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## ABSTRACT

### EXTERNAL CAVITY LASER POWER STABILIZER

by  
Jie Ding

In this thesis, we want to design a stabilizer which does not depend on laser source--with external cavity.

In the first part of the thesis ( Chapter 2 and 3 ), we discuss the wave propagation in crystals and the modulation of optical radiation. From the main two types electro-optic modulations, Phase Modulation and Transverse Modulation, we know that the transverse modulation shows an increase in the frequency limit or useful crystal length of  $(1 - c_0/nc_m)^{-1}$ , and we will use this type modulation in this thesis.

The second part of the thesis is the procedure of design and experiment. Because we used an electrooptic modulator which characteristic is  $L/d = 3$ , its halfwave voltage is 1200V, we have to find an amplifier circuit which can output high voltage as feedback signal to control the electrooptic modulator. We tried two types power transistors, and finally got the output voltage of -500v--+500v. To compare the output power intensity of the stabilizer with the input power intensity, we can find that the power stability is increased by 50 times.

**EXTERNAL CAVITY LASER POWER  
STABILIZER**

by  
**Jie Ding**

**A Thesis  
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in Partial Fulfillment of the Requirements for the Degree of  
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**Department of Physics**

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**APPROVAL PAGE**

**EXTERNAL CAVITY LASER POWER  
STABILIZER**

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This thesis is dedicated to my husband

Peng Li

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# CHAPTER 1

## INTRODUCTION

Of all the artificial light sources, the laser ( acronym for "Light Amplification by Stimulated Emission and Radiation") was invented most recently. But it is probably the most important one.

There are several ways in which we can classify different types of lasers. First, lasers can be classified according to the material or element responsible for the light amplification. Secondly, we consider whether the laser operates in pulse mode or as a continuous wave (CW).

Because of its coherence, high intensity and excellent orientation, the laser has been used in many fields since it was invented. In applications of lasers, more and more requirements were raised as regards to its stability. Usually, frequency stability is the first priority. But in some cases, power stability is also needed. Such as in research of light scattering in the air or in many experiments with CW laser as pump light. This is especially true for cases where one cannot eliminate errors with linear corrections. For example, in the Photorefractive effect, diffraction efficiency is a function of optical path and the ratio of optical path. It does not only depend on the intensity at time  $t$ , but also depends on the case before time  $t$ . Obviously, for this process, it is impossible to use linear corrections to delete the fluctuation errors caused by laser power.

The output stabilized laser is commercially available in a form which incorporates the stabilization mechanism in the cavity of the laser. In this thesis, we propose to improve the laser power stability by a standing-alone external stabilizer. This stabilizer is suitable for visible or near infrared CW laser. With the laser beam passing through the external stabilizer, the intensity of laser can be

stabilized. The fluctuation of the laser intensity can be reduced by a factor of 50 times. Because it contains a polarizer, the stabilizer also makes the laser output beam more highly polarized. The direction of the output beam polarization can be adjusted arbitrarily, so it can meet some kinds of special applications.

## CHAPTER 2

### WAVE PROPAGATION IN CRYSTALS THE INDEX ELLIPSOID

In this thesis, we will consider the problem of propagation of optical radiation in anisotropic crystal media. In Chapters 2 and 3, we discuss the wave propagation in crystals and the modulation of optical radiation. In this chapter, we start with Maxwell's equations and obtain expressions of wave propagation in material media. We consider in some detail the phenomenon of birefringence, in which the phase velocity of a plane wave in a crystal depends on its direction of polarization. The two allowed modes of propagation in uniaxial crystals--the "ordinary" and "extraordinary" rays--are discussed using the formalism of the index ellipsoid.

#### 2.1 Wave Propagation in Isotropic Media

First we consider the propagation of electromagnetic plane waves in homogeneous and isotropic media, so that  $\epsilon$  and  $\mu$  are scalar constants. Vacuum is the best example of such a "medium". Liquids and glasses are material media that can be treated as homogeneous and isotropic. Let us begin with Maxwell's equations:

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial}{\partial t} \mathbf{D} \quad (2.1-1)$$

$$\nabla \times \mathbf{E} = - \frac{\partial}{\partial t} \mathbf{B} \quad (2.1-2)$$

We choose the direction of propagation as  $z$  and, taking the plane wave to be uniform in the  $x-y$  plane, put  $\partial/\partial x = \partial/\partial y = 0$  in (2.1-1) and (2.1-2). Assuming a lossless ( $\sigma = 0$ ) medium, equations (2.1-1) and (2.1-2) become

$$\nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t} \mathbf{H} \quad (2.1-3)$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial}{\partial t} \mathbf{E} \quad (2.1-4)$$

We can rewrite the foregoing in component form:

$$\frac{\partial E_y}{\partial z} = \mu \frac{\partial H_x}{\partial t} \quad (2.1-5)$$

$$\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_x}{\partial t} \quad (2.1-6)$$

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \quad (2.1-7)$$

$$\frac{\partial H_x}{\partial z} = \varepsilon \frac{\partial E_y}{\partial t} \quad (2.1-8)$$

$$0 = \mu \frac{\partial H_z}{\partial t} \quad (2.1-9)$$

$$0 = \varepsilon \frac{\partial E_z}{\partial t} \quad (2.1-10)$$

From (2.1-9) and (2.1-10) it follows that  $H_z$  and  $E_z$  are both zero; therefore, a uniform plane wave in a homogeneous isotropic medium can have no longitudinal field components. We can obtain a self-consistent set of equations from (2.1-5) through (2.1-10) by taking  $E_y$  and  $H_x$  ( or  $E_x$  and  $H_y$  ) to be zero. In this case the last set of equations reduces to (2.1-6) and (2.1-7). Taking the derivative of (2.1-7) with respect to  $z$  and using (2.1-6), we obtain

$$\frac{\partial^2 E_x}{\partial z^2} = \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} \quad (2.1-11)$$

A reversal of the procedure will yield a similar equation for  $H_y$ . Since our main interest is in harmonic (sinusoidal) time variation we postulate a solution in the form of

$$E_x^\pm = E_x^\pm e^{i(\omega t \mp kz)} \quad (2.1-12)$$

where  $E_x^\pm \exp(\mp ikz)$  are the complex field amplitudes at  $z$ . Before substituting (2.1-12) into the wave equation (2.1-11), we may consider the nature of the two functions  $E_x^\pm$ . Taking first  $E_x^+$ : if an observer were to travel in such a way as to always experience the same field value, it would have to satisfy the condition

$$\omega t - kz = \text{constant}$$

where the constant is arbitrary and determines the field value "seen" by the observer. By differentiation of the last result, it follows that the observer must travel in the  $+z$  direction with a velocity

$$c = \frac{dz}{dt} = \frac{\omega}{k} \quad (2.1-13)$$

This is the *phase velocity* of the wave. If the wave were frozen in time, the separation between two neighboring field peaks--that is, the wavelength--is

$$\lambda = \frac{2\pi}{k} = 2\pi \frac{c}{\omega} \quad (2.1-14)$$

The  $E_x^-$  solution differs only in the sign of  $k$ , and according to (2.1-13), it corresponds to a wave traveling with a phase velocity  $c$  in the  $-z$  direction.

The value of  $c$  can be obtained by substituting the assumed solution (2.1-12) into (2.1-11), which results in

$$c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}} \quad (2.1-15)$$

or

$$k = \omega \sqrt{\mu\varepsilon}$$

The phase velocity in vacuum is

$$c_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

whereas in material media it has the value



$$c = \frac{c_0}{n}$$

where  $n \equiv \sqrt{\varepsilon/\varepsilon_0}$  is the *index of refraction*.

Turning to the magnetic field  $H_y$ , we can express it, in a manner similar to (2.1-12), in the form of

$$H_y = \mathbf{H}_y^\pm e^{i(\omega t \mp kz)} \quad (2.1-16)$$

Substitution of this equation into (2.1-6) and using (2.1-12) gives

$$-ik\mathbf{H}_y^\pm e^{i(\omega t - kz)} = -i\omega\varepsilon\mathbf{E}_x^\pm e^{i(\omega t - kz)}$$

Therefore, from (2.1-15),

$$\mathbf{H}_y^\pm = \frac{\mathbf{E}_x^\pm}{\eta} \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} \quad (2.1-17)$$

In vacuum  $\eta_0 = \sqrt{\mu_0/\varepsilon_0} \approx 377$  ohms. Repeating the same steps with  $\mathbf{H}_y^-$  and  $\mathbf{E}_x^-$  gives

$$\mathbf{H}_y^- = -\frac{\mathbf{E}_x^-}{\eta} \quad (2.1-18)$$

so that in the case of negative (  $-z$  ) traveling waves the relative phase of the electric and magnetic fields is reversed with respect to the wave traveling in the  $+z$  direction. Since the wave equation (2.1-11) is a linear differential equation, we can take the solution for the harmonic case as a linear superposition of  $\mathbf{E}_x^+$  and  $\mathbf{E}_x^-$

$$E_x(z, t) = \mathbf{E}_x^+ e^{i(\omega t - kz)} + \mathbf{E}_x^- e^{i(\omega t + kz)} \quad (2.1-19)$$

and, similarly,

$$H_y(z, t) = \frac{1}{\eta} [\mathbf{E}_x^+ e^{i(\omega t - kz)} - \mathbf{E}_x^- e^{i(\omega t + kz)}]$$

where  $\mathbf{E}_x^+$  and  $\mathbf{E}_x^-$  are arbitrary complex constants.

## 2.2 Wave Propagation in Crystals

In the discussion of electromagnetic wave propagation above, we have assumed that the medium was isotropic. This causes the induced polarization to be parallel to the electric field and to be related to it by a (scalar) factor that is independent of the direction along which the field is applied. This situation does not apply in the case of dielectric crystals. Since the crystal is made up of a regular periodic array of atoms( or ions ), we may expect that the induced polarization will depend, both in its magnitude and direction, on the direction of the applied field. Instead of the simple relation linking  $\mathbf{P}$  and  $\mathbf{E}$ ,

$$\begin{aligned}\mathbf{P} &= \varepsilon_0 \chi_e \mathbf{E} \\ \varepsilon &= \varepsilon_0 (1 + \chi_e)\end{aligned}$$

we have

$$\begin{aligned}P_x &= \varepsilon_0 (\chi_{11}E_x + \chi_{12}E_y + \chi_{13}E_z) \\ P_y &= \varepsilon_0 (\chi_{21}E_x + \chi_{22}E_y + \chi_{23}E_z) \\ P_z &= \varepsilon_0 (\chi_{31}E_x + \chi_{32}E_y + \chi_{33}E_z)\end{aligned}\tag{2.2-1}$$

where the capital letters denote the complex amplitudes of the corresponding time-harmonic quantities. The  $3 \times 3$  array of the  $\chi_{ij}$  coefficients is called the electric susceptibility tensor. The magnitude of the  $\chi_{ij}$  coefficients depends on the choice of the  $x, y, z$  axes relative to that of the crystal structure. It is always possible to choose  $x, y, z$  in such a way that the off-diagonal elements vanish, leaving

$$\begin{aligned}P_x &= \varepsilon_0 \chi_{11}E_x \\ P_y &= \varepsilon_0 \chi_{22}E_y \\ P_z &= \varepsilon_0 \chi_{33}E_z\end{aligned}\tag{2.2-2}$$

These directions are called the *principal dielectric axes of the crystal*. We use only the principal coordinate system and, instead of using (2.2-2), describe the dielectric response of the crystal by means of the electric permeability tensors  $\varepsilon_{ij}$ , defined by

$$\begin{aligned}
D_x &= \varepsilon_{11}E_x \\
D_y &= \varepsilon_{22}E_y \\
D_z &= \varepsilon_{33}E_z
\end{aligned}
\tag{2.2-3}$$

From (2.2-2) and the relation

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

we have

$$\begin{aligned}
\varepsilon_{11} &= \varepsilon_0(1 + \chi_{11}) \\
\varepsilon_{22} &= \varepsilon_0(1 + \chi_{22}) \\
\varepsilon_{33} &= \varepsilon_0(1 + \chi_{33})
\end{aligned}
\tag{2.2-4}$$

### 2.3 Birefringence

One of the most important consequences of the dielectric anisotropy of crystals is the phenomenon of birefringence in which the phase velocity of an optical beam propagating in the crystal depends on the direction of polarization of its  $\mathbf{E}$  vector. In an isotropic medium the induced polarization is independent of the field direction so that  $\chi_{11} = \chi_{22} = \chi_{33}$  and, using (2.2-4),  $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = \varepsilon$ . Since  $c = (\mu\varepsilon)^{-1/2}$ , the phase velocity is independent of the direction of polarization. In an anisotropic medium the situation is different. Consider a wave propagating along  $z$ . If its electric field is parallel to  $x$ , it will induce, according to (2.2-2), only  $P_x$  and will consequently "see" an electric permeability  $\varepsilon_{11}$ . Its phase velocity will thus be  $c_x = (\mu\varepsilon_{11})^{-1/2}$ . If the wave is polarized parallel to  $y$  it will propagate with a phase velocity  $c_y = (\mu\varepsilon_{22})^{-1/2}$ .

Birefringence has some interesting consequences. Consider a wave propagating along the crystal  $z$  direction and having in some plane,  $z = 0$ , a linearly polarized field with equal components along  $x$  and  $y$ . Since  $k_x \neq k_y$ , as the wave propagates into the crystal the  $x$  and  $y$  components get out of phase and the wave becomes elliptically polarized.

A wave propagating along the crystal  $z$  direction, assume, as in Section 2.1, that the only non vanishing field components are  $E_x$  and  $H_y$ . Maxwell's curl equations (2.1-6) and (2.1-8) reduce to

$$\begin{aligned}\frac{\partial E_x}{\partial z} &= -\mu \frac{\partial H_y}{\partial t} \\ \frac{\partial H_y}{\partial z} &= -\epsilon_{11} \frac{\partial E_x}{\partial t}\end{aligned}\tag{2.3-1}$$

Taking the derivative of the first of equation(2.3-1) with respect to  $z$  and then substituting the second equation for  $\partial H_y / \partial z$  gives

$$\frac{\partial^2 E_x}{\partial z^2} = \mu \epsilon_{11} \frac{\partial^2 E_x}{\partial t^2}\tag{2.3-2}$$

If we postulate, as in (2.1-12), a solution in the form

$$E_x = \mathbf{E}_x e^{i(\omega t - k_x z)}\tag{2.3-3}$$

then equation (2.3-2) becomes

$$k_x^2 \mathbf{E}_x = \omega^2 \mu \epsilon_{11} \mathbf{E}_x$$

Therefore, the propagation constant of a wave polarized along  $x$  and traveling along  $z$  is

$$k_x = \omega \sqrt{\mu \epsilon_{11}}\tag{2.3-4}$$

Repeating the derivation but with a wave polarized along the  $y$  axis, instead of the  $x$  axis, yields

$$k_y = \omega \sqrt{\mu \epsilon_{22}}\tag{2.3-5}$$

## 2.4 The Index Ellipsoid

As shown above, in a crystal the phase velocity of a wave propagating along a given direction depends on the direction of its polarization. For propagation along  $z$ , Maxwell's equations admit two solutions: one with its linear polarization along  $x$  and the second along  $y$ . If we consider the propagation along some arbitrary

direction in the crystal the problem becomes more difficult. We have to determine the directions of polarization of the two allowed waves, as well as their phase velocities. This is done most conveniently using the so-called index ellipsoid

$$\frac{x^2}{(\varepsilon_{11}/\varepsilon_0)} + \frac{y^2}{(\varepsilon_{22}/\varepsilon_0)} + \frac{z^2}{(\varepsilon_{33}/\varepsilon_0)} = 1 \quad (2.4-1)$$

This is the equation of a generalized ellipsoid with major axes parallel to  $x$ ,  $y$ , and  $z$  whose respective lengths are  $2\sqrt{\varepsilon_{11}/\varepsilon_0}$ ,  $2\sqrt{\varepsilon_{22}/\varepsilon_0}$ , and  $2\sqrt{\varepsilon_{33}/\varepsilon_0}$ . The procedure for finding the polarization directions and the corresponding phase velocities for a *given* direction of propagation is as follows: Determine the ellipse formed by the intersection of a plane through the origin and normal to the direction of propagation and the index ellipsoid (2.4-1). The directions of the major and minor axes of this ellipse are those of the two allowed polarizations and the lengths of these axes are  $2n_1$  and  $2n_2$ , where  $n_1$  and  $n_2$  are the indices of the refraction of the two allowed solutions. The two waves propagate, thus, with phase velocities  $c_0/n_1$  and  $c_0/n_2$ , respectively, where  $c_0 = (\mu_0\varepsilon_0)^{-1/2}$  is the phase velocity in vacuum.

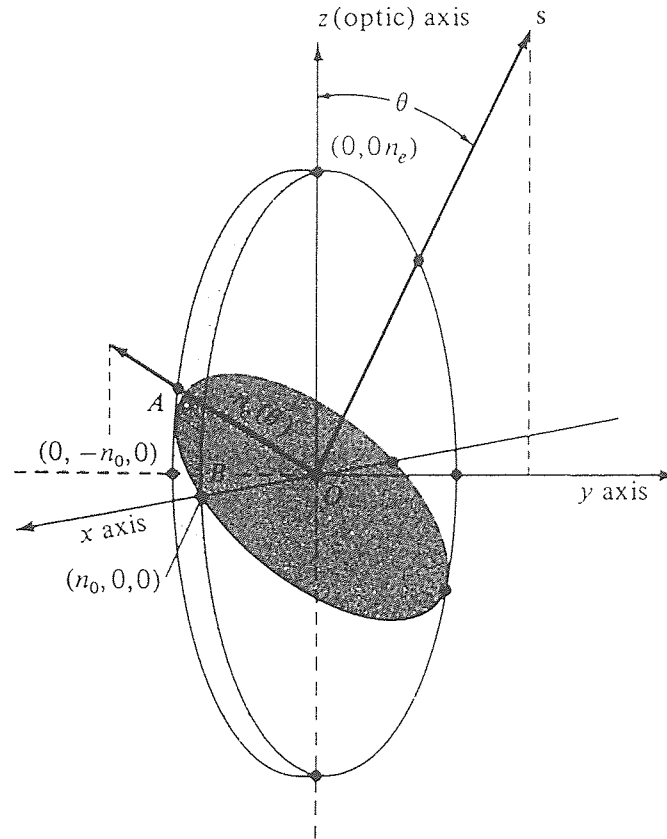
To illustrate the use of the index ellipsoid, consider the case of a uniaxial crystal (that is, a crystal which possesses a single axis of threefold, fourfold, or six fold symmetry). Taking the direction of this axis as  $z$ , symmetry considerations dictate that  $\varepsilon_{11} = \varepsilon_{22}$ . Defining the principal indices of refraction  $n_o$  and  $n_e$  by

$$n_o^2 \equiv \frac{\varepsilon_{11}}{\varepsilon_0} = \frac{\varepsilon_{22}}{\varepsilon_0} \quad n_e^2 \equiv \frac{\varepsilon_{33}}{\varepsilon_0} \quad (2.4-2)$$

the equation of the index ellipsoid (2.4-1) becomes

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1 \quad (2.4-3)$$

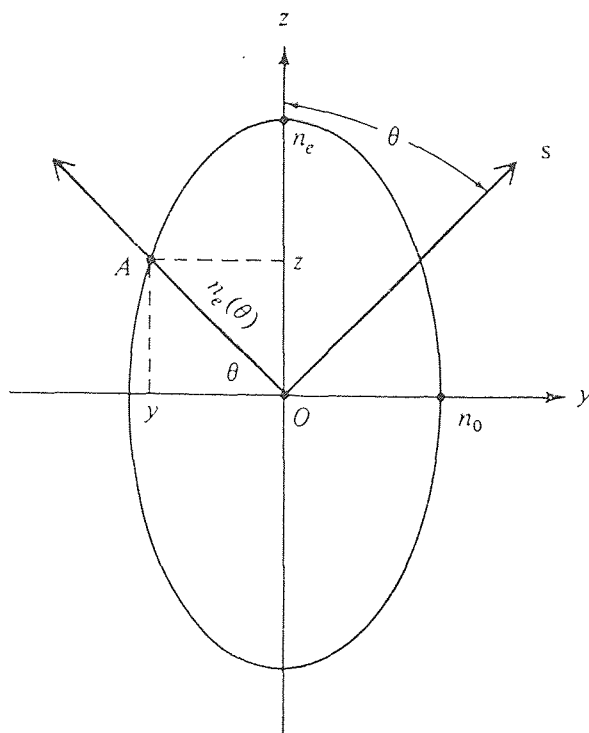
This is an ellipsoid of revolution with  $z$ -axis being the circular symmetry axis. The  $z$  major axis of the ellipsoid is of length  $2n_e$ , whereas that of the  $x$  and  $y$  axes is  $2n_o$ . The procedure of using the index ellipsoid is illustrated by Figure 2-1.



**Figure 2-1** Construction for finding indices of refraction and allowed polarization for a given direction of propagation  $s$ . The figure shown is for a uniaxial crystal with  $n_x = n_y = n_o$ .

The direction of propagation is along  $s$  and is at an angle  $\theta$  to the (optic)  $z$  axis. Because of the circular symmetry of (2.4-3) about  $z$  we can choose, without any loss of generality, the  $y$  axis to coincide with the projection of  $s$  on the  $x-y$  plane. The intersection ellipse of the plane normal to  $s$  with the ellipsoid is crosshatched in the figure. The two allowed polarization directions are parallel to the axes of the ellipse and thus correspond to the line segments  $OA$  and  $OB$ . They are consequently perpendicular to  $s$  as well as to each other. The two waves

polarized along these directions have indices of refraction given by  $n_e(\theta) = |OA|$  and  $n_o = |OB|$ . The first of these two waves, which is polarized along  $OA$ , is called the *extraordinary wave*. Its direction of polarization varies with  $\theta$  following the intersection point  $A$ . Its index of refraction is given by the length of  $OA$ . It can be determined using Figure 2-2, which shows the intersection of the index ellipsoid with the  $y-z$  plane.



**Figure 2-2** Intersection of the index ellipsoid with the  $z-y$  plane.  $|OA| = n_e(\theta)$  is the index of refraction of the extraordinary wave propagating in the direction  $s$ .

Using the relations

$$n_e^2(\theta) = z^2 + y^2$$

$$\frac{z}{n_e(\theta)} = \sin \theta$$

and the equation of the ellipse

$$\frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$$

we obtain

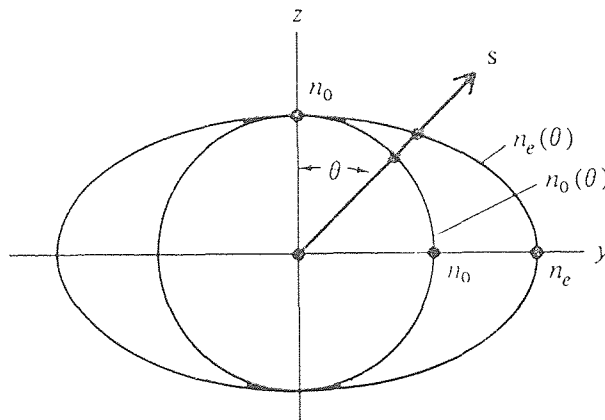
$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2} \quad (2.4-4)$$

Thus, for  $\theta = 0^\circ$ ,  $n_e(0^\circ) = n_o$  and for  $\theta = 90^\circ$ ,  $n_e(90^\circ) = n_e$ .

The ordinary wave remains, according to Figure 2-1, polarized along the same direction  $OB$  independent of  $\theta$ . It has an index of refraction  $n_o$ . The amount of birefringence  $n_e(\theta) - n_o$  thus varies from zero for  $\theta = 0^\circ$  (that is, propagation along the optic axis) to  $n_e - n_o$  for  $\theta = 90^\circ$ .

## 2.5 The Normal Index Surfaces

Consider a surface in which the distance of a given point from the origin is equal to the index of refraction of a wave propagating along this direction. This surface is called the normal index surface.



**Figure 2-3** Intersection of  $s-z$  plane with normal surfaces of a positive uniaxial crystal ( $n_e > n_o$ ).



The normal surface of the ordinary wave is a sphere, since the index of refraction is  $n_o$  and is independent of the direction of propagation. The normal surface of the extraordinary wave is an ellipsoid. In a uniaxial crystal it becomes an ellipsoid of revolution about the optic ( $z$ ) axis in which the distance  $n_e(\theta)$  to the origin is given by (2.4-4). The intersection of the normal surfaces of a positive ( $n_e > n_o$ ) uniaxial crystal with the  $s$ - $z$  plane is shown in figure 2-3.

## CHAPTER 3

### THE MODULATION OF OPTICAL RADIATION

In Chapter 2 we treated the propagation of electromagnetic waves in anisotropic crystal media. It was shown how the properties of the propagating wave can be determined from the index ellipsoid surface.

In this chapter we consider the problem of propagation of optical radiation in crystals in the presence of an applied electric field. In certain types of crystals it is possible to effect a change in the index of refraction which is proportional to the field. This is the linear electro-optic effect. It affords a convenient and widely used means of controlling the intensity or phase of the propagation radiation.

#### 3.1 The Electro-optic Effect

Given a direction in a crystal, in general two possible linearly polarized modes exist; the so-called rays of propagation. Each mode possesses a unique direction of polarization ( that is, direction of  $\mathbf{D}$  ) and a corresponding index of refraction (that is, a velocity of propagation). The mutually orthogonal polarization directions and the indices of the two rays are found most easily by using the index ellipsoid

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad (3.1-1)$$

where the directions  $x, y,$  and  $z$  are the principal dielectric axes--that is, the directions in the crystal along which  $\mathbf{D}$  and  $\mathbf{E}$  are parallel. The existence of two "ordinary" and "extraordinary" rays with different indices of refraction is called birefringence.

The linear electro-optic effect is the change in the indices of the ordinary and extraordinary rays that is caused by and is proportional to an applied electric field. This effect exists only in crystals that do not possess inversion symmetry. This statement can be justified as follows: Assume that in a crystal possessing an inversion symmetry, the application of an electric field  $E$  along some direction causes a change  $\Delta n_1 = sE$  in the index, where  $s$  is a constant characterizing the linear electro-optic effect. If the direction of the field is reversed, the change in the index is given by  $\Delta n_2 = s(-E)$ , but because of the inversion symmetry the two directions are physically equivalent, so  $\Delta n_1 = \Delta n_2$ . This requires that  $s = -s$ , which is possible only for  $s = 0$ , so no linear electro-optic effect can exist. The division of all crystal classes into those that do and those that do not possess an inversion symmetry is an elementary consideration in crystallography and this information is widely tabulated.

Since the propagation characteristics in crystals are fully described by means of the index ellipsoid (3.1-1), the effect of an electric field on the propagation is expressed most conveniently by giving the changes in the constants  $1/n_x^2, 1/n_y^2, 1/n_z^2$  of the index ellipsoid.

Following convention, we take the equation of the index ellipsoid in the presence of an electric field as

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right)_6 xy = 1 \quad (3.1-2)$$

If we choose  $x, y,$  and  $z$  to be parallel to the principal dielectric axes of the crystal, then with zero applied field, equation (3.1-2) must reduce to (3.1-1); therefore,

$$\left(\frac{1}{n^2}\right)_1 \Big|_{E=0} = \frac{1}{n_x^2}, \quad \left(\frac{1}{n^2}\right)_2 \Big|_{E=0} = \frac{1}{n_y^2}, \quad \left(\frac{1}{n^2}\right)_3 \Big|_{E=0} = \frac{1}{n_z^2}$$

$$\left(\frac{1}{n^2}\right)_4 \Big|_{E=0} = \left(\frac{1}{n^2}\right)_5 \Big|_{E=0} = \left(\frac{1}{n^2}\right)_6 \Big|_{E=0} = 0$$

The linear change in the coefficients

$$\left(\frac{1}{n^2}\right)_i \quad i = 1, \dots, 6$$

due to an arbitrary electric field  $\mathbf{E}(E_x, E_y, E_z)$  is defined by

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_{j=1}^3 r_{ij} E_j \quad (3.1-3)$$

where in the summation over  $j$  we use the convention  $1 = x, 2 = y, 3 = z$ . Equation

(3.1-3) can be expressed in a matrix form as

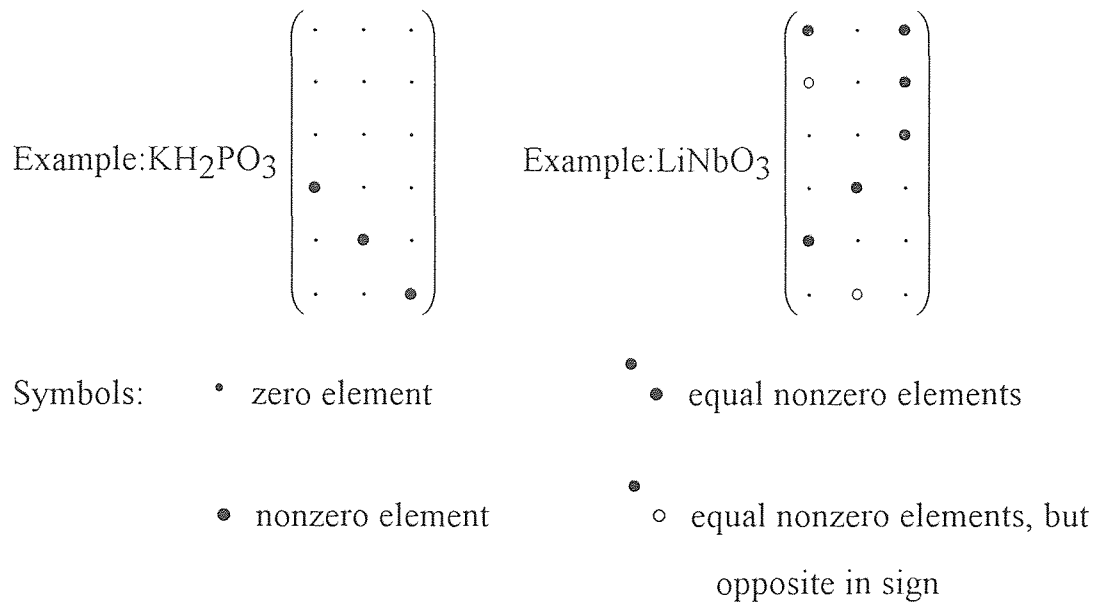
$$\begin{pmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^2}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}$$

(3.1-4)

where, using the rules for matrix multiplication, we have, for example,

$$\Delta\left(\frac{1}{n^2}\right)_6 = r_{61}E_1 + r_{62}E_2 + r_{63}E_3$$

The  $6 \times 3$  matrix with elements  $r_{ij}$  is called the electro-optic tensor. In the figure (3-1) we give the form of the electro-optic tensor for two noncentrosymmetric crystals we will discuss and use in this thesis.



**Figure 3-1** The examples of the form of the electro-optic tensor

Consider the crystal KDP ( $\text{KH}_2\text{PO}_4$ ). The crystal has a fourfold axis of symmetry, which by strict convention is taken as the  $z$  (optic) axis, as well as two mutually orthogonal twofold axes of symmetry that lie in the plane normal to  $z$ . These are designated as the  $x$  and  $y$  axes. Using Figure (3-1), we take the electro-optic tensor in the form of

$$r_{ij} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{vmatrix} \quad (3.1-5)$$

so the only nonvanishing elements are  $r_{41} = r_{52}$  and  $r_{63}$ . Using (3.1-1), (3.1-4) and (3.1-5), we obtain the equation of the index ellipsoid in the presence of a field  $\mathbf{E}(E_x, E_y, E_z)$  as

$$\frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_e^2} + 2r_{41}E_x yz + 2r_{41}E_y xz + 2r_{63}E_z xy = 1 \quad (3.1-6)$$

where the constants involved in the first three terms do not depend on the field and, since the crystal is uniaxial, are taken as  $n_x = n_y = n_0, n_z = n_e$ . We thus find that the application of an electric field causes the appearance of "mixed" terms in the equation of the index ellipsoid. These are the terms with  $xy, xz$  and  $yz$ . This means that the major axes of the ellipsoid, with a field applied, are no longer parallel to the  $x, y$ , and  $z$  axes. It becomes necessary to find the directions and magnitudes of the new axes, in the presence of  $\mathbf{E}$ , so that we may determine the effect of the field on the propagation. To be specific we choose the direction of the applied field parallel to the  $z$  axis, so (3.1-6) becomes

$$\frac{x^2 + y^2}{n_0^2} + \frac{z^2}{n_e^2} + 2r_{63}E_z xy = 1 \quad (3.1-7)$$

The problem is one of finding a new coordinate system-- $x', y', z'$ --in which the equation of the ellipsoid (3.1-7) contains no mixed terms; that is, it is of the form

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1 \quad (3.1-8)$$

$x', y'$  and  $z'$  are then the directions of the major axes of the ellipsoid in the presence of an external field applied parallel to  $z$ . The length of the major axes of the ellipsoid is, according to (3.1-8),  $2n_{x'}$ ,  $2n_{y'}$  and  $2n_{z'}$  and these will depend on the applied field.

In the case of (3.1-7) it is clear from inspection that in order to put it in a diagonal form we need to choose a coordinate system  $x', y', z'$ , where  $z'$  is parallel to  $z$ , and because of the symmetry of (3.1-7) in  $x$  and  $y$ ,  $x'$  and  $y'$  are related to  $x$  and  $y$  by a  $45^\circ$  rotation. The transformation relations from  $x, y$  to  $x', y'$  are thus

$$\begin{aligned} x &= x' \cos 45^\circ - y' \sin 45^\circ \\ y &= x' \sin 45^\circ + y' \cos 45^\circ \end{aligned}$$

which, upon substitution in (3.1-7), yield

$$\left( \frac{1}{n_o^2} + r_{63} E_z \right) x'^2 + \left( \frac{1}{n_o^2} - r_{63} E_z \right) y'^2 + \frac{z^2}{n_e^2} = 1 \quad (3.1-9)$$

Equation (3.1-9) shows that  $x', y'$  and  $z$  are indeed the principal axes of the ellipsoid when a field is applied along the  $z$  direction. According to (3.1-9), the length of the  $x'$  axis of the ellipsoid is  $2n_{x'}$ , where

$$\frac{1}{n_{x'}^2} = \frac{1}{n_o^2} + r_{63} E_z$$

which, assuming  $r_{63} E_z \ll n_o^{-2}$  and using the differential relation  $dn = -(n^3/2)d(1/n^2)$ , gives

$$n_{x'} = n_o - \frac{n_o^3}{2} r_{63} E_z \quad (3.1-10)$$

and, similarly,

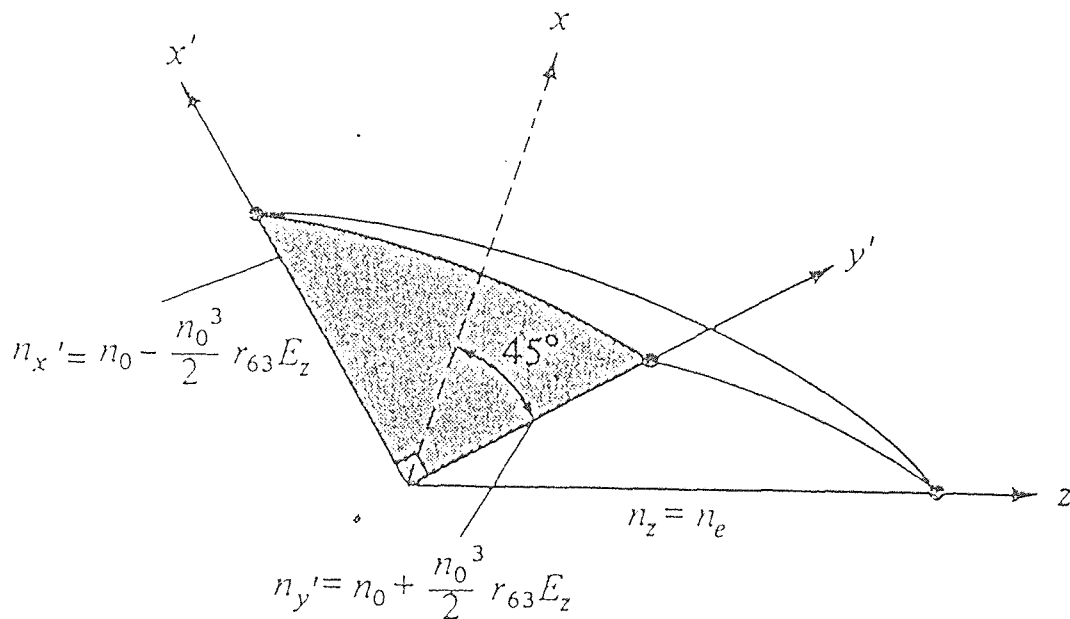
$$n_{y'} = n_o + \frac{n_o^3}{2} r_{63} E_z \quad (3.1-11)$$

$$n_z = n_e \quad (3.1-12)$$

### 3.2 Electro-optic Retardation

Assume the index ellipsoid for KDP ( $\text{KH}_2\text{PO}_4$ ) with  $\mathbf{E}$  applied parallel to  $z$ . It is shown in figure (3-2).

If we consider propagation along the  $z$  direction, then we need to determine the ellipse formed by the intersection of the plane  $z = 0$  (in general, the plane that contains the origin and is normal to the propagation direction) and the ellipsoid.



**Figure 3-2** A section of the index ellipsoid of KDP, showing the principal dielectric axes  $x'$ ,  $y'$ , and  $z'$  due to an electric field applied along the  $z$  axis.



The equation of this ellipse is obtained from (3.1-9) by putting  $z = 0$  and is

$$\left(\frac{1}{n_o^2} + r_{63}E_z\right)x'^2 + \left(\frac{1}{n_o^2} - r_{63}E_z\right)y'^2 = 1 \quad (3.2-1)$$

One quadrant of the ellipse is shown in figure (3-1), along with its minor and major axes, which in this case coincide with  $x'$  and  $y'$ , respectively. The two allowed directions of polarization are  $x'$  and  $y'$  and that their indices of refraction are  $n_{x'}$  and  $n_{y'}$ , which are given by (3.1-10) and (3.1-11).

We consider an optical field which is incident normally on the  $x'y'$  plane with its  $\mathbf{E}$  vector along the  $x$  direction. We can resolve the optical field at  $z = 0$  into two mutually orthogonal components polarized along  $x'$  and  $y'$ . Figure (3-3) shows  $E_{x'}(z)$  and  $E_{y'}(z)$  at some moment in time. The  $x'$  component propagates as

$$e_{x'} = Ae^{i[\omega t - (\omega/c_0)n_{x'}z]}$$

which using (3.1-10), becomes

$$e_{x'} = Ae^{i\left\{\omega t - (\omega/c_0)\left[n_o - \left(n_o^3/2\right)r_{63}E_z\right]z\right\}} \quad (3.2-2)$$

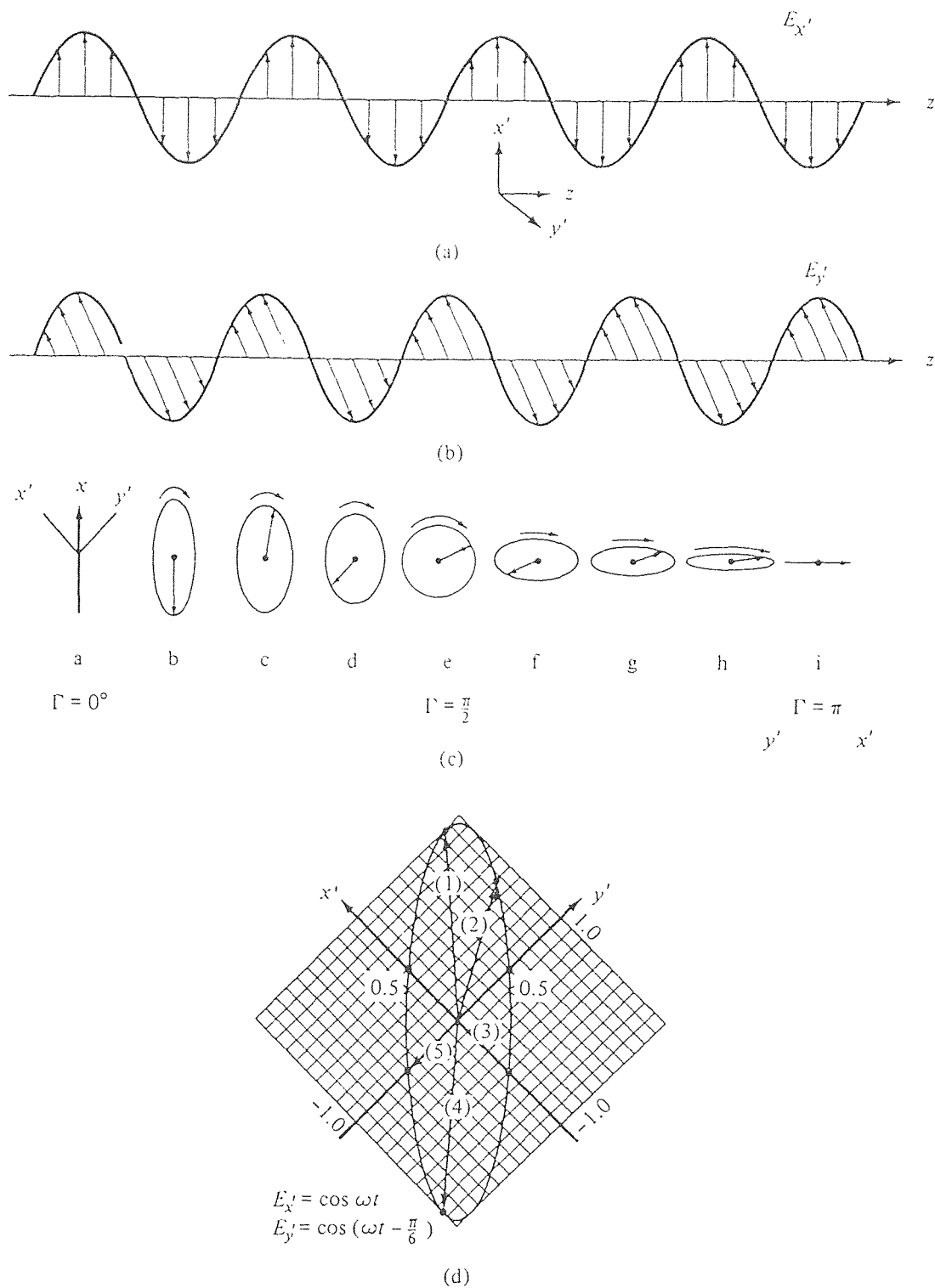
while the  $y'$  component is given by

$$e_{y'} = Ae^{i\left\{\omega t - (\omega/c_0)\left[n_o + \left(n_o^3/2\right)r_{63}E_z\right]z\right\}} \quad (3.2-3)$$

the phase difference at the output plane  $z = l$  between the two components is called the retardation. It is given by the difference of the exponents in (3.2-2) and (3.2-3) and is equal to

$$\Gamma = \Phi_{x'} - \Phi_{y'} = \frac{\omega n_o^3 r_{63} V}{c_0} \quad (3.2-4)$$

where  $V = E_z l$  and  $\Phi_{x'} = (\omega n_{x'}/c_0)l$ .



**Figure 3-3** An optical field that is linearly polarized along  $x$  is incident on an electrooptic crystal having its electrically induced principal axes along  $x'$  and  $y'$ .

In figure 3-3, (a) shows the component  $e_{x'}$  at some time  $t$  as a function of the position  $z$  along the crystal. (b) shows  $e_{y'}$  as a function of  $z$  at the same value of  $t$  as in (a). (c) shows the ellipsoid in the  $x'-y'$  plane traversed by the tip of the optical electric field at various points ( a through i ) along the crystal during one optical cycle. The arrow shows the instantaneous field vector at time  $t$ , while the curved arrow gives the sense in which the ellipsoid is traversed. (d) is a plot of the polarization  $e_{y'} = \cos(\omega t - \pi/6)$ . Also shown are the instantaneous field vectors at (1)  $\omega t = 0^0$ , (2)  $\omega t = 60^0$ , (3)  $\omega t = 120^0$ , (4)  $\omega t = 210^0$ , and (5)  $\omega t = 270^0$ .

In Figure 3-3 (a), at point  $a$  the retardation is  $\Gamma = 0$  and the field is linearly polarized along  $x$ . At point  $e$ ,  $\Gamma = \pi/2$ , omitting a common phase factor, we have

$$\begin{aligned} e_{x'} &= A \cos \omega t \\ e_{y'} &= A \cos\left(\omega t - \frac{\pi}{2}\right) = A \sin \omega t \end{aligned} \quad (3.2-5)$$

and the electric field vector is circularly polarized in the clockwise sense. At point  $i$ ,  $\Gamma = \pi$  and thus

$$\begin{aligned} e_{x'} &= A \cos \omega t \\ e_{y'} &= A \cos(\omega t - \pi) = -A \cos \omega t \end{aligned}$$

and the retardation is again linearly polarized, but this time along the  $y$  direction--that is, at  $90^0$  to its input direction of polarization.

The retardation as given by (3.2-4) can also be written as

$$\Gamma = \pi \frac{E_z l}{V_\pi} = \pi \frac{V}{V_\pi} \quad (3.2-6)$$

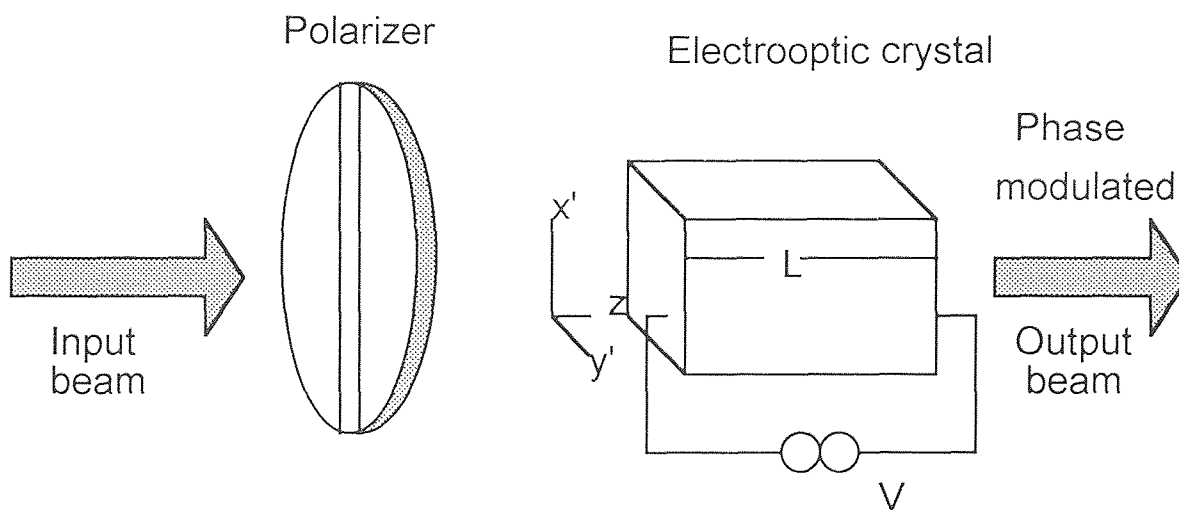
where  $V_\pi$ , the voltage yielding a retardation  $\Gamma = \pi$ , is

$$V_{\pi} = \frac{\lambda_0}{2n_o^3 r_{63}}$$

where  $\lambda_0 = 2\pi c_0/\omega$  is the free space wavelength.

### 3.3 Phase Modulation of Light

In the preceding section we saw how the modulation of the state of polarization, from linear to elliptic, of an optical beam by means of the electrooptic effect can be converted, using polarizers, to intensity modulation. Here we consider the situation depicted by Figure (3-4) below:



**Figure 3-4** An electro-optic phase modulator. The crystal orientation and applied directions are appropriate to KDP. The optical polarization is parallel to an electrically induced principal dielectric axis ( $x'$ )

Suppose instead of there being equal components along the induced birefringent axes, the incident beam is polarized parallel to one of them. In this case the application of the electric field does not change the state of polarization, but merely changes the output phase by

$$\Delta\Phi_x' = -\frac{\omega l}{c_0} \Delta n_x'$$

where, from (3.1-10)

$$\Delta\Phi_x' = -\frac{\omega n_o^3 r_{63}}{2c_0} E_z l \quad (3.3-1)$$

If the bias field is sinusoidal and is taken as

$$E_z = E_m \sin \omega_m t \quad (3.3-2)$$

then an incident optical field which, at the input ( $z = 0$ ) face of the crystal varies as  $e_{in} = A \cos \omega t$ , will emerge according to (3.2-2) as

$$e_{out} = A \cos \left[ \omega t - \frac{\omega}{c_0} \left( n_o - \frac{n_o^3}{2} r_{63} E_m \sin \omega_m t \right) l \right]$$

where  $l$  is the length of the crystal. Dropping the constant phase factor, which is of no consequence here, we rewrite the equation as

$$e_{out} = A \cos[\omega t + \delta \sin \omega_m t] \quad (3.3-3)$$

where

$$\delta = \frac{\omega n_o^3 r_{63} E_m l}{2c_0} = \frac{\pi n_o^3 r_{63} E_m l}{\lambda_0} \quad (3.3-4)$$

is referred to as the phase modulation index. The optical field is thus phase-modulated with a modulation index  $\delta$ .

### 3.4 Transverse Electro-optic Modulators

In the cases of electro-optic retardation discussed in preceding section, the electric field was applied along the direction of light propagation. This is so-called longitudinal mode of modulation. A more desirable mode of operation is the transverse one, in which the field is applied normal to the direction of propagation. The reason is that in this case the field electrodes do not interfere with the optical beam, and the retardation, being proportional to the product of the field times the crystal length, can be increased by the use of longer crystals. In the longitudinal case the retardation, according to (3.2-4), is proportional to  $E_z l = V$  and is independent of the crystal length  $l$ . How transverse retardation can be obtained using a KDP crystal is shown by an actual arrangement in figure (3-5). The light propagates along  $y'$ , and its polarization is in the  $x'-z$  plane at  $45^\circ$  from the  $z$  axis. The retardation, with a field applied along  $z$ , is,

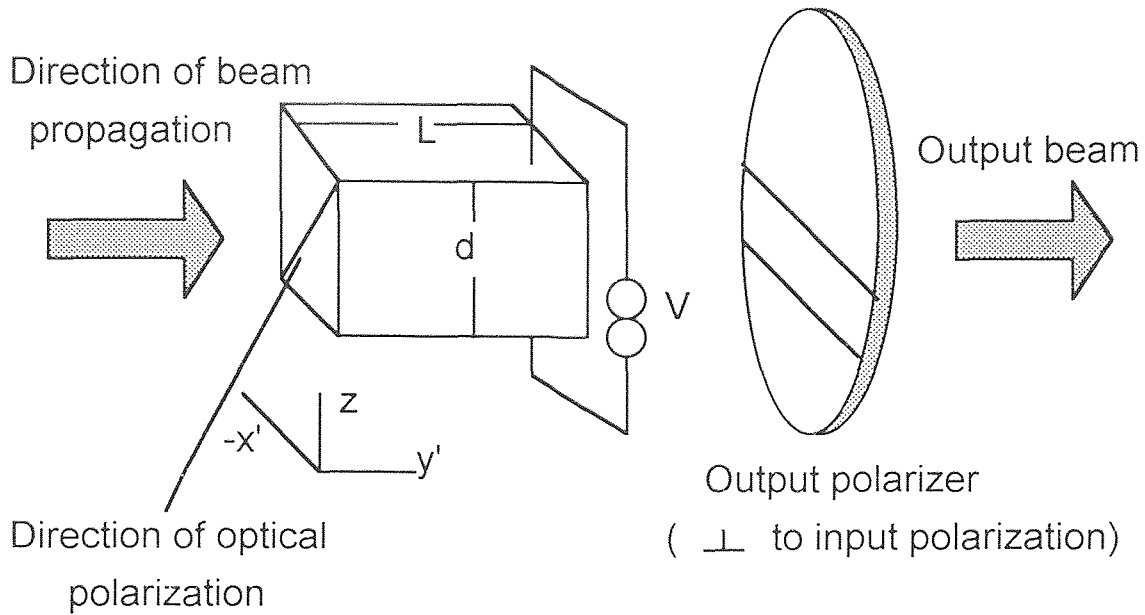
$$\Gamma = \Phi_z - \Phi_{x'} = \frac{\omega l}{c_0} \left[ (n_o - n_e) - \frac{n_o^3}{2} r_{63} \left( \frac{V}{d} \right) \right] \quad (3.4-1)$$

where  $d$  is the crystal dimension along the direction of the applied field.  $\Gamma$  contains a term that does not depend on the applied voltage.

### 3.5 High-Frequency Modulation Considerations

#### 3.5.1 Transit-Time Limitations to High-Frequency Electro-optic Modulation

According to Equation (3.2-4) the electro-optic retardation due to a field  $E$  can be written as



**Figure 3-5** A transverse electro-optic amplitude modulator using a KDP crystal in which the field is applied normal to the direction of propagation.

$$\Gamma = aEl \quad (3.5-1)$$

where  $a = n_o^3 r_{63}/c_0$  and  $l$  is the length of the optical path in the crystal. If the field  $E$  changes appreciably during the transit time  $\tau_d = l/c$  of light through the crystal, we must replace (3.5-1) by

$$\Gamma(t) = a \int_0^l e(t') dz = ac \int_{t-\tau_d}^t e(t') dt' \quad (3.5-2)$$

where  $c$  is the velocity of light and  $e(t')$  is the instantaneous electric field. In the second integral we replace integration over  $z$  by integration over time, recognizing that the portion of the wave which reaches the output face  $z = l$  at time  $t$  entered the crystal at time  $t - \tau_d$ . We also assumed that at any given moment the field  $e(t')$  has the same value throughout the crystal.

Taking  $e(t')$  as a sinusoid

$$e(t') = E_m e^{i\omega t'}$$

we obtain from (3.5-2)

$$\begin{aligned} \Gamma(t) &= acE_m \int_{t-\tau_d}^t e^{i\omega t'} dt' \\ &= \Gamma_0 \left[ \frac{1 - e^{-i\omega\tau_d}}{i\omega_m\tau_d} \right] e^{i\omega_m t} \end{aligned} \quad (3.5-3)$$

where  $\Gamma_0 = ac\tau_d E_m = dlE_m$  is the peak retardation, which obtains when  $\omega_m\tau_d \ll 1$ . The factor

$$r = \frac{1 - e^{-i\omega_m\tau_d}}{i\omega_m\tau_d} \quad (3.5-4)$$

gives the decrease in peak retardation resulting from the finite transit time. For  $r \approx 1$  (that is, no reduction), the condition  $\omega_m\tau_d \ll 1$  must be satisfied, so the transit time must be small compared to the shortest modulation period.

If we take the highest useful modulation frequency as that for which  $\omega_m\tau_d = \pi/2$  and use the relation  $\tau_d = l/n/c_0$ , we obtain

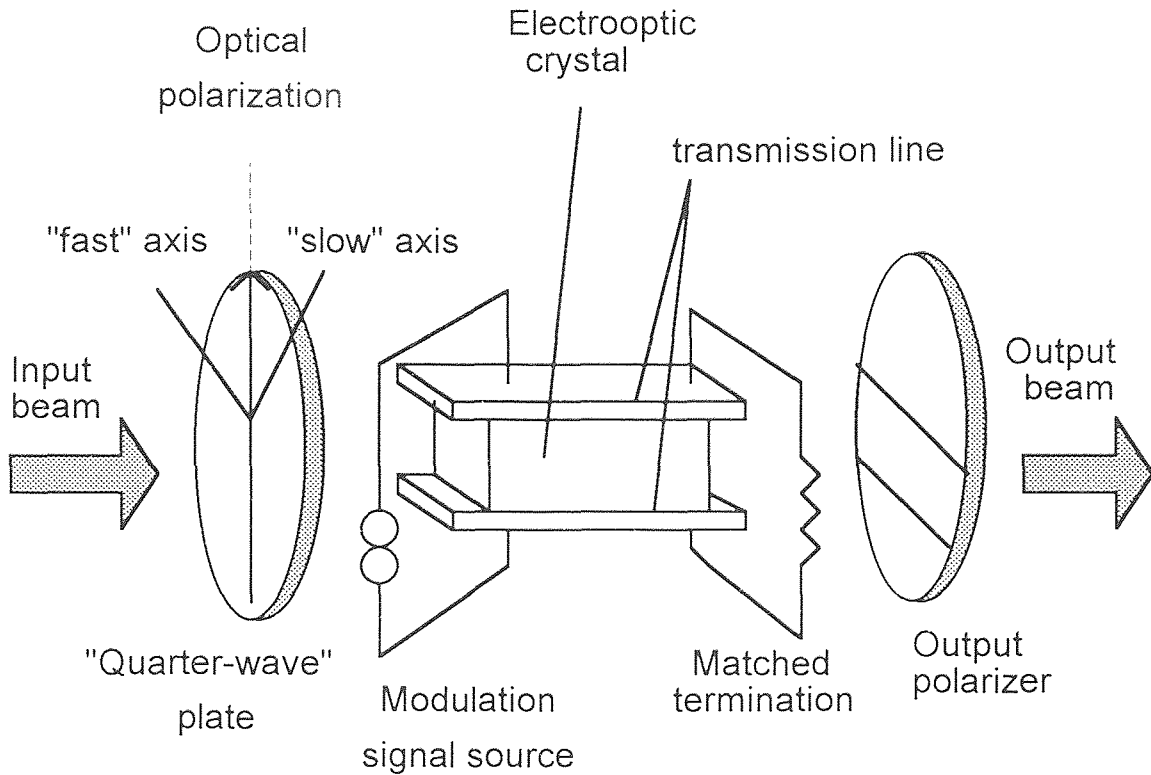
$$(\nu_m)_{\max} = \frac{c_0}{4l \cdot n} \quad (3.5-5)$$

### 3.5.2 Traveling-Wave Modulators

One method that can overcome the transit-time limitation, involves applying the modulation signal in the form of a traveling wave, as shown in figure (3-6).

If the optical and modulation field phase velocities are equal to each other, then a portion of an optical wave front will exercise the instantaneous electric field, which corresponds to the field it encounters at the entrance face, as it propagates through the crystal and the transit-time problem discussed above is eliminated.





**Figure 3-6** A traveling-wave electro-optic modulator.

Consider an element of the optical wave front that *enters* the crystal at  $z = 0$  at time  $t$ . The position  $z$  of this element at some later time  $t'$

$$z(t') = c(t' - t) \quad (3.5-6)$$

where  $c = c_0/n$  is the optical phase velocity. The retardation exercised by this element is given similarly to (3.5-2) by

$$\Gamma(t) = ac \int_t^{t+\tau_d} e[t', z(t')] dt' \quad (3.5-7)$$

where  $e[t', z(t')]$  is the instantaneous modulation field as seen by an observer traveling with the phase front. Taking the traveling modulation field as

$$e(t', z) = E_m e^{i[\omega_m t' - k_m z]}$$

we obtain, using (3.5-6)

$$e[z(t'), z(t')] = E_m e^{i[\omega_m t' - k_m c(t' - t)]} \quad (3.5-8)$$

Recalling that  $k_m = \omega_m / c_m$ , where  $c_m$  is the phase velocity of the modulation field, we substitute (3.5-8) in (3.5-7) and , carrying out the simple integration, obtain

$$\Gamma(t) = \Gamma_0 e^{i\omega_m t} \left[ \frac{e^{i\omega_m \tau_d (1 - c/c_m)} - 1}{i\omega_m \tau_d (1 - c/c_m)} \right] \quad (3.5-9)$$

where  $\Gamma_0 = a l E_m = a c \tau_d E_m$  is the retardation that would result from a dc field equal to  $E_m$ .

The reduction factor

$$r = \frac{e^{i\omega_m \tau_d (1 - c/c_m)} - 1}{i\omega_m \tau_d (1 - c/c_m)} \quad (3.5-10)$$

is of the same form as that of the lumped-constant modulator (3.5-4) except that  $\tau_d$  is replaced by  $\tau_d (1 - c/c_m)$ . If the two phase velocities are made equal so that  $c = c_m$ , then  $r = 1$  and maximum retardation is obtained regardless of the crystal length.

The maximum useful modulation frequency is taken, as in the treatment leading to (3.5-5), as that for which  $\omega_m \tau_d (1 - c/c_m) = \pi/2$ , yielding

$$(\nu_m)_{\max} = \frac{c_0}{4l \cdot n(1 - c_0/n c_m)} \quad (3.5-11)$$

which, upon comparison with (3.5-5), shows an increase in the frequency limit or useful crystal length of  $(1 - c_0/n c_m)^{-1}$ .

## CHAPTER 4

### THE PRINCIPLE AND STRUCTURE OF THE STABILIZER

#### 4.1 Background

In order to reduce the fluctuation of laser power output and improve the stability, there are many ways one could use. Among them are the following:

1. Improve the stability of open loop system

To get the output stability, the first method is to stabilize every unit of the laser.

( a ) Power source stability: for most CW lasers excitation is provided by a DC power supply. To improve stability one can use a stabilized DC supply, and additionally, incorporate filters to suppress spikes and other transients in the 60Hz AC source.

( b ) Thermal stability: the change of the cavity length is one of the main sources of laser output variations. This is primarily due to thermal dilation of the cavity. For a high precision laser, we always use Invar as main frame of cavity and use Quartz as gas discharge tube.

( c ) Reduce the effect of external conditions: in the laser cavity, one part of optical path is air. Isolating the flux of air and reducing the vibration of cavity can improve the stability.

2. Closed loop system control

To further increase the stability of laser output, closed loop control must be used. A very popular technique is to stabilize both frequency and power. At present, the way of stabilizing power is to monitor the magnitude of output beam continuously and obtain the deviation between the output signal and a given reference signal. After amplification , the deviation can be used to control laser

output power. As usually implemented this method needs to send the feedback signal to a stabilizing device inside the laser cavity. Our method is rather different; we employ an *external* cavity stabilizer. This stabilizer lets the laser output beam pass through it without any connection with the laser itself.

#### 4.2 The structure of the stabilizer

The laser power stabilizer uses optical polarizer components and an electro-optic modulator as a basic system. It is shown in Figure 4-1.

Laser source: we use an argon ion laser as the source in this experiment. Its operating wavelength can be adjusted to certain wavelength in the range of 457nm-514nm.

Polarizer: establishes a certain direction of linear polarization of the beam from the laser.

Electro-optic Modulator: changes state of polarization of optical beam according to applied voltage as discussed in Chapter 3.

Analyzer: selects one component along a given direction of input beam from modulator and produces a linearly polarized output.

Wave-plate: it is a rotatable  $\lambda/2$  waveplate . It is used to adjust the passing beam's polarization. Its operating wavelength can be adjusted according to the laser wavelength.

Beam splitter: directs 1% of the output beam to detector.

Detector: we use a Silicon Photodiode as the detector. It monitors the power of the output beam.

Reference signal and Comparer: we use the output voltage of a precision low voltage supply as the reference signal, and compare it with the voltage signal from the detector and send the difference to an amplifier.

Amplifier: to amplify the difference signal, we use a voltage amplifier, the output of which serves to control the electro-optic modulator.

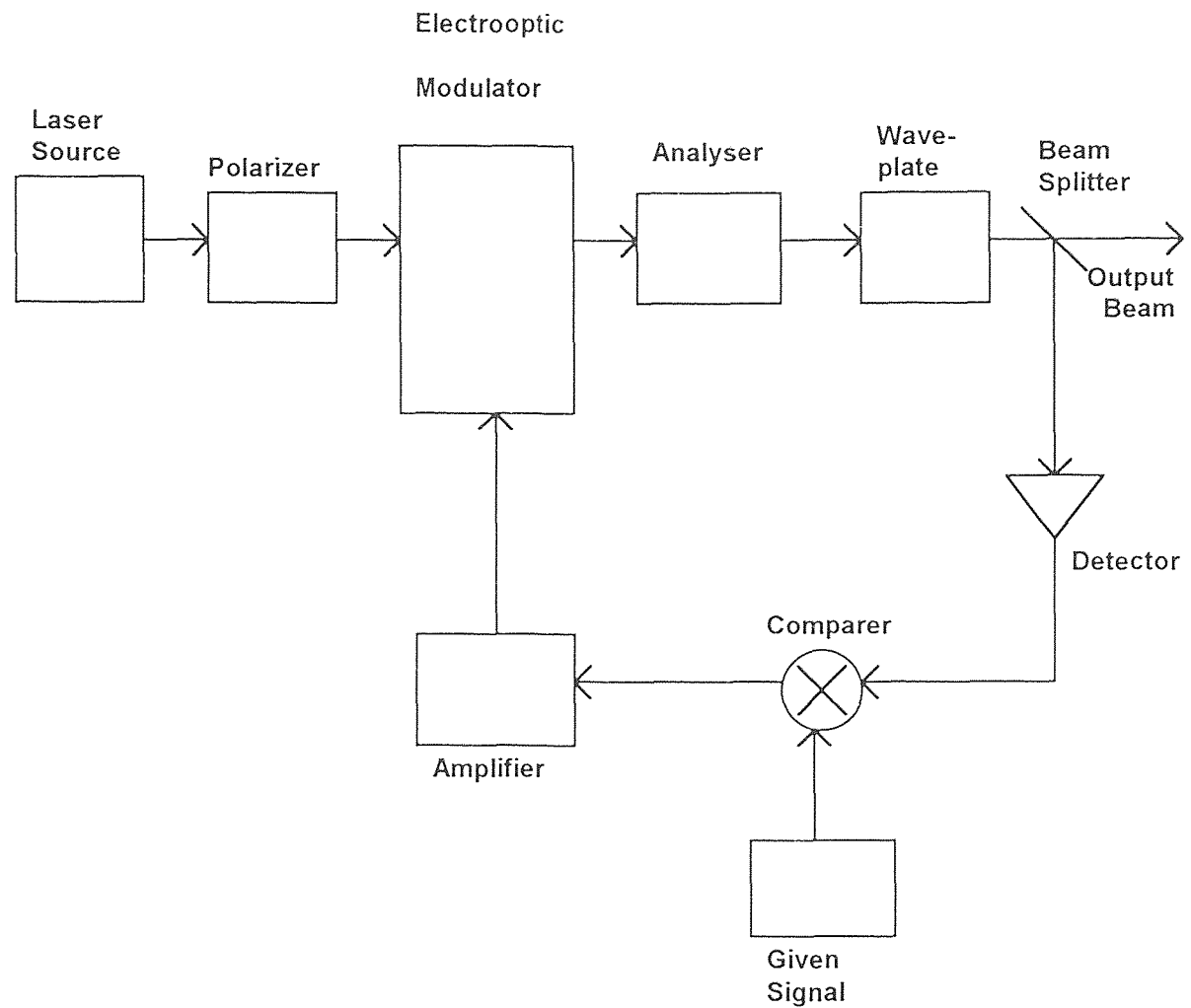


Figure 4-1 The structure of laser power stabilizer

### 4.3 Electro-optic Modulator

There are a variety of factors which will cause the fluctuation of laser power. No matter which is the principal factor, the stabilizer needs only to monitor and control the output intensity. So a high speed response is needed. From the discussion of Chapter 3, we already know that there are two main methods of electro-optic modulation, Phase Modulation and Transverse Modulation. To compare these two methods, we refer to Equations (3.5-5) and (3.5-11). From (3.5-11), we know that transverse modulation shows an increase in the frequency limit or useful crystal length of  $(1 - c_0/nc_m)^{-1}$ . Here,  $c_m$  is the propagation speed of electromagnetic field;  $n$  is the refractive index of electrooptic crystal;  $c_0$  is the light velocity in vacuum. So, we elect to use the Transverse Electro-optic Modulation as the basis of our stabilizer. ( as shown in figure 3.5 )

We use an electro-optic modulator made with crystal  $\text{LiNbO}_3$ , its crystal length is  $L = 20\text{mm}$ , and its width is  $d = 6\text{mm}$ . Its modulation half-wave voltage is  $V = 1200\text{V}$ . These parameters require that the amplifier produce a high voltage output.

### 4.4 Design of the Amplifier

After choosing a modulator, we need to design an amplifier in accordance with the characteristics of the modulator. The configuration of the detector, comparer and amplifier is shown in figure 4-2.

The detector which we use in this work is Silicon Photodiode S1337-33BR (HAMAMATSU COMPANY). For this detector we select OP-AMP CA3130 as the pre-amplifier. For the reference signal, we choose the output of MC1403, a precision low voltage supply. The comparer is an op-amp OP27. The two-step amplifiers are two op-amps OP07. The most important and most difficult section is the power amplifier. Because the halfwave voltage of the modulator is 1200V, the

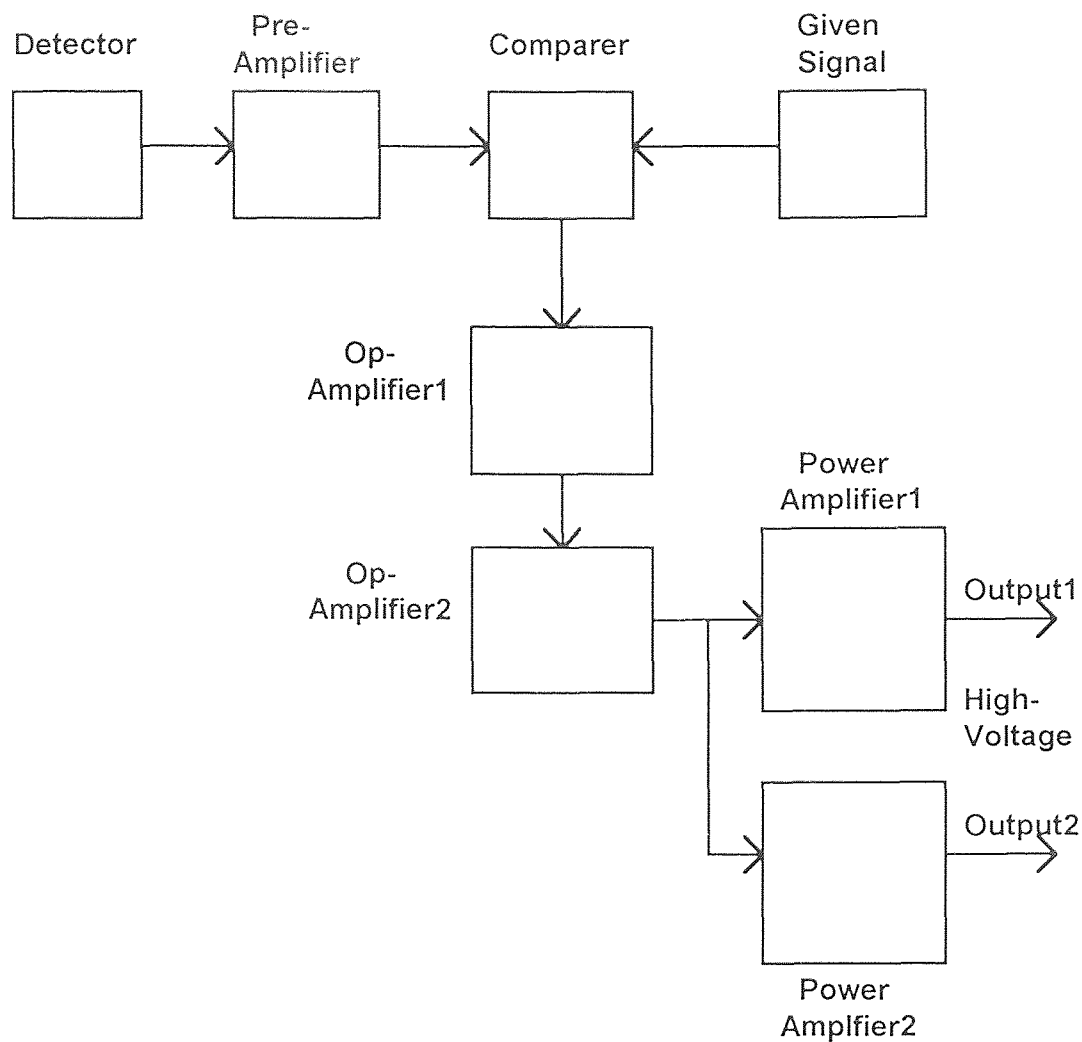


Figure 4-2 The structure of the detector, comparer and amplifier

output voltage difference of power amplifier must reach 1200V.

In general, we can use power transistors in the design of the power amplifiers. For example, the output range of the op-amp is 0V--5V. When the output of op-amp is 0V, the power amplifier should output voltage 0V; and when the output of op-amp is 5V, it should output voltage 1200V. As shown below:

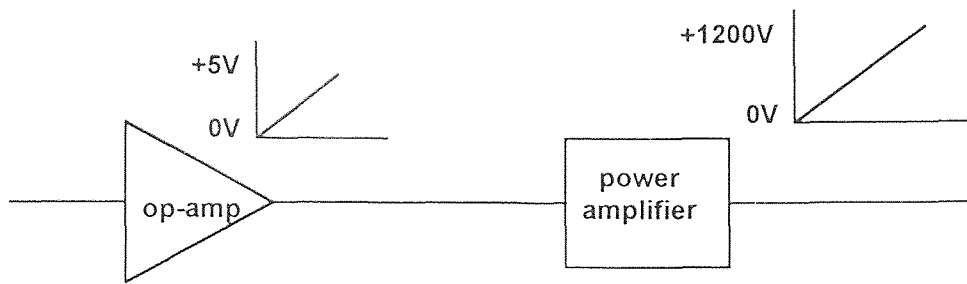


Figure 4-3 The single power amplifier configuration

Because we need a high voltage output but only a small current  $I_C$ , there is a problem in which we cannot get a power transistor for which  $V_{CE0}$  is 1200V and  $I_C$  is 1A or smaller. So, we have to find another way. We use a pair of op-amps to output signals with same magnitude and opposite sign and send them to two power amplifiers:

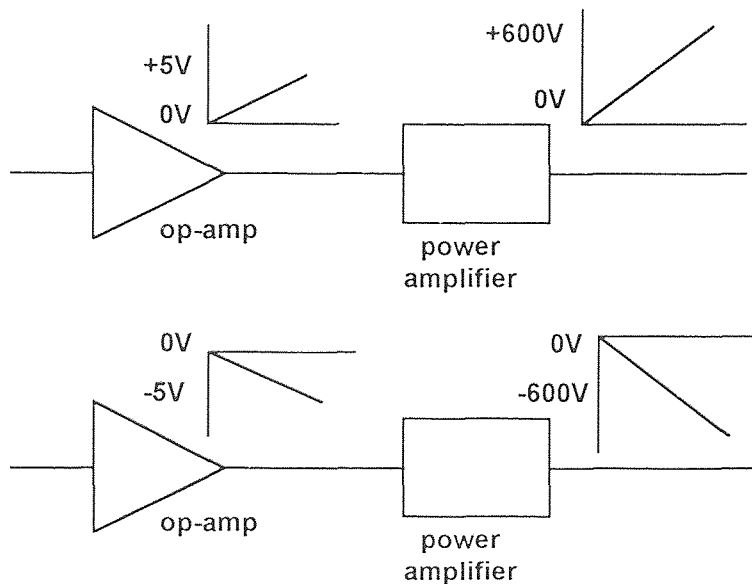


Figure 4-4 The power amplifiers in pair configuration



In order to implement this structure, we tried several circuits. Finally, we decided to use the OCL circuit ( Output Capacitorless ) and improve the structure as shown below:

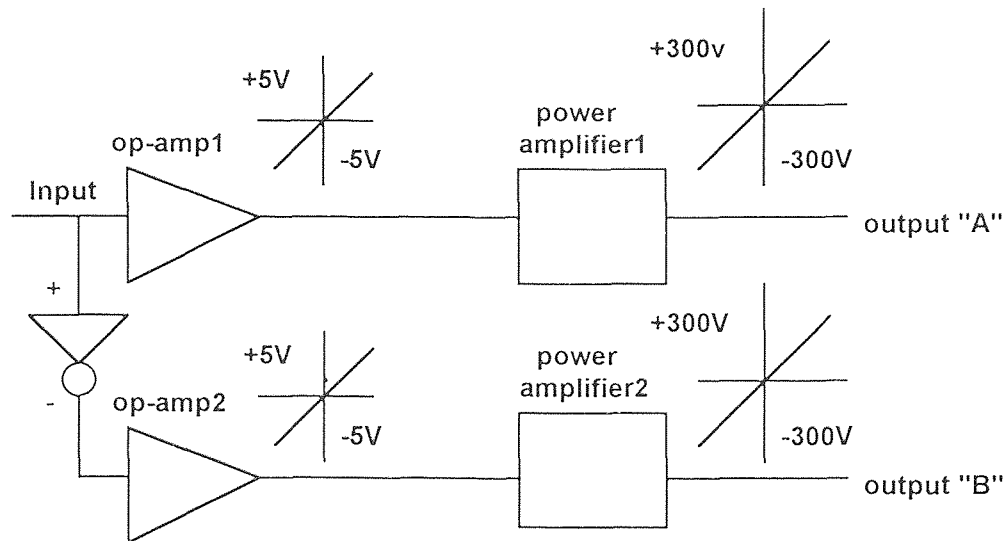


Figure 4-5 The structure of OCL circuit

The circuit is shown on Figure 4-6.

The working procedure of this circuit is: when the input is 0V, the output of the two op-amps are both 0V, so the output "A" and output "B" are also 0V, there is no voltage output. When the input is positive voltage, the output of op-amp1 is positive and the output "A" is positive voltage ( maximum +300V), because there is a inverse gate, the output of op-amp2 is negative and the output "B" is negative voltage (minimum -300V), the voltage difference of these two points "A" and "B"  $V_A - V_B$  is positive and it can reach to +600V. When the input is negative voltage, the output of op-amp1 is negative and output "A" is negative ( minimum -300V ), the output of op-amp2 is positive since the inverse gate and output "B" is positive

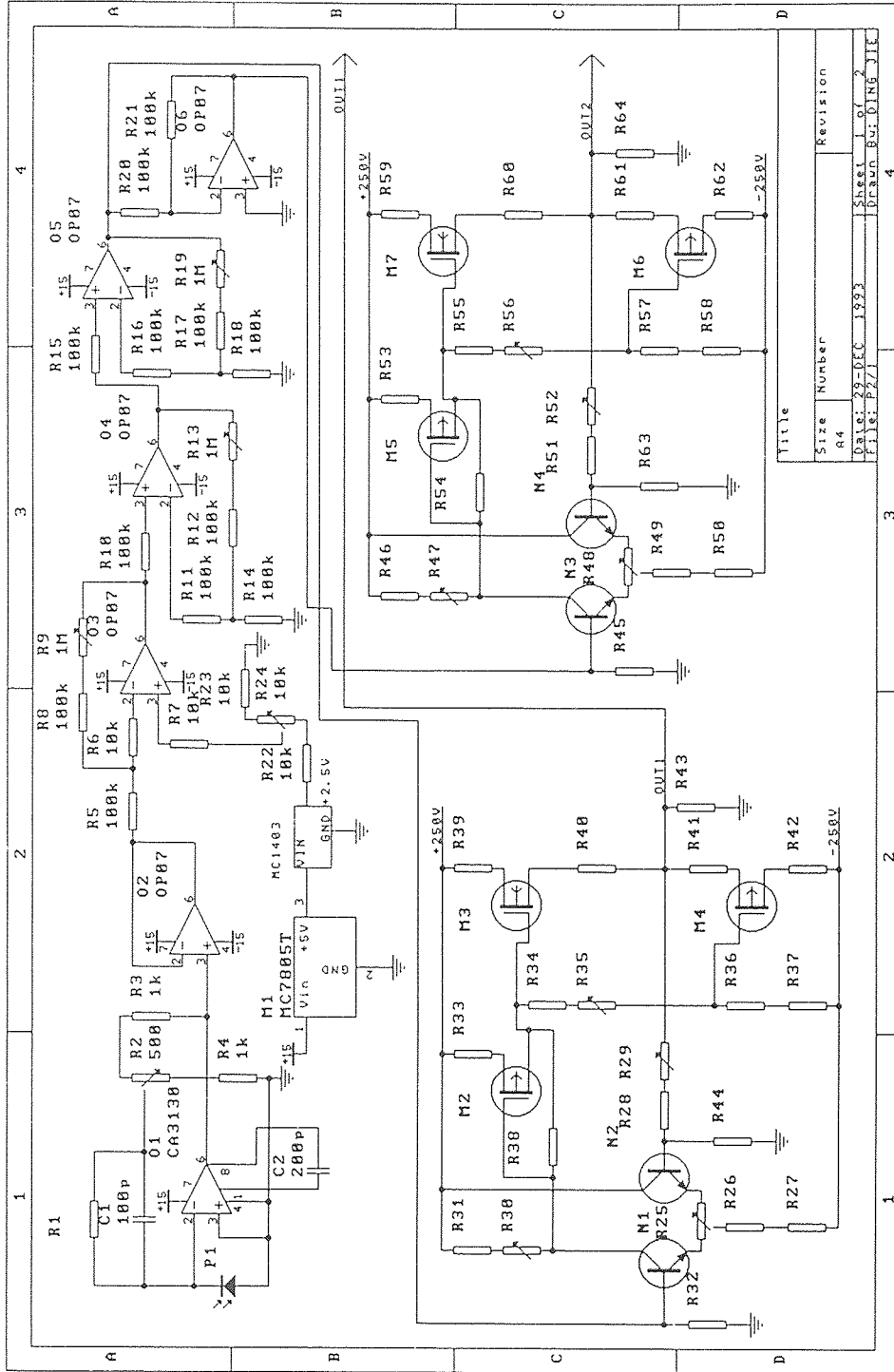


Figure 4-6 The circuit of the stabilizer amplifier

(maximum +300V ), the voltage difference of these two points is negative and it can get to -600V. For these two cases, the maximum voltage difference is 1200V and satisfies the halfwave voltage of the modulator.

The OCL circuit need a pair of power transistor, one PNP and one NPN. These two power transistors should have  $V_{CE0}=600V$  and  $I_C$  as small as possible. Because it is difficult to get a PNP transistor with high voltage, we select a pair of TMOS power FET as power amplifier. They are MTP2N50 and MTP2P50. The characteristics of these two FETs are: maximum  $V_{DSS}$  is 500V ; maximum  $I_D$  is 2.0A . So, the actual voltage range of this experiment is +500V to -500V. Although this voltage difference does not reach to 1200V, it can be used in this stabilizer.

## CHAPTER 5

### EXPERIMENT AND RESULT

#### 5.1 Modulator Working Range

There are several main factors involved in the stability of the output power. Besides the choice of appropriate detector and polarization components, another important factor is the electro-optic modulator working range. The modulator should operate in a linear voltage range. So, the first step of the experiment is to measure the ratio of output to input power intensity of the electro-optic modulator as a function of the voltage between its two polar plates. We placed the modulator between two polarizers, which have a fixed angle difference  $\Phi$  between their polarization direction. If we choose  $\Phi$  as parameter, when we change the magnitude and direction of the voltage of the modulator polar plates continuously, the transmission coefficient will be changed also. We can get the ratio curves of output power intensity  $I$  and input power intensity  $I_0$ . Some of the data can be seen in figures 5-1, 5-2, 5-3 and 5-4. Figure 5-1 is the ratio of  $\Phi = 35^0, \Phi = 40^0, \Phi = 45^0$ ; Figure 5-2 is  $\Phi = 50^0, 55^0, 60^0$ ; Figure 5-3 is  $\Phi = 65^0, 70^0, 75^0$ ; and Figure 5-4 is the ratio of  $\Phi = 80^0, \Phi = 85^0, \Phi = 90^0$ .

According to these curves, we can select a rotational angle  $\Phi$  of the two polarizers and let the modulator work in a linear range. For different laser sources or different controlling voltage range, the angle  $\Phi$  will be different.

From the curves, we find that if the electro-optic modulator works in the voltage range  $-500V$  to  $+500V$ , the angle  $\Phi$  should be selected from  $\Phi = 35^0$  to  $\Phi = 45^0$ . In this angle range, the ratio of output to input power intensity is almost linear.

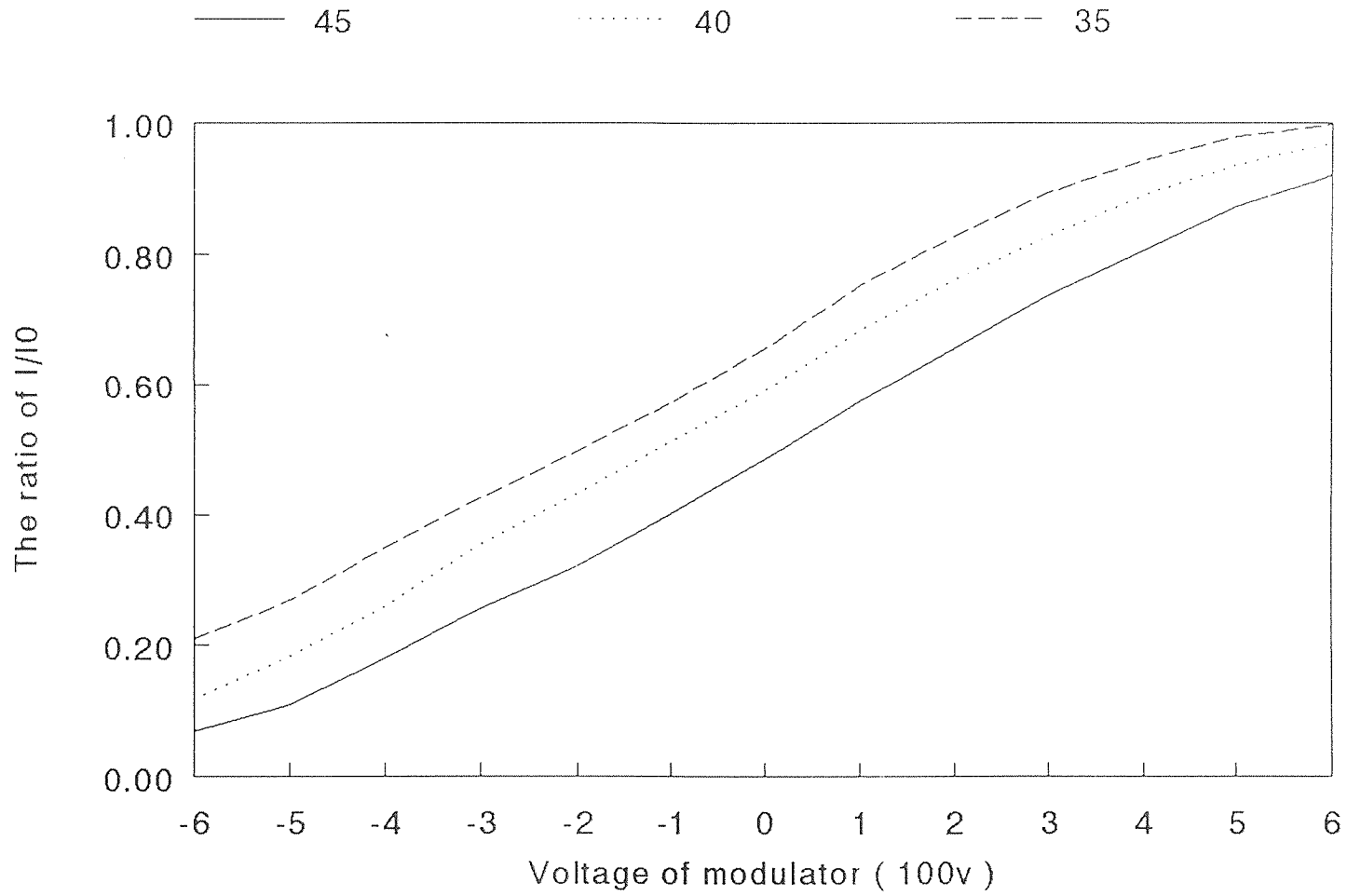


Figure 5-1 The ratio of output intensity  $I$  and input intensity  $I_0$  ( $I$ )

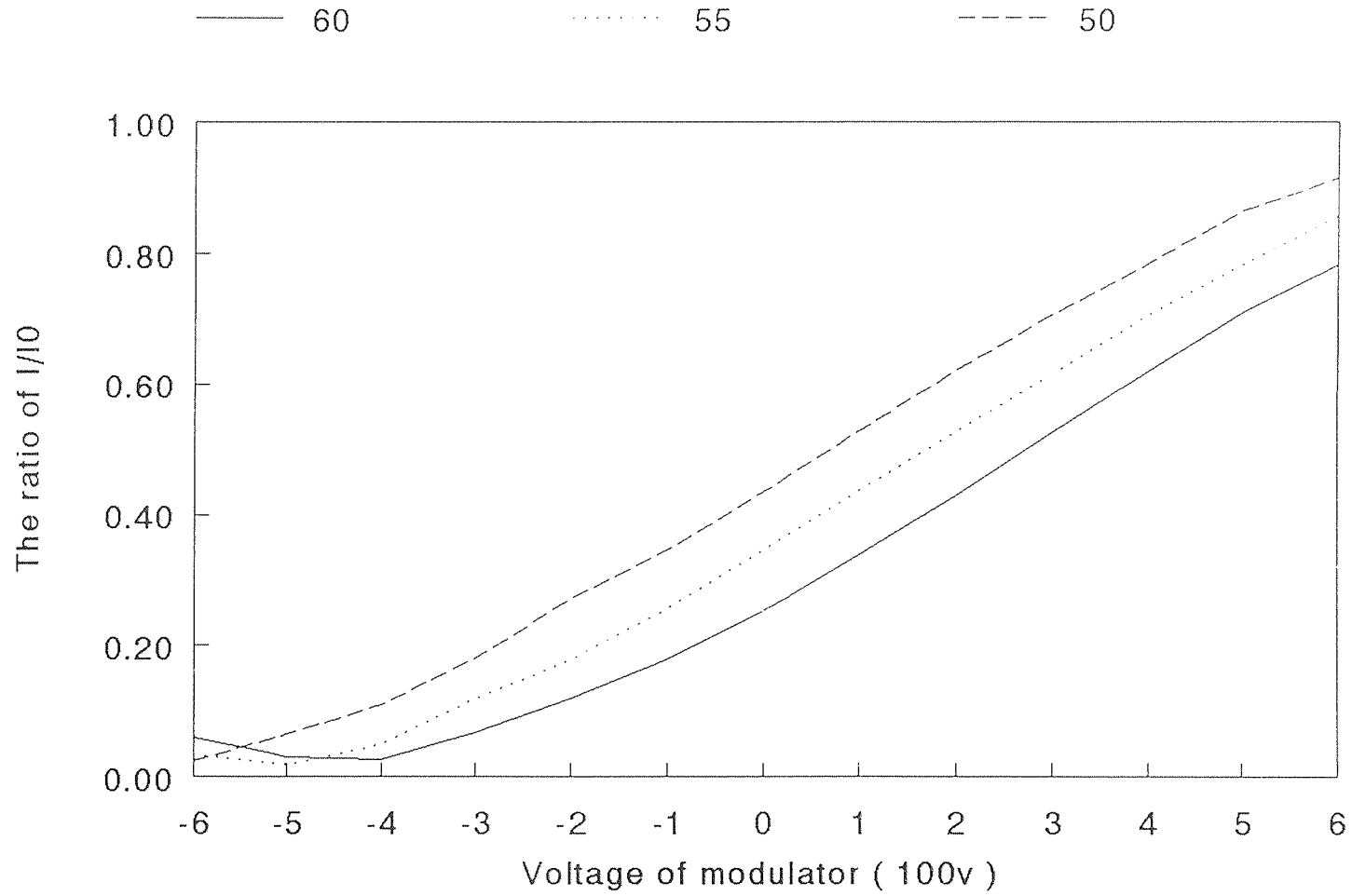


Figure 5-2 The ratio of output intensity  $I$  and input intensity  $I_0$  ( II )

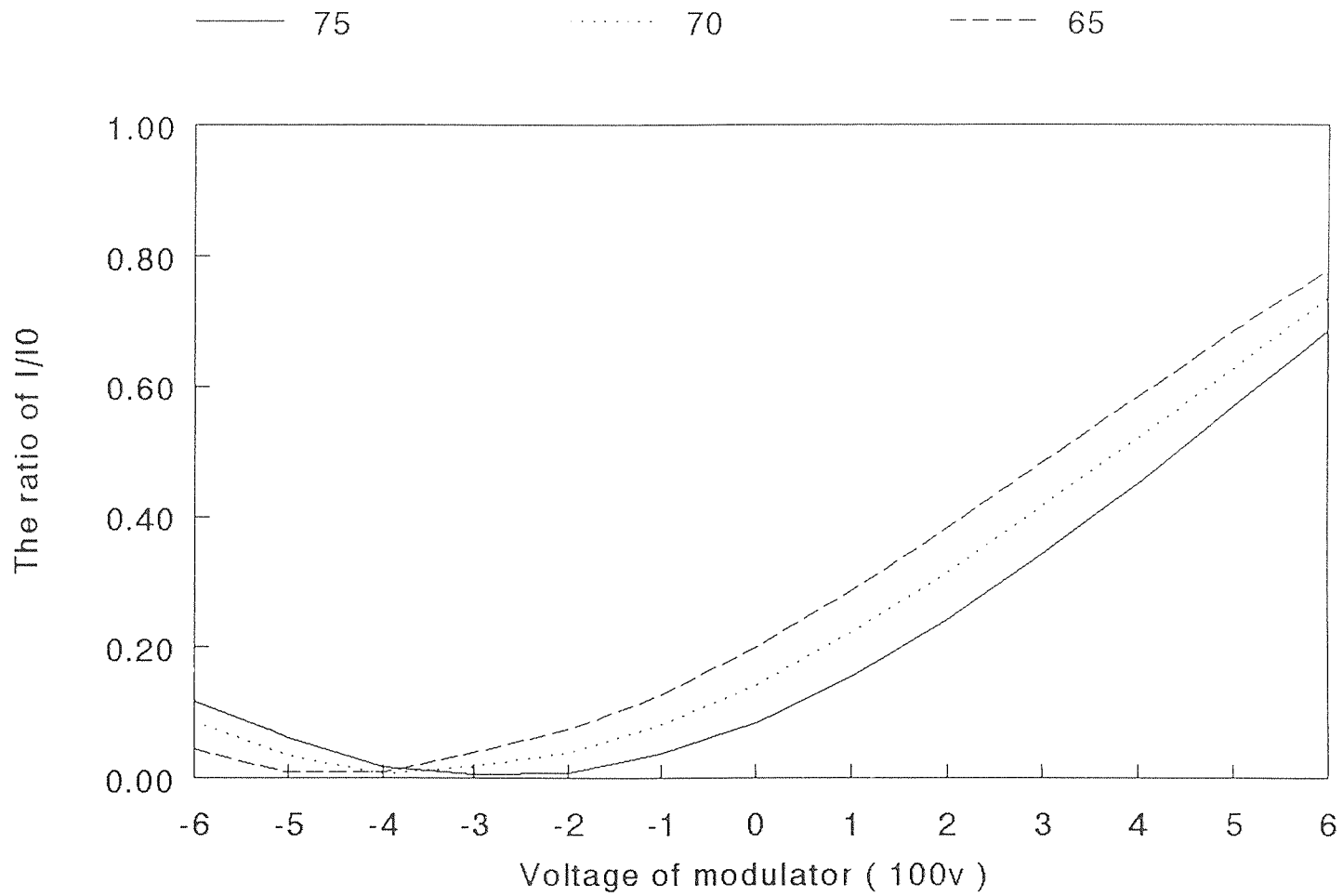


Figure 5-3 The ratio of output intensity  $I$  and input intensity  $I_0$  ( III )

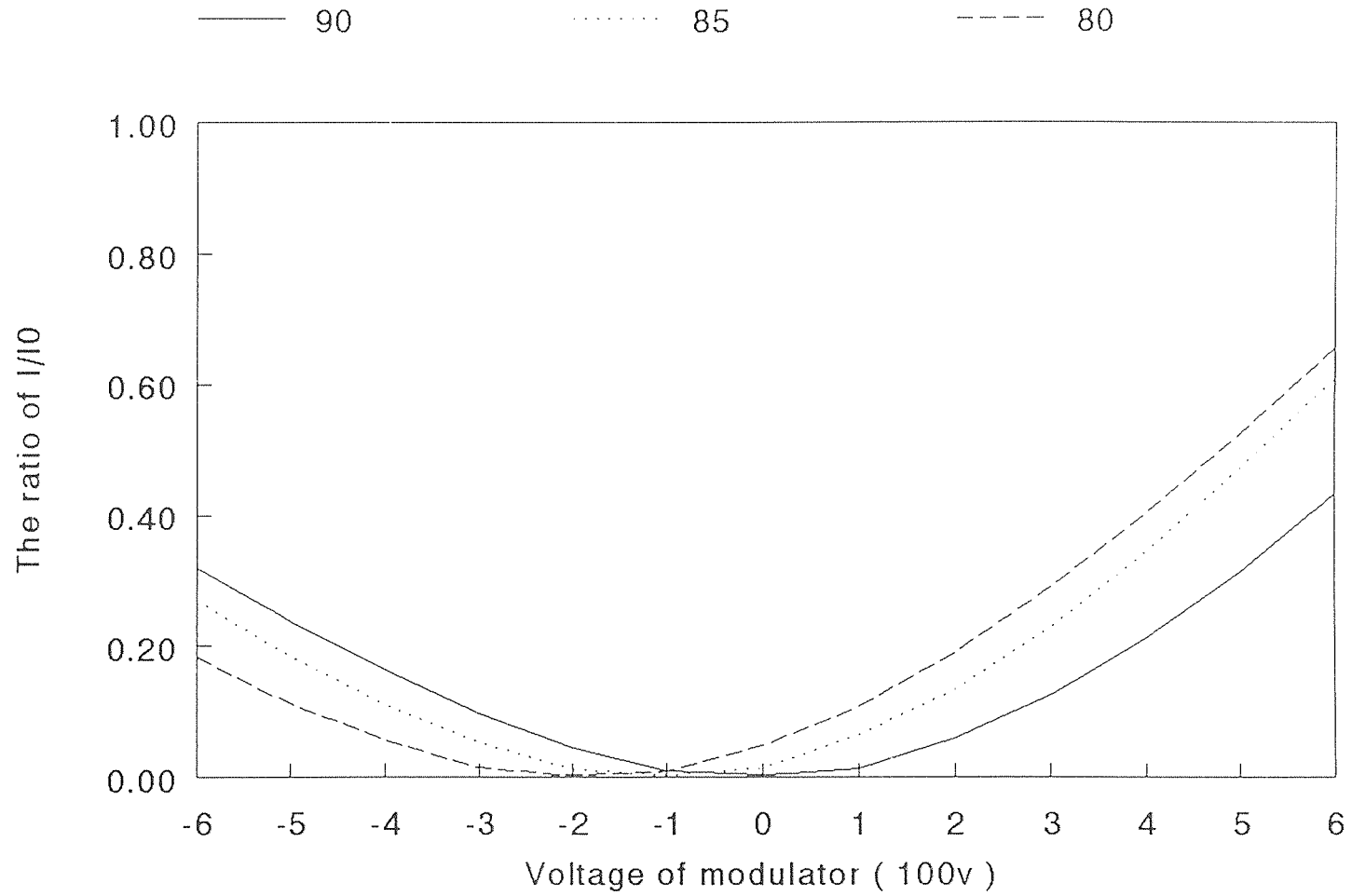


Figure 5-4 The ratio of output intensity  $I$  and input intensity  $I_0$  (IV)



## 5.2 Result

The experimental runs were carried out over a long period of time. We tested the argon ion laser without and with the stabilizer separately. First, we tested the laser without the stabilizer for 100 minutes. We used a detector with pre-amplifier to send the power signal to a 195A Digital Multimeter, and from the IEEE488 interface of the Multimeter to send the digital signal to a computer. Then we used a BASIC program to process the data. ( This BASIC program can be found in the Appendix ). Figure 5-5 shows the output of the laser source over 100 minutes. During this period, the highest point of output power is 7.7964, the lowest point is 7.0865; the fluctuation about the mean is approximately 5%. After turning on the laser power stabilizer, we tested the output for another 100 minutes. The result is shown in Figure 5-6. The highest point is 5.4326, the lowest point is 5.4012, the fluctuation about the mean is approximately 0.3% .

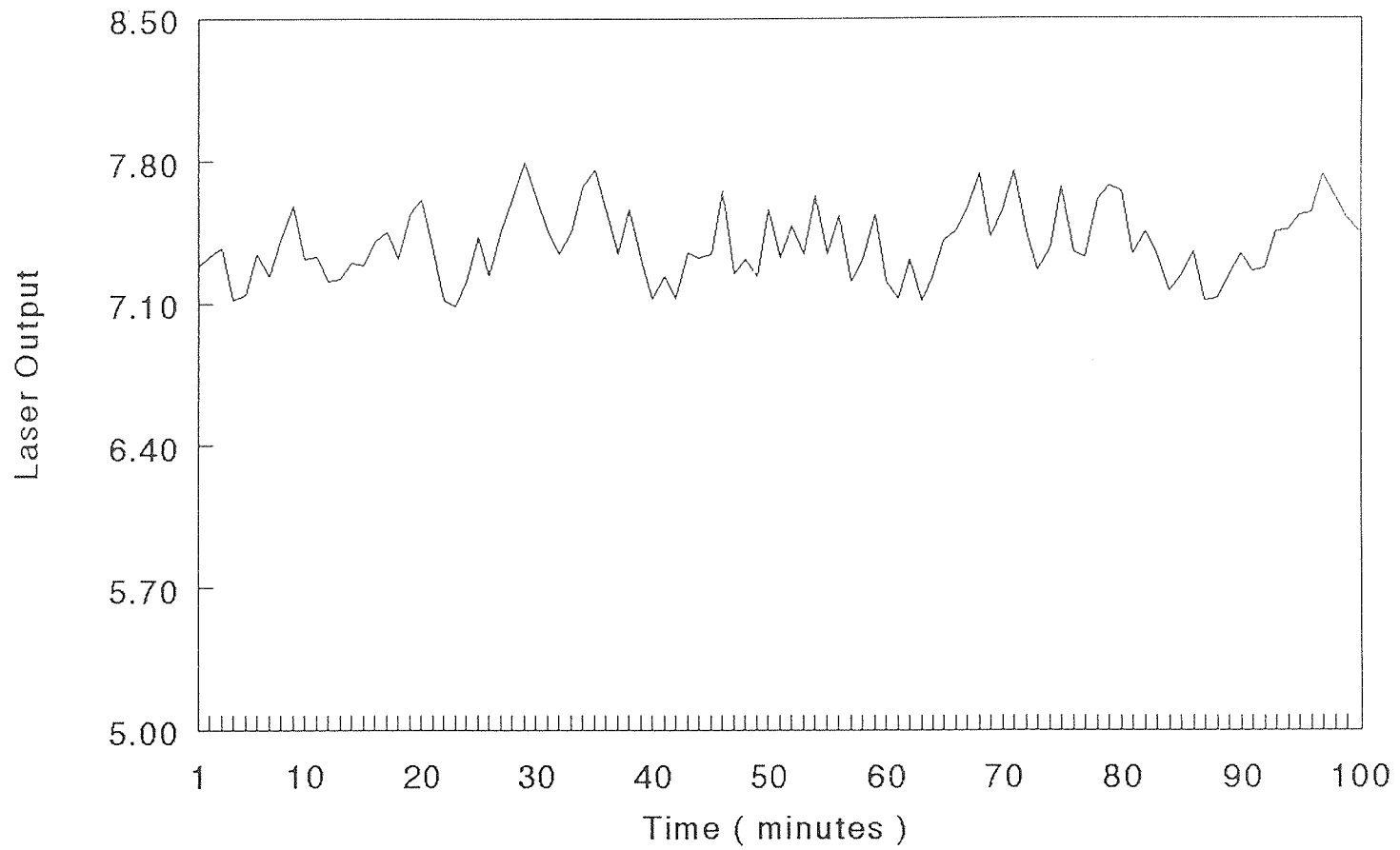


Figure 5-5 The output intensity of laser source

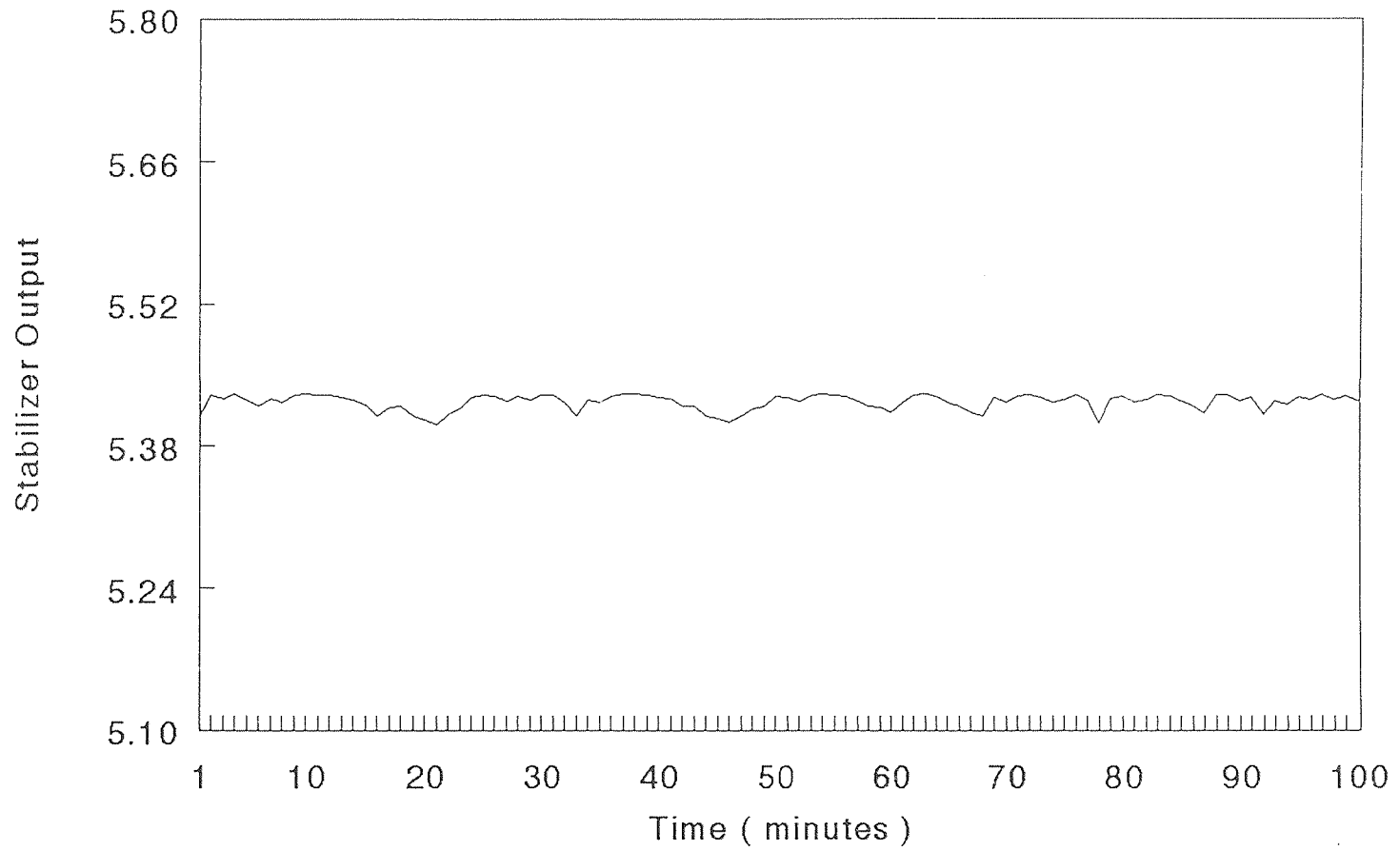


Figure 5-6 The output intensity of the stabilizer

## CHAPTER 6

### CONCLUSION AND ANALYSIS

From our results, we find that after using the laser power stabilizer the fluctuation can be reduced from 5% to 0.3%, which satisfies our expectations.

However, when we use the stabilizer, we will lose some power. So, this stabilizer can be used in some special applications which need the maximum in stability at expense in maximum power. In research of light scattering in air frequency stability of 1 part in  $10^{-8}$  or  $10^{-9}$  is required which is readily achieved in conventional laser system., so frequency stability is seldom a problem. But for its correlation over long distances, we can not correct the received scattering signal with respect to reference light intensity. In this case, laser power stability is the paramount factor. In many experiments with CW laser as pump light, there are a lot of trouble brought by the fluctuation of laser power. In the measurement of spectra by scanning a monochromator, laser output stability must be maintained during the scanning time. Sometimes, we can monitor the pump light intensity to cancel the error raised by the fluctuation of pump light. But if there are any nonlinearities present in the optical path, it is impossible to correct it with a linear procedure. For this case, the laser power stabilizer is very useful .

In this experiment, there are several factors which require further consideration. First, there is the loss of power. The main loss of power is in the two polarizers. If we do not want to lose too much power, we need to select better ones. They are also more expensive. Second, the electro-optic modulator working range is another important factor. If we select the range in a nonlinear area, the modulator can not trigger the change of the output power. The power amplifier should also work in its linear amplification area. In this experiment, we got the

modulator which halfwave voltage is high (1200V). But it is difficult to get a pair power transistor (PNP and NPN). We only use the voltage range between -500V and +500V and the power TMOS FET works in the nonlinear range.

To further improve this stabilizer, we need to change the modulator with low halfwave voltage, get polarizers with less loss, and design the power amplification circuit with power transistors which can work in a linear area. We also need a detector with lower dark current.

## APPENDIX A

### BASIC PROGRAM FOR DATA ACQUISITION

```
30 DEF FNT ( X ) = 3600 * VAL (MID$ (TIMES$, 1, 2 ) ) + 60 * VAL (MID$
(TIMES$,4, 2 )) + VAL (MID$ ( TIMES$, 7, 2 ))
40 OPEN "\DEV\IEEEEOUT" FOR OUTPUT AS #1
50 OPEN "\DEV\IEEEEIN" FOR INPUT AS #2
60 IOCTL #1, "BREAK"
70 PRINT #1, "RESET"
80 PRINT #1, "CLEAR"
90 CLS
100 OPTION BASE 1
110 DIM V1 (6000)
120 PRINT #1, "REMOTE 25"
130 PRINT #1, "OUTPUT 25; F0X"
140 PRINT #1, "OUTPUT 25; R0X"
150 PRINT #1, "OUTPUT 25; P1X"
160 PRINT #1, "OUTPUT 25; G1X"
200 S = 60
210 N = 6000 : T0 = FNT ( 1 )
220 FOR I = 1 TO N
230 T$ = TIMES$
240 FOR K = 1 TO S
250 IF T$ = TIMES$ THEN 250
260 T$ = TIMES$
270 NEXT K
280 REP = 15 : V1 = 0 : V2 = 0
290 FOR J=1 TO REP
300 PRINT #1, "ENTER 25"
310 INPUT #2, Y
320 V1 = V1 + Y
330 NEXT J : V1 ( I ) = V1 / REP
340 TT = FNT ( 1 ) - T0
350 PRINT I, V1 ( I ), TT
360 BEEP
370 IF HH$ = "STOP" THEN 390
380 NEXT I
390 CLOSE 1 : CLOSE 2
400 INPUT "DATAFILE NAME" ; A$
410 OPEN A$ + ".DAT" FOR OUTPUT AS #1
420 FOR J = 1 TO I - 1
430 PRINT #1, J, V1 ( J )
```

```
440 NEXT J  
450 CLOSE #1  
460 END
```

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