ABSTRACT

Torque and Current Response Due to Singularity Changes of Induction Machine Slip

by

Jessie G. Orpilla

Recognizing that the prime purpose of the 3 phase induction motor is to deliver torque with minimum line current. This thesis studies the torque as well as the current response produced by the singularity changes in the induction machine slip. It is desirable that the torque to be maximized for a given value of line current at the corresponding load slip.

This thesis analyzes the torque and current response, when a slip at no-load is changed to a unit-step, Qu(t), a unit-impulse, Qδ(t), a pulse characterized by Q[u(t)-u(t-T)], a unit-ramp, Qt, and a sinusoidal, Qsint, slip. The responses are analyzed againts the time. All of the computations are based on the approximate equivalent circuit of the induction motor, with the stator resistance, R_s, and the stator reactive reactance, jX_s, being neglected. It is done to simplify the complexity of the computations and the computer programming, and it is a real and practical assumption.

The Laplace Transformation method is used in solving the differential equations for torque and for the current. A computer program using FORTRAN language is simulated based on the final equation for both torque and current. The program calculates and graphs the computed results againts the time.

In addition, this thesis work analyzed the motor performance if it is running at high slip, as in the case of a low-drive induction motor, and when it is operating at negative slip, as an induction generator.

It is seen that the torque and current response of the motor and the induction generator are almost identical in nature. They follow the pattern of their slip changes, with the exception when the final slip is sinusoidal. The only difference is

that the induction generator will absorb power, which is, in effect, supplying power, that is mostly of leading power-factor or almost unity power-factor.

The torque to current ratio, being one of the most important data in the performance of induction motor, is also computed and plotted againts the time based on the previous calculations. Moreover, the torque response is also compared with respect to its maximum or breakdown torque.

It is seen that the most positively effective and practical among the slip functions is the unit-step. Because the response is constant at any period of time and the torque response will never exceed the breakdown torque as long as the slip does not exceed the slip at maximum torque. In the case of a sinusoidal and a unit-ramp function, the maximum or breakdown torque is reached at a certain time, t, due to their time-dependent functions. While the unit-impulse and the rectangular pulse are also practical functions since they are time-independent fuctions. But the responses reached their maximum when operating as a low-drive motor, with high machine slip.

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APPROVAL PAGE

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This thesis is dedicated to

my parents, Rufino and Lydia,

and to my brothers, Oscar, Rufino Jr., Pepito, and Wilson

and most especially to my wife, Marlyn.

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CHAPTER 1
INTRODUCTION

1.1 General Introduction

The induction motor[1] was invented by Nikola Tesla in 1886. It resembles like an electric transformer whose magnetic circuit is separated by an air-gap into two relatively movable parts. It has the primary winding which is held stationary and is called the stator winding and the secondary winding which is rotating and is called the rotor winding. The rotor circuit has no connection to the supply circuit and its field is only induced by the currents thru the stator, the principle of induction. That's why it is called the induction motor. It is either single-phase or polyphase machines.

1.2 General Types

There are two general types of 3 phase induction motor. One is the squirrel cage rotor type in which the rotor is self-contained and resembles a bird cage. The rotor conductors are permanently short-circuited by the end-rings. It has an adjustable, nearly constant and variable speed. In general, it has a medium starting torque. It is widely used in pump, compressor, blower, prime mover of dc generators, etc. The other type is the wound rotor type. The rotor has insulated coils of wire and resembles like a dc armature. The rotor conductors are connected to a slip rings and shorted thru the brushes. It has a variable speed and high starting torque. It is used in hoist, crane, elevator, escalator, pump, conveyor, etc.

The polyphase induction machine is the lowest-cost and most widely used motor in the industrial world. There are also a fractional-horsepower motors which are single-phase. They are used for electric fans,refrigerators,washing-machines, and other appliances. The advantages of the induction motor over the other types are its simplicity, reliability, low-cost, good efficiency, good over-load capacity, and minimal service requirement.

1.3 The Rating System

The nameplate ratings of the motor are the bases of its performance capabilities. It gives the overall performance of the motor itself. It defines the starting, continuos starting and overload torque; the temperature endurance; low starting current; and other factors. The purpose of motor rating, therefore, is to define accurately its performance in a way that is advantageous to the costumer.

The rating[2] of the induction motor gives the following: the supply voltage, the number of phases, the frequency of the power supply, the horsepower rating, speed and the temperature rise. The horsepower rating together with the rated speed gives the torque the motor can deliver. The rating also states the type of service whether continuos, intermittent, or varying duty.

1.4 General Purpose Motor

To maintain service reliability and flexibility of application, the American Standards require that a general purpose[3] polyphase motor shall have maximum or breakdown torque of not less than 2 times as its nameplate rating. The purpose of choosing 200 percent for the breakdown torque are the following: First, it is desired that it will continue delivering normal output during temporary low-voltage. Secondly, it is required for general purpose motor that it will operate continuously without injury at an output equal to 115 percent of its rating. And finally, a factor of 1.25 is given to provide for occasional overloads and unforseen contingencies in the operation of motor.

It is desireable for general-purpose motor to be started at full voltage. With the voltage being reduced at starting, the torque is reduced as the square of the voltage. Any loads requiring high torque will not be started. With a high current during starting[1] causes a reduced voltage. So it is a must to hold this current as low as possible, a problem usually encounter by the designer.

1.6 Temperature Limitation

Temperature[5] is also one of the factors to be controlled in the operation of any motor. Excessive temperature tends to deteriorate the performance capability and life of electric motor. So it is a must for a design engineer to have an insulation with greater endurance and good mechanical properties. Temperature rise is also proportional to the losses. Overloads are destructive to the motor but occasional overloads of moderate value can be tolerated without great effect in its life.

1.7 The Equivalent Circuit

The performance of a polyphase induction motor can be analyzed by the phasor or circle diagram. But for the purpose of repetitive calculation, it is not advisable to use these methods. Since the introduction of computers gives us a convenient and more efficient way of calculating the performance of a motor, the equivalent cicuit[6,7,8] provides us a sound basis for calculation. It is shown in Fig. 1.1. It is almost similar to a transformer circuit except for the replacement of the transformer load by $R_r(1-s)/s$. Because of the large number of ampere-turns required to force the flux across the motor air-gap, the magnetizing reactance, X_m, is larger for the induction motor compare to the transformer. The primary voltage E_1 is equal to the line voltage divided by $\sqrt{3}$ for a Y-connected winding. A voltage drop occurs in the primary due to resistance, R_s, and leakage reactance, X_s.

At speed below synchronism, it produces a current I_r which creates a secondary resistance drop I_rR_r/s and a rotor leakage reactance drop jI_rX_r. The secondary resistance drop multiplied by the rotor current defines the power delivered to the motor shaft, commonly called as the output mechanical power. It is to be multiplied by the number of phases to get the total output. To obtain the net power, a friction and windage must be subtracted from the total output.

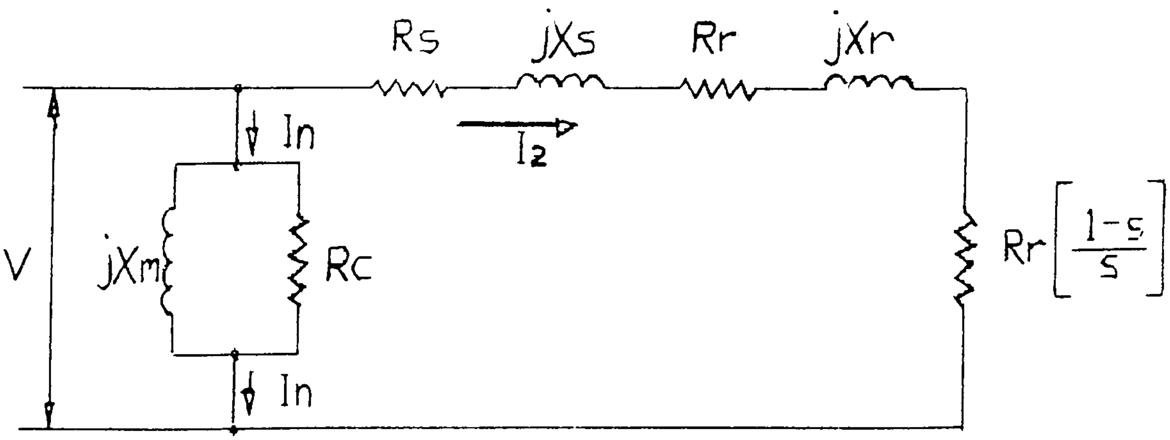

Figure 1.1 The Per Phase Equivalent Circuit of Three Phase Induction Motor

1.8 Approximate Equivalent Circuit

Most induction motors operate at low values of slip, it usually ranges from 2 to 5 percent at full load. With this proof, it shows that the value of $R_r(1-s)/s$ is larger than either to R_s or R_r. Additionally, the stator reactance X_s and rotor reactance X_r, which are seldom two times the value of R_s and X_r, are very small compared to $R_r(1-s)/s$. With these conditions, it is possible to modify the circuit of Fig. 1.1 by moving the shunt path directly across the supply voltage as shown in Fig. 1.2, the approximate equivalent circuit of induction motor[9,10].

This method will cause to raise the voltage in the shunt circuit. This means a higher flux and a higher copper loss will be developed in the machine. Additionally, the copper loss in the stator will be reduced because at no-load, there will be no stator copper loss because their is no current flowing through it. The addition of core loss will then be neutralize by the reduction of stator loss. By this reason, the approximate equivalent is commonly used in the analysis of an induction motor.

This is the basis of all computations that will be presented in the remaining chapters of this thesis.

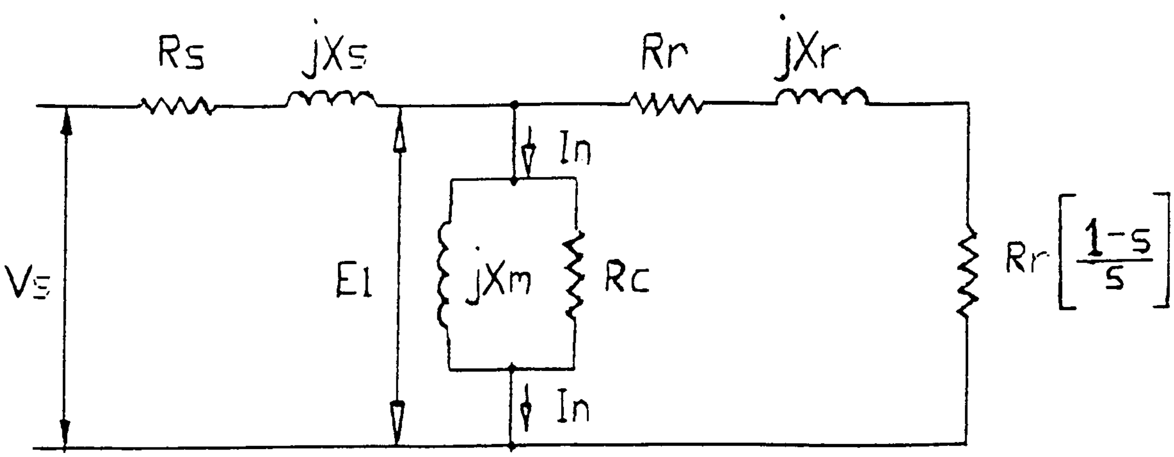

Figure 1.2 The Per Phase Approximate Equivalent Circuit of Three Phase Induction Motor

1.9 The Circle Diagram

One of the graphical methods used in the analysis of induction motor characteristics is the circle diagram[11.12]. It was introduced by Dr. A. S. McAllister. It is based on the approximate equivalent circuit of an induction motor. As stated before, the approximate equivalent circuit neglects the effect of the exciting current causing a voltage drop at no-load because it's been neutralized by the increase of core loss. The method used in this section assumed that it has a constant resistance throughout its speed range.

Referring to Fig. 1.2. with constant voltage applied to the circuit

$$I_r = \frac{V}{\sqrt{(R_s + R_r/s)^2 + (X_r + X_s)^2}} \tag{1.1}$$

This current is out of phase with the voltage by an angle θ_r where

$$\theta_r = \frac{X_r + X_s}{\sqrt{(R_s + R_r/s)^2 + (X_r + X_s)^2}} \tag{1.2}$$

Combining Eqns. 1.1 and 1.2. it gives

$$I_r = \frac{V}{X_r + X_s} sin\theta_r \tag{1.3}$$

Eqn. 1.3 is obvious to be a polar equation of a circle having a diameter of $V/(X_r+X_s)$. This is the starting point of the analysis of this method. This method makes use of the circle diagram in finding the characteristics of an induction motor such as the following: the no-load current. rotor-blocked current. stator and rotor current. power input to stator. stator and rotor copper losses. power transferred across the air-gap. useful output. power factor. torque. slip. efficiency. maximum power factor at which the motor can operate. etc.

The data needed to construct the circle are the magnitude of the no-load current. OB. the rotor blocked current. OH. and their phase angles with reference to the line voltage. OA. A circle is then drawn to pass B and H. Each line on the diagram is a measure of current in amperes. To find the power in watts. it has to be multiplied by the number of phases and by the phase voltage. Line KH signifies the total motor input with blocked rotor. while KI represents the primary I^2R loss. Line HI is the power input at standstill. To find the synchronous speed. HI should be divided by the synchronous speed.

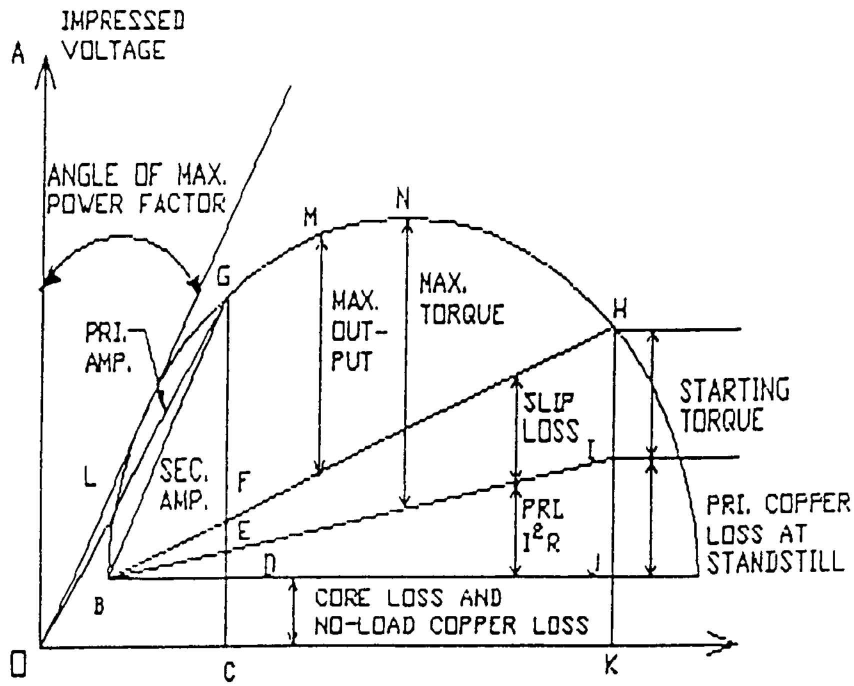

Figure 1.3 The Circle Diagram

Given any load, say at point G, OG is then the primary current. BG is the secondary current and GC represent the motor input. GE represents the motor output. FE is the secondary I^2R loss. DE the primary I^2R loss and CD the no-load copper loss. To find the maximum power-factor, a line is drawn from O tangent to the circle at point L. Similiarly, the maximum output and the maximum torque points M and R are located by tangent lines parallel to BH and BI. The diameter of a circle is equal to the voltage divided by the stanstill reactance, or equivalent to the block-rotor current with the resistances of both windings being neglected. The maximum torque in synchronous watts, is equal to a little less than the product of the radius of the circle, the voltage, OA, and the number of phases.

The circle diagram gives a convinient way of checking the overall performance of a motor with a minimum test data.

1.10 Thesis Overview

This thesis makes use of the approximate equivalent circuit in the analysis of the performance of induction motor. Because it provides an easiness in formulating a computer program that can make a repetitive calculations. Circle diagram is not advisable to use because one has to draw a different diagram for a different motor rating.

This thesis studied the torque and the current response of an induction motor produced by singularity changes of machine slip. Special attention is also given for an induction generator, when the motor is driven slightly above synchronous speed by supplying power to the shaft. The slip in this case is negative. The ratio of the torque to the rotor current, being one of the important data in the electric machine operation, is also treated in this thesis. It is always desired that the torque is maximized at a minimum line current.

CHAPTER 2
COMPUTATIONAL BASES

2.1 Introduction

This chapter deals with the bases of computations that will be used throughout this thesis. The step-by-step solutions of both current and torque response will be shown here. As stated in the previous chapter, all of the computations are based on the approximate equivalent circuit of induction motor shown in Fig. 1.2. Q is now used to stand for the slip so that it will not be mistaken in s of F(s) commonly used in Laplace Transformation. The stator resistance, R_s, and the stator reactance, jX_s are both neglected. This is a practical assumption because

$$\left|\frac{jX_m R_c}{R_c + jX_m}\right| \geq 10\sqrt{R_s^2 + X_s^2}$$

Since the only parameter present in the circuit is of the rotor side, R will now be used to stand for the rotor resistance and X for the rotor inductive reactance.

2.2 Torque Calculation

Looking back in Fig. 1.2, the approximate equivalent circuit of a 3 phase induction motor. The total resistance, R_t, excluding the magnetizing branch, and the total inductive reactance, X_t, are given by

$$R_t = R + R\frac{(1-Q)}{Q} \tag{2.1}$$

$$R_t = \frac{R}{Q} \tag{2.2}$$

$$X_t = X \tag{2.3}$$

Also the rotor current, I, is equal to

$$I = \frac{V}{\sqrt{(R/Q)^2 + X^2}} \tag{2.4}$$

in ampere per phase. The torque[13], T, is equal to

$$T = I^2 \frac{R}{Q} \tag{2.5}$$

in synchronous watts per phase. Substitute Eqn. 2.4 to Eqn. 2.5,

$$T = \frac{V^2}{X^2 + (R/Q)^2} \times \frac{R}{Q} \tag{2.6}$$

$$T = \frac{V^2 QR}{R^2 + Q^2 X^2} \tag{2.7}$$

in synchronous watts per phase.

2.3 Differentiation

The equation of torque, Eqn. 2.7, is differentiated with respect to the slip, Q.

$$\frac{\partial T}{\partial Q} = \frac{V^2[(R^2 + Q^2X^2)R - QR(2QX^2)]}{[R^2 + Q^2X^2]^2} \tag{2.8}$$

$$\frac{\partial T}{\partial Q} = V^2[\frac{R}{(R^2 + Q^2X^2)} - \frac{2Q^2RX^2}{(R^2 + Q^2X^2)^2}] \tag{2.9}$$

To find $\partial T/\partial t$, $\partial T/\partial Q$ is multiplied by $\partial Q/\partial t$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial Q} \times \frac{\partial Q}{\partial t} \tag{2.10}$$

Additionally, $\partial y/\partial t = y'$, as always been used in differential equation, hence the differential equation for solving torque becomes

$$T' = V^2[\frac{RQ'}{(R^2 + Q^2X^2)} - \frac{2Q^2RX^2Q'}{(R^2 + Q^2X^2)^2}] \tag{2.11}$$

2.4 Approximate Torque Equation

Eqn. 2.12 is very hard or even unsolvable using Laplace Transformation. It is further simplified by the method of Long Division. Separating Eqn. 2.12 into two parts, by letting

$$A = \frac{RQ'}{R^2 + Q^2X^2} \tag{2.12}$$

$$B = \frac{2RX^2Q^2Q'}{(R^2 - Q^2X^2)^2} \tag{2.13}$$

After Long Division, using up to the third term, they become

$$A \approx \frac{Q'}{R} - \frac{Q'Q^2X^2}{R^3} + \frac{Q'Q^4X^4}{R^5} \tag{2.14}$$

$$B \approx \frac{2Q'Q^2X^2}{R^3} - \frac{4Q'Q^4X^4}{R^5} - \frac{6Q'Q^6X^6}{R^7} \tag{2.15}$$

Substitute Eqn. 2.14 and Eqn. 2.15 to Eqn. 2.11, the approximate torque differential equation

$$\begin{aligned} T' \quad \approx \quad & V^2\Big(\frac{Q'}{R} - \frac{Q'Q^2X^2}{R^3} + \frac{Q'Q^4X^4}{R^5} - \frac{2Q'Q^2X^2}{R^3} - \\ & \frac{4Q'Q^4X^4}{R^5} - \frac{6Q'Q^6X^6}{R^7}\Big) \end{aligned} \tag{2.16}$$

$$T' \approx V^2\Big(\frac{Q'}{R} - \frac{3X^2Q'Q^2}{R^3} + \frac{5X^4Q'Q^4}{R^5} - \frac{6X^6Q'Q^6}{R^7}\Big) \tag{2.17}$$

Eqn. 2.18 is the final equation for torque in synchronous watts per phase. This differential equation is now solvable using the Laplace Transformation which will be presented in the next chapter.

2.5 Current Calculation

The method used in torque calculation is the same method that will be used in this section. Again, looking back in Fig. 1.2, the rotor current, I, is equal to

$$I = \frac{V}{\sqrt{X^2 + (R/Q)^2}} \tag{2.18}$$

Differentiate this with respect to slip, Q

$$\frac{\partial I}{\partial Q} = \frac{VR^2}{Q^3(X^2 + (R/Q)^2)^{3/2}} \tag{2.19}$$

To find $\partial I/\partial t$, Eqn. 2-19 is multiplied by $\partial Q/\partial t$. Simplify by letting $\partial I/\partial t$ equal to I' and $\partial Q/\partial t$ equal to Q'. It becomes

$$I' = \frac{VR^2Q'}{(X^2Q^2 + R^2)^{3/2}} \tag{2.20}$$

But then their is no way Laplace Transform can be applied to solve the differential equation above because of the complexity of the denominator. It is simplified using one of the well known formula in the number series, which is equal to

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \ldots \tag{2.21}$$

provided that $b \ll a$.

Using Eqn. 2.21 to simplify the denominator of Eqn. 2.20, and by letting $a = R^2$ and $b = X^2Q^2$. These are assumed because X^2Q^2 is much less than R^2 since slip is a very small value. The denominator of Eqn. 2.21 becomes

$$(R^2 + X^2Q^2)^{3/2} \approx R^3 + \frac{3}{2}RX^2Q^2 \tag{2.22}$$

The term beyond the second is neglected in Eqn. 2.23. This is done to simplify the current calculation. The current differential equation becomes

$$I' \approx V[\frac{R^2}{R^3 + 3/2RX^2Q2}] \tag{2.23}$$

Again, by the Method of Long Division, this becomes

$$I' \approx V[\frac{Q'}{R} - \frac{3X^2Q^2Q'}{2R^3}] \tag{2.24}$$

in amperes. The term beyond the third is neglected after division. This current is also based in per phase calculation.

2.6 Numerical Calculation

Eqn. 2.5 gives the formula for solving the torque in synchronous watts. To solve for the total torque[14] in a 3 phase system

$$T = 3|I|^2\frac{R}{Q} \tag{2.25}$$

It is a common practice to use lb-ft or N-m as a unit of measurement for torque. It is converted by

$$T = 3|I|^2\frac{R}{Q} \times \frac{33,000}{746 \times 2\pi N_s} \tag{2.26}$$

in lb-ft. N_s is the synchronous speed in rev/min. of the induction motor.

$$T = \frac{7.04}{N_s} \times \frac{3|I|^2 R}{Q} \tag{2.27}$$

To convert it in N-m. a conversion factor is used. 1 lb-ft is equal to 1.356 N-m. So the torque in N-m for a three phase system is equal to

$$T = \frac{7.04 \times 1.356 \times 3}{N_s} \times \frac{|I|^2 R}{Q} \tag{2.28}$$

where the rotor current. I. is equal to

$$|I| = \frac{V}{\sqrt{(R/Q)^2 + X^2}} \tag{2.29}$$

in ampere. per phase.

2.7 Maximum Torque

To find the maximum or breakdown torque. T_{max}. the motor can developed

$$T_{max} = \frac{7.04 \times 1.356}{N_s} \times \frac{3|I|^2 R}{Q_{max}} \tag{2.30}$$

in N-m for a three phase system. Q_{max} is the slip of induction motor at maximum torque. It can be solved by equating $\partial T/\partial Q = 0$. and solve for the slip. Q. From Eqn. 2.7

$$T = \frac{V^2 Q R}{R^2 + Q^2 X^2} \tag{2.31}$$

Solving for $\partial T/\partial Q = 0$

$$\frac{\partial T}{\partial Q} = 0 = \frac{V^2[(R^2 + Q^2X^2)R - 2Q^2RX^2]}{(R^2 + Q^2X^2)} \tag{2.32}$$

$$0 = R^3 + Q^2RX^2 - 2Q^2RX^2 \tag{2.33}$$

$$R^3 = Q^2RX^2 \tag{2.34}$$

$$Q_{max} = \frac{R}{X} \tag{2.35}$$

This is the slip for maximum torque in this case. with the stator resistance and the stator inductive reactance being neglected.

CHAPTER 3

UNIT-STEP, u(t)

3.1 Introduction

This chapter analyzed the torque-time and current-time response of a 3 phase induction motor, when a very small value of slip at no-load is suddenly changed to a unit-step value, $Qu(t)$. All of the computations are based on the previous chapter. The Table of the Laplace Transforms is presented in Appendix A of this thesis. This chapter also analyzed the torque to current ratio of an induction machine.

3.2 Applying Laplace Transformation

Going back to the differential equation for calculating torque presented in Eqn.2.17.

$$T' \approx V^2[\frac{Q'}{R} - \frac{3X^2Q^2Q'}{R^3} + \frac{5X^4Q^4Q'}{R^5} - \frac{6X^6Q^6Q'}{R^7}] \tag{3.1}$$

with the following boundary conditions

$$\begin{array}{ll} \text{at } t = 0^- & Q = Q_o \\ \text{at } t = 0^+ & Q = Qu(t) \end{array}$$

Q_o is the initial value of slip at no-load. Laplace Transform can be applied in Eqn. 3.1, term-by-term. By using the derivative property of Laplace Transform.

$$\mathcal{L}[\frac{d^n f(t)}{dt^n}] = s^n F(s) - s^{n-1} f(0) - s^{n-2}\frac{df(0)}{dt} ... - \frac{d^{n-1} f(0)}{dt^{n-1}} \tag{3.2}$$

The final slip is $Qu(t)$ and the transform of a unit-step, $u(t)$, is $1/s$. So

$$\mathcal{L}[Q] = \mathcal{L}[Qu(t)] = Q(s) = Q/s \tag{3.3}$$

$$\mathcal{L}[Q'] = sQ(s) - Q_o \tag{3.4}$$

$$\mathcal{L}[Q'] = Q - Q_o \tag{3.5}$$

$$\mathcal{L}[T'] = sT(s) - T_o \tag{3.6}$$

$$Q^2 = Q^2 u(t) \tag{3.7}$$

$$\mathcal{L}[Q^2] = \frac{Q^2}{s} \tag{3.8}$$

To find the transform of $[Q^2Q']$, a product of two functions, a Laplace Transform is given in the Appendix A, which states

$$\mathcal{L}[f_1(t)f_2(t)] = \sum_{k=1}^{q} \frac{A_1(s_k)}{B_1'(s_k)} F_2(s - s_k) \tag{3.9}$$

provided that $F_1(s)$ is a rational function. $F_1(s)$ is equal to A_1/B_1 and has only q first-order poles. S_k are the poles of $F_1(s)$. In this case, it is assumed that $f_1(t) = Q^2$ because its transform is a rational function. Q' is assumed to be $f_2(t)$.

$$\mathcal{L}[Q^2Q'] = \frac{Q^2}{1}[Q - Q_o] \tag{3.10}$$

$$\mathcal{L}[Q^2Q'] = Q^2[Q - Q_o] \tag{3.11}$$

To solve for the transform of $\mathcal{L}[Q^4Q']$, it is assumed that $f_1(t) = Q^4$ and $f_2(t) = Q'$.

$$Q^4 = Q^4 u(t) \tag{3.12}$$

$$\mathcal{L}[Q^4] = \frac{Q^4}{s} \tag{3.13}$$

$$\mathcal{L}[Q^4Q'] = Q^4(Q - Q_o) \tag{3.14}$$

Similarly,

$$\mathcal{L}[Q^6Q'] = Q^6[Q - Q_o] \tag{3.15}$$

Substitute Eqns. 3.10, 3.12, and 3.13 to Eqn. 3.1.

$$\begin{aligned} sT(s) - T_o \approx\; & V^2\left(\frac{1}{R}[Q - Q_o] - \frac{3X^2}{R^3}Q^2[Q - Q_o] + \frac{5X^4Q^4}{R^5}[Q - Q_o]\right. \\ & \left.- \frac{6X^6Q^6}{R^7}[Q - Q_o]\right) \end{aligned} \tag{3.16}$$

$$T(s) = V^2\left([Q - Q_o]\frac{1}{Rs} - \frac{3X^2Q^2}{R^3 s} + \frac{5X^4Q^4}{R^5 s} - \frac{6X^6Q^6}{R^7 s}\right) + \frac{T_o}{s} \tag{3.17}$$

Take the inverse Laplace Transform, gives,

$$T(t) = V^2u(t)([Q - Q_o][\frac{1}{R} - \frac{3X^2Q^2}{R^3} + \frac{5X^4Q^4}{R^5} - \frac{6X^6Q^6}{R^7}]) + T_ou(t) \tag{3.18}$$

in synchronous watts per phase.

3.3 Current Calculation

Using the same technique as in Sec. 3.2 in solving the differential equation for current calculation. Eqn. 2.24 states

$$I' \approx V[\frac{Q'}{R} - \frac{3X^2Q^2Q'}{2R^3}] \tag{3.19}$$

subjected to the following boundary conditions

at $t = 0^-$	$Q = Q_o$
at $t = 0^+$	$Q = Qu(t)$

Using the derivative property of Laplace Transform,

$$\mathcal{L}[I'] = sI(s) - I_o \tag{3.20}$$

Substitute Eqns. 3.5, 3.11 and 3.20 to Eqn. 3.19,

$$sI(s) - I_o = V[Q - Q_o](\frac{1}{R} - \frac{3X^2Q^2}{2R^3}) \tag{3.21}$$

$$I(s) \approx V[Q - Q_o](\frac{1}{Rs} - \frac{3X^2Q^2}{2R^3s}) + \frac{I_o}{s} \tag{3.22}$$

Take the inverse Laplace Transform,

$$I(t) \approx V[Q - Q_o]u(t)(\frac{1}{R} - \frac{3X^2Q^2}{2R^3}) + I_ou(t) \tag{3.23}$$

in ampere per phase.

3.4 Nameplate Rating

All of the case studies that will be presented in this thesis are based on one specific induction motor with the following parameters:

- 3 hp. 440 volts line-to-line. 3 phase. Y-connected. 60 hz. 4 pole. 1750 rpm induction motor
- Power Factor=0.858 lagging
- Efficiency=0.81
- Full Load Slip=0.03
- Synchronous Speed=1800 rpm
- Stator Resistance=2.69 ohms
- Rotor Resistance=2.14 ohms
- Stator Inductive Reactance=4.36 ohms
- Rotor Inductive Reactance=4.5 ohms
- Magnetizing Resistance=3.66 ohms
- Magnetizing Reactance=103 ohms

To solve for the full-load current. Eqn. 2.4 is used using the motor ratings given above

$$I = \frac{V}{\sqrt{(R/Q)^2 + X^2}} \tag{3.24}$$

$$I = \frac{440/\sqrt{3}}{\sqrt{(2.14/0.03)^2 + (4.5)^2}} = 3.55 \tag{3.25}$$

amperes.

From Eqn. 2.31 the full load torque is equal to

$$T_{FL} = \frac{7.04 \times 1.356 \times 3}{N_s} \times \frac{|I|^2 R}{Q} \tag{3.26}$$

$$T_{FL} = \frac{7.04 \times 1.356 \times 3}{N_s} \times \frac{(3.55)^2(2.14)}{0.03} \tag{3.27}$$

$$T_{FL} = 14.337 \tag{3.28}$$

in Newton-meter.

To solve for the maximum torque, the slip for maximum torque, Q_{max}, is given by Eqn. 2.35.

$$Q_{max} = \frac{R}{X} \tag{3.29}$$

$$Q_{max} = \frac{2.14}{4.5} = 0.476 \tag{3.30}$$

The rotor current at maximum torque is equal to

$$I_{max} = \frac{440/\sqrt{3}}{\sqrt{(2.14/0.476)^2 + 4.5}} \tag{3.31}$$

$$I = 39.94 \tag{3.32}$$

amperes. T_{max} then becomes

$$T_{max} = \frac{7.04 \times 1.356 \times 3}{1800} \times \frac{(39.94)^2(2.14)}{0.475} \tag{3.33}$$

$$T_{max} = 114.34 \tag{3.34}$$

in N-m.

3.5 Case Studies

A computer program for solving torque has been developed using $FORTRAN$ language[15,16]. It is based in the final equation for torque calculation given by Eqn. 3.18. The program is shown in Appendix B of this thesis. The inputs needed in the program are the following: the per phase voltage in volts, the frequency of the supply system in hertz, the number of poles of the motor, the number of phases, the rotor resistance in ohms, the rotor inductive reactance in ohms, the final slip, the initial slip, the initial torque in N-m, and the time in seconds, respectively.

A computer program has also been developed for solving the rotor current. It is based in Eqn. 3.23 and is shown in Appendix C. The inputs needed in this program are the following: the per phase supply voltage, the rotor resistance, the rotor inductive reactance, the final slip, the initial slip, the initial current in amperes, and the time, respectively. The case studies shown in this section are based in the motor ratings given in Sec. 2.4. In addition, the initial torque is equal to 0.36 N-m, which is 3 percent of the full-load torque, and the initial current is 0.3 Amp. The cases being studied are the following: when the slip at no-load is suddenly changed to $0.03u(t)$, when it is running as induction generator with negative slip, and when it is running at a very high slip as in the case of a low-drive motor.

Fig. 3.1 shows the torque-time response when the initial slip of 0.001, is suddenly changed to a unit-step, $0.03u(t)$. The response is also a unit-step with magnitude of 13.75 N-m. It is very close to the full-load torque computed in Sec. 2-4, which is equal to 14.34 N-m. In the case of induction generator, with initial slip of -0.001 and a final slip of $-0.03u(t)$, the response is almost equal to the motor response except that it is negative. It is shown in Fig. 3.2. In the case of a low-drive motor with a final slip, $0.3u(t)$, the response is also a unit-step with magnitude of approximately 2.5 times the full-load. It does not reach the maximum torque due to the fact that the slip is constant at any time, t, and it is way below the slip at maximum torque, Q_{max}. It is shown in Fig. 3.3. The unit for torque is N-m, since the conversion factors are already set in the computer program based in Eqn. 2.30.

Figs. 3.4 to 3.6 show the current responses for the three cases stated before. For the first case, the response is $3.72u(t)$, it is very close to the full-load current computed in Sec. 2.4, which is equal to 3.55 Amp. This is because the slip is maintained at a constant level equivalent to the full-load. In the case of an induction generator, the current response is equal to $-3.12u(t)$. For a low drive motor, shown in Fig. 3.6, the current response is approximately 3 times the full-load current.

Figs. 3.7 to 3.9 show the computed ratio of torque over the current. The torque to current ratio of a low-drive motor is the lowest. This is due to the greater increase of the current than the torque with the same change of machine slip.

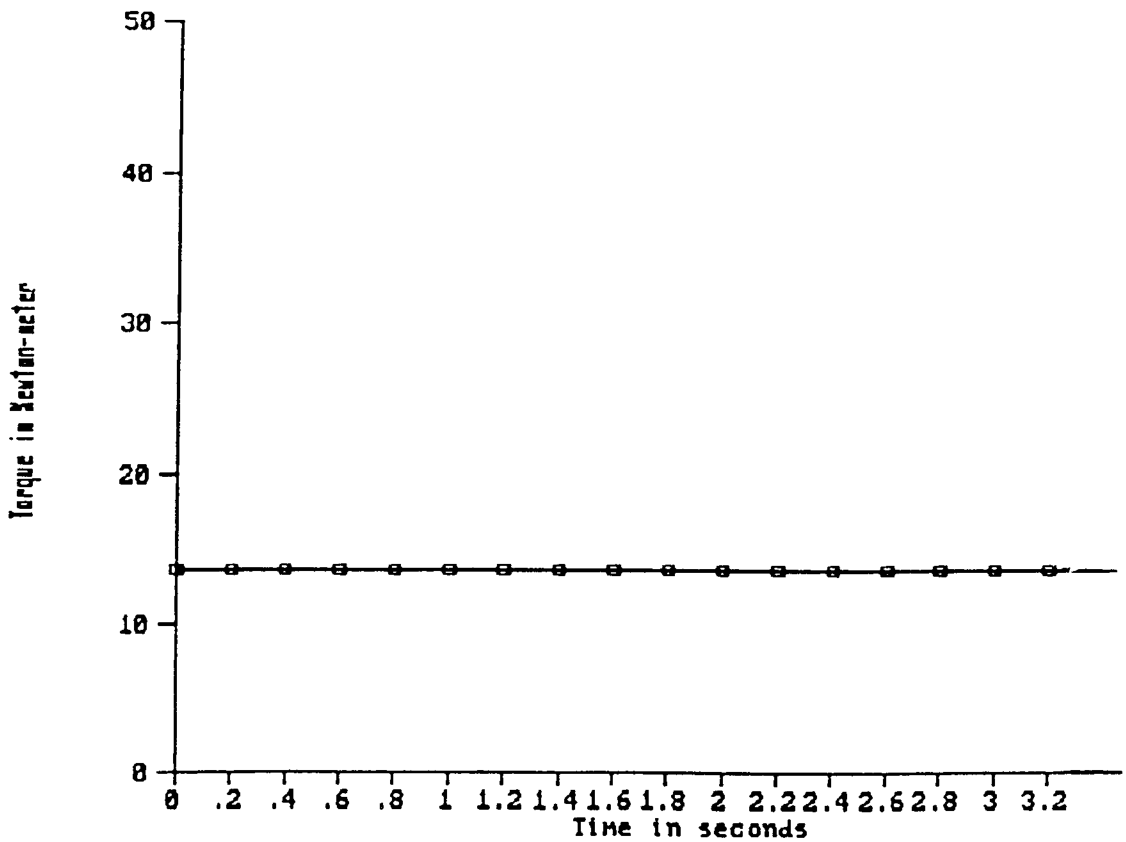

Figure 3.1 Torque Response If The Final Slip Is $0.03u(t)$

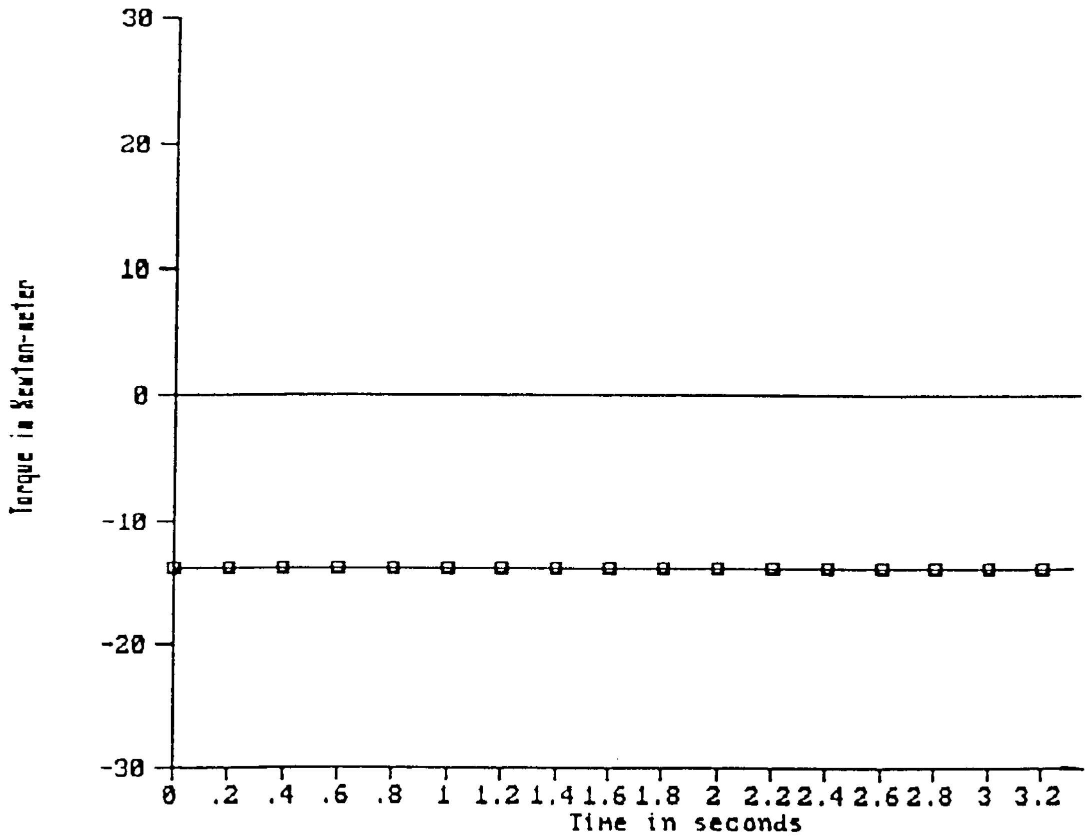

Figure 3.2 Torque Response If The Final Slip Is $-0.03u(t)$

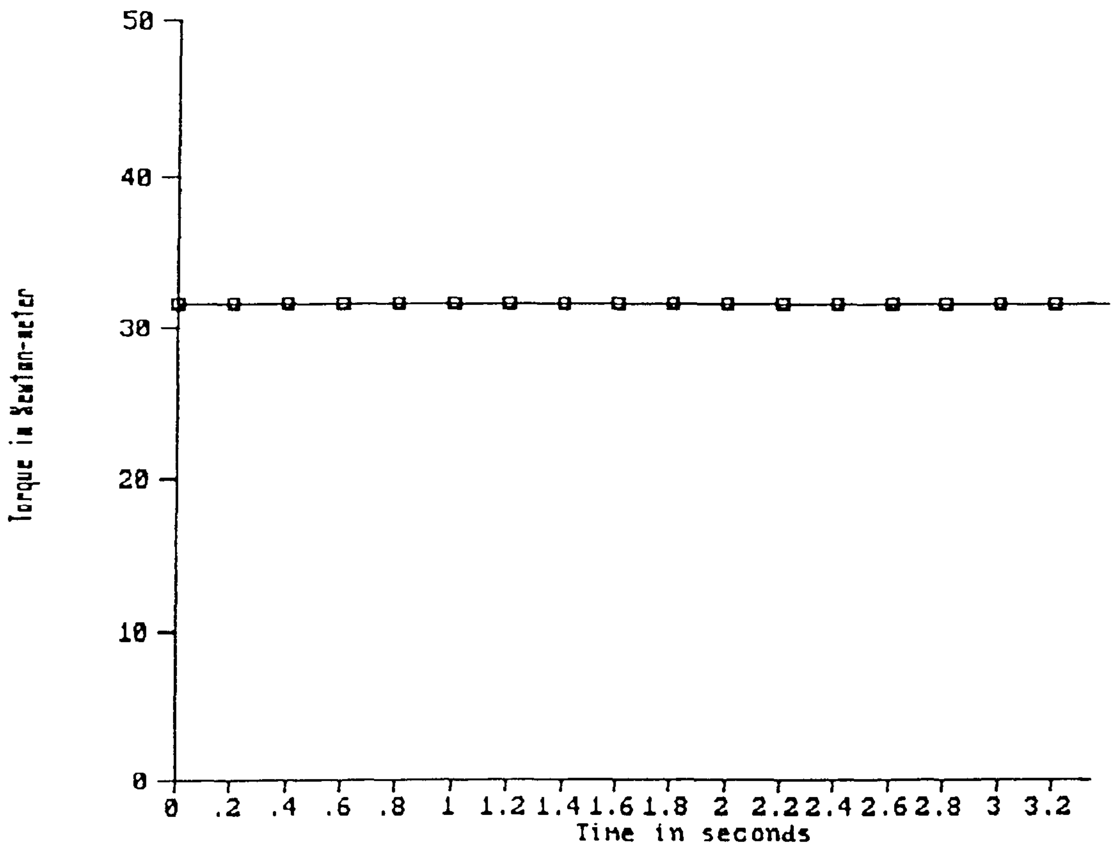

Figure 3.3 Torque Response If The Final Slip Is $0.3u(t)$

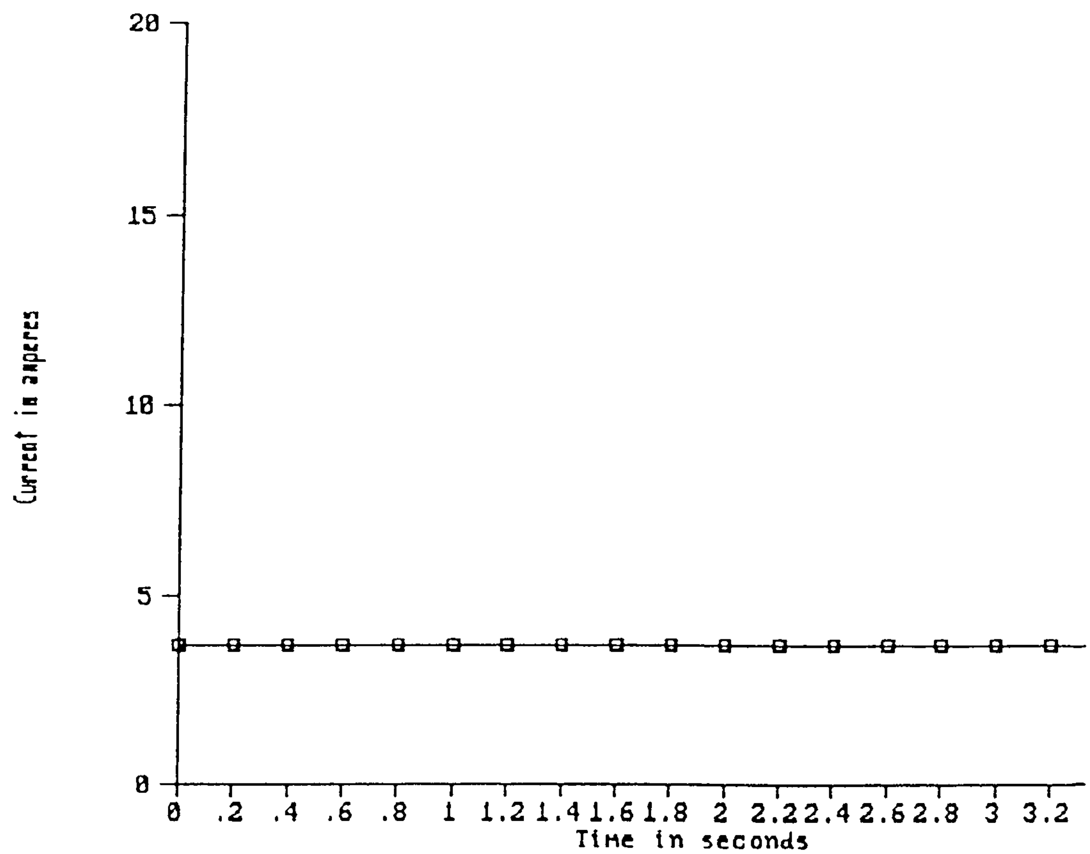

Figure 3.4 Current Response If The Final Slip Is $0.03u(t)$

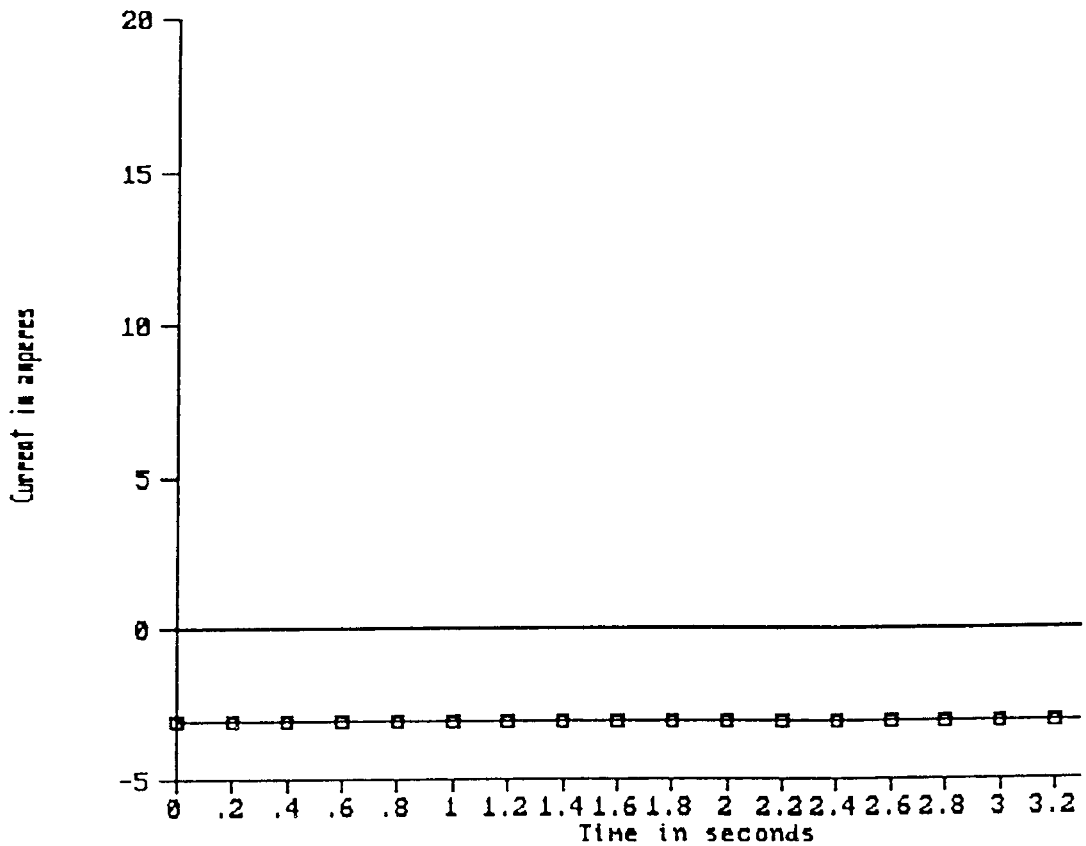

Figure 3.5 Current Response If The Final Slip Is $-0.03u(t)$

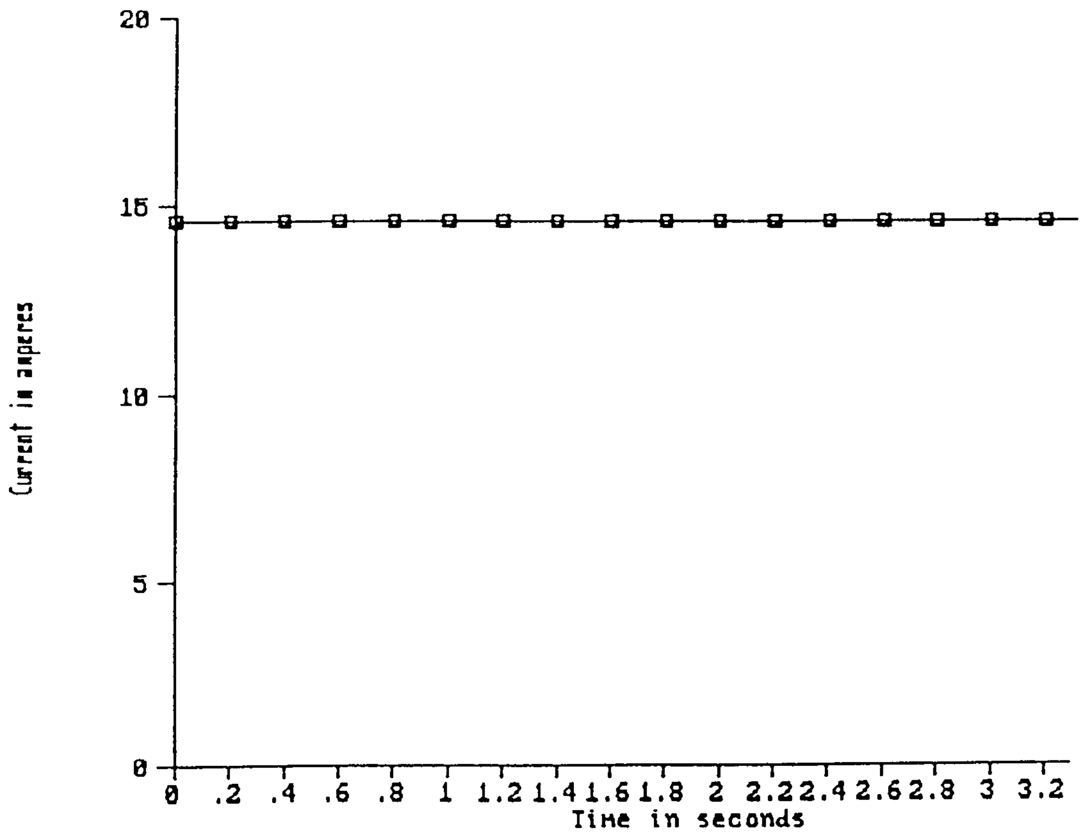

Figure 3.6 Current Response If The Final Slip Is $0.3u(t)$

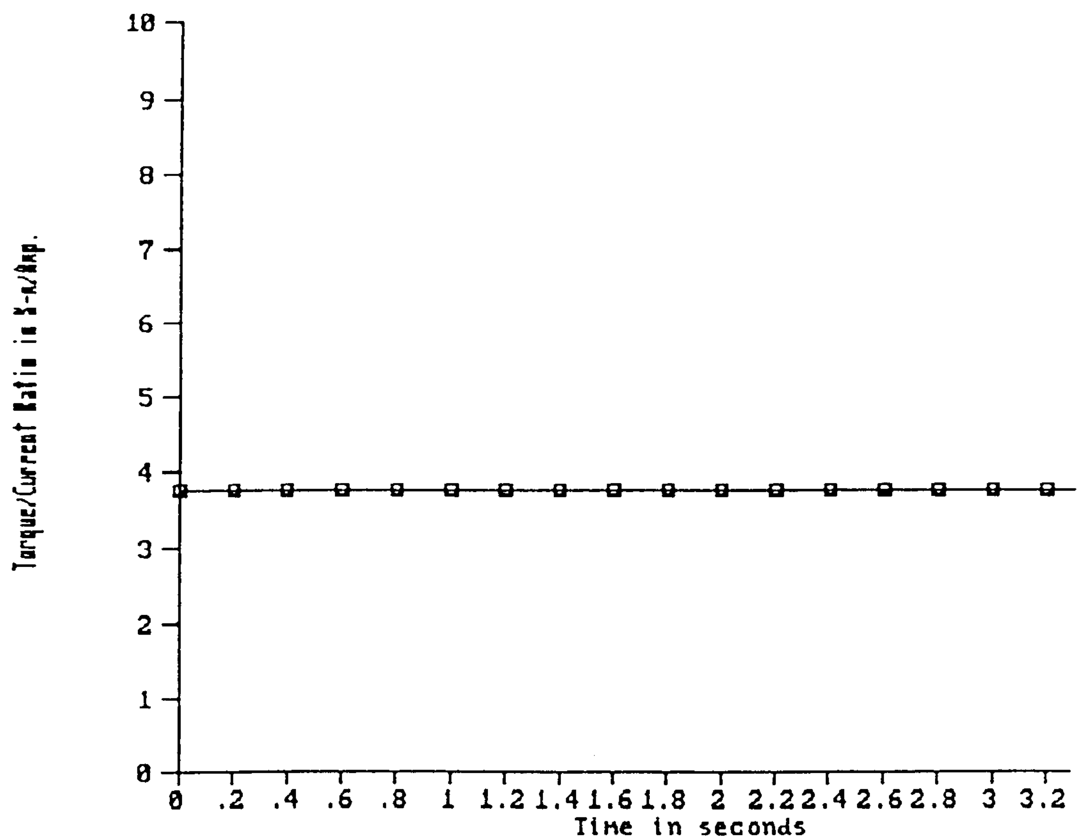

Figure 3.7 Torque to Current Ratio If The Final Slip Is $0.03u(t)$

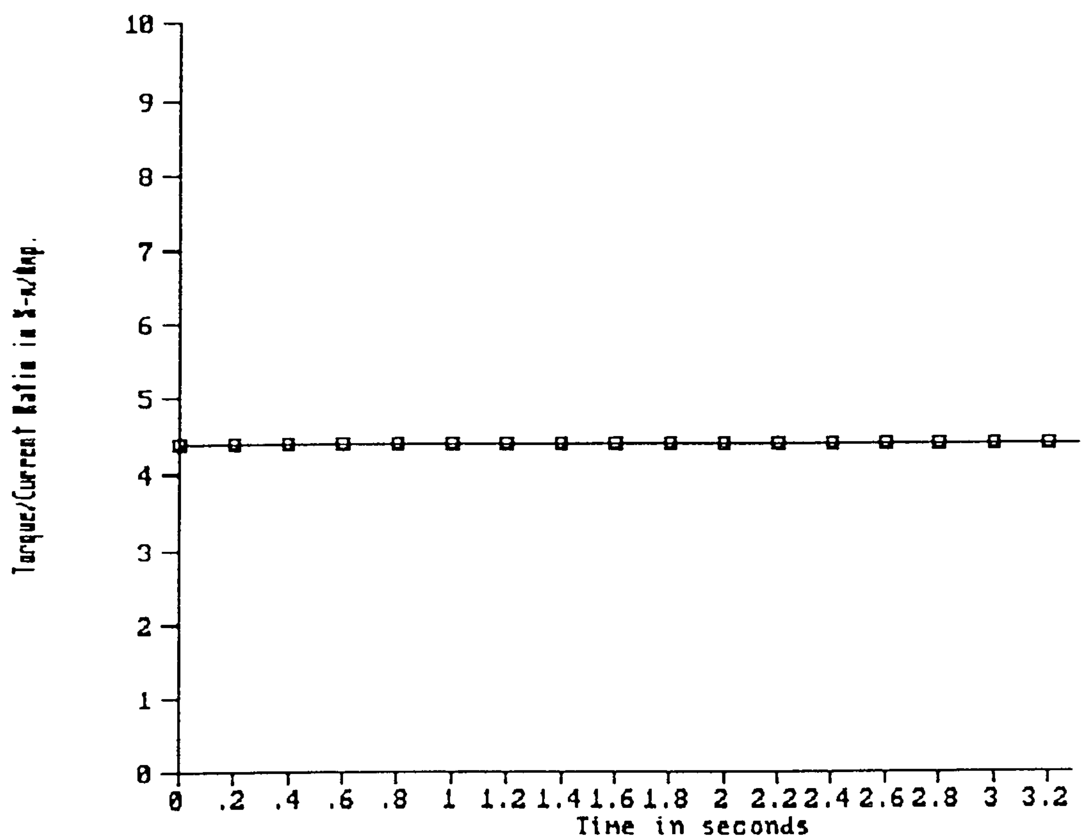

Figure 3.8 Torque to Current Ratio If The Final Slip Is $-0.03u(t)$

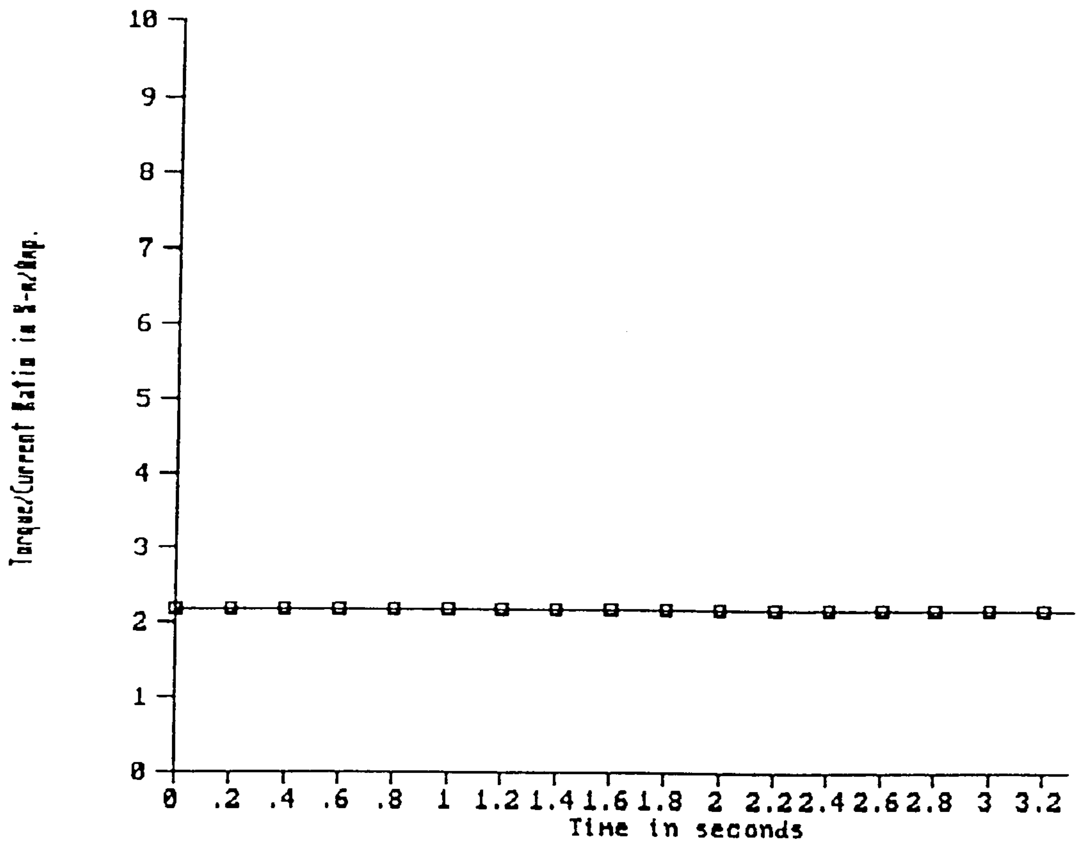

Figure 3.9 Torque to Current Ratio If The Final Slip Is $0.3u(t)$

CHAPTER 4

UNIT-IMPULSE, $Q\delta(t)$[20]

4.1 Introduction

This chapter deals with the torque-time and current-time response of induction motor, when a slip at no-load is changed to a unit-impulse, $Q\delta(t)$. Computations are all based in Chapter 2. This chapter also gives attention with the response of a low-drive induction motor which operates with a high slip, and when it is operating at negative slip as in the case of induction generator

4.2 Applying Laplace Transform

Recall the differential equation for torque given in Eqn. 2.17

$$T' \approx V^2[\frac{Q'}{R} - \frac{3X^2}{R^3}Q'Q^2 + \frac{5X^4}{R^5}Q'Q^4 - \frac{6X^6}{R^7}Q'Q^6] \tag{4.1}$$

subjected with the following boundary conditions

$$\text{at } t = 0^- \qquad Q = Q_o$$
$$\text{at } t = 0^+ \qquad Q = Q\delta(t)$$

To solve this equation, Laplace Transformation is again used. Refer to the Tables of Laplace Transform in Appendix A, the derivative property gives

$$\frac{d^n f(t)}{dt^n} = s^n F(s) - s^{n-1} f(0) - s^{n-2}\frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}} \tag{4.2}$$

Solve the Laplace Transform of Eqn. 4.1 term-by-term gives

$$\mathcal{L}(T') = sT(s) - T_o \tag{4.3}$$

$$\mathcal{L}[Q\delta(t)] = Q(s) = Q \tag{4.4}$$

$$\mathcal{L}[Q'] = sQ(s) - Q_o \tag{4.5}$$

$$\mathcal{L}[Q'] = sQ - Q_o \tag{4.6}$$

$$\mathcal{L}[Q^2] = \mathcal{L}[Q^2\delta(t)] = Q^2 \tag{4.7}$$

To find $\mathcal{L}[Q^2Q']$, a product of two functions, a formula is given in Appendix A

$$\mathcal{L}[f_1(t)f_2(t)] = \sum_{k=1}^{?} \frac{A_1 s(k)}{B_1' s(k)} F_2(s - s_k) \tag{4.8}$$

It is assumed that $F_1(s) = Q^2$ and $F_2(s) = sQ - Q_o$. But Eqn. 4.8 cannot be applied directly to solve for $\mathcal{L}[Q^2Q']$ because neither $F_1(s)$ nor $F_2(s)$ is a rational function. To satisfy the requirement, $F_1(s)$ is multiplied by s/s.

$$\mathcal{L}[Q^2] = \frac{sQ^2}{s} \tag{4.9}$$

$$\mathcal{L}[Q^2Q'] = \frac{sQ^2}{1}(sQ - Q_o) \tag{4.10}$$

$$\mathcal{L}[Q^2Q'] = s^2Q^3 - sQ^2Q_o \tag{4.11}$$

Similarly,

$$\mathcal{L}[Q^4] = \frac{sQ^4}{s} \tag{4.12}$$

$$\mathcal{L}[Q^4Q'] = s^2Q^5 - sQ^4Q_o \tag{4.13}$$

$$\mathcal{L}[Q^6] = \frac{sQ^6}{s} \tag{4.14}$$

$$\mathcal{L}[Q^6Q'] = s^2Q^7 - sQ^6Q_o \tag{4.15}$$

Substitute these in Eqn. 4.1

$$\begin{aligned} sT(s) - T_o \approx\ & \frac{V^2}{R}(sQ - Q_o) - \frac{3V^2X^2}{R^3}(s^2Q^3 - sQ^2Q_o) + \\ & \frac{5V^2X^4}{R^5}(s^2Q^5 - sQ^4Q_o) - \frac{6V^2X^6}{R^7}(s^2Q^7 - sQ^6Q_o) \end{aligned} \tag{4.16}$$

Simplify and get

$$\begin{aligned} T(s) \approx\ & \frac{V^2}{R}(Q - \frac{Q^o}{s} - \frac{3V^2X^2}{R^3}[sQ^3 - Q^2Q_o] + \frac{5V^2X^4}{R^5}[sQ^5 - Q^4Q_o] \\ & - \frac{6V^2X^6}{R^7}[sQ^7 - Q^6Q_o] + \frac{T_o}{s} \end{aligned} \tag{4.17}$$

Finding the inverse Laplace Transform.

$$\begin{aligned} T(t) \approx\ & u(t)[T_o - \frac{V^2 Q_o}{R}] + \frac{V^2 Q \delta(t)}{R}[1 + \frac{3X^2 Q Q_o}{R^2} - \frac{5X^4 Q^3 Q_o}{R^4} + \\ & \frac{6X^6 Q^5 Q_o}{R^6}] - \frac{V^2 X^2 Q^3 \delta'(t)}{R^3}[3 - \frac{5X^2 Q^2 Q_o}{R^2} + \frac{6X^4 Q^4 Q_0}{R^4}] \end{aligned} \tag{4.18}$$

in synchronous watts.

4.3 Current Calculation

Going back to Eqn. 2.24

$$I \approx V[\frac{Q'}{R} - \frac{3X^2 Q^2 Q'}{2R^3}] \tag{4.19}$$

with the following boundary conditions

at $t = 0^-$	$Q = Q_o$
at $t = 0^+$	$Q = Q\delta(t)$

Using the same method as in Sec. 4.2

$$\mathcal{L}[I'] = sI(s) - I_o \tag{4.20}$$

From Sec. 4.2

$$\mathcal{L}[Q'] = sQ(s) - Q_o \tag{4.21}$$

$$\mathcal{L}[Q^2] = \frac{sQ^2}{s} \tag{4.22}$$

$$\mathcal{L}[Q^2 Q'] = s^2 Q^3 - sQ^2 Q_o \tag{4.23}$$

Substitute these in Eqn. 4.20

$$sI(s) - I_o \approx \frac{V}{R}(sQ - Q_o) - \frac{3VX^2}{2R^3}(s^2 Q^3 - sQ^2 Q_o) \tag{4.24}$$

$$I(s) \approx \frac{V}{R}(Q - Q_o) - \frac{3VX^2}{2R^3}(sQ^3 - Q^2 Q_o) + \frac{I_o}{s} \tag{4.25}$$

Taking the inverse Laplace Transform

$$I(t) = [I_o - \frac{VQ_o}{R}]u(t) + \frac{VQ}{R}[1 + \frac{3X^2 Q Q_o}{2R^3}]\delta(t) - \frac{3VX^2 Q^3}{2R^3}\delta'(t) \tag{4.26}$$

It is seen here and also for the torque equation of Eqn. 4.18 that the response for both of them are composed of 3 different terms: a unit-step, a unit-impulse and a doublet.

4.4 Case Studies

The case studies that will be presented in this section are the same as those in Sec. 3.4. These are the following: when the slip is changed from no-load to $0.03\delta(t)$, when it is running as induction generator, and when it is running at high slip as in the case of a low-drive induction motor.

A computer program for torque calculation and for current calculation are p-resented in Appendix D and E, respectively. The inputs needed in the computer program for torque are the following: the per phase supply voltage, the frequency of the system, the number of poles of the motor, the number, of phases, the rotor resistance, the final slip, the initial slip, and the initial torque. All the units are the same as in Sec. 3.4.

Fig. 4.1 shows the response when the final slip is $0.03\delta(t)$ and the initial slip is 0.001 while Fig. 4.2 shows the torque response for an induction generator with final slip os $-0.003\delta(t)$. Their responses are almost identical except that they have opposite sign. The magnitude of the unit-impulse response is almost equal to the full-load torque. For a low-drive induction motor, the magnitude of the unit-impulse response goes beyond the maximum torque.

The responses for the current are shown in Figs. 4.4 to 4.6. The current respons-es are almost identical in shape with the torque responses, with the unit-impulse response very close to the full-load. In the case of a low-drive induction motor, the magnitude of the unit-impulse response is almost 10 times the full-load current, this is very dangerous to the motor if maintained at a longer time. The magnitude of the unit-impulse is the most practical response among the 3 functions present, it is because both the current and the torque response follows the pattern of the slip changes.

The torque to current ratio for the 3 cases are given in Figs. 4.7 to 4.9. The ratio for the impulse is the same for the 3 cases, with a magnitude of 4.04. For the doublet response, it decreases with the low-drive motor. It is due to the greater increase of the current in proportion to its torque. The torque to current ratio decreases with an increase of machine slip.

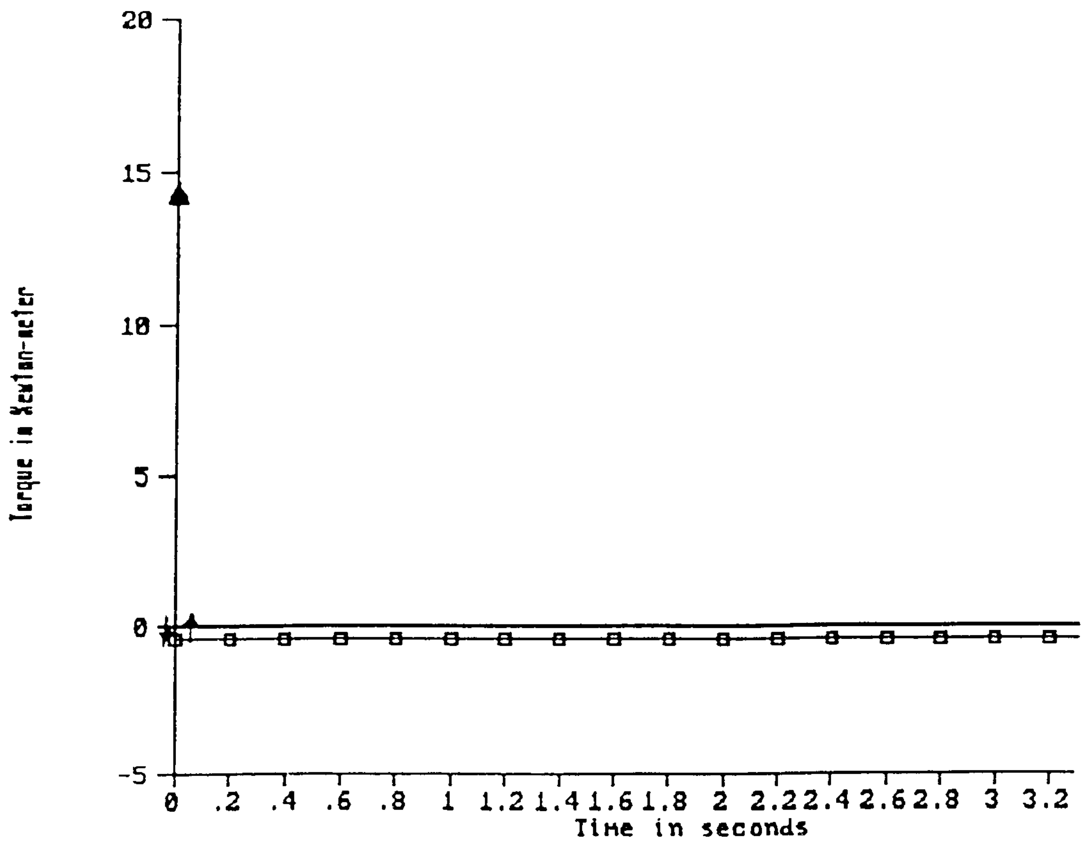

Figure 4.1 Torque Response If The Final Slip Is $0.03\delta(t)$

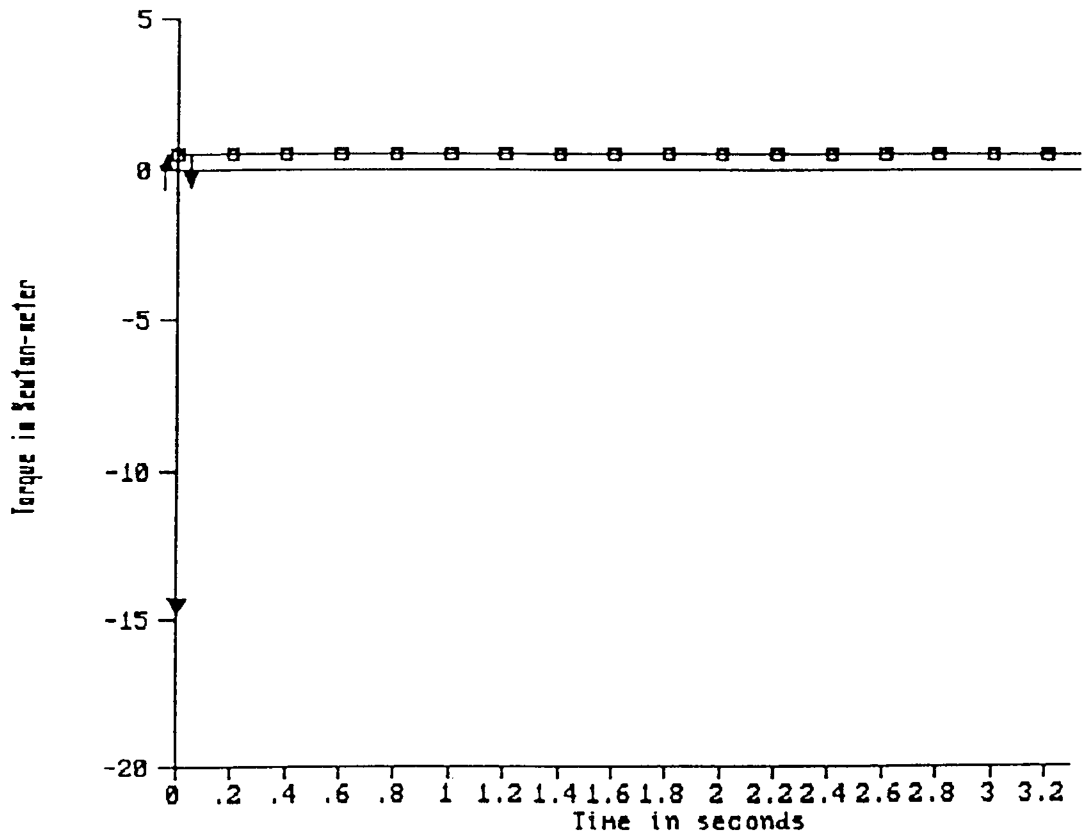

Figure 4.2 Torque Response If The Final Slip Is $-0.03\delta(t)$

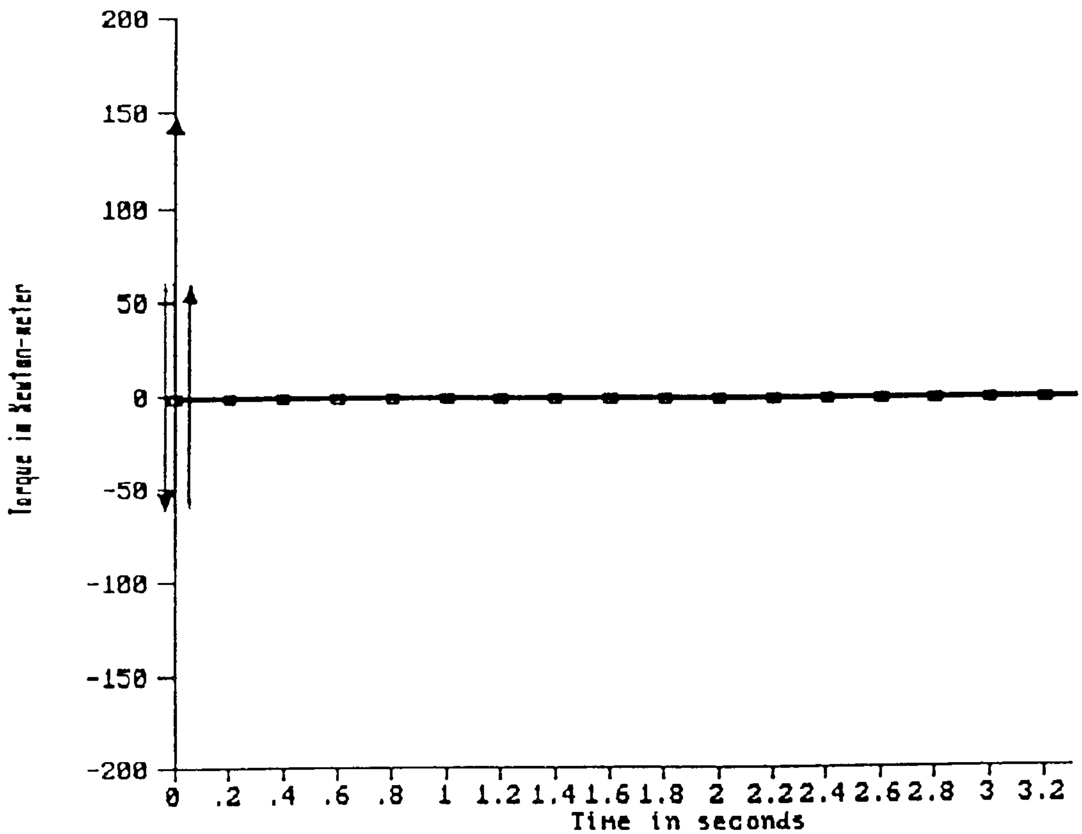

Figure 4.3 Torque Response If The Final Slip Is $0.3\delta(t)$

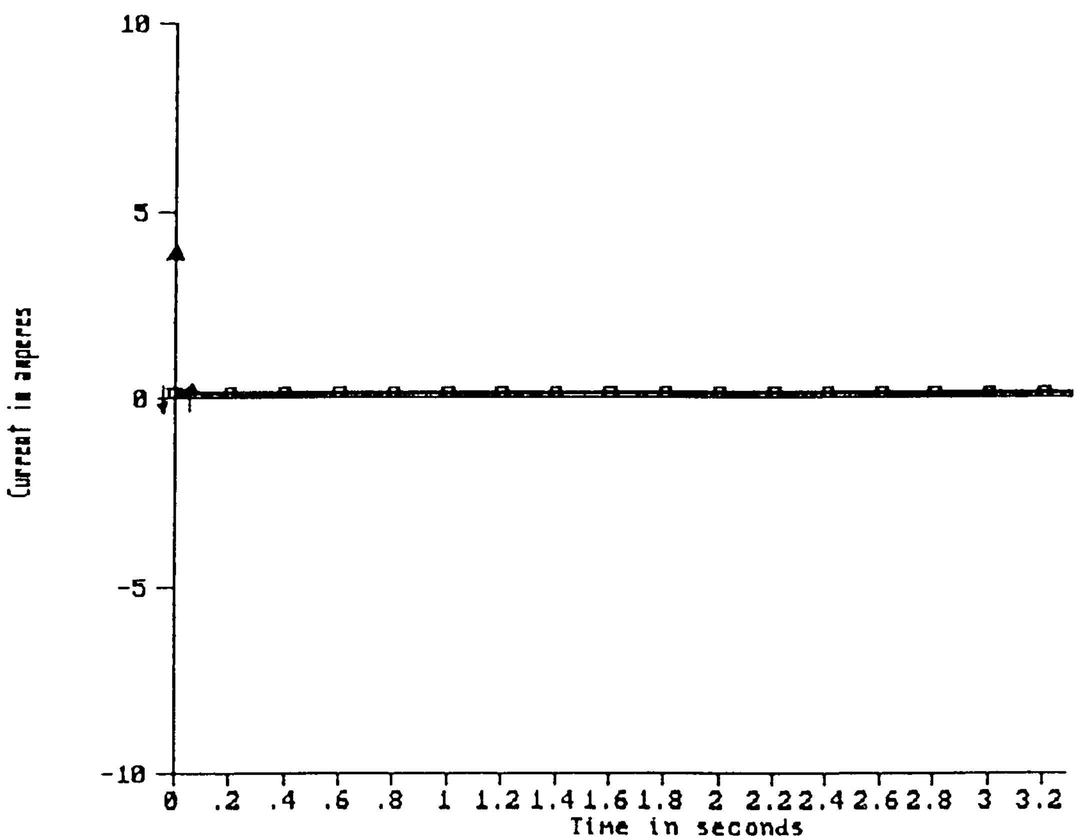

Figure 4.4 Current Response If The Final Slip Is $0.03\delta(t)$

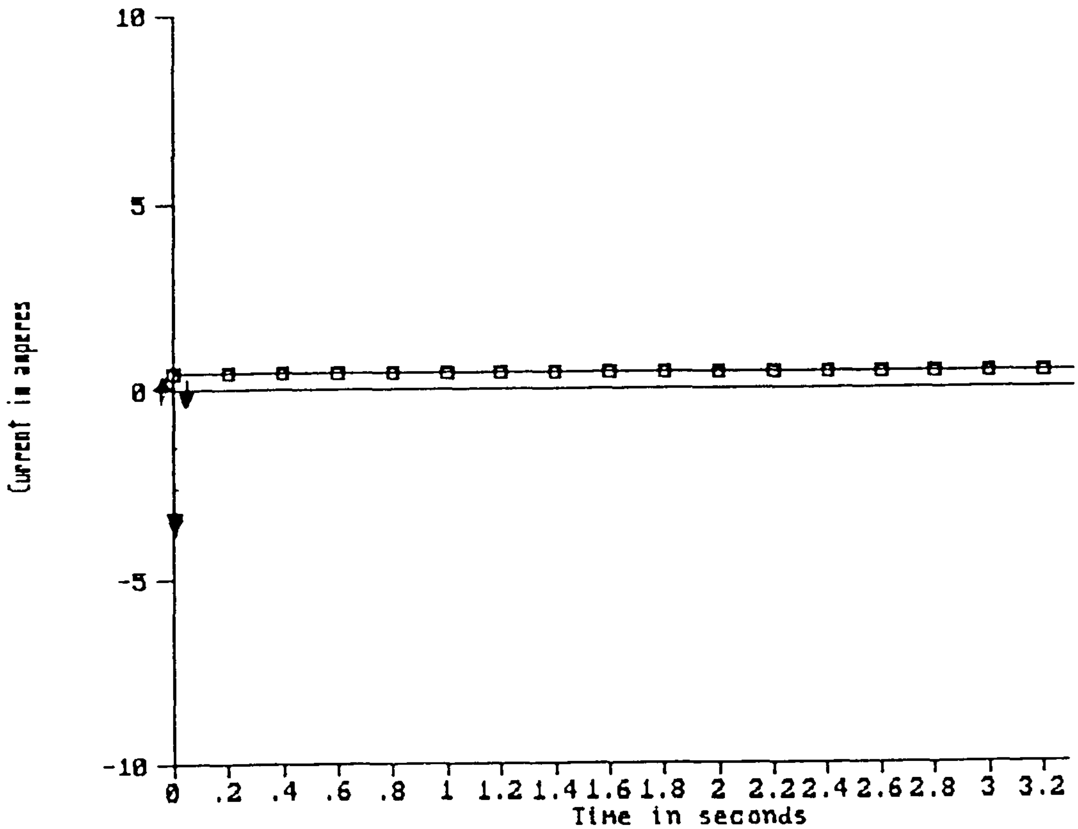

Figure 4.5 Current Response If The Final Slip Is $-0.03\delta(t)$

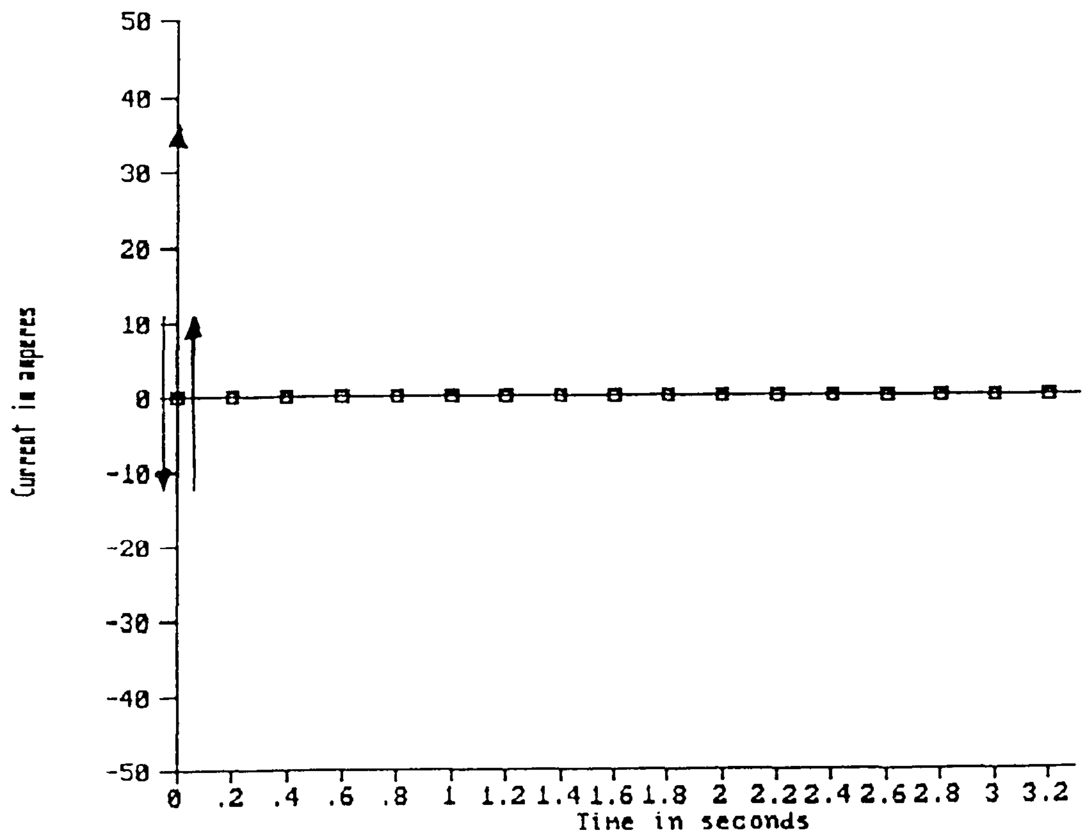

Figure 4.6 Current Response If The Final Slip Is $0.3\delta(t)$

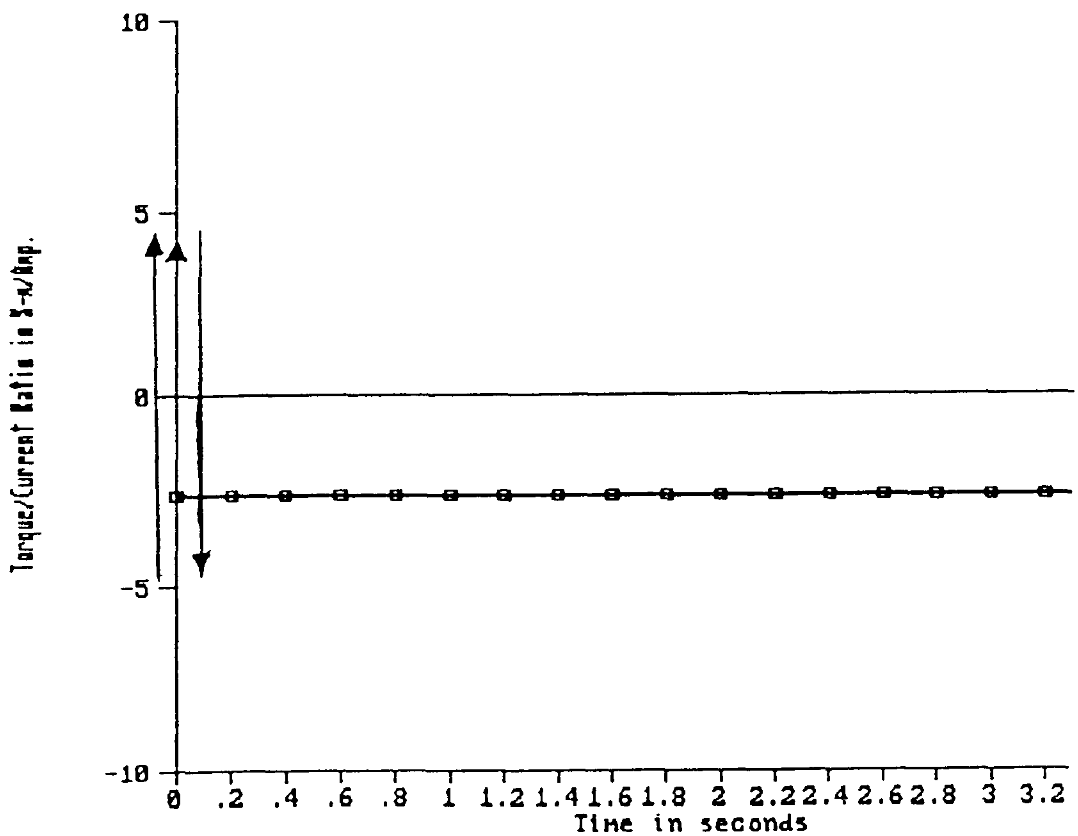

Figure 4.7 Torque/Current Ratio If The Final Slip Is $0.03\delta(t)$

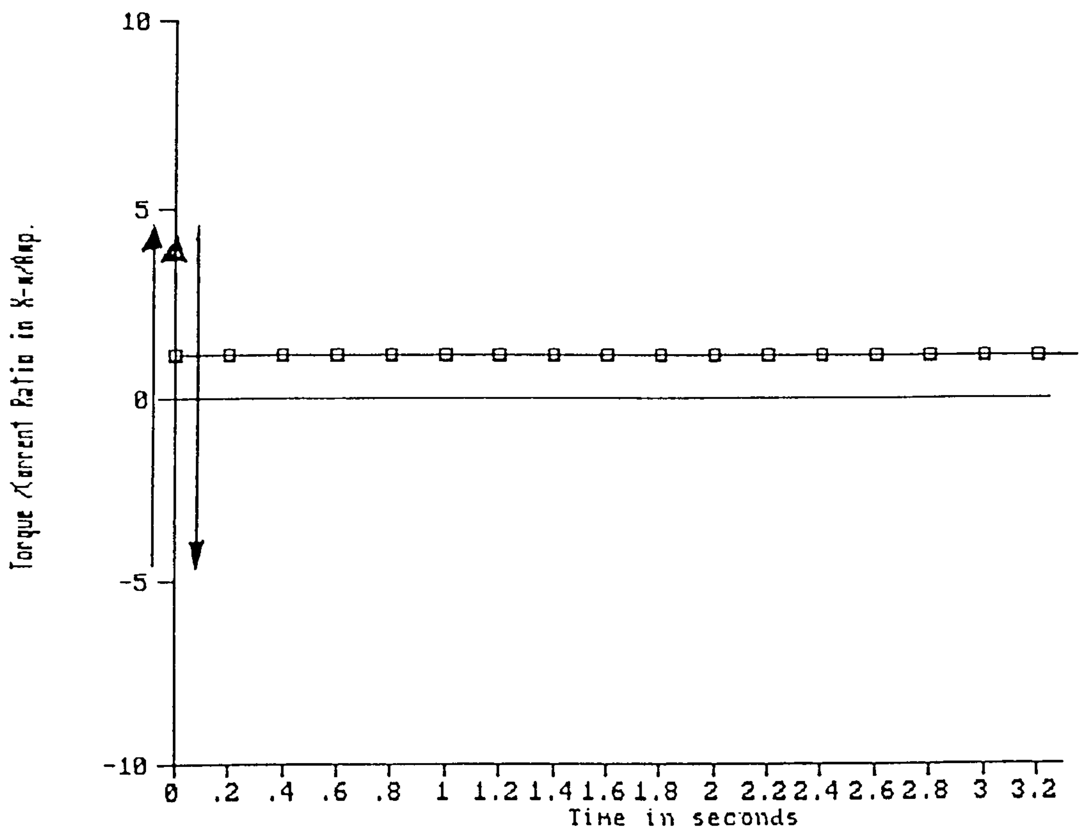

Figure 4.8 Torque/Current Ratio If The Final Slip Is $-0.03\delta(t)$

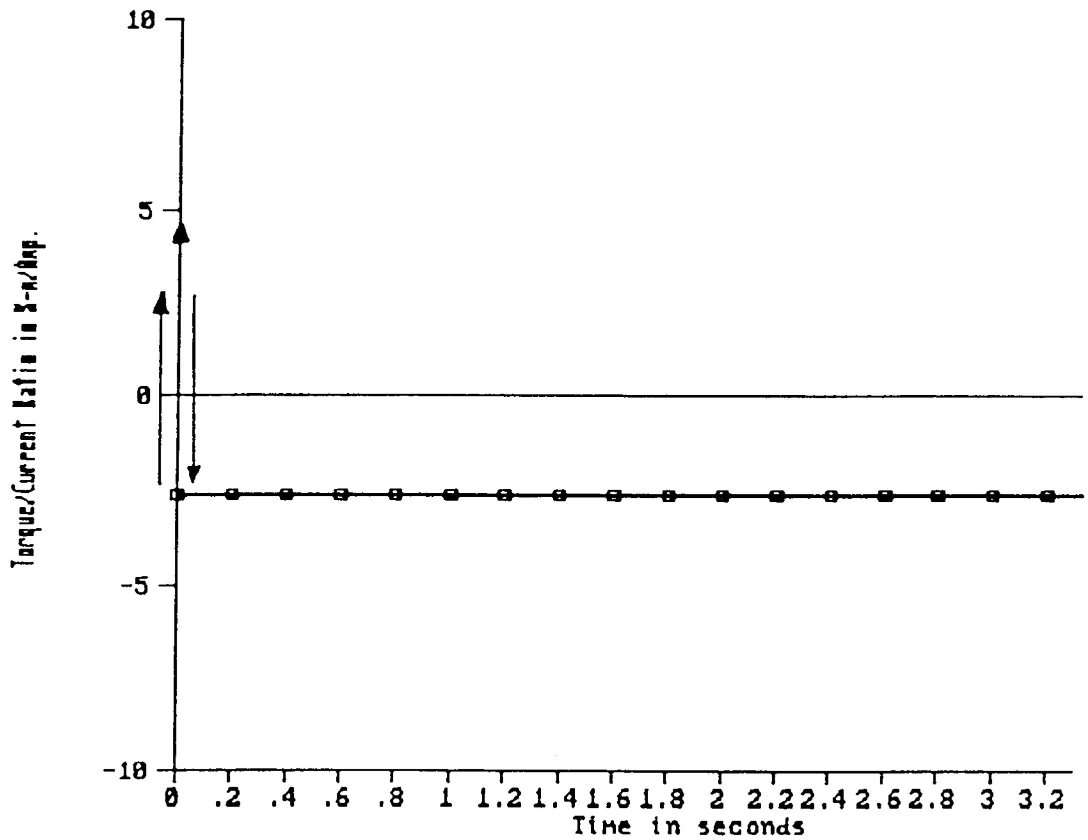

Figure 4.9 Torque/Current Ratio If The Final Slip Is $0.3\delta(t)$

CHAPTER 5

RECTANGULAR PULSE[20]

5.1 Introduction

One of the well-known waveform in the field of electrical engineering is the rectangular pulse. This chapter analyzed the response, both the torque and the current, when the slip, Q, of a 3 phase induction motor at no-load is suddenly changed to a pulse characterized by $Q[u(t) - u(t-T)]$. A case study is presented at the end of the chapter. In addition, the case of induction generator with negative slip and the case of a low drive induction motor are also given consideration.

3.2 Torque Calculation

Again, recall the differential equation for solving torque in Eqn. 2.17.

$$T' \approx V^2[\frac{Q'}{R} - \frac{3X^2}{R^3}Q'Q^2 + \frac{5X^4}{R^5}Q'Q^4 - \frac{6X^6}{R^7}Q'Q^6] \tag{5.1}$$

subjected to the following boundary conditions

at $t = 0^-$	$Q = Q_o$
at $t = 0^+$	$Q = Q[u(t) - u(t-T)]$

To solve Eqn. 5.1, the following Laplace Transform are needed

$$\mathcal{L}[u(t-T)] \leftrightarrow \epsilon^{-Ts} \tag{5.2}$$

$$\mathcal{L}[u(t-2T)] \leftrightarrow \frac{\epsilon^{-2Ts}}{s} \tag{5.3}$$

By applying the derivative property of Laplace Transform

$$\mathcal{L}[T'] = sT(s) - T_o \tag{5.4}$$

$$\mathcal{L}[Q'] = sQ(s) - Q_o \tag{5.5}$$

$$\mathcal{L}[Q] = Q(s) = \mathcal{L}[Qu(t) - Qu(t-T)] \tag{5.6}$$

$$\mathcal{L}[Q] = Q[\frac{1}{s} - \frac{\epsilon^{-Ts}}{s}] \tag{5.7}$$

Substitute Eqn. 5.7 to Eqn. 5.5

$$\mathcal{L}[Q'] = Q(1 - \epsilon^{-Ts}) - Q_o \tag{5.8}$$

Now

$$Q^2(t) = Q^2[u(t) - u(t-T)] \tag{5.9}$$

$$\mathcal{L}[Q^2] = \frac{Q^2}{s}(1 - \epsilon^{-Ts}) \tag{5.10}$$

to find $\mathcal{L}[Q^2Q']$. the transform of the product of two transform is used given in Eqn. 3.9. It is set that $F_1(s) = \mathcal{L}[Q^2]$ and $F_2(s) = \mathcal{L}[Q']$.

$$\mathcal{L}[Q^2Q'] = \frac{Q^2(1 - \epsilon^{-Ts})}{1}[Q(1 - \epsilon^{-Ts} - Q_o] \tag{5.11}$$

$$\mathcal{L}[Q^2Q'] = Q^3(1 - \epsilon^{-Ts})^2 - Q^2Q_o(1 - \epsilon^{-Ts}) \tag{5.12}$$

Similarly.

$$Q^4(t) = Q^4[u(t) - u(t-T)] \tag{5.13}$$

$$\mathcal{L}[Q^4] = \frac{Q^4}{s}(1 - \epsilon^{-Ts}) \tag{5.14}$$

$$\mathcal{L}[Q^4Q'] = Q^5(1 - \epsilon^{-Ts})^2 - Q^4Q_o(1 - \epsilon^{-Ts}) \tag{5.15}$$

$$Q^6(t) = Q^6[u(t) - u(t-T)] \tag{5.16}$$

$$\mathcal{L}[Q^6Q'] = Q^7(1 - \epsilon^{-Ts})^2 - Q^6Q_o(1 - \epsilon^{-Ts}) \tag{5.17}$$

Substitute these transforms In Eqn. 5.1.

$$\begin{aligned} sT(s) - T_o \quad \approx \quad & \frac{V^2}{R}[Q(1 - \epsilon^{-Ts}) - Q_o] - \frac{V^2X^2}{R^3}[Q^3(1 - \epsilon^{-Ts})^2 \\ & -Q^2Q_o(1 - \epsilon^{-Ts})] + \frac{5X4V^2}{R^5}[Q^5(1 - \epsilon^{-Ts})^2 - Q^4Q_o(1 - \epsilon^{-Ts})] \\ & -\frac{6V^2X^6}{R^7}[Q^7(1 - \epsilon^{-Ts})^2 - Q^6Q_o(1 - \epsilon^{-Ts})] \end{aligned} \tag{5.18}$$

$$
\begin{aligned}
T(s) \;\approx\;& \frac{V^2}{R}\left[\frac{Q}{s} - \frac{\epsilon^{-Ts}}{s} - \frac{Q_o}{s}\right] - \frac{3V^2X^2}{R^3}\left[Q^3\left(\frac{1}{s} + \frac{2\epsilon^{-Ts}}{s} + \frac{\epsilon^{-2Ts}}{s}\right)\right. \\
& -Q^2Q_o\left(\frac{1}{s} - \frac{\epsilon^{-Ts}}{s}\right)\Big] + \frac{5V^2X^4}{R^5}\Big[Q^5\left(\frac{1}{s} - \frac{2\epsilon^{-Ts}}{s} + \frac{\epsilon^{-2Ts}}{s}\right) - \\
& Q^4Q_o\left(\frac{1}{s} - \frac{\epsilon^{-Ts}}{s}\right)\Big] - \frac{6V^2X^6}{R^7}\Big[Q^7\left(\frac{1}{s} - \frac{2\epsilon^{-Ts}}{s} + \frac{\epsilon^{-2Ts}}{s}\right) - \\
& Q^6Q_o\left(\frac{1}{s} - \epsilon^{-Ts}\right)\Big] + \frac{T_o}{s}
\end{aligned}
\tag{5.19}
$$

Taking the inverse Laplace Transform and simplify

$$
\begin{aligned}
T(t) \;\approx\;& \frac{V^2}{R}\Big(\Big[Q - Q_o - \frac{3X^2Q^3}{R^2} + \frac{X^2Q^2Q_o}{R^2} + \frac{5X^4Q^5}{R^4} - \frac{5X^4Q^4Q_o}{R^4} - \frac{6X^6Q^7}{R^6} \\
& + \frac{6X^6Q^6Q_o}{R^6}\Big] + T_o\Big)u(t) + \frac{V^2}{R}\Big(\Big[-Q + \frac{6X^2Q^3}{R^2} - \frac{X^2Q^2Q_o}{R^2} - \frac{10X^4Q^5}{R^4} + \\
& \frac{5X^4Q^4Q_o}{R^4} + \frac{12X^6Q^7}{R^6} - \frac{6X^6Q^6Q_o}{R^6}\Big)u(t-T) + \frac{V^2X^2Q^3}{R^3}\Big[-3 + \\
& \frac{5X^2Q^2}{R^2} - \frac{6X^4Q^4}{R^7}\Big]u(t-2T)
\end{aligned}
\tag{5.20}
$$

in synchronous watts per phase.

5.3 Current Calculation

Looking back in the current differential equation of Eqn. 2.24

$$
I' \approx V\left[\frac{Q'}{R} - \frac{3X^2Q^2Q'}{2R^3}\right]
\tag{5.21}
$$

with the following boundary conditions

$$
\begin{array}{ll}
\text{at } t = 0^- & Q = Q_o \\
\text{at } t = 0^+ & Q = Q[u(t) - u(t-T)]
\end{array}
$$

Solve this using Laplace Transform by substituting the transforms derived in Eqns. 5.8 and 5.12 to Eqn. 5.21. It gives.

$$
\begin{aligned}
sI(s) - I_o \;\approx\;& \frac{V}{R}[Q(1-\epsilon^{-Ts}) - Q_o] - \frac{3X^2}{2R^3}[Q^3(1-\epsilon^{-Ts})^2 \\
& -Q^2Q_o(1-\epsilon^{-Ts})]
\end{aligned}
\tag{5.22}
$$

$$
\begin{aligned}
I(s) \approx\ & \frac{V}{R}[\frac{Q}{s} - \frac{Qe^{-Ts}}{s} - \frac{Q_o}{s}] - \frac{3X^2}{2R^3}[Q^3(\frac{1}{s} - \frac{2e^{-Ts}}{s} + \frac{e^{-2Ts}}{s}) - \\
& Q^2Q_o(\frac{1}{s} - \frac{e^{-Ts}}{s} + \frac{I_o}{s}
\end{aligned} \tag{5.23}
$$

Taking the inverse Laplace Transform and simplify

$$
\begin{aligned}
I(t) \approx\ & (\frac{V}{R}[Q - Q_o] - \frac{3X^2V}{2R^3}[Q^3 - Q^2Q_o] + I_o)u(t) + \frac{VQ}{R}[-1 - \frac{3X^2Q^2}{R^2} - \\
& \frac{3X^2QQ_o}{2R^2}]u(t-T) - \frac{3VX^2Q^3}{2R^3}u(t-2T)
\end{aligned} \tag{5.24}
$$

in ampere per phase.

5.4 Case Studies

A computer program has been developed based in Eqn. 5.20 for torque and Eqn. 5.24 for current calculation. They are shown in Appendix F and G, respectively. The inputs needed in the computer program are the same as in the previous chapter, except for the addition of parameter A. This is the time when the slip, $Q[u(t) - u(t-T)]$, goes to zero. A is used for both the current and the torque computer program.

Fig. 5.1 shows the torque response when the final slip is $0.03[u(t) - u(t-T)]$. At time, $t = 0^+$, up to time, $t = A^-$, which is 1 second in this case, the torque is very close to its full-load torque. It is because the sudden increase in slip is its full-load slip. It goes to a very small value at $t = A^+$ and beyond because the slip at this time suddenly drops to zero. The same thing happens in the case of induction generator, with the exception that, its response has an opposite sign as the induction motor. In the case of a low-drive induction motor, the peak of the torque response occurs at time, $t = A^+$ up to time, $t = 2$ sec. It even goes up to almost the maximum torque even the slip at this time duration is very very small.

The same pattern occurs in current response as shown by Figs. 5.7 to 5.9. In the case of a low drive induction motor, it also reaches its peak value at the same time duration as the torque response. The peak value of the rotor current is almost 2 times the computed current at maximum torque given by Eqn. 3.32. This is because the current increases more rapidly than the torque at the same increase of the machine slip. It is further satisfied by the graphs illustrating the torque to current ratio againts the time shown in Figs. 5.10 to 5.12.

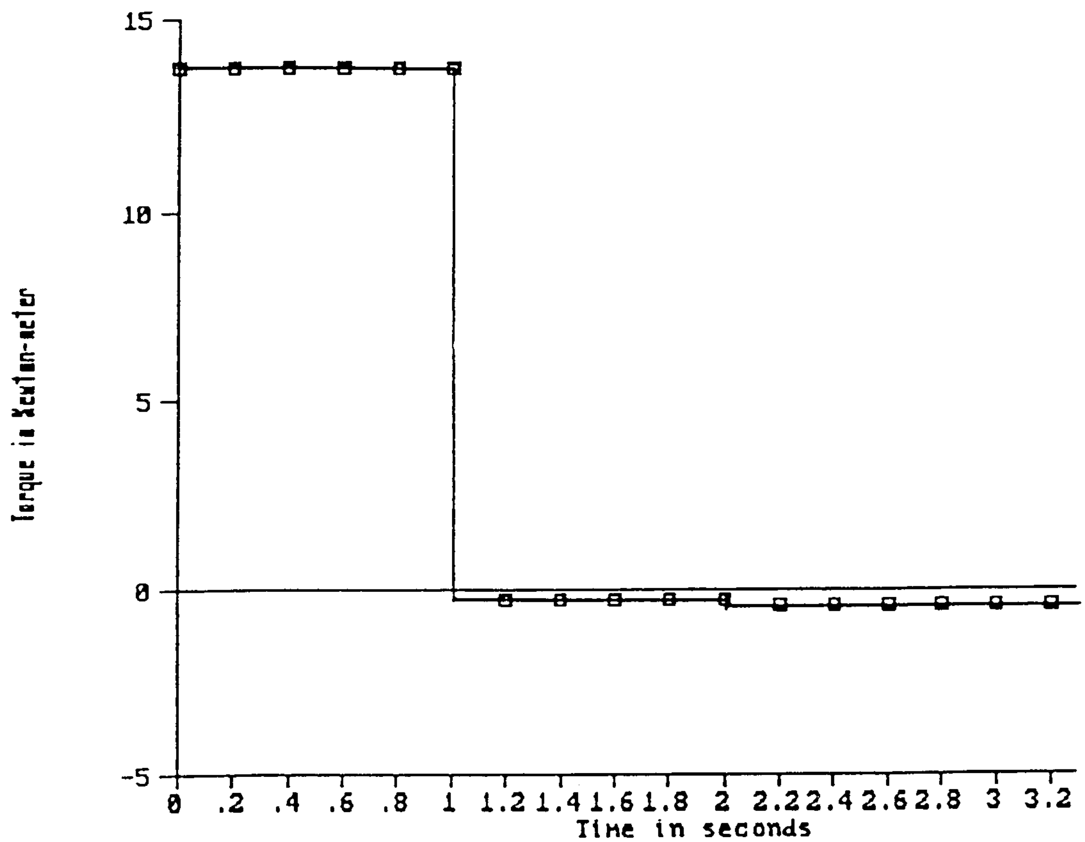

Figure 5.1 Torque Response If The Final Slip Is $0.03[u(t) - u(t-T)]$

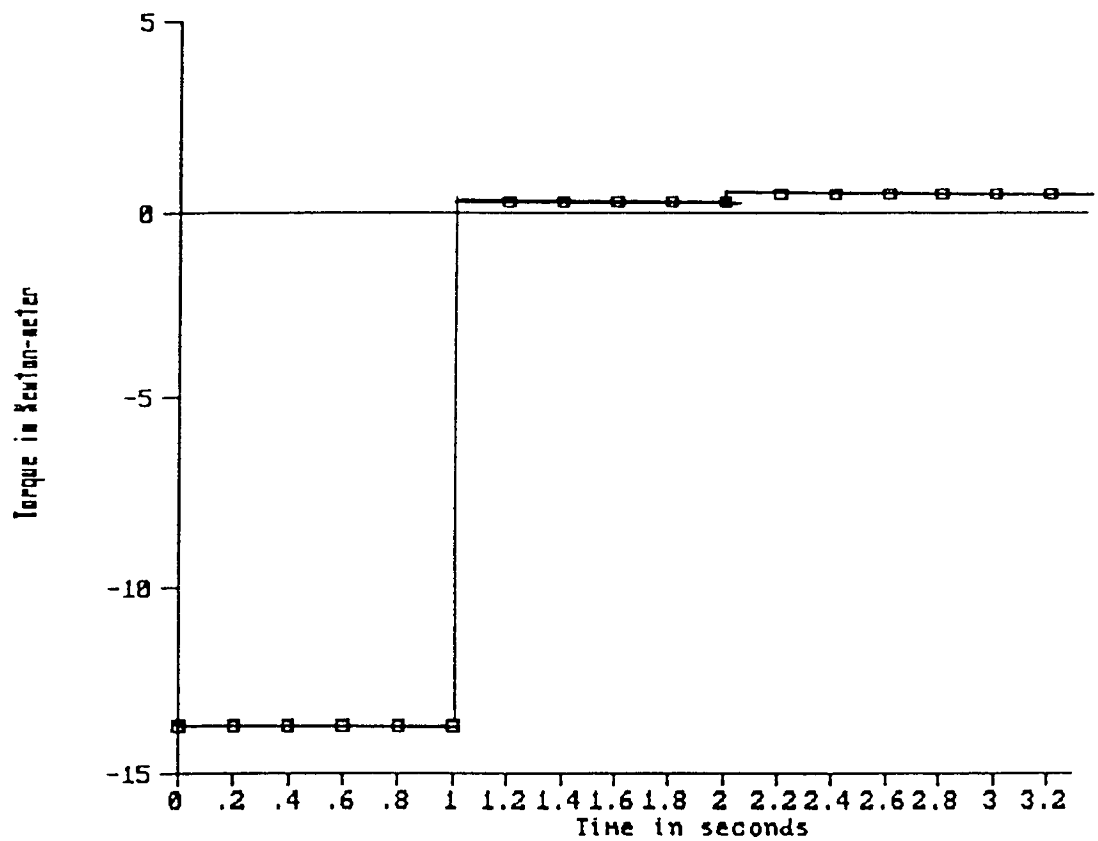

Figure5.2 Torque Response If The Final Slip Is $-0.03[u(t) - u(t - T)]$

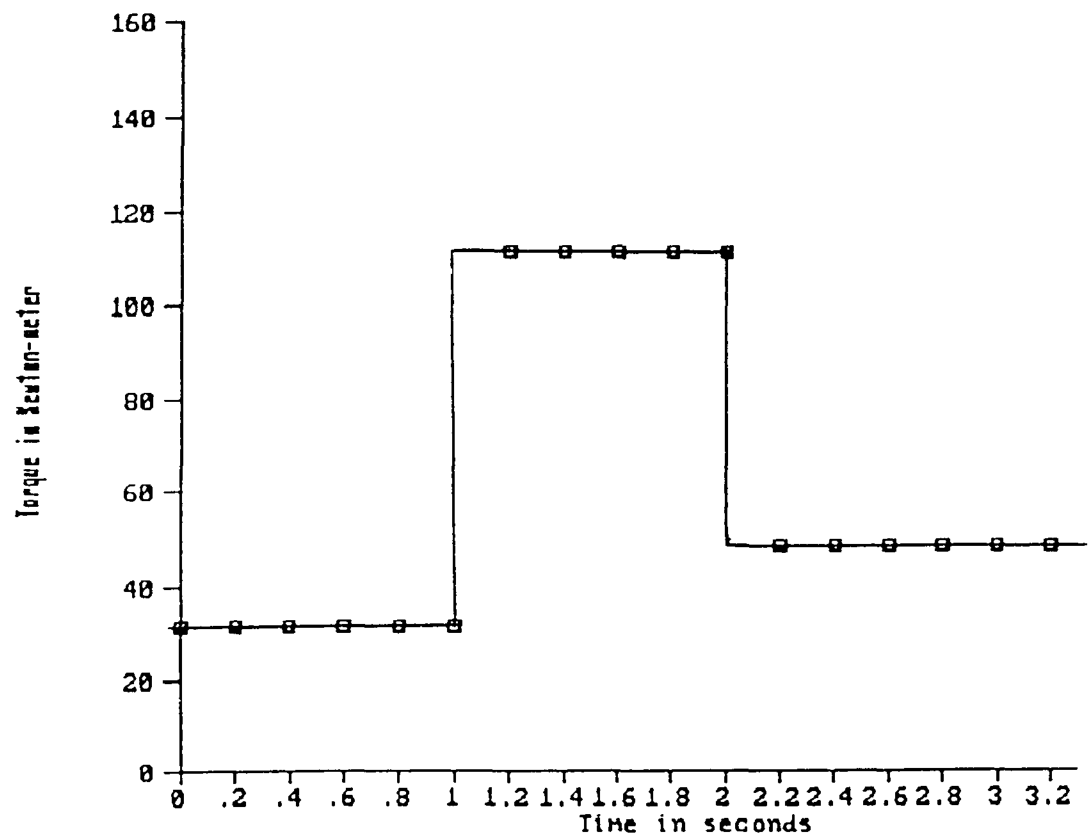

Figure 5.3 Torque Response If The Final Slip Is $0.3[u(t) - u(t-T)]$

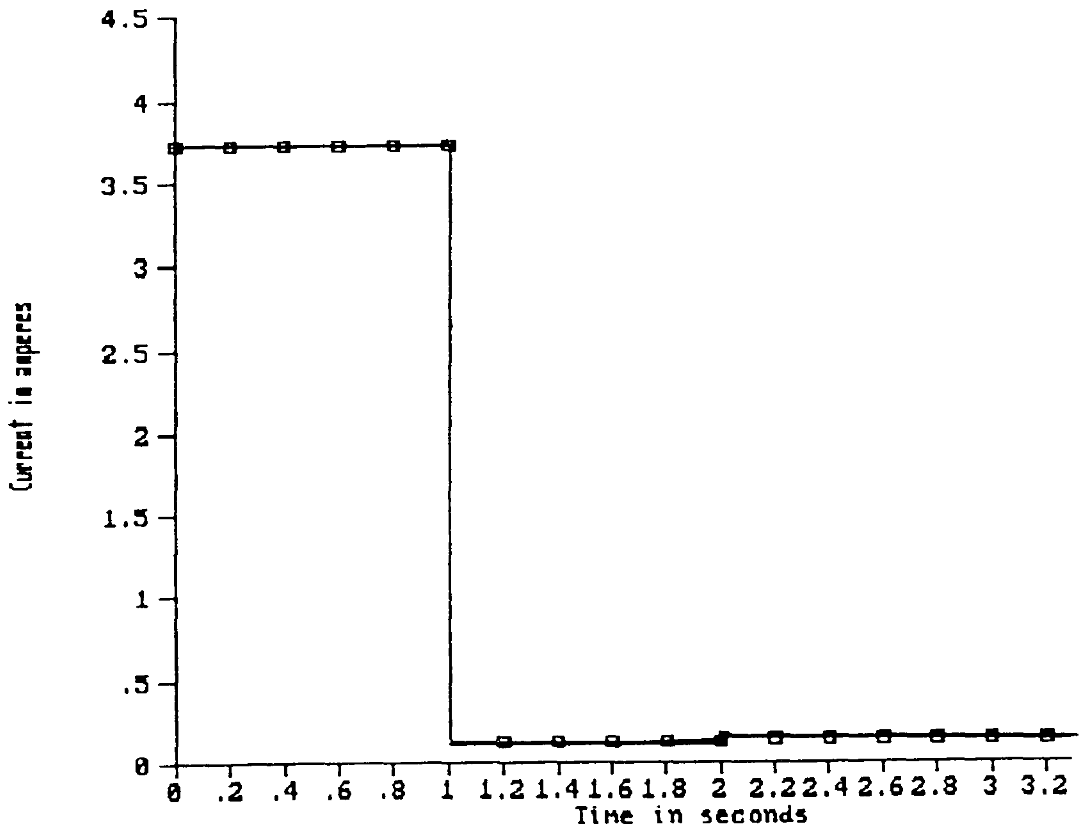

Figure 5.4 Current Response If The Final Slip Is $0.03[u(t) - u(t - T)]$

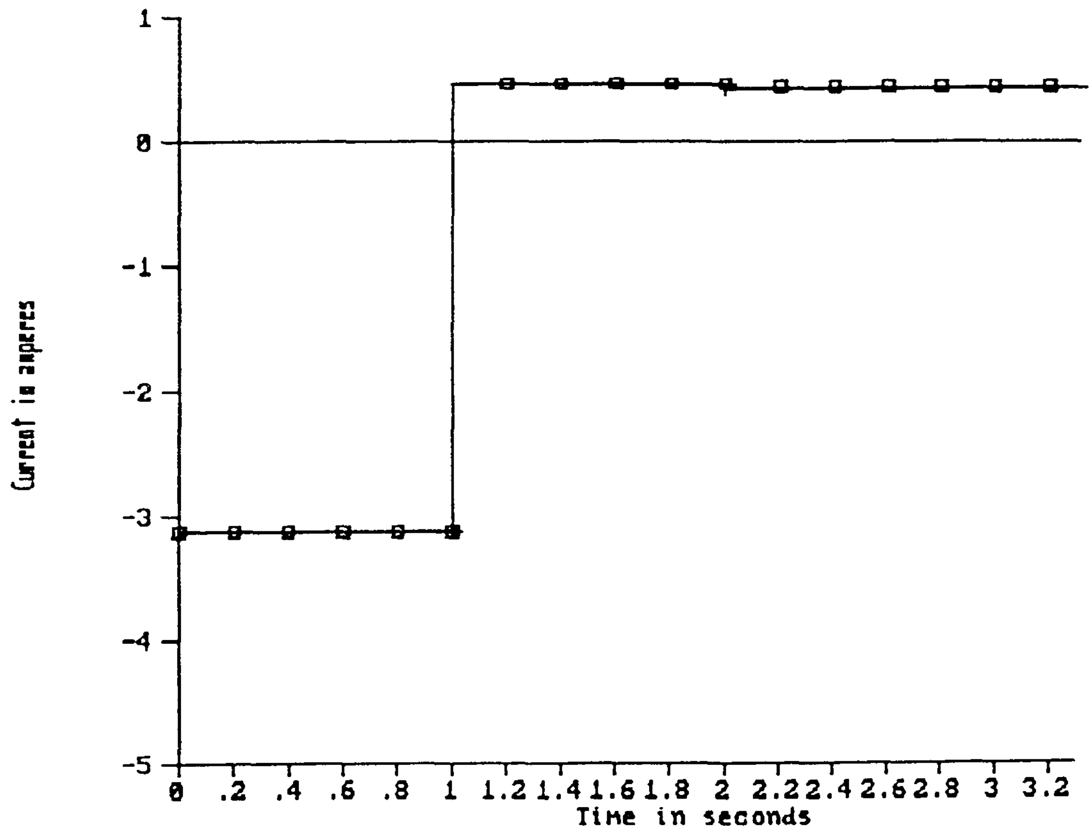

Figure 5.5 Current Response If The Final Slip Is $-0.03[u(t) - u(t-T)]$

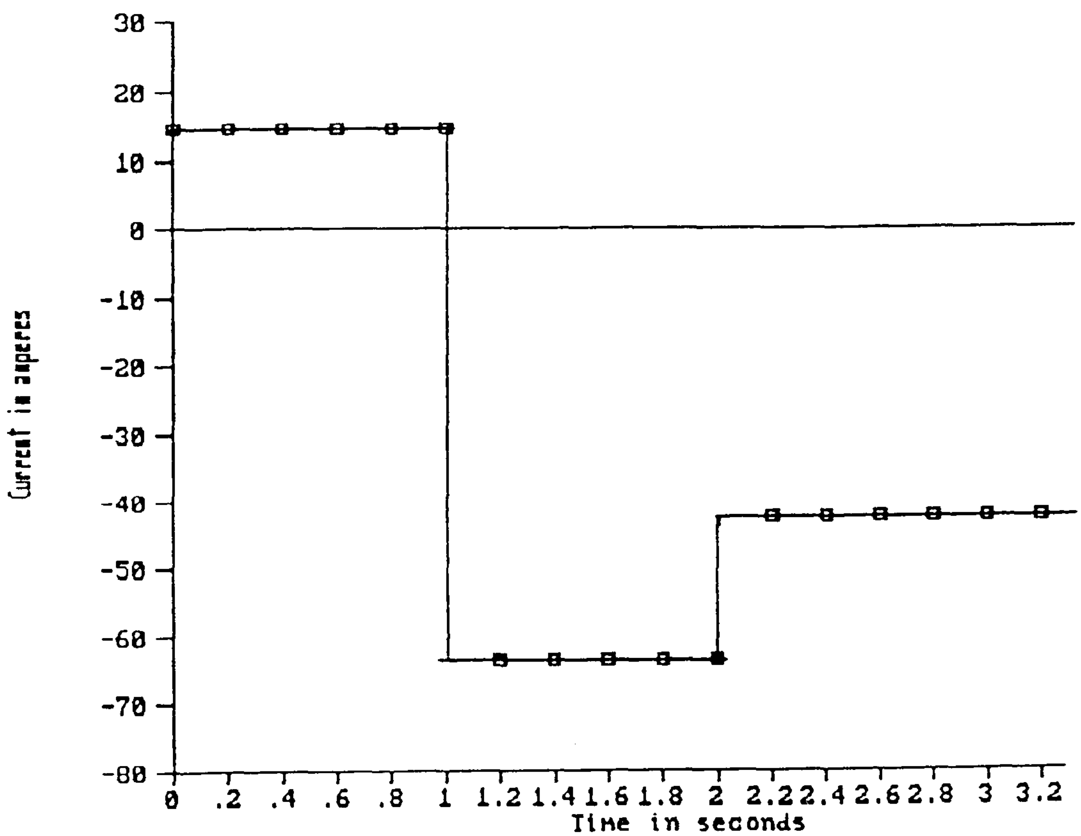

Figure 5.6 Current Response If The Final Slip Is $0.3[u(t) - u(t - T)]$

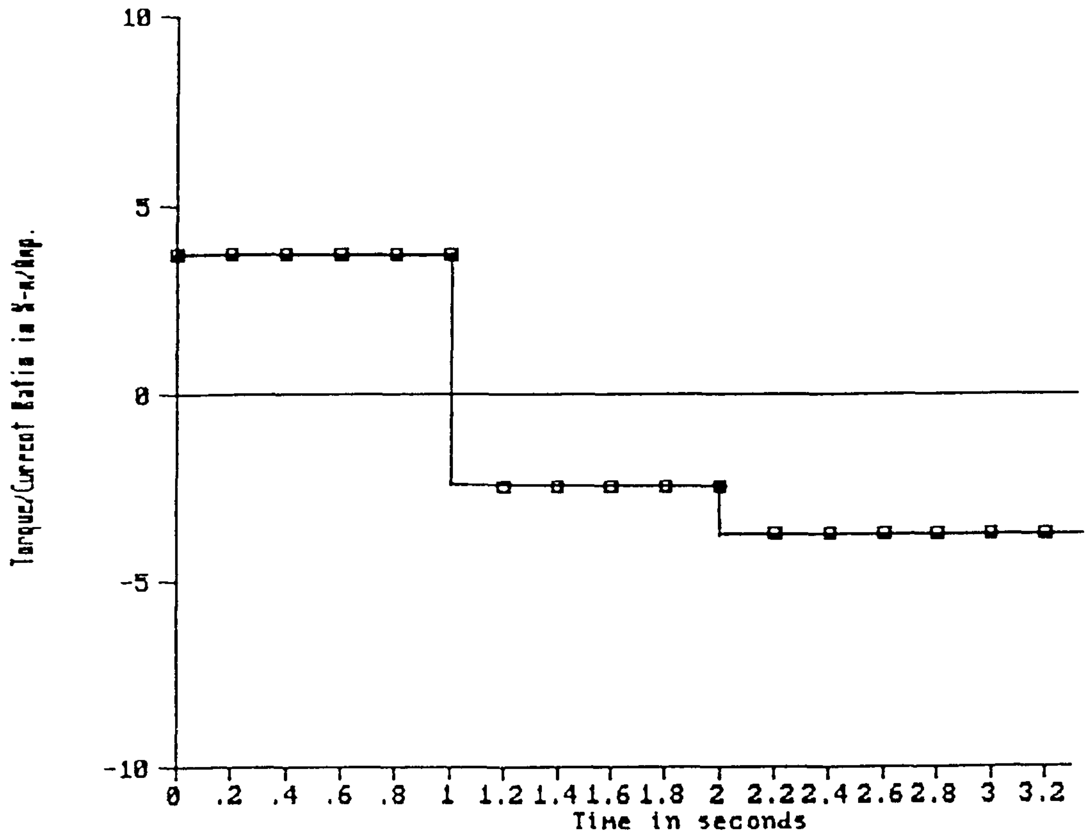

Figure 5.7 Torque/Current Ratio If The Final Slip Is $0.03[u(t) - u(t - T)]$

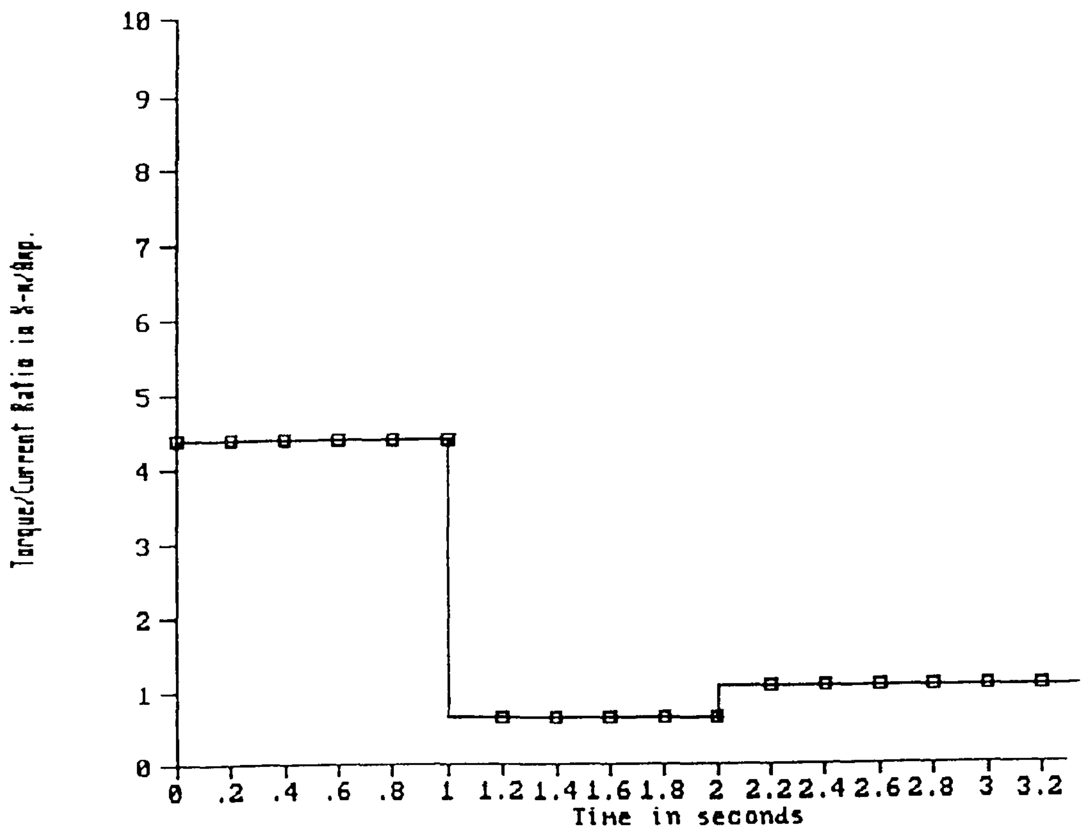

Figure 5.8 Torque/Current Ratio If The Final Slip Is $-0.03[u(t) - u(t-T)]$

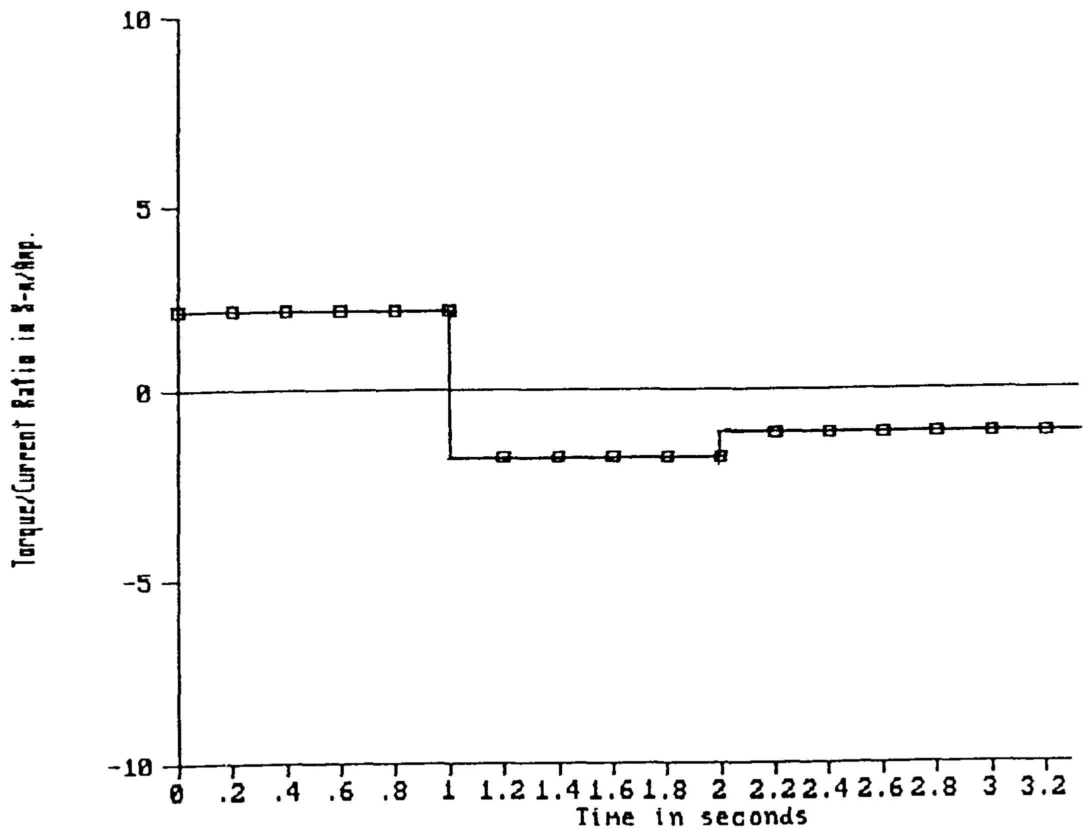

Figure 5.9 Torque/Current Ratio If The Final Slip Is $0.3[u(t) - u(t - T)]$

CHAPTER 6

UNIT-RAMP, Qt[20]

6.1 Introduction

This chapter deals with the current-time and torque-time response of a three phase induction motor, when the slip at no-load, Q_o, is suddenly changed to a unit-ramp, Qt. Again, the computations are based in the per phase approximate equivalent circuit shown in Fig. 1.2. Unlike in the previous chapter, the slip, Q, in this section is dependent on time, t. So it is expected that the maximum torque will be reached at a certain time.

6.2 Torque Calculation

From Eqn. 2.17, the torque differential equation,

$$T' \approx V^2[\frac{Q'}{R} - \frac{3X^2}{R^3}Q'Q^2 + \frac{5X^4}{R^5}Q'Q^4 - \frac{6X^6}{R^7}Q'Q^6] \tag{6.1}$$

with the following boundary conditions

at $t = 0^-$	$Q = Q_o$
at $t = 0^+$	$Q = Qt$

This is solved, once again, using Laplace Transform method. By referring to Appendix A, take the transform of Eqn. 6.1 term-by-term. Applying the derivative property,

$$\mathcal{L}[T'] = sT(s) - T_o \tag{6.2}$$

In addition, this transform is also needed in this section

$$\mathcal{L}[\frac{t^{n-1}}{(n-1)!}] \leftrightarrow \frac{1}{s^n} \tag{6.3}$$

$$\mathcal{L}[Q(t)] = \mathcal{L}[Qt] = Q(s) = \frac{Q}{s^2} \tag{6.4}$$

$$\mathcal{L}[Q'] = sQ(s) - Q_o \tag{6.5}$$

Therefore,

$$\mathcal{L}[Q'] = \frac{Q}{s} - Q_o \tag{6.6}$$

$$Q^2 = (Qt)^2 = Q^2t^2 \tag{6.7}$$

$$\mathcal{L}[Q^2] = \frac{2Q^2}{s^3} \tag{6.8}$$

To solve $\mathcal{L}[Q^2Q']$, Eqn. 3.9 is used, the Laplace Transform of the product of two functions. It is assumed that $F_1(s) = \mathcal{L}[Q^2]$ and $F_2(s) = \mathcal{L}[Q']$. Hence

$$\mathcal{L}[Q^2Q'] = \frac{2Q^3}{3s^3} - \frac{2Q^2Q_o}{3s^2} \tag{6.9}$$

To solve for $\mathcal{L}[Q^4Q']$,

$$Q^4 = Q^4t^4 \tag{6.10}$$

$$\mathcal{L}[Q^4] = \frac{24Q^4}{s^5} \tag{6.11}$$

It is assumed $F_1(s) = L[Q^4]$ and $\mathcal{L}[Q']$. Hence,

$$\mathcal{L}[Q^4Q'] = \frac{24Q^5}{5s^5} - \frac{24Q^4Q_o}{5s^4} \tag{6.12}$$

Similarly,

$$Q^6 = Q^6t^6 \tag{6.13}$$

$$\mathcal{L}[Q^6] = \frac{720Q^6}{s^7} \tag{6.14}$$

Substitute these equations to Eqn. 6.1

$$\begin{aligned} sT(s) - T_o \approx\; & \frac{V^2}{R}[\frac{Q}{s} - Q_o] - \frac{V^2}{R^3}[\frac{2Q^3}{3s^3} - \frac{2Q^2Q_o}{3s^2}] + \frac{5X^4V^2}{R^5}[\frac{24Q^5}{s^5} \\ & - \frac{24Q^4Q_o}{5s^4}] - \frac{6V^2X^6}{R^7}[\frac{720Q^7}{7s^7} - \frac{720Q^6Q_o}{7s^6} \end{aligned} \tag{6.15}$$

$$\begin{aligned} T(s) \approx\; & \frac{V^2}{R}[\frac{Q}{s^2} - \frac{Q_o}{s}] - \frac{V^2}{R^3}[\frac{2Q^3}{3s^4} - \frac{2Q^2Q_o}{3s^3}] + \\ & \frac{5X^4V^2}{R^5}[\frac{24Q^5}{5s^6} - \frac{24Q^4Q_o}{5s^5} - \frac{6V^2X^6}{R^7}[\frac{720Q^7}{7s^8} - \frac{720Q^6Q_o}{7s^7}] + T_o \end{aligned} \tag{6.16}$$

Take the inverse Laplace Transform and simplify.

$$
\begin{aligned}
T(t) \approx\ & u(t)[T_o - \frac{Q_o V^2}{R}] + \frac{V^2 Q t}{R} + \frac{V^2 Q^2 Q_o X^3 t^2}{3R^3} - \frac{2V^2 Q^3 X^3 t^3}{9R^3} - \\
& \frac{V^2 Q^4 Q_o X^4 t^4}{R^5} + \frac{V^2 Q^5 X^4 t^5}{5R^5} + \frac{6V^2 Q^6 Q_o X^6 t^6}{7R^7} - \frac{6V^2 Q^7 X^6 t^7}{19R^7}
\end{aligned}
\tag{6.17}
$$

in synchronous watts. This is the final equation for torque where the computer program is based upon.

6.3 Current Calculation

Recall the current differential equation of Eqn. 2.24.

$$I' = V[\frac{Q'}{R} - \frac{3X^2Q^2Q'}{2R^3}] \tag{6.18}$$

subjected to the following boundary conditions

$$
\begin{aligned}
&\text{at } t = 0^- \qquad\qquad Q = Q_o \\
&\text{at } t = 0^+ \qquad\qquad Q = Qt
\end{aligned}
$$

From Sec. 6.2, it is derived that

$$\mathcal{L}[Q'] = \frac{Q}{s} - Q_o \tag{6.19}$$

$$\mathcal{L}[Q^2Q'] = \frac{2Q^3}{3s^3} - \frac{2Q^2Q_o}{3s^2} \tag{6.20}$$

By using the derivative property of Laplace Transform

$$\mathcal{L}[I'] = sI(s) - I_o \tag{6.21}$$

Substitute these in Eqn. 6.18

$$sI(s) - I_o \approx \frac{V}{R}[\frac{Q}{s} - Q_o] - \frac{3VX^2}{2R^3}[\frac{2Q^3}{3s^3} - \frac{2Q^2Q_o}{3s^2}] \tag{6.22}$$

$$I(s) \approx \frac{V}{R}[\frac{Q}{s^2} - \frac{Q_o}{s}] - \frac{3VX^2}{2R^3}[\frac{2Q^3}{3s^4} - \frac{2Q^2Q_o}{3s^3}] + \frac{I_o}{s} \tag{6.23}$$

Take the inverse Laplace Transform and simplify

$$I(t) \approx [I_o - \frac{VQ_o}{R}]u(t) + \frac{VQt}{R} + \frac{VX^2Q^2Q_ot^2}{2R^3} - \frac{VX^2Q^3t^3}{6R^3} \qquad (6.21)$$

in ampere per phase.

6.4 Case Studies

A computer program for solving the torque and for the current are shown in Appendix H and I, respectively. Their inputs are the same as in Chapters 3 and 4. Again, this thesis analyzed the same motor ratings as before. The torque is again converted to N-m using the conversion factor shown in Eqn. 2.28.

Figs. 6.1 to 6.3 show the torque responses for the three cases stated as in previous chapters, except that the final slip right now is a unit-ramp. Fig. 6-1, shows an induction motor with a final slip of $0.03t$ and an induction generator with final slip, $-0.03t$. They have an identical pattern but of opposite sign. This is because the induction generator is supplying electrical power. The torque increases with time, t. They reached their maximum torque at time approximately between 8.5 to 9 seconds. In the case of a low-drive induction motor, it reached its maximum at approximately 0.9 second. The motor should not be operated beyond that time. After $t = 1.6$ second, a rapid drop in the torque is shown, that goes infinitely. This characterized a breakdown of the machine.

Figs. 6.4 to 6.6 show the graphs of the current responses. They have the same pattern as the torque responses. While Figs. 6.7 to 6.9 show the torque to current ratio for the cases stated above. For a motor with a final slip, $0.03t$, the increase of the ratio is steady until 4.5 second and then decreases continuosly with a very small proportion. It means that the current is now increasing more rapidly than the torque. In the case of induction generator, with negative slip, a continuos decrease of the ratio is shown. While the low-drive induction motor experienced a decrease in the ratio of torque over time only after 0.8 sec. This is because of a high slip that is dependent with the time. Then it goes negatively after time, $t = 2.4$ sec. with a very rapid rate. This is due to the breakdown of the machine.

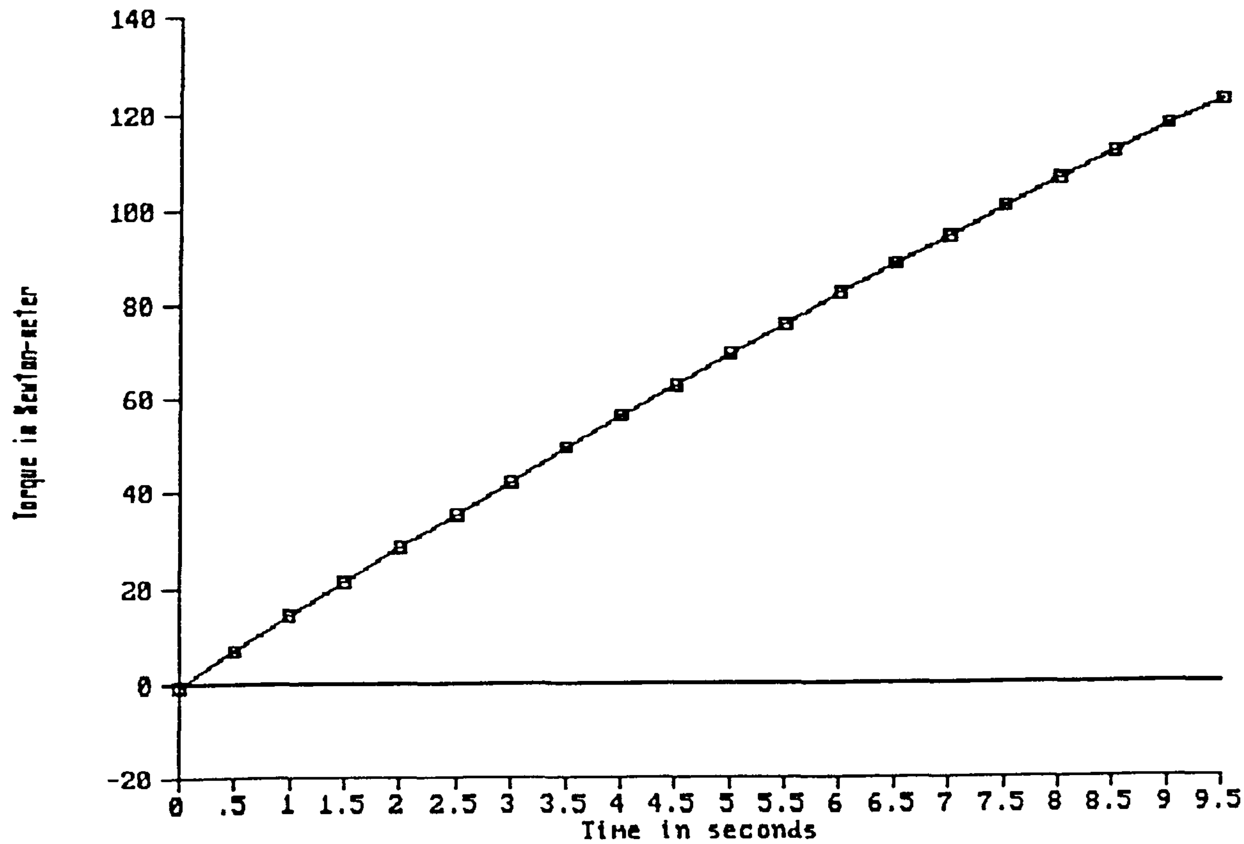

Figure 6.1 Torque Response If The Final Slip Is $0.03t$

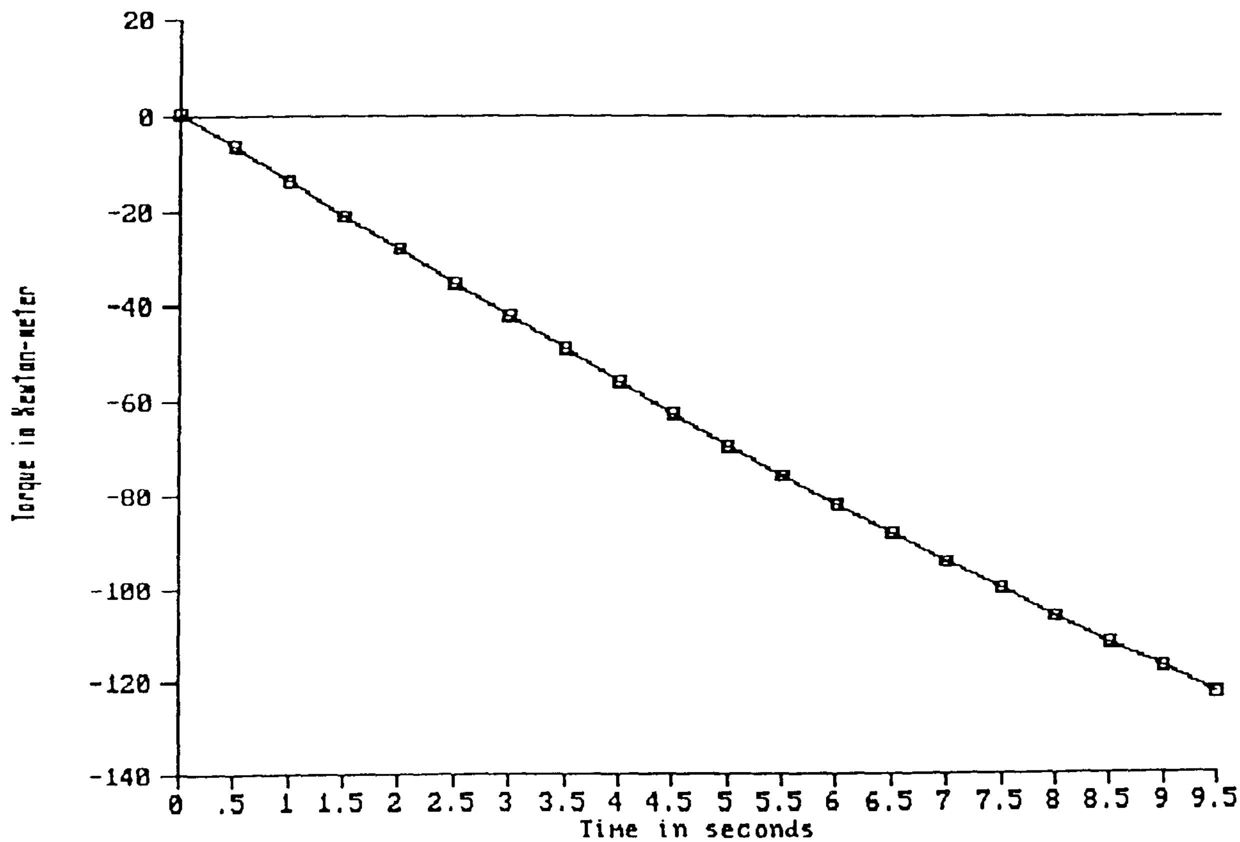

Figure 6.2 Torque Response If The Final Slip Is $-0.03t$

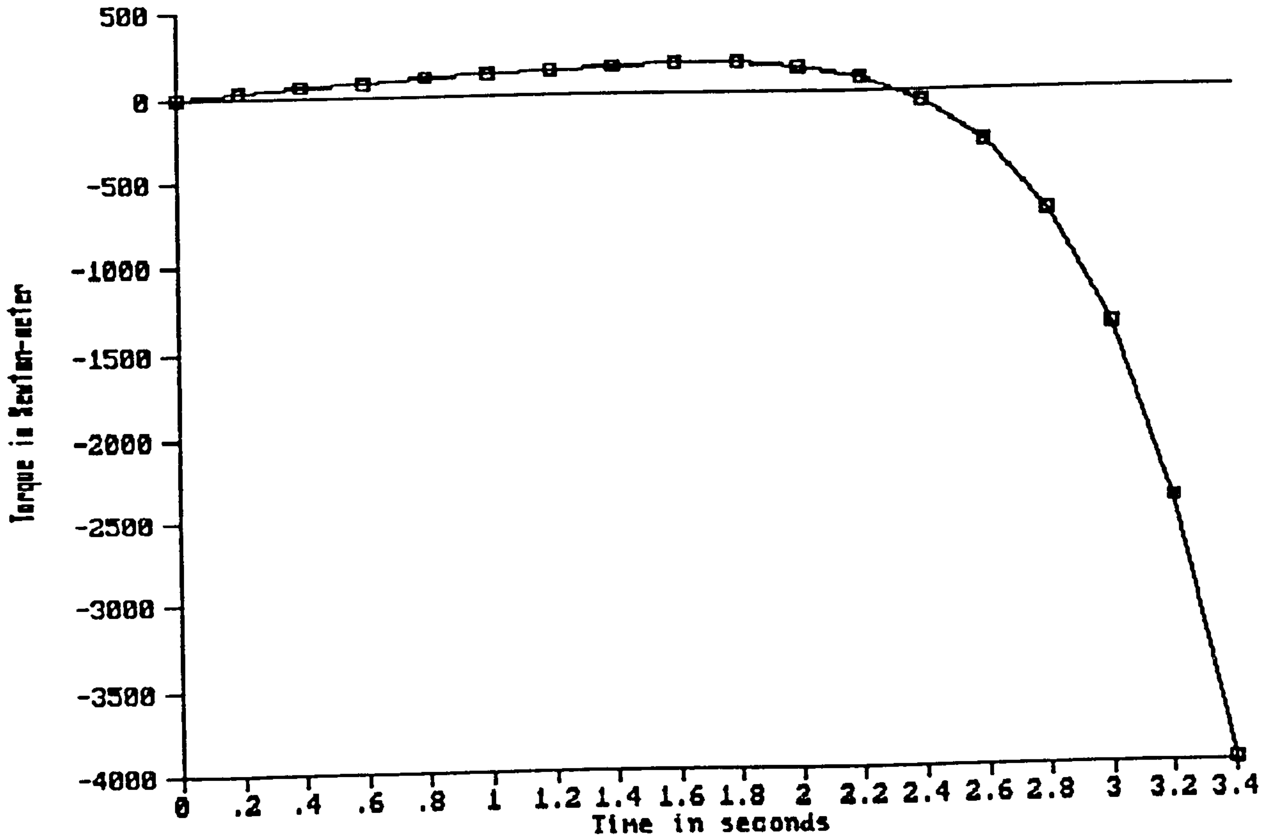

Figure 6.3 Torque Response if the Final Slip is 0.3t

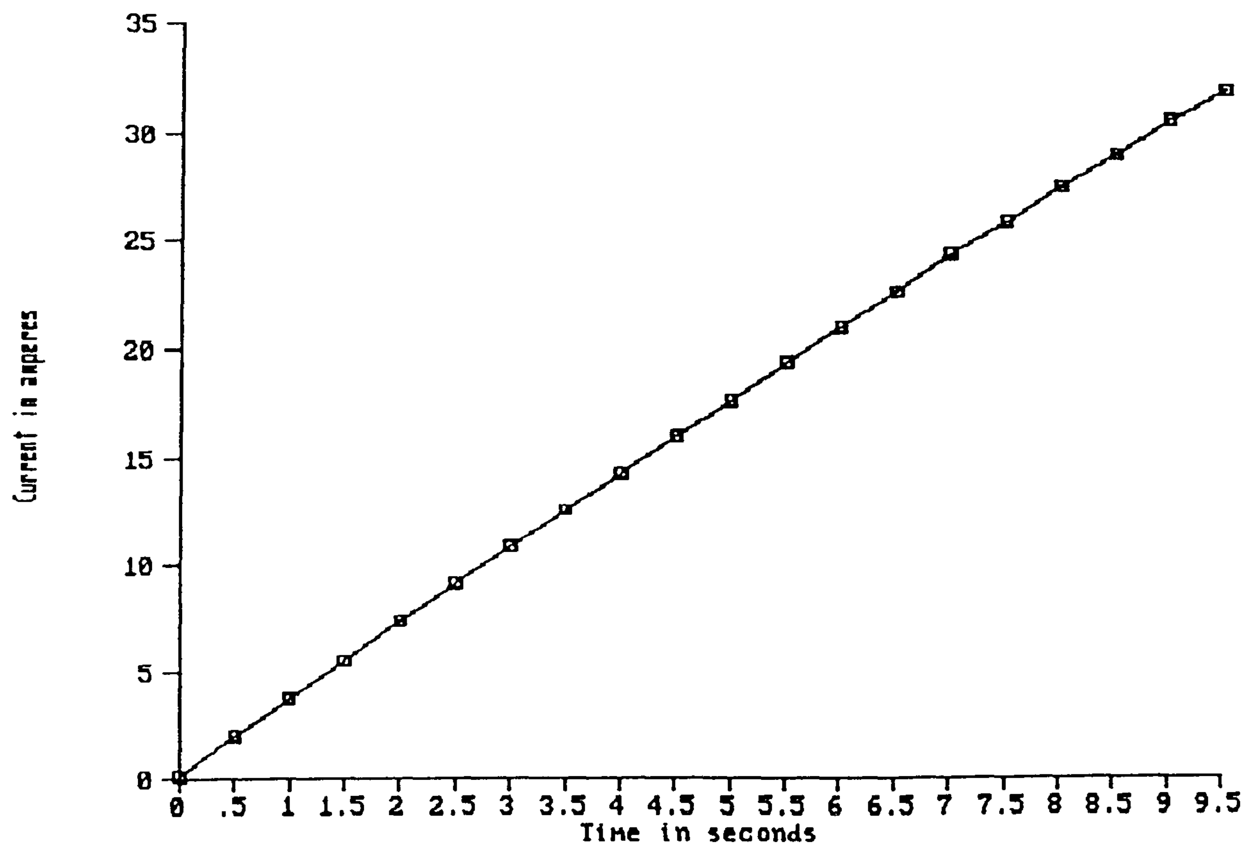

Figure 6.4 Current Response If The Final Slip Is $0.03t$

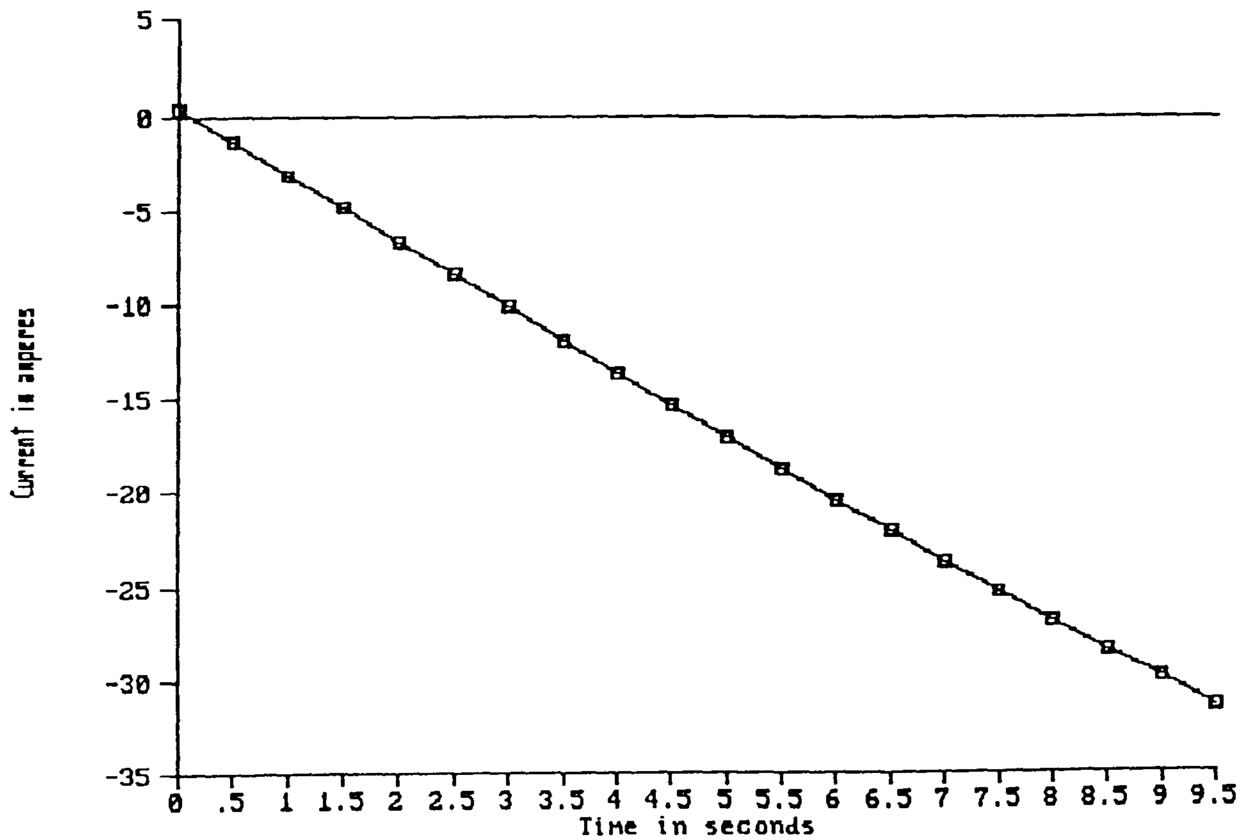

Figure 6.5 Current Response if The Final Slip Is $-0.03t$

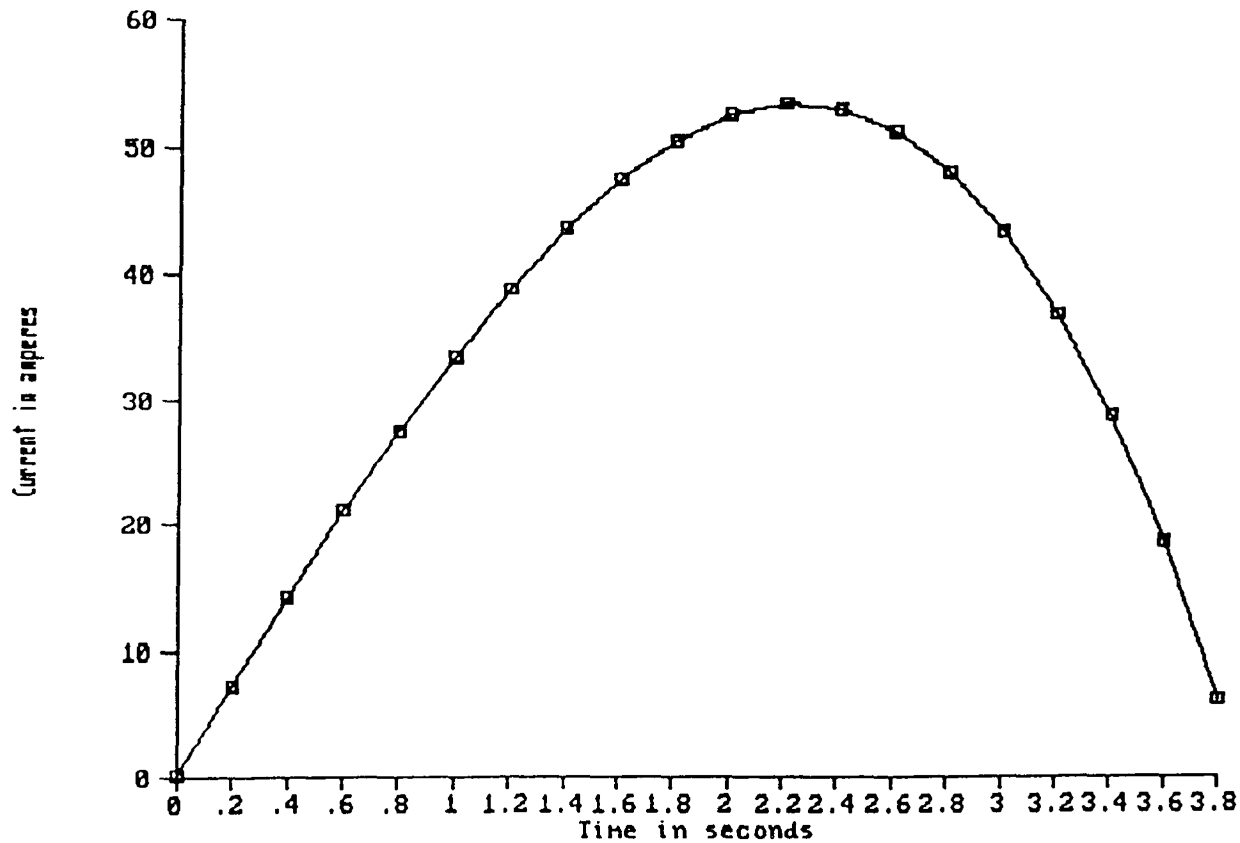

Figure 6.6 Current Response If The Final Slip Is $0.3t$

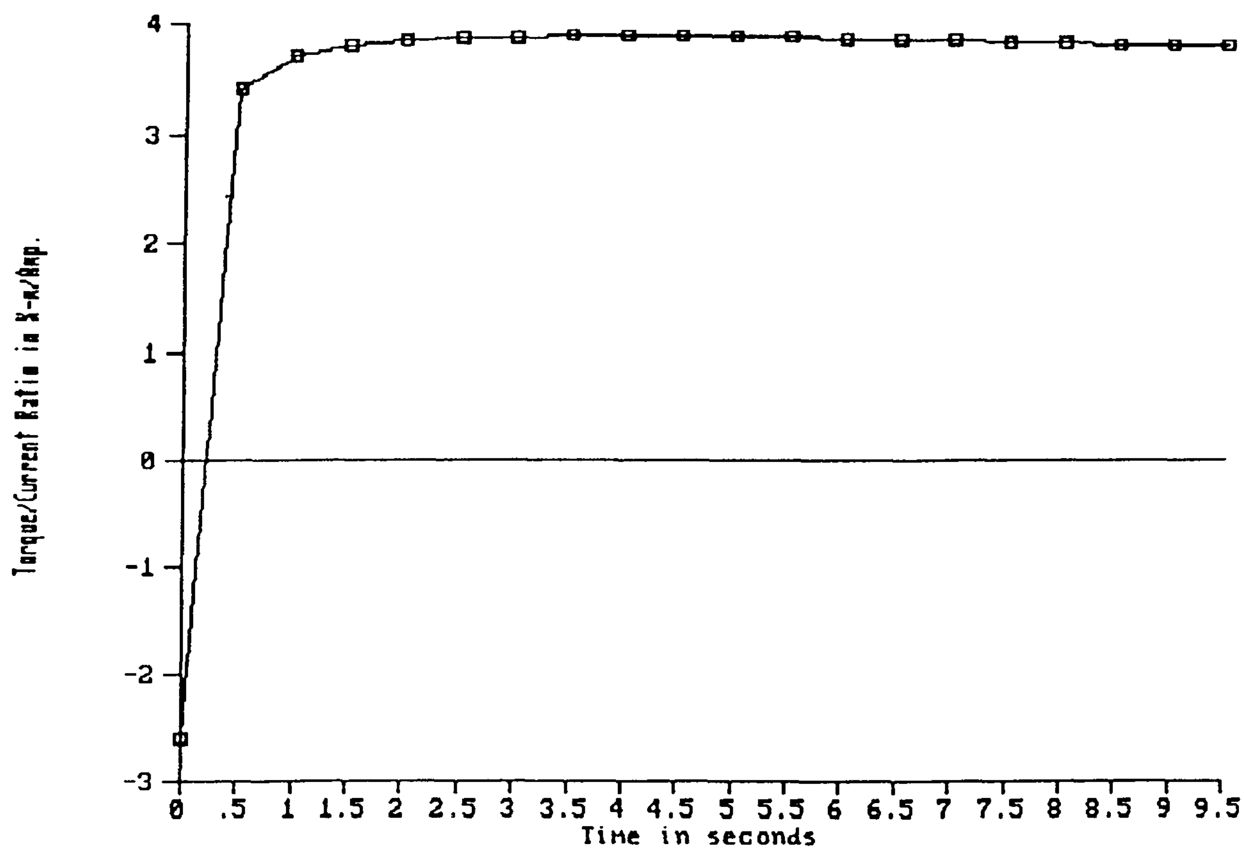

Figure 6.7 Torque/Current Ratio If The Final Slip. Is $0.03t$

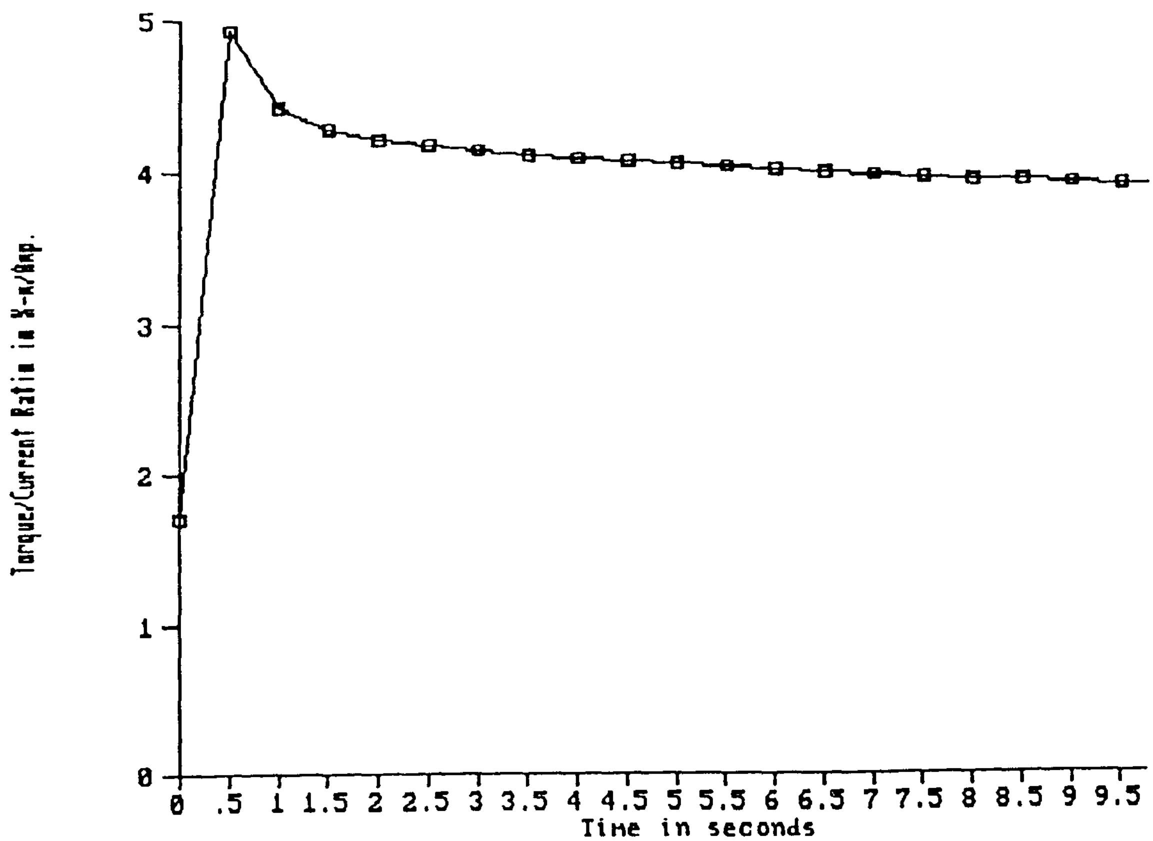

Figure 6.8 Torque/Current Ratio If The Final Slip Is $-0.03t$

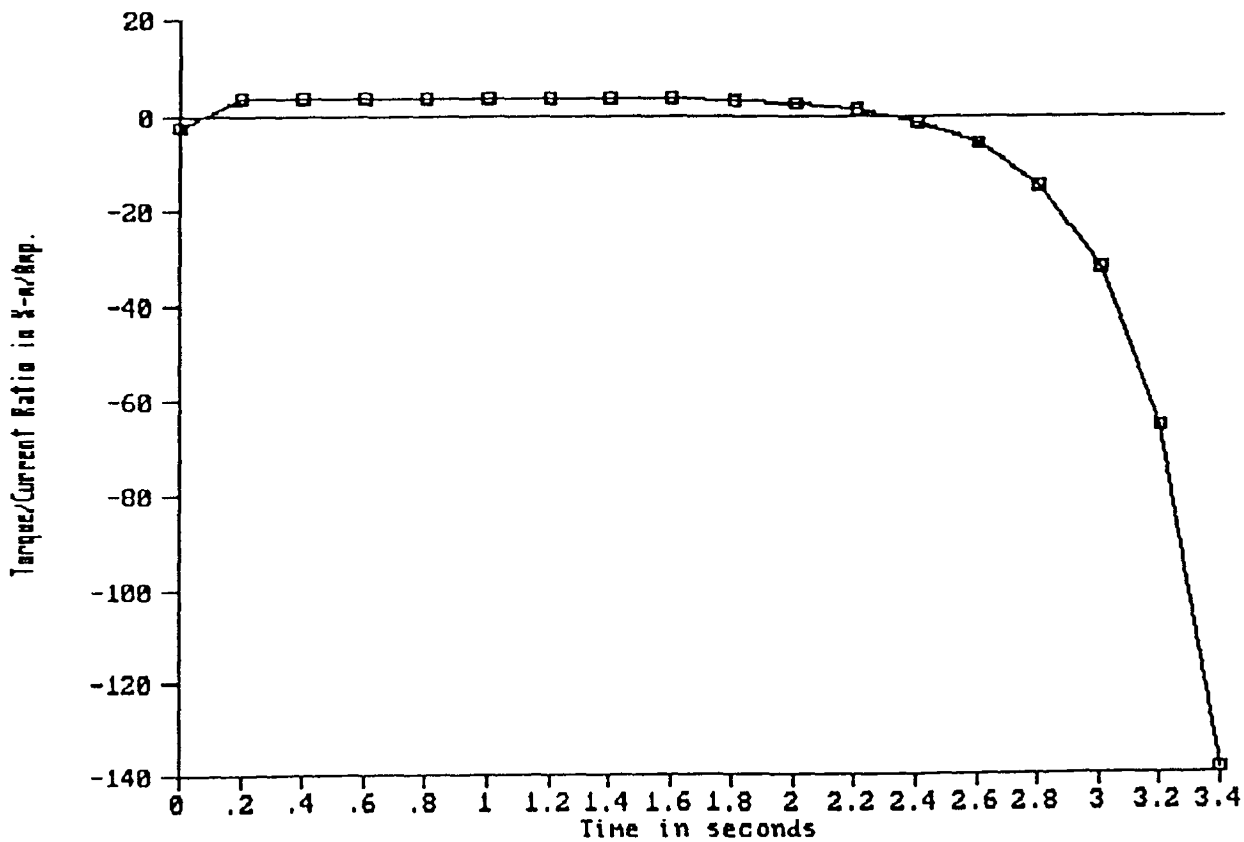

Figure 6.9 Torque/Current Ratio If The Final Slip Is $0.3t$

CHAPTER 7

SINUSOIDAL, sin t[20]

7.1 Introduction

This chapter deals with the current-time and torque-time responses of a three phase induction motor, when the slip at no-load is changed to a sinusoidal slip, $Qsint$. This pattern of machine slip is also a time dependent function. Like the previous chapters, computations are based in Chapter 2 of this thesis.

7.2 Torque Calculation

From Eqn. 2.17, the differential equation for solving the torque response of induction motor,

$$T' \approx V^2[\frac{Q'}{R} - \frac{3X^2}{R^3}Q'Q^2 + \frac{5X^4}{R^5}Q'Q^4 - \frac{6X^6}{R^7}Q'Q^6] \qquad (7.1)$$

subjected to the following boundary conditions,

$$\text{at } t = 0^- \qquad Q = Q_o$$

$$\text{at } t = 0^+ \qquad Q = Qsint$$

This differential equation is again solved by the Laplace Transformation method. By using the derivative property of the Laplace Transform,

$$\mathcal{L}[T'] = sT(s) - T_o \qquad (7.2)$$

The Laplace Transform of a sine function is given by

$$\mathcal{L}[sinat] \leftrightarrow \frac{a}{s^2 + a^2} \qquad (7.3)$$

To find $\mathcal{L}[sint]$ is easy using Eqn. 3.3, but a problem is encountered of finding the transform of a higher function of $sint$, which is not usually given in many Laplace

Transform Tables. To solve this, a well-known trigonometric series for sine function is used.

$$sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... \qquad (7.4)$$

To simplify the computation, it considers only up to the second term of Eqn. 3.4. Hence,

$$Q(t) = Qsint = Q(t - \frac{t^3}{3!}) \qquad (7.5)$$

$$\mathcal{L}[Qsint] = Q(s) = Q(\frac{1}{s^2} - \frac{1}{s^4}) \qquad (7.6)$$

Using the derivative property of Laplace Transform,

$$\mathcal{L}[Q'] = sQ(s) - Q_o \qquad (7.7)$$

$$\mathcal{L}[Q'] = \frac{Q(s^2 - 1)}{s^3} - Q_o \qquad (7.8)$$

$$Q^2(t) = (Qsint)^2 = Q^2(t^2 - \frac{t^4}{3} + \frac{t^6}{36}) \qquad (7.9)$$

$$\mathcal{L}[Q^2] = Q^2(\frac{2}{s^3} - \frac{8}{s^5} + \frac{20}{s^7}) \qquad (7.10)$$

To find $\mathcal{L}[Q'Q^2]$, Eqn. 3.9 is used, the transform of the product of two functions. It is assumed that $F_1(s) = \mathcal{L}[Q']$ and $F_2(s) = \mathcal{L}[Q^2]$.

$$\mathcal{L}[Q'Q^2] = [\frac{Q(s^2 - 1) - Q_o s^3}{3s^2}][\frac{2s^4 - 8s^2 + 20}{s^7}] \qquad (7.11)$$

$$\begin{aligned}\mathcal{L}[Q'Q^2] &= \frac{2Q^3}{3s^3} - \frac{10Q^3}{3s^5} + \frac{28Q^3}{3s^7} - \frac{20Q^3}{3s^9} - \frac{2Q^2Q_o}{3s^2} \\ &+ \frac{8Q^2Q_o}{3s^4} - \frac{20Q^2Q_o}{3s^6} \qquad (7.12)\end{aligned}$$

To find $\mathcal{L}[Q'Q^4]$,

$$Q^4(t) = t^4 - \frac{2t^6}{3} + \frac{t^8}{6} - \frac{t^{10}}{54} + \frac{t^{12}}{1,296} \qquad (7.13)$$

$$\mathcal{L}[Q^4] = \frac{24}{s^5} - \frac{480}{s^7} + \frac{6,720}{s^9} - \frac{67,200}{s^{11}} + \frac{369,600}{s^{13}} \qquad (7.14)$$

Again, Eqn. 3.9 is used to find $\mathcal{L}[Q'Q^4]$. It is assumed $F_1(1) = \mathcal{L}[Q']$ and $F_2(s) = \mathcal{L}[Q^4]$.

$$\mathcal{L}[Q'Q^4] = \frac{Q(s^2 - 1)Q_o s^3}{3s^2}[\frac{24s^8 - 480s^6 + 6,720s^4 - 67,200s^2 + 369,600}{s^{13}}] \qquad (7.15)$$

$$\begin{aligned}\mathcal{L}[Q'Q^4] &= \frac{Q^5}{s^5}+\frac{300Q^5}{s^9}-\frac{30.80Q^5}{s^{11}}+\frac{18.200Q^5}{s^{13}}-\frac{15.400Q^5}{s^{15}}-\frac{Q^4Q_o}{s^4}\\ &+\frac{20Q^4Q_o}{s^6}-\frac{280Q^4Q_o}{s^8}+\frac{2.800Q^4Q_o}{s^{10}}-\frac{15.400Q^4Q_o}{s^{12}}\end{aligned} \tag{7.16}$$

Similarly,

$$Q^6(t) = t^6 - t^8 + \frac{15t^{10}}{36} - \frac{5t^{12}}{54} + \frac{15t^{14}}{1.296} - \frac{3t^{16}}{1.944} + \frac{t^{18}}{46.656} \tag{7.17}$$

$$\begin{aligned}\mathcal{L}[Q^6] &= \frac{720}{s^7}-\frac{40.320}{s^9}+\frac{1.512.000}{s^{11}}-\frac{44.352.000}{s^{13}}+\frac{1.009.008.000}{s^{15}}\\ &-\frac{32.288.256.000}{s^{17}}+\frac{137.225.088.000}{s^{19}}\end{aligned} \tag{7.18}$$

It is also assumed that $F_1(s) = \mathcal{L}[Q']$ and $F_2(s) = \mathcal{L}[Q^2]$.

$$\begin{aligned}\mathcal{L}[Q'Q^6] &= \frac{Q(s^2-1)-Q_o s^3}{3s^2}\Big[\frac{s^{12}-56s^{10}+2.100s^8-61.600s^6+1.401.400s^4}{s^{19}}\\ &-\frac{44.844.800s^2+190.590.400}{s^{19}}\Big]\end{aligned} \tag{7.19}$$

$$\begin{aligned}\mathcal{L}[Q'Q^6] &= \frac{Q^7}{3s^7}-\frac{19Q^7}{s^9}+\frac{718.67Q^7}{s^{11}}-\frac{21.233.33Q^7}{s^{13}}+\frac{487.666.67Q^7}{s^{15}}-\\ &\frac{15.415.400Q^7}{s^{17}}+\frac{78.478.400Q^7}{s^{19}}-\frac{63.530.133.33Q^7}{s^{21}}-\frac{Q^6Q_o}{3s^6}+\\ &\frac{18.67Q^6Q_o}{s^8}-\frac{700Q^6Q_o}{s^{10}}+\frac{20.533.33Q^6Q_o}{s^{12}}-\frac{467.133.33Q^6Q_o}{s^{14}}\\ &+\frac{14.448.266.67Q^6Q_o}{s^{16}}-\frac{63.530.133.33Q_o}{s^{18}}\end{aligned} \tag{7.20}$$

Substitute these transforms to Eqn. 7-1 and simplify.

$$\begin{aligned}T(s) \approx\ & \frac{V^2}{R}\Big[\frac{Q}{s^2}-\frac{Q}{s^4}-\frac{Q_o}{s}\Big]-\frac{3V^2X^2}{R^3}\Big[\frac{2Q^3}{3s^4}-\frac{10Q^3}{3s^6}+\frac{28Q^3}{3s^8}-\frac{20Q^3}{3s^{10}}-\frac{2Q^2Q_o}{3s^3}\\ &+\frac{8Q^2Q_o}{3s^5}-\frac{20Q^2Q_o}{3s^7}\Big]+\frac{5V^2X^4}{R^5}\Big[\frac{Q^5}{s^6}-\frac{21Q^5}{s^8}+\frac{300Q^5}{s^{10}}-\frac{3.080Q^5}{s^{12}}+\\ &\frac{18.200Q^5}{s^{14}}-\frac{15.400Q^5}{s^{16}}-\frac{Q^4Q_o}{s^5}+\frac{20Q^4Q_o}{s^7}-\frac{280Q^4Q_o}{s^9}+\frac{2.800Q^4Q_o}{s^{11}}\\ &-\frac{15.400Q^4Q_o}{s^{13}}\Big]-\frac{6V^2X^6}{R^7}\Big[\frac{Q^7}{3s^8}-\frac{19Q^7}{s^{10}}+\frac{718.67Q^7}{s^{12}}-\frac{21.233.33Q^7}{s^{14}}\\ &+\frac{487.666.67Q^7}{s^{16}}-\frac{15.415.400Q^7}{s^{18}}+\frac{78.478.400Q^7}{s^{20}}-\frac{63.530.133.33Q^7}{s^{22}}\\ &-\frac{Q^6Q_o}{3s^7}+\frac{18.67Q^6Q_o}{s^9}-\frac{700Q^6Q_o}{s^{11}}+\frac{20.533.33Q^6Q_o}{s^{13}}-\frac{467.133.33Q^6Q_o}{s^{15}}\\ &+\frac{14.948.266.67Q^6Q_o}{s^{17}}-\frac{63.530.133.33Q_o}{s^{19}}\Big]+T_o\end{aligned} \tag{7.21}$$

Take the inverse Laplace Transform and simplify.

$$\begin{aligned} T \approx\ & [T_o - \frac{V^2Q_o}{R}]u(t) + V^2[\frac{Qt}{R} + \frac{0.33X^2Q^2Q_ot^2}{R^3} - \frac{Qt^3}{R}[0.17 + \frac{0.33X^2}{R^2}] \\ & - \frac{X^2Q^2Q_ot^4}{R^3}[0.11 + \frac{0.21X^2Q^2}{R^2}] + \frac{X^2Q^3t^5}{R^3}[0.03 + \frac{0.04X^2Q^2}{R^2}] + \\ & \frac{X^2Q^2Q_ot^6}{R^3}[9.3 \times 10^{-3} + \frac{0.14X^2Q^2}{R^2} + \frac{2.8 \times 10^{-3}X^4Q^4}{R^4}] - \frac{X^2Q^3t^7}{R^3} \\ & [1.9 \times 10^{-3} + \frac{0.02X^2Q^2}{R^2} + \frac{4 \times 10^{-4}X^4Q^4}{R^4}] - \frac{X^4Q^4Q_ot^8}{R^5}[0.03 + \\ & \frac{2.8 \times 10^{-3}X^2Q^2}{R^2} + \frac{X^2Q^3t^9}{R^3}[1.84 \times 10^{-5} + \frac{4.1 \times 10^{-3}X^2Q^2}{R^2} + \\ & \frac{3 \times 10^{-4}X^4}{R^4}] + \frac{X^4Q^4Q_ot^{10}}{R^5}[3.9 \times 10^{-3} + \frac{1.2 \times 10^{-3}X^2Q^2}{R^2}] - \frac{X^4Q^5t^{11}}{R^5} \\ & [4 \times 10^{-4} + \frac{1 \times 10^{-4}X^2Q^2}{R^2}] - \frac{X^4Q^4Q_ot^{12}}{R^5}[2 \times 10^{-4} + \frac{3 \times 10^{-4}X^2Q^2}{R^2}] \\ & + \frac{X^4Q^5t^{13}}{R^5}[1.46 \times 10^{-5} + \frac{2.04 \times 10^{-5}X^2Q^2}{R^2}] + \frac{3.22 \times 10^{-5}X^6Q^6Q_ot^{14}}{R^7} \\ & - \frac{X^4Q^5t^{15}}{R^5}[5.89 \times 10^{-8} + \frac{2.24 \times 10^{-6}X^2Q^2}{R^2}] - \frac{4.29 \times 10^{-6}X^6Q^6Q_ot^{16}}{R^7} \\ & + \frac{2.6 \times 10^{-7}X^6Q^7t^{17}}{R^7} + \frac{5.95 \times 10^{-8}X^6Q^6Q_ot^{18}}{R^7} - \frac{3.87 \times 10^{-9}X^6Q^7t^{19}}{R^7} \\ & + \frac{7.46 \times 10^{-12}X^6Q^7t^{21}}{R^7}] \end{aligned} \tag{7.22}$$

in synchronous watts per phase.

7.3 Current Calculation

Recall the current differential equation of Eqn. 2.24

$$I' \approx V[\frac{Q'}{R} - \frac{3X^2Q^2Q'}{2R^3}] \tag{7.23}$$

subjected to the following boundary conditions

at $t = 0^-$	$Q = Q_o$
at $t = 0^+$	$Q = Qsint$

Solve the current differential equation by substituting the transforms of Eqns. 7.8 and 7.11.

$$sI(s) - I_o \approx \frac{V}{R}([\frac{Qs^2 - Q - Q_os^3}{s^3}] - \frac{3X^2}{2R^2}[\frac{2Q^3}{3s^4} - \frac{10Q^3}{3s^6} + \frac{28Q^3}{3s^8}$$

$$-\frac{20Q^3}{3s^{10}} - \frac{2Q^2Q_o}{3s^3} + \frac{8Q^2Q_o}{3s^5} - \frac{20Q^2Q_o}{3s^7}]) \qquad (7.24)$$

Take the inverse Laplace Transform and simplify.

$$\begin{aligned} I \approx & [I_o - \frac{VQ_o}{R}]u(t) + V(\frac{Qt}{R} + \frac{0.195X^2Q^2Q_ot^2}{R^3} - \frac{Qt^3}{R}[0.167 + \frac{0.495X^2Q^2}{R^2}] \\ & - \frac{0.165X^2Q^2Q_ot^4}{R^3} + \frac{0.042X^2Q^3t^5}{R^3} + \frac{0.014X^2Q^2Q_ot^6}{R^3} - \frac{0.029X^2Q^3t^7}{R^3} \\ & + \frac{2.76 \times 10^{-5}X^2Q^3t^9}{R^3} \end{aligned} \qquad (7.25)$$

in ampere per phase.

7.4 Case Studies

A computer program for torque calculation and for current calculation based on the final equation. Eqns. 7.22 and 7.25 respectively, are shown in Appendix J and K.The calculated torque is converted once again to N-m.The inputs needed are the same as in the previous chapters.

Figs. 7.1 to 7.3 show the torque response for the three cases stated before. The first two cases has almost the same pattern but of opposite signs. They reached their maximum torque at approximately 3.9 sec. In the case of a low- drive induction motor, with final slip, $0.3sint$, it reached its maximum torque at approximately between 1.6 to 1.8 sec. This kind of slip is time-dependent function.

Figs. 7.4 to 7.6 show the current responses for these three cases. The shape of the graph is almost identical to the torque responses. The graphs for the torque to current ratio are shown in Figs. 7.7 to 7.9. The ratio for the third case, the low-drive induction motor, has the lowest. It is because the current increases more rapidly than the torque at the same level of machine slip.

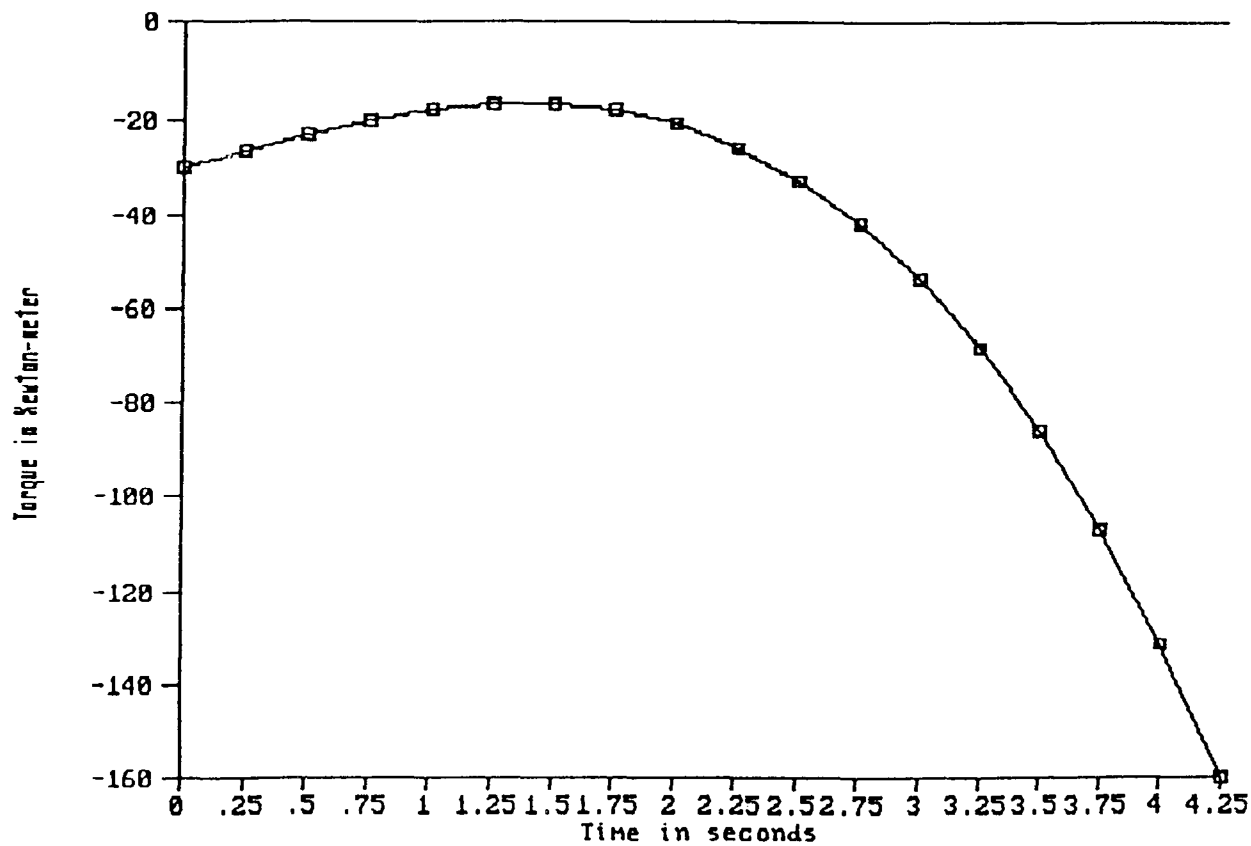

Figure 7.1 Torque Response If The Final Slip Is $0.03 sint$

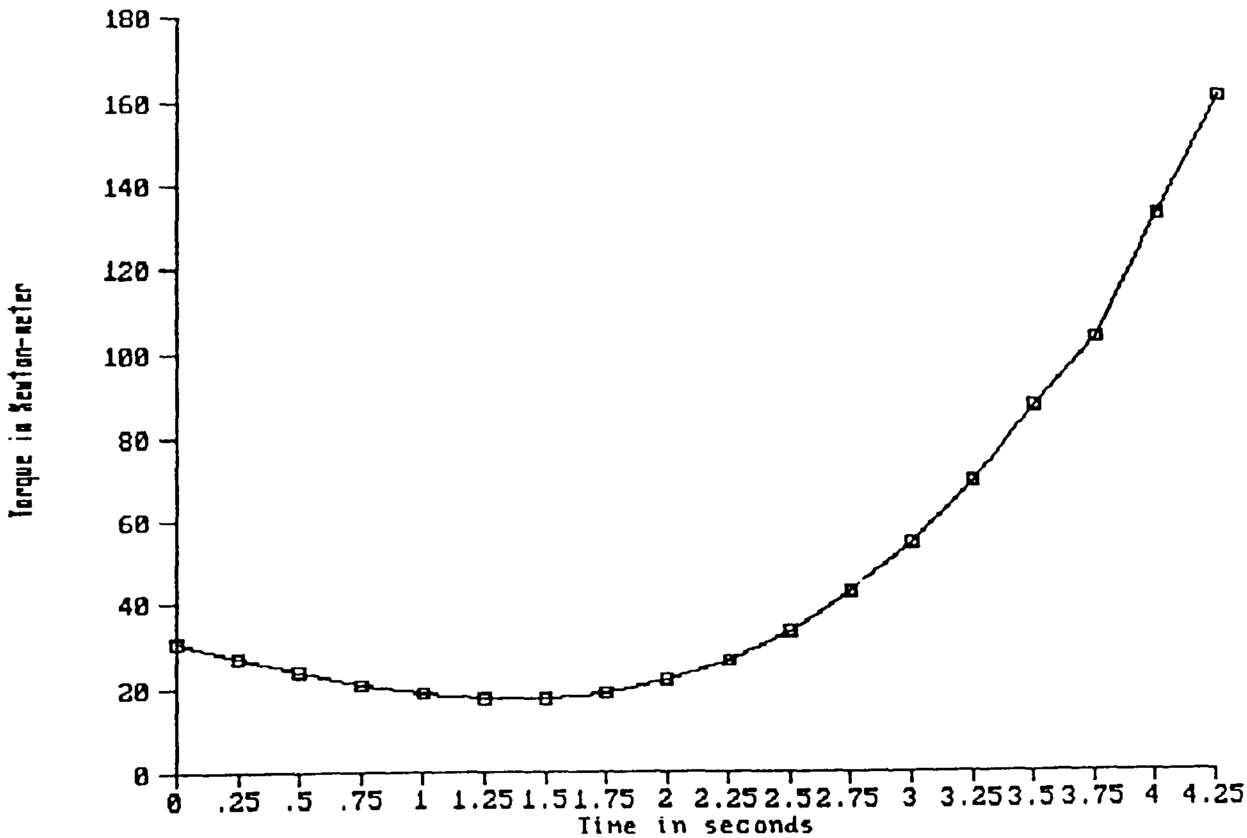

Figure 7.2 Torque Response If The Final Slip Is $-0.03sint$

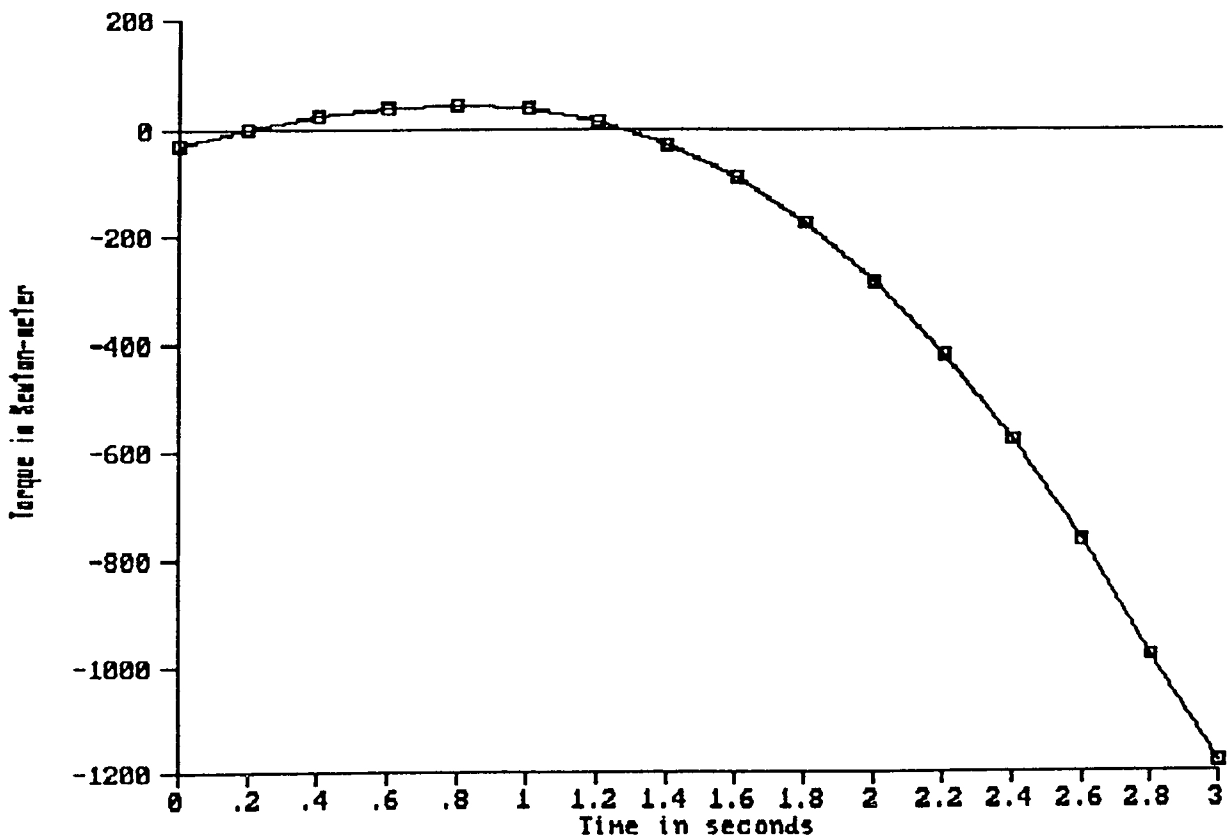

Figure 7.3 Torque Response If The Final Slip Is $0.3 sin t$

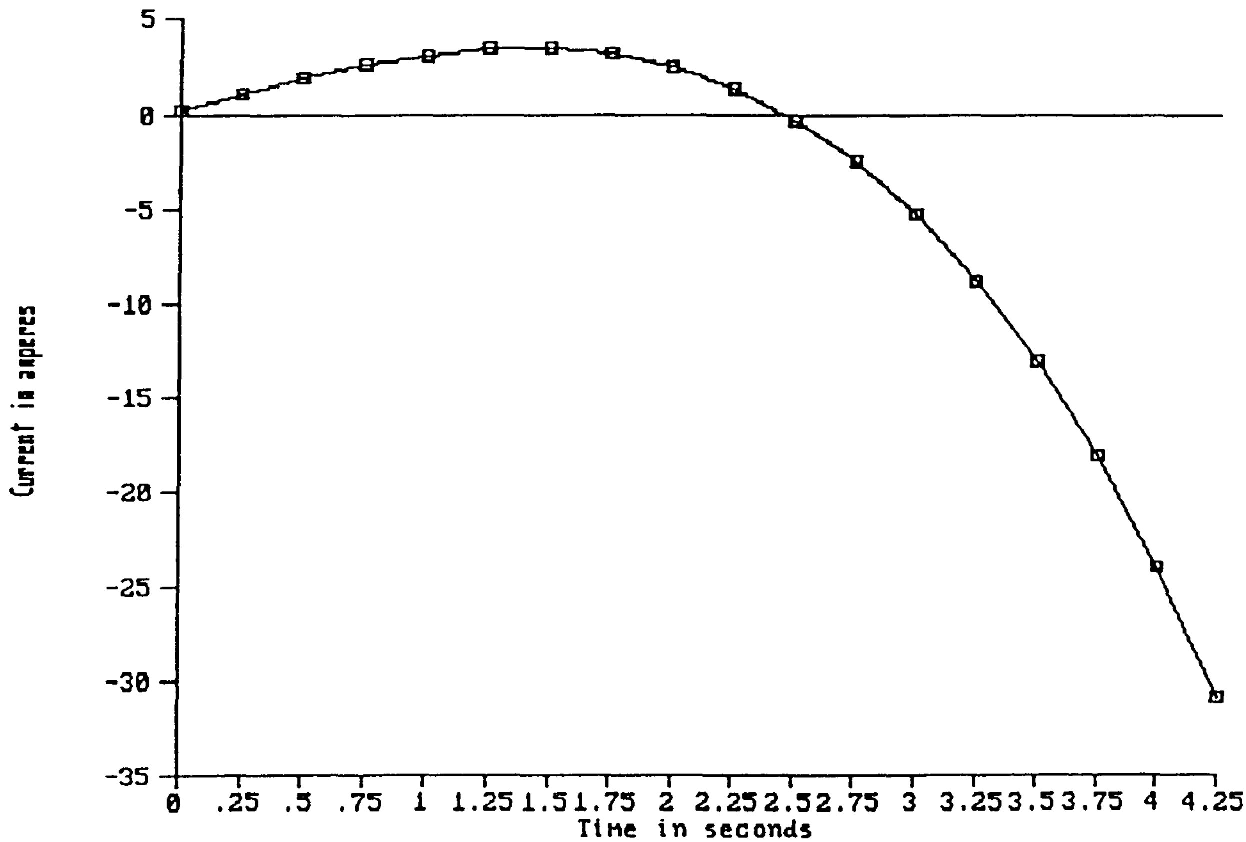

Figure 7.4 Current Response If The Final Slip Is $0.03 sint$

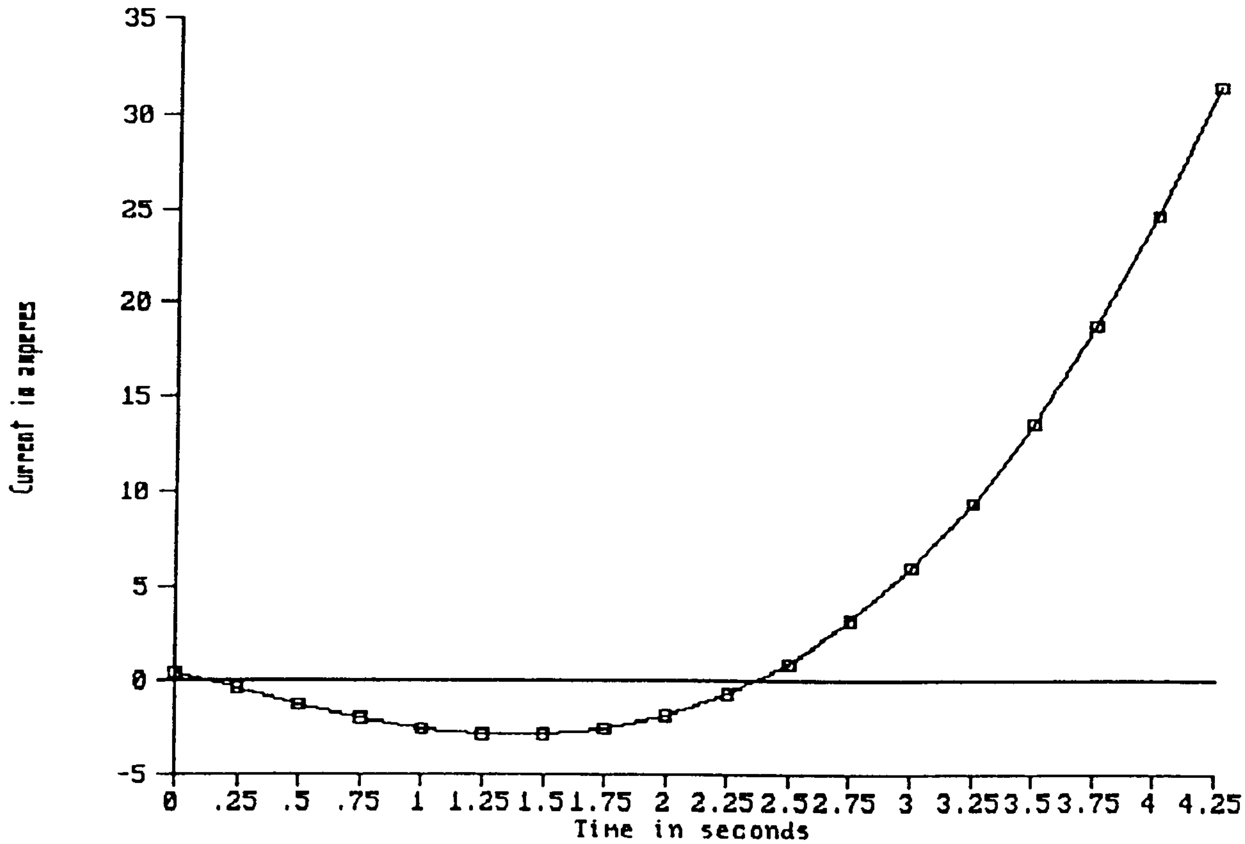

Figure 7.5 Current Response If The Final Slip Is $-0.03sint$

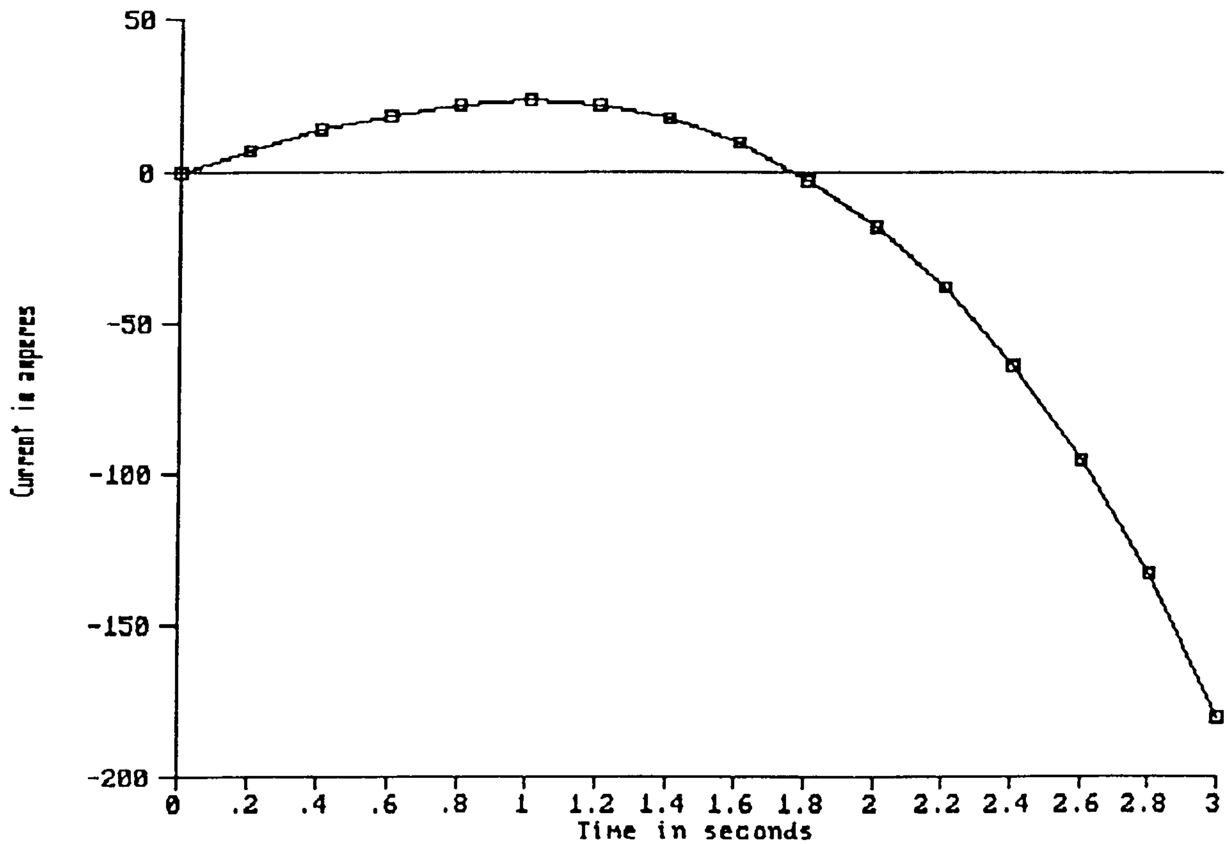

Figure 7.6 Current Response If The Final Slip Is $0.3 sint$

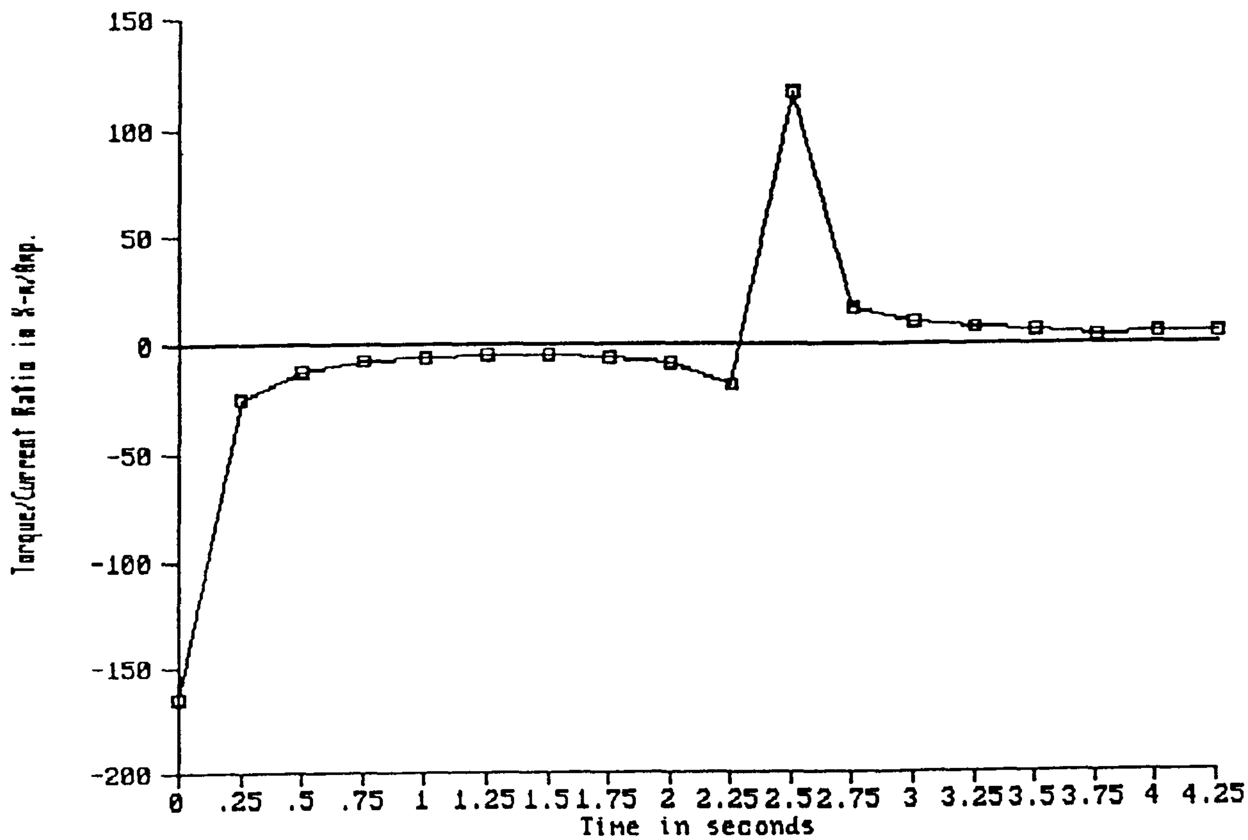

Figure 7.7 Torque/Current Ratio If The Final Slip Is $0.03 sin t$

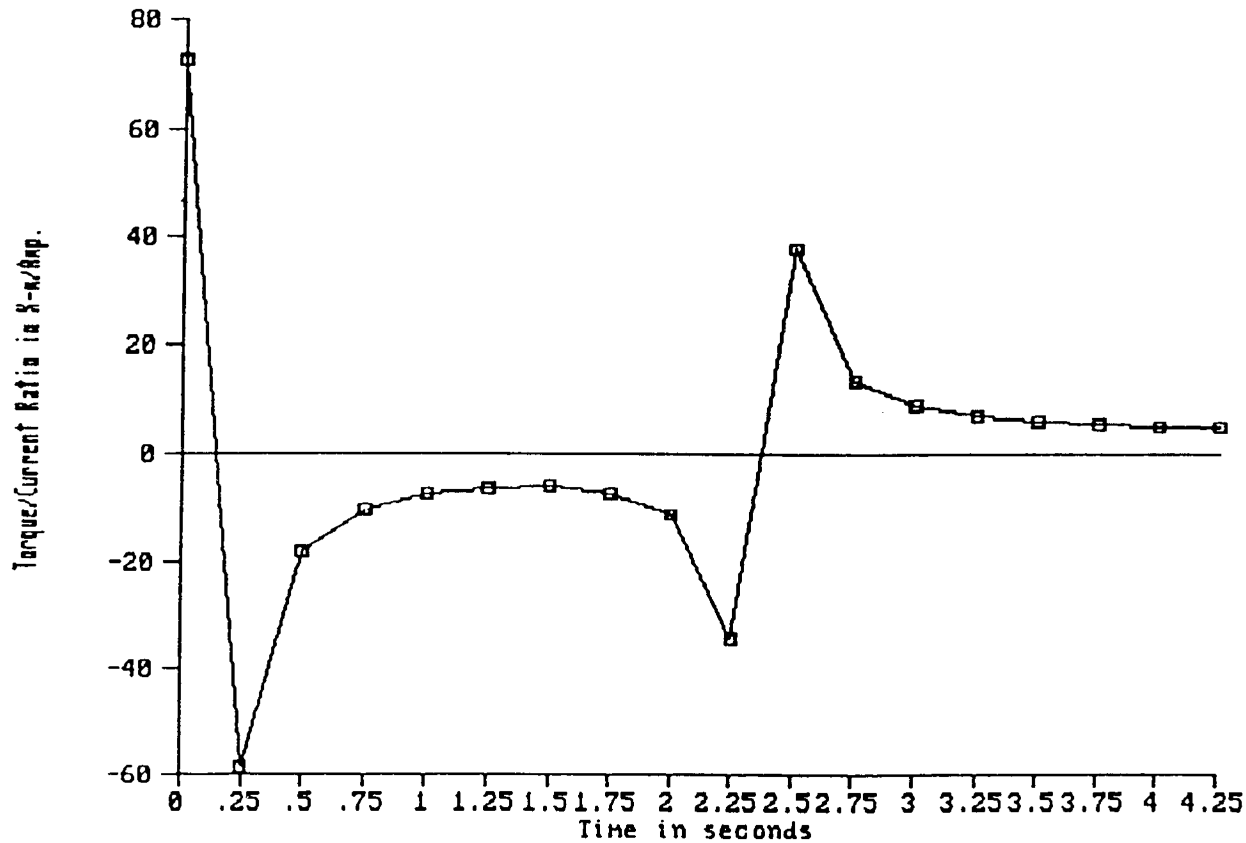

Figure 7.8 Torque/Current Ratio If The Final Slip Is $-0.03sint$

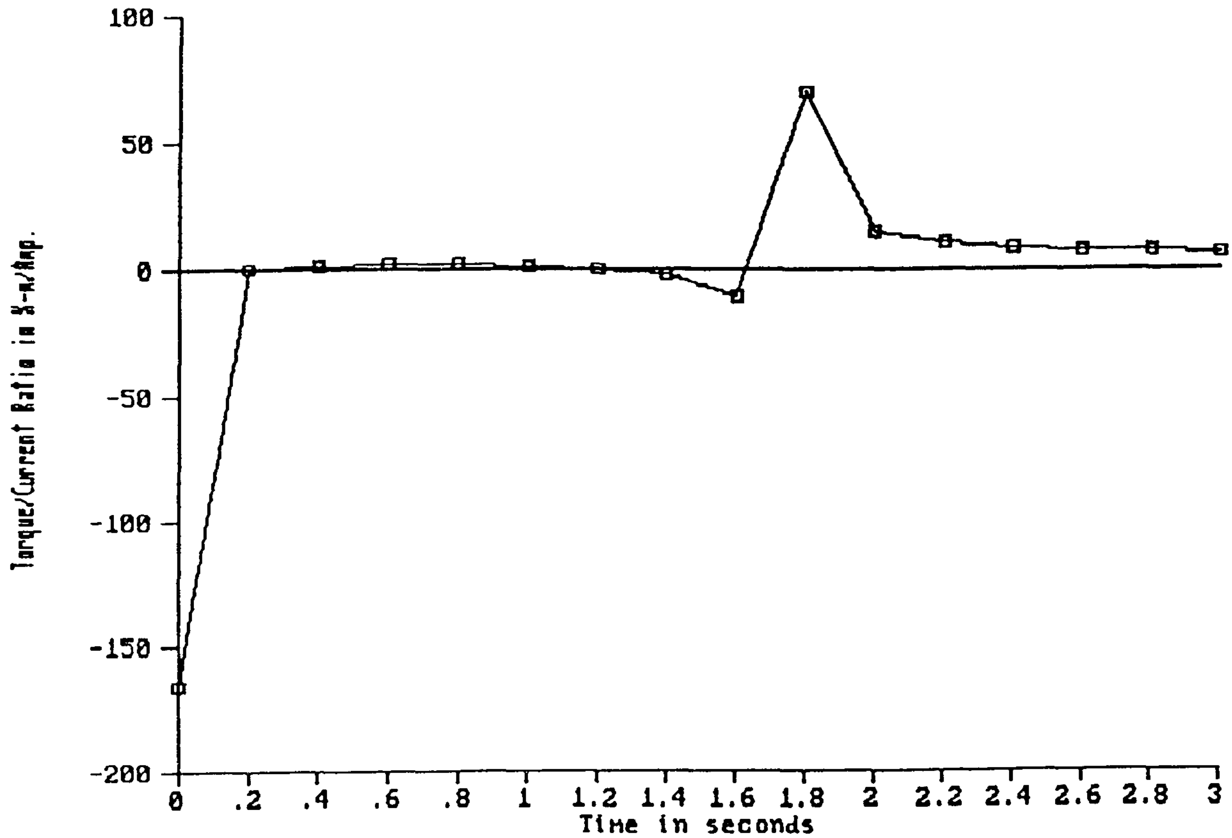

Figure 7.9 Torque/Current Ratio If The Final Slip Is $0.3sint$

Appendix A

Laplace Transform Pairs[17,18,19]

$F(s)$	$f(t)$
s^n	$\delta^n(t)$
s	$\delta'(t)$
1	$\delta(t)$
$\frac{1}{s}$	$u(t)$
$\frac{1}{s^2}$	t
$\frac{n!}{s^{n+1}}$	t^n
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$
$\frac{1}{s+a}$	e^{-at}
e^{-Ts}	$u(t-T)$
$\frac{e^{-2Ts}}{s}$	$u(t-2T)$
$\frac{a}{s^2+a^2}$	$sin(at)$
$\frac{s}{s^2+a^2}$	$cos(at)$
$\frac{a}{s^2-a^2}$	$sinh(at)$
$\frac{s}{s^2-a^2}$	$cosh(at)$
$\frac{1}{(s-a)^2}$	te^{at}
$\frac{n!}{(s-a)^{n+1}}$	t^ne^{at}
$\frac{a^2}{s(s^2+a^2)}$	$1-cos(at)$
$\frac{1}{(s-a)^2}$	te^{at}

$\frac{a^3}{s^2(s^2+a^2)^2}$	$at - sin(at)$
$s\mathcal{L}(f) - f(0)$	f'
$s^n\mathcal{L}(f) - s^{n-1}f(0)$ $-s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$	$f^{(n)}$
$(-1)^n F^{(n)}(s)$	$t^n f(t)$
$\sum_{k=1}^{q} \frac{A_1(s_k)}{B_1'(s_k)} F_2(s - s_k)$	$f_1(t)f_2(t)$

provided that $F_1(s)$ is a rational function. $F_1(s)$ is equal to A_1/B_1 and has only q first-order poles.

s_k are the poles of $F_1(s)$.

Appendix B

Appendix B

```
      This is a program to calculate the torque response of an
      induction motor when a small value of slip at no load
      is changed to a higher value of unit step type, Qu(t).

      Variable Declaration
      TOR- current in (Ampere)
      VS- per phase voltage in (volts)
      Q- slip of the motor
      QO- initial slip of the motor
      X- rotor inductive reactance of the motor in (Ohms)
      R- rotor resistance of the motor in (Ohms)
      TO- initial value of current in (Ampere)
      F- the frequency of the system in (Hz.)
      P- the number of poles of the motor
      PH- the number of phases

**********************************************************************
      DIMENSION ISCALE(7)
      PARAMETER (N=20)
      REAL TOR,VS,Q,QO,X,R,TO,F,P,SRPM,PH,T(N)
      INTEGER I,GRAPHS(70),BL,STAR,DOT
      DATA GRAPHS/70*" "/,BL/" "/,STAR/"*"/
      DATA DOT/"."/
      READ *,VS,F,P,PH,R,X,Q,QO,TO
      OPEN(UNIT=6,FILE="marl.dat",STATUS="NEW")
      WRITE(6,1)VS
 1    FORMAT(10X,"The per phase voltage is",2X,F5.1,1X,
     + "volts.")
      WRITE(6,2)F
 2    FORMAT(10X,"The frequency of the system is",1X,F4.1,1X,
     + "hertz.")
      WRITE(6,3)P
 3    FORMAT(10X,"The number of poles is",1X,F4.1,".")
      WRITE(6,4)PH
 4    FORMAT(10X,"The number of phases is",1X,F4.1,".")
      WRITE(6,5)R
 5    FORMAT(10X,"The rotor resistance is",F6.2,2X,"ohms.")
      WRITE(6,6)X
 6    FORMAT(10X,"The total inductive reactance is",F6.2,1X,"ohms.")
      WRITE(6,7)Q
 7    FORMAT(10X,"The final slip is",1X,F6.3,"u(t).")
      WRITE(6,8)QO
 8    FORMAT(10X,"The initial slip is",1X,F6.3,".")
      WRITE(6,9)TO
 9    FORMAT(10X,"The initial torque is",F5.2,1X,"N-m.")
      DO 10 I=1,7
      K=1000*I-4000
      ISCALE(I)=K
10    CONTINUE
      WRITE(6,12)ISCALE
12    FORMAT(30X,"Torque in N-m",//10X,7I10)
      WRITE(6,21)
      WRITE(6,22)
      DO 13 I=1,N
      READ*,T(I)
     Compute the torque.
      SRPM=120*F/P
      TOR1=VS**2*(Q-QO)*(1/R-3*X**2*Q**2/R**3+5*X**4*Q**4/R**5
     + -6*X**6*Q**6/R**7)+TO
      TOR=7.04/SRPM*PH*1.356*TOR1
      TORQ=TOR/100+34
```

```
      WRITE(6,20)DOT
13    CONTINUE
      STOP
15    FORMAT(1X,F3.1,1X,E9.2,2X,70A1,T4,A1)
20    FORMAT(T50,A1)
21    FORMAT("Time",3X,"Torque")
22    FORMAT("(Sec.)",1X,"(N-m)")
      END
```

4

Appendix C

```
*    This is a program to calculate the current response of an
*    induction motor when a small value of slip at no load
*    is changed to a higher value of unit step type, Qu(t).
*
*    Variable Declaration
*    AMP- current in (Ampere)
*    VR- per phase voltage in (volts)
*    Q- slip of the motor
*    QO- initial slip of the motor
*    X- rotor inductive reactance of the motor in (Ohms)
*    R- rotor resistance of the motor in (Ohms)
*    IO- initial value of current in (Ampere)
*
***********************************************************************
      DIMENSION ISCALE(7)
      PARAMETER (N=20)
      REAL AMP,VR,Q,QO,X,R,IO,T(N)
      INTEGER I,GRAPHS(70),BL,STAR,DOT
      DATA GRAPHS/70*" "/,BL/" "/,STAR/"*"/
      DATA DOT/"."/
      READ *,VR,R,X,Q,QO,IO
      OPEN(UNIT=6,FILE="mar1.dat",STATUS="NEW")
      WRITE(6,1)VR
 1    FORMAT(10X,"The per phase voltage is",2X,F5.1,2X,
     + "volts.")
      WRITE(6,5)R
 5    FORMAT(10X,"The rotor resistance is",F6.2,2X,"ohms.")
      WRITE(6,6)X
 6    FORMAT(10X,"The rotor inductive reactance is",F6.2,"ohms.")
      WRITE(6,7)Q
 7    FORMAT(10X,"The final slip is",1X,F6.3,"u(t).")
      WRITE(6,8)QO
 8    FORMAT(10X,"The initial slip is",1X,F6.3,".")
      WRITE(6,9)IO
 9    FORMAT(10X,"The initial current is",F5.2,1X,"Amp.")
      DO 10 I=1,7
      K=100*I-400
      ISCALE(I)=K
10    CONTINUE
      WRITE(6,11)ISCALE
11    FORMAT(30X,"Current in Ampere",//10X,7I10)
      WRITE(6,12)
      WRITE(6,13)
12    FORMAT("Time",3X,"Current")
13    FORMAT("(Sec.)",2X,"(Amp.)")
      DO 14 I=1,N
      READ*,T(I)
*     Compute the current.
      AMP1=VR*((Q-QO)/R-3*X**2*Q**2*(Q-QO)/(2*R**3))
      AMP=(AMP1+IO)
      AMPS=AMP/10+36
      M=AMPS
      GRAPHS(M)=STAR
      WRITE(6,15)T(I),AMP,GRAPHS
      GRAPHS(M)=BL
      WRITE(6,20)DOT
14    CONTINUE
      STOP
15    FORMAT(F3.1,2X,F7.2,2X,70A1,T4,A1)
20    FORMAT(T50,A1)
      END
```

Appendix D

```
*   This is a program to calculate and graph the torque response
*   of an induction motor when a small value of slip at no load
*   is changed to a higher value of delta function.
*
*   Variable Declaration
*   VS-per phase voltage in (volts)
*   Q- final slip of the motor
*   QO- initial slip
*   X- total inductive reactance of the motor in (ohms)
*   R- rotor resistance of the motor in (ohms)
*   TO- initial torque in (N-m)
*   SRPM- synchronous speed in (RPM)
*   F- frequency of the system in(Hz)
*   P- the number of poles of the motor
*   PH- the number of phases
*
*****************************************************************
      DIMENSION ISCALE(7)
      PARAMETER (N=20)
      REAL VS,Q,QO,X,R,TO,PH,T(N),SRPM,M,F,P
      INTEGER I,GRAPHS(70),BL,STAR,DOT
      DATA GRAPHS/70*" "/,BL/" "/,STAR/"*"/
      DATA DOT/"."/
      READ *,VS,F,P,PH,R,X,Q,QO,TO
      OPEN(UNIT=6,FILE="marl.dat",STATUS="OLD")
      WRITE(6,1)VS
  1   FORMAT(10X,"The per phase voltage is",2X,F5.1,2X,
     + "volts.")
      WRITE(6,2)F
  2   FORMAT(10X,"The frequency of the system is",1X,F4.1,1X,
     + "hertz.")
      WRITE(6,3)P
  3   FORMAT(10X,"The number of poles is",1X,F4.1,".")
      WRITE(6,4)PH
  4   FORMAT(10X,"The number of phase is",1X,F4.1,".")
      WRITE(6,5)R
  5   FORMAT(10X,"The rotor resistance is ",F6.2,2X,"ohms.")
      WRITE(6,6)X
  6   FORMAT(10X,"The total inductive reactance is",F6.2,1X,"ohms.")
      WRITE(6,7)Q
  7   FORMAT(10X,"The final slip is",1X,F6.3,".")
      WRITE(6,8)QO
  8   FORMAT(10X,"The initial slip is",1X,F6.3,".")
      WRITE(6,9)TO
  9   FORMAT(10X,"The initial torque is",F5.2,1X,"N-m.")
      SRPM=120*F/P
*    Compute the torque.
      TOR1=7.04/SRPM*PH*1.356*(TO-VS**2*QO/R)
      TOR2=7.04/SRPM*PH*1.356*VS**2*Q/R*(1+3*X**2*Q*QO/R**2-5*X**4
     + *Q**3*QO/R**4+6*X**6*QO*Q**5/R**6)
      TOR3=7.04/SRPM*PH*1.356*VS**2*X**2*Q**3/R**3*(-3+5*X**2*Q**2
     + /R**2-6*X**4*Q**4/R**4)
*   Write the output.
      WRITE(6,10)TOR1
 10   FORMAT(10X,"The unit step response (u(t)) is",F7.2,1X,"N-m.")
      WRITE(6,11)TOR2
 11   FORMAT(10X,"The delta response is",F7.2,1X,"N-m.")
      WRITE(6,12)TOR3
 12   FORMAT(10X,"The doublet response is",1X,E9.2,1X,"N-m.")
      DO 15 I=1,7
      K=1000*I-4000
      ISCALE(I)=K
```

```
      WRITE(6,17)ISCALE
 17   FORMAT(30X,"Torque in N-m",//7I
      WRITE(6,18)
      WRITE(6,19)
 18   FORMAT("Time")
 19   FORMAT(1X,"(Sec.)")
      DO 20 I=1,N
      READ*,T(I)
      TORQ=TOR1/100+27
      M=TORQ
      GRAPHS(M)=STAR
      WRITE(6,22)T(I),GRAPHS
      GRAPHS(M)=BL
      WRITE(6,23)DOT
 20   CONTINUE
      STOP
 22   FORMAT(2X,F5.2,7X,70A1,T4,A1)
 23   FORMAT(T40,A1)
      END
```

Appendix E

```fortran
*   This is a program to calculate and graph the current response
*   of an induction motor when a small value of slip at no load
*   is changed to a higher value of delta function.
*
*   Variable Declaration
*   VR-per phase voltage in (volts)
*   AMP- current in (Ampere)
*   Q- final slip of the motor
*   QO- initial slip
*   X- rotor inductive reactance of the motor in (ohms)
*   R- rotor resistance of the motor in (ohms)
*   IO- initial current in (Amp.)
*   PH- the number of phases
*
******************************************************************
      DIMENSION ISCALE(7)
      PARAMETER (N=20)
      REAL VR,Q,QO,X,R,IO,T(N)
      INTEGER I,GRAPHS(70),BL,STAR,DOT
      DATA GRAPHS/70*" "/,BL/" "/,STAR/"*"/
      DATA DOT/"."/
      READ *,VR,R,X,Q,QO,IO
      OPEN(UNIT=6,FILE="marl.dat",STATUS="OLD")
      WRITE(6,1)VR
  1   FORMAT(10X,"The per phase voltage is",2X,F5.1,2X,
     + "volts.")
      WRITE(6,5)R
  5   FORMAT(10X,"The rotor resistance is ",F6.2,2X,"ohms.")
      WRITE(6,6)X
  6   FORMAT(10X,"The rotor inductive reactance is",F6.2,1X,"ohms.")
      WRITE(6,7)Q
  7   FORMAT(10X,"The final slip is",1X,F6.3,".")
      WRITE(6,8)QO
  8   FORMAT(10X,"The initial slip is",1X,F6.3,".")
      WRITE(6,9)IO
  9   FORMAT(10X,"The initial current is",F5.2,1X,"Amp.")
*    Compute the current.
      AMP1=(IO-VR*QO/R)
      AMP2=(VR*Q/(R)*(1+3*X**2*Q*QO/(2*R**2)))
      AMP3=(-3*VR*X**2*Q**3/(2*R**3))
*   Write the output.
      WRITE(6,10)AMP1
 10   FORMAT(10X,"The unit step response (u(t)) is",F7.2,1X,"Amp.")
      WRITE(6,11)AMP2
 11   FORMAT(10X,"The delta response is",F7.2,1X,"Amp.")
      WRITE(6,12)AMP3
 12   FORMAT(10X,"The doublet response is",F7.2,1X,"Amp.")
      DO 15 I=1,7
      K=100*I-400
      ISCALE(I)=K
 15   CONTINUE
      WRITE(6,17)ISCALE
 17   FORMAT(30X,"Current in Ampere",//7I10)
      WRITE(6,18)
      WRITE(6,19)
 18   FORMAT("Time")
 19   FORMAT(1X,"(Sec.)")
      DO 20 I=1,N
      READ*,T(I)
      AMPS=AMP1/10+26
      M=AMPS
      GRAPHS(M)=STAR
      WRITE(6,23)DOT
 20   CONTINUE
      STOP
 22   FORMAT(2X,F5.2,7X,70A1,T4,
 23   FORMAT(T40,A1)
      END
```

Appendix F

```
*     This is a program to calculate the torque of an induction
*     motor when a small value of slip at no load is
*     changed to a higher value of a pulse type, Qu(t-T).
*
*     Variable Declaration
*     TORQUE- torque in (Newton-meter)
*     VS- per phase voltage in (Volts)
*     Q- slip of the motor
*     QO- initial slip of the motor
*     R- rotor resistance of the motor in (ohms)
*     X- total equivalent inductive reactance of the motor in (Ohms)
*     TOR1- is the unit step response of the motor
*     TOR2- is the pulse response (u(t-T) of the motor
*     TOR3- is the pulse response (u(t-2T) of the motor
*     P- the number of poles
*     PH-the number of phases
*     SRPM- the synchronous speed in (rpm)
*     A- is the time when the rectangular pulse goes to zero in (sec.)
      DIMENSION ISCALE(7)
      PARAMETER (N=20)
      INTEGER N,GRAPHS(70),BL,STAR,DOT
      REAL TORQ,VS,Q,QO,R,X,TOR1,TOR2,TOR3,TO,SRPM,F,P,PH,T(N),A
      DATA GRAPHS/70*" "/,BL/" "/,STAR/"*"/
      DATA DOT/"."/
      READ*,VS,F,P,PH,R,X,Q,QO,TO,A
      OPEN(UNIT=6,FILE="marl.dat",STATUS="NEW")
      WRITE(6,2)VS
  2   FORMAT(10X,"The per phase voltage is",1X,F5.1,1X,"volts.")
      WRITE(6,4)F
  4   FORMAT(10X,"The frequency of the system is",1X,F4.1,2X,
     + "herts.")
      WRITE(6,6)P
  6   FORMAT(10X,"The number of poles is",1X,F4.1,".")
      WRITE(6,8)PH
  8   FORMAT(10X,"The number of phase is",1X,F4.1,".")
      WRITE(6,10)R
 10   FORMAT(10X,"The rotor resistance is",F6.2,1X,"ohms.")
      WRITE(6,12)X
 12   FORMAT(10X,"The total inductive reactance is",F6.2,1X,
     + "ohms.")
      WRITE(6,14)Q
 14   FORMAT(10X,"The final slip is",1X,F6.3,"u(t-T).")
      WRITE(6,16)QO
 16   FORMAT(10X,"The initial slip is",1X,F6.3,".")
      WRITE(6,18)TO
 18   FORMAT(10X,"The initial torque is",1X,F5.2,1X,"N-m.")
 22   FORMAT(30X,"Torque in N-m",//10X,7I10)
 23   FORMAT(10X,"The unit step response (u(t)) of the torque is",1X,
     + E9.2,1X,"N-m.")
 24   FORMAT(10X,"The pulse response (u(t-T)) of the torque is",
     + 1X,E9.2,1X,"N-m.")
 25   FORMAT(10X,"The pulse response (u(t-2T)) of the torque is",
     + 1X,E9.2,1X,"N-m.")
*    Compute the synchronous speed.
      SRPM=120.*F/P
      TOR1=(VS**2/R*(Q-QO-3*X**2*Q**3/(R**2)+X**2*Q**2*QO/(R**2)+5*
     + X**4*Q**5/(R**4)-5*X**4*Q**4*QO/(R**4)-6*X**6*Q**7/(R**6)
     + +6*X**6*Q**6*QO/(R**6))+TO)*7.04/(SRPM)*PH*1.356
      TOR2=(VS**2/R*(-Q+6*X**2*Q**3/(R**2)-X**2*Q**2*QO/(R**2)-10*
     + X**4*Q**5/(R**4)+5*X**4*Q**4*QO/(R**5)+12*X**6*Q**7/(R**6)-
     + 6*X**6*Q**6*QO/(R**6)))*7.04/(SRPM)*PH*1.356
      TOR3=(VS**2*X**2*Q**3/(R**3)*(-3+5*X**2*Q**2/(R**2)-6*X**4
     + *Q**4/(R**7)))*7.04/(SRPM)*PH*1.356
      WRITE(6,23)TOR1
      WRITE(6,24)TOR2
      WRITE(6,25)TOR3
      DO 27 I=1,7
      K=1000*I-4000
      ISCALE(I)=K
 27   CONTINUE
      WRITE(6,22)ISCALE
      WRITE(6,28)
      WRITE(6,29)
 28   FORMAT("Time",5X,"Torque")
 29   FORMAT("(Sec.)",3X,"(N-m)")
      DO 38 I=1,N
      READ*,T(I)
      IF(T(I).GE.2*A) THEN
        GOTO 34
      ELSE IF(T(I).GE.A) THEN
        GOTO 32
      ELSE IF(T(I).GE.0) THEN
        GOTO 30
      ELSE IF(T(I).LT.0) THEN
        GOTO 39
      ENDIF
 30   TORQUE=TOR1
      GOTO 35
 32   TORQUE=TOR1+TOR2
      GOTO 35
 34   TORQUE=TOR1+TOR2+TOR3
 35   TORQ=TORQUE/100+34
      M=TORQ
      GRAPHS(M)=STAR
      WRITE(6,40)T(I),TORQUE,GRAPHS
      GRAPHS(M)=BL
      WRITE(6,45)DOT
 38   CONTINUE
 39   STOP
 40   FORMAT(1X,F3.1,2X,E9.2,2X,70A1,T4,A1)
 45   FORMAT(T50,A1)
      END
```

Appendix G

```
*     This is a program to calculate the current of an induction
*     motor when a small value of slip at no load is
*     changed to a higher value of a pulse type, Qu(t-T).
*
*     Variable Declaration
*     AMP- current in (Ampere)
*     VR- per phase voltage in (Volts)
*     Q- slip of the motor
*     QO- initial slip of the motor
*     R- rotor resistance of the motor in (ohms)
*     X- rotor inductive reactance of the motor in (Ohms)
*     AMP1- is the unit step response of the motor
*     AMP2- is the pulse response (u(t-T) of the motor
*     AMP3- is the pulse response (u(t-2T) of the motor
*     PH-the number of phases
*     A- is the time when the rectangular pulse goes to zero in (sec.)
      DIMENSION ISCALE(7)
      PARAMETER (N=20)
      INTEGER N,GRAPHS(70),BL,STAR,DOT
      REAL AMPS,VR,Q,QO,R,X,AMP1,AMP2,AMP3,IO,PH,T(N),A
      DATA GRAPHS/70*" "/,BL/" "/,STAR/"*"/
      DATA DOT/"."/
      READ*,VR,R,X,Q,QO,IO,A
      OPEN(UNIT=6,FILE="mar1.dat",STATUS="NEW")
      WRITE(6,2)VR
 2    FORMAT(10X,"The per phase voltage is",1X,F5.1,1X,"volts.")
      WRITE(6,10)R
 10   FORMAT(10X,"The rotor resistance is",F6.2,1X,"ohms.")
      WRITE(6,12)X
 12   FORMAT(10X,"The rotor inductive reactance is",F6.2,1X,
     + "ohms.")
      WRITE(6,14)Q
 14   FORMAT(10X,"The final slip is",1X,F6.3,"u(t-T).")
      WRITE(6,16)QO
 16   FORMAT(10X,"The initial slip is",1X,F6.3,".")
      WRITE(6,18)IO
 18   FORMAT(10X,"The initial current is",1X,F5.2,1X,"N-m.")
 22   FORMAT(30X,"Current in Ampere",//10X,7I10)
 23   FORMAT(10X,"The unit step response (u(t)) of the current is",1X,
     + F6.2,1X,"Amp.")
 24   FORMAT(10X,"The pulse response (u(t-T)) of the current is",
     + 1X,F6.2,1X,"Amp.")
 25   FORMAT(10X,"The pulse response (u(t-2T)) of the current is",
     + 1X,F6.2,1X,"Amp.")
      AMP1=(VR*(Q-QO)/R-3*X**2*VR*(Q**3-Q**2*QO)/(2*R**3))+IO
      AMP2=(VR*Q*(-1-3*X**2*Q**2/(R**2)-3*X**2*Q*QO/(2*R**2))/R)
      AMP3=(3*VR*X**2*Q**3/(2*R**3))
      WRITE(6,23)AMP1
      WRITE(6,24)AMP2
      WRITE(6,25)AMP3
      DO 27 I=1,7
      K=100*I-400
      ISCALE(I)=K
 27   CONTINUE
      WRITE(6,22)ISCALE
      WRITE(6,28)
 28   FORMAT("Time",5X,"Current")
      WRITE(6,29)
 29   FORMAT("(Sec.)",3X,"(Amp.)")
      DO 38 I=1,N
      READ*,T(I)
      IF(T(I).GE.2*A) THEN
      ELSE IF(T(I).GE.A) THEN
        GOTO 32
      ELSE IF(T(I).GE.0) THEN
        GOTO 30
      ELSE IF(T(I).LT.0) THEN
        GOTO 39
      ENDIF
 30   AMPS=AMP1
      GOTO 35
 32   AMPS=AMP1+AMP2
      GOTO 35
 34   AMPS=AMP1+AMP2+AMP3
 35   AMPE=AMPS/10+36
      M=AMPE
      GRAPHS(M)=STAR
      WRITE(6,40)T(I),AMPS,GRAPHS
      GRAPHS(M)=BL
      WRITE(6,45)DOT
 38   CONTINUE
 39   STOP
 40   FORMAT(1X,F3.1,2X,F7.2,2X,70A1,T4,A1
 45   FORMAT(T50,A1)
      END
```

Appendix H

```
*    This is a program to calculate the torque of an induction
*    motor when a very small value of slip at no load
*    is changed to a ramp type, Qt.
*    Variable Declaration
*     TORQUE- torque in (Newton-meter)
*     VS- the per phase voltage in (Volts)
*     Q- the slip of the motor
*     QO- the initial slip of the motor
*     R- the rotor resistance of the motor in (Ohms)
*     X- the rotor inductive reactance of the motor in (Ohms)
*     TO- the initial torque in (Newton-meter)
*     T(I)- time in (Seconds)
*     PH- the number of phases
*     F- frequency of the system in (Hz)
*     P- the number of poles
*
**************************************************************
      DIMENSION ISCALE(7)
      PARAMETER (N=20)
      REAL TORQ,VS,Q,QO,X,R,TO,TORQ1,TOR1,TOR2,TOR3,TOR4,TOR5,
     +TOR6,TOR7,SRPM,P,F,PH,T(N)
      INTEGER I,GRAPHS(70),BL,STAR,DOT
      DATA GRAPHS/70*" "/,BL/" "/,STAR/"*"/
      DATA DOT/"."/
      READ*,VS,F,P,PH,R,X,Q,QO,TO
      OPEN(UNIT=6,FILE="marl.dat",STATUS="NEW")
      WRITE(6,1)VS
  1   FORMAT(10X,"The per phase voltage is",2X,F5.1,2X
     +"volts.")
      WRITE(6,2)F
  2   FORMAT(10X,"The frequency of the system is",2X,F4.1,1X,"hertz.")
      WRITE(6,3)P
  3   FORMAT(10X,"The number of poles is",2X,F4.1,".")
      WRITE(6,4)PH
  4   FORMAT(10X,"The number of phases of the motor is",1X,F4.1,".")
      WRITE(6,5)R
  5   FORMAT(10X,"The rotor resistance of the motor is",F6.2,2X,"ohms.")
      WRITE(6,6)X
  6   FORMAT(10X,"The total inductive reactance of the motor is",F6.2,
     +1X,"ohms.")
      WRITE(6,7)Q
  7   FORMAT(10X,"The final slip is",1X,F6.3,"t.")
      WRITE(6,8)QO
  8   FORMAT(10X,"The initial slip is",1X,F6.3,".")
      WRITE(6,9)TO
  9   FORMAT(10X,"The initial torque is",1X,F5.2,1X,"N-m.")
      DO 15 I=1,7
      K=1000*I-4000
      ISCALE(I)=K
 15   CONTINUE
      WRITE(6,20)ISCALE
 20   FORMAT("1",30X,"Torque in Newton-meter",//10X,7I10)
      WRITE(6,21)
 21   FORMAT("Time",5X,"Torque")
      WRITE(6,22)
 22   FORMAT("(Sec.)",3X,"(N-m)")
      DO 25 I=1,N
      READ*,T(I)
      SRPM=120*F/P
      TORQ1=7.04/SRPM*PH*1.356*(TO-VS**2*QO/R)
      TOR1=VS**2*Q*T(I)/R
      TOR2=VS**2*X**2*Q**2*QO*T(I)**2/(R**3)
      TOR3=VS**2*X**2*Q**3*T(I)**3/(3*R**3)
      TOR4=VS**2*X**4*Q**4*QO*T(I)**4/(R**5)
      TOR5=VS**2*X**4*Q**5*T(I)**5/(5*R**5)
      TOR6=6*VS**2*X**6*Q**6*QO*T(I)**6/(7*R**7)
      TOR7=6*VS**2*X**6*Q**7*T(I)**7/(49*R**7)
      TORQUE=7.04/(SRPM)*PH*1.356*(TOR1+TOR2-TOR3-TOR4+
     &+TOR6-TOR7)+TORQ1
      TORQ=TORQUE/100+34
      M=TORQ
      GRAPHS(M)=STAR
      WRITE(6,27)T(I),TORQUE,GRAPHS
      GRAPHS(M) BL
      WRITE(6,30)DOT
 25   CONTINUE
      STOP
 27   FORMAT(1X,F3.1,F10.2,2X,70A1,T4,A1)
 30   FORMAT(T50,A1)
      END
```

Appendix I

```
*   This is a program to calculate the current of an induction
*   motor when a very small value of slip at no load
*   is changed to a ramp type, Qt.
*   Variable Declaration
*    AMP- the current in (Ampere)
*    VR- the per phase voltage in (Volts)
*    Q- the slip of the motor
*    QO- the initial slip of the motor
*    R- the rotor resistance of the motor in (Ohms)
*    X- the rotor inductive reactance of the motor in (Ohms)
*    IO- the initial current in (Ampere)
*    T(I)- time in (Seconds)
*
**************************************************************
      DIMENSION ISCALE(7)
      PARAMETER (N=20)
      REAL AMP,VR,Q,QO,X,R,IO,AMP1,AMP2,AMP3,AMP4,T(N)
      INTEGER I,GRAPHS(70),BL,STAR,DOT
      DATA GRAPHS/70*" "/,BL/" "/,STAR/"*"/
      DATA DOT/"."/
      READ*,VR,R,X,Q,QO,IO
      OPEN(UNIT=6,FILE="marl.dat",STATUS="OLD")
      WRITE(6,1)VR
  1   FORMAT(10X,"The per phase voltage is",2X,F5.1,2X
     +"volts.")
      WRITE(6,5)R
  5   FORMAT(10X,"The rotor resistance of the motor is",F6.2,2X,"ohms.")
      WRITE(6,6)X
  6   FORMAT(10X,"The rotor inductive reactance of the motor is",F6.2,
     +1X,"ohms.")
      WRITE(6,7)Q
  7   FORMAT(10X,"The final slip is",1X,F6.3,"t.")
      WRITE(6,8)QO
  8   FORMAT(10X,"The initial slip is",1X,F6.3,".")
      WRITE(6,9)IO
  9   FORMAT(10X,"The initial current is",1X,F5.2,1X,"Amp.")
      DO 15 I=1,7
      K=100*I-400
      ISCALE(I)=K
 15   CONTINUE
      WRITE(6,20)ISCALE
 20   FORMAT(" ",30X,"Current in Ampere",//10X,7I10)
      WRITE(6,21)
      WRITE(6,22)
 21   FORMAT("Time",3X,"Current")
 22   FORMAT("(Sec.)",2X,"(Amp.)")
      DO 25 I=1,N
      READ*,T(I)
      AMP1=IO-VR*QO/R
      AMP2=VR*Q*T(I)/R
      AMP3=VR*X**2*Q**2*QO*T(I)**2/(2*R**3)
      AMP4=VR*X**2*Q**3*T(I)**3/(6*R**3)
      AMP=(AMP1+AMP2+AMP3-AMP4)
      AMPS=AMP/10+36
      M=AMPS
      GRAPHS(M)=STAR
      WRITE(6,27)T(I),AMP,GRAPHS
      GRAPHS(M)=BL
      WRITE(6,30)DOT
 25   CONTINUE
      STOP
 27   FORMAT(1X,F3.1,2X,F7.2,2X,70A1,T4,A1)
 30   FORMAT(T50,A1)
      END
```

Appendix J

```
*    This is a program to calculate the torque of an induction
*    motor when a small value of slip at no load is
*    changed to a higher value of a sinusoidal type, Qsint.
*
*    Variable Declaration
*    TORQUE- torque in (Newton-meter)
*    VS- per phase voltage in (Volts)
*    Q- final slip of the motor
*    QO- initial slip of the motor
*    R- rotor resistance of the motor
*    X- total inductive reactance of the motor in (Ohms)
*    SRPM- the synchronous speed in (rpm)
*    P- the number of poles
*    PH- the number of phases
***********************************************************************
        DIMENSION ISCALE(7)
        PARAMETER (N=20)
        INTEGER N,GRAPHS(70),BL,STAR,DOT
        REAL T(N),TORQ,VS,Q,QO,R,X,TOR,TOR1,TOR2,TOR3,TOR4,TOR5,TOR6,TOR7,
     +  TOR8,TOR9,TOR10,TOR11,TOR12,TOR13,TOR14,TOR15,TOR16,TOR17,TOR18,
     +  TOR19,TOR20,TOR21,W,SRPM,F,P,PH
        DATA GRAPHS/70*" "/,BL/" "/,STAR/"*"/
        DATA DOT/"."/
        READ*,VS,F,P,PH,R,X,Q,QO,TO
        OPEN(UNIT=6,FILE="marl.dat",STATUS="NEW)
        WRITE(6,1)VS
  1     FORMAT(10X,"The per phase voltage is",1X,F5.1,1X,"volts.")
        WRITE(6,2)F
  2     FORMAT(10X,"The frequency of the system is",1X,F4.1,1X,
     +  "hertz.")
        WRITE(6,3)P
  3     FORMAT(10X,"The number of poles of the motor is",1X,F4.1,".")
        WRITE(6,4)PH
  4     FORMAT(10X,"The number of phase is",1X,F4.1,".")
        WRITE(6,5)R
  5     FORMAT(10X,"The rotor resistance is",F6.2,1X,"ohms.")
        WRITE(6,6)X
  6     FORMAT(10X,"The total inductive reactance is",F6.2,1X,"ohms.")
        WRITE(6,7)Q
  7     FORMAT(10X,"The final slip is",1X,F6.3,"sin t.")
        WRITE(6,8)QO
  8     FORMAT(10X,"The initial slip is",1X,F6.3,".")
        WRITE(6,9)TO
  9     FORMAT(10X,"The initial torque is",F5.2,1X,"N-m.")
        DO 15 I=1,7
        K=1000*I-4000
        ISCALE(I)=K
 15     CONTINUE
        WRITE(6,18)ISCALE
 18     FORMAT(30X,"Torque in N-m",//10X,7I10)
        WRITE(6,20)
        WRITE(6,21)
 20     FORMAT("Time",5X,"Torque")
 21     FORMAT("(Sec.)",3X,"(N-m)")
*    Compute the synchronous speed.
        SRPM=120.*F/P
        DO 25 I=1,N
        READ*,T(I)
*    Calculate the torque
        TOR=TO-VS**2*QO/R
        TOR1=Q*T(I)/R
        TOR2=X**2*Q**2*OO*T(I)**2/R**3
        TOR4=X**2*Q**2*QO*T(I)**4/(R**3)*(0.33+0.21*X**2*Q**2
     +  /(R**2))
        TOR5=X**2*Q**3*T(I)**5/(R**3)*(0.08+0.04*X**2*Q**2/R**2)
        TOR6=X**2*Q**2*QO*T(I)**6/(R**3)*(0.0279+0.14*X**2*Q**2/
     +  (R**2)+2.8E-3*X**4*Q**4/(R**4))
        TOR7=X**2*Q**3*T(I)**7/(R**3)*(5.7E-3+0.02*X**2*Q**2/
     +  (R**2)+4E-4*X**4*Q**4/(R**4))
        TOR8=X**4*Q**4*QO*T(I)**8/(R**5)*(0.03+2.8E-3*X**2*Q**2
     +  /(R**2))
        TOR9=X**2*Q**3*T(I)**9/(R**3)*(1.00E-4+4.1E-3*X**2*Q**2/
     +  (R**2)+3E-4*X**4*Q**4/(R**4))
        TOR10=X**4*Q**4*QO*T(I)**10/(R**5)*(3.9E-3+1.2E-3*X**2*
     +  Q**2/(R**2))
        TOR11=X**4*Q**5*T(I)**11/(R**5)*(4E-4+1E-4*X**2*Q**2
     +  /(R**2))
        TOR12=X**4*Q**4*QO*T(I)**12/(R**5)*(2.0E-4+3E-4*X**2
     +  *Q**2/(R**2))
        TOR13=X**4*Q**5*T(I)**13/(R**5)*(1.46E-5+2.04E-5*X**2
     +  *Q**2/(R**2))
        TOR14=3.22E-5*X**6*Q**6*QO*T(I)**14/(R**7)
        TOR15=X**4*Q**5*T(I)**15/(R**5)*(5.89E-8+2.24E-6*Q**2
     +  *X**2/(R**2))
        TOR16=4.29E-6*X**6*Q**6*QO*T(I)**16/(R**7)
        TOR17=2.6E-7*X**6*Q**7*T(I)**17/(R**7)
        TOR18=5.95E-8*X**6*Q**6*QO*T(I)**18/(R**7)
        TOR19=3.87E-9*X**6*Q**7*T(I)**19/(R**7)
        TOR21=7.46E-12*X**6*Q**7*T(I)**21/(R**7)
        TORQUE=(((TOR1+TOR2-TOR3-TOR4+TOR5+TOR6-TOR7-TOR8+TOR9+TOR10-
     +  TOR11-TOR12+TOR13+TOR14-TOR15-TOR16+TOR17+TOR18-TOR19+
     +  +TOR21)*VS**2)*7.04/(SRPM)*PH*1.356)+TOR
        TORQ=TORQUE/100+34
        M=TORQ
        GRAPHS(M)=STAR
        WRITE(6,27)T(I),TORQUE,GRAPHS
        GRAPHS(M)=BL
        WRITE(6,28)DOT
 25     CONTINUE
        STOP
 27     FORMAT(1X,F3.1,2X,F10.2,2X,70A1,T4,A1)
 28     FORMAT(T50,A1)
        END
```

Appendix K

```
*    This is a program to calculate the current of an induction
*    motor when a small value of slip at no load is
*    changed to a higher value of a sinusoidal type, Qsint.
*
*    Variable Declaration
*    AMPS- ampere in (Ampere)
*    VR- per phase voltage in (Volts)
*    Q- final slip of the motor
*    QO- initial slip of the motor
*    R- rotor resistance of the motor
*    X- rotor inductive reactance of the motor in (Ohms)
*    PH- the number of phases
**********************************************************************
      DIMENSION ISCALE(7)
      PARAMETER (N=20)
      INTEGER N,GRAPHS(70),BL,STAR,DOT
      REAL T(N),TORQ,VR,Q,QO,R,X,AMP1,AMP2,AMP3,AMP4,AMP5,AMP6,AMP7,
     + AMP8,AMP9,W,IO
      DATA GRAPHS/70*" "/,BL/" "/,STAR/"*"/
      DATA DOT/"."/
      READ*,VR,R,X,Q,QO,IO
      OPEN(UNIT=6,FILE="marl.dat",STATUS="NEW")
      WRITE(6,1)VR
 1    FORMAT(10X,"The per phase voltage is",1X,F5.1,1X,"volts.")
      WRITE(6,5)R
 5    FORMAT(10X,"The rotor resistance is",F6.2,1X,"ohms.")
      WRITE(6,6)X
 6    FORMAT(10X,"The rotor inductive reactance is",F6.2,1X,"ohms.")
      WRITE(6,7)Q
 7    FORMAT(10X,"The final slip is",1X,F6.3,"sin t.")
      WRITE(6,8)QO
 8    FORMAT(10X,"The initial slip is",1X,F6.3,".")
      WRITE(6,9)IO
 9    FORMAT(10X,"The initial current is",F5.2,1X,"Amp.")
      DO 15 I=1,7
      K=100*I-400
      ISCALE(I)=K
 15   CONTINUE
      WRITE(6,18)ISCALE
 18   FORMAT(30X,"Current in Ampere ",//10X,7I10)
      WRITE(6,20)
      WRITE(6,21)
 20   FORMAT("Time",3X,"Current")
 21   FORMAT("(Sec.)",2X,"(Amp.)")
      DO 25 I=1,N
      READ*,T(I)
*    Calculate the current.
      AMP1=IO-VR*QO/R
      AMP2=VR*Q*T(I)/R
      AMP3=VR*Q*T(I)**3/R*(-0.167-0.495*X**2*Q**2/R**2)
      AMP4=0.165*VR*X**2*Q**2*QO*T(I)**4/R**3
      AMP5=0.042*VR*X**2*Q**3*T(I)**5/R**3
      AMP6=1.4E-2*VR*X**2*Q**2*QO*T(I)**6/R**3
      AMP7=2.9E-3*VR*X**2*Q**3*T(I)**7/R**3
      AMP8=0.495*VR*X**2*Q**2*QO*T(I)**2/R**3
      AMP9=2.76E-5*VR*X**2*Q**3*T(I)**9/R**3
      AMP=(AMP1+AMP2+AMP3-AMP4+AMP5+AMP6-AMP7+AMP8+AMP9)
      AMPS=AMP/10+36
      M=AMPS
      GRAPHS(M)=STAR
      WRITE(6,27)T(I),AMP,GRAPHS
      GRAPHS(M)=BL
      WRITE(6,28)DOT
 25   CONTINUE
      STOP
 27   FORMAT(F3.1,2X,F7.2,2X,70A1,T4,A1)
 28   FORMAT(T50,A1)
      END
```

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