ABSTRACT

A mathematical model is developed to study the effect of extrusion speed and the bearing length on the surface temperature along the die billet interface. The surface temperature distribution resulting from the plastic work done on the die billet interface is studied. Finite element analysis is done on the model incorporating the mass transport effect occurring during the extrusion process and the interdependence of various parameters are presented. Die wearing problem is analyzed considering the effect of diffusion. Thermal analogy of the diffusion problem is utilized in the development of the mathematical model for diffusion problem and finite element analysis is done on the model to study the effect of controlling parameters in the extrusion process.

A FINITE ELEMENT THERMAL AND DIFFUSION ANALYSIS AT BEARING METAL INTERFACE IN EXTRUSION PROCESSES

by

BIJU DENNIS

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APPROVAL PAGE

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CHAPTER 1

1.1 INTRODUCTION

Extrusion is a commercially important manufacturing process to make intricately shaped products. The requirements like close dimensional tolerance, good surface properties and mechanical properties are functions of extrusion speed, bearing length etc. Productivity and optimum extrusion speed for stringent requirements are conflicting parameters and the drive of the process analysis is to achieve the maximum productivity meeting the requirements.

The prediction of the external force required to cause the metal flow and deform to the desired shape is limited by the uncertainties introduced by frictional effect, non-homogeneous deformation and the true manner in which strain hardening occur during complex deformations. The plastic work done over the area of contact with a die bearing can be simulated by replacing an equivalent heat source.

The exit temperature determines the surface quality and it increases with extrusion speed. This implies that there exist limits to extrusion speed for a particular alloy or product. The exit temperature is a function of the speed of extrusion, the reduction ratio, initial billet temperature, the mechanical properties of the material and the friction at billet tool interface. Complex interaction of these parameters prevent an analytical approach. The problem is non-linear involving thermo-mechanical coupling. Other approximation methods such as Upper Bound and Slip Line techniques are too simplistic. Therefore, numerical solution is the only way to unravel the interaction of the complex parameters.

1.2 REVIEW OF THE ANALYTICAL TECHNIQUES

1.2.1 Introduction:

The problem under study, thermal analysis at the die billet interface in extrusion process, is similar to the thermal analysis of the metal flow through an orifice. This can be analyzed using different techniques, but all of them have serious limitations.

The exact values of force requirements to cause the plastic flow are seldom predictable. Exact solutions require that both stress equilibrium and a geometrically self-consistent pattern of flow are satisfied simultaneously everywhere throughout the deforming body and on its surface. Limits theorem provide values that are either lower (Lower Bound) or higher (Upper Bound) than the actual values. Several methodologies have been developed using pertinent assumptions to calculate an approximate load requirements. To varying degrees, each of the uncertainties involved in the analysis are introduced into these analytical methods, thereby permitting an estimation of the deformation forces, the constraining forces, and the manner in which metal flow occur. These techniques do not provide a means to predict the mechanical properties of the deformed material, the maximum possible deformation up to fracture, or any variation in frictional effects during deformation.

1.2.2 Uniform Energy or Ideal Work Method:

In this simplest method the force requirement is calculated using the concept of energy or work balance. The external work is equated to the energy consumed in deforming the work piece. The process is assumed to be ideal in the sense that the external work done is solely used to cause deformation. Frictional effect and nonhomogeneous effects are neglected. The prediction done neglecting these facts is a serious mistake in the case of extrusion process where large non homogeneous deformation occur along with frictional heating.

1.2.3 Free Body Equilibrium Approach:

This method also known as slab analysis, gives results by force balance on a slab of metal of differential thickness. This analysis formulate a differential equation where the variations are considered in one direction only. Using pertinent boundary conditions, an integration of this equation then provide a solution. The assumptions involved in these are following:

(1) The direction of applied load and the planes perpendicular to the direction define principal directions, and the principal stresses do not vary in these planes.

(2) Even if the surface friction is included in the force balance, its effect on the internal distortion of the metal or the orientation of the principal directions are not taken into consideration.

(3) Deformation is assumed to be homogeneous in regard to the determination of induced strain.

All the above assumptions will seriously affect the applicability of the technique to the study of temperature distribution along the bearing metal interface.

1.2.4 Upper Bound Analysis:

In metal deforming operation we require the prediction of a force that will definitely cause the plastic deformation. This analysis focus on satisfying the yield criterion keeping the geometric self-consistency. This analysis look for a kinematically admissible solution. Following are the assumptions made in the analysis:

(1) Specimen is isotropic and homogeneous.

(2) The effect of strain hardening and strain rate on flow stress are neglected.

(3) Frictionless or constant shear stress conditions are imposed on the die billet interface.

(4) Flow is assumed to be two dimensional (plane strain). If the shear is occurring in intersecting planes that are not orthogonal, the sected plane may not be the plane of maximum shear stress.

All the above assumptions limit the use of upper bound analysis for the thermal analysis along the bearing metal interface.

1.2.5 Slip-line Field Theory:

This analysis is based upon a deformation field that is geometrically consistent with the shape change. The strain within the fields should be kinematically admissible. Following are the assumptions made in the analysis:

(1) The metal is rigid-perfectly plastic. This assumes the flow stress as constant without work hardening and neglect the effect of elastic strains.

(2) Deformation is plane strain only.

(3) Effects of stain rate and temperature changes due to deformation.

(4) Constant shear is assumed in the interface.

Above assumptions made in the technique limit the use of these methods for the extrusion analysis.

CHAPTER 2

2.1 MATHEMATICAL MODELING

The heat generation due to the plastic work dissipation and the stain rate and temperature dependence of the yield strength of the material should be included for the process analysis. Replacing the plastic work by an equivalent heat source over the moving work piece will simulate the process. The mathematical model reduces the complex process to the solution of a thermal problem with transport phenomena. The transport process is time dependent at the onset of the process, when the material started emerging from the die. However, at a longer time it can be treated as a steady state convective circumstance. This is due to the fact that a time factor is involved to attain a steady flow situation, because of the end effects, stemming from the energy losses at the end, do not affect transport over the moving surface. So assuming a typical values of surface convective heat transfer co-efficient, the temperature distribution of the moving plate can be calculated.

The 3-D problem is reduced to 2-D problem of a flat rectangular section moving with a velocity and heat is supplied at constant rate over the area of contact with the die bearing. It is assumed that there exists sticking friction and the plastic work is done due to the shearing of aluminium.

The power density of heat generation $P = \tau * V$ [W/m^2]

τ = shear stress [Pa]

V = velocity [m/s]

The plastic work done (τ.V) along the bearing billet interface is equated to a heat input Q there. To find Q, a reasonable value of τ is taken for the extrusion velocity. Shear stress is a function of temperature and strain rate and we have experimental curves for that relationship (refer section 2.2.2).

$\sigma_y = \sigma_o \; f_1(T) * f_2(\dot{e})$

σ_y = Yield stress

σ_o = Static yield stress

T = temperature

$\dot{e}$ = strain rate

With an assumed τ value, Q is calculated for different extrusion speeds and is applied along the die billet interface. FEA analysis provide the temperature distribution along the surface. For the known temperature, we have $f_1(T)$ from the curve (Fig. 1.3 (a)) and the only unknown is $f_2(\dot{e})$ now. From the equation $f_2(\dot{e})$ is calculated and the corresponding strain rate from curve (Fig. 1.3 (b)) is taken. These strain rates from the model analysis match with the strain rates from a separate flow modeling experiment for different speeds. This agreement support the validity of the assumed model.

2.2 FINITE ELEMENT ANALYSIS OF THE MODEL

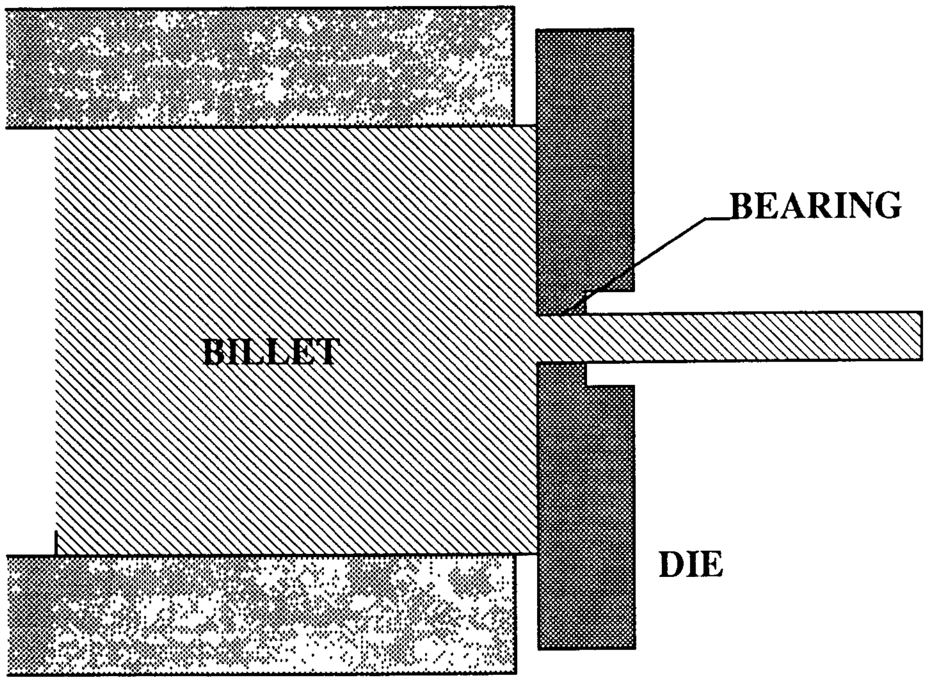
2.2.1 Introduction:

A schematic of the extrusion process is shown in Fig.1.1. The basic model is a flat rectangular section (Fig 1.2). The section has prescribed thickness and velocity and is approaching the die with a prescribed initial temperature. In order to study the effect of back conduction of the extruding section, a 10 mm model length is extended upstream of the die and to study the effect of cooling same length is extended downstream of the die. Only half of the system is analyzed to reduce the computational requirements, taking the advantage of the plane of symmetry of the model.

2.2.2 Material Properties:

Material properties of aluminium and steel are listed in table 1.1.

The thermal properties of the materials are taken to be independent of temperature, ignoring the very small error involved in it at the concerned temperature ranges. Due to the large strain rate, yield stress is taken as a function of strain rate and temperature.



Schematic of Extrusion Process

FIGURE 1.1

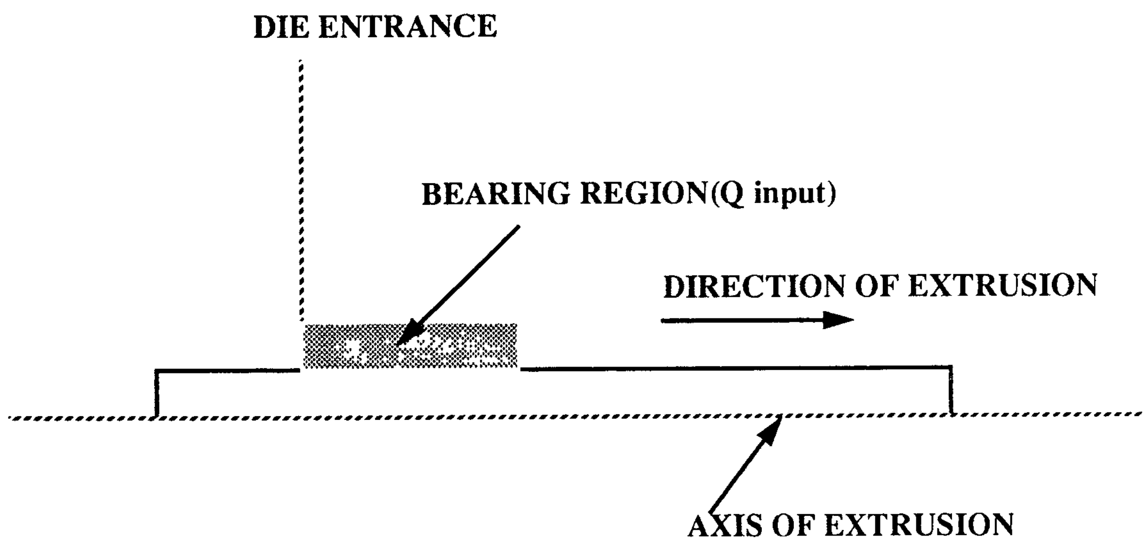

DIE ENTRANCE
BEARING REGION(Q input)
DIRECTION OF EXTRUSION
AXIS OF EXTRUSION

FIGURE 1.2

TABLE 1

Thermal Properties of Aluminium and Steel

	Aluminium	Steel
Thermal Conductivity	215 W/m C	48 W/m C
Density	2600 Kg/m^3	7800 Kg/m^3
Specific Heat	1100 J/Kg C	450 J/Kg C

Fig. 1.3 shows a typical curves for the yield stress as a function of temperature and strain rate for 6063 alloy, based on the results of Akeret[3].

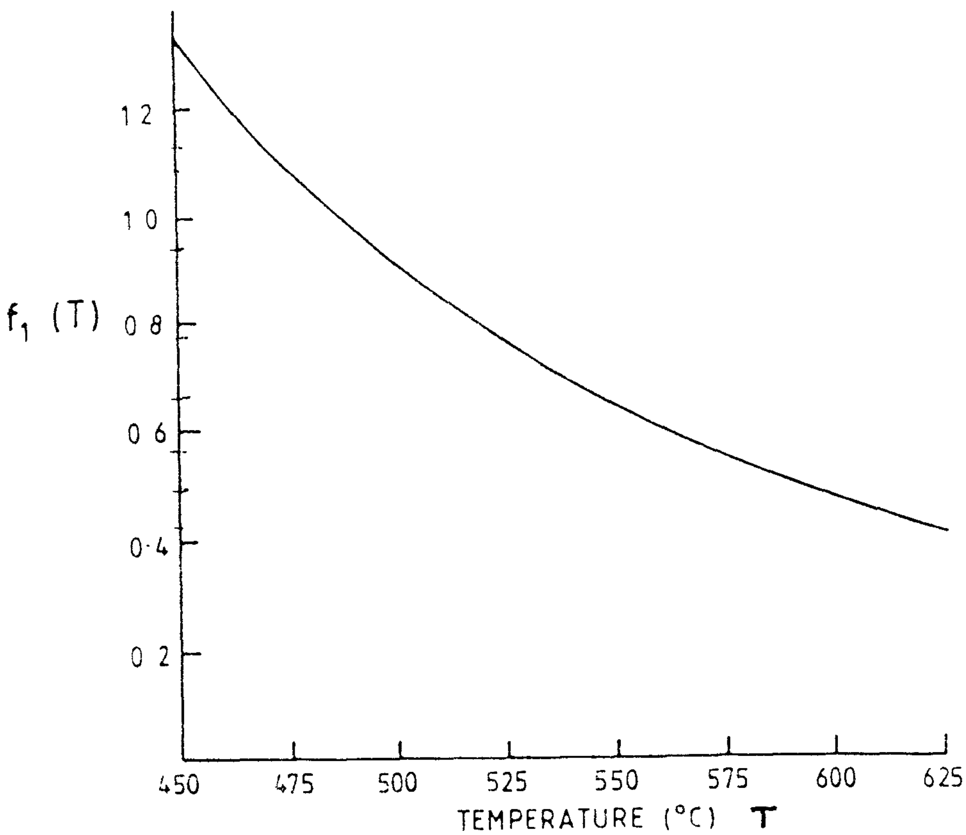
Strain rate is unknown and to pitch the correct value of shear stress the results from the study of Jowett and Coupland was taken. A value of 6.2 MPa at 500 deg. C is taken for the shear stress. This implies a yield stress of 32.4 MPa. A flow modelling study indicate that in the presence of sticking friction, with an extrusion speed of 1 m/s, the strain rate near the surface is of the order of 200 per sec. For that amount of strain rate, $f_2(e)$ is about 2.5, which gives the explanation for the high value of shear stress. This value is applied throughout the modelling.

2.2.3 FEA Element:

The finite element selected is STIFF 55 of ANSYS[9], 4-node isoparametric quadrilateral. The meshing of the element is done with KEYOPT(8) = 1, to supports mass transport effect.

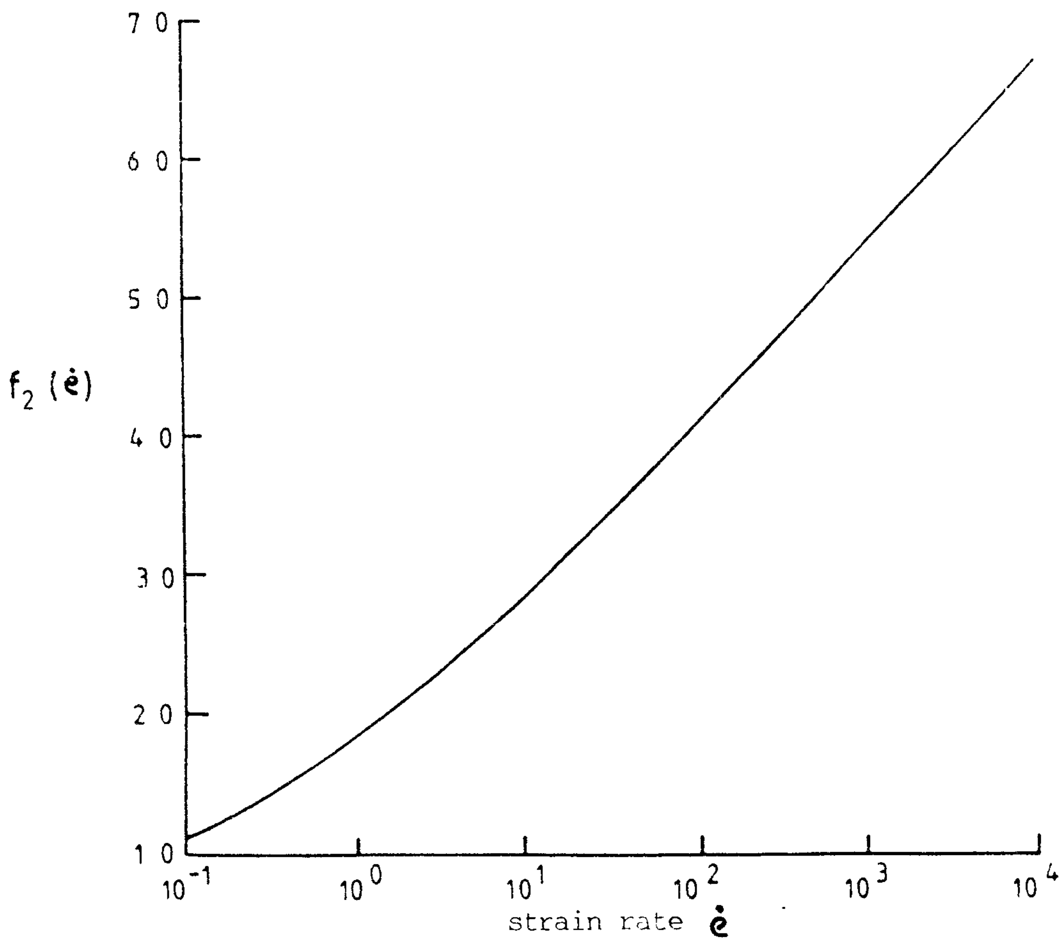
Theory:

The temperature distribution for the element is obtained from the numerical solution of the following equation (for plane analysis).



VARIATION OF f_1 (T) WITH TEMPERATURE

FIGURE 1.3 (a)



VARIATION OF $f_2(\dot{e})$ WITH STRAIN RATE FIGURE 1.3 (b)

$$\rho Cp\,(\frac{\partial}{\partial t}T + Vx\frac{\partial}{\partial x}T + Vy\frac{\partial T}{\partial y}) = \frac{\partial}{\partial x}(Kxx\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(Kyy\frac{\partial T}{\partial y})$$

where ρ = density

C = specific heat

K = thermal conductivity

q = internal heat generation rate

V = velocity of mass transport

T = temperature

Since the analysis is steady state

$$\frac{\partial T}{\partial t} = 0$$

2.2.4 Boundary Conditions:

The problem is reduced to steady state thermal analysis, and the meshing is done with an element supporting mass transport property. The section upstream is kept at a temperature of 500 deg. C. Under steady state condition heat conduction into the die is very small and has very small impact on the temperature distribution. The bearing surface is considered adiabatic to simplify the problem. This will accommodate the most severe conditions for heat generation in the section. The boundary conditions of the section downstream of the die bearing is taken as adiabatic. In actual case there will be heat radiation from the surface or cooling from an external source. An adiabatic boundary condition will simulate the most severe case possible, where the heat generated is completely absorbed by the extruding material.

CHAPTER 3

3.1 COMPARISON OF FEA SOLUTION WITH ANALOGOUS ANALYTICAL SOLUTION

Carslow & Jaeger presents an analytical solution for the surface temperature generated when an infinite strip supplies heat at the rate Q per unit time per unit area over the strip, and the surrounding medium moves across it with velocity V. This semi-infinite block analysis is analogous to the problem of die bearing friction as modelled. The solution can be expressed by the following dimensionless quantities: x/b

$$\pi . \Delta T . K V / 2aQ$$

$$B = V b/2a$$

b = half width of the heat source

x = position relative to the center of the heat source

ΔT = temperature increase

K = thermal conductivity

V = velocity of the heat source relative to the slab

a = thermal diffusivity

Q = power intensity of heat source

The boundary conditions for the analytical solution are simulated in the model by taking a slab of sufficient thickness and neglecting the heat penetration to the surface remote from the heat source.

Using the thermal properties of the aluminium, for b=3 mm, a value for B=10 could be obtained by taking the velocity V = 0.5 m/s. Q is set at 6,000,000 W/m^2 in order to obtain a reasonable temperature increase of 30 deg.C at the peak temperature on the curve for B=10.

The ANSYS solution for B=10 is superimposed on the analytical solution in Fig. 1.4. The deviations of both solutions are practically nil, which implies the validity of the FEA model implemented.

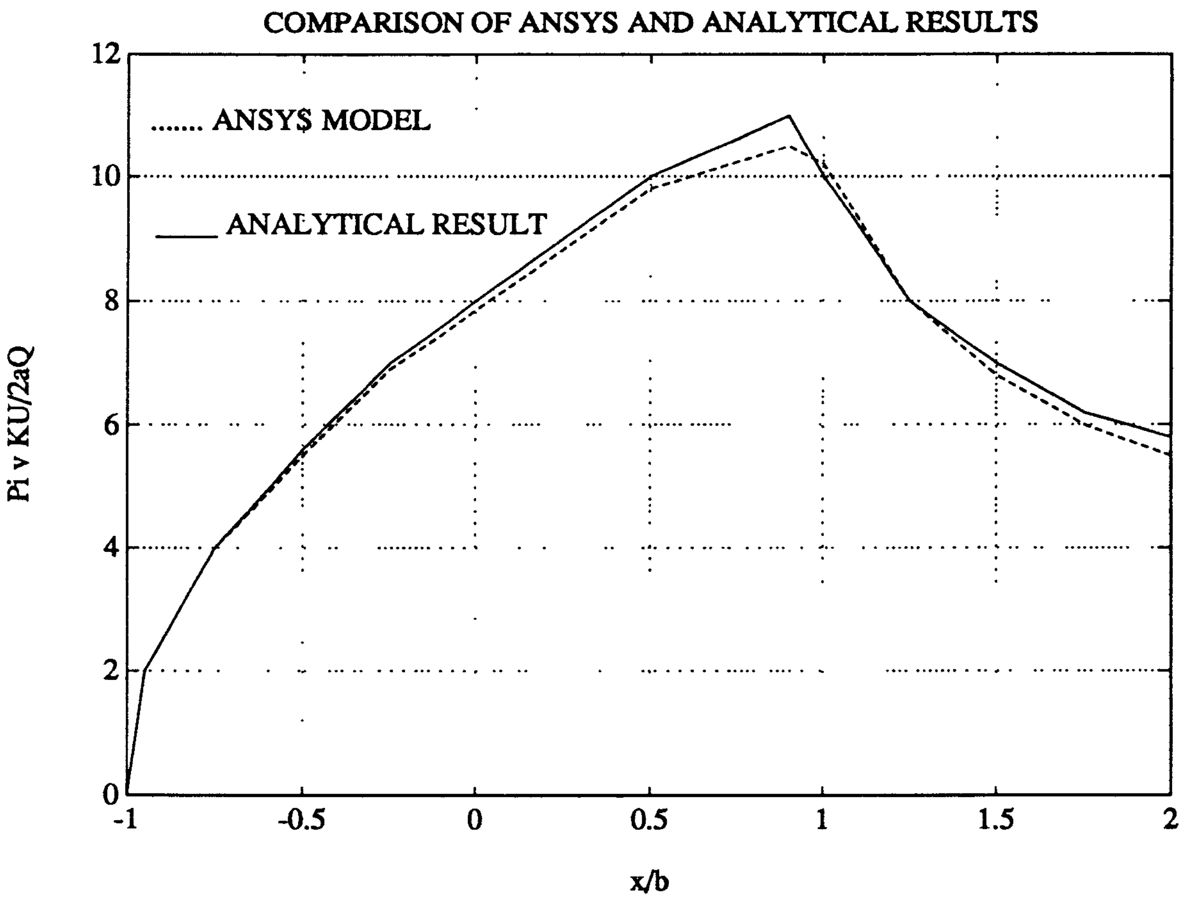

COMPARISON OF ANSYS AND ANALYTICAL RESULTS
.... ANSYS MODEL
____ ANALYTICAL RESULT
Pi v KU/2aQ
x/b
12
10
8
6
4
2
0
-1
-0.5
0
0.5
1
1.5
2

Figure 1.4

From the Carslow and Jaeger's analytical solution for surface temperature rise (section 3.1), we have for large values of B (Ub/2a) the max. temperature occur near x = b and is approximately

$$\frac{Qb}{K\sqrt{\pi B}}$$

This implies Temperature maxima,

$$T_{\max.} \propto \frac{Q}{\sqrt{B}}$$

Heat supplied

$$Q = q(t)\,U$$

This implies

$$Q \propto U$$

$$B = \frac{Ub}{2a}$$

Temperature maxima,

$$T_{\max.} \propto \frac{Q}{\sqrt{B}} \propto \frac{U}{\sqrt{U}} \propto \sqrt{U}$$

This proportionality from the analytical solution is in agreement with FEA result.

3.2 RESULTS AND DISCUSSIONS

3.2.1 Temperature Distribution in Section:

Short residence time in bearing:

Fig. 1.5 gives the temperature distribution at various depth of the section, when the velocity is 0.5 m/s and the bearing length is 6 mm. The residence time is so small that there is no significant temperature rise in the midplane. During these heating face the section behave like a semi-infinite body.

(1) There is no significant heat conduction upstream of the section.

(2) Mass transport is the dominant characteristic.

(3) Peak temperature is a surface effect and is attenuated even 0.3 mm below the surface.

(4) Peak temperature decays very steeply once the section leaves the bearing, and hence the measurement made even a few mm. downstream of the bearing will not reflect the actual temperature.

This study cover a wide range of operating conditions, and where the die bearing heating limit the extrusion speed.

Fig.1.6 (a) and Fig. 1.6 (b) shows the ANSYS contour display of temperature for 0.25 m/s and 0.75 m/s extrusion velocity.

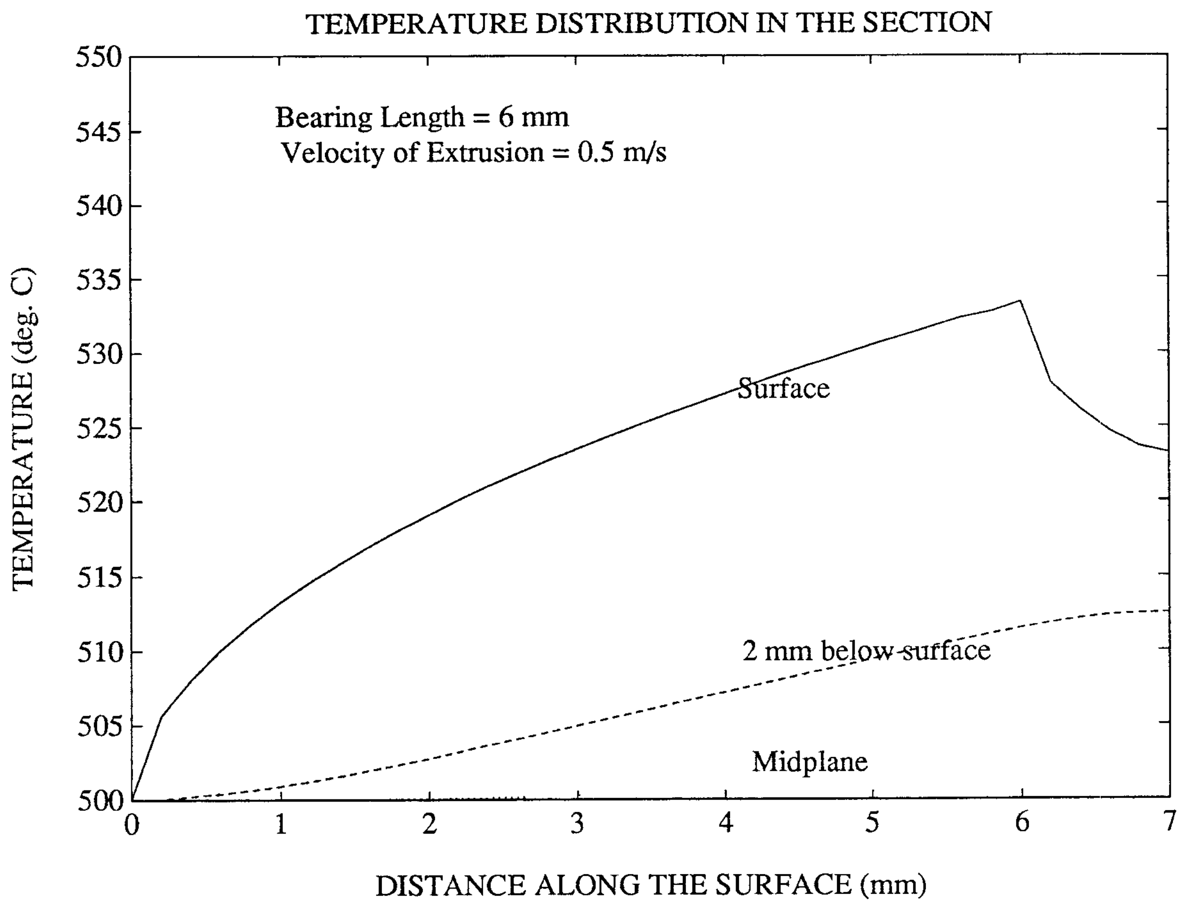

TEMPERATURE DISTRIBUTION IN THE SECTION
Bearing Length = 6 mm
Velocity of Extrusion = 0.5 m/s
TEMPERATURE (deg. C)
550
545
540
535
530
525
520
515
510
505
500
Surface
2 mm below surface
Midplane
0
1
2
3
4
5
6
7
DISTANCE ALONG THE SURFACE (mm)

Figure 1.5

Extended residence time in the bearing:

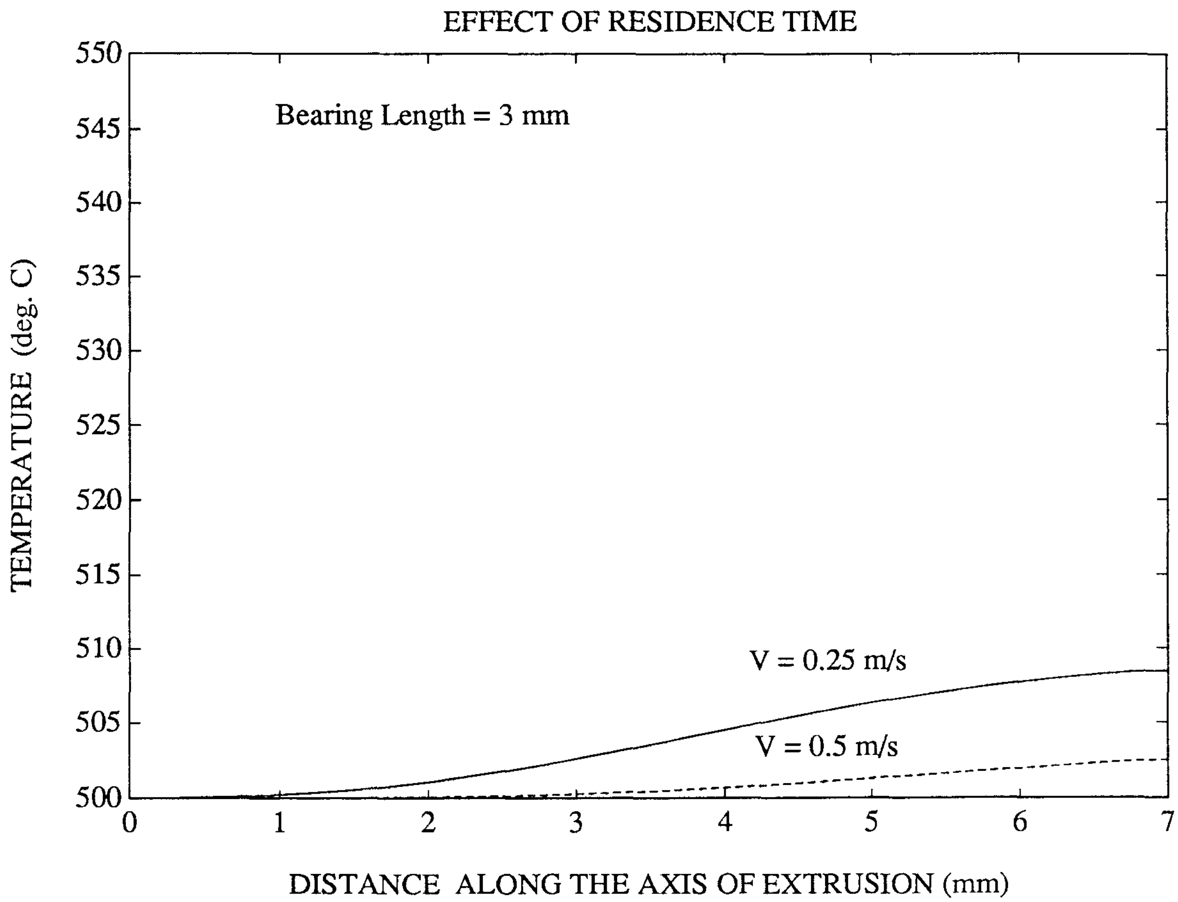
Fig. 1.7 shows the temperature history of the midplane with two different extrusion speeds. Once the residence time is increased with slow extrusion there is significant heating in the midplane. This is not of practical importance sine this condition of midplane heating occur when a thin section extrude through a long bearing with a slow extrusion speed.

3.2.2 Effect of velocity:

Fig. 1.8 shows the temperature distribution along the surface of the section for different velocities of 0.25, 0.50, 0.75 m/s. From the figure it is apparent that the peak temperature rise is proportional to the square root of the velocity.

For the conditions satisfying the section to behave as a semi infinite body, *the temperature rise is proportional to the square root of the velocity*.

As the residence time increases there is greater diffusion of heat to the middle of the section, and the temperature rise dependence of the velocity becomes less and less. In the limiting condition it became independent of the velocity. The rise in the average temperature of the section is substantially independent of velocity. This is consistent with the thermodynamic relations, since the power input is linearly related to the velocity and hence the mass flow of metal to which the power is applied is also linearly related.



EFFECT OF RESIDENCE TIME
Bearing Length = 3 mm
V = 0.25 m/s
V = 0.5 m/s
TEMPERATURE (deg. C)
550
545
540
535
530
525
520
515
510
505
500
0
1
2
3
4
5
6
7
DISTANCE ALONG THE AXIS OF EXTRUSION (mm)

Figure 1.7

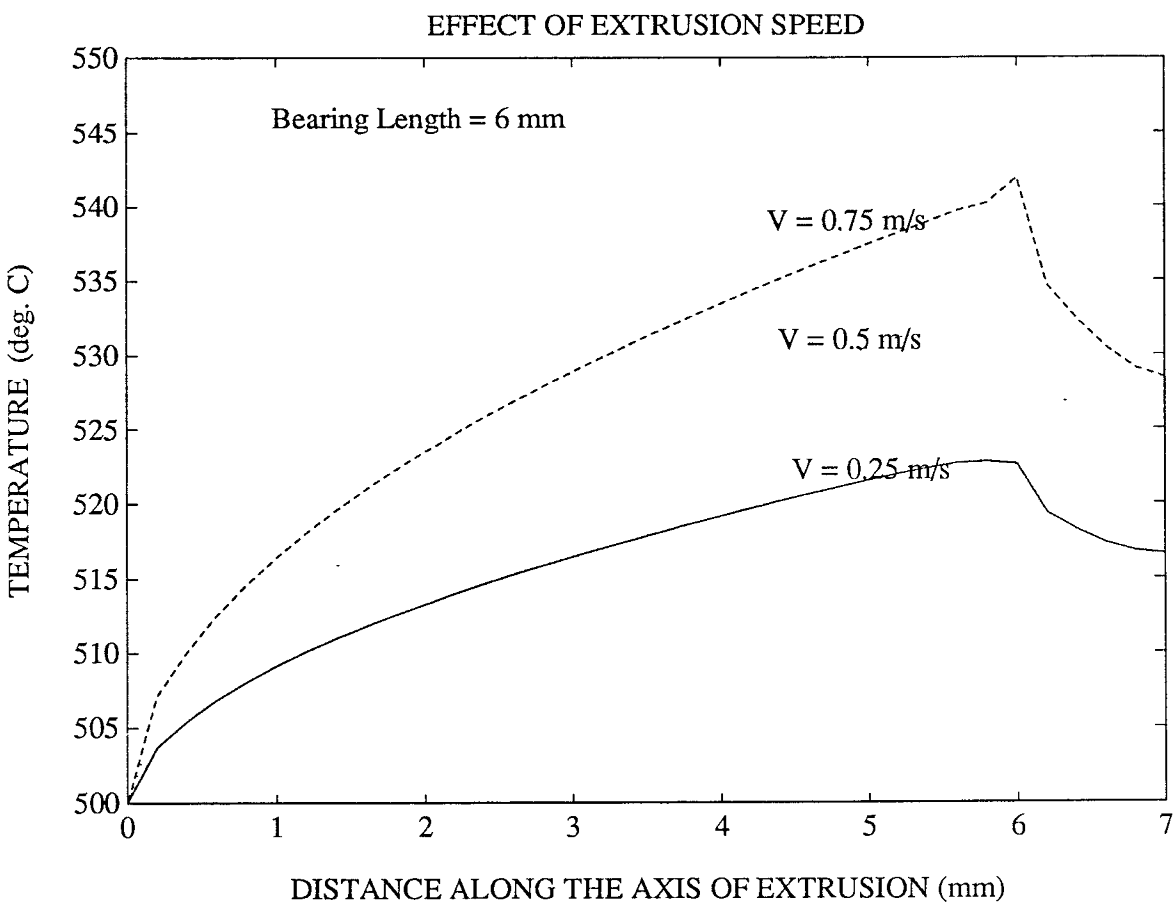

EFFECT OF EXTRUSION SPEED
Bearing Length = 6 mm
V = 0.75 m/s
V = 0.5 m/s
V = 0.25 m/s
TEMPERATURE (deg. C)
550
545
540
535
530
525
520
515
510
505
500
0
1
2
3
4
5
6
7
DISTANCE ALONG THE AXIS OF EXTRUSION (mm)

FIGURE 1.8

3.2.3 Effect of Bearing Length:

Fig. 1.9 shows the surface temperature history for various bearing length. It is clear from the figure that the *peak temperature rise is proportional to the square root of the bearing length.*

This relationship is valid for the cases where the section behaves like a semi infinite body. Otherwise in the limiting case the average temperature rise is proportional to the bearing length.

3.2.4 Combined Effect of Velocity and Bearing Length:

Peak surface temperature rise is correlated for all cases where the section behaves as a semi infinite body and the following relation is obtained.

Peak temperature rise $= 17.7\ (BV)^{0.5}$

where B = bearing length, mm

V = extrusion speed, m/s

The average temperature rise in the section is directly proportional to the bearing length and is independent of the velocity of extrusion.

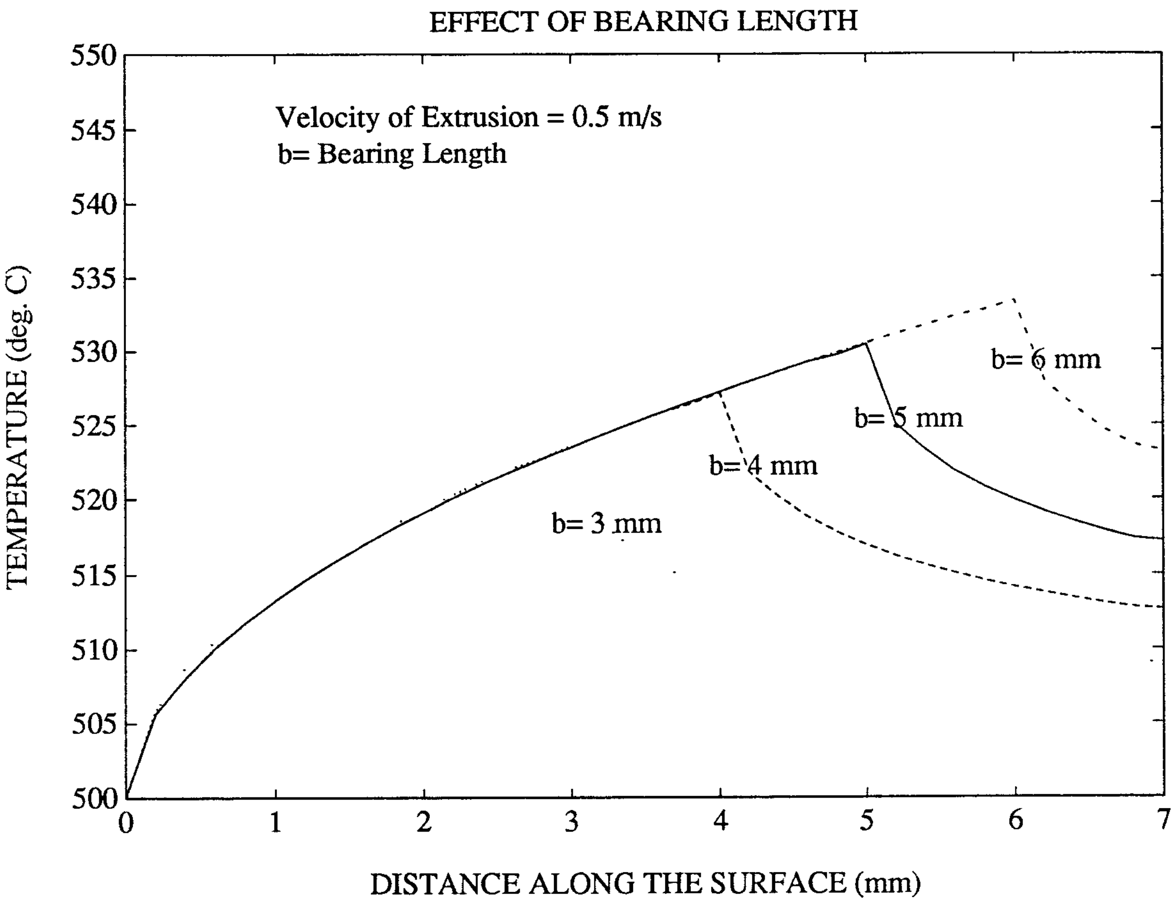
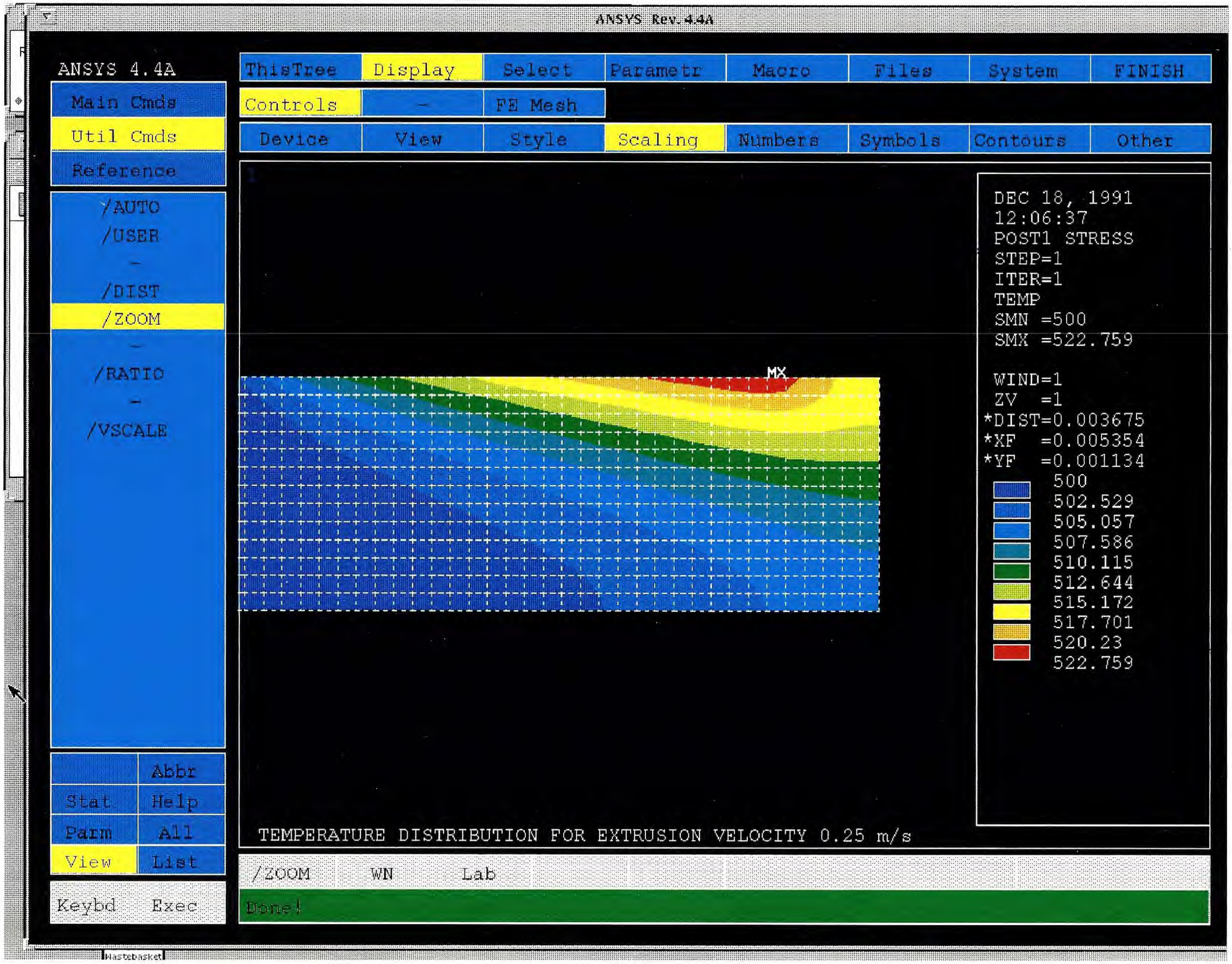

EFFECT OF BEARING LENGTH
Velocity of Extrusion = 0.5 m/s
b= Bearing Length
b= 6 mm
b= 5 mm
b= 4 mm
b= 3 mm
TEMPERATURE (deg. C)
DISTANCE ALONG THE SURFACE (mm)
550
545
540
535
530
525
520
515
510
505
500
0
1
2
3
4
5
6
7

Figure 1.9


ANSYS Rev. 4.4A
ANSYS 4.4A
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Util Cmds
Device View Style Scaling Numbers Symbols Contours Other
Reference
/AUTO
/USER
/DIST
/ZOOM
/RATIO
/VSCALE
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ITER=1
TEMP
SMN =500
SMX =522.759
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ZV =1
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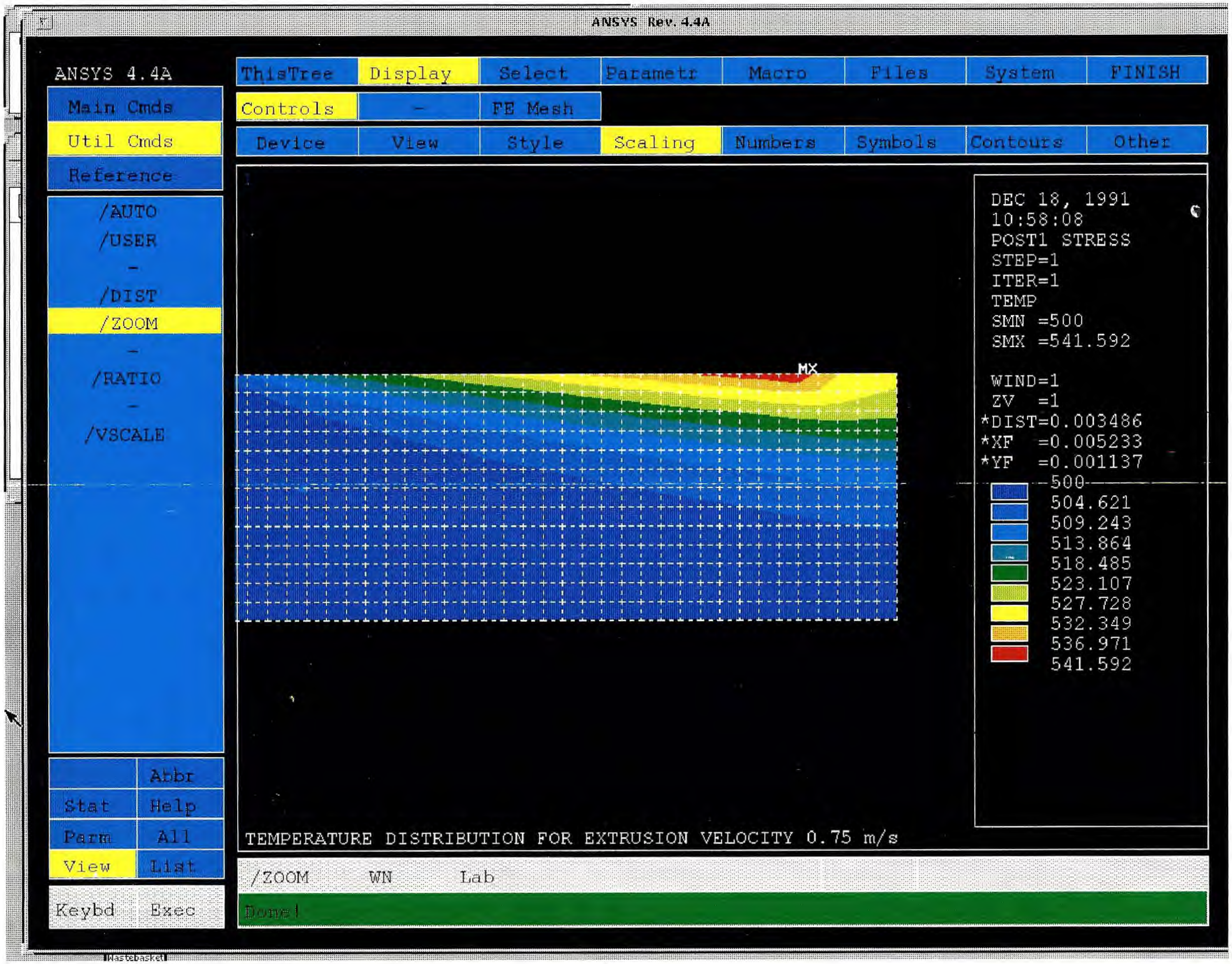
*XF =0.005354
*YF =0.001134
500
502.529
505.057
507.586
510.115
512.644
515.172
517.701
520.23
522.759
MX
TEMPERATURE DISTRIBUTION FOR EXTRUSION VELOCITY 0.25 m/s
Abbr
Stat Help
Parm All
View List
/ZOOM WN Lab
Keybd Exec
Done!
Wastebasket



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ANSYS 4.4A
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System
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Main Cmds
Util Cmds
Reference
Controls
FE Mesh
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Scaling
Numbers
Symbols
Contours
Other
/AUTO
/USER
/DIST
/ZOOM
/RATIO
/VSCALE
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ZV =1
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*XF =0.005233
*YF =0.001137
500
504.621
509.243
513.864
518.485
523.107
527.728
532.349
536.971
541.592
MX
TEMPERATURE DISTRIBUTION FOR EXTRUSION VELOCITY 0.75 m/s
Abbr
Stat
Help
Parm
All
View
List
Keybd
Exec
/ZOOM
WN
Lab
Done!

3.2.5 Boundary Layer Thickness:

There is a boundary layer formation in between the extruding material and the die. The plastic work done on the interface is simulated by applying a heat input of $Q = 6 \times 10^6$ W/m^2 sec.

$$Q = q\,V$$

For $V = 0.25$ m/s ,

$$q = Q / V = 24 \times 10^6 \text{ Pa}$$

We have the relation,

$$\text{Yield stress} = \text{Static Yield Stress} \quad f_1(t)\ f_2(\dot{e})$$

$$\text{Yield Stress} = 48 \text{ MPa}$$

$$\text{Static Yield Stress} = 10 \text{ MPa}$$

From curve 1.3 (a), $f_1(522.7) = 0.74$

Solving the equation for $f_2(e)$, we have

$$f_2(\dot{e}) = 6.48$$

From the curve 1.3 (b) for $f_2(\dot{e}) = 6.48$ we have,

$$\text{strain rate} = 10^4 \text{ sec}^{-1}$$

$$\text{Strain rate} = dV / dx$$

Since the die is stationary and the billet is moving with a velocity of V, we have $dV = V$ and dx is the thickness of boundary layer.

Solving for dx, we have

$$dx = 0.25 * 10^{-4} \ \text{m}$$

Boundary Layer Thickness = 0.025 mm

CHAPTER 4

4.1 DIFFUSION EFFECT ON DIE WEARING

4.1.1 Introduction:

The die wearing in the extrusion process can be related to the erosion of the die material due to the diffusion effect. The driving parameters acting along the die billet interface for the diffusion are the strong concentration gradient and the temperature dependent diffusion coefficient.

Solid containing initially uniform dilute concentration of constituent elements upon the application of mechanical and thermal loading, will develop regions of high constituent concentration . This will result in local degradation of material properties. Process such as hydrogen embrittlement and stress corrosion cracking are two examples of degradation process resulting from constituent mass transport within solid materials. The analysis of mass transport problem in solid diffusion has been limited to the solution of differential equation that are formulated as a function of concentration, stress and temperature.

At the die billet interface, there exists severe concentration and temperature gradient which will accelerate the diffusion effect. Diffusion problem is analogous to the thermal problem and thermal analysis will simulate the analogous diffusion problem and the controlling parameters involved are studied by changing the extrusion speed and bearing length.

4.1.2 Review of Diffusion theory:

Fick's Laws of Diffusion:

This law is similar to the Fourier equation in the classical heat flow analysis. Fick's first law states that atoms moves from regions of high concentration to that of lower concentration. The equation for the flow of matter which is consistent with these condition is

$$J = - D \frac{\partial c}{\partial x}$$

where J is the flux of atoms across unit are of the plane at any instant, and the concentration gradient is normal to the plane at the same instant.

Fick's second law called the continuity equation stems from the concept of the conservation of matter. It states that the rate of change of concentration with respect to the time is equal to the rate of change of the flux.

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}[D. \frac{\partial c}{\partial x}]$$

$$\frac{\partial c}{\partial t} = D.\nabla^2 c$$

Analytical solution for transient state Fickian diffusion and for heat conduction in solid phase are well known when the diffusion coefficient is constant. The solutions have been reviewed in detail by Carslaw and Jaeger (1959), Crank (1975) and others. However the solution becomes extremely complicated when the thermo-mechanical coupling is existing along with the concentration gradient. A number of solutions have been done by Crank (1975) and they are quite complex due to the non-linearity of the partial differential equations.

CHAPTER 5

5.1 MATHEMATICAL MODELING OF THE DIFFUSION PROBLEM

5.1.1 Thermal analogy of diffusion:

Fourier equation for heat conduction is:

$$Flux = -K\frac{\partial T}{\partial x}$$

$$\rho Cp\left(\frac{\partial}{\partial t}T + Vx\frac{\partial}{\partial x}T + Vy\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left(Kxx\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(Kyy\frac{\partial T}{\partial y}\right)$$

where K is thermal conductivity, C the specific heat and ρ the density.

Fick's equation for diffusion is

$$J = -D\frac{\partial c}{\partial x}$$

$$\frac{\partial c}{\partial t} + Vx\frac{\partial c}{\partial x} + Vy\frac{\partial c}{\partial y} = \frac{\partial}{\partial x}\left(Dxx\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(Dyy\frac{\partial c}{\partial y}\right)$$

where D is the diffusion coefficient and c the concentration. For the extrusion process Vy = 0 , Vx = extrusion velocity and Dxx = Dyy.

k = K/ρC, This term in thermal analysis is similar to D in diffusion problem.

The above relations imply that the heat flux in thermal analysis is similar to the mass flux in diffusion analysis. Diffusion coefficient D is analogous to k.

The problem to analyze is the dependence of various parameters involved in the extrusion process to the mass flux going into the billet from die. This will provide the informations about the die wearing in the extrusion process. Mass flux is directly proportional to the diffusion coefficient and the concentration gradient.

5.1.2 Variation of D with temperature:

The diffusion coefficient is a very strong function of temperature and it is virtually always represented by Arrhenius law

$$D = D_0 \, Exp(-Q/RT)$$

where D_0 is a constant and Q is called the activation energy. The value of D is always quoted in cgs units and the units of D are in cm^2/sec. The Arrhenius law is approximately valid and the activation energy given is within 2% by the empirical correlations. Due to the mass transport effect there exists temperature gradient along the surface and hence the diffusion coefficient is also varying along the surface.

Due to the mass transport effect concentration gradient between the die and the billet is also varying along the die surface. The two parameters, concentration gradient and the diffusion coefficient variation along the surface have opposing trend and the resultant effect will produce a flux gradient along the surface. The peaking of flux value along the die billet interface point out the region where maximum die erosion or wear occur. The total flux going into the billet for different velocities gives the idea of the amount of die wear dependence on extrusion speed.

5.2 FEA MODELING OF DIFFUSION PROBLEM

5.2.1 Model geometry and the Element type:

The same model geometry for the thermal analysis is taken for the diffusion analysis. The finite element meshing is done with ANSYS element STIFF 55, with KEYOPT(8) equal to 1 to incorporate the transport effect.

5.2.2 Boundary conditions:

In order to simulate the diffusion process in extrusion, following boundary conditions are taken. The flux flow occur from the die to the billet and can be idealized as the flux flow across a boundary from concentration $c=1$ to concentration $c=0$ initially. The concentration c in the die is assumed to remain constant with respect to time due to the fact that the ratio of concentration c in die to that at any time in billet is very large. Due to the mass transport, concentration of material from die in the billet is varying along the interface between die and billet and is increasing down stream of the die.

There is a temperature variation along the die billet interface. This will have an impact on the diffusion coefficient, because D is a function of temperature. In order to incorporate this effect D values corresponding to the temperatures along the surface is calculated. Keeping density and specific heat as constants and the temperature dependent D, which is similar to k , values are given to the concerned elements as a material property. D is equivalent to k (thermal diffusivity) in thermal analysis.

Temperature rise in the thermal analysis is in the range of 500 ^{0}C - 540 ^{0}C. This range is divided into 8 regions and the corresponding D values are calculated by the Arrhenius equation

$$D = D_0 \, Exp\,(-Q/RT)$$

Approximate D value for aluminium in this temperature range is 1 x e-10 cm^2/sec. D_0 and Q/R values are set to get the D value in the same range as that of aluminium. D is similar to k (thermal diffusivity). Specific heat and density is assumed to remain constant and the thermal conductivity (K) corresponding to the D values for different temperature range is given in the table 2.1. Fig 2.1 shows the variation of D with respect to the temperature.

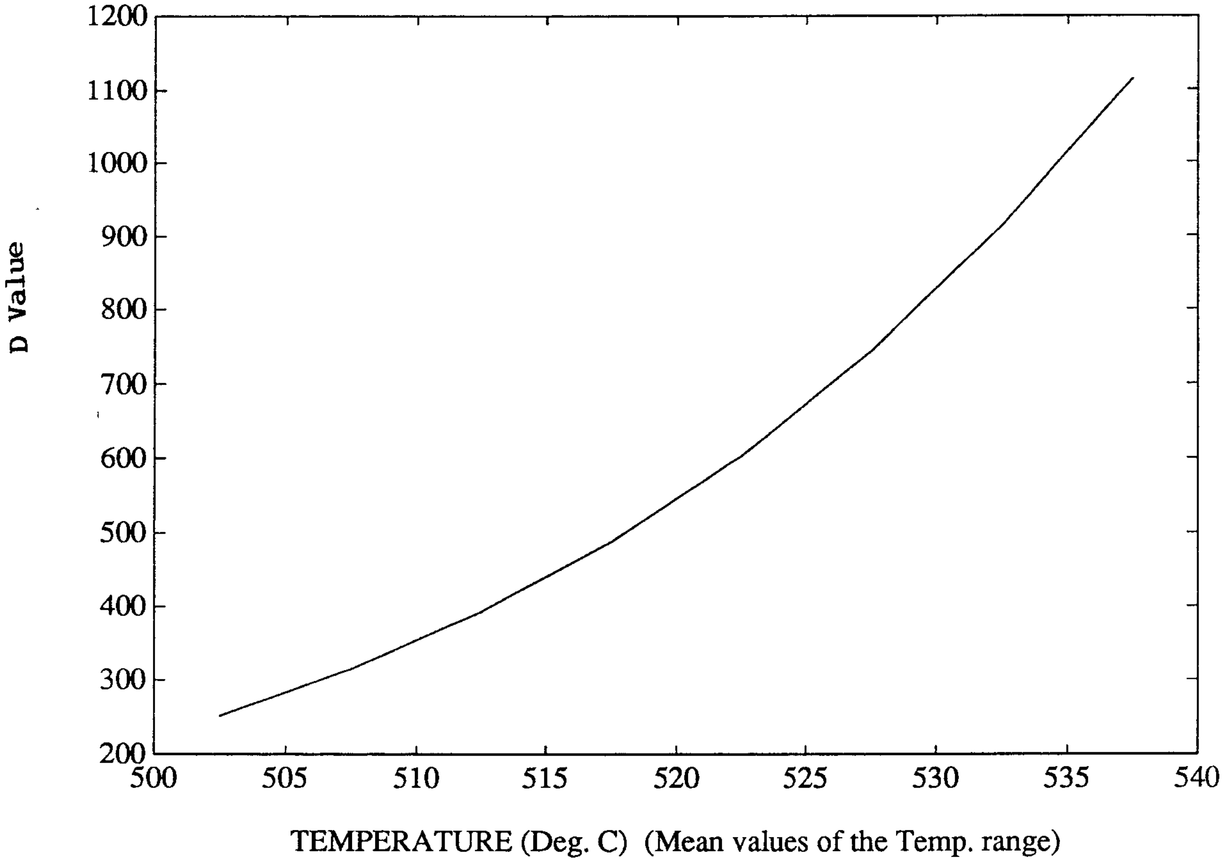
5.2.3 Validity of the model:

The governing equation in diffusion is

$$J = - D \frac{\partial C}{\partial x}$$

This implies heat flux is proportional to the diffusion coefficient. Figure 2.2 and 2.3 shows the variation of flux with two sets of D values which are linearly related. From the figure it is clear that the flux values also vary linearly following the D sets. This result support the validity of the model.

FIGURE 2.1



D Value
1200
1100
1000
900
800
700
600
500
400
300
200
500
505
510
515
520
525
530
535
540
TEMPERATURE (Deg. C) (Mean values of the Temp. range)

CHAPTER 6

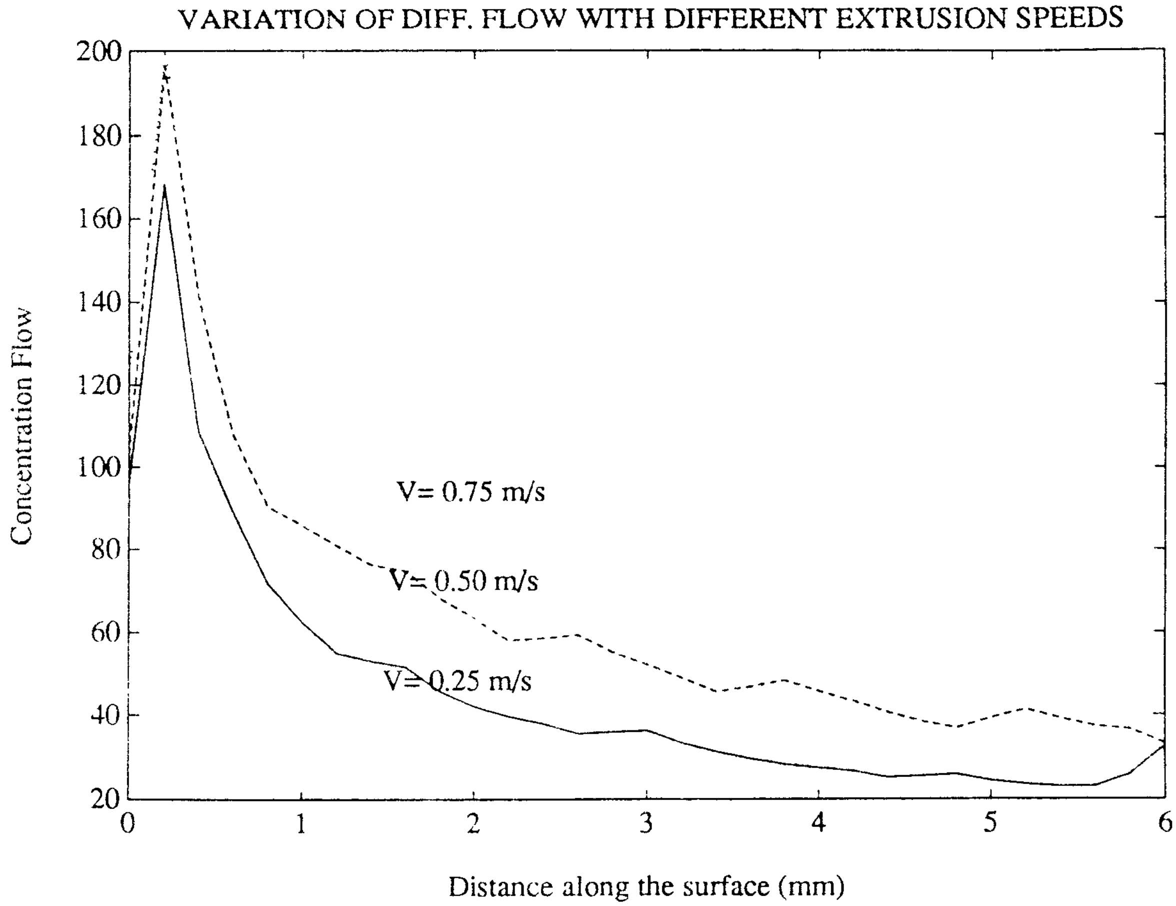
6.1 RESULTS AND DISCUSSION

6.1.1 Variation of Total Flux with Velocity:

Total flux along the surface gives an idea about the amount of matter gone out of the die and hence the wear. For three speeds 0.25 m/s, 0.50 m/s and 0.75 the total flux along the surface is calculated. Table 2.2, 2.3, 2.4 gives the values of the flux. Figure 2.4 shows the flux curves plotted for different speeds. Following are the total and normalized flux values for three different speeds.

V (m/s)	Flux (total)	Normalized Flux
0.25	1339.18	1
0.50	1891.30	1.41
0.75	2395.31	1.7

From the above values it is clear that the total flux varies as the square root of velocity of extrusion.


VARIATION OF DIFF. FLOW WITH DIFFERENT EXTRUSION SPEEDS
200
180
160
140
120
100
80
60
40
20
0
1
2
3
4
5
6
Concentration Flow
Distance along the surface (mm)
V= 0.75 m/s
V= 0.50 m/s
V= 0.25 m/s

6.1.2 Effect of Concentration Gradient:

From the flux curves it is apparent that there is severe die erosion at the billet entrance. This heavy flux flow at the entrance is due to the severe concentration gradient existing there. This fact is in agreement with the observed results. The severe influence of the concentration gradient at the die entrance override the effect of D variation along the surface. This is the limitation of the model presented.

6.1.3 Die life and Extrusion Velocity:

The life of the die is inversely proportional to the total flux going out from the die.

$$T = C_i \times Flux^{-1}$$

where T is the die life and C_i is a constant.

We have from the result

$$Flux = C_j \times V^{0.5}$$

This implies

$$T \times V^{0.5} = C_k$$

The above observation is in agreement with the result from the thermal analysis.

CHAPTER 7

CONCLUSIONS

A flat section model has been taken to study the temperature distribution along the surface and inside the section. The boundary conditions are applied to satisfy the practical operating conditions of the extrusion process. The finite element meshing is done with an element having the mass transport property. The finite element analysis results are validated by comparing with the analytical solution of an analogous problem.

For slow extrusion where the residence time is large, there is significant temperature penetration into the section and the model is no more a semi-infinite body. In this situation the temperature rise due to the bearing friction is independent of the extrusion speed and is directly proportional to the bearing length.

For the ideal operating condition, with high extrusion speed, thicker section and shorter bearing length, there is no significant temperature penetration into the section and the section behaves like a semi-infinite body. In this case the temperature rise is a surface phenomena and is proportional to the square root of the extrusion velocity and bearing length.

From thermal analysis, the peak temperature rise is proportional to the $V^{0.5}$. Since the temperature gradient curve from thermal analysis and the concentration gradient curve from the diffusion analysis have the same nature, it can be assumed that both hold the same relationship with the velocity. This implies that the concentration gradient is proportional to the square root of velocity. From the finite element analysis of the diffusion problem, total flux is proportional to $V^{0.5}$. Combining the above two statements, we have flux proportional to the concentration gradient. This conclusion from the analysis is in perfect agreement with the classical Fick's first law of diffusion and support the validity of the model analyzed.

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APPENDIX I

PREP 7 OF ANASYS FINITE ELEMENT THERMAL ANALYSIS FOR EXTRUSION

****

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BEARING LENGTH = 6 mm.
EXTRUSION SPEED = 0.25 m/s

DISTANCE (mm)	TEMPERATURE deg. C
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0.2	503.70043
0.4	505.42284
0.6	506.87628
0.8	508.08061
1.0	509.14444
1.2	510.10316
1.4	510.98320
1.6	511.80084
1.8	512.56769
2.0	513.29221
2.2	513.98081
2.4	514.63851
2.6	515.26930
2.8	515.87645
3.0	516.46267
3.2	517.03020
3.4	517.58110
3.6	518.11690
3.8	518.63900
4.0	519.14870
4.2	519.64690
4.4	520.13441
4.6	520.61113
4.8	521.07634
5.0	521.52707
5.2	521.95664
5.4	522.34525
5.6	522.66324
5.8	522.75139
6.0	522.59428
6.2	519.35563
6.4	518.25378
6.6	517.38153
6.8	516.83118
7.0	516.62188

TEMPERATURE DISTRIBUTION ALONG SURFACE

BEARING LENGTH = 6 mm.
EXTRUSION SPEED = 0.75 m/s

DISTANCE (mm)	TEMPERATURE deg. C
0.0	500.0000
0.2	507.13631
0.4	510.05565
0.6	512.55824
0.8	514.6125
1.0	516.42668
1.2	518.06065
1.4	519.56063
1.6	520.95442
1.8	522.26178
2.0	523.49694
2.2	524.67064
2.4	525.79121
2.6	526.86524
2.8	527.89808
3.0	528.89413
3.2	529.8570
3.4	530.7899
3.6	531.6954
3.8	532.5758
4.0	533.4330
4.2	534.2688
4.4	535.0848
4.6	535.8822
4.8	536.6624
5.0	537.4258
5.2	538.1765
5.4	538.9018
5.6	539.6626
5.8	540.1818
6.0	541.8643
6.2	534.6376
6.4	532.4119
6.6	530.5334
6.8	529.1108
7.0	528.4581

TEMPERATURE DISTRIBUTION ALONG SURFACE

BEARING LENGTH = 3 mm.
EXTRUSION SPEED = 0.5 m/s

DISTANCE (mm)	TEMPERATURE deg. C
0.0	500.0000
0.2	505.86233
0.4	508.80375
0.6	510.55964
0.8	512.20417
1.0	513.63711
1.2	514.94433
1.4	516.14802
1.6	517.27024
1.8	518.32514
2.0	519.32313
2.2	520.27359
2.4	521.17506
2.6	522.05506
2.8	522.73777
3.0	523.63740
3.2	518.39280
3.4	516.83360
3.6	515.53819
3.8	514.59424
4.0	513.82230
4.2	513.17845
4.4	512.62626
4.6	512.14468
4.8	511.71883
5.0	511.33818
5.2	510.99493
5.4	510.68311
5.6	510.39809
5.8	510.13619
6.0	509.89444
6.2	509.67049
6.4	509.46273
6.6	509.27163
6.8	509.10766
7.0	509.03481

TEMPERATURE DISTRIBUTION ALONG SURFACE

BEARING LENGTH = 4 mm.
EXTRUSION SPEED = 0.5 m/s

DISTANCE (mm)	TEMPERATURE deg. C
0.0	500.00000
0.2	505.64641
0.4	508.05376
0.6	510.10495
0.8	511.79205
1.0	513.28142
1.2	514.62260
1.4	515.85343
1.6	516.99677
1.8	518.06891
2.0	519.08158
2.2	520.04363
2.4	520.96194
2.6	521.84193
2.8	522.68810
3.0	523.50367
3.2	524.29295
3.4	525.05064
3.6	525.80135
3.8	526.36698
4.0	527.15996
4.2	521.81769
4.4	520.16863
4.6	518.79020
4.8	517.76927
5.0	516.92573
5.2	516.21509
5.4	515.60042
5.6	515.06028
5.8	514.57943
6.0	514.14706
6.2	513.75518
6.4	513.39826
6.6	513.07491
6.8	512.80050
7.0	512.67917

TEMPERATURE DISTRIBUTION ALONG THE SURFACE

BEARING LENGTH = 6 mm.
EXTRUSION SPEED = 0.5 m/s

DISTANCE (mm)	TEMPERATURE deg. C
0.0	500.0000
0.2	505.6464
0.4	508.0537
0.6	510.1049
0.8	511.7929
1.0	513.2814
1.2	514.6226
1.4	515.8534
1.6	516.9967
1.8	518.0689
2.0	519.0815
2.2	520.0436
2.4	520.9619
2.6	521.8419
2.8	522.6881
3.0	523.5039
3.2	524.2925
3.4	525.0565
3.6	525.7980
3.8	526.5188
4.0	527.2208
4.2	527.9052
4.4	528.5734
4.6	529.2264
4.8	529.8654
5.0	530.4909
5.2	531.1049
5.4	531.7004
5.6	532.3006
5.8	532.7259
6.0	533.3878
6.2	527.9233
6.4	526.1613
6.6	524.6879
6.8	523.6447
7.0	523.2024

TEMPERATURE DISTRIBUTION AT A DEPTH OF 0.2 mm BELOW SURFACE

BEARING LENGTH = 6 mm.
EXTRUSION SPEED = 0.5 m/s

DISTANCE (mm)	TEMPERATURE deg. C
0.0	500.0000
0.2	500.0316
0.4	500.2236
0.6	500.4006
0.8	500.6313
1.0	500.9095
1.2	501.2277
1.4	501.5782
1.6	501.9544
1.8	502.3507
2.0	502.7623
2.2	503.1858
2.4	503.6181
2.6	504.0567
2.8	504.4999
3.0	504.9463
3.2	505.3945
3.4	505.8437
3.6	506.2931
3.8	506.7422
4.0	507.1904
4.2	507.6375
4.4	508.0831
4.6	508.5270
4.8	508.9691
5.0	509.4092
5.2	509.8469
5.4	510.2809
5.6	510.7075
5.8	511.1190
6.0	511.5035
6.2	511.8473
6.4	512.1390
6.6	512.3665
6.8	512.4621
7.0	512.5215

TEMPERATURE DISTRIBUTION ALONG MIDPLANE

BEARING LENGTH = 6 mm.
EXTRUSION SPEED = 0.5 m/s

DISTANCE (mm)	TEMPERATURE deg. C
0.0	500.0000
0.2	500.0014
0.4	500.0005
0.6	500.0013
0.8	500.0029
1.0	500.0057
1.2	500.0104
1.4	500.0175
1.6	500.0279
1.8	500.0425
2.0	500.0622
2.2	500.0877
2.4	500.1198
2.6	500.1594
2.8	500.2069
3.0	500.2631
3.2	500.3282
3.4	500.4026
3.6	500.5801
3.8	500.6834
4.0	500.7964
4.2	500.9192
4.6	501.1932
4.8	501.3441
5.0	501.5041
5.2	501.6728
5.4	501.8501
5.6	501.0356
5.8	502.2290
6.0	502.4298
6.2	502.6367
6.4	502.8457
6.6	503.0381
6.8	503.1262
7.0	503.1920

TEMPERATURE DISTRIBUTION ALONG MIDPLANE

BEARING LENGTH = 3 mm.
EXTRUSION SPEED = 0.25 m/s

DISTANCE (mm)	TEMPERATURE deg. C
0.0	500.00000
0.2	500.01621
0.4	500.04355
0.6	500.08557
0.8	500.14593
1.0	500.22813
1.2	500.33527
1.4	500.46990
1.6	500.63386
1.8	500.82820
2.0	501.05320
2.2	501.30832
2.4	501.59222
2.6	501.90279
2.8	502.23720
3.0	502.59192
3.2	502.96284
3.4	503.34546
3.6	503.73505
3.8	504.12688
4.0	504.51648
4.2	504.89977
4.4	505.27325
4.6	505.63405
4.8	505.97998
5.0	506.30944
5.2	506.62141
5.4	506.91530
5.6	507.19087
5.8	507.44800
6.0	507.68647
6.2	507.90539
6.4	508.10224
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6.8	508.39608
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APPENDIX II

PREP 7 OF ANSYS - FOR DIFFUSION PROBLEM

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TABLE 2.1

TEMPERATURE (deg. C)	K
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505 - 510	315.0
510 - 515	393.0
515 - 520	488.0
520 - 525	603.6
525 - 530	743.6
530 - 535	912.4
535 - 540	1115.2

TABLE 2.2

FLUX ALONG THE SURFACE

BEARING LENGTH = 6 mm

EXTRUSION SPEED = 0.25 m/s

DISTANCE (mm)	FLUX
0.0	95.543
0.2	168.219
0.4	108.773
0.6	89.123
0.8	71.732
1.0	62.464
1.2	55.173
1.4	53.154
1.6	51.638
1.8	45.783
2.0	42.118
2.2	39.751
2.4	37.972
2.6	35.583
2.8	36.012
3.0	36.326
3.2	33.396
3.4	31.335
3.6	29.579
3.8	28.279
4.0	27.421
4.2	26.624
4.4	25.129
4.6	25.502
4.8	25.922
5.0	24.307
5.2	23.441
5.4	22.938
5.6	23.052
5.8	25.855
6.0	32.576

TABLE 2.3

FLUX ALONG THE SURFACE

BEARING LENGTH = 6 mm

EXTRUSION SPEED = 0.5 m/s

DISTANCE (mm)	FLUX
0.0	103.332
0.2	196.531
0.4	141.351
0.6	107.976
0.8	90.455
1.0	85.840
1.2	80.890
1.4	76.227
1.6	74.699
1.8	68.011
2.0	63.352
2.2	58.011
2.4	58.443
2.6	59.284
2.8	55.297
3.0	52.232
3.2	49.061
3.4	45.748
3.6	46.972
3.8	48.492
4.0	45.866
4.2	43.429
4.4	40.657
4.6	38.491
4.8	36.918
5.0	39.349
5.2	41.362
5.4	39.158
5.6	37.349
5.8	36.600
6.0	33.244

TABLE 2.4

FLUX ALONG THE SURFACE

BEARING LENGTH = 6 mm

EXTRUSION SPEED = 0.75 m/s

DISTANCE (mm)	FLUX
0.0	112.597
0.2	199.387
0.4	159.711
0.6	147.917
0.8	119.624
1.0	104.587
1.2	102.415
1.4	99.199
1.6	93.345
1.8	90.226
2.0	81.607
2.2	77.013
2.4	72.047
2.6	72.197
2.8	72.272
3.0	66.969
3.2	63.108
3.4	64.404
3.6	64.739
3.8	59.704
4.0	56.649
4.2	54.676
4.4	52.478
4.6	54.125
4.8	54.807
5.0	51.466
5.2	49.917
5.4	49.023
5.6	48.541
5.8	52.021
6.0	61.087

TEMPERATURE DISTRIBUTION ALONG MIDPLANE

BEARING LENGTH = 3 mm.
EXTRUSION SPEED = 0.5 m/s

DISTANCE (mm)	TEMPERATURE deg. C
0.0	500.00000
0.2	500.00014
0.4	500.00051
0.6	500.00132
0.8	500.00291
1.0	500.00570
1.2	500.01036
1.4	500.01750
1.6	500.02795
1.8	500.04256
2.0	500.06219
2.2	500.08768
2.4	500.11984
2.6	500.15937
2.8	500.20687
3.0	500.26282
3.2	500.32756
3.4	500.40124
3.6	500.48386
3.8	500.57519
4.0	500.67483
4.2	500.78218
4.4	500.89648
4.6	501.01682
4.8	501.14222
5.0	501.27165
5.2	501.40408
5.4	501.53850
5.6	501.67397
5.8	501.80963
6.0	501.94471
6.2	502.07845
6.4	502.20996
6.6	502.33698
6.8	502.44990
7.0	502.50093