ABSTRACT

Simulation Study for the Effect of Dependencies in Queueing System

by
Yitao Bai

Few theoretical results have been obtained in the literature for the effects of dependencies between random variables on the performance of queueing systems. This thesis aims at investigating this issue via simulation. Several dependencies are studied in detail, including dependencies between interarrival times, between interarrival time and service time, between service times and dependencies between different stages in networks of queues. We define several classes of dependent random variables and study their correlation coefficients, then we apply them to single and multiple station service systems. Comparisons with the independent case, for which the explicit form solution are available, are made and characterized by figures. The main contribution of this thesis is that it disproves the monotonicity properties of effect of dependencies on system performance in both single and multiple service stations queueing systems. These results may be helpful in evaluating the performance of both telecommunication and manufacturing systems.

SIMULATION STUDY FOR THE EFFECT OF DEPENDENCIES IN QUEUEING SYSTEMS

by
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APPROVAL PAGE

Simulation Study for the Effect of Dependencies in Queueing Systems

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CHAPTER 1

INTRODUCTION AND PRELIMINARIES

This thesis is motivated by the need to provide a guideline for the theoretical work on dependency effect. Normally, when dealing with queueing models, we assume that interarrival process and service process are independent. Study on the effect of dependency seems to be so difficult that little work has been done in this area. In this case, computer simulation on these models seems to be the first step. The simulation results are relatively accurater. Thus they helps pointing out the direction of research in this subject.

Here we use computer to generate random numbers. By using inverse transform method, we can generate variables of various kinds of distribution. In this way, arrival and service process can be simulated under any dependency condition.

This thesis presents the simulation results of different dependency conditions on single node, single server queues and 2-server tandem queues.

1.1 SIMULATION LANGUAGES

Since most realistic simulations must be done by computer because of the number of calculations required, the analyst should choose a computer programming language of communicating the essence of the model to the computer. Common computer languages such as FORTRAN, C, COBOL, BASIC or Assembler can be used to write simulation model. This is sometimes done when the simulation model is not complicated, only a small computer system available, or the analyst is thoroughly conversant with the language. Nevertheless, a analyst who wants to minimize the portion of model construction which have been expressly design for simulation. These languages imbed in the compiler contain function necessary for every simulation: establishing and updating a time clock, generating random occurrences and initializing, incrementing and printing system statistics such as utilization of service facilities and waiting time. Dozens of simulation languages currently exist: GPSS (General-purpose System Simulation), SIMAN and SIMSCRIPT for discrete event simulation, CSMP (Continuous Simulation Modeling Package) and DYNAMO continuous simulation, and GASP and SLAM for hybrid simulation.

Simulation analysts are increasingly attempting to interface formal statistical routines with simulation models. The structure of GPSS, SIMAN and SIMSCRIPT permit the user to call a FORTRAN, C or Assembler subprogram for its purpose, although in practice, discovering the correct way in which to implement this feature through the operating system of a particular computer can be tedious.

The simulation models constructed in this paper are relatively simple after analytical effort has been made. Simulation languages used in dependency effect analysis must be flexible so that different structure of dependency can be simulated without much difficulty. In this case, common language such as FORTRAN, C, COBOL can be used for simulation purpose. Also, because C is a update common language and has more effective functions, the simulation programs used for analyzing dependency effect are written by C language.

1.2 THE INVERSE TRANSFORMATION METHOD

Simulating G/G/1 queueing model requires the simulation model can simulate virtually any kind of distribution for either customer interarrival time or service time . A general method - called the Inverse Transformation Method - is used throughout this paper to simulate random variables with a continuous distribution. This method is based on the following proposition:

Proposition 1.1 Let U be a uniform (0,1) random variable. For any continuous distribution function F if we define the random variable X by

$$X = F^{-1}(U)$$

then the random variable X has distribution function F. (F^{-1} (u) is defined to equal that value x for which $F(x) = u$)

Hence we can simulate a random variable X from the continuous distribution F, when F is computable, by simulating a random number U and then setting $X = F^{-1}(U)$.

For example, simulate an exponential random variable :

$$F(x) = 1 - e^{-px},$$

then F (u) is that value of x such that

$$1 - e^{-px} = u,$$

or

$$x = -\log(1 - u) / p.$$

Hence if u is a uniform (0,1) variable , then

$$F^{-1}(U) = -\log(1 - U) / p$$

is exponentially distributed with mean p.

1.3 RANDOM NUMBER GENERATOR

When the analyst speaks of a "random" process, then, the process as a whole that may be categorized and relative frequencies of those attributes occurring may be tabulated. The purpose of random number generation in a simulation model is to convey to the model the nature of statistical distribution to be modeled and to create the impression that the value of the next draw from the distribution cannot be guessed.

Several attributes of random number generators are considered desirable:

1. Efficiency -that is, the generator produces random numbers at relatively little cost for computer time and computer workspace.
2. Uniformity - that is, approximately equal percentage of the data will be distributed in each equal length area.
3. Conformity to the desired type of statistical distribution, with mean, variance and range as stipulated.
4. Independence - that is, the inability to predict the value of the $(N+1)$th random number based on the value of the Nth random number except by examining the computer code.
5. Absence of trends - that is, generation of ascending or descending strings of values which are neither excessively long nor excessively short.
6. Long cycle length -that is, a relatively large number of numbers which can be generated before the algorithm produces a sequence identical to the previous sequence.

The Computer-based method of generating random numbers requires the initial definition of one or more constants called *seeds* which affect the magnitude of the random numbers produced. These seeds actually create pseudo random numbers instead of truly random ones. Most random number generators start with an initial value X , which is seed, and then recursively compute values by specifying positive integers a, c, and m, and then letting

$$x_{n+1} = (ax_n + c) \text{ modulo } m, \; n>=0,$$

where the above means that $ax_n + c$ is divided by m and the remainer is taken as the value of x_{n+1}. Thus each x_n is either $0,1,\ldots,m-1$ and the quantity x_n/m is taken as an approximation to a uniform (0,1) random variable. It can be shown the subject to suitable choice for a, c, m, the above gives rise to a sequence of number that looks as if it was generated from independent uniform (0,1) random variable.

Before using the random number generator of the available computer system, SUN-UNIX, to provide (0,1) uniform distribution random variables for simulating a random variable of desirable continuous distribution, we have to confirm it has approximately the same cumulative distribution function for a (0,1) uniform distibution. This is done by using the random number generator to create a large number of random data that fall in each divided area between 0 and 1.

Table 1.1 show the result after generate 5,000 (0,1) uniform distributed random data.

Table 1.1 Frequency of Random Data in Each Area

Random Data Value	Number of Data fall in this range	Percentage %
0.0 - 0.1	492	0.0984
0.1 - 0.2	503	0.1006
0.2 - 0.3	507	0.1014
0.3 - 0.4	499	0.0998
0.4 - 0.5	497	0.0994
0.5 - 0.6	500	0.1000
0.6 - 0.7	510	0.1020
0.7 - 0.8	495	0.0990
0.8 - 0.9	503	0.1006
0.9 - 1.0	494	0.0998

From table 1.1, we can see that the numbers of the random data fall in each area are almost identically distributed. That approximately follows the (0,1) uniform distribution function.

CHAPTER 2

EFFECTS OF DEPENDENCIES BETWEEN INTERARRIVAL TIME AND SERVICE TIME ON SINGLE NODE QUEUEING SYSTEMS

The queueing model analyzed in this chapter is G/G/1 single node queueing systems.

Normally, the basic assumption for this system are:

1. Customers individually and immediately enter the queueing system.
2. The interarrival times are independently and identically distributed.
3. There is only one server in the system.
4. The server completely serves on customer at a time without interruption.
5. Service time are independently and identically distributed.
6. A customer always remains in the queueing system until its waiting time in the queue (if any) and service are completed, at which time it immediately leaves and its server immediately begins serve another customer (if any had been waiting in the queue).
7. The queue discipline is first-come-first-served.

There are cases in which service time and customer interarrival time are not independent. There may be some kind of correlation between customer interarrival time and service time, between interarrival times, or between service times. Few results have been reached in this subject, because it is very difficult to get theoretical result in general situation. In this case, simulation of this system can provide a guideline for the theoretical research.

In this chapter, we provide simulation result for effects of dependencies between interarrival time and service time on performance of single queueing systems. In the first section we analyze the case in which interarrival and service considers the model in which interarrival time and service time are negatively correlated ($r<=0$).

2.1 EFFECT OF POSITIVE CORRELATION ON SYSTEM PERFORMANCE

2.1.1 Effect of Positive Correlation Generated by Variables with Bivariate Distribution

First we consider a bivariate exponential distribution for the correlated interarrival and service time. Suppose three independent exponential distributions are:

$$P[U_1>t] = e^{-p_1 t};$$
$$P[U_2>t] = e^{-p_2 t};$$
$$P[U_{12}>t] = e^{-p_{12} t}.$$

The customer interarrival time is expressed as

$$T = \min(U_1, U_{12}),$$

while service time satisfies:

$$S = \min(U_2, U_{12}).$$

Hence the exponential marginal distribution function is given by:

$$F_1(t_1) = P[T > t_1] = e^{-(p_1+p_{12})t_1};$$
$$F_2(t_2) = P[S > t_2] = e^{-(p_2+p_{12})t_2}.$$

The correlation coefficient between the interarrival time and service time then can be expressed as:

$$r = p_{12}/(p_1+p_2+p_{12}). \tag{2.1}$$

We are now considering the following queueing model: customer interarrival time follows exponential distribution with parameter $u_1 = p_1+p_{12}$, while the service time follows exponential distribution with parameter $u_2 = p_2+p_{12}$.

Further more, we fix u_1 and u_2 so that the marginal distribution of the customer interarrival time and service time will not change no matter how correlation coefficient varies.

If p_{12} increases by c (ie $p_{12}' = p_{12}+c$), then $p_1' = p_1 - c$ and $p_2' = p_2 - c$. Here $-p_{12} <= c$, because normally service rate exceeds interarrival rate; and $c <= p_1$, because the exponential distribution parameter cannot be negative.

So the correlation coefficient becomes:

$$r' = (p_{12}+c) / (p_1+p_2+p_{12}-c). \qquad (2.2)$$

$r'_{min} = 0$ when $c = -p_{12}$; $r'_{max} = (p_1+p_{12}) / (p_2+p_{12})$, when $c = p_1$.

By changing c from $-p_{12}$ to p_1, we obtain correction coefficients from 0 to $(p_1+p_{12})/(p_2+p_{12})$.

Intuitively, a bivariate exponential distribution relationship between interarrival and service time improves system performance by decreasing the average waiting time, because positive correlation coefficient means that the interarrival time decreases as service time decreases and increases when service time increases.

Fig. 2.1 is the simulation of the model described above, with $p_1=4$, $p_2=6, p_3=7$, and simulation running time for each r is 400. Horizontal axis represents correlation coefficient while vertical axis represents the average waiting time in the queue for each customer. The figure shows that average waiting time in the queue has a trend of decreasing, but not monotonously decreasing with the increment of correlation coefficient.

2.1.2 Effect of Positive Correlation Generated by Variables with Uniform Distribution

In this chapter, we try to construct another queueing model which has positive correlation between customer interarrival time and service time.

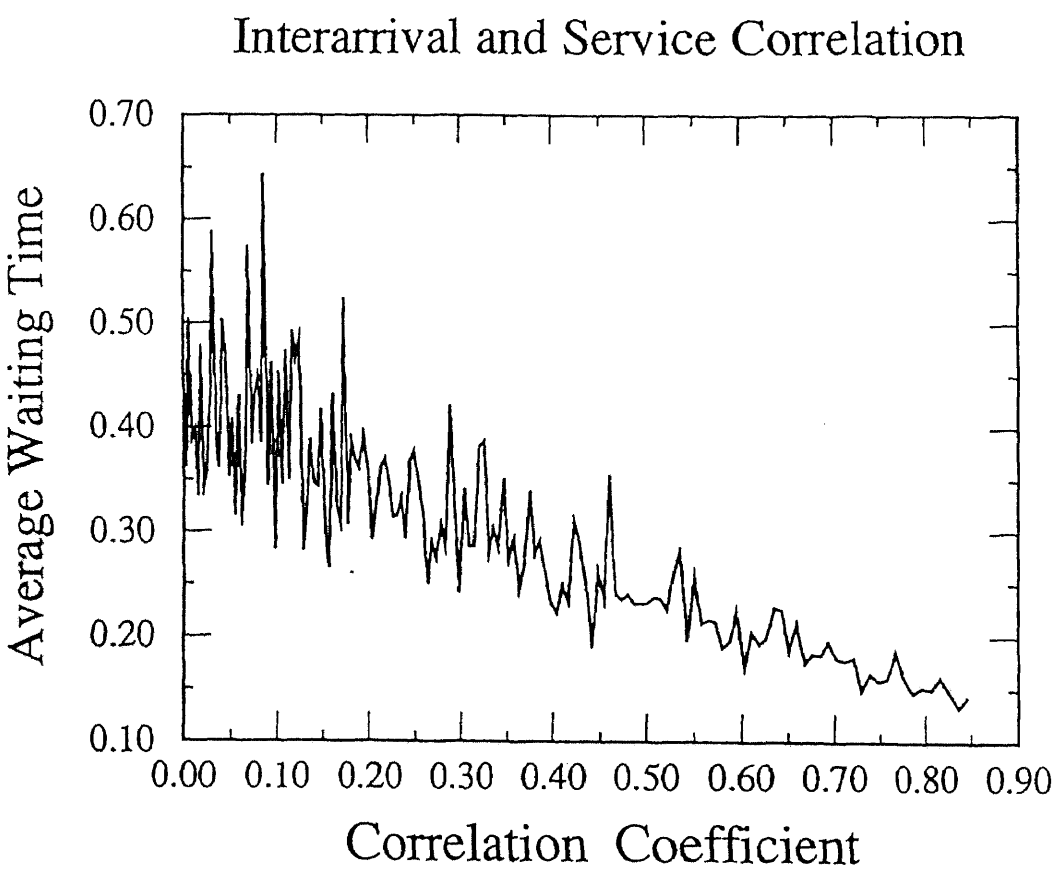

Figure 2.1 Positive Correlation between Interarrival and Service Generated by Variables with Bivariate Distribution

Let $\{X_n\}$ follows (0,3) uniform distribution;

$\{Y_n\}$ follows (0,2) uniform distribution;

$\{Z_n\}$ follows (0,1) uniform distribution.

"a" is a factor which ranges from 0 to 1.

For ith customer, the interarrival time is defined as

$$T_i = X_i + (1 - a)Y_i,$$

and service time is defined as

$$S_i = Z_i + (1 - a)Y_i.$$

The correlation coefficient for this model is calculated as:

$$r = a(1-a) / [(13a^2-8a+4)(5a^2-2a+1)]^{0.5}. \qquad (2.3)$$

For the purpose of comparison, we also establish a similar queueing model which has no correlation between customer interarrival time and service time.

Let $\{X_n'\}$ follows (0,3) uniform distribution;

$\{Y_n'\}$ follows (0,2) uniform distribution;

$\{W_n'\}$ follows (0,2) uniform distribution;

$\{Z_n'\}$ follows (0,1) uniform distribution.

The ith customer interarrival time is defined as:

$$T_i' = X_i' + (1\text{-}a)Y_i',$$

while service time for ith customer is defined as

$$S_i' = Z_i' + (1\text{-}a)W_i'.$$

Fig. 2.2 is the simulation result of the models described above with simulation running time 1500. The solid line represents the correlation model, while the dash curve represents the respective comparison model. It shows that: when the correlation coefficient ranges from 0 to 0.3, the average waiting time doesn't have any trend over comparison model; when it ranges from 0.3 to 0.5, the decreasing trend is obvious, the solid curve is always below the dash curve. The conclusion is that in positive correlated model generated by uniform distribution variables, the average waiting time in the queue does not have a decreasing trend until the correlation coefficient between customer interarrival time and service time reaches certain value.

2.2 EFFECT OF NEGATIVE CORRELATION ON SYSTEM PERFORMANCE

Negative correlation represents opposite trends of two variable. Increasing trend of one variable results the decreasing trend of the other and verse-vise. Most of researches on dependency deal with positive correlation. In this section, we are trying to establish a negative correlation between customer interarrival time and

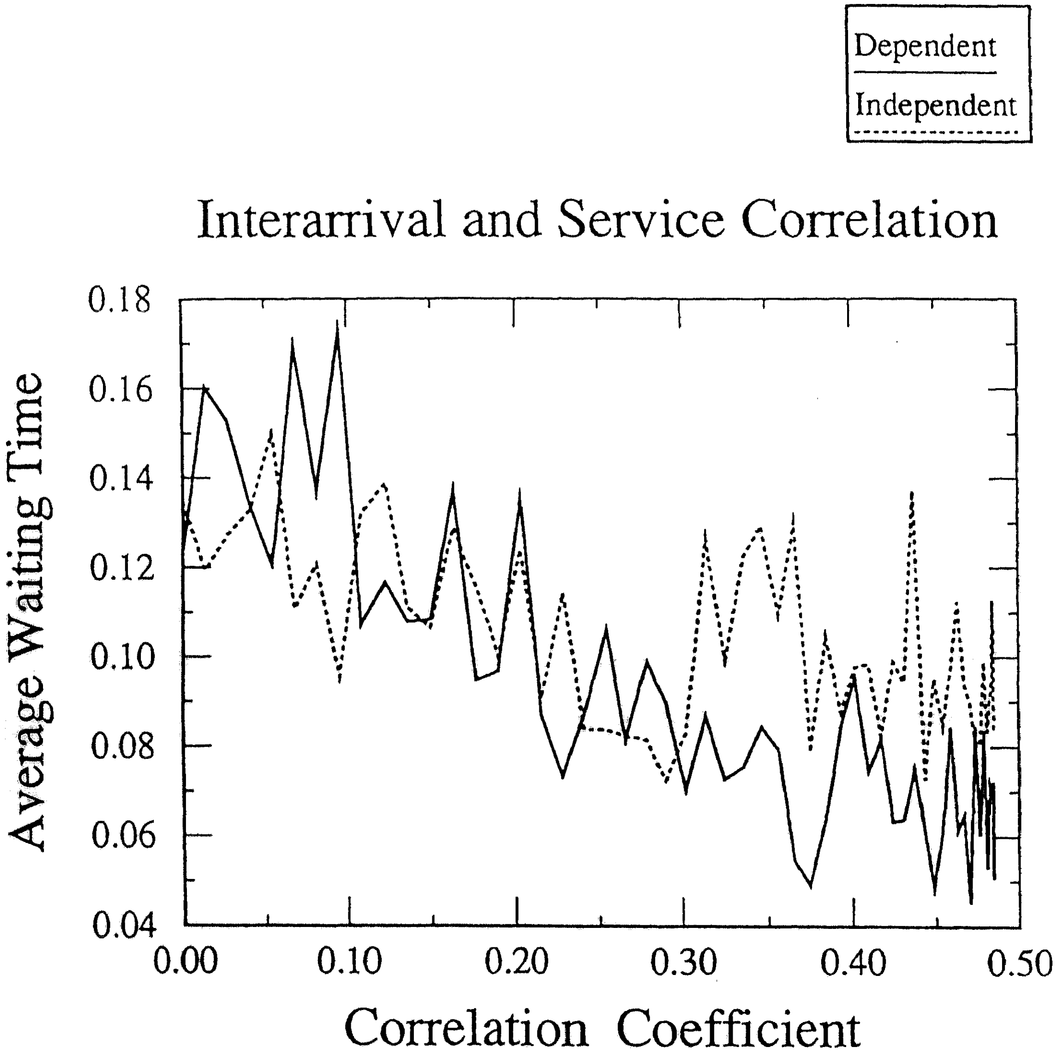

Figure 2.2 Positive Correlation between Interarrival and Service Generated by Variables with Uniform Distribution

service time to see how this kind of correlation affects the system performance measures.

One of the negative correlated model constructed as following:

Let $\{X_n\}$ follows (0,3) uniform distribution;

$\{Y_n\}$ follows (0,1) uniform distribution;

$\{Z_n\}$ follows (0,2) uniform distribution.

The *i*th customer interarrival time is defined as:

$$T_i = X_i + Y_i - (1\text{-}a)Y_{i+1}.$$

The *i*th customer service time is defined as:

$$S_i = Z_i + Y_{i+1} - (1-a)Y_{i+2}.$$

The $(i+1)$th customer interarrival time is defined as :

$$T_{i+1} = X_{i+1} + aY_{i+3} - (1-a)Y_{i+4}.$$

The $(i+1)$th customer service time is defined as

$$S_{i+1} = Z_{i+1} + aY_{i+4} - (1-a)Y_{i+5}.$$

Correlation coefficient between T and S is calculated as:

$$r = -a(1-a) / (4a^2+22a+24)^{0.5}. \tag{2.4}$$

The customer interarrival and service distribution and correlation between interarrival and service time changes coresponding to the variance of a. In this case, a comparison model is necessary. The comparison model should be similar to the dependency model but has independent customer interarrival time distribution and service time distribution.

Let $\{X_n'\}$ follows (0,3) uniform distribution;

$\{Y_n'\}$ follows (0,1) uniform distribution;

$\{W_n'\}$ follows (0,1) uniform distribution;

$\{Z_n'\}$ follows (0,2) uniform distribution.

The *i*th customer interarrival time is defined as:

$$T_i' = X_i' + aY_i' - (1-a)Y_{i+1}'.$$

The *i*th customer service time is defined as:

$$S_i' = Z_i' + aW_i' - (1-a)W_{i+1}'.$$

The $(i+1)$th customer interarrival time is defined as :

$$T_{i+1}' = X_{i+1}' + aY_{i+2}' - (1-a)Y_{i+3}'.$$

The $(i+1)$th customer service time is defined as:

$$S_{i+1}' = Z_{i+1}' + aW_{i+2}' - (1-a)W_{i+3}'.$$

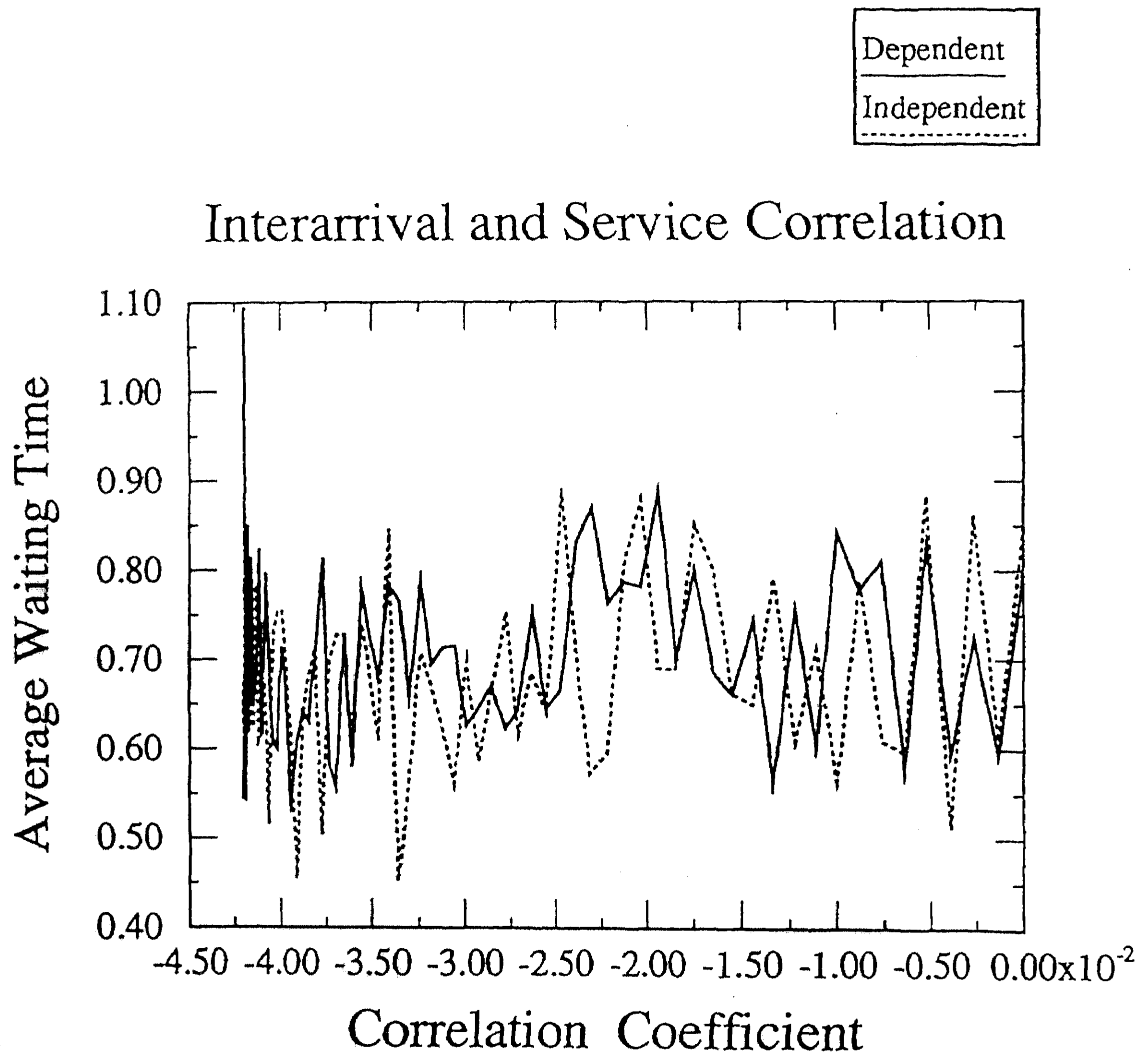

Figure 2.3 Effect of Negative Correlation between Interarrival and service

Fig. 2.3 is the simulation result of the models described above with simulation running time 1500. The dash curve represents the comparison model with respective while the solid line represents the correlation model.From the figure, we can not see any trend of how negative correlation coefficient between the interarrival time and service time will affect the system. The conclusion is that generally, negative coefficient between interarrival time and service time does not have any trend of its effect on system performance.

CHAPTER 3

EFFECTS OF DEPENDENCIES BETWEEN ADJACENT INTERARRIVALS ON SINGLE NODE QUEUEING SYSTEMS

In this chapter, we develop simulation results for effects of dependencies between adjacent customer interarrivals on performance of single node queueing systems. The correlation model used in this chapter is somewhat similar to that of Chapter 2. In the first section we analyze cases in which customer interarrivals are positively correlated (ie. correlation coefficient $r>=0$); section 2 consider the model in which interarrivals are negatively correlated ($r<=0$). In each case, the simulation result is displayed on a figure on which the result can be directly analyzed.

3.1 EFFECTS OF POSITIVE CORRELATION BETWEEN ADJACENT INTERARRIVALS

3.1.1 Effects of Positive Correlation Generated by Variables with Bivariate Distribution

A bivariate exponential distribution for the adjacent customer interarrival time is constructed as dependent model.

Let $\{X_n\}$ follows exponential distribution with parameter p_1;

$\{Y_n\}$ follows exponential distribution with parameter p_2.

The distribution function of each variable are shown as:

$$P[X>t] = e^{-p_1 t};$$

$$P[Y>t] = e^{-p_2 t}.$$

The *ith* customer interarrival time is expressed as:

$$T_i = \min(X_i, Y_i, Y_{i+1}).$$

The interarrival time then follows exponential distribution with parameter $u_1 = p_1+2p_2$.

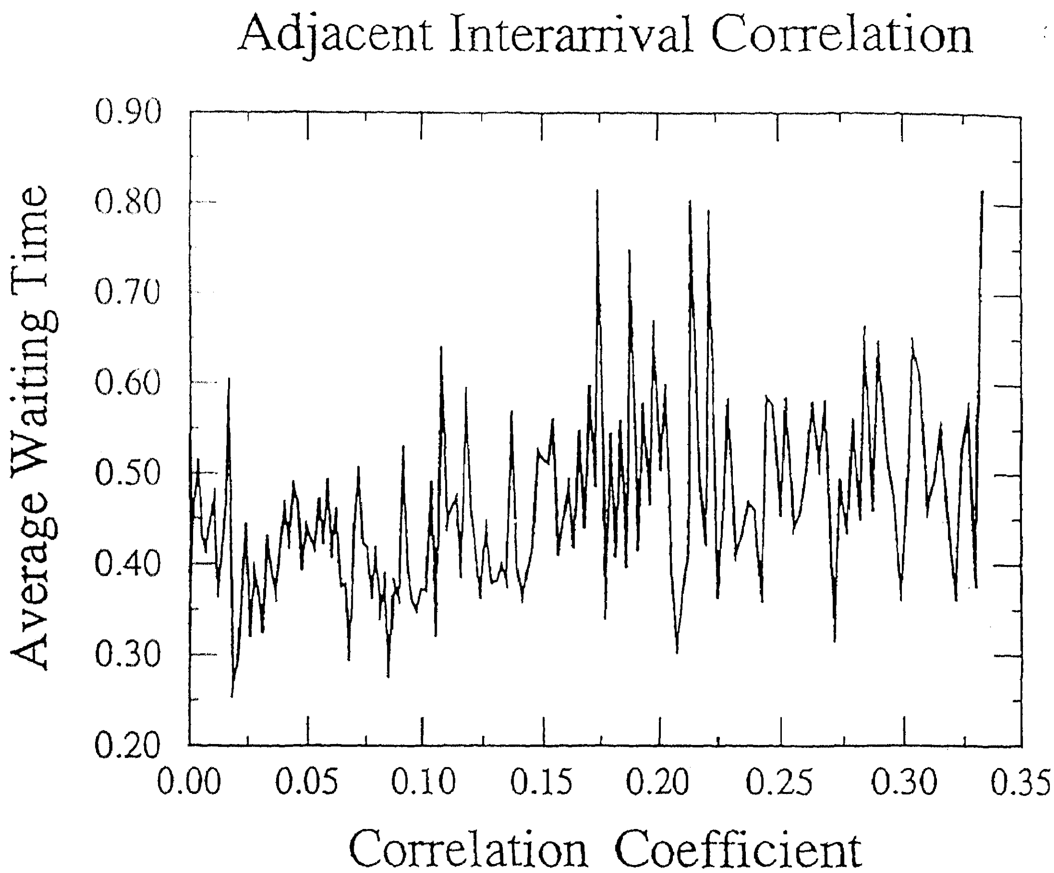

Figure 3.1 Positive Correlation between Adjacent Interarrival Generated by Variables with Bivariate Distribution

Hence the exponential marginal distribution function is given by:

$$P[T > t] = e^{-(p_1+2p_2)t}.$$

Service time follows exponential distribution with parameter u_2 ($u_2 >= u_1$).

The correlation coefficient between the adjacent customer interarrival time can be expressed as

$$r = p_2 / (2p_1 + 3p_2) \tag{3.1}$$

Here, we fix u_1, the marginal distribution of the customer interarrival time will not be changed no matter how correlation coefficient varies.

If p_2 decrease by c, the new correlation coefficient is

$$r' = (p_2 - c) / (2p_1 + 3p_2 + c) \tag{3.2}$$

$$r' = (p_2 - c) / (2p_1 + 3p_2 + c) \quad (3.2)$$

By changing c from $-p_1/2$ to p_2, we are able to obtain correction coefficients from 0 to 1/3.

Intuitively, a bivariate exponential distribution relationship between adjacent customer interarrivals worses system performance by increasing the average waiting time in the queue.

Fig. 3.1 is the simulation of the model described above, with p_1=2, p_2=4, u_2=12, and simulation running time for each r is 400. The figure shows thatthe average waiting time tends to increase, but not monotonously increases when the correlation coefficient increases.

3.1.2 Effect of Positive Correlation Generated by Variables with Uniform Distribution

In this section, we try to construct another queueing model which has positive correlation between customer interarrival time and service time.

Let $\{X_n\}$ follows (0,2) uniform distribution;

"a" is a factor which ranges from 0 to 1.

For ith customer, the interarrival time is defined as

$$T_i = X_i + (1-a)\, X_{i+1},$$

and service time follows (0,1) uniform distribution.

The correlation coefficient can be calculated as

$$r = a(1-a) / (2a_2 - 2a + 1). \quad (3.3)$$

Let $y = a(1-a)$,

then

$$r = y / (1-2y).$$

Because y ranges from 0 to 0.25,

$$r_{max} = 0.5, \qquad \text{when } a = 0.5;$$

$$r_{min} = 0, \qquad \text{when } a = 0 \text{ or } 1.$$

That means the maximum correlation happens when new variable takes both factors equally and the minimum correlation coefficient happens when new variable exclusively depends on one factor.

For comparison, we also establish a similar queueing model which has no correlation between adjacent customer interarrivals.

Let $\{X_n'\}$ follows (0,2) uniform distribution.

The ith customer interarrival time is defined as

$$T_i' = X_i' + (1\text{-}a)X_{i+1}',$$

while $i+1$st customer interarrival time is defined as

$$T_{i+1}' = X_{i+2}' + (1\text{-}a)X_{i+3}'.$$

Service time follows (0,1) uniform distribution.

Fig. 3.2 is the simulation result of the models described above with simulation running time 1500. It shows that: when the correlation coefficient ranges from 0 to 0.35, the average waiting time doesn't have any trend over comparison model; when it ranges from 0.35 to 0.5, the decreasing trend is obvious, the solid curve is always above the dash curve. The conclusion is that in model described above, the average waiting time in the queue will not have a increasing trend until the correlation coefficient between adjacent customer interarrival time reaches certain value.

3.2 EFFECT OF NEGATIVE CORRELATION BETWEEN ADJACENT INTERARRIVALS

In this section, we are trying to establish a negative correlation between adjacent customer interarrival time to see whether there is any trend of the effect over system performance when the correlation coefficient between the adjacent interarrival time increases.

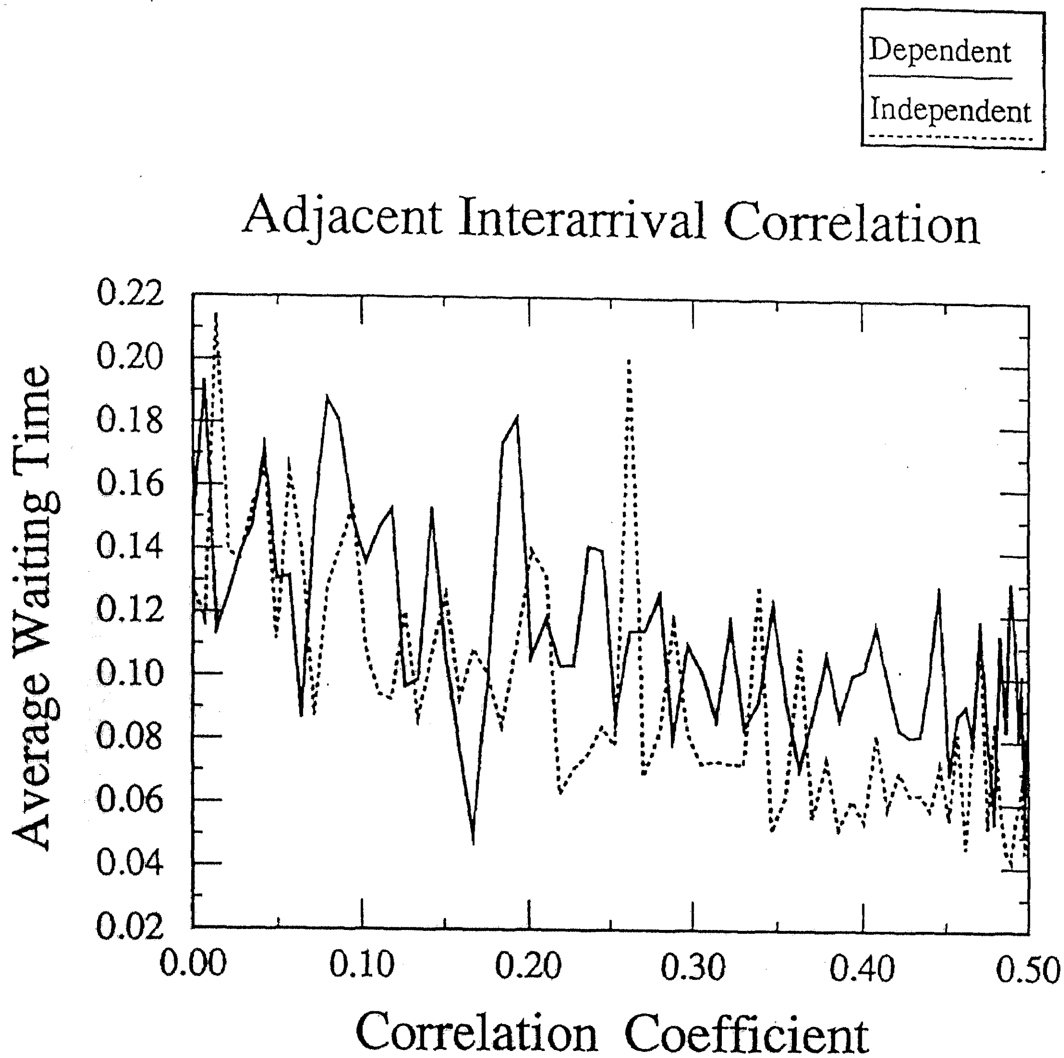

Figure 3.2 Positive Correlation between Adjacent Interarrival Generated by Variables with Uniform Distribution

The following model is one of the negative correlation model between adjacent interarrival:

Let $\{X_n\}$ follows (0,3) uniform distribution;

$\{Y_n\}$ follows (0,1) uniform distribution.

The *i*th customer interarrival time is defined as

$$T_i = X_i + aY_i - (1-a)Y_{i+1}.$$

The $(i+1)$th customer interarrival time is defined as

$$T_{i+1} = X_{i+1} + aY_{i+1} - (1-a)Y_{i+2}.$$

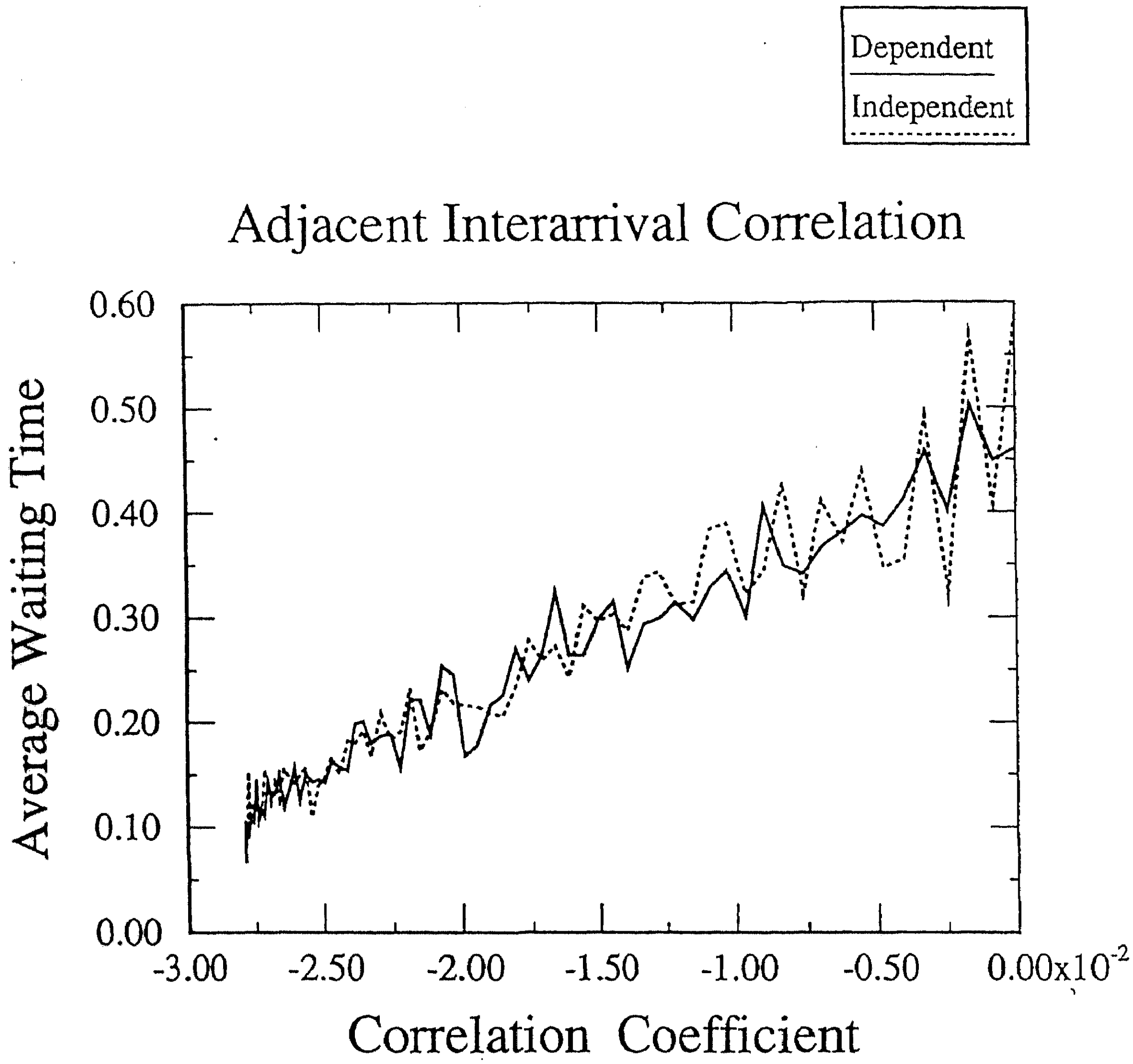

Figure 3.3 Effect of Negative Correlation Between Adjacent Interarrivals

The customer service time follows (0,1) uniform distribution.

The respective correlation coefficient is:

$$r = - a(1-a) / (2a+8). \tag{3.3}$$

The customer interarrival distribution of this model changes correspond to the variance of a. The comparison model is similar to the dependency model but has independent interarrival time between adjacent customer.

Let $\{X_n'\}$ follows (0,3) uniform distribution;

$\{Y_n'\}$ follows (0,1) uniform distribution.

The ith customer interarrival time is defined as:

$$T_i' = X_i' + aY_i' + (1\text{-}a)Y_{i+1}'.$$

The i+1st customer interarrival time is defined as :

$$T_{i+1}' = X_{i+1}' + aY_{i+2}' + (1\text{-}a)Y_{i+3}'.$$

The customer service time also follows (0,1) uniform distribution.

Fig. 3.3 is the simulation result of the models described above with simulation running time 1500. Compared with independent model, the negative correlation between the adjacent interarrival does not have a trend to increase or decrease the average waiting time in the queue.

CHAPTER 4

EFFECTS OF DEPENDENCIES BETWEEN ADJACENT SERVICES ON SYSTEM PERFORMANCE

This chapter is divided into two sections: section 1 develops simulation results for positive correlation; section 2 shows how negative correlation effect the system performance.

4.1 EFFECT OF POSITIVE CORRELATION BETWEEN ADJACENT SERVICES

4.1.1 Effect of Positive Correlation Generated by Variables with Bivariate Distribution

Let $\{X_n\}$ follows exponential distribution with parameter p_1;

$\{Y_n\}$ follows exponential distribution with parameter p_2.

The *i*th customer service time is expressed as:

$$S_i = \min (X_i,\ Y_i,\ Y_{i+1}).$$

The service time then follows exponential distribution with parameter $u_1 = p_1 + 2p_2$.

Customer interarrival time follows exponential distribution with parameter u_2.

The correlation coefficient between the adjacent service time can be expressed as:

$$r = p_2 \,/\, (2p_1 + 3p_2). \tag{4.1}$$

Here, we fix u_1, then the marginal distribution of the service time will not be changed no matter how correlation coefficient varies.

If p_2 decrease by c, the new correlation coefficient becomes:

$$r' = (p_2 - c) \,/\, (2p_1 + 3p_2 + c). \tag{4.2}$$

By changing from $-p_1/2$ to p_2 , we are able to obtain correction coefficients from 0 to 1/3.

Fig. 4.1 is the simulation of the model described above, with simulation running time t=400, p_1=3, p_2=2, u_2=5. After running simulation 150 times for different value of correlation coefficient, the result shows that this kind of positive correlation between adjacent service time has a trend to increase, but not monotone increase, the average waiting time in the queue.

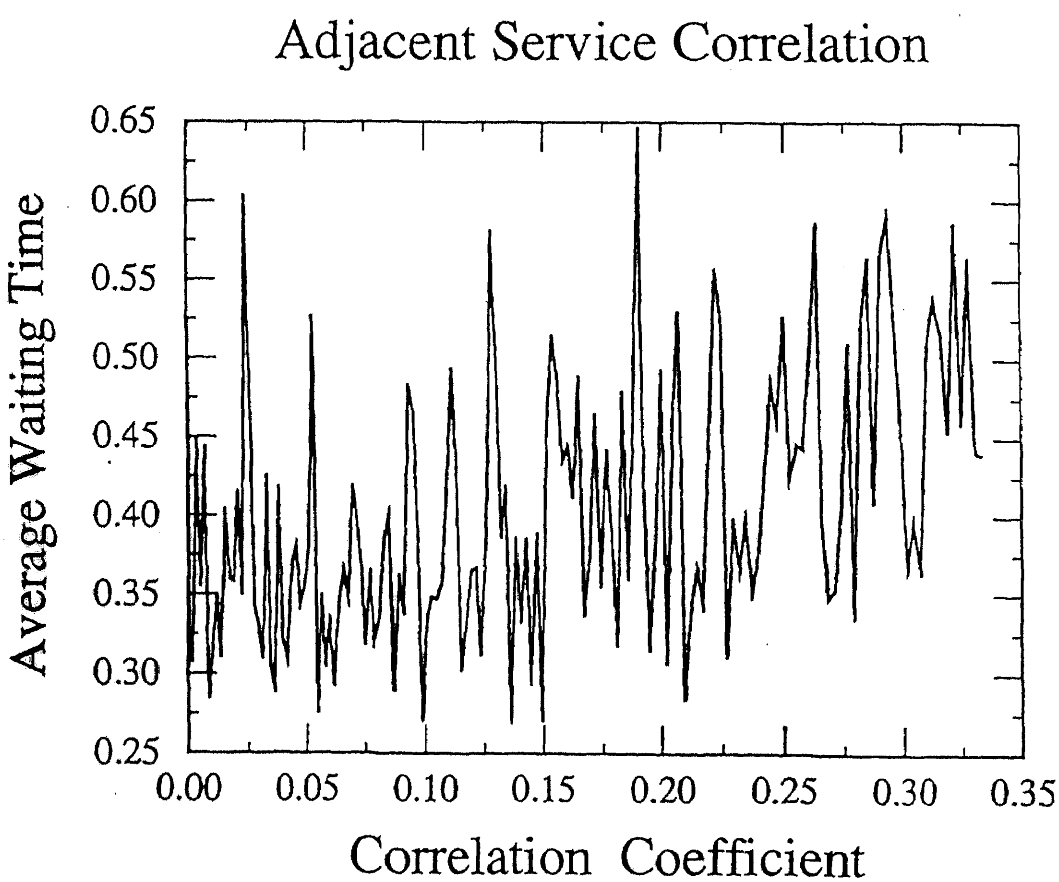

Figure 4.1 Positive Correlation between Adjacent Services Generated by Variables with Bivariate Distribution

4.1.2 Effect of Positive Correlation Generated by Variables with Uniform Distribution

Let $\{X_n\}$ follows (0,1) uniform distribution. "a" is a factor which ranges from 0 to 1.

For ith customer, the service time is defined as:

$$S_i = X_i + (1\text{-}a)\, X_{i+1}.$$

and the customer interarrival time follows (0,2) uniform distribution.

The correlation coefficient can be expressed as:

$$r = a(1\text{-}a) \,/\, (2a^2 \text{-}2a + 1). \qquad (4.3)$$

Let $y = a(1\text{-}a)$,

then

$$r = y \,/\, (1\text{-}2y).$$

Because y ranges from 0 to 0.25,

$$r_{\max} = 0.5, \qquad \text{when } a = 0.5;$$

$$r_{\min} = 0, \qquad \text{when } a = 0 \text{ or } 1.$$

The comparison model is a similar queueing model which has no correlation between adjacent customer services.

Let $\{X_n'\}$ follows (0,1) uniform distribution.

The ith customer service time is defined as:

$$S_i' = X_i' + (1\text{-}a)X_{i+1}'.$$

The $i+1$st customer service time is defined as:

$$S_{i+1}' = X_{i+2}' + (1\text{-}a)X_{i+3}'.$$

The customer interarrival time follows (0,2) uniform distribution.

Fig. 4.2 is the simulation result of the models described above with simulation running time 1500. It shows that: mostly, the correlation model has a trend to increase, not monotone increase, the average waiting time in the queue. When r ranges from 0.35 to 0.5 the increasing trend is obvious.

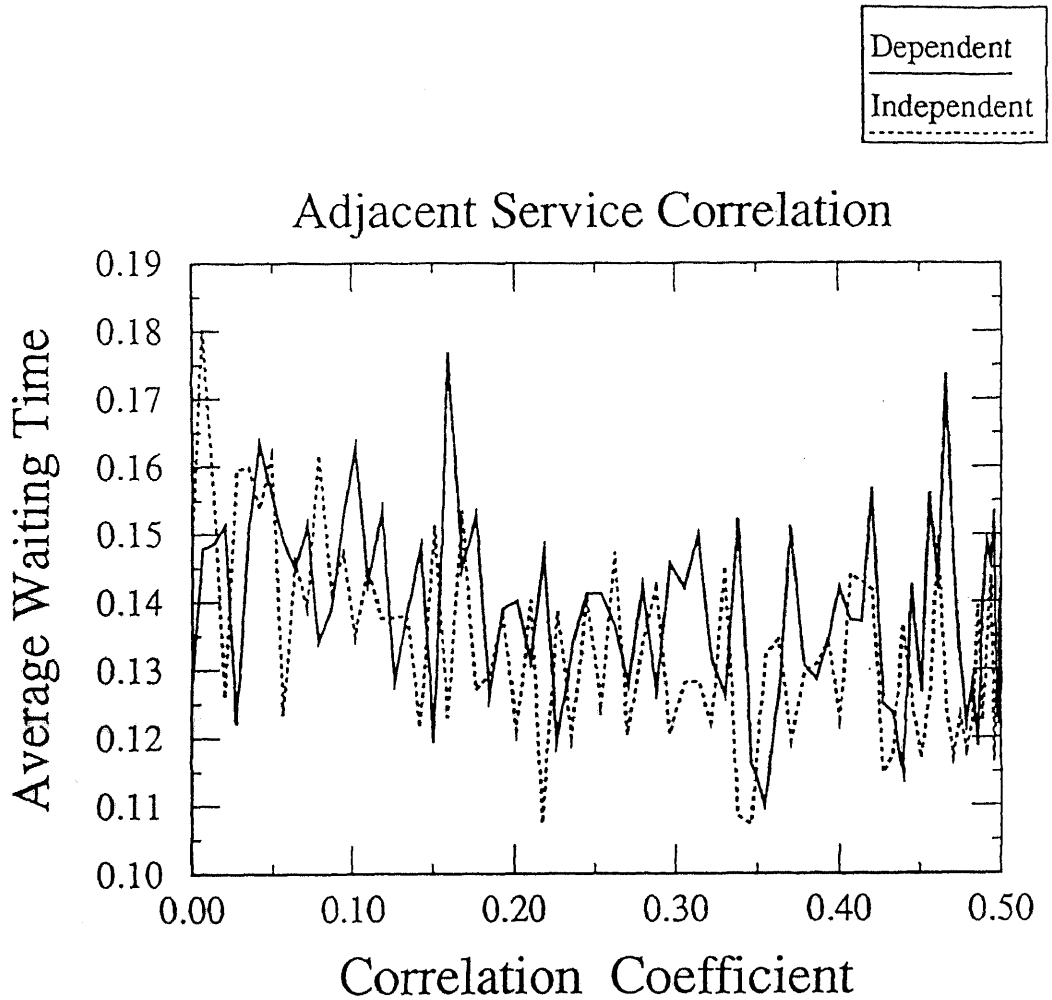

Figure 4.2 Positive Correlation between Adjacent Services Generated by Variables with Uniform Distribution

4.2 EFFECT OF NEGATIVE CORRELATION BETWEEN ADJACENT SERVICES

In this section, we are trying to establish a negative correlation between adjacent service time to see the effect over system performance when the correlation coefficient between the adjacent customer service time increases.

The following model is one of the negative correlation model between adjacent interarrivals:

Let $\{X_n\}$ follows (0,2) uniform distribution;

$\{Y_n\}$ follows (0,1) uniform distribution.

The ith customer service time is defined as

$$S_i = X_i + aY_i + (1\text{-}a)Y_{i+1}.$$

The $i+1$st customer service time is defined as

$$S_{i+1} = X_{i+1} + aY_{i+1} + (1\text{-}a)Y_{i+2}.$$

The customer interarrival time follows (0,4) uniform distribution.

The correlation coefficient can be expressed as

$$r = - a(1\text{-}a) / (2a+3). \tag{4.3}$$

The customer service time distribution of this model changes corresponding to the variance of a. The comparison model should be similar to the dependency model but has independent service time between adjacent customer.

Let $\{X_n'\}$ follows (0,2) uniform distribution;

$\{Y_n'\}$ follows (0,1) uniform distribution;

The ith customer interarrival time is defined as

$$S_i' = X_i' + aY_i' - (1\text{-}a)Y_{i+1}'.$$

The $i+1$st customer interarrival time is defined as

$$S_{i+1}' = X_{i+1}' + aY_{i+2}' - (1\text{-}a)Y_{i+3}'.$$

Customer interarrival time also follows (0,4) uniform distribution.

Figure 4.3 is the simulation result of the models described above with simulation running time 1500. Compared with independent model, the negative correlation between the adjacent service time has no trend of increasing or decreasing the average waiting time in the queue.

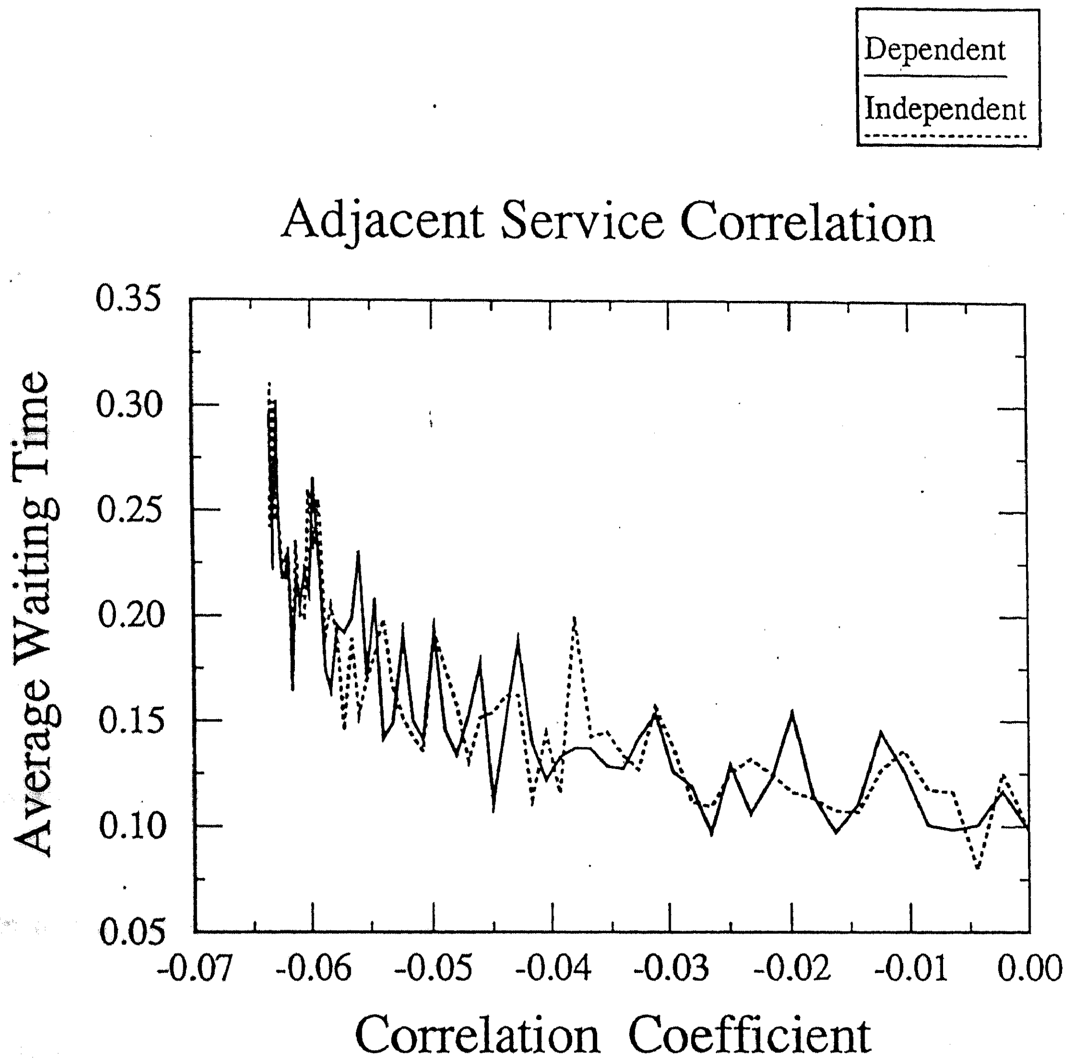

Figure 4.3 Effect of Negative Correlation Between Adjacent Services

CHAPTER 5

EFFECTS OF DEPENDENCIES ON TANDEM QUEUES

Consider a two server system in which customers arrive at a Poisson rate at server 1. After being served by server 1 they then join the queue in front of server 2. Here, we suppose there is infinite waiting space at both servers. Each server serves on customer at a time with server i taking an exponential time with rate u_i for a service, i=1,2. Such a system is called a tandem queues.

In this chapter, we are trying to find out how dependency affect the system performance in tandem queues.

5.1 EFFECT OF POSITIVE DEPENDENCY BETWEEN INTERARRIVAL AND SERVICE AT SERVER 1

Suppose three independent exponential distribution are:

$$P[U_1 > t] = e^{-p_1 t};$$

$$P[U_2 > t] = e^{-p_2 t};$$

$$P[U_{12} > t] = e^{-p_{12} t}.$$

The customer interarrival time is expressed as:

$$T = \min(U_1, U_{12}),$$

while service time at server 1 satisfies:

$$S = \min(U_2, U_{12}).$$

Service time at server 2 is a Poisson process with rate u_3.

$$u_3 >= p_2 + p_{12} >= p_1 + p_{12}.$$

Hence the exponential marginal distribution function is given by:

$$F_1(t_1) = P[T > t_1] = e^{-(p_1 + p_{12})t_1};$$

$$F_2(t_2) = P[S > t_2] = e^{-(p_2 + p_{12})t_2};$$

$$F_3(t_3) = P[Q > t_2] = e^{-u_3 t_2}.$$

So customer interarrival time follows exponential distribution with parameter $u_1 = p_1+p_{12}$, while the service time at server 1 follows exponential distribution with parameter $u_2 = p_2+p_{12}$.

The correlation coefficient between the interarrival time and service time at server 1 is:

$$r = p_{12} / (p_1+p_2+p_{12}). \tag{5.1}$$

We fix u_1 and u_2 so that the marginal distribution of the customer interarrival time and service time at server 1 won't change no matter how correlation coefficient varies.

If p_{12} increases by c, then the new correlation coefficient becomes:

$$r' = (p_{12}+c) / (p_1+p_2+p_{12}-c) \tag{5.2}$$

$r'_{min} = 0$ when $c = -p_{12}$; $r'_{max} = (p_1+p_{12})/(p2+p_{12})$, when $c = p_1$.

By changing c from $-p_{12}$ to p_1, we obtain correction coefficient from 0 to $(p_1+p_{12})/(p_2+p_{12})$.

Figure 5.1 is the simulation result of the model above, with p_1=3, p_2=6, p_{12}=4, u_3=12. It shows that the average waiting time in the queue has a slow trend of decreasing, but not monotonously decreasing, with respect to the increment of correlation coefficient.

5.2 EFFECT OF POSITIVE DEPENDENCY BETWEEN SERVICES AT SERVER 1 AND SERVER 2

Suppose three independent exponential distribution are:

$$P[U_1>t] = e^{-p_1 t};$$

$$P[U_2>t] = e^{-p_2 t};$$

$$P[U_{12}>t] = e^{-p_{12} t}.$$

The customer interarrival time is expressed as:

$$S = \min(U_1,U_2),$$

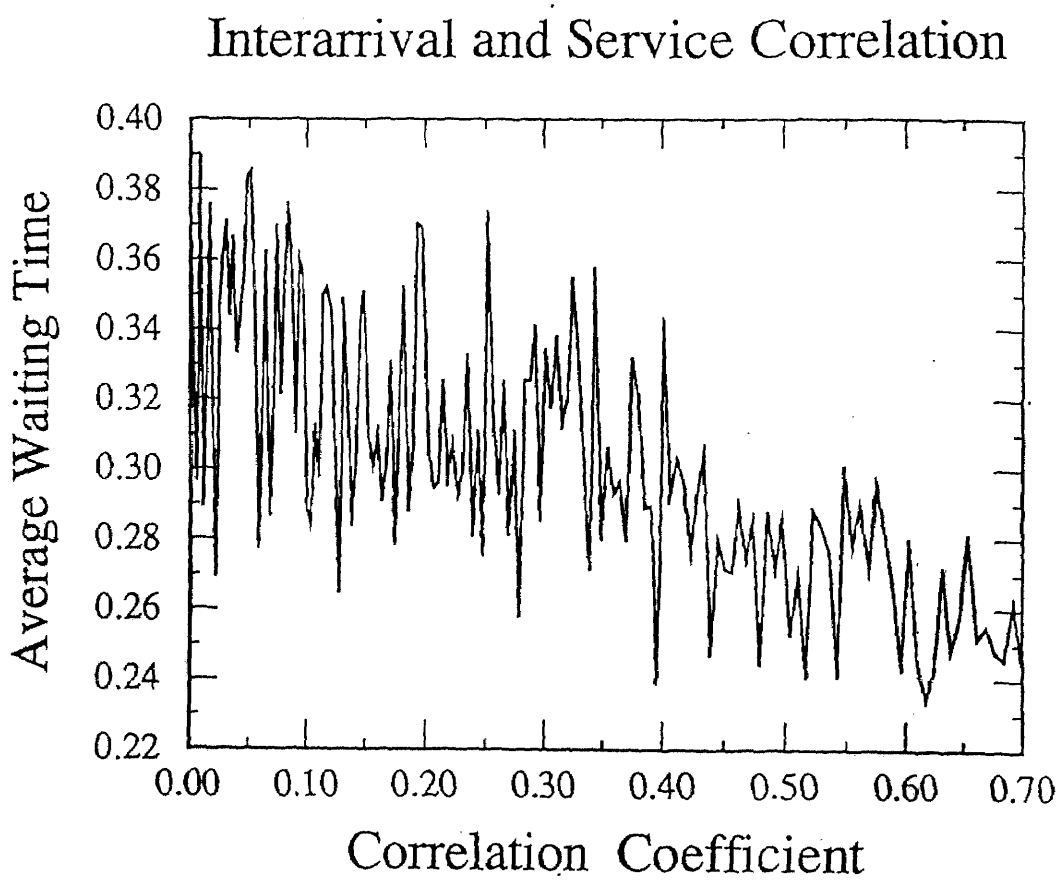

Figure 5.1 Effect of Positive Correlation Between Interarrival and Service at Server 1 in Tandem Queue

while service time at server 1 satisfies:

$$Q = \min(U_2, U_{12}).$$

Customer interarrival time is a Poisson process with rate u_3.

The correlation coefficient between the service time at server 1 and server 2 is:

$$r = p_{12} / (p_1 + p_2 + p_{12}) \tag{5.3}$$

We fix u_1 and u_2 so that the marginal distribution of the service time at server 1 and server 2 will not be changed no matter how correlation coefficient varies.

From previous analysis, we know that r will change from 0 to $(p_1+p_{12})/(p_2+p_{12})$ according to the variance of p_{12}.

Figure 5.2 is the simulation result of the model above, with running time 400, $p_1=5$, $p_2=7$, $p_{12}=6$, $u_3=4$. It shows that the average waiting time in the queue has a slow trend of increasing, but not monotonously increasing, with respect to the increment of correlation coefficient.

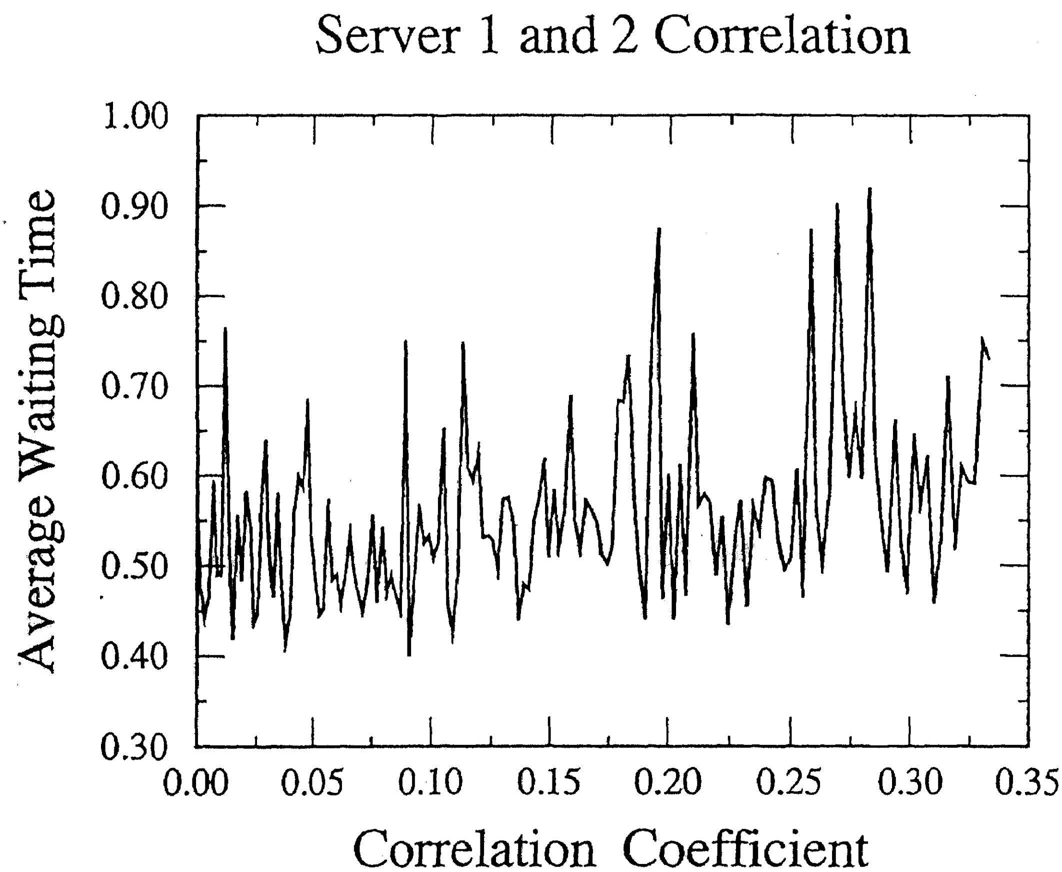

Figure 5.2 Effect of Positive Correlation Between Service time at Server 1 and Server 2 in Tandem Queue

5.3 EFFECT OF POSITIVE CORRELATION BETWEEN ADJACENT INTERARRIVALS

Let $\{X_n\}$ follows exponential distribution with parameter p_1;

$\{Y_n\}$ follows exponential distribution with parameter p_2.

The distribution function of each variable are shown as:

$$P[X>t] = e^{-p_1 t};$$

$$P[Y>t] = e^{-p_2 t}.$$

The ith customer interarrival time is expressed as:

$$T_i = \min(X_i,\ Y_i,\ Y_{i+1}).$$

The interarrival time then follows exponential distribution with parameter $u_1 = p_1 + 2p_2$.

Service time at server 1 and server 2 follows exponential distribution with parameter u_2 and u_3 respectively.

The correlation coefficient between the adjacent customer interarrival time can be expressed as:

$$r = p_2 \,/\, (2p_1 + 3p_2). \tag{5.4}$$

Here, we fix u_1, the marginal distribution of the customer interarrival time will not be changed no matter how correlation coefficient varies.

If p_2 decrease by c, the new correlation coefficient is:

$$r' = (p_2 - c) \,/\, (2p_1 + 3p_2 + c). \tag{5.5}$$

By changing c from $-p_1/2$ to p_2, we are able to obtain correction coefficients from 0 to 1/3.

Figure 5.4 is the simulation result of the model above, with running time 400, $p_1=2$, $p_2=3$, $u_2=10$, $u_3=14$. It shows that the average waiting time in the queue has a slow trend of increasing, but not monotonously increasing, with respect to the increment of correlation coefficient.

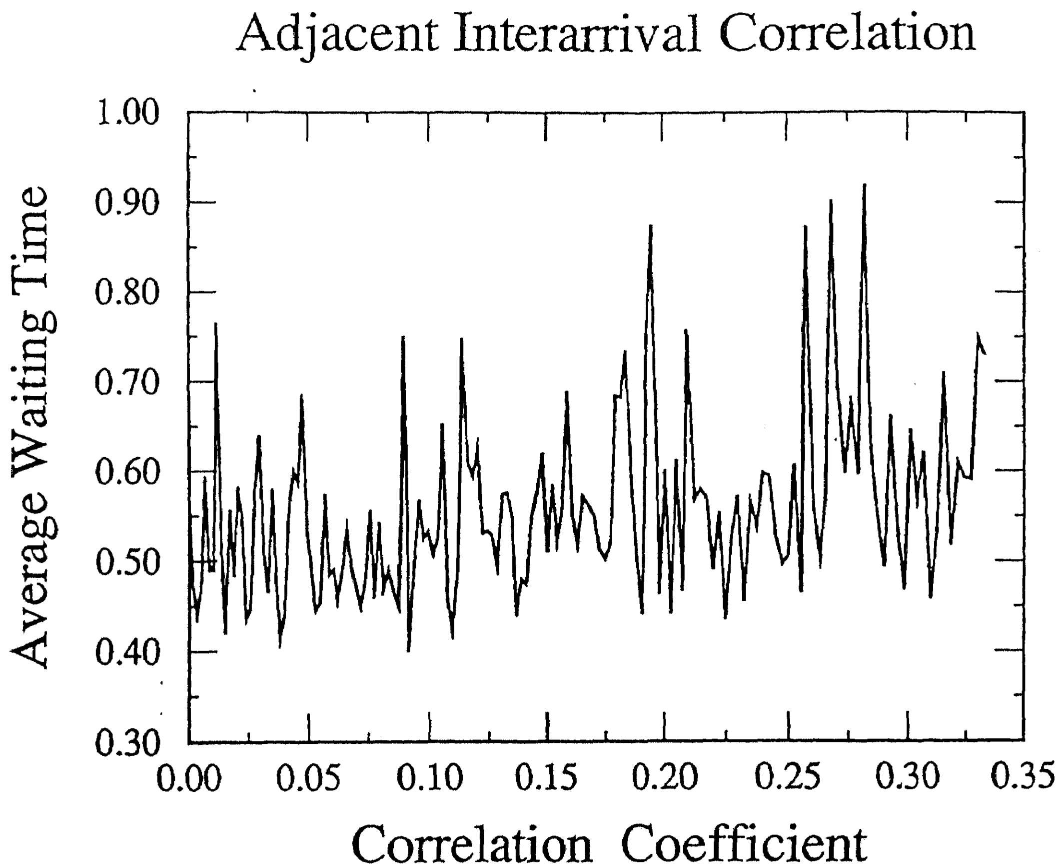

Figure 5.3 Effect of Positive Correlation Between Adjacent Interarrivals in Tandem Queue

5.4 EFFECT OF POSITIVE CORRELATION BETWEEN ADJACENT SERVICES AT SERVER 1

Let $\{X_n\}$ follows exponential distribution with parameter p_1;

$\{Y_n\}$ follows exponential distribution with parameter p_2.

The distribution function of each variable are shown as:

$$P[X>t] = e^{-p_1 t};$$

$$P[Y>t] = e^{-p_2 t}.$$

The ith customer service time at server 1 is expressed as:

$$S_i = \min(X_i,\ Y_i,\ Y_{i+1}).$$

The service time at server 1 then follows exponential distribution with parameter $u_1 = p_1 + 2p_2$.

Hence the exponential marginal distribution function is given by:

$$P[P > t] = e^{-(p_1+2p_2)t}.$$

Customer interarrival time and service time at server 2 follow exponential distribution with parameter u_2 and u_3 respectively.

The correlation coefficient between the adjacent customer interarrival time can be expressed as:

$$r = p_2 / (2p_1 + 3p_2) \qquad (5.6)$$

From previous analysis, the correction coefficients can be changed from 0 to 1/3.

Figure 5.4 is the simulation result of the model above, with running time 400, p_1=2, p_2=3, u_2=5, u_3=12. It shows that the average waiting time in the queue has a slow trend of increasing, but not monotonously increasing, with respect to the increment of correlation coefficient.

5.5 EFFECT OF POSITIVE CORRELATION BETWEEN ADJACENT SERVICES AT SERVER 2

Let $\{X_n\}$ follows exponential distribution with parameter p_1;

$\{Y_n\}$ follows exponential distribution with parameter p_2.

The distribution function of each variable are shown as:

$$P[X>t] = e^{-p_1 t};$$

$$P[Y>t] = e^{-p_2 t}.$$

The *i*th customer service time at server 2 is expressed as:

$$Q_i = \min(X_i, Y_i, Y_{i+1}).$$

The service time at server 1 then follows exponential distribution with parameter $u_1 = p_1 + 2p_2$.

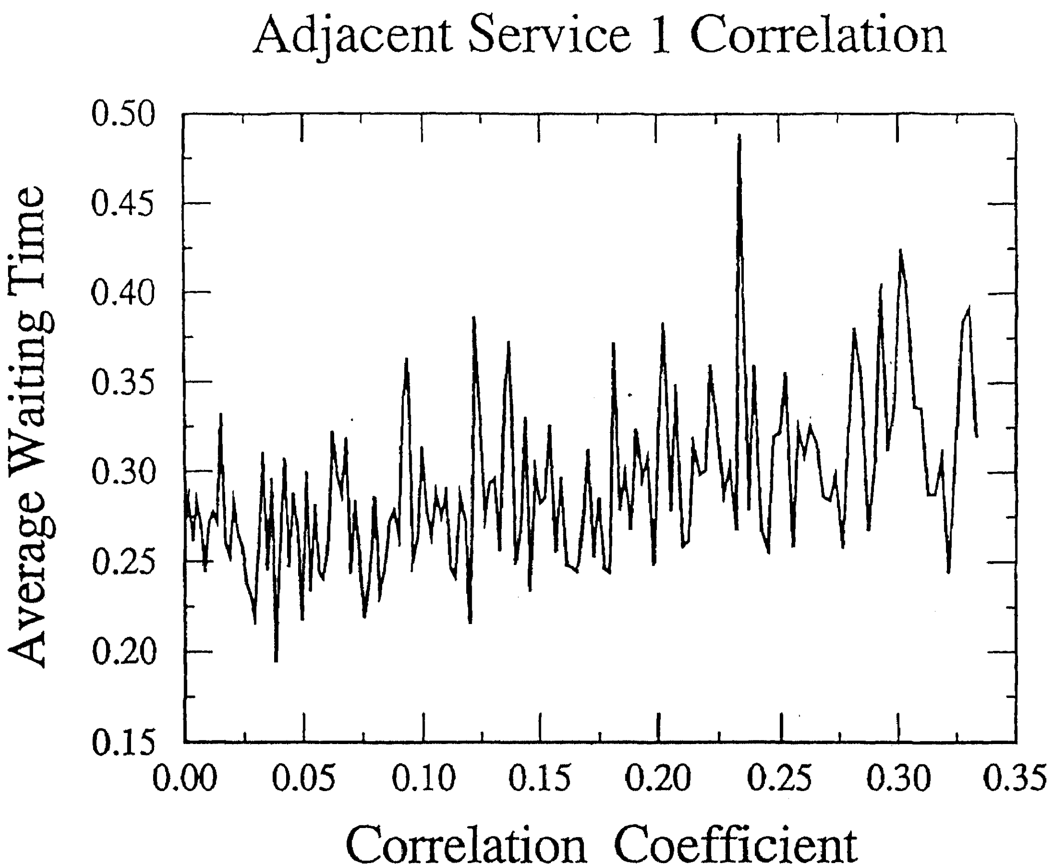

Figure 5.4 Effect of Positive Correlation Between Adjacent Services at Server 1 in Tandem Queue

Hence the exponential marginal distribution function is given by:

$$P[Q > t] = e^{-(p_1+2p_2)t}.$$

Customer interarrival time and Service time at server 1 follow exponential distribution with parameter u_2 and u_3 respectively.

The correlation coefficient between the adjacent customer interarrival time can be expressed as:

$$r = p_2 \,/\, (2p_1 + 3p_2) \tag{5.7}$$

The correction coefficient can be changed from 0 to 1/3.

Figure 5.5 is the simulation result of the model above, with running time 400, p_1=3, p_2=3, u_2=5, u_3=7. It shows that the average waiting time in the queue almost has no trend of increasing or decreasing, with respect to the increasing of correlation coefficient.

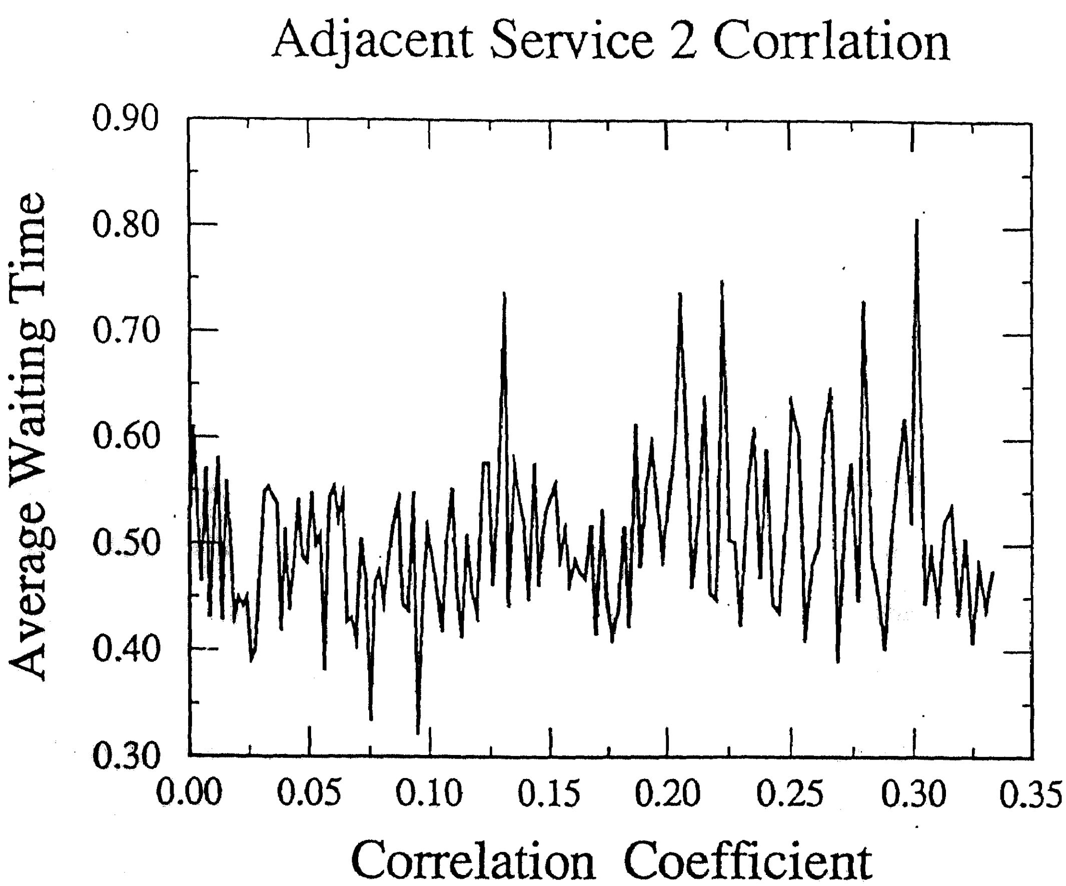

Figure 5.5 Effect of Positive Correlation Between Adjacent Services at Server 2 in Tandem Queue

CHAPTER 6

CONCLUSION

In the previous chapters, we analyzed the effects of dependencies on several G/G/1 queueing systems and 2-server tandem queues. Simulation results for each model are shown on figures and conclusions are drawn from analyzing the curves obtained by simulation. This chapter is the summary of these results.

In a single node, single server queue, positive correlation between interarrival time and service time has a trend of improving system performance measures, by decreasing the average waiting time in the queue or etc., with the increment of correlation coefficient. On the other hand, positive correlations between adjacent interarrivals and positive correlation between adjacent services have trends of worsening the system performance by increasing the average waiting time in the queue or etc., with the increment of correlation coefficient. Nevertheless, none of these effects has monotone property of effect of dependency on system performance. Negative correlation between interarrival time and service time, negative correlation between adjacent interarrivals and negative correlation between adjacent services do not have any trend or certain effect on system performance.

In the 2-server tandem system, positive correlation between interarrival time and service time at server 1 shows a trend of improving the system performance. Positive correlation between server 1 and server 2, positive correlation between adjacent interarrival time and positive correlation between adjacent service time at server 1 have trends of worsening system performance measures, by increasing the average waiting time in the queue with the increment of correlation coefficient. The positive correlation between adjacent service time at server 2 has almost no certain trend of effect on system performance. Again, none of these effects has monotone property of effect of dependency on system performance.

APPENDIX A
DATA

Positive Correlation between
Interarrival and Service
Generated by Bivariate Variables
in A Single Node Queue

Correlation Coefficient	Average Waiting Time in the Queue
0.000000	0.352907
0.003065	0.522851
0.006149	0.384457
0.009251	0.394570
0.012373	0.337386
0.015515	0.475894
0.018676	0.343482
0.021856	0.362042
0.025057	0.436016
0.028278	0.590071
0.031519	0.374040
0.034780	0.362420
0.038062	0.506607
0.041365	0.483176
0.044689	0.349757
0.048035	0.410109
0.051402	0.317168
0.054790	0.430969
0.058201	0.302783
0.061634	0.369379
0.065089	0.567848
0.068566	0.383583
0.072067	0.440497
0.075590	0.459728
0.079137	0.388192
0.082707	0.626548
0.086301	0.343815
0.089918	0.465843
0.093560	0.283837
0.097226	0.451053
0.100917	0.348195
0.104633	0.474589
0.108374	0.349887
0.112141	0.489496
0.115933	0.466603
0.119751	0.491533
0.123595	0.282196
0.127466	0.325254

0.131364	0.390263
0.135289	0.348932
0.139241	0.345692
0.143220	0.418540
0.147228	0.300611
0.151263	0.265932
0.155327	0.428655
0.159420	0.325212
0.163542	0.305268
0.167694	0.529815
0.171875	0.303122
0.176086	0.387070
0.180328	0.374856
0.184600	0.361285
0.188904	0.396124
0.193238	0.359581
0.197605	0.293370
0.202003	0.323406
0.206434	0.364097
0.210898	0.369711
0.215395	0.346091
0.219925	0.316456
0.224490	0.317920
0.229088	0.331815
0.233722	0.292352
0.238390	0.368053
0.243094	0.378426
0.247834	0.344857
0.252610	0.319151
0.257422	0.251359
0.262272	0.291541
0.267159	0.276079
0.272085	0.308634
0.277049	0.285296
0.282051	0.429516
0.287093	0.323788
0.292175	0.240538
0.297297	0.343450
0.302460	0.285154
0.307664	0.278798
0.312910	0.381372
0.318198	0.390131
0.323529	0.277664
0.328904	0.303699
0.334322	0.285539
0.339784	0.352823
0.345291	0.275592
0.350844	0.294449
0.356443	0.248830
0.362089	0.270300
0.367781	0.340660
0.373522	0.282093
0.379310	0.293268
0.385148	0.258411

0.391036	0.232939
0.396973	0.225414
0.402962	0.247194
0.409002	0.233242
0.415094	0.314374
0.421240	0.295396
0.427439	0.247719
0.433692	0.189930
0.440000	0.264692
0.446364	0.237173
0.452785	0.356970
0.459262	0.238856
0.465798	0.235553
0.472393	0.242438
0.479047	0.232948
0.485761	0.230127
0.492537	0.232285
0.499375	0.268564
0.506276	0.239890
0.513241	0.227463
0.520270	0.263753
0.527365	0.283412
0.534527	0.199067
0.541756	0.260715
0.549053	0.214456
0.556420	0.219368
0.563857	0.217920
0.571366	0.189755
0.578947	0.197566
0.586602	0.230070
0.594331	0.172100
0.602136	0.207190
0.610018	0.194291
0.617978	0.199653
0.626016	0.234557
0.634135	0.229325
0.642336	0.188191
0.650619	0.213855
0.658986	0.175090
0.667439	0.186101
0.675978	0.185412
0.684605	0.197692
0.693321	0.181202
0.702128	0.179762
0.711027	0.185247
0.720019	0.150812
0.729107	0.166293
0.738291	0.160686
0.747573	0.162189
0.756955	0.192996
0.766438	0.163910
0.776024	0.148390
0.785714	0.155931
0.795511	0.153050

0.805416	0.165300
0.815431	0.149936
0.825558	0.134815
0.835798	0.145871
0.846154	0.128024

APPENDIX B
PROGRAM

```
#include        <string.h>

#include        <math.h>

#include        <stdio.h>

#define     max(x,y)  (x>=y)?x:y

#define     min(x,y)  (x<=y)?x:y

/*SINGLE NODE QUEUEING SYSTEM*/

/*analyze bivariate dependency between interarrival time and

service time*/

main()

{

  double  drand48(), rand_t, rand_u, rand_s;

  float t,p1,p2,p3,p10,p20,p30;

  float a[10000],b[10000],c[10000],d[10000],z[10000];

  float dt[10000],du[10000],ds[10000];

  int i,j,k,n,m,l,num;

  int x,y,ql,tql;

  float e;

  float g,s1;

  float w[10000],s[10000],wt[200],aql[200],z1[200];

  FILE  *fp,*fq;

  printf(" input sumulation running time\n");
```

```
scanf("%f",&t);
/* input T series exponential distribution parameter */
printf(" input T series parameter\n");
scanf("%f",&p10);
/* input S series exponential distribution parameter */
printf("%s\n", "input S series parameter");
scanf("%f",&p20);
/* input U series exponential distribution parameter */
printf(" input U series parameter\n");
scanf("%f",&p30);
for (l=0;l<=150;l++) {
  e=(p10+p30)*l/150-p30;
  printf("e=%f\n",e);
  p1=p10-e;
  p2=p20-e;
  p3=p30+e;
  z1[l]=p3/(p1+p2+p3);
  a[0]=0.0;
  b[0]=0.0;
  c[0]=0.0;
 /* generate arrivals, compute arrival time, service
time*/
   for(i=1;i<=10000;i++) {
     rand_t=drand48();
```

```
    dt[i]=-(log(1-rand_t))/p1;    /*generate T series
data*/
    rand_s=drand48();
    ds[i]=-(log(1-rand_s))/p2;    /*generate S series
data*/
    rand_u=drand48();
    du[i]=-(log(1-rand_u))/p3;    /*generate U series
data*/
    d[i]=min(dt[i],du[i]);
    a[i]=a[i-1]+d[i];          /*compute arrival time*/
    b[i]=min(ds[i],du[i]);      /*compute service time*/
    if(a[i]>t) {
      n=i-1;
      break;
    }
  }
  /*compute service completion time if it doesn't exceed
simultion running time t*/
  for (j=1;10000;j++) {
    z[j]=max(c[j-1],a[j]);
    c[j]=z[j]+b[j];
    if (c[j]>t) {
      m=j-1;
      break;
```

```
    }
  }
  if(m<n) {
    for (i=m+1;i<=n;i++) c[i]=t;
  }
/*caculation of waiting time and server idle time*/
 g=0.0;
  for(i=1;i<=m;i++) {
    w[i]=max(0,c[i-1]-a[i]);
    g=g+w[i];
  }
  wt[l]=g/m;
  printf("%d\n",l);
 }
  if((fp=fopen("result99","w"))==NULL){
  printf("cannot open file\n");
  exit(0);
  }
  for(i=0;i<=150;i++) {
   fprintf(fp," %f                    %f\n",z1[i],wt[i]);
  }
  fclose(fp);
}
```

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