ABSTRACT

A METHODOLOGY FOR SOLVING THE NETWORK TOLL DESIGN PROBLEM

by
Mei Chen

Congestion pricing has been regarded as an efficient method to reduce network-wide travel cost. In this dissertation, a methodology for toll design is developed to provide policy-makers with suggestions on both where to charge tolls and how much the tolls should be. As opposed to the traditional approach of marginal social cost pricing, this methodology is capable of dealing with the more realistic case, in which only a small number of links can be tolled. Furthermore, this methodology is expanded to accommodate multiple user groups.

The toll design problem can be formulated using both deterministic and stochastic route choice models. The most natural formulation of this problem in both cases is a bilevel formulation. Such formulations are very difficult to solve because of the nonconvexity and nondifferentiability of the constraint set. In this dissertation, the problem is converted into a single level, standard nonlinear optimization problem by making certain simplifying assumption. This single-level version of the toll design problem can be solved using a variety of well-developed algorithms.

Tests show that this approach can be used to generate reasonable results and provide valuable decision support to policy-makers.

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To my beloved family

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CHAPTER 1

INTRODUCTION

1.1 Background

Increasing traffic congestion in metropolitan areas has become a significant concern to policy-makers in recent years. From 1975 to 1987, the fraction of peak-period miles traveled on interstate highways with volume/capacity (V/C) ratios higher than 80 percent increased from 42 to 63 percent. In just two years, from 1985 to 1987, the rush hour traffic classified as congested by the US Department of Transportation rose from 61 percent to 63 percent (Small et al. 1989). Statistics show that in 1997, the urban interstate system had 55 percent of its peak period travel with V/C ratio higher than 0.8 (Federal Highway Administration 1997). Commuters are stuck in traffic for longer periods of time at rush hours, causing huge losses of time and fuel. At the same time, emissions from idling and accelerating engines accompanying the stop-and-go traffic greatly degrade air quality. The cost of delay in U.S urban areas, based on estimates of motorists' value of time and wasted fuel, totaled $43 billion in 1990 (Shrank 1993). The cost would be even higher if externalities such as reduced air quality were also included.

Even though billions of dollars have been spent expanding urban freeways and public transportation systems, traffic congestion remains a problem. Various demand management policies, including carpool or vanpool, high occupancy vehicle (HOV) lanes, elimination of employer-subsidized parking, and implementation of flextime work schedules, have been suggested to ease congestion. Unfortunately none of them have offered substantial relief. For example, in the case of HOV, a 1992 survey showed that only 28 percent of commuters occasionally used the HOV system in Southern California. Ridesharing has increased slightly, but not even close to the extent anticipated. The one-way average travel time savings of 14 minutes one way has not been sufficient to persuade a higher proportion of solo drivers to forgo the convenience, flexibility, and comfort of driving alone (Bhatt 1994). A recent study on the evaluation of HOV lanes on I-80 in New Jersey also showed only a slight increase in peak period percentages of HOVs and people in HOVs. From 1980 to 1990, the carpooling percentages have dropped from 19.7 percent to 13.4 percent nationally, and from 18.3 percent to 12.4 percent in New Jersey (Parsons Brinckerhoff Quade & Douglas, Inc. 1997). Besides, it is argued that the HOV facilities are likely to produce only a small reduction in traffic, if any, along the mixed traffic lanes, and such reductions are likely to last for a short period only until latent demand fills up the roads again (Bhatt 1994). Under the circumstances of tighter restrictions on expanding highway capacity, the enforcement of stringent air quality standards, and the development of advanced technology, congestion pricing, as an efficient and practically feasible tool, is becoming the focus of transportation planners.

The central idea of congestion pricing is to charge different tolls on different facilities at different times of day in order to reduce congestion. Examples include tolling roads and bridges, charging fees for entering congested areas, HOV buy-in, and changing parking and transit pricing. In practical application, there are several basic forms of congestion pricing (Gomez-Ibanez and Small 1994):

(1) Point pricing, in which a traveler passing a point at a specific time is charged a fee;

(2) Cordon pricing, in which a traveler entering a congested area is charged a fee at each entry point into the area;

(3) Zone pricing, in which road users traveling within a cordoned area pay a fee;

(4) Parking pricing, in which higher parking charges are applied in congested areas during the most congested periods;

(5) Charges for distance traveled within a congested area or on a congested route.

Two forms of congestion pricing in operation are cordon pricing in Singapore and charges for distance traveled on the A1 toll road outside Paris. Additional examples of cordon pricing can be found in three Norwegian cities where the tolls do not vary by time of day. In the United States, a hybrid version of point pricing, HOV buy-in, in which solo drivers would be provided the option of paying to travel on the underused HOV lanes, is actively proposed.

Through changing road users' route choice, time of travel, and mode choice, congestion pricing is considered as an effective method of relieving congestion. Furthermore, the recent development in the electronic toll collection technology has made congestion pricing technically feasible. However, congestion pricing has been a matter of public policy debate in the United States for years because people question its political feasibility. Two issues are involved here, public acceptance of direct payment for road use, which is widely viewed as a free good, and the fairness to those unable to pay congestion fees without incurring economic hardship. Particularly, the social equity issue is a significant political barrier to the enactment of congestion pricing. The congestion fee may constitute a larger proportion of expense for low-income people, so they are more likely to be "tolled off" the road. These issues contribute to the difficulties of implementing congestion pricing policies thus far in the United States. Therefore, when designing a congestion pricing policy, the social equity issue has to be addressed and a reasonable revenue distribution package has to be developed in order to overcome the political barrier.

It is believed that using revenues to fund transportation improvements and broad economic benefits to road users through reduced taxes, rebates, or community programs may provide the greatest overall benefit and earn the widest political support. Small (1992) suggests that the revenue generated from congestion pricing may be used in various places, such as reducing taxes, and fund capacity expansion. Based on some case studies, it is concluded by Small (1992) that the efficiency of the transportation system would be improved if congestion pricing strategies are carefully designed and properly implemented, and the benefits would outweigh the costs.

In addition to all of these theoretical discussions of congestion pricing policy, there have been several practical experiments.

Since 1975, Singapore has been charging a fee to motorists who wish to enter the congested central business district (CBD) during morning and evening peaks (Hau 1992). Shortly after this system was imposed, the number of vehicles entering the CBD during the restricted hours declined by 44 percent. Twenty years later, after some modifications to the program, vehicle trips into the CBD during morning peak hours remain some 25 percent below the 1974 level (Gomez-Ibanez and Small 1994).

In France, a simple congestion pricing system has been implemented on a toll road outside Paris to deal with the weekend peak demand caused by the return of city residents on Sunday afternoons from weekend retreats. The time-varied price has caused enough travelers to change their travel schedule to avoid the excessive delay on the road (Gomez-Ibanez and Small 1994).

In Hong Kong, an extended experiment with electronic road pricing technology was conducted from 1983 to 1985. The system performed quite well, but because of political difficulties the congestion pricing project was not implemented (Dawson and Catling 1986). In 1990, a so-called "Revenue Neutral Peak Pricing Policy" was implemented on the mass transit railway system in Hong Kong. A differential pricing system was established to reroute transit passengers travelling from Kowloon to the CBD during the morning peak. Studies showed this policy did not have a highly significant effect on route choice of passengers, even though the cost difference between the two alternatives was large in terms of magnitude (Li and Wong 1994).

In Norway, the cities of Bergen, Oslo, and Trondheim have put in place "toll rings" around each city since 1986, 1990 and 1991, respectively. Among them, Oslo and Trondheim have implemented electronic toll technology (Larson 1988, Lauridsen 1990, Medland and Polak 1993). About 80 percent of the revenue earned in Trondheim is collected electronically, and the toll collection technology in both Oslo and Trondheim costs about 15 percent of the total revenue earned. Even though these pricing practices were initiated for revenue collection rather than congestion alleviation, they have been getting increasing support from the public because of their effects on congestion management, and their experience could be readily adapted to congestion pricing.

There are also several other cities that are considering congestion pricing proposals. A toll ring proposal in Stochholm has been filed in which motorists would be charged for entering the central city via its inner ring road. In the Netherlands, a proposal is being developed for the Randstad region that would impose a charge for entering the primary road system during the morning peak. Also, in England, congestion pricing is being actively considered in London and Cambridge. There has been a study for London focusing on the public and political acceptance of various pricing proposals (May 1975, May and Gardner 1990).

In the United States, there are several proposals for congestion pricing in the San Francisco Bay Area and in Southern California. In order to shift some peak period traffic demand on the San Francisco-Oakland Bay Bridge to the off-peak and to transit, the congestion pricing pilot project of the San Francisco Bay Area suggested doubling or tripling of the existing bridge toll during peak hours (Dittmar et al. 1994). In the San Diego area, it has been proposed that solo drivers pay a toll to use an existing underused HOV lane, which is the so-called "HOV buy-in", while carpools are still exempted from this charge (Duve 1994). There is also a private toll road, the Riverside Freeway (SR 91) in Southern California, on which peak-period fees are imposed on solo drivers using the newly constructed express lanes paralleled to the existing freeway. Similar to the proposal in the San Diego area, carpools will use this express lane without charge (Fielding 1994). However, the California congestion pricing projects aim at reducing the congestion only on the tolled road instead of the whole area-wide transportation network. Given the high level connectivity of the highway system, these pricing strategies certainly have built-in flaws.

1.2 Theoretical Basis of Congestion Pricing

Economists have proposed the concept of congestion pricing to allocate scarce highway capacity, especially under congested road conditions, such as Pigou (1920), Knight (1924), Walters (1961), and Vickrey (1963). By making road users pay the full marginal social cost of using the highways, congestion pricing could significantly reduce traffic congestion, thus reduce time lost, reduce air pollution and save energy.

1.2.1 General Idea

The economic theory behind the congestion pricing can be explained graphically, as shown in Figure 1-1.

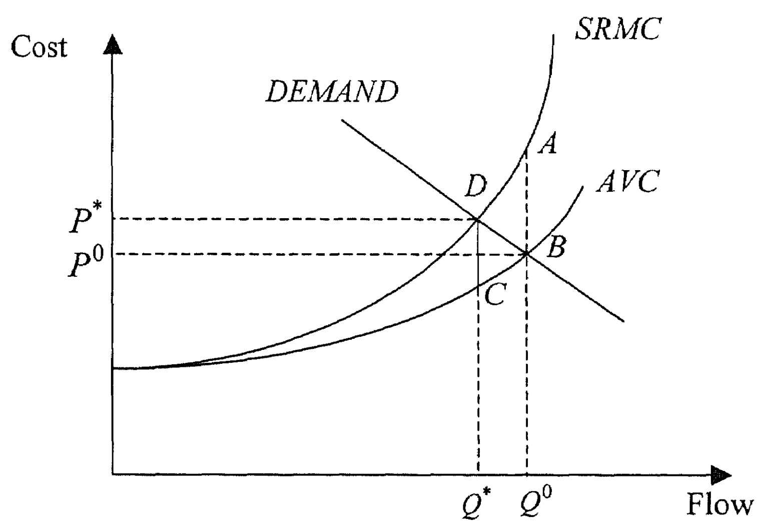

Figure 1-1 Marginal Social Cost Congestion Toll

In the absence of a congestion toll, the quantity of traffic flow will be that which occurs at the point where the average user cost curve intersects the demand function. As can be seen, short-run marginal cost is considerably higher than the average cost at this point. This is because the marginal cost curve represents the cost to all drivers of adding one more driver to the traffic stream during the same time period, while the average cost curve gives the average cost experienced by individual drivers. Perceiving only this average cost, an additional driver is likely to join a congested traffic stream. In the absence of a congestion toll, since he/she may only be aware of the average cost he/she will experience and is largely unaware of the increased cost, called external cost shown in Figure 1-1 as *AB*, that he/she is imposing on all other drivers.

If the road users are made to pay the full marginal social costs of driving during the peak period, urban traffic congestion will be fairly alleviated. This can be accomplished by charging a congestion toll in the amount of *CD* as shown in Figure 1-1, to all users driving on that specific roadway during peak period. At this point, each driver will experience the short-run marginal cost P^* instead of average individual cost P^0, thus the traffic demand would decrease from Q^0 to Q^*.

However, this theoretical model has some limitations. McMullen (1993) indicates this model is simplistic in that it ignores the fact that different vehicle types might impose different costs on the roadway. For example, large trucks obviously inflict more damage and thus higher road costs. Also, this theoretical model applies only to one section of a specific roadway. Furthermore, this model assumes that vehicle drivers are fully cognizant of the private costs of a vehicle trip while studies show that highway users usually significantly underestimate the cost of vehicle operation as well as the value of drivers' time spent in traffic jams.

Gomez-Ibanez (1992) points out that the above principle for road pricing does not apply well in cases when more than one road is involved and there are toll-free substitutes which could significantly reduce the effectiveness of tolls in managing demand. Since in most practical situations the availability of toll-free competition may be crucial to secure the political support for toll programs, the calculation of the optimal toll level becomes much more complicated.

In the following section, two categories of research on congestion pricing, route choice modeling and departure time choice modeling, which constitute the theoretical basis of congestion pricing policy will be discussed.

1.2.2 Route Choice Modeling

In the context of route choice models involving congestion tolls, Dafermos and Sparrow (1971) introduce a method to determine the congestion toll on each link that makes the resultant user equilibrium (UE) flow pattern identical to that of the system optimum (SO). This equilibrium flow pattern will generate the minimum total travel time for the whole network. Specifically, the system optimization problem is first solved for the system optimal flow pattern, then the link toll vector can be obtained by solving the user equilibrium under this toll pattern which produces the system optimal flow pattern obtained previously.

Dafermos (1973) extends the above method for networks with multiple user groups. It is shown that the system-optimizing flow pattern can still be obtained in this case by charging tolls. Also, link-based as well as route-based tolling policies are discussed.

Smith (1979) generalizes this method of marginal social cost charge for networks with link interaction and variable demand. It is proved that if the cost and the demand functions satisfy certain weak smoothness conditions, the local optimal marginal social cost charges on the transportation network could be found.

Usually in the network traffic equilibrium problem, route cost is considered as the summation of the costs of those links defining the route, while this is often not practically true. Gabriel and Bernstein (1997) point out that there are many situations in which the additive assumption is inappropriate, including nonlinear valuation of time, non-additive transit fares, and non-additive tolls (such as the practice on the New Jersey Turnpike). The existence and uniqueness conditions as well as convergence results for a generic nonlinear complementarity method are established. In a later work by Bernstein and Gabriel (1996), a route generation method based on the non-smooth equation/sequential quadratic programming (NE/SQP) algorithm is developed for solving this traffic equilibrium problem with non-additive route costs.

1.2.3 Departure Time Choice Modeling

In the context of departure time choice models involving congestion tolls, Vickrey (1969) discusses the bottleneck congestion and the use of congestion tolls vs. capacity expansion to provide optimal adjustment. It is shown that a time-varying congestion toll during peak hours could be an important element in developing an efficient transportation system, even in the long run.

Arnott et al. (1990) extend the previous studies by proposing a coarse toll, which is defined as a one-step fee paid at the front of the queue over a time interval. Equilibrium is reached when all travelers have the same travel costs, which consists of queuing time and schedule delay. It is found the optimal toll would generate substantially greater efficiency gains than those obtained from previous estimates, which ignored the change in frequency distribution of departure time. In addition, a significant portion of this efficiency gain can be achieved by applying the one-step coarse toll, which involves much lower implementation cost than the more complex time-varying toll.

Arnott et al. (1992) present a method of charging the user-group-specific congestion tolls to improve the temporal and spatial separation of user types over roads. A model with two groups of drivers choosing between two parallel routes is established, in which the trip cost of a driver in either group is assumed to be a linear function of the number of drivers in each group taking the same route. Also, through the discussion on the interaction between the spatial and the temporal separation of user groups, it is found that it may be more efficient to separate user groups temporally rather than spatially.

Bernstein (1993) proposes a congestion pricing scheme involving continuous tolls and subsidies on a simple two route network. The impact of this pricing scheme under various implementations is examined. It is shown that with the help of electronic toll collection technology, this pricing scheme with time-varying tolls and subsidies may overcome some public objections to congestion pricing.

Chen and Bernstein (1995) extend the one-directional bottleneck model to study the AM/PM commuting. It is indicated that with the help of toll collection technology, the time-varying congestion toll can be applied to influence the departure time of travelers in both directions, when tolls can only be collected in one direction. Particularly, such tolling scheme combining tolls and subsidies has to be network-varied.

1.3 Finding Appropriate Tolls

How to design an effective tolling scheme has long been an issue of focus in planning congestion pricing policy. In particular, what is the appropriate toll on the network has been discussed in many studies recently.

Ferrari (1995) shows that when the roadway network has links with physical capacity constraints, the deterministic static traffic equilibrium model with elastic demand may have no solution at all. This results from the fact that some links may bear flows higher than their capacities. By charging congestion tolls on these links, the equilibrium can be reached. The computation method for these additional costs is based on the use of modified link cost functions. Compared with the congestion tolls derived from the traditional marginal social cost pricing, the congestion tolls obtained by using this method are markedly less. However, there is no proof that this method is general and can be extended to large networks.

Bergendorff et al. (1996) present a tolling methodology for transportation networks that also ensures the resultant equilibrium flows to be the system optimum. In this model, various optimization criteria can be specified, such as minimizing the total amount of tolls collected from users, or minimizing the number of toll plazas. With the marginal social cost as one element of the possible tolls, this model would generate a toll pattern involving less tolled links, compared to the ordinary marginal social cost tolls.

Based on the concept of bi-criterion equilibrium traffic assignment, Dial (forthcoming) presents a model, an algorithm and heuristic for link-based toll design. In this model, the travelers' value of time is considered as a stochastic variable. The optimal toll will induce a user equilibrium flow pattern that is system optimal. However, for a more practical case where link tolls are bounded, the computation demand becomes formidable. Test results of a heuristic on a mid-sized network show that tolling only some congested links can greatly reduce total travel time and increase the travel speed.

Based on the theory of marginal social cost pricing, Wie and Tobin (1998) present two types of dynamic congestion pricing model. Two types of time-varying congestion tolls can be determined by solving a convex control formulation of the dynamic system optimal traffic assignment. It is shown that the equilibria under tolls for both cases are identical to the result of such assignment.

Yang and Meng (1998) introduce an optimization model to obtain the departure time and the schedule delay of commuters as well as the optimal time-varying tolls of bottlenecks, given the elastic demand functions and schedule delay costs associated with each destination. This model is constructed over a combined application of the space-time expanded network (STEN) representation of time-varying traffic flow and the conventional network equilibrium modeling techniques.

1.4 Objectives of This Research

From a theoretical perspective, these papers are quite interesting. However, either they are not network-oriented (such as Vickrey 1969, Arnott et al. 1992, and Chen and Bernstein 1995, etc.), or, they presume marginal social cost pricing (such as Dafermos and Sparrow 1971, Dafermos 1973, Bergendorff et al. 1996, Dial forthcoming, Wie and Tobin 1998, and Yang and Meng 1998). Congestion pricing has a profound impact on people's route choice decision on the whole network, not only those who travel on the tolled roads. So, it is very important to study route choice behavior over the entire network. Furthermore, in practice, it is clear that not all links are tollable. In fact, only very few links of the network can be tolled (such as bridges and tunnels, etc.). Little has been done to solve this "real world" congestion toll design problem. Hence, the primary objective of this dissertation is to study this more realistic toll design problem. More specifically, assuming that only a subset of the links in the network can be tolled and that there are multiple user groups over the network, the toll design problem will be formulated for both deterministic and stochastic route choices.

Intuitively, the toll design problem can be formulated as a bilevel optimization problem with nonlinear objective function and the solution set of another optimization problem as its constraint. Unfortunately, this constraint is nondifferentiable and nonconvex, and, hence this kind of optimization problem is very difficult to solve. To simplify this problem, it is proposed that the toll design problem be formulated as a single-level optimization problem by making certain simplifying assumptions. This simplified problem will have a set of differentiable, nonlinear equality constraints. Compared to the original problem, this problem can be solved much more easily using the algorithms for solving a standard nonlinear programming problem.

Another contribution of this dissertation is the extension of this toll design problem to accommodate multiple user groups over the network. It is well known that congestion pricing will have a significant impact on all road users. Since there are various types of vehicles, each with different operating characteristics (such as cars and trucks), they have different congestion externalities. So, it may be appropriate to charge different tolls to different vehicles. In addition, since the income level differs among travelers who may have different route choice behavior, charging the identical toll may force some travelers with lower income to switch to other routes, which would raise the social equity issue. Therefore, incorporating these factors into the congestion toll design model is necessary for planning an effective and widely accepted tolling policy. In this dissertation, a toll design model with multiple user groups will be developed. However, the nonconvexity of the problem, which is mostly attributable to the presence of multiple user groups, makes even the simplified toll design problem very complicated. Various solution techniques will be used to find the best solution to the problem on a practical network.

The plan of this dissertation is as follows. Chapter 2 presents a review of the literature related to the toll design problem, as well as the very similar but well-studied continuous network design problem. Chapter 3 presents the formulation of the toll design problem with deterministic route choice. The relationship between the bilevel formulation and the single level formulation for deterministic toll design problem is also discussed. Chapter 4 presents the formulation of the stochastic toll design problem. Also discussed is the relationship between the deterministic and stochastic toll design problems. In Chapter 5, several small examples are presented to illustrate how charging tolls on certain links would decrease the total travel cost. Chapter 6 introduces various algorithms for solving the simplified toll design problem, and discusses the applicability of these algorithms to toll design problems with deterministic and stochastic route choices. Chapter 7 presents a case study for the toll design problem, in which a practical network is chosen to test the methodology developed in Chapters 3 and 4. The dissertation concludes with Chapter 8, which contains conclusions and direction of future research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The toll design problem is to find the network toll pattern that would minimize or maximize some objectives subject to constraints on the tolls and a constraint or constraints that ensure(s) that the resulting flow pattern is in equilibrium. An important underlying assumption of this problem is that the set of links in the network on which tolls could be placed is known.

In this problem, there are basically two types of decision-makers with different objectives. The transportation planner wishes to find a network toll pattern to minimize the total network-wide travel time, while the road users choose the routes that minimize their individual travel costs. Figuratively speaking, the transportation planner evaluates the total travel time obtained from the equilibrium flow pattern under various toll schemes and determines the best tolling plan. Ignoring the details for the moment, suppose that the planner knows that a particular link toll pattern, τ, can give rise to any number of route flow patterns $F \in E_\tau$, where E_τ denotes the set of equilibrium route flow patterns of the network under the network toll pattern τ. Then, if the route travel cost under the route flow pattern F is denoted by $C(F)$, the planner's problem is

$$\min_{\tau} \; C(F)^T \cdot F \tag{2.1a}$$

subject to

$$F \in E_\tau \tag{2.1b}$$

Now, since travelers are "competing" with each other, one would expect that for any given toll pattern, any traffic pattern that would result would be an equilibrium of same kind. Because of the dependency of travel cost on the flow, these two decision-making processes actually interact with each other. In order to minimize the total travel time, the planner's decision on how much to charge and where on the network to impose the tolls is based on the user equilibrium flow pattern. This pattern in turn results from the users' route choice decision under the network toll pattern set by the planner.

For problem (2.1), suppose E_τ is a singleton. Then the planner's problem can be written as

$$\min_{\tau} \quad C(E_\tau)^T \cdot E_\tau \tag{2.2}$$

However, in general this is not the case. It is commonly known that the equilibrium route flow is usually not unique even though the equilibrium link flow is. Therefore, E_τ, as the set of equilibrium route flow patterns, most likely contains multiple elements. So, considering that the route flow is a decision variable in the road user's problem, problem (2.1) can be treated as if the planner chooses τ and F. But that F must be constrained to be an appropriate behavioral response to τ. That is

$$\min_{\tau,F} \quad C(F)^T \cdot F \tag{2.3a}$$

subject to

$$F \in E_\tau \tag{2.3b}$$

Generally, users' route choice decisions can be deterministic (in which case users will always choose the minimum cost route), or stochastic (in which case users choose their routes probabilistically). Therefore, the toll design problem can be divided into two categories, the one with deterministic route choice and the one with stochastic route choice. In this chapter, the existing literature on both types of toll design problem will be reviewed. Previous studies of the continuous network design problem will also be reviewed because of the similarity between that problem and the toll design problem.

2.2 Toll Design Problem with Deterministic Route Choice

In the deterministic route choice case, for any given toll pattern, road users will assign themselves to the route that minimizes their individual travel cost (*inclusive of tolls*). In this case, let Ψ denote the route toll vector, and Ω denote the feasible route flow patterns of the network, then the travelers' problem is to find the optimal route flow pattern $F^* \in \Omega$ that satisfies

$$[C(F^*)+\Psi]^T(F-F^*) \geq 0 \qquad F \in \Omega, \tag{2.4}$$

which is the variational inequality formulation of the deterministic equilibrium problem.

Consider a transportation network comprised of a finite set of links, A, (with cardinality $|A|$) and nodes, N (with cardinality $|N|$). Let W (with cardinality $|W|$) denote the set of origin-destination (O-D) pairs, P_w denote the set of routes that connect O-D pair $w \in W$, $P = \underset{w \in W}{\cup} P_w$ denote the complete set of routes, and D_w denote the travel demand between O-D pair $w \in W$. The relationship between routes and links is described by an indicator function, δ_{ap}. That is, $\delta_{ap} = 1$ if link a is in route p and $\delta_{ap} = 0$ otherwise. Then the incidence matrix is defined as $\Delta = (\delta_{ap} : a \in A, p \in P)$. Further, let F_p denote the flow on route p. The set of feasible route flow patterns can be given by

$$\Omega = \left\{ F \in \Re_+ : \sum_{p \in P_w} F_p = D_w, w \in W \right\}. \tag{2.5}$$

Associated with each link and route is a travel time function and a travel cost function. The vector of link travel times is denoted by t, the vector of link travel costs (*exclusive of tolls*) is denoted by c, and the vector of link tolls is denoted by τ. Similarly, the vector of route travel times is denoted by T, the vector of route travel costs (*exclusive of tolls*) is denoted by C, and the vector of route tolls is denoted by Ψ. It is assumed that all route travel times and costs are additive, i.e., $T_p = \sum_{a \in p} t_a$, $C_p = \sum_{a \in p} c_a$, and $\Psi_p = \sum_{a \in p} \tau_a$.

The set of equilibrium route flows is then given by

$$E_\tau = \left\{ F^* \in \Omega : [C(F^*) + \Psi]^T (F - F^*) \geq 0, \quad F \in \Omega \right\} \tag{2.6}$$

which is simply the set of solutions to the variational inequality formulation of the equilibrium problem. And, if f denotes a vector of link flows, the set of equilibrium link flows is then given by

$$e_\tau = \left\{ f^* \in \Re_+ : f^* = \Delta^T F, \quad F \in E_\tau \right\}. \tag{2.7}$$

Based on the deterministic route choice concept, Tan (1997) presents an link-based bilevel formulation of the toll design problem, as shown in equation system (2.8).

$$\min_{\tau, f} \quad c(f)^T f \tag{2.8a}$$

subject to

$$f \in e_\tau \tag{2.8b}$$

$$\tau_a \geq 0 \qquad a \in I \tag{2.8c}$$

$$\tau_a = 0 \qquad a \notin I \tag{2.8d}$$

where $I \subseteq A$ denotes the set of links that can be tolled.

The objective of the toll design problem is to minimize the total travel time, which is a nonlinear differentiable function. Its convexity depends on the form of the link cost functions. If the standard Bureau of Public Road (BPR) function is chosen, the objective function is then convex. However, the constraint set of the toll design problem raises serious questions. From the above formulation, it is observed that the constraint $f \in e_\tau$ involves the solution set of another optimization problem (a variational inequality problem in this case). Therefore, this constraint is nondifferentiable and nonconvex, which means descent direction method can not be used to find even a local optimum.

Tan (1997) proposes two heuristics for solving this toll design problem. One is the probabilistic search method, in which trial solutions of a network toll vector are repeatedly generated and evaluated by comparing the total travel time induced by the equilibrium flow under these toll patterns. It is expected that this method may give acceptable results at a much lower computational cost. However, generating trial solutions of the toll vector is a pure random process. The other one is a descent method, in which the gradient information is obtained numerically to facilitate the search for descent direction. However, due to the inaccuracy of numerical results, this method can not guarantee convergence before the objective value starts to increase.

However, it can be observed that the above bilevel formulation of the toll design problem is very similar to the well-known continuous transportation network design problem. Although the toll design problem has not been studied extensively, the continuous network design problem has been studied for decades. Thus, the continuous network design problem and its algorithms will be discussed next.

2.2.1 Continuous Network Design Problem with Deterministic Equilibrium Constraints

Continuous network design problems generally deal with continuous investment decision variables. They were proposed because of the computational difficulties experienced with mixed integer programming algorithms for discrete network design problems with a large number of 0-1 variables (Dantzig et al. 1976 and Morlok et al. 1973).

If ω denotes the set of feasible equilibrium link flow patterns, then

$$\omega = \left\{ f \in \Re_+ : f = \Delta^T F, \; F \in \Omega \right\}. \tag{2.9}$$

Let y denote the vector of link investment decision variables, B denote the budget limit, and h denote the vector of unit costs of link improvements. The basic formulation of the continuous network design problem is as follows:

$$\min_{y} \; t(f)^T f \tag{2.10a}$$

subject to

$$f \in e_y \tag{2.10b}$$

$$h^T y \le B \tag{2.10c}$$

where e_y is the equilibrium link flow pattern that is given by

$$e_y = \left\{ f^* \in \omega : c(f^*, y) \cdot (f - f^*) \ge 0, \; f \in \omega \right\}. \tag{2.11}$$

The objective of the continuous network design problem is to minimize the total travel cost by choosing y_a, $a \in A$, which is a measure of the increase in link capacity.

Equation (2.10b) is the deterministic equilibrium constraint, equation (2.10c) is the budget constraint that ensures that the total cost of improvements does not exceed the budget allowed.

2.2.2 Algorithms for Deterministic Network Design Problems

Many papers have been written about the continuous network design problem focusing on the development of algorithms giving exact, global solutions as well as being computationally efficient.

Based on previous formulations and solution algorithms for the network design problem, it is proposed by Dantzig et al. (1979) that continuous investment decision variables be used and the system optimal traffic assignment is performed in the lower-level in place of the more reasonable user equilibrium traffic assignment. This greatly simplifies the solution process by avoiding the mixed integer programming and nonconvex programming. When there is no budget constraint, a decomposition method is used to solve this network design problem. For the case in which a budget constraint is used, a Lagrange multiplier technique is used to obtain the solution by solving a series of traffic assignment problems, one for each value of the multiplier.

Abdulaal and LeBlanc (1979) propose a nonlinear unconstrained optimization formulation of the network design problem with continuous investment variables. Powell's method and the Hook and Jeeves's method are employed to solve the problem with convex and concave investment functions, respectively. Tests show that the convex investment costs result in minor additions to existing capacities for almost all links proposed for improvements, while on the other hand, concave investment costs result in large increases in existing capacities for only those links with high V/C ratios, and negligible additions to other links. However, for each link improvement update at the upper level, a user equilibrium problem needs to be solved at the lower level. Therefore, these algorithms have high complexity with respect to computation time that prohibits their application for solving real-world problems that usually have large magnitude.

It is demonstrated in Marcotte (1983) that the lower level user equilibrium of the network design problem can be formulated as a variational inequality for each link of the network provided that the cost functions are separable, twice continuously differentiable and strictly increasing. This would make use of the gradient information in the solution procedure of the lower level problem, and thus would improve the efficiency of those solution methods for the network design problem. The constraint accumulation (relaxation) algorithm and the iterative optimization assignment method, which are an exact algorithm and heuristic respectively, are used to solve the problem. The iterative optimization assignment method is very sensitive to the choice of the starting solution. In both cases, the network design problem remains computationally prohibitive.

In LeBlanc and Abdulaal (1984), it is suggested to substitute the user equilibrium problem with the more easily solved system optimal problem. This is based on test results indicating that the system optimal model produces solutions as good as those from the user equilibrium model. The reason is that although the network design problem with user equilibrium constraints is much more realistic, normally it can't be solved optimally. However, the network design problem with system optimal constraints can be solved optimally, provided that linear investment cost functions are used. Hence, for realistic-sized networks, the distinction between the quality of the solutions obtained is vague.

LeBlanc and Boyce (1986) present a bilevel programming approach which can give an exact solution for a moderate-sized network design problem. In this method, the link travel cost functions are approximated by piecewise linear functions, and then the grid search algorithm (Bard 1983) for solving linear bilevel programs could be applied to obtain the exact solution for networks with fewer than 200 nodes. For larger networks, near-optimal solutions can be obtained by solving an approximating nonlinear program that is very similar to a user equilibrium model. The objective function is made convex to ensure a global optimal solution. However, it is shown in Ben-Ayed and Blair (1989) that the above algorithm for solving the linear bilevel program may not always give optimal solutions.

Suwansirikul et al. (1987) propose another heuristic for finding an approximate solution to the continuous equilibrium network design problem. Numerical tests indicate that this heuristic is much more efficient than the Hook-Jeeves method, when applied on networks with significant congestion. This efficiency results from the decomposition of the original problem into a set of interacting optimization subproblems. And at each iteration, only one user equilibrium needs to be solved to update the improvement variables of all links of the network, since all decomposed subproblems are solved simultaneously. In addition, it is also noted that the use of the iterative optimization assignment method for solving the network design problem is not appropriate. The reason is that the iterative optimization assignment method is for a Cournot-Nash game, while the network design problem is in fact a Stackelberg game.

Friesz et al. (1992) present a simulated annealing method for solving the network design problem with variational inequality constraints. By not always rejecting the intermediate solutions worsening the objective value, this method, rooted from the physical annealing of solids, is expected to be able to reach the global optimal solution for such nonconvex problems. However, the computation effort involved with the simulated annealing method is highly excessive due to the large number of equilibrium assignment problems to be solved, even though it yields better a solution compared to those calculated with other well know methods.

Waller et al. (1998) introduce a linear formulation of the network design problem based on a dynamic traffic assignment model. This model presumes the system optimal traffic assignment and time-dependent demand. Then the complicated network design problem can just be solved using classic linear programming algorithms. However, this method is limited to single destination networks.

The concept of the deterministic equilibrium is based on certain behavior assumption that travelers are fully aware of the travel cost over the entire network, and they always choose the least cost route connecting their origin and destination pairs. While many argue that this deterministic route behavior representation is more accurate when travelers have perfect information on travel cost, it ignores the fact that people may have intermediate stopping places on their trip, such as dropping children off at the daycare center on the way to work and then picking them up on the way home, as well as seeing a doctor, stopping by at supermarket, etc. In reality, a traveler's decision on which route to choose is more likely based on these intermediate points. Even when there isn't any intermediate stopping points, travelers may not always choose the least cost routes. Thus the stochastic route choice model was established to describe this choice behavior.

2.3 Toll Design Problem with Stochastic Route Choice

A stochastic route choice model has been established based on the discrete choice models that describe individuals' choices between competing alternatives (for example, see Sheffi and Daganzo 1978). The underlying assumption of the discrete choice models is that when faced with a choice situation, an individual's preferences toward each alternative can be described by an "attractiveness" or "utility" measure associated with each alternative. This utility is a function of the attributes of the alternatives as well as the decision-maker's characteristics. It is presumed that the decision-maker chooses the alternative that yields the highest utility. However, because of the randomness associated with the utility (i.e., the uncertainty of the attributes that influence an individual's utility), the discrete choice model can give only the probability with which alternatives are chosen, not the choice itself. Therefore, different from the deterministic route choice model, the stochastic route choice model determines the probability of a particular route, connecting a particular O-D pair, being chosen by a decision-maker who is randomly selected from a given population. Use the most widely employed logit model, and let $\overline{P}_p$ denote the probability of choosing route $p \in P_w$ connecting O-D pair $w \in W$ being chosen, and Θ_p denote the utility of this route, then

$$\overline{P}_p = \frac{e^{-\mu\Theta_p}}{\sum_{i \in P_w} e^{-\mu\Theta_i}} \qquad w \in W, p \in P_w \tag{2.12}$$

in which μ is the cost scaling constant that represents users' sensitivity to the cost differences among alternative routes. Furthermore, this probability should be understood as the fraction of individuals in this large population. Given the utility function of individual travelers on route $p \in P_w$ connecting O-D pair $w \in W$ as the route travel cost (*inclusive of tolls*), i.e.,

$$\Theta_p = C_p(F) + \Psi_p, \tag{2.13}$$

where $\Psi_p = \sum_{a \in p} \tau_a$, as discussed earlier, the number of travelers choosing route $p \in P_w$ can be found as

$$F_p = D_w \cdot \overline{P}_p = D_w \cdot \frac{e^{-\mu[C_p(F)+\Psi_p]}}{\sum_{i \in P_w} e^{-\mu[C_i(F)+\Psi_i]}} \qquad w \in W, p \in P_w. \tag{2.14}$$

Hence, the set of equilibrium route flows, E_τ, can be given by

$$E_\tau = \left\{ F \in \Re_+ : F_p = D_w \cdot \frac{e^{-\mu[C_p(F)+\Psi_p]}}{\sum_{i \in P_w} e^{-\mu[C_i(F)+\Psi_i]}}, w \in W, p \in P_w \right\}, \tag{2.15}$$

and the set of equilibrium link flow, e_τ, can be given by

$$e_\tau = \{ f \in \Re_+ : f = \Delta^T F, \ F \in E_\tau \}. \tag{2.16}$$

This actually defines the stochastic user equilibrium problem, which is to find the route flow pattern $F \in E_\tau$, through solving the equality system in (2.15). Alternatively, the stochastic route choice model can be formulated as a mathematical program with flow conservation constraint to obtain the set of equilibrium link flows, e_τ (Sheffi and Powell 1978). Powell and Sheffi (1982) suggest the use of the method of successive average by providing a convergence proof of this method. This method involves a predetermined move size on the basis of some characteristics of the current solution, and thus it is very slow in convergence.

Consequently, under this formulation, routes with lower costs are more likely to be chosen and vice versa. More importantly, every route in the network will have a nonzero probability of being chosen, provided that $\mu < \infty$. Furthermore, the formulation of (2.15) also implicitly satisfies the flow conservation constraints. Another very important feature of this stochastic equilibrium problem is the uniqueness of its solution. This uniqueness will affect the solution of the toll design problem with stochastic route choice, and this will be addressed in Chapter 4.

Based upon the above discussion of the stochastic route choice model, the toll design problem can be written as

$$\min_{\tau,F} \; C(F)^T \cdot F \tag{2.17a}$$

subject to

$$F \in E_\tau \tag{2.17b}$$

$$\tau_a \geq 0 \qquad a \in I \tag{2.17c}$$

$$\tau_a = 0 \qquad a \notin I \tag{2.17d}$$

Problem (2.17) is a bilevel program, similar to the case of the deterministic based toll design problem, in the sense that (2.17b) can be alternatively formulated as a mathematical program, even though in this case it is written as a fixed point problem involving a series of nonlinear equations as shown in (2.15). Again, this problem is very similar to the network design problem with stochastic equilibrium constraints.

2.3.1 Continuous Network Design Problem with Stochastic Equilibrium Constraints

If equation (2.11) is replaced with (2.15) and (2.16), then the network design problem (2.10) becomes a network design problem with stochastic route choice. This is still a bilevel problem, in the sense that the lower level stochastic equilibrium can be formulated as a mathematical programming problem.

Davis (1994) finds that if the continuous network design problem is formulated based on the logit-based stochastic user equilibrium (SUE), the constraints will become differentiable, and at the same time, the number of constraints will be manageable. Compared to the deterministic network design problem, which has nonconvex and nondifferentiable constraints, this problem should be much easier to solve.

Let $\overline{P}_{wp}$ denote the probability of route $p \in P_w$ between O-D pair $w \in W$ being chosen by users, and $\overline{Q}_{wa}$ denote the probability that a trip between O-D pair $w \in W$ uses link $a \in A$. Also, let δ_{wpa} denote the indicator of O-D, route, and link relationship, $\delta_{wpa} = 1$ when link $a \in A$ lies on route $p \in P_w$ between O-D pair $w \in W$, and $\delta_{wpa} = 0$ otherwise. Then in the multinomial Logit-based stochastic user equilibrium, the probability for a route $p \in P_w$ connecting O-D pair $w \in W$ being chosen under the link flow pattern f and investment pattern y, $\overline{P}_{wp}(f,y)$, is given by

$$\overline{P}_{wp}(f,y) = \frac{e^{-\sum_{a \in p} c_a(f,y)}}{\sum_{i \in P_w} e^{-\sum_{a \in i} c_a(f,y)}}. \tag{2.18}$$

Then the probability that a trip between O-D pair $w \in W$ uses link $a \in A$, $\overline{Q}_{wa}(f,y)$, can be expressed as

$$\overline{Q}_{wa}(f,y) = \sum_{p \in P_w} \delta_{wpa} \overline{P}_{wp}(f,y). \tag{2.19}$$

Under the above conditions, Daganzo (1982) shows that the stochastic user equilibrium link flows, $f_a, a \in A$, are characterized as the solution to

$$f_a - \sum_{w \in W} D_w \overline{Q}_{wa}(f, y) = 0 \qquad a \in A, \tag{2.20}$$

which is a set of nonlinear equations.

Hence Davis (1994) concludes that the bilevel network design problem can be converted to (2.21), which is a standard nonlinear programming problem with a nonlinear objective function and nonlinear constraints (both equalities and inequalities). Particularly, the number of the nonlinear equality constraints of problem (2.21) equals to the number of links of the network, while that of the problem (2.10) with constraints (2.15) and (2.16) equals to the number of routes of the network. Since it is commonly acknowledged that the number of routes would be much larger than the number of links in a real network, it is believed that problem (2.21) should be easier to solve than problem (2.10) with the fixed point constraints (2.15) and (2.16).

$$\min_{y,f} \; c(f)^T f \tag{2.21a}$$

subject to

$$f_a - \sum_{w \in W} D_w \overline{Q}_{wa}(f, y) = 0 \qquad a \in A \tag{2.21b}$$

$$h^T y \leq B \tag{2.21c}$$

$$f \geq 0 \tag{2.21d}$$

$$y \geq 0 \tag{2.21e}$$

Note that (2.21) can actually be considered as a single level problem, with a more manageable number of nonlinear equality constraints. This single level problem can be solved relatively easier than the bilevel formulation.

2.3.2 Algorithms for Stochastic Network Design Problems

In order to solve the above network design problem with stochastic equilibrium constraint, a procedure for calculating the derivatives is given by Davis (1994). It involves applying the Dial's algorithm, in which the very time consuming process for computing route choice probabilities is eliminated. By using the gradient information, two algorithms for solving the constrained nonlinear programming problem, the generalized reduced gradient method and the sequential quadratic programming method, are tested on several networks with various sizes, which shows the SUE-constrained version of the network design problem can be solved within an acceptable time frame.

2.4 Shortcomings of the Existing Research

All the above work, both the toll design problem and the continuous network design problem with either deterministic or stochastic equilibrium constraints, consider all the road users as one group with identical characteristics. This is certainly unrealistic because road users are indeed different (for example, cars and trucks have different acceleration features, and their effects on each other are also different). Hence, in order to design a politically viable congestion toll scheme, the different features of road users should be taken into consideration. It is common practice in road pricing that trucks are charged more than cars. Therefore, it is more realistic to accommodate various user groups into the toll design models.

Unlike the stochastic based network design problem, the deterministic based toll design problem (or the continuous network design problem) has been formulated as a bilevel program, which is very difficult to solve because of the nondifferentiability and nonconvexity of the equilibrium constraint. Solving this problem would involve the iterative solution of both the upper level and the lower level optimization problems. This could involve extremely intensive computation for real-sized networks.

In the next chapter, the toll design problem will be formulated to deal with multiple user groups, based on both deterministic and stochastic route choices. In addition, both formulations will be converted into single level programs, which can be solved using standard nonlinear programming algorithms. This conversion, particularly for the deterministic based toll design problem, which is on the basis of a simplifying assumption, will greatly reduce the computation difficulty for solving these problems.

CHAPTER 3

TOLL DESIGN PROBLEM WITH DETERMINISTIC ROUTE CHOICE

3.1 Introduction

In this chapter, the toll design problem with multiple user groups assuming deterministic route choice will be discussed. In this case, the toll design problem will first be formulated as a bilevel program. The BPR function is used to represent the link travel cost. Hence, the bilevel toll design problem has a nonlinear objective function and nonconvex constraint, which is the solution set of equilibrium traffic assignment. Because of the difficulty in solving the toll design problem under bilevel formulation, it is later converted into a single level problem, under certain conditions. This simplified problem is a standard nonlinear programming problem, thus can be solved relatively easily. In addition, the single level toll design problem can be further simplified by using a linear cost function instead of the nonlinear BPR function.

3.2 Assumptions

The toll design problem with multiple user groups is defined as: given the characteristics of a network, the set of links that can be tolled, the travel demand for each user group between each O-D pair, and the link performance functions, find the optimal link toll pattern of the network that would induce the lowest total travel cost (*exclusive of tolls*). Similar to that with single user group discussed by Chen et al. (forthcoming), the toll design problem with multiple user groups has equilibrium constraints. In this case, a user chooses his/her route based on the route choices of other users in the same group as well as users of different groups. The equilibrium with multiple user groups can be defined as follows: at equilibrium, no user from any group will be able to further decrease his/her travel cost (*inclusive of tolls*) by unilaterally changing routes. The following general assumptions are made in the toll design problem:

(1) The set of links in the network that can be tolled is known;

(2) Travel demand for each user group between each origin-destination pair is fixed and known;

(3) Travelers have multiple route choices available to them; and

(4) All route travel times and costs are additive.

Particularly for the toll design problem based on deterministic equilibrium, the underlying assumption is that road users choose the routes that minimize their individual cost of traveling (*inclusive of tolls*). It is also assumed that travelers have perfect information on travel times and costs on all alternative routes.

3.3 Bilevel Toll Design Problem with Deterministic Route Choice

The toll design problem with multiple user groups can be formulated as a bilevel programming problem, similar to the toll design model for single user group presented by Tan (1997). Specifically, the toll design problem is formulated as a mathematical program with another optimization problem embedded in it as a constraint.

Consider a transportation network comprised of a finite set of links, A, (with cardinality $|A|$) and nodes, N (with cardinality $|N|$). Let W (with cardinality $|W|$) denote the set of O-D pairs, P_w denote the set of routes that connecting O-D pair $w \in W$, then $P = \bigcup_{w \in W} P_w$ denotes the complete set of routes in the network. Let U denote the set of user groups on the network, and then D_{uw} denote the travel demand of type u users between O-D pair $w \in W$. The relationship between routes and links is described by an indicator function, δ_{ap}. That is, $\delta_{ap} = 1$ if link a is in route p and $\delta_{ap} = 0$ otherwise. Further, let f denote a vector of link flows, thus $f = (f_{ua} : u \in U, a \in A)$, in which f_{ua} denotes the flow of type u user on link A. Let F denote a vector of route flows, thus $F = (F_{up} : u \in U, p \in P)$, in which F_{up} represents the flow of type u user on route p, and let $F_u = (F_{up} : p \in P)$ denote the vector of route flows of users from group $u \in U$. Hence the set of feasible route flow patterns, Ω, can be given by

$$\Omega = \left\{ F \in \Re_+ : \sum_{p \in P_w} F_{up} = D_{uw}, u \in U, w \in W \right\}, \tag{3.1}$$

and the set of feasible link flow patterns, ω, can be given by

$$\omega = \left\{ f \in \Re_+ : f = \Delta^T F, \; F \in \Omega \right\}. \tag{3.2}$$

Associated with each link and route is a travel time function and a travel cost function for each user group. The vector of link travel times is denoted by $t = (t_{ua} : u \in U, a \in A)$, the vector of link travel costs (*exclusive of tolls*) is denoted by $c = (c_{ua} : u \in U, a \in A)$, and the vector of link tolls is denoted by $\tau = (\tau_{ua} : u \in U, a \in A)$. Similarly, the vector of route travel times is denoted by $T = (T_{up} : u \in U, p \in P)$, the vector of route travel costs (*exclusive of tolls*) is denoted by $C = (C_{up} : u \in U, p \in P)$, the vector of route travel cost (*exclusive of tolls*) of users from group $u \in U$ is denoted by $C_u = (C_{up} : p \in P)$, the vector of route tolls is denoted by $\Psi = (\Psi_{up} : u \in U, p \in P)$, and the vector of route tolls on users from group $u \in U$ is denoted by $\Psi_u = (\Psi_{up} : p \in P)$. It is assumed that all route travel times and costs are additive, i.e., $T_{up} = \sum_{a \in p} t_{ua}$, $C_{up} = \sum_{a \in p} c_{ua}$, and $\Psi_{up} = \sum_{a \in p} \tau_{ua}$ for all $u \in U$.

Then the set of equilibrium route flows is given by

$$E_\tau = \left\{ F^* \in \Omega : [C_u(F^*) + \Psi_u]^T (F_u - F_u^*) \geq 0, \;\; F \in \Omega \right\} \tag{3.3}$$

which is simply the set of solutions to the variational inequality formulation of the equilibrium problem. So, the set the equilibrium link flows can be given by

$$e_\tau = \left\{ f^* \in \omega : \; f^* = \Delta^T F, \; F \in E_\tau \right\}. \tag{3.4}$$

Hence, a route-based bilevel toll design problem can be formulated as following:

$$\min_{\tau, F} \quad \sum_{u \in U} C_u(F)^T \cdot F_u \tag{3.5a}$$

subject to

$$F \in E_\tau \tag{3.5b}$$

$$\tau_{ua} \geq 0 \qquad u \in U, a \in I \tag{3.5c}$$

$$\tau_{ua} = 0 \qquad u \in U, a \notin I \tag{3.5d}$$

where $I \subseteq A$ denotes the set of links that can be tolled.

In this problem, there are basically two classes of decision-makers, the transportation planner chooses the toll that minimizes the total travel cost (*exclusive of tolls*), while each individual driver chooses the route that minimizes his or her travel cost (*inclusive of tolls*), given the behavior of other drivers. The upper level of this bilevel toll design problem can be viewed as the “transportation planner’s problem”, in which the transportation planner tries to find a toll pattern that minimizes the total travel cost. It has a nonlinear objective function, which is the total travel cost. When the BPR function is used to describe the link travel cost, the objective function is then differentiable while its convexity would depend on the parameters chosen in the BPR function. The lower level of the toll design problem can be viewed as the "road users' problem", in which each road user tries to minimize his or her travel cost under the influence of congestion tolls. There is no equivalent mathematical optimization problem for the equilibrium with multiple user groups. The diagonalization method has to be used to solve the lower level problem because of the asymmetric interaction between the cost functions of various user groups. This results in solving a series of diagonalized subproblems with nonlinear objective functions that is the total individual travel time, and linear constraints that are the flow conservation constraint and nonnegativity constraints.

By incorporating multiple user groups into the toll design problem, this model is capable of dealing with the characteristics of various user groups. For example, in terms of the travel time, charging group-varying tolls would help modeling the different operating features of different types of vehicles. In such cases, road users are divided into groups like cars and trucks. In terms of the travel cost, charging congestion tolls would result in different route choice behaviors among different income groups. It is likely that low-income travelers tend to switch to toll-free alternatives or change the travel schedule if they could, while high-income travelers may be more willing to pay for a less congested trip. The effect of congestion tolls on the route choice behavior can be evaluated through grouping road users according to their income levels. This method is crucial for the study of the effects of congestion tolls on various income groups, and for the design of a convincing revenue distribution package, which could ease the arguments on the social equity consequences of the congestion pricing plan.

Solving this deterministic based toll design problem with multiple user groups is very difficult because of the nondifferentiability and nonconvexity of the equilibrium constraint. Moreover, the solution procedure of the lower level problem is further complicated by the asymmetric interactions among multiple user groups.

3.4 Single Level Toll Design Problem with Deterministic Route Choice

In order to simplify the toll design problem, we consider a slightly different, but closely related formulation of the equilibrium subproblem. In particular, we will assume that, based on the pre-toll equilibrium, a set of "reasonable" routes that will be used after the tolls are put in place can be found. There are many different ways to define "reasonable" routes in this context. Perhaps the simplest definition is

$$R = \{p \in P : F_p^* \geq 0\} \tag{3.6}$$

Another definition may be

$$R_w = \left\{p \in P_w : C_p(f) = \min_{q \in P_w} C_q(f)\right\} \qquad w \in W \tag{3.7}$$

and

$$R = \bigcup_{w \in W} R_w . \tag{3.8}$$

Alternatively, one could define the "reasonable" route set as either the set of minimum-cost routes associated with the most probable route flow pattern (Larsson et al. 1998) that is consistent with f^*.

Similarly, one could also define R as the largest (in terms of its cardinality) set of used or minimum-cost routes that in consistent with f^*.

In any event, we assume that there is some way, given the pre-toll equilibrium, to determine the set of routes that will be used after the tolls are put in place. While this is a somewhat strong assumption from a theoretical standpoint, we do not think that it is at all unreasonable in practice. Indeed, in practice it seems unlikely that policy-makers would be interested in charging tolls that would change the set used routes dramatically. Instead, it seems like they would be most interested in tolls that would shift people from some congested routes to other routes.

With the above assumption, the equilibrium constraint in the toll design problem can be dramatically simplified. In particular, observe that a route flow pattern, F, is now an equilibrium if and only if

$$C_{up}(F)+\Psi_{up}=C_{uq}(F)+\Psi_{uq} \qquad u\in U, w\in W, p,q\in R_w. \tag{3.9}$$

That is, F is an equilibrium if and only if the costs on all routes connecting a particular O-D pair are equal. Hence, the toll design problem can be written as follows.

$$\min_{\tau,F} \quad \sum_{u\in U}\left(\sum_{w\in W,p\in R_w} C_{up}(F)^T\cdot F_{up}\right) \tag{3.10a}$$

subject to

$$C_{up}(F)+\Psi_{up}=C_{uq}(F)+\Psi_{uq} \qquad u\in U, w\in W, p,q\in R_w \tag{3.10b}$$

$$\sum_{p\in R_w} F_{up}=D_{uw} \qquad u\in U, w\in W \tag{3.10c}$$

$$F\geq 0 \tag{3.10d}$$

$$\tau_{ua} \geq 0 \qquad u \in U, a \in I \tag{3.10e}$$

$$\tau_{ua} = 0 \qquad u \in U, a \notin I \tag{3.10f}$$

Note that this is a single level problem. In fact, when C is linear, this is a convex quadratic minimization problem with linear equality and nonnegativity constraints. When the BPR function is used to represent the link travel cost, this problem is a nonlinear optimization problem with nonlinear and linear equality constraints, as well as nonnegativity constraints. Particularly, both the objective function and the constraints are differentiable. In such cases, this problem can be solved using a variety of algorithms for solving standard nonlinear programming problems. However, constraint (3.10b) is likely nonconvex because it is the difference between the two convex functions, $C_{up}(F)+\Psi_{up}$ and $C_{uq}(F)+\Psi_{uq}$ (e.g., when the BPR function is used to represent the link travel cost). Therefore, the single level toll design problem is usually nonconvex. For this kind of problem, a global optimal solution is not guaranteed.

Nevertheless, if we modify the formulation (3.10b) somewhat, as shown in (3.11), the solution of this problem can be facilitated. Specifically, if Θ_{uw} denotes the vector of total route cost (*inclusive of tolls*), i.e.,

$$\Theta_{uw} = C_{up}(F) + \Psi_{up} \qquad u \in U, w \in W, p \in R_w \tag{3.11}$$

then the toll design problem (3.10) can be written as

$$\min_{\tau,F,\Theta} \quad \sum_{u\in U}\left(\sum_{w\in W,p\in R_w} (\Theta_{uw} - \Psi_{up})^T \cdot F_{up} \right) \tag{3.12a}$$

subject to

$$C_{up}(F)+\Psi_{up}=\Theta_{uw} \qquad u\in U, w\in W, p\in R_w \tag{3.12b}$$

$$\sum_{p\in R_w} F_{up}=D_{uw} \qquad u\in U, w\in W \tag{3.12c}$$

$$F\geq 0 \tag{3.12d}$$

$$\tau_{ua}\geq 0 \qquad u\in U, a\in I \tag{3.12e}$$

$$\tau_{ua}=0 \qquad u\in U, a\notin I \tag{3.12f}$$

$$\Theta\geq 0 \tag{3.12g}$$

Note that Θ_{uw} with cardinality $|U\times W|$ is added into the problem as a decision variable, which is independent of all other variables. Since R_w is defined as the set of used routes between O-D pair $w\in W$, Θ_{uw} turns out to be the equilibrium route cost for each O-D pair, which is the minimum possible route cost under the equilibrium flow.

The objective function (3.12a) remains convex since Θ and Ψ are independent of each other. Constraint (3.12b) is still a nonlinear equality. However, if we relax this constraint to

$$C_{up}(F)+\Psi_{up}\leq\Theta_{uw} \qquad u\in U, w\in W, p\in R_w, \tag{3.13}$$

the toll design problem will look like the following:

$$\min_{\tau,F,\Theta} \quad \sum_{u\in U}\left(\sum_{w\in W, p\in R_w}(\Theta_{uw}-\Psi_{up})^T\cdot F_{up}\right) \tag{3.14a}$$

subject to

$$C_{up}(F)+\Psi_{up}\leq\Theta_{uw} \qquad u\in U, w\in W, p\in R_w \tag{3.14b}$$

$$\sum_{p\in R_w} F_{up} = D_{uw} \qquad u \in U, w \in W \tag{3.14c}$$

$$F_{up} \geq 0 \qquad u \in U, w \in W, p \in R_w \tag{3.14d}$$

$$\tau_{ua} \geq 0 \qquad u \in U, a \in I \tag{3.14e}$$

$$\tau_{ua} = 0 \qquad u \in U, a \notin I \tag{3.14f}$$

$$\Theta \geq 0 \tag{3.14g}$$

Even though they are both standard nonlinear programming problems, (3.14) may be easier to solve than (3.12), since most algorithms for nonlinear programming problems require a feasible solution to start with, and it is rather easy to find a feasible point that satisfies inequality constraint (3.14b) instead of equality constraint (3.12b). Furthermore, many algorithms require that this starting point lie strictly inside rather than on the boundary of the feasible region. Here, we prove that problem (3.14) and problem (3.12) have the same solution set. Note that besides the aforementioned two constraints, all other constraints are the same for the two problems. Therefore, it is just necessary to compare constraints (3.14b) and (3.12b).

Theorem Problem (3.14) and problem (3.12) have the same solution set.

Proof

Any solution (F^*, τ^*, Θ^*) of problem (3.12) that satisfies constraint (3.12b) also satisfies constraint (3.14b) since (3.12b) is a special case of (3.14b). Thus, any solution to problem (3.12) must be a solution of (3.14).

On the other hand, if there exists an optimal solution (F^*, τ^*, Θ^*) to the problem (3.14), which only satisfies

$$C_{up}(F^*) + \Psi^*_{up} < \Theta^*_{uw} \qquad u \in U, w \in W, p \in R_w \tag{3.15}$$

and at the same time satisfies all other constraints of (3.14), then

$$\Theta^*_{uw} - \Psi^*_{up} > C_{up}(F^*) \qquad u \in U, w \in W, p \in R_w \tag{3.16}$$

and then

$$\sum_{u \in U}\left(\sum_{w \in W, p \in R_w} (\Theta^*_{uw} - \Psi^*_{up}) \cdot F^*_{up} \right) > \sum_{u \in U}\left(\sum_{w \in W, p \in R_w} C_{up}(F^*) \cdot F^*_{up} \right) \tag{3.17}$$

The right-hand side of (3.17) is equivalent to the original objective of the toll design problem (3.10). Hence, (3.17) shows that the solution (F^*, τ^*, Θ^*) results in a larger objective value than the original minimization formulation (3.10). Thus the objective function can be further minimized, which contradicts the assumption that (F^*, τ^*, Θ^*) is an optimal solution to the problem (3.14). Therefore, such a solution does not exist. In other words, all optimal solutions of problem (3.14) have to satisfy (3.12b), which means that they are also solutions to the problem (3.12).

Hence we can conclude that problems (3.14) and (3.12) have the same solution set, and we can solve (3.14) instead of (3.12). ■

Note that we can not relax (3.12b) to

$$C_{up}(F) + \Psi_{up} \geq \Theta_{uw} \qquad u \in U, w \in W, p \in R_w \tag{3.18}$$

since if we do, there would be no lower bound for $\Theta_u - \Psi_u$ and thus the minimization problem would have no lower bound and would reach infinity.

Since (3.12) is equivalent to (3.10) and (3.14), it follows that (3.14) and (3.10) are also equivalent. However, the numbers of decision variables in these two problems are different. For (3.10) the total number of decision variables is $\left|U \times \bigcup_{w \in W} R_w\right| + |U \times A|$, while for (3.14) this number is $\left|U \times \bigcup_{w \in W} R_w\right| + |U \times A| + |U \times W|$ because of the addition of decision variable Θ.

3.5 Equivalence of the Two Deterministic Toll Design Problems

The single level toll design problem with deterministic route choice is based on a revised formulation of the deterministic user equilibrium, in which we assume that we are able to find the route set which will be used after placing the toll. In this section, the relationship between this single level problem and the bilevel toll design problem is discussed.

The toll design problem has an equilibrium constraint that can be formulated as a variational inequality, as shown in equation (3.3). Alternatively, the equilibrium constraint can also be formulated as a complementarity problem as follows, where, as discussed earlier, Θ_{uw} denotes the equilibrium (minimum) travel cost of users from group $u \in U$ on those used routes between O-D pair $w \in W$ at equilibrium.

$$F_{up}[C_{up}(F) + \Psi_{up} - \Theta_{uw}] = 0 \qquad u \in U, w \in W, p \in P_w \qquad (3.19a)$$

$$C_{up}(F) + \Psi_{up} - \Theta_{uw} \geq 0 \qquad u \in U, w \in W, p \in P_w \qquad (3.19b)$$

$$\sum_{p \in P_w} F_{up} = D_{uw} \qquad u \in U, w \in W \qquad (3.19c)$$

In addition, all nonnegativity constraints should be satisfied.

Looking at the bilevel toll design problem (3.5) and the single level toll design problem (3.10), one can see that they have the same objective function but different equilibrium constraints. Therefore, it is sufficient to compare their feasible regions only.

The feasible region of (3.5) consists of the solutions of the user equilibrium problem. These solutions should satisfy the complementarity condition specified in equation system (3.19).

Lemma 1 For any route flow vector $F = (F_{up} : u \in U, w \in W, p \in P_w)$ that belongs to the feasible region of (3.5), it belongs to the feasible region of (3.10).

Proof

Let R_w denote the set of used routes connecting O-D pair $w \in W$, then for any $F = (F_{up} : u \in U, w \in W, p \in P_w)$ in the feasible region of (3.5), the complementarity condition (3.19) should be satisfied. According to the definition of the deterministic equilibrium, the complementarity condition (3.19) can be expanded as system (3.20) as follows.

$$C_{up}(F) + \Psi_{up} - \Theta_{uw} = 0 \qquad u \in U, w \in W, p \in R_w \qquad (3.20a)$$

$$C_{up}(F) + \Psi_{up} - \Theta_{uw} > 0 \qquad u \in U, w \in W, p \in P_w - R_w \qquad (3.20b)$$

$$F_{up} > 0 \qquad u \in U, w \in W, p \in R_w \qquad (3.20c)$$

$$F_{up} = 0 \qquad u \in U, w \in W, p \in P_w - R_w \qquad (3.20d)$$

$$\sum_{p \in P_w} F_{up} = D_{uw} \qquad u \in U, w \in W \qquad (3.20e)$$

Equation (3.20a) is equivalent to (3.10b) if variable Θ is disregarded, and equations (3.20d) and (3.20e) imply (3.10c). Equations (3.20c) is contained in (3.10d). Since (3.10b), (3.10c) and (3.10d) constitute the feasible region for flows of problem (3.10), we can conclude that for any F that belongs to the feasible region of (3.5), it also belongs to the feasible region of (3.10). ■

Lemma 2 For any route flow vector $F=(F_{up}:u\in U, w\in W, p\in P_w)$ that belongs to the feasible region of (3.10), it belongs to the feasible region of (3.5).

Proof

For any $F=(F_{up}:u\in U, w\in W, p\in P_w)$ in the feasible region of (3.10), the following equations hold.

$$F_{up}\geq 0 \qquad u\in U, w\in W, p\in R_w \tag{3.24}$$

$$F_{up}=0 \qquad u\in U, w\in W, p\in P_w-R_w \tag{3.25}$$

$$C_{up}(F)+\Psi_{up}=C_{uq}(F)+\Psi_{uq} \qquad u\in U, w\in W, p,q\in R_w \tag{3.26}$$

$$\sum_{p\in R_w}F_{up}=D_{uw} \qquad u\in U, w\in W \tag{3.27}$$

Let

$$C_{up}(F)+\Psi_{up}=\Theta_{uw} \qquad u\in U, w\in W, p\in R_w, \tag{3.28}$$

equation (3.20a) as well as the following hold.

$$F_{up}[C_{up}(F)+\Psi_{up}-\Theta_{uw}]=0 \qquad u\in U, w\in W, p\in R_w. \tag{3.29}$$

Equation (3.25) also guarantees

$$F_{up}[C_{up}(F)+\Psi_{up}-\Theta_{uw}]=0 \qquad u\in U, w\in W, p\in P_w-R_w. \tag{3.30}$$

Now we compare the $C_{up}(F)+\Psi_{up}$ and Θ_{uw} when $w\in W, p\in P_w-R_w$. If

$$C_{up}(F)+\Psi_{up}<\Theta_{uw} \qquad u\in U, w\in W, p\in P_w-R_w, \tag{3.31}$$

route p would have a lower cost than those used routes. Based on the equilibrium principles, the following holds.

$$F_p > 0 \qquad u \in U, w \in W, p \in P_w - R_w, \qquad (3.32)$$

which is contradictory to equation (3.25). Thus, (3.31) is false and equation (3.33) holds.

$$C_{up}(F) + \Psi_{up} \geq \Theta_{uw} \qquad u \in U, w \in W, p \in P_w - R_w \qquad (3.33)$$

Equations (3.20a) and (3.33) can be combined into (3.19b), equations (3.29) and (3.30) can be combined into (3.19a), and equations (3.25) and (3.27) imply (3.19c). At the same time, all nonnegativity constraints are satisfied. Since equation system (3.19) is the complementary formulation of the equilibrium constraint of (3.5), any F which is from the feasible region of (3.10) should be also in the feasible region of (3.5). ■

Based on Lemma 1 and Lemma 2, we can conclude that the single level toll design problem (3.10) is equivalent to the bilevel toll design problem (3.5).

3.6 The Special Case of Linear Cost Functions

Usually, the BPR function is used to represent the dependence of the link travel cost on the link flow. Because this function is nonlinear, the constraint (3.12b) of the single level formulation of the toll design problem with deterministic route choice is a nonlinear equality. This causes the feasible region of the problem (3.12) remains to be nonconvex. Even though it can be relaxed to problem (3.14), this relaxation would only help in locating an internal starting point. Therefore, the simplified toll design problem (3.12) is still rather difficult to solve.

The nonconvexity of the feasible region is a major factor of preventing the algorithm from converging to the global optimum. Even though one can use various starting points and get different solutions, so as to obtain the best one, it is still unlikely to be the global optimum. Alternatively, one can also use the iterative linearization method to convert the toll design problem into a series of linearly constrained optimization problems, as will be discussed in the next chapter. However, the convergence of this method depends on the convexity of every constraint function. For the toll design problem with multiple user groups, this condition is not met except for problem (3.14). Even though sometimes this algorithm would converge when the constraint convexity condition is not met, there is no guarantee that this solution is even a local optimum.

Hence, it can be generally concluded that the nonlinear link cost function brings about the major difficulty in solving the deterministic based toll design problem. However, if a linear link cost function is used, the complexity of the single level, deterministic based toll design problem will be largely reduced. In particular, if the link cost function is in the linear format of $c_a(f) = \alpha_a + \beta_a f_a$, $a \in A$, then constraint functions of (3.10b), (3.12b), as well as (3.14b) will become linear. Moreover, because of the independence among decision variables, these functions should be convex also. Therefore, the feasible regions of toll design problems (3.10), (3.12), and (3.14) become convex. Furthermore, the objective function (3.10a) becomes quadratic. According to the characteristics of the link cost function (i.e., the travel cost increases while the link flow increases), this quadratic objective function will certainly be convex. Therefore, the single level toll design problem could be converted to a standard quadratic programming problem, which is a rather easy problem to solve. The following example shows how simple the deterministic toll design problem could become if the linear cost function is used. In this example, the simple two-link network shown in Figure 3-1 is considered. In this particular case, each route between the origin and destination contains only one link. Therefore, $F_1 = f_1$, and $F_2 = f_2$, so link flow variables can be replaced with route flow variables in the formulation. The link cost function is in the linear format, as specified in the figure, and the total demand is 5.

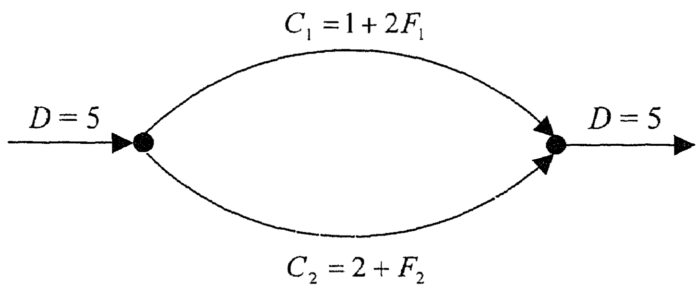

Figure 3-1 Two-Link Network

The user equilibrium solution to this problem is $F_1 = 2$ and $F_2 = 3$ with total travel time 25, while the system optimum solution is $F_1 = 1.83$ and $F_2 = 3.17$ with total travel time 24.92. From the no-toll equilibrium, one can see that both routes are used in the equilibrium, and therefore they will also be used after tolls are in place. If only Link 1 can be tolled, i.e., $I = \{1\}$, the toll design problem can be formulated as follows. For simplicity, only the single user group case is considered.

$$\min_{\tau, F} \quad (1 + 2F_1) \cdot F_1 + (F_2 + 2) \cdot F_2$$

subject to

$$1 + 2F_1 + \tau_1 = F_2 + 2 + \tau_2$$

$$F_1 + F_2 = 5$$

$$F_1, F_2 \geq 0$$

$$\tau_1 \geq 0$$

$$\tau_2 = 0$$

This problem has linear constraints and quadratic objective function, and can be solved analytically. The result is that $F_1 = 1.83$, $F_2 = 3.17$, and $\tau_1 = 0.5$. The objective value at the optimum is 24.92. Note that in this case, the solution is unique, and it is the global optimum. In addition, charging a toll valued 0.5 on Link 1 will yield the system optimal flow pattern on this network.

From the mathematical programming perspective, using linear cost function in the toll design problem would greatly simplify the solution process. Furthermore, this guarantees global optimum. One method to obtain linear cost function is through the linearization of the original nonlinear function, such as the BPR-type cost function. When solving the toll design problem, one can use a first-order approximation of the nonlinear function at the equilibrium solution as the linear cost function. It should be noted that this linearization might cause odd solutions to be obtained, such as negative flows, when minimizing the total cost. For example, in Figure 3-2, at the user

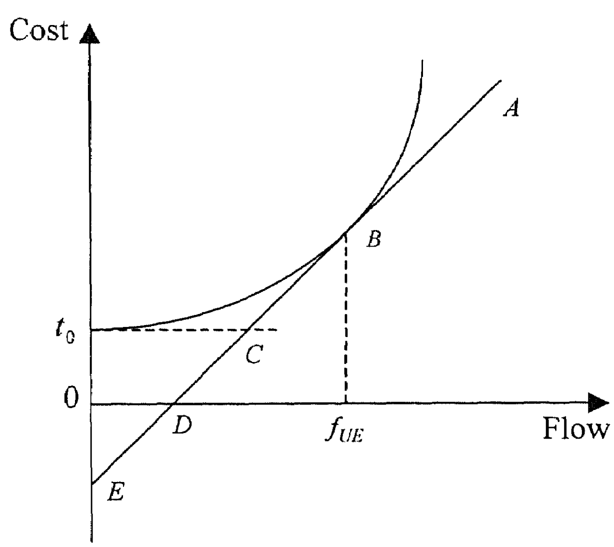

Figure 3-2 Problem with Linear Cost Function

equilibrium solution point f_{UE}, one can take the first-order approximation of the nonlinear cost function and then obtain the linear function line, indicated in Figure 3-2 as line $ABCDE$. Within the feasible region of nonnegative flows, negative cost (from D to E, for example) may arise, which is obviously an unrealistic representation of practice.

Considering that there is always a lower bound of the travel cost on the link, defined as the free-flow travel cost (denoted by t_0 in Figure 3-2), which can be found at the intersection of the nonlinear cost function with the cost axis, one should only use the portion ABC of this linear function to represent the link cost. In addition, piecewise linear approximation is recommended, if possible, in order to provide a more realistic representation of link travel cost.

CHAPTER 4

TOLL DESIGN PROBLEM WITH STOCHASTIC ROUTE CHOICE

4.1 Introduction

The deterministic route choice model is based on the assumptions that all travelers have perfect information regarding travel cost on every route of the network and that they always choose the route with least cost. However, this deterministic route choice rule overlooks the fact that some other factors may affect travelers' route choice behavior. Since there are so many factors that would affect travelers' decision, it can be assumed that there is a random component in travelers' route choice behavior. In this chapter, we consider toll design problems with logit route choice, and discuss the relationship between the deterministic and the logit toll design problem.

4.2 Bilevel Toll Design Problem with Logit Stochastic Route Choice

The stochastic route choice model is based on discrete choice models, which usually assume that an individual's preferences toward each alternative is described by a "utility" measure associated with it, and the decision-maker will always choose the alternative that yields the highest utility. However, this utility function is a random variable since there are many factors that would affect this function, so it can not be modeled precisely. If the random terms of each utility function are independent, identical and Gumbel distributed variables, the choice probability can be given by the logit model. If Γ denotes the set of alternatives to be chosen, μ denotes a positive scaling factor, and V denotes the vector of deterministic parts of utility functions, then the probability of alternative $p \in \Gamma$ being chosen should equal to $\frac{e^{\mu V_p}}{\sum_{i \in \Gamma} e^{\mu V_i}}$. In transportation-related applications, the negative route travel cost (*inclusive of tolls*) C is usually thought of as the deterministic component of the utility function. For example, in the case of binary choice, this probability equals to $\frac{e^{-\mu C_p}}{e^{-\mu C_p} + e^{-\mu C_q}}$ and can be shown in Figure 4-1. For the purpose of simplicity, we assume that in a later discussion, $\mu = 1$. Ben-Akiva and Lerman (1985) provide a detailed derivation for the most widely used multinomial logit model.

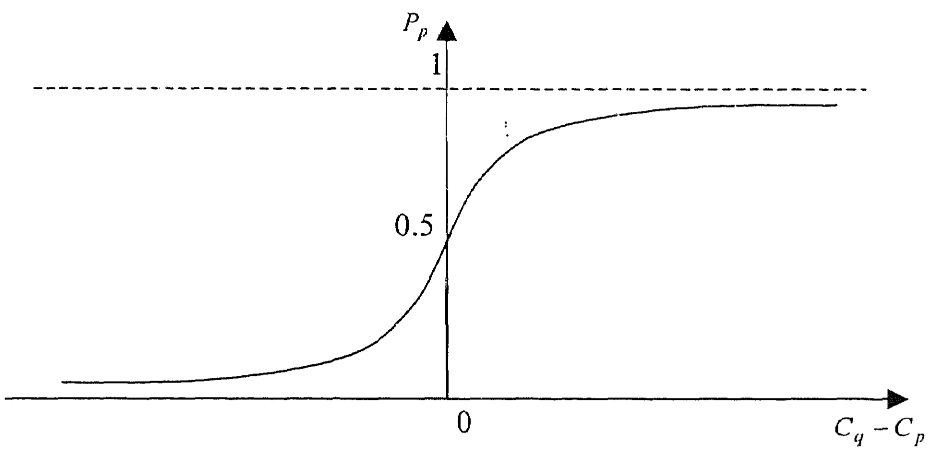

Figure 4-1 Binary Logit Model

Considering the stochastic equilibrium under tolls as an optimization problem, the toll design problem can be intuitively formulated as a bilevel program. However, for the stochastic equilibrium problem with multiple user groups, there is no equivalent mathematical program. Therefore, the stochastic equilibrium can be formulated as a fixed-point problem, which is to determine the equilibrium route flows $F \in E_\tau$, where E_τ, the set of equilibrium route flows, is given by

$$E_\tau = \left\{ F \in \Re_+ : F_{up} = D_{uw} \cdot \frac{e^{-\mu[C_{up}(F)+\Psi_{up}]}}{\sum_{i \in P_w} e^{-\mu[C_{ui}(F)+\Psi_{ui}]}}, u \in U, w \in W, p \in P_w \right\}. \tag{4.1}$$

It should be noted that this formulation implicitly contains the flow conservation constraints. Furthermore, it should also be understood that the equilibrium route flow pattern determined from (4.1) is unique, which is different from the deterministic equilibrium. Therefore, the resulting equilibrium link flow is also unique.

The bilevel stochastic toll design problem with multiple user groups can thus be formulated as follows.

$$\min_{\tau, F} \sum_{u \in U} C_u(F)^T \cdot F_u \tag{4.2a}$$

subject to

$$F \in E_\tau \tag{4.2b}$$

$$\tau_{ua} \geq 0 \qquad u \in U, a \in I \tag{4.2c}$$

$$\tau_{ua} = 0 \qquad u \in U, a \notin I \tag{4.2d}$$

The upper level problem, viewed as a "transportation planner's problem", describes the planner's objective to find an optimal toll pattern that minimizes the total travel cost. When the BPR function is used to describe the link travel cost, the objective function is differentiable while its convexity depends on the parameters chosen in the BPR functions. The equilibrium constraint is in the format of a fixed point problem that is a set of nonlinear equalities. Thus the feasible region of the toll design problem is most likely nonconvex.

The lower level problem, viewed as a "road users' problem", is different from that in the deterministic toll design model. In the stochastic equilibrium, users make route choices according to the probability of each route being chosen. This probability is based on the relative magnitude of the cost associated with a route compared with other available routes connecting the same origin-destination pair.

Similar to the deterministic toll design problem, this bilevel stochastic toll design problem is very difficult to solve. One reason is that the toll vector is not included in the objective function, and the gradient of the objective function with respect to the toll vector can not even be explicitly obtained. Besides, the equilibrium constraint is most likely nonconvex. Moreover, the number of equilibrium constraints in problem (4.2) equals to the number of routes in the network, which is usually very large for real-sized networks.

4.3 Single Level Toll Design Problem with Logit Stochastic Route Choice

In order to simplify the solution process, the method presented by Davis (1994) is applied here to convert this bilevel stochastic toll design problem into a single level problem. This link-based procedure is introduced in the following.

Let $\overline{P}_{uwp}$ denote the probability of route $p \in P_w$ between O-D pair $w \in W$ being chosen by users from group $u \in U$, and $\overline{Q}_{uwa}$ denote the probability that a trip from user group $u \in U$ between O-D pair $w \in W$ uses link $a \in A$. Also, let δ_{wpa} denote the indicator of O-D, route, and link relationship, $\delta_{wpa} = 1$ when link $a \in A$ lies on route $p \in P_w$ between O-D pair $w \in W$, and $\delta_{wpa} = 0$ otherwise.

In the multinomial Logit-based stochastic user equilibrium, the probability for a route $p \in P_w$ connecting O-D pair $w \in W$ being chosen by a user from group $u \in U$, $\overline{P}_{uwp}(f,\tau)$, is given by

$$\overline{P}_{uwp}(f,\tau) = \frac{e^{-\mu \sum_{a \in p} c_{ua}(f)+\tau_{ua}}}{\sum_{i \in P_w} e^{-\mu \sum_{i \in p} c_{ui}(f)+\tau_{ui}}}. \tag{4.3}$$

Then the probability that a trip from user group $u \in U$ between O-D pair $w \in W$ uses link $a \in A$, $\overline{Q}_{uwa}(f,\tau)$, can be expressed as

$$\overline{Q}_{uwa}(f,\tau) = \sum_{p \in P_w} \delta_{wpa} \overline{P}_{uwp}(f,\tau). \tag{4.4}$$

Under the above conditions, the result in Daganzo (1982) should also apply. That is, the stochastic user equilibrium link flows, f_{ua}, $u \in U, a \in A$, can be characterized as solutions to the set of nonlinear equations

$$f_{ua} - \sum_{w \in W} D_{uw} \overline{Q}_{uwa}(f,\tau) = 0 \qquad u \in U, a \in A. \tag{4.5}$$

Using this result, the stochastic toll design problem with multiple user groups can be formulated as follows.

$$\min_{\tau,f} \sum_{u \in U} c_u(f)^T f_u \tag{4.5a}$$

subject to

$$f_{ua} - \sum_{w \in W} D_{uw} \overline{Q}_{uwa}(f,\tau) = 0 \qquad u \in U, a \in A \tag{4.5b}$$

$$f \geq 0 \tag{4.5c}$$

$$\tau_{ua} \geq 0 \qquad u \in U, a \in I \tag{4.5d}$$

$$\tau_{ua} = 0 \qquad u \in U, a \notin I \tag{4.5e}$$

This single level problem is actually in the standard nonlinear program format. It has a nonlinear objective function, and nonlinear and linear equality constraints, as well as nonnegativity constraints. Moreover, the number of the flow constraints in the system (4.5b) equals to the number of links of the network, which makes the number of constraints of this problem more manageable, since the number of routes are usually much more than the number of links in a network. This problem can be solved using various algorithms developed for constrained nonlinear programs. However, because of the nonconvexity of nonlinear equality constraints, the global optimal solution is still not guaranteed.

4.4 Relationship between Deterministic and Logit Toll Design Problems

The toll design problem involves two decision-making processes, as discussed earlier. The difference between the toll design problem based on deterministic route choice and stochastic route choice is in the lower level decision-making process. In the deterministic toll design problem, travelers make their route choices decision based on deterministic equilibrium rules, i.e., they always choose the route which minimizes their individual travel cost (*inclusive of tolls*). This route choice behavior results in the equilibrium condition that all the used routes have the same travel cost, which is the minimum travel cost between a particular O-D pair, while those unused routes have higher travel cost than the minimum cost. The existence of these unused routes is the main reason for the nondifferentiability of the equilibrium constraint in the bilevel deterministic toll design problem. Therefore, in order to simplify the deterministic toll design problem, we assume that we can find a set of used routes after the tolls are put in place, and equalize the travel costs on these routes. Then the toll design problem with deterministic route choice can be converted into a single level mathematical program.

However, it should be noted that this effort of finding used routes is only necessary in simplifying the deterministic based toll design problem. In the stochastic toll design problem, travelers choose their routes based on certain stochastic equilibrium rules. More specifically, each route connecting a particular O-D pair has associated with it a nonzero probability of being chosen. In another words, travelers between an O-D pair will assign themselves on these routes according to the probability associated with each of them. If the multinomial logit model is used to describe this stochastic route choice behavior, then

$$F_{up} = D_{uw} \cdot \frac{e^{-\mu[C_{up}(F)+\Psi_{up}]}}{\sum_{i \in P_w} e^{-\mu[C_{ui}(F)+\Psi_{ui}]}} \qquad u \in U, w \in W, p \in P_w \qquad (4.6)$$

for any $0 < \mu < \infty$. Therefore, under the circumstance of stochastic route choice, all routes connecting a particular O-D pair will be used. It is this feature of stochastic equilibrium that enables us to simplify the stochastic toll design problem.

In the above logit based model, the choice probability of a route can be expressed as a function of the difference between the costs of this route and all other alternatives. This parameter μ is a positive constant that scales the route travel cost. The route choice probability would have depended on the units of measurement of travel cost without proper scaling. In a logit model formulation like (4.6), the scaling parameter μ actually measures the sensitivity of users to the route travel cost. A small value of μ indicates that users are relatively insensitive to the cost, and thus the route choice probability tends to be very close among all alternatives. When μ approaches zero, the extreme case would arise, in which all routes connecting a particular O-D pair have the same probability of being chosen. At this moment, the cost is actually not a factor of the route choice, since users are completely indifferent to the cost. On the other hand, a large value of μ means the high sensitivity of users toward the cost, and thus the route choice probability is very closely related to the cost difference among all alternatives. When μ approaches infinity, the extreme case can be reached, in which the users will all choose the route with minimum cost. At this moment, the stochastic route choice behavior actually converges to the deterministic route choice behavior. The following analysis proves this assertion.

Consider a particular O-D pair $w \in W$ connected by N routes. Ignore the factor of multiple user groups for now, since the following rule will be applicable to any number of user groups. Also, let's incorporate the tolls into the travel cost function (i.e., let $\Theta_p = C_p(F) + \Psi_p$). Using the multinomial Logit model, the probability of route $i \in P_w$ being chosen, $\overline{P}_i$, can be expressed as

$$\overline{P}_i = \frac{e^{-\mu\Theta_i}}{\sum_{j \in P_w} e^{-\mu\Theta_j}} = \frac{1}{1 + \sum_{j \in P_w - \{i\}} e^{-\mu(\Theta_j - \Theta_i)}} \tag{4.7}$$

When $\mu = 0$, no matter what values Θ_j and Θ_i would take, $e^{-\mu(\Theta_j - \Theta_i)} = e^0 = 1$ holds. Therefore,

$$\overline{P}_i = \frac{1}{1 + \sum_{j \in P_w - \{i\}} e^{-\mu(\Theta_j - \Theta_i)}} = \frac{1}{1 + \sum_{j=1}^{N-1} e^0} = \frac{1}{N}. \tag{4.8}$$

This implies that all routes connecting O-D pair $w \in W$ are equally likely to be chosen, and then the flows on all these routes should be the same.

On the other hand, two scenarios when $\mu = \infty$ can be considered. First, if route $i \in P_w$ is the minimum cost route between O-D pair $w \in W$, i.e.,

$$\Theta_i = \min\{\Theta_j : j \in P_w,\ j = 1, \dots, N\}, \tag{4.9}$$

then $\Theta_j \geq \Theta_i$ for all $j \in P_w$ and $j \neq i$. When $\Theta_j > \Theta_i$ for all $j \in P_w$ and $j \neq i$, $e^{-\mu(\Theta_j - \Theta_i)} = e^{-\infty} = 0$ holds for all $j \in P_w$ and $j \neq i$. Then

$$\overline{P}_i = \frac{1}{1 + \sum_{j=1}^{N-1} e^{-\infty}} = 1 \tag{4.10}$$

This implies that if there is only one route with minimum cost, then the probability of this route being chosen is 1, and hence all demand between O-D pair $w \in W$ will be traveling on this route and there will be no flows on other routes.

When there are K routes in the set $P_w - \{i\}$ with the minimum travel cost, $\Theta_k = \Theta_i$ ($k \in P_w$ and $k = 1, \dots, K$) holds. So, $e^{-\mu(\Theta_k - \Theta_i)} = e^{-\infty \cdot 0} = 1$, and

$$\overline{P}_i = \frac{1}{1 + \sum_{j=1}^{K} e^{-\infty \cdot 0}} = \frac{1}{1+K}, \tag{4.11}$$

which implies that only those minimum cost routes will carry flows and their flows will be equal.

Second, if route $i \in P_w$ is not the minimum cost route between O-D pair $w \in W$, i.e., there exists $j \in P_w$ and $j \neq i$ such that $\Theta_j < \Theta_i$. At this time, $e^{-\mu(\Theta_j - \Theta_i)} = e^{\infty} = \infty$, which leads to

$$\overline{P}_i = \frac{1}{1 + \sum_{j \in P_w - \{i\}} e^{-\mu(\Theta_j - \Theta_i)}} = \frac{1}{1+\infty} = 0, \tag{4.12}$$

which implies that the nonminimum-cost route will never be chosen when $\mu = \infty$.

Therefore, combining the above two cases together results in the description of the stochastic equilibrium under the circumstance of $\mu = \infty$, namely all used routes will have the same costs, and those unused routes will have costs higher than the minimum cost. Note that this is exactly the same description for the deterministic user equilibrium. Hence, it can be concluded that the toll design problem with deterministic route choice is actually a special case of that with stochastic route choice.

Realizing this relationship between the deterministic and stochastic equilibrium, one may conclude that the deterministic and stochastic toll design problems are very tightly related. The deterministic toll design problem is essentially a particular case of the stochastic toll design problem.

4.5 Special Treatment for Stochastic Based Toll Design Problem

From the formulation (4.6) of the stochastic equilibrium with logit route choice, one can see that the probability of a route being chosen is unique, given parameter μ. In other words, the route flow pattern of the logit equilibrium is unique, and so is the link flow pattern, which implies that the lower level equilibrium problem of the toll design problem has a single solution. This means that the feasible region of the stochastic based toll design problem is a single point, and there is no interior for this feasible region. Hence those algorithms that require a feasible interior starting point to initialize can not be applied directly to solve the stochastic toll design problem with logit route choice.

Alternatively, we consider enlarging the feasible region of this toll design problem. Specifically, the equality constraint (4.6) can be converted into the following inequality constraint, in which ε denotes a small perturbation.

$$(1-\varepsilon)\cdot D_{uw}\cdot\frac{e^{-\mu[C_{up}(F)+\Psi_{up}]}}{\sum_{i\in P_w} e^{-\mu[C_{ui}(F)+\Psi_{iu}]}} \le F_{up} \le (1+\varepsilon)\cdot D_{uw}\cdot\frac{e^{-\mu[C_{up}(F)+\Psi_{up}]}}{\sum_{i\in P_w} e^{-\mu[C_{ui}(F)+\Psi_{ui}]}} \qquad (4.13)$$
$$u\in U, w\in W, p\in P_w$$

This way, the feasible region of the toll design problem is enlarged after constraint (4.6) replaces constraint (4.13).

The value of ε can be selected to be extremely small, which is essentially not changing the optimality from the engineering perspective, while making the calculation feasible.

CHAPTER 5

NUMERICAL EXAMPLES

In previous chapters, toll design models based on both deterministic and stochastic route choices were developed. The issue of how different types of link cost functions affect the solution of the single level problem was also discussed. In this chapter, several small examples are presented to illustrate the behavior of the toll design problem.

5.1 Two-Link Network

The simplest network, shown in Figure 5-1, is used first. This two-link network has only one O-D pair with demand of 5, and the linear link cost function.

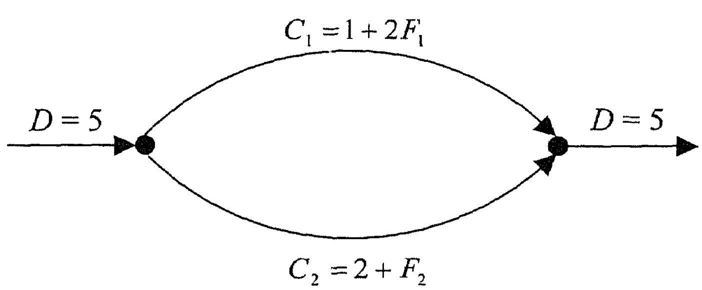

Figure 5-1 Two-Link Network

5.1.1 Deterministic Toll Design Problem

In Section 3.6, this example was discussed assuming that only Link 1 can be tolled. If both links are tollable there will be multiple solutions. For example, if the tolls are set to $\tau_1 = 1$ and $\tau_2 = 0.5$, then at equilibrium, the following has to be satisfied.

$$1 + 2F_1 + 1 = F_2 + 2 + 0.5$$

$$F_1 + F_2 = 5$$

The solution to these equations yields $F_1 = 1.83$, $F_2 = 3.17$. The results for system optimum and user equilibrium are listed in Table 5-1. Similarly, other combinations of link tolls on these two links, such as $\tau_1 = 2$, $\tau_2 = 1.5$, would generate the same flow pattern.

Table 5-1 Two-Link Deterministic Toll Design

Link	System Optimum Flows	User Equilibrium Flows	Toll Design $I = \{1\}$	
			Flows	Tolls
1	1.83	2	1.83	0.5
2	3.17	3	3.17	-
Total Cost	24.92	25	24.92	

This observation leads one to conclude that in this particular two-link network, as long as the difference in the tolls between the two routes is 0.5, regardless of the actual values of the tolls, the same network flow pattern will arise. This example also implies that in general the solution to the toll design problem might not be unique.

5.1.2 Stochastic Toll Design Problem

Based on the same set of parameters given earlier, the toll design problem can also be formulated based on logit route choice. In this case, there is no need to identify the routes used after the tolls are in place, since under the logit route choice behavior, all routes will be used. When $\mu = 1$, as stated in Chapter 4, and $I = \{1\}$, the problem becomes

$$\min_{\tau,F} \quad (1+2F_1)\cdot F_1 + (F_2+2)\cdot F_2$$

subject to

$$F_1 = 5\cdot\frac{e^{-[1+2F_1+\tau_1]}}{e^{-[1+2F_1+\tau_1]}+e^{-[F_2+2+\tau_2]}}$$

$$F_2 = 5 - F_1$$

$$F_1, F_2 \geq 0$$

$$\tau_1 \geq 0$$

$$\tau_2 = 0$$

Unlike the previous problem, this one can not be solved analytically since the stochastic equilibrium constraint is a fix-point problem itself. A simple AMPL program was written to solve this problem using LOQO, a solver employing the interior point method. The results are $F_1 = 1.83$, $F_2 = 3.17$, and $\tau_1 = 1.05$, $\tau_2 = 0$, with the total travel cost of 24.92. Table 5-2 lists the results as well as the user equilibrium and system optimal flows. Again, if both links are tollable, many combinations of link tolls would arise, such as $\tau_1 = 6.06$, $\tau_2 = 5.01$ and $\tau_1 = 2.05$, $\tau_2 = 1$. At this time, charging tolls could reach maximum system efficiency, as shown in Table 5-2.

Table 5-2 Two-Link Stochastic Toll Design

Link	System Optimum Flows	User Equilibrium Flows	Toll Design $I = \{1\}$	
			Flows	Tolls
1	1.83	2.11	1.83	1.05
2	3.17	2.89	3.17	-
Total Cost	24.92	25.14	24.92	

The user equilibrium solution of this network is $F_1 = 2.11$, $F_2 = 2.89$, and the total travel cost is 25.14. This cost is a little higher than that of the deterministic equilibrium. This is a reasonable result because of the difference in route choice behavior. Another observation is that under the logit-based stochastic route choice, it is actually possible to reach the system optimal flow pattern, which produces the lowest possible total travel cost. It is believed that this will only happen when all routes are used.

5.2 Braess Network

A slightly more complicated example is shown in Figure 5-2. There are five links, and one origin-destination pair in this network (from O to D) with the demand of 6. There are three routes in this network, route 1 consists of links 1 and 4, route 2 consists of links 3 and 2, and route 3 consists of links 3, 5 and 4. This is the classic Braess example, which is often used to explain the paradox in network design issue. It is used here to illustrate how to reduce the total travel cost by charging tolls after the new link (Link 5) is in place.

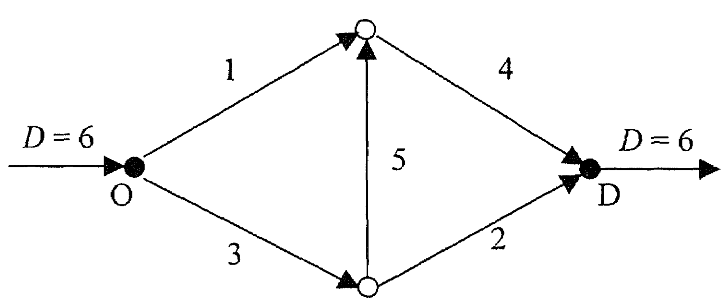

Figure 5-2 Braess Network

The cost functions for the five links are given by:

$$c_1(f_1) = 50 + f_1$$
$$c_2(f_2) = 50 + f_2$$
$$c_3(f_3) = 10f_3$$
$$c_4(f_4) = 10f_4$$
$$c_5(f_5) = 10 + f_5$$

5.2.1 Deterministic Toll Design Problem

It is well known that the no-toll equilibrium for this problem has a solution $f_1 = f_2 = f_5 = 2$, $f_3 = f_4 = 4$ with a total cost of 552, and that the system optimum for this problem (in the absence of a toll) has a solution $f_1 = f_2 = f_3 = f_4 = 3$ and $f_5 = 0$, with a total cost of 498. Thus, it is known that all routes are used in the no-toll equilibrium, and it is assumed that they will be used after tolls are in place. Assume that $I = \{5\}$, then for convenience, the toll design problem in this case can be formulated on link flow basis as follows.

$$\min_{\tau, F, \Theta} \quad \sum_{a=1}^{5} c_a(f) \cdot f_a$$

subject to

$$50 + f_1 + 10f_4 + \tau_1 + \tau_4 = \Theta$$

$$10f_3 + 50 + f_2 + \tau_2 + \tau_3 = \Theta$$

$$10f_3 + 10 + f_5 + 10f_4 + \tau_3 + \tau_5 + \tau_4 = \Theta$$

$$f_1 = F_1$$

$$f_2 = F_2$$

$$f_3 = F_2 + F_3$$

$$f_4 = F_1 + F_3$$

$$f_5 = F_3$$

$$F_1 + F_2 + F_3 = 6$$

$$F_1, F_2, F_3 \geq 0$$

$$f_1, f_2, f_3, f_4, f_5 \geq 0$$

$$\tau_5 \geq 0$$

$$\tau_1, \tau_2, \tau_3, \tau_4 = 0$$

$$\Theta \geq 0$$

Note that all constraints of this problem are linear, they form a convex feasible region. The objective function is quadratic, which ensures global optimality of the solution. The results of this problem are $f_1 = f_2 = f_3 = f_4 = 3$, $f_5 = 0$, and $\tau_5 = 13$, with a total cost of 498. These are identical with the system optimal flow pattern and total travel cost, respectively.

If there is more than one link that can be tolled, it is also possible to reach the system optimal flow pattern as indicated above. For example, when $I = \{1, 2, 5\}$, the toll pattern of $\tau_1 = 1$, $\tau_2 = 1$, and $\tau_5 = 14$ will still yield the same flow pattern as in the case of $I = \{5\}$. Again, as long as the difference of tolls among alternate routes remains the same, in this case, $\Psi_1 = \Psi_2$ and $\Psi_3 = \Psi_1 + 13$, the resulting flow patterns are all identical. The results are listed in Table 5-3.

Table 5-3 Braess Network Deterministic Toll Design

Link	System Optimum Flows	User Equilibrium Flows	Toll Design $I = \{5\}$		Toll Design $I = \{1,2,3,4,5\}$	
			Flows	Tolls	Flows	Tolls
1	3	2	3	-	3	1
2	3	2	3	-	3	1
3	3	4	3	-	3	-
4	3	4	3	-	3	-
5	0	2	0	13	0	14
Total Cost	498	552	498		498	

5.2.2 Stochastic Toll Design Problem

If the logit model is used to describe the route choice behavior of travelers, the stochastic toll design problem for the Braess network can be formulated as following:

$$\min_{\tau, F, \Theta} \quad \sum_{a=1}^{5} c_a(f) \cdot f_a$$

subject to

$$F_1 = 6 \cdot \frac{e^{-[50+f_1+10f_4+\tau_1+\tau_4]}}{e^{-[50+f_1+10f_4+\tau_1+\tau_4]} + e^{-[10f_3+50+f_2+\tau_2+\tau_3]} + e^{-[10f_3+10+f_5+10f_4+\tau_3+\tau_4+\tau_5]}}$$

$$F_2 = 6 \cdot \frac{e^{-[10f_3+50+f_2+\tau_2+\tau_3]}}{e^{-[50+f_1+10f_4+\tau_1+\tau_4]} + e^{-[10f_3+50+f_2+\tau_2+\tau_3]} + e^{-[10f_3+10+f_5+10f_4+\tau_3+\tau_4+\tau_5]}}$$

$$F_3 = 6 - F_1 - F_2$$

$$f_1 = F_1$$

$$f_2 = F_2$$

$$f_3 = F_2 + F_3$$

$$f_4 = F_1 + F_3$$

$$f_5 = F_3$$

$$F_1, F_2, F_3 \geq 0$$

$$f_1, f_2, f_3, f_4, f_5 \geq 0$$

$$\tau_5 \geq 0$$

$$\tau_1, \tau_2, \tau_3, \tau_4 = 0$$

This problem is more complicated than the previous ones since there exist nonlinear equality constraints here, which make the feasible region nonconvex. Using LOQO, we found the solution to the problem is that $f_1 = f_2 = f_3 = f_4 = 3$, $f_5 = 0$, and $\tau_5 = 28.70$, with a total cost of 498, which is also the system optimal.

If the set of links that can be tolled is $I = \{1, 2, 5\}$, one can also obtain the system optimal flows by charging tolls on these links, in the amount of $\tau_1 = 1$, $\tau_2 = 1$, and $\tau_5 = 28.07$, or $\tau_1 = 2$, $\tau_2 = 2$, and $\tau_5 = 29.71$, etc. Actually there are an infinite number of such combinations of toll values, all of which can produce a flow pattern that is system optimal. However, the differences among route tolls are not fixed, as in previous cases, because of the effect of the exponential term in this particular network. Nevertheless, it is still possible to get the system optimal flows by charging tolls under the stochastic route choice. Table 5-4 shows the results of different tolling schemes as well as the comparison with equilibrium and optimum.

Table 5-4 Braess Network Stochastic Toll Design

Link	System Optimum Flows	User Equilibrium Flows	Toll Design $I = \{5\}$		Toll Design $I = \{1,2,3,4,5\}$	
			Flows	Tolls	Flows	Tolls
1	3	2	3	-	3	2
2	3	2	3	-	3	2
3	3	4	3	-	3	-
4	3	4	3	-	3	-
5	0	2	0	28.70	0	29.71
Total Cost	498	552	498		498	

5.3 Seven-Link Network with Single User Group

Previously, only examples with linear link cost functions were discussed, because it would greatly simplify the solution process of the toll design problem. However, the widely used BPR cost function is nonlinear. In this section, the toll design problem is illustrated by using a seven-link network with the BPR cost function. This network is shown in Figure 5-3.

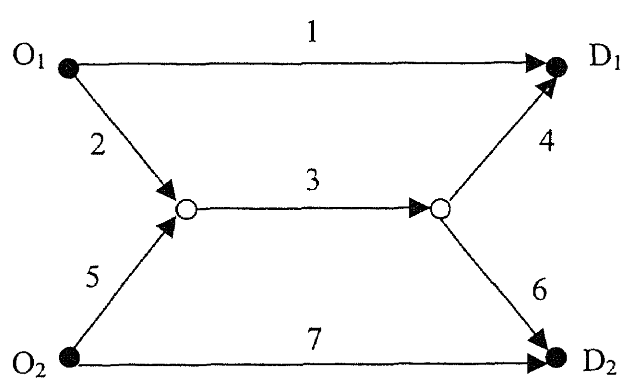

Figure 5-3 Seven-Link Network

There are four origin-destination pairs in this network, (O_1, D_1), (O_1, D_2), (O_2, D_1), and (O_2, D_2). The demands for these four O-D pairs are 500, 400, 400, and

600, respectively. There are seven links and six routes in this network. Route 1 only consists of link 1, route 2 consists links 2, 3, and 4, route 3 consists of links 2, 3, and 6, route 4 consists of links 5, 3, and 4, route 5 consists links 5, 3, and 6, and route 6 consists of link 7 only. The link cost functions are given by

$$c_1(f_1) = 1 + 0.15\left(\frac{f_1}{200}\right)^4$$

$$c_2(f_2) = 0.25\left[1 + 0.15\left(\frac{f_2}{300}\right)^4\right]$$

$$c_3(f_3) = 0.25\left[1 + 0.15\left(\frac{f_3}{700}\right)^4\right]$$

$$c_4(f_4) = 0.25\left[1 + 0.15\left(\frac{f_4}{300}\right)^4\right]$$

$$c_5(f_5) = 0.25\left[1 + 0.15\left(\frac{f_5}{300}\right)^4\right]$$

$$c_6(f_6) = 0.25\left[1 + 0.15\left(\frac{f_6}{300}\right)^4\right]$$

$$c_7(f_7) = 1 + 0.15\left(\frac{f_7}{200}\right)^4$$

5.3.1 Deterministic Toll Design Problem

Based on the given data, the solution of the user equilibrium is $f_1 = 327.56$, $f_2 = 572.45$, $f_3 = 1210.11$, $f_4 = 572.45$, $f_5 = 637.67$, $f_6 = 637.67$, and $f_7 = 362.33$, with a total travel cost (*exclusive of tolls*) of 4487.13, while the system optimal solution is $f_1 = 336.48$, $f_2 = 563.52$, $f_3 = 1194.59$, $f_4 = 563.52$, $f_5 = 631.08$, $f_6 = 631.08$, and $f_7 = 368.92$, with a total travel cost (*exclusive of tolls*) of 4479.34. In the no-toll equilibrium, all routes connecting each O-D pair are used, since one possible route flow pattern is $F_1 = 327.56$, $F_2 = 172.46$, $F_3 = 400$, $F_4 = 400$, $F_5 = 237.67$, and $F_6 = 362.33$.

The toll design problem is tested for two scenarios, $I = \{3\}$ and $I = \{1,2,3,4,5,6,7\}$. The formulations for these cases are similar to the previous examples. The only difference is that the link cost functions for this network are nonlinear. Because of the nonlinear equality constraints, a global optimum is not guaranteed in this case. The solution of the deterministic toll design problem when $I = \{3\}$ is $f_1 = 336.48$, $f_2 = 563.52$, $f_3 = 1194.59$, $f_4 = 563.52$, $f_5 = 631.08$, $f_6 = 631.08$, $f_7 = 368.92$, $\tau_3 = 0.2$, with a total travel cost (*exclusive of tolls*) of 4479.34. Note that these are identical with the system optimal flow pattern and the total travel cost respectively. With a toll pattern $\tau_1 = 0.74$, $\tau_2 = 0.32$, $\tau_3 = 0.30$, $\tau_4 = 0.32$, $\tau_5 = 0.32$, $\tau_6 = 0.32$ and $\tau_7 = 0.74$, the flow pattern resulting from these tolls when $I = \{1,2,3,4,5,6,7\}$ is the same as that of the system optimum, and therefore the resulting total travel cost reaches the system optimum too. Table 5-5 lists the solutions of toll design problem as well as the user equilibrium and the system optimum. Note that even though this problem has a nonconvex feasible region, the global optimum solution is found in both scenarios. Again, there exist multiple toll patterns that can result in the system optimal flows on the network, such as $\tau_1 = 1.56$, $\tau_2 = 0.61$, $\tau_3 = 0.54$, $\tau_4 = 0.61$, $\tau_5 = 0.61$, $\tau_6 = 0.61$ and $\tau_7 = 1.56$.

Table 5-5 Seven-Link Deterministic Toll Design

Link	System Optimum Flows	User Equilibrium Flows	Toll Design $I=\{3\}$		Toll Design $I=\{1,2,3,4,5,6,7\}$	
			Flows	Tolls	Flows	Tolls
1	336.48	327.56	336.48	-	336.48	0.74
2	563.52	572.45	563.52	-	563.52	0.32
3	1194.59	1210.11	1194.59	0.2	1194.59	0.30
4	563.52	572.45	563.52	-	563.52	0.32
5	631.08	637.67	631.08	-	631.08	0.32
6	631.08	637.67	631.08	-	631.08	0.32
7	368.92	362.33	368.92	-	368.92	0.74
Total Cost	4479.34	4487.13	4479.34		4479.34	

5.3.2 Stochastic Toll Design Problem

The stochastic toll design problem can be formulated in a similar fashion to the previous examples. For the purpose of comparison, the stochastic equilibrium without tolls is first solved. The solution is $f_1 = 305.93$, $f_2 = 594.08$, $f_3 = 1243.10$, $f_4 = 594.08$, $f_5 = 649.02$, $f_6 = 649.02$, and $f_7 = 350.98$, with a total travel cost of 4554.80. The stochastic toll design problems are formulated in two scenarios, one with $I = \{3\}$ and the other with $I = \{1,2,3,4,5,6,7\}$. The solutions of both scenarios, as well as those from stochastic user equilibrium and system optimum, are listed in Table 5-6. The table shows that when all links of the network can be tolled, the system optimal flows can be reached. However, when only Link 3 can be tolled, the resulting total travel cost could be very close to that of the system optimum. In the case, charging a toll of 0.8 on Link 3 counts for 97.5% maximum possible improvement on total travel cost. This shows that if tolls can only be placed on a very limited number of links in the network, a flow pattern that is very close to the system optimal flow could be reached. From the perspective of congestion toll policy design, it is preferred to select few links to charge and thus reduce the toll collection system cost, while maintaining very good congestion relieving effects.

Table 5-6 Seven-Link Stochastic Toll Design

Link	System Optimum Flows	User Equilibrium Flows	Toll Design $I = \{3\}$		Toll Design $I = \{1,2,3,4,5,6,7\}$	
			Flows	Tolls	Flows	Tolls
1	336.48	305.93	332.34	-	336.48	1.27
2	563.52	594.08	567.66	-	563.52	0.74
3	1194.59	1243.10	1195.04	0.8	1194.59	0.71
4	563.52	594.93	567.66	-	563.52	0.74
5	631.08	649.02	627.39	-	631.08	0.66
6	631.08	649.02	627.39	-	631.08	0.66
7	368.92	350.98	372.61	-	368.92	1.35
Total Cost	4479.34	4554.80	4481.19		4479.34	

5.4 Seven-Link Network with Two User Groups

It is clear that the existence of multiple user groups may complicate user equilibrium assignment, thus making the toll design problem more difficult to solve. In this section, a seven-link network with two user groups, cars and trucks, is considered. In this example, the BPR-type link cost function is modified a little to reflect the two user groups. Let t_{Aa}^0 and t_{Ta}^0 denote the free-flow travel costs of passenger cars and trucks on link a respectively, t_{Aa} and t_{Ta} denote the travel costs (*exclusive of tolls*) of passenger cars and trucks on link a respectively, N denote the average passenger car equivalent of a truck, and K denote the vector of link capacity. The following link cost functions are used in this example.

$$t_{Aa} = t_{Aa}^0 \left(1 + 0.15 \left(\frac{f_{Aa} + 0.5N \cdot f_{Ta}}{K_a} \right)^4 \right) \qquad a \in A \qquad (5.1a)$$

$$t_{Ta} = t_{Ta}^0 \left(1 + 0.15 \left(\frac{0.5 f_{Aa} + N \cdot f_{Ta}}{K_a} \right)^4 \right) \qquad a \in A \qquad (5.1b)$$

There are some variations of these functions from the traditional BPR function. The reason why these particular cost functions are used will be addressed in Chapter 7.

Assume that the free-flow travel time of trucks is 1.2 times of that of car, and one truck is four passenger car equivalent, i.e., $t_{Ta}^0 = 1.2 t_{Aa}^0$ and $N = 4$, the cost functions for car and truck become the following:

$$c_{A1}(f) = 1 + 0.15 \left(\frac{f_{A1} + 2 f_{T1}}{200} \right)^4$$

$$c_{A2}(f) = 0.25 \left[1 + 0.15 \left(\frac{f_{A2} + 2 f_{T2}}{300} \right)^4 \right]$$

$$c_{A3}(f) = 0.25 \left[1 + 0.15 \left(\frac{f_{A3} + 2 f_{T3}}{700} \right)^4 \right]$$

$$c_{A4}(f) = 0.25 \left[1 + 0.15 \left(\frac{f_{A4} + 2 f_{T4}}{300} \right)^4 \right]$$

$$c_{A5}(f) = 0.25 \left[1 + 0.15 \left(\frac{f_{A5} + 2 f_{T5}}{300} \right)^4 \right]$$

$$c_{A6}(f) = 0.25 \left[1 + 0.15 \left(\frac{f_{A6} + 2 f_{T6}}{300} \right)^4 \right]$$

$$c_{A7}(f) = 1 + 0.15 \left(\frac{f_{A7} + 2 f_{T7}}{200} \right)^4$$

$$c_{T1}(f) = 1.2\left[1 + 0.15\left(\frac{0.5f_{A1} + 4f_{T1}}{200}\right)^4\right]$$

$$c_{T2}(f) = 0.3\left[1 + 0.15\left(\frac{0.5f_{A2} + 4f_{T2}}{300}\right)^4\right]$$

$$c_{T3}(f) = 0.3\left[1 + 0.15\left(\frac{0.5f_{A3} + 4f_{T3}}{700}\right)^4\right]$$

$$c_{T4}(f) = 0.3\left[1 + 0.15\left(\frac{0.5f_{A4} + 4f_{T4}}{300}\right)^4\right]$$

$$c_{T5}(f) = 0.3\left[1 + 0.15\left(\frac{0.5f_{A5} + 4f_{T5}}{300}\right)^4\right]$$

$$c_{T6}(f) = 0.3\left[1 + 0.15\left(\frac{0.5f_{A6} + 4f_{T6}}{300}\right)^4\right]$$

$$c_{T7}(f) = 1.2\left[1 + 0.15\left(\frac{0.5f_{A7} + 4f_{T7}}{200}\right)^4\right]$$

Same as in the one user group case, there are four origin-destination pairs in this network, (O_1, D_1), (O_1, D_2), (O_2, D_1), and (O_2, D_2). The passenger car demands for these four O-D pairs are 475, 380, 380, and 570, respectively, and the truck demands for these four O-D pairs are 25, 20, 20, and 30, respectively. There are also six routes in this network. Route 1 only consists of link 1, route 2 consists of links 2, 3, and 4, route 3 consists of links 2, 3, and 6, route 4 consists of links 5, 3, and 4, route 5 consists of links 5, 3, and 6, and route 6 consists of link 7 only. In addition, it is assumed that trucks are charged more than passenger cars.

5.4.1 Deterministic Toll Design Problem

In order to evaluate the toll design results, the system optimum problem was solved first followed by the user equilibrium problem. For the latter, it is well known that no equivalent mathematical programming exists, so a diagonalization algorithm was used to get the equilibrium flows. The results of both problems are listed in Table 5-7.

Table 5-7 Seven-Link, Two User Groups, SO and UE

Link		System Optimal Flows	User Equilibrium Flows
1	Car	316.36	334.87
	Truck	19.00	5.59
2	Car	538.64	520.14
	Truck	26.00	39.41
3	Car	1140.65	1108.97
	Truck	55.91	78.99
4	Car	538.64	520.14
	Truck	26.00	39.41
5	Car	602.01	588.84
	Truck	29.91	39.58
6	Car	602.01	588.84
	Truck	29.91	39.58
7	Car	347.99	361.16
	Truck	20.09	10.42
Total Cost		4976.29	4984.92

Two instances of toll design problems were solved in this example, one with $I = \{3\}$, and the other with $I = \{1,2,3,4,5,6,7\}$. Table 5-8 lists resulting link flows and tolls. For comparison, the system optimal flows and no-toll user equilibrium flows are also listed. The table shows that both tolling schemes resulted in the flow pattern that is system optimal. Even though the feasible regions of both toll design problems are nonconvex, solutions that are globally optimal were still found, and there are a number of these optimal solutions. Again, it should be noticed that the optimal flows could be reached by only placing tolls on Link 3 of this network.

Table 5-8 Seven-Link, Two User Groups, Deterministic Toll Design

Link		System Optimal Flows	User Equilibrium Flows	Toll Design $I = \{3\}$		Toll Design $I = \{1,2,3,4,5,6,7\}$	
				Flows	Tolls	Flows	Tolls
1	Car	316.36	334.87	316.36	-	316.36	0.72
	Truck	19.00	5.59	19.00	-	19.00	0.72
2	Car	538.64	520.14	538.64	-	538.64	0.34
	Truck	26.00	39.41	26.00	-	26.00	0.35
3	Car	1140.65	1108.97	1140.65	0.22	1140.65	0.29
	Truck	55.91	78.99	55.91	0.35	55.91	0.36
4	Car	538.64	520.14	538.64	-	538.64	0.32
	Truck	26.00	39.41	26.00	-	26.00	0.35
5	Car	602.01	588.84	602.01	-	602.01	0.32
	Truck	29.91	39.58	29.91	-	29.91	0.35
6	Car	602.01	588.84	602.01	-	602.01	0.32
	Truck	29.91	39.58	29.91	-	29.91	0.35
7	Car	347.99	361.16	347.99	-	347.99	0.72
	Truck	20.09	10.42	20.09	-	20.09	0.72
Total Cost		4976.29	4984.92	4976.29		4976.29	

5.4.2 Stochastic Toll Design Problem

The stochastic toll design problem is also formulated for the same two scenarios, $I = \{3\}$, and $I = \{1,2,3,4,5,6,7\}$. In both cases, there exist nonlinear equality constraints, which make the feasible regions nonconvex. Therefore, the global optimal solution is not guaranteed. Table 5-9 lists the solutions of both scenarios, as well as those from stochastic user equilibrium and system optimum. The table shows that when all links of the network can be tolled, the system optimal flows can be reached. However, when only Link 3 can be tolled, the resulting total travel cost could be very close to that of the system optimum. In the case, charging the tolls of 0.79 to cars and 1.34 to trucks on Link 3 counts for 97.7% maximum possible improvement on total travel cost.

Table 5-9 Two User Groups' Stochastic Toll Design Results

Link		System Optimal Flows	User Equilibrium Flows	Toll Design $I = \{3\}$		Toll Design $I = \{1,2,3,4,5,6,7\}$	
				Flows	Tolls	Flows	Tolls
1	Car	316.36	299.89	313.90	-	316.36	0.25
	Truck	19.00	12.11	18.36	-	19.00	0.75
2	Car	538.64	555.11	541.10	-	538.64	0.32
	Truck	26.00	32.89	26.64	-	26.00	0.69
3	Car	1140.65	1163.54	1142.20	0.79	1140.65	0.53
	Truck	55.91	68.40	55.18	1.34	55.91	0.87
4	Car	538.64	555.11	541.10	-	538.64	0.32
	Truck	26.00	32.89	26.64	-	26.00	0.69
5	Car	602.01	608.44	601.10	-	602.01	0.21
	Truck	29.91	35.50	28.54	-	29.91	0.51
6	Car	602.01	608.44	601.10	-	602.01	0.21
	Truck	29.91	35.50	28.54	-	29.91	0.51
7	Car	347.99	341.56	348.90	-	347.99	0.29
	Truck	20.09	14.50	21.46	-	20.09	0.83
Total Cost		4976.29	5044.96	4977.84		4976.29	

Similarly, there are multiple toll patterns that could result in a flow that is system optimal.

5.5 Summary

In this chapter, several numerical examples are used to illustrate the behavior of the toll design problem under both deterministic and stochastic route choices. Results show that there are often multiple toll patterns that will generate the system optimal flow level, even when only a few links can be tolled. For the deterministic toll design problem with linear cost functions, the feasible region is convex, hence the global optimal solution is guaranteed. When nonlinear cost functions such as BPR functions are used, or in the case of the stochastic route choice model, where the nonlinear equality constraints exist, the feasible region becomes convex. Even though in some examples, such as that in the Subsection 5.4.1, a global optimal solution can be reached, there is no guarantee. Multiple user groups could certainly complicate the problem by providing additional nonconvexity.

The purpose of charging tolls is to alleviate traffic congestion. In a deterministic equilibrium, users choose the routes that minimize their individual travel costs. In contrast, the system optimization problem describes the scenario in which all users behave cooperatively in choosing routes in order to minimize the total travel cost. It is well known that this total travel time is the minimum that can be obtained on this particular network. Of course, this is just the ideal situation that is not a realistic representation of practice. The difference between the user equilibrium and system optimum is that road users are not experiencing the full costs of traveling. If they are made to pay the full costs through congestion tolls, the most efficient use of the network can be obtained. That is, the total travel cost (*exclusive of tolls*) could reach the system-optimized value. This is the idea behind marginal social cost pricing, which presumes that all links of the network are tollable. However, when only a subset of links can be tolled, the system optimal total travel cost may not be achieved. Nevertheless, the total travel cost under the tolls should always be no more than that in the user equilibrium. In other words, the system optimal and user equilibrium travel costs are the lower and upper bounds, respectively, of the total travel cost when tolls are in place. Let Z_{SO} denote the system optimal total travel cost, Z_{UE} denote the user equilibrium total travel cost, and Z_{TDP} denote the total travel cost of the toll design problem, then

$$Z_{SO} \leq Z_{TDP} \leq Z_{UE} \tag{5.3}$$

In some examples introduced earlier in this chapter, the system optimal flows are actually reached by only tolling certain links of the network. However, for more complicated networks, this most likely would not be the case because of the nonconvexity of the constraint set. Instead some local optima may be found if different starting points are used. Fortunately, the total travel cost from system optimum can serve as the lower bound. So the quality of these solutions can be evaluated.

For the stochastic toll design problem, the same rule applies. However, in the cases that all routes connecting an O-D pair are used, equation (5.3) holds. This is demonstrated in all stochastic based examples in this chapter. Otherwise,

$$Z_{SO} < Z_{TDP} \leq Z_{UE} \tag{5.4}$$

i.e., it is impossible to reach the system optimum by charging tolls on a network with stochastic route choice when not all routes are used.

It is also observed in these numerical examples that there exist multiple toll patterns that would generate the same flow patterns. In this case, one could choose different solutions for different purposes. For example, one may choose the solution with the fewest tolled links in order to achieve the minimum system investment costs, or choose the solution with smallest total toll amount to favor the road users while having the same traffic management effect, or choose the solution with largest total toll amount to reach the goal of maximizing the total toll revenue. This certainly gives transportation planner more flexibility in choosing the appropriate tolling scheme.

CHAPTER 6

ALGORITHMS

6.1 Introduction

In previous chapters, we formulated the toll design problems, both deterministic and stochastic, as single level optimization problems with nonlinear objective functions and nonlinear equality constraints. For toll design problems with deterministic route choice, the convexity of the objective function depends on the form of the link cost function. By carefully choosing those parameters of the cost function, one can obtain a convex objective function for the toll design problem. In the special case when the cost functions are linear, the toll design problem reduces to a quadratic program with linear constraints. However, when nonlinear cost functions (such as the popular BPR function) are used, the resulting nonlinear equality constraints will greatly increase the complexity of the toll design problem. This is because the nonlinear equalities define a feasible region that is nonconvex. This is also true for toll design problems with stochastic route choice, since the exponential term of the logit-based route choice probability function has determined the nonconvexity of the feasible region.

Tests on some small numerical examples in Chapter 5 show that there may be multiple toll patterns that would incur the same flow pattern. This suggests that the toll design problem may have multiple solutions, as well as very "deep valleys" on its objective function. In order to solve the toll design problem with such complexity, either deterministic based or stochastic based, one has to use an algorithm that is capable of handling the nonlinear constraints, particularly the nonconvex feasible region.

Many algorithms have been developed for solving constrained nonlinear optimization problems. In general, these algorithms can be categorized into types such as feasible direction methods and interior point algorithms. There have been various software packages developed using these algorithms, which are capable of solving the constrained nonlinear programming problems. Such products include MINOS, a projected Lagrangian method combined with a reduced-space approach, LANCELOT, a trust region method performed on an augmented Lagrangian, SNOPT, a sequential quadratic programming method, and LOQO, an interior point method for nonconvex nonlinear programs. Among these packages, MINOS, LANCELOT, and SNOPT use algorithms in the category of feasible direction method, and LOQO uses an interior point method. In this chapter, these algorithms and their applicability to the toll design problem is discussed.

Observing that the nonconvexity of the toll design problem arises from its nonlinear constraints, we will make an attempt to linearize these nonlinear functions using the iterative linearization method introduced by Rosen (1966). By solving a series of linearly constrained optimization problems, such effort will greatly reduce the complexity of the solution procedure.

In the next section, we first outline the procedures for solving both deterministic and stochastic toll design problems.

6.2 Procedures for Solving Toll Design Problems

6.2.1 Deterministic Toll Design Problem

As discussed in Chapter 3, the single level deterministic toll design problem (3.14) is much easier to solve than the bilevel problem (3.5), however, in order to convert the bilevel problem into the single level one, the set of routes that will be used after tolls are in place has to be identified. It is common knowledge that at equilibrium, there may exist multiple route flow patterns that would generate the same link flow pattern on the network. For example, consider the network shown in Figure 6-1, in which there are four links and four routes. Route 1 consists of links 1 and 3, route 2 consists of links 1 and 4, route 3 consists of links 2 and 3, and route 4 consists of links 2 and 4.

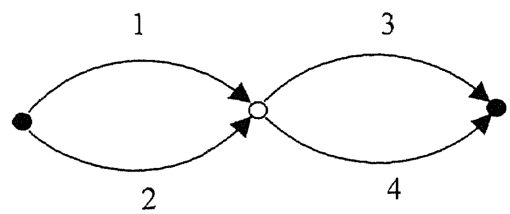

Figure 6-1 Demo of Multiple Route Flows

Let F_i, $i = 1,2,3,4$ denote the flow on route i, and f_j, $j = 1,2,3,4$ denote the flow on link j. Then

$$\begin{aligned} f_1 &= F_1 + F_2 \\ f_2 &= F_3 + F_4 \\ f_3 &= F_1 + F_3 \\ f_4 &= F_2 + F_4 \end{aligned}$$

If the equilibrium link flow pattern is $f_1 = f_2 = f_3 = f_4 = 2$, then one possible route flow pattern is $F_1 = F_2 = F_3 = F_4 = 2$, another possible route flow pattern is $F_1 = 2$, $F_2 = F_3 = 0$, and $F_4 = 2$. So, the set of used routes is not unique.

In order to identify this set, we use the method introduced by Larsson et al (1998), which can uniquely find the most likely route flows given the equilibrium link flows. In this method, the equilibrium link flow solution is defined as a macro state, which arises as the result of the route choices of the individual travelers. The travelers' route choices between minimum and equal cost routes of a particular O-D pair define a set of micro states. All micro states consistent with the macro state is argued to be equally probable, since the travelers are indifferent to which route they use, as long as it is the minimum cost route (Smith 1987 and Smith 1983). Based on well-known principles from information theory, it is argued that the most likely route flow solution is the one which corresponds to the largest number of micro states among those which are consistent with the observed macro state.

Even though this method is designed for the single user group case, it also applies to the situation with multiple user groups. Let R_w^* denote the set of equilibrium routes between O-D pair $w \in W$, that is,

$$R_w^* = \{p \in P_w : C_{up} = \Theta_{uw}, u \in U\} \tag{6.1}$$

Then, after applying Stirling's approximation, the most likely route flows can be found by solving the maximum entropy problem (6.2).

$$\max_F \quad -\sum_{u \in U} \left(\sum_{w \in W, p \in R_w^*} F_{up} \ln F_{up} \right) \tag{6.2a}$$

subject to

$$\sum_{p \in R_w^*} F_{up} = D_{uw} \qquad u \in U, w \in W \tag{6.2b}$$

$$\sum_{w\in W, p\in R_w^*} \delta_{ap} F_{up} = f_{ua} \qquad u \in U, a \in A \tag{6.2c}$$

$$F_{up} \geq 0 \qquad u \in U, w \in W, p \in R_w^* \tag{6.2d}$$

This is a standard nonlinear programming problem with linear constraints. A variety of algorithms could be applied to this problem. However, it should be mentioned that because of the entropy formulation in the objective function, $F_{up} \geq 0$ for all $u \in U$, $w \in W$ and $p \in R_w^*$. In fact, it is even difficult to get the exact solution to this entropy problem, since the objective function becomes extremely flat when approaching the optimum. However, for the purpose of identifying the most likely routes that carry flow, a near-optimum solution is good enough. In this near-optimum solution, although none of the route flows would be zero exactly, some of them could be very close to zero. Theoretically, these routes are used, while from an engineering point of view, these routes certainly should not be included in the set of used routes. A tolerance $\varepsilon > 0$ will be specified to exclude these routes, i.e., the set of "reasonable" routes between O-D pair $w \in W$ can be defined as

$$R_w = \{p \in R_w^* : F_{up} \geq \varepsilon, u \in U\} \tag{6.3a}$$

and then

$$R = \bigcup_{w\in W} R_w. \tag{6.3b}$$

It is possible that some routes that were not in the "no-toll" route set will be attractive after the tolls are in place (i.e., if the tolls are placed on links that are not in that route). Such routes should, of course, be included. While it is difficult to identify such routes *a priori,* one can discover them *ex post*. Hence, one could apply an iterative procedure in which the initial route set is based on the "no-toll" equilibrium and, after the "with-toll" equilibrium is found for this route set, new routes could be generated by solving a sequence of shortest route problems. Then the toll design problem could be re-solved. As it turns out, this issue did not arise in those examples presented in this dissertation. Specifically, let R'_w denote the set of equilibrium routes connecting O-D pair $w \in W$ after the tolls are in place, and R' denote the set of all equilibrium routes after the tolls are in place, then

$$R'_w = \{p \in P_w : C_{up} + \Psi_{up} = \Theta_{uw}, u \in U\} \tag{6.4a}$$

and

$$R' = \bigcup_{w \in W} R'_w \tag{6.4b}$$

and then restart solving the most likely route flow problem (6.2). Then one can compare R'_w with R. If there is any $p \in R'$ while $p \notin R$, this p is then added into R. The toll design problem is solved again, and the calculation of R'_w and R' is redone. The above process is then repeated until $R' \subseteq R$. Tests on some networks show that the condition of $R' \subseteq R$ is satisfied in the first place. Therefore, no further iterations would be necessary to identify the used routes. For example, if one more link is added to the network shown in Figure 5-1, with the cost function indicated in Figure 6-2, one can obtain the user equilibrium flows as $F_1 = 2$, $F_2 = 3$, and $F_3 = 0$, with a total travel cost of 25. In this case, the used route set $R = \{1,2\}$. If only link 1 can be tolled, the toll design problem is then solved based on this route set. The solution is $F_1 = 1.83$, $F_2 = 3.17$, $F_3 = 0$, and $\tau_1 = 0.5$, with a total travel cost (*exclusive of tolls*) 24.917.

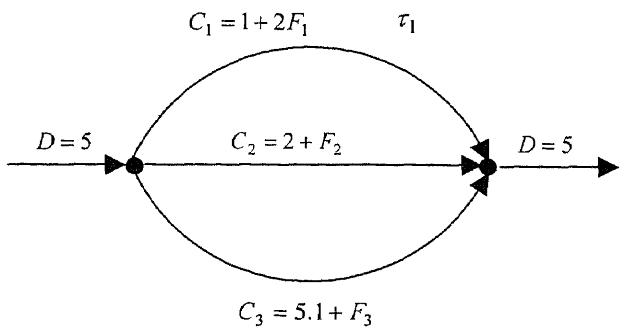

Figure 6-2 Demo Network of the Most Probable Route Procedure

At this time, however, the travel cost (*inclusive of tolls*) on these three routes are 5.17, 5.17, and 5.1, respectively (i.e., because of the toll on link 1, travel cost on route 3 becomes lower than the cost on those used routes). Therefore, route 3 should be added into the used route set, i.e., $R = \{1,2,3\}$, and then the toll design problem is solved again, based on this new set. The results are $F_1 = 1.510$, $F_2 = 3.295$, $F_3 = 0.195$, and $\tau_1 = 1.275$, with a total travel cost (*exclusive of tolls*) of 24.550. At this time, equal route travel costs (*inclusive of tolls*) will arise on all routes.

Based on the above procedure of identifying the used-route set, we can develop a flowchart, as shown in Figure 6-3, for solving deterministic toll design problem.

6.2.2 Stochastic Toll Design Problem

The solution process is much easier for the stochastic toll design problem, since there is no need to identify the set of used routes. Therefore, the toll design problem can be solved right after the stochastic user equilibrium problem. Note that, as indicated in Chapter 4, we have to enlarge the feasible region of the toll design problem in order to apply those algorithms that need a feasible starting point.

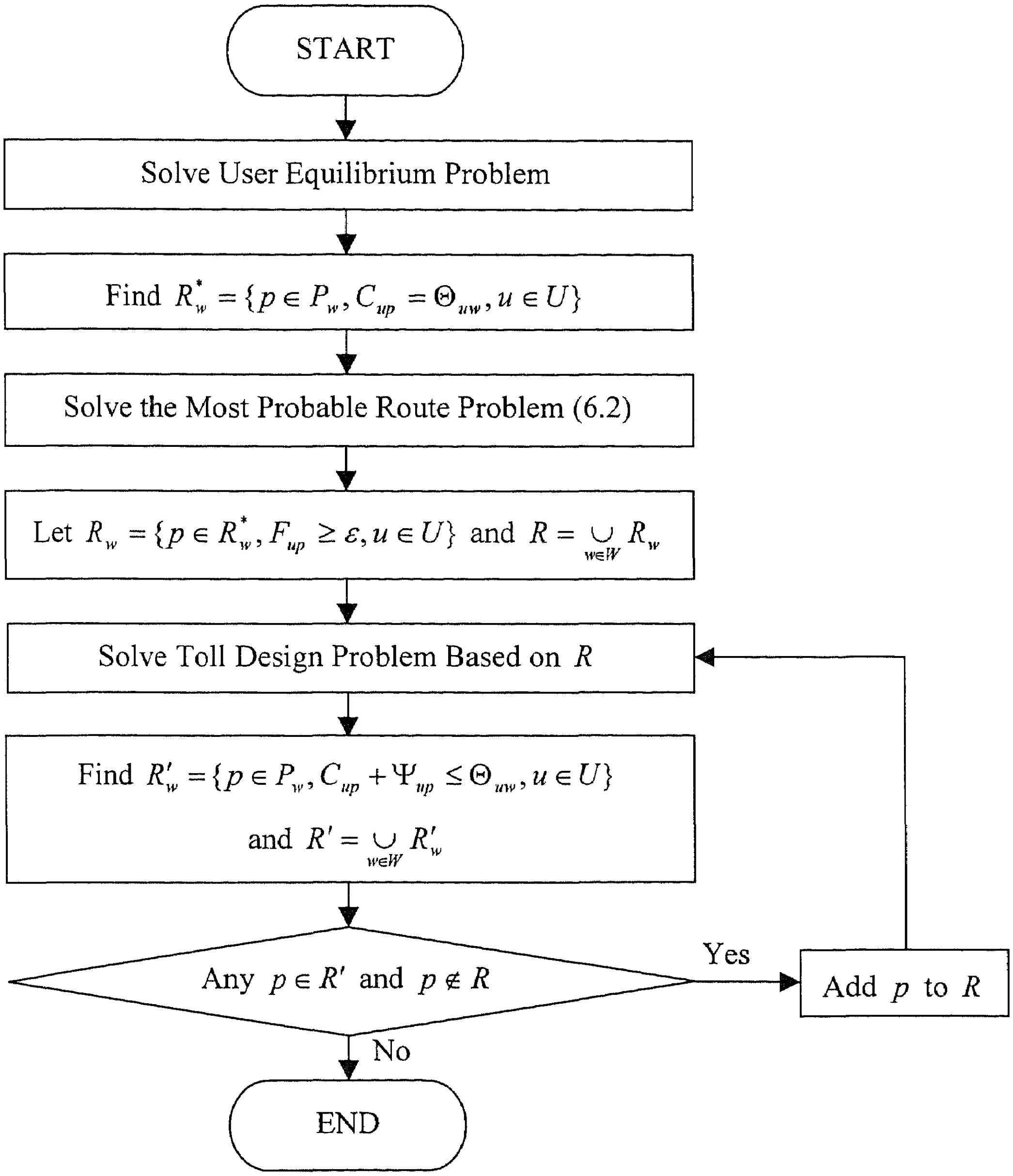

Figure 6-3 Flowchart of Solving Deterministic Toll Design Problem

6.3 Feasible Direction Methods

The feasible direction methods include those algorithms that solve a nonlinear programming problem by moving from a feasible point to an improved feasible point.

The optimization strategy in the feasible direction methods is usually the following (Bazaraa et al 1993):

(1) At a feasible point x_k, find the descent direction, d_k, of the objective function, such that for λ that is greater than zero and sufficiently small, $x_k + \lambda d_k$ is feasible and the objective value at $x_k + \lambda d_k$ is better than the objective value at x_k;

(2) On this descent direction, find $\lambda_k > 0$ by doing a line search, which is the maximum step size on d_k;

(3) Update the solution by letting $x_{k+1} = x_k + \lambda_k d_k$, and go back to step (1) until two successive points are close enough.

Among those software packages implementing the feasible direction method, LANCELOT, MINOS, and SNOPT, as well as their applicability to the toll design problems is discussed.

6.3.1 LANCELOT

LANCELOT, Large And Nonlinear Constrained Extended Lagrangian Optimization Techniques, is a package for solving large-scale nonlinearly constrained optimization problems (Conn et al. 1992). When dealing with the toll design problem, the method actually solves a series of subproblems with augmented Lagrangian functions and nonnegativity constraints. Let g represent all except the simple bound (nonnegativity) constraints in the toll design problem. Then for the deterministic based toll design problem (3.14), $g(F, \tau) = 0$ represents the equation system (3.14b), (3.14c), and (3.14f).

For this route-based formulation of the deterministic toll design problem, the number of equations in this system is $\left|U \times \bigcup_{w \in W} R_w\right| + |U \times W| + |U \times (A - I)|$. For example, for the seven-link network shown in Figure 5-3, which carries two user groups with Link 3 tollable only, there are 32 constraints in this system.

For the logit based toll design problem (4.5), $g(f,\tau) = 0$ represents the equation system (4.5b) and (4.5e). For this link-based formulation of the toll design problem, the number of equations in this system is $|U \times A| + |U \times (A - I)|$. For the same seven-link network discussed above, there are 26 constraints in this system.

Let λ denote the vector of Lagrangians, μ denote the penalty parameter, and S represent a diagonal matrix of scaling factor. Note that these parameters may vary in dimension and value between deterministic and the stochastic based toll design problems. Then using the trust-region method, the subproblems (6.5) and (6.6) are solved for deterministic and stochastic toll design problems respectively.

$$\min_{\tau, F, \Theta} \quad \sum_{u \in U} C_u(F)^T F_u + \lambda^T g(F,\tau) + \frac{1}{2\mu} g^T(F,\tau) \cdot S \cdot g(F,\tau) \tag{6.5a}$$

subject to

$$F \geq 0 \tag{6.5b}$$

$$\tau_{ua} \geq 0 \qquad u \in U, a \in I \tag{6.5c}$$

$$\Theta \geq 0 \tag{6.5d}$$

$$\min_{\tau, f} \quad \sum_{u \in U} c_u(f)^T f_u + \lambda^T g(f,\tau) + \frac{1}{2\mu} g^T(f,\tau) \cdot S \cdot g(f,\tau) \tag{6.6a}$$

subject to

$$f \geq 0 \tag{6.6b}$$

$$\tau_{ua} \geq 0 \qquad u \in U, a \in I \tag{6.6c}$$

$$\Theta \geq 0 \tag{6.6d}$$

The basic idea of the trust-region method is to model the objective function about the current point. That is, one "trust" the approximation of the objective function in the neighborhood (called "trust-region") of this point. First, the approximation of the generalized Cauchy point is obtained. This ensures the convergence of the algorithm, since the projected gradient will reach zero provided the value of the quadratic approximation of the above objective function at the final iteration is no larger than that at the generalized Cauchy point. Then, a new point that could further reduce the quadratic objective value within the intersection of the feasible region and the trust region is obtained. This can be done by fixing those variables that lie on their bounds at the approximation to the generalized Cauchy point. Then one tests to see if there is general agreement between the values of this quadratic model and the true objective function at this new point. If a good agreement is obtained, the trust region is expanded, while if the agreement is moderate, the trust region is unchanged. If the agreement is poor, the new point is discarded and the trust region is contracted.

In general, LANCELOT is recommended for large problems with many degrees of freedom (Bongartz et al. 1997). Test runs show that LANCELOT works well for those examples introduced in Chapter 5.

6.3.2 SNOPT

SNOPT, Sparse Nonlinear Optimizer, is a software package designed for solving general nonlinear programming problems using sequential quadratic programming (SQP) method, which has been proved highly effective for solving constrained optimization problems with smooth nonlinear functions in the objective and constraints (Gill et al. 1997).

Let s denote the system of decision variables of the toll design problem, $Z(s)$ denote the objective function of the toll design problem, which is the total travel cost (*exclusive of tolls*), $g(s)$ denote the system of all constraints. For the deterministic toll design problem (3.14), s is a vector with dimension $1\times\left(\left|U\times\underset{w\in W}{\cup}R_w\right|+|U\times W|+|U\times A|\right)$, while for stochastic toll design problem (4.5), s is a vector with dimension $2|U\times A|$. Then for the deterministic based toll design problem, $g(s)$ represents (3.14b) to (3.14g), with a total of $2\left|U\times\underset{w\in W}{\cup}R_w\right|+2|U\times W|+|U\times A|$ equations in this system, and for the link-based stochastic toll design problem, it represents (4.5b) to (4.5e), with a total of $3|U\times A|$ equations in the system. Let $g_L(s,s_k)$ denote the first-order approximation of $g(s)$ at the solution $s=s_k$, and $J(s_k)$ denote the Jacobian of constraints system $g(s)$. Then

$$g_L(s,s_k)=g(s_k)+J(s_k)(s-s_k)\,. \tag{6.7}$$

SNOPT is based on the augmented Lagrangian associated with the problem. At the k th iteration, the augmented Lagrangian can be written as

$$L(s,s_k,\lambda_k)=Z(s)-\lambda_k^T[g(s)-g_L(s,s_k)]\,, \tag{6.8}$$

where λ_k represents the Lagrangian multiplier on the k th iteration.

Furthermore, let H_k represent the approximation to $\nabla_s^2 L$ at the current point (s_k,λ_k). So H_k is a matrix of $\left(\left|U\times\underset{w\in W}{\cup}R_w\right|+|U\times W|+|U\times A|\right)\times$

$\left(\left|U \times \bigcup_{w \in W} R_w\right| + |U \times W| + |U \times A|\right)$ for the deterministic problem, and a matrix of $(2|U \times A|) \times (2|U \times A|)$ for the stochastic problem. For the same seven-link network, H_k is a 34×34 matrix for deterministic problem and a matrix of 28×28 for stochastic problem. The subproblem (6.9) is then solved, the solution of which actually defines the optimal searching direction.

$$\min_{s,\lambda} \quad Z(s) + \nabla Z(s_k)(s - s_k) + \frac{1}{2}(s - s_k)^T H_k (s - s_k) \tag{6.9a}$$

subject to

$$g_L(s, s_k) \geq 0 \tag{6.9b}$$

It should be noted that only constraint (3.14b) is linearized for the deterministic toll design problem, and constraint (4.5b) for the stochastic based toll design problem. Besides, all constraints should keep their original equality or inequality. Once the descent direction is obtained, a line search is performed to get the step size along this descent direction.

SNOPT is designed for problems with many thousands of constraints and variables but a moderate number of degrees of freedom (up to 2000). Test runs of SNOPT on the toll design problem indicated some difficulties. In some cases, such as the example of the seven-link network with two user groups in Subsection 5.4.1, the solutions to the toll design problem obtained from SNOPT are not as good as those from the other solver. In this particular case, SNOPT gave a solution with total travel cost (*exclusive of tolls*) of 4982.52, compared to the solution of 4976.29 from LOQO and LANCELOT.

6.3.3 MINOS

MINOS is a program designed for solving large-scale nonlinear programs whose objective and constraint functions are smooth and continuously differentiable. The algorithm behind MINOS is based on the augmented Lagrangian, as well as a series of linearized subproblems (Murtagh and Saunders 1982).

For the toll design problem, all inequality constraints should be first converted to equality constraints by adding slack variables. As before, let s denote the system of decision variables of the toll design problem, $Z(s)$ denote the objective function of the toll design problem, which is the total travel cost (*exclusive of tolls*), $g(s)$ denote the system of all constraints, which are all equality constraints after the conversion. Then for the deterministic toll design problem, $g(s)$ represents (3.14b) to (3.14g), with a total of $2\left|U \times \bigcup_{w\in W} R_w\right| + 2|U \times W| + |U \times A|$ equations in this system, and for the stochastic based toll design problem, it represents (4.5b) to (4.5e), with a total of $3|U \times A|$ equations in the system. Let $g_L(s,s_k)$ denote the first-order approximation of $g(s)$ at solution $s = s_k$, and $J(s_k)$ denote the Jacobian of constraints system $g(s)$, then

$$g_L(s,s_k) = g(s_k) + J(s_k)(s - s_k). \tag{6.10}$$

Note that $J(s_k)$ is a matrix $\left(2\left|U \times \bigcup_{w\in W} R_w\right| + 2|U \times W| + |U \times A|\right) \times \left(\left|U \times \bigcup_{w\in W} R_w\right| + |U \times W| + |U \times A|\right)$ for the deterministic problem, and a matrix of $(3|U \times W|) \times (2|U \times W|)$ for the stochastic problem. In the seven-link example discussed earlier, $J(s_k)$ is 54×34 for the deterministic problem and 52×28 for the stochastic problem. Furthermore, let ρ denote the penalty parameter, λ_k denote the Lagrangian multiplier on the k th iteration, then the following subproblem is solved.

$$\min_{s,\lambda} \; L(s,s_k,\lambda_k,\rho)=Z(s)-\lambda_k^T[g(s)-g_L(s,s_k)]+\frac{1}{2}\rho[g(s)-g_L(s,s_k)]^T[g(s)-g_L(s,s_k)] \quad (6.10a)$$

subject to

$$g_L(s,s_k)=0 \quad (6.10b)$$

in which $L(s,s_k,\lambda_k,\rho)$ is the augmented Lagrangian. This is a standard nonlinear program with linear constraints. Particularly, when $\lambda_k=0$ and $\rho=0$, the subproblem becomes identical with the iterative linearization method proposed by Rosen (1966). This method will be introduced later.

In MINOS, this linearized subproblem is solved using the reduced gradient method proposed by Wolfe (1967), which is an algorithm designed for solving linearly constrained nonlinear optimization problems. In the projected Lagragian (6.10a), the penalty term is chosen to ensure that the Hessian of $L(x,x_k,\lambda_k,\rho)$ is positive definite within an appropriate subspace. It also inhibits large discrepancies between $h(x)$ and $\bar{h}(x,x_k)$, thereby discouraging large changes in x in each sub-problem if the nonlinearities are such that the linearized constraints have little meaning far from the point of linearization. However, this method depends rather heavily on making a good choice for ρ. Heuristically, ρ is increased whenever it seems necessary to prevent nonconvergence. At the same time, a mechanism for deciding when to reduce ρ to zero is developed to benefit from the quadratic convergence introduced in Robinson (1972).

Due to the unavailability of the solver MINOS, we were unable to conduct the test of this solver on the toll design problem.

6.3.4 Application to the Toll Design Problems

Feasible direction methods are basically applicable to almost all nonlinear programming problems. These algorithms will always give feasible solutions even for the case where they terminate before reaching the solutions. And, the optimal solutions given by the feasible direction methods will be at least local minimum. Furthermore, these methods do not rely on convexity, thus are applicable to general nonlinear programming problems. However, the feasible direction methods require an additional procedure to obtain the initial feasible points. And most importantly, excessive computational difficulty will arise when nonlinear constraints are included due to the necessity to remain within the feasible region as the method progress. Even if the convergence rate of the feasible direction methods are competitive with those of other methods, they may fail to converge for certain problems.

When applied to the toll design problem, the most appealing feature of these algorithms is that they will always provide feasible solutions. This is vital to the design of a practical policy. Although these algorithms may terminate at a saddle point instead of a local optimum, one can overcome this by solving the problem from various starting points. Furthermore, the initial feasible point needed in these algorithms can be easily obtained by designating the user equilibrium flow vector and the zero toll vector as the starting points for flow and toll variables, respectively.

Among the feasible direction methods, LANCELOT and SNOPT were tested on those examples in Chapter 5. Results show that these algorithms work well in terms of providing feasible solutions. However, their capability of dealing with "very nonconvex" problems is limited. They may just terminate at a nearby local optimum because of the extremely steep "cliff". In addition, these algorithms tend to have longer running time (particularly LANCELOT).

6.4 Interior Point Algorithm

The basic idea of an interior point algorithm is to approximate constrained optimization problems by unconstrained problems. By using this type of method, the constraints are placed into the objective function via either a penalty or barrier parameter, to either penalize the violation of constraints by imposing a high cost for it, or prevent the intermediate solutions from leaving the feasible region.

Vanderbei and Shanno (1998) propose an interior point algorithm for solving nonconvex nonlinear programming problems. This is an extension of the interior point algorithm for quadratic programming problems (Vanderbei 1994). The basic idea of this algorithm is to use Newton's method to find the feasible search direction. In the case of toll design problems, the nonlinear constraints are all equalities, and the only inequalities are the nonnegativity constraints. (If one adds the constraint that the toll on trucks should be higher than the toll on cars, it can be converted into an equality constraint by adding slack variables.) The inequalities is then eliminated if they are added to the objective function as barrier terms. A Lagrangian is then formulated to incorporate those equality constraints.

Let g denote the system of all constraints in the toll design problem except those with simple bounds on variables. That is, g represents equation system of (3.14b), (3.14c) and (3.14f) for the deterministic toll design problem, and equation system of (4.5b) and (4.5e) for the stochastic toll design problem. Note that these constraints should already be converted to equality constraints by adding slack variables. Let h represent the system of simple bounds as well as bounds on those added slack variables.

Let μ denote the barrier parameter, λ denote the vector of Lagrangian multiplier, and Y denote the vector with all components being 1, then the Lagrangian L can be written as

$$L = \sum_{u \in U} C_u(F)^T F_u - \mu\left(\log[h(F,\tau)]\right)^T Y - \lambda^T g(F,\tau) \tag{6.11}$$

for the deterministic problem, and

$$L = \sum_{u \in U} c_u(f)^T f_u - \mu\left(\log[h(f,\tau)]\right)^T Y - \lambda^T g(f,\tau) \tag{6.12}$$

for the stochastic problem. By solving the Newton system of the first-order KKT condition of the Lagrangian, one can find the descent direction from the current solution, provided that a feasible starting point is used.

In order to find the step size, merit function (6.13) for the deterministic problem and (6.14) for the stochastic problem are used, in which ρ denotes the penalty parameter. Specifically, these merit functions can prevent the selection of a step length that will lead to the infeasibility of the subsequent solution point.

$$\varphi_{\rho,\mu} = \sum_{u \in U} C_u(F)^T F_u - \mu\left[\log h(F,\tau)\right]^T Y + \frac{\rho}{2}\left\|g(F,\tau)\right\|_2^2 \tag{6.13}$$

$$\varphi_{\rho,\mu} = \sum_{u \in U} c_u(f)^T f_u - \mu\left[\log h(f,\tau)\right]^T Y + \frac{\rho}{2}\left\|g(f,\tau)\right\|_2^2 \tag{6.14}$$

Tests on those examples in Chapter 5 show that this algorithm gives at least the same solution, if not better, to toll design problems, compared with LANCELOT and SNOPT.

6.5 Iterative Linearization Method

The iterative linearization method, proposed by Rosen (1966), essentially consists of Newton's method with a convex or linear programming subproblem solved at each iteration. In this method, the nonlinear constraints are linearized using a first-order approximation. For the deterministic based toll design problem, the only nonlinear constraint is (314b), and the only nonlinear constraint for the stochastic toll design problem is (4.5b).

Let $\overline{C}_{up}(F)$ denote the first-order approximation of the cost function. For the deterministic toll design problem (3.14), the following subproblem is solved.

$$\min_{\tau,F,\Theta} \sum_{u\in U}\left(\sum_{w\in W,p\in R_w}(\Theta_{uw}-\Psi_{up})^T \cdot F_{up}\right) \tag{6.15a}$$

subject to

$$\overline{C}_{up}(F)+\Psi_{up} \leq \Theta_{uw} \qquad u\in U, w\in W, p\in R_w \tag{6.15b}$$

$$\sum_{p\in R_w} F_{up} = D_{uw} \qquad u\in U, w\in W \tag{6.15c}$$

$$F \geq 0 \tag{6.15d}$$

$$\tau_{ua} \geq 0 \qquad u\in U, a\in I \tag{6.15e}$$

$$\tau_{ua} = 0 \qquad u\in U, a\notin I \tag{6.15f}$$

$$\Theta \geq 0 \tag{6.15g}$$

For the stochastic toll design problem, let $g_L(f,\tau)$ denote the linearized constraint (4.5b). Then the following subproblem is solved:

$$\min_{\tau,f} \sum_{u \in U} c_u(f)^T f_u \tag{6.16a}$$

subject to

$$g_L(f,\tau) = 0 \tag{6.16b}$$

$$f \geq 0 \tag{6.16c}$$

$$\tau_{ua} \geq 0 \qquad u \in U, a \in II \tag{6.16d}$$

$$\tau_{ua} = 0 \qquad u \in U, a \notin I \tag{6.16e}$$

In this algorithm, the next iteration uses the linearized cost function at current solution, until the convergence criteria are met. It was proved by Rosen (1966) that the above method is guaranteed to converge to a local optimum (global optimum if the objective is also convex), provided that every constraint function in the original toll design problem is convex. For the deterministic toll design problem (6.15), the constraint function of (6.15b) is convex since F, Ψ, and Θ are all decision variables of the problem and are independent of each other. Hence, in the case of the deterministic problem, convergence of this iterative linearization method is ensured. Chen et al (forthcoming) shows that this method works well in several test problems including the Sioux Falls network with single user group. However, for the stochastic toll design problem (6.16), the constraint (6.16b) is the difference between two related terms that is not always convex. Therefore, the convergence of this method is not guaranteed (although Rosen (1966) also points out that this procedure will often converge even when the convexity condition of constraints is not satisfied). However, tests of this method on both deterministic and stochastic cases show that it often fails to converge.

6.6 Summary

In this chapter, several algorithms for nonlinearly constrained optimization problems, as well as their applicability to the toll design problem are discussed. Among those algorithms that could be directly applied to the toll design problem, both feasible direction methods and interior point method will always generate a feasible solution even if it terminates at points other than local optimum. The iterative linearization method can greatly simplify the solution process by approximating the nonlinearly constrained problem using linearly constrained problems repeatedly. However, all these methods require additional effort to searching for a feasible starting point except for the interior point algorithm, which requires only that the inequality constraints be satisfied. Fortunately, for the toll design problem, the user equilibrium flow and zero tolls can be used as the starting point in these algorithms.

It should be noted that all these algorithms only provide local optimal solutions, no global optimum is guaranteed when nonconvex constraint exists. Various starting points can be alternatively used to obtain multiple optima. Also, as discussed earlier, the bounds of the toll design problem are known. Thus one can evaluate the performance of a particular algorithm by comparing the objective value it generates to the bounds of the problem.

Tests show that among those algorithms introduced earlier in this chapter, LOQO has a superior performance over both LANCELOT and SNOPT, in terms of both computation speed and results' closeness to the system optimum. In the next chapter, these algorithms will be tested on a much larger-sized network.

CHAPTER 7

CASE STUDY

7.1 Introduction

In this chapter, the toll design problem with two user groups is solved on a realistic-sized network, under both deterministic and stochastic route choices. The results can be used to evaluate the performance of various algorithms, and explore the practical use of the toll design model developed in previous chapters.

7.2 Network

The network chosen to test the toll design methodology developed in this dissertation is the road network of the Sioux Falls City, South Dakota. This network has been used for traffic assignment tests in many studies. The roadway map of Sioux Falls City is shown in Figure 7-1. Based on this map, the roadway network has been abstracted as shown in Figure 7-2.

There are 76 links, 1118 routes, and 552 O-D pairs on this network. The original data on travel demand between these O-D pairs are for the case of a single user group. In this dissertation, travelers are divided into two groups, cars and truck, according to their different operating features. We assume that there are 5% trucks in the total demand. The demand, capacity and other parameters of this network are listed in the Appendix A. A BPR-type link cost function is used. However, as described in the next section, a slight change of its parameters was made to ensure that a unique solution could be obtained.

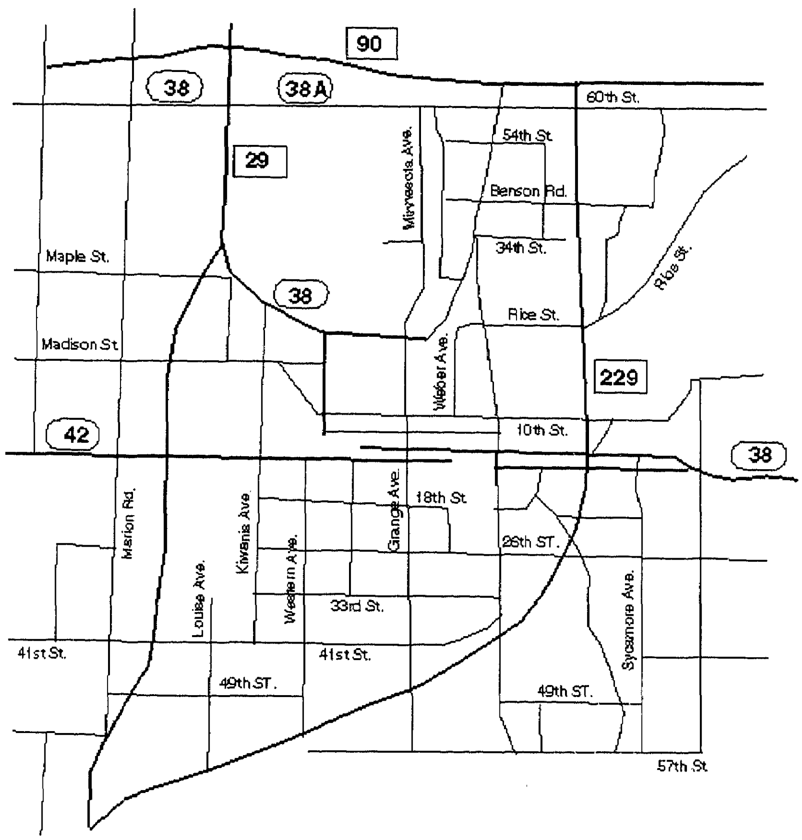

Figure 7-1 Sioux Falls City, South Dakota

7.2.1 Cost Functions

As stated before, the road users are categorized as cars and trucks. A modified BPR-type link cost function was used in Mouskos et al. (1989) and Mahmassani et al (1987) to represent the interaction of passenger cars and trucks sharing the same link. The major modification is on the enrollment of the passenger car equivalent factor of truck, as indicated in (7.1), in which t_{Aa} and t_{Ta} denote the travel time of passenger car and truck on link $a \in A$ respectively, t^0_{Aa} and t^0_{Ta} denote the free-flow travel time of passenger car and truck on link $a \in A$ respectively, K_a denotes the capacity of link $a \in A$, and α_A, α_T, β_A, and β_T are parameters.

$$t_{Aa} = t_{Aa}^0 \left(1 + \alpha_A \left(\frac{f_{Aa} + E \cdot f_{Ta}}{K_a} \right)^{\beta_A} \right) \tag{7.1a}$$

$$t_{Ta} = t_{Ta}^0 \left(1 + \alpha_T \left(\frac{f_{Aa} + E \cdot f_{Ta}}{K_a} \right)^{\beta_T} \right) \tag{7.1b}$$

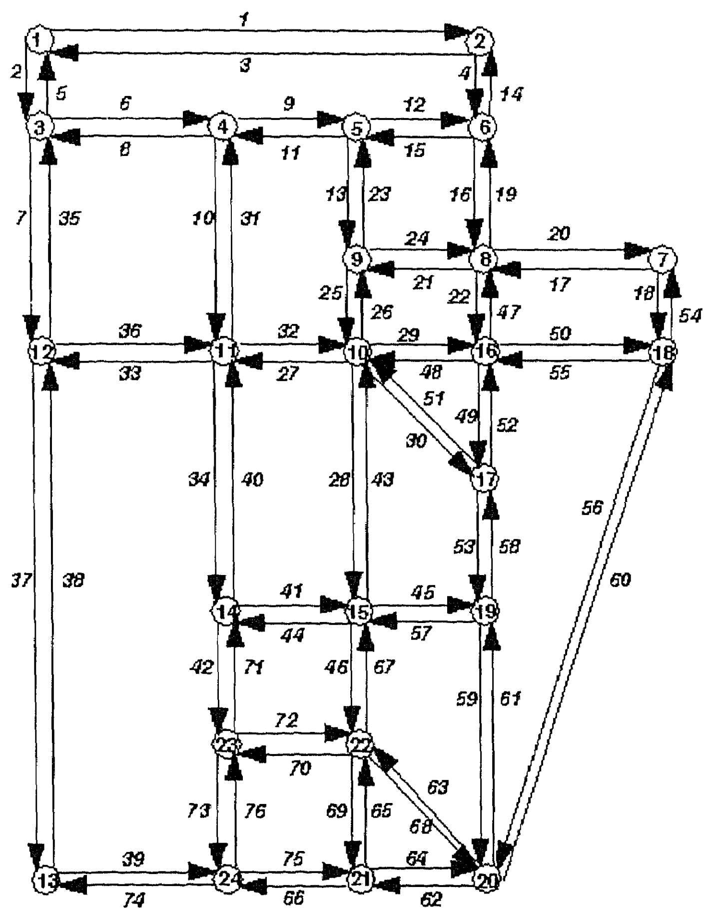

Figure 7-2 Roadway Network of Sioux Falls City

Even though this modification was not calibrated with any real data, it is consistent with the concept of passenger car equivalents for heavy vehicles as in the Highway Capacity Manual (1985 and later Edition).

Considering that in most highways, there exist lanes that are basically passenger cars only, Zeng (1998) points out that the above link cost functions tend to overestimate the impact of trucks on passenger cars, since a high percentage of passenger cars utilize the truck free lanes to avoid interaction with trucks. Due to the lack of studies to quantify this indirect impact, Zeng (1998) suggests the link cost functions, shown in (7.2), which halve the direct impact.

$$t_{Aa} = t_{Aa}^{0}\left(1+\alpha_A\left(\frac{f_{Aa}+0.5E\cdot f_{Ta}}{K_a}\right)^{\beta_A}\right) \tag{7.2a}$$

$$t_{Ta} = t_{Ta}^{0}\left(1+\alpha_T\left(\frac{0.5f_{Aa}+E\cdot f_{Ta}}{K_a}\right)^{\beta_T}\right) \tag{7.2b}$$

It is also proved by Zeng (1998) that when $E = 4$, $t_{Aa}^{0} = 1$ minute/mile, $t_{Ta}^{0} = 1.2$ minute/mile, $\alpha_A = \alpha_T = 0.15$, and $\beta_A = \beta_T = 4$, the cost functions (7.2) would generate a positive definite Jacobian matrix of link cost functions, which ensures the uniqueness of the user equilibrium solution. In this dissertation, the modified BPR-type cost function (7.2) is used.

7.2.2 Deterministic Equilibrium

The deterministic user equilibrium link flows as well as the V/C ratio on the Sioux Falls City network are shown in Table 7-1.

Table 7-1 Sioux Falls Network Deterministic Equilibrium Flows

Link	Capacity (thousand)	Flows (thousand)		V/C
		Car	Truck	
1	25.9002	6.3213	0.1925	0.2738
2	23.4035	10.1152	0.3465	0.4914
3	25.9002	6.3532	0.1925	0.2750
4	4.9582	6.1123	0.3465	1.5123
5	23.4035	10.0833	0.3465	0.4901
6	17.1105	17.0859	0.6490	1.1503
7	23.4035	12.7430	0.4455	0.6206
8	17.1105	17.0488	0.6490	1.1481
9	17.7828	20.2664	0.9680	1.3574
10	4.9088	5.5794	0.3300	1.4055
11	17.7828	20.3408	0.9680	1.3616
12	4.9480	7.5715	0.6435	2.0504
13	10.0000	17.6064	0.7040	2.0422
14	4.9582	6.1442	0.3465	1.5187
15	4.9480	7.5884	0.6435	2.0538
16	4.8986	13.0567	0.7920	3.3121
17	7.8418	13.8383	0.5115	2.0256
18	23.4035	17.7485	0.7260	0.8825
19	4.8986	13.1057	0.7920	3.3221
20	7.8418	13.7691	0.5115	2.0168
21	5.0502	7.9943	0.2090	1.7485
22	5.0458	8.3855	0.5500	2.0979
23	10.0000	17.6638	0.7040	2.0480
24	5.0502	7.9484	0.2090	1.7394
25	13.9158	23.1301	1.1330	1.9878
26	13.9158	23.2461	1.1385	1.9977
27	10.0000	18.3051	0.9295	2.2023
28	13.5120	24.9301	1.0263	2.1489
29	5.1335	11.1223	0.7502	2.7512
30	4.9935	8.4645	0.4235	2.0343
31	4.9088	5.5725	0.3355	1.4086
32	10.0000	18.2250	0.9238	2.1920
33	4.9088	8.1454	0.5390	2.0985
34	4.8765	9.6935	0.7205	2.5788
35	23.4035	12.7481	0.4455	0.6209
36	4.9088	8.1454	0.5390	2.0986
37	25.9002	14.6089	0.6760	0.6684
38	25.9002	14.6236	0.6765	0.6691

Table 7-1 (Continued)

Link	Capacity (thousand)	Flows(thousand)		V/C
		Car	Truck	
39	5.0913	10.9970	0.6160	2.6439
40	4.8765	9.7109	0.7258	2.5867
41	5.1275	9.4679	0.4127	2.1684
42	4.9248	8.5405	0.4290	2.0826
43	13.5120	24.9946	1.0307	2.1549
44	5.1275	9.5020	0.4180	2.1793
45	15.6508	19.3896	0.9427	1.4798
46	10.3150	17.6419	1.0219	2.1066
47	5.0458	8.4112	0.5500	2.1030
48	5.1335	11.1493	0.7515	2.7575
49	5.2299	11.6483	0.6655	2.7362
50	19.6799	18.9199	0.7722	1.1183
51	4.9935	8.4645	0.4235	2.0343
52	5.2299	11.6341	0.6655	2.7335
53	4.8240	9.0801	0.7700	2.5208
54	23.4035	17.8176	0.7260	0.8854
55	19.6799	18.9868	0.7735	1.1220
56	23.4035	21.6888	0.9097	1.0822
57	15.6508	19.4196	0.9468	1.4828
58	4.8240	9.0660	0.7700	2.5178
59	5.0026	9.2786	0.4389	2.2057
60	23.4035	21.7203	0.9055	1.0828
61	5.0026	9.2944	0.4430	2.2121
62	5.0599	6.7467	0.3971	1.6473
63	5.0757	7.3150	0.3850	1.7446
64	5.0599	6.6895	0.3915	1.6316
65	5.2299	8.0378	0.5710	1.9736
66	4.8854	10.1789	0.7149	2.6689
67	10.3150	17.6061	1.0220	2.1032
68	5.0757	7.3150	0.3850	1.7446
69	5.2299	8.0303	0.5708	1.9720
70	5.0000	9.7240	0.5721	2.4024
71	4.9248	8.5239	0.4290	2.0793
72	5.0000	9.6807	0.5720	2.3937
73	5.0785	8.2428	0.3136	1.8700
74	5.0913	11.0022	0.6160	2.6449
75	4.8854	10.1293	0.7095	2.6543
76	5.0785	8.1828	0.3135	1.8582

Statistics show that among all 76 links in the network, 2 links carry flows that are over three times their capacities, 38 links carry flows that are more than twice their capacities, and 26 links carry flows that are larger than their capacities. Figure 7-3 illustrates the distribution of the V/C ratio at equilibrium, which indicates that most links in the Sioux Falls City network are extremely overloaded. The most crowded arterials include $16 \rightarrow 22 \rightarrow 49 \rightarrow 53 \rightarrow 59$, $61 \rightarrow 58 \rightarrow 52 \rightarrow 47 \rightarrow 19$, and $39 \rightarrow 75$, $66 \rightarrow 74$. All these links have V/C ratios higher than 2.0. The average V/C at equilibrium is 1.84. The total travel cost at equilibrium is 111.011, compared to that at the system optimum of 108.863.

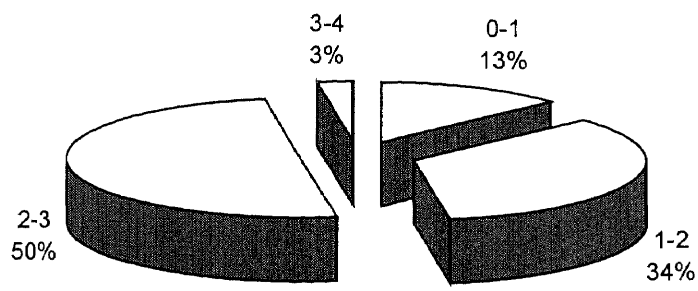

Figure 7-3 V/C Distribution at Deterministic Equilibrium

7.2.3 Stochastic User Equilibrium

The stochastic user equilibrium link flows as well as the V/C ratio on the Sioux Falls City network are shown in Table 7-2.

Table 7-2 Sioux Falls Network Stochastic Equilibrium Flows

Link	Capacity (thousand)	Flows (thousand)		V/C
		Car	Truck	
1	25.9002	6.0146	0.3120	0.2804
2	23.4035	9.0680	0.4650	0.4669
3	25.9002	5.9472	0.3093	0.2774
4	4.9582	7.0132	0.3733	1.7156
5	23.4035	9.1354	0.4677	0.4703
6	17.1105	16.1169	0.8307	1.1361
7	23.4035	13.1913	0.6811	0.6801
8	17.1105	16.5416	0.8535	1.1663
9	17.7828	19.4149	1.0035	1.3175
10	4.9088	9.7947	0.5311	2.4281
11	17.7828	19.6623	1.0026	1.3312
12	4.9480	7.2517	0.3587	1.7556
13	10.0000	17.4365	0.9024	2.1046
14	4.9582	6.9458	0.3706	1.6999
15	4.9480	7.8910	0.4001	1.9182
16	4.8986	12.9736	0.6842	3.2071
17	7.8418	16.2124	0.8580	2.5051
18	23.4035	16.4916	0.8643	0.8524
19	4.8986	13.5455	0.7228	3.3554
20	7.8418	16.0154	0.8622	2.4821
21	5.0502	10.0185	0.5588	2.4264
22	5.0458	8.1479	0.4355	1.9600
23	10.0000	17.0446	0.8602	2.0485
24	5.0502	10.4082	0.6128	2.5463
25	13.9158	22.7856	1.1797	1.9765
26	13.9158	22.8879	1.1970	1.9888
27	10.0000	18.7470	1.0108	2.2790
28	13.5120	24.6684	1.2651	2.2002
29	5.1335	11.8618	0.5862	2.7674
30	4.9935	11.9100	0.7479	2.9842
31	4.9088	10.0765	0.5602	2.5093
32	10.0000	18.7150	1.0045	2.2733
33	4.9088	8.3490	0.4540	2.0707
34	4.8765	10.8432	0.5824	2.7013
35	23.4035	12.8340	0.6610	0.6614
36	4.9088	7.9951	0.4290	1.9783
37	25.9002	15.0016	0.7899	0.7012
38	25.9002	14.2999	0.7454	0.6672

Table 7-2 (Continued)

Link	Capacity (thousand)	Flows (thousand)		V/C
		Car	Truck	
39	5.0913	11.1007	0.5800	2.6360
40	4.8765	11.5514	0.6357	2.8902
41	5.1275	9.5723	0.5082	2.2633
42	4.9248	9.7412	0.5246	2.4041
43	13.5120	24.5734	1.2593	2.1914
44	5.1275	10.1240	0.5469	2.4011
45	15.6508	14.4173	0.6816	1.0954
46	10.3150	18.0107	0.9120	2.0997
47	5.0458	8.1331	0.4243	1.9482
48	5.1335	11.9552	0.5915	2.7897
49	5.2299	16.9598	1.0358	4.0351
50	19.6799	18.4499	0.9516	1.1309
51	4.9935	11.9414	0.7666	3.0054
52	5.2299	16.7225	0.9997	3.9621
53	4.8240	11.5674	0.6482	2.9354
54	23.4035	16.6886	0.8602	0.8601
55	19.6799	18.7658	0.9817	1.1531
56	23.4035	20.2412	1.0356	1.0419
57	15.6508	15.1310	0.7380	1.1554
58	4.8240	11.3615	0.6307	2.8782
59	5.0026	11.1863	0.5734	2.6946
60	23.4035	20.6497	1.0561	1.0628
61	5.0026	11.6941	0.6123	2.8272
62	5.0599	6.9429	0.3708	1.6653
63	5.0757	11.0530	0.5923	2.6444
64	5.0599	9.3330	0.5169	2.2531
65	5.2299	6.4905	0.3100	1.4782
66	4.8854	8.7792	0.4478	2.1637
67	10.3150	17.6492	0.8830	2.0534
68	5.0757	9.4746	0.5002	2.2608
69	5.2299	7.8519	0.3937	1.8024
70	5.0000	10.3181	0.5470	2.5012
71	4.9248	9.8977	0.5393	2.4478
72	5.0000	9.7397	0.5095	2.3555
73	5.0785	9.4379	0.5008	2.2528
74	5.0913	10.3895	0.5350	2.4609
75	4.8854	9.8079	0.5102	2.4253
76	5.0785	9.0159	0.4779	2.1517

In stochastic equilibrium, as indicated in Table 7-2, of all 76 links one carries a flow that is over four times over its capacity, 4 links carry flows that are over three times their capacities, 39 links carry flows that are over two times their capacities, while only 10 links carry flows that are lower than their capacities. The average V/C ratio is 2.0035 for the stochastic equilibrium. The most crowed links include 16, 19, 49, 51, 52, etc. Figure 7-4 shows the distribution of V/C ratios at stochastic equilibrium.

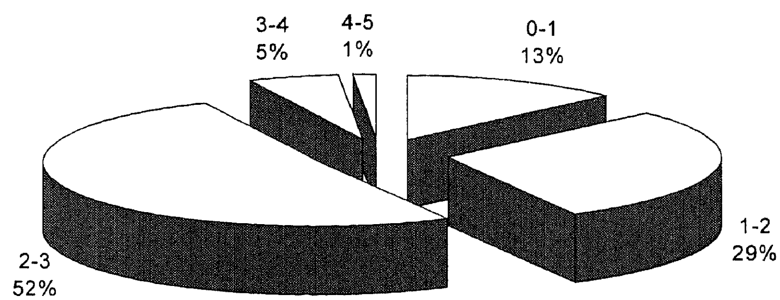

Figure 7-4 V/C Distribution at Stochastic Equilibrium

One critical assumption of the toll design problem is that the set of links that can be tolled is already known. For the purpose of illustration, several links with V/C ratios higher than certain value are selected as the tollable links. The issue of how to select the links to be tolled will be discussed later. For now, it is assumed the set of links that can be tolled is {16, 19, 22, 47, 49, 52, 53, 58, 59, 61} for both deterministic and stochastic toll design problems.

7.3 Deterministic Toll Design Problem

7.3.1 Results

Table 7-3 shows the link flows from the toll design results based on the given set of links that can be tolled. From this table, one can see that there are still 2 links with V/C higher than 3.0, 41 links with V/C between 2.0 and 3.0, 23 links with V/C between 1.0 and 2.0, and 10 links with V/C lower than 1.0. Compared to the situation in the pre-toll equilibrium, even though the number of links that fall in the higher V/C category increased, the average V/C decreased, from 1.8393 to 1.8372. Furthermore, total travel cost (*exclusive of tolls*), the most important criterion, decreased from 111.01 before tolls to 110.72 after tolls are placed, as indicated in Table 7-4. This is a 0.3% decrease based on the pre-toll total travel time, and this decrease accounts for 13.5% of the maximum possible decrease (from 111.01 to 108.86). For specific user groups, after placing the tolls as indicated in Table 7-4, the total cost decrease varies. For passenger cars, the decrease accounts for 12% of the maximum possible decrease, and for trucks, the total cost after the tolls is actually lower than that in the system optimum as well as equilibrium without tolls. This shows that heavy trucks have forced passenger cars to take routes with higher cost from a society point of view.

Table 7-3 Sioux Falls Network Deterministic Toll Design

Link	Capacity (thousand)	Flows (thousand)		Tolls		V/C
		Car	Truck	Car	Truck	
1	25.9002	6.2949	0.1925	-	-	0.2728
2	23.4035	10.0845	0.3465	-	-	0.4901
3	25.9002	6.3225	0.1925	-	-	0.2738
4	4.9582	6.0859	0.3465	-	-	1.5070
5	23.4035	10.0569	0.3465	-	-	0.4889
6	17.1105	17.3150	0.6490	-	-	1.1637
7	23.4035	12.9431	0.4455	-	-	0.6292
8	17.1105	17.2797	0.6490	-	-	1.1616
9	17.7828	20.2743	0.9680	-	-	1.3578
10	4.9088	5.6201	0.3300	-	-	1.4138
11	17.7828	20.3461	0.9680	-	-	1.3619
12	4.9480	7.5415	0.6435	-	-	2.0444
13	10.0000	17.6443	0.7040	-	-	2.0460
14	4.9582	6.1135	0.3465	-	-	1.5125
15	4.9480	7.5560	0.6435	-	-	2.0473
16	4.8986	13.0004	0.7920	0.0148	5.2391	3.3006
17	7.8418	14.1959	0.5115	-	-	2.0712
18	23.4035	17.9129	0.7260	-	-	0.8895
19	4.8986	13.0425	0.7920	0.0147	5.2387	3.3092
20	7.8418	14.1411	0.5115	-	-	2.0642
21	5.0502	8.1512	0.2090	-	-	1.7796
22	5.0458	8.1274	0.5500	0.0169	2.3189	2.0467
23	10.0000	17.7015	0.7040	-	-	2.0518
24	5.0502	8.1186	0.2090	-	-	1.7731
25	13.9158	23.2054	1.1330	-	-	1.9932
26	13.9158	23.3345	1.1385	-	-	2.0041
27	10.0000	18.3382	0.9295	-	-	2.2056
28	13.5120	24.7868	1.0089	-	-	2.1331
29	5.1335	11.2574	0.7676	-	-	2.7910
30	4.9935	8.4645	0.4235	-	-	2.0343
31	4.9088	5.6174	0.3355	-	-	1.4177
32	10.0000	18.2544	0.9249	-	-	2.1954
33	4.9088	8.1686	0.5390	-	-	2.1033
34	4.8765	9.6318	0.7205	-	-	2.5661
35	23.4035	12.9509	0.4455	-	-	0.6295
36	4.9088	8.1673	0.5390	-	-	2.1030
37	25.9002	14.8322	0.6760	-	-	0.6771
38	25.9002	14.8482	0.6765	-	-	0.6778

Table 7-3 (Continued)

Link	Capacity (thousand)	Flows (thousand)		Tolls		V/C
		Car	Truck	Car	Truck	
39	5.0913	10.9530	0.6160	-	-	2.6353
40	4.8765	9.6511	0.7269	-	-	2.5754
41	5.1275	9.4829	0.4116	-	-	2.1705
42	4.9248	8.5567	0.4290	-	-	2.0859
43	13.5120	24.8492	1.0276	-	-	2.1432
44	5.1275	9.5225	0.4180	-	-	2.1832
45	15.6508	19.3276	0.9589	-	-	1.4800
46	10.3150	17.5507	1.0890	-	-	2.1238
47	5.0458	8.1473	0.5500	0.0170	2.3190	2.0507
48	5.1335	11.3035	0.7535	-	-	2.7890
49	5.2299	10.7459	0.6655	0.0426	3.2745	2.5637
50	19.6799	19.8861	0.7896	-	-	1.1710
51	4.9935	8.4645	0.4235	-	-	2.0343
52	5.2299	10.7292	0.6655	0.0438	3.2743	2.5605
53	4.8240	8.5130	0.7700	0.0211	4.5780	2.4032
54	23.4035	17.9677	0.7260	-	-	0.8918
55	19.6799	19.9688	0.7755	-	-	1.1723
56	23.4035	22.5049	0.9271	-	-	1.1201
57	15.6508	19.3199	1.0120	-	-	1.4931
58	4.8240	8.4963	0.7700	0.0215	4.5787	2.3997
59	5.0026	9.7863	0.3544	0.0066	0.0066	2.2397
60	23.4035	22.5379	0.9075	-	-	1.1181
61	5.0026	9.7619	0.4075	0.0022	0.0022	2.2772
62	5.0599	6.5984	0.3300	-	-	1.5649
63	5.0757	7.3150	0.3850	-	-	1.7446
64	5.0599	6.5024	0.3580	-	-	1.5681
65	5.2299	8.1833	0.6045	-	-	2.0271
66	4.8854	10.1340	0.7150	-	-	2.6598
67	10.3150	17.5559	1.0555	-	-	2.1113
68	5.0757	7.3150	0.3850	-	-	1.7446
69	5.2299	8.1336	0.6380	-	-	2.0432
70	5.0000	9.7408	0.5720	-	-	2.4058
71	4.9248	8.5364	0.4290	-	-	2.0818
72	5.0000	9.6963	0.5720	-	-	2.3969
73	5.0785	8.2450	0.3135	-	-	1.8704
74	5.0913	10.9595	0.6160	-	-	2.6366
75	4.8854	10.0878	0.7095	-	-	2.6458
76	5.0785	8.1802	0.3135	-	-	1.8577

Table 7-4 Deterministic Total Travel Costs (5% Trucks)

Travel Cost	Car	Truck	Total
System Optimum	105.59	3.28	108.86
User Equilibrium	107.92	3.09	111.01
Toll Design Problem	107.64	3.08	110.72

7.3.2 Sensitivity Analysis

In this section, the sensitivity analysis of the results from the toll design problem with respect to the change of the percentage of trucks, as well as the network demand level is conducted.

When the percentage of trucks on the network increases, it is expected that the total travel cost may rapidly increase. On the Sioux Falls City network, if the percentage of trucks is increased from 5% to 10%, then the total cost is shown in Table 7-5.

Table 7-5 Deterministic Total Travel Costs (10% Trucks)

Travel Cost	Car	Truck	Total
System Optimum	113.12	8.60	121.72
User Equilibrium	115.08	8.63	123.70
Toll Design Problem	114.81	8.61	123.42

It is shown in this table that total travel cost (*exclusive of tolls*), the most important criterion, decreased from 123.70 before tolls to 123.42 after tolls are placed. This is a 0.2% decrease based on the pre-toll total travel time, and this decrease accounts for 14.1% of the maximum possible decrease (from 123.70 to 121.72). For specific user groups, placing the tolls on the arterial specified earlier would cause various changes on the total cost. For passenger cars, the decrease accounts for 14.1% of the maximum possible decrease, and for trucks, the decrease accounts for 67% of the maximum possible decrease.

Another aspect to look at is the influence of various demand levels on the effectiveness of the tolls. Here, the toll design model is tested on less congested networks, which carry 70% and 50% of original demands. Table 7-6 and Table 7-7 show the total travel costs for both cases.

Table 7-6 Deterministic Total Travel Costs (70% Demand)

Travel Cost	Car	Truck	Total
System Optimum	37.82	1.94	39.76
User Equilibrium	39.77	1.74	41.51
Toll Design Problem	39.57	1.73	41.30

Table 7-7 Deterministic Total Travel Costs (50% Demand)

Travel Cost	Car	Truck	Total
System Optimum	20.55	1.27	21.82
User Equilibrium	21.41	1.15	22.56
Toll Design Problem	21.17	1.15	22.32

In the 70% demand case, 0.5% of cost decrease accounts for 12% of maximum possible decrease, while in the 50% original demand case, 0.6% of cost decrease accounts for 32.4% of maximum possible decrease. It may be concluded that even though the absolute value of the total cost decrease remains small, the percentage of this decrease in the maximum possible decrease is increasing with lower demand levels. One reason for this to happen is that the test network with the original demand level is very congested, which can be seen from Table 7-1. When the network-wide congestion level increases, the travel cost (*exclusive of tolls*) increases very quick, which may outweigh the factor of tolls, so there will be fewer routes with less cost that a traveler can switch to after tolls are in place. However, for less congested network, travelers could be much more sensitive to the tolls, and charging tolls can effectively reroute traffic, and thus more effectively reduce the total travel cost.

It is also observed that in less congested networks, the system optimum costs for heavy trucks are higher than those of equilibrium as well as the toll design model. It is well known that trucks have more substantial effects on cars than the effects of cars on trucks, and the number of trucks is much smaller than the number of cars in the traffic stream. Therefore, it would be reasonable to increase the travel cost of trucks in order to decrease the travel cost of cars. This would explain the above observation.

7.4 Stochastic Toll Design Problem

7.4.1 Results

Table 7-8 shows the results on link flows and tolls from the stochastic toll design problem. It is observed that trucks are usually charged much higher than cars, similar to the results of the deterministic toll design problem. The average V/C ratio after placing the tolls shown in Table 7-8 is 1.9668, compared to 2.0035 before the tolls. Hence, charging tolls on Links 16, 19, 22, 47, 49, 52, 53, 58, 59, and 61 can certainly reduce the total travel time on the network.

Table 7-8 Sioux Falls Network Stochastic Toll Design

Link	Capacity (thousand)	Flows (thousand)		Tolls		V/C
		Car	Truck	Car	Truck	
1	25.9002	6.0181	0.2929	-	-	0.2776
2	23.4035	9.2159	0.4750	-	-	0.4750
3	25.9002	5.9257	0.2754	-	-	0.2713
4	4.9582	6.9359	0.3259	-	-	1.6618
5	23.4035	9.3083	0.4925	-	-	0.4819
6	17.1105	16.1567	0.8509	-	-	1.1432
7	23.4035	13.1328	0.6511	-	-	0.6724
8	17.1105	16.5977	0.8837	-	-	1.1766
9	17.7828	19.4128	1.0147	-	-	1.3199
10	4.9088	9.9136	0.5559	-	-	2.4726
11	17.7828	19.7019	1.0288	-	-	1.3393
12	4.9480	7.4166	0.3284	-	-	1.7644
13	10.0000	17.5125	1.0296	-	-	2.1631
14	4.9582	6.8435	0.3084	-	-	1.6290
15	4.9480	8.0188	0.3134	-	-	1.8740
16	4.8986	12.8139	0.5177	0	0.8547	3.0385
17	7.8418	16.4720	0.8541	-	-	2.5362
18	23.4035	16.1963	0.7786	-	-	0.8251
19	4.8986	13.3237	0.4851	0.0800	1.4231	3.1160
20	7.8418	16.1581	0.8804	-	-	2.5096
21	5.0502	10.3080	0.7830	-	-	2.6613
22	5.0458	7.7208	0.3560	-	-	1.8123
23	10.0000	17.1994	1.0588	-	-	2.1434
24	5.0502	10.5881	0.7702	-	-	2.7066
25	13.9158	23.4088	1.3160	-	-	2.0604
26	13.9158	23.4803	1.3379	-	-	2.0719
27	10.0000	18.7288	1.0246	-	-	2.2827
28	13.5120	25.0283	1.2418	-	-	2.2199
29	5.1335	11.9361	0.6576	-	-	2.8375
30	4.9935	12.1730	0.8050	-	-	3.0826
31	4.9088	10.1699	0.5801	-	-	2.5444
32	10.0000	18.7078	1.0034	-	-	2.2721
33	4.9088	8.3570	0.4668	-	-	2.0828
34	4.8765	10.8803	0.5922	-	-	2.7169
35	23.4035	12.7841	0.6358	-	-	0.6549
36	4.9088	8.0098	0.4325	-	-	1.9841
37	25.9002	15.0061	0.7950	-	-	0.7022
38	25.9002	14.3198	0.7459	-	-	0.6681

Table 7-8 (Continued)

Link	Capacity (thousand)	Flows (thousand)		Tolls		V/C
		Car	Truck	Car	Truck	
39	5.0913	11.1525	0.6104	-	-	2.6701
40	4.8765	11.5673	0.6348	-	-	2.8928
41	5.1275	9.4932	0.4778	-	-	2.2242
42	4.9248	9.9820	0.5847	-	-	2.5018
43	13.5120	24.8551	1.2528	-	-	2.2103
44	5.1275	10.0300	0.4954	-	-	2.3426
45	15.6508	14.2884	0.5291	-	-	1.0482
46	10.3150	18.6218	1.0871	-	-	2.2269
47	5.0458	7.6367	0.3625	-	-	1.8008
48	5.1335	12.0818	0.7096	-	-	2.9065
49	5.2299	12.6327	0.5016	1.0978	2.7258	2.7991
50	19.6799	18.3087	1.0650	-	-	1.1468
51	4.9935	12.1885	0.7797	-	-	3.0655
52	5.2299	12.2561	0.5731	1.0560	2.0612	2.7818
53	4.8240	10.5185	0.4695	0.1364	0.1364	2.5697
54	23.4035	16.5102	0.7524	-	-	0.8340
55	19.6799	18.7468	1.0520	-	-	1.1664
56	23.4035	20.5913	1.0945	-	-	1.0669
57	15.6508	14.6283	0.5280	-	-	1.0696
58	4.8240	10.1575	0.5157	0.2137	0.2137	2.5332
59	5.0026	10.8902	0.0660	0.3655	11.9855	2.2297
60	23.4035	21.2388	1.0497	-	-	1.0869
61	5.0026	10.8691	0.1111	0.5203	4.6007	2.2615
62	5.0599	6.7724	0.3273	-	-	1.5972
63	5.0757	11.4304	0.7134	-	-	2.8142
64	5.0599	9.1821	0.4960	-	-	2.2068
65	5.2299	6.7791	0.3874	-	-	1.5925
66	4.8854	8.8328	0.4648	-	-	2.1886
67	10.3150	18.5411	1.1113	-	-	2.2284
68	5.0757	9.5427	0.5395	-	-	2.3052
69	5.2299	8.0704	0.4541	-	-	1.8905
70	5.0000	10.4430	0.6037	-	-	2.5716
71	4.9248	10.1321	0.6097	-	-	2.5526
72	5.0000	9.7659	0.5208	-	-	2.3698
73	5.0785	9.5833	0.5415	-	-	2.3135
74	5.0913	10.4567	0.5608	-	-	2.4945
75	4.8854	9.9511	0.5668	-	-	2.5010
76	5.0785	9.0562	0.4836	-	-	2.1641

Table 7-9 shows the total travel cost of the stochastic toll design problem, compared with those of system optimum and stochastic equilibrium. There is a 6.2% decrease in the total travel cost, and it accounts for 17.3% of the maximum possible decrease.

Table 7-9 Stochastic Total Travel Costs (5% Trucks)

Travel Cost	Car	Truck	Total
System Optimum	105.59	3.28	108.86
User Equilibrium	164.80	4.03	168.83
Toll Design Problem	154.44	3.98	158.43

7.4.2 Sensitivity Analysis

When the percentage of trucks in the total demand is increased, the toll design model gives a total travel cost of 176.73, which is a 6.1% decrease from the total cost of stochastic equilibrium, which is 188.17, as shown in Table 7-10. This decrease also accounts for 17.2% of the maximum possible decrease, from 188.17 to 121.72.

Table 7-10 Stochastic Total Travel Costs (10% Trucks)

Travel Cost	Car	Truck	Total
System Optimum	113.12	8.60	121.72
User Equilibrium	175.99	12.18	188.17
Toll Design Problem	165.02	11.71	176.73

Similar to the deterministic toll design problem, the stochastic toll design model is tested on less congested networks, which carry 70% and 50% of the original demands respectively. The total travel costs for both cases are listed in Table 7-11 and Table 7-12. In the 70% demand case, a 4.7% cost decrease accounts for 16.0% of maximum possible cost decrease, from 56.39 to 39.76. In the 50% demand case, a 2.1% cost decrease accounts for 10.7% of maximum possible cost decrease, from 27.04 to 21.82.

Table 7-11 Stochastic Total Travel Costs (70% Demand)

Travel Cost	Car	Truck	Total
System Optimum	37.82	1.94	39.76
User Equilibrium	54.31	2.08	56.39
Toll Design Problem	51.66	2.07	53.73

Table 7-12 Stochastic Total Travel Costs (50% Demand)

Travel Cost	Car	Truck	Total
System Optimum	20.55	1.27	21.82
User Equilibrium	25.70	1.34	27.04
Toll Design Problem	25.14	1.34	26.48

7.5 Special Case of Linear Cost Functions

As discussed in Chapter 3, the solution procedure of the deterministic based toll design problem can be greatly simplified if linear instead of nonlinear cost function is applied. In this section, the nonlinear BPR-type cost function is replaced with its first-order approximation at the pre-toll equilibrium point. The following general steps are taken:

(1) Solve the pre-toll user equilibrium with nonlinear BPR-type cost function;

(2) Linearize the cost function at the equilibrium point;

(3) Solve for system optimum, user equilibrium, and then the toll design problem.

Assume the set of links that can be tolled remains {16, 19, 22, 47, 49, 52, 53, 58, 59, 61}, the results of the toll design problem after applying linearized cost function are listed in Table 7-13.

Table 7-13 Deterministic Toll Design with Linear Cost Function

Link	Capacity (thousand)	Flows (thousand)		Tolls		V/C
		Car	Truck	Car	Truck	
1	25.9002	6.2479	0.1925	-	-	0.2710
2	23.4035	10.0324	0.3465	-	-	0.4879
3	25.9002	6.2704	0.1925	-	-	0.2718
4	4.9582	6.0389	0.3465	-	-	1.4975
5	23.4035	10.0099	0.3465	-	-	0.4869
6	17.1105	17.5670	0.6490	-	-	1.1784
7	23.4035	13.1449	0.4455	-	-	0.6378
8	17.1105	17.5335	0.6490	-	-	1.1764
9	17.7828	20.2888	0.9680	-	-	1.3587
10	4.9088	5.6705	0.3300	-	-	1.4241
11	17.7828	20.3619	0.9680	-	-	1.3628
12	4.9480	7.4988	0.6435	-	-	2.0357
13	10.0000	17.7015	0.7040	-	-	2.0518
14	4.9582	6.0614	0.3465	-	-	1.5020
15	4.9480	7.5102	0.6435	-	-	2.0380
16	4.8986	12.9107	0.7920	0.0321	4.4910	3.2823
17	7.8418	14.6415	0.5115	-	-	2.1280
18	23.4035	18.1781	0.7260	-	-	0.9008
19	4.8986	12.9445	0.7920	0.0337	4.4903	3.2892
20	7.8418	14.5778	0.5115	-	-	2.1199
21	5.0502	8.3364	0.2090	-	-	1.8162
22	5.0458	7.7750	0.5500	0.0389	2.0037	1.9769
23	10.0000	17.7632	0.7040	-	-	2.0579
24	5.0502	8.2926	0.2090	-	-	1.8076
25	13.9158	23.2602	1.1330	-	-	1.9972
26	13.9158	23.3825	1.1385	-	-	2.0075
27	10.0000	18.3756	0.9295	-	-	2.2094
28	13.5120	24.6039	1.0224	-	-	2.1236
29	5.1335	11.4586	0.7541	-	-	2.8197
30	4.9935	8.4645	0.4235	-	-	2.0343
31	4.9088	5.6685	0.3355	-	-	1.4281
32	10.0000	18.2877	0.9263	-	-	2.1993
33	4.9088	8.1940	0.5390	-	-	2.1085
34	4.8765	9.5725	0.7205	-	-	2.5540
35	23.4035	13.1559	0.4455	-	-	0.6383
36	4.9088	8.1926	0.5390	-	-	2.1082
37	25.9002	15.0594	0.6760	-	-	0.6858
38	25.9002	15.0785	0.6765	-	-	0.6867

Table 7-13 (Continued)

Link	Capacity (thousand)	Flows (thousand)		Tolls		V/C
		Car	Truck	Car	Truck	
39	5.0913	10.9097	0.6160	-	-	2.6268
40	4.8765	9.5885	0.7283	-	-	2.5637
41	5.1275	9.5074	0.4102	-	-	2.1742
42	4.9248	8.5691	0.4290	-	-	2.0884
43	13.5120	24.6796	1.0262	-	-	2.1303
44	5.1275	9.5384	0.4180	-	-	2.1863
45	15.6508	19.2737	1.0038	-	-	1.4880
46	10.3150	17.5676	1.0890	-	-	2.1254
47	5.0458	7.7890	0.5500	0.0396	2.0040	1.9797
48	5.1335	11.4886	0.7535	-	-	2.8251
49	5.2299	9.6656	0.6655	0.0997	2.8330	2.3571
50	19.6799	21.0378	0.7761	-	-	1.2267
51	4.9935	8.4645	0.4235	-	-	2.0343
52	5.2299	9.6453	0.6655	0.0996	2.8327	2.3533
53	4.8240	7.9209	0.7700	0.0454	3.9346	2.2804
54	23.4035	18.2417	0.7260	-	-	0.9035
55	19.6799	21.1021	0.7755	-	-	1.2299
56	23.4035	23.3825	0.9136	-	-	1.1553
57	15.6508	19.3040	1.0120	-	-	1.4921
58	4.8240	7.9005	0.7700	0.0456	3.9348	2.2762
59	5.0026	10.3282	0.3679	0.0045	0.0045	2.3587
60	23.4035	23.4060	0.9075	-	-	1.1552
61	5.0026	10.3382	0.3761	0.0041	0.0041	2.3673
62	5.0599	6.3419	0.3300	-	-	1.5142
63	5.0757	7.3150	0.3850	-	-	1.7446
64	5.0599	6.2709	0.3266	-	-	1.4975
65	5.2299	8.3713	0.6359	-	-	2.0870
66	4.8854	10.0984	0.7150	-	-	2.6525
67	10.3150	17.5395	1.0869	-	-	2.1219
68	5.0757	7.3150	0.3850	-	-	1.7446
69	5.2299	8.3546	0.6380	-	-	2.0854
70	5.0000	9.7593	0.5720	-	-	2.4095
71	4.9248	8.5542	0.4290	-	-	2.0854
72	5.0000	9.7144	0.5720	-	-	2.4005
73	5.0785	8.2403	0.3135	-	-	1.8695
74	5.0913	10.9193	0.6160	-	-	2.6287
75	4.8854	10.0442	0.7095	-	-	2.6369
76	5.0785	8.1805	0.3135	-	-	1.8577

Similar to the case with nonlinear cost function, the total travel cost (*exclusive of tolls*) of the toll design problem lies between those of the system optimum and user equilibrium, as indicated in Table 7-14.

Table 7-14 Deterministic Total Travel Costs with Linear Cost Function

Travel Cost	Car	Truck	Total
System Optimum	103.54	3.45	106.99
User Equilibrium	107.92	3.09	111.01
Toll Design Problem	107.34	3.07	110.42

As discussed earlier the deterministic based toll design problem with linear cost function can be solved exactly, since it can be converted into a quadratic programming problem with linear constraints. The solution obtained from such a problem is guaranteed to be the global optimum. Comparing the results obtained from the toll design problems with nonlinear as well as linear cost functions, one can find they are quite close to each other. This provides a side-proof that the results of the toll design problem with nonlinear cost function is a reasonably good solution among all the local optima.

7.6 Determining the Tollable Links

Usually the set of links that can be tolled is preset by policy-makers. The selection of such links can be based on multiple criteria, including the category of the link (e.g. whether it is a bridge, or tunnel, or freeway, etc., or just a local road segment), as well as the congestion level of the link. In practice, the congestion level usually is the critical factor in determining whether to place tolls on the link. In many cases, bridges, tunnels, and certain freeways are very congested during the peak hours. Such examples include the Holland Tunnel, the Lincoln Tunnel, and the George Washington Bridge connecting New York and New Jersey. The toll design methodology developed in this dissertation can also help decision-makers to select the links to be tolled, among those that are most congested.

Based on the network of Sioux Falls City, various tolling schemes are tested based on the V/C on each link of the network. Specifically, those most congested links are tolled. These links are selected according to their V/C values at the pre-toll equilibrium. Table 7-15 shows the comparison of results from various sets of tollable links.

Table 7-15 Total Costs of Different Tolling Schemes

Total Cost		Car	Truck	Total
System Optimum		105.59	3.28	108.86
User Equilibrium		107.92	3.09	111.01
Toll Design				
	> 3.00	107.92	3.09	111.01
	> 2.75	107.87	3.09	110.95
	> 2.50	107.56	3.08	110.63
V/C	> 2.25	107.55	3.08	110.63
	> 2.00	106.83	3.06	109.89
	> 1.75	106.83	3.06	109.89
	> 1.50	106.83	3.06	109.89

The results in the above table confirmed the general rule that the more links are tolled, the lower the total travel cost (*exclusive of tolls*).

7.7 Performance of Algorithms

Three algorithms, LANCELOT, SNOPT and LOQO, are tested on this Sioux Falls City network, respectively. LOQO is the only one that would give solutions for both deterministic and stochastic toll design problems. LANCELOT failed to converge when solving the deterministic user equilibrium using diagonalization algorithm. While solving the toll design problem, LANCELOT stopped without giving a solution. SNOPT did well when solving the stochastic toll design problem, but terminated at a point that is even not a local optimum. With the iterative linearization method, it worked well in solving deterministic toll design problem, but failed to converge in the case of stochastic toll design problem.

CHAPTER 8

CONCLUSIONS AND FUTURE RESEARCH

8.1 Conclusions

Expanding infrastructure capacity has become impractical in many urban areas. As a result, congestion pricing has been widely suggested as a viable alternative to reduce congestion. In this dissertation, a methodology for toll design was developed to provide policy-makers with suggestions on both where to charge tolls and how much the tolls should be. This methodology is capable of dealing with situations in which only a very small number of links can be tolled. Furthermore, this methodology can accommodate multiple user groups.

The toll design problem is to minimize or maximize some objective subject to constraints on the tolls and constraints that ensure that the resulting flow pattern is in equilibrium. An important underlying assumption of this problem is that the set of links in the network on which tolls could be placed is already known. This is a political issue rather than an engineering issue.

The intuitive way of formulating the toll design problem is through bilevel programming, in which the upper level problem is the "transportation planner's problem" and the lower level problem is the "road users' problem". However, the bilevel formulation of the toll design problem is rather difficult to solve. Intuitively for each iteration of the upper level problem, an equilibrium traffic assignment problem (either deterministic or stochastic) must be solved at the lower level. This process is extremely time-consuming. Moreover, the feasible region of the upper level problem is the solution set of an equilibrium problem, which is most likely nonconvex. In the deterministic toll design problem the constraints are nondifferentiable, which prevents us from using gradient information to facilitate the solution process. Even though many attempts have been made, the bilevel problem remains difficult to solve. In this dissertation, the bilevel toll design problem is converted into a single level optimization problem by making certain simplifying assumption.

In the case of deterministic equilibrium, it is presumed that a traveler would choose the route that minimizes his or her individual travel cost, which could result in unused routes in the network. It is these unused routes that bring about the noncovexity of the equilibrium constraint. By identifying the set of routes that will be used after tolls are in place, all unused routes can be eliminated and only the used ones are kept. Hence, the equilibrium constraint can be converted into a series of equality constraints that only describe the costs and flows for the used routes, which makes the deterministic toll design problem become a single level, standard nonlinear optimization problem.

For the stochastic toll design problem, the equilibrium constraint is in the form of a fixed-point problem. Even though the constraint functions are differentiable, the derivative can not be obtained explicitly. In addition, the number of equilibrium constraints equals to the total number of routes (which is rather large for a real-sized network). So, based on a previous study, the toll design problem with stochastic route choice is reformulated on a link basis. This reduces the number of equilibrium constraints to the number of links of the network, which is much less than the number of routes.

The mathematical properties of these problems, as well as the interrelationships between them were also discussed. As was shown, the simplified toll design problem can be solved using many well-developed algorithms. However, when nonlinear cost functions are used, the constraints would still be nonconvex, which results in an "extremely nonconvex" feasible region. Some variations were made to ensure the convergence of algorithm. Among those packages tested, LOQO had the best performance on both the deterministic and stochastic toll design problems.

Tests of the toll design methodology were conducted on simple as well as real-world networks. Some obvious conclusions can be drawn from the results. It is well known that charging tolls can usually reduce the total travel cost. In addition, there could be multiple toll patterns that give the same flow pattern, as indicated in the results of the Braess network. It can be observed that tolls do not work as well on very congested networks as on less congested networks, as shown in the Sioux Falls City network.

Furthermore, some less obvious conclusions can be drawn from the test results. It can be concluded from the Braess network that it is possible to reach the system optimum by only tolling some links. For realistic networks such as the Sioux Fall City network, it is also possible to reduce the total travel cost by only tolling some links. In addition, the single level formulation appears to be a good approximation to the bilevel formulation of the toll design problem. Particularly in the deterministic toll design problem, the single level formulation with linear cost approximation appears to work well and is very easy to solve exactly. With this conclusion, the "real-world" toll design application can be greatly simplified.

8.2 Contributions of the Research

The toll design methodology developed in this dissertation is a contribution in the fields of congestion modeling and congestion pricing policy design.

This dissertation contributes to the field of traffic congestion modeling by developing a methodology that fills a void in the literature on network toll design. Compared to the conventional marginal social cost pricing, this method is more realistic because it deals with the case where only very few links of a network can be tolled. Furthermore, based on certain assumptions, the bilevel program is simplified to a single level program, which is a standard nonlinear optimization problem. Hence, there is no need to develop some special algorithm for solving this simplified version of the toll design problem. It also turns out that the simplifying assumption is quite easy to satisfy by determining the most probable routes. In addition, the relationships between different versions of the toll design problem have been explored, including the relationship between the deterministic bilevel and single level problems, and the relationship between the deterministic and stochastic toll design problems.

Multiple user groups have also been incorporated into the toll design model to ensure a more realistic representation of "real-world" network. It is well known that a pricing policy could face substantial objection if it fails to consider the variety of road users. In this dissertation, road users are grouped according to their operating characteristics, and it is possible to apply different tolls to them. Of course, users could also be grouped according to other criteria, such as income, which would provide quantified information when evaluating the impact of congestion pricing policy on various income groups, and help deal with the social equity issue, which may be one of the main hurdle of implementing congestion pricing policy.

In the aspect of congestion pricing policy design, this dissertation shows that in realistic network, only tolling some links could reduce total cost effectively. This provides transportation planners with a helpful decision support when designing network tolls. It is also shown that multiple user groups can have unexpected interactions from the Sioux Falls City case, where a much higher toll is put on trucks than cars. In addition, it is suggested that congestion level could affect the benefits of toll policies, as indicated in the Sioux Falls City case.

8.3 Direction of Future Research

The toll design methodology established in this dissertation falls into the category of a route choice behavior impact study, as discussed in Chapter 1. It focused on the fixed demand case. However it is well known that when the price increases, the demand will generally decrease. Similarly, when congestion tolls are in place, some people who do not have to travel during that tolling period may choose not to travel. Thus the travel demand may decrease. In order to make the toll design methodology more powerful, we will improve this model in the future to accommodate elastic demand.

Let $D_{uw}(\tau)$ denote the demand function of user group $u \in U$ between O-D pair $w \in W$. It is written as a function of network toll vector τ, which means that the demand would vary with tolls. Then the set of feasible route flow patterns, Ω, can be given by

$$\Omega = \left\{ F \in \Re_+ : \sum_{p \in P_w} F_{up} = D_{uw}(\tau), \quad u \in U, w \in W \right\}, \tag{8.1}$$

The set of equilibrium route flows, E_τ, is then given by (8.2) for deterministic toll design problem.

$$E_\tau = \left\{F^* \in \Omega : [C_u(F^*) + \Psi_u]^T (F_u - F_u^*) \geq 0, \;\; F \in \Omega\right\} \tag{8.2}$$

For stochastic toll design problem, this set can be given as follows.

$$E_\tau = \left\{ F \in \Re_+ : \; F_{up} = D_{uw}(\tau) \cdot \frac{e^{-\mu[C_{up}(F)+\Psi_{up}]}}{\sum_{i \in P_w} e^{-\mu[C_{ui}(F)+\Psi_{ui}]}}, u \in U, w \in W, p \in P_w \right\}. \tag{8.3}$$

Therefore, the toll design problem with elastic demand can be formulated as (8.4).

$$\min_{\tau, F} \quad \sum_{u \in U} C_u(F)^T \cdot F_u \tag{8.4a}$$

subject to

$$F \in E_\tau \tag{8.4b}$$

$$\tau_{ua} \geq 0 \qquad u \in U, a \in I \tag{8.4c}$$

$$\tau_{ua} = 0 \qquad u \in U, a \notin I \tag{8.4d}$$

where E_τ can be given by (8.2) or (8.3) for deterministic or stochastic toll design problem respectively. In the future, we will further explore the properties of the toll design problem with elastic demand, for both deterministic and stochastic toll design problems, as well as search for appropriate algorithms to solve them.

In addition, we will look into other capabilities of this toll design methodology, such as incorporating the parking fee design issue. In this case, we need to incorporate destination choice capability into the model, since each parking lot will be considered as a destination.

APPENDIX A

INPUT DATA OF THE SIOUX FALLS NETWORK

There are two user groups in the test network of Sioux Falls City, are cars and trucks. The travel demand for cars between the 552 O-D pairs is listed in Table A-1, and the travel demand for trucks is listed in Table A-2. The link cost function is of the format in equation system (7.2). The free-flow travel cost t^0 is given in Table A-3. Values of other parameters such as α and β are the same as those in Subsection 7.2.1.

Table A-1 Demand of Passenger Cars on the Sioux Falls Network

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1		0.1045	0.1045	0.5225	0.209	0.3135	0.5225	0.836	0.5225	1.3585	0.5225	0.209	0.5225	0.3135	0.5225	0.5225	0.418	0.1045	0.3135	0.3135	0.1045	0.418	0.3135	0.1045
2	0.1045		0.1045	0.209	0.1045	0.418	0.209	0.418	0.209	0.627	0.209	0.1045	0.3135	0.1045	0.1045	0.418	0.209	0	0.1045	0.1045	0	0.1045	0	0
3	0.1045	0.1045		0.209	0.1045	0.3135	0.1045	0.209	0.1045	0.3135	0.3135	0.209	0.1045	0.1045	0.1045	0.209	0.1045	0	0	0	0	0.1045	0.1045	0
4	0.5225	0.209	0.209		0.5225	0.418	0.418	0.7315	0.7315	1.254	1.463	0.627	0.627	0.5225	0.5225	0.836	0.5225	0.1045	0.209	0.3135	0.209	0.418	0.5225	0.209
5	0.209	0.1045	0.1045	0.5225		0.209	0.209	0.5225	0.836	1.045	0.5225	0.209	0.209	0.1045	0.209	0.5225	0.209	0	0.1045	0.1045	0.1045	0.209	0.1045	0
6	0.3135	0.418	0.3135	0.418	0.209		0.418	0.836	0.418	0.836	0.418	0.209	0.209	0.1045	0.209	0.9405	0.5225	0.1045	0.209	0.3135	0.1045	0.209	0.1045	0.1045
7	0.5225	0.209	0.1045	0.418	0.209	0.418		1.045	0.627	1.9855	0.5225	0.7315	0.418	0.209	0.5225	1.463	1.045	0.209	0.418	0.5225	0.209	0.5225	0.209	0.1045
8	0.836	0.418	0.209	0.7315	0.5225	0.836	1.045		0.836	1.672	0.836	0.627	0.627	0.418	0.627	2.299	1.463	0.3135	0.7315	0.9405	0.418	0.5225	0.3135	0.209
9	0.5225	0.209	0.1045	0.7315	0.836	0.418	0.627	0.836		2.926	1.463	0.627	0.627	0.627	0.9405	1.463	0.9405	0.209	0.418	0.627	0.3135	0.7315	0.5225	0.209
10	1.3585	0.627	0.3135	1.254	1.045	0.836	1.9855	1.672	2.926		4.18	2.09	1.9855	2.1945	4.18	4.598	4.0755	0.7315	1.881	2.6125	1.254	2.717	1.881	0.836
11	0.5225	0.209	0.3135	1.5675	0.5225	0.418	0.5225	0.836	1.463	4.0755		1.463	1.045	1.672	1.463	1.463	1.045	0.1045	0.418	0.627	0.418	1.1495	1.3585	0.627
12	0.209	0.1045	0.209	0.627	0.209	0.209	0.7315	0.627	0.627	2.09	1.463		1.4535	0.7315	0.7315	0.7315	0.627	0.209	0.3135	0.418	0.3135	0.7315	0.7315	0.5225
13	0.5225	0.3135	0.1045	0.627	0.209	0.209	0.418	0.627	0.627	1.9855	1.045	1.3585		0.627	0.7315	0.627	0.5225	0.1045	0.3135	0.627	0.627	1.3585	0.836	0.836
14	0.3135	0.1045	0.1045	0.5225	0.1045	0.1045	0.209	0.418	0.627	2.1945	1.672	0.7315	0.627		1.3585	0.7315	0.7315	0.1045	0.3135	0.5225	0.418	1.254	1.1495	0.418
15	0.5225	0.1045	0.1045	0.5225	0.209	0.209	0.5225	0.627	1.045	4.18	1.463	0.7315	0.7315	1.3585		1.254	1.5675	0.209	0.836	1.1495	0.836	2.717	1.045	0.418
16	0.5225	0.418	0.209	0.836	0.5225	0.9405	1.463	2.299	1.463	4.598	1.463	0.7315	0.627	0.7315	1.254		2.926	0.5225	1.3585	1.672	0.627	1.254	0.5225	0.3135
17	0.418	0.209	0.1045	0.5225	0.209	0.5225	1.045	1.463	0.9405	4.0755	1.045	0.627	0.5225	0.7315	1.5675	2.926		0.627	1.7765	1.7765	0.627	1.7765	0.627	0.3135
18	0.1045	0	0	0.1045	0	0.1045	0.209	0.3135	0.209	0.7315	0.209	0.209	0.1045	0.1045	0.209	0.5225	0.627		0.3135	0.418	0.1045	0.3135	0.1045	0
19	0.3135	0.1045	0	0.209	0.1045	0.209	0.418	0.7315	0.418	1.881	0.418	0.3135	0.3135	0.3135	0.836	1.3585	1.7765	0.3135		1.254	0.418	1.254	0.3135	0.1045
20	0.3135	0.1045	0	0.3135	0.1045	0.3135	0.5225	0.9405	0.627	2.6125	0.627	0.5225	0.627	0.5225	1.1495	1.672	1.7765	0.418	1.254		1.254	2.508	0.7315	0.418
21	0.1045	0	0	0.209	0.1045	0.1045	0.209	0.418	0.3135	1.254	0.418	0.3135	0.627	0.418	0.836	0.627	0.627	0.1045	0.418	1.254		1.881	0.7315	0.5225
22	0.418	0.1045	0.1045	0.418	0.209	0.209	0.5225	0.5225	0.7315	2.717	1.1495	0.7315	1.3585	1.254	2.717	1.254	1.7765	0.3135	1.254	2.508	1.881		2.1945	1.1495
23	0.3135	0	0.1045	0.5225	0.1045	0.1045	0.209	0.3135	0.5225	1.881	1.3585	0.7315	0.836	1.1495	1.045	0.5225	0.627	0.1045	0.3135	0.7315	0.7315	2.1945		0.7315
24	0.1045	0	0	0.209	0	0.1045	0.1045	0.209	0.209	0.836	0.627	0.5225	0.7315	0.418	0.418	0.3135	0.3135	0	0.1045	0.418	0.5225	1.1495	0.7315	

Table A-2 Demand of Trucks on the Sioux Falls Network

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1		0.0055	0.0055	0.0275	0.011	0.0165	0.0275	0.044	0.0275	0.0715	0.0275	0.011	0.0275	0.0165	0.0275	0.0275	0.022	0.0055	0.0165	0.0165	0.0055	0.022	0.0165	0.0055
2	0.0055		0.0055	0.011	0.0055	0.022	0.011	0.022	0.011	0.033	0.011	0.0055	0.0165	0.0055	0.0055	0.022	0.011	0	0.0055	0.0055	0	0.0055	0	0
3	0.0055	0.0055		0.011	0.0055	0.0165	0.0055	0.011	0.0055	0.0165	0.0165	0.011	0.0055	0.0055	0.0055	0.011	0.0055	0	0	0	0	0.0055	0.0055	0
4	0.0275	0.011	0.011		0.0275	0.022	0.022	0.0385	0.0385	0.066	0.077	0.033	0.033	0.0275	0.0275	0.044	0.0275	0.0055	0.011	0.0165	0.011	0.022	0.0275	0.011
5	0.011	0.0055	0.0055	0.0275		0.011	0.011	0.0275	0.044	0.055	0.0275	0.011	0.011	0.0055	0.011	0.0275	0.011	0	0.0055	0.0055	0.0055	0.011	0.0055	0
6	0.0165	0.022	0.0165	0.022	0.011		0.022	0.044	0.022	0.044	0.022	0.011	0.011	0.0055	0.011	0.0495	0.0275	0.0055	0.011	0.0165	0.0055	0.011	0.0055	0.0055
7	0.0275	0.011	0.0055	0.022	0.011	0.022		0.055	0.033	0.1045	0.0275	0.0385	0.022	0.011	0.0275	0.077	0.055	0.011	0.022	0.0275	0.011	0.0275	0.011	0.0055
8	0.044	0.022	0.011	0.0385	0.0275	0.044	0.055		0.044	0.088	0.044	0.033	0.033	0.022	0.033	0.121	0.077	0.0165	0.0385	0.0495	0.022	0.0275	0.0165	0.011
9	0.0275	0.011	0.0055	0.0385	0.044	0.022	0.033	0.044		0.154	0.077	0.033	0.033	0.033	0.0495	0.077	0.0495	0.011	0.022	0.033	0.0165	0.0385	0.0275	0.011
10	0.0715	0.033	0.0165	0.066	0.055	0.044	0.1045	0.088	0.154		0.22	0.11	0.1045	0.1155	0.22	0.242	0.2145	0.0385	0.099	0.1375	0.066	0.143	0.099	0.044
11	0.0275	0.011	0.0165	0.0825	0.0275	0.022	0.0275	0.044	0.077	0.2145		0.077	0.055	0.088	0.077	0.077	0.055	0.0055	0.022	0.033	0.022	0.0605	0.0715	0.033
12	0.011	0.0055	0.011	0.033	0.011	0.011	0.0385	0.033	0.033	0.11	0.077		0.0765	0.0385	0.0385	0.0385	0.033	0.011	0.0165	0.022	0.0165	0.0385	0.0385	0.0275
13	0.0275	0.0165	0.0055	0.033	0.011	0.011	0.022	0.033	0.033	0.1045	0.055	0.0715		0.033	0.0385	0.033	0.0275	0.0055	0.0165	0.033	0.033	0.0715	0.044	0.044
14	0.0165	0.0055	0.0055	0.0275	0.0055	0.0055	0.011	0.022	0.033	0.1155	0.088	0.0385	0.033		0.0715	0.0385	0.0385	0.0055	0.0165	0.0275	0.022	0.066	0.0605	0.022
15	0.0275	0.0055	0.0055	0.0275	0.011	0.011	0.0275	0.033	0.055	0.22	0.077	0.0385	0.0385	0.0715		0.066	0.0825	0.011	0.044	0.0605	0.044	0.143	0.055	0.022
16	0.0275	0.022	0.011	0.044	0.0275	0.0495	0.077	0.121	0.077	0.242	0.077	0.0385	0.033	0.0385	0.066		0.154	0.0275	0.0715	0.088	0.033	0.066	0.0275	0.0165
17	0.022	0.011	0.0055	0.0275	0.011	0.0275	0.055	0.077	0.0495	0.2145	0.055	0.033	0.0275	0.0385	0.0825	0.154		0.033	0.0935	0.0935	0.033	0.0935	0.033	0.0165
18	0.0055	0	0	0.0055	0	0.0055	0.011	0.0165	0.011	0.0385	0.011	0.011	0.0055	0.0055	0.011	0.0275	0.033		0.0165	0.022	0.0055	0.0165	0.0055	0
19	0.0165	0.0055	0	0.011	0.0055	0.011	0.022	0.0385	0.022	0.099	0.022	0.0165	0.0165	0.0165	0.044	0.0715	0.0935	0.0165		0.066	0.022	0.066	0.0165	0.0055
20	0.0165	0.0055	0	0.0165	0.0055	0.0165	0.0275	0.0495	0.033	0.1375	0.033	0.0275	0.033	0.0275	0.0605	0.088	0.0935	0.022	0.066		0.066	0.132	0.0385	0.022
21	0.0055	0	0	0.011	0.0055	0.0055	0.011	0.022	0.0165	0.066	0.022	0.0165	0.033	0.022	0.044	0.033	0.033	0.0055	0.022	0.066		0.099	0.0385	0.0275
22	0.022	0.0055	0.0055	0.022	0.011	0.011	0.0275	0.0275	0.0385	0.143	0.0605	0.0385	0.0715	0.066	0.143	0.066	0.0935	0.0165	0.066	0.132	0.099		0.1155	0.0605
23	0.0165	0	0.0055	0.0275	0.0055	0.0055	0.011	0.0165	0.0275	0.099	0.0715	0.0385	0.044	0.0605	0.055	0.0275	0.033	0.0055	0.0165	0.0385	0.0385	0.1155		0.0385
24	0.0055	0	0	0.011	0	0.0055	0.0055	0.011	0.011	0.044	0.033	0.0275	0.0385	0.022	0.022	0.0165	0.0165	0	0.0055	0.022	0.0275	0.0605	0.0385	

Table A-3 Free-Flow Link Travel Cost on the Sioux Falls Network

Link	Car	Truck
1	0.06	0.072
2	0.04	0.048
3	0.06	0.072
4	0.05	0.06
5	0.04	0.048
6	0.04	0.048
7	0.04	0.048
8	0.04	0.048
9	0.02	0.024
10	0.06	0.072
11	0.02	0.024
12	0.04	0.048
13	0.05	0.06
14	0.05	0.06
15	0.04	0.048
16	0.02	0.024
17	0.03	0.036
18	0.02	0.024
19	0.02	0.024
20	0.03	0.036
21	0.1	0.12
22	0.05	0.06
23	0.05	0.06
24	0.1	0.12
25	0.03	0.036
26	0.03	0.036
27	0.05	0.06
28	0.06	0.072
29	0.05	0.06
30	0.08	0.096
31	0.06	0.072
32	0.05	0.06
33	0.06	0.072
34	0.04	0.048
35	0.04	0.048
36	0.06	0.072
37	0.03	0.036
38	0.03	0.036

Table A-3 (Continued)

Link	Car	Truck
39	0.04	0.048
40	0.04	0.048
41	0.05	0.06
42	0.04	0.048
43	0.06	0.072
44	0.05	0.06
45	0.04	0.048
46	0.04	0.048
47	0.05	0.06
48	0.05	0.06
49	0.02	0.024
50	0.03	0.036
51	0.08	0.096
52	0.02	0.024
53	0.02	0.024
54	0.02	0.024
55	0.03	0.036
56	0.04	0.048
57	0.04	0.048
58	0.02	0.024
59	0.04	0.048
60	0.04	0.048
61	0.04	0.048
62	0.06	0.072
63	0.05	0.06
64	0.06	0.072
65	0.02	0.024
66	0.03	0.036
67	0.04	0.048
68	0.05	0.06
69	0.02	0.024
70	0.04	0.048
71	0.04	0.048
72	0.04	0.048
73	0.02	0.024
74	0.04	0.048
75	0.03	0.036
76	0.02	0.024

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