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The Van Houten library has removed some of the personal information and all signatures from the approval page and biographical sketches of theses and dissertations in order to protect the identity of NJIT graduates and faculty.
The purpose of this thesis is to provide an introduction in the design of composite columns. The design methods according to AISC and Eurocode 4 are summarized and provide a procedure to design a composite column. In addition, equation are derived to determine the nominal flexural strength of typical composite cross-sections.
COMPOSITE COLUMNS

by
Magnar Berge

A Thesis
Submitted to the Faculty of
New Jersey Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of
Master of Science in Civil Engineering

Department of Civil and Environmental Engineering

August 1998
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COMPOSITE COLUMNS

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Major: Civil Engineering
To my beloved family
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CHAPTER 1

INTRODUCTION

Composite Columns represent a combination of one or more steel sections and concrete in a compression member. The two main types are concrete-encased (either fully or partly) and concrete-filled composite columns. The advantage of a concrete encasement is to stiffen the steel section, making it more effective against both local and global buckling. In addition the encasement functions as a fireproofing. The main disadvantage is that full formwork is required. In the case of concrete filled tubes or pipes the steel section is not protected against fire, but no formwork is required.

Combinations of concrete-encased and concrete-filled composite columns are common. In concrete-encased composite columns a reinforcement cage is required to prevent the concrete cover from spalling.

In Chapter 2 the design of composite columns according to the AISC manual is given. Chapter 3 shows the design according to Eurocode 4. The two methods are compared in Chapter 4. Finally, Appendix A provides equations to determine the nominal flexural strength.
CHAPTER 2
DESIGN ACCORDING TO AISC

2.1 General

2.1.1 Limitations

The cross-sectional area of the steel shape, pipe, or tubing shall comprise at least four percent of the total composite cross section.

Concrete encasement of a steel core shall be reinforced with longitudinal load carrying bars, longitudinal bars to restrain concrete, and lateral ties. Longitudinal load carrying bars shall be continuous at framed levels; longitudinal restraining bars may be interrupted at framed levels. The spacing of ties shall be not greater than two-thirds of the least dimension of the composite cross section. The cross-sectional area of the transverse and longitudinal reinforcement shall be at least 0.007 sq. in. per inch of bar spacing. The encasement shall provide at least 1.5 in. of clear cover outside of both transverse and longitudinal reinforcement.

Concrete shall have a specified compressive strength $f'_c$ of not less than 3 ksi nor more than 8 ksi for normal weight concrete and not less than 4 ksi for light weight concrete.

The specified minimum yield stress of structural steel and reinforcing bars used in calculating the strength of a composite column shall not exceed 55 ksi.
The minimum wall thickness of structural steel pipe or tubing filled with concrete shall be equal to \( b \sqrt{F_y / 3E} \) for each face of width \( b \) in rectangular sections and \( D \sqrt{F_y / 8E} \) for circular sections of outside diameter \( D \).

2.1.2 Columns with Multiple Steel Shapes

If the composite cross section includes two or more steel shapes, the shapes shall be interconnected with lacing, tie plates, or batten plates to prevent buckling individual shapes before hardening of concrete.

2.1.3 Load Transfer

The portion of the design strength of axial loaded composite columns resisted by concrete shall be developed by direct bearing at connections. When the supporting concrete area is wider than the loaded area on one or more sides and otherwise restrained against lateral expansion on the remaining sides, the maximum design strength of concrete shall be \( 1.7 \phi_c \cdot f_c' \cdot A_{B} \), where

\[ \phi_c = 0.6 \]

\( A_{B} \) = loaded area
2.2 Design

2.2.1 Compression

Design Strength: \( \phi_c P_n = A_s F_{cr} \)  \hspace{1cm} (2.1)

where

\( A_s = \) cross-sectional area of structural steel, in\(^2\)

\[
F_{cr} = \begin{cases} 
\exp(-0.419\lambda_c^2) \lambda_c F_{my} & \text{if } \lambda_c \leq 1.5 \\
\frac{0.877}{\lambda_c^2} \lambda_c F_{my} & \text{if } \lambda_c > 1.5 
\end{cases} \]  \hspace{1cm} (2.2)

\[
F_{my} = F_y + c_1 + F_{yr} \frac{A_r}{A_s} + c_2 f'_{c} \frac{A_c}{A_s} \]  \hspace{1cm} (2.4)

\[
E_m = E + c_3 E_c \frac{A_c}{A_s} \]  \hspace{1cm} (2.5)

\[
E_c = w^{1.5} \sqrt{f'_{c}} \]  \hspace{1cm} (2.6)

\[
\lambda_c = \frac{KI}{r_m \pi \sqrt{\frac{F_{my}}{E_m}}} \]  \hspace{1cm} (2.7)

\[
r_m = \sqrt{\frac{I_x}{A_s}} \geq 0.3 \]  \hspace{1cm} \text{radius of gyration of steel shape}  \hspace{1cm} (2.8)

\( w = \) unit weight of concrete, lb / ft\(^3\)

\( E = 29,000 \) ksi

\( f'_{c} = \) specified compressive strength of concrete, ksi

\( F_{yr} = \) yield stress reinforcement, ksi

\( c_1=1.0, c_2=0.85, c_3=0.4 \) \hspace{1cm} \text{for concrete-filled tubes or pipes}

\( c_1=0.7, c_2=0.6, c_3=0.2 \) \hspace{1cm} \text{for concrete-encased shapes}
2.2.2 Combined Flexure and Compression

\[
\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left[ \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right] \leq 1.0 \quad \text{for} \quad \frac{P_u}{\phi_c P_n} \geq 0.2 \quad (2.9)
\]

\[
\frac{P_u}{2\phi_c P_n} + \frac{1}{\phi_b} \left[ \frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}} \right] \leq 1.0 \quad \text{for} \quad \frac{P_u}{\phi_c P_n} < 0.2 \quad (2.10)
\]

An approximate formula for the nominal flexural strength \( M_n \) is given in Galambos and Chapuis (1980):

\[
M_n = M_p = Z F_p + \frac{1}{2} (h_2 - 2c_r) A_r F_{yr} + \left( \frac{h_2}{2} - \frac{A_w F_{yr}}{1.7 f_c h_1} \right) A_w F_{yr} \quad (2.11)
\]

\( P_u = \) required tensile strength, kips

\( M_u = \) required flexural strength, second order

\( \phi_c = 0.85 \)

\( \phi_b = 0.9 \)

\( A_w = \) web area of encased steel shape; for concrete filled tubes, \( A_w = 0 \), in.\(^2\)

\( Z = \) plastic section modulus of steel section, in.\(^3\)

\( c_r = \) average of distance from compression face to longitudinal reinforcement in that face and distance from tension face to longitudinal reinforcement in that face, in.

\( h_1 = \) width of composite cross section perpendicular to the plane of bending, in.

\( h_2 = \) width of composite cross section parallel to the plane of bending, in.
CHAPTER 3

SIMPLIFIED DESIGN METHOD ACCORDING TO EUROCODE 4 (EC4)

Necessary Checks according to EC4 4.8.3.1 (5)

1. Control of Limitations (4.8.3.1 (3)).

2. Check for local buckling (4.8.2.4).

3. Control of Cover and Ratio for reinforcement (4.8.2.5).

4. Calculation of $N_{cr}$ and $\bar{\lambda}$ (4.8.3.7), Definition of $\gamma_{Ma}$ (4.8.3.2.)

5. Control according to 4.8.3.10, if a Calculation of the moment to the second order is necessary.

6. Check of load-capacity of the column according to 4.8.3.3, 4.8.3.8, 4.8.3.9 and 4.8.3.11 to 4.8.3.14.

7. Check of load transfer and ultimate shear strength according to 4.8.2.6 through 4.8.2.8

3.1 Control of Limitations

1. The cross-section must be double symmetric and be constant over the length.

2. The cross-section value $\delta_a$ should be between 0.2 and 0.9.

$$\delta_a = \frac{A_a \cdot f_{yd}}{N_{pl,rd}}$$

(3.1)

$A_a$ area of steel section

$f_{yd}$ design strength steel
$N_{pl,Rd}$ characteristic squash load

3. The slenderness ratio $\lambda$ should not exceed 2.0.

4. For concrete encased steel shapes the following reinforcement cover must be provided:
   - in y-direction: $40 \text{ mm} \leq c_y \leq 0.4 \, b$,  
   - in z-direction: $40 \text{ mm} \leq c_y \leq 0.3 \, h$,

A larger cover can be used but is not to be considered in calculations.

5. The cross-sectional area of the longitudinal reinforcement shall not exceed 4% of the concrete area in calculations. If the longitudinal reinforcement is not considered in calculations and the environmental influences are according to EC2 Table 4.1 Line 1, the following reinforcement is sufficient:
   - Longitudinal reinforcement with a minimum diameter of 8 mm and a maximum spacing of 250 mm,
   - Ties with a minimum diameter of 6 mm and a maximum spacing of 200 mm,
   - Mesh reinforcing with a minimum diameter of 4 mm.

3.2 Check for Local Buckling

A proof against local buckling of fully encased steel shapes is not necessary. This is also valid for other cross sections if the following limitations are fulfilled:
   - for concrete filled pipes: $d / t \leq 90 \, e^2$
• for concrete filled rectangular tubes: \( h/t \leq 52 \varepsilon \)

• for partly encased steel shapes: \( b/t_f \leq 44 \varepsilon \)

where:

\[
\varepsilon = \sqrt{\frac{235}{f_y}}
\]  

(3.2)

3.3 Control of Cover and Ratio for Reinforcement

• The cover of the flanges of fully encased I-beams should not be less than 40 mm or 1/6 of the width \( b \) of the flange.

• If the longitudinal reinforcement is considered in calculations, the ratio of reinforcement should be at least 0.3 %.

• Stirrups and spacing according to EC2

• The effective perimeter of the reinforcement is eventually to be determined and considered.

• For concrete filled tubes there is usually no need for longitudinal reinforcement.

3.4 Calculation of \( N_{cr} \) and \( \bar{\lambda} \), Definition of \( \gamma_{Ma} \)

\[
N_{cr} = \frac{\pi^2 (EI)_e}{l^2} \varepsilon
\]  

(3.3)

\[
\bar{\lambda} = \sqrt{\frac{N_{pt,R}}{N_{cr}}}
\]  

(3.4)

\( \gamma_{Ma} = 1.10 \)

where:
\[ N_{pl,R} = A_d f_{yy} + A_c \alpha_c f_{ck} + A_s f_{sk} \]  
characteristic squash load (3.5)

\[ (EI)_e = E_a I_a + 0.8 E_{cd} I_c + E_s I_s \]  
effective bending stiffness (3.6)

\[ E_{cd} = E_{cm}/\gamma_c \]  
secant modulus (see Table 3.1) (3.7)

\[ l \]  
effective length

\[ \gamma_c \]  
partial safety factor for concrete

**Table 3.1** Secant Modulus \( E_{cm} \) (KN/mm²)

<table>
<thead>
<tr>
<th>Concrete</th>
<th>C20/25</th>
<th>C25/30</th>
<th>C30/37</th>
<th>C35/45</th>
<th>C40/50</th>
<th>C45/55</th>
<th>C50/60</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{cm} )</td>
<td>29</td>
<td>30.5</td>
<td>32</td>
<td>33.5</td>
<td>35</td>
<td>36</td>
<td>37</td>
</tr>
</tbody>
</table>

The influence of the long-term behavior of the concrete on the elastic modulus of rupture should be considered, if:

- \( \bar{\lambda} > 0.8 \) for concrete encased cross-sections
- \( \bar{\lambda} > \frac{0.8}{1-\delta} \) for concrete filled cross-sections

and

- \( e/d < 2 \)

The effective elastic modulus of rupture than is:

\[ E_c = E_{cd} \left( 1 - 0.5 \frac{N_{G,St}}{N_{St}} \right) \]  
(3.8)
3.5 Check if Calculation of the Moment to the Second Order is Required

A calculation of the moment to the second order is not necessary if

- \( \frac{N_{sd}}{N_{cr}} \leq 0.1 \) or

- \( \bar{\lambda} \leq \bar{\lambda}_{cr} \)

where:

\[
\bar{\lambda}_{cr} = 0.2(2 - r)
\]

(3.9)

\( r \) absolute ratio of end moments

The moment to the second order can be simply calculated by multiplying the maximum moment with the correction factor \( k \).

\[
k = \frac{\beta}{1 - \frac{N_{sd}}{N_{cr}}} \geq 1.0
\]

(3.10)

where

\( \beta = 1.0 \) if moment at midspan governs.

\( \beta = 0.66 + 0.44r \geq 0.44 \) if moment at support governs.

(3.11)

If there is a simultaneous effect of support moments and midspan moments, \( \beta \) should not be less than 1.0.
3.6 Check of Load Capacity

3.6.1 Check for Axial Compression

\[
\frac{N_{ad}}{\chi \cdot N_{pl, Rd}} \leq 1.0
\]

(3.12)

where

\[
\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}}
\]

(3.13)

\[
\Phi = 0.5 \cdot [1 + \alpha \cdot (\lambda - 0.2) + \lambda^2]
\]

(3.14)

Table 3.2 Imperfection Factor \(\alpha\)

<table>
<thead>
<tr>
<th>Buckling curve</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperfection factor (\alpha)</td>
<td>0.21</td>
<td>0.34</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Curve a for concrete filled tubes.

Curve b for fully or partially encased I-Beams with bending about the strong axis.

Curve c for fully or partially encased I-Beams with bending about the weak axis.
3.6.2 Check for Compression and Bending about one Axis

\[
\frac{M_{sd}}{0.9 \cdot \mu \cdot M_{pl,Rd}} \leq 1.0 \tag{3.15}
\]

**Figure 3.1 Interaction Curve Bending and Compression**

where

\[
\mu = \mu_d - \mu_k \frac{\chi_d - \chi_n}{\chi - \chi_n} \tag{3.16}
\]

\[
\chi = \frac{N_{Rd}}{N_{pl,lo}} \tag{3.17}
\]

\[
\mu_k = \mu(\chi) \tag{3.18}
\]

\[
\chi_d = \frac{N_{sd}}{N_{pl,Rd}} \tag{3.19}
\]

\[
\mu_d = \mu(\chi_d) \tag{3.20}
\]

\[
\chi_n = \chi \frac{1-r}{4} \leq \chi_d \tag{3.21}
\]

Equation of interaction curve according to R. Bergman:
\[ \mu(\chi) = a_4 \cdot \chi^4 + a_3 \cdot \chi^3 + a_2 \cdot \chi^2 + a_1 \cdot \chi + 1.0 \]  
(3.22)

where

\[ a_4 = \frac{4 \cdot \mu_D \cdot (1 - \chi_c) - (2 - \chi_c)^2}{\chi_c^2 \cdot (1 - \chi_c) \cdot (1 - \frac{\chi_c}{2})^2} \]  
(3.23)

\[ a_3 = -2 \cdot a_4 \cdot \chi_c \]  
(3.24)

\[ a_2 = a_4 \cdot (\chi_c^2 + \chi_c - 1) - \frac{1}{1 - \chi_c} \]  
(3.25)

\[ a_1 = a_4 \cdot \chi_c \cdot (1 - \chi_c) + \frac{\chi_c}{1 - \chi_c} \]  
(3.26)

\[ \chi_c = \frac{A_e \cdot f_{ct}}{N_{pl,Rd}} \]  
(3.27)

\[ \mu_D = \frac{M_{D,Rd}}{M_{pl,Rd}} \]  
(3.28)

For concrete encased I-Beams with bending about the y-axis and \( \delta > 0.6 \), it could happen that for \( \chi > \chi_c \) the above equation yields too small \( \mu \)-values. The following equation should be checked to see if it yields greater values and therefore controls:

\[ \mu(\chi) = \frac{1 - \chi}{1 - \chi_c} \]  
(3.29)

### 3.6.3 Check for Compression and Biaxial Bending

\[ \frac{M_{y,rd}}{0.9 \cdot \mu_y \cdot M_{pl,y,Rd}} \leq 1.0 \]  
(3.30)
The influence of imperfection is usually only to be considered for the axis that is most endangered for failure, i.e. for the check of the axis that is less endangered $\mu_d$ can be used instead of $\mu$.

\[ \frac{M_{z,\text{ad}}}{0.9 \cdot \mu_z \cdot M_{pl,z,Rd}} \leq 1.0 \]  \hspace{1cm} (3.31)

\[ \frac{M_{y,\text{sd}}}{\mu_y \cdot M_{pl,y,Rd}} + \frac{M_{z,\text{sd}}}{\mu_z \cdot M_{pl,z,Rd}} \leq 1.0 \] \hspace{1cm} (3.32)

The influence of imperfection is usually only to be considered for the axis that is most endangered for failure, i.e. for the check of the axis that is less endangered $\mu_d$ can be used instead of $\mu$.

### 3.7 Check of Load Transfer and Ultimate Shear Strength

**Load Transfer:**

- The load transfer has to formed so that the slip in the bonding due to force transmission does not violate the assumptions for the design.
- The length of the transmission should not be greater than two times the corresponding width of the column.
- For I-Beams with concrete filled flanges stirrups must be used to provide a transfer between steel shape and concrete (Stirrups welded or continuing through the web).

**Ultimate Shear Strength:**

- Shear stresses have eventually to be considered. The stresses can be computed according to elastic calculations.
Allowable Shear Stresses:

If the shear stresses exceed the figures in Table 3.3, studs must be used.

Table 3.3 Allowable Shear Stresses N/mm²

<table>
<thead>
<tr>
<th>Description</th>
<th>Allowable Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>for concrete encased steel shapes</td>
<td>0.6 N/mm²</td>
</tr>
<tr>
<td>for concrete filled tubes</td>
<td>0.4 N/mm²</td>
</tr>
<tr>
<td>for the flanges of partly encased steel shapes</td>
<td>0.2 N/mm²</td>
</tr>
<tr>
<td>for the web of partly encased steel shapes</td>
<td>0.0 N/mm²</td>
</tr>
</tbody>
</table>

3.8 Ultimate Axial Strength

\[ N_{pl,rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd} \]  \hspace{1cm} (3.33)

where

\( A_a, A_c, A_s \) area of steel shape, concrete, and reinforcement

\( f_{yd} = \frac{f_y}{\gamma_{Mu}} \)

\( f_{cd} = \frac{f_{ck}}{\gamma_c} \)

\( f_{sd} = \frac{f_{sk}}{\gamma_s} \)

\( f_y, f_{ck}, \) and \( f_{sk} \) characteristic strength in accordance with EC3 and EC2

\( \gamma_{Mu} = 1.10 \) partial safety factor for steel

\( \gamma_c = 1.50 \) partial safety factor for concrete

\( \gamma_s = 1.15 \) partial safety factor for reinforcement
\( a_c = 1.0 \) for concrete filled tubes

\( a_c = 0.85 \) for other cross sections

For concrete filled pipes the positive effect of tying can be considered, if:

- \( \lambda \leq 0.5 \)

- \( M_{m_{ax,sd}} \leq N_{sd} \frac{d}{10} \)

The plastic ultimate normal force is then

\[
N_{pl,\text{rd}} = A_a \cdot \eta_2 \cdot f_{y,sl} + A_c \cdot f_{cd} \left( 1 + \eta_1 \cdot \frac{t}{d} \cdot \frac{f_y}{f_{ek}} \right) + A_s \cdot f_{yd}
\]  

(3.34)

where

\[
\eta_1 = \eta_{b_o} \left( 1 - \frac{10 \cdot e}{d} \right)
\]  

(3.35)

\[
\eta_2 = \eta_{2_o} + \left( 1 - \eta_{b_o} \right) \frac{10 \cdot e}{d}
\]  

(3.36)

For \( e > d / 10 \) is \( \eta_1 = 0 \) and \( \eta_2 = 1.0 \).

\[
\eta_{10} = 4.9 - 18.5 \cdot \lambda + 17 \cdot \lambda^2 \geq 0
\]  

(3.37)

\[
\eta_{10} = 0.25 \cdot (3 + 2 \cdot \lambda ) \leq 1.0
\]  

(3.38)

\[
e = \frac{M_{m_{ax,sl}}}{N_{sd}}
\]  

(3.39)

### 3.9 Plastic Ultimate Normal Force of the Concrete

\[
N_{pm,\text{rd}} = A_c \cdot f_{cd}
\]  

(3.40)
3.10 Ultimate Moment

$$M_{pl, Rd} = M_{max, Rd} - M_{n, Rd}$$ (3.41)

$$M_{max, Rd} = W_{po} \cdot f_{yd} + W_{ps} \cdot f_{yd} + 0.5 \cdot W_{pc} \cdot f_{cd}$$ (3.42)

$$M_{n, Rd} = W_{pan} \cdot f_{yd} + W_{psn} \cdot f_{sd} + 0.5 \cdot W_{pcn} \cdot f_{cd}$$ (3.43)

where

$W_{pa}$, $W_{ps}$, and $W_{pc}$ plastic section modulus of steel shape, reinforcement, and uncracked concrete.

$W_{pan}$, $W_{psn}$, and $W_{pcn}$ plastic section modulus of steel shape, reinforcement, and uncracked concrete in the area $2 h_n$.

The equations to determine the location of the neutral axis $h_n$ and the plastic section modulus are given in Appendix A.
CHAPTER 4

COMPARISON OF THE AMERICAN AND EUROPEAN METHODS

4.1. Nominal Flexural Strength $M_n$

AISC gives a very simplified equation to determine the nominal flexural strength according to Galambos and Chapuis (1980), whereas the Eurocode 4 provides a more exact approach. For the determination of the nominal flexural strength according to Eurocode 4 the actual neutral axis is computed and gives the base to compute the nominal flexural strength. In the following example the nominal flexural strength for a typical cross-section is computed according to both methods. Only bending about the strong axis is regarded. The influence of the reinforcement is neglected.

**Figure 4.1** Concrete Encased I-Section

**Section Properties**

$A_s = 96.4 \text{ in}^2$

$A_w = 36.4 \text{ in}^2$
I_{xx} = 26,800 \text{ in}^4

Z_x = 1510 \text{ in}^3

b_t = 17.91 \text{ in}

A_c = 1,077 \text{ in}^2

A_r = 1.24 \text{ in}^2

4.1.1. Nominal Flexural Strength $M_n$ according to AISC

$$M_n = M_p = Z F_y + \frac{1}{2} \left( h_2 - 2 c_r \right) A_r F_{yr} + \left( \frac{h_2}{2} - \frac{A_w F_y}{1.7 f \cdot h_1} \right) A_w F_y$$

$$M_{ns} = 1,510 \cdot 55 + \frac{1}{2} (47 - 2 \cdot 2) \cdot 1.24 \cdot 60 + \left( \frac{47}{2} - \frac{36.4 \cdot 55}{1.7 \cdot 5 \cdot 25} \right) \cdot 36.4 \cdot 55 = 112,302 k - \text{in}$$

4.1.2. Nominal Flexural Strength $M_n$ according to Eurocode 4

$$N_{pm,Rd} = A_c \cdot f_{cd}$$

$$N_{pm,Rd} = 1,077 \cdot 5 = 5,385 \text{kips}$$

$$W_{xpc} = \frac{b_c h_c^2}{4} - W_{xpa} - W_{xps}$$

$$W_{xpc} = \frac{25 \cdot 47^2}{4} - 1,510 - 0 = 12,296 \text{in}^3$$

Assume the neutral axis in the web of the steel shape: $h_{nx} \leq h / 2 - t_f$

$$h_{nx} = \frac{N_{pm,Rd} - A_{in} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd})}{2 \cdot b_c \cdot a_c \cdot f_{cd} + 2 \cdot t_w \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}$$
The results show that the ultimate moment is the same for both methods. The ultimate moment resistance occurs when the concrete is cracked to half height, i.e. the tension stresses due to bending are overpressed by compression due to the axial force so that the neutral axis is in the middle of the cross-section.

\[
\begin{align*}
h_{nx} &= \frac{5,385}{2 \cdot 25 \cdot 0.85 \cdot 5 + 0.91 \cdot (2 \cdot 55 - 0.85 \cdot 5)} = 13.3 \text{in} < 18.27 \text{in} \quad \text{Assumption OK} \\
W_{xpan} &= t_w \cdot h_{nx}^2 \\
W_{xpan} &= 0.91 \cdot 13.3^2 = 160.9 \text{in}^3 \\
W_{xpen} &= \frac{b_c h_{nx}^2}{4} - W_{xpan} - W_{xpan} \\
W_{xpen} &= \frac{25 \cdot 13.3^2}{4} - 160.9 - 0 = 944.7 \text{in}^3 \\
M_{pl,ld} &= M_{max,ld} - M_{n,ld} \\
M_{max,ld} &= W_{pa} \cdot f_{yd} + W_{ps} \cdot f_{sd} + 0.5 \cdot W_{pc} \cdot f_{cd} \\
M_{x,max,ld} &= 1,510 \cdot 55 + 0.5 \cdot 12,296 \cdot 5 = 113,790 \text{in}^2 \text{lb}, \text{equivalent to } M_{nx} \\
M_{n,ld} &= W_{pam} \cdot f_{yd} + W_{psm} \cdot f_{sd} + 0.5 \cdot W_{pcn} \cdot f_{cd} \\
M_{x,n,ld} &= 160.9 \cdot 55 + 0.5 \cdot 944.7 \cdot 5 = 11,211 \text{in} \\
M_{x,pl,ld} &= 113,790 - 11,211 = 102,579 \text{in}^2 \text{lb}
\end{align*}
\]
4.2. Interaction

AISC takes combined flexure and bending into account by a linear interaction between axial strength and flexural strength. Thus, the positive effect of the axial force on the resisting moment is not taken into account like it is in Eurocode 4. This results in over-designed columns and waste of material. The more economical approach is to determine the moment capacity corresponding to the actual axial force (Figure 3.1).

4.3. Tying Effect in Circular Tubes

In the case of concrete-filled circular tubes the compressive strength of the concrete is enhanced by the containment through the circular tube. Eurocode 4 takes this into account by increasing the ultimate axial strength. The hoop stresses in the steel tube cause a reduction of the yield strength of the steel, but are not accounted for in Eurocode 4.
APPENDIX A

PLASTIC SECTION MODULUS OF TYPICAL CROSS-SECTIONS

Cross-Section A.1

![Diagram of Cross-Section A.1]

Figure A.1 One I-Section Rectangular Encased

Bending about y-axis:

\[ W_{ypa} = \frac{t_w \cdot (h - 2 \cdot t_f)^2}{4} + b \cdot t_f \cdot (h - t_f) \]  \hspace{1cm} (A.1)

\[ W_{ypc} = \frac{b \cdot h_c^2}{4} - W_{ypa} - W_{yps} \]  \hspace{1cm} (A.2)

(a) Neutral axis in the web of the steel shape: \( h_{wy} \leq h / 2 - t_f \)

\[ h_{wy} = \frac{N_{pm,Rd} - A_{sm} \cdot (2 \cdot f_{sf} - a_c \cdot f_{cd})}{2 \cdot b_c \cdot a_c \cdot f_{cd} + 2 \cdot t_w \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  \hspace{1cm} (A.3)

\[ W_{ypaw} = t_w \cdot h_{wy}^2 \]  \hspace{1cm} (A.4)
(b) Neutral axis in the flange of the steel shape: $h/2 - t_f \leq h_{nf} \leq h/2$

$$h_{nf} = \frac{N_{pan,rd} - A_{ss} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) + (b - t_w) \cdot (h - 2 \cdot t_f) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot b_c \cdot a_e \cdot f_{cd} + 2 \cdot b \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}$$  \hspace{1cm} (A.5)

$$W_{span} = b \cdot h_{nf}^2 - \frac{(b - t_w) \cdot (h - 2 \cdot t_f)^2}{4}$$  \hspace{1cm} (A.6)

(c) Neutral axis outside of the steel shape: $h/2 \leq h_{nf} \leq h_c/2$

$$h_{nf} = \frac{N_{pan,rd} - A_{ss} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) + A_a \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot b_c \cdot a_e \cdot f_{cd}}$$  \hspace{1cm} (A.7)

$$W_{span} = W_{ypan}$$  \hspace{1cm} (A.8)

**Bending about z-axis:**

$$W_{zpa} = \frac{(h - 2 \cdot t_f) t_w^2}{4} + \frac{2 \cdot t_f \cdot b^2}{4}$$  \hspace{1cm} (A.9)

$$W_{zpc} = \frac{h_c b_c^2}{4} - W_{zpa} - W_{zps}$$  \hspace{1cm} (A.10)

(a) Neutral axis in the web of the steel shape: $h_{nw} \leq t_w/2$

$$h_{nw} = \frac{N_{pan,rd} - A_{ss} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd})}{2 \cdot h_c \cdot a_e \cdot f_{cd} + 2 \cdot h \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}$$  \hspace{1cm} (A.11)

$$W_{zpan} = h \cdot h_{nw}^2$$  \hspace{1cm} (A.12)
(b) Neutral axis in the flange of the steel shape: \( t_w / 2 \leq h_{nz} \leq b / 2 \)

\[
h_{nz} = \frac{N_{pm, Rd} - A_n \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) + t_w \left(2 \cdot t_f - h\right) \cdot \left(2 \cdot f_{yd} - a_e \cdot f_{cd}\right)}{2 \cdot h_c \cdot a_e \cdot f_{cd} + 4 \cdot t_f \cdot \left(2 \cdot f_{yd} - a_e \cdot f_{cd}\right)}
\]  
(A.13)

\[
W_{zpm} = 2 \cdot t_f \cdot h_{nz}^2 + \frac{\left(h - 2 \cdot t_f\right) \cdot t_w^2}{4}
\]  
(A.14)

(c) Neutral axis outside of the steel shape: \( h_{nz} > b / 2 \)

\[
h_{nz} = \frac{N_{pm, Rd} - A_n \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) - A_n \cdot \left(2 \cdot f_{yd} - a_e \cdot f_{cd}\right)}{2 \cdot h_c \cdot a_e \cdot f_{cd}}
\]  
(A.15)

\[
W_{zpm} = W_{zpu}
\]  
(A.16)
Cross-Section A.2

Figure A.2 Two I-Sections Rectangular Encased

Bending about the y-axis:

\[
W_{ysa} = \frac{t_{w1} \cdot \left( h_1 - 2 \cdot t_{f1} \right)^2}{4} + b_1 \cdot t_{f1} \cdot \left( h_1 - t_{f1} \right) + \frac{\left( h_2 - 2 \cdot t_{f2} \right) t_{w2}^2}{4} + \frac{2 \cdot t_{f2} \cdot b_2^2}{4}
\]  
(A.17)

\[
W_{ysc} = \frac{b_c \cdot h_c^2}{4} - W_{ysa} - W_{ysd}
\]  
(A.18)

(a) Neutral axis in the web of the steel shape 2: \( h_{ny} \leq t_{w2} / 2 \)

\[
h_{ny} = \frac{N_{pnw,Rd} - A_{sw} \left( 2 \cdot f_{sd} - a_c \cdot f_{cd} \right)}{2 \cdot b_c \cdot a_c \cdot f_{cd} + 2 \cdot \left( h_2 + t_{w1} \right) \left( 2 \cdot f_{sd} - a_c \cdot f_{cd} \right)}
\]  
(A.19)

\[
W_{yspm} = \left( h_2 + t_{w1} \right) \cdot h_{ny}^2
\]  
(A.20)
(b) Neutral axis in the flange of the steel shape 2: $t_{w2}/2 \leq h_{ny} \leq b_2/2$

$$h_{ny} = \frac{N_{pw,Rd} - A_{sn} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd}) - t_{w2} \cdot (h_2 - 2 \cdot t_{f2}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot b_c \cdot a_c \cdot f_{cd} + 2 \cdot (2t_{f2} + t_{w1}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}$$

(A.21)

$$W_{ypa2} = (2 \cdot t_{f2} + t_{w1}) \cdot h_{ny}^2 + (h_2 - 2 \cdot t_{f2}) \cdot t_{w2}^2$$

(A.22)

(c) Neutral axis in the web of the steel shape 1: $b_2/2 \leq h_{ny} \leq h_2/2 - t_{f1}$

$$h_{ny} = \frac{N_{pw,Rd} - A_{sn} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd}) - A_{a2} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot b_c \cdot a_c \cdot f_{cd} + 2 \cdot t_{w1} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}$$

(A.23)

$$W_{ypa2} = W_{zpa2} + t_{w1} \cdot h_{ny}^2$$

(A.24)

(d) Neutral axis in the flange of the steel shape 1: $h_1/2 - t_{f1} \leq h_{ny} \leq h_1$

$$h_{ny} = \frac{N_{pw,Rd} - A_{sn} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd}) - A_{a2} \cdot (b_1 - t_{w1}) \cdot (h_1 - 2 \cdot t_{f1}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot b_c \cdot a_c \cdot f_{cd} + 2 \cdot b_1 \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}$$

(A.25)

$$W_{ypa2} = W_{zpa2} + b_1 \cdot h_{ny}^2 - \frac{(b_1 - t_{w1}) \cdot (h_1 - 2 \cdot t_{f1})^2}{4}$$

(A.26)

(e) Neutral axis outside of the steel shape 1: $h_2 \leq h_{ny}$

$$h_{ny} = \frac{N_{pw,Rd} - A_{sn} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd}) - A_c \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot b_c \cdot a_c \cdot f_{cd}}$$

(A.27)

$$W_{ypa2} = W_{yza1} + W_{zpa2}$$

(A.28)
Bending about z-axis

\[ W_{zps} = \frac{t_{w2} \cdot \left( h_2 - 2 \cdot t_{f2} \right)^2}{4} + b_1 \cdot t_{f2} \cdot \left( h_2 - t_{f2} \right) + \frac{\left( h_1 - 2 \cdot t_{f1} \right) t_{w1}^2}{4} + \frac{2 \cdot t_{f1} \cdot b_1^2}{4} \]

(A.29)

\[ W_{zpc} = \frac{h_c \cdot b_c^2}{4} - W_{zps} - W_{zps} \]

(A.30)

(a) Neutral axis in the web of steel shape 1: \( h_{nz} \leq t_{w1} / 2 \)

\[ h_{nz} = \frac{N_{pnw, Rd} - A_{sn} \cdot \left( 2 \cdot f_{yd} - a_c \cdot f_{cd} \right)}{2 \cdot h_c \cdot a_c \cdot f_{cd} + 2 \cdot h_1 \cdot \left( 2 \cdot f_{yd} - a_c \cdot f_{cd} \right)} \]

(A.31)

\[ W_{zpan} = h_1 \cdot h_{nz}^2 \]

(A.32)

(b) Neutral axis in the flange of steel shape 1: \( t_{w1} / 2 \leq h_{nz} \leq b_1 / 2 \)

\[ h_{nz} = \frac{N_{pnw, Rd} - A_{sn} \cdot \left( 2 \cdot f_{yd} - a_c \cdot f_{cd} \right) - \left( h_1 - 2 \cdot t_{f1} \right) \cdot t_{w1} \cdot \left( 2 \cdot f_{yd} - a_c \cdot f_{cd} \right)}{2 \cdot h_c \cdot a_c \cdot f_{cd} + 2 \cdot \left( 2 \cdot t_{f1} + t_{w2} \right) \cdot \left( 2 \cdot f_{yd} - a_c \cdot f_{cd} \right)} \]

(A.33)

\[ W_{zpdm} = \left( 2 \cdot t_{f1} + t_{w2} \right) \cdot h_{nz}^2 + \frac{h_1 - 2 \cdot t_{f1}}{4} \cdot t_{w1}^2 \]

(A.34)

(c) Neutral axis in the web of steel shape 2: \( b_1 / 2 \leq h_{nz} \leq h_2 / 2 - t_{fl} \)

If \( b_1 \geq (h_2 + t_{w1}) \):

...
\[ h_{nz} = \frac{N_{pm,Rd} - A_{sn} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd}) + (h_2 - 2 \cdot t_{f2} - t_{w1}) \cdot (b_2 - t_{w1}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot h_c \cdot a_c \cdot f_{cd} + 2 \cdot (b_2 + 2 \cdot t_{f1}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  

(A.35)

\[ W_{zpan} = (2 \cdot t_{f1} + t_{w1}) \cdot h_{nz}^2 + \frac{(h_2 - 2 \cdot t_{f1})}{4} \cdot t_{w1}^2 \]  

(A.36)

If \( b_1 < (h_2 + t_{w1}) \):

\[ h_{nz} = \frac{N_{pm,Rd} - A_{sn} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd}) - A_{u1} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot h_c \cdot a_c \cdot f_{cd} + 2 \cdot t_{w2} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  

(A.37)

\[ W_{zpan} = W_{zpol} + t_{w2} \cdot h_{nz}^2 \]  

(A.38)

(d) Neutral axis in the flange of steel shape 2: \( h_2 / 2 \cdot t_{f1} \leq h_{nz} \leq h_2 \)

If \( b_1 \geq (h_2 + t_{w1}) \):

\[ h_{nz} = \frac{N_{pm,Rd} - A_{sn} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd}) + (b_2 - t_{w2}) \cdot (h_2 - 2 \cdot t_{f2} - t_{w1}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot h_c \cdot a_c \cdot f_{cd} + 2 \cdot (b_2 + 2 \cdot t_{f1}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  

(A.39)

\[ W_{zpan} = b_2 \cdot h_{nz}^2 - \frac{(b_2 - t_{w2}) \cdot (h_2 - 2 \cdot t_{f2})^2}{4} + 2 \cdot t_{f1} \cdot h_{nz}^2 + \frac{h_1 - 2 \cdot t_{f1}}{4} \cdot t_{w1}^2 \]  

(A.40)

If \( b_1 < (h_2 + t_{w1}) \):

\[ h_{nz} = \frac{N_{pm,Rd} - A_{sn} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd}) - \left[ A_{u1} - (b_2 - t_{w2}) \cdot \left( \frac{h_2}{2} - t_{f2} \right) \right] \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot h_c \cdot a_c \cdot f_{cd} + 2 \cdot b_2 \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  

(A.41)

\[ W_{zpan} = W_{zpol} + b_2 \cdot h_{nz}^2 - \frac{(b_2 - t_{w2}) \cdot (h_2 - 2 \cdot t_{f2})^2}{4} \]  

(A.42)
(e) Neutral axis outside of steel shape 1 and 2: $h_1 / 2 \leq h_{uc} \geq b_1 / 2$

$$h_{uc} = \frac{N_{pn,Rd} - A_{sn} \cdot \left( 2 \cdot f_{sd} - a_\epsilon \cdot f_{cd} \right) - A_{\alpha} \cdot \left( 2 \cdot f_{yd} - a_\epsilon \cdot f_{cd} \right)}{2 \cdot h_c \cdot a_\epsilon \cdot f_{cd}}$$  \hspace{1cm} (A.43)

$$W_{zpan} = W_{zpo1} + W_{zpo2}$$  \hspace{1cm} (A.44)
Cross-Section A.3

Figure A.3 Solid Section Rectangular Encased

Bending about the y-axis:

\[ W_{ypsy} = \frac{b \cdot h^2}{4} \]  \hspace{1cm} (A.45)

\[ W_{yspe} = \frac{b_c \cdot h^2}{4} - W_{ypsy} - W_{ypx} \]  \hspace{1cm} (A.46)

(a) Neutral axis inside of steel shape: \( h_{ny} \leq h / 2 \)

\[ h_{ny} = \frac{N_{pmy,rd} - A_{pm} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd})}{2 \cdot b_c \cdot a_c \cdot f_{cd} + 2 \cdot b \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  \hspace{1cm} (A.47)

\[ W_{ypam} = b \cdot h_{ny}^2 \]  \hspace{1cm} (A.48)

(b) Neutral axis outside of steel shape: \( h_{ny} > h / 2 \)
Bending about z-axis:

\[ W_{z_{po}} = \frac{h \cdot b^2}{4} \]  

\[ W_{z_{pc}} = \frac{h_c \cdot b_c^2}{4} - W_{z_{po}} - W_{z_{pi}} \]  

(a) Neutral axis inside of steel shape: \( h_{nz} \leq b / 2 \)

\[ h_{nz} = \frac{N_{pm,Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd})}{2 \cdot h_c \cdot a_c \cdot f_{cd} + 2 \cdot h \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  

\[ W_{z_{pan}} = h \cdot h_{nz}^2 \]  

(b) Neutral axis outside of steel shape: \( h_{nz} > b / 2 \)

\[ h_{nz} = \frac{N_{pm,Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd}) - A_u \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot h_c \cdot a_c \cdot f_{cd}} \]  

\[ W_{z_{pan}} = W_{z_{po}} \]
Cross-Section A.4

Figure A.4 One I-Section Circular Encased

Bending about y-axis:

\[ W_{spa} = \frac{t_w \cdot (h - 2 \cdot t_f)^2}{4} + b \cdot t_f \cdot (h - t_f) \]  \hspace{1cm} (A.57)

\[ W_{spc} = \frac{d_c^3}{6} - W_{spa} - W_{ypa} \]  \hspace{1cm} (A.58)

(a) Neutral axis in the web of the steel shape: \( h_{yw} \leq h / 2 - t_f \)

\[ h_{yw} = \frac{N_{pm, bd} - A_{sh} \cdot (2 \cdot f_{sd} - f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd} + 2 \cdot t_w \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  \hspace{1cm} (A.59)

\[ W_{ypan} = t_w \cdot h_{yw}^2 \]  \hspace{1cm} (A.60)

(b) Neutral axis in the flange of the steel shape: \( h / 2 - t_f \leq h_{yw} \leq h / 2 \)
$$h_{ny} = \frac{N_{pm, Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd}) + (b - t_w) \cdot (h - 2 \cdot t_f) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd} + 2 \cdot b \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}$$

(A.61)

$$W_{ypan} = b \cdot h_{ny}^2 - \frac{(b - t_w) \cdot (h - 2 \cdot t_f)^2}{4}$$

(A.62)

(c) Neutral axis outside of the steel shape: $h / 2 \leq h_{ny} \leq h_c / 2$

$$h_{ny} = \frac{N_{pm, Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd}) + A_a \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd}}$$

(A.63)

$$W_{ypan} = W_{ypa}$$

(A.64)

Bending about z-axis:

$$W_{zpa} = \frac{(h - 2 \cdot t_f) t_w^2}{4} + \frac{2 \cdot t_f \cdot b^2}{4}$$

(A.65)

$$W_{zpc} = \frac{d^3}{6} - W_{zpa} - W_{zps}$$

(A.66)

(a) Neutral axis in the web of the steel shape: $h_{nz} \leq t_w / 2$

$$h_{nz} = \frac{N_{pm, Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd} + 2 \cdot h \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}$$

(A.67)

$$W_{zpan} = h \cdot h_{nz}^2$$

(A.68)
(b) Neutral axis in the flange of the steel shape: \( t_w / 2 \leq h_{iz} \leq b / 2 \)

\[
h_{iz} = \frac{N_{pm, Rd} - A_{sa} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) + t_w \left(2 \cdot t_f - h\right) \cdot \left(2 \cdot f_{yd} - a_e \cdot f_{cd}\right)}{2 \cdot d_e \cdot a_e \cdot f_{cd} + 4 \cdot t_f \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \quad (A.69)
\]

\[
W_{zpan} = 2 \cdot t_f \cdot h_{iz}^2 + \frac{(h - 2 \cdot t_f) \cdot t_w^2}{4} \quad (A.70)
\]

(c) Neutral axis outside of the steel shape: \( h_{iz} > b / 2 \)

\[
h_{iz} = \frac{N_{pm, Rd} - A_{sa} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) - A_{a} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot d_e \cdot a_e \cdot f_{cd}} \quad (A.71)
\]

\[
W_{zpan} = W_{zpa} \quad (A.72)
\]
Cross-Section A.5

Figure A.5 Two I-Sections Circular Encased

Bending about the y-axis:

\[ W_{spa} = \frac{t_{w1} \cdot (h_1 - 2 \cdot t_{f1})^2}{4} + b_1 \cdot t_{f1} \cdot (h_1 - t_{f1}) + \frac{(h_2 - 2 \cdot t_{f2}) \cdot t_{w2}^2}{4} + \frac{2 \cdot t_{f2} \cdot b_2^2}{4} \]  

(A.73)

\[ W_{spc} = \frac{d_c^3}{6} - W_{pa} - W_{ps} \]  

(A.74)

(a) Neutral axis in the web of the steel shape 2: \( h_{ny} \leq t_{w2} / 2 \)

\[ h_{ny} = \frac{N_{pa, Rd} - A_{sn} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd} + 2 \cdot (h_2 + t_{w1}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  

(A.75)

\[ W_{spin} = (h_2 + t_{w1}) \cdot h_{ny}^2 \]  

(A.76)

(b) Neutral axis in the flange of the steel shape 2: \( t_{w2} / 2 \leq h_{ny} \leq b_2 / 2 \)
\[ h_{ny} = \frac{N_{pm, Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd}) - t_{w1} \cdot (h_2 - 2 \cdot t_{f2}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd} + 2 \cdot (2 \cdot f_{f2} + t_{w1}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  
(A.77)

\[ W_{ypan} = (2 \cdot t_{f2} + t_{w1}) \cdot h_{ny}^2 + (h_2 - 2 \cdot t_{f2}) \cdot t_{w1}^2 \]  
(A.78)

(c) Neutral axis in the web of the steel shape 1: \( b_2 / 2 \leq h_{ny} \leq h_2 / 2 - t_{f1} \)

\[ h_{ny} = \frac{N_{pm, Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd}) - A_{n2} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd} + 2 \cdot t_{w1} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  
(A.79)

\[ W_{ypan} = W_{zpa2} + t_{w1} \cdot h_{ny}^2 \]  
(A.80)

(d) Neutral axis in the flange of the steel shape 1: \( h_1 / 2 - t_{f1} \leq h_{ny} \leq h_1 \)

\[ h_{ny} = \frac{N_{pm, Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd}) - A_{n2} \cdot (b_1 - t_{w1}) \cdot (h_1 - 2 \cdot t_{f1}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd} + 2 \cdot b_1 \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]  
(A.81)

\[ W_{ypan} = W_{zpa2} + b_1 \cdot h_{ny}^2 - \frac{(b_1 - t_{w1}) \cdot (h_1 - 2 \cdot t_{f1})^2}{4} \]  
(A.82)

(e) Neutral axis outside of the steel shape 1: \( h_2 \leq h_{ny} \)

\[ h_{ny} = \frac{N_{pm, Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd}) - A_{u} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd}} \]  
(A.83)

\[ W_{ypan} = W_{ypa1} + W_{zpa2} \]  
(A.84)
Bending about z-axis:

\[
W_{zpa} = \frac{t_{w2} \left( h_2 - 2 \cdot t_{f2} \right)^2}{4} + b_2 \cdot t_{f2} \cdot \left( h_2 - t_{f2} \right) + \frac{\left( h_1 - 2 \cdot t_{f1} \right) t_{w1}^2}{4} + \frac{2 \cdot t_{f1} \cdot b_1^2}{4}
\]

(A.85)

\[
W_{zpc} = \frac{d_c^3}{6} - W_{zpa} - W_{zps}
\]

(A.86)

(a) Neutral axis in the web of steel shape 1: \( h_{w2} \leq t_{w1} / 2 \)

\[
h_{nz} = \frac{N_{pm,rd} - A_{su} \cdot \left( 2 \cdot f_{sd} - a_e \cdot f_{cd} \right)}{2 \cdot d_c \cdot a_e \cdot f_{cd} + 2 \cdot h_1 \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)}
\]

(A.87)

\[
W_{zpas} = h_1 \cdot h_{nz}^2
\]

(A.88)

(b) Neutral axis in the flange of steel shape 1: \( t_{w1} / 2 \leq h_{nz} \leq b_1 / 2 \)

\[
h_{nz} = \frac{N_{pm,rd} - A_{su} \cdot \left( 2 \cdot f_{sd} - a_e \cdot f_{cd} \right) - \left( h_1 - 2 \cdot t_{f1} \right) \cdot t_{w1} \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)}{2 \cdot d_c \cdot a_e \cdot f_{cd} + 2 \cdot \left( 2 \cdot t_{f1} + t_{w1} \right) \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)}
\]

(A.89)

\[
W_{zpas} = \left( 2 \cdot t_{f1} + t_{w2} \right) \cdot h_{nz}^2 + \frac{h_1 - 2 \cdot t_{f1}}{4} \cdot t_{w1}^2
\]

(A.90)

(c) Neutral axis in the web of steel shape 2: \( b_1 / 2 \leq h_{nz} \leq h_2 / 2 - t_{f2} \)

If \( b_1 \geq (h_2 + t_{w1}) \):
\[ h_{nz} = \frac{N_{pm,Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) + \left( h_2 - 2 \cdot t_{f2} - t_{w1} \right) \cdot \left( b_2 - t_{w2} \right) \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)}{2 \cdot d_e \cdot a_e \cdot f_{cd} + 2 \cdot \left( b_2 + 2 \cdot t_{f1} \right) \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)} \]  
(A.91)

\[ W_{zpon} = (2 \cdot t_{f1} + t_{w1}) \cdot h_{nz}^2 + \frac{(h_1 - 2 \cdot t_{f1})}{4} \cdot t_{w1}^2 \]  
(A.92)

If \( b_1 < (h_2 + t_{w1}) \):

\[ h_{nz} = \frac{N_{pm,Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) - A_{a1} \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)}{2 \cdot d_e \cdot a_e \cdot f_{cd} + 2 \cdot t_{w2} \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)} \]  
(A.93)

\[ W_{zpon} = W_{zpon1} + t_{w2} \cdot h_{nz}^2 \]  
(A.94)

(d) Neutral axis in the flange of steel shape 2: \( h_2 / 2 \cdot t_{f1} \leq h_{nz} \leq h_2 \)

If \( b_1 \geq (h_2 + t_{w1}) \):

\[ h_{nz} = \frac{N_{pm,Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) + \left( b_2 - t_{w2} \right) \cdot \left( h_2 - 2 \cdot t_{f2} - t_{w1} \right) \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)}{2 \cdot d_e \cdot a_e \cdot f_{cd} + 2 \cdot \left( b_2 + 2 \cdot t_{f1} \right) \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)} \]  
(A.95)

\[ W_{zpon} = b_2 \cdot h_{nz}^2 - \frac{(b_2 - t_{w2}) \cdot (h_2 - 2 \cdot t_{f2})^2}{4} + 2 \cdot t_{f1} \cdot h_{nz}^2 + \frac{h_1 - 2 \cdot t_{f1}}{4} \cdot t_{w1}^2 \]  
(A.96)

If \( b_1 < (h_2 + t_{w1}) \):

\[ h_{nz} = \frac{N_{pm,Rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) - A_{a1} - \left( b_2 - t_{w2} \right) \cdot \left( \frac{h_2}{2} - t_{f2} \right) \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)}{2 \cdot d_e \cdot a_e \cdot f_{cd} + 2 \cdot b_2 \cdot \left( 2 \cdot f_{yd} - a_e \cdot f_{cd} \right)} \]  
(A.97)
\[ W_{z_{pan}} = W_{z_{pa1}} + b_2 \cdot h_{nz}^2 - \frac{(b_2 - t_{w2}) \cdot (h_2 - 2 \cdot t_{f3})^2}{4} \]  \hspace{1cm} (A.98)

(e) Neutral axis outside of steel shape 1 and 2: \( h_2 / 2 \leq h_{nz} \geq b_1 / 2 \)

\[ h_{nz} = \frac{N_{pan,rd} - A_{an} \cdot \left(2 \cdot f_{yd} - a_c \cdot f_{cd}\right) - A_a \cdot \left(2 \cdot f_{yd} - a_c \cdot f_{cd}\right)}{2 \cdot d_c \cdot a_c \cdot f_{cd}} \]  \hspace{1cm} (A.99)

\[ W_{z_{pan}} = W_{z_{pa1}} + W_{z_{pa2}} \]  \hspace{1cm} (A.100)
Cross-Section A.6

Figure A.6 Solid Section Circular Encased

Bending about the y-axis:

\[ W_{ypa} = \frac{d^3}{6} \]  \hspace{1cm} (A.101)

\[ W_{ypc} = \frac{d^3}{6} - W_{ypa} - W_{yps} \]  \hspace{1cm} (A.102)

(a) Neutral axis in the steel shape: \( h_{ny} \leq d / 2 \)

\[ h_{ny} = \frac{N_{pma,Rd} - A_{as} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot d_e \cdot a_e \cdot f_{cd} + 2 \cdot d \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \]  \hspace{1cm} (A.103)

\[ W_{ypma} = d \cdot h_{ny}^2 \]  \hspace{1cm} (A.104)

(b) Neutral axis outside of the steel shape: \( h_{ny} > d / 2 \)
Bending about the \( z \)-axis:

\[
h_{ny} = \frac{N_{pm,Rd} - A_{ss} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) - A_a \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot d \cdot a_e \cdot f_{cd}} \quad (A.105)
\]

\[
W_{zpa} = W_{zpa} \quad (A.106)
\]

(a) Neutral axis inside of the steel shape: \( h_{nz} \leq d / 2 \)

\[
h_{nz} = \frac{N_{pm,Rd} - A_{ss} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd})}{2 \cdot d \cdot a_e \cdot f_{cd} + 2 \cdot d \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \quad (A.109)
\]

\[
W_{zpan} = d \cdot h_{nz}^2 \quad (A.110)
\]

(b) Neutral axis outside of the steel shape: \( h_{nz} > d / 2 \)

\[
h_{nz} = \frac{N_{pm,Rd} - A_{ss} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) - A_a \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot d_c \cdot a_e \cdot f_{cd}} \quad (A.111)
\]

\[
W_{zpan} = W_{zpa} \quad (A.112)
\]
Cross-Section A.7

![Diagram](image)

**Figure A.7** One I-Section Partly Encased

**Bending about y-axis:**

\[
W_{ysa} = \frac{t_w \cdot (h - 2 \cdot t_f)^2}{4} + b \cdot t_f \cdot (h - t_f)
\]  
(A.113)

\[
W_{ysc} = \frac{b h^2}{4} - W_{ysa} - W_{yps}
\]  
(A.114)

(a) Neutral axis in the web of the steel shape: \( h_{ny} \leq h / 2 - t_f \)

\[
h_{ny} = \frac{N_{pm, Rd} - A_{sa} \cdot (2 \cdot f_{sl} - a_c \cdot f_{cd})}{2 \cdot b \cdot a_c \cdot f_{cd} + 2 \cdot t_w \cdot (2 \cdot f_{sl} - a_c \cdot f_{cd})}
\]  
(A.115)

\[
W_{yswa} = t_w \cdot h_{ny}^2
\]  
(A.116)

(b) Neutral axis in the flange of the steel shape: \( h / 2 - t_f \leq h_{ny} \leq h / 2 \)
\[ h_{ny} = \frac{N_{pm,Rd} - A_{sh} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) + (b - t_w) \cdot (h - 2 \cdot t_f) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot b \cdot a_e \cdot f_{cd} + 2 \cdot b \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \]  
(A.117)

\[ W_{zan} = b \cdot h_{ny}^2 - \frac{(b - t_w) \cdot (h - 2 \cdot t_f)^2}{4} \]  
(A.118)

**Bending about z-axis:**

\[ W_{zpa} = \frac{(h - 2 \cdot t_f)^2}{4} + \frac{2 \cdot t_f \cdot b^2}{4} \]  
(A.119)

\[ W_{zpc} = \frac{hb^2}{4} - W_{zpa} - W_{zps} \]  
(A.120)

(a) Neutral axis in the web of the steel shape: \( h_{nz} \leq t_w / 2 \)

\[ h_{nz} = \frac{N_{pm,Rd} - A_{sh} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot h \cdot a_e \cdot f_{cd} + 2 \cdot h \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \]  
(A.121)

\[ W_{zan} = h \cdot h_{nz}^2 \]  
(A.122)

(b) Neutral axis in the flange of the steel shape: \( t_w / 2 \leq h_{nz} \leq b / 2 \)

\[ h_{nz} = \frac{N_{pm,Rd} - A_{sh} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) + t_w (2 \cdot t_f - h) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot h \cdot a_e \cdot f_{cd} + 4 \cdot t_f \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \]  
(A.123)

\[ W_{zan} = 2 \cdot t_f \cdot h_{nz}^2 + \frac{(h - 2 \cdot t_f) \cdot t_w^2}{4} \]  
(A.124)
Cross-Section A.8

Figure A.8 Two I-Section Partly Encased

Bending about the y-axis:

\[ W_{spa} = \frac{t_{w1} \left( h_1 - 2 \cdot t_{f1} \right)^2}{4} + b_1 \cdot t_{f1} \cdot \left( h_1 - t_{f1} \right) + \frac{\left( h_2 - 2 \cdot t_{f2} \right) f_{w2}^2}{4} + \frac{2 \cdot t_{f2} \cdot b_2^2}{4} \]

(A.125)

\[ W_{yse} = \frac{b_1 \cdot h_1^2}{4} - W_{spa} - W_{yse} \]

(A.126)

(a) Neutral axis in the web of the steel shape 2: \( h_{ny} \leq t_{w2} / 2 \)

\[ h_{ny} = \frac{N_{pm,td} - A_{sn} \cdot (2 \cdot f_{cd} - a_c \cdot f_{cd})}{2 \cdot b_1 \cdot a_c \cdot f_{cd} + 2 \cdot (h_2 + t_{w1}) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})} \]

(A.127)

\[ W_{ypan} = \left( h_2 + t_{w1} \right) \cdot h_{ny}^2 \]

(A.128)

(b) Neutral axis in the flange of the steel shape 2: \( t_{w2} / 2 \leq h_{ny} \leq b_2 / 2 \)
\[ h_{ny} = \frac{\frac{N_{pm, Rd} - A_{sn} \cdot (2 \cdot f_{sl} - a_e \cdot f_{cd}) - t_{w2} \cdot (h_2 - 2 \cdot t_{f2}) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot b_1 \cdot a_e \cdot f_{cd} + 2 \cdot (2t_{f2} + t_{w1}) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}} \]

\[ W_{span} = \left(2 \cdot t_{f2} + t_{w1}\right) \cdot h_{ny}^2 + (h_2 - 2 \cdot t_{f2}) \cdot t_{w2}^2 \]

(A.129)

(c) Neutral axis in the web of the steel shape 1: \( b_2 / 2 \leq h_{ny} \leq h_2 / 2 - t_{f1} \)

\[ h_{ny} = \frac{\frac{N_{pm, Rd} - A_{sn} \cdot (2 \cdot f_{sl} - a_e \cdot f_{cd}) - A_{a2} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot b_1 \cdot a_e \cdot f_{cd} + 2 \cdot t_{w1} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}} \]

\[ W_{span} = W_{zpm} + t_{w1} \cdot h_{ny}^2 \]

(A.130)

(A.131)

(d) Neutral axis in the flange of the steel shape 1: \( h_1 / 2 - t_{f1} \leq h_{ny} \leq h_1 \)

\[ h_{ny} = \frac{\frac{N_{pm, Rd} - A_{sn} \cdot (2 \cdot f_{sl} - a_e \cdot f_{cd}) - A_{a2} - (b_1 - t_{w1}) \cdot (h_1 - 2 \cdot t_{f1}) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{4 \cdot b_1 \cdot f_{yd}}}{4 \cdot b_1 \cdot f_{yd}} \]

\[ W_{span} = W_{zpm} + b_1 \cdot h_{ny}^2 - \frac{(b_1 - t_{w1}) \cdot (h_1 - 2 \cdot t_{f1})^2}{4} \]

(A.132)

(A.133)

(Bending about z-axis):

\[ W_{zpa} = \frac{t_{w2} \cdot (h_2 - 2 \cdot t_{f2})^2}{4} + b_2 \cdot t_{f2} \cdot (h_2 - t_{f2}) + \frac{(h_1 - 2 \cdot t_{f1})^2}{4} + \frac{2 \cdot t_{f1} \cdot b_1^2}{4} \]

(A.134)

(A.135)
(a) Neutral axis in the web of the steel shape 1: \( h_{nz} \leq t_{w1} / 2 \)

\[
h_{nz} = \frac{N_{pm,rd}}{4 \cdot h_{1} \cdot f_{yd}}
\]

(A.137)

\[
W_{zpan} = h_{1} \cdot h_{nz}^{2}
\]

(A.138)

(b) Neutral axis in the flange of the steel shape 1: \( t_{w1} / 2 \leq h_{nz} \leq b_{1} / 2 \)

\[
h_{nz} = \frac{N_{pm,rd} - A_{sn} \cdot (f_{yd} - a_{c} \cdot f_{cd}) - \left(h_{1} - 2 \cdot t_{f1}\right) \cdot t_{w1} \cdot \left(2 \cdot f_{yd} - a_{c} \cdot f_{cd}\right)}{2 \cdot h_{1} \cdot a_{c} \cdot f_{cd} + 2 \cdot \left(2 \cdot t_{f1} + t_{w2}\right) \cdot \left(2 \cdot f_{yd} - a_{c} \cdot f_{cd}\right)}
\]

(A.139)

\[
W_{zpan} = \left(2 \cdot t_{f1} + t_{w2}\right) \cdot h_{nz}^{2} + \frac{h_{1} - 2 \cdot t_{f1}}{4} \cdot t_{w1}^{2}
\]

(A.140)

(c) Neutral axis in the web of the steel shape 2: \( b_{1} / 2 \leq h_{nz} \leq h_{2} / 2 - t_{f2} \)

\[
h_{nz} = \frac{N_{pm,rd} - A_{sn} \cdot \left(2 \cdot f_{yd} - a_{c} \cdot f_{cd}\right) + \left(h_{2} - 2 \cdot t_{f2} - t_{w1}\right) \cdot \left(b_{2} - t_{w2}\right) \cdot \left(2 \cdot f_{yd} - a_{c} \cdot f_{cd}\right)}{2 \cdot h_{1} \cdot a_{c} \cdot f_{cd} + 2 \cdot \left(b_{2} + 2 \cdot t_{f1}\right) \cdot \left(2 \cdot f_{yd} - a_{c} \cdot f_{cd}\right)}
\]

(A.141)

\[
W_{zpan} = \left(2 \cdot t_{f1} + t_{w1}\right) \cdot h_{nz}^{2} + \frac{h_{1} - 2 \cdot t_{f1}}{4} \cdot t_{w1}^{2}
\]

(A.142)

(d) Neutral axis in the flange of the steel shape 2: \( h_{2} / 2 - t_{f1} \leq h_{nz} \leq h_{2} \)
\[
\hat{h}_{nc} = \frac{N_{pan, Rd} - A_{sn} \cdot (2 \cdot f_{yld} - a_c \cdot f_{cd}) - (b_2 - t_w2) \cdot (h_2 - 2 \cdot t_{f2} - t_w1) \cdot (2 \cdot f_{yld} - a_c \cdot f_{cd})}{2 \cdot h_1 \cdot a_c \cdot f_{cd} + 2 \cdot (b_2 + 2 \cdot t_{f1}) \cdot (2 \cdot f_{yld} - a_c \cdot f_{cd})}
\]
(A.143)

\[
W_{span} = b_2 \cdot \hat{h}_{nc}^2 - \frac{(b_2 - t_w2) \cdot (h_2 - 2 \cdot t_{f2})^2}{4} + 2 \cdot t_{f1} \cdot \hat{h}_{nc}^2 + \frac{h_1 - 2 \cdot t_{f1}}{4} \cdot t_{w1}^2
\]
(A.144)
Cross-Section A.9

Figure A.9 Two I-Sections Partly Encased

\[ y_0 = \frac{h_2 + t_{w1} - b_1}{2} \]  \hspace{1cm} (A.145)

\[ z_0 = \frac{h_1 - b_2}{2} \]  \hspace{1cm} (A.146)

Bending about the y-axis:

\[ W_{spa} = \frac{t_{w1} \cdot (h_1 - 2 \cdot t_{f1})^2}{4} + b_1 \cdot t_{f1} \cdot (h_1 - t_{f1}) + \frac{(h_2 - 2 \cdot t_{f2}) t_{w2}^2}{4} + \frac{2 \cdot t_{f2} \cdot b_2^2}{4} \]  \hspace{1cm} (A.147)

\[ W_{spc} = \frac{(h_2 + t_{w1}) \cdot h_i^2}{4} - 2 \cdot z_0 \cdot y_0 \cdot \left( \frac{h_1 - z_0}{3} \right) - W_{spa} - W_{spc} \]  \hspace{1cm} (A.148)

(a) Neutral axis in the web of the steel shape 2: \( h_{w2} \leq t_{w2} / 2 \)
\[ h_{ny} = \frac{N_{pu, Rd}}{4 \cdot (h_2 + t_{w1}) \cdot f_{yd}} \]  

(A.149)

\[ W_{span} = (h_2 + t_{w1}) \cdot h_{ny}^2 \]  

(A.150)

(b) Neutral axis in the flange of the steel shape 2: \( t_{w2} / 2 \leq h_{ny} \leq b_2 / 2 \)

\[ h_{ny} = \frac{N_{pu, Rd} - A_{sa} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) - t_{w2} \cdot (h_2 - 2 \cdot t_{f2}) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot h_2 \cdot a_e \cdot f_{cd} + 2 \cdot (2 \cdot t_{f2} + t_{w1}) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \]  

(A.151)

\[ W_{span} = (2 \cdot t_{f2} + t_{w1}) \cdot h_{ny}^2 + \frac{(h_2 - 2 \cdot t_{f2}) \cdot t_{w2}^2}{4} \]  

(A.152)

(c) Neutral axis in the web of the steel shape 1: \( b_2 / 2 \leq h_{ny} \leq h_2 / 2 \cdot t_{f1} \)

Iterative Solution:

\[ h_{ny} = \frac{N_{pu, Rd} - A_{sa} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) - A_{s2} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) + \left( h_{ny} - \frac{b_2}{2} \right)^2 \cdot y_0}{2 \cdot h_2 \cdot a_e \cdot f_{cd} + 2 \cdot t_{w1} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) + \frac{y_0}{z_0}} \]  

(A.153)

\[ W_{span} = W_{zpa2} + t_{w1} \cdot h_{ny}^2 \]  

(A.154)

(d) Neutral axis in the flange of the steel shape 1: \( h_1 / 2 \cdot t_{f} \leq h_{ny} \leq h_1 \)

\[ h_{ny} = \frac{N_{pu, Rd} - A_{sa} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) - A_c \cdot a_e \cdot f_{cd} - A_u \cdot 2 \cdot f_{yd} \cdot h_1}{4 \cdot b_1 \cdot f_{yd} + \frac{h_1}{2}} \]  

(A.155)
\[ W_{\text{span}} = W_{\text{p}} + b_1 \cdot h_{w}^2 - \frac{(b_1 - t_{w1}) \cdot (h_1 - 2 \cdot t_{f1})^2}{4} \]  
\hspace{6cm} (A.156)

Bonding about z-axis:

\[ W_{z_{\text{ps}}} = \frac{t_{w2} \cdot \left( h_2 - 2 \cdot t_{f2} \right)^2}{4} + b_2 \cdot t_{f2} \cdot \left( h_2 - t_{f2} \right) + \frac{\left( h_1 - 2 \cdot t_{f1} \right) t_{w1}^2}{4} + \frac{2 \cdot t_{f1} \cdot b_1^3}{4} \]  
\hspace{6cm} (A.157)

\[ W_{z_{\text{pc}}} = \frac{(h_2 + t_{w1})^2 \cdot h_1}{4} - 2 \cdot z_0 \cdot y_0 \cdot \left( \frac{h_2 + t_{w1}}{2} - \frac{y_0}{3} \right) - W_{z_{\text{ps}}} - W_{z_{\text{ps}}} \]  
\hspace{6cm} (A.158)

(a) Neutral axis in the web of the steel shape 1: \( h_{nc} \leq t_{w1} / 2 \)

\[ h_{nc} = \frac{N_{pm, Rd}}{4 \cdot h_1 \cdot f_{yd}} \]  
\hspace{6cm} (A.159)

\[ W_{z_{\text{p}}_{\text{nc}}} = h_1 \cdot h_{nc}^2 \]  
\hspace{6cm} (A.160)

(b) Neutral axis in the flange of the steel shape 1: \( t_{w1} / 2 \leq h_{nc} \leq b_1 / 2 \)

\[ h_{nc} = \frac{N_{pm, Rd} - A_{su} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd}) - t_{w1} \cdot \left( h_1 - 2 \cdot t_{f1} \right) \cdot \left( 2 \cdot f_{yd} - a_c \cdot f_{cd} \right)}{2 \cdot h_1 \cdot a_c \cdot f_{cd} + 2 \cdot \left( 2 t_{f1} + t_{w2} \right) \cdot \left( 2 \cdot f_{yd} - a_c \cdot f_{cd} \right)} \]  
\hspace{6cm} (A.161)

\[ W_{z_{\text{p}}_{\text{nc}}} = \left( 2 \cdot t_{f1} + t_{w2} \right) \cdot h_{nc}^3 + \left( h_1 - 2 \cdot t_{f1} \right) \cdot t_{w2}^3 \]  
\hspace{6cm} (A.162)

(c) Neutral axis in the web of the steel shape 2: \( b_1 / 2 \leq h_{nc} \leq h_1 / 2 \cdot t_{f1} \)
Iterative Solution:

\[
\begin{align*}
  h_{iz} &= \frac{N_{pm,rd} - A_{m} \cdot (2 \cdot f_{cd} - a_{c} \cdot f_{cd}) - A_{w} \cdot (2 \cdot f_{yd} - a_{c} \cdot f_{cd})}{2 \cdot h_{1} \cdot a_{c} \cdot f_{cd} + 2 \cdot t_{w2} \cdot (2 \cdot f_{yd} - a_{c} \cdot f_{cd})} + \frac{(h_{iz} - \frac{b_{1}}{2})^{2} \cdot z_{0}}{y_{0}} \\
  W_{zpan} &= W_{ypan1} + t_{w2} \cdot h_{iz}^{2}
\end{align*}
\]  

(A.163)  

(d) Neutral axis in the flange of the steel shape 2: \(h_{2}/2 - t_{f2} \leq h_{iz} \leq h_{2}\)

\[
\begin{align*}
  h_{iz} &= \frac{N_{pm,rd} - A_{m} \cdot (2 \cdot f_{cd} - a_{c} \cdot f_{cd}) - A_{w} \cdot a_{c} \cdot f_{cd} - A_{w} \cdot 2 \cdot f_{yd} + h_{2}}{4 \cdot b_{2} \cdot f_{yd}} \\
  W_{zpan} &= W_{ypan1} + b_{2} \cdot h_{iz}^{2} - \frac{(b_{2} - t_{w2}) \cdot (h_{2} - 2 \cdot t_{f2})^{2}}{4}
\end{align*}
\]  

(A.165)  

(A.166)
Cross-Section A.10

![Cross-Section Diagram](image)

**Figure A.10** Filled Circular Tube

\[ W_{pa} = \frac{d_a^3 - (d_a - 2 \cdot t)^3}{6} \tag{A.167} \]

\[ W_{pc} = \frac{d_a^3}{6} - W_{pa} - W_{ps} \tag{A.168} \]

\[ h_n = \frac{N_{pns,rd} - A_{sv} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd})}{2 \cdot d_a \cdot a_e \cdot f_{cd} + 4t \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd})} \tag{A.169} \]

\[ W_{pan} = 2 \cdot t \cdot h_n^2 \tag{A.170} \]
Cross-Section A.11

\[ W_{y_{pa}} = \frac{d_o^3 - (d_a - 2 \cdot t)^3 + d_i^3}{6} \]  \hspace{1cm} (A.171)

\[ W_{y_{pa}} = \frac{d_o^3}{6} - W_{y_{pa}} - W_{y_{ps}} \]  \hspace{1cm} (A.172)

(a) Neutral axis inside of the steel shape: \( h_{ny} \leq \frac{d_i}{2} \)

\[ h_{ny} = \frac{N_{pu,Ed} - A_{as} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd})}{2 \cdot d_a \cdot a_e \cdot f_{cd} + 2 \cdot (d_i + 2 \cdot t) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \]  \hspace{1cm} (A.173)

\[ W_{y_{pan}} = (d_i + 2 \cdot t) \cdot h_{ny}^2 \]  \hspace{1cm} (A.174)

(b) Neutral axis outside of the steel shape: \( h_{ny} > \frac{d_i}{2} \)

Figure A.11 Circular Encased Cross-Section in Tube
Bending about the z-axis:

\[ W_{zpa} = W_{zpa} + 2 \cdot t \cdot h_{ny}^2 \]  
(A.176)

\[ W_{zpc} = \frac{d_a^3}{6} - W_{zpa} - W_{zpx} \]  
(A.178)

(a) Neutral axis inside of the steel shape: \( h_{nz} \leq d_i / 2 \)

\[ h_{nz} = \frac{N_{pm,rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd})}{2 \cdot d_a \cdot a_e \cdot f_{cd} + 2 \cdot (d_i + 2 \cdot t) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \]  
(A.179)

\[ W_{zpan} = (d_i + 2 \cdot t) \cdot h_{ny}^2 \]  
(A.180)

(b) Neutral axis outside of the steel shape: \( h_{nz} > d_i / 2 \)

\[ h_{nz} = \frac{N_{pm,rd} - A_{sn} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd}) - A_{di} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot d_a \cdot a_e \cdot f_{cd} + 4 \cdot t \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \]  
(A.181)

\[ W_{zpan} = W_{zpol} + 2 \cdot t \cdot h_{nz}^2 \]  
(A.182)
Cross-Section A.12

Figure A.12 One I-Section in Filled Tube

Bending about the y-axis:

\[ W_{ypa} = \frac{t_w \cdot (h - 2 \cdot t_f)^2}{4} + b \cdot t_f \cdot (h - t_f) + \frac{d^3 - (d - 2 \cdot t)^3}{6} \]  \hspace{1cm} (A.183)

\[ W_{ypc} = \frac{d^3}{6} - W_{ypa} - W_{ypc} \]  \hspace{1cm} (A.184)

(a) Neutral axis in the web of the steel shape: \( h_{ny} \leq h / 2 - t_f \)

\[ h_{ny} = \frac{N_{pm,hd} - A_{mn} \cdot (2 \cdot f_{sd} - a_e \cdot f_{cd})}{2 \cdot d \cdot a_e \cdot f_{cd} + 2 \cdot (t_w + 2 \cdot t) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \]  \hspace{1cm} (A.185)

\[ W_{ypav} = (t_w + 2 \cdot t) \cdot h_{ny}^2 \]  \hspace{1cm} (A.186)
(b) Neutral axis in the flange of the steel shape: \( h / 2 \cdot t_f \leq h_n \leq h / 2 \)

\[
h_n = \frac{N_{pm,ld} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd}) + (b - t_w) \cdot (h - 2 \cdot t_f) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd} + 2 \cdot (b + 2 \cdot t) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}
\]

\((A.187)\)

\[
W_{xpan} = (b + 2 \cdot t) \cdot h_n^2 - \frac{(b - t_w) \cdot (h - 2 \cdot t_f)^2}{4}
\]

\((A.188)\)

(c) Neutral axis outside of the steel shape: \( h / 2 \leq h_n \leq d / 2 \)

\[
h_n = \frac{N_{pm,ld} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd}) + A_{a1} \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}{2 \cdot d \cdot a_c \cdot f_{cd} + 4 \cdot t \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}
\]

\((A.189)\)

\[
W_{xpan} = W_{xpan_1} + 2 \cdot t \cdot h_n^2
\]

\((A.190)\)

Bending about the z-axis:

\[
W_z = \frac{(h - 2 \cdot t_f)^2 \cdot t_w^2}{4} + \frac{2 \cdot t_f \cdot b^2}{4} + \frac{d^3 - (d - 2 \cdot t)^3}{6}
\]

\((A.191)\)

\[
W_{zpe} = \frac{d^3}{6} - W_{zpa} - W_{sz}
\]

\((A.192)\)

(a) Neutral axis in the web of the steel shape: \( h_n \leq t_w / 2 \)

\[
h_n = \frac{N_{pm,ld} - A_{sn} \cdot (2 \cdot f_{sd} - a_c \cdot f_{cd})}{2 \cdot d_c \cdot a_c \cdot f_{cd} + 2 \cdot (h + 2 \cdot t) \cdot (2 \cdot f_{yd} - a_c \cdot f_{cd})}
\]

\((A.193)\)

\[
W_{zpan} = (h + 2 \cdot t) \cdot h_n^2
\]

\((A.194)\)
(b) Neutral axis in the flange of the steel shape: \( t_w / 2 \leq h_{nz} \leq b / 2 \)

\[
h_{nz} = \frac{N_{pm, Bd} - A_{sn} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) - t_w \cdot (2 \cdot f - h) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot d_e \cdot a_e \cdot f_{cd} + 4 \cdot (t_f + t) \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}
\]  
\[\text{(A.195)}\]

\[
W_{zpnu} = 2 \cdot (t_f + t) \cdot h_{nz}^2 + \frac{(h - 2 \cdot t_f) \cdot t_w^2}{4}
\]
\[\text{(A.196)}\]

(c) Neutral axis outside of the steel shape: \( h_{nz} > b / 2 \)

\[
h_{nz} = \frac{N_{pm, Bl} - A_{sn} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd}) - A_{a1} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot d_e \cdot a_e \cdot f_{cd} + 4 \cdot t \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}
\]  
\[\text{(A.197)}\]

\[
W_{zpnu} = W_{zpnu} + 2 \cdot t \cdot h_{nz}^2
\]
\[\text{(A.198)}\]
Cross-Section A.13

![Filled Rectangular Tube Diagram](image)

**Figure A.13** Filled Rectangular Tube

**Bending about the y-axis:**

\[
W_{spa} = \frac{(b - 2t)h^2}{4} \quad (A.199)
\]

\[
W_{ype} = \frac{bh^2}{4} - W_{spa} - W_{spv} \quad (A.200)
\]

\[
h_{ny} = \frac{N_{pau,ty} - A_{en} \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})}{2 \cdot b \cdot a_e \cdot f_{cd} + 4t \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \quad (A.201)
\]

\[
W_{spyn} = 2 \cdot t \cdot h_{ny}^2 \quad (A.202)
\]
Bending about the z-axis:

\[ W_{zpa} = \frac{(h-2t)b^2}{4} \]  \hspace{1cm} (A.203)

\[ W_{zpc} = \frac{hb^3}{4} - W_{zpa} - W_{zps} \]  \hspace{1cm} (A.204)

\[ h_{nz} = \frac{N_{p_{n,ld}} - A_{sn} \cdot (2 \cdot f_{sl} - a_e \cdot f_{cd})}{2 \cdot h \cdot a_e \cdot f_{cd} + 4t \cdot (2 \cdot f_{yd} - a_e \cdot f_{cd})} \]  \hspace{1cm} (A.205)

\[ W_{zpan} = 2 \cdot t \cdot h_{nz}^2 \]  \hspace{1cm} (A.206)
REFERENCES


