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ABSTRACT

PERFORMANCE ANALYSIS OF
THE INTERFERENCE ADAPTATION DYNAMIC CHANNEL
ALLOCATION TECHNIQUE
IN WIRELESS COMMUNICATION NETWORKS

by
Ziqiang Xu

Dynamic channel allocation (DCA) problem is one of the major research topics in the wireless networking area. The purpose of this technique is to relieve the contradiction between the increasing traffic load in wireless networks and the limited bandwidth resource across the air interface. The challenge of this problem comes from the following facts: a) even the basic DCA problem is shown to be NP-complete (none polynomial complete); b) the size of the state space of the problem is very large; and c) any practical DCA algorithm should run in real-time.

Many heuristic DCA schemes have been proposed in the literature. It has been shown through simulation results that the interference adaptative dynamic channel allocation (IA-DCA) scheme is a promising strategy in Time Devision Multiple Access/Frequency Devision Multiple Access (TDMA/FDMA) based wireless communication systems. However, the analytical work on the IA-DCA strategy in the literature is nearly blank.

The performance of a DCA algorithm in TDMA/FDMA wireless systems is influenced by three factors: representation of the interference, traffic fluctuation, and the processing power of the algorithm. The major obstacle in analyzing IA-DCA is the computation of co-channel interference without the constraint of conventional channel reuse factors. To overcome this difficulty, one needs a representation pattern which can approximate the real interference distribution as accurately as desired, and is also computationally viable. For this purpose, a concept called channel reuse zone (CRZ) is introduced and the methodology of computing the area of a CRZ with
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PERFORMANCE ANALYSIS OF
THE INTERFERENCE ADAPTATION DYNAMIC CHANNEL
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To my beloved wife, expected baby, and parents
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CHAPTER 1
INTRODUCTION

1.1 Background

In recent years, intense research and development efforts are being made in two areas in communication industry, i.e., broadband networks and personal communication networks (PCN). These two revolutionary concepts have triggered the booming of the whole modern telecommunication industry. Due to the huge capacity and extremely low error rate of optical fiber lines, broadband networks are characterized by cell-relay transport, bandwidth-upon-demand, and multimedia traffic integration. On the other hand, personal communication networks are meant to offer users the convenience of person-to-person, rather than place-to-place, communications. The integration of wireless access features with broadband networks is expected in the literature.

In contrast to the comparatively mature broadband networks, PCN still has some key technical obstacles yet to be conquered. Among all the problems in PCN, one of the most important and most difficult ones might be the scarcity of radio spectrum resource across the air interface. The shortage of available bandwidth has already been the main constraint to the growth of the first and second-generation wireless communication networks, which are mostly used to deliver audio traffics. It is natural that this constraint is going to be the top challenge for the ambitious goals of offering multimedia services in the next generation PCN [74].

The endeavor to alleviate the capacity constraint has been pursued in various aspects of wireless networks [67]. For example, one may increase the system capacity by using advanced multiple access techniques. Typical multiple access strategies are Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA). Another well known method is of decreasing the coverage of cells (micro-cell/pico-cell) so that the same frequency
band can be reused within a small area. Nevertheless, no matter what multiple access scheme is used or how well the physical layer of the network is designed, there always exists a problem of allocating available network resources in the most efficient way. This problem is of our major focus in this thesis — the resource allocation problem.

1.2 Outline of the Dissertation

In the next chapter we define the resource allocation problem and introduce several performance measures. Then in Chapter 3 we give an overview of the problem. The channel allocation problem is discussed in detail. This chapter is concluded by the motivation and goals of the dissertation. Chapters 4-7 present the accomplished research work. Chapter 4 presents notation, models, and assumptions used throughout our work. A generic methodology to analyze the interference adaptation dynamic channel allocation (IA-DCA) strategy is proposed in Chapter 5. In Chapters 6 and 7, this generic method is applied to two different traffic models, and the performance bound for the IA-DCA algorithms are derived in each case. Chapter 8 briefly discusses a possible extension of this work from the single-traffic case to the heterogeneous-traffic case. The last chapter summarizes the contributions of this work and outlines the future research directions.
CHAPTER 2
PROBLEM DEFINITION

2.1 Network Infrastructure

Two different network infrastructures are considered in this dissertation. In a 1-dimensional ‘linear’ network, each cell is a portion of the straight line with a base station (BS) at the middle of it. This infrastructure can be used to model the network architecture along highways or streets [26]. In the conventional 2-dimensional ‘planar’ network, a cell is a geographic area corresponding to the radio-service area of a BS. Our research is mainly based on 2-dimensional network infrastructure1.

In both cases, a user, or mobile station (MS), communicates with an assigned BS through one or more assigned channels with an assigned transmission power level. Here a channel means either a frequency band in FDMA systems, or a time slot in TDMA systems, or a code in CDMA systems, although resource allocation in CDMA systems is not in this dissertation considered in this dissertation.

A group of BS’ are connected with each other and with a switching center, which is connected to the network backbone by wirelines.

2.2 Definition of the Resource Allocation Problem

Definition 2.1 The system capacity: the largest number of users that can be handled simultaneously by a wireless communication network [98].

In most cases it is hard to compute the system capacity directly. Instead the definition of Erlang capacity of a network is widely used [84].

1The assumption of the hexagonal structure is made in this dissertation for the sake of computational simplicity. Although the numerical results are obtained under this assumption, our analytical results are independent of the shape of cells.
**Definition 2.2** The Erlang capacity: the average traffic load, in terms of average number of users demanding service, resulting in a pre-determined blocking probability.

**Definition 2.3** The resource allocation problem: given a wireless communication system with BS set $V = \{1, 2, \ldots, V\}$ and channel set $M = \{1, 2, \ldots, M\}$. Assuming that there are $U$ users in the system, find a scheme that assigns to each user a base station from $V$, a duplex channel from $M$, a transmission power level at MS (for uplink connections) and another at BS (for downlink connections), respectively, such that the system capacity is maximized under the constraint

$$\text{(CIR)}_i \geq \gamma, \quad \forall i = 1, 2, \ldots, M,$$

(2.1)

where $(\text{CIR})_i$ is the carrier-to-interference ratio (CIR) on the $i$th channel, and $\gamma$ is the desired CIR threshold.

**Definition 2.4** The channel allocation problem: a one-dimensional subproblem of the resource allocation problem when transmission power levels and the BS are fixed for a specific MS.

### 2.3 Performance Measurements

A resource allocation algorithm can be evaluated from two perspectives: its realizability and its quality. An algorithm is called realizable if its implementation will not result in catastrophic events in the system. The realizability of an algorithm may be ruined by two reasons: the computational complexity and the instability.

Since even the subproblem of channel allocation has been shown to be NP-complete [36], the computational burden of any algorithm whose complexity depends on system-size parameters $(V, M, U)$ will increase exponentially as the values of parameter increase. As the result, unless the size of the problem is very small, centralized optimization schemes are impractical. This is the very point at which
all the centralized allocation algorithms (see below) violate the realizability criterion and only have theoretical values.

According to Ref. [75], a service interruption occurs when an allocation to a new call causes an ongoing connection to fall below the performance threshold $\gamma$; and the occurrence of successive interruptions is defined as instability. The possibility of instability is one of the main concerns for the distributed version of channel allocation algorithms. Although some simulation results have shown that with power control the probability of instability is extremely small [25], still there is no theoretical proof for the stability of distributed algorithms. Fundamentally, the stability of a system depends heavily upon the aggressiveness of the resource allocation algorithm it uses [25][40]. The aggressiveness is defined as the tendency of an algorithm to sacrifice ongoing calls while the resource assignment for a new call is about to interrupt the existing ones.

Assuming that an algorithm is realizable, its merit (quality) can be characterized by a series of measures. Among these measures following ones are broadly used in the literature.

- **Blocking probability ($P_b$):** the probability that an arriving call is rejected by the system.

- **Dropping probability ($P_d$):** the probability that an ongoing call is dropped by the system, mostly due to CIR outage resulting from arrival of new calls.

- **Probability of unsuccessful calls ($P_u$):** $P_b + P_d$.

- **Outage probability ($P_{out}$):** the expected portion of the service time of completed calls in which the actual CIR is below a specified threshold.

- **Call setup time.**
In above measures, $P_b$ and $P_u$ are of the most important, since they are related directly to the goal of the resource allocation problem. They represent the grade of service (GoS) a system provides. Apparently, the concept of $P_b$ is borrowed from the wireline telephony network where ongoing calls are never interrupted by new calls. Thus, in wireless environment, $P_u$ is more appropriate to measure the quality of a network. $P_d$ is a key measure of an algorithm's aggressiveness. $P_{out}$ represents the quality of service (QoS) the system offers. There is clearly a trade-off between $P_u$ ($P_b$) and $P_{out}$. In order to consider a balanced performance of an allocation strategy, some combine measures may be utilized.
CHAPTER 3
AN OVERVIEW OF THE RESOURCE ALLOCATION TECHNIQUES

In this chapter, we give an overview of various the resource allocation techniques. Historically the general resource allocation problem stems from the channel allocation (CA) problem. Therefore, we discuss the CA problem at first, then review the general resource allocation problem and the newly developed problem of CA with heterogeneous traffics. In this chapter, we only discuss representative strategies in each category and mention some other interesting schemes. For a more comprehensive survey, please refer to references [50] and [81].

3.1 Channel Allocation Problem

3.1.1 A General Appreciation of the CA Problem

Until recently, there does not exist an analytical optimal solution for an NP-complete optimization problem. The fact that the CA problem is NP-complete excludes the possibility of analytical derivation of a practical optimal allocation algorithm from a cost function. In face of this unconquerable obstacle, most researchers turn to heuristic approaches and show the potentials of their algorithms through simulation results. The other approach is to draw a theoretical bound, in terms of a specified performance measure, and then to show how much space left between the bound and the performance of heuristic schemes. This is useful in directing the research for new heuristic algorithms.

The assignment of a channel to an oncoming call consists of two major steps. First, checking if there are channels available. Second, if no channel is available then the call is blocked. If only one channel is available, it is assigned to the call. If more than one channel is available, then we have some design space and a selection is made under some allocation protocols. In the next subsection, we will discuss the
meaning of the availability of a channel. It becomes clear later that various channel allocation schemes can be classified according to their explanations of this meaning.

3.1.2 The Availability of a Channel

From Definition 2.3, the constraint to the assignment of a channel comes from the CIR on it. In general, a channel may experience intra-cell as well as inter-cell interferences. The latter includes co-channel interference (CCI) and adjacent channel interference (ACI). Therefore, the CIR constraint consists of three categories:

- Co-site constraint (CSC): represented by $d_{scs}$, the minimum distance between frequencies used in the same cell (number of unit channel bandwidth).

- Adjacent channel constraint (ACC): the minimum frequency separation between a channel in cell $p$ and another in cell $q$ (number of unit channel bandwidth).

- Co-channel constraint (CCC): the uplink co-channel CIR at BS $I$ from MS $j$ can be represented by $[66]$

$$
(CIR)_{JI} = \frac{G_{ji}P_j}{\sum_{s \neq j} G_{si}P_s + n},
$$

where $n$ denotes the received thermal noise power; $P_j$ is the transmission power of MS $j$; and $G_{ij}$ is called the link gain. Similarly, the downlink CIR at MS $i$ from BS $J$ is

$$
(CIR)_{IJ} = \frac{G_{ji}P_j}{\sum_{s \neq J} G_{si}P_s + n}.
$$

In systems with orthogonal waveforms, as in FDMA and TDMA, the CIR constraint might be stated in two aspects. First, $d_{scs} = 1$ and the ACI is negligible\(^1\). This

\(^1\)This statement is usually correct for microcells with uplink power control, but may not be true for macro-cells [41]. In latter case, $d_{scs} > 1$ can be added to CCC in the corresponding DCA problems [41][20].
means that one channel can only be used for one BS-MS connection in the same cell. Second, due to CCC, when traffic load and transmission power are fixed, the CIR is determined by link gains, which in turn depends on the distance and fadings. Since the CCC is too complex for real-time applications like the channel allocation problem, various approximate representations of CCC are used. A widely-used representation is the cochannel reuse distance (CRD). CRD is defined as the minimum distance at which the same radio spectrum can be reused with CIR above \( \gamma \) (in number of cells). If CRD is used to represent CCC, it means that if a channel is occupied by cell \( i \), the same channel can not be used in any other cell within the CRD of cell \( i \).

Therefore, a channel is available in a cell if it meets the three “availability conditions”: 1) The channel is not in use in that cell (by CSC); 2) the co-channel CIR satisfies the CCC or one of its approximate representations; and 3) the channel belongs to a pre-determined channel group.

Channel allocation strategies can be categorized into three classes: fixed, flexible, and dynamic [81]. In the next subsection, the fixed and flexible schemes are described briefly. We then focus on dynamic channel allocation (DCA) strategies. Some valuable analysis results are collectively surveyed in subsection 3.1.6.

### 3.1.3 Fixed and Flexible Channel Allocation

#### 3.1.3.1 FCA — Fixed Channel Allocation Strategies:

There are two common themes in all FCA algorithms [81]: the channels in the system are assigned permanently to each cell; and the assignment is based on long-term, expected statistical information of traffic and interference in each cell. In FCA, a channel is available if it satisfies conditions 1), 2) (in terms of CRD), and it belongs to the channel group of the assigned cell or its neighboring cells (under some restrictions).

The Basic FCA [16] implies that the channel set in each cell is pre-determined and fixed. An oncoming call can only be served by the idle channels of the serving
cell. If there is no idle channel available, the call is simply blocked. This strategy is able to provide high system capacity under heavy load in macro-cell infrastructure, where the traffic and interference distribution are close to the expected statistical model. However, in case of micro-cell and light to moderate traffic load, the basic FCA is far from efficient. In order to deal with traffic dynamics, several borrowing-based FCA strategies have been devised.

In the Simple Borrowing FCA strategy, if there is no idle channel in a cell, a channel may be borrowed from neighboring cells, provided that the borrowing action does not violate the CRD constraint. The Hybrid Channel Assignment (HCA) strategy [48] modifies the simple borrowing FCA by dividing the permanent channel set of a cell into two groups: a local group (with $N_l$ channels) which can only be used within the cell, and a borrowable group (with $N_b$ channels). The ratio $N_l/N_b$ is pre-determined on the basis of the statistical model. Further improvement is obtained by Borrowing with Channel Ordering (BWCO) [22], in which $N_l/N_b$ is adapted with respect to the traffic dynamics. Each channel has an adjustable probability of being borrowed and is ordered accordingly. Idle channels with higher probability are more likely to be borrowed whereas those with lower probability are more likely to be assigned in the local cell. The Borrowing with Directional Channel Locking (BDCL) strategy [99] has similar channel ordering as BWCO, and increases the system capacity by permitting channel reallocation. Channel reallocation means that when a channel is released by a terminated call, another ongoing call may switch to this channel from its original one.

The borrowing-based FCA schemes have two drawbacks. One is the channel locking problem [45]. It refers to the situation that cells out of the CRD of the lending cell but within the CRD of the borrowing cell are now prohibited to use the borrowed channel. It limits $N_b$ since a channel is borrowable only if it is idle in all the cells within the CRD of the borrowing cell. The other drawback is the necessity
of continue monitoring by the switching center (SC). It again limits $N_b$ due to the exponentially increased processing complexity. These shortages also exist in flexible CA and many DCA strategies. In order to deal with the channel locking problem, a scheme called *Channel Borrowing without Locking* (CBWL) [45] suggests that the signal power on the borrowed channel is reduced such that the signal does not interfere with other co-channel traffic within the CRD of the borrowing cell. The new problem is that because the signal power is reduced, the borrowed channel can only be used by users very close to the BS. To alleviate this problem, channel reallocation is utilized to reassign regular channels to the users which can not use borrowed channels from those which can use, and let the latter group transmit on borrowed channels.

### 3.1.3.2 Flexible Channel Allocation Strategies

The flexible CA approach is more 'flexible' than FCA schemes in that only a portion of channel set in the system is assigned permanently to each cell, which suffices under light traffic load. Another part of channel set is put in a central pool in SC which can be accessed by all BS's in the system. Channels in the central pool act as reinforcing roles, and are temporarily assigned to cells suffering heavier traffic load than their permanent channels can handle. In flexible CA, a channel is available if it satisfies availability conditions 1) and 2) (in terms of CRD), and it belongs to the channel group of the assigning cell or the central pool.

The methods of assigning reinforcing channels vary from long-term, statistical information based scheduling manner to short-term, measurement based predictive manner [78]. In fact, every DCA scheme introduced below applies to this task.

### 3.1.4 Dynamic Channel Allocation Strategies

To accommodate the increasing demand on the system capacity, the cell size in the network has to be smaller and smaller. As a result, traffic load and interference
distribution in each cell fluctuate in such a way that the long-term statistics becomes awkward or even useless. Therefore, the trend of using dynamic channel allocation (DCA) algorithms in wireless networks is inevitable in the future. In DCA schemes, there no longer exist permanent channels assigned to each cell. All of the channels in the system can be accessed by every cell as long as the CIR constraint is met. The channel allocation decision is made on a call-by-call basis according to short-term or real-time measurements. Therefore, in DCA a channel is available if it meets the first two “availability conditions”.

DCA schemes are further classified according to the CCC representation they employ. DCA strategies attempting to take advantage of the unevenness of traffic load among cells are called **Traffic Adaptation DCA (TA-DCA)** strategies. In TA-DCA the CCC is also representaed by the CRD constraint. DCA strategies adapting themselves to the traffic load fluctuation and the location change of an MS are called **Location Adaptation DCA (LA-DCA)** strategies. In the typical implementation of LA-DCA, i.e., reuse partitioning, a cell is split into a number of concentric subcells and the CCC is approximated by different CRDs for different subcell groups. The third type of DCA schemes which assign channels based on real-time interference measurement are called **Interference Adaptation DCA (IA-DCA)** strategies. There exists no formal representation of CCC in IA-DCA.

From the implementation viewpoint, each category of DCA defined above can be further classified into different groups:

- **Centralized DCA**: the assignment decision is made by a higher-level monitoring unit based upon the system-wide information.

- **Distributed DCA**: the assignment decision is made by local BS and MS. Some inter-cell communication and information distribution are required.
• **Autonomous DCA:** the channel allocation is done by local BS and MS and relies exclusively on local measurements.

Apparently there exists a tradeoff between system performance and processing complexity. For the same kind of DCA scheme, the centralized algorithm will give the performance upperbound but will be impeded from practical implementation by its complexity. On the other hand, the autonomous algorithm is the simplest in implementation, but its performance may not be as good as desired due to its “selfishness”. Three types of DCA strategies are discussed in the following subsections, respectively.

### 3.1.4.1 Traffic Adaptation DCA Strategies

In TA-DCA, the original CCC is replaced by the simpler but more stringent CRD constraint, as in FCA and flexible strategies. Using propagation models and long-term interference measurements the compatibility of a pair of cells can be determined\(^2\). This compatibility information may be represented by a binary matrix called the *compatibility matrix*. With this modification the DCA problem becomes: under constraint of the compatibility matrix, to find a pair of channels to an arriving call, such that the system capacity is maximized.

**Centralized TA-DCA**

The centralized TA-DCA is the earliest DCA strategy ever studied. The original work of Cox and Reudink [16, 17, 18, 19] in 1970's founded not only the basic idea of DCA, but the research methodology in this area. The method they used is defining the availability of a channel, and then proposing a few heuristic algorithms and comparing their performance through simulation. The DCA algorithms they proposed were of 1) First Available, 2) Nearest Neighbors, and 3) Nearest Neighbors+1. In the *First Available* approach, the first available channel

\(^2Two cells are called compatible if the same channel can be used simultaneously by them.
encountered during channel search is assigned. It saves the computation time with moderate performance. In *Nearest Neighbors*, a channel is chosen if it is already in use in cells nearest to the assigned BS but out of the CRD. If more than one channels are used in these "nearest neighbors", the one used in more "neighbors" is preferred. This strategy is designed to compress the reuse distance of a channel. *Nearest Neighbors+1* is similar to the nearest neighbors strategy except that it finds channels used in cells at the distance CRD+1. This strategy tends to allow more MS to keep their assigned channels when they move across cell boundaries (handoff calls).

Other centralized heuristic schemes were also suggested. In the *Markov Allocation* algorithm [68], each cell is assigned an ordering of channels with idle channels on the top. An arriving call receives the first channel if it is available. The call is otherwise blocked, even if there are eligible channels further along the ordering. The design problem is how to devise the particular ordering for different cells. In another algorithm called *MAXAVAIL* [77], the serving BS will compute the *systemwide channel availability* (SCA) for each available channel and assign to the MS the channel with the maximum SCA. The SCA for channel $j$ is given by $\sum_i n_{av}^j(i)$, where $n_{av}^j(i)$ is the number of available channels in the $i^{th}$ cell if channel $j$ were assigned. This algorithm seeks to maximizing the number of available channels in the system for future calls. In Ref. [20], a systematic bookkeeping procedure was suggested for the centralized DCA schemes. The authors also analytically proved the idea of the *Nearest Neighbors* strategy such that an available channel should be chosen if it is used in most cells in the clique of the assigning cell. The *clique* of a cell $p$ is a set of cells having significant interference with cell $p$. The objective functions proposed in [20] are variations of this idea.

Many researchers tried to formulate the DCA problem into a constrained optimization problem, with a closed-form objective function and a set of constraints,
then solve the problem by exhaustive searching [6, 76] or by some existed programming algorithms [31]. This kind of approaches can only apply to very small systems because of their processing time.

**AI approaches** In recent years two artificial intelligence (AI) strategies, namely the Neural Networks (NN) approach and the Genetic Algorithm (GA) approach, have attracted much interest. The introduction of AI strategies is based on the speculation that the CA problem is an NP-complete combinatorial optimization problem, and that AI approaches are efficient tools in acquiring a feasible solution for this kind of problems.

In the NN approach, the elementary unit of a network is called neuron. The inputs and outputs of neurons are inter-connected in different ways, resulting in various structures. Among them the full-connected Hopfield networks and the multi-layered feedforward NN (MFNN) are two dominant structures. The application of the Hopfield NN to the DCA problem [54, 70] includes interpreting the CIR constraints into a Lyapunov-type energy function, and recursively adapting the output of neurons along the descendent direction of the energy function until \( \frac{dE}{dt} = 0 \). In this way it is guaranteed that the solution converges at least to a local minima. The disadvantages of this approach are the network size (number of neurons = \( M \times V \)), the convergence speed, and the quality of the local minima. The approach using MFNN [7] includes two stages, i.e., the off-line training stage and the on-line mapping stage. In the training stage, numerous representative samples are used to “train” the network. In the mapping stage, the trained NN maps the inputs (new events) into its outputs (channel assignments). The essence of this approach is to find a feasible allocation according to past experiences. The advantage of the MFNN approach is of its processing speed. The cost is that its performance depends upon the quality of the training data extracted from long-term statistics.
GA emulates the evolution process in the nature. The basic idea of GA is that the combination of fitting ancestors will produce an offspring who fits the environment better than either of its parents. The GA approach to the DCA problem [44, 53, 54, 55] consists of representing CIR constraints by a fitness function and iteratively searching the solution space for the most fitting assignment. The searching process includes selecting parents, crossover (mating), mutation, updating population, and evaluating the fitness function. GA approach is a global search procedure. Its main weaknesses are heavy computational burden and possible diversity, which means the majority of the population stays away from any feasible solutions.

In AI approaches, the optimization aspect of the DCA problem is over-emphasized whereas the other two key factors, the large size and the rigid time requirement, are neglected (the same as those objective-function-driven centralized schemes). The AI approaches might become feasible only if the parallel processing technique makes breakthrough and distributed algorithms can be developed.

**Maximum Packing** Among TA-DCA schemes, the Maximum Packing (MP) policy [23] [24] is of special theoretical interest. The policy idealizes the DCA problem by assuming that there exists a super controller who knows all of the system-wide information at any time and does anything needed to accept a call, including reconfiguring the system-wide channel distribution and reallocating channels to all involved calls in process. A new call is blocked only if there is no possible channel reallocation which would make the call being carried. The significance of MP is that it is sufficient to track and record only the total number of busy channels in the system rather than individual busy channels. This reduces the state space of the problem substantially and makes the problem analytically tractable. The MP policy provides an upper performance bound for TA-DCA schemes in terms of blocking
probability. In addition, it has been shown that for linear networks the reallocation number of MP is limited [68].

**Distributed TA-DCA** There is a common philosophy behind distributed TA-DCA (TA-DDCA) strategies. Instead of system-wide knowledge as in centralized cases, a BS only needs to know the channel reuse information within its clique. Therefore, the compatibility matrix used in a BS has limited size.

The *Greedy* algorithm [26] is a DDCA scheme for linear cellular networks. The allocation is done from left to right along the line. Each cell is assigned as many channels as it requires or as possible. A remarkable feature of the work in reference [26] is that it analytically proves that the Greedy algorithm is optimum under some practical assumptions. Unfortunately, the analysis is difficult to be extended to planar cases.

The planar TA-DDCA was widely reported in publications of Bell Labs [14, 15, 38, 40]. In these schemes, the channel reallocation is permitted. However, this permission raises the concern of aggressiveness and instability of the algorithms due to the lack of central monitoring mechanisms. Therefore, the initial proposal was very “timid” (*Timid DDCA*) [15], where the calling MS probes an idle channel and seizes it if it is available. Otherwise it relinquishes that channel and turns to another one. The Timid DDCA has no danger of instability but is not efficient. It was shown in reference [14] that the capacity could be dramatically improved if a more aggressive algorithm was used and more update time is allowed. In an algorithm called *m-Persistent Polite Aggressive DDCA* (m-PPA DDCA) [38, 14], the calling MS attempts to seize a channel even when interference exists. The disturbed MS tries to find another channel using the timid DDCA. If the attempt fails *m* times the arriving call is blocked. Although the m-PPA DDCA has been shown with simulation to approach the performance of MP, it faces an implementation problem. The MS
cannot be made so smart to measure all the channels a priori and make decisions by itself. By this reason, in Local Packing DDCA proposed later [40], the allocation decisions are made by the BS. The logic behind the Local Packing DDCA is clear. If possible, the BS assigns to a new call an available channel; otherwise it tries to generate a free channel by one reallocation, that is, seizing a channel used in only one cell in its clique if the occupying cell of that channel has other free channels within its own clique. If both choices fail, the call is blocked. The Geometric DCA is another example of timid TA-DDCA strategy [4].

More sophisticated TA-DDCA strategies were also proposed in the literature. These schemes use objective functions with optimization procedures to assign available channels. Their results are, without surprise, better than simple DDCA schemes. The cost is of complexity and processing time. The TA-DDCA algorithm suggested in reference [69] does not permit channel reallocation. Each cell is assigned "favorate" channels determined by an optimum FCA. The algorithm takes advantage of long-term statistical information. A cost function is devised for each cell. The minimization of the function corresponds to the combination of a few heuristic principles. It aims at packing the occupied channel set and saving available channels for oncoming traffic. The Compact-Pattern-Based DCA [93] attempts to allocate available channels into the best "compact pattern" (CP). A CP is defined as the channel allocation pattern with the minimum average distance between co-channel cells. The "best" CP is chosen based on the blocking rate computation similar to the Erlang-B Formula.

3.1.4.2 Reuse-Partitioning DCA — Location Adaptative Strategies:
The capacity gains of TA-DCA is moderate (less than 50% over that of FCA [98]). The reason is that the CRD constraint is an inaccurate approximation of the CIR constraint. The CRD is derived by conceiving an MS at the corner of a cell and
calculating the minimum distance between the signaling BS and the interfering BS such that the CIR the MS receives is greater than $\gamma$. However, most mobiles are much closer to their BS than the MS put in the cell corner. This means that most mobiles receive higher signal power than the worst case, and they are more tolerant to the inter-cell interference.

Reuse-partitioning (RP) DCA schemes are devised in order to exploit this kind of tolerance [27, 57, 61, 64, 96, 98]. In such schemes a cell is partitioned into several concentric subcells, where each subcell has different CRD. Mobiles in inner subcells receive stronger signal power and are allocated channels from dense channel reuse plans, whereas those outer (and “weaker”) mobiles are assigned channels from a reuse plan with larger CRD.

Besides channel allocation methods, other design issues in RP-DCA include the selection of reuse factors for different cell layers, the allocation of the spectrum to different cell layers, and the policy of borrowing channels between cell layers such that performance throughout the whole cell is balanced [61].

RP-DCA schemes have been reported to provide additional capacity over corresponding TA-DCA approaches [96]. In essence, every DCA algorithm discussed in preceding subsection, no matter centralized or distributed, may be enhanced by the RP concept at the cost of complexity. For example, in [27] the Greedy algorithm is modified with the RP concept and the performance improvement is significant.

3.1.4.3 Interference Adaptation DCA Strategies: In addition to the traffic distributions and location changes of mobiles, there are other factors affecting the co-channel interference like fadings and transmission powers. These factors can be further exploited by two kinds of more efficient strategies, namely, IA-DCA and IA-DCA with power control (PC).
**IA-DCA** As its name shows, the IA-DCA strategy seeks to directly take advantage of the dynamics of real-time interference in the system. Since the interference a mobile receives and generates depends heavily upon the propagation environment and its movement, it is extremely difficult to make a pre-planned co-channel reuse pattern like seen in previous cases. In this scenario, the real-time estimation of interference at both BS and MS becomes a must. Assuming that such an estimate is available, the IA-DCA strategy may be expressed as: a) scanning idle channels in local cell and estimating their interference; b) assigning a channel to an arriving call under some allocation rules.

The strategy of IA-DCA naturally results in several phenomena:

- The constraint of CRD disappears. Allocation decision depends exclusively upon real-time interference estimation.

- The performance of an algorithm depends on the accuracy and speed of the interference estimation.

- Although there exist some centralized algorithms, e.g., MAXMIN [32], the strategy can be easily implemented in an autonomous way.

- The theoretical analysis is even more difficult than other strategies.

A *Two-Way Balanced Autonomous DCA* algorithm [11, 12] was proposed based on two observations: a) the uplink and downlink CIR of the same BS/MS pair may be different; and b) the MS’s ability of scanning and estimation is limited. In this algorithm, the BS scans all idle channels and makes a short channel list with the lowest uplink interference. The list is broadcast periodically in the cell. After receiving the list, the arriving MS scans listed channels and orders the list with the channel with lowest downlink interference at first. At link-setup time, the MS will pick a channel from top to bottom and check with the BS if the CIR on both links of
chosen channel are satisfactory. If the downlink CIR < $\gamma$ at any channel or the uplink CIR < $\gamma$ at all channels on the list, the call is blocked. Note that the allocation rule used here is the Minimum Interference (MI).

There are a body of similar heuristic rules scattered in the literature. They can be classified into two groups. Those with threshold (MI below Threshold, Highest Interference below Threshold, et. al. [8, 71]) and those without threshold (Random MI, Sequence MI, et. al. [32]). Allocation rules with threshold may have two different thresholds: CIR threshold and interference threshold. The reason of using interference threshold is that the interference measurement is easier than CIR estimation. Simulation results show that with respect to the combined performance of $P_b$ and QoS, the CIR-threshold-based rules are the best if CIR can be estimated accurately a priori. Otherwise rules without threshold outperform rules with interference threshold. In an interesting IA-DCA strategy called the Channel Segregation (CS) scheme [2], a priority function $P$ is recorded and updated for each channel in every cell. A channel with higher value of $P$ is assigned if a selection is needed. Since $P$ is usually the ratio of the number of successful accesses to the number of total trials, it records the access history of a channel in a cell. In this way, CS captures interference environments in different cells and applies them into the assignment process.

It is worth emphasizing that many DCA schemes declaring themselves as “reuse partitioning” should be classified into IA-DCA in our categorization, since they all use real-time CIR measurements to check the availability of a channel. These schemes include Autonomous RP (ARP) [49], Flexible Reuse [60], Self-Organized RP [29], All-Channel Concentric Allocation (ACCA) [80], and Distributed Control Channel Allocation (DCCA) [58], and others.

The autonomous IA-DCA is “rude” in the sense that a BS seizes a channel based exclusively upon its local CIR and neglects the interference the new connection may
generate. This selfishness results in a substantial probability of outage and ongoing call dropping [25]. The solution for this flaw is the distributed IA-DCA [3], in which a cell, before it captures a specific channel, informs its intention to its neighbors and waits for them to assess the potential CIR deterioration the expected connection may cause.

**IA-DCA/PC** The transmission power of interference sources is another factor which influences the change of interference. Furthermore, it is the only factor which can be controlled by a system designer. This is the very reason that an optimum power control (PC) scheme is so important for the resource allocation problem, in addition to its crucial role in fighting the near-far problem and in extending mobile battery life.

The purpose of introducing PC into the topic of resource allocation is multi-fold. First, from the system point of view, transmitters ought to set their power levels around the minimum required value in order to maintain their desired CIR without harming their neighbors’ CIR [25]. Second, the co-channel interference experienced in the uplink and the downlink of the same BS-MS connection are different. This phenomenon will seriously increase $P_b$ since a connection can be set up only if both links’ CIR $\geq \gamma$. It has been shown that balanced CIR can be achieved in both links with PC [13, 62]. And finally, PC is necessary to ensure an even distribution of $P_b$ among cells [83].

There are two types of power control schemes. Signal-strength-based PC adjusts the transmission power level such that the signal strength at the receiver is kept at a desired level. CIR-based PC tries to control the CIR at the receiver. The performance of IA-DCA with both PC schemes are compared in [13] through simulation. The results favor the latter one. This is expected since the CIR-based scheme achieves the objective of CA directly.
It is reported that the optimum PC alone can provide concrete capacity gains (more than 100%) over that of FCA [98]. More substantial gains have been shown in results on IA-DCA/PC schemes [13, 25, 33, 43, 75, 83]. Since DCA and PC were investigated separately in the history, how to integrate these two strategies effectively is still an interesting research topic. In Ref. [43], it is suggested that there exists a trade-off between DCA and PC on the CIR margin above the desired threshold, and that channels with maximum CIR should be selected by a DCA algorithm so that the PC algorithm may have enough working space. The relation between PC and channel reallocation is discussed in [72]. Their conclusion is that when the CIR outage is about to happen, the number of channel reallocation will be dramatically reduced if PC is executed before the reallocation.

3.1.5 DCA in Two-Tier Cellular Systems and Prioritized DCA

Although it has been shown that IA-DCA strategies may provide significant enhancement in the system capacity, most studies mentioned above did not consider the user's mobility. When an MS moves around during a call, the CIR on its channel changes. If the CIR goes below the threshold $\gamma$ (outage), the call has to be reallocated to another channel. This reallocation procedure is well-known as the handoff procedure. If the handoff occurs within the same cell, it is called intra-cell handoff. If it happens between two or more cells, it is called inter-cell handoff, or in many cases simply "handoff". If a call needs a handoff but can not find another proper channel, the call is forced to terminate (dropped). It is obvious that if an IA-DCA scheme is applied to fast mobiles, both the network control due to handoffs and the probability of call dropping will tremendously increase. One of the solutions to this problem is using IA-DCA in the two-tier (microcell/macrocell) cellular architecture.
3.1.5.1 DCA in Two-Tier Cellular Systems: The microcell/macrocell architecture was introduced in purpose of providing a balance between maximizing the system capacity and reducing the network control associated with handoffs [39, 94]. A two-tier system consists of microcell clusters overlaid with macrocells. In this system, slow users should be assigned a channel from the microcell tier and undergo inter-cell handoffs between microcells, while fast users should be assigned a channel from the umbrella macrocells and undergo inter-cell handoffs between them. When the user’s velocity changes during a call, it might be “handed up” from a microcell channel to a macrocell channel, or be “handed down” vice versa. Aside of the DCA schemes, the design problems in the two-tier systems include a) an estimation of a user's velocity and selection of the velocity threshold between microcell/macrocell users, b) channel partitioning between two tiers, and 3) handoff procedures (including hand-up and hand-down) [94].

The application of DCA to the two-tier cellular architecture is a new topic which has not been thoroughly investigated in the literature. A prominent problem here is that if both tiers use the IA-DCA strategy and both can access all channels, the microcell tier will nearly be prevented from obtaining available channels in real time [42]. The reason is that the transmission power used in a macrocell is much stronger than that used in the microcells. A simple solution may be dividing the channel set into two fixed subsets and each tier is allowed to use only one of them. In [42] this method was modified such that macrocells are excluded from the use of a subset of channels while microcells can access all. The methods of building the exclusion channel set were also proposed. Another approach is that two tiers use the channels in an opposite order [73].

3.1.5.2 Prioritized DCA: Even in cases where the intra-cell handoffs are not a major concern (as in TA-DCA), a practical DCA algorithm ought to distinguish
between originating calls and (inter-cell) handoff calls, especially in microcell systems. Since call dropping is more severe than call blocking, handoff calls should have some priority over originating calls in channel assignment. DCA schemes containing this priority are termed as Prioritized DCA. The priority of handoff calls can be realized in various ways. One way is of reserving a number of channels (guard channel) exclusively for handoff calls [37, 59]. Reference [59] also discussed the dimensioning problem, i.e., the ratio between number of guard channels and total number of channels. The second method is of queueing originating calls [34]. The third scheme is of queueing handoff calls [37, 82]. In this approach, an originating call is not assigned a channel until the handoff queue is empty. In addition, a measurement-based non-preemptive priority queueing discipline was proposed in [82] in place of usual first-in/first-out discipline. In the Sub-Rating Scheme [56], a new channel is created for a handoff call by sub-rating an existing call. Sub-rating means that a full-rate channel is temporarily divided into two half-rate channels. One of them serves the handoff request. This approach reduces the probability of dropping at the cost of deteriorated voice quality in part of ongoing calls.

All of these works emphasize on the implementation of the priority and do not include DCA. The Aggressive DRA strategy suggested in [89] is a TA-DDCA scheme which uses the concept of aggressiveness [14] in order to take care the priority of handoff calls. In this scheme, a cell with a handoff queue forces a neighboring cell to release a carrier which carries the least calls in the clique. The condition of doing this process is that the carrier is able to carry more calls than it did before.

3.1.6 Analytical Studies

Among piles of publications in CA problem, a few papers are theoretically valuable. Most of theoretical studies do not deal with finding an analytical solution to the
NP-complete problem, rather focus on traffic models, blocking probabilities, and performance bounds of algorithms.

Some analytical models have been developed for borrowing-based FCA strategies [90, 91, 92]. All of them are based upon classical Markov chain (MC) models and make some assumptions. In TA-DCA, the analysis of MP by the inventors [24] is virtually half numerical. Raymond’s analysis on MP [68] shows that in a linear network it is possible to implement MP without doing more than two reallocations, but in planar networks the worst case reallocation number grows unboundedly with the number of cells. Kelly’s work in [51] is a more comprehensive analysis for MP in planar networks.

In DDCA aspect, Frodigh’s work [26, 28] on linear networks is thorough but cannot be extended to planar case. For planar networks, Cimini et. al. [15] developed an equivalent Erlang-B approximation for computation of $P_b$ under the assumption of even traffic load among cells. In reference [38], a model for DDCA under light traffic conditions is proposed using a combinatorial approach. A common method in these studies is of decoupling a particular cell, together with its clique, from the rest of the system [15, 38, 91]. This is called the cell group decoupling analysis. Many researchers investigated the performance bound of the TA-DCA strategy. A lower bound of all TA-DCA schemes in the FDMA system was given in [30] in terms of number of required channels for a given number of calls. Whiting et. al. [88] showed that the performance of TA-DCA schemes could be bounded by the solution of a related linear programming problem. And their simulation with simple linear networks showed that the bound is tight enough. Jordan et. al. [47] model a cellular network as a general multiple-service, multiple-resource system under the assumption of even traffic load. Their conclusion is not surprising: the TA-DCA schemes are bound by MP at light load and by FCA at heavy loads.
For the RP-DCA, the performance bound is computed by Zander [96] with simplified propagation models (without shadowing, downlink-only). In reference [57], the capacity gain for the concept of reuse partitioning, not RP-DCA, over FCA is given in an elegant form under the assumption of an even traffic distribution. It also proposed an MC-based mobility model for simulation purpose. Whiting et. al. [88] also extended their work into RP case.

The performance bound of IA-DCA was roughly estimated in references [86] and [72] without details. Both of them used a modified Erlang-B formula with an estimated “effective number of servers”.

In summary, some successful analytical works exist, mainly for strategies with pre-planned reuse pattern under the assumption of even traffic loads. The systematic analysis for up-to-date strategies, i.e., IA-DCA/PC in planar infrastructure, is still an open problem. And how far one can explore this problem is a big question.

3.2 The General Resource Allocation Problem

As it is defined in Chapter 2, the general resource allocation problem is a three-dimensional optimization problem — an arriving call needs to be assigned a channel, a BS, and a transmission power level. Although the subproblem of channel allocation has been on the central stage for a long time, primitive works on other two subproblems also exist in the literature.

Zander [95] considered the optimum power distribution for the case of pre-assigned BS and only one channel. It is shown that the number of MS in the system is maximized by a PC regime which balances the CIR at all MS it supports and shuts out the others. The IA-DCA/PC strategy can be viewed as a quasi-two-dimensional allocation schemes. The word “quasi” means that in this kind of strategies people are interested in controlling the transmission power so that received CIR is fixed and without exploiting variant power levels to acquire higher system capacity. The idea
of reducing transmission power on borrowed channels in CBWL scheme might be closer to the thought of a combined channel/power allocation.

The condition (and cost) of BS assignment is that the density of the base stations is higher than necessary, so that there are significant overlapping coverage regions between cells. In Channel Sharing schemes such as Directed Retry (DR) [21] and Generalized FCA [9], channels are allocated to cells as in FCA, but a mobile can be served by one of the neighboring BS with a good CIR if its own geometric cell is full. Channel sharing with RP was also proposed and analyzed in reference [10]. In short, channel sharing is aimed at exploiting the overlapping areas between cells, and can be conceived as a simple BS allocation scheme. From this point of view, the Hybrid CBWL/DR algorithm [46] may be regarded as precedent work on the three-dimensional resource allocation problem. A basic dynamic cell allocation algorithm was suggested in reference [79], in which an MS in an overlapping area collects the congestion information from both BS and selects one with the help of a threshold calculation.

Recently, the joint resource allocation problem is formally proposed [62, 63]. The contribution of the work in [62, 63] is the formulation of the problem and some preliminary analysis. The suggested heuristic algorithm is obviously centralized.

The research on joint allocation of three resource types is in its infancy. However, it is the future of the resource allocation problem due to its potential of further capacity enhancement.

### 3.3 DCA for Heterogeneous Traffic

The idea of transmitting multimedia traffic across the air interface becomes more and more popular. As a result, some researchers have turned their attention to the topic of DCA for heterogeneous traffic (DCAHT). It is worth emphasizing that DCAHT is different from either the dynamic resource assignment problem in wireline...
ATM (asynchronous transfer mode) networks, or the conventional DCA problem in pure circuit-switching cellular networks. The resource assignment in wireline networks amounts to a one-cell FCA problem with mixed traffic types. There is not any frequency reuse involved. Therefore, we may call the problem as a queueing management problem. On the other hand, the conventional DCA problem might be viewed as a reuse management problem [87]. In DCAHT one faces a combination of reuse and queueing management problems.

Anderline [3] proposed a true distributed IA-DCA which handles circuit-switching traffics with various number of requested channels. It made an assumption that an MS can estimate the path loss to the base stations in its vicinity. This may not be practical, especially when the mobile’s mobility is considered. The Packet Dynamic Resource Allocation (Packet DRA) in [87] applied the basic concept of IA-DCA to a packet-switching wireless network though it investigated only one channel. A method which assigns channel to both voice and data traffics was the Multilevel Channel Assignment (MCA) proposed in [52]. Its generic policy of offering higher CIR to data users makes sense. In order to implement this policy, a cell is splitted into two concentric zones as in the case of RP. Data users in outer zones are assigned a higher transmission power level and are assigned channels occupied by inner-zone voice users' co-channel cells. Therefore, MCA is a TA-DCA strategy that exploits the surplus of inner-zone voice users.

The DCAHT problem is, with no doubt, an extremely important and challenging topic. The publications on this issue are rare and preliminary. We can foresee a vast research effort on this topic in the near future.

3.4 Motivation and Objectives of the Dissertation

It is clear that the trend in resource allocation area is the IA-DCA/PC strategies with considerations of mobility (handoff calls and two-tier systems). The joint
dynamic resource allocation methods will deliver further capacity enhancement. The increasing interest in transferring multimedia over wireless networks will surely boost the investigation in the DCAHT problem.

As shown in the previous literature survey, although the IA-DCA strategy has dominated the research activity in the DCA area and its performance has been confirmed by many simulation studies, systematic analysis of the strategy is still remarkably open. The main obstacle to the performance analysis of IA-DCA is the lack of a proper representation of the CIR constraint. Ideally speaking, a “proper” CIR representation ought to be moderate in computational complexity at the cost of a slight approximation, and it should be as accurate as desired if more computational complexity is permitted.

The objectives of this study can be highlighted as follows.

- Build a theoretical framework for the analysis of the IA-DCA strategy.
- Find a tight bound for the benchmark performance measure of all IA-DCA schemes.
- Investigate the possibility of extending the theoretical framework to the analysis of the problem of IA-DCA with heterogeneous traffics.

**Analytical Framework** For the theoretical framework we have to set up some tractable models, define performance measures for our purposes, find the way of calculating the co-channel interference (CCI) and outage probability in an IA-DCA environment, create a proper representation of the CIR constraint, and build a generic analytical methodology for the analysis of various IA-DCA schemes. Furthermore, in order to make our work more useful in practice, we plan to include the effect of the shadow fading in our analysis.
Performance Bound  For an optimization problem as complex as the IA-DCA strategy, it is more reasonable to calculate the performance bounds than to count every possible occupancy distribution of every channel at any moment (remember the advantage of the MP policy). A tight analytical bound can help estimating the improvement space of existing heuristic algorithms and guiding further research effort. In order to make the first step easier, we consider the asymptotic study at first by making the assumption that the channel number and user number can be arbitrarily large. Then we shall lift this assumption and investigate practical systems. Our goal is to find out theoretically how much capacity enhancement the IA-DCA strategy can gain over the FCA in real FDMA/TDMA systems.

Extensions to the DCAHT Problem   We would like to explore if a similar analytical framework can be used to find some preliminary results in analyzing the more involved DCAHT problem. We attack the problem by trying some simplified models at first. An example problem is analyzing the IA-DCA strategy with two types of circuit-switching traffics at different arrival rates.
CHAPTER 4
NOTATION, MODELS, AND ASSUMPTIONS

The notations used and assumptions made throughout this dissertation are listed in this chapter. Some other notations temporarily used in separate chapters will be introduced individually.

4.1 Notation

The CIR constraint rules that a channel $p$ can be assigned to a BS/MS connection only if $(C/I)_p \geq \gamma$, where $(C/I)_p$ is the CIR of channel $p$, and $\gamma$ is the desired CIR threshold. A downlink(uplink)-only DCA algorithm assigns a channel pair to a BS/MS connection if the CIR on the downlink (uplink) channel is above the threshold. This algorithm assumes that the CIR's on both channels are identical. This is not always true in the real world. A two-way DCA scheme assigns a pair of channels only if both downlink and uplink channels meet the CIR constraint.

Following notation is used in this study:

- $R$: radius of a cell.
- $A_c$: area of a cell.
- $D$: distance between two neighboring BS's. $D = \sqrt{3}R$.
- $r_{ij}$: light-of-sight distance between BS $I$ and MS $j$.
- $P^d(P^u)$: received downlink (uplink) local mean signal power.
- $P_0$: received local mean signal power at unit distance.
- $L_{ji}$: shadowing effect of the received signal.
- $\alpha$: the path loss exponent.
- $U_J$: number of offered calls in cell J, with mean $\xi$ (calls/cell).
• $F$: average rate of blocked and dropped calls.

• $M$: total number of channels in the system.

• $\xi$: expected traffic load per cell (Erlang).

• $\rho = \xi/M$: relative traffic load (calls/cell/channel).

• $V$: number of cells in the system.

• $P_b, P_d, P_u, P_{out}$: see definitions in Chapter 2.

4.2 Propagation Model

The propagation model used in this work is the model in reference [66]. The downlink signal power received at MS $i$ from BS $J$ is expressed as

$$P_{ji}^d = P_0 L_{ji} r_{ji}^{-\alpha}. \quad (4.1)$$

Similarly, the uplink signal power received at BS $I$ from MS $j$ is given as

$$P_{ji}^u = P_0 L_{ij} r_{ij}^{-\alpha}. \quad (4.2)$$

In our study, we assume that a) the system is interference limited, such that background noise power is negligible; b) the shadow fading $L_{ji}$’s are independent, lognormal random variables (RV) with a mean of 0 dB and a standard deviation of $\sigma$ dB$^1$; c) omnidirectional antennas are used.

4.3 Traffic Model

We assume that the traffic in the system has uniform distribution, hence $U_J$ is a random variable with statistics independent of cell index $J$. Furthermore, offered traffic in each cell is assumed to be an independent Poisson process with an arrival

\[1\]The i.i.d. assumption is made for the sake of simplicity. The result can be easily modified for the correlated case by applying formulas in Ref [1].
rate $\lambda$ (calls/sec). The duration of each call is exponentially distributed with a mean $1/\mu$ (sec). The average offered traffic in a cell is

$$\xi = \lambda/\mu \ (\text{Erlang}).$$

(4.3)

We can define the relative traffic load as $\rho = \xi/M$ [96]. The probability of assignment failure is defined as $\nu = E[F]/\xi V$. Therefore, at any snapshot in time, $U_J$’s are independent RV with a mean of $\lambda(1-\nu)/\mu$.

A keen reader may notice that the mobility of mobiles is not considered in this model. In most cases, the mobility issue makes huge difference during the design and evaluation of a practical DCA algorithm. Mobility increases the signaling load in the system. It may also lead the DCA designer to giving priorities to handoff calls at the cost of deducing the system capacity. Nevertheless, in this study the effect of mobility is neglected due to following reasons. First, in evaluating the system capacity, the impact of mobility is as follows. First, increased rate of channel seizure attempts due to mobility is accompanied by shorter channel holding times, resulting in similar Erlang traffic load to Eq. (4.3). Second, in order to find the performance bound of IA-DCA strategies, one needs an allocation algorithm which is more powerful than any other IA-DCA schemes. For this purpose, we design the idealized IAMP strategy, which is assumed to have unlimited processing power to perform all of necessary channel allocations/re-allocations within an arbitrarily short time period. Therefore, the probability of forced termination (handoff call dropping) due to processing delay is negligible. Third, in case of heavy traffic, where there always exist originating calls waiting to enter the system, the effect of probability of forced termination for a given call due to lack of available channels is still negligible. The reason is that the probability of assignment failure $\nu$, the benchmark performance measure in our study, includes both probability of new call blocking and forced termination and does not distinguish between them. Let us consider two cases in heavy traffic situation. Case 1, mobility is considered. A handoff call is dropped due to the failure of searching
an available channel by the IAMP method, but the channel left by the dropped call is immediately assigned to an originating call. Case 2, no mobility is assumed. The ongoing call will never experience forced termination, but the new call, who in Case 1 has chances to pick up the channel left by a handoff call, is blocked. These two cases have the same effect on \( \nu \). The same argument can be made for intra-cell handoffs resulting from mobile’s movement.
CHAPTER 5
A FRAMEWORK FOR ANALYZING IA-DCA STRATEGIES

To calculate the performance measures for IA-DCA schemes, one needs to know the maximum number of users the system can accommodate for a specific DCA algorithm. For this purpose, a) a representation of the CCC has to be defined in an IA-DCA environment; and b) the outage probability at every point in the cell has to be calculated for the given DCA algorithm and the given representation of the CIR constraint. The calculation of $P_{out}$ depends on the co-channel interference (CCI) computation.

The major contribution of this chapter is the introduction of a novel concept called the channel reuse zone (CRZ) and its extension in the shadowing model, the extended channel reuse zone (ECRZ). These two concepts are powerful in representing the CCC in an IA-DCA environment. Furthermore, a CRZ/ECRZ structure called concentric CRZ structure is defined. This structure represents the most compact channel reuse pattern under the CIR constraint. The general method of computing the co-channel interference in both downlink and uplink connections is described.

The idea of approaching the upper bound of the system capacity by dividing a cell into infinite number of concentric zones has appeared in the literature [57, 96, 97]. However, since the method of calculating the perimeter of an arbitrary zone for a channel reuse factor other than the classical values (1, 3, 4, 7, ...) was unknown, the authors in references [57] and [96] could only assume hexagonal zone shape and could not break the limitation of the conventional channel reuse factor. This means that the bounds they obtained were for LA-DCA rather than IA-DCA schemes. The method we present in this chapter is able to compute the concentric zone boundaries
corresponding to any non-trivial value of "channel reuse factor"\textsuperscript{1}. The result is: not only are we able to get rid of the confine of the conventional concept of the channel reuse factor, but also we can extend our investigation from the downlink-only, deterministic case to the both-link, shadow fading case.

Materials in next two sections are organized in the same way as: a) the CIR constraint, b) the representation of the constraint, and c) the computation of interference and $P_{\text{out}}$ for both links. A high-level methodology of finding the performance bound for an IA-DCA scheme is also given in Section 5.3.

5.1 Co-Channel Interference Analysis for Deterministic Case

In the deterministic case, shadow fadings defined in Eqs. (4.1) and (4.2) are set to be 0 dB. The downlink and uplink CIR constraints for the cell of interest (denoted as cell 0) are thereby defined, respectively, as

$$\frac{(C/I)_d}{\sum_{j\neq 0} r_{j0}^{-\alpha}} \geq \gamma$$ \hspace{1cm} (5.1)

and

$$\frac{(C/I)_u}{\sum_{j\neq 0} r_{j0}^{-\alpha}} \geq \gamma.$$ \hspace{1cm} (5.2)

5.1.1 Representation of the CIR Constraint – Channel Reuse Zones

Imagine the scenario that a mobile 0 on channel $p$ moves from BS 0 to the border of the cell in an arbitrary direction. At the beginning, the MS is very close to BS 0 so that Eq. (5.1) holds even if all other cells are transmitting on channel $p$. When MS 0 moves away from BS 0, its CIR decreases. At a certain point, one of the neighboring cells of cell 0 has to be forbidden from transmitting on channel $p$, in

\textsuperscript{1}A "non-trivial value" means a real number in the range of $[1, h_K]$, where $h_K$ is determined by the CIR threshold and the propagation model (see definition in the next section).
order to satisfy Eq. (5.1). On its way to the cell border, MS 0 causes more and more cells to be prohibited from using channel $p$. Since the base stations are discretely located, channel reuse zones are naturally formed.

**Definition 5.1** A downlink channel reuse zone (CRZ) is a region in a cell where Eq. (5.1) holds for the same interference distribution despite of the mobile’s location. An uplink CRZ is a region in a cell where Eq. (5.2) holds for the same interference distribution despite of the mobile’s location. A 2-way CRZ is a region in which both Eqs. (5.1) and (5.2) are simultaneously satisfied.

It is clear that the shape and area of a CRZ are determined by $\alpha, \gamma$, and the distribution of co-channel interferers. Virtually an infinite number of possible zone partitionings exist. A special partitioning of CRZs is defined below. It turns out by CCI computation (see the next chapter) that this specific CRZ partitioning consists of a set of concentric zones. Therefore, we call it as a *concentric CRZ structure*. The word “zone” in the next paragraph stands for either a downlink, uplink, or 2-way CRZ.

In the concentric CRZ structure, *zone-1* is defined as the largest CRZ in which a channel can be reused in all the cells without violating the CIR constraint. Then, the cells in the system are divided into V/2 groups. Each has two neighboring cells. *Zone-2* is the largest CRZ in which a channel can be reused in any of the other cells. By dividing the system into V/3 identical groups, with three neighboring cells in each group, we can define two CRZs: *Zone-(3/2)* is the largest CRZ in which a channel can be reused in 2 out of 3 cells. *Zone-3* is the largest CRZ in which a channel can be reused in 1 out of 3 cells. In general, *Zone-(n/l)* ($n, l$ integers, $n > l > 0$) is obtained by splitting the system into V/n identical cell groups, each with $n$ cells, and assigning a channel to $l$ cells in each group in such a way that the resulted CRZ covers the largest possible region. Let $K$ be the number of CRZs in each cell, the $k^{th}$ zone is denoted as *zone-$h_k$, $k=1, 2, \ldots, K$*, where $h_1 = 1$, $h_k > h_{k-1}$. 
The physical meaning of $h_k$ is that a channel can be reused in one out of $h_k$ cells within zone-$h_k$. Therefore, it can be viewed as an extension of the "channel reuse factor". However, it should be emphasized that $h_k$ might take improper fraction values. This implies that $K$ may go to infinity, although the value of $h_k$ is upper bounded by $\alpha$ and $\gamma$. Note that for group indices $n \geq 2$ the layout of the cell group is not unique. For example, 4 cells may be arranged in a row, or stacked in two rows. Even the shape of a cell group which gives the largest CRZ area may change for the same $k$. However, for a given $h_k$, the area of the largest CRZ is identical in every cell since the interference distributions are the same for every cell group. Since only the area of CRZs, which corresponds to the offered number of calls, is useful in our analysis, the definition of zone-$h_k$ makes sense.

Figures 5.1 and 5.2 illustrate the relationship between zones in the concentric CRZ structure and the corresponding channel reuse patterns. Numbers in cells represent the distance between the cell of interest and another BS (unit:D). Fig. 5.1 shows the most sparse co-channel user distribution for a channel used in every one out of five cells. This reuse pattern is utilized to compute the area of CRZ zone-5. Fig. 5.2 shows the most sparse channel reuse pattern corresponding to zone-13/2.

**Definition 5.2** A zone-$h_k$ user is defined as a mobile located within zone-$h_k$ and out of zone-$h_{k-1}$. A zone-$h_k$ channel is a channel assigned to a zone-$h_k$ user, which might be either an uplink or a downlink channel or a duplex channel.

The theorem below is the basis for the whole study on the performance bound of IA-DCA strategies.

**Theorem 5.1** Assume that a) traffic is uniformly distributed in each cell; and b) the allocation policy is fair for all cells. As the number of CRZs $K$ goes to infinity, the concentric CRZ structure represents the CIR constraint of the most compact channel reuse pattern in TDMA/FDMA systems.
Figure 5.1 Channel reuse pattern of concentric CRZ zone-5.

Figure 5.2 Channel reuse pattern of concentric CRZ zone-13/2.
Proof: The most compact channel usage in TDMA/FDMA systems is the one where a channel is reused in all \( V \) cells (reuse pattern 1). To use channels in an optimum way, reuse pattern 1 should contain as many users as possible. In other words, reuse pattern 1 should cover the largest possible region in a cell (by assumption a); and the area of this largest region should be identical in every cell (by assumption b). This argument results in the definition of the concentric CRZ zone-1. For the users outside zone-1, the next most compact reuse pattern is when one channel is reused in \((n-1)V/n\) cells, where \( n \) is an arbitrarily large positive integer. By the same argument as before, we obtain the definition of zone-\(n/(n-1)\). By continuing the same procedure on users outside zone-\(n/(n-1)\), one obtains the whole concentric CRZ structure in a straightforward way.

It is apparent that the concentric CRZ structure with finite \( K \) is still an approximation to the CIR constraint. However, this structure can approximate the original constraint with an unlimited accuracy by increasing \( K \). The real CIR constraint of the most compact channel reuse pattern will be reached when \( K \to \infty \).

5.1.2 Computation of Co-Channel Interference: Deterministic Model

The existing literature on the CCI computation are either for the CDMA systems [65, 85, 100] or for FCA based TDMA systems [96, 100], where CRD is used to represent the CIR constraint. In this section we describe the method of computing CCI in IA-DCA environment.

The computation of downlink CCI is explained through Fig. 5.3. To simplify notations, let us define \( r = r_{oo} \), and \( \theta \) and \( \phi_j \) as shown in Fig. 5.3. The downlink local mean interference is expressed as

\[
\bar{P}^d_{j0} = P_0 r_j r_{oo}^\alpha = P_0 \left[ w_j^2 D^2 + r^2 - 2w_j D r \cos(\phi_j - \theta) \right]^{-\frac{\alpha}{2}},
\]  

(5.3)
Figure 5.3 Computation of downlink co-channel interference.

where $w_j = 1, \sqrt{3}, \sqrt{7}, 3, \ldots$, depending on the distance between BS $J$ and BS 0. Define the set

$$T^d = \{ \text{cells sharing the downlink channel with cell 0} \}.$$ 

Then the downlink CIR at MS 0 is written as

$$ (CIR)^d = \frac{r^{-\alpha}}{\sum_{j \in T^d} \left[ w_j^2 D^2 + r^2 - 2w_j Dr \cos(\phi_j - \theta) \right]^{-\frac{\alpha}{2}}} $$

and it is a function of $r$ and $\theta$.

The computation of the uplink interference is illustrated in Fig. 5.4. Now the difficulty is that every $r_{j0}$ is a random variable independent of each other. Therefore, an equation like Eq. (5.3) which relates $P_{j0}$ with the position of MS 0 does not exist. The main difference between the uplink interference computation in CDMA (see, e.g., [65] [85]) and this study is that in CDMA the interference from another cell is
Figure 5.4 Computation of uplink co-channel interference.

due to an accumulated effect of average $\xi$ users, whereas in TDMA/FDMA there is up to one interferer in each cell. Therefore, instead of computing the expected total interference from a cell, we compute the expected interference power received at BS $0$ from MS $j$ as

$$P_{j0}^n = E \left\{ P_0 r_{j0}^{-\alpha} \right\} = P_0 E \left\{ \left( w_j^2 D^2 + r_j^2 - 2 w_j D r_j \cos \phi_j \right)^{-\frac{\alpha}{2}} \right\}, \quad (5.5)$$

where the expectation is over a specific region in which MS $j$ may appear. In cases of FCA and traffic adaptive DCA, this region covers a whole cell. Define a set

$$I^u = \{ \text{cells sharing the uplink channel with cell } 0 \}.$$ 

Then the uplink CIR at BS $0$ is expressed as

$$(CIR)^u = \frac{r^{-\alpha}}{\sum_{j \in I^u} E \left\{ \left( w_j^2 D^2 + r_j^2 - 2 w_j D r_j \cos \phi_j \right)^{-\frac{\alpha}{2}} \right\}}. \quad (5.6)$$
The computation of $P_{out}$ turns out to be simple in the concentric CRZ structure. For a specific interference distribution corresponding to $h_k$, the outage probability $P_{out}$ equals zero within zone-$h_k$ and equals one outside of it.

5.2 Co-Channel Interference Analysis for Shadowing Case

In the shadowing environment, the downlink and uplink CIR constraints are expressed as

\begin{equation}
(C/I)^d = \frac{L_0 r^{-\alpha}}{\sum_{j \neq 0} L_{j0} r_j^{-\alpha}} \geq \gamma, \tag{5.7}
\end{equation}

and

\begin{equation}
(C/I)^u = \frac{L_0 r^{-\alpha}}{\sum_{j \neq 0} L_{j0} r_j^{-\alpha}} \geq \gamma, \tag{5.8}
\end{equation}

respectively. In the deterministic case, the location of a mobile is the sole factor that decides its CIR for the given $\alpha$ and $\gamma$ parameters. This factor is mapped into channel reuse zones. In this way the CIR of a zone-$h_k$ user is guaranteed by assigning a zone-$h_k$ channel to it. The only reason left to block a call is the lack of available channels. However, in the shadowing model $P_{out}$ is no longer zero or one as in the deterministic model. It is a complicated function of $h_k$, $r$, and $\theta$. As a result, the relationship between the geometric borders (and the area) of CRZs and the number of zone-$h_k$ users is destroyed. This relationship plays a pivotal role in the study of performance bound. In order to recover this relationship, the concepts of “CRZ” and “cell” have to be revamped as follows. The new definitions are called “the extended cell” and “the extended channel reuse zone” for the consistency of terminology, although there no longer exist “zones” in a geometric sense.

5.2.1 Extended Cell and Extended CRZ

Regardless of whether there is any shadow fading or not, an active MS always has a higher CIR with one of BSs at any given moment. Since the purpose of concepts
“CRZ” and “cell” is to represent the CIR constraint rather than the geometric distance, the definitions should be modified according to Eqs. (5.7) and (5.8).

**Definition 5.3** A downlink (uplink) extended cell (EC) of BS J is a group of active mobiles, with a mean value \( \xi \), which has a higher downlink (uplink) CIR on a channel with BS J than with any other BS.

**Definition 5.4** A downlink (uplink) extended channel reuse zone (ECRZ) of \( h_k \), denoted as zone-\( h_k \), is the largest group of mobiles in a downlink (uplink) EC whose CIR \( \geq \gamma \) in a channel assigned to one out of \( h_k \) cells. A 2-way ECRZ of \( h_k \) is defined as a group of active mobiles which belong to both the downlink and uplink ECRZ zone-\( h_k \).

The word “largest” means that if there exist more than one interference distribution patterns for a specific \( h_k \), then zone-\( h_k \) corresponds to the one with the lowest \( P_{out} \) curve. Obviously, it is extremely difficult to identify ECs and ECRZs one by one, even for a snapshot. However, it is straightforward to find the “area” of an EC or an ECRZ. The “area” of an EC/ECRZ is defined as the average number of users in the EC/ECRZ at a given time instance. The “area” of ECRZ zone-\( h_k \) equals

\[
\int_{0}^{2\pi} \int_{0}^{R_k} r[1 - P_{out}(h_k, r, \theta)]dr d\theta,
\]

where \( P_{out}(h_k, r, \theta) \) depends upon the specific DCA algorithm. \( R_k \) is the upper bound of \( r \), such that \( P_o(h_k, R_k) \approx 1 \). The “area” of EC is the summation of “areas” of ECRZs from 1 to \( K \).

### 5.2.2 Computation of Co-channel Interference: Shadowing Case

**Downlink CIR:** From Eqs. (5.4) and (5.7), the downlink CIR at user 0 in shadowing case is expressed as

\[
(CIR)^d = \frac{L_0r^{-\alpha}}{\sum_{j \in \Omega} L_{j0} \left[w_j^2D^2 + r^2 - 2w_jDr\cos(\phi_j - \theta)\right]^{-\frac{\alpha}{2}}}.
\]
The primary problem in computing $P_{out}^d$ with a shadowing model is that of dealing with $L_j$'s. Let

$$e^Y = L_0 r^{-\alpha}, \quad e^{Z_j} = L_j r_j^{-\alpha}, \quad \forall j \neq 0.$$  

(5.10)

Then,

$$Y = \ln L_0 - \alpha \ln r, \quad Z_j = \ln L_j - \alpha \ln r_j.$$  

(5.11)

Since $L_j$s are assumed to be lognormal RVs, $Y$ and $Z_j$ are both Gaussian with $N(m_y, \sigma^2)$ and $N(m_j, \sigma^2)$, respectively, where $m_y = -\alpha \ln r$ and $m_j = -\alpha \ln r_j$. Moreover, let $e^{Z_d}$ be the summation of lognormal RV's as

$$e^{Z_d} = \sum_{j \in I_d} e^{Z_j}.$$  

(5.12)

Combining Eqs. (5.9), (5.10), and (5.11), we get

$$P_{out}^d = P_r \{ e^Y / e^{Z_d} < \gamma \} = P_r \{ Y - Z_d < \ln \gamma \}$$  

(5.13)

Note that $e^{Z_d}$ is a weighted summation of a group of lognormal RV's. There exist three major approaches to approximating such a summation. The comparative study of these methods was reported in references [1, 5]. We use Wilkinson's method in our investigation due to the conclusions in references [1, 5].

In Wilkinson's method, $e^{Z_d}$ is assumed to be another lognormal RV. In other words, $Z_d$ is Gaussian with $N(m_{zd}, \sigma_{zd}^2)$, which are obtained by equating first two moments of Eq. (5.12) as

$$\mu_{1d} = E(e^{Z_d}) = E\left( \sum_{j \in I_d} e^{Z_j} \right) = e^{\frac{\sigma_d^2}{2}} \sum_{j=1}^{V_d} e^{m_j} = e^{\frac{\sigma_j^2}{2}} \sum_{j=1}^{V_d} r_j^{-\alpha},$$  

(5.14)

and

$$\mu_{2d} = E(e^{2Z_d}) = E\left( \sum_{j \in I_d} e^{2Z_j} \right) \begin{equation} \begin{aligned} &= e^{2\sigma^2} \sum_{j=1}^{V_d} e^{2m_j} + 2e^{\sigma^2} \sum_{j=1}^{V_d} \sum_{s=J+1}^{V_d} e^{m_s + m_j} \\ &= e^{2\sigma^2} \sum_{j=1}^{V_d} r_j^{-2\alpha} + 2e^{\sigma^2} \sum_{j=1}^{V_d} \sum_{s=J+1}^{V_d} r_j^{-\alpha} r_s^{-\alpha}. \end{aligned} \end{equation}$$  

(5.15)
where $V_d$ is the number of cells in $T^d$. After $\mu_{1d}$ and $\mu_{2d}$ are obtained, $m_{zd}$ and $\sigma_{zd}^2$ can be found as

\begin{align}
    m_{zd} &= 2\ln \mu_{1d} - \frac{1}{2} \ln \mu_{2d} \tag{5.16} \\
    \sigma_{zd}^2 &= \ln \mu_{2d} - 2\ln \mu_{1d} \tag{5.17}
\end{align}

Since both $Y$ and $Z$ are Gaussian, $Y - Z$ is also Gaussian with

$$N \left( -m_{zd} - \alpha \ln r, \sqrt{\sigma^2 + \sigma_{zd}^2} \right).$$

Then, the downlink outage probability of an MS at $(r, \theta)$ is expressed as

$$P_{out}^d(r, \theta) = P_r\{Y - Z_d \leq \ln \gamma\}$$

$$= 1 - Q\left( \frac{\ln \gamma + \alpha \ln r + m_{zd}}{\sqrt{\sigma^2 + \sigma_{zd}^2}} \right). \tag{5.18}$$

In Eq. (5.18), $r_j$ can be represented as a function of $(r, \theta)$. As a result, $m_{zd}, \sigma_{zd}$, and eventually $P_{out}^d$ are functions of $(r, \theta)$ if the interference distribution and parameters $(\alpha, \gamma, \sigma)$ are known.

**Uplink CIR:** From Eqs. (5.6) and (5.8), the uplink CIR at BS 0 is

$$(CIR)^u = \frac{L_0 r^{-\alpha}}{\sum_{j \in T^u} L_{ij} E\left\{ \left( w_j^2 D_j^2 + r_j^2 - 2 w_j D_j r_j \cos \phi_j \right)^{\frac{\alpha}{2}} \right\}}, \tag{5.19}$$

where the expectation is taken over the region in which MS $j$ may appear. The region depends upon the specific DCA algorithm. Assuming that

$$\overline{r}_{j0} \triangleq \left[ E \left\{ \left( w_j^2 D_j^2 + r_j^2 - 2 w_j D_j r_j \cos \phi_j \right)^{\frac{\alpha}{2}} \right\} \right]^{-\frac{1}{\alpha}} \tag{5.20}$$

is known for a given DCA algorithm, $P_{out}^u$ can be found in the same way as in the downlink case with obvious changes on indices,

$$P_{out}^u(r, \overline{r}_{j0}) = 1 - Q\left( \frac{\ln \gamma + \alpha \ln r + m_{zu}}{\sqrt{\sigma^2 + \sigma_{zu}^2}} \right), \tag{5.21}$$
where

\[ m_{zu} = 2 \ln \mu_1^u - 0.5 \ln \mu_2^u \]  
\[ \sigma_{zu}^2 = \ln \mu_2^u - 2 \ln \mu_1^u \]  

and

\[ \mu_1^u = e^{\sigma^2/2} \sum_{j=1}^{V_u} \bar{r}_{j0}^{-\sigma} \]  
\[ \mu_2^u = e^{2\alpha^2} \sum_{j=1}^{V_u} \bar{r}_{j0}^{-2\alpha} + 2e^{\alpha^2} \sum_{j=1}^{V_u} \sum_{s=j+1}^{V_u} \bar{r}_{j0}^{-\alpha} \bar{r}_{s0}^{-\alpha}. \]  

After both \( P_{out} \) and \( P_{out}^u \) are calculated, the 2-way outage probability is computed after the definition,

\[ P_{out}(r, \theta, \bar{r}_{j0}) = P_{out}^d(r, \theta) \cup P_{out}^u(r, \bar{r}_{j0}). \]  

5.3 A Framework for Finding the Performance Bound of an IA-DCA Scheme

A framework of analyzing the performance bound for a given IA-DCA algorithm includes following the steps:

1) For each "reuse factor" \( h_k = n/l \) (\( n, l \) integers, \( n > l > 0 \)), split the network into \( V/n \) identical cell groups.

2) Find the most sparse interference distribution under a given DCA algorithm. In most cases this can be done with a few trials.

3) Compute \( P_{out}(h_k, r, \theta) \).

4) Change \( k \). Repeat steps 1) - 3) until sufficient number of CRZ(ECRZ) are defined such that the desired accuracy is satisfied.

5) Find the outage probability for an arbitrary "reuse factor" \( h \) through interpolation.

6) Usual performance measures can be obtained from these outage probabilities. The asymptotic study below is just an example.
Since we use the concentric CRZ structure in this framework, what we get through above procedure is the upper bound of a performance measure for IA-DCA schemes due to Theorem 5.1. Note that this framework applies to any DCA algorithm. The concept of CRZ and ECRZ and the computation of $P_{out}$ can be used to either downlink, uplink or two-way balanced connections. In the next chapter, however, we use a theoretical algorithm called IAMP to calculate the upper-bound for all practical IA-DCA strategies.
CHAPTER 6
THE ASYMPTOTIC PERFORMANCE ANALYSIS FOR INTERFERENCE ADAPTATION DCA STRATEGIES

In order to study the performance bound for IA-DCA schemes, every channel is expected to be “packed” as densely as the CIR constraint allows. Due to the limited processing capability of a real system, no practical DCA scheme is able to take full advantage of the channel reuse pattern represented by the concentric CRZ structure. For the purpose of drawing the upper bound for all practical IA-DCA schemes, the idea of the well-known Maximum Packing (MP) approach [24] is employed in this chapter. As we mentioned in Chapter 3, the basic idea of MP is of assuming that there exists a central controller who knows all the system-wide information and has an unlimited processing power. The controller will do all of the necessary channel reallocations in the whole system in order to accommodate a new call. Therefore, the only reason of blocking a call is the CIR constraint. Since MP is originally used in traffic adaptive DCA, the method proposed here is called the Interference Adaptive MP (IAMP) method to avoid any confusion.

In this chapter, the IAMP method is defined in Section 1. In Section 2 the CCI and outage probability are calculated for IAMP for both links. The result of the asymptotic analysis is presented in Section 3. Numerical results and some discussions are presented in the last section.

6.1 The Interference Adaptative Maximum Packing (IAMP) Method

6.1.1 The IAMP Method

The following algorithm might be viewed as either downlink/uplink-only or 2-way IA-DCA, depending on what type of CRZ zones is used.
Whenever a new call arrives, the IAMP method allocate channels in the system in following way:

**Step 1:** Set \( k = 1 \).

**Step 2:** Pick one zone-\( h_k \) user from every group of \( h_k \) cells, do all the necessary channel reallocation, and pack them into a single channel (or a channel pair in 2-way case). The cells occupying the same channel should be in the same location within their own groups\(^1\). In a 2-way case, uplink and downlink channels are assigned such that cells using same uplink channels are assigned with same downlink channels and vise versa. Continue this procedure until all zone-\( h_k \) users are packed or available channels are used up.

**Step 3:** Increase \( k \) by 1; go back to step 2 if \( k \leq K \), otherwise stop the algorithm.

### 6.1.2 Discussions on the IAMP Method

**Intra-cell fairness and possible call dropping** The purpose of ordering the channel packing is to increase efficiency. Since a zone-\( h_k \) channel can only be reused in \( V/h_k \) cells, the maximum number of users a channel can support decreases as \( k \) increases. Therefore, it is more efficient if channels are assigned to zone-\( h_1 \) users first, then zone-\( h_2 \) users, zone-\( h_3 \) users, and so on.

This extremely aggressive policy has some side effects. First, it raises the issue of intra-cell fairness. It is clear that in the IAMP method users close to the cell periphery have much less chance to access a channel than inside users. This unfairness may not be allowed in practical systems, especially in static or low-tier wireless networks. Furthermore, since the IAMP method may reallocate channels in the system (in favor of inside calls) when each new call arrives, ongoing outside calls

\(^{1}\)see Figs. 5.1 and 5.2 for illustration.
are prone to be dropped. In practical algorithms, this kind of call dropping is more serious than call blocking due to unfairness.

Nonetheless, due to following consideration, we keep the aggressiveness in IAMP at the sacrifice of intra-cell unfairness and the possibility of call dropping. The goal of this work is to find the upper bound of all IA-DCA schemes in terms of system capacity. Therefore we do not exclude the possibility that in some cases the DCA algorithm is required to increase the system capacity at the cost of slightly increasing the blocking/dropping probability of periphery calls. Moreover, in regard to the system capacity, we do not need to distinguish between call blocking and call dropping, although keeping the probability of dropping low is a very important QoS requirement. In conclusion, in the IAMP method we view the intra-cell fairness issue as one of the implementation details and push it to the extreme, just as we deal with processing complexity. This, of course, makes the algorithm more impractical. It should be emphasized that the intra-cell fairness problem does not exist in the original MP method in TA-DCA category, in which all users in a cell are assumed under the same interference situation.

Obviously, if the intra-cell fairness is enforced, the system capacity will decrease. This point has been discussed in reference [88].

Channel binding  The significance of binding uplink and downlink channels together can be conceived through the following inequality,

\[ P_{out}^{nb} = 1 - (1 - P_{out}^u) \left( 1 - P_{out}^d \right) \geq \max \left[ P_{out}^u, P_{out}^d \right] = P_{out}^{bd}, \]

where \( P_{out}^u, P_{out}^d, P_{out}^{bd}, \) and \( P_{out}^{nb} \) are outage probabilities of uplink channel, downlink channel, 2-way channels assigned with binding, and 2-way channels assigned without binding, respectively.
6.1.3 The Optimum of the IAMP Method

**Theorem 6.1** Under the assumptions in Theorem 5.1, the IAMP method applied to the concentric CRZ structure becomes the upper bound (in terms of channel use efficiency) for IA-DCA strategies as $K$ goes to infinity.

**Proof:** From Theorem 5.1, when $K$ goes to infinity, the concentric CRZ structure represents the most compact channel reuse pattern. In other words, it gives the largest set of available channels under the CIR constraint. The assumption of unlimited processing power of IAMP means that if there is an available channel in the system, the IAMP method is able to assign it within arbitrarily short time period. As the result, the combination of these two factors gives the optimal channel use efficiency under assumptions of evenly distributed traffic and fair assignment policy among cells.

6.2 Interference Analysis for the IAMP Method

6.2.1 CCI Computation for the IAMP Method: Deterministic Model

**Downlink CRZ:** Under the IAMP method, the border of a downlink CRZ zone-$h_k$ can be found by solving the radius of the downlink zone-$h_k$ $R_{kd}(\theta)$ for various $\theta$'s from the following equation

$$\frac{R_{kd}}{\sum_{j \in T^d_k} \left[ w_j^2 D^2 + R_{kd}^2 - 2w_j Dr \cos(\phi_j - \theta) \right]^{-\frac{1}{2}}} = \gamma.$$  \hspace{1cm} (6.1)

This equation is obtained by a) defining the most sparse co-channel user distribution corresponding to $h_k$; b) inserting Eq. (5.4) into Eq. (5.1); c) setting the inequality in Eq. (5.1) to an equality and setting $r = R_{kd}$. The subscript $k$ of $T^d_k$ emphasizes that $T^d_k$ changes with different $h_k$. The concentric CRZ structure is formed due to the fact that interferers move farther as $h_k$ increases.
Uplink CRZ: In the case of IAMP, all of the uplink CCI in a zone-$h_k$ channel is from zone-$h_k$ users in other-cells. This is the most important distinction between the CCI computation in this work and those for other systems. As a result, the expectation in Eq. (5.5) becomes

$$\bar{P}_j^u = P_0 \int_0^{2\pi} \int_{R_{j-1}}^{R_k} (w_j^2 D^2 + r_j^2 - 2w_j D r_j \cos \phi_j)^{-\frac{\alpha}{2}} p_k(r_j, \phi_j) dr_j d\phi_j$$  \hspace{1cm} (6.2)$$

where $p_k(r_j, \phi_j)$ is the probability density function (pdf) of a zone-$h_k$ user being at $(r_j, \phi_j)$ (uniform by assumption). Then the radius of uplink zone-$h_k$, $R_{ku}$, can be solved from

$$\frac{\pi (R_{ku}^2 - R_{ku-1}^2) r^{-\alpha}}{\sum_{j=0}^{2\pi} \int_{R_{ku-1}}^{R_k} (w_j^2 D^2 + r_j^2 - 2w_j D r_j \cos \phi_j)^{-\frac{\alpha}{2}} r_j dr_j d\phi_j} = \gamma.$$  \hspace{1cm} (6.3)$$

Eq. (6.3) is obtained by following a similar procedure as in the downlink case. Note that the denominator of Eq. (6.3) is independent of $r$. This means that $R_{ku}$ is independent of $\theta$, and the uplink CRZs are circles.

Figure 6.1 shows the concentric CRZ structure of the IAMP method in deterministic case, where $h_k$'s are integers, and $\alpha = 4.0, \gamma = 18$ dB. By definition, a 2-way zone-$h_k$ is the intersection region between downlink zone-$h_k$ and uplink zone-$h_k$.

6.2.2 CCI Computation for IAMP Method: Shadowing Model

Under IAMP method, the interference on every downlink/uplink zone-$h_k$ channel is from other downlink/uplink zone-$h_k$ channels, and downlink and uplink channels are "bound" together. Thus, for a zone-$h_k$ user in cell 0, $T_k^d = T_k^u \triangleq T_k$ with number of interferers $V_d = V_u \triangleq V_k = V/h_k - 1$. To emphasize the fact that in IAMP method the outage probability depends on $h_k$, we write $P_{out}^d$ as $P_{out}^d(h_k, r, \theta)$ and $P_{out}^u$ as $P_{out}^u(h_k, r, r_{j0})$ below.
Figure 6.1 Concentric CRZ structure with integer $h_k$'s.

**Downlink CIR:** With the new notation, $P^d_{out}$ of a zone-$h_k$ user at $(r, \theta)$ can be obtained from Eq. (5.18) with $\mu_{1d}$ and $\mu_{2d}$ changed to

$$\mu_{1d} = e^{\sigma^2/2} \sum_{J=1}^{V_k} t_{J0}^{-\alpha},$$

and

$$\mu_{2d} = e^{2\sigma^2} \sum_{J=1}^{V_k} t_{J0}^{-2\alpha} + 2e^{\alpha^2} \sum_{J=1}^{V_k} \sum_{J+1}^{V_k} r_{J0}^{-\alpha} r_{S0}^{-\alpha},$$

respectively.

**Uplink CIR** In the uplink case, the expected distance between BS 0 and a zone-$h_k$ user in cell $j$ is calculated from Eq. (5.20) by taking the expectation over ECRZ zone-$h_k$ in cell $j$ (assume uniformly distributed traffic)

$$\bar{r}_{j0}(h_k)$$

$$= \left\{ \int_0^{2\pi} \int_0^{R_k} \left( w_j^2 D^2 + r_j^2 - 2w_j Dr_j \cos \phi_j \right)^{-\frac{3}{2}} p_k(r_j, \phi_j) dr_j d\phi_j \right\}^{-\frac{1}{\lambda}}$$

(6.6)
Note that $R_k$ is the upper bound for $r_j$ such that $P_{out}(h_k, r_j) \approx 1$.

Eqs. (6.7), and (5.20)-(5.25) form a functional equation with unknown function $P_{out}^u$. Even $R_k$ in Eq. (6.7) is unknown a priori. One way to solve this functional equation is of searching the solution recursively. A good guess for the initial $P_{out}^u$ is the corresponding $P_{out}^d$ found in Eq. (5.18). The numerical experiment shows that $P_{out}^u$ can be reached within 5 to 6 iterations if $P_{out}^d$ is chosen as the initial function$^2$.

After both $P_{out}^d$ and $P_{out}^u$ for IAMP are obtained, the 2-way outage probability is computed as the following due to the “bound” downlink/uplink channels in IAMP

$$P_{out}(h_k, r, \theta, \bar{r}_{j0}) = \max \left\{ P_{out}^d(h_k, r, \theta), P_{out}^u(h_k, r_j, \bar{r}_{j0}) \right\}. \quad (6.7)$$

Figures 6.2 and 6.3 illustrate the outage probability as a function of normalized distance $r/R$ for various $h_k$ with the same $\alpha, \gamma$ and $\sigma = 8$ dB. Fig. 6.2 shows both $P_{out}^d$ and $P_{out}^u$ for $h_k$'s whose interference distribution is symmetric and therefore $\theta$ has no influence on $P_{out}$. Note that $P_{out}^u$ is higher than $P_{out}^d$ in most cases. To appreciate the effect of shadowing, notice that in deterministic cases, with $\sigma = 0$, zone-1, zone-3, and zone-7 are circles with normalized radii being 0.357, 0.622, and 0.951, respectively. From Fig. 6.2 it is observed that due to shadow fading of the signal, some mobile’s CIR is no longer guaranteed. For example, a zone-1 user at the distance 0.3$R$ to its BS has an outage probability around 0.5. On the other side, thanks to the shadow fading of the interference strength, some users at the distance as far as 0.9 $\sim$ 1.0$R$ might have a chance to be packed into zone-1 channels. This distance spreading of users in the same CRZ zone increases as $h_k$ increases. An interesting task here is to check if we gain or lose capacity due to the shadow fading. The change of expected number of users in each zone will be calculated later.

$^2$The stop condition is that the maximum absolute error between two consecutive iterations is less than $10^{-4}$. 
Figure 6.2 Probability of outage under shadowing (symmetric interference).

For most of CRZs with $h_k$ other than conventional channel reuse factors (1, 3, 4, 7, 9, etc.), CCI is not symmetrically distributed. Thus, mobiles with different angles to the BS have different $P_{out}^d$. The effect of $\theta$ on the downlink outage probability for asymmetric interference distributions is displayed in Fig. 6.3. The figure displays that the influence of $\theta$ decreases for either less $r$, because of higher CIR, or larger $h_k$, due to farther interferer positions.

6.3 An Asymptotic Performance Bound for IA-DCA

As an important application of the CCI computation obtained in the previous sections, a lower bound of a performance measure called “the asymptotic probability of assignment failure” is derived for both propagation models. Throughout this section we assume that a) $\xi$, $M$ are arbitrarily large although $\rho$ is kept finite; and b) $V$ is large so that the boundary effect is negligible. Since we are considering the asymptotic situation, this section may be called as “the asymptotic analysis”.
6.3.1 Definitions

Definition 6.1 The probability of assignment failure is defined as $\nu = E[F]/\xi V$.

The asymptotic probability of assignment failure is defined as $\nu^* = \lim_{\xi,M \to \infty} \nu$.

Definition 6.2 The area increment of CRZ is defined as

$$A_k = (\text{area of zone-}h_k) - (\text{area of zone-}h_{k-1})$$
where $R_0 \triangleq 0$. Let $\Delta h_k = h_k - h_{k-1}$. The area change of CRZ is defined as

$$a(h_k) = \lim_{\Delta h_k \to 0} \frac{A_k - A_{k-1}}{\Delta h_k}.$$  \hfill (6.9)

**Definition 6.3** The extended area increment of ECRZ is defined as

$$A_k^e = \int_0^{2\pi} \int_{R_{k-1}(\theta)}^{R_k(\theta)} [P_{\text{out}}(h_{k-1},r,\theta) - P_{\text{out}}(h_k,r,\theta)] r \, dr \, d\theta,$$  \hfill (6.10)

where $P_{\text{out}}(h_0,r) \triangleq 1$, and $R_k$ is the upper bound of $r$, such that $P_{\text{out}}(h_k,R_k) \approx 1$ (s stands for “shadowing”). The extended area change of ECRZ is expressed as

$$a^e(h_k) = \lim_{\Delta h_k \to 0} \frac{A_k^e - A_{k-1}^e}{\Delta h_k}.$$  \hfill (6.11)

In our simulation studies, $R_k$ is chosen such that

$$\frac{\ln \gamma + \alpha \ln R_k + m_z(R_k)}{\sqrt{\sigma^2 + \sigma_z^2(R_k)}} = 3.25,$$  

which corresponds to $P_{\text{out}} \approx 0.9995$.

### 6.3.2 An Asymptotic Performance Bound for IA-DCA: Deterministic Case

**Theorem 6.2** The lower bound of the asymptotic probability of assignment failure for IA-DCA in a deterministic case can be approximately represented as

$$\nu_{\text{min}}^* = \left\{ \begin{array}{ll} 0 & \rho < 1/\eta(K) \\ \frac{1}{\eta(q-1)} \left[ \eta(q-1) - \frac{1}{\rho} \right] + \zeta(q) & 1/\eta(q) \leq \rho < 1/\eta(q-1) \\ 1 - \frac{1}{\rho} & \rho > 1/\eta(1) \end{array} \right.$$  \hfill (6.12)

where $q = 2, 3, \ldots, K$,

$$\eta(q) = \frac{1}{A_c} \sum_{k=1}^{q} h_k A_k, \quad \zeta(q) = \frac{1}{A_c} \sum_{k=q}^{K} A_k.$$  \hfill (6.13)
**Proof:** Since $U_j$ is uniformly distributed over a cell,

\[(\text{Avg \ # of CRZ zone-$h_k$ users in the system}) = \xi V A_k / A_c. \quad (6.14)\]

The maximum number of zone-$h_k$ users a channel supports is $V / h_k$. Hence, the average number of channels needed to support zone-$h_k$ users is $\xi h_k A_k / A_c$. Thus,

\[
\text{total \ # \ of \ channels \ needed} = \sum_{k=1}^{K} \frac{\xi h_k A_k}{A_c} = \xi \eta(K)
\]

Let us define $F_k = \text{the number of failed zone-$h_k$ users.}$ Due to the ordering of channel packing in IAMP algorithm, a zone-$h_k$ user can not fail unless all zone-$h_l (l > k)$ users have already failed. Therefore, when the number of available channels $M \geq \xi \eta(K)$, $\nu^*_\min = 0$. And when $\xi \eta(K) > M \geq \xi \eta(K-1),$

\[
E[F] = E[F_k]
\]

\[
= (\text{zone-$h_K$ channel deficit})(\text{# of zone-$h_K$ users per channel})
\]

\[
= \left( \frac{\xi}{A_c} \sum_{k=1}^{K} h_k A_k - M \right) \frac{V}{h_K}
\]

\[
= \xi V \left\{ \left( \frac{1}{A_c} \sum_{k=1}^{K-1} h_k A_k - \frac{M}{\xi} \right) \frac{1}{h_K} + \frac{A_K}{A_c} \right\}
\]

\[
= \xi V \left\{ \frac{1}{h_K} \left( \eta(K-1) - \frac{1}{\rho} \right) + \zeta(K) \right\}.
\]

For the case where $\xi \eta(K-1) > M \geq \xi \eta(K-2),$

\[
E[F] = E[F_{K-1}] + E[F_K]
\]

\[
= \left( \frac{\xi}{A_c} \sum_{k=1}^{K-1} h_k A_k - M \right) \frac{V}{h_{K-1}} + \xi V \frac{A_K}{A_c}
\]

\[
= \xi V \left\{ \left( \frac{1}{A_c} \sum_{k=1}^{K-2} h_k A_k - \frac{M}{\xi} \right) \frac{1}{h_{K-1}} + \frac{A_K + A_{K-1}}{A_c} \right\}
\]

\[
= \xi V \left\{ \frac{1}{h_{K-1}} \left( \eta(K-2) - \frac{1}{\rho} \right) + \zeta(K-1) \right\}.
\]

In general, when $\xi \eta(q) > M \geq \xi \eta(q-1), \ q = 2, 3, \ldots, K,$

\[
E[F] = \sum_{k=q}^{K} E[F_k] = \xi V \left\{ \frac{1}{h_q} \left( \eta(q-1) - \frac{1}{\rho} \right) + \zeta(q) \right\}.
\]
Finally, when $M < \xi \eta(1)$,

$$E[F] = \sum_{k=1}^{K} E[F_k]$$

$$= \xi V \left\{ \left( \frac{A_1}{A_e} - \frac{M}{\xi} \right) \frac{1}{1 + \frac{A_2 + \ldots + A_K}{A_e}} \right\}$$

$$= \xi V \left( 1 - \frac{1}{\rho} \right).$$

Eq. (6.12) is obtained via the definition of $\nu^\ast$.

**Theorem 6.3** The lower bound for the asymptotic probability of assignment failure of IA-DCA in deterministic case is

$$\nu_{\min}^\ast = \begin{cases} 0 & , \rho < 1/\eta(K) \\ \zeta(q) & , 1/\eta(K) \leq \rho < 1/\eta(1) \\ 1 - \frac{1}{\rho} & , \rho \geq 1/\eta(1) \end{cases} \quad (6.15)$$

where

$$\zeta(q) = \frac{1}{A_e} \int_{h_q}^{h_K} a(x) \, dx, \quad \eta(q) = \frac{1}{A_e} \left[ A_1 + \int_{1}^{h_q} x a(x) \, dx \right], \quad (6.16)$$

and $h_K$ and $h_q$ are defined, respectively, by

$$\int_{1}^{h_K} a(x) \, dx = A_e - A_1, \quad \int_{1}^{h_q} x a(x) \, dx = \frac{A_e}{\rho} - A_1. \quad (6.17)$$

**Proof:** Using the definition of the area change of CRZ and noticing that $\eta(k) - \eta(k-1) \to 0$ as $K \to \infty$, one can prove the theorem in a straightforward way.

### 6.3.3 An Asymptotic Performance Bound for IA-DCA: Shadowing Case

The major impact of shadow fading on performance analysis is the damage to the relationship of Eq. (6.14). However, whenever $P_{out}$ is found, this relationship is recovered by the introduction of ECRZ as shown in the following theorem.

**Theorem 6.4** The lower bound for the asymptotic probability of assignment failure of IA-DCA under lognormal shadowing can be approximately represented as

$$\nu_{\min}^\ast = \begin{cases} 0 & , \rho < 1/\eta(K) \\ \frac{1}{h_q} \left[ \eta(q-1) - \frac{1}{\rho} \right] + \zeta(q) & , 1/\eta(q) \leq \rho < 1/\eta(q-1), \\ 1 - \frac{1}{\rho} & , \rho \geq 1/\eta(1) \end{cases} \quad (6.18)$$
where \(q = 2, 3, \ldots, K\),

\[
\eta(q) = \frac{1}{A_c} \sum_{k=1}^{q} h_k A_k^s, \quad \zeta(q) = \frac{1}{A_c} \sum_{k=q}^{K} A_k^s. \tag{6.19}
\]

**Proof:** A tiny region in which \(P_{out}(h_k, r, \theta)\) is approximately constant for a fixed \(h_k\) has an area \(\Delta A = r \Delta r \Delta \theta\). Since \(U_J\) is uniformly distributed over a cell, the average number of active users in \(\Delta A\) is \(\xi \Delta A / A_c\). Therefore, average number of zone-\(h_k\) users in the system is expressed as

\[
\lim_{\Delta A \to 0} \frac{\xi V}{A_c} \Delta A \left[ 1 - P_{out}(h_k, r, \theta) - (1 - P_{out}(h_{k-1}, r, \theta)) \right]
= \frac{\xi V}{A_c} \int_0^{2\pi} \int_0^{R_k} [P_{out}(h_{k-1}, r, \theta) - P_o(h_k, r, \theta)] r \, dr \, d\theta
= \xi V A_k^s / A_c. \tag{6.20}
\]

The rest of the proof follows a similar way to the one used to prove Theorem 6.2 in the deterministic case, by making obvious changes on \(A_k\) and \(a(h_k)\).

**Theorem 6.5** The lower bound for the asymptotic probability of assignment failure of IA-DCA for shadowing model is

\[
\nu_{\min}^* = \left\{ \begin{array}{ll}
0, & \rho < 1/\eta(K) \\
\zeta(q), & 1/\eta(K) \leq \rho < 1/\eta(1) \\
1 - \frac{1}{\rho}, & \rho \geq 1/\eta(1)
\end{array} \right. \tag{6.21}
\]

where

\[
\zeta(q) = \frac{1}{A_c} \int_{h_q}^{h_k} a^s(x) \, dx, \quad \eta(q) = \frac{1}{A_c} \left[ A_1^s + \int_1^{h_q} x a^s(x) \, dx \right], \tag{6.22}
\]

and \(h_K\) and \(h_q\) are defined, respectively, by

\[
\int_1^{h_K} a^s(x) \, dx = A_c - A_1^s, \quad \int_1^{h_q} x a^s(x) \, dx = \frac{A_c}{\rho} - A_1^s. \tag{6.23}
\]

**Proof:** The theorem can be obtained by introducing the definition of \(a^s(h_k)\) and letting \(K \to \infty\), as in the deterministic case.
6.4 Numerical Results and Discussions

Theorems 6.3 and 6.5 give concise expressions for the lower bounds of \( \nu^* \) for the IA-DCA schemes. However, computation of \( \nu^*_{\text{min}} \) through these propositions is not easy due to the calculation of \( a(h_k) \) and \( a^*(h_k) \). A closed-form solution for \( a(h_k) \) and \( a^*(h_k) \) is still a topic for future study. The results shown in this section are obtained numerically. All of bounds are computed for \( \gamma = 18 \) dB and \( \alpha = 4.0 \). In shadowing model, \( \sigma \) is assumed to be 8 dB.

In order to compare with the lower bound for reuse partitioning DCA and \( \nu^* \) of FCA proposed by Zander and Eriksson [96] (with a channel reuse factor \( N = 9 )^{3} \), we depict the downlink lower bound for \( \nu^* \) in Fig. 6.4. Two approximate bounds obtained from Theorem 6.2 are also shown. The lower bound is calculated from Theorem 6.3, and \( a(h_k) \) is obtained by interpolation. It is fairly understandable that the two bounds are very close when \( \rho \) is low, since in [96] the assumption of continuous reuse partitioning is used, which is impractical in IA-DCA, but is implemented by IA-DCA. For a higher value of \( \rho \) the Zander-Eriksson's bound is too loose, in the sense that \( \nu^* \) goes up to about 0.65 rather than 1 (as in the proposed bound) when \( \rho \) goes to infinity. It should be pointed out that \( \xi \) is pushed to infinity in the definition of \( \nu^* \). As a result, the traffic dynamics among cells are eliminated. On the other hand, it is well known that FCA outperforms all TA-DCA schemes when \( \xi \) is very large. Therefore, it is appropriate to view the difference between the bound of IA-DCA and FCA in Fig. 6.4 as the gain from more accurate representation of the CIR constraint in IA-DCA than in TA-DCA.

The lower bounds of \( \nu^* \) for the deterministic and the shadowing cases are displayed in Fig. 6.5 and Fig. 6.6, respectively. Comparison of the lower bounds in these figures shows that lognormal shadowing has some positive impact on the

\(^{3} \gamma=18 \) dB and \( \alpha=4.0 \) corresponds to \( h_k \approx 7.22 \); Only downlink bound in deterministic case is available for comparison.
system capacity. For example, if we observe two downlink bounds at $\nu^* = 1\%$, the average traffic load $\rho$ (user/chanel/cell) of the system is approximately 0.36 in Fig. 6.6 and 0.29 in Fig. 6.5. This implies that 20% $\sim$ 25% capacity gain can be obtained by utilization of shadowing with a typical value of variance ($\sigma = 8$ dB) for downlink-only IA-DCA strategies. Even with 2-way bounds, there still exists about 8% $\sim$ 10% capacity gain. This theoretical conclusion is in agreement with the simulation results reported in reference [35].

An explanation of the performance difference is given via Fig. 6.7. In this figure, the areas of downlink CRZ(ECRZ) zone-$h_k$ ($\sum_{n=1}^{k} A_n$, or $\sum_{n=1}^{k} A_n^s$, or $\sum_{n=1}^{k} A_n^r$) are depicted as a function of $h_k$. Since the number of users in zone-$h_k$ equals $\xi A_k/A_c$, the area $\sum_{n=1}^{k} A_n$ reflects the number of users in zone-1 through zone-$h_k$. Therefore, $(\sum_{n=1}^{K} A_n)/A_c = 1$ implies that in order to handle the group of users with average number $\xi$, one needs up to $K$ CRZs with the largest “reuse factor” being $h_K$. From Fig. 6.7 it is known that the area of zones in ECRZ is always larger than the one in CRZ at the same $h_k$. The value of $h_K$ decreases from about 7.22 in the deterministic

![Figure 6.4 Comparison of deterministic downlink lower bound with Zander-Eriksson's result.](image-url)
Figure 6.5 Lower bound of $\nu^*$ for deterministic model.

Figure 6.6 Lower bound of $\nu^*$ for shadowing model.
Figure 6.7 Area change in CRZ and ECRZ.

model to 5.56 in the shadowing case. This increase in $A_k$ (or decrease in $h_K$) results in the improvement of the bound for $\nu^*$.  

Other conclusions that can be drawn from these figures include: 1) In the deterministic case, three bounds are very close to each other. This implies that interference on downlink and uplink channels are very close if channels are allocated by IAMP method. 2) In the shadowing case, the uplink co-channel interference is the main limitation to the capacity of the TDMA/FDMA system using IA-DCA schemes.
CHAPTER 7

PERFORMANCE BOUND FOR IA-DCA WITH FINITE TRAFFIC AND FINITE NUMBER OF CHANNELS

The utilization efficiency of available channels is affected by three factors: 1) the interference factor, in which some available channels are regarded unavailable by a loose approximation of the CIR constraint (as in the TA-DCA case); 2) the traffic factor, in which a channel carries less users than the CIR constraint permits, due to a different traffic load among cells; and 3) the limited processing power. An example of the traffic factor is that a zone-$h_k$ channel, which can carry up to $V/h_k$ calls under the CIR constraint, carries calls much less than $V/h_k$ due to the lack of zone-$h_k$ users at a specific moment. The smaller $\xi$ is, the more frequently this situation occurs.

Throughout our study, we use the IAMP method to exclude the influence of the limited processing power. In the asymptotic analysis of last chapter, $\xi$ and $M$ are assumed to be arbitrarily large. As a result, the randomness of the traffic load in a cell disappears. Since the average number of active users in the system equals $\xi(1 - \nu)$, for every zone-$h_k$ user in cell $i$, one can always find a corresponding zone-$h_k$ user in other cells and pack them into the same zone-$h_k$ channel. In this way, we excluded the effect of the traffic factor and thus focused on the influence of the interference factor. However, when both $\xi$ and $M$ are finite, the variance of the Poisson RV for the traffic load has to be taken into consideration.

In this chapter, we first define a set of new concepts in Section 7.1. The next step is to modify the IAMP policy such that it can take care of the effect of the traffic change in different cells. The refined IAMP method is presented in Section 7.2. Some crucial quantities have to be calculated in Section 7.3 before we can find the closed-form expression for the lower bound of assignment failure, which is presented in in Section 7.4. Numerical results and discussion are presented in Section 7.5.
7.1 Definitions

Definition 7.1 The interference clique of a channel $p$ — The cell of interest who uses channel $p$, and co-channel cells making significant co-channel interference on channel $p$. The interference clique of an $h_k$ channel is called the $h_k$ clique.

Even though the numbers of users in all cells are assumed i.i.d. Poisson RV’s with mean $\xi$, the instant number of users in different cells is obviously not identical. The concept of “zone-$h_k$ remainder” is useful for describing this scenario.

Definition 7.2 When all zone-$h_1$ users in one of the cells are assigned a channel, zone-$h_1$ users remained in other cells within its $h_1$ clique are called zone-$h_1$ remainders.

In the refined IAMP method (defined below), zone-$h_1$ remainders are mixed with zone-$h_2$ users and will be assigned with a zone-$h_2$ channel. We may define zone-$h_k$ remainders by continuing to assign channels in the same way.

Definition 7.3 When all zone-$h_k$ users and zone-$h_{k-1}$ remainders in one of the cells are assigned a channel, zone-$h_k$ users and zone-$h_{k-1}$ remainders remained in other cells in its $h_k$ clique are called zone-$h_k$ remainders.

Definition 7.4 The common zone-$h_k$ user — a zone-$h_k$ user or zone-$h_{k-1}$ remainder which can be packed into a zone-$h_k$ channel together with $(V/h_k) - 1$ zone-$h_k$ users or zone-$h_{k-1}$ remainders from other cells.

In the last CRZ zone, after all common zone-$h_K$ users are assigned zone-$h_K$ channels, there are a few zone-$h_K$ remainders left, but we can not push them further outside of a cell. Nevertheless, we still need to “pack” these users as compactly as possible. The following two concepts turn out to be helpful in dealing with zone-$h_K$ remainders.

Definition 7.5 The common zone-$h_{K,t}$ remainder — a zone-$h_K$ remainder who may be packed into a channel together with other $t-1$ zone-$h_K$ remainders in its interference clique, where $t = N_K - 1, N_K - 2, \ldots, 1$. 
Definition 7.6 The zone-$h_{K,t}$ remainder — a zone-$h_{K}$ remainder who have not been assigned a channel after the assignment of all common zone-$h_{K,t+1}$ remainders.

Following notations are needed to quantify the study in this chapter. Here "zone-$h_k$" may be either a CRZ zone or an ECRZ zone.

- $N_k$: the number of cells in an $h_k$ clique.
- $\xi_k$: average number of offered users in zone-$h_k$. By assumption, $\xi_k = \xi A_k / A_c$.
- $\Delta \xi_k$: average number of offered users inside zone-$h_k$ and outside zone-$h_{k-1}$ (zone-$h_k$ users). $\Delta \xi_k = \xi_k - \xi_{k-1}$ for $k > 1$ and $\Delta \xi_k = \xi_1$ for $k = 1$.
- $u_k$: the number of offered zone-$h_k$ users. By assumption, $u_k$ is a Poisson RV with mean $\Delta \xi_k$.
- $C_k$: the total number of offered zone-$h_k$ users and zone-$h_{k-1}$ remainders in a cell.
- $H_k$: the number of common zone-$h_k$ users in a cell.
- $C_{K,t}$: the number of zone-$h_{K,t}$ remainders in a cell.
- $H_{K,t}$: the number of common zone-$h_{K,t}$ remainders in a cell.

7.2 The Modified IAMP Method

The original IAMP method has to be modified to handle zone-$h_k$ remainders. One may call the following algorithm as the IAMP method – version 2 (IAMP-V2).

Whenever a new call arrives, the IAMP method allocate channels in the system in following way:

Step 1: Set $k = 1$. 
Step 2: Pick one of zone-$h_1$ users from each cell and pack them into a zone-$1$ channel. Continue the process until zone-$h_1$ users in one cell are used up.

Step 3: Increase $k$ by 1.

Step 4: Pick one of zone-$h_k$ users or possible zone-$h_{k-1}$ remainders from each one of $h_k$ cells, and pack them into a zone-$h_k$ channel. Continue the process until zone-$h_k$ users/zone-$h_{k-1}$ remainders in a cell are used up.

Step 5: If available channels in the system are used up, stop. If there are available channels left and $k < K$, go to Step 3; otherwise goto Step 6.

Step 6: Go to the auxiliary algorithm.

An auxiliary algorithm for the IAMP-V2 method is needed to process zone-$h_K$ remainders. It can be expressed as

Step 1: Set $t = N_K - 1$.

Step 2: Pick one common zone-$h_K,t$ remainder from each one of $t$ cells and pack them into a zone-$h_K$ channel. Continue the process until common zone-$h_K,t$ users or available channels are used up.

Step 3: Decrease $t$ by 1. Go to Step 2.

7.3 Average Number of Common Users

From the IAMP-V2 method, we may easily find some basic relations between these variables $C_k, H_k, \text{ and } u_k$:

\[ C_1 = u_1, \quad (7.1) \]
\[ C_k = u_k + (C_{k-1} - H_{k-1}), \quad k > 1, \quad (7.2) \]
\[ C_{K,N_K-1} = C_K - H_K, \quad (7.3) \]
\[ C_{K,t} = C_{K,t+1} - H_{K,t+1}. \quad (7.4) \]
7.3.1 Computation of the Average Number of $H_k$

From the derivation in the asymptotic analysis, we notice that the key step in finding the probability of assignment failure ($\nu$) is of computing the average number of common users a CRZ can contain ($H_k$). In the asymptotic analysis, the average number of common zone-$h_k$ users and the average number of zone-$h_k$ users in an individual cell are the same, due to the assumption of "unlimited $M$". In this chapter, however, the "limited $M$" condition makes the calculation of the average number of $H_k$ extremely difficult.

In this chapter we denote a Poisson RV with mean $\xi$ as

$$p_x(\xi) = \xi^x e^{-\xi} / x!,$$  \hspace{1cm} (7.5)

and "the tail of the Poisson distribution" as

$$Q_s(\xi) = \sum_{x=s+1}^{\infty} \xi^x e^{-\xi} / x! = 1 - \sum_{x=0}^{s} \xi^x e^{-\xi} / x!.$$  \hspace{1cm} (7.6)

Note that

$$Q_{s-1}(\xi) = p_x(\xi) + Q_s(\xi) = 1 - \sum_{x=0}^{s-1} \xi^x e^{-\xi} / x!.$$  \hspace{1cm} (7.7)

**Theorem 7.1** The average number of common zone-$h_1$ users is

$$\overline{H}_1 = \sum_{s=1}^{\infty} s P_r(H_1 = s),$$  \hspace{1cm} (7.8)

where

$$P_r(H_1 = s) = [Q_{s-1}(\xi_1)]^{N_1} - [Q_s(\xi_1)]^{N_1}.$$  \hspace{1cm} (7.9)

**Proof:** By the definition of $H_1$,

$$P_r(H_1 = s) = \begin{cases} \begin{aligned} & P_r(u_1 = s \text{ in 1 cell, } u_1 > s \text{ in } N_1 - 1 \text{ cells}) \\ & \quad + P_r(u_1 = s \text{ in 2 cells, } u_1 > s \text{ in } N_1 - 2 \text{ cells}) \\ & \quad \cdots + P_r(u_1 = s \text{ in } N_1 \text{ cells}) \end{aligned} \end{cases} = \begin{aligned} & N_1 P_r(C_1 = s) [P_r(C_1 > s)]^{N_1 - 1} + \binom{2}{N_1} [P_r(C_1 = s)]^2 [P_r(C_1 > s)]^{N_1 - 1} \end{aligned}$$
\[
+ \left( \begin{array}{c} 3 \\ N_1 \end{array} \right) [P_r(C_1 = s)]^3 [P_r(C_1 > s)]^{N_1-3} + \cdots + [P_r(C_1 = s)]^{N_1} \\
= [P_r(C_1 = s) + P_r(C_1 > s)]^{N_1} - [P_r(C_1 > s)]^{N_1} .
\]

(7.10)

Using the fact that \( u_1 \) is Poisson and noting the relationship of Eq. (7.1), we have

\[
P_r(C_1 = s) = p_s(\xi_1), \quad P_r(C_1 > s) = Q_s(\xi_1).
\]

(7.11)

And Eq. (7.9) is obtained by inserting Eq. (7.11) into Eq. (7.10).

In order to find the distribution of the number of common zone-\( h_k \) users (\( H_k \)), one has to know the distribution of \( C_k \), which equals the number of zone-\( h_k \) users plus zone-\( h_{k-1} \) remainders. From Eq. (7.2) we can see that in addition to the i.i.d. Poisson RV \( u_k \), \( C_k \) includes another identical, but no longer independent nor Poisson RV: \( C_{k-1} - H_{k-1} \). The correlation among \( C_k \)'s in different cells comes from the mechanism of the IAMP-V2 method. After \( C_{k-1} \) is processed, \( C_{k-1} - H_{k-1} \) equals zero in at least one cell within the interference clique. This correlation is not very significant, especially when the clique size is large. However, it turns out that it is extremely difficult to express the correlation exactly, especially after the convolution between \( P_r(C_{k-1} - H_{k-1}) \) and \( P_r(u_k) \). In the following theorem, we give an approximate expression for \( P_r(H_k) \) by assuming that \( C_k \)'s are independent. Then we discuss the possible error range caused by the independence assumption. The proof of the theorem is given in Appendix A.

**Theorem 7.2** Assume that \( C_k \)'s in different cells are independent of one another.

The distribution of common zone-\( h_k \) users (\( k > 1 \)) can be expressed as

\[
P_r(H_k = s) = \left[ P_r(C_k \geq s) \right]^{N_k} - \left[ P_r(C_k > s) \right]^{N_k} ,
\]

(7.12)

where

\[
P_r(C_k \geq s) = 1 - \sum_{x=0}^{s-1} P_r(C_k = x), \quad P_r(C_k > s) = 1 - \sum_{x=0}^{s} P_r(C_k = x),
\]

(7.13)
\[ Pr(C_k = s) = p_s(\Delta \xi_k) \sum_{y=0}^{\infty} Pr(C_{k-1} = y) P_{r}^{N_{k-1}-1}(C_k \geq s) + \sum_{x=1}^{s} \{ p_{x-1}(\Delta \xi_k) \} \] (7.14)

\[ \sum_{y=0}^{\infty} Pr(C_{k-1} = x+y) \left[ P_{r}^{N_{k-1}-1}(C_{k-1} \geq y) - P_{r}^{N_{k-1}-1}(C_{k-1} > y) \right] \}

The average number of common zone-\( H_k \) users is

\[ \overline{H}_k = \sum_{s=1}^{\infty} s \, Pr(H_k = s). \] (7.15)

**Proof:** See Appendix A.

### 7.3.1.1 Discussions on Theorem 7.2:

Since the average number of \( H_k \) is the most important parameter in analyzing the performance of IA-DCA in systems with finite \( M \), it is worthwhile to make some insightful discussion on Theorem 7.2.

**The Exact Expression of** \( Pr(H_k = s) \)

It should be noticed that in Theorem 7.2 the distribution of \( C_k \) (Eq. (7.15)) is accurate. However, even though Eq. (7.15) applies to the individual \( C_k \), it does not describe the dependence between different \( C_k \)'s. This makes Eq. (7.12) an approximate representation of \( Pr(H_k = s) \). At this point, it is natural to raise some questions regarding to the exact expression of \( Pr(H_k = s) \). Is it possible to find an exact formula? How difficult is it to find it? How significant, in either theoretical or practical sense, is the exact expression? To answer these questions, we have to estimate the size (span) of the solution space of \( C_k \). Let us first estimate the size of the state space of remainders \( C_{k-1} - H_{k-1} \), which is a subspace of \( C_k \) by setting \( u_k \) to be 0.

Let \( s_{km} \) be the maximum number of \( C_k \)^1. Then the distribution of zone-\( h_{k-1} \) remainders is

\[ Pr \left( C_{k-1} - H_{k-1} = 0 \text{ in } n_0 \text{ cells}, C_{k-1} - H_{k-1} = 1 \text{ in } n_1 \text{ cells}, \ldots, C_{k-1} - H_{k-1} = s \text{ in } n_s \text{ cells} \right) \]

^1In case of Poisson RV, \( s_{km} \) may go to infinity theoretically. But in the simulation we can define \( s_{km} \) such that \( Pr(C_k = s_{km}) = \epsilon \), with an \( \epsilon \) very small.
\[ \begin{aligned}
\binom{n_0}{N_{k-1}} \binom{n_1}{N_{k-1}-n_0} \cdots \binom{n_s}{N_{k-1}-\sum_{i=0}^{s-1} n_i} \sum_{x=0}^{s(k-1)m} [P_r^{n_0}(C_{k-1}=x) \cdot P_r^{n_1}(C_{k-1}=x+1) \cdots P_r^{n_s}(C_{k-1}=x+s)].
\end{aligned} \] 

(7.16)

where \(n_0 + n_1 + \ldots + n_s = N_{k-1}\), and, by the IAMP-V2 method, \(n_0 \geq 1\). Eq. (7.16) represents only one point (state) in the solution space of the number of zone-\(h_{k-1}\) remainders. The maximum number of the states in this space is

\[ N_{k-1} \times s_{(k-1)m}. \]

In order to find a useful probability, e.g., \(P_r(C_{k-1}-H_{k-1} = 0 \ in \ n_0 \ cells)\), one has to let other parameter (in this case \(n_1, n_2, \ldots, n_s\)) change in their ranges and sum all the possible states together. Please refer to Theorem 7.3 below for a good demonstration.

Let us consider the range of \(N_{k-1}\) and \(s_{km}\). \(N_{k-1} = 19\) for an interference clique including the cell of interest and its co-channel neighbors within three layers. For an AMPS system \((M = 416)\), \(\xi \ (\text{calls/cell}) \approx 0.1M \sim 0.3M \approx 42 \sim 120\). Let \(k = 2\), when \(\alpha = 4\) and \(\gamma = 18\)dB, the average users in zone-\(h_1 \ \xi_1 \approx 0.2 \xi = 9 \sim 25\). If we choose \(\xi_1 = 17\) and truncate the Poisson distribution at \(\epsilon = 10^{-5}\), \(s_{1m} = 35\). Consequently, we are dealing with a solution space with at least \(19 \times 35^{18}\) states.

Now consider the solution space of \(H_k\). A state in this space is

\[ P_r(C_k = 0 \ in \ n_0 \ cells, \ C_k = 1 \ in \ n_1 \ cells, \ldots, C_k = s \ in \ n_s \ cells). \]

From Eq. (7.2), each term in this probability is no longer as simple as \(P_r(C_k = x+s)\), but a convolution between the pdf of \(u_k\) and the pdf of \(C_{k-1} - H_{k-1}\). The size of the space is approximately \(s_{km}\) to the power of \(N_k\), where \(N_k\) may or may not be equal to \(N_{k-1}\), but \(s_{km}\) should be much larger than \(s_{(k-1)m}\).

From above discussions, we may draw the following conclusions in regards to the exact representation of \(P_r(H_k = s)\). 1) It is possible, yet very much difficult, to find an exact representation. 2) This kind of representation will be too complicated.
and will require too much computational power to be used in the analysis of any practical system with IA-DCA schemes. 3) If the exact representation is found, it may be useful in estimating the error range of the approximate expression Eq. (7.12) when both the exact and the approximate ones are applied to a fairly small system.

The Approximate Representation  As mentioned earlier, Eq. (7.12) is obtained by assuming the independence of \( C_k - H_k \). The questions for this equation are: How good is this approximation in the range of interest? Is it optimistic or pessimistic in terms of \( \bar{H}_k \)? In this case, a pessimistic approximation is preferred, since we know the upper bound from the asymptotic analysis.

Since neither exact formula nor simulation result is available, we are not able to give satisfactory answers to these questions at this time. The following discussion merely includes some points which make sense to the author.

The effect of the assumption of the independence of \( C_k - H_k \) includes two aspects. First, the "wear out" effect. In general, after assigning channels to common zone-\( h_k \) users, the probability decreases dramatically that the number of remainders \( C_k - H_k \) remains high. However, this is not true for those extremely uneven distributions, in which \( C_k \) equals 0 in few cells but equals very high numbers in others. These high values of \( C_k \) are not affected by the assignment of zone-\( h_k \) channels, and they may generate high values for \( H_{k+1} \) if \( u_{k+1} \) in those zero-\( C_k \) cells is large. Since the probability of those uneven distributions of \( C_k \) does not remain unchanged in Eq. (7.12) as it should be, these potential high values are "weared out" by the current approach. Second, the "phantom states". Since the rule that \( C_k - H_k = 0 \) in at least one cell is neglected, Eq. (7.12) generates some "phantom states" in the solution space of \( C_k - H_k \). These phantom states will add some false states on the higher side of \( H_{k+1} \). One approach to alleviate the influence of phantom states is of forcing \( P_r(H_k > u_{km}) = 0 \), where \( u_{km} \) is the maximum number of \( u_k \).
It is obvious that the "wear out" effect makes the estimation of $\overline{H}_k$ pessimistic, and the "phantom state" effect makes it optimistic. In addition, the larger $N_k$ and larger ratio $u_k$ in $C_k$ will make the approximation better. In our numerical result, $N_k$ is around 30. Based upon some indirect observations, the author believes that Eq. (7.12) is a good approximation at such a large $N_k$. Nevertheless, more quantitative analysis is needed and will be one of the research issues in the near future.

### 7.3.2 Computation of the Expected Value of $H_{k,t}$

In the previous section we proposed an auxiliary algorithm to handle users remained after common zone-$h_K$ users are assigned channels. This algorithm groups these remaining users into common zone-$h_{K,t}$ remainders and assign them channels accordingly. In this subsection, we give the exact expression of the expected value of the number of common zone-$h_{k,t}$ users. The reason for deriving an exact formula here is that, without randomizing from new $u_k$s, the approximate result of Eq. (7.12) becomes too pessimistic. On the other hand, without $u_k$, it is "easier" to find out an exact representation.

**Theorem 7.3** The expected value of $h_{K,N_k-t}$, $t = 1, 2, ..., N_k - 1$, is

$$
\overline{H}_{K,N_k-t} = \sum_{s=1}^{m} s P_r(H_{K,N_k-t} = s),
$$

where

$$P_r(H_{K,N_k-t} = s) = \frac{N_k!}{(N_k-t)!} \{\text{term}_1 + \text{term}_2 + \ldots + \text{term}_t\},
$$

and

$$
\text{term}_1 = \sum_{x_1=0}^{s_m-s} \frac{1}{t!} P_r^t(C_k = x_1) \left[ P_r^{N_k-t}(C_k \geq s+x_1) - P_r^{N_k-t}(C_k > s+x_1) \right],
$$

$$
\text{term}_2 = \sum_{x_1=1}^{s_m-s} \sum_{x_2=0}^{s_m-s-x_1} \left\{ \sum_{q_1=1}^{t-1} \frac{1}{q_1! (t-q_1)!} P_r^{q_1}(C_k = x_2) P_r^{t-q_1}(C_k = x_1 + x_2) \right\}.
$$
\[
\begin{align*}
\text{term}_3 &= \sum_{x_1=1}^{s_m-s} \sum_{x_2=1}^{s_m-s-x_1} \sum_{x_3=0}^{s_m-s-x_1-x_2} \left\{ \left( \sum_{q_1=1}^{t-2} \sum_{q_2=1}^{t-2-q_1} \frac{1}{q_1!q_2!(t-q_1-q_2)!} \right) P_r^{q_1}(C_K=x_3) \right\} P_r^{q_2}(C_K=\sum_{i=2}^{3} x_i) \left[ P_r^{N_K-t}(C_K \geq s + \sum_{i=1}^{3} x_i) - P_r^{N_K-t}(C_K > s + \sum_{i=1}^{3} x_i) \right],
\end{align*}
\]

......

\[
\begin{align*}
\text{term}_{t} &= \sum_{x_1=1}^{s_m-s} \sum_{x_2=1}^{s_m-s-x_1} \cdots \sum_{x_t=0}^{s_m-s-x_1-x_2-x_{t-1}} \left\{ P_r(C_K=x_t) P_r(C_K=\sum_{i=1}^{t} x_i) \right\} P_r(C_K=\sum_{i=1}^{t} x_i) \left[ P_r^{N_K-t}(C_K \geq s + \sum_{i=1}^{t} x_i) - P_r^{N_K-t}(C_K > s + \sum_{i=1}^{t} x_i) \right].
\end{align*}
\]

\textbf{Proof:} See Appendix B.

Note that Eq. (7.18) does not apply for the case of \( s = 0 \). The form of \( P_r(H_{k,t} = 0) \) is similar to Eq. (7.18) but a little more complex. Since the computation of \( \overline{H}_{k,t} \) does not need this probability, it is not included in this section.

### 7.4 The Lower Bound for IA-DCA with Limited Number of Channels

After the expected values of \( H_k \) and \( H_{K,t} \) are calculated according to the IAMP-V2 method and its auxiliary algorithm, the lower bound for performance of the IA-DCA strategies can be derived in a similar way to that in the asymptotic analysis. In this case, however, the bound is drawn in terms of the probability of assignment failure. Since the propagation models and interference computation are the same as in the last chapter, we do not specify the propagation models in theorems below. In other words, the following theorems apply to either uplink, downlink, or two-way connections with either propagation model, depending on which one is used to calculate the expected values of \( H_k \) and \( H_{K,t} \).
Theorem 7.4  The lower bound of the probability of assignment failure for IA-DCA can be approximately represented as

\[
\nu_{\min} = \begin{cases} 
0 & , \rho \leq \frac{1}{\eta(K) + h_K N_K \psi(1)} \\
\frac{q}{h_K N_K} \left[ \eta(K) - \frac{1}{\rho} \right] + q \psi(q) + \chi(q) & , \rho \in \left( \frac{1}{\eta(K) + h_K N_K \psi(q)}, \frac{1}{\eta(K) + h_K N_K \psi(q+1)} \right), q \leq N_K - 1 \\
\frac{1}{h_q} \left[ \eta(q-1) - \frac{1}{\rho} \right] + \zeta(q) + \chi(N_K) & , \rho \in \left(1/\eta(q), 1/\eta(q-1)\right), q = 2, \ldots, K \\
1 - \frac{1}{\rho} & , \rho > 1/\eta(1)
\end{cases}
\]  

(7.19)

where

\[
\eta(q) = \frac{1}{\xi} \sum_{k=1}^{q} h_k \overline{H}_k, \quad \zeta(q) = \frac{1}{\xi} \sum_{k=q}^{K} \overline{H}_k, 
\]

(7.20)

\[
\psi(q) = \frac{1}{\xi} \sum_{t=q}^{N_K-1} \frac{\overline{H}_{K,t}}{t}, \quad \chi(q) = \left\{ \begin{array}{ll}
\frac{1}{\xi} \sum_{t=1}^{q-1} \overline{H}_{K,t} & , q = 2, \ldots, N_K \\
0 & , q = 1
\end{array} \right. 
\]

(7.21)

Proof: The proof of this theorem is very similar to that for Theorem 6.2, except that we have to take care of the \( H_{K,t} \)'s. The detailed proof is given in Appendix C.

As in the asymptotic case, more accurate representation can be obtained by letting the number of CRZ zones goes to infinity. Define \( \Delta h_k = h_k - h_{k-1} \) and the increment of the number of common users as

\[
d_h(h_k) = \lim_{\Delta h_k \to 0} \frac{\overline{H}_k}{\Delta h_k}. 
\]

(7.22)

Then we get the following result.

Theorem 7.5  The lower bound for the probability of assignment failure for IA-DCA can be expressed as

\[
\nu_{\min} = \begin{cases} 
0 & , \rho \leq \frac{1}{\eta(K) + h_K N_K \psi(1)} \\
\frac{q}{h_K N_K} \left[ \eta(K) - \frac{1}{\rho} \right] + q \psi(q) + \chi(q) & , \rho \in \left( \frac{1}{\eta(K) + h_K N_K \psi(q)}, \frac{1}{\eta(K) + h_K N_K \psi(q+1)} \right), q \leq N_K - 1 \\
\frac{1}{h_q} \left[ \eta(q-1) - \frac{1}{\rho} \right] + \zeta(q) + \chi(N_K) & , \rho \in \left(1/\eta(q), 1/\eta(q-1)\right), q = 2, \ldots, K \\
1 - \frac{1}{\rho} & , \rho > 1/\eta(1)
\end{cases}
\]  

(7.23)

where \( \psi(q) \) and \( \chi(q) \) are the same as those in Eq. (7.21), and

\[
\zeta(q) = \frac{1}{\xi} \int_{h_q}^{h_K} d_h(x) \, dx, \quad \eta(q) = \frac{1}{\xi} \left[ \overline{H}_1 + \int_{1}^{h_q} x \, d_h(x) \, dx \right]. 
\]

(7.24)
And \( h_K \) and \( h_q \) are defined, respectively, by

\[
\int_1^{h_K} d_h(x) \, dx = \xi \left( 1 - \chi(N_K) \right) - \bar{H}_1, \quad \int_1^{h_q} x \, d_h(x) \, dx = \frac{\xi}{\rho} - \bar{H}_1.
\]  

(7.25)

**Proof:** Using the definition of \( d_h \) and noting that \( \eta(k) - \eta(k-1) \to 0 \) as \( K \to \infty \), one can prove the theorem in a straightforward way.

### 7.5 Numerical Results and Discussions

The loss of number of common users due to the fluctuation of traffic among cells is illustrated in Figure 7.1. The numbers are normalized by the corresponding values of \( \xi \) (expected number of users/cell). The string of stars shows the ideal case discussed in Chapter 6. When \( \xi \) is infinitely large, user numbers in all cells are identical at any time. Therefore, the \( H_k \) is equal to the number of users \( (u_k) \) in each CRZ zone, and the total of expected common zone-\( h_k \) users is equal to \( \xi \). On the other hand, when \( \xi \) equals 100, the total number of common users is only about 80% of it. As \( \xi \) increases, \( H_k \) increases. When \( \xi \) reaches 500, more than 90% of users are common users.

Figure 7.2 displays the lower bound for the probability of assignment failure for some example TDMA/FDMA systems with IA-DCA strategies. Note that the system with the number of channels \( M = 415 \) is very close to the AMPS system with 12.5 MHz bandwidth \( (M = 416) \). The asymptotic lower bound obtained in Chapter 6 is also depicted for comparison.

From Figure 7.2, we can see that due to the traffic change among cells, performance of practical systems are not able to reach the performance of the asymptotic bound. When the number of channels in a system increases, the DCA schemes get larger performance space. As the result, the performance bound gets improved in some degree. However, as Figure 7.2 shows, this kind of improvement is limited.
Figure 7.1 Number of common users in CRZ zones as the function of traffic load.

Figure 7.2 Lower bound of $v$ with limited number of channels (two-way connection, deterministic model)
The results in this chapter are useful for estimating the maximum Erlang capacity of a TDMA/FDMA system when interference adaptation DCA is employed. For example, assuming a BS has unlimited number of transceivers and no sectorization is used, if the effect of shadow fading is not considered and the desired \( \nu \) is chosen as 2%, the maximum expected traffic an AMPS system (with IA-DCA) can handle is approximately 0.21 Erlang/channel/cell, or about 87 Erlang/cell. If we use FCA and assume channel reuse factor being 7, following the calculations in reference [84], the Erlang capacity of an AMPS system at \( P_b = 2\% \) is about 57 Erlang/cell. This gives IAMP 53% capacity enhancement over the FCA policy in the AMPS system. Note that the channel reuse factor corresponding to the parameters used in our calculation is 7.22 rather than 7. The capacity enhancement by IAMP should be above 55%. If we add 8% to 10% capacity increase resulted from the shadow fading effect, we have 95 Erlang/cell and about 70%.

For the IS-136 system, we have 832 carriers on each link. Each carrier is divided into 6 time slots. A user is assigned 2 slots at the same time. Therefore, each carrier can support 3 calls simultaneously. From Fig. 7.2, the curve corresponding to \( M = 830 \) gives \( \rho \approx 0.23 \) at \( \nu = 2\% \). As a result, the maximum Erlang capacity of the IS-136 system with IA-DCA in the shadowing case is about

\[
0.23 \times 832 \times 3 \times 1.09 = 626 \text{ (Erlang/cell)}.
\]

Following the calculations in reference [84], the Erlang capacity of an IS-136 system using FCA (at \( P_b = 2\% \)) is 342 (Erlang/cell). This means approximately 83% increase in the system capacity. Note that, in this example, we only use IA-DCA among carriers. More increase is expected if IA-DCA is used among time slots within the same carrier (in this case \( M = 832 \times 3 \)).
CHAPTER 8
THE PROBLEM OF DYNAMIC CHANNEL ALLOCATION WITH HETEROGENEOUS TRAFFICS

8.1 Problem Formulation

There exist two circuit-switching mobile subscriber classes in the system. Type-1 users require 1 duplex channel for each call at the cost of $w_1$. Type-2 users require $m(m \geq 2)$ duplex channels for each call at the cost of $w_2$.

Assume that the offered traffic is uniformly distributed in every cell for both user types. Type-1 traffic has Poisson arrival rate with a mean of $\lambda_1$ (calls/sec) in each cell. The call duration of type-1 user is exponentially distributed with a mean of $1/\mu_1$ (sec). Type-2 traffic has the same distribution type of arrival rate and call duration but with the means of $\lambda_2$ and $1/\mu_2$, respectively. Therefore, the offered type-1 traffic is a Poisson RV with mean $\xi_1 = \lambda_1/\mu_1$. It is the same for the type-2 traffic, but with $\xi_2 = \lambda_2/\mu_2$.

Define:

- $F_i$: the rate of failed (blocked and dropped) type-$i$ calls, $i = 1, 2$.
- $\nu_i \triangleq E[F_i]/\xi_i$: the probability of type-$i$ call failure, $i = 1, 2$.
- $\nu \triangleq (w_1 \nu_1 + w_2 \nu_2)/(w_1 + w_2) = (\nu_1 + w \nu_2)/(1 + w)$: combined probability of call failure, where $w = w_2/w_1$.
- $\nu^* = \lim_{\xi_i, M \to \infty} \nu$: the asymptotic combined probability of call failure.

In addition, we define the Combined Erlang Capacity as the combined offered traffic load $w_1 \xi_1 + w_2 \xi_2$ at which $\nu$ equals a pre-determined value.

Now the problem of dynamic channel allocation with two traffic classes can be defined as the following.
Definition 8.1 Assume that the serving BS and the transmitter power levels have been assigned to an arriving call. Given two different traffic types and their parameters \( (\xi_1, \xi_2, w_1, w_2, m) \) find a channel allocation algorithm which maximizes the combined Erlang capacity, under the constraint \((CIR)_p \geq \gamma, \forall p = 1, 2, ..., M\).

8.2 The Performance Bound of DCA Strategies with Two Traffic Types

The problem of finding the performance bound of DCA strategies with two different traffic types might be formulated as the following.

Given \( \xi_1, \xi_2, w_1, w_2, m, M \), and a certain propagation model, find the minimum value of \( \nu \) as a function of above parameters

\[
\nu_{\min} = f(\xi_1, \xi_2, w_1, w_2, m, M),
\]

where \( f(.) \) is determined by the propagation model and the DCA algorithm.

If we find \( \nu_{\min}^{*} \) instead of \( \nu_{\min} \), it is the asymptotic bound.

Similar to the method we used in the previous chapters, the performance bound for DCA schemes with two traffic types can be found through the following steps.

1) Define and represent the CIR constraint. We need to add possible influence of \( w_1, w_2, \) and \( m \).

2) According to the CIR calculation, we can find the expected number type-1 and type-2 users which each CRZ/ECRZ zone contains.

3) Modify the IAMP method. We definitely have to add some extra steps in order to deal with the new user type, just as we did for the zone-\( h_{K,t} \) remainders in the previous chapter.

4) Using the results in Steps 2) and 3), calculate the expected number of channels needed for each user type in each CRZ/ECRZ zone.

5) Find \( \nu_{\min} \) or much more easily, \( \nu_{\min}^{*} \).
In conclusion, the author foresees that, based upon our current work on the single-traffic case, the asymptotic analysis for the performance of IA-DCA with two circuit-switching traffic types is not as hard as the extension of the work from the asymptotic case to the limited-\(M\) case. However, more complicated cases cannot be predicted at this point, especially when packet-switching traffic and the queueing problem are involved.
CHAPTER 9
SUMMARY OF ACCOMPLISHED WORK

In this last chapter, we briefly summarize the work completed in this dissertation.

As we mentioned at the beginning of the previous chapter, in order to calculate any one of performance measures for a DCA algorithm, one has to deal with three factors: the interference factor, the traffic factor, and the DCA algorithm itself.

For the interference analysis of the IA-DCA strategy, we proposed two novel concepts in "the channel reuse zone" (CRZ) and "the extended channel reuse zone" (ECRZ). In addition, we defined a special CRZ/ECRZ structure called the concentric CRZ/ECRZ structure and showed that it represents the lowest co-channel interference pattern. The method of computing the concentric CRZ zones extends the conventional concept of channel reuse factor to an arbitrary real number larger than 1. Based on the new concepts, we found the way to compute the co-channel interference and the outage probability in the IA-DCA environment. This computation is carried out for two different propagation models.

In order to analyze the influence of the dynamics of traffic load on the IA-DCA strategy, we defined two concepts in "common users" and "common remainders". Furthermore, we derived an approximate expression for the expected value of common users and an exact expression for the expected value of common remainders.

On the aspect of the DCA algorithm, since our purpose is analyzing generally the whole class of IA-DCA, we proposed two optimal but impractical IA-DCA schemes in IAMP and IAMP-V2. These algorithms substantially reduce the complexity of the analysis, and give the upper performance bound of all of the IA-DCA algorithms.

The work in these three aspects provides us a sound basis for the derivation of lower bounds of the asymptotic probability of assignment failure ($\nu^*$) and the probability of assignment failure ($\nu$). These two bounds are significant in estimating
the design space between an existed heuristic IA-DCA algorithm and the upper performance bound. They are also useful in estimating the maximum enhancement the IA-DCA strategy can provide over the traditional fixed channel allocation method.
APPENDIX A
PROOF OF THEOREM 7.2

In Chapter 7, since \( u_k \) and \( C_k \) are all identically distributed in the cells, it is not necessary to use cell indices to distinguish \( u_k \) in cell \( i \) from \( u_k \) in cell \( j \) (the same as \( C_k \)). In this appendix, however, when we need to emphasize this distinction, we use \( u_{ki} \) (\( C_{ki} \)) to denote the value of \( u_k \) (\( C_k \)) in the cell \( i \). On the other hand, \( u_k \) and \( C_k \) are used to represent corresponding values which apply to every cell. In the following, we derive Eqs. (7.13) and (7.15) first, then prove Eq. (7.12).

For all \( i = 2, 3, ..., K \),

\[
Pr(C_{ki} = s) = Pr(u_{ki} + C_{(k-1)i} - H_{k-1} = s) = \sum_{x=0}^{s} Pr(u_{ki} = s-x) Pr(C_{(k-1)i} - H_{k-1} = x) = \sum_{x=0}^{s} [p_{s-x}(\Delta \xi_k) \sum_{y=0}^{\infty} P(C_{(k-1)i} = x+y, H_{k-1} = y)]. \tag{A.1}
\]

When \( x > 0 \),

\[
Pr(C_{ki} = x+y, H_k = y) = Pr(C_{ki} = x+y, C_k = y \text{ in 1 cell, } C_k > y \text{ in } N_k - 2 \text{ cells}) + Pr(C_{ki} = x+y, C_k = y \text{ in 2 cells, } C_k > y \text{ in } N_k - 3 \text{ cells}) + \cdots + Pr(C_{ki} = x+y, C_k = y \text{ in other } N_k - 1 \text{ cells})
\]

\[
= (N_k - 1) Pr(C_{ki} = x+y, C_{kj} = y, C_{kl} > y, \forall l \neq i, j) + \binom{2}{N_k - 1} Pr(C_{ki} = x+y, C_{kj} = y, C_{kp} = y, C_{kl} > y, \forall l \neq i, j, p) + \cdots + Pr(C_{ki} = x+y, C_{kj} = y, \forall j \neq i)
\]

\[
= Pr(C_k = x+y) \left[ (N_k - 1) Pr(C_k = y) Pr^{N_k-2}(C_k > y) + \binom{2}{N_k - 1} Pr^2(C_k = y) Pr^{N_k-3}(C_k > y) + \cdots + Pr^{N_k-1}(C_k = y) \right] = Pr(C_k = x+y) \left[ Pr^{N_k-1}(C_k > y) - Pr^{N_k-1}(C_k > y) \right]. \tag{A.2}
\]
When \( x = 0 \),

\[
P(C_{ki} = y, H_k = y) = P_r(C_{ki} = y, C_{kj} > y, \forall j \neq i) + (N_k - 1) P_r(C_{ki} = y, C_{kj} = y, C_{kl} > y, \forall l \neq i, j)
\]

\[
+ \binom{2}{N_k - 1} P_r(C_{ki} = y, C_{kj} = y, C_{kp} = y, C_{kl} > y, \forall l \neq i, j, p)
\]

\[
+ \cdots + P_r(C_{ki} = y, C_{kj} = y, \forall j \neq i)
\]

\[
= P_r(C_k = y) \left[ P_r^{N_k - 1}(C_k > y) + (N_k - 1) P_r(C_k = y) P_r^{N_k - 2}(C_k > y) + \cdots + P_r^{N_k - 1}(C_k = y) \right]
\]

\[
= P_r(C_k = y) P_r^{N_k - 1}(C_k = y).
\]  \hspace{1cm} (A.3)

Inserting Eqs. (A.2) and (A.3) into Eq. (A.1) yields Eq. (7.15). The derivation of Eq. (7.13) is straightforward as

\[
P_r(C_k > s) = \sum_{x=s+1}^{\infty} P(C_k = x) = 1 - \sum_{x=0}^{s} P(C_k = x), \hspace{1cm} (A.4)
\]

\[
P_r(C_k \geq s) = \sum_{x=s}^{\infty} P(C_k = x) = 1 - \sum_{x=0}^{s-1} P(C_k = x). \hspace{1cm} (A.5)
\]

The proof of Eq. (7.12) is similar to that of Eq. (A.2), except that in current case \( C_{ki} \) is not fixed.

\[
P_r(H_k = s)
\]

\[
= P_r(C_k = s \text{ in 1 cell, } C_k > s \text{ in } N_k - 1 \text{ cells})
\]

\[
+ P_r(C_k = s \text{ in 2 cells, } C_k > s \text{ in } N_k - 2 \text{ cells})
\]

\[
+ \cdots + P_r(C_k = s \text{ in } N_k \text{ cells})
\]

\[
= N_k P_r(C_{ki} = s, C_{kj} > s, \forall j \neq i) + \binom{2}{N_k} P_r(C_{ki} = s, C_{kj} = s, C_{kl} > s, \forall l \neq i, j)
\]

\[
+ \cdots + P_r(C_{ki} = s, \forall i)
\]

\[
= N_k P_r(C_k = s) P_r^{N_k - 1}(C_k > s) + \binom{2}{N_k - 1} P_r^2(C_k = s) P_r^{N_k - 2}(C_k > s)
\]

\[
+ \cdots + P_r^{N_k}(C_k = s)
\]
\[ \begin{align*}
= & \left[ P_r(C_k = y) + P_r(C_k > y) \right]^N_k - P_r^{N_k}(C_k > y) \\
= & P_r^{N_k}(C_k \geq s) + P_r^{N_k}(C_k > s). 
\end{align*} \]

(A.6)
APPENDIX B

PROOF OF THEOREM 7.3

The derivation of Theorem 7.3 is a typical inductive process. Here we only describe the way we find the probability of $H_{K,N_{K-t}} = s$. Note that

$$P_r(H_{K,N_{K-t}} = s) = \sum_{m=1}^{N_{K-t}} P_r(C_{K,N_{K-t}} = 0 \text{ in } t \text{ cells, } C_{K,N_{K-t}} = s \text{ in } m \text{ cells,}}$$

$$C_{K,N_{K-t}} > s \text{ in } N_{K-t-m} \text{ cells).}$$  \hspace{1cm} (B.1)

Eq. (B.1) includes probabilities of all possible assignments all the way back to the assignment of common zone-$h_{K}$ users. If we put all of the probabilities for $C_{Ki} = x$, $\forall i = 1, ..., N_{K}$, $x = 0, ..., s_m$ in layer 1, $C_{Ki,N_{K-1}} = x$, $\forall i, x$ in layer 2, etc., and $P_r(H_{K,N_{K-t}} = s)$ in the bottom layer, we have to count all of the possible ways from one point in layer 1 (a vector $[C_{K1}, C_{K2} \ldots C_{K(N_{K})}]$) to the bottom layer and sum them up. It may have only one step from layer 1 down to the bottom. This gives the term$_1$ in Eq. (7.18). It may consist of two steps, e.g. from one point in layer 1 to one point in layer 2 ($[C_{K1,N_{K-1}}, C_{K2,N_{K-1}} \ldots C_{K(N_{K},N_{K-1})}]$) then to the bottom. This gives the term$_2$ in Eq. (7.18). One may get term$_3$, term$_4$, etc., by continuing on this procedure. Apparently, the larger $t$ becomes, the more possible roads from top to bottom have to be counted. Fortunately, there is a clear path to follow and this path yields the form of Eq. (7.18).
APPENDIX C
PROOF OF THEOREM 7.4

The average number of common zone-$h_k$ users in the system is $\bar{H}_k V$. The maximum number of zone-$h_k$ users a channel supports is $V/h_k$. Hence, the average number of channels needed to support zone-$h_k$ users is $h_k \bar{H}_k$.

For zone-$h_K$ users and remainders, one zone-$h_K$ channel can carry $V/h_K$ common zone-$h_k$ users or $V t/h_K N_K$ common zone-$h_{K,t}$ remainders. Therefore, the average number of zone-$h_K$ channels needed is

$$h_K \bar{H}_K + \sum_{t=1}^{N_K-1} \frac{h_K N_K \bar{H}_{K,t}}{t}.$$

Thus, total number of channels needed

$$= \sum_{k=1}^{K} h_k \bar{H}_k + h_K N_K \sum_{t=1}^{N_K-1} \frac{\bar{H}_{K,t}}{t} = \xi [\eta(K) + h_K N_K \psi(1)].$$

Due to the ordering of channel packing in the IAMP algorithm, a zone-$h_k$ user can not fail unless all zone-$h_{l_t}(l > k)$ users have already failed. Similarly, due to the ordering of channel packing in the auxiliary algorithm, a zone-$h_{K,t}$ remainder can not fail unless all zone-$h_{K,t-1}$ remainders have failed. Therefore, when the number of available channels $M \geq \xi [\eta(K) + h_K N_K \psi(1)]$, $\nu_{\text{min}} = 0$. When

$$\xi [\eta(K) + h_K N_K \psi(1)] > M \geq \xi [\eta(K) + h_K N_K \psi(2)],$$

the expected number of failed users

$$E[F] = (\text{zone-$h_K$ channel deficit})(\# \text{ of zone-$h_K$ remainders/channel})$$

$$= \left( \sum_{k=1}^{K} h_k \bar{H}_k + h_K N_K \sum_{t=1}^{N_K-1} \frac{\bar{H}_{K,t}}{t} - M \right) \frac{V}{h_K N_K}$$

$$= V \left\{ \frac{1}{h_K N_K} \left( \sum_{k=1}^{K} h_k \bar{H}_k - M \right) + \sum_{t=1}^{N_K-1} \frac{\bar{H}_{K,t}}{t} \right\}.$$

Hence,

$$\nu_{\text{min}} = \frac{E[F]}{\xi V} = \frac{1}{h_K N_K} \left[ \eta(K) - \frac{1}{\rho} \right] + \psi(1) + \chi(1).$$

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When

\[ \xi [\eta(K) + h_K N_K \psi(q)] > M \geq \xi [\eta(K) + h_K N_K \psi(q + 1)], \quad q = 2, 3, \ldots, N_K - 1, \]

\[
E[F] = \left( \sum_{k=1}^{K} h_k \overline{H}_k + h_K N_K \sum_{t=q}^{N_K-1} \frac{\overline{H}_{K,t}}{t} - M \right) \frac{V}{h_K N_K} + \sum_{t=1}^{q-1} V \overline{H}_{K,t}
\]

\[
= V \left\{ \frac{q}{h_K N_K} \left( \sum_{k=1}^{K} h_k \overline{H}_k - M \right) + q \sum_{t=q}^{N_K-1} \frac{\overline{H}_{K,t}}{t} + \sum_{t=1}^{q-1} \overline{H}_{K,t} \right\}.
\]

Hence,

\[
\nu_{\text{min}} = \frac{q}{h_K N_K} \left[ \eta(K) - \frac{1}{\rho} \right] + q \psi(q) + \chi(q).
\]

For the case where \( \xi \eta(K) > M \geq \xi \eta(K - 1) \),

\[
E[F] = \left( \sum_{k=1}^{K} h_k \overline{H}_k - M \right) \frac{V}{h_K} + V \sum_{t=1}^{N_K-1} \overline{H}_{K,t}
\]

\[
= V \left\{ \frac{1}{h_K} \left( \sum_{k=1}^{K} h_k \overline{H}_k - M \right) + \overline{H}_K + \sum_{t=1}^{N_K-1} \overline{H}_{K,t} \right\}.
\]

Hence,

\[
\nu_{\text{min}} = \frac{1}{h_K} \left[ \eta(K - 1) - \frac{1}{\rho} \right] + \eta(K) + \chi(N_K).
\]

Since the number of zone-\( h_{K,t} \) remainders has been dealt with, the rest of the proof follows exactly the same way as the proof of Theorem 6.2, except some obvious changes on the notation (\( A_k/A_c \rightarrow \overline{H}_k \)).
REFERENCES


