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Theoretical evaluation of the Break Off Test for concrete

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ABSTRACT

THEORETICAL EVALUATION OF THE BREAK OFF TEST FOR CONCRETE

by

Arjuna Priyara Ranasinghe

Strength of concrete is normally measured using the standard cylinder or cube. The measured strength is used for design. The accuracy of concrete strength is frequently challenged, particularly in large concrete structures where size effect of the test specimens is attributed for the differences. Many nondestructive tests were developed to evaluate concrete strengths. In recent years, it was obvious that these tests are unreliable. As the infrastructure decays, more nondestructive tests are required to evaluate the existing structures.

The Break Off Test is a recently developed nondestructive test. Although substantial amount of experimental investigations have been carried out on this test, no in-depth theoretical evaluation has yet been done to date.

In this study the behavior of the break off test specimen was investigated and the potential theoretical basis of this test explored.

Based on linear elastic fracture mechanics, a model to predict the compressive strength of concrete-manometer reading relationship of the break off tester was proposed and compared with experimental results

with good correlation. Both flexural and shear failure modes were considered and the effect of aggregate interlock was investigated.

The stress distribution of the deep-beam cantilever core was obtained using finite elements. It also confirmed the experimentally established minimum thickness of structural members for which this test method could be used. The study also found that the American Concrete Institute's recommendation on the modulus of rupture is an extremely conservative value, especially for members with widths less than 6". The modulus of rupture of a rectangular beam is different from that observed from a circular cross section such as the break off test specimen. These findings strengthen the concerns over the size effects on various recommended concrete strength parameters. In this study, new modulus of rupture values were suggested for small rectangular beams and members with circular cross sections.

The study confirmed the existence of a theoretical basis for the break off test and showed that it can be a simple and reliable nondestructive test for measuring the compressive strength of concrete.

THEORETICAL EVALUATION OF THE BREAK OFF TEST FOR CONCRETE

by
Arjuna Priyara Ranasinghe

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This dissertation is dedicated to
the author's wife Chintha Ranasinghe

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CHAPTER 1

INTRODUCTION

Ever increasing use of concrete in the construction industry necessitates developing reliable quality assurance practices such as measuring strength of concretes to ensure safety. Traumatic construction failures such as " Cooling Tower Failure in West Virginia " in 1978 (1) and " Skyline Plaza Collapse " in Connecticut (2) have raised doubts on reliability of current quality assurance practices, to assess strength of concrete structures.

A popular method of measuring strength of concrete in structures is the " Cylinder Test " (3). This test was developed many years ago as the industry needed a simple way to measure the strength of concrete. In this test, a representative sample from a batch of concrete, in the form of a cylinder, is tested to assess the potential compressive strength of the batch.

The actual compressive strength of the concrete in the structure (in-situ strength) is not given by the test cylinder. Many researchers have repeatedly observed discrepancies between the strength measured in the concrete structure and the standard strength determined on cylinder specimens cast with the same concrete mix (4-10). Such discrepancies should be expected as the in-situ concrete is placed, compacted, and cured in a different manner than the cylinder specimen concrete. Further, it is unusual for the concrete in a structure to have the same maturity as a standard-cured cylinder and it is difficult and often impossible to assure identical bleeding. The cylinder test is often susceptible to abuse. Improper handling or inappropriate storage of these cylinders may result in misleading data for critical operations.

The best way to measure the accumulated effects of all the variables that would influence the concrete strength in a structure is the use of an in-situ method.

It is increasingly being recognized by the industry that strength of concrete in structures should be measured by in-place testing (11). Referring to construction failures, former president of the American Concrete Institute (ACI), R. E. Phillieo, stated : " I am not aware of an example where collapse followed the verification of concrete quality by in-situ testing " (12).

Construction practices have changed over the years and today a contractor may want to remove the formwork as soon as possible after casting. A knowledge of the in-situ strength and other properties is essential for this purpose.

Determination of accurate in-situ strength is most critical in prestress and post-tension force release operations, because the structural element should not be stressed before a certain level of in-situ strength is achieved.

The concrete in nuclear reactor systems, are subject to various degradation modes related to irradiation and thermal effects. This results in a loss in concrete strength and shielding efficiency (13). It is important to determine the in-situ strength of concrete to assess the accumulated damage in concrete in order to assure the safety and integrity of nuclear reactor concrete structures.

The use of in-situ testing becomes very important when the responsibility for the concrete is divided. In disputes, it is essential to determine the performance of each party. The concrete supplier is responsible for delivering adequate quality concrete to the site which is tested by the standard cylinder. The contractor is responsible for handling, forming, stripping and curing the concrete which can be tested by an in-situ method.

When structures of historical importance are to be preserved or restored, nondestructive tests are carried out in order to obtain the information needed without destroying or damaging the structure with respect to its historic or artistic character (14).

Millions of concrete highways, bridges, buildings, dams, sewage and water works, flood walls, locks, harbor works, and airports, around the world need constant repair and maintenance. As they age, most of them have to be rehabilitated. In such projects one could use nondestructive testing methods to assess the degree of deterioration and evaluate concrete characteristics such as compressive strength. This will invariably reduce the project cost and the completion time.

For decades, fire damaged concrete structures were evaluated by visual inspection or auditory methods like using a hammer, a metal chain or an archaeological pick. With the advent of reliable nondestructive test methods, a much more comprehensive assessment of damage is possible (15).

It is no secret that, although standard cylinder lends it self readily as a standard to measure compressive strength of concrete specimen, it no way gives the actual strength of concrete in a structure. Therefore, if one is interested in the actual strength of the concrete in a structure, whether it is for quality control, precast and prestress concrete operations, evaluation and repair, restoration, and rehabilitation one has to resort to in-situ, nondestructive test methods.

CHAPTER 2

LITERATURE SURVEY

2.1 Existing Nondestructive Test Methods

2.1.1 Introduction

Several nondestructive methods are available to predict in-situ characteristics of concrete such as compressive strength, Poisson's ratio, modulus of elasticity, modulus of rupture, voiding, honey combing, micro and macro-cracking, loss of cement matrix, and loss of bond to aggregate etc. The most widely used methods are as follows.

1. Hardness Test (also known as Rebound Hammer, Schmidt Hammer or Swiss Hammer)
2. Probe Penetration Test (also known as Windsor Probe)
3. Resonant Frequency Method
4. Mechanical Sonic Pulse Velocity Method
5. The Ultrasonic Pulse Velocity Method
6. The Maturity Method
7. Pull Out Test
8. Break Off Test
9. Cast In Place Cylinder
10. Core Cylinders

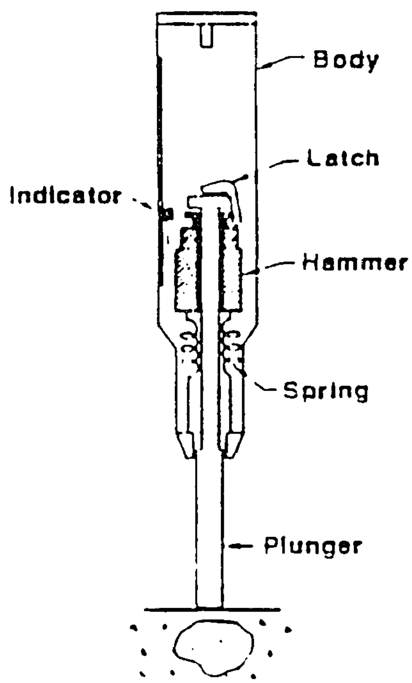


Figure 1 Rebound Hammer

In this study special emphasis will be given to the measurement of compressive strength of concrete in a structure. The underlying principles of some of these tests and background information is given by Malhotra (16), Bungey (17), and ACI Committee 228 (18). An excellent review of in-situ and nondestructive testing is given in ACI SP-82 (19).

The following section discusses the underlying principles, advantages, and the shortcomings of the above nondestructive tests.

2.2 Hardness Test (Rebound Hammer, Swiss Hammer, Schmidt Hammer, Sclerometer, Impact Hammer)

This test developed in 1948 by Ernst Schmidt, (20-23) is based on the principle that the rebound of an elastic mass depends on the hardness of the surface against which it impinges.

Figure 1 shows the components of the Rebound Hammer (18). To perform the test, the plunger is brought in to contact with concrete by extending the body of the instrument. At this position a latching mechanism engages the hammer to the upper end of the plunger. Then the body of the instrument is pushed towards the concrete surface. This extends the spring connecting the hammer to the body and subsequently the latch releases and the spring pulls the hammer towards the plunger. The hammer hits the plunger and rebounds. The rebounding hammer

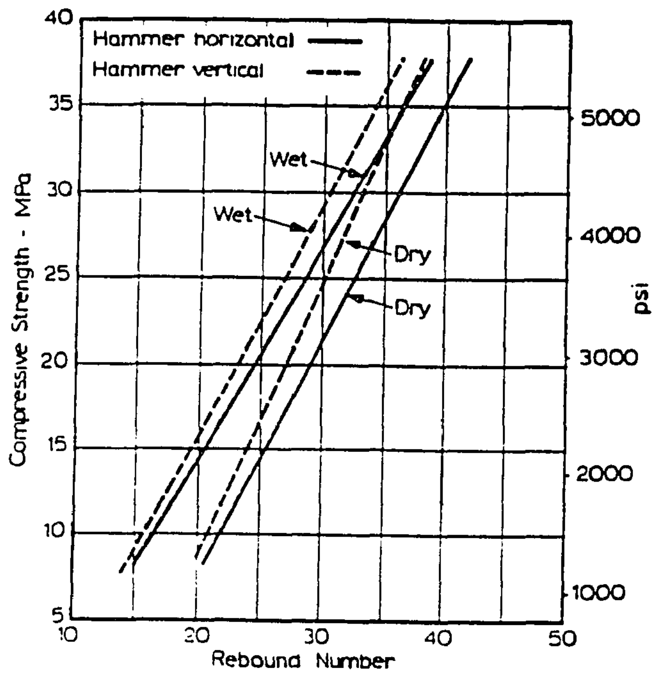


Figure 2 Compressive Strength vs Rebound Number

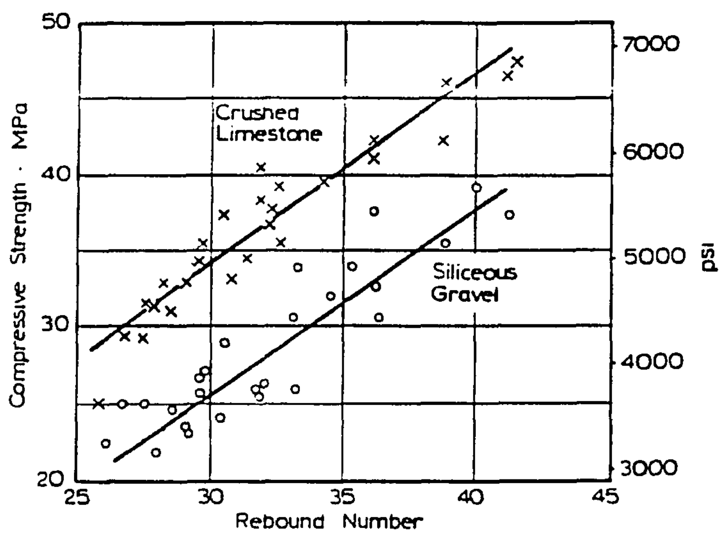


Figure 3 Effect of Aggregates on Rebound Test

moves the slide indicator which records the rebound distance. The distance traveled by the hammer expressed as a percentage of the initial extension of the spring is called the rebound number.

The rebound number is related to the energy absorbed by the concrete. This depends on the stress-strain curve of the concrete. Therefore, the rebound number is related to the strength and stiffness of concrete. There are relations developed between the rebound number and concrete strength properties. Kolek (24), has attempted to establish a correlation between the rebound number and hardness test as measured by Brinell method. Figures 2 and 3 show the relation between compressive strength and rebound number as observed by Willets (25) and Grieb (26).

The major advantage of this test is its simplicity, speed, and low cost.

2.2.1 Limitations of the Test

1. It is possible for a concrete to have the same strength but different stiffnesses. Since,the rebound number is related to both strength and stiffness, this will give two different rebound numbers. Also, it is possible for two concretes with different strengths to give the same rebound number if the stiffness of the low strength concrete is greater than the

stiffness of the high strength concrete. This can be disastrous in a critical operation.

2. Since, the rebound hammer test probes only the near-surface layer of concrete, the rebound number may not be representative of interior concrete. The presence of a layer of carbonation can result in higher readings than uncarbonated concrete surface. A dryer surface will result in a higher rebound number than for the moist-interior concrete. Slightly absorptive oiled plywood will absorb moisture from concrete and produce a harder surface layer than the interior concrete. Similarly, curing conditions also have a greater effect on the strength of surface layer than the interior concrete. On the other hand if bleeding occurs, the surface layer can be weaker than the concrete elsewhere on the structure, and result in misleading rebound numbers.

3. The aggregate type has an effect on the rebound number and therefore it is necessary to develop correlation relationships on concrete made with the same materials that will be used for the concrete in the structure. Klieger (27), has found that for equal compressive strength of concrete, crushed lime stone coarse aggregate show rebound numbers 7 points lower than those for concrete with gravel coarse aggregate. Green (28), has observed widely varying results when Schmidt hammer was used on light weight concrete.

4. The surface texture influences the rebound number. On rough textured concrete crushing occurs under the plunger and the indicated strength may be lower than the true value. Rough surfaces have to be ground before testing. Kolek (24), and Green (28), have found that troweled surfaces or surface made against metal forms yield rebound numbers 5-25% higher than surfaces made against wooden forms.

5. If the concrete section or specimen to be tested is small, any movement under the impact will lower the rebound number.

6 Although, the test can be conducted horizontally, vertically upward or downward, or at an intermediate angle, the rebound number is different at each angle for the same concrete and will require separate calibration or correction charts.

7. The degree of saturation of the concrete and the presence of surface moisture have a decisive effect on the results. Zoldners (29), has found that well cured, air dried specimens, when soaked in water and tested in saturated surface dried condition, show rebound readings 5 points lower than when tested dry.

8. It has been proved by Zoldners (29), and Victor (30), that for equal strength, higher rebound values are obtained on 7 days old cylinders than 28 days old cylinders. The use of the test hammer for low strength at early ages or where the strength is less than 1000 psi, is discouraged by Mitchell and Hoagland (31).

9. According to Kolek (32), the type of cement also affects the rebound number. High-alumina cement and super sulphate cement can give 100% higher and 50% lower values respectively than those obtained from ordinary portland cement concrete calibration charts. Polymer-impregnated concrete has been reported to give up to a 70% higher rebound number than unimpregnated concrete (33).

10. The test is sensitive to the local conditions where the test is performed. If the plunger is located over an aggregate, an unusually high rebound number will be given and, over an air void a very low rebound number will result. To take these possibilities in to account, ASTM C 805, requires at least 10 rebound numbers to be taken for a test (34).

Although, the rebound test is very easy to perform, it is seen that there are many factors other than concrete strength, that influence the test results. Malhotra (16), discourages the prediction of the strength of structural concrete by using calibration charts based on laboratory results.

2.3 Other Surface Hardness Methods

2.3.1 Williams Testing Pistol

In 1936, Williams (35) reported the use of a pistol that uses a ball as an indenter. The diameter of the impression made by the ball is measured by a magnifying scale. Williams established the relationship ;

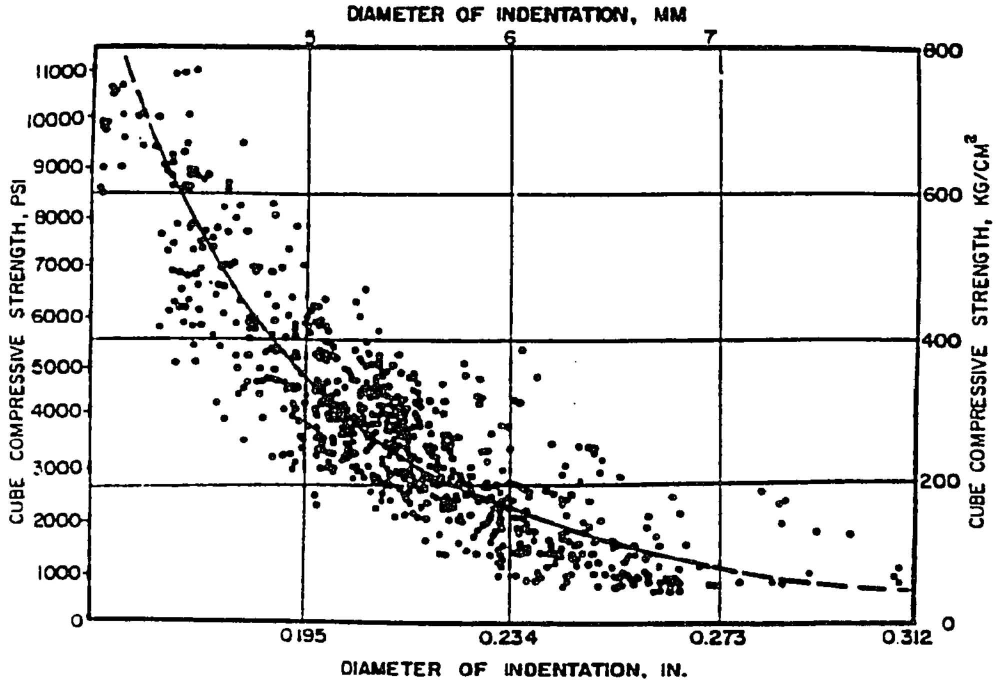


Figure 4 Compressive Strength vs Diameter of Indentation for Frank Spring Hammer

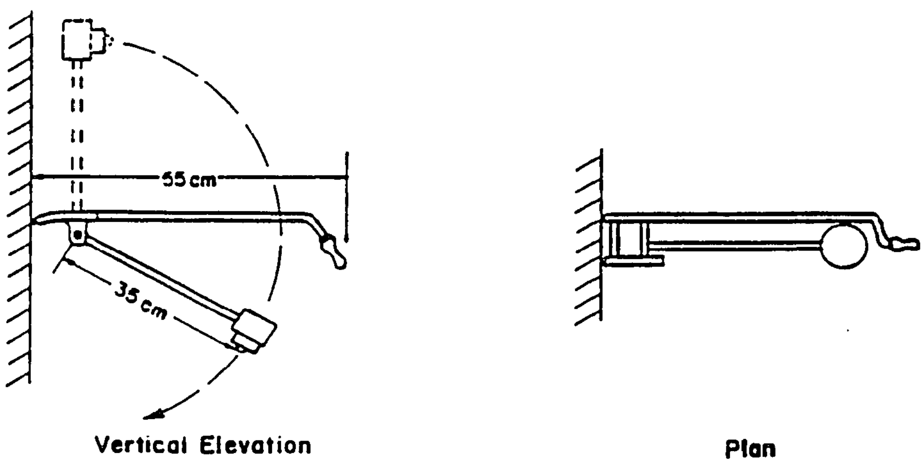


Figure 5 Einbeck Pendulum Hammer

f'_c is proportional to $1/Z$, where f'_c is the compressive strength and Z is the curved surface area of indentation.

Scramtaev and Leshchinzy (36) also have reported the use of a pistol in the testing of concrete in the USSR.

2.3.2 Frank Spring Hammer

The equipment consists of a spring controlled mechanism, housed in a tubular frame. The tip of the hammer can be fitted with different diameter balls and impact is achieved by placing the hammer against the surface under test and manipulating the spring mechanism. The diameter of indentation is measured, and this is correlated with the compressive strength of concrete. Figure 4 depicts the relation between compressive strength and diameter of indentation (37).

2.3.3 Einbeck Pendulum Hammer

Einbeck Pendulum Hammer is as shown in Figure 5 (37). It consists of a horizontal leg at the end of which an arm is pivoted with a pendulum head weighing about 5 lbs. The indentation is made by holding the horizontal leg against the concrete and allowing the pendulum head to fall and strike the concrete. The diameter and the depth of indentation is measured and these are correlated with the compressive strength of concrete. This hammer can be used for concrete with vertical surfaces only (37).

2.3.4 Limitations of Surface Hardness Tests

Weil (38) and others (39), have pointed out the need for extreme care in the use of these tests. Frequent calibration and checking of the hammers and the equipment are required. Almost all the limitations of Rebound Hammer discussed earlier are valid for these methods as well.

2.4 Probe Penetration Test (Windsor Probe)

Windsor Probe was developed from 1964 to 1966 by the Port Authority of New York and Windsor Machinery Co., Connecticut. The results of Ports Authority investigations were reported by Cantor (40). A number of other organizations and individuals have carried out exploratory investigations and prepared reports (41-46).

A specially designed gun is used to drive a hardened steel rod in to the concrete. The amount of penetration of the probe is used as an indicator of the concrete strength. The principle behind this test is, that the initial kinetic energy of the probe is absorbed by the concrete. An essential requirement of the test is, that the probe should have a consistent value of initial energy. To satisfy this condition ASTM C 803

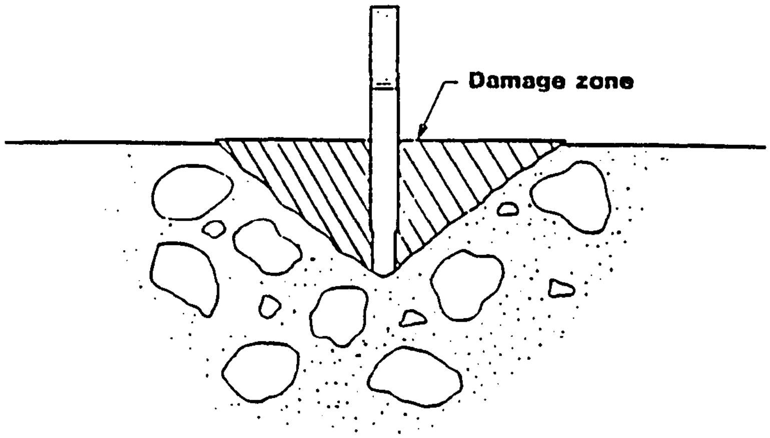


Figure 6 Failure of Concrete During Probe Test

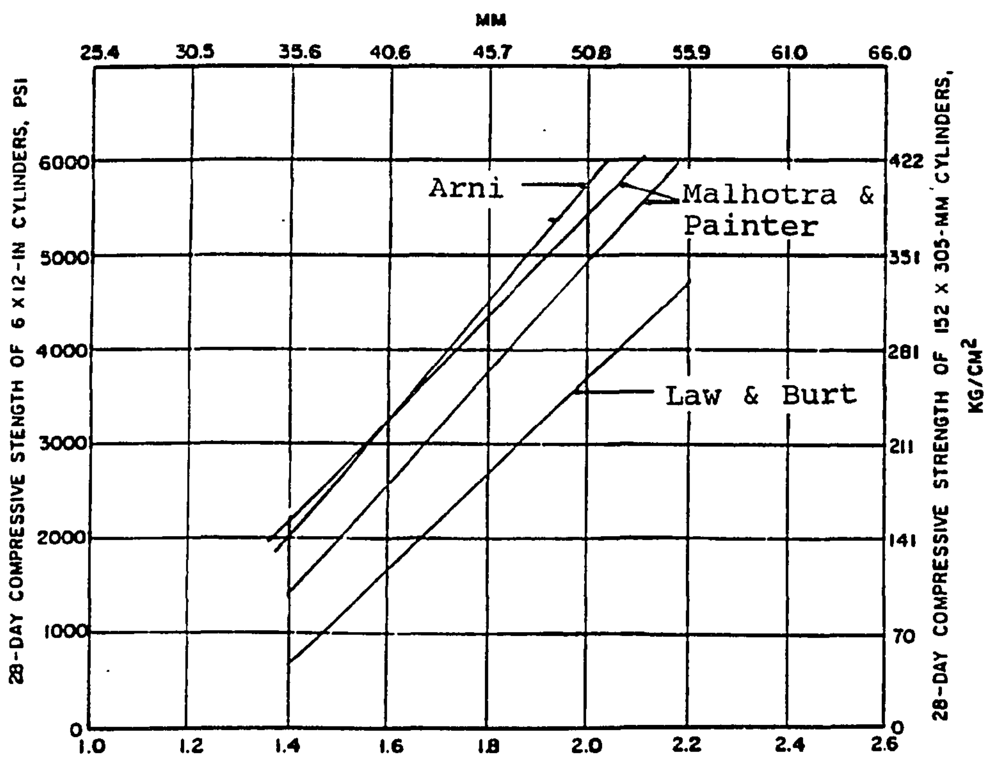


Figure 7 Compressive Strength vs Exposed Probe Length for Probe Test

requires that the exit velocities of probes should not have a coefficient of variation greater than 3% , based on 10 tests by approved ballistic methods (47).

Figure 6 depicts the approximately coned-shaped fracture zone, where most of the probe energy is absorbed (18). The cracks in the fracture zone are through the mortar matrix and the coarse aggregates. Hence, the strength properties of both materials influence the penetration distance. This contrasts with the behavior of concrete in compression in a compression test, where the strength of the mortar matrix is the most predominant factor. Thus, the type of aggregate has a very strong influence on the penetration tests. This is depicted by Figure 7, which is based on the investigations of Law and Burt (45), Arni (48), and Malhotra (49).

Low cost and speed compared to coring are the main advantages of this method. The Windsor Probe equipment is simple and within grasp of a lab technician. It is made rugged and needs little maintenance.

2.4.1 Limitations of the Test

1. Since the penetration test is strongly influenced by the type of aggregate, the manufacturer of Windsor Probe equipment provides calibration tables, that give different compressive strengths for each probe value depending on the hardness of the aggregate as measured on

the mohs' scale of hardness. Investigations carried out by Gaynor (44), Arni (48), and Malhotra (50) and Others (45) show that the manufacturer's tables can not be used with satisfactory results. Therefore, it is imperative for each user of the probe to calibrate his probe test results with the type of aggregate being used.

2. According to Malhotra (16), the within-batch variation of the Windsor Probe is at least two or three times as high as in the compression test. The following statement by Malhotra regarding this is worth noting;

"Because of the large variability in the probe test results,the usefulness of this approach lies in determining the relative quality of concrete in place rather than in its use as a means of quantitatively predicting the 28-day compressive strength of concrete".

3. Test results are not affected by local surface conditions such as moisture content, carbonation and texture. However, a harder surface layer as would occur in trowel finishing, can result in low penetration values.

4. The probe should be driven perpendicular to the surface. Whether the probe is driven horizontally, vertically up or down, does not affect the results.

This test is basically a hardness test and should not be expected to yield absolute values of strength of concrete in a structure. However, the

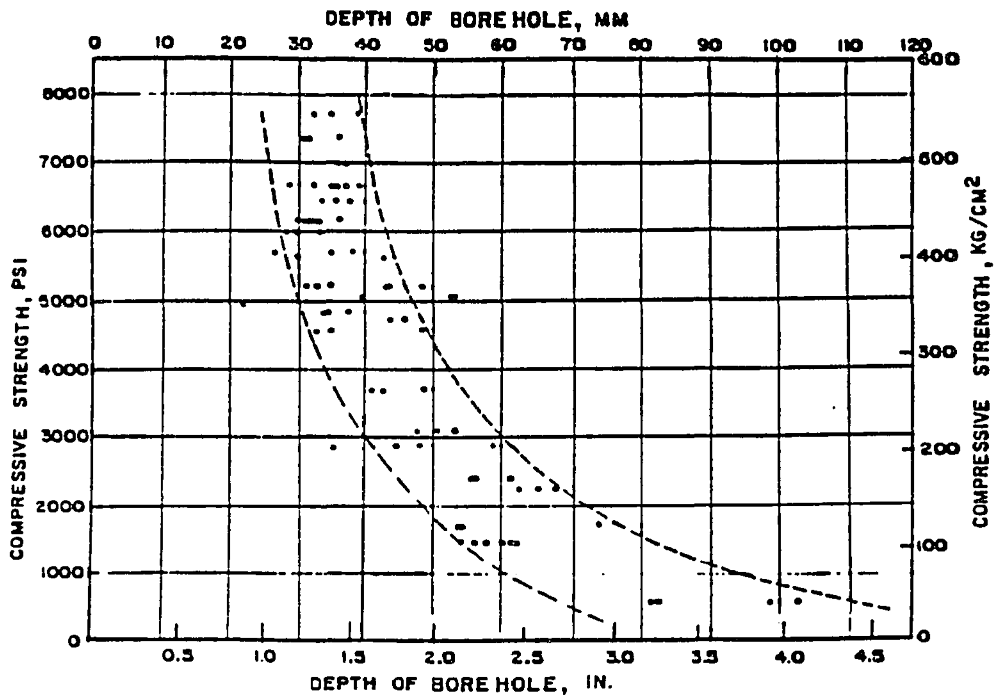


Figure 8 Compressive Strength vs Depth of Bore Hole for Simbi Hammer

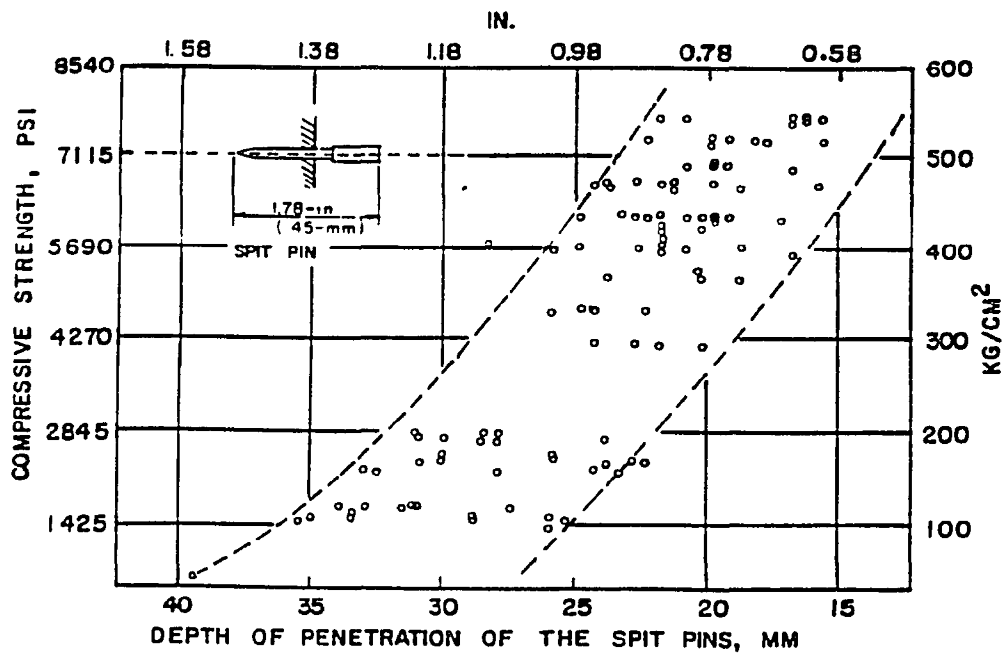


Figure 9 Compressive Strength vs Depth of Penetration of Spit Pins

probe test can be used to determine the relative strength of concrete in the same structure.

2.5 Other Penetration Techniques

2.5.1 Simbi Hammer

Voellmy (51) in 1954 used this hammer to perforate concrete and the depth of borehole was correlated to compressive strength of concrete (Figure 8). The results of this test was affected by the type and the arrangement of the coarse aggregate.

2.5.2 Split Pins

In this method the probing of concrete was achieved by blasting with spit pins, and the depth of penetration of the pins was correlated with the compressive strength of concrete as depicted by Figure 9 (51). The results of this test was affected by the type and arrangement of the coarse aggregate.

These tests appear to have received little acceptance. The introduction of rebound method may be one reason.

2.6 Dynamic or Vibration Method

The principles on which these methods are based were given by Rayleigh (52) as early as 1877. According to him, the natural frequency n of a long thin rod, vibrating in flexure is given by equations 2.1 and 2.2.

$$n = (k\ V\ m^2) / (2\ \pi\ L^2) \qquad (2.1)$$

$$V = (E / p)^{1/2} \qquad (2.2)$$

Where

V = Velocity of Sound

L = Length of Specimen

k = Radius of Gyration of the Section about an axis perpendicular to the plane of bending

m = A constant (4.73 for the fundamental mode of vibration)

E = Modulus of Elasticity

p = Density of the medium

The dynamic testing techniques can be divided into two principal methods; namely, Resonant Frequency Method and Pulse Velocity Method. The Pulse Velocity Method can be further subdivided into mechanical sonic pulse velocity method and ultrasonic pulse velocity method.

2.6.1 Resonant Frequency Method

This method was developed by Powers in the United States in 1938 (53), and improved by Hornibrook (54) by using electronic equipment to measure resonance. This method is based upon the determination of the fundamental resonant frequency of vibration of a specimen. The vibrations are continuously generated electromechanically. The equipment used is usually known as a sonometer.

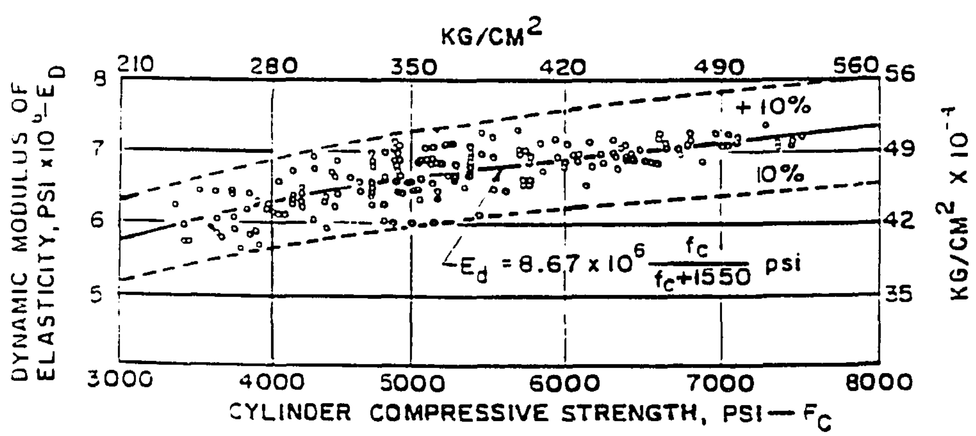


Figure 10 Dynamic Modulus of Elasticity vs Compressive Strength from Resonance Frequency Method

Figure 10 shows the relationship between the dynamic modulus of elasticity and cylinder compressive strength (55). The dynamic modulus of elasticity E_D is related to f'_c as follows;

$$E_D = 8.67\text{x}10^6 \, [\, f'_C \,] \,/\, [\, f'_C + 1550 \,] \text{ psi} \tag{2.3}$$

The following equations are given by ASTM C215-60 to calculate, transverse or flexural dynamic modulus of elasticity, longitudinal dynamic modulus of elasticity and dynamic modulus of rigidity (56).

1. The transverse or flexural modulus of elasticity,

$$E_R = C\,W\,n^2 \tag{2.4}$$

where, E_R = Dynamic modulus of elasticity in psi

W = Weight of specimen in lbs

n = Fundamental transverse frequency in cycles per sec

C = $0.00416\ L^3T/d^4$, sec^2/sq.in (for a cylinder)

= $0.00245\ L^3T/bt^3$, sec^2/sq.in (for a prism)

L = Length of specimen in inches

d = Diameter of cylinder in inches

t,b = Dimensions of cross section of prism in inches

T = A correction factor

2. The longitudinal dynamic modulus elasticity,

$$E^R = D\,W\,(n')^2 \tag{2.5}$$

where, E^R = Dynamic modulus of elasticity in psi

W = Weight of specimen in lbs

n' = Fundamental longitudinal frequency in cycles per second

D = 0.01318 L/d^2, sec^2/sq.inches (for a cylinder)

= 0.1035 L/bt, sec^2/sq.inches (for a prism)

L = Length of specimen in inches

t,b = Dimensions of cross section of prism in inches

3. Dynamic modulus of rigidity,

$$G_R = B\,W\,(n'')^2 \tag{2.6}$$

where, G_R = Dynamic modulus of rigidity in psi

W = Weight of specimen in lbs

n'' = Fundamental torsional frequency in cycles per second

B = $4\ L\ R\ /\ g\ A$, sec^2/sq.inches

L = Length of specimen in inches

R = Shape factor (1.0 for a cylinder 1.183 for a square section)

g = gravitational acceleration (386.4 in./sec^2)

A = Cross sectional area of specimen in square inches

4. Poisson's ratio of a small regular shaped specimen,

$$u = E_R\ /\ 2\ G_R \tag{2.7}$$

where, u = Dynamic Poisson's ratio

E_R = Dynamic modulus of elasticity

G_R = Dynamic modulus of rigidity

2.6.1.1 Limitations of the Test A number of factors affect the resonant frequency measurements, the dynamic modulus of elasticity. Some of them are discussed as follows.

1. According to Jones (57) the dynamic modulus of concrete is affected by the moduli of its constituent materials.

2. Obert and Duval (58) have showed that when specimens of different sizes are made from the same concrete, and tested by flexural resonance methods, different values of dynamic modulus are obtained.

3. The effect of curing conditions on the resonance frequency and dynamic modulus of elasticity is rather critical.

There are other factors that limit the usefulness of this method.

1. This test is normally carried out in the laboratory. It is difficult to perform this test in the field. The possibility of vibrating structural members at resonance is not practical and desirable.

2. The equations for the calculation of dynamic modulus involve shape factor corrections and thus limit the shape of the specimens to cylindrical or prismatic shapes.

2.6.2 Mechanical Sonic Pulse Velocity Method

This method was first applied by Long et al. (59). The principle of this method is that a longitudinal or compressional wave is initiated by a single hammer blow, and the time taken to travel between two points on the surface is electronically measured.

Mitchel (60), Anderson and Nevenst (61) have done considerable work on this method. Inspite of good correlation between flexural strength and the pulse modulus reported by Long et al. (59), there are many possible sources of error in this method as discussed below.

2.6.2.1 Limitations of the Method The following are the limitations of the test;

1. The method measures only the surface conditions of the concrete in situ and not the whole structure.
2. Errors are likely to be included because of the assumed value of poisson's ratio.
3. The measurement of travel time may be affected by the intensity and direction of the hammer blow.
4. There is a possible reduction in the amplitude of the pulse as it travels through the concrete. This can result in incorrect estimates of travel time between the pick up points.

2.6.3 Ultra Sonic Pulse Velocity Test

This method was developed in Canada in 1945 by Leslie and Cheeseman (62) and in England by Jones (56,63,64).

Parker (65), Sturrup (66), Philleo (67), Batchelder and Lewis (68), Whitehurst (69-73), Klieger (74), Mather (75), Meyer (76) have made significant contributions to the advancement of this method.

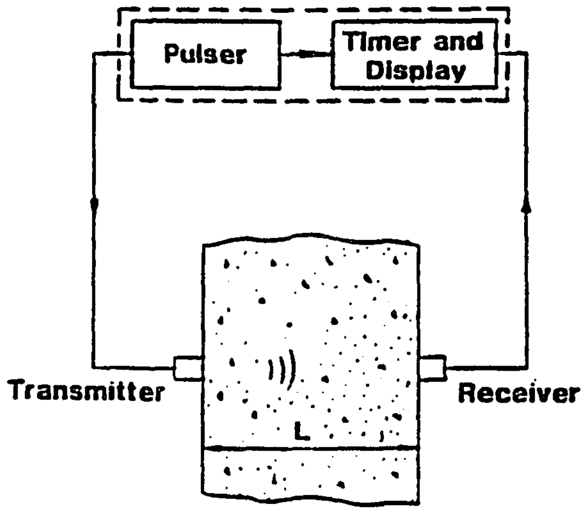


Figure 11 Principle of Operation of Apparatus for Measuring Ultrasonic Pulse Velocity

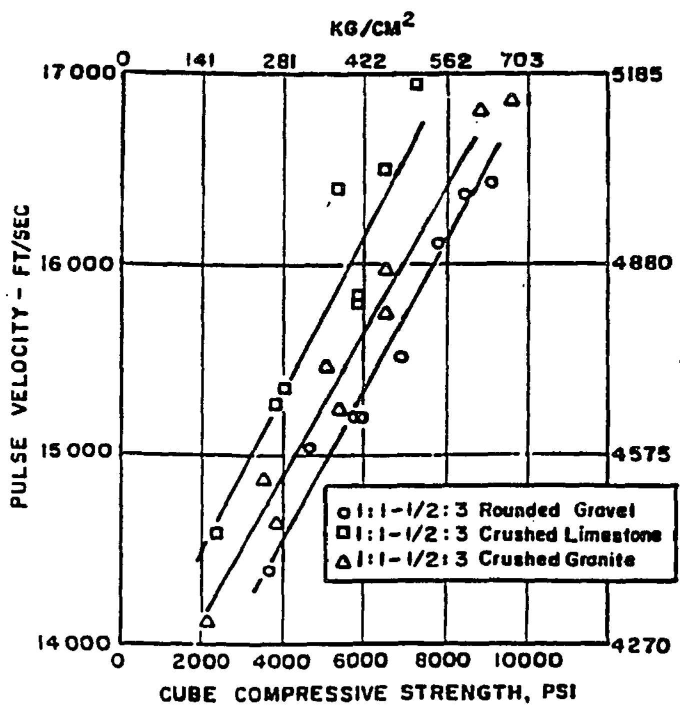


Figure 12 Pulse Velocity vs Compressive Strength

In this method, the time of travel of an ultrasonic pulse passing through the concrete is measured. The operational principle is shown by Figure 11 (18). A pulser sends a high voltage signal to the transducer causing it to vibrate at its resonant frequency. These vibrations are transferred to the concrete by a viscous coupling fluid. A receiving transducer coupled to the opposite concrete surface detects the pulse travelling through the concrete. The time taken by the pulse to travel through the concrete is electronically measured, and the direct path length is divided by this time to obtain the pulse velocity. ASTM C 597 has standardized this test (77).

This method has been used to, establish uniformity of concrete (65), establish acceptance criteria (62), determine pulse modulus of elasticity, study setting characteristics of concrete (73), durability of concrete (62,78-80), estimate strength (57), measure and detect cracks (57,62,66). Figure 12 shows the relationship of pulse velocity and compressive strength of concrete (57).

2.6.3.1 Limitations of the Method The measurements of the pulse velocity are affected by a number of factors. Some are given below.

1. The pulse velocity increases with increased moisture content of concrete. The pulse velocity of saturated concrete may be 2% higher than that of similar dry concrete (81).

2. Pulse velocity taken near steel bars is higher and will not represent the true velocity in concrete (82).

3. At temperatures between 86^0 and 140^0 F, there is up to 5% reduction in pulse velocity. At 25^0 F, an increase of up to 7.5% in the pulse velocity through water saturated concrete has been reported (82).

4. It is important to maintain good acoustical contact between the surface of concrete and the face of each transducer.

5. Roshore (83) and Varghese (84) have reported comparison of pulse velocity measurements through concrete specimens of varying length cast from the same batch of concrete.

6. Age of concrete. Facaoaru (85) has found that for a given pulse velocity, the compressive strength is higher for higher ages.

7. Presence of cracks and voids affect the pulse velocity through concrete.

2.7 Maturity Method

The basic principle of this method is that the strength varies as a function of both time and temperature. The thermal history of the concrete and a so-called maturity function are used to compute a maturity value that quantifies the combined effects of time and temperature. The strength of

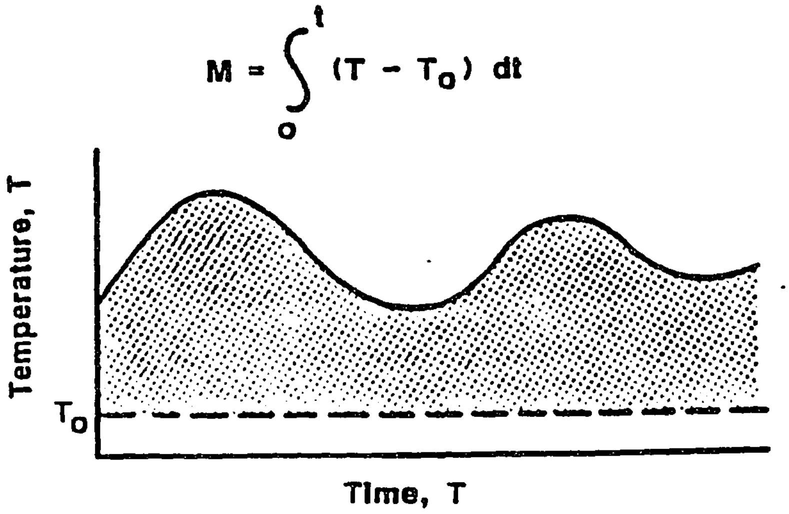


Figure 13 Maturity Function

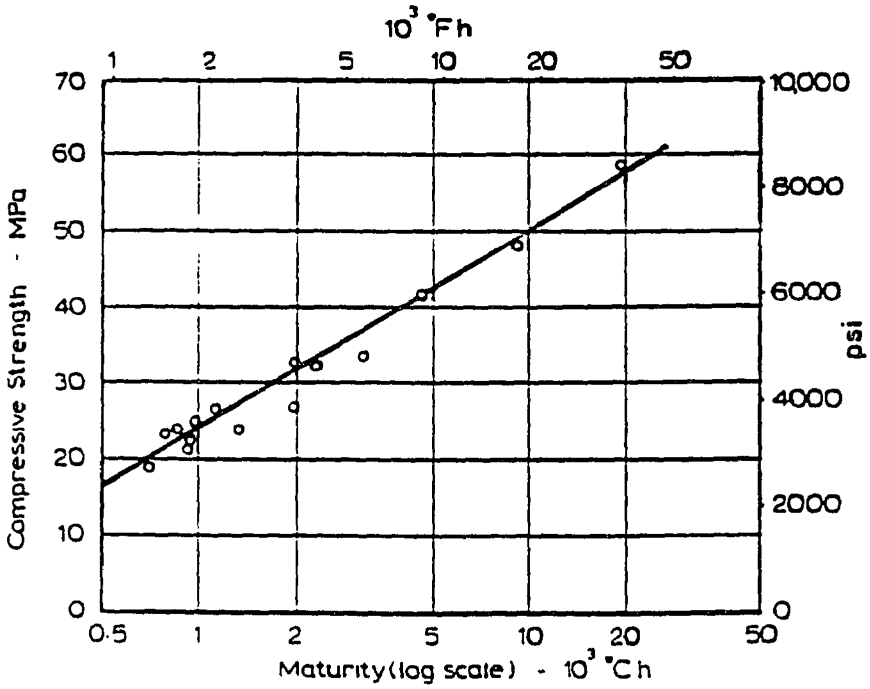


Figure 14 Compressive Strength vs Maturity

a particular concrete mixture is expressed as a function of its maturity by means of a strength maturity relationship.

Figure 13 shows a commonly used maturity function. Several such functions have been proposed and reviewed by Malhotra (86) and RILEM (87). Malhotra (88), has prepared an excellent review of the maturity concept. Figure 14 shows the relationship between maturity and compressive strength (89). Maturity of in-situ concrete is monitored by thermocouples or by instruments called maturity meters. Disposable maturity meters of Danish origin are also available (90). ASTM C 1074 gives the procedure for using the maturity method (91).

Hulslizer et.al (92) have found this method effective in reducing form removal time in a tunnelling project. Naik (93), Carino (94,95) and others (96) also have investigated
the maturity concept.

2.7.1 Limitations of the Maturity Method

1. The major limitation of this technique is that it can not be used in existing structures.

2. To utilize the maturity method requires establishing of strength-maturity relationship for the concrete that will be used in the structure.

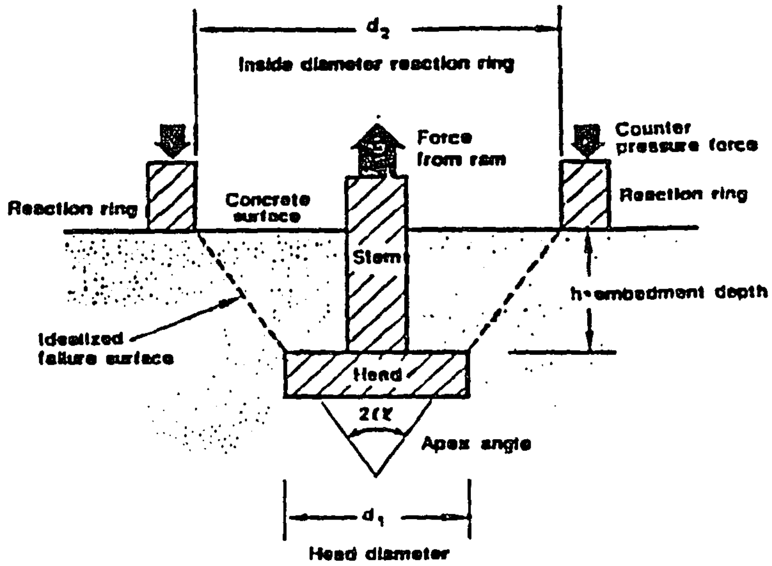


Figure 15 Schematic of Pull Out Test

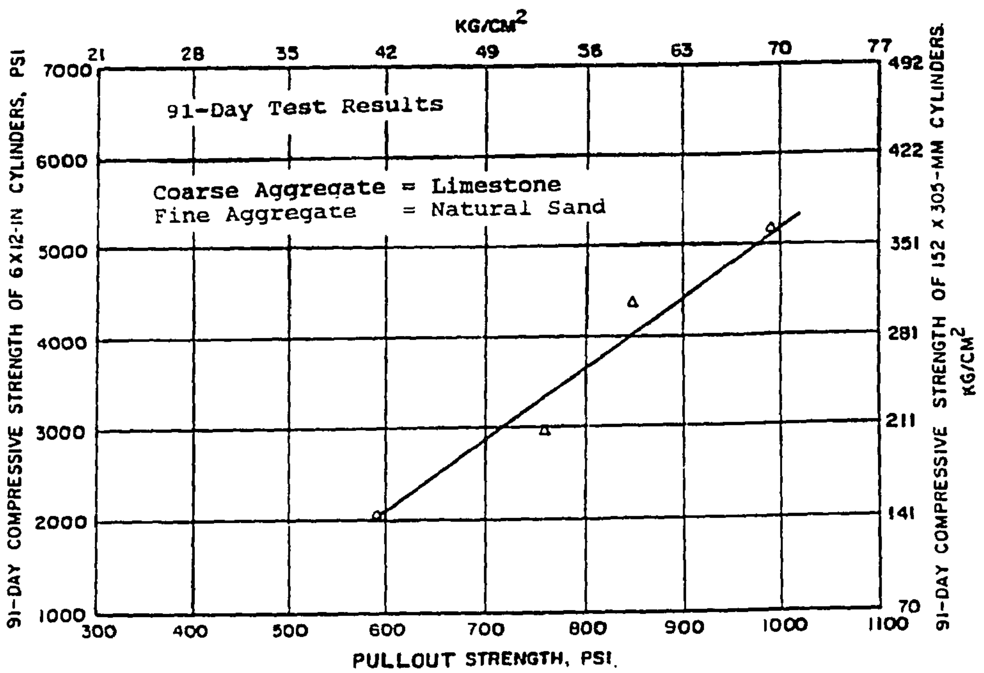


Figure 16 Compressive Strength vs Pull Out Strength

3. As observed by Klieger (97), the strength-maturity relation depends on the properties of the cement and on the general quality of concrete, and is valid only within a range of temperatures.

2.8 Pull Out Test

According to Skramatajew (98), in-situ testing of concrete including pull out test has been developed in USSR since 1934. Tremper (99), in 1944 reported the results of pull out tests and concluded that these tests can be reproduced within limits of that are nearly as close as compression test and a high degree of correlation exists between the pull out and compression test.

After a lapse of a few decades, Richard (100), has advocated the use of pull out test in USA. Malhotra has used this test in Canada (101).

This test measures the ultimate load required to pull an embedded insert with an enlarged head from the concrete. Figure 15, shows the schematic of the pull out test (18). The requirements for the test configuration is given by ASTM C 900 (103).

Figure 16 Shows how the pull out force is correlated to the compressive strength (102). Using finite element methods, the stress in the concrete in a pull out test has been evaluated by Stone and Carino (104), Ottosen (105), and Hellier et al. (106). A series of analytical and experimental studies have been carried out to determine the failure mechanism of the pull out test, some of which has been reviewed by Yener and Chen (107). Hellier et al. (106) have concluded that ultimate failure does not occur because of a compressive failure of concrete. Ballarani et al. (108) have used linear elastic fracture mechanics and a two dimensional model and concluded that the ultimate load is governed by fracture toughness. There is no agreement on the nature of the ultimate pull out load.

Khoo (109), has concluded that pull out technique is an effective method for evaluation of in-situ strength of concrete. This test has been used by Parson and Naik to determine early age concrete strength (110).

2.8.1 Limitations of the Pull Out Test

1. The standard pull out test requires preplanning the location of the inserts on the formwork. The test can not be performed on structures that do not have embedded inserts.
2. Commercial inserts are about 30mm. Since the pull out strength is governed by the concrete located adjacent to the conic frustum defined by the insert head and reaction ring, only a small concrete volume is tested. Due to this reason the within batch variation of the results of this tests are about two times higher than the standard cylinder compression test.

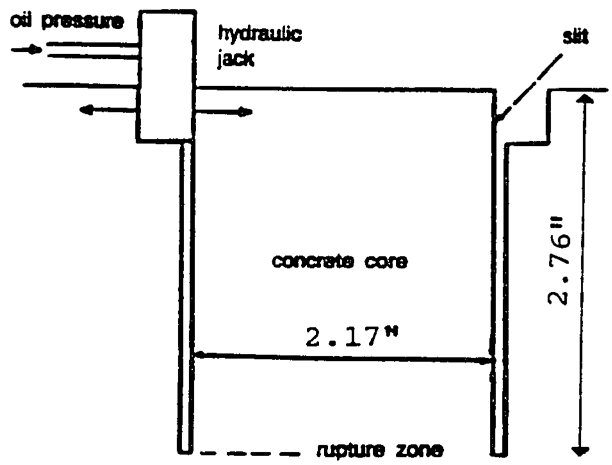


Figure 17 Schematic of Break Off Test Specimen

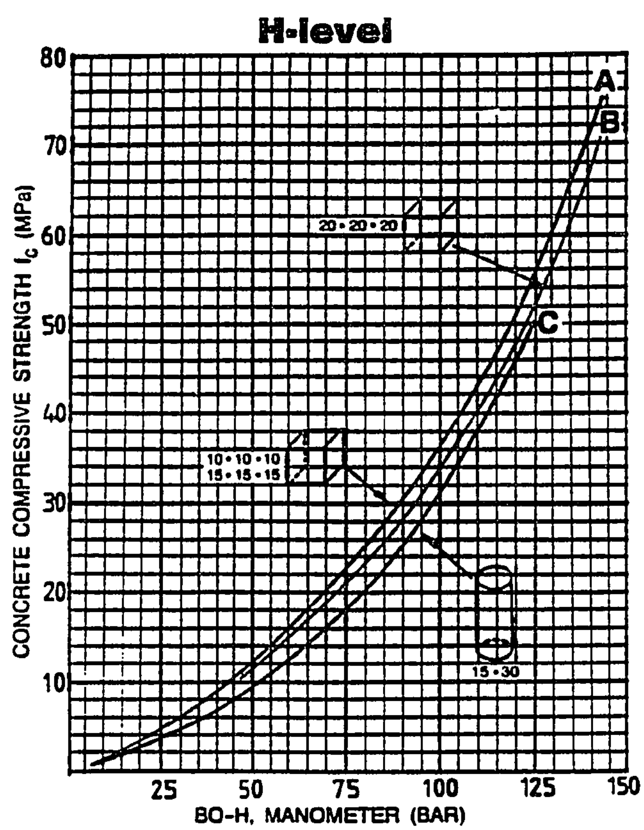


Figure 18 Concrete Compressive Strength vs Break Off Manometer Reading

3. Since there is no consensus of the static strength property the pull out test measures, it is necessary to develop an empirical correlation relationship between the pull out strength and the compressive strength of concrete.

2.9 Break Off Test

The break off test was developed in Norway (111). It consists of breaking off an in-place cylindrical concrete specimen at a failure plane parallel to the finished surface of the concrete. Figure 17, shows a schematic of the break off test specimen (112). The break off stress at failure can be related to the compressive strength of concrete using a predetermined relationship which relates the compressive strength of concrete measured by conventional test specimens, cylinders or cores to the break off strength for that particular concrete. Figure 18 depicts such a relationship as given by the manufacturer of the tester (112).

In 1977, the break off tester was developed and patented as a method for determination of the compressive strength of the in-place concrete by researchers at the Norwegian Technical University (NTH) (113,114). In 1981-82, the instrument was further developed by NTH and A/S Scancem Company (112). A/S Scancem is a company in Norway which provides technical support for the tester.

Johansen published the first paper on the break off tester in 1976 and indicated this test as a very efficient way of determining the in-place concrete strength for form removal (115).

In 1979, Johansen and Dahl-Jorgensen published a paper on the use of the break off method to detect variation in the concrete strength and curing conditions (116). A comparison was made between the break off method and the pull out test method. The compressive strength of cores obtained from the break off tests were compared with the standard cube compressive strength. They have found that both the break off and pull out test methods are very suitable for testing young concrete. Further, they have concluded that the pull out test method and the cores compressive strength values obtained from the break off test have a better ability to differentiate between concrete qualities than the standard cube test. On the other hand, the break off test results demonstrated their ability in detecting variations in curing conditions, while the pull out test method did not register some of the curing differences demonstrated by the break off and the core results.

Johansen, in 1979 published another paper (111) on the use of the break off method, with particular reference to airport pavements made of vacuum concrete. He concluded that variation of the concrete strength detected by the break off method is of the same order of magnitude as the variation detected by conventional flexure beam test. Furthermore, the break off strength was about 30% higher than conventional modulus of rupture because of deviations in the load configurations and geometric parameters between the two testing methods. He detected a high sensitivity of the break off method to sense the influence of the ambient air temperature on early strength. He also obtained a good relationship between the break off test readings and the compressive strength of the concrete obtained by standard cube testing.

In 1980, using the break off method, Byfors tested concrete at early stages (117). He tested concrete with different water to cement ratios and different aggregate sizes (5/16", 5/8", 11/4"). He concluded that the break off method is well suited for low strength concrete.

In 1982, Dahl-Jorgensen used the improved break off tester (116,118) and investigated the use of new equipment in testing epoxy to concrete bond strength and compared the results of break off and pull out methods. He concluded that the break off test provided results with smaller variation between individual tests than the pull out method and fewer tests were rejected.

In 1983, Nishikawa investigated the use of break off method for determining flexural strength of concrete (119). He concluded that the relationship between break off test results, and compressive strength tests is complex and practically useless. However, other researchers have found data contrary to this conclusion (120,121,122).

In 1983, Dahl-Jorgensen investigated the influence of curing conditions on the strength development of concrete (123). He observed a difference of 30% in strength between the least and the most favorable curing condition both for young and mature concrete. Tests on two construction sites demonstrated that field cured and especially laboratory cured standard test specimens provide strength results with little relevance to the actual in-situ concrete strength, mainly due to differences in curing and placing. An in-situ testing showed larger within-test variations than a standard cube or cylinder test. He concluded that the reduced accuracy of the testing apparatus can however be compensated for by taking a few additional tests.

In a paper published in 1984, Carlsson, Eeg and Jahren have discussed the field experiences with the use of the break off tester with six case histories (124). They have concluded that there is a trend towards greater acceptance of the break off test method in the field as the need for in place testing increases in the future.

The break off test method has been standardized recently in Norway (125), Sweden (126), England (127), New Zealand (128) and USA (129).

In 1987, Hashida et. al., used the break off test method for determination of the fracture toughness of concrete in a structure (130,131,132). The testing procedure involves breaking a notched cylindrical core that is drilled in to the concrete. The break off tester was used to apply a load to the concrete core. The J-integral procedure combined with an acoustic emission technique was employed to determine the fracture toughness of the concrete toughness of the concrete. They have concluded that the break off method developed gives reasonable fracture toughness values for concrete.

In 1988, Naik et al., have investigated the sensitivity of the break off method to different types of concrete (120). Several parameters such as concrete strength, aggregate shape, age of concrete, slab thickness and method of obtaining cylindrical break off test specimens were considered. Their evaluation of results have indicated that the break off test readings show a similar trend of strength development versus age as that for the standard cured specimen. They have found that the break off test results for crushed aggregates concrete were 10% higher than that for rounded aggregate concrete. Slab thickness of 5 and 7 inches did not have any significant effect on the variability or the average value of the break off reading. The drilled cores break off test results were on the average 9% higher than the inserted sleeves Break off test results. A regression analysis showed a high degree of correlation between the break off readings and the compressive strength of concrete. Finally, they have concluded that the break off test is an accurate, fast and easy way of determining the in-place compressive strength of concrete.

In 1988, Baker and Ramirez (121,122), have investigated the correlations of break off test results with those of the ASTM compressive strength cylinder and the ASTM modulus of rupture beam tests. The variables investigated were the water cement ratio, the aggregate type, and the maximum aggregate size. They have found that the break off test is less influenced by aggregate effects than the modulus of rupture beam. The inherent variability of the beam test was not evident in the break off test. They observed that the Break off test better correlates with the compression cylinder. The tests results show that estimating in-place compressive strengths using the break off tester seems promising for aggregate sizes up to at least one-half inch (13 mm) maximum.

Choy (133) has concluded that the break off tester can be used in concrete with maximum aggregate size 3/4" and the test method is reliable for concrete strengths in between 2500 psi and 5000 psi.

Naik (134) has given details of factors affecting the break off test method, and the practical use of this method for laboratory and site investigations. He points out that the concept of deep beam analysis should be applied for theoretical considerations of the test and concludes that the test is reproducible to an acceptable degree of accuracy and does correlates well with the compressive strength of concrete. He reports the use of the break off test for safe form removal for two buildings in Oslow, Norway and other applications in England and Norway.

2.9.1 Limitations of the Test

1. There is no theoretical relationship developed todate, between f'_c and the break off value.

2. Current specimen size can not be used for concrete with large size aggregates.

3. As this test is relatively new, its applicability to different types of concrete such as polymer concrete and fiber reinforced concrete is unknown.

These deficiencies were investigated within the scope of this study.

2.10 Cast-in-Place Cylinder

The object of this test is to obtain a sample which has been subjected to the same curing as the concrete in the structure. This method is described in ASTM C 873 (135) and uses the mold shown in Figure 19 to obtain cylindrical concrete specimens from newly cast slabs without drilling cores (18).

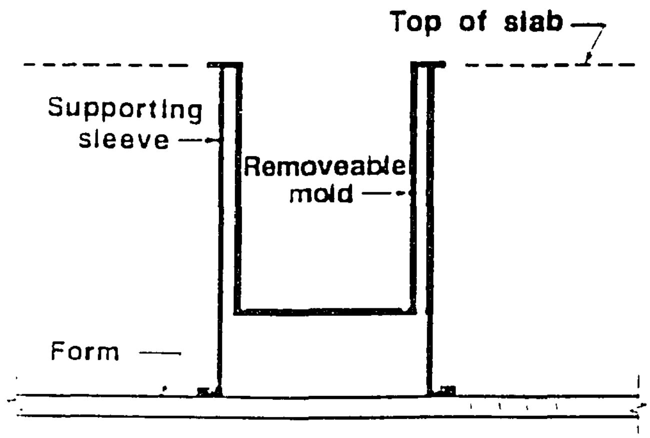


Figure 19 Mold Used to Obtain Cast-in Place Cylinders

2.10.1 Limitations of the Method

1.This test can not be applied to existing structures.

2. Although the specimens have the same thermal history as the concrete structure, the effects of compaction, bleeding etc. are not the same.

2.11 Core Cylinders

ASTM6 C 42-84a (136) has standardized this method. Munday and Dhir have assessed this technique (137). The disadvantage of this testing procedure is its high cost, and time consumption. The presence of reinforcements and their orientation also affect the results.

CHAPTER 3

OBJECTIVE

It is evident from the previous chapter that, the existing nondestructive test methods used to predict the strength parameters of concrete are far from perfect, even with their ever increasing importance in the field of Structural Engineering. However, the advent of sensational new techniques is also unlikely. Therefore, it appears that any improvements on the existing methods or a better understanding of the principles and mechanics involved would be a significant contribution.

With such knowledge and improvements, it is possible for such tests like the break off test to be accepted as more reliable standard tests. This will no doubt enable the practicing Engineers to employ nondestructive testing of concrete with more accuracy, reliability, safety and confidence.

All investigations, conclusions and the final acceptance of the break off test have been based on experimental work. There is no attempt made so far to, theoretically link the break off value and the compressive strength of concrete. Hence, the objective of this study was to establish a theoretical basis for the break off test.

In this study, the behavior of the break off test specimens was investigated. Inorder to present a theoretical relationship between the break off value and the compressive strength of concrete, fracture mechanics, finite element analysis and an approximate method based on experiments were used. The theoretical relationship thus obtained was compared with the experimental results obtained by the author and others.

3.1 Fracture Mechanics Approach

Based on linear elastic fracture mechanics, a model to predict the compressive strength of concrete - break off manometer reading relationship was obtained and compared with experimental results. Both flexural (Mode I) and shear (Mode II) failure modes were considered. Also the effect of aggregate interlock on the break off test results was investigated.

3.2 Approximate method

Center-point load tests were carried out on specimens with both rectangular and circular cross sections to find a relationship between compressive strength of concrete and the break off manometer reading.

Using this method, a new Modulus of Rupture was defined for concrete specimens smaller than 6 inches. The use of break off tester to obtain this new Modulus of Rupture for small concrete elements with both rectangular and circular cross sections was investigated.

3.3 Finite Element Analysis

A finite element analysis was used to obtain the flexural stress distribution at the fixed end of the break off specimen. This was used with a numerical integration technique to obtain the relationship between compressive strength and the break off manometer reading. The above stress distribution was also used together with known expressions in solid mechanics, numerical integration to obtain the stresses behind the break off test specimens.

The finite element analysis was also used to study the effect of the slab thickness on the break off test.

3.4 Prediction of the Strength of Plain Concrete Deep Beams

The investigations carried out on the Break off test specimens led to a method to predict the capacity of unreinforced, concrete deep beams with varying support conditions and aspect ratios for both rectangular and circular cross sections. Based on the results thus obtained, a set of design curves were developed for unreinforced concrete beams with various length to depth ratios, cross sections, support and loading conditions.

3.5 Materials, Experimental Methods

The materials, experimental and theoretical methods used are explained in detail in Chapter 4.

3.6 Theoretical Formulations, Results and Discussions

The theoretical formulations, results obtained and the discussions are given in Chapter 5.

CHAPTER 4

MATERIALS, EXPERIMENTAL AND THEORETICAL METHODS

4.1 Fracture Mechanics Approach

4.1.1 Theoretical Relationship between Compressive Strength of Concrete and Break Off Value using Fracture Mechanics

Linear elastic fracture mechanics concepts were used to model the break off specimens. Based on the fictitious crack model (FCM) a relationship between the compressive strength of concrete and the break off value was developed. This was compared with experimental relationships obtained by the manufacturer of the break off tester and other researchers.

Both flexural (Mode I) and shear (Mode II) failure modes were considered. The effects of specimen length to depth ratio (size effects) and aggregate interlock on the break off test results were investigated. The maximum aggregate size considered were, 3/8", 1/2" and 3/4".

The formulations and results are given in Chapter 5. The computer programs used are given in Appendix A.

4.2 Approximate Method

4.2.1 Introduction

An approximate method was used to find a relationship between the compressive strength of concrete and the break off reading, based on experiments done in this study and other researchers. A new modulus of rupture for concrete was also defined for concrete specimens smaller than 6 inches with both rectangular and circular cross sections. The formulations are given in Chapter 5.

4.2.2 Experimental Program

An experimental program was carried out to find the shape effects, and the breaking forces of specimens loaded in a similar manner to the break off test specimens (i.e. center-point load tests). In order to verify the validity of the theoretical formulations developed in the approximate method of this study, the experimental program was extended to find the breaking force of cantilevered cylindrical specimens loaded with a point load at the free end of the cantilever.

4.2.2.1 Investigation of the Shape Effect on Specimens Loaded in a Similar Manner to the Break Off Specimens In order to find the shape effect on deep beams, a center-point load test was carried out on 3" diameter, 8" long cylinders and 3" x 1.8" x 8" solid specimens. Three specimens were tested for each compressive strength of concrete and cross section type.

The mix proportions are as given in Table 1. Three 3" x 6" cylinders were also prepared and tested as per ASTM C 39-86 (3) for each mix to ascertain the compressive strength of concrete. It should be noted that all specimens had the same length to depth ratio and same moment of inertia.

Table 1 Mix Proportions

Design Compressive Strength (psi)	Water (lb/cy)	Cement (lb/cy)	Coarse Aggregate (lb/cy)	Sand (lb/cy)
2000	350	427	1242	1821
3000	350	515	1242	1733
4000	350	614	1242	1634
5000	350	729	1242	1513
6000	350	854	1242	1394

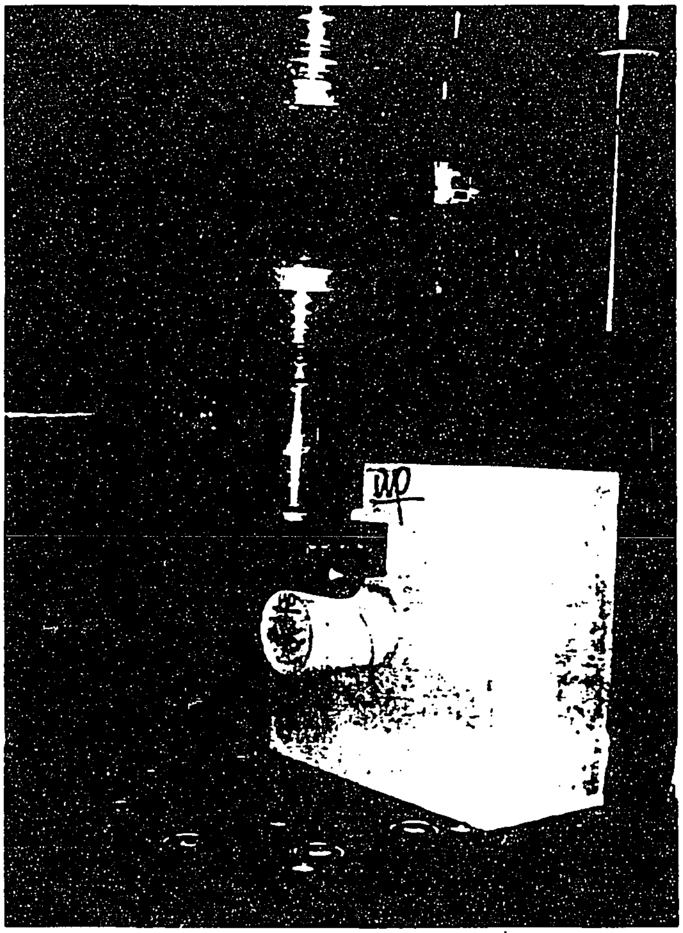


Figure 20 Test Setup for Cantilevered Cylindrical Specimens

4.2.2.2 Breaking Force of Cantilevered Cylinders Loaded by a Point Load at the Free End of the Cantilever The test specimen consists of a 3" Diameter and 3.8" long cylinder cantilevered from a 6" x 12" x 12" slab. The slab was held fixed and the cylinder was loaded with a point load at the free end of the cantilever. Three test specimens for a particular concrete strength was tested at 28 days. The concrete strength was varied from 2000 psi to 6000 psi and the mixes were designed as per ACI 211.1-77 (138). The mix proportions are as given in Table 1. Three 3" x 6" cylinders were also prepared and tested as per ASTM C 39-86 [3] for each mix in order to ascertain the compressive strength of concrete. Figures 20 shows the test setup.

4.2.3 Theoretical Relationship between Compressive Strength of Concrete and Break Off Value Using Approximate Method

4.2.3.1 Break Off Test Specimens The stress distribution at the fixed end across the test specimen at failure, was obtained from available test results and experiments. A relationship between the compressive strength of concrete and the break off value was obtained for each of the above mentioned stress distributions. These were compared with experimental relationships obtained by the manufacturer of the break off tester and other researchers.

Relationships between the modulus of rupture and the break off value were also found for specimen with both rectangular and circular cross sections. The formulations are given in Chapter 5.

4.2.3.2 Cylindrical Cantilevered Specimens A relationship between compressive strength and theoretical breaking force was obtained for each stress distribution from experiments. These were compared with experimental results.

4.3 Finite Element Modelling

4.3.1 Introduction

A finite element analysis was carried out to investigate the following.

1. Flexural Stress distribution at the fixed end of a break off test specimen.

2. The effect of slab thickness on the break off test results.

The flexural stress (Fr_{CORE}) distribution at the fixed end of the cylindrical portion of the finite element model and numerical integration was used to develop a relationship between the break off value and the compressive strength of concrete. The computer program used is given in Appendix A.

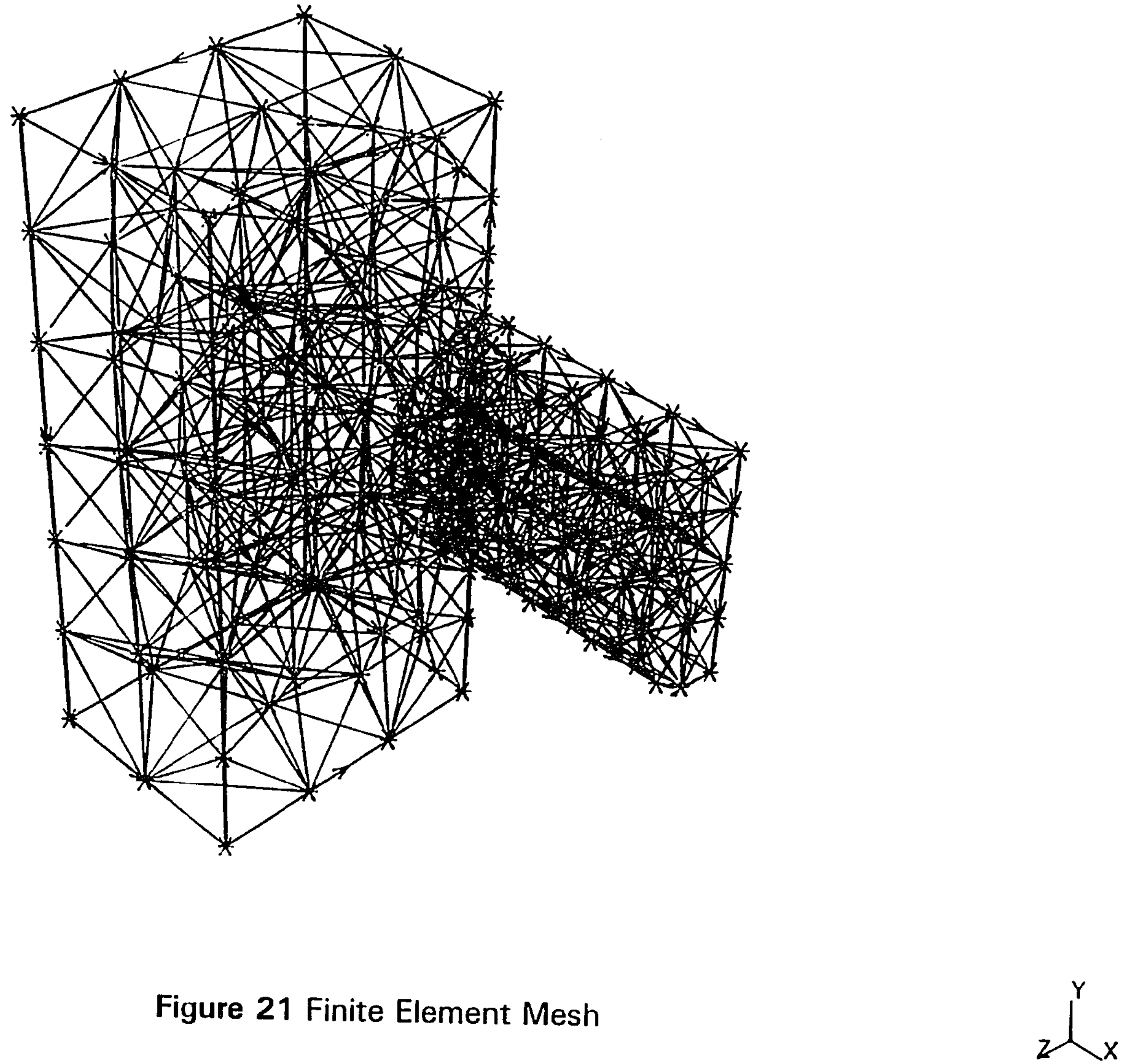


Figure 21 Finite Element Mesh

The effect of slab thickness on the break off test results was studied by varying the slab thickness of the finite element model and comparing the maximum flexural stress (Fr_{CORE}) at the fixed end of the cylindrical portion of the finite element model.

4.3.2 Finite Element Model

The computer codes IDEAS (139) and SUPERTAB (140) were utilized for finite element modelling. IDEAS was the pre and post processor used and SUPERTAB was the finite element program. A four-node, isoparametric tetrahedral element was used to model the break off test specimens. 11 nodes were used along the diameter at the fixed end of the cantilevered specimen. The rest of the nodes and the elements of the finite element model were created by automatic mesh generation. Figure 21, shows a typical finite element model.

4.3.3 Theoretical Relationship between Compressive Strength of Concrete and Break Off Value using Finite Element Method

The stress distribution at the fixed end across the test specimen at failure, was obtained using finite element analysis. A relationship between the compressive strength of concrete and the break off value was obtained for the above mentioned stress distribution. This relationship was compared with experimental relationships obtained by the manufacturer of the break off tester and other researchers.

The program used is given in Appendix A.

4.4 Stress Field In the Vicinity of the Break Off Specimens

From the finite element analysis, the stress distribution at the fixed end of the cantilevered break off specimen was obtained. Based on the experimental results and numerical integration, the maximum load applied at the free end was obtained. Using these with classical equations available in solid mechanics and numerical integration, the stresses in the vicinity of the break off specimens is obtained for concrete with compressive strength varying from 1000 psi to 9000 psi. The formulations are given in Chapter 5 and stress distributions are given in Appendix B. The programs used are given in Appendix A.

4.5 Prediction of Breaking Force of Plain Concrete Deep Beams

As an extension of the study of the break off tester, the flexural stress distributions available for beams with other length to depth ratios were considered. Leonhart and Walther (141) have found that for deep beams (When the length to depth ratio is less than 2) the flexural stress distribution at the center span of simply supported (also at the fixed end of cantilevered) and continuous beams is not linear. Hence Navier's simple bending equation can not be used. Therefore, it is very tedious if at all, to predict the strength (i.e. the maximum load the beam can carry) of any deep beam since manual integration techniques can not be used.

In order to develop a design aid, using the stress distributions available and numerical integration, the breaking loads were obtained for various supporting conditions, length to depth ratios and compressive strengths of concrete. Both point loads and distributed loads were considered. The results are presented as design charts for various length to depth ratios and for beams with both circular and rectangular cross sections. The computer programs used are given in Appendix A. The design charts are given in Appendix C.

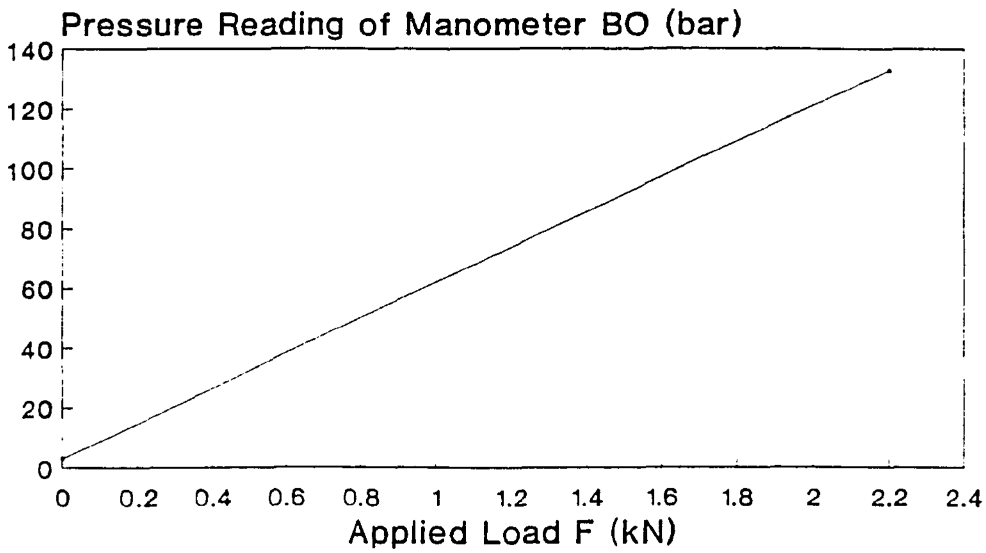


Figure 22 Break Off Reading vs Applied Force (Hashida 1987)

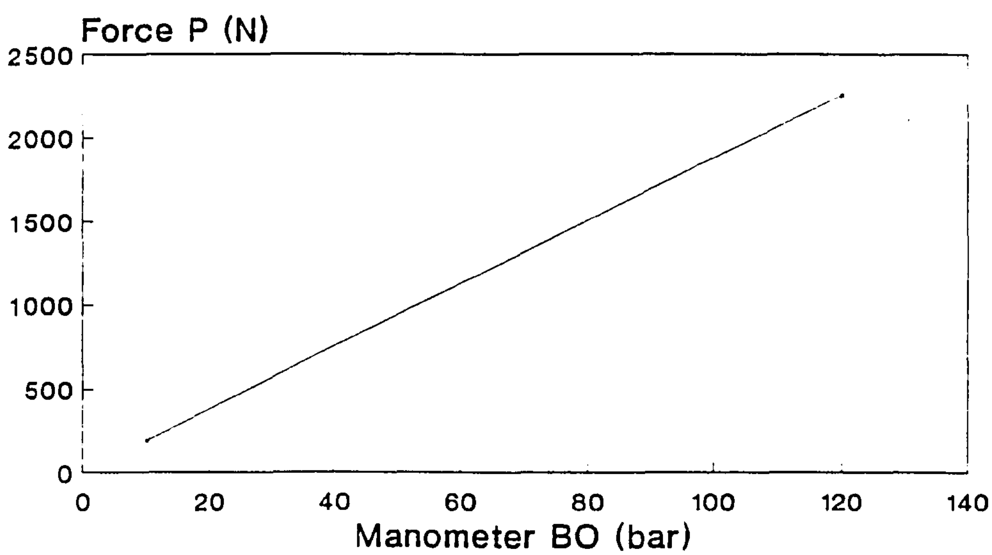


Figure 23 Break Off Manometer vs Applied Force (Dahl-Jorgensen 1991)

CHAPTER 5

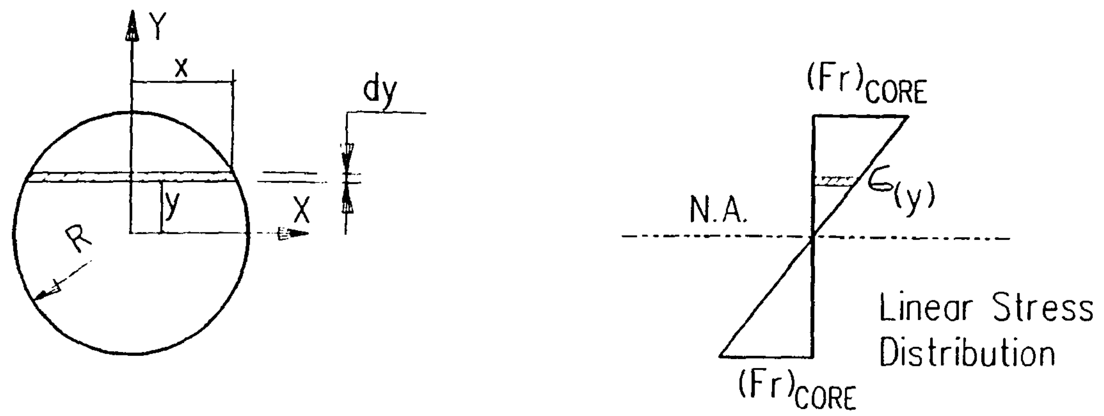
THEORETICAL FORMULATIONS, RESULTS AND DISCUSSIONS

5.1 Introduction

5.1.1 Theoretical Basis for the Break Off Test

As mentioned in earlier chapters, all investigations, conclusions and the final acceptance of the break off test have been based on experimental work. There has been no attempt so far been made to find a theoretical basis for the test other than the simple conclusion that break off test specimen fails in flexure.

The first step in developing a theoretical model to predict the relationship between the compressive strength of concrete and manometer reading of the break off tester is to find the relationship between the manometer reading and the force applied at the free end of the break off specimen. The relationship between the manometer reading and the force applied at the free end of the specimen has been reported by Hashida et al. (130,131,132) as shown in Figure 22. Figure 23 shows a similar relation obtained by Dahl-Jorgensen (142). It is seen that the load vs B.O. relation is as follows :



where:
Fr = Modulus of Rupture
fc' · Ultimate Compressive Strength
$(Fr)_{CORE}$ · Critical Stress at Outermost Fiber at Failure

Figure 24 Stiess Distribution for Approximate Method

$$P = 3.81 \, (BO - 2.973) \tag{5.1}$$

where, P = Applied load in lbs

BO = Manometer reading in bars

As a first step assuming a linear stress distribution across the critical section just prior to fracture as shown in Figure 24, the outer most fiber has a stress of $(Fr)_{CORE}$.

The value of $(Fr)_{CORE}$, based on linear elastic theory can be easily determined from :

$$q = M / S \tag{5.2}$$

where, q = Flexural Stress in psi

M = External Moment in lb-in

S = Section Modulus in in^3

Using a cantilever beam concept with a circular cross section, the stress at the critical section is :

$$(Fr)_{CORE} = 32 \, (PL) / \pi D^3 \tag{5.3}$$

where, D = Diameter of the Break off core

L = Length of the Break off core

P = Applied load provided by the Break off tester

It should be noted that, although the specimen length is 2.75", the load is assumed to be applied at 2.46" from the fixed end. Hence, the value of L in equation (5.3) is taken as 2.46". Substituting the known values of D and L in inches, equation (5.3) can be written as follows :

$$(Fr)_{CORE} = 2.4688\ P \quad or \quad P = 0.4051\ (Fr)_{CORE} \tag{5.4}$$

Combining equations (5.1) and (5.4) leads to :

$$(Fr)_{CORE} = 9.4060\ (\ BO - 2.973\ ) \tag{5.5}$$

In equations (5.4) and (5.5), $(Fr)_{CORE}$, P and BO are in psi, lbs and bars respectively. Equation (5.5) allows us to determine the maximum bending stress of the break off core provided that the break off manometer reading is known.

To relate the break off manometer reading with the ultimate compressive strength (f'_c), the relation between modulus of rupture and compressive strength is needed. American Concrete Institute (143) recommends the relation of modulus of rupture versus (f'_c),

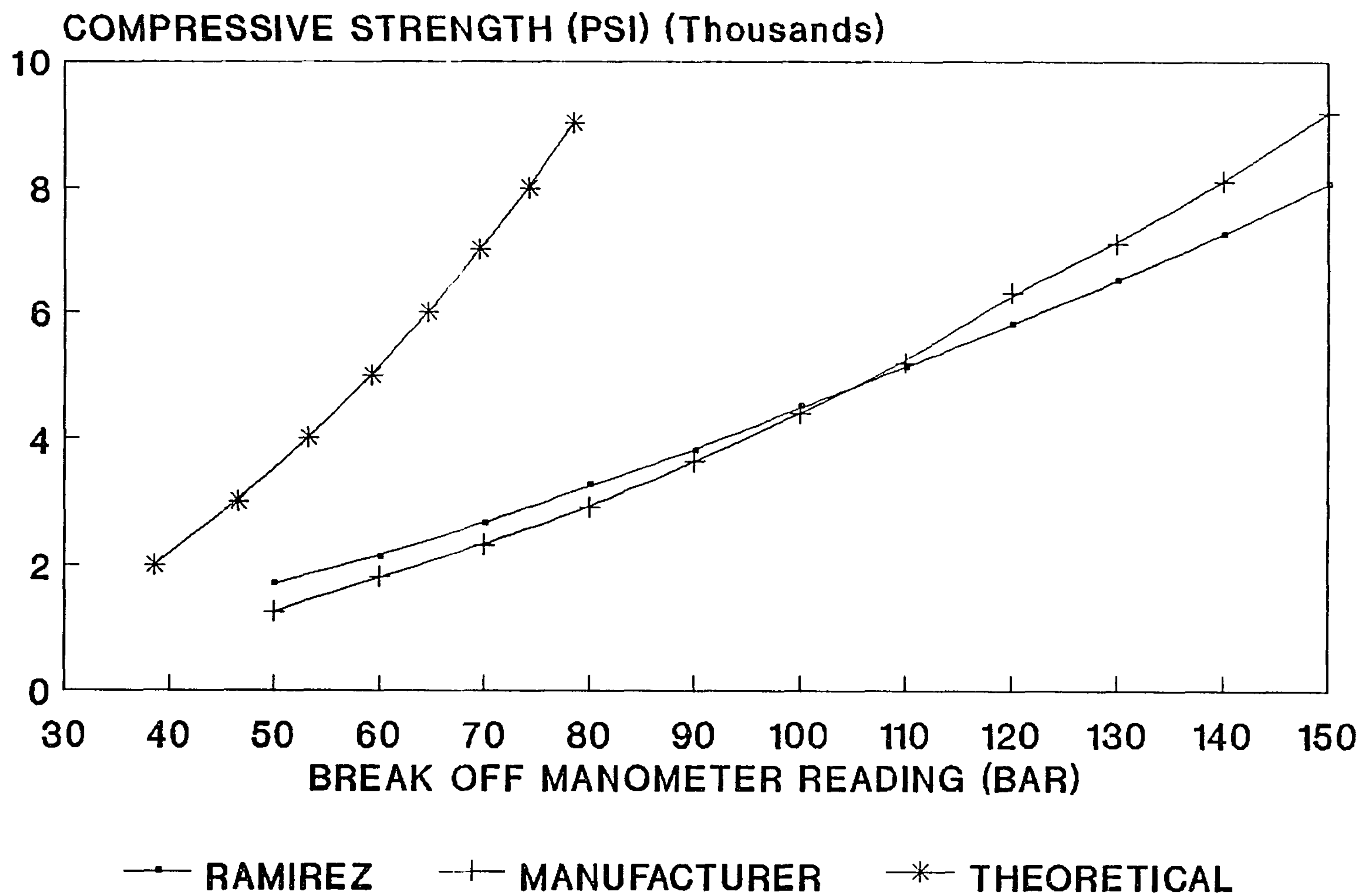


Figure 25 Compressive Strength of Concrete vs Break Off Reading

$$(Fr)_{BEAM} = 7.5\ (f'_c)^{1/2} \qquad (5.6)$$

where, $(Fr)_{BEAM}$ and f'_c are in psi.

If one assumes that $(Fr)_{CORE}$ is equal to $(Fr)_{BEAM}$, then equations (5.5) and (5.6) give,

$$(f'_c)^{1/2} = 1.2541\ (\ BO - 2.973\ ) \qquad (5.7)$$

where, f'_c and BO are in psi and bars respectively.

The relationship between the compressive strength of concrete and break off manometer reading as given by equation (5.7) is depicted by Figure 25. This is compared with the experimental results given by Ramirez (121) and the manufacturer of the break off tester (114).

From Figure 25 it is seen that the relationship between compressive strength of concrete and manometer reading do not agree well with experimental results. Therefore, simple mechanics based on flexural theory is inadequate to explain the theoretical basis of the break off test. Hence, linear elastic fracture mechanics, an approximate method and finite elements were utilized for this purpose in this study.

5.2 Fracture Mechanics Approach

5.2.1 Introduction

It has been very common to assume the tensile strength of concrete to be zero in modelling concrete. Although this makes the analysis very simple, it can be very conservative in some cases, especially in the design of unreinforced concrete beams. It is well known that concrete can withstand significant tensile stress and tensile damage. When concrete cracks, the stress strain relationship is not linear in the vicinity of the crack and to study the behavior of concrete fracture mechanics concepts are used.

When a uniaxial tension specimen fails, a reduction in strength is observed as microcracks develop and form in to a single macrocrack. The region where this reduction in strength is observed is known as the strain softening region. Based on this phenomenon Hillerborg et al. (144) in 1976, introduced the fictitious crack model (FCM). The fictitious crack model is very useful in understanding the fracture and failure of concrete structures. Hence it was used to investigate the failure of the break off test specimens in this study. The fictitious crack model assumes that the fracture process zone (FPZ) at the tip of a crack is long and narrow.

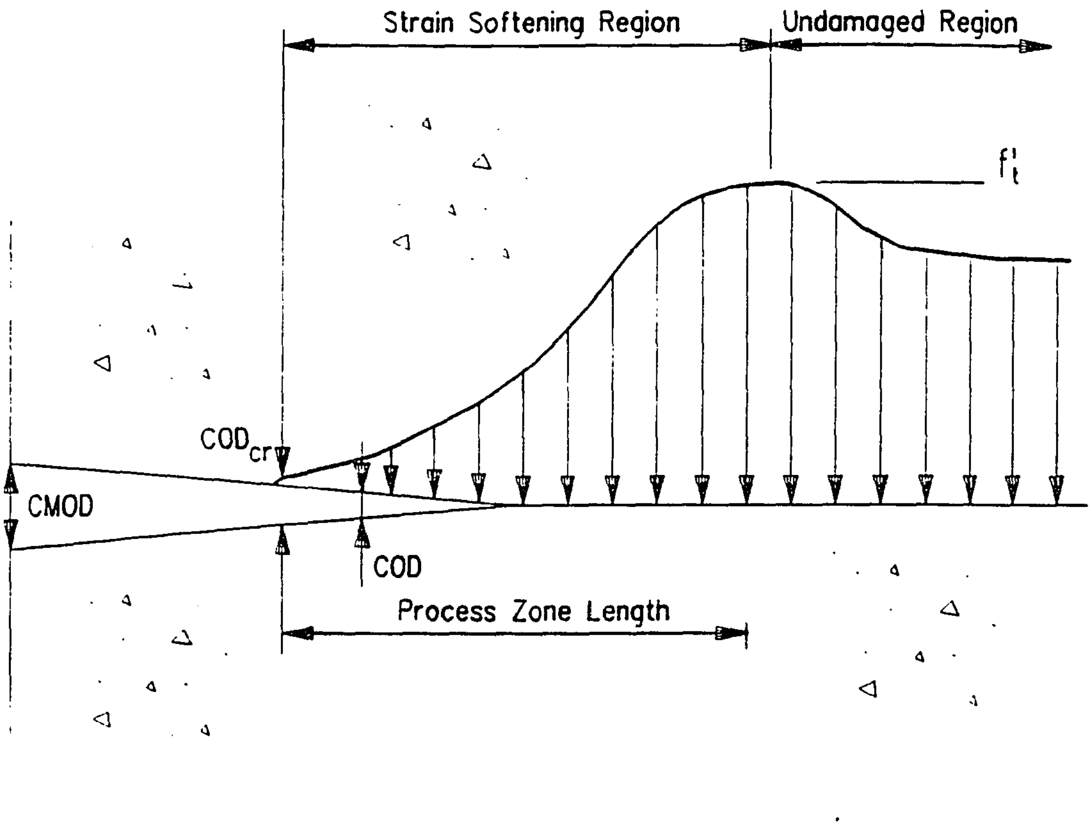


Figure 26 Terminology in Fictitious Crack Model

Figure 26 shows the terminology used in the fictitious crack model (145,146,147).

Figure 27 shows the relationship between the normal stress and displacement which characterize the fracture process zone (148).

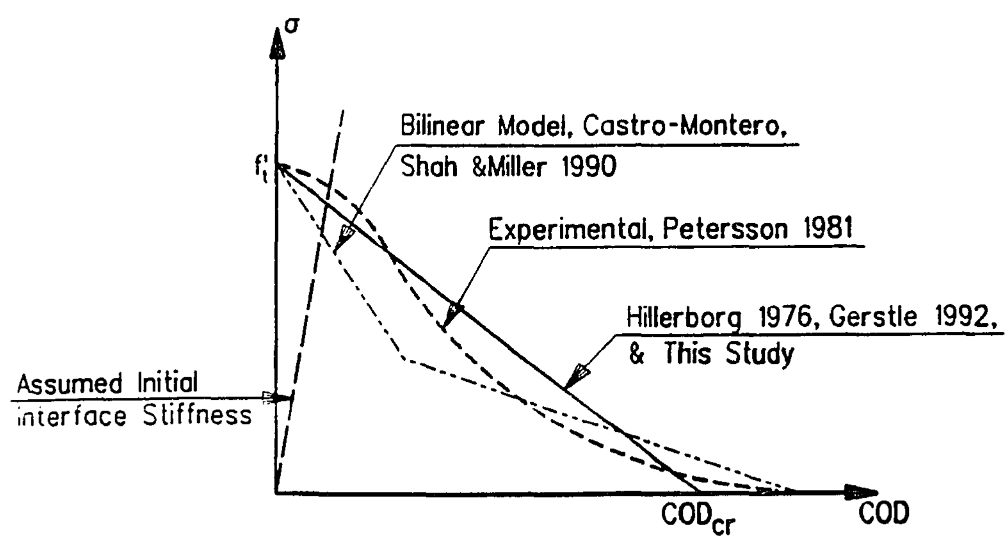
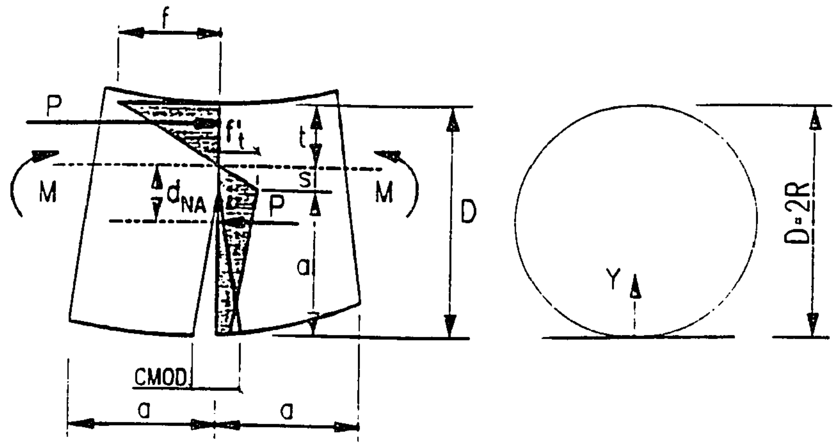


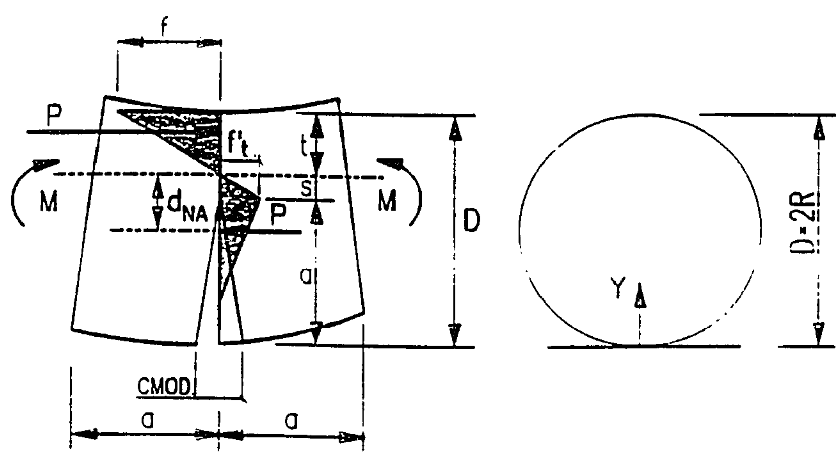
Figure 27 Relationship between Normal Stress and Crack Opening Displacement

5.2.2 Flexural Cracking Model

The break off test method assumes that the ultimate flexural strength of concrete is reached at the extreme outside fiber at the base of the break



CASE 1



CASE II

Figure 28 Schematic of Cracked Concrete Break Off Specimen off test specimen. The circular cross section restricts the ultimate fiber stress to a point, and a crack is initiated at this point (134).

Figure 28 shows an idealized and magnified deformed shape of the break off specimen used in this study. Two cases are considered: Case I, in which the fictitious crack has not yet opened far enough to relieve the normal stress at its mouth (CMOD < COD_{cr}), and Case II, in which CMOD > COD_{cr}.

5.2.2.1 Modeling Assumptions Gerstle et al. (148) have used the fictitious crack model to analyze reinforced and unreinforced concrete beams with rectangular cross sections in bending. The concrete members considered were without an initial crack. The following assumptions made by Gerstle et al. (148) for beams with rectangular cross sections are assumed to be valid for the break off specimen with a circular cross section. A finite element analysis has verified that their simplified assumptions are reasonable.

1. At a horizontal distance equal to the crack length a (See Figure 28) from the crack, plane sections of the beam remain plane after deformation (Bernoulli's beam assumption).
2. Fictitious crack surfaces remain plane after deformation.

3. Normal closing tractions acting on the fictitious crack follow the linear stress-COD curve shown in Figure 27.

4. Fiber bending stress in the concrete along the bottom of the beam is equal to the traction normal to the crack mouth at the bottom of the beam.

5. The concrete is linear elastic.

5.2.2.2 Normalization of Parameters Using the stress distributions shown in Figure 28, the maximum moment capacity of the circular section was obtained. In order to achieve this and simplify the algebra, the parameters in Figures 27 and 28 are normalized as follows:

5.2.2.2.1 Geometric Parameters The geometric parameters used are as follows;

Crack mouth opening displacement $C=\frac{CMOD}{COD_{cr}}$

Crack length A = a/D

Distance from crack tip to neutral axis S = s/D

Distance from neutral axis to top of beam T = t/D

5.2.2.2.2 Material Parameters Two material parameters are needed here for concrete;

a scale parameter for concrete
$$\beta = \frac{f_t' D}{E_c COD_{cr}}$$

where f'_t represents the tensile strength and E_c is the Young's modulus of concrete.

a strength ratio
$$k = \frac{f_t'}{f_c'}$$

where f'_c is the compressive strength of concrete.

5.2.2.2.3 Stress parameters The stress parameters used are as follows;

Stress at crack mouth opening
$$\sigma_{CMOD} = f_t'(1-C)$$

Stress in top fiber of beam $F = f/f'_t$

Applied moment
$$M = \frac{m}{f_t' D^3}$$

where m is the internal resisting moment.

5.2.2.3 Determination of Maximum Moment From the circular cross section shown in Figure 28, the depth of the section leads to,

$$T + S + A = 1 \tag{5.8}$$

Considering the linear elastic region of the stress distribution given in Figure 28, from similar triangles,

$$T = (F)(S) \tag{5.9}$$

From equations (5.8) and (5.9),

$$S = \frac{(1-A)}{(1+F)} \tag{5.10}$$

Stress strain relation in concrete gives the strain in the top fiber,

$$\epsilon_t = \frac{f}{E_c} \tag{5.11}$$

Two cases as described earlier and depicted by Figure 28, are considered,

Case I

Strain in the bottom fiber,

$$\epsilon_b = \frac{(1-C)(f_t')}{E_c} \tag{5.12}$$

Gerstle et al. (148) have obtained the following for a rectangular beam, which is also valid for members with circular sections:

$$C = \frac{2A^2\beta(1+F)}{(1-A)(1-2A\beta)} \tag{5.13}$$

It can be shown that for a cantilevered beam, C is half the above value. The expressions of forces and moments obtained for circular cross sections are more complex than those of rectangular sections studied by Gerstle et al. (148). The compressive force on the circular section developed in this study is the integral,

$$P_c = \int_{D-t}^{D} 2F\left(\sqrt{R^2-(Y-R)^2}\right)\left(\frac{t-2R+Y}{t}\right)dY \tag{5.14}$$

The tensile forces on the circular section are the integrals,

$$P_{t1} = \int_{a}^{(a+s)} 2\left(\sqrt{R^2-(Y-R)^2}\right)\left(\frac{2R-t-Y}{s}\right)dY \tag{5.15}$$

$$P_{t2} = \int_{0}^{a} 2\left(\sqrt{R^2-(Y-R)^2}\right)\left(\frac{C}{a}Y+1.0-C\right)dY \tag{5.16}$$

Horizontal force equilibrium dictates that,

$$P_c = P_{t1} + P_{t2} \tag{5.17}$$

Internal moment due to compression on the circular section,

$$M_c = \int_{D-t}^{D} 2F\left(\sqrt{R^2-(Y-R)^2}\right)\left(\frac{t-2R+Y}{t}\right)Y.dY \tag{5.18}$$

Internal moments due to tension on the circular section,

$$M_{t1}=\int_{a}^{(a+s)}2(\sqrt{R^2-(Y-R)^2})(\frac{2R-t-Y}{s})Y.dY \tag{5.19}$$

$$M_{t2}=\int_{0}^{a}2(\sqrt{R^2-(Y-R)^2})(\frac{C}{a}Y+1.0-C)YdY \tag{5.20}$$

Total moment acting on the circular section of the break off specimen can be derived from equations (5.18), (5.19) and (5.20) as,

$$M=\frac{m}{f_t^{\prime}D^3}=\frac{1}{D^3}[M_c-M_{t1}-M_{t2}] \tag{5.21}$$

<u>Case II</u>

Since crack starts to propagate in this case the strain in the bottom fiber is,

$$\epsilon_b=0 \tag{5.22}$$

It can be shown that,

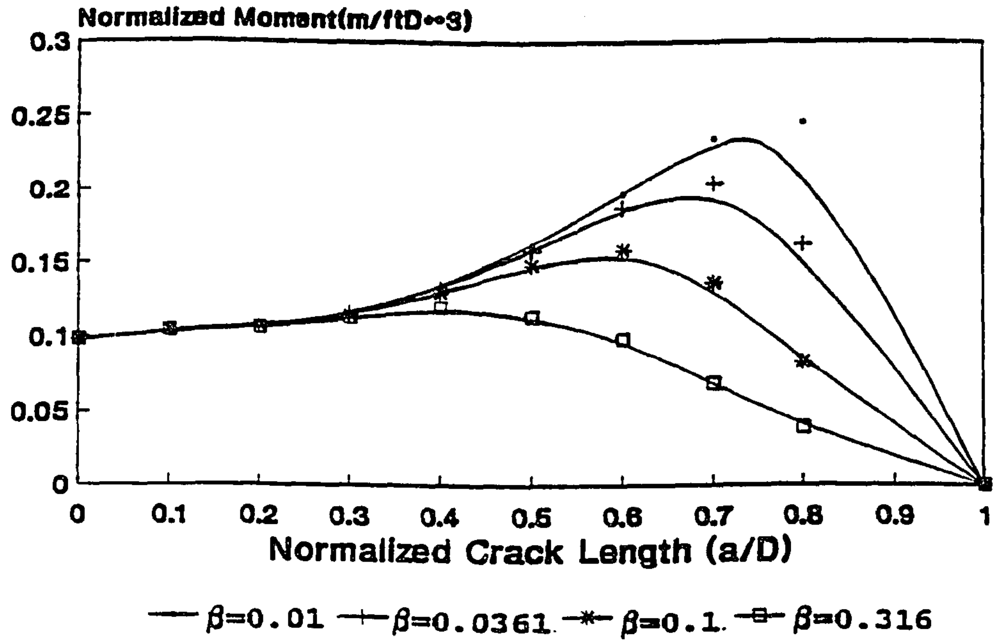


Figure 29 Normalized Moment vs Normalized Crack Length

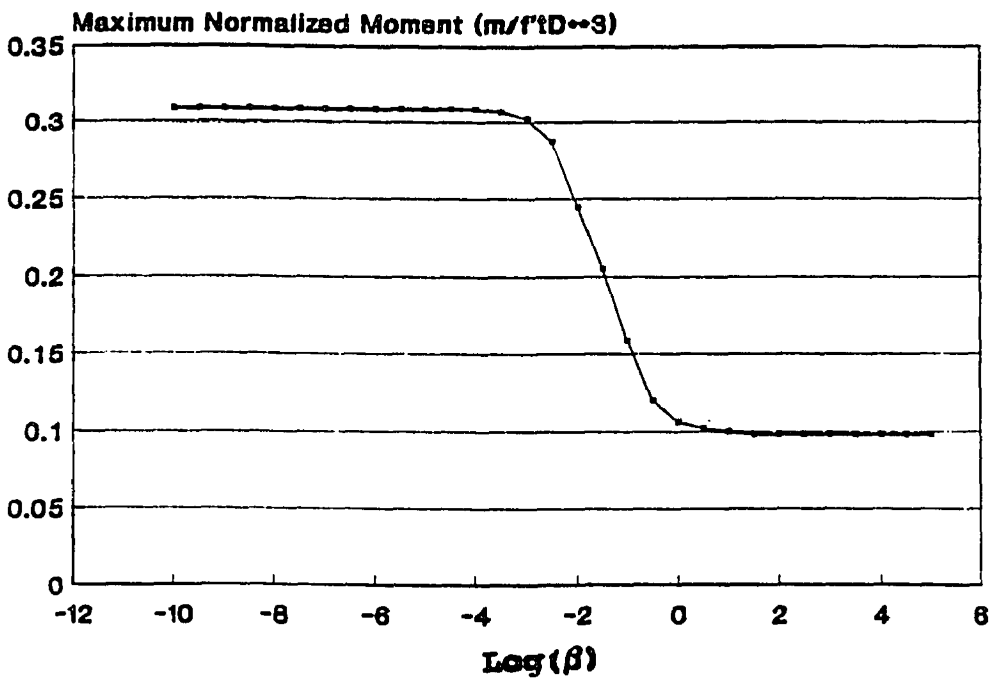


Figure 30 Normalized Peak Moment vs Log(β)

$$C=\frac{A\beta(1+AF)}{(1-A)} \tag{5.23}$$

Tensile force on the circular section,

$$P_{t2}=\int_{(a-a/C)}^{a}2\left(\sqrt{R^2-(Y-R)^2}\right)\left(\frac{C}{a}Y+1.0-C\right)dY \tag{5.24}$$

Internal moment due to tension on the circular section,

$$M_{t2}=\int_{(a-a/C)}^{a}2\left(\sqrt{R^2-(Y-R)^2}\right)\left(\frac{CY}{a}+1.0-C\right)Y\,dY \tag{5.25}$$

The P_c, P_{t1}, M_c and M_{t1} are the same as in Case I. To obtain the total moment M, using equations (5.8) through (5.25), numerical programming was needed. A FORTRAN program was written for this purpose (See Appendix A). The moments for various material-scale parameters (β values), and crack lengths (a values) were obtained. Figure 29 shows the best fit curves of the relationship between normalized moment and normalized crack length for various β values, as obtained by the computer program developed. It is seen that the total normalized

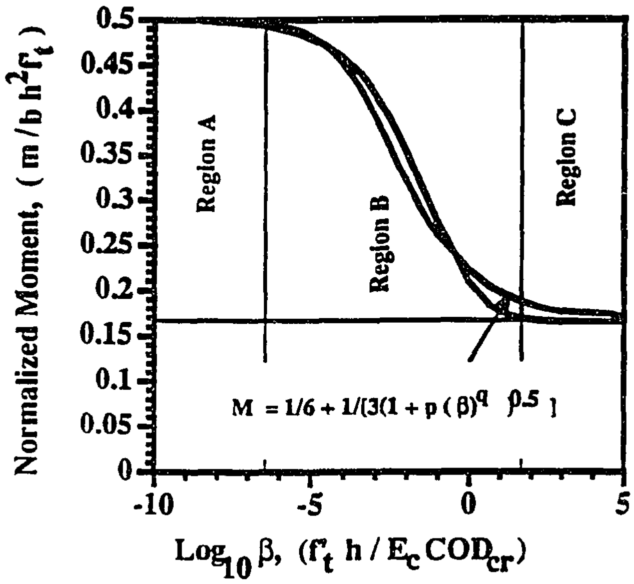
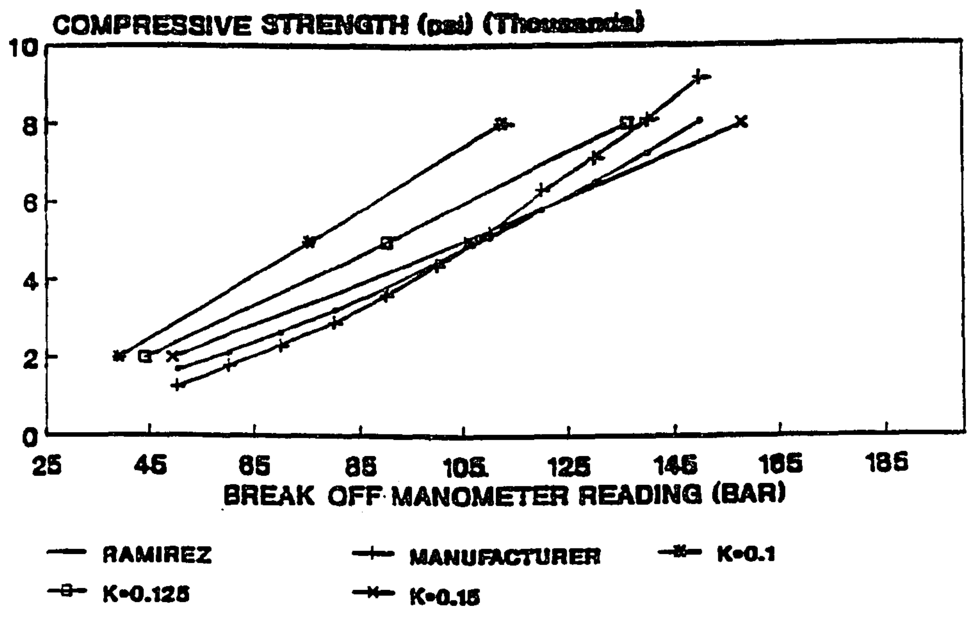


Figure 31 Normalized Peak Moment vs Normalized Crack (Gerstle et al.)

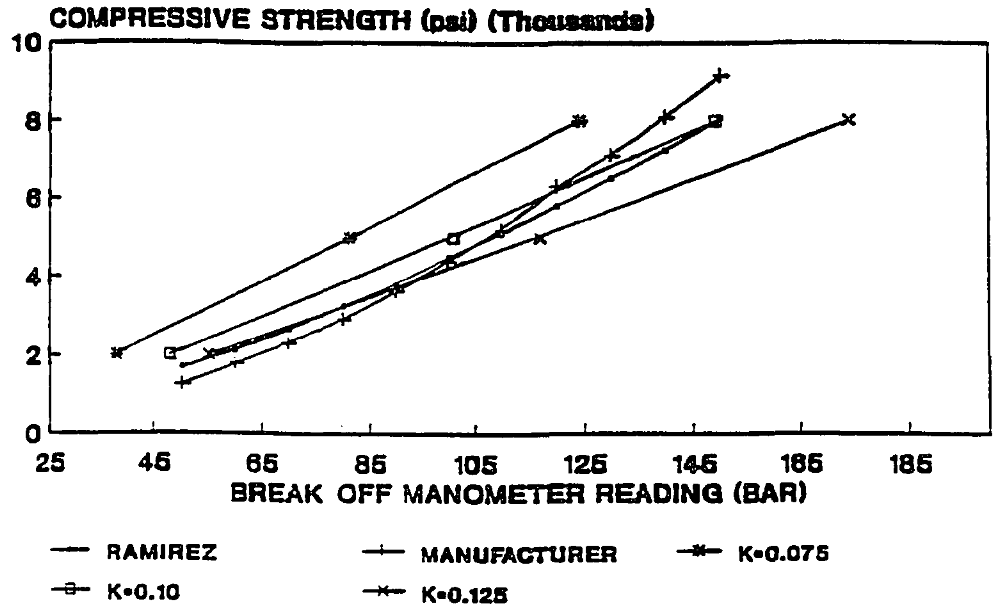


$COD_{cr}=0.001$, $k=f'_t/f'_c$

Figure 32 Compressive Strength vs Break Off Reading Using Flexural Model moment increases and then decreases as the crack propagates. Figure 30 shows the best fit curve of the maximum normalized moment versus Log(β) values for a specimen with a circular cross section. It should be noted that it was not necessary to use CMOD, COD_{cr}, f'_t, and f'_c values to obtain Figures 29 and 30, due to normalization.

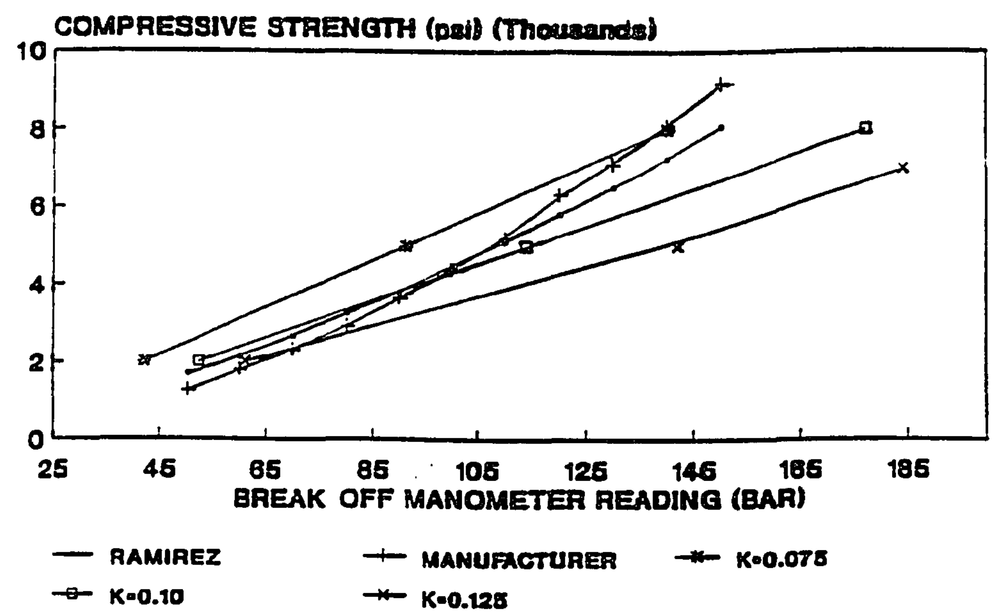
Figure 31 depicts the relationship between Normalized peak moment and Log(β) as obtained by Gerstle et al.(148) for beams with rectangular cross sections.

5.2.2.4. Relationship between Compressive Strength of Concrete and Break Off Manometer Reading (Break Off Value) Equating the maximum moment as given in Figure 30 to the externally applied moment, one can get the force applied at the end of the specimen. From equation (5.1), the corresponding manometer reading (break off value) was obtained. Since the maximum moment was obtained for a particular β value (hence the compressive strength is known) the corresponding break off value can be predicted for a particular compressive strength. This relationship is shown on Figures 32 through 34 for various COD_{cr} and k ($k = f'_t/f'_c$) values. On Figures 32 through 34, the predicted break off values for various compressive strengths of concrete are compared with experimental results of Ramirez (121) and the manufacturer of break off tester (114).



COD_{cr}=0.005, k=f'$_t$/f'$_c$

Figure 33 Compressive Strength vs Break Off Reading Using Flexural Model



COD_{cr}=0.01, k=f'$_t$/f'$_c$

Figure 34 Compressive Strength vs Break Off Reading Using Flexural Model

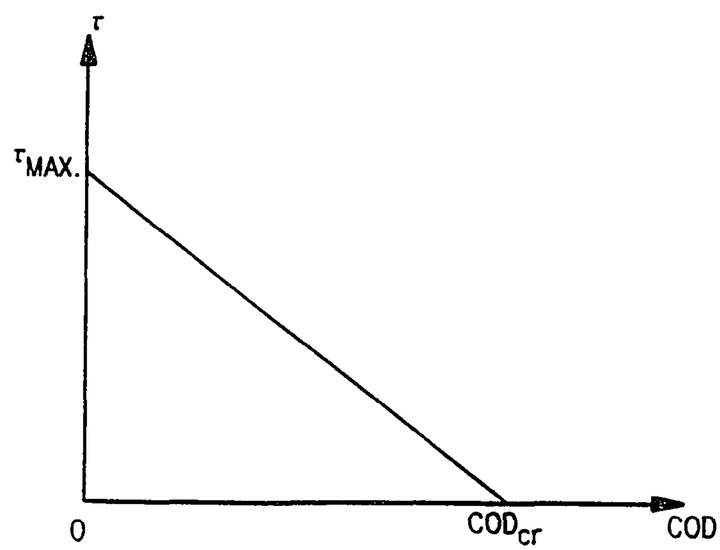


Figure 35 Relationship Between Shear Stress and Crack Opening Displacement

It is evident from Figures 32 through 34, that the theoretical results obtained in this study show the same trend as the experimental results, indicating the existence of a theoretical relationship between the compressive strength of concrete and the break off manometer reading. The apparent linearity of results is probably due to the assumption of a linear relationship between the compressive and tensile strengths of concrete.

5.2.3 Shear Model

5.2.3.1 Modeling Assumptions In addition to the assumptions made for the flexural crack model, it is assumed that the shear acting on the fictitious crack follow the linear stress-COD curve shown in Figure 35.

5.2.3.2 Normalization of Parameters The following normalized parameter was used in the shear model in addition to the normalized parameters used in the flexural model.

Applied shear force $P = p / (\tau_{MAX} D^2)$

where p is the internal shear force and τ_{MAX} is the shear strength of concrete.

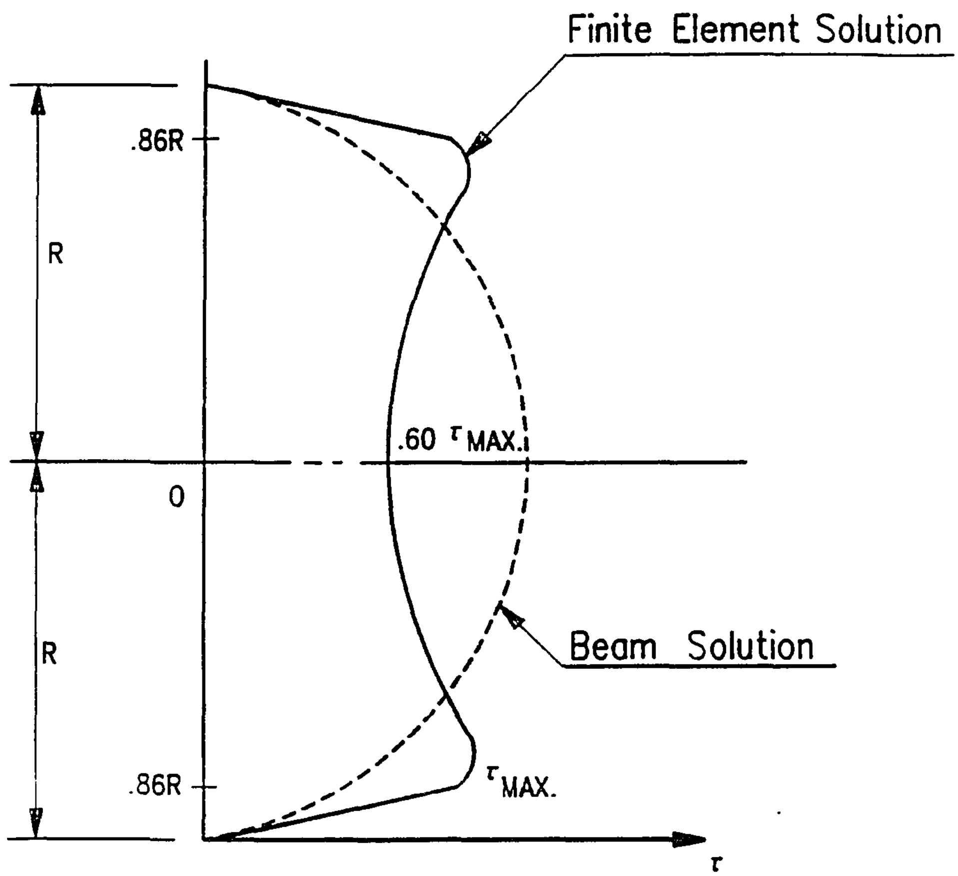


Figure 36 Shear Stress Distribution at Fixed End

5.2.3.3 Determination of the Normalized Shear Force Figure 36 shows the shear stress distribution of a cantilever at the fixed end given by Gerstle (149). This is used in the development of the shear model.

Based on Figure 35, it is seen that the stress at crack mouth opening is given by,

$$\tau_{CMOD} = \tau_{MAX}(1\text{-}C) \tag{5.26}$$

Figure 37 shows an idealized and magnified deformed shape of an unreinforced cylindrical concrete cantilever beam. Two cases are considered: Case I, in which the fictitious crack has not yet opened far enough to relieve the normal stress at the mouth (CMOD < COD_{cr}), and Case II, in which CMOD > COD_{cr}.

<u>case I</u>

From Figure 37, it is seen that the shear force components on the circular section are the integrals,

$$P_1 = \int_{1.86R}^{D} 14.2864\tau_{MAX}\left(\sqrt{R^2-(Y-R)^2}\right)\left(\frac{2R-Y}{R}\right)dY \tag{5.27}$$

$$P_2 = \int_{(1.86R+a)/2}^{1.86R} 2\tau_{MAX}\left(\sqrt{R^2-(Y-R)^2}\right)\left(\frac{0.96Y+0.0744R-a}{(1.86R-a)}\right)dY \tag{5.28}$$

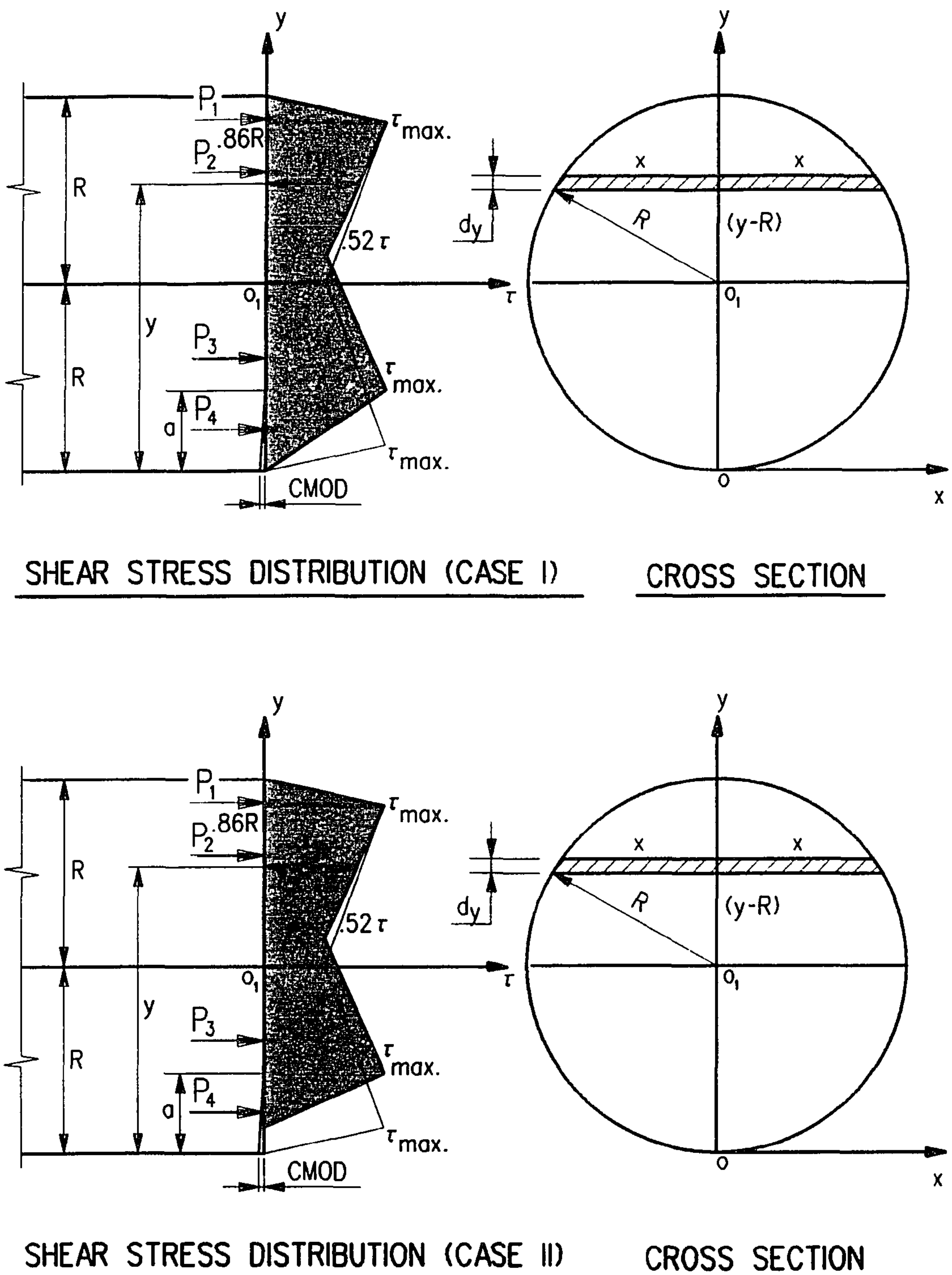


Figure 37 Shear Model

$$P_3 = \int_a^{(1.86R+a)/2} 2\tau_{MAX}(\sqrt{R^2-(Y-R)^2})\left(\frac{0.96Y+0.04a-1.86R}{(a-1.86R)}\right)dY \tag{5.29}$$

$$P_4 = \int_0^a 2\tau_{MAX}(\sqrt{R^2-(Y-R)^2})[Y-(a-\frac{a}{C})](\frac{C}{a})dY \tag{5.30}$$

The normalized shear force P is the sum of the internal shear forces as follows,

$$P = p / (\tau_{MAX}D^2) = (P_1 + P_2 + P_3 + P_4) / (\tau_{MAX}D^2) \tag{5.31}$$

<u>Case II</u>

For case II, P_1, P_2 and P_3 are as given by equations (5.27) through (5.29).

$$P_4 = \int_{(a-\frac{a}{C})}^{a} 2\tau_{MAX}(\sqrt{R^2-(Y-R)^2})[Y-(a-\frac{a}{C})](\frac{C}{a})dY \tag{5.32}$$

To obtain the total vertical Force P, using equations (5.26) through (5.32), numerical programming was needed. A FORTRAN program was written for this purpose (See Appendix A). The force for various material-

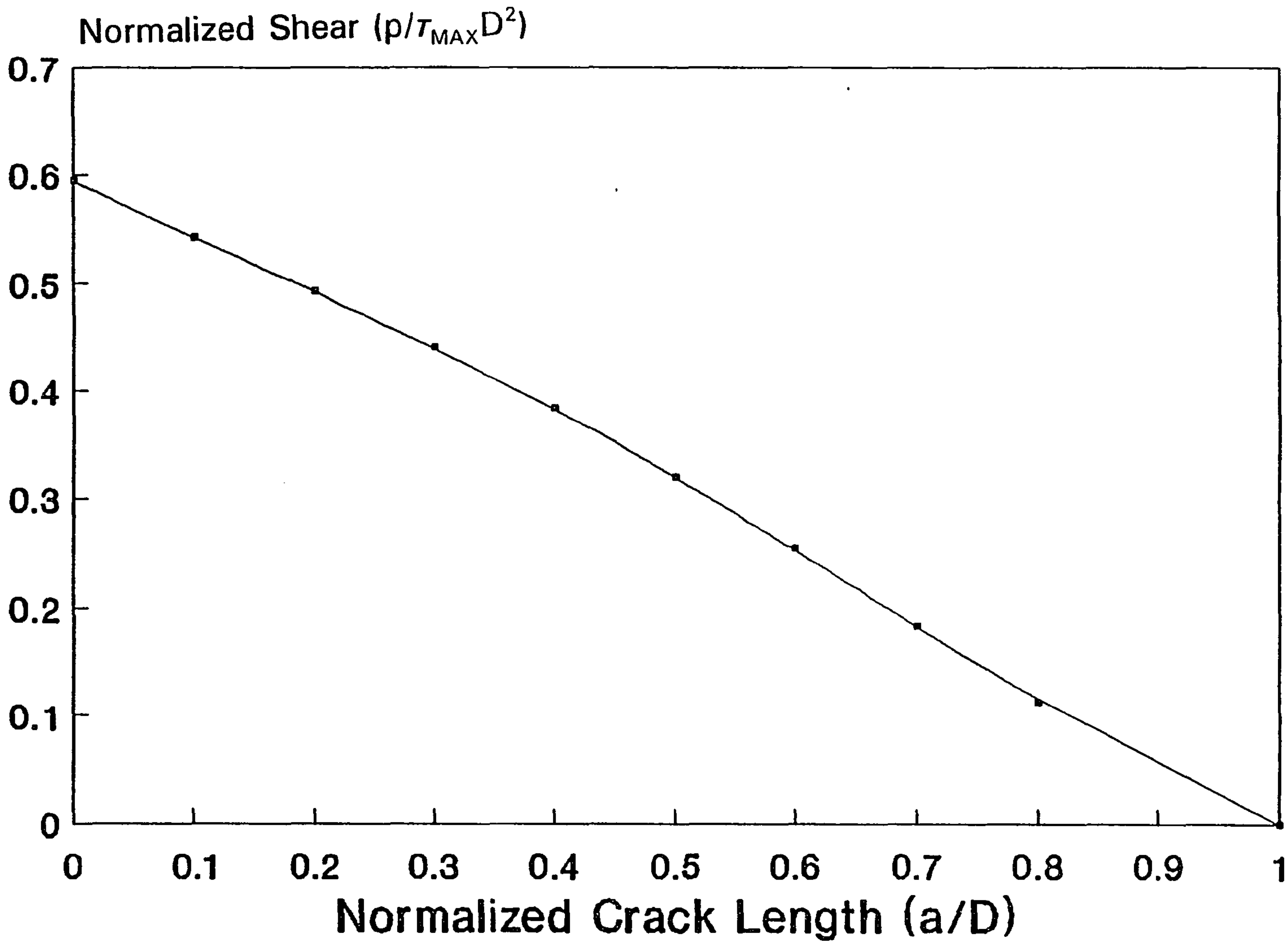


Figure 38 Normalized Shear Force vs Normalized Crack Length

scale parameters (β values), and crack lengths (a values) were obtained. Figure 38 shows the best fit curves of the relationship between normalized force and normalized crack length, as obtained by the computer program developed. It is seen that the total normalized force decreases as the crack propagates. It should be noted that it was not necessary to use CMOD, COD_{cr}, f'_t, and f'_c values to obtain Figure 38, due to normalization.

5.2.3.4 Relationship between Compressive Strength of Concrete and Break Off Manometer Reading (Break Off Value) From the maximum internal shear, the force applied at the end of the specimen was obtained. From equation (5.1), the corresponding manometer reading (break off value) was obtained. Since the maximum force was obtained for a particular strength of concrete (f'_c), the corresponding break off value can be predicted for a particular compressive strength. This relationship is shown on Figure 39. On Figure 39, the predicted break off values for various compressive strengths of concrete are compared with experimental results of Ramirez (121) and the manufacturer of the break off tester (114).

Figure 40 shows the relationship between compressive strength of concrete and the break off values, if the shear force applied is assumed to create an average shear stress acting across the circular cross section.

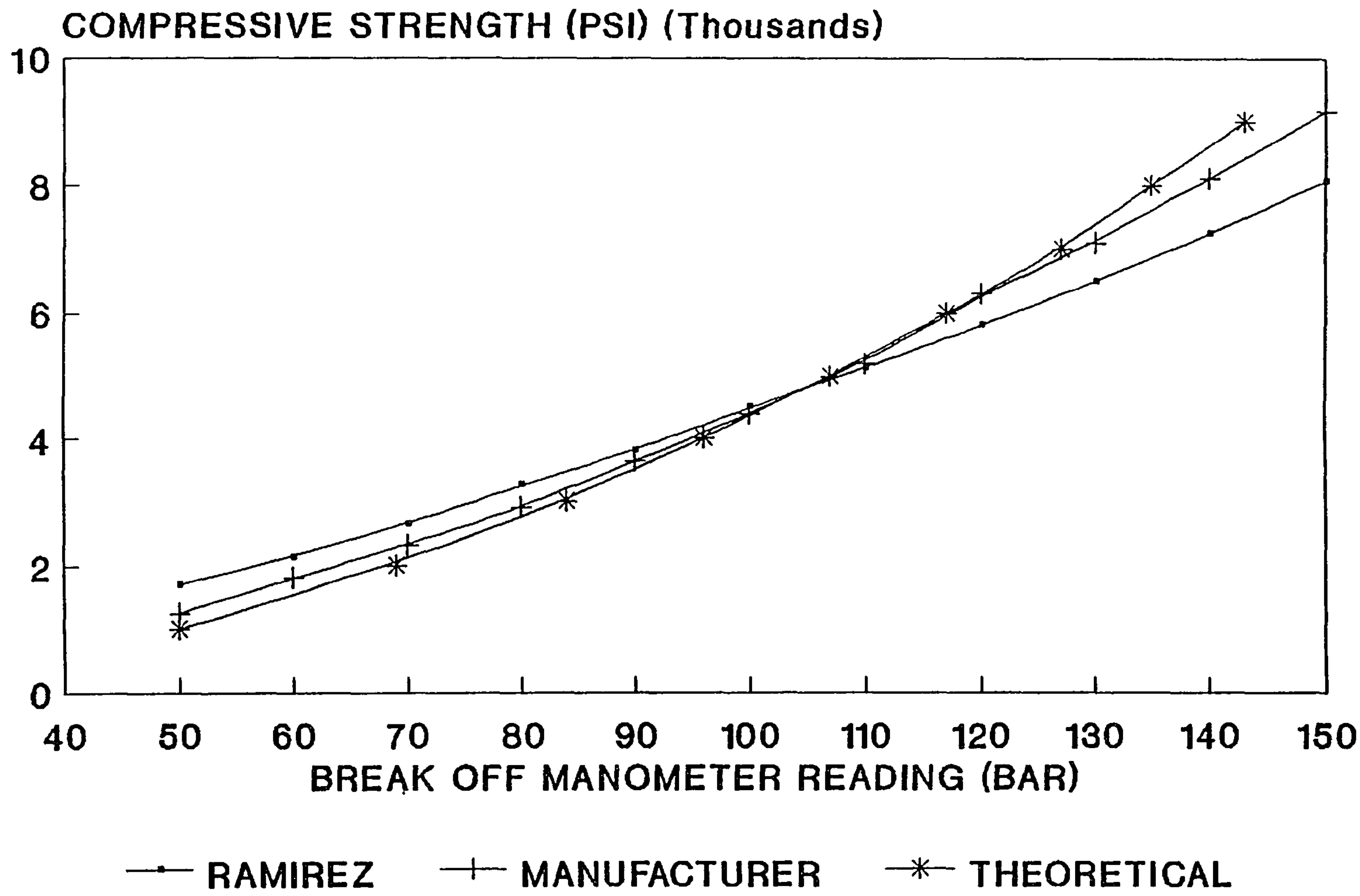


Figure 39 Compressive Strength vs Break Off Reading Using Shear Model

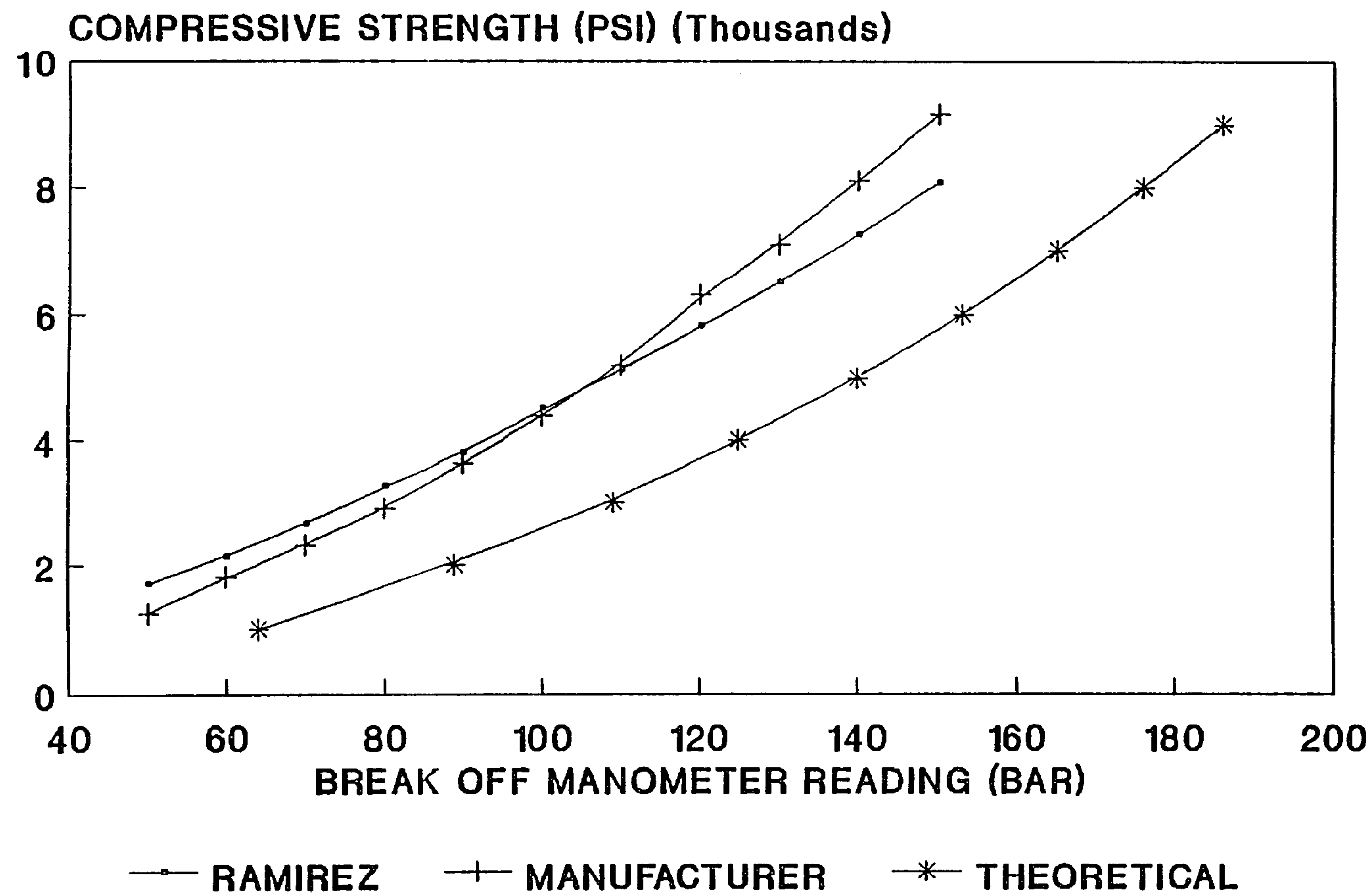


Figure 40 Compressive Strength vs Break Off Reading Using Average Shear

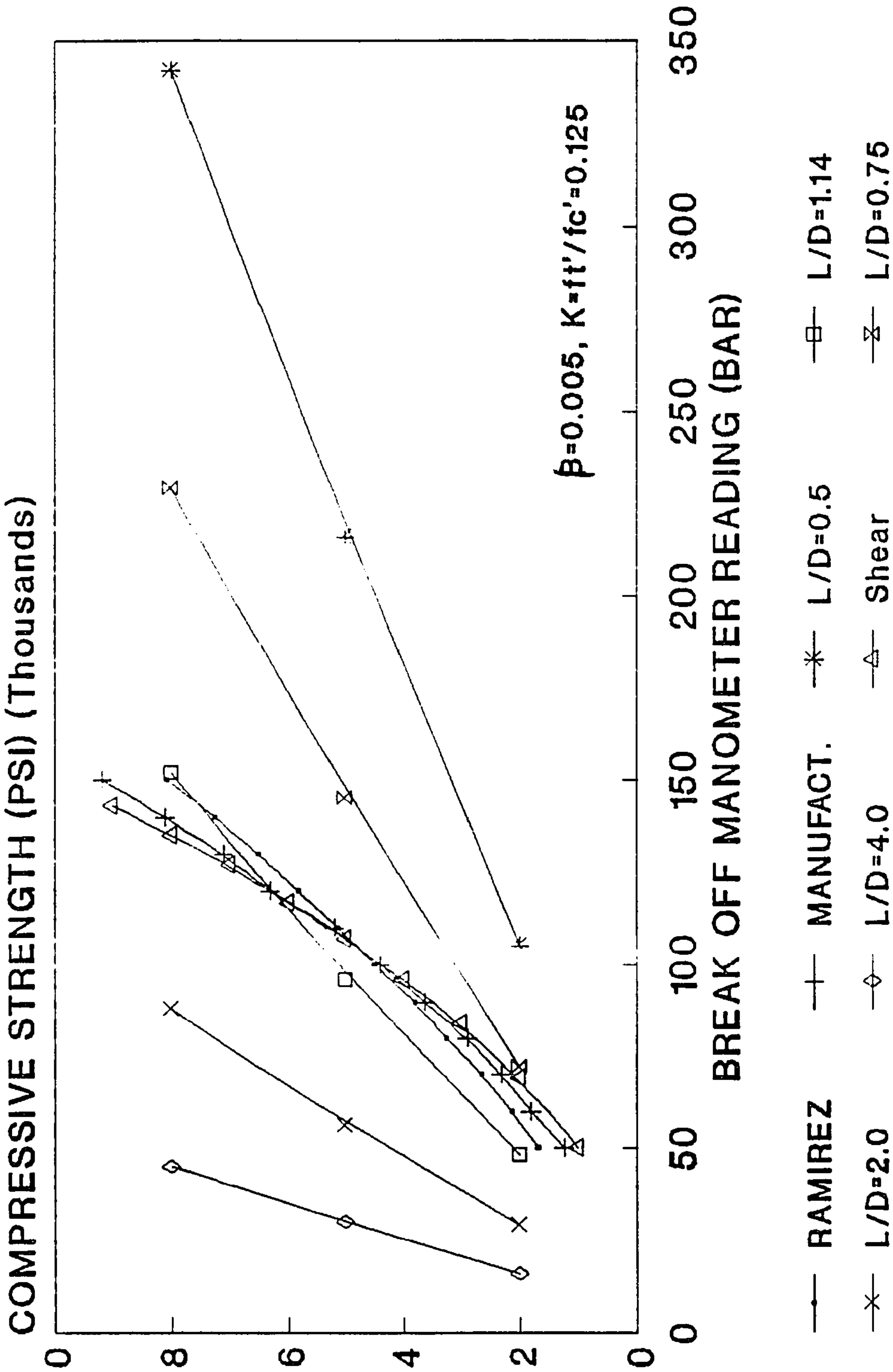


Figure 41 Effect of Shear Span to Depth Ratio on Break Off Reading

It is compared with experimental results of Ramirez (121) and the manufacturer of the break off tester (114). It is seen that the predicted relationship does not match with the experimental results indicating the simple method of taking an average shear across the circular cross section does not predict the relationship between the compressive strength of concrete and the break off value.

It is evident from Figures 39, that the theoretical results obtained in this study show the same trend as the experimental results, indicating the existence of a theoretical relationship between the compressive strength of concrete and the break off manometer reading.

Figure 41 shows the effect of shear span to depth ratio, on the compressive strength of concrete to break off manometer reading, obtained from the flexural model. On it, the theoretical relationship obtained from shear model and the experimental results of Ramirez (121) and the manufacturer of the break off tester (114) are also shown. It is seen that the theoretical curve from flexural model agrees well with experimental results for the shear span to depth ratio of the break off test specimen which is 1.14. It is seen that both flexural and shear models give theoretical relationships between compressive strength and break off manometer readings that correlate well with experimental results.

The above phenomenon can be explained by the work done by Bresler et al. (150). Figure 42 shows the variation in shear strength with

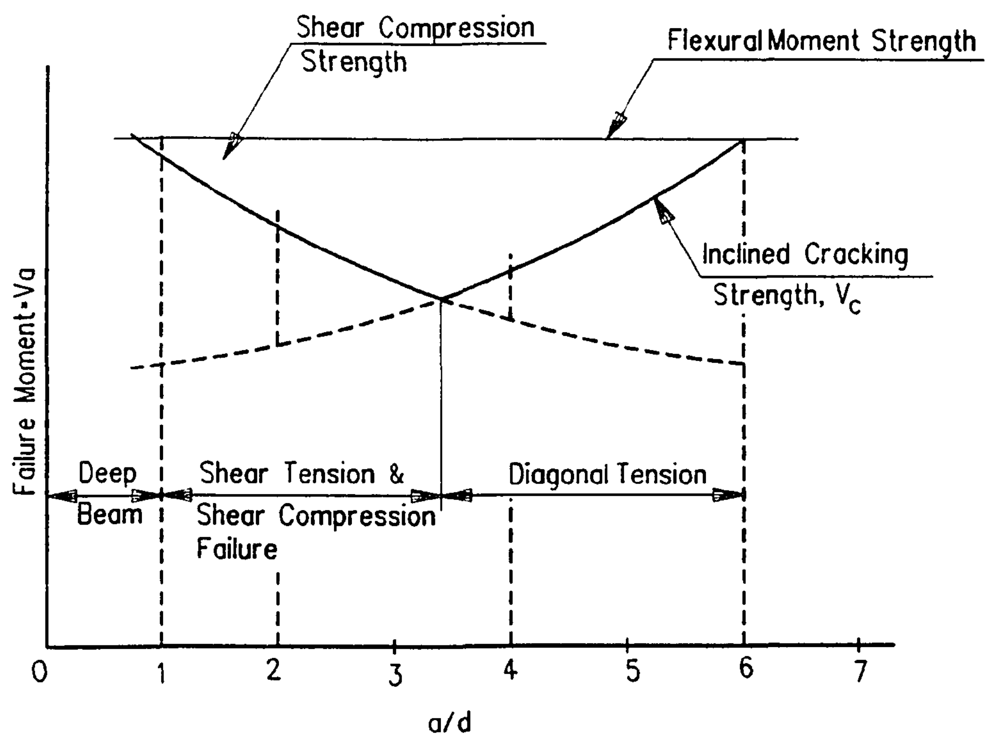


Figure 42 Variation in Shear Strength with Shear Span to Depth Ratio

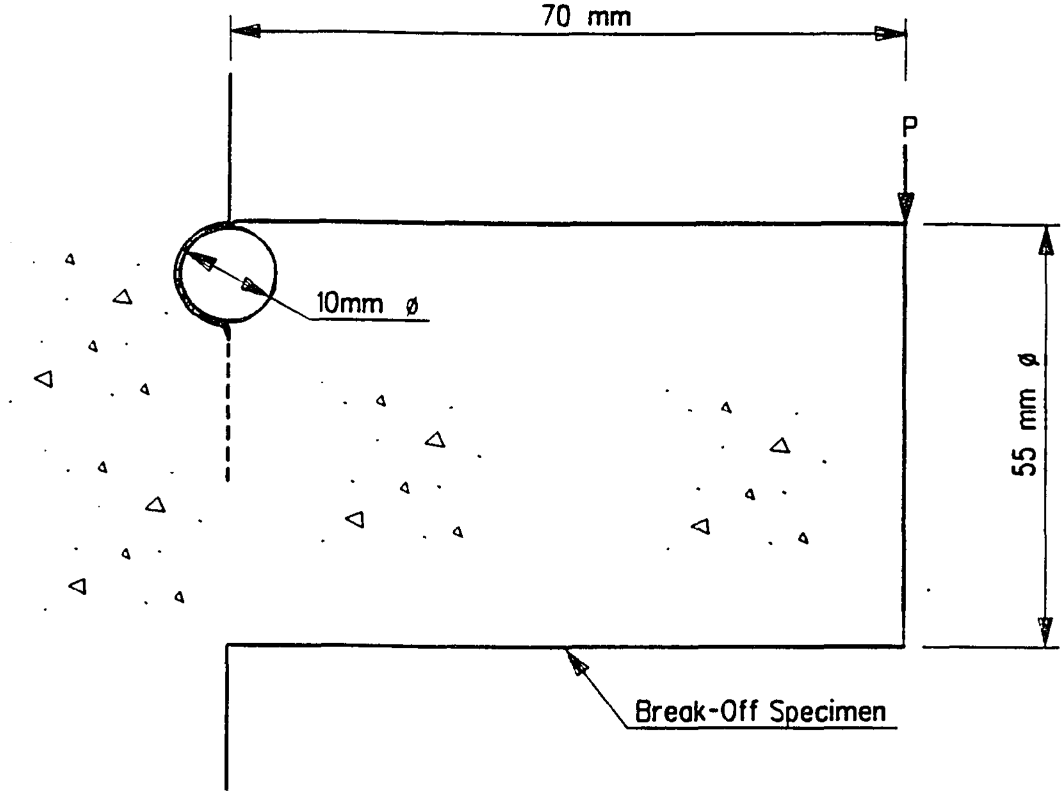


Figure 43 Model for Aggregate Effects on Break Off Test

shear span to depth ratio as given by Bresler et al. (150). For the break off test specimen the shear span to depth ratio is 1.1. It is seen from Figure 42 that when shear span ratio to depth ratio is around 1.0, the shear strength and the flexural moment strength are almost the same. Hence it can be concluded that the theoretical basis of the break off test can be explained by either flexure or shear.

5.2.4 Effects of Aggregate Size and Aggregate Interlocking

The break off test is generally recommended for concrete with maximum aggregate size of 10mm (3/8"). To study the effects of aggregate size and aggregate interlocking, the shear model described earlier was used in combination with the model shown in Figure 43.

It is assumed that the maximum size aggregate occurs at the crack. As the load is increased, the crack will propagate around the aggregate. This will increase the shear area and the ultimate load. The additional shear force the section can resist due to aggregate interlocking is given by,

$$P_{ADDITIONAL} = \tau_{MAX} [{}_0\int^{\pi} ({}_0\int^{\pi} r^2 \sin\theta \, d\theta) \, d\alpha - \pi r^2] \qquad (5.33)$$

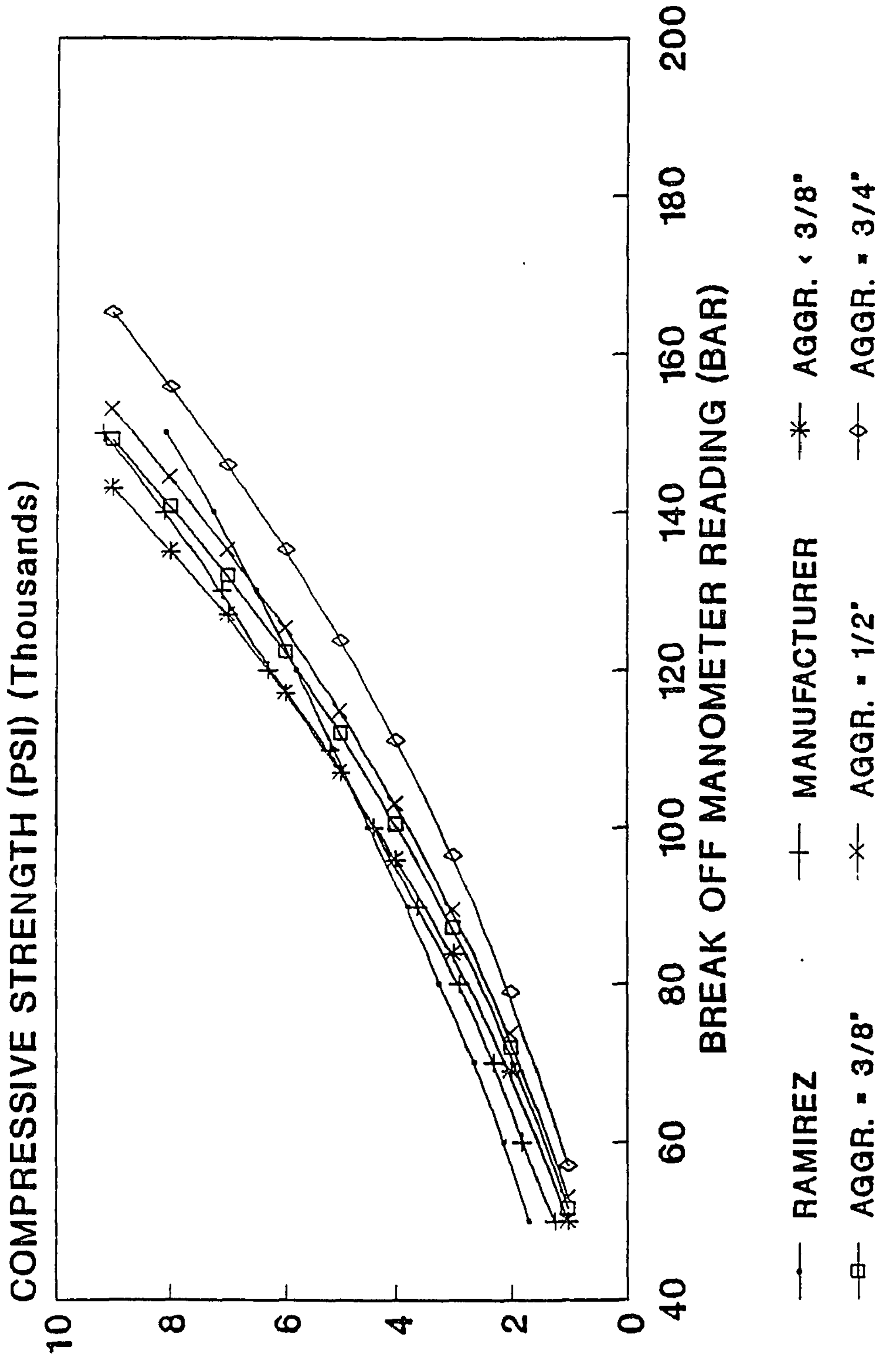


Figure 44 Effect of Maximum Aggregate Size on Break Off Reading

where, τ_{MAX} = Shear strength of concrete in psi

$= 2\ (f'_c)^{1/2}$

r = Radius of aggregate in inches

Simplifying equation (5.33) gives,

$$P_{ADDITIONAL} = \tau_{MAX}(2\pi r^2 - \pi r^2) \tag{5.34}$$

The above additional shear force was found for various compressive strengths of concrete and added to the shear force obtained from equation (5.31). From the total shear force, using equation (5.1) the corresponding break off number was obtained. Figure 44 shows the relationship between the compressive strength of concrete and the break off number for concretes with maximum size aggregates of 3/8", 1/2" and 3/4". This is compared with results given by the manufacturer of the break off equipment (114) , Ramirez (121) and the theoretical relationship developed earlier with no aggregate interlocking considerations. It is seen that for concrete with maximum size aggregates up to 1/2", aggregate interlocking has no significant effect on the relationship between concrete strength and break off number. This indicates that the break off test is more suitable for concrete with maximum size aggregate up to 1/2".

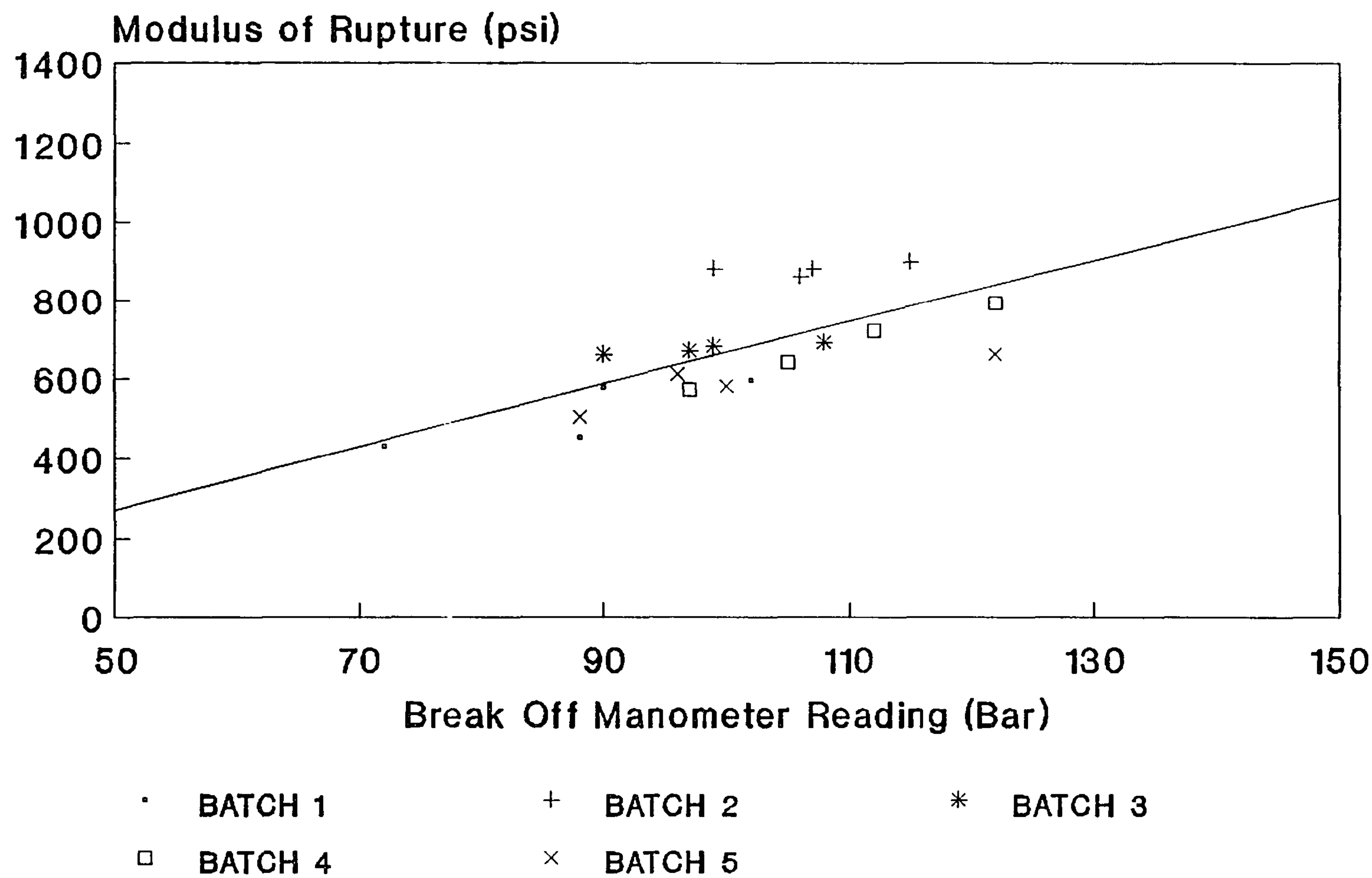


Figure 45 Modulus of Rupture vs Break Off Manometer Reading

It should be noted that only one aggregate of a particular maximum size was used in this model, since to include more maximum size aggregates, the crack has to propagate further and as seen by Figure 38, this will reduce the shear load capacity and the single maximum aggregate condition will govern.

5.3 Approximate Method

5.3.1 Introduction

An approximate method was used to find a relationship between the compressive strength of concrete and the break off reading. A new modulus of rupture for concrete beams with both rectangular and circular cross sections was also defined using this method.

5.3.2 Relationship between Compressive Strength of Concrete and Break Off Value

Equation (5.5) allows us to determine the maximum bending stress of the break off core provided that the break off manometer reading is known.

Ramirez et al. (121,122) conducted a series of break off tests on concrete and reported the relationships of modulus of rupture (Fr) and the ultimate compressive strength (f'_c) with corresponding break off manometer reading (BO). See Figures 45 and 46.

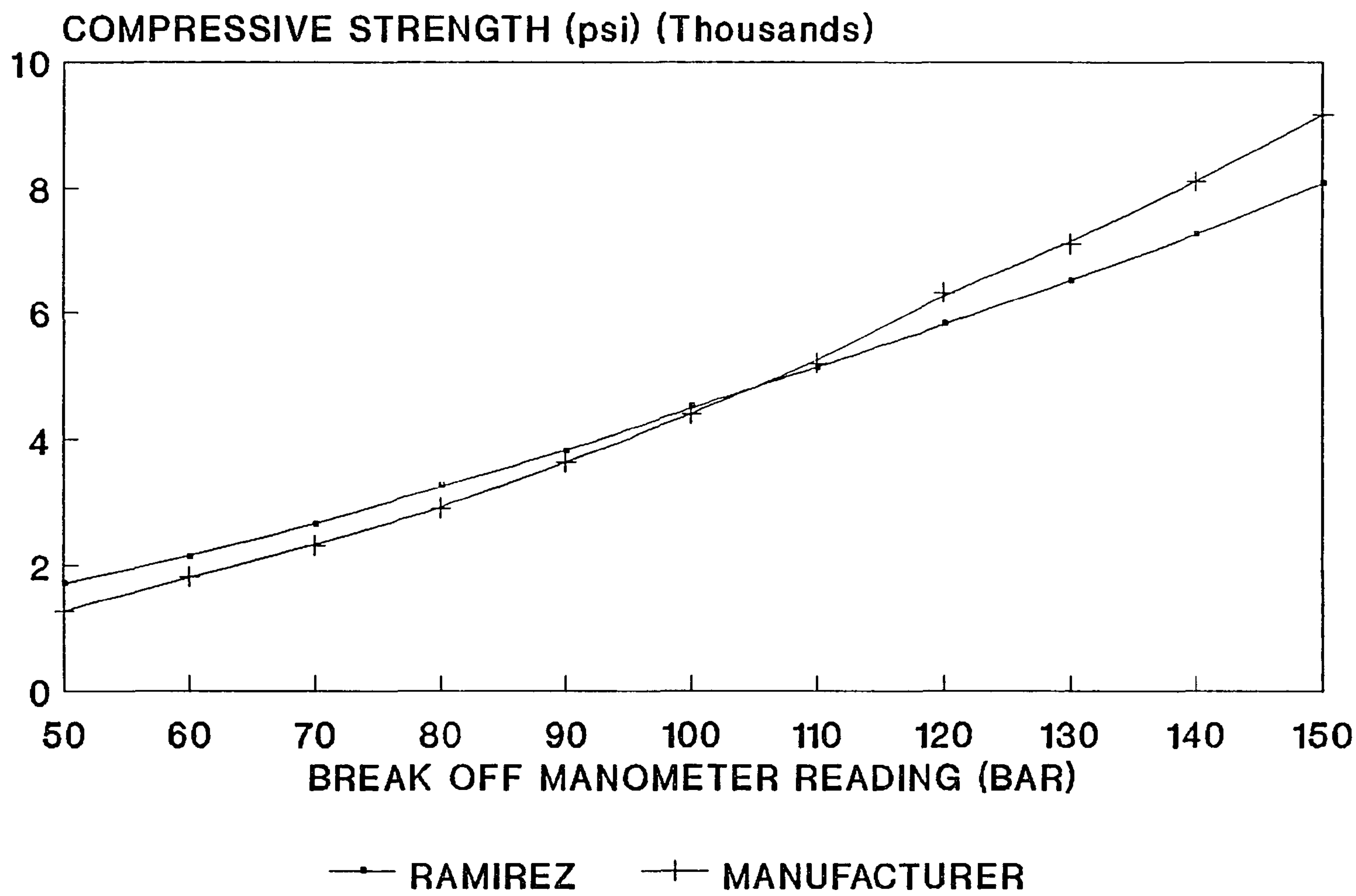


Figure 46 Compressive Strength vs Break Off Manometer Reading from Experiments

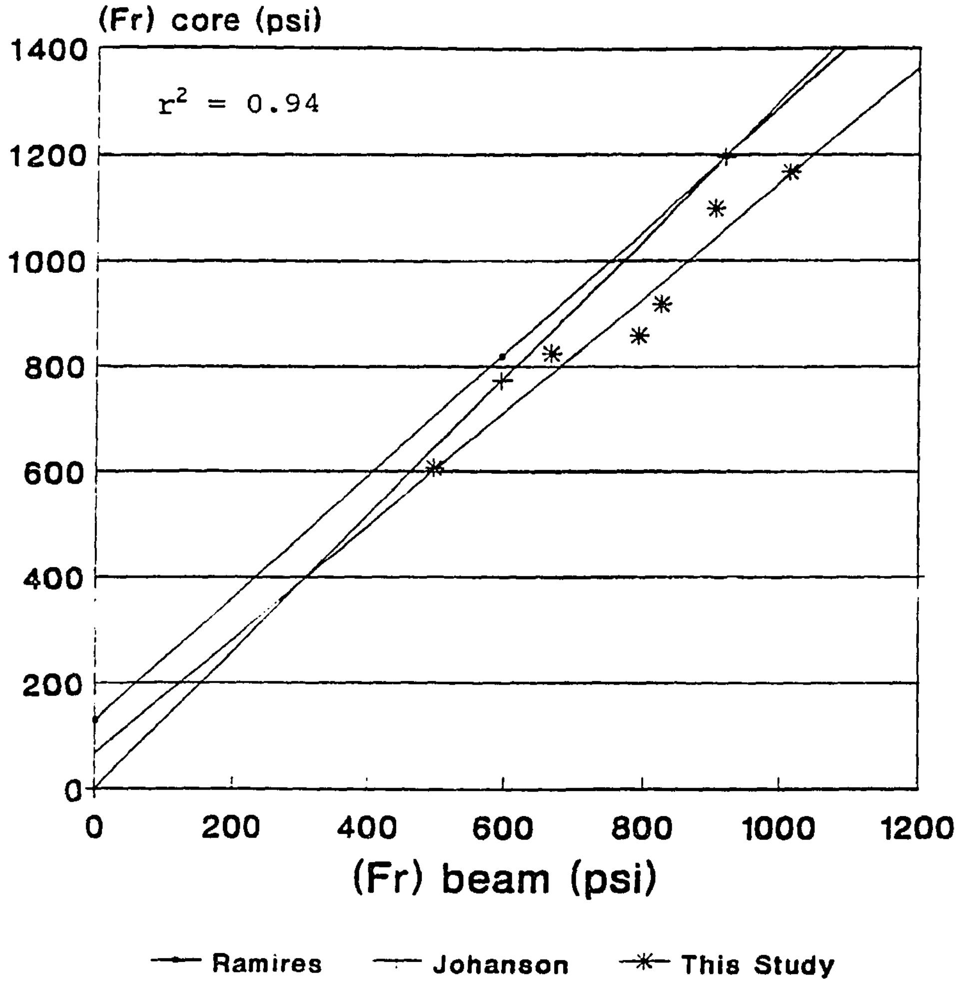


Figure 47 Modulus of Rupture - Core vs Beam

Table 2 shows the results of Center-Point Loading tests carried out on rectangular and circular beams as described in section 4.2.2.1.

Table 2 Center-Point Load Test Results

Compressive Strength of Concrete (psi)	Breaking Force (lbs)		Fr_{BEAM}(psi)	Fr_{CORE}(psi)
	Rectangular	Circular		
1900	660	806	497	607
3100	884	1095	666	825
4050	1051	1140	792	860
5243	1095	1221	825	920
6030	1202	1457	905	1099
7169*	1346	1549	1014	1167

* Mix # 5 tested at 90 days.

The relationship between $(Fr)_{CORE}$ and $(Fr)_{BEAM}$ as shown by Figure 47 is as follows :

$$(Fr)_{CORE} = 1.08\ (Fr)_{BEAM} + 70 \qquad (5.34)$$

where, $(Fr)_{CORE}$ and $(Fr)_{BEAM}$ are in psi.

Using equation (5.5) and experimental results of Ramirez (121), the variation between $(Fr)_{CORE}$ and $(Fr)_{BEAM}$ was also plotted on Figure 47. An approximate linear equation between $(Fr)_{CORE}$ and $(Fr)_{BEAM}$ is obtained from Figure 47 as follows :

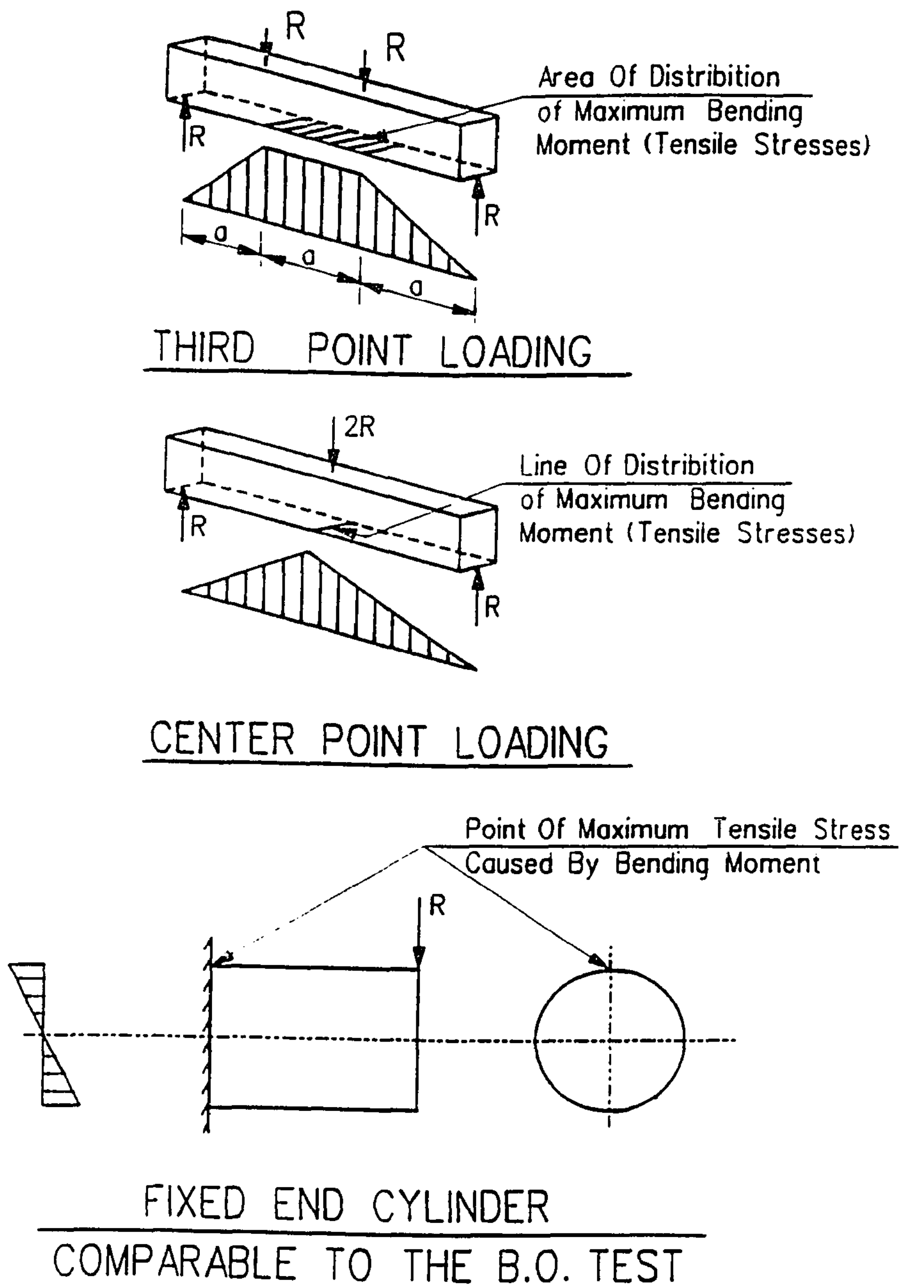


Figure 48 Flexural Stresses of Different Beams

$$(Fr)_{CORE} = 1.16\ (Fr)_{BEAM} + 130 \tag{5.35}$$

where, $(Fr)_{CORE}$ = The maximum bending stress of the break off test in psi

$(Fr)_{BEAM}$ = The modulus of rupture determined experimentally from the 6" x 6" x 18" in psi

Johansen (111), has reported the relationship,

$$(Fr)_{CORE} = 1.30\ (Fr)_{BEAM} \tag{5.36}$$

where, $(Fr)_{CORE}$ and $(Fr)_{BEAM}$ are in psi. The above relationship is also shown on Figure 47.

It is seen that the $(Fr)_{CORE}$ value is higher than $(Fr)_{BEAM}$ value. Johansen (111), Ramirez (121,122), and other researchers have concluded that this is due to low probability of a weak point occurring at the lowest point of a circular cross section where as cracking can initiate at any point across the rectangular cross section. This is illustrated by Figure 48.

Substituting equation (5.5) into equations (5.35) and (5.36) provides the relationship between the break off manometer reading and the modulus of rupture as,

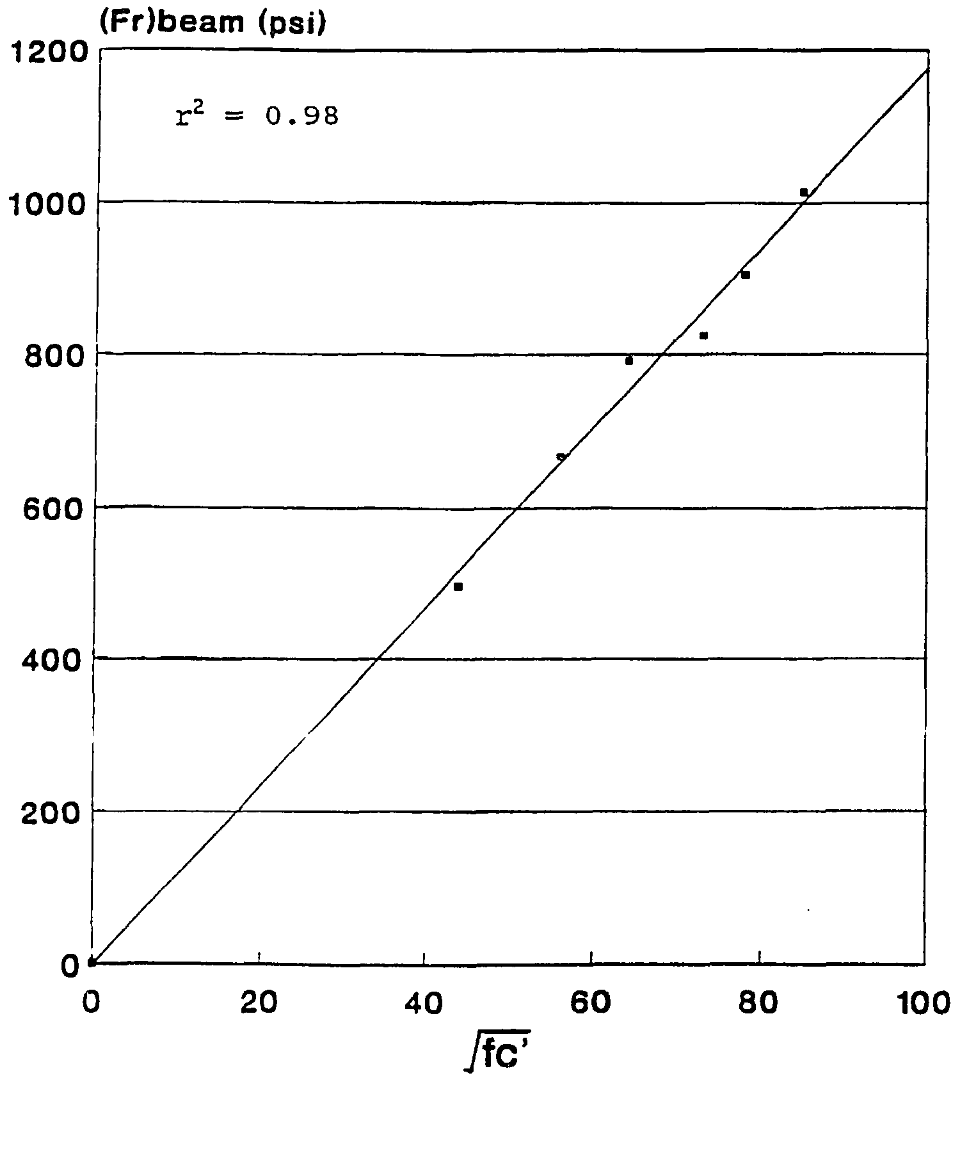


Figure 49 $(Fr)_{BEAM}$ vs $(f'_c)^{1/2}$

$$9.4060\ (BO - 2.973) = 1.16\ (Fr)_{BEAM} + 130 \tag{5.37}$$

$$9.4060\ (BO - 2.973) = 1.30\ (Fr)_{BEAM} \tag{5.38}$$

where, BO is in bars and $(Fr)_{BEAM}$ is in psi.

Substituting equation (5.6) into equations (5.37) and (5.38), the relation between f'_C and the break off manometer reading (BO) can be obtained as follows :

$$(f'_C)^{1/2} = 1.0811\ (BO) - 18.1568 \tag{5.39}$$

$$(f'_C)^{1/2} = 0.9641\ (BO) - 2.8681 \tag{5.40}$$

where, f'_C = Compressive Strength of Concrete in psi

BO = Break Off Manometer Reading in bars

Figure 49 shows the variation of modulus of rupture $(Fr)_{BEAM}$ with compressive strength of concrete as given in Table 2. These results yield the relation of modulus of rupture versus f'_C as,

$$(Fr)_{BEAM} = 11.9\ (f'_C)^{1/2} \tag{5.41}$$

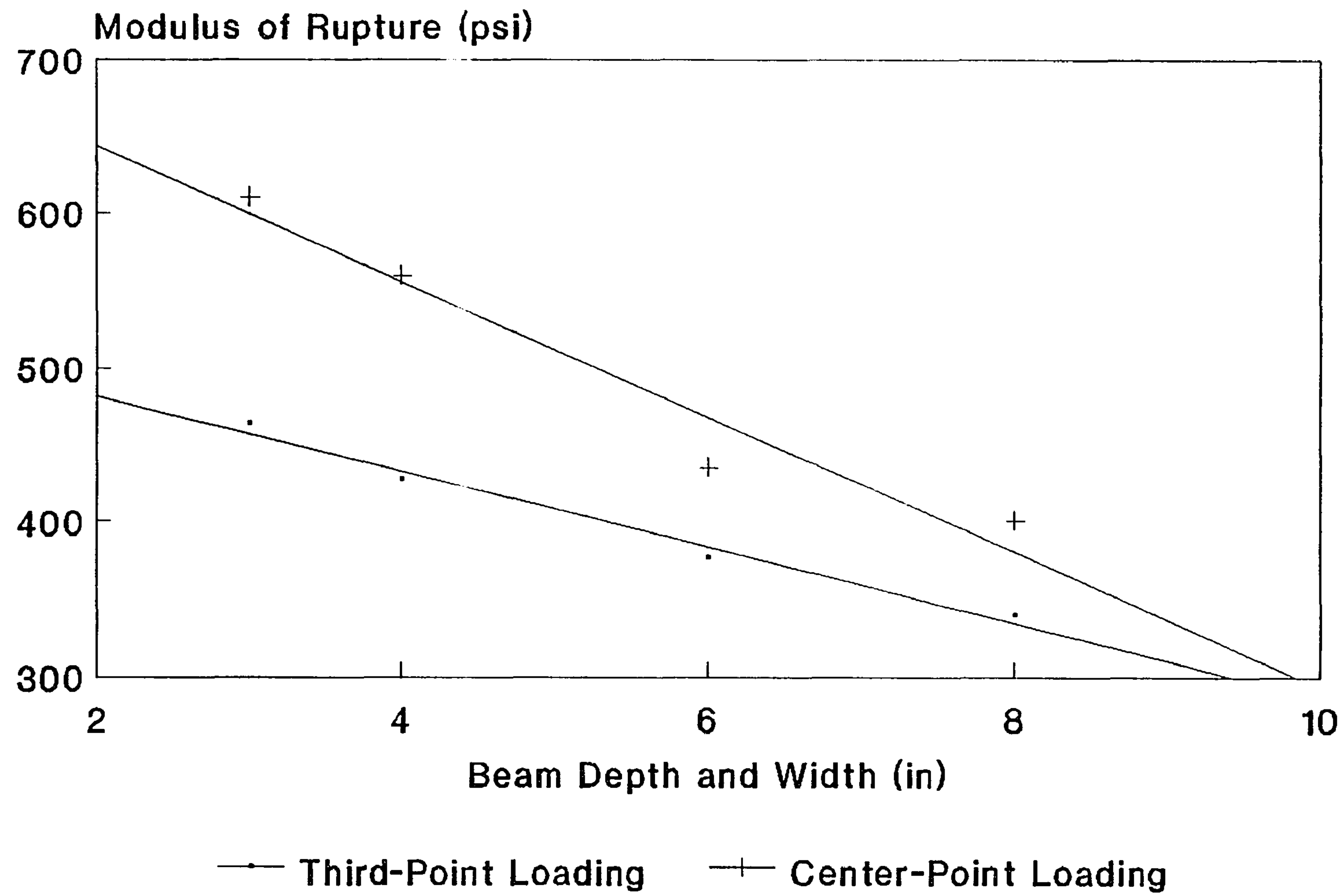


Figure 50 Modulus of Rupture of Beams of Different Sizes

where, $(Fr)_{BEAM}$ and f'_c are in psi.

It should be noted that compared to the modulus of rupture value specified by American Concrete Institute (143), and as given by equation (5.6), the value given by equation (5.41) is high. The American Concrete Institute value is based on beams with 6" x 6" cross sections. According to Wright, as Reported by Neville (151), the smaller the beams tested in cross sectional area, higher the modulus of rupture values. This is depicted by Figure 50. Substituting equations (5.34) and (5.41) in Equation (5.5) gives,

$$(f'_C)^{1/2} = 0.7319\ (BO) - 7.6225 \tag{5.42}$$

where, f'_C is in psi and BO is in bars.

Relationships between compressive strength of concrete and break off manometer reading as given by equations (5.39), (5.40) and (5.42) together with experimental data by the manufacturer of break off test are shown in Figure 51. It is seen that the theoretical curve obtained in this study agrees well with experimental results given by the manufacturer of the break off tester (114). The theoretical curves based on work carried out by Ramirez et al. (121,122) and Johansen (111) also show a similar trend. In Figure 52, the theoretical results are compared with experimental results published by Ramirez et al (121,122) and the

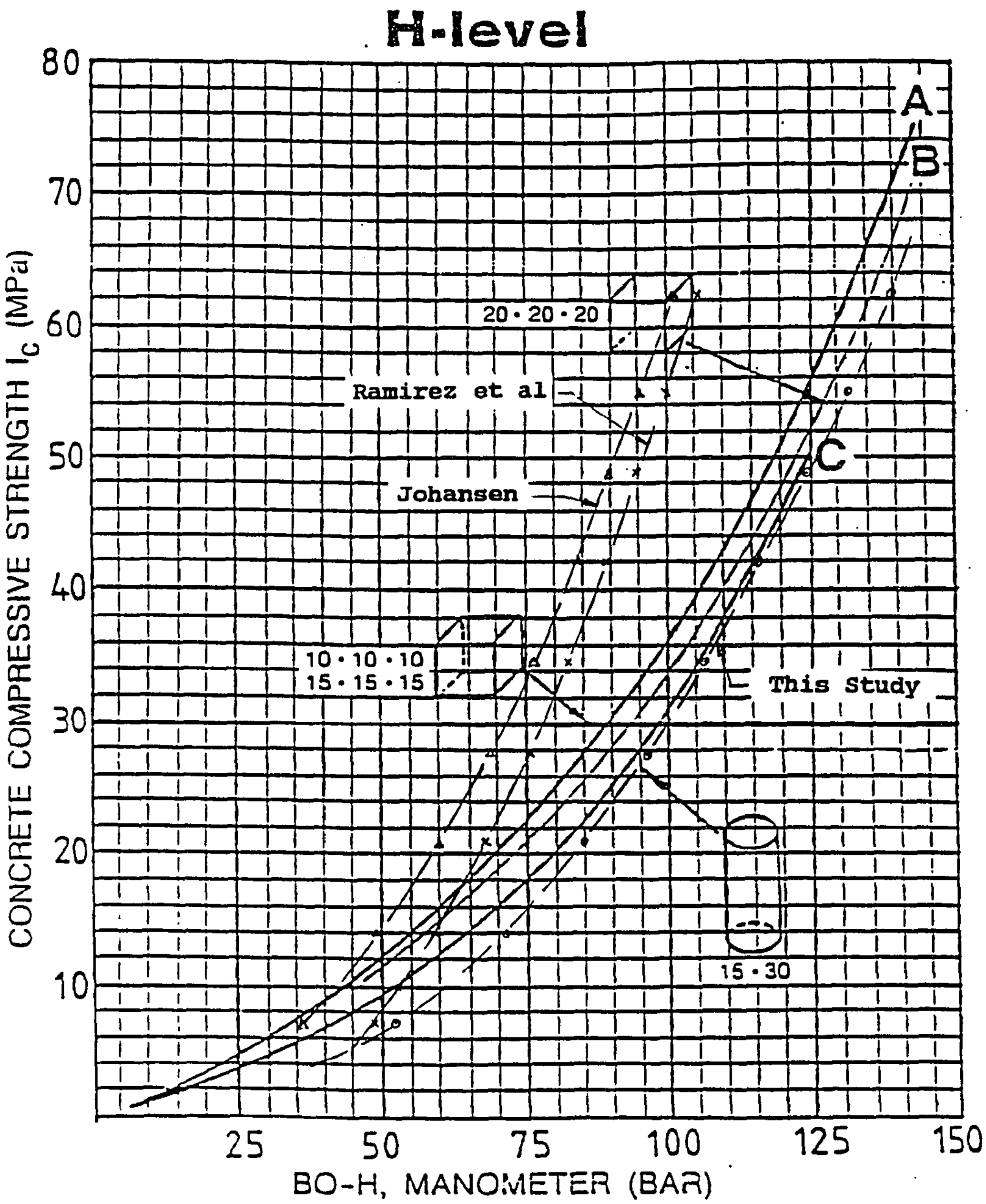


Figure 51 Compressive Strength of Concrete vs Break Off Value from Approximate Method

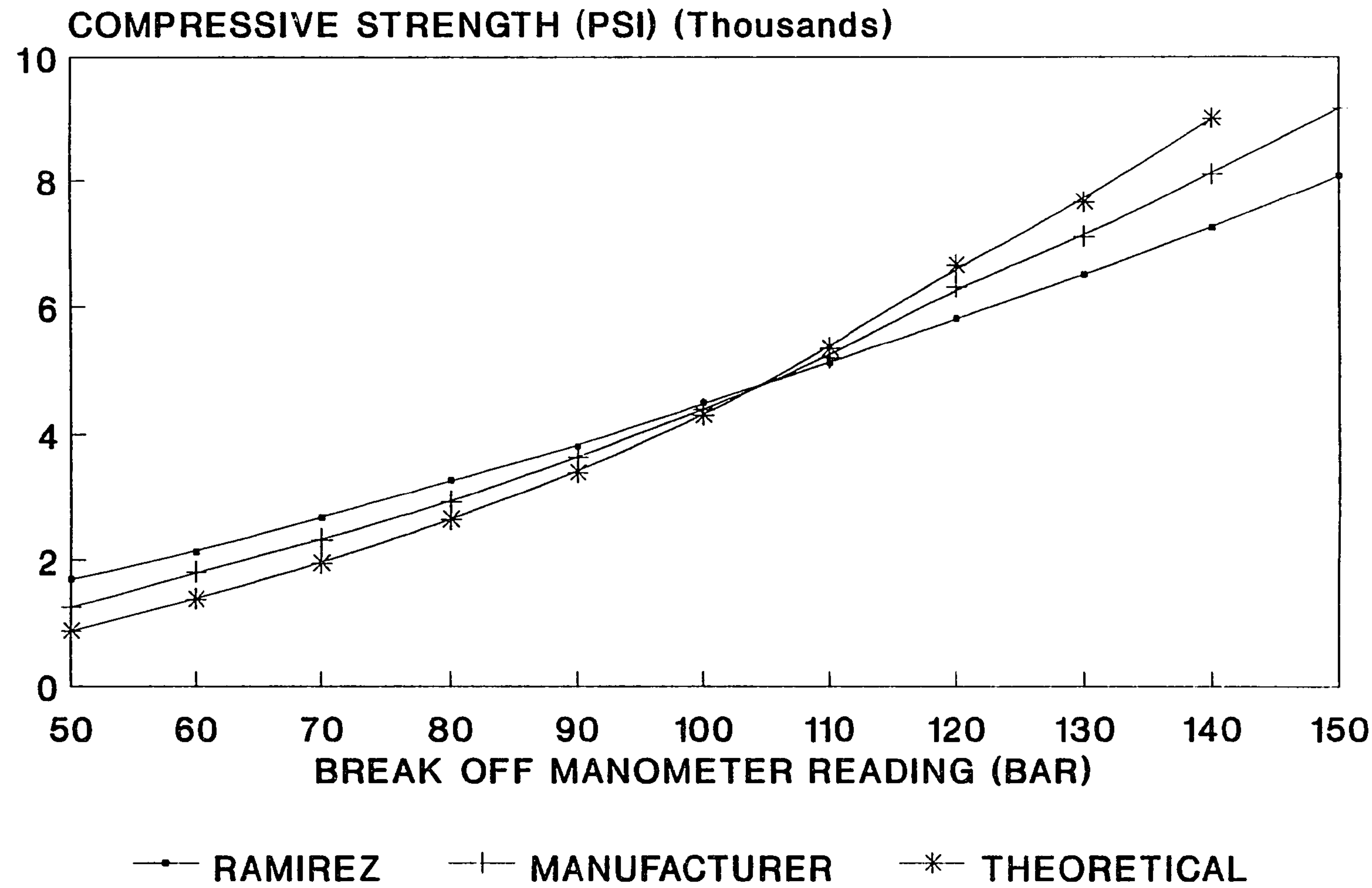


Figure 52 Compressive Strength vs Break off Value from Approximate Method

manufacturer of the break off tester (114). It is seen that the theoretical results in this study agrees well with the experimental results. The slight variation observed may be attributed to the residual stresses in concrete and other experimental errors.

5.3.3 Breaking Force of Cylindrical Cantilever Specimens Loaded with a Point Load at the Free End

Inorder to check the validity of the approximate method, cylindrical cantilever specimens were tested and compared with the theoretical values given by the approximate method. The specimens were tested as described in section 4.2.2.2 and the experimental results are given in Table 3.

Table 3 Theoretical and Experimental Breaking Forces of Cantilevered Specimen

Compressive Strength of Concrete (psi)	Breaking Force (lbs)	
	Experimental	Theoretical
2210	519	496
2607	554	534
3774	580	632
4613	675	694
6100	711	791

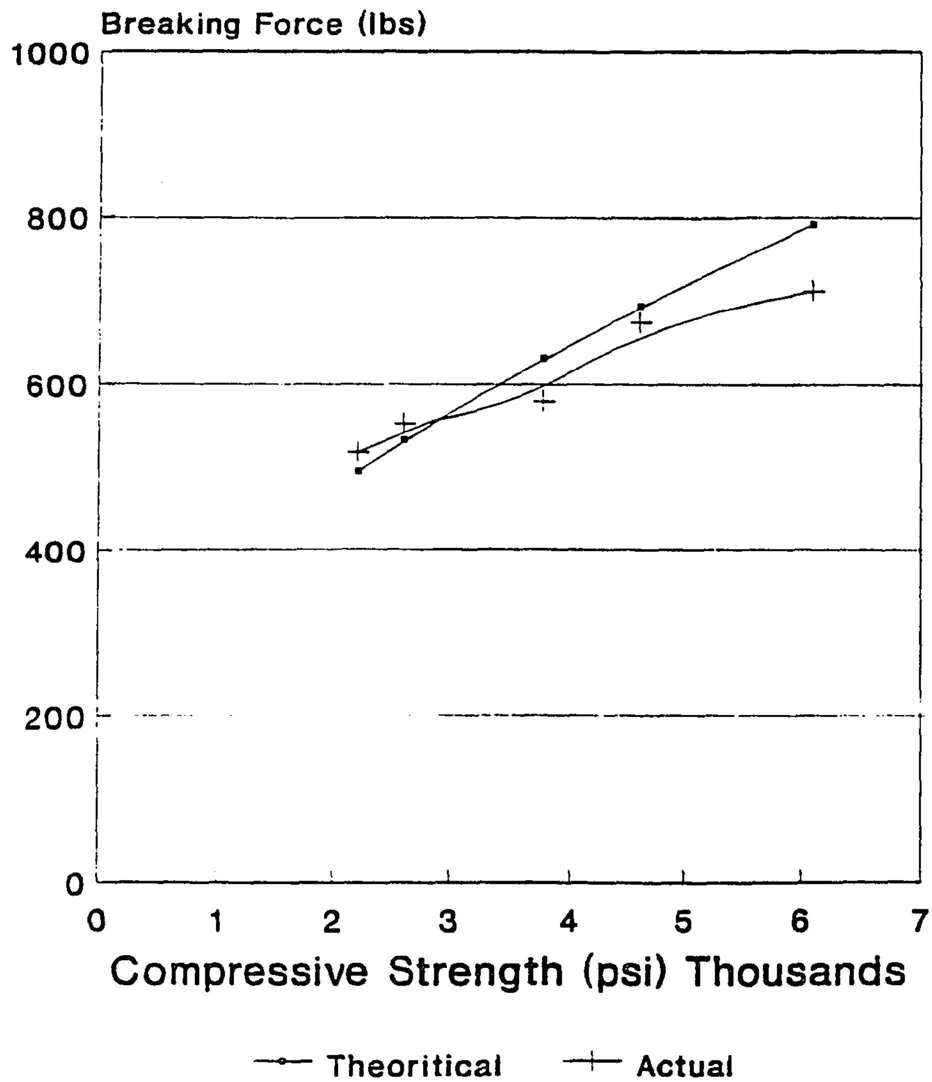


Figure 53 Force vs Compressive Strength

A theoretical relationship between the compressive strength of concrete and the breaking force was obtained by using equations (5.3), (5.34) and (5.41) as follows :

$$(f'_c)^{1/2} = 0.1115\,(P) - 5.4466 \tag{5.43}$$

where, f'_c is in psi and P is in lbs.

The experimental and theoretical results are plotted in Figure 53. It is seen that the experimental results and theoretical results agree well. The small discrepancy may be due to the residual stresses of concrete and experimental errors.

5.3.4 Modulus of Rupture for Structural Elements Smaller than Six Inches

As noted earlier in this study and depicted by Figure 50, the modulus of rupture value (MOR) specified by American Concrete Institute (143) and given by equation (5.6), is not suitable for rectangular beams with depth and width smaller than six inches and for members with circular cross sections. Since break off specimens are smaller than the 6" x 6" specimens used to find the modulus of rupture, the break off tester is ideal for the determination of the modulus of rupture for structural elements smaller than six inches.

For circular members, equation (5.5) gives,

$$\text{Modulus of Rupture (MOR)} = 9.4060\ (\ BO - 2.973\ ) \qquad (5.44)$$

where, (MOR) is in psi and BO is in bars.

Equations (5.34) and (5.41) give,

$$\text{Modulus of Rupture (MOR)} = 12.85\ (\ f'_c)^{1/2} + 70 \qquad (5.45)$$

where, (MOR) and f'_c are in psi.

For small rectangular members equations (5.5) and (5.34) yield,

$$\text{Modulus of Rupture (MOR)} = 8.709\ (\ BO - 10.415\ ) \qquad (5.46)$$

where, (MOR) is in psi and BO is in bars.

From equation (5.41),

$$\text{Modulus of Rupture (MOR)} = 11.9\ (\ f'_c)^{1/2} \qquad (5.47)$$

where, (MOR) and f'_c are in psi.

The break off tester is already well known and accepted by the American Society for Testing of Materials (129) as an apparatus used in the determination of in-situ strength of concrete. Also, the testing procedure is easy to perform and quick. Hence, for structural elements

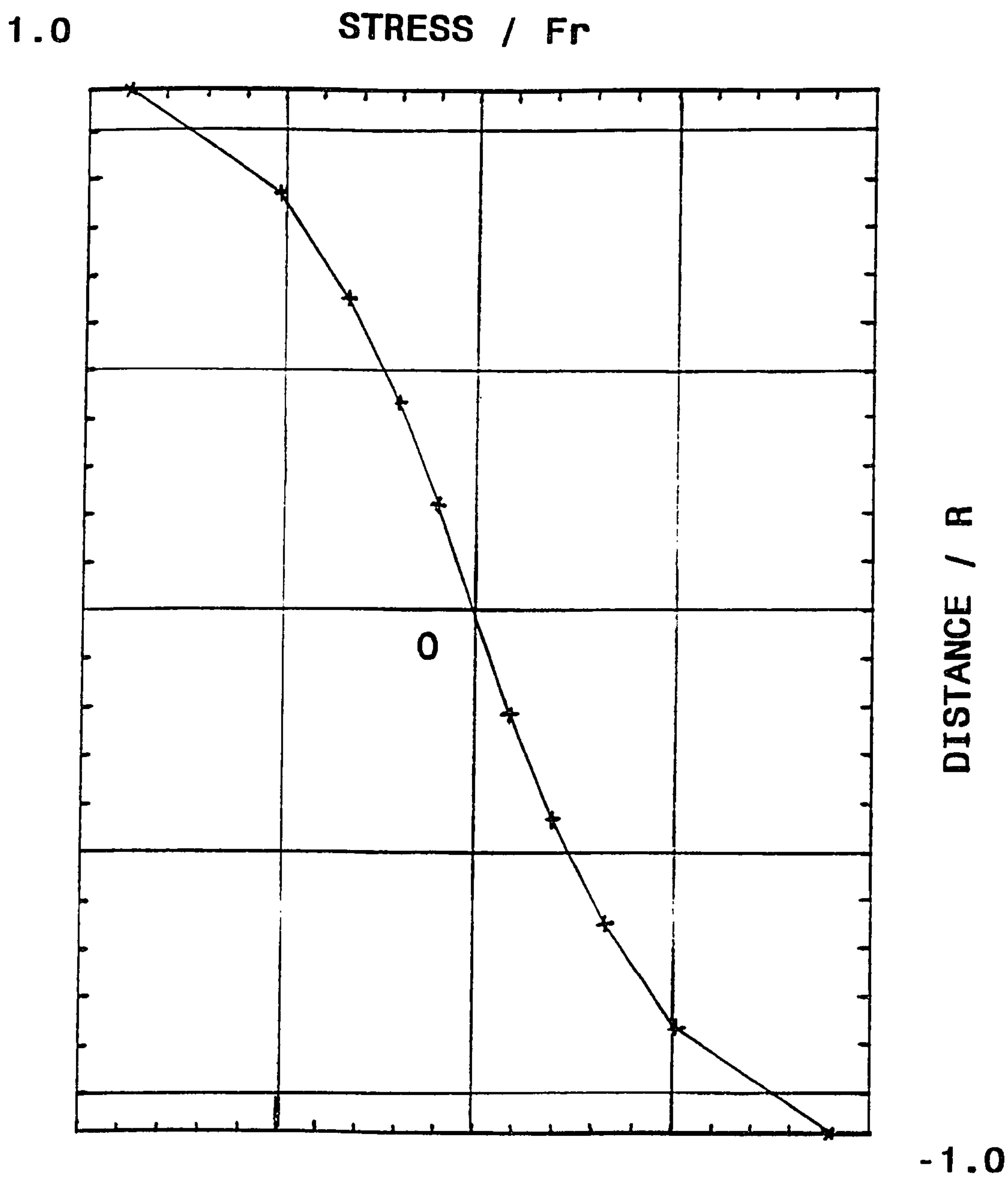


Figure 54 Stress Distribution at the Fixed End from Finite Elements

smaller than six inches, such as thin slabs with metal forms sometimes used in parking garages, the break off tester can be used to find the modulus of rupture. It will result in more meaningful values than one would obtain from the current American Concrete Institute (143) method.

5.4 Finite Element Analysis

5.4.1 Flexural Stress Distribution at the Fixed End of the Cantilevered Break Off Test Specimens

The flexural stress distribution was obtained as described in section 4.3 and shown in Figure 54. It is seen that the stress distribution is nonlinear. Figure 55 shows a comparison of experimental relationships between compressive stress of concrete and the break off value obtained by Ramirez et al (121,122) and the manufacturer of the break off tester (114), and the theoretical relationships obtained using the approximate method described earlier in section 5.3 and the finite element method.

The theoretical relationship based on the finite element method was obtained by using the stress distribution shown in Figure 54, with Fr calculated from equations (5.34) and (5.41) for a particular compressive strength of concrete. The above stress distribution and a numerical integration technique was used to find the internal moment. Computer program used is given in Appendix A. Equating the internal moment at the

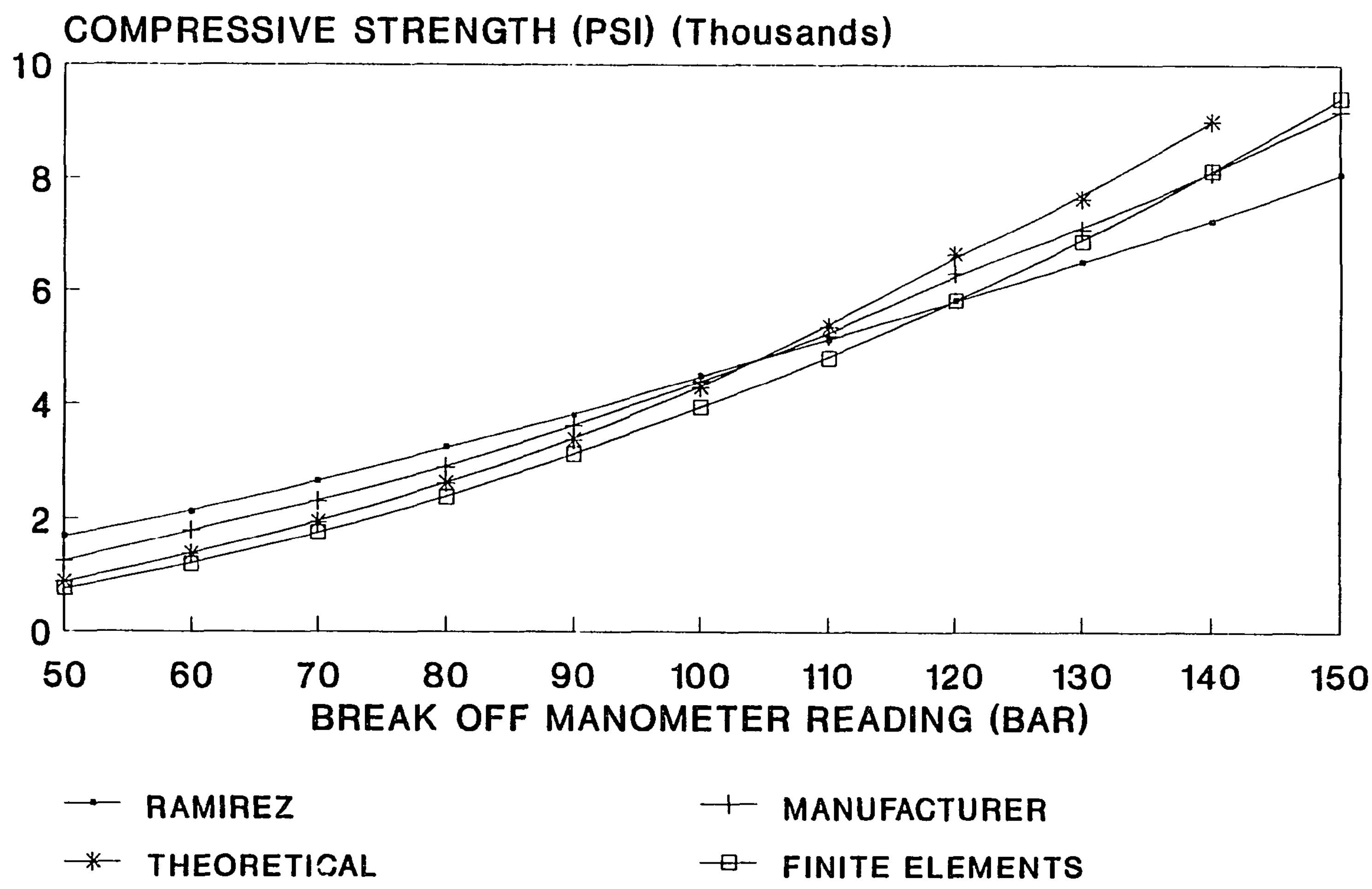


Figure 55 Compressive Strength vs Break Off Value Using Finite Elements

fixed end of the specimen to the external moment created by the point load, the value of the point load can be found. Using equation (5.1), the corresponding break off manometer reading was obtained. It is seen that the results agree well with the experimental results. The small discrepancies are due to the residual stresses of concrete, experimental errors and the assumption of the linear elastic behavior of concrete in the finite element method used.

5.4.2 The Effect of Slab Thickness on the Break Off Test Results

For different slab thicknesses of the finite element model, the maximum flexural stress $(Fr)_{CORE}$ was obtained as described in section 4.2. From Equation (5.5) it seen that $(Fr)_{CORE}$ is almost proportional to the break off value (BO).

Therefore,

$$(Fr)_{CORE} \,/\, (Fr)_{CORE5} = (BO) \,/\, (BO)_5 \qquad (5.48)$$

where,

$(Fr)_{CORE5}$ = $(Fr)_{CORE}$ value when slab thickness is 5"

$(BO)_5$ = Break off value when slab thickness is 5"

In Figure 56, $(BO)/(BO)_5$ is plotted against the thickness of the slab (t). It is seen that when slab thickness is 5" or more there is no change in the Break off values indicating that the break off test is not sensitive to the slab thickness beyond 5". It is interesting to note that Naik et al. (120) have arrived at the same conclusion from their experiments.

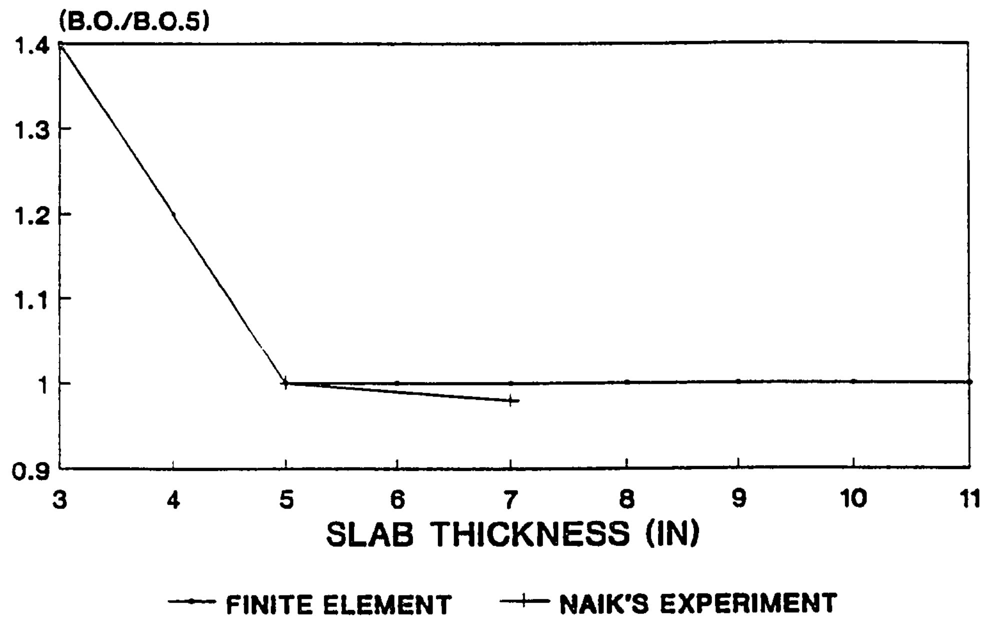


Figure 56 Effect of Slab Thickness on the Break Off Value

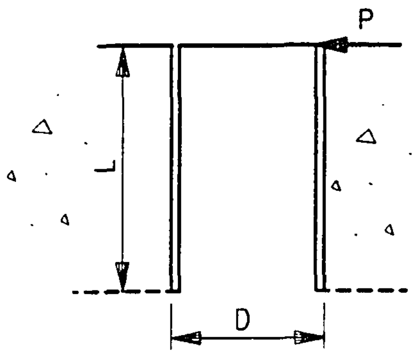


FIG. (a)

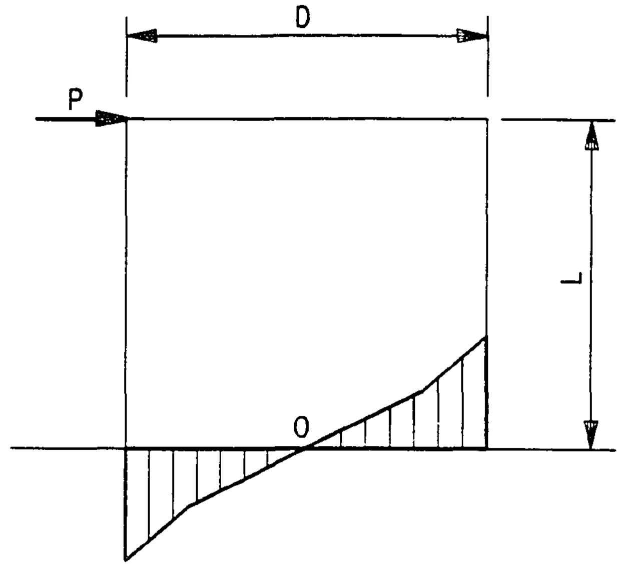


FIG. (b)

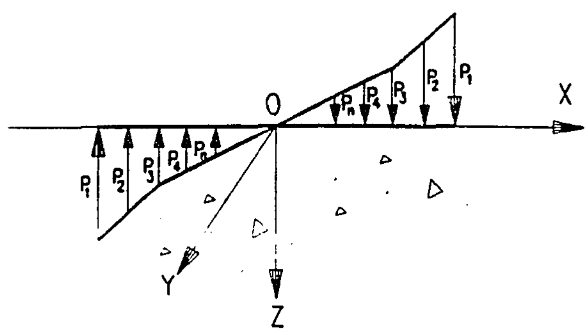


FIG. (c)

+

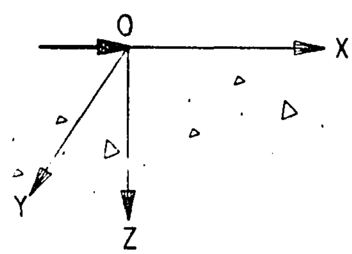


FIG. (d)

Figure 57 Idealized Break Off Specimen to Obtain Stresses

5.5 Stresses in the Vicinity of the Break Off Specimen

5.5.1 Introduction

The stresses in the vicinity of the break off specimen were obtained by the flexural stress distribution from finite element analysis and the equations available for stresses in elastic half space. The stresses were obtained for break off specimens of concrete with compressive strength varying from 1000 psi to 9000 psi.

Figures 57(a) and 57(b) show the idealized break off specimen. The forces acting at the fixed end of the cantilever specimen are the vertical forces due to flexure and a shear force due to the point load as depicted by Figures 57(c) and 57(d). The vertical force is assumed to be a collection of small vertical forces (see figure 57(c)).

5.5.2 Expressions for Stresses

Boussinesq (152) has derived the expressions for stresses due to a vertical point load as given by equations (5.49) through (5.51). Equations (5.52) through (5.54) give the stresses due to a horizontal point load as derived by Little (153).

Stresses due to a vertical point load are given by,

$$\sigma_{xx}=\frac{3P\cos^2\beta}{2\pi Z^2}\{\sin^2\beta\cos\beta\sin^2W-\frac{1-2\mu}{3}[\frac{2+\cos\beta}{(1+\cos\beta)^2}\sin^2\beta\sin^2W-\frac{1}{1+\cos\beta}+\cos($$

$$-\frac{1}{1+\cos\beta}+\cos\boldsymbol{\beta}]\} \tag{5.49}$$

$$\sigma_{yy}=\frac{3P\cos^2\beta}{2\pi Z^2}\{\sin^2\beta\cos\beta\cos^2W-\frac{1-2\mu}{3}[\frac{2+\cos\beta}{(1+\cos\beta)^2}\sin^2\beta\sin^2W-\frac{1}{1+\cos\beta}+\cos$$

$$-\frac{1}{1+\cos\beta}+\cos\boldsymbol{\beta}]\} \tag{5.50}$$

$$\sigma_{zz}=\frac{3P\cos^5\beta}{2\pi Z^2} \tag{5.51}$$

where,

μ = Poisson's ratio

$R=\sqrt{x^2+y^2+z^2}$ $\qquad$ $r=\sqrt{x^2+y^2}$

$\cos\beta=\frac{z}{R}$ $\quad$ $\sin\beta=\frac{r}{R}$ $\quad$ $\sin W=\frac{x}{r}$ $\quad$ $\cos W=\frac{y}{r}$

$\sin^2\beta\sin^2W=\frac{x^2}{R^2}$ $\qquad$ $\sin^2\beta\cos^2W=\frac{y^2}{R^2}$

Stresses due to a horizontal point load are given by,

$$\sigma_{xx}=\frac{Px}{2\pi R^3}\{\frac{-3x^2}{(R^2)}+\frac{(1-2\mu)}{(R+z)^2}(R^2-y^2-\frac{2Ry^2}{R+z})\} \tag{5.52}$$

$$\sigma_{yy}=\frac{Px}{2\pi R^3}\{-\frac{3y^2}{(R^2)}+\frac{(1-2\mu)}{(R+z)^2}(3R^2-x^2-\frac{2Rx^2}{R+z})\} \tag{5.53}$$

$$\sigma_{zz}=-\frac{3}{2\pi}\frac{Pxz^2}{R^5} \tag{5.54}$$

where,

$R=\sqrt{x^2+y^2+z^2}$

μ = Poisson's ratio

Using the principle of superposition, the stresses at a point due to above forces were obtained by adding the corresponding expressions for stresses given by Boussinesq (152) and Little (153). The total effect of the vertical point loads was taken into account by integrating over the cross sectional area of the break off specimen. Numerical integration was

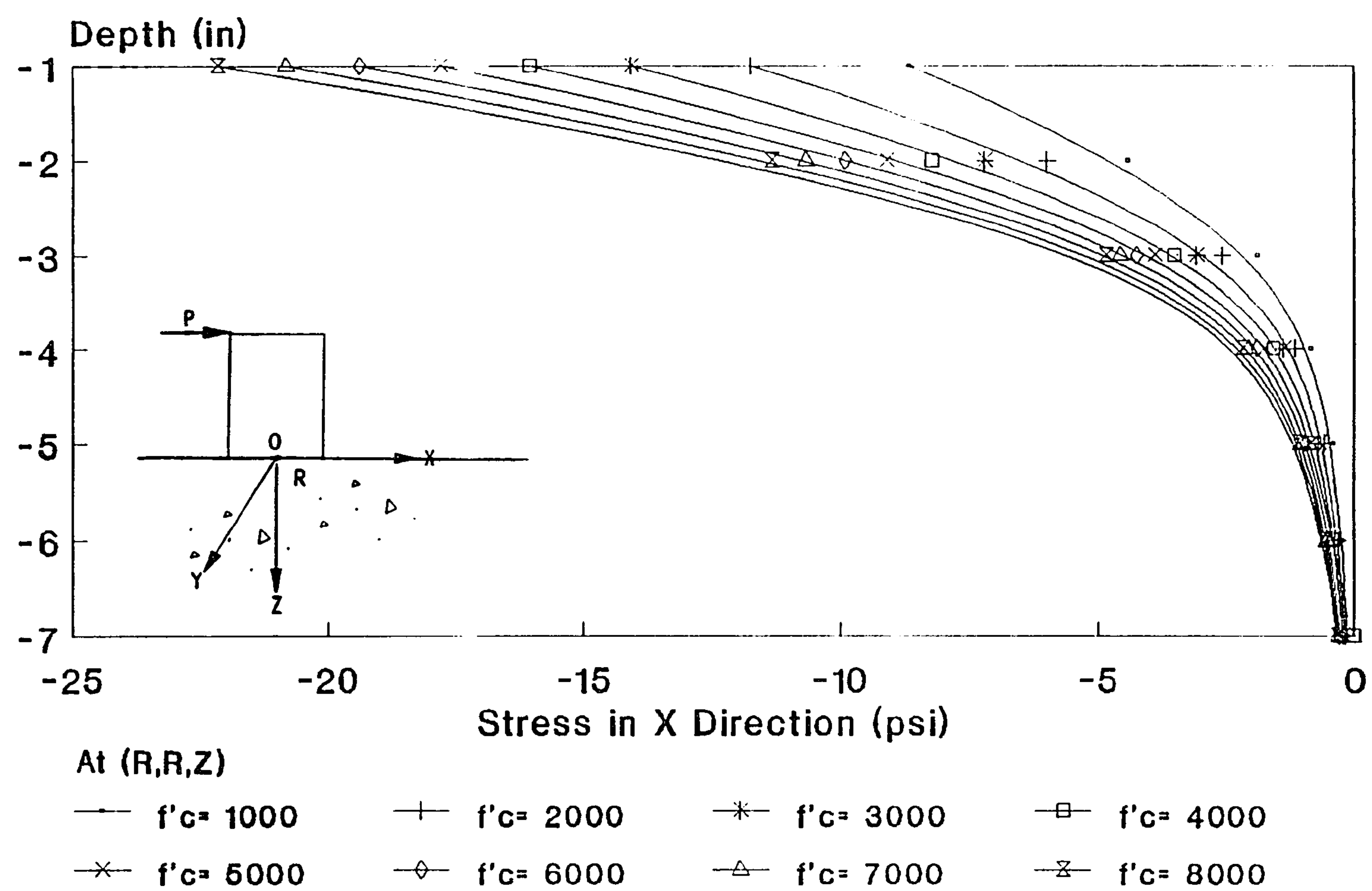


Figure 58 Distribution of Stress σ_{xx} Along Slab Depth

used for this purpose, and the computer program used for this is given in Appendix A. Figures 58 to 60 show the stresses in X,Y and Z direction along an axis parallel to the Z axis and going thorough the point (1.08",1.08",0.0"). Stresses at other locations are given in Appendix B.

5.6 Capacity of Unreinforced Concrete Deep Beams

5.6.1 Introduction

Leonhart and Walther (141), have obtained the stress distributions on beams with various support conditions and loaded with uniform and point loads. The stress distributions are available for beams with different length to depth ratios. Using numerical integration techniques and these stress distributions, the capacity of these beams were computed for various compressive strengths of concrete. These capacities are compared with the values one could obtain using the conventional equations assuming linear stress distributions. The computer programs used are given in Appendix A.

It was found that the capacities obtained using the actual stress distributions and numerical integration techniques differ significantly with those obtained with conventional equations. Therefore, to realistically predict the capacity of the deep beams considered in this study, one has to use the actual stress distributions and numerical integration

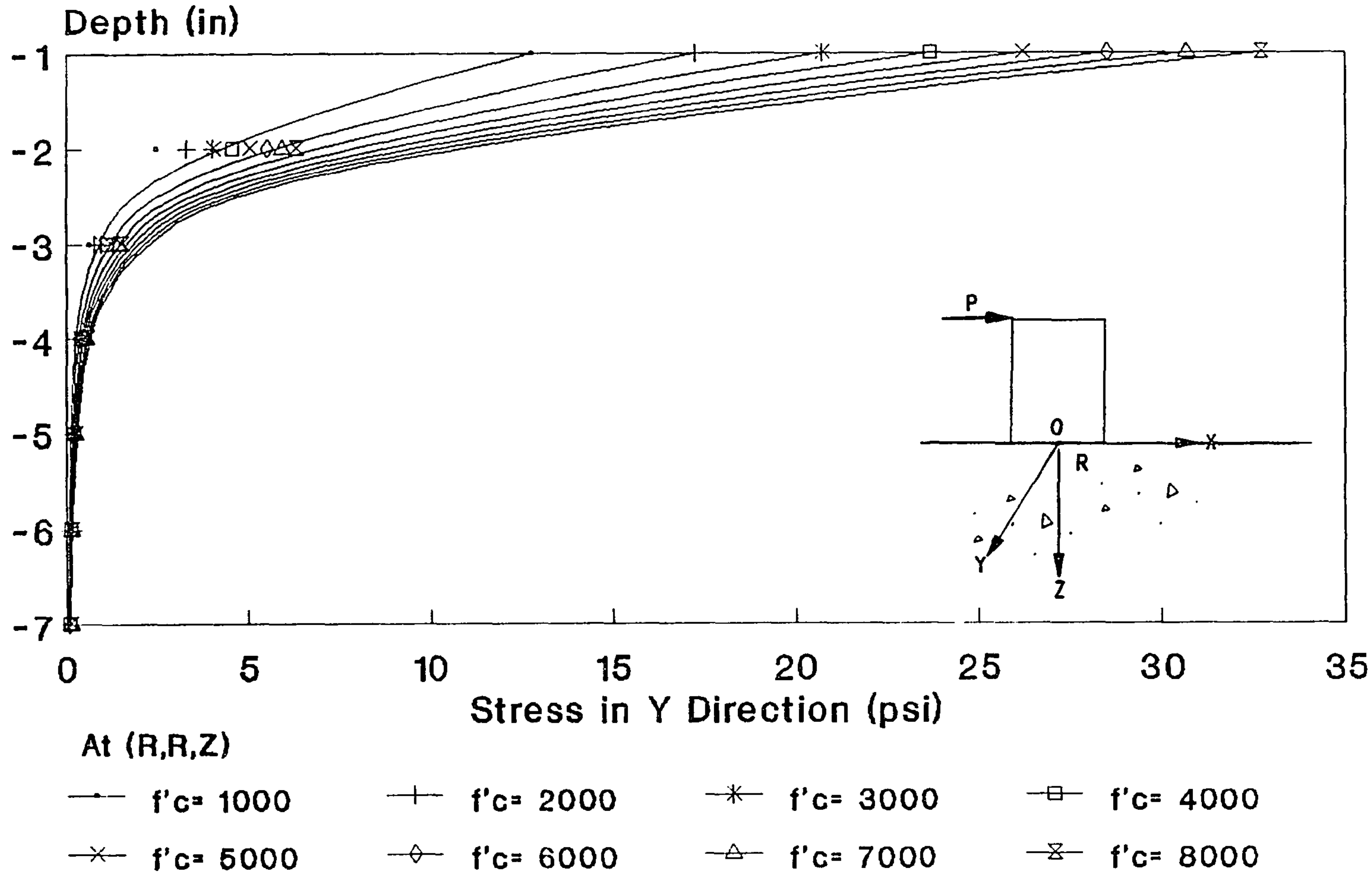


Figure 59 Distribution of Stress σ_{yy} Along Slab Depth

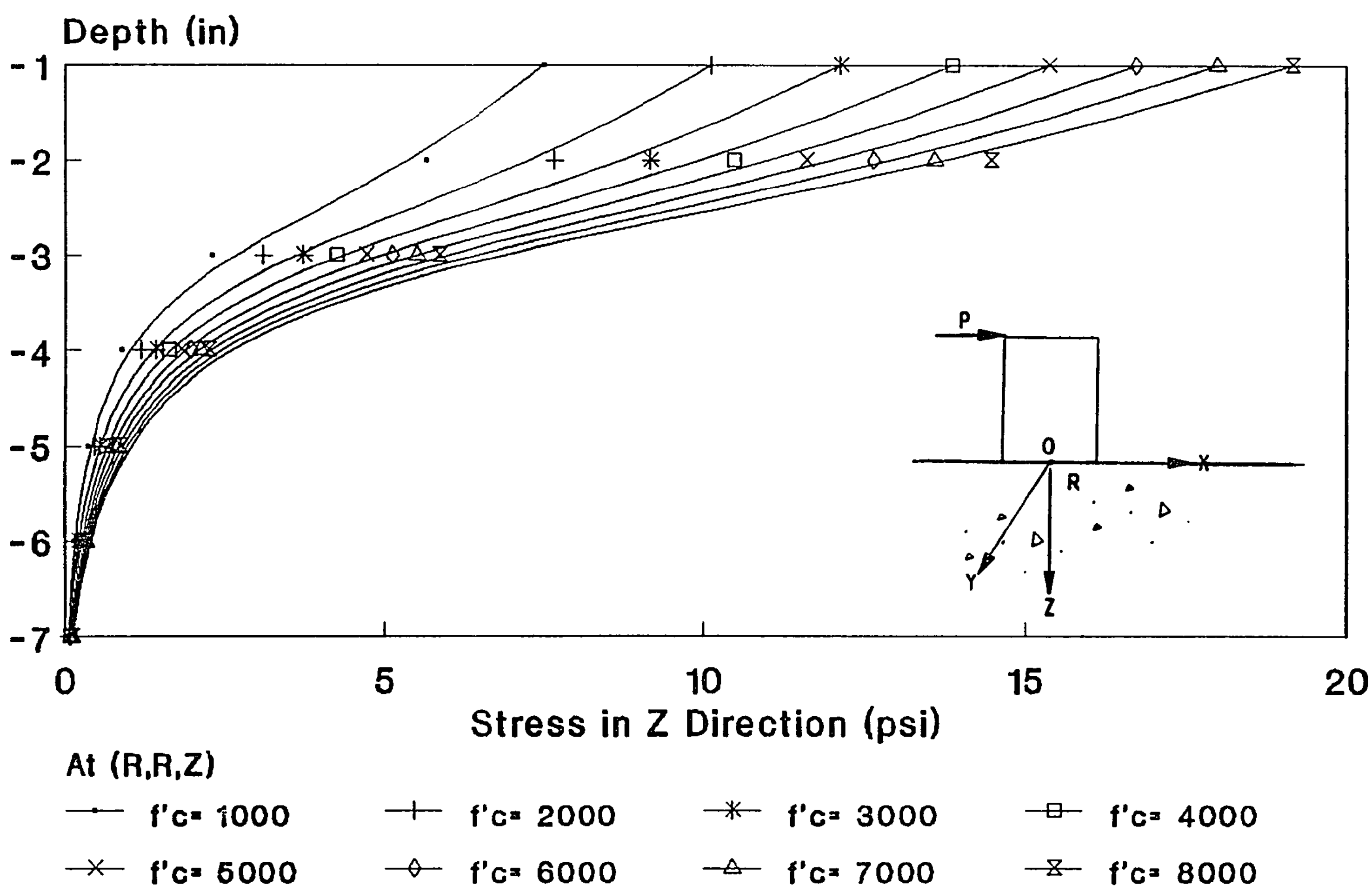


Figure 60 Distribution of Stress σ_{zz} Along Slab Depth

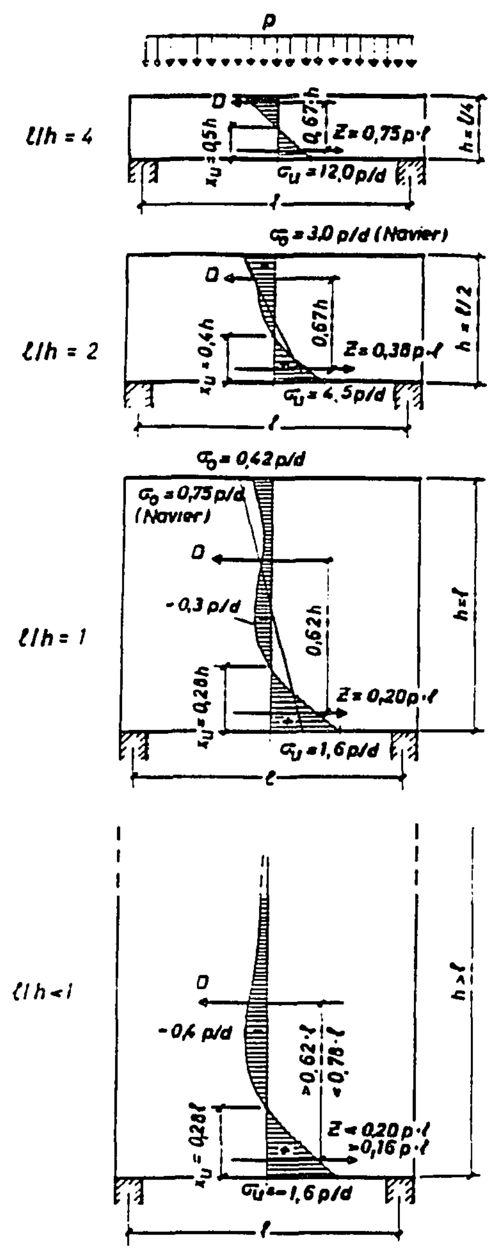


Figure 61 Stress Distribution of Deep Beams Loaded with a Uniform Load

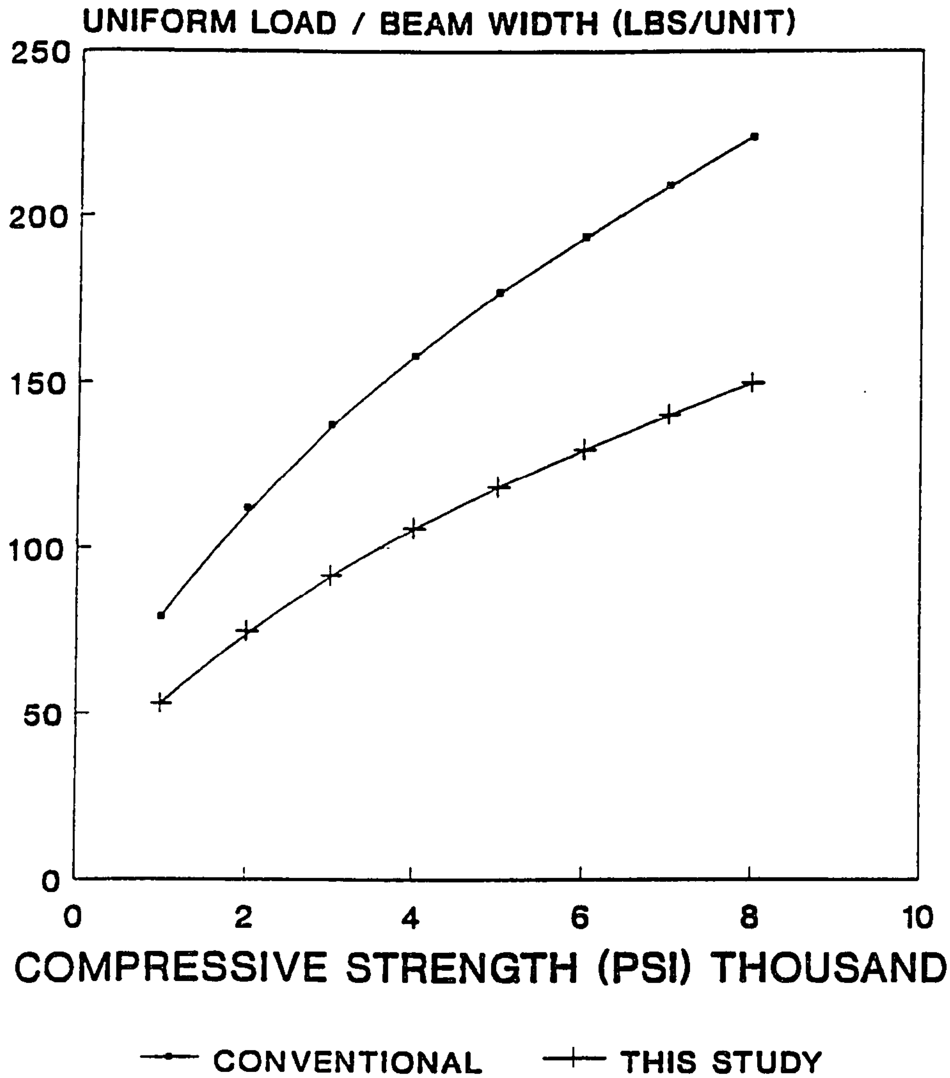
techniques. This is very cumbersome and inorder to help the Practicing Engineers, the capacities are given in the form of design charts in Appendix C.

5.6.2 Simply Supported and Cantilevered Beams with Uniform Load

Figure 61, shows the stress distribution obtained by Leonhart and Walther (141) for beams loaded with a uniform load. Figure 62 gives the capacity (i.e. uniform load/ width of beam) for various compressive strengths of concrete. The length to depth ratio of the beam is 2.0 with a rectangular cross section. The capacities computed with conventional equations are also shown. It is seen that capacities calculated based on conventional equations are overly conservative. Design charts for other aspect ratios and circular cross sections are given in Appendix C.

5.6.3 Simply Supported and Cantilevered Beams with Point Loads

Figure 63 shows the stress distribution obtained by Leonhart and Walther (141). The design curves for both rectangular and circular sections with various aspect ratios are given in Appendix C. It should be noted that the design curve given for simply supported beams with an aspect ratio 1.0, is of academic interest only, since there is arch action taking place in such beams.



ASPECT RATIO = 2.0

Figure 62 Capacity of Deep Beams - Rectangular Section

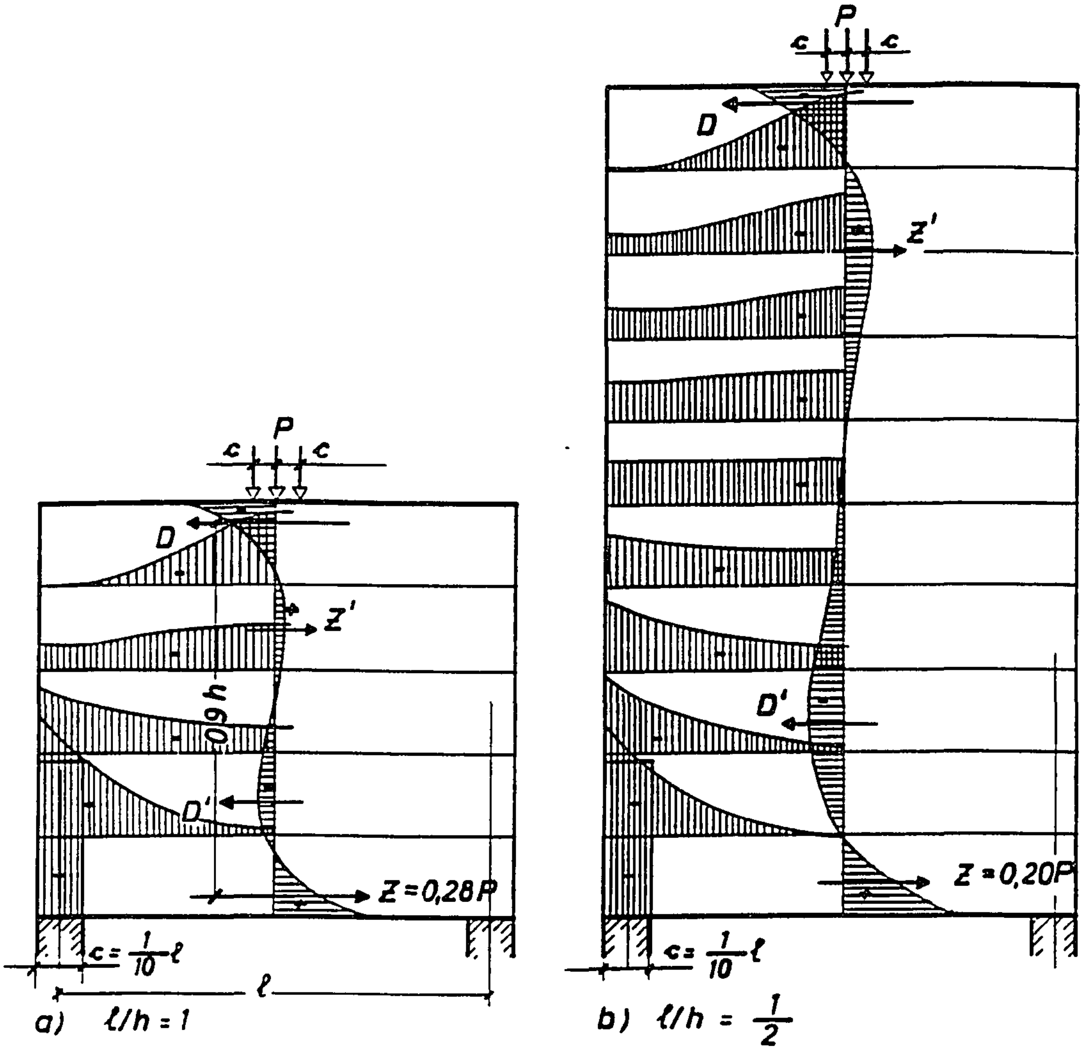


Figure 63 Stress Distribution of Deep Beams Loaded with a Point Load

CHAPTER 6

CONCLUSIONS AND SUGGESTIONS

6.1 Conclusions

1. It is seen that the relationship between the compressive strength of concrete and the theoretical break off manometer reading agrees well with the results obtained by the manufacturer of the break off tester and other researchers. Further, for the cylindrical cantilevered specimens, theoretical and experimental relations obtained between the compressive strength of concrete and breaking force closely agree. It is seen that there is a definite theoretical relationship between the compressive strength of concrete and the break off value of the break off test method. All three approaches used in this study ; the fracture mechanics approach, approximate method and finite element method reinforced this conclusion. This should not only make this test method more credible, but also install confidence in the mind of the practicing Engineer.

2. Based on the fracture mechanics approach, it was found that for concrete with maximum size aggregates up to 1/2", aggregate

interlocking has no significant effect on the relationship between concrete strength and break off manometer reading.

3. From the finite element analysis, it is seen that the effect of slab thickness on the break off test results is insignificant for slabs thicker than 5 inches. It is interesting to note that other researchers also have arrived at the same conclusion from their experiments.

4. As concluded earlier the stress distribution at the fixed end of the break off specimen plays a significant role in predicting the compressive strength of a concrete. Some structural members are subject to large prestress values either due to loads or residual stresses such as creep and shrinkage. Break off test specimens made on these members may have stress distributions at the fixed end that can give very large or very small break off values. These in turn can result in erroneous compressive strengths. Neville (143) has reported the occurrence of a 1400 psi stress due to differential shrinkage in a 6 inch, mortar slab after 200 days. Therefore, it is essential that one avoids highly stressed regions of structural members when performing the break off test ensuring as much as possible that the specimens will fail only due to the force applied by the break off tester at the free end.

5. The break off tester has been calibrated for normal strength concrete. The theoretical relationship introduced in this study was developed for normal concrete. For a given compressive strength, Polymer Impregnated Concrete and Fiber Reinforced Concrete have different Modulus of Rupture values from normal concrete. Hence, the theoretical relationship between the compressive strength of concrete and the break off manometer reading developed in this study or the experimental correlations obtained by the manufacturer of the break off tester and other researchers may not be valid for Polymer Impregnated Concrete and Fiber Reinforced concrete. Therefore, the use of the break off tester to ascertain the compressive strength of any concrete other than normal concrete is not recommended.

6. The current American Concrete Institute method of testing the modulus of rupture may be inadequate for structural elements with cross sectional dimensions smaller than six inches. The break off tester can be used to determine a new modulus of rupture, for rectangular concrete beams with depths and widths smaller than six inches, and members with circular cross sections.

7. Due to the inherent size of the test specimens, the break off test is normally recommended for concrete with a maximum size aggregate of

10 mm. Since there is a good theoretical basis for the break off test as evident in this study, the test apparatus and the specimen size can easily be modified to test concrete with larger maximum size aggregates.

8. The design charts given in this study may be used to obtain the breaking point load or uniformly distributed load for a given support condition, length to depth ratio of a beam, and compressive strength of concrete. Charts are provided for beams with both rectangular and circular cross sections.

6.2 Suggestions

1. The use of break off tester to ascertain the compressive strength of fiber reinforced concrete should be investigated.

2. The use of break off tester to ascertain the compressive strength of polymer impregnated concrete should be investigated.

3. The nature of the break off test is very favorable to be used in the testing of rock. This will be very useful in Rock Mechanics.

4. The effect of residual stresses of concrete on the break off test results should be investigated.

5. The break off tester should be modified to test concrete with maximum aggregates larger than 10 mm.

APPENDIX A

COMPUTER PROGRAMS

The following FORTRAN programs were used in this study.

Program 1 was written to obtain a relationship between compressive strength of concrete and the break off manometer reading, using fracture mechanics and the flexural model.

Program 2 was written to obtain a relationship between compressive strength of concrete and the break off manometer reading, using fracture mechanics and the shear model.

Program 3 was written to obtain a relationship between compressive strength of concrete and the break off manometer reading, using the stress distribution obtained from finite elements.

Program 4 was written to obtain the stresses in the vicinity of the break off specimen.

Programs 5 through 15 were written to obtain the breaking force of unreinforced concrete beams with various strengths, aspect ratios, support conditions, load types and cross sections.

APPENDIX A
(Continued)

A.1 Program 1

```
C     PROGRAM FOR THE DEVELOPMENT OF A THEORETICAL
C     EQUATION
C     FOR THE BREAK OFF TESTER
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      R = 1.083
      D = 2.0*R
      ZZ = -6.0
      DO 500 L=1,23
      B = 10.0**ZZ
      WRITE(6,20) L,B
   20 FORMAT(/,2X,'TABLE',I4,3X,'B=',F16.7,/,5X,'A/D',
     %4X,'MOMENT',/)
      E=0.0
      DO 350 M=1,9
      A=E*D
      CALL LIMITS(T1,T2,D,A,B)
      EPS=0.0000001
      CALL BISE(T1,T2,T0,D,A,B,EPS)
```

APPENDIX A
(Continued)

```
      S = D-T0-A
      F = T0/S
      E = A/D
      Q = D-T0
      H1 = T0/100.0
      H2 = S/100.0
C
C     CALCULATE M1
C
      SUM4=F4(Q,A,T0,F)+F4(D,A,T0,F)
      DO 90 JJ=1,99
       SUM4=SUM4+2.0*F4(Q+DFLOAT(JJ)*H1,A,T0,F)
   90  CONTINUE
      OM1=H1*SUM4/2.0
C
C     CALCULATE M2
C
      SUM5=F5(A,A,T0,S,F)+F5(A+S),A,T0,S,F)
      DO 100 KK=1,99
      SUM5=SUM5+2.0*F5(A+DFLOAT(KK)*H2,A,T0,S,F)
```

APPENDIX A
(Continued)

```
 100  CONTINUE
      OM2=H2*SUM5/2.0
C
C     CALCULATE M3
C
      IF(1.0-2.0*E*B .LE. 0.0 )THEN
       C= 2.0*E*B*(1.0+E*F)/(1.0-E)
      ELSE
       C=2.0*(E**2)*B*(1.0+F)/((1.0-E)*(1.0-2.0*E*B))
       IF(C.GT.1.0) THEN
         C = 2.0*E*B*(1.0+E*F)/(1.0-E)
       END IF
      END IF
      CC=0.0
      IF(C.GT.1.0) CC=A-A/C
      SUM6=F6(CC,A,T0,S,F,C)+F6(A,A,T0,S,F,C)
      DO 200 NN=1,99
      SUM6=SUM6+2.0*F6(CC+DFLOAT(NN)*H3,A,T0,S,F,C)
 200  CONTINUE
      H3=(A-CC)/100.0
```

APPENDIX A
(Continued)

```
      OM3=H3*SUM6/2.0
C
C     CALCULATE MOMENT
C
      OM=OM1-OM2-OM3
      OUM=OM/(D**3)
      WRITE(6,300) E,OUM
 300  FORMAT(/2X,2F12.7)
      E=E+0.1
 350  CONTINUE
C      WRITE(6,400) TO
 400  FORMAT(/,2X,F11.7)
      ZZ=-6.0+DEFLOAT(L)/2.0
 500  CONTINUE
      STOP
      END

      SUBROUTINE LIMITS(T1,T2,D,A,B)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
       T1 = 0.01*D
```

APPENDIX A
(Continued)

```
      E1 = EQUIL(T1,D,A,B)
      DO 520 I=1,1000
        T2 = T1 + I*0.98*D/1000.0
        E2 = EQUIL(T2,D,A,B)
        IF(E1*E2 .LT. 0.) GOTO 530
520   CONTINUE
530   CONTINUE
      RETURN
      END

      SUBROUTINE BISE(T1,T2,T0,D,A,B,EPS)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      E1 = EQUIL(T1,D,A,B)
      E2 = EQUIL(T2,D,A,B)
      IF( E1*E2 .GT. 0.0 ) THEN
      WRITE(6,*) "Starting value incorrect"
      WRITE(6,*) "el,e2=",e1,e2
      END IF
      DELTA = T2-T1
```

APPENDIX A
(Continued)

```
      DO 1200 WHILE (DELTA .GT. EPS)
      DELTA = (T2-T1)/2.0
       T3 = T1+DELTA
       E3 = EQUIL(T3,D,A,B)
       IF( E1*E3 .GT. 0.00) THEN
         T1 =T3
       ELSE
         T2 = T3
       END IF
1200   CONTINUE
       T0 = T1+DELTA/2.0
      RETURN
      END

      FUNCTION PC(T,D,A)
C
C       CALCULATE PC
C
      implicit double precision (a-h,o-z)
       H1=T/100.0
```

APPENDIX A
(Continued)

```
      F = T/(D-T-A)
      SUM1=F1(D-T,A,T,F)+F1(D,A,T,F)
      DO 50 J=1,99
      SUM1=SUM1+2.0*F1(D-T+DFLOAT(J)*H1,A,T,F)
 50   CONTINUE
      PC= H1*SUM1/2.0
      RETURN
      END
C
C      CALCULATE PT1
C
      FUNCTION PT1(T,D,A)
      implicit double precision (a-h,o-z)
      S = D-T-A
      F = T/(D-T-A)
      H2=S/100.0
      SUM2=F2(A,A,T,S,F)+F2(A+S),A,T,S,F)
      DO 60 K=1,99
      SUM2=SUM2+2.0*F2(A+DFLOAT(K)*H2,A,T,S,F)
 60   CONTINUE
```

APPENDIX A
(Continued)

```
      PT1=H2*SUM2/2.0
      RETURN
      END
C
C      CALCULATE PT2
C
      FUCTION PT2(T,D,A,B)
      implicit double precision (a-h,o-z)
      E=A/D
      F = T/(D-T-A)
      S = D-T-A
      IF(1.0-2.0*E*B .LE. 0.0 )THEN
        C= 2.0*E*B*(1.0+E*F)/(1.0-E)
      ELSE
        C=2.0*(E**2)*B*(1.0+F)/((1.0-E)*(1.0-2.0*E*B))
        IF(C.GT.1.0) THEN
          C = 2.0*E*B*(1.0+E*F)/(1.0-E)
        END IF
      END IF
      BB = 0.0
```

APPENDIX A
(Continued)

```
      IF (C.GT.1.0) BB = A - A/C
      H3=(A-BB)/100.0
      SUM3=F3(BB,A,T,S,F,C)+F3(A,A,T,S,F,C)
      DO 70 N=1,99
        SUM3=SUM3+2.0*F3(BB+DFLOAT(N)*H3,A,T,S,F,C)
 70     CONTINUE
      PT2=H3*SUM3/2.0
      RETURN
      END
C
C      CALCULATE PT
C
      FUNCTION PT(T,D,A,B)
      implicit double precision (a-h,o-z)
        PT = PT1(T,D,A) + PT2(T,D,A,B)
      RETURN
      END
      FUNCTION EQUIL(T,D,A,B)
      implicit double precison (a-h,o-z)
        EQUIL = PC(T,D,A) - PT(T,D,A,B)
```

APPENDIX A
(Continued)

```
RETURN
END
FUNCTION F1(Y,A,T,F)
implicit double precision (a-h,o-z)
R=1.083
F1=F*2.0*(DSQRT(R*R-(Y-R)**2))*(T-2.0*R+Y)/T
END
FUNCTION F2(X,A,T,S,F)
implicit double precision (a-h,o-z)
R=1.083
F2=1.0*2.0*(DSQRT(R*R-(X-R)**2))*(2.0*R-T-X)/S
END
FUNCTION F3(Z,A,T,S,F,C)
implicit double precision (a-h,o-z)
R=1.083
F3=0.0
IF(Z.LE.0.0) THEN
  F3 = 0.0
ELSE
   IF(A.NE.0.0) F3=2.0*(DSQRT(R*R-(Z-R)**2))
```

APPENDIX A
(Continued)

```
%    *((Z*C/A)+1.0-C)
 END IF
 END
 FUNCTION F4(U,A,T,F)
 implicit double precision (a-h,o-z)
 R=1.083
 F4=F*2.0*(DSQRT(R*R-(U-R)**2))*(T-2.0*R+U)*U/T
 END
 FUNCTION F5(V,A,T,S,F)
 implicit double precision (a-h,o-z)
 R=1.083
 F5=1.0*2.0*(DSQRT(R*R-(V-R)**2))*(2.0*R-T-V)*V/S
 END
 FUNCTION F6(W,A,T,S,F,C)
 implicit double precision (a-h,o-z)
 R=1.083
 F6=0.0
 IF(A.NE.O.) F6=2.0*(DSQRT(R*R-(W-R)**2))
%*((W*C/A+1.0-C)
 END
```

APPENDIX A
(Continued)

A.2 Program 2

```
C     PROGRAM FOR THE DEVELOPMENT OF A THEORETICAL
C     EQUATION FOR THE BREAK OFF TESTER (SHEAR CONDITION)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      R = 1.083
      D = 2.0*R
      ZZ = -6.0
      DO 500 L=1,23
        B = 10.0**ZZ
        WRITE(6,20) L,B
   20   FORMAT(/,2X,'TABLE',I4,3X,'B=',F16.7,/,5X,'A/D',
     %4X,'MOMENT',/)
      E=0.0
      DO 350 M=1,9
      A=E*D
      CALL LIMITS(T1,T2,D,A,B)
      EPS=0.0000001
      CALL BISE(T1,T2,T0,D,A,B,EPS)
      S = D-T0-A
```

APPENDIX A
(Continued)

```
      F = TO/S

      E = A/D

      Q = D-TO

      SS = (1.86*R+A)/2.0

         H1 = 0.14*R/100.0

         H2 = (1.86*R-A)/200.0


         IF (E.LT.O.14) THEN

           A = 0.14*R

         ELSE

           A = E*R

      END IF
C
C     CALCULATE P1
C
      SUM4=F4(1.86*R,A,TO,F)+F4(D,A,TO,F)

      DO 90 JJ=1,99

       SUM4=SUM4+2.0*F4(1.86*R+DFLOAT(JJ)*H1,A,TO,F)

  90   CONTINUE

      P1=H1*SUM4/2.0
```

APPENDIX A
(Continued)

```
C
C    CALCULATE P2
C
      SUM5=F5(SS,A,TO,S,F)+F5(1.86*R,A,TO,S,F)
      DO 100 KK=1,99
      SUM5=SUM5+2.0*F5(SS+DFLOAT(KK)*H2,A,TO,S,F)
 100  CONTINUE
      PS=HS*SUM5/2.0
C
C    CALCULATE P3
C
      SUM6=F6(A,A,TO,S,F)+F6(SS,A,TO,S,F)
      DO 250 KK=1,99
      SUM6=SUM6+2.0*F6(A+DFLOAT(KK)*H2,A,TO,S,F)
      250 CONTINUE
      P3=H2*SUM6/2.0

C
C    CALCULATE P4
C
```

APPENDIX A
(Continued)

```
   IF(1.0-2.0*E*B .LE. 0.0 )THEN
  C=E*B*(1.0+E*F)/(1.0-E)
 ELSE
  C=(E**2)*B*(1.0+F)/((1.0-E)*(1.0-2.0*E*B))
  IF(C.GT.1.0) THEN
    C = E*B*(1.0+E*F)/(1.0-E)
  END IF
 END IF
 CC=0.0
 IF(C.GT.1.0) CC=A-A/C
   H3 = (A-CC)/100.0
   SUM7=F7(CC,A,T0,S,F,C,CC)+F7(A,A,T0,S,F,C,CC)
 DO 200 NN=1,99
 SUM7=SUM7+2.0*F7(CC+DFLOAT(NN)*H3,A,T0,S,F,C)
200 CONTINUE
 P4=H4*SUM7/2.0
C
C   CALCULATE FORCE
C
 P=P1+P2+P3+P4
```

APPENDIX A
(Continued)

```
      PN=P/(D**2)
      WRITE(6,300) E,PN
 300  FORMAT(/2X,2F12.7)
      E=E+0.1
 350  CONTINUE
C      WRITE(6,400) T0
 400  FORMAT(/,2X,F11.7)
      ZZ=-6.0+DEFLOAT(L)/2.0
 500  CONTINUE
      STOP
      END


      SUBROUTINE LIMITS(T1,T2,D,A,B)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
       T1 = 0.01*D
       E1 = EQUIL(T1,D,A,B)
       DO 520 I=1,1000
          T2 = T1 + I*0.98*D/1000.0
          E2 = EQUIL(T2,D,A,B)
          IF(E1*E2 .LT. 0.) GOTO 530
```

APPENDIX A
(Continued)

```
520       CONTINUE

530       CONTINUE

      RETURN

      END


      SUBROUTINE BISE(T1,T2,T0,D,A,B,EPS)

      IMPLICIT DOUBLE PRECISION(A-H,O-Z)

      E1 = EQUIL(T1,D,A,B)

      E2 = EQUIL(T2,D,A,B)

      IF( E1*E2 .GT. 0.0 ) THEN

       WRITE(6,*) "Starting value incorrect"

       WRITE(6,*) "el,e2=",e1,e2

      END IF

      DELTA = T2-T1

      DO 1200 WHILE (DELTA .GT. EPS)

       DELTA = (T2-T1)/2.0

       T3 = T1+DELTA

       E3 = EQUIL(T3,D,A,B)

       IF( E1*E3 .GT. 0.00) THEN

         T1 =T3
```

APPENDIX A
(Continued)

```
      ELSE
       T2 = T3
      END IF
1200  CONTINUE
      T0 = T1+DELTA/2.0
      RETURN
      END

      FUNCTION PC(T,D,A)
C
C      CALCULATE PC
C
      implicit double precision (a-h,o-z)
      H1=T/100.0
      F = T/(D-T-A)
      SUM1=F1(D-T,A,T,F)+F1(D,A,T,F)
      DO 50 J=1,99
      SUM1=SUM1+2.0*F21(D-T+DFLOAT(J)*H1,A,T,F)
  50  CONTINUE
      PC= H1*SUM1/2.0
```

APPENDIX A
(Continued)

```
      RETURN
      END
C
C      CALCULATE PT1
C
      FUNCTION PT1(T,D,A)
      implicit double precision (a-h,o-z)
      S = D-T-A
      F = T/(D-T-A)
      H2=S/100.0
      SUM2=F2(A,A,T,S,F)+F2(A+S),A,T,S,F)
      DO 60 K=1,99
      SUM2=SUM2+2.0*F2(A+DFLOAT(K)*H2,A,T,S,F)
 60   CONTINUE
      PT1=H2*SUM2/2.0
      RETURN
      END
C
C      CALCULATE PT2
C
```

APPENDIX A
(Continued)

```
      FUNCTION PT2(T,D,A,B)
      implicit double precision (a-h,o-z)
      E=A/D
      F = T/(D-T-A)
      S = D-T-A
      IF(1.0-2.0*E*B .LE. 0.0 )THEN
        C= E*B*(1.0+E*F)/(1.0-E)
      ELSE
        C=(E**2)*B*(1.0+F)/((1.0-E)*(1.0-2.0*E*B))
        IF(C.GT.1.0) THEN
          C = E*B*(1.0+E*F)/(1.0-E)
        END IF
      END IF
      BB = 0.0
      IF (C.GT.1.0) BB = A - A/C
      H3=(A-BB)/100.0
      SUM3=F3(BB,A,T,S,F,C)+F3(A,A,T,S,F,C)
      DO 70 N=1,99
        SUM3=SUM3+2.0*F3(BB+DFLOAT(N)*H3,A,T,S,F,C)
70    CONTINUE
```

APPENDIX A
(Continued)

```
      PT2=H3*SUM3/2.0
      RETURN
      END
C
C      CALCULATE PT
C
      FUNCTION PT(T,D,A,B)
      implicit double precision (a-h,o-z)
        PT = PT1(T,D,A) + PT2(T,D,A,B)
      RETURN
      END
      FUNCTION EQUIL(T,D,A,B)
      implicit double precison (a-h,o-z)
        EQUIL = PC(T,D,A) - PT(T,D,A,B)
      RETURN
      END2
      FUNCTION F1(Y,A,T,F)
      implicit double precision (a-h,o-z)
      R=1.083
          F1=F*2.0*(DSQRT(R*R-(Y-R)**2))*(T-2.0*R+Y)/T
```

APPENDIX A
(Continued)

```
END
FUNCTION F2(X,A,T,S,F)
implicit double precision (a-h,o-z)
R=1.083
   F2=1.0*2.0*(DSQRT(R*R-(X-R)**2))*(2.0*R-T-X)/S
END
FUNCTION F3(Z,A,T,S,F,C)
implicit double precision (a-h,o-z)
R=1.083
F3=0.0
   IF(Z.LE.0.0) THEN
 F3 = 0.0
ELSE
  IF(A.NE.0.0) F3=2.0*(DSQRT(R*R-(Z-R)**2))
%   *((Z*C/A)+1.0-C)
END IF
END
FUNCTION F4(U,A,T,F)
implicit double precision (a-h,o-z)
R=1.083
```

APPENDIX A
(Continued)

```
  F4=14.286*(2.0*R-U)*(DSQRT(R*R-(U-R)**2))/R
     END
  FUNCTION F5(V,A,T,S,F)
  implicit double precision (a-h,o-z)
  R=1.083
  F5=2.0*(0.96*V+0.0744*R-A)*(DSQRT(R*R-(V-
%  R)**2))/(1.86*R-A)
  END
  FUNCTION F6(W,A,TO,S,F)
  implicit double precision (a-h,o-z)
  R=1.083
  F6=2.0*(0.96*W+0.04*A-1.86*R)*(DSQRT(R*R-(W-R)**2))
  /(A-1.86*R)
  END
     FUNCTION F7(XX,A,C,CC)
     implicit double precision (a-h,o-z)
     R=1.083
     F7=2.0*(XX-CC)*(DSQRT(R*R-(XX-R)**2))/(A/C)
     END
```

APPENDIX A
(Continued)

A.3 Program 3

```
C     PROGRAM FOR BREAK OFF TESTER
C     PROGRAM MAIN
      REAL M,MT,MT1
      OPEN(UNIT=10,FILE="P4.OUT")
      R=1.083
      DO 2000 FC=1000.0,9000.0,1000.0
        FR=12.852*SQRT(FC) +70.0
C
C      CALCULATE MT
C
          H1=R/50.0
          SUM1=F1(FLOAT(0),R,FR)+F1(R,R,FR)
          DO 50 J=1,49
            SUM1=SUM1+2.0*F1(FLOAT(J)*H1,R,FR)
50          CONTINUE
          MT=H1*SUM1/2.0
C
C       CALCULATE P
```

APPENDIX A
(Continued)

```
C
         M=2.0*MT
         P=M/2.46
         BO=P/3.81=2.973
         WRITE(10,4000)FC,P,BO
4000       FORMAT(5X,F10.4,2X,F10.4,2X,F10.4)
2000      CONTINUE
        STOP
        END


        REAL FUNCTION F1(Y,R,FR)
        REAL Y,R,FR
        F1=0.86*FR*Y*Y*(.039*(Y**4)-1.099*(Y**3)+1.78*
     %   (Y**2)-1.428*Y+1.786)*2.0*SQRT(R*R-Y*Y)/R
        RETURN
        END
```

APPENDIX A
(Continued)

A.4 Program 4

```
C      PROGRAM FOR STRESSES IN THE VICINITY OF
C      BREAK OFF TEST SPECIMEN
C
        R=1.083
        DO 2000 FC = 1000.0,9000.0,1000.0
        FR=12.852*SQRT(FC)+70.0
C
C       CALCULATE P
C
        H=R/50.0
        SUM=F(FLOAT(O),R,FR)+F(R,R,FR)
        DO 5 JJ=1,49
          SUM=SUM+2.0*F(FLOAT(JJ)*H,R,FR)
5       CONTINUE
        TM=H*SUM/2.0
        TM2=2.0*TM
        P=TM2/2.46
C
```

APPENDIX A
(Continued)

```
C
      DO 10 NN=1,3
      Y=(2.0-FLOAT(NN))*R
       DO 15 I=1,3
       X2=(2.0-FLOAT(I))*R
    Z=1.0
         DO 20 KK=1,10
      RR=SQRT((X2**2)+Y*Y+Z*Z)
C     RR=ABS(RR)
C     CALCULATE S1
      H1=R/50.0
      SUM1=F1(FLOAT(0),R,FR,X2,Y,Z)+F1(R,R,FR,X2,Y,Z)
      DO 50 J=1,49
      SUM1=SUM1+2.0*F1(FLOAT(J)*H1,R,FR,X2,Y,Z)
50    CONTINUE
      S11=H1*SUM1/2.0
      SUM2=F2(FLOAT(0),R,FR,X2,Y,Z)+F2(R,R,FR,X2,Y,Z)
      DO 60 K=1,49
      SUM2=SUM2+2.0*F2(FLOAT(K)*H1,R,FR,X2,Y,Z)
60    CONTINUE
```

APPENDIX A
(Continued)

```
      S22=H1*SUM2/2.0
      S1=S11+S22+P*X2*((-3.0*(X2)**2/(RR)**2)+0.6*(RR*RR-
   %   Y*Y-2*RR*Y*Y/(RR+Z))/(RR+Z)**2)/(2*3.142*(RR)**3)
C
C     CALCULATE S2
C
      H2=R/50.0
      SUM3=F3(FLOAT(0),R,FR,X2,Y,Z)+F3(R,R,FR,X2,Y,Z)
      DO 70 L-1,49
      SUM3=SUM3+2.0*F3(FLOAT(L)*H2,R,FR,X2,Y,Z)
70    CONTINUE
      S33=H2*SUM3/2.0
      SUM4=F4(FLOAT(0),R,FR,X2,Y,Z)+F4(R,R,FR,X2,Y,Z)
      DO 80 M=1,49
      SUM4=SUM4+2.0*F4(FLOAT(M)*H2,R,FR,X2,Y,Z)
80    CONTINUE
      S44=H2*SUM4/2.0
      S2=S33+S44+P*X2*((-3.0*(Y)**2)+0.6*(3.0*RR*RR-
   %   (X2)*(X2)2*RR*X2*X2/(RR+X))/(RR+Z)**2)/(2*3.142*
   %   (RR)**3)
```

APPENDIX A
(Continued)

```
C
C      CALCULATE S3
C
       H3=R/50.0
       SUM5=F5(FLOAT(0),R,FR,X2,Y,Z)+F5(R,R,FR,X2,Y,Z)
       DO 90 N=1,49
       SUM5=SUM5+2.0*F5(FLOAT(N)*H3,R,FR,X2,Y,Z)
90     CONTINUE
       S55=H3*SUM5/2.0
       SUM6=F6(FLOAT(0),R,FR,X2,Y,Z)+F6(R,R,FR,X2,Y,Z)
       DO 100 JJ =1,49
       SUM6=SUM6+2.0*F6(FLOAT(JJ)*H3,R,FR,X2,Y,Z)
100    CONTINUE
       S66=H3*SUM6/2.0
       S3=S55+S66-3.0*P*X2*Z*Z/(2.0*3.142*(RR)**5)
C
C
       WRITE(6,400)FC,X2,Y,Z,S1,S2,S3,P
400    FORMAT(/,2X,7F12.7)
C
```

APPENDIX A
(Continued)

```
C
      Z=Z+1.0
20    CONTINUE
15    CONTINUE
10    CONTINUE
2000  CONTINUE
      STOP
      END
C
C
C
      REAL FUNCTION F(W,R,FR)
      F=0.86*FR*W*W*(0.039*(W**4)-
     %  1.099*(W**3)+1.78*(W**2)-
     %   1.428*W+1.786)*2.0*SQRT(R*R-
     %   W*W)/R
       RETURN
       END
C
C
```

APPENDIX A
(Continued)

```
      REAL FUNCTION F1(X,R,FR,X2,Y,Z)
      X3=X2-X
      R1=SQRT(X3*X3+Y*Y+Z*Z)
      R2=SQRT(X3*X3+Y*Y)
      CB=Z/R1
      SB=R2/R1
      SW=X3/R2
      F1=3.0*F(X,R,FR)*(CB**2)*((X3**2)*(CB)/(R1**2)-
     % 0.2*((2.0+CB)*(((X3/R1/(1.0+CB))**2)-1.0/(1.0+CB)
     % +CB))/(2.0*3.142*Z*Z)
      RETURN
      END
C
C
      REAL FUNCTION F2(X,R,FR,X2,Y,Z)
      X3=X2+X
      R1=SQRT(X3*X3+Y*Y+Z*Z)
      R2=SQRT(X3*X3+Y*Y)
      CB=Z/R1
      SB=R2/R1
```

APPENDIX A
(Continued)

```
      SW=X3/R2
      F2=-3.0*F(X,R,FR)*(CB**2)*((X3**2)*(CB)/(R1**2)-
   %   0.2*((2.0+CB)*(((X3/R1)/(1.0+CB))**2)-1.0/(1.0+CB)
   %   +CB))/(2.0*3.142*Z*Z)
      RETURN
      END
C
C
      REAL FUNCTION F3(X,R,FR,X2,Y,Z)
      X3=X2-X
      R1=SQRT(X3*X3+Y*Y+Z*Z)
      R2=SQRT(X3*X3+Y*Y)
      CB=Z/RQ
      SB=R2/R1
      SW=X3/R2
      F3=3.0*F(X,R,FR)*(CB**2)*((Y**2)*(CB)/R1**2)-0.2*
   %   ((2.0+CB)*(((X3/R1)/(1.0+CB))**2)-
   %   1.0/(1.0+CB)+CB))/(2.0*3.142*Z*Z)
      RETURN
      END
```

APPENDIX A
(Continued)

```
C
C
      REAL FUNCTION F4(X,R,FR,X2,Y,Z)
      X3=X2+X
      R1=SQRT(X3*X3+Y*Y+Z*Z)
      R2=SQRT(X3*X3+Y*Y)
      CB=Z/R1
      SB=R2/R1
      SW=X3/R2
      F4=-3.0*F(X,R,FR)*(CB**2)*((Y**2)*(CB)/(R1**2)-
    %  0.2*((2.0+CB)*(((X3/R1)/(1.0+CB))**2)-
    %  1.0/(1.0+CB)+CB))/(2.0*3.142*Z*Z)
      RETURN
      END
C
C
      REAL FUNCTION F5(X,R,FR,X2,Y,Z)
      X3=X2-X
      R1=SQRT(X3*X3+Y*Y+Z*Z)
      CB=Z/R1
```

APPENDIX A
(Continued)

```
      F5=3.0*F(X,R,FR)*(CB**5)/(2.0*3.142*Z*Z)
      RETURN
      END
C
C
      REAL FUNCTION F6(X,R,FR,X2,Y,Z)
      X3=X2+X
      R1=SQRT(X3*X3+Y*Y+Z*Z)
      CB=Z/R1
      F6=-3.0*F(X,R,FR)*(CB**5)/(2.0*3.142*Z*Z)
      RETURN
      END
```

A.5 Program 5

```
C
C     PROGRAM FOR SIMPLY SUPPORTED BEAMS
C        ASPECT RATIO = 4.0, UNIFORMLY
C        DISTRIBUTED LOAD AS PER LINEAR
C        STRESS DISTRIBUTION
```

APPENDIX A
(Continued)

```
C
C
C
      REAL L,M,MT


      OPEN(UNIT=10, FILE = "P3.OUT")
      H = 0.0
      DO 1000 N=1,7
       H = 3.0 + H
       R = H/2.0
        DO 2000 FC = 1000.0, 8000.0, 1000.0
          FR = 7.5 * SQRT(FC)
          L = 4.0 *H
C
C     CALCULATE MT
C
           H1 = R/50.0
           SUM1 = F1(0,R,FR) + F1(R,R,FR)


           DO 50 J=1,49
```

APPENDIX A
(Continued)

```
          SUM1 = SUM1 +2.0*F1(FLOAT(J)*H1,R,FR)
50        CONTINUE
         MT = H1*SUM1/2.0
C
C    CALCULATE W
C
         M = 2.0*MT
         W = M * 8.0 / (L*L)
         WRITE(10,4000) H,FC,W
4000       FORMAT(5X,F10.4,2X,F10.4,2X,F10.4)

2000      CONTINUE
1000     CONTINUE
       STOP
       END

       FUNCTION F1(Y,R,FR)
          F1 = FR*Y*Y/R
       RETURN
       END
```

APPENDIX A
(Continued)

A.6 Program 6

```
C     PROGRAM FOR SIMPLY SUPPORTED BEAMS
C         ASPECT RATIO = 4.0, POINT LOAD AS PER LINEAR
C         STRESS DISTRIBUTION
C
              REAL  L,M,MT
              OPEN(UNIT=10, FILE = "P4.OUT")
              H  =  0.0
              DO   1000  N=1,7
               H = 3.0+H
               R = H/2.0
                  DO 2000 FC= 1000.0,8000.0,1000.0
                      FR = 7.5 * SQRT(FC)
                      L = 4.0 * H
C
C         CALCULATE MT
C
            H1 = R/50.0
            SUM1 = F1(0,R,FR) + F1(R,R,FR)
```

APPENDIX A
(Continued)

```
          DO 50 J=1,49
           SUM1 = SUM1 + 2.0*F1(FLOAT(J)*H1,R,FR)
50         CONTINUE
          MT = H1*SUM1/2.0
C
C     CALCULATE P
C
              M =2.0*MT
              P =4.0*M/L
              WRITE(10,4000) H,FC.P
4000          FORMAT(5X,F10.4,2X,F10.4,2X,F10.4)
2000    CONTINUE
1000    CONTINUE
        STOP
        END
        FUNCTION F1(Y,R,FR)
           F1= FR*Y*Y/R
        RETURN
        END
```

APPENDIX A
(Continued)

A.7 Program 7

```
C         PROGRAM FOR SIMPLY SUPPORTED BEAMS
C         ASPECT RATIO  =4.0, UNIFORMLY
C         DISTRIBUTED LOAD AS PER LINEAR STRESS
C        DISTRIBUTION,CIRCULAR SECTION
C
C
            REAL  L,M,MT
            OPEN(UNIT=10,FILE="P5.OUT)
            H=0.0
            DO 1000N=1,7
             H = 3.0+H
             R =H/2.0
             DO 2000 FC= 1000.0, 8000.0, 1000.0
              FR = 7.5*SQRT(FC)
              L = 4.0 * H
C
C         CALCULATE  MT
C
```

APPENDIX A
(Continued)

```
      H1=R/50.0
      SUM1=F1(0,R,FR)+F1(R,R,FR)
      DO 50 J=1,49
    SUM1 = SUM1+2.0*F1(FLOAT(J)*H1,R,FR)
50    CONTINUE
                 MT =H1*SUM1/2.0
C
C        CALCULATE W
                 M=2.0*MT
                 W=M*8.0/(L*L)
                 WRITE 910,4000)H,FC,W
4000             FORMAT(5X,F10.4,2X,F10.4,2X,F10.4)
2000     CONTINUE
1000     CONTINUE
       STOP
         END
         FUNCTION F1(Y,R,FR)
                 F1=2.0*SQRT(R*R-Y*Y)*FR*Y*Y/R
         RETURN
         END
```

APPENDIX A
(Continued)

A.8 Program 8

```
C         PROGRAM FOR SIMPLY SUPPORTED BEAMS
C          ASPECT RATIO=4.0, POINT LOAD AS PER LINEAR
C          STRESS DISTRIBUTION
C          CIRCULAR SECTION
C
           REAL L,M,MT
           OPEN(UNIT=10,FILE="P6.OUT")
           H=0.0
           DO 1000 N=1,7
             H=H+3.0
             R=H/2.0
             DO 2000 FC= 1000.0,8000.0,1000.0
                FR=1.08*7.5*SQRT(FC) + 70.0
                L=4.0*H
C
C         CALCULATE MT
C
                H1=R/50.0
```

APPENDIX A
(Continued)

```
              SUM1=F1(0,R,FR)+F1(R,R,FR)
              DO 50 J=1,49
                SUM1=SUM1+2.0*F1(FLOAT(J)*H1,R,FR)
50            CONTINUE
              MT=H1*SUM1/2.0
C
C       CALCULATE P
C
           M= 2.0*MT
           P=4.0*M/L
           WRITE (10,4000) H,FC,P
4000          FORMAT(5X,F10.4,2X,F10.4,2X,F10.4)
2000          CONTINUE
1000    CONTINUE
        STOP
        END
        FUNCTION F1(Y,R,FR)
              F1=2.0*SQRT(R*R-Y*Y)*FR*Y*Y/R
        RETURN
        END
```

APPENDIX A
(Continued)

A.9 Program 9

```
C      PROGRAM FOR SIMPLY SUPPORTED AND CANTILEVERED
C     BEAMS
C           ASPECT RATIO = 2.0, UNIFORMLY DISTRIBUTED LOAD
C       AS PER WALTHER et. al. STRESS DISTRIBUTION
C       CIRCULAR SECTION
C
      REAL L,M,MC,MT
      OPEN(UNIT=10, FILE = "P7.OUT")
      H = 0.0
      DO 1000 N=1,7
        H = 3.0 + H
        R = H/2.0
        DO 2000 FC = 1000.0, 8000.0, 1000.0
          FR = 7.5 * SQRT(FC)
          L = 2.0 * H
C
C     CALCULATE MT
C
```

APPENDIX A
(Continued)

```
                H1 = 0.8*R/50.0
          SUM1 = F1(0.2*R,R,FR) + F1(R,R,FR)
          DO 50 J=1,49
           SUM1 = SUM1 + 2.0*F1(0.2*R +FLOAT(J)*H1,R,FR)
50         CONTINUE
          MT = H1*SUM1/2.0
C
C    CALCULATE MC
C
          H2 = 1.2*R/100.0
                SUM2 = F2(-0.2*R,R,FR) + F2(R,R,FR)
          DO 60 J = 1,99
           SUM2 = SUM2 + 2.0*F2(-0.2*R+FLOAT(J)*H2,R,FR)
60         CONTINUE
          MC = H2*SUM2/2.0
C
C    CALCULATE W
C    M = MT +MC
      W = M * 8.0 / (L*L)
      WRITE(10,4000) H,FC,W
```

APPENDIX A
(Continued)

```
4000   FORMAT(5X,F10.4,2X,F10.4,2X,F10.4)
2000   CONTINUE
1000   CONTINUE
       STOP
          END
       FUNCTION F1(Y,R,FR)
        F1 = 2.0*SQRT(R*R-Y*Y)*FR*(Y-0.2*R)*(1.25*Y/R-0.25)
       RETURN
          END
          FUNCTION F2(Y,R,FR)
        Z = (5.0*Y +R)/(6.0*R)
        F2 = 2.0*SQRT(R*R-Y*Y)*ABS(Y)*0.67*FR*(-0.6211*(Z**3)+
     %   1.1925*(Z**2) + 0.4286*Z)
       RETURN
       END
```

APPENDIX A
(Continued)

A.10 Program 10

```
C     PROGRAM FOR SIMPLY SUPPORTED AND CANTILEVERED BEAMS
C          ASPECT RATIO = 1.0, UNIFORMLY DISTRIBUTED LOAD
C          AS PER WALTHER et. al. STRESS DISTRIBUTION
C          RECTANGULAR SECTION
C
C
        REAL L,M,MC,MT
        OPEN(UNIT=10, FILE = "P8.OUT")
        H = 0.0
        DO 1000 N=1,7
          H = 3.0 +H
          R = H/2.0
          DO 2000 FC = 1000.0, 8000.0, 1000.0
            FR = 7.5 * SQRT(FC)
            L = H
C
C     CALCULATE MT
C
```

APPENDIX A
(Continued)

```
      H1 = 0.56*R/50.0
      SUM1 = F1(0.44*R,R,FR) + F1(R,R,FR)
        DO 50 J=1,49
        SUM1 = SUM1 +2.0*F1(0.44*R +FLOAT(J)*H1,R,FR) 50
    CONTINUE
      MT = H1*SUM1/2.0
C
C     CALCULATE MC
C
      H2 = 1.44*R/100.0
      SUM2 = F2(-0.44*R,R,FR) + F2(R,R,FR)
            DO 60 J = 1,99
      SUM2 = SUM2 + 2.0*F2(0.44*R+FLOAT(J)*H2,R,FR)
60        CONTINUE
            MC = H2*SUM2/2.0
C
C      CALCULATE W
C
      M = MT + MC
      W = M * 8.0 / (L*L)
```

APPENDIX A
(Continued)

```
      WRITE(10,4000) H,FC,W
4000     FORMAT(5X,F10.4,2X,F10.4,2X,F10.4)
2000    CONTINUE
1000    CONTINUE
         STOP
         END
      FUNCTION F1(Y,R,FR)
       F1 = FR*(Y-0.44*R)*(1.786*Y/R-0.786)
      RETURN
      END
      FUNCTION F2(Y,R,FR)
        Z = (Y + 0.44*R)/(1.44*R)
        F2 = ABS(Y)*0.26*FR*(-0.8805*(Z**5)
%        -2.9*(Z**4) + 20.0867*(Z**3)
%        -23.2371*(Z**2) + 7.93066*Z)
      RETURN
      END
```

APPENDIX A
(Continued)

A.11 Program 11

```
C     PROGRAM FOR SIMPLY SUPPORTED AND CANTILEVERED BEAMS
C          ASPECT RATIO = 1.0, UNIFORMLY DISTRIBUTED LOAD
C          AS PER WALTHER et. al. STRESS DISTRIBUTION
C          CIRCULAR SECTION
C
        REAL L,M,MC,MT
        OPEN(UNIT=10, FILE = "P9.OUT")
          H = 0.0
        DO 1000 N=1,7
         H = 3.0 + H
         R = H/2.0
         DO 2000 FC = 1000.0, 8000.0, 1000.0
          FR = 7.5*SQRT(FC)
          L = H
C
C      CALCULATE MT
C
          H1 = 0.56*R/50.0
```

APPENDIX A
(Continued)

```
      SUM1 = F1(0.44*R,R,FR) + F1(R,R,FR)
      DO 50 J=1,49
      SUM1 = SUM1 + 2.0*F1(0.44*R+FLOAT(J)*H1,R,FR)
50      CONTINUE
      MT = H1*SUM1/2.0
C
C   CALCULATE MC
C
      H2 = 1.44*R/100.0
      SUM2 = F2(-0.44*R,R,FR) + F2(R,R,FR)
      DO 60 J =1,99
      SUM2 = SUM2 +2.0*F2(-0.44*R+FLOAT(J)*H2,R,FR)  60
    CONTINUE
      MC = H2*SUM2/2.0
C
C   CALCULATE W
C
      M = MT + MC
      W = M + 8.0 / (L*L)
      WRITE(10,4000) H,FC,W
```

APPENDIX A
(Continued)

```
4000         FORMAT(5X,F10.4,2X,F10.4,2X,F10.4)
2000        CONTINUE
1000       CONTINUE
        STOP
        END
        FUNCTION  F1(Y,R,FR)
         F1 = 2.0*SQRT(R*R-Y*Y)*FR*(Y-0.44*R)*
     %         (1.786*Y/R-0.786)
       RETURN
       END
        FUNCTION  F2(Y,R,FR)
         Z  = (Y + 0.44*R)/(1.44*R)
         F2 = 2.0*SQRT(R*R-Y*Y)*
     %         ABS(Y)*0.26*FR*(-0.8805*(Z**5)
     %         -2.9*(Z**4) + 20.0867*(Z**3)
     %         -23.2371*(Z**2) + 7.93066*Z)
       RETURN
       END
```

APPENDIX A
(Continued)

A.12 Program 12

```
C     PROGRAM FOR SIMPLY SUPPORTED AND CANTILEVERED BEAM
C     ASPECT RATIO = 2.5, POINT LOAD, RECTANGULAR SECTION
C
      REAL L,M,MT
      OPEN(UNIT=10, FILE = "P10.OUT")
       DO 5000 N=1,6
       R = 6.0 + 3.0*FLOAT(N-1)
       DO 2000 FC = 1000.0, 9000.0, 1000.0
        FR = 7.5*SQRT(FC)
C
C     CALCULATE MT
C
        H1 = R/50.0
        SUM1 = F1(0.0,R.FR) + F1(R,R,FR)
        DO 50 J=1,49
         SUM1 = SUM1 + 2.0*F1(FLOAT(J)*H1,R,FR)
50       CONTINUE
        MT = H1*SUM1/2.0
```

APPENDIX A
(Continued)

```
C
C     CALCULATE P
C
         L = 5.0*R
         M = 2.0*MT
         P = 4.0*M/L
         WRITE(10,4000) R,FC,P
4000        FORMAT(5X,F10.4,2X,F10.4,2X,F15.4)
2000       CONTINUE
5000       CONTINUE
      STOP
      END
      FUNCTION F1(Y,R,FR)
       Z = Y/R
       F1 = FR*Y*(0.4484*(Z**2) + 0.5716*Z)
      RETURN
      END
```

APPENDIX A
(Continued)

A.13 Program 13

```
C     PROGRAM FOR SIMPLY SUPPORTED AND CANTILEVERED BEAM
C      ASPECT RATIO = 2.5, POINT LOAD, CIRCULAR SECTION
C
        REAL L,M,MT
        OPEN(UNIT=10, FILE = "P11.OUT")
          DO 5000 N=1,6
          R = 6.0 +3.0*FLOAT(N-1)
          DO 2000 FC = 1000.0, 9000.0, 1000.0
            FR = 1.08*7.5*SQRT(FC) +70.0
C
C      CALCULATE MT
C
            H1 = R/50.0
            SUM1 = F1(0.0,R,FR) + F1(R,R,FR)
            DO 50 J=1,49
              SUM1 = SUM1 + 2.0*F1(FLOAT(J)*H1,R,FR)
50            CONTINUE
            MT = H1*SUM1/2.0
```

APPENDIX A
(Continued)

```
C
C     CALCULATE P
C
        L = 5.0*R
        M = 2.0*MT
        P = 4.0*M/L
        WRITE(10,4000) R,FC,P
4000      FORMAT(5XF10.4,2X,F10.4,2X,F15.4)
2000     CONTINUE
5000     CONTINUE
      STOP
      END
      FUNCTION F1(Y,R,FR)
       Z = Y/R
       F1 = FR*Y*(0.4484*(Z**2) +
    %  0.5716*Z)*2.0*SQRT(R*R-Y*Y)
      RETURN
      END
```

APPENDIX A
(Continued)

A.14 Program 14

```
C     PROGRAM FOR SIMPLY SUPPORTED AND CANTILEVERED BEAM
C     ASPECT RATIO = 1.0, POINT LOAD, RECTANGULAR SECTION
C
        REAL L,M,MT
        OPEN(UNIT=10, FILE = "P12.OUT")
         DO 5000 N=1,6
         R = 6.0 + 3.0*FLOAT(N-1)
         DO 2000 FC = 1000.0, 9000.0,1000.0
          FR = 7.5*SQRT(FC)
C
C      CALCULATE MT
C          H1 = R/50.0
          SUM1 = F1(0.0,R,FR) + F1(R,R,FR)
          DO 50 J=1,49
           SUM1 = SUM1 = 2.0*F1(FLOAT(J)*H1,R,FR)
50         CONTINUE
          MT = H1*SUM1/2.0
C
```

APPENDIX A
(Continued)

```
C     CALCULATE P
C
         L = 5.0*R
         M = 2.0*MT
         P = 4.0*M/L
         WRITE(10,4000) R,FC,P
4000        FORMAT(5X,F10.4,2X,F10.4,2X,F15.4)
2000      CONTINUE
5000      CONTINUE
       STOP
       END
       FUNCTION F1(Y,R,FR)
        Z = Y/R
        F1 = FR*Y*(2.5*(Z**3)-1.3*(Z**2)-0.2*Z)
       RETURN
       END
```

APPENDIX A
(Continued)

A.15 Program 15

```
C     PROGRAM FOR SIMPLY SUPPORTED AND CANTILEVERED BEAM
C      ASPECT RATIO = 1.0, POINT LOAD, CIRCULAR SECTION
C
        REAL  L,M,MT
        OPEN (UNIT=10, FILE = "P13.OUT")
          DO 5000  N=1,6
          R = 6.0 + 3.0*FLOAT(N-1)
          DO 2000  FC = 1000.0, 9000.0, 1000.0
            FR = 1.08*7.5*SQRT(FC) + 70.0
C
C      CALCULATE MT
C
            H1 = R/50.0
            SUM1 = F1(0.0,R,FR) +F1(R,R,FR)
            DO 50 J+1,49
                SUM1 = SUM1 + 2.0*F1(FLOAT(J)*H1,R,FR)
50            CONTINUE
            MT = H1*SUM1/2.0
```

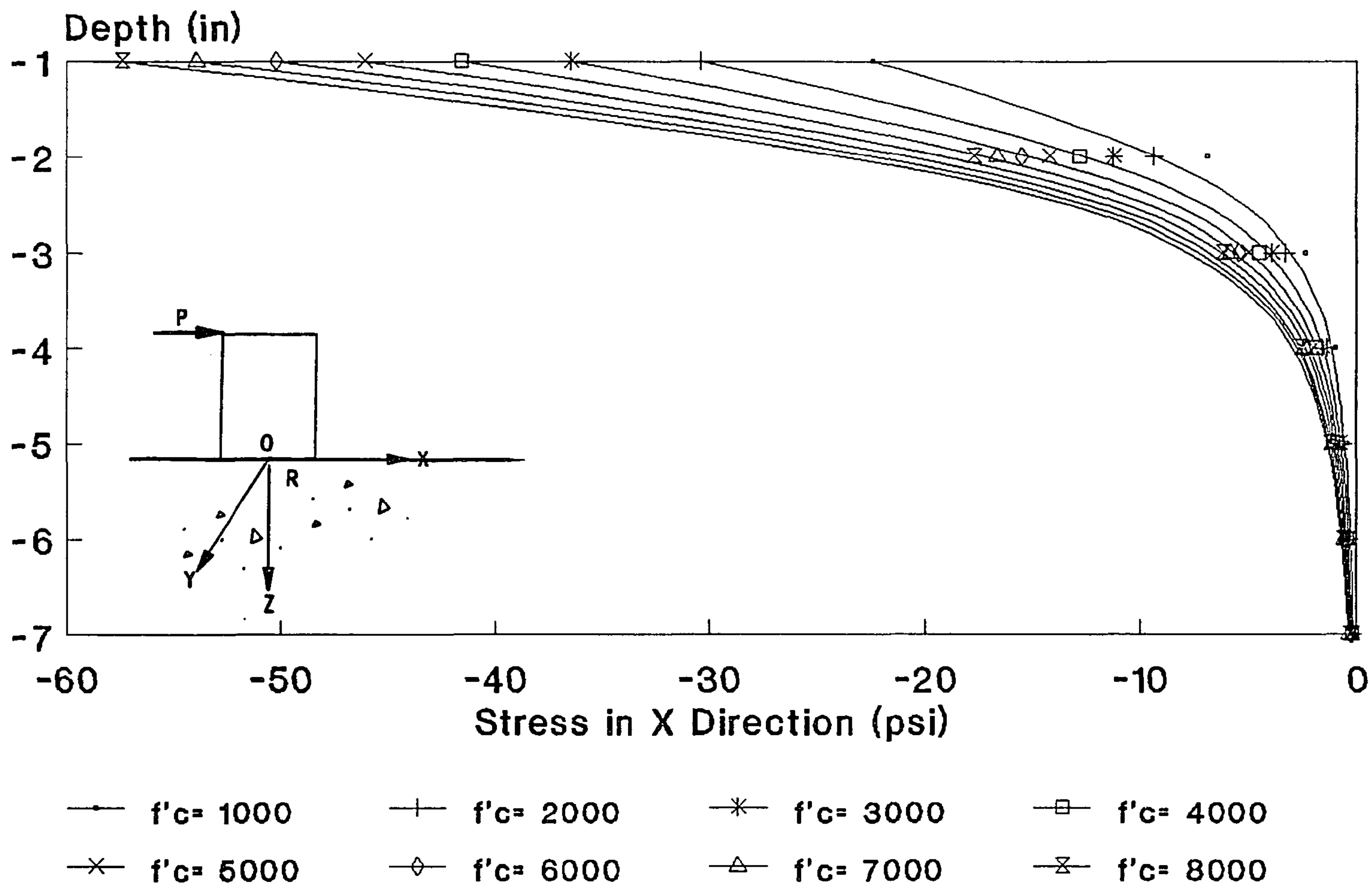
APPENDIX A
(Continued)

```
C
C     CALCULATE P
C
          L = 5.0*R
          M = 2.0*MT
          P = 4.0*M/L
          WRITE(10,4000) R,FC,P
4000        FORMAT(5X,F10.4,2X,F10.4,2XF15.4)
2000       CONTINUE
5000       CONTINUE
            STOP
      END
            FUNCTION F1(Y,R,FR)
         Z = Y/R
         F1 = FR*Y*2.0*SQRT(R*R-Y*Y)*(2.5*(Z**3)-
     %    1.3*(Z**2)-0.2*Z)
        RETURN
        END
```

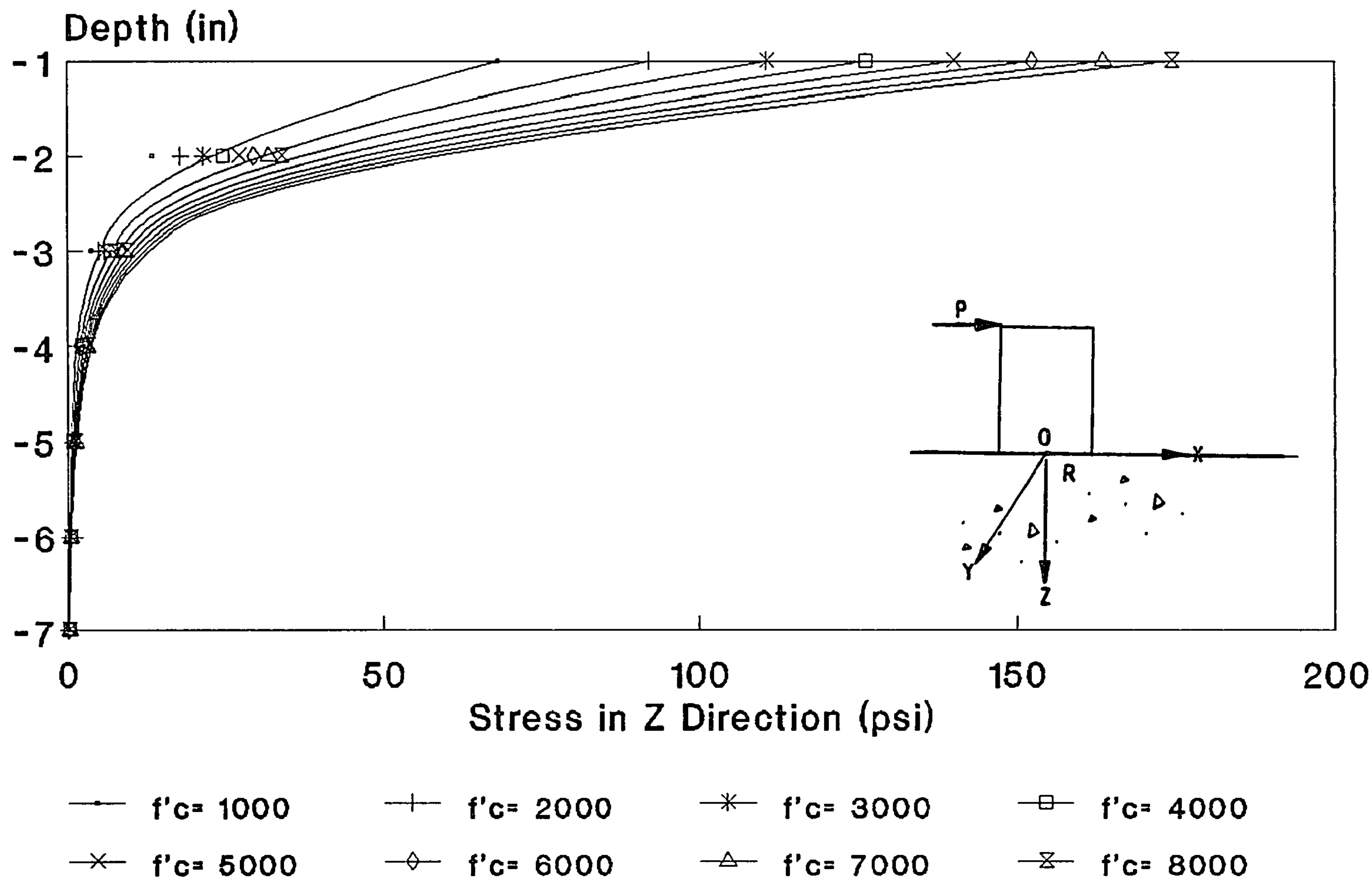
APPENDIX B

STRESSES IN THE VICINITY OF BREAK OFF TEST SPECIMEN

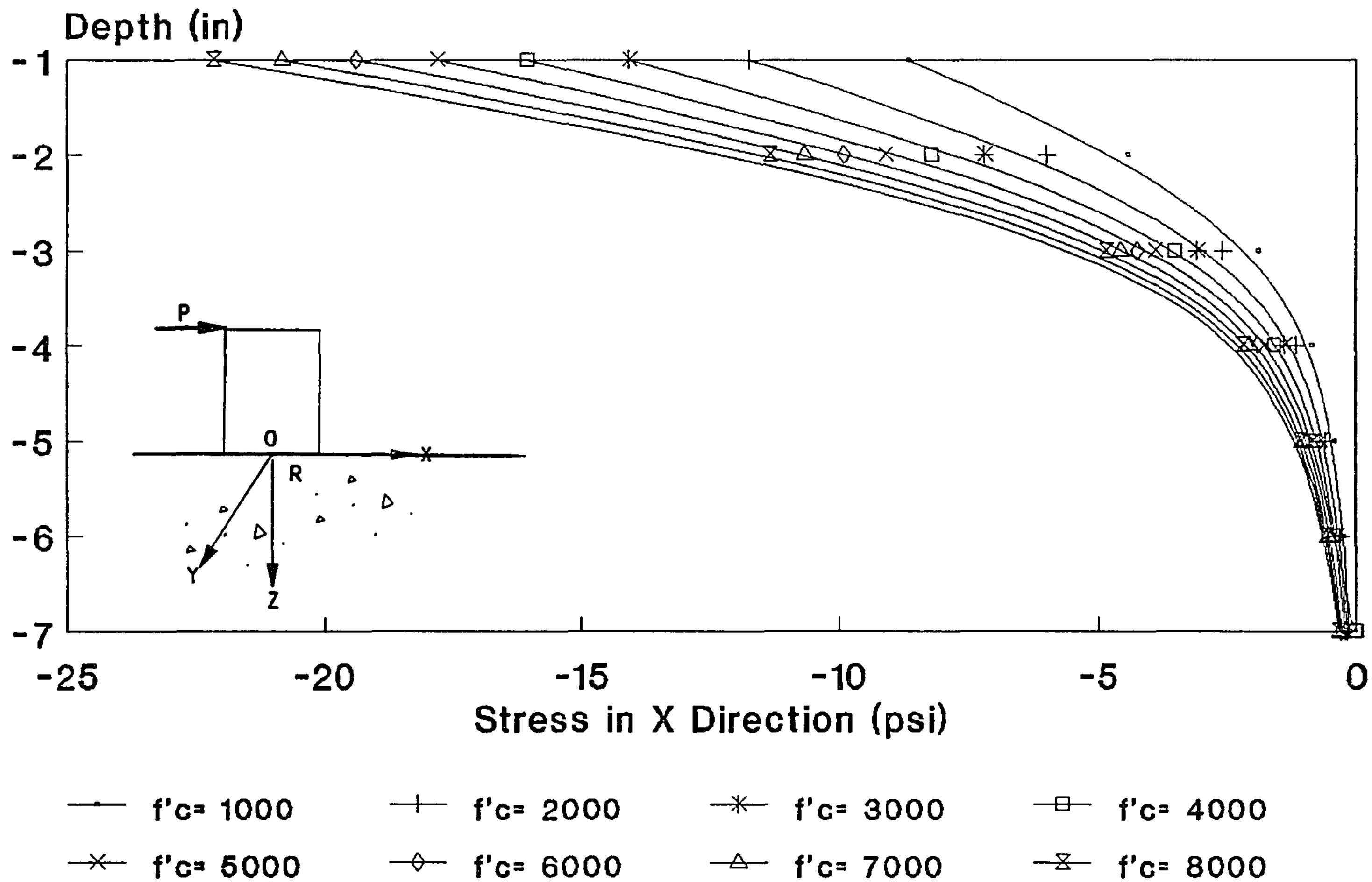
In this section, the stress distributions in the vicinity of the break off test specimen are given.


Depth (in)
-1
-2
-3
-4
-5
-6
-7
-60
-50
-40
-30
-20
-10
0
Stress in X Direction (psi)
P
O
X
R
Y
Z
f'c= 1000
f'c= 2000
f'c= 3000
f'c= 4000
f'c= 5000
f'c= 6000
f'c= 7000
f'c= 8000

At (R,0,Z)

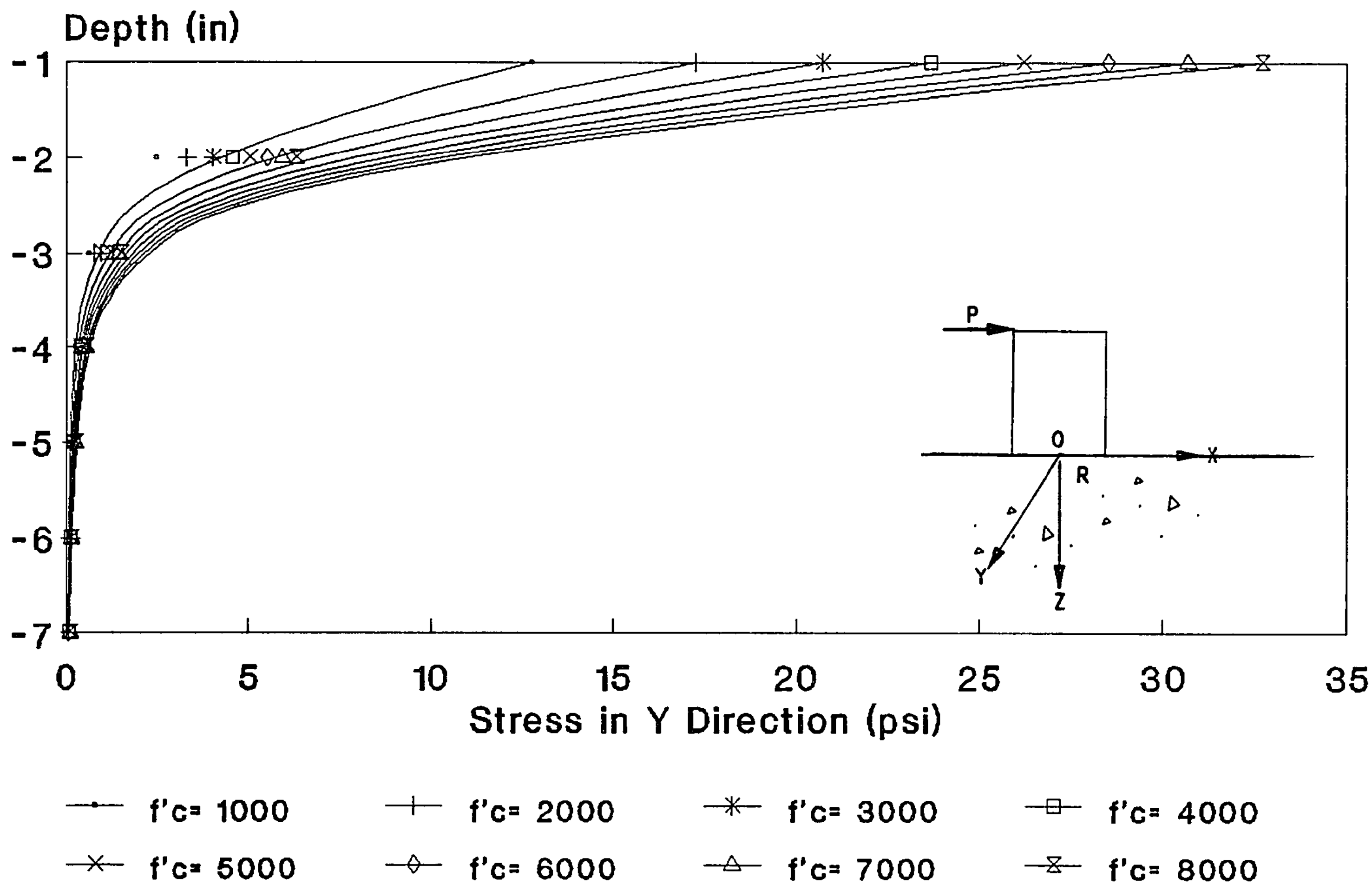

Depth (in)
-1
-2
-3
-4
-5
-6
-7
0
50
100
150
200
Stress in Z Direction (psi)
P
O
R
X
Y
Z
f'c= 1000
f'c= 2000
f'c= 3000
f'c= 4000
f'c= 5000
f'c= 6000
f'c= 7000
f'c= 8000

At (R,0,Z)

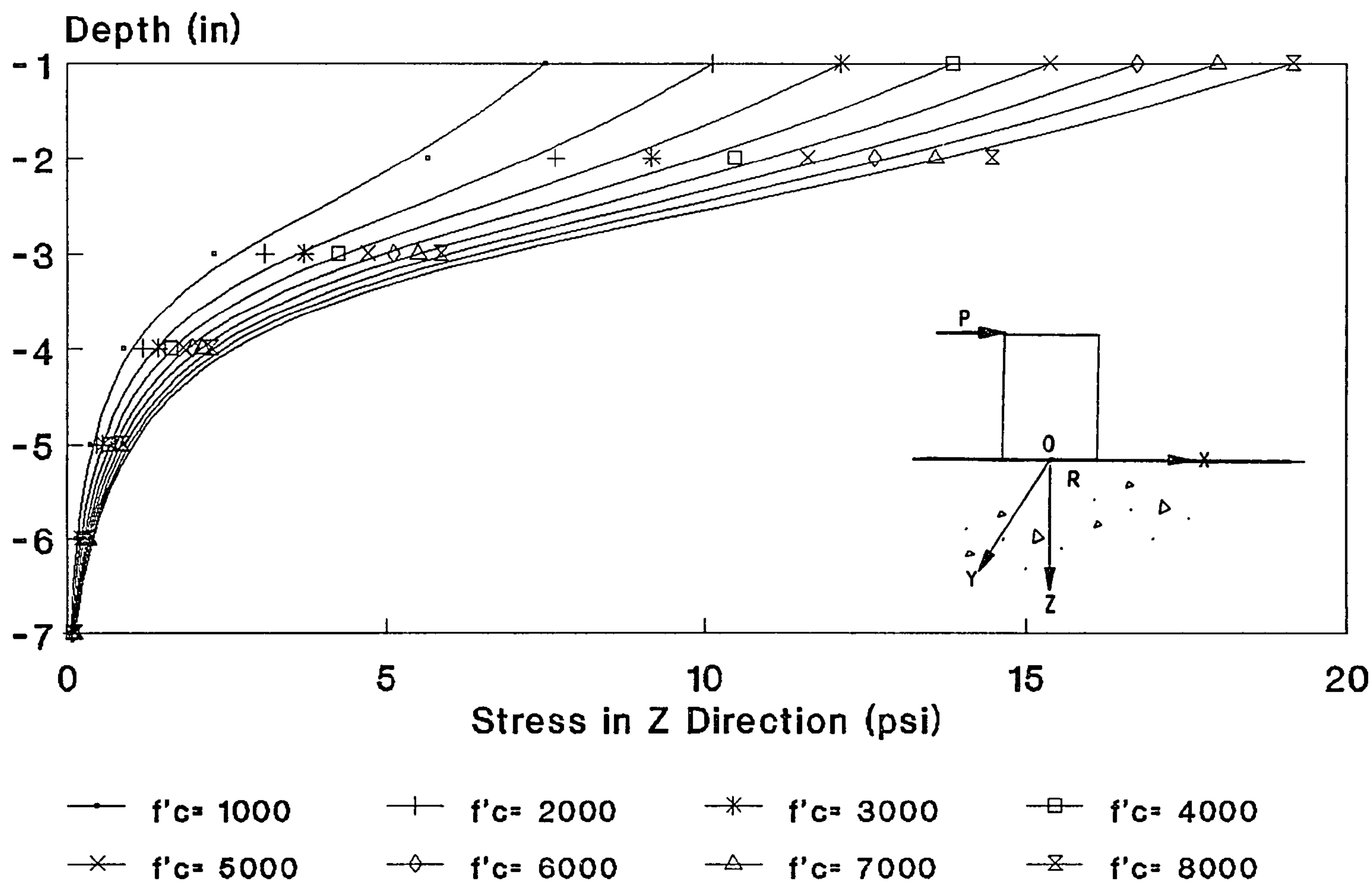


Depth (in)
-1
-2
-3
-4
-5
-6
-7
-25
-20
-15
-10
-5
0
Stress in X Direction (psi)
P
O
X
R
Y
Z
f'c= 1000
f'c= 2000
f'c= 3000
f'c= 4000
f'c= 5000
f'c= 6000
f'c= 7000
f'c= 8000

At (R,-R,Z)


Depth (in)
-1
-2
-3
-4
-5
-6
-7
0
5
10
15
20
25
30
35
Stress in Y Direction (psi)
P
O
X
R
Y
Z
f'c= 1000
f'c= 2000
f'c= 3000
f'c= 4000
f'c= 5000
f'c= 6000
f'c= 7000
f'c= 8000

At (R,-R,Z)



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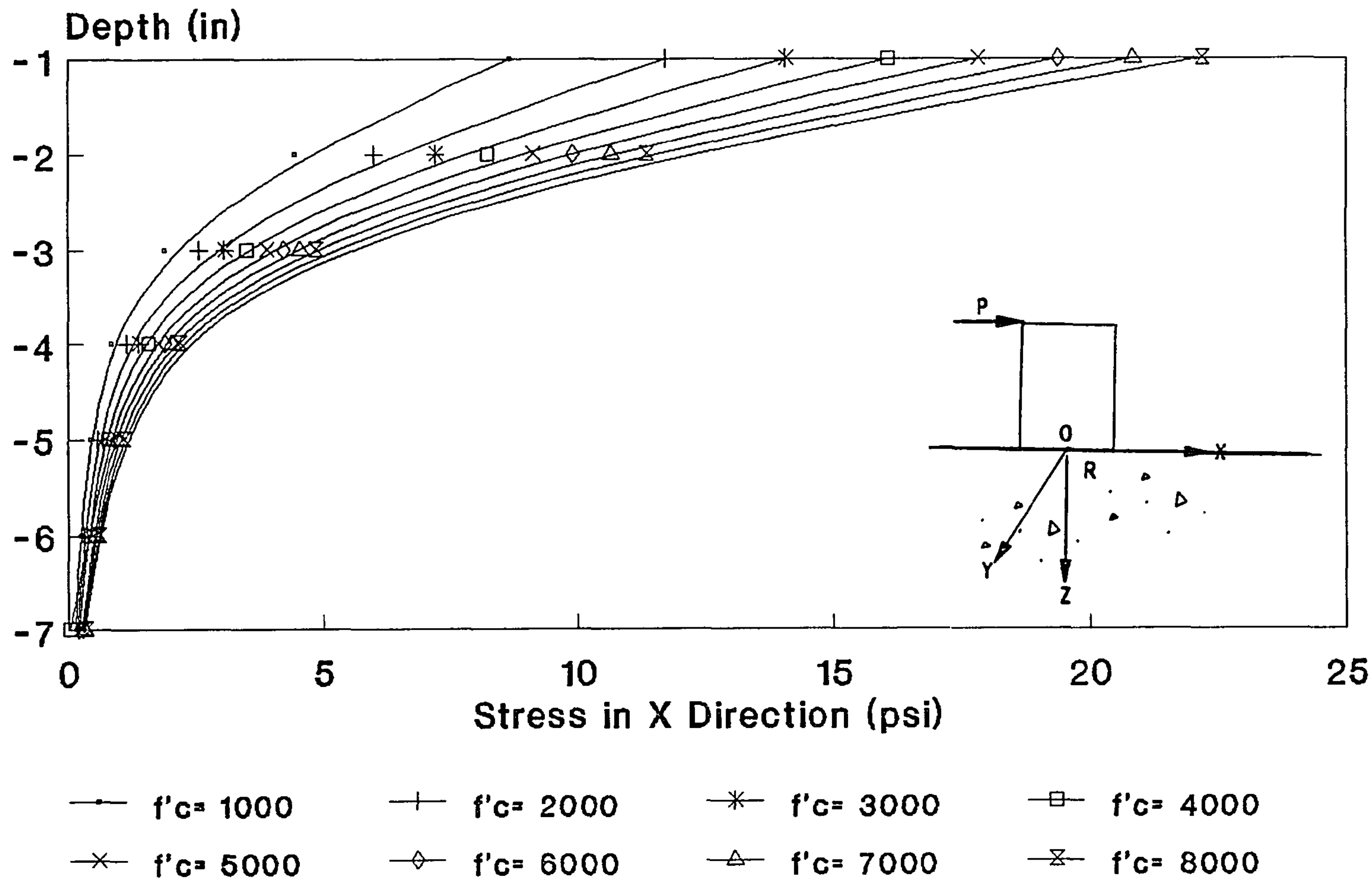
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f'c= 1000
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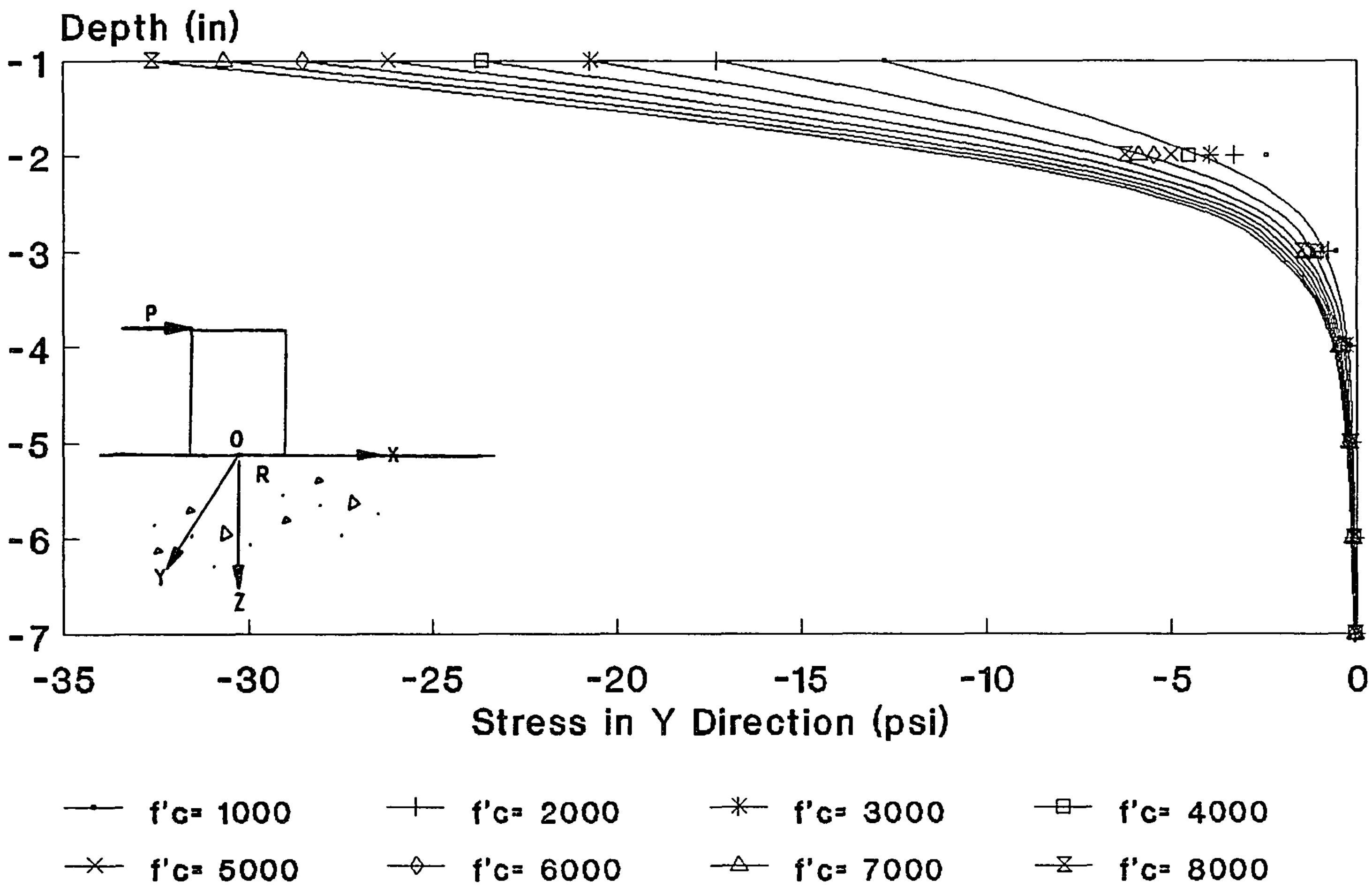
f'c= 8000

At (-R,R,Z)



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Stress in Y Direction (psi)
P
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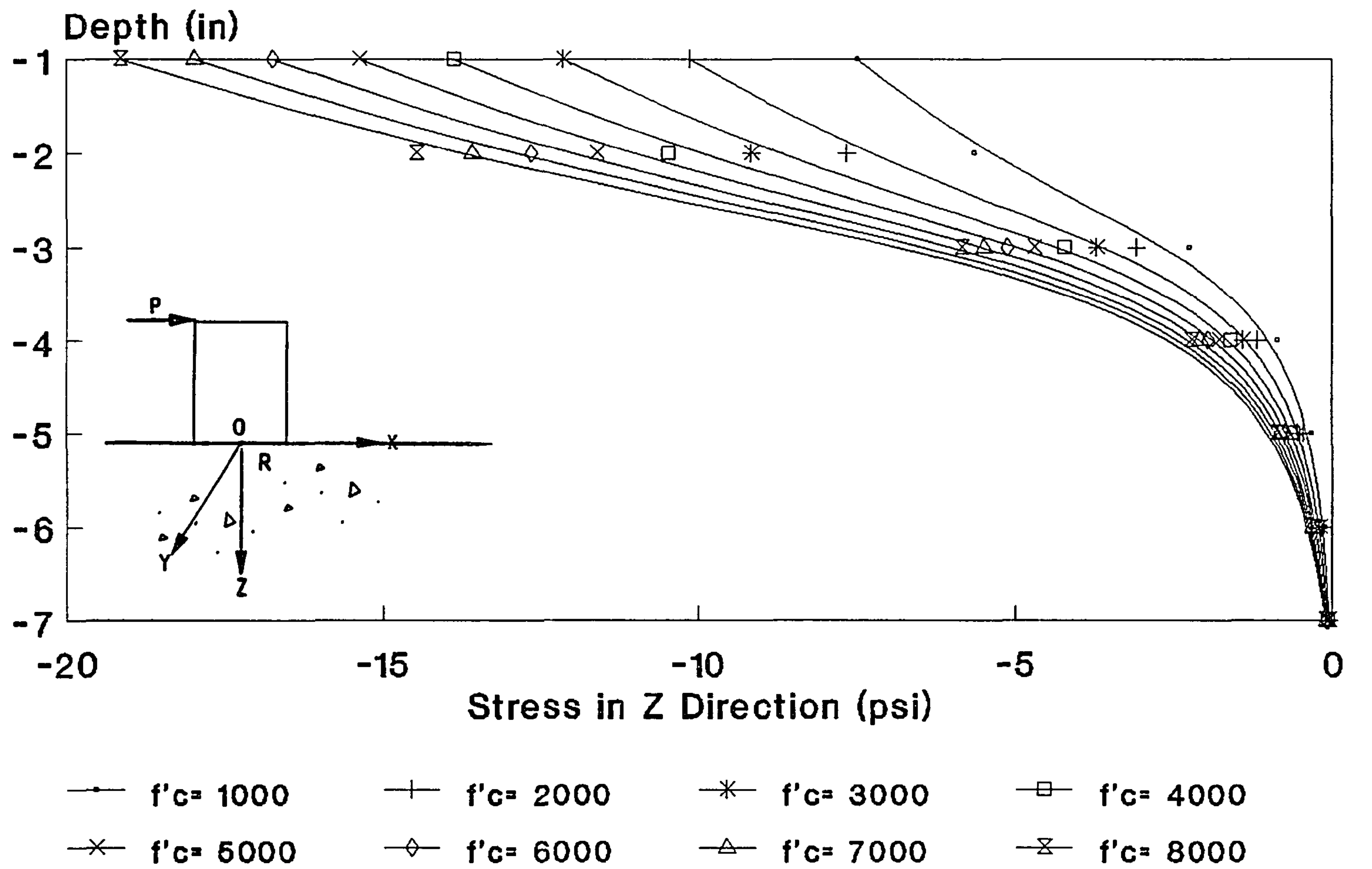
At (-R,R,Z)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f'c= 1000
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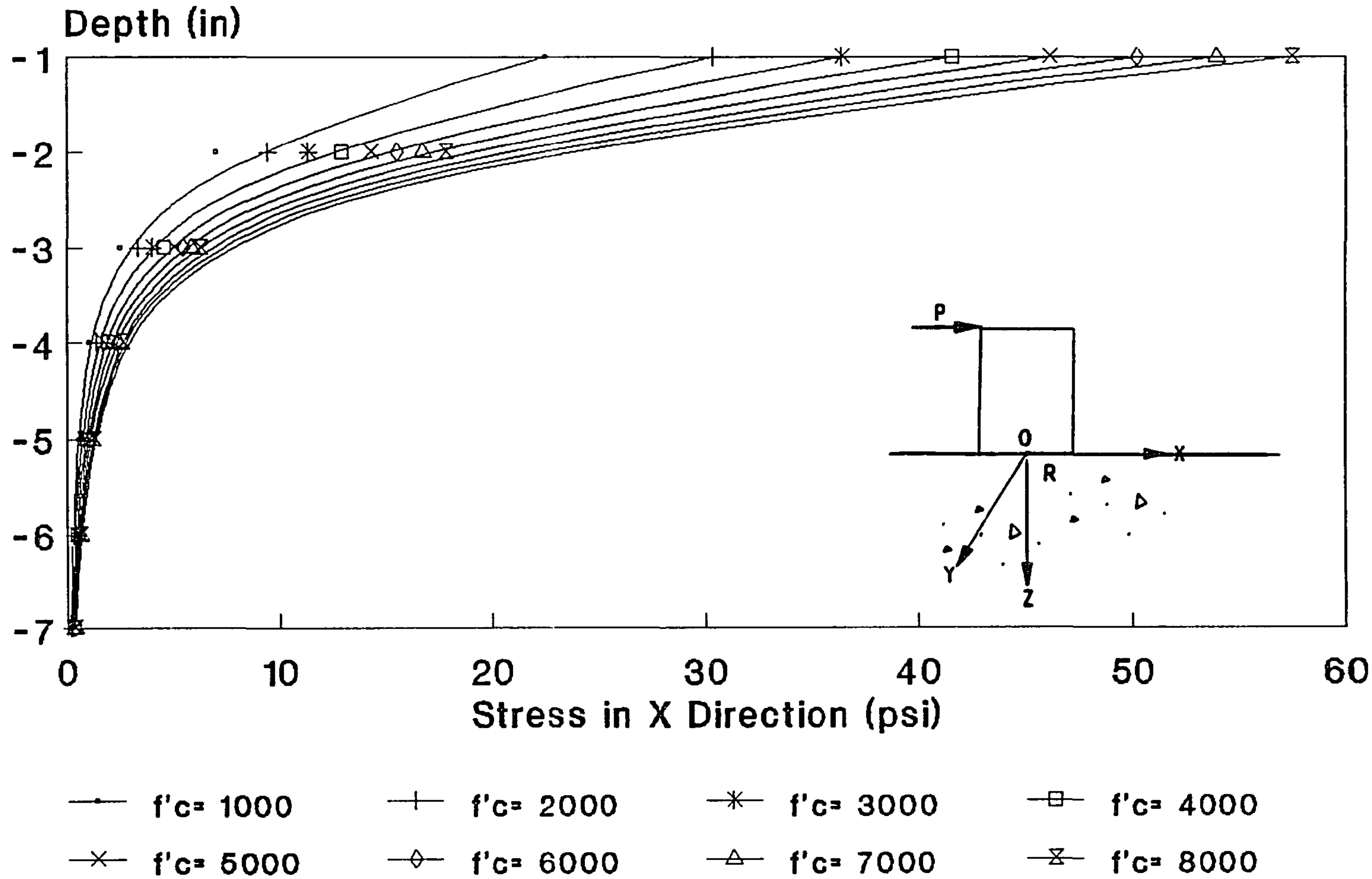
At (-R,R,Z)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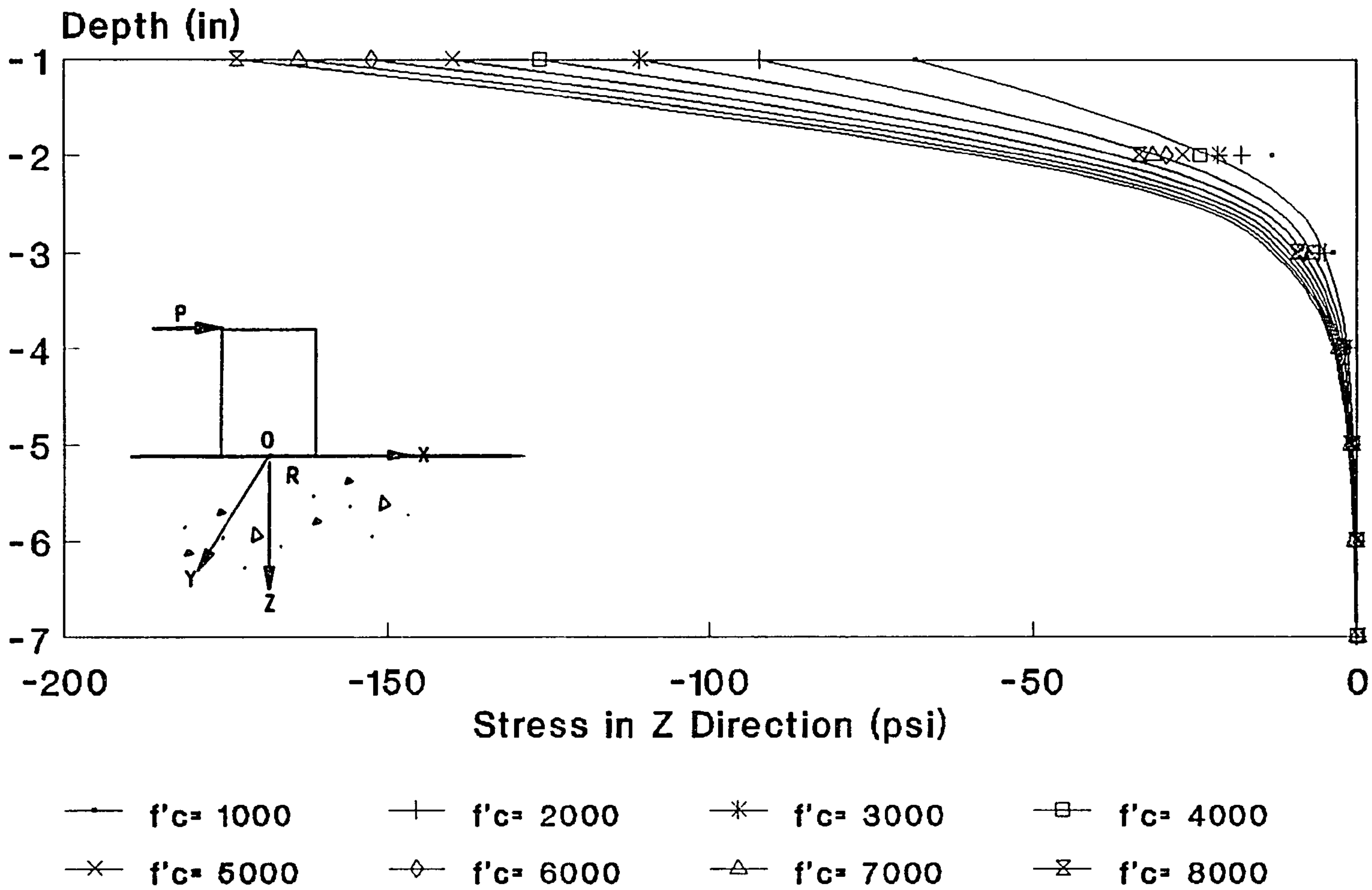
At (-R,0,Z)



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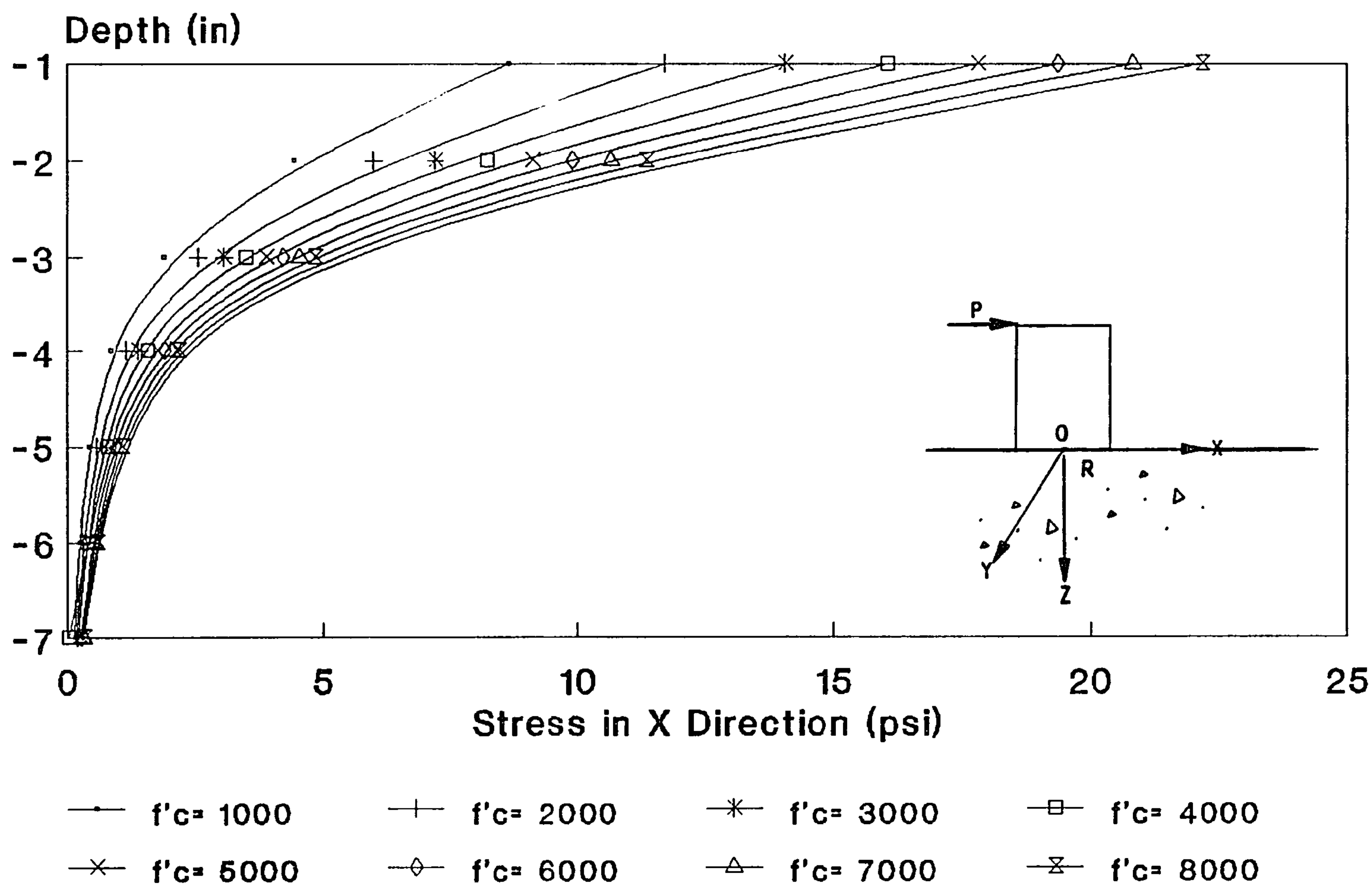
At (-R,0,Z)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Stress in X Direction (psi)
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f'c= 5000
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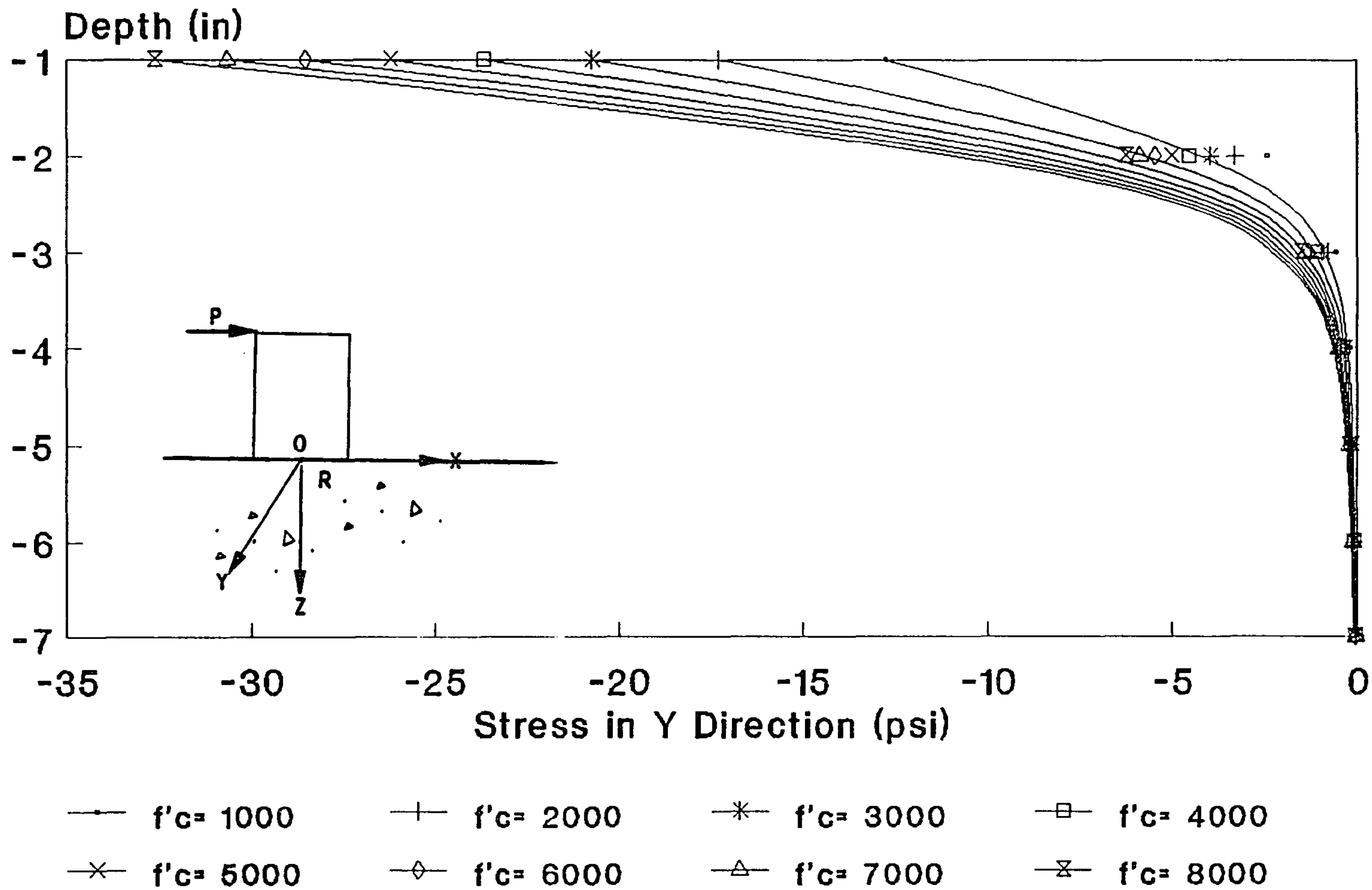
f'c= 7000
f'c= 8000

At (-R,-R,Z)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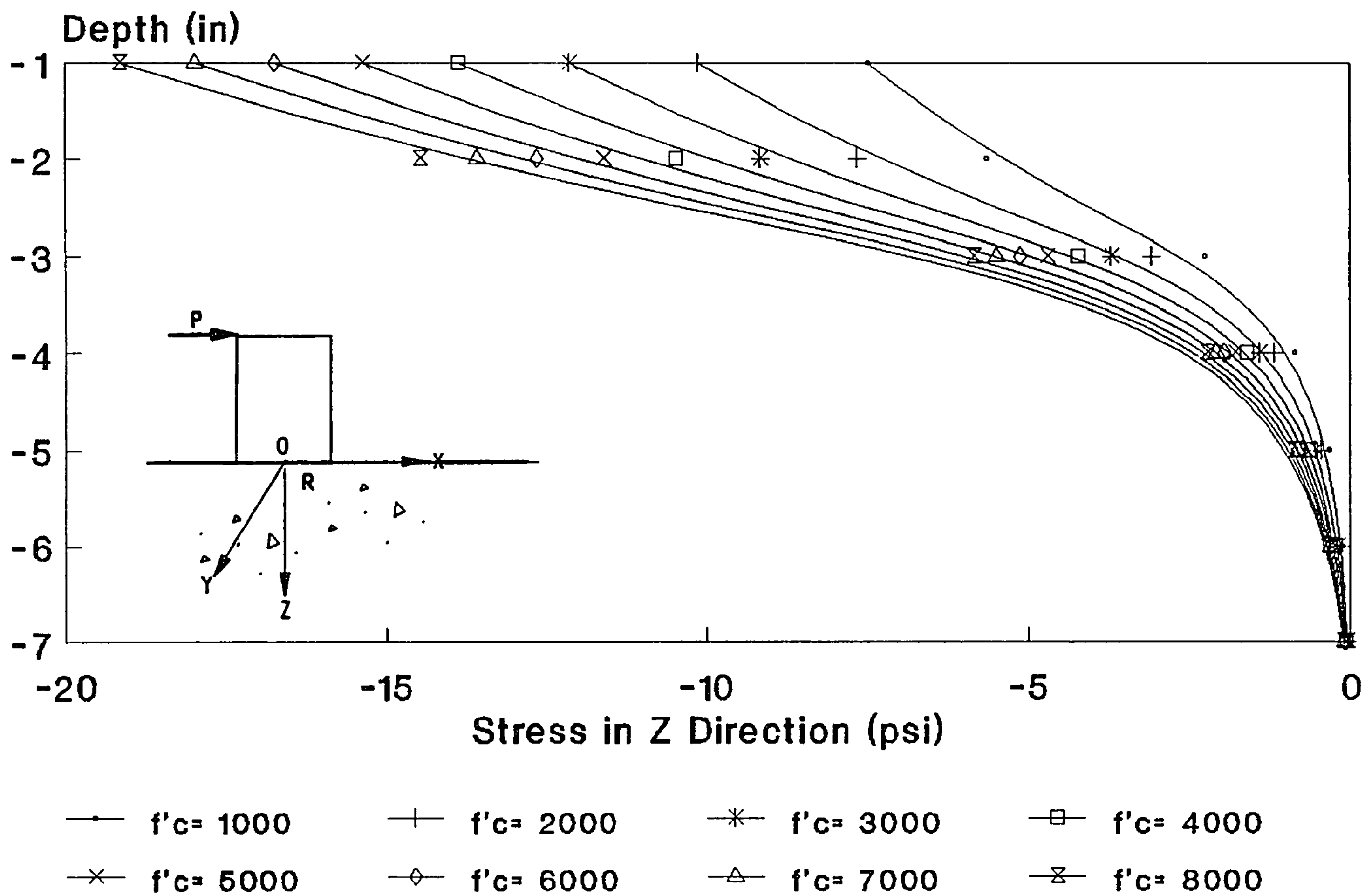
At (-R,-R,Z)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X
R
Y
Z
f'c= 1000
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f'c= 3000
f'c= 4000
f'c= 5000
f'c= 6000
f'c= 7000
f'c= 8000

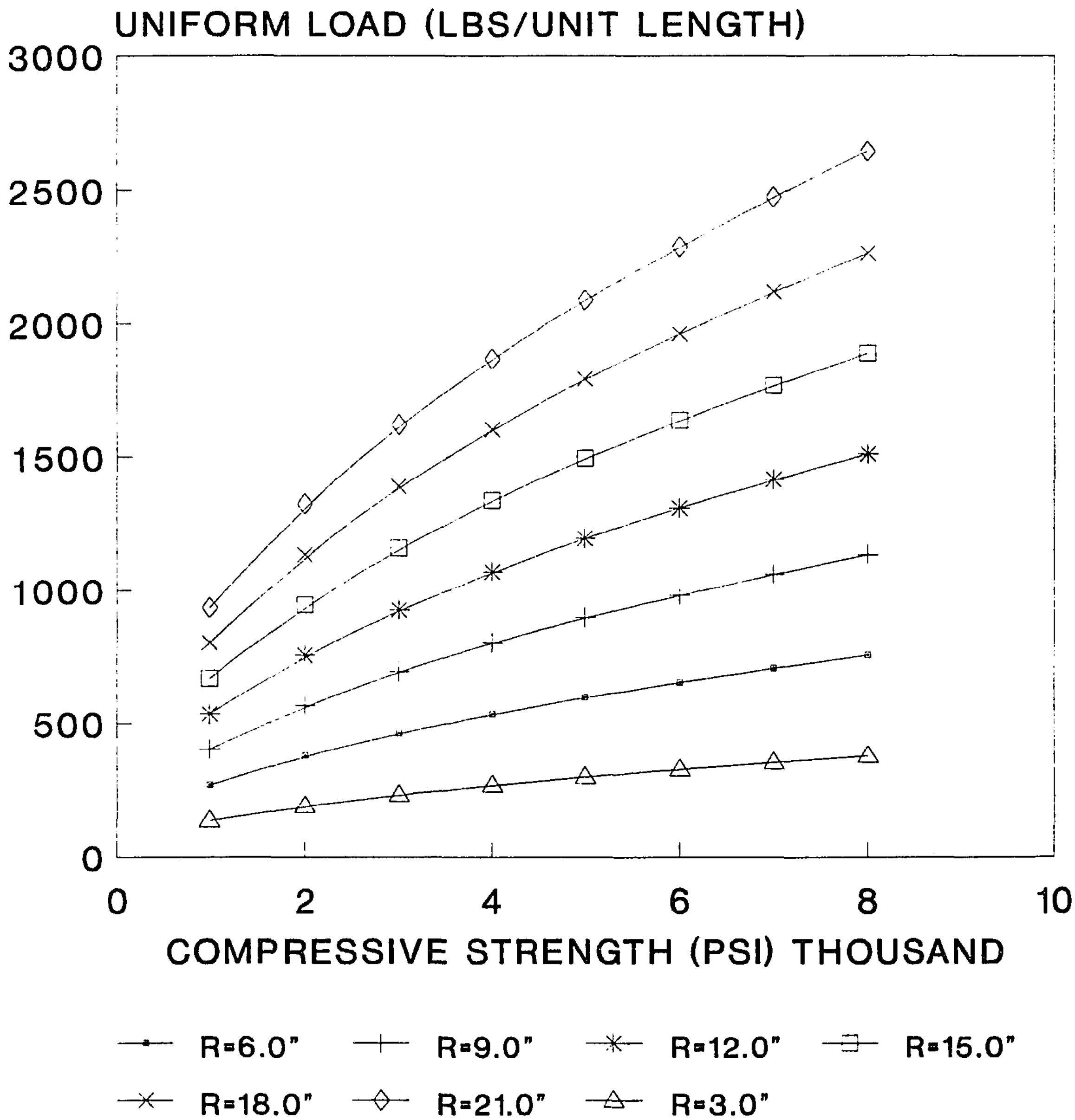
At (-R,-R,Z)

APPENDIX C

CAPACITIES OF DEEP BEAMS

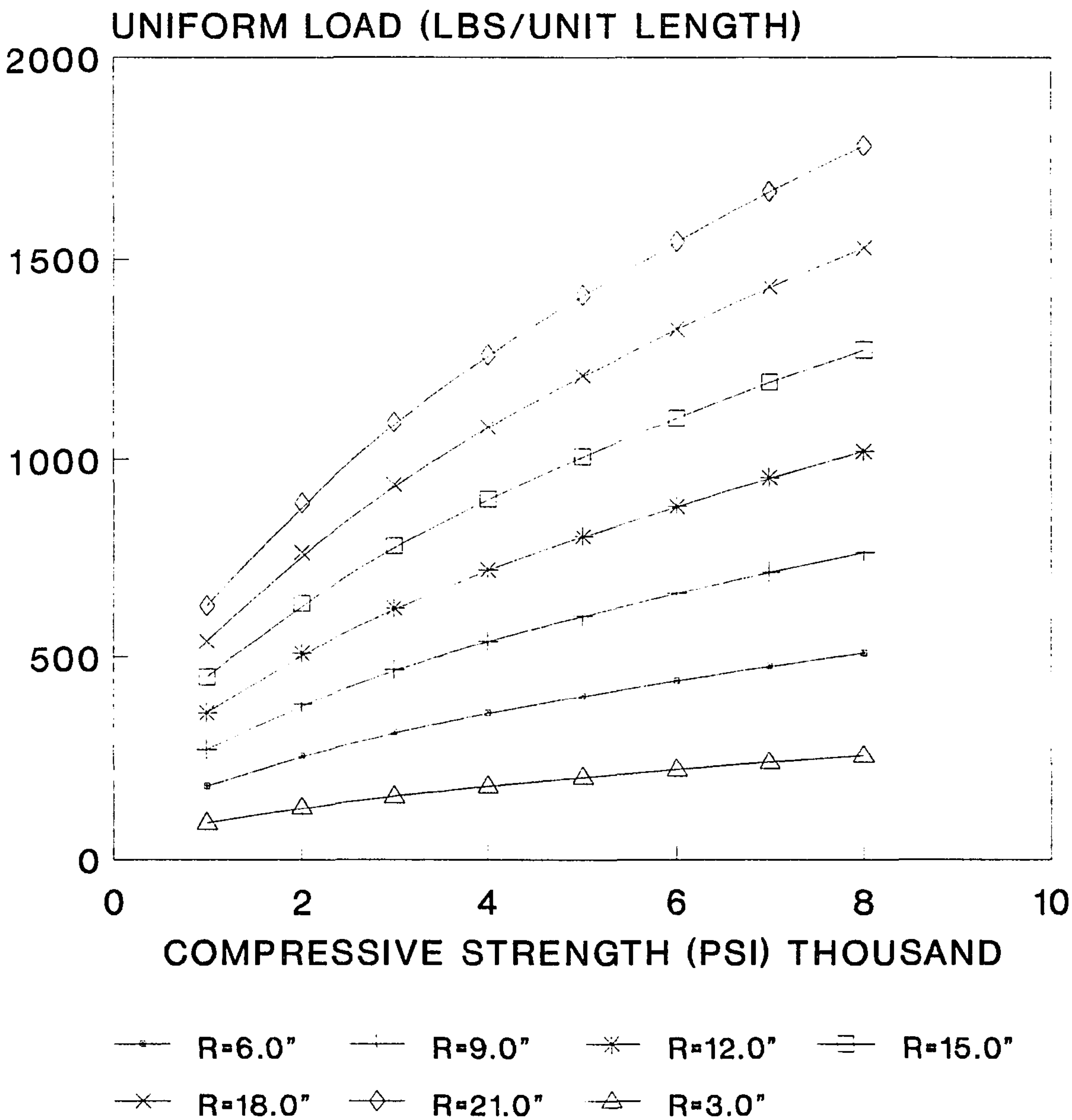
In this section, the capacities of deep beams with both rectangular and circular cross sections are given for various concrete strengths, support and loading conditions.

CAPACITY OF DEEP BEAMS CIRCULAR SECTION



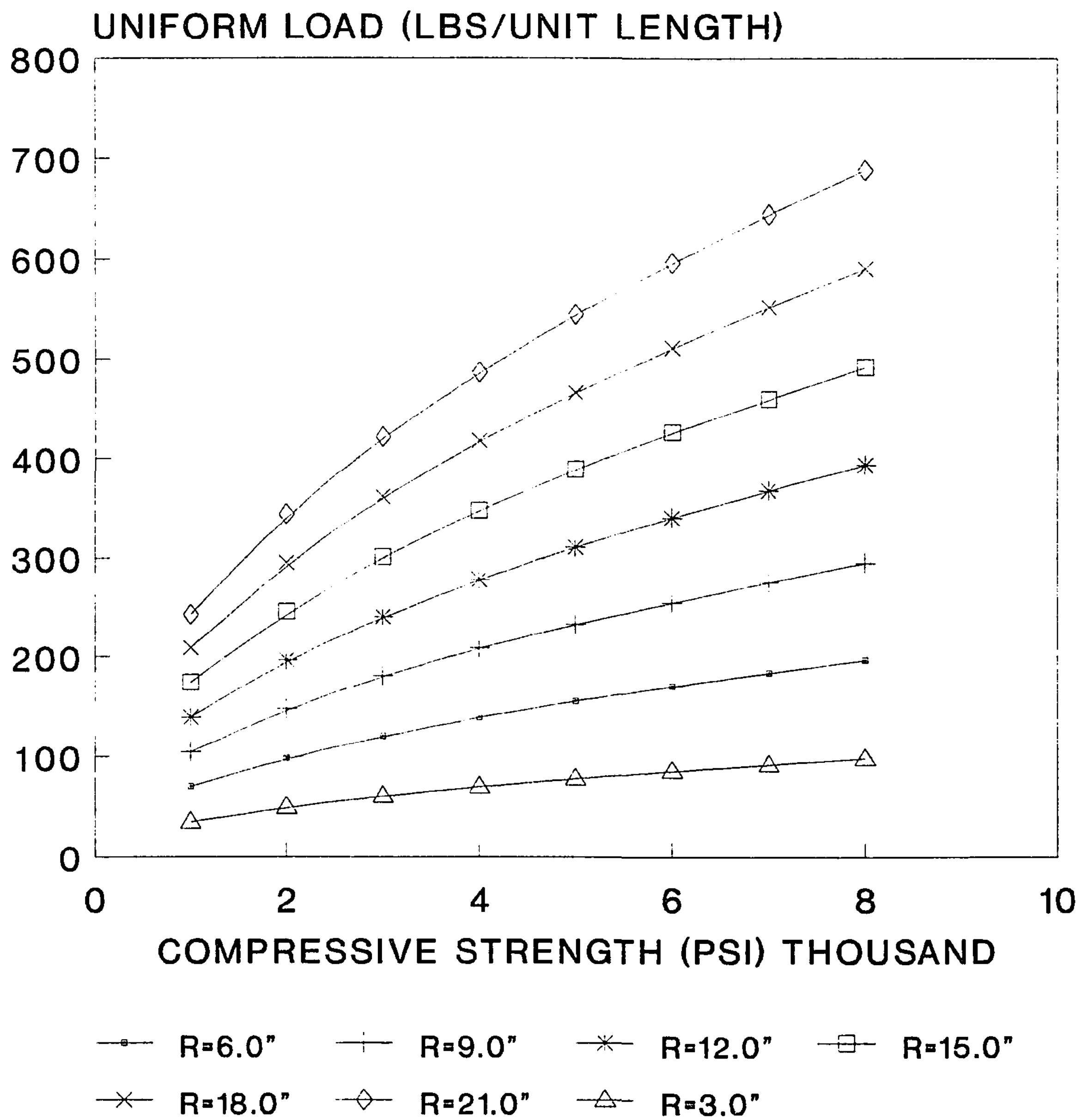
SIMPLY SUPPORTED, A.R=1.0

CAPACITY OF DEEP BEAMS
CIRCULAR SECTION



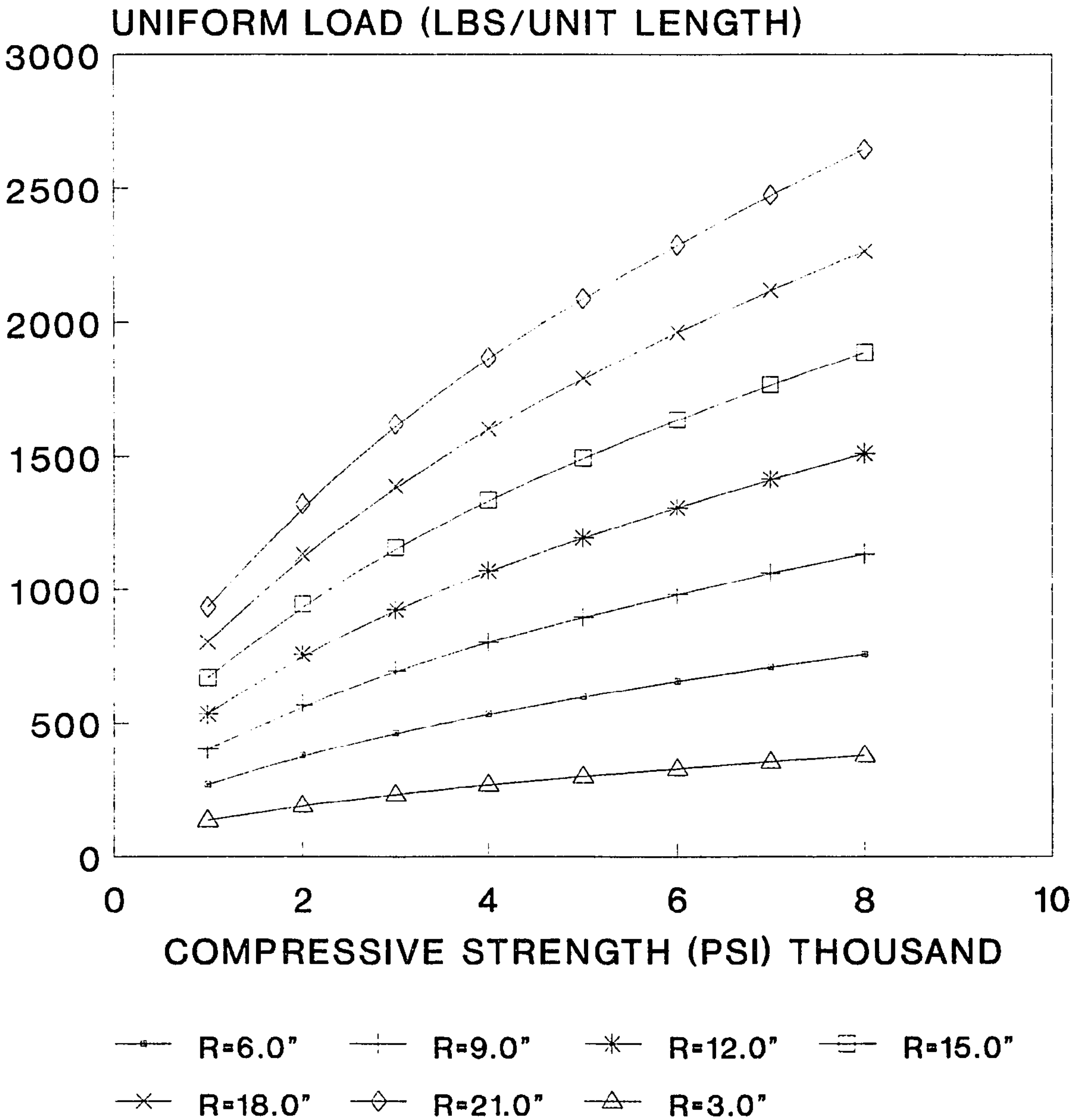
SIMPLY SUPPORTED, A.R=2.0

CAPACITY OF DEEP BEAMS CIRCULAR SECTION



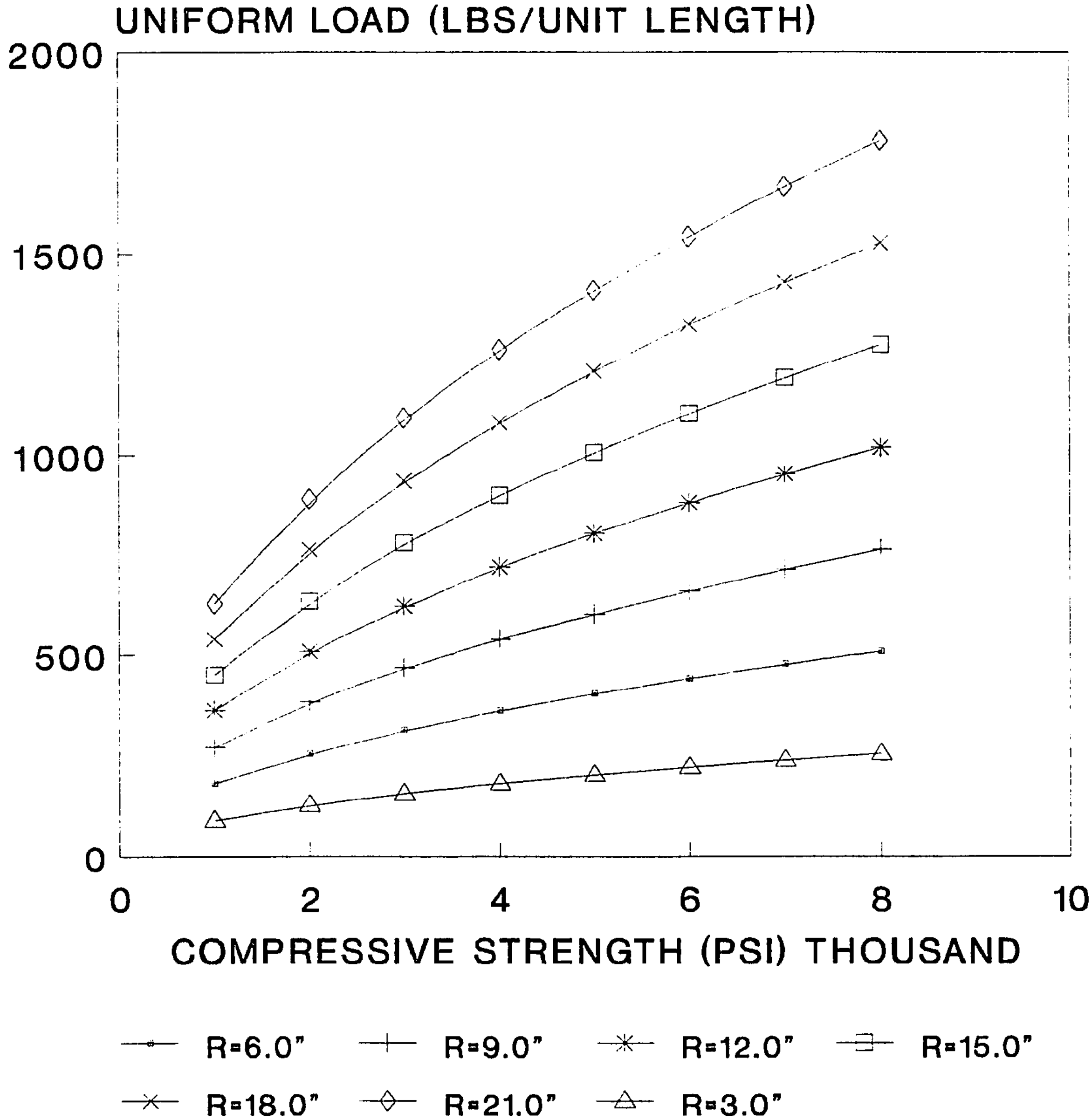
SIMPLY SUPPORTED, A.R=4.0

CAPACITY OF DEEP BEAMS CIRCULAR SECTION



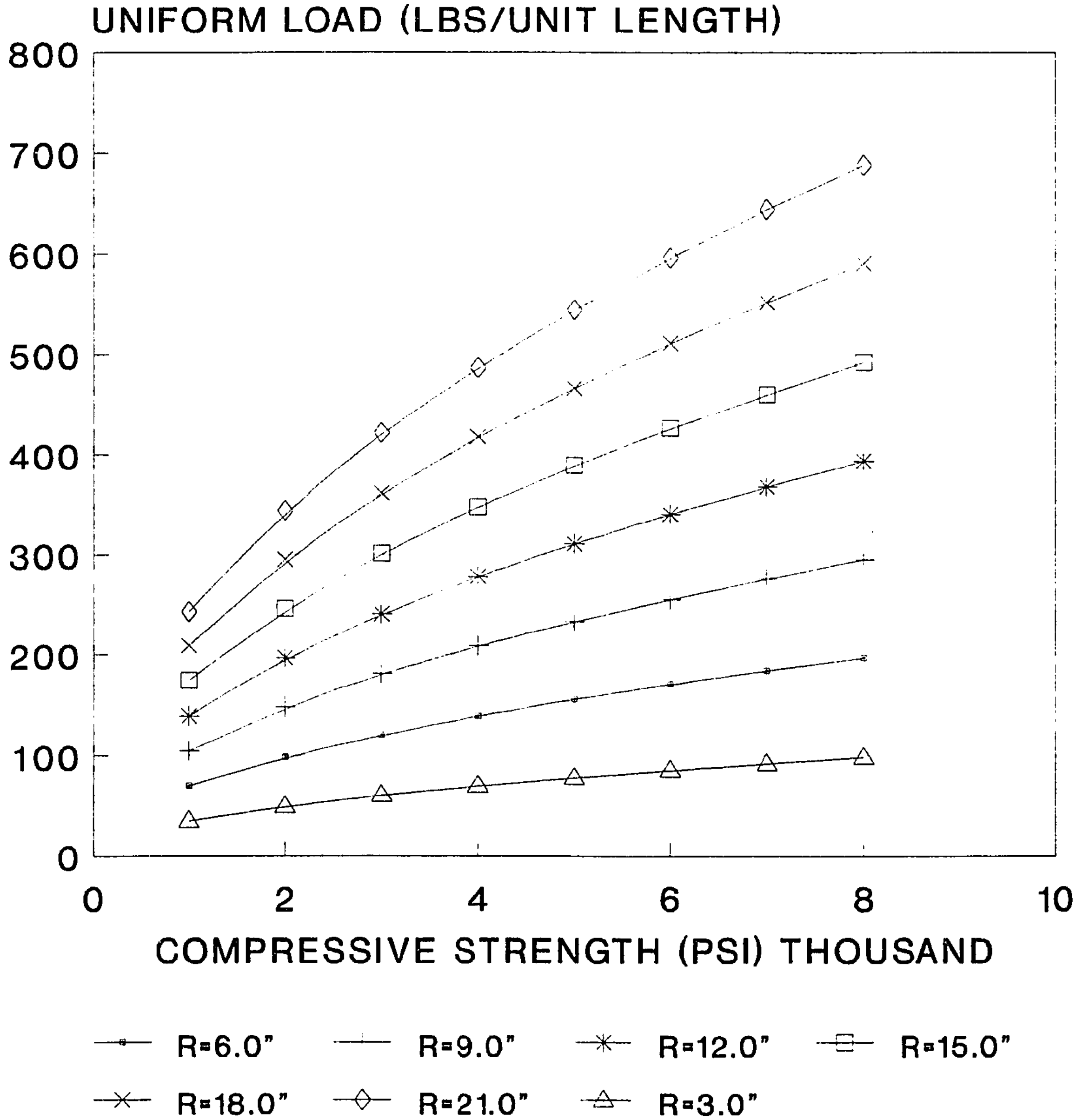
CANTILEVERED, A.R=0.5

CAPACITY OF DEEP BEAMS
CIRCULAR SECTION



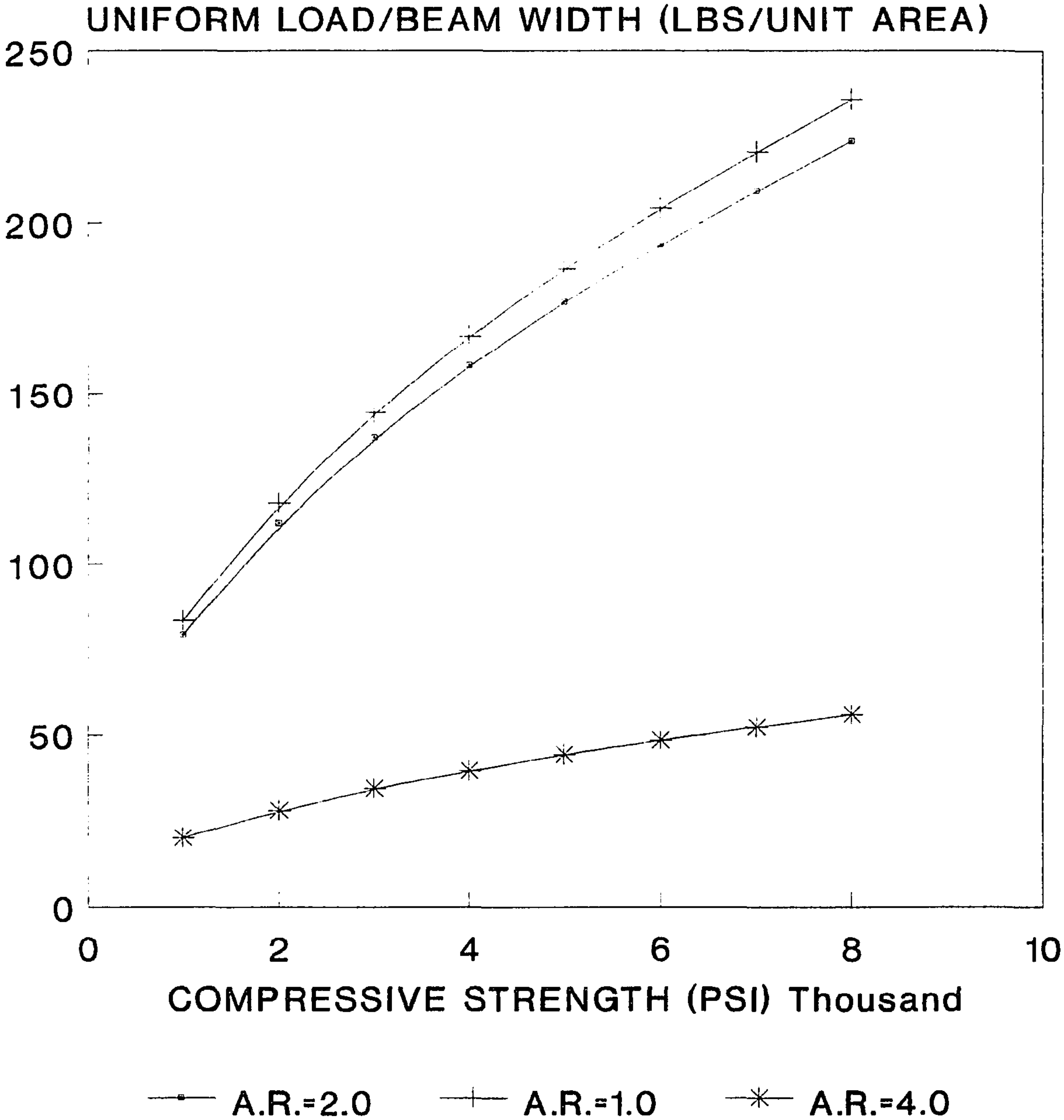
CANTILEVERED, A.R=1.0

CAPACITY OF DEEP BEAMS CIRCULAR SECTION



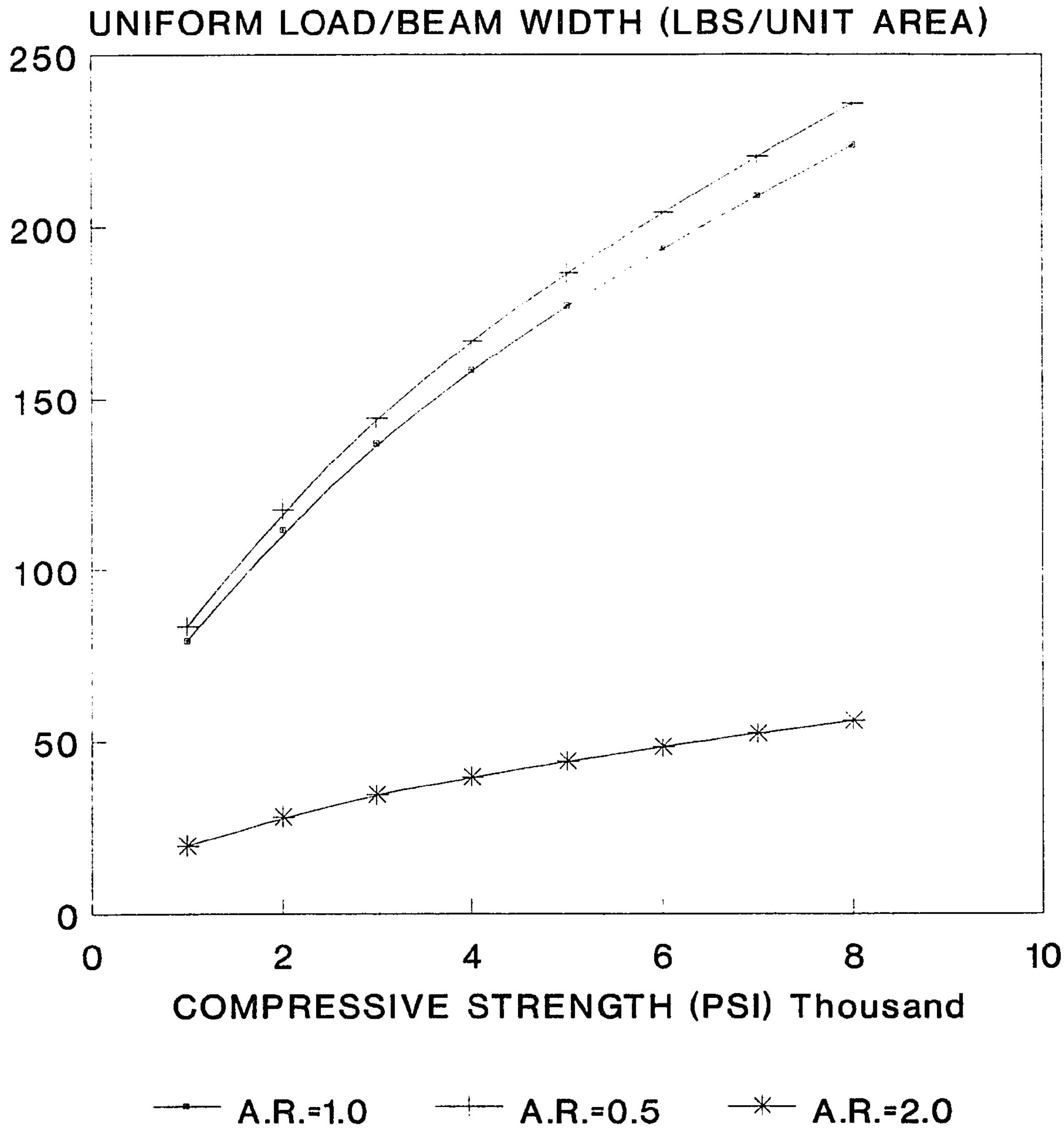
CANTILEVERED, A.R=2.0

CAPACITY OF DEEP BEAMS RECTANGULAR SECTION



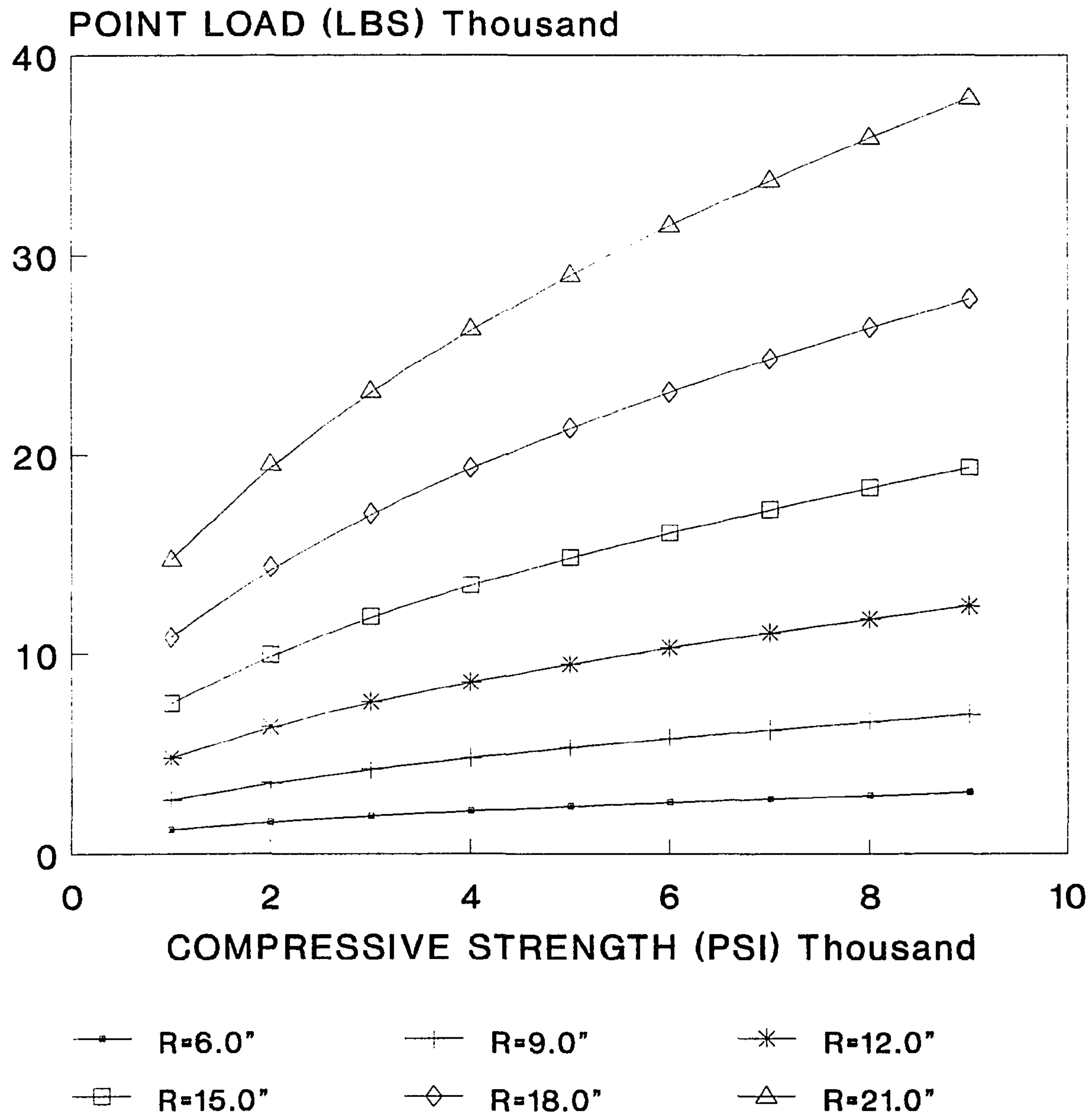
SIMPLY SUPPORTED

CAPACITY OF DEEP BEAMS RECTANGULAR SECTION



CANTILEVERED

CAPACITY OF DEEP BEAMS
CIRCULAR SECTION



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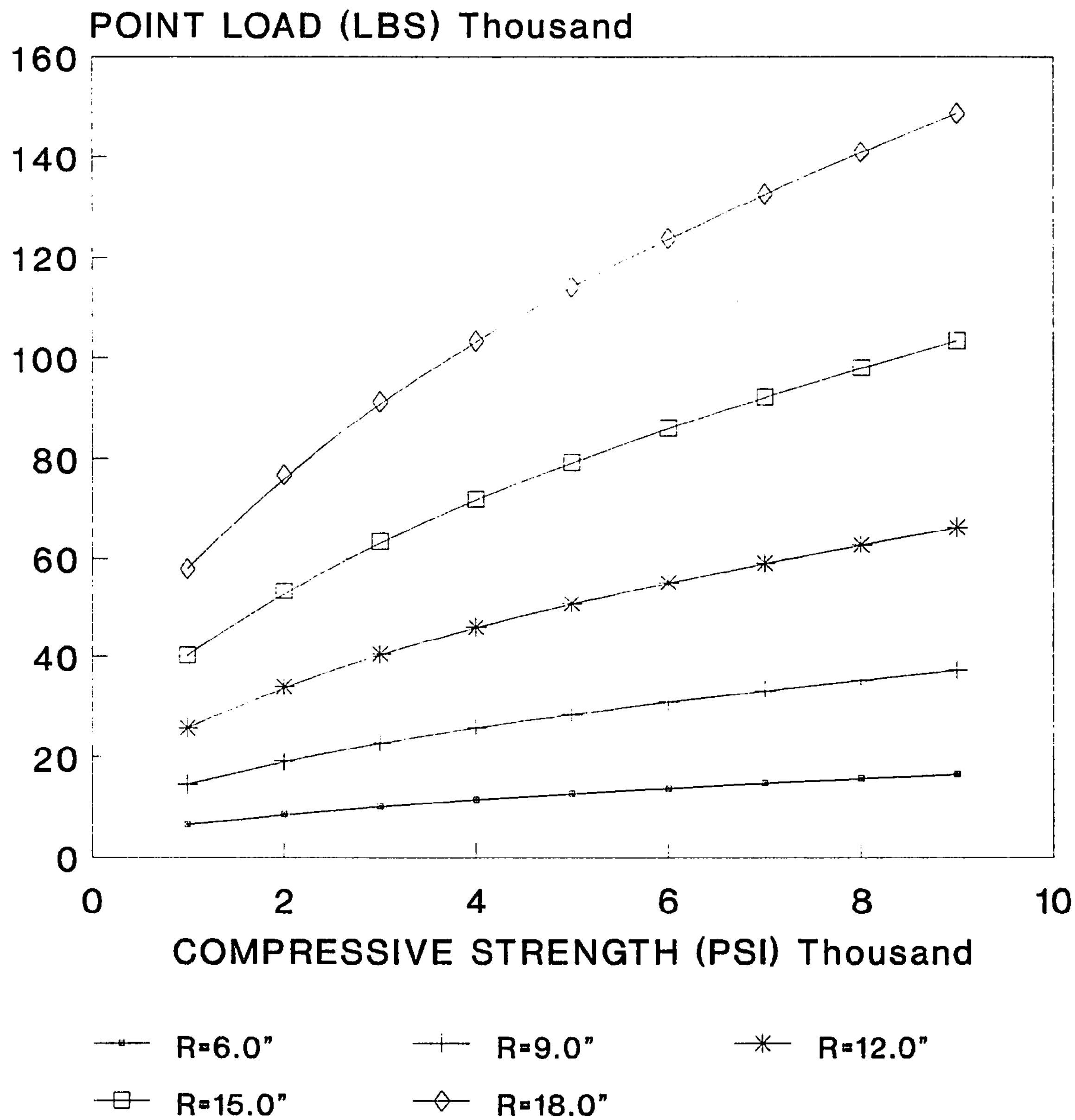
CAPACITY OF DEEP BEAMS CIRCULAR SECTION



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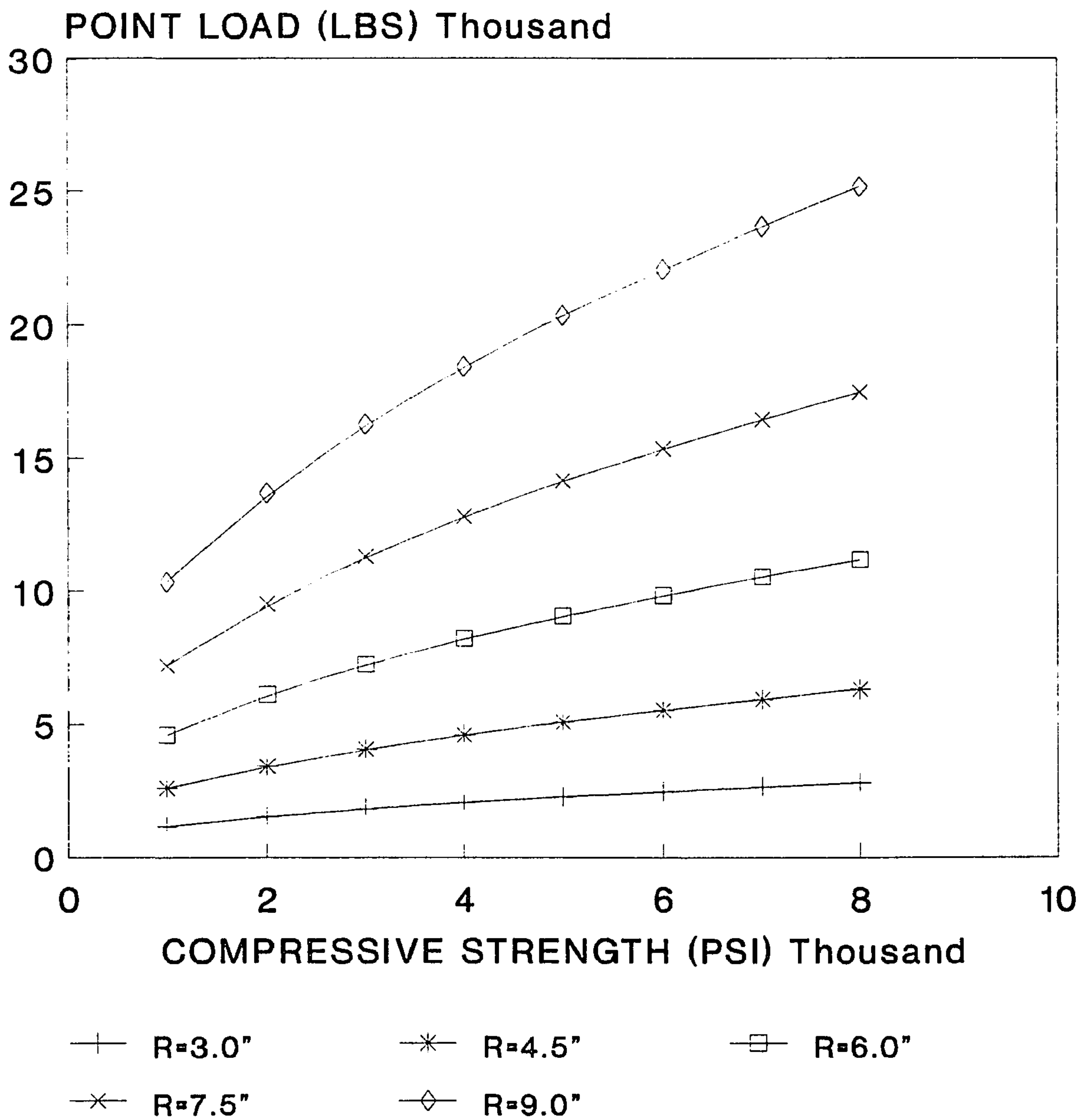
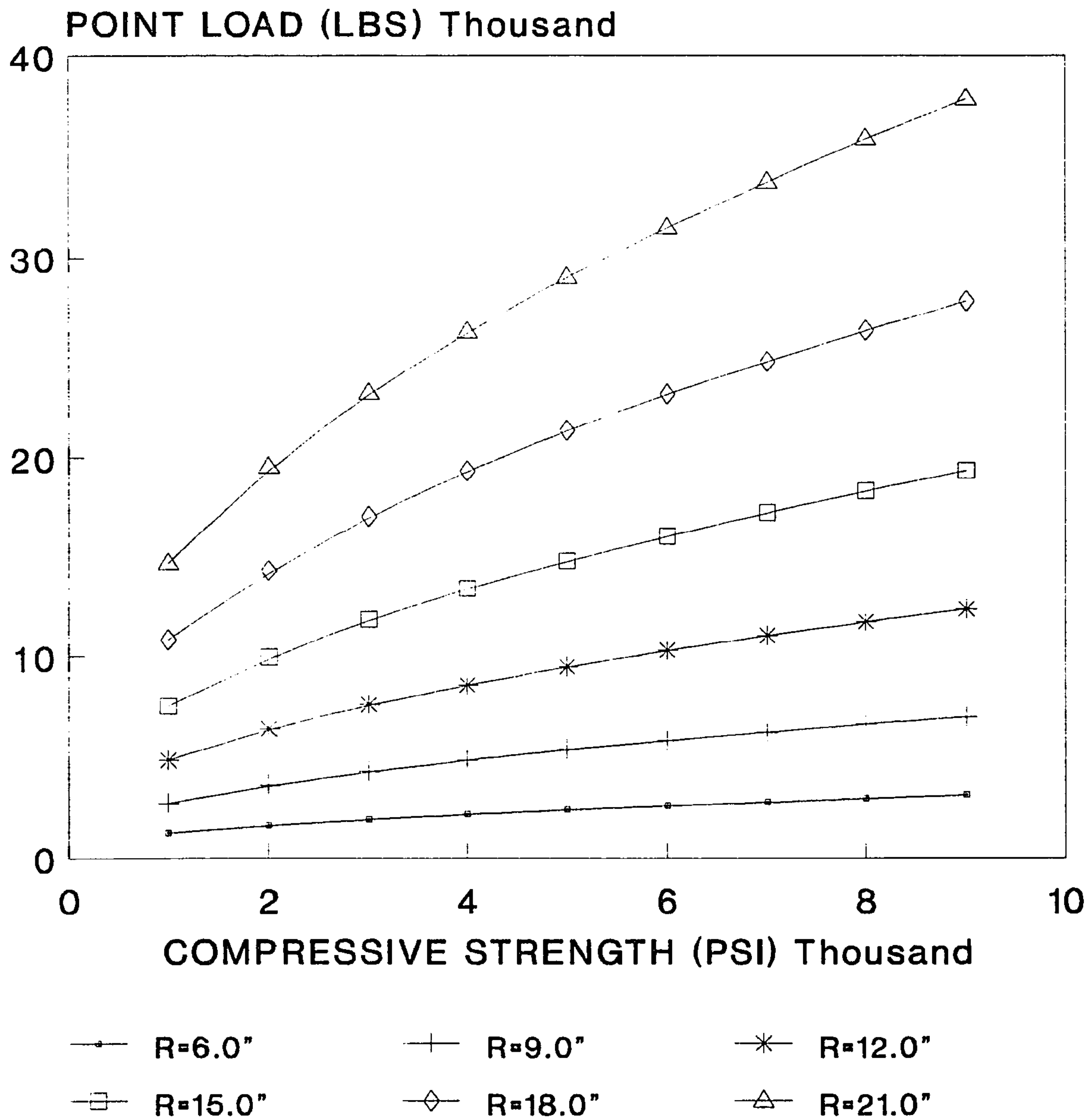
CAPACITY OF DEEP BEAMS
CIRCULAR SECTION



SIMPLY SUPPORTED, A.R.=4.0

CAPACITY OF DEEP BEAMS CIRCULAR SECTION



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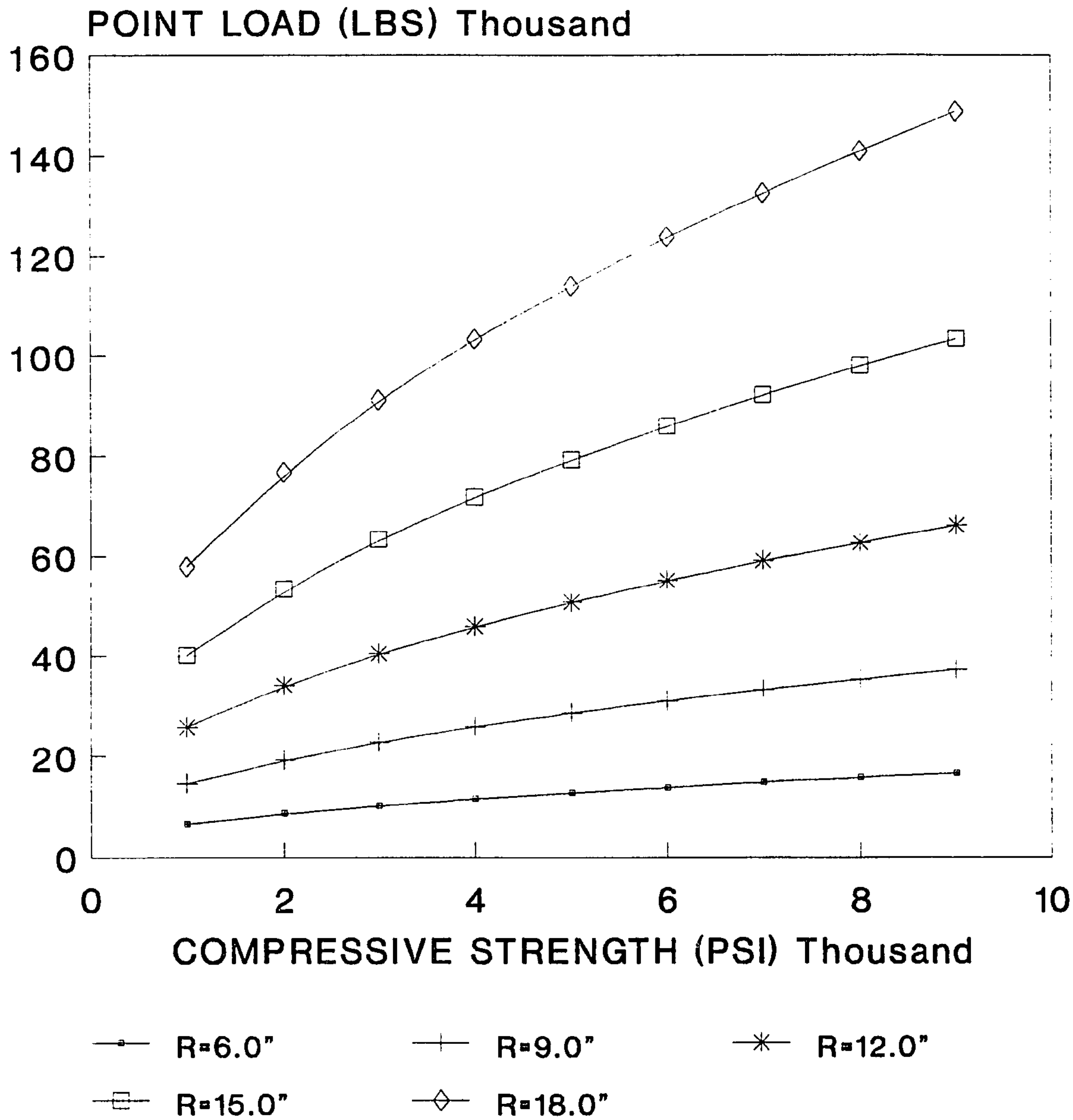
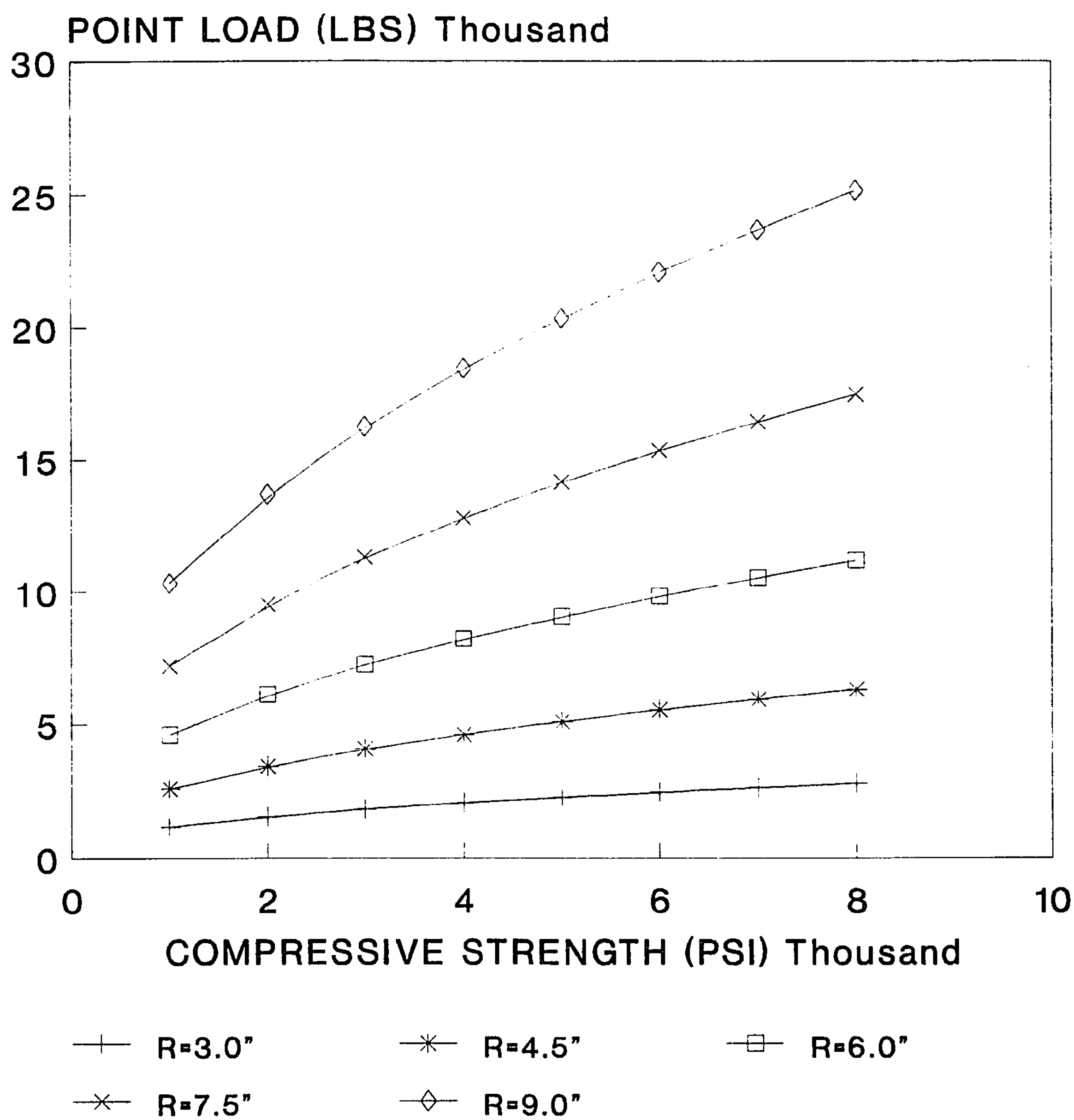
CAPACITY OF DEEP BEAMS
CIRCULAR SECTION



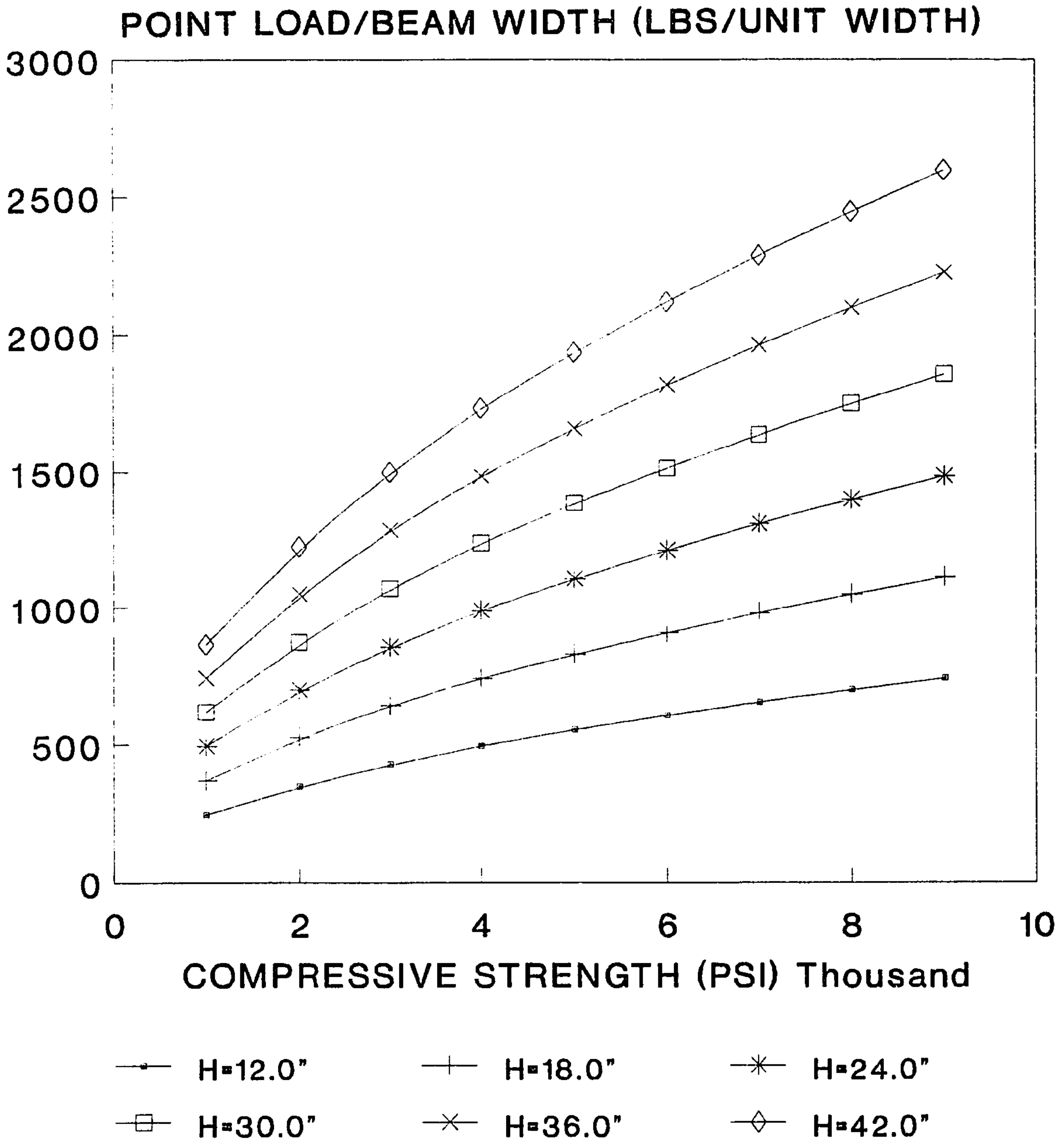
CANTILEVERED, A.R.=0.625

CAPACITY OF DEEP BEAMS CIRCULAR SECTION



CANTILEVERED, A.R.=1.0

CAPACITY OF DEEP BEAMS RECTANGULAR SECTION



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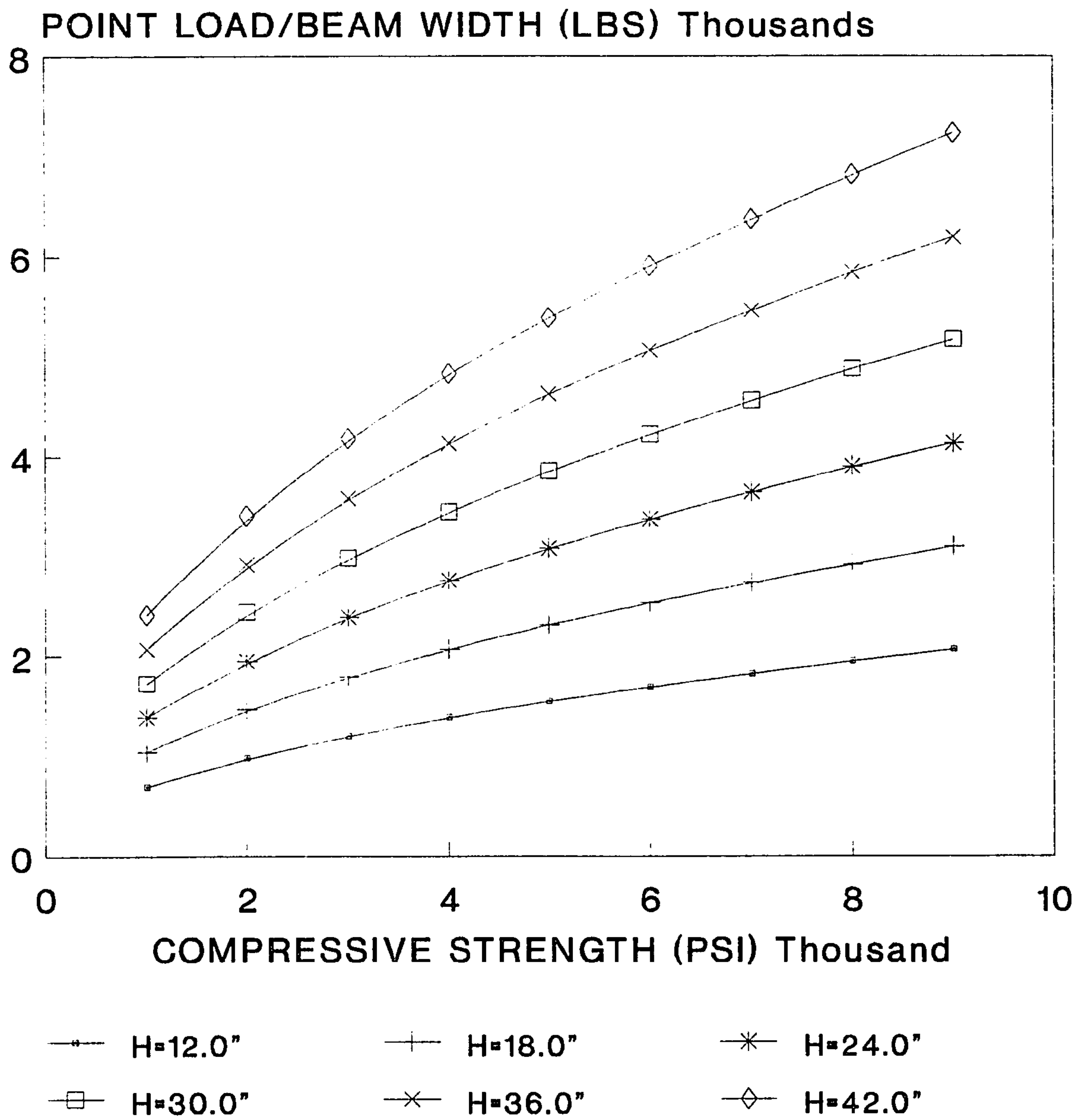

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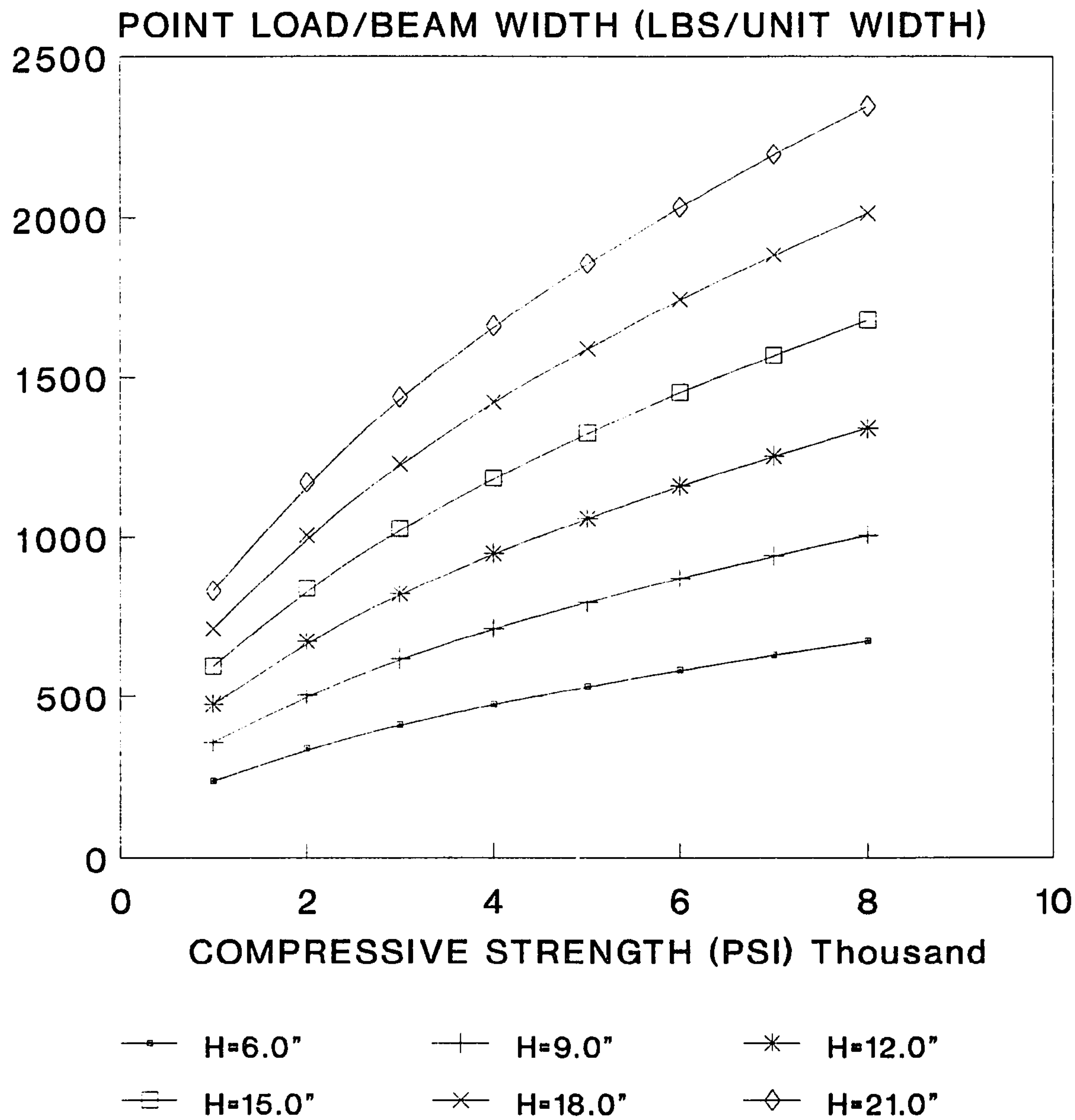

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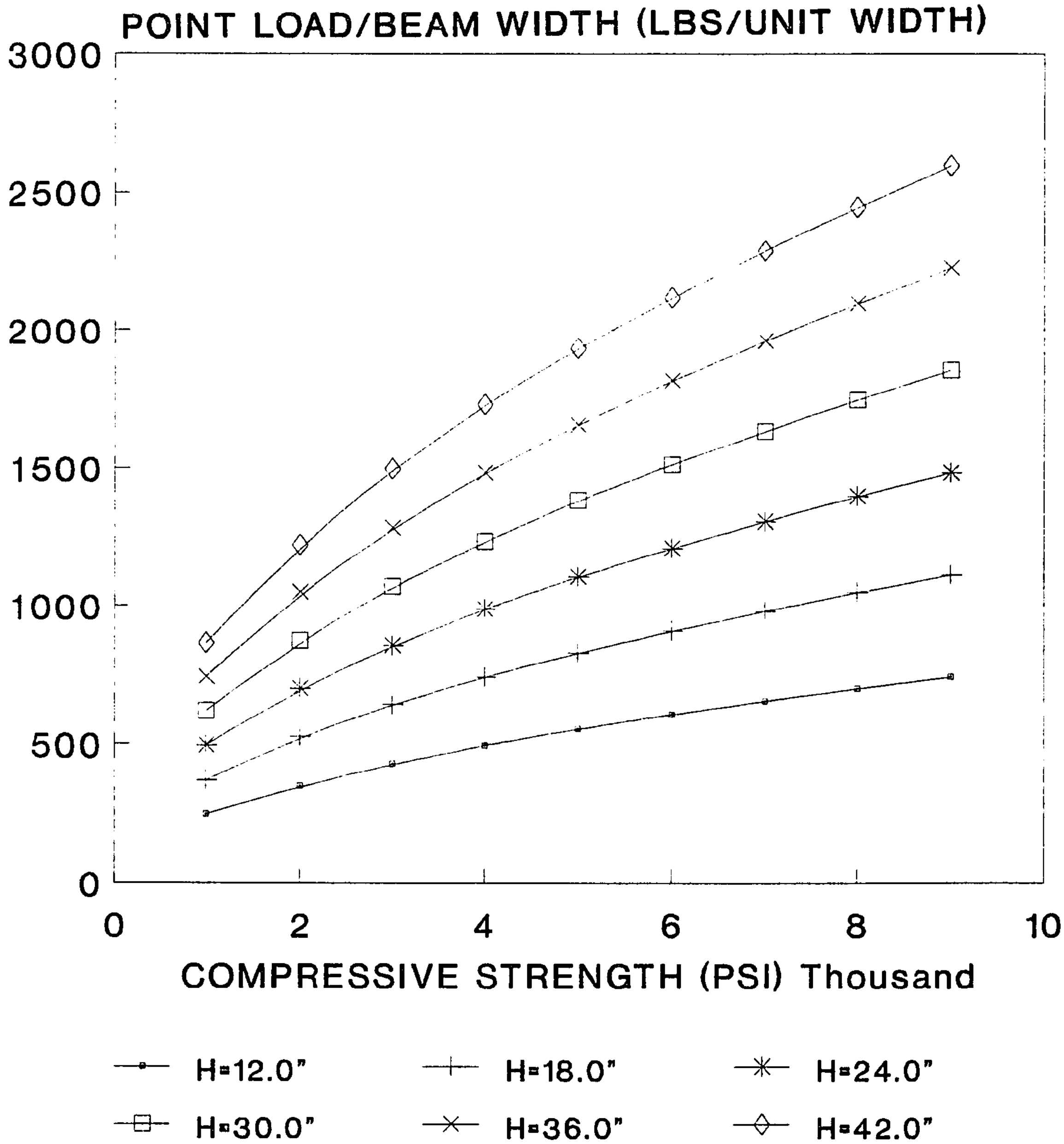

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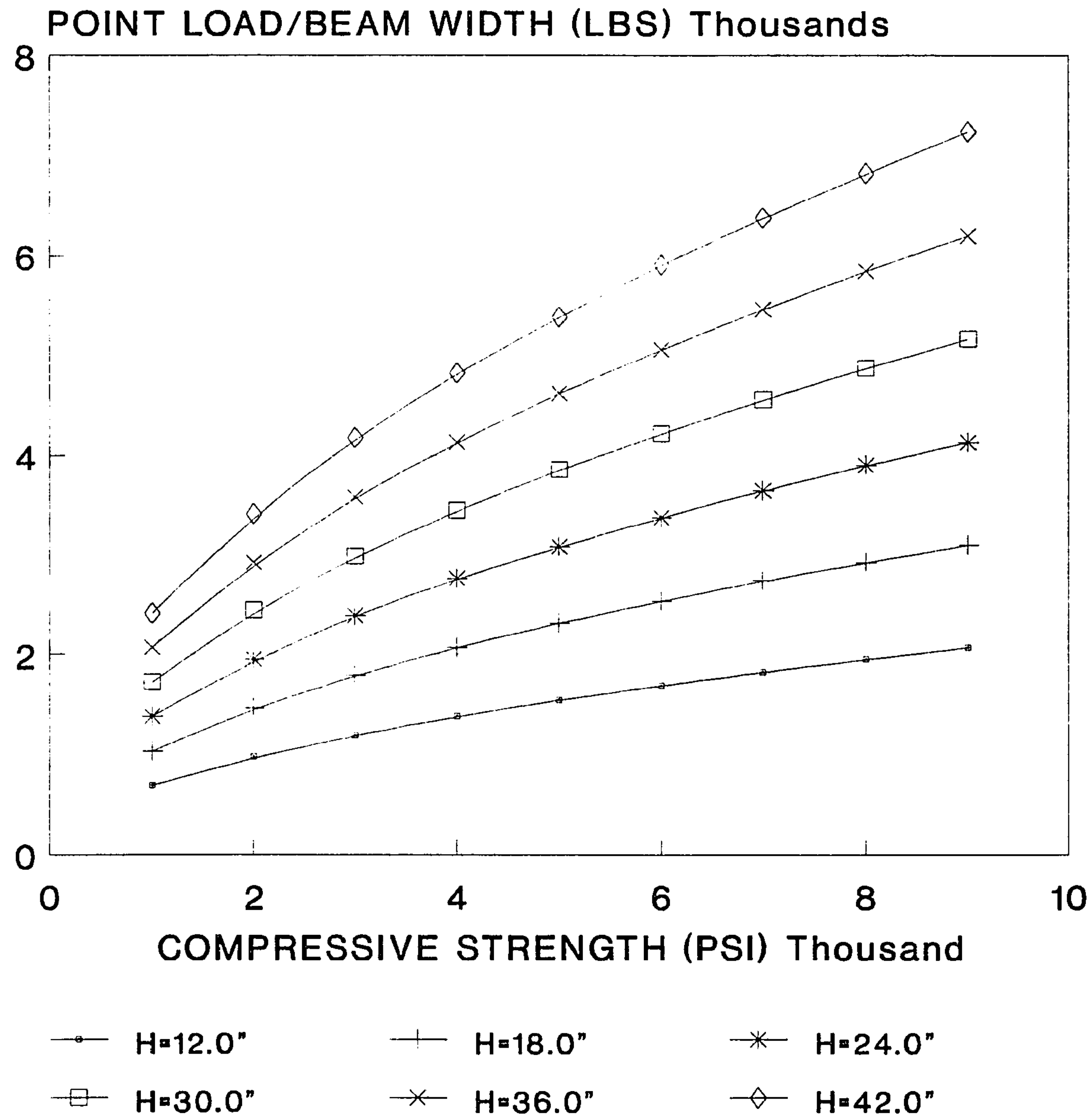
CAPACITY OF DEEP BEAMS
RECTANGULAR SECTION



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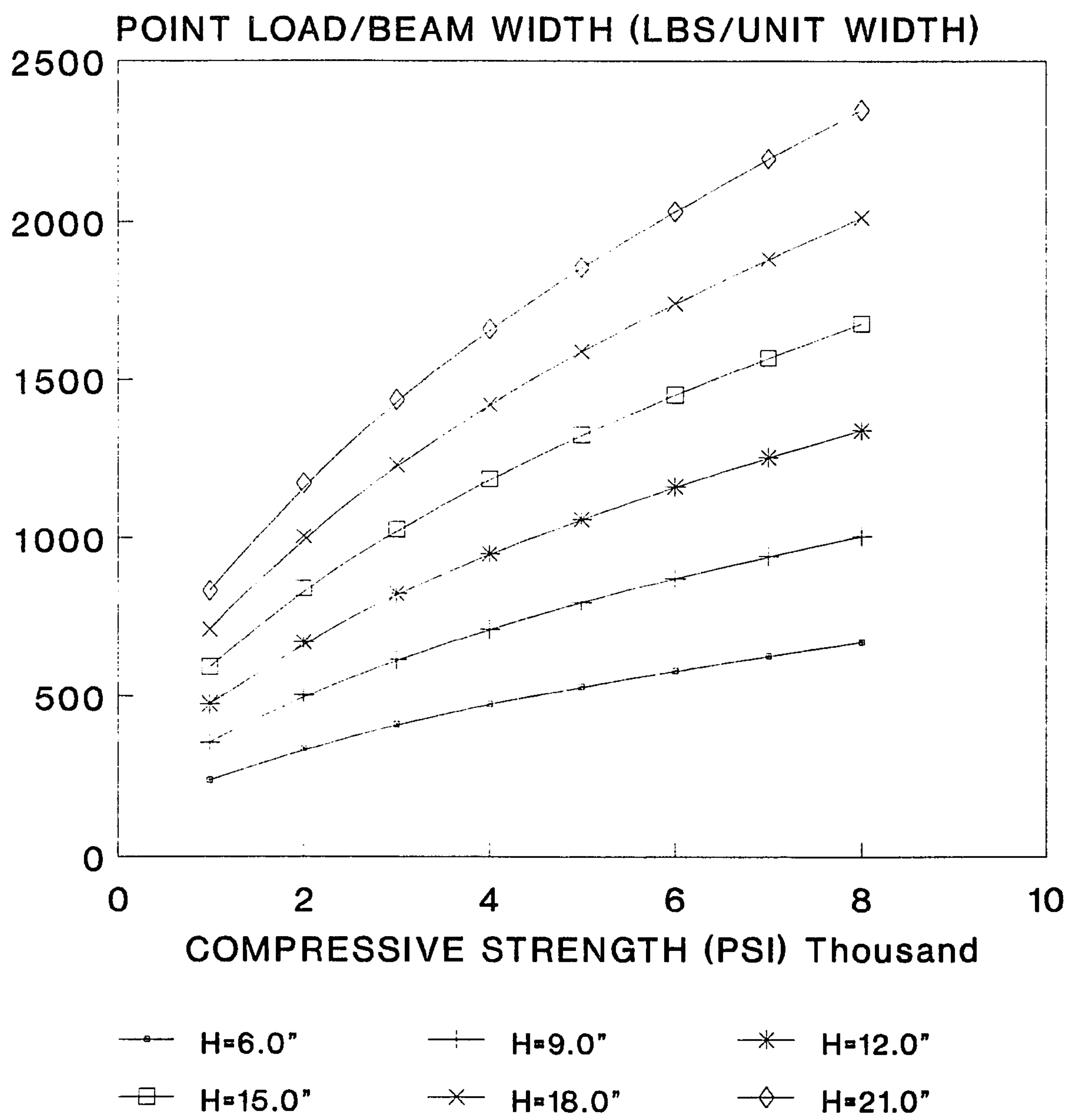
CAPACITY OF DEEP BEAMS RECTANGULAR SECTION



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