ABSTRACT

Forward Kinematics Solution of a Special Class of General Stewart Platforms

by
Jian Li

A Stewart platform is a fully parallel, six-degree-of-freedom manipulator mechanism. A platform manipulator has a fixed platform acting as base, a mobile platform on which end-effector is mounted, and in-parallel kinematics chains (legs) between the two platforms. Although some direct position kinematics solutions for Stewart platforms of simplified geometry have been represented, to the best of our knowledge, no close-form direct kinematics solution for the general Stewart platform is available yet.

A common feature of six-degree-of-freedom Stewart platform of simplified geometry is the use of pairs of concentric ball joints. Due to inevitable manufacturing and assembly errors, practically there are no perfect concentric ball joints. This thesis presents an efficient method for solving the forward kinematics of this class of general Stewart platforms. The approach described in this thesis first finds an initial solution by approximating the original platform with a platform of simplified geometry, then improve the solution with a Jacobian based method. A numerical example is used to illustrate the method.

FORWARD KINEMATICS SOLUTION OF A SPECIAL CLASS OF GENERAL STEWART PLATFORMS

by
Jian Li

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APPROVAL PAGE

Forward Kinematics Solution of A Special Class of General Stewart Platforms

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$\mathbf{P}_r$ = the position of R in base coordinate

$\mathbf{P}_s$ = the position of S in base coordinate

$\mathbf{P}_t$ = the position of T in base coordinate

$\mathbf{P}_{or}$ = the position vector of O_r coordinate

$\mathbf{P}_{os}$ = the position vector of O_s coordinate

$\mathbf{P}_{ot}$ = the position vector of O_t coordinate

$\mathbf{W}_r$ = the unite vector along links O_rR

$\mathbf{W}_s$ = the unite vector along links O_sS

$\mathbf{W}_t$ = the unite vector along links O_tT

m_r = the length of O_rR

m_t = the length of O_tS

m_s = the length of O_sT

b_1 = the length of RS

b_2 = the length of ST

b_3 = the length of RT

PA_1 = the position A_1 in base coordinate

PA_2 = the position A_2 in base coordinate

PB_1 = the position B_1 in base coordinate

PB_2 = the position B_2 in base coordinate

PC_1 = the position C_1 in base coordinate

PC_2 = the position C_2 in base coordinate

β_r = the angles made by the lines perpendicular to C_1C_2

β_s = the angles made by the lines perpendicular to B_1B_2

β_t = the angles made by the lines perpendicular to A_1A_2

ϕ_r = the angle line O_rR and its projection on base coordinate

ϕ_t = the angle line O_tT and its projection on base coordinate

ϕ_s = the angle line O_sS and its projection on base coordinate

L_a = the approximate leg lengths

L_e = the exact leg lengths

L_j = the leg lengths using the Jacobian method

ΔL = the error between the approximate and the exact leg lengths

ΔL_j = the error between the L_j and the exact leg lengths (method 1)

ΔL_{j1} = the error between the L_j and the exact leg lengths (method 2, first iteration)

ΔL_{j2} = the error between the L_j and the exact leg lengths (method 2, second iteration)

CHAPTER 1
INTRODUCTION

Industrial robots have traditionally been used as general-purpose positioning devices and are anthro-promotion open-chain mechanism that generally has the links connected end to end or serially. A radical departure from conventional robot design is to connect the links side by side, in parallel. The advantage of a parallel linkage is the increased rigidity because it doesn't have cantilever-link structure and high force/torque capacity as the actuators are arranged in parallel and near the base. Therefore they are good candidate as high accuracy positioning devices under high loads. The most notable configuration of this kind is a six-degree-of-freedom Stewart platform (Figure 1), which has been originally designed as an aircraft simulator, and later as a robot wrist. A manipulator consists of a base platform, six extensible links, and a moving platform on which end-effector is mounted. The moving platform is connected to the links by the means of spherical joints, the other ends of the links are connected to the base platform through universal joints. Sometime spherical joints are used at both ends of a link. By altering the lengths of the six limbs, the moving platform can be manipulated with respect to the base platform through all six degree of freedom.

Manipulator kinematics is concerned with distances and angles and transitional and angular velocities and accelerations, but not with forces, masses, torques and moments of inertial that are the province of dynamics. The fundamental problem of robot kinematics deal with mapping between vectors in two spaces: joint space θ and Cartesian space $\mathbf{x}$, where θ represents the joints of a robot manipulator and $\mathbf{x}$ represent the position and orientation of the robot end effector. The mapping from joint space to Cartesian is referred to as "direct

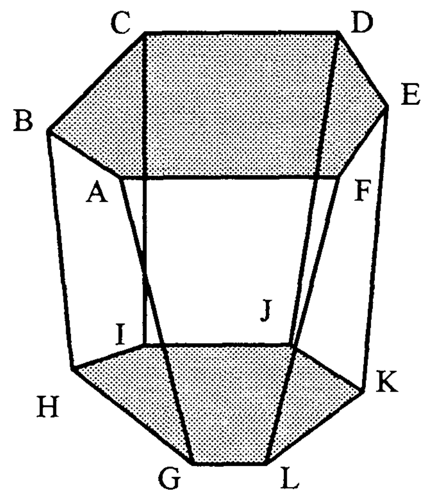


Figure 1. Stewart platform of general geometry

Waldron and Hunt showed that kinematics behavior of parallel manipulator has many dual characteristics to that of serial manipulators. Several examples of such duality were discussed in (4). While inverse kinematics of a serial manipulator is much more difficult than its direct kinematics, the opposite is true for a parallel manipulator. The direct kinematics of Stewart platform can be stated as follow: given the values of the six lengths of the legs (AG, BH, CI, DJ, EK and FL in Figure 1), compute the position and orientation of the moving platform. Due to the difficult in solving direct kinematics of Stewart platform with general geometry, the direct kinematics of simple geometry was studied first. The most simplified form of the mechanism contains six legs which meet in a pair-wise fashion at three points in both the top and base platform. This form of the mechanism is called the "3-3 Stewart Platform". Its direct kinematics was solved by Griffis and Duffffy. Significant progress has been now made in solving direct mechanism is called the "3-3 Stewart Platform". Its direct kinematics was solved by Griffis and Duffffy. Significant progress has been now made in solving direct kinematics problem of platform manipulators, as evidenced by the large number of published reports. Closed-form solutions, expressed as single-variable polynomials, have been obtained for various special platform configurations, such as "3-6", "4-4", "4-5", "4-6" and "5-5" platforms (5-10), where the numbers show the configuration of the platforms. For general "6-6" platforms (Figure 1), Raghavan (11) concluded that there are forty solutions to the direct kinematics, based on a numerical technique known as polynomial continuation. With a mono-dimensional-search algorithm, Innocenti and Parenti-Castelli (12) developed a numerical approach to find all the real solutions to the direct kinematics of the most general Stewart platforms, based on their position analysis of "5-5". To the best of our knowledge, no closed-form direct kinematics solution for the general Stewart platform is available yet.

For all the simplified Stewart platforms, at least one concentric ball joint is used. Due to inevitable manufacturing and assembly errors, practically there is no perfect concentric ball joint between cointersecting connecting legs. Therefore the actual platforms are all of general geometry. Since it is not possible to solve this direct kinematics in closed form, a new method needs to be developed. In the thesis, we present a method for a class of general Stewart platforms whose moving plate has at least one of very closely placed ball joints (Figure 2) either purposely to alleviate design difficulty or as the result of manufacturing and assembly errors. Our approach has three major steps. First, the pair of closely placed joints are approximated by concentric joint, and the position and the orientation of the coordinate frame fixed to the moving plate is obtained for the resulted "Simplified" platform, using the closed-form solution is available, which will bring us quickly to a solution close to the exact solution of the original lengths is then mapped to obtain the position and orientation error of the moving plate. Finally, a correction to the approximation is made to improve the solution.

To illustrate our new method, we implemented our approach with a near "3-6" Stewart platform whose six joints of moving platform from three pairs of closely placed joins shown in Figure 3. In Figure 3, points A, B, C, D, E and F reprsent the three concentric joints of the approximate platform. For a given set of leg lengths, the position and orientation of the moving platform is approximated using the triangle RST. The resulted errors in leg lengths are then used to improve the solution with a Jacobian based method.

The results using Jacobian method is listed at the end of the thesis. The data show the better tendency to modify the approximate leg lengths using jocobian method. It also shows the better results of the Jacobian method when just using a fraction of the error.

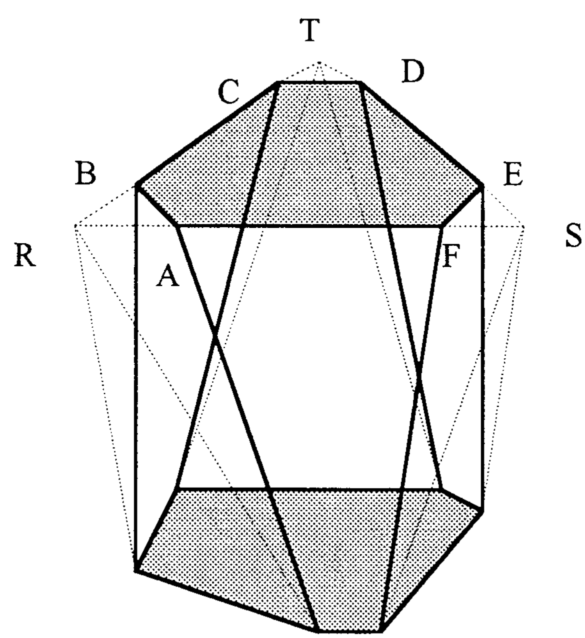


Figure 2. Equivalent form of general Stewart platform

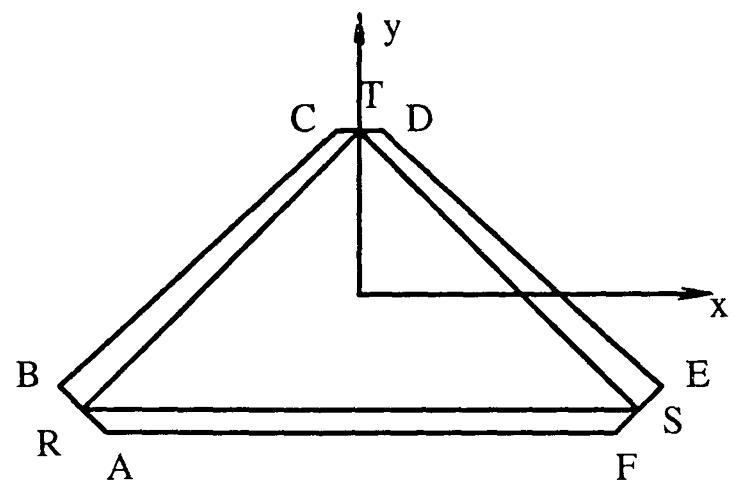


Figure 3. The approximate configuration of "6-6" moving platform

CHAPTER 2

TRIANGLE APPROXIMATION

2.1 General Description of the Approximation

In this chapter, the equations for the direct position kinematics of the "3-6" Stewart platform has been presented, and the solution has been show to be reducible to a 16th-order polynomial equation in $\tan(\phi_r/2)$. This result implies that for a given set of link lengths, the Stewart platform can be assembled in at almost 16 different configurations.

2.2 Equations for Satisfying the Geometry Conditions

First we simplify this problem, assuming that the joint center of pairs of adjacent limbs of the upper platform is coincident. (Figure 4)

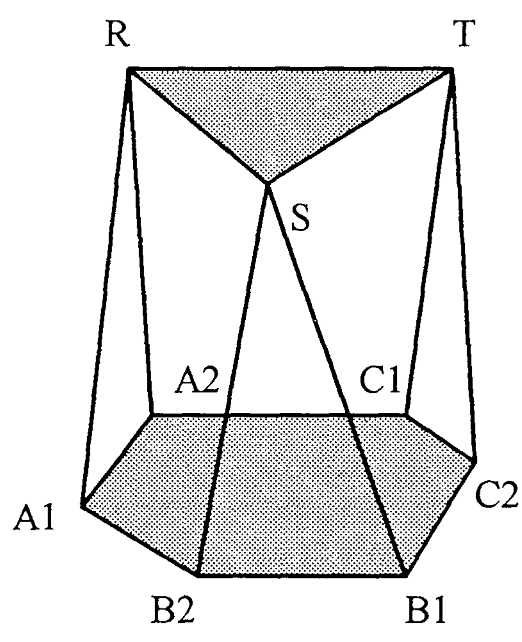


Figure 4. The structure of "3-6" platform

It is shown that the direct kinematics equation for this form of Stewart platform can be put into a form that is similar to those of the triple arm mechanism. Then, we think it without moving platform for the moment. In the triangle A_1RA_2, because R is the joint of two limbs RA_1 and RA_2, according to the geometry of the structure, R must be located either on a sphere with its center at A_1 and radius equal to L_1 or on a sphere with its center at A_2 and radius equal to L_2. Thus the locus of R will be at the intersection of the two spheres, which is a circle with its center located on the line joining the centers of the two spheres. Thus, the triangular structure A_1RA_2 can be replaced by a single link with a revolute joint at one end and a spherical joint at the other end, which is passing through R and perpendicular to the line A_1A_2. The analyses are the same to those of B_1SB_2 and C_1TC_2. We use O_rR, O_sS, O_sT standing for the triangular A_1RA_2, B_1SB_2 and C_1TC_2, see Figure 5.

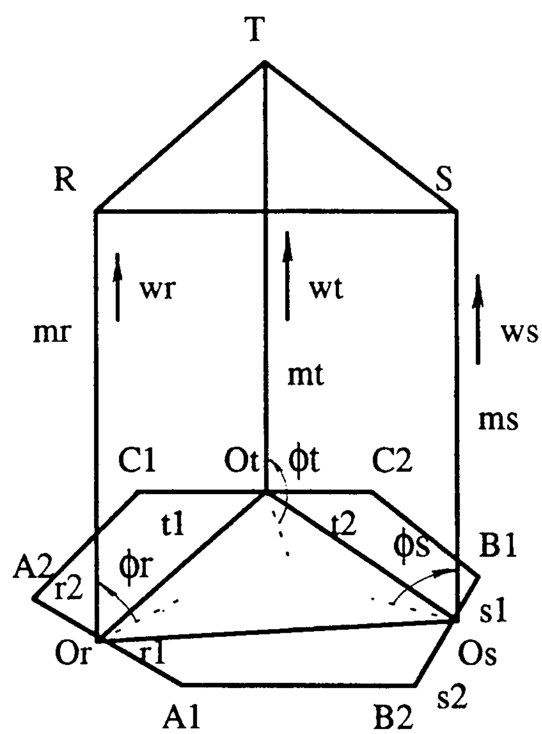


Figure 5. Base and the links with the moving member

We place the fixed coordinate system x-y plane in the base platform, the position vector of R,S,T are determined as following:

$$\begin{aligned}\mathbf{P_r} &= \mathbf{P_{or}} + m_r \cdot \mathbf{W_r}\\ \mathbf{P_s} &= \mathbf{P_{os}} + m_s \cdot \mathbf{W_s}\\ \mathbf{P_t} &= \mathbf{P_{ot}} + m_t \cdot \mathbf{W_t}\end{aligned} \tag{2.1}$$

Still, we should add some constrained equation for R, S, T's moving.

$$\begin{aligned}|\mathbf{P_r} - \mathbf{P_s}|^2 &= b_1^2\\ |\mathbf{P_s} - \mathbf{P_t}|^2 &= b_2^2\\ |\mathbf{P_t} - \mathbf{P_r}|^2 &= b_3^2\end{aligned} \tag{2.2}$$

Let us use the notation to express the limb length as the figure before.

$$r_1 = \frac{A_1^2 + L_1^2 - L_2^2}{2 \cdot A_2}$$

$$r_2 = A_1 - r_1$$

$$s_1 = \frac{A_2^2 + L_6^2 - L_5^2}{2 \cdot A_2}$$

$$s_2 = A_2 - s_1$$

$$t_1 = \frac{A_3^2 + L_4^2 - L_3^2}{2 \cdot A_3}$$

$$t_2 = A_3 - t_1$$

$$m_r = \left(L_1^2 - r_1^2\right)^{1/2}$$

$$m_s = \left(L_6^2 - s_1^2\right)^{1/2}$$

$$m_t = \left(L_4^2 - t_1^2\right)^{1/2}$$

Next

$$\mathbf{PO_r = PA_1 + A_1O_r}$$
$$\mathbf{PO_s = PB_1 + B_1O_s}$$
$$\mathbf{PO_t = PC_1 + C_1O_t}$$

$$\mathbf{A_1O_r} = r_1 \cdot \frac{\mathbf{PA_2 - PA_1}}{|\mathrm{PA_2 - PA_1}|}$$

$$\mathbf{B_1O_s} = s_1 \cdot \frac{\mathbf{PB_2 - PB_1}}{|\mathrm{PB_2 - PB_1}|}$$

$$\mathbf{C_1O_t} = t_1 \cdot \frac{\mathbf{PC_2 - PC_1}}{|\mathrm{PC_2 - PC_1}|}$$

At last,

$$W_r = \cos\beta_r \cos\phi_r i + \sin\beta_r \cos\phi_r j + \sin\phi_r k$$
$$W_s = \cos\beta_s \cos\phi_s i + \sin\beta_s \cos\phi_s j + \sin\phi_s k$$
$$W_t = \cos\beta_t \cos\phi_t i + \sin\beta_t \sin\phi_t j + \sin\phi_t k$$

$$\beta x = \cos^{-1}\left[\frac{\{(P_{Y2} - P_{Y1}) \times k\} \cdot i}{|(P_{Y2} - P_{Y1})|}\right]$$

or

$$\beta x = \sin^{-1}\left[\frac{\{(P_{Y2} - P_{Y1}) \times k\} \cdot i}{|(P_{Y2} - P_{Y1}) \times k\}|}\right]$$

Where (x, y) = (R, A), (S, B) or (T, C).

2.3 Solve the Polynomial Equation

So far, we have all the kinematics equations to solving our problem.

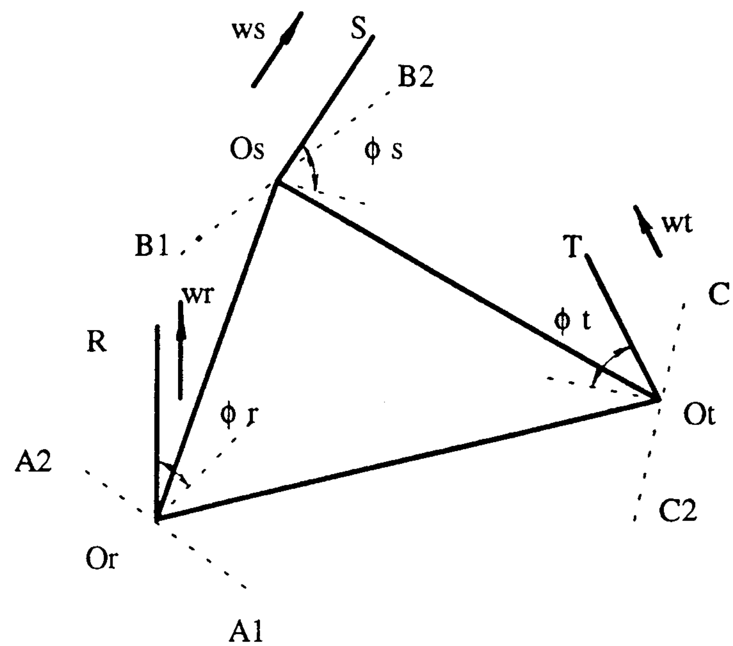


Figure 6. Description of vector

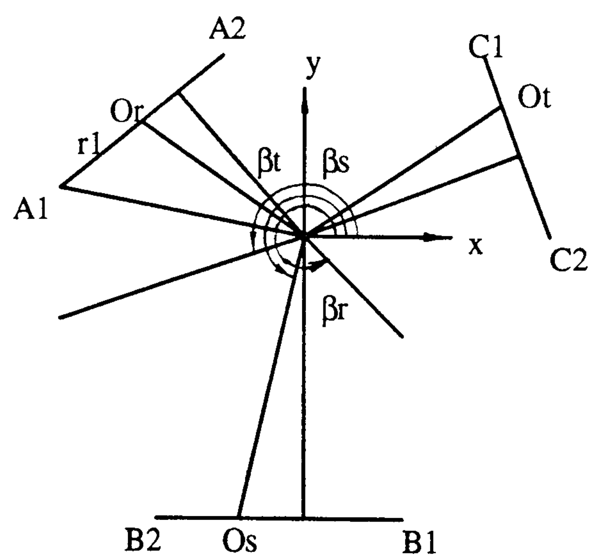


Figure 7. Description of angle

Substitute of the expressions for **Pr** and **Ps**, **Ps** and **Pt**, **Pr** and **Pt** from (2.1) into (2.2), we simplify them and get the following equations.

$$D(1)\cdot\cos\phi_r + D(2)\cdot\cos\phi_s + D(3)\cdot\cos\phi_r\cdot\cos\phi_s + D(4)\cdot\sin\phi_r\cdot\sin\phi_s + D(5)$$
$$E(1)\cdot\cos\phi_s + E(2)\cdot\cos\phi_t + E(3)\cdot\cos\phi_t\cdot\cos\phi_s + E(4)\cdot\sin\phi_t\cdot\sin\phi_s + E(5)$$
$$F(1)\cdot\cos\phi_t + F(2)\cdot\cos\phi_r + F(3)\cdot\cos\phi_t\cdot\cos\phi_r + F(4)\cdot\sin\phi_t\cdot\sin\phi_r + F(5)$$

In between:

$$D(1) = 2\cdot m_r\cdot\cos\beta_r\{(P_{or})_X - (P_{os})_X\} + 2\cdot m_r\cdot\sin\beta_r\cdot\{(P_{or})_Y - (P_{os})_Y\}$$
$$D(2) = -2\cdot m_s\cdot\cos\beta_s\{(P_{or})_X - (P_{os})_X\} - 2\cdot m_s\cdot\sin\beta_s\cdot\{(P_{or})_Y - (P_{os})_Y\}$$
$$D(3) = -2\cdot m_r\cdot m_s\cdot\cos(\beta_r - \beta_s)$$
$$D(4) = -2\cdot m_r\cdot m_s$$
$$D(5) = \{(P_{or})_X - (P_{os})_X\}^2 + \{(P_{or})_Y - (P_{os})_Y\}^2 + m_r^2 + m_s^2 - b_1^2$$

$$E(1) = 2\cdot m_s\cdot\cos\beta_s\{(P_{os})_X - (P_{ot})_X\} + 2\cdot m_s\cdot\sin\beta_s\cdot\{(P_{os})_Y - (P_{ot})_Y\}$$
$$E(2) = -2\cdot m_t\cdot\cos\beta_t\{(P_{os})_X - (P_{ot})_X\} - 2\cdot m_t\cdot\sin\beta_t\cdot\{(P_{os})_Y - (P_{ot})_Y\}$$
$$E(3) = -2\cdot m_s\cdot m_t\cdot\cos(\beta_s - \beta_t)$$
$$E(4) = -2\cdot m_s\cdot m_t$$
$$E(5) = \{(P_{os})_X - (P_{ot})_X\}^2 + \{(P_{os})_Y - (P_{ot})_Y\}^2 + m_s^2 + m_t^2 - b_2^2$$

$$F(1) = 2\cdot m_t\cdot\cos\beta_t\{(P_{ot})_X - (P_{or})_X\} + 2\cdot m_t\cdot\sin\beta_t\cdot\{(P_{ot})_Y - (P_{or})_Y\}$$
$$F(2) = -2\cdot m_r\cdot\cos\beta_r\{(P_{ot})_X - (P_{or})_X\} - 2\cdot m_r\cdot\sin\beta_r\cdot\{(P_{ot})_Y - (P_{or})_Y\}$$
$$F(3) = -2\cdot m_t\cdot m_r\cdot\cos(\beta_t - \beta_r)$$
$$F(4) = -2\cdot m_t\cdot m_r$$
$$F(5) = \{(P_{ot})_X - (P_{or})_X\}^2 + \{(P_{ot})_Y - (P_{or})_Y\}^2 + m_t^2 + m_r^2 - b_3^2$$

Let $$Xi = \tan\left(\frac{\phi_i}{2}\right) \qquad (i = R, S, T)$$

$$\cos\phi_i = \frac{1 - X_i^2}{1 + X_i^2}$$

$$\sin\phi_i = \frac{2 \cdot X_i^2}{1 + X_i^2}$$

Solve D, E, F simultaneously to determine the possible values of $\phi r, \phi s, \phi t$.

We get:

$$[G(1)\cdot X_r^2 + G(2)]\cdot X_s^2 + [G(3)\cdot X_r)]\cdot X_s + [G(4)\cdot X_r^2 + G(5)] = 0$$
$$[H(1)\cdot X_t^2 + H(2)]\cdot X_s^2 + [H(3)\cdot X_t)]\cdot X_s + [H(4)\cdot X_t^2 + H(5)] = 0$$
$$[I(1)\cdot X_r^2 + I(2)]\cdot X_t^2 + [I(3)\cdot X_r)]\cdot X_t + [I(4)\cdot X_r^2 + I(5)] = 0$$

$$G(1) = -D(1) - D(2) + D(3) + D(5)$$
$$G(2) = D(1) - D(3) - D(3) + D(5)$$
$$G(3) = 4 \cdot D(4)$$
$$G(4) = -D(1) + D(2) - D(3) + D(5)$$
$$G(5) = D(1) + D(2) + D(3) + D(5)$$

$$H(1) = -E(1) - E(2) + E(3) + E(5)$$
$$H(2) = E(1) - E(3) - E(3) + E(5)$$
$$H(3) = 4 \cdot E(4)$$
$$H(4) = -E(1) + E(2) - E(3) + E(5)$$
$$H(5) = E(1) + E(2) + E(3) + E(5)$$

$$
\begin{aligned}
I(1) &= -F(1) - F(2) + F(3) + F(5) \\
I(2) &= F(1) - F(3) - F(3) + F(5) \\
I(3) &= 4 \cdot F(4) \\
I(4) &= -F(1) + F(2) - F(3) + F(5) \\
I(5) &= F(1) + F(2) + F(3) + F(5)
\end{aligned}
$$

Using Bezout's method (13), we eliminate Xs from equations of G(i) and H(i) (i=1,......5) to get:

$$
\begin{vmatrix}
\begin{vmatrix} G(1)\cdot X_r^2 + G(2) & G(4)\cdot X_r^2 + G(5) \\ H(1)\cdot X_t^2 + H(4) & H(2)\cdot X_r^2 + H(5) \end{vmatrix} & \begin{vmatrix} H(3)\cdot X_t & G(3)\cdot X_r \\ H(1)\cdot X_t^2 + H(4) & G(1)\cdot X_r^2 + G(2) \end{vmatrix} \\
\begin{vmatrix} G(3)\cdot X_r & G(4)\cdot X_r^2 + G(5) \\ H(3)\cdot X_t & H(2)\cdot X_t^2 + H(5) \end{vmatrix} & \begin{vmatrix} G(1)\cdot X_r^2 + G(2) & G(4)\cdot X_r^2 + G(5) \\ H(1)\cdot X_t^2 + H(4) & H(2)\cdot X_t^2 + H(5) \end{vmatrix}
\end{vmatrix} = 0
\qquad (2.3)
$$

We simplify equation (2.3) as:

$$
\begin{aligned}
& J_1 \cdot X_t^4 + J_2 \cdot X_t^3 + J_3 \cdot X_t^2 + J_4 \cdot X \cdot J_5 = 0 \\
& J_1 = K_1 \cdot X_r^4 + K_2 \cdot X_r^2 + K_3 \\
& J_2 = K_4 \cdot X_r^3 + K_5 \cdot X_r \\
& J_3 = K_6 \cdot X_r^4 + K_7 \cdot X_r^2 + K_8 \\
& J_4 = K_9 \cdot X_r^3 + K_{10} \cdot X_r \\
& J_5 = K_{11} \cdot X_r^4 + K_{12} \cdot X_r^2 + K_{13}
\end{aligned}
$$

K1, K2, ..., K13 are constants computer from about G's and H's. We can rewrite the equation about I's as:

$$M_1 = I_1 \cdot Xr^2 + I_4$$
$$M_2 = I_3 \cdot Xr$$
$$M_3 = I_2 \cdot Xr^2 + I_5$$

Using the same method of Bezout's, eliminate Xt from J's and M's equation:

$$\begin{vmatrix} J_2 \cdot M_1 - J_1 \cdot M_2 & J_3 \cdot M_1 - J_1 \cdot M_3 & J_4 \cdot M_1 & J_5 \cdot M_1 \\ J_3 \cdot M_1 - J_1 \cdot M_3 & J_3 \cdot M_2 - J_2 \cdot M_3 + J_4 \cdot M_1 & J_4 \cdot M_2 + J_5 \cdot M_1 & J_5 \cdot M_2 \\ M_1 & M_2 & M_3 & 0 \\ 0 & M_1 & M_2 & M_3 \end{vmatrix} = 0 \qquad (2.4)$$

Then we get the equation as following:

$$\begin{aligned} &A(9) \cdot Xr^{16} + A(8) \cdot Xr^{14} + A(7) \cdot Xr^{12} + A(6) \cdot Xr^{10} + A(5) \cdot Xr^{8} \\ &+ A(4) \cdot Xr^{6} + A(3) \cdot Xr^{4} + A(2) \cdot Xr^{2} + A(1) = 0 \end{aligned} \qquad (2.5)$$

The coefficients A(1), A(2), ..., A(9) are computed from K1, K2,......., K10, I1, I2, ..., I6.

We evaluate the determinations in equation (2.3) and (2.4) with "MATHEMATICA" in symbolic forms. The symbolic solution is then used in our program to get the coefficient of equation (2.5).

The polynomial equation (2.5) have some complex roots, we just disregard those conjugate complex results of Xr's.

As we described before

$$\cos\phi_i = \frac{1 - X_i^2}{1 + X_i^2}$$

$$\sin \phi_i = \frac{2 \cdot X_i^2}{1 + X_i^2}$$

Substitute above values into F's, E's equation, we can change the equation form into as below:

$$P \cdot \cos\phi + Q \cdot \sin\phi + R = 0$$

It is easy to solve this equation as

$$\cos\phi = \frac{-P \cdot R \pm Q \cdot (P^2 + Q^2 - R^2)^{1/2}}{P^2 + Q^2}$$

$$\sin\phi = \frac{-Q \cdot R \pm P \cdot (P^2 + Q^2 - R^2)^{1/2}}{P^2 + Q^2}$$

Now we get the results of ϕ_s, ϕ_t consequently.

2.4 Some Comments on Solving "3-6" Platform

So far, the three pairs of closely joints are approximated by three pairs of concentric joints. The position and the orientation of the coordinate frame. The position fixed to the moving plate can be obtained for the resulted of "3-6" platform, using the close-form solution discussed above.

After we find all the real solutions of "3-6" platform, we can substitute them into the equation (2.1) to get the final positions of R, S and T. Except those of the values not satisfying the constrained equation (2.2). The approximated position and orientation of the moving platform is now obtained.

2.5 About the Computer Program

The program consists of main and sub-programs. In main program, we solve the simple parameter such as D's, E's, F's and G's, H's, I's, and calculate the coefficients of the polynomial from the symbolic equations obtained with " MATHEMATICA."

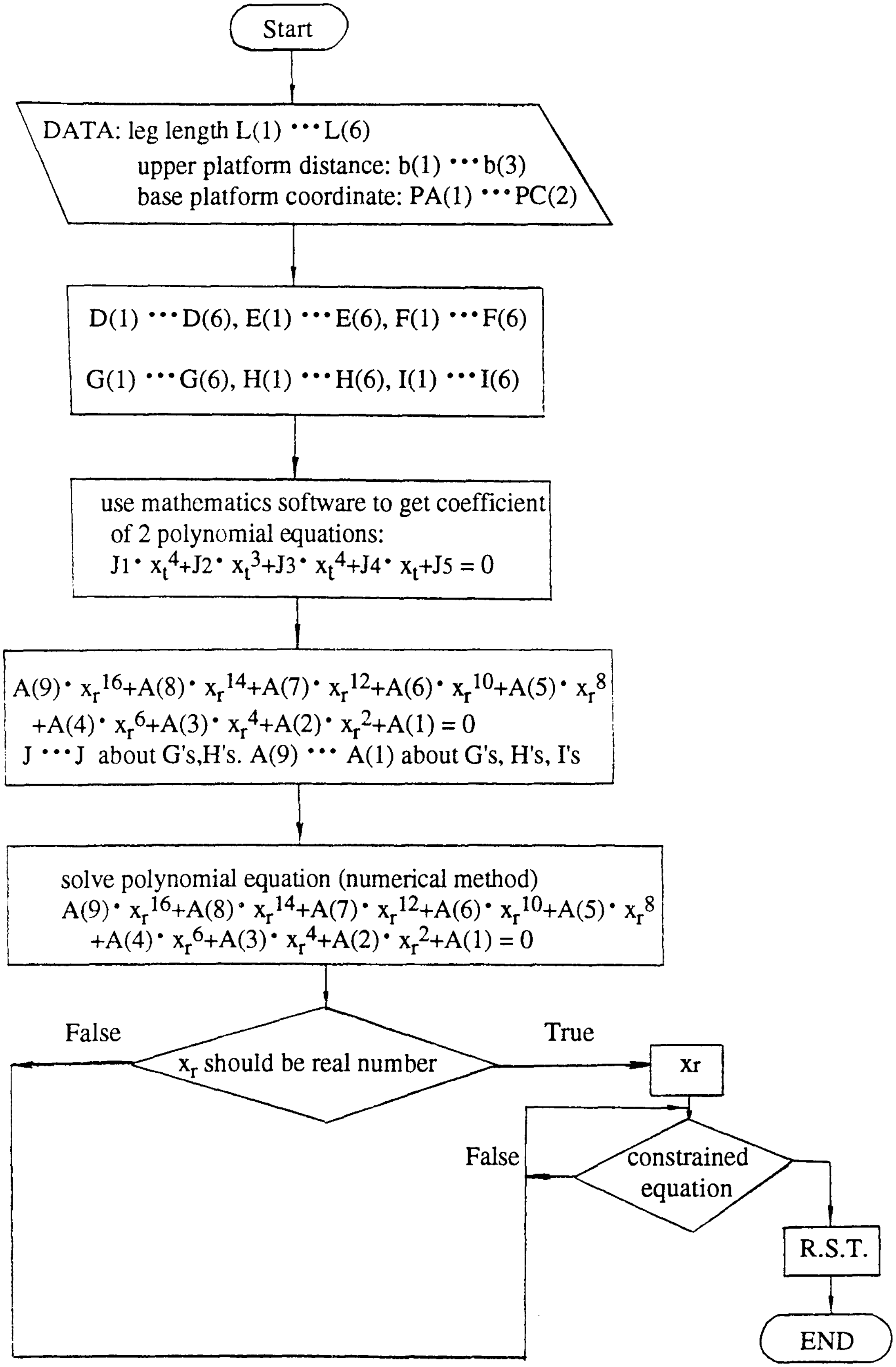
The sub-programs include solving the polynomial equation and checking the computing angle which quadrant is located, check the value of the angle whether it satisfies the constrained equation (2.2).

In order to increase the precision, "Double Precision" is used for all real number.

Because of the existence of complex numbers, we use FORTRAN language to implement our method.

2.6 Computer Diagram

Following is the computer program diagram we used to solve the moving position of "3-6" platform. The inputs to the program include the side lengths of triangle, and the coordinates of joints on the base platform. Through the procedure described in the chapter, we get the polynomial equation of degree sixteen, using the numerical method to get the results of the equation and at last reach the position of the moving platform.


Start
DATA: leg length L(1) ••• L(6)
upper platform distance: b(1) ••• b(3)
base platform coordinate: PA(1) ••• PC(2)
D(1) ••• D(6), E(1) ••• E(6), F(1) ••• F(6)
G(1) ••• G(6), H(1) ••• H(6), I(1) ••• I(6)
use mathematics software to get coefficient
of 2 polynomial equations:
$J1 \cdot x_t^4 + J2 \cdot x_t^3 + J3 \cdot x_t^4 + J4 \cdot x_t + J5 = 0$
$A(9) \cdot x_r^{16} + A(8) \cdot x_r^{14} + A(7) \cdot x_r^{12} + A(6) \cdot x_r^{10} + A(5) \cdot x_r^8$
$+ A(4) \cdot x_r^6 + A(3) \cdot x_r^4 + A(2) \cdot x_r^2 + A(1) = 0$
J ••• J about G's,H's. A(9) ••• A(1) about G's, H's, I's
solve polynomial equation (numerical method)
$A(9) \cdot x_r^{16} + A(8) \cdot x_r^{14} + A(7) \cdot x_r^{12} + A(6) \cdot x_r^{10} + A(5) \cdot x_r^8$
$+ A(4) \cdot x_r^6 + A(3) \cdot x_r^4 + A(2) \cdot x_r^2 + A(1) = 0$
False
True
x_r should be real number
xr
False
constrained equation
R.S.T.
END

CHAPTER 3

IMPROVEMENT OF APPROXIMATE SOLUTION

3.1 Jacobian Method

For the description of the relative configuration of the two plates, a base coordinates frame B(O, x, y, z) and a moving frame M(P, x', y', z') are defined on the two plates respectively, as shown in Figure 8. The origins of the two frames are located at the centers of the corresponding plates. With this definition, the configuration of the moving plate with respect to the base plate can be described by a vector **P** directed to point P from O, and a rotation matrix T representing the orientation of the moving frame M with respect to the base frame B. The geometries of the two plates can be described by vector $\mathbf{u}_i$ and $\mathbf{r}_i$ here i=1, 2, ..., 6, as indicated in figure 1. Clearly, each of the two vectors sets $\{\mathbf{u}_i\}$ and $\{\mathbf{r}_i\}$ is coplanar. Moreover, vector $\mathbf{u}_i$ has constant components in frame B, while the components of vector $\mathbf{r}_i$ are orientation-dependent in B.

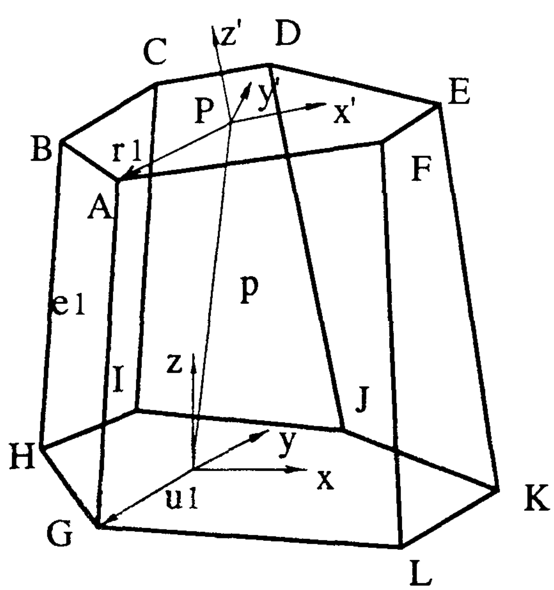


Figure 8. Illustration of position vector

Where **ri**' is the representation of vector **ri** in the moving plate. With the above definitions, it is a simple matter to derive basic kinematics constraints of the manipulator, namely,

$$(\mathbf{P}+\mathbf{ri} -\mathbf{ui})^T (\mathbf{P}+\mathbf{ri} -\mathbf{ui}) = q_i \tag{3.1}$$

Here q_i denotes the coordinate representing the leg length of the *i*th leg. Furthermore, to treat the orientation and translation in the same frame, a 6-D twist vector of the end-effector is defined as:

$$\mathbf{t} = \begin{bmatrix} \mathbf{w} \\ \mathbf{p} \end{bmatrix}$$

Upon differentiating the constraint equations with respect to time, we get the desired relation between the end-effector twist and the joint velocity, the form is

$$\mathbf{At} = \mathbf{Bq}$$

Where

$$A = \begin{bmatrix} \mathbf{r1} \times (\mathbf{p} - \mathbf{u1}) & \cdot & \cdot & \cdot & \mathbf{r6} \times (\mathbf{p} - \mathbf{u6}) \\ \mathbf{r1} \times \mathbf{p} - \mathbf{u1} & \cdot & \cdot & \cdot & \mathbf{r6} \times \mathbf{p} - \mathbf{u6} \end{bmatrix}^T$$

$$q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \end{bmatrix}^T$$

$$B = \mathrm{diag}\,(q_1, q_2, ..., q_6)$$

We define the Jacobian matrix of the manipulators under study as the mapping from the end-effector twist vector to the joint velocity vector, namely,

$$\mathbf{Jt} = \mathbf{q}$$

$$\mathbf{J} = \mathbf{B}^{-1}\mathbf{A}$$

$$\mathbf{J} = \begin{bmatrix} \mathbf{r1} \times \mathbf{e1} & \cdot & \cdot & \cdot & \mathbf{r6} \times \mathbf{e6} \\ \mathbf{e1} & \cdot & \cdot & \cdot & \mathbf{e6} \end{bmatrix}^T \tag{3.2}$$

Since the exact solution to the position and orientation of the moving frame is unknown, we use the closed-form solution of "3-6" that is easily to find "6-6"

Since the exact solution to the position and orientation of the moving frame is unknown, we use the closed-form solution of "3-6" that is easily to find "6-6" platform as the exact leg lengths (figure 2). Then use three pairs of concentric joints to approximate. For a given set of leg lengths L_i (i = 1, ..., 6), represented by a rotation matrix R_a and position vector P_a in the reference frame {B} attached to the base frame.

Let us express the relation between the exact solution and the approximate solution as

$$R = \Delta R_a \quad \text{and} \quad \mathbf{P} = \mathbf{P_a} + \Delta \mathbf{P} \tag{3.3}$$

Where

$$\Delta = \begin{bmatrix} 1 & -\delta\theta z & \delta\theta y \\ \delta\theta z & 1 & -\delta\theta x \\ -\delta\theta y & \delta\theta x & 1 \end{bmatrix} \tag{3.4}$$

and Δ is a differential displacement:

$$\Delta\mathbf{P} = [\delta x, \delta y, \delta z] \tag{3.5}$$

The error for the *i*th legs is

$$\Delta L_i = L_i - L_a \tag{3.6}$$

By defining two 6 x 1 vectors

$$\Delta t = [\delta\theta x \quad \delta\theta y \quad \delta\theta z \quad \delta x \quad \delta y \quad \delta z]^T$$

And

$$\Delta L = [\Delta L_1 \quad \Delta L_2 \quad \Delta L_3 \quad \Delta L_4 \quad \Delta L_5 \quad \Delta L_6]^T$$

$$J \cdot \Delta t = \Delta L \tag{3.7}$$

First, from Eqs.(3.1), (3.6) and (3.3) and the approximate solution represented by R_a and P_a, we can evaluate the error in leg lengths ΔL and matrix J. Second, $\Delta \mathbf{t}$ is solved from Eq.(3.7). In this step, we find if the magnitude of vector ΔL is not small enough, then we should divided one step of correction into two or more. Third, correction in the form of matrix Δ and the vector Δp are calculated from Eqs (3.4), (3.5). Finally, the improved solution is computed from Eq. (3.2).

3.2 The Idea of Solving the Problem

As described before, general Stewart platform with moving plate has three pairs of very closely placed ball joints, either purposely to alleviate design difficulty or as the result of manufacturing and assembly errors. The forward kinematics of this class of Stewart platform is much complex than that of the platform with simplified geometry, which usually include three steps.

(1) The three pairs of closely placed joints are approximated by three pairs of concentric joints, and the position and the orientation of the coordinate frame fixed to the moving plate is obtained for the resulted "3-6" platform, using the closed-form solution (first part).

(2) The actual leg lengths corresponding to this approximation are now calculated and compared with the given leg lengths. The difference in the leg lengths mapped to obtain the position and the orientation error of the moving plate.

(3) Correction to the approximate solution is made to obtain new solution.

First step:

Assuming a set of leg lengths, with base and moving platform (six points) in Figure 1 which represented by the solid lines. There also exist three points on the moving platform such that, when these three points are used as three pairs of concentric ball joints, they should be the same corresponding leg lengths with the concentric ball joints, they should be the same corresponding leg lengths with the same position and orientation of a frame {M} attached to the moving plate, but since there is no prior knowledge of the position and orientation of the moving plate, it is impossible to locate these three points on the moving plate, furthermore, these three points could be anywhere if they satisfy the moving platform (six points) position and orientation. However, since each pair of joints of two points is closely placed together in the platforms under our discus, they can be approximated as three pairs of concentric joints. Solution based on the approximate "3-6" platform will be close to the exact solution and is therefore a good initial data for further improvement.

Since the exact solution to the position and orientation of the moving plate is unknown, the error of the approximation can only be measured in terms of the leg lengths.

Following are the procedure to solve this problem. For a given set of leg lengths Li (i = 1,2, ..., 6), we use forward kinematics ("3-6") method to find the mid-points of each pair of joint respectively, thus alternatively, we can get this each pair of joint with simple mathematical calculation. That is to say, we use the triangle approximation for obtaining an initial solution of leg lengths for known parameter. Since the exact solution to the position and orientation of the moving frame is unknown only the error of the leg lengths can be measured. The detail of the calculation is described as following.

3.3 The Way to Solve the Problem

For known the leg lengths Li (i = 1,2,...6), the moving platform's distances, the coordination of the base platform, we use the program described in chapter one and get the positions' results of R, T, S (the points of triangle of moving platform). From these three points, we calculate corresponding relative six points that have

From these three points, we calculate corresponding relative six points that have the same position and orientation frame attached to the moving platform of these three points. Now we use six points' leg lengths Lei (calculated by using Te)and Te (same as the above three points) as a reference. Let Re, Pe, represent an exact solution's (six points) rotation matrix and original position. Now choose the middle point of each pair of closely joints of these six points as the approximate triangle. Using the same "3-6" platform program to get the R', T', S' when knowing leg lengths (now are equal to the six points) and distances of the moving platform, also can get the corresponding rotate matrix Ta and original matrix Pa. Then use the same way to calculate the corresponding relative six points (we choose the same six points as before). Then calculate the approximate leg lengths according to the Ta and Lai.

Now let us express the relation between the exact solution and the approximate solution as

$$R = \Delta \cdot R_a$$

and

$$\mathbf{P} = \mathbf{P}_a + \Delta\mathbf{P}$$

Where Δ is a differential rotation matrix:

$$\Delta = \begin{bmatrix} 1 & -\delta\theta z & \delta\theta y \\ \delta\theta z & 1 & -\delta\theta x \\ -\delta\theta y & \delta\theta x & 1 \end{bmatrix}$$

and $\Delta\mathbf{P}$ is a differential displacement:

$$\Delta\mathbf{P} = [\delta x, \delta y, \delta z]$$

With some mathematical manipulation to equation, we get the following relation by neglecting terms that contain product of $\delta\theta x$, $\delta\theta y$, $\delta\theta z$, δx, δy, δz, we get the equation:

$$J \cdot \Delta t = \Delta L$$

$$\Delta t = [\delta\theta x \quad \delta\theta y \quad \delta\theta z \quad \delta x \quad \delta y \quad \delta z]^T$$

And

$$\Delta L = [\Delta L_1 \quad \Delta L_2 \quad \Delta L_3 \quad \Delta L_4 \quad \Delta L_5 \quad \Delta L_6]^T$$

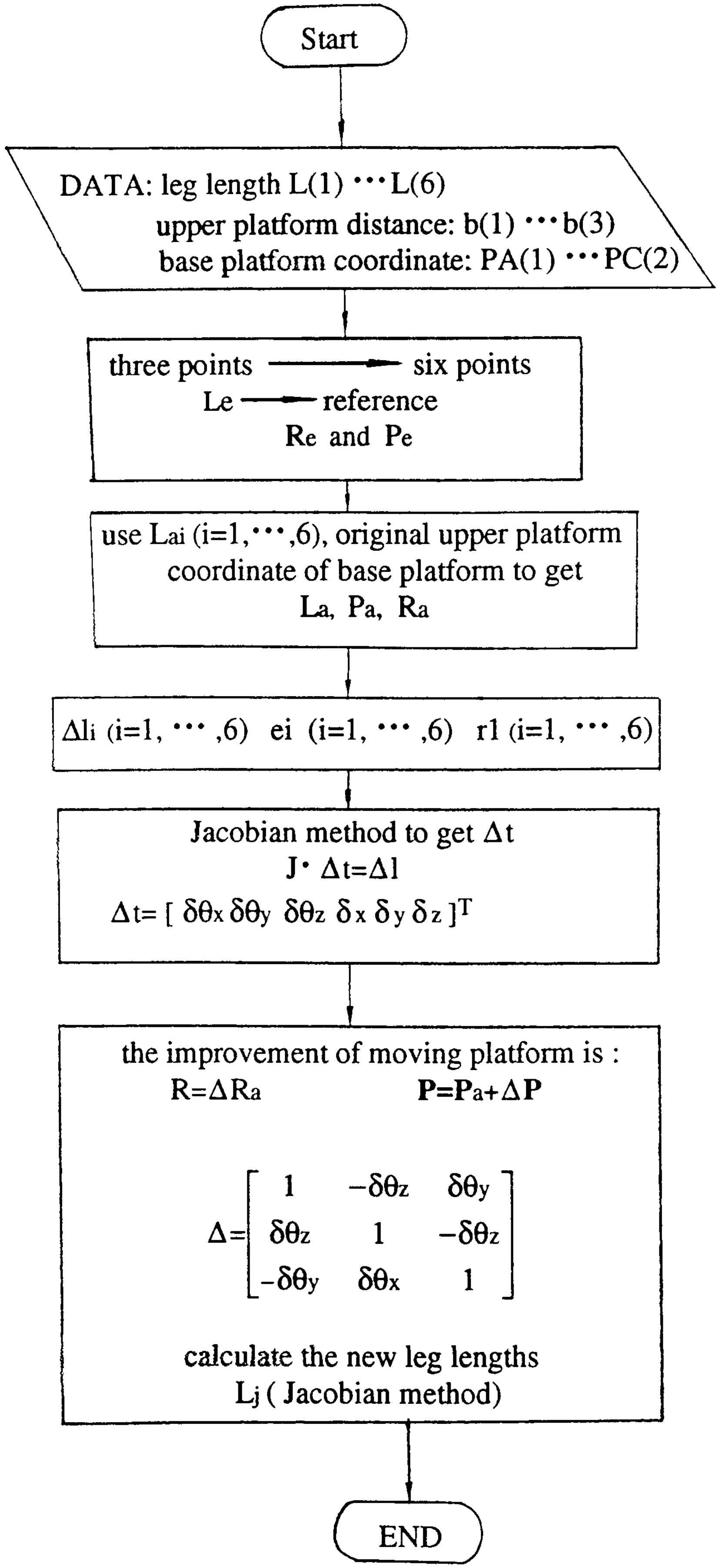
For J $\Delta t = \Delta L$, knowing J matrix and ΔL, we can get Δt directly and that provides a way to estimate how the initial approximation should be connected. Following is how to get improving solution for correcting.

When we get Δ from above procedure, the improved solution then should be computed.

$$\begin{cases} \mathbf{R} = \Delta \mathbf{R_a} \\ \mathbf{P} = \mathbf{P_e} + \Delta \mathbf{P} \end{cases}$$

The procedure may be repeated if necessary to better the solutions.

3.4 The Computer Program Diagram


Start
DATA: leg length L(1) ••• L(6)
upper platform distance: b(1) ••• b(3)
base platform coordinate: PA(1) ••• PC(2)
three points → six points
Le → reference
Re and Pe
use Lai (i=1,•••,6), original upper platform
coordinate of base platform to get
La, Pa, Ra
Δli (i=1, ••• ,6) ei (i=1, ••• ,6) r1 (i=1, ••• ,6)
Jacobian method to get Δt
J• Δt=Δl
Δt= [δθx δθy δθz δx δy δz]T
the improvement of moving platform is :
R=ΔRa
P=Pa+ΔP
Δ=
1 −δθz δθy
δθz 1 −δθz
−δθy δθx 1
calculate the new leg lengths
Lj (Jacobian method)
END

CHAPTER 4

NUMERICAL EXAMPLE

4.1 The Geometry and Dimension of the "6-6" Platform

The "6-6" platform (Figure 9) consists of the base platform (Figure 10) and the moving platform (Figure 11), also six extensible link lengths with spherical or universal joints at both ends of the link.

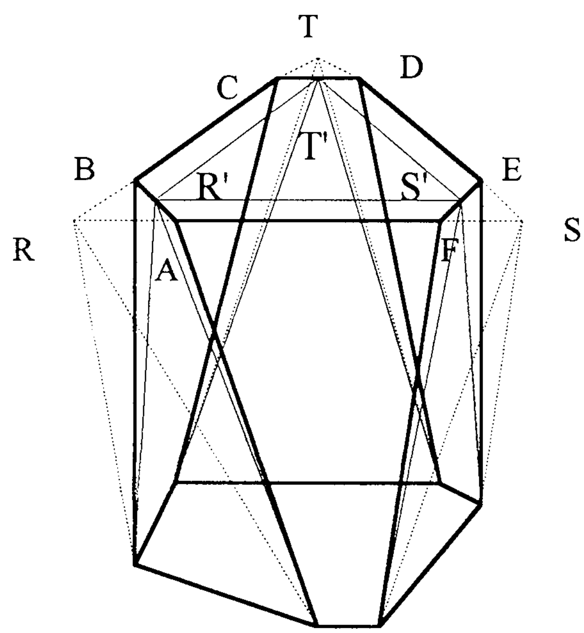


Figure 9. Description of "6-6" platform

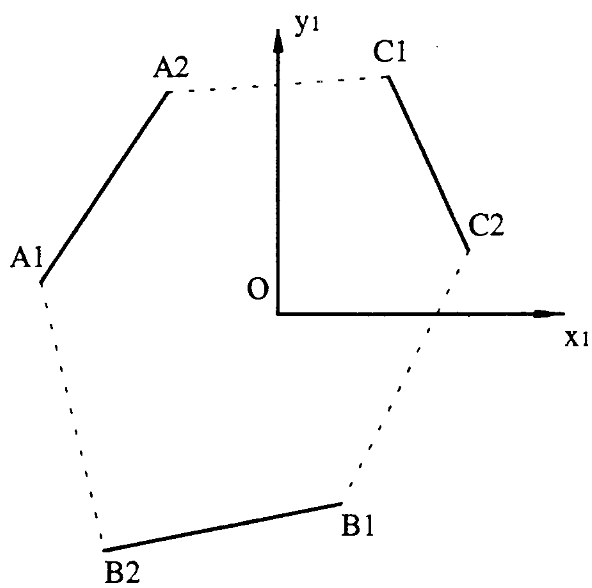


Figure 10. The geometry relation about the base platform

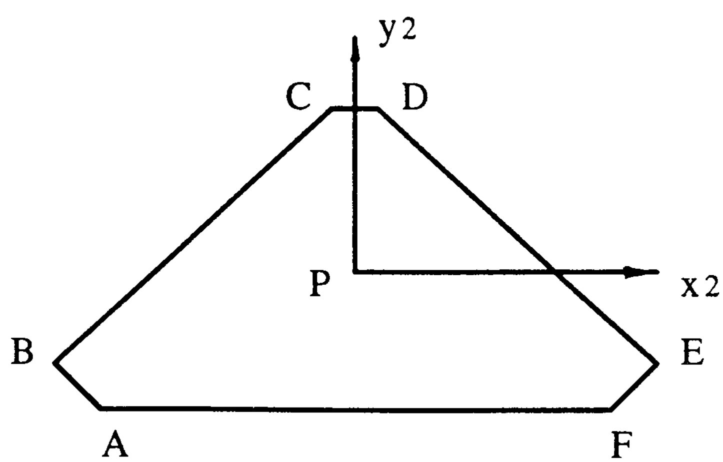


Figure 11. The geometry relation about the moving platform

When we use "3-6" platform to find the initial solution (or the exact solution) of T_e and L_e, its dimension and geometry is following (Figure 3 and 9).

The initial data include leg lengths Li:

L_1=5.0 L_2=4.5 L_3=5.0 L_4=5.5 L_5=5.5 L_6=5.7

The distance of the moving platform:

RT=2.5 TS=2.5 RS=2.5

The position of A, B, C, D,E, F in moving platform's coordinate:

Suppose we give AR=d=0.001

F_1= -d/2	F_2= -2.5 • sin(60) • 2/3+d • sin(60)	F_3=0.0
E_1=d/2	E_2=F_2	E_3=0.0
D_1=2.5/2 - d • cos(60)	D_2=1/3 • 2.5 • sin(60)-d • sin(60)	D_3=0.0
C_1=2.5/2 - d	C_2=2.5/6sin(60)	C_3=0.0
B_1=-C_1	B_2=C_2	B_3=0.0
A_1=-D_1	A_2=D_2	A_3=0.0

The distance of the triangle of R'T'S':

R'T'=2.4958 T'S'=2.4958 R'S'=2.4958

The dimension of base platform are:

PA_1(-2.9, -0.9, 0.0) PA_2(-1.2, 3.0, 0.0)

PB_1(1.3, -2.3, 0.0) PB_2(-1.2, -3.7, 0.0)

PC_1(2.5, 4.1, 0.0) PC_2(3.2, 1.0, 0.0)

4.2 The Results

Let suppose

$$\Delta L = L_a - L_e$$

$$\Delta L_j = L_j - L_e$$

$$\Delta L_{j1} = L_{j1} - L_e$$

$\Delta Lj2 = Lj2 - Le$

For the first result of polynomial equation:

$\phi r = 97.98$
$\phi s = 51.06$
$\phi t = 56.67$

Le	4.713	5.422	5.627	6.731	5.509	5.682
La	4.709	5.433	5.609	6.748	5.536	5.719
Lj	4.713	5.429	5.574	6.716	5.504	5.676
ΔL	-0.004	0.011	-0.018	0.017	0.027	0.037
ΔLj	0.000	0.007	-0.053	-0.015	-0.005	-0.006

Le	4.713	5.422	5.627	6.731	5.509	5.682
La	4.709	5.433	5.609	6.748	5.536	5.719
Lj1	4.709	5.434	5.560	6.705	5.488	5.660
Lj2	4.711	5.431	5.592	6.733	5.521	5.698
ΔL	-0.004	0.011	-0.018	0.017	0.027	0.037
ΔLj1	0.000	0.012	-0.067	-0.026	-0.021	-0.022
ΔLj2	-0.002	0.009	-0.035	0.002	-0.012	0.016

The second result of polynomial equation

$\phi r = 101.02$
$\phi s = 28.40$
$\phi t = 54.91$

Le	5.186	5.612	5.531	6.629	5.522	5.675
La	5.179	5.623	5.515	6.645	5.564	5.695
Lj	5.223	5.588	5.516	6.621	5.569	5.680
ΔL	-0.007	0.011	-0.016	0.016	0.044	0.020
ΔLj	0.037	-0.024	-0.015	-0.008	0.047	0.020

Le	5.186	5.612	5.531	6.629	5.522	5.675
La	5.179	5.623	5.515	6.645	5.564	5.695
Lj1	5.206	5.623	5.505	6.621	5.568	5.694
Lj2	5.249	5.632	5.524	6.612	5.575	5.732
ΔL	-0.007	0.011	-0.016	0.016	0.042	0.020
ΔLj1	0.020	0.011	-0.026	-0.008	0.046	0.001
ΔLj2	0.070	0.020	-0.007	-0.033	0.053	0.057

For the third results of the polynomial equation:

ϕr = 103.92
ϕs = 38.35
ϕt = 18.23

Le	4.990	4.506	4.986	5.509	6.691	6.247
La	4.991	4.517	4.966	5.531	6.264	6.023
Lj	4.888	4.487	4.900	5.519	6.115	5.900
ΔL	0.001	0.011	-0.020	0.022	-0.427	-0.224
ΔLj	-0.102	-0.019	-0.086	0.010	-0.576	-0.347

Le	4.990	4.506	4.986	5.509	6.691	6.247
La	4.991	4.517	4.966	5.531	6.264	6.023
Lj1	4.941	4.502	4.924	5.516	6.184	5.952
Lj2	4.802	4.491	4.830	5.484	5.994	5.788
ΔL	0.001	0.011	-0.020	0.022	-0.427	0.224
ΔLj1	-0.049	-0.004	-0.062	0.007	-0.507	0.290
ΔLj2	-0.188	-0.015	-0.156	-0.025	-0.697	-0.459

For the forth results of polynomial equation

$\phi r = 56.59$
$\phi s = 20.88$
$\phi t = 48.73$

Le	5.106	5.345	6.285	6.856	5.510	5.698
La	5.093	5.337	6.283	6.847	6.302	6.824
Lj	5.124	5.428	6.283	6.778	6.313	6.875
ΔL	-0.013	-0.008	-0.002	-0.009	0.792	0.126
ΔLj	0.018	0.083	-0.002	-0.078	0.803	0.177

Le	5.106	5.345	6.285	6.856	5.510	5.698
La	5.093	5.337	6.283	6.847	6.302	5.824
Lj1	5.125	5.428	6.283	6.778	6.313	5.875
Lj2	5.236	5.589	6.194	6.563	6.316	5.937
ΔL	-0.013	-0.008	-0.002	-0.009	0.792	0.126
ΔLj1	0.019	0.083	-0.002	-0.078	0.803	0.177
ΔLj2	0.130	0.244	-0.091	-0.293	0.806	0.239

CHAPTER 5

CONCLUSIONS

This thesis deals with the direct kinematics of a special class of general Stewart platforms, whose moving plates contains some closely placed joints. This class of Stewart platforms can be approximated with platform of simpler geometry so that closed-form solution can be found as initial solutions. A Jacobian based correction method is then used to improve the initial solutions.

Since the correction with Jacobian method is based on the linear relation. It is sensitive to the initial approximation errors. Multi-step correction may be needed when the errors in the initial approximation is large.

A numerical example for a Stewart platform that can be approximated by a "3-6" platform is presented.

The effectiveness of the method developed in the thesis is supported by the results obtained from the numerical example.

APPENDIX PROGRAM FOR CALCULATION OF SPECIAL CLASS OF GENERAL STEWART PLATFORM

```
C  MAIN  PROGRAM ( METHOD 1 )
     CALL  LEG ( L, R, S, T )
     DATA     L/5.0, 4.5, 5.0, 5.5, 5.5, 5.7/
     CALL  SEL1( R, S, T, RR1, SS1, TT1 )
     CALL  NEWPS ( RR1, TT1, SS1, LE, PE, RE )
     CALL  DATA3 ( RR2, TT2, SS2 )
     CALL  TTF ( RR2, CA, CB, CC, PE, RR )
     CALL  TTF ( TT2, CA, CB, CC, PE, TT )
     CALL  TTF (  SS2, CA, CB, CC, PE, SS )
     CALL  DD1 ( SS, PB2, LA(5) )
     CALL  DD1 ( SS, PB1, LA( 6) )
     CALL  DD1 ( TT, PC2, LA(3) )
     CALL  DD1 ( TT, PC1, LA(4) )
     CALL  DD1 ( RR, PA1, LA(1) )
     CALL  DD1 ( RR, PA2, LA(2) )
     DO 5  I = 1, 6
     LA1(I) = LA(I)
   5 CONTINUE
     ID1 = (LA1 - LE)/2.0
     CALL  WPS (  ID1, RE, PE, WP )
     CALL  IP ( WP, C1 )
     DO 10 I=1, 3
     DO 10 J=1,3
     TA(I,J)=RE( I,J )
```

```
10 CONTINUE
   TA(1,4)=PE(1)
   TA(2,4)=PE(2)
   TA(3,4)=PE(3)
   TA(4,4) = 1.0
   TA(4,1)=0.0
   TA(4,2)=0.0
   TA(4,3)=0.0
   CALL  MUL(C1, TA, TAA)
   PAC=PE+WP
   DO 14  I=1,3
   DO 14  J=1,3
   RAA(I,J)=TAA(I,J)
   CALL  WPP ( PAC, RAA, LAA )
   ID2=LAA - LE
   CALL  WPS (ID2, RAA, PAC, WP2 )
   CALL IP ( WP2, C2 )
   DO 15 I=1,3
   DO 15 J=1,3
   TAA(I,J)=RAA(I,J)
15 CONTINUE
   TAA(4,1)=0.0
   TAA(4,2)=0.0
   TAA(4,3)=0.0
   TAA(4,4)=1.0
   TAA(1,4)=PAC(1)
   TAA(2,4)=PAC(2)
```

```
      TAA(3,4)=PAC(3)
      PAC2=PAC+WP2
      CALL  MUL( C2, TAA, TAA2 )
      DO 20 I=1,3
      DO 20 J=1,3
      RAA2(I,J)=TAA2(I,J)
   20 CONTINUE
      CALL  WPP(  OAC2, RAA2, LAA2)
      ID3 = LAA2 - LE
      STOP
      END
```

```
    C MAIN PROGRAM ( METHOD 2 )
      CALL  LEG ( L, R, S, T)
      DATA  L/5.0, 4.5, 5.0, 5.5, 5.5, 5.7/
      CALL  SEL1 ( R, S, T, RR1, SS1, TT1 )
      CALL  NEWPS ( RR1, TT1, SS1, LE, PE, RE )
      CALL  DATA3 ( RR2, TT2, SS2 )
      CALL  TTF ( RR2, CA, CB, CC, PE, RR )
      CALL  TTF ( TT2, CA, CB, CC, PE, TT )
      CALL  TTF (  SS2, CA, CB, CC, PE, SS )
      CALL  DD1 ( SS, PB2, LA(5) )
      CALL  DD1 ( SS, PB1, LA( 6) )
      CALL  DD1 ( TT, PC2, LA(3) )
      CALL  DD1 ( TT, PC1, LA(4) )
      CALL  DD1 ( RR, PA1, LA(1) )
      CALL  DD1 ( RR, PA2, LA(2) )
```

```
      DO 5  I= 1, 6
      LA1(I) = LA(I)
    5 CONTINUE
      ID = LA1 - LE
      CALL  WPS (  ID , RE, PE, WP )
      CALL  IP ( WP, C1 )
      DO 10 I=1, 3
      DO 10 J=1,3
      TA(I,J)=RE(I,J)
   10 CONTINUE
      TA(1,4)=PE(1)
      TA(2,4)=PE(2)
      TA(3,4)=PE(3)
      TA(4,4)=1.0
      TA(4,1)=0.0
      TA(4,2)=0.0
      TA(4,3)=0.0
      CALL  MUL(C1, TA, TAA)
      PAC=PE+WP
      DO 14  I=1,3
      DO 14  J=1,3
      RAA(I,J)=TAA(I,J)
   14 CONTINUE
      CALL  WPP ( PAC, RAA, LAA )
      ID1=LAA - LE
      STOP
      END
```

```
SUBROUTINE  NEWPS
PE(1) = ( R(1)+S(1)+T(1))/3.0
PE(2) = ( R(2)+S(2)+T(2))/3.0
RE(3) = ( R(3)+S(3)+T(3))/3.0
L1=T(1)-R(1)
M1=T(2)-R(2)
N1 T(3)-R(3)
RE(1,1)=L1/(L1^2+M1^2+N1^2)^1/2
RE(2,1)=M1/(L1^2+M1^2+N1^2)^1/2
RE(2,1)=L2/(L1^2+M1^2+N1^2)^1/2
RE(2,2)=M2/(L1^2+M1^2+N1^2)^1/2
RE(2,3)=N2/(L1^2+M1^2+N1^2)^1/2
RE(3,1)=L3/(L1^2+M1^2+N1^2)^1/2
RE(3,2)=M3(L1^2+M1^2+N1^2)^1/2
RE(3,3)=N3/(L1^2+M1^2+N1^2)^1/2
DO 5  I=1,3
CA( I,1)=RE(I,1)
CB(I,2)=RE(I,2)
CC(I,3)=RE(I,3)
RETURN
END
```

```
SUBROUTINE TF
Z(1)=PE(1)+CA(1)*X(1)+CA(2)*X(2)+CA(3)*X(3)
Z(2)=PE(2)+CB(1)*X(1)+CB(2)*X(2)+CB(3)*X(3)
```

```
Z(3)=PE(3)+CC(1)*X(1)+CC2)*X(2)+CC(3)*X(3)
RETURN
END
```

```
SUBROUTINE LEG
P1=(1-RROOTS)**2/(1+RROOTS**2)
P2=2*RROOTS/(1+RROOTS**2)
CALL FF( F, P1, P2, Q1, Q2, Q11, Q22 )
CALL FF( E, Q1, Q2, Z1, Z2, Z11, Z22 )
CALL FFF (D, P1, P2, Z1, Z2, ZZ1 )
CALL DD( P2, P1 , P )
CALL DD( Q2, Q1, Q )
CALL DD( Z2, Z1, Z )
WR= P/PI*180
WT= Q/PI*180
WS=Z/PI*180
CALL DDD( POR, MR, CSR, P, PR(1), PR(2), PR(3) )
CALL DDD( POT, MT, CST, Q, PT(1), PT(2), PT(3) )
CALL DDD( POS, MS, CSS, Z, PS(1), PS(2), PS(3) )
RETURN
END
```

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