ABSTRACT

Improving the Jitter Performance of Timing Recovery by Employing Delay Shifts

by

Xin De Lu

Many methods have been developed for evaluating the jitter performance of timing recovery circuits for binary synchronous transmission systems. These circuits consist of a nonlinear device followed by a narrowband filter tuned to the pulse repetition frequency. This thesis introduce delay shifts in the timing recovery circuits to improve the jitter performance. The function of the delay shifts is analyzed and mathematical proof of jitter performance improvement is given for the system using square-law device. Finally, the numerical results obtained from the specific examples serve to illustrate the significant improvement between the system with and without delay shifts. That is, one delay shift introduced in the system can gain nearly 3 db in rms jitter performance.

IMPROVING THE JITTER PERFORMANCE OF TIMING RECOVERY BY EMPLOYING DELAY SHIFTS

by

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APPROVAL PAGE

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by Employing Delay Shifts

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This thesis is dedicated to

my wife, my parents and my sisters

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CHAPTER 1
INTRODUCTION

Synchronization is a basic issue in communication and control. In digital communication systems, synchronization problems can be classified into three levels: phase, symbol, and frame. Most digital communication systems using coherent modulation require all three of these levels of synchronization. The knowledge of synchronization information at all three levels is the key to successfully detect the transmitted signal.

Phase Synchronization: phase-locked loop (PLL)

Phase synchronization is needed in coherent modulation and demodulation. For example, in the case of coherent phase modulation (PM), the receiver is required to generate a reference signal whose phase is identical to the phase of incoming signal. Generally, at the heart of all phase synchronization circuits is the some version of a phase-locked loop. Figure 1.1 is a basic phase-locked loop. The details of phase-locked loop are discussed in several textbooks [1] [2].

Frame Synchronization:

Frame synchronization is required when the information is organized in blocks. This will occur, for example, if a block code is used for forward error control, or if the communications channel being time shared on a regular basis, by several users

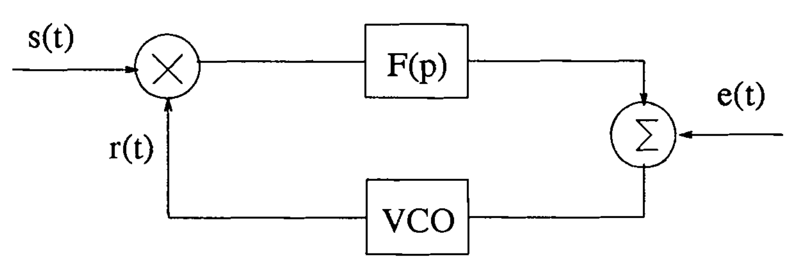


Figure 1.1 Basic Phase-Locked Loop

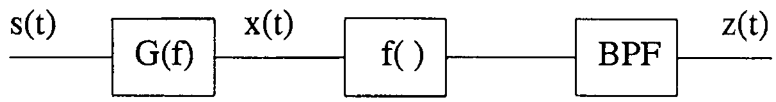


Figure 1.2 STR Block Diagram

(TDMA). Frame synchronization is usually accomplished with the aid of some special signal procedure from the transmitter. A simple example of frame synchronization aid is the frame marker, which is a single bit, or a short pattern of bits that the transmitter injects periodically into the data sequences. Although adding additional synchronization bits will reduce the information rates transmitted in a given channel, the correlation should be nearly perfect.

Symbol Synchronization:

All digital receivers need to have demodulation synchronized to the incoming digital symbol transition in order to achieve optimum demodulation. The main objective of this thesis is to design a symbol timing recovery circuits (STR) to extract synchronization information or timing wave from incoming signal. Fig. 1.2 is a basic STR block diagram used to illustrate the principle of STR circuits.

where x(t) is the incoming binary sequences and can be expressed as

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k g(t - kT) \tag{1.1}$$

where a_k is data sequences (± 1), assumed to be statistically independent, g(t) is impulse response of the prefilter. T is the data period or data clock. Clearly after even-law device, the output waveform will contain a Fourier components at the fundamental frequency of the data clock ($\frac{1}{T}$). This frequency component is isolated from its harmonics by a bandpass filter (BPF). Thus, timing recovery is achieved.

Based on the principle above, many methods such as absolute value, square law, full wave rectifier, high order non-linear device, half bit delay etc., have been developed to extract timing wave and their timing jitter are thoroughly analyzed by [3] [4] [5] [6] [7] [8]. This thesis developed a new method to improve jitter performance of STR circuits. That is, for the nonlinear device keep the same, every delay shift introduced, the resulted jitter performance is improved by nearly 3 db.

1.1 Previous Work Review

Timing recovery has been the subject for the research in synchronous digital communication for long time. There are a number of strategies developed to achieve this purpose by transmitting extra power and using additional bandwidth. For example, one can insert synchronization bits in data sequences or use a separate sub-channel for transmitting synchronization information.

Synchronization by using extra power and bandwidth is not economical and timing recovery using no extra power and bandwidth have been developed, which extracts timing information directly from incoming data sequence. In this thesis, we restrict our discussion in this class of synchronization strategies.

The performance of timing recovery circuits employing simply a narrowband filter tuned to a harmonic of pulse repetition frequency was analyzed by Bennett [1]. His results showed that, this scheme works only when incoming data sequence having a nonzero mean value and Fourier transform of the data pulse not vanish at pulse repetition frequency. Therefore, this scheme requires a strong restriction on channel bandwidth and signal power.

In the interest of meeting power and bandwidth limitations, a nonlinear device must be inserted before the narrowband filter. Franks and Bubrouski [2] considered a timing circuits involving a square-law device followed by a narrowband filter, their results showed that the jitter performance depends on the excess bandwidth of the input pulse and it is satisfactory for medium and large values of rolloff factor of incoming pulse.

When the incoming pulse is strongly bandlimited (rolloff factor goes to zero), the pulse overlap or intersymbol interference is significant and timing recovery fails, but for small rolloff factor, it is possible to use high order nonlinear device or other nonlinear device to achieve satisfactory jitter performance. Yegal Barness [3] developed a new method (moments method) to evaluate jitter performance of STR circuits employing high order nonlinear device, his method is suitable particularly for the case of incoming pulse having small rolloff factor.

1.2 System Model

From previous work, it is recognizable that square-law device works well for incoming pulse with medium and large rolloff factor, for small rolloff factor incoming pulse, other nonlinear device must be considered in order to achieve satisfactory jitter performance. This thesis developed a new method to improve jitter performance when prefilter, nonlinear device and postfilter are kept the same.

Fig 1.3 is the system model for extracting the timing information. Where G(f) is prefilter used to reshape the pulse and reduce the influence of noise. f() is nonlinear device such as absolute value, square law, the fourth-law rectifiers etc. These nonlinear device must be symmetric. $k_1T...k_nT$ are delay shifts. k_i is an integer. H(f) is a narrowband filter used to eliminate high order harmonic components. H(f) is centered at the pulse repetition frequency $1/T$ and satisfies the band-limiting condition.

$$H(f) = 0 \qquad for \quad \left| |f| - \frac{1}{T} \right| > \frac{1}{2T}$$

The output of postfilter is timing wave used as reference signal for demodulation. Due to the intersymbol interference and noise in channel, this timing wave is nearly

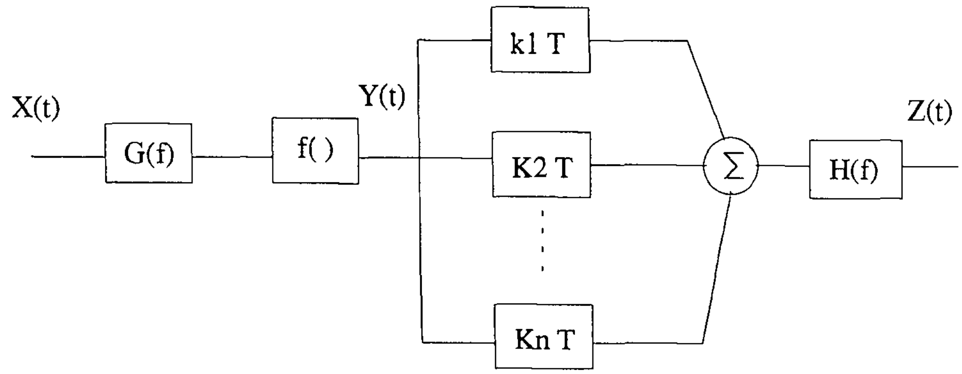


Figure 1.3 System Model With Delay Shifts

sinusoidal wave which is fluctuated at zero crossing time t_0. No matter what kind of modulation scheme is used in transmitting the data sequence, the system model above is suitable because one can select proper demodulation scheme to get the received signal as described in equation 1.1.

There are five chapters in this thesis. The first chapter states what kind of STR circuits we are interested in and we review the previous work related to this subject. In chapter 2, we follow Frank and Bubrouski, Y. Barness in analyzing the STR circuit employing a square-law device followed by a narrowband filter tuned to the pulse repetition frequency. Although the results are the same as [4] [5], the analytical approach is different, which helps us in analyzing the STR circuits with delay shifts.

In chapter 3, we evaluate the jitter performance of STR circuits with delay shifts. For comparison purpose and for mathematical reason, we give the final rms jitter performance expression only for the STR circuits with one delay shift. The numerical results are given in chapter 4 and the property of STR circuits (jitter performance and bandwidth relation) are also discussed in this chapter. In the last chapter, we draw our conclusions from the study of this subject.

CHAPTER 2

JITTER PERFORMANCE OF STR CIRCUITS WITHOUT DELAY SHIFTS

The symbol timing recovery (STR) circuits that employed only a square-law device followed by a narrowband filter were investigated by many people [4] [5]. No matter what kinds of analytical approaches were used in evaluating the jitter performance of STR circuits, it is desirable to consider square-law device at first. Because mathematically, a square-law device is ease to deal with, and many properties of STR circuits can be uncovered through this analysis.

It is possible that the timing circuit might be simply a narrowband filter tuned to the pulse repetition frequency. This scheme requires that the data sequence have a nonzero mean value and that the Fourier transform of the data pulse not vanish at the pulse repetition frequency. However, in many communication systems, these conditions do not hold. So other STR circuits employing nonlinear devices were developed, such as absolute value, square-law, full wave rectifier, high order nonlinear device etc. Previous work [4] [5] showed that the STR circuits involving only a square-law device followed by a narrowband filter will give satisfactory jitter performance for medium and large rolloff factor. For a rolloff factor that goes to zero, high order nonlinear device must be employed to get a satisfactory jitter performance.

In this thesis, a different analytical method is used. This chapter emphases on the STR circuits employed only square-law device followed by a narrowband filter. Although the mathematical expression for rms jitter is slightly different from those given by [4] [5], the simulation results are exact the same.

2.1 Definition of Timing Jitter

The jitter of timing wave ($\Delta\tau$) is defined as the deviation between the timing

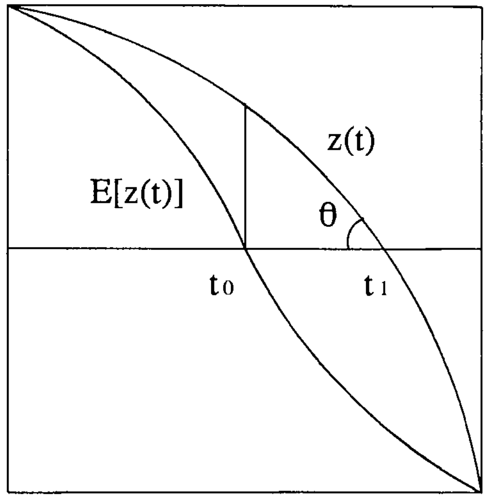


Figure 2.1 Timing Wave Sample Functions

wave's actual zero crossing time and its mean timing wave zero crossing time.

Suppose we have a cyclostationary process (CT process) z(t) and its mean waveform E[z(t)]. One sample of these two waveforms is sketched in Fig. 2.1

In Fig.2.1, t_0 is zero crossing of mean wave E[z(t)] and t_1 is zero crossing of random process z(t), from the definition of timing jitter, we have

$$\Delta\tau = t_1 - t_0$$

Assuming the timing jitter is relatively small compared to the period of mean timing wave T, approximately,

$$\tan\theta = \frac{z(t_0)}{\Delta\tau} = E[\dot{z}\ (t_0)]$$

or

$$\Delta\tau = \frac{z(t_0)}{E[\dot{z}\ (t_0)]}$$

Clearly $\Delta\tau$ is random variable. Normalized it to the period T and measure its root mean square value, thus

$$\left(\frac{\Delta\tau}{T}\right)_{rms} = \frac{1}{T}\frac{(E[z^2(t_0)])^{1/2}}{E[\dot{z}\,(t_0)]} \tag{2.1}$$

The equation 2.1 is the criteria for evaluating jitter performance of STR circuits. This thesis analyzed different scheme only based on this criteria. The main objective of this thesis is just to improve the rms jitter value by using more complicated scheme.

2.2 Evaluation of the Mean Value of the Timing Wave

As mentioned previously, the incoming data $\{a_n\}$ is random variable, so the input signal for nonlinear device is random process. Here we chose square-law device as nonlinear device, therefore we get

$$y(t) = x^2(t) = \sum_m \sum_n a_m a_n g(t - mT)g(t - nT) \tag{2.2}$$

Obviously $y(t)$ is a random process. $H(f)$ is a linear narrowband filter, and its output $z(t)$ is a random process too. According to the properties of linear filter, the expected value of $z(t)$ equals the response of the system to the mean waveform $E[y(t)]$ of the input, thus;

$$E[z(t)] = \int_{-\infty}^{+\infty} E[y(\alpha)]h(t - \alpha)d\alpha \tag{2.3}$$

and

$$\begin{aligned} E[y(t)] &= E\left[\sum_m \sum_n a_m a_n g(t - mT)g(t - nT)\right] \\ &= \sum_m \sum_n E[a_m a_n]g(t - mT)g(t - nT) \qquad (2.4) \end{aligned}$$

The data sequence a_n is assumed to be statistically independent binary sequence taking value of ± 1, with equal probability, hence

$$E[a_m a_n] = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases} \tag{2.5}$$

Substitute equation (2.5) in (2.4), the expected value of $y(t)$ as follows;

$$E[y(t)] = \sum_n g^2(t - nT) \tag{2.6}$$

Examining the equation above, it is not difficult to find that this expected waveform is periodic with period T. As it is known that any periodic signal can be expressed as Fourier series, and the spectrum of this periodic signal has frequency components only at fundamental frequency $\frac{1}{T}$ and its higher order harmonics. Thus, if we use a narrowband filter to pick up frequency component at $\frac{1}{T}$, we can get a nearly sinusoidal wave with period T at output.

Defining $g_2(t) = g^2(t)$, equation 2.6 can be written as

$$E[y(t)] = \sum_n g_2(t - nT) \tag{2.7}$$

Taking Fourier transform of equation 2.7

$$\begin{aligned} F\{E[y(t)]\} &= \frac{1}{T}\sum_n G_2(\frac{n}{T})e^{\frac{j2\pi nt}{T}} \\ &= \frac{1}{T}\sum_n G_2(\frac{n}{T})\delta(f - \frac{n}{T}) \end{aligned} \tag{2.8}$$

taking the Fourier transform of equation 2.3 on both side and substitute equation 2.8 into it,

$$F\{E[z(t)]\} = \frac{1}{T}\sum_n G_2(\frac{n}{T})\delta(f-\frac{n}{T})H(f)$$

$$= \frac{1}{T}\sum_n G_2(\frac{n}{T})H(\frac{n}{T})\delta(f-\frac{n}{T}) \quad (2.9)$$

$H(f)$ is a narrowband filter centered at pulse repetition frequency $\frac{1}{T}$ and satisfies the bandlimiting condition, we get $H(\frac{n}{T}) \neq 0$ only for $n = \pm 1$. therefore,

$$F\{E[z(t)]\} = \frac{1}{T}G_2(-\frac{1}{T})H(-\frac{1}{T})e^{-\frac{j2\pi nt}{T}}$$

$$+ \frac{1}{T}G_2(\frac{1}{T})H(\frac{1}{T})e^{\frac{j2\pi nt}{T}} \quad (2.10)$$

Defining $|u_1| = \frac{1}{T}|G_2(\frac{1}{T})H(\frac{1}{T})|$, and noting that $G_2(-\frac{1}{T})H(-\frac{1}{T}) = \left[G(\frac{1}{T})H(\frac{1}{T})\right]^*$, taking the inverse Fourier transform, we obtain;

$$E[z(t)] = 2|u_1|\cos(\frac{2\pi t}{T} + \phi) \quad (2.11)$$

Where ϕ is the phase of $G_2(\frac{1}{T})H(\frac{1}{T})$. If the bandwidth of $G_2(f)$ is greater than $\frac{1}{T}$, then, $E[z(t)] \neq 0$. That means this STR circuits works, but we can not know how well this system works up to this point. The next section, we will evaluate the mean square value of timing wave $z(t)$ at zero crossing time t_0, and get the exact expression of rms of jitter value, the jitter performance of different prefilter and postfilter can be discussed thoroughly.

From equation 2.11, zero crossing time of timing wave satisfies

$$\frac{2\pi t_0}{T} + \phi = (2n-1)\frac{\pi}{2} \quad (2.12)$$

and

$$t_0 = T(\frac{2n-1}{4} - \frac{\phi}{2\pi}) \tag{2.13}$$

where n is an integer.

2.3 Evaluation of the Mean Square Value of the Timing Wave

For any non stationary process, we can write

$$\begin{aligned} E[z^2(t)] &= E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} y(t-\alpha)y(t-\beta)h(\alpha)h(\beta)d\alpha d\beta\right] \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E[y(t-\alpha)y(t-\beta)]h(\alpha)h(\beta)d\alpha d\beta \end{aligned} \tag{2.14}$$

Similar as equation 2.2, $y(t-\alpha)\ and\ y(t-\beta)$ can be expressed as

$$y(t-\alpha) = \sum_m\sum_n a_m a_n g(t-\alpha-mT)g(t-\alpha-nT) \tag{2.15}$$

$$y(t-\beta) = \sum_i\sum_j a_i a_j g(t-\beta-iT)g(t-\beta-jT) \tag{2.16}$$

Equation 2.15 multiplies equation 2.16 and taking the expected value, yield

$$\begin{aligned} E[y(t-\alpha)(t-\beta)] &= \sum_m\sum_n\sum_i\sum_j E[a_m a_n a_i a_j]g(t-\alpha-mT) \\ & g(t-\alpha-nT)g(t-\beta-iT)g(t-\beta-jT) \end{aligned} \tag{2.17}$$

The only random variable here is data sequence $\{a_n\}$ which is statistically independent and having values ± 1 with equal probability. The expected value of right side of equation 2.17 can be done by directly using following relation [6].

$$E[a_m a_n a_i a_j] = \delta_{mn}\delta_{ij} + \delta_{mi}\delta_{nj} + \delta_{mj}\delta_{ni} - 2\delta_{mnij} \tag{2.18}$$

Where δ_{mnij} means it equals 1 only when $m = n = i = j$, Substitute equation 2.18 into 2.17 and rewrite it as

$$\begin{aligned} E[y(t-\alpha)(t-\beta)] &= \sum_m g^2(t-\alpha-mT)\sum_i g^2(t-\beta-iT) \\ &+ 2\left[\sum_m g(t-\alpha-mT)g(t-\beta-mT)\right]^2 \\ &- 2\sum_m g^2(t-\alpha-mT)g^2(t-\beta-mT) \end{aligned} \tag{2.19}$$

There are three terms on the right side of equation 2.19. Later on, we will show that the first term has no contribution on the jitter performance. Only the last two terms play an important role on the rms jitter performance of timing wave. Substitute equation 2.19 back into equation 2.14 and evaluate the influence of the first term of equation

$$\begin{aligned} E_1[z^2(t)] &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\sum_m g^2(t-\alpha-mT)\sum_i g^2(t-\beta-iT)h(\alpha)h(\beta)d\alpha d\beta \\ &= \int_{-\infty}^{+\infty}\sum_m g^2(t-\alpha-mT)h(\alpha)d\alpha\int_{-\infty}^{+\infty}\sum_i g^2(t-\beta-iT)h(\beta)d\beta \\ &= \left[\int_{-\infty}^{+\infty}\sum_m g^2(t-\alpha-mT)h(\alpha)d\alpha\right]^2 \end{aligned} \tag{2.20}$$

From equation 2.6, equation 2.20 can be written as the square of the convolution of the mean wave of input and impulse of response

$$E_1[z^2(t)] = \{E[y(t)] * h(t)\}^2 = \{E[z(t)]\}^2 \qquad (2.21)$$

This term evaluated at zero crossing time (t_0), obviously it equals zero

$$E_1[z^2(t)] \equiv 0 \qquad \text{for } t = t_0$$

The last two terms can be combined together, and this can be explained as intersymbol interference. But for mathematical ease, we still write them separately

$$\begin{aligned} E_2[z^2(t)] &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\sum_m g(t-\alpha-mT)g(t-\beta-mT)\right]^2 h(\alpha)h(\beta)d\alpha d\beta \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\sum_m\sum_n g(t-\alpha-mT)g(t-\beta-mT) \\ &\quad \times g(t-\alpha-nT)g(t-\beta-nT)h(\alpha)h(\beta)d\alpha d\beta \\ &= \sum_m\sum_n\left[\int_{-\infty}^{+\infty} g(t-\alpha-mT)g(t-\alpha-nT)h(\alpha)d\alpha\right]^2 \\ &= \sum_m\sum_k\left[\int_{-\infty}^{+\infty} g(t-\alpha-mT)g(t-\alpha-mT-kT)h(\alpha)d\alpha\right]^2 \qquad (2.22) \end{aligned}$$

Where k=n-m, if we denote

$$\int_{-\infty}^{+\infty} g(t-\alpha)g(t-\alpha-kT)h(\alpha)d\alpha = P_k(t) \qquad (2.23)$$

and

$$\sum_k P_k^2(t) = Q(t)$$

then the expected value $E_2[z^2(t)]$ could be rewritten as

$$E_2[z^2(t)] = \sum_m [\sum_k P_k^2(t - mT)] = \sum_m Q(t - mT) \tag{2.24}$$

Similarly, the influence of the last term of equation 2.19 can be evaluated as

$$\begin{aligned} E_3[z^2(t)] &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sum_m g^2(t - \alpha - mT) g^2(t - \beta - mT) h(\alpha) h(\beta) d\alpha d\beta \\ &= \sum_m \left[\int_{-\infty}^{+\infty} g^2(t - \alpha - mT) h(\alpha) d\alpha \right]^2 \end{aligned} \tag{2.25}$$

Using the notation of equation 2.23, hence k=0

$$\int_{-\infty}^{+\infty} g^2(t - \alpha) h(\alpha) d\alpha = P_0(t) \tag{2.26}$$

then

$$E_3[z^2(t)] = \sum_m P_0^2(t - mT) = \sum_m S(t - mT) \tag{2.27}$$

where S(t) is defined as $P_0^2(t)$.

Now, checking all three terms of mean square value of the timing wave, it is interesting that they are all periodic waveform with period T. Combine all three terms together and note that $E_1[z^2(t_0)] = 0$,

$$\begin{aligned} E[z^2(t_0)] &= E_2[z^2(t_0)] + E_3[z^2(t_0)] \\ &= \sum_m [2Q(t_0 - mT) - \sum_m 2P_0^2(t_0 - mT)] \end{aligned} \tag{2.28}$$

According to the definition, the rms jitter performance is evaluated at zero crossing time instant. Therefore the final expression of rms jitter at time domain is obtained

$$\left(\frac{\Delta t}{T}\right)_{rms} = \frac{\frac{1}{T}E[z^2(t_0)]^{1/2}}{E[\dot{z}\ (t_0)]}$$

$$= \frac{\left[\sum_m 2Q(t_0 - mT) - \sum_m 2P_0^2(t_0 - mT)\right]^{1/2}}{4\pi|u_1|} \tag{2.29}$$

If we know h(t) which is the impulse response of postfilter (narrowband filter), then $P_k(t)$ can be evaluated by numerical integration, and it is possible to write a program to evaluate the final rms jitter performance based on the equation 2.29. But in order to understand the behavious of the postfilter especially the band-limiting condition. It is better to derive the final rms jitter expression in frequency domain, and this is discussed in next section.

2.4 Evaluation of the Mean Square Value of the Timing Wave in Frequency Domain

The jitter expression in equation 2.29 is determined by the term $P_k^2(t)$ only. Taking the Fourier transform on equation 2.23, we have

$$P_k(f) = H(f)\int_{-\infty}^{+\infty} G(\beta)G(f-\beta)e^{-j2\pi\beta kT}d\beta \tag{2.30}$$

in the case of k=0, and noting $g_2(t) = g^2(t)$, or

$$G_2(f) = \int_{-\infty}^{+\infty} G(\beta)G(f-\beta)d\beta \tag{2.31}$$

then

$$P_0(f) = H(f)\int_{-\infty}^{+\infty} G(\beta)G(f-\beta)d\beta = H(f)G_2(f) \tag{2.32}$$

Using the properties of Fourier transform, the Fourier transform of Q(t) can easily be obtained

$$Q(f) = \sum_k P_k(f) \bigotimes P_k(f)$$

$$= \sum_k \int_{-\infty}^{+\infty} P_k(\alpha) P_k(f-\alpha) d\alpha \quad (2.33)$$

Substitute equation 2.23 into 2.33, then

$$Q(f) = \sum_k \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} H(\alpha) H(f-\alpha) G(u) G(\alpha-u) G(v)$$

$$G(f-\alpha-v) e^{-j2\pi(u+v)kT} du dv d\alpha \quad (2.34)$$

Using the famous relation

$$\sum_m e^{j2\pi tmT} = \frac{1}{T} \sum_m \delta(t - \frac{m}{T}) \quad (2.35)$$

then 2.34 becomes

$$Q(f) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} H(\alpha) H(f-\alpha) G(u) G(\alpha-u) G(v) G(f-\alpha-v)$$

$$\times \sum_k \frac{1}{T} \delta(u+v-\frac{k}{T}) du dv d\alpha$$

$$= \sum_k \frac{1}{T} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} H(\alpha) H(f-\alpha) G(v) G(\frac{k}{T}-v)$$

$$\times G(f-\alpha-v)G(\alpha+v-\frac{k}{T})dvd\alpha \tag{2.36}$$

Since g(t) is band-limiting prefilter response, therefore

$$G(f)=0 \qquad for|f|\geq\frac{1}{T}$$

that is

$$G(v)G(\frac{k}{T}-v)\equiv 0 \quad fork\geq 2$$

finally, the expression becomes

$$Q(f)=\frac{1}{T}\sum_{k=-1}^{1}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}H(\alpha)H(f-\alpha)G(v)G(\frac{k}{T}-v)G(f-\alpha-v)G(\alpha+v-\frac{k}{T})dvd\alpha \tag{2.37}$$

2.5 Final Rms Jitter Expression

Using Fourier series, the intersymbol interference term of equation 2.24 and equation 2.27 can be expressed as

$$E_2[z^2(t)]=\frac{1}{T}\sum_m Q(\frac{m}{T})e^{\frac{j2\pi mt}{T}} \tag{2.38}$$

and

$$E_3[z^2(t)]=\frac{1}{T}\sum_m S(\frac{m}{T})e^{\frac{j2\pi mt}{T}} \tag{2.39}$$

where $Q(\frac{m}{T})$ and $S(\frac{m}{T})$ are given by equation 2.40 and 2.41 respectively

$$Q(\frac{m}{T})=\frac{1}{T}\sum_{k=-1}^{1}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}H(\alpha)H(\frac{m}{T}-\alpha)G(v)G(\frac{k}{T}-v)G(\frac{m}{T}-\alpha-v)G(\alpha+v-\frac{k}{T})dvd\alpha \tag{2.40}$$

and

$$S(\frac{m}{T}) = \int_{-\infty}^{+\infty} H(\alpha)H(\frac{m}{T}-\alpha)G_2(\alpha)G_2(\frac{m}{T}-\alpha)d\alpha \tag{2.41}$$

Using the band-limiting condition for H(f), clearly

$$H(\alpha)H(\frac{m}{T}-\alpha) \neq 0 \quad only for \quad m = 0, \pm 2;$$

So

$$E[z^2(t)] = \sum_{m=0,\pm 2} [Q(\frac{m}{T})e^{\frac{j2\pi mt}{T}} - S(\frac{m}{T})e^{\frac{j2\pi mt}{T}}]$$

$$= \sum_{m=0,\pm 2} v_m e^{\frac{j2\pi mt}{T}} \tag{2.42}$$

Noting that $v_{-2} = v_2^*$, then

$$E[z^2(t)] = v_0 + |v_2|\cos(\frac{4\pi t}{T} + \theta) \tag{2.43}$$

where

$$v_0 = \frac{2}{T}[Q(0) - S(0)]$$

$$|v_2| = \frac{2}{T}[Q(\frac{2}{T}) - S(\frac{2}{T})]$$

$$\theta = \arctan[Q(\frac{2}{T}) - S(\frac{2}{T})]$$

Evaluating equation 2.43 at zero crossing time t_0 we have

$$E[z^2(t_0)] = v_0 + |v_2|\cos(n\pi - 2\phi + \theta) \quad n\ odd \tag{2.44}$$

the minimum rms jitter occurs when $2\phi = \theta$

$$\left(\frac{\Delta\tau}{T}\right)_{rms,min} = \frac{(v_0 - 2|v_2|)^{1/2}}{4\pi u_1} \tag{2.45}$$

where

$$v_0 = \frac{2}{T^2}[\sum_{k=-1}^{1}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} H(\alpha)H(-\alpha)G(v)G(\frac{k}{T}-v)G(-\alpha-v)G(\alpha+v-\frac{k}{T})dvd\alpha$$

$$-T\int_{-\infty}^{+\infty} H(\alpha)H(-\alpha)G_2(\alpha)G_2(-\alpha)d\alpha] \quad (2.46)$$

and

$$v_2 = \frac{2}{T^2}[\sum_{k=-1}^{1}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} H(\alpha)H(\frac{2}{T}-\alpha)G(v)G(\frac{k}{T}-v)G(\frac{2}{T}-\alpha-v)$$

$$G(\alpha+v-\frac{k}{T})dvd\alpha - T\int_{-\infty}^{+\infty} H(\alpha)H(\frac{2}{T}-\alpha)G_2(\alpha)G_2(\frac{2}{T}-\alpha)d\alpha] \quad (2.47)$$

2.6 Computer Simulation

The C program has been written to explore the jitter performance of STR circuits described in this chapter. We investigate two parameters which are related to jitter performance of STR circuits. One is the magnitude of timing wave, this is an important parameter since small magnitude result in poor synchronization (noise is unavoidable and magnitude value can not be increased by using amplifier), the other one is minrms jitter performance defined by equation 2.45. For mathematical easiness, we use ideal lowpass filter as prefilter and the bandwidth of this prefilter is described by rolloff factor γ. For a postfilter, we used a single tuned circuits with resonant frequency $\frac{1}{T}$ and quality factor Q, that is

$$H(f) = H_0(f-\frac{1}{T}) + H_0(f+\frac{1}{T}) \quad (2.48)$$

with $H_0(f)$, the lowpass equivalent, given by

$$H_0(f) = \begin{cases} \frac{1}{1+jfTQ} & |f| < \frac{1}{2}T \\ 0 & otherwise \end{cases} \quad (2.49)$$

Table 2.1: Rms Jitter as a Function of Q, for an Ideal Band-limiting Filter Response

Q	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
25	0.034571	0.017257	0.011615	0.009292
50	0.019658	0.009874	0.006668	0.005291
75	0.013985	0.007059	0.004783	0.003786
100	0.010946	0.005489	0.003772	0.002987
125	0.009035	0.004598	0.003136	0.002484
150	0.007715	0.003941	0.002696	0.002137

The program is written based on equation 2.45, 2.46, 2.47. Table 2.1 shows the rms jitter of timing wave z(t) as a function of quality factor Q. These datas are plotted in Fig. 2.2. (Rolloff factor γ as parameter.)

The quality factor Q reflects the characteristic of postfilter. As quality factor Q increase, the postfilter functions more and more like a narrowband filter. Therefore, the jitter level decrease as Q increase. The results shown in Fig. 2.2 are exactly the same as expected.

In order to view the influence of rolloff factor, we plot the rms jitter of timing wave as function of rolloff factor in Fig. 2.4, where Q is selected as parameter.

2.7 Property of STR Circuits

The final rms jitter expression (2.45) shows that the rms jitter performance depend on the impulse response of prefilter and postfilter if the nonlinear device is chosen. The function of postfilter is clear, it simply pick up the frequency component at pulse repetition frequency. The character of postfilter is described by quality factor Q, large Q leads single tuned circuit much close to an ideal narrowband filter. Therefore, the jitter performance is better. The simulation results in Fig.2.1 demonstrate this analysis.

We are much interested in the impulse response of prefilter. There are two reason

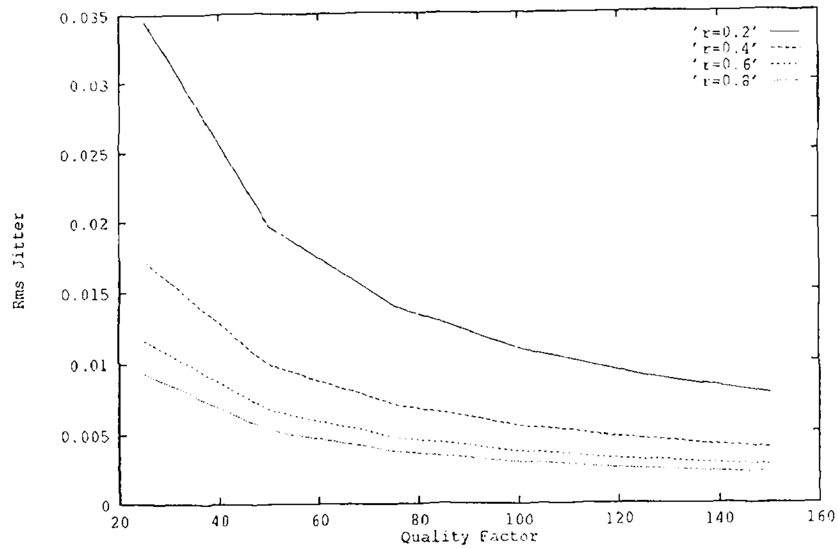


Figure 2.2 Jitter Performance as Function of Quality Factor

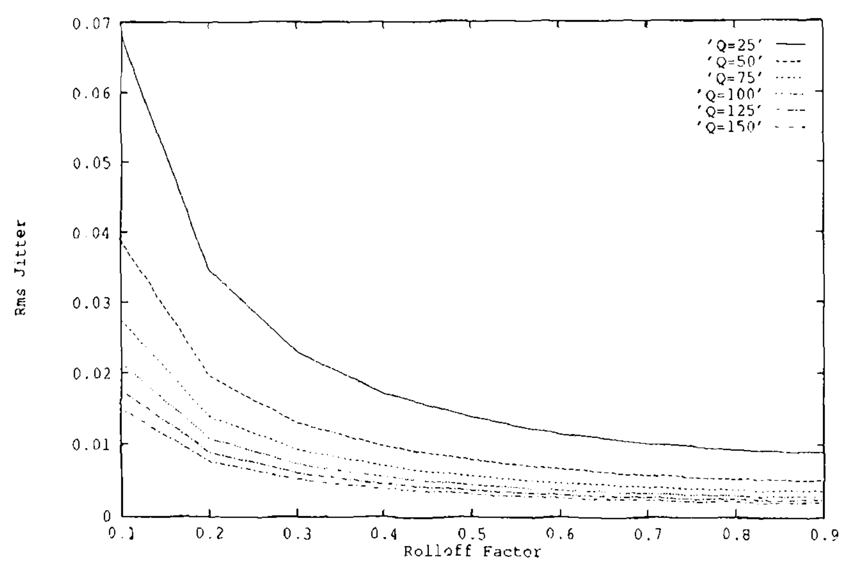


Figure 2.3 Jitter Performance as Function of Rolloff Factor

for this, one is that the channel can be charactlized by a prefilter, the influence of channel bandwidth can be uncovered by exploring the function of prefilter. The other reason is that reshaping the incoming wave by using prefilter can improve the jitter performance of timing wave, in particular, will result in a error-free timing recovery [3]. The condition for achieving error-free timing wave is not easy to get and sometime impractical [6]

The result in Fig. 2.2 show that the jitter level decrease as γ increase. That is to say, if the channel has wider bandwidth, the jitter performance of timing recovery circuits is better. While the channel bandwidth decrease, the jitter performance is worsen. The decrease of channel bandwidth will increase the intersymbol interference. Therefore, the jitter of timing recovery is primarily due to the intersymbol interference when the noise is small.

CHAPTER 3

EVALUATION OF JITTER PERFORMANCE FOR STR CIRCUITS WITH DELAY SHIFTS

The evaluation procedure for jitter performance of timing wave z(t) is exactly the same as it was in chapter 2. But it is more complicated since we introduce the delay shifts. Here only one delay shifts is considered, from this simplest case, we can see the improvement by using delay shifts. For the system using nonlinear device other than square-law, this scheme also works. Although the close mathematical expression is hard to tract, one can use computer simulation package such as BOSS to see the jitter performance improvement for the system using different kinds of nonlinear device and different number of delay shifts.

The simplest and most mathematically tractable system is as follows:
where x(t), G(f), H(f) are described the same as in chapter 2.
lT is delay time, l must be an integer.

3.1 Evaluation of Rms Jitter on Time Domain

From Fig. 3.1, $y_2(t)$ equals

$$y_2(t) \;\; = \;\; y(t) + y(t - lT)$$

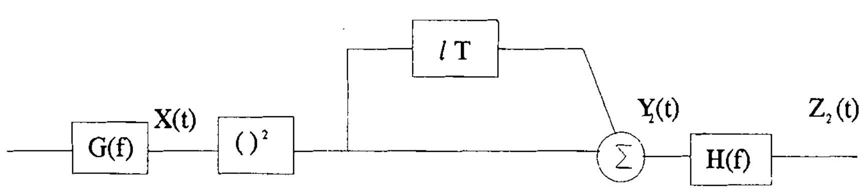


Figure 3.1 Sample Function

$$= \sum_m \sum_n a_m a_n g(t - mT) g(t - nt)$$

$$+ \sum_m \sum_n a_m a_n g(t - mT - lT) g(t - nt - lT)$$

$$= \sum_m \sum_n (a_m a_n + a_{m+l} a_{n+l}) g(t - mT) g(t - nt) \tag{3.1}$$

Taking the expected value of equation 3.1, we have

$$E[y_2(t)] = \sum_m \sum_n E[a_m a_n + a_{m+l} a_{n+l}] g(t - mT) g(t - nt) \tag{3.2}$$

Substitute equation 2.5 into 3.2, we get

$$E[y_2(t)] = \sum_m \sum_n 2\delta_{mn} g(t - mT) g(t - nt)$$

$$= 2 \sum_m g^2(t - mT) \tag{3.3}$$

Therefore, the expected value of the output of postfilter H(f);

$$E[z_2(t)] = \int_{-\infty}^{+\infty} E[y_2(\alpha)] h(t - \alpha) d\alpha \tag{3.4}$$

Substituting equation 3.3 into equation 3.4 and comparing it to equation 2.3, then

$$E[z_2(t)] = 2E[z(t)] \tag{3.5}$$

This result is obvious since the postfilter H(f) is time invariant, and the mean value of output is the sum of mean value of inputs. Coefficient 2 is due to two inputs, one is the output of square device and the other one is the delayed version of it.

Using the results in chapter 2, the Fourier transform of $E[z_2(t)]$ is easily obtained, and finally

$$E[z_2(t)] = 2E[z(t)] = 4|u_1|\cos(\frac{2\pi t}{T} + \phi) \tag{3.6}$$

and zero crossing time is the same as before, that is

$$t_0 = T(\frac{2n-1}{4} - \frac{\phi}{2\pi}) \tag{3.7}$$

where n is an integer.

To evaluate the variance of time wave $Z_2(t)$, we follow all the procedure in chapter 2 and write

$$y_2(t-\alpha) = \sum_m \sum_n [a_m a_n + a_{m+l} a_{n+l}] g(t-\alpha-mT) g(t-\alpha-nt) \tag{3.8}$$

$$y_2(t-\beta) = \sum_i \sum_j [a_i a_j + a_{i+l} a_{j+l}] g(t-\beta-iT) g(t-\beta-jt) \tag{3.9}$$

Multiplying two equation above and taking the expected value, yield

$$E[y_2(t-\alpha) y_2(t-\beta)] = \sum_m \sum_n \sum_i \sum_j E[(a_m a_n + a_{m+l} a_{n+l})(a_i a_j + a_{i+l} a_{j+l})]$$

$$g(t-\alpha-mT) g(t-\alpha-nT) g(t-\beta-iT) g(t-\beta-jT) \tag{3.10}$$

The results of equation 3.10 is given by (see Appendix A)

$$E[y_2(t-\alpha) y_2(t-\beta)] = 2E[y(t-\alpha)(t-\beta)] + 2\sum_m g^2(t-\alpha-mT) \sum_i g^2(t-\beta-iT)$$

$$+ \quad 2\left[\sum_m g(t-\alpha-mT)g(t-\beta-(m+l)T)\right]^2$$

$$+ \quad 2\left[\sum_m g(t-\alpha-mT)g(t-\beta-(m-l)T)\right]^2$$

$$- \quad 2\sum_m g^2(t-\alpha-mT)g^2(t-\beta-(m+l)T)$$

$$- \quad 2\sum_m g^2(t-\alpha-mT)g^2(t-\beta-(m-l)T) \qquad (3.11)$$

After filtering, and noting that equation 2.20 and 2.21, we have

$$E[z_2^2(t)] = 2E[z^2(t)] + 2Z_1(t) - 2Z_2(t) \qquad (3.12)$$

where $Z_1(t)$ and $Z_2(t)$ are defined as

$$Z_1(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\left(\sum_m g(t-\alpha-mT)g(t-\beta-(m+l)T)\right)^2 + \left(\sum_m g(t-\alpha-mT)g(t-\beta-(m-l)T)\right)^2\right] h(\alpha)h(\beta)d\alpha d\beta \qquad (3.13)$$

and

$$Z_2(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\sum_m g^2(t-\alpha-mT)g^2(t-\beta-(m+l)T) + \sum_m g^2(t-\alpha-mT)g^2(t-\beta-(m-l)T)\right] h(\alpha)h(\beta)d\alpha d\beta \qquad (3.14)$$

We look $Z_1(t)$, and $Z_2(t)$ as error terms, and if we can prove these error terms are small comparing to mean square value of timing wave $E[z^2(t)]$ at zero crossing time t_0, then these terms can be neglected and approximately, we have

$$\left(\frac{\Delta\tau_2}{T}\right)_{rms} = \frac{1}{T}\frac{(E[z_2^2(t_0)])^{1/2}}{E[\dot{z}_2\,(t_0)]}$$

$$= \frac{1}{T}\frac{(2E[z^2(t_0)])^{1/2}}{2E[\dot{z}\,(t_0)]}$$

$$= \frac{\sqrt{2}}{2}\left(\frac{\Delta\tau}{T}\right)_{rms} \tag{3.15}$$

The result above is equivalent to 3 db jitter performance improvement. But for real system, the error terms $Z_1(t)$, and $Z_2(t)$ are small number which depends on the delay time lT and on what kind of prefilter and postfilter the system uses. In the next section, we write the error terms expression on frequency domain and simplify it to get final rms jitter performance expression on frequency domain.

3.2 Frequency Domain Expression for Error Terms

Checking equation 3.13 carefully, we can find that $m+l$ and $m-l$ are symmetric. So, we only simplify the term containing $(m+l)T$ delay time and use symmetric property to get time domain simplified expression for $Z_1(t)$.

First, we have

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\sum_m g(t-\alpha-mT)g(t-\beta-(m+l)T)\right)^2 h(\alpha)h(\beta)d\alpha d\beta$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\sum_m\sum_n g(t-\alpha-mT)g(t-\beta-(m+l)T)g(t-\alpha-nT)$$

$$g(t-\beta-(n+l)T)h(\alpha)h(\beta)d\alpha d\beta$$

$$= \sum_m \sum_n \int_{-\infty}^{+\infty} g(t-\alpha-mT)g(t-\alpha-nT)h(\alpha)d\alpha$$

$$\times \int_{-\infty}^{+\infty} g(t-\beta-(m+l)T)g(t-\beta-(n+l)T)h(\beta)d\beta$$

$$= \sum_m \sum_k \int_{-\infty}^{+\infty} g(t-\alpha-mT)g(t-\alpha-mT-kT)h(\alpha)d\alpha$$

$$\times \int_{-\infty}^{+\infty} g(t-\beta-(m+l)T)g(t-\beta-(m+l)T-kT)h(\beta)d\beta$$

$$= \sum_m \sum_k P_k(t-mT)P_k(t-mT-lT) \tag{3.16}$$

Where k=m-n, and P_k is defined in equation 2.23.

Considering the $m-l$ term together, we have

$$Z_1(t) = \sum_m \sum_k P_k(t-mT)[P_k(t-mT-lT)+P_k(t-mT+lT)] \tag{3.17}$$

Similar for $Z_2(t)$, we have

$$Z_2(t) = \sum_m P_0(t-mT)[P_0(t-mT-lT)+P_0(t-mT+lT)] \tag{3.18}$$

Clearly, $Z_1(t)$ and $Z_2(t)$ are periodic with period T. Define A(t) and B(t) is one period of $Z_1(t)$ and $Z_2(t)$ respectively, we have

$$Z_1(t) = \sum_m A(t-mT) \tag{3.19}$$

$$Z_2(t) = \sum_m B((t - mT) \tag{3.20}$$

Where

$$A(t) = \sum_k P_k(t)[P_k(t - lT) + P_k(t + lt)] \tag{3.21}$$

and

$$B(t) = P_0(t)[P_0(t - lT) + P_0(t + lT)] \tag{3.22}$$

Taking the Fourier transform of equation 3.19, then

$$\begin{aligned} A(f) &= \sum_k P_k(f) \bigotimes [P_k(f)e^{-j2\pi flT} + P_k(f)e^{+j2\pi flT}] \\ &= \sum_k P_k(f) \bigotimes [P_k(f) 2\cos(2\pi flT)] \\ &= \sum_k \int_{-\infty}^{+\infty} 2\cos(2\pi\alpha lT) P_k(\alpha) P_k(f - \alpha) d\alpha \end{aligned} \tag{3.23}$$

Comparing equation with equation 2.33, the only difference is $2\cos(2\pi\alpha lT)$, which is due to delay, therefore

$$\begin{aligned} A(f) &= \sum_k \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} 2\cos(2\pi\alpha lT) H(\alpha) H(f - \alpha) G(u) G(\alpha - u) G(v) \\ & G(f - \alpha - v) e^{-j2\pi(u+v)kT} du dv d\alpha \end{aligned} \tag{3.24}$$

Using the same procedure and condition as in page 16, we have

$$A(f) = \frac{1}{T} \sum_{k=-1}^{1} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} 2\cos(2\pi\alpha lT) H(\alpha) H(f - \alpha) G(v)$$

$$G(\frac{k}{T} - v)G(f - \alpha - v)G(\alpha + v - \frac{k}{T})dv d\alpha \tag{3.25}$$

Similarly, we have

$$\begin{aligned} B(f) &= P_0(f) \bigotimes [P_0(f)e^{-j2\pi f lT} + P_0(f)e^{+j2\pi f lT}] \\ &= P_0(f) \bigotimes [2\cos(2\pi f lT)P_0(f)] \\ &= \int_{-\infty}^{+\infty} 2\cos(2\pi \alpha lT)P_0(\alpha)P_0(f - \alpha)d\alpha \end{aligned} \tag{3.26}$$

Substitute equation 2.32 into equation 3.26

$$B(f) = \int_{-\infty}^{+\infty} 2\cos(2\pi \alpha lT)H(\alpha)H(f - \alpha)G_2(\alpha)G_2(f - \alpha)d\alpha \tag{3.27}$$

Using the band-limiting condition for H(f) as in chapter 2, we finally have

$$\begin{aligned} E_e[z_2^2(t)] &= 2Z_1(t) - 2Z_2(t) \\ &= v_{0e} + |v_{2e}|\cos(\frac{4\pi t}{T} + \vartheta) \end{aligned} \tag{3.28}$$

Where

$$v_{0e} = \frac{2}{T}[A(0) - B(0)]$$

$$|v_2| = \frac{2}{T}[A(\frac{2}{T}) - B(\frac{2}{T})]$$

$$\vartheta = \arctan[A(\frac{2}{T}) - B(\frac{2}{T})]$$

3.3 Final Rms Jitter Expression for STR Circuits with Delay Shifts

According to the definition of rms jitter in equation 2.1, the rms jitter for the STR circuits with delay shifts can be expressed as

$$\left(\frac{\Delta\tau_2}{T}\right)_{rms} = \frac{1}{T}\frac{(E[z_2^2(t_0)])^{1/2}}{E[\dot{z_2}\,(t_0)]} \tag{3.29}$$

where $z_2(t)$ is the timing wave.

Substitute equation 3.28 into equation 3.12

$$E[z_2^2(t)] = 2E[z^2(t)] + v_{0e} + |v_{2e}|\cos(\frac{4\pi t}{T} + \vartheta) \tag{3.30}$$

Using 2.43, we have

$$\begin{aligned} E[z_2^2(t_0)] &= 2[v_0 + |v_2|\cos(\frac{4\pi t}{T} + \theta)] \\ &+ v_{0e} + |v_{2e}|\cos(\frac{4\pi t}{T} + \vartheta) \end{aligned} \tag{3.31}$$

Substituting equation 3.6, 3.31 into equation 3.29, and the minimum rms jitter occurs when $2\phi = \theta$, therefore

$$\left(\frac{\Delta\tau_2}{T}\right)_{rms,min} = \frac{[2(v_0 - 2|v_2|) + v_{0e} - 2|v_{2e}|\cos(\theta - \vartheta)]^{1/2}}{8\pi u_1} \tag{3.32}$$

CHAPTER 4

NUMERICAL RESULTS OF JITTER PERFORMANCE FOR THE SYSTEM WITH DELAY SHIFTS

The numerical results for the system without delay elements is given in section 2.6, which is exactly the same as the results given by previous work [3] [5]. In this chapter, we will present the numerical results for the system with delay shifts, we also explore the function of delay time. The simulation is based on the simplest system model described in chapter 3. For the comparison purpose, we keep the prefilter, postfilter and nonlinear device the same as in chapter 2.

4.1 Evaluating the Error Terms of Rms Jitter

In equation 3.15, we assume that the error terms are relatively small comparing to the mean square value of timing wave $E[z^2(t)]$, here we use computer simulation to check the above assumption.

For the comparison convenience, we normalized the error terms by 2, that is , we compare the term $Z_1(t) - Z_2(t)$ with mean square value $E[z^2(t)]$.

Table 4.1 show the mean square value $E[z^2(t)]$ for different combination of quality factor Q and rolloff factor γ.

We are interested in the case of small rolloff factor, since the error terms due to the intersymbol interference are big when rolloff factor is small. From the data in three tables, we find that the error term are roughly 1/10 of mean square value when $l = 5$ and 1/20 of mean square value when $l = 10$. Therefore, we can conclude that the assumption in equation 3.15 is correct even the delay is not big.

Table 4.2 and 4.3 show the value of error term $Z_1(t) - Z_2(t)$ with respect to delay parameter $l = 10$ and $l = 5$.

Table 4.1 Mean Square Value $E[z^2(t)]$ for Different Q and γ

Q	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
25	1.819666e-03	9.793564e-04	5.851355e-04	4.156549e-04
50	5.883400e-04	3.206290e-04	1.928212e-04	1.347514e-04
75	2.977757e-04	1.638756e-04	9.922098e-05	6.909118e-05
100	1.824048e-04	1.012535e-04	6.170691e-05	4.295134e-05
125	1.242815e-04	6.953538e-05	4.264187e-05	2.970768e-05
150	9.063022e-05	5.108282e-05	3.151357e-05	2.198970e-05

Table 4.2 The Value of Error Term $Z_1(t) - Z_2(t)$ for $l = 10$

Q	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
25	6.907442e-05	3.310523e-05	1.707011e-05	8.895217e-06
50	4.276068e-05	2.721557e-05	1.824837e-05	1.326071e-05
75	4.483332e-05	2.768433e-05	1.816317e-05	1.285515e-05
100	3.745621e-05	2.316030e-05	1.521574e-05	1.073553e-05
125	3.058372e-05	1.903430e-05	1.257130e-05	8.882619e-06
150	2.516757e-05	1.579015e-05	1.049565e-05	7.438703e-06

It is interesting that for a fixed quality factor Q, the ratio of error term to the mean square value are almost the same, that is to say, the improvement of jitter performance has nothing to do with rolloff factor, even for large rolloff factor, the STR circuits with delay shifts can gain 3 db jitter improvement no matter the intersymbol interference is relative small.

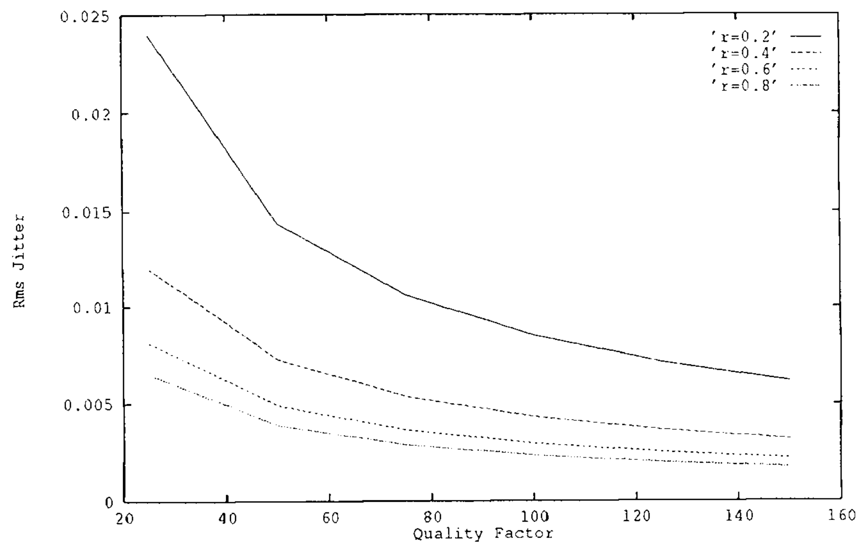
As the quality factor increase, the ratio of error terms to mean square value decrease and jitter improvement is not significant, especially for high Q (Q=150), the ratio is only 1/4, this occurs because when the quality of postfilter is large, the spectrum of timing wave much more look like a line spectrum, therefore the jitter is small and improvement is not significant.

Table 4.3 The Value of Error Term $Z_1(t) - Z_2(t)$ for $l = 5$

Q	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
25	1.747496e-04	1.008501e-04	6.231426e-05	4.117055e-05
50	1.452411e-04	8.357826e-05	5.176824e-05	3.453624e-05
75	9.694547e-05	5.637615e-05	3.525106e-05	2.372943e-05
100	6.850740e-05	4.027034e-05	2.541502e-05	1.723578e-05
125	5.107493e-05	3.033249e-05	1.931039e-05	1.318172e-05
150	3.966331e-05	2.378646e-05	1.526749e-05	1.048484e-05

Table 4.4 Rms Jitter as a Function of Q for l=10

Q	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
25	0.023973	0.011995	0.008092	0.006500
50	0.014394	0.007272	0.004933	0.003921
75	0.010605	0.005397	0.003679	0.002917
100	0.008496	0.004349	0.002978	0.002361
125	0.007130	0.003670	0.002523	0.002002
150	0.006166	0.003189	0.002201	0.001748



'igure 4.1 Rms Jitter as Function of Quality Factor for l=10

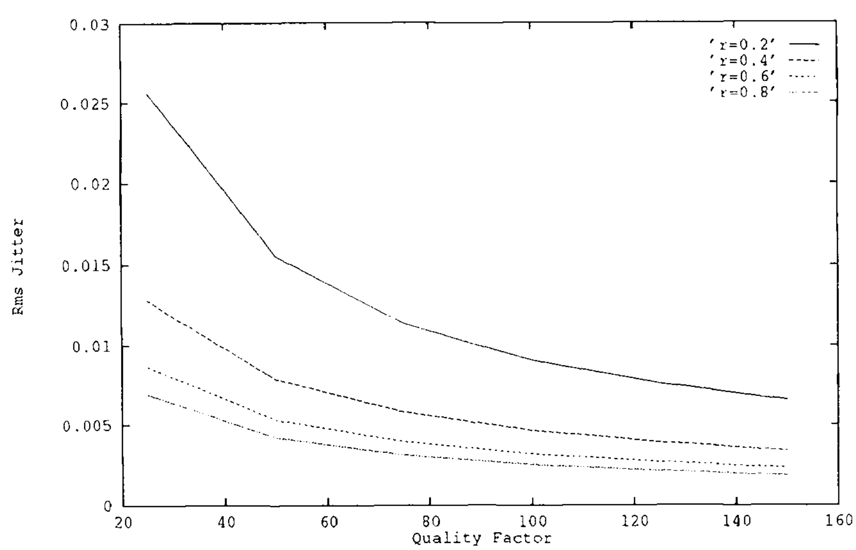


Figure 4.2 Rms Jitter as Function of Quality Factor for l=5

Table 4.5 Rms Jitter as a Function of Q for l=5

Q	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
25	0.025588	0.012816	0.008639	0.006888
50	0.015518	0.007839	0.005310	0.004193
75	0.011383	0.005787	0.003937	0.003105
100	0.009076	0.004639	0.003169	0.002500
125	0.007587	0.003897	0.002672	0.002111
150	0.006540	0.003374	0.002322	0.001836

4.2 The Final Rms Results

The final numerical results of rms jitter is given in Table 4.4 when delay parameter $l = 10$. The data are drawn in Fig. 4.1. We also observe the jitter performance for small delay time $l = 5$. The data and figure are shown in table 4.5 and Fig. 4.2 respectively.

Fig 4.3 shows three curves for $l = 10, l = 5$, and no delay when rolloff factor γ=0.2. Clearly, for small delay time l=5, the jitter performance is improved comparing the system without delay elements. When l becomes larger, the jitter performance is better, for l=10, the jitter performance improvement is nearly 3 db, which is consistent with theoretical results given by (3.15).

Since if we increase rolloff factor, we can get better jitter performance, in other words, the jitter performance improvement is equivalent to bandwidth saving. This concept is important since when we design STR circuits, the jitter performance (error probability) is given and if we design STR circuits with delay elements, we can require small rolloff factor for the channel (small bandwidth) to achieve the desired jitter performance.

Fig.4.4 and 4.5 show three curves of rms jitter as function of rolloff factor for small quality factor Q equals 25 and 50 respectively. As previous mentioned, for large quality factor Q, the jitter is relatively small and jitter improvement is not significant, therefore, we explore the bandwidth save for the case of small quality factor.

Referring to Fig.4.4, the rms jitter for the STR circuits with delay shifts is 0.024 when γ=0.2 and l = 10. For the STR circuits without delay shifts, if we want the same jitter level, the rolloff factor for the prefilter is approximately 0.3, that is to say, the bandwidth save is about 30%. For the case of Q=50, we can get the same result.

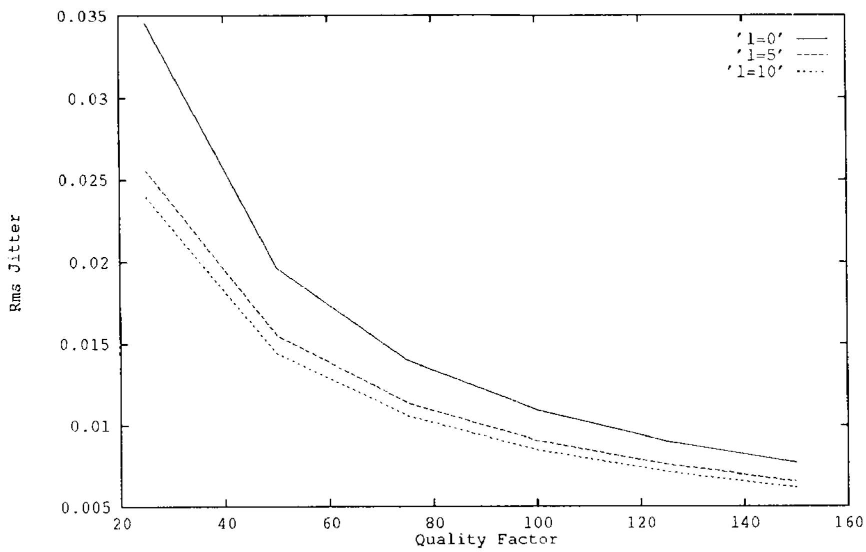


Figure 4.3 Rms Jitter Comparison

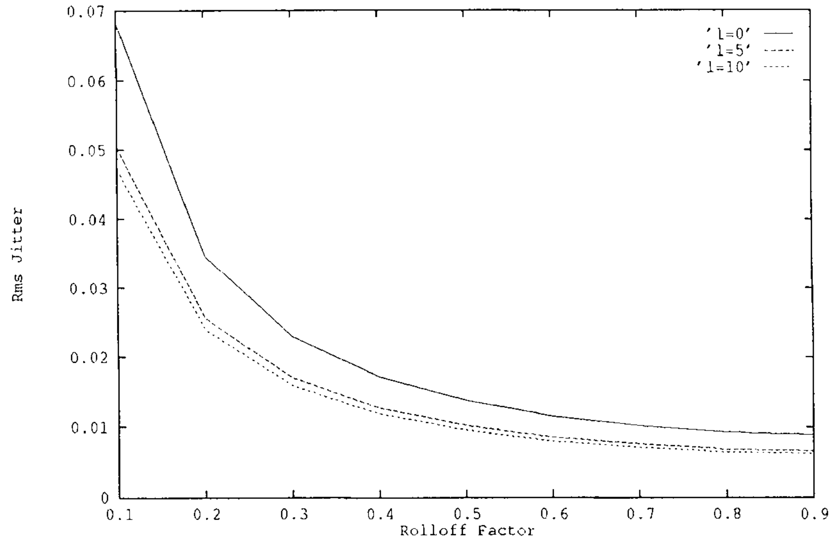


Figure 4.4 Comparison of Rms Jitter as Function of Rolloff Factor for Q=25

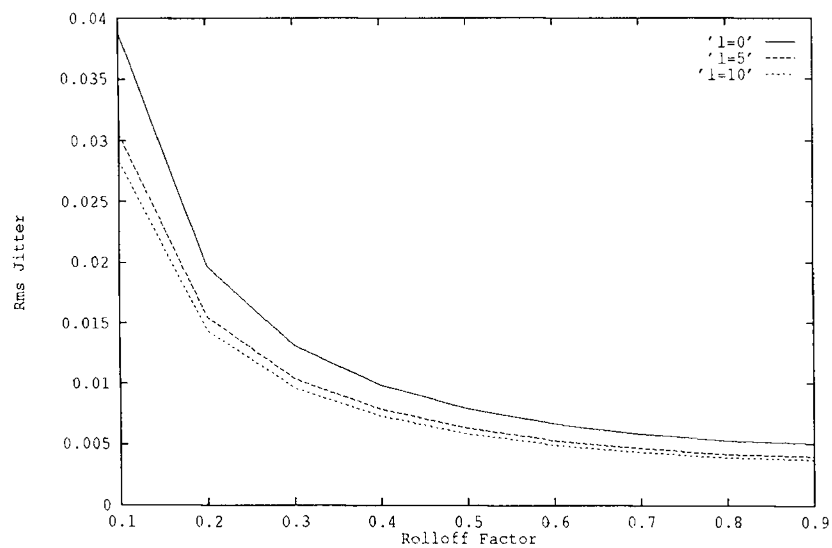


Figure 4.5 Comparison of Rms Jitter as Function of Rolloff Factor for Q=50

CHAPTER 5

CONCLUSION

The symbol timing recovery (STR) circuits using no extra power and bandwidth have been studied for long time. There are many strategies that have been developed so far to achieve the timing recovery. All the strategies are based on the fact that the incoming data sequence for the receiver is CT process, and there are some frequency components of power spectrum at pulse repetition frequency, if we use some kinds of nonlinear device. The function of narrowband filter is simply to pick up this frequency component.

Generally speaking, the principle of different strategies are the same, the difference is mainly at the use of different nonlinear device. In this thesis, we did not explore a new nonlinear to achieve the better jitter performance, however, we use delay shifts to achieve the jitter performance improvement. Because of the mathematical difficulties, we only show the simplest case of one delay shift, and the simulation is based on this simple model. But we can extend this simple model to more complex model by using more than one delay shifts, that is the subject for further research.

From the study of STR circuits with or without delay shifts, we can draw the following conclusion:

1. Rms jitter decreases as the bandwidth of narrowband filter decrease.

2. Rms jitter decreases as rolloff factor increase.

3. The use of delay shifts can improve the jitter performance or save the bandwidth for the channel.

APPENDIX A

EVALUATING THE MEAN VALUE OF A DATA SEQUENCE

Rewrite equation 3.10, we have

$$E[y_2(t-\alpha)y_2(t-\beta) \;=\; \sum_m\sum_n\sum_i\sum_j E[a_m a_n a_i a_j + a_{m+l}a_{n+l}a_{i+l}a_{j+l}$$

$$+a_m a_n a_{i+l}a_{j+l} + a_{m+l}a_{n+l}a_i a_j]$$

$$g(t-\alpha-mT)g(t-\alpha-nT)g(t-\beta-iT)g(t-\beta-jT) \tag{A-1}$$

Comparing the first two terms of equation B-1 with equation 2.17, we have

$$E[y_2(t-\alpha)y_2(t-\beta)] \;=\; E[y(t-\alpha)y(t-\beta)]$$

$$+E[a_m a_n a_{i+l}a_{j+l} + a_{m+l}a_{n+l}a_i a_j]$$

$$g(t-\alpha-mT)g(t-\alpha-nT)g(t-\beta-iT)g(t-\beta-jT) \tag{A-2}$$

Similar as equation 2.18, we have

$$E[a_m a_n a_{i+l}a_{j+l}] \;=\; \delta_{mn}\delta_{ij} + \delta_{m+l,i}\delta_{n+l,j} + \delta_{m+l,j}\delta_{n+l,i} - 2\delta_{m+l,n+l,,j} \tag{A-3}$$

and

$$E[a_{m+l}a_{n+l}a_i a_j] \;=\; \delta_{mn}\delta_{ij} + \delta_{m,i+l}\delta_{n,j+l} + \delta_{m,j+l}\delta_{n,i+l} - 2\delta_{m,n,i+l,j+l} \tag{A-4}$$

The output term results from $\delta_{mn}\delta_{ij}$ can be neglected since it is zero when evaluated at t_0. Therefore, combine remaining positive terms yielt

$$\sum_m \sum_n \sum_i \sum_j (\delta_{m+l,i}\delta_{n+l,j} + \delta_{m+l,j}\delta_{n+l,i} + \delta_{m,i+l}\delta_{n,j+l} + \delta_{m,j+l}\delta_{n,i+l}$$

$$\times g(t-\alpha-mT)g(t-\alpha-nT)g(t-\beta-iT)g(t-\beta-jT)$$

$$= 2\left[\sum_m g(t-\alpha-mT)g(t-\beta-(m+l)T)\right]^2$$

$$+2\left[\sum_m g(t-\alpha-mT)g(t-\beta-(m-l)T)\right]^2 \qquad \text{(A-5)}$$

Combine all negative terms, yield

$$E[\delta_{m+l,n+l,j} + \delta_{m,n,i+l,j+l}]g(t-\alpha-mT)g(t-\alpha-nT)g(t-\beta-iT)g(t-\beta-jT)$$

$$= \sum_m g^2(t-\alpha-mT)g^2(t-\beta-(m+l)T)$$

$$+\sum_m g^2(t-\alpha-mT)g^2(t-\beta-(m-l)T) \qquad \text{(A-6)}$$

Add equation A-5, A-6 into A-2, the final result is given by equation 3.11.

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