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## ACKNOWLEDGMENT

I would like to express my deepest appreciation to Dr. George Abdou, who not only served as my research advisor, providing valuable and countless resources, insight, and intuition, but also constantly gave me support, encouragement, and reassurance. Special thanks are given to Dr. A. Bladikas and Dr. O. Jeng for actively participating in my committee.

Many of my fellow graduate students in the Industrial and Manufacturing Engineering Department are deserving of recognition for their support.

## **ABSTRACT**

### **PARAMETERS' ANALYSIS OF 3D RANDOM STACKING PALLETIZATION**

**by  
Najeeb Syed**

Palletization is a very important element of production, distribution and warehousing activities. The 3D random stacking palletization method is based on volume utilization instead of surface area optimization. Random stacking provides interlocking among the boxes and hence it improves the stability of the pallet load. Volumetric Pallet Utilization is normally the prime concern of any palletization process, with Work in Process and Palletization time being also important. This research uses data generated from a previous heuristic to establish mathematical relationships between the three mentioned performance indices and also three additional indirect variables, namely: Total Number of Sub Volumes, Partitioned Remaining Volume Load Capacity and Total Zero Count. An Estimation method for Volumetric Pallet Utilization is also developed by using the established mathematical relationships.

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**PARAMETERS' ANALYSIS OF 3D RANDOM STACKING PALLETIZATION**

by  
**Najeeb Syed**

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Master of Science in Manufacturing Systems Engineering  
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**To my beloved Parents**

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## NOMENCLATURE

<b>Box<sub>C1</sub></b>	The box at the pick up location at the end of the conveyor.
<b>Box<sub>C2</sub></b>	The box following box <sub>C1</sub> in the random sequence.
<b>Box<sub>WIP</sub></b>	Best Box in the Work-In-Process area.
<b>h<sub>BTi</sub></b>	Height of box type i.
<b>H<sub>SVj</sub></b>	Height of sub volume j.
<b>ISVLC</b>	Individual Box Type Sub Volume Load Capacity.
<b>l<sub>BTi</sub></b>	Length of box type i.
<b>L<sub>SVj</sub></b>	Length of sub volume j.
<b>MCBT</b>	Maximum Capacity of Box Type.
<b>N</b>	The number of sub volumes that can fit a particular box, ( $N < M$ ).
<b>NBOP</b>	Number of Boxes On the Pallet.
<b>NF</b>	Negative Flexibility.
<b>P</b>	Number of additional sub volume added after a box loading process.
<b>PF</b>	Positive Flexibility
<b>PRVLC</b>	Partitioned Remaining Volume Load Capacity.
<b>RPT</b>	Robotic Palletization Time.
<b>TNBT</b>	Total Number of Box Types.
<b>TNSV, M</b>	Total Number of Sub Volume.
<b>TZC</b>	Total Zero Count.
<b>VPU</b>	Volumetric Pallet Utilization.
<b>w<sub>BTi</sub></b>	Width of box type i.

**NOMENCLATURE**  
**(continued)**

<b>WIP</b>	Work-In-Process.
<b>W<sub>svj</sub></b>	Width of sub volume j.
<b>ZCBT</b>	Zero Count of <b>B</b> ox Type

# CHAPTER 1

## INTRODUCTION

The importance of using unit loads to make material movements efficient is well known. The most commonly used unit loads are those that use pallets for unitization. The pallet loading problem, as discussed here, addresses the geometrical composition of the unit load.

Pallet loading has applications in both production and distribution environments. Since the production environment is likely to involve palletization of identical items, pallet packing of the same size items is called manufacturer's problem. On the other hand, distribution channels often require placing of non uniform items on a single pallet. The pallet packing problem for non-uniform boxes is therefore called distributor's problem. Most often, individual items considered for palletization are rectangular boxes.

There are three techniques by which boxes can be loaded on a pallet: the layer by layer method which produces a layered pallet load, the column stacking method which results in a column stacked pallet load, and thirdly, the random stacking pallet load method. This last method results in interlocking of the boxes and hence provides greater stability. This study will use the random stacking method for generation of data.

A palletization model has several basic objectives. They are:

- minimization of palletization time
- minimization of Work In Process

- maximization of pallet utilization
- maximization of stability of boxes

### **1.1 Problem Definition**

Researchers have used different methods to solve the Palletization problem. For the 3D Random Stacking problem, Abdou and El-Masry (1997) developed an algorithm to solve a palletization case with random arrival of boxes. There were three direct outputs of the algorithm, namely the Volumetric Pallet Utilization, Work In Process, and Robotic Palletization Time. To get to these values they used three more variables called Total No. Of Sub-Volumes, Partitioned Remaining Volume Load Capacity and Total Zero Count. The heuristic maximizes the volumetric utilization while minimizing work in process and palletization time. . These variables are explained later in chapter 2.

These six variables or parameters generate a lot of data during a palletization run and the relationships between them are also not clear. So as a next step in the research, data analysis is necessary to get meaningful conclusions and establish relationships between the parameters, that will help in optimizing palletization performance indices.

One way to establish relationships is to get a mathematical equation representing the connection between parameters. Following thesis includes the data generation using the heuristic by Abdou and El-Masry, analysis of this data and the establishing of mathematical relationship between the parameters discussed above.

## **1.2 Literature Review**

The proposed research has two aspects as explained briefly in problem definition, one aspect concerns the optimization of Palletization performance indices. And for this data analysis has to be done, which constitutes the second aspect of the research. The following literature review is hence divided into two parts, first part covers the palletization literature and the second reviews some of the data analysis literature.

### **1.2.1 Palletization**

Interest in palletization research goes back to as early as the end of the 60's. Until that time, as simple as the palletization problem seemed to be, as much attention was given to it. But as quality standards increased, the palletization problem required extra attention and this can be observed in the many research studies that have tackled the problem in the last two decades. While the past was mainly interested in maximizing the coverage of the pallet's area, the current trend is to focus on three-dimension problems that are concerned with the volume of the pallet rather than the coverage of its area. Thus only 3D palletization and related works are reviewed in the literature so as to continue the progress in developing new approaches serving today's needs.

Heuristics are generally developed to satisfy special needs of manufacturers and warehousing industries. To state simply that the problem considered is that of pallet loading, is not sufficient to define it. This general description fits many situations with quite different characteristics. In fact, there exists a wide variety of different problems, each being constrained by its own problem definition, input requirements, and variations in the packing approaches

adopted. A problem definition can vary from loading identical boxes on a pallet to loading a mixed combination of box types on the same pallet. Input requirement can vary from having all packaged goods lying on warehouse shelves to having boxes follow a path on conveyor belts in pre-defined or completely random order. As for the different packing approaches, boxes may sometimes require special attention such as being stacked with a certain face up. In other cases some goods are not allowed to be stacked in proximity and sometimes packages may contain fragile contents. Moreover, pallet fragility and material handling aspects are further considerations which play a dominant role. The list could be easily expanded, but these few examples are perhaps sufficient to illustrate that pallet loading can involve quite different - and often multiple - objectives and constraints of which some may be very difficult to define in precise terms.

Han *et al.* (1989) proposed a heuristic to load packaged goods into vehicles or cargo containers. The dynamic programming approach was used to solve the 3D cargo-loading problem. The heuristic is designed to solve for different problems but the restriction in the algorithm is that the packages must be constant in both size and shape in a particular problem. The loading procedure is based on the layer by layer style with no constraints with regard to rotation about any of the three coordinate axis of the boxes.

Haessler *et al.* (1990) came up with a computer-based heuristic procedure for sizing customer orders and developing 3D load diagrams for rail and truck shipment of low density products. The loading heuristic develops an actual load plan that specifies what and how boxes should be loaded in vehicles. The objective of the model is to maximize the size of the order while at the same time avoid the risk of product damage, limit the time required to load the

vehicle, and keep each product in the order as close together as possible to minimize the effort in stack building, and in unloading inventory and storing the product at the receiving location. This last consideration would be relatively obsolete if material handling were to be executed by mechanical equipment or robots. This is worth mentioning, since customers could order any mix of products. Furthermore, it is very difficult to know at the time of order entry, if any order will completely utilize the cubic volume of the shipment vehicle. Therefore, an extra input to the heuristic was information regarding how the order can be adjusted up and down so as to, if needed, help maximizing the volumetric utilization of the vehicle while still meeting customer product needs. Communication between the customer and the material handling and shipping departments plays a valuable role in optimizing the overall process and should not be ignored.

Gehring *et al.* (1990) designed a computer-based heuristic to pack rectangular boxes of different size in a shipping container of known dimensions. The objective is to determine positions for placing the boxes in the container so as to minimize the waste space which is sometimes inevitable. For this problem which can be considered a 3D cutting stock problem, various sub optimal solutions are generated using the proposed computer-based heuristic. The proposed solution procedure is based on sorting the boxes in a linear list with decreasing volume of the elements from the beginning to the end of the list. The idea behind starting packing with high volume boxes is that it tends to result in good container volume utilization. Regarding the manner by which the boxes are stacked in the container, the boxes are stacked in vertical layers and no box is permitted to straddle neighboring layers. The solution to the heuristic enables the user to generate different loading patterns for a given problem. The user may then select the most appropriate pattern.

Abdou and Lee (1991) introduced the 3D palletization problem and highlighted some ideas about how to solve such problem. They proposed to use the layer by layer loading approach and that once a box height is loaded onto a new pallet layer, this layer must maintain this height for the rest of the process. The way to achieve that was to restrict the particular layer to boxes of the considered height. This procedure may be efficient if all boxes are present and known. However, if boxes randomly arrive at the pallet loading station, this may lead to a very high number of boxes in the WIP which will automatically increase the palletization time. Based on Loschau's stability criterion model (1989), Abdou and Lee discussed some box selection rules for the loading of the pallets but did not pursue the application of the rules to actual problems. Hence no results are reported.

Dowsland (1991) examined some of the solution approaches and strategies which could be used in the development/improvement of heuristics that provide packings which are volumetric (3D) efficient. He discussed five strategies that might be adopted in the search for improvements of an already existing heuristic that is successfully developed, implemented, and tested rather than building a new heuristic from scratch. He emphasized the importance of an interactive approach to algorithm improvement which incorporates a two way dialogue between the algorithm developer and the end user while allowing both to make full use of the ideas of other researches in this area and the considerable number-crunching power available on today's technology.

Following Abdou and Lee's work (1991), Abdou and Yang (1995) tackled the problem with the same layer by layer approach. However, while restricting the considered layer to a specific box height, Abdou and Yang proposed to define blocks consisting of at least one box



such that the block height is equal to the height of the layer being filled. They proposed two models to solve the problem. Both are characterized by the fact that boxes with similar heights are grouped together. The first model is as much mathematical as heuristic and is characterized by the pre-knowledge of the availability and quantity of each type of boxes. Furthermore, because the model is mathematically oriented, it was able to determine all possible layer combinations and pallet patterns and finally choose the pattern with best pallet utilization and relative loading stability according to the results obtained from a finite element software that simulated stresses, friction forces, and external forces to the different pallet loads. The second heuristic model added random arrival of boxes into the system. However load stability was not considered in the second model because utilization was the objective of the proposed model.

Arghavani and Abdou (1996) have developed optimization procedures to solve the problem with random arrival of boxes using the stacked column method to build the pallet. As in the case of the Abdou and Yang's second heuristic model (1994), Arghavani and Abdou did not consider any loading stability in his model. His heuristic is a base ILP model. The optimal layout on the pallet (2D cutting stock) with respect to the boxes base area is first determined using LINDO. Then each optimal layout on the pallet is stacked up providing layers with different heights in each column stack. It seemed that Arghavani and Abdou followed Dowsland's advice (1991) regarding improving already existing research work and using tools found in the literature rather than building from scratch. In fact, Arghavani and Abdou were successful enough to understand and modify Yang's C program to solve for stacked instead of layered palletization. Again, the concept of building blocks is used and then each block is treated as a box and loaded as a whole. Though the model was restricted to loading boxes or

blocks on a new column stack only after the previous stack is finished, its results showed better volumetric utilization of the pallet and more efficient palletization time when compared to those of Abdou and Yang's layered palletization (1994) using the same box types and same random sequences. However, the load stacking stability is generally much lower than that of layered palletization.

G. Abdou and J. Arghavani (1997) developed interactive ILP procedures for stacking optimization for the 3D palletization problem. The model deals with different complexity levels of the 3D stacking palletization, (i.e. the pallet volume is optimized using different stacking columns of rectangular and/or square boxes of multiple dimensions). The interactive ILP procedures considered the exact location of the boxes to be stacked on the pallet and incorporated two successive models with different objectives; the first one is the optimization of the base area of the pallet, and the second is the optimization of each stacking height for the different sub-areas generated from the first model. The resulting optimum solution is only of the column stacking type.

Abdou and El-Masry (1997) developed two heuristics to solve the 3D random stacking palletization problem. The heuristics maximize the volumetric pallet utilization, while minimizing the work-in-process as well as the robotic palletization time. They also established new performance indices: Total Number of SubVolume, Partitioned Remaining Volume Load Capacity and Total Zero Count. The results from this heuristic will be used in the study, and above defined variables will be accounted for in the data analysis.

### 1.2.2 Data Analysis

Data, and the innate curiosity of humans, generate a need to understand the data. An understanding of the data and, it is hoped, as understanding of the situation that generated the data are developed through analysis. An understanding of the real world is the ultimate purpose of a data analysis.

A data analysis always begins with data. The data may already be collected or they may be only conceptual. If they have not been collected, the data analyst may be able to influence how they are collected and recorded. A plan for collecting and recording data should not be confused with experimental design.

Regression techniques seek to establish a relationship between a response variable and one or more explanatory (or predictor) variables  $x_1, x_2, \dots$ . The approach is widely used and is useful. The simplest case with one predictor variable, say  $x$ , where the data indicates a linear relationship. The conditional distribution of the response variable,  $y$ , a random variable, for a given fixed value of  $x$  has a mean value. The regression curve is the line joining these conditional expectations and is here assumed to be of the form

$$y = a + b x$$

A linear model may be inappropriate for external reasons or because the eye detects non-linearity. One possibility is to transform one or both variables so that the transformed relationship is linear. The alternative is to fit a non-linear curve directly.

Data analysis has vast applications, from the field of physical sciences to social sciences. Data analysis is used in medicine to ascertain the effect of a certain drug. In the field of biology it could be used to determine the average lifetime of a virus. Or in the field

of nuclear physics to find out the magnetic properties of a subatomic particle. Some examples of application of data analysis are given below.

Kadas (1995) describes Health Care Data Analysis Systems (HCDAS) available to measure the resource utilization of almost any defined population. HCDAS are decision support applications designed for the easy and efficient comparison, analysis, and presentation of health care information. In an article titled "Cube utilization with racks: Analyzing the data" in the journal called *Material Handling Engineering* (1995), the anonymous author use data analysis of storage system requirements to develop best storage configuration. Bures, Henderson, (1995) and others used data analysis to find the effects of spousal support and gender on worker's stress and job satisfaction. They analyzed data collected in a cross national investigation of dual career couples.

A important aspect of data analysis is its presentation and visualization. Visualization tools are becoming more and more popular and also necessary. Studt, Tim (1995) reporting a survey by *R&D Magazine* says that data acquisition and analysis applications have become the largest use of visualization tools, easily exceeding that of other applications such as CAD, mechanical modeling, image analysis, and mathematical functions. Almost half the survey respondents use visualization tools for data acquisition and analysis functions. In analytical instrumentation, automotive, computer, test and measurement, and medical industries alone, 80 % of the researchers indicated the use of visualization tools for their data analysis work.

### 1.3 Limitations of Palletization Algorithms in the Literature

All the above studies have approached the 3D packing problem, especially the pallet loading problem through various objectives and constraints and obtained interesting results. However, they still have not covered all aspects of the problem. And with the rapidly changing demands of the manufacturing, production and other industries that need palletization, researchers must continue finding solutions to such demands and problems. The lacunae revealed as result of the literature review are as follows:

1. Most of the studies tried to solve the 3D problem through 2D approaches.
2. The pallet loading process was restricted to either layer by layer or stacking columns.
3. In layered type models, upper layers can not be loaded with boxes unless the lower layer is completely filled.
4. Similarly, models using the stacking columns approach considered sequential build-up. That is no simultaneous column stack built-up was considered.
5. Most have considered the pallet volumetric utilization as the only objective of their models.
6. Very few have given attention to the loading stability issue.
7. None have designed for boxes with dimensions of non integral proportions.
8. There is no extensive study of the established performance measures and indices.
9. No empirical or analytical relationship has been developed between the various palletization parameters.

### 1.4 Research Objective

Previous research in this field has ignored the relationships between the various palletization variables . These relationships can be useful in developing production rules, which would help in improving the heuristics and also facilitate the real life implementation in the field of robotic palletization.

Hence, the main objective is to optimize the Palletization process by increasing Volumetric palletization utilization, and also to examine the values of Work in process and Robotic palletization time. For this purpose, the empirical (mathematical) relationships between these parameters and the other indirect parameters, mentioned in the literature review and explained in the following chapters, are to be determined using appropriate Data Analysis Techniques.

#### *Procedures*

In pursuit of the above objective the following general steps will be followed:

1. Collection and tabulation of data from the algorithm developed by Abdou and El-Masry (1997).
2. Application of an appropriate technique or techniques of data analysis.
3. Development of relationships between the direct and indirect variables.
4. Development of production rules for established significant relationships, such as the Estimation of VPU based on the established relationships of the variables.

## CHAPTER 2

### PERFORMANCE CRITERIA

It is of great importance to develop a set of criteria meeting the requirements of the practical situation of the palletization problem and measure the efficiency of the palletization task. Traditionally, the pallet loading problem was tackled by attempting to maximize the number of boxes that can be fitted on the pallet or in other words, pallet volumetric utilization was the one criterion to which attention was given. There are other practical considerations in the geometric composition of pallet loads and in the palletization task in general. Among such considerations are the maintaining of integrity of the load during transport that is the pallet load stability, load clampability, approval of different aspects as loading and unloading times, and WIP (Work-In-Process).

#### 2.1 Volumetric Pallet Utilization, VPU

When talking about pallet volume, what is meant is the product of the pallet's base area with the maximum stacking height allowed for the pallet. Volumetric Pallet Utilization, VPU is the percentage of the total pallet volume that is actually filled with boxes. Initially, the VPU is 0% and as boxes are loaded on the pallet, the VPU increases and can reach 100% in optimal conditions. The volumetric pallet utilization can be generally expressed as in equation 2.1.1.

$$VPU (\%) = \frac{\sum_{i=1}^N l_i \times w_i \times h_i}{L_p \times W_p \times H_p} \times 100 \quad (2.1.1)$$

where,  $N$  = the number of boxes on the pallet

$l_i, w_i, h_i$  = the dimensions of box  $i$

$L_p, W_p, H_p$  = the dimensions of the pallet

## 2.2 Work-in-Process, WIP

Work-in-process is defined as the temporary storage space to which boxes are taken in between pallet loading operations. The need for such space is clearly understood when some of the boxes arriving at the end of the conveyor line, are required by the palletization algorithm to lay aside for a while until the time is right for them to be loaded onto the pallet. This adds some flexibility to the decision making process of the algorithm to improve the quality of the palletization with respect to performance criteria. In some situations, these criteria may be the volumetric pallet utilization and/or load stability. It is clear that all decisions of this type are executed on-line during the actual palletization process. While the use of the WIP may help achieving better volumetric pallet utilization and pallet load stability, it is definitely a burden on the palletization time since it will require more number of pick-and-place operations by the robot arm. Palletization time is discussed in more detail in the next section.



### 2.3 Robotic Palletization Time, RPT

Considering that a robot arm is in charge of moving boxes from the conveyor to the pallet or to the WIP or from the WIP to the pallet, one may define '*Robotic Palletization Time, RPT.*' as the total elapsed time taken by the robot to perform all the above mentioned motions until boxes are no longer loaded on the pallet. Assuming that before any action, the robot is always at its home position, the path describing the motion will include going from the home position to the "pick-up" location then, after arrival at "place" location or destination, the robot is to go back to its home position. Therefore one should add the times taken by the robot to go back and forth to and from its home position and the various system locations that are the conveyor, the pallet, and the WIP.

Therefore if the robot command is to pick up a box from the conveyor and place it on the pallet, the overall elapsed time for the command,  $T_{C-P}$  is

$$T_{C-P} = t_{H-C} + t_{C-P} + t_{P-H} \quad (2.3.1)$$

where,  $t_{H-C}$  = robot motion time from the home position to the conveyor,  
 $t_{C-P}$  = time taken by the robot to move a box from the conveyor to the pallet,  
 $t_{P-H}$  = robot motion time from the pallet to the home position,

Similarly, for a conveyor/WIP operation,  $T_{C-WIP}$  is

$$T_{C-WIP} = t_{H-C} + t_{C-WIP} + t_{WIP-H} \quad (2.3.2)$$

and for a WIP/pallet operation,  $T_{WIP-P}$  is

$$T_{WIP-P} = t_{H-WIP} + t_{WIP-P} + t_{P-H} \quad (2.3.3)$$

Furthermore, if

$T_{idle}$  = total idling time in which the robot at its home position, waits for a motion command.

$N_{C-P}$  = the number of times the robot performs a conveyor/pallet operation,

$N_{C-WIP}$  = the number of times the robot performs a conveyor/WIP operation,

$N_{WIP-P}$  = the number of times the robot performs a WIP/pallet operation.

The robotic palletization Time, RPT is calculated by formula:

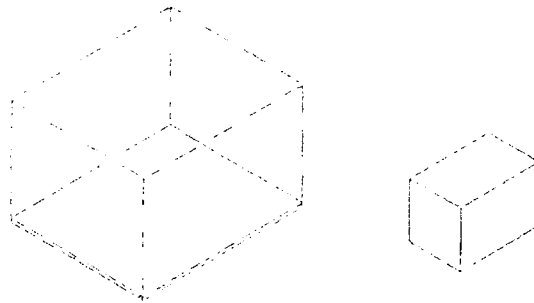
$$RPT = N_{C-P} \times T_{C-P} + N_{C-WIP} \times T_{C-WIP} + N_{WIP-P} \times T_{WIP-P} + T_{idle} \quad (2.3.4)$$

The idling time of the robot is mainly affected by the irregular time intervals between the arrival of boxes on the conveyor at the pick-up location. There are some other physical factors that may increase the idling time of the robot arm which are the computation time required by the palletization algorithm, the response times of hardware components such the conveyor, vision system used for box identification and its operation times. The presence of above factors is essential to the palletization problem and their effect on  $T_{C-P}$ ,  $T_{C-WIP}$ ,  $T_{WIP-P}$ , and  $T_{idle}$  is uncontrollable unless better equipment are used. However conceptually, lower RPT is a result of efficient palletization algorithms which work on minimizing  $N_{C-WIP}$  that is the number of times a box needs to be taken from the conveyor to the WIP. Since for every conveyor/WIP

motion, there is a WIP/pallet equivalent, one can visibly notice the relation between the WIP and the RPT. The less boxes are required to go to the WIP, the lower the overall palletization time and better. Thus, the performance of palletization algorithms is measured by such indexes as WIP and palletization time. Furthermore, these indices are important when comparing palletization algorithms together.

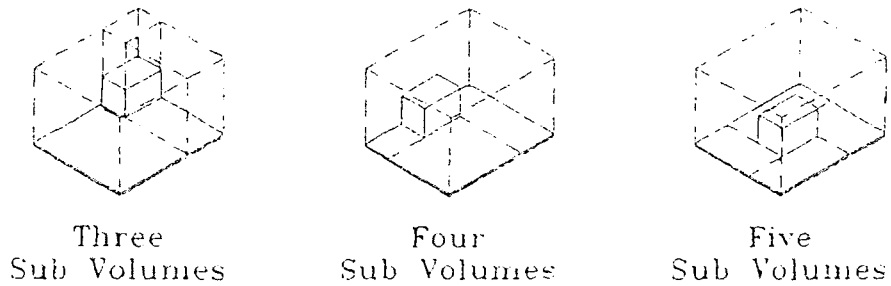
#### 2.4 Total Number of Sub Volumes, TNSV

A *sub volume* is the three-dimensional equivalent of a subarea which is commonly found in the literature of two dimensional palletization algorithms. To introduce the term sub volume, one can look at how the remaining empty space of a pallet of length  $L$ , width  $W$ , and height  $H$  is partitioned when a box of length  $l$ , width  $w$ , and height  $h$  is loaded in it as shown in figures 2-4-1.



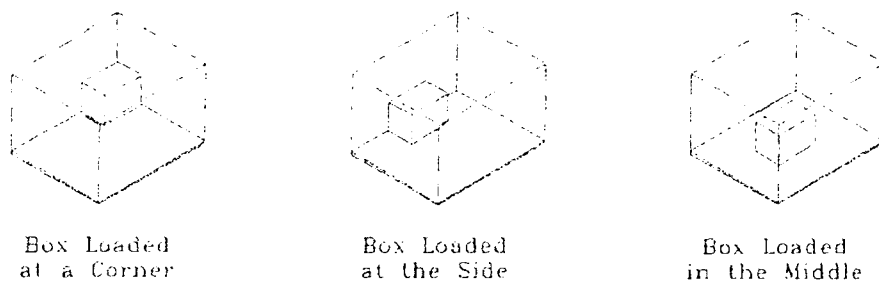
**Figure 2-4-1** Box being loaded on an Empty Pallet

One can load the box within the boundaries of the pallet's base in three typical locations. It can be loaded right in the middle where it does not touch any of the sides of the pallet's base, it can be put such that it touches only one side of the base, and it can touch two sides of the base of the pallet (i.e. in one of the four corners). Figure 2-4-2. represent these three cases.



**Figure 2-4-2** Typical Box Locations on the Pallet

The volumes of the remaining empty space in the three cases are equal. They are the volume of empty pallet minus the volume of the box. However, the way to partition them into small empty rectangular volumes is different. Moreover, the number of these small empty volumes is different in each situations. Refer to figure 2-4-3.



**Figure 2-4-3** Different Number of Sub Volumes for Different Box Locations

Each of these small empty rectangular volumes is called a *sub volume (SV)*. Therefore a sub volume can be defined as: "*a rectangularly shaped three dimensional empty space found*

anywhere at any given time on a pallet." The sum of all such volumes is referred to as Total Number of Sub Volumes, (TNSV).

### 2.5 Partitioned Remaining Volume Load Capacity, PRVLC

If the total number of box types is **TNBT**, Individual Box Type Sub Volume Load Capacity, **ISVLC** is the maximum integer number of a box type that fits in each sub volume. For instance, the ISVLC of sub volume,  $SV_j$  for box type,  $BT_i$  is:

$$ISVLC_{ij} = \text{Max} \left[ \text{integ} \left( \frac{L_{SV_j}}{l_{BT_i}} \right) \times \text{integ} \left( \frac{W_{SV_j}}{w_{BT_i}} \right), \text{integ} \left( \frac{L_{SV_j}}{w_{BT_i}} \right) \times \text{integ} \left( \frac{W_{SV_j}}{l_{BT_i}} \right) \right] \times \text{integ} \left( \frac{H_{SV_j}}{h_{BT_i}} \right)$$

$$\forall i = 1, 2, \dots TNSV$$

$$\forall j = 1, 2, \dots TNBT$$
(2.5.1)

The Maximum Capacity of each Box Type for all sub volumes, **MCBT** is the sum of all ISVLCs for each box type. For instance, the MCBT of  $BT_i$  for all sub volumes is:

$$MCBT_i = \sum_{j=1}^{TNSV} ISVLC_{ij}$$
(2.5.2)

**Partitioned Remaining Volume Load Capacity, PRVLC** defines the maximum filling of all sub volumes with all box types. It is expressed as the sum of the MCBTs for all box types

that can be expected in the system. For instance, for a box among others being tested, its PRVLC is:

$$PRVLC_{TestBox} = \sum_{i=1}^{TNBT} MCBT_i \quad (2.5.3)$$

Regarding the relation between the MCBTs and their PRVLC, It is of great value to discover that it is not only a matter of summation but also a matter of finding the proportion of each box type to the PRVLC. Positive Flexibility of box type  $i$ ,  $PF_i$  is defined as the flexibility of the partitioned pattern to accommodate box type  $i$ . It is determined as follows:

$$PF_i = \frac{MCBT_i}{PRVLC} \quad (2.5.4)$$

Since the proportions of all box types to the PRVLC are independent and mutually exclusive and only one situation may occur at any state analysis, the probability of all situations can be formulated as follows:

The sum of all  $PF_i$ 's = 1.

$$P( PF_1, PF_2, \dots OR PF_{TNBT} ) = P(PF_1) + P(PF_2) + \dots + P(PF_{TNBT}) \quad (2.5.5)$$

When a particular  $PF_i = 0$ , the probability of the partitioned pattern to accommodate box type  $i$  is nil meaning that if a box of type  $i$  comes into the system while this pattern is the actual

layout of the pallet, none of the available sub volumes on the pallet would be large enough for the box to fit into. At this stage, the only feasible solution would be to take the box to the WIP.

## 2.6 Total Zero Count, TZC

If the value of ISVLC of each sub volume for each box type is zero. As opposed to a non zero positive value for ISVLC, **Zero-Count**,  $ZC_{ij} = 1$  whenever  $ISVLC_{ij} = 0$  meaning that box type  $i$  does not fit in sub volume  $j$ . When the  $ZC$ s of all sub volumes for a particular box type are added together, a new parameter called **Zero-Count for a Box Type**, **ZCBT** is produced.

$$ZCBT_i = \sum_{j=1}^{TNSV} ZC_{ij} \quad \forall i = 1, 2, \dots, TNBT \quad (2.6.1)$$

ZCBT can reach a maximum value equal to the TNSV. In such case, none of the TNSVs can accommodate the box type. Furthermore, if the demand for this box type is greater or equal to 1, then the tested box leading to such outcome is eliminated from the race to the pallet.

On another hand, if all the ZCBTs for all box types are added together the resultant is called the **Total Zero-Count**, **TZC** for the partitioned pattern of the among others being tested for loading on the pallet.

$$TZC_{Test\ Box} = \sum_{i=1}^{TNBT} ZCBT_i \quad (2.6.2)$$

It is clear that the smaller the TZC, the better performance will the pattern have in accommodating more box types in a larger number of its sub volumes. Therefore, the box which results in a smaller total zero-count is selected for loading on the pallet. To complete the quantitative analysis of the TZC, it is important to compare it to the maximum value it can possibly take,  $TZC_{max}$ .  $TZC_{max}$  is reached only when the number of remaining sub volumes on the pallet is  $> 0$ , i.e. the pallet is not 100% full and all of the remaining sub volumes are too small to fit any of the box types. For each of the boxes tested for loading priority,  $TZC_{max}$  is expressed as:

$$TZC_{max, Box} = TNBT \times TNSV_{Box} \quad (2.6.3)$$

The proportion of each TZC for each box considered to  $TZC_{max}$  is named **Negative Flexibility, NF** of every box. It is expressed as follows:

$$NF_{Test\ Box} = \frac{TZC_{Test\ Box}}{TZC_{max}} = \frac{TZC_{Test\ Box}}{TNBT \times TNSV} \quad (2.6.4)$$

The concept of negative flexibility is another way of looking at the total zero-count in which one is able to study the effect of the box loading process on the flexibility of the pallet



layout to accommodate box types. One is able to recognize the progressive diminishing of flexibility as the volumetric pallet utilization increases. This is due to the decreasing volume of the remaining empty space on the pallet as boxes are loaded on it.

## CHAPTER 3

### PROPOSED METHODOLOGY

The heuristic by Abdou and El-Masry (1997), which provides the result or in other words the data for the analysis is explained in the next section. Data analysis which forms an important part of this research will be discussed after the previous heuristic followed by the proposed methodology.

#### 3-1 Summary of Previous Heuristic

The previous model developed by Abdou and El-Masry used three new indirect parameters, Total Number of Sub Volumes, Partitioned Remaining Volume Load Capacity and Total Zero Count. These parameters are used according to decision theory, explained later in this section, for box selection. This box selection then generate values of Volumetric Pallet Utilization, Work In Process and Robotic Palletization Time.

##### 3.1.1 Model Inputs and Assumptions

As any other solution to the palletization problem, this heuristic requires some input information and assumptions.

**3.1.1.1 Pallet Dimensions:** The pallet dimensions and the allowable pallet height must be predetermined and given as input to the heuristic.

**3.1.1.2 Stacking and Sub Volume Height:** All sub volumes on the pallet have maximum possible height. That is the maximum allowable height of the pallet minus their respective individual stack height.

**3.1.1.3 Box Dimensions:** The heuristic is flexible to accommodate any box types provided that their dimensions are given as input to the heuristic. If box types are added or removed from the existing set, an appropriate update input should be given to the heuristic.

**3.1.1.4 Box Availability:** There is no upper or lower limit regarding the quantity of each box type involved in the palletization process. As boxes come into the system, they are tested for loading on the pallet.

**3.1.1.5 Box Density and Stackability:** All boxes are assumed to have the same density and the mass is uniformly distributed. In other words, all boxes are stackable and box crushing is assumed not possible.

**3.1.1.6 Nature of Incoming Box Arrival:** Boxes arrive into the system one at a time. The distance between two consecutive boxes is not taken into account. However it is assumed

there are always two boxes on the conveyor, one at the pick up place and one at the vision system location.

### 3.1.2 Model Constraints

The model requires some physical constraints of which some are somehow trivial but important to consider in order to develop a practical solution.

**3.1.2.1 Box Overhanging and Clampability:** In some cases, boxes are not loaded directly off the production line or off the storage shelves onto pallets but are stacked and transported in this form to another area for storage or loading. The normal transportation method employed in these instances is that of a clamp truck -- which clamps the load on two opposite faces. To allow such an operation to be successfully carried out, the stack must possess at least one pair of perfectly flat opposite faces. This requirement is termed '*clampability*.' Carpenter *et al.* (1985) proposed a clampability criterion which demands that at least two opposite stack sides be flat, and set a lower limit of percentage of the length of all box edges parallel to the plane of the clamping (apart from those edges that form the perimeter of a layer) must be in contact with other box edges.

No box overhanging is allowed by this heuristic. That is 100% of the base of any box must have support from boxes underneath. However, since smaller boxes may be loaded on top of larger ones, the clampability requirement is not guaranteed especially when small boxes are loaded near the perimeter of the pallet.

**3.1.2.2 Box Rotation:** Boxes are not allowed to rotate about the X or the Y axis. Hence boxes can have one of two orientations on the pallet according to the rotation about the Z axis.

**3.1.2.3 Sub Volume Constraints:** Since box rotation is permissible, the orientation of a box in a sub volume need not be length to length and width to width as some boxes may have their widths lying along the lengths and perpendicular to the widths of their respective sub volumes. The following equations represent the loading constraints of box  $i$  in sub volume  $j$ .

1. For normal box orientation, i.e.  $l_{BTi}$  is parallel to  $L_{SVj}$ , the box must have an equal or smaller length than the length of the sub volume. The box width must also be less than or equal to the width of the sub volume

$$L_{SVj} - l_{BTi} \geq 0 \tag{3.1.2.1}$$

$$W_{SVj} - w_{BTi} \geq 0$$

2. For the box that has been turned  $90^\circ$  from its normal orientation, i.e.  $l_{BTi}$  is perpendicular to  $L_{SVj}$ , it must have an equal or smaller length than the width of the sub volume and have an equal or smaller width than the length of the sub volume.

$$L_{SVj} - w_{BTi} \geq 0 \tag{3.1.2.2}$$

$$W_{SVj} - l_{BTi} \geq 0$$

3. In both cases, the height of box  $i$ ,  $h_{BTi}$  must always be parallel to the height of the sub volume  $j$ ,  $H_{SVj}$  and:

$$H_{SVj} - h_{BTi} \geq 0 \quad (3.1.2.3)$$

**3.1.2.4 WIP Constraints:** There can be a maximum of one (1) box of every box type in the WIP area.

### 3.1.3 Decision Making Theory

In order to better understand the heuristic, it is of major importance to look into special features of the heuristic such as how to decide about the loading priority of the boxes or how to choose the best partitioning pattern when the available empty space can be partitioned in more than one way. Also, how to determine the sub volume that best suits a particular box.

**3.1.3.1 Priority Levels:** Generally, box selection is based on three main comparison levels. These three priority levels decide which of several boxes is given priority over the others. First, the boxes are compared through the first priority level. If one box is given priority in one level, it is immediately chosen as the best box and is loaded onto the pallet. However, if a tie occurs among several boxes when compared in a priority level, the decision making is postponed to the following level. Finally, if the tie reaches priority level 3 and a decision is not yet made, a box is randomly selected. The three priority levels are individually explained in the following sections.

**3.1.3.2 Priority Level 1, TNSV:** As discussed earlier, among the many positions where one can load a box, one of the heuristic rules forces the boxes to be loaded in one of the corners of the empty space. As the number of sub volumes decreases, their relative individual volumes increases and the larger their volume is, the bigger the chance of loading large boxes as well as smaller sized boxes. Moreover when if a box can completely fill one of the available sub volumes, the number of remaining sub volumes is reduced by one. The application of the discussed rules increases significantly the volumetric pallet utilization. Therefore, the new Total Number of Sub Volumes, (TNSV) remaining if each of the boxes being tested were loaded is chosen as index for priority level 1. It is worth mentioning that sometimes, early in the test, two boxes may equal TNSV. However, in the case of one box, surface leveling may occur and some sub volumes may merge together resulting in a lower new TNSV for that box which gives it advantage over its rival. Priority level 1 is summarized in that the box with smallest index value (TNSV) is the box chosen to load on the pallet.

**3.1.3.3 Priority Level 2, PRVLC:** Priority level 2 is sought only when there is a tie between two boxes in level 1. Since the boxes come into the system in random order, optimization is used to solve for an unknown future. Thus, the procedure used in these circumstances can only be qualitative. Consequently, since each box considered for optimality results in a distinct partitioning pattern of the remaining volume if this box were loaded, the total capacities of all sub volumes to accommodate larger varieties of box types, defines the second level of comparison.

The larger the value of the PRVLC of a box, the more flexible is its corresponding pallet layout to accept a larger variety of box types. Therefore, the box is given better chances of being chosen to load on the pallet. When two boxes result in equal values of PRVLCs, the tie break is postponed to the third comparison level to decide on their loading priority.

**3.1.3.4 Priority Level 2, TZC:** As a final level in which loading priority is decided, the priority level 3 takes into account the handicap of each sub volume of the pattern in terms of not being able to accommodate particular box types. The index for this handicap is called Total Zero Count. And the box which results in a smaller total zero-count is selected for loading on the pallet.

**3.1.3.5 Choosing Between Partitioning Patterns:** When a box is loaded onto the pallet and the remaining empty volume can be partitioned in more than one way. In order to choose the best (optimal) partitioning pattern, their respective PRVLCs are computed and compared at priority level 2. The pattern with higher value is the one chosen. However, if there is a tie among partitioning patterns, the tie break is made at level 3 as discussed above.

**3.1.3.6 Choosing between Sub Volumes:** When there are many sub volumes that can fit a box, there is a need to find which one of them should be chosen to load the box into in order to obtain best results. Since the empty space on the pallet has been previously partitioned into an old pattern made of  $M$  available old sub volumes, the following is the procedure to consider for finding the best loading position for box <sub>$i$</sub> .



- Among the  $M$  old sub volumes, there are  $N$  sub volumes that can accommodate  $\text{box}_i$ ,  
 $N \leq M$ .
- Consider loading  $\text{box}_i$  in each one of the  $N$  sub volumes at a time.
- Starting at  $N = 1$
- The best box orientation in the sub volume is found.
- The remaining empty space of the sub volume used can be partitioned in 2 ways, thus 2 sub patterns.
- Compare the 2 sub patterns at priority levels 2 and 3 if needed.
- The chosen new sub pattern is composed of  $P$  new sub volumes, ( $P_{\max} = 3$ ).
- The  $M-1$  old sub volumes that are intact, when added to the  $P$  new sub volumes, they form one new pattern composed of a  $\text{TNSV} = M - 1 + P$  sub volumes
- If one or some of the  $P$  new sub volumes can be combined with any of the  $M-1$  old sub volumes thus forming  $K$  additional new patterns each having a  $\text{TNSV} \leq M - 1 + P$
- The best of the  $K+1$  patterns is selected according to priority levels 2 and 3.
- Repeat the above procedure for all the  $N$  sub volumes that can accommodate  $\text{box}_i$ , thus  $\text{box}_i$  will have  $N$  PRVLCs and TZC corresponding to the  $N$  sub volumes.
- Select the optimal sub volume to accommodate  $\text{Box}_i$  according to priority level 2 and 3.

### 3.1.4 Overview of the Heuristic

As a new box enters the system, it is given the title of Boxc2 when it arrives under the camera. If no box is present at the end of the conveyor in the pick up place, boxc2 moves to the pick up place and its name changes to Boxc1. Afterward, a new box enters the system and becomes the new Boxc2.

Assuming that there are many boxes in the WIP, they are all compared among each other and the best box in the WIP,  $Box_{WIP}$  is selected according to priority levels 1, 2, and then 3. If Boxc1 and  $Box_{WIP}$  are of the same type, Boxc1 is loaded right away on the pallet since there can only be a maximum of one of each box type in the WIP. Not only will this decision contribute to maximizing the VPU, but will also minimize the RPT since it avoids the possibility of loading  $Box_{WIP}$  first then being obliged to carry Boxc1 to the WIP afterwards in case Boxc2 has priority over it. However, if  $Box_{WIP}$  and Boxc1 are not of the same type, they are compared using priority levels 1, 2, and then 3. If  $Box_{WIP}$  is found to have priority over Boxc1, it is loaded right away on the pallet thus working on reducing the WIP. Then the cycle loops back to find a new  $Box_{WIP}$  among the remaining boxes in the WIP, if any. On the other hand, if Boxc1 has priority over  $Box_{WIP}$  or if there is no boxes left in the WIP, Boxc1 is compared with Boxc2 using priority level 1 only. If the index (new TNSV) for Boxc1 is smaller or equal to that of Boxc2, Boxc1 is loaded on the pallet. Otherwise, boxc1 is carried to the WIP. Boxc2 advances on the conveyor and becomes the new Boxc1 and new box becomes Boxc2. The reason why Boxc1 is only compared with Boxc2 at priority level 1 only is to minimize the number of times Boxc1 must go to the WIP which will have a negative impact upon the RPT as well as the WIP performance measures.

The cycle continues until one of three situations occurs; the pallet is 100% filled, the remaining sub volumes on the pallet are too small for any of the boxes in the system, or boxes stopped entering the system.

### **3-2 Data Analysis**

Data Analysis is an attempt to apply statistics to practical problems. It is a constant effort to transform data analysis situations into situations covered by statistical theory and to adapt statistical methodology for use with practical problems. No assumptions about data are permissible without an analysis.

Two purposes for a data analysis are the comparisons of two or more samples and the exploration of the relationship between two or more variables. The first step in any data analysis is to organize the data in tables that can easily be explained. This usually means listing the data for each sample separate from the other samples and for each relationship explored.

#### **3.2.1 Curve Fitting**

Whenever possible, we try to express, or approximate, relationships between known quantities and quantities that are to be predicted in terms of mathematical equations. This has been very successful in the natural sciences, where it is known, for instance, that at a constant temperature the relationship between the volume,  $y$ , and the pressure,  $x$ , of a gas is given by the formula

$$y = k/x$$

where  $k$  is a numerical constant. Also, it has been shown that the relationship between the size of a culture bacteria,  $y$ , and the length of time,  $x$ , it has been exposed to certain environmental conditions is given by the formula

$$y = a \cdot b^x$$

where  $a$  and  $b$  are numerical constants. More recently, equation like these have also been used to describe relationships in the behavioral sciences, the social sciences, and other fields. For instance, the first equation above is often used in economics to describe the relationship between price and demand, and the second has been used to describe the growth of one's vocabulary or the accumulation of wealth.

Whenever we use observed data to arrive to a mathematical equation which describes the relationship between two variables - a procedure known as **curve fitting** - we must face three kinds of problem

1. We must decide what kind of an equation we want to use ( for instance, that of a straight line or that of a special kind of curve,
2. We must find the particular equation which is "best" in some sense, and
3. We must investigate certain questions regarding the merits of the particular equation, and the of the predictions made from it.

The first kind of problem is sometimes decided by theoretical considerations, but more often by direct inspection of data. We plot the data on graph paper, sometimes on

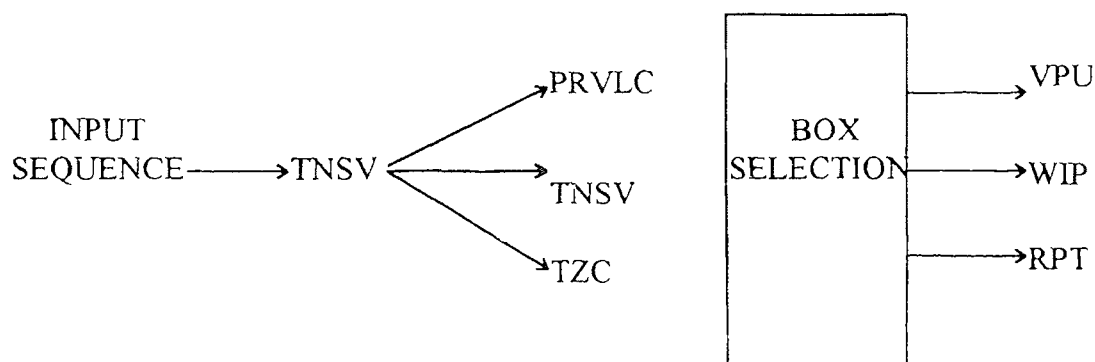
special graph paper with special scales, and we judge visually what kind of curve best describes their overall pattern.

Although there is virtually a limitless choice of forms that can be used for fitting equations to data, actual usage is limited to a very small number of curve types. Unless the observed trends exhibit some special quirk, it is possible to represent the data adequately with one of a relatively repertoire of fairly simple equations.

The second kind of problem, namely finding the equation which in some sense provides the best possible fit. The fit quality criteria include Coefficient of Determination ( $r^2$ ), DOF adjusted  $r^2$ , etc. Coefficient of Determination " $r^2$ " is the square of the correlation between the observed and fitted values of 'y'. Its value ranges from 0 to 1, the better the fit, the closer will  $r^2$  lie towards one.

### 3-3 Proposed Methodology

The previous heuristic by Abdou and El-Masry, explained earlier in the chapter, gives the pattern of data flow as shown in the following figure,

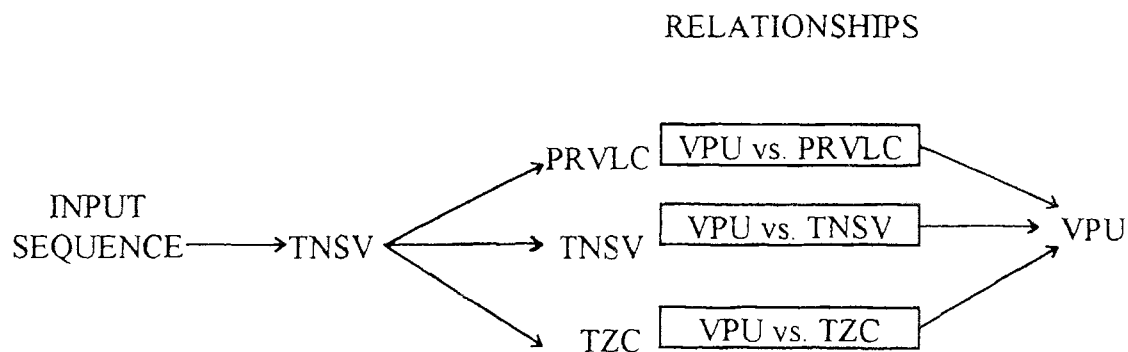


**Figure 3-3-1** Previous Heuristic's Data Flow

The incoming sequence determines the value of TNSV which in turn is used to calculate PRVLC and TZC. Using these three values and going through the decision process a box is selected and loaded on the pallet. This loading creates the value of VPU, WIP and RPT. The process is repeated for each box and at the end of the sequence the final values of VPU, WIP and RPT is determined.

So to select a sequence of arrival for the boxes based on the performance measure such as VPU and other direct parameters. The whole process has to be repeated for each sequence going through all the iterations.

A simpler way to estimate is developed in this research, by using the data obtained from previous heuristic. It uses the values of TNSV, PRVLC and TZC and estimates the value of VPU. This method uses the equation developed for the relationships between VPU and the three above mentioned parameters for estimation. The following figure graphically depicts the proposed data flow.



**Figure 3-3-2** Proposed Data Flow

Following equation shows the estimation procedure,

$$VPU_e = 1/M \sum_{j=1}^3 VPU_j * r_j^2 \quad (3.3.1)$$

where,  $VPU_e$  = estimated VPU

$M$  = summation of  $r^2$  values for the relationships between VPU and the TNSV, PRVLC and TZC.

$j$  = value of  $j$  represent the three combinations of VPU with the indirect variables.

$VPU_j$  = represent the function max. value of VPU for a particular relationship.

$r_j^2$  = represents the corresponding value of  $r^2$  of the relationship.

The estimated values of VPU are checked with the following,

$$\text{mean average deviation} = \text{MAD} = 1/n \sum |e_i| \quad (3.3.2)$$

$$\text{mean square error} = 1/n \sum (e)^2 \quad (3.3.3)$$

$$\text{mean average percentage error} = 1/n \sum |e_i / VPU_a| \quad (3.3.4)$$

where,  $n$  = no. of sequences

$e_i$  = the absolute value of error for each sequence. =  $VPU_e - VPU_a$

$VPU_a$  = actual value of VPU from the previous heuristic.

## CHAPTER 4

### CASE STUDY AND RESULTS

#### 4.1 Output of Previous Heuristic

As explained in the previous chapter TNSV is first calculated and then the values of PRVLC and TZC are obtained using the value of TNSV. After these values are calculated for the options available a box is selected based on the decision theory also explained in the previous chapter.

The box selected this way is loaded on the pallet and then the values of VPU , WIP and RPT are calculated. This procedure is repeated for the whole sequence. The heuristic was simulated for ten sequences and data was collected. The outputs of ten sequences in tabulated form can be found in appendix A-1. An example of the results is shown below,

Table 4-1 Output of Sequence 1

SEQ. 1						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
4	3	244	8	7.03	0	40
5	3	233	8	10.94	0	80
7	5	176	18	20.7	0	120
2	4	160	15	28.52	60	200
6	5	157	21	30.08	0	240
4	4	154	15	31.64	180	320
2	6	119	29	38.67	0	360
8	6	89	32	54.14	0	400
6	8	59	49	61.95	0	440
5	8	51	50	65.86	0	480
1	7	48	44	68.2	60	560
3	6	27	40	77.97	60	600
7	5	10	37	85.78	60	640
6	6	9	44	86.59	60	680
1	6	8	45	87.34	0	720
5	7	0	56	91.25	0	760



Below is the summary of all the outputs for each sequence,

**Table 4-2** Summary of Outputs

Seq #	Sequence	VPU (%)	TNSV Remaining	WIP Remaining	Max. WIP (inches <sup>2</sup> )	RPT (s)
1	4-5-7-2-6-4-2-8-6-5-1-3- 7-6-1-5	91.25	7	0	180	760
2	6-4-5-8-2-8-4-6-4-1-3-5- 1-5-3-2-5-2-2	87.92	7	300	600	880
3	3-2-6-5-4-2-7-4-3-7-2-1- 5-3-6-2-6-3-2-3-1-2-2	89.06	4	0	360	1080
4	5-1-6-7-3-1-5-6-7-3-2-4- 6-2-7-2-3-3-5-1-4-2-2-1	94.92	4	300	375	1120
5	6-2-8-4-6-4-8-1-3-5-7-6- 1-4-2-3-7-5-2-3-3	92.03	5	876	876	960
6	4-5-7-6-7-2-2-4-8-6-1-5- 3-1-6-5	91.25	7	0	375	760
7	8-8-7-4-4-1-2-5-2-2-2-3- 2-7-3-1-5-1	87.19	8	0	60	760
8	4-5-7-2-2-1-7-7-2-2-1-3- 6-5-1-7-2-7-7-2-1	96.09	1	0	375	920
9	2-2-1-8-8-6-1-2-6-3-3-4- 3-8-5-3-3-1	91.72	4	0	300	760
10	4-3-7-7-2-1-6-2-4-1-1-2- 4-6-5-2-3-2-1-3-2-5-6	91.41	4	0	300	1000

## 4.2 Analysis of the Data

The output shows that there are six parameters namely, TNSV, PRVLC, TZC, VPU, WIP and RPT. The total possible combination of these variables is fifteen which are as follows,

TNSV vs. PRVLC	PRVLC vs. TZC	TZC vs. VPU
vs. TZC	vs. VPU	vs. WIP
vs. VPU	vs. WIP	vs. RPT
vs. WIP	vs. RPT	
vs. RPT		
VPU vs. WIP	RPT vs. WIP	
vs. RPT		

This no. of combinations doubles if the parameters are switched between x and y axes. Fifteen of these thirty combinations were selected using the following logic. Since TNSV decides the values of PRVLC and TZC, so TNSV was selected as x-variable in the equation of following form,

$$y = f(x)$$

for the two possible relationships. Since the values of PRVLC show a definite decreasing trend over the iterations, hence it was set as the x-variable in the comparison TZC vs. PRVLC.

As the three direct parameters VPU, WIP and RPT are obtained based on the values of the previous three variables hence they were set as y-variables for the relationship analysis between the two groups.

For the relationships in between the direct variables all the options were analyzed and the significant results were kept for further analysis.

The output and of each sequence and there decided combinations were then analyzed using Table Curve. TABLE CURVE is a software which combines a powerful automated curve fitter with the ability to find the ideal equation to describe an empirical data. Table Curve automatically select and rank best fit equations for a X,Y data set.

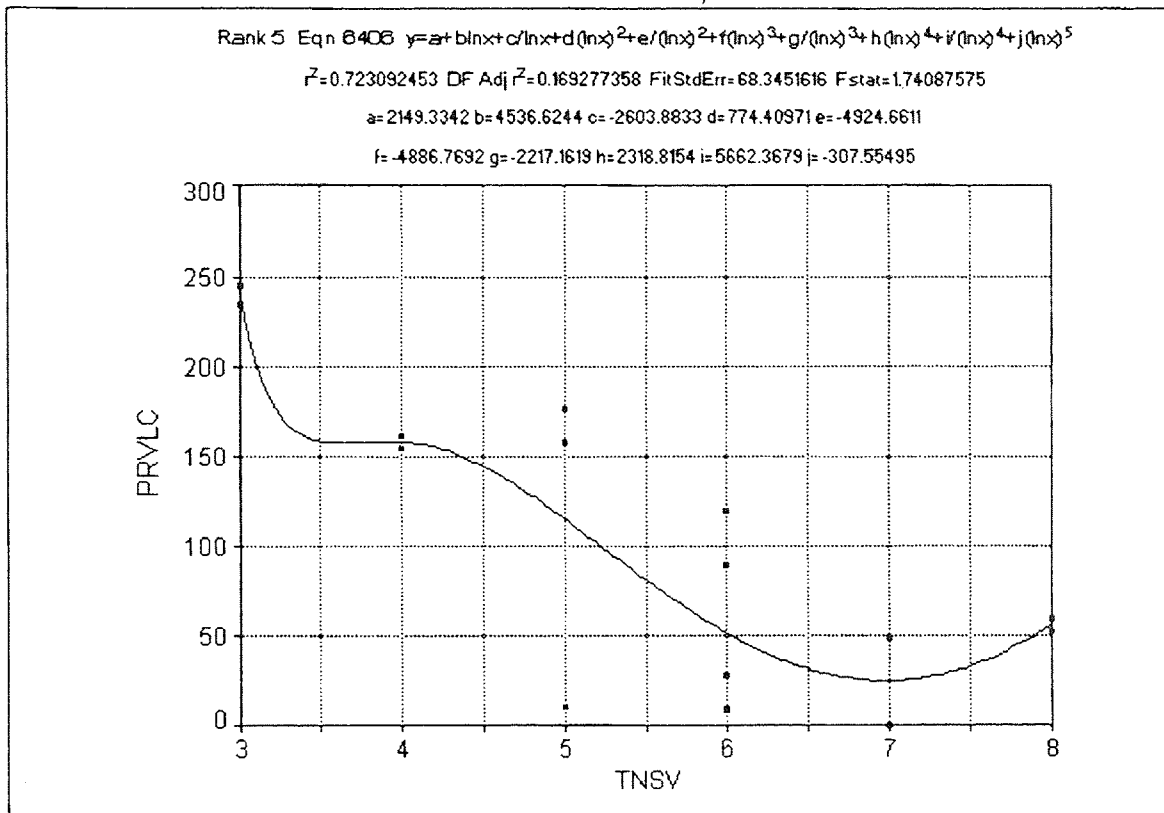
Table Curve has over 3600 built-in equations which it can fit to a data set. It then ranks them according to the criteria selected. The fit quality criteria options include Coefficient of Determination ( $r^2$ ), DOF adjusted  $r^2$ , root MSE and F statistic. It also displays the fitted graphs and equation for review. Numeric summary and other information can also be obtained.

This software is made by JANDEL Scientific Software which is based in California.

### **4.3 Results Obtained from Data Analysis**

Using the data obtained from the previous heuristic and the decided combinations, and then analyzing them using Table Curve a equation list was obtained. The list was ranked on the basis of the values of  $r^2$ . Since Table Curve fit thousands of Equations to dataset hence it is impossible to include the lists in this thesis write-up, as it runs into hundreds of pages for all the relationships. A sample of results for sequence 1, which shows the curve

of an equation and its numeric summary, is shown in appendix C. The results shown are only for one equation selected for each relationship. A sample curve is shown below, it shows the relation between PRVLC and TNSV. The selection of equation is discussed in the next chapter.



**Figure 4-3-1** Curve between PRVLC and TNSV.

## CHAPTER 5

### ANALYSIS OF RESULTS

#### 5.1 Selection of an Equation

As mentioned in the previous chapter there is a list of thousand of Equations for each sequence in each relationship. To decide on a single equation for a relationship a simple rationale was used. Firstly those equations were selected which have ranked first in at least one sequence, and then from these equations one which has better average  $r^2$  value over the ten sequences was selected.

For example see the following table,

**Table 5-1** Table for selection of Equation for PRVLC VS. TNSV

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
6303	0.723	0.947	0.484	0.748	0.781	0.81	0.791	0.511	0.798	0.765
6308	0.723	0.947	0.501	0.748	0.779	0.821	0.79	0.511	0.798	0.777
6407	0.723	0.947	0.503	0.748	0.793	0.821	0.791	0.511	0.798	0.792
6202	0.721	0.946	0.477	0.748	0.761	0.81	0.779	0.494	0.708	0.762
6604	0.723	0.947	0.502	0.748	0.796	0.809	0.792	0.511	0.798	0.783
6403	0.723	0.947	0.503	0.748	0.789	0.821	0.791	0.511	0.798	0.791
6307	0.723	0.947	0.501	0.748	0.775	0.821	0.79	0.511	0.798	0.772
<b>6406</b>	<b>0.723</b>	<b>0.947</b>	<b>0.503</b>	<b>0.748</b>	<b>0.793</b>	<b>0.821</b>	<b>0.791</b>	<b>0.511</b>	<b>0.798</b>	<b>0.792</b>
6303	$y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5$									
6307	$y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5+g(\ln x)^6+h(\ln x)^7+i(\ln x)^8+j(\ln x)^9$									
<b>6406</b>	<b><math>y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^3+f(\ln x)^4+g/(\ln x)^5+h(\ln x)^6+i/(\ln x)^7+j(\ln x)^8</math></b>									

**Table 5-1** (continued)

6407	$y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3+h(\ln x)^4+i/(\ln x)^4+j(\ln x)^5+k/(\ln x)^5$
6403	$y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3$
6308	$y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5+g(\ln x)^6+h(\ln x)^7+i(\ln x)^8+j(\ln x)^9+k(\ln x)^{10}$
6604	$y=a+b/\ln x+c/(\ln x)^2+d/(\ln x)^3+e/(\ln x)^4+f/(\ln x)^5$
6202	$y=a+bx+c/x+dx^2+e/x^2+fx^3+g/x^3$

The number before each equation comes from the Table Curve List of Equations. Of the equations in the above table the highest average  $r^2$  over the ten sequence is of equation no. 6406. Hence this equation was selected for further analysis. The rest of the tables are presented in appendix A-2.

A listing of the selected equations is tabled below,

**Table 5-2** List of Selected Equations

RELATIONSHIP	EQUATION
PRVLC & TNSV	$y=a+b \ln x+c/\ln x+d \ln x^2+e/\ln x^2+f \ln x^3+g/\ln x^3+h \ln x^4+i/\ln x^4+j \ln x^5$
TZC & TNSV	$y=a+b \ln x+c/\ln x+d \ln x^2+e/\ln x^2+f \ln x^3+g/\ln x^3+h \ln x^4+i/\ln x^4+j \ln x^5$
TZC & PRVLC	$y=a+b x+c x^{2.5}+d x^{0.5}+e e^{-x}$
VPU & TNSV	$y=a+b \ln x+c/\ln x+d \ln x^2+e/\ln x^2+f \ln x^3+g/\ln x^3+h \ln x^4+i/\ln x^4+j \ln x^5$
VPU & PRVLC	$y=a+b x+c x^2+d x^{2.5}+e x^3$
VPU & TZC	$y=a+b/\ln x+c/\ln x^2+d/\ln x^3+e/\ln x^4+f/\ln x^5$
RPT & TZC	$y=a+b x+c x^{2.5}+d x^{3.0}+e e^{-x}$
RPT & TNSV	$y=a+b \ln x+c/\ln x+d \ln x^2+e/\ln x^2+f \ln x^3+g/\ln x^3+h \ln x^4+i/\ln x^4+j \ln x^5$
RPT & PRVLC	$y=a+b x+c x^2+d x^{2.5}+e x^3$
WIP & PRVLC	$y=a+b \exp(-\exp(-((x-c)/d)))-((x-c)/d)+1$ [ExtrVal]

## 5.2 Analysis of Equations and their Parameters

The following is an example of the table listing the equation selected for a relationship with its parameters for each sequence. It also includes the  $r^2$  values, the rest of the tables are in appendix B.

**Table 5-3** Selected Equation for PRVLC vs. TNSV

SEQ.	1	2	3	4	5	6	7	8	9	10
6406	$y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3+h(\ln x)^4+i/(\ln x)^4+j(\ln x)^5$									
PARAMETER										
a	2149.3342	-1136.539	18192.188	-539.3867	233114.39	-24672.55	-1918.44	-920.89	-3394.09	10622.35
b	4536.6244	1168.569	-24908.603	-481.9936	-65074.53	49896.42	-15219.08	-833.70	-3382.21	-10660.14
c	-2603.8832	-1561.414	32536.521	47.28686	144575.02	-56960.62	12269.79	-43.59	116.55	15005.46
d	774.40970	1415.878	-24998.173	214.7247	-258093.21	39593.71	-7184.52	356.49	859.88	-14144.60
e	-4924.6611	610.6598	-3062.8733	728.8548	-171444.18	-448.07	14309.60	935.73	3834.31	-2984.69
f	-4886.7692	-1916.727	35371.283	632.1314	223093.33	-70578.88	18715.52	1220.81	4287.36	17161.63
g	-2217.1619	2422.446	-40399.895	750.6003	-263601.49	66826.67	2779.45	991.20	3881.72	-18863.97
h	2318.8154	768.0789	-13739.836	-488.2629	-69542.79	31203.37	-8534.14	-816.66	-2621.90	-6147.00
i	5662.3678	-1373.752	19196.406	-683.0986	222160.55	-29627.37	-17445.27	-870.36	-3485.74	9371.55
j	-307.55494	-105.6594	1779.6070	97.69814	7719.81	-4561.09	1218.41	133.58	410.17	731.37
R <sup>2</sup>	0.723	0.947	0.503	0.748	0.793	0.821	0.791	0.511	0.798	0.792

Including ten such tables for the purpose of analysis in this chapter would take too much space. Hence relationships which have the same equation selected were combined together to make a table. The first table (Table 5-4) gives information of relationships

having equation 6406, which is written in table 5-3. As mentioned earlier separate tables for each equation for each relationship are presented in appendix B.

The analysis of the table 5-4 for PRVLC vs. TNSV, shows an average  $r^2$  of about 0.742 which shows there is relationship between PRVLC and TNSV though not very strong but still significant. The equation can be used to establish relationship between the two variables. There is a lot of variation in the values of the parameters over the ten sequence hence no common parameter list can be generated.

The relationship between TZC vs. TNSV as shown by the  $r^2$  of the selected equation is quite good. The value shows that TNSV has a better relationship with TZC as compared to PRVLC. But the value of the parameters vary a lot over the ten sequences here also.

The values of  $r^2$  in the table for RPT vs. TNSV is small for most of the sequences, this indicates that the relationship between these two variables is weaker. The parameter values also vary a lot, hence not much could be done with this equation.

The relationship between VPU and TNSV is not very strong as shown by some very low values of  $r^2$  in the following table. The large variation of the parameter values over the ten sequence also preclude any general parameter list.



Table 5-4 Parameters' Values of Relationships using Equation 6406

SEQ.	PARAMETER	1	2	3	4	5	6	7	8	9	10
a	PRVLC vs. TNSV	2149.33	-1136.54	18192.19	-539.39	233114.39	-24672.55	-1918.44	-920.89	-3394.09	10622.35
b		4536.62	1168.57	-24908.60	-481.99	-65074.53	49896.42	-15219.08	-833.70	-3382.21	-10660.14
c		-2603.88	-1561.41	32536.52	47.29	144575.02	-56980.62	12269.79	-43.59	116.55	15005.46
d		774.41	1415.88	-24998.17	214.72	-258093.21	39593.71	-7184.52	356.49	859.88	-14144.60
e		-4924.66	610.66	-3062.87	728.85	-171444.18	-448.07	14309.60	935.73	3834.31	-2984.69
f		-4886.77	-1916.73	35371.28	632.13	223093.33	-70578.88	18715.52	1220.81	4287.36	17161.63
g		-2217.16	2422.45	-40399.90	750.60	-263601.49	66826.67	2779.45	991.20	3881.72	-18863.97
h		2318.82	768.08	-13739.84	-488.26	-69542.79	31203.37	-8534.14	-816.66	-2621.90	-6147.00
i		5662.37	-1373.75	19196.41	-683.10	222160.55	-29827.37	-17445.27	-870.36	-3485.74	9371.55
j		-307.55	-105.66	1779.61	97.70	7719.81	-4561.09	1218.41	133.58	410.17	731.37
R <sup>2</sup>		0.723	0.947	0.503	0.748	0.793	0.821	0.791	0.511	0.798	0.792
a	TZC vs. TNSV	1083.04	5992.40	-3342.31	134.93	-118093.62	3865.27	-2041.97	52.49	262.51	-1451.85
b		1620.85	10017.39	6089.45	73.58	47981.66	-7174.55	-12214.45	43.93	257.87	1514.00
c		-590.07	-11536.85	-7094.16	48.35	-83789.60	8449.58	9200.07	9.43	0.44	-2080.67
d		-162.93	9303.68	5181.31	-101.41	139330.63	-5972.83	-5314.50	-20.97	-64.60	1965.01
e		-1611.92	629.92	152.25	-89.32	79048.04	-82.28	11115.04	-41.79	-280.21	411.20
f		-2167.91	-15285.57	-8794.44	-154.95	-135967.62	10073.16	15297.94	-64.25	-322.57	-2433.80
g		-924.65	13349.49	8364.31	-129.02	135106.24	-10020.67	2426.58	-48.62	-288.72	2621.72
h		1443.18	6652.56	3840.37	175.15	46726.46	-4297.53	-7192.39	49.06	201.33	889.04
i		1625.85	-6107.57	-3792.96	89.67	-105631.78	4522.98	-13134.16	39.81	255.63	-1302.75
j		-270.52	-969.97	-559.87	-43.19	-5724.42	604.99	1050.50	-8.78	-32.11	-107.93
R <sup>2</sup>		0.871	0.950	0.733	0.858	0.851	0.975	0.562	0.845	0.880	0.933

Table 5-4 (continued)

a	VPU vs. TNSV	79.91	1118.62	-5517.85	271.95	-24233.45	10298.07	1092.70	417.05	1291.30	-4322.03
b		-295.14	-1454.71	7240.61	230.11	-923.72	-20786.98	7999.76	370.17	1272.26	4414.13
c		420.56	1846.52	-9616.77	20.99	-9553.44	23778.16	-6329.59	50.90	-4.98	-6134.75
d		-336.29	-1536.28	7488.30	-94.56	22445.47	-16488.24	3723.28	-144.00	-318.24	5816.08
e		447.99	-278.14	1049.40	-285.05	21959.82	226.21	-7439.87	-367.27	-1389.91	1237.17
f		54.34	2266.74	-10209.13	-285.47	-11513.25	29429.12	-9843.81	-516.72	-1597.77	-7072.14
g		72.65	-2319.53	12082.89	-316.18	26675.84	-27825.71	-1473.94	-407.12	-1427.24	7746.90
h		246.09	-929.67	3850.48	219.38	1416.01	-13019.86	4508.57	344.03	975.78	2534.92
i		-646.14	1146.26	-5790.68	275.50	-26832.80	12402.79	9036.74	347.46	1269.57	-3852.85
j		-85.32	128.61	-481.67	-43.67	107.73	1904.36	-645.68	-56.24	-152.50	-301.71
R <sup>2</sup>		0.634	0.903	0.456	0.715	0.716	0.811	0.654	0.588	0.727	0.787
a	RPT vs. TNSV	2822.79	-3011.38	-47815.48	-702.38	91932.72	99752.05	21054.85	5705.90	9368.50	-60482.51
b		-1430.86	40684.56	59164.03	-181.08	-167311.22	-198521.84	148875.64	5055.67	9269.27	92175.02
c		4865.37	-51081.22	-80677.02	-499.30	157783.19	228169.05	-116936.37	684.16	-58.52	-86154.68
d		-4724.14	42710.80	63709.91	772.60	-182655.79	-158660.89	68477.26	-2003.37	-2239.33	81542.30
e		3629.43	5735.16	9991.88	87.88	8381.34	1355.39	-138034.08	-5058.90	-10107.35	17138.79
f		-2597.75	-61502.60	-82654.33	943.20	303040.17	280809.73	-183904.55	-7103.86	-11552.50	-99680.34
g		-493.82	61742.27	102331.31	456.11	-117554.52	-267733.89	-27786.39	-5618.69	-10361.28	108555.3
h		5260.67	24724.77	29776.13	-871.95	-134536.62	-123722.93	84614.72	4744.84	6996.57	35850.88
i		-6461.45	-29297.84	-49415.76	-54.54	18766.70	119652.43	166944.37	4713.95	9234.06	-53888.79
j		-1459.40	-3342.36	-3500.15	174.64	19834.75	18035.62	-12153.06	-777.58	-1087.32	-4278.64
R <sup>2</sup>		0.655	0.732	0.422	0.688	0.645	0.805	0.458	0.461	0.729	0.752

The next combined table is for relationships using Equation 4341, which is

$$y=a+bx+cx^2+dx^{2.5}+ex^3$$

VPU vs. PRVLC has very high values of  $r^2$  as shown in following table, this indicates a strong relationship. The parameter values also show a definite trend and the curve resulting is very much linear. The graphs for all these relationships for sequence 1 are tabulated in appendix C.

Table 5-5 also shows the relationship between RPT and PRVLC. The values of  $r^2$  are quite high showing a strong relation between the two variables. The parameter values has a definite trend, some near zero values of the last three parameters gives the curve some linearity.

Table 5-5 Parameters' Values of Relationships using Equation 4341

SEQ.		1	2	3	4	5	6	7	8	9	10
PARAMETER											
a	RPT vs. PRVLC	763.73	887.80	1038.32	1077.4	944.58	731.66	722.30	903.23	716.4	1046.84
b		-10.61	-10.85	-10.48	-10.22	-13.39	-0.64	-0.60	-12.34	-4.95	-10.78
c		0.26	-0.04	0.08	0.05	0.12	-0.16	-0.50	0.56	0.03	0.24
d		-0.03	0.01	-0.00	0.00	-0.00	0.02	0.07	-0.07	-0.00	-0.03
e		0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00
R^2		0.978	0.984	0.985	0.988	0.988	0.977	0.971	0.979	0.980	0.975
a	VPU vs. PRVLC	91.59	87.94	87.39	93.51	90.99	89.66	86.76	96.31	91.76	92.20
b		-0.60	-0.45	-0.33	-0.27	-0.43	-0.19	-0.33	-0.75	-0.66	-0.79
c		-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.02	0.02	0.00	0.02
d		-0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
e		0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00
R^2		0.998	0.999	0.996	0.997	0.999	0.996	0.999	0.998	0.999	0.997

The rest of the relationships to be discussed have separate equations. So they are combined in one table (Table 5-6). They include following equations,

$$4649 \quad y=a+b x+c x^{2.5}+d x^{0.5}+e e^{-x}$$

$$6604 \quad y=a+b/\ln x+c/\ln x^2+d/\ln x^3+e/\ln x^4+f/\ln x^5$$

$$4594 \quad y=a+b x+c x^{2.5}+d x^{3.0}+e e^{-x}$$

$$8006 \quad y=a+b \exp(-\exp(-((x-c)/d))-((x-c)/d)+1) [\text{ExtrVal}]$$

The next relationship analyzed is between TZC and PRVLC as shown in the following table, the values of  $r^2$  is quite high implying a good relationship between the two variables. The values of some parameters specially value of parameter 'c' is very small.

The relationship between VPU and TZC is also quite strong as shown in table 5-6. The parameter values however vary a lot over the different sequences.

On the other hand the relationship between RPT and TZC though not very strong as shown in the table, shows a trend in the values of the parameters. These value keep the curve linear for most part of it.

The relationship between WIP and PRVLC has varying value of  $r^2$ . The values range from 0.209 to 0.999, this shows the relationship is random and very much depends on the sequence of arrival of boxes.

The other relationships for example WIP and TNSV were quite insignificant, hence they are not included in this analysis.

Table 5-6 Parameters' Values of Relationships using Different Equations

SEQ.	PARAMETER	1	2	3	4	5	6	7	8	9	10
a	TZC vs. PRVLC	19.98	63.49	33.81	55.45	49.21	51.19	33.48	25.61	44.48	15.15
b		-0.93	-21816282.00	-0.02	0.15	-0.17	-0.21	-1.81	0.41	0.18	-0.85
c		0.00	0.00	-0.00	-0.00	-0.00	0.00	0.00	-0.00	-0.00	0.00
d		9.97	-1.92	-0.23	-5.27	1.20	-1.23	16.16	-4.45	-2.93	9.09
e		36.02	-63.22	-33.53	-52.51	-49.28	-51.19	-27.17	-24.75	-42.68	-15.15
R <sup>2</sup>		0.942	0.953	0.816	0.898	0.945	0.944	0.889	0.575	0.951	0.877
a	VPU vs. TZC	-8377.22	144.25	94.03	114.20	356.86	178.73	-125.90	2118.09	410.55	87.42
b		98033.92	2702.31	280.62	992.14	12300.67	4187.76	-9530.44	94496.19	15526.14	-159.15
c		-390266.20	-29063.33	-559.06	-6641.00	-157409.30	-45870.34	1446561.82	-1172900.00	-154013.62	3672.19
d		540849.79	102587.00	-6306.78	11172.95	672500.76	168025.86	-713519.29	4897300.00	491056.01	-19837.99
e		115807.60	-157099.30	16057.43	-4515.78	-1240100.00	-266178.17	1472430.00	-8787000.00	-617758.83	32245.66
f		-577290.43	88789.10	-9819.41	-1688.45	840359.25	154563.63	-1093500.00	5792940.00	239909.62	-14018.44
R <sup>2</sup>		0.889	0.958	0.745	0.870	0.812	0.930	0.779	0.552	0.822	0.889
a	RPT vs. TZC	102.70	53.27	-51.54	133.99	-72.43	44.87	240.08	-3392.08	-55.66	-123.88
b		2.38	4.21	28.08	10.02	14.21	10.58	-22.84	608.82	18.11	25.89
c		0.13	0.05	0.07	0.28	-0.04	0.14	0.22	-13.18	-0.21	0.07
d		-0.02	-0.00	-0.02	-0.04	0.00	-0.02	-0.02	1.89	0.04	-0.01
e		-231448.53	826.94	1136.78	987.33	1032.45	715.43	519.90	4312.08	814.33	1123.85
R <sup>2</sup>		0.818	0.867	0.665	0.872	0.677	0.892	0.673	0.564	0.768	0.867

Table 5-6 (continued)

a	WIP vs. PRVLC	20.00	1.54	49.09	189.44	68.66	-0.03	-176.23	-0.67	-23.76
b		2325.71	564.32	426.09	3578.14	2736.77	645.22	622.83	10123.99	247.57
c		151.25	17.15	197.59	-307.41	36.18	38.97	74.76	103.23	79.34
d		-1.64	15.22	0.67	109.93	6.48	0.61	61.30	4.44	34.48
R <sup>2</sup>		0.666	0.902	0.209	0.652	0.478	0.999	0.691	0.839	0.528

### 5.3 VPU Estimation

Using the results and the analysis done above, a method is developed to estimate the value of VPU based on  $r^2$  values and the variables TNSV, PRVLC and TZC. For each sequence a ratio is assigned to each relationships based on the  $r^2$  values. This ratio is multiplied by the maximum value of VPU in the function. However if the maximum function value of VPU is greater than 100, it is assigned the value of 100%. The results of those multiplication for the three relationships are then added giving the estimated VPU.

For example, for sequence 1 the summation of  $r^2$  values for the three relationships are,

$$0.634 + 0.889 + 0.998 = 2.521$$

Now the ratio assigned to each relationship is as follows,

$$\text{VPU vs. TNSV} = 0.634/2.521 = 0.2515$$

$$\text{VPU vs. TZC} = 0.889/2.521 = 0.3526$$

$$\text{VPU vs. PRVLC} = 0.998/2.521 = 0.3959$$

Multiplying these ratios with the respective peak values of VPU gives,

$$\text{VPU vs. TNSV} : 0.2515 * 79.74 = 20.05461$$

$$\text{VPU vs. TZC} : 0.3526 * 94.04 = 33.158504$$



$$\text{VPU vs. PRVLC : } 0.3959 * 91.59 = 36.260481$$

Adding the above values gives the estimated value of VPU as 89.47 % which is quite close to the actual VPU of 91.25 %.

The values for all the sequences are tabulated on the below with the absolute difference between the actual VPU and estimated VPU.

**Table 5-7** Table for Estimation of VPU

SEQ	VPU	max VPU	max VPU	max VPU	R <sup>2</sup>	R <sup>2</sup>	R <sup>2</sup>	VPU	Absolute
	(actual)	w.r.t TNSV	w.r.t TZC	w.r.t PRVLC	VPU vs. TNSV	VPU vs. TZC	VPU vs. PRVLC	estimated	VPUa-VPUE
1	91.25	79.74	94.04	91.59	0.634	0.889	0.998	89.47	1.78
2	87.92	87.97	100.00	87.94	0.903	0.958	0.999	91.99	4.07
3	89.06	100.00	76.44	87.39	0.456	0.745	0.996	86.29	2.77
4	94.92	100.00	95.47	93.51	0.715	0.87	0.997	95.97	1.05
5	92.03	76.41	100.00	90.99	0.716	0.812	0.999	89.75	2.28
6	91.25	88.12	100.00	89.63	0.811	0.93	0.996	92.71	1.46
7	87.19	72.24	69.79	86.76	0.654	0.779	0.999	77.42	9.77
8	96.09	100.00	100.00	96.31	0.588	0.552	0.998	98.28	2.19
9	91.72	100.00	100.00	91.76	0.727	0.822	0.999	96.77	5.05
10	91.41	100.00	100	92.2	0.788	0.889	0.997	97.09	5.68
								MAD	3.61
								MAPE	0.04
								MSE	19.37

The values of MAD, MSE and especially MAPE is very low showing that the estimated value is quite good. This estimated value hence can be used to distinguish between the box sequences.

## CHAPTER 6

### CONCLUSIONS AND RECOMMENDATIONS

This study includes three direct parameters, Volumetric Pallet Utilization (VPU), Work in Process (WIP), and Robotic Palletization Time (RPT). Also it includes three indirect variables which are Total Number of Sub Volumes (TNSV), Partitioned Remaining Volume Load Capacity (PRVLC), and Total Zero Count (TZC). These parameters have thirty possible combinations for comparisons, of which fifteen logical combinations were selected. Using the Table curve software these relationships were analyzed and data was fitted to the equations. Of these ten relationships were found to be significant and were used for further analysis. A single equation was selected for each relationship. The mathematical equations established for each combination gives an insight into parameter relationship. Four important conclusions can be made from this study.

Firstly, the relationships between TNSV and PRVLC, TZC, VPU and RPT shows an interesting result, they all follow the same pattern and is represented by the same equation eqn. no. 6406, but with different parameters. Similarly relationships of PRVLC with RPT and VPU also have the same and is represented by the same equation eqn. no. 4341. In addition to that, in these two relationships the parameters' values also have a conspicuous trend. The values for parameter '*c*', '*d*' and '*e*' are almost zero, this gives the resultant graph a linear trend. The other relationships are represented by different equations.

Secondly, with these results and using the mathematical relationships of VPU with TNSV, PRVLC and TZC, and the values of  $r^2$  for these combinations, an estimation method has been developed. In the previous heuristic all sequences have to be simulated to obtain the final values of VPU for an input box sequence. However, by applying the proposed method VPU value can be estimated accurately, as shown by the low MAPE value of 0.04, for all sequences and the optimal one can be selected for the pallet loading, without using extensive simulation.

These equations and rest of the analysis also show that VPU and PRVLC have a very strong relationship having  $r^2$  values ranging from 0.996 - 0.999. So instead of having TNSV as first priority check in a heuristic, PRVLC could be used as first priority level and the resultant data can be analyzed for improvement.

It is also concluded that the relationship of WIP with rest of the parameters was quite random, mainly because it very much depends on the incoming sequence of the boxes, which is randomly generated. Future studies could test different incoming sequences to ascertain sequences which can reduce WIP.

This research work has generated new ideas in palletization research, with respect to practical problems that often use the heuristic approach. The inspiration from real-life palletization scenarios whether in warehousing or transportation, can only bring more ideas that need to be pursued and developed to improve and go beyond what has already been achieved.

Future study may include the following:

- 1- Physically implement the proposed models.

- 2- Improve and generalize the current models for more robust industrial applications.
- 3- Study extensively the newly established performance measures and indices.
- 4- Incorporate CAD simulations to existing or new models. This will have a great impact on the visualization part of the research as well as enhance new design concepts.
- 5- Develop estimation models for Robotic Palletization Time and Work in Process based on further data analysis.
- 6- The estimation model for VPU could be further improved by taking into account the inter-relationship of TNSV, PRVLC, and TZC.

## APPENDIX A-1

### TABLES OF VARIABLES FROM THE PREVIOUS HEURISTIC

**Table A-1-1**      Output of Sequence 1

SEQ. 1						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
4	3	244	8	7.03	0	40
5	3	233	8	10.94	0	80
7	5	176	18	20.7	0	120
6	4	160	15	28.52	60	200
2	5	157	21	30.08	0	240
2	4	154	15	31.64	180	320
4	6	119	29	38.67	0	360
8	6	89	32	54.14	0	400
6	8	59	49	61.95	0	440
5	8	51	50	65.86	0	480
3	7	48	44	68.2	60	560
7	6	27	40	77.97	60	600
6	5	10	37	85.78	60	640
1	6	9	44	86.59	60	680
1	6	8	45	87.34	0	720
5	7	0	56	91.25	0	760

**Table A-1-2**      Output of Sequence 2

SEQ. 2						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
6	3	240	7	7.81	0	40
4	3	226	9	14.84	0	80
5	2	217	5	18.75	0	120
8	4	150	15	34.22	0	160
8	4	95	20	46.69	60	240
2	6	75	34	51.25	0	280
4	7	66	42	58.28	0	320
4	7	43	43	65.31	300	400
3	7	39	43	67.66	360	480
5	7	31	44	71.56	360	520
1	8	30	51	72.34	360	560
1	8	29	52	73.13	300	600
3	7	26	46	75.47	600	680
2	6	23	40	77.03	600	720
5	7	15	51	80.94	600	760
5	7	6	54	84.84	300	800
2	8	3	62	86.41	300	840
2	10	0	0	87.97	300	880

**Table A-1-3**      Output of Sequence 3

SEQ. 3						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
3	3	253	6	2.34	0	40
6	3	235	7	10.16	60	120
4	2	223	3	17.19	360	200
2	3	217	11	18.75	360	240
2	4	197	13	20.31	300	280
5	4	190	14	24.22	0	320
7	6	139	23	33.98	0	360
4	8	111	38	41.02	0	400
3	7	108	32	43.36	0	440
7	6	87	28	53.13	0	480
2	5	84	22	54.69	0	520
5	5	73	24	58.59	60	600
3	5	69	26	60.94	60	640
1	5	68	26	61.72	0	680
6	6	38	35	69.53	0	720
6	5	21	32	77.34	60	800
3	5	16	33	79.69	60	840
2	6	13	41	81.25	60	880
2	5	10	35	82.81	0	920
3	4	7	29	85.16	0	960
1	3	6	22	85.94	0	1000
2	5	3	38	87.5	0	1040
2	10	0	0	89.06	0	1080

Table A-1-4 Output of Sequence 4

SEQ. 4						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
5	3	247	6	3.91	0	40
6	3	229	9	11.27	60	120
1	4	227	17	12.5	0	160
3	3	224	11	14.84	375	240
1	2	223	4	15.63	375	280
5	3	203	6	19.53	375	320
6	3	185	9	27.34	375	360
7	5	134	18	37.11	375	400
7	4	120	13	46.88	0	440
3	4	116	15	49.22	0	480
4	4	102	17	56.25	60	560
6	4	76	17	64.06	60	600
2	5	73	27	65.63	60	640
7	4	52	23	75.39	60	680
2	5	43	31	76.95	60	720
3	4	40	25	79.3	60	760
2	4	37	27	80.86	0	800
3	6	20	41	83.2	0	840
4	5	8	37	90.23	360	960
1	5	7	37	91.02	300	1000
2	6	4	45	92.58	300	1040
2	5	1	39	94.14	300	1080
1	10	0	0	94.92	300	1120



**Table A-1-5**      Output of Sequence 5

SEQ. 5						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
6	3	240	7	7.81	0	40
2	4	234	15	9.38	0	80
4	4	220	17	16.41	396	160
6	5	193	20	24.22	396	200
4	7	158	30	31.25	396	240
8	9	86	48	46.72	396	280
1	9	87	50	47.5	396	320
3	8	85	44	49.84	396	360
5	8	76	47	53.75	396	400
6	7	59	44	61.56	771	480
1	6	58	37	62.34	771	520
7	8	36	54	72.11	396	560
7	7	15	50	81.88	816	720
3	7	13	50	84.22	636	760
2	8	10	58	85.78	576	800
2	7	7	52	87.34	876	880
3	6	4	46	89.69	876	920
3	10	0	0	92.03	876	960

**Table A-1-6**      Output of Sequence 6

SEQ. 6						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
4	3	244	8	7.03	0	40
5	3	233	8	10.94	0	80
6	2	217	5	18.75	375	160
7	4	160	15	28.52	375	200
7	3	139	11	38.28	0	240
2	4	136	17	39.84	0	280
2	3	133	11	41.41	0	320
4	5	98	25	48.44	0	360
8	5	68	28	63.91	0	400
5	5	60	29	67.81	360	520
3	4	57	23	70.16	360	560
1	5	56	30	70.94	360	600
6	7	26	47	78.75	360	640
6	6	9	44	86.56	60	680
1	6	8	45	87.34	0	720
5	10	0	0	91.25	0	760

**Table A-1-7** Output of Sequence 7

SEQ. 7						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
8	3	187	8	15.47	0	40
8	3	152	13	30.94	0	80
7	5	112	27	40.7	0	120
4	7	85	42	47.73	0	160
4	7	63	43	54.77	0	200
1	8	61	51	55.55	0	240
2	8	58	52	57.11	0	280
5	9	50	60	61.02	0	320
2	11	47	74	62.58	0	360
2	10	44	68	64.14	0	400
3	10	41	68	66.48	60	480
2	11	38	76	68.05	60	520
2	10	35	70	69.61	0	560
7	9	14	66	79.38	0	600
3	8	11	60	81.72	0	640
1	7	10	53	82.5	0	680
5	7	1	55	86.41	0	720
1	10	0	0	87.19	0	760

**Table A-1-8** Output of Sequence 8

SEQ. 8						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
4	3	244	8	7.03	0	40
5	3	233	8	10.94	0	80
7	5	176	18	20.7	0	120
2	6	173	24	22.27	0	160
2	5	170	18	23.83	0	200
7	4	149	14	33.59	60	280
1	5	147	20	34.38	0	320
2	5	144	21	35.94	375	400
2	6	141	27	37.5	375	440
1	5	140	20	38.28	375	480
3	4	137	14	40.63	375	520
6	4	119	17	48.44	375	560
5	3	113	12	52.34	375	600
1	3	111	12	53.13	375	640
7	4	77	18	62.89	375	680
2	3	74	12	64.45	375	720
7	3	54	16	74.22	375	760
7	3	25	18	83.98	375	800
7	2	4	14	93.75	0	840
2	2	1	15	95.31	0	880
1	30	0	0	96.09	0	920

**Table A-1-9**      Output of Sequence 9

SEQ. 9						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
2	3	270	8	1.56	0	40
2	2	267	2	3.13	0	80
1	3	265	8	3.91	0	120
8	5	188	18	19.38	0	160
8	6	127	30	34.84	0	200
1	6	126	30	35.63	300	280
2	5	123	24	37.19	300	320
6	6	95	34	45	300	360
6	5	78	31	52.81	0	400
3	5	74	31	55.16	0	440
3	5	70	32	57.5	0	480
4	5	45	31	64.53	0	520
3	5	42	31	66.88	0	560
8	5	17	34	82.34	0	600
5	5	10	36	86.25	0	640
3	5	4	37	88.59	0	680
3	4	1	31	90.94	0	720
1	30	0	0	91.72	0	760

**Table A-1-10**      Output of Sequence 10

SEQ. 10						
SEQ.	TNSV	PRVLC	TZC	VPU	WIP	RPT
4	3	244	8	7.03	0	40
3	2	241	2	9.38	0	80
7	4	184	12	19.14	0	120
7	3	163	8	28.91	0	160
2	4	160	14	30.47	0	200
1	4	158	15	31.25	0	240
6	4	140	16	39.06	0	280
2	5	137	22	40.63	0	320
1	4	136	15	41.41	180	400
1	4	134	16	42.19	180	440
2	5	131	22	43.75	180	480
4	7	100	36	50.78	180	520
6	6	83	33	58.59	180	560
4	6	63	35	65.63	0	600
2	5	60	29	67.19	300	680
3	5	57	29	69.53	300	720
5	6	34	37	73.44	0	760
2	8	31	53	75	0	800
1	7	30	46	75.78	0	840
3	6	27	40	78.13	0	880
2	5	24	34	79.69	0	920
5	5	17	35	83.59	0	960
6	30	0	0	91.41	0	1000

## APPENDIX A-2

### TABLES FOR SELECTION OF EQUATION

**Table A-2-1** Table for Selection of Equation for PRVLC vs. TNSV

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
6303	0.723	0.947	0.484	0.748	0.781	0.81	0.791	0.511	0.798	0.765
6308	0.723	0.947	0.501	0.748	0.779	0.821	0.79	0.511	0.798	0.777
6407	0.723	0.947	0.503	0.748	0.793	0.821	0.791	0.511	0.798	0.792
6202	0.721	0.946	0.477	0.748	0.761	0.81	0.779	0.494	0.708	0.762
6604	0.723	0.947	0.502	0.748	0.796	0.809	0.792	0.511	0.798	0.783
6403	0.723	0.947	0.503	0.748	0.789	0.821	0.791	0.511	0.798	0.791
6307	0.723	0.947	0.501	0.748	0.775	0.821	0.79	0.511	0.798	0.772
6406	0.723	0.947	0.503	0.748	0.793	0.821	0.791	0.511	0.798	0.792

$$6303 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5$$

$$6307 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5+g(\ln x)^6+h(\ln x)^7+i(\ln x)^8+j(\ln x)^9$$

$$6406 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3+h(\ln x)^4+i/(\ln x)^4+j(\ln x)^5$$

$$6407 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3+h(\ln x)^4+i/(\ln x)^4+j(\ln x)^5+k/(\ln x)^5$$

$$6403 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3$$

$$6308 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5+g(\ln x)^6+h(\ln x)^7+i(\ln x)^8+j(\ln x)^9+k(\ln x)^{10}$$

$$6604 \quad y=a+b/\ln x+c/(\ln x)^2+d/(\ln x)^3+e/(\ln x)^4+f/(\ln x)^5$$

$$6202 \quad y=a+bx+c/x+dx^2+e/x^2+fx^3+g/x^3$$

**Table A-2-2** Table for Selection of Equation for TZC vs. TNSV

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
6402	0.871	0.95	0.733	0.858	0.851	0.975	0.562	0.845	0.88	0.933
6406	0.871	0.964	0.766	0.858	0.907	0.982	0.563	0.845	0.88	0.944
6308	0.871	0.964	0.768	0.858	0.903	0.982	0.5771	0.845	0.88	0.935
6604	0.871	0.928	0.716	0.858	0.819	0.981	0.555	0.845	0.88	0.931
6403	0.871	0.964	0.764	0.858	0.849	0.982	0.561	0.845	0.88	0.943
4235	0.871	0.961	0.749	0.858	0.884	0.966	0.573	0.842	0.854	0.9307
6307	0.871	0.964	0.768	0.858	0.881	0.982	0.57	0.845	0.88	0.933

$$6402 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3$$

$$6406 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3+h(\ln x)^4+i/(\ln x)^4+j(\ln x)^5$$

$$6307 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5+g(\ln x)^6+h(\ln x)^7+i(\ln x)^8+j(\ln x)^9$$

$$6308 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5+g(\ln x)^6+h(\ln x)^7+i(\ln x)^8+j(\ln x)^9+k(\ln x)^{10}$$

$$6604 \quad y=a+b/\ln x+c/(\ln x)^2+d/(\ln x)^3+e/(\ln x)^4+f/(\ln x)^5$$

$$6403 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3$$

$$4235 \quad y=a+bx+cx^{1.5}+de^x+e(\ln x)^2$$



**Table A-2-3** Table for Selection of Equation for TZC vs. PRVLC

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
6604	0.964	0.958	0.772	0.766	0.887	0.96	0.959	0.35	0.494	0.881
6403	0.919	0.958	0.831	0.952	0.925	0.933	0.645	0.372	0.826	0.872
4649	0.942	0.953	0.816	0.898	0.945	0.944	0.889	0.575	0.951	0.877
4224	0.946	0.867	0.772	0.817	0.918	0.944	0.879	0.663	0.849	0.877
4288	0.939	0.952	0.816	0.898	0.944	0.945	0.882	0.536	0.956	0.877
6401	0.946	0.956	0.828		0.907	0.955	0.349	0.368		0.881

$$6401 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2$$

$$6604 \quad y=a+b/\ln x+c/(\ln x)^2+d/(\ln x)^3+e/(\ln x)^4+f/(\ln x)^5$$

$$4288 \quad y=a+bx+cx^{1.5}+dx^{0.5}+ee^{-x}$$

$$4224 \quad y=a+bx+cx^{1.5}+dx^3+ex^{0.5}$$

$$4649 \quad y=a+bx+cx^{2.5}+dx^{0.5}+ee^{-x}$$

$$6403 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3$$

**Table A-2-4** Table for Selection of Equation for VPU vs. TNSV

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
6303	0.634	0.902	0.439	0.715	0.713	0.794	0.653	0.588	0.727	0.749
6308	0.634	0.903	0.454	0.715	0.71	0.811	0.652	0.588	0.727	0.767
6406	0.634	0.903	0.456	0.715	0.716	0.811	0.654	0.588	0.727	0.787
6206	0.632	0.899	0.427	0.715	0.709	0.799	0.629			0.719
6407	0.634	0.903	0.456	0.715	0.716	0.811	0.654	0.588	0.727	0.787
6403	0.634	0.903	0.456	0.715	0.716	0.811	0.654	0.588	0.727	0.787
6604	0.634	0.903	0.453	0.715	0.711	0.792	0.654	0.588	0.727	0.776
6307	0.634	0.903	0.455	0.715	0.709	0.811	0.652	0.588	0.727	0.76

$$6406 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3+h(\ln x)^4+i/(\ln x)^4+j(\ln x)^5$$

$$6407 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3+h(\ln x)^4+i/(\ln x)^4+j(\ln x)^5+k/(\ln x)^5$$

$$6403 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3$$

$$6604 \quad y=a+b/\ln x+c/(\ln x)^2+d/(\ln x)^3+e/(\ln x)^4+f/(\ln x)^5$$

$$6308 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5+g(\ln x)^6+h(\ln x)^7+i(\ln x)^8+j(\ln x)^9+k(\ln x)^{10}$$

$$6307 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5+g(\ln x)^6+h(\ln x)^7+i(\ln x)^8+j(\ln x)^9$$

$$6206 \quad y=a+bx+c/x+dx^2+e/x^2+fx^3+g/x^3+hx^4+i/x^4+jx^5+k/x^5$$

$$6303 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5$$

**Table A-2-5** Table for Selection of Equation for VPU vs. PRVLC

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
4740	0.998	0.995	0.995	0.997	0.996	0.995	0.995	0.994	0.999	0.995
4341	0.998	0.999	0.996	0.997	0.999	0.996	0.999	0.998	0.999	0.997
4179	0.998	0.997	0.995	0.997	0.997	0.995	0.997	0.995	0.999	0.995

$$4179 \quad y=a+bx+cx^{1.5}+dx^2+ex^{0.5}$$

$$4740 \quad y=a+bx+cx^3+dx^{0.5}+ee^{-x}$$

$$4341 \quad y=a+bx+cx^2+dx^{2.5}+ex^3$$

**Table A-2-6** Table for Selection of Equation for VPU vs. TZC

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
4790	0.952									
6604	0.889	0.958	0.745	0.87	0.812	0.93	0.779	0.552	0.822	0.889
4598	0.891	0.917	0.757	0.872	0.809	0.899	0.787	0.466	0.804	0.849
4373	0.887	0.925	0.755	0.874	0.81	0.904	0.784	0.484	0.807	0.863
4594	0.863	0.947	0.733	0.869	0.809	0.908	0.8	0.555	0.825	0.891
4174	0.862	0.922	0.753	0.869	0.811	0.892	0.762	0.558	0.788	0.805
4740	0.869	0.946	0.739	0.87	0.81	0.908	0.791	0.545	0.825	0.892

$$4594 \quad y=a+bx+cx^{2.5}+dx^3+ee^{-x}$$

$$6604 \quad y=a+b/\ln x+c/(\ln x)^2+d/(\ln x)^3+e/(\ln x)^4+f/(\ln x)^5$$

$$4740 \quad y=a+bx+cx^3+dx^{0.5}+ee^{-x}$$

$$4373 \quad y=a+bx+cx^2+de^x+ex^{0.5}$$

$$4598 \quad y=a+bx+cx^{2.5}+de^x+ex^{0.5}$$

$$4174 \quad y=a+bx+cx^{1.5}+dx^2+ex^3$$

$$4790 \quad y=a+bx+ce^x+d(\ln x)^2+ex^{0.5}$$

**Table A-2-7** Table for Selection of Equation for RPT vs. TZC

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
4790	0.941									
4698	0.852	0.869	0.669	0.867	0.677	0.895	0.609	0.469	0.768	0.866
4173	0.817	0.849	0.678	0.869	0.675	0.886	0.654	0.562	0.733	0.813
6604	0.852	0.858	0.677	0.875	0.676	0.913	0.629	0.545	0.765	0.863
4585	0.817	0.863	0.676	0.874	0.678	0.892	0.667	0.563	0.762	0.858
4594	0.818	0.867	0.665	0.872	0.677	0.892	0.673	0.564	0.768	0.867
4649	0.826	0.867	0.668	0.874	0.676	0.892	0.649	0.558	0.768	0.867
4369	0.819	0.867	0.665	0.872	0.677	0.892	0.67	0.563	0.768	0.867

$$4594 \quad y=a+bx+cx^{2.5}+dx^3+ee^{-x}$$

$$4369 \quad y=a+bx+cx^2+dx^3+ee^{-x}$$

$$4649 \quad y=a+bx+cx^{2.5}+dx^{0.5}+ee^{-x}$$

$$4173 \quad y=a+bx+cx^{1.5}+dx^2+ex^{2.5}$$

$$6604 \quad y=a+b/\ln x+c/(\ln x)^2+d/(\ln x)^3+e/(\ln x)^4+f/(\ln x)^5$$

$$4585 \quad y=a+bx+cx^{2.5}+dx^3+ex^{0.5}$$

$$4698 \quad y=a+bx+cx^3+de^x+ee^{-x}$$

$$4790 \quad y=a+bx+ce^x+d(\ln x)^2+ex^{0.5}$$

**Table A-2-8** Table for Selection of Equation for RPT vs. TNSV

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
6402	0.655	0.731	0.419	0.688	0.642	0.781	0.458	0.461	0.729	0.715
6406	0.655	0.732	0.422	0.688	0.645	0.805	0.458	0.461	0.729	0.752
6403	0.655	0.732	0.422	0.688	0.641	0.805	0.458	0.461	0.729	0.753
6206	0.647	0.731	0.404	0.688	0.644	0.787	0.408			0.666
4234	0.646	0.726	0.383	0.678	0.646	0.786	0.41	0.425	0.582	0.68
6303	0.655	0.731	0.411	0.688	0.645	0.781	0.458	0.461	0.729	0.704
6307	0.655	0.732	0.42	0.688	0.644	0.805	0.457	0.461	0.729	0.719
6308	0.655	0.732	0.419	0.688	0.644	0.805	0.457	0.461	0.729	0.727
6405	0.655	0.732	0.422	0.688	0.643	0.805	0.458	0.461	0.729	0.753

$$6406 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3+h(\ln x)^4+i/(\ln x)^4+j(\ln x)^5$$

$$6308 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5+g(\ln x)^6+h(\ln x)^7+i(\ln x)^8+j(\ln x)^9+k(\ln x)^{10}$$

$$6307 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5+g(\ln x)^6+h(\ln x)^7+i(\ln x)^8+j(\ln x)^9$$

$$6405 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3+h(\ln x)^4+i/(\ln x)^4$$

$$6403 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3$$

$$6206 \quad y=a+bx+c/x+dx^2+e/x^2+fx^3+g/x^3+hx^4+i/x^4+jx^5+k/x^5$$

$$6303 \quad y=a+b(\ln x)+c(\ln x)^2+d(\ln x)^3+e(\ln x)^4+f(\ln x)^5$$

$$4234 \quad y=a+bx+cx^{1.5}+de^x+ex^{0.5}\ln x$$

$$6402 \quad y=a+b(\ln x)+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3$$

**Table A-2-9** Table for Selection of Equation for RPT vs. PRVLC

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
4341	0.978	0.984	0.985	0.988	0.988	0.977	0.971	0.979	0.98	0.975
6604	0.971	0.988	0.963	0.908	0.985	0.977	0.972	0.9	0.907	0.973
4585	0.975	0.984	0.987	0.989	0.989	0.974	0.963	0.974	0.983	0.972
4594	0.973	0.984	0.986	0.989	0.988	0.974	0.96	0.966	0.985	0.972
8011	0.969	0.979	0.974	0.971	0.979	0.974	0.978	0.969	0.98	0.971
4179	0.975	0.982	0.986	0.987	0.988	0.974	0.958	0.958	0.985	0.971

$$4179 \quad y=a+bx+cx^{1.5}+dx^2+ex^{0.5}$$

$$8011 \quad y=a+b/(1+\exp(-(x-c)/d)) \text{ [Sigmoid]}$$

$$4594 \quad y=a+bx+cx^{2.5}+dx^3+ee^{-x}$$

$$4585 \quad y=a+bx+cx^{2.5}+dx^3+ex^{0.5}$$

$$6604 \quad y=a+b/\ln x+c/(\ln x)^2+d/(\ln x)^3+e/(\ln x)^4+f/(\ln x)^5$$

$$4341 \quad y=a+bx+cx^2+dx^{2.5}+ex^3$$

**Table A-2-10** Table for Selection of Equation for WIP vs. PRVLC

SEQ.	1	2	3	4	5	6	7	8	9	10
EQUATIONS										
8006	0.666	0.902	0.209		0.652	0.478	0.999	0.691	0.839	0.528
8004	0.003	0.911	0.81	0.194	0.695	0.026	0.987	0.677	0.798	0.481
8005	0.666	0.779	0.878	0.236	0.358		1	0.617	0.843	0.535
4594	0.163	0.689	0.428	0.624	0.761	0.076	0.097	0.748	0.318	0.53
4341	0.154	0.804	0.582	0.621	0.763	0.619	0.165	0.749	0.479	0.514
8003	0.666	0.894	0.684	0.229	0.635	0.028	1	0.708	0.844	0.298
8008	0.075	0.885	0.688	0.261	0.001	0.028	0.999	0.746	0.849	0.495
4173	0.195	0.842	0.505	0.616	0.755	0.404	0.135	0.738	0.409	0.538

$$4341 \quad y=a+bx+cx^2+dx^{2.5}+ex^3$$

$$4594 \quad y=a+bx+cx^{2.5}+dx^3+ee^{-x}$$

$$8008 \quad y=a+\text{berfc}(((x-c)/d)2) \text{ [Erfc Peak]}$$

$$4173 \quad y=a+bx+cx^{1.5}+dx^2+ex^{2.5}$$

$$8003 \quad y=a+b \exp(-0.5((x-c)/d)^2) \text{ [Gaussian]}$$

$$8006 \quad y=a+b \exp(-\exp(-(x-c)/d)-((x-c)/d)+1) \text{ [ExtrVal]}$$

$$8005 \quad y=a+b \exp(-0.5(\ln(x/c)/d)^2) \text{ [Log-Normal]}$$

$$8004 \quad y=a+b/(1+((x-c)/d)^2) \text{ [Lorentzian]}$$



**APPENDIX B**

**TABLES OF SELECTED EQUATION WITH PARAMETER LIST**

Table B-1 Selected Equation for PRVLC vs. TNSV

SEQ.	1	2	3	4	5	6	7	8	9	10
PARAMETER										
a	2149.3342	-1136.539	18192.188	-539.3867	233114.39	-24672.55	-1918.44	-920.89	-3394.09	10622.35
b	4536.6244	1168.569	-24908.603	-481.9936	-65074.53	49896.42	-15219.08	-833.70	-3382.21	-10660.14
c	-2603.8832	-1561.414	32536.521	47.28686	144575.02	-56980.62	12269.79	-43.59	116.55	15005.46
d	774.40970	1415.878	-24998.173	214.7247	-258093.21	39593.71	-7184.52	356.49	859.88	-14144.60
e	-4924.6611	610.6598	-3062.8733	728.8548	-171444.18	-448.07	14309.60	935.73	3834.31	-2984.69
f	-4886.7692	-1916.727	35371.283	632.1314	223093.33	-70578.88	18715.52	1220.81	4287.36	17161.63
g	-2217.1619	2422.446	-40399.895	750.6003	-263601.49	66826.67	2779.45	991.20	3881.72	-18863.97
h	2318.8154	768.0789	-13739.836	-488.2629	-69542.79	31203.37	-8534.14	-816.66	-2621.90	-6147.00
i	5662.3678	-1373.752	19196.406	-683.0986	222160.55	-29827.37	-17445.27	-870.36	-3485.74	9371.55
j	-307.55494	-105.6594	1779.6070	97.69814	7719.81	-4561.09	1218.41	133.58	410.17	731.37
R^2	0.723	0.947	0.503	0.748	0.793	0.821	0.791	0.511	0.798	0.792

$$6406 \quad y = a + b(\ln x) + c(\ln x)^2 + e(\ln x)^2 + f(\ln x)^3 + g(\ln x)^3 + h(\ln x)^3 + i(\ln x)^4 + j(\ln x)^4 + (\ln x)^5$$

Table B-2 Selected Equation for TZC vs. TNSV

SEQ.	1	2	3	4	5	6	7	8	9	10
PARAMETER										
a	1083.0415	5992.401	-3342.3099	134.9287	-118093.62	3865.27	-2041.97	52.49	262.51	-1451.85
b	1620.8465	10017.39	6089.4536	73.58470	47981.66	-7174.55	-12214.45	43.93	257.87	1514.00
c	-590.06705	-11536.85	-7094.1629	48.35432	-83789.60	8449.58	9200.07	9.43	0.44	-2080.67
d	-162.92503	9303.678	5181.3103	-101.4102	139330.63	-5972.83	-5314.50	-20.97	-64.60	1965.01
e	-1611.9238	629.9156	152.25110	-89.32240	79048.04	-82.28	11115.04	-41.79	-280.21	411.20
f	-2167.9135	-15285.57	-8794.4404	-154.9517	-135967.62	10073.16	15297.94	-64.25	-322.57	-2433.80
g	-924.64839	13349.48	8364.3143	-129.0192	135106.24	-10020.67	2426.58	-48.62	-288.72	2621.72
h	1443.1797	6652.558	3840.3694	175.1496	46726.46	-4297.53	-7192.39	49.06	201.33	889.04
i	1625.8475	-6107.571	-3792.9621	89.66559	-105631.78	4522.98	-13134.16	39.81	255.63	-1302.75
j	-270.51969	-969.9688	-559.86715	-43.18647	-5724.42	604.99	1050.50	-8.78	-32.11	-107.93
R <sup>2</sup>	0.871	0.950	0.733	0.858	0.851	0.975	0.562	0.845	0.880	0.933

$$6406 \quad y = a + b \ln x + c (\ln x)^2 + e / (\ln x)^2 + f (\ln x)^3 + g / (\ln x)^3 + h (\ln x)^4 + i / (\ln x)^4 + j (\ln x)^5$$

Table B-3 Selected Equation for TZC VS. PRVLC

SEQ.	1	2	3	4	5	6	7	8	9	10
PARAMETER										
a	19.977737	63.49297	33.809910	55.44962	49.211443	51.18779	33.480450	25.61050	44.484023	15.14957
b	-0.9307606	-21816282	-0.0218465	0.146618	-0.1743078	-0.214066	-1.8076840	0.411530	0.1755826	-0.84840
c	6.38E-04	2.83E-04	-2.20E-04	-7.48E-06	-1.82E-04	3.00E-04	0.0001912	-5.21E-04	-3.24E-04	0.00006
d	9.9687923	-1.920933	-0.2331189	-5.267918	1.2012760	-1.234139	16.157585	-4.449276	-2.9301233	9.08778
e	36.023756	-63.22062	-33.525673	-52.50890	-49.283850	-51.18831	-27.167035	-24.75035	-42.682966	-15.14957
R^2	0.942	0.953	0.816	0.898	0.945	0.944	0.889	0.575	0.951	0.877

$$4649 \quad y = a + bx + cx^{2.5} + dx^{0.5} + ee^{-x}$$

**Table B-4** Selected Equation for VPU vs. TNSV

SEQ.	1	2	3	4	5	6	7	8	9	10
PARAMETER										
a	79.91	1118.62	-5517.85	271.95	-24233.45	10298.07	1092.70	417.05	1291.30	-4322.03
b	-295.14	-1454.71	7240.61	230.11	-923.72	-20786.98	7999.76	370.17	1272.26	4414.13
c	420.56	1846.52	-9616.77	20.99	-9553.44	23778.16	-6329.59	50.90	-4.98	-6134.75
d	-336.29	-1536.28	7488.30	-94.56	22445.47	-16488.24	3723.28	-144.00	-318.24	5816.08
e	447.99	-278.14	1049.40	-285.05	21959.82	226.21	-7439.87	-367.27	-1389.91	1237.17
f	54.34	2266.74	-10209.13	-285.47	-11513.25	29429.12	-9843.81	-516.72	-1597.77	-7072.14
g	72.65	-2319.53	12082.89	-316.18	26675.84	-27825.71	-1473.94	-407.12	-1427.24	7746.90
h	246.09	-929.67	3850.48	219.38	1416.01	-13019.86	4508.57	344.03	975.78	2534.92
i	-646.14	1146.26	-5790.68	275.50	-26832.80	12402.79	9036.74	347.46	1269.57	-3852.85
j	-85.32	128.61	-481.67	-43.67	107.73	1904.36	-645.68	-56.24	-152.50	-301.71
R <sup>2</sup>	0.634	0.903	0.456	0.715	0.716	0.811	0.654	0.588	0.727	0.787

$$6406 \quad y = a + b \ln x + c(\ln x)^2 + e/(\ln x)^2 + f(\ln x)^3 + g/(\ln x)^3 + h(\ln x)^4 + i/(\ln x)^4 + j(\ln x)^5$$

**Table B-5** Selected Equation for VPU vs. PRVLC

SEQ.	1	2	3	4	5	6	7	8	9	10
PARAMETER										
a	91.58727384	87.941059	87.3880158	93.513704	90.990230	89.663121	86.760300	96.306087	91.763887	92.204076
b	-0.59739831	-0.448017	-0.3277677	-0.265228	-0.429697	-0.192171	-0.331914	-0.745186	-0.663251	-0.788209
c	-0.00412899	-0.007013	-0.0063516	-0.006086	-0.009033	-0.009201	-0.016910	0.017661	0.003393	0.017741
d	-0.00035430	0.001148	0.0008429	0.000692	0.001342	0.001036	0.002597	0.002120	0.000180	-0.001988
e	0.00001002	-0.000043	-0.0000279	-0.000021	-0.000048	-0.000031	-0.000101	0.000070	0.000003	0.000062
R^2	0.998	0.999	0.996	0.997	0.999	0.996	0.999	0.998	0.999	0.997

4341  $y=a+bx+cx^2+dx^{2.5}+ex^3$

Table B-6 Selected Equation for VPU vs. TZC

SEQ.	1	2	3	4	5	6	7	8	9	10
PARAMETER										
a	-8377.22	144.25	94.03	114.20	356.86	178.73	-125.90	2118.09	410.55	87.42
b	98033.92	2702.31	280.62	992.14	12300.67	4187.76	-9530.44	94496.19	15526.14	-159.15
c	-390266.20	-29063.33	-559.06	-6641.00	-157409.30	-45870.34	1446561.82	-1172900.00	-154013.62	3672.19
d	540849.79	102587.00	-6306.78	11172.95	672500.76	168025.86	-713519.29	4897300.00	491056.01	-19837.99
e	115807.60	-157099.30	16057.43	-4515.78	-1240100.00	-266178.17	1472430.00	-8787000.00	-617758.83	32245.66
f	-577290.43	88789.10	-9819.41	-1688.45	840359.25	154563.63	-1093500.00	5792940.00	239909.62	-14018.44
R^2	0.889	0.958	0.745	0.870	0.812	0.930	0.779	0.552	0.822	0.889

$$6604 \quad y = a + b/\ln x + c/(\ln x)^2 + d/(\ln x)^3 + e/(\ln x)^4 + f/(\ln x)^5$$

Table B-7 Selected Equation for RPT vs. TZC

SEQ.	1	2	3	4	5	6	7	8	9	10
PARAMETER										
a	102.70455	53.26786	-51.53745	133.98505	-72.4327	44.8650	240.0836	-3392.084	-55.6603	-123.8792
b	2.38360	4.21310	28.08379	10.02450	14.2113	10.5753	-22.8351	608.821	18.1054	25.8865
c	0.13089	0.04653	0.06905	0.27656	-0.0409	0.1424	0.2221	-13.177	-0.2080	0.0665
d	-0.01518	-0.00361	-0.01528	-0.03667	0.0055	-0.0195	-0.0211	1.895	0.0353	-0.0121
e	-231448.52500	826.94130	1136.78289	987.33372	1032.4468	715.4303	519.9008	4312.081	814.3322	1123.8493
R^2	0.818	0.867	0.665	0.872	0.677	0.892	0.673	0.564	0.768	0.867

4594  $y=a+bx+cx^2+dx^3+ee^x$



Table B-8 Selected Equation for RPT vs. TNSV

SEQ.	1	2	3	4	5	6	7	8	9	10
PARAMETER										
a	2822.79	-3011.38	-47815.48	-702.38	91932.72	99752.05	21054.85	5705.90	9368.50	-60482.51
b	-1430.86	40684.56	59164.03	-181.08	-167311.22	-198521.84	148875.64	5055.67	9269.27	92175.02
c	4865.37	-51081.22	-80677.02	-499.30	157783.19	228169.05	-116936.37	684.16	-58.52	-86154.68
d	-4724.14	42710.80	63709.91	772.60	-182655.79	-158660.89	68477.26	-2003.37	-2239.33	81542.30
e	3629.43	5735.16	9991.88	87.88	8381.34	1355.39	-138034.08	-5058.90	-10107.35	17138.79
f	-2597.75	-61502.60	-82654.33	943.20	303040.17	280809.73	-183904.55	-7103.86	-11552.50	-99680.34
g	-493.82	61742.27	102331.31	456.11	-117554.52	-267733.89	-27786.39	-5618.69	-10361.28	108555.39
h	5260.67	24724.77	29776.13	-871.95	-134536.62	-123722.93	84614.72	4744.84	6996.57	35850.88
i	-6461.45	-29297.84	-49415.76	-54.54	18766.70	119652.43	166944.37	4713.95	9234.06	-53888.79
j	-1459.40	-3342.36	-3500.15	174.64	19834.75	18035.62	-12153.06	-777.58	-1087.32	-4278.64
R <sup>2</sup>	0.655	0.732	0.422	0.688	0.645	0.805	0.458	0.461	0.729	0.752

$$6406 \quad y = a + b \ln x + c / \ln x + d (\ln x)^2 + e / (\ln x)^2 + f (\ln x)^3 + g / (\ln x)^3 + h (\ln x)^4 + i / (\ln x)^4 + j (\ln x)^5$$

**Table B-9** Selected Equation for RPT vs. PRVLC

SEQ.	1	2	3	4	5	6	7	8	9	10
PARAMETER										
a	763.73121	887.79760	1038.31963	1077.43607	944.584560	731.66025	722.29709	903.22878	716.43688	1046.84303
b	-10.61020	-10.84816	-10.47975	-10.21867	-13.393648	-0.64250	-0.60201	-12.34035	-4.94518	-10.77951
c	0.25670	-0.03795	0.08316	0.05026	0.123487	-0.16253	-0.50113	0.55654	0.02715	0.23564
d	-0.02650	0.01381	-0.00310	0.00176	-0.005056	0.01919	0.06918	-0.06786	-0.00207	-0.02643
e	0.00077	-0.00061	-0.00003	-0.00022	-0.000020	-0.00060	-0.00247	0.00221	0.00006	0.00084
R <sup>2</sup>	0.978	0.984	0.985	0.988	0.988	0.977	0.971	0.979	0.980	0.975

4341  $y=a+bx+cx^2+dx^{2.5}+ex^3$

**Table B-10** Selected Equation for WIP vs. PRVLC

SEQ.	1	2	3	4	5	6	7	8	9	10
PARAMETER										
a	20.000	1.5382	49.0909		189.441	68.662	-0.0276	-176.2306	-0.67	-23.7641
b	2325.709	564.3205	426.0902		3578.137	2736.775	645.2168	622.8322	10123.99	247.5688
c	151.249	17.1549	197.5889		-307.410	36.182	38.9740	74.7564	103.23	79.3383
d	-1.640	15.2240	0.6699		109.932	6.481	0.6067	61.2988	4.44	34.4823
R^2	0.666	0.902	0.209		0.652	0.478	0.999	0.691	0.839	0.528

8006  $y=a+b \exp(-\exp(-(x-c)/d))-((x-c)/d)+1$  [ExtrVal]

# APPENDIX C

## GRAPHICAL REPRESENTATION OF EQUATIONS FOR SEQUENCE 1

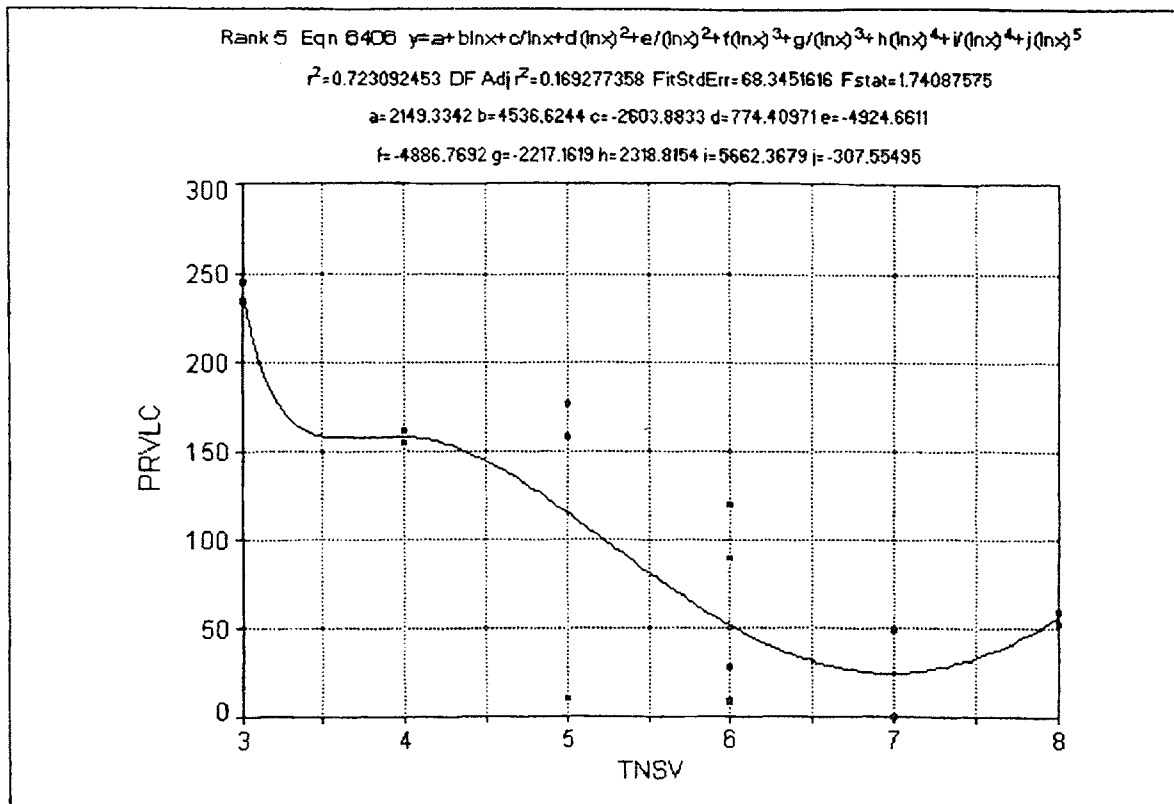


Figure C-1 Curve between PRVLC and TNSV

### Figure C-1 (continued)

#### Numeric Summary

Rank 5 Eqn 6406  $y=a+blnx+c/lnx+d(lnx)^2+e/(lnx)^2+f(lnx)^3+g/(lnx)^3+h(lnx)^4+i/(lnx)^4+j(lnx)^5$

r2	Coef Det	DF	Adj r2	Fit Std Err	F-value
0.7230924528		0.1692773584		68.345161578	1.7408757547

Parm	Value	Std Error	t-value	99% Confidence Limits	
a	2149.334209	28604.65958	0.075139304	-103907.431	108206.0994
b	4536.624435	46921.99245	0.096684395	-169434.848	178508.0966
c	-2603.88329	20258.97587	-0.12852986	-77717.5703	72509.80375
d	774.4097059	1904.732266	0.406571422	-6287.71744	7836.536849
e	-4924.66110	48203.04963	-0.10216493	-183645.876	173796.5540
f	-4886.76922	59287.48695	-0.08242497	-224705.469	214931.9309
g	-2217.16192	25866.52216	-0.08571550	-98121.8050	93687.48117
h	2318.815432	36600.72780	0.063354353	-133384.768	138022.3984
i	5662.367868	50423.16576	0.112296953	-181290.315	192615.0508
j	-307.554947	6480.646317	-0.04745745	-24335.6817	23720.57177

Area Xmin-Xmax	Area Precision		
457.81456485	1.148868e-13		
Function min	X-Value	Function max	X-Value
23.985722539	6.9779804689	238.49776101	3.0000048028
1st Deriv min	X-Value	1st Deriv max	X-Value
-466.1826369	3.0000048028	60.382325126	8.0000000000
2nd Deriv min	X-Value	2nd Deriv max	X-Value
-81.80701450	4.1670871791	2193.1355754	3.0000048028

Soln Vector	Covar Matrix	SVD Cond		
SVDecomp	SVDecomp	9.489506e+21		
r2	Coef Det	DF	Adj r2	
0.7230924528		0.1692773584		
		Fit Std Err		
		68.345161578		
Source	Sum of Squares	DF	Mean Square	F
Regr	73185.633	9	8131.737	1.74088
Error	28026.367	6	4671.0611	
Total	101212	15		

#### X Variable: TNSV

Xmin: 3.0000000000 Xmax: 8.0000000000 Xrange: 5.0000000000  
 Xmean: 5.5625000000 Xstd: 1.5478479684 Xmedian: 6.0000000000  
 X@Ymin: 7.0000000000 X@Ymax: 3.0000000000 X@Yrange: 4.0000000000

#### Y Variable: PRVLC

Ymin: 0.0000000000 Ymax: 244.00000000 Yrange: 244.00000000  
 Ymean: 96.500000000 Ystd: 82.142964803 Ymedian: 74.000000000  
 Y@Xmin: 244.00000000 Y@Xmax: 51.000000000 Y@Xrange: 193.00000000

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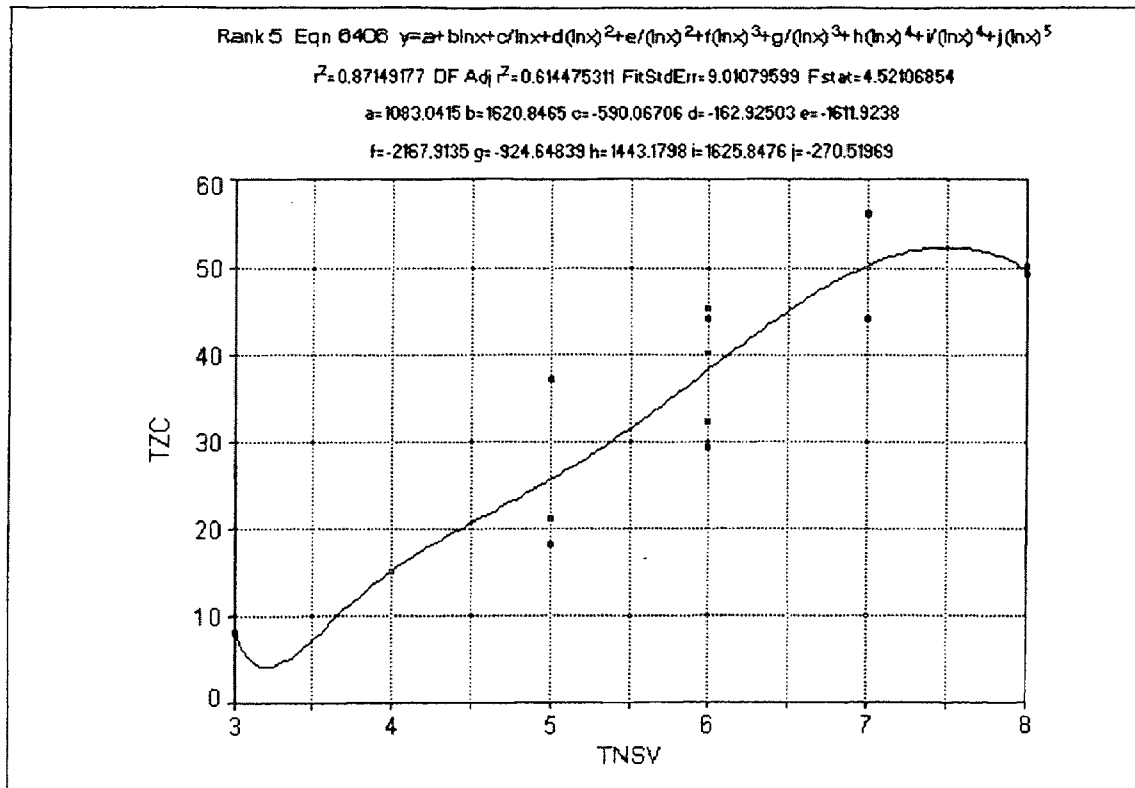


Figure C-2 Curve between TZO and TNSV

Figure C-2 (continued)

## Numeric Summary

Rank 5 Eqn 6406  $y=a+blnx+c/lnx+d(lnx)^2+e/(lnx)^2+f(lnx)^3+g/(lnx)^3+h(lnx)^4+i/(lnx)^4+j(lnx)^5$ 

r2	Coef Det	DF	Adj r2	Fit Std Err	F-value
0.8714917704		0.6144753112		9.0107959939	4.5210685369

Parm	Value	Std Error	t-value	99% Confidence Limits	
a	1083.041517	3771.309424	0.287179172	-12899.7460	15065.82908
b	1620.846510	6186.312123	0.262005291	-21315.9848	24557.67786
c	-590.067059	2670.993738	-0.22091668	-10493.2423	9313.108157
d	-162.925033	251.1246368	-0.64878156	-1094.01343	768.1633643
e	-1611.92381	6355.209886	-0.25363817	-25174.9731	21951.12546
f	-2167.91352	7816.609656	-0.27734704	-31149.3574	26813.53032
g	-924.648390	3410.306580	-0.27113351	-13568.9549	11719.65815
h	1443.179768	4825.530934	0.299071706	-16448.3179	19334.67748
i	1625.847551	6647.915515	0.244565014	-23022.4589	26274.15400
j	-270.519694	854.4245199	-0.31661041	-3438.44765	2897.408261

Area Xmin-Xmax	Area Precision		
155.74079140	9.147139e-14		
Function min	X-Value	Function max	X-Value
3.9356485032	3.2075364663	52.106742780	7.5000000000
1st Deriv min	X-Value	1st Deriv max	X-Value
-10.92107402	8.0000000000	17.098002370	3.5887321824
2nd Deriv min	X-Value	2nd Deriv max	X-Value
-24.71692295	8.0000000000	365.11288548	3.0000048028

Soln Vector	Covar Matrix	SVD Cond		
SVDdecomp	SVDdecomp	9.489506e+21		
r2	Coef Det	DF	Adj r2	
0.8714917704		0.6144753112		
		Fit Std Err		
		9.0107959939		
Source	Sum of Squares	DF	Mean Square	F
Regr	3303.7708	9	367.08565	4.52107
Error	487.16667	6	81.194444	
Total	3790.9375	15		

X Variable: TNSV

Xmin: 3.0000000000 Xmax: 8.0000000000 Xrange: 5.0000000000  
 Xmean: 5.5625000000 Xstd: 1.5478479684 Xmedian: 6.0000000000  
 X@Ymin: 3.0000000000 X@Ymax: 7.0000000000 X@Yrange: 4.0000000000

Y Variable: TZC

Ymin: 8.0000000000 Ymax: 56.0000000000 Yrange: 48.0000000000  
 Ymean: 31.9375000000 Ystd: 15.897457868 Ymedian: 34.5000000000  
 Y@Xmin: 8.0000000000 Y@Xmax: 50.0000000000 Y@Xrange: 42.0000000000

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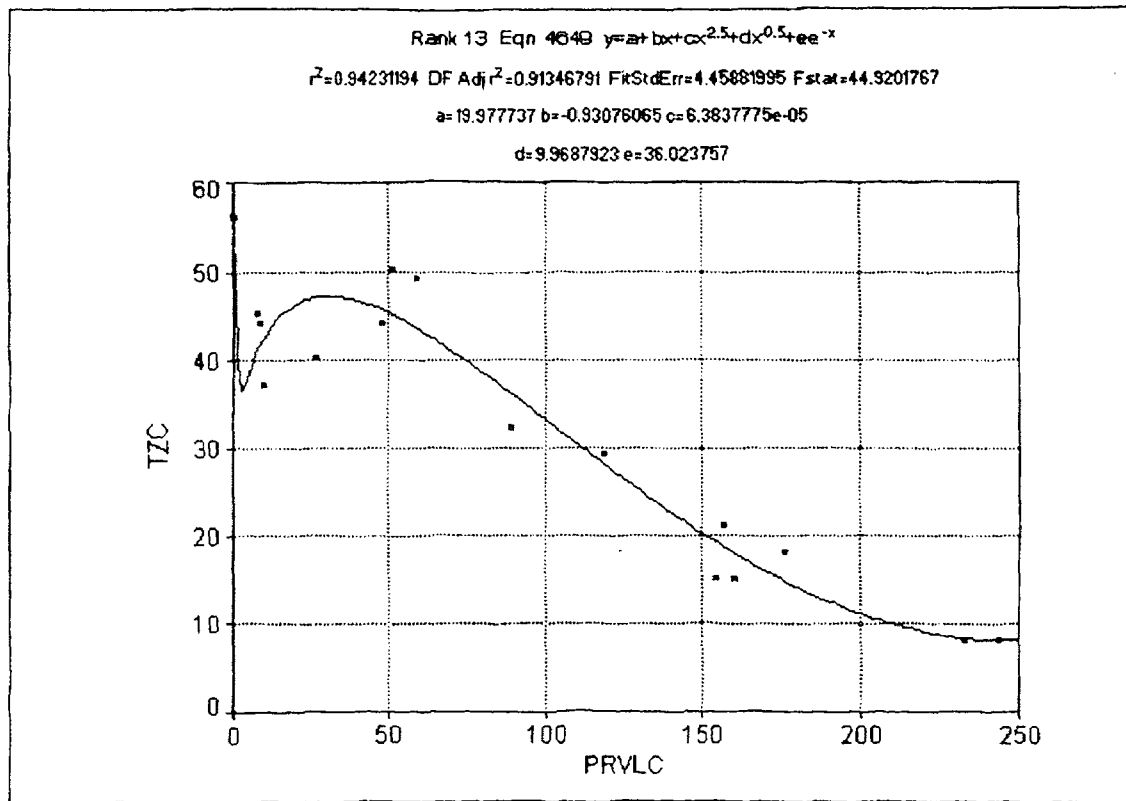


Figure C-3 Curve between TZC and PRVLC



### Figure C-3 (continued)

#### Numeric Summary

Rank 13 Eqn 4649  $y=a+bx+cx^2.5+dx^{0.5}+ee^{-x}$

r2 Coef Det	DF Adj r2	Fit Std Err	F-value
0.9423119403	0.9134679105	4.4588199491	44.920176733

Parm	Value	Std Error	t-value	99% Confidence Limits	
a	19.97773702	8.596634808	2.323902023	-6.72190922	46.67738326
b	-0.93076065	0.235710508	-3.94874485	-1.66283632	-0.19868499
c	6.38378e-05	2.32508e-05	2.745617252	-8.3751e-06	0.000136051
d	9.968792317	2.990012337	3.334030497	0.682336228	19.25524841
e	36.02375662	9.685096251	3.719504245	5.943539244	66.10397399

Area Xmin-Xmax	Area Precision		
6672.5540542	1.105144e-08		
Function min	X-Value	Function max	X-Value
7.9576365554	244.00000000	46.972024109	30.400346696
1st Deriv min	X-Value	1st Deriv max	X-Value
-0.272825786	102.03209431	1.0586238141	5.1246258336
2nd Deriv min	X-Value	2nd Deriv max	X-Value
-0.101461084	7.1986005666	0.0030855377	244.00000000

Soln Vector	Covar Matrix			
GaussElim	LUDecomp			
r2 Coef Det	DF Adj r2	Fit Std Err		
0.9423119403	0.9134679105	4.4588199491		
Source	Sum of Squares	DF	Mean Square	F
Regr	3572.2457	4	893.06142	44.9202
Error	218.69183	11	19.881075	
Total	3790.9375	15		

X Variable: PRVLC

Xmin: 0.000000000 Xmax: 244.0000000 Xrange: 244.0000000  
 Xmean: 96.500000000 Xstd: 82.142964803 Xmedian: 74.000000000  
 X@Ymin: 233.0000000 X@Ymax: 0.000000000 X@Yrange: 233.0000000

Y Variable: TZC

Ymin: 8.000000000 Ymax: 56.000000000 Yrange: 48.000000000  
 Ymean: 31.937500000 Ystd: 15.897457868 Ymedian: 34.500000000  
 Y@Xmin: 56.000000000 Y@Xmax: 8.000000000 Y@Xrange: 48.000000000

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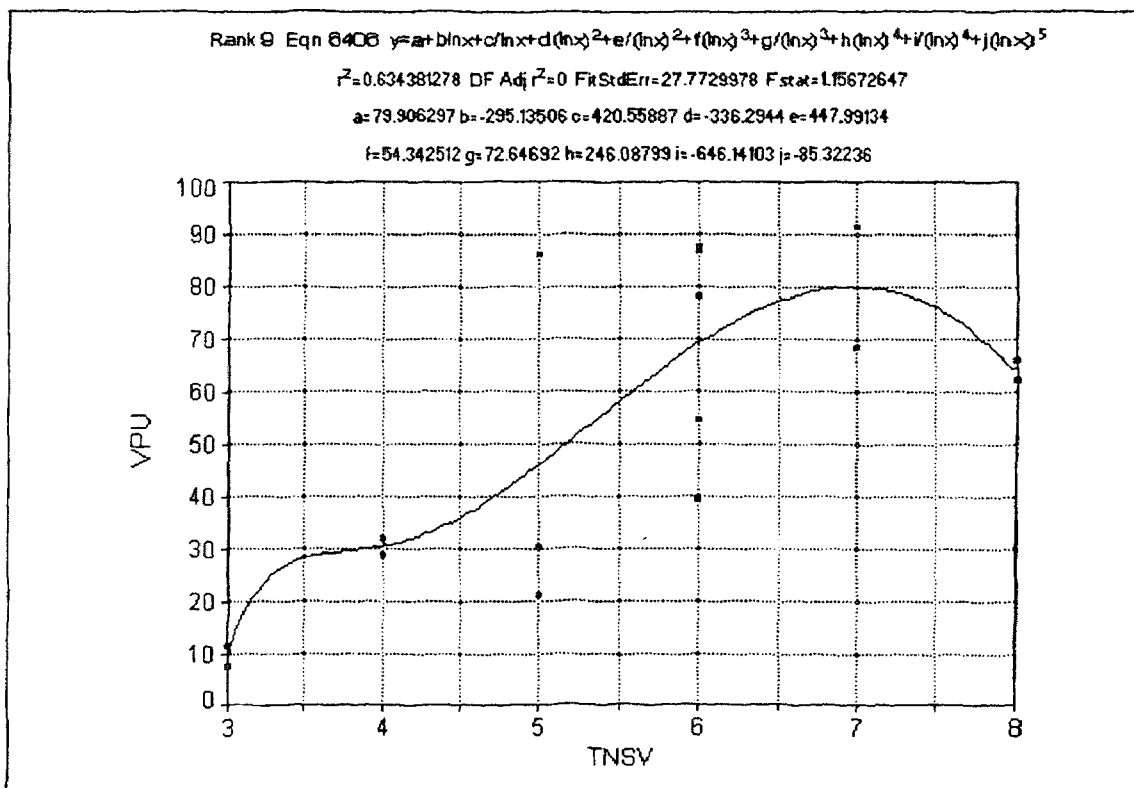


Figure C-4 Curve between VPU and TNSV

Figure C-4 (continued)

## Numeric Summary

Rank 9 Eqn 6406  $y=a+blnx+c/lxn+d(lnx)^2+e/(lnx)^2+f(lnx)^3+g/(lnx)^3+h(lnx)^4+i/(lnx)^4+j(lnx)^5$ 

r2	Coef Det	DF	Adj r2	Fit Std Err	F-value
0.6343812784		0.0000000000		27.772997768	1.1567264673

Parm	Value	Std Error	t-value	99% Confidence Limits	
a	79.90629686	11623.89741	0.006874312	-43017.7210	43177.53363
b	-295.135058	19067.39792	-0.01547852	-70990.8390	70400.56887
c	420.5588684	8232.513883	0.051085109	-30102.9224	30944.04018
d	-336.294404	774.0141915	-0.43448093	-3206.08704	2533.498231
e	447.9913426	19587.97315	0.022870735	-72177.8361	73073.81875
f	54.34251186	24092.28693	0.002255598	-89272.0140	89380.69899
g	72.64691988	10511.21755	0.006911371	-38899.5258	39044.81959
h	246.0879881	14873.20986	0.016545721	-54898.9321	55391.10807
i	-646.141030	20490.14792	-0.03153423	-76616.9388	75324.65670
j	-85.3223600	2633.499893	-0.03239885	-9849.48260	9678.837885

Area Xmin-Xmax	Area Precision		
269.75128145	2.421736e-14		
Function min	X-Value	Function max	X-Value
8.9854605409	3.0000048028	79.736952666	6.9702006213
1st Deriv min	X-Value	1st Deriv max	X-Value
-32.06994209	8.0000000000	95.889830686	3.0000048028
2nd Deriv min	X-Value	2nd Deriv max	X-Value
-402.9212053	3.0000048028	21.393161964	4.2557664657

Soln Vector	Covar Matrix	SVD Cond		
SVDdecomp	SVDdecomp	9.489506e+21		
r2	Coef Det	DF	Adj r2	Fit Std Err
0.6343812784		0.0000000000		27.772997768

Source	Sum of Squares	DF	Mean Square	F
Regr	8030.0583	9	892.22871	1.15673
Error	4628.0364	6	771.3394	
Total	12658.095	15		

## X Variable: TNSV

Xmin: 3.0000000000	Xmax: 8.0000000000	Xrange: 5.0000000000
Xmean: 5.5625000000	Xstd: 1.5478479684	Xmedian: 6.0000000000
X@Ymin: 3.0000000000	X@Ymax: 7.0000000000	X@Yrange: 4.0000000000

## Y Variable: VPU

Ymin: 7.0300000000	Ymax: 91.2500000000	Yrange: 84.2200000000
Ymean: 52.9162500000	Ystd: 29.049491992	Ymedian: 58.0450000000
Y@Xmin: 10.9400000000	Y@Xmax: 61.9500000000	Y@Xrange: 51.0100000000

Date	Time	File Source
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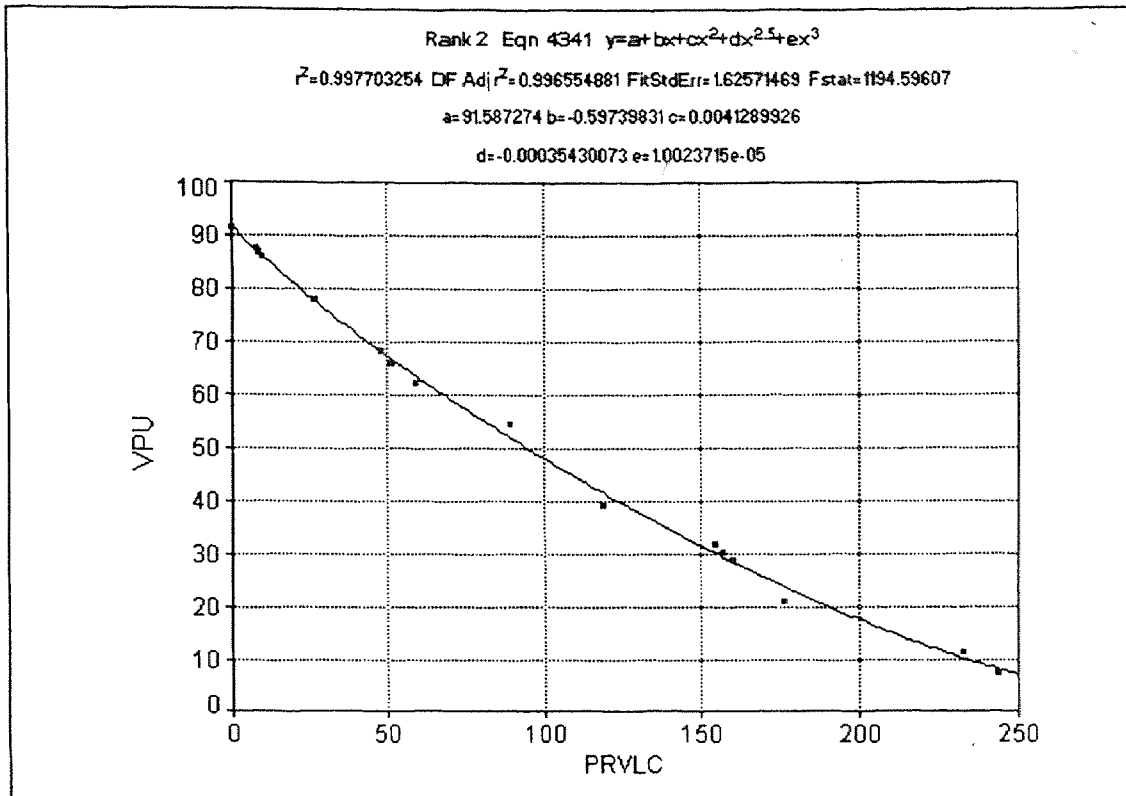


Figure C-5 Curve between VPU and PRVLC

## Figure C-5 (continued)

## Numeric Summary

Rank 2 Eqn 4341  $y=a+bx+cx^2+dx^2.5+ex^3$ 

r2 Coef Det	DF Adj r2	Fit Std Err	F-value
0.9977032538	0.9965548808	1.6257146856	1194.5960726

Parm	Value	Std Error	t-value	99% Confidence Limits
a	91.58727384	1.236479134	74.07102257	87.74698561 95.42756207
b	-0.59739831	0.139996975	-4.26722298	-1.03220446 -0.16259216
c	0.004128993	0.005902171	0.699571798	-0.01420212 0.022460106
d	-0.00035430	0.000658393	-0.53812934	-0.00239916 0.001690554
e	1.00237e-05	2.02704e-05	0.494501195	-5.2932e-05 7.29799e-05

Area Xmin-Xmax	Area Precision		
10469.563951	3.544404e-09		
Function min	X-Value	Function max	X-Value
7.7648022528	244.00000000	91.587273843	1.046656e-10
1st Deriv min	X-Value	1st Deriv max	X-Value
-0.597398311	1.046656e-10	-0.168090498	244.00000000
2nd Deriv min	X-Value	2nd Deriv max	X-Value
0.0009201709	122.01816530	0.0082501585	8.669783e-10

Soln Vector	Covar Matrix	
GaussElim	LUDecomp	
r2 Coef Det	DF Adj r2	Fit Std Err
0.9977032538	0.9965548808	1.6257146856

Source	Sum of Squares	DF	Mean Square	F
Regr	12629.022	4	3157.2556	1194.6
Error	29.072431	11	2.6429482	
Total	12658.095	15		

## X Variable: PRVLC

Xmin: 0.000000000 Xmax: 244.0000000 Xrange: 244.0000000  
Xmean: 96.50000000 Xstd: 82.142964803 Xmedian: 74.00000000  
X@Ymin: 244.0000000 X@Ymax: 0.000000000 X@Yrange: 244.0000000

## Y Variable: VPU

Ymin: 7.030000000 Ymax: 91.25000000 Yrange: 84.22000000  
Ymean: 52.91625000 Ystd: 29.049491992 Ymedian: 58.04500000  
Y@Xmin: 91.25000000 Y@Xmax: 7.030000000 Y@Xrange: 84.22000000

Date	Time	File Source
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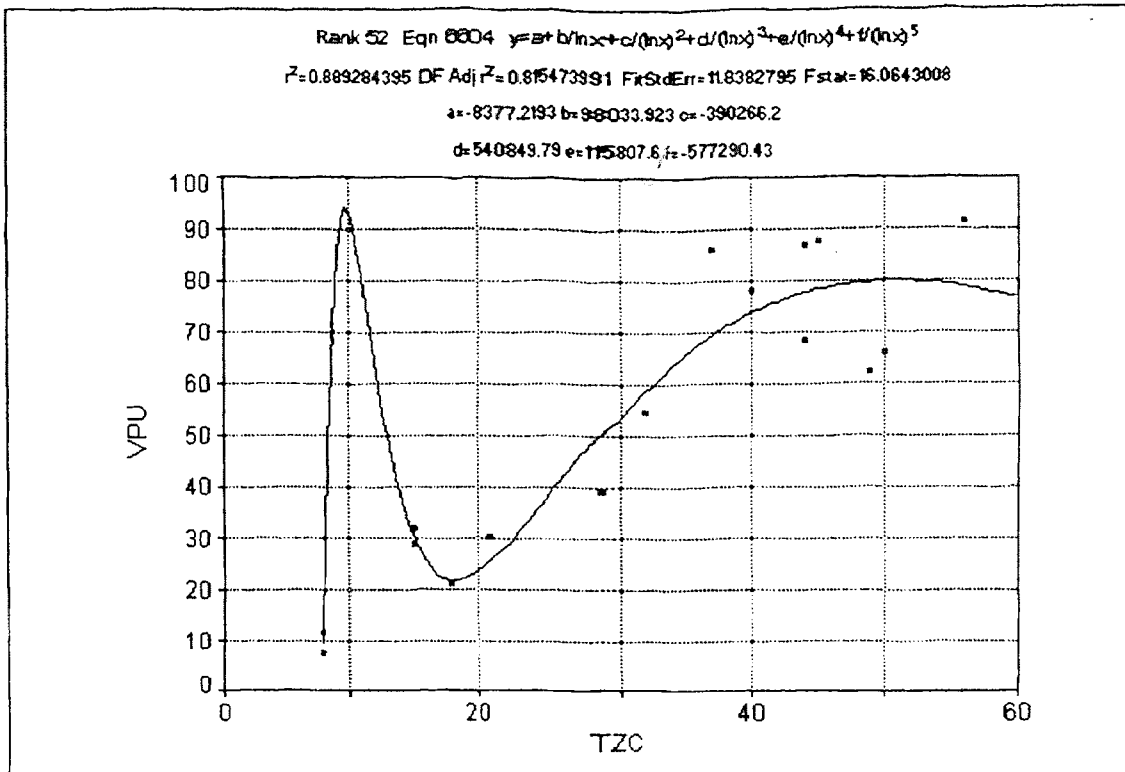


Figure C-6 Curve between VPU and TZC

## Figure C-6 (continued)

## Numeric Summary

Rank 52 Eqn 6604  $y=a+b/\ln x+c/(\ln x)^2+d/(\ln x)^3+e/(\ln x)^4+f/(\ln x)^5$ 

r2	Coef Det	DF Adj r2	Fit Std Err	F-value
0.8892843947		0.8154739912	11.838279540	16.064300828

Parm	Value	Std Error	t-value	99% Confidence Limits	
a	-8377.21926	5123.709315	-1.63499113	-24616.0937	7861.655223
b	98033.92286	57586.32055	1.702382127	-84477.8032	280545.6490
c	-390266.204	223945.2512	-1.74268577	-1.1e+06	319496.7046
d	540849.7902	305388.0706	1.771024615	-427034.655	1.50873e+06
e	115807.5974	67540.88219	1.714629623	-98253.7087	329868.9035
f	-577290.433	324217.5927	-1.78056480	-1.6049e+06	450271.5285

Area Xmin-Xmax	Area Precision		
2831.9321839	1.671539e-12		
Function min	X-Value	Function max	X-Value
21.378936775	18.167333945	94.042439306	9.7072975823
1st Deriv min	X-Value	1st Deriv max	X-Value
-16.54692488	11.460507248	3.2873847808	25.297546434
2nd Deriv min	X-Value	2nd Deriv max	X-Value
-187.6754350	8.0000096832	3.4679677147	13.427905538

Soln Vector	Covar Matrix	SVD Cond
SVDdecomp	SVDdecomp	1.741744e+14
r2	Coef Det	DF Adj r2
0.8892843947		0.8154739912
		Fit Std Err
		11.838279540

Source	Sum of Squares	DF	Mean Square	F
Regr	11256.646	5	2251.3292	16.0643
Error	1401.4486	10	140.14486	
Total	12658.095	15		

X Variable: TZC

Xmin: 8.000000000	Xmax: 56.000000000	Xrange: 48.000000000
Xmean: 31.937500000	Xstd: 15.897457868	Xmedian: 34.500000000
X@Ymin: 8.000000000	X@Ymax: 56.000000000	X@Yrange: 48.000000000

Y Variable: VPU

Ymin: 7.030000000	Ymax: 91.250000000	Yrange: 84.220000000
Ymean: 52.916250000	Ystd: 29.049491992	Ymedian: 58.045000000
Y@Xmin: 10.940000000	Y@Xmax: 91.250000000	Y@Xrange: 80.310000000

Date	Time	File Source
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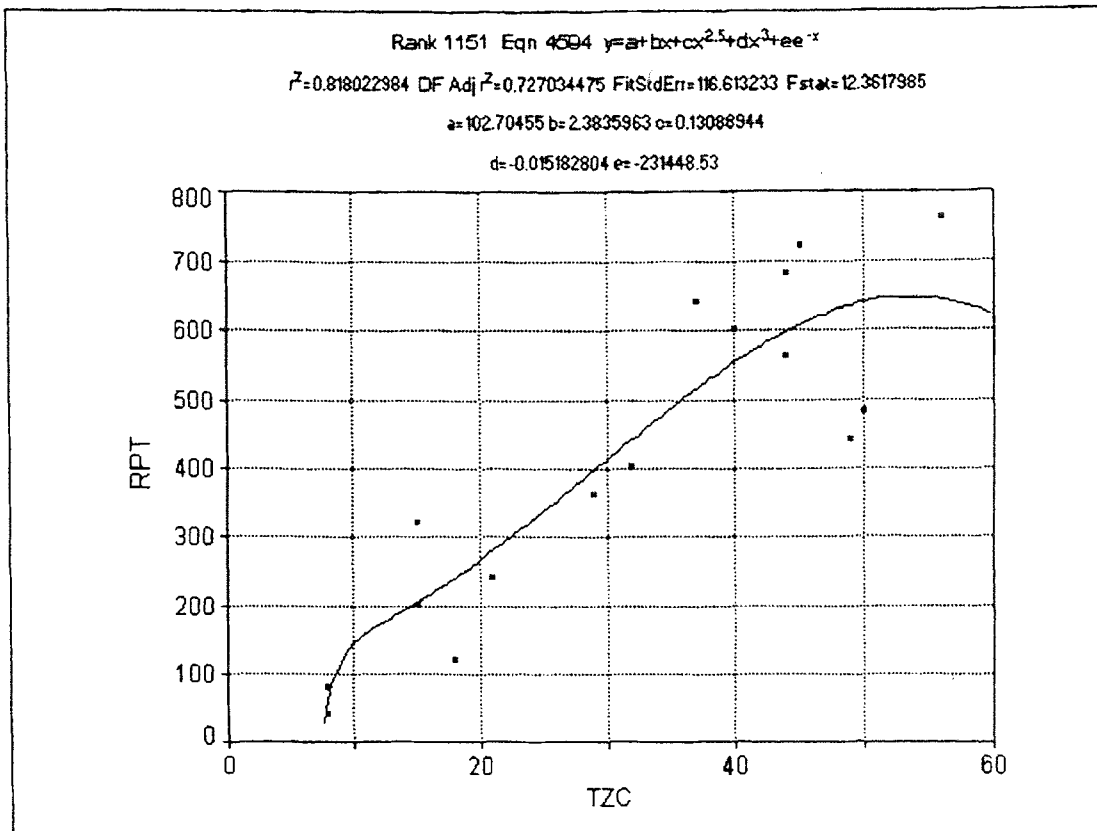


Figure C-7 Curve between RPT and TZC



## Figure C-7 (continued)

## Numeric Summary

Rank 1151 Eqn 4594  $y=a+bx+cx^2.5+dx^3+ee-x$ 

r2 Coef Det	DF Adj r2	Fit Std Err	F-value
0.8180229835	0.7270344753	116.61323312	12.361798476

Parm	Value	Std Error	t-value	99% Confidence Limits
a	102.7045527	558.9413239	0.183748362	-1633.26958 1838.678681
b	2.383596279	47.45887779	0.050224455	-145.015389 149.7825816
c	0.130889443	0.386926497	0.338279865	-1.07083668 1.332615564
d	-0.01518280	0.040177761	-0.37789075	-0.13996791 0.109602304
e	-231448.525	742390.6924	-0.31176108	-2.5372e+06 2.07429e+06

Area Xmin-Xmax	Area Precision		
20292.698646	7.675116e-11		
Function min	X-Value	Function max	X-Value
60.051734996	8.0000096832	645.52290243	53.547788603
1st Deriv min	X-Value	1st Deriv max	X-Value
-3.328059453	56.000000000	84.514308054	8.0000096832
2nd Deriv min	X-Value	2nd Deriv max	X-Value
-76.98206125	8.0000096832	0.4799811099	15.875446464

Soln Vector	Covar Matrix	
GaussElim	LUDecomp	
r2 Coef Det	DF Adj r2	Fit Std Err
0.8180229835	0.7270344753	116.61323312

Source	Sum of Squares	DF	Mean Square	F
Regr	672414.89	4	168103.72	12.3618
Error	149585.11	11	13598.646	
Total	822000	15		

## X Variable: TZC

Xmin: 8.000000000	Xmax: 56.000000000	Xrange: 48.000000000
Xmean: 31.937500000	Xstd: 15.897457868	Xmedian: 34.500000000
X@Ymin: 8.000000000	X@Ymax: 56.000000000	X@Yrange: 48.000000000

## Y Variable: RPT

Ymin: 40.000000000	Ymax: 760.000000000	Yrange: 720.000000000
Ymean: 415.000000000	Ystd: 234.09399821	Ymedian: 420.000000000
Y@Xmin: 80.000000000	Y@Xmax: 760.000000000	Y@Xrange: 680.000000000

Date	Time	File Source
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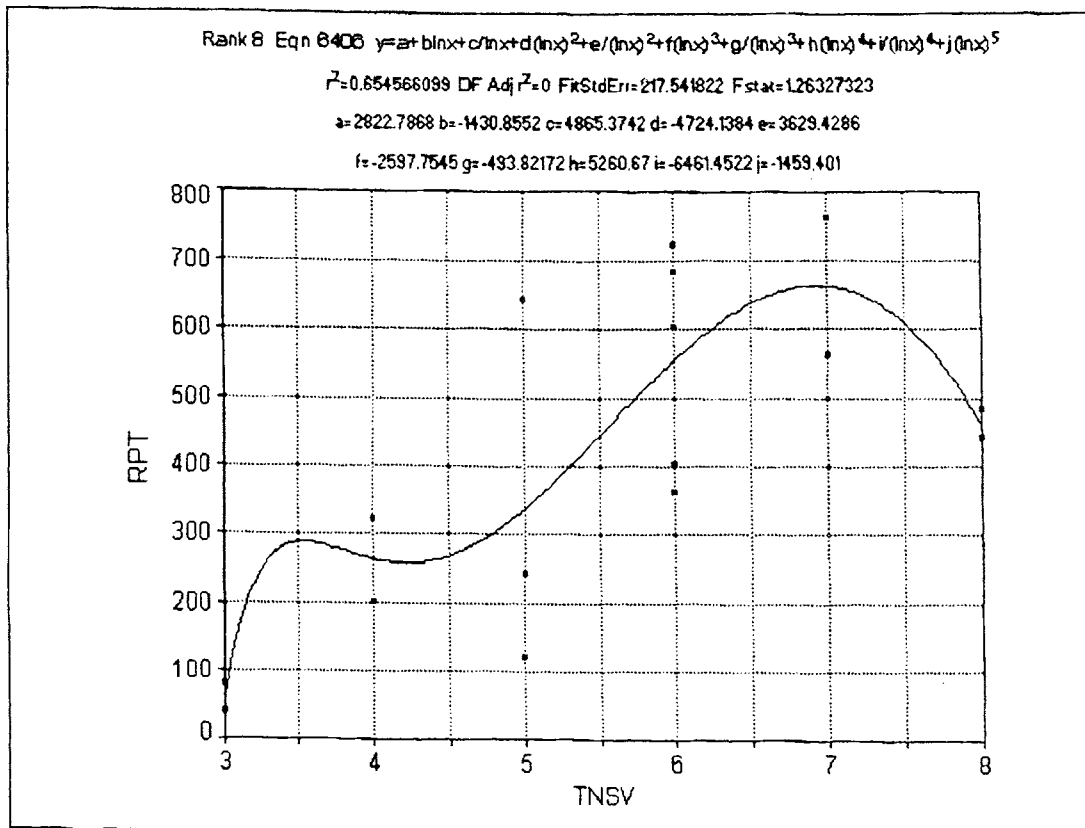


Figure C-8 Curve between RPT and TNSV

## Figure C-8 (continued)

## Numeric Summary

Rank 8 Eqn 6406  $y=a+b\ln x+c/\ln x+d(\ln x)^2+e/(\ln x)^2+f(\ln x)^3+g/(\ln x)^3+h(\ln x)^4+i/(\ln x)^4+j(\ln x)^5$ 

r2	Coef Det	DF	Adj r2	Fit Std Err	F-value
0.6545660989		0.0000000000		217.54182229	1.2632732281

Parm	Value	Std Error	t-value	99% Confidence Limits	
a	2822.786796	91048.28530	0.031003185	-334754.614	340400.1872
b	-1430.85521	149352.1342	-0.00958041	-555179.946	552318.2355
c	4865.374165	64484.07504	0.075450786	-234220.583	243951.3311
d	-4724.13841	6062.739756	-0.77920851	-27202.8037	17754.52683
e	3629.428634	153429.7237	0.023655316	-565238.036	572496.8938
f	-2597.75448	188711.3536	-0.01376576	-702278.017	697082.5079
g	-493.821720	82332.82702	-0.00599787	-305757.140	304769.4967
h	5260.670032	116499.6737	0.045156092	-426682.193	437203.5326
i	-6461.45223	160496.3265	-0.04025919	-601529.580	588606.6754
j	-1459.40098	20627.81881	-0.07074917	-77940.6376	75021.83560

Area Xmin-Xmax	Area Precision		
2181.3401481	3.161499e-14		
Function min	X-Value	Function max	X-Value
60.005949893	3.0000048028	660.66428037	6.9343089388
1st Deriv min	X-Value	1st Deriv max	X-Value
-399.1506648	8.0000000000	1238.8349071	3.0000048028
2nd Deriv min	X-Value	2nd Deriv max	X-Value
-5301.548005	3.0000048028	277.01073435	4.3632923000

Soln Vector	Covar Matrix	SVD Cond		
SVDecomp	SVDecomp	9.489506e+21		
r2	Coef Det	DF	Adj r2	Fit Std Err
0.6545660989		0.0000000000		217.54182229
Source	Sum of Squares	DF	Mean Square	F
Regr	538053.33	9	59783.704	1.26327
Error	283946.67	6	47324.444	
Total	822000	15		

## X Variable: TNSV

Xmin:	3.0000000000	Xmax:	8.0000000000	Xrange:	5.0000000000
Xmean:	5.5625000000	Xstd:	1.5478479684	Xmedian:	6.0000000000
X@Ymin:	3.0000000000	X@Ymax:	7.0000000000	X@Yrange:	4.0000000000

## Y Variable: RPT

Ymin:	40.0000000000	Ymax:	760.0000000000	Yrange:	720.0000000000
Ymean:	415.0000000000	Ystd:	234.09399821	Ymedian:	420.0000000000
Y@Xmin:	80.0000000000	Y@Xmax:	440.0000000000	Y@Xrange:	360.0000000000

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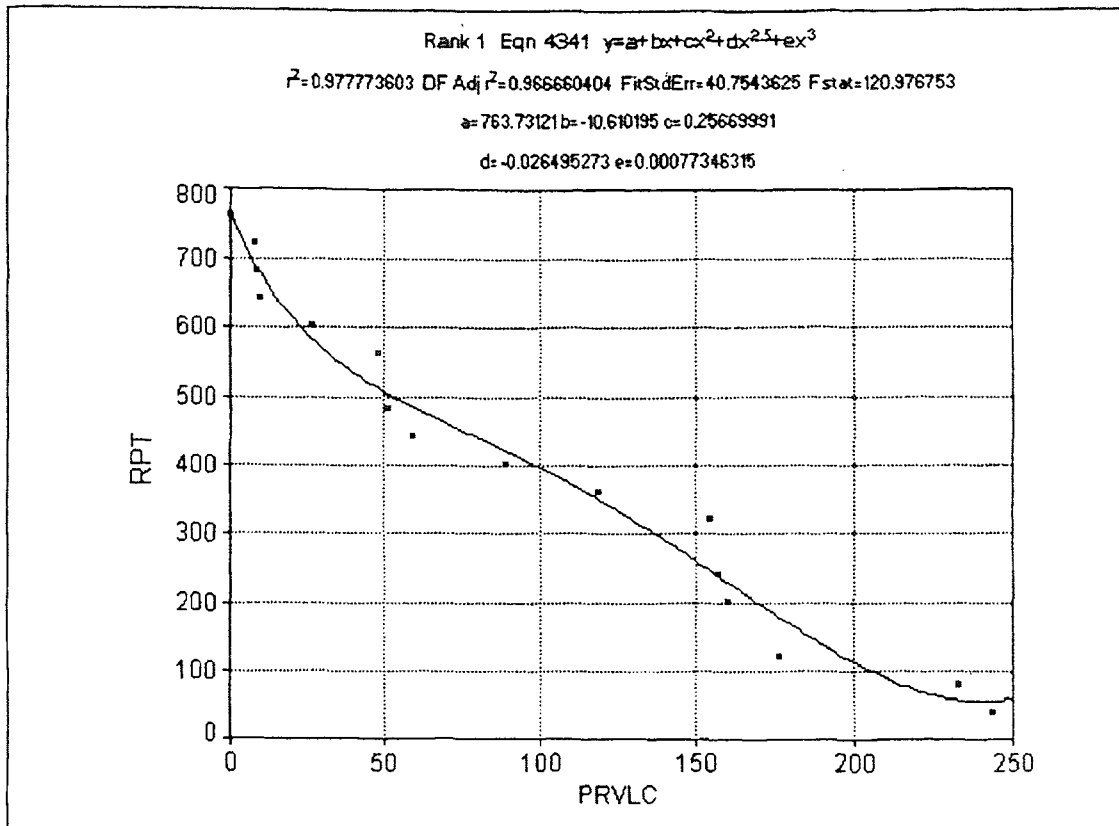


Figure C-9 Curve between RPT and PRVLC

## Figure C-9 (continued)

## Numeric Summary

Rank 1 Eqn 4341  $y=a+bx+cx^2+dx^2.5+ex^3$ 

r2	Coef Det	DF	Adj r2	Fit Std Err	F-value
0.9777736025		0.9666604038		40.754362531	120.97675347

Parm	Value	Std Error	t-value	99% Confidence Limits	
a	763.7312135	30.99677904	24.63905080	667.4606316	860.0017953
b	-10.6101950	3.509525711	-3.02325610	-21.5101689	0.289778891
c	0.256699909	0.147959068	1.734938672	-0.20283511	0.716234932
d	-0.02649527	0.016504986	-1.60528905	-0.07775688	0.024766330
e	0.000773463	0.000508149	1.522118653	-0.00080476	0.002351685

Area Xmin-Xmax	Area Precision			
81137.098662	2.249394e-09			
Function min	X-Value	Function max	X-Value	
52.883059053	240.19983834	763.73121348	1.046656e-10	
1st Deriv min	X-Value	1st Deriv max	X-Value	
-10.61019505	1.046656e-10	0.3457372976	244.00000000	
2nd Deriv min	X-Value	2nd Deriv max	X-Value	
-0.018400338	114.59382606	0.5128145183	1.311557e-10	

Soln Vector	Covar Matrix			
GaussElim	LUDecomp			
r2	Coef Det	DF	Adj r2	Fit Std Err
0.9777736025		0.9666604038		40.754362531
Source	Sum of Squares	DF	Mean Square	F
Regr	803729.9	4	200932.48	120.977
Error	18270.099	11	1660.9181	
Total	822000	15		

X Variable: PRVLC

Xmin: 0.000000000 Xmax: 244.0000000 Xrange: 244.0000000  
Xmean: 96.500000000 Xstd: 82.142964803 Xmedian: 74.000000000  
X@Ymin: 244.00000000 X@Ymax: 0.000000000 X@Yrange: 244.00000000

Y Variable: RPT

Ymin: 40.000000000 Ymax: 760.0000000 Yrange: 720.0000000  
Ymean: 415.00000000 Ystd: 234.09399821 Ymedian: 420.00000000  
Y@Xmin: 760.00000000 Y@Xmax: 40.000000000 Y@Xrange: 720.00000000

Date	Time	File Source
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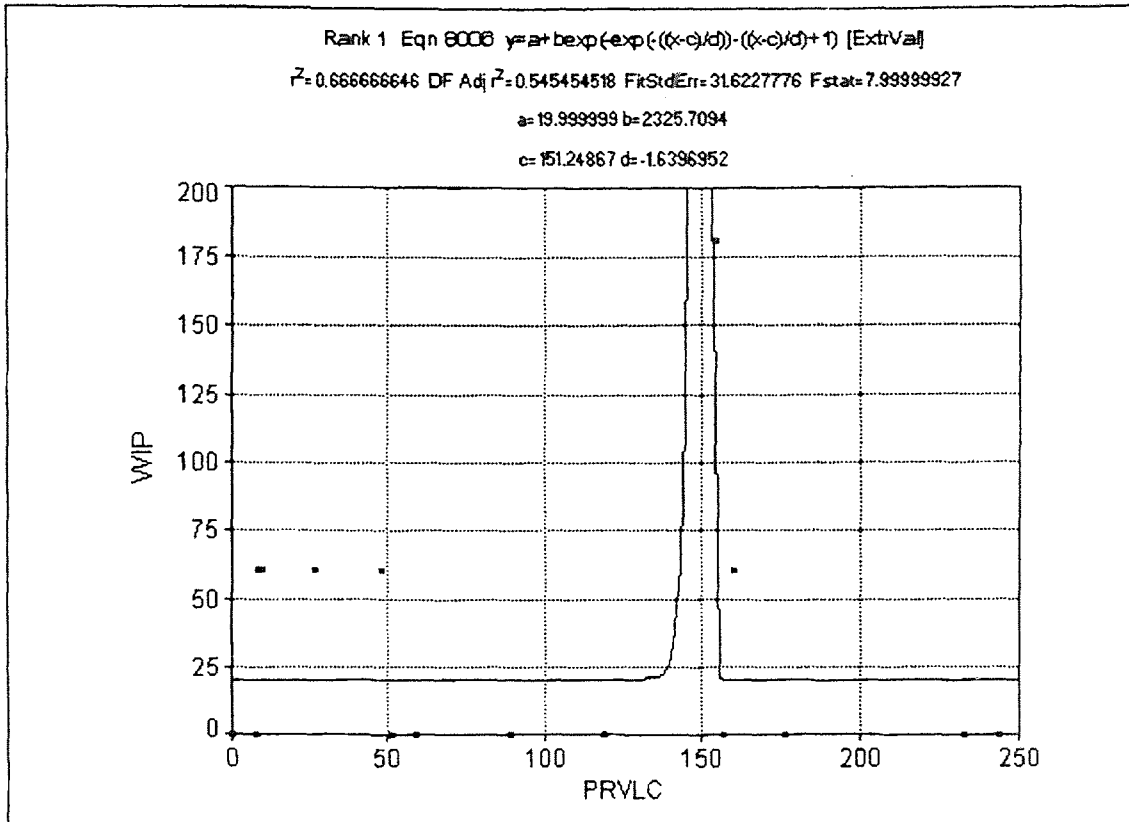


Figure C-10 Curve between WIP and PRVLC

**Figure C-10 (continued)**

## Numeric Summary

Rank 1 Eqn 8006  $y=a+b\exp(-\exp(-(x-c)/d))-((x-c)/d)+1$  [ExtrVal]

r2 Coef Det	DF Adj r2	Fit Std Err	F-value
0.6666666464	0.5454545178	31.622777563	7.9999992702

Parm	Value	Std Error	t-value	99% Confidence Limits	
a	19.99999898	8.461129263	2.363750553	-5.84455700	45.84455495
b	2325.709449	1.94003e+12	1.1988e-09	-5.9258e+12	5.92582e+12
c	151.2486736	1.81887e+08	8.31551e-07	-5.5558e+08	5.55576e+08
d	-1.63969523	7.87953e+07	-2.081e-08	-2.4068e+08	2.40681e+08

Area Xmin-Xmax	Area Precision		
15248.836837	0.0318963309		
Function min	X-Value	Function max	X-Value
19.999998977	1.046656e-10	331.92132806	146.40000000
1st Deriv min	X-Value	1st Deriv max	X-Value
0.0000000000	1.046656e-10	180.34465961	146.40000000
2nd Deriv min	X-Value	2nd Deriv max	X-Value
0.0000000000	1.046656e-10	98.240960054	146.40000000

r2 Coef Det	DF Adj r2	Fit Std Err
0.6666666464	0.5454545178	31.622777563

Source	Sum of Squares	DF	Mean Square	F
Regr	23999.999	3	7999.9998	8
Error	12000.001	12	1000.0001	
Total	36000	15		

X Variable: PRVLC

Xmin: 0.0000000000 Xmax: 244.00000000 Xrange: 244.00000000  
 Xmean: 96.500000000 Xstd: 82.142964803 Xmedian: 74.000000000  
 X@Ymin: 0.0000000000 X@Ymax: 154.00000000 X@Yrange: 154.00000000

Y Variable: WIP

Ymin: 0.0000000000 Ymax: 180.00000000 Yrange: 180.00000000  
 Ymean: 30.000000000 Ystd: 48.989794856 Ymedian: 0.0000000000  
 Y@Xmin: 0.0000000000 Y@Xmax: 0.0000000000 Y@Xrange: 0.0000000000

Date	Time	File Source
Apr 19, 1997	8:19:14 PM	CLIPBRD.PRN

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## GLOSSARY

Block	A combination of several boxes forming a larger three dimensional rectangular.
Box Type Demand	The number of boxes of that type that are required to go on the pallet.
Boxes in the System	All the boxes that can be found on the conveyor, in the WIP, and on the pallet.
Box Orientation	It is the way the length and width of the box are directed with respect to the length and width of the pallet.
Column Stacking	A pallet loading technique by which boxes are loaded column by column.
Guillotine Cut Heuristic	Separation between two box surface adjacent to each other. Different criteria, methods, or principles for deciding which among several alternative courses of action promises to be the most effective in order to achieve some goal.
Individual Box Type Sub Volume Load Capacity, ISVLC:	The maximum integer number of that box type that the sub volume can accommodate.
Layer Palletization	A pallet loading technique by which boxes are loaded layer by layer.
Mathematical Model	Approximate representation of a concept, an object, a system, or a process in mathematical terms.
Maximum Capacity of Box Type, MCBT:	The sum of all the ISVLCs that relate to the particular box type.
Negative Flexibility	The proportion of the TZC of a particular box type to the maximum TZC.
New Pattern	The overall configuration of the empty volume after the box is loaded onto one sub volume on the pallet. Sub volume combining is considered to produce new patterns.

New Sub Volume	A sub volume that still exists or is newly created right after the box is loaded onto the pallet.
Old Pattern	The way the empty volume is partitioned before the box is loaded onto the pallet.
Old Sub Volume	A sub volume that existed right before the box was loaded onto the pallet.
Pallet Stability	State of the pallet in which the load is supported.
Partitioned Remaining Volume Load Capacity, PRVLC:	The sum of all the MCBTs that relate to the particular pattern.
Pick up Place	The location at the end of the conveyor from which the robot picks up the boxes.
Positive Flexibility	The flexibility of a partitioned pattern to accommodate a particular box type.
Priority Level	A comparison stage where boxes, sub patterns, partitioning patterns are compared.
Random Stacking	Pallet loading technique by which boxes can be loaded anywhere on the pallet upon decision making.
Sub Pattern	One of the two ways that the empty volume is partitioned into sub volumes after a box is loaded onto the pallet. It is related to the two possible box orientation in its sub volume.
Sub Volume	Rectangularly shaped three dimensional empty space found anywhere at any given time on a pallet.
Surface Leveling	Loading boxes on adjacent columns so that they have a common height.
System	The palletization environment composed of the conveyor, the WIP, and the pallet.

Total Number of Sub Volumes, TNSV	The number of old sub volumes. (Before the box is loaded onto the pallet).
Work-In-Process	The holding area in which boxes that are not immediately loaded onto the pallet are temporarily stored.
Z-Axis of a Box	The vertical axis of a box.
Zero Count	A computed parameter by which ties of patterns are broken.