ABSTRACT

REDUCTION OF INERTIA-INDUCED FORCES IN A GENERAL SPATIAL MECHANISM

by
Sahidur Rahman

A computer-aided design procedure has been developed for minimizing the adverse effects of the inertia-induced forces by optimum mass redistribution amongst the links of high speed general spatial linkages. The evaluation of an optimality criterion for the mass redistribution of the mechanism will be carried out with the aid of a quadratic programming technique. This has been found to be successful in minimizing inertia-induced forces and torques. The validity of the optimization procedure will be demonstrated by application to one kind of spatial linkage.

No literature has been found on the balancing of a general spatial mechanism, since its kinematic equations are highly non-linear and therefore, are very difficult to solve. This is the first analysis of inertia-induced forces and torques in a general spatial mechanism. This method allows for the trade-offs necessary to achieve optimum dynamic response of the linkage in design stage. These trade-offs involve a balance among the shaking force, shaking moment, bearing reactions, and input torque fluctuations by mass distribution of the moving links. The results will be reduced to design procedures and guidelines. These have been outlined in a step-by-step fashion suitable for the non-specialist.

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by
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REDUCTION OF INERTIA-INDUCED FORCES IN A GENERAL SPATIAL MECHANISM

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To my father

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CHAPTER 1

INTRODUCTION

Balancing of shaking forces and moments in high-speed machinery has been a challenging problem for mechanism and machine designers. In recent years machines have been operated at higher and higher speeds. Smoothness of operation is frequently a dominant consideration in the design of high speed machines, but most mechanisms are not naturally smooth in their operation. The objective of balancing a mechanism is to eliminate or reduce the effect of the shaking force and shaking moment the mechanism exerts upon its frame and surroundings, in order that the mechanism will attain improved dynamic, wear, noise, precision of operation properties and extended fatigue life. The results of this study will provide the designer an enhanced control over dynamic properties of reciprocating machinery in the design stage. By this procedure, three sets of shaking force, shaking moment, bearing force, bearing moment and input torque for main directions X, Y, Z will be derived. The balancing condition is to be developed by combining the effects of all the inertia-induced forces and torques. The objective function is the summation of non-dimensionalized mean squared inertia-induced forces and torques with some weight factors. The designer will be given enough flexibility to adjust the weight factors depending upon different situations. The masses of the moving links will be kept constant. A quadratic programming technique will be developed and numerical example will be used to illustrate the methodology.

The dynamic balancing of machinery is essential for good high-speed performance. A considerable amount of research on balancing of shaking force and shaking moment in planar mechanisms has been carried out in recent years [22-30]. In contrast to rapid progress in balancing theory and techniques for planar linkages, the understanding of shaking force and shaking moment balancing of spatial linkages is very limited. Because of their complexity, it is generally not practical to perform an analysis of spatial linkages by hand computation or by graphical methods. Spatial linkages have therefore attracted much research interest in recent years following the advent of the high speed digital computers. The complete shaking force and shaking moment balancing of spatial linkages is a very difficult problem.

When operated at high speeds, the mass distribution in the links of a mechanism give rise to forces and moments which are transmitted to the ground link of the machine. These forces and moments shake the foundation upon which the machine is mounted, causing vibration disturbing people and doing structural damage to the floor and often to the entire building.

Our objective is to optimally distribute a given amount of mass within a link so to reduce the shaking forces which disturb the foundation of a machine. Essentially what we are doing is to represent the moment of inertia of the links by a collection of point masses whose magnitudes are optimized to achieve the reduction in inertia-induced forces and moments.

The methodology is novel and has the advantage over previous methods in that it can be applied to spatial mechanisms rather than just to planar mechanisms. As an example, it is applied to a generalized slider crank mechanism, which contains different kinds of joints such as the cylindrical, spherical and prismatic types.

1.1 Background

In this chapter, existing techniques used for balancing high-speed mechanisms and machinery are discussed. There has been a need to develop the optimum balancing of general three-dimensional mechanisms.

An unbalanced linkage running at high speed transmits shaking forces and shaking moments to its foundation (frame). The shaking force is the resultant inertia force exerted on the frame and is equal to the vector sum of the inertia forces associated with the moving links of the mechanism. The shaking moment about an axis in the frame is the vector sum of the inertia torques and the moments of the inertia forces about this axis. These forces and moments cause vibrations, fluctuations in the input torque and stresses, and therefore impose limitations on the performance of high-speed machinery.

1.1.1 Complete Balancing Techniques

Much literature [31] is available on the balancing of planar linkages. Complete shaking force and shaking moment balancing is important in the dynamic balancing of mechanism, both theoretically and practically. The major goal in this is full shaking-force balancing. Complete balancing of shaking forces can be achieved if the center of mass of the mechanism remains stationary. Various techniques have been developed for this purpose. "The static balancing method" consists of replacing the masses of the links by a statically equivalent system of point masses. By adding counterweights to the links, the center of mass of all the moving links can be brought to rest, i.e. to coincide with a point in the frame. "The method of principal vectors" consists of describing the motion of the center of mass of a mechanism analytically and then determining the parameters by which the total center of mass can be located at a stationary point. "The method of linearly-independent vectors" by Berkof [4] requires the ability to redistribute the masses of the links in such a way that the total center of mass becomes stationary. Lowen, Tepper, and Walker et. al. further developed this theory to a higher degree [39, 40, 43, 44]. They solved the problem of full shaking force balancing of general planar linkages by the method of inertial mass distribution [39, 43, 44]. Ning-Xin Chen extended this method to spatial linkages [6, 7]. Bagci made a special contribution on the "irregular force transmission mechanism" for both planar and spatial mechanisms [1, 2]. "The method of linearly-independent vectors" has been the most suitable method for full shaking force balancing of mechanisms and has been applied to both planar and spatial mechanisms. Therefore, the study of full shaking force balancing of mechanisms is satisfactory.

Nowadays, the complete shaking force and shaking moment balancing still remains a problem for some special planar mechanisms [22-30]. The complete shaking force and shaking moment balancing is much more complicated than the full shaking force balancing of a mechanism, and so only some special planar mechanisms could be completely balanced. When the shaking force of a mechanism is fully balanced, the shaking moment of the mechanism becomes a pure torque which is only relative to the rotations of the moving links of the mechanism, but not to the translations of mass centers of the links. "The method of linearly-independent vectors" of Berkof and Lowen is extended by Elliott and Tesar [9] to the shaking moment and driving torque functions. These tools are combined to completely eliminate shaking force and shaking moment with the addition of a physical negative mass. In addition to redistributing the masses, additional moving elements (cams, balance weights, etc.) can be introduced to eliminate shaking force and moments.

Investigation of the complete shaking force and shaking moment balancing of spatial mechanism has been very limited. In fact Yue-Qing's research [46-48] appears to be the only study in this field, and an encouraging achievement dealing with some types of mechanisms by the method of addition of balancing dyads. This paved the way to achieve the complete shaking force and shaking moment balancing of various kinds of spatial linkages.

1.1.2 Partial Balancing Techniques

Shaking force, shaking moment, inertia-induced joint reactions (bearing reactions) and input torque fluctuation are dynamic characteristics of mechanisms. Complete balancing of any one of these may result in an increased unbalance in the others. Hence partial balancing techniques permit desirable design trade-offs. In high speed mechanisms this is very essential. Some of the previously investigated techniques are described below.

In 1971, Berkof and Lowen [5] have presented a least-square theory for the optimization of the shaking moment of fully force-balanced planar four-bar linkages running at constant angular velocity. Sherwood [37] has used equivalent masses to minimize the kinetic-energy fluctuation of the coupler of a planar four-bar linkage having drag-linkage and crank-and-rocker proportions. Hockey [19] later presented an approach for the distribution of mass in the coupler to approximate a constant energy level for the four-bar linkage, which implies the driving torque remains near zero. Tricamo and Lowen [41, 42] introduced a two and three counterweight technique for simultaneously minimizing the maximum values of such dynamic reactions as the bearing force, the input moment and the shaking moment of a constant input-speed planar four-bar linkage, while additionally obtaining a prescribed maximum value of shaking force. In 1991 Kochev [29] performed optimum balancing of a well-known class of complex planar mechanisms which remain kinematically invariant (function cognates) with respect to the angular rearrangement of their sub linkages. His research revealed the potential of function cognate transformation for optimum balancing of such mechanisms. Providing complete shaking force balancing, he discussed two basic objectives: (i) minimization of the total balancing mass and (ii) minimization of shaking moment. However, the concept is rather general and may well contribute to other optimization problems, like minimization of a given joint reaction, balancing of flexibly mounted machines, etc.

Relatively little research has been devoted to techniques for the partial balancing of spatial linkages. Hockey [18] has minimized the fluctuation of kinetic energy and inertia forces of a spatial slider crank (RSKP) mechanism by optimizing the mass distribution. Symbol K denotes the universal joint. The exact solution of the optimized set of equations, which were obtained by assuming ten point masses in a particular configuration (to represent a three dimensional coupler), showed that for balancing purposes the coupler ideally should be a perfectly thin rod (an impractical proportion) rather than a three dimensional body. Hockey also obtained an approximate solution for a three dimensional coupler. Sherwood [36] dealt with the distribution of mass in the links of a simple harmonic spatial slider crank mechanism in order to achieve constancy of total kinetic energy and inertia force and torque balance during the motion cycle. He replaced the coupler by three in-line point masses. For constancy of kinetic energy and inertia force balance, Sherwood obtained the condition that the center of mass of the piston and connecting rod should lie on the center of the crank pin. The inertia couple, however, was not completely balanced.

Very few method [15, 16] can allow a trade-off among the shaking force, the shaking moment, bearing reactions, and the input torque in three dimensional mechanism. This method is limited to spherical mechanisms only. There is no general method which can solve this problem for general linkages, especially for spatial mechanisms. Both kinematic and dynamic properties of spatial mechanisms are much more complicated than those of planar mechanisms. Many balancing methods for planar mechanisms can not be applied to spatial linkages. Therefore, techniques for shaking force and shaking moment balancing of general spatial mechanisms are still unavailable.

1.2 Motivation, Objective and Scope of Work

Kinematic and dynamic analyses of the generalized slider crank mechanism for a single cylinder engine were accomplished by Fischer and Rahman [11, 12] in 1993. In this mechanism the joint between frame and crank is cylindrical having one translational and one rotational degree of freedom Both the joints between crank and connecting rod and connecting rod and slider (piston, in case of an engine) are spherical (ball) having three rotational degrees of freedom, one of which is passive, i.e. rotation about the connecting rod longitudinal axis. The joint between slider and frame is prismatic, having one translational degree of freedom. While conducting the dynamic analysis it is observed that due to the presence of offsets, the forces and torques acting on the joints deviate from the ideal case. Dynamic force and torque reactions in the mechanism were obtained using dual-number ($\varepsilon \neq 0$, but $\varepsilon^2=0$) techniques as developed by Yang [45] in 1971. The particular formulation used in that study was developed by Pennock and Yang [33] in 1983.

The motivation for this research is to overcome certain difficulties involved in balancing the inertia effects occurring in high speed mechanisms. An analysis or design procedure should allow for trade-offs among various quantities and thus requires a new formulation of the dynamic problem. This is likely to involve lengthy calculations, such as matrix inversion or solution of simultaneous equations. Consequently for effective modeling of a linkage, efficient numerical procedures are required. An optimality criterion which can truly represent the dynamic characteristics of a linkage has to be developed and subsequently, an efficient optimization technique is required to yield a solution. The linkage balancing problem, although considered to be an old problem, certainly faces new challenges, particularly in light of the rational design of linkages. It therefore warrants an investigation from a global perspective, that is, a balancing of combined shaking force, shaking moment, bearing reactions and torque fluctuations in high speed linkages. The purpose of this investigation is to develop a balancing method which is capable of carrying out the trade-offs that are necessary to achieve optimum dynamic response of the linkage.

The objectives of the research are as follows:

(i) Determination of the inertia force and inertia torque associated with each moving link of the mechanism.

(ii) Determination of shaking force, shaking moment, bearing reactions and input torque as a function of joint variables.

(iii) Optimization of the mass distribution with respect to shaking force, shaking moment, bearing reactions and input torque fluctuation.

(iv) Application of these techniques for balancing a CSSP mechanism. This includes determination of inertia forces and torques due to the entire mechanism and optimization of mass distribution for minimization of shaking force, shaking moment, bearing reactions and input torque fluctuation.

(v) Development of suitable computer-aided design procedures with the help of IMSL routines for the optimum mass distribution of high speed CSSP mechanism.

The result of this investigation will demonstrate that this method offers several advantages. The procedure is so general that it is applicable to many linkages with no restrictions. The method is efficient and can be easily utilized by practicing engineers without requiring any specialized skills.

1.3 Summary of Research

In this research a computer-aided design procedure has been developed for minimization of inertia-induced forces in an CSSP mechanism. Kinematic analysis data and dynamic force and torque equations used are from Fischer and Rahman [11, 12].

In Chapter 2 the kinematics of the mechanism are developed. For this purpose, one fixed coordinate system, three moving coordinate systems (each attached to a moving link) and four dual number transformation matrices are established. The mass distribution of each moving link (crank and connecting rod) is replaced by a dynamically equivalent system of four point masses. Vector coordinates of point masses relative to the fixed frame are determined by using "principle of transference" [21] and then direction cosines of the principal axes with respect to distal frame attached to each moving link. Vector coordinates of center of mass of the slider are determined by using the "principle of transference" only. Acceleration of point masses relative to the fixed frame are determined by the method as discussed in Fu, Gonzalez and Lee [14].

In Chapter 3 shaking forces, shaking moments, bearing reactions and input torques are determined as a function of crank rotation. All the forces and moments are expressed with respect to the fixed coordinate frame.

Chapter 4 deals with the minimization of inertia-induced forces in the mechanism. A quadratic objective function consisting of summation of non-dimensionalized, squared shaking force, shaking moment, bearing reactions and input torque is formulated over one complete cycle of rotation. Point masses are considered as designed variables. Design constraints are formulated as a set of equations linear in the design variables. The optimization of mass distribution is obtained by the application of IMSL routines.

In chapter 5 an example is presented to demonstrate the feasibility of the technique. Results are discussed with the help of tables and graphs.

Finally, Chapter 6 describes the general conclusions of this study and outlines the goals of future research.

CHAPTER 2

KINEMATICS OF THE MECHANISM

2.1 Coordinate Systems

Each link of the mechanism will be characterized by the relationship between the axes of its joints. As seen in figure 2.1, the link connecting axes n and $n+1$ can be characterized by its length a_n, the shortest distance between axes n and $n+1$, and twist angle α_n, the angle between axes n and $n+1$. On the distal end of each link n, there is a fixed coordinate frame {n+1} such that the i_{n+1} axis is aligned with a line of length a_n and the k_{n+1} axis is aligned with the axis of joint $n+1$. The displacements at each joint n are the rotation θ_n, representing the angles between the i-axes of frames {n} and {n+1} and the translation s_n representing the shortest distance between those i-axes.

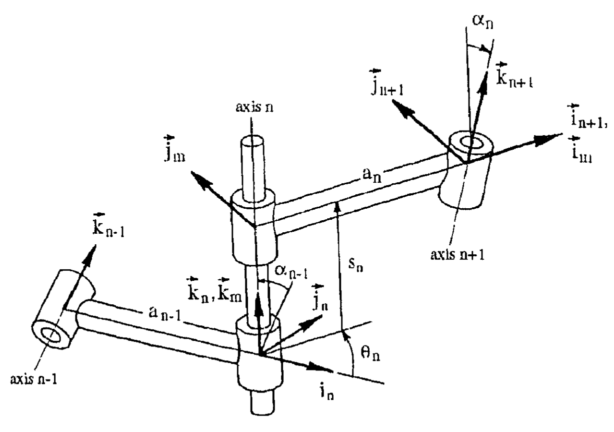

Figure 2.1 Generalized model of a link connecting two joints which are either cylindrical, prismatic or revolute.

As seen in figure 2.2, the crank of the generalized slider crank, designated as link 1, has a length a_1 and zero twist angle. The connecting rod is link 2, has length a_2 and also has zero twist angle. Link 3 is the slider, or piston, and it has zero length with a twist angle $\alpha_3 = \pi/2$. The frame, or ground link, has a length a_4, the offset, and the twist angle α_4 which would have a value of $3\pi/2$ for the planar case. At the joint between the crank and the frame, there occurs rotation through angle θ_1 and translation through distance s_1. At each end of the connecting rod is a ball joint where the displacements are specified by two rotations, angles θ_2 and η_2 at the connection with the crank, and angles θ_3 and η_3 at the connection with the slider. The rotation of the connecting rod about its own axis is a redundant degree of freedom which is neglected. The displacement of the slider is a translation through distance s_4.

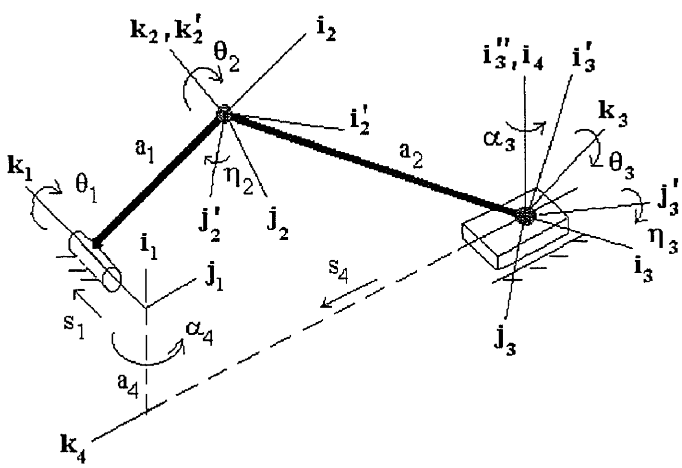

Figure 2.2 The generalized slider-crank mechanism in which the cylinder and crankshaft axes are offset and non-perpendicular.

2.1.1 Fixed Coordinate Frame

As seen in figure 2.2, coordinate frame {1} is the fixed coordinate frame, it does not move when the mechanism works. All forces and torques are expressed in terms of frame {1}.

2.1.2 Moving Coordinate Frames

The coordinate frame {2} is located at the distal end of the crank, frame {3}is located at the distal end of the connecting rod and frame {4} is located at the distal end of the slider. These are the moving coordinate frames. They move with respect to the fixed frame {1}.

2.1.3 Coordinate Transformation Matrices

A 3×3 dual-number matrix can be formulated to express the transformation between coordinate frames fixed on the distal ends of links comprising a mechanism. Referring to figure 2.1, one can trace the path from the position of frame {n-1} to the position of frame {n} as a rotation through the angle θ_n and translation through distance s_n about the k_n axis followed by rotation through angle α_n and translation through distance along the i_m or i_{n+1} axis. These displacements can be combined into the dual angles $\hat{\alpha}_n = \alpha_n + \varepsilon a_n$ and $\hat{\theta}_n = \theta_n + \varepsilon s_n$ where letter ε represents the dual number $(\varepsilon^2 = 0, \varepsilon \neq 0)$. The transformation between the coordinate frames can be considered as a screw motion through dual angle $\hat{\theta}_n$ with respect to a k-axis followed by a screw motion through dual angle $\hat{\alpha}_n$ about an i-axis. All dual-number coordinate transformations are explained in detail in Appendix A. These screw motions $\hat{Z}(\hat{\theta}_n)$ and $\hat{X}(\hat{\alpha}_n)$ can be combined into a matrix $\hat{M}_n$ and expressed in 3×3 form as

$$
{}^{n}_{n+1}\hat{M} = [Z(\hat{\theta}_n)][X(\hat{\alpha}_n)] = \begin{bmatrix} c\hat{\theta}_n & -s\hat{\theta}_n & 0 \\ s\hat{\theta}_n & c\hat{\theta}_n & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\hat{\alpha}_n & -s\hat{\alpha}_n \\ 0 & s\hat{\alpha}_n & c\hat{\alpha}_n \end{bmatrix} \tag{2.1}
$$

which expands to

$$
{}^{n}_{n+1}\hat{M} = \begin{bmatrix} c\theta_n & -c\alpha_n s\theta_n & s\alpha_n s\theta_n \\ s\theta_n & c\alpha_n c\theta_n & -s\alpha_n c\theta_n \\ 0 & s\alpha_n & c\alpha_n \end{bmatrix}
$$

$$+\varepsilon\begin{bmatrix} -s_n s\theta_n & a_n s\alpha_n s\theta_n - s_n c\alpha_n c\theta_n & a_n c\alpha_n s\theta_n + s_n s\alpha_n c\theta_n \\ s_n c\theta_n & -a_n s\alpha_n c\theta_n - s_n c\alpha_n s\theta_n & -a_n c\alpha_n c\theta_n + s_n s\alpha_n s\theta_n \\ 0 & a_n c\alpha_n & -a_n s\alpha_n \end{bmatrix} \tag{2.2}$$

For the crank, link *1*, $\alpha_1 = 0$, this specializes to

$${}^1_2\hat{M} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \varepsilon\begin{bmatrix} -s_1 s\theta_1 & -s_1 c\theta_1 & a_1 s\theta_1 \\ s_1 c\theta_1 & -s_1 s\theta_1 & -a_1 c\theta_1 \\ 0 & a_1 & 0 \end{bmatrix} \tag{2.3}$$

and since the slider is constrained from rotating so that $\theta_4 = 0$, we obtain

$${}^4_1\hat{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha_4 & -s\alpha_4 \\ 0 & s\alpha_4 & c\alpha_4 \end{bmatrix} + \varepsilon\begin{bmatrix} 0 & -s_4 c\alpha_4 & s_4 s\alpha_4 \\ s_4 & -a_4 s\alpha_4 & -a_4 c\alpha_4 \\ 0 & a_4 c\alpha_4 & -a_4 s\alpha_4 \end{bmatrix} \tag{2.4}$$

There are ball joints on the proximal ends of the connecting rod and the slider. Thus the motion at those joints requires an additional rotation through an angle η_n for its description so that the complete transformation through the joint and link takes the form

$${}^{n}_{n+1}\hat{L} = [Z(\hat{\theta}_n)][Y(\hat{\eta}_n)][X(\hat{\alpha}_n)] \tag{2.5}$$

For the connecting rod, link *2*, lengths s_2=0, e_2=0 and twist angle α_2=0, so that

$${}^2_3\hat{L} = \begin{bmatrix} c\theta_2 c\eta_2 & -s\theta_2 & c\theta_2 s\eta_2 \\ s\theta_2 c\eta_2 & c\theta_2 & s\theta_2 s\eta_2 \\ -s\eta_2 & 0 & c\eta_2 \end{bmatrix} + \varepsilon\begin{bmatrix} 0 & a_2 c\theta_2 s\eta_2 & a_2 s\theta_2 \\ 0 & a_2 s\theta_2 s\eta_2 & -a_2 c\theta_2 \\ 0 & a_2 c\eta_2 & 0 \end{bmatrix} \tag{2.6}$$

and for the slider, link *3*, where lengths s_3=0, e_3=0, a_3=0 and twist angle α_3=π/2, we have

$${}^3_4\hat{L} = \begin{bmatrix} c\theta_3 c\eta_3 & c\theta_3 s\eta_3 & s\theta_3 \\ s\theta_3 c\eta_3 & s\theta_3 s\eta_3 & -c\theta_3 \\ -s\eta_3 & c\eta_3 & 0 \end{bmatrix} \tag{2.7}$$

All the joint variables, their derivatives and their second derivatives were developed by Fischer and Rahman [11].

2.2 Replacement of the Mass Distribution of a Link by Four Point Masses

The mass distribution of each moving link of the mechanism can be represented by four point masses [18, 38]. Magnitudes and the locations of the point masses are determined on the basis of dynamical equivalence. For systems to be dynamically equivalent they must have the same mass, the same center of mass, the same principal axes and same principal moments of inertia about the center of mass. The details of the replacement of the mass distribution of a moving link by four point masses is described below.

Let symbol m denotes the mass of the moving link and symbols I_{XX}, I_{YY} and I_{ZZ} the moments of inertia of the moving link about principal axes through the center of mass.

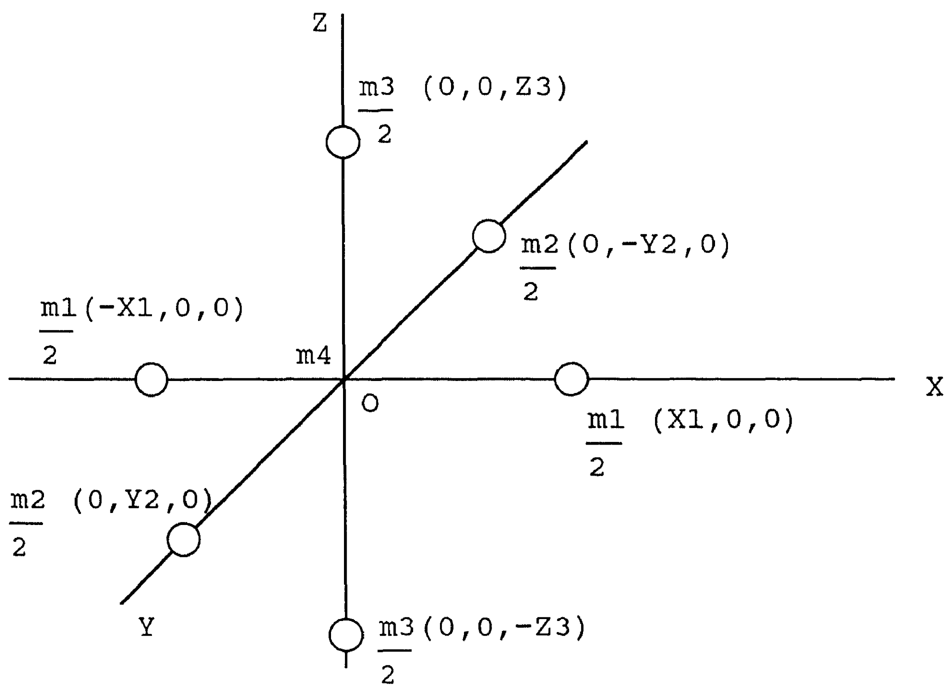

Figure 2.3 The mass distribution of the moving link

Let symbols m_1, m_2, m_3 and m_4 represent the point masses equivalent to the mass distribution of the moving link. In order to have same center of mass before and after mass distribution we use half point masses, each placed on the negative and positive side of each of the principal axes at equal distance from the origin. As shown below in figure 2.3, half point masses $m_1/2$, $m_2/2$ and $m_{3/2}$ are respectively located on the principal axes of the link at distances X_1, Y_2 and Z_3 and at distances $-X_1$, $-Y_2$ and $-Z_3$ from the center of mass. Mass m_4 lies at the center of mass of the moving link. We call this orientation a four-point mass system because of the symmetrical nature of location of the masses.

For the two systems to be dynamically equivalent, we obtain the following relations:

$$m = m_1 + m_2 + m_3 + m_4 \tag{2.8}$$

$$\left.\begin{aligned} I_{XX} &= m_2 Y_2^2 + m_3 Z_3^2 \\ I_{YY} &= m_1 X_1^2 + m_3 Z_3^2 \\ I_{ZZ} &= m_1 X_1^2 + m_2 Y_2^2 \end{aligned}\right\} \tag{2.9}$$

We have four equations with seven unknowns (four point masses m_1, m_2, m_3, m_4 and three distances X_1, Y_2 and Z_3). A solution of the system can be obtained by selecting values for three of the seven unknowns.

From equations (2.9) we obtain

$$\left.\begin{aligned} m_1 X_1^2 &= \frac{1}{2}(-I_{XX} + I_{YY} + I_{ZZ}) \\ m_2 Y_2^2 &= \frac{1}{2}(I_{XX} - I_{YY} + I_{ZZ}) \\ m_3 Z_3^2 &= \frac{1}{2}(I_{XX} + I_{YY} - I_{ZZ}) \end{aligned}\right\} \tag{2.10}$$

If we assume values for m_1, m_2 and m_3 then equation (2.10) determines X_1, Y_2 and Z_3 and vice versa.

Let

$$m_1 = m_2 = m_3 = \frac{m}{4} \tag{2.11}$$

Then from equation (2.10) we obtain

$$\left.\begin{aligned} X_1 &= \sqrt{\frac{2}{m}(-I_{XX} + I_{YY} + I_{ZZ})} \\ Y_2 &= \sqrt{\frac{2}{m}(I_{XX} - I_{YY} + I_{ZZ})} \\ Z_3 &= \sqrt{\frac{2}{m}(I_{XX} + I_{YY} - I_{ZZ})} \end{aligned}\right\} \tag{2.12}$$

and from equation (2.8)

$$m_4 = \frac{m}{4} \tag{2.13}$$

The mass distribution of the crank, link 1, is replaced by four point masses m_{11}, m_{12}, m_{13} and m_{14}. Point mass m_{14} lies at the center of mass of the crank and the half point masses $m_{11}/2$, $m_{12}/2$ and $m_{13}/2$ lie on the negative and positive side of the principal axes attached to the crank at distances l_{11}, l_{12} and l_{13}, respectively from its center of mass. The values of point masses m_{11}, m_{12}, m_{13} and m_{14} and distances l_{11}, l_{12} and l_{13} can be evaluated from the equations (2.11), (2.12), and (2.13) for the known values of moments of inertia of the crank.

The mass distribution of the connecting rod is replaced by four point masses m_{21}, m_{22}, m_{23} and m_{24}. Point mass m_{24} lies at the center of mass of the connecting rod and the half point masses $m_{21}/2$, $m_{22}/2$ and $m_{23}/2$ lie on the negative and positive side of the principal axes attached to the connecting rod at distances l_{21}, l_{22} and l_{23}, respectively from its center of mass. The values of point masses m_{21}, m_{22}, m_{23} and m_{24} and distances l_{21}, l_{22} and l_{23} can be evaluated from the equations (2.11), (2.12) and (2.13) for the known values of moments of inertia of the connecting rod.

The mass distribution of the piston is replaced by a single point mass, m_4. Point mass m_4 lies at the center of the mass of the piston.

2.3 Kinematics of the Point Masses

Now for each moving link, four point masses and their locations with respect to center of mass can be computed using equations (2.11), (2.12) and (2.13). Having done that, the equations for position, velocity and acceleration of each point mass with respect to fixed coordinate frame can be formulated by the following method.

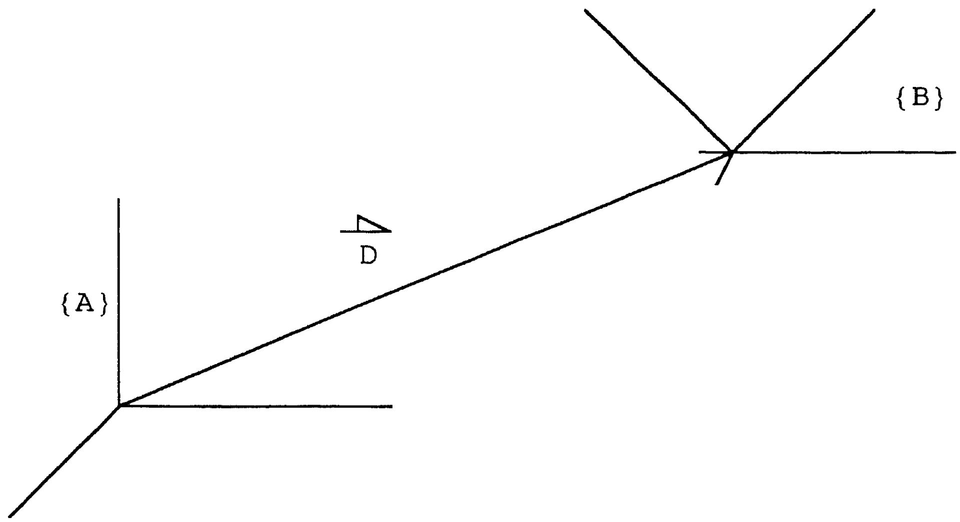

Figure 2.4 Distance between two frames {A} and {B}

If the transformation matrix ${}^{A}_{B}\hat{T}$ describes unit vectors of frame {B} in terms of the unit vectors of frame {A} as shown in figure 2.4 and can be decomposed into

$$ {}^{A}_{B}\hat{T} = T_p + \varepsilon T_d \tag{2.14}$$

then using the "principle of transference" developed by Hsia and Yang [21] the location of distal coordinate frame on each moving link with respect to the fixed frame can be found:

$$[D] = [T_d][T_p]^T \tag{2.15}$$

Where the vector D is the 3×1 primary-number column matrix describing displacement of origin of frame {B} relative to origin of frame {A} such that the displacement matrix

$$[D]=\begin{bmatrix} 0 & -d_3 & d_2 \\ d_3 & 0 & -d_1 \\ -d_2 & d_1 & 0 \end{bmatrix} \tag{2.16}$$

is the 3×3 form of vector

$$D=\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} \tag{2.17}$$

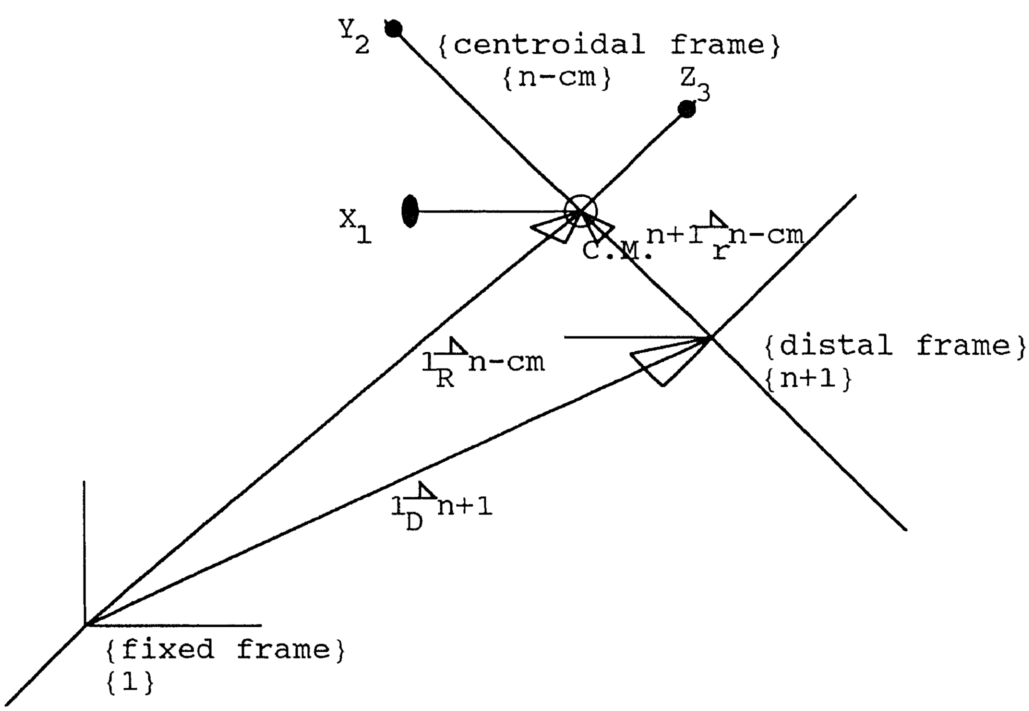

Figure 2.5 Location of distal frame {n+1} of the n-th link and four-point mass system of the moving link n with respect to fixed frame {1}.

As shown in figure 2.5 let the position vector of the distal frame {n} relative to the fixed frame {1} be

$$ {}^{1}D^{n+1} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} \tag{2.18} $$

The time derivative of the position vector, i.e. the velocity vector, is

$$ {}^{1}\dot{D}^{n+1} = \begin{Bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{Bmatrix} \tag{2.19} $$

The second time derivative of the position vector, i.e. the acceleration vector, is

$$ {}^{1}\ddot{D}^{n+1} = \begin{Bmatrix} \ddot{d}_1 \\ \ddot{d}_2 \\ \ddot{d}_3 \end{Bmatrix} \tag{2.20} $$

If the coordinates representing the location of center of mass of the moving link with respect to the distal coordinate frame{n+1} is

$$ {}^{n+1}r^{n-cm} = \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} \tag{2.21} $$

and the direction cosines between the centroidal principal coordinate frame{n-cm} and the distal frame{n+1} are represented by the matrix

$$ {}^{n+1}_{n-cm}L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \tag{2.22} $$

then the following derivations give the location vectors of the point masses with respect to the distal coordinate frame{n+1}.

Let us consider the point masses m_1. The vector locating the half point mass $m_1/2$, placed on the positive side of X-axis, relative to frame {n-cm} is

$$^{n-cm}P^{1,P} = \begin{Bmatrix} X_1 \\ 0 \\ 0 \end{Bmatrix} \tag{2.23a}$$

$$^{n+1}l_{1,P} = {}^{n+1}\bar{r}^{n-cm} + {}^{n+1}_{n-cm}L\, {}^{n-cm}P^{1,P} = \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} + \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} X_1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} X_0 + X_1 L_{11} \\ Y_0 + X_1 L_{21} \\ Z_0 + X_1 L_{31} \end{Bmatrix} \tag{2.24a}$$

Abbreviations P and N respectively represent positive and negative.

The vector locating the half point mass $m_1/2$, placed on the negative side of X-axis, relative to frame {n-cm} is

$$^{n-cm}P^{1,N} = \begin{Bmatrix} -X_1 \\ 0 \\ 0 \end{Bmatrix} \tag{2.23b}$$

$$^{n+1}l_{1,N} = {}^{n+1}\bar{r}^{n-cm} + {}^{n+1}_{n-cm}L\, {}^{n-cm}P^{1,N} = \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} + \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} -X_1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} X_0 - X_1 L_{11} \\ Y_0 - X_1 L_{21} \\ Z_0 - X_1 L_{31} \end{Bmatrix} \tag{2.24b}$$

We now consider the point masses m_2. The vector locating the half point mass $m_2/2$, placed on the positive side of Y-axis, relative to frame {n-cm} is

$$^{n-cm}P^{2,P} = \begin{Bmatrix} 0 \\ Y_2 \\ 0 \end{Bmatrix} \tag{2.25a}$$

$$^{n+1}l_{2,P} = {}^{n+1}\bar{r}^{n-cm} + {}^{n+1}_{n-cm}L\, {}^{n-cm}P^{2,P} = \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} + \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ Y_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} X_0 + Y_2 L_{12} \\ Y_0 + Y_2 L_{22} \\ Z_0 + Y_2 L_{32} \end{Bmatrix} \tag{2.26a}$$

The vector locating the half point mass $m_2/2$, placed on the negative side of Y-axis, relative to frame {n-cm} is

$$ {}^{n-cm}P^{2,N} = \begin{Bmatrix} 0 \\ -Y_2 \\ 0 \end{Bmatrix} \tag{2.25b} $$

$$ {}^{n+1}l_{2,N} = {}^{n+1}\bar{r}^{n-cm} + {}^{n+1}_{n-cm}L\, {}^{n-cm}P^{2,N} = \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} + \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ -Y_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} X_0 - Y_2 L_{12} \\ Y_0 - Y_2 L_{22} \\ Z_0 - Y_2 L_{32} \end{Bmatrix} \tag{2.26b} $$

Let us now consider the point masses m_3. The vector locating the half point mass $m_3/2$, placed on the positive side of Z-axis, relative to frame {n-cm} is

$$ {}^{n-cm}P^{3,P} = \begin{Bmatrix} 0 \\ 0 \\ Z_3 \end{Bmatrix} \tag{2.27a} $$

$$ {}^{n+1}l_{3,P} = {}^{n+1}\bar{r}^{n-cm} + {}^{n+1}_{n-cm}L\, {}^{n-cm}P^{3,P} = \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} + \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ Z_3 \end{Bmatrix} = \begin{Bmatrix} X_0 + Z_3 L_{13} \\ Y_0 + Z_3 L_{23} \\ Z_0 + Z_3 L_{33} \end{Bmatrix} \tag{2.28a} $$

The vector locating the half point mass $m_3/2$, placed on the negative side of Z-axis, relative to frame {n-cm} is

$$ {}^{n-cm}P^{3,N} = \begin{Bmatrix} 0 \\ 0 \\ -Z_3 \end{Bmatrix} \tag{2.27b} $$

$$ {}^{n+1}l_{3,N} = {}^{n+1}\bar{r}^{n-cm} + {}^{n+1}_{n-cm}L\, {}^{n-cm}P^{3,N} = \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} + \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -Z_3 \end{Bmatrix} = \begin{Bmatrix} X_0 - Z_3 L_{13} \\ Y_0 - Z_3 L_{23} \\ Z_0 - Z_3 L_{33} \end{Bmatrix} \tag{2.28b} $$

We consider now point mass m_4. The vector locating the point mass m_4 relative to frame {n-cm} is

$$ {}^{n-cm}P^4 = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2.29} $$

$$ {}^{n+1}l_4 = \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} \tag{2.30} $$

The position vectors of the point masses m_1, m_2, m_3 and m_4 with respect to frame {1} are respectively,

$$ \{L_{1,P}\} = \{D\} + \left[{}^{1}_{n+1}T_R\right]\left\{{}^{n+1}l_{1,P}\right\} \tag{2.31a} $$

$$ \{L_{1,N}\} = \{D\} + \left[{}^{1}_{n+1}T_R\right]\left\{{}^{n+1}l_{1,N}\right\} \tag{2.31b} $$

$$ \{L_{2,P}\} = \{D\} + \left[{}^{1}_{n+1}T_R\right]\left\{{}^{n+1}l_{2,P}\right\} \tag{2.31c} $$

$$ \{L_{2,N}\} = \{D\} + \left[{}^{1}_{n+1}T_R\right]\left\{{}^{n+1}l_{2,N}\right\} \tag{2.31d} $$

$$ \{L_{3,P}\} = \{D\} + \left[{}^{1}_{n+1}T_R\right]\left\{{}^{n+1}l_{3,P}\right\} \tag{2.31e} $$

$$ \{L_{3,N}\} = \{D\} + \left[{}^{1}_{n+1}T_R\right]\left\{{}^{n+1}l_{3,N}\right\} \tag{2.31f} $$

$$ \{L_{4}\} = \{D\} + \left[{}^{1}_{n+1}T_R\right]\left\{{}^{n+1}l_{4}\right\} \tag{2.31g} $$

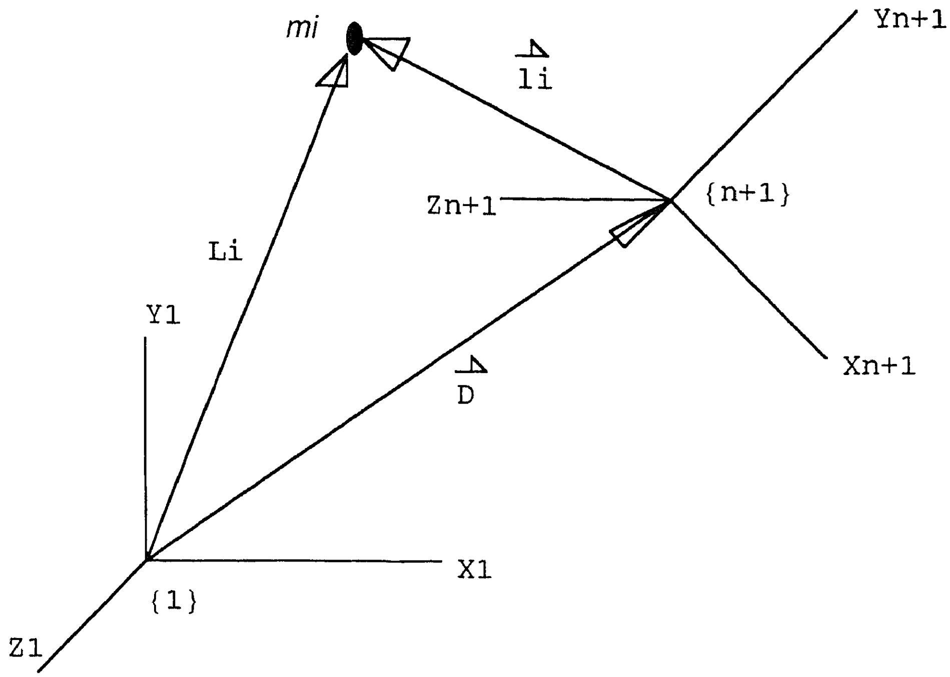

Figure 2.6 Inertial and moving (translating and rotating) coordinate systems and moving point mass m_i.

As shown in figure 2.6, symbol l_i (i=1, 2, 3 and 4) denotes the position vectors of the moving masses m_i (i=1, 2, 3 and 4) which are at rest in distal coordinate system {n+1} which is moving (translating and rotating) relative to inertial coordinate system {1}. Symbol L_i (i=1, 2, 3 and 4) denotes the position vectors of the moving masses m_i (i=1, 2, 3 and 4) relative to inertial coordinate system {1}. The acceleration of a moving mass m_i relative to coordinate frame {1} can be expressed as

$$a = \ddot{l}_i + 2\omega \times \dot{l}_i + \omega \times (\omega \times l_i) + \frac{d\omega}{dt} \times l_i + \ddot{D} \tag{2.32}$$

where symbol ω is the angular velocity vector of the coordinate system $\{X_{n+1}Y_{n+1}Z_{n+1}\}$ with respect to fixed coordinate system $\{X_1Y_1Z_1\}$ (see texts such as Fu, Gonzalez and Lee [14]). The first term on the right-hand side of the equation (2.32) is the acceleration relative to the coordinate system $\{X_{n+1}Y_{n+1}Z_{n+1}\}$. The second term is called the Coriolis acceleration. The third term is called the centripetal (toward the center) acceleration. The fourth term points directly toward and perpendicular to the axis of rotation. The last term is the linear acceleration of the frame {n+1} relative to the inertial frame {1}. The 3×3 skew-symmetric matrix expression of vector ω can be found from the following equation as derived in Nikravesh [32].

$$\tilde{\omega} = \left[{}_{n+1}^{1}\dot{T}_R\right]\left[{}^{n+1}_{1}T_R\right] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{2.33}$$

where $\left[{}^{n+1}_{1}T_R\right]$ and $\left[{}_{n+1}^{1}\dot{T}_R\right]$ are respectively the rotation matrix and its derivative with respect to time. The components of vector ω may be expressed as

$$\omega = \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \tag{2.33}$$

2.3.1 Kinematics of the Crank

A crank of mass M_1 is replaced by four equal point masses, m_{11}, m_{12}, m_{13} and m_{14}, such that

$$m_{11} = m_{12} = m_{13} = m_{14} = \frac{M_1}{4} \tag{2.35}$$

The position vector of the point mass m_{11} is

$${}^{1\text{-}cm}p^{11} = \begin{Bmatrix} X_{11} \\ 0 \\ 0 \end{Bmatrix} \tag{2.36}$$

where

$$X_{11} = \sqrt{\frac{1}{2m_{11}}(I_{1Y} + I_{1Z} - I_{1X})} \tag{2.37}$$

The position vector of the point mass m_{12} is

$$^{1-cm}P^{12} = \begin{Bmatrix} 0 \\ Y_{12} \\ 0 \end{Bmatrix} \tag{2.38}$$

where

$$Y_{12} = \sqrt{\frac{1}{2m_{12}}(I_{1X} + I_{1Z} - I_{1Y})} \tag{2.39}$$

The position vector of the point mass m_{13} is

$$^{1-cm}P^{13} = \begin{Bmatrix} 0 \\ 0 \\ Z_{13} \end{Bmatrix} \tag{2.40}$$

where

$$Z_{13} = \sqrt{\frac{1}{2m_{13}}(I_{1X} + I_{1Y} - I_{1Z})} \tag{2.41}$$

Mass m_{14} is located at the center of mass of the crank, so that

$$^{1-cm}P^{14} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2.42}$$

As shown in figure 2.7, using the "principle of transference" we can find the location of the origin of the distal coordinate frame $\{2\}$ with respect to the fixed frame $\{1\}$.

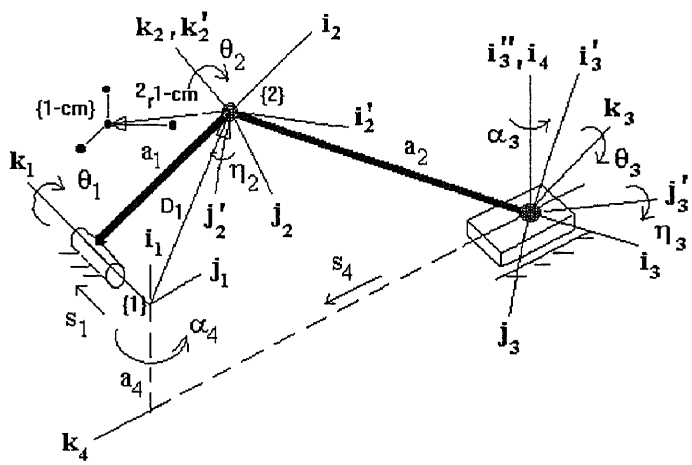

Figure 2.7 Crank is replaced by four point masses; D_1 is the position vector representing distance of distal frame {2} of the crank from frame {1}.

$$[D_1] = [{}^1_2M_D][{}^1_2M_R]^T$$

$$= \begin{bmatrix} -S_1 s\theta_1 & -S_1 c\theta_1 & a_1 s\theta_1 \\ S_1 c\theta_1 & -S_1 s\theta_1 & -a_1 c\theta_1 \\ 0 & a_1 & 0 \end{bmatrix} \begin{bmatrix} c\theta_1 & s\theta_1 & 0 \\ -s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -S_1 & a_1 s\theta_1 \\ S_1 & 0 & -a_1 c\theta_1 \\ -a_1 s\theta_1 & a_1 c\theta_1 & 0 \end{bmatrix} \tag{2.43}$$

Thus, the position vector $\{D_1\}$ representing the distance from origin of fixed frame {1} to the origin of the coordinate frame at point 2 on the crank can be written as

$$\{D_1\} = \begin{Bmatrix} a_1 c\theta_1 \\ a_1 s\theta_1 \\ S_1 \end{Bmatrix} \tag{2.44}$$

The time derivative of the position vector, i.e. the velocity vector, is

$$\{\dot{D}_1\} = \begin{Bmatrix} -a_1 s\theta_1 \dot{\theta}_1 \\ a_1 c\theta_1 \dot{\theta}_1 \\ \dot{S}_1 \end{Bmatrix} \tag{2.45}$$

The second time derivative of the position vector, i.e. the acceleration vector, is

$$\{\ddot{D}_1\} = \begin{Bmatrix} -a_1(c\theta_1\dot{\theta}_1^2 + s\theta_1\ddot{\theta}_1) \\ a_1(c\theta_1\ddot{\theta}_1 - s\theta_1\dot{\theta}_1^2) \\ \ddot{S}_1 \end{Bmatrix} \tag{2.46}$$

The vector representing the location of the center of mass of the crank with respect to the distal coordinate frame {2} is

$$^{2}r^{1-cm} = \begin{Bmatrix} X_{10} \\ Y_{10} \\ Z_{10} \end{Bmatrix} \tag{2.47}$$

and the direction cosines between the centriodal principal coordinate frame{1-cm} and the distal coordinate frame{2} are represented by the matrix

$$^{2}_{1-cm}L = \begin{bmatrix} L_{1,11} & L_{1,12} & L_{1,13} \\ L_{1,21} & L_{1,22} & L_{1,23} \\ L_{1,31} & L_{1,32} & L_{1,33} \end{bmatrix} \tag{2.48}$$

Point masses m_{11}, m_{12}, m_{13} and m_{14} are located on the principal axes X_1, Y_1, Z_1 and origin of the centroidal coordinate frame respectively. Therefore, coordinates of half point masses $m_{11}/2$, $m_{12}/2$, $m_{13}/2$ and point mass m_{14} are respectively $(\pm X_{11},0,0)$, $(0,\pm Y_{12},0)$, $(0,0,\pm Z_{13})$ and $(0,0,0)$.

The location of half point mass $m_{11}/2$, placed on the positive side of X-axis, with respect to the distal coordinate frame {2} is

$$\left\{{}^{2}d_{11,P}\right\} = {}^{2}r^{1-cm} + {}_{1-cm}^{\;\;2}L^{1-cm}p^{11,P} = \begin{Bmatrix} X_{10} \\ Y_{10} \\ Z_{10} \end{Bmatrix} + \begin{bmatrix} L_{1,11} & L_{1,12} & L_{1,13} \\ L_{1,21} & L_{1,22} & L_{1,23} \\ L_{1,31} & L_{1,32} & L_{1,33} \end{bmatrix} \begin{Bmatrix} X_{11} \\ 0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} X_{10} + L_{1,11} X_{11} \\ Y_{10} + L_{1,21} X_{11} \\ Z_{10} + L_{1,31} X_{11} \end{Bmatrix} \tag{2.49a}$$

Similarly, the location of half point mass $m_{11}/2$, placed on the negative side of X-axis, with respect to the distal coordinate frame {2} is

$$\left\{{}^{2}d_{11,N}\right\} = \begin{Bmatrix} X_{10} - L_{1,11} X_{11} \\ Y_{10} - L_{1,21} X_{11} \\ Z_{10} - L_{1,31} X_{11} \end{Bmatrix} \tag{2.49b}$$

The location of half point mass $m_{12}/2$, placed on the positive side of Y-axis, with respect to the distal coordinate frame {2} is

$$\left\{{}^{2}d_{12,P}\right\} = {}^{2}r^{1-cm} + {}_{1-cm}^{\;\;2}L^{1-cm}p^{12,P} = \begin{Bmatrix} X_{10} \\ Y_{10} \\ Z_{10} \end{Bmatrix} + \begin{bmatrix} L_{1,11} & L_{1,12} & L_{1,13} \\ L_{1,21} & L_{1,22} & L_{1,23} \\ L_{1,31} & L_{1,32} & L_{1,33} \end{bmatrix} \begin{Bmatrix} 0 \\ Y_{12} \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} X_{10} + L_{1,12} Y_{12} \\ Y_{10} + L_{1,22} Y_{12} \\ Z_{10} + L_{1,32} Y_{12} \end{Bmatrix} \tag{2.50a}$$

Similarly, the location of half point mass $m_{12}/2$, placed on the negative side of Y-axis, with respect to the distal coordinate frame {2} is

$$\{{}^{2}d_{12,N}\}=\begin{Bmatrix} X_{10}-L_{1,12}Y_{12} \\ Y_{10}-L_{1,22}Y_{12} \\ Z_{10}-L_{1,32}Y_{12} \end{Bmatrix} \tag{2.50b}$$

The location of half point mass $m_{13}/2$, placed on the positive side of Z-axis, with respect to the distal coordinate frame {2} is

$$\{{}^{2}d_{13,P}\}={}^{2}r^{1-cm}+{}_{1-cm}^{\;\;2}L^{1-cm}P^{13,P}=\begin{Bmatrix} X_{10} \\ Y_{10} \\ Z_{10} \end{Bmatrix}+\begin{bmatrix} L_{1,11} & L_{1,12} & L_{1,13} \\ L_{1,21} & L_{1,22} & L_{1,23} \\ L_{1,31} & L_{1,32} & L_{1,33} \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ Z_{13} \end{Bmatrix}$$

$$=\begin{Bmatrix} X_{10}+L_{1,13}Z_{13} \\ Y_{10}+L_{1,23}Z_{13} \\ Z_{10}+L_{1,33}Z_{13} \end{Bmatrix} \tag{2.51a}$$

Similarly, the location of half point mass $m_{13}/2$, placed on the negative side of Z-axis, with respect to the distal coordinate frame {2} is

$$\{{}^{2}d_{13,N}\}=\begin{Bmatrix} X_{10}-L_{1,13}Z_{13} \\ Y_{10}-L_{1,23}Z_{13} \\ Z_{10}-L_{1,33}Z_{13} \end{Bmatrix} \tag{2.51b}$$

The location of point mass m_{14} with respect to the distal coordinate frame {2} is

$$\{{}^{2}d_{14}\}={}^{2}r^{1-cm}+{}_{1-cm}^{\;\;2}L^{1-cm}P^{14}=\begin{Bmatrix} X_{10} \\ Y_{10} \\ Z_{10} \end{Bmatrix} \tag{2.52}$$

The position vectors of the half point masses $m_{11}/2$, $m_{12}/2$, $m_{13}/2$ and point mass m_{14} with respect to frame $\{1\}$ are respectively,

$$\{D_{11,P}\} = \{D_1\} + [{}^1_2T_R]\{{}^2d_{11,p}\} \tag{2.53a}$$

$$\{D_{11,N}\} = \{D_1\} + [{}^1_2T_R]\{{}^2d_{11,N}\} \tag{2.53b}$$

$$\{D_{12,P}\} = \{D_1\} + [{}^1_2T_R]\{{}^2d_{12,P}\} \tag{2.53c}$$

$$\{D_{12,N}\} = \{D_1\} + [{}^1_2T_R]\{{}^2d_{12,N}\} \tag{2.53d}$$

$$\{D_{13,p}\} = \{D_1\} + [{}^1_2T_R]\{{}^2d_{13,p}\} \tag{2.53e}$$

$$\{D_{13,N}\} = \{D_1\} + [{}^1_2T_R]\{{}^2d_{13,N}\} \tag{2.53f}$$

$$\{D_{14}\} = \{D_1\} + [{}^1_2T_R]\{{}^2d_{14}\} \tag{2.53g}$$

The acceleration of half point masses $m_{11}/2$, $m_{12}/2$, $m_{13}/2$ and point mass m_{14} can respectively be written as follows. Since they are stationary with respect to distal coordinate frame $\{2\}$, first and second time derivatives of their distances from $\{2\}$ are zero.

$$a_{11,P} = \omega_1 \times (\omega_1 \times {}^2d_{11,P}) + \frac{d\omega_1}{dt} \times {}^2d_{11,P} + \ddot{D}_1 \tag{2.54a}$$

$$a_{11,N} = \omega_1 \times (\omega_1 \times {}^2d_{11,N}) + \frac{d\omega_1}{dt} \times {}^2d_{11,N} + \ddot{D}_1 \tag{2.54b}$$

$$a_{12,P} = \omega_1 \times (\omega_1 \times {}^2d_{12,P}) + \frac{d\omega_1}{dt} \times {}^2d_{12,P} + \ddot{D}_1 \tag{2.54c}$$

$$a_{12,N} = \omega_1 \times (\omega_1 \times {}^2d_{12,N}) + \frac{d\omega_1}{dt} \times {}^2d_{12,N} + \ddot{D}_1 \tag{2.54d}$$

$$a_{13,P} = \omega_1 \times (\omega_1 \times {}^2d_{13,P}) + \frac{d\omega_1}{dt} \times {}^2d_{13,p} + \ddot{D}_1 \tag{2.54e}$$

$$a_{13,N} = \omega_1 \times (\omega_1 \times {}^2d_{13,N}) + \frac{d\omega_1}{dt} \times {}^2d_{13,N} + \ddot{D}_1 \tag{2.54f}$$

$$a_{14} = \omega_1 \times (\omega_1 \times {}^2d_{14}) + \frac{d\omega_1}{dt} \times {}^2d_{14} + \ddot{D}_1 \tag{2.54g}$$

where vector $\omega_1 = \begin{Bmatrix} \omega_{1x} \\ \omega_{1y} \\ \omega_{1z} \end{Bmatrix}$ represents the angular velocity of frame {2} with respect to frame {1} and as previously discussed, the elements of this vector can be determined using the relationship

$$\tilde{\omega}_1 = [{}^1_2\dot{T}_R][{}^2_1T_R] = \begin{bmatrix} -s\theta_1\dot{\theta}_1 & -c\theta_1\dot{\theta} & 0 \\ c\theta_1\dot{\theta}_1 & -s\theta_1\dot{\theta}_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c\theta_1 & s\theta_1 & 0 \\ -s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.55}$$

where

$$\tilde{\omega}_1 = \begin{bmatrix} 0 & -\omega_{1z} & \omega_{1y} \\ \omega_{1z} & 0 & -\omega_{1x} \\ -\omega_{1y} & \omega_{1x} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.56}$$

Since crank is rotating only about the Z-axis, other rotation components are zero.

2.3.2 Kinematics of Connecting Rod

A connecting rod of mass M_2 is replaced by four equal point masses, m_{21}, m_{22}, m_{23} and m_{24}, such that

$$m_{21} = m_{22} = m_{23} = m_{24} = \frac{M_2}{4} \tag{2.57}$$

The position vector of the point mass m_{21} is

$${}^{2-cm}P^{21} = \begin{Bmatrix} X_{21} \\ 0 \\ 0 \end{Bmatrix} \tag{2.58}$$

where

$$X_{21} = \sqrt{\frac{1}{2m_{21}}(I_{2Y} + I_{2Z} - I_{2X})} \tag{2.59}$$

The position vector of the point mass m_{22} is

$$^{2-cm}p^{22} = \begin{Bmatrix} 0 \\ Y_{22} \\ 0 \end{Bmatrix} \tag{2.60}$$

where

$$Y_{22} = \sqrt{\frac{1}{2m_{22}}(I_{2X} + I_{2Z} - I_{2Y})} \tag{2.61}$$

The position vector of the point mass m_{23} is

$$^{2-cm}p^{23} = \begin{Bmatrix} 0 \\ 0 \\ Z_{23} \end{Bmatrix} \tag{2.62}$$

where

$$Z_{23} = \sqrt{\frac{1}{2m_{23}}(I_{2X} + I_{2Y} - I_{2Z})} \tag{2.63}$$

Mass m_{24} is located at the center of mass of the connecting rod.

$$^{2-cm}p^{24} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2.64}$$

First the position vectors from the origin of the fixed coordinate frame at point 1 to the moving point masses m_{21}, m_{22}, m_{23}, m_{24} will be formulated. Since point 3 on the connecting rod coincides with the point 4 of slider, the position of the distal coordinate frame on the connecting rod can be obtained from matrix ${}^{4}_{1}\hat{M}$.

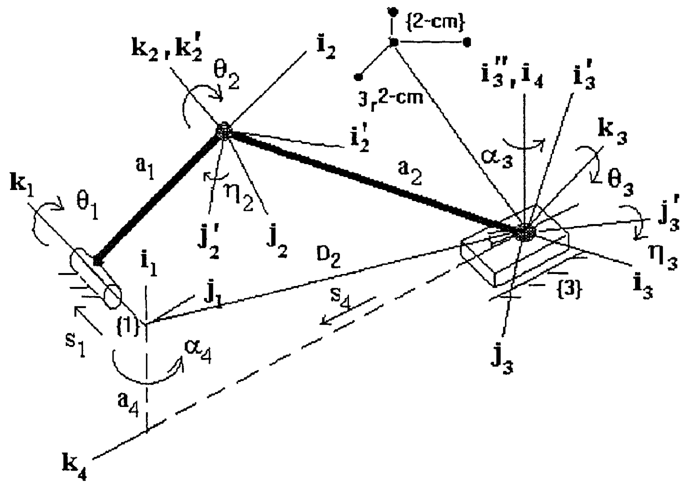

Figure 2.8 Connecting rod is replaced by four point masses; D_2 is the position vector representing distance of distal frame {3} of the connecting rod from frame {1}.

Using the "principle of transference" we can find the location of origin of distal coordinate frame {3} with respect to the fixed frame {1}.

$$[D_3]=[{}_4^1M_D][{}_4^1M_R]^T=\begin{bmatrix} 0 & S_4 & 0 \\ -S_4c\alpha_4 & -a_4s\alpha_4 & a_4c\alpha_4 \\ S_4s\alpha_4 & -a_4c\alpha_4 & -a_4s\alpha_4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha_4 & s\alpha_4 \\ 0 & -s\alpha_4 & c\alpha_4 \end{bmatrix}^T$$

$$=\begin{bmatrix} 0 & S_4c\alpha_4 & -S_4s\alpha_4 \\ -S_4c\alpha_4 & 0 & a_4 \\ S_4s\alpha_4 & -a_4 & 0 \end{bmatrix} \tag{2.65}$$

Position vector $\{D_2\}$ representing the distance from origin of fixed frame {1} to the origin of the coordinate frame at point 3 on the connecting rod or at point 4 on the slider can be written as

$$\{D_2\}=\begin{Bmatrix} -a_4 \\ -S_4s\alpha_4 \\ -S_4c\alpha_4 \end{Bmatrix} \tag{2.66}$$

The velocity vector of the origin of the coordinate frame at point 3 with respect to the fixed frame {1} can be obtained by differentiating equation (2.66) with respect to time. Therefore,

$$\{\dot{D}_2\} = \begin{Bmatrix} 0 \\ -\dot{S}_4 s\alpha_4 \\ -\dot{S}_4 c\alpha_4 \end{Bmatrix} \tag{2.67}$$

Differentiating equation (2.67) with respect to time, the acceleration vector can be obtained.

Therefore,

$$\{\ddot{D}_2\} = \begin{Bmatrix} 0 \\ -\ddot{S}_4 s\alpha_4 \\ -\ddot{S}_4 c\alpha_4 \end{Bmatrix} \tag{2.68}$$

The location of the center of mass of the connecting rod with respect to the coordinate frame {3} is represented by vector

$$^{3}r^{2-cm} = \begin{Bmatrix} X_{20} \\ Y_{20} \\ Z_{20} \end{Bmatrix} \tag{2.69}$$

and the direction cosines between the centroidal principal coordinate frame{2-cm} and frame {3} are represented by the matrix

$$_{2-cm}^{\;\;3}L = \begin{bmatrix} L_{2,11} & L_{2,12} & L_{2,13} \\ L_{2,21} & L_{2,22} & L_{2,23} \\ L_{2,31} & L_{2,32} & L_{2,33} \end{bmatrix} \tag{2.70}$$

Point masses m_{21}, m_{22}, m_{23} and m_{24} are located on the principal axes X_2, Y_2, Z_2 and origin of the centroidal coordinate frame respectively. Therefore, coordinates of half point masses $m_{21}/2$, $m_{22}/2$, $m_{23}/2$ and point mass m_{24} are respectively $(\pm X_{21},0,0)$, $(0,\pm Y_{22},0)$, $(0,0,\pm Z_{23})$ and $(0,0,0)$.

The location of half point mass $m_{21}/2$, placed on the positive side of X-axis, with respect to the distal coordinate frame {3} is

$$\left\{{}^{3}d_{21,P}\right\} = {}^{3}r^{2-cm} + {}^{3}_{2-cm}L^{2-cm}P^{21,P} = \begin{Bmatrix} X_{20} \\ Y_{20} \\ Z_{20} \end{Bmatrix} + \begin{bmatrix} L_{2,11} & L_{2,12} & L_{2,13} \\ L_{2,21} & L_{2,22} & L_{2,23} \\ L_{2,31} & L_{2,32} & L_{2,33} \end{bmatrix} \begin{Bmatrix} X_{21} \\ 0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} X_{20} + L_{2,11}X_{21} \\ Y_{20} + L_{2,21}X_{21} \\ Z_{20} + L_{2,31}X_{21} \end{Bmatrix} \tag{2.71a}$$

Similarly, the location of half point mass $m_{21}/2$, placed on the negative side of X-axis, with respect to the distal coordinate frame {3} is

$$\left\{{}^{3}d_{21,N}\right\} = \begin{Bmatrix} X_{20} - L_{2,11}X_{21} \\ Y_{20} - L_{2,21}X_{21} \\ Z_{20} - L_{2,31}X_{21} \end{Bmatrix} \tag{2.71b}$$

The location of half point mass $m_{22}/2$, placed on the positive side of Y-axis, with respect to the distal coordinate frame {3} is

$$\left\{{}^{3}d_{22,P}\right\} = {}^{3}r^{2-cm} + {}^{3}_{2-cm}L^{2-cm}P^{22,P} = \begin{Bmatrix} X_{20} \\ Y_{20} \\ Z_{20} \end{Bmatrix} + \begin{bmatrix} L_{2,11} & L_{2,12} & L_{2,13} \\ L_{2,21} & L_{2,22} & L_{2,23} \\ L_{2,31} & L_{2,32} & L_{2,33} \end{bmatrix} \begin{Bmatrix} 0 \\ Y_{22} \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} X_{20} + L_{2,12}Y_{22} \\ Y_{20} + L_{2,22}Y_{22} \\ Z_{20} + L_{2,32}Y_{22} \end{Bmatrix} \tag{2.72a}$$

Similarly, the location of half point mass $m_{22}/2$, placed on the negative side of Y-axis, with respect to the distal coordinate frame $\{3\}$ is

$$\left\{ {}^{3}d_{22,N} \right\} = \begin{Bmatrix} X_{20} - L_{2,12}Y_{22} \\ Y_{20} - L_{2,22}Y_{22} \\ Z_{20} - L_{2,32}Y_{22} \end{Bmatrix} \tag{2.72b}$$

The location of half point mass $m_{23}/2$, placed on the positive side of Z-axis, with respect to the distal coordinate frame $\{3\}$ is

$$\left\{ {}^{3}d_{23,P} \right\} = {}^{3}r^{2-cm} + {}^{3}_{2-cm}L^{2-cm}p^{23,P} = \begin{Bmatrix} X_{20} \\ Y_{20} \\ Z_{20} \end{Bmatrix} + \begin{bmatrix} L_{2,11} & L_{2,12} & L_{2,13} \\ L_{2,21} & L_{2,22} & L_{2,23} \\ L_{2,31} & L_{2,32} & L_{2,33} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ Z_{23} \end{Bmatrix}$$

$$= \begin{Bmatrix} X_{20} + L_{2,13}Z_{23} \\ Y_{20} + L_{2,23}Z_{23} \\ Z_{20} + L_{2,33}X_{23} \end{Bmatrix} \tag{2.73a}$$

Similarly, the location of half point mass $m_{23}/2$, placed on the negative side of Z-axis, with respect to the distal coordinate frame $\{3\}$ is

$$\left\{ {}^{3}d_{23,N} \right\} = \begin{Bmatrix} X_{20} - L_{2,13}Z_{23} \\ Y_{20} - L_{2,23}Z_{23} \\ Z_{20} - L_{2,33}X_{23} \end{Bmatrix} \tag{2.73b}$$

The location of point mass m_{24} with respect to the distal coordinate frame $\{3\}$ is

$$\left\{ {}^{3}d_{24} \right\} = {}^{3}r^{2-cm} + {}^{3}_{2-cm}L^{2-cm}p^{24} = \begin{Bmatrix} X_{20} \\ Y_{20} \\ Z_{20} \end{Bmatrix} \tag{2.74}$$

The position vectors of the half point masses $m_{21}/2$, $m_{22}/2$, $m_{23}/2$ and point mass m_{24} with respect to frame {1} are respectively,

$$\{D_{21,P}\} = \{D_2\} + [{}^1_3T_R]\{{}^3d_{21,P}\} \tag{2.75a}$$

$$\{D_{21,N}\} = \{D_2\} + [{}^1_3T_R]\{{}^3d_{21,N}\} \tag{2.75b}$$

$$\{D_{22,P}\} = \{D_2\} + [{}^1_3T_R]\{{}^3d_{22,P}\} \tag{2.75c}$$

$$\{D_{22,N}\} = \{D_2\} + [{}^1_3T_R]\{{}^3d_{22,N}\} \tag{2.75d}$$

$$\{D_{23,P}\} = \{D_2\} + [{}^1_3T_R]\{{}^3d_{23,P}\} \tag{2.75e}$$

$$\{D_{23,N}\} = \{D_2\} + [{}^1_3T_R]\{{}^3d_{23,N}\} \tag{2.75f}$$

$$\{D_{24}\} = \{D_2\} + [{}^1_3T_R]\{{}^3d_{24}\} \tag{2.75g}$$

where

$$[{}^1_3T_R] = [{}^1_2T_R][{}^2_3T_R] = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 c\eta_2 & -s\theta_2 & c\theta_2 s\eta_2 \\ s\theta_2 c\eta_2 & c\theta_2 & s\theta_2 s\eta_2 \\ -s\eta_2 & 0 & c\eta_2 \end{bmatrix}$$

$$= \begin{bmatrix} c\eta_2 c(\theta_1+\theta_2) & -s(\theta_1+\theta_2) & s\eta_2 c(\theta_1+\theta_2) \\ c\eta_2 s(\theta_1+\theta_2) & c(\theta_1+\theta_2) & s\eta_2 s(\theta_1+\theta_2) \\ -s\eta_2 & 0 & c\eta_2 \end{bmatrix} \tag{2.76}$$

The accelerations of half point masses $m_{21}/2$, $m_{22}/2$, $m_{23}/2$, point mass m_{24} can respectively be written as follows. Since they are stationary with respect to distal coordinate frame {3}, first and second time derivatives of their distances from {3} are zero.,

$$a_{21,P} = \omega_2 \times (\omega_2 \mathrm{x}\, {}^3d_{21,P}) + \frac{d\omega_2}{dt} \mathrm{x}\, {}^3d_{21,P} + \ddot{D}_2 \tag{2.77a}$$

$$a_{21,N} = \omega_2 \times (\omega_2 \mathrm{x}\, {}^3d_{21,N}) + \frac{d\omega_2}{dt} \mathrm{x}\, {}^3d_{21,N} + \ddot{D}_2 \tag{2.77b}$$

$$a_{22,P} = \omega_2 \times (\omega_2 \mathrm{x}\, {}^3d_{22,P}) + \frac{d\omega_2}{dt} \mathrm{x}\, {}^3d_{22,P} + \ddot{D}_2 \tag{2.77c}$$

$$a_{22,N} = \omega_2 \times (\omega_2 \times {}^3d_{22,N}) + \frac{d\omega_2}{dt} \times {}^3d_{22,N} + \ddot{D}_2 \tag{2.77d}$$

$$a_{23,P} = \omega_2 \times (\omega_2 \times {}^3d_{23,P}) + \frac{d\omega_2}{dt} \times {}^3d_{23,P} + \ddot{D}_2 \tag{2.77e}$$

$$a_{23,N} = \omega_2 \times (\omega_2 \times {}^3d_{23,N}) + \frac{d\omega_2}{dt} \times {}^3d_{23,N} + \ddot{D}_2 \tag{2.77f}$$

$$a_{24} = \omega_2 \times (\omega_2 \times {}^3d_{24}) + \frac{d\omega_2}{dt} \times {}^3d_{24} + \ddot{D}_2 \tag{2.77g}$$

where, $\omega_2 = \begin{Bmatrix} \omega_{2x} \\ \omega_{2y} \\ \omega_{2z} \end{Bmatrix}$ represents the angular velocity of the origin of frame $\{3\}$ with respect to frame $\{1\}$ and the elements of this vector can be determined using the relationship

$$\tilde{\omega}_2 = [{}^1_3\dot{T}_R][{}^3_1T_R]$$

$$= \begin{bmatrix} -c\eta_2 s(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) - s\eta_2\dot{\eta}_2 c(\theta_1+\theta_2) & -c(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) & -s\eta_2 s(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) + c\eta_2\dot{\eta}_2 c(\theta_1+\theta_2) \\ c\eta_2 c(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) - s\eta_2\dot{\eta}_2 s(\theta_1+\theta_2) & -s(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) & s\eta_2 c(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) + c\eta_2\dot{\eta}_2 s(\theta_1+\theta_2) \\ -c\eta_2\dot{\eta}_2 & 0 & -s\eta_2\dot{\eta}_2 \end{bmatrix} \begin{bmatrix} c\eta_2 c(\theta_1+\theta_2) & c\eta_2 s(\theta_1+\theta_2) & -s\eta_2 \\ -s(\theta_1+\theta_2) & c(\theta_1+\theta_2) & 0 \\ s\eta_2 c(\theta_1+\theta_2) & s\eta_2 s(\theta_1+\theta_2) & c\eta_2 \end{bmatrix} \tag{2.78}$$

where

$$\tilde{\omega}_2 = \begin{bmatrix} 0 & -\omega_{2z} & \omega_{2y} \\ \omega_{2z} & 0 & -\omega_{2x} \\ -\omega_{2y} & \omega_{2x} & 0 \end{bmatrix} \tag{2.79}$$

After multiplying the matrices in equation (2.78) and then equating with the matrix in equation (2.79) angular velocity components can be determined.

2.3.3 Kinematics of the Slider

The slider motion is a translation along a straight line, no rotation is involved. Therefore, we can consider it as a single point mass.

The position vector locating the center of mass of the slider with respect to the fixed frame $\{1\}$ is

$$\{D_3\} = \begin{Bmatrix} -a_4 \\ -S_4 s\alpha_4 \\ -S_4 c\alpha_4 \end{Bmatrix} \tag{2.80}$$

The velocity of the slider is

$$\{\dot{D}_3\} = \begin{Bmatrix} 0 \\ -\dot{S}_4 s\alpha_4 \\ -\dot{S}_4 c\alpha_4 \end{Bmatrix} \tag{2.81}$$

The acceleration of the slider is

$$\{\ddot{D}_3\} = \begin{Bmatrix} 0 \\ -\ddot{S}_4 s\alpha_4 \\ -\ddot{S}_4 c\alpha_4 \end{Bmatrix} \tag{2.82}$$

CHAPTER 3

DYNAMICS OF THE MECHANISM

3.1 Dynamics of the Point Masses

The inertia forces and torques exerted on the frame link by the moving links of the mechanism will be determined. By considering the inertia forces and external forces as applied forces acting on the system it is possible to apply d'Alembert's principle and reduce the analysis to the application of static equilibrium conditions. The mass distribution of the moving links will be replaced by a dynamically equivalent system of point masses. After calculation of their vector coordinates and accelerations, the inertia forces and torques will be obtained as well.

3.1.1 Definition of the Inertia Force

Newton's law of motion for a particle is given by

$$F = m\ddot{P} \tag{3.1}$$

where symbol F represents the sum of the external forces acting on the particle, symbol m is its mass and $\ddot{P}$ is the acceleration of the particle with respect to an inertial coordinate system. We can write the above equation in the form

$$F - m\ddot{P} = 0 \tag{3.2}$$

If we consider the term $-m\ddot{P}$ to represent the inertia force, then equation (3.2) states that the vector sum of external and internal forces vanishes (d'Alembert's principle).

3.1.2 Definition of the Inertia Torque

If a particle (point mass) moves relative to a fixed point, then the moment of inertia force about the fixed point is given by

$$T = P \times (-m\ddot{P}) \tag{3.3}$$

where T is the inertia torque and P is the position vector from the fixed point to the particle m.

3.2 Inertia Forces and Torques Exerted by the Moving Links on the Frame

In the CSSP mechanism moving links are crank, connecting rod and slider. When the mechanism runs in high speed the moving links exert huge amount of inertia-induced force and torque on the frame.

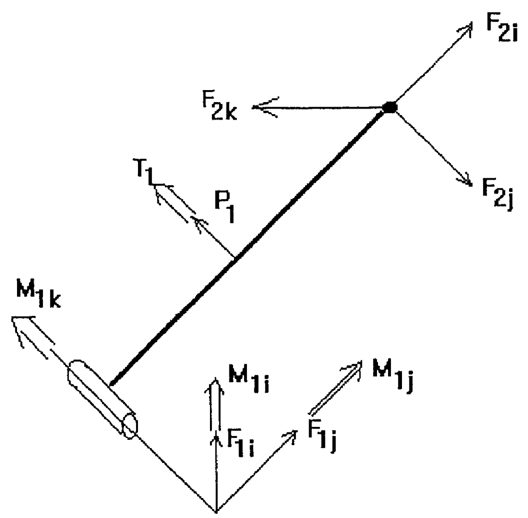

Figure 3.1 Free body diagram of the crank

3.2.1 Inertia Force and Torque Calculation for the Crank

The inertia force of the crank is given by

$$P_1 = \frac{m_{11}(a_{11,P} + a_{11,N}) + m_{12}(a_{12,P} + a_{12,N}) + m_{13}(a_{13,P} + a_{13,N})}{2} + m_{14}a_{14} \tag{3.4}$$

and the inertia torque of the crank is given by

$$T_1 = \frac{(D_{11,P} \times m_{11}a_{11,P}) + (D_{12,P} \times m_{12}a_{12,P}) + (D_{13,P} \times m_{13}a_{13,P})}{2}$$

$$+ \frac{(D_{11,N} \times m_{11}a_{11,N}) + (D_{12,N} \times m_{12}a_{12,N}) + (D_{13,N} \times m_{13}a_{13,N})}{2} + (D_{14} \times m_{14}a_{14}) \tag{3.5}$$

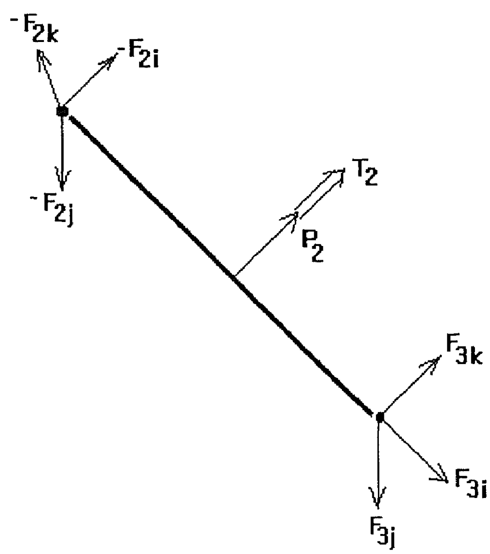

Figure 3.2 Free body diagram of the connecting rod

3.2.2 Inertia Force and Torque Calculation for the Connecting Rod

The inertia force of the connecting rod is given by

$$P_2 = \frac{m_{21}(a_{21,P} + a_{21,N}) + m_{22}(a_{22,P} + a_{22,N}) + m_{23}(a_{23,P} + a_{23,N})}{2} + m_{24}a_{24} \tag{3.6}$$

The inertia torque of the connecting rod is given by

$$T_2 = \frac{(D_{21,P} \times m_{21}a_{21,P}) + (D_{22,P} \times m_{22}a_{22,P}) + (D_{23,P} \times m_{23}a_{23,P})}{2}$$

$$+ \frac{(D_{21,N} \times m_{21}a_{21,N}) + (D_{22,N} \times m_{22}a_{22,N}) + (D_{23,N} \times m_{23}a_{23,N})}{2} + (D_{24} \times m_{24}a_{24}) \tag{3.7}$$

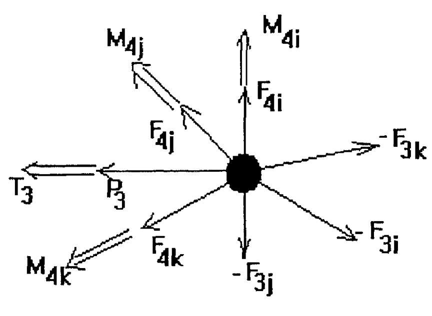

Figure 3.3 free body diagram of the slider (one point mass)

3.2.3 Inertia Force and Torque Calculation for the Slider

The inertia force of the slider is

$$P_3 = m_3 \ddot{D}_3 \tag{3.8}$$

The inertia torque of the slider is

$$T_3 = D_3 \times m_3 \ddot{D}_3 \tag{3.9}$$

3.3 Determination of Shaking Force and Shaking Moment

The shaking force is the sum of the forces exerted upon the frame by each moving link

$$\begin{aligned} F &= P_1 + P_2 + P_3 \\ &= \frac{m_{11}(a_{11,P} + a_{11,N}) + m_{12}(a_{12,P} + a_{12,N}) + m_{13}(a_{13,P} + a_{13,N})}{2} + m_{14}a_{14} \\ &= \frac{m_{21}(a_{21,P} + a_{21,N}) + m_{22}(a_{22,P} + a_{22,N}) + m_{23}(a_{23,P} + a_{23,N})}{2} + m_{24}a_{24} \\ &+ m_3\ddot{D}_3 \end{aligned} \tag{3.10}$$

The shaking moment is the sum of the torque exerted upon the frame by each moving link

$$\begin{aligned} T &= T_1 + T_2 + T_3 \\ &= \frac{(D_{11,P} \times m_{11}a_{11,P}) + (D_{12,P} \times m_{12}a_{12,P}) + (D_{13,P} \times m_{13}a_{13,P})}{2} \\ &+ \frac{(D_{11,N} \times m_{11}a_{11,N}) + (D_{12,N} \times m_{12}a_{12,N}) + (D_{13,N} \times m_{13}a_{13,N})}{2} + (D_{14} \times m_{14}a_{14}) \\ &+ \frac{(D_{21,P} \times m_{21}a_{21,P}) + (D_{22,P} \times m_{22}a_{22,P}) + (D_{23,P} \times m_{23}a_{23,P})}{2} \\ &+ \frac{(D_{21,N} \times m_{21}a_{21,N}) + (D_{22,N} \times m_{22}a_{22,N}) + (D_{23,N} \times m_{23}a_{23,N})}{2} + (D_{24} \times m_{24}a_{24}) \\ &+ D_3 \times m_3\ddot{D}_3 \end{aligned} \tag{3.11}$$

where all the variables except point masses m_{11}, m_{12}, m_{13}, m_{14}, m_{21}, m_{22}, m_{23} m_{24}, and m_3 are known from kinematic analysis.

3.4 Determination of Bearing Force and Bearing Moment

The inertia forces and inertia torques of the moving links with respect to the fixed coordinate frame {1} were formulated in section 3.2. They are to be converted with respect to the distal coordinate frame of each moving link to determine the bearing reaction forces and moments using the formulas developed by Fischer and Rahman [11, 12]. Then these reaction forces and torques will be expressed with respect to the moving coordinate frames located at the distal end of the moving links. To express those forces and torques with respect to fixed coordinate frame {1} we shall premultiply the force and torque vectors by the rotational part of the transformation matrix expressed in terms of frame {1}. All these operations are mathematically expressed in the following equations.

Step 1: Inertia forces and torques are expressed in terms of the distal coordinate frame:

$$\begin{aligned} \{{}^2_1F\} &= [{}^1_2T_R]^T\{P_1\} \\ \{{}^3_2F\} &= [{}^1_3T_R]^T\{P_2\} \\ \{{}^4_3F\} &= [{}^1_4T_R]^T\{P_3\} \end{aligned} \tag{3.12}$$

$$\begin{aligned} \{{}^2_1M\} &= [{}^1_2T_R]^T\{T_1\} \\ \{{}^3_2M\} &= [{}^1_3T_R]^T\{T_2\} \\ \{{}^4_3M\} &= [{}^1_4T_R]^T\{T_3\} \end{aligned} \tag{3.13}$$

Step 2: Equations developed by Fischer and Rahman [12] are used to determine the bearing reaction forces F_1, F_2, F_3 and F_4 and torques M_1, M_2, M_3 and M_4 in terms of the distal coordinate frames.

Step. 3: The bearing reaction forces and torques determined in step 2 in terms of frame $\{1\}$ are

$$\begin{aligned}
\{{}^1_1F\} &= \{F_1\} \\
\{{}^1_2F\} &= [{}^1_2T_R]\{F_2\} \\
\{{}^1_3F\} &= [{}^1_3T_R]\{F_3\} \\
\{{}^1_4F\} &= [{}^1_4T_R]\{F_4\}
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
\{{}^1_1M\} &= \{M_1\} \\
\{{}^1_2M\} &= [{}^1_2T_R]\{M_2\} \\
\{{}^1_3M\} &= [{}^1_3T_R]\{M_3\} \\
\{{}^1_4M\} &= [{}^1_4T_R]\{M_4\}
\end{aligned} \tag{3.15}$$

Therefore, the sum of all bearing forces in terms of frame $\{1\}$ can be expressed as

$$R = \{{}^1_1F\} + \{{}^1_2F\} + \{{}^1_3F\} + \{{}^1_4F\} \tag{3.16}$$

Similarly, the sum of all bearing moments in terms of frame $\{1\}$ can be expressed as

$$M = \{{}^1_1M\} + \{{}^1_2M\} + \{{}^1_3M\} + \{{}^1_4M\} \tag{3.17}$$

3.5 Determination of Input Torque

The external torque required to operate the mechanism is the Z-component of moment $\{{}^1_1M\}$. Let T_0 represent the input torque. Then

$$T_0 = {}^1_1M_Z \tag{3.18}$$

CHAPTER 4

MINIMIZATION OF INERTIA-INDUCED FORCES IN THE MECHANISM

4.1 Problem Formulation

An objective function is to be formulated for the purpose of minimizing the adverse effect of inertia-induced forces and torques. A quadratic objective function consisting of shaking forces, shaking moments, bearing reaction forces, bearing reaction torques and input torque is minimized by optimum mass redistribution of the links of the mechanism. The objective function involves the sum of the squared non-dimensionalized shaking force, shaking moment, bearing reactions and input torque over one cycle of operation of the mechanism. Then the magnitudes of the active point masses are chosen as design variables, and design constraint equations are formulated which are linear in the point masses.

4.2 Objective Function

Let $\overline{F}$, $\overline{T}$, $\overline{R}$, $\overline{M}$ and $\overline{T_0}$ denote the non-dimensionalized mean squared values of the shaking force, shaking moment, bearing forces, bearing moments and input torque, respectively for one complete cycle of operation (360 degrees rotation of input crank). Then

$$\overline{F} = \frac{1}{2\pi m_1^2 a_1^2 \dot{\theta}_1^4} \int_0^{2\pi} (F.F) d\theta_1$$
$$\overline{T} = \frac{1}{2\pi m_1^2 a_1^4 \dot{\theta}_1^4} \int_0^{2\pi} (T.T) d\theta_1$$
$$\overline{R} = \frac{1}{2\pi m_1^2 a_1^2 \dot{\theta}_1^4} \int_0^{2\pi} (R_1.R_1 + R_2.R_2 + R_3.R_3 + R_4.R_4) d\theta_1$$
$$\overline{M} = \frac{1}{2\pi m_1^2 B_l^2 a_1^2 \dot{\theta}_1^4} \int_0^{2\pi} [(M_1 - T_0).(M_1 - T_0) + M_2.M_2 + M_3.M_3 + M_4.M_4) d\theta_1$$
$$\overline{T_0} = \frac{1}{2\pi m_1^2 a_1^4 \dot{\theta}_1^4} \int_0^{2\pi} (T_0^2) d\theta_1 \tag{4.1}$$

In this analysis, integration over a complete cycle has been performed numerically by dividing the cycle into L equal intervals. The non-dimensionalized values then become as follows:

$$\overline{F} = \frac{1}{L m_1^2 a_1^2 \dot{\theta}_1^4} \sum_{i=1}^{L} (F_i.F_i)$$
$$\overline{T} = \frac{1}{L m_1^2 a_1^4 \dot{\theta}_1^4} \sum_{i=1}^{L} (T_i.T_i)$$
$$\overline{R} = \frac{1}{L m_1^2 a_1^2 \dot{\theta}_1^4} \sum_{i=1}^{L} [R_{1i}.R_{1i} + R_{2i}.R_{2i} + R_{3i}.R_{3i} + R_{4i}.R_{4i}]$$
$$\overline{M} = \frac{1}{L m_1^2 B_l^2 a_1^2 \dot{\theta}_1^4} \sum_{i=1}^{L} [(M_1 - T_0)_i.(M_1 - T_0)_i + M_{2i}.M_{2i} + M_{3i}.M_{3i} + M_{4i}.M_{4i}]$$
$$\overline{T_0} = \frac{1}{L m_1^2 a_1^4 \dot{\theta}_1^4} \sum_{i=1}^{L} (T_{0i}^2) \tag{4.2}$$

where distance B_l denotes the length of each bearing. Expressions for the shaking force, shaking moment, bearing forces, bearing moments and input torque have been determined earlier.

In the formulation of the objective function, weight factors W_1, W_2, W_3, W_4 and W_5 are assigned to the shaking force, shaking moment, input torque, bearing force and bearing moment respectively. Weight factors are adjusted according to the designer's will, depending upon different circumstances and applications. Let the symbol G represent the objective function which can be optimized using an IMSL package (described in the appendix B).

$$G = W_1\overline{F} + W_2\overline{T} + W_3\overline{T_0} + W_4\overline{R} + W_5\overline{M} \tag{4.3}$$

This quadratic function optimization algorithm QPROG is based on M.J.D. Powell's implementation of the Goldfarb and Idnani [17] dual quadratic programming (QP) algorithm for convex QP problems subject to general linear equality/inequality constraints, i.e., a problem of the form

$$\min_{x \varepsilon R^n} \quad g^T x + \frac{1}{2} x^T H x$$

subject to

$$A_1 x = b_1$$
$$A_2 x \geq b_2$$

given the vectors b_1, b_2 and g and the matrices H (Hessian Matrix), A_1 and A_2. Matrix H is required to be positive definite. In this case, a unique vector x solves the problem or the constraints are inconsistent. If H is not positive definite, a positive definite perturbation of H is used in place of H. For more details, see Powell [34, 35].

4.3 Design Variables

The mass distribution of each moving link is replaced by four point masses. One point mass lies at center of mass. The remaining three point masses are termed active point masses. The objective function consists only of active point masses and these will be chosen as design variables. By varying these point masses systematically it is possible to minimize the inertia-induced forces. The column vector of the design variables is then given as follows:

$$X = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ -- \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ -- \\ x_9 \end{Bmatrix} \tag{4.4}$$

where $x_1=m_{11}$, $x_2=m_{12}$, $x_3=m_{13}$, $x_4=m_{14}$, $x_5=m_{21}$, $x_6=m_{22}$, $x_7=m_{23}$, $x_8=m_{24}$ and $x_9=m_3$.

4.4 Design Constraints

The design constraints will be a set of equations, linear in the design variables, which will allow the point masses to vary within prescribed limits. Each active point mass will be allowed to decrease a certain percentage of its original magnitude. The sum of the optimized active point masses of each link will be kept either less than or equal to the sum of the original active point masses for the same link. The constraint equations are given in their general form. For a particular problem these equations can be modified depending upon the characteristics of the linkage. The constraints are formulated as follows:

$$x_1 \geq m_{11}(1-w') \tag{4.5a}$$

$$x_2 \geq m_{12}(1-w') \tag{4.5b}$$

$$x_3 \geq m_{13}(1-w') \tag{4.5c}$$

$$x_4 \geq m_{14}(1-w') \tag{4.5d}$$

$$x_5 \geq m_{21}(1-w') \tag{4.5e}$$

$$x_6 \geq m_{22}(1-w') \tag{4.5f}$$

$$x_7 \geq m_{23}(1-w') \tag{4.5g}$$

$$x_8 \geq m_{24}(1-w') \tag{4.5h}$$

$$x_9 = m_3 \tag{4.5i}$$

$$x_1 + x_2 + x_3 \leq m_{11} + m_{12} + m_{13} \tag{4.5j}$$

$$x_5 + x_6 + x_7 \leq m_{21} + m_{22} + m_{23} \tag{4.5k}$$

where $0 < w' < 1$.

CHAPTER 5

RESULTS

5.1 Example

A numerical example will be presented to demonstrate the effectiveness of the balancing method developed in this investigation. The designer has been given enough flexibility to adjust the weight factors involved in the objective function G and to change the magnitude of the point masses of the crank and connecting rod, depending upon different circumstances and applications. The following numerical results are calculated setting all weight factors to 1. In this process the magnitude of the optimized point masses change (decrease or increase) slightly within the limits of the design constraints.

5.2 Dimensions of Example CSSP Mechanism and Other Necessary Data

Length of the crank a_1 = 2.0 inches

Mass of the crank M_1 = 1.9 lbs

Center of mass is at the midpoint of the crank

Length of the connecting rod a_2 = 8.0 inches

Mass of the connecting rod M_2 = 7.6 lbs

Center of mass is at the midpoint of the connecting rod

Mass of the slider m_3 =6.0 lbs and its center of mass is at the joint between itself and connecting rod

Offset a_4 = 1.0 inch

Offset s_1 = 0.4 inch

Offset α_4 = 250 degrees

Acceleration due to gravity = 386.4 inches/second^2

Crank speed = 3000 RPM

Mass moments of Inertia (lb-sec^2-inch) of the crank about its center of mass

I_{1XX} = 2.048826E-04, I_{1YY} = 1.741503E-03, I_{1ZZ} = 1.741503E-03

Mass moments of Inertia (lb-sec^2-inch) of the connecting rod about its center of mass

I_{2XX} = 8.195304E-04, I_{2YY} = 0.105310, I_{2ZZ} = 0.105310

Mass moments of Inertia (lb-sec^2-inch) of the slider about its center of mass

I_{3XX} = 4.20548E-03, I_{3YY} = 4.20548E-03, I_{3ZZ} = 7.76397E-03

The active point masses of the crank are $m_{11} = m_{12} = m_{13} = m_{14}$ = 1.9/4 lbs.

The active point masses of the connecting rod are $m_{21} = m_{22} = m_{23} = m_{24}$ = 7.6/4 lbs.

The magnitude of the active point masses of the crank and connecting rod were allowed to decrease by five percent while the sum of the point masses associated with each moving link was kept constant. The mass of the slider is considered as one point mass and kept constant.

5.3 Discussion of Results

Results are given in the form of tables and graphs. The variations of the objective function, shaking force, shaking moment, bearing force, bearing torque and input torque are shown. The improvements in the magnitude of inertia-induced forces are described below in detail.

Comparative values of the mass properties of the moving links before and after optimization, are tabulated in table 5.1. The crank and the connecting rod are modified to match the design characteristics obtained from the optimized values of point masses. It has been found that after optimization I_{XX} of the crank and connecting rod slightly decreases and I_{YY} and I_{ZZ} slightly increase.

Comparative values of the objective functions before and after optimization, are tabulated in table 5.2. Percentage variations of the objective function over a complete rotation of the crank are given in the form of a graph in figure 5.1. As shown in figure 5.1 the decrement of objective function G varies from +4.59% to -0.128%. It increases only at crank angle 20 degrees. The average optimized value of the objective function has been decreased by 2.283%.

The shaking forces before and after optimization are compared and are tabulated in table 5.3. Percentage variations of the shaking force over a complete rotation of the crank are given in the form of a graph in figure 5.2. As shown in figure 5.2 the decrement of shaking force $\overline{F}$ varies from +0.7248% to +0.732%. It never increases and decreases very steadily. The average optimized value of the shaking force has been decreased by 0.728%.

Comparisons of the shaking moments before and after optimization are tabulated in table 5.4. Over a complete rotation of the crank percentage variations of the shaking moment are given in the form of a graph in figure 5.3. As shown in figure 5.3 the decrement of shaking moment $\overline{T}$ varies from +22.8% to -26.355%. Most of the time the decrement is in positive direction, it only goes negative a few times. The average optimized value of the shaking moment has been decreased by 5.389%.

Comparative values of the bearing forces before and after optimization, are tabulated in table 5.5. Over a complete rotation of the crank percentage variations of the bearing force are given in the form of a graph in figure 5.4. As shown in figure 5.4 the decrement of bearing force $\overline{R}$ varies between +2.936% and -1.33%. It increases only at an angle of 110 degrees. The average optimized value of the bearing force has been decreased by 1.3%.

The bearing moments before and after optimization are tabulated in table 5.6 to demonstrate the comparison. A graph in figure 5.5 shows the percentage variations of the bearing torque over a complete rotation of the crank. It is found that the decrement of bearing torque $\overline{M}$ varies from +3.535% to -0.26%. Most of the time the decrement is in positive direction, it only goes negative a few times. The average optimized value of the bearing torque has been decreased by 1.19%.

Comparative values of the input torques before and after optimization, are tabulated in table 5.7. Percentage variations of the input torque over a complete rotation of the crank are given in the form of a graph in figure 5.6. As shown in figure 5.6 the decrement of input torque $\overline{T}_0$ varies from +35.552% to -2.725%. Most of the time the decrement is in positive direction, it only goes negative a few times. The average optimized value of the input torque has been decreased by 2.157%.

The average values of the objective function, shaking force, shaking moment, bearing force, bearing torque and input torque always decrease after optimization while the mass of the moving links remains unchanged. This demonstrates the effectiveness of this optimum balancing method.

The weight factors used in the objective function are $W_1 = W_2 = W_3 = W_4 = W_5 =$ 1.0

Table 5.1

Mass moment of inertia (lb-sec^2-inch)	Before optimization	After optimization
Crank		
I_{XX}	2.048826E-04	1.946666E-04
I_{YY}	1.741503E-03	1.899999E-03
I_{ZZ}	1.741503E-03	1.899999E-03
Connecting rod		
I_{XX}	8.195304E-04	7.783330E-04
I_{YY}	0.105310	0.115802
I_{ZZ}	0.105310	0.115802

Comparative values of the mass properties before and after optimization

Table 5.2

Crank angle (degrees)	Before optimization	After optimization
0	10.670883	10.180821
10	28.359000	28.270926
20	47.249014	47.309470
30	45.508196	45.419730
40	37.900430	37.683624
50	26.849201	26.613307
60	17.079143	16.856237
70	13.005502	12.772193
80	14.755094	14.487766
90	15.272538	15.038194
100	10.053970	9.931598
110	8.420173	8.216053
120	24.990859	24.128653
130	58.059051	55.988413
140	86.648568	83.427017
150	95.701605	91.965717
160	91.394264	87.743183
170	78.125154	75.025988
180	25.443404	24.280701
190	2.062062	2.021815
200	8.214458	8.078112
210	24.620364	24.049713
220	33.528715	32.601859
230	31.787401	30.815644
240	28.471811	23.696789
250	16.912072	16.400460
260	11.818102	11.515033
270	9.668070	9.485531
280	10.297789	10.161332
290	14.054016	13.894905
300	21.725833	21.452218
310	33.417686	32.909414
320	47.184986	46.335525
330	58.468741	57.261149
340	61.003306	59.579073
350	48.494153	47.188659
360	10.670883	10.180821

Comparative values of the objective functions before and after optimization

Table 5.3

Crank angle (degrees)	Before optimization (lbs)	After optimization (lbs)
0	3099.388258	3076.830267
10	4635.095971	4601.482231
20	5781.337225	5739.414084
30	5962.427821	5919.183360
40	6206.192680	6161.172538
50	6562.647144	6515.033022
60	7026.045556	6975.060344
70	7540.065702	7485.341719
80	7983.161538	7925.215606
90	8233.565698	8173.801370
100	8225.679136	8165.977758
110	7949.340452	7891.656783
120	7438.673855	7384.712598
130	6773.218081	6724.102242
140	6086.169716	6042.049809
150	5550.444030	5510.211404
160	5288.091484	5249.752908
170	5105.093191	5068.068468
180	3854.506059	3826.543209
190	3161.939012	3138.925873
200	3414.219109	3389.330760
210	3753.027874	3725.612994
220	4117.481055	4087.373741
230	4436.924639	4404.467266
240	4692.866943	4658.530489
250	4886.549827	4850.792879
260	5024.927163	4988.155764
270	5115.038661	5077.606489
280	5160.970215	5123.200602
290	5162.038924	5124.259961
300	5111.967111	5074.552727
310	4999.097532	4962.507022
320	4807.765822	4772.573514
330	4520.841527	4487.748298
340	4123.210973	4093.031545
350	3608.123100	3581.731929
360	3099.388258	3076.830267

Comparative values of the shaking forces before and after optimization

Table 5.4

Crank angle (degrees)	Before optimization (lb-in)	After optimization (lb-in)
0	5039.187267	4665.623716
10	6330.150772	5938.667031
20	6560.050713	6159.998457
30	6016.932033	5662.362265
40	5687.040104	5374.276958
50	5342.042478	5072.316978
60	4878.863267	4657.443846
70	4239.657846	4073.767555
80	3420.556277	3316.518078
90	2457.133427	2818.983390
100	1393.313383	1423.417473
110	331.225814	418.521638
120	973.759370	831.523493
130	2110.606046	1897.616807
140	3168.005695	2896.533259
150	4064.936671	3742.045786
160	4780.183988	4410.185893
170	5343.114970	4936.789780
180	5500.814700	5129.849155
190	5950.426284	5608.086387
200	5901.083511	5547.409987
210	5713.303856	5376.559658
220	5334.542624	5030.833209
230	4818.038260	4555.563347
240	8199.925194	3984.282275
250	3507.755362	3343.005509
260	2763.903874	2652.964113
270	1986.262955	1931.125630
280	1189.910736	1191.575202
290	402.954655	456.625697
300	481.819104	371.960595
310	1269.449366	1096.038651
320	2052.914043	1824.886629
330	2805.910043	2528.243745
340	3516.277275	3195.648331
350	4191.590709	3836.822548
360	5039.187267	4665.623716

Comparative values of the shaking moments before and after optimization

Table 5.5

Crank angle (degrees)	Before optimization (lbs)	After optimization (lbs)
0	4818.165352	4746.038558
10	5662.175071	5566.946313
20	6735.796959	6633.176259
30	7188.526004	7092.245537
40	7557.524300	7465.486683
50	7667.467644	7580.964162
60	7354.020493	7276.052303
70	6529.350434	6464.435739
80	5337.113183	5289.514945
90	4028.706407	4001.393480
100	2610.484637	2606.349589
110	1108.617449	1123.388280
120	1749.457304	1701.812355
130	3507.717862	3419.265434
140	4616.748989	4500.302883
150	4819.158583	4690.501084
160	4396.018468	4272.358673
170	3484.619713	3382.313047
180	2089.492959	2050.586062
190	4347.560539	4311.843354
200	6428.682323	6346.527083
210	8055.307193	7947.614061
220	8434.472145	8317.160523
230	7990.046870	7877.373301
240	7110.634962	7013.522042
250	6116.724903	6040.279817
260	5233.419938	5178.274837
270	4580.639885	4542.795687
280	4179.270264	4153.460318
290	3977.529618	3958.407089
300	3875.827531	3858.636554
310	3755.305505	3736.349455
320	3523.522855	3500.201981
330	3181.113797	3150.752421
340	2915.052534	2872.925620
350	3185.321390	3126.609204
360	4818.165352	4746.038558

Comparative values of the bearing forces before and after optimization

Table 5.6

Crank angle (degrees)	Before optimization (lb-in)	After optimization (lb-in)
0	5839.550869	5685.426140
10	9861.576982	9873.119249
20	12980.229748	13014.013037
30	12451.089925	12467.810435
40	10878.745631	10879.836862
50	8464.746618	8459.624771
60	5858.150434	5842.874067
70	4825.801084	4784.069144
80	6012.721210	5957.951164
90	6481.704897	6438.427068
100	4754.018594	4740.452504
110	3912.213656	3851.302770
120	8788.174345	8613.498415
130	14197.910955	13924.841522
140	17564.286822	17217.514954
150	18483.583607	18101.162375
160	18012.398516	17630.990836
170	16560.777872	16213.109726
180	9207.085839	8971.416183
190	585.994252	565.281262
200	4642.398522	4611.826266
210	9190.782607	9078.510952
220	10859.193934	10699.841534
230	10534.091207	10362.331431
240	9130.628454	8975.205351
250	7431.564347	7309.891672
260	6051.322028	5968.522286
270	5406.485749	5356.777636
280	5698.015983	5667.787205
290	6918.532536	6890.414777
300	8875.663837	8832.473137
310	11217.959107	11146.135290
320	13469.212386	13361.555643
330	15071.688245	14929.000711
340	15413.488560	15245.127579
350	13670.554078	13495.726699
360	5839.550869	5685.426140

Comparative values of the bearing torques before and after optimization

Table 5.7

Crank angle (degrees)	Before optimization (lb-in)	After optimization (lb-in)
0	-2818.377026	-2745.318356
10	4474.134067	4484.028263
20	5727.618873	5750.326562
30	5190.198720	5209.394603
40	4137.555697	4153.164485
50	2628.177235	2641.403952
60	787.212608	808.662738
70	-1103.244484	-1077.045376
80	-2518.137877	-2490.807790
90	-2856.414871	-2836.856151
100	-1806.105371	-1807.146193
110	442.720797	408.132681
120	3232.808145	3156.583379
130	5694.420829	5577.441362
140	7157.472382	7010.462378
150	7545.023226	7384.540466
160	7341.948578	7183.608216
170	6744.511384	6602.050696
180	3782.044012	3683.868960
190	99.086368	63.859355
200	-1878.216703	-1863.682742
210	-3787.426955	-3738.193055
220	-4507.351907	-4437.850594
230	-4401.762853	-4326.556288
240	-3844.629266	-3776.054987
250	-3159.963961	-3105.807650
260	-2609.211236	-2572.144366
270	-2378.568569	-2356.558607
280	-2568.412403	-2555.811176
290	-3188.666217	-3177.733611
300	-4160.348369	-4142.977912
310	-5322.837210	-5292.257731
320	-6447.565166	-7399.790839
330	-7258.676605	-7193.493159
340	-7450.217835	-7371.649489
350	-6617.333858	-6534.421796
360	-2818.377026	-2745.318356

Comparative values of the input torques before and after optimization

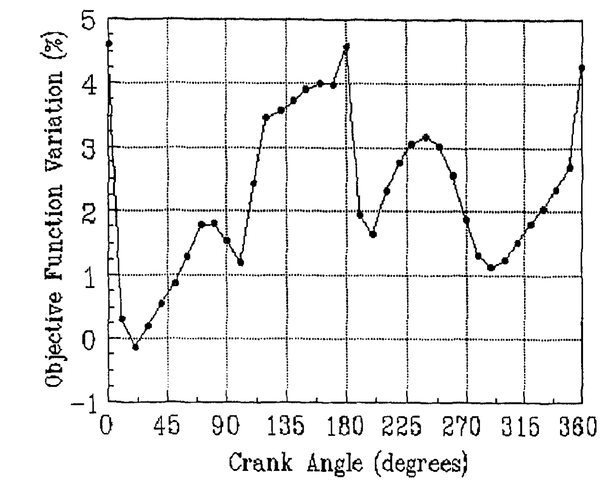

Figure 5.1 Variation of Objective Function vs. Crank Angle

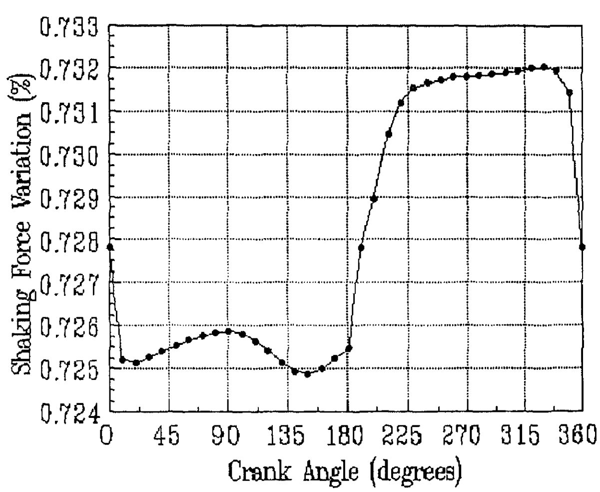

Figure 5.2 Variation of Shaking Force vs. Crank Angle

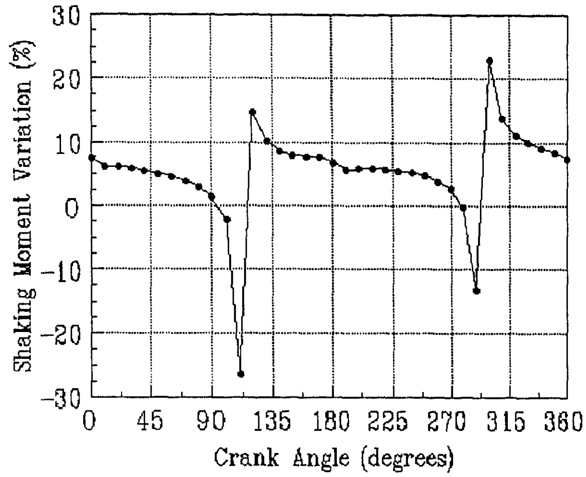

Figure 5.3 Variation of Shaking Moment vs. Crank Angle

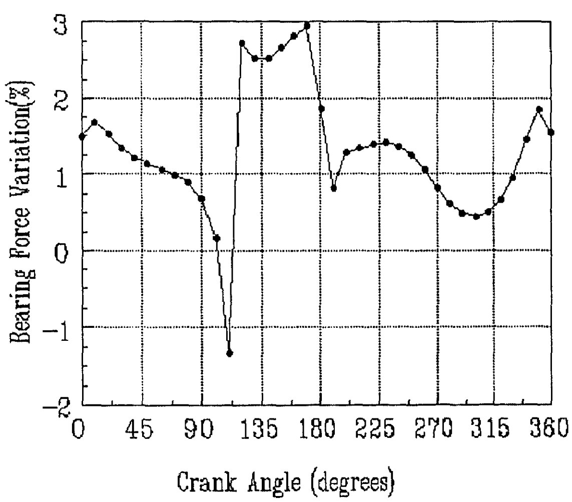

Figure 5.4 Variation of Bearing Force vs. Crank Angle

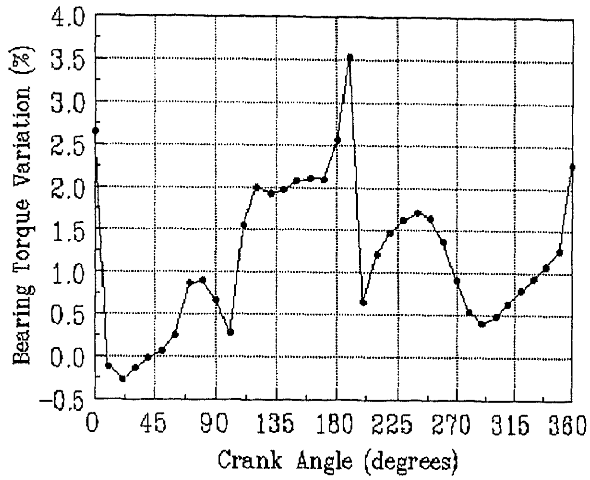

Figure 5.5 Variation of Bearing Torque vs. Crank Angle

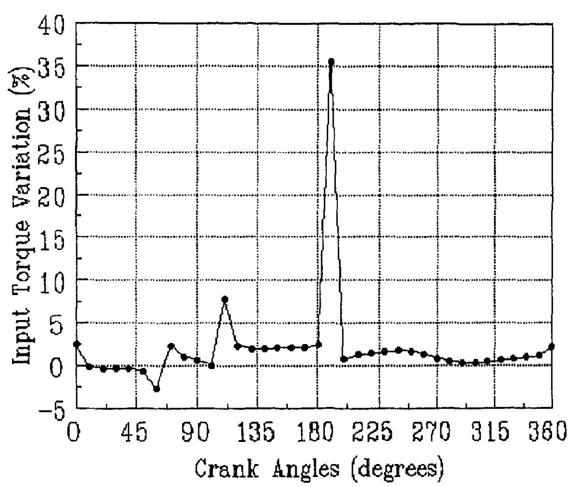

Figure 5.6 Variation of Input Torque vs. Crank Angle

CHAPTER 6

CONCLUSION

6.1 Conclusions

The principal objective of this dissertation has been the development of computer-aided design procedures for minimizing the adverse effect of the inertia-induced forces in a high-speed general spatial mechanism by optimum redistribution of the mass of the links.

To achieve this objective, the mass distribution of each moving link has been replaced by a dynamically equivalent system of point masses. Having calculated the vector coordinates with the help of the "principle of transference" and the accelerations, the shaking forces, shaking moments, bearing reactions and input torque are obtained. A quadratic objective function consisting of shaking forces, shaking moments, bearing reactions and input torque is then formulated. This function is generally a convex function. Choosing active point masses as design variables and forming the constraints as linear in the design variables, the optimum mass distribution is obtained using the IMSL routine.

The optimality criterion for the mass distribution of the links by using a quadratic programming technique has been found to be successful in minimizing the inertia-induced forces in a high-speed CSSP mechanism. An average decrease of 2.283% was achieved in the value of the objective function by allowing point masses to decrease upto five percent of their magnitudes while total mass of the links remained constant.

Much work has been conducted by many researchers to balance wide variety of mechanisms. Most of them [1, 4, 9, 22-30, 31, 39, 40, 43, 44] are on complete balancing of planar mechanisms. Relatively little research [2, 6, 7, 46-48] have been done on the complete balancing of spatial mechanisms because of its complicated kinematic and dynamic properties. Complete balancing of shaking forces can be achieved if the center of mass of the mechanism remains stationary as developed by Berkof. Most of the time a stationary center of mass is obtained by adding counterweights to the moving links or by introducing additional moving elements as for example cams, etc. As a result of that, other dynamic characteristics deteriorate. Therefore, partial balancing techniques by optimum distribution of mass caught many researchers interest.

In the existing literature, optimal-mass distribution criteria have been used to minimize a few of the inertia-induced forces and has been limited to specific mechanisms. The least-square technique developed by Berkof and Lowen [5] for the optimization of the shaking moment of fully force-balanced four-bar linkages is applicable to planar mechanisms only.

Tricamo and Lowen's [41, 42] method for optimization of dynamic reactions such as the bearing force, the input moment and the shaking moment with prescribed maximum shaking force is restricted to only planar mechanisms. Tricamo and Lowen [41] describes a two-counterweight method for partially force balancing a four-bar linkage which allows the realization of a prescribed value for the maximum shaking force anywhere between zero and an inherent upper limit. In that paper it was found that a 50 percent reduction of the shaking force results in small amount of increases in bearing forces, shaking moments and input moments over a considerable portion of the design range. Tricamo and Lowen [42] introduces simultaneous optimization of the maximum values of the bearing forces, input moment and shaking moment of a constant speed four-bar linkage while additionally obtaining a prescribed maximum shaking force. The optimization technique determines the parameters of the three counterweights which must be attached to input link , coupler and output link.

Gill and Freudenstein [15,16] minimized the inertia-induced forces in general spherical four-bar mechanisms. Their method allows an optimum trade-off among shaking forces, shaking moments, bearing reactions and input torque fluctuations; but it is limited to one particular type i.e. spherical mechanism in which kinematic and dynamic analyses are relatively simple.

Hockey [18, 19] and Sherwood's [36, 37] research on the optimum distribution of mass in the link of a spatial slider crank mechanism and a planar four-bar mechanism dealt with the minimization of fluctuation of kinetic energy and inertia force and torque balance during the motion cycle. Such an analysis is inadequate for high-speed machinery because it does not permit the trade-offs necessary for an effective design.

This is the first analysis which is capable of minimizing combined effects of all the inertia-induced forces and torques in a general spatial mechanism without any restriction, by optimum mass redistribution and allows for trade-offs among different inertia-induced forces and torques. In case of a general spatial mechanism kinematic and dynamic analyses are very complicated. Use of the dual number method and the "principle of transference" make this method generalized so that it can be used to design any spatial mechanism.

The results presented here, it is hoped, will be an aid to practicing engineers in the rational design of high-speed three dimensional mechanisms.

6.2 Future Work

Since this is the most generalized method of balancing of three dimensional mechanism, any other three dimensional mechanisms can be analyzed by similar means. Other types of design constraints, e.g. based on the yield strength of the link may be developed. Computer programs may be developed for synthesizing realistic proportions from optimized point mass distribution.

APPENDIX A

Dual-number Coordinate Transformations

The trigonometric functions of dual angle $\hat{\theta}$ can be obtained by using the Taylor expansion.

$$\begin{aligned} \sin\hat{\theta} &= \sin\theta + \varepsilon s\cos\theta \\ \cos\hat{\theta} &= \cos\theta - \varepsilon s\sin\theta \\ \tan\hat{\theta} &= \tan\theta + \varepsilon s\sec^2\theta \end{aligned} \tag{A.1}$$

where $\hat{\theta} = \theta + \varepsilon s$, symbol θ being the primary component of dual angle $\hat{\theta}$ denotes rotational displacement and symbol s being the dual component denotes translational displacement. All formal operations, except division by pure dual number, of dual numbers are the same as those of ordinary algebra followed by the setting $\varepsilon^2 = \varepsilon^3 = \varepsilon^4 = \ldots\ldots\ldots.0$. All identities for ordinary trigonometry hold true for dual angle. Sines and cosines are respectively abbreviated as s and c.

Screw motion through dual angle $\hat{\alpha}_n$ ($\hat{\alpha}_n = \alpha_n + \varepsilon a_n$) about the X axis can be written in a 3×3 transformation matrix form as

$$[X(\hat{\alpha}_n)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\hat{\alpha}_n & -s\hat{\alpha}_n \\ 0 & s\hat{\alpha}_n & c\hat{\alpha}_n \end{bmatrix} \tag{A.2}$$

which, by separating primary and dual components, expands to

$$[X(\hat{\alpha}_n)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha_n & -s\alpha_n \\ 0 & s\alpha_n & c\alpha_n \end{bmatrix} + \varepsilon \begin{bmatrix} 0 & 0 & 0 \\ 0 & -a_n s\alpha_n & -a_n c\alpha_n \\ 0 & a_n c\alpha_n & -a_n s\alpha_n \end{bmatrix} \tag{A.3}$$

Screw motion through dual angle $\hat{\eta}_n$ ($\hat{\eta}_n = \eta_n + \varepsilon e_n$) about Y axis can be written in a 3×3 transformation matrix form as

$$[Y(\hat{\eta}_n)] = \begin{bmatrix} c\hat{\eta}_n & 0 & s\hat{\eta}_n \\ 0 & 1 & 0 \\ -s\hat{\eta}_n & 0 & c\hat{\eta}_n \end{bmatrix} \tag{A.4}$$

which, by separating primary and dual components, expands to

$$[Y(\hat{\eta}_n)] = \begin{bmatrix} c\eta_n & 0 & s\eta_n \\ 0 & 1 & 0 \\ -s\eta_n & 0 & c\eta_n \end{bmatrix} + \varepsilon \begin{bmatrix} -e_n s\eta_n & 0 & e_n c\eta_n \\ 0 & 0 & 0 \\ -e_n c\eta_n & 0 & -e_n s\eta_n \end{bmatrix} \tag{A.5}$$

Screw motion through dual angle $\hat{\theta}_n$ ($\hat{\theta}_n = \theta_n + \varepsilon s_n$) about Z axis can be written in a 3×3 transformation matrix form as

$$[Z(\hat{\theta}_n)] = \begin{bmatrix} c\hat{\theta}_n & -s\hat{\theta}_n & 0 \\ s\hat{\theta}_n & c\hat{\theta}_n & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.6}$$

which, by separating primary and dual components, expands to

$$[Z(\hat{\theta}_n)] = \begin{bmatrix} c\theta_n & -s\theta_n & 0 \\ s\theta_n & c\theta_n & 0 \\ 0 & 0 & 1 \end{bmatrix} + \varepsilon \begin{bmatrix} -s_n s\theta_n & -s_n c\theta & 0 \\ s_n c\theta_n & -s_n s\theta_n & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{A.7}$$

Screw motions $\hat{Z}(\hat{\theta}_n)$ followed by screw motion $\hat{X}(\hat{\alpha}_n)$ can be combined into a matrix $\hat{M}_n$ and expressed in 3×3 form as

$${}^{n}_{n+1}\hat{M} = [Z(\hat{\theta}_n)][X(\hat{\alpha}_n)] = \begin{bmatrix} c\hat{\theta}_n & -s\hat{\theta}_n & 0 \\ s\hat{\theta}_n & c\hat{\theta}_n & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\hat{\alpha}_n & -s\hat{\alpha}_n \\ 0 & s\hat{\alpha}_n & c\hat{\alpha}_n \end{bmatrix} \tag{A.8}$$

which, by observing that $\varepsilon^2 = 0$, expands to

$$
{}_{n+1}^{\;\;n}\hat{M} = \begin{bmatrix} c\theta_n & -c\alpha_n s\theta_n & s\alpha_n s\theta_n \\ s\theta_n & c\alpha_n c\theta_n & -s\alpha_n c\theta_n \\ 0 & s\alpha_n & c\alpha_n \end{bmatrix}
$$

$$
\varepsilon \begin{bmatrix} -s_n s\theta_n & a_n s\alpha_n s\theta_n - s_n c\alpha_n c\theta_n & a_n c\alpha_n s\theta_n + s_n s\alpha_n c\theta_n \\ s_n c\theta_n & -a_n s\alpha_n c\theta_n - s_n c\alpha_n s\theta_n & -a_n c\alpha_n c\theta_n + s_n s\alpha_n s\theta_n \\ 0 & a_n c\alpha_n & -a_n s\alpha_n \end{bmatrix} \tag{A.9}
$$

The motion at ball joint requires an additional rotation through an angle $\hat{\eta}_n$ for its description so that the complete transformation through the joint and link takes the form

$$
{}_{n+1}^{\;\;n}\hat{L} = [Z(\hat{\theta}_n)][Y(\hat{\eta}_n)][X(\hat{\alpha}_n)] \tag{A.10}
$$

which can also be expanded and simplified for different link-joint parameters.

APPENDIX B

FORTRAN Program for optimum balancing of the CSSP mechanism

```
C**************************MAIN PROGRAM**********************
C  PROGRAM FOR OPTIMUM BALANCING OF THE GENERALIZED SLIDER
C  CRANK MECHANISM (CSSP). KINEMATICS IS PERFORMED USING
C  UICKER-DENAVIT-HARTENBERG METHOD.
C************************************************************

C*************Declaration of the variables******************
C  Joint variables
   REAL*8 THT1,THT2,ETA,ETA2,THT3,ETA3,ALP4,S4
C  Derivatives of joint variables
   REAL*8 THT1D,THT2D,ETA2D,THT3D,ETA3D,S4D
C  Double derivatives of joint variables
   REAL*8 THT2DD,ETA2DD,THT3DD,ETA3DD,S4DD
C  Transformation matrices
   REAL*8 M1R(3,3),M1D(3,3),L2R(3,3),L2D(3,3),
   1 L3R(3,3),L3D(3,3),M4R(3,3),M4D(3,3)
C  Differential operators of transformation matrices
C      Q1=operator of M1, L2(wrt THT2) & L3(wrt THT3)
C      Q2=operator of L2(wrt ETA2)
C      Q3=operator of M4
C      Q4=operator of L3(wrt ETA3)
   REAL*8 Q1R(3,3),Q1D(3,3),Q2R(3,3),Q2D(3,3),
   1 Q3R(3,3),Q3D(3,3),Q4R(3,3),Q4D(3,3)

C  Derivatives, transpose, multiplications, etc of
C          transformation matrices
   REAL*8 M1Q1R(3,3),M1Q1D(3,3),TM1R(3,3),TM1D(3,3),
   1 H2R(3,3),H2D(3,3),M1Q2R(3,3),M1Q2D(3,3),H2DR(3,3),
   1 H2DD(3,3),M1L2R(3,3),M1L2D(3,3),TM1L2R(3,3),
   1 TM1L2D(3,3),H6R(3,3),H6D(3,3),H3R(3,3),H3D(3,3),
   1 H7R(3,3),H7D(3,3),H3DR(3,3),H3DD(3,3),H5R(3,3),
   1 H5D(3,3),H5Q3R(3,3),H5Q3D(3,3),TH5R(3,3),TH5D(3,3),
   1 H4DDR(3,3),H4DDD(3,3),BR(3,3),BD(3,3),A(6,5),V(6),
   1 AT(5,6),ATA(5,5),IATA(5,5),DELTA(5),IA(5,6),
   1 L3M4R(3,3),L3M4D(3,3),TL3M4R(3,3),TL3M4D(3,3),
   1 TL3R(3,3),TM4R(3,3),L3RD(3,3),L3RDD(3,3),
   1 TM4L3R(3,3),TM4L3RD(3,3),TM4L3RDD(3,3)
C  L3RD & L3RDD are the diff and double diff of TL3R
C  Joint angles in degrees
```

```
      REAL*8 TH1,TH2,TH3,ET2,ET3,AL4
C     Velocity components
      REAL*8 V2XP,V2XD,V2YP,V2YD,V2ZP,V2ZD
C     Velocity and acceleration matrices
      REAL*8 VM(6,5),VMT(5,6),VMTVM(5,5),IVMTVM(5,5),
     1 IVM(5,6),VV(6),AV(6),DOT(5),DDOT(5)

C     Mass properties (Radii of gyration, C.G.'s, masses & MOI)
C           of the moving links
      REAL*8 K1X,K1Y,K1Z,K2X,K2Y,K2Z,K3X,K3Y,K3Z,
     1 K1XCG,K1YCG,K1ZCG,K2XCG,K2YCG,K2ZCG,
     1 G1,G2,G3,M1,M2,M3,I1X,I1Y,I1Z,I2X,I2Y,I2Z,
     1 I3X,I3Y,I3Z,NUI1X,NUI1Y,NUI1Z,NUI2X,NUI2Y,NUI2Z
C     Momentum components and their derivatives
      REAL*8 H2XP,H2XD,H2YP,H2YD,H2ZP,H2ZD,
     1 H2DXP,H2DXD,H2DYP,H2DYD,H2DZP,H2DZD
C     Inertia force and torque components
      REAL*8 R1P(3),R2P(3),R3P(3),R1D(3),R2D(3),R3D(3),
     1 M1R1P(3),M1R1D1(3),M1R1D2(3),M1R1D(3),
     1 M1L2R2P(3),M1L2R2D1(3),M1L2R2D2(3),M1L2R2D(3),
     1 TM4R3P(3),NUR1XP,NUR2XP,NUR1YP,NUR2YP,NUR1ZP,NUR2ZP,
     1 NUR1XD,NUR2XD,NUR1YD,NUR2YD,NUR1ZD,NUR2ZD
C     Reaction force and torque components
      REAL*8 T1I,T1J,T1K,F1I,F1J,F1K,F2I,F2J,F2K,
     1 F2K1,F2K2,F3I,F3J,F3K,T4I,T4J,T4K,F4I,F4J,F4K

C     Location of point masses and their derivatives
      REAL*8 D(3),DD(3),DDD(3),LL(3),LL1(3),LL2(3),
     1 L11DD(3),L12DD(3),L13DD(3),L14(3),L14D(3),L14DD(3),
     1 L111(3),L11D1(3),temp,
     1 L11DD1(3),L121(3),L12D1(3),L12DD1(3),L131(3),
     1 L13D1(3),L13DD1(3),L112(3),L11D2(3),
     1 L11DD2(3),L122(3),L12D2(3),L12DD2(3),L132(3),
     1 L13D2(3),L13DD2(3),L21DD(3),
     1 L22DD(3),L23DD(3),L24(3),L24D(3),L24DD(3),
     1 L211(3),L21D1(3),L21DD1(3),L221(3),L22D1(3),
     1 L22DD1(3),L231(3),L23D1(3),L23DD1(3),
     1 L212(3),L21D2(3),L21DD2(3),L222(3),L22D2(3),
     1 L22DD2(3),L232(3),L23D2(3),L23DD2(3)
      REAL*8 X10,Y10,Z10,X1,Y1,Z1,L1(3,3),
     1      X20,Y20,Z20,X2,Y2,Z2,L2(3,3)
      REAL*8 M1RD(3,3),M1RDD(3,3)
      REAL*8 PI,DEGRAD
      real*8 m11,m12,m13,m14,m21,m22,m23,m24,m(9)
```

```
C   Variables required for optimization
    real*8 px(9),fx(9),fxsq(9,9),py(9),fy(9),fysq(9,9),
    1     pz(9),fz(9),fzsq(9,9),qx(9),tx(9),txsq(9,9),
    1     qy(9),ty(9),tysq(9,9),qz(9),tz(9),tzsq(9,9),
    1     f1x(9),f1xsq(9,9),f1y(9),f1ysq(9,9),
    1     f1z(9),f1zsq(9,9),t1x(9),t1xsq(9,9),
    1     t1y(9),t1ysq(9,9),t1z(9),t1zsq(9,9),
    1     f2x(9),f2xsq(9,9),f2y(9),f2ysq(9,9),
    1     f2z(9),f2zsq(9,9),f3x(9),f3xsq(9,9),
    1     f3y(9),f3ysq(9,9),f3z(9),f3zsq(9,9),
    1     f4x(9),f4xsq(9,9),f4y(9),f4ysq(9,9),
    1     t4x(9),t4xsq(9,9),t4y(9),t4ysq(9,9),
    1     t4z(9),t4zsq(9,9),f4z(9),f4zsq(9,9),
    1     fsq(9,9),tsq(9,9),rsq(9,9),msq(9,9),t0sq(9,9),
    1     F(9,9),OF(9,9),C1,C2,C3,OBJFUN,NUOBJFUN,
    1     mfx,nufx,mfy,nufy,mfz,nufz,mtx,nutx,mty,nuty,
    1     mtz,nutz,mf1x,nuf1x,mf1y,nuf1y,mf1z,nuf1z,
    1     mt1x,nut1x,mt1y,nut1y,mt1z,nut1z,mf2x,nuf2x,
    1     mf2y,nuf2y,mf2z,nuf2z,mf3x,nuf3x,mf3y,nuf3y,
    1     mf3z,nuf3z,mf4x,nuf4x,mf4y,nuf4y,mt4x,nut4x,
    1     mt4y,nut4y,mt4z,nut4z,shf,nushf,sht,nusht,
    1     brf,nubrf,brt,nubrt,int,nuint,DOBJFUN,dshf,
    1     dsht,dbrf,dbrt,dint
      integer lda,ldh,ncon,neq,nvar
      parameter (ncon=11,neq=3,nvar=9,lda=ncon,ldh=nvar)
      integer iact(nvar),k,nact,nout
      real aa(lda,nvar),alamda(nvar),b(ncon),diag,gg(nvar),
    1      h(ldh,ldh),sol(nvar)
      external qprog,umach
      data aa/1.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
    1        1.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,0.0,0.0,
    1        1.0,0.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,0.0,
    1        0.0,0.0,0.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,
    1        0.0,1.0,0.0,0.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,
    1        0.0,1.0,0.0,0.0,0.0,0.0,0.0,0.0,1.0,0.0,0.0,
    1        0.0,1.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,1.0,0.0,
    1        0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,1.0,
    1        0.0,0.0,1.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0/
      data b/0.003688,0.01475,0.01553,4*0.001168,4*0.00467/
      data gg/9*0.0/

C   Weight factors and bearing length
    PARAMETER (W1=1,W2=1,W3=1,W4=1,W5=1,BL=0.4)
C   LINK PARAMETERS ARE AS FOLLOWS
    PARAMETER (A1=2.0,A2=8.0,A4=1.0,AL4=250.0,S1=0.4)
```

```
      PARAMETER (G=386.4,RPM=3000.0,THT1DD=0.0,
     1          S1D=0.0,S1DD=0.0)
        OPEN (6,FILE='dint.dat',STATUS='new')
          PI=4.0*ATAN(1.0)
          DEGRAD=PI/180.0

C   Mass properties of the original links

      G1=-1.0
      G2=-4.0
      G3=0.0
        M1=1.9/G
        M2=7.6/G
        M3=6.0/G

C   Radii of gyration about C.G. & about distal coordinate frame
      K1XCG=0.2041241
      K1YCG=0.595119
      K1ZCG=0.595119
           K1X=0.2041241
           K1Y=1.1636867
           K1Z=1.1636867
      K2XCG=0.2041241
      K2YCG=2.3139072
      K2ZCG=2.3139072
           K2X=0.2041241
           K2Y=4.6210569
           K2Z=4.6210569
      K3X=0.5204165
      K3Y=0.5204165
      K3Z=0.7071067

C   Mass moment of inertia about C.G.
         I1X=M1*K1XCG**2
         I1Y=M1*K1YCG**2
         I1Z=M1*K1ZCG**2
         I2X=M2*K2XCG**2
         I2Y=M2*K2YCG**2
         I2Z=M2*K2ZCG**2
         I3X=M3*K3X**2
         I3Y=M3*K3Y**2
         I3Z=M3*K3Z**2
            X10=G1
            X20=G2
```

```
C  Magnitude of point masses
      m11=M1/4.0
      m12=m11
      m13=m11
      m14=M1-(m11+m12+m13)
      m21=M2/4.0
      m22=m21
      m23=m21
      m24=M2-(m21+m22+m23)

        m(1)=m11
        m(2)=m12
        m(3)=m13
        m(4)=m14
        m(5)=m21
        m(6)=m22
        m(7)=m23
        m(8)=m24
        m(9)=M3

C  Direction cosines between distal & principal coordinate frames
         L1(1,1)=1.0
         L1(2,2)=1.0
         L1(3,3)=1.0
         L2(1,1)=1.0
         L2(2,2)=1.0
         L2(3,3)=1.0

C  Location of point masses
       X1=((I1Y+I1Z-I1X)/(2.0*m11))**0.5
       Y1=((I1X+I1Z-I1Y)/(2.0*m12))**0.5
       Z1=((I1X+I1Y-I1Z)/(2.0*m13))**0.5
       X2=((I2Y+I2Z-I2X)/(2.0*m21))**0.5
       Y2=((I2X+I2Z-I2Y)/(2.0*m22))**0.5
       Z2=((I2X+I2Y-I2Z)/(2.0*m23))**0.5

C     Initialization of the objective function OF(9,9)
          DO 100 I2=1,9
          DO 110 J2=1,9
          OF(I2,J2)=0
 110        CONTINUE
 100        CONTINUE

        do 41 I3=1,2
C  INITIAL ESTIMATE FOR DISPLACEMENTS
```

```
      S4=(A2**2-(A1+A4)**2)**0.5
        temp=S4/(A4+A1)
        THT2=PI-ATAN(temp)
C       THT2=PI-ATAN2(S4,(A4+A1))
      THT3=2*PI-THT2

      DO 400 J=1,37
      TH1=J*10.0-10.0
      THT1=TH1*DEGRAD
      ALP4=AL4*DEGRAD
      THT1D=6.0*RPM*DEGRAD
C     STEP 1
      I=1
   10 IF (I.LE.150) THEN

C     SPECIFICATION OF M1,L2,L3,M4 MATRICES RESPECTIVELY
      M1R(1,1)=COS(THT1)
      M1R(1,2)=-SIN(THT1)
      M1R(1,3)=0.0
      M1R(2,1)=SIN(THT1)
      M1R(2,2)=COS(THT1)
      M1R(2,3)=0.0
      M1R(3,1)=0.0
      M1R(3,2)=0.0
      M1R(3,3)=1.0
           M1D(1,1)=-S1*SIN(THT1)
           M1D(1,2)=-S1*COS(THT1)
           M1D(1,3)=A1*SIN(THT1)
           M1D(2,1)=S1*COS(THT1)
           M1D(2,2)=-S1*SIN(THT1)
           M1D(2,3)=-A1*COS(THT1)
           M1D(3,1)=0.0
           M1D(3,2)=A1
           M1D(3,3)=0.0
C
      L2R(1,1)=COS(THT2)*COS(ETA2)
      L2R(1,2)=-SIN(THT2)
      L2R(1,3)=COS(THT2)*SIN(ETA2)
      L2R(2,1)=SIN(THT2)*COS(ETA2)
      L2R(2,2)=COS(THT2)
      L2R(2,3)=SIN(THT2)*SIN(ETA2)
      L2R(3,1)=-SIN(ETA2)
      L2R(3,2)=0.0
      L2R(3,3)=COS(ETA2)
           L2D(1,1)=0.0
```

```
      L2D(1,2)=A2*COS(THT2)*SIN(ETA2)
      L2D(1,3)=A2*SIN(THT2)
      L2D(2,1)=0.0
      L2D(2,2)=A2*SIN(THT2)*SIN(ETA2)
      L2D(2,3)=-A2*COS(THT2)
      L2D(3,1)=0.0
      L2D(3,2)=A2*COS(ETA2)
      L2D(3,3)=0.0
C
   L3R(1,1)=COS(THT3)*COS(ETA3)
   L3R(1,2)=COS(THT3)*SIN(ETA3)
   L3R(1,3)=SIN(THT3)
   L3R(2,1)=SIN(THT3)*COS(ETA3)
   L3R(2,2)=SIN(THT3)*SIN(ETA3)
   L3R(2,3)=-COS(THT3)
   L3R(3,1)=-SIN(ETA3)
   L3R(3,2)=COS(ETA3)
   L3R(3,3)=0.0
      L3D(1,1)=0.0
      L3D(1,2)=0.0
      L3D(1,3)=0.0
      L3D(2,1)=0.0
      L3D(2,2)=0.0
      L3D(2,3)=0.0
      L3D(3,1)=0.0
      L3D(3,2)=0.0
      L3D(3,3)=0.0
C
   M4R(1,1)=1.0
   M4R(1,2)=0.0
   M4R(1,3)=0.0
   M4R(2,1)=0.0
   M4R(2,2)=COS(ALP4)
   M4R(2,3)=-SIN(ALP4)
   M4R(3,1)=0.0
   M4R(3,2)=SIN(ALP4)
   M4R(3,3)=COS(ALP4)
      M4D(1,1)=0.0
      M4D(1,2)=-S4*COS(ALP4)
      M4D(1,3)=S4*SIN(ALP4)
      M4D(2,1)=S4
      M4D(2,2)=-A4*SIN(ALP4)
      M4D(2,3)=-A4*COS(ALP4)
      M4D(3,1)=0.0
      M4D(3,2)=A4*COS(ALP4)
```

```
        M4D(3,3)=-A4*SIN(ALP4)
C
C   SPECIFICATION OF PARTIAL DERIVATIVE OPERATORS
      Q1R(1,2)=-1.0
      Q1R(2,1)=1.0
         Q2R(1,3)=COS(THT2)
         Q2R(2,3)=SIN(THT2)
         Q2R(3,1)=-COS(THT2)
         Q2R(3,2)=-SIN(THT2)
      Q3D(1,2)=-1.0
      Q3D(2,1)=1.0
         Q4R(1,3)=COS(THT3)
         Q4R(2,3)=SIN(THT3)
         Q4R(3,1)=-COS(THT3)
         Q4R(3,2)=-SIN(THT3)
C
      CALL PRODUCTDUAL (M1R,M1D,Q1R,Q1D,M1Q1R,M1Q1D)
        CALL TRANSPOSE (M1R,M1D,TM1R,TM1D)
      CALL PRODUCTDUAL (M1Q1R,M1Q1D,TM1R,TM1D,H2R,H2D)
      CALL PRODUCTDUAL (M1R,M1D,Q2R,Q2D,M1Q2R,M1Q2D)
      CALL PRODUCTDUAL (M1Q2R,M1Q2D,TM1R,TM1D,H2DR,H2DD)
      CALL PRODUCTDUAL (M1R,M1D,L2R,L2D,M1L2R,M1L2D)
        CALL TRANSPOSE (M1L2R,M1L2D,TM1L2R,TM1L2D)
      CALL PRODUCTDUAL (M1L2R,M1L2D,Q1R,Q1D,H6R,H6D)
      CALL PRODUCTDUAL (H6R,H6D,TM1L2R,TM1L2D,H3R,H3D)
      CALL PRODUCTDUAL (M1L2R,M1L2D,Q4R,Q4D,H7R,H7D)
      CALL PRODUCTDUAL (H7R,H7D,TM1L2R,TM1L2D,H3DR,H3DD)
      CALL PRODUCTDUAL (M1L2R,M1L2D,L3R,L3D,H5R,H5D)
      CALL PRODUCTDUAL (H5R,H5D,Q3R,Q3D,H5Q3R,H5Q3D)
        CALL TRANSPOSE (H5R,H5D,TH5R,TH5D)
      CALL PRODUCTDUAL (H5Q3R,H5Q3D,TH5R,TH5D,H4DDR,H4DDD)
      CALL PRODUCTDUAL (H5R,H5D,M4R,M4D,BR,BD)
      CALL PRODUCTDUAL (L3R,L3D,M4R,M4D,L3M4R,L3M4D)
        CALL TRANSPOSE (L3M4R,L3M4D,TL3M4R,TL3M4D)

C   SPECIFICATION OF A & V MATRICES
      A(1,1)=H2R(1,2)
      A(1,2)=H2DR(1,2)
      A(1,3)=H3R(1,2)
      A(1,4)=H3DR(1,2)
      A(1,5)=H4DDR(1,2)
         A(2,1)=H2R(1,3)
         A(2,2)=H2DR(1,3)
         A(2,3)=H3R(1,3)
         A(2,4)=H3DR(1,3)
```

```
          A(2,5)=H4DDR(1,3)
      A(3,1)=H2R(2,3)
      A(3,2)=H2DR(2,3)
      A(3,3)=H3R(2,3)
      A(3,4)=H3DR(2,3)
      A(3,5)=H4DDR(2,3)
          A(4,1)=H2D(1,2)
          A(4,2)=H2DD(1,2)
          A(4,3)=H3D(1,2)
          A(4,4)=H3DD(1,2)
          A(4,5)=H4DDD(1,2)
      A(5,1)=H2D(1,3)
      A(5,2)=H2DD(1,3)
      A(5,3)=H3D(1,3)
      A(5,4)=H3DD(1,3)
      A(5,5)=H4DDD(1,3)
          A(6,1)=H2D(2,3)
          A(6,2)=H2DD(2,3)
          A(6,3)=H3D(2,3)
          A(6,4)=H3DD(2,3)
          A(6,5)=H4DDD(2,3)
C
      V(1)=BR(2,1)+BR(1,1)+BR(2,2)-2.0
      V(2)=BR(3,1)+BR(1,1)+BR(3,3)-2.0
      V(3)=BR(3,2)+BR(2,2)+BR(3,3)-2.0
      V(4)=BD(2,1)
      V(5)=BD(3,1)
      V(6)=BD(3,2)

C   Determination of joint variables
      CALL TRANS (A,AT,6,5)
      CALL MULTI (AT,A,ATA,5,5,6)
      CALL INVERSE (ATA,IATA)
      CALL MULTI (IATA,AT,IA,5,6,5)
      CALL MULMAVEC (IA,V,DELTA,5,6)
C
        IF ( ABS(DELTA(1)) .LE. 0.0000001 .AND.
     1       ABS(DELTA(2)) .LE. 0.0000001 .AND.
     1       ABS(DELTA(3)) .LE. 0.0000001 .AND.
     1       ABS(DELTA(4)) .LE. 0.0000001 .AND.
     1       ABS(DELTA(5)) .LE. 0.0000001 ) THEN

      TH2=THT2/DEGRAD
      TH3=THT3/DEGRAD
      ET2=ETA2/DEGRAD
```

```
      ET3=ETA3/DEGRAD

C  Derivative of joint variables
          VM(1,1)=-SIN(ETA2)
          VM(1,3)=SIN(THT2)*COS(ETA2)
          VM(2,5)=SIN(THT3)
          VM(3,3)=COS(THT2)
          VM(3,4)=1.0
          VM(4,1)=A2*COS(ETA2)
          VM(4,3)=A2*SIN(THT2)*SIN(ETA2)
          VM(4,5)=-COS(THT3)
          VM(5,1)=COS(ETA2)
          VM(5,2)=1.0
          VM(5,3)=SIN(THT2)*SIN(ETA2)
          VM(6,3)=-A2*COS(THT2)

      VV(1)=THT1D*SIN(ETA2)
      VV(2)=S1D*SIN(ETA2)-A1*THT1D*SIN(THT2)*COS(ETA2)
      VV(4)=-THT1D*(A1*COS(THT2)+A2*COS(ETA2))
      VV(5)=-THT1D*COS(ETA2)
      VV(6)=-THT1D*A1*SIN(THT2)*SIN(ETA2)-S1D*COS(ETA2)

        CALL TRANS (VM,VMT,6,5)
        CALL MULTI (VMT,VM,VMTVM,5,5,6)
        CALL INVERSE (VMTVM,IVMTVM)
        CALL MULTI (IVMTVM,VMT,IVM,5,6,5)
        CALL MULMAVEC (IVM,VV,DOT,5,6)

       THT2D=DOT(1)
       THT3D=DOT(2)
       ETA2D=DOT(3)
       ETA3D=DOT(4)
       S4D=DOT(5)

C  Double derivative of joint variables
      AV(1)=THT1DD*SIN(ETA2)+ETA2D*(THT1D+THT2D)*COS(ETA2)
     1 -ETA2D*(THT2D*COS(THT2)*COS(ETA2)-ETA2D*
     1 SIN(THT2)*SIN(ETA2))
      AV(2)=A1*(THT1D*SIN(THT2)*SIN(ETA2)*ETA2D-THT1DD*
     1 SIN(THT2)*COS(ETA2)-THT1D*THT2D*COS(THT2)*COS(ETA2))
     1 +S1DD*SIN(ETA2)+S1D*ETA2D*COS(ETA2)-S4D*THT3D*COS(THT3)
      AV(3)=THT2D*ETA2D*SIN(THT2)
      AV(4)=A1*SIN(THT2)*THT1D*THT2D+A2*SIN(ETA2)*ETA2D*
     1 (THT1D+THT2D)-THT1DD*(A1*COS(THT2)+A2*COS(ETA2))
     1 -A2*ETA2D*(COS(THT2)*SIN(ETA2)*THT2D+SIN(THT2)*
```

```
     1    COS(ETA2)*ETA2D)-SIN(THT3)*THT3D*S4D
      AV(5)=THT1D*ETA2D*SIN(ETA2)-THT1DD*COS(ETA2)+SIN(ETA2)*
     1    ETA2D*THT2D-ETA2D*(COS(THT2)*SIN(ETA2)*THT2D+
     1    SIN(THT2)*COS(ETA2)*ETA2D)
      AV(6)=S1D*SIN(ETA2)*ETA2D-A1*THT1DD*SIN(THT2)*SIN(ETA2)
     1    -A1*THT1D*THT2D*COS(THT2)*SIN(ETA2)-S1DD*COS(ETA2)
     1    -A1*SIN(THT2)*COS(ETA2)*THT1D*ETA2D
     1    -A2*THT2D*ETA2D*SIN(THT2)

       CALL MULMAVEC (IVM,AV,DDOT,5,6)
       THT2DD=DDOT(1)
       THT3DD=DDOT(2)
       ETA2DD=DDOT(3)
       ETA3DD=DDOT(4)
       S4DD=DDOT(5)

C  Inertia forces and torques of original links
      V2XP=0.0
      V2XD=-S4D*SIN(THT3)
      V2YP=-ETA3D
      V2YD=S4D*COS(THT3)
      V2ZP=-THT3D
      V2ZD=0.0
      H2XP=-S4D*SIN(THT3)
      H2XD=0.0
      H2YP=-G2*THT3D+S4D*COS(THT3)
      H2YD=-(K2Y**2)*ETA3D
      H2ZP=ETA3D*G2
      H2ZD=-(K2Z**2)*THT3D+G2*S4D*COS(THT3)
C
      H2DXP=-S4DD*SIN(THT3)-S4D*THT3D*COS(THT3)
      H2DXD=0.0
      H2DYP=-G2*THT3DD-THT3D*S4D*SIN(THT3)+
     1S4DD*COS(THT3)
      H2DYD=-(K2Y**2)*ETA3DD
      H2DZP=G2*ETA3DD
      H2DZD=-(K2Z**2)*THT3DD+G2*(S4DD*COS(THT3)-SIN(THT3)*
     1THT3D*S4D)
C
      R1P(1)=-M1*(THT1D**2)*(A1+G1)
      R1D(1)=0.0
      R1P(2)=M1*THT1DD*(A1+G1)
      R1D(2)=-M1*G1*S1DD
      R1P(3)=M1*S1DD
      R1D(3)=M1*THT1DD*((K1Z**2)+G1*A1)
```

```
      CALL MULMAVEC (M1R,R1P,M1R1P,3,3)
      CALL MULMAVEC (M1R,R1D,M1R1D1,3,3)
      CALL MULMAVEC (M1D,R1P,M1R1D2,3,3)
      CALL ADDVEC (M1R1D1,M1R1D2,M1R1D)

C
      R2P(1)=M2*(H2DXP-V2ZP*H2YP+V2YP*H2ZP)
      R2D(1)=M2*(H2DXD-V2ZP*H2YD-V2ZD*H2YP+V2YP*H2ZD+V2YD*H2ZP)
      R2P(2)=M2*(H2DYP-V2XP*H2ZP+V2ZP*H2XP)
      R2D(2)=M2*(H2DYD-V2XP*H2ZD-V2XD*H2ZP+V2ZP*H2XD+V2ZD*H2XP)
      R2P(3)=M2*(H2DZP-V2YP*H2XP+V2XP*H2YP)
      R2D(3)=M2*(H2DZD-V2YP*H2XD-V2YD*H2XP+V2XP*H2YD+V2XD*H2YP)
C       CALL MULMAVEC (M1L2R,R2P,M1L2R2P,3,3)
C       CALL MULMAVEC (M1L2R,R2D,M1L2R2D1,3,3)
C       CALL MULMAVEC (M1L2D,R2P,M1L2R2D2,3,3)
C       CALL ADDVEC (M1L2R2D1,M1L2R2D2,M1L2R2D)
      CALL MULMAVEC (TL3M4R,R2P,M1L2R2P,3,3)
      CALL MULMAVEC (TL3M4R,R2D,M1L2R2D1,3,3)
      CALL MULMAVEC (TL3M4D,R2P,M1L2R2D2,3,3)
      CALL ADDVEC (M1L2R2D1,M1L2R2D2,M1L2R2D)

      R3P(1)=0.0
      R3D(1)=0.0
      R3P(2)=0.0
      R3D(2)=M3*G3*S4DD
      R3P(3)=-M3*S4DD
      R3D(3)=0.0
      CALL TRANS (L3R,TL3R,3,3)
      CALL TRANS (M4R,TM4R,3,3)
      CALL MULMAVEC (TM4R,R3P,TM4R3P,3,3)
      CALL MULTI (TM4R,TL3R,TM4L3R,3,3,3)

C     Joint forces and torques of the original links
      F3J=-(R2P(2)+R2D(3)/A2)
      F3K=R2D(2)/A2-R2P(3)
      T4I=-R3D(1)
      T4J=-R3D(2)
      T4K=-R3D(3)
      F3I=(F3J*COS(THT3)-F4K-R3P(3))/SIN(THT3)
      F4J=-(COS(THT3)*SIN(ETA3)*F3I+SIN(THT3)*SIN(ETA3)*F3J+
     1     COS(ETA3)*F3K+R3P(2))
      F4I=-(COS(THT3)*COS(ETA3)*F3I+SIN(THT3)*COS(ETA3)*F3J-
     1     SIN(ETA3)*F3K+R3P(1))
      F2J=-(F3J+R2P(2))*COS(THT2)-R2D(2)*SIN(ETA2)*SIN(THT2)/A2
     1   -(F3I+R2P(1))*SIN(THT2)*COS(ETA2)
```

```
      F2I=(F3J+R2P(2))*SIN(THT2)-R2D(2)*SIN(ETA2)*COS(THT2)/A2
     1 -(F3I+R2P(1))*COS(THT2)*COS(ETA2)
      F2K1=-(F2I*COS(THT2)*SIN(ETA2)+F2J*SIN(THT2)*SIN(ETA2)+
     1   F3K+R2P(3))
      F2K2=(F2I*COS(THT2)*COS(ETA2)+F2J*SIN(THT2)*COS(ETA2)+
     1   F3I+R2P(1))
        temp=F2K2/F2K1
        ETA=ATAN(temp)
C       ETA=ATAN2(F2K2,F2K1)
      if ( ABS(COS(ETA)) .GT. ABS(SIN(ETA)) ) then
      F2K=F2K1/COS(ETA)
      else
      F2K=F2K2/SIN(ETA)
      endif
      F1K=-(F2K+R1P(3))
      F1I=-(F2I+R1P(1))*COS(THT1)+(F2J+R1P(2))*SIN(THT1)
      F1J=-(F2I+R1P(1))*SIN(THT1)-(F2J+R1P(2))*COS(THT1)
      T1K=COS(THT1)*A1*F1J-SIN(THT1)*A1*F1I-R1D(3)
      T1J=S1*F1I-R1D(1)*SIN(THT1)-(A1*F1K+R1D(2))*COS(THT1)
      T1I=-S1*F1J-R1D(1)*COS(THT1)+(A1*F1K+R1D(2))*SIN(THT1)

C*************************************************************
C     USE OF POINT MASSES
C*************************************************************
C   For Crank (link 1)
      D(1)=A1*COS(THT1)
      DD(1)=-A1*SIN(THT1)*THT1D
      DDD(1)=-A1*(COS(THT1)*THT1D**2+SIN(THT1)*THT1DD)
      D(2)=A1*SIN(THT1)
      DD(2)=A1*COS(THT1)*THT1D
      DDD(2)=A1*(COS(THT1)*THT1DD-SIN(THT1)*THT1D**2)
      D(3)=S1
      DD(3)=S1D
      DDD(3)=S1DD
C   Derivative and double derivative of M1R
        M1RD(1,1)=-SIN(THT1)*THT1D
        M1RD(1,2)=-COS(THT1)*THT1D
        M1RD(2,1)=COS(THT1)*THT1D
        M1RD(2,2)=-SIN(THT1)*THT1D
        M1RDD(1,1)=-SIN(THT1)*THT1DD-COS(THT1)*(THT1D**2)
        M1RDD(1,2)=-COS(THT1)*THT1DD+SIN(THT1)*(THT1D**2)
        M1RDD(2,1)=COS(THT1)*THT1DD-SIN(THT1)*(THT1D**2)
        M1RDD(2,2)=-SIN(THT1)*THT1DD-COS(THT1)*(THT1D**2)

C   Point mass m11
```

```
      LL1(1)=X10+X1*L1(1,1)
      LL1(2)=Y10+X1*L1(1,2)
      LL1(3)=Z10+X1*L1(1,3)
      LL2(1)=X10-X1*L1(1,1)
      LL2(2)=Y10-X1*L1(1,2)
      LL2(3)=Z10-X1*L1(1,3)
      CALL KINEMATIC (M1R,M1RD,M1RDD,D,DD,DDD,LL1,
     1             L111,L11D1,L11DD1)
      CALL KINEMATIC (M1R,M1RD,M1RDD,D,DD,DDD,LL2,
     1             L112,L11D2,L11DD2)
      CALL ADDVEC (L11DD1,L11DD2,L11DD)

C     Point mass m12
      LL1(1)=X10+Y1*L1(2,1)
      LL1(2)=Y10+Y1*L1(2,2)
      LL1(3)=Z10+Y1*L1(2,3)
      LL2(1)=X10-Y1*L1(2,1)
      LL2(2)=Y10-Y1*L1(2,2)
      LL2(3)=Z10-Y1*L1(2,3)
      CALL KINEMATIC (M1R,M1RD,M1RDD,D,DD,DDD,LL1,
     1             L121,L12D1,L12DD1)
      CALL KINEMATIC (M1R,M1RD,M1RDD,D,DD,DDD,LL2,
     1             L122,L12D2,L12DD2)
      CALL ADDVEC (L12DD1,L12DD2,L12DD)

C     Point mass m13
      LL1(1)=X10+Z1*L1(3,1)
      LL1(2)=Y10+Z1*L1(3,2)
      LL1(3)=Z10+Z1*L1(3,3)
      LL2(1)=X10-Z1*L1(3,1)
      LL2(2)=Y10-Z1*L1(3,2)
      LL2(3)=Z10-Z1*L1(3,3)
      CALL KINEMATIC (M1R,M1RD,M1RDD,D,DD,DDD,LL1,
     1             L131,L13D1,L13DD1)
      CALL KINEMATIC (M1R,M1RD,M1RDD,D,DD,DDD,LL2,
     1             L132,L13D2,L13DD2)
      CALL ADDVEC (L13DD1,L13DD2,L13DD)

C     Point mass m14
      LL(1)=X10
      LL(2)=Y10
      LL(3)=Z10
      CALL KINEMATIC (M1R,M1RD,M1RDD,D,DD,DDD,LL,
     1             L14,L14D,L14DD)
```

```
C   New inertia forces and torques of crank using point masses
    NUR1XP=(-m11*L11DD(1)-m12*L12DD(1)-m13*L13DD(1))/2
    1      -m14*L14DD(1)
    NUR1YP=(-m11*L11DD(2)-m12*L12DD(2)-m13*L13DD(2))/2
    1      -m14*L14DD(2)
    NUR1ZP=(-m11*L11DD(3)-m12*L12DD(3)-m13*L13DD(3))/2
    1      -m14*L14DD(3)
    NUR1XD=-m11*(L111(2)*L11DD1(3)-L111(3)*L11DD1(2))/2
    1     -m11*(L112(2)*L11DD2(3)-L112(3)*L11DD2(2))/2
    1     -m12*(L121(2)*L12DD1(3)-L121(3)*L12DD1(2))/2
    1     -m12*(L122(2)*L12DD2(3)-L122(3)*L12DD2(2))/2
    1     -m13*(L131(2)*L13DD1(3)-L131(3)*L13DD1(2))/2
    1     -m13*(L132(2)*L13DD2(3)-L132(3)*L13DD2(2))/2
    1     -m14*(L14(2)*L14DD(3)-L14(3)*L14DD(2))
    NUR1YD=-m11*(L111(3)*L11DD1(1)-L111(1)*L11DD1(3))/2
    1     -m11*(L112(3)*L11DD2(1)-L112(1)*L11DD2(3))/2
    1     -m12*(L121(3)*L12DD1(1)-L121(1)*L12DD1(3))/2
    1     -m12*(L122(3)*L12DD2(1)-L122(1)*L12DD2(3))/2
    1     -m13*(L131(3)*L13DD1(1)-L131(1)*L13DD1(3))/2
    1     -m13*(L132(3)*L13DD2(1)-L132(1)*L13DD2(3))/2
    1     -m14*(L14(3)*L14DD(1)-L14(1)*L14DD(3))
    NUR1ZD=-m11*(L111(1)*L11DD1(2)-L111(2)*L11DD1(1))/2
    1     -m11*(L112(1)*L11DD2(2)-L112(2)*L11DD2(1))/2
    1     -m12*(L121(1)*L12DD1(2)-L121(2)*L12DD1(1))/2
    1     -m12*(L122(1)*L12DD2(2)-L122(2)*L12DD2(1))/2
    1     -m13*(L131(1)*L13DD1(2)-L131(2)*L13DD1(1))/2
    1     -m13*(L132(1)*L13DD2(2)-L132(2)*L13DD2(1))/2
    1     -m14*(L14(1)*L14DD(2)-L14(2)*L14DD(1))

C   For Connecting rod (link 2)
     D(1)=-A4
     D(2)=-S4*SIN(ALP4)
     D(3)=-S4*COS(ALP4)
     DD(1)=0.0
     DD(2)=-S4D*SIN(ALP4)
     DD(3)=-S4D*COS(ALP4)
     DDD(1)=0.0
     DDD(2)=-S4DD*SIN(ALP4)
     DDD(3)=-S4DD*COS(ALP4)

C*******Derivative of (TM4R)(TL3R)=TM4L3RD***************
    L3RD(1,1)=-COS(THT3)*SIN(ETA3)*ETA3D
    1       -SIN(THT3)*COS(ETA3)*THT3D
    L3RD(1,3)=-COS(ETA3)*ETA3D
    L3RD(1,2)=COS(THT3)*COS(ETA3)*THT3D
```

```
1       -SIN(THT3)*SIN(ETA3)*ETA3D
 L3RD(2,1)=COS(THT3)*COS(ETA3)*ETA3D
1       -SIN(THT3)*SIN(ETA3)*THT3D
 L3RD(2,3)=-SIN(ETA3)*ETA3D
 L3RD(2,2)=COS(THT3)*SIN(ETA3)*THT3D
1       +SIN(THT3)*COS(ETA3)*ETA3D
 L3RD(3,1)=COS(THT3)*THT3D
 L3RD(3,3)=0.0
 L3RD(3,2)=SIN(THT3)*THT3D
       CALL MULTI (TM4R,L3RD,TM4L3RD,3,3,3)
C********Double derivative of (TM4R)(TL3R)=TM4L3RDD*********
 L3RDD(1,1)=2*SIN(THT3)*SIN(ETA3)*THT3D*ETA3D
1       -COS(THT3)*COS(EAT3)*((THT3D**2)+(ETA3D**2))
1       -SIN(THT3)*COS(ETA3)*THT3DD
1       -COS(THT3)*SIN(ETA3)*ETA3DD
 L3RDD(1,3)=SIN(ETA3)*(ETA3D**2)-COS(ETA3)*ETA3DD
 L3RDD(1,2)=-2*SIN(ETA3)*COS(THT3)*THT3D*ETA3D
1       -SIN(THT3)*COS(EAT3)*((THT3D**2)+(ETA3D**2))
1       -SIN(THT3)*SIN(ETA3)*ETA3DD
1       +COS(THT3)*COS(ETA3)*THT3DD
 L3RDD(2,1)=-2*SIN(THT3)*COS(ETA3)*THT3D*ETA3D
1       -COS(THT3)*SIN(EAT3)*((THT3D**2)+(ETA3D**2))
1       +COS(THT3)*COS(ETA3)*ETA3DD
1       -SIN(THT3)*SIN(ETA3)*THT3DD
 L3RDD(2,3)=-COS(ETA3)*(ETA3D**2)-SIN(ETA3)*ETA3DD
 L3RDD(2,2)=2*COS(THT3)*COS(ETA3)*THT3D*ETA3D
1       -SIN(THT3)*SIN(EAT3)*((THT3D**2)+(ETA3D**2))
1       +COS(THT3)*SIN(ETA3)*THT3DD
1       +SIN(THT3)*COS(ETA3)*ETA3DD
 L3RDD(3,1)=-SIN(THT3)*(THT3D**2)+COS(THT3)*THT3DD
 L3RDD(3,3)=0.0
 L3RDD(3,2)=COS(THT3)*(THT3D**2)+SIN(THT3)*THT3DD

       CALL MULTI (TM4R,L3RDD,TM4L3RDD,3,3,3)

C*****************************************************
C  Point mass m21
 LL1(1)=X20+X2*L2(1,1)
 LL1(2)=Y20+X2*L2(1,2)
 LL1(3)=Z20+X2*L2(1,3)
 LL2(1)=X20-X2*L2(1,1)
 LL2(2)=Y20-X2*L2(1,2)
 LL2(3)=Z20-X2*L2(1,3)
 CALL CORRLS (TM4L3R,TM4L3RD,TM4L3RDD,D,DD,DDD,LL1,
1           L211,L21D1,L21DD1)
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      CALL CORRLS (TM4L3R,TM4L3RD,TM4L3RDD,D,DD,DDD,LL2,
     1            L212,L21D2,L21DD2)
      CALL ADDVEC (L21DD1,L21DD2,L21DD)

C     Point mass m22
      LL1(1)=X20+Y2*L2(2,1)
      LL1(2)=Y20+Y2*L2(2,2)
      LL1(3)=Z20+Y2*L2(2,3)
      LL2(1)=X20-Y2*L2(2,1)
      LL2(2)=Y20-Y2*L2(2,2)
      LL2(3)=Z20-Y2*L2(2,3)
      CALL CORRLS (TM4L3R,TM4L3RD,TM4L3RDD,D,DD,DDD,LL1,
     1            L221,L22D1,L22DD1)
      CALL CORRLS (TM4L3R,TM4L3RD,TM4L3RDD,D,DD,DDD,LL2,
     1            L222,L22D2,L22DD2)
      CALL ADDVEC (L22DD1,L22DD2,L22DD)

C     Point mass m23
      LL1(1)=X20+Z2*L2(3,1)
      LL1(2)=Y20+Z2*L2(3,2)
      LL1(3)=Z20+Z2*L2(3,3)
      LL2(1)=X20-Z2*L2(3,1)
      LL2(2)=Y20-Z2*L2(3,2)
      LL2(3)=Z20-Z2*L2(3,3)
      CALL CORRLS (TM4L3R,TM4L3RD,TM4L3RDD,D,DD,DDD,LL1,
     1            L231,L23D1,L23DD1)
      CALL CORRLS (TM4L3R,TM4L3RD,TM4L3RDD,D,DD,DDD,LL2,
     1            L232,L23D2,L23DD2)
      CALL ADDVEC (L23DD1,L23DD2,L23DD)

C     Point mass m24
      LL(1)=X20
      LL(2)=Y20
      LL(3)=Z20
      CALL CORRLS (TM4L3R,TM4L3RD,TM4L3RDD,D,DD,DDD,LL,
     1            L24,L24D,L24DD)

C     New inertia forces and torques of connecting rod using point masses
      NUR2XP=(-m21*L21DD(1)-m22*L22DD(1)-m23*L23DD(1))/2
     1      -m24*L24DD(1)
C       NUR2XP=-M2*L24DD(1)
      NUR2YP=(-m21*L21DD(2)-m22*L22DD(2)-m23*L23DD(2))/2
     1      -m24*L24DD(2)
C       NUR2YP=-M2*L24DD(2)
      NUR2ZP=(-m21*L21DD(3)-m22*L22DD(3)-m23*L23DD(3))/2
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     1      -m24*L24DD(3)
C     NUR2ZP=-M2*L24DD(3)
      NUR2XD=-m21*(L211(2)*L21DD1(3)-L211(3)*L21DD1(2))/2
     1      -m21*(L212(2)*L21DD2(3)-L212(3)*L21DD2(2))/2
     1      -m22*(L221(2)*L22DD1(3)-L221(3)*L22DD1(2))/2
     1      -m22*(L222(2)*L22DD2(3)-L222(3)*L22DD2(2))/2
     1      -m23*(L231(2)*L23DD1(3)-L231(3)*L23DD1(2))/2
     1      -m23*(L232(2)*L23DD2(3)-L232(3)*L23DD2(2))/2
     1      -m24*(L24(2)*L24DD(3)-L24(3)*L24DD(2))
      NUR2YD=-m21*(L211(3)*L21DD1(1)-L211(1)*L21DD1(3))/2
     1      -m21*(L212(3)*L21DD2(1)-L212(1)*L21DD2(3))/2
     1      -m22*(L221(3)*L22DD1(1)-L221(1)*L22DD1(3))/2
     1      -m22*(L222(3)*L22DD2(1)-L222(1)*L22DD2(3))/2
     1      -m23*(L231(3)*L23DD1(1)-L231(1)*L23DD1(3))/2
     1      -m23*(L232(3)*L23DD2(1)-L232(1)*L23DD2(3))/2
     1      -m24*(L24(3)*L24DD(1)-L24(1)*L24DD(3))
      NUR2ZD=-m21*(L211(1)*L21DD1(2)-L211(2)*L21DD1(1))/2
     1      -m21*(L212(1)*L21DD2(2)-L212(2)*L21DD2(1))/2
     1      -m22*(L221(1)*L22DD1(2)-L221(2)*L22DD1(1))/2
     1      -m22*(L222(1)*L22DD2(2)-L222(2)*L22DD2(1))/2
     1      -m23*(L231(1)*L23DD1(2)-L231(2)*L23DD1(1))/2
     1      -m23*(L232(1)*L23DD2(2)-L232(2)*L23DD2(1))/2
     1      -m24*(L24(1)*L24DD(2)-L24(2)*L24DD(1))

C*****************OPTIMIZATION EQUATIONS*******************
        px(1)=-L11DD(1)
        px(2)=-L12DD(1)
        px(3)=-L13DD(1)
        px(4)=-L14DD(1)
        px(5)=-L21DD(1)
        px(6)=-L22DD(1)
        px(7)=-L23DD(1)
        px(8)=-L24DD(1)
        px(9)=0.0
            py(1)=-L11DD(2)
            py(2)=-L12DD(2)
            py(3)=-L13DD(2)
            py(4)=-L14DD(2)
            py(5)=-L21DD(2)
            py(6)=-L22DD(2)
            py(7)=-L23DD(2)
            py(8)=-L24DD(2)
            py(9)=M3*S4DD*SIN(ALP4)
        pz(1)=-L11DD(3)
        pz(2)=-L12DD(3)
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      pz(3)=-L13DD(3)
      pz(4)=-L14DD(3)
      pz(5)=-L21DD(3)
      pz(6)=-L22DD(3)
      pz(7)=-L23DD(3)
      pz(8)=-L24DD(3)
      pz(9)=M3*S4DD*COS(ALP4)

 qx(1)=-(L111(2)*L11DD1(3)-L111(3)*L11DD1(2))/2
1     -(L112(2)*L11DD2(3)-L112(3)*L11DD2(2))/2
 qx(2)=-(L121(2)*L12DD1(3)-L121(3)*L12DD1(2))/2
1     -(L122(2)*L12DD2(3)-L122(3)*L12DD2(2))/2
 qx(3)=-(L131(2)*L13DD1(3)-L131(3)*L13DD1(2))/2
1     -(L132(2)*L13DD2(3)-L132(3)*L13DD2(2))/2
 qx(4)=-(L14(2)*L14DD(3)-L14(3)*L14DD(2))
 qx(5)=-(L211(2)*L21DD1(3)-L211(3)*L21DD1(2))/2
1     -(L212(2)*L21DD2(3)-L212(3)*L21DD2(2))/2
 qx(6)=-(L221(2)*L22DD1(3)-L221(3)*L22DD1(2))/2
1     -(L222(2)*L22DD2(3)-L222(3)*L22DD2(2))/2
 qx(7)=-(L231(2)*L23DD1(3)-L231(3)*L23DD1(2))/2
1     -(L232(2)*L23DD2(3)-L232(3)*L23DD2(2))/2
 qx(8)=-(L24(2)*L24DD(3)-L24(3)*L24DD(2))
 qx(9)=0.0
      qy(1)=-(L111(3)*L11DD1(1)-L111(1)*L11DD1(3))/2
1           -(L112(3)*L11DD2(1)-L112(1)*L11DD2(3))/2
      qy(2)=-(L121(3)*L12DD1(1)-L121(1)*L12DD1(3))/2
1           -(L122(3)*L12DD2(1)-L122(1)*L12DD2(3))/2
      qy(3)=-(L131(3)*L13DD1(1)-L131(1)*L13DD1(3))/2
1           -(L132(3)*L13DD2(1)-L132(1)*L13DD2(3))/2
      qy(4)=-(L14(3)*L14DD(1)-L14(1)*L14DD(3))
      qy(5)=-(L211(3)*L21DD1(1)-L211(1)*L21DD1(3))/2
1           -(L212(3)*L21DD2(1)-L212(1)*L21DD2(3))/2
      qy(6)=-(L221(3)*L22DD1(1)-L221(1)*L22DD1(3))/2
1           -(L222(3)*L22DD2(1)-L222(1)*L22DD2(3))/2
      qy(7)=-(L231(3)*L23DD1(1)-L231(1)*L23DD1(3))/2
1           -(L232(3)*L23DD2(1)-L232(1)*L23DD2(3))/2
      qy(8)=-(L24(3)*L24DD(1)-L24(1)*L24DD(3))
      qy(9)=A4*fz(9)
 qz(1)=-(L111(1)*L11DD1(2)-L111(2)*L11DD1(1))/2
1     -(L112(1)*L11DD2(2)-L112(2)*L11DD2(1))/2
 qz(2)=-(L121(1)*L12DD1(2)-L121(2)*L12DD1(1))/2
1     -(L122(1)*L12DD2(2)-L122(2)*L12DD2(1))/2
 qz(3)=-(L131(1)*L13DD1(2)-L131(2)*L13DD1(1))/2
1     -(L132(1)*L13DD2(2)-L132(2)*L13DD2(1))/2
 qz(4)=-(L14(1)*L14DD(2)-L14(2)*L14DD(1))
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 qz(5)=-(L211(1)*L21DD1(2)-L211(2)*L21DD1(1))/2
1     -(L212(1)*L21DD2(2)-L212(2)*L21DD2(1))/2
 qz(6)=-(L221(1)*L22DD1(2)-L221(2)*L22DD1(1))/2
1     -(L222(1)*L22DD2(2)-L222(2)*L22DD2(1))/2
 qz(7)=-(L231(1)*L23DD1(2)-L231(2)*L23DD1(1))/2
1     -(L232(1)*L23DD2(2)-L232(2)*L23DD2(1))/2
 qz(8)=-(L24(1)*L24DD(2)-L24(2)*L24DD(1))
 qz(9)=A4*fy(9)

     DO 30 K=1,4
     fx(K)=TM1R(1,1)*px(K)+TM1R(1,2)*py(K)+TM1R(1,3)*pz(K)
     fy(K)=TM1R(2,1)*px(K)+TM1R(2,2)*py(K)+TM1R(2,3)*pz(K)
     fz(K)=TM1R(3,1)*px(K)+TM1R(3,2)*py(K)+TM1R(3,3)*pz(K)
     tx(K)=TM1R(1,1)*qx(K)+TM1R(1,2)*qy(K)+TM1R(1,3)*qz(K)
     ty(K)=TM1R(2,1)*qx(K)+TM1R(2,2)*qy(K)+TM1R(2,3)*qz(K)
     tz(K)=TM1R(3,1)*qx(K)+TM1R(3,2)*qy(K)+TM1R(3,3)*qz(K)
30    CONTINUE
  DO 40 K1=5,8
  fx(K1)=TM1L2R(1,1)*px(K1)+TM1L2R(1,2)*py(K1)+TM1L2R(1,3)*pz(K1)
  fy(K1)=TM1L2R(2,1)*px(K1)+TM1L2R(2,2)*py(K1)+TM1L2R(2,3)*pz(K1)
  fz(K1)=TM1L2R(3,1)*px(K1)+TM1L2R(3,2)*py(K1)+TM1L2R(3,3)*pz(K1)
  tx(K1)=TM1L2R(1,1)*qx(K1)+TM1L2R(1,2)*qy(K1)+TM1L2R(1,3)*qz(K1)
  ty(K1)=TM1L2R(2,1)*qx(K1)+TM1L2R(2,2)*qy(K1)+TM1L2R(2,3)*qz(K1)
  tz(K1)=TM1L2R(3,1)*qx(K1)+TM1L2R(3,2)*qy(K1)+TM1L2R(3,3)*qz(K1)
40 CONTINUE
      fx(9)=M4R(1,1)*px(9)+M4R(1,2)*py(9)+M4R(1,3)*pz(9)
      fy(9)=M4R(2,1)*px(9)+M4R(2,2)*py(9)+M4R(2,3)*pz(9)
      fz(9)=M4R(3,1)*px(9)+M4R(3,2)*py(9)+M4R(3,3)*pz(9)
      tx(9)=M4R(1,1)*qx(9)+M4R(1,2)*qy(9)+M4R(1,3)*qz(9)
      ty(9)=M4R(2,1)*qx(9)+M4R(2,2)*qy(9)+M4R(2,3)*qz(9)
      tz(9)=M4R(3,1)*qx(9)+M4R(3,2)*qy(9)+M4R(3,3)*qz(9)

  f3y(5)=-(fy(5)+tz(5)/A2)
  f3y(6)=-(fy(6)+tz(6)/A2)
  f3y(7)=-(fy(7)+tz(7)/A2)
  f3y(8)=-(fy(8)+tz(8)/A2)
        f3z(5)=-(fz(5)+ty(5)/A2)
        f3z(6)=-(fz(6)+ty(6)/A2)
        f3z(7)=-(fz(7)+ty(7)/A2)
        f3z(8)=-(fz(8)+ty(8)/A2)
  f3x(5)=cos(THT3)*f3y(5)/sin(THT3)
  f3x(6)=cos(THT3)*f3y(6)/sin(THT3)
  f3x(7)=cos(THT3)*f3y(7)/sin(THT3)
  f3x(7)=cos(THT3)*f3y(7)/sin(THT3)
  f3x(9)=fz(9)/sin(THT3)
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 f4x(5)=(sin(ETA3)*f3z(5)-cos(THT3)*cos(ETA3)*f3x(5)
1     -sin(THT3)*cos(ETA3)*f3y(5))
 f4x(6)=(sin(ETA3)*f3z(6)-cos(THT3)*cos(ETA3)*f3x(6)
1     -sin(THT3)*cos(ETA3)*f3y(6))
 f4x(7)=(sin(ETA3)*f3z(7)-cos(THT3)*cos(ETA3)*f3x(7)
1     -sin(THT3)*cos(ETA3)*f3y(7))
 f4x(8)=(sin(ETA3)*f3z(8)-cos(THT3)*cos(ETA3)*f3x(8)
1     -sin(THT3)*cos(ETA3)*f3y(8))
 f4x(9)=(-cos(THT3)*cos(ETA3)*f3x(9)-fx(9))
      f4y(5)=-(cos(ETA3)*f3z(5)+cos(THT3)*sin(ETA3)*f3x(5)
1          +sin(THT3)*sin(ETA3)*f3y(5))
      f4y(6)=-(cos(ETA3)*f3z(6)+cos(THT3)*sin(ETA3)*f3x(6)
1          +sin(THT3)*sin(ETA3)*f3y(6))
      f4y(7)=-(cos(ETA3)*f3z(7)+cos(THT3)*sin(ETA3)*f3x(7)
1          +sin(THT3)*sin(ETA3)*f3y(7))
      f4y(8)=-(cos(ETA3)*f3z(8)+cos(THT3)*sin(ETA3)*f3x(8)
1          +sin(THT3)*sin(ETA3)*f3y(8))
      f4y(9)=(cos(THT3)*sin(ETA3)*f3x(9)+fy(9))
  t4x(9)=T4I
  t4y(9)=T4J
  t4z(9)=T4K

 f2x(5)=((-f3x(5)-fx(5))*cos(THT2)*cos(ETA2)+(f3y(5)+fy(5))
1       *sin(THT2)-ty(5)*sin(ETA2)*cos(THT2)/A2)
 f2x(6)=((-f3x(6)-fx(6))*cos(THT2)*cos(ETA2)+(f3y(6)+fy(6))
1       *sin(THT2)-ty(6)*sin(ETA2)*cos(THT2)/A2)
 f2x(7)=((-f3x(7)-fx(7))*cos(THT2)*cos(ETA2)+(f3y(7)+fy(7))
1       *sin(THT2)-ty(7)*sin(ETA2)*cos(THT2)/A2)
 f2x(8)=((-f3x(8)-fx(8))*cos(THT2)*cos(ETA2)+(f3y(8)+fy(8))
1       *sin(THT2)-ty(8)*sin(ETA2)*cos(THT2)/A2)
 f2x(9)=-f3x(9)*cos(THT2)*cos(ETA2)
  f2y(5)=((-f3x(5)-fx(5))*sin(THT2)*cos(ETA2)-(f3y(5)+fy(5))
1       *cos(THT2)-ty(5)*sin(ETA2)*sin(THT2)/A2)
  f2y(6)=((-f3x(6)-fx(6))*sin(THT2)*cos(ETA2)-(f3y(6)+fy(6))
1       *cos(THT2)-ty(6)*sin(ETA2)*sin(THT2)/A2)
  f2y(7)=((-f3x(7)-fx(7))*sin(THT2)*cos(ETA2)-(f3y(7)+fy(7))
1       *cos(THT2)-ty(5)*sin(ETA2)*sin(THT2)/A2)
  f2y(8)=((-f3x(8)-fx(8))*sin(THT2)*cos(ETA2)-(f3y(8)+fy(8))
1       *cos(THT2)-ty(8)*sin(ETA2)*sin(THT2)/A2)
  f2y(9)=-f3x(9)*sin(THT2)*cos(ETA2)
   if ( ABS(COS(ETA)) .GT. ABS(SIN(ETA)) ) then
 f2z(5)=-(fz(5)+f3z(5)+f2y(5)*sin(THT2)*sin(ETA2)+
1       f2x(5)*cos(THT2)*sin(ETA2))/cos(ETA2)
 f2z(6)=-(fz(6)+f3z(6)+f2y(6)*sin(THT2)*sin(ETA2)+
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1       f2x(6)*cos(THT2)*sin(ETA2))/cos(ETA2)
 f2z(7)=-(fz(7)+f3z(7)+f2y(7)*sin(THT2)*sin(ETA2)+
1       f2x(7)*cos(THT2)*sin(ETA2))/cos(ETA2)
 f2z(8)=-(fz(8)+f3z(8)+f2y(8)*sin(THT2)*sin(ETA2)+
1       f2x(8)*cos(THT2)*sin(ETA2))/cos(ETA2)
 f2z(9)=-(f2y(9)*sin(THT2)*sin(ETA2)+
1       f2x(9)*cos(THT2)*sin(ETA2))/cos(ETA2)
   else
 f2z(5)=(fz(5)+f3z(5)+f2y(5)*sin(THT2)*cos(ETA2)+
1       f2x(5)*cos(THT2)*cos(ETA2))/sin(ETA2)
 f2z(6)=(fz(6)+f3z(6)+f2y(6)*sin(THT2)*cos(ETA2)+
1       f2x(6)*cos(THT2)*cos(ETA2))/sin(ETA2)
 f2z(7)=(fz(7)+f3z(7)+f2y(7)*sin(THT2)*cos(ETA2)+
1       f2x(7)*cos(THT2)*cos(ETA2))/sin(ETA2)
 f2z(8)=(fz(8)+f3z(8)+f2y(8)*sin(THT2)*cos(ETA2)+
1       f2x(8)*cos(THT2)*cos(ETA2))/sin(ETA2)
 f2z(9)=(f2y(9)*sin(THT2)*cos(ETA2)+
1       f2x(9)*cos(THT2)*cos(ETA2))/sin(ETA2)
   endif

  DO 50 K2=5,8
  f3x(K2)=M1L2R(1,1)*f3x(K2)+M1L2R(1,2)*f3y(K2)+M1L2R(1,3)*f3z(K2)
  f3y(K2)=M1L2R(2,1)*f3x(K2)+M1L2R(2,2)*f3y(K2)+M1L2R(2,3)*f3z(K2)
  f3z(K2)=M1L2R(3,1)*f3x(K2)+M1L2R(3,2)*f3y(K2)+M1L2R(3,3)*f3z(K2)
50 CONTINUE
     DO 60 K3=5,8
     f2x(K3)=M1R(1,1)*f2x(K3)+M1R(1,2)*f2y(K3)+M1R(1,3)*f2z(K3)
     f2y(K3)=M1R(2,1)*f2x(K3)+M1R(2,2)*f2y(K3)+M1R(2,3)*f2z(K3)
     f2z(K3)=M1R(3,1)*f2x(K3)+M1R(3,2)*f2y(K3)+M1R(3,3)*f2z(K3)
60    CONTINUE
  DO 70 K4=5,8
  f4x(K4)=TM4R(1,1)*f4x(K4)+TM4R(1,2)*f4y(K4)
  f4y(K4)=TM4R(2,1)*f4x(K4)+TM4R(2,2)*f4y(K4)
  f4z(K4)=TM4R(3,1)*f4x(K4)+TM4R(3,2)*f4y(K4)
70 CONTINUE
  t4x(9)=TM4R(1,1)*t4x(9)+TM4R(1,2)*t4y(9)+TM4R(1,3)*t4z(9)
  t4y(9)=TM4R(2,1)*t4x(9)+TM4R(2,2)*t4y(9)+TM4R(2,3)*t4z(9)
  t4z(9)=TM4R(3,1)*t4x(9)+TM4R(3,2)*t4y(9)+TM4R(3,3)*t4z(9)

  f1z(1)=-fz(1)
  f1z(2)=-fz(2)
  f1z(3)=-fz(3)
  f1z(4)=-fz(4)
  f1z(5)=-f2z(5)
  f1z(6)=-f2z(6)
```

```
f1z(7)=-f2z(7)
f1z(8)=-f2z(8)
f1z(9)=-f2z(9)
      f1x(1)=(-fx(1)*cos(THT1)+fy(1)*sin(THT1))
      f1x(2)=(-fx(2)*cos(THT1)+fy(2)*sin(THT1))
      f1x(3)=(-fx(3)*cos(THT1)+fy(3)*sin(THT1))
      f1x(4)=(-fx(4)*cos(THT1)+fy(4)*sin(THT1))
      f1x(5)=(-f2x(5)*cos(THT1)+f2y(5)*sin(THT1))
      f1x(6)=(-f2x(6)*cos(THT1)+f2y(6)*sin(THT1))
      f1x(7)=(-f2x(7)*cos(THT1)+f2y(7)*sin(THT1))
      f1x(8)=(-f2x(8)*cos(THT1)+f2y(8)*sin(THT1))
      f1x(9)=(-f2x(9)*cos(THT1)+f2y(9)*sin(THT1))
f1y(1)=-(fx(1)*sin(THT1)+fy(1)*cos(THT1))
f1y(2)=-(fx(2)*sin(THT1)+fy(2)*cos(THT1))
f1y(3)=-(fx(3)*sin(THT1)+fy(3)*cos(THT1))
f1y(4)=-(fx(4)*sin(THT1)+fy(4)*cos(THT1))
f1y(5)=-(f2x(5)*sin(THT1)+f2y(5)*cos(THT1))
f1y(6)=-(f2x(6)*sin(THT1)+f2y(6)*cos(THT1))
f1y(7)=-(f2x(7)*sin(THT1)+f2y(7)*cos(THT1))
f1y(8)=-(f2x(8)*sin(THT1)+f2y(8)*cos(THT1))
f1y(9)=-(f2x(9)*sin(THT1)+f2y(9)*cos(THT1))

t1z(1)=(f1y(1)*A1*cos(THT1)-f1x(1)*A1*sin(THT1)-tz(1))
t1z(2)=(f1y(2)*A1*cos(THT1)-f1x(2)*A1*sin(THT1)-tz(2))
t1z(3)=(f1y(3)*A1*cos(THT1)-f1x(3)*A1*sin(THT1)-tz(3))
t1z(4)=(f1y(4)*A1*cos(THT1)-f1x(4)*A1*sin(THT1)-tz(4))
t1z(5)=(f1y(5)*A1*cos(THT1)-f1x(5)*A1*sin(THT1))
t1z(6)=(f1y(6)*A1*cos(THT1)-f1x(6)*A1*sin(THT1))
t1z(7)=(f1y(7)*A1*cos(THT1)-f1x(7)*A1*sin(THT1))
t1z(8)=(f1y(8)*A1*cos(THT1)-f1x(8)*A1*sin(THT1))
t1z(9)=(f1y(9)*A1*cos(THT1)-f1x(9)*A1*sin(THT1))
      t1x(1)=(-f1y(1)*S1-tx(1)*cos(THT1)+f1z(1)*A1+
1           ty(1)*sin(THT1))
      t1x(2)=(-f1y(2)*S1-tx(2)*cos(THT1)+f1z(2)*A1+
1           ty(2)*sin(THT1))
      t1x(3)=(-f1y(3)*S1-tx(3)*cos(THT1)+f1z(3)*A1+
1           ty(3)*sin(THT1))
      t1x(4)=(-f1y(4)*S1-tx(4)*cos(THT1)+f1z(4)*A1+
1           ty(4)*sin(THT1))
      t1x(5)=t1z(5)*A1-f1y(5)*S1
      t1x(6)=t1z(6)*A1-f1y(6)*S1
      t1x(7)=t1z(7)*A1-f1y(7)*S1
      t1x(8)=t1z(8)*A1-f1y(8)*S1
      t1x(9)=t1z(9)*A1-f1y(9)*S1
t1y(1)=f1x(1)*S1-f1z(1)*A1-tx(1)*sin(THT1)-ty(1)*cos(THT1)
```

```
t1y(2)=f1x(2)*S1-f1z(2)*A1-tx(2)*sin(THT1)-ty(2)*cos(THT1)
t1y(3)=f1x(3)*S1-f1z(3)*A1-tx(3)*sin(THT1)-ty(3)*cos(THT1)
t1y(4)=f1x(4)*S1-f1z(4)*A1-tx(4)*sin(THT1)-ty(4)*cos(THT1)
t1y(5)=f1x(5)*S1-f1z(5)*A1
t1y(6)=f1x(6)*S1-f1z(6)*A1
t1y(7)=f1x(7)*S1-f1z(7)*A1
t1y(8)=f1x(8)*S1-f1z(8)*A1
t1y(9)=f1x(9)*S1-f1z(9)*A1

    CALL SQUARE (fx,fxsq)
    CALL SQUARE (fy,fysq)
    CALL SQUARE (fz,fzsq)
    CALL SQUARE (tx,txsq)
    CALL SQUARE (ty,tysq)
    CALL SQUARE (tz,tzsq)
    CALL SQUARE (f1x,f1xsq)
    CALL SQUARE (f1y,f1ysq)
    CALL SQUARE (f1z,f1zsq)
    CALL SQUARE (t1x,t1xsq)
    CALL SQUARE (t1y,t1ysq)
    CALL SQUARE (t1z,t1zsq)
    CALL SQUARE (f2x,f2xsq)
    CALL SQUARE (f2y,f2ysq)
    CALL SQUARE (f2z,f2zsq)
    CALL SQUARE (f3x,f3xsq)
    CALL SQUARE (f3y,f3ysq)
    CALL SQUARE (f3z,f3zsq)
    CALL SQUARE (f4x,f4xsq)
    CALL SQUARE (f4y,f4ysq)
    CALL SQUARE (t4x,t4xsq)
    CALL SQUARE (t4y,t4ysq)
    CALL SQUARE (t4z,t4zsq)

      C1=37*(M1**2)*(A1**2)*(THT1D**4)
      C2=C1*(A1**2)
      C3=C1*(BL**2)
        DO 80 I1=1,9
        DO 90 J1=1,9
fsq(I1,J1)=(fxsq(I1,J1)+fysq(I1,J1)+fzsq(I1,J1))/C1
tsq(I1,J1)=(txsq(I1,J1)+tysq(I1,J1)+tzsq(I1,J1))/C2
rsq(I1,J1)=(f1xsq(I1,J1)+f1ysq(I1,J1)+f1zsq(I1,J1)
1       +f2xsq(I1,J1)+f2ysq(I1,J1)+f2zsq(I1,J1)
1       +f3xsq(I1,J1)+f3ysq(I1,J1)+f3zsq(I1,J1)
1       +f4xsq(I1,J1)+f4ysq(I1,J1)+f4zsq(I1,J1))/C1
msq(I1,J1)=(t1xsq(I1,J1)+t1ysq(I1,J1)
```

```
     1      +t4xsq(I1,J1)+t4ysq(I1,J1)+t4zsq(I1,J1))/C3
      t0sq(I1,J1)=t1zsq(I1,J1)/C2
      F(I1,J1)=W1*fsq(I1,J1)+W2*tsq(I1,J1)+W3*rsq(I1,J1)
     1     +W4*msq(I1,J1)+W5*t0sq(I1,J1)
      OF(I1,J1)=OF(I1,J1)+F(I1,J1)
      h(I1,J1)=2*OF(I1,J1)
   90 CONTINUE
   80 CONTINUE

C          write (6,1) F(1,1)
C          write (6,1) OF(6,8),OF(8,6),OF(9,9)
C         write (6,1) fx(1),fx(2),fx(3)
C         write (6,1) L11DD(1),L12DD(1),L13DD(1)
C         write (6,1) fx(4),fx(5),fx(6)
C         write (6,1) fx(7),fx(8),fx(9)
C         write (6,1) fxsq(1,1),fxsq(1,2),fxsq(1,3)
C         write (6,1) fxsq(1,4),fxsq(1,5),fxsq(1,6)
C         write (6,1) tsq(1,7),tsq(1,8),tsq(1,9)
C         write (6,1) NUR2XP,NUR2YP,NUR2ZP
C         write (6,1) M1L2R2P(1),M1L2R2P(2),M1L2R2P(3)
        ELSE
        THT2=THT2+DELTA(1)
        ETA2=ETA2+DELTA(2)
        THT3=THT3+DELTA(3)
        ETA3=ETA3+DELTA(4)
        S4=S4+DELTA(5)
        I=I+1
        GO TO 10
        ENDIF
        ELSE
        WRITE(6,2) I
        STOP
        ENDIF
C  1 FORMAT('SLIDER VELOCITY=',F10.5,2X,'WHEN CRANK ANGLE='
C    1,F7.2,1X,'S1=',F7.2,1X,'NO OF ITERATIONS=',I3)
   1 FORMAT(3F20.5)
   2 FORMAT ('THE METHOD FAILS AFTER',I3,'ITERATIONS')

C    Determination of the objective function
        OBJFUN=0.0
        NUOBJFUN=0.0
      if (I3 .EQ. 2) then
        do 42 I4=1,9
        do 43 J4=1,9
           if (I4 .NE. J4) then
```

```
          F(I4,J4)=2.0*F(I4,J4)
          endif
        OBJFUN=OBJFUN+m(I4)*m(J4)*F(I4,J4)
        NUOBJFUN=NUOBJFUN+sol(I4)*sol(J4)*F(I4,J4)
 43     continue
 42     continue

        call dotprod(m,fx,mfx)
C         call dotprod(sol,fx,nufx)
        call dotprod(m,fy,mfy)
C         call dotprod(sol,fy,nufy)
        call dotprod(m,fz,mfz)
C         call dotprod(sol,fz,nufz)
        call dotprod(m,tx,mtx)
C         call dotprod(sol,tx,nutx)
        call dotprod(m,ty,mty)
C         call dotprod(sol,ty,nuty)
        call dotprod(m,tz,mtz)
C         call dotprod(sol,tz,nutz)
        call dotprod(m,f1x,mf1x)
C         call dotprod(sol,f1x,nuf1x)
        call dotprod(m,f1y,mf1y)
C         call dotprod(sol,f1y,nuf1y)
        call dotprod(m,f1z,mf1z)
C         call dotprod(sol,f1z,nuf1z)
        call dotprod(m,t1x,mt1x)
C         call dotprod(sol,t1x,nut1x)
        call dotprod(m,t1y,mt1y)
C         call dotprod(sol,t1y,nut1y)
        call dotprod(m,t1z,mt1z)
C         call dotprod(sol,t1z,nut1z)
        call dotprod(m,f2x,mf2x)
C         call dotprod(sol,f2x,nuf2x)
        call dotprod(m,f2y,mf2y)
C         call dotprod(sol,f2y,nuf2y)
        call dotprod(m,f2z,mf2z)
C         call dotprod(sol,f2z,nuf2z)
        call dotprod(m,f3x,mf3x)
C         call dotprod(sol,f3x,nuf3x)
        call dotprod(m,f3y,mf3y)
C         call dotprod(sol,f3y,nuf3y)
        call dotprod(m,f3z,mf3z)
C         call dotprod(sol,f3z,nuf3z)
        call dotprod(m,f4x,mf4x)
C         call dotprod(sol,f4x,nuf4x)
```

```
      call dotprod(m,f4y,mf4y)
C       call dotprod(sol,f4y,nuf4y)
      call dotprod(m,t4x,mt4x)
C       call dotprod(sol,t4x,nut4x)
      call dotprod(m,t4y,mt4y)
C       call dotprod(sol,t4y,nut4y)
      call dotprod(m,t4z,mt4z)
C       call dotprod(sol,t4z,nut4z)

      shf=(mfx**2+mfy**2+mfz**2)**0.5
      sht=(mtx**2+mty**2+mtz**2)**0.5
        brf=((mf1x+mf2x+mf3x+mf4x)**2
     1       +(mf1y+mf2y+mf3y+mf4y)**2
     1       +(mf1z+mf2z+mf3z)**2)**0.5
        brt=((mt1x+mt4x)**2
     1       +(mt1y+mt4y)**2
     1       +(mt1z+mt4z)**2)**0.5
      int=mt1z

C      write (6,*) OBJFUN,NUOBJFUN
C     write (6,*) mfx,nufx
C     write (6,*) mfy,nufy
C     write (6,*) mfz,nufz
C      write (6,*) f1x(3),f1x(1),m(3)*fx(3),sol(3)*fx(3)

      nufx=0.0
      nufy=0.0
      nufz=0.0
      nutx=0.0
      nuty=0.0
      nutz=0.0
        nuf1x=0.0
        nuf1y=0.0
        nuf1z=0.0
        nut1x=0.0
        nut1y=0.0
        nut1z=0.0
          nuf2x=0.0
          nuf2y=0.0
          nuf2z=0.0
            nuf3x=0.0
            nuf3y=0.0
            nuf3z=0.0
              nuf4x=0.0
              nuf4y=0.0
```

```
                nut4x=0.0
                nut4y=0.0
                nut4z=0.0

      do 233 i10=1,9
         nufx=nufx+sol(i10)*fx(i10)
         nufy=nufy+sol(i10)*fy(i10)
         nufz=nufz+sol(i10)*fz(i10)
         nutx=nutx+sol(i10)*tx(i10)
         nuty=nuty+sol(i10)*ty(i10)
         nutz=nutz+sol(i10)*tz(i10)
           nuf1x=nuf1x+sol(i10)*f1x(i10)
           nuf1y=nuf1y+sol(i10)*f1y(i10)
           nuf1z=nuf1z+sol(i10)*f1z(i10)
           nut1x=nut1x+sol(i10)*t1x(i10)
           nut1y=nut1y+sol(i10)*t1y(i10)
           nut1z=nut1z+sol(i10)*t1z(i10)
          nuf2x=nuf2x+sol(i10)*f2x(i10)
          nuf2y=nuf2y+sol(i10)*f2y(i10)
          nuf2z=nuf2z+sol(i10)*f2z(i10)
            nuf3x=nuf3x+sol(i10)*f3x(i10)
            nuf3y=nuf3y+sol(i10)*f3y(i10)
            nuf3z=nuf3z+sol(i10)*f3z(i10)
          nuf4x=nuf4x+sol(i10)*f4x(i10)
          nuf4y=nuf4y+sol(i10)*f4y(i10)
            nut4x=nut4x+sol(i10)*t4x(i10)
            nut4y=nut4y+sol(i10)*t4y(i10)
            nut4z=nut4z+sol(i10)*t4z(i10)
233      continue

      nushf=((nufx**2)+(nufy**2)+(nufz**2))**0.5
      nusht=(nutx**2+nuty**2+nutz**2)**0.5
          nubrf=((nuf1x+nuf2x+nuf3x+nuf4x)**2
     1             +(nuf1y+nuf2y+nuf3y+nuf4y)**2
     1             +(nuf1z+nuf2z+nuf3z)**2)**0.5
          nubrt=((nut1x+nut4x)**2
     1           +(nut1y+nut4y)**2
     1           +(nut1z+nut4z)**2)**0.5
       nuint=nut1z

        DOBJFUN=(OBJFUN-NUOBJFUN)*100/OBJFUN
        dshf=(shf-nushf)*100/shf
        dsht=(sht-nusht)*100/sht
        dbrf=(brf-nubrf)*100/brf
        dbrt=(brt-nubrt)*100/brt
```

```
      dint=(int-nuint)*100/int

      write(6,*) TH1,dint
     endif
 400 CONTINUE

C       write (6,1) OF(6,8),OF(8,6),OF(9,9)
      call qprog (nvar,ncon,neq,aa,lda,b,gg,h,ldh,diag,sol,
   1            nact,iact,alamda)
      call umach (2,nout)
C       write (nout,999) (sol(k),k=1,nvar)
C 999   format ('the solution vector is',/,'sol=(',9F12.6,')')
C       write(6,*) m(1),m(2),m(3),m(4),m(5),m(6),m(7),m(8),m(9)
  41      continue

           NUI1X=(Y1**2)*sol(2)+(Z1**2)*sol(3)
           NUI2X=(Y2**2)*sol(6)+(Z2**2)*sol(7)
           NUI1Y=(X1**2)*sol(1)+(Z1**2)*sol(3)
           NUI2Y=(X2**2)*sol(5)+(Z2**2)*sol(7)
           NUI1Z=(X1**2)*sol(1)+(Y1**2)*sol(2)
           NUI2Z=(X2**2)*sol(5)+(Y2**2)*sol(6)
C         write(6,*) I1X,NUI1X
C         write(6,*) I1Y,NUI1Y
C         write(6,*) I1Z,NUI1Z
C         write(6,*) I2X,NUI2X
C         write(6,*) I2Y,NUI2Y
C         write(6,*) I2z,NUI2Z


     STOP
       END

C********************SUBROUTINES START HERE*****************

     SUBROUTINE PRODUCTDUAL (AR,AD,BR,BD,RESLTR,RESLTD)
     real*8 AR(3,3),AD(3,3),BR(3,3),BD(3,3),TEMP1(3,3)
     real*8 RESLTR(3,3),RESLTD(3,3),TEMP2(3,3)
        CALL MULTI (AR,BR,RESLTR,3,3,3)
        CALL MULTI (AD,BR,TEMP1,3,3,3)
        CALL MULTI (AR,BD,TEMP2,3,3,3)
        CALL ADDMAT (TEMP1,TEMP2,RESLTD)
     RETURN
     END
 C
     SUBROUTINE CORRLS (MTRX,MTRXD,MTRXDD,D,DD,DDD,
```

```
 1                 LL,L,LD,LDD)
 real*8 MTRX(3,3),MTRXD(3,3),MTRXDD(3,3),D(3),DD(3),
 1      DDD(3),LL(3),L(3),LD(3),LDD(3),
 1      TMTRX(3,3),TMTRXD(3,3),OMEGA(3,3),
 1      ALPHA1(3,3),ALPHA2(3,3),ALPHA(3,3),
 1      WX,WY,WZ,ALPX,ALPY,ALPZ,MTRXL(3),MTRXDL(3)
 CALL KINEMATIC (MTRX,MTRXD,MTRXDD,D,DD,DDD,
 1                 LL,L,LD,LDD)
 CALL MULMAVEC (MTRX,LL,MTRXL,3,3)
 CALL MULMAVEC (MTRXD,LL,MTRXDL,3,3)

 CALL TRANS (MTRX,TMTRX,3,3)
 CALL TRANS (MTRXD,TMTRXD,3,3)
 CALL MULTI (MTRXD,TMTRX,OMEGA,3,3,3)
 CALL MULTI (MTRXDD,TMTRX,ALPHA1,3,3,3)
 CALL MULTI (MTRXD,TMTRXD,ALPHA2,3,3,3)
 CALL ADDMAT (ALPHA1,ALPHA2,ALPHA)

           WX=OMEGA(3,2)
           WY=OMEGA(1,3)
           WZ=OMEGA(2,1)
          ALPX=ALPHA(3,2)
          ALPY=ALPHA(1,3)
          ALPZ=ALPHA(2,1)

 LDD(1)=WY*(WX*MTRXL(2)-WY*MTRXL(1))
 1      -WZ*(WZ*MTRXL(1)-WX*MTRXL(3))
C  1      +2*WY*MTRXDL(3)-2*WZ*MTRXDL(2)
 1      +ALPY*MTRXL(3)-ALPZ*MTRXL(2)
 1      +DDD(1)
 LDD(2)=WZ*(WY*MTRXL(3)-WZ*MTRXL(2))
 1      -WX*(WX*MTRXL(2)-WY*MTRXL(1))
C  1      +2*WZ*MTRXDL(1)-2*WX*MTRXDL(3)
 1      +ALPZ*MTRXL(1)-ALPX*MTRXL(3)
 1      +DDD(2)
 LDD(3)=WX*(WZ*MTRXL(1)-WX*MTRXL(3))
 1      -WY*(WY*MTRXL(3)-WZ*MTRXL(2))
C  1      +2*WX*MTRXDL(2)-2*WY*MTRXDL(1)
 1      +ALPX*MTRXL(2)-ALPY*MTRXL(1)
 1      +DDD(3)

 RETURN
 END
C
 SUBROUTINE KINEMATIC (MTRX,MTRXD,MTRXDD,D,DD,DDD,
```

```
     1                  LL,L,LD,LDD)
      real*8 MTRX(3,3),MTRXD(3,3),MTRXDD(3,3),D(3),DD(3),
     1      DDD(3),LL(3),L(3),LD(3),LDD(3),
     1      MTRXL(3),MTRXDL(3),MTRXDDL(3)
         CALL MULMAVEC (MTRX,LL,MTRXL,3,3)
         CALL ADDVEC (D,MTRXL,L)
         CALL MULMAVEC (MTRXD,LL,MTRXDL,3,3)
         CALL ADDVEC (DD,MTRXDL,LD)
         CALL MULMAVEC(MTRXDD,LL,MTRXDDL,3,3)
         CALL ADDVEC (DDD,MTRXDDL,LDD)
      RETURN
      END

      SUBROUTINE SQUARE (A,ASQ)
      real*8 A(9),ASQ(9,9)
      DO 10 I=1,9
          DO 20 J=1,9
          ASQ(I,J)=A(I)*A(J)
C      IF (I .NE. J) THEN
C      ASQ(I,J)=2*ASQ(I,J)
C      ENDIF
   20     CONTINUE
   10 CONTINUE
      RETURN
      END
C
      SUBROUTINE ADDMAT (A,B,C)
      real*8 A(3,3),B(3,3),C(3,3)
       DO 20 I=1,3
         DO 10 J=1,3
          C(I,J)=A(I,J)+B(I,J)
   10     CONTINUE
   20   CONTINUE
      RETURN
      END
C
      SUBROUTINE ADDVEC (A,B,C)
      REAL*8 A(3),B(3),C(3)
       DO 10 I=1,3
       C(I)=A(I)+B(I)
   10 CONTINUE
      RETURN
      END
C
      SUBROUTINE TRANSPOSE (AR,AD,TANSAR,TANSAD)
```

```
      real*8 AR(3,3),AD(3,3),TANSAR(3,3),TANSAD(3,3)
      CALL TRANS (AR,TANSAR,3,3)
      CALL TRANS (AD,TANSAD,3,3)
      RETURN
      END
C
      SUBROUTINE TRANS (A,AT,m,n)
      real*8 A(m,n),AT(n,m)
       DO 1, I=1,m
       DO 1, J=1,n
  1      AT(J,I)=A(I,J)
      RETURN
      END
C
      SUBROUTINE MULTI (A,B,C,l,m,n)
      real*8 A(l,n),B(n,m),C(l,m)
       DO 1, I=1,l
        DO 1, J=1,m
          C(I,J)=0.0
         DO 1, K=1,n
  1        C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
C
      SUBROUTINE MULMAVEC (A,B,C,l,m)
      real*8 A(l,m),B(m),C(l)
        DO 1, I=1,l
          C(I)=0.0
        DO 1, J=1,m
  1       C(I)=C(I)+A(I,J)*B(J)
      RETURN
      END

      subroutine dotprod(a,b,c)
      real*8 a(9),b(9),c
           c=0.0
           do 10 i=1,9
           c=c+a(i)*b(i)
  10  continue
      return
      end
C
      SUBROUTINE INVERSE (S,B)
      REAL*8 S(5,5),B(5,5),C(5,10)
      REAL*8 TEMP
```

```
C   SPECIFICATION OF C MATRIX(AUGMENTED ATA MATRIX)
      DO 10 L=1,5
       DO 20 J=1,5
        C(L,J)=S(L,J)
   20 CONTINUE
   10  CONTINUE
      DO 11 I=1,5
      C(I,I+5)=1.0
   11 CONTINUE
C   TO CHECK WHETHER PIVOT C(1,1) IS NONZERO
       IF (C(1,1).EQ.0.00000000)THEN
        DO 30 J=1,10
        TEMP=C(1,J)
        C(1,J)=C(2,J)
        C(2,J)=TEMP
   30 CONTINUE
      ELSE
      DO 61 I=2,5
      DO 71 J=2,5
      C(I,J)=C(I,J)-(C(I,1)*C(1,J)/C(1,1))
   71 CONTINUE
   61 CONTINUE
      GO TO 75
      END IF
      IF (C(1,1).EQ.0.00000000)THEN
      DO 31 J=1,10
      TEMP=C(1,J)
      C(1,J)=C(3,J)
      C(3,J)=TEMP
   31 CONTINUE
      ELSE
      DO 62 I=2,5
      DO 72 J=2,5
      C(I,J)=C(I,J)-(C(I,1)*C(1,J)/C(1,1))
   72 CONTINUE
   62 CONTINUE
      GO TO 75
      END IF
      IF (C(1,1).EQ.0.00000000)THEN
      DO 32 J=1,10
      TEMP=C(1,J)
      C(1,J)=C(4,J)
      C(4,J)=TEMP
   32 CONTINUE
      ELSE
```

```
      DO 63 I=2,5
      DO 73 J=2,5
      C(I,J)=C(I,J)-(C(I,1)*C(1,J)/C(1,1))
   73 CONTINUE
   63 CONTINUE
      GO TO 75
      END IF
      IF (C(1,1).EQ.0.00000000)THEN
      DO 33 J=1,10
      TEMP=C(1,J)
      C(1,J)=C(5,J)
      C(5,J)=TEMP
   33 CONTINUE
      ELSE
      DO 64 I=2,5
      DO 74 J=2,5
      C(I,J)=C(I,J)-(C(I,1)*C(1,J)/C(1,1))
   74 CONTINUE
   64 CONTINUE
      GO TO 75
      END IF
      IF (C(1,1).EQ.0.00000000)THEN
      WRITE(6,*)'MATRIX IS SINGULAR'
      STOP
      ELSE
C     DETERMINATION OF NEW C(I,J)
      DO 60 I=2,5
      DO 70 J=2,5
      C(I,J)=C(I,J)-(C(I,1)*C(1,J)/C(1,1))
   70 CONTINUE
   60 CONTINUE
      END IF
   75 C(2,6)=-C(2,1)/C(1,1)
      C(3,6)=-C(3,1)/C(1,1)
      C(4,6)=-C(4,1)/C(1,1)
      C(5,6)=-C(5,1)/C(1,1)
C     TO CHECK PIVOT C(2,2)
      IF( C(2,2).EQ.0.00000000)THEN
      DO 100 J=2,10
      TEMP=C(2,J)
      C(2,J)=C(3,J)
      C(3,J)=TEMP
  100 CONTINUE
      ELSE
      DO 111 I=3,5
```

```
      DO 121 J=3,6
      C(I,J)=C(I,J)-(C(I,2)*C(2,J)/C(2,2))
  121 CONTINUE
  111 CONTINUE
      GO TO 114
      END IF
      IF( C(2,2).EQ.0.00000000)THEN
      DO 80 J=2,10
      TEMP=C(2,J)
      C(2,J)=C(4,J)
      C(4,J)=TEMP
   80 CONTINUE
      ELSE
      DO 112 I=3,5
      DO 122 J=3,6
      C(I,J)=C(I,J)-(C(I,2)*C(2,J)/C(2,2))
  122 CONTINUE
  112 CONTINUE
      GO TO 114
      END IF
      IF( C(2,2).EQ.0.00000000)THEN
      DO 90 J=2,10
      TEMP=C(2,J)
      C(2,J)=C(5,J)
      C(5,J)=TEMP
   90 CONTINUE
      ELSE
      DO 113 I=3,5
      DO 123 J=3,6
      C(I,J)=C(I,J)-(C(I,2)*C(2,J)/C(2,2))
  123 CONTINUE
  113 CONTINUE
      GO TO 114
      END IF
      IF( C(2,2).EQ.0.00000000)THEN
      WRITE(6,*)'MATRIX IS SINGULAR'
      STOP
      ELSE
C   DETERMINATION OF NEW C(I,J)
      DO 110 I=3,5
      DO 120 J=3,6
      C(I,J)=C(I,J)-(C(I,2)*C(2,J)/C(2,2))
  120 CONTINUE
  110 CONTINUE
      END IF
```

```
  114 C(3,7)=-C(3,2)/C(2,2)
      C(4,7)=-C(4,2)/C(2,2)
      C(5,7)=-C(5,2)/C(2,2)
C     TO CHECK PIVOT C(3,3)
      IF (C(3,3).EQ.0.00000000)THEN
      DO 150 J=3,10
      TEMP=C(3,J)
      C(3,J)=C(4,J)
      C(4,J)=TEMP
  150 CONTINUE
      ELSE
      DO 161 I=4,5
      DO 171 J=4,7
      C(I,J)=C(I,J)-(C(I,3)*C(3,J)/C(3,3))
  171 CONTINUE
  161 CONTINUE
      GO TO 430
      END IF
      IF (C(3,3).EQ.0.00000000)THEN
      DO 130 J=3,10
      TEMP=C(3,J)
      C(3,J)=C(5,J)
      C(5,J)=TEMP
  130 CONTINUE
      ELSE
      DO 162 I=4,5
      DO 172 J=4,7
      C(I,J)=C(I,J)-(C(I,3)*C(3,J)/C(3,3))
  172 CONTINUE
  162 CONTINUE
      GO TO 430
      END IF
      IF (C(3,3).EQ.0.00000000)THEN
      WRITE(6,*)'MATRIX IS SINGULAR'
      STOP
      ELSE
C     DETERMINATION OF NEW C(I,J)
      DO 160 I=4,5
      DO 170 J=4,7
      C(I,J)=C(I,J)-(C(I,3)*C(3,J)/C(3,3))
  170 CONTINUE
  160 CONTINUE
      END IF
  430 C(4,8)=-C(4,3)/C(3,3)
      C(5,8)=-C(5,3)/C(3,3)
```

```
C   TO CHECK PIVOT C(4,4)
      IF (C(4,4).EQ.0.00000000)THEN
      DO 200 J=4,10
      TEMP=C(4,J)
      C(4,J)=C(5,J)
      C(5,J)=TEMP
  200 CONTINUE
      ELSE
      DO 220 J=5,8
      C(5,J)=C(5,J)-(C(5,4)*C(4,J)/C(4,4))
  220 CONTINUE
      GO TO 190
      END IF
      IF (C(4,4).EQ.0.00000000)THEN
      WRITE(6,*)'MATRIX IS SINGULAR'
      STOP
      ELSE
C   DETERMINATION OF NEW C(I,J)
      DO 180 J=5,8
      C(5,J)=C(5,J)-(C(5,4)*C(4,J)/C(4,4))
  180 CONTINUE
      END IF
  190 C(5,9)=-C(5,4)/C(4,4)
C   TO CHECK PIVOT C(5,5)
      IF (C(5,5).EQ.0.00000000)THEN
      WRITE(6,*)'MATRIX IS SINGULAR'
      STOP
      ELSE
      C(5,10)=1.0
      C(4,10)=-C(4,5)/C(5,5)
      C(4,9)=1.0-(C(5,9)*C(4,5)/C(5,5))
      C(4,8)=C(4,8)-(C(5,8)*C(4,5)/C(5,5))
      C(4,7)=C(4,7)-(C(5,7)*C(4,5)/C(5,5))
      C(4,6)=C(4,6)-(C(5,6)*C(4,5)/C(5,5))
      C(3,10)=-C(3,5)/C(5,5)
      C(3,9)=-C(5,9)*C(3,5)/C(5,5)
      C(3,8)=1.0-(C(5,8)*C(3,5)/C(5,5))
      C(3,7)=C(3,7)-(C(5,7)*C(3,5)/C(5,5))
      C(3,6)=C(3,6)-(C(5,6)*C(3,5)/C(5,5))
      C(2,10)=-C(2,5)/C(5,5)
      C(2,9)=-C(5,9)*C(2,5)/C(5,5)
      C(2,8)=-C(5,8)*C(2,5)/C(5,5)
      C(2,7)=1.0-(C(5,7)*C(2,5)/C(5,5))
      C(2,6)=C(2,6)-(C(5,6)*C(2,5)/C(5,5))
      C(1,10)=-C(1,5)/C(5,5)
```

```
      C(1,9)=-C(5,9)*C(1,5)/C(5,5)
      C(1,8)=-C(5,8)*C(1,5)/C(5,5)
      C(1,7)=-C(5,7)*C(1,5)/C(5,5)
      C(1,6)=1.0-(C(5,6)*C(1,5)/C(5,5))
      DO 250 I=3,1,-1
      DO 260 J=5,10
      C(I,J)=C(I,J)-(C(I,4)*C(4,J)/C(4,4))
  260 CONTINUE
  250 CONTINUE
      DO 270 I=2,1,-1
      DO 280 J=4,10
      C(I,J)=C(I,J)-(C(I,3)*C(3,J)/C(3,3))
  280 CONTINUE
  270 CONTINUE
      DO 300 J=3,10
      C(1,J)=C(1,J)-(C(1,2)*C(2,J)/C(2,2))
  300 CONTINUE
C  TO DETERMINE ELEMENTS OF B
      DO 310 I=1,5
      DO 320 J=1,5
      B(I,J)=C(I,J+5)/C(I,I)
  320 CONTINUE
  310 CONTINUE
      END IF
      RETURN
      END
```

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