ABSTRACT

Recursive Soft Morphological Filters

by
Padmaja Puttagunta

Mathematical morphology which is based on set-theoretic concept, extracts object features by choosing a suitable structuring shape as a probe. Morphological filters are set operations that transform an image into a quantitative description of its geometrical structure. Appropriately used, they can eliminate noises or irrelevancies while preserving the details of the original image. The applications of morphological filters in image processing and analysis are numerous, which include shape recognition, industrial parts inspection, nonlinear filtering, and biomedical image processing.

Soft morphological filters are used for smoothing signals with the advantage of being less sensitive to additive noises and to small variations in the shape of the objects to be filtered as compared to standard morphological filters. These filters, along with many other transformations, such as Fourier transform, averaging, median, and ranked order filters, are considered as parallel or non-recursive transformations. In the field of signal and image processing, however, apart from these parallel transformations, a class of recursive transformations such as sequential block labeling, predictive coding, adaptive dithering, and sequential distance transforms, are widely used. In chapter two, we introduce recursive soft morphological filters which provide better smoothing capabilities and consume less computational time to reach the root signal. The properties of recursive soft morphological filters, the cascade combination of these filters, and idempotent recursive soft morphological filters are presented. These properties allow problems in the implementation of cascaded recursive soft morphological filters to be reduced to the equivalent problems of a single recursive standard morphological filter.

Ever since Zadeh introduced the concept of fuzzy set theory in 1965, it has found many applications in a variety of fields. The significance of the the fuzzy logic is primarily because it is based on a very intuitive, although somewhat subtle, idea capable of generating many intellectually appealing results that provide new insights to old, often-debated questions. The fuzzy logic has been developed in order to capture the uncertainties associated with human cognitive processes such as in thinking, reasoning, perception, etc.

In Chapter one, we survey the role of fuzzy sets, namely for object extraction and pattern recognition, and assess the strength and limitations. Our survey covers the various algorithms presented, applications, and mathematical foundations.

It is possible to generalize and develop fuzzy approaches for many well-known imaging operations, such as skeleton extraction, enhancement etc., all of which can be used effectively in processing an image pattern. Previous research has accomplished major tasks by utilizing the concepts of fuzzy set theory. In Chapter three, we present an algorithm for skeleton extraction which also ensures connectivity in the resulting skeleton. This algorithm uses the center of gravity measure effectively. In chapter f our, we present an innovative algorithm for polygonal curve fitting by efficiently using the adjacency feature of fuzzy sets. In Chapter five, the future research to perform Image Enhancement by minimizing the degree of adjacency is outlined.

RECURSIVE SOFT MORPHOLOGICAL FILTERS

by
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This thesis is dedicated to
the supreme power who has guided my life to this stage

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CHAPTER 1

INTRODUCTION

1.1 Recursive Soft Morphological Filters

Mathematical morphology which is based on set-theoretic concept, extracts object features by choosing a suitable structuring shape as a probe [4,10,11,14]. Morphological filters are set operations that transform an image into a quantitative description of its geometrical structure. Appropriately used, they can eliminate noises or irrelevancies while preserving the details of the original image. The applications of morphological filters in image processing and analysis are numerous, which include shape recognition, industrial parts inspection, nonlinear filtering, and biomedical image processing.

Mathematical morphology which is based on set-theoretic concept, extracts object features by choosing a suitable structuring shape as a probe [4,10,11,14]. Morphological filters are set operations that transform an image into a quantitative description of its geometrical structure. Appropriately used, they can eliminate noises or irrelevancies while preserving the details of the original image. The applications of morphological filters in image processing and analysis are numerous, which include shape recognition [1,13], industrial parts inspection [12,16], nonlinear filtering [3,8,9], and biomedical image processing [10,15].

The structuring element in morphological filters can be regarded as a template which is translated to each pixel location in an image. These filters can be implemented in parallel due to the fact that each pixel's value in the transformed image is only a function of its neighboring pixels in the given image [5]. Also, the sequence in which the pixels are processed is completely irrelevant. Thus, these parallel image operations can be applied to each pixel simultaneously if a suitable parallel architecture is available. Parallel image transformations are also referred to as *non-recursive transformations*.

In contrast to non-recursive transformations, a class of *recursive transformations* are also widely used in signal and image processing, for example, sequential block labeling, predictive coding, and adaptive dithering [5,10]. The main distinction between these two classes of transformations is that in the recursive transformations, the pixel's value of the transformed image depends upon the pixel's values of both the input image and the transformed image itself. Due to this reason, some partial order has to be imposed on the underlying image domain, so that the transformed image can be computed recursively according to this imposed partial order.

Koskinen *et al.* [6,7] introduced soft morphological filters which possess the desirable property of being less sensitive to additive noises and to small variations in the shape of the objects to be filtered. The structuring element in soft morphological filters is divided into two parts: one being the "hard center" and the other being the "soft boundary." Soft morphological filters can also be viewed as a special class of the weighted order statistic filters [2,8], where only two weights are given to the two parts of the structuring element. A greater weight of the assigned order index is given to the hard center and a weight of one is given to the remaining elements which lie outside the hard center and within the boundary. In chapter two, we introduce recursive soft morphological filters and their properties. A novel idea of reducing the cascaded recursive soft morphological filters to a single recursive standard morphological filter is presented. A new class of idempotent recursive soft morphological filters and their properties are also presented.

1.2 Object Extraction and Pattern Classification Using Fuzzy Sets

1.2.1 Introduction To Fuzzy Theory

Ever since Zadeh introduced the concept of fuzzy set theory in 1965, it has found many applications in a variety of fields. The significance of the the fuzzy logic is primarily because it is based on a very intuitive, although somewhat subtle, idea capable of generating many intellectually appealing results that provide new insights to old, often-debated questions. The fuzzy logic has been developed in order to capture the uncertainties associated with human cognitive processes such as in thinking, reasoning, perception, etc. A conventional set is defined as a collection of elements which have some common properties. The sets are known as *crisp sets* and are defined by a characteristic function as

$$f(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \textit{ does not belong } A \text{ ,} \end{cases}$$

where A is any finite set. However, the object classes generally encountered in the real world are not so "precisely" or "crisply" defined. In most cases, several ambiguities arise in the determination of whether a particular element belongs to a set or not. A good example mentioned by Zadeh is a class of animals. This class clearly includes dogs, cats, tigers, etc. and excludes rocks, plants, houses, etc. However, an ambiguity arises in the context of objects such as bacteria and starfish with respect to the class of animals.

Unlike a crisp set in which each object is assigned a value of 0 or 1, each object in a fuzzy set is given a certain "degree of membership" which denotes the degree of belongingness of the object to the set. The notion of the plausibility of set membership leads to the generalization of the degree of membership in a set, and from this generalization comes a varient of the crisp set theory, called the fuzzy set theory. Therefore, a fuzzy set can be considered as a class of objects with a continuum of membership grades. The membership function assigns to each object a degree of membership value ranging from 0 to 1. The closer the membership value is to 1, the more the object belongs to the set, and vice versa.

Over the years, fuzzy set theory has received more and more attention from researchers in a wide range of scientific areas, especially in the past few years. System-Oriented problems such as Decision-Making, Fuzzy Control, Learning Systems, Pattern Classification, Fuzzy Diagnosis, Identification of structures etc., utilize the fuzzy approach. The applications also concern the following fields: Artificial Intelligence and robotics, Image processing, speech recognition, biological and medical sciences, appl ied operations research, economics and geography, sociology, psychology, linguistics, semiotics, and some more restricted topics.

In this chapter, we survey the role of fuzzy sets, namely for object extraction and pattern recognition, and assess the strength and limitations. Our survey covers the various algorithms presented in this field, applications, and mathematical foundations.

1.2.2 Role Of Fuzzy Geometry In Image Analysis

Image processing inherently bears some fuzziness in nature. This is due to the fact that the regions in the image are not always crisply defined. The approach to the analysis or interpretation of an image requires traditionally to segment the image into meaningful regions, extract the features of each region, and finally construct the relationships among the regions. However, due to the fuzzy behavior of the images, it is convenient to regard the regions of the image as fuzzy subsets of the image. Rozenfeld generalized the standard geometrical properties to the properties of fuzzy image geometry, such as topological connectedness, adjacency, surroundedness, area, perimeter, etc. based on which many significant algorithms utilizing the concepts of fuzzy geometry in the field of image analysis have been developed.

Definitions of Fuzzy sets

A fuzzy set of a grey-scale image S is a mapping μ from S into the values ranging [0, 1]. For any pixel $p \in S$, $\mu(p)$ is called *the degree of membership* of p.

The mapping function μ is any function which follows the properties of symmetry and ambiguity around the cross-over point. From the definition above, one may assume that the fuzzy set is identical to the probability function. However, there are significant differences between these two. For example, the summation of all the values of the probability function should be equal to 1, which is not the case in fuzzy sets. The summation of all the values of the degree of membership in a fuzzy set need not be equal to 1.

Several definitions in fuzzy sets are given below.

(a) *Empty set*: A fuzzy set μ is said to be empty, denoted by $A = \varnothing$, iff $\mu(x) = 0$, *oppA* $x \in S$.

(b) *Equality*: Two fuzzy sets μ and ν are said to be equal, denoted by $\mu = \nu$, iff $\mu(x) = \nu(x)$, *oppA* $x \in S$.

(c) *Support*: The support of a fuzzy set μ is an ordinary subset of S, i.e., Supp (μ) = { $x \in S \mid \mu(x) > 0$ }.

(d) *Cross-over points*: The elements $x \in S$ such that $\mu(x) = 1/2$ are called cross-over points of μ.

(e) *Normalized fuzzy set*: μ is said to be normalized iff *oppE* $x \in S$, $\mu(x) = 1$.

(f) *Height of* μ : The height of a fuzzy set μ is defined as hgt(μ) = sup { $\mu(x)$ }, i.e., the height of μ is defined to be the least upper bound of $\mu(x)$. It is therefore obvious that for a normalized fuzzy set μ, hgt(μ) = 1.

(g) *Cardinality of a fuzzy set*: When X is a finite set, the cardinality $|A|$ of a fuzzy set A is defined as $|A| = \Sigma\, \mu\,(x)$ $|A|$ is also called the power of A.

(h) *Convexity of a fuzzy set*: There are several alternative definitions to the convexity property of fuzzy sets. Listed below are three of major ones.

Definition 1. A fuzzy set μ is said to be convex if for every p, q belonging to S, and all r on the line segment pq, we have $\mu\,(r)$ >= min[$\mu\,(p)$, $\mu\,(q)$]. This implies that μ is fuzzily convex, if it is convex in the ordinary sense.

Definition 2. A fuzzy set μ of X is convex iff the sets T defined by $T_\alpha = \{ x \in S \mid \mu(x) >= \alpha \}$ are convex for all in the interval (0,1]. The sets T_α are called the "level sets" of μ.

Definition 3. A direct definition of convexity is the following: μ is convex iff

$$\mu[\lambda x_1 + (1-\lambda) x_2] \geq \min[\mu(x_1), \mu(x_2)]$$

for every x_1, x_2 belonging to S and in [0,1].

(i) *Cross-Section*: The cross-section of μ by a line l is defined as the restriction of μ to l.

(j) *Star-shapedness*: μ is star-shaped from p if its cross-sections by lines through p are all convex.

Set Theoretic Operations

(a) *Union*: The union of two fuzzy sets A and B with their respective membership functions $f_A(x)$ and $f_B(x)$ is a fuzzy set C, denoted as $C = A \cup B$, whose membership function is

$$f_C(x) = \max[f_A(x), f_B(x)], \ oppA \ x \in X$$

The union of two fuzzy sets A and B can be interpreted as the smallest fuzzy set containing both A and B. Let D be any fuzzy set containing both A and B. Since $f_D \geq f_A$ and $f_D \geq f_B$, we have $f_D \geq \max[f_A, f_B] = f_C$.

(b) *Intersection*: The intersection of two fuzzy sets A and B with their respective membership functions f_A and f_B is a fuzzy set C, denoted as $C = A \cap B$, whose membership function is $f_C(x) = \min [\, f_A(x), f_B(x)\,], x \in X$. The intersection of two fuzzy sets can also be interpreted as the largest fuzzy set which is contained in both.

Corollary: A and B are disjoint if their intersection C is empty, i.e., $f_C(x) = 0$.

(c) *Complement*: The complement of a fuzzy set μ is denoted by $\vec{\mu}$ and is defined as $\vec{\mu}(x) = 1 - \mu(x)$, $oppA\ x \in S$.

(d) *Containment*: A fuzzy set μ is said to be contained in another fuzzy set ν iff $\mu(x) \leq \nu(x)$, $oppA\ X \in S$.

Properties:

(a) *Commutativity*: $A \cup B = B \cup A$; $A \cap B = B \cap A$.

(b) *Associativity*: $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$.

Note: the above two properties follow immediately for union and intersection because their corresponding operators of Max and Min are associative and commutative.

(c) *Idempotency*: $A \cup A = A$; $A \cap A = A$.

(d) *Distributivity*: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(e) $A \cap \varnothing = \varnothing$; $A \cup X = X$.

(f) *Identity*: The identity element with respect to union $\cup$ is $\varnothing$ and with respect to intersection is X.

(g) *Absorption*: $A \cup (A \cap B) = A$; $A \cap (A \cup B) = A$.

(h) *De Morgans laws*: $(A \cup B) = A \cap B$; $(A \cap B) = A \cup B$.

(i) *Involution*: $A = A$.

(j) *Equivalence formula*: $(A \cup B) \cap (A \cup B) = (A \cap B) \cup (A \cap B)$.

(k) *Symmetrical difference formula*: $(A \cap B) \cup (A \cap B) = (A \cup B)\ (A \cup B)$.

Alternative Operators

(a) *Probabilistic operators*:

1. *Intersection*: For every $x \in X$, $\mu_{A\ mdot\ B}\ (x) = \mu_A(x)\ mdot\ \mu_B(x)$.

2. *Union*: For every $x \in X$, $\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\ mdot\ \mu_B(x)$.

1.2.3 Old Definitions And New Versions

Pal *et.al* [18], have modified some of the old definitions and formulated a set of new definitions. Based on these definitions, a number of algorithms for object extraction have been presented. The new definitions are as follows:

The height and width of a fuzzy set μ as given by Rozenfeld have been modified by the authors above, yielding the definitions for the length and the breadth of a fuzzy set. Whereas the height was defined as the summation of the maximum of the membership values along each column, the new definition defines the length to be the maximum of the summation along each column. That is, the height and length are given by

$$height: h(\mu) = \max \mu(x,y) dy$$

$$length: l(\mu) = max\, \mu(x,y) dy$$

The width is defined as the summation of the maximum membership values along each row, and the corresponding new definition of breadth computes it to be the maximum of the summation of the membership values along each row. In other words, the width and breadth are defined as

$$width: w(\mu) = \max \mu(x,y) dx$$

$$breadth: b(\mu) = max\, \mu(x,y) dx$$

Remarks: According to the definitions stated above, the new definitions do not provide significant improvement over the old definitions. Also, when applied to any image processing algorithm, they do not exhibit any outstanding improvement over the previous definitions.

However, the definition of the index of area coverage (IOAC) which is an indicator of the fraction of the maximum area actually covered by the object, and which is somewhat similar to the compactness measure(since both these functions are used to determine the same goal) is used efficiently in object extraction algorithms. Based on this property, the authors have also formulated an algorithm for object extraction which incorporates the idea of optimizing the IOAC. The definition of IOAC is

$$IOAC: \quad Ioac(\mu) = a(\mu) \, / \, l(\mu) \, b(\mu)$$

The authors have also modified the definition of ''adjacency'' and ''degree of adjacency'' have also been modified. The new definitions fit very well with the geometric notion of adjacency. The adjacency measure is therefore directly proportional to the length of the common border between the two fuzzy sets and is inversely proportional to two factors: the distance between the regions, and the difference of the grey levels. Two peicewise constant fuzzy regions μ and τ are said to be more adjac ent to each other if the difference in the membership values of μ and τ is less.

The degree of adjacency as given by Rozenfeld is a measure of the physical adjacency between the two regions, but does not take into account the membership values. The new definition, however overcomes this drawback and defines a function for the degree of adjacency which combines the new adjacency definition with the old definition of the degree of adjacency. In other words, it considers both the physical distance as well as the difference between the grey levels of the two fuzzy regions. These new functions defined take into account account the local information.

An object extraction algorithm has been presented which uses the foundation of optimum degree of adjacency. This is due to the definition of the degree of adjacency that the greater the difference in grey levels between pixels, the greater distance between them, and hence the lower degree of adjacency. Therefore, a lower value of adjacency implies that the segments formed are more separable considering both their grey levels and the physical distance.

This algorithm works as follows. The algorithm starts with selecting a threshold 's' from the grey level range of the picture. Next, we calculate the degree of adjacency. A coouccurance matrix is used to compute the degree of adjacency. This matrix is a two dimensional array of size L x L (where L is the number of gray levels in the picture), and the value of each location (i,j) gives the number of times gray level j follows grey level i. The meaning of j is a neighbor of grey level i, where 'neighbor' can be considered as either 8 neighbor or 4 neighbor. The degree of adjacency is computed for different values of the threshold 's'. The value of optimal segmented version of the image.

This algorithm which optimizes the degree of adjacency between two regions with respect to the grey levels, takes into account the spatial information. Although this algorithm can be applied to any kind of image, computation of the co-occurance matrix can be time consuming.

The author formulates similar functions to extract the object by optimizing the compactness/IOAC. The method involves using the standard 'S' function to extract a bright image. Later, the cross over point is iteratively changed, and in each such iteration, the COMP/IOAC is computed. The $\mu(x)$ plane having the optimum COMP/IOAC is treated as the optimum fuzzy segmented version. This algorithm is based on local information and also takes the shape of the object into account.

Skeleton Extraction By Using IOAC/COMPACTNESS:

The fuzzy geometric concept of IOAC/Compactness is used in an algorithm which extracts the skeleton of an image.

The fuzzy version of the image is fed as input to the algorithm. New membership values are then assigned to the pixels denoting the degree of belongingness of the pixel to the skeleton of the object. These membership values are assigned depending upon two factors, namely, the maximum intensity, and the property of occupying vertically and horizontally middle positions from the edges. Finally, the skeleton is chosen from one of the α-cuts of the new fuzzy plane, i.e., that fuzzy plane which yields a minimum value of the IOAC/comp value. For any other value of α, comp(μ) would be greater.

This paper has made an attempt to implement the concept of fuzzy geometry in image processing/analysis problems. Many new geometric functions such as minor and major axis, center of gravity and density, length etc., have been introduced. While no algorithm has been developed as yet which incorporates many of these new features, however, the scope of these functions in the field of image processing/analysis has to be further investigated since this promises to be a fruitful field.

1.2.4 Breast Cancer Detection Based On Fuzzy Entropy Thresholding

Similar to the numerous types of cancers in the human body, such as lung cancer, kidney cancer etc., breast cancer, which effects almost a ninth of the female population in the United States of America, can be treated effectively with the existing methods of treatment, if detected at an early stage. Therefore, detecting the cancer tubules(one type of breast cancer lesions) at an early stage is of primary importance. Xueqin Li et.al, at the Utah State University have done significant research in this area and have developed a computer-aided diagnose system for processing a digitized microscopic slide image to compute tubule estimation. The algorithm is based on maximizing the fuzzy entropy to thereby select the threshold of the image. T he entropy of an image is used as a parameter for measuring the degree of ambiguity of an image.

Definition of Entropy

Entropy of a system (Shannon's Definition): The entropy of a system is a measure of uncertainty about its actual structure.

Shannon's definition of entropy of an 'n' state system is

$$H = -\sum_{i=0}^{i=n} p_i \, log\,(p_i)$$

where p_i is the probability of occurence of event i and the summation of the total probability for all n states is 1.

Entropy of an image: The entropy of an image, based on the above definition, is given by

$$H = - \sum_{i=0}^{i=L-1} p_i \log(p_i); \qquad p_i = N_i / N$$

Aposteriori Entropy of the Image: Aposteriori entropy of the image is defined as

$$H_L(S) = -P_S \log(P_S) - (1-P_S) \log(1-P_S)$$

where S is the threshold.

Criteria for Detection of Tubules:

The process of detecting tubules in the microscopic slide image is a difficult task primarily due to the rich variations of grey level intensities, and also due to the presence of noise and the very rich image textures consisting of glands and fatty tissues which increase the background variations of tubule areas. To solve this difficulty, the computer-aided diagnose system incorporates the diagnosis strategy of the physicians in the early stage, namely, tubules are detected in a medical image due to thre e reasons:

a) brightness-tubules are brighter than the surrounding tissues and the background;

b) homogeneous region-most tubule areas have uniform density;

c) dark boundary-tubules are surrounded by dark edges.

The algorithm works as follows. First, the standard "S" function is used to compute a bright image plane with a particular band width. Next, the amount of entropy is computed for a selected value of the grey level of the image within the bandwidth. The entropy value is then computed iteratively by varying the selected grey level. Finally, the image plane with the maximum entropy value is regarded as the fuzzy segmented version of the image.

The method suggested in this paper has been proved experimentally to work well with noisy and vague images. However, it involves thresholding the image twice, firstly to separate the object from the background, and secondly to separate tubule regions from glands and fatty tissue areas within the extracted object. The threshold values are chosen from the experience of medical doctors. Thus, fuzzy theory plays an important role in the field of medical sciences also.

1.2.5 The Role Of Fuzzy Sets In Image Description And Primitive Extraction

The theory of fuzzy sets has been applied extensively to the understanding of an image. Image understanding differs significantly from Image processing. In fact, the field of pattern recognition can be considered as a two fold task, namely, image processing and image understanding. Image processing takes as input an image and also produces as output an improved version of the input image. The process between taking the input and producing the output typically involves operations such as enhancement, re storation, smoothing, sharpening and other noise detection techniques in order to extract the objects in the picture. In contrast, the process of image understanding involves inputing an image but producing as output an interpretation and description of the image, rather than an image itself. Pal *et al.* [18], have done a significant amount of research in applying the theory of fuzzy sets to the understanding of an image.

The application of fuzzy sets for image understanding

The model presented in the paper assumes an edge-detected image to be the input. It then produces as output the automatic interpretation and description of the image. The major steps involved in the process consist of:

1. Encoding the image contours to represent them by octal-coded strings

2. Smoothing of chains to remove spurious wiggles in the contours

3. Segmentation and assignment of degree of arcness to each segmented smoothed chain.

Fuzzy membership functions are used both in the process of assigning the octal codes and in obtaining the arcness of the image. Initially, each line segment of the image is assigned four memberships corresponding to four different fuzzy sets, namely a vertical fuzzy set, horizontal fuzzy set, oblique fuzzy set, and an 'arc' fuzzy set. The vertical fuzzy set depicts the degree of belongingness of the line segment to being a vertical line. Similar definitions follow for the other fuzzy sets. An octal co de is generated for each line segment depending upon the maximum of the vertical, horizontal, and oblique membership values. This code is then fed to smoothers which remove any spurious wiggles in the contour.

Although the method involves employing the fuzzy set theory in assigning the octal codes, the main drawback of this method sprouts from the application of smoothers which tend to vary the original contours of the image in the process of removing spurious wiggles.

1.3 Conclusions

In this chapter, several properties of fuzzy sets and their applications in the field of image processing and pattern recognition have been presented. Fuzzy set theory is helpful in weakening crisp decisions (as is done in the case of conventional techniques). Rozenfeld [19], had explained various concepts of fuzzy geometry in a grey image, many of which are generalizations of crisp properties of, and relationships between, regions in an image. These extensions include the topological concepts of connectedness, adjacency and surroundness, star-shapedness and convexity, area, perimeter, compactness, height, width, extent, diameter, etc.

Many new geometric functions such as minor and major axis, center of gravity and density, length etc., have been introduced. While no algorithm has been developed as yet which incorporates many of these new features, however, the scope of these functions in the field of image processing/analysis has to be further investigated since this promises to be a fruitful field.

CHAPTER 2

RECURSIVE SOFT MORPHOLOGICAL FILTERS

2.1 Definitions of Soft Morphological Filters

The underlying strategy in morphological transformations is to expose the characteristics of an object by probing its microstructure with various forms which are known as *structuring elements*. The analysis is geometrical in nature and it approaches to image processing from the vantage point of human perception. This unique feature, however, is accompanied by some drawbacks such as being highly sensitive to additive noises and to defects in the image. The structure of the soft morphological filters was designed in such a manner that the filter is more tolerant to the defects in shape and to additive noises [6,7].

Let A and B be two finite convex sets in the N-dimensional Euclidean space E^N. The constraint on the two sets is $A \subseteq B$, and B is divided into two subsets: the *hard center set* (A) and the *soft boundary set* $(B \setminus A)$, where "\" denotes the *set difference*. Also let k be a positive integer, such that $1 \le k \le \min \{ Card(B)/2, Card(B \setminus A) \}$, where $Card(B)$ denotes the *cardinality* of the set B. The translation of a set A by a vector $z \in E^N$ is defined by

$$A_z = \{ a + z \mid a \in A \}$$

A collection set of pixels where repetition is applied is called a *multiset*. The repetition k times of $f(a)$ is represented by $\{k \diamond f(a)\} = \{f(a), f(a), \cdots, f(a)\}$ (k times). The formal definitions of soft morphological transformations are given as follows, where the input image f is gray scale and the structuring element B is binary (in other words, A is also binary since $A \subseteq B$). That is no gray value is assigned to the structuring element, or the structuring element is flat on the top.

Definition 1: The soft morphological dilation of f by $[B, A, k]$ is

$$(f \oplus [B, A, k])(x) = k\text{th largest of multiset} \tag{1}$$

$$\{ k \diamond f(a) \mid a \in A_x \} \cup \{ f(b) \mid b \in (B-A)_x \}.$$

Definition 2: The soft morphological erosion of f by $[B, A, k]$ is

$$(f \ominus [B, A, k])(x) = k\text{th smallest of multiset} \tag{2}$$

$$\{ k \diamond f(a) \mid a \in A_x \} \cup \{ f(b) \mid b \in (B-A)_x \}.$$

Definition 3: The soft morphological opening of f by $[B, A, k]$ is

$$f \circ [B, A, k] = (f \ominus [B, A, k]) \oplus [B, A, k]. \tag{3}$$

Soft morphological opening is defined as a soft morphological erosion followed by a soft morphological dilation. Soft morphological closing in the opposite sequence of dilation and erosion is called the *morphological dual* to opening.

Definition 4: The soft morphological closing of f by $[B, A, k]$ is

$$f \bullet [B, A, k] = (f \oplus [B, A, k]) \ominus [B, A, k]. \tag{4}$$

Note that when $k = 1$, the soft morphological operations correspond to the standard morphological operations.

Since soft morphological filters adopt order statistics [2] and mathematical morphology [4,10,11], they can be viewed as weighted order statistic filters which apply set union operation and more weights are given to the pixels in the hard center than to the pixels in the soft boundary. Since general recursive structures usually provide better smoothing capabilities and take less computational time even though at the expense of increased detailed distortion [5,10], the recursive soft morphological filters with the advantage of preserving the fine details of shape are worthy of investigation. In the next section, the definitions and the basic properties of recursive soft morphological filters are presented.

2.2 Definitions And Properties Of Recursive Soft Morphological Filters

Recursive filters are the filters which use previously filtered outputs as their inputs. Let x_i and y_i denote the input and output values at location i, respectively, where $i = \{0, 1, \cdots, N-1\}$. Let the domain of the structuring element be $< -L, \cdots, -1, 0, 1, \cdots, R >$, where L is the left margin and R is the right margin. Hence, the structuring element has the size of $L+R+1$. Start up and end effects are accounted for by appending L samples to the beginning and R samples to the end of the signal sequence. The L appended samples are given the value of the first signal sample; similarly, the R appended samples receive the value of the last sample of the signal.

Definition 5: The recursive counterpart of a nonrecursive filter Ψ given by

$$y_i = \Psi\,(x_{i-L}, \ \cdots, x_{i-1}, x_i, x_{i+1}, \ \cdots, x_{i+R}) \tag{5}$$

is defined as

$$y_i = \Psi\,(y_{i-L}, \ \cdots, y_{i-1}, x_i, x_{i+1}, \ \cdots, x_{i+R}) \tag{6}$$

by assuming that the values of $y_{i-L}, \ \cdots, y_{i-1}$ are already given.

The ordering relationship that exists between standard morphological operations and soft morphological operations is

$$f \ominus B \le f \ominus [B,A,k] \le f \oplus [B,A,k] \le f \oplus B. \tag{7}$$

However, in the case of recursive soft morphological filters, no underlying ordering relationship can be given with respect to standard morphological filters. They can be greater than or smaller than the corresponding standard morphological filters depending upon the input signals.

Example 1: Let $B = < -2\ -1\ 0\ 1\ 2 >$, $A = < -1\ 0\ 1 >$, and $k = 2$. Let the input signal $f = \{\ 4\ 7\ 2\ 9\ 6\ 8\ 5\ 4\ 7\ \}$. We have

$$f \oplus B = \{\ 7\ 9\ 9\ 9\ 9\ 9\ 8\ 8\ 7\ \},$$

$$f \oplus [B,A,k] = \{\ 7\ 7\ 9\ 9\ 9\ 8\ 8\ 7\ 7\ \},$$

$$f \oplus_r [B,A,k] = \{\ 7\ 7\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ \},$$

where " $\oplus_r$ " denotes the recursive soft morphological dilation.

From the above example, it is evident that recursive soft morphological filters are neither completely greater than or smaller than standard morphological filters. However, the ordering relationship that exists between soft morphological filters and their recursive counterpart is

$$f \ominus_r [B,A,k] \leq f \ominus [B,A,k] \leq f \oplus [B,A,k] \leq f \oplus_r [B,A,k]. \tag{8}$$

Definition 6: A filter Ψ is said to be *idempotent* if $\Psi(\Psi(f)) = \Psi(f)$ for any input signal f.

Definition 7: A filter Ψ is said to be *extensive* if $\Psi(f(x)) \geq f(x)$ for every x. Otherwise, if $\Psi(f(x)) \leq f(x)$, the filter is said to be *anti-extensive*.

Definition 8: A filter Ψ is said to be *increasing* if for any two input signals f and g, such that $f(x) \le g(x)$ for every x, the resultant outputs satisfy the relationship $\Psi(f(x)) \le \Psi(g(x))$.

We prove that recursive soft morphological filters are increasing by first proving that if a soft morphological filter is increasing, then its recursive counterpart is also increasing.

Theorem 1: If a soft morphological filter is increasing, then the recursive soft morphological filter is also increasing.

Proof: Let two input signals be $f(x) = \{x_0, x_1, \cdots, x_{N-1}\}$ and $g(x') = \{x_0', x_1', \cdots, x_{N-1}'\}$ and have the ordering relation of $f(x) \le g(x')$ for every x and x'. We need to prove that for every y_i and y_i',

$$y_i = \Psi(y_{i-L}, \cdots, y_{i-1}, x_i, x_{i+1}, \cdots, x_{i+R})$$

$$\le \Psi(y_{i-L}', \cdots, y_{i-1}', x_i', x_{i+1}', \cdots, x_{i+R}') = y_i'$$

That is $y_i \le y_i'$ for every i. We prove the theorem by induction. Since the recursive and non-recursive filters of y_0 and y_0' only depend upon the values of the input pixels $\{x_0, \cdots, x_R\}$ and $\{x_0', \cdots, x_R'\}$, respectively, which have the ordering of $\Psi(f(x)) \le \Psi(g(x'))$, the initial condition of $i = 0$ is satisfied Assume that the condition is true for $i = L-1$. That is the output values at locations $\{0, \cdots, L-1\}$ satisfy the condition. Now for $i = L$, we have

$$y_L = \Psi(y_0, \cdots, y_{L-1}, x_L, x_{L+1}, \cdots, x_{L+R})$$

$$\le \Psi(y_0', \cdots, y_{L-1}', x_L', x_{L+1}', \cdots, x_{L+R}') = y_L'$$

simply because the output values at locations $\{0, \cdots, L-1\}$ and the input values at locations $\{L, \cdots, L+R\}$ both satisfy the condition. This implies that the recursive counterpart of Ψ also has the increasing property. □

Based on the above theorem, in order to prove that recursive soft morphological operations are increasing, it is sufficient to prove that the non-recursive operations are increasing.

Theorem 2: Soft morphological dilation and erosion are increasing.

Proof: Let f and g be two input signals such that $f(x) \leq g(x)$ for every x. According to the definition of soft morphological dilation, we have

$$f \oplus [B,A,k] = k\text{th largest of } \left[\{ k \diamond f(a) \,|\, a \in A_x \} \cup \{ f(b) \,|\, b \in (B-A)_x \} \right]$$

$$\leq k\text{th largest of } \left[\{ k \diamond g(a) \,|\, a \in A_x \} \cup \{ g(b) \,|\, b \in (B-A)_x \} \right] = g \oplus [B,A,k].$$

Thus for every x,

$$(f \oplus [B, A, k])(x) \leq (g \oplus [B, A, k])(x).$$

The proof for soft morphological erosion can be similarly derived. □

Theorem 3: Soft morphological closing and opening are increasing.

Proof: Let f and g be two input signals such that $f(x) \leq g(x)$ for every x. Let D and E denote soft morphological dilation and erosion, respectively. According to the increasing property of dilation, we have $D[f(x)] \leq D[g(x)]$. Taking the soft morphological erosion of the dilated results yields $E[D[f(x)]] \leq E[D[g(x)]]$ for every x, according to the increasing

property of soft morphological erosion. This result implies that soft morphological closing is increasing. By changing the above ordering of soft morphological dilation and erosion, we obtain the result that the soft morphological opening is also increasing. □

Preposition 1: From Theorems 1-3, we can state that recursive soft morphological dilation, erosion, opening and closing are all increasing.

Example 2: Let two input signals $f = \{ 7\ 2\ 8\ 3\ 4\ 9\ 6\ 1 \}$ and $g = \{ 8\ 5\ 9\ 7\ 5\ 9\ 6\ 2 \}$. Consider a structuring element $[B,A,k]$, such that $B = < -2\ -1\ 0 >$, $A = < -1\ 0 >$, and $k = 2$. Applying the recursive soft morphological dilation on f and g yields

$$f \oplus_r [B,A,2] = \{ 7\ 7\ 8\ 8\ 8\ 9\ 9\ 9 \}$$

$$g \oplus_r [B,A,2] = \{ 8\ 8\ 9\ 9\ 9\ 9\ 9\ 9 \}$$

Theorem 4: Recursive soft morphological dilation is extensive, and recursive soft morphological erosion is anti-extensive.

Proof: According to the definition of soft morphological dilation, the values of the multiset $\{ k \diamond f(a) \mid a \in A_x \} \cup \{ f(b) \mid b \in (B-A)_x \}$ are sorted in the descendent order and the kth largest is selected. If $f(x)$ is the maximum value in the set B, it is selected as the output after the repetition k times. If $f(x)$ is not the maximum value in B, the selected kth largest value must be greater than or equal to $f(x)$ after $f(x)$ is repeated k times. This implies that for every x, soft morphological dilation is extensive. We can similarly derive that the recursive soft morphological erosion is anti-extensive. □

Example 3: Let $f = \{ 3\ 4\ 2\ 4\ 7\ 6\ 9\ 5 \}$, $B = < -2\ -1\ 0\ 1\ 2 >$, $A = < -1\ 0\ 1 >$, and $k = 2$. Applying the recursive soft morphological dilation on f yields

$$f \oplus_r [B,A,2] = \{ 4\ 4\ 4\ 7\ 7\ 9\ 9\ 9 \}.$$

Applying the recursive soft morphological erosion on f yields

$$f \ominus_r [B,A,2] = \{ 3\ 2\ 2\ 2\ 2\ 2\ 2\ 2 \}.$$

Theorem 5: Recursive soft morphological opening and closing are neither extensive nor anti-extensive.

Proof: Let f be any input signal. Also let $D[f(x)]$ and $E[f(x)]$ denote the dilated and eroded values at any point x belonging to f, respectively. If we perform a recursive soft morphological erosion first on f, we have $E[f(x)] \leq f(x)$ for every x belonging to f. Performing a recursive soft morphological dilation on these values, i.e., an opening which is denoted by O, yields $O[f(x)] \geq E[f(x)]$ for every x belonging to f. This implies that the final output $O[f(x)]$ may be greater than, equal to, or less than the initial input value $f(x)$. The proof can be similarly derived for the recursive soft morphological closing. □

Example 4: Given $f = \{ 2\ 3\ 4\ 1\ 0\ 2\ 3 \}$. Let $B = < -2\ -1\ 0\ 1\ 2 >$, $A = < -1\ 0\ 1 >$, and $k = 2$. After performing a recursive soft morphological erosion, we have the result to be $\{ 2\ 2\ 1\ 0\ 0\ 0\ 0 \}$. Applying a recursive soft morphological dilation on this result yields $\{ 2\ 2\ 2\ 2\ 2\ 2\ 2 \}$, which is neither completely greater than nor less than the input signal f.

From the definition of recursive soft morphological dilation and the properties of scalar multiplication, the result obtained by multiplying the dilated value of the input signal by a positive constant is equal to the result obtained by initially performing a recursive soft morphological dilation and then multiplying it by a positive number. We give the property without proof.

Property 1: Recursive soft morphological filters are scaling-invariant.

Example 5. Consider $f = \{\ 2\ 3\ 4\ 0\ 1\ 2\ 3\ \}$. Let $B = <-2\ -1\ 0\ 1\ 2>$, $A = <-1\ 0\ 1>$, and $k = 2$. The output after applying a recursive soft morphological erosion is $\{\ 2\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \}$. Multiplying this by 2 gives $\{\ 4\ \ 4\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \}$. Now multiplying f by the scale 2, we have $\{\ 4\ \ 6\ \ 8\ \ 0\ \ 2\ \ 4\ \ 6\ \}$. Applying a recursive soft morphological erosion on this, we have $\{\ 4\ \ 4\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \}$.

2.3 Idempotent Recursive Soft Morphological Filters

An idempotent filter in Definition 6 maps an arbitrary input signal into an associated set of root sequences. Each of these root signals is invariant to additional filter passes and is the result of the repeated filter passes on one or more of the input signals. The standard morphological opening and closing have the property of being idempotent. In contrast, recursive soft morphological opening and closing are not idempotent in the general case. However, when the filter is designed in a specific way, recursive soft morphological opening and closing are also idempotent. In this section, we will describe idempotent recursive soft morphological filters in one dimension along with a few examples.

If $B = < -n, -n+1, \cdots, -1, 0, 1, \cdots, n-1, n >$ and $A = < -n+1, \cdots, -1, 0, 1, \cdots, n-1 >$, where $n \geq 1$, then we denote the structuring element $[B,A,k]$ by $[n, n-1, k]$.

Property 2: Recursive soft morphological dilation and erosion are idempotent for the structuring element: B of length three, A the central point, and $k = 2$. That is the structuring element $[1,0,2]$.

Example 6: Given $f = \{\ 2\ 3\ 4\ 0\ 1\ 2\ 3\ \}$. Let $B = < -1\ 0\ -1 >$, $A = < 0 >$, and $k = 2$. Applying a recursive soft morphological dilation on f by [B,A,k] gives $\{\ 2\ 3\ 4\ 1\ 1\ 2\ 3\ \}$. Repeatedly applying the dilation yields the same result. Applying a recursive soft morphological erosion on f by $[B,A,k]$ gives $\{\ 2\ 3\ 3\ 0\ 1\ 2\ 3\ \}$. Repeatedly applying the erosion yields the same result.

According to the above property of the dilation and erosion, we can easily derive the following property for opening and closing.

Property 3: Both recursive soft morphological opening and closing are idempotent for the structuring element: B of any length and A of no point to the right hand side of the center.

Example 7: Let $B = < -2\ -1\ 0\ 1\ 2 >$, $A = < -1\ 0 >$, and $k = 2$. Given $f = \{\ 7\ 3\ 9\ 2\ 6\ 3\ 5\ 8\ \}$. Performing a recursive dilation yields $\{\ 7\ 7\ 9\ 9\ 9\ 9\ 9\ 9\ \}$. Followed by a recursive erosion yields the closing result $\{\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ \}$. Performing the closing operation again gives the same result.

Property 4: Recursive soft morphological closing and opening by $[n,\ n-1,\ k]$ are idempotent, where $k = 1$ or 2.

Example 8: Let $B = < -3\ -2\ -1\ 0\ 1\ 2\ 3 >$, $A = < -2\ -1\ 0\ 1\ 2 >$ and $k = 2$. Given $f = \{\ 6\ 9\ 4\ 8\ 2\ 8\ 1\ 5\ \}$. Applying a recursive erosion by [3,2,2] gives $\{\ 4\ 4\ 2\ 2\ 1\ 1\ 1\ 1\ \}$. Followed by a recursive dilation yields the opening result $\{\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ \}$. Performing the opening operation repeatedly yields the same result.

Property 5: If the maximum positive impulse (for recursive soft morphological dilation) or minimum positive impulse (for recursive soft morphological erosion) of the input signal lies within the hard center window of the filter during the scanning of the first pixel of the input signal, then the filter is idempotent.

Example 9: Let $B = < -2\ -1\ 0\ 1\ 2 >$, $A = < -1\ 0\ 1 >$ and $k = 2$. Given $f = \{\ 3\ 9\ 8\ 7\ 2\ 5\ 6\ 3\ \}$. Applying a recursive dilation by [2,1,2] gives $\{\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ \}$. Repeatedly applying the filter yields the same result.

Example 10: Let $f = \{\ 4\ 3\ 9\ 6\ 9\ 7\ 3\ 5\ \}$ and [B,A,k] be the same as the previous example. Applying a recursive erosion gives $\{\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ \}$. Repeatedly applying the filter yields the same result.

Corollary 1: If the kth largest (smallest) value selected during the scanning of the first pixel of the input signal for a recursive soft morphological dilation (erosion) happens to be the maximum (minimum) positive impulse in the input signal, then the filter is idempotent.

Example 11: Let $B = < -3\ -2\ -1\ 0\ 1\ 2\ 3 >$, $A = < -1\ 0\ 1 >$ and $k = 2$. Given $f = \{\ 4\ 5\ 1\ 1\ 9\ 7\ 6\ 8\ \}$. Applying a recursive soft erosion yields $\{\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ \}$. Repeatedly applying the filter yields the same result.

2.4 Cascaded Recursive Soft Morphological Filters

Cascaded weighted median filters were introduced by Yli-Harja *et al.* [17] in which several interesting properties of weighted cascaded filters were discussed. We now present some properties of cascaded recursive soft morphological filters. By cascade connection of filters F and G, we mean that the original input signal f is filtered by the filter F to produce an intermediate signal g. Then g is filtered by the filter G to produce the output signal h. Cascaded filters F and G can also be presented as a single filter H which produces the output h directly from the input f. We now discuss the properties of cascaded recursive soft morphological filters.

Property 6: The cascaded recursive soft morphological filters are not commutative.

The above property describes that the result of applying cascaded recursive soft morphological operations are dependent upon the ordering of the cascaded operators.

Property 7: The cascaded recursive soft morphological filters are associative.

The above property describes that the result of applying cascaded recursive soft morphological operations is independent of the grouping of the cascaded operators as long as the ordering relationship remains the same.

Observation 1: For any input signal being applied to different cascaded recursive soft morphological filters, irrespective of the intermediate results produced, the final outputs may be the same.

Example 12: Let two filters of $[B_1, A_1, k]$, $[B_2, A_2, k]$ be the first cascade combination and another two filters of $[B_3, A_3, k]$, $[B_4, A_4, k]$ be the second cascade combination, where $k = 2$, $A_1 = < -1\ 0 >$, $B_1 = < -2\ -1\ 0 >$, $A_2 = < -1\ 0\ 1 >$, $B_2 = < -2\ -1\ 0\ 1\ 2 >$, $A_3 = < 0 >$, $B_3 = < -1\ 0\ 1 >$, $A_4 = < -1\ 0\ 1 >$, and $B_4 = < -2\ -1\ 0\ 1\ 2 >$. Let $f = \{ 9\ 2\ 1\ 3\ 7\ 9\ 9 \}$. Applying the recursive soft morphological erosions on f by the first cascade combination yields the intermediate result $\{ 9\ 2\ 1\ 1\ 1\ 1\ 1 \}$ and the final result $\{ 2\ 1\ 1\ 1\ 1\ 1 \}$. Applying the recursive soft morphological erosions on f by the second cascade combination yields the intermediate result $\{ 9\ 2\ 1\ 3\ 7\ 9\ 9 \}$ and the final result $\{ 2\ 1\ 1\ 1\ 1\ 1 \}$.

Property 8: A cascade combination of recursive soft morphological filters can be equivalently represented as a single recursive standard morphological filter.

Proof: The output of recursive soft morphological filters depends upon two factors, namely, the order index k and the length of the hard center. Although the technique to combine any two cascaded filters F and G into a single filter H is the same, no direct formula can be given to produce the combined filter H for different cascade combinations. Due to the many variations present, we can only present the essential idea in constructing the combined filter and illustrate it on some examples.

We will prove the cascade combination by considering two filters $[B_1, A_1, k]$ and $[B_2, A_2, k]$ of length five and having a hard center of length three, where $k \geq 2$. Let $B_1 = B_2 = < -2\ -1\ 0\ 1\ 2 >$ and $A_1 = A_2 = < -1\ 0\ 1 >$. Denote the input signal f to be $\{x_0, x_1, \cdots, x_{N-1}\}$, the output of the first filter to be $\{y_0, y_1, \cdots, y_{N-1}\}$, and the output of the second filter to be $\{z_0, z_1, \cdots, z_{N-1}\}$. Since $k \geq 2$, we have $y_0 =$ kth largest of multiset $\{k \diamond x_0, k \diamond x_1\}$, where the multiset need not consider x_2. It means that y_0 depends upon (x_0, x_1). Again, $y_1 =$ kth largest of multiset $\{k \diamond y_0, k \diamond x_1, k \diamond x_2\}$. It means that y_1 depends upon (y_0, x_1, x_2). Proceeding in this way, we obtain y_i to be y_i = kth largest of multiset $\{k \diamond y_{i-1}, k \diamond x_i, k \diamond x_{i+1}\}$. It means that y_i depends upon (y_{i-1}, x_i, x_{i+1}).

According to the definition of cascaded filters, the intermediate output $\{y_0, y_1, \cdots, y_i, \cdots, y_{N-1}\}$ is used as the input further processed by the filter $[B_2, A_2, k]$. Similarly, we obtain the final output z_i to be z_i = kth largest of multiset $\{z_{i-1}, y_i, y_{i+1}\}$. It means that z_i depends upon $(z_{i-1}, x_i, x_{i+1}, x_{i+2})$.

The above observation suggests that the result of the cascade combination of filters can be equivalently obtained by a single recursive standard morphological filter of $< -1\ 0\ 1\ 2 >$ for this example. Other examples of reducing the cascaded filters into a single recursive standard morphological filter can be similarly derived. □

Note: The number of computations required also reduces significantly when we use the combined filter obtained by the above result. For example, if we have two filters of size 5 each having a hard center of length 3 and the order index k, then the number of computations required for an input signal of length N is equal to $Nk(5k+3)$, whereas according to Table 1 in the case of the combined filter of size 4, we have only $3N$.

Example 13: Consider the input signal $f = \{2\ 4\ 6\ 5\ 8\ 3\ 2\ 3\ 5\ 8\ 3\}$. Let the two recursive soft morphological filters with $A_1 = < 0 >$, $B_1 = < -1\ 0\ 1 >$, $A_2 = < -1\ 0\ 1 >$, $B_2 = < -2\ -1\ 0\ 1\ 2 >$, and the order index $k = 2$. By applying a recursive soft morphological dilation by $[B_1, A_1, 2]$ first, we obtain $\{2\ 4\ 6\ 6\ 8\ 3\ 3\ 3\ 5\ 8\ 3\}$. Followed by applying the dilation by $[B_2, A_2, 2]$ yields $\{4\ 6\ 6\ 8\ 8\ 8\ 8\ 8\ 8\ 8\ 8\}$. Now according to Table 1, the combined recursive standard morphological filter is $< -1\ 0\ 1 >$. Applying this filter directly to the input signal f gives $\{4\ 6\ 6\ 8\ 8\ 8\ 8\ 8\ 8\ 8\ 8\}$ which is exactly the same as the

Table 1. Different Cascade Combination and Their Combined Filters

(for $2 \le k \le \min \{ Card(B)/2, Card(B \backslash A) \}$)

B_1	B_2	A_1	A_2	Combined Filter
$< -1\ 0\ 1 >$	$< -1\ 0\ 1 >$	$< 0 >$	$< -1\ 0 >$	$< -1\ 0 >$
$< -2\ -1\ 0 >$	$< -1\ 0\ 1 >$	$< -1\ 0 >$	$< 0 >$	$< -2\ -1\ 0 >$
$< -1\ 0\ 1 >$	$< -2\ -1\ 0\ 1\ 2 >$	$< 0 >$	$< -1\ 0\ 1 >$	$< -1\ 0\ 1 >$
$< -2\ -1\ 0 >$	$< -2\ -1\ 0\ 1\ 2 >$	$< -1\ 0 >$	$< -1\ 0\ 1 >$	$< -1\ 0\ 1 >$
$< -1\ 0\ 1 >$	$< -2\ -1\ 0\ 1\ 2 >$	$< -1\ 0 >$	$< -1\ 0\ 1 >$	$< -1\ 0\ 1 >$
$< -2\ -1\ 0\ 1\ 2 >$	$< -2\ -1\ 0\ 1\ 2 >$	$< -1\ 0\ 1 >$	$< -1\ 0\ 1 >$	$< -1\ 0\ 1\ 2 >$

output of the cascaded filters.

The following property provides the reduction for the cascaded filters when $k = 1$.

Property 9: If the cascade combination of two filters of any lengths, denoted by $< -L_1 \cdots -1\ 0\ 1 \cdots R_1 >$ and $< -L_2 \cdots -1\ 0\ 1 \cdots R_2 >$, and the order index $k = 1$, their combined recursive standard morphological filter will be $< -1\ 0\ 1 \cdots (R_1 + R_2) >$.

Example 14: Let the two recursive soft morphological filters with $A_1 = < 0 >$, $B_1 = < -2\ -1\ 0\ 1\ 2 >$, $A_2 = < -1\ 0\ 1 >$, $B_2 = < -2\ -1\ 0\ 1\ 2 >$, and the order index $k = 1$.

Consider the input signal $f = \{\ 5\ 2\ 3\ 6\ 9\ 3\ 6\ 1\ 7\ \}$. Applying a recursive soft morphological erosion on f by $[B_1, A_1, 1]$ gives $\{\ 2\ 2\ 2\ 2\ 2\ 1\ 1\ 1\ 1\ \}$. Now, followed by the erosion on the result by $[B_2, A_2, 1]$ yields $\{\ 2\ 2\ 2\ 1\ 1\ 1\ 1\ 1\ 1\ \}$. According to Property 9, the combined recursive standard morphological filter is $< -1\ 0\ 1\ 2\ 3\ 4 >$. Performing a recursive standard morphological erosion on f by this combined filter gives the same final result.

2.5 Conclusions

In this chapter, several properties of recursive soft morphological filters have been presented. Recursive filters, in general, provide better smoothing capabilities at the expense of increased distortion. However, in the case of recursive soft morphological filters, due to the adaptability of soft morphological filters to noise-based images, the increased distortion aspect usually associated with recursive filters is overcome by recursive soft morphological filters. Similar to the cascade combination of median filters, the cascade combination of recursive soft morphological filters can be combined into a single recursive standard morphological filter with a smaller window size. This crucial property reduces the number of computations significantly. A new class of idempotent recursive soft morphological filters has also been introduced.

CHAPTER 3

SKELETON EXTRACTION BY USING THE CENTER OF GRAVITY

3.1 Introduction

Skeleton extraction was performed by Pal *et.al.* by using the compactness/IOAC measure. The process involved assigning new membership values to the fuzzy segmented version of the image. The membership values were assigned depending upon three factors. These include the property of possesing the maximum intensity, occupying vertically and horizontally middle positions from the edges of the object. Finally, the optimum skeleton is extracted by minimizing the COMP/IOAC measures. However, this proces s involves many computations. Our proposed skeleton extraction method reduces the number of computations involved significantly and also eliminates the iterations.

The proposed skeleton extraction method involves mainly the centre of gravity measure of fuzzy sets. The centre of gravity is defined as the position of the pixel for which the area over a given radius '*r*' is a maximum. The direct implication of this definition is that the centre of gravity must belong to the core of the image. Therefore, we employ the centre of gravity measure effectively in our algorithm to extract the skeleton of the image.

3.2 Algorithm for Skeleton Extraction

Geometrical properties play an important role in picture analysis and discription. One of those is the connectivity property. The proposed method for the skeleton extraction also ensures that connectivity is preserved in the output skeleton. Rozenfeld [19], had extented the topological concepts of connectedness and surroundness to fuzzy sets and and developed some of the basic properties of these generalized concepts.

Connectivity is defined as:

Two points P and Q in a fuzzy set are connected if there exists a path $P = P_0, P_1, ..., P_n = Q$such that $(P_i) >= \min((P), (Q))$, $1<=i<n$.

The algorithm for skeleton extraction is

1. First, we compute the centre of gravity of the image by fixing a certain radius 'r'. If the centre of gravity computed is not unique, in other words, if two or more pixels possess the maximum energy, then we vary the radius r and compute the centre of gravity.

2. Secondly, depending upon this value of the centre of gravity, we fix a certain tolerance limit. This is a user defined parameter.

3. Third, we compute the area (energy) of each pixel by taking the same radius r. If the energy value lies within the tolerance limit, we assume that the pixel belong's to the core (skeleton) of the image. Store all the selected points in an array.

4. Next, for each selected pixel, we determine the connectivity with respect to the other selected vertices. This is done in the following way:

a) For each of the pixels, we determine whether any of its eight neighbours exist in the selected array. If exists, this pixel is determined to be connected to the rest of the pixels.

b) Else, we check the array for any pixel which has the same vertical alignment with the current pixel. If exists, we store all the points lying between the two pixels in a 'store' array.

c) If there are no pixels in the initial selected array which have the same vertical alignment, we check for any pixel which has the same horizontal allignment with the current pixel in the initial selected array. If exists, we store all the points lying between the two pixels in the 'store' array

d) Else, we discard the pixel.

5. Finally, we plot all the pixels in the initial selected array, which have not been discarded, and the pixels in the store array. This gives a skeleton which is connected.

Example: Let the fuzzy version of the image be

0.8	0.4	0.3	0.9	0.9
0.6	0.7	0.2	0.6	0.4
0.4	0.3	0.5	0.7	0.3
0.6	0.8	0.6	0.5	0.4
0.8	0.1	0.5	0.9	0.7

We compute the area (energy) of each pixel. The area is computed by considering the four neighbours of each pixel. Boundary pixels are assumed to have a neighbour of membership value 0. The values of the areas of each pixel are

1.8	2.2	1.8	2.7	2.2
2.5	2.2	2.3	2.8	2.2

1.9	2.7	2.3	2.6	1.8
2.6	2.4	2.9	**3.1**	1.7
1.5	2.2	2.1	2.6	2.0

The maximum area is 3.1. By setting a tolerance range of $2.5<x<3.1$, we obtain the skeleton to be

0	0	0	*	0
0	0	0	*	0
0	*	0	*	0
*	0	*	*	0
0	0	0	*	0

Note: It is to be noted that by varying the lower limit of the tolerance range, one can obtain skeletons of varying degree. That is, as the lower limit of the tolerance range reduces, we obtain a thicker skeleton and vice versa. The upper limit, however, is fixed to the value of the centre of gravity.

CHAPTER 4

CONTOUR FITTING BY USING THE ADJACENCY MEASURE

The Adjacency measure of two fuzzy regions gives us an idea of how adjacent (or close) the two regions. That is, the greater the degree of adjacency, the closer the regions are assumed to be, and vice-versa. Adjacency is dependent on two factors, namely, the length of the common border between the two sets, and the difference in the membership values. As can be seen, adjacency increases as the length of the common border increases and decreases as the difference in the membership values increases. That is, it is inversely proportional to the difference in the membership values and is directly proportional to the length of their common border.

The algorithm proposed utilizes the adjacency measure to determine which pixels belong to the egde and which pixels to discard.

Assumptions: The assumptions made for the algorithm are

1. The input to the algorithm is an edge detected fuzzy image.

2. Each pixel in the image is a peicewise constant fuzzy set.

The main steps involved in the algorithm can be summarized as follows.

Steps:

a) Initial vertices are located by computing the center of gravity in the edge detected image. A certain tolerance limit is assumed and all pixels within the tolerance limit are taken to be the initial vertices.

b) One of the vertices is considered as the "initial" vertex. The adjacency between the initial vertex and one of its adjacent vertices is computed. Let this vertex be called the "end vertex". Let this be some value p. Mark the initial vertex as the "current" vertex.

c) Consider the pixel closest to the current pixel. Compute the adjacency between the pixel and the initial vertex. Let this be q_1.

d) If the value of q_1 is greater than or equal to p, assign the pixel to the edge and draw a connection between the current pixel and the pixel under consideration. Mark this pixel as the 'current' pixel.

e) Otherwise, if the value of q_1 is less than p, compute the adjacency between the pixel and the end vertex. Let this be q_2. If the value of q_2 is greater than or equal to p, assign the pixel to the edge and draw a connection between the current pixel and the pixel under consideration. Mark this pixel as the 'current' pixel. Else, discard the pixel.

f) Repeat steps (c) through (e) for every pixel lying in between the initial and the end vertex.

g) Repeat step's (b) through (f) for every pair of vertices.

Example: An example which incorporates the above algorithm is presented.

Consider an edge detected fuzzy image to be

0	.5	0	.3	0	.7	0	0
0	.6	.4	0	.6	0	.8	0
0	.7	0	0	0	0	0	.3
0	.4	0	0	0	0	.4	0
0	.6	0	0	0	0	0	.7
0	.6	0	0	0	0	.3	0
0	.4	0	0	0	0	.6	0
0	0	0	0	0	0	0	.5

0 0 0 0 0 0 6 0

0 **.9** .2 **.6** **.8** **.7** 0 0

The initial vertices located by the vertex detection routine are 0.6, 0.8, 0.7, and 0.9 (shown as bold face in the figure above). The algorithm is started with o.6 as the initial vertex. 0.8 is taken to be the end vertex. The iterations for the above input for the edges of 0.6 to 0.8 and 0.8 to 0.7 are shown below.

The final curve is obtained by connecting the verteces with all the points which have a status of ''current''.

The advantages of this method are that it is simple to implement and involves no complicated mathematical formulae. The computations are kept at a very minimum by eleminating extra calculations. For example, q_2 is computed only if q_1 is less than p. This approach also has the advantage of reducing the time taken to implement the algorithm.

Table 2. Iteration Sequence Of Algorithm

p	*Points*	q_1	q_2	*Status*
10 / 12	0.5	10 / 11		Current
	0.4	10 / 12		Current
	0.3	10 / 13	10 / 15	Discard
	0.6	1		Current
	0.7	10 / 11		Current
10 / 11	0.3	10 / 15	10 / 14	Discard
	0.4	10 / 14	10 / 13	Discard
	0.3			Discard
	0.7	10 / 11		Current
	0.5	10 / 13	10 / 12	Discard
	0.6	10 / 12	10 / 11	Current

CHAPTER 5

FUTURE RESEARCH

5.1 Image Enhancement By Minimizing the Degree of Adjacency

Many of the definitions of fuzzy sets presented by Rozenfeld have been modified later by Pal *et al.* However, not all of these modified definitions provide any significant improvement over the existing definitions. In contrast to this, the new definitions of adjacency and degree of adjacency provide a more meaningfull definition which fits very well to the geometric meaning of adjacency.

According to the new definition, any two fuzzy sets μ and τ are more adjacent if the difference in the membership values of μ and τ is less, and vice-versa. For example, if the two fuzzy sets have the values of 0.3 and 0.35 in the first case, and 0.8 and 0.85 in the second case, then they are expected to have the same adjacency because their difference in membership values is the same in both cases. However, according to Rozenfeld's definition, the value's of adjacency differ's signif icantly in both cases. Hence, we agree with the new definition of adjacency, since it fit's very well with the meaning of adjacency defined in the general sense.

Both *compactness* and *degree of adjacency* measures take into account fuzziness in the spatial domain. Previous research has accomplished image enhancement by optimizing fuzzy compactness. However, the noticeable point regarding the degree of adjacency is that the higher the contrast between two regions and the greater the physical distance between them, the lower the degree of adjacency. This implies that the lower value of the degree of adjacency indicates that the segments formed are more v alid and separable considering both their grey level and physical distances. Therefore, the degree of adjacency can be utilized for enhancing the contrast in an image. That is, the degree of adjacency can be minimized and that value can be taken to be the threshold value for image enhancement.

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