ABSTRACT

KINEMATIC ANALYSIS OF A SWASH-PLATE MECHANISM USING TRANSFORMATION MATRICES

by
David Edgar Evans

The swash-plate mechanism is used to transfer rotational motion into translational motion and vice versa. Using transformation matrices to describe each joint, the displacements of the swash-plate mechanism were analyzed. The general transformation matrix model for the plane joint, not previously developed using transformation matrices, was presented. The solution was tested using a computer program. Examples of three different swash-plate angles; 10, 45, and 25° were considered. The solution was found to agree with previous solutions while offering a more complete description of the mechanism. Future work in this area could include a complete static, velocity, and/or dynamic analysis of the swash-plate mechanism based on the solution presented.

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by
David Edgar Evans

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BIOGRAPHICAL SKETCH

Author: David Edgar Evans

Degree: Master of Science in Mechanical Engineering

Date: January 1998

Undergraduate and Graduate Education:

- Master of Science in Mechanical Engineering,
  New Jersey Institute of Technology, Newark, NJ
  1998

- Bachelor of Science in Mechanical Engineering,
  New Jersey Institute of Technology, Newark, NJ
  1995

Major: Mechanical Engineering

APPROVAL PAGE

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by
David Edgar Evans

Dr. Ian S. Fischer, Thesis Advisor — Date
Associate Professor of Mechanical Engineering,
New Jersey Institute of Technology

Dr. R. Sodhi, Committee Member — Date
Associate Professor of Mechanical Engineering,
New Jersey Institute of Technology

Dr. K.A. Narh, Committee Member — Date
Assistant Professor of Mechanical Engineering,
New Jersey Institute of Technology

This thesis is dedicated to
Lois Speiden, M.D.
and
Edgar Speiden

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CHAPTER 1

INTRODUCTION

The swash-plate mechanism is comprised of four links and four joints: The ground link is connected to the shaft/swash link by a revolute joint (R). The shaft/swash link is mated with the coupler by a plane joint (E). The coupler is connected to the follower (also referred to as a piston) with a ball joint (S), and the follower (piston) is mated with the ground link by a prismatic joint (P). The mechanism is defined by its joints as an RESP mechanism.

The appeal of the swash-plate mechanism is that its pistons and shaft of rotation are parallel, thus facilitating a compact design with low weight and high-energy transfer efficiency. The configuration is robust because it is easily possible to completely balance the mechanism by choosing a thickness of the angled plane to offset the momentum of the pistons when there are three or more odd number of pistons as observed by Galin and Harris. [3]

The swash-plate mechanism has been described previously by Galin and Harris [3] and Mitchell [11] using an old style methodology. While these previous solutions were effective in finding a specific solution for an output translation given an input rotation, they did not fully describe the displacements and/or rotations of all the links in the mechanism before proceeding with the dynamic and/or force analysis.

This work completely describes the mechanism using dual number transformation matrices. In considering the kinematics of this mechanism, a plane (E) joint is modeled for the first time using transformation matrices. Transformation matrix solutions can now be utilized to describe other mechanisms with plane and/or spherical joints, such as the Rzeppa constant-velocity coupling, which may have no other current means of exhaustive solution.

CHAPTER 2

PREVIOUS WORK AND SCOPE

James Watt invented the swash-plate mechanism in the late nineteenth century. The purpose of the mechanism was to translate rotating motion into reciprocating motion or reciprocating motion into rotating motion. It's compact and simple design made the swash-plate appealing to designers at the time it was invented. However, Maki and DeHart report that it was not until the 1920's that the mechanism was rendered operational enough to be considered a viable alternative to the more widely used reciprocating/rotating mechanisms.[1]

In the early 1920's, A. G. M. Mitchell made the first application of the swash-plate mechanism that could compete with the crank and connecting rod mechanism. [11] According to Galin and Harris, Mitchell's first application for the swash-plate mechanism was an air compressor. [2] However, the swash-plate was soon used in an internal combustion engine for airplanes. Other applications for the swash plate include gas compressors and air motors. Its most notable application (because of shear volume of production) is in the automotive air-conditioning compressor. Weibel and Mantey provide an elegant description of the swash-plate mechanism, as it is used in the automotive air-conditioning compressor. [6] Because of the mechanism's wide range of utilization, it is important that its kinematics be well defined for reference of machine designers.

Using transformation matrices, the equation that describes the position of the piston in a swash-plate mechanism will be derived. The equation should be of such form that the output displacement s_4 is a sole function of the input variable θ_1 and the mechanism's dimensions.

To properly design a balanced swash-plate type mechanism, the thickness of the swash plate can be calculated to produce a balancing moment. The balancing moment counteracts the moment produced by the cyclical force of the pistons reciprocating at a distance from the center of the plate. This configuration of swash-plate/piston balance only works for a three or more piston mechanism. Only one piston is shown in Figure 1.

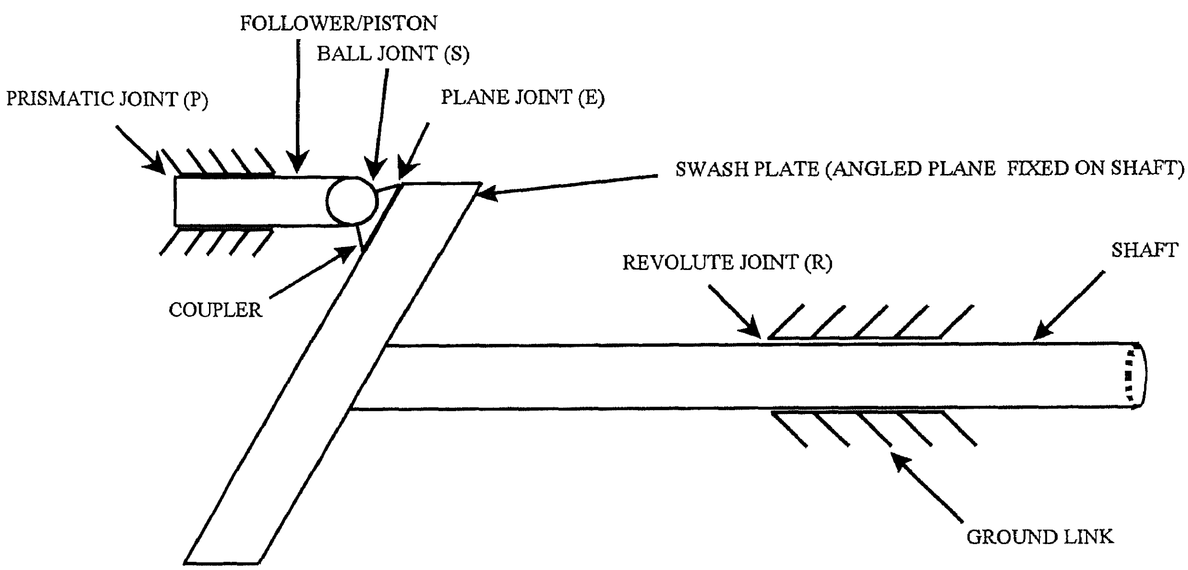

Figure 1

Swash Plate Mechanism with One Follower/Piston Shown

Before the moment produced by the reciprocating pistons can be found, the position equation for one piston assembly must be calculated relative to the input angle of the shaft. Let us call that the first position equation. Using the first position equation, a general expression can be made to describe the position of any piston in the assembly. Also, the velocity and change of momentum equations can be found by taking the first and second derivatives of the position equation respectively.

The position equation describes the displacement of a designated piston from a defined origin based on a given shaft angle of rotation. The only variable in the first displacement equation should be the shaft angle of rotation θ_1.

The kinematic solution for a swash-plate mechanism will be developed as follows:

In chapter three, the transformation matrix method of solving the mechanism will be explained (including a full solution of a generalized plane joint). In the same chapter, the variables used in the calculation and model of the mechanism will also be presented and explained. In chapter four, the solution of the basic model will be presented. Multiple iterations of data output will be listed and graphed. Finally, in chapter five, the results of the analysis will be discussed, recommendations for the use of the data in mechanism design will be made, future work using the general solution will be highlighted and conclusions based on the presented solution will be drawn.

CHAPTER 3

ANALYSIS

3.1 Description of Methodology

The motion of the swash plate mechanism is modeled using coordinate transformation matrices. Therefore, the essential elements of the Transformation Method will now be discussed.

Clifford introduced the concept of the dual number in 1873 and the name of "dual number" was coined by Study in 1903. The definition of the dual number (ε) is:

$$\begin{aligned} &\varepsilon \neq 0 \\ &\varepsilon^2 = 0 \end{aligned} \tag{1}$$

In analogy with complex numbers, a duplex number can be written as:

$$\hat{a} = a + \varepsilon(a_o) \tag{2}$$

Where:

a = primary (or real) component of $\hat{a}$
a_o = dual component of $\hat{a}$

Dual numbers follow all the rules of regular algebra keeping in mind that a dual number raised to any power is equal to zero.

A dual angle can be used to specify the relative displacement and orientation between two lines in space.

The dual angle is written as:

$$\hat{\theta} = \theta + \varepsilon(s) \tag{3}$$

Where θ is the primary rotational part (projected angle between the lines) and s is the dual translation part (shortest distance between the lines) of the dual angle $\hat{\theta}$.

The trigonometric functions of dual angles are obtained by using Taylor Series expansions:

$$\begin{aligned}\sin\hat{\theta} &= \sin\theta + \varepsilon(s)\cos\theta \\ \cos\hat{\theta} &= \cos\theta - \varepsilon(s)\sin\theta \\ \tan\hat{\theta} &= \tan\theta + \varepsilon(s)\sec^2\theta\end{aligned} \tag{4}$$

All trigonometric identities hold for dual trigonometric functions.

Using coordinate transformation matrices, dual numbers can be used to describe a mechanism. This is accomplished by describing the succession of joints and links found in the mechanism with the descriptive transformation matrix of each joint. Then, the joint parameters used are determined and shown in a Denavit-Hartenberg table. Finally, the mechanism displacements are evaluated by performing a series of transformations through the joints and links specific to the mechanism. In this problem, the mechanism being considered is an RESP (Revolute, Plane, Spherical, and Prismatic) swash-plate mechanism. There are four joints and four links in the swash-plate mechanism.

. For the solution presented, all transformations are relative to moving coordinate systems. Therefore the order of multiplication will be to the right.

The first joint encountered in the swash-plate mechanism is the revolute joint. The revolute, cylindrical and prismatic joints can each be described in general as a transformation through the dual angle $\hat{\theta}_n = \theta_n + \varepsilon(s_n)$ about the z-axis followed by a dual angle transformation about the x-axis through the angle $\hat{\alpha}_n = \alpha_n + \varepsilon(a_n)$. In the case of the revolute joint, there is one degree of rotational freedom. Also, in the prismatic joint there is one translational degree of freedom along the shaft.

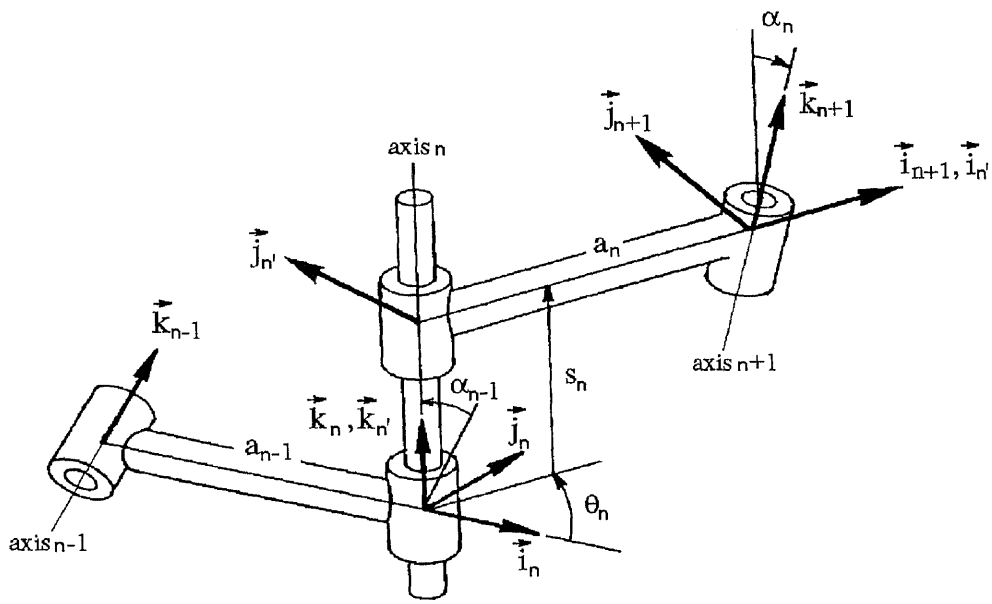

Figure 2

General Transformations for a Revolute/Prismatic Joint
(Drawn by Dr. Ian S. Fisher)

Note that expression (5) represents the relation of frame {n} fixed on the distal end of link "n-1" to frame {n+1} fixed on the distal end of link "n". Letting a matrix $\hat{\mathbf{M}}$ represent the general form of the revolute/prismatic transformation, this is expressed as follows:

Note: s = sine, c = cosine

$$ {}_{n+1}^{\;\;n}\hat{\mathbf{M}} = \hat{\mathbf{Z}}(\hat{\theta}_n)\hat{\mathbf{X}}(\hat{\alpha}_n) = \begin{bmatrix} c\hat{\theta}_n & -s\hat{\theta}_n & 0 \\ s\hat{\theta}_n & c\hat{\theta}_n & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\hat{\alpha}_n & -s\hat{\alpha}_n \\ 0 & s\hat{\alpha}_n & c\hat{\alpha}_n \end{bmatrix} \tag{5} $$

Multiplying these matrices and separating the primary and dual components gives:

$$
{}^{n}_{n+1}\hat{\mathbf{M}}(\hat{\alpha}_n,\hat{\theta}_n) = \begin{bmatrix} c\theta_n & -c\alpha_n s\theta_n & s\alpha_n s\theta_n \\ s\theta_n & c\alpha_n c\theta_n & -s\alpha_n c\theta_n \\ 0 & s\alpha_n & c\alpha_n \end{bmatrix}
$$

$$
+\varepsilon \begin{bmatrix} -s_n s\theta_n & a_n s\alpha_n s\theta_n - s_n c\theta_n & a_n c\alpha_n s\theta_n + s_n s\alpha_n c\theta_n \\ s_n c\theta_n & -a_n s\alpha_n c\theta_n - s\alpha_n c\alpha_n s\theta_n & -a_n c\alpha_n c\theta_n + s_n s\alpha_n s\theta_n \\ 0 & a_n c\alpha_n & -a_n s\alpha_n \end{bmatrix} \tag{6}
$$

The next joint encountered in the swash-plate is the plane joint denoted by E after the German word for plane, ebene (pronounced ehb-ih-nah). This joint has three degrees of freedom; two in translation and one in rotation. The transformation that describes the general form of the plane joint consists of a translation $\hat{r}_n = 0 + \varepsilon(r_n)$ about the x-axis followed by a translation $\hat{s}_n = 0 + \varepsilon(s_n)$ along the y-axis followed by a rotation $\hat{\theta}_n = \theta_n + \varepsilon(0)$ about the z-axis, and concluding with a translation $\hat{a}_n = 0 + \varepsilon(a_n)$ about the x-axis.

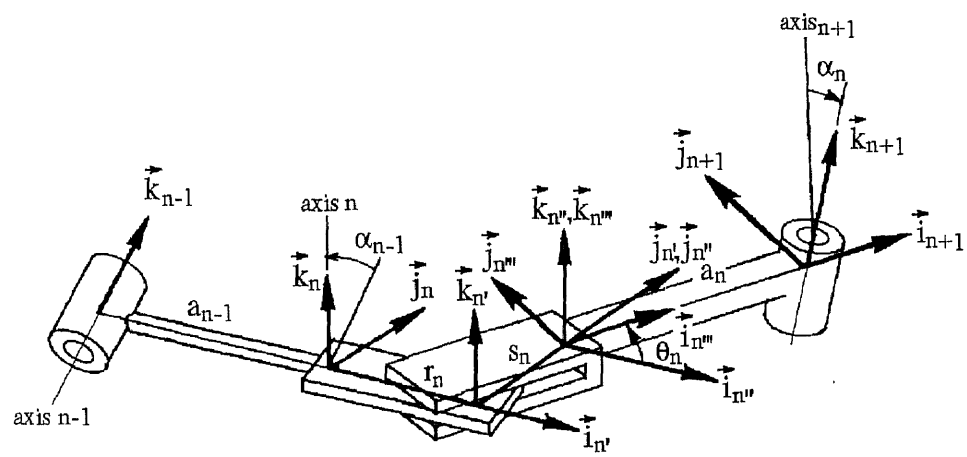

Figure 3

General Transformations for a Plane Joint
(Drawn by Dr. Ian S. Fisher)

Start at system {i} fixed on the distal end of link n-1. Translate distance r_n along the i_n-axis. Translate distance s_n in the direction of the j_n-axis. Then rotate through θ_n about $k_{n'}$-axis into alignment with system {n'}. Then make a screw motion through translation a_n and rotation α_n into alignment with frame {n+1} on the distal end of link n.

Let $\hat{\mathbf{P}}$ represent the general form of the plane joint:

Note: c = cosine, s = sine

$$
\begin{aligned}
{}^{n}_{n+1}\hat{\mathbf{P}} &= \hat{\mathbf{X}}(\hat{r}_n)\hat{\mathbf{Y}}(\hat{s}_n)\hat{\mathbf{Z}}(\hat{\theta}_n)\hat{\mathbf{X}}(\hat{a}_n) \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\hat{r}_n & -s\hat{r}_n \\ 0 & s\hat{r}_n & c\hat{r}_n \end{bmatrix}
\begin{bmatrix} c\hat{s}_n & 0 & s\hat{s}_n \\ 0 & 1 & 0 \\ -s\hat{s}_n & 0 & c\hat{s}_n \end{bmatrix}
\begin{bmatrix} c\hat{\theta}_n & -s\hat{\theta}_n & 0 \\ s\hat{\theta}_n & c\hat{\theta}_n & 0 \\ 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\hat{a}_n & -s\hat{a}_n \\ 0 & s\hat{a}_n & c\hat{a}_n \end{bmatrix}
\end{aligned}
\tag{7}
$$

Multiplying these matrices and separating the primary from the dual components gives:

$$
{}^{n}_{n+1}\hat{\mathbf{P}}(\hat{r}_n,\hat{s}_n,\hat{\theta}_n,\hat{a}_n) =
\begin{bmatrix} c\theta_n & -s\theta_n & 0 \\ s\theta_n & c\theta_n & 0 \\ 0 & 0 & 1 \end{bmatrix} + \varepsilon \begin{bmatrix} 0 & 0 & a_n s\theta_n + s_n \\ 0 & 0 & -a_n c\theta_n - r_n \\ r_n s\theta_n - s_n c\theta_n & r_n c\theta_n + s_n s\theta_n + a_n & 0 \end{bmatrix} \tag{8}
$$

The final joint in need of description for this mechanism is a spherical or ball joint. This joint has three degrees of rotational freedom about all three axes. The general form of transformations for this mechanism will be denoted as $\hat{\mathbf{L}}$. Because there are no natural axes, the motions of the ball joint are expressed with respect to arbitrarily chosen axis. The order of motion is:

First, a screw motion about the z-axis $\hat{\theta}_n = \theta_n + \varepsilon(s_n)$ followed by a screw motion about the y-axis $\hat{\eta}_n = \eta_n + \varepsilon(N_n)$, then a screw motion about the x-axis $\hat{\zeta}_n = \zeta_n + \varepsilon(Z_n)$.

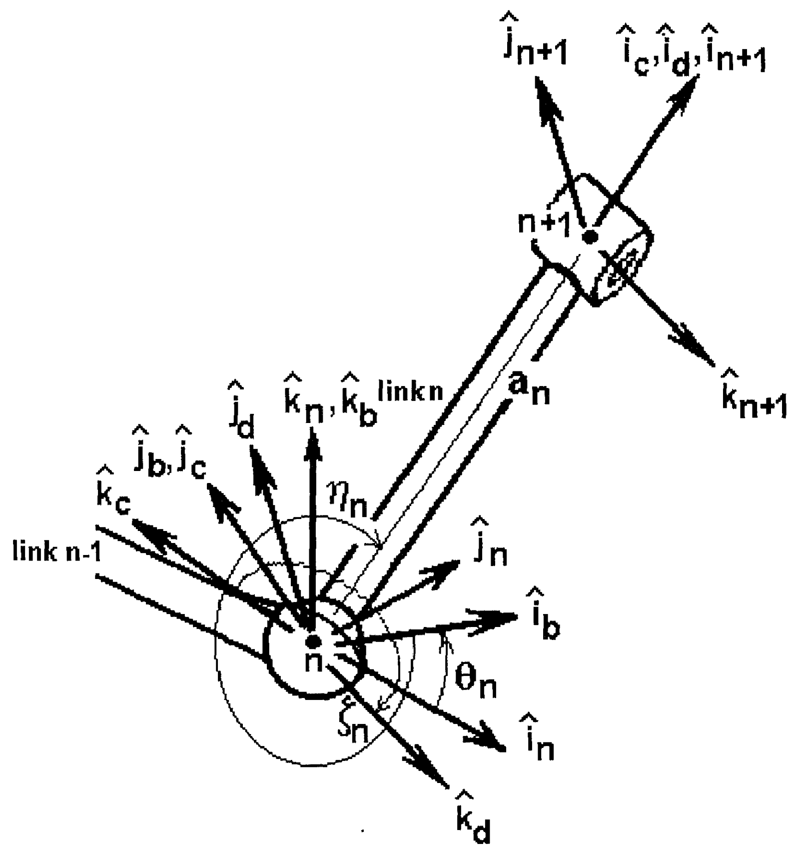

Figure 4

General Transformations for a Spherical Joint
(Drawn by Dr. Ian S. Fisher)

The expression represents transformation from frame {n} fixed on link "n-1" to frame {n+1} fixed on the distal end of link "n".

Let $\hat{\mathbf{L}}$ represent the general form of the plane joint:

Note: c = cosine, s = sine

$$
{}^{n}_{n+1}\hat{\mathbf{L}} = \hat{\mathbf{Z}}(\hat{\theta}_n)\hat{\mathbf{Y}}(\hat{\eta}_n)\hat{\mathbf{X}}(\hat{\zeta}_n)\hat{\mathbf{X}}(\hat{\alpha}_n)
$$

$$
= \begin{bmatrix} c\hat{\theta}_n & -s\hat{\theta}_n & 0 \\ s\hat{\theta}_n & c\hat{\theta}_n & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\hat{\eta}_n & 0 & s\hat{\eta}_n \\ 0 & 1 & 0 \\ -s\hat{\eta}_n & 0 & c\hat{\eta}_n \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\hat{\zeta}_n & -s\hat{\zeta}_n \\ 0 & s\hat{\zeta}_n & c\hat{\zeta}_n \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\hat{\alpha}_n & -s\hat{\alpha}_n \\ 0 & s\hat{\alpha}_n & c\hat{\alpha}_n \end{bmatrix} \qquad (9)
$$

Multiplying these matrices and separating the primary from the dual components gives:

$$
\begin{aligned}
&{}_{n+1}^{\;\;n}\hat{\mathbf{L}}(\hat{\theta}_n,\hat{\eta}_n,\hat{\zeta}_n,\hat{\alpha}_n)\\
&=\begin{bmatrix}
c\eta_n c\theta_n & s\eta_n c\theta_n c\zeta_n - s\theta_n c\zeta_n & s\eta_n c\theta_n c\zeta_n + s\theta_n s\zeta_n\\
c\eta_n s\theta_n & s\eta_n s\theta_n s\zeta_n + c\theta_n c\zeta_n & s\eta_n s\theta_n c\zeta_n - c\theta_n s\zeta_n\\
-s\eta_n & c\eta_n s\zeta_n & c\eta_n c\zeta_n
\end{bmatrix}\\
&+\varepsilon\begin{bmatrix}
\begin{array}{l} s_n c\eta_n s\theta_n \\ - e_n s\eta_n c\theta_n \end{array} &
\begin{array}{l} e_n c\eta_n c\theta_n s\zeta_n \\ + s\eta_n c\theta_n (z_n + a_n) c\zeta_n \\ - s_n s\eta_n s\theta_n s\zeta_n \\ - s_n c\theta_n c\zeta_n \\ + s\theta_n (z_n + a_n) s\zeta_n \end{array} &
\begin{array}{l} e_n c\eta_n c\theta_n c\zeta_n \\ - s\eta_n (z_n + a_n) s\zeta_n \\ - s\eta_n s_n s\theta_n c\zeta_n \\ + s_n c\theta_n s\zeta_n \\ + s\theta_n (z_n + a_n) c\zeta_n \end{array}\\[2ex]
\begin{array}{l} s_n c\eta_n c\theta_n \\ - e_n s\eta_n s\theta_n \end{array} &
\begin{array}{l} e_n c\eta_n s\theta_n s\zeta_n \\ + s\eta_n s_n c\theta_n s\zeta_n \\ + s\eta_n s\theta_n (z_n + a_n) c\zeta_n \\ - c\theta_n (z_n + a_n) s\zeta_n \\ - s_n s\theta_n c\zeta_n \end{array} &
\begin{array}{l} e_n c\eta_n s\theta_n c\zeta_n \\ + s\eta_n s_n c\theta_n c\zeta_n \\ + s\eta_n s\theta_n (z_n + a_n) s\zeta_n \\ - c\theta_n (z_n + a_n) c\zeta_n \\ + s_n s\theta_n s\zeta_n \end{array}\\[2ex]
- e_n c\eta_n &
\begin{array}{l} c\eta_n (z_n + a_n) c\zeta_n \\ - e_n s\eta_n s\zeta_n \end{array} &
\begin{array}{l} c\eta_n (z_n + a_n) s\zeta_n \\ - e_n s\eta_n c\zeta_n \end{array}
\end{bmatrix}
\end{aligned}
\qquad (10)
$$

3.2 Definition of Variables

Table 1 Variables

VARIABLE	DESCRIPTION OF VARIABLES
θ_1	Input shaft rotation in revolute joint
α_1	swash-plate twist about i_1-axis
r_2	plane joint translation in j_2-axis direction
s_2	plane joint translation in i_2-axis direction
θ_2	plane-joint Rotation about k_2-axis
η_3	ball-joint rotation about j_3-axis
ζ_3	ball-joint rotation about i_3-axis
s_4	cylindrical sliding along k_4-axis (piston oscillation)
c_4	frame offset along i_4-axis (perpendicular distance from geometric axis of the swash plate to the center of the piston)
α_4	frame twist about i_4-axis

3.3 Formulation of Solution

The first step in the transformation matrix solution process is defining and arranging the parameters that define the swash-plate mechanism. The arrangement of the joint variables and link dimensions in a table of parameters as suggested by Denavit and Hartenberg [13] is shown in Table 1.

Table 2 Denavit-Hartenburg Parameter Table

Index	Name	θ_i	s_i	α_i	$r_i c_i$	η_i	N_i	ζ_i	Z_i
1	Input Shaft	θ_1	0	α_1	0	0	0	0	0
2	Swash-Plate	θ_2	s_2	0	r_2	0	0	0	0
3	Ball coupler	0	0	0	0	η_3	0	ζ_3	0
4	Follower	0	s_4	α_4	c_4	0	0	0	0

In the solution of this problem, a pictorial representation mixed with a mathematical description will be used. The order of solution will be as follows: First, the position of the coordinate system on the swash plate at each transformation will be shown with the corresponding transformation matrix. Second, the transformation matrices will be arranged according to their order of multiplication. Thirdly, the matrices will be rearranged for the solution. Fourthly and finally, mathematical manipulation software (Derive) will be used to multiply the matrices together and then set their components equal to each other for the final solution equation.

The starting point of the swash plate transformations lies on the neutral axis. The neutral axis can be thought of as a line lying on the face of the swash-plate that is perpendicular to the input shaft regardless of the swash-plate angle. The piston/follower assembly is at mid-stroke when the center of the coupler contacts the swash-plate at the neutral axis. See figures 5 and 6.

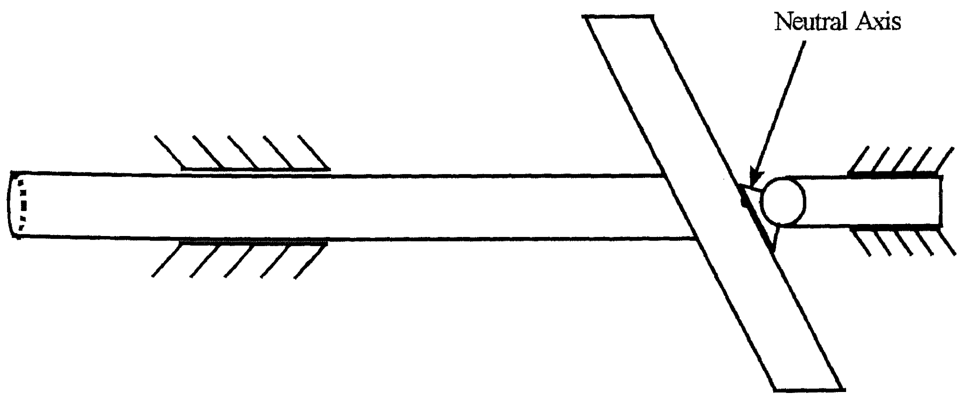

Figure 5

Top View of Swash-Plate Mechanism Showing Neutral Axis as a Point

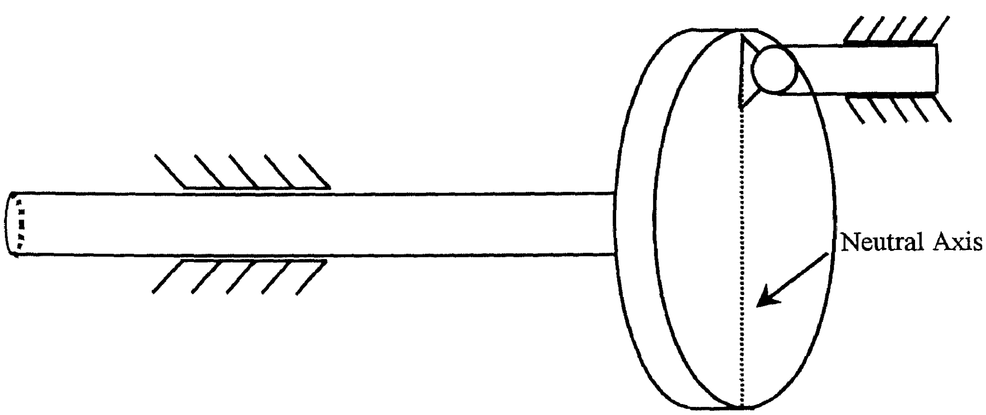

Figure 6

Side View of Swash-Plate Mechanism Showing Neutral Axis

3.4 Swash Plate Transformations

3.4.1 Transformation #1

For this transformation, the input variable θ_1 is shown as a rotation about the k-axis. Note the intermediate position of the coordinate axis denoted i', j' and k'. The constant swash-angle α_1 about the i-axis is also shown. These transformation matrices will be represented as:

$$\hat{\mathbf{M}}(\hat{\theta}_1, \hat{\alpha}_1) = \hat{\mathbf{Z}}(\theta_1)\hat{\mathbf{X}}(\alpha_1) \tag{11}$$

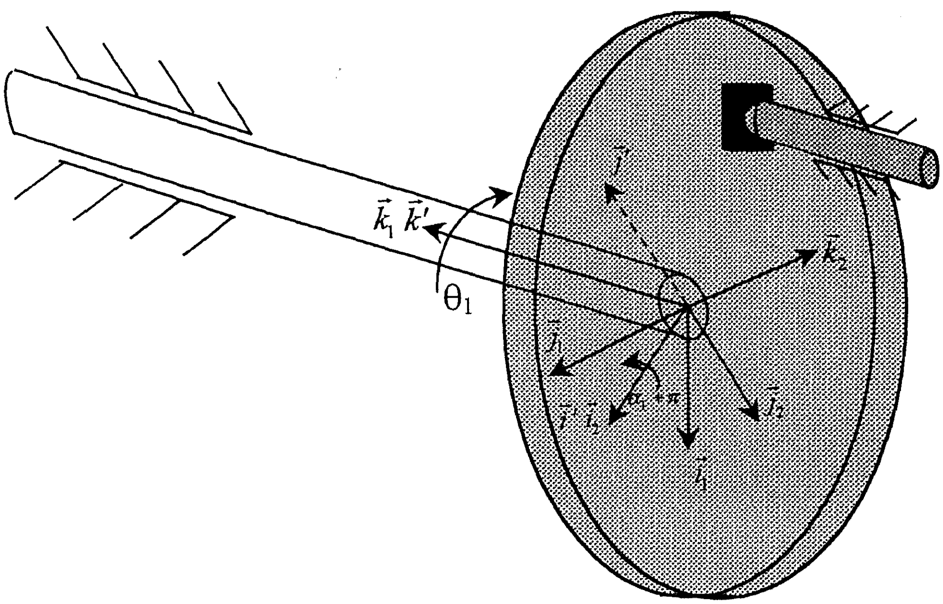

Figure 7

Swash-Plate Showing Transformation #1

3.4.2 Transformation #2

For this transformation, the plane joint is described with the intermediate variable translations of r_2 and s_2 along the i-axis and j-axis respectively. Also described is the intermediate variable transformation θ_2 acting about the k-axis. This transformation is shown as follows:

$$\hat{\mathbf{P}}(\hat{r}_2, \hat{s}_2, \hat{\theta}_2) = \hat{\mathbf{X}}(r_2)\hat{\mathbf{Y}}(s_2)\hat{\mathbf{Z}}(\theta_2) \tag{12}$$

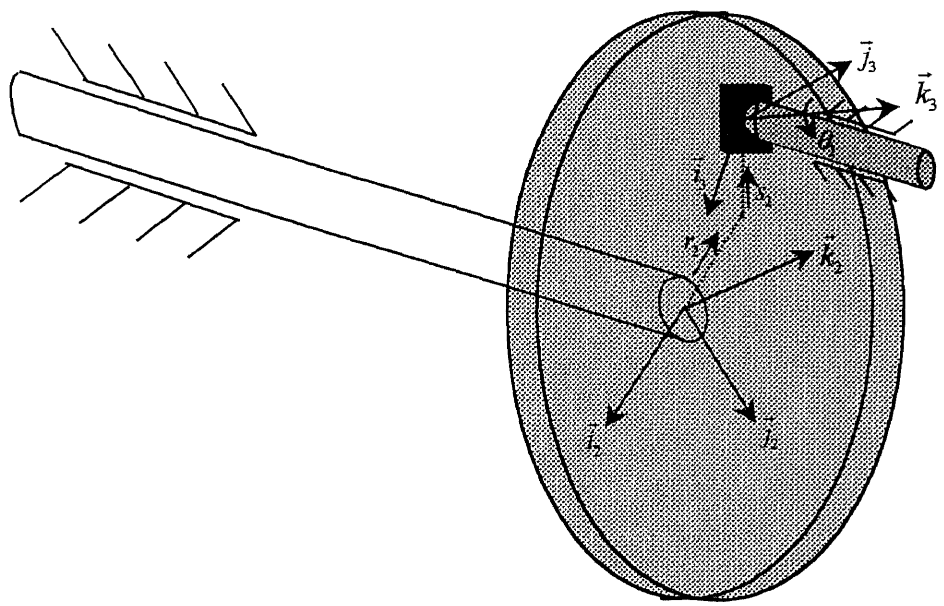

Figure 8

Swash-Plate Showing Transformation #2

3.4.3 Transformation #3

For this transformation, the ball or spherical joint will be described using the intermediate rotational variables η_3 and ζ_3 about the j-axis and i-axis respectively. The transformation is shown as follows:

$$\hat{\mathbf{L}}(\hat{\zeta}_3, \hat{\eta}_3) = \hat{\mathbf{Y}}(\eta_3)\hat{\mathbf{X}}(\zeta_3) \tag{13}$$

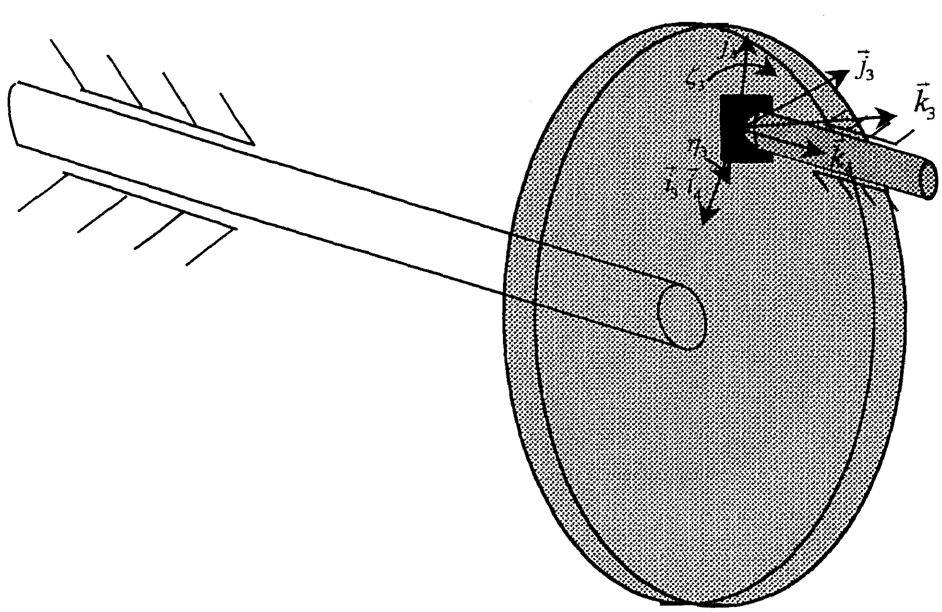

Figure 9

Swash-Plate Showing Transformation #3

3.4.4 Transformation #4

In this transformation, the prismatic joint is described by a translation through the output variable s_4 along the k-axis. There is also a screw motion through intermediate variable α_4 and constant c_4 about the i-axis. The coordinate transformation is as follows:

$$\hat{\mathbf{M}}(\alpha_4, c_4, s_4) = \hat{\mathbf{Z}}(s_4)\hat{\mathbf{X}}(\alpha_4, c_4) \tag{14}$$

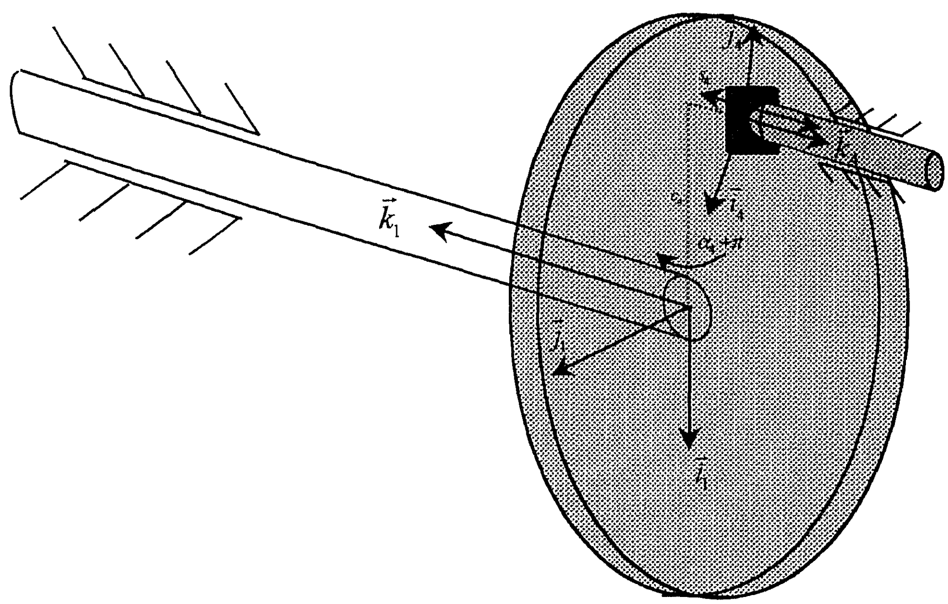

Figure 10

Swash-Plate Showing Transformation #4

Multiplying these four transformations together forms one transformation equation:

$$\hat{\mathbf{M}}(\theta_1, \alpha_1)\hat{\mathbf{P}}(r_2, s_2, \theta_2)\hat{\mathbf{L}}(\eta_3, \zeta_3)\hat{\mathbf{M}}(\alpha_4, c_4, s_4) = \hat{\mathbf{I}} \tag{15}$$

For purposes of the solution, the input variables from the output variables are segregated before performing the multiplication:

$$\hat{\mathbf{P}}(r_2, s_2, \theta_2)\hat{\mathbf{L}}(\eta_3, \zeta_3) = \hat{\mathbf{M}}^{-1}(\theta_1, \alpha_1)\hat{\mathbf{M}}^{-1}(\alpha_4, c_4, s_4) \tag{16}$$

3.5 Solution of the Swash-Plate Mechanism

3.5.1 Initial Results

Multiplying the matrices described in section 3.4 together with some simplification gives:

$$
\begin{bmatrix}
\begin{array}{l} c\eta_3 c\theta_2 \\ -\varepsilon s_2 s\eta_3 \end{array} &
\begin{array}{l} s\eta_3 c\theta_2 s\zeta_3 \\ -s\theta_2 c\zeta_3 \\ +\varepsilon s_2 c\eta_3 s\zeta_3 \end{array} &
\begin{array}{l} s\eta_3 c\theta_2 c\zeta_3 \\ +s\theta_2 s\zeta_3 \\ +\varepsilon s_2 c\eta_3 c\zeta_3 \end{array} \\[2ex]
\begin{array}{l} c\eta_3 s\theta_2 \\ +\varepsilon r_2 s\eta_3 \end{array} &
\begin{array}{l} s\eta_3 s\theta_2 s\zeta_3 \\ +c\theta_2 c\zeta_3 \\ -\varepsilon r_2 c\eta_3 s\zeta_3 \end{array} &
\begin{array}{l} s\eta_3 s\theta_2 c\zeta_3 \\ -c\theta_2 s\zeta_3 \\ -\varepsilon r_2 c\eta_3 c\zeta_3 \end{array} \\[2ex]
\begin{array}{l} -s\eta_3 \\ \varepsilon c\eta_3 \begin{pmatrix} r_2 s\theta_2 - \\ s_2 c\theta_2 \end{pmatrix} \end{array} &
\begin{array}{l} c\eta_3 s\zeta_3 \\ +\varepsilon \begin{pmatrix} s\eta_3 \begin{pmatrix} r_2 s\theta_2 s\zeta_3 - \\ s_2 c\theta_2 s\zeta_3 \end{pmatrix} \\ + r_2 c\theta_2 c\zeta_3 \\ + s_2 s\theta_2 c\zeta_3 \end{pmatrix} \end{array} &
\begin{array}{l} c\eta_3 c\zeta_3 \\ +\varepsilon \begin{pmatrix} s\eta_3 \begin{pmatrix} r_2 s\theta_2 c\zeta_3 \\ - s_2 c\theta_2 c\zeta_3 \end{pmatrix} \\ - r_2 c\theta_2 s\zeta_3 \\ - s_2 s\theta_2 s\zeta_3 \end{pmatrix} \end{array}
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
\begin{array}{l} c\theta_1 \\ -\varepsilon s_4 c\alpha_4 s\theta_1 \end{array} &
\begin{array}{l} c\alpha_4 s\theta_1 \\ -\varepsilon \begin{pmatrix} c_4 s\alpha_4 s\theta_1 \\ + s_4 c\theta_1 \end{pmatrix} \end{array} &
\begin{array}{l} s\alpha_4 s\theta_1 \\ +\varepsilon c_4 c\alpha_4 s\theta_1 \end{array} \\[2ex]
\begin{array}{l} c\alpha_1 s\theta_1 \\ +\varepsilon \begin{pmatrix} s_4 s\alpha_1 s\alpha_4 \\ - c\alpha_1 s_4 c\alpha_4 c\theta_1 \end{pmatrix} \end{array} &
\begin{array}{l} \begin{pmatrix} c\alpha_1 c\alpha_4 c\theta_1 \\ - s\alpha_4 s\alpha_1 \end{pmatrix} \\ -\varepsilon \begin{pmatrix} c_4 c\alpha_4 s\alpha_1 \\ + c\alpha_1 c_4 s\alpha_4 c\theta_1 \\ + s_4 s\theta_1 c\alpha_1 \end{pmatrix} \end{array} &
\begin{array}{l} \begin{pmatrix} c\alpha_4 s\alpha_1 \\ + s\alpha_4 c\theta_1 c\alpha_1 \end{pmatrix} \\ +\varepsilon \begin{pmatrix} + c_4 c\alpha_4 c\theta_1 c\alpha_1 \\ - c_4 s\alpha_4 s\alpha_1 \end{pmatrix} \end{array} \\[2ex]
\begin{array}{l} s\theta_1 s\alpha_1 \\ +\varepsilon \begin{pmatrix} s_4 c\alpha_4 c\theta_1 s\alpha_1 \\ + s_4 c\alpha_1 s\alpha_4 \end{pmatrix} \end{array} &
\begin{array}{l} -\begin{pmatrix} c\alpha_4 c\theta_1 s\alpha_1 \\ + s\alpha_4 c\alpha_1 \end{pmatrix} \\ -\varepsilon \begin{pmatrix} c_4 c\alpha_4 c\alpha_1 \\ - c_4 s\alpha_4 c\theta_1 s\alpha_1 \\ - s_4 s\theta_1 s\alpha_1 \end{pmatrix} \end{array} &
\begin{array}{l} \begin{pmatrix} c\alpha_4 c\alpha_1 \\ - s\alpha_4 c\theta_1 s\alpha_1 \end{pmatrix} \\ -\varepsilon \begin{pmatrix} c_4 c\alpha_4 c\theta_1 s\alpha_1 \\ + c_4 s\alpha_4 c\alpha_1 \end{pmatrix} \end{array}
\end{bmatrix}
\tag{17}
$$

The solution matrix primary and dual elements are set equal to each other. This gives eighteen equations that can be used to solve for six unknown variables ($r_2, s_2, s_4, \theta_2, \eta_3, \zeta_3$).

3.5.2 Primary Elements

Equation of the primary components of the elements in Equation (15) yield

primary matrix element 1-1:

$$c\eta_3 c\theta_2 = c\theta_1 \tag{18}$$

primary matrix element 1-2:

$$s\eta_3 c\theta_2 s\zeta_3 - s\theta_2 c\zeta_3 = c\alpha_4 s\theta_1 \tag{19}$$

primary matrix element 1-3:

$$s\eta_3 c\theta_2 c\zeta_3 + s\theta_2 s\zeta_3 = s\alpha_4 s\theta_1 \tag{20}$$

primary matrix element 2-1:

$$c\eta_3 s\theta_2 = -c\alpha_1 s\theta_1 \tag{21}$$

primary matrix element 2-2:

$$s\eta_3 s\theta_2 s\zeta_3 + c\theta_2 c\zeta_3 = c\alpha_1 c\alpha_4 c\theta_1 - s\alpha_1 s\alpha_4 \tag{22}$$

primary matrix element 2-3:

$$s\eta_3 s\theta_2 c\zeta_3 - c\theta_2 s\zeta_3 = c\alpha_1 s\alpha_4 c\theta_1 + s\alpha_1 c\alpha_4 \tag{23}$$

primary matrix element 3-1:

$$-s\eta_3 = s\alpha_1 s\theta_1 \tag{24}$$

primary matrix element 3-2:

$$c\eta_3 s\zeta_3 = -c\alpha_1 s\alpha_4 - s\alpha_1 c\alpha_4 c\theta_1 \tag{25}$$

and primary matrix element 3-3:

$$c\eta_3 c\zeta_3 = c\alpha_1 c\alpha_4 - s\alpha_1 s\alpha_4 c\theta_1 \tag{26}$$

3.5.3 Dual Elements

Equation of the dual components of the elements of Equation (15) yield

dual matrix element 1-1:

$$-s_2 s\eta_3 = -s_4 c\alpha_4 s\theta_1 \tag{27}$$

dual matrix element 1-2:

$$s_2 c\eta_3 s\zeta_s = -c_4 s\alpha_4 s\theta_1 + s_4 c\theta_1 \tag{28}$$

dual matrix element 1-3:

$$s_2 c\eta_3 c\zeta_3 = c_4 c\alpha_4 s\theta_1 \tag{29}$$

dual matrix element 2-1:

$$r_2 s\eta_3 = s_4 s\alpha_1 s\alpha_4 - c\alpha_1 s_4 c\alpha_4 c\theta_1 \tag{30}$$

dual matrix element 2-2:

$$-r_2 c\eta_3 s\zeta_3 = -c\alpha_1 c_4 s\alpha_4 c\theta_1 - c\alpha_1 s_4 s\theta_1 - s\alpha_1 c_4 c\alpha_4 \tag{31}$$

dual matrix element 2-3:

$$-r_2 c\eta_3 c\zeta_3 = c\alpha_1 c_4 c\alpha_4 c\theta_1 - s\alpha_1 c_4 s\alpha_4 \tag{32}$$

dual matrix element 3-1:

$$c\eta_3 r_2 s\theta_2 - c\eta_3 s_2 c\theta_2 = s_4 c\alpha_1 s\alpha_4 + s\alpha_1 s_4 c\alpha_4 c\theta_1 \tag{33}$$

dual matrix element 3-2:

$$\begin{aligned} &s\eta_3 (r_2 s\theta_2 s\zeta_3 - s_2 c\theta_2 s\zeta_3) + r_2 c\theta_2 c\zeta_3 + s_2 s\theta_2 c\zeta_3 \\ &= -c\alpha_1 c_4 c\alpha_4 + s\alpha_1 (c_4 s\alpha_4 c\theta_1 + s_4 s\theta_1) \end{aligned} \tag{34}$$

and dual matrix element 3-3:

$$s\eta_3 r_2 s\theta_2 c\zeta_3 - s\eta_3 s_2 c\theta_2 c\zeta_3 - r_2 c\theta_2 s\zeta_3 - s_2 s\theta_2 s\zeta_3 = c_4 s\alpha_4 - s\alpha_1 c_4 c\alpha_4 c\theta_1 \tag{35}$$

3.5.4 Derivation of the Solution in its Final Form

Using equation (19):

$$c\eta_3 = \frac{-c\alpha_1 s\theta_1}{s\theta_2} \tag{36}$$

Using equation (22):

$$s\eta_3 = -s\alpha_1 s\theta_1 \tag{37}$$

Using equations (34) and (35), rotation angle η_3 can be determined in its proper quadrant.

Substituting equation (34) into equation (16) gives:

$$\frac{-c\alpha_1 s\theta_1 c\theta_2}{s\theta_2} = c\theta_1$$

$$\frac{s\theta_2}{c\theta_2} = \frac{-c\alpha_1 s\theta_1}{c\theta_1} \tag{38}$$

This reveals angle θ_2 as per its proper quadrant.

Substituting equation (34) into equation (23) gives:

$$\frac{-c\alpha_1 s\theta_1}{s\theta_2} s\zeta_3 = -c\alpha_1 s\alpha_4 - s\alpha_1 c\alpha_4 c\theta_1$$

$$s\zeta_3 = \frac{s\theta_2 (c\alpha_1 s\alpha_4 + s\alpha_1 c\alpha_4 c\theta_1)}{c\alpha_1 s\theta_1} \tag{39}$$

Substituting equation (34) into equation (24) gives:

$$\frac{-c\alpha_1 s\theta_1}{s\theta_2} c\zeta_3 = c\alpha_1 c\alpha_4 - s\alpha_1 s\alpha_4 c\theta_1$$

$$c\zeta_3 = -\frac{s\theta_2 \left(c\alpha_1 c\alpha_4 - s\alpha_1 s\alpha_4 c\theta_1\right)}{c\alpha_1 s\theta_1} \tag{40}$$

Rotation ζ_3 can now be determined in its proper quadrant using equations (37) and (38).

Using equation (27):

$$s_2 = \frac{c_4 c\alpha_4 s\theta_1}{c\eta_3 c\zeta_3} \tag{41}$$

Using equation (30):

$$r_2 = \frac{-c\alpha_1 c_4 c\alpha_4 c\theta_1 + s\alpha_1 c_4 s\alpha_4}{c\eta_3 c\zeta_3} \tag{42}$$

Using equation (25):

$$s_4 = \frac{s_2 s\eta_3}{c\alpha_4 s\theta_1} \tag{43}$$

These equations represent a full solution for the displacements and orientations of the swash-plate mechanism's links.

CHAPTER 4

RESULTS

To represent the kinematic behavior of the swash-plate mechanism according to the solution developed in chapter 3, the variation of the output variable s_4 with respect to the input variable θ_1 will be shown and explained first. Secondly, the variation of the intermediate variable θ_2 will be considered. Thirdly, the intermediate variables r_2 and s_2 will be described and explained. Finally, the intermediate variables ζ_3 and η_3 will each be described, explained and graphed with respect to the input angle θ_1.

Several standard dimensions were used in the example of the results. These include a frame offset c_4 of 1" and a standard frame twist angle α_4 of 180°. To show the displacement behavior of the swash-plate mechanism, the equations (34) through (41) derived in chapter 3 were incorporated in a FORTRAN computer program called "SWASHP" (see Appendix A). The step increment of 10° was used for the input angle θ_1.

The input for the first data set is shown in Table 2 and the output of the program "SWASHP" with a swash angle α_1 of 10° and an input angle θ_1 step increment of 10° can be found in Table 3.

Table 3 Input for First Data Set ($\alpha_1 = 10°$)

INPUT VARIABLE	VALUE	DESCRIPTION	UNITS
θ_1 initial value	0°	Initial input angle value	degrees
θ_1 step increment	10°	The step increment at which the program SWASHP will evaluate all intermediate and output variables	degrees
α_1	10°	The swash-plate angle	degrees
α_4	180°	The angle between the input shaft axis of rotation and the output axis of translation	degrees
c_4	1	Perpindicular distance (or offset) between the input shaft axis of rotation and the output axis of translation	inches

Table 4 FORTRAN Program "SWASHP" Output for First Data Set (α_1 = 10°)

THETA1	THETA2	ETA3	ZETA3	S2	R2	S4
0	360.0000	180.0000	190.0000	0	1.00000	0
10	350.1489	181.7280	189.8511	-0.17633	0.98481	-0.03062
20	340.2803	183.4049	189.4081	-0.34730	0.93969	-0.06031
30	330.3784	184.9809	188.6822	-0.50771	0.86603	-0.08816
40	320.4313	186.4087	187.6926	-0.65270	0.76604	-0.11334
50	310.4325	187.6443	186.4664	-0.77786	0.64279	-0.13507
60	300.3813	188.6492	185.0384	-0.87939	0.50000	-0.15270
70	290.2836	189.3913	183.4512	-0.95419	0.34202	-0.16569
80	280.1511	189.8466	181.7538	-1.00000	0.17365	-0.17365
90	270.0000	190.0000	180.0000	-1.01543	0	-0.17633
100	259.8489	189.8466	178.2462	-1.00000	-0.17365	-0.17365
110	249.7165	189.3913	176.5488	-0.95419	-0.34202	-0.16569
120	239.6188	188.6492	174.9616	-0.87939	-0.50000	-0.15270
130	229.5676	187.6443	173.5336	-0.77786	-0.64279	-0.13507
140	219.5687	186.4087	172.3074	-0.65270	-0.76604	-0.11334
150	209.6217	184.9809	171.3178	-0.50771	-0.86603	-0.08816
160	199.7198	183.4049	170.5919	-0.34730	-0.93969	-0.06031
170	189.8511	181.7280	170.1489	-0.17633	-0.98481	-0.03062
180	180.0000	180.0000	170.0000	0	-1.00000	0
190	170.1489	178.2720	170.1489	0.17633	-0.98481	0.03062
200	160.2803	176.5951	170.5919	0.34730	-0.93969	0.06031
210	150.3784	175.0191	171.3178	0.50771	-0.86603	0.08816
220	140.4313	173.5914	172.3074	0.65270	-0.76604	0.11334
230	130.4325	172.3557	173.5336	0.77786	-0.64279	0.13507
240	120.3813	171.3508	174.9616	0.87939	-0.50000	0.15270
250	110.2836	170.6087	176.5488	0.95419	-0.34202	0.16569
260	100.1511	170.1534	178.2462	1.00000	-0.17365	0.17365
270	90.00002	170.0000	180.0000	1.01543	0	0.17633
280	79.84895	170.1534	181.7538	1.00000	0.17365	0.17365
290	69.71647	170.6087	183.4512	0.95419	0.34202	0.16569
300	59.61877	171.3508	185.0384	0.87939	0.50000	0.15270
310	49.56757	172.3557	186.4664	0.77786	0.64279	0.13507
320	39.56872	173.5913	187.6926	0.65270	0.76604	0.11334
330	29.62169	175.0191	188.6822	0.50771	0.86603	0.08816
340	19.71978	176.5951	189.4081	0.34730	0.93969	0.06031
350	9.85112	178.2720	189.8511	0.17633	0.98481	0.03062
360	0.00004	180.0000	190.0000	0	1.00000	0

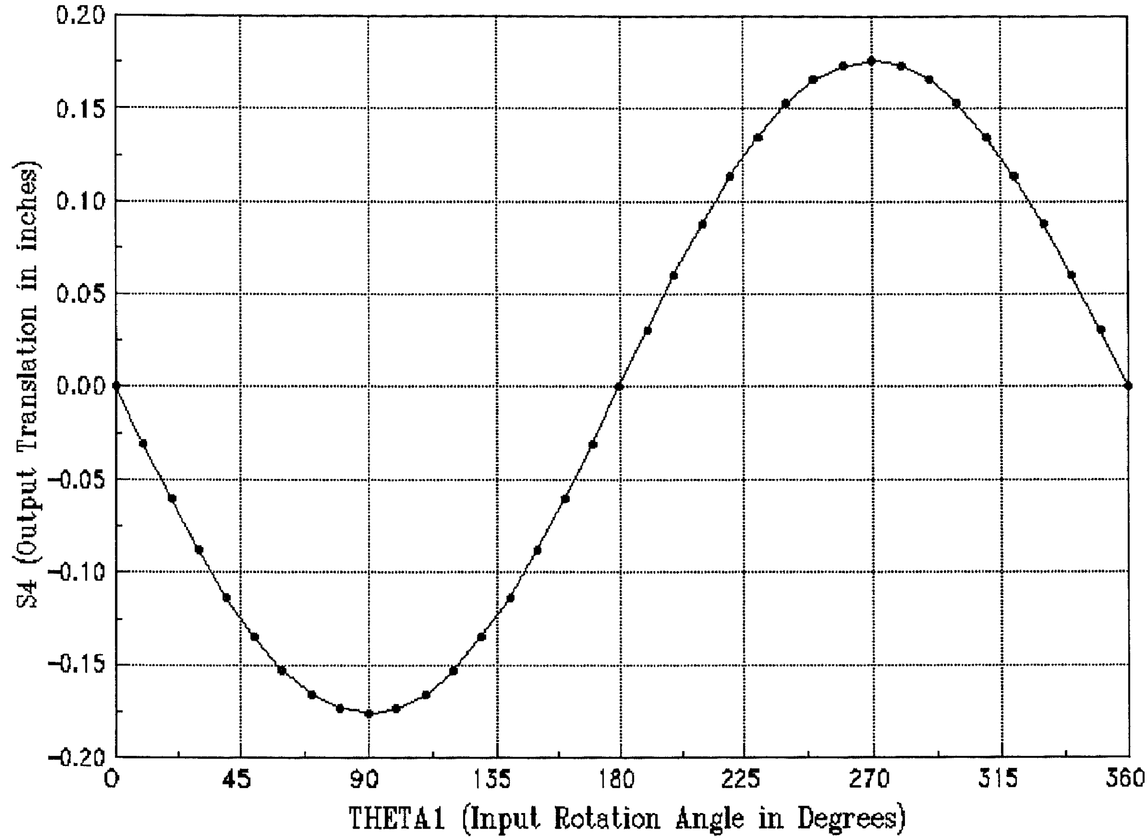

Figure 11

Output Translation as a Function of Input Rotation ($\alpha_1 = 10°$)

Figure 11 indicates that the output translation is periodic in nature given a constant input shaft rotation. This graph represents the physical oscillation of the piston as a result of input shaft rotation.

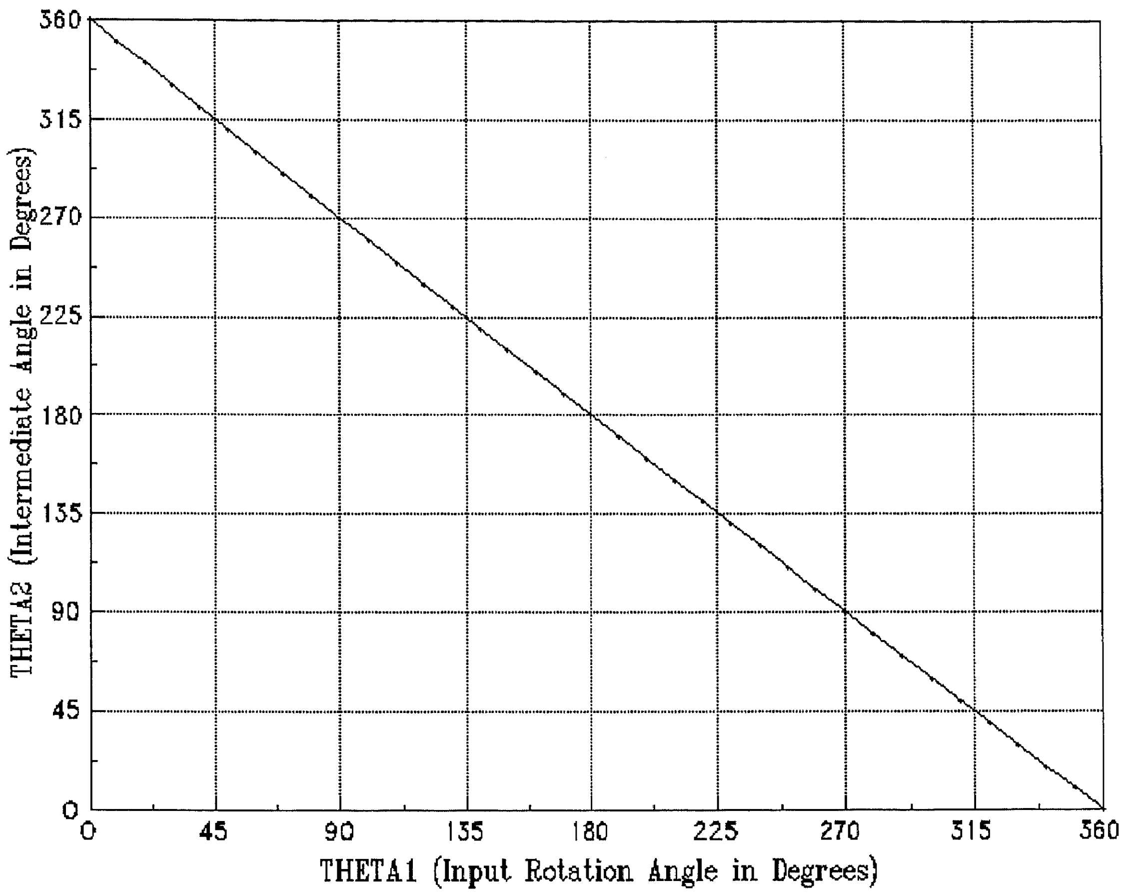

Figure 12

Intermediate Angle θ_2 and Input Rotation θ_1 (α_1 = 10°)

Figure 12 represents the relationship between input rotation θ_1 and intermediate rotation θ_2. While not immediately apparent, upon close examination, it is observed that the relationship is not linear. This will be accentuated with the second data set with a larger value for α_1.

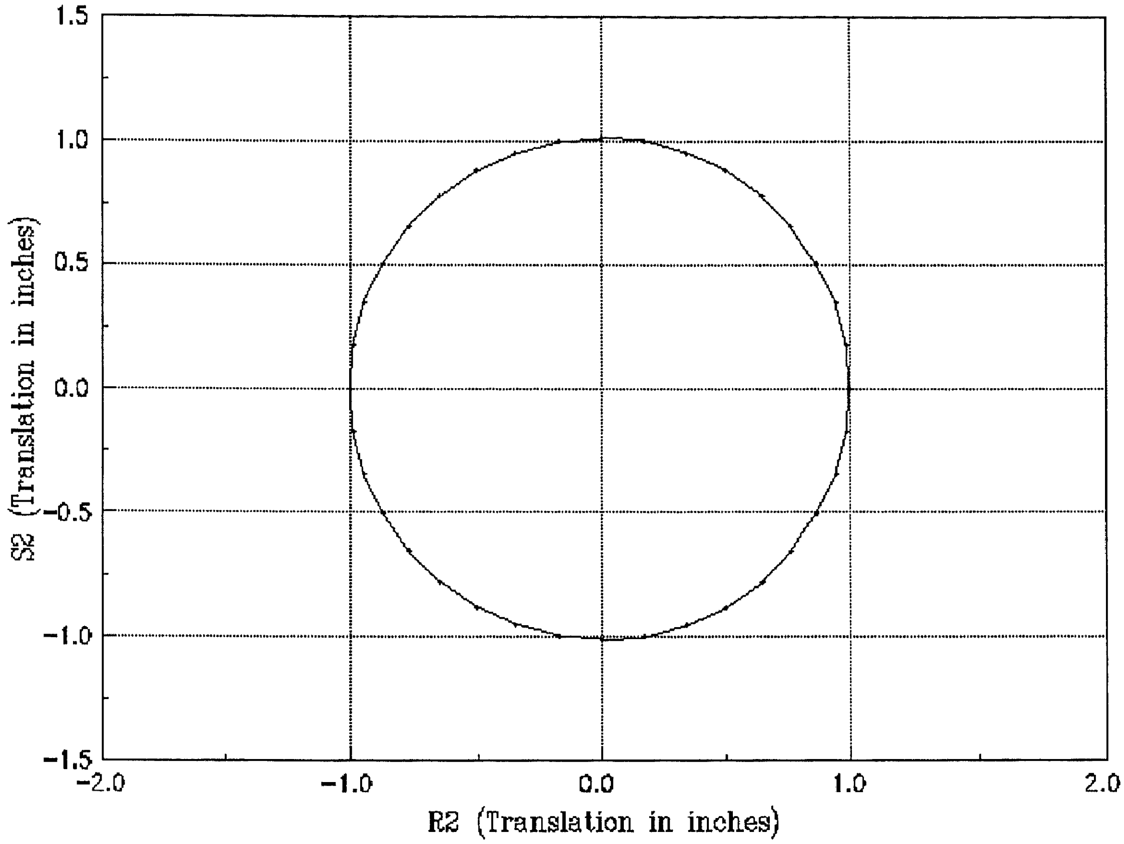

Figure 13

Intermediate Translations s_2 and r_2 ($\alpha_1 = 10°$)

Figure 13 represents the path of the coupler traced on the face of the swash plate. Upon close examination, it is observed that the path is elliptical. This shape can be accentuated with a larger swash angle α_1. This accentuated shape will be shown in the second and following data sets.

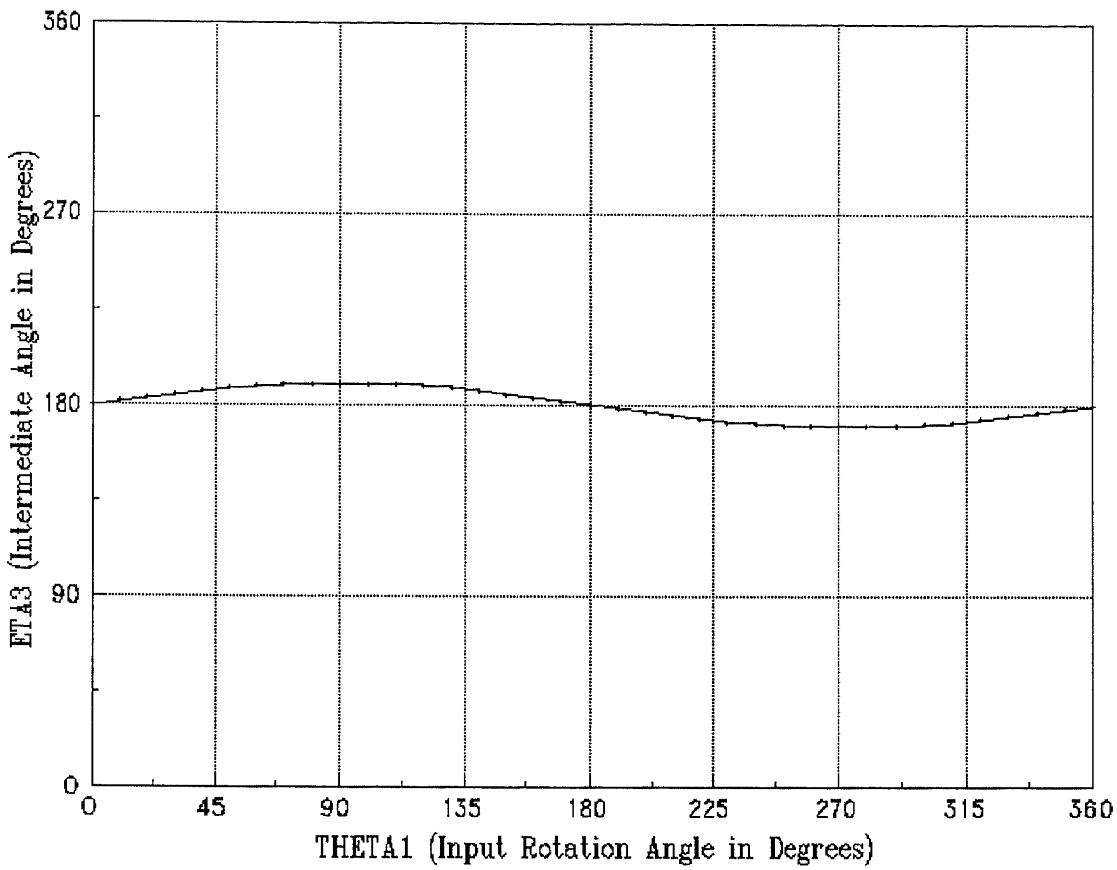

Figure 14

Intermediate Angle η_3 and Input Angle θ_1 ($\alpha_1 = 10°$)

The physical meaning of the curve graphed in Figure 14 is a representation of the forward rocking motion or "pitch" of the spherical joint given the input rotation θ_1. A single frequency fluctuation of small amplitude is observed.

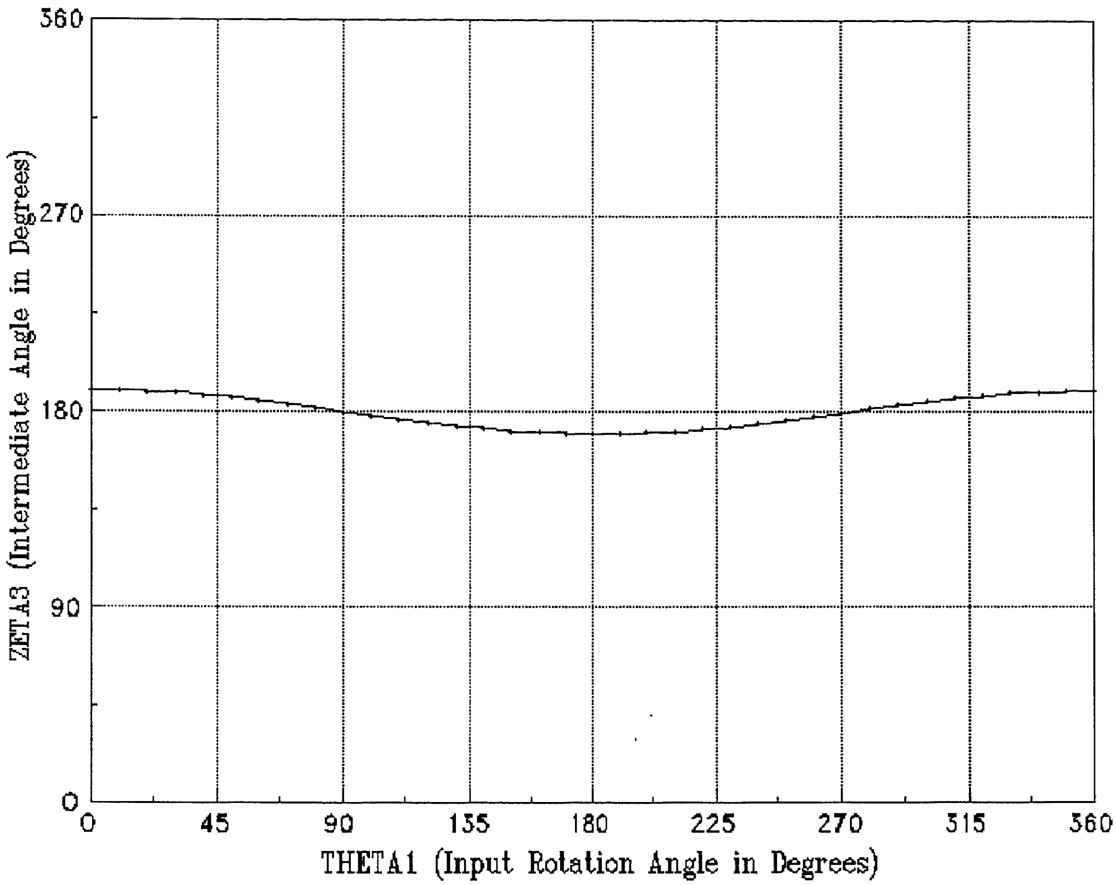

Figure 15

Intermediate Variable ζ_3 and Input Variable θ_1 (α_1 = 10°)

The side-to-side rotation or "yaw" of the spherical joint on the end of the follower is described by angle ζ_3. It is noted that the behavior of angle ζ_3 is similar in nature to the behavior of angle η_3. The functions that describe the single frequency fluctuation evident in these two variables are identical except for a 90° shift. This corresponds to the angle between the coordinate axis from which each is measured.

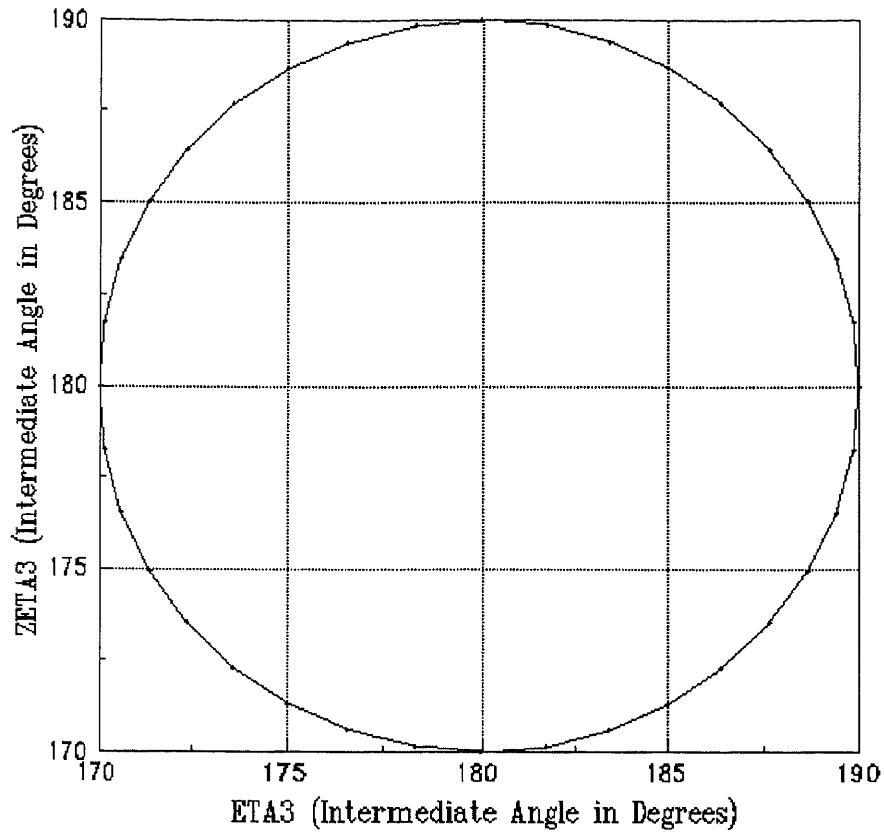

Figure 16

Intermediate Variable ζ_3 and Intermediate Variable η_3 ($\alpha_1 = 10°$)

Figure 16 is a graph describing the pattern traced by the center of the coupler pad in two-dimensional space as the swash-plate rotates. It is a perfect circle as expected.

For the second and third data sets, the swash angle α_1 is varied. In the second data set it is set at 45° to represent an extreme value thus accentuating the behavior of the intermediate variables for observation. In the third data set, α_1 is set 25° to show a trend of α_1 between 10° and 45°. The results of all three examples are combined in figures 17-22.

Table 5 Input for Second Data Set ($\alpha_1 = 45°$)

INPUT VARIABLE	VALUE	DESCRIPTION	UNITS
θ_1 initial value	0°	Initial input angle value	degrees
θ_1 step increment	10°	The step increment at which the program SWASHP will evaluate all intermediate and output variables	degrees
α_1	45°	The swash-plate angle	degrees
α_4	180°	The angle between the input shaft axis of rotation and the output axis of translation	degrees
c_4	1	Perpindicular distance (or offset) between the input shaft axis of roation and the output axis of translation	inches

Table 6 FORTRAN Program "SWASHP" Output for Second Data Set ($\alpha_1 = 45°$)

THETA1	THETA2	ETA3	ZETA3	S2	R2	S4
0	360.0000	180.0000	225.0000	0	1.00000	0
10	352.8929	187.0530	224.5614	-0.24558	0.98481	-0.17365
20	345.5673	193.9955	223.2192	-0.48369	0.93969	-0.34202
30	337.7924	200.7048	220.8934	-0.70711	0.86603	-0.50000
40	329.3180	207.0340	217.4537	-0.90904	0.76604	-0.64279
50	319.8793	212.7978	212.7324	-1.08335	0.64279	-0.76604
60	309.2315	217.7612	206.5650	-1.22474	0.50000	-0.86603
70	297.2363	221.6412	198.8817	-1.32893	0.34202	-0.93969
80	284.0020	224.1360	189.8511	-1.39273	0.17365	-0.98481
90	270.0000	225.0000	180.0000	-1.41421	0	-1.00000
100	255.9981	224.1360	170.1489	-1.39273	-0.17365	-0.98481
110	242.7637	221.6412	161.1183	-1.32893	-0.34202	-0.93969
120	230.7685	217.7613	153.4350	-1.22475	-0.50000	-0.86603
130	220.1208	212.7978	147.2676	-1.08335	-0.64279	-0.76604
140	210.6821	207.0340	142.5463	-0.90904	-0.76604	-0.64279
150	202.2077	200.7048	139.1066	-0.70711	-0.86603	-0.50000
160	194.4328	193.9955	136.7808	-0.48369	-0.93969	-0.34202
170	187.1071	187.0531	135.4386	-0.24558	-0.98481	-0.17365
180	180.0000	180.0000	135.0000	0	-1	0
190	172.8929	172.9470	135.4385	0.24558	-0.98481	0.17365
200	165.5673	166.0046	136.7808	0.48369	-0.93969	0.34202
210	157.7924	159.2952	139.1066	0.70711	-0.86603	0.50000
220	149.3180	152.9660	142.5463	0.90904	-0.76604	0.64279
230	139.8793	147.2023	147.2676	1.08335	-0.64279	0.76604
240	129.2315	142.2388	153.4349	1.22474	-0.50000	0.86603
250	117.2363	138.3589	161.1183	1.32893	-0.34202	0.93969
260	104.0020	135.8640	170.1489	1.39273	-0.17365	0.98481
270	90.00003	135.0000	180.0000	1.41421	0	1.00000
280	75.99809	135.8640	189.8510	1.39273	0.17365	0.98481
290	62.76373	138.3588	198.8817	1.32893	0.34202	0.93969
300	50.76851	142.2387	206.5650	1.22475	0.50000	0.86603
310	40.12077	147.2022	212.7324	1.08335	0.64279	0.76604
320	30.68209	152.9659	217.4537	0.90904	0.76604	0.64279
330	22.20768	159.2952	220.8934	0.70711	0.86603	0.50000
340	14.43279	166.0045	223.2192	0.48369	0.93969	0.34202
350	7.10711	172.9469	224.5614	0.24558	0.98481	0.17365
360	0.00003	180.0000	225.0000	0	1.00000	0

Table 7 Input for Third Data Set ($\alpha_1 = 25°$)

INPUT VARIABLE	VALUE	DESCRIPTION	UNITS
θ_1 initial value	0°	Initial input angle value	degrees
θ_1 step increment	10°	The step increment at which the program SWASHP will evaluate all intermediate and output variables	degrees
α_1	25°	The swash-plate angle	degrees
α_4	180°	The angle between the input shaft axis of rotation and the output axis of translation	degrees
c_4	1	Perpindicular distance (or offset) between the input shaft axis of roation and the output axis of translation	inches

Table 8 FORTRAN Program "SWASHP" Output for Third Data Set ($\alpha_1 = 25°$)

THETA1	THETA2	ETA3	ZETA3	S2	R2	S4
0	360.0000	180.0000	205	0	1	0
10	350.9205	184.2085	204.6657	-0.19160	0.98481	-0.08097
20	341.7439	188.3109	203.6624	-0.37738	0.93969	-0.15949
30	332.3789	192.1991	201.9906	-0.55169	0.86603	-0.23315
40	322.7477	195.7627	199.6574	-0.70924	0.76604	-0.29974
50	312.7949	198.8895	196.6854	-0.84524	0.64279	-0.35721
60	302.4986	201.4690	193.1243	-0.95555	0.50000	-0.40383
70	291.8802	203.3990	189.0616	-1.03684	0.34202	-0.43819
80	281.0097	204.5948	184.6293	-1.08662	0.17365	-0.45922
90	270.0000	205.0000	180.0000	-1.10338	0	-0.46631
100	258.9904	204.5948	175.3706	-1.08662	-0.17365	-0.45922
110	248.1198	203.3990	170.9384	-1.03684	-0.34202	-0.43819
120	237.5014	201.4690	166.8757	-0.95555	-0.50000	-0.40383
130	227.2052	198.8895	163.3146	-0.84524	-0.64279	-0.35721
140	217.2524	195.7627	160.3426	-0.70924	-0.76604	-0.29974
150	207.6212	192.1991	158.0094	-0.55169	-0.86603	-0.23315
160	198.2562	188.3109	156.3376	-0.37738	-0.93969	-0.15949
170	189.0795	184.2086	155.3343	-0.19160	-0.98481	-0.08097
180	180.0000	180.0000	155.0000	0	-1.00000	0
190	170.9205	175.7915	155.3343	0.19160	-0.98481	0.08097
200	161.7439	171.6891	156.3376	0.37738	-0.93969	0.15949
210	152.3789	167.8009	158.0094	0.55169	-0.86603	0.23315
220	142.7477	164.2373	160.3426	0.70924	-0.76604	0.29974
230	132.7949	161.1105	163.3146	0.84524	-0.64279	0.35721
240	122.4986	158.5310	166.8757	0.95555	-0.50000	0.40383
250	111.8802	156.6010	170.9384	1.03684	-0.34202	0.43819
260	101.0097	155.4052	175.3706	1.08662	-0.17365	0.45922
270	90.00002	155.0000	180.0000	1.10338	0	0.46631
280	78.99037	155.4052	184.6293	1.08662	0.17365	0.45922
290	68.11980	156.6010	189.0616	1.03684	0.34202	0.43819
300	57.50145	158.5310	193.1243	0.95555	0.50000	0.40383
310	47.20515	161.1105	196.6854	0.84524	0.64279	0.35721
320	37.25239	164.2373	199.6574	0.70924	0.76604	0.29974
330	27.62116	167.8009	201.9906	0.55169	0.86603	0.23315
340	18.25616	171.6891	203.6624	0.37738	0.93969	0.15949
350	9.07951	175.7914	204.6657	0.19160	0.98481	0.08097
360	0.00004	180.0000	205.0000	0	1.00000	0

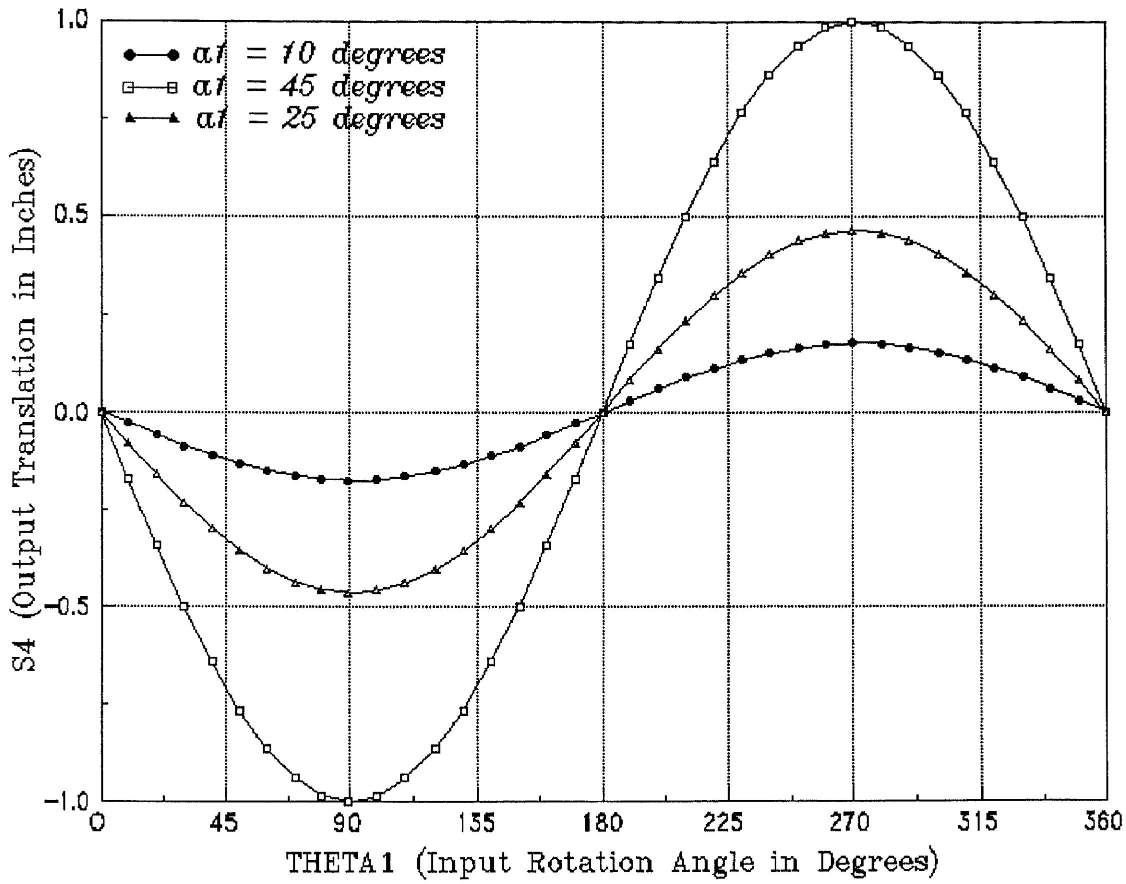

Figure 17

Output Translation s_4 and Input Rotation θ_1

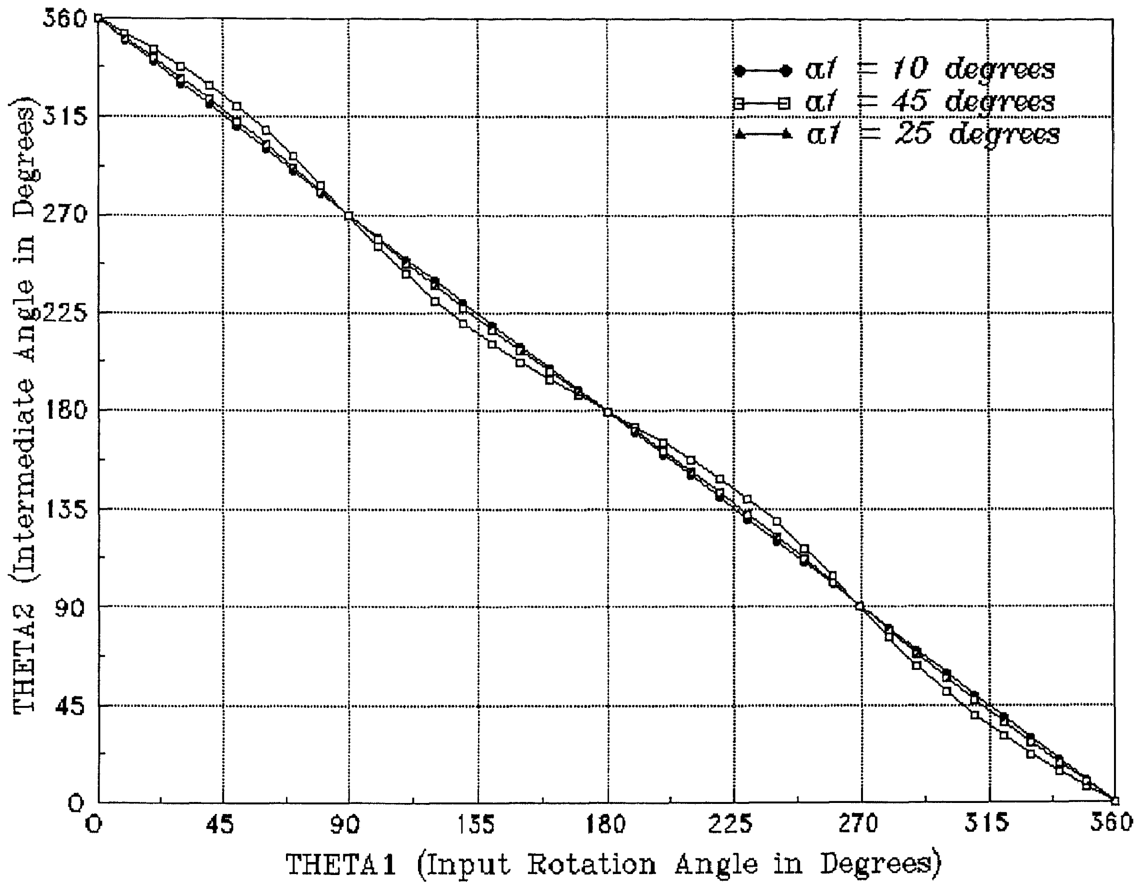

Figure 18

Intermediate Angle θ_2 and Input Rotation θ_1

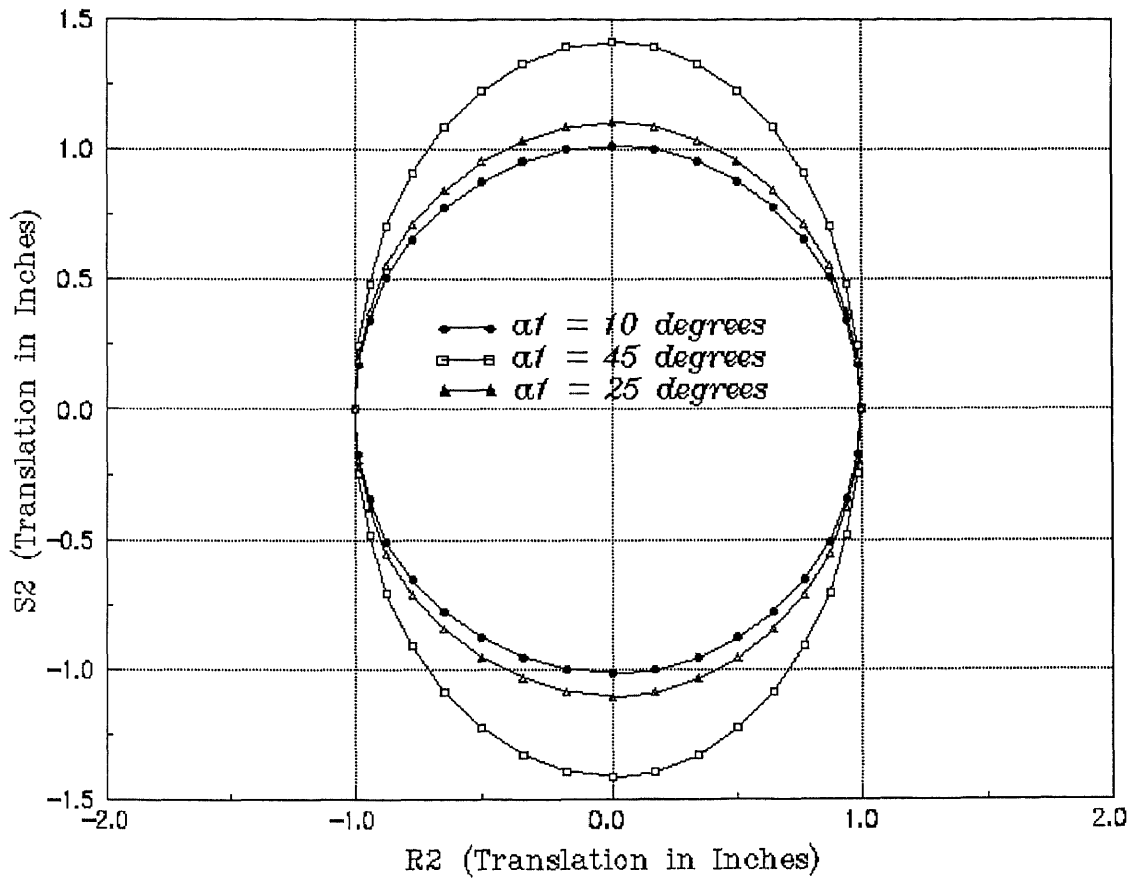

Figure 19

Intermediate Translations s_2 and r_2

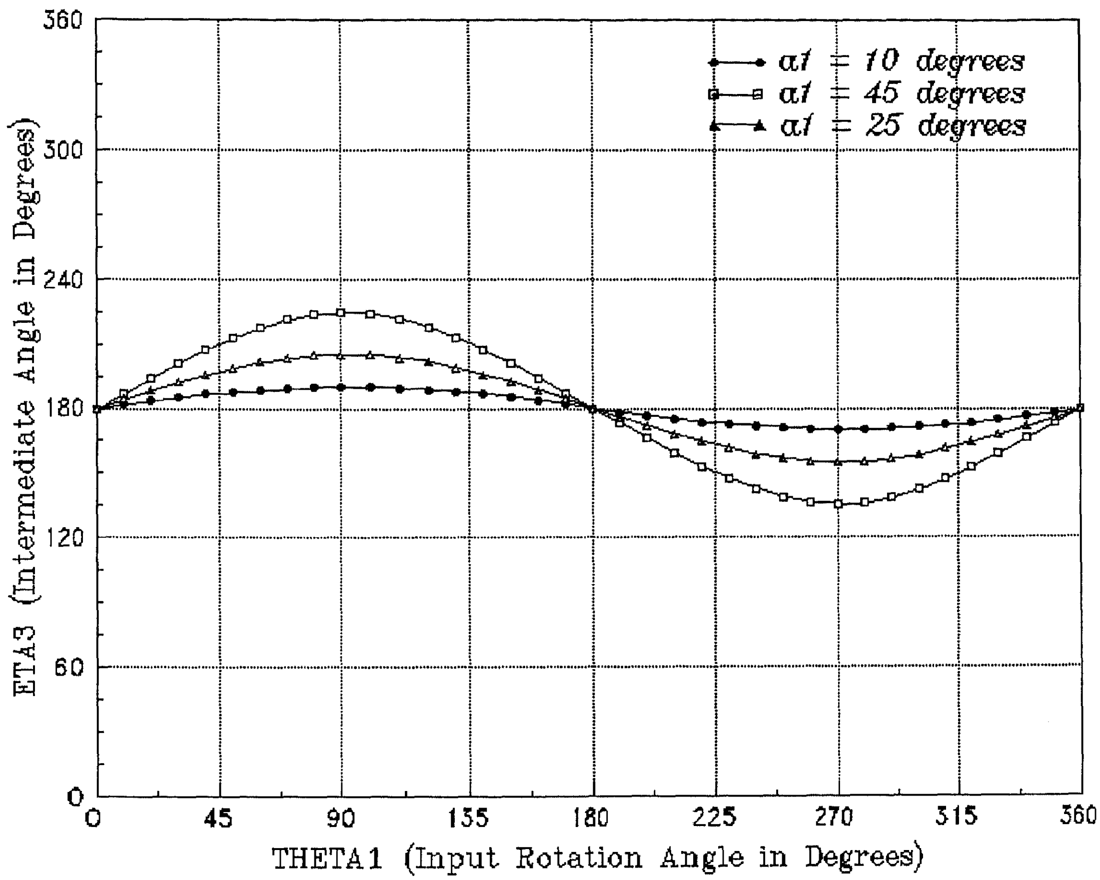

Figure 20

Intermediate Variable η_3 and Input Variable θ_1

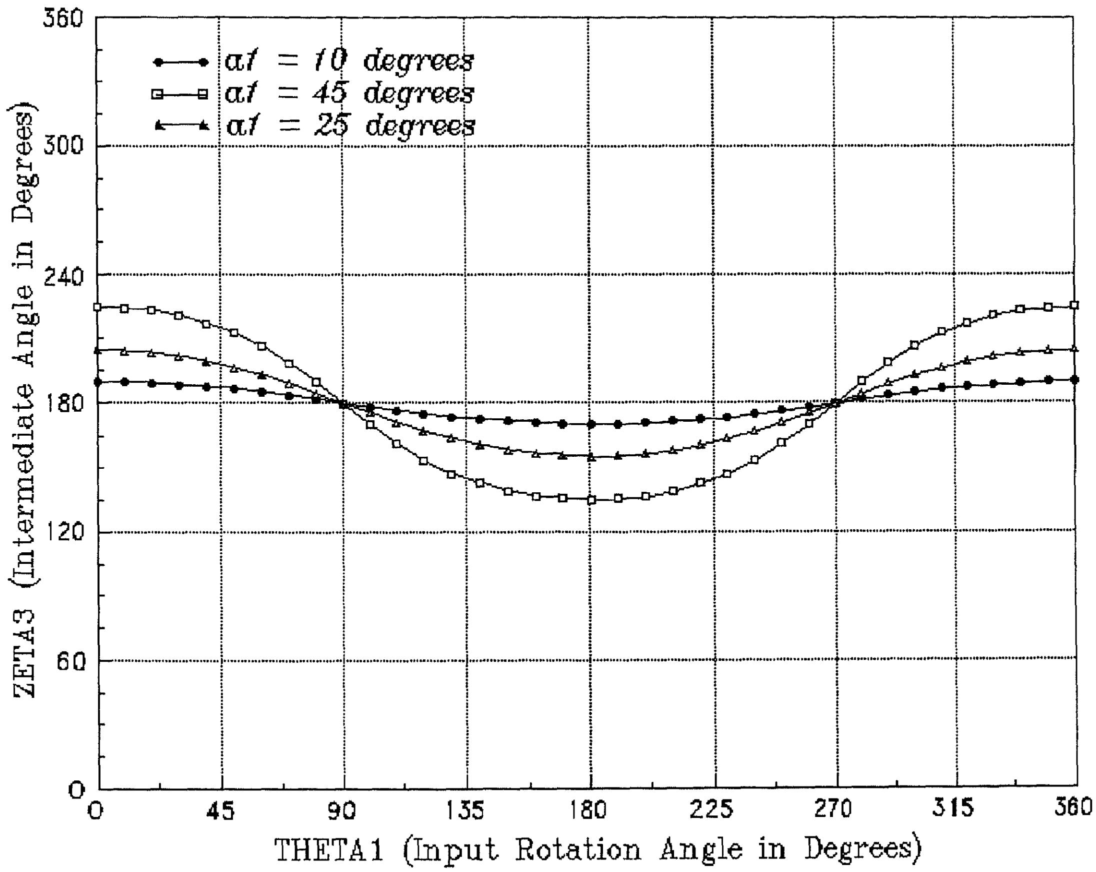

Figure 21

Intermediate Variable ζ_3 and Input Variable θ_1

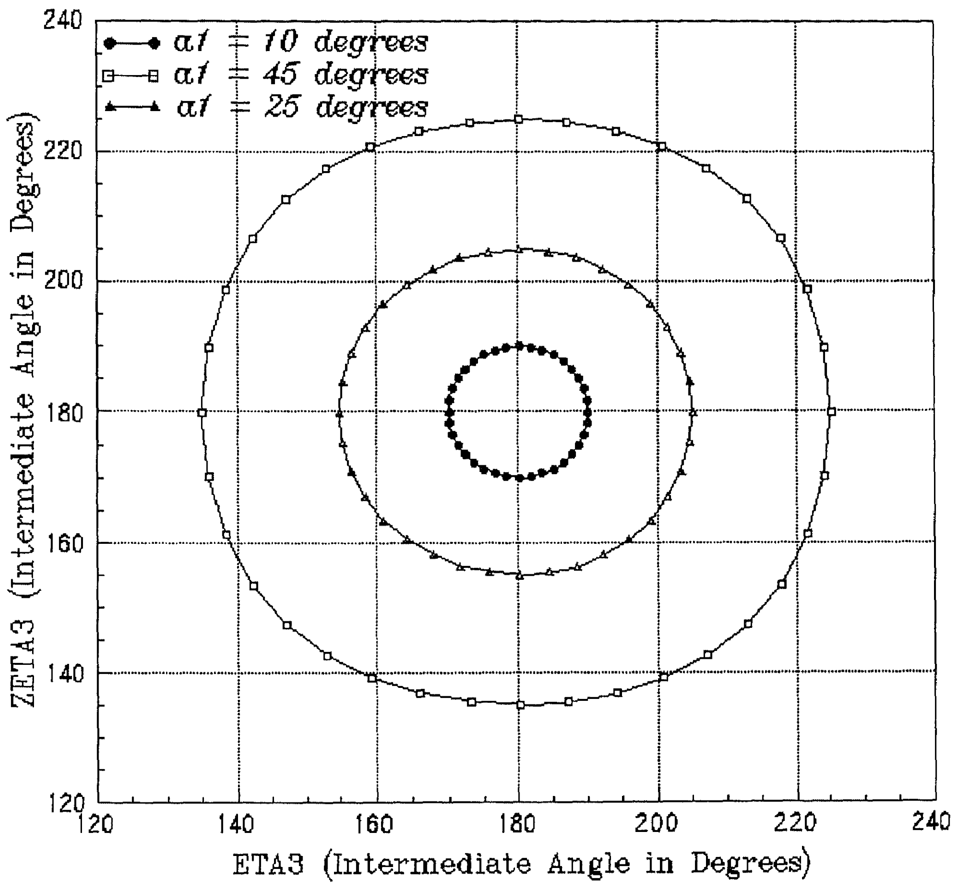

Figure 22

Intermediate Variable η_3 and Intermediate Variable ζ_3

CHAPTER 5

DISCUSSION AND CONCLUSION

In the previous chapters, a study to analyze the kinematic behavior of the swash-plate mechanism was presented. Upon examination of the data provided in Chapter 4, the solution presented seemed to be exhaustive and complete. The position and/or orientation of any link in the swash-plate mechanism was found given any input angle. This solution can be expanded to describe the position and/or orientation of any number of followers simply by adding the corresponding phase angle describing the location of that follower with respect to the reference follower at which the solution originated.

Furthermore, it was proven that the general transformation matrix solutions for the ebene and spherical joints provided accurate results in the solution of the swash-plate mechanism.

In figure 17, the trend over the single frequency produced in 360° of rotation is a non-linear increase in magnitude. This can be confirmed by checking the data. Figure 18 is perhaps the most interesting graph in chapter 4. It shows how input angle θ_1 and intermediate angle θ_2 are related with a double frequency. The variation in magnitude is observed as non-linear. Figure 19 is perhaps the most dramatic example of the nonlinear relationship between the data sets. It is obvious that the magnitude of deviation from a circle for α_1 = 10° is not just under a fourth of the magnitude of deviation from a circle when α_1 = 45°. The single frequency relationships of intermediate angles η_3 and ζ_3 and input angle θ_1 in figures 20 and 21 show the direct relationship between the swash-plate angle and magnitude of those intermediate variables. The relationship is 1:1 with a 90° phase shift because of the axis used to measure the angles being different. In figure 22, the trend for intermediate variables ζ_3 and η_3 is to increase in magnitude proportionally to the swash-plate angle α_1. This trend is shown by the larger circles corresponding to the larger swash-plate angles.

The figures in chapter 4 showing the relationships of the intermediate, input and output variables indicate that there may be some new information available to machine designers. This solution could assist in optimizing the swash-plate mechanism as it is configured in the automobile air-conditioning compressor. For example, figures 12 and 18 show a double frequency relationship between θ_1 and θ_2. This could explain a critical vibration that might affect the fatigue life of the mechanism. Cursory observation indicates that the critical area affected by this double frequency vibration is the junction between the input rotation shaft and the swash-plate plane.

Future work with this solution could be extensive. The equations derived in chapter 3 offer much more potential information than was presented in chapter 4. For example, the swash angle α_1 could be held at zero and the angle between the input axis and the piston α_4 could be varied. This would show a different configuration for the mechanism that could be compared to the original configuration already presented. A combination of varying the swash-plate angle α_1, the angle between the input shaft and the output follower/piston assembly α_4 could also be studied and used for a possible mechanism configuration optimization in the future. Another possibility for future work is a dynamic analysis of the swash-plate mechanism. By taking the first and second derivatives, the velocity and change in momentum of each link in the mechanism could be studied in detail.

In conclusion, the aforementioned work presents a reasonable, accurate and complete solution for the swash-plate mechanism. The solution is consistent with previous geometric solutions in its explanation of the extreme positions of the swash plate and yet it provides a simple way to understand the motion of the mechanism more completely. This information could prove useful to mechanism designers interested in optimizing the behavior of the swash-plate mechanism for any of its current applications or possibly for any new application being explored.

APPENDIX A

FORTRAN Computer Program "SWASHP"

```
PROGRAM SWASHP

C     This program is designed to solve the equations produced
C     by a transformation matrix analysis of the swash-plate mechanism.
C     Also, a data file will be produced for plotting the function
described by the last equation.

            PARAMETER (PI=3.14159265359)


        CHARACTER *1 ANSWER
        CHARACTER *10 FILNAM

            INTEGER     K,K2

      REAL  ZTA3CT,ZTA3CB
      REAL  THTA2S,THTA2C,ETA3S,ETA3C,ZETA3S,ZETA3C
            REAL  ALPH1,ALPHA1, ALPH4,ALPHA4, C4, R2, S2
      REAL  THETA1, THETA2, ETA3, ZETA3, S4

C     This statement is asking for the swash-angle as defined by the
user
C

      PRINT *,'Enter the swash-angle, in degrees:'
      READ*, ALPH1

C     ALPH1 is converted to radians and changed into its true name
"ALPHA1"
C

            ALPHA1=(ALPH1/180.0)*PI

C     Swash-Plate offset is now entered. This value is in inches
C

        PRINT *, 'Enter the offset of the piston from the center of the
     + swash-plate axis, in inches:'
      READ *, C4

C     The angle between the piston axis and the axis of the swash-plate
is entered here
C

        PRINT *,'Angle between piston and swash-plate axis, in degrees:'
      READ*,ALPH4

C     ALPHA4 is derived through the conversion of ALPH4 to radians
C

      ALPHA4=(ALPH4/180.0)*PI
```

```
C     This statement initializes the angle THETA1
C

        THETA1 =(0.000001/180)*PI

C     This statement determines the input step  (# of data points)
between
C     0 and 360 degrees of rotation
C
      PRINT*, 'Enter input angle step increment:'
      READ *,K2


C     This statement asks the user if an output file should be created.
C     It also prompts the user for the name of the output file.
C
        PRINT *, 'Output file? y/n (Surround answer with parenthesis)?'
      READ *,ANSWER

C     This statement is used to open the output file in which the
C     data for the graph is placed.

        IF(ANSWER .EQ. 'y') THEN
                PRINT *,'Name of Output file? (In parenthesis please)'
                READ *,FILNAM
                OPEN(UNIT=2,FILE=FILNAM,STATUS='NEW')
                WRITE(2,40)'THETA1','THETA2','ETA3',
     +          'ZETA3','S2','R2','S4'
      ENDIF

      PRINT 40, 'THETA1',' THETA2', 'ETA3','ZETA3', 'S2','R2','S4'

40    FORMAT(7(A6,1X))
50    FORMAT(7(F10.5,1X))

C     This loop is used to actually perform the calculations of the
output formulas
C     for the swash-plate transform.

       DO 100 K = 0,360,K2

      THTA2S=-COS(ALPHA1)*SIN(THETA1)

      THTA2C= COS(THETA1)

      THETA2= ATAN2(THTA2S,THTA2C)

         IF (THETA2 .LT. 0) THEN

               THETA2 = THETA2 + 2*PI

         ENDIF

      ETA3S=-SIN(ALPHA1)*SIN(THETA1)

      ETA3C= COS(ALPHA1)*SIN(THETA1)/SIN(THETA2)

      ETA3= ATAN2(ETA3S,ETA3C)
```

```
      IF (ETA3 .LT. 0) THEN

              ETA3 = ETA3 + 2*PI

      ENDIF


     ZETA3S=(SIN(THETA2)*(COS(ALPHA1)*SIN(ALPHA4)+
    +            SIN(ALPHA1)*COS(ALPHA4)*COS(THETA1)))
    +            /(COS(ALPHA1)*SIN(THETA1))

     ZTA3CT=-SIN(THETA2)*(COS(ALPHA1)*COS(ALPHA4)-
    +            SIN(ALPHA1)*SIN(ALPHA4)*COS(THETA1))

     ZTA3CB= COS(ALPHA1)*SIN(THETA1)

     ZETA3C=ZTA3CT/ ZTA3CB

     ZETA3= ATAN2(ZETA3S,ZETA3C)

      IF (ZETA3 .LT. 0) THEN

              ZETA3 = ZETA3 + 2*PI

      ENDIF


             S2= (C4*COS(ALPHA4)*SIN(THETA1))/(COS(ETA3)*COS(ZETA3))


           R2= (C4*
    +        (SIN(ALPHA1)*SIN(ALPHA4)-
COS(ALPHA1)*COS(ALPHA4)*COS(THETA1) ) )
    +              /(COS(ETA3)*COS(ZETA3))


             S4= R2*SIN(ETA3)/(SIN(ALPHA1)*SIN(ALPHA4)-
    +            COS(ALPHA1)*COS(ALPHA4)*COS(THETA1))




       IF (ANSWER .EQ. 'y') THEN
C            This statement is used to write the program's output to the
C            data files for use later by the graphing package.


              WRITE(2,50) THETA1*(180.0/PI), THETA2*(180.0/PI),
    +         ETA3*(180.0/PI), ZETA3*(180.0/PI), S2,R2,S4

     ENDIF

       PRINT 50, THETA1*(180.0/PI), THETA2*(180.0/PI),ETA3*(180.0/PI),
    +  ZETA3*(180.0/PI), S2,R2,S4

      THETA1=THETA1+(K2/180.0)*PI

100  CONTINUE
```

```
      STOP
C     End the Program
      END
```

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