ABSTRACT

Performance Of The Stirling Cycle Thermal Regenerator

by
Cesar A. Cravo

Most of the methods developed to analyze the performance of regenerators make assumptions which are not valid in Stirling cycle regenerators. To more adequately describe the conditions of an actual Stirling cycle regenerator a more complex method has been investigated. This method takes into account the time dependence of the mass flow and pressure fluctuations and considers the temperature dependence of the thermophysical properties.

The solution is accomplished by finite difference techniques. The solution determines the temperature distributions of the gas and matrix along the length of the regenerator and calculates the effectiveness over a cycle. A wide range of parameters can be varied in the analysis including pressure, mass flow rate, speed of operation and size. In general it was found that the effectiveness decreased with an increase in the mass flow rate but increased with an increase in the speed of operation. Variations in the pressure and phase angle had little influence on the effectiveness. An increase in the matrix size resulted in an increase in the effectiveness of the regenerator.

PERFORMANCE OF THE STIRLING CYCLE THERMAL REGENERATOR

by
Cesar A. Cravo

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Performance Of The Stirling Cycle Thermal Regenerator

Cesar A. Cravo

Dr. Rajesh N. Dave, Thesis Adviser   (date)
Associate Professor of Mechanical Engineering, NJIT

Dr. Avraham Harnoy, Committee Member   (date)
Associate Professor of Mechanical Engineering, NJIT

Dr. Rong-Yaw Chen, Committee Member   (date)
Professor of Mechanical Engineering, NJIT

BIOGRAPHICAL SKETCH

Author: Cesar A. Cravo

Degree: Master of Science in Mechanical Engineering

Date: October 1993

Undergraduate and Graduate Education:

- Master of Science in Mechanical Engineering,
  New Jersey Institute of Technology, Newark, NJ, 1993

- Bachelor of Science in Mechanical Engineering,
  New Jersey Institute of Technology, Newark, NJ, 1992

Major: Mechanical Engineering

This thesis is dedicated to my parents
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CHAPTER 1

INTRODUCTION

The regenerator is in many ways an ideal heat exchanger. It is able to effect relatively large rates of heat transfer at very high effectiveness values (97% or better are very common) using a very small package and very little material. It is easy to maintain due to the fact that it is highly resistant to fouling. In addition, because of its simple construction and optimum use of material it is often much more economical than a typical counterflow type heat exchanger.

A regenerator is commonly referred to as a thermodynamic 'sponge' (Walker, 1983) because of its ability to alternately absorb and release heat. A simple explanation of its operation can be described as follows. Initially a cold fluid stream is passed through the device, releasing heat from the regenerator packing and reducing its temperature. After a fixed period of time, the cold fluid stream is stopped and is replaced by a warm fluid stream flowing in the opposite direction. The packing absorbs heat from the warm stream and thereby increases its temperature. The process then repeats itself so that the regenerator cyclically absorbs and releases heat. Ideally, the amount of heat absorbed and released by the packing will be equal.

The key to the regenerator's high efficiency is in its construction. Unlike typical counterflow heat exchangers which use separate flow

passages to effect heat transfer across a solid surface (tube wall, etc.), regenerators use a common flow passage and surface (regenerator packing) which is responsible for all heat transfer. The energy to be transferred from one flow to the other must be stored and released from this surface - commonly referred to as the regenerator matrix. Therefore, for high transfer rates and thus efficient operation, a regenerator must consist of a vessel packed with a material of high heat capacity and large surface area.

In the case of a regenerative cryocooler, the regenerator is typically constructed of a thin walled stainless steel or plastic cylinder tightly packed with metallic wires, spheres or mesh. Because of their high heat capacity (relative to the gas flow), lead, brass, phosphor bronze and stainless steel are the most commonly used materials. To achieve a large surface area, high efficiency devices have regenerators with particle sizes that are usually 50-200 μm and wire mesh sizes of number 100-200. For example, a regenerator matrix of 200-mesh screen has a surface area of approximately 7500 square feet per cubic foot of volume.

Such high area densities, unheard of in typical counterflow heat exchangers, imply very small flow passages. As a result, the pressure drop can be significant. Therefore the amount of material in a device and particle size used usually represents a compromise between obtaining high rates of heat transfer and reasonably low pressure drops.

There are a number of theories and correlations developed for the design of regenerators. Unfortunately most of the methods that have been developed are for cases of steady unidirectional flow at a constant gas pressure throughout the cycle - which is typical of operation of regenerators in gas turbines, air liquefaction plants, and air preheaters for boilers. The regenerators in most small to intermediate size cryocoolers operate on cycles (Stirling, Gifford McMahon, Solvay and Vuilleumier) in which the flow is varying in both magnitude and the direction while the gas undergoes large pressure fluctuations. Consequently the methods developed for regenerators in steady flow and constant pressure are not applicable to regenerators used in such devices. Until recently there were no general methodologies and no accepted correlations that are fully appropriate to the design of Stirling regenerators. Methods and correlations from the steady flow, constant pressure model had to be utilized.

CHAPTER 2

LITERATURE SURVEY

2.1 Ideal Regenerator

Ideal regenerators can be thought of as a thermodynamic "black box" (Walker, 1983) accepting a gas at temperature T_c and leaving at a temperature T_h. After a period of time the flow would be reversed and would enter at T_h and leave at T_c. No fluid would remain in the regenerator in the transition from one flow to the other. The amount of heat rejected and absorbed by the regenerator would equal. The pressure drop across the regenerator would be zero.

The ideal regenerator is clearly an impossible achievement. Constant inlet/outlet temperatures are difficult to obtain in many machines and would require either infinitely slow operation or the heat transfer coefficient and/or heat transfer area to be infinite. Alternatively, the heat capacity of the fluid would have to be zero or that of the matrix to be infinite. A pressure drop of zero would require frictionless flow.

2.2 Classical Regenerator Theory (Hausen Model)

A real regenerator operates in a manner far different from that of the ideal regenerator. A real regenerator requires that there be a temperature difference between the gas and the matrix in order for heat to flow from one to the other. As a result, the gas will leave the regenerator cold end with a

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temperature slightly above the expansion space temperature and in the reverse flow direction, will leave the hot end at a temperature slightly below the compression space temperature. In most cases the temperature difference is the same at both ends, because the amount of heat stored in the regenerator and the amount withdrawn are very nearly the same.

A more practical analysis of the operation of a regenerator was made by Hausen (1976). This model assumes that the only method of heat transfer between the flowing gases and the regenerator matrix is by forced convection. The operation of the Hausen regenerator proceeds in the following manner.

A hot gas flow enters at constant temperature T_h, passes through the matrix giving up part of its heat and leaves the regenerator at a temperature lower than at the inlet. The hot gas flow is stopped and all the gas is ejected from the matrix. A cold gas flow entering from the opposite end at a constant temperature T_c, absorbs heat from the matrix given up during the first part of the cycle and leaves at a temperature higher than at the inlet. The cold gas flow is stopped and all gas is ejected from the matrix. This cycle is repeated until a steady state condition is reached such that the temperature at any one point in the regenerator is the same as it was a cycle earlier.

For a regenerator, the average efficiency or heat transfer effectiveness ε for the half cycle during which the matrix is warming up, is given by

$$\varepsilon = (T_h - T_{out})/(T_h - T_c)$$

where T_{out} is the average bulk temperature of the gas after it has passed through the regenerator. Similarly for the half cycle when the matrix is cooling down, the effectiveness is given by

$$\varepsilon = (T_{out} - T_c)/(T_h - T_c)$$

Typically, regenerators used in cryocoolers have effectiveness values of 95% and greater and it is not uncommon to find values approaching 1. As a result a commonly used measure of the performance of a regenerator is the ineffectiveness I, given by the relation:

$$I = 1 - \varepsilon$$

As a result of the ineffectiveness of the regenerator a certain amount of warm fluid enters the expansion space and decreases the amount of refrigeration. The heat leak into the cold volume of the regenerator due to the regenerator ineffectiveness is given by the expression

$$\dot{Q}_e = \dot{m}C_g(T_h - T_c)I$$

where C_g is the specific heat of the gas. For small powerful coolers which operate at high speeds and pressures the mass flow rates can be quite large and thus even a small change in the ineffectiveness, I can have a large effect on the heat leak. This heat leak represents a decrease in the amount of

available refrigeration. In a sample calculation performed by West (1986) he found that a 2% decrease in the effectiveness resulted in a 4% decrease in thermal efficiency. It is evident that good regenerator design is critical to the design of a high efficiency device.

To derive the differential equations for the Hausen model, a number of assumptions must be made. These assumptions are necessary to reduce the resulting equations to form a set of first order partial differential equations. Although greatly simplified, no general analytical solutions for the equations exist, although approximate numerical solutions for particular cases have been calculated by several authors. The idealizations made by the Hausen model are as follows.

1. Thermal conductivity of the matrix is infinite perpendicular to the flow and zero in the direction of the flow.

2. The specific heats of the fluids and of the matrix material do not change with the temperature.

3. Inlet temperatures are constant both over the flow section and with time.

4. The heat-transfer coefficients and fluid velocities are constant with time and space.

5. The mass flow rates of both fluids, although they may be different, are constant with time during the blow period.

6. The blow time is long compared to the time required for a gas particle to pass through the matrix.

7. The entire matrix participates in the heat transfer process.

Assumption one is a very good approximation to the real regenerator for many reasons. For wire screen matrices stacked axially to the flow, heat conduction normal to the flow is fairly good due to the high conductivity of the metals used. Then by making the assumption that the conductivity is infinite normal to the flow, the equations can be made one-dimensional in space. The conduction in the direction of flow is negligible in most cases if care is taken to reduce conduction paths. If wire screen matrices are used the conductivity in the direction of the flow is poor due to the contact resistance between the stacked elements. However, longitudinal heat conduction can be quite significant in the wall of the regenerator and care must be taken to use a material of very low conductivity. In an experimental investigation by Gifford, Acharya and Ackerman (1968) they found a 74% increase in the ineffectiveness in using stainless steel walls over the low thermal conductivity phenolic walls.

Assumption two has been investigated by Saunders and Smoleniec (1951) and they have found that the error in the effectiveness is less than 1% for most cases in assuming average values for the specific heats of the matrix and fluid. Their results are geared to much higher temperatures where the specific heats of the matrix and fluid are fairly linear and do not suffer from large changes. In another theoretical study by Rios and Smith (1968) they found that the variable specific heat of the regenerator matrix

can have a significant effect on the effectiveness. This is due in large part to the large changes in specific heat for most metals at low (cryogenic) temperatures. At low temperatures a decrease in temperature results in a decrease in the specific heat of solids but an increase in the specific heat of gases. Helium, which is the only possible working fluid at low temperatures (helium only liquefies at 4 K), has a high specific heat compared to most metals and increases rapidly at low temperatures. On the other hand, many common matrix materials (copper, bronze, stainless steel, etc.) have a specific heat which decreases with temperature. As a result the heat capacity of the matrix may become comparable or even less than the heat capacity of the gas. When this occurs the matrix is said to be thermally 'saturated.' There must be a sufficient heat capacity in the matrix material so that most of the heat (ideally all) is absorbed during the hot gas blow period. If there isn't sufficient thermal mass in the regenerator, the regenerator will become 'saturated' and large temperature swings within the matrix will result.

Assumptions three, four and five parallel those commonly made in conventional heat exchanger design theory. In most heat exchanger applications the temperature of the inlet fluid is fairly constant once steady state conditions have been reached. However, in many cryocoolers the inlet temperatures to the regenerator can vary considerably although the degree

of the variation and its effect on the performance of the regenerator is not known.

Assumptions four and five complement each other to some extent. That is, for a given mass flow rate the heat transfer coefficient between the matrix and the fluid does not change much. This assumption is fairly consistent with many regenerator applications such as gas turbines where the flow rates are fairly constant. If there are large changes in the mass flow rate, as is encountered in the Stirling cycle cryocooler, the heat transfer coefficient can change considerably.

In addition to the time dependence of the mass flow rate, Jones (1989) has shown that there are large changes in the flow along the cross section of the regenerator for many types of matrix material. In this experimental analysis the outlet flow of the regenerator was examined using a laminar flow profile at the inlet. The difference between the two profiles for common screen matrices is striking. The inlet flow profile is typical of fully developed flow in a pipe - i.e. a smooth parabolic shape. The outlet, however, shows a very large increase in the flow rate near the outer edge of the regenerator. The reason for this disparity is unknown, but to obtain a more even flow profile Jones recommends some remedial measures. No experiments were conducted to determine the effect of using a turbulent flow profile at the inlet.

The time dependence of the gas flow has another effect that can have a significant effect on the performance of the regenerator. Since the flow moves continually back and forth in an oscillatory motion within the regenerator there exists a frequency above which the time between flow reversals is too short to permit establishment of the boundary layer or flow pattern familiar from unidirectional steady flow experiments. The effect is similar to the well known behavior of a fluid stream entering a pipe, where it takes some time to establish the regular boundary layer. This entry flow region has different pressure drop and heat transfer characteristics than the well established flow further down the pipe. Fortunately, the boundary layer establishes itself in less time the smaller the diameter of the flow passage Thus for the fine wire mesh matrices where the flow passages are on the order of 1/1000 of an inch the effect of this entry flow can be considered negligible for all but the highest speed applications.

Assumption six is made in order to neglect the effects of 'carryover' leakage. Carryover leakage occurs in the transition from one flow to another. In this transition period a certain amount of fluid known as the 'carryover leakage' remains in the void spaces of the regenerator. If the blow time is much longer than the time required for a gas particle to move through the regenerator the assumption is that most of the fluid moves through the regenerator and only a small amount of the flow becomes part of the carryover leakage and can be considered negligible. Shah (1981)

presents a highly idealized theory on the losses associated with carryover leakage although it is not known what effect this may have on regenerators in cryocoolers. As will be shown later, in many cryocoolers a large part of the flow is responsible for carryover leakage.

Assumption seven is needed to insure that all the matrix material is utilized in heat transfer. In some regenerators there may be some flow bypass that creates dead regions within the regenerator which are not involved in the heat transfer. Also at high engine speeds where changes in flow direction occur very quickly a limited amount of material may be involved in the regeneration. In such cases the flow rates are so high that the matrix is unable to absorb/release heat at a very quick rate and only a small surface layer of the matrix is involved in heat transfer. To ensure that this does not happen, the matrix material should be made as fine as possible and of a high conductivity so that the entire matrix mass is utilized in the heat absorption/release.

On the basis of these idealizations made before the following set of partial differential equations may be expressed. Applying the first law of thermodynamics to a differential element of gas, we find

$$\frac{hA}{L}(T_g - T_m)dx + Wc_p\frac{\partial T_g}{\partial x}dx + \rho_g c_p A_o\frac{\partial T_g}{\partial t}dx = 0$$

where h = heat transfer coefficient between the gas and the matrix

   A = heat transfer surface area

   L = length of regenerator

T_g = temperature of the gas

T_m = temperature of the matrix

W = mass flow rate

c_p = specific heat of the gas

ρ_g = density of the gas

A_o = cross-sectional area of the regenerator open to the flow

dx = differential element of axial space coordinate x

dt = differential element of time coordinate t

The first term represents the heat transfer from the gas to the matrix over the length dx. The second term represents the change in enthalpy of the gas in the length dx. The third term represents the change in the energy stored in the differential element of gas.

Applying the first law to differential element of the matrix, we find

$$\frac{hA}{L}(T_g - T_m)dx = \frac{M}{L}c_m\frac{\partial T_m}{\partial t}$$

where M = mass of the matrix

c_m = specific heat of the matrix

As before, the term on the left hand side represents the heat transfer from the gas to the matrix. The term on the right hand side represents the change in the energy stored in the differential element of matrix.

In the Hausen theory, the above set of differential equations are written for the hot and cold blow periods, respectively. The above

equations can be simplified by introducing the following dimensionless variables:

$$\xi = \tfrac{x}{L} \qquad \eta = \tfrac{t}{P}$$

where P is the heating or cooling period for the regenerator. Hausen then introduced the dimensionless variables called the reduced length Λ, and the reduced period Π defined as follows:

$$\Lambda = \tfrac{hA}{Wc_p} \qquad \Pi = \tfrac{hAP}{Mc_m}$$

This results in two equations for hot gas flow period and two for the cold gas flow period. The effectiveness is then defined in terms of the set of dimensionless variables Λ_h, Λ_c, Π_h, and Π_c, where the subscripts h and c denote the hot and cold flows respectively.

An alternate method of regenerator analysis using the same assumptions above was proposed by Coppage and London (1953). It defines the effectiveness in terms of the dimensionless variables NTU_o, C_c/C_h, C_r/C_h, $(hA)_c/(hA)_h$ where NTU_o is defined as follows

$$NTU_o = NTU_c \left[\tfrac{1}{1+(hA)_c/(hA)_h} \right]$$

C_c and C_h are the flow heat capacity rates (Wc_p) for the hot and cold sides, respectively. C_r is the capacity rate of the matrix (Mc_m). This method (NTU-ε method) and the method by Hausen ($\Lambda-\Pi$ method) were shown to be equivalent by Shah (1981). A number of numerical solutions (finite difference, finite elements, method of lines, etc.) have been found and

tabulated results are presented for a wide range of cases (Kays and London, 1984, Baclic and Dragutinovic, 1990).

The solutions found using the above theory have proven successful for regenerators found in air liquefaction and gas separation plants, air preheaters, and gas turbines. This is because the assumptions made in the theory are amenable to the operating conditions of the regenerator in such devices. In many small to intermediate size cryocoolers which work on regenerative cycles such as the Stirling, Vuilleumier, Solvay, etc. the flow and heat transfer conditions are much more complex. Attempts to analyze such devices using the $\Lambda-\Pi$ or NTU-ε method requires one to choose average values for all properties of the flow. The results of such an analysis are questionable and Walker (1983) warns that "the performance determined in this way appears so unrealistic that attempts to develop this procedure have been abandoned." Clearly a more complete regenerator theory is required to adequately analyze the regenerator in such cryocoolers.

The more complete analyses are performed with particular reference to those conditions found in a Stirling cycle cryocooler. It is helpful to begin with a simple review of the Stirling thermodynamic cycle.

2.3 Stirling Cycle

The Stirling cycle was invented in 1816 by Robert Stirling. Originally intended for the production of power, it was soon realized that such a

machine could be run in reverse for the purpose of refrigeration. While the Stirling engine has enjoyed little commercial success, the Stirling cycle cooler has been in production for a number of years. It is the preferred cycle for the production of cryogenic temperatures for small scale applications (under 1 kW cooling).

2.3.1 Ideal Cycle

The ideal refrigeration cycle follows a very simple thermodynamic path. The cycle proceeds as follows. See Figure 1.

<u>Process 1 to 2</u>: The working fluid (typically helium) is compressed isothermally in the (hot) compression space and the heat is removed at constant temperature T_h through the use of heat exchangers to the surroundings.

<u>Process 2 to 3</u>: The compressed gas is then transferred into the (cold) expansion space at temperature Tc through a regenerator and a cold heat exchanger in such a way that the net working volume is kept constant.

<u>Process 3 to 4</u>: The gas in the expansion space at T_c is then expanded and the refrigeration achieved is used to extract heat at constant T_c.

<u>Process 4 to 1</u>: The cold gas in the expansion space is transferred back into the compression space at constant volume through the same cold heat exchanger and regenerator and is warmed to T_h.
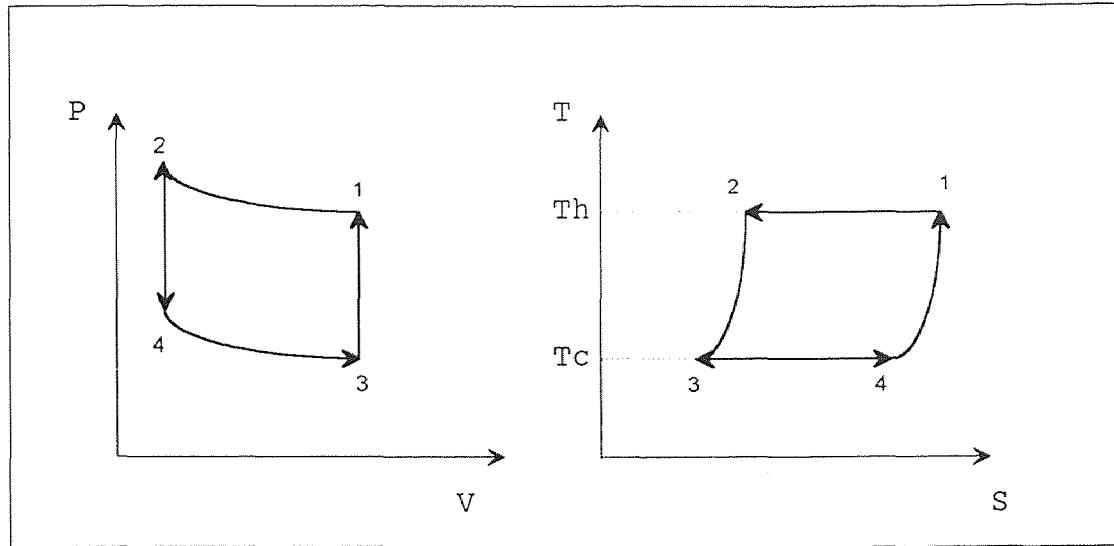
Figure 1 P-V and T-S diagrams for Ideal Stirling Cycle

While the coefficient of performance (COP) for the ideal Stirling cycle is equal to the Carnot cycle, actual COP's are usually only 17% of that value. This is due to the fact that in the translation from theory to actuality, as in most machines, the theoretical cycle looks quite different than the actual.

The ideal cycle assumes that all the working fluid is concentrated in either the compression or expansion spaces. This would require the regenerator matrix and the connecting ducts and associated heat exchangers to have zero void volumes. Some working fluid will always be in the void spaces of the heat exchangers. This fluid has a significant effect on the performance of the cycle because any increase in the void volume results in a reduction of the compression ratio. This in turn reduces the amount of refrigeration capacity.

The ideal cycle also demands that the volume variations occur in a discontinuous motion. This would require a complex mechanical linkage to effect such variations. Instead a common crank-connecting-rod linkage is used and this results in a simple harmonic motion for the volume variations. The change in piston motion from discontinuous to a continuous sinusoidal motion has profound effects on the Stirling cycle. To better approximate the sinusoidal volume motion of an actual cycle a more realistic analysis of the Stirling cycle was developed by Schmidt in 1871.

2.3.2 Schmidt Cycle

Although it retains many of the assumptions of the ideal cycle, the Schmidt cycle gives a better idea of how various parameters affect the performance of a real Stirling cycle device. The theory assumes a sinusoidal motion for the reciprocating elements, but retains the assumption of isothermal compression and expansion and of perfect regeneration. The major assumptions of the Schmidt cycle are as follows (Walker,1983):

1. The regenerator operates ideally, i.e. $\varepsilon = 1$.

2. The instantaneous pressure is the same throughout the system.

3. The working fluid obeys the ideal gas law, $PV = RT$.

4. There is no leakage and the amount of fluid mass remains constant.

5. The volume variations in system occur sinusoidally.

6. The temperature in the compression and expansion spaces is constant.

7. There is perfect mixing of the cylinder contents.

8. The speed of the machine is constant.

9. Steady state conditions have been established.

Using the Schmidt analysis, the performance of the cycle still yields Carnot efficiency since it neglects a number of important losses in the cycle. These include losses attributed to the small regenerator inefficiency, which can be a significant fraction of the net useful cooling power. Also neglected are the thermal conduction losses arising from heat transfer from the hot to the cold space. These losses can be significant especially in small coolers where regenerators tend to be small and compact and the hot and the cold ends are separated by relatively short distance. In systems where a piston reciprocates inside a cylinder, there exists a heat-pumping mechanism, known as shuttle heat transfer, whereby heat is exchanged between the hot and cold spaces, thus introducing a further loss to the cycle.

Furthermore, the assumption of isothermality in the compression and expansion spaces can never be achieved in practice. It would require either infinite rates of heat transfer in the compression and expansion spaces or the engine running at very low speeds. However, in real engines running at realistic speeds (900-3000 RPM), conditions in the cylinders are closer to adiabatic (no heat transfer) than isothermal (infinite heat transfer). The departure from isothermal conditions in the compression and expansion

spaces results in a decrease of refrigerating capacity and an increase in the input work , thereby reducing the COP.

Another effect not accounted for in the Schmidt cycle are pressure losses. There is power lost in moving the gas through the working spaces, the largest contribution is generally from the regenerator which, as explained earlier, is usually made up of fine passages that provide good heat transfer but also have a large resistance to the gas flow.

Nevertheless, the Schmidt cycle analysis provides a good tool in the design of a Stirling cooler because it provides insight into the effect of various parameters on the performance of the cooler and gives an indication of the complex flow patterns that take place. There are a number of parameters defined by the Schmidt Cycle analysis which are used to evaluate the performance of the Stirling cycle (note: the letters c and e denote the compression and expansion spaces, respectively). They are:

1. The temperature ratio, $\tau = Tc/Te$, the ratio of the temperatures in the compression and expansion spaces.

2. The swept volume ratio, $\kappa = Vc/Ve$, the ratio of swept volume in the compression and expansion spaces.

3. The dead volume ratio, $X = Vd/Ve$, the ratio of the total dead volume of heat exchangers and associated ducts to the swept volume of the expansion space.

4. The phase angle α by which volume variations in the expansion space lead those in the compression space.

5. The pressure of the working fluid expressed as the maximum or mean pressure, P_{max} or P_{mean}.

6. The speed of the engine N in cycles per second.

The Schmidt cycle is useful to explore the effects of variation of the above parameters. Using the analysis it can be seen that the refrigerating capacity is a linear function of the engine speed N, the maximum pressure of the working fluid P_{max} and the combined swept volume $Vt=Vc+Ve$.

According to the Schmidt theory, to double the refrigerating capacity one simply doubles the speed N, the maximum pressure P_{max}, or the combined swept volume Vt. In practice, the results are not as straightforward. Increase in the pressure and speed will certainly increase the refrigerating capacity but the improvement occurs at a progressively diminishing rate because of increasing friction losses and thermal saturation of the regenerative matrix.

The effect on performance of the parameters τ, κ, α , and X is less obvious. It is not clear which combinations should be used to achieve the optimum performance. This is important because these parameters are determined at the design stage and except for the temperature ratio τ, cannot be easily changed without a structural change to the engine.

Optimal combinations for several hundred cases were determined and consolidated design charts prepared for refrigerating machines (Walker, 1983). The design chart was prepared for the optimum design in terms of heat lifted Qe, expressed in dimensionless units based on the maximum pressure and the combined swept volume. According to Walker this is the preferred basis for optimization because the maximum pressure is indicative of the weight of an engine and the combined swept volume is indicative of the size. Thus, optimization generates an engine design having the maximum refrigerating capacity for a given size and weight. To use the chart one must choose a temperature ratio, and an appropriate value of the dead-volume ratio and then read off the values of κ, α , and $Qe/P_{max}V_t$.

Using the Schmidt cycle analysis the results for a typical Stirling cryocooler were evaluated. See details in the Appendix.

The Schmidt cycle assumes that the volume variations are sinusoidal functions separated by a phase angle α. This assumption is very good since the volumes are usually displaced using a crank-connecting-rod mechanism which operates in a sinusoidal fashion. The volume variations are represented by the equations below.

$$Vcomp(\phi) = \tfrac{1}{2}\kappa Ve(1 + \cos(\phi - \alpha)) \qquad\qquad Vexp(\phi) = \tfrac{1}{2}Ve(1 + \cos\phi)$$

Again α represents the phase angle by which the expansion space volume leads the compression space volume, ϕ represents the crank angle. The volume variations in the compression and expansion spaces is shown in

Figure 2. Using the pressure function derived by the analysis, P-V diagrams for the expansion and compression spaces can be generated. See Figures 3 and 4. As can be clearly seen, the more realistic P-V diagram looks very little like the box-like diagram of the ideal thermodynamic cycle.
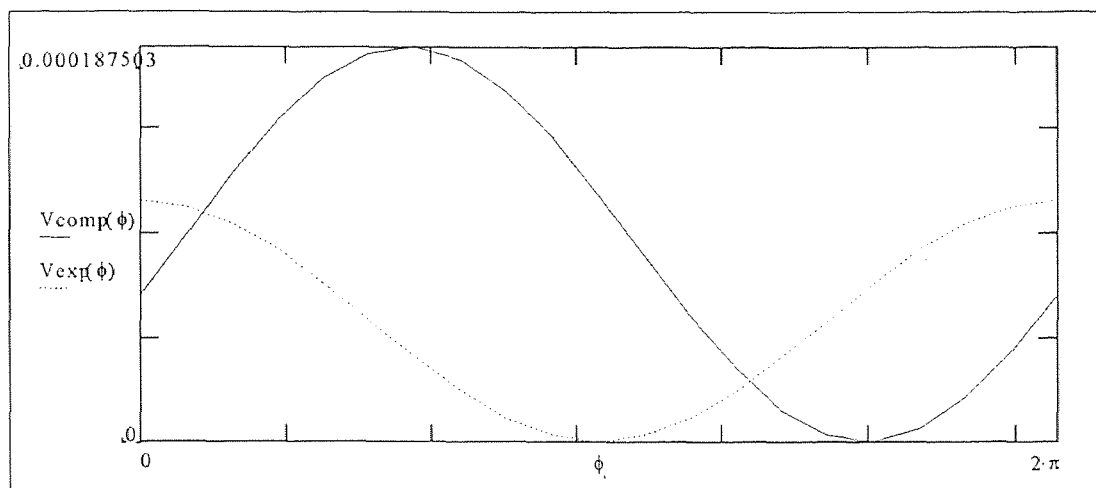
Figure 2 Volume Variations in Compression and Expansion Spaces
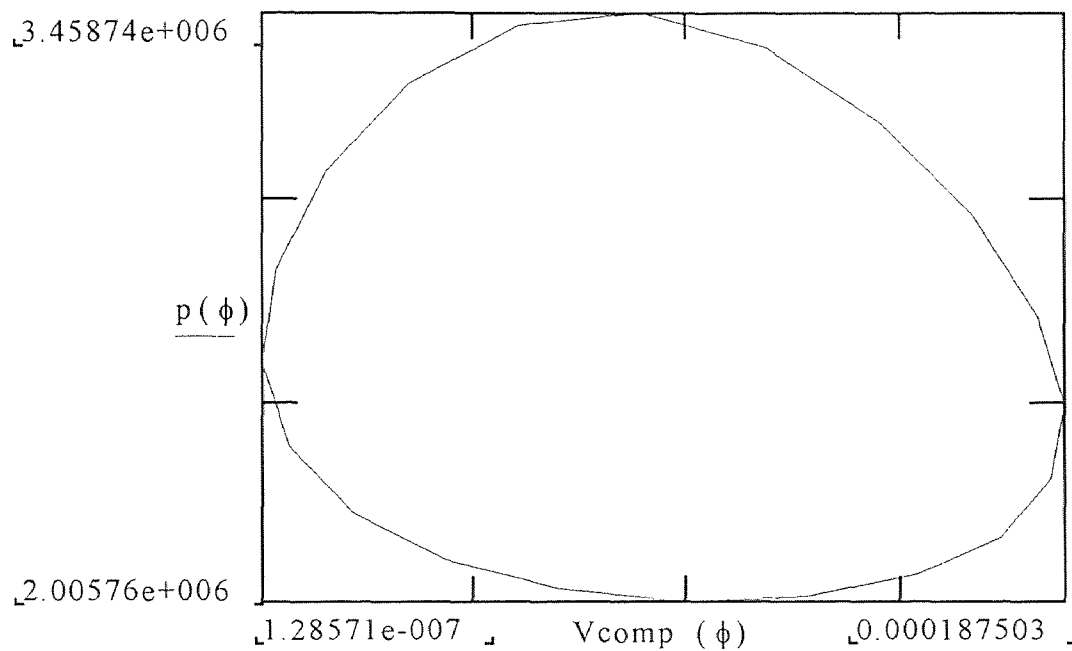
Figure 3 P-V Diagram for Expansion Space
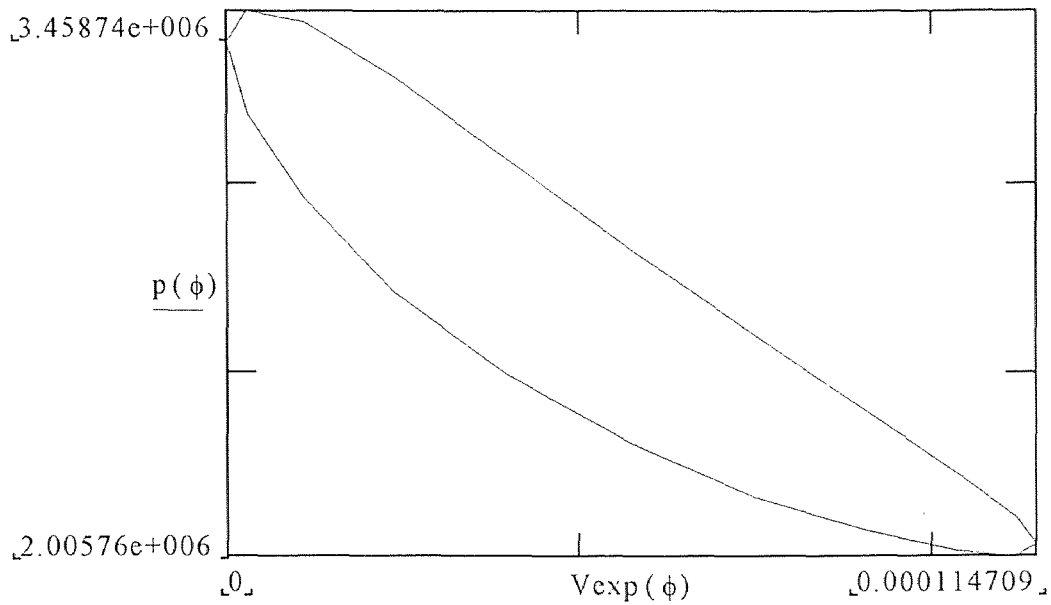
Figure 4 P-V Diagram for Compression Space
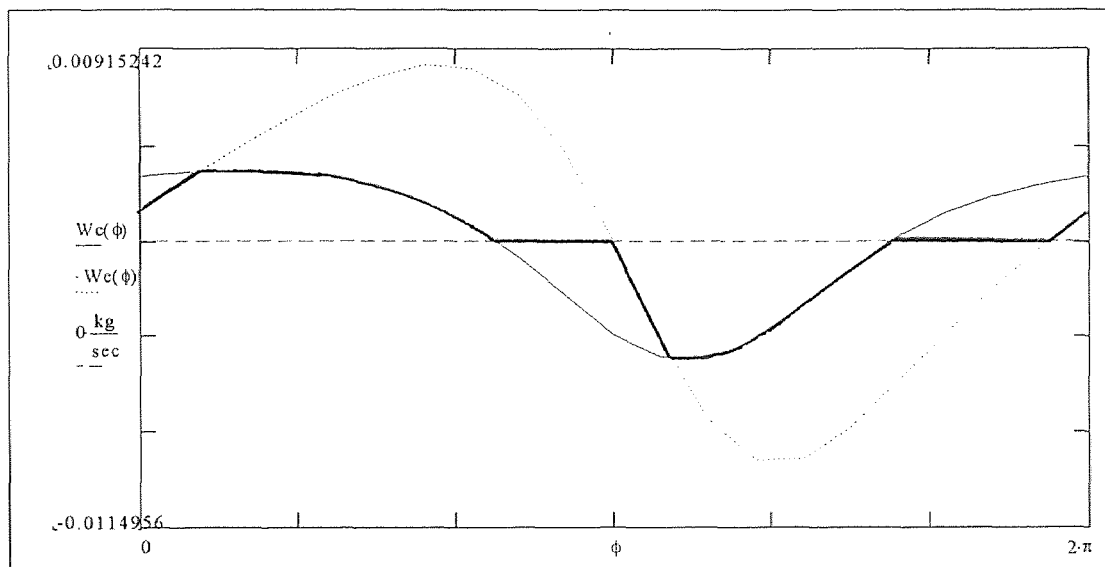
Figure 5 Mass Flow Rates of the Compression and Expansion Spaces

An analysis of the mass flow rates for the compression and expansion spaces is shown in Figure 5. It should be noted that positive mass flow rates (lines above the datum) depicted in the diagram represent the net flow into the compression space or out of the expansion space. Similarly negative

mass flow rates (below the datum) represent flow into the expansion space and out of the compression space. The net flow across the regenerator is represented by dark solid line in which positive flow is denoted by flow out of the expansion space and into the compression space.

As shown by the diagram, the time dependence of the gas flow in and out of the regenerator is very complicated. First, all of the gas in the engine does not flow right through the regenerator. As evidenced by the graph the net flow into the expansion space and out of the compression space (or vice versa) are rarely equal. This is due in large part to the large void volume of the regenerator. In many machines with a relatively long regenerator and short blow times there is gas that never leaves the regenerator altogether, but simply flows back and forth within it. Walker (1983) refers to this motion as 'tidal' flow.

Another peculiarity of the gas flow is that although during much of the cycle the flow is traversing from one end to the other, there exists a period when the gas flow is entering from both ends of the regenerator. Similarly there is a period when flow is exiting from both ends of the regenerator. This is due to the expansion and compression of the gas in the regenerator.

Clearly, the flow through the regenerator is very complex. The time dependence of the pressure and mass flow rates, as well as the temperature dependence of thermophysical properties, cannot be adequately evaluated

using the Hausen regenerator theory. More complex theories are needed to evaluate the performance of a regenerator operating under the conditions found in Stirling cycle machines.

2.4 Enthalpy-Flux Method

The enthalpy-flux method, developed by Qvale and Smith (1969), is an approximate closed form solution for the performance of a Stirling cycle regenerator. The theory assumes a sinusoidal flow rate and sinusoidal pressure variation with a phase angle between them. By assuming a second order polynomial for the temperature distribution in the regenerator a closed form solution is obtained for the net change in enthalpy or enthalpy flux. The theory assumes the gas and matrix temperatures are constant with time, and neglects the effect of fluid friction. The major assumptions of the theory are:

1. Regenerator is one-dimensional, there are no variations normal to the flow.

2. The effect of longitudinal conduction is negligible in both the matrix and the gas.

3. The gas behaves ideally, i.e. $PV=RT$

4. The thermophysical properties are constant throughout the regenerator.

5. The free flow area of the regenerator is constant.

6. The gas and matrix temperatures at a particular location in the regenerator are constant with time.

The assumptions made above parallel many of those made in the Hausen theory. As was shown before, many of these idealizations can have a significant effect on the performance of the regenerator. But unlike the Hausen theory this method takes into account the cyclic variation of the pressure and mass flow rates, an important factor that is probably the greatest drawback to the Hausen theory. Additionally, to more accurately describe the large change in heat transfer rates with flow rate, heat transfer coefficients between the gas and the matrix are based upon an experimental correlation dependent on the mass flow rate and temperature.

This more accurate model of the heat transfer in the regenerator yields fairly good results in comparison with some limited experimental results obtained by Rea (1967). A reasonably good correlation between this method and the experimental data was found. However, because the experimental data was obtained at relatively slow cycle speeds (84 and 158 RPM), it is not known how this method compares with more practical devices operating at speeds of 900 to 3000 RPM, where the flow conditions are much different.

To extend the accuracy of the enthalpy-flux method another analysis was performed by Harris, Rios and Smith (1970). Using the procedure developed above, separate equations were developed to evaluate the axial

conduction losses and pressure drop losses not accounted for in the enthalpy-flux method. A computer program was written to calculate the losses of a regenerator (imperfect heat transfer, conduction, pressure) for a particular refrigerator using a large number of different matrices. Optimum values for the length to diameter ratio were presented along with the losses associated with different types of matrices. In general the overall losses decreased as the particle size and wire diameter decreased. In addition, pressure drop losses and conduction losses were shown to be comparable to the losses due to imperfect heat transfer, especially at higher mesh values (200-400 mesh).

2.5 Experimental Research

Very little has been published about the experimental performance of regenerators operating under conditions found in Stirling cycle devices. One simple experimental investigation performed by Walker (1961) found that a reduction in wire diameter increases the regenerator effectiveness. To give an indication of the performance of the regenerator, the quantity of liquid air produced by a cryocooler (operating at constant speed and mean pressure) was measured. The results of this experiment are consistent with basic thinking since a reduction in wire diameter results in an increase in the surface area for heat transfer.

An experiment which is most representative of the conditions in a regenerative cryocooler was performed by Gifford, Acharya and Ackermann (1969). Their test apparatus was set up to accurately determine the effectiveness (which they term efficiency) of a regenerator to within 0.02%. A large number of different matrix materials were used including mesh screens of stainless steel and bronze (sizes 100, 150 and 200) as well as very small lead balls. Helium was used as the working fluid and was operated with the end temperatures of 300 and 78K. Flow rates were varied from 4 to 24 cubic feet per minute and speeds were varied from 60 to 150 cycles per minute. A constant regenerator diameter of 0.75 inches was used while the length was varied from 2 to 4 inches. Wall material was either stainless steel or phenolic plastic.

As with the experiments conducted by Walker (1961), they found that the effectiveness increased with an increase in mesh number (100-200). As would be expected, an increase in length resulted in an increase in effectiveness, although at a progressively diminishing rate. The increase in effectiveness in both cases is due to greater heat transfer surface area.

Regenerators that operated with higher speeds (150 RPM) and thus smaller blow times, showed higher effectiveness values, especially at higher flow rates. An optimum flow rate was found for many matrix materials (typically around 13 CFM) where effectiveness is a maximum. Effectiveness tends to decrease at flow rates both below and above this optimum value.

The exception is fine mesh materials (150-200 mesh) which at high speeds (over 100 RPM) the effectiveness changed little with an increase in the flow rate.

The increase in conduction losses at low flow rates was blamed for the decrease in effectiveness. When comparing stainless steel (high conductivity) to the low conductivity phenolic plastic, it was found that conduction losses are a considerable part of the overall losses, especially at lower flow rates. The ineffectiveness resulting from the use of stainless steel resulted in conduction losses that contribute almost 50% to the total ineffectiveness in some cases.

CHAPTER 3

PRESENT ANALYSIS

The analysis presented in this paper is derived from the model of Atrey, Bapat and Narayankhedkar (1991). Their mathematical model provides a high level of simulation to the actual operating conditions of Stirling cycle regenerators. To accomplish this, the model makes very few idealizations about the flow and heat transfer. Like the Enthalpy-flux method of Smith (1969), this model allows for sinusoidal mass and pressure functions. Axial conduction of the gas and matrix are also taken into account. In addition, the numerical simulation also allows for temperature dependent thermophysical properties and flow dependent heat transfer coefficients. Once coded, the model is easily modified to suit a wide variety of operating conditions. Because of this adaptability this model may easily be applied to any of the regenerative cryocoolers.

The key to the model's high level of simulation is due to the fact that it makes very few assumptions. They are:

1. The gas obeys the ideal gas law, i.e. $PV=RT$.

2. The pressure drop across the regenerator is negligible.

3. The flow at any instant is one-directional.

4. The conductivity in the direction normal to the flow is infinite.

5. The temperature at the inlet is constant.

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Usually hydrogen and helium exhibit very good ideal gas properties for most heat transfer applications and assumption one is a very good one. However, many cryocoolers operate at high pressures (above 20 atm) and at very low temperatures. Under these conditions the fluids do not behave ideally and the deviation from ideal gas behavior is significant. Fortunately, the error involved in assuming ideal gas behavior is not more than a few percent. More exact equations may be used in approximating the gas behavior such as the Clausius equation or the Beattie-Bridgeman equation of state but this would greatly add to the mathematical complexity of the model.

Assumption two requires that the pressure be same throughout the regenerator. It neglects the change in pressure in the direction of the flow rate. Fortunately, in high pressure devices this pressure drop is only a small fraction of the time dependent pressure change and has a negligible impact on the effectiveness. The pressure drop losses however, can have a significant impact on the performance of the cooler and should be assessed separately.

Together assumptions three and four are required to make the resulting equations one dimensional in space. This serves to reduce the computational effort considerably. Assumption four is a fairly good approximation since the conductivity normal to the flow is high for most materials - such as wire screen matrices. Unfortunately, assumption three

may not be a good representation of the actual flow. As pointed out in the experimental analysis by Jones (1989), the flow can be a strong function in the direction normal to the flow. Whether such flow profiles exist in real operating regenerators is questionable. Nevertheless, the remedial measures recommended by Jones should be used to smooth the flow profile as best as possible since any inhomogeneity of the fluid flow will most certainly cause a decrease in the effectiveness of the regenerator.

Assumption five is commonly made in the design of conventional counterflow heat exchangers. It is one of the principle assumptions of the Schmidt cycle. However, as was shown before, the actual conditions in the compression and expansion spaces of the Stirling cryocooler are more adiabatic than isothermal. Therefore the inlet conditions to the regenerator will undergo fluctuations in temperature. It is difficult to predict these fluctuations and little is known about how they will effect the performance of the regenerator.

With these assumptions the governing equations are derived by taking energy and mass balances for a differential element of gas and matrix. For a differential element of gas:

$$\frac{\partial}{\partial x}(Wh) - \left[\frac{\partial}{\partial x}(k_g A_o \frac{\partial T_g}{\partial x})dx\right] + H_T A_T (T_g - T_m)dx + \frac{\partial}{\partial t}(\rho_g u A_o dx) = 0$$

where W = mass flow rate

h = enthalpy

k_g = thermal conductivity of the gas

T_g = temperatue of the gas for the differential element dx

T_m = temperature of the matrix for the differential element dx

H_T = heat transfer coefficient between the matrix and gas

A_T = heat transfer surface area per unit length

ρ_g = density of the gas

u = internal energy of the gas

A_o = cross sectional area of regenerator open to the flow

The first term in the equation above represents the net flux of energy associated with the flow. The second term represents the net flux of energy due to conduction of the gas. The third term represents the energy transferred from the gas to the matrix over the differential length dx. The final term represents the change in internal energy of the differential gas element.

Assuming the gas behaves ideally, the following approximations can be made:

$$h = c_p T_g \qquad u = h - PV = c_p T_g - R T_g = (c_p - R) T_g \qquad \rho_g = \frac{P}{R T_g}$$

Substituting the expressions above into the gas energy equation and simplifying:

$$W c_p \frac{\partial T_g}{\partial x} + c_p T_g \frac{\partial W}{\partial x} - k_g A_o \frac{\partial^2 T_g}{\partial x^2} + H_T A_T (T_g - T_m) + (\frac{c_p}{R} - 1) A_o \frac{\partial P}{\partial t} = 0$$

Additionally, writing an expression for gas continuity:

$$\frac{\partial W}{\partial x} = -A_o \frac{\partial}{\partial t} \rho_g = -\frac{A_o}{R} \frac{\partial}{\partial t} (\frac{P}{T_g}) = -\frac{A_o}{R T_g^2} (T_g \frac{\partial P}{\partial t} - P \frac{\partial T_g}{\partial t})$$

Substituting this relation into the gas energy equation we find:

$$W c_p \frac{\partial T_g}{\partial x} - \frac{A_o c_p}{R T_g} (T_g \frac{\partial P}{\partial t} - P \frac{\partial T_g}{\partial t}) - k_g A_o \frac{\partial^2 T_g}{\partial x^2} + H_T A_T (T_g - T_m) + (\frac{c_p}{R} - 1) A_o \frac{\partial P}{\partial t} = 0$$

Writing an energy balance for a differential element of matrix:

$$H_T A_T (T_g - T_m) + k_m A \frac{\partial^2 T_m}{\partial t} = M_m c_m \frac{\partial T_m}{\partial t}$$

where k_m = thermal conductivity of matrix in direction of flow

c_m = specific heat of matrix

A = cross sectional area of matrix

M_m = mass of the matrix per unit length

As before, the first term represents the energy transferred from the gas to the matrix over the length dx. The second term represents the energy flux due to axial conduction in the matrix. The term on the right hand side represents the change in the internal energy of a differential matrix element.

To simplify the two equations above, a number of non-dimensional parameters were derived. They are:

1. $T_g^* = \frac{T_g - T_e}{T_c - T_e}$

2. $T_m^* = \frac{T_m - T_e}{T_c - T_e}$

3. $W^* = W/W_a$, where W_a is the amplitude of the sinusoidal mass flow variation.

4. $P^* = P/P_a$, where P_a is the amplitude of the sinusoidal pressure variation.

5. $x^* = x/L$

6. $t^* = \omega t$, where ω is the angular speed

7. $E = \frac{k_g A_o}{W_a c_p L}$

8. $B = \frac{A_o P_a \omega L}{W_a R (T_c - T_e)}$

9. $NTU = \frac{H_T A_T L}{W_a c_p}$

10. $C = \frac{k_m A}{M_m C_m L^2 \omega}$

11. $D = \frac{H_T A_T}{M_m c_m \omega}$

Substituting these relations into the gas and matrix energy equations we get:

$$W^* \frac{\partial T_g^*}{\partial x^*} - B \frac{\partial P^*}{\partial t^*} + \frac{BP^*}{[T_g^* + \frac{T_e}{T_c - T_e}]} \frac{\partial T_g^*}{\partial t^*} - E \frac{\partial^2 T_g^*}{\partial (x^*)^2} + NTU(T_g^* - T_m^*) + (1 - \frac{R}{c_p})B \frac{\partial P^*}{\partial t^*} = 0$$

and

$$D(T_g^* - T_m^*) + C \frac{\partial^2 T_m^*}{\partial (x^*)^2} = \frac{\partial T_m^*}{\partial t^*}$$

To more accurately simulate the conditions in a real Stirling cycle regenerator the following functions for the mass flow rate and pressure variations were used. They differ slightly from the functions used by Atrey, Bapat and Narayankhedkar.

$$W = W_a \sin \omega t \qquad \text{or} \qquad W^* = \sin t^*$$
$$P = P_m + P_a \cos \omega t \qquad \text{or} \qquad P^* = P_m/P_a - \cos t^*$$

Atrey, Bapat and Narayankhedkar assumed fixed temperatures for the matrix at both ends of the matrix and fixed the temperature of the gas at the inlet. In this analysis, to give a better estimate of the effect of reversing flow the following boundary conditions were used:

When W is (+) $\qquad T_g = T_e$ at x=0 $\qquad$ or $\qquad T_g^* = 0$ when $x^* = 0$

When W is (-) $\qquad\quad T_g = T_c$ at x=L $\qquad$ or $\qquad T_m^* = 1$ when $x^* = 1$

To arrive at a steady state solution with fewer iterations, the initial temperature distribution for the gas and the matrix were assumed to vary linearly from the hot end to the cold end or:

$$T_g = T_m = T_e + (T_c - T_e)\frac{x}{L} \qquad \text{or} \qquad T_g^* = T_m^* = x^*$$

Now that the model is properly defined it is only necessary to choose an appropriate method to arrive at a solution.

CHAPTER 4

COMPUTATIONAL SOLUTION

The solution to the set of partial differential equations above was performed using a finite difference routine. See Appendix. The equations were made explicit by rearranging terms and solving for the partial time derivatives of T_m and T_g. The solution then typically proceeds by solving for the temperature of the gas, T_g and matrix, Tm at a time $t+dt$ using the solution at time t. The solution begins at the cold end of the matrix and marches along at space step dx to the hot end of the regenerator.

The mass flow rate and pressure is evaluated at each time step. Thermophysical properties for the gas and matrix are evaluated at every space step and hence at the local temperature of the gas and matrix. Functions for the thermal conductivity, specific heat and viscosity of the working fluid were obtained by curve fitting data found in the TPRC Data Book (1970). Unfortunately, the equations obtained are functions of temperature only and at atmospheric pressure despite the fact that many thermophysical properties, notably viscosity, are strong functions of pressure as well. Curve fitting was done using the AMVAC v. 1.2 curve fitter developed by Dudley Benton and most functions were within a few percent of the experimental data.

For the matrix, functions for the thermal conductivity and specific heat for phosphor bronze were obtained from the model developed by Atrey *et. al.* (1991). The function obtained for the conductivity is a rough estimate at best since the method of regenerator construction has an important impact on the thermal conductivity of the matrix. In many cases stacks of wire mesh are compressed and sintered together to form a continuous piece for easy machinability. The thermal conductivity of the matrix in the axial (flow) direction is then dependent on the contact resistance between the layers of the matrix.

Experimental heat transfer data was obtained from Walker and Vasishta (1971). They conducted experiments to measure the heat transfer and flow friction characteristics for a wide range of wire mesh materials using air as the working fluid. Their results are presented in a series of charts relating the Nusselt number, Nu and Fanning friction factor, f to the Reynolds number, Re. An approximate curve fit to the Nu vs. Re chart for the 200 mesh phosphor bronze wire screen was evaluated using the AMVAC program. The approximate relation is as follows:

$$Nu = .3721 Re^{.75}$$

where the Reynolds number, Re and Nusselt number, Nu are defined as follows:

$$Nu = \frac{4 r_h H_T}{k_g} \qquad \text{and} \qquad Re = \frac{4 r_h W}{\mu_g p A}$$

where $r_h = \frac{A L p}{A_T}$, the hydraulic radius

where p = porosity

μ_g = viscosity of the gas

Initially a forward difference in time, central difference in space finite difference scheme was used. It involves the calculation of the solution at time $t+dt$ using the solution at time t. The space step was discretized using the standard central difference method except at the boundaries. At the cold end of the matrix a first order forward difference was used. Similarly a first order backward difference was used to discretize the differential equations at the hot end of the regenerator.

Commonly the size of dt and dx necessary to achieve stability are determined by performing a stability analysis on the differential equations. Unfortunately, because of the nonlinearity of the equations it is very difficult if not impossible to perform such an analysis. Instead, the values of dt and dx were continually adjusted until stability was obtained. To achieve stability using the method above, a rather severe limit on the time step dt of 1/100,000 was required. The space step on the other hand, had little effect on the stability of the solution and a space step as large as 1/5 was utilized.

To reduce the stability requirement a higher order method was used to discretize the time step. A three time level scheme was utilized such that the solution for the time step $t+dt$ is found using the solution at time t and time $t-dt$. The standard explicit method described above was used to start the solution. Additionally second order forward and backward differences were used to discretize the terms at the boundary conditions. This yields a

method which is order $O(dt^2)$ in time and $O(dx^2)$ in space as opposed to the $O(dt)$ and $O(dx^2)$ of the previous method. Using this method stability was achieved with a time step as large as 1/40,000.

The initial space step dx of 1/5 was found to be inadequate to properly discretize the space step. Temperatures were found to go above the hot end temperatures and below the cold end temperatures - in violation of the second law of thermodynamics. More points were needed to yield a more accurate solution. To do this the number of points used in the discretization was consecutively doubled until four digit accuracy was achieved. Reasonably good results were found using a space step of 1/40.

CHAPTER 5

RESULTS AND DISCUSSION

The regenerator was analyzed for a wide range of operating conditions to determine what effect if any the parameters such as mass flow rate, pressure and length had on the effectiveness. First, baseline operating conditions were defined as follows: DIA=3 inches, L = 3 inches, P_m = 2500 kPa, ϕ = 30 deg, Wa = .01 kg/s @ 1400 RPM. Then one parameter such as the mass flow rate was varied to determine its effect on the performance of the regenerator. For the operating conditions described the following trends were found:

Variation of the mass flow rate had a significant impact on the effectiveness of the regenerator. Flow rates were evaluated over the region of .001 to .04 kg/s. See Figure 6. The graph shows that the effectiveness decreases with an increase in the mass flow rate. This agrees with the experimental results obtained by Gifford *et. al.* (1969). Apparently the increase in the heat capacity of the fluid results in a decrease in the effectiveness despite the fact that heat transfer coefficients are larger at the higher flow rates. The drop in effectiveness at low flow rates that Gifford *et. al.* found in their experiments was not predicted by the present method.

The speed of regenerator operation was also shown to have a significant impact on the performance. Speeds were evaluated over the range of 100 to 1400 RPM. See Figure 7.
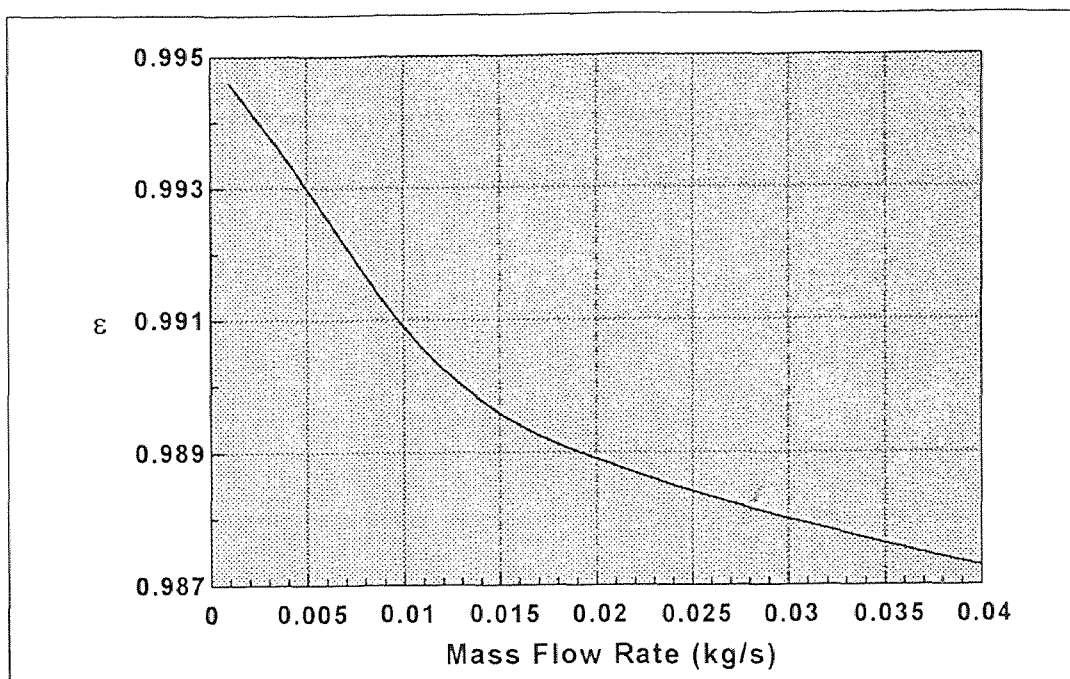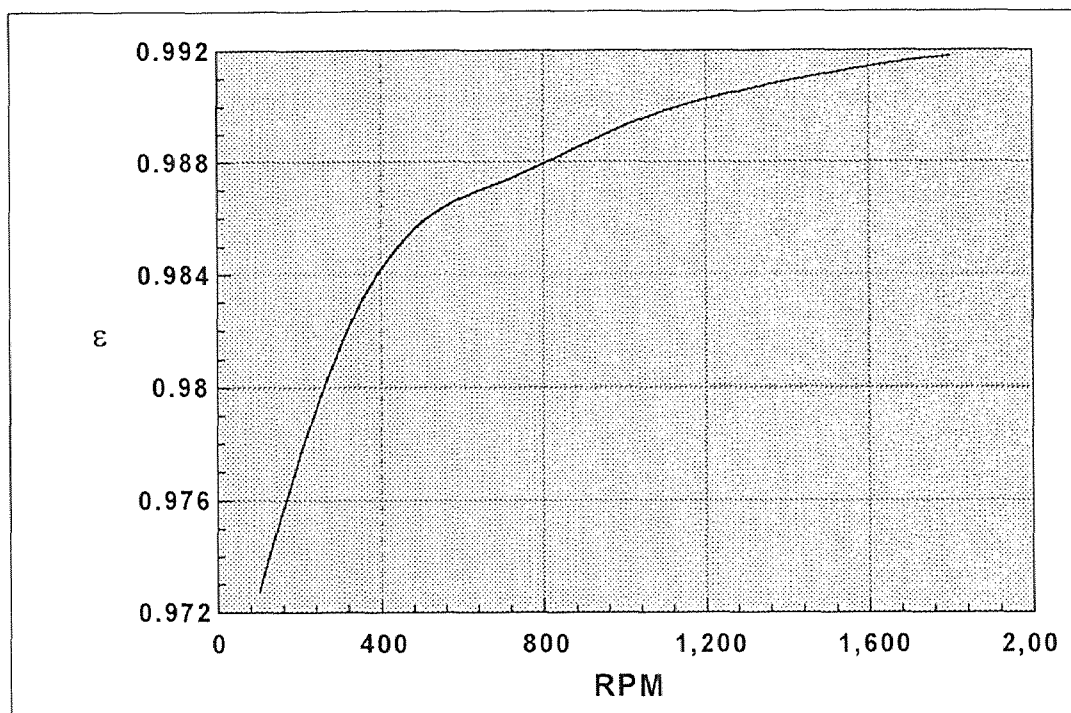
41

Figure 6 Effectiveness vs. Mass Flow Rate

Figure 7 Effectiveness vs. Operating Speed

The present method shows an increase in effectiveness as the speed of operation is increased. Again, these results are in agreement with the results obtained by Gifford *et. al.* Apparently the drop in effectiveness is a result of the increase in the heat capacity of the fluid at low speeds. For a given flow rate the heat capacity will increase as a result of the longer blow times at slower speeds. A regenerator effectiveness as high as .9918 was predicted for a speed of 1800 RPM with a low of .9727 at 100 RPM.

The pressure and phase angle were found to have little impact on the effectiveness of the regenerator. The effectiveness was a very weak function of the phase angle. An increase in the effectiveness of only .001 was found in the range of 0 to 40 degrees. The effect of the pressure on the effectiveness was found to be even smaller. A decrease in effectiveness of only .0003 was predicted over the range from 1 to 40 bar. This correlates with the results found by Atrey *et. al.* It should be noted that if the pressure dependence of the thermophysical properties and the deviation from ideal gas properties were taken into account a greater variation in the effectiveness is expected to occur.

The effect of the matrix volume, as would be expected, showed large changes in the effectiveness. See Figure 8. To evaluate these volume variations the diameter of the regenerator was fixed at 1 inch while the length of the regenerator was varied from 0.5 to 4 inches. Thus varying the length of the regenerator had a direct impact on the amount of heat transfer

surface area and heat capacity of the matrix. As a result it was found that an increase in the length resulted in an increase in the effectiveness, although at a diminishing rate. A regenerator effectiveness as high as .993 was predicted using a length of 4 inches.
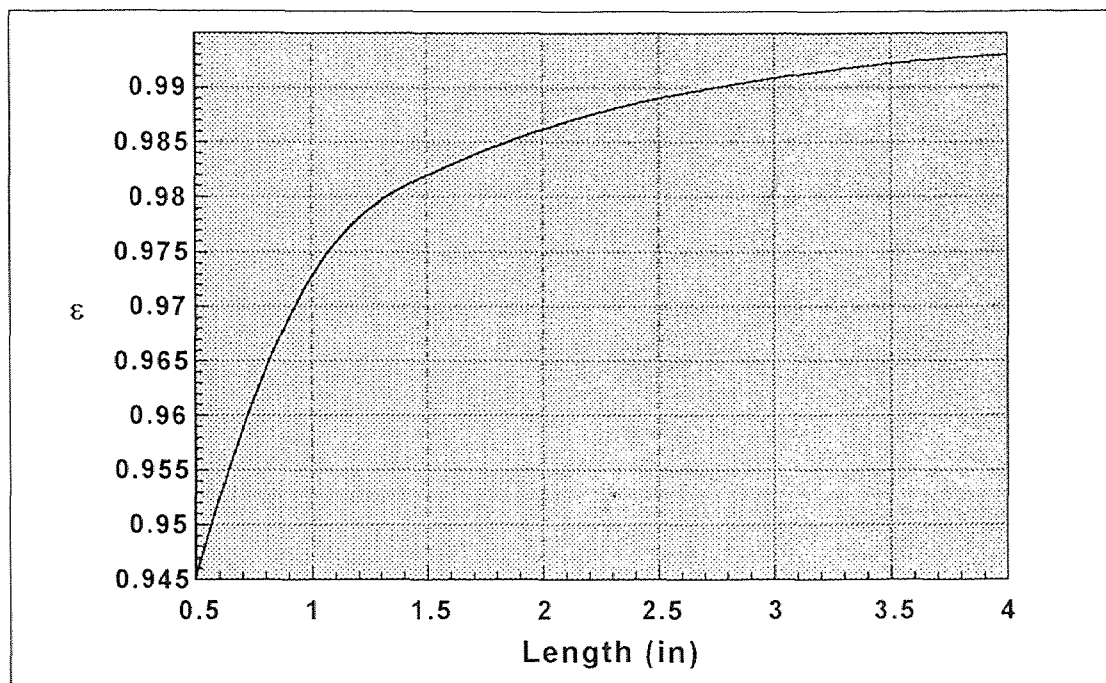
Figure 8 Effectiveness vs. Length

An additional analysis was made of the variation of the temperature distributions in the matrix. See Figure 9. For this particular case ($\varepsilon = .873$) the temperature within the matrix undergoes large variations with time. Such cyclic thermal variations can have an adverse effect on the matrix material. To minimize this effect the regenerator should be made as efficient a possible.
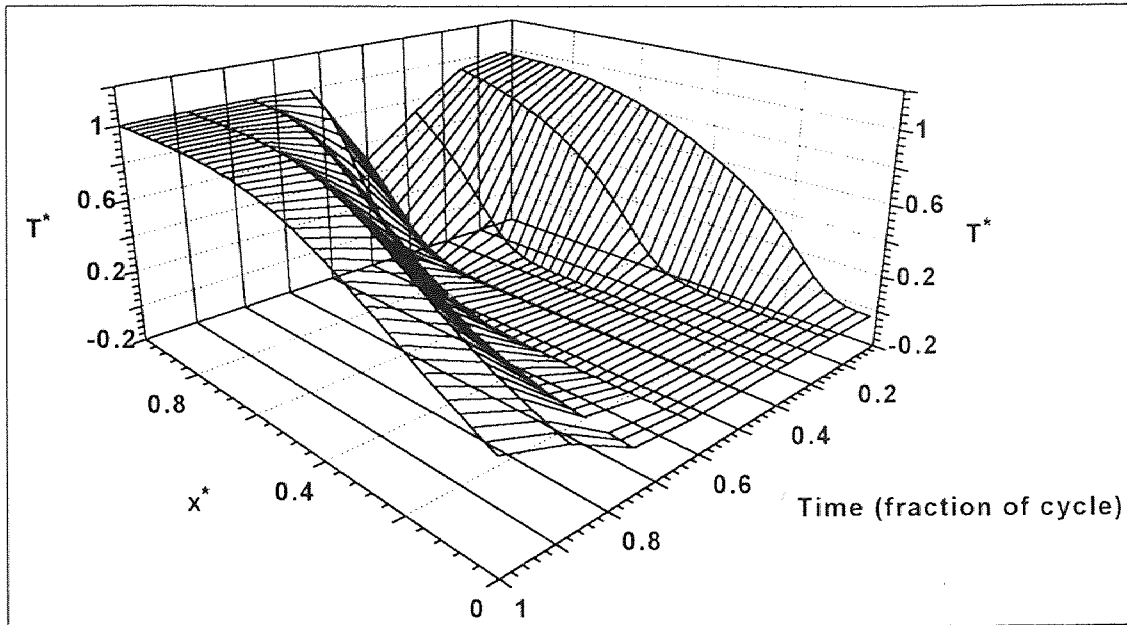
Figure 9 Cyclic Temperature Variations in the Matrix

CHAPTER 6

CONCLUSIONS AND SUGGESTIONS

The method presented in this paper provides one of the most complete descriptions of the operation of a thermal regenerator of a Stirling cycle device. Among the other methods, this model makes the fewest assumptions about the flow and heat transfer. There is an allowance for sinusoidal mass flow rate and pressure variations along with a phase difference between them. Thermophysical properties such as conductivity, viscosity and specific heat are all temperature dependent. Heat transfer coefficients between the gas and matrix are flow dependent. Also, thermal conduction of both the gas and matrix in the direction of flow are accounted for. Because of this high degree of simulation the precision of the model is mostly limited by the accuracy of the functions for the thermophysical properties and heat transfer coefficient.

Unfortunately there is little experimental evidence available to corroborate the results found in this model. This is due to the fact that it is very difficult to measure the performance of a regenerator operating under normal flow conditions typical of a Stirling cooler. Thus the results obtained using this model are questionable. Further experimental work needs to be done to measure the performance of regenerators operating under conditions more typical of efficient Stirling coolers, i.e. high pressure

(20-40 atm), high speed (600-1800 RPM) with sinusoidally varying pressure and mass flow rates.

The method is not without merit however. Since this method provides one of the highest levels of simulation to date, it can be very useful as a design tool. Although in this study the analysis was limited to the performance of a regenerator operating with a matrix consisting of 200 mesh phosphor bronze with helium as the working fluid operating between the temperatures of 75 and 300K the method can easily be adapted to wide range of materials, fluids and temperature ranges. All that is needed is accurate data for heat transfer coefficients and thermophysical properties. Also the pressure and mass flow rate variations can easily be modified to evaluate the performance of regenerators operating under other regenerative cycles such as the Gifford McMahon and Vuilleumier cycles.

Particularly important, this method provides a means to investigate the effect various parameters have on the performance of the regenerator. Regenerator dimensions, mean pressure, matrix materials, working fluid, phase angle, flow rates, etc. can all be varied. This allows various different combinations to be tried to determine which set of parameters will yield the best results. Coupled with an effective analysis of the pressure drop loss this method can provide a reasonably good means of performing an optimization study.

In general the following results were obtained:

1. The effectiveness decreases with an increase in the mass flow rate.

2. The effectiveness increases with an increase in the frequency of operation.

3. Pressure and phase angle have little or no effect on the effectiveness.

4. An increase in the volume of the matrix results in an increase in the effectiveness.

Thus to design an efficient regenerator the matrix should be made as large as possible (optimized against pressure losses), and made to operate with low flow rates and high speeds. Since the model was programmed to only evaluate the temperature distributions of the gas and matrix, as well as the effectiveness, further work can be done to evaluate other properties important in the design of the regenerator. These include the pressure drop, void volume and effects of wall conduction on the performance of the regenerator. These factors can have a significant effect on the performance of the regenerator and should be assessed in the process of design. The analysis of these factors are beyond the scope of this paper.

APPENDIX A

SCHMIDT CYCLE ANALYSIS

Temperature of Compression and Expansion Spaces

$Tc = 300 \cdot K$ $Te = 70 \cdot K$

Temperature Of Dead Space (average): $Td = \dfrac{Tc + Te}{2}$ $Td = 185 \cdot K$

Volume of Compression and Expansion Spaces

$Vc = 11.45 \cdot in^3$ $Vc = 1.876 \cdot 10^{-4} \cdot m^3$ $Ve = 7 \cdot in^3$ $Ve = 1.147 \cdot 10^{-4} \cdot m^3$

Phase angle between Compression and Expansion Spaces

$\alpha = 105 \cdot deg$ $\alpha = 1.833 \cdot rad$

Temperature Ratio: Gas constant for Helium

$\tau := \dfrac{Tc}{Te}$ $\tau = 4.286$ $Ru = \dfrac{8314 \cdot joule}{kg \cdot mole \cdot K}$ $M = 2.016 \cdot \dfrac{kg}{kg \cdot mole}$ $R = \dfrac{Ru}{M}$ $R = 4.124 \cdot 10^3 \cdot \dfrac{joule}{kg \cdot K}$

Swept Volume Ratio:

$\kappa := \dfrac{Vc}{Ve}$ $\kappa = 1.636$

Crank Angle: $\phi = 0, \dfrac{\pi}{10} .. 4 \cdot \pi$

Dead Volume Ratio (assumed): $X = 3$

Constants:

$S := \dfrac{2 \cdot X \cdot \tau}{\tau + 1}$ $S = 4.865$ $\delta = \dfrac{\sqrt{\tau^2 + \kappa^2 + 2 \cdot \tau \cdot \kappa \cdot \cos(\alpha)}}{\tau + \kappa + 2 \cdot S}$ $\delta = 0.267$

$\theta := atan\left(\dfrac{\kappa \cdot \sin(\alpha)}{\tau + \kappa \cdot \cos(\alpha)}\right)$ $\theta = 22.248 \cdot deg$

Mean Pressure: $Pm = 26 \cdot atm$ $Pm = 382.095 \cdot psi$

Instantaneous Pressure: $p(\phi) = \dfrac{Pm \cdot \sqrt{1 - \delta^2}}{1 + \delta \cdot \cos(\phi - \theta)}$
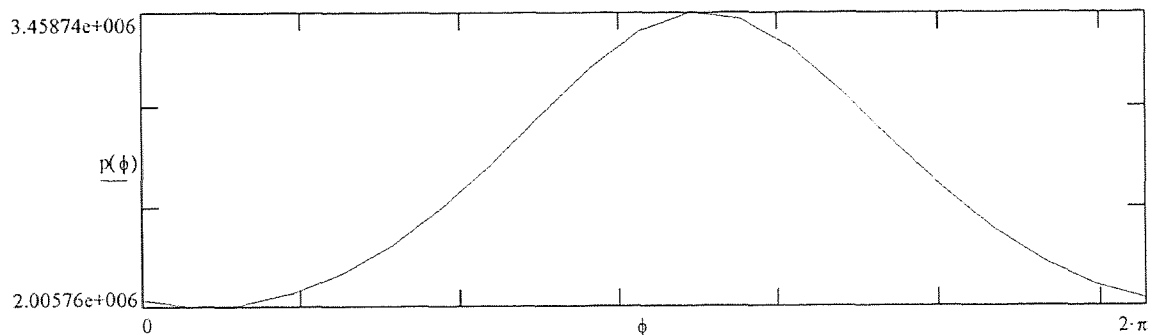
Expansion Space Volume: $\quad Vexp(\phi) = \dfrac{1}{2} \cdot Ve \cdot (1 + \cos(\phi))$

Instantaneous mass of fluid in Expansion Space:

$$Me(\phi) = \dfrac{p(\phi) \cdot Vexp(\phi)}{R \cdot Te}$$

Compression Space Volume: $\quad Vcomp(\phi) = \dfrac{1}{2} \cdot \kappa \cdot Ve \cdot (1 + \cos(\phi - \alpha))$

Instantaneous mass of fluid in Compression Space

$$Mc(\phi) = \dfrac{p(\phi) \cdot Vcomp(\phi)}{R \cdot Tc}$$

Dead Space: $\qquad\qquad Vd = X \cdot Ve \qquad\quad Vd = 3.441 \cdot 10^{-4} \cdot m^3$

$$Md(\phi) = \dfrac{p(\phi) \cdot Vd}{R \cdot Td}$$

Volume Variations in the Compression and Expansion Spaces
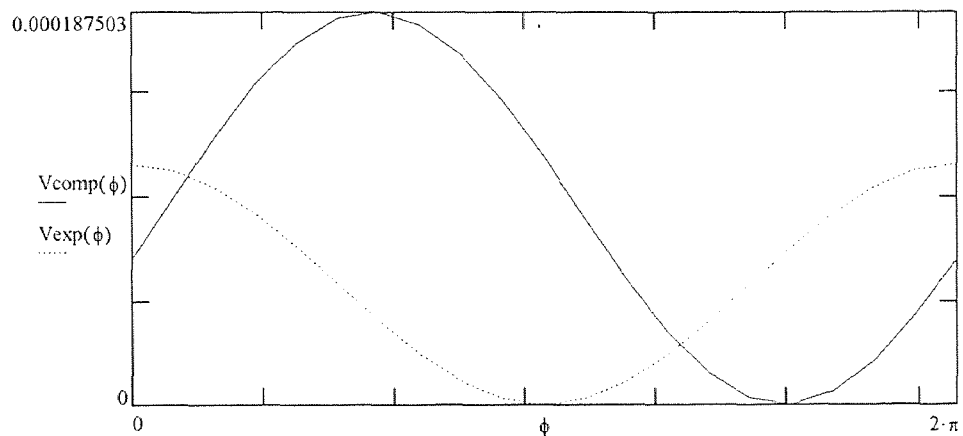
P-V Diagrams for expansion and compression spaces
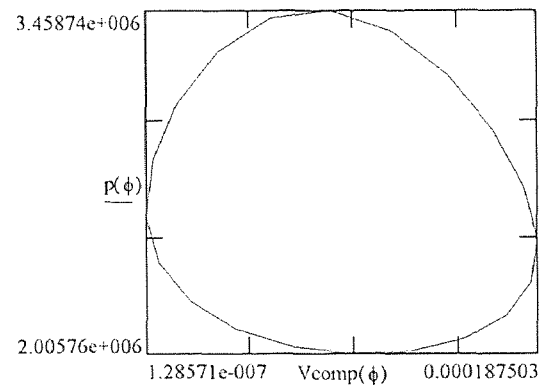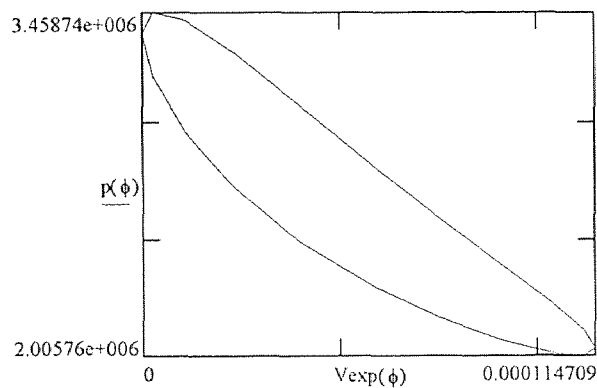
Mass Variations in the Compression, Expansion and Dead Spaces
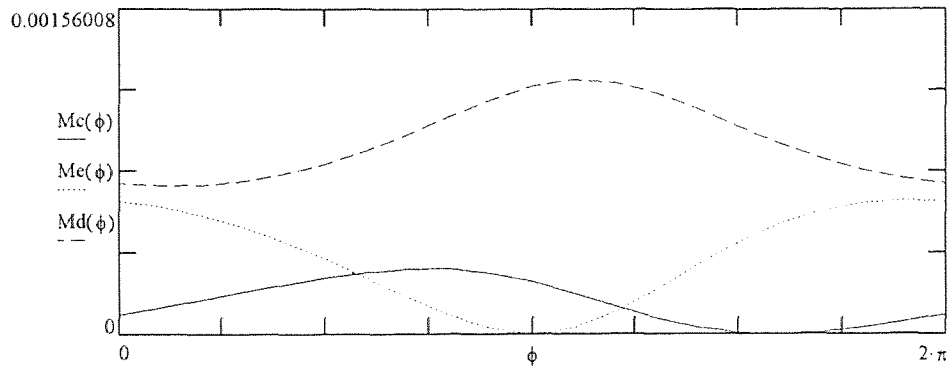
Speed $= 24 \cdot Hz$

$$We(\phi) := \frac{Ve \cdot Pm \cdot \sqrt{1 - \delta^2} \cdot (\delta \cdot (\sin(\phi - \theta) - \sin(\theta)) - \sin(\phi))}{2 \cdot R \cdot Te \cdot (1 + \delta \cdot \cos(\phi - \theta))^2} \cdot Speed$$

$$Wc(\phi) := \frac{\kappa \cdot Ve \cdot Pm \cdot \sqrt{1 - \delta^2} \cdot (\delta \cdot (\sin(\phi - \theta) - \sin(\theta - \alpha)) - \sin(\phi - \alpha))}{2 \cdot R \cdot Tc \cdot (1 + \delta \cdot \cos(\phi - \theta))^2} \cdot Speed$$

Instantaneous Mass Flow Rates of Compression and Expansion Spaces
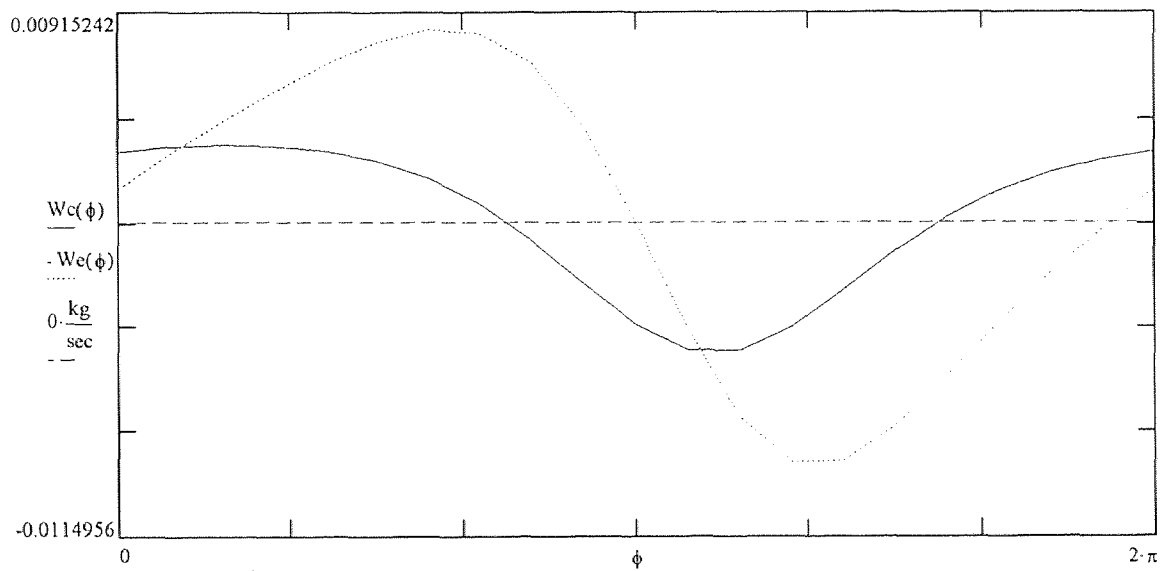
APPENDIX B

PROGRAM LISTING

```
*    Solution of the partial differential equations for heat
*    transfer of a regenerator in a Stirling engine/refrigerator

     PARAMETER (PI = 3.14159265359D0, N=40, CYCLES=5 )

*    Dimensionless variables
     DOUBLE PRECISION BETA,B,C,D,E,NTU,W,P,RE

*    Finite Difference
     DOUBLE PRECISION DX,DT
     INTEGER START,I,J,REV

*    Regenerator Thermodynamic Parameters
     DOUBLE PRECISION CM,CG,HT,KG,KM,VI,TE,TC,RU,R,MOLAR
     DOUBLE PRECISION H,MFLOW
     DOUBLE PRECISION PRESS,PPRIME,DERIVP,PM,PA,PHASE,WA

*    Regenerator Physical Parameters
     DOUBLE PRECISION MASS,L,DIA,A,AT,AO,POROS,RH

*    Misc Parameters
     DOUBLE PRECISION SPEED,RPM,SUM,EFFECT,TEMP
     CHARACTER TDATA*20

*    Dimension grid
     DOUBLE PRECISION TM(0:N,2),TMNEW(0:N),TG(0:N,2),TGNEW(0:N)

*    Grid Spacing
     DX = 1.D0/N
     DT = 1.D0/40000.D0
     M  = NINT(2.D0*PI/DT)

*    Define Constants (SI Units)
     WRITE (*,*) 'Enter diameter of regenerator (in)'
     READ  (*,*) TEMP
     DIA = TEMP*.0254D0
     WRITE (*,*) 'Enter length of regenerator (in)'
     READ  (*,*) TEMP
     L  = TEMP*.0254D0
     A  = PI*DIA**2/4.D0
     RU = 8314.D0
     MOLAR = 4.003D0
     R  = RU/MOLAR
     TE = 75.D0
     TC = 300.D0
     BETA = TE/(TC-TE)

     WRITE (*,*) 'Enter mean pressure (bar)'
     READ  (*,*) TEMP
     PM = TEMP*1.D5
     PA = PM/3.D0
     WRITE (*,*) 'Enter the phase angle (deg)'
     READ  (*,*) TEMP
     PHASE = PI*TEMP/180.D0
```

```
      WRITE (*,*) 'Enter the maximum mass flow rate (kg/s)'
      READ  (*,*) WA
      WRITE (*,*) 'Enter the speed of operation (RPM)'
      READ  (*,*) RPM
      SPEED = 2.D0*PI*RPM/60.D0

*     Matrix Geometry
      CALL MGEOM(L,A,MASS,AT,AO,POROS,RH)
      B = AO*L*PA*SPEED/(R*WA*(TC-TE))

*     Initialize array for Initial Conditions
      DO 10 I = 0,N
         TG(I,1) = (1.D0*I)/N
         TM(I,1) = TG(I,1)
10    CONTINUE
      SUM = 1.D0

*     Store results to the file: temp.dat
      WRITE (*,*) 'Enter name of data file'
      READ (*,*) TDATA
      OPEN (1, FILE = TDATA)

*     Perform standard explicit method for first iteration
      J = 1

*     Evaluate Pressure and mass flow rate
      P = PRESS(J*DT,PM,PA,PHASE)
      PPRIME = DERIVP(J*DT,PHASE)
      W = MFLOW(J*DT)

*     Evaluate boundary conditions
      TG(0,2) = 0.D0

      CALL THERMO(CG,CM,KG,KM,VI,TG(0,1),TM(0,1),TC,TE)
      RE  = 4.D0*RH*ABS(W)*WA/(VI*POROS*A)
      H   = HT(RH,KG,RE)
      D   = H*AT/(MASS*CM*SPEED)
      C   = KM*A/(MASS*CM*SPEED*L**2)
      E   = KG*AO/(WA*CG*L)
      NTU = H*AT*L/(WA*CG)

      TM(0,2) = TM(0,1) + DT*D*(TG(0,1)-TM(0,1)) + C*DT/(DX**2)
      C    * (-2.D0*TM(0,1)+ 5.D0*TM(1,1)- 4.D0*TM(2,1)+ TM(3,1))

      CALL THERMO(CG,CM,KG,KM,VI,TG(N,1),TM(N,1),TC,TE)
      RE  = 4.D0*RH*ABS(W)*WA/(VI*POROS*A)
      H   = HT(RH,KG,RE)
      D   = H*AT/(MASS*CM*SPEED)
      C   = KM*A/(MASS*CM*SPEED*L**2)
      E   = KG*AO/(WA*CG*L)
      NTU = H*AT*L/(WA*CG)

      TM(N,2) = TM(N,1) + DT*D*(TG(N,1)-TM(N,1)) + DT/(DX**2)*C
      C    * (2.D0*TM(N,1)-5.D0*TM(N-1,1)+4.D0*TM(N-2,1)-TM(N-3,1))
```

```
      TG(N,2) = TG(N,1) - W*DT/(B*P)*(TG(N,1)+BETA)
C    * (TG(N-2,1) - 4.D0*TG(N-1,1) + 3.D0*TG(N,1))/(2.D0*DX)
C    + R*DT / (CG*P) * (TG(N,1)+BETA) * PPRIME
C    + E*DT / (B*P) *  (TG(N,1)+BETA)
C    * (2.D0*TG(N,1)-5.D0*TG(N-1,1)+4.D0*TG(N-2,1)-TG(N-3,1))
C    / (DX**2)- NTU*DT/(B*P)* (TG(N,1)+BETA)* (TG(N,1)-TM(N,1))

      SUM = SUM + TG(N,2)

      DO 4 I = 1,N-1

      CALL THERMO(CG,CM,KG,KM,VI,TG(I,1),TM(I,1),TC,TE)
      RE  = 4.D0*RH*ABS(W)*WA/(VI*POROS*A)
      H   = HT(RH,KG,RE)
      D   = H*AT/(MASS*CM*SPEED)
      C   = KM*A/(MASS*CM*SPEED*L**2)
      E   = KG*AO/(WA*CG*L)
      NTU = H*AT*L/(WA*CG)

*     Finite Difference Solution - First Iteration
*     Central difference in space, forward in time

      TM(I,2) = TM(I,1) + DT*D*(TG(I,1)-TM(I,1)) + DT/(DX**2)*C
C    * (TM(I+1,1) - 2.D0*TM(I,1) + TM(I-1,1))

      TG(I,2) = TG(I,1)
C    - W*DT/(B*P)*(TG(I,1)+BETA)*(TG(I+1,1)-TG(I-1,1))/(2.D0*DX)
C    + R*DT / (CG*P) * (TG(I,1)+BETA) * PPRIME
C    + E*DT / (B*P) *  (TG(I,1)+BETA)
C    *  (TG(I+1,1) - 2.D0*TG(I,1) + TG(I-1,1)) / (DX**2)
C    - NTU*DT / (B*P) * (TG(I,1)+BETA) * (TG(I,1)-TM(I,1))

4     CONTINUE

      START = 2
      DO 100 REV = 1,CYCLES

*     Finite Difference Solution
      DO 1 J = START,M

*     Evaluate Pressure and mass flow rate
      P = PRESS(J*DT,PM,PA,PHASE)
      PPRIME = DERIVP(J*DT,PHASE)
      W = MFLOW(J*DT)

*     Determine direction of flow to establish gas boundary cond.
      IF (W .GE. 0.D0) THEN
        TGNEW(0) = 0.D0

        CALL THERMO(CG,CM,KG,KM,VI,TG(N,2),TM(N,2),TC,TE)
        RE  = 4.D0*RH*ABS(W)*WA/(VI*POROS*A)
        H   = HT(RH,KG,RE)
        D   = H*AT/(MASS*CM*SPEED)
```

```
      C   = KM*A/(MASS*CM*SPEED*L**2)
      E   = KG*AO/(WA*CG*L)
      NTU = H*AT*L/(WA*CG)


      TGNEW(N) = (-TG(N,1) + 4.D0*TG(N,2)
C   - W*DT/(B*P*DX)*(TG(N,2)+BETA)
C   * (TG(N-2,2) - 4.D0*TG(N-1,2) + 3.D0*TG(N,2))
C   + 2.D0*R*DT / (CG*P) * (TG(N,2)+BETA) * PPRIME
C   + 2.D0*E*DT / (B*P*DX**2) *      (TG(N,2)+BETA)
C   * (2.D0*TG(N,2)- 5.D0*TG(N-1,2)+ 4.D0*TG(N-2,2)- TG(N-3,2))
C   - 2.D0*NTU*DT/ (B*P)* (TG(N,2)+BETA)* (TG(N,2)-TM(N,2)))
C   /3.D0


      SUM = SUM + TGNEW(N)

      ELSE

      TGNEW(N) = 1.D0

      CALL THERMO(CG,CM,KG,KM,VI,TG(0,2),TM(0,2),TC,TE)
      RE  = 4.D0*RH*ABS(W)*WA/(VI*POROS*A)
      H   = HT(RH,KG,RE)
      D   = H*AT/(MASS*CM*SPEED)
      C   = KM*A/(MASS*CM*SPEED*L**2)
      E   = KG*AO/(WA*CG*L)
      NTU = H*AT*L/(WA*CG)


      TGNEW(0) = (-TG(0,1) + 4.D0*TG(0,2)
C   - W*DT/(B*P*DX)*(TG(0,2)+BETA)
C   * (-TG(2,2) + 4.D0*TG(1,2) - 3.D0*TG(0,2))
C   + 2.D0*R*DT / (CG*P) * (TG(0,2)+BETA) * PPRIME
C   + 2.D0*E*DT / (B*P*DX**2) *  (TG(0,2)+BETA)
C   * (-2.D0*TG(0,2)+ 5.D0*TG(1,2)- 4.D0*TG(2,2)+ TG(3,2))
C   - 2.D0*NTU*DT/ (B*P)* (TG(0,2)+BETA)* (TG(0,2)-TM(0,2)))
C   /3.D0


      SUM = SUM + 1.D0 - TGNEW(0)

      ENDIF

      CALL THERMO(CG,CM,KG,KM,VI,TG(0,2),TM(0,2),TC,TE)
      RE  = 4.D0*RH*ABS(W)*WA/(VI*POROS*A)
      H   = HT(RH,KG,RE)
      D   = H*AT/(MASS*CM*SPEED)
      C   = KM*A/(MASS*CM*SPEED*L**2)
      E   = KG*AO/(WA*CG*L)
      NTU = H*AT*L/(WA*CG)


      TMNEW(0) = (-TM(0,1) + 4.D0*TM(0,2)
C   + 2.D0*DT*D*(TG(0,2)-TM(0,2)) + 2.D0*C*DT/(DX**2)
C   * (-2.D0*TM(0,2)+ 5.D0*TM(1,2)- 4.D0*TM(2,2)+ TM(3,2))
C   )/3.D0

      CALL THERMO(CG,CM,KG,KM,VI,TG(N,2),TM(N,2),TC,TE)
```

```
      RE  = 4.D0*RH*ABS(W)*WA/(VI*POROS*A)
      H   = HT(RH,KG,RE)
      D   = H*AT/(MASS*CM*SPEED)
      C   = KM*A/(MASS*CM*SPEED*L**2)
      E   = KG*AO/(WA*CG*L)
      NTU = H*AT*L/(WA*CG)


      TMNEW(N) = (-TM(N,1) + 4.D0*TM(N,2)
   C   + 2.D0*DT*D*(TG(N,2)-TM(N,2)) + 2.D0*C*DT/(DX**2)
   C   * (2.D0*TM(N,2)- 5.D0*TM(N-1,2)+ 4.D0*TM(N-2,2)- TM(N-3,2))
   C   )/3.D0


      DO 2 I = 1,N-1


      CALL THERMO(CG,CM,KG,KM,VI,TG(I,2),TM(I,2),TC,TE)
      RE  = 4.D0*RH*ABS(W)*WA/(VI*POROS*A)
      H   = HT(RH,KG,RE)
      D   = H*AT/(MASS*CM*SPEED)
      C   = KM*A/(MASS*CM*SPEED*L**2)
      E   = KG*AO/(WA*CG*L)
      NTU = H*AT*L/(WA*CG)


*     Finite Difference Solution - Central difference in space


      TMNEW(I) = (-TM(I,1) + 4.D0*TM(I,2)
   C   + 2.D0*D*DT*(TG(I,2)-TM(I,2)) + 2.D0*C*DT/(DX**2)
   C   * (TM(I+1,2) - 2.D0*TM(I,2) + TM(I-1,2)))/3.D0


      TGNEW(I) = (-TG(I,1) + 4.D0*TG(I,2)
   C   - W*DT/(B*P*DX)*(TG(I,2)+BETA)* (TG(I+1,2) - TG(I-1,2))
   C   + 2.D0*R*DT / (CG*P) * (TG(I,2)+BETA) * PPRIME
   C   + 2.D0*E*DT / (B*P*DX**2) *       (TG(I,2)+BETA)
   C   * (TG(I+1,2) - 2.D0*TG(I,2) + TG(I-1,2))
   C   - 2.D0*NTU*DT / (B*P) * (TG(I,2)+BETA)
   C   * (TG(I,2) - TM(I,2)))/3.D0

2     CONTINUE

*  Reset variables for next time step
      DO 5 I = 0,N
        TG(I,1) = TG(I,2)
        TM(I,1) = TM(I,2)
        TG(I,2) = TGNEW(I)
        TM(I,2) = TMNEW(I)
5     CONTINUE

1     CONTINUE

*  End of cycle - print out temp, calculate effectiveness
      DO 8 I = 0,N
        WRITE (1,30) TGNEW(I),TMNEW(I)
8     CONTINUE
      WRITE(1,*)'TIME = ', (J+(REV-1)*M)*DT/SPEED
      WRITE(1,*)
```

```fortran
      EFFECT = (SUM*(TC-TE)+TE)/(M*(TC-TE))
      WRITE(1,*) 'Avg effectiveness for cycle #',REV,' is ',EFFECT
      WRITE(1,*)

*     Reset variables for next cycle
      START = 1
      SUM = TGNEW(N)


100   CONTINUE

30    FORMAT (2F10.6)
40    FORMAT (4F15.12)
      CLOSE (1)
      END


      SUBROUTINE MGEOM(L,A,MASS,AT,AO,POROS,RH)
      PARAMETER (PI = 3.14159265359D0, CONV = 39.3700787402)
      DOUBLE PRECISION L,A,MASS,AT,AO,N,MESH,THICK,VOL,POROS,RH,DENS
      MESH  = 200.D0
      THICK = .0021D0/CONV
      DENS  = 8874.23
      N     = 1.D0/(2.D0*THICK)
      AT    = N * (PI*THICK*CONV*MESH*2.D0*A)
      AO    = A*(1 - THICK*CONV*MESH*2.D0 + MESH**2*(THICK*CONV)**2)
      VOL   = N*L * PI*(THICK*CONV)**2/2.D0*MESH*A/CONV
      MASS  = (VOL * DENS)/L
      POROS = 1 - VOL/(A*L)
      RH    = A*L*POROS/(AT*L)
      END


      SUBROUTINE THERMO(CG,CM,KG,KM,VI,TG,TM,TC,TE)
      DOUBLE PRECISION  CG,CM,KG,KM,VI,TG,TM,TC,TE
      DOUBLE PRECISION SHG,SHM,TCG,TCM,VISC
        CG = SHG(TG*(TC-TE)+TE)
        CM = SHM(TM*(TC-TE)+TE)
        KG = TCG(TG*(TC-TE)+TE)
        KM = TCM(TM*(TC-TE)+TE)
        VI = VISC(TG*(TC-TE)+TE)
      END


      DOUBLE PRECISION FUNCTION HT(RH,KG,RE)
      DOUBLE PRECISION RH,KG,RE,NU,K1
      K1 = 3.72097D-1
      NU = K1*RE**(3./4.)
      HT = NU*KG/(4.D0*RH)
      END


      DOUBLE PRECISION FUNCTION VISC(TG)
      DOUBLE PRECISION TG,K1,K2
      K1 = 4.7744D0
      K2 = 0.65671D0
      VISC = K1*TG**K2 * 1D-7
      END
```

```
DOUBLE PRECISION FUNCTION SHG(TG)
DOUBLE PRECISION TG
SHG = (5.1967D0+TG*0.D0)*1000.D0
END

DOUBLE PRECISION FUNCTION TCG(TG)
DOUBLE PRECISION TG,K1,K2,K3,K4
K1 = 2.03174D-3
K2 = 2.69044D-19
K3 = 1.10309D-1
K4 = -1.96402D1
TCG = K1*TG**(3./4.)+K2*TG**5+K3*TG**(-2./3.)+K4*TG**(-3)
END

DOUBLE PRECISION FUNCTION SHM(TM)
DOUBLE PRECISION TM,K1,K2,K3,K4,K5
K1 =  0.6356D-8
K2 = -0.4708D-5
K3 =  0.1231D-2
K4 = -0.02438D0
K5 =  4188.0D0
SHM = (K1*TM**3 + K2*TM**2 + K3*TM + K4)*K5
END

DOUBLE PRECISION FUNCTION TCM(TM)
DOUBLE PRECISION TM,K1,K2,K3
K1 = -0.2245D-3
K2 =  0.2776D0
K3 =  0.0208D0
TCM = K1*TM**2 + K2*TM + K3
END

DOUBLE PRECISION FUNCTION PRESS(WT,PM,PA,PHASE)
DOUBLE PRECISION PM,PA,WT,PHASE
PRESS  = PM/PA - COS(WT-PHASE)
END

DOUBLE PRECISION FUNCTION DERIVP(WT,PHASE)
DOUBLE PRECISION WT,PHASE
DERIVP = SIN(WT - PHASE)
END

DOUBLE PRECISION FUNCTION MFLOW(WT)
DOUBLE PRECISION WT
MFLOW  = SIN(WT)
END
```

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