ABSTRACT

An Improved
Human Gait Model Incorporating
The Effects of Muscle Activity and Joint Friction

by
Tae Ho Choi

In designing an artificial leg for an amputee, it is important to find those underlying principles which determine the normal human gait. For this purpose we have developed a model of human walking, in which it is possible to predict an optimal gait at any given speed of walking based on the principle of minimum mechanical energy consumption.

Our model is an extension of the model proposed by Mochon and McMahon (1980). Their model assumes that during the swing phase of walking mechanical energy is conserved. Non-conservative forces due to muscle activity are assumed to occur during the double support phase when both legs are in contact with the ground. We have applied these idealizations and have extended their model to calculate the energy required to maintain any periodic walking motion consistent with their model. A new constraint arises when the heel of the swing leg strikes the ground making the end of the swing phase. This constraint that after heel strike the heel of the swing leg remains on the ground produces a loss of energy to the system that must be resupplied by muscle activity to maintain a periodic motion. This allows us to uniquely determine an optimal gait for any given speed of walking which minimizes the mechanical energy loss per unit length of motion.

We propose that this energy minimizing walking motion is selected during normal periodic walking and therefore is an underlying principle determining the normal human gait. This hypothesis is tested by comparing our predicted gait with that actually observed experimentally.

AN IMPROVED
HUMAN GAIT MODEL INCORPORATING
THE EFFECTS OF MUSCLE ACTIVITY AND JOINT FRICTION

by
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APPROVAL PAGE

An Improved
Human Gait Model Incorporating
The Effects of Muscle Activity and Joint Friction

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This thesis is dedicated to
Jesus Christ

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Finally I must say that it was a great fun to think together, to fight together during discussions and enjoy together the results obtained in this thesis. Thanks again to the members of the team for creating this type of congenial atmosphere.

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CHAPTER 1

INTRODUCTION

1.1 Literature Survey

There are many documents which try to explain human walking. Some say that the swing leg acts as a free pendulum(1). Some say that there are many other forces and moments acting on the swing leg in addition to gravity(2,6). And some say that it seems reasonable to expect that the movement of the legs would be made in such a way as to minimize the amount of mechanical work that is done(3,4). Under some constraints, every statement is correct and has its own point of view. As early as 1836, the brothers Wilhelm and Eduard Weber studied the mechanism of walking and running and concluded that the motion during the swing phase of a step was pure pendulum motion(1). Fenn(5,6), in the period around 1930, studied the changes in kinetic and potential energies of the body during walking and running. Nubar and Contini(3) contend that the individual will determine his motion to reduce the muscular effort to a minimum consistent with imposed conditions of constraint. Experimentally it is found that the energy consumption per unit distance is a minimum at a particular chosen frequency(7). This result led Inman(4) to describe locomotion as the translation of the center of mass through space along a path requiring the least expenditure of energy. Beckett and Chang(2) include joint moment effects in the swing leg to produce motion that is consistent with the geometrical constraint and in such a way to give a minimum expenditure of energy. The analysis gives the motion of the leg and foot, the equivalent moments in the hip and knee to produce the motion, and the energy expended in the swing phase of the leg. But no theoretical work attempting to predict the form of swing period vs. speed relationship has yet been reported. Therefore Mochon and McMahon(8) have developed a mathematical model to predict the form of swing period vs. speed relationship. In this model the body is represented by three limbs, one for the stance leg and two for the thigh and shank of the swing leg, respectively. It is assumed that the muscles act only to establish an initial configuration and velocity of the limbs at the beginning of swing phase. The swing leg and the rest of the body then moves through the remainder of the swing phase entirely under the action of gravity. This model assumes initial energy is conserved during the swing phase. In this model many aspects of walking at normal speed, from a prediction of the foot forces to an understanding of the relationship between walking cadence and body stature, are well represented by a model which completely disregards the action of muscles, except for setting the initial positions and velocities of the limbs at the beginning of the swing phase.

As explained above, Mochon's model includes both the swing leg and the stance leg, and Beckett's model includes only the swing leg. In Mochon's model energy is conserved, and in Beckett's model energy is not conserved, during the swing phase. Beckett's idea that forces and moments are imposed at the joints of the leg improves the performance of Mochon's model. Therefore, in this thesis, we have modified Mochon's model with Beckett's idea, and extended it by an algorithm which calculates the energy loss at heel strike. As the algorithm produces the amount of energy loss which should be re-supplied during the double support, it will make it possible to develop a model which includes the double support phase as well as the swing phase in the future.

1.2 Objective

A model of human walking is applicable for improved understanding of rehabilitation medicine and legged robots. There are complaints about artificial legs, such as heavy weight even though they are made of very light materials. If we can design an artificial leg to fit the individual person, those complaints may be minimized. When we design legged robots, it will improve the efficiency if robots could move with minimum energy expenditure. Therefore, in designing artificial legs for amputees and formulating control laws for robots, it is important to find those underlying principles which determine the normal human gait.

Our purpose in this thesis is to develop a mathematical model of human walking which can be used for understanding the effects of parameter changes, and predicting the optimal human gait with minimal energy loss. There are many papers which explain human gait experimentally, as we showed above, but unfortunately no result can be used directly to predict the optimal gait. As the swing phase is regarded as the more important period when the majority of step length is attained, almost every paper touches only the swing phase, but the dominant energy loss occurs at heel strike which occurs between the swing phase and the double support phase. The swing phase means that one leg is on the ground, and the other is off the ground. The double support phase means that both legs are on the ground. A true model of human walking should include both phases. Until now no model that incorporates both phases, has been reported. Here our purpose is focused on the calculation of the energy loss at heel strike and preparing a method for the prediction of the optimal human gait based on this energy loss.

We have selected Mochon's free pendulum model as our base model. It operates on the assumption that energy is conserved during the swing phase. As artificial legs move like free pendulums, this model effectively predicts an amputee's gait. But it is limited in predicting the swing time of the normal gait during the swing phase. It predicts only high speed normal walking reasonably accurately. At low speed walking, the model produces an abnormal walking gait because the shank kicks too high. It is well known that there is excessive knee flexion in amputees during swing time, and thus is consistent with our result. As the model acts like a free compound pendulum under gravity, and neglects the joint resistance during walking, the shank must swing higher to stay in the air longer at slow walking. Another limitation is that it represents only the swing phase. Because of these limitations, we can not apply Mochon's model directly to calculate the mechanical work that is done when walking, and the energy that must be re-supplied during the double support phase. We have modified and extended Mochon's model as follows to calculate the energy loss when walking.

First we have tried to match theoretical data with experimental data by including frictional effects. At this point we consider friction at joints is a source of resistance. When we include frictional effects, we can manipulate the swing time with small amount of energy loss, and the model can match experimental data better than Mochon's model does. Without this frictional effect, our model becomes a free compound pendulum model, and the model cannot predict experimental data effectively because of the high kicking of the shank. We have regarded friction of joints as representative of every resistance, and developed our model. There may be many other sources of resistance which retard the movement of the limbs, like fluctuation of the limbs, air resistance, muscle activity, etc. But even though we include only one source of resistance, it does not affect the form of the mathematical model. As every source of resistance acts to reduce the effect of gravity, the friction of joints can represent all the resistances combined. The three joints are the ankle joint of the swing leg, hip joint of both legs, and the knee joint of the swing leg. The coefficient of friction of joints is assumed the same because the lubricant material in the joint is the same. These frictional forces reduce the effect of gravity. Under the reduced gravity the shank can stay longer in the air without kicking up too high. Our model, which includes frictional effects, does not produce abnormal high kicking of the shank. This frictional effect is one source of energy loss during the swing phase.

Even though we have modified Mochon's model by including the frictional effects, it is impossible to calculate energy loss during walking with this model because the model does not consider energy losses. We think there are three sources of energy loss. The first is friction as explained above. The second is the energy loss at knee lock. At the end of the swing phase, the knee locks just before the heel strikes. When knee lock occurs, the thigh and the shank of the swing leg moving at different velocities, become one unit that move with the same velocity. This produces an energy loss. The third energy loss occurs at heel strike, and after heel strike the swing leg no longer moves like a free pendulum. Both legs must stay on the ground, which is a new constraint. This constraint produces energy loss at heel strike because the legs must suddenly change their velocities. We have developed an algorithm based on the new constraint, which calculates the energy loss at heel strike.

After modifying and extending Mochon's model, we have calculated energy loss component of walking, compared them, and found the energy loss at heel strike is dominant energy loss when walking. These energy losses give us a criterion to determine the optimal gait of human walking, and we propose that the translation trajectories of body masses which produces the minimal energy loss, is the optimal gait.

CHAPTER 2

MATHEMATICAL MODEL

2.1 Assumptions

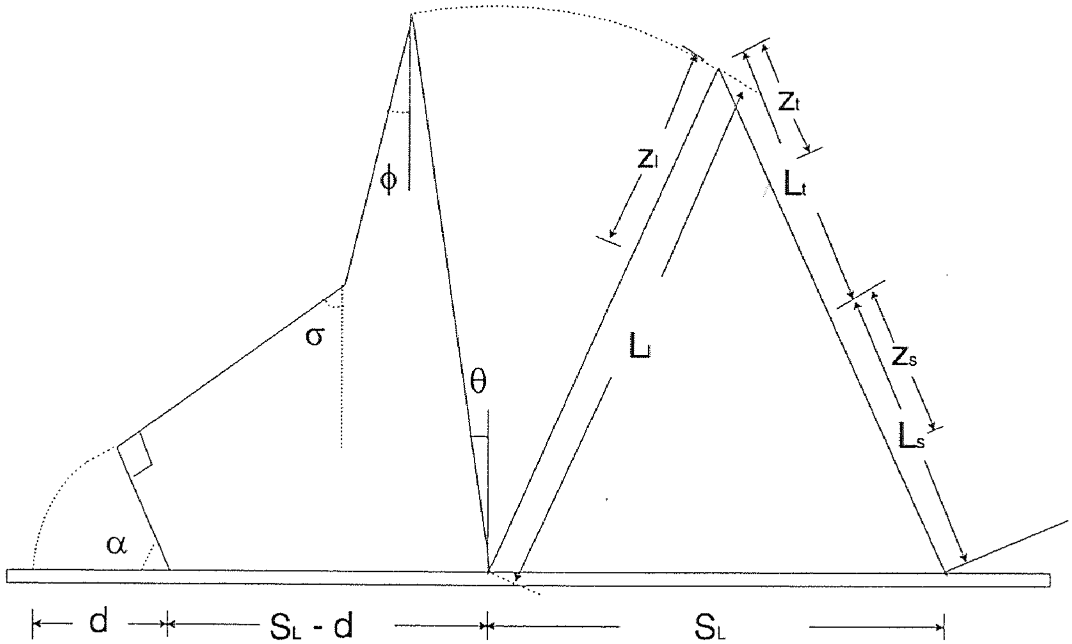

Figure 1 Schematic representation of the model during the swing phase. S_L is the step length, and d is the foot length. Meaning of other symbols are in Appendix A.

The model is shown schematically in Figure 1. It consists of 3 links; one representing the stance leg and two representing the thigh and shank of the swing leg. The foot of the swing leg is rigidly attached to the distal link, and does not constitute a separate link. Each link is assumed to have a distributed mass. The moment of inertia and location of the center of mass of each link is taken from Dempster's anthropometric data(10). The mass of the foot is lumped into the shank. The mass of the trunk, head and arms is represented by a point mass at the hip joint. The lengths, positions of the center of mass and angles of each limb are shown in Figure 1.

During the swing phase, muscle activity is not as prominent as during the double support phase, but some muscle groups accelerate, decelerate and control the limbs during the swing phase. For example, muscle activity of quadriceps and hamstring groups during the swing phase are shown in Figure 2, which is quoted from Beckett's paper(2). At present we do not know exactly how much force is produced by each muscle group, how it changes, and how many muscle groups are actually active, during the swing phase. Therefore we assume that during the swing phase muscle activity is continuous, that the magnitude of muscle forces is constant, and that these forces oppose gravity. The resultant effect of these forces is postulated to reduce the gravitational constant because there is acceleration after toe-off, and deceleration before heel-strike. To easily establish our mathematical model of walking, we assume that energy is supplied only at the beginning of the swing phase, and the rest of the body moves through the remainder of the swing phase under the action of gravity whose effect on the body is reduced by muscle groups.

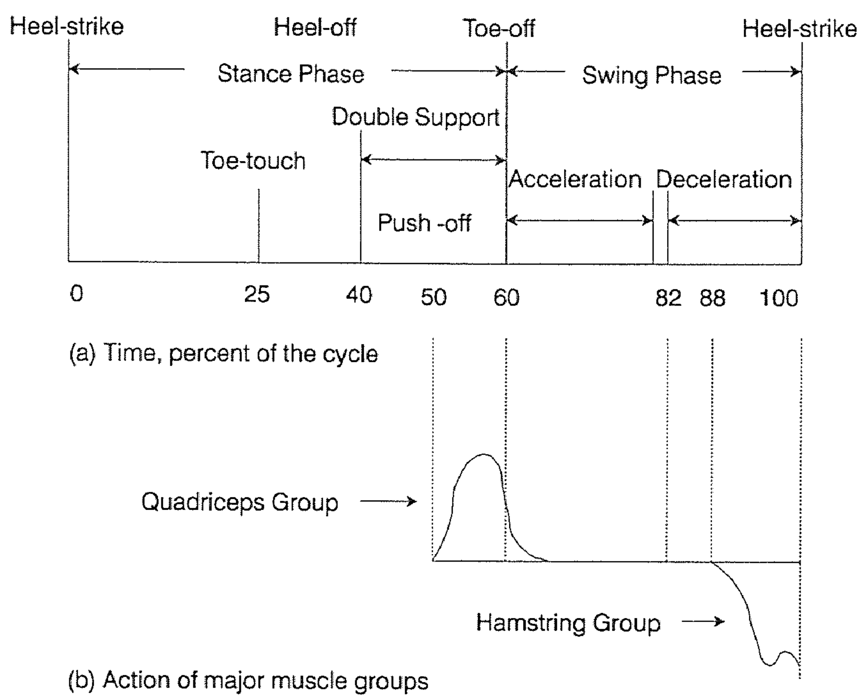

Figure 2 One complete cycle in level walking

Joint friction produces a frictional moment which is proportional to the angular speed of the joint. There are three joints in our model - ankle, hip and knee. But, in reality, the hip joint consists of two separate joints instead of one. One is for the stance leg, and one is for the swing leg. The angular speed of the stance leg(θ') is much less than the angular speed of the thigh(ϕ') and the shank(σ') of the swing leg. In normal walking, the angular speed of the stance leg is roughly more than five times smaller than the angular speed of the thigh and the shank of the swing leg. Therefore, the ankle joint and the hip joint of the stance leg produce less frictional moment than the knee joint and the hip joint of the swing leg. In this thesis we consider frictional effects in only the two joints of the swing leg to simplify the problem.

2.2 Mathematical Model

Our mathematical model is expressed with the following equations based on the assumption of free pendulums.

$$
\begin{aligned}
c_{11}\ddot{\theta} + c_{12}\ddot{\phi} + c_{13}\ddot{\sigma} + c_{12}^{*}\dot{\phi}^{2} + c_{13}^{*}\dot{\sigma}^{2} &= w_1 \sin\theta \\
c_{21}\ddot{\theta} + c_{22}\ddot{\phi} + c_{23}\ddot{\sigma} + c_{21}^{*}\dot{\theta}^{2} + c_{23}^{*}\dot{\sigma}^{2} &= w_2 \sin\phi \\
c_{31}\ddot{\theta} + c_{32}\ddot{\phi} + c_{33}\ddot{\sigma} + c_{31}^{*}\dot{\theta}^{2} + c_{32}^{*}\dot{\phi}^{2} &= w_3 \sin\sigma
\end{aligned}
\tag{2.1}
$$

where the meaning of the coefficients as well as the derivation of the equations is given in APPENDIX A.

As explained above, the resultant effect of muscle activity is to reduce the effect of gravity, and can be expressed as -kg(where k is a coefficient between 0 and 1, and **g** is the force of gravity). Without this effect, the output of our mathematical model, which is derived on the assumption of free pendulums, can not match the experimental data. In particular the free pendulum model will produce an abnormally high kick of the shank. Therefore, in equation(2.1), the gravity term **g** must be changed to (1-k)**g** to incorporate the effect of muscle activity. In equation(2.1) the effect of gravity is included in the coefficients w_n (See Appendix A).

The frictional moments of the hip joint and the knee joint of the swing leg are expressed in equation (2.2). The derivation process and the definition of the symbols are in

Appendix B. If we include frictional moments in our model, the equations of our model become equation(2.3).

Equations of frictional moment:

$$
\begin{aligned}
F_\phi &= -b\dot{\phi}\{1+\frac{L_t}{r}\cos(\phi-\alpha)\} - c\dot{\beta}\{Z_t/(L_t-Z_t)+\frac{L_t}{Z_s}\cos\beta\} \\
F_\sigma &= -c\dot{\beta} - b\dot{\phi}\frac{Z_s}{r}\cos(\sigma-\alpha)
\end{aligned}
\tag{2.2}
$$

Equations of our model including joint frictional effect are

$$
\begin{aligned}
c_{11}\ddot{\theta}+c_{12}\ddot{\phi}+c_{13}\ddot{\sigma}+c_{12}^{*}\dot{\phi}^2+c_{13}^{*}\dot{\sigma}^2 &= w_1\sin\theta \\
c_{21}\ddot{\theta}+c_{22}\ddot{\phi}+c_{23}\ddot{\sigma}+c_{21}^{*}\dot{\theta}^2+c_{23}^{*}\dot{\sigma}^2 &= w_2\sin\phi - F_\phi \\
c_{31}\ddot{\theta}+c_{32}\ddot{\phi}+c_{33}\ddot{\sigma}+c_{31}^{*}\dot{\theta}^2+c_{32}^{*}\dot{\phi}^2 &= w_3\sin\sigma - F_\sigma
\end{aligned}
\tag{2.3}
$$

2.3 Solution Method

Our equations are non-linear equations. We formulate a two point boundary value problem and solve it by the shooting method(11). This method employs a 4th order Runge-Kutta algorithm. If we assume the initial angles of leg position at toe-off and guess the initial velocities of the legs at toe-off, our program solves for the final angles of leg position at a specific swing time, which is also given. If discrepancies occur between the calculated final angles and the chosen configuration at heel-strike, our program guesses new initial velocities and recalculates the final angles. This process continues until the program finds the correct initial velocities, with which it can produce the final angles of leg position which agree with the desired final angles which correspond to heel-strike. If the program finds the correct initial velocities of the legs, it can produce the trajectory of the legs, and know the position and velocities of the legs at any instant during the swing time. With these data, the program calculates energy losses, and displays the gait on the screen.

2.4 Calculation of Energy Losses

From the output of the program, we know the position and velocities of the legs at any instant. Therefore we can calculate the kinetic and potential energy of the legs at any instant. The total mechanical energy at any instant is expressed in equation(2.4), and the derivation of potential and kinetic energy in Appendix A.

$$\begin{aligned} TE &= PE + KE \\ &= \sum_{i=1}^{3} w_i \cos \varpi_i + \frac{1}{2} \sum_{i,j=1}^{3} c_{ij} \dot{\varpi}_i \dot{\varpi}_j \end{aligned} \tag{2.4}$$

Frictional energy loss during the swing time is the energy difference between the initial and the final energy. Initial energy means the energy at toe-off, and the final energy is the energy at heel-strike. Because we know the angles and velocities of the limbs at toe-off and at heel-strike, we can calculate the energy at toe-off and at heel-strike from the equation(2.4), and frictional energy loss with these calculated energy. Energy loss at knee-lock is the energy difference between the energy before knee-lock and the energy after knee-lock. It can also be calculated with equation(2.4) because we know the angles and velocities of the limbs before and after knee-lock. Energy loss at heel-strike can be calculated as follows. During the swing phase the swing leg clears the ground, but after heel strike the swing leg and the stance leg must remain on the ground. Because of this constraint the legs change their direction of motion, and lose energy. The derivation of angles and velocities of the limbs before and after heel-strike are in Appendix C. Similarly energy loss at heel-strike can be calculated with equation(2.4).

CHAPTER 3

PROGRAM

3.1 Flowchart

The flowchart of the program is shown below. It provides an overview of the process of arriving at a solution for our model. The next sections will explain each step in more detail.

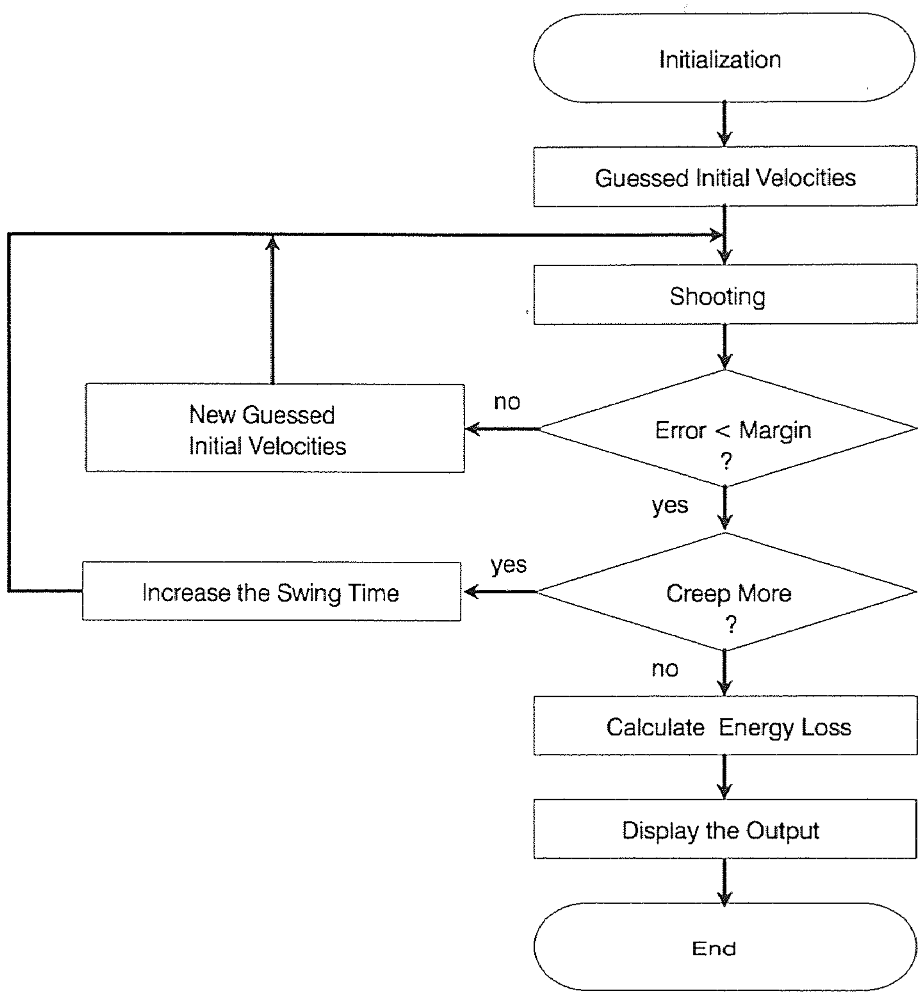

Figure 3 Flowchart of the program.

3.2 Initialization

The program requires the specification of initial parameters. With these parameters the program initializes every coefficient of our mathematical model of walking. The required parameters are listed below.

* parameters of the stance leg	... weight, length, center of mass
* parameters of the shank of the swing leg	... weight, length, center of mass
* parameters of the thigh of the swing leg	... weight, length, center of mass
* parameter of the foot	... length
* parameters of the body	... weight
* parameters of walking	... step length, swing time, angles at toe-off and at heel-strike

The weight of the shank of the swing leg includes the weight of the foot. In addition to parameters, arbitrarily assumed initial velocities of the stance leg, of the shank and of the thigh ,of the swing leg, are also to be satisfied.

3.3 Shooting Method

With angles at toe-off and initial velocities of each limb, the program calculates angles of each limb at the time of heel-strike, and compares these calculated angles with the angles given at initialization. If there are discrepancies between the calculated and the given, the program tries to guess new initial velocities based on the discrepancies. With these new initial velocities, the program calculates angles of each limb at heel-strike again. The program continues this process until every discrepancy is less than the given discrepancy margin. In our program the discrepancy margin is 10^{-5}.

The process will be explained with an example. Table 1 shows the guessed initial velocities and calculated final angles at heel strike of every limb during the shooting process of the example. The given initial angles of each limb at toe off are (θi, ϕi, σi) = (10, -9.4, -55), and the given final angles of each limb at heel strike are (θf, ϕf, σf) =

(-22.02, 22.02, 22.02). At first the program tries to calculate final angles with arbitrarily guessed initial velocities (θ', ϕ', σ') = (-102.04, 256.82, -317.34). The calculated final angles are (θ, ϕ, σ) = (-36.99, 10.32, 77.32). This result does not satisfy the given final angles. Therefore the program finds new guessed initial velocities (θ', ϕ', σ') = (-74.33, 334.62, -689.72) by the shooting method based on the discrepancies between the calculated and the given, and tries to calculate final angles again. At the seventh trial, the program finally finds true initial velocities, with which it can produce the given final angles. The true initial velocities are (θ', ϕ', σ') = (-74.43, 271.63, -523.88).

Table 1 The guessed Initial Velocities and the Calculated Final Angles During the Shooting Process in the Example. (θi, ϕi, σi) = (10, -9.4, -55); (θf, ϕf, σf) = (-22.02, 22.02, 22.02).

shoot number	1	2	3	4	5	6	7
theta_dot	-102.04	-74.33	-75.39	-74.59	-74.48	-74.43	-74.43
phi_dot	256.82	334.62	254.96	277.49	271.1	271.63	271.63
sigma_dot	-317.34	-689.72	-464.12	-545.06	-521.62	-523.89	-523.88
theta_final	-36.99	-22.19	-22.8	-22.04	-22.06	-22.02	-22.02
phi_final	10.32	52.08	9.74	27.73	21.44	22.03	22.02
sigma_final	77.32	-26.52	48.25	9.36	23.38	22.02	22.02

The outputs of each shooting are shown in Figure 4. Sixth and seventh output look almost the same, and sixth output is omitted. In Figure 3, the sign of angle of the stance leg is reversed($-\theta$) to make three curves end at the same point(-22.02) finally. The heel-strike configuration is $-\theta(T) = \phi(T) = \sigma(T)$.

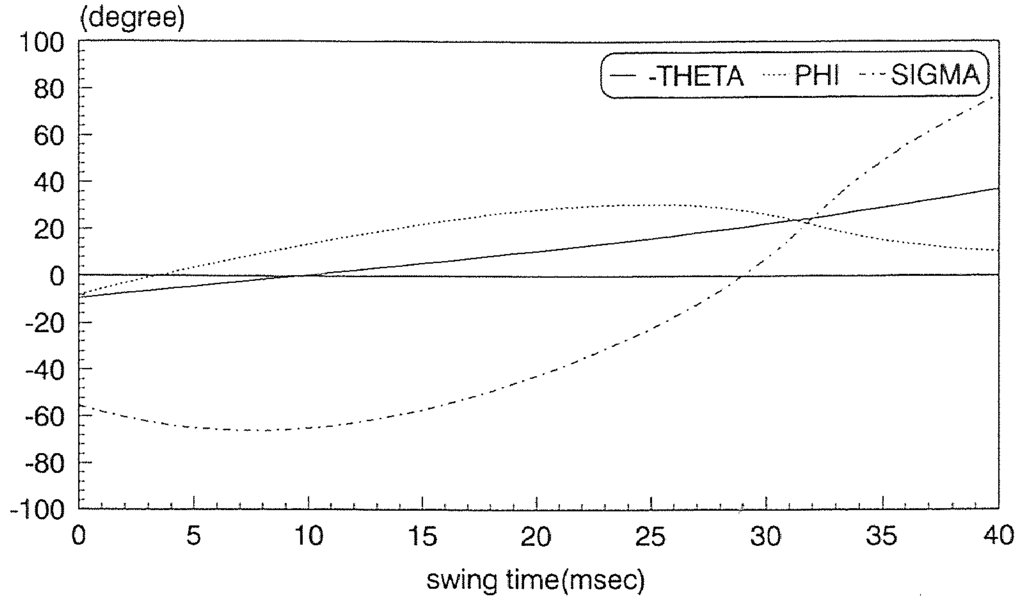

Figure 4(a) The output of the first shooting. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-102.04, 256.82, -317.34), and calculated final angles (θf, ϕf, σf) = (-36.99, 10.32, 77.32).

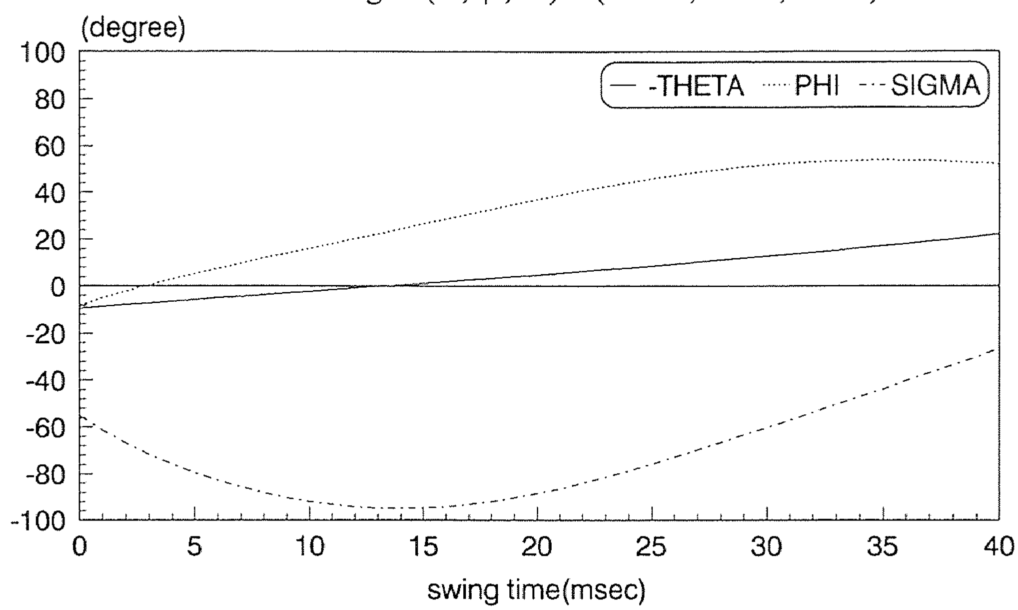

Figure 4(b) The output of the second shooting. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-74.33, 334.62, -689.72), and calculated final angles (θf, ϕf, σf) = (-22.19, 52.08, -26.52).

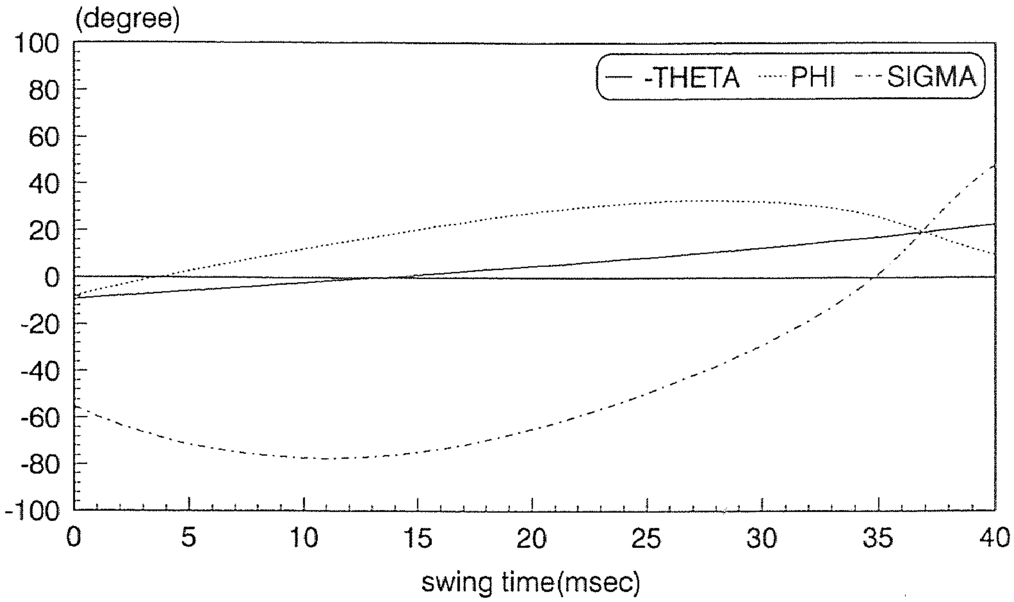

Figure 4(c) The output of the third shooting. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-75.39, 254.96, -464.12), and calculated final angles (θf, ϕf, σf) = (-22.8, 9.74, 48.25).

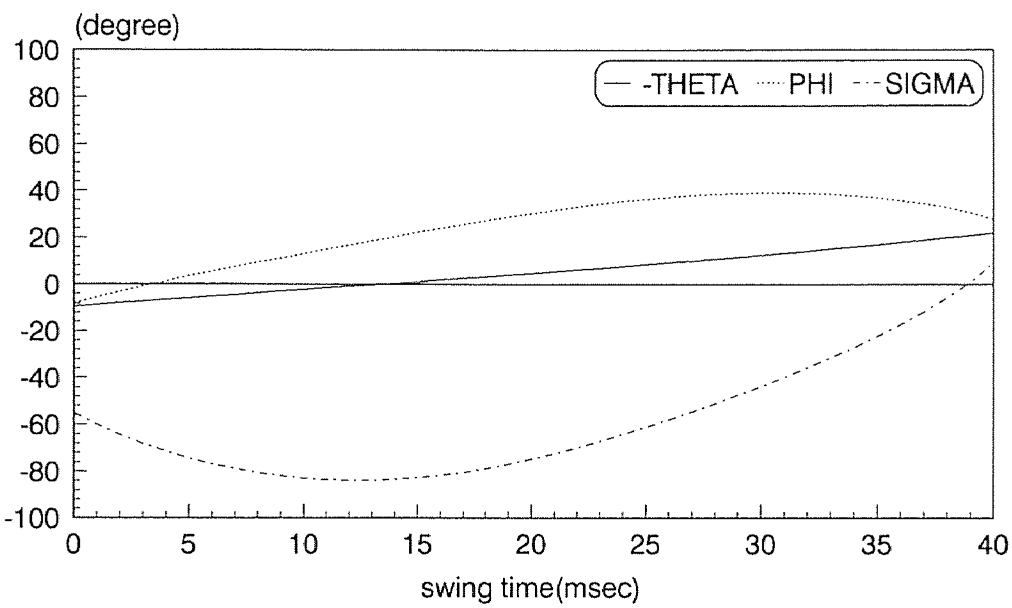

Figure 4(d) The output of the fourth shooting. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-74.59, 277.49, -545.06), and calculated final angles (θf, ϕf, σf) = (-22.04, 27.73, 9.36).

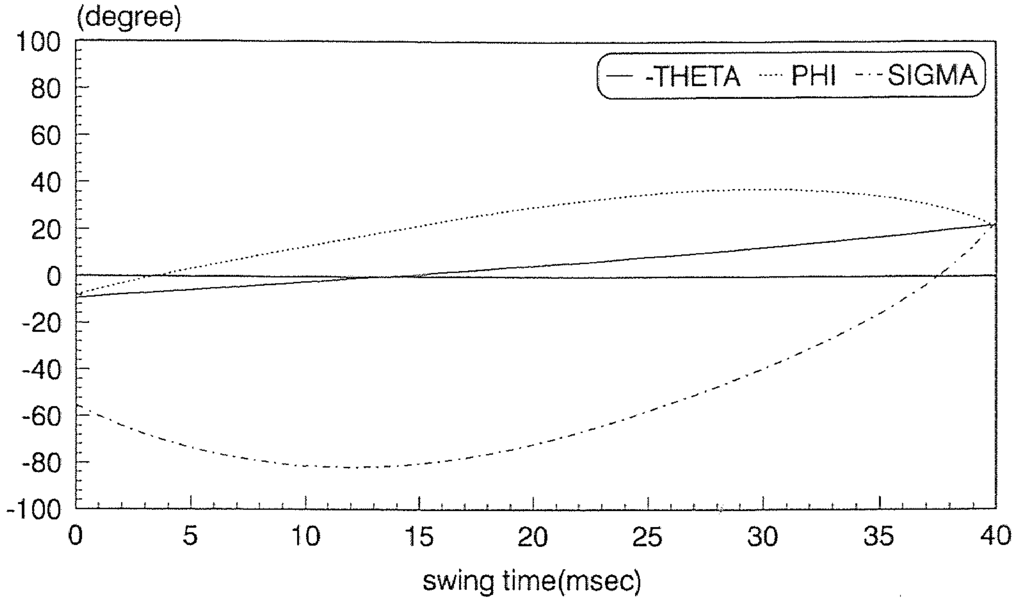

Figure 4(e) The output of the fifth shooting. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-74.48, 271.1, -521.62), and calculated final angles (θf, ϕf, σf) = (-22.06, 21.44, 23.38).

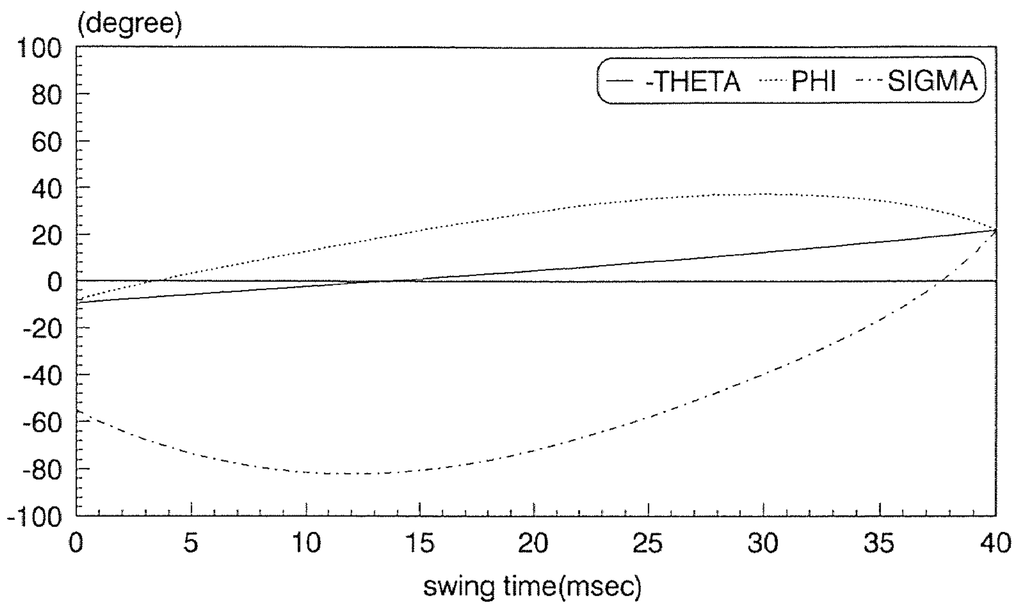

Figure 4(f) The output of the seventh shooting. Initial angles (θi, ϕi, σi) =(10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-74.43, 271.63, -523.88), and calculated final angles (θf, ϕf, σf) = (-22.02, 22.02, 22.02).

3.4 Creeping Method

As the program starts with arbitrary initial velocities of each limb, these velocities may not satisfy the initial and final angles of each limb even though we use the shooting method. This usually occurs when the swing time is long. In that event we first select a short swing time, and allows the swing time to increase(creep) until we achieve the desired final angles at the desired swing time. For example, at some step length, we want to know the gait with a swing time of 0.4 second. At first we do not know the exact initial velocities which satisfy the boundary conditions. Therefore one example of arbitrary initial velocities(-86.01, -114.5, 598.51) may produce a solution which is not feasible as shown in figure 5. Another example (-95.58, -540.19, 1881.41) may blow up before it produces the final result as shown in figure 6.

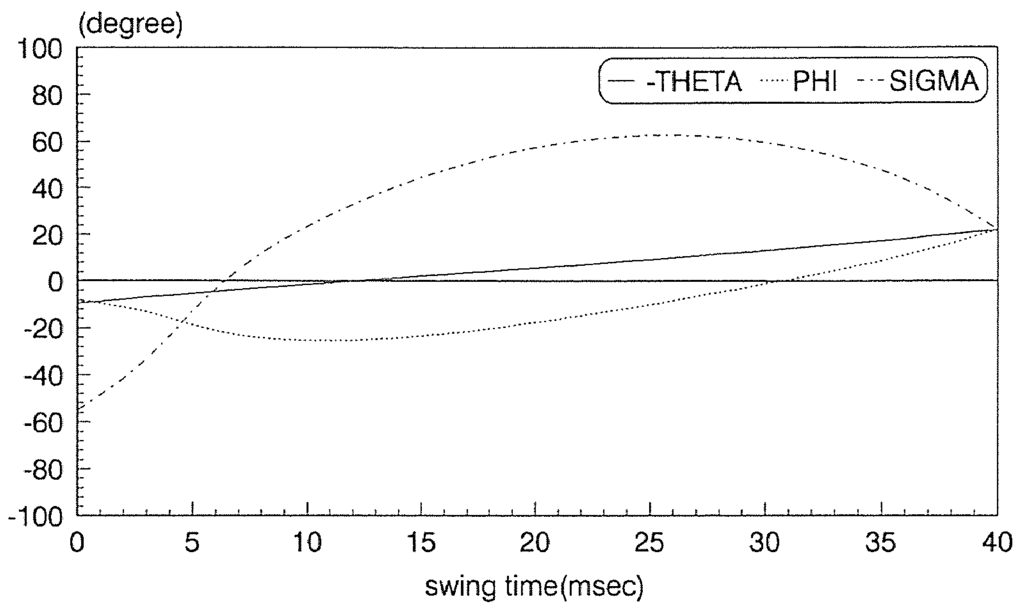

Figure 5 The output that is not feasible. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-86.01, -114.5, 598.51), and calculated final angles (θf, ϕf, σf) = (-22.02, 22.02, 22.02).

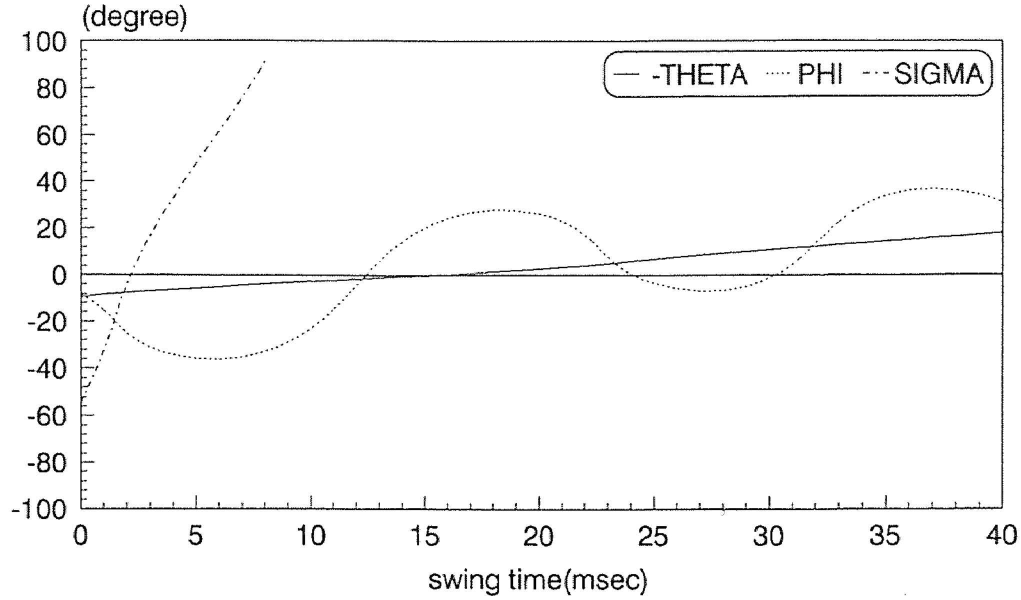

Figure 6 The output that blows up. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-95.58, -540.19, 1881.41).

But if we creep the final swing time, we can find the exact initial velocities which give feasible solutions, or which do not blow up. Table 2 shows initial velocities which satisfy the final angles at each swing time during the creeping process. The outputs at each swing time are shown in Figure 7.

Table 2 The Initial Velocities at Each Swing Time During the Creeping Process. (θi, ϕi, σi) = (10, -9.4, -55); (θf, ϕf, σf) = (-22.02, 22.02, 22.02).

swing time	25	30	35	40
theta_dot	-122.36	-101.15	-85.92	-74.43
phi_dot	261.61	257.42	261.65	271.63
sigma_dot	-159.3	-294.98	-415.43	-523.88

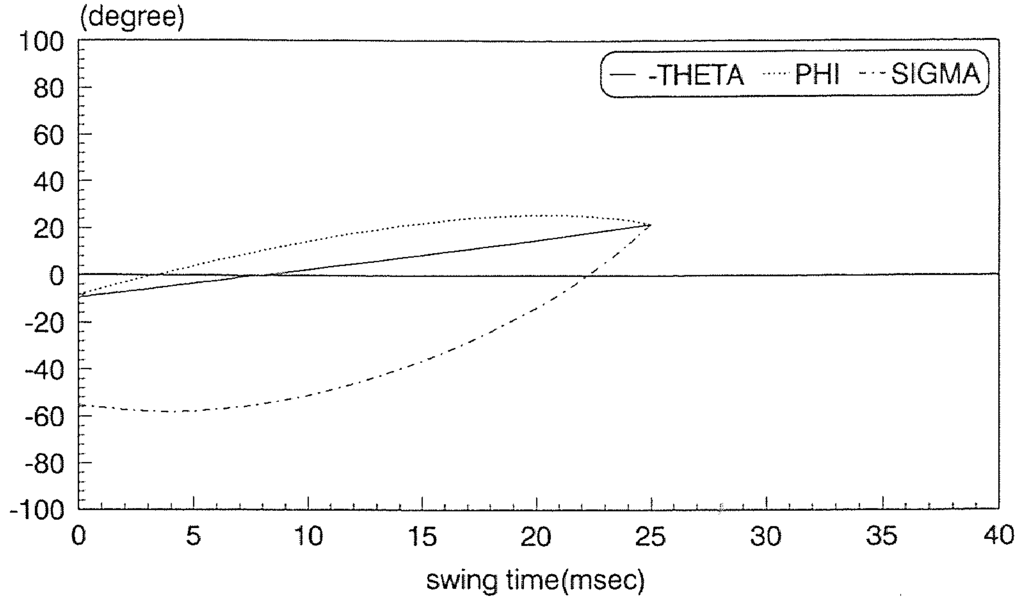

Figure 7(a) The output at swing time 25 msec. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-122.36, 261.61, -159.3), and calculated final angles (θf, ϕf, σf) = (-22.02, 22.02, 22.02).

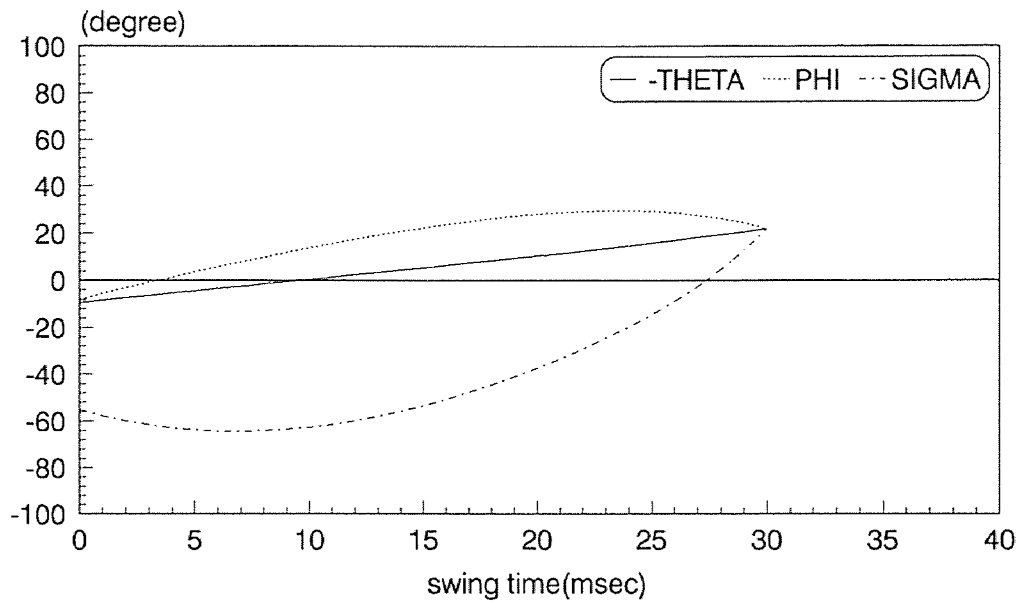

Figure 7(b) The output at swing time 30 msec. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-101.15, 257.42, -294.98), and calculated final angles (θf, ϕf, σf) = (-22.02, 22.02, 22.02).

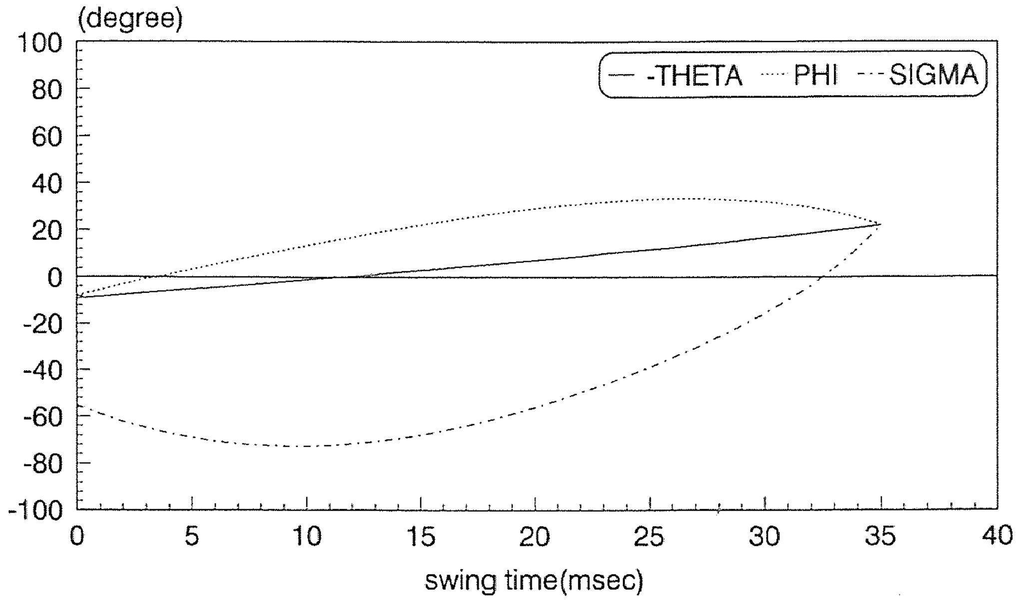

Figure 7(c) The output at swing time 35 msec. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-85.92, 261.65, -415.43), and calculated final angles (θf, ϕf, σf) = (-22.02, 22.02, 22.02).

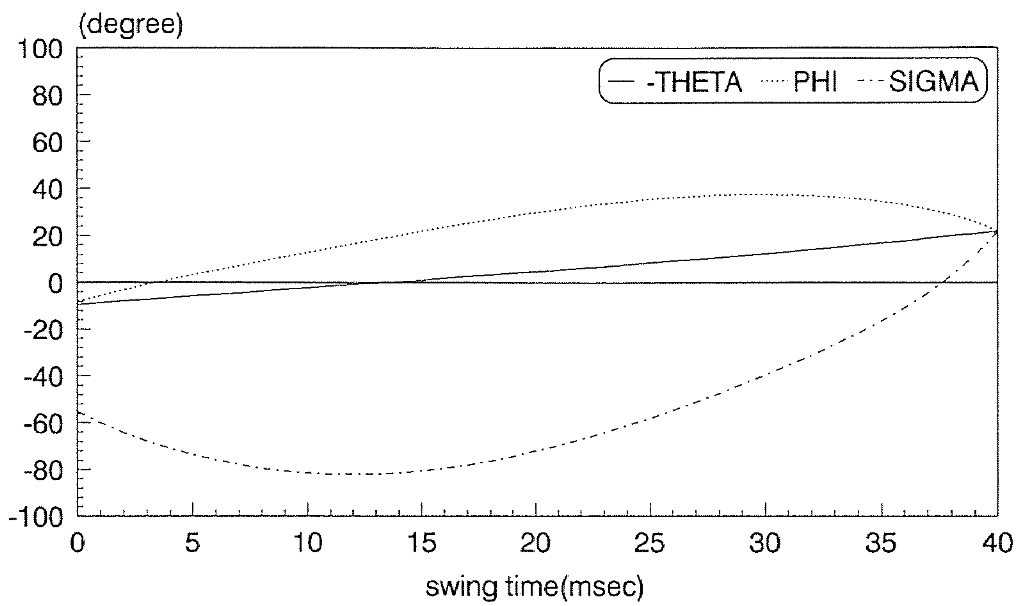

Figure 7(d) The output at swing time 40 msec. Initial angles (θi, ϕi, σi) = (10, -9.4, -55), initial velocities (θ', ϕ', σ') = (-74.43, 271.63, -523.88), and calculated final angles (θf, ϕf, σf) = (-22.02, 22.02, 22.02).

3.5 Calculation of Energy Losses

As the result of shooting and creeping, the program knows the exact initial velocities which satisfy the initial and final angles of each limb. If the program knows the exact initial velocities, it can calculate the final velocities also. With these exact initial and final velocities, energy losses, of friction, at knee-lock, and at heel-strike are calculated.

CHAPTER 4

RESULTS

4.1 Effect of Parameter Variation

We has introduced three new parameters to improve the gait of the model. They are muscle activity, friction of the hip joint, and knee joint of the swing leg. Now we do not know the exact amount of muscle activity and coefficient of friction of joints. The program is useful to understand the effect of parameter variations. With the program we can simulate the walking motion by changing parameters. In Figure 8, we can see the effect of *k* parameter. These figures are outputs when we apply the same value of *k* parameter to all three limbs. When *k* increases, the effect is to reduce the angle variation of all three limbs. The effect of *k* on the thigh and shank is much greater than the effect on the stance leg.

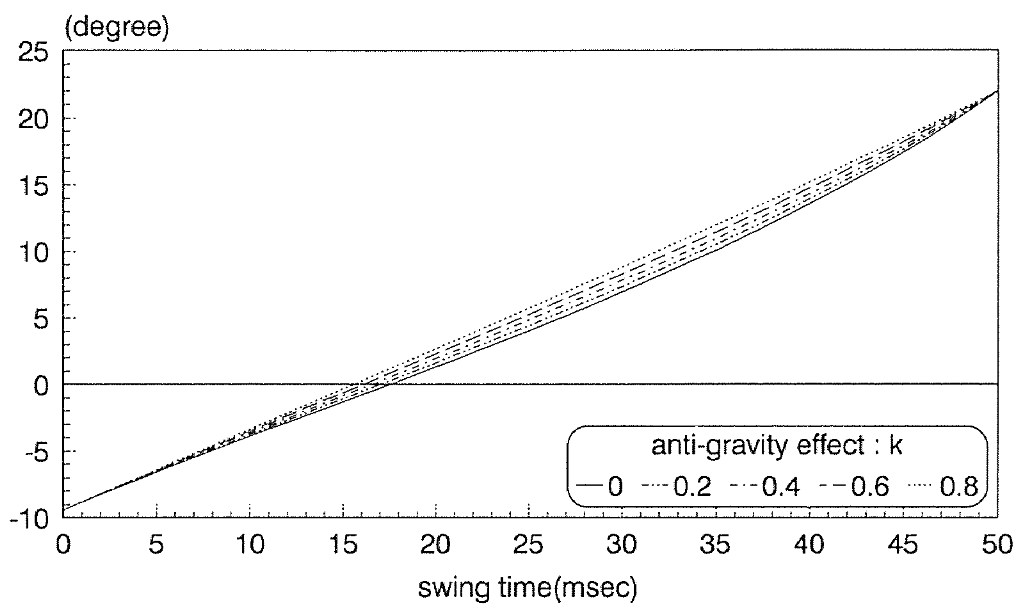

Figure 8(a) Effect of anti gravity k on the stance leg. The curve becomes more linear as k increases. The change of the curve is relatively small.

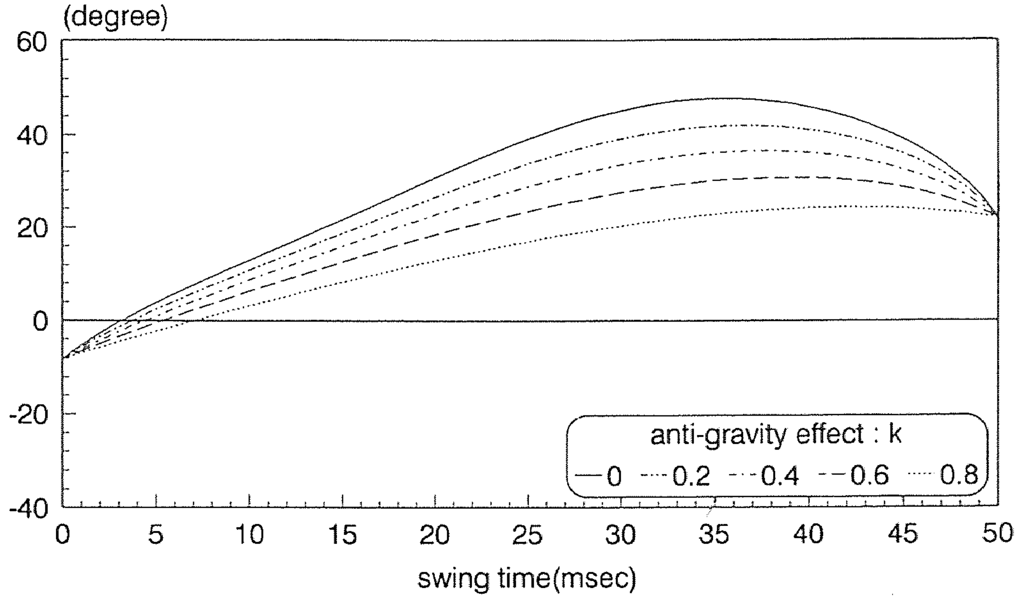

Figure 8(b) Effect of anti gravity k on the thigh. The curve becomes more linear as k increases. The change of the curve is much greater than the case of the stance leg.

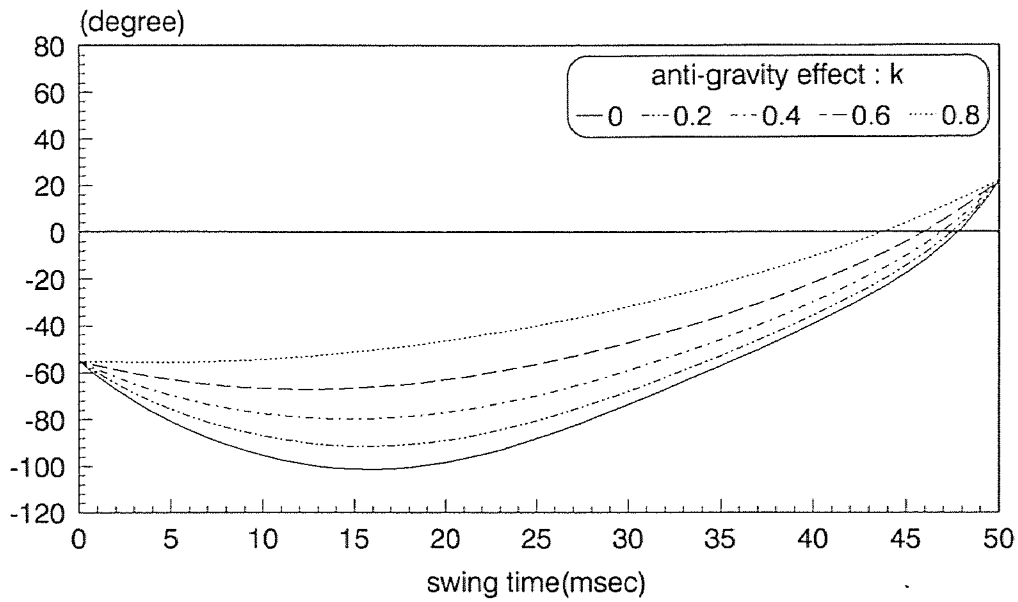

Figure 8(c) Effect of anti gravity k on the shank. The curve becomes more linear as k increases. The change of the curve is also much greater than the case of the stance leg.

In Figure 9, we can see the effect of anti gravity which is applied only to one limb at a time. The effect of anti gravity of the stance leg is negligible to all three limbs. The anti gavity of the thigh has much effect on the thigh, and small effect on the shank. But the effect on the shank makes the shank kick higher, which is undesired effect. It has negligible effect on the stance leg. The effect of anti gravity of the shank has great effect only on the shank, and effect on other limbs are negligible. Here we may say that only the anti gravity effect on the shank can make the shank not kick high.

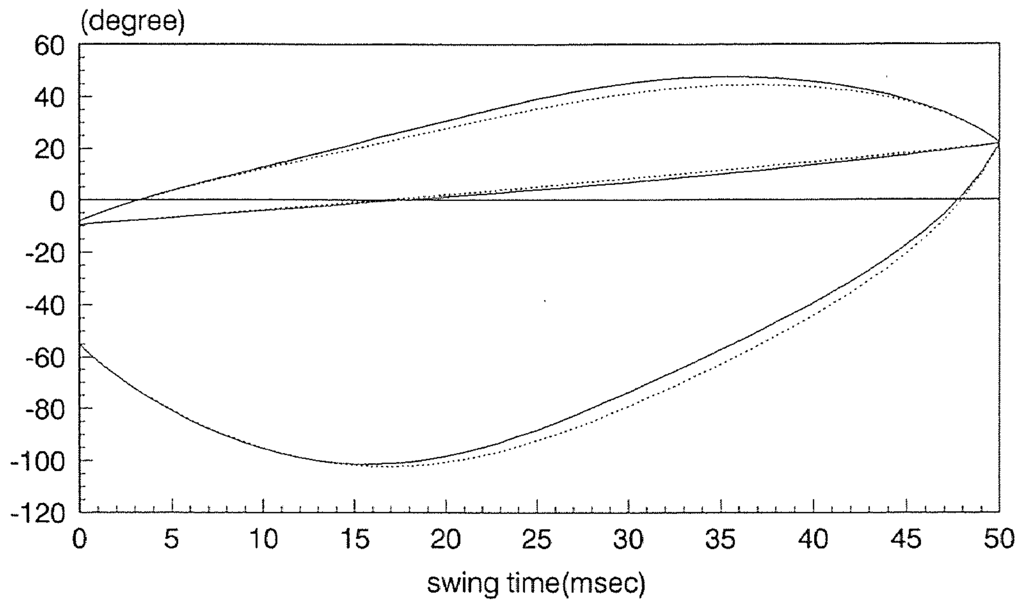

Figure 9(a) Effect of anti gravity k. In this figure, only the stance leg has the effect of anti gravity. The line is the curve when k=0, and the dotted line is the curve when k=0.8. The effect on the limbs is negligible.

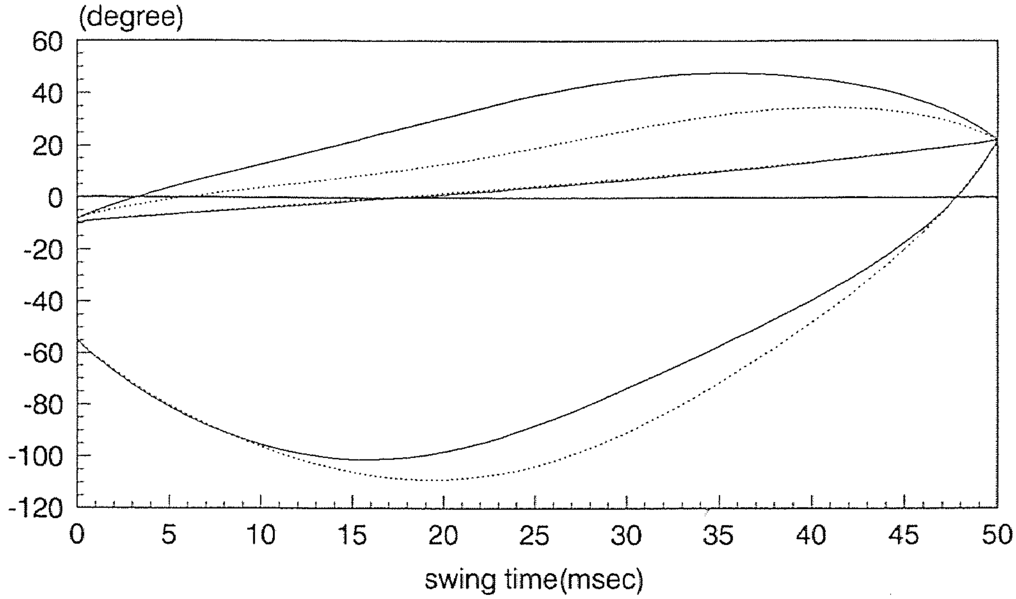

Figure 9(b) Effect of anti gravity k. In this figure, only the thigh has the effect of anti gravity. The line is the curve when k=0, and the dotted line is the curve when k=0.8. The effect on the stance leg is negligible. The effect on the thigh is great. The effect on the shank is much, but smaller than the effect on the thigh.

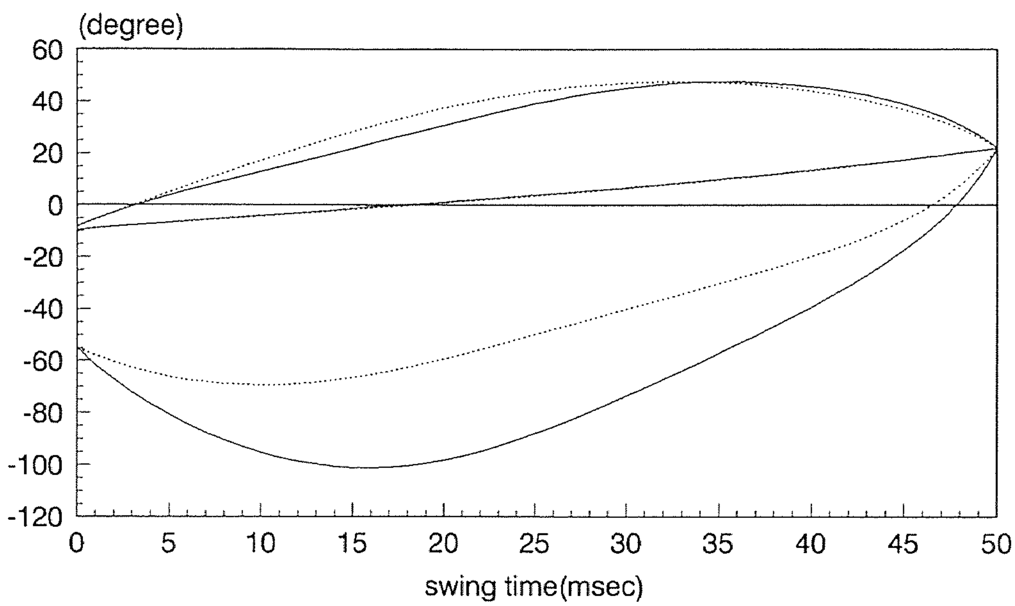

Figure 9(c) Effect of anti gravity k. In this figure, only the shank has the effect of anti gravity. The line is the curve when k=0, and the dotted line is the curve when k=0.8. The effect on the stance leg is negligible. The effect on the shank is great. The effect on the thigh is negligible.

Friction of joints also has much effect on the thigh and on the shank. The effect of knee joint friction is shown in Figure 10. The effect of hip joint friction is shown in Figure 11.

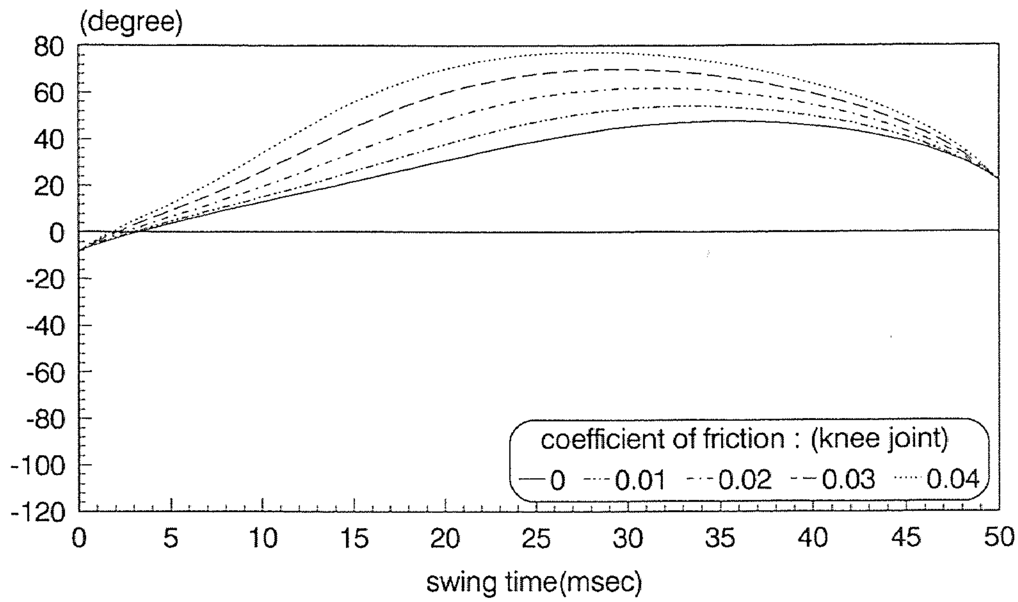

Figure 10(a) The effect of friction at the knee joint on the thigh. As the friction increases, the thigh goes higher.

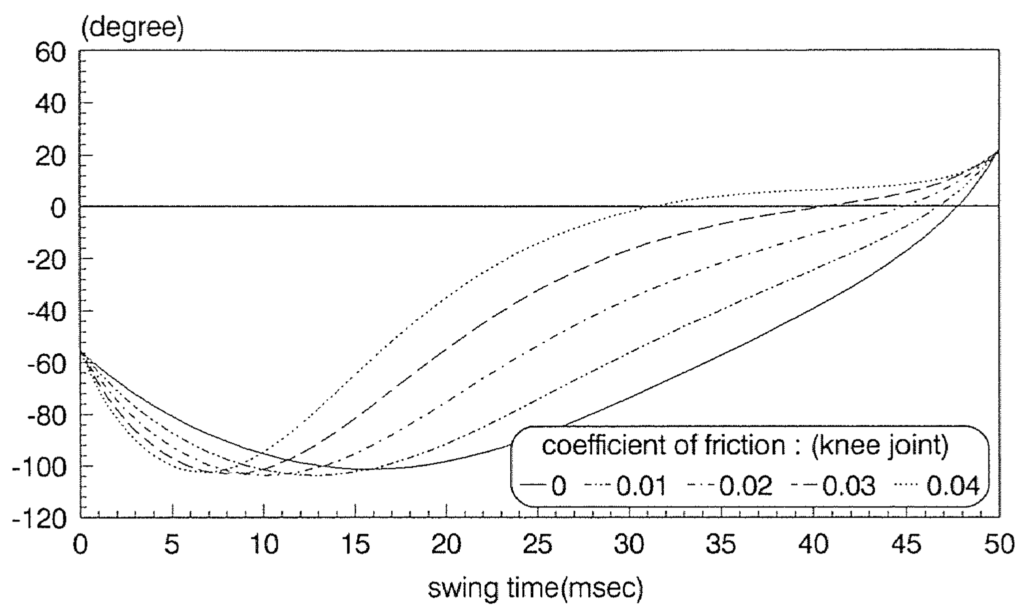

Figure 10(b) The effect of friction at the knee joint on the shank. As the friction increases, the height of shank's kicking decreases.

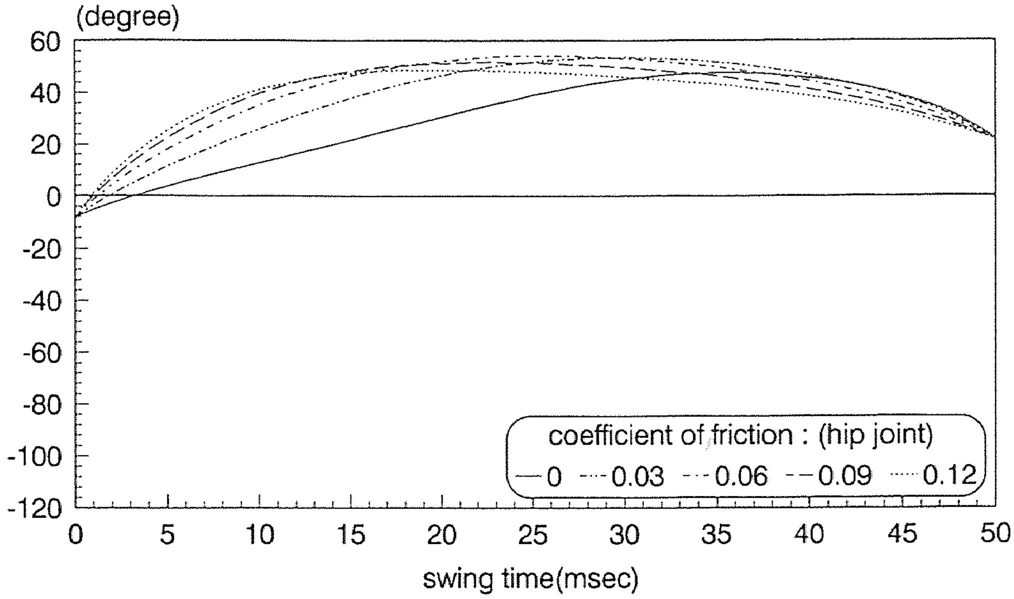

Figure 11(a) The effect of friction at the hip joint on the thigh.

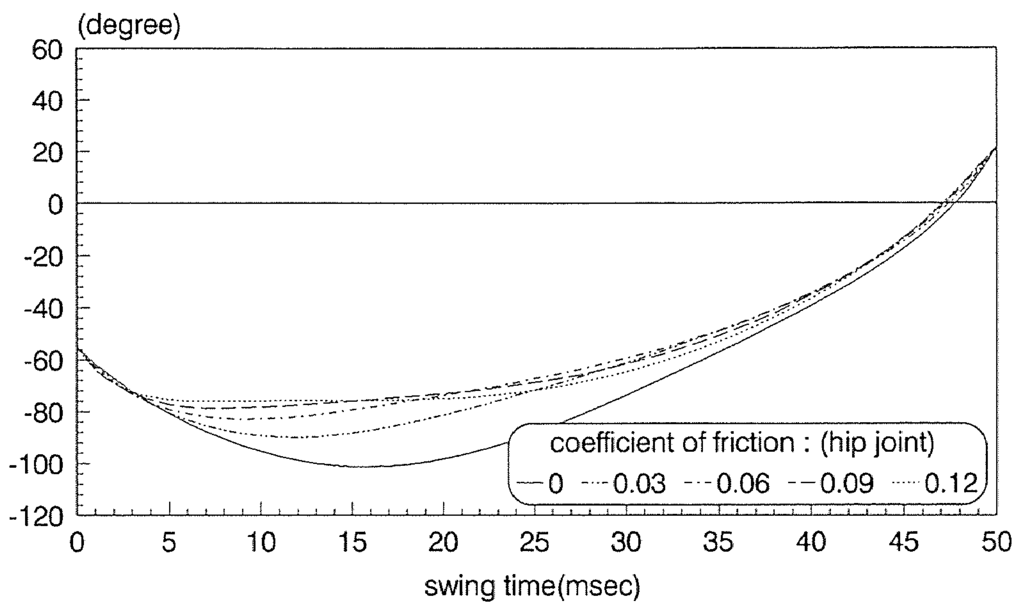

Figure 11(b) The effect of friction at the hip joint on the shank.

4.2 Comparison with Real Data

Even though we do not know the exact value of parameters, we understand which parameter has what kind of effect. At present we have small amount of real data, and do not know the parameters of the subject which is required to run the program. Therefore we use Dempster's anthropometric data(10). If we manage the parameters of anti gravity and coefficients of friction, we can produce a simulation output which is similar to the real data as you see in Figure 14. Our model has three angles, but real data shows 4 angles because the knee of the swing leg is bended at heel strike. There are some discrepancies between the simulated and the real. But, at this time, we can have hope that some day in the future we can build a walking model which produces same data as the real. It may come when we know the exact amount of muscle activity and friction effect, and improves our model based on the knowledge of these.

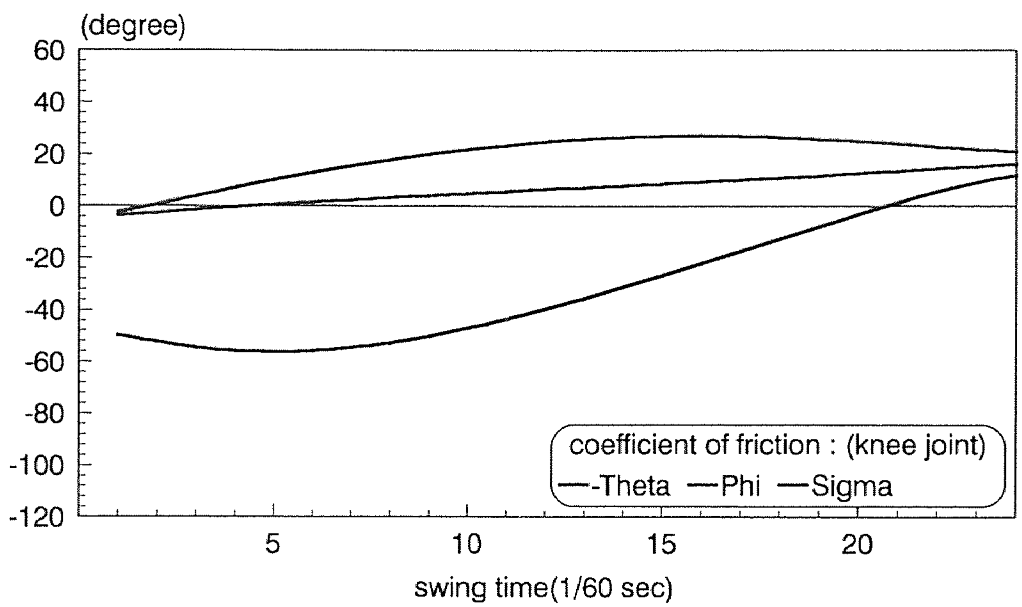

Figure 12 Experimental output

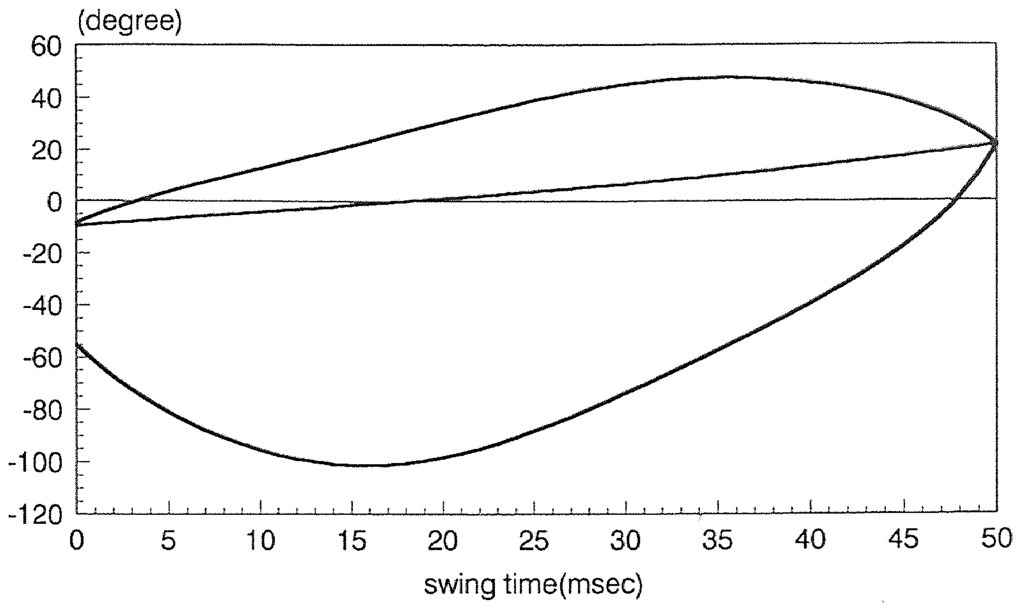

Figure 13 Simulated output for the free pendulum model. (b=c=0.0, k=0.0)

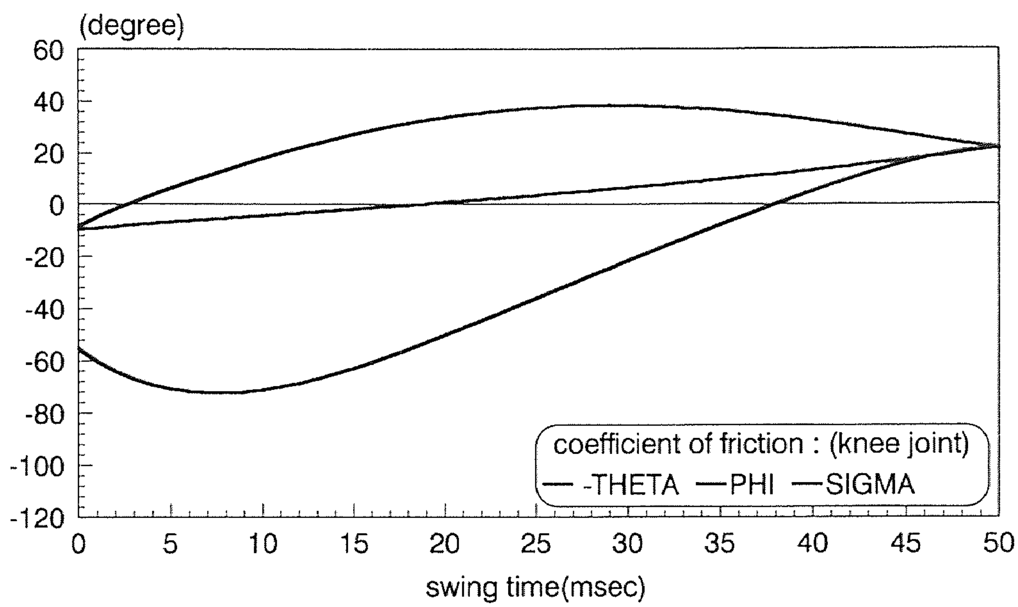

Figure 14 Simulated output of the improved model. (b=0.02, c=0.015, k(thigh)=0.2, k(shank)=0.7)

Table 3 Comparison of Energy Losses(N·m) from Each Model.

	Friction Loss	Knee-lock Loss	Heel-strike Loss	Remained Energy	Initial Energy
Free Pendulum	0.0	0.02	1.17	7.93	9.12
Improved Model	0.18	0.0	1.12	7.85	9.15

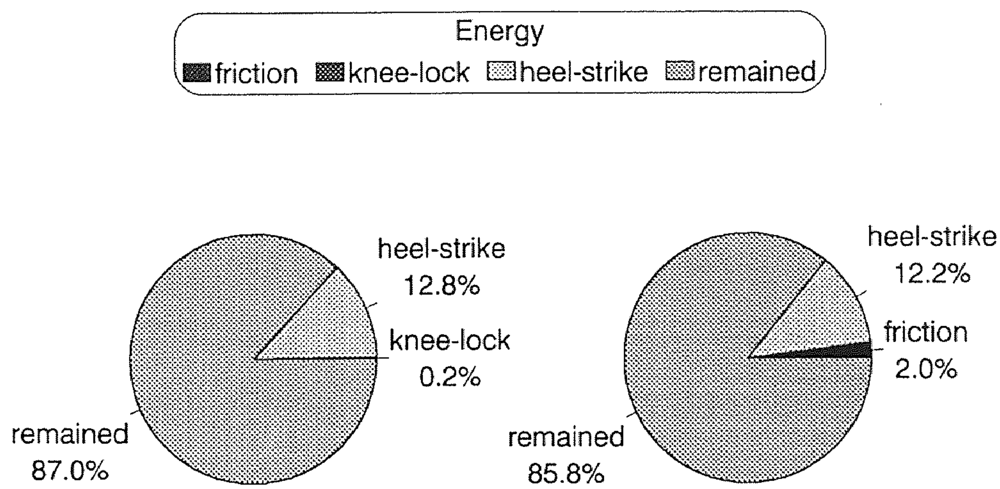

Figure 15 Comparison of energy distribution: left pie(free pendulum) and right pie(improved model).

CHAPTER 5

DISCUSSION

* Model

As a starting point, we selected Mochon's free pendulum model(8) as our base model because this model included the stance leg as well as the swing leg. When we designed a program to solve this model and obtained results of the program, we found that the swing time of the swing leg was abnormally short to produce normal movement of the shank, and that the shank kicked up too high if we made the swing time normal. After extensive searching for errors in the computer algorithms and finding none, we concluded that the assumption in Mochon's model that there is no energy input during the swing phase is an appropriate assumption for the model of an above-knee amputee. An artificial leg is a passive element. After toe off, it moves as a free pendulum until heel strike. There is no energy input to the artificial leg during the swing phase. The output of the program that the shank swings high when walking is consistent with the well-known observation that amputees kick high when walking. To solve this problem, we have introduced frictional forces into our model, which produce frictional moments to limbs. This idea is based on Beckett's model(2). There are papers which say that the swing leg is a free pendulum(1,8), or that muscle activity is reasonably quiescent in the swing leg during the swing phase(9). But the muscle activities in the swing leg are not absolutely quiescent. There is a relatively small amount of muscle activity in the swing leg, which is used for balancing mechanisms. Even though it is small compared with the amount of muscle activity of the stance leg, we understand that this small amount of muscle activity is enough to improve the shank's control considerably. It is interesting to note that such a small small amount of activity improves the gait dynamics so much. We think that this is because the shank's speed is high, and high speed produces more frictional force. When we have introduced a velocity dependent term like friction, the effect is greatest on the shank motion, and the shank does not kick too high when walking. Because a free pendulum model cannot predict experimental data, we can hypothesize that the swing leg does not act like a free pendulum, and that the stance leg is not a free reverse pendulum, either.

*** program**

With our program we can analyze the effect of every parameters on the human gait. When we design an artificial leg, we can predict the walking motion of the amputee, and improve the performance of the artificial leg by changing parameters.

*** coefficient of friction**

We have manipulated the coefficient of friction until the computer output matches the experimental data. But the source of the friction is not clear. It may come from muscle, joint, or other sources. At present, we do not know how much friction is contributed by each source. Since the resultant effect of any resistance is that of decreasing the effect of the gravity, we considerate all the joint frictional forces. If we know how much muscle activity exists during the swing phase, we can better understand the effect of each source. At this time we have introduced only frictional forces at joints to improve the model. In the future we hope to include each muscle's activity to improve the model. Even though it is relatively small, it may play a large role in controlling the movement of the shank during the swing phase.

*** parameters for theoretical data**

When we calculate the theoretical data, we must identify parameters of the legs such as the weight of the foot, thank, and thigh. At this time, we do not have a method of measuring these parameters of the subject such as the center of mass and weight of each limb, and do not know the values of the parameters that match the experimental data; we use Dempster's data which was used in the Mochon's model. To use the theoretical model to predict the human gait, it is necessary to have a method to measure the subject's parameters-for example, the weight of the shank.

*** energy loss at heel strike**

One major contribution of this thesis is that it proposes an algorithm to calculate the energy loss at heel strike. As a transitional stage to design a model of walking which includes the double support phase as well as the swing phase, we have tried to calculate the energy loss that occurs at heel strike and that must be re-supplied during the double support. Having calculated the energy loss at heel strike, we now know that this energy loss is the major energy loss.

*** knee lock and 4 angles model**

Our model is the 3 component model which Mochon used. In the model the thigh and shank of the swing leg becomes a straight line at the end of the swing phase. In practice the thigh and shank do not become a straight line at the end of the swing phase. That is because the knee is locked before the thigh and shank become straight, and at heel strike the knee is slightly bent. The theoretical result shows that the thigh angular velocity is much more negative than the experimental data shows, near the end of the swing phase. To solve this problem, if we introduce knee-bend angles as boundary conditions, our model can produce an output of gait dynamics with a knee bend in the swing leg. However it is not correct result at this time. If we want to use a bent knee model of the swing leg, the stance leg must be also bent. This means that we must use 4 components and 4 angles in our model. If the knee is bent, the energy loss at heel strike may be smaller than the energy loss when the knee is stiff. In the future we shall improve our model by using a four-component, four-angle model.

*** muscle activity.**

In our model energy is obtained only at toe-off. Energy is assumed lost due to friction, but no energy is added during the swing phase. Therefore our model is a passive model. To construct a continuous periodic walking model, the model must resupply the lost energy by muscle activities. In the future we shall include muscle activities in our model, and convert it an active model.

*** future model**

Our future model will be a 4 angles model, including muscle activities, that will predict the double support phase as well as the swing phase. The conclusions of this thesis may be incorporated in our future model, but the method of finding the optimum gait will not be changed.

CHAPTER 6

CONCLUSIONS

We have developed a mathematical model of human walking, and a computer algorithm for the model. The model and algorithm offers a method for better insight into the mechanics of walking, and makes it possible to determine the optimum human gait and to analyze the effect of various parameters that affect the gait.

We have found that the swing leg does not move like a free pendulum, that there exists some resistance which acts to decrease the effect of the gravity, and that this resistance improves the motion of the swing leg of the model and produces energy loss during the swing phase. This energy loss is smaller than the energy loss at heel strike, and we can say that swing leg is controlled during the swing phase at the expense of a relatively small energy expenditure.

In our model there are three sources of energy loss. They are the energy loss of friction during the swing phase, energy loss at knee lock, and energy loss at heel strike. We have developed algorithms to calculate these energy losses, and have determined that the energy loss at heel strike is predominant. Therefore, we believe that optimum gait dynamics and the energy loss at heel strike are closely interrelated.

APPENDIX A

Equations of the Mathematical Model

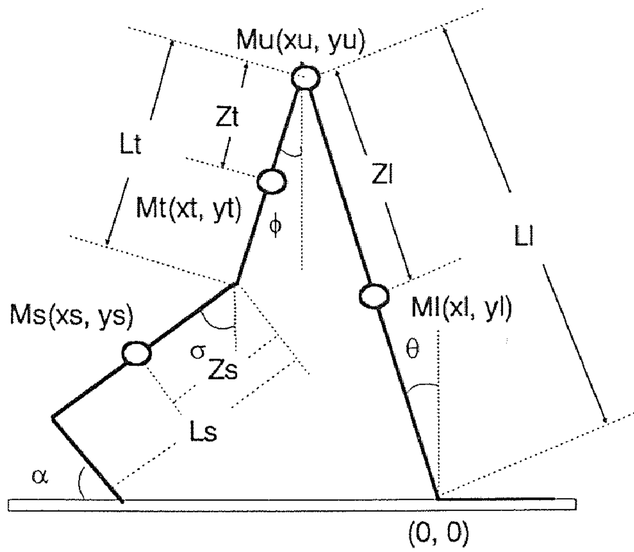

Figure A Configuration of the leg, thigh and shank at toe-off

$$x_l = -(L_l - Z_l)\sin\theta \qquad \dot{x}_l = -(L_l - Z_l)\cos\theta \cdot \dot{\theta}$$
$$y_l = (L_l - Z_l)\cos\theta \qquad \dot{y}_l = -(L_l - Z_l)\sin\theta \cdot \dot{\theta}$$
$$x_u = -L_l \sin\theta \qquad \dot{x}_u = -L_l\cos\theta \cdot \dot{\theta}$$
$$y_u = L_l\cos\theta \qquad \dot{y}_u = -L_l \sin\theta \cdot \dot{\theta}$$

$$x_t = -L_l\sin\theta + Z_t\sin\phi \qquad \dot{x}_t = -L_l\cos\theta\cdot\dot{\theta} + Z_t\cos\phi\cdot\dot{\phi}$$
$$y_t = L_l\cos\theta - Z_t\cos\phi \qquad \dot{y}_t = -L_l\sin\theta\cdot\dot{\theta} + Z_t\sin\phi\cdot\dot{\phi}$$
$$x_s = -L_l\sin\theta + L_t\sin\phi + Z_s\sin\sigma \qquad \dot{x}_s = -L_l\cos\theta\cdot\dot{\theta} + L_t\cos\phi\cdot\dot{\phi} + Z_s\cos\sigma\cdot\dot{\sigma}$$
$$y_s = L_l\cos\theta - L_t\cos\phi - Z_s\cos\sigma \qquad \dot{y}_s = -L_l\sin\theta\cdot\dot{\theta} + L_t\sin\phi\cdot\dot{\phi} + Z_s\sin\sigma\cdot\dot{\sigma}$$

L_l, L_t, L_s	Lengths of the leg, thigh and shank
Z_l, Z_t, Z_s	Distances of the center of mass of the leg, thigh and shank
M_l, M_t, M_s	Masses of the leg, thigh and shank
M_u, M_T	Masses of the upper body and the total body
θ, ϕ, σ	Angles that the leg, thigh and shank make with the vertical line
$\dot{\theta}, \dot{\phi}, \dot{\sigma}$	Velocities of the leg, thigh and shank

Potential energy is expressed as follows.

$$
\begin{aligned}
PE &= (M_l y_l + M_U y_u + M_t y_t + M_s y_s)g \\
&= (M_l L_l - M_l Z_l + M_u L_l + M_t L_l + M_s L_l)g\cos\theta - (M_t Z_t + M_s L_t)g\cos\phi \\
&\quad - M_s Z_s g\cos\sigma \\
&= (M_T L_l - M_l Z_l)g\cos\theta - (M_t Z_t + M_s L_t)g\cos\phi - M_s Z_s g\cos\sigma
\end{aligned}
$$

$$
\therefore PE = \sum_{i=1}^{3} w_i \cos\omega_i
$$

where

$$
(w_1,\ w_2,\ w_3) = (M_T L_l g - M_l Z_l g,\ -M_t Z_t g - M_s L_t g,\ -M_s Z_s g)
$$

$$
(\omega_1,\ \omega_2,\ \omega_3) = (\theta,\ \phi,\ \sigma)
$$

Kinetic energy is expressed as follows.

$$
\begin{aligned}
KE_l &= \frac{1}{2}M_l(\dot{x}_l^2 + \dot{y}_l^2) = \frac{1}{2}M_l(L_l - Z_l)^2\dot{\theta}^2 \\
KE_U &= \frac{1}{2}M_U(\dot{x}_U^2 + \dot{y}_U^2) = \frac{1}{2}M_U L_l^2\dot{\theta}^2 \\
KE_t &= \frac{1}{2}M_t(\dot{x}_t^2 + \dot{y}_t^2) = \frac{1}{2}M_t\{L_l^2\dot{\theta}^2 + Z_t^2\dot{\phi}^2 - 2L_l Z_t\cos(\theta-\phi)\cdot\dot{\theta}\cdot\dot{\phi}\} \\
KE_s &= \frac{1}{2}M_s(\dot{x}_s^2 + \dot{y}_s^2) \\
&= \frac{1}{2}M_s(L_l^2\dot{\theta}^2 + L_t^2\dot{\phi}^2 + Z_s^2\dot{\sigma}^2) \\
&\quad + M_s\{-L_l L_t\cos(\theta-\phi)\cdot\dot{\theta}\cdot\dot{\phi} - L_l Z_s\cos(\theta-\sigma)\cdot\dot{\theta}\cdot\dot{\sigma} + L_t Z_s\cos(\phi-\sigma)\cdot\dot{\phi}\cdot\dot{\sigma}\}
\end{aligned}
$$

$$
\begin{aligned}
KE &= KE_l + KE_u + KE_t + KE_s \\
&= \frac{1}{2}(M_T L_l^2 - 2M_l L_l Z_l + M_l Z_l^2)\dot{\theta}^2 + \frac{1}{2}(M_t Z_t^2 + M_s L_t^2)\dot{\phi}^2 + \frac{1}{2}M_s Z_s^2\dot{\sigma}^2 \\
&\quad - (M_t L_l Z_t + M_s L_l L_t)\cos(\theta-\phi)\cdot\dot{\theta}\cdot\dot{\phi} - M_s L_l Z_s\cos(\theta-\sigma)\cdot\dot{\theta}\cdot\dot{\sigma} \\
&\quad + M_s L_t Z_s\cos(\phi-\sigma)\cdot\dot{\phi}\cdot\dot{\sigma}
\end{aligned}
$$

$$
\therefore KE = \frac{1}{2}\sum_{i,j=1}^{3} c_{ij}\dot{\varpi}_i\dot{\varpi}_j
$$

where

$$c_{ij} = \bar{c}_{ij}\cos(\varpi_i - \varpi_j)$$
$$(\bar{c}_{11},\ \bar{c}_{22},\ \bar{c}_{33}) = (M_T L_l^2 - 2M_l L_l Z_l + M_l Z_l^2,\ M_t Z_t^2 + M_s L_t^2,\ M_s Z_s^2)$$
$$(\bar{c}_{12},\bar{c}_{13},\ \bar{c}_{23}) = (-M_t L_l Z_t - M_s L_l L_t,\ -M_s L_l Z_s,\ M_s L_t Z_s)$$
$$(\bar{c}_{21},\ \bar{c}_{31},\ \bar{c}_{32}) = (-M_t L_l Z_t - M_s L_l L_t,\ -M_s L_l Z_s,\ M_s L_t Z_s)$$

Lagrange's equations of motion for our model will be derive as follows .

$$L = KE - PE = \frac{1}{2}\sum_{i,j=1}^{4} c_{ij}\omega_i\omega_j - \sum_{i=1}^{4} w_i \cos\omega_i$$

$$\frac{\partial L}{\partial \dot{\omega}_k} = \frac{1}{2}\sum_{\substack{i=1\\ i\neq k}}^{3} c_{ik}\dot{\omega}_i + \frac{1}{2}\sum_{\substack{j=1\\ j\neq k}}^{3} c_{kj}\dot{\omega}_j + c_{kk}\dot{\omega}_k$$
$$= \sum_{\substack{i=1\\ i\neq k}}^{3} c_{ik}\dot{\omega}_i + c_{kk}\dot{\omega}_k\ (\because c_{ik} = c_{kj})$$
$$= \sum_{i=1}^{3} c_{ik}\dot{\omega}_i$$

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\omega}_k}\right) = \sum_{i=1}^{3}\frac{d}{dt}(c_{ik})\dot{\omega}_i + \sum_{i=1}^{3} c_{ik}\ddot{\omega}_i = -\sum_{i=1}^{3} c_{ik}^{*}(\dot{\omega}_i - \dot{\omega}_k)\dot{\omega}_i + \sum_{i=1}^{3} c_{ik}\ddot{\omega}_i$$

where

$$c_{ik}^{*} = \bar{c}_{ik}\sin(\varpi_i - \varpi_k)$$

$$\frac{\partial L}{\partial \omega_k} = -\frac{1}{2}\sum_{i,j=1}^{3} c_{ij}^{*}\dot{\omega}_i\dot{\omega}_j \frac{d}{d\,\omega_k}(\omega_i - \omega_j) + w_k \sin\varpi_k$$
$$= -\frac{1}{2}\sum_{j=1}^{3} c_{kj}^{*}\dot{\omega}_k\dot{\omega}_j + \frac{1}{2}\sum_{i=1}^{3} c_{ik}^{*}\dot{\omega}_i\dot{\omega}_k + w_k \sin\varpi_k$$
$$= \sum_{i=1}^{3} c_{ik}^{*}\dot{\omega}_i\dot{\omega}_k + w_k \sin\varpi_k\ (\because c_{ik}^{*} = -c_{ki}^{*})$$

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\omega}_k}\right) - \frac{\partial L}{\partial \omega_k} = \sum_{i=1}^{3} c_{ik}\ddot{\omega}_i - \sum_{i=1}^{3} c_{ik}^{*}\dot{\omega}_i^{2} - w_k \sin\varpi_k = 0$$

Equations of our model:

$$\therefore \sum_{i=1}^{3} c_{ki}\ddot{\omega}_i + \sum_{i=1}^{3} c_{ki}^{*}\dot{\omega}_i^{\,2} = w_k \sin \varpi_k \; (k = 1,2,3)$$

or

$$c_{11}\ddot{\theta} + c_{12}\ddot{\phi} + c_{13}\ddot{\sigma} + c_{12}^{*}\dot{\phi}^2 + c_{13}^{*}\dot{\sigma}^2 = w_1 \sin \theta$$
$$c_{21}\ddot{\theta} + c_{22}\ddot{\phi} + c_{23}\ddot{\sigma} + c_{21}^{*}\dot{\theta}^2 + c_{23}^{*}\dot{\sigma}^2 = w_2 \sin \phi$$
$$c_{31}\ddot{\theta} + c_{32}\ddot{\phi} + c_{33}\ddot{\sigma} + c_{31}^{*}\dot{\theta}^2 + c_{32}^{*}\dot{\phi}^2 = w_3 \sin \sigma$$

APPENDIX B

Frictional Effect

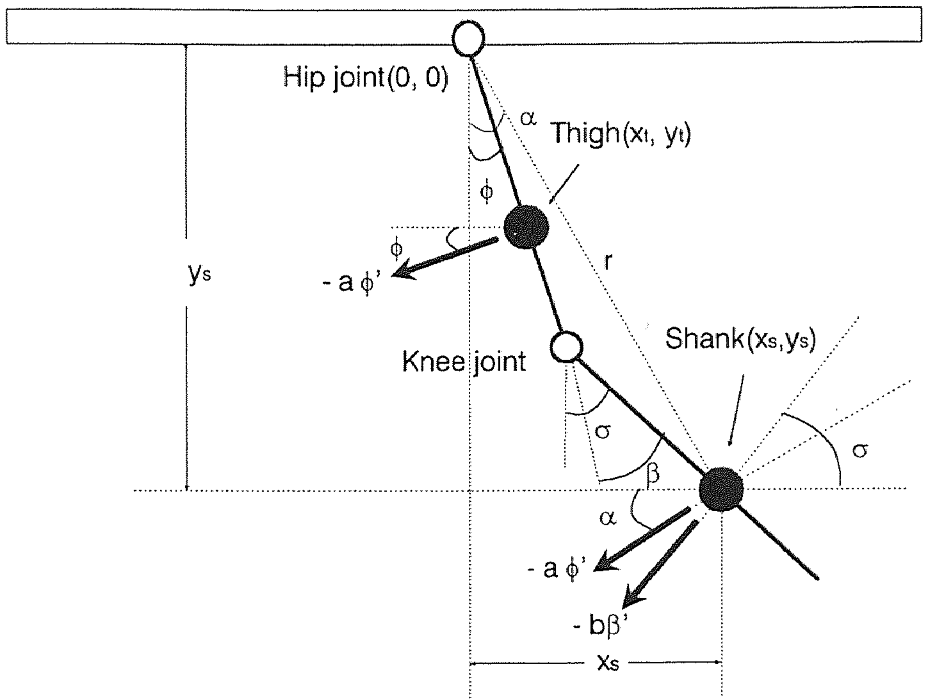

Figure B Frictional moments on the thigh and the shank from the hip and knee joints. Coefficients of friction for the hip joint and the knee joint are a and b .

Equations of coordinates (xt, yt) and (xs, ys) of the center of mass of the thigh and the shank and their derivatives:

$$x_t = Z_t \sin\phi, \qquad y_t = -Z_t \cos\phi$$
$$x_s = L_t \sin\phi + Z_s \sin\sigma, \qquad y_s = -L_t \cos\phi - Z_s \cos\sigma$$

$$\frac{dx_t}{d\phi} = Z_t \cos\phi, \qquad \frac{dy_t}{d\phi} = Z_t \sin\phi \qquad \frac{dx_s}{d\phi} = L_t \cos\phi, \qquad \frac{dy_s}{d\phi} = L_t \sin\phi$$
$$\frac{dx_t}{d\sigma} = 0, \qquad \frac{dy_t}{d\sigma} = 0, \qquad \frac{dx_s}{d\sigma} = Z_s \cos\sigma, \qquad \frac{dy_s}{d\sigma} = Z_s \sin\sigma$$

Equations of force components in the x and y directions:

$$Fx_t = -\{\frac{b\dot{\phi}}{Z_t} + c\dot{\sigma} / (L_t - Z_t)\}\cos\phi, \qquad Fy_t = -\frac{b\dot{\phi}}{Z_t}\sin\phi$$

$$Fx_s = -\frac{c\dot{\sigma}}{Z_s}\cos\sigma - \frac{b\dot{\phi}}{r}\cos\alpha, \qquad Fy_s = -\frac{c\dot{\sigma}}{Z_s}\sin\sigma - \frac{b\dot{\phi}}{r}\sin\alpha$$

where

$$\alpha = \frac{x_s}{|y_s|}, \text{ and } \quad r = \sqrt{x_s^2 + y_s^2}$$

The generalized forces of friction can be obtained by substitution into the formula below :

$$F_\phi = Fx_t \frac{dx_t}{d\phi} + Fy_t \frac{dy_t}{d\phi} + Fx_s \frac{dx_s}{d\phi} + Fy_s \frac{dy_s}{d\phi}$$

$$F_\sigma = Fx_t \frac{dx_t}{d\sigma} + Fy_t \frac{dy_t}{d\sigma} + Fx_s \frac{dx_s}{d\sigma} + Fy_s \frac{dy_s}{d\sigma}$$

The results yields:

$$F_\phi = -b\dot{\phi}\{1 + \frac{L_t}{r}\cos(\phi - \alpha)\} - c\dot{\beta}\{Z_t / (L_t - Z_t) + \frac{L_t}{Z_s}\cos\beta\}$$

$$F_\sigma = -c\dot{\beta} - b\dot{\phi}\frac{Z_s}{r}\cos(\sigma - \alpha)$$

APPENDIX C

Angular Velocities after Knee-Lock

Angular velocities of the thigh and shank of the swing leg after knee-lock are calculated from the conservation law of momentum. The definition of variables are in Appendix A. The calculation process is as follows.

The x - component of momentum of each mass at any instant :

$$\begin{cases} M_l\dot{x}_l = -M_l(L_l - Z_l)\cos\theta\cdot\dot{\theta} \\ M_u\dot{x}_u = -M_u L_l\cos\theta\cdot\dot{\theta} \\ M_t\dot{x}_t = -M_t L_l\cos\theta\cdot\dot{\theta} + M_t Z_t\cos\phi\cdot\dot{\phi} \\ M_s\dot{x}_s = -M_s L_l\cos\theta\cdot\dot{\theta} + M_s L_t\cos\phi\cdot\dot{\phi} + M_s Z_s\cos\sigma\cdot\dot{\sigma} \end{cases}$$

The total x - component of momentum:

$$\begin{aligned} \therefore M_x &= M_l\dot{x}_l + M_u\dot{x}_u + M_t\dot{x}_t + M_s\dot{x}_s \\ &= (-M_T L_l + M_l Z_l)\cos\theta\cdot\dot{\theta} + (M_t Z_t + M_s L_t)\cos\phi\cdot\dot{\phi} + M_s Z_s\cos\sigma\cdot\dot{\sigma} \end{aligned}$$

The y - component of momentum of each mass at any instant :

$$\begin{cases} M_l\dot{y}_l = -M_l(L_l - Z_l)\sin\theta\cdot\dot{\theta} \\ M_u\dot{y}_u = -M_u L_l\sin\theta\cdot\dot{\theta} \\ M_t\dot{y}_t = -M_t L_l\sin\theta\cdot\dot{\theta} + M_t Z_t\sin\phi\cdot\dot{\phi} \\ M_s\dot{y}_s = -M_s L_l\sin\theta\cdot\dot{\theta} + M_s L_t\sin\phi\cdot\dot{\phi} + M_s Z_s\sin\sigma\cdot\dot{\sigma} \end{cases}$$

The total y - component of momentum:

$$\begin{aligned} \therefore M_y &= M_l\dot{y}_l + M_u\dot{y}_u + M_t\dot{y}_t + M_s\dot{y}_s \\ &= (-M_T L_l + M_l Z_l)\sin\theta\cdot\dot{\theta} + (M_t Z_t + M_s L_t)\sin\phi\cdot\dot{\phi} + M_s Z_s\sin\sigma\cdot\dot{\sigma} \end{aligned}$$

Let angular variables after knee lock as $(\overline{\theta}, \overline{\phi}, \overline{\sigma}, \dot{\overline{\theta}}, \dot{\overline{\phi}}, \dot{\overline{\sigma}})$.

The x - components of momemtum before and after knee lock must be the same .

$$\begin{aligned} &(-M_T L_l + M_l Z_l)\cos\theta\cdot\dot{\theta} + (M_t Z_t + M_s L_t)\cos\phi\cdot\dot{\phi} + M_s Z_s\cos\sigma\cdot\dot{\sigma} \\ &= (-M_T L_l + M_l Z_l)\cos\overline{\theta}\cdot\dot{\overline{\theta}} + (M_t Z_t + M_s L_t)\cos\overline{\phi}\cdot\dot{\overline{\phi}} + M_s Z_s\cos\overline{\sigma}\cdot\dot{\overline{\sigma}} \end{aligned}$$

The y - components of momemtum before and after knee lock must be the same.

$$(-M_T L_l + M_l Z_l)\sin\theta\cdot\dot{\theta} + (M_t Z_t + M_s L_t)\sin\phi\cdot\dot{\phi} + M_s Z_s \sin\sigma\cdot\dot{\sigma}$$
$$= (-M_T L_l + M_l Z_l)\sin\overline{\theta}\cdot\dot{\overline{\theta}} + (M_t Z_t + M_s L_t)\sin\overline{\phi}\cdot\dot{\overline{\phi}} + M_s Z_s \sin\overline{\sigma}\cdot\dot{\overline{\sigma}}$$

We know the value of $(\theta, \phi, \sigma, \dot{\theta}, \dot{\phi}, \dot{\sigma})$ before knee lock, and $\theta = \overline{\theta},\ \phi = \overline{\phi},\ \sigma = \overline{\sigma},\ \overline{\phi} = \overline{\sigma}$ and $\dot{\overline{\phi}} = \dot{\overline{\sigma}}$.

Let

$$p = (-M_T L_l + M_l Z_l)\cos\theta\cdot\dot{\theta} + (M_t Z_t + M_s L_t)\cos\phi\cdot\dot{\phi} + M_s Z_s \cos\sigma\cdot\dot{\sigma}$$
$$q = (-M_T L_l + M_l Z_l)\sin\theta\cdot\dot{\theta} + (M_t Z_t + M_s L_t)\sin\phi\cdot\dot{\phi} + M_s Z_s \sin\sigma\cdot\dot{\sigma}$$

$$a = (-M_T L_l + M_l Z_l)\cos\overline{\theta}$$
$$b = (M_t Z_t + M_s L_t + M_s Z_s \cos\overline{\phi}$$

$$c = (-M_T L_l + M_l Z_l)\sin\overline{\theta}$$
$$d = (M_t Z_t + M_s L_t + M_s Z_s)\sin\overline{\phi}$$

Then, from the equations of x and y components of momentum :

$$\therefore \begin{cases} p = a\dot{\overline{\theta}} + b\dot{\overline{\phi}} \\ q = c\dot{\overline{\theta}} + d\dot{\overline{\phi}} \end{cases}$$

Therefore, angular velocities after knee lock :

$$\therefore \begin{cases} \dot{\overline{\theta}} = \dfrac{dp - bq}{ad - bc} & \text{(angular velocity of the stance leg)} \\ \dot{\overline{\phi}} = \dfrac{aq - cp}{ad - bc} & \text{(angular velocities of the thigh and shank of the swing leg)} \end{cases}$$

APPENDIX D

Energy Loss at Heel-Strike

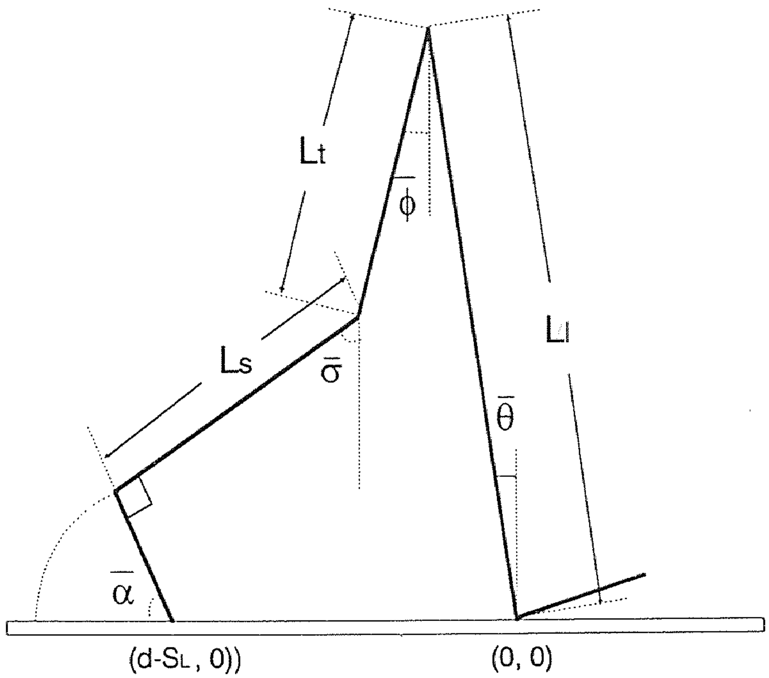

Figure C Configuration of legs during the double support phase.

Equations of the coordinate (x, y) of the swing leg's toe and their derivatives:

$$
\begin{aligned}
x(t) = d - S_L \quad &= -L_l \sin\overline{\theta} + L_t \sin\overline{\phi} + L_s \sin\overline{\sigma} + d\cos\overline{\alpha} \\
y(t) = 0 \quad &= \ L_l \cos\overline{\theta} - L_t \cos\overline{\phi} - L_s \cos\overline{\sigma} + d\sin\overline{\alpha} \\
x'(t) = 0 \quad &= -L_l \cos\overline{\theta}\cdot\dot{\overline{\theta}} + L_t \cos\overline{\phi}\cdot\dot{\overline{\phi}} + L_s \cos\overline{\sigma}\cdot\dot{\overline{\sigma}} - d\sin\overline{\alpha}\cdot\dot{\overline{\alpha}} \\
y'(t) = 0 \quad &= -L_l \sin\overline{\theta}\cdot\dot{\overline{\theta}} + L_t \sin\overline{\phi}\cdot\dot{\overline{\phi}} + L_s \sin\overline{\sigma}\cdot\dot{\overline{\sigma}} + d\cos\overline{\alpha}\cdot\dot{\overline{\alpha}}
\end{aligned}
$$

where

$t > Ts$ (after heel - strike)

We assume the following three vectors.

$$
\begin{aligned}
\vec{v}_1 &= (-L_l \cos\overline{\theta},\ \ L_t \cos\overline{\phi},\ \ L_s \cos\overline{\sigma},\ \ -d\sin\overline{\alpha}) \\
\vec{v}_2 &= (-L_l \sin\overline{\theta},\ \ L_t \sin\overline{\phi},\ \ L_s \sin\overline{\sigma},\ \ d\cos\overline{\alpha}) \\
\vec{V} &= (\dot{\overline{\theta}}, \dot{\overline{\phi}}, \dot{\overline{\sigma}}, \dot{\overline{\alpha}})
\end{aligned}
$$

Then, from the derivative equations, we can get the next equations.

$$\vec{v}_1 \bullet \vec{V} = 0$$
$$\vec{v}_2 \bullet \vec{V} = 0$$

Both vectors $\bar{v}_1$ and $\bar{v}_2$ are orthogonal to the vector $\bar{V}$. With this condition, energy loss at heel - strike can be calculated as follows.

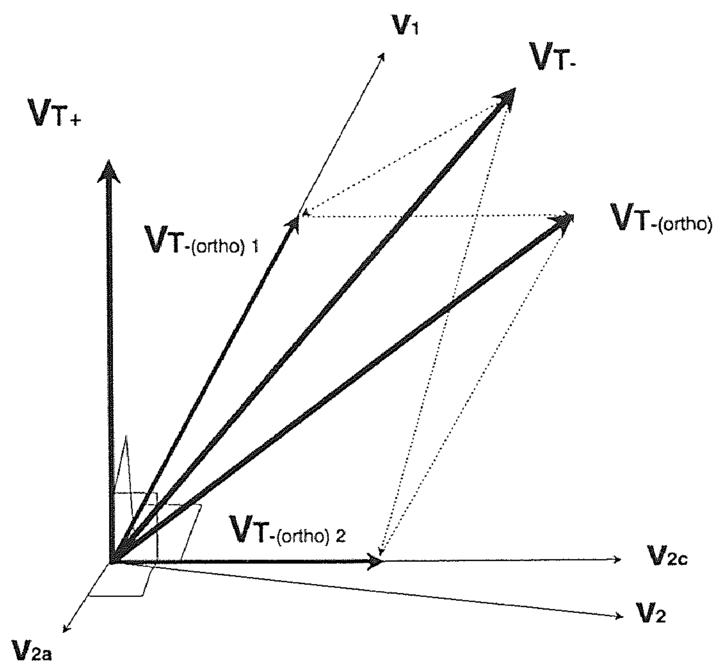

Figure D Angular velocity vector diagram at heel strike

$$\vec{v}_1 = (a_1, a_2, a_3, a_4)$$
$$\vec{v}_2 = (b_1, b_2, b_3, b_4)$$

The unit vector in $\bar{v}_1$ direction:

$$\vec{u}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = (a_1', a_2', a_3', a_4')$$

The component of $\bar{v}_2$ in $\bar{v}_1$ direction:

$$\vec{v}_{2a} = \frac{\vec{v}_2 \bullet \vec{v}_1}{\|\vec{v}_1\|^2}\vec{v}_1 = A\vec{v}_1 \qquad (A = \frac{\vec{v}_2 \bullet \vec{v}_1}{\|\vec{v}_1\|^2} = \frac{b_1a_1 + b_2a_2 + b_3a_3 + b_4a_4}{a_1^2 + a_2^2 + a_3^2 + a_4^2})$$

The component of $\bar{v}_2$ in the direction which is orthogonal to $\bar{v}_1$:

$$\begin{aligned}\vec{v}_{2c} &= \vec{v}_2 - \vec{v}_{2a} \\ &= (b_1 - Aa_1,\ b_2 - Aa_2,\ b_3 - Aa_3,\ b_4 - Aa_4) \\ &= (c_1, c_2, c_3, c_4)\end{aligned}$$

The unit vector in $\bar{v}_{2c}$ direction:

$$\vec{u}_{2c} = \frac{\vec{v}_{2c}}{\|\vec{v}_{2c}\|} = (c_1', c_2', c_3', c_4')$$

$$\vec{V} = (\dot{\bar{\theta}}, \dot{\bar{\phi}}, \dot{\bar{\sigma}}, \dot{\bar{\alpha}})$$
$$\vec{V}_{T^-} = (\dot{\bar{\theta}}, \dot{\bar{\phi}}, \dot{\bar{\sigma}}, \dot{\bar{\alpha}})_{T^-} = (\dot{\phi}, \dot{\theta}, \dot{\theta}, \dot{\alpha})_{after\ kneelock}$$

where

$$\begin{cases}\dot{\phi} \text{ is the angular velocity of thigh of the swing leg before heelstrike} \\ \dot{\theta} \text{ is the angular velocity of the swing leg before heelstrike}\end{cases}$$

$$\vec{V}_{T^-}(\vec{u}_1) = \frac{\vec{V}_{T^-} \bullet \vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1 = B\vec{u}_1 \qquad (B = \frac{\vec{V}_{T^-} \bullet \vec{u}_1}{\|\vec{u}_1\|^2} = \dot{\sigma}_f a_1' + \dot{\theta}_f a_2' + \dot{\theta}_f a_3' + \dot{\alpha}_f a_4')$$

$$\vec{V}_{T^-}(\vec{u}_{2c}) = \frac{\vec{V}_{T^-} \bullet \vec{u}_{2c}}{\|\vec{u}_{2c}\|}\vec{u}_{2c} = C\vec{u}_{2c} \qquad (C = \frac{\vec{V}_{T^-} \bullet \vec{u}_{2c}}{\|\vec{u}_{2c}\|^2} = \dot{\sigma}_f c_1' + \dot{\theta}_f c_2' + \dot{\theta}_f c_3' + \dot{\alpha}_f c_4')$$

$$\begin{aligned}\therefore \vec{V}_{T^-}(ortho) &= \vec{V}_{T^-}(\vec{u}_1) + \vec{V}_{T^-}(\vec{u}_{2c}) \\ &= B\vec{u}_1 + C\vec{u}_{2c} \\ &= (Ba_1' + Cc_1',\ Ba_2' + Cc_2',\ Ba_3' + Cc_3',\ Ba_4' + Cc_4') \\ &= (d_1, d_2, d_3, d_4)\end{aligned}$$

Therefore the remained vector of angular velocities is as follow.

$$
\begin{aligned}
\vec{V}_{T^+} &= (\dot{\overline{\theta}}, \dot{\overline{\phi}}, \dot{\overline{\sigma}}, \dot{\overline{\alpha}})_{T^+} \\
&= \vec{V}_{T^-} - \vec{V}_{T^-}(ortho) \\
&= (\dot{\sigma}_f - d_1,\ \dot{\theta}_f - d_2,\ \dot{\theta}_f - d_3,\ \dot{\alpha}_f - d_4)
\end{aligned}
$$

Now that we know angles and angular velocities of each limb, we can calculate the energy loss at heel strike with the result of Appendix A. As potential energy does not change at heel-strike, we consider only kinetic energy.

$$
\begin{aligned}
E_{T^-} &= KE(\theta, \phi, \sigma, \dot{\theta}, \dot{\phi}, \dot{\sigma})_{T^-} \\
E_{T^+} &= KE(\overline{\theta}, \overline{\phi}, \overline{\sigma}, \dot{\overline{\theta}}, \dot{\overline{\phi}}, \dot{\overline{\sigma}})_{T^+} \\
E_{loss}(T) &= E_{T^-} - E_{T^+}
\end{aligned}
$$

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