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ABSTRACT

STATISTICAL AND FINITE ELEMENT ANALYSIS OF THE EFFECTS OF RANDOM INPUT PARAMETERS ON THE MECHANICAL CHARACTERISTICS OF SHEETMETAL FORMING PROCESSES USING QUADRATURE METHOD

by
Yu Quan

In this thesis, the computer aided finite element analysis is implemented in the analysis of sheetmetal forming process. To rationalize the experimentation and analysis, the statistical method is introduced to study the variability of sheetmetal deformation process and final shape with respect to the variation of different parameters.

In Chapter 2, the basic characteristics and considerations of deep drawing operation is described with emphasis on the study of geometrical and physical variables of the forming process.

Chapter 3 discussed the three phases which are encountered in the computer aided finite element analysis. As a typical representative of professional finite element software packages, ABAQUS is used through this work as the main programming environment and its language characteristics and programming techniques are investigated in great detail. As an example of its application, the modeling and analysis of a deep drawing part is specified.

Chapter 4 delved with the application of statistical approaches in the experimentation of sheetmetal forming. A comparison of different traditional methods for statistical analysis is given among which the Taguchi's method is discussed in detail because of its simplicity, practicality and relatively high accuracy. However, on the basis of analysis of its theory, it is found that the degree of central moment it can approximate accurately is too limited. Therefore, a new method for analysis of complex system - the quadrature method is introduced which can obtain the highest possible degree of accurate central moment of a distribution. Finally, by using the quadrature approach, the maximum stresses and reaction forces occurred during the deep drawing process is studied with respect to the change of three geometrical and interface parameters. The results of the experimentation proved the validity and applicability of this method.

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New Jersey Institute of Technology*

**A Thesis
Submitted to the Faculty of
New Jersey Institute of Technology
in Partial Fulfillment of the Requirement for the Degree of
Master of Science in Manufacturing Systems Engineering**

Manufacturing Engineering Division

January 1995

APPROVAL PAGE

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This thesis is dedicated to
my parents and my younger brother

ACKNOWLEDGMENT

The author wishes to express his sincere gratitude to his thesis advisor, Dr. Nouri Levy, for his professional guidance, friendship, and moral support throughout this research.

Special thanks to Dr. Billatos and Dr. Rong-Yaw Chen for serving as members of the committee.

The author gratefully acknowledge Dr. Nouri Levy, Dr. Raj Sodhi and Dr. Rong-Yaw Chen for their financial supports throughout this work.

The author is thankful to his parents and his younger brother for their encouragement and support.

And finally, the author appreciates the valuable help from all his friends.

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CHAPTER 1

INTRODUCTION

Historically, the evolution of a sheetmetal stamping from conception through part design to die design to the final die tryout has been a slow, cautious process based on the trial-and error experience and the skill of the artisan [1]. The press-shop today is successful only because of the technological skill developed over the years by the artisans. Under the present condition, it would be very difficult to duplicate the capability of the artisan in producing part by use of the scientific approach only. What we can provide so far limits to only an approximate analytical solution to the anticipated process performance of sheetmetal forming of only the simplest parts.

However, a breakdown of the artisan system is beginning to occur for the following reasons [2]:

- (1) breakdown of the long-term apprenticeship train system;
- (2) the trend toward earlier retirement;
- (3) reduced lead time for die tryout and development;
- (4) increased complexity of parts;
- (5) introduction of new, unfamiliar materials;
- (6) greater emphasis on cost-effectiveness; and
- (7) the rapid development of computer aided design.

It would be very difficult for the average die maker to function properly without the backup support from the process engineer. New developments are occurring at an ever-increasing rate. The typical tool maker is no longer able to cope with these developments. There is definitely a need to replace the present experience-based, trial-and-error techniques with cost-effective, knowledge-based, analytical techniques in sheetmetal forming. Productivity increases in sheetmetal forming can be achieved if the part geometry, the fabrication method, the die design, and the material property parameters are correctly specified at the design stage.

According to Keeler [1], the system of the future to replace the artisan should meet eight requirements. It must:

- (1) be an interactive system;
- (2) be modeled with known and unknown variables;
- (3) incorporate the material properties of real materials;
- (4) not be biased by historical rules of thumb;
- (5) provide predictive capability;
- (6) improve the interaction between design and manufacturing functions;
- (7) be responsive to new in-service requirements; and
- (8) be attuned to end-product economy.

To meet the above requirements, it is necessary to develop the analytical models describing the material behavior under various forming conditions of temperature, strain, and strain rate, and the mathematical models simulating each specific sheetmetal forming process of interest [3]. The analytical models for material behavior should enable the

calculation of the limits to which the material can be deformed. The mathematical model for the processes should describe the local states of the stresses and strains in the material during forming [4].

Table 1.1 Summary of Various Analysis Methods [5]

Method	Input		Output				Comments
	Flow Stress	Friction	Velocity	Stress Field	Temperature	Stress on Tools	
Slab	Average	a, b	No	Yes	No	Yes	Ignore redundant work
Uniform Energy	Average	b	No	No	No	Average	Redundant work can be included approximately
Slip Line	Average	a, b	Yes	Yes	No	Yes	Valid for plain-strain problem
Upper-bound	Distributed	b	Yes	No	No	Average	Gives upper-bound loads, can determine free boundary
Hill's	Distributed	a, b	Yes	No	Yes	Average	Can treat 3D problems
Finite Difference	Distributed	a, b	Yes	No	Yes	Yes	Require considerable computer time
Finite Element	Distributed	a, b	Yes	No	Yes	Yes	Require considerable computer time
Matrix Weighted Residuals	Distributed	a, b	Yes	No	Yes	Yes	Treats rigid-plastic material very general approach

Note: a. $\tau = \mu \cdot \sigma_n$; b. $\tau = \frac{m}{\sqrt{3}} \cdot \sigma$.

As the basis of the metalworking process, plasticity theory is a macroscopic phenomenological theory based on mathematically described, large-scale behavior of a material continuum during plastic deformation. A brief summary of the most commonly used method of analysis is given in Table 1.1 [5].

In this thesis, the finite element method is utilized as a powerful tool for modeling and analysis of sheetmetal forming process by using a sophisticated software package -- ABAQUS. Different variables of sheetmetal forming operations are studied, and their influences on the deformation procedure and the final shape of the work piece are investigated. In order to rationalize the experimentation of sheetmetal forming processes, the statistical approaches of quadrature method are introduced to minimize the number of experiments of searching for the optimal conditions. It is basically a method of discretization and approximation of a symmetric distribution. The index for the accuracy evaluation is the highest degree of central moment it can approximate accurately. This method is used when the statistical distribution of each factor is known and all-combination experiments are required to be conducted. Comparing to other statistical approaches, this method has greater significance in practice in that it can save a lot of efforts of experimentation when the experiments are either costly or time-consuming.

CHAPTER 2

ANALYSIS OF SHEETMETAL FORMING PROCESSES

2.1 General Review of Sheetmetal Forming Processes

In a manufacturing process, a given material, usually shapeless or of a simple geometry, is transformed into a useful part. This part usually has a complex geometry with well defined shape, size, accuracy and tolerances, appearances and properties [7].

There are five main characteristics of any manufacturing process -- namely geometry, tolerances, production, and human and environmental factors. The manufacture of metal parts and assemblies can be classified into five general areas:

- (1) Primary shaping processes, such as casting, melt extrusion, die casting and pressing of metal powder. In all these processes the material initially has no shape but through the process obtains a well defined geometry.
- (2) Metal forming process, such as rolling, extrusion, cold and hot forging, bending and deep drawing, where metal is formed by plastic deformation.
- (3) Metal cutting processes, such as sawing, turning, milling and broaching, where a new shape is generated by removing the metal.
- (4) Metal treatment processes, such as heat treating, anodizing and surface hardening, where the part remains essentially unchanged in shape but undergoes change in shape.

(5) Joining processes, such as welding, diffusion bonding, mechanical joining as riveting, shrink fitting, where two parts of the same or different metals are joined together.

In metal forming, an initially simple part, for example, a billet or a sheet blank is plastically deformed between dies to obtain the desired final configuration. Forces are applied to the sheetmetal blank to cause permanent change of contour: Metal forming processes usually produce little or no scrap and generate the final part geometry in a very short time, usually in one or a few strokes of a press or hammer. During forging, one area of the blank is usually held stationary on the die as the punch forces the other up or down to complete the change in contour. A given shape of a workpiece is converted into another shape without change in mass or composition of the material of the work piece. For a given weight, parts produced by metal forming exhibit better mechanical and metallurgical properties and reliability than do those manufactured by casting or machining. There are various forming processes such as rolling, extrusion, cold and hot forging, bending and deep drawing, where metal is formed by plastic deformation.

Metal flow is influenced mainly by:

- (1) Tool geometry;
- (2) Friction condition;
- (3) Characteristics of the stock material;
- (4) Thermal conditions existing in the deformation zone.

The details of metal flow influence the quality and properties of the formed product and the force and energy requirements of the process.

2.2 Characteristics and Considerations of Sheetmetal Drawing Processes

2.2.1 Basic Process Description

Sheetmetal forming processes, broadly classified as deep drawing or stamping operations, include a wide spectrum of operations and flow conditions [2]. At one end of the spectrum is the forming of flat-bottomed cylindrical cups by radial drawing or cupping. In this case one of the principle strains in the plane of the sheet is positive and the other is negative with the change in thickness, if any, being small. At the other end of the spectrum are operations involving biaxial stretching of the sheet, at which two principle strains are tensile and thinning is required. The distinction between shallow drawing and deep drawing is arbitrary, although shallow drawing generally refers to the forming of a cup no deeper than one half of its diameter, with little thinning of the metal. In deep drawing, the cup is deeper than one-half of its diameter, and wall thinning, although not necessary intentional, may be more than in shallow drawing.

In this thesis, conventional deep drawing of a cylindrical cup from a thin, flat, circular blank with a flat-bottom punch as shown in Fig. 2.1 will be discussed primarily.

During the deep drawing, a planar disk is transformed into a cup with flat bottom, cylindrical walls, and open top [8]. As shown in Fig. 2.1 (a) and (b), the disk is placed over the opening in the die and forced to deform by a moving punch. As the punch moves downward, it pulls the flange toward the center. The flange is held between the die and the blank holder, with the purpose of preventing the flange from folding upward. The

flange moves inward radically while its inner side bends over the rounded corner of the die and transformed from a flat disk to a circular tube.

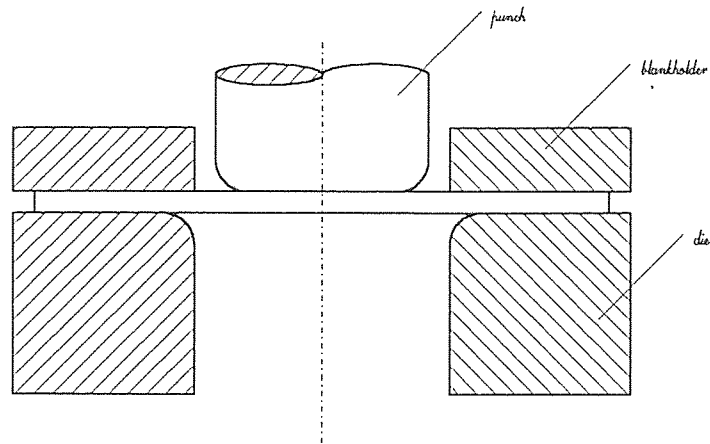


Figure 2.1(a) Beginning Stage of Deep Drawing

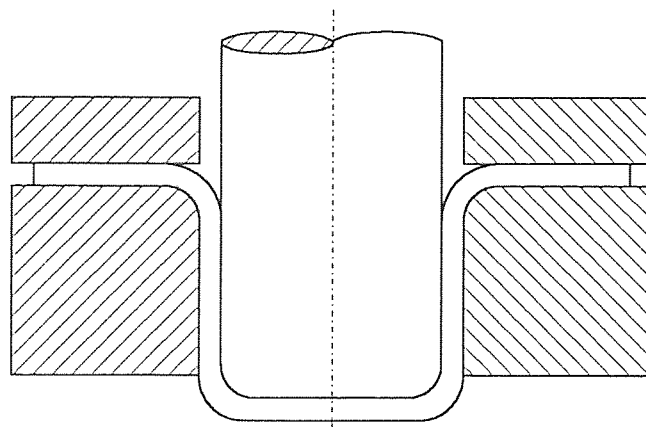


Figure 2.1(b) End Stage of Deep Drawing

At this point, the bottom is not deformed, while the cylinder is already deformed but is not undergoing further deformation. Friction prevails on the top and bottom flat

surfaces of the flange, and on the surface of the corner radii of the female die. Punch force supplies the motive power to overcome the deformation and friction resistance. The punch force is normally transmitted from the punch to the cup through pressure on the bottom of the cup. The punch pressure is transmitted from the bottom of the cup to the deformation region through tension on the wall of the cup. This tension must remain elastic.

The punch is forced down through the die, pulling the blank inward while converting the flat disk into a cylindrical shape. When classical deep-drawing is performed, the gap between the punch and the die is designed to be larger than the thickest part of the wall of the cup, which occurs at the open top. Although modification of the process from classical deep drawing to stretching and wall ironing allow for smaller gaps, and other changes sometimes eliminate the hold-down ring, this always is addressed to the basic process as described above.

2.2.2 Analysis of Drawing Operation

For use in the analysis of deep drawing, the flat blank may be divided into three zones: X, Y, and Z as shown in Fig. 2.2 [9].

The outer annular zone X consists of material in contact with the die, the inner annular zone Y is initially not in contact with either the punch or the die, and the circular zone Z is in contact with the flat bottom of the punch only.

During the course of deep drawing, the following five processes take place [9]:

- (1) Pure radial drawing between the die and the blankholder;
- (2) Bending and sliding over the die profile;
- (3) Stretching between the die and punch;
- (4) Bending and sliding over the punch profile radius;
- (5) Stretch and sliding over the punch nose.

Various parts of zone X may go through some or all of the process 1, 2, and 3; those of Y through 2, 3, and 4; and those of Z through 3, 4, and 5.

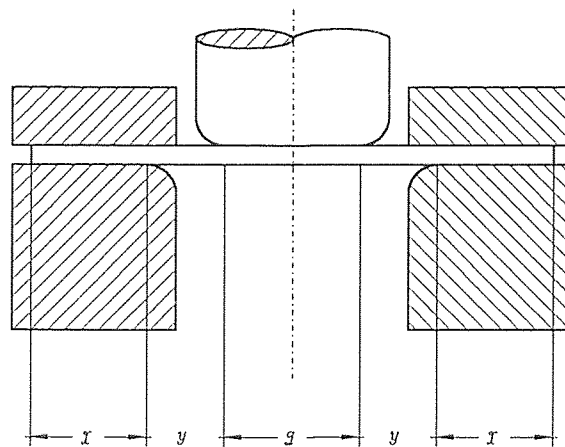


Figure 2.2 Three Zones of Flat Blank

To get a better understanding of deep drawing processes, it is necessary to find out those variables affecting the deformation of the workpiece. In the paper of Sirkirk [10], he identified thirty sheetmetal forming process variables as listed in Table 2.1. These variables can be roughly divided into three categories:

- (1) tooling variables, including the geometry and hardness of tools;

- (2) workpiece variables, including the geometry and material property of the blank;
- (3) interface variables, including the friction and lubrication condition between the tools and workpiece.

Table 2.1 Major Sheetmetal Forming Process Variables

Blank Variables	1. Dimensions
	2. Positive in die (gaging location)
	3. Edge condition
Lubrication	4. Type
	5. Coating thickness and distribution
Press Variables	6. Punch guidance
	7. Punch speed (as a function of press stroke)
	8. Binder force and its variation around the binder ring
	9. Counterbalance pressure where it can affect press load
Work Material Variables	10. Rigidity
	11. Thickness (normal and thickness profile)
	12. Mechanical properties
	13. Surface topography
Die Variables	14. Coating and surface chemistry
	15. Guidance
	16. Alignment; i.e., positive in press
	17. Surface finish
	18. Material
	19. Draw beads (change with wear)
	20. Hardness
	21. Punch radii (change with wear)
	22. Surface coating and surface chemistry
	23. Profile radii (change with wear)
	24. Rigidity
Miscellaneous Variables	25. Dirt in the die or on the blank
	26. Blank pre-bend position (affects blank location)
Interactive Variables	27. Working material temperature
	28. Die temperature
	29. Atmospheric conditions (temperature, humidity)
	30. Shims on stop block (affect binder load distribution)

What we are interested here is the forces which occur during drawing and the punch force as a function of the independent process variables.

Various forces which are generated during the process of drawing are:

- (1) Bending at radii P_b ;
- (2) Friction between
 - (a) blank holder and sheetmetal, F_B ;
 - (b) die steel and sheetmetal, F_d ;
 - (c) punch steel and sheetmetal, F_p ;
- (3) Compression at the flange area or extremity of the cup, P_c .

The punch force is the sum of the above forces:

$$P_p = P_b + (F_B + F_d + F_p) + P_c \quad (2.1)$$

As can be observed from the study by Avitzur [11] and from the findings in other processes, when the material is not sensitive to the strain-rate effects, the punch force becomes independent of the punch speed v_f . Furthermore, the solution can be presented in dimensionless form. In symbolic terms the function relating the punch force (the dependent parameter) to the independent parameters can be represented as follows:

$$P_p = \sigma_0 \cdot R_t^2 \cdot f\left(\frac{r_i}{R_t}, \frac{t_0}{R_t}, \frac{r_0}{R_t}, m, P_H\right) \quad (2.2)$$

where t_0 and R_0 are the thickness and outer radius of the flange (Fig. 2.3); R_t is the radius of the surface separating the flange (region I) from the toroidal bending of region II; r_i is the corner radius of the die, which is also the inner radius of the bead in the toroidal region

II; m is the friction factor acting between the workpiece and the tools; P_H is the hold-down force acting on the hold-down ring.

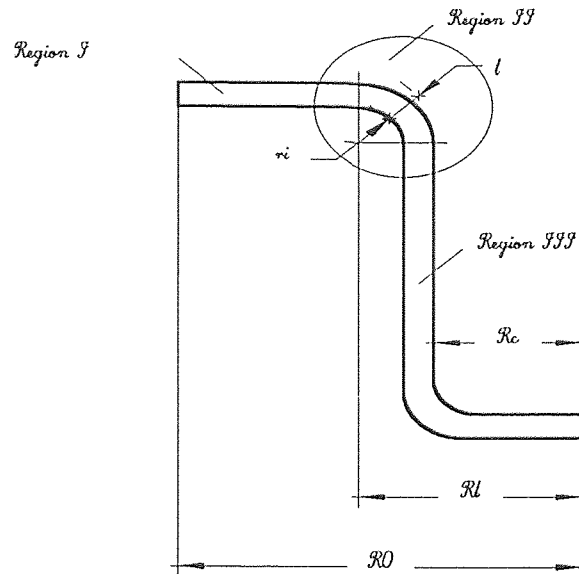


Figure 2.3 The Parameters of the Workpiece

In the Chapter 4 of this thesis, experimentation of the effects of various factors on punch force are conducted using statistical methods.

CHAPTER 3

COMPUTER AIDED FINITE ELEMENT MODELING AND ANALYSIS

3.1 General Introduction of Computer Aided Finite Element Analysis

The finite element method has been used in engineering analysis for many years. However, its use throughout the entire engineering design process has been limited by the cost of both the computing resources and the workforce to synthesize and manage the data needed for the multiple analyses required by design modifications [12]. The last two decades have witnessed an explosive growth in computer technology. The introduction of these new computing systems has made a strong impact on finite-element technology. The drastically reduced expense of computing resources and the development of more integrated design software are increasing the cost-effectiveness of finite element methods to the point that they can become an integral part of the design process, not just a check on the final design or a tool for post-failure analysis.

In computer-aided design (CAD) the determination of the performance (e.g., stress or deformation) of a device using the finite element method during its design process is accomplished by analysis of the partial differential equations which describe the given system. This involves the following three steps:

- (1) preprocessing: the description of the geometry, the physical characteristics and the mesh;

- (2) problem solving: the application of the finite element method;
- (3) postprocessing: the visualization and interpretation of the results of the simulation.

3.1.1 Preprocessor

The generation of a finite element model ready for input to the desired analysis program can be considered a four-step process as shown in Fig. 3.1 [12].

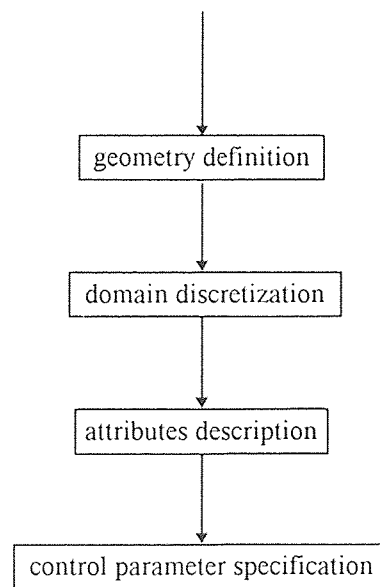


Figure 3.1 The Process of Pre-processing

The first two steps are the most general since the definition and discretization of geometry are independent of the particular class of problem to be solved or the analysis

program to be used. The type of element geometries and the way a mesh is graded are dependent on the problem to be solved, the type of element to be selected, and the level of accuracy desired.

Traditionally, the need to generate element meshes has been a drawback of using the finite element method. However, there are a number of software packages and methods available today to aid in the generation of finite element meshes. The mesh generation consists of collection of finite elements which form an acceptable discretization of the domain. Such a discretization must respect the boundaries of the domain and interfaces between two regions. The shape of the finite elements must not be too irregular (elongated) and should as much as possible resemble the reference elements (equilateral triangle or tetrahedra, squares or cubes, etc.), i.e., their aspect ratio should near 1:1.

The element aspect ratio is a measure of the shape of an element. It is defined simply as the ratio of the length of the longest element side to the length of the shortest element side. The importance of this measure is that numerical ill-conditioning of the element stiffness matrix may result when an element becomes too elongated and/or the vertex angle become too small or too large. The degree of sensitivity to aspect ratio is a function of the element type and number of digits of accuracy available.

The nodes are defined by their coordinates while the elements are characterized by their type and a list of their nodes. Certain formulations involve boundary integrals, not only interior finite elements (volume elements in three dimensions, surface in two), but also boundary finite elements in three dimensions (surface elements in dimensions, line elements in two) or the corresponding boundaries must be constructed.

The description of attributes include the specification of physical characteristics such as the material properties (e.g., Young's modules), sources (e.g., distributed or concentrated loads), and boundary conditions (for time dependent or time independent problems).

3.1.2 Problem Solver

The solver computes the unknowns in a finite element problem, i.e., it solves the linear or non-linear system of equations coming from the variational or the projective formulation. Its input is the domain discretization, the physical characteristics and the boundary conditions. The output is the value of the unknown force or displacement at each of the node of the grid (Fig. 3.2 and 3.3).

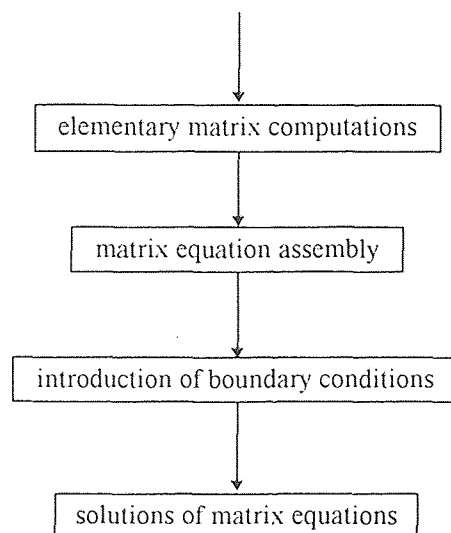


Figure 3.2 Solver: Operation for a Linear Static Problem

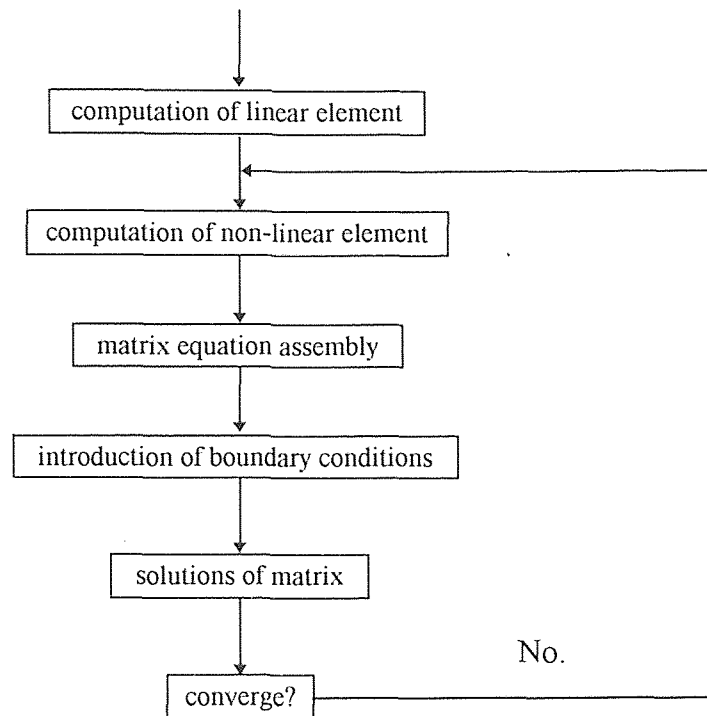


Figure 3.3 Solver: Operation for a Non-linear Static Problem

Two large classes of methods are used to solve these sets of equations: point or block method of relaxation or global matrix methods. The latter, more popular today, requires several steps:

- (1) creation of sub-matrices and sub-vectors corresponding to each individual finite element;
- (2) assembly of these elementary matrices and vectors to build the system matrix and right hand vector, the bigger the system assembly matrix, the more powerful and expensive the software;

- (3) solution of the linear system of equations.

3.1.3 Postprocessor

The reduction of finite element results to a manageable level of useful information is the function of a post-processor [12]. In general, postprocessing is concerned with the two separate questions of reducing and presenting results in an understandable fashion, and of ensuring that the results used are the most accurate results the given model can produce.

To enhance the engineer's capabilities to interpret the results of a finite element analysis there are a number of ways to distill and present graphically the information obtained, e.g., displaced shapes, contour maps, vector display maps, animation, thresholding of results, automatic checks against design codes, and automatic generation of result reports.

Displaced shapes are typical of composite vector displays in that all components of a vector response are shown by an exaggerated, deformed mesh. Contour maps can be used to display scalars such as temperatures and concentrations of pollutants, or one component of vectors or tensors such as maximum principle stress or horizontal flow. Color is widely used in the display of stress contours because it not only adds impact to the display of results but can also be used as an added dimension to transmit additional information. The understanding of time-dependent or harmonic results can be greatly enhanced with animation. For example, the dynamic display of natural-vibration modes gives an improved understanding of vibrational tendencies of a structure, and the time-

history display of displaced shapes or temperature contours greatly improves the understanding of the time-dependent results.

Since the major function of postprocessing is to give the user an understanding of the results, which is done best in a pictorial manner, and to allow the user to interpret the results as desired, interactive computer graphics is the ideal medium for postprocessing. An interactive-graphic postprocessor allows the user to obtain quickly plots of deformed shapes, stress, contours, and other desired parameters. Instead of scanning columns of numbers, the user can use overall display to determine the basic trends and then can concentrate on the critical areas to determine the desired values. The user also can exercise real-time control over all the factors that affect perception of the results, including viewing directions, magnifications, parameters, displayed, scale factors, contour intervals, and color maps.

3.2 Finite Element Programming in ABAQUS

3.2.1 An Overview of Finite Element Software Packages and ABAQUS

The significant advances made in finite element technology, coupled with the rapid developments in computer hardware and software, provided the foundation from which general-purpose finite element programs have evolved. After many years of development, a wide variety of finite element programs are currently being used in government and industry for the solution of a widely variety of practical problems. The analysis

capabilities and user features vary considerably from one code to the other, and, therefore, it is often difficult to identify the proper code that meets a specific need. A number of factors which affect the selection of a code are enumerated as follows [13, 14]:

- (1) Analysis capabilities: include the range of applications and limitations of the code, which include both those implied by the formulation aspects and numerical selection procedures adopted by the code as well as the element library available in the code;
- (2) Adequacy of user-oriented features: such as automatic (or semi-automatic) mesh (or model) generation, error checks, and displays of original model and of various intermediate results;
- (3) Maintainability: include updating the computational modules, extending the capabilities of the code, and improving its performance;
- (4) Adequacy of user-support facilities, such as users' manuals, sample problems and help interactive commands, etc.;
- (5) Portability: compatibility between different computer systems.

Among numerous finite element software packages, ABAQUS, a general-purpose finite element system developed by the Hibbit, Karlsson and Sorensen, Inc., is chosen as the tool of analysis in this work because of its advantages of large library of capabilities, including large element library and wide range of non-linear feature, ease of use (very simple, readable keyword and parameter input, automatic time stepping for non-linear application, extensive graphical output), efficiency and high level support.

ABAQUS is designed as a production tool of maximum generality. Its major capabilities are focused on reliability in practical cases. Since much of the program's use is in the nonlinear range, it has an extensive library of nonlinear features that will provide solutions for wide range of problem parameters, with minimum guidance from the user. For example, a strong emphasis is placed on automatic incrementation schemes for static case (including unstable postbuckling response), dynamics (including impact), fully coupled stress problems, pore-fluid-flow-porous-medium deformation cases, etc. Four principal ideas form the basis of the user interface in ABAQUS: simple input, careful documentation, extensive plotting capability, and automatic time stepping. Input is organized by keywords and "sets". Keywords introduce blocks of data; keyword cards often include parameters. The set concept is an effective data organizer for the user, especially in large models. It allows collection of nodes or elements to be addressed by a user-defined name. Sets can be assembled into other sets, to any level. Sets are used for most specifications -- material properties, loading and boundary conditions, output edits, etc.

ABAQUS is designed for advanced applications, especially in the nonlinear range. Because it is one of the easiest large-scale finite element programs to learn and use, and it is one of the most computationally efficient, even in simple, linear applications, ABAQUS is used as the main media of finite element analysis in this work.

3.2.2 ABAQUS Programming Techniques

ABAQUS runs as a batch application. The main input is a file which options are required and gives the data associated with these options [15, 16]. There may also be supplementary files, such as restart or results files from previous analyses. The main input file is discussed in the terminology of a card image file (a "card file"). For analysis, the file consists of two sections: model input and history input. For postprocessing in batch mode, the file contains output commands.

Since ABAQUS is a batch program, the objective is to assemble a "data deck" which describes a problem so that ABAQUS can provide an analysis. Data decks for complex simulations can be large, but can be managed without too much difficulty by using the convenient features built into the program's input structure.

A data deck for ABAQUS contains model data and history data. Model data define a finite element model: the elements, nodes, element properties, material definitions, and so on - - any data that specify the model itself. History data define what happens to the model -- the sequence of event to loadings for which the model's response is sought.

- Model input

A finite element model consists of a geometric description, which is given by the element and their nodes (the "bulk data") and a set of properties associated with the elements, describing their attributes. These properties include material definitions, cross-section definitions in the case of structural elements like beam and shells, and other parameters for interface elements, springs, dashpots, etc. There may also be constraints

that must be included in the model -- "multi-point constraints" or "equations" (linear or nonlinear equations involving several of the fundamental solution variables in the model), or simple "boundary conditions" that are to be imposed throughout the analysis. Environment properties, such as attributes of a fluids surrounding the model, must also be defined in some cases. Non-zero initial conditions, such as initial stresses, temperatures or velocities, may also be required. All of these are classified in ABAQUS as "model definition data" and are given as the first part of an ABAQUS analysis data deck.

The data structure of model input in ABAQUS is shown in Fig. 3.4.

One of the most useful features of the ABAQUS data definition method is the availability of "sets". A set can be a set of nodes or a set of elements. The user provides a name for each set. That name then provides a means of referencing all of the members of the sets. Sets are the basic reference throughout ABAQUS, and the use of sets is recommended. Choosing meaningful set names makes it simple to identify which data belong to which part of the model. By using sets and some other options provided in ABAQUS, it is possible to define and mesh a complex geometry shape without much difficulty.

The material library in ABAQUS is intended to provide comprehensive coverage of both linear and nonlinear, isotropic and anisotropic material models. The use of numerical integration in the elements, especially the numerical integration across the cross-section of shells, means that this flexibility in material modeling can be used to full advantage to analyze the most complex composite structures.

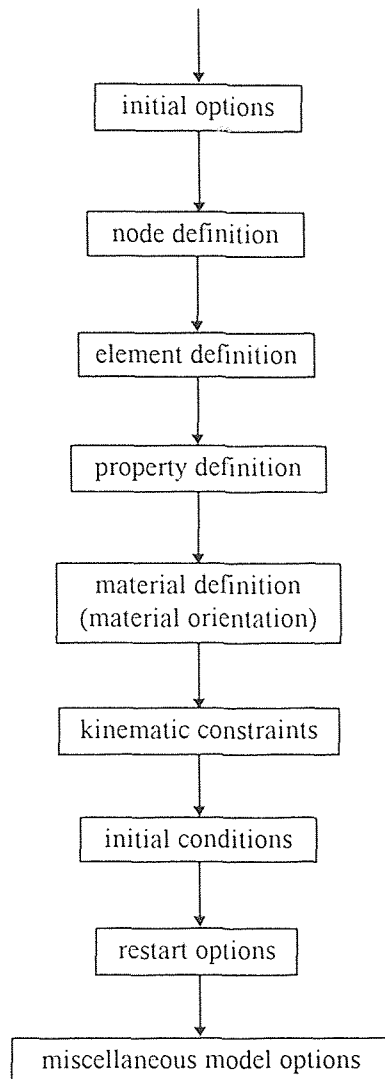


Figure 3.4 Data Structure of Model Input

Most materials of engineering interest initially respond elastically. Elastic behavior means that the deformation is fully recoverable, so that, when the load is removed, the specimen returns to its initial shape. In the case of linear elasticity, the total stress is defined from the total elastic strain as:

$$\sigma = D^{el} \cdot \varepsilon^{el} \quad (3.8)$$

where σ is the total stress; D^{el} is the elasticity matrix; and ε^{el} is the total elastic strain. The simplest form of linear elasticity is the isotropic case: D^{el} is defined by giving Young's modulus and Poisson's ratio. If the load exceeds some limit (the "yield load") the deformation is no longer fully recoverable. Some part of the deformation will remain when the load is removed, as, for example, when a paperclip is bent too much, or when a billet of metal is rolled or forged in a manufacturing process.

Plasticity theories model the material's mechanical response as it undergoes such nonrecoverable deformation in a ductile fashion.

Most of the plasticity models in ABAQUS are "incremental" theory, in which the mechanical strain rate is decomposed into an elastic part and a plastic (inelastic) part. For example, the stress-strain relation for a certain kind of material is shown in Fig. 3.5. The first yield occurs at 200 MPa. The material then hardens to 300 MPa at one percent strain, after which it is perfectly plastic. Assuming the Young's modulus is 2×10^5 MPa, the plastic strain at the one percent strain point is

$$0.01 - \frac{300}{2 \times 10^5} = 0.0085$$

Thus, the plasticity model for the material can be listed as Table 3.1. Note that plastic strain values, not the total strain values, are used in define the hardening behavior. Note also that the yield stress remain constant for plastic strains exceeding the last value given.

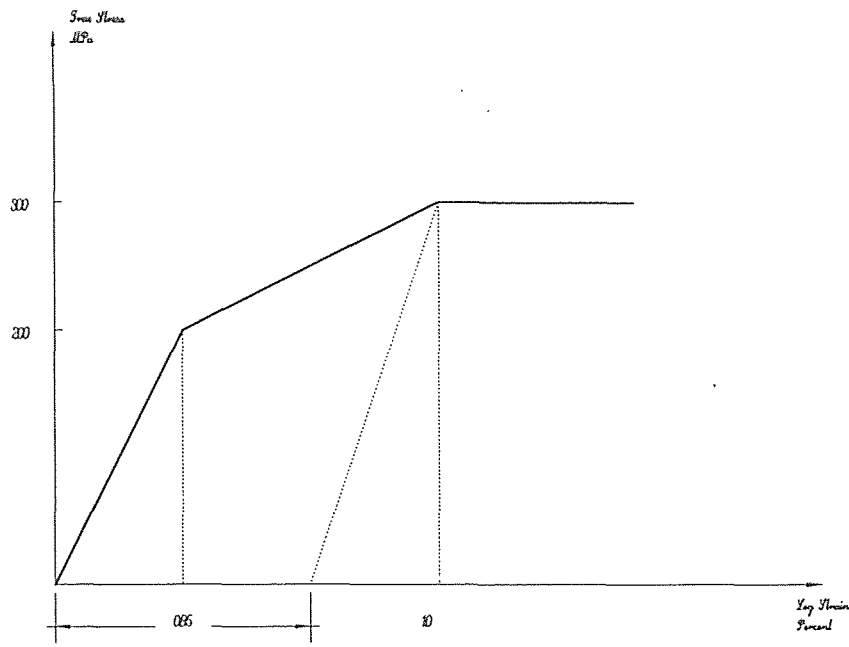


Figure 3.5 The Stress-strain Relation for a Certain Kind of Material

Table 3.1 Plasticity Model for a Certain Material

yield stress (N)	plastic strain (mm)
200.	0.
300.	0.0085

- History Input

The purpose of an analysis is to predict the response of a model to some form of external loading or to some non-equilibrium initial conditions. ABAQUS is designed as a flexible tool for finite element modeling. An important aspect of this flexibility is the manner in which ABAQUS allows the user to step through the history to be analyzed.

A basic concept in ABAQUS is the division of the problem history into steps. A step is any convenient phase of the history -- a thermal transient, a creep hold, a dynamic transient, etc. In each step the user choose a procedure, thus defining the type of analysis to be performed during the step: dynamic stress analysis, eigenvalue buckling, transient heat transfer analysis, etc. The procedure choice may be changed from step to step in any meaningful way, so that the user has great flexibility in performing analysis.

The data structure of history input in ABAQUS is shown in Fig. 3.6.

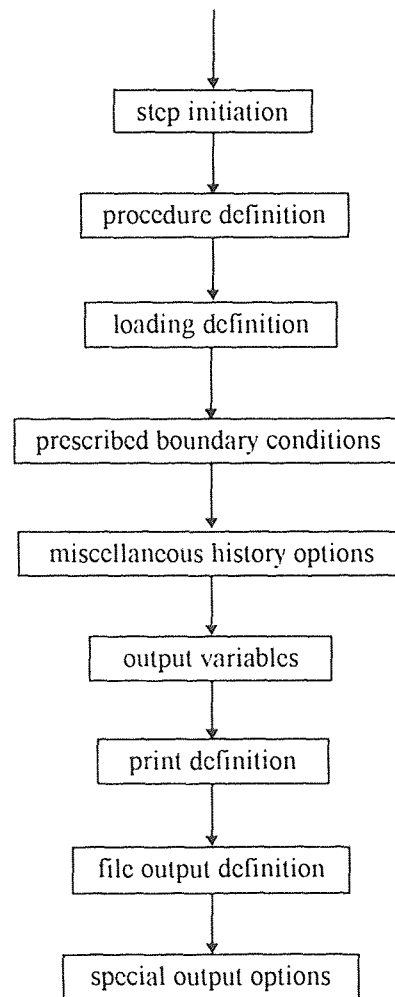


Figure 3.6 Data Structure of History Input

ABAQUS provides both linear and nonlinear response options. The problem is truly integrated, so that linear analysis is always considered as linear perturbation analysis about the state at the time when the linear analysis procedure is introduced. This linear perturbation approach allows general application of linear analysis techniques in cases where the linear response depends on preloading, or the nonlinear response history of the model.

In nonlinear problems the challenge is always to a convergent solution at a minimum cost. The nonlinear procedure in ABAQUS offer two approaches to this. Direct user control of increment size is one choice, whereby the user specifies the incrementation scheme. This is sometimes useful in repetitive analyses, where the user has a good "feel" for the problem. Automatic control is the alternate approach: the user defines the step and specifies certain tolerances or error measures. ABAQUS then automatically selects the increments as it develops the response in the step. This approach is usually more efficient, because the user cannot predict the response ahead of time. Automatic control may sometimes increase the cost of analysis over the cost when the response is essentially predictable and direct user specification of increments is adopted, but automatic control can save enormously over repeated user controlled running of a problem to obtain a satisfactory incrementation scheme. Automatic control is particularly valuable in cases where the time or load increment varies widely through the step. Ultimately, automatic control allows nonlinear problem to be run with confidence without extensive experience with the problem.

3.2.3 Case Study

- Finite Element Modeling

The geometry of the workpiece before deforming is very simple which can be represented by two parallel straight lines with a certain distance apart. However, the shape of the tools is relatively complex. The blankholder consists of two straight lines while the die is made up of two straight lines with an arc between them. Since relatively larger displacement occurs between the punch and the workpiece than the interface of die and blankholder, the shape of the punch is designed so that one straight line and two arcs which are connected smoothly.

The basic configuration and geometry of the deep drawing problem is shown in Fig. 3.7.

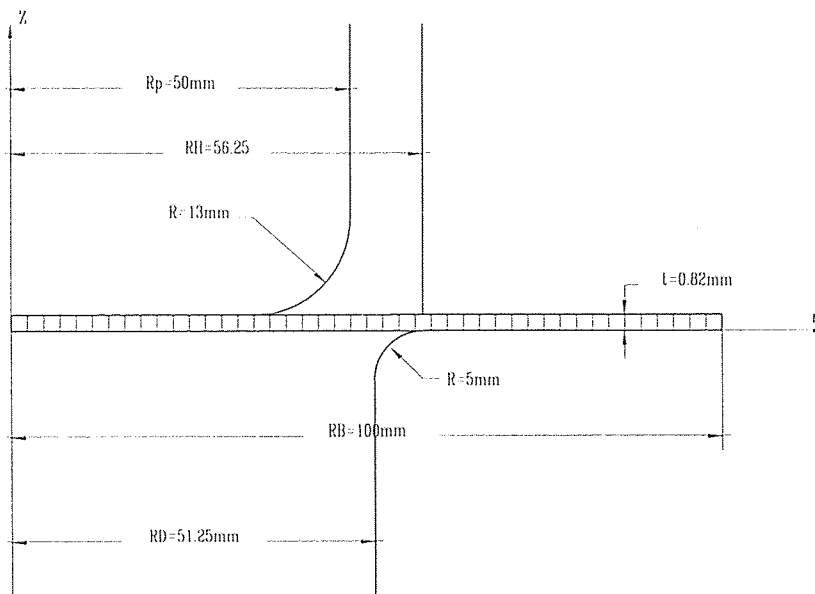


Figure 3.7 Configuration and Geometry for the Deep Drawing Problem

The basic configuration and geometry of the deep drawing problem is shown in Fig. 3.7.

The definition of nodes and elements are shown in Fig. 3.8.

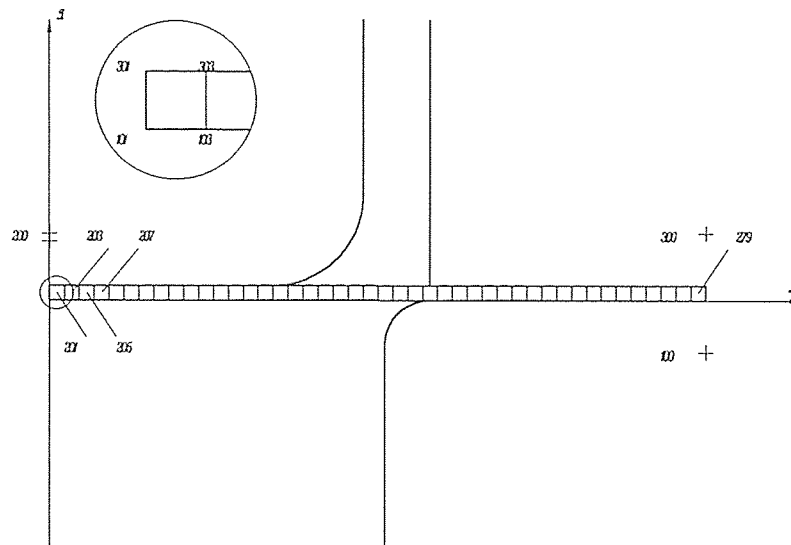


Figure 3.8 Definition of Nodes and Elements

- Finite Element Analysis

This project is a problem of large deformation whose process cannot be determined before hand. Therefore, the automatic control method is implemented in this project. The ABAQUS program for the deep drawing process is listed in Appendix I.

The analysis process is divided into five steps:

- (1) push the blank holder down by a prescribed displacement;
- (2) apply the prescribed force on the blank holder and release the displacement;
- (3) move the punch down;
- (4) fix all nodes and remove the IRS elements;
- (5) replace the boundary conditions by the regular set.

Detailed analysis and statistical experimentation of this problem will be discussed in Chapter 5.

CHAPTER 4

APPROACHES FOR STATISTICAL ANALYSIS OF COMPLEX SYSTEM

4.1 Traditional Methods for Statistical System Analysis

Statistical analysis is widely used by engineers and scientists to study the effect that component variations have on the output variability of a mechanism or system. Two classes of methods have been applied for variability analysis, i.e., high/low method which specifies threshold by high/low or go/no-go values, and statistical method which specifies threshold by probability distributions. There is a critical difference between these two kinds of methods [17]: namely, except for the most simple mechanisms high/low method poses analytical difficulties in principle as well as in practice; statistical method does not generally pose a problem in principle, although it may very well do so in practice. The essence behind this statement is that since in statistical method component variations are given as probability distributions, the response of a mechanism has a well defined probability distribution under conditions common in practice, and further, one can write down expressions for the moments of this distributions and thus characterize it. On the other hand, one does not have this machinery or some equivalent available when working with high/low method.

The important technical problem is the determination of the probability distribution of the response of a mechanism for a given set of component distributions [17-19]. There is

a known relationship between the response of a mechanism and the value of the component in the mechanism:

$$Y = f(X_1, X_2, \dots, X_n) \quad (4.1)$$

where $f(\cdot)$ is some known function and X_1, X_2, \dots, X_n are the values of the n components. The component values, X_1, X_2, \dots, X_n are usually assumed to be statistically independent random variables, and the problem is to determine (or approximate) the probability distribution of Y . The relationship (4.1) may exist in any form of which it is possible to ascertain a value for Y for given values of the X_i ; as an analytical expression -- either implicit or explicit, or engineering calculations of an involved sort may be required, or Y may have to be determined by experiment or by analog, or so on. In other words, f is a function of the X_k in the most general sense of the word. The form in which it is expressed has a great bearing on the way a complex problem can be handled.

The normalized r th central moments of the system response can be computed as follows:

$$M_r = \sum_{i=1}^N (Y_i - M_1)^r \cdot p(Y_i), \quad r = 1, 2, \dots, n \quad (4.2)$$

where $Y_i, i=1, 2, \dots, N$ are N values of Y ; $p(Y_i)$ is the probability density function, which is often taken as $\frac{1}{N}$; M_1 , the first moment, is the mean of Y which is given by

$$M_1 = \sum_{i=1}^n Y_i \cdot p(Y_i) \quad (4.3)$$

According to Evans [20, 21], if the moments are known with sufficient precision for $Y = 1, 2, 3, 4$, then the Pearson system can be used for the distribution of Y . Therefore, a principle problem of statistical analysis research is to find methods of approximating the moments for a distribution.

In [18], different methods for estimating these moments have been described, i.e.,

- (1) The linear case;
- (2) The extended Taylor series approximation;
- (3) Approximation by numerical integration (quadrature methods); and
- (4) Monte Carlo methods.

The linear case which is often called stack tolerancing is the problem in which n components are X_i and the response Y is given by the weighted sum of the X_i ,

$$Y = a_0 + a_1 \cdot X_1 + \dots + a_n \cdot X_n \quad (4.4)$$

where the a_k are constants. The mean and variance of Y can be obtained by

$$\mu(Y) = a_0 + a_1 \cdot \mu_1 + \dots + a_n \cdot \mu_n \quad (4.5)$$

and

$$\sigma^2(Y) = a_1^2 \cdot \sigma_1^2 + a_2^2 \cdot \sigma_2^2 + \dots + a_n^2 \cdot \sigma_n^2 \quad (4.6)$$

where μ_i and σ_i are the mean and standard deviation of X_i , $i = 1, 2, \dots, n$. The distribution of X can usually be treated as normal with mean and variance as shown. The linear case

can also be used for the determination of derivative, $\frac{\partial f}{\partial X_k}$, in which the linearized Taylor series expansion can be implemented [18].

When the linear approximation is not accurate enough, more advanced techniques are needed. If the functional relationship (4.1) can be expressed in analytical form, the extended Taylor series approximation can be used. The basic idea is to start with the Taylor series expansion for f of (4.1) about the mean of the X_i up to the sixth order:

$$\begin{aligned}
Y &= f(X_1, X_2, \dots, X_n) \\
&= f(\mu_1, \mu_2, \dots, \mu_n) + \sum_a (X_a - \mu_a) \cdot f_a \\
&\quad + \frac{1}{2!} \cdot \sum_{ab} (X_a - \mu_a) \cdot (X_b - \mu_b) \cdot f_{ab} + \dots \\
&\quad + \frac{1}{5!} \cdot \sum_{abcde} (X_a - \mu_a) \cdot (X_b - \mu_b) \cdot (X_c - \mu_c) \cdot (X_d - \mu_d) \cdot (X_e - \mu_e) f_{abcde} + o[(X - \mu)^6]
\end{aligned} \tag{4.7}$$

where f_k , f_{jk} , etc., are the partial derivatives of f with respect to the X_k , X_j , etc., all evaluated at the point $X_i = \mu_i$, $i = 1, 2, \dots, n$, and the sums are over all indices from 1 to n , sufficient differentiability is assumed. The last term represents the neglected terms and it is understood that it includes all monomials of the sixth order and higher. In [23], [24] and [25], Turkey gave an algorithm by which the first four central moment of Y can be calculated without much effort.

On the other hand, however, when f cannot be written in analytical expressions, the above method is inapplicable and the Taylor series approximation cannot be used. In this case, for any given values of X_i , there is a way of finding the Y , e.g., by measuring it, by analog computation, by numerical calculations, by engineering methods, etc. For any function $f(X_1, X_2, \dots, X_n)$, the expected value of f is given by the integral

$$E(Y) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} f(X_1, X_2, \dots, X_n) \prod_{i=1}^n p_i(X_i) dX_i \quad (4.8)$$

where the X_i , $i = 1, 2, \dots, n$ are independent random variables with known density $p_i(X_i)$. It can be shown that $E(Y)$ can be approximated by the quadrature expression. The basic technique developed for this circumstance is described explicitly in [20 - 22].

Theoretically, any mechanism which is to be analyzed on a statistical basis can be solved by one of the method mentioned above [18]. However, except for some stack tolerancing analysis, in most cases their precision is too limited, the linear case most of all. Therefore, the most popular method is the Monte Carlo method because it allows unlimited precision.

The way Monte Carlo analysis is done is to have a population of numbers available for each X_i , $i = 1, 2, \dots, n$, which are true to the distribution assumed for X_i , draw a random sample $\{X_1, X_2, \dots, X_n\}$ from this set of populations, evaluate $Y = f(X_1, X_2, \dots, X_n)$ for this sample, and then replicate this procedure N times. This gives a random sample for Y , $\{Y_1, Y_2, \dots, Y_n\}$. Standard method of statistics are then available to analyze the distribution of Y . Since the usual behavior is for the precision of the statistical analysis to get better as N increases -- usually proportionally to \sqrt{N} -- the precision of the method is unlimited.

Suppose the probability density function of X_i is $p(X_i)$ which is given via elementary function or is specified graphically (Fig. 4.1) [26].

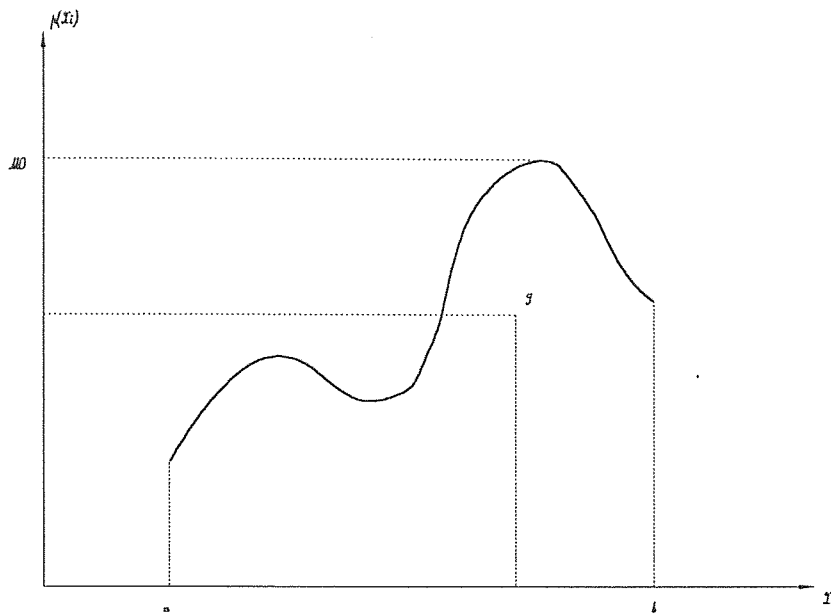


Figure 4.1 Probability Density Function for X_i

Let us assume that the value of X_i limited within a finite interval (a, b) and its density is limited, i.e.,

$$p(X_i) \leq M_0 \quad (4.9)$$

The value of X_i may be drawn as follows:

(1) Select two values γ' and γ'' of the random variable γ and generate a random point $T(\eta', \eta'')$ with coordinate

$$\begin{cases} \eta' = a + \gamma' \cdot (b - a) \\ \eta'' = \gamma'' \cdot M_0 \end{cases} \quad (4.10)$$

(2) If the point T lies below the curve $p(X)$, assume $X_i = \eta^i$; otherwise reject the pair (γ^i, γ^{i+1}) and select a new pair (γ^i, γ^{i+1}) .

(3) Repeat procedure (1) and (2) N times, thus obtain N samples of X_i .

Although the Monte Carlo method is relatively simple and straightforward, it will require a lot of experiments or calculations. In most applications, the number of random values must be very large (usually more than 100,000) to obtain satisfactory approximation. This makes it too costly or time consuming to be practical for some kinds of situation. Therefore, a more recent method, the Taguchi's method, has become widely used in engineering applications, which will be investigated in the rest of this work.

4.2 Taguchi's Method of Statistical Analysis

For a given mechanism with a set of components X_1, X_2, \dots, X_n , the response of the mechanism Y with respect to the input of the components can be determined by Equ. (4.1) if the function f can be expressed in analytical form or by N experiments from which a series of values of Y: Y_1, Y_2, \dots, Y_N can be obtained.

Taguchi's method is an experimental design approach which is to design the levels of each components so that one can approximate the moments of the probability distribution of the system response efficiently through a limited number of experiments.

To apply the method, a three-level factorial experiment is created. The mean of each component μ_i is the center level and $\mu_i \pm \sqrt{\frac{3}{2}} \cdot \sigma_i$ are the high and low levels, respectively, with σ_i as the standard deviation of the i th component. Note that each level has equal probability $\frac{1}{3}$. The function (4.1) is evaluated at all $N = 3^n$ combinations of level, giving the values Y_i , $i = 1, 2, \dots, N$ (Fig. 4.3). The first r th moments of the system response can be computed directly from the N data points according to Equ. (4.2) and (4.3). The mathematical justification of Taguchi's method is given in the next chapter.

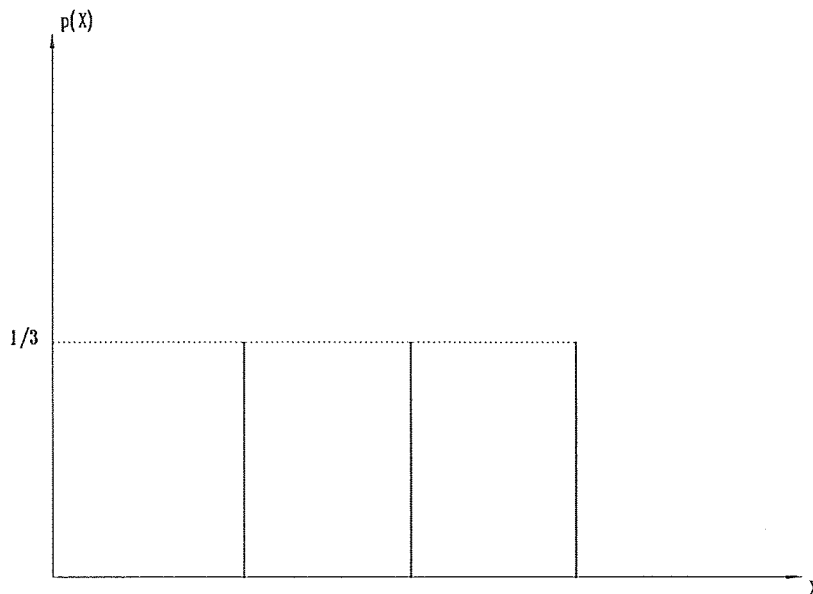


Figure 4.2 Distribution of Taguchi's Method

Taguchi's method has the following advantages over the more traditional statistical methods that we described in Section 4.1.

- (1) Taguchi's method does not required the function f to be expressed in analytical form;
- (2) The method does not require the use of partial derivatives of the function;
- (3) The method is easily described to scientists and engineers;
- (4) The total number of evaluations of the function is significantly less than that required by a Monte Carlo simulation.

However, based on the justification of Taguchi's method as explained in the next chapter, it is found that this method can match the third central moment only which significantly affect the approximation accuracy. Thus, a new method for statistical analysis - quadrature method is introduced to obtain the highest possible accuracy of approximation with a limited number of experiments.

CHAPTER 5

THE QUADRATURE METHOD AND THE EXPERIMENTATION OF SHEETMETAL FORMING PROCESSES

5.1 The Quadrature Method in Statistical Analysis

5.1.1 The Theory of Quadrature Method

Good quality and reliable performance are generally associated with small variability of the desired features of products or systems. As a rule, reducing the variability of the input variables, reduces the variability of the output, but generally, this results in higher costs. The trade-off between high quality at higher cost and vice versa is an ever present problem in the design of products. As mentioned in the last chapter, Taguchi introduced the concept of “loss function”, and established a quantitative method to deal with the economics of this tradeoff.

Quality improvement can also be achieved by reducing the effect of the variability of input variables on the variability of the output, by proper design of the system. Taguchi refers to the approach as “quality improvement by proper values of design parameters”. The approach leads to the concept of “robust design” whereby a design is robust with respect to a parameter, if it conserves good performance for a wide range in the value of a parameter.

A simple example shows the difference between the two approaches used to reduce the output variability. Figure 5.1 shows a product of height H made of three parts, A, B, and C. The tolerance of H is affected by the tolerances of the parts through the tolerance accumulation (variability propagation) and is always larger than each of the tolerances of A, B and C. If we need to reduce the tolerance of H we have two options: The first is to reduce all the tolerances of A, B and C, but this involves higher costs. Another way is to design the part H in two pieces or one piece (if acceptable): the tolerance of H can be reduced and at the same time the cost is decreased. The two options gradually reduce the tolerance of H by reducing the accumulation or the variability propagation in the product H . This designing with proper design parameters is essentially an approach to control the propagation of the variability of the input by changing the design of the product.

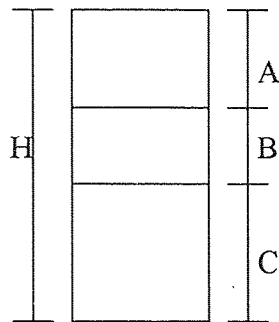


Figure 5.1 A Product Made of Three Parts

In more complex products, it is often necessary to study the effects of the variability of the components in an assembled product, on the variability of the final product to find economical ways of improving quality, or of increasing robustness of performance.

In many systems where the relations between input and output are well defined in a mathematically closed form, it is possible to apply the method of Taylor series expansion. The Monte Carlo method is also applicable although the number of input-output instances must be very large (usually more than 100,000), to obtain good approximation. However, in most real world systems the input-output relation is not known and many variables are known to affect the output. Many chemical processes involve many input variables that interact in very complicated ways. To study the effect of an input variable, a simulation run often requires large computation if carried out. Other examples are the complex structures interacting with randomly distributed external forces and for their analysis we need finite element simulation which involves heavy computation. Obviously, in such cases both of the above mentioned methods are not applicable. The Taylor Series method is not applicable because the mathematical relation is not available. The Monte Carlo method which, as mentioned earlier, requires a large number of cases and hence a significantly large amount of computations will not be practical. The quadrature method, on the other hand, can be very useful for the analysis of such systems.

Generally, a system with the function $f(X_1, X_2, \dots, X_n)$ given by Equ. [4.1] is not known explicitly. The output f is a random continuous variable which is either calculated through simulation or through computational methods such as Finite Element techniques which will be discussed later this chapter, or numerical solution of differential equations. Information about the average value of f and its variance is always required. Often, the probability density function of f is also required, and moments of higher ranks are required.

For simplicity, we begin with a function with one symmetrically distributed component X (not necessarily normally distributed), by definition, the k th moment (if it exists) about the origin of the system response function is

$$E(f^k) = \int_{-\infty}^{+\infty} [f(X)]^k \cdot p(X) dX \quad (5.1)$$

where $p(X)$ is the probability density function of X .

When the centered moment integration are evaluated, the function f is $(X-\mu)$, and $E(f^k)$ becomes

$$M_k = \int_{-\infty}^{+\infty} (X - \mu)^k \cdot p(X) dX \quad (5.2)$$

where μ is the average given by equation the first central moment to zero:

$$M_1 = \int_{-\infty}^{+\infty} (X - \mu) \cdot p(X) dX \quad (5.3)$$

Now the problem is to find the most efficient numerical quadrature formula for Equ. (5.2).

Since the probability density function of X is symmetric about its average, the distribution of X after discretization will also be symmetric, with equal number of discrete levels located around the average. If there are m levels on each side, the total number of levels of X is

$$N = \begin{cases} 2m & \text{without a level at the average} \\ 2m + 1 & \text{with a level at the average} \end{cases} \quad (5.4)$$

Because of the symmetric distribution, each level on the one side of the average has a corresponding level located on the other side of the average with equal distance from the average and same probability (except the level coincident with the average when it is used). The discrete levels can be denoted by X_j with j negative if the level is on the left side of the average and positive if it is on the right side. Similarly, their weights and distance to the average can be represented by w_j and A_j , respectively. Therefore, the expressions of the discrete levels and their probabilities are shown as follows

$$\begin{array}{rcl}
 X_{-m} = \mu - A_m & w_{-m} & \\
 \vdots & \vdots & \\
 X_{-1} = \mu - A_1 & w_{-1} & \\
 X_0 = \mu & w_0 & \\
 X_{+1} = \mu + A_1 & w_1 & \\
 \vdots & \vdots & \\
 X_{+m} = \mu + A_m & w_m &
 \end{array} \tag{5.5}$$

Note that for each pair of levels, their weights as well as their distances to the average are the same.

The polynomial order of a method is defined as the highest-order polynomial for which all polynomial of that order or less are integrated exactly by a given method. To determine the unknowns (the locations and the probabilities) in Equ. (5.5), we require that the moments up to a certain rank of the discrete distribution must be equal to the moment of the same rank of the corresponding continuous distribution, i.e.,

$$\sum_{j=-m}^m (X_j - \mu)^k \cdot w_j = \int_{-\infty}^{+\infty} (X - \mu)^k p(X) d(X) \tag{5.6}$$

Substitution in Equ. (5.6) from Equ.(5.5) and taking into account the symmetry and that $A_0=0$, we obtain

$$\sum_{j=1}^m A_j^k \cdot w_j = \int_{-\infty}^{+\infty} (X - \mu)^k p(X) d(X) \quad (5.7)$$

Note that for odd ranks, the equation reduce to zero because of symmetry. These equations are not used to calculate the unknowns because they have no informational content beyond the symmetry condition.

In the following we show the detail of the generation of a two-level discrete random variable distribution to replace a normally distributed continuous variable with average μ and variance σ^2 . Since we have two unknowns, i.e., $w_{-1}(=w_{+1})$ and A_1 , two equations are needed to get the solutions. The first equation expresses the condition that the sum total of the probabilities or weights of the discrete variables is equal to unity. The second equation expresses the requirement that the moments up to the second rank of the discrete distribution must be equal to the moment of the same rank of the corresponding continuous distribution. (The central moments of the uniform distribution and normal distribution are given in Table 5.1.) The equations are shown as follows:

$$\begin{aligned} w_{-1} + w_{+1} &= 1 \\ w_{-1}A^2 + w_{+1}A^2 &= \sigma^2 \end{aligned} \quad (5.8)$$

The solution of the above equations is

$$\begin{aligned} w_{-1} = w_{+1} &= \frac{1}{2} \\ A &= \sigma \end{aligned} \quad (5.9)$$

From the solution the two discrete levels are

$$\begin{aligned} X_{-1} &= \mu - \sigma \\ X_{+1} &= \mu + \sigma \end{aligned} \quad (5.10)$$

Now we derive the case of using a three-level discrete random variable to replace a normal distribution. Since in this case we have three unknowns, i.e., $w_{-1}(=w_{+1})$, w_0 , and A , one more equation is needed to express the requirement that the moments up to the fourth rank of the discrete distribution must be equal to the moment of the same rank of the corresponding continuous distribution. The equations are shown as follows:

$$\begin{aligned} w_{-1} + w_{+1} + w_0 &= 1 \\ w_{-1}A^2 + w_{+1}A^2 &= \sigma^2 \\ w_{-1}A^4 + w_{+1}A^4 &= 3\sigma^4 \end{aligned} \quad (5.8)$$

The solution of the above equations is

$$\begin{aligned} w_{-1} &= w_{+1} = \frac{1}{6} \\ w_0 &= \frac{4}{6} \\ A &= \sqrt{3}\sigma \end{aligned} \quad (5.9)$$

From the solution the three discrete levels are expressed as

$$\begin{aligned} X_{-1} &= \mu - \sqrt{3}\sigma \\ X_0 &= \mu \\ X_{+1} &= \mu + \sqrt{3}\sigma \end{aligned} \quad (5.10)$$

Table 5.1 Central Moments of Two Symmetric Distributions

Distribution	Uniform Distribution	Normal Distribution
Central Moment	$M_k = \int_{-a}^{+a} (X - \mu)^k \cdot \frac{1}{2a} dX$	$M_k = \int_{-\infty}^{+\infty} (X - \mu)^k \cdot \phi\left(\frac{X - \mu}{\sigma}\right) dX$
Central Moment Value	$\frac{a^k}{k+1}$	$\frac{k!}{2^{k/2} \cdot (k/2)!} \cdot \sigma^k$

In Chapter 4, the solution of Taguchi's method is

$$w_{-1} = w_0 = w_{+1} = \frac{1}{3} \quad (5.11)$$

$$A = \frac{\sqrt{3}}{2} \sigma$$

Engel showed that (5.6) must hold true for each value of k from 0 to the order of the method. The polynomial order of Taguchi's method is shown in Table 5.1.

It is found that the discrete distribution in Fig. 4.3 have the same first three centered moments as the normal distribution of component X but have the fourth moment of $\frac{3}{2}\sigma^4$ instead of $3\sigma^4$. On the contrast, the derivation of the quadrature method mentioned above ensures that this discrete distribution will reproduce correctly the first five moments of a normal distribution with mean μ and variance σ^2 (Figure 5.3).

Table 5.2 Approximation of Centered Moment Using Taguchi's Method

k	Exact value of centered moment	Taguchi's approximation value
1	0	0
2	σ^2	σ^2
3	0	0
4	$3\sigma^4$	$\frac{3}{2}\sigma^4$

If the central moments of the continuous distribution up to the k th rank are known, it is easy to obtain from Equ. (5.6) the values for the set of parameters $\{w_{-m}, \dots, w_{-1}, w_0, w_1, \dots, w_m, A_0, A_1, \dots, A_m\}$. For example, assuming X is normal distributed, Table 5.3 gives the different values of the parameter set when X is discretized into 2, 3 or 4 levels.

Table 5.3 Discretization of a Normal Distribution at Different Levels

No. of Levels	Probability	Distance to the average
2	$\frac{1}{2}$	σ
3	$\frac{1}{6}, \frac{4}{6}, \frac{1}{6}$	$-\sqrt{3}\sigma, 0, \sqrt{3}\sigma$
4	$\frac{3-\sqrt{6}}{12}, \frac{3+\sqrt{6}}{12}, \frac{3+\sqrt{6}}{12}, \frac{3-\sqrt{6}}{12}$	$-\sqrt{3+\sqrt{6}}\sigma, -\sqrt{3-\sqrt{6}}\sigma, \sqrt{3-\sqrt{6}}\sigma, \sqrt{3+\sqrt{6}}\sigma$

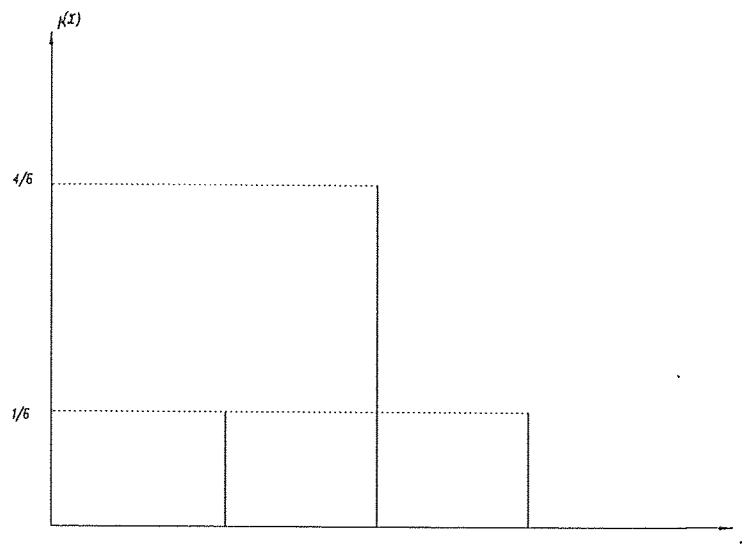


Figure 5.2 Distribution of Quadrature Method

5.1.2 Discussion

(1) The formulation above can be extended to the case of n components which are independent and normally distributed. The expectation becomes an integral over n dimensions:

$$E(f^k) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} [f(X_1, X_2, \dots, X_n)]^k \cdot \phi\left(\frac{X_i - \mu_i}{\sigma_i}\right) dX_i \quad (5.12)$$

(2) We should note that all the central moments with odd rank are automatically equal to zero due to the symmetry condition. Therefore, equations that express this fact have no information content and cannot be used to help obtain the solution.

(3) As can be seen from comparing the two and three level problems, that the addition of one level coincide with the average increased the rank of the central moment

from two to four. Non-centrally located levels have to be added in pairs to conserve the symmetry. An addition of a pair generates two unknowns: the distance from the average A and a weight w . Two added equations will guarantee the matching of two additional ranks of the central moments.

(4) If an input variable is generated through measurements or other means, there is no need to know exactly its distribution. Suffice to calculate from data samples the moments to ranks that are needed by the analysis. In other words the method of generating the discrete levels and their weights is general and is applicable to any distribution provided, of course, that the distribution is symmetric or reasonably close to one.

(5) It should be noted that to solve Equ. (5.7), we can assume some of the weights or some of the locations of the discrete levels as long as we abide by the constraints on the sum of the weights or the symmetry condition. However, any such assumption will reduce the number of the equations that can be posed, and hence the highest rank of the central moment that can be marched. It is clear that the efficiency of the discretization is diminished, because we end up with the same number of discrete levels and consequently with the same number of computations or experiments while we have reduced the accuracy of the output statistical characteristics. In other words, the 'natural solution' of Equ. (5.7) is the most efficient solution in that it gives moments with the highest rank for a given number of discrete variables. For example, Table 5.4 gives the three-level approximation of centered moment using quadrature method where the highest rank matched is fifth.

Table 5.4 Approximation of Centered Moment Using Quadrature Method

k	Exact value of centered moment	Quadrature approximation value
1	0	0
2	σ^2	σ^2
3	0	0
4	$3\sigma^4$	$3\sigma^4$
5	0	0
6	$15\sigma^6$	$27\sigma^6$

(6) The relation between the number of levels n and the highest rank of the central moment M is given by

$$M = 2 \cdot (n - 1) \quad (5.13)$$

Equ. (5.13) shows the relationship between the rank of the moment that is matched and the number of the discrete variables for one continuous random variable. But the main concern is to determine accurately the statistical characteristics of the output. The average, and the variance of the output are often all the required information, but occasionally higher ranks of the central moments are required to fit known distributions to the output statistical distribution. The problem is to find the relation between the highest rank matched by an input variable and the accuracy of the output statistical characterization.

Let P_{ST} be a polynomial of degrees S and T in the variables X and Y respectively. The highest term will be the term $X^S Y^T$. The moment of rank U of the polynomial is given by

$$M_U(P_{ST}) = \iint P_{ST}^U \cdot g(X)h(Y)dXdY \quad (5.14)$$

The integrand contains a highest term which is given by

$$\text{Highest Term} = \iint X^{SU} \cdot Y^{TV} \cdot g(X)h(Y)dXdY \quad (5.15)$$

It is clear that to match the moment of rank U of the polynomial, we need to match the moments of rank $S \cdot U$ for X and of rank $T \cdot U$ for Y. Reference to Equ.(5.13) shows that the required numbers N_X and N_Y for X and Y respectively are given by

$$\begin{aligned} N_X &= S \cdot \frac{U}{2} + 1 \\ N_Y &= T \cdot \frac{U}{2} + 1 \end{aligned} \quad (5.16)$$

For example, for a function with two variables $f(X_1, X_2) = X_1^2 \cdot X_2$, if we require the moment of f of rank k, the required moments for X_1 and X_2 are 2k and k, respectively.

5.2 Application of Quadrature Method in the Experimentation of Sheet Metal Forming Process

5.2.1 Description of Experiments

In Section 2.2, the characteristics of the sheetmetal drawing operation and various variables that affect the processes have been identified. For the three categories of variables, the geometry of the tools and the material property of the blank is usually

designed according to the geometric specification of the workpiece, and thus, is generally considered as fixed variables at the early stage of design. Therefore, the variables that are adjustable during the design process are the interface variables which include the friction and lubrication condition between the tools and the workpiece. Repetitive experiments also indicated that the thickness of the blank is another important variable which to a great extent will affect the quality of the deep drawing so that deformation defects such as cracks, orange-peel, etc. can be avoided.

Compared to the sliding between the punch and the workpiece, the sliding displacements between the holder and the workpiece as well as between the die and the workpiece are relatively small and about the same value. The friction coefficients of these two interfaces are set equal values in this experimentation. Therefore, three variables are chosen in the experiments which include the thickness of the blank (t), the friction coefficient between the die (the blank holder) and the workpiece (df), and the friction coefficient between the punch and the workpiece (pf).

5.2.2 Statistical Experimentation

The results of the five steps deep drawing are shown in Fig. 5.2. During the statistical experimentation, the standard deviation of thickness σ_t is defined as 5% of the mean of thickness μ_t . The standard deviation of the friction, i.e., σ_{df} and σ_{pf} , are defined 10% of their respective means, i.e., μ_{df} and μ_{pf} (Table 5.4). According to the quadrature method,

the normal distribution is approximated with the set of parameters $\{\frac{1}{6}, \frac{4}{6}, \frac{1}{6}, -\sqrt{3}\sigma, 0, \sqrt{3}\sigma\}$. The three-levels for the three meshing parameters are listed in Table 5.5.

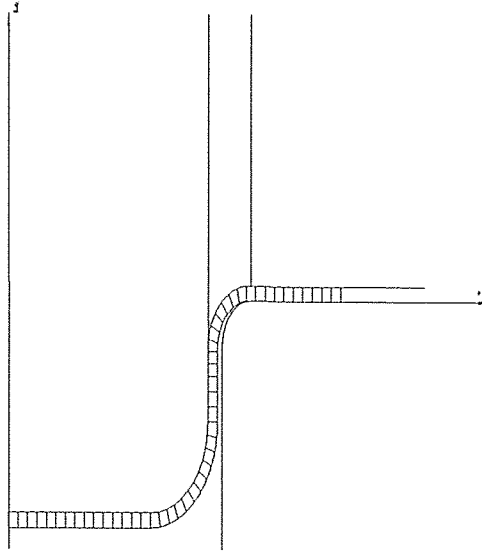


Figure 5.3 The Result of Deep Drawing

Table 5.5 Three-levels for the Meshing Parameters

	μ	σ	$\mu - \sqrt{3}\sigma$	μ	$\mu + \sqrt{3}\sigma$
thickness (t)	0.00082	0.000041	0.0007490	0.00082	0.0008910
die (blank holder) friction (df)	0.1	0.01	0.08268	0.1	0.1173
punch friction (pf)	0.25	0.025	0.2067	0.2500	0.2933

There are $3 \times 3 \times 3 = 27$ different combinations of the three parameters, which can be developed into 27 experiments of deep drawing (Appendix II). These experiments can be easily simulated on SUN workstation using ABAQUS. The purpose of the study is to study the effect of different parameters on the final shape of workpiece as well as the distribution of the maximum principle stresses and the reaction forces on the tools, so that the weak points on the workpiece can be located and the requirements for the tooling can be determined.

The results of experiment is listed in Table A2.1 of Appendix II.

5.2.3 Analysis of Experimentation Results

In order to find the effect of different parameters on the maximum principle stresses and the reaction force applied on the tools during deformation, it is necessary to separate them and study them individually. The analysis is performed by isolating the experimental data for the each level of each parameter and finding the average, which gives the value that combines the information of different levels of the other two parameters, and thus, can approximate the relative realistic value under that specific deformation condition.

For example, the sequence of finding the maximum principle stress (+) for the parameter $t = 0.0008910$ is explained as follows:

- (1) find the nine cases of experiments in which the thickness of sheet metal is 0.0008910, i.e., case 1 through 9;

- (2) multiply the nine values of maximum principle stress (+) with their respect weights;
- (3) add up the nine weighted values;
- (4) since the weight for the thickness value 0.0008910 is $\frac{1}{6}$, the sum obtained from (3) has to be divided by $\frac{1}{6}$, the result is the average of the maximum principle stress (+) for the parameter $t = 0.0008910$.

Table 5.6 Statistical Characteristics of Stresses and Reaction Forces

	max principle stress (+) ($\times 10^8$)	max principle stress (-) ($\times 10^8$)	R reac force on the punch ($\times 10^4$)	Z reac force on the punch ($\times 10^4$)	R reac force on the die (holder) ($\times 10^4$)	Z reac force on the die (holder) ($\times 10^4$)
μ	3.150597	-4.16708	-6.15157	17.5442	6.53129	-7.5441
M_2	0.061218	0.003795	0.11685	0.32400	0.22349	0.3239
M_3	0.006193	-0.00028	0.01439	-0.0876	0.04769	0.0874
M_4	0.022852	0.000071	0.04701	0.26862	0.17035	0.2684
Skewness	0.408889	-1.20497	0.36017	-0.4750	0.45134	0.4744
Kurtosis	2.99237	3.000024	2.98426	2.9072	2.9422	2.9073

By the same token, the maximum principle stresses (-) as well as the tool reaction forces for the nine parameters levels can also be calculated, which are listed in Table A2.2 of Appendix II. The variation of stresses and reaction forces with respect to the three parameters are listed in the Figure A3.1 - A3.3.

In order to study in depth the variation of stresses and forces, their changes with respect to the friction of die (holder) and the friction of punch are displayed at the same time in three-dimensional drawings as shown in Figure A3.4(a) - (f), in which the x-axis is the die (holder) friction and the y-axis is the punch friction.

In the following analysis, only the absolute values of the stresses and forces will be considered, and since the Z force on the holder is always zero it will not be discussed.

(1) With the decrease of the thickness of sheet metal, only the positive maximum principle stresses will increase, all the rest stresses and forces will decrease except for the R force on the holder which gets a minimum point at the average of thickness. It is also found that most of the stresses and forces change nearly linearly with respect to the thickness. From these results, it is suggested that the thickness of raw material should be made as thin as possible provided that the positive maximum principle stresses will not exceed the strength limit of the material and the geometrical and physical characteristic requirements of the final workpiece should be satisfied.

(2) For the friction coefficient between the die (the blank holder) and the workpiece, the negative maximum principle stress gets a minimum value at its average while the R force on the die gets a maximum value at its average, all the rest parameters will decrease with the decrease of the friction coefficients. This implies that the surface finish of the die and blank holder should be made smooth enough to reduce the stress within workpiece and the reaction force on the tools, and facilitate the flow of sheet metal during deformation. At the same time, however, it should be

controlled to ensure that the workpiece can be held firmly between the die and the blankholder.

(3) Due to the relatively large sliding between the punch and the workpiece, with the decrease of the punch friction, the behaviors of the stresses and the reaction forces are quite different from the case when die (blank holder) friction changes: most of these parameters get a maximum point at their respect average while the negative maximum principle stress and the R force on the punch will increase continuously. This fact suggests that the friction coefficient between the punch and the workpiece will have to be kept to a certain value to allow relatively smooth metal flow during the deformation process.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

This thesis mainly consists of three parts, i.e., analysis of sheetmetal forming processes, computer aided finite element modeling and analysis of sheetmetal forming, and the introduction of quadrature method in the experimentation of sheet metal forming. The last part is one of the major contributions of this work.

Sheetmetal forming is basically a process of converting a given shape of a sheetmetal, often as cold rolled steel strip, to the required shape of the workpiece through multi-station dies, where operations such as cutting, bending and drawing are performed sequentially to produce the part. From metal flow point of view, this is a process of large deformation, which is influenced by many factors such as geometry of tools, friction coefficients, physical property of the stock material and the thermal conditions of the deformation zone, etc. With all these variables are concerned, it is very difficult to obtain the part with right dimensions at one time, which in the machining shop environment entails a slow process of repeated fine readjustment of the dies. The process is often time consuming and labor intensive [30].

To acquire sufficient understanding of the process, the finite element analysis is introduced to simulate the metal forming process. There are many commercially available

software packages for finite element analysis. However, as far as the characteristics of the sheet metal forming processes are concerned, the program should have the following capabilities:

- (1) the ability to deal with large displacement and large strain;
- (2) the ability to deal with metal-tool contact and relative sliding;
- (3) the ability to deal with friction;
- (4) the ability to accept user-defined material property; and
- (5) the ability to calculate the energy dissipation and to solve coupled thermal-elastic-plastic analysis.

Based on the above requirements for the software, the ABAQUS program is chosen as the primary tool to model and analyze the sheetmetal forming process. The ABAQUS program proved to be capable of dealing with almost all the complexities of the process. Moreover, the program is very efficient in dealing with metal-tool interactions. Its algorithm is very efficient and results in very short CPU time for reasonably complex process.

One of the major objectives of finite element analysis is to rationalize specifications of the model and the process. A good understanding of the process could help avoid too stringent or too loose specifications and thus strike an economically sound balance between the two extremes. In this work, the study is mainly concentrated on the changes of the maximum principle stresses in the workpiece and the maximum reaction forces on the tools with respect to the variations of the sheetmetal thickness and the metal-tool frictions during deformation. In fact, it is possible to find the most important physical and

geometrical properties of the workpiece through the study of parameters mentioned above.

(1) Material ductility: Material ductility is measured by the magnitude of the tensile strain in a tensile test that the material can sustain before breaking. This can be used as a basis to predict the behavior of the material in the forming cup. The strain builds up with the progress of the drawing. If the accumulated strain increase beyond the ductility of the material, it is expected that cracks may occur during the process and which will appear in the final product.

(2) Material strain hardening index: This material property has an important effect on the deformation distribution in the part. High strain hardening index prevents localization of the deformation and instead, tends to spread the strain. Thus, because the strain is more homogeneous and less localized, the overall attainable deformation can be larger. In general, higher strain rate hardening are preferable.

(3) Uniformity of sheetmetal thickness: Non-uniform thickness, combined with low strain hardening index can promote localization of strain a premature cracking or localized orange-peel.

(4) Friction at metal-tool interface: Friction has a major role in shaping the part. When friction is high, the relative movement between the deformed metal and the tools will become more difficult and could thus change the final dimensions and shape of the part.

Since this study is to relate the variability of the system response (i.e., stresses, reaction forces) to the variability of the input (i.e., thickness, friction) through computer

simulation, it is not possible to express the relations between input and output variables in closed mathematical form. The Taylor series method is not possible and the Monte Carlo method will require too many experiments and calculations to be practical. The Taguchi's method, however, is suitable for such cases since it reduces the number of interests to be considered in the statistical analysis to a manageable level [31]. It does so by replacing the continuous random variables with corresponding probabilities known usually as weights. The discrete distribution matches some of the global statistical features of the original variables distributions, such as symmetry, and moments of chosen ranks.

Taguchi's method is basically a discretization of a normal distribution with a three-level discrete distribution, i.e., $\mu - \sqrt{\frac{2}{3}}\sigma$, μ , $\mu + \sqrt{\frac{2}{3}}\sigma$, each with weight $\frac{1}{3}$. The highest rank of centered moment that Taguchi's method can reach is three. The quadrature method, with three levels at $\mu - \sqrt{3}\sigma$, μ , and $\mu + \sqrt{3}\sigma$, and with weight of $\frac{1}{6}$, $\frac{4}{6}$, and $\frac{1}{6}$, respectively, can match up to the fifth rank of centered moment. And thus, the approximation errors are greatly reduced in the quadrature method than in Taguchi's. Moreover, with the method suggested in this work, the improved set of parameters can be expanded to N-level variables, which will give up to $2 \cdot (N - 1)$ rank of the centered moments.

The quadrature method is implemented in the experimentation of sheetmetal forming process. It is a set of three-level factorial experiments. Therefore, the total number of experiments is $3^3 = 27$.

In order to study the influence of one individual input on a specific output, it is necessary to isolate the output from the effects of other inputs. This is achieved through weighted isolation which has been explained in detail in Chapter 4. The analysis of experiment results is also discussed.

6.2 Future Work

Up to now the discussion is based on the assumption that all of the input variables are normally distributed. In the actual world, however, there are a lot of variables whose distribution is not normal, instead, their distribution may be other symmetrical or non-symmetrical ones. In the future study, ways will have to be found to deal with these classes of problems.

APPENDIX I

THE ABAQUS PROGRAM FOR DEEP DRAWING

```
*HEADING, UNSYMM
DEEP DRAWING OF CYLINDRICAL CUP WITH CAX4R
*RESTART, WRITE, FREQ=25
*NODE
101,
181, 0.1
301, 0.0, 0.00082
381, 0.1, 0.00082
*NGEN, NSET=BOT
101, 181, 2
*NGEN, NSET=TOP
301, 381, 2
*NSET, NSET=WRKPC
BOT, TOP
*NODE, NSET=DIE
100, 0.1, -0.05
*NODE, NSET=PUNCH
200, 0.0, 0.05
*NODE, NSET=HOLDER
300, 0.1, 0.05
*NSET, NSET=TOOLS
PUNCH, DIE, HOLDER
*NSET, NSET=CENTER
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201, 101, 103, 303, 301
*ELGEN, ELSET=BLANK
201, 40, 2, 2
*ELEMENT, TYPE=IRS21A
441, 341, 343, 300
*ELGEN, ELSET=HOLDER
441, 20, 2, 2
*ELEMENT, TYPE=IRS21A
131, 131, 133, 100
*ELGEN, ELSET=DIE
131, 25, 2, 2
*ELEMENT, TYPE=IRS21A
301, 301, 303, 200
*ELGEN, ELSET=PUNCH
301, 25, 2, 2
*ELSET, ELSET=TOOLS
PUNCH, DIE, HOLDER
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BLANK, TOOLS
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*ORIENTATION, NAME=LOCAL
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0, 0.0
*MATERIAL, NAME=STEEL
*ELASTIC
2.1E11, 0.3
*PLASTIC, HARD=ISO
0.91294E+08, 0.00000E+00
0.10129E+09, 0.21052E-03
0.11129E+09, 0.52686E-03
0.12129E+09, 0.97685E-03
0.13129E+09, 0.15923E-02
0.14129E+09, 0.24090E-02
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0.50129E+09, 0.89928E+00
0.51129E+09, 0.98274E+00
0.52129E+09, 0.10721E+01
```

```

*RIGID SURFACE, TYPE=SEGMENTS, ELSET=DIE
START, 0.05125, -0.060
LINE, 0.05125, -0.005
CIRCL, 0.05625, 0.0, 0.05625, -0.005
LINE, 0.1, 0.0
*RIGID SURFACE, TYPE=SEGMENTS, ELSET=HOLDER
START, 0.1, 0.00082
LINE, 0.05630, 0.00082
CIRCL, 0.05625, 0.00087, 0.05630, 0.00087
LINE, 0.05625, 0.06
*RIGID SURFACE, TYPE=SEGMENTS, ELSET=PUNCH, SMOOTH=0.013
START, 0.05, 0.060
LINE, 0.05, 2.250782E-3
CIRCL, 0.0, 0.001, 0.0, 1.001
*INTERFACE, ELSET=DIE
*FRICTION
0.1
*INTERFACE, ELSET=HOLDER
*FRICTION
0.1
*INTERFACE, ELSET=PUNCH
*FRICTION
0.25
*STEP, INC=10, NLGEOM
PUSH THE BLANKHOLDER DOWN BY A PRESCRIBED DISPLACEMENT
*STATIC
1.0, 1.0
*BOUNDARY
CENTER, 1, 1
DIE, 1, 1
DIE, 2, 2
DIE, 6, 6
PUNCH, 1, 1
PUNCH, 2, 2
PUNCH, 6, 6
HOLDER, 1, 1
HOLDER, 2, 2, -1.75E-8
HOLDER, 6, 6
*MONITOR, NODE=200, DOF=2
*PRINT, CONTACT=YES
*NODE PRINT, NSET=TOOLS, FREQ=100
COORD, U, RF
*EL PRINT, ELSET=ALL, FREQ=500
S, E
*EL PRINT, ELSET=ALL, FREQ=500
PRIN
*NODE FILE, NSET=TOOLS, FREQ=10
U, RF
*END STEP
*STEP, INC=10, NLGEOM
APPLY THE PRESCRIBED FORCE ON THE BLANKHOLDER AND RELEASE
THE DISPLACEMENT
*STATIC
1.0, 1.0

```

```
*BOUNDARY, OP=NEW
CENTER, 1, 1
DIE, 1, 1
DIE, 2, 2
DIE, 6, 6
PUNCH, 1, 1
PUNCH, 2, 2
PUNCH, 6, 6
HOLDER, 1, 1
HOLDER, 6, 6
*CLOAD
HOLDER, 2, -100000.0
*END STEP
*STEP, INC=500, NLGEOM
MOVE THE PUNCH DOWN
*STATIC
0.01, 1.0, 1.0E-6
*CONTROLS, ANALYSIS=DISCONTINUOUS
*BOUNDARY, OP=NEW
CENTER, 1, 1
DIE, 1, 1
DIE, 2, 2
DIE, 6, 6
PUNCH, 1, 1
PUNCH, 2, 2, -0.06
PUNCH, 6, 6
HOLDER, 1, 1
HOLDER, 6, 6
*CLOAD
HOLDER, 2, -100000.0
*END STEP
*STEP, INC=100, NLGEOM
FIX ALL NODES AND REMOVE THE IRS ELEMENTS
*STATIC
1.0, 1.0, 1.0, 1.0
*BOUNDARY, FIXED
WRKPC, 1, 2
*MODEL CHANGE, REMOVE
TOOLS
*CLOAD, OP=NEW
HOLDER, 2, 0.0
*END STEP
*STEP, INC=50, NLGEOM
REPLACE THE BOUNDARY CONDITIONS BY THE REGULAR SET
*STATIC
0.1, 1.0, 1.0E-6
*BOUNDARY, OP=NEW
181, 2
CENTER, 1, 1
*END STEP
```

APPENDIX II

EXPERIMENTAL DATA

Table A2.1 Different Combinations of Three Parameters

case	thickness	die (holder) friction	punch friction	weights
1	0.0008910	0.1173	0.2933	1/216
2	0.0008910	0.1173	0.2500	4/216
3	0.0008910	0.1173	0.2067	1/216
4	0.0008910	0.1000	0.2933	4/216
5	0.0008910	0.1000	0.2500	16/216
6	0.0008910	0.1000	0.2067	4/216
7	0.0008910	0.08268	0.2933	1/216
8	0.0008910	0.08268	0.2500	4/216
9	0.0008910	0.08268	0.2067	1/216
10	0.0008200	0.1173	0.2933	4/216
11	0.0008200	0.1173	0.2500	16/216
12	0.0008200	0.1173	0.2067	4/216
13	0.0008200	0.1000	0.2933	16/216
14	0.0008200	0.1000	0.2500	64/216
15	0.0008200	0.1000	0.2067	16/216
16	0.0008200	0.08268	0.2933	4/216
17	0.0008200	0.08268	0.2500	16/216
18	0.0008200	0.08268	0.2067	4/216
19	0.0007490	0.1173	0.2933	1/216
20	0.0007490	0.1173	0.2500	4/216
21	0.0007490	0.1173	0.2067	1/216
22	0.0007490	0.1000	0.2933	4/216
23	0.0007490	0.1000	0.2500	16/216
24	0.0007490	0.1000	0.2067	4/216
25	0.0007490	0.08268	0.2933	1/216
26	0.0007490	0.08268	0.2500	4/216
27	0.0007490	0.08268	0.2067	1/216

Table A2.2 Results of Experiments

case	maximum principle stress (+) ($\times 10^8$)	maximum principle stress (-) ($\times 10^8$)	R, Z reaction forces on the punch ($\times 10^4$)	R, Z reaction forces on the die ($\times 10^4$)
1	3.2010	-4.1201	-6.9938, 18.253	7.0688, -8.2528
2	3.0307	-4.2071	-6.8538, 17.835	7.6134, -7.8352
3	3.2359	-4.1061	-6.7454, 18.366	8.0365, -8.3663
4	3.1237	-4.1548	-6.3787, 18.187	6.5922, -8.1867
5	3.1324	-4.1500	-6.6577, 18.219	6.9627, -8.2195
6	3.1326	-4.1316	-6.8930, 18.224	7.2875, -8.2242
7	2.6934	-4.3486	-6.3472, 17.155	6.0866, -7.1554
8	2.7422	-4.3571	-6.2952, 17.280	6.3336, -7.2804
9	2.8178	-4.3745	-6.1808, 17.486	6.6813, -7.4864
10	3.0474	-4.1956	-6.3420, 17.186	6.6959, -7.1865
11	3.3113	-4.1685	-6.2196, 17.808	7.2058, -7.8075
12	3.9585	-4.0282	-5.9682, 18.814	7.7628, -8.8139
13	3.2589	-4.1460	-6.1297, 17.819	6.2714, -7.8190
14	3.2590	-4.1371	-6.2588, 17.835	6.5617, -7.8346
15	2.6462	-4.1829	-6.3106, 16.364	6.4347, -6.3643
16	2.8477	-4.2917	-5.7699, 16.938	5.7453, -6.9382
17	2.8949	-4.2902	-5.6828, 17.063	5.9475, -7.0634
18	2.9511	-4.2235	-5.7608, 17.200	6.3214, -7.2002
19	3.3753	-4.1684	-5.7948, 17.081	6.3361, -7.0808
20	3.6392	-4.1413	-5.6724, 17.703	6.8460, -7.7018
21	4.2864	-4.0010	-5.4210, 18.709	7.4030, -8.7082
22	3.2834	-4.1216	-5.7836, 17.171	5.8329, -7.1714
23	3.1878	-4.1096	-5.8335, 16.967	6.0482, -6.9667,
24	3.0469	-4.1954	-5.7329, 16.652	6.4177, -6.6521
25	3.0169	-4.2341	-5.0478, 16.697	5.3440, -6.6971
26	3.0327	-4.2184	-5.2348, 16.731	5.5554, -6.7313
27	3.0778	-4.1417	-5.5621, 16.830	5.9116, -6.8304

APPENDIX III

THE RELATIONS BETWEEN DIFFERENT PARAMETERS AND THE MECHANICAL CHARACTERISTICS OF THE PART

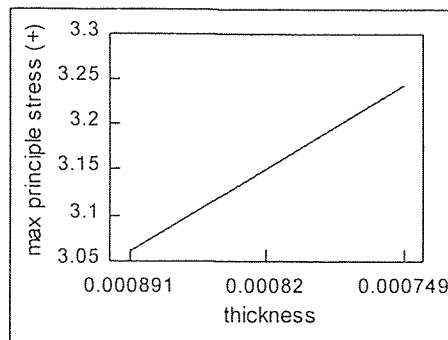


Figure A3.1(a) Maximum Principle Stresses (+) vs. Thickness

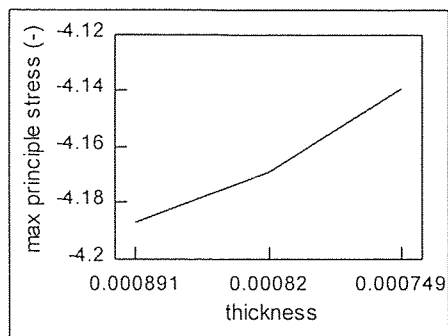


Figure A3.1(b) Maximum Principle Stresses (-) vs. Thickness

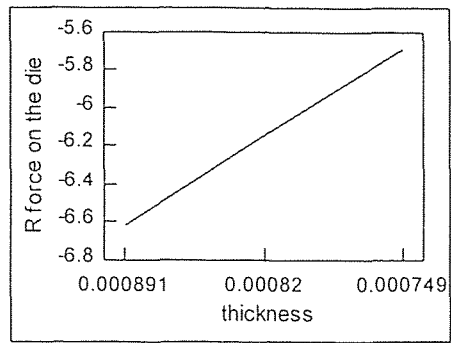


Figure A3.1(c) R Force on the Die vs. Thickness

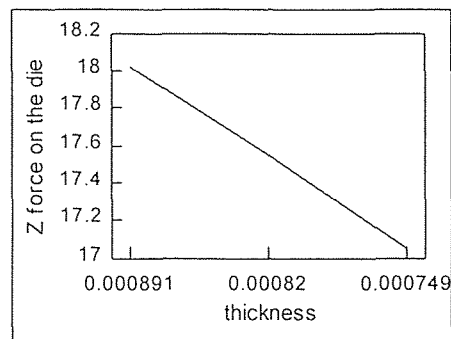


Figure A3.1(d) Z Force on the Die vs. Thickness

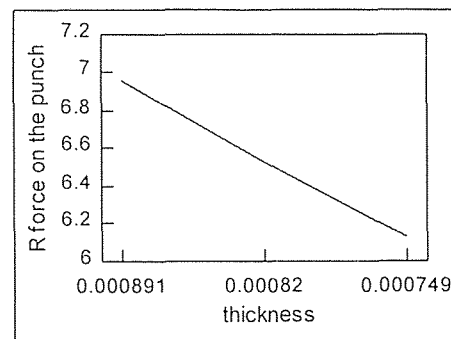


Figure A3.1(e) R Force on the Punch vs. Thickness

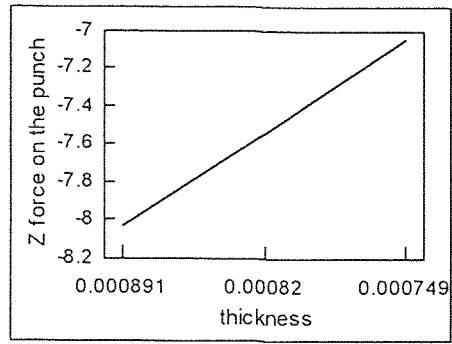


Figure A3.1(f) Z Force on the Punch vs. Thickness

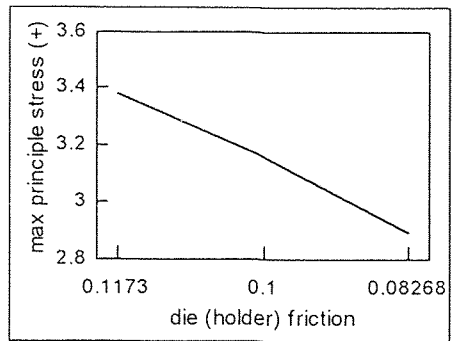


Figure A3.2(a) Maximum Principle Stresses (+) vs. Die Friction

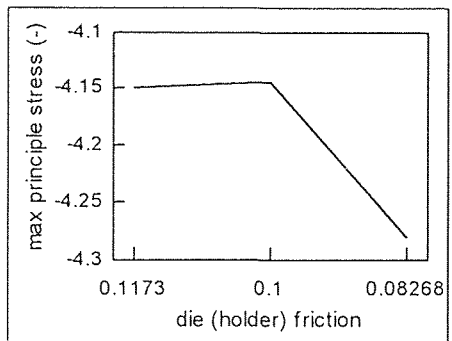


Figure A3.2(b) Maximum Principle Stresses (-) vs. Die Friction

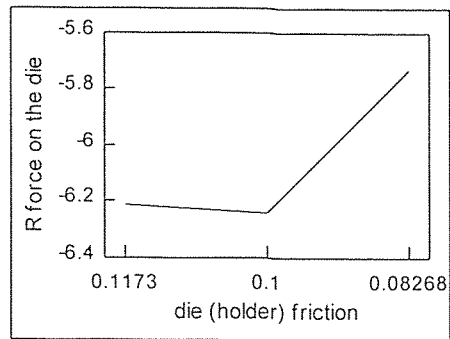


Figure A3.2(c) R Force on the Die vs. Die Friction

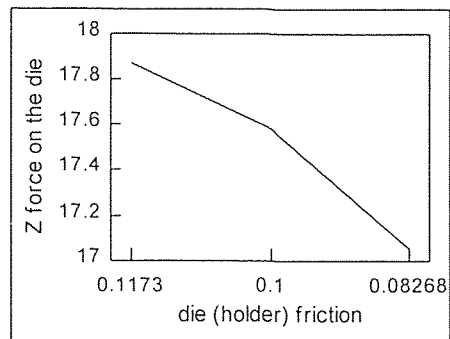


Figure A3.2(d) Z Force on the Die vs. Die Friction

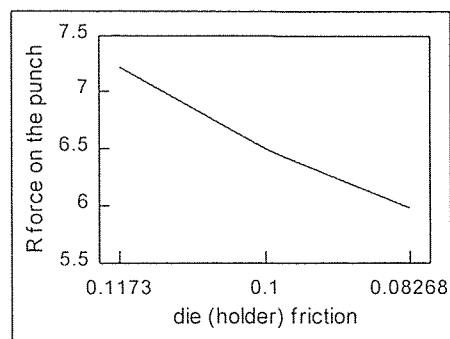


Figure A3.2(e) R Force on the Punch vs. Die Friction

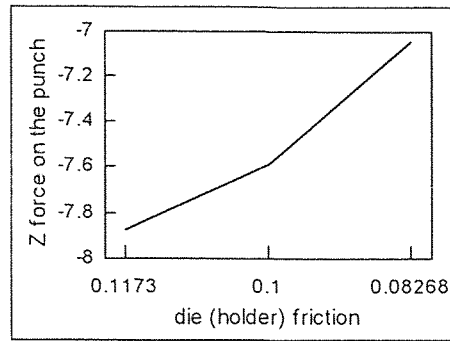


Figure A3.2(f) Z Force on the Punch vs. Die Friction

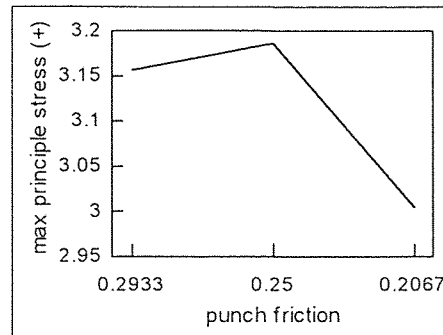


Figure A3.3(a) Maximum Principle Stresses (+) vs. Punch Friction

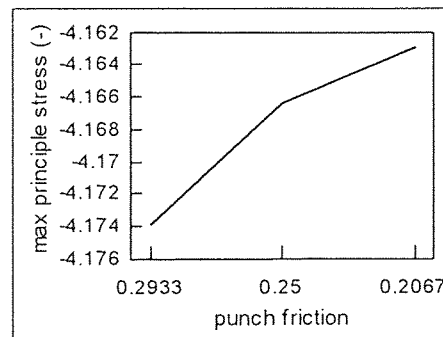


Figure A3.3(b) Maximum Principle Stresses (-) vs. Punch Friction

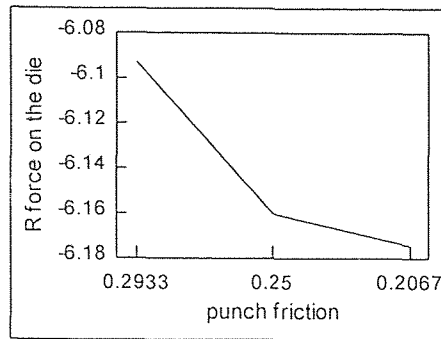


Figure A3.3(c) R Force on the Punch vs. Punch Friction

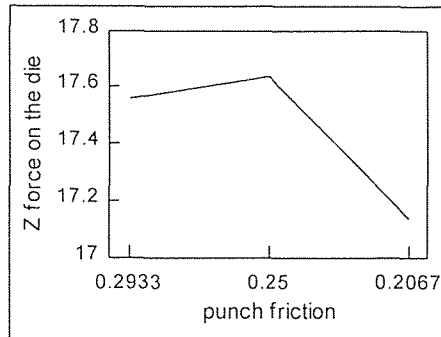


Figure A3.3(d) Z Force on the Punch vs. Punch Friction

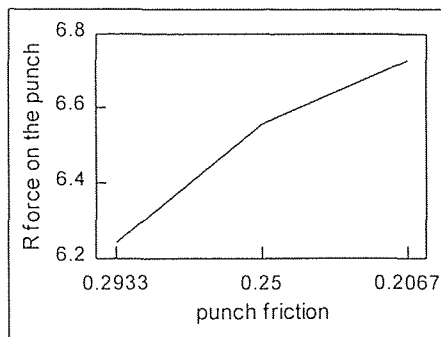


Figure A3.3(e) R Force on the Punch vs. Punch Friction

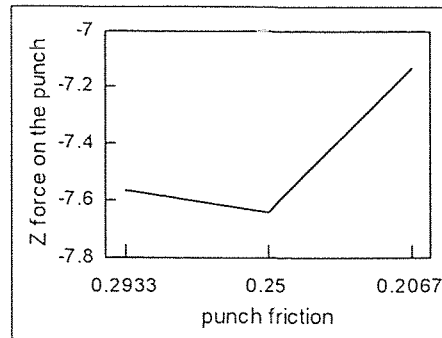
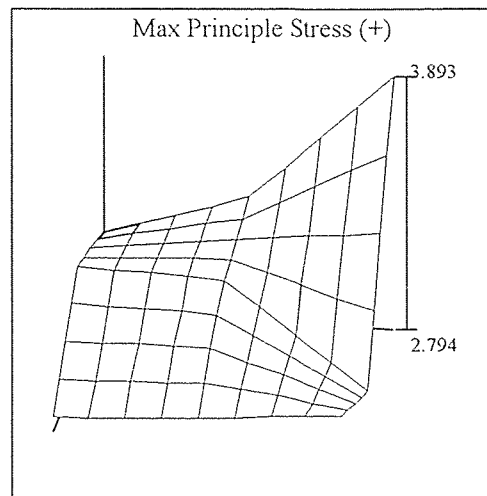
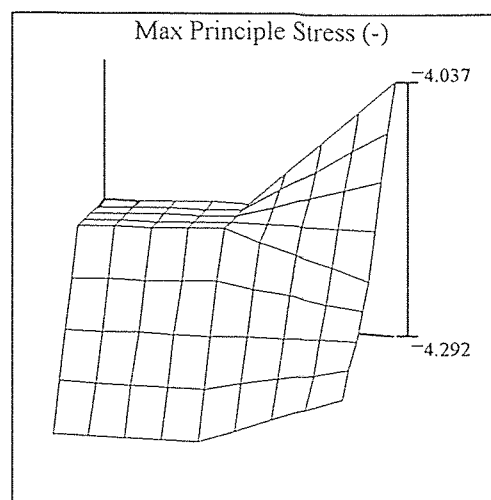


Figure A3.3(f) Z Force on the Punch vs. Punch Friction



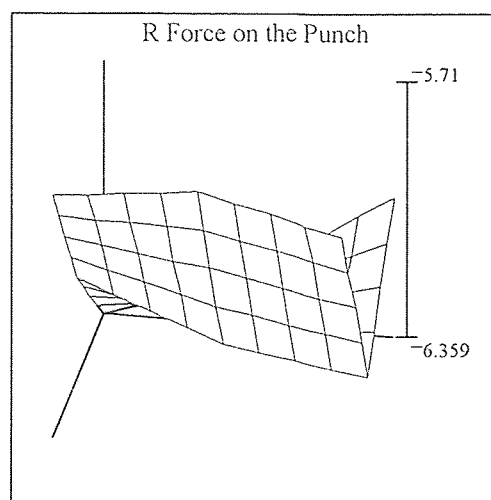
MA1

Figure A3.4(a) Maximum Principle Stresses (+) vs. Die Friction (X) and Punch Friction (Y)



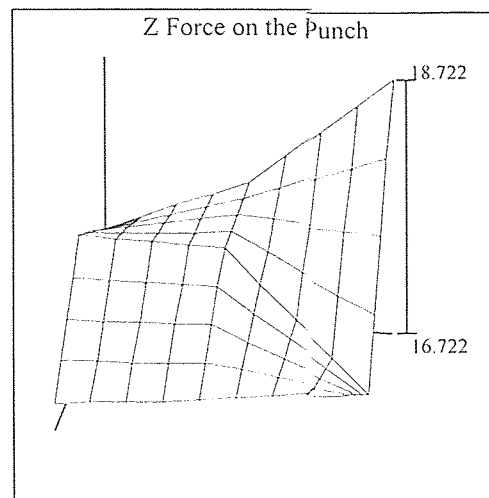
MA2

Figure A3.4(b) Maximum Principle Stresses (-) vs. Die Friction (X) and Punch Friction (Y)



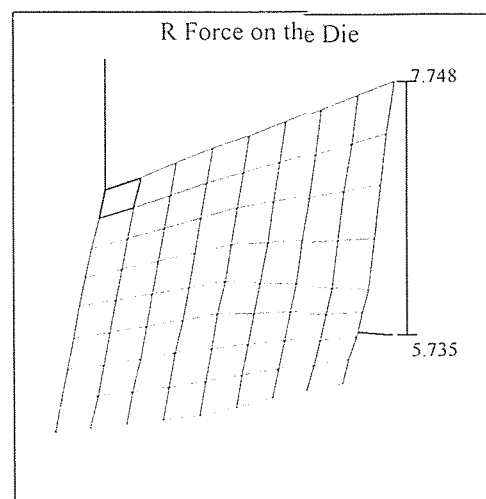
MA3

Figure A3.4(c) R Force on the Die vs. Die Friction (X) and Punch Friction (Y)



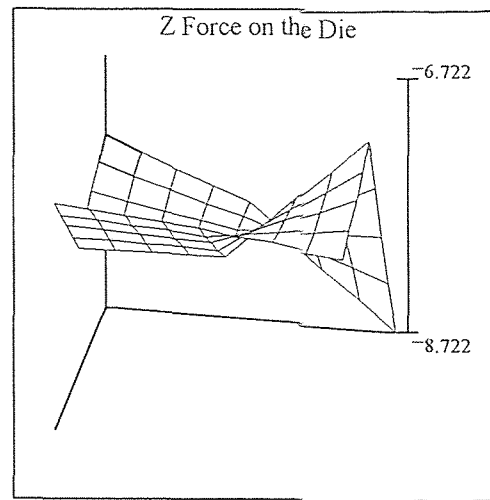
MA4

Figure A3.4(d) Z Force on the Die vs. Die Friction (X) and Punch Friction (Y)



MA5

Figure A3.4(e) R Force on the Punch vs. Die Friction (X) and Punch Friction (Y)



MA6

Figure A3.4(f) Z Force on the Punch vs. Die Friction (X) and Punch Friction (Y)

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