-evaluation) time, the optimal holding time for each coordinated vehicle can be determined numerically.

To remember the arrival and departure time of each vehicle as well as the transfer demand information, a dynamic database will be accessed and updated with real time information such as transfer demand from one vehicle to another, vehicle arrival and departure times, and locations of late incoming vehicles. The dynamic dispatching procedure for each coordinated vehicle is activated at the time that a coordinated vehicle arrives at the transfer station. The step by step procedure for dynamic dispatching of coordinated vehicles is shown in Figure 4-6 and is as follows.

- Step 1: As the dispatching decision time for vehicle v approaches, estimate the transfer demand from delivery vehicles to the holding one.
- Step 2: For all late vehicles, estimate their means and the standard deviations of

arrival times to the transfer station according to their current locations. If all coordinated vehicles have arrived, dispatch vehicle v immediately; otherwise, go to Step 3.

Step 3: Optimize the holding time of vehicle v by minimizing the objective total cost function formulated in Eq. 4-1.

Step 4: If the optimal holding time (t_v^h) is less than or equal to the evaluation interval (Δ), dispatch vehicle v at the end of the optimal holding time; otherwise wait for the next dispatching (reevaluation) time and go to Step 1.

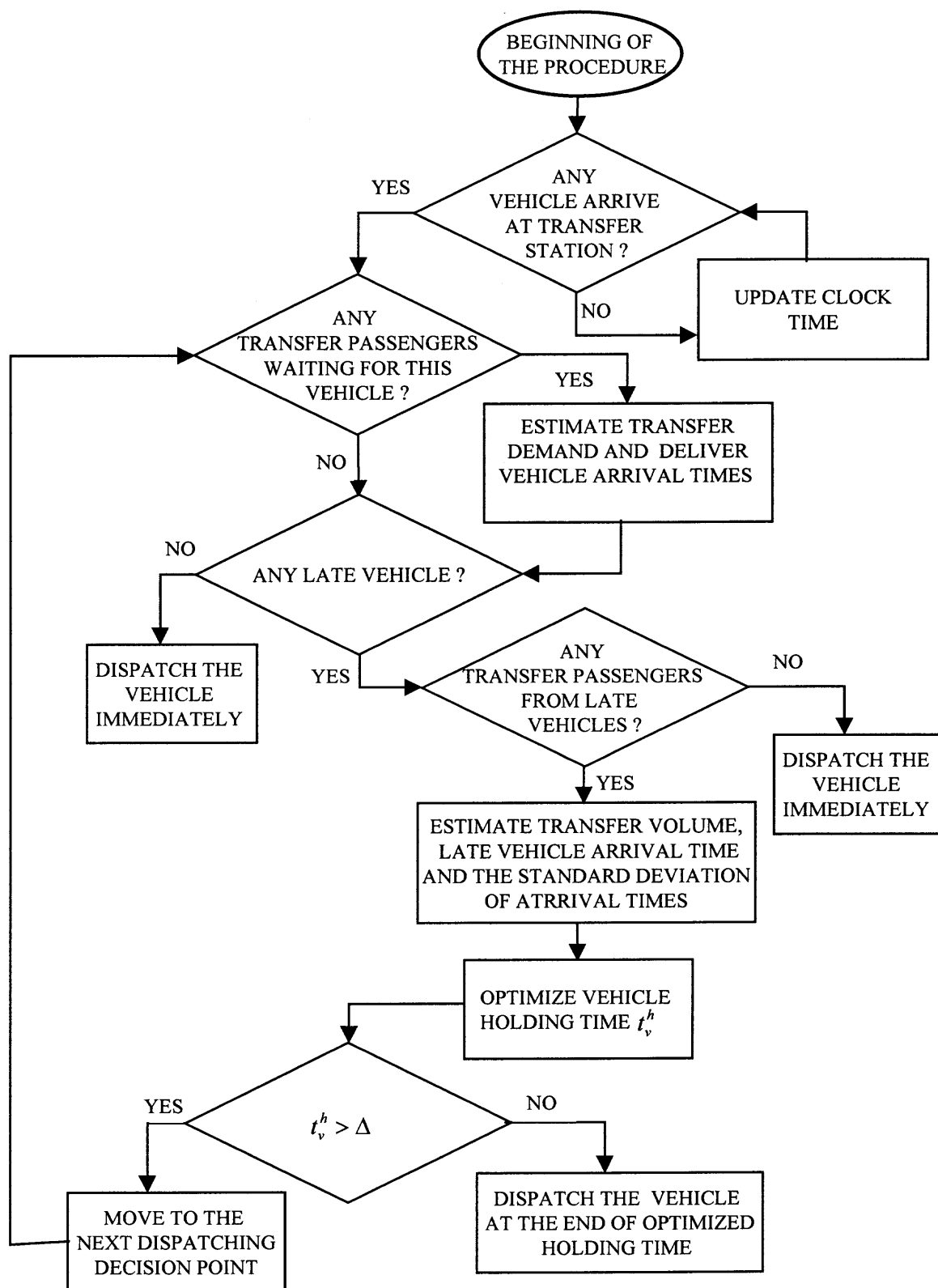


Figure 4-6 Procedure for Dynamic Vehicle Dispatching

CHAPTER 5

THE ENHANCEMENT OF CORSIM

5.1 Introduction

To demonstrate the application of the model developed in Chapter 4 for dynamically optimizing holding times, the bus operations in a hypothetical transit network are simulated using the microscopic traffic simulation model CORSIM. Since CORSIM does not generate vehicle arrival or departure time information, it is not capable of generating data. Moreover, CORSIM has some deficiencies in simulating bus operations, especially in calculating bus dwell time [Chien, Chowdhury, Mouskos, and Ding, 1999]. Thus the transit operations can not be simulated properly.

CORSIM deals with bus dwell times by simply relying on mean dwell times specified by users and embedded statistical distributions rather than the loading and unloading demand. Thus, the bus dwell times determined in CORSIM are extracted from a distribution and can be regarded as a random variable. This deficiency may generate unreasonable simulation results. In this study, CORSIM is enhanced by modifying bus dwell time and generating additional outputs (i.e., vehicle arrival and departure times at stations) to simulate reasonable bus operations and to collect the required data.

5.2 Enhancement of CORSIM

CORSIM simulates transit operations by representing the movements and driving behavior of individual transit vehicles, such as buses. Buses are generated onto a network recognized by CORSIM based on their dispatching headway. The actual bus dwell time is

determined by the mean dwell time and a statistical distribution embedded in CORSIM. This is a very simplistic approach, where the current estimate of the dwell time can not reflect the actual bus operation at stops due to the lack of consideration of the number of boarding and alighting passengers.

In the enhanced CORSIM, the deficiencies mentioned above are removed. Bus dwell times are determined by the time dependent passenger arrival rate and the headway between consecutive buses. Buses will be released based on the user-specified timetable (the posted schedule) for each bus route. This feature allows for the dispatching of feeder buses based on time tables to maintain the coordinated schedule at transfer station.

The required inputs corresponding to the proposed enhancements include passenger arrival rates and distributions at stops, average passenger boarding times, bus stops and route locations, bus schedules, traffic volumes along the routes, and traffic control devices and signal timing (if any) at intersections that affect transit operations. A description of stationary nodes, providing bus route information, is still required by the enhanced model as by CORSIM. In developing the enhanced model, passenger arrivals at stops follow Poisson distributions, while passenger demand at different stops may vary. Other empirical passenger arrival distributions can be programmed and linked with the enhanced model, given the corresponding passenger flow data at bus stops.

The additional simulation output related to transit operations produced by the enhanced model include arrival/departure times at stops, number of waiting passengers, average passenger waiting times, estimated bus dwell times, and the mean and variance of headways at bus stops. All variables used to formulate models for dwell and wait time are defined in appendix A.

5.3 Modules for Simulating Transit Vehicle Operations

Based on the discussion in the previous section, four new modules were developed for the enhanced model to simulate buses operating in urban networks. The functions of the four new modules are to

1. dispatch buses based upon time tables,
2. determine number of passengers waiting at bus stops,
3. calculate bus dwell times at stops, and
4. generate real-time information for calculating transit related MOEs.

Table 5-1 shows the functional differences between the original and enhanced CORSIM. The original CORSIM does not estimate the number of waiting passengers and does not calculate passenger waiting times. Thus, the bus dwell time is arbitrarily determined by user specified mean dwell times at stops and embedded distributions. Due to the new feature (estimating the numbers of passengers waiting at stops) established in the enhanced model, the bus dwell time can be determined by the passenger arrival rates and headway. Additionally, in the enhanced model, the headway based vehicle dispatching in CORSIM, which may restrict the model to simulate nonheadway based dispatching, has been abandoned and replaced by a time based dispatching logic.

The implementation of the new features and new modules is shown in Figure 5-1. A description of the newly added or modified features in CORSIM, which are shown in thick solid boxes in Figure 5-1, is presented below.

TABLE 5-1 Differences between CORSIM and the Enhanced Model

Module	CORSIM	Enhanced Model
Dispatching buses	Headway based schedules	Time based schedules
Estimating number of waiting passengers	N/A	Headway and passenger arrival rates
Determining dwell time	Mean dwell time and distribution tables	A function of boarding passengers
Calculating average wait time	N/A	Headway and passenger arrival distribution

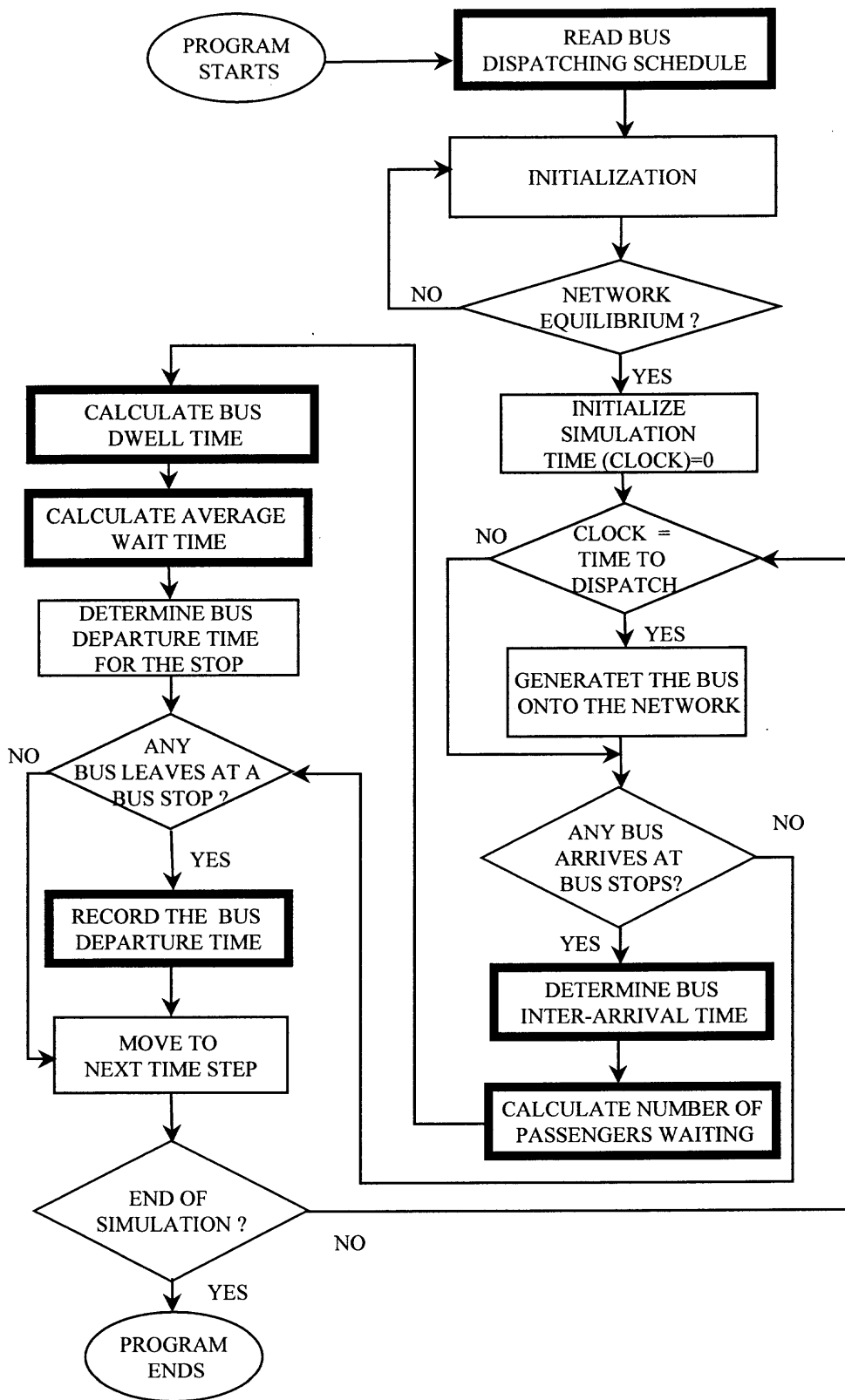


Figure 5-1 Configuration of the Proposed Model
(NOTE: The thick solid boxes represent new modules.)

1. Bus dispatching schedules (timetables) can be specified by users based on bus departure times at all bus terminals. The schedule of each bus route is specified in an input file named SCHEDULE.INP. Once the simulation starts, the schedule information in file SCHEDULE.INP will be retrieved for dispatching buses.
2. Mean dwell times at stops specified in card type 185 of CORSIM are replaced by passenger arrival rates. The function determining the mean dwell time in CORSIM is disabled in the proposed model. Instead, the bus dwell time is determined by the average passenger boarding time, the number of passengers waiting at the stop, and the bus headway.
3. All features related to dwell time distributions are abandoned because the bus dwell time is no longer determined by the user specified mean values and the embedded statistical distributions.
4. The number of passengers waiting and the duration of bus dwelling at each stop are determined by the corresponding bus headway (e.g., the duration of bus arrival time and departure time of its leading bus). Thus, the bus departure time is required to be updated as a bus departs from a stop.

Real-time information, such as bus arrival/departure times, number of passengers waiting, bus dwell times at stops are generated from the enhanced model, which can be used to determine the mean and variance of headways at each stop. The formulas for calculating the duration of bus dwell time and the average passenger wait time are presented below.

Bus Dwell Time

As shown in Figure 5-2, the dwell time of bus v at stop s , called $d_{v,s}$, is affected by the number of passengers arriving during a period between the departure time of bus $v-1$, called T_1 ; the arrival time of bus v , called T_2 ; and the departure time of bus v , called T_3 . Note, that bus $v-1$ is the bus operating prior to bus v . Thus, $d_{v,s}$ is equal to $(T_3 - T_2)$. Since T_3 is also the point in time where the queue for boarding bus v vanishes, it can be ascertained by solving Eq. 5-1.

$$(T_3 - T_2) = b_s \int_{T_1}^{T_3} q_s(t) dt \quad \forall v, s \quad (5-1)$$

where b_s and $q_s(t)$ represent the average passenger boarding time and the passenger arrival distribution at stop s , respectively. While T_3 is the only unknown variable in Eq. 5-1, it can be easily solved. The resulting value of T_3 can be used to compute $d_{v,s}$.

Passenger Wait Time

In addition to T_3 obtained from solving Eq. 5-1, if the passenger arrival distribution $q_s(t)$ and the headway between buses v and $v-1$ at stop s are known, the total number of boarding passengers $Q_{v,s}$ (the shaded area in Figure 5-2) can be obtained from Eq. 5-2.

$$Q_{v,s} = \int_{T_1}^{T_3} q_s(t) dt \quad \forall v, s \quad (5-2)$$

Similarly, the total passenger wait time $W_{v,s}$ for bus v at stop s can be obtained from Eq. 5-3.

$$W_{v,s} = \int_{T_1}^{T_3} (T_3 - t) q_s(t) dt \quad \forall v, s \quad (5-3)$$

Therefore, the average wait time w_s at stop s can be obtained from the total wait time divided by the total number of passengers served by all buses operating during the service time period as

$$w_s = \frac{\sum_v W_{v,s}}{\sum_v Q_{v,s}} \quad \forall s \quad (5-4)$$

In an urban environment, the bus dispatching frequency is high during peak periods to meet the demand. Therefore, the scheduled bus headway is short, and passenger arrivals at stops tend to be random. Thus, a Poisson typed passenger arrival generator is developed and built into the enhanced model.

If passenger arrivals follow a Poisson distribution, Eq. 5-1 can be simplified as

$$d_{v,s} = b_s \lambda_s (T3 - T1) \quad \forall v, s \quad (5-5)$$

where λ_s and b_s represent the mean passenger arrival rate and the average passenger boarding time at stop s , respectively. Before analyzing bus operations using simulation, the model has to be calibrated and validated [Chien, Chowdhury, Mouskos, and Ding, 1999].

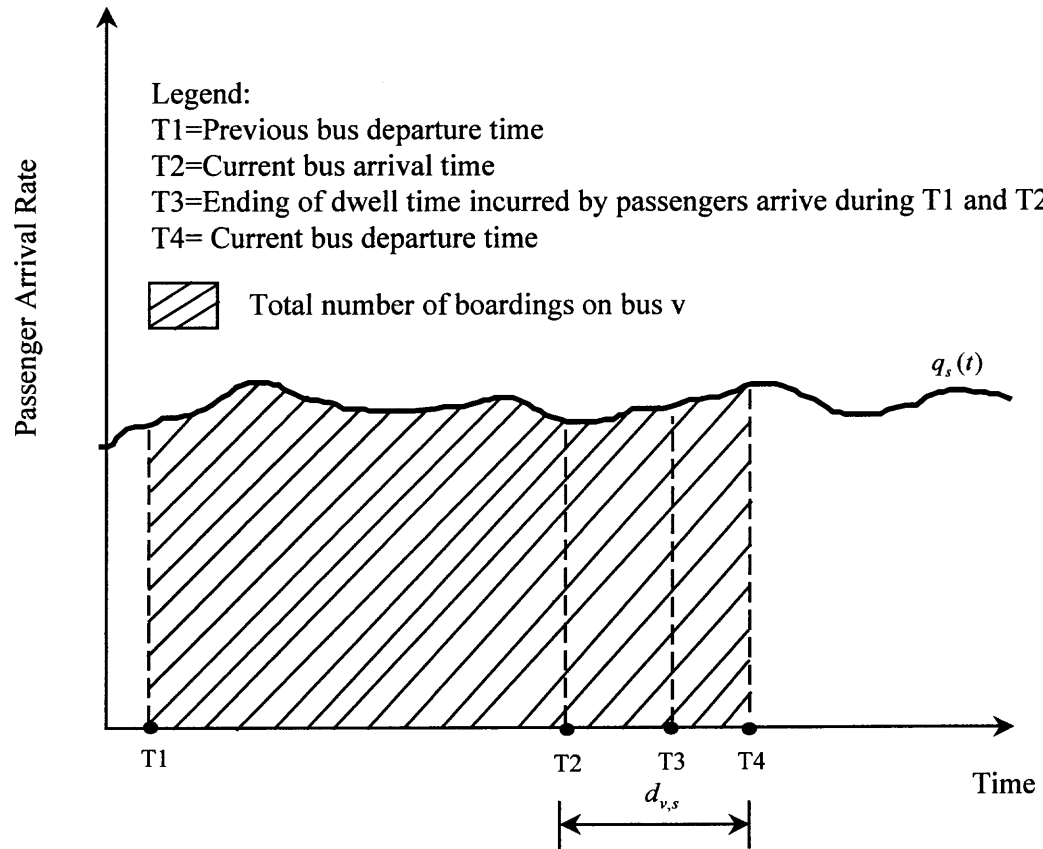


Figure 5-2 Passenger Arrival Distribution and Bus Arrival/Departure Times at Stop s

CHAPTER 6

EVALUATION OF COORDINATION AND DISPATCHING MODELS

In this chapter the application of both the intermodal transit system coordination and the dynamic vehicle dispatching models is discussed. This Chapter contains 3 sections. Sections 6.1 and 6.2 illustrate the hypothetically created numerical examples for coordination and dynamic dispatching models, respectively, while Section 6.3 summarizes the results obtained from Sections 6.1 and 6.2.

6.1 Coordination Model

The major purpose of this section is to test the coordination model formulated in Section 3.4 and optimize a coordinated intermodal transit network with the developed four-stage procedure. The sensitivity analysis of the total cost and decision variables (e.g., headways and slack times) with respect to various parameters (e.g., value of user wait time, vehicle operating cost, standard deviation of vehicle arrival times, and transfer demand multiplier) are also presented. The optimization and sensitivity analysis results are obtained from a computer program coded in FORTRAN.

6.1.1 Intermodal Transit Network

The studied network, as shown in Figure 1-1, contains a 16-mile rail line serving 11 stations. Stations 1, 2, 5, and 11 are transfer stations connecting with 6, 4, 4 and 5 feeder bus routes, respectively. Bus arrival times at transfer stations follow normal distributions. The probability density function of a normal distribution with mean μ and standard deviation σ is shown in Appendix D.

The baseline values of design variables and model parameters (e.g., value of user wait time, value of operator cost, acceleration rate, deceleration rate, speed, etc) are listed in Appendix A. The vehicle capacities for buses operating on feeder routes and one-car trains operating on the rail line are assumed to be 80 pass/bus and 250 pass/car (including standees), respectively. The length of feeder routes and the standard deviation of vehicle arrival times for each bus route are summarized in Table 6-1, while the rail station spacings are shown in Table 6-2.

The transfer demand among transit routes is one of the major determinants affecting coordination benefit. Thus, three sets of transfer demand are examined and shown from Tables 6-3 through 6-8. The first set of input demand data assumes that rail demand is low to medium (536 pass/hr), while transfer demand both from rail-to-bus (91, 103, 104 and 80 pass/hr at stations 1, 2, 5, and 11, respectively) and bus-to-rail (87, 50, 127 and 96 pass/hr at stations 1, 2, 5, and 11, respectively) is medium at all four transfer stations. The second set of input demand data assumes that rail demand (2745 pass/hr) is high while transfer demand from both rail-to-bus (297, 107, 337 and 448 pass/hr at stations 1, 2, 5, and 11, respectively) and bus-to-rail (212, 26, 355 and 260 pass/hr at stations 1, 2, 5, and 11, respectively) is also high at stations 1, 5, and 11. Unlike the first and second sets of demand, the third one assumes that both rail demand (2745 pass/hr) and transfer demand from bus-to-rail (197, 24, 333, and 243 pass/hr at stations 1,2,5, and 11, respectively) is high. However, the demand transferring from rail-to-bus (145, 36, 138, and 189 pass/hr at stations 1,2,5, and 11, respectively) is low.

Table 6-1 Input for Bus Routes

$i^{(1)}$	$j^{(2)}$	$BL^{(3)}$	$SD^{(4)}$
1	1	5	0.025
	2	9	0.050
	3	9	0.045
	4	7	0.035
	5	5	0.030
	6	5	0.020
2	1	9	0.035
	2	7	0.025
	3	6	0.025
	4	4	0.020
5	1	10	0.040
	2	9	0.030
	3	7	0.020
	4	7	0.020
11	1	8	0.030
	2	10	0.025
	3	9	0.030
	4	4	0.025
	5	4	0.020

(1) = index of rail stations

(2) = index of bus routes

(3) = bus route length (miles)

(4) = standard deviation of bus arrival times at the transfer station (hr)

Table 6-2 Input for the Rail Line

Station to Station Index	Station to Station Spacing (mile)	Station to Station Index	Station to Station Spacing (mile)
1-2	2.0	6-7	1.5
2-3	1.0	7-8	2.0
3-4	1.5	8-9	1.5
4-5	2.5	9-10	2.0
5-6	1.0	10-11	1.0

Table 6-3 Demand for Bus Routes (Data Set 1)

$i^{(1)}$	$j^{(2)}$	$I_{ij1}^{(3)}$	$I_{ij2}^{(4)}$	Route to Route Demand Matrix (U_{ikj}) (pass/hr)						Buses to Rail Demand (U_{ijd}) (pass/hr)	
				1	2	3	4	5	6	d1	d2
1	1	20	11	0	0	0	0	0	9	11	0
	2	80	82	1	0	26	26	7	9	11	0
	3	76	110	1	19	0	23	6	8	19	0
	4	130	111	1	33	43	0	10	13	30	0
	5	180	201	2	5	7	13	0	144	9	0
	6	220	195	4	9	13	22	165	0	7	0
2	1	25	32	0	2	2	4	-	-	13	4
	2	80	75	4	0	31	36	-	-	7	2
	3	85	92	3	31	0	43	-	-	6	2
	4	80	124	2	26	36	0	-	-	12	4
5	1	70	90	0	25	10	14	-	-	7	14
	2	110	103	33	0	17	22	-	-	13	25
	3	170	171	17	25	0	111	-	-	6	11
	4	210	172	12	21	126	0	-	-	18	33
11	1	40	93	0	9	10	4	4	-	0	13
	2	95	97	19	0	24	9	10	-	0	33
	3	110	100	25	30	0	8	11	-	0	36
	4	250	238	10	15	17	0	200	-	0	8
	5	280	232	19	20	22	213	0	-	0	6

(1) = index of rail stations

(2) = index of bus routes

(3) = the inflow demand of bus route j at station i in direction 1 (pass/hr)(4) = the inflow demand of bus route j at station i in direction 2 (pass/hr)

Table 6-4 Demand for the Rail Line (Data Set 1)

Rail Station Index (i)	Bus Route (j)	$U_{ij}^{(1)}$	$U_{ij}^{(2)}$	$I_{i1}^{(3)}$	$I_{i2}^{(4)}$	$Q_{i1}^{(5)}$	$Q_{i2}^{(6)}$
1	1	0	2	90	0	0	172
	2	0	16				
	3	0	21				
	4	0	27				
	5	0	12				
	6	0	13				
2	1	14	8	50	18	70	37
	2	11	6				
	3	14	8				
	4	28	14				
3	-	-	-	5	6	7	10
4	-	-	-	6	5	6	9
5	1	13	15	65	115	55	55
	2	15	17				
	3	10	8				
	4	14	12				
6	-	-	-	9	10	8	6
7	-	-	-	6	7	5	3
8	-	-	-	7	6	3	2
9	-	-	-	12	8	4	5
10	-	-	-	7	10	7	6
11	1	19	0	0	120	92	0
	2	24	0				
	3	27	0				
	4	4	0				
	5	6	0				

(1) = transfer demand from rail direction 1 to bus route j at station i (pass/hr)

(2) = transfer demand from rail direction 2 to bus route j at station i (pass/hr)

(3) = the rail inflow demand at station i in direction 1 (pass/hr)

(4) = the rail inflow demand at station i in direction 2 (pass/hr)

(5) = the rail outflow demand at station i in direction 1 (pass/hr)

(6) = the rail outflow demand at station i in direction 2 (pass/hr)

Table 6-5 Demand for Bus Routes (Data Set 2)

$i^{(1)}$	$j^{(2)}$	$I_{ij1}^{(3)}$	$I_{ij2}^{(4)}$	Route to Route Demand Matrix (U_{ikj}) (pass/hr)						Buses to Rail Demand (U_{ijd}) (pass/hr)	
				1	2	3	4	5	6	D1	D2
1	1	30	27	0	1	2	1	2	6	18	0
	2	50	82	1	0	10	11	6	6	16	0
	3	60	97	1	12	0	15	5	7	20	0
	4	100	115	2	20	21	0	10	10	37	0
	5	110	118	2	6	9	11	0	16	66	0
	6	130	128	4	13	15	17	26	0	55	0
2	1	40	59	0	7	10	14	-	-	7	2
	2	240	272	24	0	101	110	-	-	4	1
	3	260	301	13	102	0	140	-	-	4	1
	4	300	289	12	120	162	0	-	-	5	2
5	1	90	82	0	4	6	9	-	-	25	46
	2	110	131	6	0	11	13	-	-	28	52
	3	130	114	5	12	0	23	-	-	32	58
	4	150	138	4	9	23	0	-	-	40	74
11	1	60	130	0	6	8	7	5	-	0	37
	2	90	164	12	0	16	6	8	-	0	48
	3	120	140	21	24	0	6	10	-	0	59
	4	90	103	6	7	6	0	18	-	0	53
	5	100	111	6	9	7	15	0	-	0	63

(1) = index of rail station

(2) = index of bus route

(3) = the inflow demand of bus route j at station i in direction 1 (pass/hr)(4) = the inflow demand of bus route j at station i in direction 2 (pass/hr)

Table 6-6 Demand for the Rail Line (Data set 2)

Rail Station Index (<i>i</i>)	Bus route (<i>j</i>)	$U_{ij}^{(1)}$	$U_{ij}^{(2)}$	$I_{il}^{(3)}$	$I_{i2}^{(4)}$	$Q_{il}^{(5)}$	$Q_{i2}^{(6)}$
1	1	0	16	300	0	0	300
	2	0	30				
	3	0	40				
	4	0	59				
	5	0	69				
	6	0	83				
2	1	4	6	250	200	180	200
	2	22	22				
	3	14	14				
	4	13	12				
3	-	-	-	100	80	80	75
4	-	-	-	110	90	70	80
5	1	31	35	220	280	240	220
	2	50	55				
	3	41	33				
	4	50	42				
6	-	-	-	90	95	70	90
7	-	-	-	80	90	80	85
8	-	-	-	85	50	75	90
9	-	-	-	100	75	90	110
10	-	-	-	90	80	80	70
11	1	85	0	0	280	460	0
	2	118	0				
	3	103	0				
	4	71	0				
	5	71	0				

(1) = transfer demand from rail direction 1 to bus route *j* at station *i* (pass/hr)

(2) = transfer demand from rail direction 2 to bus route *j* at station *i* (pass/hr)

(3) = the rail inflow at station *i* in direction 1 (pass/hr)

(4) = the rail inflow at station *i* in direction 2 (pass/hr)

(5) = the rail outflow at station *i* in direction 1 (pass/hr)

(6) = the rail outflow at station *i* in direction 2 (pass/hr)

Table 6-7 Demand for Bus Routes (Data Set 3)

$i^{(1)}$	$j^{(2)}$	$I_{ij1}^{(3)}$	$I_{ij2}^{(4)}$	Route to Route Demand Matrix (U_{ikj}) (pass/hr)						Buses to Rail Demand (U_{jda}) (pass/hr)	
				1	2	3	4	5	6	D1	D2
1	1	30	23	0	1	2	1	2	6	18	0
	2	50	65	1	0	10	11	6	6	16	0
	3	60	73	1	12	0	15	5	7	20	0
	4	90	80	2	18	19	0	9	9	33	0
	5	100	78	2	5	8	10	0	15	60	0
	6	120	79	4	12	14	16	24	0	50	0
2	1	40	49	0	7	10	14	-	-	7	2
	2	200	224	20	0	84	92	-	-	3	1
	3	240	256	12	93	0	130	-	-	4	1
	4	280	243	11	112	151	0	-	-	4	2
5	1	90	30	0	4	6	9	-	-	25	46
	2	100	54	5	0	10	12	-	-	26	47
	3	120	81	5	11	0	22	-	-	29	54
	4	140	91	4	9	21	0	-	-	37	69
11	1	60	66	0	6	8	4	5	-	0	37
	2	90	71	12	0	16	6	8	-	0	48
	3	110	73	20	22	0	6	8	-	0	54
	4	80	77	6	6	5	0	16	-	0	47
	5	90	89	5	8	6	14	0	-	0	57

(1) = index of rail stations

(2) = index of bus routes

(3) = the inflow demand of bus route j at station i in direction 1 (pass/hr)(4) = the inflow demand of bus route j at station i in direction 2 (pass/hr)

Table 6-8 Demand for the Rail Line (Data set 3)

Rail Station Index (<i>i</i>)	Bus route (<i>j</i>)	$U_{ij}^{(1)}$	$U_{ij}^{(2)}$	$I_{il}^{(3)}$	$I_{i2}^{(4)}$	$Q_{il}^{(5)}$	$Q_{i2}^{(6)}$
1	1	0	13	300	0	0	300
	2	0	17				
	3	0	20				
	4	0	26				
	5	0	33				
	6	0	36				
2	1	4	2	250	200	180	200
	2	5	6				
	3	7	4				
	4	4	4				
3	-	-	-	100	80	80	75
4	-	-	-	110	90	70	80
5	1	7	9	220	280	240	220
	2	17	13				
	3	24	20				
	4	26	22				
6	-	-	-	90	95	70	90
7	-	-	-	80	90	80	85
8	-	-	-	85	50	75	90
9	-	-	-	100	75	90	110
10	-	-	-	90	80	80	70
11	1	24	0	0	280	460	0
	2	28	0				
	3	38	0				
	4	47	0				
	5	52	0				

(1) = transfer demand from rail direction 1 to bus route *j* at station *i* (pass/hr)

(2) = transfer demand from rail direction 2 to bus route *j* at station *i* (pass/hr)

(3) = the rail inflow demand at station *i* in direction 1 (pass/hr)

(4) = the rail inflow demand at station *i* in direction 2 (pass/hr)

(5) = the rail outflow demand at station *i* in direction 1 (pass/hr)

(6) = the rail outflow demand at station *i* in direction 2 (pass/hr)

6.1.2 Results of Intermodal Transit System Coordination

Based on the baseline values of parameters while considering different levels of demand, the optimal headways and slack times for the analyzed intermodal transit network can be obtained by applying the developed four-stage procedure, as discussed next.

Stage I Optimization

At Stage I, decision variables (i.e., bus and rail headways) are optimized without considering coordination. The bus operator and user costs (i.e., wait, transfer, and in-vehicle costs) at each transfer station are obtained. The optimized bus and rail headways (under the different demand conditions) are shown in Tables 6-9 and 6-10, while the corresponding cost components are shown in Table 6-11. The minimum total costs for data sets 1, 2, and 3 are found to be 13,119.10 \$/hr, 16,030.11 \$/hr, and 14,154.82 \$/hr, respectively. The values of various cost components obtained from the Stage I procedure will be compared with the values of cost components obtained in the subsequent stages.

Stage II Optimization

At Stage II, bus coordination at each transfer station is considered independently. The optimal results are obtained after going through the proposed Stage II procedure discussed in Chapter 3. The results (e.g., coordinated bus groups, common headway, and bus operator and user costs) obtained from various iterations at Stage II considering various demand situations addressed in data sets 1, 2, and 3 are presented in Tables 6-12, 6-13 and 6-14, respectively, while the optimal coordinated groups at all transfer stations are highlighted.

Table 6-9 Optimal Bus Headways – Stage I

Rail Station (i)	Bus Route (j)	Bus Headway H_j (hr)		
		Data Set 1	Data Set 2	Data Set 3
1	1	0.564	0.417	0.433
	2	0.330	0.366	0.392
	3	0.306	0.335	0.365
	4	0.236	0.251	0.283
	5	0.157	0.206	0.233
	6	0.149	0.192	0.220
2	1	0.559	0.423	0.447
	2	0.296	0.158	0.175
	3	0.256	0.139	0.149
	4	0.194	0.111	0.118
5	1	0.349	0.336	0.402
	2	0.285	0.268	0.337
	3	0.196	0.234	0.258
	4	0.184	0.214	0.240
11	1	0.342	0.285	0.352
	2	0.317	0.274	0.347
	3	0.287	0.257	0.308
	4	0.123	0.200	0.222
	5	0.119	0.191	0.207

Table 6-10 Optimal Rail Headway – Stage I

Data Set	Rail Headway (hr)
1	0.270
2	0.121
3	0.121

Table 6-11 Various Cost Components (Stage I)

Routes	Rail Station Index (<i>i</i>)	Wait Cost (\$/hr)	Transfer Cost (\$/hr)	In-vehicle Cost (\$/hr)	UC_i (\$/hr)	SC_i (\$/hr)	SUC_i (\$/hr)
Data Set 1							
Buses	1	535.87	552.73	1177.01	2265.61	1231.09	3496.7
Buses	2	262.78	309.95	452.17	1024.90	631.62	1656.52
Buses	5	448.28	447.28	1122.84	2018.42	1020.79	3039.21
Buses	11	489.37	529.69	1173.88	2192.94	1183.61	3376.55
Rail	-	119.11	412.57	475.03	1006.70	543.42	1550.12
Total	-	1855.41	2252.22	4400.93	8508.57	4610.53	13119.1
Data Set 2							
Buses	1	433.44	538.01	886.94	1858.40	1071.13	2929.53
Buses	2	436.18	510.38	1364.17	2310.74	1147.61	3458.35
Buses	5	428.82	420.07	990.00	1838.90	953.20	2792.10
Buses	11	384.57	565.77	1046.09	1996.44	1068.39	3064.63
Rail	-	805.68	361.49	1382.80	2549.98	1235.52	3785.5
Total	-	2488.69	2395.72	5670.00	10554.46	5475.85	16030.11
Data Set 3							
Buses	1	454.42	426.15	721.85	1602.43	961.24	2563.67
Buses	2	426.75	458.59	1177.62	2062.98	1057.41	3120.39
Buses	5	471.58	257.71	728.93	1458.22	808.34	2266.56
Buses	11	430.30	374.36	726.80	1531.48	887.22	2418.70
Rail	-	828.80	338.37	1382.80	2549.98	1235.52	3785.5
Total	-	2611.85	1855.18	4738.00	9205.09	4949.73	14154.82

SUC_i = supplier and user cost at station *i*

UC_i = user cost at station *i*

SC_i = supplier cost at station *i*

Table 6-12 Optimal Common Headway Stage - II Optimization (Data Set 1)

Rail Station (i)	Coordinated Group	Common Headway (hr)	$BOUC_i$ (\$/hr)
1	1,2	0.376	3510.18
	1,2,3	0.348	3504.68
	1,2,3,4	0.325	3490.08
	1,2,3,4,5	0.270	3569.52
	1,2,3,4,5,6	0.290	3439.03
	2,3	0.322	3487.93
	2,3,4	0.305	3469.67
	2,3,4,5	0.252	3532.16
	2,3,4,5,6	0.273	*3413.98
	3,4	0.277	3490.92
	3,4,5	0.226	3530.99
	3,4,5,6	0.239	3436.65
	4,5	0.192	3511.83
	4,5,6	0.207	3436.13
	5,6	0.166	3460.11
2	1,2	0.385	1682.04
	1,2,3	0.365	1660.93
	1,2,3,4	0.360	1599.31
	2,3	0.294	1640.69
	2,3,4	0.310	*1576.97
	3,4	0.240	1646.49
5	1,2	0.325	3028.62
	1,2,3	0.276	3045.81
	1,2,3,4	0.297	*2897.82
	2,3	0.237	3048.20
	2,3,4	0.258	2934.23
	3,4	0.225	2960.02
11	1,2	0.335	3368.71
	1,2,3	0.350	*3322.11
	1,2,3,4	0.252	3486.65
	1,2,3,4,5	0.262	3359.28
	2,3	0.310	3365.06
	2,3,4	0.223	3486.05
	2,3,4,5	0.232	3382.92
	3,4	0.187	3455.04
	3,4,5	0.193	3376.81
	4,5	0.129	3343.30

$BOUC_i$ = bus operator and user costs only for bus routes connected at transfer station i

Table 6-13 Optimal Common Headway - Stage II Optimization (Data Set 2)

Rail Station (<i>i</i>)	Coordinated Group	Common Headway (hr)	$BOUC_i$ (\$/hr)
1	1,2	0.383	2929.75
	1,2,3	0.365	2929.51
	1,2,3,4	0.335	2925.22
	1,2,3,4,5	0.306	2944.46
	1,2,3,4,5,6	0.301	2920.13
	2,3	0.352	2925.02
	2,3,4	0.322	2922.99
	2,3,4,5	0.293	2938.80
	2,3,4,5,6	0.286	*2918.25
	3,4,	0.292	2930.43
	3,4,5	0.264	2938.78
	3,4,5,6	0.252	2928.98
	4,5	0.230	2929.91
	4,5,6	0.222	2921.23
5,6	0.201	2923.82	
2	1,2	0.221	3553.74
	1,2,3	0.198	3549.58
	1,2,3,4	0.204	3440.88
	2,3	0.151	3441.22
	2,3,4	0.169	*3350.04
	3,4	0.130	3440.73
5	1,2	0.299	2795.80
	1,2,3	0.278	2797.55
	1,2,3,4	0.268	2785.70
	2,3	0.253	2789.98
	2,3,4	0.246	*2778.94
	3,4	0.226	2785.40
11	1,2	0.280	3061.33
	1,2,3	0.279	3045.56
	1,2,3,4	0.266	3049.96
	1,2,3,4,5	0.261	*3038.82
	2,3	0.268	3057.88
	2,3,4	0.252	3062.67
	2,3,4,5	0.243	3060.18
	3,4,	0.235	3068.53
	3,4,5	0.224	3065.57
	4,5	0.197	3060.55

$BOUC_i$ = bus operator and user costs only for bus routes connected at transfer station *i*

Table 6-14 Optimal Common Headway - Stage II Optimization (Data Set 3)

Rail Station (i)	Coordinated Group	Common Headway (hr)	$BOUC_i$ (\$/hr)
1	1,2	0.40	2563.41
	1,2,3	0.39	2557.86
	1,2,3,4	0.369	2549.99
	1,2,3,4,5	0.342	2560.36
	1,2,3,4,5,6	0.356	*2508.43
	2,3	0.382	2558.38
	2,3,4	0.357	2550.24
	2,3,4,5	0.329	2559.10
	2,3,4,5,6	0.339	2517.30
	3,4	0.326	2561.59
	3,4,5	0.296	2566.66
	3,4,5,6	0.287	2550.27
	4,5	0.260	2563.35
	4,5,6	0.254	2551.55
	5,6	0.229	2557.38
2	1,2	0.243	3199.95
	1,2,3	0.221	3189.18
	1,2,3,4	0.226	3075.89
	2,3	0.165	3105.03
	2,3,4	0.186	*3002.87
	3,4	0.140	3101.56
5	1,2	0.368	2266.89
	1,2,3	0.330	2274.64
	1,2,3,4	0.326	*2249.53
	2,3	0.297	2268.60
	2,3,4	0.288	2253.66
	3,4	0.252	2259.14
11	1,2	0.354	2413.82
	1,2,3	0.366	2380.34
	1,2,3,4	0.341	2388.91
	1,2,3,4,5	0.329	*2372.88
	2,3	0.335	2410.4
	2,3,4	0.305	2420.68
	2,3,4,5	0.295	2414.16
	3,4	0.273	2425.02
	3,4,5	0.255	2424.25
	4,5	0.217	2414.56

$BOUC_i$ = bus operator and user costs only for bus routes connected at transfer station i

For example, in Table 6-12 (shown in bold), the optimal coordinated group at station 1 includes bus routes 2, 3, 4, 5, and 6, whose optimal common headway is 0.273 hrs that achieves the minimum total cost of 34,13.98 \$/hr.

The optimal common headway of each coordinated group and slack time of each coordinated route are summarized in Table 6-15. The slack times of coordinated routes vary from 0 to 0.047 hrs, depending on the standard deviation of vehicle arrival times and transfer demand. In general, the increasing standard deviation of vehicle arrival times and transfer demand (sum of the demand transferring from a coordinated route to other coordinated routes) may cause the increase of slack time. The relation between slack time and standard deviation of vehicle arrival time will be discussed further in Section 6.1.3.

Various cost components (e.g., wait, transfer, in-vehicle and supplier costs) associated with both the rail line and bus routes at different transfer stations are computed and shown in Table 6-16. It is shown that the minimum total costs of the analyzed network for different demand levels (e.g., data sets 1, 2, and 3) are 12,760.99 \$/hr, 15,871.54 \$/hr, 13,919.21 \$/hr, respectively. All of them are lower than that obtained from Stage I. Comparing results obtained from Stages I (Table 6-11) and II (Table 6-16) optimization, it was found that the bus user wait and in-vehicle costs are higher with coordinated service (Stage II), while there are compensations savings from transfer and supplier costs.

Table 6-15 Optimal Slack Times and Common Headways– Stage II Optimization

Data Set	Rail Station (i)	Coordinated Group	Common Headway (hr)	Optimal Slack Time (hr)					
				Route 1	Route 2	Route 3	Route 4	Route 5	Route 6
1	1	2,3,4,5,6	0.273	-	0.033	0.034	0.040	0.043	0.034
	2	2,3,4	0.310	-	0.038	0.039	0.033	-	-
	5	1,2,3,4	0.297	0.043	0.041	0.034	0.036	-	-
	11	1,2,3	0.350	0.027	0.036	0.043	-	-	-
2	1	2,3,4,5,6	0.286	-	0.000	0.006	0.028	0.030	0.029
	2	2,3,4	0.169	-	0.029	0.030	0.027	-	-
	5	2,3,4	0.246	-	0.000	0.011	0.016	-	-
	11	1,2,3,4,5	0.261	0.000	0.009	0.023	0.016	0.020	-
3	1	1,2,3,4,5,6	0.356	0.023	0.045	0.047	0.047	0.041	0.035
	2	2,3,4	0.186	-	.0032	0.032	0.030	-	-
	5	1,2,3,4	0.326	0.019	0.032	0.030	0.029	-	-
	11	1,2,3,4,5	0.329	0.028	0.034	0.041	0.033	0.030	-

Stage III Optimization

The optimal common headways for the rail-bus coordination optimization in Stage III are obtained and shown in Table 6-17, while the optimal coordination for isolated transfer stations are identified in Table 6-18. Various cost components with optimal rail-bus coordination at isolated transfer stations are shown in Table 6-19. For the demand of data set 1, due to the rail-bus coordination at each transfer station, the total cost is reduced as shown in Tables 6-16 and 6-17. It is also found that the optimal common headway of 0.336 hr at station 11 yields the minimum total cost of 12,530.01 \$/hr (lower than that obtained from Stage II).

For demand of data set 2, the rail-bus coordination at both stations 2 (direction 1) and 11 produces a total cost (Table 6-17) that is lower than the total cost obtained at Stage II (Table 6-16). It is also found that the coordination of bus routes and both rail service directions at station 11 yields the minimum total cost of 15,789.31 \$/hr (lower than that obtained from Stage II), while the optimal coordinated headway is 0.192 hrs.

Table 6-16 Various Cost Components - Stage II Optimization

Routes	Rail Station (<i>i</i>)	Wait Cost (\$/hr)	Transfer Cost (\$/hr)	In-vehicle Cost (\$/hr)	UC_i (\$/hr)	SC_i (\$/hr)	SUC_i (\$/hr)
Data Set 1							
Buses	1	695.37	398.79	1203.08	2297.24	1116.74	3413.98
Buses	2	315.23	237.85	457.10	1010.19	566.78	1576.97
Buses	5	583.71	274.82	1142.07	2000.61	897.21	2897.81
Buses	11	525.52	478.5	1176.76	2180.78	1141.33	3322.11
Rail	-	119.11	412.57	475.03	1006.70	543.42	1550.12
Total	-	2238.94	1802.53	4454.04	8495.52	4265.48	12760.99
Data Set 2							
Buses	1	494.55	486.12	893.34	1874.02	1044.23	2918.25
Buses	2	532.78	404.73	1386.58	2324.10	1025.94	3350.04
Buses	5	442.35	402.50	991.42	1836.28	942.65	2778.93
Buses	11	420.68	526.83	1048.70	1996.21	1042.61	3038.82
Rail	-	805.68	361.49	1382.80	2549.98	1235.52	3785.5
Total	-	2696.04	2181.67	5702.84	10580.59	5290.95	15871.54
Data Set 3							
Buses	1	561.45	319.40	729.37	1610.23	898.20	2508.43
Buses	2	532.59	338.60	1198.41	2069.61	933.26	3002.87
Buses	5	513.52	215.97	733.03	1462.54	786.99	2249.53
Buses	11	495.43	303.89	731.29	1530.57	842.31	2372.88
Rail	-	828.80	338.37	1382.80	2549.98	1235.52	3785.5
Total	-	2931.79	1516.23	4774.9	9222.93	4696.28	13919.21

SUC_i = supplier and user cost at station *i*

UC_i = user cost at station *i*

SC_i = supplier cost at station *i*

For demand of data set 3, the total cost (Table 6-17) obtained while considering the rail-bus coordination at all transfer stations are higher than the corresponding minimum total costs (Table 6-16) obtained at Stage II. Due to low transfer volume from trains to buses coordination is not cost-effective at this situation.

Table 6-17 Costs and Common Headways-Stage III Optimization

Rail Station Index (<i>i</i>)	Coordinated Rail Direction	$BOUC_i$ (\$/hr)	$ROUC_i$ (\$/hr)	TC (\$/hr)	Common Headway (hr)
Data Set 1					
1	Both	3327.17	1489.11	12613.175	0.292
2	1	1522.44	1532.31	12599.73	0.299
2	2	1549.16	1545.97	12640.11	0.292
5	1	2843.94	1514.6	12582.67	0.297
5	2	2843.87	1478.61	12546.61	0.303
11	Both	3243.54	1486.63	12530.01	0.336
Data Set 2					
1	Both	2907.52	3922.95	15998.27	0.188
2	1	3334.12	3792.67	15862.8	0.132
2	2	3390.01	3792.88	15918.9	0.132
5	1	2823.55	3850.43	15981.09	0.163
5	2	2829.18	3810.39	15946.68	0.166
11	Both	2851.96	3890.12	15789.31	0.192
Data Set 3					
1	Both	2691.65	3971.84	14288.77	0.196
2	1	3073.55	3799.01	14003.40	0.135
2	2	3078.15	3798.51	14007.50	0.135
5	1	2452.22	3935.86	14272.26	0.181
5	2	2455.39	3887.01	14226.58	0.186
11	Both	2476.55	3907.87	14145.25	0.193

$BOUC_i$ = Bus operator and user costs for bus routes connected at transfer station *i*

$ROUC_i$ = Rail operator and user costs

TC = The total cost

Table 6-18 Optimal Common Headways and Slack Times – Stage III Optimization

Data Set	Rail Station (<i>i</i>)	Coordinated Rail Direction	Common Headway (hr)	Slack Time (hr)					
				Route 1	Route 2	Route 3	Route 4	Route 5	Route 6
1	11	Both	.336	.0446	.0461	.0530	-	-	-
2	11	Both	.192	.0386	.0376	.0434	.0382	.0343	-
3	2	1	.135	-	0	.0038	.0137	-	-

Table 6-19 Various Cost Components – Stage III Optimization

Rail Station (<i>i</i>)	Wait Cost (\$/hr)	Transfer Cost (\$/hr)	In-vehicle Cost (\$/hr)	User Cost (\$/hr)	Supplier Cost (\$/hr)
Data Set 1					
11 (both)	513.83	382.11	1175.80	2071.74	1171.80
Rail	148.24	424.29	475.74	1048.28	438.35
Data Set 2					
11 (both)	309.69	76.26	1036.22	1422.17	1429.79
Rail	1275.7	421.35	1395.78	3092.84	797.28
Data Set 3					
2 (d=1)	404.73	349.47	1173.67	1927.87	1145.68
Rail	926.27	376.96	1385.42	2688.66	1110.35

Stage IV Optimization

Finally, at Stage IV the rail-bus coordination considering multiple transfer stations is analyzed and the optimal results for various demand levels are shown in Table 6-20. For demand of data set 1, the minimum total cost of 12,244.80 \$/hr is achieved when transfer stations 1, 2, 5, and 11 (chosen for coordination at Stage III) are jointly coordinated with the rail service. However, for demand of data set 2, coordination at stations 1, 5 and 11 with an optimal headway of 0.247 hr can minimize the total cost. Under demand of data set 3, rail-bus coordination is not recommended because of higher minimum cost. The minimum total cost of 12,244.80 \$/hr and 15,311.74 \$/hr for demand of data sets 1 and 2 respectively at Stage IV optimization is better than that of 12,530.01 \$/hr and 15,789.31 \$/hr obtained at Stage III. For demand of data set 3, the minimum total cost obtained at Stage II is lower than that obtained at both Stages III and IV. Table 6-21 shows various cost components associated with rail and bus routes at different transfer stations.

Table 6-20 Optimal Common Headways – Stage IV Optimization

Stations containing coordinated rail-bus	BOUC at Station 1 (\$/hr)	BOUC at Station 2 (\$/hr)	BOUC at Station 5 (\$/hr)	BOUC at Station 11 (\$/hr)	ROUC (\$/hr)	TC (\$/hr)	Common Headway (hr)
Data Set 1							
5,11	3413.98	1576.97	2847.56	3244.16	1398.03	12480.7	0.334
2,5,11	3413.98	1517.52	2848.52	3242.95	1371.49	12394.46	0.337
1,2,5,11	3338.59	1517.61	2846.53	3246.48	1295.56	12244.8	0.3297
2,5	3413.98	1519.47	2843.48	3322.11	1455.53	12554.57	0.313
1,2,5	3330.52	1519.45	2843.84	3322.11	1384.87	12400.79	0.312
1,2	3328.38	1521.14	2897.82	3322.11	1465.91	12533.36	0.304
Data Set 2							
11,1	2742.19	3350.04	2778.94	2698.36	3988.12	15557.65	0.233
11,1,5	2708.12	3350.04	2673.86	2662.82	3916.97	15311.81	0.247
11,1,5,2	2742.89	3378.39	2684.98	2699.39	3838.74	15344.40	0.233
1,5	2807.64	3350.04	2711.28	3038.82	3925.22	15833.00	0.212
1,5,2	2839.34	3337.39	2726.01	3038.82	3876.88	15818.44	0.203
5,2	2922.99	3321.33	2821.69	3038.82	3812.64	15917.47	0.167
Data Set 3							
1,11	2512.05	3002.87	2249.53	2297.45	4051.07	14112.97	0.242
1,11,5	2444.17	3002.87	2230.79	2238.74	4018.67	13935.24	0.268
1,11,5,2	2477.44	3046.91	2251.86	2267.39	3937.99	13981.6	0.254
11,5	2508.43	3002.87	2296.81	2325.24	3969.27	14099.62	0.229
11,5,2	2508.43	3009.96	2318.40	2352.06	3917.69	14106.54	0.223
5,2	2508.43	2992.33	2452.29	2372.88	3884.50	14210.43	0.186

BOUC = bus operator and user costs

ROUC= rail operator and user costs

TC= total cost

The benefit of coordination in terms of the cost savings for demand situations specified in data sets 1, 2 and 3 are summarized in Table 6-22, while the optimized decision variables and coordinated status of each route are shown in Tables 6-23, 6-24, and 6-25. It is shown in Table 6-22 that for all three sets of demand data, the wait and in-vehicle costs increase and the transfer and supplier costs decrease when coordination is applied.

Table 6-21 Various Cost Components - IV Optimization

Routes	Rail Station (i)	Wait Cost (\$/hr)	Transfer Cost (\$/hr)	In-vehicle Cost (\$/hr)	User Cost (\$/hr)	Supplier Cost (\$/hr)
Data Set 1						
Buses	1	831.35	315.04	1220.33	2366.72	971.85
Buses	2	331.77	182.73	458.42	972.92	544.67
Buses	5	646.40	226.09	1149.52	2022.01	824.51
Buses	11	508.15	379.50	1175.33	2062.98	1183.48
Rail	-	233.51	139.403	475.67	848.58	446.97
Total	-	2551.18	1242.76	4479.27	8273.21	3971.48
Data Set 2						
Buses	1	434.04	168.89	887.77	1490.70	1217.41
Buses	2	532.78	404.73	1386.58	2324.10	1025.94
Buses	5	444.26	270.84	991.61	1706.71	967.13
Buses	11	398.86	79.89	1046.25	1525.00	1137.82
Rail	-	1642.99	238.69	1405.93	3287.61	629.35
Total	-	3452.93	1163.04	5718.14	10334.12	4977.65
Data Set 3						
Buses	1	400.68	121.53	719.70	1241.91	1235.51
Buses	2	703.67	353.32	1231.52	2288.51	758.38
Buses	5	400.68	112.15	725.99	1238.82	1013.02
Buses	11	382.87	74.32	724.29	1181.48	1085.90
Rail	-	1735.59	181.16	1407.15	3323.90	614.07
Total	-	3623.49	842.48	4808.65	9274.62	4706.88

Table 6-22 Cost Comparison between Coordination and not Coordination

Costs (\$/hr)	Without Coordination			With Coordination			Benefit (\$/hr)		
	Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 1	Set 2	Set 3
Wait	1855.41	2488.69	2611.85	2551.18	3452.93	2931.79	-695.77	-964.24	-319.94
Transfer	2252.22	2395.72	1855.18	1242.76	1163.04	1516.23	1009.46	1232.68	338.95
In-vehicle	4400.93	5670.00	4738.00	4479.27	5718.14	4774.90	-78.34	-48.14	-36.90
User	8505.57	10554.46	9205.09	8273.21	10334.12	9222.93	232.36	220.34	-17.84
Supplier	4610.53	5475.85	4949.73	3971.48	4977.65	4696.28	639.05	498.20	253.45
Total	13119.1	16030.31	14154.82	12244.80	15311.81	13919.21	874.41	718.50	235.61

Overall it can be concluded that (1) coordination is beneficial in intermodal transit operation for routes with large headways and large demand transfer among routes, (2) coordination is beneficial if the increase of wait and in-vehicle costs can be compensated from the savings in transfer cost.

Table 6- 23 Optimal Results of Coordinated Intermodal Transit System (Data Set 1)

Rail Station (i)	Bus Route (j)	*Coordination Status (0, 1)	Common Headway (hr)	Slack Time (hr)
1	1	0	0.33	-
	2	1		0.0621
	3	1		0.0623
	4	1		0.0564
	5	1		0.0495
	6	1		0.0371
	Rail	1 (1,2)**		-
2	1	0	0.33	-
	2	1		0.0411
	3	1		0.0413
	4	1		0.0360
	Rail	1 (1)		-
5	1	1	0.33	0.0562
	2	1		0.0505
	3	1		0.0373
	4	1		0.0386
	Rail	1 (2)		-
11	1	1	0.33	0.0443
	2	1		0.0459
	3	1		0.0527
	4	0		-
	5	0		-
	Rail	1 (both)		-

* 0= not coordinated, 1= coordinated

** 1 = rail direction 1, 2 = rail direction 2

Table 6-24 Optimal Results of Coordinated Intermodal Transit System (Data Set 2)

Rail Station (i)	Bus Route (j)	*Coordination Status (0, 1)	Common Headway (hr)	Slack Time (hr)
1	1	0	0.247	-
	2	1		0.0430
	3	1		0.0469
	4	1		0.0491
	5	1		0.0512
	6	1		0.0366
	Rail	1 (1,2)**		-
2	1	0	0.169	-
	2	1		0.029
	3	1		0.030
	4	1		0.027
	Rail	0		-
5	1	0	0.247	-
	2	1		0.0498
	3	1		0.0379
	4	1		0.0397
	Rail	1 (2)		-
11	1	1	0.247	0.0467
	2	1		0.0432
	3	1		0.0498
	4	1		0.0438
	5	1		0.0384
	Rail	1 (1,2)		-

* 0= not coordinated, 1= coordinated

** 1 = rail direction 1, 2 = rail direction 2

Table 6-25 Optimal Results of Coordinated Intermodal Transit System (Data Set 3)

Rail Station (i)	Bus route (j)	*Coordination Status (0, 1)	Common Headway (hr)	Slack Time (hr)
1	1	1	0.356	0.0233
	2	1		0.0448
	3	1		0.0474
	4	1		0.0467
	5	1		0.0412
	6	1		0.0351
	Rail	0		-
2	1	0	0.186	-
	2	1		0.0315
	3	1		0.0324
	4	1		0.0295
	Rail	0		-
5	1	1	0.326	0.0193
	2	1		0.0317
	3	1		0.0296
	4	1		0.0291
	Rail	0		-
11	1	1	0.329	0.028
	2	1		0.0343
	3	1		0.0408
	4	1		0.0332
	5	1		0.0296
	Rail	0		-

* 0= not coordinated, 1= coordinated

6.1.3 Sensitivity Analysis

The sensitivity analysis of optimal decision variables (i.e., headway and slack times) with respect to various values of parameters (i.e., standard deviation of vehicle arrival times and value of user's time) conducted in this section confirms the relationship established in the model developed in Chapter 3. Unless otherwise specified, all sensitivity analyses are performed based on demand of data set 1.

In general, the optimal bus and rail headways decrease, while the value of users' wait time increases because of increased user costs (e.g., wait and transfer costs). Figure 6-1 shows how the optimal bus and rail headways vary with the value of users' wait time. The figure shows that the optimal headway increases as the value of users' wait time decreases. However, the bus headway (station 1, route 6) is constant at 0.36 hr if the value of users' wait time is less than 0.75 \$/pass-hr, subject to the service capacity constraint.

Figure 6-2 shows the relationship between the optimal common headway obtained from Stage IV optimization and the value of users' wait time. As in Figure 6-1, the optimal common headway increases while the value of users' wait time decreases. However, subject to capacity constraints (bus route 6 at station 1), the common headway remains constant at 0.36 hr if the value of users' wait time is less than 5.75 \$/pass-hr.

In Figure 6-3, it is shown that the total cost is convex with respect to common headway. The figure also shows that total cost increases, as the common headway deviates from its optimum (i.e., 0.333 hr). Due to the service capacity constraint, the optimal common headway cannot exceed 0.36 hr.

Figure 6-4 shows the relationship between various cost components and the common headway. The wait, transfer and in-vehicle costs increase and the supplier cost decreases with the increase of common headway. In addition, this figure shows that the minimum total cost is a trade off between the supplier and user (wait, transfer and in-vehicle cost) costs.

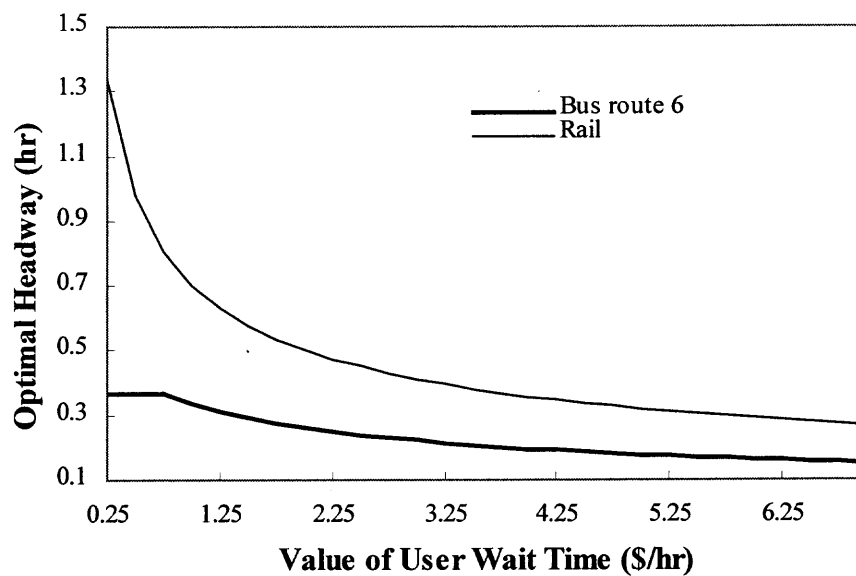


Figure 6-1 Optimal Headway vs. Value of Users' Wait Time

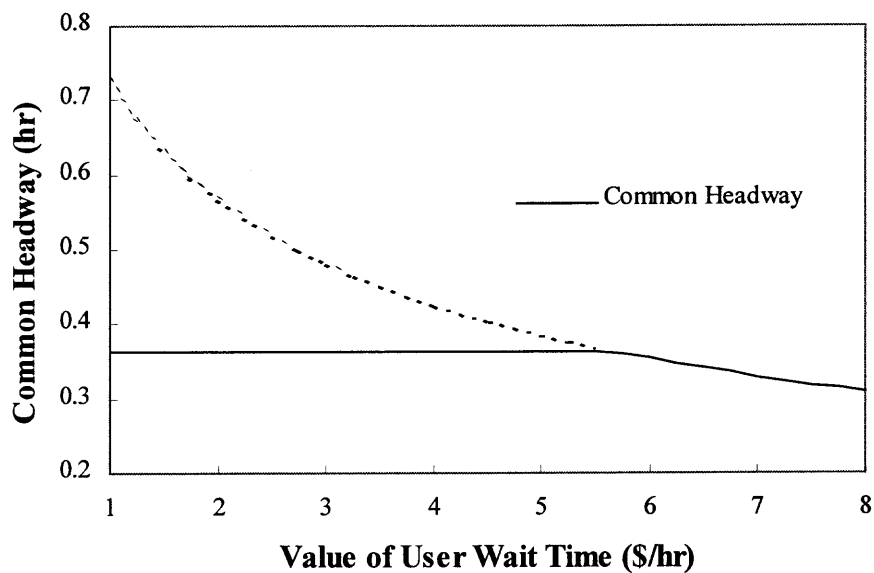


Figure 6-2 Common Headway vs. Value of Users' Wait Time

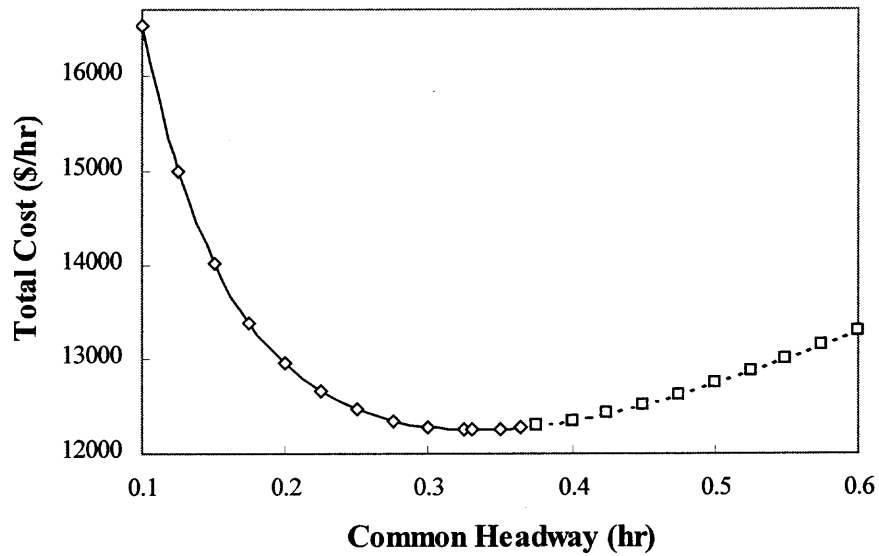


Figure 6-3 Total Cost vs. Common Headway

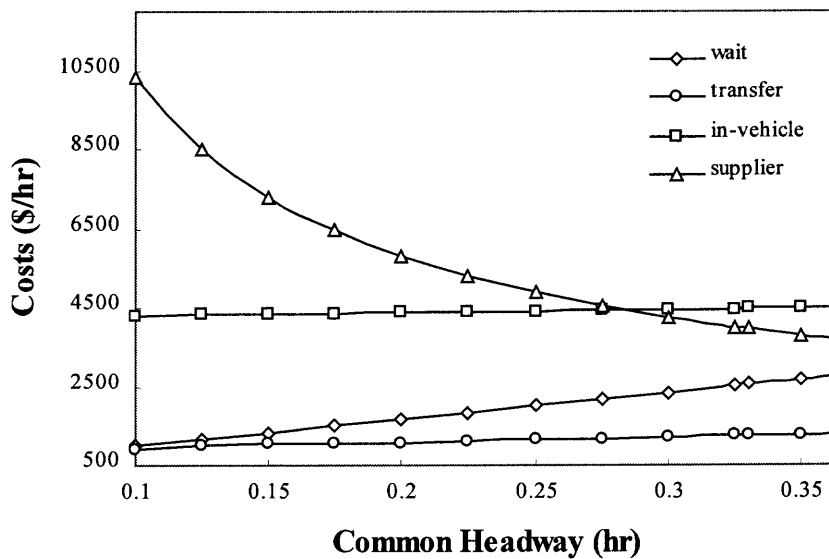


Figure 6-4 Various Cost Components vs. Common Headway

Figure 6-5 shows the relationship between the total cost and the slack time for bus route 3 at station 1. It is shown that the total cost curve is convex with respect to slack time and yields a minimum value when the slack time is 0.0623 hr.

Figure 6-6 shows how the standard deviation of vehicle arrival times affects the total cost both with and without coordination. The standard deviation of vehicle arrival time of routes 2 to 6 (coordinated group) at station 1 are varied from 0.001 to 0.065 hr and the corresponding total cost with (Stage IV) and without (Stage I) coordination are computed. From the figure, it is shown that as the standard deviation of vehicle arrival times increases the benefit from coordination decrease. Figures 6-7 and 6-8 show how the standard deviation of vehicle arrival times affects bus operator and user cost (BOUC) on bus routes at stations 1 and 5 both with (Stage III) and without (Stage I) coordination when demand of data set 2 is used. From these figures, it is observed that if the standard deviation (SD) of vehicle arrival time increases, bus operator and user cost increases faster with coordination than without it. It is also observed that the threshold standard deviation (0.04 hr) of vehicle arrival time at station 1 (Figure 6-7) is relatively higher than that (0.0075) of station 5 (Figure 6-8). This is mainly due to higher demand among coordinated routes at station 1 than station 5. In general the larger the common headway and transfer demand the higher the threshold vehicle arrival time standard deviation.

Figure 6-9 shows that the optimal slack time could be zero if the common headway is small. The figure also shows that the optimal slack time increases monotonically at a decreasing rate beyond certain critical headway ($h=0.125$ approximately).

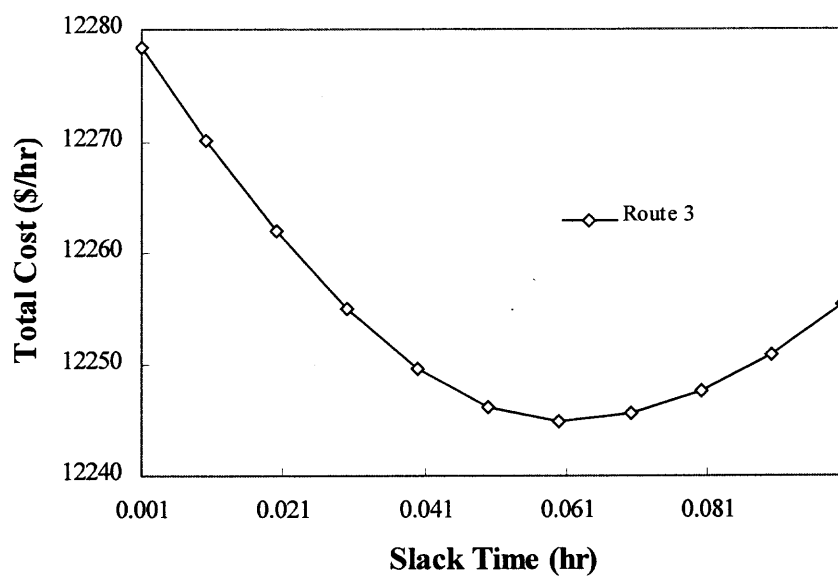


Figure 6-5 Total Cost vs. Slack Time

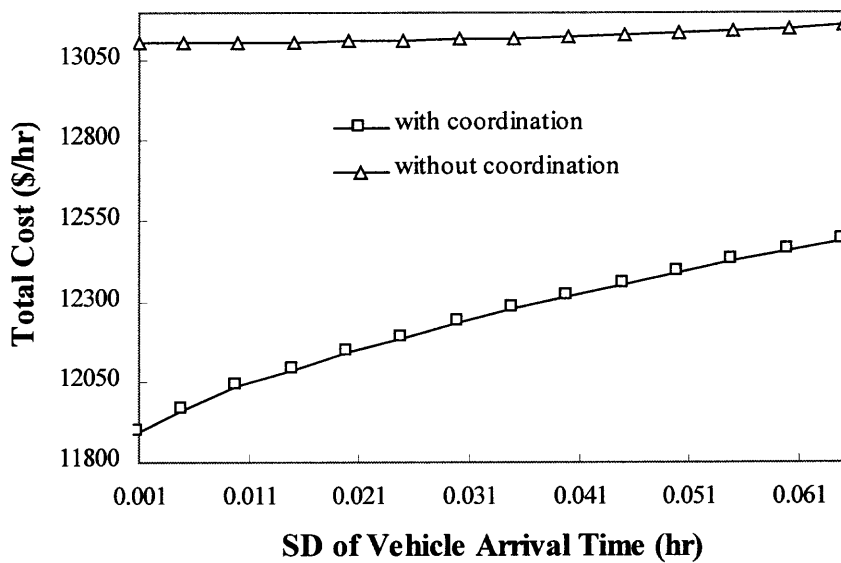


Figure 6-6 Standard Deviation (SD) of Vehicle Arrival Time
(Route 2 to 6 at Station 1) vs. Total Cost

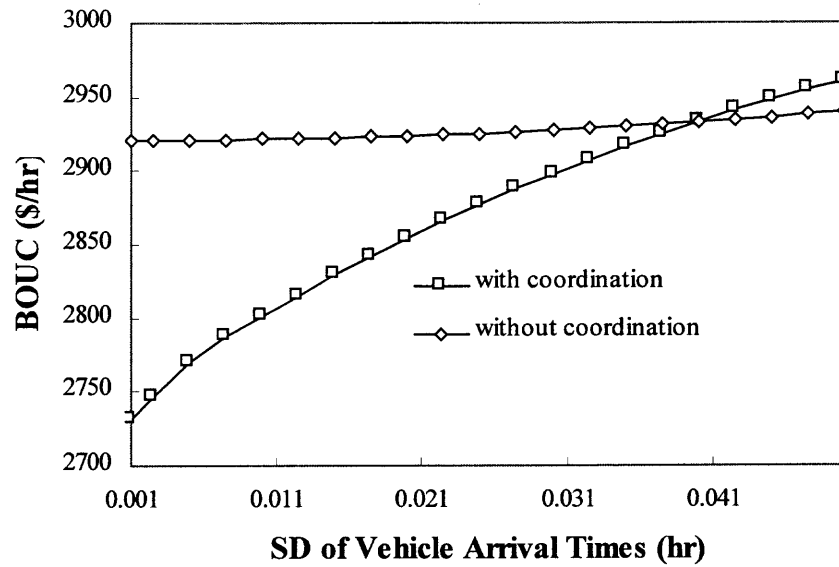


Figure 6-7 Standard Deviation (SD) of vehicle Arrival Time vs. Bus Operator and User Cost (BOUC) (Route 2 to 6 at Station 1)

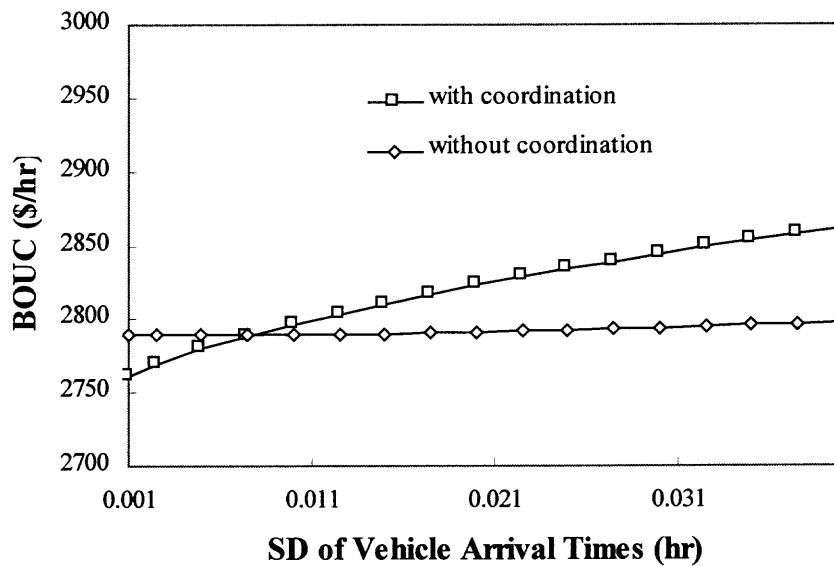


Figure 6-8 Standard Deviation (SD) of vehicle Arrival Time vs. Bus Operator and User Cost (BOUC) (Route 2 to 4 at Station 5)

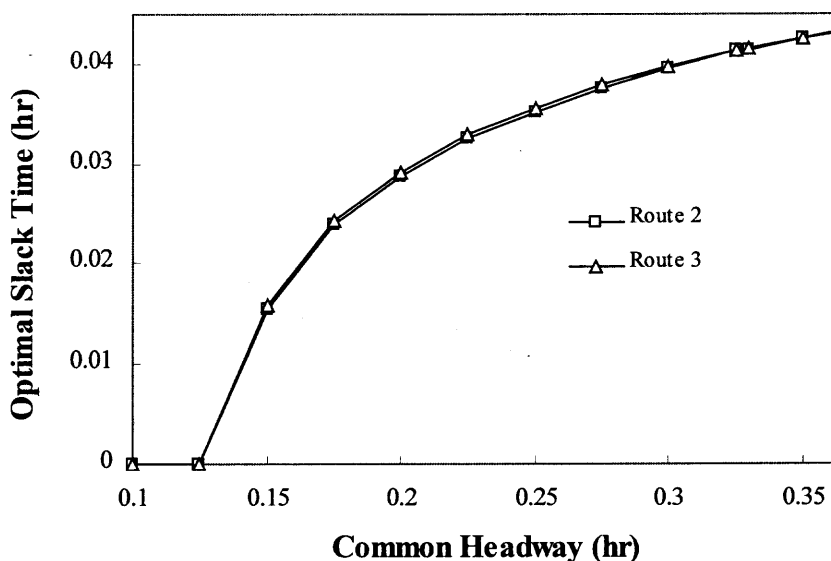


Figure 6-9 Optimal Slack Time vs. Common Headway

Figures 6-10 and 6-11 show the relationship between the optimal slack time and the standard deviation of vehicle arrival times at station 2 when demand of data sets 1 and 2 are used, respectively. First the optimal slack time increases as the standard deviation of vehicle arrival times increases'. However, as the standard deviation becomes a significant fraction of the headway, and increases further, the optimal slack time declines rapidly. The optimal slack time could be zero if the standard deviation of vehicle arrival time is high. The value of the slack time depends on four factors: (1) the sum of the demand transferring to coordinated bus routes, (2) demand transferring to the coordinated rail direction (3) the optimal common headway and (4) the standard deviation of vehicle arrival time. In general, systems with higher transfer demand favor higher slack time to increase the probability of a successful connection. The slack time also increases as the common headway increases. The influence of the above four factors on slack time can be observed in Figures 6-10 and 6-11. Note that in Figure 6-10 optimal common headway

varies from 0.331 to 0.277 hr, while in Figure 6-11, the common headway varies from 0.233 to 0.199 hr.

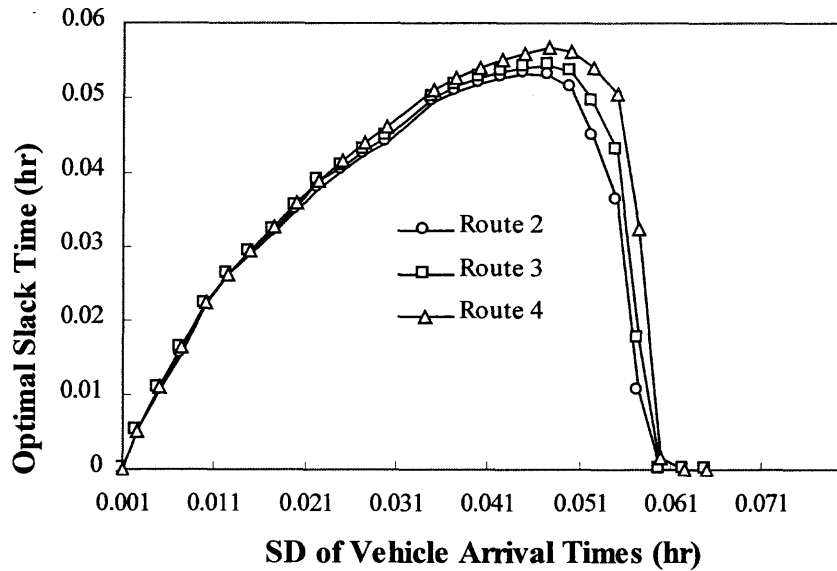


Figure 6-10 Optimal Slack Time vs. Standard Deviation (SD) of Vehicle Arrival Times (Demand Data set 1, Station 2)

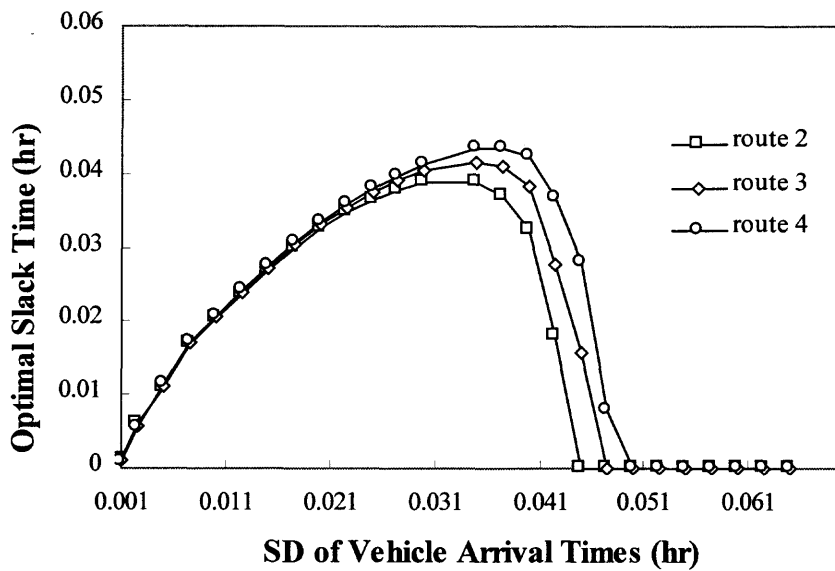


Figure 6-11 Optimal Slack Time vs. Standard Deviation (SD) of Vehicle Arrival Times (Demand Data set 2, Station 2)

6.2 Dynamic Dispatching Model

The purpose of the dynamic dispatching model discussed in this section is (1) to establish the relationship between distance and travel time from any vehicle location to the transfer station, (2) to optimize vehicle holding time through minimizing the total cost developed in Chapter 4 and (3) to explore the relationship between the decision variable (i.e., holding time) and various parameters (i.e., transfer demand, SD of vehicle arrival times, mean arrival times, value of user wait time and operating cost). The evaluation of the dispatching model is discussed in the following subsections.

- Network description
- Simulation and data collection
- Estimation of vehicle arrival times
- Optimization of vehicle holding time
- Sensitivity analysis

6.2.1 Network Description

To evaluate the application of the dynamic vehicle dispatching model at transfer stations along a rail line shown in Figure 6-12, transit routes connecting at station #2 are selected for the analysis. According to the intermodal transit network used for evaluating the coordination model discussed in section 6.1, it was found that (at station # 2) three feeder routes (routes # 2, #3, and #4) and rail direction 1 should be coordinated with a common headway of 19.8 minutes (0.33 hours) for demand of data set 1. Therefore, the network consists of three feeder routes and a rail line.

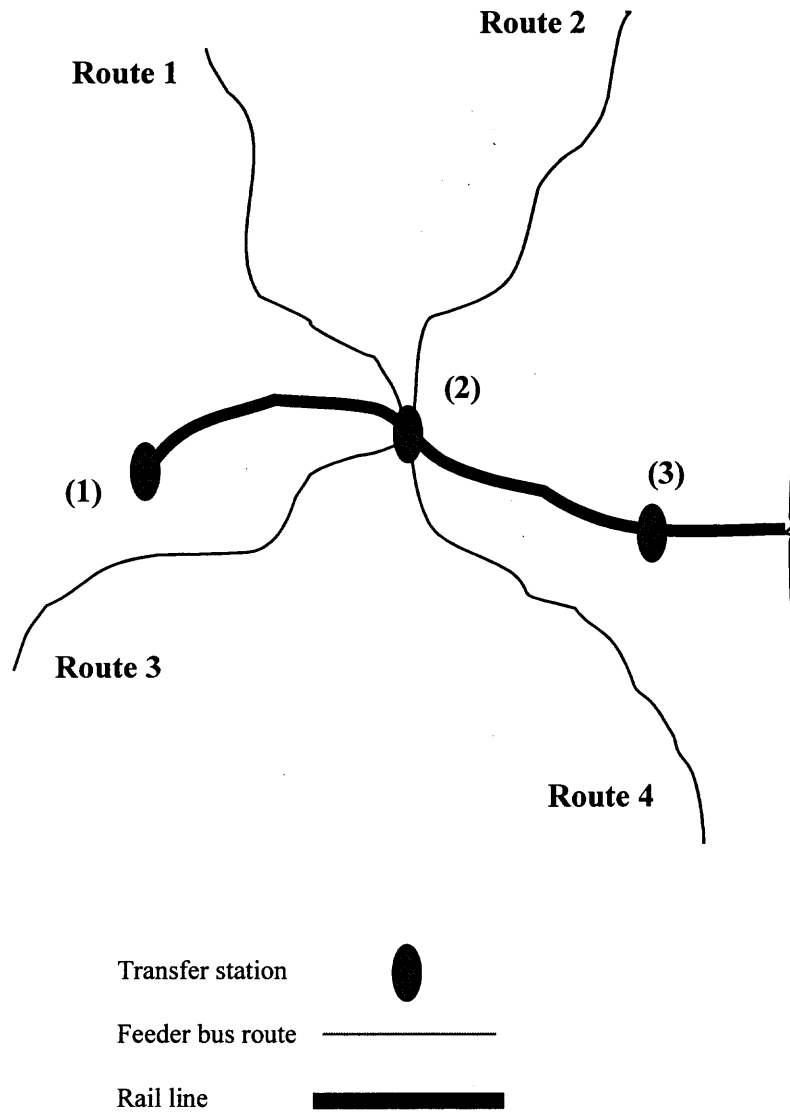


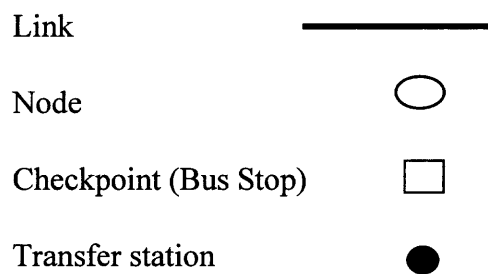
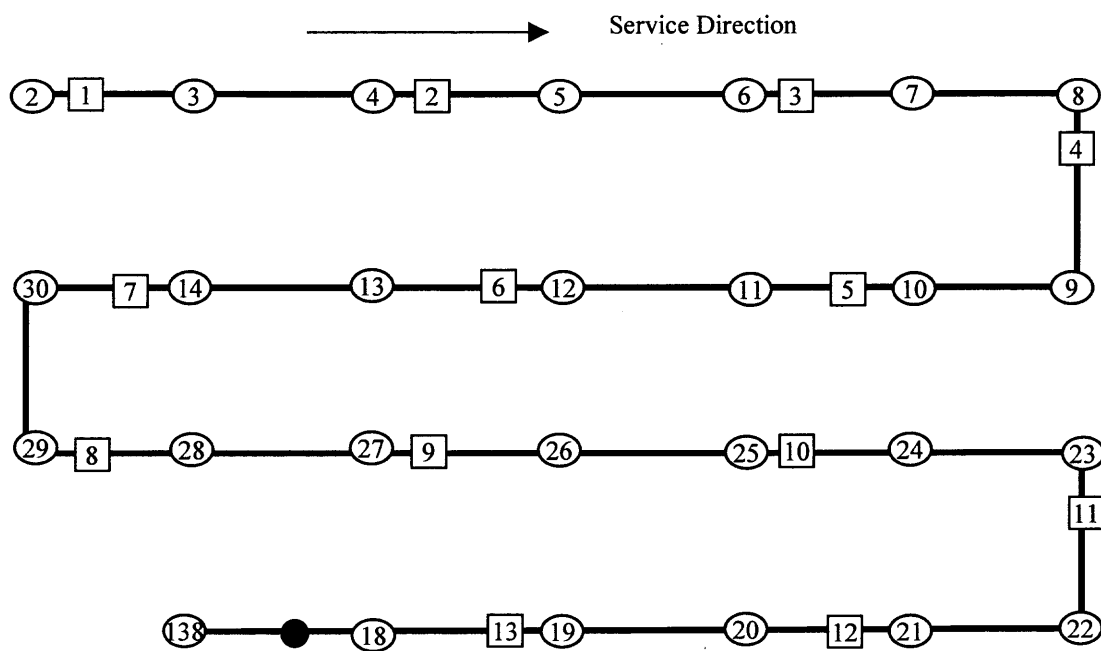
Figure 6-12 Configuration of Rail Station 2 and Its Feeder Bus Routes

The input information required by the dispatching model includes the hourly transfer demand for all routes, which can be obtained from Tables 6-3 and 6-4. To determine the numbers of passenger transferring from one vehicle to another, the hourly demand is converted to headway-based demand (i.e., the product of the hourly demand and headway). The baseline values of parameters (i.e., value of user wait time and average bus operating cost) used in this analysis are listed in Appendix A. Since the rail headway is assumed to be deterministic, train arrival times at station #2 are always known.

6.2.2 Simulation and Data Collection

The purpose of performing simulation is to emulate bus operations and monitor vehicle travel times from various locations to the transfer station. Thus, the vehicle arrival times can be estimated. In reality, vehicles arrive at a transfer station stochastically. Generally, the variation of vehicle arrival times reduces as the distance from the vehicle location to the transfer station decreases. Since the enhanced CORSIM model can emulate bus operations and generate second by second vehicle information (vehicle speed and locations), the means and standard deviations of vehicles travel times from various locations along the bus route to the transfer station can be determined.

A 2.22-mile segment of a bus network (as shown in Figure 6-13), consisting of 26 links and 27 signalized intersections, is designed for simulation experiments. Thirteen checkpoints, as shown in Figure 6-13 checkpoints are located at all bus stops, are selected for monitoring vehicle departure times and the corresponding vehicle arrival times at the transfer station.



Note:

All nodes represent signalized intersections

Each link is 900 feet long

Distance between upstream node and check point (bus stop) is 274 feet

Figure 6-13 Link Node Diagram for the Simulated Bus Route

Among the thirteen checkpoints, checkpoint #13 is the nearest checkpoint, while checkpoint #1 is the farthest checkpoint from the transfer station. The vehicle departure times from all checkpoints and their corresponding arrival times at the transfer station are collected during simulation. The departure and arrival times of 10 consecutive vehicles collected from various checkpoints and the transfer station are shown in Table 6-26.

6.2.2 Estimation of Vehicle Arrival Times

The vehicle travel time from a checkpoint to the transfer station is simply equal to the difference between the departure time from the checkpoint and the arrival time at the transfer station. The mean travel time from each checkpoint to the transfer station and the corresponding standard deviation of travel times are calculated by using Eqs. 4-12, and 4-13, respectively. The shortest travel times from all checkpoints to the transfer station are also observed. The distances between checkpoints to the transfer station and the corresponding travel time information is summarized in Table 6-27, while the relationship between travel times and the distance to the transfer station is shown in Figures 6-14 and 6-15. It was found that both the mean and standard deviation of travel times increase as the travel distance to the transfer station increases.

To find the vehicle travel time from any point (other than checkpoints) to the transfer station, it is assumed that both the mean and the standard deviation of travel times between any adjacent pairs of checkpoints have a linear relationship. If the distance between the checkpoints is relatively short such an assumption is quite reasonable. Therefore, the mean and standard deviation of travel times from any point to the transfer station can be approximated by linear interpolation.

Table 6-26 Departure and Arrival Times Information from Various Checkpoints

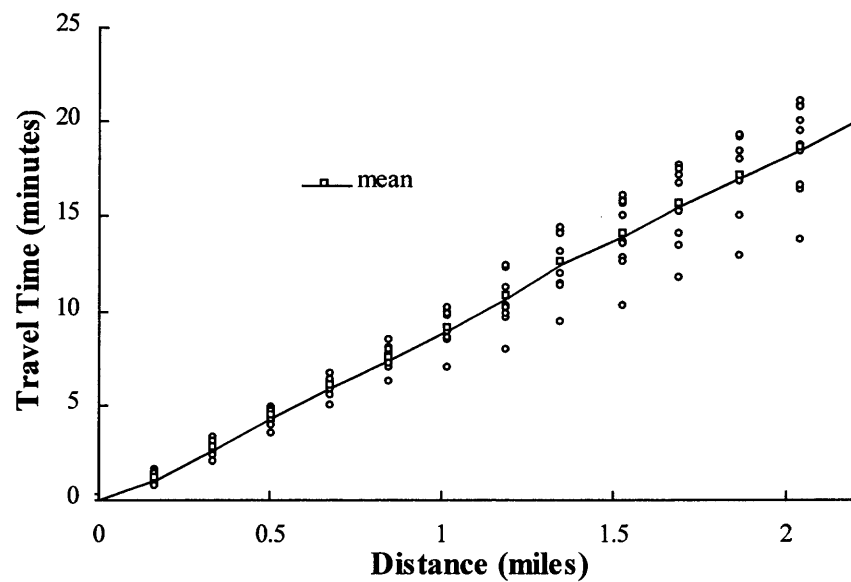
Checkpoints (Bus Stop)	Departure and Arrival Times (Simulation Second)									
	Bus	Bus	Bus	Bus	Bus	Bus	Bus	Bus	Bus	Bus
	# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	# 10
** 1	66	342	679	958	1242	1577	1854	2141	2482	2755
2	169	430	784	1048	1332	1677	1942	2233	2576	2848
3	269	515	874	1150	1422	1772	1993	2332	2684	2947
4	369	569	929	1251	1519	1866	2063	2424	2791	3049
5	470	650	1015	1355	1617	1919	2154	2525	2896	3149
6	572	728	1106	1451	1717	1994	2202	2626	2998	3265
7	685	836	1213	1553	1845	2083	2292	2730	3100	3375
8	821	907	1349	1630	1996	2164	2346	2796	3249	3519
9	936	1001	1449	1731	2097	2212	2393	2894	3363	3620
10	1037	1051	1557	1821	2199	2303	2471	2991	3459	3719
11	1128	1138	1661	1923	2294	2401	2558	3087	3563	3818
12	1229	1246	1762	2026	2395	2501	2645	3189	3658	3912
13	1335	1350	1862	2129	2494	2599	2691	3289	3757	4009
*ATATS	1414	1411	1950	2166	2574	2670	2761	3335	3836	4044

*ATATS = arrival time at the transfer station

** Departure time from check-point #1

Table 6-27 Travel Distance and Time from Checkpoints to the Transfer Station

Check point	Distance (mile)	Travel Time (min)		³ SD (min)
		¹ MTT	² STT	
1	2.22	20.10	15.12	2.42
2	2.05	18.54	13.65	2.38
3	1.875	17.00	12.8	2.17
4	1.70	15.55	11.63	2.02
5	1.53	14.02	10.12	1.90
6	1.35	12.50	9.32	1.64
7	1.19	10.75	7.82	1.50
8	1.02	8.97	6.92	0.94
9	0.85	7.44	6.13	0.65
10	0.68	5.92	4.83	0.51
11	0.51	4.32	3.38	0.47
12	0.34	2.66	1.93	0.41
13	0.17	1.07	0.58	0.32

¹MTT= Mean Travel Time²STT=Shortest Travel Time³SD= Standard Deviation of Vehicle Arrival Time**Figure 6-14** Travel Time Distribution

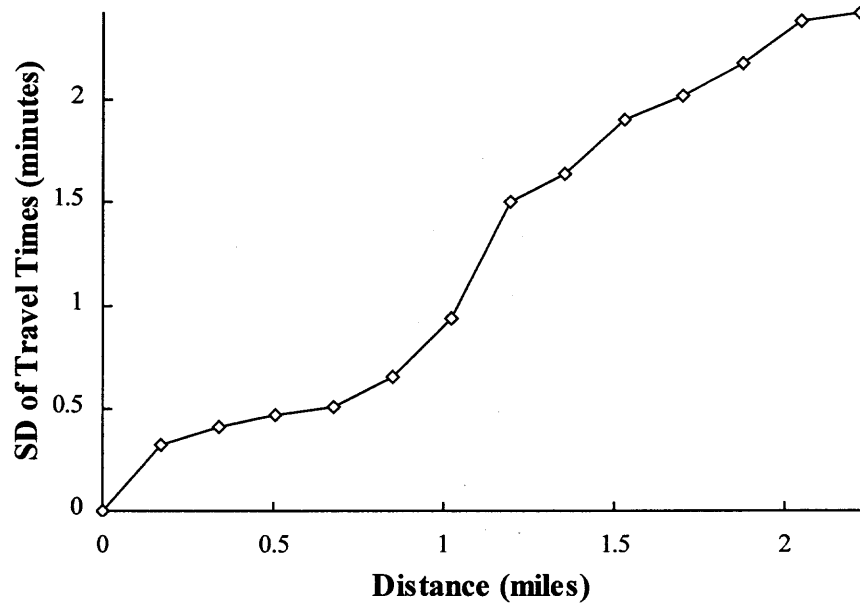


Figure 6-15 Standard Deviation (SD) of Travel Times vs. Travel Distance

The standard deviation of vehicle arrival times from any location to the transfer station can also be obtained by a linear interpolation of the standard deviation of travel time at adjacent (up-and down-stream) check-points. For optimizing the vehicle holding time, both normal [Taylor, 1982; Knoppers and Muller, 1995; Senevirate, 1990] and lognormal [Tunquist, 1978] vehicle arrival distributions are examined. The probability density function of vehicle travel time from various checkpoints to the transfer station for normal and lognormal vehicle arrival distributions (see appendix D) are shown in Figures 6-16 and 6-17, respectively.

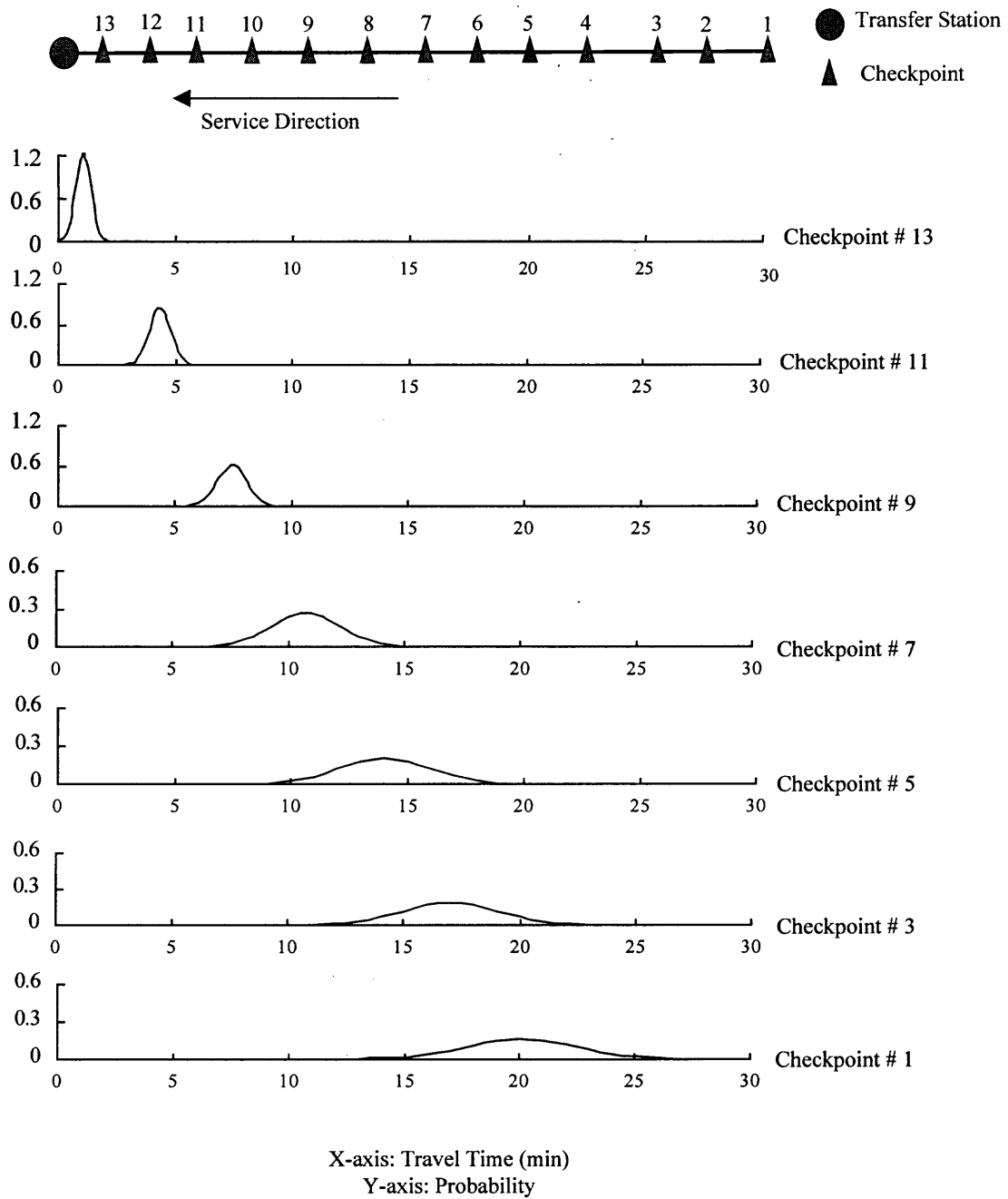


Figure 6-16 Normal Probability Density Functions of Vehicle Arrivals from Various Checkpoints to the Transfer Station

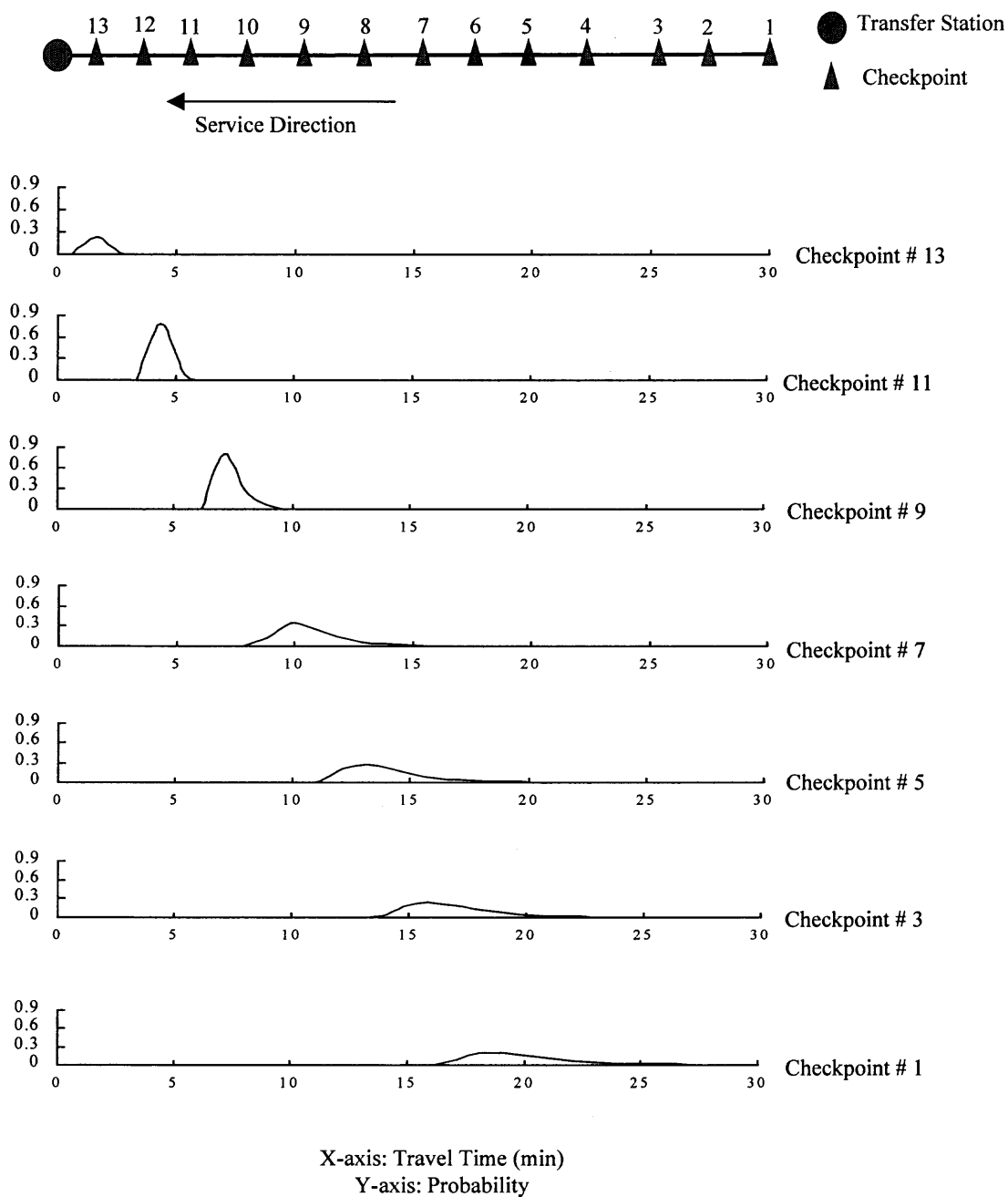


Figure 6-17 Lognormal Probability Density Functions of Vehicle Arrivals from Various Checkpoints to the Transfer Station

6.2.4 Optimization of Vehicle Holding Times

To optimize the vehicle-holding time using the dynamic dispatching model, the following we assumed to be known:

- Vehicle arrival probability density function at the transfer station,
- Transfer demand from one vehicle to another,
- Arrival times (i.e., mean and standard deviation) of all late vehicles from their current locations to the transfer station, and
- The headway of the coordinated routes

For example, vehicle a on route # 2 arrives at the transfer station on time, while vehicles b and c on routes # 3 and # 4 are late (see Figure 6-18). The operating headways for all routes (e.g., # 2, # 3 and # 4) are identical and equal to 19.8 minutes. If the reference point of time is $00:00$ (zero minute and zero second), the schedule time for the next arrival on route #2 is $19:48$. Given that, at the dispatching decision time of vehicle a , vehicles b and c are 0.51 and 1.02 miles away from the transfer station, respectively. The travel times of vehicles b and c from their location to the transfer station are estimated from Figures 6-14 and 6-15. The resulting mean and standard deviations of travel times are shown in Table 6-28.

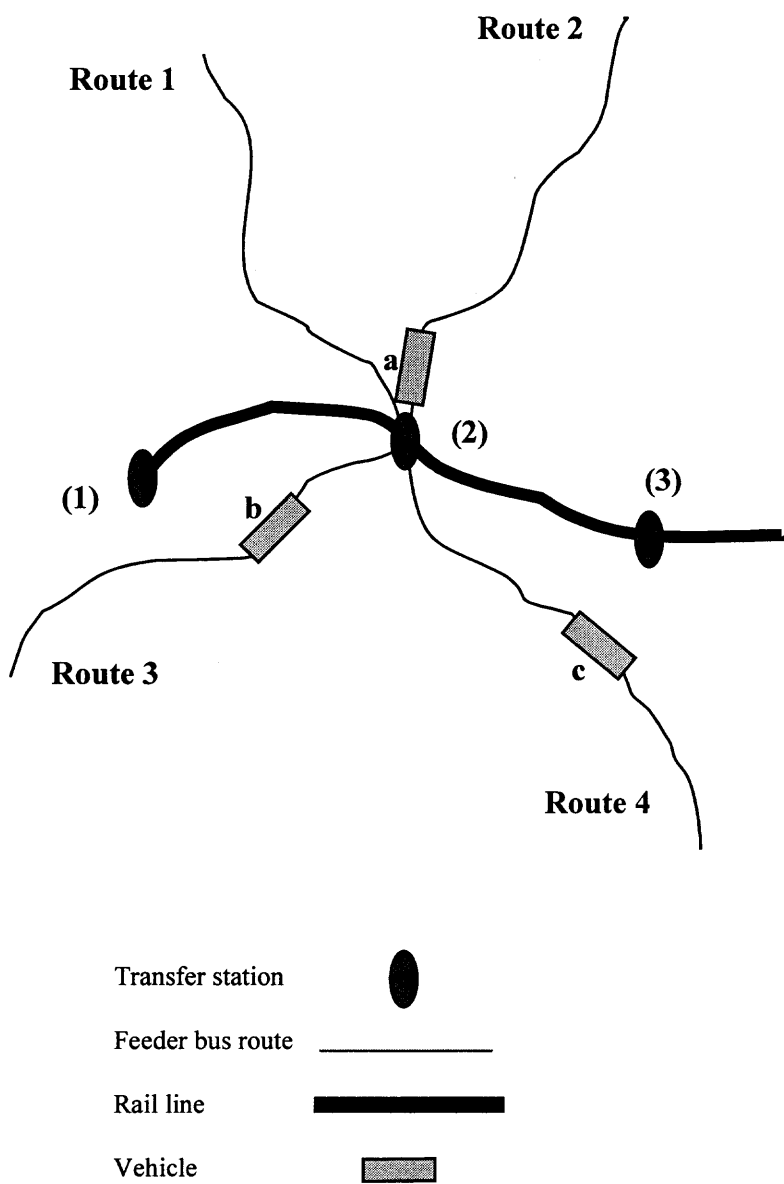


Figure 6-18 Coordinated Bus Routes with Vehicle Locations

Table 6-28 Late Vehicles Travel Time

Veh ID	Route #	¹ VL (mile)	Travel Time		⁴ SD of Travel Time (min)
			² MTT (min)	³ STT (min)	
<i>b</i>	3	0.51	4.32	3.38	.47
<i>c</i>	4	1.02	8.97	6.92	.94

¹VL= Vehicles location (distance from Transfer station)

²MAT = Mean Travel Time

³EAT = Shortest Travel Time

⁴SD = Standard Deviation

According to the given information (e.g., travel times and transfer demand), the holding time for dispatching vehicle *a* at the first dispatching decision time (i.e. 00:00) can be optimized by minimizing the total cost function formulated in Chapter 4. The maximum holding time can be determined by considering that both vehicles *b* and *c* are successfully connected with vehicle *a*. Based on an incremental line search procedure (set size = 2 seconds), the total costs for different holding times are calculated. The global minimum total cost and the corresponding holding time can be identified.

At the first dispatching decision time, the optimal holding time of vehicle *a* is found to be 5.16 and 5.08 minutes considering normal and lognormal arrival distributions of vehicles *b* and *c*, which yield the global minimum total costs of \$20.84, and \$21.19, respectively. However, without holding, the total costs are \$ 29.85 and \$ 29.88, respectively. Despite the difference between the normal and lognormal distribution functions, the optimal holding times and the minimum total costs in both cases are quite close. Due to the small standard deviation of travel times, the shapes of the normal and lognormal distributions are very similar to each other (see Figures 6-16 and 6-17 at checkpoint 11). Note that the probability density of the lognormal distribution is skewed with a long tail to the right, while the probability density of the normal distribution is

symmetric with respect to its mean. Therefore, the departure time of vehicle a is either 05:10 or 05:05 for normal and lognormal vehicle arrival distributions, and the relationship between the total cost and holding time considering both types of distributions is shown in Figures 6-19 and 6-20, respectively. It is obvious from the figures, that the total cost with respect to holding time is a non-convex function. The components of total cost include the connection delay cost incurred by passengers transferring from rail direction 1, the connection delay and missed connection costs incurred by passengers transferring from vehicles b and c , and the operator cost caused by holding vehicle a . The relationship between various cost components and holding time are shown in Figures 6-21 and 6-22.

To better understand the relationship between the total cost and holding time, the curve of the total cost function is partitioned into five zones (i.e., zone A-B bounded by lines A and B, and zones B-C, C-D, D-E, and E-F). In zone A-B, the total cost increases as the holding time increases because both the operator cost caused by holding vehicle a and the connection delay cost incurred by transfer passengers from rail direction 1 increase.

In zone B-C, the sharp decrease of total cost with the increase of holding time is caused by the increase of successful connection probability. From both Figures 6-21 and 6-22, it can be seen that the decrease of total cost accounts for the decrease of missed connection costs incurred by transfer passengers from vehicle b to a , if the holding time of vehicle a increases.

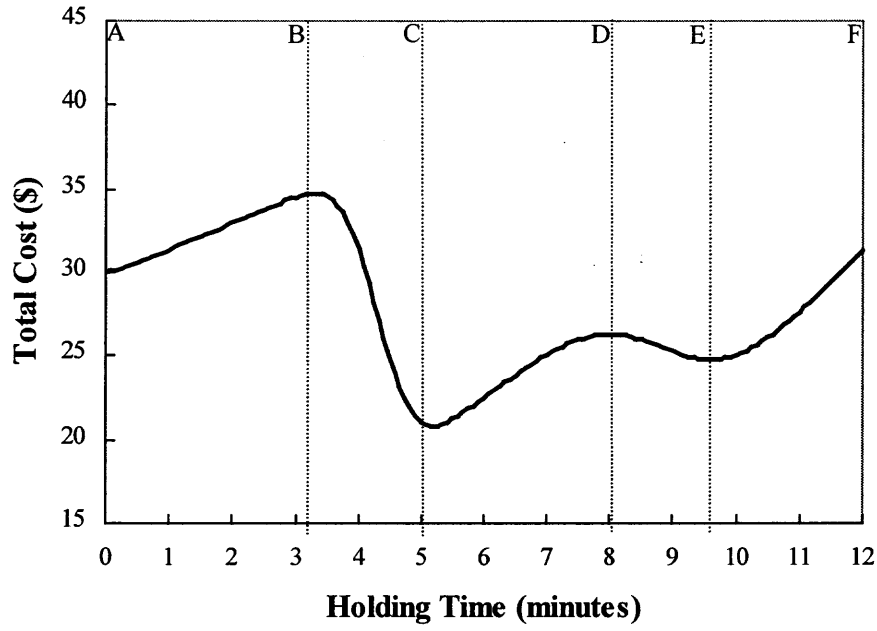


Figure 6-19 Total Cost vs. Holding Time (Normal Distribution)

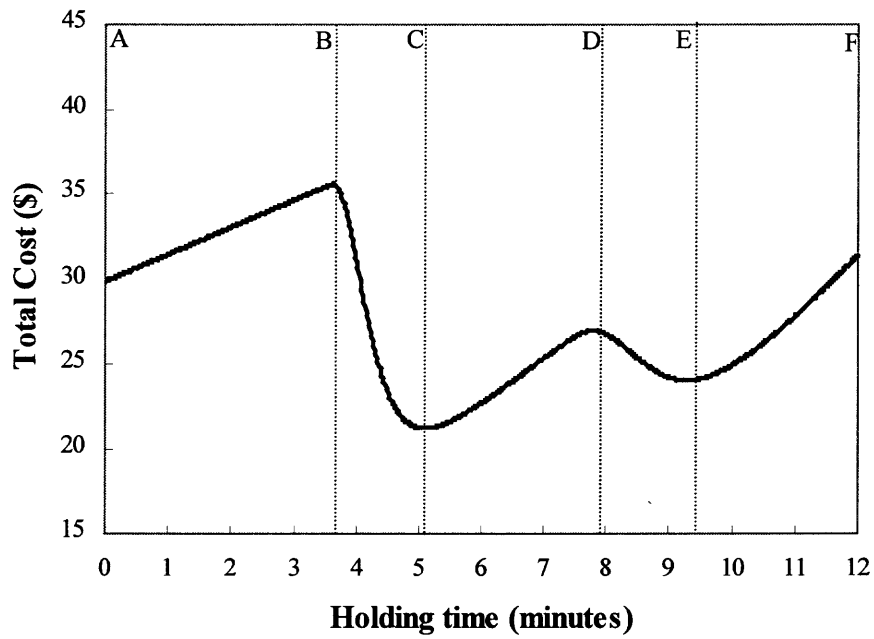


Figure 6-20 Total Cost vs. Holding Time (Lognormal Distribution)

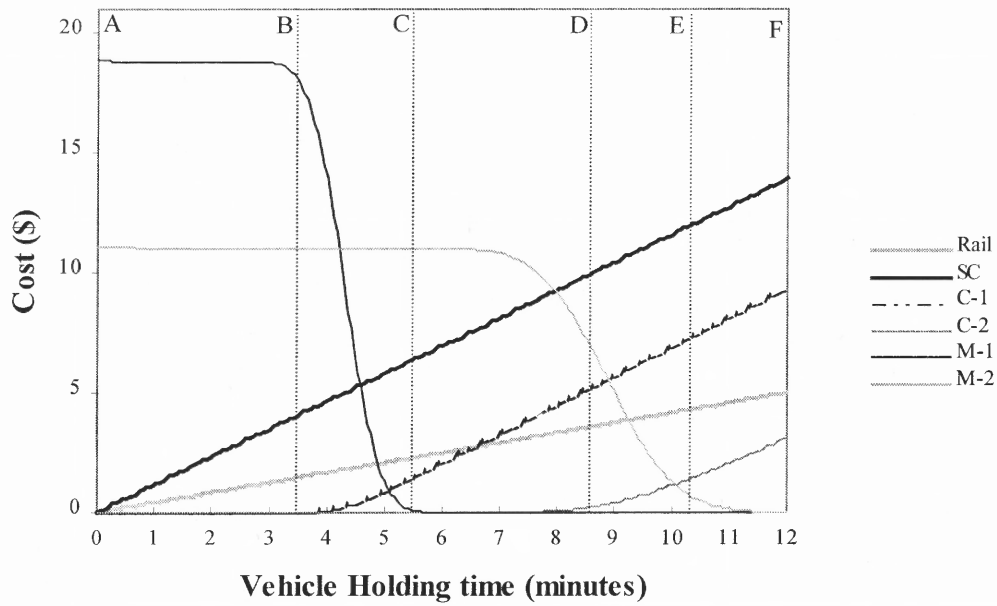


Figure 6-21 Various Cost Components vs. Holding Time (Normal Distribution)

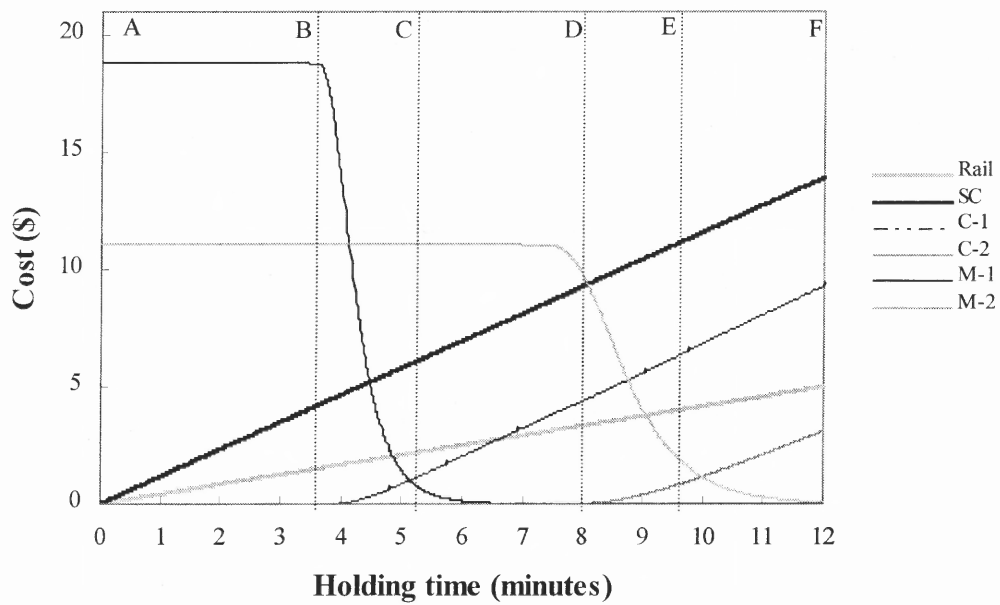


Figure 6-22 Various Cost Components vs. Holding Time (Lognormal Distribution)

- SC=dispatching delay cost
- Rail=connection delay cost incurred by passengers from rail direction 1
- C-1= connection delay cost incurred by passengers from vehicle b
- C-2= connection delay cost incurred by passengers from vehicle c
- M-1= missed connection cost incurred by passengers from vehicle b
- M-2= missed connection cost incurred by passengers from vehicle c

Like zone A-B, the total cost in zone C-D increases as the holding time increases because both the connection delay cost incurred by transfer passengers from trains and vehicle *b* to vehicle *a* and the operating cost of holding vehicle *a* increase. Similar characteristics can be found in zones D-E and E-F.

From both Figures 6-19 or 6-20, three local minimum points at the intersection points with lines A, C and E and the total cost curve can be identified, while point C is the global minimum point, presenting the optimal holding time of vehicle *a*.

Since vehicle arrival times may vary from time to time due to incidents (e.g. passenger demand, vehicle breakdown and roadway congestion), the dispatching decision should be re-evaluated during the holding period to reflect the real-time situation. Assuming that the dispatching decision will be re-evaluated at a 30-second interval, the second dispatching decision (re-evaluation) time will be 00:30. At the second dispatching decision time, it is required to update every late vehicle location, and re-estimate the vehicle arrival times to the transfer station. To examine the sensitivity of the optimal holding time, three situations are considered at the second dispatching decision time and discussed next.

Three situations with different vehicle locations are summarized in Table 6-29. The first situation shows that vehicle *b* moves toward the transfer station during the re-evaluation interval, while vehicle *c* does not move due to predictable reasons. The second situation shows that vehicle *c* moves toward the transfer station during the re-evaluation interval, but vehicle *b* does not move. The third situation shows that neither vehicle *b* nor *c* moves toward the transfer station during the re-evaluation interval.

At the second dispatching decision time, the holding time of vehicle *a* is re-optimized based on the three situations shown in Table 6-29. The optimal solutions of holding and departure times of vehicle *a* are summarized in Table 6-30. The relationship between various cost components and holding times for the three situations are discussed below.

Table 6-29 Vehicle Travel Times at the Second Decision Time

Veh ID	Route #	¹ VL (mile)	² Travel Time (min)		⁵ SD of Travel Time (min)
			³ MTT	⁴ STT	
Situation 1					
<i>b</i>	3	0.34	2.66	1.93	0.41
<i>c</i>	4	1.02	8.97	6.92	0.94
Situation 2					
<i>b</i>	3	0.51	4.32	3.38	0.47
<i>c</i>	4	0.85	7.44	6.13	0.65
Situation 3					
<i>b</i>	3	0.51	4.32	3.38	0.47
<i>c</i>	4	1.02	8.97	6.92	0.94

¹VL= vehicle location (distance to the transfer station)

²Travel Time: from current location to the transfer station

³MAT = mean travel time

⁴SAT = shortest travel time

⁵SD = standard deviation

Table 6-30 Optimal Holding and Departure Time of Vehicle a
(at the Second Dispatching Decision Time 00:30)

Situation	Optimal Holding Time (min)		Departure Time of Vehicle a	
	Normal	Lognormal	Normal	Lognormal
1	3.44	3.44	03:56	03:56
2	8.15	7.98	08:39	08:29
3	5.16	5.08	05:40	05:35

Situation 1

The optimal holding times considering normal and lognormal vehicle arrival distributions are both found to be around 3.44 minutes (Figures 6-23 and 6-24), which yield the minimum total costs of \$17.44 and \$17.87, respectively. Therefore, the new departure time of vehicle a evaluated at 00:30 is updated from the original 05:10 (or 05:05) for a normal (or lognormal) vehicle arrival distribution to 03:56 for both distribution types. Since the standard deviation of late vehicle b 's arrival time is small (0.41 min), the optimal holding time is insensitive to vehicle b 's arrival distribution. Therefore, the resulting optimal holding times for both distributions of vehicle arrival times are very close. The new departure time of vehicle a is earlier than that was determined at the first dispatching decision time because at this situation vehicle b is expected to arrive earlier than it was expected at the first dispatching decision time.

The relationship between various cost components and holding time considering both normal and lognormal vehicle arrival distributions is shown in Figures 6-25 and 6-26, respectively. Although, under situation 1, the optimal holding times are found to be close for both normal and lognormal vehicle arrival distributions, the total cost for the lognormal case is slightly higher than that for a normal vehicle arrival distribution (see Figures 6-23 and 6-24) due to the higher missed connection cost (See Figures 6-25 and 6-26).

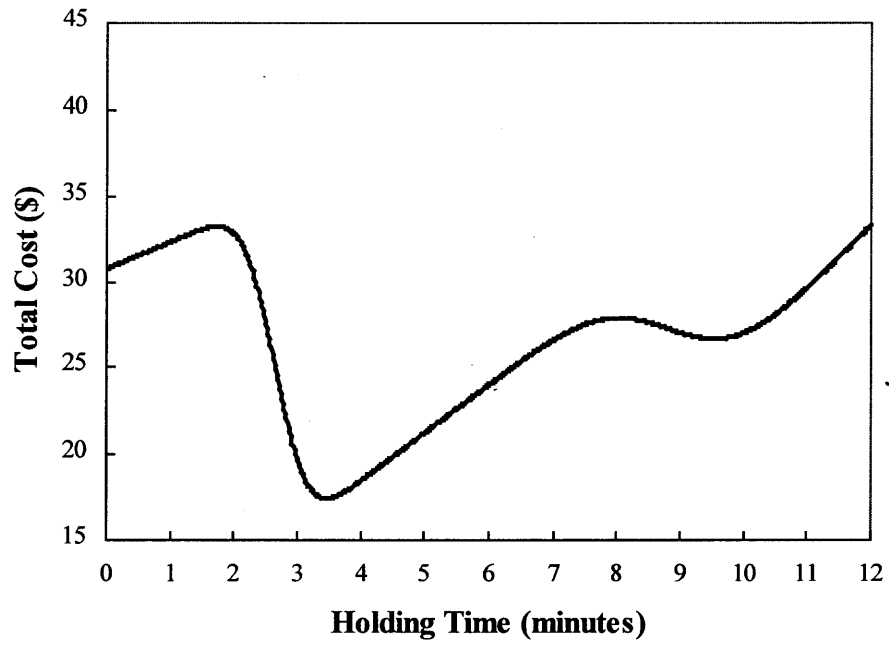


Figure 6-23 Total Cost vs. Holding Time
(Situation 1, Normal Distribution)

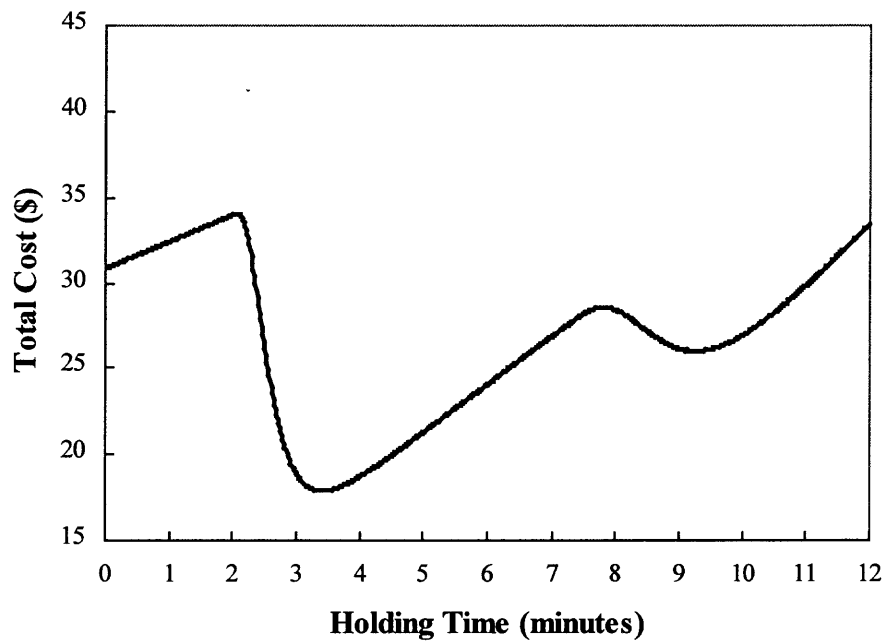


Figure 6-24 Total Cost vs. Holding Time
(Situation 1, Lognormal Distribution)

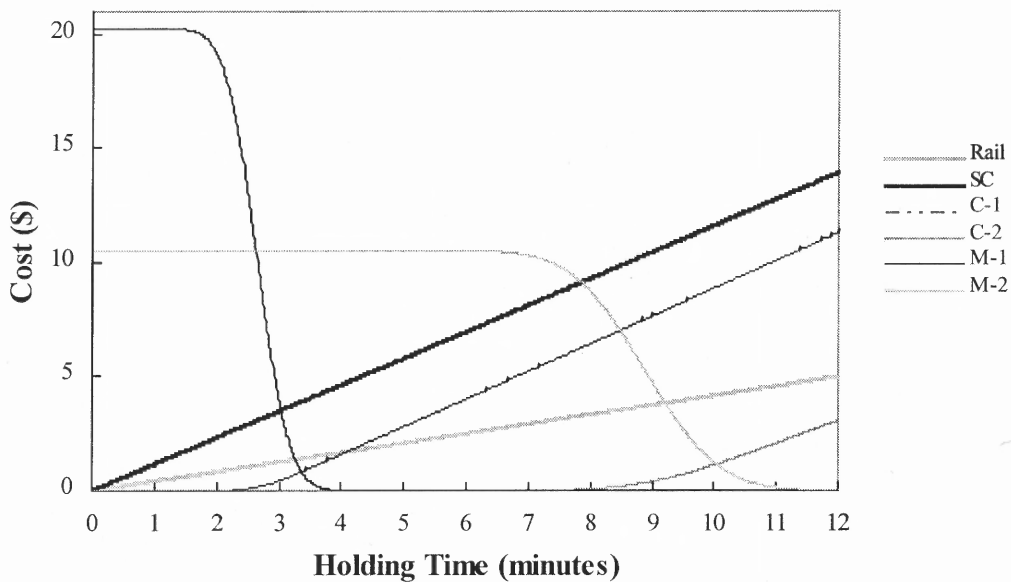


Figure 6-25 Various Cost Components vs. Holding Time
(Situation 1, Normal Distribution)

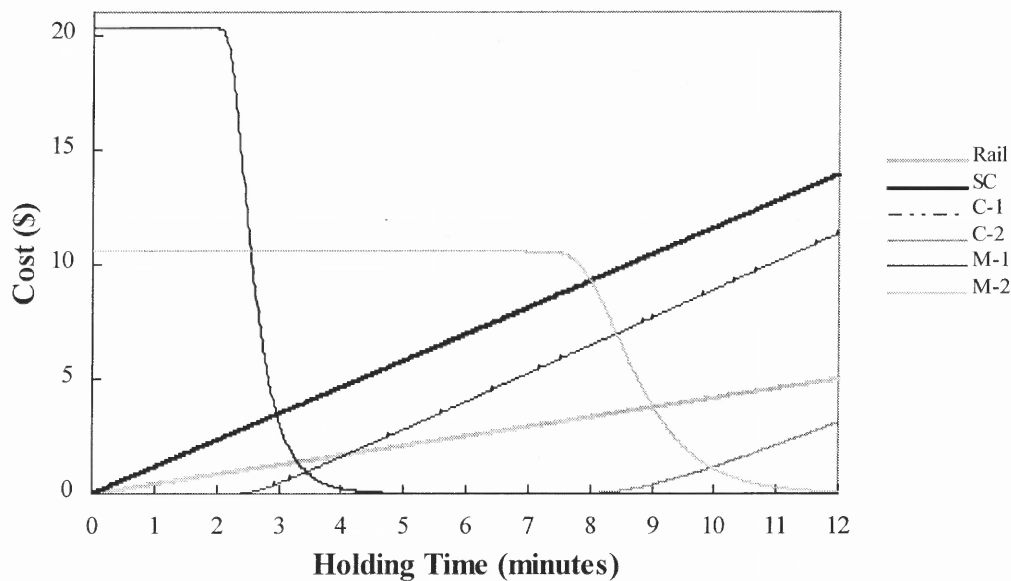


Figure 6-26 Various Cost Components vs. Holding Time
(Situation 1, LogNormal Distribution)

SC=dispatching delay cost
 Rail=connection delay cost incurred by transfer passengers from rail direction 1
 C-1= connection delay cost incurred by transfer passengers from vehicle b
 C-2= connection delay cost incurred by transfer passengers from vehicle c
 M-1= missed connection cost incurred by transfer passengers from vehicle b
 M-2= missed connection cost incurred by transfer passengers from vehicle c

Situation 2

The optimal holding times under situation 2 considering both normal and lognormal vehicle arrival distributions are found to be 8.15 and 7.98 minutes (see Figures 6-27 and 6-28), and yield the minimum total costs of \$19.77 and \$19.43, respectively. The new departure time of vehicle a reevaluated at 00:30 is updated from the original 05:10 (or 05:05) for normal (or lognormal) vehicle arrival distribution to 08:39 (or 08:29). The new departure time of vehicle a is later than that was determined at the first dispatching decision time due to late arrival of vehicle b and earlier arrival of vehicle c . Both the optimal holding times and the minimum total costs considering normal and lognormal vehicle arrival distributions slightly varies because of the variation of the probability density function of vehicle arrival times, which affects both the connection and missed connection delay costs. The relationship between various cost components and holding time is shown in Figures 6-29 and 6-30, respectively.

Situation 3

The optimal holding times under situation 3 considering normal and lognormal vehicle arrival distributions are found to be 5.16 and 5.08 minutes (See Figures 6-31, and 6-32), and yield the minimum total costs of \$20.31 and \$20.64, respectively. Therefore, the new departure time of vehicle a evaluated at 00:30 is updated from the original 05:10 (or 05:05) for normal (or lognormal) vehicle arrival distribution to 05:40 (or 05:35). The departure time of vehicle a is 30 seconds later than it was determined at the first dispatching decision time because vehicles b and c did not move during the past evaluation interval. The relationship between various cost components and holding times for both vehicle arrival distributions are shown in Figures 6-33 and 6-34, respectively.

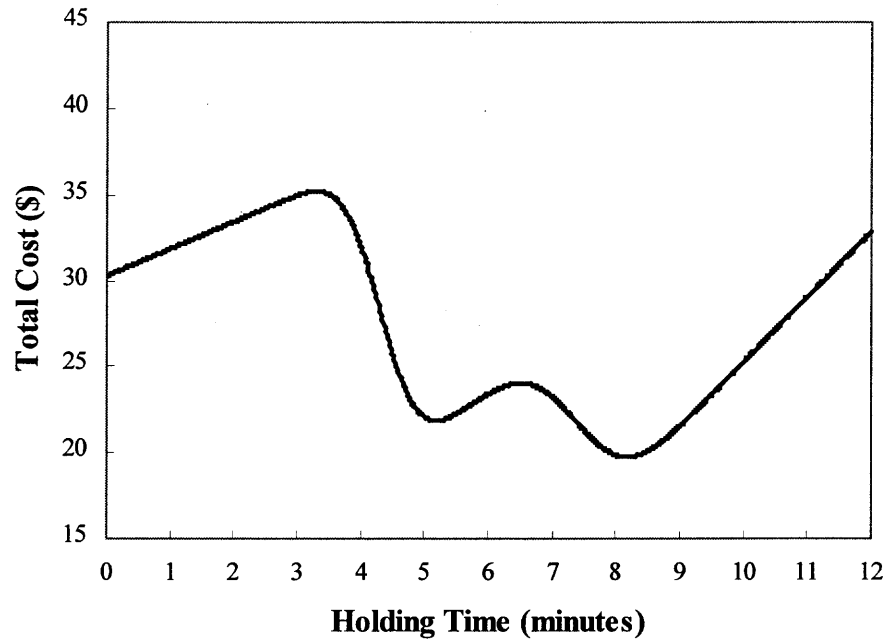


Figure 6-27 Total Cost vs. Holding Time
(Situation 2, Normal Distribution)

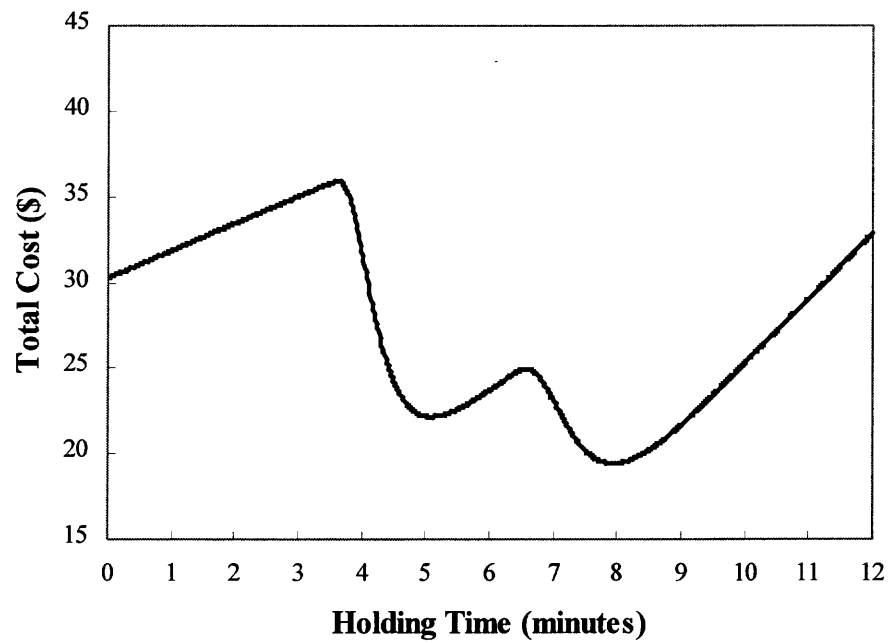


Figure 6-28 Total Cost vs. Holding Time
(Situation 2, Lognormal Distribution)

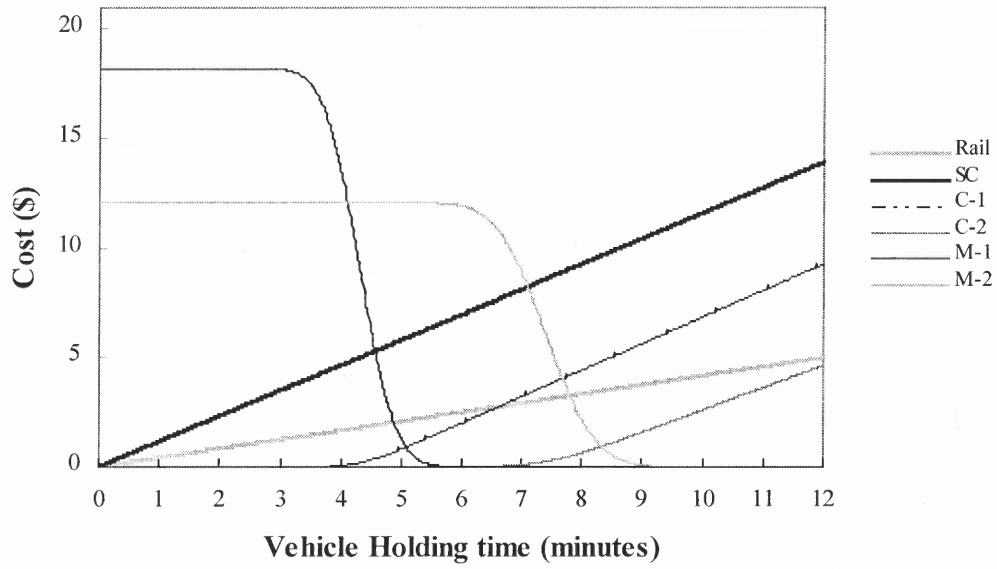


Figure 6-29 Various Cost Components vs. Holding Time
(Situation 2, Normal Distribution)

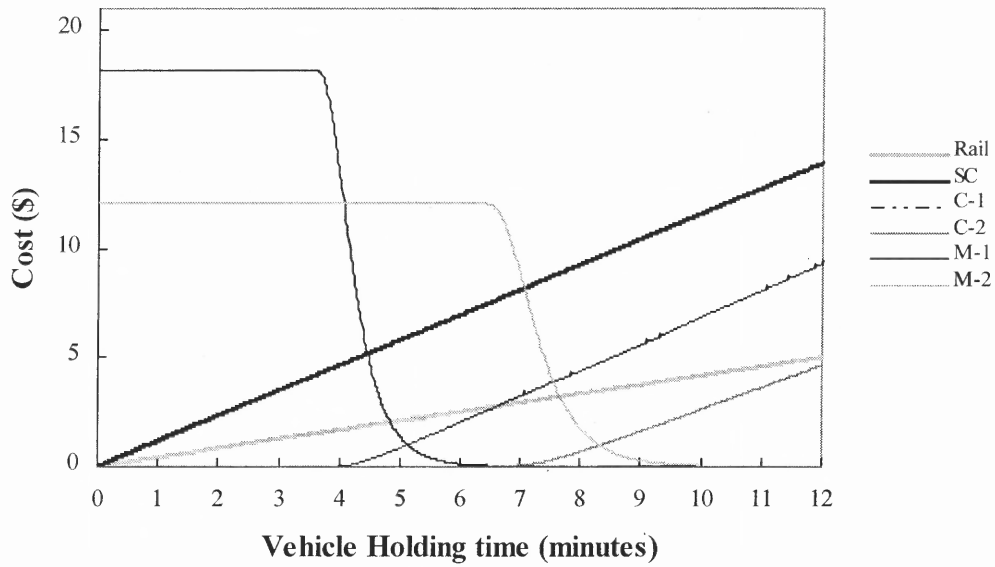


Figure 6-30 Various Cost Components vs. Holding Time
(Situation 2, Lognormal Distribution)

SC=dispatching delay cost
 Rail=Connection delay cost of passengers from rail direction 1
 C-1= connection delay cost of passengers from vehicle b
 C-2= connection delay cost of passengers from vehicle c
 M-1= missed connection cost of passengers from vehicle b
 M-2= missed connection cost of passengers from vehicle c

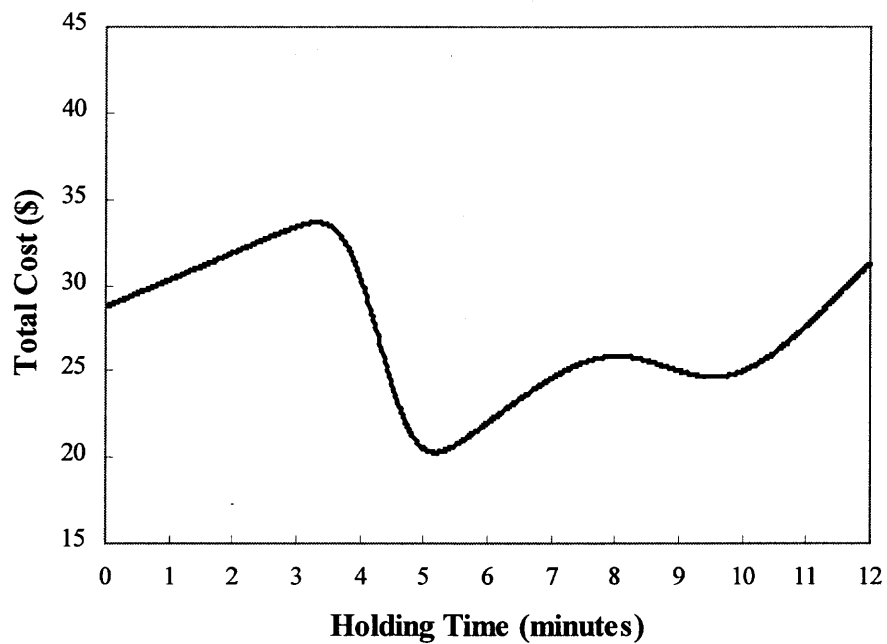


Figure 6-31 Total Cost vs. Holding Time
(Situation 3, Normal Distribution)

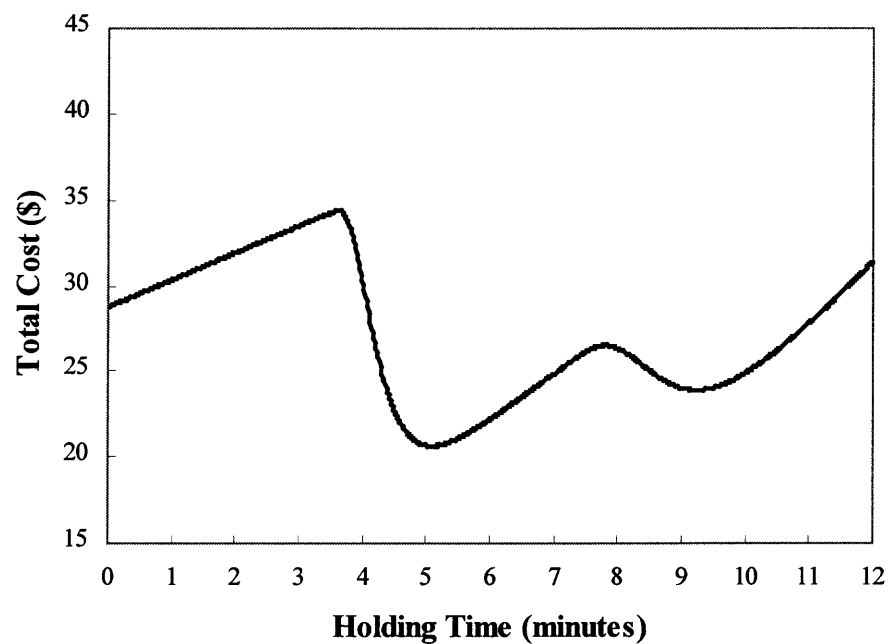


Figure 6-32 Total Cost vs. Holding Time
(Situation 3, Lognormal Distribution)

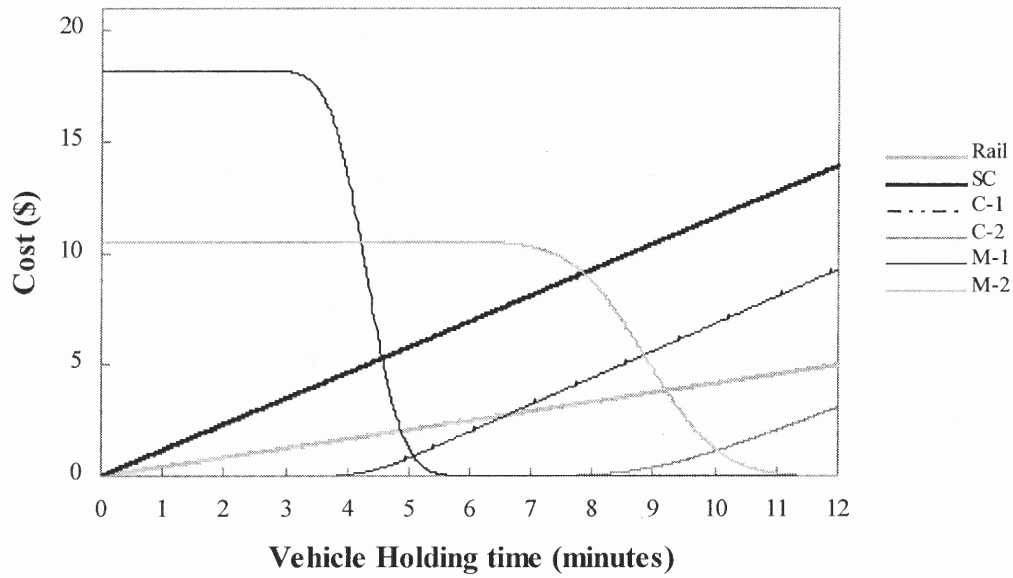


Figure 6-33 Various Cost Components vs. Holding time
(Situation 3, Normal Distribution)

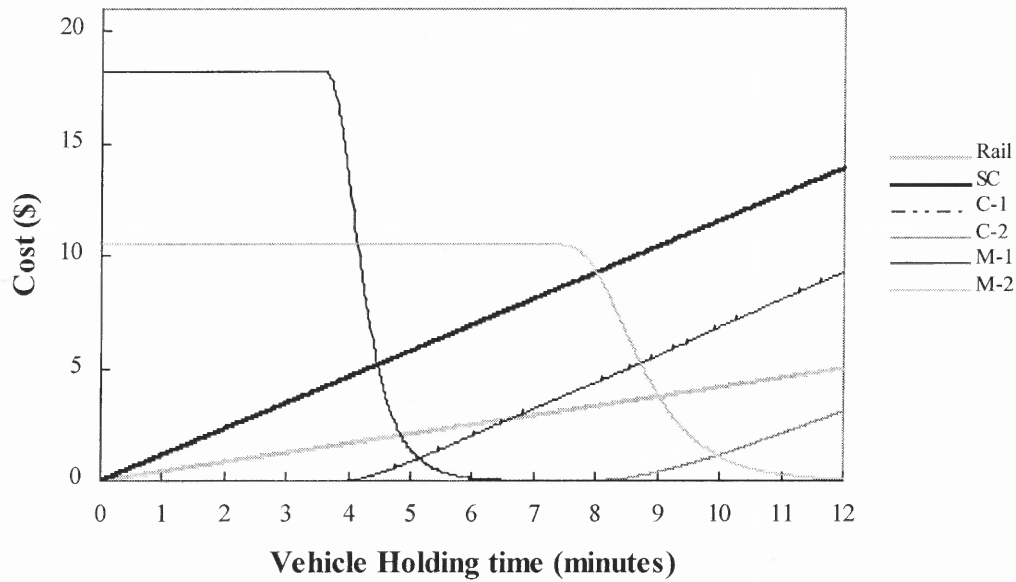


Figure 6-34 Various Cost Components vs. Holding Time
(Situation 3, Lognormal Distribution)

SC=dispatching delay cost
 Rail=connection delay cost incurred by transfer passengers from rail direction 1
 C-1= connection delay cost incurred by transfer passengers from vehicle b
 C-2= connection delay cost incurred by transfer passengers from vehicle c
 M-1= missed connection cost incurred by transfer passengers from vehicle b
 M-2= missed connection cost incurred by transfer passengers from vehicle c

The iterative procedure will be performed at all subsequent dispatching decision times until the holding vehicle is dispatched.

6.2.5 Sensitivity Analysis

The purpose of conducting sensitivity analysis in this section is to explore the relationship between the decision variable (e.g., holding time) and various model parameters (e.g., the mean and standard deviation of vehicle arrival times, transfer demand, headway, value of users' time and vehicle operating cost). The sensitivity of holding time to the variation of model parameters may vary from one situation to another.

For situation 2, addressed in the previous section, the sensitivity of optimal holding time to various standard deviation (SD) of vehicle c 's arrival times is conducted, and the results are shown in Figure 6-35. As the SD of vehicle arrival times increases from 0 to 0.8 minutes, the optimal holding time also increases. Then, the optimal holding time decreases as the SD of vehicle arrival times increases from 0.8 to 1.2 minutes. This indicates that a higher standard deviation of vehicle arrival times may disfavor vehicle-holding depending on the relative delay of late vehicles. Figure 6-36 shows that the vehicle holding cost increases as the SD of vehicle arrival time increases. In addition, as the SD of vehicle arrival time increases, the total cost for holding vehicle a increases. This indicates that as the SD of vehicle arrival time is small the holding of vehicle a is more productive.

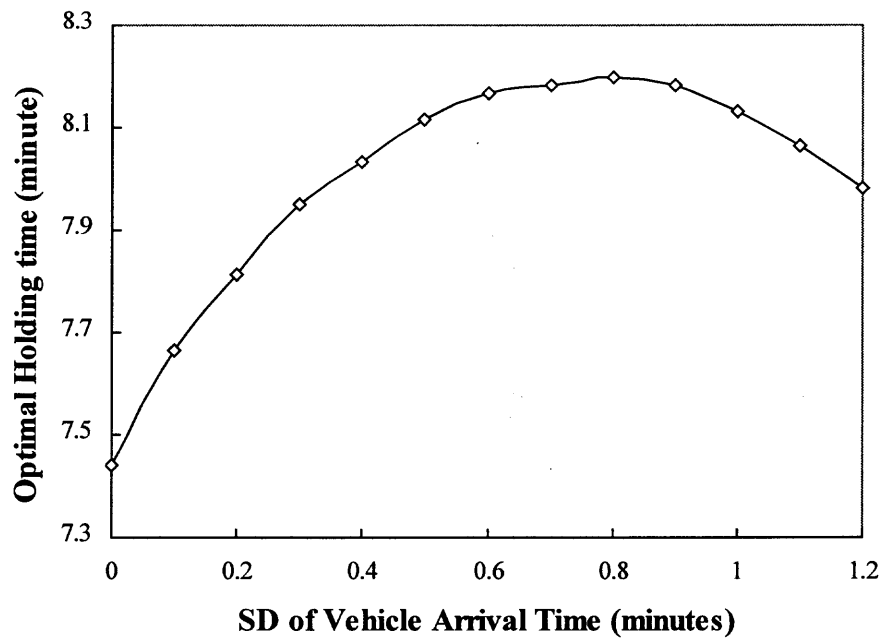


Figure 6-35 Optimal Holding Time vs. Standard Deviation (SD) of Vehicle c's Arrival Time (Situation 2)

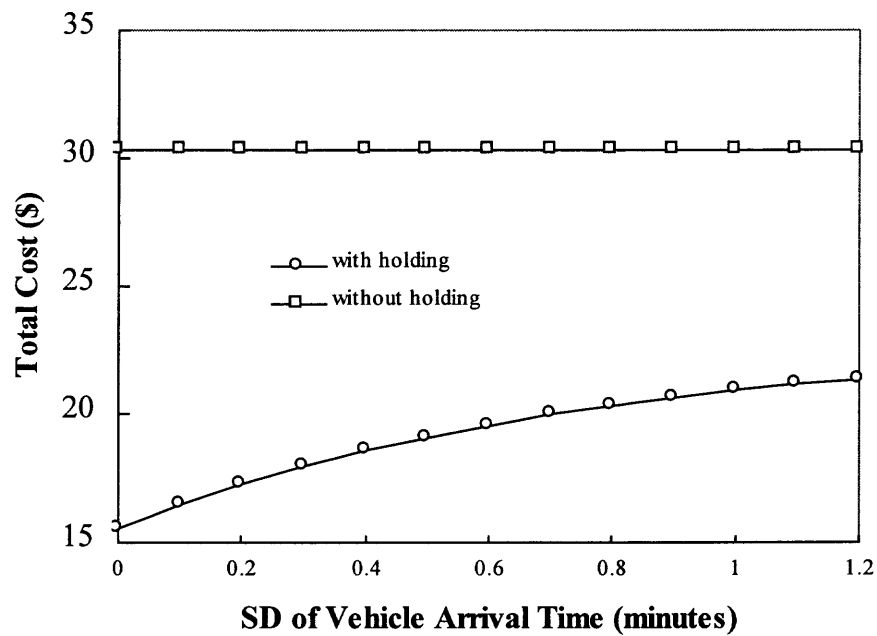


Figure 6-36 Total Cost vs. Standard Deviation (SD) of Vehicle c's Arrival Time (Situation 2)

Figure 6-37 shows the relationship between optimal holding time and the transfer demand from vehicle c to a under situation 3. If the transfer demand from c to a varies from 0 to 12 passengers, holding of vehicle a is cost effective only to pick up passengers from vehicle b . At this situation connection delay and missed connection cost of transfer passengers from vehicle b to a are insensitive to the transfer demand from vehicle c to a . Thus the optimal holding time of vehicle a remains constant at 5.17 minutes. However, if the transfer demand from vehicle c to a increases from 12 to 14 passengers, additional holding time for vehicle a is justified. Thus, the optimal holding time has a relatively sharp increase. Figure 6-38 shows the relationship between the total costs with and without holding vehicle a and the transfer demand from vehicle c to a under situation 3. The total costs both with and without holding increase proportionally as the transfer demand increases from 0 to 12. However, if transfer demand is more than 12, the cost difference between holding and not holding increases because of the decreased missed connection cost saved from holding vehicle a for vehicle c .

The relationship between optimal holding time and the value of users' wait time for situation 2 is shown in Figure 6-39. The figure shows that at low values of user wait time (0.025 \$/minute), the holding of vehicle a is not justified, however as the value of user wait time increases, first the holding of vehicle a for vehicle b and then for both vehicles b and c is justified. Figure 6-40 shows that the higher the value of users' wait time the greater the benefit that can be obtained from vehicle holding. Figure 6-41 and 6-42 show that a higher operating cost will discourage the holding of vehicle a . Thus, if vehicle operating cost increases, the optimal holding time and the benefit that can be obtained from vehicle holding decrease.

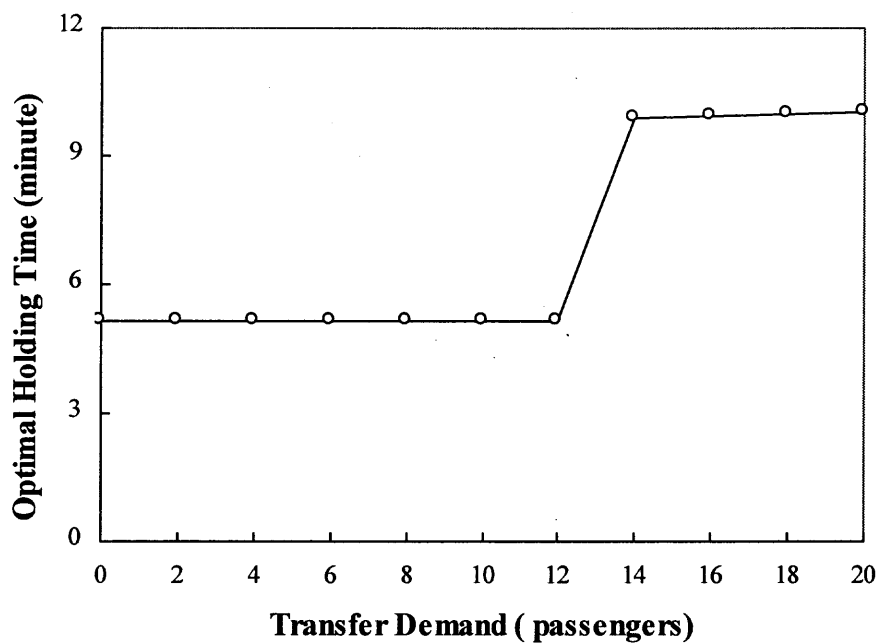


Figure 6-37 Optimal Holding Time vs. Transfer Demand from Vehicle c to a
(Situation 3)

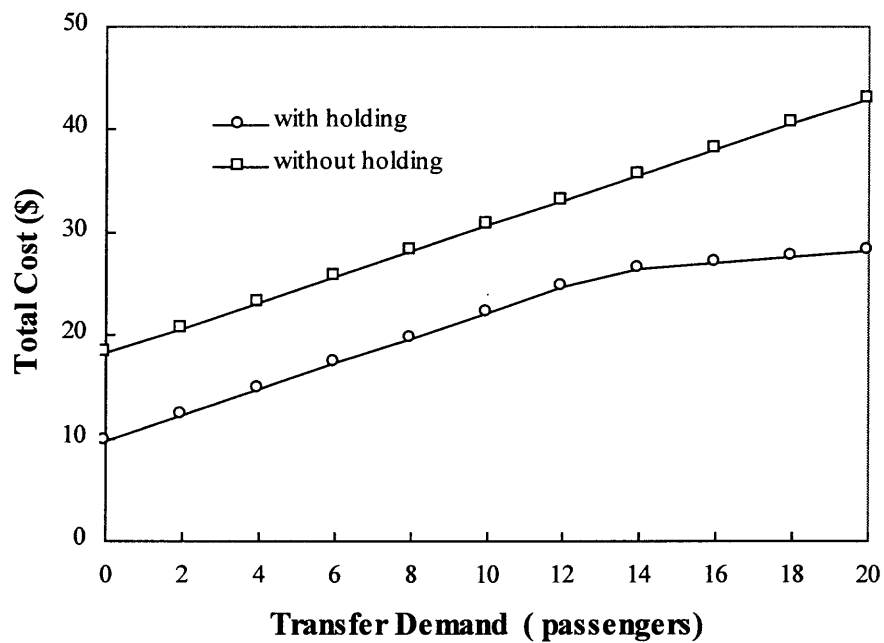


Figure 6-38 Total Cost vs. Transfer Demand from Vehicle c to a
(Situation 3)

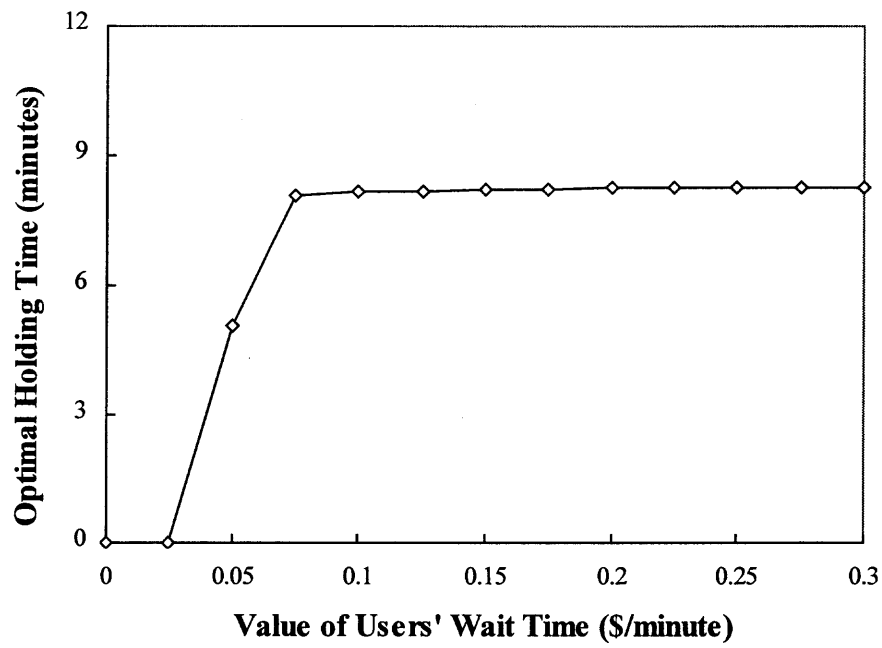


Figure 6-39 Optimal Holding time vs. Value of user wait time (Situation 2)

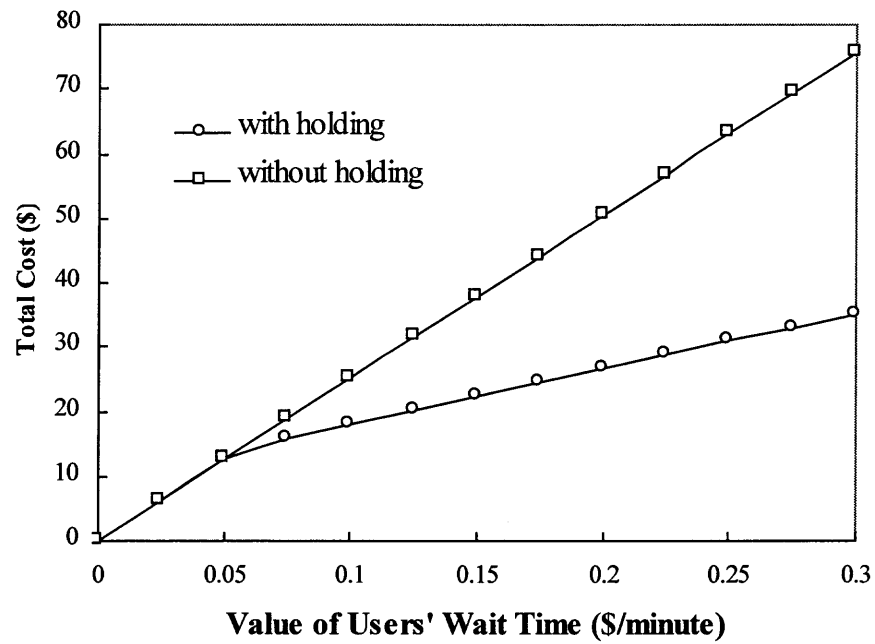


Figure 6-40 Total Cost vs. Value of Users' Wait Time (Situation 2)

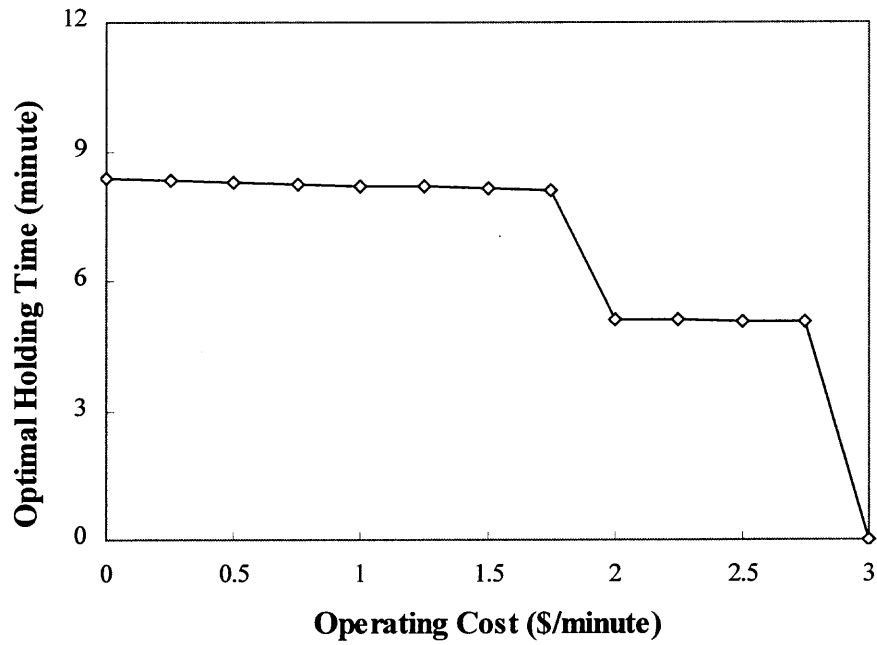


Figure 6-41 Optimal Holding Time vs. Vehicle Operating Cost (Situation 2)

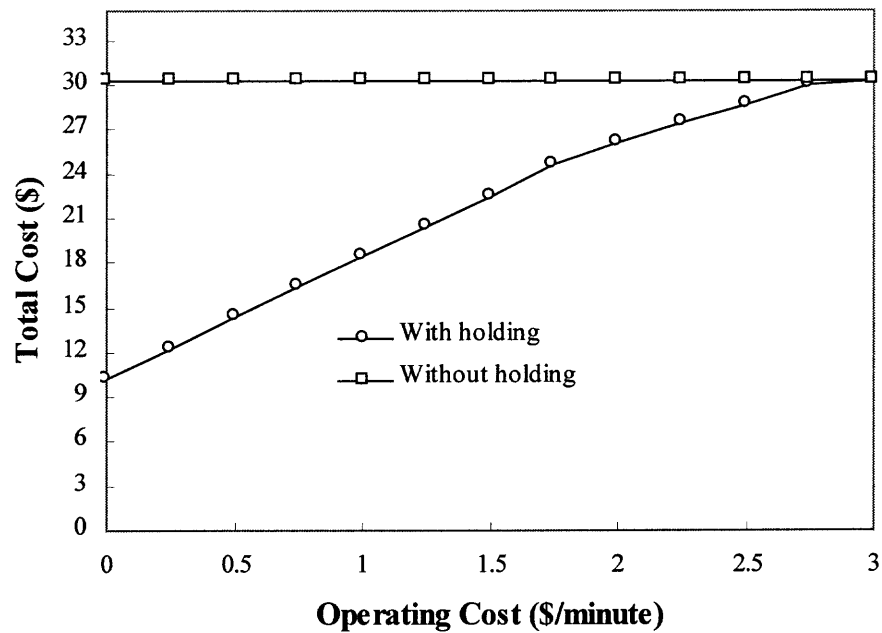


Figure 6-42 Total Cost vs. Vehicle Operating Cost (Situation 2)

To quantify the impact of late vehicle arrivals to the decision on holding time, a sensitivity analysis of the optimal holding time with respect to the arrival delay of vehicle *b* is conducted for situation 3. Figure 6-43 shows that the optimal holding time increases as the arrival delay of vehicle *b* increases from 1 to 5.5 minutes. However, the linear relation between holding time and arrival delay of vehicle *b* changes when the arrival delay is over 5.5 minutes, because the impact of holding vehicle *a* for vehicle *c* is involved.

Figure 6-44 shows the relationship between the total cost of the holding decision and the arrival delay of vehicle *b*. In general, the total cost decreases as the delay of vehicle *b*'s arrival increases, while holding of vehicle *a* is not considered. If the arrival delay of vehicle *b* increases from 1 to 6 minutes, the total cost for holding vehicle *a* increases. As the arrival delay of vehicle *b* increases from 1 to 6 minutes, the increased total cost is caused by the increased optimal holding time. Thus, both the operator cost of holding vehicle *a* and the connection delay cost increase. If the arrival delay of vehicle *b* exceeds 6 minutes, but is less than 9 minutes, the total cost decreases because of decreased missed connection cost incurred by transfer passengers from vehicle *c* to *a*. If the arrival delay of vehicle *b* exceeds 9 minutes, but is less than 10 minutes, the total cost with holding decision increases again, because of increased connection delay cost incurred by passengers transferring from *c* to *a*, and operator cost.

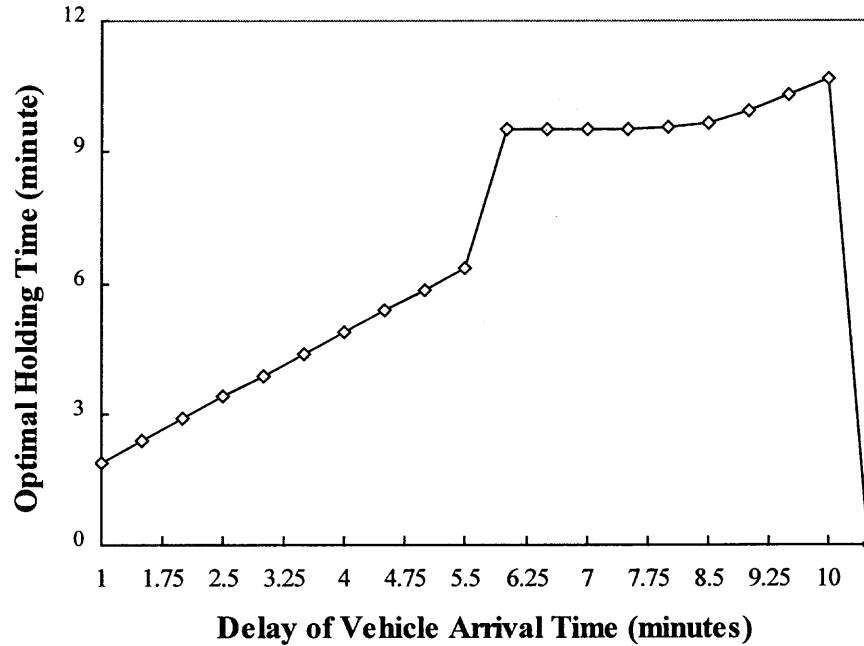


Figure 6-43 Optimal Holding Time vs. Delay of Vehicle b's Arrival Time
(Situation 3)

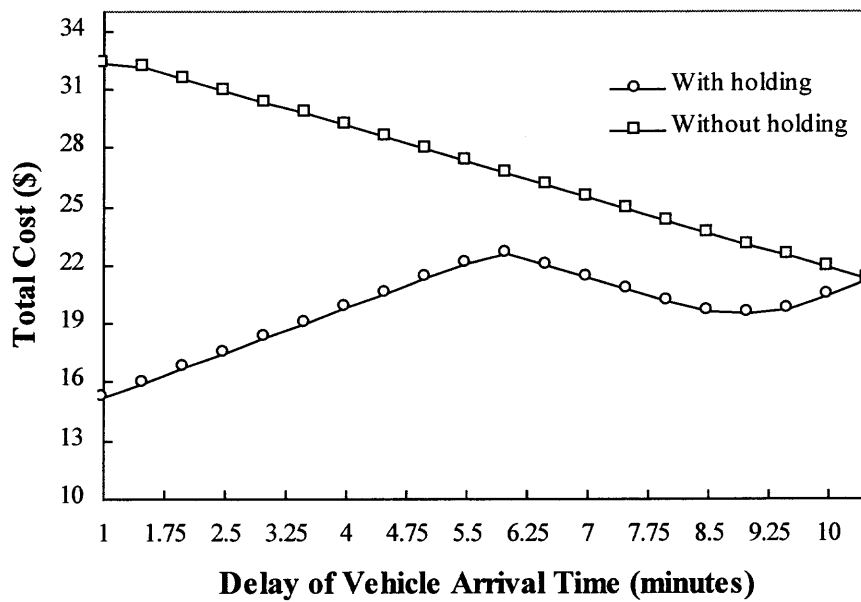


Figure 6-44 Total Cost vs. Delay of Vehicle b's Arrival Time
(Situation 3)

6.3 Summary

The numerical examples given in this chapter demonstrated how the problems of transfer coordination and dynamic vehicle dispatching in intermodal transit can be resolved. The models developed in Chapters 3 and 4 can be successfully implemented to solve the optimization problem by both using analytical and numerical approaches. In addition, the enhanced simulation model can be applied to generate travel time information for each transit vehicle, and thus to be used for arrival time estimation. A sensitivity analysis has been conducted for various important control variables and parameters used in both the coordination and dispatching models.

To summarize, the models developed in this study have the ability to (1) deal with different intermodal transit networks with multiple transfer stations on a major transit line, (2) examine trade offs among conflicting objectives, and (3) simultaneously handle real time dispatching decisions at different transfer stations based on real-time information.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

A complex transfer coordination problem has been solved with sufficient accuracy by employing basic calculus and the optimization algorithms developed in this work. The coordination optimization with both fixed slack time and dynamic dispatching for coordinated vehicles operating in an intermodal transit system consisting of rail and feeder bus service has been fully discussed in this study. An analytical approach is used to optimize headways for routes without coordination, while a numerical search algorithm (the Powell's algorithm) is used to find the optimal coordinated headways (common headways) and slack times for coordinated routes. Numerical integrations are applied to calculate connection delay and missed connection costs. The coordination optimization model developed in this study can efficiently be applied to a real intermodal transit network containing multiple feeder routes connecting at multiple transfer stations along a rail line.

The route coordination model and optimization procedure were developed in Chapter 3, while a methodology for dynamic dispatching of vehicles on coordinated routes was presented in Chapter 4. Both the coordination and the dynamic dispatching models were coded in FORTRAN to search the optimal decision variables. In Chapter 5, the microscopic simulation model CORSIM was enhanced for estimating dwell time and emulating an AVL system. The enhanced CORSIM was used to collect data for the dynamic dispatching model. Based on the numerical example presented in Chapter 6, the optimal results for intermodal transit system coordination and dynamic dispatching

models were obtained. The possibilities and limitations of coordination were shown through numerical examples. Finally, a sensitivity analysis of the decision variables with respect to various model parameters was conducted, which confirmed the relationships in the models developed in Chapters 3 and 4.

7.1 Conclusions

The development of the models themselves in this study is a significant contribution for assessing the benefits of intermodal transit system coordination both from theoretical and practical point of views. From a theoretical point of view, the models have capability to examine the possibilities and limitations of intermodal transit system coordination considering various demand, capacity and vehicle arrival situations. From a practical point of view, a realistic tool is developed and becomes available for scheduling coordinated transfers in intermodal transit systems. General conclusions for the developed coordination model are discussed below.

- (1) Coordination is beneficial in intermodal transit operations for routes with long headway, low standard deviation of vehicle arrival time and large transfer demand among routes.
- (2) Coordination is beneficial if the increase of wait and in-vehicle costs can be compensated from the savings in transfer cost.

Due to stochastic vehicle arrivals at the transfer station, a slack time is required to increase the probability of successful connection. The slack times on coordinated routes depend on the joint effect of transfer volume, standard deviation of vehicle arrival times and headway. Since the optimal slack time is a trade off among slack delay, connection

delay and missed connection costs. A coordinated route with long headway and standard deviation of vehicle arrival times requires a relatively large slack time. However, as the standard deviation exceeds a significant fraction of the headway, it becomes preferable to reduce the slack time, because at that situation, slack delay and connection delay cost can not be compensated from the savings of missed connection cost. Furthermore, if the standard deviation is too big, the optimal slack time becomes zero. In such a situation route coordination may not be cost effective.

Optimization of dynamic vehicle dispatching on coordinated routes at transfer stations is implemented based on predicted arrival distributions, the delay of vehicle arrivals, and transfer demand. An iterative algorithm is developed to evaluate the decision of holding time periodically. The enhanced CORSIM model is used to emulate bus operations and monitor vehicle travel times from various locations to the transfer station.

Based on the results obtained from the dynamic vehicle dispatching model, the conclusions may be summarized as follows:

1. The optimal holding time is a trade off among several cost components including transfer, wait, in-vehicle and operator costs.
2. Dynamic vehicle dispatching can significantly improve the transfer efficiency and fine-tune the optimal slack times obtained from the route coordination model.
3. At each vehicle dispatching decision point of time, the optimal holding time of a coordinated vehicle depends on the arrival time of other connecting vehicles and transfer demand. In general, holding for a late vehicle is

preferred if that vehicle carries a large enough number of transfer passengers.

4. Vehicle holding is preferable when uncertainty in the arrivals of late vehicles is small. As the standard deviation of vehicle arrival times increases, the holding cost also increases. Thus, accurate predictions of vehicle arrival times are important for increasing transfer efficiency.
5. Excessive fluctuation in predicted arrival times and transfer demand may reduce the benefit from holding a vehicle, specially when the delay of vehicle arrival is large and the standard deviation of vehicle arrival times is high.
6. As the standard deviation of vehicle arrival times decreases, the influence of the arrival distribution functions (e.g., normal and lognormal) on the total cost and optimal holding time is reduced.

7.2 Recommendations for Future Research

The possible areas where the analyses and mathematical models developed in this study can be further extended are summarized below. It would be noted that some extensions may significantly complicate the current models.

(1) Demand Estimation

Transfer demand is a critical factor in optimizing coordination for an intermodal transit system. In the developed dynamic dispatching model, transfer demand should be accurately predicted to evaluate transit ridership and operator costs. Therefore, further

research should focus on dynamic demand estimation based on data collected from automatic passenger counter systems and/or intelligent fare payment system.

(2) Demand Elasticity

Passenger demand is assumed to be inelastic in this study. However, a model without demand elasticity can not properly address fare policy or optimize system objectives that include consumer surplus. Therefore, passenger demand should vary with the level of service provided by the intermodal transit system. Due to service variation (i.e., variation of headway and transfer time) demand may also vary. The total cost minimization model developed in this study may be integrated with an elastic demand model to iteratively optimize the intermodal transit system coordination. A model that analytically integrates our supply system optimization with a demand equilibrium approach (e.g., as in Kocur and Hendrickson [1995] or Chien and Spasovic [1999]) would be a desirable extension.

(3) Probability Distribution of Vehicle Arrivals

Normal and lognormal distributions are used in the numerical examples, which may not be the actual vehicle arrival distribution in many real world situations. Therefore, future research should focus on developing empirical vehicle arrival distributions at transfer stations.

(4) Value of User Time

A nonlinear wait cost may to be introduced to capture the changing marginal disutility of waiting. A travel behavior study conducted by Liu et. al [1997], suggests the value of intermodal transfer penalty is more than twice the wait time by assuming that wait time is one-half the transit headway. This issue needs to be investigated further.

(5) Externalities

Schedule coordination will induce a ridership increase by lowering the overall transfer wait cost, and may be a very cost effective way to improve transit service quality [Nelson, Brand, and Mandel, 1982; Becker and Spielberg 1999]. Therefore, additional ridership in transit may reduce roadway congestion and thus may improve air quality. However, in this study such factors were not considered. In the future, benefits from air quality improvement should be incorporated in the model.

(6) Mixed Size Fleets

Different vehicle sizes may be used in different routes to keep headways among routes reasonably close to each other. It is found that if headways (without coordination) of connecting routes are close to each other, coordination is more productive. Therefore, a mixed size fleet should be considered in the system so that vehicles can be switched among routes to minimize the difference in headways among routes at different time periods.

(7) Objective Function

In the route coordination model, we formulated total cost including supplier and user costs, which is minimized in the four-Stage procedure. Other objective functions such as profit maximization, or social welfare maximization can also be considered in a future study.

APPENDIX A

NOTATION

b_s	=	Average passenger boarding time (sec/pass);
C_o	=	Total supplier cost (\$/hr);
C_v^o	=	Operator cost caused by holding vehicle v (\$);
C_v^c	=	Connection delay cost for holding vehicle v (\$);
C_v^M	=	Missed Connection cost for holding vehicle v (\$);
$C_{b,v}^c$	=	Bus-to-bus connection delay cost for holding vehicle v (\$);
$C_{r,v}^c$	=	Rail-to-bus connection delay cost for holding vehicle v (\$);
$C_{r,v}^M$	=	Rail-to-bus missed connection cost for holding vehicle v (\$);
$C_{b,v}^M$	=	Bus-to-bus missed connection cost for holding vehicle v (\$);
C_{or}	=	Rail operator cost (\$/hr);
C_{ob}	=	Bus operator cost (\$/hr);
C_w	=	Total wait cost (\$/hr);
C_{wb}	=	Wait-bus cost (\$/hr);
C_{wr}	=	Wait-train cost (\$/hr);
C_t	=	Total transfer cost (\$/hr);
C_T	=	Total cost for coordination model (\$/hr);
C_{tbb}	=	Bus-to-bus transfer cost (\$/hr);
C_{tbr}	=	Bus-to-rail transfer cost (\$/hr);
C_{trb}	=	Rail-to-bus transfer cost (\$/hr);
C_v	=	Total in-vehicle cost (\$/hr);
C_U	=	Total user cost (\$/hr);
C_{vb}	=	Total in-bus cost (\$/hr);

- C_{vr} = Total in-train cost (\$/hr);
 C_{vr1} = In-train cost due to train motion time (\$/hr);
 C_{vr2} = In-train cost due to dwell times of train at stations (\$/hr);
 C_T = The total cost of intermodal transit system (\$/hr);
 D_{ikj} = Connection delay time from route k to j at station i (hr);
 d = Index of rail service directions;
 $d_{v,s}$ = Dwell time of bus v at stop s (sec);
 D_{ij} = Maximum bus load of route j at station i (pass./hr);
 D_r = Maximum train load (pass/hr);
 F_{ij} = Bus fleet size (buses);
 F_r = Rail fleet size (cars);
 H_{ij} = Headway of bus route j at station i (hr);
 H_r = Rail headway (hr);
 $I_{ij\delta}$ = Demand of bus route j at station i in direction δ (pass/hr);
 i = Index of rail stations;
 I_{id} = Rail demand in direction 1 at station i (pass./hr);
 j = Index of bus route;
 K = Index of bus route;
 K_{ij} = Slack time of bus route j at station i (hr);
 L_{ij} = Length of bus route j at station i (mile);
 l_i = Interstation spacing between station i and $i+1$ (mile);
 l = Index of rail stations;
 m = Index of rail stations;
 m_i = Number of bus routes at station i ;
 M_{ikj} = Missed connection delay time from route k to j (hr);

- M_{ijd} = Missed connection delay time from route j to rail direction d (hr);
 n = Total number of rail stations (station);
 N = Sample size;
 P_{ij} = Service capacity of buses of route j at station i (pass.);
 P_r = Service capacity of train (pass.);
 Q_{id} = Rail outflows at station i in direction d (pass./hr);
 $Q_{v,s}$ = Number of passengers boarding on bus v at stop s (pass.);
 $q_s(t)$ = Passenger arrival rate at stop s time t (pass./sec);
 q_b = Bus passengers boarding and alighting rate (1800 pass/hr);
 q_r = Rail passengers boarding and alighting rate (21600 pass/hr);
 S_{ij} = Average bus operating speed of route j at station i (20 mph);
 S_p = Standard deviation of vehicle travel time from checkpoint p to transfer station i (hr);
 s = Index of bus stops (stop);
 $T1$ = Departure time of bus $v-1$ at stop s (hr);
 $T2$ = Arrival time of bus v at stop s (hr);
 $T3$ = Departure time of bus v at stop s (hr);
 T_{idj}^N = Transfer time from rail direction d to route j without coordination (hr);
 T_{idj}^C = Transfer time from rail direction d to coordinated bus route j (hr);
 T_{ikj}^C = Transfer time from bus route k to j if they are coordinated (hr);
 T_{ikj}^N = Transfer time from coordinated route k to j (hr);
 T_{ijd}^C = Transfer time from coordinated bus route j to rail direction d (hr);
 T_{ijd}^N = Transfer time from bus route j to rail direction d without coordination (hr);
 t_{ij} = Vehicle arrival time of bus route j at station i (hr);
 T_{ij}^R = Round trip time of bus route j at station i (hr);

- T_r^M = Train motion time (hr);
 T_r^D = Rail dwell time (hr);
 T_r^S = Rail stop delay time (hr);
 T_r^R = Rail round trip time (hr);
 TC_v = Total cost for dispatching vehicle v on route j at stop s (\$);
 t_b^e = Earliest arrival time of vehicle b at station i (hr);
 t_v^h = Holding time of vehicle v (hr);
 t_{v-1}^d = Departure time of vehicle $v-1$ (hr);
 t_v^d = Departure time of vehicle v (hr);
 t_v^{dd} = Dispatching decision time for vehicle v (hr);
 t_r^a = Arrival time of train from coordinated direction (hr);
 t_{v+1}^a = Arrival time of vehicle $v+1$ (hr);
 t_v^a = Arrival time of vehicle v (hr);
 $t_{b,v}^c$ = Connection delay time of passengers from late vehicle b to v (hr);
 $t_{b,v}^M$ = Missed connection delay time of passengers from late vehicle b to v (hr);
 \bar{t}_p = Mean travel time from checkpoint p to the transfer station (hr);
 $t_{b,p}$ = Travel time of vehicle b from checkpoint p to the transfer station i (hr);
 $U_{r,v}$ = Transfer demand from train r to vehicle v (pass);
 $U_{b,v}$ = Transfer demand from vehicle b to vehicle v (pass);
 u_w = Value of users' wait time (7 \$/hr);
 u_b = Average bus operating cost (70 \$/hr);
 u_r = Average train operating cost (180 \$/hr);
 u_v = Value of users' in-vehicle time (5 \$/hr);
 U_{ijd} = Transfer demand from route j to rail direction d at station i (pass/hr);

- U_{ikj} = Transfer demand from route k to route j at station i (pass/hr);
 U_{idj} = Transfer demand from rail direction d to route j at station i (pass/hr);
 V_i = Average train operating speed between station i and $i+1$ (mph);
 V_r = Average rail cruising speed (40 mph);
 $\text{var}(H_{ij})$ = Variance of headway of vehicle route j at station i (hr);
 $W_{v,s}$ = Total passenger wait time for bus v at stop s (sec);
 w_s = Average passenger wait time at stop s (sec/pass);
 y_{idj} = (0 or 1) indicating that train direction d and vehicle route j at transfer station i are (without or with) coordination;
 y_{ikj} = (0 or 1) indicating that route k and j at transfer station i are (without or with) coordination;
 α_{id} = Rail demand in direction d walk to station i (pass/hr);
 β_{id} = Demand destining at station i from rail direction d (pass/hr);
 δ = Bus route direction (1 or 2);
 λ_s = Mean passenger arrival rate at stop s (pass/sec);
 Δ = Evaluation interval for dispatching vehicle (sec);

APPENDIX B

RAIL ROUND TRIP TRAVEL TIME

The round trip time for trains is formulated in this section. In this study trains serve a series of stations along a rail line. The rail round trip time is formulated based on three regimes of motion between a pair of stations consisting of (1) motion time: the time that trains move at a constant speed, (2) stop delay time: the time that trains spend accelerating and decelerating and (3) dwell time: the time that trains spend at stations for loading and unloading passengers. Figure B-1 shows the inter-station travel time and speed diagram. Further details on vehicle characteristics and motion can be found in “Urban Public Transportation: Systems and Technology” [Vuchic, 1981].

Motion Time

The train motion time t_i^c between stations i and $i+1$, is equal to the cruising distance l_i^c divided by the cruising speed V_r . Thus t_i^c is

$$t_i^c = \frac{l_i^c}{V_r} \quad (\text{B-1})$$

where l_i^c can be obtained from Eq. B-2.

$$l_i^c = l_i - l_i^a - l_{i+1}^b \quad (\text{B-2})$$

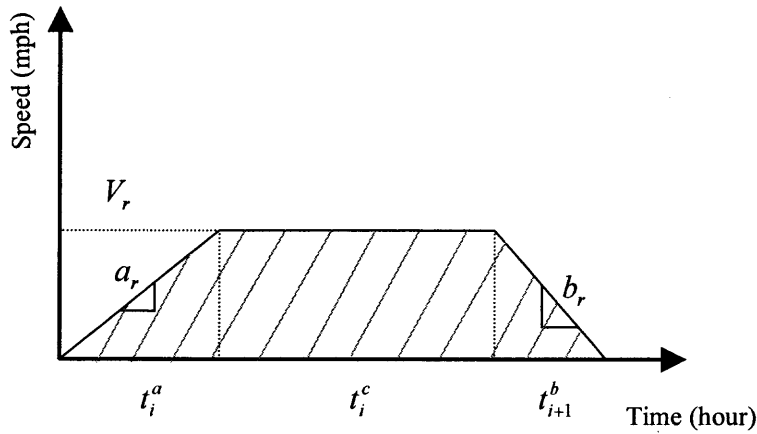
In Eq. B-2, l_i , l_i^a , and l_{i+1}^b represent the spacing between stations i and $i+1$, acceleration distance for departing from station i , and deceleration distance for approaching to station $i+1$, respectively.

The distances required for acceleration and deceleration are formulated as

$$l_i^a = \frac{(V_r)^2}{2a_r} \quad (\text{B-3})$$

$$l_{i+1}^b = \frac{(V_r)^2}{2b_r} \quad (\text{B-4})$$

where a_r , and b_r represent the acceleration and deceleration rates, respectively.



V_r Cruise Speed

t_i^a Duration of acceleration interval

t_{i+1}^b Duration of deceleration interval

t_i^c Duration of interval trains travel at cruise speed


 Station Spacing

Figure B-1 Travel Time and Speed Diagram

The total motion time T_r^M (both directions) is the summation of all inter-station motion times. If the rail line consists of n stations T_r^M can be obtained from Eq. B-5.

$$T_r^M = 2 \sum_{i=1}^{n-1} t_i^c \quad (\text{B-5})$$

Stop Delay Time

The stop delay time is caused by deceleration and acceleration when trains arrive and depart from stations.

The stop delay time t_r^i at station i is the summation of acceleration and deceleration intervals, which can be formulated as

$$t_r^i = t_i^a + t_i^b \quad (\text{B-6})$$

From Figure B-1, t_i^a and t_i^b can be obtained as follows

$$t_i^a = \frac{V_r}{a_r} \quad (\text{B-7})$$

$$t_i^b = \frac{V_r}{b_r} \quad (\text{B-8})$$

Thus, the stop delay time T_r^S (both directions) is the summation of stop delay times incurred by trains at all stations:

$$T_r^S = 2 \sum_{i=1}^{n-1} t_r^i \quad (\text{B-9})$$

Dwell Time

The train dwell time is assumed to be a linear function of the total number of boarding and alighting passengers t_d^i represents the dwell time at a station, which is equal to the boarding and alighting demand at that station multiplied by train headway and divided by the passenger boarding/alighting rate. Thus, the total dwell time R_r^D incurred by trains (both directions) is the summation of dwell times at all stations:

$$T_r^D = \sum_{i=1}^n \sum_{d=1}^2 (I_{id} + Q_{id}) \frac{H_r}{q_r} \quad (\text{B-10})$$

where i varies from 1 to n . I_{id} , Q_{id} and H_r represent boarding and alighting demand at station i , and headway, respectively.

Average Operating Speed

The average train operating speed V_i between station i and $i+1$ is equal to interstation spacing between station i and $i+1$ divided by train travel time t_i , which is

$$V_i = \frac{l_i}{t_i} \quad (\text{B-11})$$

where $t_i = t_i^a + t_{i+1}^b + t_i^c$

APPENDIX C
SERVICE CAPACITIES OF BUSES AND RAIL ROUTES

The rail route hourly service capacity is the maximum loaded passengers per train (train capacity) divided by the rail headway. Thus, the rail capacity can be formulated as

$$P_r = \frac{C_r n_c}{H_r} \quad (C-1)$$

where C_r is car capacity and n_c is the number of cars per train. If the maximum rail transit load is called D_r , it can be calculated from the given rail inflow and outflow demand at stations.

$$D_r = \text{Max.}(D_{id}) \quad \forall i, d \quad (C-2)$$

where D_{id} is the train load at station i in direction d and is formulated as

$$D_{i1} = \sum_{i=1}^i (I_{i1} - Q_{i1}) \quad (C-3)$$

$$D_{i2} = \sum_{m=i+1}^n (I_{m2} - Q_{m2}) \quad (C-4)$$

In Eq. C-3 and C-4, I_{i1} , Q_{i1} , I_{m2} , and Q_{m2} represent inflow and outflow demand at station l and m in direction 1 and 2, respectively. If the maximum rail transit load is greater than the rail service capacity H_r , will be adjusted while optimizing headways and slack times. For example, the maximum train headway can be obtained from the train capacity divided by the maximum load of the rail system, as shown in Eq. C-5.

$$H_r = \frac{C_r n_c}{D_r} \quad (C-5)$$

For feeder bus routes, the maximum headway can be similarly derived.

APPENDIX D
PROBABILITY DENSITY FUNCTION, NORMAL AND LOGNORMAL
DISTRIBUTIONS

To calculate the connection delay and missed connection costs in the dynamic dispatching model formulated in Chapter 4, the vehicle arrival distribution should be known. From the previous study normal and lognormal vehicle arrival distributions were observed in most transit systems. The probability density functions for both normal and lognormal vehicle arrival distributions are required for estimating connection delay and missed connection costs, as formulated in Eqs. D-1 and D-2. Additional information on normal and lognormal distribution can be found in Hines and Montgomery [1990].

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-((x-\mu)^2/2\sigma^2)} \quad \text{for } \forall x \quad (\text{D-1})$$

where,

x = vehicle arrival time (random variable)

π = 3.14 (constant)

σ = standard deviation of vehicle arrival time

μ = vehicle arrival time (mean)

$$f(x) = \begin{cases} \frac{1}{x\sigma_y\sqrt{2\pi}} e^{-(1/2)[(\ln x - \mu_y)/\sigma_y]^2} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (\text{D-2})$$

where,

$y = \ln x$ is normally distributed with mean μ_y and standard deviation σ_y

x = vehicle arrival time (random arrival)

$\pi = 3.14$ (constant)

The mean $E(x)$ and variance $V(x)$ of x are,

$$E(x) = e^{\mu_y + (1/2)\sigma_y^2} \quad (\text{D-3})$$

$$V(x) = e^{2\mu_y + \sigma_y^2} (e^{\sigma_y^2} - 1) \quad (\text{D-4})$$

REFERENCES

1. Abdelfattah, A. M., and Khan, A. M. (1998) "Models for Predicting Bus Delays," *Transportation Research Record* 1623, 8-15.
2. Abkowitz, M., and Engelstein, I. (1984) "Method for Maintaining Transit Service Regularity," *Transportation Research Record* No. 961, 1-8.
3. Abkowitz, M., Eiger, A., and Engelstein, I. (1986) "Optimal Control of Headway Variation on Transit Routes," *Journal of Advanced Transportation*, 20(1), 73-88.
4. Abkowitz, M., Josef, R., and Driscoll, M. (1987) "Operational Feasibility of Time Transfer in Transit System," *Journal of Transportation Engineering*, ASCE, 113(2), 168-177.
5. Abkowitz M., and Tozzi J. (1986) "Transit Route Characteristics and Headway-Based Reliability Control," *Transportation Research Record* 1078, 11-16.
6. Adamski, A. (1992) "Probabilistic Model of Passenger Service Processes at Bus Stops," *Transportation Research*, 26B(4), 253-259.
7. Avriel M. (1976) "Nonlinear Programming Analysis and Methods," *Prentice-Hall Inc*, 244-265.
8. Bakker, J. J., Calkin, J., and Sylvester, S. (1988) "Multi-Centered Time Transfer System for Capital Metro, Austin, Texas," *Transportation Research Record* No. 1202, 22-28.
9. Bazaraa M. S., Sherali H. D. and Shetty C. M. (1993) "Nonlinear Programming: Theory and Algorithm," *Second Edition*, John Wiley & Sons, Inc.
10. Becker, A., and Spielberg F. (1999) "Implementation of a Timed Transfer Network at Norfolk Virginia," *Preprint, the 78th Annual Meeting*, Transportation Research Board, Washington, D.C.
11. Bookbinder, J.H. and Desilets, A. (1992) "Transfer Optimization in a Transit Network," *Transportation Science*, 26(2), 106-118.

12. Bowman L. A., and Turnquist, M.A. (1981) "Service Frequency, Schedule Reliability and Passenger Wait Times at Transit Stops," *Transportation Research*, 15A(6), 465-471.
13. Ceder, A., and Tal, O. (1999) "Designing Maximal Synchronization for Bus Timetables," Preprint, the 78th Annual Meeting, Transportation Research Board, Washington, D.C.
14. Chowdhury, M. and Chien S. (2000) "Intermodal Transit System Coordination," Preprint, the 79th Annual Meeting, Transportation Research Board, Washington D.C.
15. Chien, S. (1995) "Optimization of Coordinated Intermodal Transit Networks," Ph.D. dissertation, Department of Civil Engineering, University of Maryland at College Park, College Park, MD 20742.
16. Chien, S. and Chowdhury, M. (1997) "Scheduling of Coordinated Transfers for Public Transit Services," The Office of Sponsored Programs, New Jersey Institute of Technology, Acc. # 421580, Final Report.
17. Chien, S., and Ding, Y. (1999) "Application of Artificial Neural Networks in Prediction of Transit Arrival Times," The 9th Annual Conference of ITS America, Washington D.C.
18. Chien, S., Chowdhury, M., Mouskos, K., and Ding, Y. (2000) "Enhancement of the CORSIM Model in Simulating Transit Vehicle Operations," the *Journal of Transportation Engineering*, American Society of Civil Engineers (Forthcoming).
19. Chien, S., Mouskos, K., and Chowdhury, M. (1999) "Generating Driver Population for Microscopic Simulation Model," Submitted for Publication, *Simulations*.
20. Chien S., and Schonfeld P. (1997) "Joint Optimization of a Rail Transit Line and Its Feeder Bus System," *Journal of Advance Transportation*, 251-284.
21. Chien S., and Spasovic, L. (1999) "Optimization of Grid Transit System with Elastic Demand," submitted to *Journal of Advanced Transportation*.
22. Clever, R. (1997) "Integrated Timed Transfer - A European Perspective," the 76th Annual Meeting, Transportation Research Board, Washington D.C.

23. Dessouky M, Hall R., Nowroozi A., and Mourikas K. (1999) "Bus dispatching at timed transfer transit stations using bus tracking technology," *Transportation Research* 7C, 187-208.
24. Engeln-Mullges, G., and Uhlig, F. (1996) "Numerical Algorithm with FORTRAN," Springer-Verlog, Berlin, Heidelberg.
25. Federal Highway Administration. (1996) "CORSIM User Manual," Contact No. DTFH61-92-Z-00074, U.S. Department of Transportation.
26. Federal Transit Administration. (1998) "Advanced Public Transportation Systems: The State of Art, Update' 98," U.S. Department of Transportation.
27. Gray G., and Hoel, L. (1992) "Public Transportation," Prentice Hall, 159-161.
28. Guenther, R. P. and Hamat, K. (1988a) "Transit Dwell Time under Complex Fare Structure," *Journal of Transportation Engineering*, ASCE, 114(3) 367-379.
29. Guenther, R. P. and Hamat, K. (1988b) "Distribution of Bus Transit On-time Performance," *Transportation Research Record* No. 1202, 1-8.
30. Guenther, R. P. and Sinha, K. C. (1983) "Modeling Bus Delays Due to Passenger Boardings and Alightings," *Transportation Research Record* No. 915, 7-13.
31. Hall, R. W. (1985) "Vehicle Schedule at a Transportation Terminal with Random Delay en Route," *Transportation Science*, 19, 308-320.
32. Hickey, T.R. (1992) "Coordination of Intermodal Transfers at LRT Stations," *Transportation Research Record* 1361, 136-140.
33. Highway Capacity Manual. (1992) "Transit Capacity: Chapter 12," TRR Special Report 209, Second Edition (Revised), TRB, National Research Council, Washington, D.C.
34. Hillier S. F., and Lieberman J. G. (1995) "Introduction to Operation Research," McGraw-Hill, Inc., 599-603.
35. Himmelblan, M. D. (1972) "Applied Nonlinear Programming," McGraw-Hill Inc., 167-217.

36. Hines W. W., and Montgomery, D. C. (1990) "Probability and Statistics in Engineering and Management Science," John Willy and Sons, Inc., Third Edition.
37. Hurdle V. F. (1973) "Minimum Cost Locations for Parallel Public Transit Lines," *Transportation Science*, 7, 340-350.
38. Jolliffe, J. K., and Hutchinson, T. P. (1975) "A Behavioral Explanation of the Association Between Bus and Passenger Arrivals at a Bus Stop," *Transportation Science*, 9(3), 248-282.
39. Kalapatapu, R., and Demetsky, M. (1995) "Application of Artificial Neural Networks and Automatic Vehicle Location Data for Bus Transit Schedule Behavior Modeling," *Transportation Research Record* 1497, 44-52.
40. Knoppers P., and Muller T. (1995) "Optimized Transfer Opportunities in Public Transport," *Transportation Science*, 29(1), 101-105.
41. Kocur, G. and Hendrickson, C. (1995) "Design of Local Bus Service with Demand Equilibrium," *Transportation Science* 16(2), 101-105.
42. Koffman, D. (1978) "A Simulation Study of Alternative Real-time Bus Headway Control Strategies," *Transportation Research Record* 663, 41-46.
43. Kraft, W. H. and Deutschman, H. (1977) "Bus Passenger Service Time distributions," *Transportation Research Record* No. 625, 37-43.
44. Kuah, G. K., and Perl, J. (1988) "Optimization of Feeder Bus Routes and Bus Stop Spacing," *Journal of Transportation Engineering*, ASCE, 114(3), 341-354.
45. Kulash, D. (1970) "Routing and Scheduling in Public Transportation Systems," Ph.D. Thesis (Civil Engr.), Massachusetts Institute of Technology, Cambridge.
46. Larson, R.C. and Odoni, A.R. (1976) "Urban Operations Research," Printice-Hall, Inc.
47. Lee, K. K. T., and Schonfeld, P.M. (1991) "Optimal Slack Times for Timed Transfers at a Transit Terminal," *Journal of Advanced Transportation*, 25(22), 281-308.

48. Lee, K. K. T. and Schonfeld, P.M. (1994) "Real-Time Dispatching Control for Coordinated Operation in Transit Terminals," *Transportation Research Record*. 1433, 3-9.
49. Levine, J., Hong, Q., Hug, G.E., and Rodriguez. D. (2000) "Impacts of an Advanced Public Transportation System Demonstration Project," the 79th Annual Meeting, Transportation Research Board, Washington D.C.
50. Lin, W. and Zeng, J. (1998) "An Experimental Study on Real Time Bus Arrival Time Prediction with GPS Data," Preprint, the 77th Annual Meeting, Transportation Research Board, Washington, D.C.
51. Liu, R., Pendyala, R.M., and Polzin, S.E. (1997) "An Assessment of Intermodal Transfer Penalties Using Stated Preference Data," the 76th Annual Meeting, Transportation Research Board, Washington D.C.
52. Maxwell, R. (1999) "Intercity Rail Fixed-Interval Timed-Transfer System: Applicability of the Integrated *Taktfahrplan* Strategy to North America," Preprint, the 78th Annual Meeting, Transportation Research Board, Washington D.C.
53. Miller M. A. and Tsao C. (2000) "Passenger Intermodal Operations and Services: A Synthesis of the Challenges and Opportunities for Successful Implementation," the 79th Annual Meeting, Transportation Research Board, Washington D.C.
54. Nelder, J.A., and Mead, R. (1964) "A Simplex Method for Function Minimization," *The Computer Journal*, 7, 308-313.
55. Nelson M., Brand, D., and Mandel, M. (1982) "State of the Art of Current Bus Transfer Practices," *Transportation Research Record* 854, 6-11.
56. O'Flaherty, C. A., and Mangan, D. O. (1970) "Bus Passenger Waiting Times in Central Areas," *Traffic Engineering and Control*, 11(9), 419-421.
57. Osuna, E. E., and Newell, G. F. (1972) "Control Strategies for An Idealized Public Transportation System," *Transportation Science*, 6(1), 52-72.
58. Pfeiffer, P. (1990) "Probability for Application," Springer-Verlag, New York Inc.

59. Powell M. J. D. (1964) "An Efficient Method for Finding the Minimum of a Function of Several Variables without Calculating Derivatives," *The Computer Journal*, 7, 155-162.
60. Press, H. William et. al. (1992) "Numerical Recipes in FORTRAN," Second Edition, Cambridge University Press, 406-411.
61. Rapp, M. H. and Gehner, C. D. (1976) "Transfer Optimization in an Interactive Graphic System for Transit Planning," *Transportation Research Record* 619, 22-29.
62. Reynolds, M. M. and Hixson, C. D. (1992) "Transit Vehicle Meets System: A Method for Measuring Transfer Times between Transit Routes," *Transportation Research Record* 1349, 35-41.
63. Salzbom, F. J. M. (1972) "Optimum Bus Scheduling," *Transportation Science*, 6(2), 137-148.
64. Salzbom F. J. M. (1980) "Scheduling Bus System with Interchanges," *Transportation Science*, 14, 211-220.
65. Seddon, P. A., and Day, M. P. (1974) "Bus Passenger Waiting Times in Greater Manchester," *Traffic Engineering and Control*, 15(9), 442-445.
66. Seneviratne, P.N. (1990) "Analysis of On-time Performance of Bus Service Using Simulation," *Journal of Transportation Engineering*, ASCE, 116, 517-531.
67. Spasovic L. N., and Schonfeld, P. (1993) "Method for Optimizing Transit Service Coverage," *Transportation Research Record* 1402, 28-38.
68. Strathman et al. (1999) "Automated Bus Dispatching, Operations Control, and Service Reliability: Baseline Analysis," Preprint, the 78th Annual Meeting, Transportation Research Board, Washington D.C.
69. Talley, W. K. and Becker, A. J. (1987) "On-time Performance and the Exponential Probability Distribution," *Transportation Research Record* 108, 22-26.
70. Taylor, M. A. P. (1982) "Travel Time Variability- The Case of Two Public Modes," *Transportation Science*, 16(4), 507-521.

71. Turnquist, M. A. (1978) "A Model for Investigating the Effects of Service Frequency and Reliability on Bus Passenger Waiting Time," *Transportation Research Record* 663, 70-73.
72. Turnquist, M. A. (1981) "Strategies for Improving Reliability of Bus Transit Service," *Transportation Research Record* 818, Transportation Research Board, 8-11.
73. Turnquist, M. A. and Blume, S. W. (1980) "Evaluating Potential Effectiveness of Headway Control Strategies for Transit Systems," *Transportation Research Record* 746, 25-29.
74. U.S. Department of Transportation. (1991) "A Summary: Intermodal Surface Transportation Efficiency Act of 1991," Federal Highway Administration.
75. Vandbona, U., and Richardson, A. J. (1985) "TRAMS: Transit Route Animation and Modelling by Simulation," *Journal of Advanced Transportation*, 19(2), 153-177.
76. Vuchic, R. V. (1981) "Urban Public Transportation System and Technology," Prentice-Hall Inc.
77. Vuchic R. V. (2000) "Transfers in Transit Networks: Theory and Practice," the 79th Annual Meeting, Transportation Research Board, Washington D.C.
78. Wall, Z., and Dailey, D. J. (1998) "An Algorithm for Predicting the Arrival Time of Mass Transit Vehicles Using Automatic Vehicle Location Data," the 77th Annual Meeting, Transportation Research Board, Washington D.C.
79. Welding, P.I. (1957) "The Instability of Close Interval Service," *Operation Research Quarterly*, 8(3), 133-148.
80. Whitney, D., and Brill, J. (1998) "Development of an Intermodal Transit Simulation and It's Application to the Frankford Transportation Center," Preprint, the 77th Annual Meeting, Transportation Research Board, Washington D.C.
81. Wirasinghe, S. C. (1980) "Nearly Optimal Parameters for a Rail/Feeder-Bus System on a Rectangular Grid," *Transportation Research*, 12A, 33-40.
82. Wirasinghe, S. C., and Ho, H. H. (1982) "Analysis of Radial Bus System for CBD Commuters Using Auto Access Models," *Journal of Advanced Transportation*, 16(2), 189-208.
83. Wirasinghe, S. C., Hurdle, B. F., and Newell, G. F. (1977) "Optimal Parameters for a Coordinated Rail and Bus Transit System," *Transportation Science*, 11(4), 369-374.

84. Zeng, J., and Lin W. (1999) "An Experimental Study on Real Time Bus Arrival Time Prediction with GPS Data," Preprint, the 78th Annual Meeting, Transportation Research Board, Washington D.C.
85. Zografos, K. G., and Levinson H. S. (1986) "Passenger Service Time for a No-Fare Bus System," Transportation Research Record 1051, 42-48.