ABSTRACT

Cause and Effect Diagrams for
Process Control and Analysis

by
Philip Thomas

Cause and effect diagrams can be used as an effective tool for identifying the possible causes of a quality problem. Using this diagram one can go deep into the causes of the problem and take proper remedial action. Several graphical techniques like matrix diagrams, systematic diagrams, etc., can be combined with this tool to better represent the cause effect relationships. While a quantitative approach of this technique is feasible but limited, there are useful applications based on qualitative approach to this technique. The objective of this thesis is to propose a quantitative approach to the cause and effect diagrams especially for trouble shooting in quality control. We use Analysis of Means(ANOM), to further investigate the problem after its possible causes have been identified and to conduct a formal analysis of the main and interaction effects between different factors. Ease of interpretation and graphical presentation are considered as the major advantages of this technique. A case study is included to illustrate these concepts.

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FOR PROCESS ANALYSIS AND CONTROL

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Cause and Effect Diagrams for
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This thesis is dedicated to
my parents

ACKNOWLEDGEMENT

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CHAPTER 1

INTRODUCTION

1.1 Process Control

A process control system is essentially a feed back system consisting of four elements which include the process itself, information about the process, action taken on the process and action taken on the output from the process. Figure 1 shows how these elements are related each other. The process can be considered as a combination of man, machine, material, methods and environment that work together to produce a desired output. The performance information is obtained, in part, from evaluation of the process output. The output of the process includes products and also information about the operating state of the process. Based on the performance information, we take action on either process or output to improve the performance of the system. In a process control system, action taken on a process is future oriented in the sense that it will affect output yet to come, whereas action on the output is past oriented because it involves detecting out-of-specification output that has already been produced.

There has been a tendency in the past to concentrate attention on the past oriented strategy of inspection. With this approach, we wait until output has been produced, then the output is inspected and either accepted or rejected. It is obvious that this does not prevent substandard output in the future. With the new approach, statistical process control, we concentrate attention more on the process, which is future oriented. The process improvement involves reducing or removing the effect of potential causes of variability by modifying the process, to make it less sensitive to these causes. With most practical

applications, the number of possible causes for any problem is very large, thus making the analysis complex.

```
┌──────────────┐      ┌──────────────┐      ┌──────────────┐
│ Action on    │      │ Performance  │      │ Action on    │
│ the process  │──────│ Information  │──────│ the output   │
└──────────────┘      └──────────────┘      └──────────────┘
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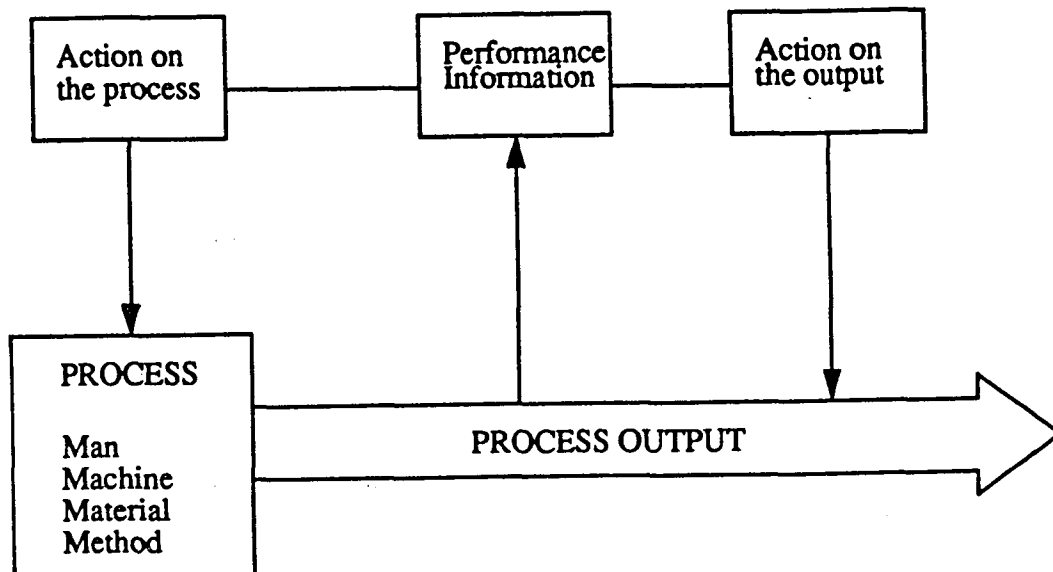
Figure 1 A Process Control System

In a continuous process, investigating every lot or large sample size is unnecessary. This approach is also not efficient because one may not have optimal guidelines for determining what sample size is large enough. If we start investigating every number that is larger than the average of the group, we will be chasing random fluctuation. This approach will cause embarrassment for the management and not good for the morale of the workman/repair crew. Whenever there is large fluctuation of an attribute due to a random cause, we will be searching for an `assignable cause' that does not exists and the employees will be annoyed by an investigation that yields nothing. The above reasons highlight the intricacy of investigating a process for finding the real cause for an abnormality.

Various methods like brainstorming, cause and effect analysis, check sheet, acceptance sampling, etc., are used in process control. These tools can

Problem Recognized by Management

Expected Time - 59 + 44 days
Expected Cost - $8520 + $6135

Reward for Good Job

Orders Come Down 'Fix it'

Series of Meetings
Bottom Line Test it out

Simple test is done

Analysis of Test Data And report

p=.3   Did a new problem result due to the 1st solution   p=.3   Did the right problem get solved   p=.2   Is the problem solved   No

No   Yes   Yes

Yes

Is problem solved forever   p=.2
No   Still more testing

Figure 2 System Flow Diagram for Jump on the Problem Method

be classified into two categories based on the purpose for which it is used. One category consists of techniques which are used for identification of a cause, whereas other category focuses on monitoring a process. Cause and effect analysis which belongs to the first category is a simple but powerful tool for identifying the cause of a quality problem. Generally it is used together with brainstorming to organize the information generated during the pre-experimental investigation period. It includes gathering and organizing possible reasons or causes of a problem, selecting the most probable cause and verifying possible causes until a valid cause and effect relationship, leading to a solution is established or leading to the choice of most appropriate experimental set-up.

There is a large amount of qualitative literature on how this tool can be used for process improvement. However, literature on `how a quantitative treatment can be done using this diagram' is limited. In addition to, just enumerating the possible causes and their inter-relationships, we are equally concerned about pinpointing the exact cause of a quality problem and the applicable remedies. This generally requires a formal scientific treatment of available data in addition to the informal qualitative approaches.

1.2 Need For Pre-experimental Investigation

Most quality control supervisors do not realize that a small investment in a good experiment is actually more cost effective than a series of `shot' in the dark attempts to fix problems. Barker[1985] compared the usual traditional approaches of dealing with quality problems by quality control supervisors and the statistical experimental design methods and reported the findings as in Figure 2 and Figure 3. He enhanced these examples by assigning chances of success based on actual experience. As seen in the figures, the result show

Problem
Recognized by
Management
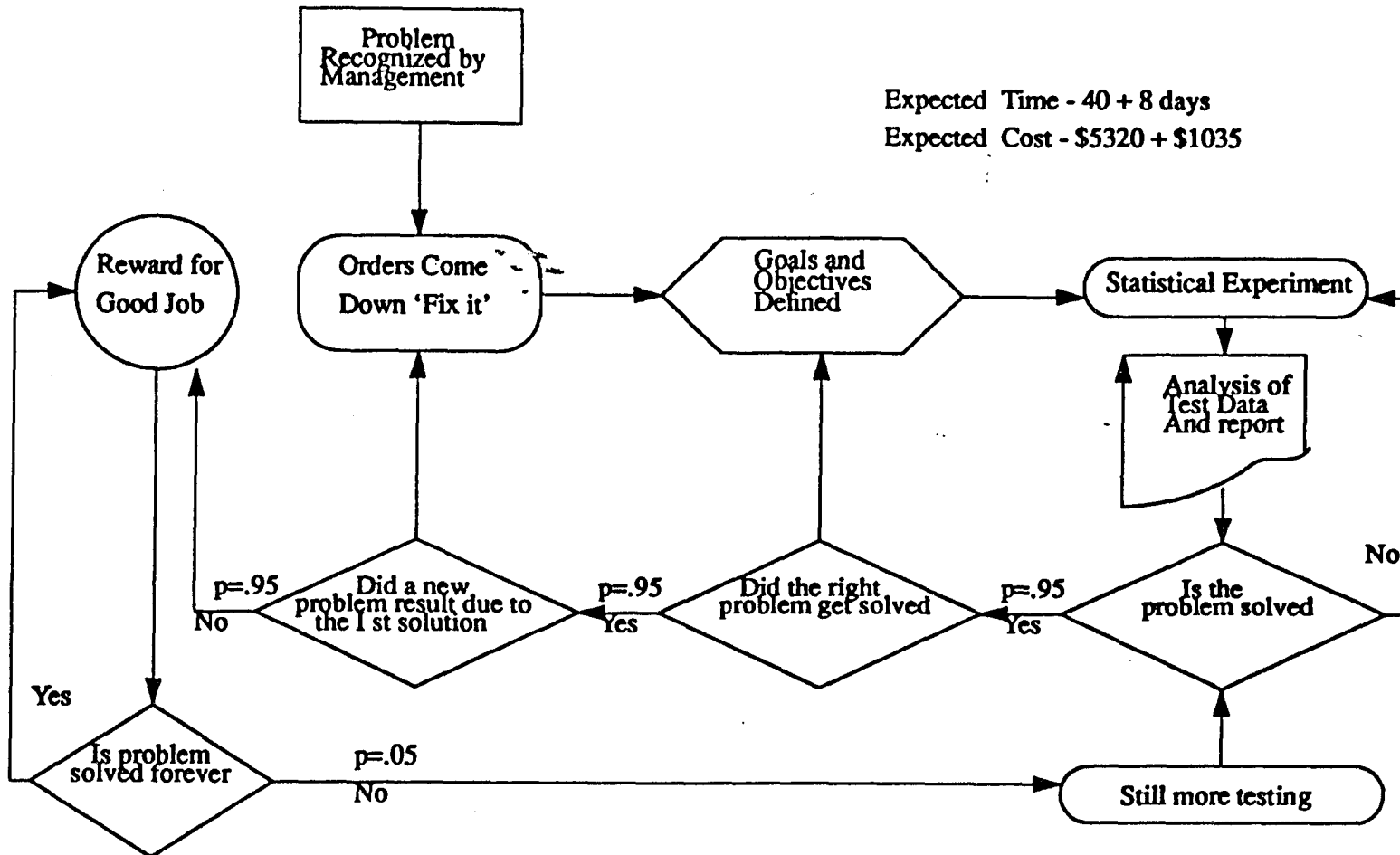
Expected Time - 40 + 8 days

Expected Cost - $5320 + $1035

Reward for
Good Job

Orders Come
Down 'Fix it'

Goals and
Objectives
Defined

Statistical Experiment

Analysis of
Test Data
And report

$p=.95$ Did a new
problem result due to
the 1 st solution

$p=.95$ Did the right
problem get solved

$p=.95$ Is the
problem solved

No

No

Yes

Yes

No

Yes

Is problem
solved forever

$p=.05$

No

Still more testing

Figure 3 System Flow Diagram for Statistical Experimental Design Method

that there is much better control and lower overall cost with the statistical approach to the design and analysis of experiments.

As seen in Figure 2, the chance of solving any problem is only about 20% and out of these 20%, the chances of solving the right problem is only 30%. Furthermore, in this traditional approach, there is also a high probability that a new problem will emerge as a result of the solution to the first problem. A computer simulation of this·`jump on the problem' method shows that there is a high cost in time (59 days) with a very high variance of 44 days by using this trial and error approach to quality problem solving.

In Figure 3 shows the simulated result for a statistically designed experiment as the investigation methodology. With this approach, the chances are much higher for solving the problem the first time. In fact, there is almost guaranteed (95%) chance to solve the right problem in only one try. This is because the statistical approach considers the whole situation and uses acceptable scientific procedures. With this approach, the time to do the job is reduced to 40 days and the variation is reduced to only 8 days.

1.3 Summary

In a process control system, action needs to be taken on process rather than on outputs so that it will prevent defect occurrence in future. This has not been the tendency in the past, and now more attention is being paid to actions on process to improve the performance of the system. A major hurdle in this strategy is identifying the exact cause of the problem. In this chapter, the significance of cause and effect analysis in determining the possible causes of a quality problem is discussed. Apart from a qualitative approach, we can also give a quantitative treatment to this technique. This calls for statistical experimental design method. We explained the significant difference between

decisions made on the basis of facts and data rather than on gut-feel and intuition.

The remaining part of this thesis is organized as follows: In chapter 2 we review the literature on cause and effect diagrams, and briefly introduce several types of cause and effect diagrams and their applications in various fields. Chapter 3 presents how cause and effect diagrams can be used as a decision tool. Three such cases are discussed. Procedure for a systematic analysis of cause and effect diagrams is presented in Chapter 4. Analysis of means which is an alternative to the analysis of variance for experimental design is discussed in detail in this chapter. Application of this techniques is illustrated in chapter 5 using a case study. Summary and conclusions are provided in chapter 6.

CHAPTER 2

CAUSE AND EFFECT DIAGRAMS - A REVIEW

2.1 Review of the Literature

Ishikawa [1976] suggested three methods for deriving the cause and effect diagrams depending on how we want to organize and arrange the various relationships between the causes and the effects. He also suggested the strong and weak points of each method. He has pointed out the limitation of this diagram in representing the problems that might occur due to the interaction of various causes.

A variant to cause and effect analysis was suggested by Ryuji Fukuda [1983]. In this technique, instead of expressing various factors such as skill, machine, material, etc., with a single definition, he proposed using short sentences to express the information. A diagram indicating the present problem will be displayed on a large poster and anyone who wants to contribute something towards a specific problem can write out his ideas and know-how on small cards, and put them on the diagrams. This information is available to be read by anyone at any time. Thus the use of cards enable all persons concerned to participate in solving problems without holding any specific meetings.

Kindlarski [1984] gave good illustration to this technique using various examples and case studies. He attempted to generate a quantitatively weighted graphical representation of the bonds of causation. He assigned special codes to individual twigs or branches of the diagram. Next, each nonconformity of the final product was carefully analyzed and assigned to the code matching one of the arrows of the diagram. After a period of time, the number of nonconformities assigned to each code was added up, and the thickness of the individual twigs

was made proportional to the number of nonconformities. While this technique gives likely sources of variation, there is a limitation that we cannot examine every rejected item for probable source of variation as it is uneconomical. Moreover in many cases, the source of variation would not be apparent in the examination.

Donald L. Dewar [1987] suggested a new problem solving technique called `The Murder Board", which uses a strategy that is somewhat different from cause and effect analysis. When using cause and effect analysis, the group first identifies the problem and then brainstorm to suggest for as many causes as possible. Then it assembles groupings of possible causes. Group participants offer free flowing ideas that form logical and interconnected patterns. The order of importance of causes often is determined by voting. In contrast, the `murder board' assumes that each idea is guilty until proven innocent. In this process, every possible negative feature of a cause is listed until the final consideration is presented. Participants on the murder board attempt to `murder' all possible causes, until one emerges beaten but not destroyed. The drawbacks of this strategy is that it requires a lot of time for implementation and its insensitivity to the person who makes a suggestion. However adding murder board techniques can energize cause and effect analysis.

Ophir [1988] used a case study approach to show how a quality problem can be solved using cause and effect analysis. A team was set up to investigate the root cause of `shots' in Nickel Cadmium cells. The team used (1) a Pareto chart to determine the most common types of the problem, (2) a cause and effect diagram to determine the factors to be examined and (3) experimental designs to determine the best way to manufacture the nickel cadmium cells.

Many other investigators such as Skrabec [1986], Kane [1989], Sarazen [1990], Jaehn [1990] etc. have described the potential usefulness of this tool for quality improvement. However their contributions are limited to few basic qualitative introductory concepts.

2.2 Types of Cause and Effect Diagrams

Cause and effect diagram was originally introduced by Prof. Kaoru Ishikawa of the University of Tokyo in 1943. It is basically a modified type of flow diagram where the specific problem called effect being investigated is placed in a box on the extreme right, and the potential contributing factors or causes are placed on the left. A large arrow pointing directly to the problem connects the suspected causes with the problem to be solved. Branches of the main arrow represent the contributing causes.

There are various methods for making cause and effect diagrams depending on how one organizes and arranges the causes in them. These methods can be divided into three following types: the dispersion analysis type, the production process classification type and the cause enumeration type.

2.2.1 Dispersion Analysis Type

The dispersion analysis diagram shown in Figure 4 is the most widely used of the three types of diagrams. The preparation of a dispersion analysis diagram first requires recording the main group of factors that influence the problem by their general category such as work methods, materials, equipment, measurements and environment. The next step is to enumerate all the detailed factors that would contribute to variability in each of the categories. These factors are then listed on the smaller connecting branches. The strong point of this type is that, since it involves the breaking of dispersion, it helps organize

and relate the factors for dispersion. The week point is that the form the diagram takes often depends on the person making it, thus, sometimes even small causes are not picked up.
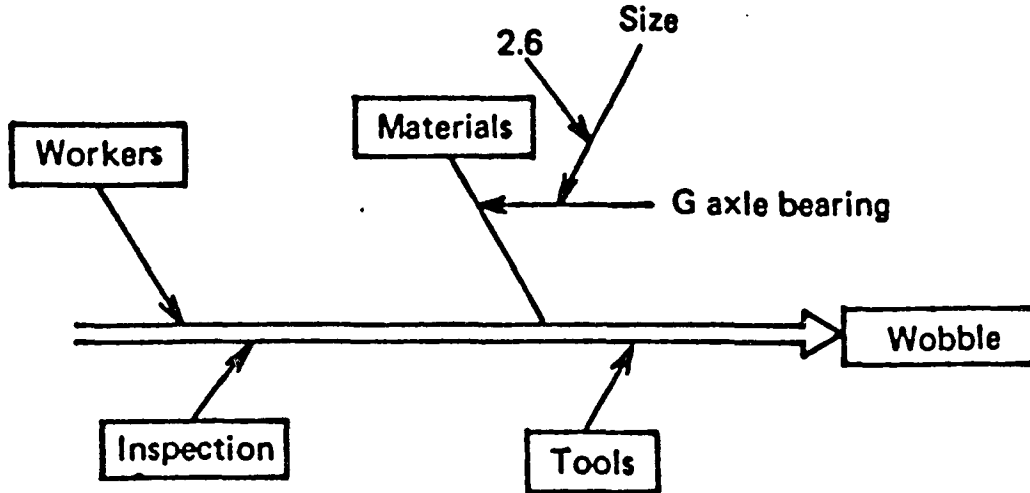
Figure 4 Dispersion Analysis Diagram
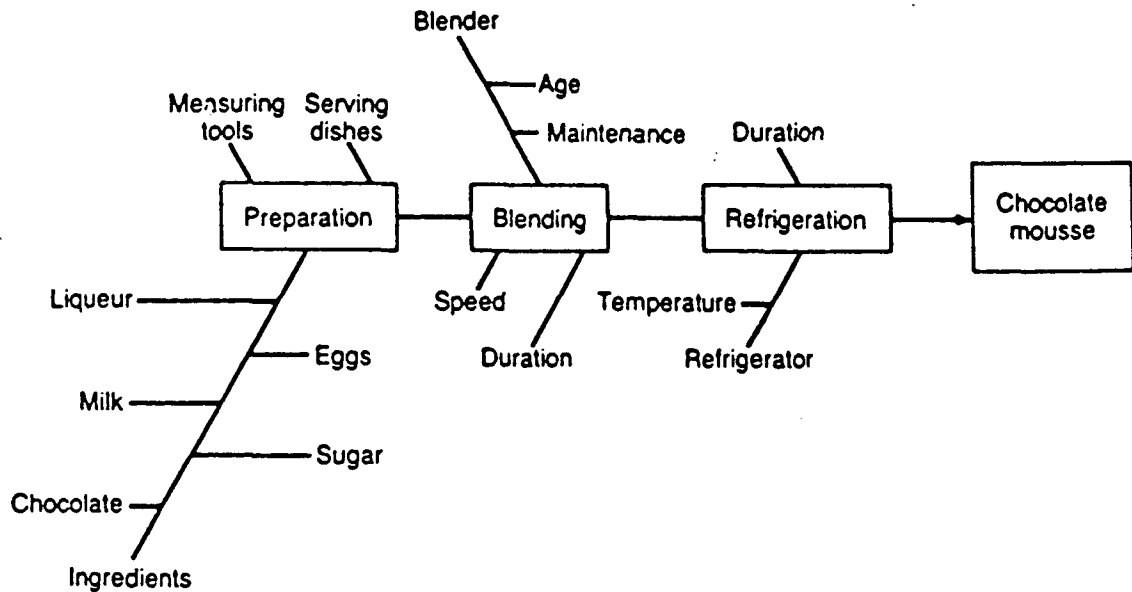
Figure 5 Production Process Classification Diagram

2.2.2 Production Process Classification Type

In this method, each processing area is identified and connected by arrows going from left to right, with the final arrow pointing to the description of the problem. At each step in the process, items that could influence the problem at that point are listed. The strong point of this type is that, since it follows the sequence of the production process, it is easy to make and understand. Its weak point is that similar causes may appear many times, and the causes due to the combination of more than one factor (ie., interaction effects) are difficult to explain. An example of the process classification type is shown in Figure 5. It illustrates the cause and effect diagram of a process for making chocolate mousse.

2.2.3 Cause Enumeration Type

In this type, all the possible causes are simply listed. These causes are then organized in accordance with the product quality showing the relationship between the cause and the effect and then a cause and effect diagram can be made. The advantage of this type is that all causes are listed and thus no major causes are missed. Also by considering the relationship between the cause and the effect, the diagram is quite complete. Its disadvantage is that it is difficult to relate the twig causes to the result and thus the diagram is difficult to draw.

2.3 Some Areas of Application of Cause and Effect Diagrams

Cause and effect diagrams are mainly used in brain storming sessions in which a group of problem solvers is seeking a solution to a quality related problem. The diagram provides a structured approach (ie., a road map) for listing all possible causes. This list is then analyzed and rationalized so that the most

likely causes emerge. It can also be used to estimate the expected resistance to a proposed problem remedy.

Cause and effect diagrams can be used in the development of new products. After the quality desired by users is defined based on research, market and engineering efforts, a planned quality is identified as the primary goal. However, we may not be able to achieve this planned quality due to several resource constraints. In such cases, cause and effect diagrams can be used to relate various resources to obtain the desired output.

Another worthwhile application of the cause and effect diagram is in planning activity. For this use, the main arrow points to what is expected to occur, while the branches show the activities required to achieve the result. These diagrams can also be helpful in finding the source of sudden, unexpected and unexplained good and bad process variations, and thus helps to achieve product improvement. In this case, instead of seeking solution to a problem, we are taking advantage of an unexpected opportunity to study the reason for the sudden improvement in the performance.

2.4 Limitations of Cause and Effect Diagrams

One of the main weakness of this diagram is that it assumes all the causes are independent and known. In practice, this may not be true. All the causes may not be known at the initial stage, especially when the problem is complex. Furthermore, it is not easy to establish whether the problem is due to assignable factors or random factors. Also these diagrams do not emphasize the relative weights of individual causes in the final effect.

As stated earlier, some quality related problem may be due to the effect of interaction between two or more factors. A primitive way to address this problem is to vary one factor, while keeping all the others constant. Similar

experiments are conducted with the remaining variables and the results are analyzed to find better levels of each variable. This method is generally inefficient since one may have to run the experiment several times to locate and confirm the exact cause. This procedure is expensive and time consuming in most cases. Also the result that we obtain may not be applicable in a noisy situation. For example, in certain cases, the number of factors involved may be large, and the causes may be due to interactions between two or more factors.

To study the effect of such interactions, one would need to conduct factorial experiments with multiple factors and multiple levels, and then compare the significance of each variable/interaction using statistical methods. Once the significant interaction is identified, it is possible to identify the cause/causes for a specific effect.

2.6 Graphical Representations

It is said that a picture is worth a thousand words. To understand a language, one must know some basic rules regarding its structuring, without which the language is totally incomprehensible. With a picture, the number of such constraints can be reduced to a manageable size, since it is easier for most people to understand pictures. Thus, it is evident that tools anchored in graphics will emerge as powerful techniques in the promotion of company wide total quality control. Two diagrams that could be used in conjunction with cause and effect diagram are systematic diagrams and matrix diagrams [Mizuno 1988].

2.6.1 The Systematic Diagram

A cause and effect diagram may be simple and effective. However, it is possible to encounter minor inconveniences in the following cases:

1. When causes at the sample level need to be compared, examined and evaluated.

2. When the influence of each cause is quantified and expressed in a diagram.

3. When the number of causes is very large.

In such cases, one solution is to arrange causes and effects in the form of a systematic diagram. A systematic diagram that expresses causes and their effects is called a cause and effect systematic diagram. Figure 6 shows how a cause and effect diagram and a cause and effect systematic diagram expresses an identical event or process in two different forms.
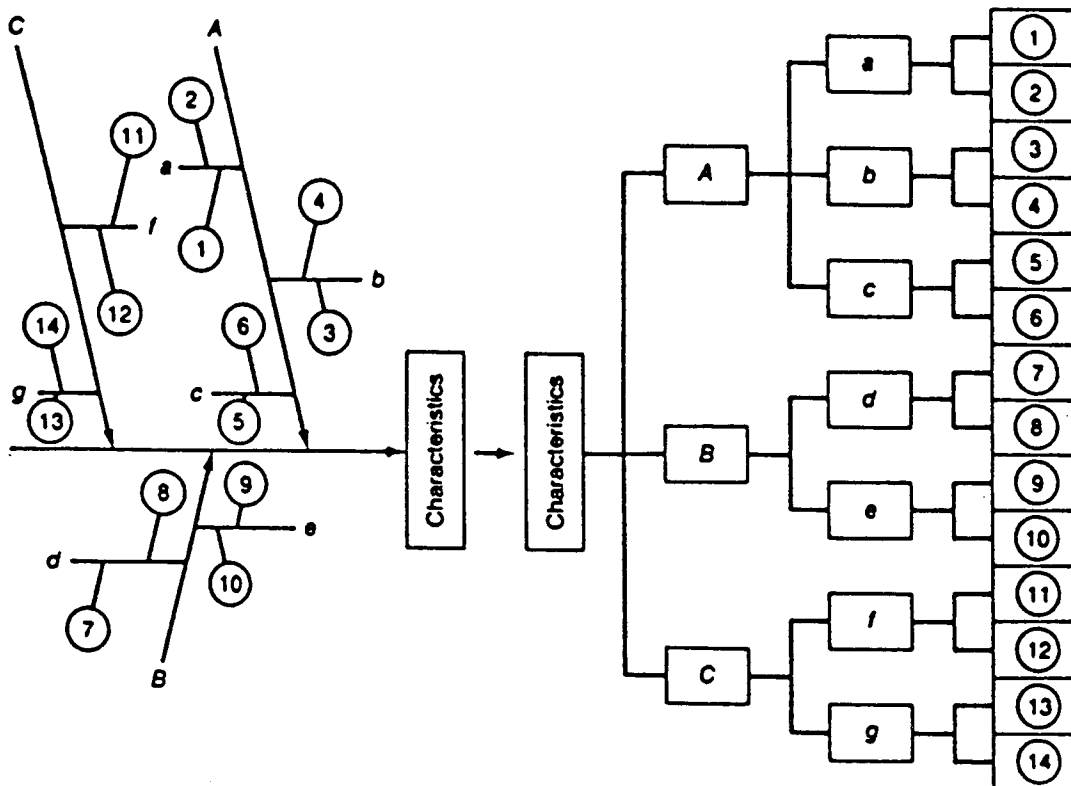
Figure 6 Cause and Effect Diagram and Systematic Diagram
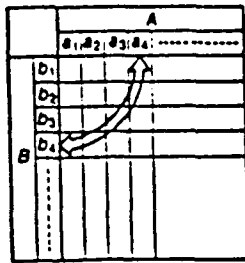
L - Type
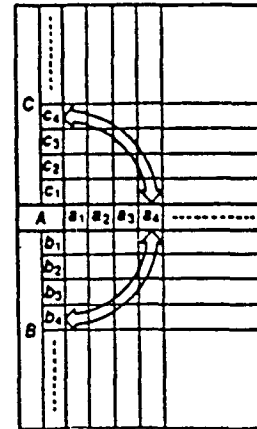
T - Type
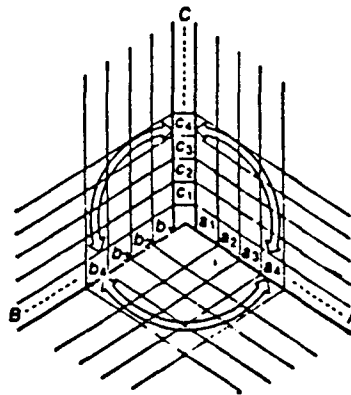
Y - Type
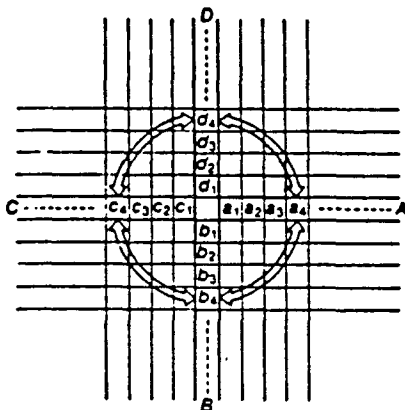
X - Type
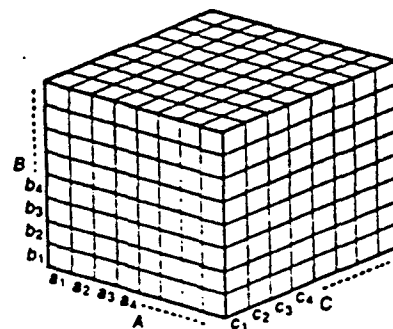
C - Type

Figure 7 Different Types of Matrix Diagrams

2.6.2 Matrix Diagram

The systematic diagram method is used to clarify a problem when upto two causes are involved in the problem relationships of which can be explained in one dimension. When there are more than two causes, we have we have to use matrix diagrams to correlate these causes each other. The matrix diagram helps to expedite the process of problem solving by indicating the presence and degree of the strength of relationships between two sets of factors. It identifies corresponding elements involved in a problem situation. These elements are arranged in rows and columns in a chart. On the basis of the pattern, matrix diagrams can be classified into different groups like L type matrix, T type matrix, Y type matrix , X type matrix and C type matrix. Figure 7 depicts the different types of matrix diagrams that can be used for representing the relationship between various factors.
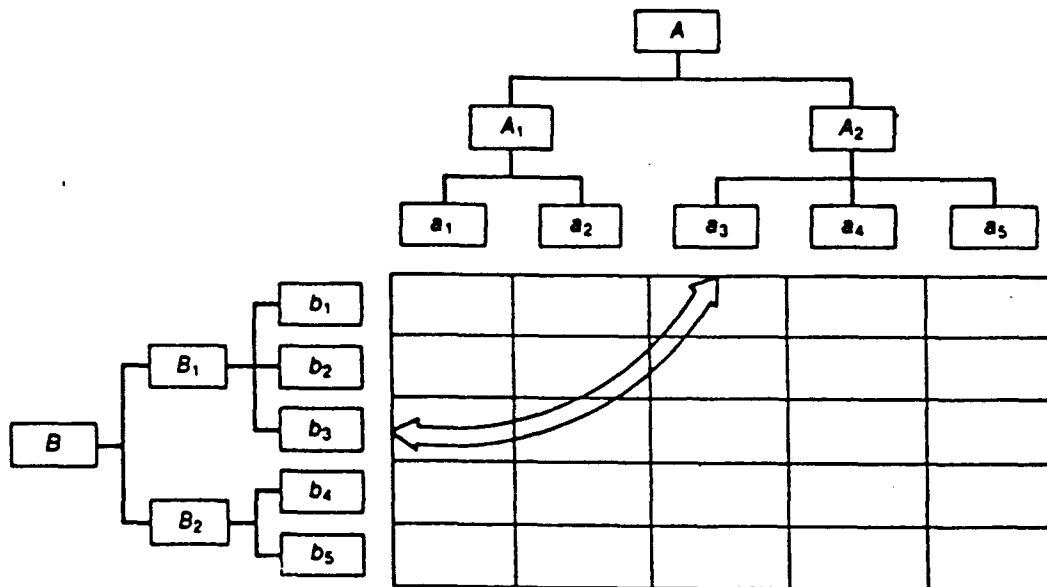
Figure 8 Combination of Systematic Diagram and Matrix Diagram

L type matrix is a basic matrix diagram with one set of data expressed in two dimensions employing rows and columns. The T type matrix combines a

matrix of A and B factors with a matrix of A and C factors. Y type matrix is a combination of three L type matrices. X type matrix shows the correspondence of four sets of factors. C type matrix is expressed in a rectangular cube whose sides are represented by three factors A, B, and C in a three dimensional space.

The most important point in designing a matrix is deciding how to combine the sets of phenomena and the factors that correspond to them. There is no single way to combine sets of phenomena because the combinations depend on the nature of the problem under study. Using a matrix diagram, the correspondence between the phenomena is shown; then the relevant factors are developed to illustrate their levels of significance. The systematic diagram is used to develop these factors. Figure 8 illustrates how a systematic diagram and a matrix diagram can be combined to represent the relationships more effectively.

2.7 Summary

In this chapter, we reviewed the literature on cause and effect diagrams and their applications in quality control area. We have highlighted the need for giving a quantitative approach to this technique especially for trouble shooting in quality control. Different methods of drawing cause and effect diagrams and their limitations are discussed. Representation of cause-effect relationships in the form of matrix diagrams helps easy comprehension by all concerned. Such matrix diagrams are also presented. In the next chapter, we discuss how cause and effect diagrams can be used as a decision tool.

CHAPTER 3

CAUSE AND EFFECT DIAGRAM AS A DECISION TOOL

3.1 Introduction

Cause and effect diagrams can be used as decision tools in many situations. Three such examples discussed in this chapter are Fault Tree Analysis, Decision Tree Analysis and Information Tree analysis. Fault tree analysis which is generally used in complex circuit analysis mainly focuses on assessing the probability of fault occurrence of each component in a system, and on locating the component with maximum likelihood of being faulty. Decision tree is a technique employed in decision theory to display the expected pay-off or penalties for each course of action in a complex decision making process. Using this method, one can explore the best course of action to get the desired effect. In this case, the best effect is maximum pay-off. In an information processing organization, the input requests are transformed into output responses through a set of procedures or processing rules. Information trees model such transformations in the organization.

3.2 Fault Tree Analysis

Fault tree analysis (FTA) is a graphical technique applied to products and systems for modeling the various faults that lead to the occurrence of a defined, undesired hazardous event. The analysis allows a product or system to be analyzed in the contexts of its environment and operation to find all possible ways in which the event of interest can occur. The fault tree also provides a visual representation of the logical inter-relationships between a specific failure events, and the ultimate effect it has upon the subject item. A basic fault reflects a specific part or component hardware failure(s), human error(s) or any other

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pertinent event(s) that can lead to a higher undesired event.

A key part of FTA is the definition of the top undesired event to focus on. The top event gradually reflects a complete or catastrophic failure of the item under consideration. The power of the fault tree structure comes from its quantitative and qualitative ability to asses criticality. A fault tree structure reflects a series of logic gates that serve to permit or inhibit the passage of fault logic up the tree to the top event. Fault trees use qualitative and/or quantitative evaluation, to provide an assessment of the probability of reaching the top undesired event. This analysis procedure enables corrective recommendations and actions to be readily formulated and prioritized to enhance product or system safety cost effectively.
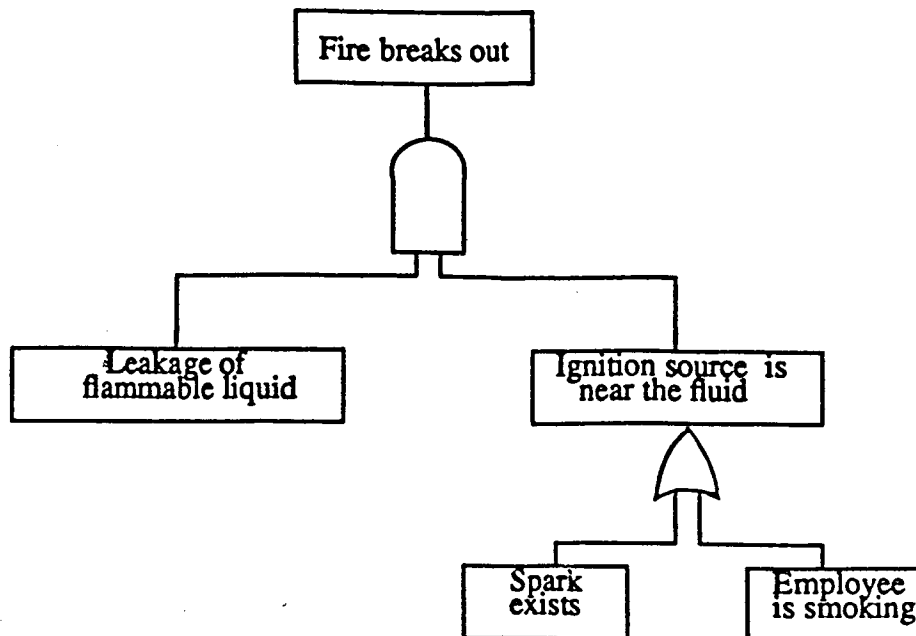
Figure 9 Fault Tree Analysis

A simplified example of a fault tree diagram is seen in Figure 9. The event `fire breaks out' happens when two events, `leakage of flammable fluid', and `ignition source is near the fluid' occur simultaneously. The latter event happens when either one of the two events, `spark exists' or `employee is

smoking' occurs. The cause and effect relationships between various elements can be either deterministic or stochastic. Also a particular top event may be one of many possible undesired events in the system.

3.3 Decision Tree Analysis

Decision tree is a technique used in decision analysis when a decision maker is faced with several decision alternatives and an uncertain or risk-filled pattern of future events. It provides a graphical representation of the decision making process in the form of a tree with numerous branches and twigs. It contains points that can be decided on by the decision makers indicated by [] and points that are decided on by outside elements by O. These points are connected one by one to form twigs and branches that represent various phases of the process, leading to a final result. If the probability of the events and the gain derived from the anticipated results can be quantified, the most suitable decision can be calculated.

A decision tree is suitable for analyzing decisions that extend over several related decision phases. It indicates the best course of action for the current decision. As time progresses, some uncertainties may be reduced and new ones may arise. Acts previously identified as optimal may turn out to be obviously poor choices, and brand new candidates may be determined. The relevant portion of the decision tree can be updated and revised prior to each new immediate decision. But each such decision is analyzed in the same general manner, using the best information available at the time the choice is made. Although a decision tree is analyzed using a backward process, the analysis is really forward looking because it indicates the optimal course of action to take when future decision points are reached.
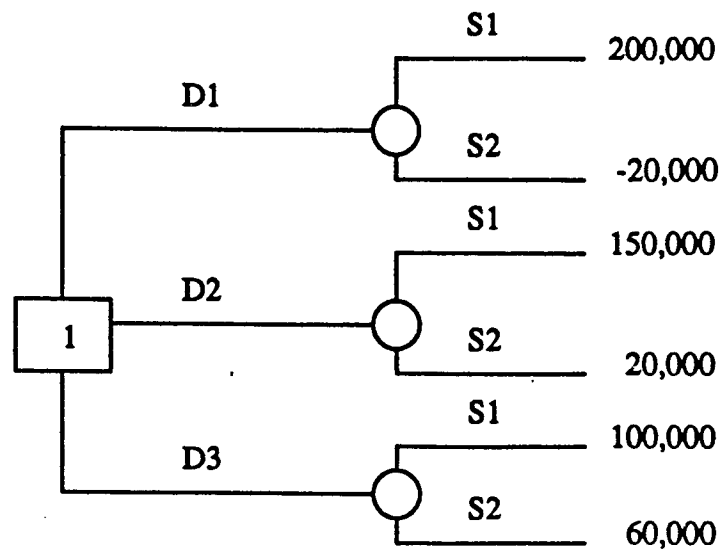
Figure 10 Decision Tree Analysis

Figure 10 shows a decision tree for a computer leasing problem. A decision has to be made for selecting a computer system for a firm. The firm has decided on a computer manufacturer, it is currently attempting to determine the size of the computer system that would satisfy the functional requirements and is economical to lease. The three decision alternatives, denoted by D_1, D_2, and D_3 are as follows:

D_1 = lease a large computer system

D_2 = lease a medium sized computer system

D_3 = lease a small computer system

The future events that might occur referred as states of nature in each case is uncertain. However it is defined so that one and only one of the listed state of nature will occur. The two possible alternatives in each case are high acceptance and low acceptance by the customer. Then the state of nature denoted by S_1 and S_2 are as follows:

S_1 = high customer acceptance

$S_2 =$ low customer acceptance

These decision alternatives and states of nature are represented in the decision tree using the symbols [] and O. First the firm must make a decision (D_1, D_2 or D_3) then, once the decision is implemented, the state of nature of S_1 or S_2 will occur. The number at each end point of the tree represents the pay-off associated with a particular choice of events. In many decision making situations, it is possible to obtain probability estimates for each of the possible states of nature. When such probabilities are available, the expected value approach can be used to identify the best decision alternative.

3.4 Information Tree

A complex organization can be viewed as an information processing system. Such organizations are collections of people, equipment, activities and procedures, that receive information inputs from the surrounding environment and produce information outputs to other organizations or individuals. Generally, an information tree represents the information flow through the organization as inputs are transformed into outputs [Feinstein 1988].

Information trees consist of three elements: information entities, information transformation functions, and procedures. Examples of information entities are the people and files of an organization that can communicate to store information. It is natural to characterize an entity by its state of information. The state of information specifies what an entity knows at any point in time. Information transformation functions are mechanisms for updating entity information states. Information trees use three such functions for changing such entity states. They are communication, observation and creation. Communication refers to the relationship between two entities for updating the information. Observation occurs when an entity in an organization receives

some information from outside the organization. Creation occurs when a new information is created based on some previous information. The three information transformation functions are distinguished by the origin of the information that updates an entity state. In communication, the information comes from some other entity within the organization, in observation the information comes from outside the organization and in creation, the information is self generated.

The third and final element in the information tree model is procedure. A procedure specifies how information transformation functions are connected in an actual organization to enable the organization to generate specific outputs in response to a given set of inputs.

3.5 Summary

Three examples in which cause and effect diagram is used as a decision tool is presented in this chapter. They are Fault Tree Analysis(FTA), Decision Tree Analysis(DTA) and Information Tree Analysis. In all the above cases we showed how to systematically analyze the factors or components leading to a specific effect. In the next chapter, we present different procedures for analyzing a cause and effect diagram.

CHAPTER 4

PROCEDURE TO ANALYZE CAUSE & EFFECT DIAGRAMS

4.1 Introduction

Methods to analyze cause & effect diagram can be classified into two groups. These are the group using a qualitative approach and the group using an experimental approach. The question of following a particular method depends on how deep one wants to go to analyze the cause and effect relationships. As mentioned in section 1.2, a qualitative approach to this problem is easy but the likelihood of arriving at the correct solution without opening a new problem is very small. However this is the widely used technique. The two methods other than the qualitative approach to analyze cause and effect diagrams are discussed in the following paragraphs.

4.2 Experimental Design Method

Design of experiments can be successfully used to trouble shoot a problem by varying the factors. In this method failure possibilities are induced at will in order to understand the sources of failure or variations. The steps to be followed are: (1) Identify the important variables such as product or process parameters, raw materials or subassemblies from suppliers and factors associated with the measuring equipments. (2) Determine the significance of these variables-generally no more than one to four - important variables. (3) Vary the operating levels of these significant factors and determine the optimal level (lowest defect rate). Now, different statistical techniques can be used to determine the significance of each variable. Analysis of Variance (ANOVA) and Analysis of Means (ANOM) are the two techniques which are generally used for this purpose.

4.2.1 Factorial Experiment

A factor is a variable which may or may not have effect on the characteristics to be plotted, but which has been selected as an object of study in the experiment. We run the experiment to discover the possible effect of one or more factors. An experiment containing n variables is called a n-factorial experiment.

Factors are generally classified as fixed or random. When there is only one factor, the classification does not have any effect on how the data are analyzed but it does make a difference when there are two or more factors. A factor is fixed if the levels of a factor are predetermined, and the experimenter is interested only in those particular levels. A factor is classified as random if the levels are selected at random from a number of levels, and the inference is to apply to this population of levels rather than the particular levels used in the experiment. This is the case with the fixed factors.

A level is a particular condition or state of one of the factors being studied. The different conditions, or states of the same variable, are called its different levels. The presence of a certain condition may be considered as one level, and its absence may be considered as another level.

A main effect is a simple, direct and consistent effect on the characteristic being investigated. Sometimes variables may not have a particular effect when acting alone, but they produce significant effect when acting in combination with other variables. Such variations are said to exhibit 'interactions' rather than main effects.

A factorial experiment is one in which various levels of a given factor are combined with various levels of every other factor in the experiment. A factorial experiment can be either full a factorial experiment or it can be a fractional factorial experiment. In a full factorial experiment, every one of the chosen variables is tested with all levels of every other variable. Thus all possible
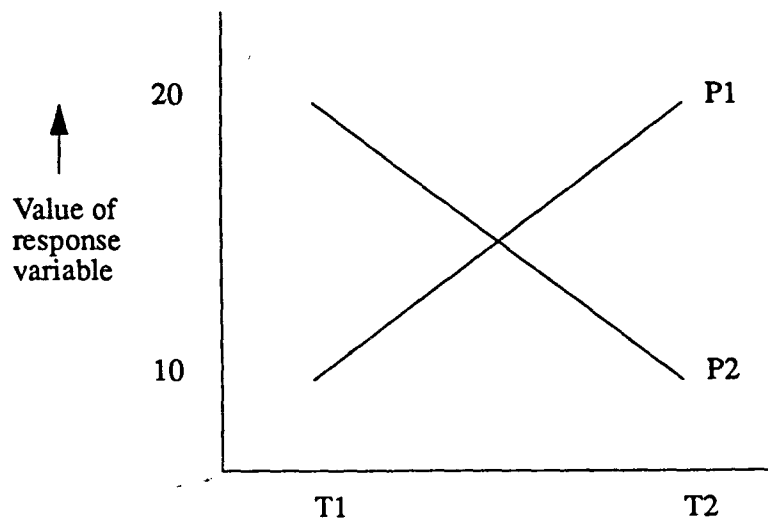
Figure 11 Main Effect Illustration
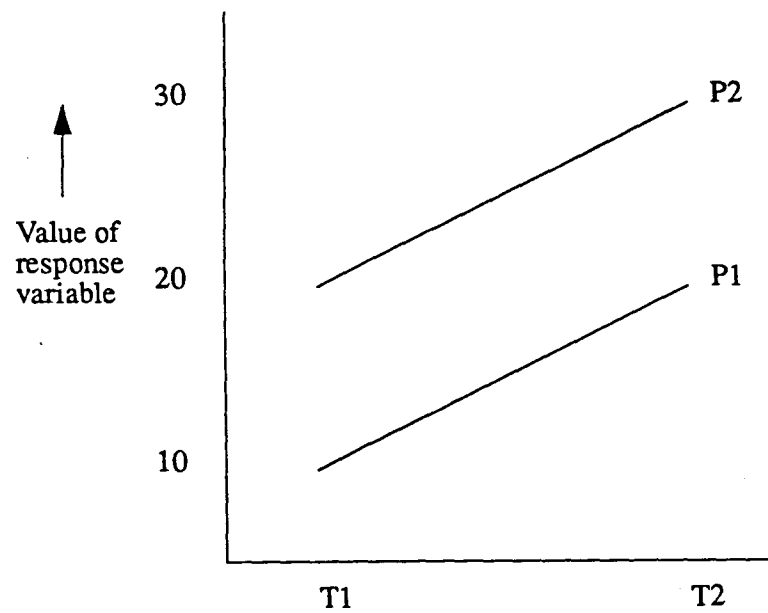
Figure 12 Interaction Effect Illustration

combinations of factors and levels are tested, allowing for the systematic separation and quantification of all main effects as well as interaction effects at multi-orders. In a fractional factorial experiment only a fraction of the total combinations will be tried to estimate the main effects and interaction effects.

Factors can be grouped into columns in such a way that we can extract much more precise information than if the experiments were conducted with a single factor alone. The matrix developed in such a way is termed as `orthogonal array'. In every pair of such orthogonal columns, all combinations of different factor levels occur at same number of times. In the orthogonal array method, many factors are studied simultaneously. In successive tests, we change the values of many variables. A suitable orthogonal array is used to determine how these values should be changed, so that the effect of each variable can be separated from the others at the end of the study. Furthermore, some needed information on `interaction' between variables can also be obtained from this type of experiment.

4.2.2 Effect Estimates

An effect of a factor can be defined as the change in the response variable that results from a change in the level of that factor. Estimation of such an effect can be explained using Figure 11. As can be seen from this figure, if the pressure is set at P_1 and the temperature is increased from T_1 to T_2, the value of the response variable increases by 10 units. On the other hand, if the pressure is set at P_2 and the temperature is increased from T_1 to T_2, the value of the response variable decreases by 10 units. Then the average of these two changes can be considered as the temperature effect given by,

$$T = 1/2 \left(T_2 P_1 - T_1 P_1 + T_2 P_2 - T_1 P_2 \right)$$

Similarly the pressure effect would be

$$P = 1/2 \{ T_1P_2 - T_1P_1 + T_2P_2 - T_2P_1 \}$$

The interaction effect can be explained using Figure 12. While Figure 12 may look quite different from Figure 11, only one change has been made, namely T_2P_2 was changed from 10 to 30. We know that interaction results from nonparallelism of these two lines. Thus the extent of interaction depends on

$$(T_2P_2 - P_2T_1) - \{T_1P_2 - T_1P_1\}.$$

Accordingly, the interaction effect can be estimated as

$$TP = 1/2[(T_2P_2 - T_2P_1) - (T_1P_2 - T_1P_1)],$$
$$TP = 1/2\{T_2P_2 - T_2P_1 - T_1P_2 + T_1P_1\}.$$

4.2.3 Analysis of Variance

The purpose of ANOVA is to separate the total variability of the data, which is measured by the sum of the squared deviations from an overall mean value, into contributions by each of the factors and the error. To see which of the factors have a significant effect, F tests are performed. In performing the standard F test, we assume that the errors are normally distributed with equal variance and are independent. The result of the F tests are indicated by the significance level. When we say that a factor is significant at 5% level, we mean that there is 5% or less chance that, if we change the level of the factor, the response will remain the same. If the F test indicates that a factor is not significant at the 5% level, it means that if we change the level of the factor, there is more than 5% chance that the response will remain unchanged. It is mentioned that 5% is traditional, although arbitrary. Other levels generally used include 1% and 10%.

The levels of factors which are identified as significant are then set to obtain the best performance. The levels of the other factors can be set at any level within the experimental range. If the assumptions of the F test are not

completely satisfied, the calculated significance will not be accurate. However, the standard F test is relatively insensitive to deviations from the assumptions used in its derivations.

In studying the cause and effect relationships, we can consider the main causes as `factors' and the subcauses as `levels'. An ANOVA analysis should indicate the most significant cause for a specific effect. This main cause could be a factor at a particular level, or it could be an effect due to `interaction' of two or more factors.

4.3 Graphical Method - Analysis of Mean

Analysis of Means (ANOM) was developed by E.R Ott [1967]. The original concept has been extended by Schilling[1973] to what he called Analysis of Means for Treatment Effects [ANOME]. It is an alternative to the analysis of variance for experimental design with factors at fixed levels. Ease of interpretation and graphical presentation of results is considered as a major advantage of this method. An ANOM chart is conceptually similar to a control chart but uses decision lines instead of control lines so that magnitude differences and statistical significance of the treatments can be assessed simultaneously.

Control charts are designed to control process quality as a process is run. This is done by examining each successive point in the context of a fixed standard and the preceding pattern of points. By contrast, ANOM examines a group of points in an effort to find underlying causes of problems. In ANOM, there is usually no fixed or known standard; the mean and variability are estimated from the sample data collected during the experiment. Ramig [1983] has given a six step procedure for performing ANOM with either variables or attribute data. These steps are discussed in section 4.3.3

4.3.1 Comparing ANOM & ANOVA

Both ANOM and ANOVA compare the effects observed in designed experiments. These designed experiments modify simultaneously different factors (suspected of contributing to the variation of a process) in a planned fashion to observe and obtain data about the response of the population to the modifications. The standard method of comparing such effects is ANOVA. ANOM could be considered as a simplification of ANOVA which include a graphical comparison of effects as part of the experimental results. Following are the advantages of using ANOM :

1. It provides a direct study of possible effects of the factors by dealing directly with the means instead of the variances. Thus ANOM provides a comparison of the relative importance and magnitude of the factors as well as their statistical significance.

2. It provides a graphical comparison of effects. A graphical presentation of data is almost a necessity when interpreting the meaning of 'interactions' whose presents have been indicated by an ANOVA. A primary function of industrial experimentation is not only to obtain information, but to present it in a way which will be accepted as a basis for decision and action by appropriate technical and administrative personnel.

3. It provides a pin-pointing of sources of non randomness. An analysis of variance may indicate certain factors which affect the response being studied; this analysis must usually be followed by some supplementary analysis to pin-point the important factors.

4. ANOM is more sensitive in detecting the non randomness of a single mean than ANOVA. When (k-1) sample averages are pooled together while the kth sample average differs considerably from the (k-1) averages, the F value in

ANOVA would likely be relatively small (thus indicating that population means are equal). Such a difference would most likely be detected using ANOM. However, if the differences between the adjacent sample averages are both sizable and similar, the likely difference in the population means is more apt to be detected with ANOVA than with ANOM.

4.3.2 Degrees of Freedom

The number of degrees of freedom (DF) for a sample is simply the number of independent comparisons that can be made with any one of the values in the sample or (n-1). For multiple samples in a single factor experiment DF = k(n-1), where k is the number of comparisons and n is the sample size. For multiple factor experiments, where each level of treatment is represented by sets of readings or replications as subsamples, each subsample is reduced by one (or r-1) in calculating degrees of freedom. For an experiment in which there are 3 factors, a factor consisting of 4 levels and there are 6 readings at each level, then degrees of freedom DF = 3x4x5 = 60.

4.3.3 Procedure for doing ANOM when no standard is given

In a real investigation of a process problem, we usually do not have and probably do not want a previously developed standard. The data points themselves are used to develop the mean and standard deviation. Under this circumstance, the normal distribution is no longer applicable; instead we turn to the student's t distribution for t values corresponding to the z values. To provide the decision limits for a given α risk, we would like values similar to the z values in the normal table, but based on the t distribution. The procedure and analysis to obtain these values is considerably more complex than the usual z value

determination. However, this has been done, and the values published by L.S. Nelson[1983] for use in ANOM trouble shooting are given Appendix.

a) ANOM with Variables Data

Let us consider k groups (ie., samples) of equal size n. All samples are assumed to come from normal populations with the same variance. Let the overall grand mean of the means be x and the pooled estimate of the common but unknown variance be s^2. These quantities are defined mathematically by

$$\overline{\overline{X}} = (X_1 + X_2 + ... + X_k)/k \qquad (4.1)$$

$$s^2 = (s_1^2 + s_2^2 + ... + s_k^2)/k \qquad (4.2)$$

$$s_i^2 = \sum_{j=1}^{n}(x_{ij} - \overline{X})^2/(n-1) \qquad (4.3)$$

$X_{ij} = j^{th}$ observation from population i.

The steps to carry out ANOM are as follows:

1. Compute the group means X_i (i=1,2,...k)

2. Compute the grand mean $\overline{\overline{X}}$, using the equation (4.1)

3. Compute s, an estimate of the standard deviation of an individual observation. This is the square root of s^2 where s^2 is computed using the equations(4.2) and (4.3).

4. Obtain the value of h_α from the Appendix. h_α is a constant to be taken from Appendix which depends on risk level α, the number of means k and the degrees of freedom (n-1)k.

5. Compute the upper and lower decision lines using the expressions:

$$UDL = \overline{\overline{x}} + h_\alpha s[(k-1)/(kn)]^{1/2} \qquad (4.4)$$

$$LDL = \overline{\overline{x}} - h_\alpha s[(k-1)/(kn)]^{1/2} \qquad (4.5)$$

6.   Plot the mean against the decision lines. Any of the means lying outside the decision lines is an indication that there is a statistically significant difference

among the samples. If all the points are within the decision limits, then it indicates that there is no statistical difference among the samples.

b) ANOM with Attribute Data

i) Proportions Data

When the data consists of the number of proportion of units having a particular attribute, it can often be represented with a binomial distribution. The normal approximation to the binomial distribution is considered to be adequate if $np>5$ and $n(1-p)>5$, where p is the proportion of items in the population having the attribute of interest. The ANOM procedure for proportions data is similar to that for variables data with quantitative measurements replaced by proportions. The procedural steps are outlined below.

1. Obtain samples of equal size n from each of k populations. Let the number of units having the attribute of interest in each of the k samples be denoted by X_1, X_2, X_3....., X_k.

2. Compute the proportions for each sample i = 1,2,...,k

$$p_i = (X_1 + X_2 + ...X_k)/n$$

3. Compute the overall average proportion

$$\bar{p} = (p_1 + p_2 + ...+ p_k)/k.$$

4. Compute an estimate of the standard deviation of the proportion using

$$s = [p(1-p)/n]^{1/2}$$

5. Determine the decision lines at risk α using

$$DL = \bar{p} \pm h_\alpha s[(k-1)/k]^{1/2} \tag{4.6}$$

where h_α is the value from Appendix for k means and infinite degrees of freedom. Infinite degrees of freedom is used because one is approximating the binomial distribution with a normal distribution.

6. Plot each proportions p_i against the decision lines. If all proportions fall between the decision lines accept the hypothesis that all of the k proportions are equal ie., there is no significant difference between the proportions at α% level. Otherwise, conclude `assignable causes' exist.

Using the analysis of means for proportions data is particularly applicable in multiple machine situations where p charts are generally employed. In addition to this regular use, p charts can be reviewed periodically at formal quality meetings. Presentation of percent defective accumulated over a chosen time period on an ANOM chart provides a concise comparison of machines that quickly reveals those with more of a tendency to run out of control. Also production employees who are familiar with control charts, have no difficulty in interpreting the ANOM for percent defective data, which is a logical extension of the p chart.

ii) Count Data (Non Measurable Data)

When the attribute of interest can appear potentially an infinite number of times, then one has to use Poisson distribution. For example, this would be the case when one counts the number of nonconformities rather than the number of non conforming units. The Poisson distribution can be adequately approximated by the normal, and hence the ANOM technique can be applied, if the mean of the Poisson distribution is at least five. The procedure for the Poisson model is essentially the same as for proportions. Proportions are replaced by counts c and the standard deviations of counts given by

$$s = (\bar{c})^{1/2}$$

where $\bar{c}$ is the overall average and is expressed mathematically as

$$\bar{c} = (c_1 + c_2 + ... + c_k)/k \tag{4.7}$$

Thus decision lines are computed using

$$DL = \bar{c} \pm h_\alpha[c(k-1)/k]^{1/2} \tag{4.8}$$

where c is the overall average, k is the number of means and ha is the constant to be taken from Appendix for a given number of means and degrees of freedom.

c) ANOM For Factorial Design

Steps similar to those outlined for the one way classification model are to be followed in the case of factorial design. Here, we need to consider the means of each level of each factor as well as the effect of any interaction between the two factors.

For main effects, the decision lines are given by

$$UDL = \bar{x} + h_\alpha \, s \, (q/N)^{1/2} \tag{4.9}$$

$$LDL = \bar{x} - h_\alpha \, s \, (q/N)^{1/2} \tag{4.10}$$

where `q' is the number of degrees of freedom associated with that factor (the number of levels of the factor less one) and N is the total number of observations. For interaction effects, the decision lines are given by

$$UDL = 0 + h_\alpha{}^* s(q/N)^{1/2} \tag{4.11}$$

$$LDL = 0 - h_\alpha{}^* s(q/N)^{1/2} \tag{4.12}$$

Whenever ANOM is applied to a 2^k or 2^{k-p} design, $h_\alpha{}^*$ can be computed as the upper $\alpha^*/2$ percentage point of the students t distribution and

$$\alpha^* = 1 - (1-\alpha)^{1/k}$$

where α is the desired significance level and k is the number of means (based on two factors combination). Values of $h_\alpha{}^*$ can be determined either by interpolating the t values or by using the nomogram given by L.S Nelson[1975]. Once the decision lines are obtained, effects of different combinations of factors can be plotted and the significant interaction of these factors at different risk levels if any, can be determined. Interaction effects are calculated as:

$$T_{ij} = X_{ij} - X_i - X_j + \bar{X}$$

where X_{ij} is the cell mean for the i^{th} row and j^{th} column, X_i is the i^{th} row mean, X_j is the j^{th} column mean and $\bar{\bar{X}}$ is the overall mean. When ANOM is applied to factorial design, we assume normality and equality of variances for every treatment combination. Equality of variance can be easily tested using an R chart. Test for normality may not be easy for smaller number of observations. However, since 't' values are relatively insensitive to slight to moderate departure from normality, the assumption of normality is not very critical.

Analysis of main effect and interaction effect with factors at 2 levels is straight forward. But as the number of factors and levels increases, determination of these effects become complex. Ott[1975] has suggested the following procedure to estimate the interaction effect with factors at more than two levels.

1.   Arrange the data in a factorial form and plot the points for variables at different levels. Determine the most similar and most dissimilar pair of levels.

2.   Determine the difference between the cell averages in each pair.

3.   Plot the difference between the cell averages.

4.   Decision lines may be determine as

$$\bar{D} \pm h_\alpha \, s[2(k\text{-}1)/nk])^{1/2} \qquad\qquad (4.13)$$

where D is the middle line given by the average of D_is, hα is the constant from Appendix, s is the standard deviation, n is the number of observations from which each average is computed and k is the number of means compared.

This is an adhoc procedure, but it provides important information. If the difference formed from the most similar machines are judged to be different, then the overall interaction is significant. Conversely, if the differences formed from the least similar machines are not significant, then the overall interaction is probably not significant.

4.3.4 Procedure for doing ANOM When Standards are Given

i) ANOM with Variables Data

The decision lines given in equation (4.4) and (4.5) were obtained under the assumption that population mean and variance are unknown, so they were estimated by x and s respectively. If both population mean and variance are known, decision lines can be obtained from

$$\mu \pm hz \left(\frac{\sigma}{n}\right)$$

ii) ANOM with Attribute Data

a) Proportions Data

Analysis assumes that samples of size n are drawn from a process whose known average is $\bar{p}$; n and $\bar{p}$ are such that the distribution of p_i in samples of size n is essentially normal. Decision lines in this case can be obtained as:

$$DL = \bar{p} \pm h_{\alpha}[p(1-p)/n]^{1/2} \tag{4.14}$$

where n is the number of items and h_{α} is the value obtained from Appendix for specified values of α and k, with infinity for the degrees of freedom.

b) Count Data (Non Measurable Data)

When ANOM is applied to count data and a standard is to be used, the decision lines are obtained from

$$DL = \bar{c} \pm h_{\alpha}[c]^{1/2} \tag{4.15}$$

where c is the standard acceptable count and h_{α} is the value obtained from Appendix for specified values of α with infinity for the degrees of freedom.

4.4 Using ANOM In Trouble Shooting Tasks

A quality problem like excessive rejects can be due to effect of components purchased from different vendors or difference between machines intended to produce the same items, or materials, or difference between machines, or difference between shifts of operators. A study of these effects, which could be either main effect or interaction effect, is often called "trouble shooting". Its purpose is to improve either the product, the process, or both.

The presence and nature of any assignable cause in a process can be determined using a control chart. Generally a 3 sigma limits around the mean are used in control charts. When a point falls outside these limits, we investigate the process to identify the presence of any assignable cause. It is well established that the risk of an unwarranted investigation from such a signal is very small - about three in a thousand.

In trouble shooting, it is necessary to make an investigation of the process with a greater chance(risk) of an unwarranted investigation than the typical three in thousand. Thus lines drawn at $\bar{p} \pm 2sigma$ will be more sensitive to the presence of an assignable cause. In such a case, the risk of making an unwarranted investigation is about 1 in 20 ie., about 5%. However, there is now a smaller risk of missing an important opportunity to investigate the cause of a problem.

A trouble shooting project often begins with the study of possible differences in the quality output of different machines, method of operation, operators or other operating parameters. When important differences are established, a careful study of the sources of better and worse performance is initiated. A key to making adjustments and improvements is in knowing that actual differences do exist, and in being able to pinpoint the sources of the differences. Analysis of means can be successfully used for this purpose.

4.4 Summary

In this chapter, we introduced different methods of analyzing a cause and effect diagram. Application of Analysis of Means (ANOM) for both attribute data and variable data is discussed. A method developed by Ott[1967] for estimating interaction effect when the factors are at more than two levels is also presented. Application of Analysis of Means (ANOM) with attribute data is Illustrated using a case study in the next chapter.

CHAPTER 5

CASE STUDY

The following problem is based on a case history of defective glass bottles reported by Ott [1975; pages 140-142]. Our objective is to analyze the data provided by the author to illustrate how ANOM can be used to provide evidence of important differences in the production system and indicate sources to be investigated for improvements. The author did not analyze the data and the solution to this problem is not available anywhere.

According to the author, a meeting was arranged by the telephone with a quality control inspector from a company whose only product was glass bottles. As the plant was a hundred miles away and the client seemed intelligent, they had a sensible discussion when they met.

The following points were established during the discussion:

1. There were too many rejects: the process was producing about 10% rejects of different kinds.

2. Data on the process performance of rejects was obtained from a 100% inspection of the bottles. The inspection station was in a warehouse separate from the production areas; the usual purpose of inspection was to cull out the rejects before shipping the bottles to their customers. The information was not of much value for any process improvement effort as they are obtained a week after bottles were made, and too late to be considered for current production problems.

3. Large quantities of glass bottles were being produced. Several machines were operating continuously on three shifts, for seven days per week. Each machine had many cavities producing bottles.

The inspector returned to his plant and made the following arrangements:

Table 1 Day-Machine-Shift wise Defectives

Day	Shift	Machine		
		1	2	3
Day-1	A	1	4	4
	B	4	0	4
	C	12	6	9
Day-2	A	3	6	30
	B	2	8	46
	C	2	7	27
Day-3	A	2	1	1
	B	8	11	15
	C	8	7	17
Day-4	A	4	11	10
	B	5	7	11
	C	4	6	11
Day-5	A	10	8	9
	B	6	12	10
	C	7	15	19
Day-6	A	7	11	15
	B	12	9	19
	C	24	8	18
Day-7	A	8	6	16
	B	10	12	17
	C	8	19	15

- A plant committee was organized representing production, inspection and industrial engineering to study causes and solutions to the problem of defects.

- An initial sampling procedure was planned for some quick information. From the most recent production, samples of 15 per hour were to be chosen at random from : (1) each of three machines (on the hot end), on (2) each of three shifts, and (3) over seven days.

- The sample bottles were to be placed in slots in an egg-carton type box marked to indicate the time of sampling as well as machine number, shift and date.

- After the bottles were collected and inspected, the number and type of various defects were recorded. The data in Table 1 show the total of all rejects.

Major conclusions that may be reached using cause and effect supported by ANOM for such a problem include:

1. Whether there were fundamental differences between the three machines and differences between shifts.

2. Whether there was a general deterioration of the machines, or possibly in raw materials, over the seven days. A comparison of this performance pattern with the scheduled maintenance may suggest changes in the maintenance schedule.

3. Whether each machine showed a general uptrend in rejects; one machine is best and another is consistently the worst.

4. Whether there is an unusual increase in rejects on all shifts on Aug 13 on one machine only. Manufacturing records should indicate whether there was any change in the raw material going to that one machine. The records should show what adjustments were made.

Our discussion will include:

1. Analysis of the effect of the days. All machines and shifts combined. Is there a significant difference observed over the seven days?

2. Analysis of the effect of the machines. Each machine with three shifts combined. What is the behavior pattern of each machine over the seven days?

3. Analysis of the effect of shifts. Each shift with three machines combined. What is the behavior pattern of each shift over the seven days?

4. Analysis of (a) main effects (b) shift-machine interaction (c) machine-day interaction using the data from the last four days.

ANALYSIS

There are three factors to be considered in this analysis These are the main factors: machine, shift, and day at 3, 3, and 7 levels respectively. The cause and effect diagram for this case is given at Figure 13. The three factors and their relationships can be illustrated using a C-type matrix diagram as shown in Figure 14. Each cell in this diagram represents the effect, ie., the number of rejects with two factors at the specified level. In a real situation, color coding of each cell for example, using red for excessive rejects, yellow for above average rejects and green for acceptable number of rejects can be employed to make the interpretation of the data easier.
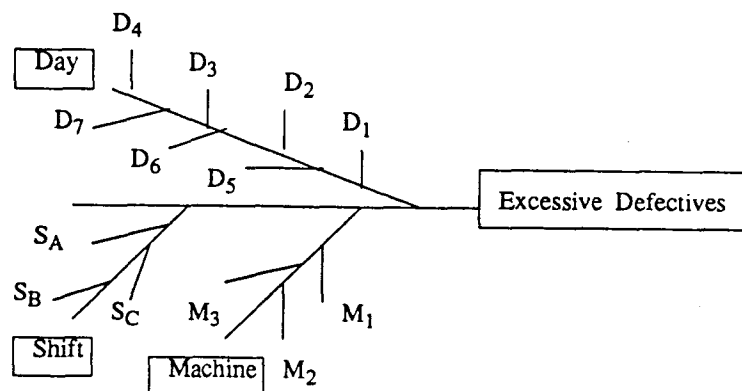
Figure 13 Cause and Effect Diagram for Excessive Defectives

Shift

Machine
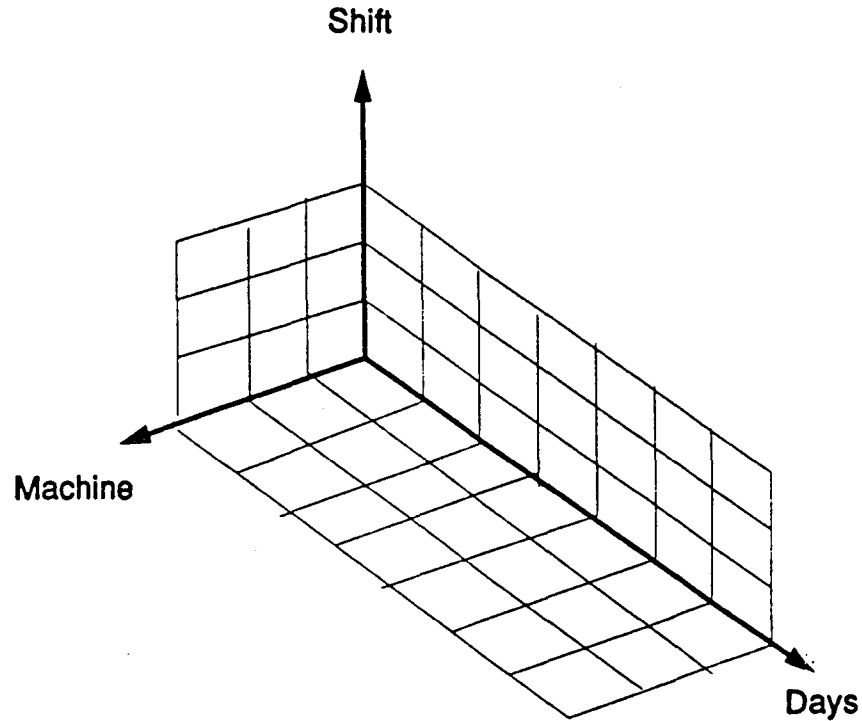
Days

Figure 14 Y-Type Matrix Representation for Excessive Defectives

Total number of rejects during the 7 days operation = 644

Total number of bottles produced during these 7 days = 7560

Percentage defective = 0.0852

Standard deviation of each sample can be calculated using the equation $[p(1-p)/N]^{1/2}$, where N is the sample size, p is the proportion of defectives.

(1) Analysis of effect of days.

Table 2 indicates the data with both machines and shifts combined.

Standard deviation $= [(0.0852 \times 0.9148)/(9 \times 120)]^{1/2}$

$= 0.0085$

In this case, the number of means to be compared (k) is 7, and the degrees of freedom is infinity. Decision lines are calculated using the following equation:

$$DL_\alpha = 0.0852 \pm h_\alpha(0.0085) \, (6/7)^{1/2}$$
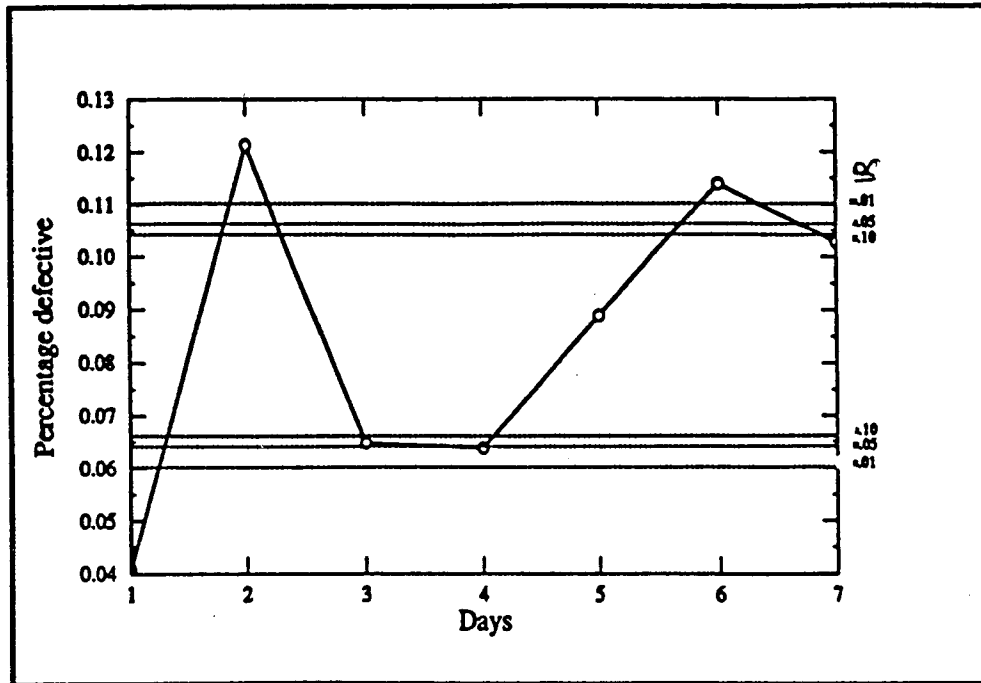
Figure 15 Days Effect - 7 days
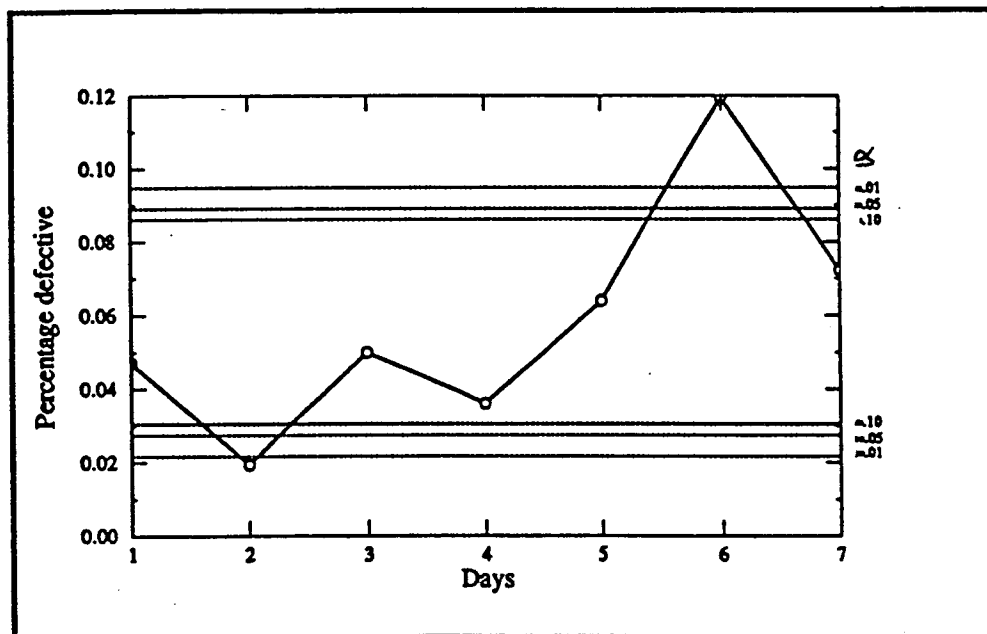
Figure 16 Machine Effect (Machine-1) - 7 days

Table 2 Day wise Percentage Defective (7 days)

Date	No. of rejects	Percentage defective
· Day-1	44	0.0407
Day-2	131	0.1213
Day-3	70	0.0648
Day-4	69	0.0639
Day-5	96	0.0889
Day-6	123	0.1139
Day-7	111	0.1028
Average	644	0.0852

Table 3 Decision Lines to Determine Days Effect (7 days)

Value of α	UDL	LDL
$\alpha = 0.01$ ($h_\alpha = 3.18$)	0.1102	0.0602
$\alpha = 0.05$ ($h = 2.68$)	0.1063	0.0641
$\alpha = 0.10$ ($h_\alpha = 2.42$)	0.1042	0.0662

Decision lines calculated based on the above equation is given in Table 3. Their graphical representation is given in Figure 15. It can be seen from the figure that the number of rejects on days day-2 and day-6 are statistically higher than of other days. Similarly, the number of rejects on day-1 is statistically lower than other days'.

(2) Analysis of effect of machines

Table 4 indicates the data with three shifts combined. Columns (5), (6), and (7) indicate the percentage rejects in each machine.
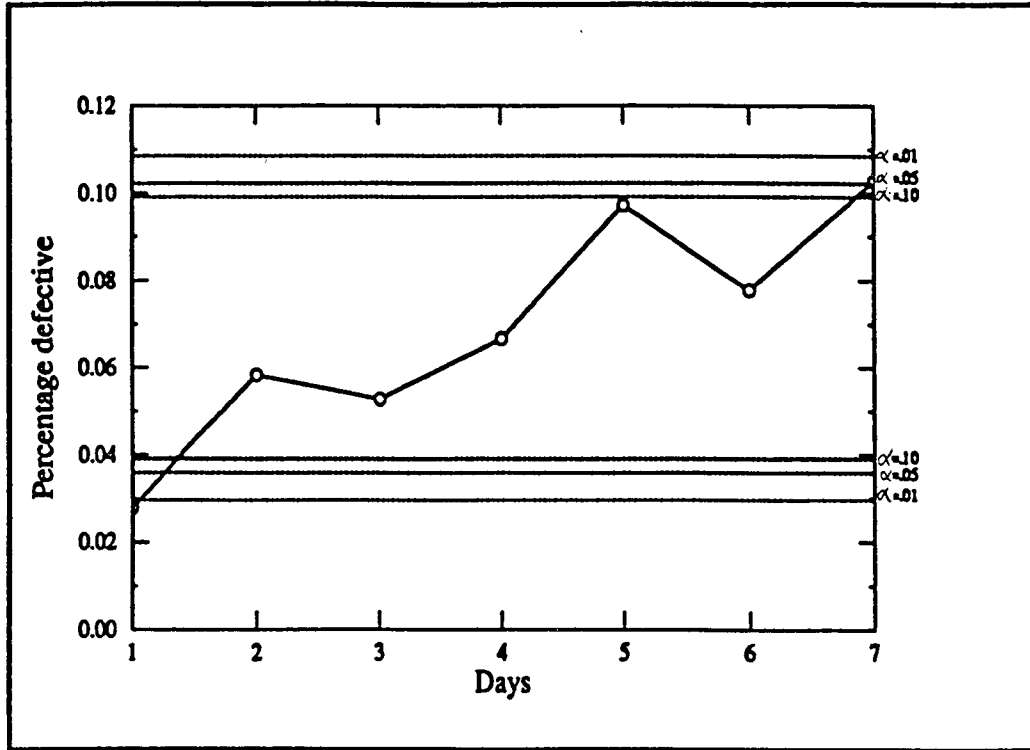
Figure 17 Machine Effect (Machine-2) - 7 days
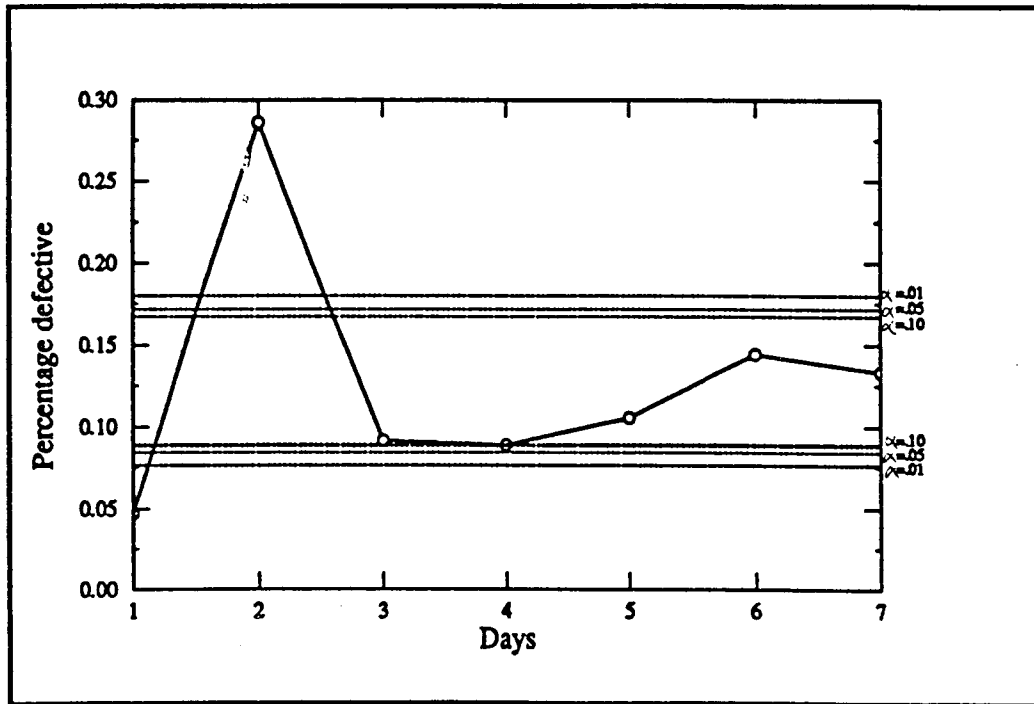
Figure 18 Machine Effect (Machine-3) - 7 days

Table 4 Machinewise Percentage Defective (7 days)

Day	Machine #1	Machine #2	Machine #3	% defective P_1	% defective P_2	% defective P_3
Day-1	17	10	17	0.0472	0.0278	0.0472
Day-2	7	21	103	0.0194	0.0583	0.2861
Day-3	18	19	33	0.0500	0.0528	0.0917
Day-4	13	24	32	0.0361	0.0667	0.0889
Day-5	23	35	38	0.0639	0.0972	0.1056
Day-6	43	28	52	0.1194	0.0778	0.1444
Day-7	26	37	48	0.0722	0.1028	0.1333
Average	147	174	323	0.0583	0.0691	0.1282

For machine-1;

Percentage defective P_1 = 0.0583

Standard deviation S_1 = $[(0.0583 \times 0.9417)/360]^{1/2}$

= 0.0124

For machine-2;

Percentage defective P_2 = 0.0691

Standard deviation S_2 = $[(0.0691 \times 0.9309)/360]^{1/2}$

= 0.0134

For machine-3;

Percentage defective P_3 = 0.1282

Standard deviation S_3 = $[(0.1282 \times).8718)/360]^{1/2}$

= 0.0176

In this case, the number of means compared (k) is 7, and the degrees of freedom is infinity. Decision lines can calculated as follows:

$$DL_1 = 0.0583 \pm h_\alpha (0.0124) (6/7)^{1/2}$$

$$DL_2 = 0.0691 \pm h_\alpha (0.0134) (6/7)^{1/2}$$

$$DL_3 = 0.1282 \pm h_\alpha (0.0176) (6/7)^{1/2}$$

Table 5 Decision Lines to Determine Machine Effect (7 days)

Machine	For $\alpha = 0.01$; $h_\alpha = 3.18$		For $\alpha = 0.05$; $h_\alpha = 2.68$		For $\alpha = 0.10$; $h_\alpha = 2.42$	
	UDL	LDL	UDL	LDL	UDL	LDL
M_1	0.0948	0.0218	0.0999	0.0275	0.0861	0.0305
M_2	0.1086	0.0296	0.1023	0.0359	0.0991	0.0391
M_3	0.1800	0.0764	0.1719	0.0845	0.1676	0.0888

Table 5 gives the decision lines calculated using the above equations. Graphical representation of these values for machines 1, 2 and 3 are given in Figure 16, Figure 17 and Figure 18 respectively. Number of rejects on day-6 in machine-A has statistically higher than the others. There is a noticeable increasing trend for the rejects on machine-2 over the 7 days. However, it is not statistically significant at $\alpha=0.01$. This needs to be monitored for assignable causes. Number of rejects in machine-3 on day-2 is statistically significant as seen in Figure 18. Note that this trend gets somewhat better at later period.

(3) Analysis of effect of shifts

Table 6 indicates the data with three machines combined. Columns (5), (6), and (7) indicate the percentage rejects in each shift.

For shift A;

Percentage defective	P_A	=	0.0663
Standard deviation	S_A	=	$[(0.0663 \times 0.9337)/360]^{1/2}$
		=	0.0131

For shift B;

Percentage defective	P_B	=	0.0905
Standard deviation	S_B	=	$[(0.0905 \times 0.9095)/360]^{1/2}$
		=	0.0151
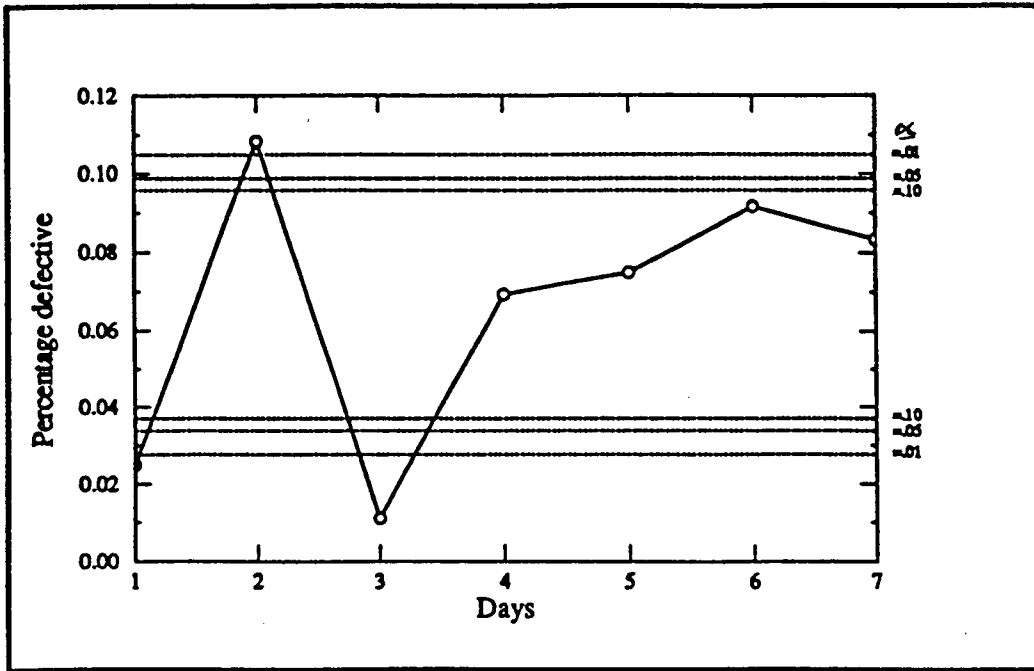
Figure 19 Shift Effect (Shift-A) - 7 days
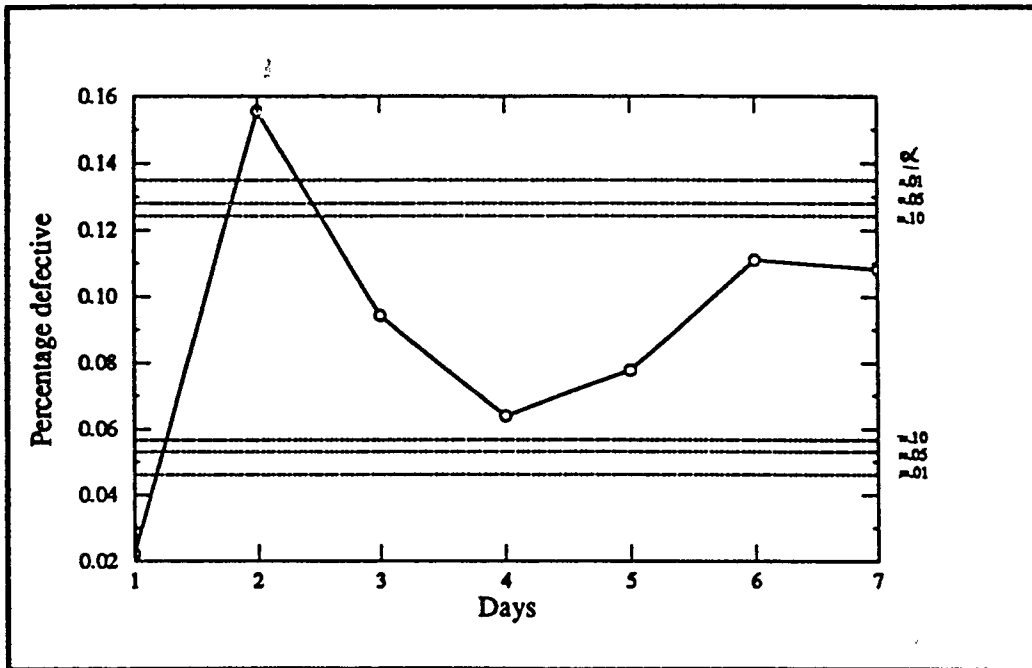
Figure 20 Shift Effect (Shift-B) - 7 days

For shift C;

Percentage defective P_C =   0.0988

Standard deviation S_C = $[(0.0988 \times 0.9012)/360]^{1/2}$

                                            =   0.0157

Table 6   Shift wise Percentage Defective (7 days)

Day	Shift A	Shift B	Shift C	% defective P_A	% defective P_B	% defective P_C
Day-1	9	8	27	0.0250	0.0222	0.0750
Day-2	39	56	36	0.1083	0.1556	0.1000
Day-3	4	34	32	0.0111	0.0944	0.0889
Day-4	25	23	21	0.0694	0.0639	0.0583
Day-5	27	28	41	0.0750	0.0778	0.1139
Day-6	33	40	50	0.0917	0.1111	0.1389
Day-7	30	39	42	0.0833	0.1083	0.1167
	167	228	249	0.0663	0.0905	0.0988

In this case the number of means to be compared (k) is 7, and the number of degrees of freedom is infinity. Decision lines can then be calculated as follows:

$$DL_A = 0.0663 \pm h_\alpha(0.0131)(6/7)^{1/2}$$

$$DL_B = 0.0905 \pm h_\alpha(0.0905)(6/7)^{1/2}$$

$$DL_C = 0.0988 \pm h_\alpha(0.0157)(6/7)^{1/2}$$

Table 7 gives the decision lines calculated using the above equations. Graphical representation of these values for shift-A, shift-B and shift-C are given in Figure 19, Figure 20 and Figure 21 respectively. Number of rejects in shift-A and shift-B are statistically significant, as seen in Figure 19 and Figure 20 respectively. Note that all points in shift-C are within the decision limits.
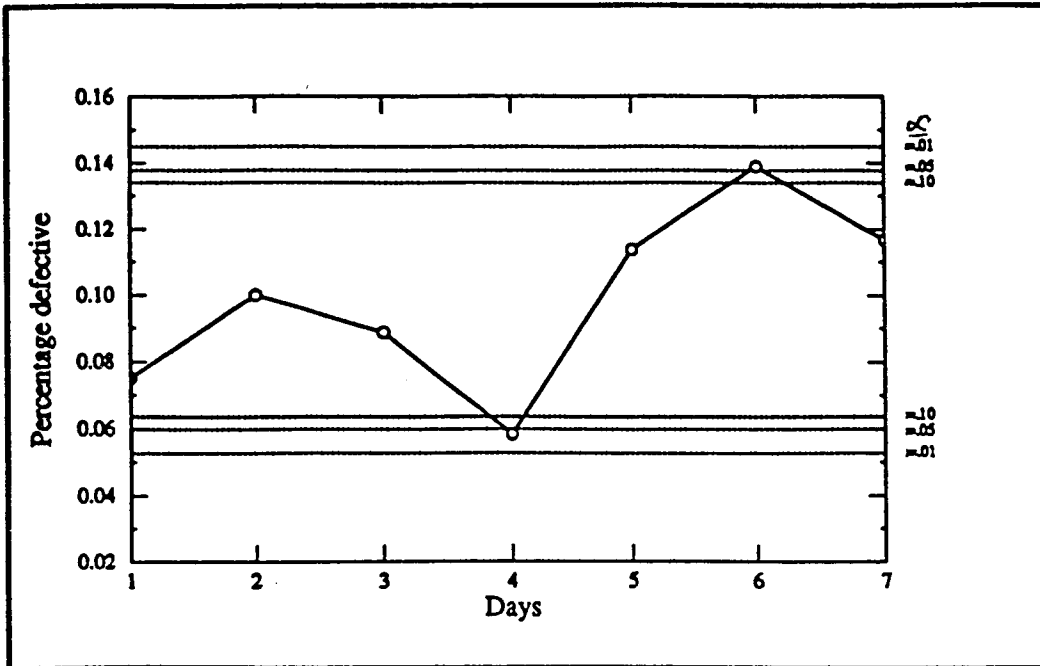
Figure 21 Shift Effect (Shift-C) - 7 days
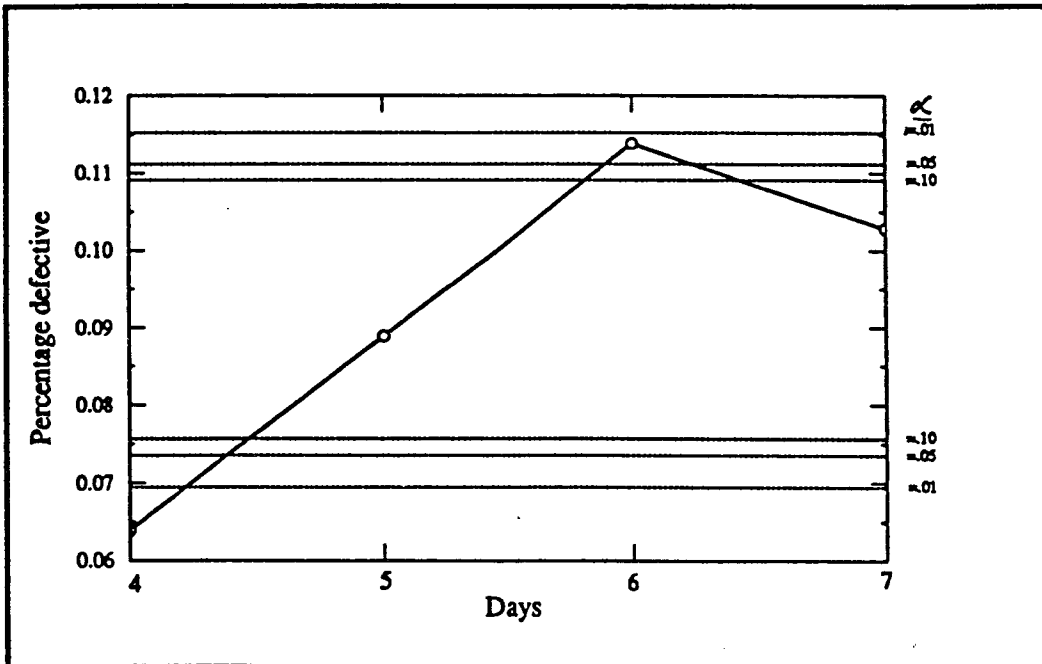
Figure 22 Days Effect - 4 days

Table 7 Decision Lines to Determine Shift Effect (7 days)

Shift	For α= 0.01; h_α=3.18		For α= 0.05; h_α= 2.68		For α= 0.10; h_α= 2.42	
	UDL	LDL	UDL	LDL	UDL	LDL
Shift A	0.1049	0.0277	0.0988	0.0338	0.0957	0.0370
Shift B	0.1350	0.0460	0.1280	0.0530	0.1243	0.0567
Shift C	0.1450	0.0526	0.1378	0.0599	0.1340	0.0636

4 (a) Analysis of main effect

(1) Effect of days

Table 8 gives the data with both machines and shifts combined for the last four days.

Table 8 Day wise Percentage Defective (4 days)

Day	Number of defectives	Percentage defective
Day-4	69	0.0639
Day-5	96	0.0889
Day-6	123	0.1139
Day-7	111	0.1028
	399	0.0924

Percentage defective   =   0.0924

Standard deviation   = $[(0.0924 \times 0.9076)/1080]^{1/2}$

The number of means to be compared (k) is 4, and the degrees of freedom is infinity. Decision lines are calculated using the following equation:

$$DL_\alpha = 0.0924 + h_\alpha (0.0088)(3/4)^{1/2}$$

Decision lines calculated based on the above equation is given in Table 9. Graphical representation of percentage defectives over the last four days is

given in Figure 22. Note that the points corresponding to day-4 and day-6 are falling outside the decision limits.

Table 9 Decision Lines to Determine Days Effect (4 days)

Value of 'α'	UDL	LDL
α = 0.01 ; h_α= 3.01	0.1153	0.0695
α = 0.05 ; h_α = 2.47	0.1112	0.0736
α = 0.10 ; h_α = 2.19	0.1091	0.0757

(2) Effect of machines

Table 10 indicates the data with three shifts combined. Columns (5), (6), and (7) indicate the percentage rejects in each machine.

For machine-1;

Percentage defective P_1 =   0.0729

Standard deviation S_1 = $[(0.0729 \times 0.9271)/360]^{1/2}$

=   0.0137

For machine-2;

Percentage defective P_2 =   0.0861

Standard deviation S_2 = $[(0.0861 \times 0.9139)/360]^{1/2}$

=   0.0148

For machine-3;

Percentage defective P_3 =   0.1181

Standard deviation S_3 = $[(0.1181 \times 0.8819)/360]^{1/2}$

=   0.0170

The number of means to be compared (k) is 4, and the degrees of freedom is infinity. Decision lines can be calculated as follows:

$$DL_1 = 0.0729 \pm h_\alpha(0.0137)(3/4)^{1/2}$$

$$DL_2 = 0.0861 \pm h_\alpha(0.0148)(3/4)^{1/2}$$
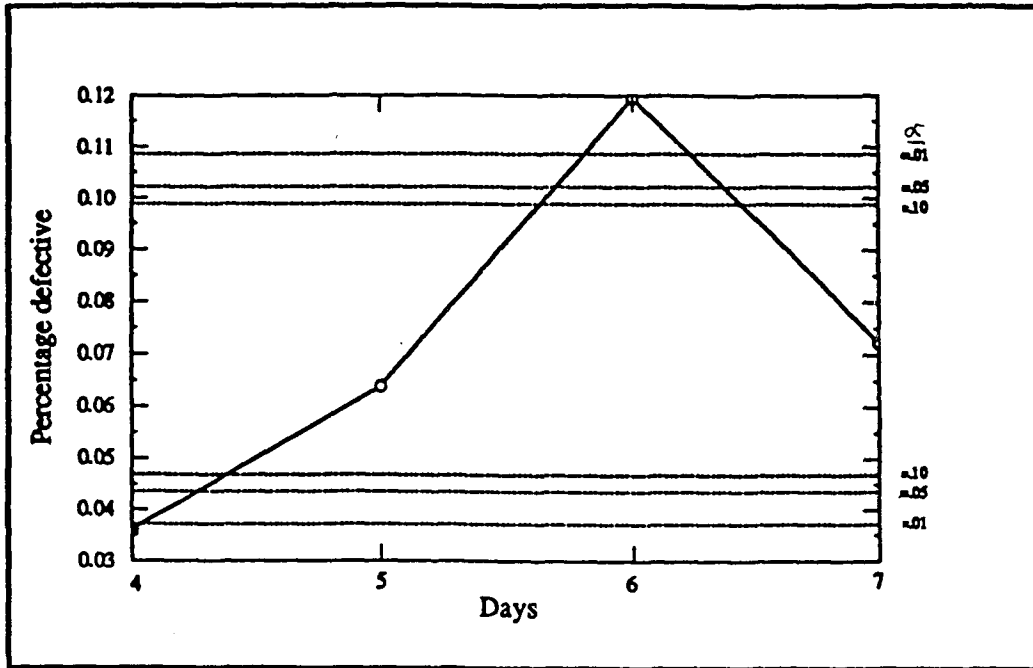
Figure 23 Machine Effect (Machine-1) - 4 days
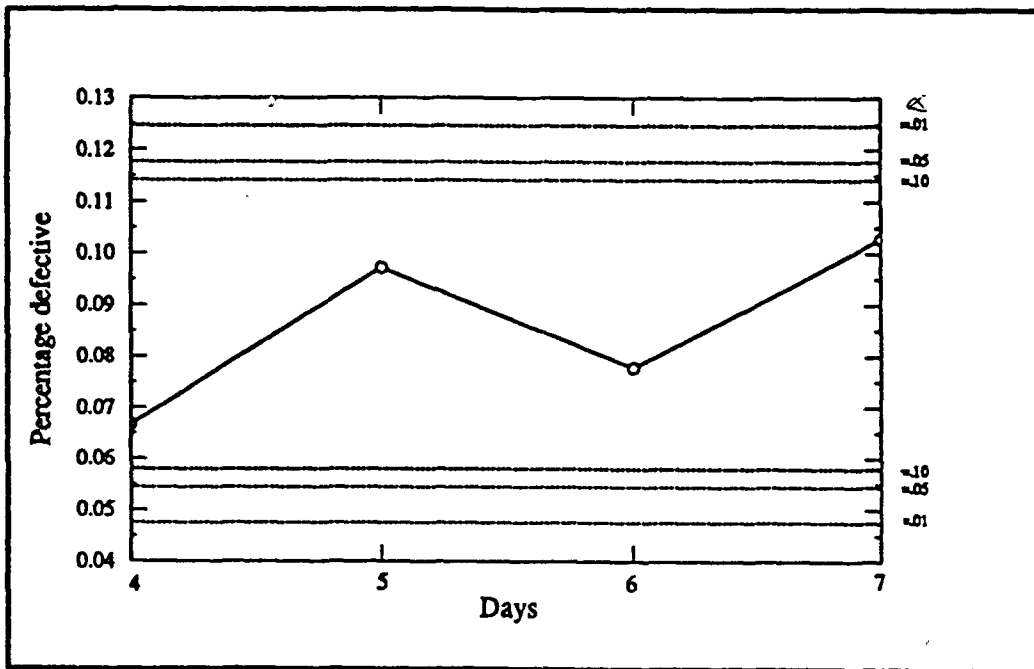
Figure 24 Machine Effect (Machine-2) - 4 days

$$DL_3 = 0.1181 \pm h_\alpha (0.0170)(3/4)^{1/2}$$

Table 10 Machine wise Percentage Defective (4 days)

Day	Machine # 1	Machine # 2	Machine # 3	% defe-ctive P_1	% defe-ctive P_2	% defe-ctive P_3
Day-4	13	24	32	.0361	0.0667	0.0889
Day-5	23	35	38	0.0639	0.0972	0.1056
Day-6	43	28	52	0.1194	0.0778	0.1444
Day-7	26	37	48	0.0722	0.1028	0.1333
	105	124	170	0.0729	0.0861	0.1181

Table 11 Decision Lines to Determine Machine Effect (4 days)

Machine	For α = 0.01; h_α= 3.01		For α = 0.05; h_α= 2.47		For α = 0.10; h_α = 2.19	
	UDL	LDL	UDL	LDL	UDL	LDL
M_1	0.1086	0.0372	0.1022	0.0436	0.0989	0.0469
M_2	0.1247	0.0475	0.1178	0.0544	0.1142	0.0580
M_3	0.1624	0.0738	0.1545	0.0817	0.1503	0.0859

Table 11 gives the decision lines calculated using the above equations. Graphical representation of percentage defectives over the last four days for machine-1, machine-2 and machine-3 are given in Figure 23, Figure 24 and Figure 25 respectively. Figure 23 shows that defectives produced by machine-1 on day-6 is statistically higher than that of the others.Note that the points corresponding to machine-2 and machine-3 are all within the limits.

(3) Effect of shift

Table 12 gives the data with three machines combined. Columns (5), (6), and (7) indicate the percentage rejects in each shift.
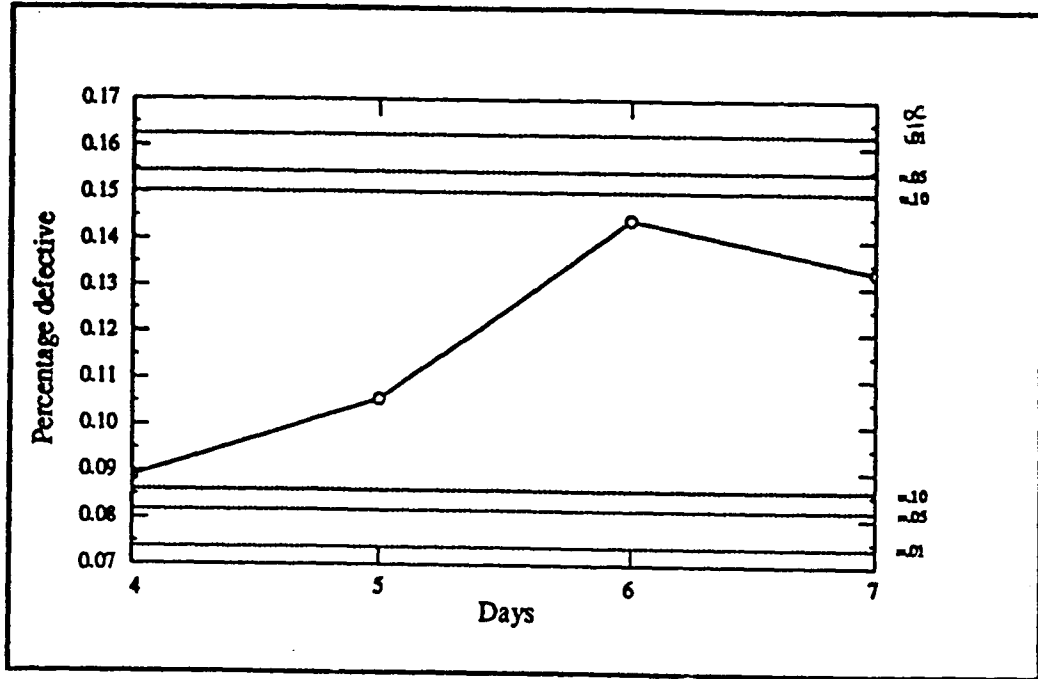
Figure 25 Machine Effect (Machine-3) - 4 days
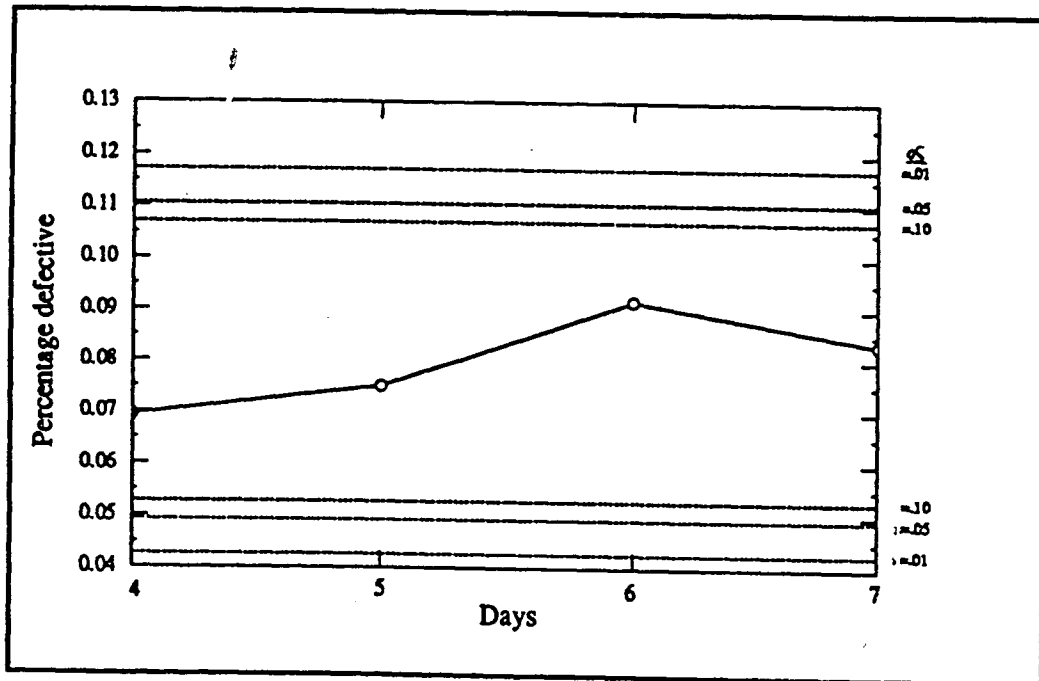
Figure 26 Shift Effect (Shift-A) - 4 days

Table 12   Shift wise Percentage Defective   (4 days)

Day	Shift A	Shift B	Shift C	% defective P_A	% defective P_B	% defective P_C
Day-4	25	23	21	0.0694	0.0639	0.0583
Day-5	27	28	41	0.0750	0.0778	0.1139
Day-6	33	40	50	0.0917	0.1111	0.1389
Day-7	30	39	42	0.0833	0.1083	0.1167
Average	115	130	154	0.0799	0.0903	0.1070

For shift-A;

Percentage defective	P_A	=	0.0799
Standard deviation	S_A	=	$[(0.0799 \times 0.9201)/360]^{1/2}$
		=	0.0143

For shift-B;

Percentage defective	P_B	=	0.0903
Standard deviation	S_B	=	$[(0.0903 \times 0.9097)/360]^{1/2}$
		=	0.0151

For shift-C;

Percentage defective	P_C	=	0.1070
Standard deviation	S_C	=	$[(0.1070 \times 0.8930)/360]^{1/2}$
		=	0.0163

The number of means to be compared (k) is 4, and the number of degrees of freedom is infinity. Decision lines can be calculated as follows:

$$DL_A = 0.0799 \pm h_\alpha(0.0143)(3/4)^{1/2}$$

$$DL_B = 0.0903 \pm h_\alpha(0.0151)(3/4)^{1/2}$$

$$DL_C = 0.1070 \pm h_\alpha(0.0163)(3/4)^{1/2}$$
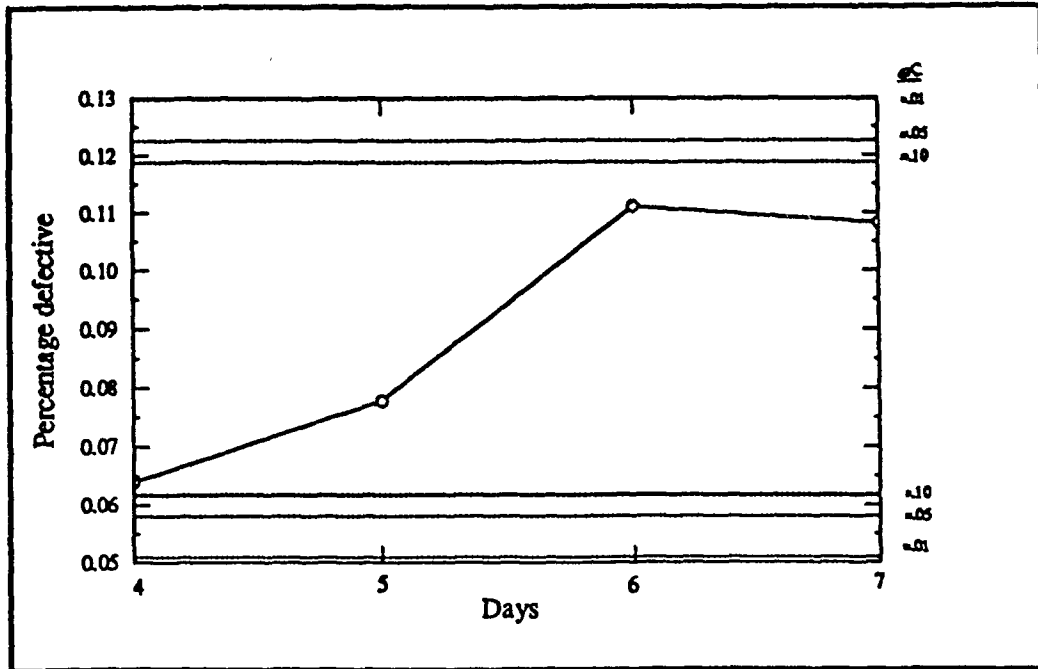
Figure 27 Shift Effect (Shift-B) - 4 days
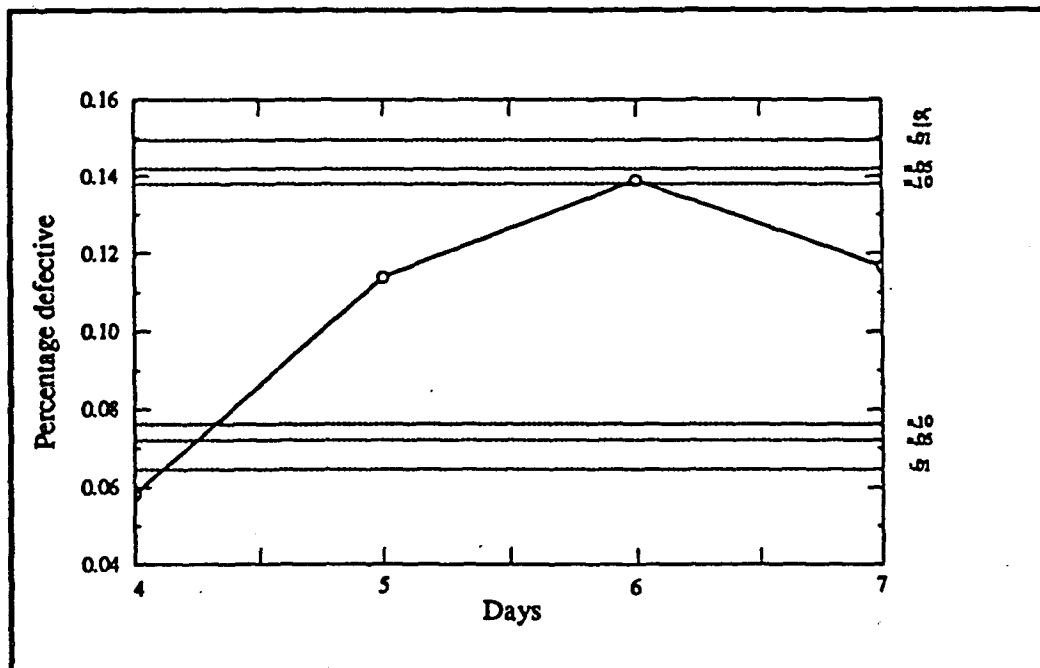
Figure 28 Shift Effect (Shift-C) - 4 days

Table 13 Decision Lines to Determine Shift Effect (4 days)

Shift	For $\alpha = 0.01$; $h_\alpha = 3.01$		For $\alpha = 0.05$; $h_\alpha = 2.47$		For $\alpha = 0.10$; $h_\alpha = 2.19$	
	UDL	LDL	UDL	LDL	UDL	LDL
Shift A	0.1172	0.0426	0.1105	0.0493	0.1070	0.0528
Shift B	0.1297	0.0509	0.1226	0.0580	0.1189	0.0617
Shift C	0.1495	0.0645	0.1419	0.0721	0.1379	0.0761

Table 13 gives the decision lines calculated using the above equations. Graphical representation of these values for Shift-A, Shift-B and Shift-C are given in Figure 26, Figure 27 and Figure 28 respectively. These figures indicate that all points are generally within the limits except percentage defective in Shift-B during day-4 is significantly low at $\alpha = 0.01$.

4 (b) Shift-machine interaction

The analysis of entire set of interactions is a complex procedure [Ott 1975]. Our analysis is limited to two-way interactions only. Table 14 presents the number of rejects shift-machine category for the last four days. A graphical representation of these values is given in Figure 29.

Table 14 Shift x Machine Interaction (4 days)

Shift	Machine # 1	Machine # 2	Machine # 3	Total
A	29	36	50	115
B	33	40	57	130
C	43	48	63	154
Total	105	124	170	399
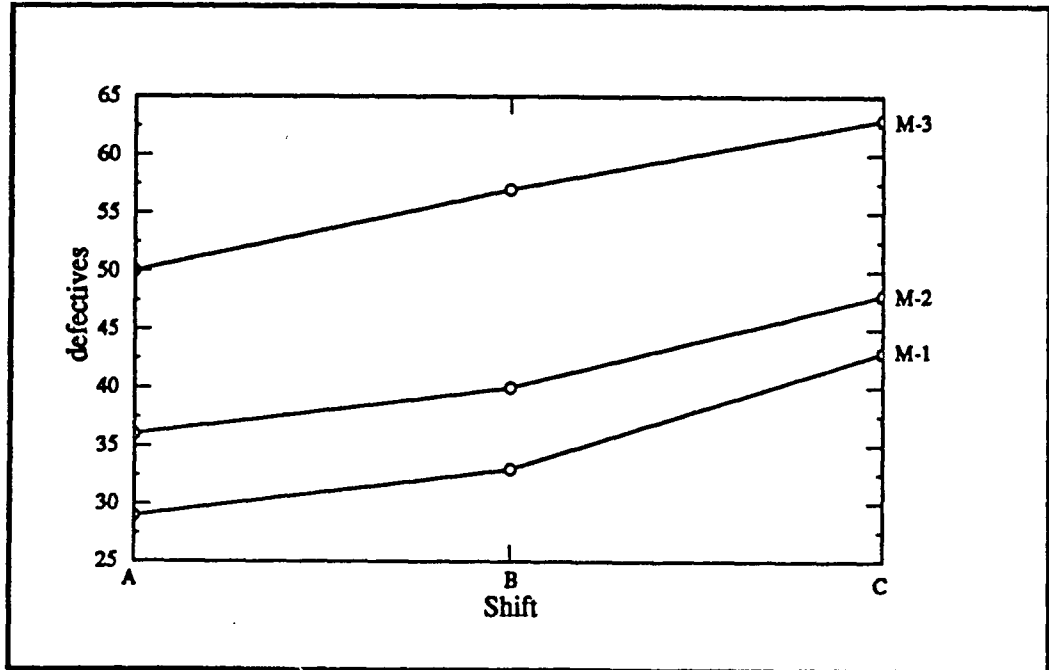
Figure 29 Shift x Machine Interaction (4 days)
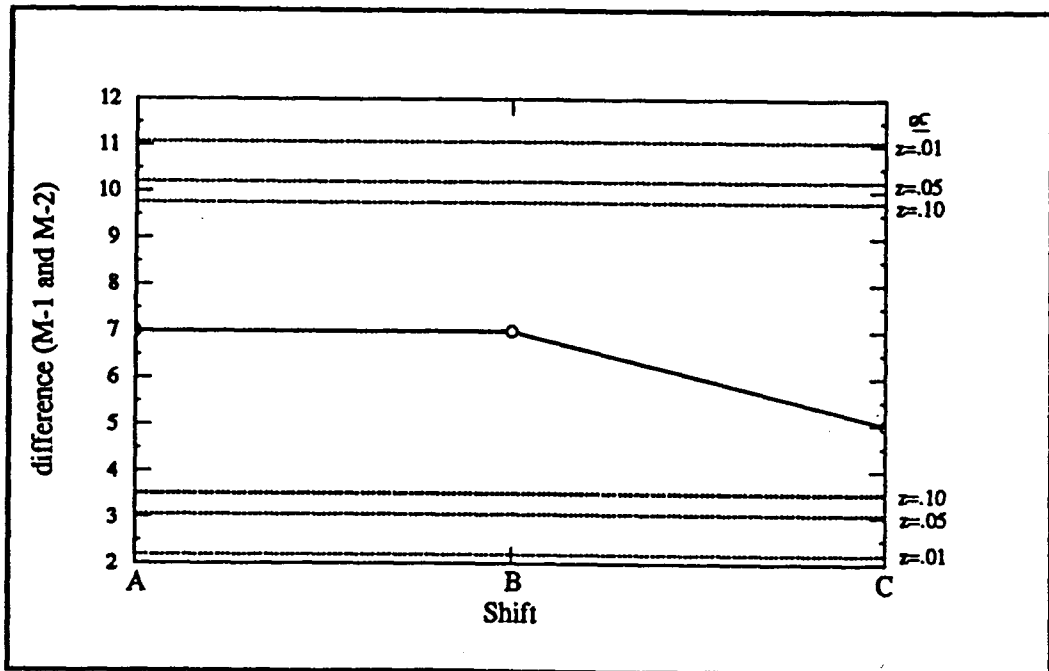
Figure 30 Shift x Machine Interaction (M-1 and M-2)

It can be seen that pattern in machine-1 and machine-2 is similar and the pattern in machine-1 and machine-3 least similar. Hence machine-1 and machine-2 can be treated as a similar pair and machine-1 and machine-3 can be treated as a dissimilar pair. These pairs can be compared as follows:

1.   The average differences, $\bar{D}$ for similar pair is shown in Table-15. Graphical representation of average differences along with the decision lines is shown in Figure-30. Decision lines are calculated as under:

$$DL = \bar{\bar{D}} \pm h_\alpha\ (2)^{1/2}\ s\ [(k-1)/k]^{1/2}$$

Standard deviation $s = [(9.24 \times 90.76)/480]^{1/2} = 1.32$

Table 15 Shift x Machine Interaction (M-1 and M-2)

Machine	Shift		
	A	B	C
M-1	29	33	43
M-2	36	40	48
Difference	7	7	5

Substituting the corresponding values in the above equation, we get,

For $\alpha = 0.01$, DL = 6.63 $\pm$ 4.44

For $\alpha = 0.05$, DL = 6.63 $\pm$ 3.57

For $\alpha = 0.10$, DL = 6.63 $\pm$ 3.12

Here the number of means to be compared (k) is 3, and the average difference D is 6.33. It can be seen that the three points are all within the decision lines and we decide that the patterns of machine-1 and machine-2 are not statistically different.

2.   The average difference for the dissimilar pair is shown in Table-16. Graphical representation of these values along with the decision lines is given in Figure-31. Decision lines calculated using as under:

Figure 31 Shift x Machine Interaction (M-1 and M-3)
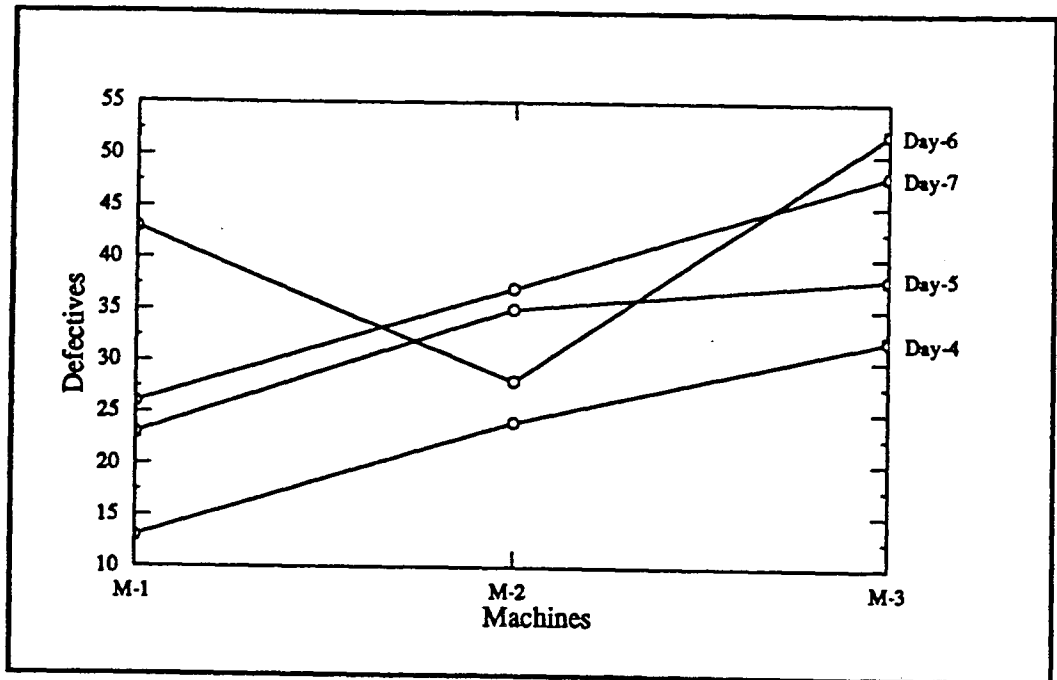
Figure 32 Machine x Day Interaction (4 days)

$$DL = \bar{\bar{D}} \pm h_\alpha \, (2)^{1/2} \, s \, [(k-1)/k]^{1/2}$$

Standard deviation $s = [(9.24 \times 90.76)/480]^{1/2} = 1.32$

Substituting the corresponding values in the above equation, we get,

For $\alpha = 0.01$, DL $= 21.67 \pm 4.44$

For $\alpha = 0.05$, DL $= 21.67 \pm 3.57$

For $\alpha = 0.10$, DL $= 21.67 \pm 3.12$

Table 16 Shift x Machine Interaction (M-1 and M-3)

Machine	Shift		
	A	B	C
M-1	29	33	43
M-3	50	57	63
Difference	21	24	20

Here the number of means to be compared (k) is 3, and the average difference $\bar{D}$ is 21.67. As all the points are within the decision lines, we conclude that the difference in the patterns of machine-1 and machine-3 are not statistically significant.

4 (c) Machine-day interaction

Table 17 presents the number of rejects day-machine category wise for the last four days. The graphical representation of these values is given in Figure 32. It can be seen from the graph that the patterns of defectives on day-4 and day-5 are similar where as the pattern of day-6 and day-7 are least similar. These pairs can be compared as follows:

1. The average difference for day-4 and day-5 is shown in Table-18. Here the mean of the difference is 9 and the decision lines are calculated as follows:

$$DL = \bar{\bar{D}} \pm h_\alpha \, (2)^{1/2} \, s \, [(k-1)/k]^{1/2}$$
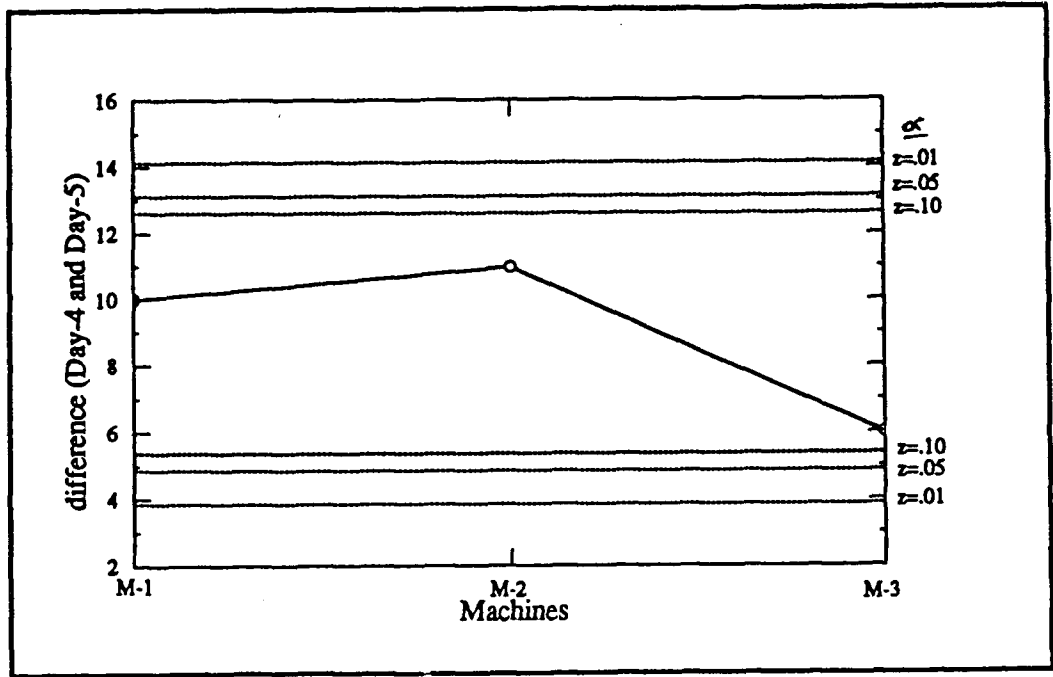
Figure 33 Machine x Day Interaction (Day-4 and Day-5)
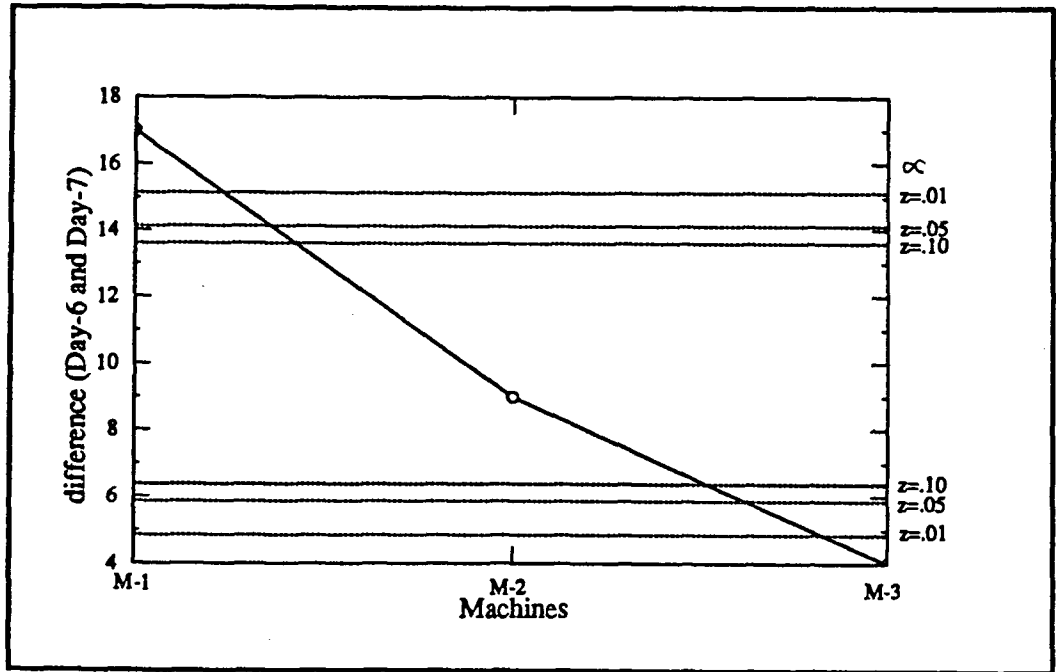
Figure 34 Machine x Day Interaction (Day-6 and Day-7)

Standard deviation s = $[(9.24 \times 90.76)/360]^{1/2}$ = 1.53

Substituting the corresponding values in the above equation, we get,

For α = 0.01, DL = 9 $\pm$ 5.14

For α = 0.05, DL = 9 $\pm$ 4.13

For α = 0.10, DL = 9 $\pm$ 3.62

Table 17 Machine x Day Interaction (4 days)

Machine	Days				Total
	Day-4	Day-5	Day-6	Day-7	
M-1	13	23	43	26	105
M-2	24	35	28	37	124
M-3	32	38	52	48	170
Total	69	96	123	111	399

Table 18 Machine x Day Interaction (Day-4 and Day-5)

Day	Machine		
	M-1	M-2	M-3
Day-4	13	24	32
Day-5	23	35	38
Difference	10	11	6

Graphical representation of average differences along with the decision lines is given in Figure 33. It can be seen that the three points are all within the decision lines and we conclude that the difference observed in the pattern on day-4 and day-5 are not statistically different.

2.   The pattern on day-6 and day-7 appear to be least similar. The average difference is shown in Table 19. Here the number of means compared k is 3,

and D is 10. The graphical representation of these values along with the decision lines is given in Figure 34. Decision lines are calculated as follows:
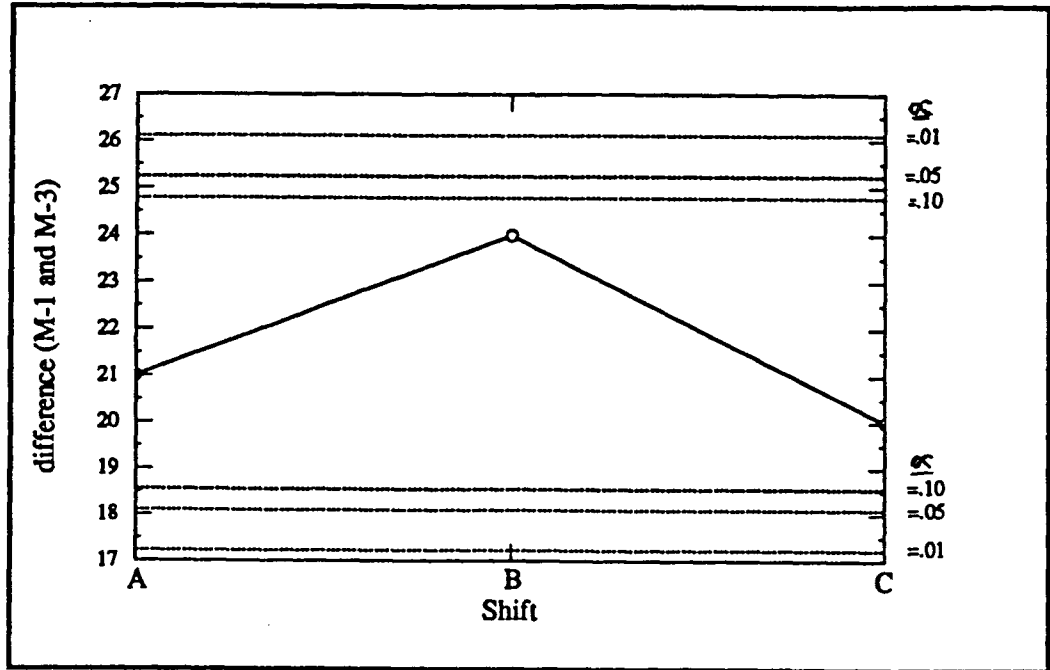
$$DL = \bar{\bar{D}} \pm h_\alpha \, (2)^{1/2} \, s \, [(k\text{-}1)/k]^{1/2}$$

Standard deviation $s = [(9.24 \times 90.76)/360]^{1/2} = 1.53$

Substituting the corresponding values in the above equation, we get,

For $\alpha = 0.01$, DL $= 10 \pm 5.14$

For $\alpha = 0.05$, DL $= 10 \pm 4.13$

For $\alpha = 0.10$, DL $= 10 \pm 3.62$

Table 19 Machine x Day Interaction (Day-6 and Day-7)

Day	Machine		
	M-1	M-2	M-3
Day-6.	43	28	52
Day-7	26	37	48
Difference	17	9	4

It can be seen from the graph that the point corresponding to M-1 is above the decision line and the point corresponding to M-3 is below the decision line. Hence, we can conclude that there is statistically significant difference at $\alpha=0.01$. Adjustments made on the machines or other days effect may be the possible reason.

Wherever the points fall outside the decision limits, it indicates that there is statistical difference between that point and other points. In other words, it indicates the presence of an assignable cause. We need to check the record or other relevant data to analyze such differences. For example, a corrective action taken at the end of shift-B on day-2 must have brought down the percentage defectives to normal during the subsequent days.

CHAPTER 6

CONCLUSION

Simple graphical problem-solving techniques can be applied to support the continuous improvement of product and processes. One of the most useful and powerful graphical tools for use in the problem solving process is the cause and effect diagram. In this thesis, we presented how this tool can be used for trouble shooting in quality control. We showed how the likely causes for the problem and its relationship are analyzed to locate the major cause. Also, we presented how different matrix diagrams can be used for representing the cause effect relationships.

We also proposed an advanced graphical technique for further analysis of cause effect relationship. This technique is based on the Analysis of Means (ANOM) developed by Prof. Ott. While main effect and interaction effect of 2-level factorial designs are easy using this method, it becomes complicated as the number of factors and levels increases. However, the procedure using ANOM provides good practical application. The structuring of cause effect diagrams, and corresponding matrix diagrams with application of ANOM have been illustrated using a case study.

APPENDIX

Exact critical values h_α for $\alpha = 0.01$

DF	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	DF
							Number of means, k													
3	5.841	7.51																		3
4	4.604	5.74	6.21																	4
5	4.032	4.93	5.29	5.55																5
6	3.707	4.48	4.77	4.98	5.16															6
7	3.499	4.18	4.44	4.63	4.78	4.90														7
8	3.355	3.98	4.21	4.38	4.52	4.63	4.72													8
9	3.250	3.84	4.05	4.20	4.33	4.43	4.51	4.59												9
10	3.169	3.73	3.92	4.07	4.18	4.28	4.36	4.43	4.49											10
11	3.106	3.64	3.82	3.96	4.07	4.16	4.23	4.30	4.36	4.41										11
12	3.055	3.57	3.74	3.87	3.98	4.06	4.13	4.20	4.25	4.31	4.35									12
13	3.012	3.51	3.68	3.80	3.90	3.98	4.05	4.11	4.17	4.22	4.26	4.30								13
14	2.977	3.46	3.63	3.74	3.84	3.92	3.98	4.04	4.09	4.14	4.18	4.22	4.26							14
15	2.947	3.42	3.58	3.69	3.79	3.86	3.92	3.98	4.03	4.08	4.12	4.16	4.19	4.22						15
16	2.921	3.38	3.54	3.65	3.74	3.81	3.87	3.93	3.98	4.02	4.06	4.10	4.14	4.17	4.20					16
17	2.898	3.35	3.50	3.61	3.70	3.77	3.83	3.89	3.93	3.98	4.02	4.05	4.09	4.12	4.14	4.17				17
18	2.878	3.33	3.47	3.58	3.66	3.73	3.79	3.85	3.89	3.94	3.97	4.01	4.04	4.07	4.10	4.12	4.15			18
19	2.861	3.30	3.45	3.55	3.63	3.70	3.76	3.81	3.86	3.90	3.94	3.97	4.00	4.03	4.06	4.08	4.11	4.13		19
20	2.845	3.28	3.42	3.53	3.61	3.67	3.73	3.78	3.83	3.87	3.90	3.94	3.97	4.00	4.02	4.05	4.07	4.09	4.12	20
24	2.797	3.21	3.35	3.45	3.52	3.58	3.64	3.69	3.73	3.77	3.80	3.83	3.86	3.89	3.91	3.94	3.96	3.98	4.00	24
30	2.750	3.15	3.28	3.37	3.44	3.50	3.55	3.59	3.63	3.67	3.70	3.73	3.76	3.78	3.81	3.83	3.85	3.87	3.89	30
40	2.704	3.09	3.21	3.29	3.36	3.42	3.46	3.50	3.54	3.58	3.60	3.63	3.66	3.68	3.70	3.72	3.74	3.76	3.78	40
60	2.660	3.03	3.14	3.22	3.29	3.34	3.38	3.42	3.46	3.49	3.51	3.54	3.56	3.59	3.61	3.63	3.64	3.66	3.68	60
120	2.617	2.97	3.07	3.15	3.21	3.26	3.30	3.34	3.37	3.40	3.42	3.45	3.47	3.49	3.51	3.53	3.55	3.56	3.58	120
Inf.	2.5758	2.91	3.01	3.08	3.14	3.18	3.22	3.26	3.29	3.32	3.34	3.36	3.38	3.40	3.42	3.44	3.45	3.47	3.48	Inf.

Exact critical values h_α for $\alpha = 0.05$

DF	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	DF
							Number of means, k													
3	3.182	4.18																		3
4	2.776	3.56	3.89																	4
5	2.571	3.25	3.53	3.72																5
6	2.447	3.07	3.31	3.49	3.62															6
7	2.365	2.94	3.17	3.33	3.45	3.56														7
8	2.306	2.86	3.07	3.21	3.33	3.43	3.51													8
9	2.262	2.79	2.99	3.13	3.24	3.33	3.41	3.48												9
10	2.228	2.74	2.93	3.07	3.17	3.26	3.33	3.40	3.45											10
11	2.201	2.70	2.88	3.01	3.12	3.20	3.27	3.33	3.39	3.44										11
12	2.179	2.67	2.85	2.97	3.07	3.15	3.22	3.28	3.33	3.38	3.42									12
13	2.160	2.64	2.81	2.94	3.03	3.11	3.18	3.24	3.29	3.34	3.38	3.42								13
14	2.145	2.62	2.79	2.91	3.00	3.08	3.14	3.20	3.25	3.30	3.34	3.37	3.41							14
15	2.131	2.60	2.76	2.88	2.97	3.05	3.11	3.17	3.22	3.26	3.30	3.34	3.37	3.40						15
16	2.120	2.58	2.74	2.86	2.95	3.02	3.09	3.14	3.19	3.23	3.27	3.31	3.34	3.37	3.40					16
17	2.110	2.57	2.73	2.84	2.93	3.00	3.06	3.12	3.16	3.21	3.25	3.28	3.31	3.34	3.37	3.40				17
18	2.101	2.55	2.71	2.82	2.91	2.98	3.04	3.10	3.14	3.18	3.22	3.26	3.29	3.32	3.35	3.37	3.40			18
19	2.093	2.54	2.70	2.81	2.89	2.96	3.02	3.08	3.12	3.16	3.20	3.24	3.27	3.30	3.32	3.35	3.37	3.40		19
20	2.086	2.53	2.68	2.79	2.88	2.95	3.01	3.06	3.11	3.15	3.18	3.22	3.25	3.28	3.30	3.33	3.35	3.37	3.40	20
24	2.064	2.50	2.65	2.75	2.83	2.90	2.96	3.01	3.05	3.09	3.13	3.16	3.19	3.22	3.24	3.27	3.29	3.31	3.33	24
30	2.042	2.47	2.61	2.71	2.79	2.85	2.91	2.96	3.00	3.04	3.07	3.10	3.13	3.16	3.18	3.20	3.22	3.25	3.27	30
40	2.021	2.43	2.57	2.67	2.75	2.81	2.86	2.91	2.95	2.98	3.01	3.04	3.07	3.10	3.12	3.14	3.16	3.18	3.20	40
60	2.000	2.40	2.54	2.63	2.70	2.76	2.81	2.86	2.90	2.93	2.96	2.99	3.02	3.04	3.06	3.08	3.10	3.12	3.14	60
120	1.980	2.37	2.50	2.59	2.66	2.72	2.77	2.81	2.84	2.88	2.91	2.93	2.96	2.98	3.00	3.02	3.04	3.06	3.08	120
Inf.	1.9600	2.34	2.47	2.56	2.62	2.68	2.72	2.76	2.80	2.83	2.86	2.88	2.90	2.93	2.95	2.97	2.98	3.00	3.02	Inf.

Exact critical values h_α for $\alpha = 0.10$

DF	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	DF
3	2.353	3.16																		3
4	2.132	2.81	3.10																	4
5	2.015	2.63	2.88	3.05																5
6	1.943	2.52	2.74	2.91	3.03															6
7	1.895	2.44	2.65	2.81	2.92	3.02														7
8	1.860	2.39	2.59	2.73	2.85	2.94	3.02													8
9	1.833	2.34	2.54	2.68	2.79	2.88	2.95	3.01												9
10	1.812	2.31	2.50	2.64	2.74	2.83	2.90	2.96	3.02											10
11	1.796	2.29	2.47	2.60	2.70	2.79	2.86	2.92	2.97	3.02										11
12	1.782	2.27	2.45	2.57	2.67	2.75	2.82	2.88	2.93	2.98	3.02									12
13	1.771	2.25	2.43	2.55	2.65	2.73	2.79	2.85	2.90	2.95	2.99	3.03								13
14	1.761	2.23	2.41	2.53	2.63	2.70	2.77	2.83	2.88	2.92	2.96	3.00	3.03							14
15	1.753	2.22	2.39	2.51	2.61	2.68	2.75	2.80	2.85	2.90	2.94	2.97	3.01	3.04						15
16	1.746	2.21	2.38	2.50	2.59	2.67	2.73	2.79	2.83	2.88	2.92	2.95	2.99	3.02	3.05					16
17	1.740	2.20	2.37	2.49	2.58	2.65	2.72	2.77	2.82	2.86	2.90	2.93	2.97	3.00	3.03	3.05				17
18	1.734	2.19	2.36	2.47	2.56	2.64	2.70	2.75	2.80	2.84	2.88	2.92	2.95	2.98	3.01	3.03	3.06			18
19	1.729	2.18	2.35	2.46	2.55	2.63	2.69	2.74	2.79	2.83	2.87	2.90	2.94	2.96	2.99	3.02	3.04	3.06		19
20	1.725	2.18	2.34	2.45	2.54	2.62	2.68	2.73	2.78	2.82	2.86	2.89	2.92	2.95	2.98	3.00	3.03	3.05	3.07	20
24	1.711	2.15	2.32	2.43	2.51	2.58	2.64	2.69	2.74	2.78	2.82	2.85	2.88	2.91	2.93	2.96	2.98	3.00	3.02	24
30	1.697	2.13	2.29	2.40	2.48	2.55	2.61	2.66	2.70	2.74	2.77	2.81	2.84	2.86	2.89	2.91	2.93	2.96	2.98	30
40	1.684	2.11	2.27	2.37	2.45	2.52	2.57	2.62	2.66	2.70	2.73	2.77	2.79	2.82	2.85	2.87	2.89	2.91	2.93	40
60	1.671	2.09	2.24	2.34	2.42	2.49	2.54	2.59	2.63	2.66	2.70	2.73	2.75	2.78	2.80	2.82	2.84	2.86	2.88	60
120	1.658	2.07	2.22	2.32	2.39	2.45	2.51	2.55	2.59	2.62	2.66	2.69	2.71	2.74	2.76	2.78	2.80	2.82	2.84	120
Inf.	1.6448	2.05	2.19	2.29	2.36	2.42	2.47	2.52	2.55	2.59	2.62	2.65	2.67	2.69	2.72	2.74	2.76	2.77	2.79	Inf.

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