ABSTRACT

MATHEMATICAL MODEL OF HUMAN SIT-TO-STAND AND STAND-TO-SIT MOTION ANALYSIS

by
Jayeshkumar Gandhi

In designing a device for an amputee, it is important to find those underlying principles which determine the normal human sit-to-stand task. For this purpose we have developed a mathematical model of human sit-to-stand movement, in which it is possible to predict the minimum mechanical energy consumption to move from the sit-to-stand position.

To the best of the author's knowledge, this thesis represents the first time that the periodic motion of stand-to-sit and sit-to-stand movements have been mathematically modeled by a simple mechanical system. A complex model, such as the one used by Seireg and Arvikar (1973)[1] that contained 31 muscles per leg, is certainly impressive from a mathematical point of view alone. However, biomechanists should always reduce as much as possible the complexity in their models. The discussions of what the appropriate level of complexity to model, this biomechanical process will probably never end.

Our purpose, for this thesis, is to develop a simple mathematical model of sit-to-stand motion, which can be used to understand the effects of parameter changes, and to predict the human motion that minimizes energy expenditure. This knowledge can be used to design a mechanical device for this purpose. There are very few papers which explain mechanical and muscular dynamics of rising from a seated position, but unfortunately, no one has successfully constructed a model to solve the motion by forward dynamics.

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by
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APPROVAL PAGE

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To my beloved family
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CHAPTER 1

BACKGROUND AND RESEARCH OBJECTIVES

In order to appreciate fully the contribution of a muscle or muscle groups to a movement of interest, that movement must be fully evaluated and studied. Usually, several moving segments are involved in each human motion. One segment moves on an adjacent segment, which moves on another. This is similar to engineering links that involve overlapping segments held together by pins (joints) that serve as an axes of rotation. In general, overlapping segments do not occur in the human body except in a few places such as the ankle and the C-1-Odentoid articulation.

For our purpose, a link is assumed to be a straight line (e.g. a rigid rod) of constant length. Such a system of links can serve as a geometric model to analyze motion. If power is to be transmitted, the links of a machine must form a closed system in which each link has a particular relation to every other link in the system. The closed system is such that no movement of one segment can be made that does not affect the other links (segments) in a predictable manner. This guarantees that forces are transmitted in predictable manner.

Examples of early twentieth century interest in muscles mechanics can be found in the works of Lombard (1903), Fischer (1906), and Lombard and Abbott (1907) [2]. Lombard and Abbott were concerned with the contribution of lower extremity muscles to hip and knee joint movement in the frog. Lombard (1903) argued that those muscles which cross both hip and knee joints have better leverage as extensors than as flexors. Sit-to-stand movements involve concurrent extensions of hip and knee joints that are produced by the hamstrings (prevailing across the back of the knee joint) and the rectus femoris (across the front of the knee joint). The phenomenon of complex co-contractions involving both extensors and flexors has been recognized for some time as being a normal function under certain conditions. Although, there has been scant interest since the early twentieth century in studying the sit-to-stand movement, interest in co-contraction and the mechanics of two-joint muscles has prevailed up to the present time. Landsmear (1961), Molbech (1965), Carlsoo and Molbech (1966), and Carlsoo, Fohhin, and Skoglund (1973)[2].

D.L.Kelley, A. Dainis, and D. K. Wood (1975)[2] studied the mechanics and muscular dynamics of rising from a seated position, as performed by male and female subjects of different body sizes. In particular they were interested in the functions of selected one- and two-joints muscles crossing the hip and knee joints. They concluded that there were many more similarities than differences in the EMG and motion patterns of these muscles. Once the body left the seat, co-contractions of the quadriceps and hamstrings were observed throughout the movement in all but one subject, and with one other exception, the gluteus maximus was also active throughout the entire period of hip extension. The major observable movement pattern differences occurred in the first part of the movement. These being the variation among subjects in hip and shoulder flexion before loss of seat support. In the six subjects investigated, no size or sex related variation was detected.

There are many documents that have been studied to explain human walking. A very recent human walking model has been developed by Lacker, et al.[Personal

Communication] In this thesis we will try a model similar to Lacker, et al., to predict human sit-to-stand motion. A complete sit-to-stand and stand-to-sit motion, we call a "cyclic squat." We will develop an inverted pendulum system as a first attempt to model a cyclic squat.

A model of human cyclic squat is applicable to rehabilitation medicine. There are many complaints from patients relating to the difficulties in transferring themselves from a sitting to standing position. If we can design a device to facilitate patient rising from a sitting position, those complaints may be reduced. Therefore, to design such a device for amputees, we believe it is important to find those underlying principles which determine the normal human motion.

CHAPTER 2

MATHEMATICAL POINT OF VIEW

An individual is considered to be a collection of joined body segments, and segment movement involves displacement, velocity, acceleration, external forces, and forces of interaction between segments. In application to biomechanics, the mutual interaction between segments are the most troublesome. For example, in the analysis of human locomotion, we must know the forces in all involved muscles of the legs. But the human musculoskelatel system is highly redundant (more muscles than the available degrees of freedom of motion) and determination of the forces in the muscle is one of the most difficult problems in biomechanics. Hence there is a need for a method that can reduce such detailed information. The method of Joseph Louis Lagrange (1736-1813), offers such an alternative in terms of work and energy. If the kinetic and potential energies of the system are known as a function of the generalized coordinates and their derivatives with respect to time, and if the work done by external forces can be computed when a generalized coordinate changes, then the equations of motion can be written.

The Lagrangian method is characterized by simplicity and is applicable in any suitable coordinate system. In Lagrangian dynamics equations of constraint arise to limit the dynamics variables; but unlike the Newtonian approach, the forces required to maintain the constraints do not have to be explicitly considered in the formulation of the equations of motion. These constraint forces are already implicit in the geometric equations of constraint. Instead of the explicit use of force terms to derive the equations of motion, the Lagrangian method expresses work and energy in terms of the generalized coordinates to obtain a set of second-order dynamic differential equations in those coordinates [5].

We model a standing human as an inverted pendulum system. The simplest model consists of three body segments, representing the shank, thigh, and trunk respectively. (fig. 2). The segments lengths are represented by (L). The location of the segment centers of mass are represented by (z). (All measured from the distal end), and the segments masses by (m).

With the heel considered to be fixed on the ground, the system has three dynamics ($\theta 1$, $\theta 2$, $\theta 3$) variables, that are defined as the angles made by the lower end of each joint segment with respect to the horizontal (counter-clockwise is defined positive). These angles are referred to as segments angles. Therefore three differential equations are expected. Note that the individual joints angles can be easily expressed in terms of the segments angles. For example (Fig.1), the knee joint angle is similar the difference between the thigh and shank angles.

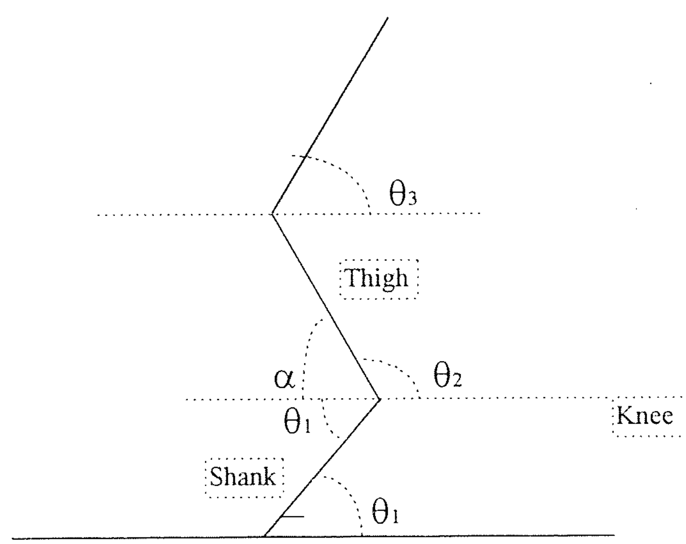

Figure 1 : Example of calculating knee joint angle

Therefore,

$$\theta_{knee} = \theta_1 + \alpha$$

$$\alpha = \pi - \theta_2$$

$$\theta_{knee} = (\theta_1 - \theta_2 + \pi)$$

As previously explained the Lagrangian approach does not require explicit formulation of reactions forces at the joints. They are already implicitly taken into account in the chosen coordinate system.

We assume that dissipation joint force terms like joint frictional force (viscosity) are not significant.

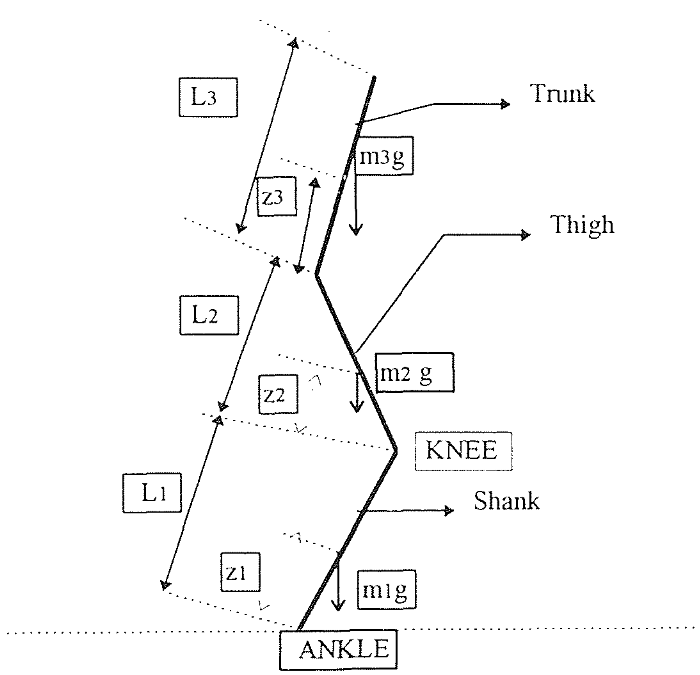

Figure : 2 Three-segment model of the standing human includes shank, thigh, and trunk

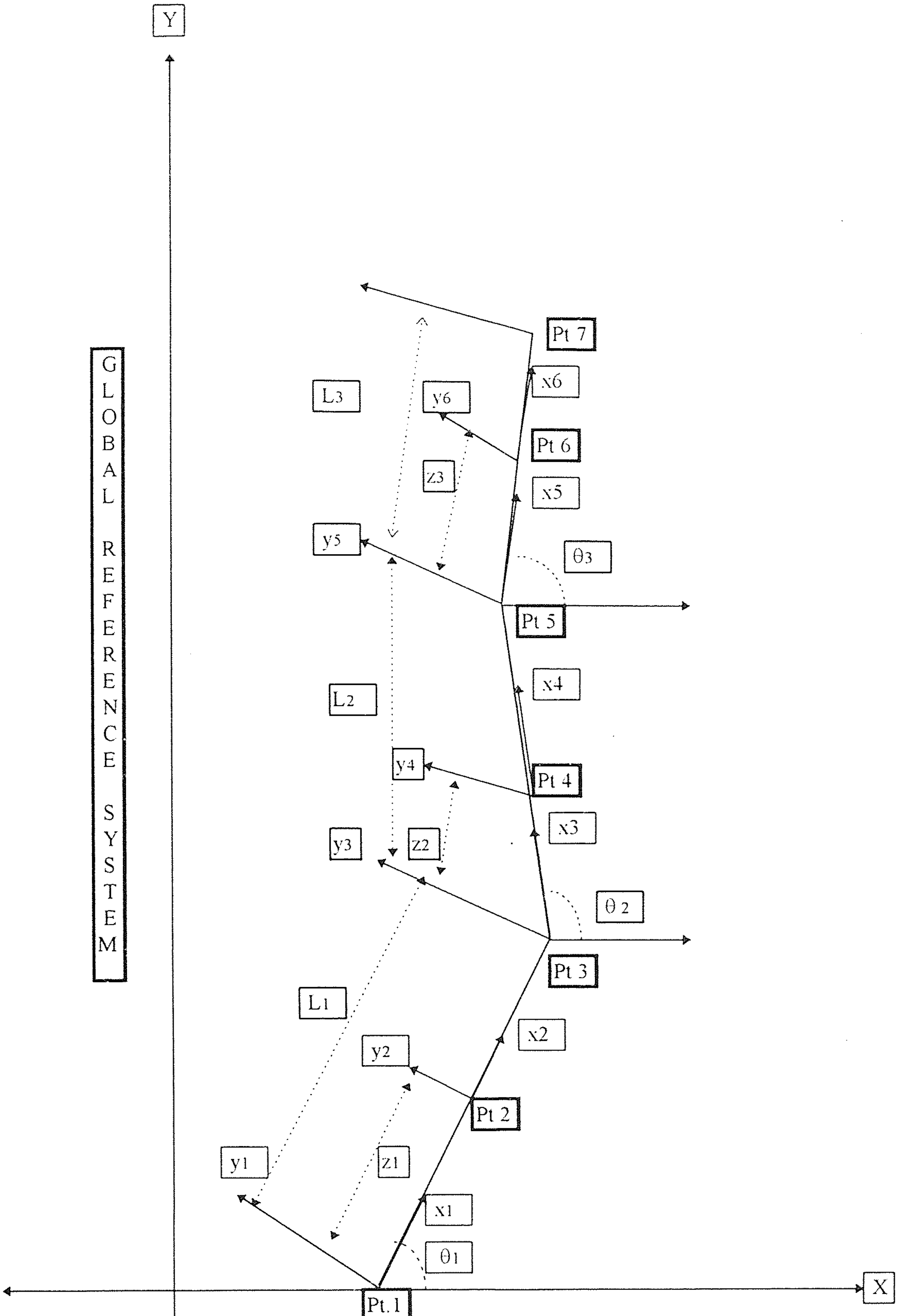

Figure:3 Link-segment model

Where,

L_1 = Length of shank

L_2 = Length of thigh

L_3 = Length of trunk

z_1 = Distance to center of mass of shank

z_2 = Distance to center of mass of thigh

z_3 = Distance to center of mass of trunk

m_1 = mass of shank

m_2 = mass of thigh

m_3 = mass of trunk

G = Gravity

θ_1 = Angle of shank horizontal axis

θ_2 = Angle of thigh with horizontal axis

θ_3 = Angle of trunk with horizontal axis

θ_1' = Velocity of shank

θ_2' = Velocity of thigh

θ_3' = Velocity of trunk

2.1 Explicit Formulation of the Equations of Motion the Cyclic Squat Model
(Refer to Fig. 3)

2.1.1 Displacements:

At point : 1

$$x_1 = 0$$

$$y_1 = 0$$

At point : 2

$$x_2 = z_1 \cos \theta_1$$

$$y_2 = z_1 \sin \theta_1$$

At point : 3

$$x_3 = L_1 \cos \theta_1$$

$$y_3 = L_1 \sin \theta_1$$

At point : 4

$$x_4 = L_1 \cos \theta_1 + z_2 \cos \theta_2$$

$$y_4 = L_1 \sin \theta_1 + z_2 \sin \theta_2$$

At point : 5

$$x_5 = L_1 \cos \theta_1 + L_2 \cos \theta_2$$

$$y_5 = L_1 \sin \theta_1 + L_2 \sin \theta_2$$

At point : 6

$$x_6 = L_1 \cos \theta_1 + L_2 \cos \theta_2 + z_3 \cos \theta_3$$

$$y_6 = L_1 \sin \theta_1 + L_2 \sin \theta_2 + z_3 \sin \theta_3$$

2.1.2 Velocity Vectors:

At point: 1

$$x_1' = 0$$

$$y_1' = 0$$

At point: 2

$$x_2' = -z_1\,\theta_1'\sin\theta_1$$

$$y_2' = z_1\,\theta_1'\cos\theta_1$$

At point: 3

$$x_3' = -L_1\theta_1'\sin\theta_1$$

$$y_3' = L_1\,\theta_1'\cos\theta_1$$

At point: 4

$$x_4' = -L_1\,\theta_1'\sin\theta_1 - z_2\,\theta_2'\sin\theta_2$$

$$y_4' = L_1\,\theta_1'\cos\theta_1 + z_2\,\theta_2'\cos\theta_2$$

At point: 5

$$x_5' = -L_1\,\theta_1'\sin\theta_1 - L_2\,\theta_2'\sin\theta_2$$

$$y_5' = L_1\,\theta_1'\cos\theta_1 + L_2\,\theta_2'\cos\theta_2$$

At point: 6

$$x_6' = -L_1\,\theta_1'\sin\theta_1 - L_2\,\theta_2'\sin\theta_2 - z_3\,\theta_3'\sin\theta_3$$

$$y_6' = L_1\,\theta_1'\cos\theta_1 + L_2\,\theta_2'\cos\theta_2 + z_3\,\theta_3'\cos\theta_3$$

2.1.3 Potential Energy:

$$PE = g\,(m_2 y_2 + m_4 y_4 + m_6 y_6)$$

$$= g\,[\,m_2 (z_1 \sin\theta_1) + m_4 (L_1 \sin\theta_1 + z_2 \sin\theta_2)$$

$$+ m_6 (L_1 \sin\theta_1 + L_2 \sin\theta_2 + z_3 \sin\theta_3)$$

$$= g\,[\,(m_2 z_1 + m_4 L_1) \sin\theta_1 + (m_4 z_2 + m_6 L_2) \sin\theta_2$$

$$+ m_6 z_1 \sin\theta_3$$

Therefore,

$$PE = g \sum_{i-1}^{3} a_i \sin\theta_i$$

where,

$$a_1 = (m_2 z_1 + m_4 L_1)$$

$$a_2 = (m_4 z_2 + m_6 L_2)$$

$$a_3 = (m_6 z_1)$$

2.1.4 Kinetic Energy:

$$\text{Segment KE} = \tfrac{1}{2}\, m\,(x'^2 + y'^2)$$

$$KE_2 = \tfrac{1}{2}\, m_2\, [\,x_2'^2 + y_2'^2\,]$$

$$= \tfrac{1}{2}\, m_2\, [(-\,z_1\, \theta_1' \sin\theta_1)^2\,] + (z_1\, \theta_1' \cos\theta_1)^2$$

$$= \tfrac{1}{2} m_2 [z_1^2 \theta_1'^2 \sin^2 \theta_1 + z_1^2 \theta_1'^2 \cos^2 \theta_1]$$

$$= \tfrac{1}{2} m_2 [z_1^2 \theta_1'^2]$$

$$KE_4 = \tfrac{1}{2} m_4 [x_4'^2 + y_4'^2]$$

$$= \tfrac{1}{2} m_4 [(- L_1 \theta_1' \sin \theta_1 - z_2 \theta_2' \sin \theta_2)^2$$

$$+ (L_1 \theta_1' \cos \theta_1 + z_2 \theta_2' \cos \theta_2)^2]$$

$$= \tfrac{1}{2} m_4 [(L_1^2 \theta_1'^2 \sin^2 \theta_1 + z_2^2 \theta_2'^2 \sin^2 \theta_2 + 2 L_1 \theta_1' \sin \theta_1 z_2 \theta_2' \sin \theta_2)$$

$$+ (L_1^2 \theta_1'^2 \cos^2 \theta_1 + z_2^2 \theta_2'^2 \cos^2 \theta_2 + 2 L_1 \theta_1' \theta_2' z_2 \cos \theta_1 \cos \theta_2)]$$

$$= \tfrac{1}{2} m_4 [L_1^2 \theta_1'^2 + z_2^2 \theta_2'^2 + 2 L_1 \theta_1' \theta_2' z_2 (\sin \theta_1 \sin \theta_2 +$$

$$\cos \theta_1 \cos\theta_2)]$$

USING FORMULA: $\cos (\theta_1 - \theta_2) = \sin \theta_1 \sin \theta_2 + \cos \theta_1 \cos \theta_2$

$$= \tfrac{1}{2} m_4 [L_1^2 \theta_1'^2 + z_2^2 \theta_2'^2 + 2 L_1 z_2 \theta_1' \theta_2' \cos (\theta_1 - \theta_2)]$$

$$KE_6 = \tfrac{1}{2} m_6 [x_6'^2 + y_6'^2]$$

$$= \tfrac{1}{2} m_6 [(- L_1\theta_1' \sin \theta_1 - L_2 \theta_2' \sin \theta_2 - z_3 \theta_3' \sin \theta_3)^2$$

$$+ (L_1 \theta_1' \cos \theta_1 + L_2 \theta_2' \cos \theta_2 + z_3 \theta_3' \cos \theta_3)^2$$

$$= \tfrac{1}{2} m_6 [L_1^2 \theta_1'^2 \sin^2 \theta_1 + L_2^2 \theta_2'^2 \sin^2 \theta_2 + z_3^2 \theta_3'^2 \sin^2 \theta_3$$

$$+ 2 L_1 L_2 \theta_1' \theta_2' \sin \theta_1 \sin \theta_2 + 2 L_1 z_3 \theta_1' \theta_3' \sin \theta_1 \sin \theta_3$$

$$+ 2 L_2 z_3 \theta_2' \theta_3' \sin \theta_2 \sin \theta_3$$

$$+ L_1^2\, \theta_1'^2 \cos^2\theta_1 + L_2^2\, \theta_2'^2 \cos^2\theta_2 + z_3^2\, \theta_3'^2 \cos^2\theta_3$$

$$+ 2\, L_1\, \theta_1'\, L_2\, \theta_2' \cos\theta_1 \cos\theta_2 + 2\, L_2\, z_3\, \theta_2'\, \theta_3' \cos\theta_2 \cos\theta_3$$

$$+ 2\, L_1\, z_3\, \theta_1'\, \theta_3' \cos\theta_1 \cos\theta_3$$

$$= ½\, m_6\, [L_1^2\, \theta_1'^2 + L_2^2\, \theta_2'^2 + z_3^2\, \theta_3'^2$$

$$+ 2\, L_1\, L_2\, \theta_1'\, \theta_2' \,(\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2)$$

$$+ 2\, L_2\, z_3\, \theta_2'\, \theta_3' \,(\sin\theta_2 \sin\theta_3 + \cos{}_2 \cos\theta_3)$$

$$+ 2\, L_1\, z_3\, \theta_1'\, \theta_3' \,(\sin\theta_1 \sin\theta_3 + \cos\theta_1 \cos\theta 3)\,]$$

$$= ½\, m_6\, [L_1^2\, \theta_1'^2 + L_2^2\, \theta_2'^2 + z_3^2\, \theta_3'^2$$

$$+ 2\, L_1\, L_2\, \theta_1'\, \theta_2' \cos(\theta_1 - \theta_2)$$

$$+ 2\, L_2\, z_3\, \theta_2'\, \theta_3' \cos(\theta_2 - \theta_3)$$

$$+ 2\, L_1\, z_3\, \theta_1'\, \theta_3' \cos(\theta_1 - \theta_3)\,]$$

Therefore,

$$\text{TOTAL}\quad KE = KE_2 + KE_4 + KE_6$$

$$\text{TOTAL}\quad KE = ½\, m_2\, [\, z_1^2\, \theta_1'^2\,]$$

$$+ ½\, m_4\, [L_1^2\, \theta_1'^2 + z_2^2\, \theta_2'^2 + 2\, L_1\, z_2\, \theta_1' \theta_2' \cos(\theta_1 - \theta_2)\,]$$

$$+ ½\, m_6\, [L_1^2\, \theta_1'^2 + L_2^2\, \theta_2'^2 + z_3^2\, \theta_3'^2$$

$$+ 2\, L_1\, L_2\, \theta_1'\, \theta_2' \cos(\theta_1 - \theta_2)$$

$$+ 2\, L_2\, z_3\, \theta_2'\, \theta_3' \cos(\theta_2 - \theta_3)$$

$$+ 2\, L_1\, z_3\, \theta_1'\, \theta_3' \cos(\theta_1 - \theta_3)$$

Arrange together,

$$KE = \tfrac{1}{2}\,[\,(z_1^2 m_2 + L_1^2 m_4 + L_1^2 m_6)\,\theta_1'^2$$

$$+ (z_2^2 m_4 + L_2^2 m_6)\,\theta_2'^2 + (z_3^2 m_6)\,\theta_3'^2$$

$$+ (m_4 z_2 L_1 + m_6 L_1 L_2)\,\theta_1'\,\theta_2' \cos(\theta_1 - \theta_2)$$

$$+ (m_6 z_3 L_2)\,\theta_2'\,\theta_3' \cos(\theta_2 - \theta_3)$$

$$+ (m_6 z_3 L_1)\,\theta_1'\,\theta_3' \cos(\theta_1 - \theta_3)$$

Therefore,

$$KE = \tfrac{1}{2}\,\vec{\theta_i}'^{\,t}\; M_{ij}\,\vec{\theta_i}'$$

$$PE = g \sum_{i=1}^{3} a_i \sin\theta_i$$

Now in vector matrix notation,

$$KE = \tfrac{1}{2}\,\vec{\theta_i}'^{\,t}\; M_{ij}\,\vec{\theta_i}'$$

where $\mathbf{M_{ij}} = C_{ij} \cos(\theta_i - \theta_j)$

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

where

$$C_{11} = z_1^2\, m_2 + L_1^2\, m_4 + L_1^2\, m_6$$

$$C_{22} = z_2^2\, m_4 + L_2^2\, m_6$$

$$C_{33} = z_3^2\, m_6$$

$$C_{12} = C_{21} = m_4\, L_1\, z_2 + m_6\, L_1\, L_2$$

$$C_{31} = C_{13} = L_1\, m_6\, z_3$$

$$C_{23} = C_{32} = L_2\, m_6\, z_3$$

Therefore the Lagrangian ,

$$L = KE - PE$$

$$L = \underbrace{\tfrac{1}{2} \sum_{i,j=1}^{3} M_{ij}\, \theta_i'\, \theta_j'}_{①} - \underbrace{g \sum_{i=1}^{3} a_i \sin \theta_i}_{②}$$

Now the equation of motion of each segment angle θ is,

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \theta'}\right] - \frac{\partial L}{\partial \theta} = 0$$

Now take partial derivatives with respect to another variable k, for part ①

$$\frac{\partial L}{\partial \theta_k'} = \tfrac{1}{2}\sum_{i=1}^{3} M_{ik}\,\theta_i' + \tfrac{1}{2}\sum_{j=1}^{3} M_{kj}\,\theta_j' + M_{kk}\,\theta_k'$$

but $M_{ik} = M_{kj}$

$$\frac{\partial L}{\partial \theta_k'} = \sum_{i=1}^{3} M_{ik}\,\theta_i'$$

now

$$M_{ik} = C_{ik}\cos(\theta_i - \theta_k)$$

because

$$M_{ij} = C_{ij}\cos(\theta_i - \theta_j)$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \theta_k'}\right] = \sum_{i=1}^{3} \frac{d}{dt}(M_{ik})\,\theta_i' + \sum_{i=1}^{3} M_{ik}\,\theta_i''$$

now

$$M_{ik} = C_{ik}\cos(\theta_i - \theta_k)$$

$$\frac{d}{dt}\left[M_{ik}\right] = -\,C_{ik}\sin(\theta_i - \theta_k)$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \theta'_k}\right] = -\sum_{i=1}^{3} C_{ik}\sin(\theta_i - \theta_k)(\theta'_i - \theta'_k)\,\theta'_i$$

$$+\sum_{i=1}^{3} C_{ik}\cos(\theta_i - \theta_k)\,\theta_i''$$

For part ②

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \theta_k'}\right] = -\sum_{i=1}^{3} C_{ik} \sin(\theta_i - \theta_k)(\theta_i' - \theta_k')\,\theta_i'$$

$$+\sum_{i=1}^{3} C_{ik} \cos(\theta_i - \theta_k)\,\theta_i''$$

Now $$\frac{\partial L}{\partial \theta_k} = -\tfrac{1}{2}\sum_{i,j=1}^{3} C_{ij} \sin(\theta_i - \theta_j)\,\theta_i'\,\theta_j' \frac{d}{d\theta_k}(\theta_i - \theta_j)$$

$$= \tfrac{1}{2}\sum_{i,j=1}^{3} C_{ik} \sin(\theta_i - \theta_k)\,\theta_i'\,\theta_k' - \tfrac{1}{2} C_{kj} \sin(\theta_k - \theta_j)\,\theta_k'\,\theta_j'$$

$$- g \sum_{i-1}^{3} a_k \cos\theta_k$$

but $C_{ik} = C_{kj}$

$C_{ik} = -C_{ki}$

$$= \sum_{i=1}^{3} C_{ik} \sin(\theta_i - \theta_k)\,\theta_i'\,\theta_k'$$

$$- g \sum_{i-1}^{3} a_k \cos\theta_k$$

Now,

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \theta_k'}\right] - \frac{\partial L}{\partial \theta_k} = 0$$

Therefore,

$$0 = - \sum_{i=1}^{3} C_{ik} \sin (\theta_i - \theta_k) (\theta_i' - \theta_k') \theta_i'$$

$$+ \sum_{i=1}^{3} C_{ik} \cos (\theta_i - \theta_k) \theta_i''$$

$$- \sum_{i=1}^{3} C_{ik} \sin (\theta_i - \theta_k) \theta_i' \theta_k'$$

$$+ g \sum_{i=1}^{3} a_k \cos \theta_k$$

Therefore,

$$0 = \sum_{i=1}^{3} C_{ik} \cos (\theta_i - \theta_k) \theta_i''$$

$$- \sum_{i=1}^{3} C_{ik} \sin (\theta_i - \theta_k) \theta_i'^2$$

$$+ \sum_{i=1}^{3} g\, a_k \cos \theta_k$$

Therefore,

$$0 = C_{11} \cos (\theta_1 - \theta_1) \theta_1'' + C_{12} \cos (\theta_1 - \theta_2) \theta_2'' + C_{13} \cos (\theta_1 - \theta_3) \theta_3''$$

$$- C_{11} \sin (\theta_1 - \theta_1) \theta_1'^2 - C_{12} \sin (\theta_1 - \theta_2) \theta_2'^2 - C_{13} \sin (\theta_1 - \theta_3) \theta_3'^2$$

$$+ g\, a_1 \cos \theta_1 \qquad \text{(Eq. 1)}$$

$$0 = C_{21} \cos(\theta_2 - \theta_1)\, \theta_1'' + C_{22} \cos(\theta_2 - \theta_2)\, \theta_2'' + C_{23} \cos(\theta_2 - \theta_3)\, \theta_3''$$
$$- C_{21} \sin(\theta_2 - \theta_1)\, \theta_1'^2 - C_{22} \sin(\theta_2 - \theta_2)\, \theta_2'^2 - C_{23} \sin(\theta_2 - \theta_3)\, \theta_3'^2$$
$$+ g\, a_2 \cos\theta_2 \qquad \text{(Eq. 2)}$$

$$0 = C_{31} \cos(\theta_3 - \theta_1)\, \theta_1'' + C_{32} \cos(\theta_3 - \theta_2)\, \theta_2'' + C_{33} \cos(\theta_3 - \theta_3)\, \theta_3''$$
$$- C_{31} \sin(\theta_3 - \theta_1)\, \theta_1'^2 - C_{32} \sin(\theta_3 - \theta_2)\, \theta_2'^2 - C_{33} \sin(\theta_3 - \theta_3)\, \theta_3'^2$$
$$+ g\, a_3 \cos\theta_3 \qquad \text{(Eq. 3)}$$

We refer to the derived equations (1, 2, 3) as " ballistic equations," since these equations of motion do not explicitly contain sources or sinks of mechanical energy that arise from muscle forces acting on the segments. These muscle forces arise from model output as described below.

We propose to find cyclic squat trajectories by breaking the movement into discrete connected phases. Each phase is solved as a two point boundary value problem. Each two point boundary value problem consists of an initial and final configuration and a specified time for moving from the designated initial to final configuration.

CHAPTER 3

NUMERICAL SOLUTION OF TWO POINT BOUNDARY VALUE PROBLEM BY SHOOTING MATHOD

The motion for each model considered in this thesis is obtained by solving one or several linked two points boundary value problem. Each boundary value problem consists of an initial and final target configuration and a specified duration for moving from the initial to target configuration.

The shooting method is used to solve the two point boundary value problem. The shooting method is an iteration method. Each iteration is a solution of an initial value problem in which the initial configuration and tentative guess for the initial velocity are given. If we assume the initial angles of the shank, the thigh, and the trunk position at the sitting configuration and guess the initial velocities of the segments, our program solves for the final angles of the shank, the thigh, and the trunk at the standing configuration at a specific duration of the time, which is also given. If discrepancies occur between the calculated final angles and chosen standing configuration, our program guesses new initial velocities and recalculates the final angles. This process continues until the program finds the correct initial velocities, with which it can produce the final angles at standing configuration which agree with the desired final angles. If the program finds the correct initial velocities, it can produce the trajectory of the segments, and know the position and velocities at any instant during the squat.

A 4^{th} order Runga-Kutta initial value solver is used with adaptive time step to insure that the local truncation error is less than a specified value. In each of the numerical methods for solving the initial value problem, the points in the solution are in general, only approximation to the true value. The errors associated with each computation will come principally from truncation of formulas and from round off of numbers in the computation. The value of the error in the computation for any one point in the solution can be controlled by the choice of the method, by the choice of spacing, and by the number of significant digits used in the calculation. The method and the spacing can be chosen so that the error in any computation is very small, but it generally cannot be reduced to zero. The stability behavior of the predictor-corrector methods is dependent upon the value of the product of the spacing (h) and the partial derivative of the function $f(x, y)$ with respect to y. The product is designated $\bar{h}$.

$$\bar{h} = h \frac{\partial f(x, y)}{\partial y}$$

Ordinarily the user will have no control over the function $f(x, y)$. So that only h can be varied to alter $\bar{h}$. The most effective way of avoiding instability is to select a method that is stable for all expected values of $\bar{h}$. The 4th order Runga-Kutta method has been proved stable for small value of $\bar{h}$. Experience of many users indicates that it is stable even for rather large value of $\bar{h}$.

The initial value problem is solved up to the specified duration for the boundary value problem. The configuration of the model at this time is compared to the target final configuration and the difference between is used to find the next new guess for the initial velocity using a multidimensional root-finding algorithm. We use a multidimensional Newton's method. The iteration process continues until the error vector defined as the difference between the target configuration of the boundary value problem and the final configuration of the initial value problem is less than a pre-selected magnitude.

When the separate phases that comprise each boundary value solution are pieced together, a complete continuous cyclic squat solution obtained. It is continuous because the end-configuration of each phase is the start-configuration of the next phase. However, in general, the solution has discontinuous velocities at each of the boundary point configurations, separating phases. More precisely, the terminal velocity that solves one phase is not in general, the starting velocity that solves the next phase. This means that impulsive forces must be acting at each time that the solution is at a boundary configuration. At each of these times, mechanical energy must be supplied to or removed from the segments. We interpret these sources and sinks of mechanical energy to arise from the action of muscles. We assume that an insignificant amount of mechanical energy is stored in elastic tissues in this cyclic human squat motion. Because there are relatively small losses of energy from joint dissipation forces, we have in this model also ignored the small amount of energy dissipated into heat as a result of joint friction and viscosity in the connective tissues. Mechanical energy is continuously flowing into and out of muscles and from segment to segment, but in our model muscle activity is restricted to the discrete interface times between phases. In the model the number of such times of muscle activity and the configuration at which muscle activity occurs is arbitrary. During each phase no external energy source is supplied and therefore each phase is a ballistic solution. In the next section, we will focus on three examples of cyclic squats with the same overall motion pattern. Each of these three examples, however, will consists of a different number of distinct phases and interface boundary configurations where muscle activity can occur.

CHAPTER 4

THREE CYCLIC SQUAT MODELS

We will consider three distinct cyclic squat models. Each model has three degrees of freedom and involves three segments (trunk, thigh, and shank) and no model involves raising the heel off the ground during the squat. (This would add an additional segment and degree of freedom to the model). Each model has a different number of phases and boundary points, where muscle activity can occur.

The 1st model is shown in fig.4and 5. In this model there is only one phase, and only one boundary configuration (sitting configuration). Muscle activity only occurs in this sitting configuration and the energy supplied by this activity is sufficient to carry the body through the entire cycle.

Configuration 1 : sitting

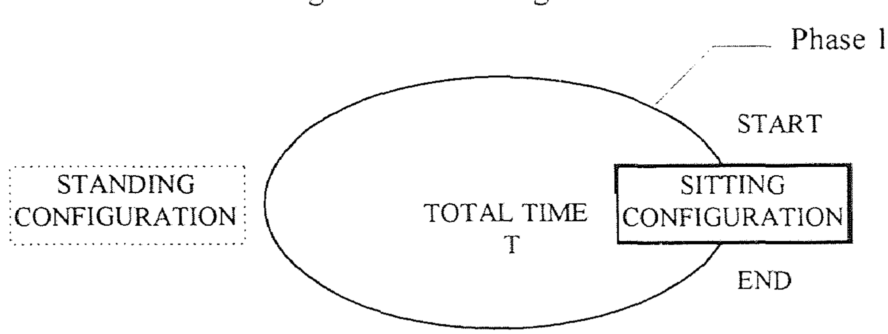

Figure 4: One configuration block diagram

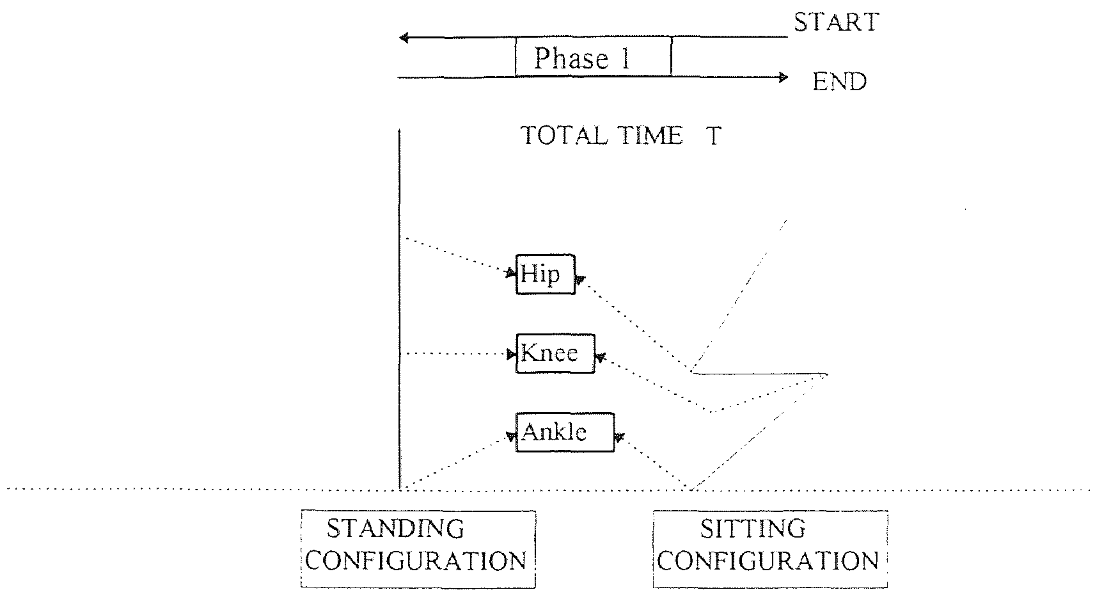

Figure 5: One configuration model

T total = T   Where, T total is total time period for the squat

The 2nd model is shown in fig.6 and 7. In this model there are two phases and two boundary configurations. Muscle activity can occur at initial standing and final sitting configuration and the energy supplied by these activities is sufficient to carry the body for each phase through entire cycle.

Configuration 1 : standing

Configuration 2 : sitting

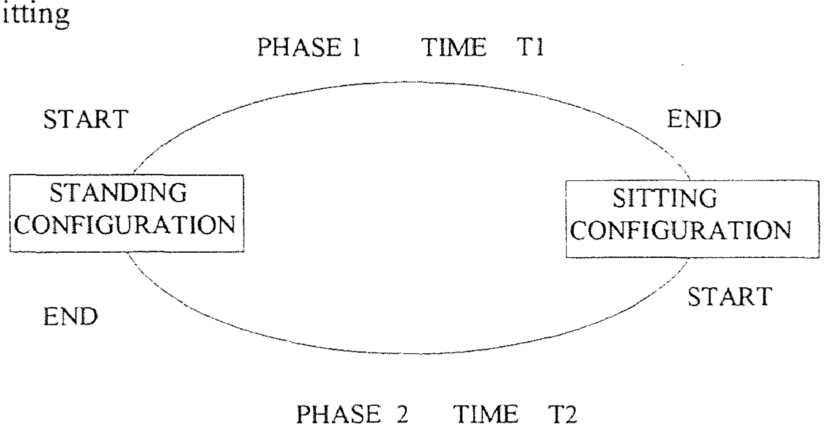

Figure 6: Two configuration block diagram

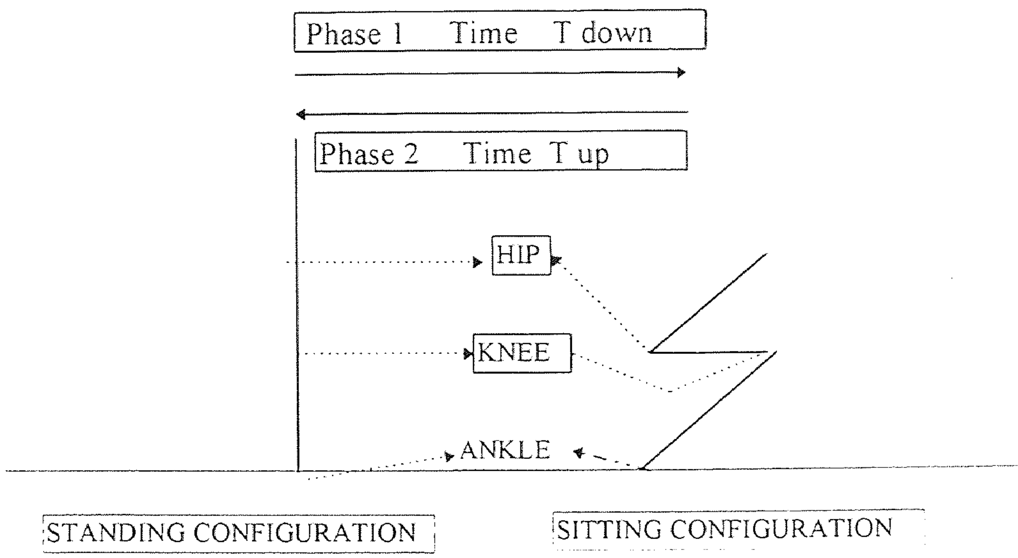

Figure 7: Two configuration model

$$T\ total = T\ down + T\ up$$

If T total is symmetric, then

$$T\ down = T\ up$$

where, T total = the squat period

T up = time from the sitting configuration to the standing configuration

T down = time from the standing configuration to the sitting configuration

We considered a 3rd model, in which we implement a 3rd intermediate configuration. At least fifteen major muscles are responsible for the sagital plane movements of the ankle, knee, and hip joints. During a squat, all of the three segments (trunk, thigh, and shank) involve multi-joint muscles that control the knee angles. Thus there is considerable indeterminacy when relating knee angle changes to any single movement pattern or to any unique combination of muscle activities.

For example, at the knee, there are nine muscles whose forces create the net movement. The line of action of each of these muscles is different and continuously changes with time. Therefore, it would seems reasonably to consider a model of the cycle which allows muscle activity to occur between the sitting and standing positions.

Introducing muscle activity at an intermediate configuration during the cyclic squat, may have an effect on

(1) The impact forces that must be absorbed by the knee in the sitting configuration

(2) The ability to control cyclic squat without falling over

(3) The distribution of mechanical energy and overall mechanical energy cost of the cycle.

It is our goal to measure and compare the three models proposed in this thesis with respect to

(1) Joint impact and ground reaction forces

(2) Dynamic stability and

(3) Mechanical energy cost.

In addition, these models will be compared to data collected from a normal subject performing cyclic squat. (see chapter 5 Experimental data)

The 3rd model is shown in fig.8 and 9. In this model there are four phases and three boundary configurations. Muscle activity can occur at the standing configuration, the intermediate configuration, and the sitting configuration during the cyclic squat and the energy supplied by these activities are sufficient to carry the body for each phases through entire cycle.

Configuration 1 : standing

Configuration 2 : intermediate

Configuration 3 : sitting

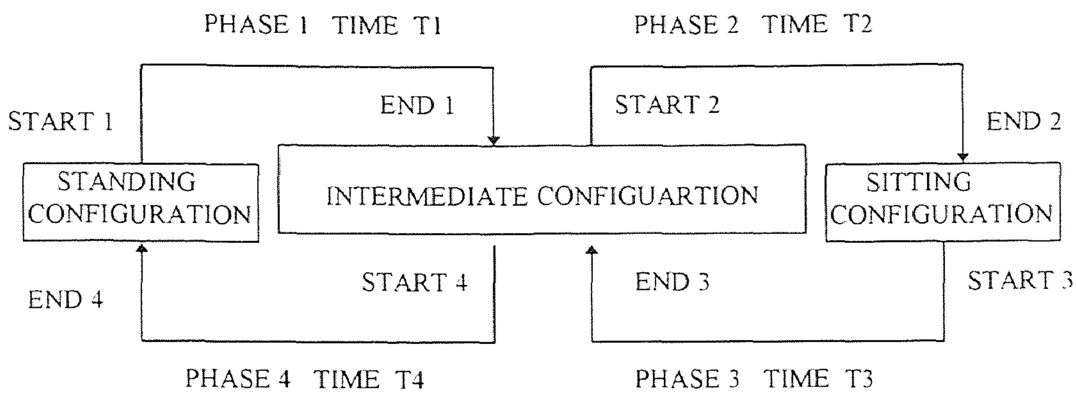

Figure 8: Three configuration block diagram

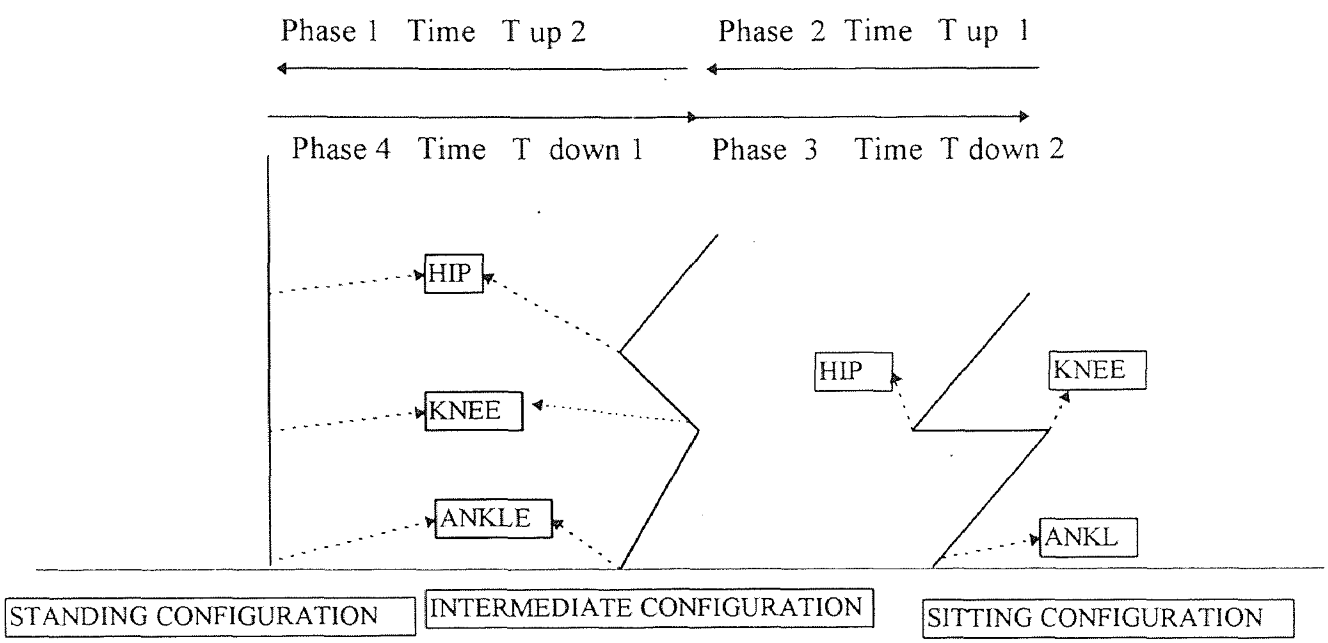

Figure 9: Three configuration model

$$T_{total} = T_{up\ 1} + T_{up\ 2} + T_{down\ 1} + T_{down\ 2}$$

Where,

T up1 = time from the sitting configuration to the intermediate configuration

T up2 = time from the intermediate configuration to the standing configuration

Tdown1 = time from the standing configuration to the intermediate configuration

Tdown2 = time from the intermediate configuration to the sitting configuration

We start by considering a symmetric case where,

$$T\ down\ 1 = T\ up\ 2$$

$$T\ down\ 2 = T\ up\ 1$$

Therefore, $2(T\ down\ 1) + 2(T\ down\ 2) = T\ total$

If the structural parameters (mass, length, and center of mass) are given then, 3 standing configuration segment angles, 3 intermediate configuration segment angles, 3 sitting configuration segment angles (total 9 angles) and the times T down 1 and T down 2 (2 time variable) need to specified (a total 11 parameters) to solve the equations of motion for this model. The solution is obtained numerically using the shooting method (see chapter 3) (Press, 1992)[6].

CHAPTER 5

EXPERIMENTAL DATA

The experimental data were taken in the Motion Analysis Laboratory of the Kessler Institute for Rehabilitation in West Orange, NJ using the VICON 370 Movement Analysis System. The VICON system generates the three dimensional coordinates of markers attached to critical points on the standing subject. The system output are the x-, y-, and z- coordinates of each of the markers which are attached to the subject. The markers are attached to the Anterior Iliac Spine(ASIS), hip, thigh, knee, tibia, ankle, heel and toe on left and right sides and an additional marker is attached to the sacrum. They are shown in figure 10.

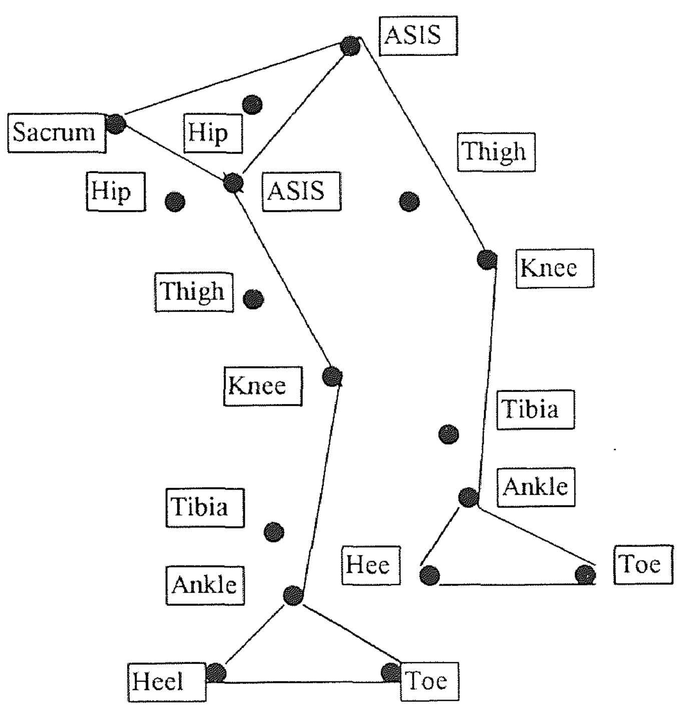

Figure 10 : Positions of the marker to get raw data

For this model only ASIS, thigh, Knee, tibia and ankle of the right side of the subject are recorded. The circular squat model is a two dimensional model. Therefore, it is necessary to project the data collected from a three dimensional coordinate system onto the two dimensional sagital plane. The laboratory frame coordinate system is such that the x-direction is parallel to the floor and passing through the anterior-posterior axes of the subject (anterior is positive), the y-direction is parallel to the floor and perpendicular to the anterior-posterior plane of the subject, and the z-direction is orthogonal to the floor plane and up.

Since there is relatively little motion recorded in the x-direction, the required sagital plane projection described above is obtained by deleting the x-coordinate and utilizing only the raw data of the y- and the z-coordinate. For normal individual, the experimental values of the three segments angles describe in the model are shown as a function of time for cyclic squat in figure.11. (Refer to figure 10)

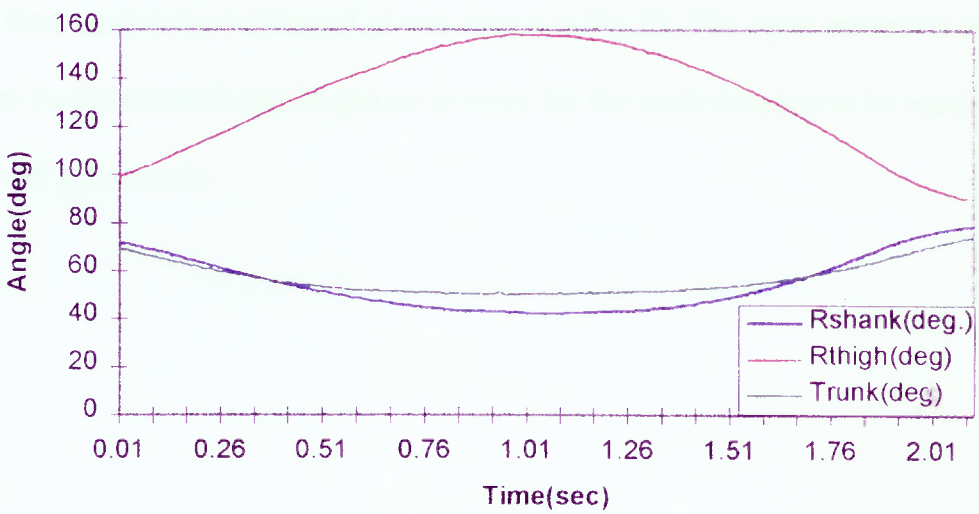

Figure 11: Experimental data

CHAPTER 6

RESULTS AND CONCLUSION

In this section, we will compare model results with experimental data collected from the VICON 370 motion analysis system. Our studies indicate that the period of the cyclic squat can be adjusted theoretically by varying the number of separate ballistic phases that comprise the cycle. At each interface time, separating two consecutive ballistic phases, there will be a discontinuity in the momentum that corresponds to a time of muscle action. Therefore, a cyclic squat with more phases corresponds to a squat where there are more instants of muscle activity. Our studies shows that, the period of time during the cyclic squat was different for all three models and for the experimental squat of a normal individual. For example, we found that the cyclic squat for the one phase and one configuration model is 0.60 seconds, the two phase and two configuration model is 0.48 seconds, the four phase and three configuration model is 0.83 seconds, while the normal individual experimental data is 2.2 seconds (Figures: 12,13,14,15,16,17). The cycle time against the number of phases shown in Fig.28. The squat maneuver would have to be fractionated into 28 phases in order for the cycle duration to be equal to the observed 2.2 seconds.

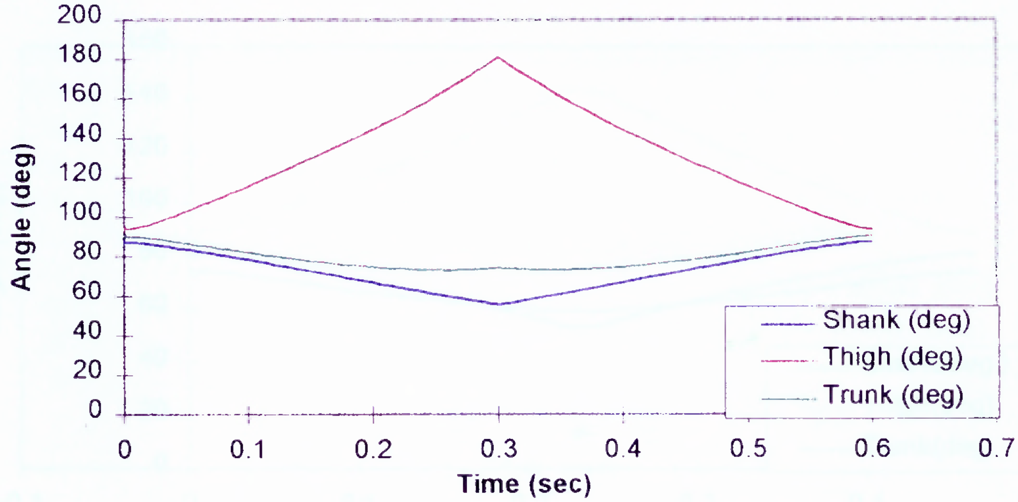

Figure:12 Angle: One phase circular squat

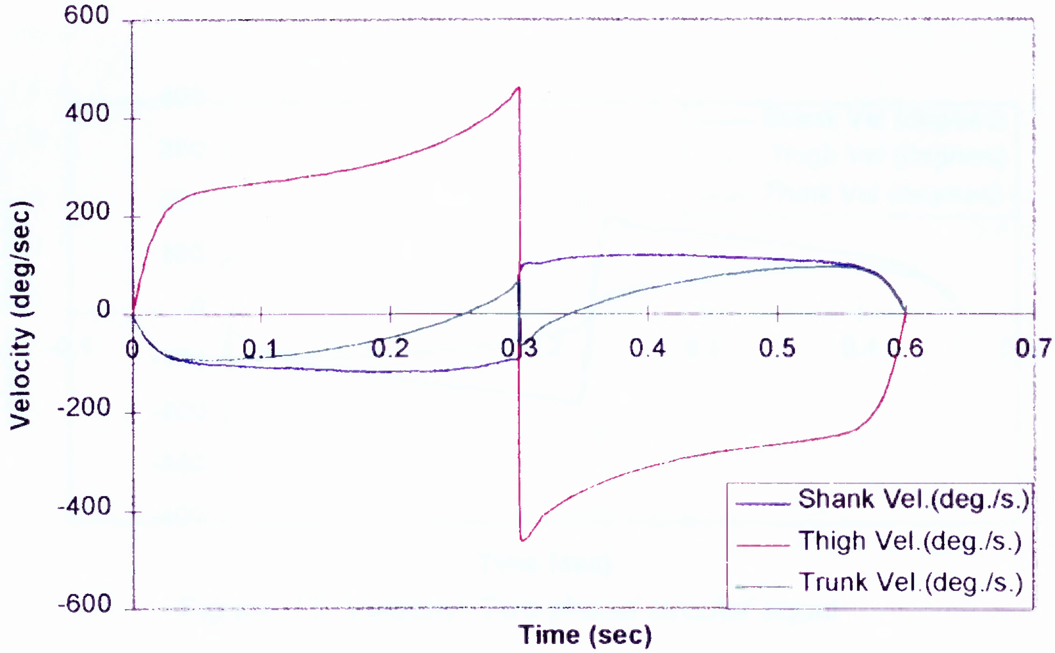

Figure: 13 Velocity: One phase circular squat

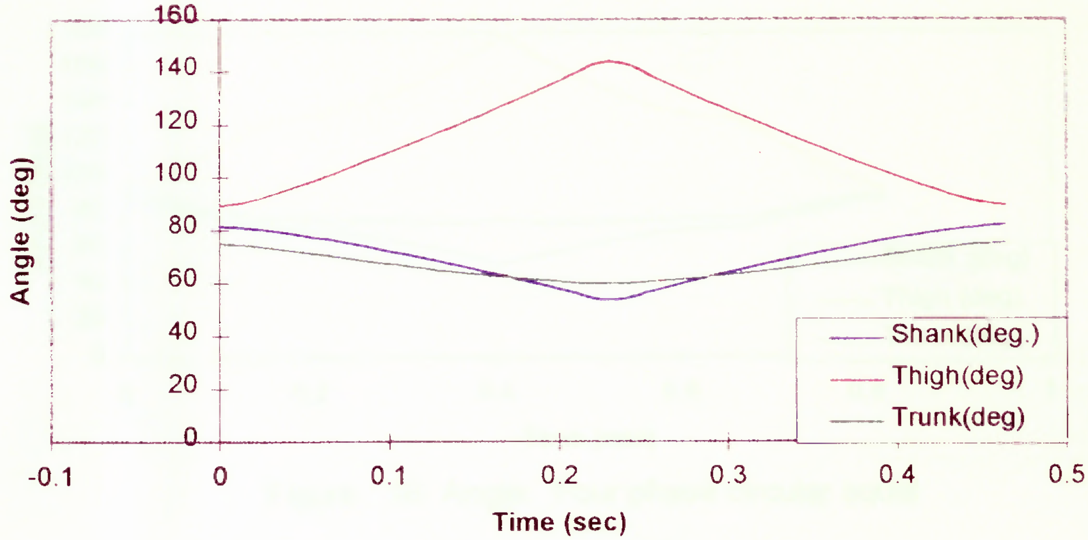

Figure: 14 Angle: Two phase circular squat

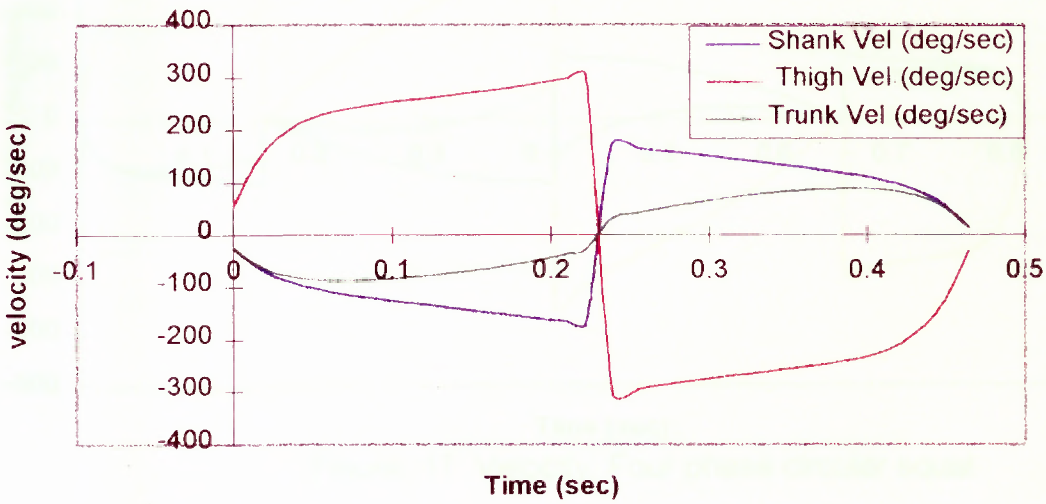

Figure: 15 Velocity: Two phase circular squat

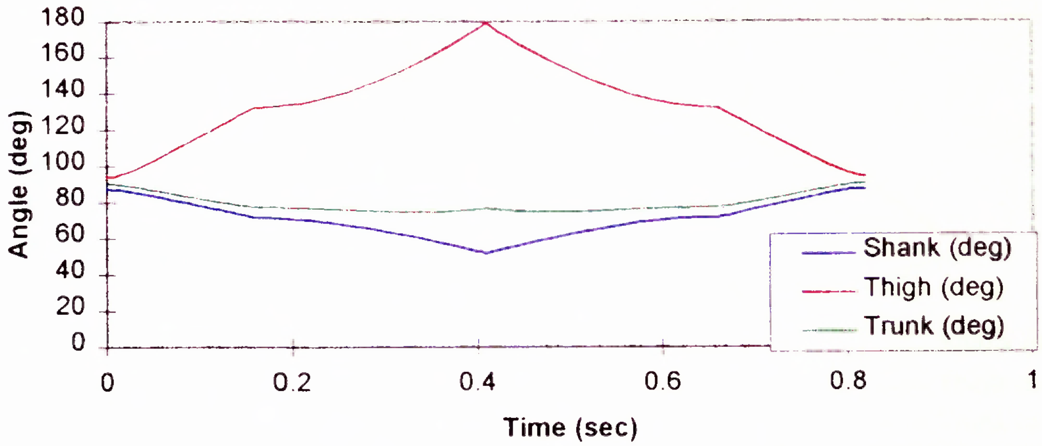

Figure : 16 Angle: Four phase circular squat

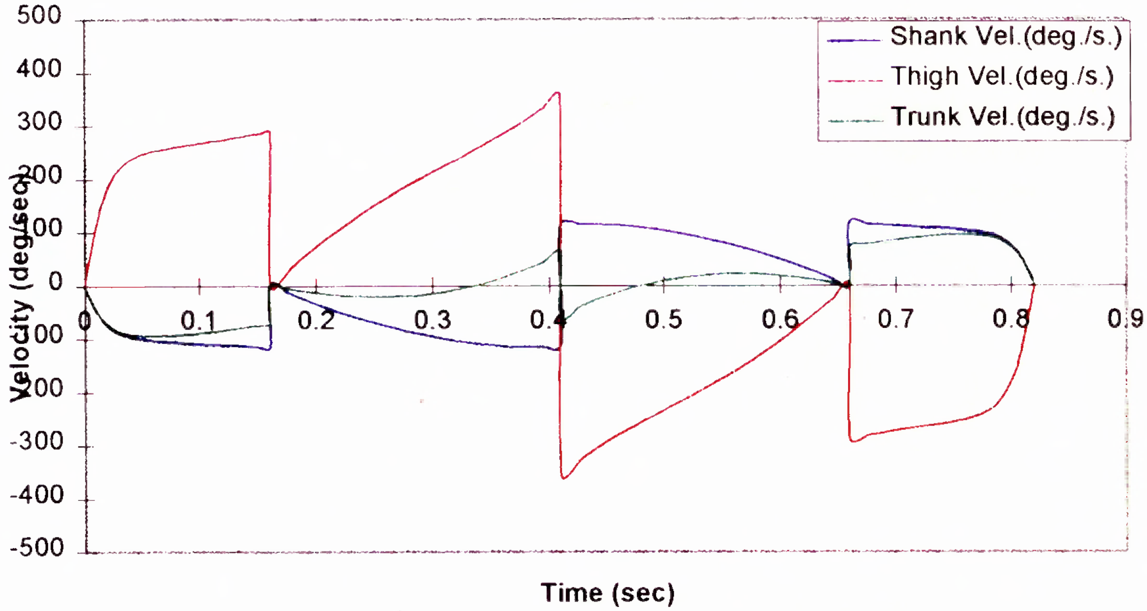

Figure: 17 Velocity: Four phase circular squat

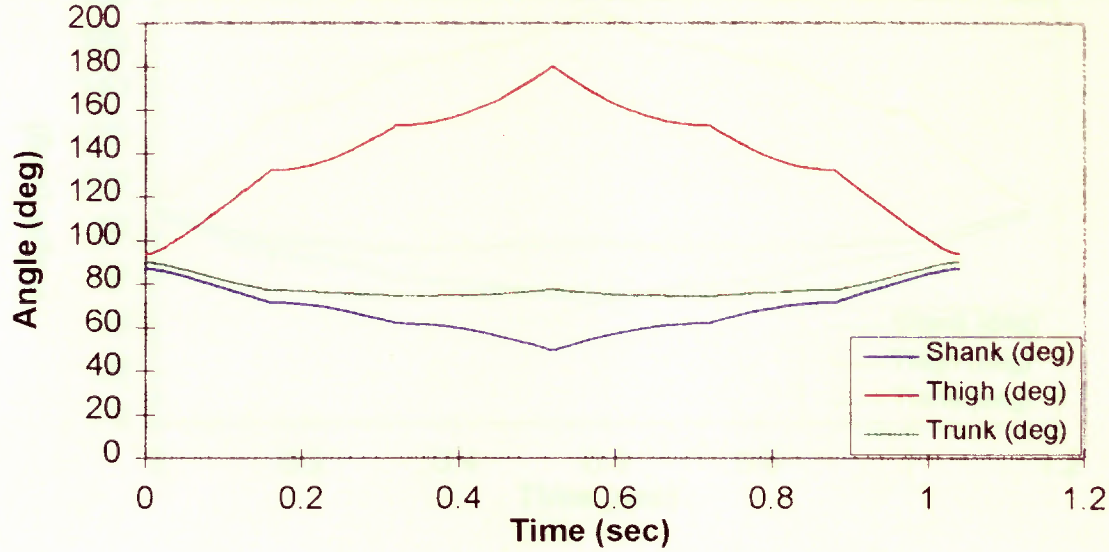

Figure: 18 Angle: Six phase circular squat

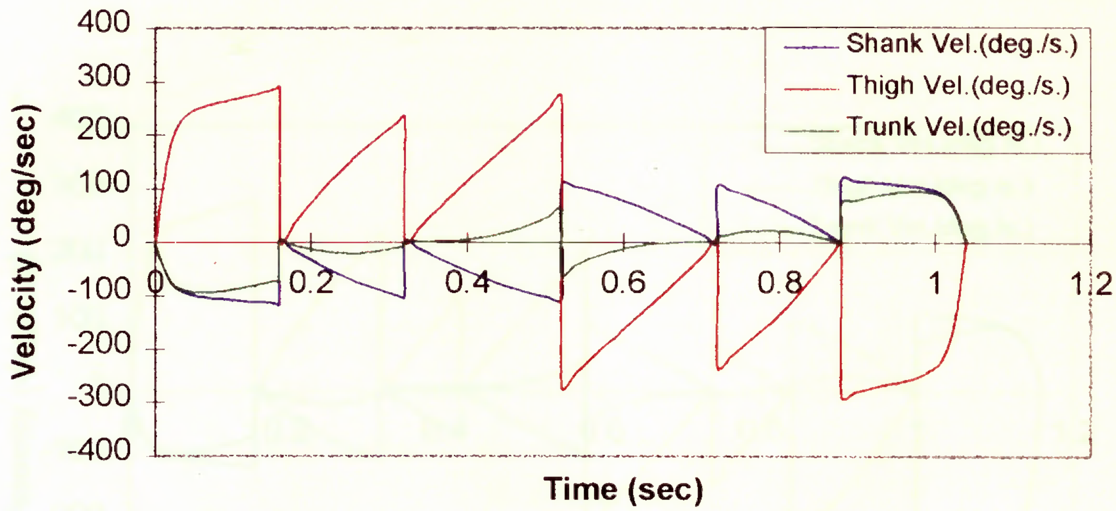

Figure: 19 Velocity: Six phase circular squat

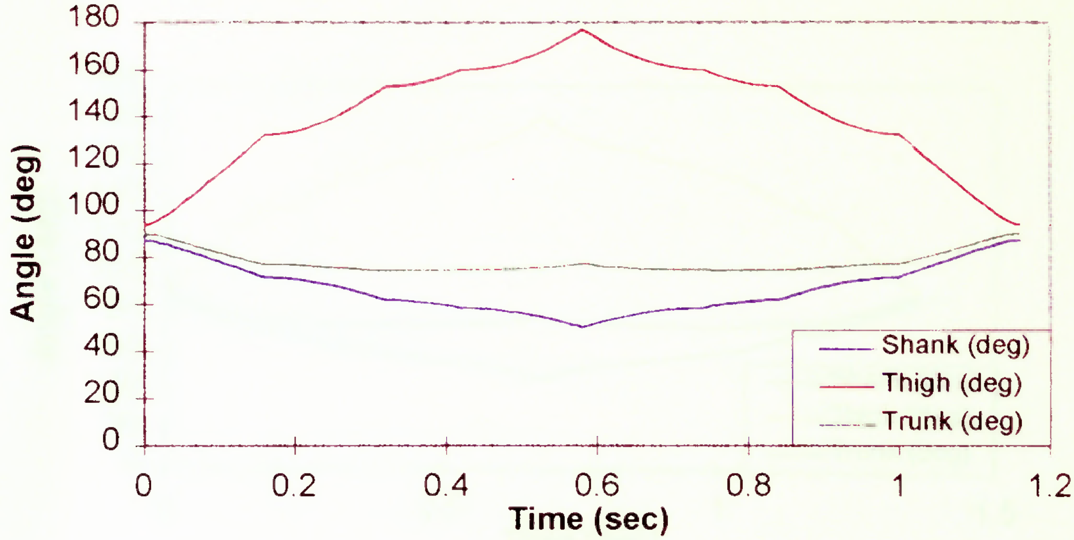

Figure: 20 Angle: Eight phase circular squat

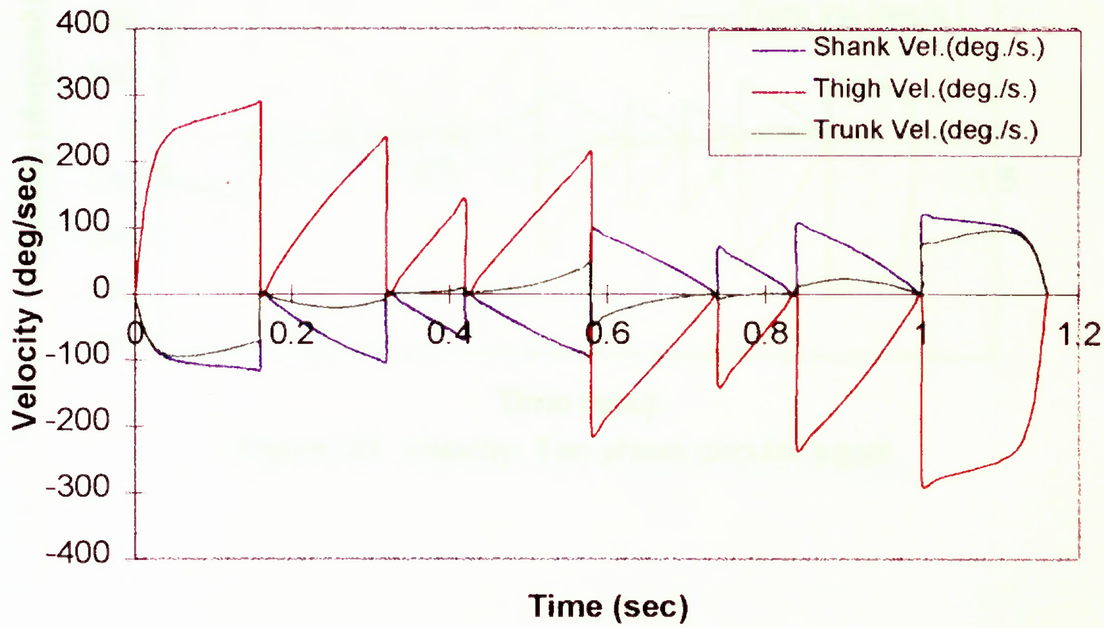

Figure: 21 Velocity: Eight phase circular squat

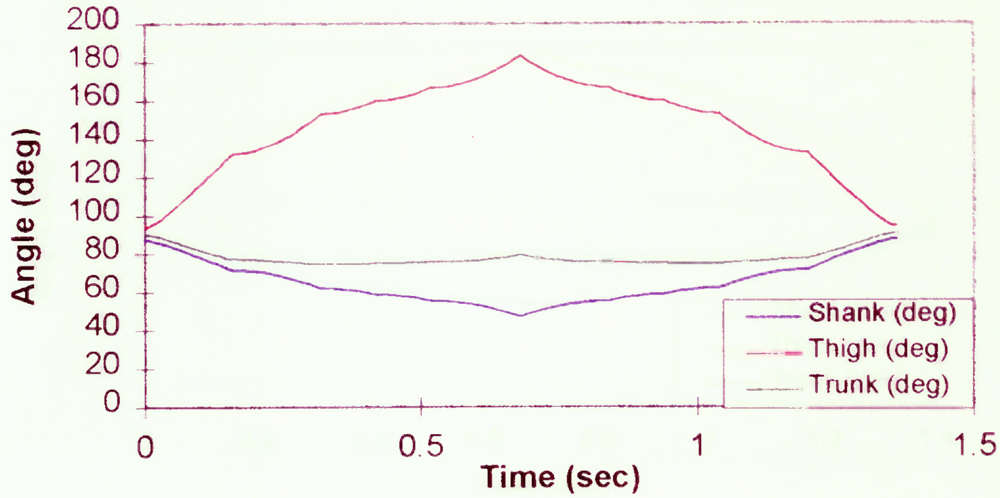

Figure: 22 Angle: Ten phase circular squat

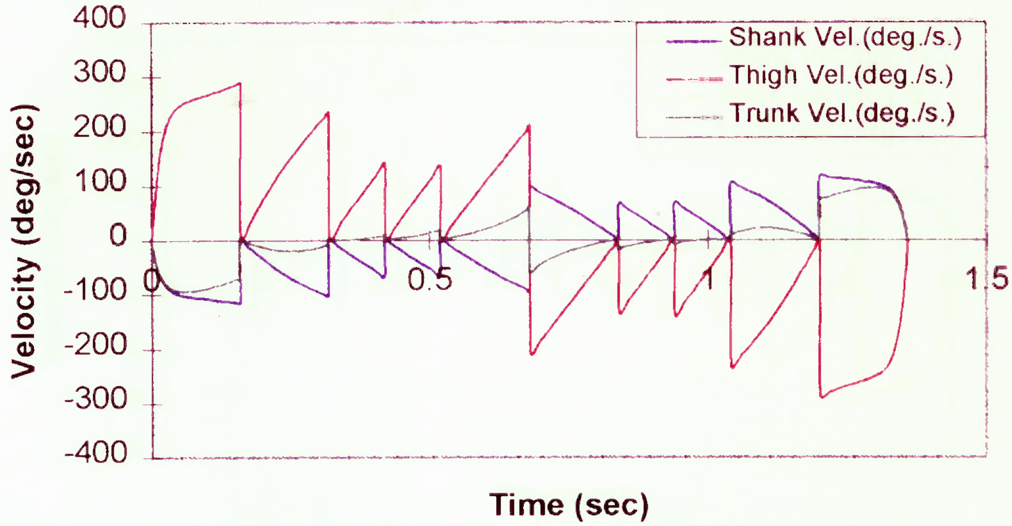

Figure: 23 Velocity: Ten phase circular squat

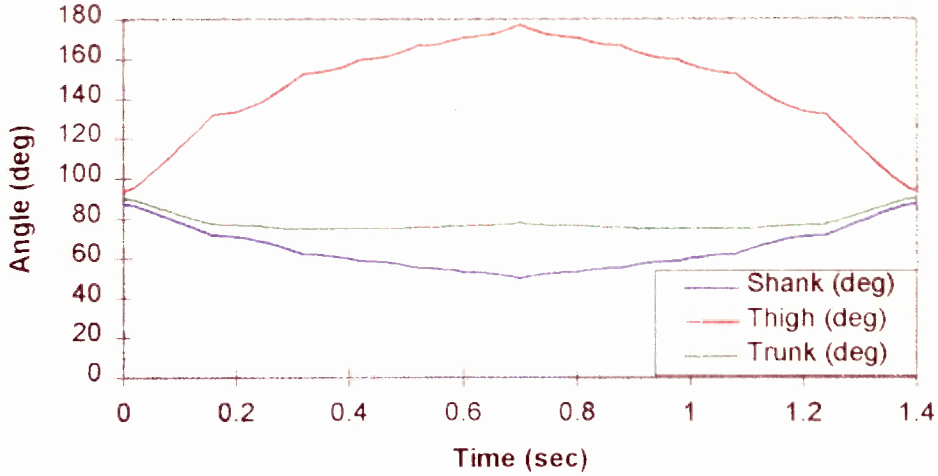

Figure: 24 Angle : Twelve phase circular squat

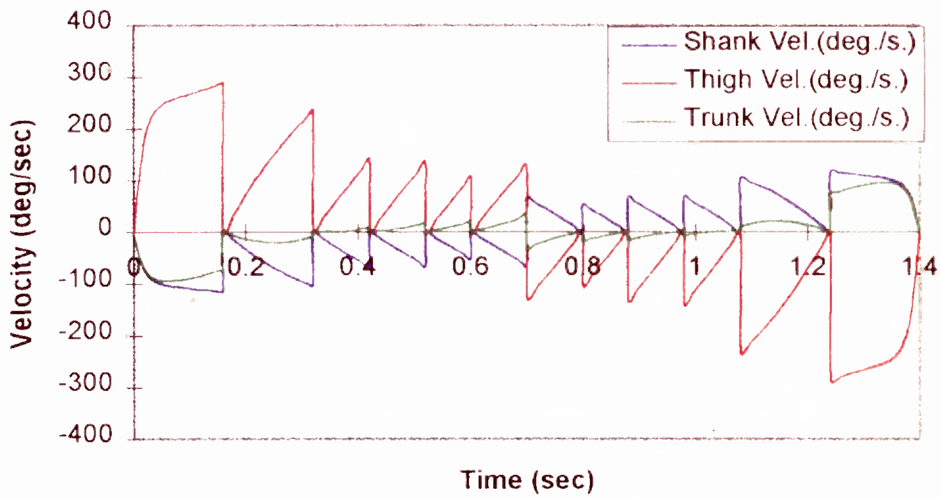

Figure :25 Velocity: Twelve phase circular squat

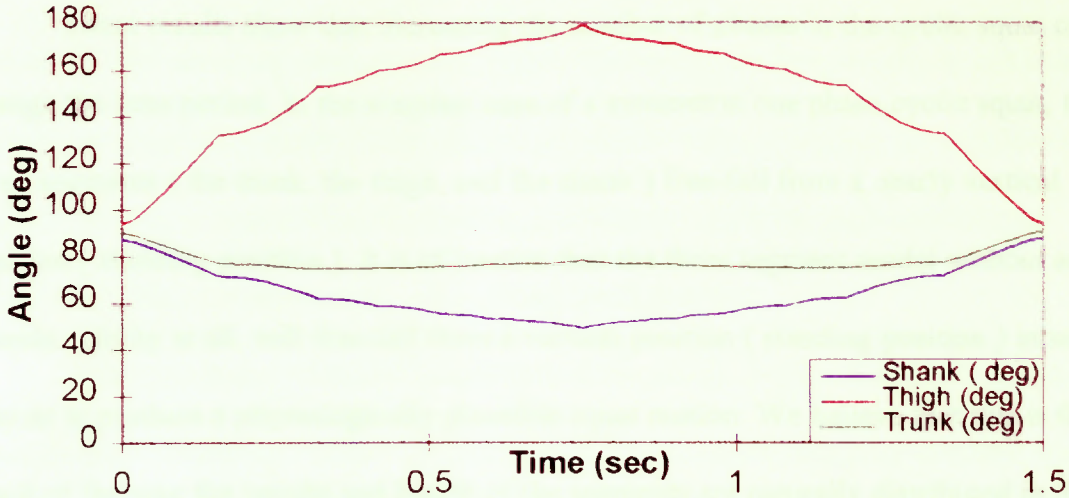

Figure : 26 Angle: Fourteen phase circular squat

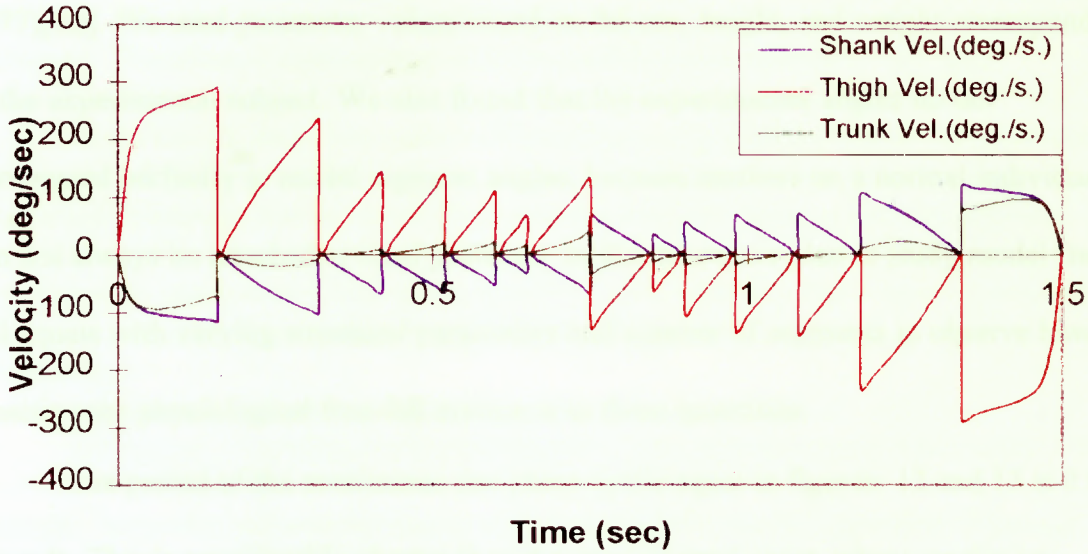

Figure: 27 Velocity: Fourteen phase circular squat

These results show that increasing the number of phases in the cyclic squat can change the time period. In the simplest case of a symmetric one phase cyclic squat, the three segments (the trunk, the thigh, and the shank) free-fall from a nearly vertical position (standing position). It is of interest that the three segment model without any muscle activity at all, will free-fall from a vertical position (standing position) in such a way as to produce a physiologically plausible squat motion. We believe that this is the result of the way the weight and length of the segments are naturally distributed in the human structure. The structural parameters (the length, the masses, and the center of masses) used in the model were obtained from data published by Dempster (Veau 1977)[10]. We used parameter values based on the sex, height, and weight corresponding to the experimental subject. We also found that the experimental angles do not correspond perfectly to model segment angles, because markers on a normal individual can not always be attached at ideal positions. In the future, we plan to study model free-fall squats with varying structural parameters and number of segments to observe how sensitive the physiological free-fall motion is to these quantities.

The period of the continuous one phase cyclic squat in figures: 12 and 13 is 0.60 seconds. This is considerably shorter than the experimental squat, whose angles are illustrated in figure: 11. Both theoretical and experimental curves are symmetric in time about the vertical line that represents the time at the end of the half squat (see figure: 11, approximately 1.1 second). We will call this midway configuration the sitting configuration as the thigh segment angle is horizontal with respect to the ground at this time. The symmetry in the theoretical curve is obtained by reversing the segment velocities at the sitting configuration (joint viscosity are assumed to be zero). Figure: 13

shows that the reversing segment velocities at the sitting position, results in achieving nearly zero segment velocity in the standing position. More precisely, no additional muscle activity is required in the standing position to initiate the next cyclic squat. Muscle activity is required only at the sitting configuration.

More precisely, when the model falls to the sitting configuration, the muscles must exert a " breaking " action to stop the fall. Thus, kinetic energy is removed from the system (body) at this time. In addition the muscles must now give a propulsive impulse to the system to bring the segment velocities to equal but opposite values to the previous falling velocities at the sitting position so that the symmetric rising phase can be achieved. Therefore, kinetic energy must be added by the muscles at this time.

In the model, these two effects of breaking and propulsion occur instantaneously and result in impulse forces on all joints and in particular figure: 13 shows that the instantaneous changes in velocity, are greatest in the segments angle representing the knee joint.

In figures: 14 and 15, the model has two phase, and the cyclic squat takes 0.48 seconds. Since the segment velocities are not zero in the standing position of the two phase squat, the breaking and propulsive muscle activities that were described in the sitting position for the one phase squat now must also occur in the standing position of the two phase squat.

Therefore, one may consider the one phase model as a limiting instance of the two phase model, where the muscle activity in the standing configuration has been reduced to zero. This one phase squat represents the minimum mechanical energy cost of all possible two phase squats. The one phase squat has the smallest momentum reversal in the sitting position (least knee impact) of all two phase squats and in addition, as we have already mentioned, the one phase squat has no mechanical energy cost in the standing position. Thus, the one phase solution represent the minimum power needed to complete a two phase squat, since it maximize the time and minimize the mechanical work.

Figures: 16 through 26 shows four phase squat, six phase squat, eight phase squat, ten phase squat, twelve phase squat, and fourteen phase squat, respectively. These periodic squat solutions were obtained in the following manner. At the beginning of the each falling phase, the initial velocity of all segments are set to zero. Each phase is solved as a ballistic motion. That is, there are assumed to be no external forces acting on the system, except gravitational forces. Symmetry of the solution is enforced by reversing the falling velocities at the end of each falling phase to obtain the corresponding solutions of the rising phase. This allows the model body to rise in a symmetric fashion. Therefore, in this squat solution, a zero velocity is achieved at the end of each rising phase.

Our model shows that the period of the squat, depends on the number of phases of the cyclic squat. As we add more phases in this manner, the period of the squat increases, the number of sudden changes in momentum increases but the momentum change of the each phase decreases and therefore impact on each joint is reduced. The maximum impact on the joints can be reduced by adding more phases to the model. Also, the curve of the shank segment angle as a function of time during each phase of the squat is concave up; however as more phases are added, the overall shape of the curve becomes concave down and appears to qualitatively match the experimental curve (see figure:11). Similar observation can be made for the other segment angle curves as a function of time.

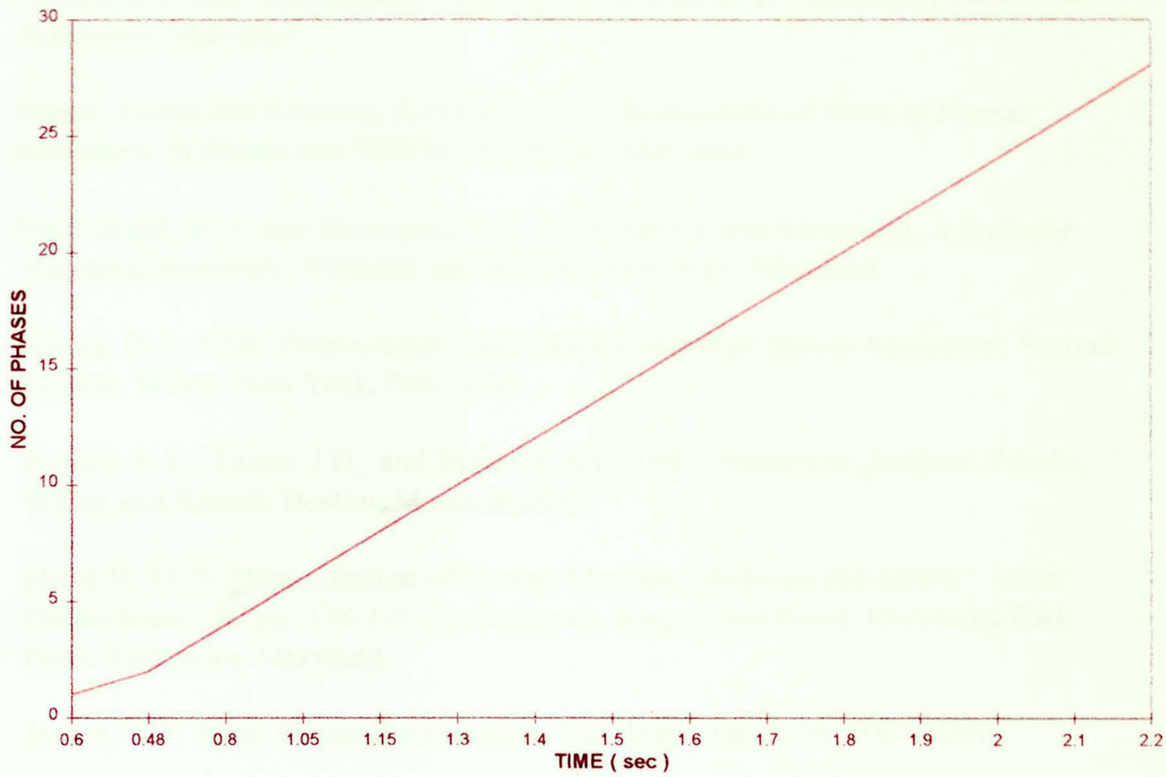

Figure: 28 Number of phases vs. Cycle time

In summary, these results suggest that, the free-fall squat minimizes the mechanical energy cost for all squats; however, adding additional phases can reduce the impact force on the joints and allow for better control of the motion through the intervention of muscular activity. The free-fall technique appears to be used by professionally trained athletes and in particular in those athletes, who train for weight lifting [11].

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