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# Deposition of particles in a convergent channel 

Chiou, Hsi-Chung, Ph.D.

New Jersey Institute of Technology, 1993

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## DEPOSITION OF PARTICLES IN A CONVERGENT CHANNEL

by<br>Hsi-Chung Chiou


#### Abstract

A Dissertation Submitted to the Faculty of New Jersey Institute of Technology in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

Department of Mechanical and Industrial Engineering


October 1993

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## APPROVAL PAGE

# DEPOSITION OF PARTICLES IN A CONVERGENT CHANNEL 

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# ABSTRACT <br> Deposition of Particles in a Convergent Channel 

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The deposition of suspensions for uniform flow in convergent channels under the combined influences of inertia, viscous, gravity and electrostatic image forces were studied theoretically. A two dimensional, steady, incompressible, laminar flow with uniform velocity profile was assumed.

It was found that when the ratio of charge parameter to gravity parameter, $\mathrm{Q} / \mathrm{G}$, is greater than 5 , the deposition is dominated by the electrostatic image force alone and that the gravity effect can be neglected. When the ratio of charge parameter to gravity parameter, $\mathrm{Q} / \mathrm{G}$, is less than 0.001 , the image force can then be neglected. It was also observed that when the value of the inertia parameter $S$ is less than 0.01 , the inertia effect on the deposition can also be neglected.

Results under the influence of inertia, viscous, gravity and electrostatic image forces indicate that for a constant convergent channel angle and fixed inertia and gravity effect (less than 1), the deposition increases as the image force increases. If gravity is greater than 10 , the deposition increases at the entrance and then decreases as the image force increases. The deposition decreases at the entrance region of the convergent channel as the convergent angle increases. The deposition then decreases as the axial displacement X increases. However, when Q is greater than 10 , the deposition increases as the convergent angle increases. In all cases the deposition increases as the gravity effect increases.

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This dissertation is dedicated to my parents, my wife and my two children, for their patience and encouragement.

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## NOMENCLATURE

Symbol
a particle radius
D Brownian diffusion coefficient
f Stoke's drag force
$F_{x}, F_{y} \quad$ particle image force in the axial and vertical components
$\mathrm{F}_{\mathrm{x}}{ }^{*}, \mathrm{~F}_{\mathrm{y}}{ }^{*} \quad$ dimensionless particle image force in the axial and verticalcomponents for the convergent channel
g gravity acceleration constant
G dimensionless gravitational force parameter
h half the channel width
$\mathrm{h}_{0} \quad$ half the channel width at the entrance
H half of the dimensionless channel width
mass of a particle
particle concentration
number of image pairs
$\mathrm{n}_{0}$ inlet particle concentration
N dimensionless particle concentration
total number of image pairs
electric charge per particle
distance between the point charge to the primary image $n$
distance between the point charge to the secondary image $\mathrm{n}^{\prime}$
dimensionless radial distance
R radius of image circleS
dimensionless inertia parameter
time variable

| T | dimensionless time variable |
| :---: | :---: |
| u, v | $x, y$ component of fluid velocity |
| $\mathrm{u}_{\mathrm{p}}, \mathrm{v}_{\mathrm{p}}$ | $\mathrm{x}, \mathrm{y}$ component of particle velocity |
| $\mathrm{u}_{0}$ | inlet velocity of fluid (uniform) |
| $\mathrm{u}_{\mathrm{po}}$ | inlet velocity of particle |
| $\mathrm{v}_{\mathrm{g}}$ | settling velocity of the particle |
| U, V | dimensionless X , Y component of fluid velocity |
| $\mathrm{U}_{\mathrm{P}}, \mathrm{V}_{\mathrm{P}}$ | dimensionless $\mathrm{X}, \mathrm{Y}$ component of particle velocity |
| U | dimensionless inlet velocity of fluid (uniform) |
| $\mathrm{U}_{\mathrm{P}_{\mathrm{o}}}$ | dimensionless inlet velocity of particle |
| $\mathrm{X}_{\mathrm{m}}$ | horizontal distance from the charge point to the center of image circle |
| $\mathrm{x}, \mathrm{y}$ | axial, vertical and horizontal coordinates |
| X, Y | dimensionless vertical and horizontal coordinates |
| $\mathrm{y}_{0}$ | inlet particle coordinate |
| $\mathrm{y}_{1}$ | inlet particle coordinate where particle deposit on the lower channel wall |
| $\mathrm{y}_{u}$ | inlet particle coordinate where particle deposit on the upper channel wall |
| Y | dimensionless inlet particle coordinate where particle deposit on the lower channel wall |
| $\mathrm{Y}_{u}$ | dimensionless inlet particle coordinate where particle deposit on the upper channel wall |
| $\mathrm{Y}_{0}$ | dimensionless inlet coordinate |

## Greek Letters

$\varepsilon_{0} \quad$ permittivity of free space
$\theta$ polar angle coordinate of inlet point
$\rho_{\mathrm{p}} \quad$ particle density
$\alpha_{0} \quad$ angle between the center line and the line from the charge to the center
$\alpha_{n} \quad$ half the angle subtended by the chord connecting the charge to the image $n$
$\alpha_{n^{\prime}} \quad$ half the angle subtended by the chord connecting the charge to image $\mathrm{n}^{\prime}$
$\beta_{\mathrm{n}} \quad$ angle between the y -axis and the chord connecting the charge to the primary image $n$
 the secondary image $\mathrm{n}^{\prime}$
dynamic viscosity of the fluid of suspension
$\pi$
3.14159...
$\nu \quad$ kinematic viscosity of the fluid

## Superscripts

| " dimensionless quantities as defined |  |
| :--- | :--- |
| 1 | secondary image as defined |

## Subscripts

$i \quad$ the $i^{\text {th }} \operatorname{step}(i=1,2,3 \ldots)$
o initial or inlet condition
p particle phase
$w \quad$ condition at wall
$\mathrm{x}, \mathrm{y} \quad$ axial and vertical components

## CHAPTER 1

## INTRODUCTION

The flow of a two phase fluid-solid particle suspension in a conduit has been studied by many researchers for many years. This is due to its close relationship to a variety of practical phenomena and its various engineering and scientific applications. Some examples are aerosol and paint sprays, air scrubbing systems, blood flows, dust collectors, pneumatic conveyors, rocket exhausts, respiratory tracts and many others. It is also known that solid suspension in a fluid stream is the primary cause of contamination and air pollution in the environment. This has been a major concern of industries and extensive investigation has recently been initiated.

A detailed knowledge on the mechanism of two-phase fluid-solid particle suspension flow would be helpful in understanding the deposition process. Therefore, a successful technique in controlling pollution depends upon the knowledge of the flow characteristics of the suspensions and the ability to predict the amount of particle deposition.

Recently, most of the investigations have concentrated on the significance of parameters that affect the deposition process in fully developed laminar flow in a constant area channel only. Although some of the investigations were on convergent or divergent channels, the studies were limited to the entrance region with high particle concentration.

The parameters that affect the motion of a particle are the inertia or the mass of the particle, the gravity acting on the particle, the force exerted on the particle due to the difference in velocities between the particle and the fluid, and the forces due to electrostatic charges.

For particles with electrostatic charges, the deposition forces can be divided into space charge repulsive forces and image forces. When particles having the same charge polarity are distributed in a space, they tend to repel one another and drift toward the channel wall. This type of forces are called the space charge repulsive forces. When a charged particle is placed in front of a grounded wall, an attractive electrostatic force, called the image force, between the particle and the wall is induced. When a charged particle is placed in a two-dimensional channel, image forces are presented from both walls.

It has been observed that when the particle number density of an aerosol flow is sufficiently low (less than 1.0 E 5 per $\mathrm{cm}^{3}$ ). the predominant effect of the electrostatic force on deposition is due to the image force.

In a variety of cases of practical importance, it is possible to treat the particle cloud as a continuum. The flow of suspension may be regarded, at least for the purpose of this study, as a mixture of two interpenetrating continuous flows.

The objective of this study is to investigate the deposition of particles from a uniform laminar flow in a convergent two-dimensional channel under the influence of, the inertia force of the particle, the viscous force due to the fluid, the gravity force of the particle, and the image force due to the electrostatic charges on the particle. Many practical applications involving suspension of particles have relatively low particle number density. Therefore the electrostatic effect due to the repulsive force between particles can be neglected. In this study, it is assumed that the particle number density in the channel is low enough such that the image force becomes the dominant effect on the particle deposition.

As mentioned above the image force is due to attractive force between the particle charge and the grounded wall. The image force is a function of the particle position with respect to the channel wall. The image force equations have been derived by previous researchers for a convergent channel.

The fluid phase is assumed to be two-dimensional, steady, incompressible, and uniform throughout the entire flow field. The governing equations with initial conditions are solved by using the fourth order Runge-Kutta method. The characteristics of the deposition curve for different flow parameters such as the inertia parameter $S$, gravity parameter $G$, image force parameter Q , and convergent angle $\theta$, will be investigated. A convergent channel with angles of $5^{\circ}, 10^{\circ}$ and $15^{\circ}$ has been considered. Trajectories of particles at 115 locations of initial position at the entrance of the channel are traced and the fractions of deposition are calculated.

In chapter two, a summarized literature survey on the deposition of suspension in two-phase flow is presented. Governing equations together with image force equations are presented in chapter three. In chapter four, the trajectory of particles due to the above mentioned forces are solved numerically by the fourth order Runge-Kutta method. The deposition analysis for various parameters $\theta, S, G$ and $Q$ are discussed in chapter five. The parameters discussed are $2 \theta=5^{\circ}, 10^{\circ}$ and $15^{\circ} ; S=100,10,1,0.1$, $0.01 ; G=100,10,1,0.1,0.01$; and $Q=10000,1000,100,10,1,0.1$, $0.01,0.0010 .0001$ and 0.00001 . Conclusions and recommendations for further investigation are presented in chapters six and seven.

## CHAPTER 2

## LITERATURE SURVEY

Investigations of particles deposition in a suspension fluid have been carried out by many scholars both theoretically and experimentally. In this chapter, some of the studies related to the particles deposition for laminar flow and turbulent flow from internal flow of suspensions are surveyed.

As early as 1938, Kalinske and Van Driest [1] studied the transport of suspended particles by means of turbulent stream of water. In 1953, Longwell and Weiss [2] studied the mixing and distribution of liquid droplets in high velocity gas streams.

Friedlander and Johnstone [3] found that when a stream of gas carrying suspended particles flow in turbulent motion past a surface, the particles are deposited due to the radial fluctuating component of velocity. They performed an experimental study of the rate of deposition of dust particles on the wall of a tube with an analysis of the mechanism of particle transport in a turbulent stream.

They found that the net rate of deposition depends on both the rate of transport of the particles to the wall and the rate of re-entrainment; the second effect was reduced to a minimum by allowing only a single layer of particles to accumulate on the surface and by taking precautions to ensure adherence of all particles that struck the wall.

Soo and Rodgers [4] studied the occurrence of deposition due to field forces. They defined a sticking probability, $\sigma$, which depends on material properties. This sticking probability is related to the force of adhesion of particles to a surface. It is found that $\sigma=1$ when all particles drifting to
the wall stick to or settle at the wall ; and $\sigma=0$ for complete reentrainment.

Corn [5] studied the adhesion of solid particles to solid surfaces and showed that adhesive forces are either due to electrical effects or liquid effect (viscosity and surface tension) in origin. The electrical forces include contact potential difference and dipole effect, space charge and electronic structure. The liquid forces include re-entrainment of particles from the surface, the adhesion of particles to the wall, and lifting of particles away from the wall in the shear flow field.

Thomas [6] has determined the minimum transport velocity, which is defined as the mean stream velocity required to prevent the accumulation of a layer of stationary or sliding particles on the bottom of a horizontal conduit for flocculated thorium oxide and kaolin suspensions in gas flowing in glass pipes. The pipe sizes ranged from 1 to 4 inches in diameter, and the concentration was varied from 0.01 to 0.17 volume fraction of the suspensions. In the first flow rigime the suspension was sufficiently concentrated to be in the compaction zone and hence had an extremely low setting rate. The second flow regime was observed with more dilute suspensions which were in the hindered-settling zone and settled ten to one hundred times faster than slurries which were in compaction.

The concentration for transition from one regime to the other was dependent on both the tube diameter and the degree of flocculation. When the suspended particles were smaller than the thickness of the laminar sublayer, they settled according to Stoke's drag law.

Stukel and Soo [7] conducted an experimental investigation on hydrodynamics of a suspension of $10 \mu$ magnesia particles in air in a parallel-plate channel at various flow velocities, plate gap widths, and
mass flow ratios of solid particles to air. They found that the nature of the developing turbulent boundary layer for dilute suspensions is such that the density of particles is higher at the wall than at the core because of the presence of charge on the particles induced by surface contacts. Furthermore, as analogous to rarefied gas motions, a particle slip velocity brought about by the lack of particle-to-particle collisions in the suspension was observed at the wall.

It was concluded that similarity laws for the scaling of equipment for air pollution control should include the momentum transfer number, the electroviscous number and the Reynolds number. The electroviscous number is the most important parameter when particles possess large charge-to-mass ratios.

Wang et al [8] has derived a series solution for the distribution and deposition of particles in a stationary fluid between two horizontal plane surfaces under the influence of diffusion and settling (or gravity).

Numerical investigations have shown that it is better to approximate deposition by the larger of the two separate effects for pure diffusion and for pure settling than by combining them as if they were independent effects.

The continuum theory of solid-fluid suspensions including solid-phase viscosity was discussed by Yang and Peddieson [9] and was used to solve three steady flow problems: (1)plane Poiseuille flow, (2) plane Couette flow, and (3) vertical film flow. They obtained closed-form solutions and calculated the velocity profiles, skin-friction coefficients, and flow rates of both phases for several numerical values of the parameters arising in the problem. Their results showed that the inclusion of solid-phase viscosity and the amount of particle slip allowed at the channel walls have important
consequences in the problems solved.
Soo and Tung [10] analyzed the general case of fully developed pipe flow of a suspension in a turbulent fluid with electrically charged particles and gravity effect. Parameters they used to define the state of motion are: pipe flow Reynolds number, Froude number, electro-diffusion number, diffusion-response number, momentum-transfer number and particle Knudson number.

Comparison with experimental results were made for both gas-solid and liquid-solid suspensions. It was shown that the gravity effect becomes significant in the case of large diameters and large particle concentrations.

Soo and Tung [11] extended their previous studies [10] to include the effects of deposition and entrainment of particles. Additional considerations from previous studies were, diffusion and settling under field forces, and the sticking probability of a particle at the wall and that to a bed of similar particles. The transient conditions gave the rate of build-up of deposited particles. The method is applicable to pipes at any inclination to the direction of gravity.

In the study of fully developed pipe flow of a particulate suspension. Soo [12] defined four dimensionless parameters of particle-fluid interactions in addition to the Reynolds number of the flow.

These dimensionless parameters resulted from the gravity effect, the Magnus effect due to fluid shear, electrostatic repulsion due to electric charges on the particles, and Brownian diffusion. In the case of negligible gravity and shear effects, the distribution in particle density and the velocity distribution were presented in figures. The influences of particle slip velocity at the wall and electrostatic repulsive forces on particle density distribution were shown.

Pich [13] derived an equi-penetration curve of particles at the inlet plane by means of the concept of the particle trajectory function. The particles below the equi- penetration curve are considered deposited on the wall. The deposition efficiency is found by integrating the product of particle velocity and the area between the equi-penetration curve and the boundary of the channel wall.

A new approach based on the concepts of the particle trajectory function and the limiting trajectory is developed by Wang [14] for calculating the precipitation efficiency of channels of different crosssection. The trajectory function he used is equivalent to the stream function of a virtual flow field where the motion of particles are considered. His analysis was limited to two dimensional motions of small particles and solutions were derived for gravitational deposition of particles from laminar flows in inclined flat channels and circular tubes. According to Wang's study, the use of the trajectory function provides a simple way for calculating the flow rate of particles through any area. This is particularly useful in the analysis of the deposition in the inclined channels of which the inlet and outlet cross-sections are not vertical.

The motion of a two-phase (dust-carrier gas) suspension in the vicinity of a sphere or a circular cylinder was studied by Peddieson [15]. The problem of analyzing the flow of a fluid containing solid particles or droplets past such bodies has been of interest for a long time because such flow exist in several situations of engineering interest. These include the formation of ice on airplane wings, the erosion of missile surfaces due to high speed raindrop impacts, and the collection and sampling of dust for the purpose of monitoring and controlling air pollution.

Comparin et al. [16] conducted experiments on contamination effects
in a laminar proportional amplifier and found that contamination may cause deterioration of the amplifier.

Eldighidy et al. [17] performed some theoretical analysis and investigations on deposition in the entrance of a channel and in a diffuser. In these analyses, effects of diffusion, electrostatic charge repulsive force and adhesive force were considered and the results showed that the electrostatic charge effect played an important role in the deposition of particles. They further found that the surface adhesion has a smaller effect on the rate of deposition than the electric charge. Moreover, it was found that the rate of deposition was greatly affected by the divergence angle in a diffusive flow. The pressure gradient as well as the rate of deposition increases as the diffuser angles increases. However, because the separation occurs earlier at larger diffuser angles, the rate of deposition increases rapidly in the presence of electric charge. In the limiting case of the absence of electric charge, the rate of deposition decreases rapidly with increasing diffuser angle. The effect of electrostatic image force was not investigated in these analyses.

Ingham [18,19] investigated theoretically the simultaneous diffusion and sedimentation of aerosol particles in two dimensional channels. Both plug (uniform) and fully developed (Poiseuille) flow were considered with the emphasis on the case when diffusion effects are larger than or are of the same order of magnitude as sedimentation effects due to gravity.

A similar investigation for both plug and Poiseuille flow was conducted by Taulbee and Yu [20]. They found that the fractional penetration depends on a parameter $\mathrm{q}^{\prime \prime}=\mathrm{h} \mathrm{V}_{\mathrm{g}} / \mathrm{D}$ where h is the channel half height, $\mathrm{V}_{\mathrm{g}}$ is the settling velocity of a particle and D is the Brownian diffusion coefficient. The results shows that for $q^{\prime \prime}<0.1$, the particle loss
was practically due to diffusion alone while for $q^{\prime \prime}>200$, the deposition was mainly due to the settling. The deposition due to the combined mechanism in the range $0.1<q^{\prime \prime}<200$ was significantly smaller than the algebraic sum of deposition due to two independent mechanisms.

When electrostatically charged particles are suspended in a fluid, the deposition is due to both the space charge repulsive force and the image force. Yu and Chandra [21] analyzed these forces separately and compared the results with existing experimental data. It was found that at 1.0 E5 particles per cubic centimeter, the space charge force can only lead to a small effect on the deposition and the predominant effect is due to image forces exerted on the particles. In the image force model, the interactive forces between particles is neglected and the deposition is independent of the particle density. They investigated theoretically the deposition of charged particles by their image forces from laminar flows in rectangular and cylindrical channels. They considered the case in which particles of $1 \mu \mathrm{~m}$ diameter and $100 \mathrm{el} . /$ particle were breathed into the lung with 1000 cc tidal volume and 12 respirations per minute. It was found that the image force contributes approximately the same amount of deposition as the gravitational force does.

Precipitation of charged particles under the influence of image force from laminar flows in rectangular and cylindrical channels was investigated theoretically by Yu and Chandra [22]. They neglected the gravity, inertia and diffusion (Brownian motion) effects of particles and studied the effect of electrostatic image force on the deposition of particles from laminar flows in a rectangular (parallel plate) channel and that in a circular tube. The equation of image force for rectangular channel is introduced in the following chapter on which the derivation of the equation of image force
for a convergent channel in based.
Yu and Chandra's numerical calculations were based on the analysis of limiting trajectories of particles that enabled the determination of the precipitation efficiency for channels of different dimensions. The results for cylindrical tubes are applicable to the deposition of charged particles in human lung airways.

Chen [23] studied the deposition of aerosols in a long channel due to diffusive and electrostatic charge effects. The diffusion equation and the Poisson equation for flow of aerosol particles with electrostatic charge field force were solved with an integral method based on flow of gas with a uniform or parabolic velocity profile. He found that the inverse of the centerline particle density increased linearly with the product of the electrostatic parameter and the axial distance for the flow near the channel inlet. The centerline particle density, the penetration, and the electric field force decreased exponentially with the axial distance for flow far from the channel inlet.

Thiagarajan and Yu [24] studied the deposition of aerosol from laminar flow in parallel plate and cylindrical channels due to the simultaneous effect of gravitational and electrostatic image forces. The equations of particle motion are solved numerically to obtained the particle trajectories in 3-dimensions. It was found that the deposition is considerably lower than that obtained by adding the deposition due to gravity alone and that due to the image force alone.

Ingham [25] considered the deposition of a steady flow of suspensions due to electrostatic charge field force near the entrance of a cylindrical tube neglecting the axial diffusions in the steady state transport equation and Poisson's equation for electrostatic field. He solved these equations
analytically.
Laminar flow of particles in a parallel plate channel with electrostatic charge field force, diffusion, and gravitational effects was studied by Chen and Gelber [26]. Variations in deposition were determined by using a dimensionless parameter (charge-diffusion parameter) which is a ratio of the space electrostatic charge effect to the diffusion effect. They found that when this parameter was greater than 50 , the diffusion effect may be neglected. When gravity acting in the direction of flow was considered, a velocity ratio (terminal velocity of the particle to the mean velocity of the fluid flow) was introduced. The space electrostatic charge field force effect and the gravity effect were considered in this case, and the velocity profile was either uniform or fully developed.

In the theoretical investigations on deposition in channels carried out by Yu [27], Chen [23], Ingham [25] and Chen and Gelber [28], the axial diffusive term in the continuity equation was neglected in the analyses and, thus, the axial velocity of the fluid was considered to be the same as that of the particle. In other words, the inertia of the particle was totally neglected.

Chen et al [29] extended the study to include the effect of particle inertia on the deposition of aerosol in a parallel plate channel. Highly charged fine particles of sizes less than 20 micrometer had been analyzed numerically for both uniform and fully developed flows using a trajectory method. They considered the deposition to be primarily due to space charge alone and the image and gravity forces were not included in the analysis. They also defined a charged-inertia parameter n (ranging from 0 to 1) to characterize the flow deposition phenomena. They found that for $n$ less than 0.1 , the effect of inertia forces may be neglected and that the
fraction of deposition near the entrance of the channel deviated substantially from the result that neglected the inertia effect.

Chen et al [30] derived the image force equation for the twodimensional convergent channels by using the concept of image circle. The deposition analysis due to the image force alone for the $15^{\circ}, 10^{\circ}$, and $5^{\circ}$ of convergent channel was studied for uniform flow. They concluded that in comparison with the parallel-plate channel, the depositions were found to be smaller near the entrance in the convergent channels, but they increased as the axial distance was increased.

Chen et al [31] studied the deposition of particles in a parallel-plate channel under the influence of inertia, viscous, gravity and electrostatic image force by using a trajectory method. A laminar fluid flow of uniform and fully-developed velocity profiles is assumed and the fluid is suspended with particles having the same velocity as the fluid at the inlet plane.

They found that the deposition increases with increasing particle size, but decreases with increasing viscous force. They concluded that the image force effects dominate the deposition when the ratio of the charge parameter Q to the gravity parameter G is greater than 5 and diminishes when the ratio becomes less than 0.001 .

A matter of further investigation would be to study theoretically the deposition of particles from a laminar flow in a convergent channel due to inertia, gravity and image forces.

## CHAPTER 3

## ANALYSIS

In this chapter, our main interest is focused on the analysis of efficiency of deposition due to inertia, viscous, gravity and electrostatic charge forces for the suspension flow in a convergent channel. As we discussed in the previous chapter, when the particle number density of an aerosol flow is sufficiently low, the predominant electrostatic charge effect is mainly due to the image force.

### 3.1 Image Force in a Parallel-Plate Channel

The deposition of charged particles from laminar flow in a parallel-plate channel due to the image forces has been studied by Yu and Chandra [23]. For a channel which is made of two parallel plates placed $2 h_{0}$ apart with walls that are conducting and grounded, the image forces on a charged particle may be expressed as follows:

$$
\begin{align*}
& F_{x}=0 \\
& F_{y}=\frac{q^{2} h_{0} y}{4 \pi \varepsilon_{0}}\left\{\frac{1}{\left(h_{0}{ }^{2}-y^{2}\right)^{2}}+\sum_{n=1}^{\infty} \frac{2 n+1}{\left[(2 n+1)^{2} h_{0}{ }^{2}-y^{2}\right]^{2}}\right\} \tag{3-1}
\end{align*}
$$

where the flow direction is along the x direction and the x -axis coincides with the centerline of the channel.
$y:$ the vertical distance from the centerline of the channel to
the point charge.
q : the charge per particle
$\varepsilon_{0}$ : the permitivity of space
The coordinate system for a parallel-plate channel is shown in figure 3.1(Figures may be found in Appendix C). Let $2 \mathrm{~h}_{0}$ be the width of the channel where $y=h_{0}$ and $-h_{0}$ represent the upper and lower walls of the channel respectively.

Equation (3-1) can be expanded to be the summation of infinite pairs of images as:

$$
\begin{align*}
& \mathrm{F}_{\mathrm{x}}= 0 \\
& \mathrm{~F}_{\mathrm{y}}=\frac{\mathrm{q}^{2} \mathrm{~h}_{0} \mathrm{y}}{4 \pi \varepsilon_{0}}\left\{\frac{1}{\left(\mathrm{~h}_{0}{ }^{2}-\mathrm{y}^{2}\right)^{2}}+\frac{3}{\left(3^{2} \mathrm{~h}_{0}{ }^{2}-\mathrm{y}^{2}\right)^{2}}\right. \\
&+\frac{5}{\left(5^{2} \mathrm{~h}_{0}{ }^{2}-\mathrm{y}^{2}\right)^{2}}+\frac{7}{\left(7^{2} \mathrm{~h}_{0}{ }^{2}-\mathrm{y}^{2}\right)^{2}} \\
&\left.+\cdots+\frac{2 \mathrm{n}-1}{\left((2 \mathrm{n}-1)^{2} \mathrm{~h}_{0}{ }^{2}-\mathrm{y}^{2}\right)^{2}}\right\} \tag{3-2}
\end{align*}
$$

where the first, second, third, ... term represents the image force due to the first, third, fifth, ... pair of image respectively. This equation can be further expanded out as follows.

$$
\begin{align*}
& \mathrm{F}_{x}= 0 \\
& \mathrm{~F}_{y}=\frac{\mathrm{q}^{2}}{4 \pi \varepsilon_{0}}\left\{\frac{1}{\left[2\left(\mathrm{~h}_{0}-\mathrm{y}\right)\right]^{2}}+\frac{-1}{\left[2\left(\mathrm{~h}_{0}+\mathrm{y}\right)\right]^{2}}\right. \\
&+\frac{1}{\left[2\left(3 \mathrm{~h}_{0}-\mathrm{y}\right)\right]^{2}}+\frac{-1}{\left[2\left(3 \mathrm{~h}_{0}+\mathrm{y}\right)\right]^{2}} \\
&\left.+\frac{1}{\left[2\left(5 \mathrm{~h}_{0}-\mathrm{y}\right)\right]^{2}}+\frac{-1}{\left[2\left(5 \mathrm{~h}_{0}+\mathrm{y}\right)\right]^{2}}+\cdots\right\} \tag{3-3}
\end{align*}
$$

where the summation forces of the odd terms are due to all the images that produce positive forces in the $y$-direction on the particle. The even terms represent the summation forces due to all the images that produce negative forces in the negative $y$ direction on the particle.

It is also worth noticing that for the first two terms, the value inside the bracket of the denominators in equation (3-3) represent the distance between the point charge (particle) and the first pair of images of the point charge. For example, $2\left(\mathrm{~h}_{0}-\mathrm{y}\right)$ in the first term is the distance between the point charge and the image 1 (first primary image of the point charge located above the upper channel wall). $2\left(h_{0}+y\right)$ in the second term is the distance between the point charge and the image 1' (first secondary image of the point charge located below the lower channel wall). $2\left(3 h_{0}-\mathrm{y}\right)$ in
the third term is the distance between the point charge and the image 3 (third primary image of the point charge located above the upper channel wall). $2\left(3 \mathrm{~h}_{0}+\mathrm{y}\right)$ in the fourth term is the distance between the point charge and the image 3 ' (third secondary image of the point charge located below the lower channel wall). In the same analysis, for the ( $2 \mathrm{n}-1)^{\text {lh }}$ pair of image, the value inside the bracket of the denominators represents the distance between the point charge and the corresponding ( $2 \mathrm{n}-1)^{\text {th }}$ and ( $2 \mathrm{n}-$ $1)^{\mathrm{th}}\left((2 \mathrm{n}-1)^{\mathrm{th}}\right.$ primary and secondary) image.

Therefore equation (3-3) is actually the form of

$$
\begin{align*}
& \mathrm{F}_{\mathrm{x}}=0 \\
& \begin{aligned}
\mathrm{F}_{y}= & \frac{\mathrm{q}^{2}}{4 \pi \varepsilon_{0}}\left\{\frac{1}{\mathrm{q}_{1}{ }^{2}}+\frac{-1}{\mathrm{q}_{\mathrm{r}}{ }^{2}}+\frac{1}{\mathrm{q}_{3}{ }^{2}}+\frac{-1}{\mathrm{q}_{3}{ }^{2}}\right. \\
& \left.+\frac{1}{\mathrm{q}_{5}{ }^{2}}+\frac{-1}{\mathrm{q}_{s^{2}}{ }^{2}}+\cdots+\frac{1}{\mathrm{q}_{(2 \mathrm{n}-1)}{ }^{2}}+\frac{-1}{\mathrm{q}_{(2 \mathrm{n}-1)^{2}}{ }^{2}}\right\}
\end{aligned}
\end{align*}
$$

where $\mathrm{q}_{1}{ }^{2}, \mathrm{q}_{3}{ }^{2}, \mathrm{q}_{5}{ }^{2}, \ldots \mathrm{q}_{2 n-1}{ }^{2}$ and $\mathrm{q}_{1}{ }^{2}, \mathrm{q}_{3}{ }^{2}, \mathrm{q}_{5}{ }^{2}, \ldots \mathrm{q}_{(2 n-1)}{ }^{{ }^{2}}$ represent the distance between the point charge and the image $1,3,5, \ldots, 2 n-1$ above the upper channel and the image $1^{\prime}, 3^{\prime}, 5^{\prime}, \ldots(2 n-1)^{\prime}$ below the lower channel respectively. It is interesting to note the fact that the image forces between the pairs of $2,4,6, \ldots 2 n$ cancel out due to the same distances on both sides of the point charge and the same polarity of the image pair.

### 3.2 Image Forces in a Convergent Channel

The same approach is also applied for a convergent channel to obtain the governing image force equations. In order to study the image forces on a convergent channel, it is necessary to first understand the characteristics of the system of image forces due to conducting and grounded walls of convergent channels so that the appropriate assumptions may be applied in the derivations of the image force equations.

A convergent channel can be assumed to be a parallel plate channel with its lower and upper walls tilted at angle $\theta$ and $-\theta$ with respect to the x - axis respectively as shown in figure 3.2.

The coordinate system for convergent channels is also shown in figure 3.2. The flow is along the x -direction where the x -axis coincides with the center line of the convergent channel. Let $2 h_{0}$ be the width of the convergent channel at the entrance where $y=h_{0}$ and $y=-h_{0}$ refer to the upper and lower walls of the channel at the entrance respectively. As we can see from the figure, $h$ decreases as it moves further toward the exit of the convergent channel; i.e. when $\mathrm{x}>0$, the channel half width $\mathrm{h}(\mathrm{h}<$ $h_{0}$ ) is not a constant but a function of $x$. The channel walls are also assumed to be conducting and grounded.

As we mentioned before, the image force for a parallel plate contains an infinite pair of images. Because of this tilted angle of the channel walls, the image of a point particle between convergent channel walls which are conducting and grounded is no longer two infinite sequence as that shown in figure 3.1. The first pair of images ( 1 and $1^{\prime}$ in figure 3.2 ) of the particle are shifted ahead of the point particle through an angle $\theta$. The second pair of images ( 2 and $2^{\prime}$ in figure 3.2 ), which are actually resulting from the first pair, are now shifted further ahead of the first pair
of images also through an angle $\theta$. The third and fourth pairs of images ( 3 and $3^{\prime}, ~ 4$ and $4^{\prime}$ ) and so on can then be obtained in a similar manner until the $\mathrm{n}^{\text {th }}$ pair of images is reached.

From basic electromagnetic theory $[32,33]$, it is noted that for two conducting planes intersecting at an angle $2 \theta$, the system of images for a point charge particle is finite and confined on a circle which is the image circle. This image circle is defined by having its center at the extended intersecting point of the two conducting planes, with the distance from this intersecting point to the point charge particle as the radius. As the particle moves toward the exit of the convergent channel, the smaller the image circle will be.

The total number of pairs of images, N , confined on the image circle is related to the intersecting angle of the two conducting planes, the convergent channel angle. The equation for the pair of images is defined as follows.

$$
\begin{equation*}
\mathrm{N}=\frac{\pi}{2 \theta} \tag{3-5}
\end{equation*}
$$

As can be seen from the equation, N is inversely proportional to the convergent channel angle. Examples of image pairs confined on an image circle is shown in figure 3.3 to figure 3.9 for convergent angles of 90 , $60,45,42,30,15$ and 10 degrees. The number of image pairs confined on the image circle is calculated as $2,3,4,5,6,12$ and 18 respectively.

As we can see from these figures, if N is an integer, the two images of the last pair will coincide i.e., there will be a total of N pairs of images confined on the image circle. If N is a non integral real number,
we assume that the two images of the last pair will coincide as well, but the total number of pairs will be the integral part of N plus 1.

From equation (3-5), It is seen that as the angle of the convergent channel decreases, the number of image pairs will increase. If we consider an extreme case of a zero degree convergent channel, The number of image pairs becomes infinite. This actually becomes the case of the parallel plate channel.

From the graphical analysis, the center of image circle is located at the intersection of the two tilted plates. As the angle of convergence becomes smaller, the intersection will be further away from the entrance point and the larger the image circle will be. This results in more path needed to reach the other side of the circle, that is, an increase in the number of image pairs.

The concept of equation (3-4) can be also applied to a convergent channel, which results in the following form.

$$
\begin{align*}
\mathrm{F}_{\mathrm{x} \text { local }}= & \frac{\mathrm{q}^{2}}{4 \pi \varepsilon_{0}}\left\{\frac{1}{\mathrm{q}_{1}{ }^{2}}+\frac{1}{\mathrm{q}_{1^{2}}{ }^{2}}+\frac{-1}{\mathrm{q}_{2}{ }^{2}}+\frac{-1}{\mathrm{q}_{2^{\prime}}{ }^{2}}\right. \\
& \left.+\frac{1}{\mathrm{q}_{3}{ }^{2}}+\frac{1}{\mathrm{q}_{3^{\prime}}{ }^{2}}+\cdots+\frac{(-1)^{\mathrm{n+1}}}{\mathrm{q}_{\mathrm{n}}{ }^{2}}+\frac{(-1)^{\mathrm{n+1}}}{\mathrm{q}_{n^{\prime}}{ }^{2}}\right\} \\
\mathrm{F}_{\mathrm{y} \text { local }} & =\frac{\mathrm{q}^{2}}{4 \pi \varepsilon_{0}}\left\{\frac{1}{\mathrm{q}_{1}{ }^{2}}+\frac{1}{\mathrm{q}_{1^{\prime}}{ }^{2}}+\frac{-1}{\mathrm{q}_{2}{ }^{2}}+\frac{-1}{\mathrm{q}_{2^{2}}{ }^{2}}\right. \\
& \left.+\frac{1}{\mathrm{q}_{3}{ }^{2}}+\frac{1}{\mathrm{q}_{3^{\prime}}{ }^{2}}+\cdots+\frac{(-1)^{\mathrm{n+1}}}{\mathrm{q}_{\mathrm{n}}{ }^{2}}+\frac{(-1)^{n^{n+1}}}{\mathrm{q}_{\mathrm{n}^{\prime}}{ }^{2}}\right\} \tag{3-6}
\end{align*}
$$

where $q_{n}$ and $q_{n}$, represent the distance between the charged particle and image pairs on the image circle. $F_{y}$ local is the force in the $q_{n}$ and $\mathrm{q}_{\mathrm{n}}, \mathrm{s}$ direction and $\mathrm{F}_{\mathrm{x} \text { local }}$ is the force in the direction perpendicular to the $\mathrm{q}_{\mathrm{n}}$ and $\mathrm{q}_{\mathrm{n}}, \mathrm{s}$ direction.

Before we continue to derive the equations for the image force on the convergent channel, the following definitions are given here for better understanding. (Refer to figure 3.10 )

R : radius of the image circle

$$
\mathrm{R}=\sqrt{\mathrm{y}^{2}+\mathrm{x}_{\mathrm{m}}{ }^{2}}
$$

$\mathrm{x}_{\mathrm{m}}$ : horizontal distance from the charge point to the center of the image circle

$$
\mathrm{x}_{\mathrm{m}}=\mathrm{h} / \tan \theta
$$

$\theta$ : half of the convergent channel angle

$$
\theta=\tan ^{-1}\left(\mathrm{y} / \mathrm{x}_{\mathrm{m}}\right)
$$

$h$ : local half width of the convergent channel $\mathrm{h}=\mathrm{h}_{0}-\mathrm{x} \tan \theta$
$h_{0}$ : half width of the convergent channel at the entrance
$\alpha_{0}$ : angle between the center line and the line from the charge to the center
$\alpha_{\mathrm{n}}$ : half the angle subtended by the chord connecting the charge to image $n$

$$
\alpha_{\mathrm{n}}=\mathrm{n} \theta-\frac{1}{2}\left[1-(-1)^{\mathrm{n}}\right] \alpha_{0}
$$

$\alpha_{\mathrm{n}}$ : half the angle subtended by the chord connecting the charge
to image $\mathrm{n}^{\prime}$

$$
\alpha_{\mathrm{n}^{\prime}}=\mathrm{n} \theta+\frac{1}{2}\left[1-(-1)^{\mathrm{n}}\right] \alpha_{0}
$$

$\beta_{\mathrm{n}}$ : angle between the y -axis and the chord connecting the charge to the primary image $n$
$\beta_{\mathrm{n}}=\alpha_{\mathrm{n}}+\alpha_{0}$
$\beta_{n}: \quad$ angle between the $y$-axis and the chord connecting the charge to the secondary image $\mathrm{n}^{\prime}$
$\beta_{n}=\alpha_{n}, \alpha_{0}$
$\mathrm{q}_{\mathrm{n}}$ : distance between the point charge to the primary image n $\mathrm{q}_{\mathrm{n}}=2 \mathrm{R} \sin \alpha_{\mathrm{n}}$
$\mathrm{q}_{\mathrm{n}^{\prime}}$ : distance between the point charge to the secondary image $n^{\prime}$ $\mathrm{q}_{\mathrm{n}}=2 \mathrm{R} \sin \alpha_{\mathrm{n}}$,
A proof of these formulas appears in [31] and is shown in Appendix A.

The y-component image force in equation (3-6) is a local value since the attraction or repulsion force acting between the point charge and its image are all in the direction with respect to their distance $q_{1}, q_{1}, q_{2}, q_{2}$, $\ldots, q_{n}$ and $q_{n^{\prime}}$, respectively. All these forces have to be resolved into the global x and y coordinates of the two-phase flow field for analysis. From figure 3.10, it can be deduced that the distances $q_{1}, q_{1^{\prime}}, q_{2}, q_{2^{\prime}}, \ldots, q_{n}$ and $\mathrm{q}_{\mathrm{n}}$, are inclined with the y -axis through an angle $\beta_{1}, \beta_{1^{\prime}}, \beta_{2}, \beta_{2^{\prime}}, \ldots, \beta_{\mathrm{n}}$ and $\beta_{\mathrm{n}}$, respectively where these angles have been defined and proved previously.

They are derived as

$$
\begin{aligned}
& \beta_{1}=\alpha_{1}+\alpha_{0} \\
& \beta_{1^{\prime}}=\alpha_{1^{\prime}}-\alpha_{0}
\end{aligned}
$$

$$
\begin{aligned}
& \beta_{2}=\alpha_{2}+\alpha_{0} \\
& \beta_{2^{\prime}}=\alpha_{2^{\prime}}-\alpha_{0} \quad \ldots \text { and so on }
\end{aligned}
$$

From vector analysis, all the forces due to the primary images above the upper channel wall have to be multiplied by the corresponding $\cos \beta_{\mathrm{n}}$ or $\sin \beta_{\mathrm{n}}$ to be resolved into the global fluid flow coordinates. Similarly, the forces due to the secondary images below the lower channel wall have to be multiplied by the corresponding $\cos \beta_{\mathrm{n}}$ or $\sin \beta_{\mathrm{n}}$, to be transferred to the global coordinates. Inserting these factors into equation (3-6) and arranging them in the x and y components separately yields,

$$
\begin{align*}
\mathrm{F}_{\mathrm{x}}= & \frac{\mathrm{q}^{2}}{4 \pi \varepsilon_{0}}\left\{\frac{\sin \beta_{1}}{\mathrm{q}_{1}{ }^{2}}+\frac{\sin \beta_{\mathrm{r}}}{\mathrm{q}_{\mathrm{r}^{2}}{ }^{2}}+\frac{-\sin \beta_{2}}{\mathrm{q}_{2}{ }^{2}}+\frac{-\sin \beta_{2^{\prime}}}{\mathrm{q}_{2^{\prime}}{ }^{2}}\right. \\
& \left.+\cdots+\frac{(-1)^{\mathrm{n}^{+1} \sin \beta_{\mathrm{n}}}}{\mathrm{q}_{\mathrm{n}}{ }^{2}}+\frac{(-1)^{\mathrm{n}^{+1} \sin \beta_{n^{\prime}}}}{\mathrm{q}_{\mathrm{n}^{\prime}}{ }^{2}}\right\}  \tag{3-7a}\\
\mathrm{F}_{\mathrm{y}}= & \frac{\mathrm{q}^{2}}{4 \pi \varepsilon_{0}}\left\{\frac{\cos \beta_{1}}{\mathrm{q}_{1}{ }^{2}}-\frac{\cos \beta_{\mathbf{r}^{\prime}}}{\mathrm{q}_{r^{\prime}}{ }^{2}}+\frac{-\cos \beta_{2}}{\mathrm{q}_{2}{ }^{2}}-\frac{-\cos \beta_{2^{\prime}}}{\mathrm{q}_{2^{\prime}}{ }^{2}}\right. \\
& \left.+\cdots+\frac{(-1)^{\mathrm{n}+1} \cos \beta_{\mathrm{n}}}{\mathrm{q}_{\mathrm{n}}{ }^{2}}-\frac{(-1)^{\mathrm{n}+1} \cos \beta_{n^{\prime}}}{\mathrm{q}_{\mathrm{n}^{\prime}}{ }^{2}}\right\} \tag{3-7b}
\end{align*}
$$

or in the summation forms

$$
\begin{align*}
& \mathrm{F}_{\mathrm{x}}=\frac{-\mathrm{q}^{2}}{4 \pi \varepsilon_{0}}\left\{\sum_{\mathrm{n}=1}^{\mathrm{N}}(-1)^{\mathrm{n}} \frac{\sin \beta_{\mathrm{n}}}{\mathrm{q}_{\mathrm{n}}{ }^{2}}+\sum_{\mathrm{n}=1}^{\mathrm{N}-1}(-1)^{\mathrm{n}} \frac{\sin \beta_{\mathrm{n}^{\prime}}}{\mathrm{q}_{\mathrm{n}^{\prime}}{ }^{2}}\right\}  \tag{3-8a}\\
& \mathrm{F}_{\mathrm{y}}=\frac{-\mathrm{q}^{2}}{4 \pi \varepsilon_{0}}\left\{\sum_{\mathrm{n}=1}^{\mathrm{N}}(-1)^{\mathrm{n}} \frac{\cos \beta_{\mathrm{n}}}{\mathbf{q}_{\mathrm{n}}{ }^{2}}-\sum_{\mathrm{n}=1}^{\mathrm{N}-1}(-1)^{\mathrm{n}} \frac{\cos \beta_{\mathrm{n}^{\prime}}}{\mathrm{q}_{\mathrm{n}^{\prime}}{ }^{2}}\right\} \tag{3-8b}
\end{align*}
$$

These are the final image force equations for convergent channels. As we stated before the number of image pairs will be determined by the degree of the convergent angle. As the convergent angle becomes smaller the number of image pairs will become larger. Consider the extreme case of two parallel plates channel. The angle of $\beta_{\mathrm{n}}$ and $\beta_{\mathrm{n}}$ on the above equations become $0^{\circ}$ (i.e. $\sin \beta_{\mathrm{n}}$ and $\sin \beta_{n^{\prime}}=0$ ). The image force in the $x$-direction will then become zero because of the sine function. The equations then become

$$
\begin{align*}
\mathrm{F}_{\mathrm{x}} & =0 \\
\mathrm{~F}_{\mathrm{y}} & =\frac{\mathrm{q}^{2}}{4 \pi \varepsilon_{0}}\left\{\frac{1}{\mathrm{q}_{1}{ }^{2}}+\frac{-1}{\mathrm{q}_{r^{2}}{ }^{2}}+\frac{1}{\mathrm{q}_{2}{ }^{2}}+\frac{-1}{\mathrm{q}_{2}{ }^{2}}\right. \\
& \left.+\frac{1}{\mathrm{q}_{3}{ }^{2}}+\frac{-1}{\mathrm{q}_{3}{ }^{2}}+\cdots+\frac{1}{\mathrm{q}_{\mathrm{n}}{ }^{2}}+\frac{-1}{\mathrm{q}_{\mathrm{n}^{2}}{ }^{2}}\right\} \tag{3-9}
\end{align*}
$$

where $\mathrm{q}_{1}=2\left(\mathrm{~h}_{0}-\mathrm{y}\right), \mathrm{q}_{1^{\prime}}=2\left(\mathrm{~h}_{0}+\mathrm{y}\right)$ and $\mathrm{q}_{2}=\mathrm{q}_{1}+\mathrm{q}_{1^{\prime}}=\mathrm{q}_{2^{\prime}}=4 \mathrm{~h}_{0}$. Thus, the second pair cancels out. The third gives $q_{3}=2\left(3 h_{0}-y\right), q_{3^{\prime}}=$ $2\left(3 h_{0}+y\right)$ and equation (3-9) matches the equation of image forces for two parallel plates as shown in equation (3-4).

### 3.3 Governing Equations

In this chapter, the numerical method is applied to study the deposition of suspended particles for uniform flow in a two-dimensional convergent channel due to the effect of image charge, inertia force, viscous force and gravitational force.

The analysis is divided into two categories.
(1) Deposition due to image force, inertia force and viscous force.
(2) Deposition due to image force, inertia force, viscous force and gravitational force.
In this numerical method, equi-penetration points or curves over the inlet plane are constructed, and the fraction of deposition is obtained by integrating the product of particles' initial velocity and their concentration over the area enclosed by the equi-penetration curve and the boundary wall.

For flow of suspension in the convergent channel, the following assumptions are made.
(1) Incompressible, steady flow.
(2) Two dimensional, uniform flow. The boundary layer buildup in a convergent channel is very small and, therefore, this assumption is justified.
(3) Dilute suspension. When the particle concentration is less than 1.0 E 05 , the suspension may be treated as dilute and the dominant force is the image force.
(4) Fluid-particle interaction is negligible.
(5) Particle-particle interaction is negligible.
(6) Negligible diffusion force in comparison to image force.
(7) Convergent conducting channel walls are assumed to be grounded and may be extended to intersect at a point.
(8) Thickness of the layer of deposit is much smaller than the channel width.
(9) Negligible lift force on the particle.
(10) Material density of the fluid phase in comparison to that of the solid particles is negligible.
(11) No chemical reactions considered.
(12) Particle number density is sufficiently low so that mutual electrostatic repulsion between particles is negligible.
(13) Electrostatic charge is uniformly distributed on surface of particle.
(14) Particles behave like uniform spheres of radius "a".
(15) Negligible temperature effects.

The rectangular Cartesian coordinate system as shown in figure 3.2 is employed in this analysis. The x -axis is along the centerline of the convergent channel with the positive direction coinciding with the stream wise direction, and the $y$-axis is parallel to the direction of gravity.

The fluid flowing in this channel is treated as a continuum with uniform velocity profile in the $x$-direction and the fluid velocity in the $y$ direction is such that the continuity and momentum equation are satisfied by the fluid.

Based on Newton's second law of motion, which states that the product of mass and acceleration of a particle in any direction is proportional to the force acting on the particle in that direction, the governing equation for a particle moving in a fluid subject to viscous
forces, image forces, inertia forces and gravity forces can be expressed in a rectangular Cartesian coordinate system which incorporates the assumptions made above as follows.

$$
\begin{align*}
& m \frac{d u_{p}}{d t}=f\left(u-u_{p}\right)+F_{x}  \tag{3-10a}\\
& m \frac{d v_{p}}{d t}=f\left(v-v_{p}\right)-m g+F_{y} \tag{3-10b}
\end{align*}
$$

where

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{x}}=\text { image force per particle in x-direction (refer to eq. (3-9) ) } \\
& \mathrm{F}_{\mathrm{y}}=\text { image force per particle in y-direction (refer to eq. (3-9) ) } \\
& \mathrm{f}=6 \pi \mu \mathrm{a} \quad \text { (Stoke's drag law) } \\
& \mathrm{m}=\text { mass of the particle } \\
& \mathrm{g}=\text { gravity acceleration coefficient } \\
& \mathrm{t}=\text { time unit } \\
& \mathrm{u}=\text { velocity of fluid in the x-direction } \\
& \mathrm{v}=\text { velocity of fluid in the y-direction } \\
& \mathrm{u}_{\mathrm{p}}=\text { velocity of particle in the x-direction } \\
& \mathrm{v}_{\mathrm{p}}=\text { velocity of particle in the y-direction } \\
& \mathrm{a}=\text { radius of the particle } \\
& \mu=\text { dynamic viscosity of the fluid }
\end{aligned}
$$

In equations (3-10a and 3-10b), the left-hand-side term is the product of the particle mass and acceleration, which is the inertia term. On the right hand side is the sum of the forces, which are the viscous forces based on Stoke's law when the Reynolds number is low, the image force which acts on the given charge $q$ and is induced by its interaction with the conducting channel plate, and finally the gravitational force.

For a spherical particle moving in a fluid, the drag force experienced by the particle depends on the Reynolds number which is the product of the fluid velocity relative to the particle and the particle diameter divided by the kinematic viscosity of the fluid or $2 \mathrm{a}\left(\mathrm{u}_{\mathrm{p}}-\mathrm{u}\right) / \nu$. When the Reynolds number is low (i.e. of order 1 or less) the drag force is given by the Stokes drag law, $6 \pi \mu \mathrm{a}\left(\mathrm{u}_{\mathrm{p}}-\mathrm{u}\right)$ or $\mathrm{f}\left(\mathrm{u}_{\mathrm{p}}-\mathrm{u}\right)$. Since the particles in this study are very small (order of one micro-meter) and the relative velocity is also small, we may apply the Stokes drag law in equation (3-10a and 3-10b).

These equations can also be non-dimensionalized by using the following dimensionless terms.

Let

$$
\begin{aligned}
& T=\frac{t u_{0}}{h_{0}} \\
& U=\frac{u}{u_{0}} \\
& V=\frac{v}{u_{0}} \\
& Y=\frac{y}{h_{0}} \\
& X=\frac{x}{h_{0}} \\
& S=\frac{m u_{0}}{h_{0} f}=\frac{\text { inertia force }}{\text { viscous force }}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{Q}=\frac{\mathrm{q}^{2}}{4 \pi \varepsilon_{0} \mathrm{~h}_{0}{ }^{2} \mathrm{f} \mathrm{u}_{0}}=\frac{\text { charge force }}{\text { viscous force }} \\
& \mathrm{G}=\frac{\mathrm{mg}}{\mathrm{f} \mathrm{u}_{0}}=\frac{\text { gravity force }}{\text { viscous force }}
\end{aligned}
$$

Substituting these dimensionless terms into equations (3-10a and 310b), we have

$$
\begin{align*}
& m \frac{u_{0}{ }^{2}}{h_{0}} \frac{d^{2} X}{d T^{2}}=f u_{0}\left(U-\frac{d X}{d T}\right) \\
& \quad+\frac{-q^{2}}{4 \pi \varepsilon_{0}}\left\{\sum_{\mathrm{n}=1}^{N}(-1)^{\mathrm{n}} \frac{\sin \beta_{\mathrm{n}}}{\mathrm{q}_{\mathrm{n}}{ }^{2}}+\sum_{\mathrm{n}=1}^{\mathrm{N}-1}(-1)^{\mathrm{n}} \frac{\sin \beta_{\mathrm{n}^{\prime}}}{\mathrm{q}_{\mathrm{n}^{2}}{ }^{2}}\right\} \tag{3-11a}
\end{align*}
$$

and

$$
\begin{align*}
& \mathrm{m} \frac{\mathrm{u}_{0}{ }^{2}}{\mathrm{~h}_{0}} \frac{\mathrm{~d}^{2} Y}{d T^{2}}=\mathrm{f} \mathrm{u}_{0}\left(\mathrm{~V}-\frac{\mathrm{dY}}{\mathrm{dT}}\right)-\mathrm{mg} \\
& \quad+\frac{-\mathrm{q}^{2}}{4 \pi \varepsilon_{0}}\left\{\sum_{\mathrm{n}=1}^{N}(-1)^{\mathrm{n}} \frac{\cos \beta_{\mathrm{n}}}{\mathrm{q}_{\mathrm{n}}{ }^{2}}-\sum_{\mathrm{n}=1}^{\mathrm{N}-1}(-1)^{\mathrm{n}} \frac{\cos \beta_{\mathrm{n}^{\prime}}}{\mathrm{q}_{\mathrm{n}^{\prime}}{ }^{2}}\right\} \tag{3-11b}
\end{align*}
$$

Dividing the whole equations by $\mathrm{fu}_{0}$, these equations become

$$
\begin{align*}
& \frac{\mathrm{mu}_{0}}{\mathrm{~h}_{0} f} \frac{\mathrm{~d}^{2} \mathrm{X}}{\mathrm{dT} T^{2}}=\mathrm{U}-\frac{\mathrm{d} X}{\mathrm{dT}} \\
& \quad+\frac{-\mathrm{q}^{2}}{4 \pi \varepsilon_{0} \mathrm{f} u_{0}}\left\{\sum_{\mathrm{n}=1}^{\mathrm{N}}(-1)^{\mathrm{n}} \frac{\sin \beta_{\mathrm{n}}}{\mathrm{q}_{\mathrm{n}}{ }^{2}}+\sum_{\mathrm{n}=1}^{\mathrm{N}-1}(-1)^{\mathrm{n}} \frac{\sin \beta_{\mathrm{n}^{\prime}}}{\mathrm{q}_{\mathrm{n}^{\prime}}}\right\} \tag{3-12a}
\end{align*}
$$

and

$$
\begin{align*}
& \frac{m u_{0}}{h_{0} f} \frac{d^{2} Y}{d T^{2}}=V-\frac{d Y}{d T}-\frac{m g}{h_{0} f} \\
& \quad+\frac{-q^{2}}{4 \pi \varepsilon_{0} f u_{0}}\left\{\sum_{n=1}^{N}(-1)^{n} \frac{\sin \beta_{n}}{q_{n}{ }^{2}}-\sum_{n=1}^{N-1}(-1)^{n} \frac{\sin \beta_{n^{\prime}}}{q_{n^{\prime}}{ }^{2}}\right\} \tag{3-12b}
\end{align*}
$$

Finally we have

$$
\begin{align*}
& S \frac{d^{2} X}{d T^{2}}=U-\frac{d X}{d T}+Q F_{x}^{*}  \tag{3-13a}\\
& S \frac{d^{2} Y}{d T^{2}}=V-\frac{d Y}{d T}-G+Q F_{y}^{*} \tag{3-13b}
\end{align*}
$$

where

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{x}}^{*}=-\mathrm{h}_{0}{ }^{2}\left\{\sum_{\mathrm{n}=1}^{\mathrm{N}}(-1)^{\mathrm{n}} \frac{\sin \beta_{\mathrm{n}}}{\mathrm{q}_{\mathrm{n}}{ }^{2}}+\sum_{\mathrm{n}=1}^{\mathrm{N}-1}(-1)^{\mathrm{n}} \frac{\sin \beta_{\mathrm{n}^{\prime}}}{\mathrm{q}_{\mathrm{n}^{\prime}}{ }^{2}}\right\} \\
& \mathrm{F}_{\mathrm{y}^{*}}^{*}=-\mathrm{h}_{0}{ }^{2}\left\{\sum_{\mathrm{n}=1}^{\mathrm{N}}(-1)^{\mathrm{n}} \frac{\cos \beta_{\mathrm{n}}}{\mathrm{q}_{\mathrm{n}}{ }^{2}}-\sum_{\mathrm{n}=1}^{\mathrm{N}-1}(-1)^{\mathrm{n}} \frac{\cos \beta_{\mathrm{n}^{\prime}}}{\mathrm{q}_{\mathrm{n}^{\prime}}{ }^{2}}\right\}
\end{aligned}
$$

Equations (3-13a) and (3-13b) are the governing equations for the deposition of suspension flow in a convergent channel due to the effects of inertia, viscosity, gravity and image forces. If the gravity effect is neglected, i.e. $G=0$, the governing equations then become

$$
\begin{align*}
& S \frac{d^{2} X}{d T^{2}}=U-\frac{d X}{d T}+Q F_{x}^{*}  \tag{3-14a}\\
& S \frac{d^{2} Y}{d T^{2}}=V-\frac{d Y}{d T}+Q F_{y}^{*} \tag{3-14b}
\end{align*}
$$

## CHAPTER 4

## METHOD OF SOLUTION

A fourth order Runge-Kutta Method is applied here to solve the governing equations which we derived in the previous chapter.

$$
\begin{align*}
& S \frac{\mathrm{~d}^{2} X}{\mathrm{dT}^{2}}=\mathrm{U}-\frac{\mathrm{dX}}{\mathrm{dT}}+Q \mathrm{~F}_{\mathrm{x}}^{*}  \tag{3-13a}\\
& S \frac{\mathrm{~d}^{2} Y}{\mathrm{dT}^{2}}=\mathrm{V}-\frac{\mathrm{dY}}{\mathrm{dT}}-G+Q \mathrm{~F}_{\mathrm{y}}^{*} \tag{3-13b}
\end{align*}
$$

In the case of neglecting the gravity effect, we have

$$
\begin{align*}
& S \frac{d^{2} X}{d T^{2}}=U-\frac{d X}{d T}+Q F_{x}^{*}  \tag{3-14a}\\
& S \frac{d^{2} Y}{d T^{2}}=V-\frac{d Y}{d T}+Q F_{y}^{*} \tag{3-14b}
\end{align*}
$$

### 4.1 Initial Condition

In our analysis, we have considered a constant area parallel channel be connected to the entrance of the convergent channel such that a laminar uniform flow velocity distribution can be obtained at the entrance region of the convergent channel, and the continuity equation can be satisfied throughout the entire region of the channel.

Based on the geometry of a convergent channel and the continuity equation of the flow, we have
$\mathrm{h}=\mathrm{h}_{0}-\mathrm{x} \tan \theta, \quad$ where $\theta$ is the half convergent angle
$u=\frac{1}{h}=\frac{1}{h_{0}-x \tan \theta} ;$ for uniform flow
$\mathrm{v}=-\mathrm{u} y \tan \theta ;$ by appling the continuity theorem
Which can be expressed in dimensionless form as

$$
\begin{aligned}
& \mathrm{H}=1-\mathrm{X} \tan \theta \\
& \mathrm{U}=\frac{1}{\mathrm{H}}=\frac{1}{1-\mathrm{X} \tan \theta} \\
& \mathrm{~V}=-\frac{\mathrm{Y}}{\mathrm{H}^{2}} \tan \theta
\end{aligned}
$$

The initial conditions are as follows.

$$
\begin{aligned}
\text { At } t & =0 \\
x & =0 \text { and } y=y_{0} ; \text { for }-h_{0}<y_{0}<h_{0} \\
v_{p} & =\frac{d y}{d t}=0 ; \text { for }-h_{0}<y_{0}<h_{0} \\
u & =u_{p}=\frac{d x}{d t}=u_{0} \\
h & =h_{0}
\end{aligned}
$$

In dimensionless form these conditions become

$$
\begin{aligned}
& \text { At } \mathrm{T}=0 \\
& \qquad \begin{array}{l}
\mathrm{X}=0 \text { and } \mathrm{Y}=\mathrm{Y}_{0} ; \text { for }-1<\mathrm{Y}_{0}<1 \\
\mathrm{~V}_{\mathrm{p}}=\frac{\mathrm{dY}}{\mathrm{dT}}=0 ; \text { for }-1<\mathrm{Y}_{0}<1 \\
\mathrm{U}=\mathrm{U}_{\mathrm{p}}=\frac{\mathrm{dX}}{\mathrm{dT}}=1 ; \text { for uniform flow } \\
H=1
\end{array}
\end{aligned}
$$

### 4.2 Runge-Kutta Method

The second-order differential equations can be rewritten in the form of two equivalent first order differential equations as follows:
let

$$
\begin{equation*}
\mathrm{A}=\frac{\mathrm{dX}}{\mathrm{dT}} \tag{4-1}
\end{equation*}
$$

be the particle velocity in the x-direction, therefore,

$$
\frac{\mathrm{dA}}{\mathrm{dT}}=\frac{\mathrm{d}}{\mathrm{dT}}\left(\frac{\mathrm{dX}}{\mathrm{dT}}\right)=\frac{\mathrm{d}^{2} \mathrm{X}}{\mathrm{dT}^{2}}
$$

Which is the particle acceleration in the x -direction. By substituting these into equation (3-13a), we have

$$
\mathrm{S} \frac{\mathrm{dA}}{\mathrm{dT}}=\mathrm{U}-\mathrm{A}+\mathrm{Q} \mathrm{~F}_{\mathrm{x}}^{*}
$$

or

$$
\begin{equation*}
\frac{\mathrm{dA}}{\mathrm{dT}}=\frac{\mathrm{U}-\mathrm{A}+\mathrm{Q} \mathrm{~F}_{\mathrm{x}}^{*}}{\mathrm{~S}} \tag{4-2}
\end{equation*}
$$

In a similar manner,
let

$$
\begin{equation*}
B=\frac{d Y}{d T} \tag{4-3}
\end{equation*}
$$

be the particle velocity in the $y$-direction. Hence,

$$
\frac{\mathrm{dB}}{\mathrm{dT}}=\frac{\mathrm{d}}{\mathrm{dT}}\left(\frac{\mathrm{dY}}{\mathrm{dT}}\right)=\frac{\mathrm{d}^{2} \mathrm{Y}}{\mathrm{dT}^{2}}
$$

which is the particle acceleration in the y-direction. By substituting these into equation (3-13b), we have

$$
\mathrm{S} \frac{\mathrm{~dB}}{\mathrm{dT}}=\mathrm{V}-\mathrm{B}-\mathrm{G}+\mathrm{Q} \mathrm{~F} \mathrm{~F}_{\mathrm{y}}^{*}
$$

or

$$
\begin{equation*}
\frac{\mathrm{dB}}{\mathrm{dT}}=\frac{\mathrm{V}-\mathrm{B}-\mathrm{G}+\mathrm{Q} \mathrm{~F}_{\mathrm{y}}^{*}}{\mathrm{~S}} \tag{4-4a}
\end{equation*}
$$

or, by neglecting the gravity effect, equation (4-4a) becomes

$$
\begin{equation*}
\frac{\mathrm{dB}}{\mathrm{dT}}=\frac{\mathrm{V}-\mathrm{B}+\mathrm{QF}_{\mathrm{y}}^{*}}{\mathrm{~S}} \tag{4-4b}
\end{equation*}
$$

Therefore the second-order differential equations (3-13a), (3-13b) or (3-14a), (3-14b) can then be transformed to the first-order differential equations (4-1), (4-2), (4-3) and (4-4).

The initial conditions become

$$
\begin{aligned}
& \text { At } T=0 \\
& \text { A }=\frac{\mathrm{dX}}{\mathrm{dT}}=1 \text { at } \mathrm{X}=0 ; \text { for uniform flow } \\
& \mathrm{U}=\mathrm{U}_{0}=1 \text { at } \mathrm{X}=0 ; \text { for uniform flow } \\
& B=\frac{\mathrm{dY}}{\mathrm{dT}}=0 \text { at } \mathrm{X}=0 \\
& V=-\frac{Y}{\mathrm{H}^{2}} \tan \theta
\end{aligned}
$$

A description of the scheme for the fourth order Runge-Kutta numerical method has been listed in appendix B.

The parameters that were used to solve these equations to study the trajectory of the suspended particles are:

$$
\begin{aligned}
\theta= & 7.5^{\circ}, 5.0^{\circ}, 2.5^{\circ} \\
S= & 100,10,1,0.1,0.01 \\
G= & 100,10,1,0.1,0.01 \\
Q= & 10000,1000,100,10,1,0.1,0.01,0.001,0.0001, \\
& 0.00001
\end{aligned}
$$

A particle is considered deposited on the wall when it reaches the channel wall or $\mathrm{Y}=1.0$ or $\mathrm{Y}=-1.0$. (In our computer program, we have considered that the particle has been deposited on the wall when $\mathrm{Y} \geq$ 0.999 or $\mathrm{Y} \leq-0.999$.) To find the trajectory of a particle, a set of 115 initial particle position at the inlet plane (ranging from $\mathrm{Y}_{0}=0.998$ to $\mathrm{Y}_{0}=$ -0.998 ) was used.

### 4.3 Fraction of Deposition

The main purpose of solving these equations is to determine the fraction of deposition of the particles on the convergent channel wall due to the combination of the forces.

In a two-dimensional plate channel, the fraction of deposition is defined as the ratio of total number of particles deposited on the wall at the given distance to the total number of particles entering the channel.

$$
\text { Fraction of deposition }=\frac{\text { total number of particles deposited on the wall }}{\text { total number of particles entering the channel }}
$$

The distribution of particle deposition is not symmetric due to the particle gravity effect. Consider a particle $p_{1}$ starting at $x=0$ and $y=y_{u}$ ( $0<y_{u}<h_{0}$ ). The particle is considered to have been deposited on the wall as soon as it reaches the wall at $x=x_{1}$. Similarly another particle $p_{2}$ starting at $x=0$ and $y=y_{1}\left(0>y_{1}>-h_{0}\right)$ deposits at the lower wall at $x$ $=x_{1}$. Those particles which enter the inlet plane (i.e. $x=0$ ) of the convergent channel between $y=y_{u}$ and $y=h_{0}$ would have been deposited on the upper wall between $x=0$ and $x=x_{1}$. Similarly all the particles that enter the channel between $\mathrm{y}=\mathrm{y}_{1}$ and $\mathrm{y}=-\mathrm{h}_{0}$ would have been
deposited on the lower wall between $x=0$ and $x=x_{1}$. The fraction of deposition and penetration at the specific axial distance can be calculated from the inlet section. The fraction of deposition, which is the fraction of particle deposited on a section at any given axial distance can be obtained from particle properties at $\mathrm{x}=0$.

$$
\frac{\int_{-b_{0}}^{y_{p}} u_{p} \rho_{p} d y+\int_{y_{p}}^{b_{p}} u_{p} \rho_{p} d y}{\int_{-h_{0}}^{h_{p}} u_{p} \rho_{p} d y}
$$

This can be rewritten as the dimensionless form as

$$
\frac{\int_{-1}^{Y_{1}} U_{p} \rho_{p} d Y+\int_{Y_{1}}^{1} U_{p} \rho_{\mathrm{p}} d Y}{\int_{-1}^{1} U_{p} \rho_{\mathrm{p}} d Y}=1-\frac{1}{2}\left(Y_{0}-Y_{1}\right)
$$

where $Y_{1}=y_{1} / h_{0}, Y_{u}=y_{u} / h_{0} U_{p}=1$ and $\rho_{p}$ is a constant at $X=0$.

## CHAPTER 5

## RESULTS AND DISCUSSION

The deposition of suspended flow particles in a convergent channel due to the effects of image forces, gravity forces, viscous forces and inertia forces has been solved numerically. In this chapter, the trajectories of particle will be investigated and the influence on the deposition from the above effects will be discussed. As we mentioned in the previous chapters, the values of dimensionless parameters investigated in this analysis are 100 , $10,1,0.1$, and 0.01 for both $S$ (inertia parameter) and $G$ (gravity parameter). Q (the image parameter) is investigated in the range from 10000 to 0.00001 . The half convergent channel angles investigated in this study are $7.5,5$ and 2.5 degrees.

The convergent channel will intersect at a point if the upper and lower walls of the convergent channel are extended. Theoretically, all the particles entering the convergent channel will deposit either on the upper wall or the lower wall on or before the intersection of the two walls. However, as the width of the convergent channel becomes smaller, the gas velocity at the channel will be increased as well, and finally reaches infinity at the intersection of the upper wall and lower wall. Near the intersection, this will result in a tremendous high velocity and the flow will become compressible. However, this is beyond the scope of our investigation. Therefore, in this study, the range of the convergent channel is set to a length such that the width of the exit of the convergent channel is equal to $20 \%$ of the entrance width. This means that the lengths of the convergent channels analyzed here are equal to $6.07,9.14$ and 18.3 for
half convergent channel of $7.5^{\circ}, 5^{\circ}$ and $2.5^{\circ}$ respectively. However, the calculation of the particle trajectories and the fraction of deposition have been investigated thoroughly in order to check the correctness of the calculated results.

In this chapter, the influence of deposition of particles due to the effects of inertia forces, viscous forces, electrostatic image forces and gravity forces will be discussed separately (by setting some of the dimensionless parameters equal to zero). The combined effects from the above forces on the deposition will be discussed thereafter. The influence from different convergent channel angles together with various force effects will be discussed as well.

In this study, the particle and the fluid velocity at the inlet plane of the convergent channel are assumed to be identical.

### 5.1 Electrostatic Image Force Distribution

Tables 5.1 to 5.4 (Tables and Figures are in Appendix C) give the electrostatic image forces in the x and y direction acting on particles at various vertical positions from 0.01 (farthest from the channel wall) to 0.999 (closest to the channel wall) at half convergent channel angle of 7.5, 5, 2.5 and zero degrees respectively.

As discussed in chapter 3, for a particle suspension flow in a convergent channel, the images are confined on the image circle. This image circle is defined by having its center at the extended intersection point of the two channel walls, with the distance from this intersecting point to the point charge particle as the radius. The total number of pairs of images is related to the convergent channel angle as given in equation (3.5). Hence, depending on the convergent channel angle and the vertical
position $Y$ of the particle, the total number of image pairs and then the magnitude of the image forces in the X and Y directions are defined.

As listed in tables 5.1 to 5.3 , the total number of image pairs are 12, 18 , and 36 for half convergent channel angles of $7.5,5$ and 2.5 degrees respectively. It is shown that for any fixed convergent angle, the image force increases with increasing Y. That is, the closer the particle is to the channel wall, the larger the image force it will experience. The data shows that the X and Y component image forces are of 1.0 E 6 to 1.0 E 7 times larger at $\mathrm{Y}=0.999$ than that at $\mathrm{Y}=0.01$.

It is also noted that the X component image force is slightly greater than the $Y$ component force at $Y=0.01$ for the convergent angle of 7.5, 5.0 and 2.5 degrees. This is due to the Y component image force of a particle near the center line of the channel balancing out. For all cases investigated, the force in the Y direction is greater than that in the X direction for all particles located at $\mathrm{Y}>0.1$, and then becomes $1 / \tan \theta$ times larger. For example, $F_{y}$ is $7.6,11.4$ and 22.9 times larger than $F_{x}$ as Y reaches 0.99 for $\theta=7.5,5.0$ and 2.5 degrees respectively.

Furthermore, at a constant Y , results reveal that the larger the convergent angle, the larger the image force the particle will experience. However, the order of this increment is moderate for the Y component force which implies that the change of the image force in the Y direction is less sensitive to the convergent angle than that in the X component. For example, when $\theta=2.5^{\circ}$ and at $Y_{0}=0.9$, table 5.3 shows that $F_{x}$ and $F_{y}$ are equal to 1.091985 and 25.00874 . As $\theta$ increases to $5^{\circ}$, table 5.2 shows $\mathrm{F}_{\mathrm{x}}$ and $\mathrm{F}_{y}$ are 2.194417 and 25.08049. That is a $101 \%$ and a $0.3 \%$ increase in the magnitude of the forces in the X and Y directions as the half convergent channel angle increases from $2.5^{\circ}$ to $5^{\circ}$. It is also observed that
$\mathrm{F}_{\mathrm{x}}$ and $\mathrm{F}_{\mathrm{y}}$ increase by $51 \%$ and $0.5 \%$ respectively as the half convergent channel angle increases from $5^{\circ}$ to $7.5^{\circ}$.

It can thus be concluded that for a particle under the influence of an electrostatic image force, regardless of the convergent channel angle and at a certain vertical position, the particle will experience almost the same amount of image force in the Y direction. With increasing convergent channel angles, because of the corresponding larger increase in the $X$ component image force, the particle will have a higher acceleration in the horizontal direction.

### 5.2 Trajectory Analysis of the Particle in the Convergent Channel

Figures 5.1 to 5.3 show the stopping distance, which is defined as the distance that the particles will travel before its deposit on the wall for the half convergent channel angel of $2.5^{\circ}$ due to the effects of viscosity, inertia, electrostatic image and gravity forces. The Y-axis represents the initial position of the particles entering the convergent channel and the X axis represents the distance that the particles will travel before the particle deposition has taken place. Let's consider the case of $S=100, G=0.01$ as shown in figure 5.1. The longest stopping distance is observed as 2.61 , $7.58,18.29,22.81$ and 22.82 for $\mathrm{Q}=1000,100,10.001$ and 0.00001 respectively. $\mathrm{Y}_{0}$, the initial position that results in the particle entering the channel having the longest stopping distance, is observed as 0.0 when Q is greater than 1 . This is because the gravity effect on the particles is very small when compared to the image force. The particles entering the convergent channel at the position nearest to the center of the channel will encounter the image force from both the upper and lower walls with almost the same magnitude of force. This will keep the particle traveling in the
channel near the center of the channel and traveling a longer distance before its deposit at the channel wall. For the case of $\mathbf{G}=0$, the particle entering the channel at the center of the entrance will not deposit on the wall before exiting the channel ( X displacement is equal to 22.904 times the half-inlet width for the case of a convergent angle of $2.5^{\circ}$ ). However, as we mentioned before, all the deposition will take place theoretically at the point where the upper wall and the lower wall intersect.

As the gravity effect increases, the initial $\mathrm{Y}_{0}$ position, where the particle entering the channel resulting in the longest stopping distance, will shift closer to the upper channel wall. This is because of the gravity effect being added to the system. The particle with the zero force balance will not occur at the center of the convergent channel. It is shown on figure 5.3, the longest stopping distance is $1.2,3.1,2.42,1.98$ and 1.97 for $\mathrm{Q}=$ $10000,100,10.001$ and 0.00001 , where $\mathrm{Y}_{0}$ is located at $0.01,0.5,0.94$, 0.995 and 0.998 . It is shown that because of the increasing gravity effect, $\mathrm{Y}_{0}$ is shifted closer to the upper wall. The stopping distance is also decreased with increasing gravity effect for a fixed $S$ and $Q$. It is also observed from the figures that for a fixed gravity force, $\mathrm{Y}_{0}$ will shift even closer to the upper channel wall for the particle with the smaller image force effect .

The same phenomena are also observed for the case of half convergent channel angle of $5^{\circ}$ and $7.5^{\circ}$. It is also found that as the convergent angle decreases, the stopping distance will increase. As the convergent angle becomes zero degrees, which is the case of a parallel channel, the stopping distance will be increased, and will then become infinite for the case when $G$ is equal to zero.

### 5.3 X Displacement Effect on the Particle Deposition

In all cases, the particle deposition increases with increasing $X$ displacement. This is true for all the convergent channel angles with various values of $\mathrm{S}, \mathrm{Q}$ and G parameters. The results are shown in figures 5.10 to 5.93 . Examples are shown on figure 5.13 . The deposition of particles for the case of half convergent channel angle equal to $2.5^{\circ}$ with $\mathrm{S}=0.01, \mathrm{G}=0.01$ and $\mathrm{Q}=10000$ is $4.77 \%, 17.12 \%$ and $41.61 \%$ when the X displacement is equal to $0.01,0.1$ and 1 . The same results are obtained for different Q parameters. Theoretically the deposition will reach $100 \%$ on or before the point where the upper and lower channel walls intersect.

### 5.4 Deposition due to the Inertia Force

The deposition due to inertia force alone can be obtained by setting $G$ and $Q$ equal to zero. Figure 5.11 shows the deposition as the $S$ parameter ranges from 100 to 0.01 for the half convergent channel angle equal to $2.5^{\circ}$

It is observed from these results that the particle deposition increases with increasing inertia force (i.e. for higher $S$ value). This is because a particle with higher inertia force will have a tendency to move straight, toward the exit of the convergent channel. This will make the particle hit the channel wall easily and will result in increasing deposition of the particle.

Consider the case where the convergent channel angle equals $2.5^{\circ}$. The deposition at X displacement equal to 1 is $4.3 \%, 4.1 \%, 2.8 \%, 0.5 \%$ and $0.1 \%$ for $S$ equals $100,10,1.0,0.1$ and 0.01 respectively. When X is equal to 18.3 (at the exit of the convergent channel), the deposition is
observed as $77.69 \%, 50.31 \% 9.3 \%, 0.5 \%$ and $0.1 \%$ for $S$ is equal to 100 , $10,1.0,0.1$, and 0.01 respectively. These show that the deposition increases with increasing inertia force. It is also observed that when the value of $S$ is equal to 0.01 , the deposition is only $0.1 \%$ at the exit of the channel. The same results can also be obtained when the half convergent channel angles are $5^{\circ}$ and $7.5^{\circ}$

### 5.5 Deposition due to Electrostatic Image Force

In this section, the results on particle deposition due to the electrostatic image force alone will be discussed. This can be obtained by setting both $S$ and $G$ equal to zero. Figure 5.12, 5.40 and 5.68 show the deposition of particles for the half convergent channel angles of $2.5^{\circ}, 5^{\circ}$ and $7.5^{\circ}$ respectively with Q ranging from 10000 to 0.00001 . The depositions at X displacement equal to $0.01,0.1,1$ and 5 are observed as $22.6 \%, 97.9 \%$ $100 \%$ and $100 \%$ for $\mathrm{Q}=100$. Deposition reaches $14.2 \%, 39.4 \%, 82.9 \%$ and $100 \%$ for $\mathrm{Q}=1 ; 7.9 \%, 19 \%, 42.2 \%$ and $73.9 \%$ for $\mathrm{Q}=0.1$, while for $\mathrm{Q}=0.0001$ the deposition is only $0.01 \%, 1.9 \%, 4.3 \%$ and $7.9 \%$. It is shown that for a fixed convergent channel angle, the fraction of particle deposition increases with increasing electrostatic image force. It is also shown on these figures that no deposition is obtained in the channel for various convergent channel angles when there is no image force (i.e. when $\mathrm{Q}=0$ ). This is true since the gravity force G is also set to zero in this case. Any particle having no image force and gravity force acting on it will simply flow through the channel without hitting the channel wall and result in zero particle deposition. As we mentioned earlier, theoretically, the deposition will still reach $100 \%$ as the extended channel walls intersect at a point.

### 5.6 Deposition due to Gravity Force

When both Q and S are set to zero, the deposition of particles increases with increasing gravity force. Effect on deposition due to the gravity force alone are shown on figures $5.10,5.38$ and 5.66 for $\theta=2.5^{\circ}, 5^{\circ}$ and $7.5^{\circ}$ respectively with $G$ varying from 100 to 0.01 . As shown on figure 5.11 , the deposition reaches $50 \%$ when X is equal to $0.01,0.1,1,10$ and 22.9 for $\mathrm{G}=100,10,1,0.1$ and 0.01 respectively. The deposition is increased to $100 \%$ as X increases to $0.02,0.2,2,20$ and 22.9 . The results reveal that the deposition increases with increasing gravity effect for a fixed $\theta$, when Q and S are both equal to zero.

When both image force and inertia force are set to zero, the governing equation (3-13) can then be reduced to first order differential equations.

These are:

$$
\begin{align*}
& \frac{d X}{d T}=U \\
& \frac{d Y}{d T}=V-G \tag{5.1}
\end{align*}
$$

After eliminating dT and substituting $\mathrm{U}=1 / \mathrm{H}, \mathrm{V}=-\mathrm{Y} \tan \theta / \mathrm{H}^{2}$, equations (5.1) become

$$
\begin{equation*}
\frac{d \bar{Y}}{d X}=-G \tag{5.2}
\end{equation*}
$$

where $\bar{Y}=Y / H$
The solution for equation (5.2) is $\mathrm{X}=\left(1+\mathrm{Y}_{0}\right) / \mathrm{G}$ with the fraction
of deposition obtained as GX/2. It is revealed that the deposition due to the gravity force alone in a convergent channel is independent of the convergent angle. Which also indicates that the curves for deposition due to the gravity effect alone for $\theta=2.5^{\circ}, 5^{\circ}$ and $7.5^{\circ}$ as shown on figure 5.11, 5.39 and 5.67 are all identical. All of these curves can be plotted as a single linear curve if the fraction of deposition is plotted against the dimensionless axial distance GX.

In this study, we have assumed that the gravity effect coincides with the y - axis. The gravity force will act on the particles in the negative y direction. The larger gravity force the particle experiences, the faster the particle will deposit on the lower channel wall and consequently increase the deposition of the particles. However, this is only for the case that the image force is neglected. If the image force is present in this case, the trajectories for the particles entering the channel at the upper half of the entrance will be determined by the effect of the image force from the upper channel wall(in the positive $y$-direction) and the gravity force(in the negative $y$-direction) as well. The deposition for this case will be discussed later.

### 5.7 Deposition due to the Combined Effects of Inertia Force, Electrostatic Image Force and Gravity Force

The purpose of this study is to investigate the particle deposition due to the combined effect of inertia force, electrostatic image force and the gravity force. Figure 5.13 to 5.37 depict the deposition of particles in a $2.5^{\circ}$ half convergent channel angle as $S$ and $G$ range from 100 to 0.01 for various $Q$ ranges from 10000 to 0.00001 . It is observed from the above figures that for a constant convergent channel angle and fixed inertia parameter $S$ and
gravity parameter G, the particle deposition increases with increasing electrostatic image force $\mathbf{Q}$. This is true for all cases of $S$ when $G$ is less than 0.1. One example is shown in figure 5.33 for $\theta=2.5^{\circ}, \mathrm{S}=100$ and $\mathrm{G}=0.01$. Deposition obtained for X displacement equal to 0.5 is $97 \%$ for Q equal to 10000 , and $71.2 \%, 36.7 \%, 18 \%$ and $9 \%$ for $Q$ equal to 1000 , 100, 10 and 1 respectively. As $X$ displacement increases to 5 , the deposition increases to $100 \%$ for $Q>1000$, and $99.6 \%, 81.6 \%$ and $48.6 \%$ for $Q$ equal to 100,10 and 1 . Another example is shown on figure 5.14 , for $\theta=2.5^{\circ}, \mathrm{S}=0.01$ and $\mathrm{G}=0.1$. It is also observed that at any X displacement, the deposition increases with increasing image force effect. These results show that for a fixed convergent channel angle, $S$ and $G$ (where $\mathbf{G}$ is less than 0.1 ), the deposition increases with increasing electrostatic image force, which is the same as we discussed earlier.

As the gravity effect becomes greater than 1 , the same phenomena is also observed. However, as X displacement increases, the deposition for particles with a small image force Q will experience a greater deposition than that of a particle with a larger Q effect. An example is shown on figure 5.35 , for $\theta=2.5^{\circ}, S=100$ and $G=10$. The deposition at $X$ equal to 0.5 is observed as $97.3 \%, 37.7 \%, 9.3 \%, 2.6 \%$ and $2.3 \%$ for Q equal to $10000,100,1,0.001$ and 0.00001 . However as X increases to 6 , the deposition is observed as $100 \%, 100 \%, 96.5 \%, 100 \%$ and $100 \%$ respectively. That is, as the X displacement increases to 6 , the deposition for smaller $Q(0.001$ and 0.00001$)$ is greater than that of larger $Q(1)$. The same result can also be obtained when $\theta=5^{\circ}$ and $7.5^{\circ}$. The deposition will be increased with increasing image force $Q$ for a fixed inertia $S$ and gravity G , where G is less than 0.1 .

It is also shown that for a fixed value of $S$, the deposition remains the
same for any value of $G$ when $Q$ is large ( $Q$ is greater than 1000). However, as Q becomes smaller ( 0.001 for example), the deposition of particles increases with increasing G.

For the case of small $\mathbf{Q}$, the increase in the gravity effect will increase the deposition of the particle to the lower channel wall resulting in a higher fraction of deposition. As the gravity effect becomes greater, more particles will deposit on the lower channel wall even faster, which in turn increases the particle deposition. When the particles move closer to the exit of the channel, due to the decreasing of the channel width the image effect on the particle will be increased. The image force from the upper channel wall will balance the gravity effect. This phenomena will take place even earlier in the entrance region of the channel if Q and G are of the same order of magnitude. Therefore, the particle will not deposit on the wall until it reaches further downstream, which consequently results in decreasing the deposition. The deposition of the particle will be delayed due to the balance of the image force and the gravity force downstream in the channel.

It can be observed from figures 5.13 to 5.37 for a fixed convergent channel angle, inertia force $S$ and image force Q , the deposition increases with increasing gravity force.

Consider the case of $\theta=2.5^{\circ}, \mathrm{S}=10, \mathrm{G}=0.01$ as shown in figure 5.28. The deposition at X-displacement equals 1 is $100 \%, 93.2 \%, 29.1 \%$, $5.6 \%$ and $4.2 \%$ for $Q=10000,100,1,0.001$ and 0.00001 respectively. As the gravity effect increases to 1 , the deposition becomes $100 \%, 93.2 \%$, $29.1 \%, 6.8 \%$ and $5.3 \%$ respectively. When $G$ increases to 100 , the deposition is $100 \%$ for all values of Q . It is shown that the deposition for various $G$ values remain the same for $Q=10000$ and 1000 , deposition
increases as $G$ increases for $Q=0.00001$, and the deposition increases for $Q=100$ when $G$ increases from 1 to 100 but deposition is not changed when $G$ increases from 0.01 to 1 . This data reveals that when Q is large the deposition will not be affected by the changing of gravity, however, when Q is small, the deposition increases with increasing gravity. In fact, numerical analysis has shown that for $Q / G$ greater or equal to five, the gravity effect can be neglected. Numerical results also show that we may neglect the charge effect when $\mathrm{Q} / \mathrm{G}$ is less than 0.001 .

There are three groups of curves shown in figure 5.94 for $\mathbf{G}=100$, 1.0 and 0.01 at the case of $\theta=2.5^{\circ}, \mathrm{S}=1$. The first curve of each group is $Q / G=100$, and then $Q / G=10,1.0,0.1$ and 0.01 for the other curves. The deposition for $\mathrm{Q} / \mathrm{G}>10$ were found to be the same as those presented in figure 5.23. For example, the first two curves in the first group (i.e. $Q=10000$ and 1000 for $G=100$ ) coincide with the curve in figure 5.23 for $\mathrm{Q}=10000$ and 1000. The first two curves in the second group (i.e. $Q=100$ and 10 for $G=1$ ) coincide with the curve in figure 5.23 for $\mathrm{Q}=100$ and 10 . The first two curves in the third group (i.e. $\mathrm{Q}=$ 1 and 0.1 for $G=0.01$ ) coincide with the curve in figure 5.23 for $\mathrm{Q}=1$ and 0.1 (i.e. in all cases, the gravity effect can be neglected when the ratio of Q/G is greater than 5 ). It is also observed from figure 5.27 that when $G=100$, all the deposition curve coincide with each other when Q is less than 0.1 . That is the deposition will not be affected by the values of an image force Q when $\mathrm{Q} / \mathrm{G}$ is less than 0.001 .

Figures 5.13, 5.23 and 5.33 show the deposition for various image forces $\mathbf{Q}$ at $\mathrm{G}=0.01$ for $S=0.01,1$ and 100 respectively. The deposition at $X$ displacement equal to 0.1 for $S=0.01$ is observed as $95.7 \%, 92 \%$, $35.4 \% 4 \%$ and $1 \%$ for $Q$ equal to $10000,100,10.001$ and 0.00001 . The
depositions are also found as $91.2 \%, 49 \%, 12.3 \% 1.5 \%$ and $0.6 \%$ for $\mathrm{S}=$ 1 ; and $49.2 \%, 12.4 \%, 3 \%, 0.6 \%$ and $0.4 \%$ for $S=100$. It is revealed from the above data that the deposition decreases with increasing $S$ for all Q. However, as the X displacement increases to 5, the depositions for the above $Q$ parameter are found to be $100 \%, 100 \%, 100 \%, 17.1 \%$ and $4.8 \%$ for $S=0.01 ; 100 \%, 100 \%, 98 \%, 17.9 \%$ and $5.9 \%$ for $S=1$; and $100 \%$, $\mathbf{9 9 . 8 \%}, 50.7 \%, 22.3 \%$ and $21.8 \%$ for $\mathrm{S}=100$. It is found that the deposition decreases with increasing $S$ for all the cases except that when $Q$ is less than 0.001 . This is actually the case we discussed in section 5.4 , where the gravity force and the image force are set to zero.

If $G$ increases to 0.1 , the results from figure $5.14,5.24$ and 5.34 , show the deposition for various image force $Q$ at $G=0.1$ for $S=0.01,1$ and 100 respectively. It is observed from these figures the deposition increases with decreasing inertia effect. The same phenomenon is also observed for the half convergent channel angle of $5^{\circ}$ and $7.5^{\circ}$.

If the image force is large, most of the deposition will occur at the entrance region of the convergent channel. However, by increasing the inertia effect on the particle which consequently increases the particle velocity in the X -direction, the particle will travel further downstream before it can be attracted by the channel wall due to the image force. In other words, increasing the inertia effect on particles for the case of a large image force effect actually helps the particle to prevent deposition on the channel wall at the entrance region, which results in the decrease of particle deposition. In contrast, in the case of small image forces and gravity effects, most depositions take place near the exit of the channel. The increase in inertia effect on the particle actually forces the particle to move straight forward in the X -direction and brings the particle closer to
the channel wall. As we discussed in section 5.1, the closer the particle is to the channel wall, the larger the image force the particle will experience. Because of the increase in the image force, the deposition will happen earlier, and therefore, increase the particle deposition. However, this phenomena only happens when both the $G$ and Q effects are very small. As we discussed in section 5.4, the deposition increases with increasing inertia force when both $\mathbf{G}$ and Q are equal to zero.

Since the dimensionless variables $S, G$ and $Q$ are inversely proportional to the viscous force, an increase in the viscosity of the fluid will decrease the $S, G$ and $Q$ parameters. The deposition due to the effect from the viscosity of the fluid can also be determined.

### 5.8 Deposition due to the Convergent Channel Angles

The main purpose of this section is to investigate the influence of the convergent channel angle to the deposition of the particles for various inertia, gravity and image parameters. The effect of the convergent channel angle on the fraction of deposition are shown in figures 5.10 to 5.93 for half convergent channel angles equal to $2.5^{\circ}, 5.0^{\circ}$ and $7.5^{\circ}$

It is observed from these figures, in general, for a fixed inertia and gravity parameter, the deposition decreases with an increase in the convergent channel angle for a very small axial distance. For example, when the half convergent channel angle is equal to $2.5^{\circ}$ and with $\mathrm{S}=0.1$, $G=0.1$, and the $X$ displacements are equal to $0.01,0.1$ and 1 , the deposition for $\mathrm{Q}=100$ are observed as $15.1 \%, 77.4 \%$ and $100 \%$ respectively. The deposition for $Q$ equal to 1 and 0.01 are observed as $5.3 \%, 23.9 \%$ and $85.2 \%$, and $1.3 \%, 5.9 \%$ and $50.2 \%$ respectively. As the half convergent channel angle increases to $5^{\circ}$ with the same value of $S$ and

G , the deposition for $\mathrm{Q}=100$ are observed as $10.7 \%, 64.6 \%$ and $100 \%$ respectively. The depositions for $Q$ equal to 1 and 0.01 are observed as $4.9 \%, 22.7 \%$ and $85.2 \%$, and $1.3 \%, 6 \%$ and $50.6 \%$ respectively. It is shown that as the half convergent channel angle is changed from $2.5^{\circ}$ to $5^{\circ}$, the deposition decreases for $\mathrm{Q}=100$ and $\mathrm{Q}=1$. However, the deposition is observed to decrease for $\mathrm{Q}=0.01$ at $\mathrm{X}=0.01$ and then increase after $\mathrm{X}=0.1$.

As the convergent channel angle increases, the magnitude of the fluid moving away from the wall (or - V ) increases as well. This tends to push the particles away from the channel wall. Consequently resulting in the decreasing fraction of deposition. Meanwhile, an increase in the convergent angle increases the image force in the X -direction. The attraction from this image force will accelerate the particle moving in the X-direction. The particle may travel further downstream before it deposits on the wall.

As the X distance increases, the channel wall moves closer to the center line of the convergent channel, resulting in increasing image forces (especially in the Y-direction as shown in table 5.1 to 5.3 ) causing the particles to deposit on the wall. This is also shown in the above example. As the X distance increases to 1 , the deposition for $\mathrm{Q}=0.01$ increases from $50.2 \%$ to $50.6 \%$ as the half convergent channel angle increases from $2.5^{\circ}$ to $5^{\circ}$.

It is then observed from these figures that, in general, for a fixed inertia parameter and gravity force parameter, the particle deposition is higher for smaller half convergent angles. This is true for Q , which is greater or equal to 10 , when $S$ is less or equal to 0.01 , and for $Q$, which is greater or equal to 1000 , when $S$ is equal to 100 . For all $Q$ which are
less than the above said range, for example $\mathrm{Q}=10$ when $\mathrm{S}=100$, the higher deposition for small half convergent angle is only valid at the entrance region.

### 5.9 Deposition due to the Effect from Particle Size

In this section, the effect of particle size on the deposition of the particle will be discussed. The increase in the particle size will increase the mass of the particle. That means that the inertia force as well as the gravity force parameter will be increased by increasing the particle size. The electrostatic charge on a particle may be considered as proportional to the surface area of the particle. Increasing the particle size will actually increase the surface area of the particle. That is the electrostatic image force will be increased by increasing the particle size.

Consider the case of half convergent angle of $5^{\circ}$, with $\mathrm{S}=0.1, \mathrm{G}=$ 0.1 and $\mathrm{Q}=1$ as shown in Figure 5.47. It is found that the deposition for $\mathrm{X}=0.01,0.1$ and 1 is $4.6 \%, 22.6 \%$ and $76.6 \%$ respectively. If we increase the particle size (for example, the diameter of the particle) by a factor of 10 , then both the $S$ and $G$ will increase by the same factor of 100. When we increase the size of a spherical particle by a factor of 10 , its Q parameter will increase by a factor of 1000 . It is shown in figure 5.59 for $S=10, G=10$ and $Q=1000$, the deposition for $X=0.01,0.1$ and 1 is $7.4 \%, 43 \%$ and $99.7 \%$ respectively. Increasing the particle size will both increase the image force deposition, and the gravity deposition. The same phenomena can also be observed for the cases of half convergent angles of $7.5^{\circ}$ and $2.5^{\circ}$. It is then concluded that increasing the particle size will increase the particle deposition.

As an example, consider a gas fluid with particle density 10 E5
particles $/ \mathrm{cm}^{3}$ and a charge electron density of 1 electron per $1.18 \mathrm{E}-10$ $\mathrm{cm}^{2}$ (i.e. 29 electrons on a $0.165 \mu \mathrm{~m}$ radius of particle as used in [25] and [29] ).

Assume a fluidic device with air as the fluid phase with the velocity of $30 \mathrm{~cm} / \mathrm{s}$ and $\mathrm{h}_{0}$ equal to 0.1 cm . Let the pressure of the air fluid be equal to 1 atm , and the temperature equal to $25^{\circ} \mathrm{C}$. The particle specific gravity is assumed to be 1 . and $\mu=1.8 \mathrm{E}-4$ dyne $\mathrm{sec} / \mathrm{cm}^{2}, \varepsilon_{0}=8.85434 \mathrm{E}-21$ coulomb ${ }^{2} /$ dyne $\mathrm{cm}^{2}$.

First consider the particle size of $\mathrm{a}=5 \mu \mathrm{~m}$, then

$$
\begin{aligned}
& \mathrm{f}=6 \pi \mu \mathrm{a}=1.69646 \times 10^{-6} \mathrm{dyne} \mathrm{sec} / \mathrm{cm} \\
& \mathrm{q}=26629 \text { electrons } \\
& \mathrm{m}=5.236 \times 10^{-10} \mathrm{gram} \\
& \mathrm{~S}=\frac{\mathrm{m} \mathrm{u}}{\mathrm{u}_{0}} \\
& \mathrm{~h}_{0} \mathrm{f}
\end{aligned}=0.093 \mathrm{C}=\frac{\mathrm{q}^{2}}{4 \pi \varepsilon_{0} \mathrm{~h}_{0}^{2} \mathrm{fu}_{0}}=0.00032 \mathrm{Q}=\frac{\mathrm{mg}}{\mathrm{f} \mathrm{u}_{0}}=0.01 \quad .
$$

As the particle size increases to $50 \mu \mathrm{~m}$, then

$$
\begin{aligned}
& \mathrm{f} \quad=6 \pi \mu \mathrm{a}=1.69646 \times 10^{-5} \mathrm{dyne} \mathrm{sec} / \mathrm{cm} \\
& \mathrm{q}=2662900 \text { electrons } \\
& \mathrm{m}=5.236 \times 10^{-7} \text { gram }
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{S}=\frac{\mathrm{m} \mathrm{u}_{0}}{\mathrm{~h}_{0} \mathrm{f}}=9.3 \\
& \mathrm{Q}=\frac{\mathrm{q}^{2}}{4 \pi \varepsilon_{0} \mathrm{~h}_{0}{ }^{2} \mathrm{f} \mathrm{u}_{0}}=0.32 \\
& \mathrm{G}=\frac{\mathrm{mg}}{\mathrm{f} \mathrm{u}_{0}}=1
\end{aligned}
$$

The above example has shown that in practical application of a convergent channel flow, the gravity effect can not be neglected if the particle size is of the order of one micro-meter. The electrostatic image force is also an important factor on deposition when the channel width is very small (less than 1 cm .).

## CHAPTER 6

## CONCLUSION

When a particle enters a parallel-plate channel with a uniform velocity profile, the fluid velocity in the x-direction remains constant throughout the flow field and there is no velocity component in the y-direction(i.e. $\mathbf{u}=$ constant and $\mathrm{v}=0$ ). This is not the case when a particle enters a convergent channel. The fluid velocity in the x -direction continues to increase along the axial distance and the fluid near the wall moves toward the center-line of the channel. These fluid motions tend to reduce the deposition of particles.

Based on the geometrical consideration the convergent angle brings the channel wall closer to the channel center-line which in turn promotes the particle deposition.

The gravity force tends to accelerate the particle to move downward and increase the deposition on the bottom wall of the channel. This force is, therefore, expected to increase the deposition.

Higher particle mass increases the $S$ parameter. The high inertia force of the particle tends to slow down the directional change and keep the particle moving in a straight line (or horizontal direction). This will make the particle less affected by the motion of the fluid.

The image force has two components, $\mathrm{F}_{\mathrm{x}}$ and $\mathrm{F}_{\mathrm{y}}$. With convergent angle there is a $F_{x}$ force which tends to increase $u_{p}$ and, thus, increase the distance traveled by the particle before deposition. The vertical image force, $\mathrm{F}_{\mathrm{y}}$, is increased by the convergent wall getting closer to the centerline. High $\mathrm{F}_{\mathrm{y}}$ will increase the deposition.

The forces mentioned above are involved in this analysis and based on the discussions in chapter 5, certain conclusions are drawn as follows:
(1) The image force equation for a convergent channel was derived and shown in eq. (3-8). The equation is reduces to that given by $\mathrm{Yu}[22]$, for the case when the convergent channel angle is zero. (parallel plate channel)
(2) The ratio of $F_{y} / F_{x}$ is greater than 1 when $Y$ is greater than 0.1 . This value increases to $1 / \tan \theta$ as the position of the particle becomes closer to the channel wall. For example, when $\theta=2.5^{\circ}$, the ratio of $\mathrm{F}_{y} / \mathrm{F}_{\mathrm{x}}$ approaches to $1 / \tan (2.5)=22.9$ when the particle position, Y , is greater than 0.9 .
(3) In the absence of the electrostatic image force and gravity force, the deposition will increase with increasing inertia force $S$. It is also found that only $0.1 \%$ of deposition was observed at the exit of the convergent channel for $S=0.01$. That is, the inertia effect can be neglected if $S$ is less than 0.01 .
(4) Deposition increases with decreasing inertia force for a fixed convergent channel angle with constant electrostatic image force and gravity force. However, as the electrostatic image force becomes smaller than 0.001 , and when the gravity force is smaller than 0.01 , the deposition increases with increasing inertia force. If both the electrostatic image force and the gravity force are set to zero, the conclusion is the same as (3).
(5) For any fixed convergent channel angle with constant inertia force and electrostatic image force, the deposition increases with increasing gravity force. This is true for all cases when the ratio of $Q / G$ is less than 5 .
(6) For any fixed convergent channel angle with constant gravity force (less than 1) and constant inertia force, the deposition increases with increasing electrostatic image force. When the gravity force is greater than 10, the deposition increases at the entrance region as Q is increased, and then decreases as the X distance increases.
(7) When the ratio of $Q / G$ is greater than 5 , the deposition is dominated by the electrostatic image force alone and the gravity effects can be neglected.
(8) When the ratio of $\mathrm{Q} / \mathrm{G}$ is less than 0.001 , the image force can be neglected.
(9) Deposition increases at the entrance of the channel if the convergent channel angle decreases. However, as the axial distance increases, the deposition was found to be greater for a greater convergent channel angle. As $\mathbf{Q}$ becomes greater than 10 , the deposition increases with decreasing convergent channel angle.
(10) The deposition increases with increasing particle size.

## CHAPTER 7

## RECOMMENDATIONS

The deposition of solid particles of uniform, steady and incompressible flow in a convergent channel due to the effects of inertia, viscous, electrostatic image and gravity was investigated. The following should be included in future investigations.
(1) Flow pattern other than uniform flow should be investigated.
(2) The deposition of particles in a divergent channel subject to the same effects should be studied.
(3) Although the gravity effect was included in this study, the gravity is set in the negative $y$-direction. Future study should consider different gravity directions (for example, an inclined convergent channel).

## APPENDIX A

## DERIVATION OF IMAGE CIRCLE

We derive the formula for the variables for $\alpha_{n}, \alpha_{n}, q_{n}, q_{n}, \beta_{n}, \beta_{n}$, in term of $\alpha_{n}, \alpha_{n}$, as follows.

Let o be the center of the image circle and $p$ represents the point of the particle charge. Then from figure (2),

$$
\alpha_{1}=\theta-\alpha_{0} \quad \text { and } \quad \alpha_{r}=\theta+\alpha_{0}
$$

Where $\theta$ is defined as the half angle of the convergent channel . Again, from figure(2), we have

$$
2 \alpha_{2}=\angle \mathrm{PO} 2=\angle \mathrm{PO} 1+\angle 102=2 \alpha_{1}+\angle 102
$$

By reflecting across the upper wall, we have

$$
\angle 1 \mathrm{O} 2=\angle \mathrm{PO} 1^{\prime}=2 \alpha_{1^{\prime}}
$$

Therefore

$$
\begin{aligned}
2 \alpha_{2} & =2 \alpha_{1}+2 \alpha_{1} \\
\alpha_{2} & =\alpha_{1}+\alpha_{r}
\end{aligned}
$$

Again from figure (2), we have

$$
\alpha_{1}+\alpha_{r^{\prime}}=2 \theta
$$

Therefore

$$
\alpha_{2}=2 \theta
$$

Again, from the diagram

$$
2 \alpha_{2}=\angle \mathrm{PO} 2+\angle 2 \mathrm{O} 3=2 \alpha_{2}+\angle 2 \mathrm{O} 3
$$

By reflecting across the upper wall, we have

$$
\angle 2 \mathrm{O} 3=\angle 1^{\prime} \mathrm{O} 2 \prime
$$

Again by reflecting across the lower wall,

$$
\angle 1^{\prime} \mathrm{O} 2^{\prime}=\angle \mathrm{PO1}=2 \alpha_{1}
$$

and thus

$$
\begin{gathered}
2 \alpha_{3}=2 \alpha_{2}+2 \alpha_{1} \\
\alpha_{3}=\alpha_{2}+\alpha_{1}
\end{gathered}
$$

Substituting $\alpha_{2}=2 \theta$ and $\alpha_{1}=\theta-\alpha_{0}$ into the above equation, we have

$$
\alpha_{3}=3 \theta-\alpha_{0}
$$

Similarly, We can derive

$$
\begin{aligned}
\alpha_{4}=\alpha_{3}+\alpha_{2} & =3 \theta-\alpha_{0}+2 \alpha_{r} \\
& =3 \theta-\alpha_{0}+\theta+\alpha_{0} \\
& =4 \theta
\end{aligned}
$$

The same procedure can also be done on the image pair upon the lower wall.

$$
\begin{aligned}
& \alpha_{2^{\prime}}=2 \theta \\
& \alpha_{3^{\prime}}=3 \theta-\alpha_{0} \\
& \alpha_{4^{\prime}}=4 \theta \quad \ldots \text { and so on }
\end{aligned}
$$

In general, for any number of $\mathbf{n}, \mathbf{n}>1$, we can derive $\alpha_{\mathrm{n}}$ as

$$
\alpha_{\mathrm{n}}=\alpha_{1}+\alpha_{(\mathrm{n}-1)}=\mathrm{n} \theta-\frac{1}{2}\left[1-(-1)^{\mathrm{n}}\right] \alpha_{0}
$$

and

$$
\alpha_{\mathrm{n}}=\alpha_{1}+\alpha_{(\mathrm{n}-1)^{\prime}}=\mathrm{n} \theta-\frac{1}{2}\left[1-(-1)^{\mathrm{n}}\right] \alpha_{0}
$$

## APPENDIX B

## THE RUNGE-KUTTA METHOD

The family of Runge-Kutta solutions has various order of accuracy, but they are all the same in that the differential equation has its solution extended forward from known conditions by an increment of the independent variable without using information outside of this increment. It is particularly suitable in the case when the computation of higher derivatives is complicated. The method used in this analysis is of order four. The calculations for the first increment, for example, are exactly the same as for any other increment.

$$
\text { Let } \begin{array}{r}
\quad y^{\prime}=f(x, y) \\
y\left(x_{0}\right)=y_{0}
\end{array}
$$

denote any first order differential equation connecting the variables x and $y$, and let $h$ denote any increment $\Delta x$ in the independent variable $x$. Then if the initial values of the variables are $x_{0}$ and $y_{0}$, the first increment in $y$ is computed from the formulas

$$
\begin{aligned}
& \mathrm{k}_{1}=\mathrm{f}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right) \mathrm{h} \\
& \mathrm{k}_{2}=\mathrm{f}\left(\mathrm{x}_{0}+\mathrm{h} / 2, \mathrm{y}_{0}+\mathrm{k}_{1} / 2\right) \mathrm{h} \\
& \mathrm{k}_{3}=\mathrm{f}\left(\mathrm{x}_{0}+\mathrm{h} / 2, \mathrm{y}_{0}+\mathrm{k}_{2} / 2\right) \mathrm{h} \\
& \mathrm{k}_{4}=\mathrm{f}\left(\mathrm{x}_{0}+\mathrm{h}, \mathrm{y}_{0}+\mathrm{k}_{3}\right) \mathrm{h} \\
& \Delta \mathrm{y}=\left(\mathrm{k}_{1}+2 \mathrm{k}_{2}+2 \mathrm{k}_{3}+\mathrm{k}_{4}\right) / 6
\end{aligned}
$$

taken in the order given. Then

$$
\begin{aligned}
& \mathrm{x}_{1}=\mathrm{x}_{0}+\mathrm{h} \\
& \mathrm{y}_{1}=\mathrm{y}_{0}+\Delta \mathrm{y}
\end{aligned}
$$

The increment in $y$ for the second interval is computed in a similar manner by mean of the formula

$$
\begin{aligned}
& \mathrm{k}_{1}=\mathrm{f}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right) \mathrm{h} \\
& \mathrm{k}_{2}=\mathrm{f}\left(\mathrm{x}_{1}+\mathrm{h} / 2, \mathrm{y}_{1}+\mathrm{k}_{1} / 2\right) \mathrm{h} \\
& \mathrm{k}_{3}=\mathrm{f}\left(\mathrm{x}_{1}+\mathrm{h} / 2, \mathrm{y}_{1}+\mathrm{k}_{2} / 2\right) \mathrm{h} \\
& \mathrm{k}_{4}=\mathrm{f}\left(\mathrm{x}_{1}+\mathrm{h}, \mathrm{y}_{1}+\mathrm{k}_{3}\right) \mathrm{h} \\
& \Delta \mathrm{y}=\left(\mathrm{k}_{1}+2 \mathrm{k}_{2}+2 \mathrm{k}_{3}+\mathrm{k}_{4}\right) / 6
\end{aligned}
$$

and so on for the succeeding intervals.
It will be noticed that the only change in the formula for the different intervals is in the values of $x$ and $y$ to be substituted. Thus, to find $\Delta y$ in the $n^{\text {th }}$ interval, one should have to substitute $x_{(n-1)}, y_{(n-1)}$ in the expressions for $k_{1}, k_{2}$, etc.

The Runge-Kutta solution can be applied to the initial value problem of our present particle deposition analysis. First, for our simultaneous governing equations

$$
\begin{aligned}
& x^{\prime \prime}=f\left(x^{\prime}, x, t\right) \\
& y^{\prime \prime}=g\left(y^{\prime}, y, t\right)
\end{aligned}
$$

we set $x^{\prime}=A, y^{\prime}=B$ and then obtain the following system of simultaneous first-order equations

$$
\begin{aligned}
\mathrm{x}^{\prime} & =\mathrm{A}(\mathrm{x}, \mathrm{t}) \\
\mathrm{A}^{\prime} & =\mathrm{f}(\mathrm{~A}, \mathrm{x}, \mathrm{t}) \\
\mathrm{y}^{\prime} & =\mathrm{B}(\mathrm{y}, \mathrm{t}) \\
\mathrm{B}^{\prime} & =\mathrm{g}(\mathrm{~B}, \mathrm{y}, \mathrm{t})
\end{aligned}
$$

where $x$ and $y$ are functions of $t$
Then to integrate them, we compute the increments in x and y for the first interval by mean of the formulas

$$
\begin{aligned}
& \mathrm{k}_{1}=\mathrm{A}\left(\mathrm{x}_{0}, \mathrm{t}\right) \mathrm{h} \\
& \mathrm{p}_{1}=\mathrm{f}\left(\mathrm{~A}_{0}, \mathrm{x}_{0}, \mathrm{t}\right) \mathrm{h} \\
& \mathrm{k}_{2}=\mathrm{A}\left(\mathrm{x}_{0}+\mathrm{p}_{1} / 2, \mathrm{t}+\mathrm{h} / 2\right) \mathrm{h} \\
& \mathrm{p}_{2}=\mathrm{f}\left(\mathrm{~A}+\mathrm{p}_{1} / 2, \mathrm{x}_{0}+\mathrm{k}_{1} / 2, \mathrm{t}+\mathrm{h} / 2\right) \mathrm{h} \\
& \mathrm{k}_{3}=\mathrm{A}\left(\mathrm{x}_{0}+\mathrm{p}_{2} / 2, \mathrm{t}+\mathrm{h} / 2\right) \mathrm{h} \\
& \mathrm{p}_{3}=\mathrm{f}\left(\mathrm{~A}+\mathrm{p}_{2} / 2, \mathrm{x}_{0}+\mathrm{k}_{2} / 2, \mathrm{t}+\mathrm{h} / 2\right) \mathrm{h} \\
& \mathrm{k}_{4}=\mathrm{A}\left(\mathrm{x}_{0}+\mathrm{p}_{3}, \mathrm{t}+\mathrm{h}\right) \mathrm{h} \\
& \mathrm{p}_{4}=\mathrm{f}\left(\mathrm{~A}+\mathrm{p}_{3}, \mathrm{x}_{0}+\mathrm{k}_{3}, \mathrm{t}+\mathrm{h}\right) \mathrm{h} \\
& \Delta \mathrm{x}=\left(\mathrm{k}_{1}+2 \mathrm{k}_{2}+2 \mathrm{k}_{3}+\mathrm{k}_{4}\right) / 6 \\
& \Delta \mathrm{~A}=\left(\mathrm{p}_{1}+2 \mathrm{p}_{2}+2 \mathrm{p}_{3}+\mathrm{p}_{4}\right) / 6
\end{aligned}
$$

and

$$
\begin{aligned}
& m_{1}=B\left(y_{0}, t\right) h \\
& q_{1}=g\left(B_{0}, y_{0}, t\right) h \\
& m_{2}=B\left(y_{0}+q_{1} / 2, t+h / 2\right) h \\
& q_{2}=g\left(B+q_{1} / 2, y_{0}+m_{1} / 2, t+h / 2\right) h \\
& m_{3}=B\left(y_{0}+q_{2} / 2, t+h / 2\right) h \\
& q_{3}=g\left(B+q_{2} / 2, y_{0}+m_{2} / 2, t+h / 2\right) h \\
& m_{4}=B\left(y_{0}+q_{3}, t+h\right) h \\
& q_{4}=g\left(B+q_{3}, y_{0}+m_{3}, t+h\right) h \\
& \Delta y=\left(m_{1}+2 m_{2}+2 m_{3}+m_{4}\right) / 6 \\
& \Delta B=\left(q_{1}+2 q_{2}+2 q_{3}+q_{4}\right) / 6
\end{aligned}
$$

The increments for the succeeding intervals are computed in exactly the same way except that $x_{0}, y_{0}, A_{0}, B_{0}$ and $t_{0}$ are replaced by $x_{1}, y_{1}, A_{1}, B_{1}$ and $t_{1}$. as the computation proceeds, until $x_{n}, y_{n}, A_{n}, B_{n}$ and $t_{n}$ are obtained.

## APPENDIX C <br> TABLES AND FIGURES FOR CHAPTERS 3 AND 5

Table 5.1 Image force for particle at various vertical positions for $\boldsymbol{\theta}=\mathbf{7 . 5 ^ { \circ }}$

| $\mathrm{Y}_{0}$ | $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathrm{y}}$ |
| :---: | :---: | :---: |
| 0.010 | 0.04595711 | 0.01063478 |
| 0.050 | 0.04646312 | 0.05341781 |
| 0.100 | 0.04806962 | 0.10838280 |
| 0.200 | 0.05489936 | 0.22987370 |
| 0.300 | 0.06791493 | 0.38199120 |
| 0.400 | 0.09045012 | 0.59411260 |
| 0.450 | 0.10730770 | 0.73890830 |
| 0.500 | 0.12985740 | 0.92481070 |
| 0.550 | 0.16067090 | 1.17156600 |
| 0.600 | 0.20403440 | 1.51190400 |
| 0.650 | 0.26751130 | 2.00342200 |
| 0.700 | 0.36547130 | 2.75539200 |
| 0.750 | 0.52800170 | 3.99645500 |
| 0.800 | 0.82713880 | 6.27387000 |
| 0.850 | 1.47312600 | 11.1846800 |
| 0.900 | 3.31800700 | 25.2009900 |
| 0.940 | 9.22022000 | 70.0349000 |
| 0.980 | 82.9905200 | 630.386200 |
| 0.990 | 331.969800 | 2521.60300 |
| 0.995 | 1327.90000 | 10086.5700 |
| 0.999 | 33200.6300 | 252188.000 |

Table 5.2 Image force for particle at various vertical positions for $\boldsymbol{\theta}=\mathbf{5}^{\circ}$

| $Y_{0}$ | $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathrm{y}}$ |
| :---: | :---: | :---: |
| 0.010 | 0.03042571 | 0.01057068 |
| 0.050 | 0.03075995 | 0.05309655 |
| 0.100 | 0.03182114 | 0.10773370 |
| 0.150 | 0.03364766 | 0.16557030 |
| 0.200 | 0.03633308 | 0.22851970 |
| 0.250 | 0.04002298 | 0.29892420 |
| 0.300 | 0.04493328 | 0.37980030 |
| 0.350 | 0.05137990 | 0.47520620 |
| 0.400 | 0.05982743 | 0.59081970 |
| 0.450 | 0.07097111 | 0.73489000 |
| 0.500 | 0.08587927 | 0.91987940 |
| 0.550 | 0.10625320 | 1.16544200 |
| 0.600 | 0.13492750 | 1.50415000 |
| 0.650 | 0.17690500 | 1.99333000 |
| 0.700 | 0.24168970 | 2.74173500 |
| 0.750 | 0.34918070 | 3.97691100 |
| 0.800 | 0.54702130 | 6.24351700 |
| 0.850 | 0.97426030 | 11.1309700 |
| 0.900 | 2.19441700 | 25.0804900 |
| 0.940 | 6.09795200 | 69.7002900 |
| 0.980 | 54.8875900 | 627.378600 |
| 0.990 | 219.546400 | 2509.47000 |
| 0.995 | 878.150100 | 10037.4700 |
| 0.999 | 21948.6500 | 250878.500 |

Table 5.3 Image force for particle at various vertical positions for $\boldsymbol{\theta}=2 . \mathbf{5}^{\circ}$

| $\mathrm{Y}_{0}$ | $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathrm{y}}$ |
| :---: | :---: | :---: |
| 0.010 | 0.01514972 | 0.01053253 |
| 0.050 | 0.01531592 | 0.05290548 |
| 0.100 | 0.01584359 | 0.10734760 |
| 0.150 | 0.01675189 | 0.16498110 |
| 0.200 | 0.01808734 | 0.22771420 |
| 0.250 | 0.01992243 | 0.29788290 |
| 0.300 | 0.02236463 | 0.37849600 |
| 0.350 | 0.02557116 | 0.47360030 |
| 0.400 | 0.02977330 | 0.58885850 |
| 0.450 | 0.03531699 | 0.73249640 |
| 0.500 | 0.04273396 | 0.91694130 |
| 0.550 | 0.05287089 | 1.16179200 |
| 0.600 | 0.06713834 | 1.49953000 |
| 0.650 | 0.08802597 | 1.98731600 |
| 0.700 | 0.12026330 | 2.73359800 |
| 0.750 | 0.17375260 | 3.96527200 |
| 0.800 | 0.27220250 | 6.22543700 |
| 0.850 | 0.48480650 | 11.0989800 |
| 0.900 | 1.09198500 | 25.0087400 |
| 0.940 | 3.03448600 | 69.5015000 |
| 0.980 | 27.3137000 | 625.597200 |
| 0.990 | 109.255700 | 2502.40900 |
| 0.995 | 437.030200 | 10009.8000 |
| 0.999 | 10926.7100 | 250267.000 |

Table 5.4 Image force for particle at various vertical positions for $\boldsymbol{\theta}=\boldsymbol{0}^{\circ}$

| $Y_{0}$ | $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathrm{y}}$ |
| :---: | :---: | :---: |
| 0.010 | 0 | 0.01044995 |
| 0.050 | 0 | 0.05276921 |
| 0.100 | 0 | 0.10714310 |
| 0.150 | 0 | 0.16470580 |
| 0.200 | 0 | 0.22736350 |
| 0.250 | 0 | 0.29745050 |
| 0.300 | 0 | 0.37797300 |
| 0.350 | 0 | 0.47297390 |
| 0.400 | 0 | 0.58811050 |
| 0.450 | 0 | 0.73160150 |
| 0.500 | 0 | 0.91586240 |
| 0.550 | 0 | 1.16047300 |
| 0.600 | 0 | 1.49788400 |
| 0.650 | 0 | 1.98520500 |
| 0.700 | 0 | 2.73077700 |
| 0.750 | 0 | 3.96128200 |
| 0.800 | 0 | 6.21930300 |
| 0.850 | 0 | 11.0882400 |
| 0.900 | 0 | 24.9848100 |
| 0.940 | 0 | 69.4354800 |
| 0.980 | 0 | 624.998500 |
| 0.990 | 0 | 2500.00200 |
| 0.995 | 0 | 9999.89600 |
| 0.999 | 0 | 249985.600 |



Figure 3.1 Coordinate System for a Parallel-plate Channel


Figure 3.2 Coordinate System for a Convergent Channel


Figure 3.3 Image Pairs for a Convergent Channel with $2 \theta=90^{\circ}$


Figure 3.4 Image Pairs for a Convergent Channel with $2 \theta=60^{\circ}$


Figure 3.5 Image Pairs for a Convergent Channel with $2 \theta=45^{\circ}$


Figure 3.6 Image Pairs for a Convergent Channel with $2 \theta=42^{\circ}$


Figure 3.7 Image Pairs for a Convergent Channel with $2 \theta=30^{\circ}$


Figure 3.8 Image Pairs for a Convergent Channel with $2 \theta=15^{\circ}$


Figure 3.9 Image Pairs for a Convergent Channel with $2 \theta=10^{\circ}$


Figure 3.10 Image Force for a convergent channel with $2 \theta=60^{\circ}$

Figure 5.1 Particle Stopping Distance for $\theta=2.5^{\circ}, S=100, G=0.01$



Figure 5.4 Particle Stopping Distance for $\theta=5^{\circ}, S=100, G=0.01$



Figure 5.7 Particle Stopping Distance for $\theta=7.5^{\circ}, \mathrm{S}=100, \mathrm{G}=0.01$






$=0.1$









Figure 5.23 Effect on Deposition for Various Image Force Parameters at $\theta=2.5^{\circ}, \mathrm{S}=1, \mathrm{G}=0.01$












Figure 5.36 Effect on Deposition for Various Image Force Parameters at $\theta=2.5^{\circ}, S=100, G=10$

Figure 5.37 Effect on Deposition for Various Image Force Parameters at $\theta=2.5^{\circ}, \mathrm{S}=100, \mathrm{G}=100$


Figure 5.39 Effect on Deposition for Various Inertia Parameters at $\theta=5.0^{\circ}, \mathrm{G}=0, \mathrm{Q}=0$


Figure 5.41 Effect on Deposition for Various Image Force Parameters at $\theta=5.0^{\circ}, \mathrm{S}=0.01, \mathrm{G}=0.01$

Figure 5.42 Effect on Deposition for Various Image Force Parameters at $\theta=5.0^{\circ}, \mathrm{S}=0.01, \mathrm{G}=0.1$















































Figure 5.92 Effect on Deposition for Various Image Force Parameters at $\theta=7.5^{\circ}, \mathrm{S}=100, \mathrm{G}=10$


Figure 5.94 Comparison of Deposition for Various Gravity Parameters at $\theta=2.5^{\circ}, S=1$

Figure 5.95 Comparison of Deposition for Various Gravity Parameters at $\theta=5.0^{\circ}, S=1$


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