## Copyright Warning \& Restrictions

The copyright law of the United States (Title 17, United States Code) governs the making of photocopies or other reproductions of copyrighted material.

Under certain conditions specified in the law, libraries and archives are authorized to furnish a photocopy or other reproduction. One of these specified conditions is that the photocopy or reproduction is not to be "used for any purpose other than private study, scholarship, or research." If $a$, user makes a request for, or later uses, a photocopy or reproduction for purposes in excess of "fair use" that user may be liable for copyright infringement,

This institution reserves the right to refuse to accept a copying order if, in its judgment, fulfillment of the order would involve violation of copyright law.

Please Note: The author retains the copyright while the New Jersey Institute of Technology reserves the right to distribute this thesis or dissertation

Printing note: If you do not wish to print this page, then select "Pages from: first page \# to: last page \#" on the print dialog screen

The Van Houten library has removed some of the personal information and all signatures from the approval page and biographical sketches of theses and dissertations in order to protect the identity of NJIT graduates and faculty.

## INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be reniōved, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality $6^{\prime \prime} \times 9^{\prime \prime}$ black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

# On local stresges and spring constants of pipe-nozzle connection 

Sun, Hson-Chih, Ph.D.

New Jersey Institute of Technology, 1991

300 N. Zeeb Rd.
Ann Arbor, MI 48106

## NOTE TO USERS

THE ORIGINAL DOCUMENT RECEIVED BY U.M.I. CONTAINED PAGES WITH SLANTED AND POOR PRINT. PAGES WERE FILMED AS RECEIVED.

THIS REPRODUCTION IS THE BEST AVAILABLE COPY.

# ON LOCAL STRESSES AND SPRING CONSTANTS OF <br> PIPE-NOZZLE CONNECTION 

by<br>Hson-Chih Sun

Dissertation submitted to the Faculty of the Graduate school of the New Jersey Institute of Technology in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
1990

## APPROVAL SHEET



Name: Hson-Chih Sun

Degree and date to be conferred: Doctor of Philosophy, 1991

| Collegiate institutions attended | Dates | Degree Date of Degree |  |
| :--- | :--- | :--- | :--- | :--- |
| New Jersey Institute of Tech. | $86-90$ | Ph.D. | Dec., 1990 |
| New Jersey Institute of Tech. | $84-86$ | M.S.M.E. May, 1986 |  |
| National Chiou-Tung University | $77-81$ | B.S. | May, 1981 |

Major: Mechanical Engineering

## Publications:

1."On Radial and Rotational Spring Constants of Piping with a Square Tubing Attachment." ASME, Pressure Vessel and Piping Conference, San Diego, CA., June, 1987. (with B. C. Sun)
2."Radial Rotational Spring Constants of PIping-Lug Attach ment", ASME, PVPD Conference, San Diego, CA., June, 1987. (with B. C. Sun)
3."Finite Element Analysis of Piping-Nozzle Connections, Part I - Spring Constants." Proceeding, PVP - Vol. 194, ASME PVPD Conference, Nashville, Tenn., June 1990. (with B. C. Sun, H. Herman).
4." Finite Element Analysis of Piping-Nozzle Connections, Part II - Localized Stresses." Proceeding, PVp - Vol. 194, ASME, PVPD Conference, Nashville, Tenn., June 1990. (with B. C. Sun, H. Herman).
5."Local Stresses of Piping-Nozzle due to shear forces and Torsional Moment by Finite Element Method." to be published in ASME, PVPD Conference, San Diego, CA., June 1991. (with B. C. Sun, H. Herman)

Position held: Mechanical Engineer
Formosa Plastics Corporation, USA
9, Peach Tree Hill Road, Livingston, New Jersey 07032

## ABSTRACT

# of Thesis: On Local Stresses and Spring Constants of Pipe-Nozzle Connection 

Hson-Chih Sun, Doctor of Phisolophy, 1991

Thesis directed by: Dr. Benedict C. Sun Associate Processor

This thesis presents a comprehensive study of local stresses and spring coefficients of pipe with a nozzle connection, analyzed by the finite element method (FEM). Six types of loading are discussed: radial force, circumferential moment, longitudinal morient, circumferential shear force, longitudinal shear force, and torsional moment.

For the local stresses, the bending and membrane stress factors due to each of these loadings are presented in a series of plots with various gamma (piping radius/thickness) and beta (nozzle radius/pipe radius) values. These stress factors will readily replace those previously published in WRC No. 107. This work not only gives more accurate results, but also provides an extended range of beta for large combinations of previously unavailable nozzle-pipe geometries. Comparisons with data from available literature sources show that the finite element results from the thin shell model are very reasonable.

In the study of spring coefficients, three types of spring constant coefficients are presented: the coefficients in the radial direction and rotational coefficients in the circumferential and longitudinal directions. This study was previously conducted by Murad \& Sun, and Sun \& Sun. They used Bijlaard's double Fourier series solutions of displacement due to a distributed square load on the surface of a closed cylinder to derive the spring coefficients at the nozzle-pipe connection.

Due to the convergence problem in the double Fourier solution, the beta value in all the previous work on locai stresses and spring coefficients is limited to a maximum of 0.55. Using the ANSYS FEM code, the maximum beta has been extended from 0.55 to 0.9 , the gamma's (pipe radius/pipe thickness) range is 5 to 200 , while the alpha (pipe length/pipe radius) has been taken as 8.0 to isolate thr effect of the pipe end conditions. The use of the double Fourier series solution for radial direction deflection used in previous studies represents neither the real geometry nor the real loading conditions. The finite element method used in this thesis does describe the real piping-nozzle geometry and actual loading conditions and hence produces results that provide a significant improvement over previous studies. Discussion and comparison of data with various literature are included in this thesis.

## ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks and appreciation to his advisor, Dr. Benedict $C$. Sun, for his useful comments and advise, and for supplying valuable research information, much of which has been integrated into this dissertation. He also wished to thank the members of the dissertation committee: Dr. Bernard Koplik, Dr. Harry Herman, and Dr. Rong-Yaw Chen, all of the Department of Mechanical and Industrial Engineering, Dr. Thomas Hsu of the Department of Civil and Environmental Engineering, and Prof. Harry Kountouras of the Department of Engineering Technology for their kindly suggestions and support. Also, the author would like to thank his wife, Xiaolin, who with her love and patience supported the writing of this dissertation. To his parents, the author like to dedicate this work to them with gratitude.
Chapter Page
TABLE OF CONTENTS ..... iv
LIST OF TABLES ..... vi
LIST OF FIGURES ..... vii
NOMENCLATURE ..... x
I. INTRODUCTION ..... 1
II. LITERATURE SURVEY ..... 4
III. BASIC EQUATION ..... 11
3.1 Derivation of Equations for
Deflections, Bending Moment, and
Unit Membrane Forces ..... 13
3.2 Equation for Load Factor $Z_{m n}$ ..... 18
3.3 Stress Factors ..... 31
3.4 Spring Constants ..... 36
3.5 Stress Factors Summary ..... 36
IV. FINITE ELEMENT MODEL ..... 42
4.1 Geometry of the Finite Element Model ..... 44
4.2 Loading and Boundary Condition forShell Models47
4.3 Loading for Three Dimensional Iso-
Parametric Solid Model ..... 53
V. DATA COMPARISON ..... 58
VI. NUMERICAL EXAMPLE ..... 75
VII. CONCLUSIONS ..... 81
APPENDIX A. PLOTS OF STRESS FACTORS ..... 84
APPENDIX B. PLOTS OF SPRING COEFFICIENTS ..... 106
APPENDIX C. ANSYS FINITE ELEMENT MODEL AND PROGRAM ..... 110
APPENDIX D. FORTRAN PROGRAM OF SQUARE TUBING ATTACHMENT ..... 152
REFERENCES ..... 162

## LIST OF TABLES

TABLE Page

1. Computation sheet for Local Stresses ..... in Cylindrical Shells ..... 38
2. Computation and Sign Notation Sheet for Local Stresses of Pipe-Nozzle Model ..... 41
3. Pipe-Nozzle Geometry of Model ORNL-3 [19] and model $C-1$ [6] ..... 63
4. Comparison of Stress Factors with Steele [6], ORNL-3, C-1, and WRC 107 [1] ..... 63
5. Data Comparison for WRC 297, FEM, and WRC 107.. ..... 64
6. Spring Coefficients Comparison of WRC 297 and
FEM ..... 73
7. Computing Stresses ..... 77
Figure. ..... Page
8. Typical Configuration of Pipe with a Nozzle Attachment Subjected to six Components of Loadings ..... 2
9. Cylindrical Coordinates Applied to a Cylindri- cal Vessel with Displacements, $u, v, w, i n X$, Y, \& Z Direction Respectively ..... 12
10. External Loadings on a Square Tubing Attachment of Piping ..... 20
11. Shear Forces and Torsional Moment Acting on the Pipe-Nozzle Model ..... 30
5(a). Stress direction for pipe-nozzle connection model ..... 33
5(b). Juncture Points of Nozzle-Pipe Connection ..... 33
12. Loadings and Boundary Conditions for Shell Model Due to Radial Force, Circumferential and Longitudinal Moments ..... 45
13. Loadings and Boundary Conditions for Shell Model Due to Circumferential and Longitudinal Shear Forces and Torsional Moment ..... 52
14. Loading for Three Dimensional Iso-parametric Solid Element Model ..... 55
15. Comparison of Bending stress Factors in the $\phi$ Direction Due to Longitudinal Moment from 1. WRC 107 [1], 2. FEM Shell, 3. Decock [21] ..... 59
16. Comparison of Bending Stress Factors in the x Direction Due to Longitudinal Moment from 1. WRC 107 [1], 2. FEM Shell, 3. Decock [21] ..... 60
17. Comparison of Membrane Stress Factors in the $\phi$ Direction Due to Longitudinal Moment from 1. WRC 107 [1], 2. FEM Shell, 3. Decock [21] ..... 61
18. Comparison of Membrane Stress Factors in the $x$ Direction Due to Longitudinal Moment from 1. WRC 107 [1], 2. FEM Shell, 3. Decock [21] ..... 62
19. Comparison of Radial Spring Coefficients from 1. FEM 3D, 2. FEM Shell, 3. Sun \& Sun [15], 4. Murad \& Sun [14] ..... 69
20. Comparison of Circumferential Spring Coefficients from l. FEM 3D, 2. FEM Shell, 3. Sun \& Sun [15], 4. Murad \& Sun ..... 70
21. Comparison of Longitudinal Spring Coefficient from 1. FEM 3D, 2. FEM SHell, 3. Sun \& Sun [15] 4. Murad \& Sun [14] ..... 71
A-1A. Moment $M_{\phi} /\left(M_{c} / R_{m} \beta\right)$ Due to an External Circumferential Moment $M_{C}$ on a Nozzle- Pipe Connection ..... 85
A-2A. Moment $M_{X} /\left(M_{C} / R_{m} \beta\right)$ Due to an External Circumferential Moment $\mathrm{M}_{\mathrm{C}}$ on a Nozzle- Pipe Connection ..... 86
A-3A. Membrane force $N_{\phi} /\left(M_{c} / R_{m}{ }^{2} \beta\right)$ Due to an External Circumferential Moment $M_{C}$ on a Nozzle-Pipe Connection ..... 87
A-4A. Membrane force $N_{x} /\left(M_{C} / R_{m}^{2} \beta\right)$ Due to an External Circumferential Moment $M_{c}$ on $a$ Nozzle-Pipe Connection ..... 88
A-1B. Moment $M_{\phi} /\left(M_{L} / R_{m} \beta\right)$ Due to an External Longitudinal Moment $\mathrm{M}_{\mathrm{L}}$ on a Nozzle- Pipe Connection ..... 89
A-2B. Moment $M_{x} /\left(M_{L} / R_{m} \beta\right)$ Due to an External Longitudinal Moment $\mathrm{M}_{\mathrm{L}}$ on a Nozzle- Pipe Connection ..... 90
$A-3 B$. Membrane force $N_{\phi} /\left(M_{L} / R_{m}^{2} \beta\right)$ Due to an External Longitudinal Moment $M_{L}$ on a Nozzle-Pipe Connection ..... 91
A-4B. Membrane force $N_{x} /\left(M_{I} / R_{m}^{2} \beta\right)$ Due to an External Longitudinal Moment $M_{L}$ on a Nozzle-Pipe Connection ..... 92
A-1C. Bending Moment $M_{\phi} / P$ Due to an External Radial Load $P$ on a Nozzle-to-Pipe Connection ..... 93
$A-2 C$. Bending Moment $M_{x} / P$ Due to an External Radial Load $P$ on a Nozzle-to-Pipe connection ..... 94
A-3C. Membrane force $N_{\phi} /\left(P / R_{m}\right)$ Due to an External Radial Load $P$ on a Nozzle-to-Pipe Connection ..... 95
$A-4 C$. Membrane force $N_{x} /\left(P / R_{m}\right)$ Due to an External Radial Load $P$ on a Nozzle-to-Pipe connection ..... 96
A-1D. Bending Force $M_{\phi} / V_{c}$ Due to a Circumferential Shear Force $V_{C}$ ..... 97
A-2D. Bending Force $M_{X} / V_{C}$ Due to a Circumferential Shear Force $V_{C}$ ..... 98
A-3D. Membrane force $N_{\phi} /\left(V_{C} / R_{m}\right)$ Due to a Circum- ferential Shear Force $V_{C}$ ..... 99
A-4D. Membrane force $N_{x} /\left(V_{C} / R_{m}\right)$ Due to a Circum- ferential Shear Force $V_{C}$ ..... 100
A-5D. Shear Force $N_{\phi x} /\left(V_{C} / R_{m}\right)$ Due to a Circumferential Shear Force $V_{C} \ldots . .$. ..... 101
A-2E. Bending Force $M_{X} / V_{L}$ Due to a Longitudinal Shear Force $V_{L}$ ..... 102
A-4E. Membrane force $N_{x} /\left(V_{I} / R_{m}\right)$ Due to a Longi- tudinal Shear Force $\vec{V}_{L}$ ..... 103
A-5E. Shear Force $N_{x \phi} /\left(V_{C} / R_{m}\right)$ Due to a Longitudinal Shear Force $V_{L}$ ..... 104
A-1F. Shear Force $N_{\phi x} /\left(M_{T} / R_{m}{ }^{2} \beta\right)$ Due to a Torsional Moment $\mathrm{M}_{\mathrm{T}}$ ..... 105
B-1. Radial Spring Coefficient of Nozzle-to-Pipe Connection with $t / T=1, q=0$ psi ..... 107
B-2. Circumferential Spring Coefficient of Nozzle- to-Pipe Connection with $t / T=1, q=0 \mathrm{psi}$ ..... 108
B-3. Longitudinal Spring Coefficient of Nozzle-to- Pipe Connection with $t / T=1, q=0$ psi ..... 109

## NOMENCLATURE

```
    ca = mean radius of cylindrical shell; in.
a
    b= coordinate x of center of loading surface
bo.bm}= coefficients of Fourier series in longitudinal directio
    c = mean radius of nozzle; in.
    c}=\mp@code{inside half-length of loading surface; in.
    co
    D=}\frac{E\mp@subsup{T}{}{3}}{12(1-\mp@subsup{v}{}{2})
    d = mean diameter of nozzle; in.
d, d
D.. D 吕 = inside diameter and mean diameter of cylindrical vessel; in.
    E = modulus of elasticity; psi.
    K
    K
    Kc = rotational spring constant in circumferential direction;
        in.-lb/rad.
    KL = rotational spring constant in longitudinal direction; in.-lb/rad.
    K}\mp@subsup{R}{R}{}= linear spring constant in radial direction; lb./in.
        L=2\piRm
        l= length of cylindrical shell; in.
        l}=\frac{l}{2
        m,n = integer number
```

$M=$ concentrated external overturning moment; in-lb.
$M_{c}=$ circumferential moment; in.-lb.
$M_{L}=$ longitudinal moment; in. - lb
$M_{T}=$ torsional moment: in. -lb .
$M_{1}=$ bending moment per unit length in i direction; in. - lb/in.
$M_{1}=$ bending moment in shell in longitudinal direction; in-lb/in.
$M_{x}=$ bending moment in shell in circumferential direction; in-lb/in.
$N_{1}=$ membrane force per unit length in i direction; lb./in.
$N_{4}=$ membrane force in shell in circumferential direction: lb./in.
$N_{x}=$ membrane force in shell in longitudinal direction; lb./in.
$P=$ radial force loading; lb.
$p=$ uniformly distributed load intensity; lbs /in. ${ }^{2}$
$P=$ concentrated radial load or total distributed radial load; lb.
$P_{0}=$ maximum normal load intensity for longitudinal or circumferential moment: lb./in. ${ }^{2}$
$q=$ internal pressure; psi
$R_{m}=$ mean radius of cylindrical shell; in.
$S=$ stress intensity; psi
$T=$ thickness of shell; in.
$t=$ thickness of nozzle; in.
$V=$ concentrated external shear load; lb.
$V_{c}=$ concentrated shear load in circumferential direction; in.
$V_{L}=$ concentrated shear load in longitudinal direction; lb.
$u, u, w=$ displacements in the $X, Y$, and $Z$ directions
Y.Y.Z = coordinate axis
$Z_{\sigma}=$ radial load per unit surface
$\alpha=\frac{l}{R_{m}}, \quad$ alpha
$\alpha^{\prime}=\frac{\alpha}{2}$
$\beta=\frac{c}{R_{m}}$. beta
$\beta_{1}=\frac{c_{l}}{R_{m}}$
$\beta_{o}=\frac{c_{0}}{R_{m}}$
$y=\frac{R_{m}}{T}, \quad$ gamma
$\theta=$ angle around attachment: degree
$\lambda=\frac{n \pi R_{m}}{l}$
$\lambda^{\prime}=2 \lambda$
$\bar{\lambda}=d /\left(D_{m} T\right)^{1 / 2}$, as defined in WRC 297
$A=U\left(D_{m} T\right)^{1 / 2}$
$v=$ Poisson's ratio
$\phi=$ angle rotation; rad.
$\sigma_{0}=$ Normal stress in circumferential direction; psi.
$\sigma_{x}=$ Normal stress in longitudinal direction: psi.
$\tau_{x}=$ Shear stress on the plane perpendicular to $x$ axis in $\phi$ direction; psi.
$\tau_{0 x}=$ Shear stress on the plane perpendicular to $\phi$ axis in $x$
direction; psi.
$\nabla^{4} w=\left(\frac{\partial^{2} w}{\partial x^{2}}+\frac{\partial^{2} w}{R_{m}^{2} \partial \phi^{2}}\right)^{2}$
$\nabla^{8} w=\left(\frac{\partial^{8} w}{\partial x^{8}}+\frac{4}{R_{m}^{2}} \cdot \frac{\partial^{8} w}{\partial x^{6} \partial \phi^{2}}+\frac{6}{R_{m}^{4}} \cdot \frac{\partial^{8} w}{\partial x^{4} \partial \phi^{4}}+\frac{4}{R_{m}^{6}} \cdot \frac{\partial^{8} w}{\partial x^{2} \partial \phi^{6}}+\frac{\partial^{8} w}{R_{m}^{\partial} \partial \phi^{8}}\right)$

## CHAPTER I INTRODUCTION

For a nozzle-to-pipe connection due to external loadings, the local stresses along the intersecting juncture and the spring constants are always major concerns for pressure vessel designers. The external loadings on a pressure vessel are comprised of the vessel's own weight, thermal expansion load, activation of safety/relief valves, earthquake, wind load, water or steam hammer phenomenon and other effects. These loadings contribute to six different generic load components at the nozzle-to-pipe connection. They are: radial force, $P$, circumferential moment, $M_{C}$, longitudinal moment, $M_{L}$, circumferential shear force, $V_{C}$, longitudinal shear force, $\mathrm{V}_{\mathrm{L}}$, and torsional moment, $\mathrm{M}_{\mathrm{T}}$ respectively as shown in Figure 1. These loads produce peak stresses at the juncture of the vessel nozzle, and contribute to the fatigue of the material. This is particularly important in the design of a nuclear power plant piping system for safety.

Also, the stiffness of the piping at the nozzle connection associated with the deformations due to the six load components represent six degrees of constraint; it is very crucial to the accuracy of the piping-system stress analysis. This thesis presents a comprehensive study of the local stresses as well as the spring constants at the pipe-nozzle juncture.


Figure 1. Typical configuration of pipe with a nozzle attachment subjected to six components of loadings.

The Welding Research Council (WRC) Bulletin No. 107 [1] and Bulletin No. 297 [2] are two of the most important design guides ever published for local stress calculation in the design of pressure vessels. Stress factors and spring coefficients presented in these two Bulletins are based on Bijlaard's [3] [4] [5] and Steele's [6] works. In these publications, stresses and displacements are calculated based on thin shell theory and double Fourier series solutions to specific loads. There are two major deficiencies in these treatments. First, due to mathematical difficulties, the stress factors and spring coefficients can not be obtained for beta values (nozzle radius/pipe radius) larger than 0.55 in Bulletins No. 107 and No. 297. Second, the original theoretical work by Bijlaard was based on square or rectangular-shaped uniform loads acting on a closed cylinder, and does not represent the real pipe-nozzle connection.

In this thesis, the finite element method is being applied to re-evaluate those results and extend the beta value from 0.55 to 0.9 .

## CHAPTER II LITERATURE SURVEY

There are many important references related to the local stresses factors and spring coefficients at the nozzle-pipe connection.

## a) Stress factor:

1. WRC Bulletin No. 107: K.R. Wichman, A.G. Hopper and J.L. Mershon [1] published this bulletin in 1965. It suggests the method to calculate the local stresses in spherical and cylindrical shells due to external loadings. The theoretical base of this Bulletin is based on a study by Bijlaard published in 1955 [3]. Bijlaard's work is based on thin-shell theory and provided double Fourier series solution. Bulletin No. 107 has undergone several revisions since its first publication. The latest revision which was made in March 1979, in which some of the curves were relabeled, did not discuss the stiffness factors or the spring coefficients. Due to the mathematical limitation of Bijlaard's work, Bulletin No. 107 can only apply to problems in which there is a lug or a solid trunnion attached to the pipe or sphere. The bulletin does not recommend any specific method in analyzing an actual nozzle connection to a pressure vessel, either cylindrical or spherical. The induced normal stresses are reported as membrane and bending
stresses factors in biaxial directions. The shear stresses are reported through approximate formulas, since Bijlaard's work [3] for the pressure vessel does not provide analytical solutions due to external shear forces and torsional moments. Only the approximate formulas are used to calculate the shear stresses by these loadings. Stresses from various local loads are summed in their respective directions before the principal stresses and stress intensity are calculated.
2. WRC Bulletin No. 198: Rodabaugh et al. [7] used the same approach as Bijlaard, and introduced the stress index concept, which was defined as the ratio of maximum stress intensity to the nominal stress in the pipe. The authors considered the problem of localized loading on a cylindrical vessel. The stress indices are based on the calculated normal stresses at the edge or outside of the loaded area. Without specifying stress direction, this bulletin only provides outside skin stress indices. The indices for radial load and bending moment are based on an extensive parameter study and are represented by simple formulas that may be directly incorporated into the ASME Pressure Vessel and Boiler Code.
3. Mirza and Gupgupoglu [8] [9] in 1988 introduced 17-node doubly curved shell finite elements to simulate the case of longitudinal moments applied at discrete points
around the circumference of the vessel. The results from the finite element method are in good agreement with WRC 198 [7] and WRC 107 [1]. Spring coefficients are not discussed here.
4. WRC Bulletin No. 297: WRC No. 297 was published by J.L. Mershon et al. in August 1984 [2]. It is a supplement to WRC 107 and is specifically applicable to round nozzles on cylindrical vessels. This bulletin was based on Professor Steele's [6] theoretical work with data for larger gamma ( $D / T$ ) values than in WRC 107. It also provides better readability for small beta values. This new theory considers an opening on the shell together with the restraining effect of the nozzle wall. Data for larger beta values are still not available here.
5. Sadd and Avent [10] in 1982 studied a trunnion pipe anchor by the finite element method. The model is analyzed for the case of internal pressure and various end moment loadings. Primary and secondary stress indices are provided. The Georgia Tech ICES STRUDL finite element package using a quadrilateral element with six degree of freedom at each of the four corner nodes was utilized. The alpha value (pipe length/pipe radius) is taken as 8.0 for their models. However, data provided in this paper are for a beta (trunnion size/pipe size) range from 0.5 to 1.0 , and a gamma (pipe radius/pipe thickness) range from 5 to 20 only.
6. Tabone and Mallett [11] in 1987 established a finite element model of a nozzle in a cylindrical shell subjected to internal pressure, out-of-plane moment, and a combination of pressure plus out-of-plane moment. The model used ANSYS three-dimensional finite elements and the analysis considered inelastic behavior at small displacements. Two elements along thickness direction of the nozzle and vessel were employed in this geometrical model. The purpose of this paper is to obtain an estimation of limit loads based on extrapolation of the load-versus-inverse-displacement curves. An expression is given for the effect of the combined loading, for a case in which the internal pressure reduces the moment capability of the nozzle by 35 percent.
7. Brooks [12] [13] in $1988 \& 1990$ developed an integral equation formulation for the problem of a loaded rigid attachment on a cylindrical shell. The integral equation formulation is simplified by modifying existing Green's functions for the unbounded shell to account for simply supported boundary conditions at the ends of the vessel. Numerical examples for circular shape and rectangular shape attachments were presented and showed good agreement with experimental data.

## b) Spring coefficient:

In regard to spring coefficients, Steele [6] in 1984 discussed the flexibility of nozzle-to-cylinder connection with limited data. Murad and Sun [14] in 1984 reported the radial and rotational spring coefficients at the piping-nozzle juncture. Sun and Sun, in 1987, presented spring coefficients of a pipe with a square tubing attachment [15] and with a solid lug attachment [16]. These results were based on Bijlaard's deflection solution, in double-Fouri-er-series form, to the eighth-order governing differential equation. There are two major difficulties with Bijlaard's double Fourier series solution: First, due to the convergence problem in the computations, the beta value (nozzle radius/pipe radius) can not be extended beyond 0.55 . Second, the governing differential equation only describes a closed thin cylinder without any opening and there is no actual nozzle geometry considered. The radial and moment loadings and the nozzle are simulated by applying a distributed regional loading on the surface of the cylinder.

Chiou and Sun [17], in 1987, applied the finite element method to study the torsional spring coefficients of the cylinder at a nozzle connection. Chen [18], in 1988, used the finite element method to study the shear spring coefficients at the pipe-nozzle connection in both the longitudinal and the circumferential directions of the pipe.

This thesis uses the finite element method to study the remaining three spring coefficients. They are: radial spring coefficients, circumferential rotational spring coefficients, and longitudinally rotational spring coefficients.

## c) Experimental Data:

Gwaltney et al. [19] in 1976 published some experimental data for cylinder-to-cylinder shell models and compared them with theoretical predictions from a thin-shell finite element analysis. Four carefully machined cylin-der-to-cylinder shell models were tested and the agreement between these particular finite element predictions and the experimental results is reasonably good.
Brown et al. [20] in 1977 presented a comparison of
stress results using a FESAP three-dimensional finite
element program with results obtained from experimental
testing. The comparison shows good agreement for the case
of internal pressure applied to a cylinder-to-cylinder
structure with a variable shell thickness at the juncture
with a beta (nozzle radius/pipe radius) value of 0.625 .

Decock [21] in 1980 provided experimental test data of external loadings on a nozzle-pipe connection with gamma values of 25 and 40 , and beta values of $0.19,0.37,0.69$ and
1.0. The test models were subjected to radial load, longitudinal and circumferential moments. From the experimental results, he suggested that extrapolation of beta value from 0.55 to 1.0 is permissible, although WRC 107 specifically stated that extrapolation should not be used. There is also a brief discussion of the influence of nozzle thickness.

Historically, experimental work using strain gages or the photoelastic method was considered not only to be realistic but also a reliable indication of the stress distribution at the nozzle-to-cylinder intersections. However, accuracy of the testing data largely depends on idealized material properties and good instrumentation. Experimental work is expensive, tedious, and time-consuming. There are always some uncertainties introduced in the preparation of experimental specimens, materials quality control, and instrumentation set up. Therefore, experimental data of nozzle-to-cylinder structure are only available in some specific cases for data-proofing purposes. In industry, most of the design data are still provided by analytical methods.

## ChAPTER III BASIC EQUATIONS

Analytical solutions for local stresses in cylindrical shells with loadings over a rectangular area were developed by Bijlaard [3]. The basic theory involved in this thesis has been established for some time. It is the thin-shell theory, as given by Timoshenko [22] for a radial loading $Z$ per unit surface. The resultant three basic thin-wall cylinder equations contribute an eighth order partial differential equation system as follows:

$$
\begin{align*}
& \begin{array}{l}
\frac{\partial^{2} u}{\partial x^{2}}+\frac{1-v}{2 R_{m}^{2}} \cdot \frac{\partial^{2} u}{\partial \phi^{2}}+\frac{1+v}{2 R_{m}} \frac{\partial^{2} v}{\partial x \partial \phi}-\frac{v}{R_{m}} \frac{\partial w}{\partial x}=0 \\
\begin{aligned}
\frac{1+v}{2 R_{m}} \frac{\partial^{2} u}{\partial x \partial \phi} & +\frac{1-v}{2} \frac{\partial^{2} v}{\partial x^{2}}+\frac{1}{R_{m}^{2}} \frac{\partial^{2} v}{\partial \phi^{2}}-\frac{1}{R_{m}^{2}} \frac{\partial w}{\partial \phi}+\frac{T^{2}}{12 R_{m}^{2}}\left(\frac{\partial^{3} w}{\partial x^{2} \partial \phi}+\frac{\partial^{3} w}{R_{m}^{2} \partial \phi^{3}}\right) \\
& +\frac{T^{2}}{12 R_{m}^{2}}\left[(1-v) \frac{\partial^{2} v}{\partial x^{2}}+\frac{\partial^{2} v}{R_{m}^{2} \partial \phi^{2}}\right]=0
\end{aligned} \\
v \frac{\partial u}{\partial x}+\frac{\partial v}{R_{m} \partial \phi}-\frac{w}{R_{m}}-\frac{R_{m} T^{2}}{12} \nabla^{+} w-\frac{T^{2}}{12}\left(\frac{2-v}{R_{m}} \frac{\partial^{3} v}{\partial x^{2} \partial \phi}+\frac{\partial^{3} v}{R_{m}^{3} \partial \phi^{3}}\right)+\frac{\left(1-v^{2}\right) R_{m}}{E T} Z=0
\end{array}
\end{align*}
$$

Here $u, v$, and $w$ are the displacements in the $X, Y(\varnothing)$, and $Z$ directions respectively as shown in Figure 2; the eighth-order uncoupled governing differential equation is:

$u: x$-direction displacement

Figure 2. Cylindrical coordinate applied to a cylindrical vessel with displacements, $u, v, w, i n X, Y, \& Z$ direction respectively.

$$
\begin{gathered}
\nabla^{8} w+\frac{12\left(1-v^{2}\right)}{R_{m}^{2} T^{2}} \cdot \frac{\partial^{4} w}{\partial x^{4}}+\frac{1}{R_{m}^{2}}\left[2 \cdot \frac{\partial^{6} w}{R_{m}^{6} \partial \phi^{6}}+\left(6+v-v^{2}\right) \cdot \frac{\partial^{6} w}{R_{m}^{2} \partial x^{4} \partial \phi^{2}}\right. \\
+(7+v) \frac{\partial^{6} w}{R_{m}^{4} \partial x^{2} \partial \phi^{4}}-\frac{1}{D} \nabla^{4} Z=0
\end{gathered}
$$

### 3.1 Derivation of Equations for Deflections, Bending Moment, and Unit Membrane Forces.

The eighth order equation can be solved for radial displacement under a radial load, and it can also be used to compute an angular displacement under an imposed moment in a circumferential or in a longitudinal direction. According to Bijlaard's method, the membrane forces, caused by the internal pressure $q$ on the vessel, are:

$$
N_{x}=\frac{q \cdot R_{m}}{2} ; N_{\phi}=q \cdot R_{m}
$$

and the changes of curvature of the vessel due to an internal pressure, if it exists, would be taken into account and be derived an extra term as:
$Z_{q}=\frac{q R_{m}}{2}\left(\frac{\partial^{2} w}{\partial x^{2}}+2 \cdot \frac{w}{R_{m}^{2}}+2 \cdot \frac{\partial^{2} w}{R_{m}^{2} \partial \phi^{2}}\right)$
(4)

This term should be included in the $Z$ term of the eighth order differential equation as $Z+Z_{q}$. This equation, according to Bijlaard's method [3], can be solved by developing the deflection $w$ as well as the external load $Z$ in double Fourier series:
$w=\sum \sum w_{m n} \cdot \cos m \phi \cdot \sin \frac{\lambda}{R_{m}} x$
$Z=\sum \sum Z_{m n} \cdot \cos m \phi \cdot \sin \frac{\lambda}{R_{m}} x$
where

$$
\lambda=\frac{n \pi R_{m}}{l}
$$

To solve the eight order differential equation

$$
R_{m}^{4}\left(m^{2}+\lambda^{2}\right)^{2} Z_{m n}
$$

$\left[D\left(\left(m^{2}+\lambda^{2}\right)^{4}+12\left(1-v^{2}\right) \cdot \lambda^{4} R_{m}^{2} / T^{2}-m^{2}\left[2 m^{4}+\left(6+v-v^{2}\right) \lambda^{4}+(7+v) m^{2} \lambda^{2}\right]\right\}+\right.$

$$
\begin{equation*}
\left.\left(m^{2}-1+\frac{\lambda^{2}}{2}\right) \cdot\left(m^{2}+\lambda^{2}\right)^{2} \cdot q R_{m}^{3}\right] \tag{7}
\end{equation*}
$$

If there is no internal pressure, $q=0$ psi., this equation can be simplified as:

$$
w_{m n}=\phi_{m n} Z_{m n} \frac{l^{4}}{2 D}
$$

where

$$
2\left(m^{2} \alpha^{2}+n^{2} \pi^{2}\right)^{2}
$$

$\phi_{m n}=$

$$
\left(m^{2} \alpha^{2}+n^{2} \pi^{2}\right)^{4}+12\left(1-v^{2}\right) n^{4} \pi^{4} \alpha^{4} y^{2}-m^{2} \alpha^{4}\left[2 m^{4} \alpha^{4}+\left(6+v-v^{2}\right) n^{4} \pi^{4}+(7+v) m^{2} \alpha^{2} n^{2} \pi^{2}\right]
$$

## so that from Equation (5)

$w=\frac{l^{4}}{2 D} \sum \sum \phi_{m n} Z_{m n} \cos m \phi \sin \frac{\lambda}{R_{m}} x$

The other displacements, $u$ and $v$, can be expressed in terms of $w_{m n}$ by Bijlaard's method as:

$$
\begin{align*}
u & =\sum \sum u_{m n} \cos m \phi \cos \frac{\lambda}{R_{m}} x \\
& =\sum \sum \frac{\lambda\left(m^{2}-v \lambda^{2}\right)}{\left(\lambda^{2}+m^{2}\right)^{2}} w_{m n} \cos m \phi \cos \frac{\lambda}{R_{m}} x  \tag{11}\\
v & =\sum \sum v_{m n} \sin m \phi \sin \frac{\lambda}{R_{m}} x \\
& =\sum \sum \frac{m\left[(2+v) \lambda^{2}+m^{2}\right]}{\left(\lambda^{2}+m^{2}\right)^{2}} w_{m n} \sin m \phi \sin \frac{\lambda}{R_{m}} x \tag{12}
\end{align*}
$$

From Timoshenko [22], the bending moments are:
$M_{x}=-D\left(\chi_{x}+v \chi_{\theta}\right)$ and $M_{\theta}=-D\left(\chi_{0}+v \chi_{x}\right)$
since,
$\chi_{x}=\frac{\partial^{2} w}{\partial x^{2}}, \quad \chi_{\phi}=\frac{1}{R_{m}^{2}}\left(w+\frac{\partial^{2} w}{\partial \phi^{2}}\right)$
so that,

$$
\begin{aligned}
M_{x} & =-\frac{D}{R_{m}^{2}}\left[R_{m}^{2} \frac{\partial^{2} w}{\partial x^{2}}+v\left(w+\frac{\partial^{2} w}{\partial \phi^{2}}\right)\right] \\
& =\frac{1}{2} \alpha^{2} l^{2} \sum \sum \phi_{m n} Z_{m n}\left[\left(\frac{n^{2} \pi^{2}}{\alpha^{2}}\right)+v\left(m^{2}-1\right)\right] \cdot \cos m \phi \sin \frac{\lambda}{R_{u}} x
\end{aligned}
$$

$$
\begin{aligned}
M_{\phi} & =-\frac{D}{R_{m}^{2}}\left[w+\frac{\partial^{2} w}{\partial \phi^{2}}+v R_{m}^{2} \frac{\partial^{2} w}{\partial x^{2}}\right] \\
& =\frac{1}{2} \alpha^{2} l^{2} \sum \sum \phi_{m n} Z_{m n}\left[m^{2}-1+\left(v n^{2} \pi^{2} / \alpha^{2}\right)\right] \cdot \cos m \phi \sin \frac{\lambda}{R_{m}} x
\end{aligned}
$$

## and the unit membrane forces are

$$
\begin{align*}
N_{x} & =\frac{E T}{1-v^{2}}\left[\frac{\partial u}{\partial x}+v\left(\frac{\partial v}{R_{m} \partial \phi}-\frac{w}{R_{m}}\right)\right] \\
= & -6 \pi^{2}\left(1-v^{2}\right) \alpha^{6} \gamma^{2} R_{m} \sum \sum \phi_{m n} Z_{m n} \\
\therefore & \cdot \frac{m^{2} n^{2}}{\left(m^{2} \alpha^{2}+n^{2} \pi^{2}\right)^{2}} \cdot \cos m \phi \sin \frac{\lambda}{R_{m}} x  \tag{17}\\
N_{\phi}= & \frac{E T}{1-v^{2}}\left[\frac{\partial v}{R_{m} \partial \phi}-\frac{w}{R_{m}}+v \frac{\partial u}{\partial x}\right] \\
= & -6 \pi^{4}\left(1-v^{2}\right) \alpha^{4} \gamma^{2} R_{m} \sum \sum \phi_{m n} Z_{m n} \\
& \cdot \frac{n^{4}}{\left(m^{2} \alpha^{2}+n^{2} \pi^{2}\right)^{2}} \cdot \cos m \phi \sin \frac{\lambda}{R_{m}} x \tag{18}
\end{align*}
$$

Here only the $Z_{m n}$ term is left undefined, which should be derived by the following conditions according to each external loading.

### 3.2 Equations for Load Factor $\mathrm{Z}_{\mathrm{mn}}$

## a) Radial load:

The solution for the above equation due to a radial load is given by Bijlaard [3]:
$w=\sum_{m=0,1,2 \ldots n}^{m=0} \sum_{n=1,3,5, \ldots}^{n=\infty} R_{m} \cdot W_{m n} Z_{m n} \cos m \phi \sin \frac{\lambda x}{R_{m}}$
where

$$
\left(m^{2}+n^{2} \pi^{2} / \alpha^{2}\right)^{2}
$$



$$
\begin{aligned}
& \left\{E /\left(12\left(1-v^{2}\right) y^{3}\right)\left\{\left(m^{2}+n^{2} \pi^{2} / \alpha^{2}\right)^{4}+\left(2\left(1-v^{2}\right) \cdot n^{4} \pi^{4} y^{2} / \alpha^{4}-m^{2}\left[2 m^{4}+\left(6+v-v^{2}\right)\right.\right.\right.\right. \\
& \left.\left.\left.\cdot n^{4} \pi^{4} / \alpha^{4}+(7+v) m^{2} n^{2} \pi^{2} / \alpha^{2}\right]\right\}+\left(m^{2}-1+n^{2} \pi^{2} / 2 \alpha^{2}\right) \cdot\left(m^{2}+n^{2} \pi^{2} / \alpha^{2}\right)^{2} \cdot q\right\}
\end{aligned}
$$

and
$Z_{m n}=(-1)^{\frac{n-1}{2}} \cdot\left(\frac{4 \beta_{0} P}{\pi^{2} n}\right) \sin \frac{n \pi \beta_{0}}{\alpha}$

$$
\begin{align*}
& \text { when } m=0 ; \mathrm{n}=1,3,5, \ldots \\
& =(-1)^{\frac{n-1}{2}} \cdot\left(\frac{8 P}{\pi^{2} m n}\right) \sin m \beta_{0} \sin \frac{n \pi \beta_{0}}{\alpha} \\
& \text { when } m=1,2,3, \ldots ; \quad \mathrm{n}=1,3,5, \ldots \tag{21}
\end{align*}
$$

Let $\varnothing=0$, and $x=1 / 2$
and using the superposition method [15] in Figure 3 (a) to simulate a square-tubing insert instead of a rigid insert as shown in WRC 107, the new displacement and stress equations become as follows:

$$
\begin{aligned}
w & =\sum_{m=0,1,2 \ldots n}^{m=0} \sum_{n=1,3,5, \ldots}^{n=0} R_{m} \cdot W_{m n} Z_{m n} \cos m \phi \sin \frac{\lambda x}{R_{m}} \\
& =\frac{4 \alpha p}{\pi^{2}} \sum_{m=0,1,2 \ldots \ldots}^{m=\infty} \sum_{n=1,3.5 \ldots}^{n=0} W_{m n} \bar{Z}_{m n} \sin \frac{n \pi}{2}
\end{aligned}
$$

and
$M_{x}=\frac{D P}{R_{m} \pi^{2}\left(c_{0}^{2}-c_{l}^{2}\right)} \cdot \sum_{m=1,2,3 \ldots n=1,3,5, \ldots}^{m=0} W_{m n} \bar{Z}_{m n}\left[\frac{n^{2} \pi^{2}}{\alpha^{2}}\right.$

$$
\left.+v\left(m^{2}-1\right)\right] \cos m \phi \cdot \sin \frac{\lambda x}{R_{m}}
$$

$M_{\theta}=\frac{D P}{R_{m} \pi^{2}\left(c_{0}^{2}-c_{l}^{2}\right)} \cdot \sum_{m=0,1,2 \ldots n}^{m=0} \sum_{n=1,3,5, \ldots}^{n=0} w_{m a}^{\prime} \bar{Z}_{m a}\left[m^{2}-1\right.$

$$
\begin{equation*}
\left.+\frac{v \pi^{2} \pi^{2}}{\alpha^{2}}\right] \cos m \phi \cdot \sin \frac{\lambda x}{R_{m}} \tag{24}
\end{equation*}
$$



Figure 3. External loadings on a square tubing attachment of piping.

$$
\begin{gather*}
N_{x}=-\frac{E T \alpha^{2} P}{c_{0}^{2}-c_{l}^{2}} \sum_{m=1,2.3 \ldots n=1,0.5}^{m=0} \sum_{m n}^{n=0} W_{m n} \frac{m^{2} n^{2}}{\left(m^{2} \alpha^{2}+n^{2} \pi^{2}\right)^{2}} \\
\cdot \cos m \phi \cdot \sin \frac{\lambda x}{R_{m}} \\
N_{\phi}=-\frac{E T \pi^{2} \bar{P}}{c_{0}^{2}-c_{i}^{2}} \sum_{m=1.2,3 \ldots n=1.3 .5 \ldots}^{m=0} \sum_{m n}^{n=\infty} W_{m n} \frac{n^{4}}{\left(m^{2} \alpha^{2}+n^{2} \pi^{2}\right)^{2}} \\ \tag{26}
\end{gather*}
$$

where

$$
\begin{align*}
\bar{Z}_{m n}= & \frac{(-1)^{\frac{n-1}{2}} \beta_{0}}{n} \cdot \sin \frac{n \pi \beta_{0}}{\alpha}+\frac{(-1)^{\frac{n-1}{2}} \beta_{i}}{n} \cdot \sin \frac{n \pi \beta_{i}}{\alpha} \\
& \text { when } m=0 ; \mathrm{n}=1,3,5, \ldots \\
\bar{Z}_{m n}= & \frac{2(-1)^{\frac{n-1}{2}}}{m n} \cdot \sin m \beta_{0} \sin \frac{n \pi \beta_{0}}{\alpha}+\frac{2(-1)^{\frac{n-1}{2}}}{m n} \cdot \sin m \beta_{i} \sin \frac{n \pi \beta_{i}}{\alpha} \\
& \text { when } m=1,2,3, \ldots ; \quad \mathrm{n}=1,3,5, \ldots \tag{27}
\end{align*}
$$

and the radial spring constant would be
$K_{R}=\frac{\pi^{2} \cdot\left[c_{0}^{2}-c_{l}^{2}\right]}{R_{m} \cdot \sum_{m=0,1,2 \ldots \ldots n}^{m=0} \sum_{n, 3, \mathrm{~s} \ldots}^{n-\infty} W_{m n} \cdot \bar{Z}_{m n} \cdot \sin \left(\frac{n \pi}{2}\right)}$

## b) Circumferential moment:

According to the solution by Bijlaard [3], the deflection of the cylinder due to a circumferential moment can be expressed as:

$$
w=\sum_{m=1,2,3, \ldots n=1,3,5 \ldots}^{m=0} R_{m}^{n=0} \cdot W_{m n} Z_{m n} \sin m \phi \sin \frac{\lambda x}{R_{m}}
$$

where

$$
\lambda=\frac{n \pi R_{m}}{l}
$$

and

$$
\left.\left.\left.W_{m n}=\frac{\left(m^{2}+n^{2} \pi^{2} / \alpha^{2}\right)^{2}}{\left\{E /\left(12\left(1-v^{2}\right) \gamma^{3}\right)\left\{\left(m^{2}+n^{2} \pi^{2} / \alpha^{2}\right)^{4}+12\left(1-v^{2}\right) \cdot n^{4} \pi^{4} \gamma^{2} / \alpha^{4}-m^{2}\left[2 m^{4}+\left(6+v-v^{2}\right)\right.\right.\right.}, ~\left(n^{4} \pi^{4} / \alpha^{4}+(7+v) m^{2} n^{2} \pi^{2} / \alpha^{2}\right]\right\}+\left(m^{2}-1+n^{2} \pi^{2} / 2 \alpha^{2}\right) \cdot\left(m^{2}+n^{2} \pi^{2} / \alpha^{2}\right)^{2} \cdot q\right\}\right)
$$

and,

$$
\begin{gather*}
Z_{m \beta}=(-1)^{\frac{(m-1)}{2}} \cdot\left(\frac{8 P_{0}}{\pi^{2} \beta_{0} m^{2} n}\right) \cdot\left(\sin m \beta_{0}-m \beta_{0} \cos m \beta_{0}\right) \cdot \sin \frac{n \pi \beta_{0}}{\alpha} \\
\text { when } m=1,2,3, \ldots ; \quad n=1,3,5, \ldots \tag{31}
\end{gather*}
$$

Using the superposition method again, the stress equations and displacement equation for a square-tubing type
attachment without an opening on the cylindrical vessel subjected to a circumferential moment as shown in Figure 3 (b) would be:

$$
\begin{aligned}
M_{x}=\frac{6 D}{R_{m} \pi^{2}} \cdot \frac{M_{c}}{c_{0}^{3}-\frac{c_{1}^{i}}{c_{0}^{\prime}}} \sum_{m=0,1,2, \ldots n=1,3,5 \ldots}^{m-0} & \sum_{m n}^{n=\infty} \bar{Z}_{m n}\left[\frac{n^{2} \pi^{2}}{\alpha^{2}}\right. \\
& \left.+v\left(m^{2}-1\right)\right] \sin m \phi \cdot \sin \frac{\lambda x}{R_{m}}
\end{aligned}
$$

$$
M_{\phi}=\frac{6 D}{R_{m} \pi^{2}} \cdot \frac{M_{c}}{c_{0}^{3}-\frac{c_{1}^{4}}{c_{0}^{\prime}}} \sum_{m=0,1,2, \ldots n}^{m=0} \sum_{n=1,3.5 \ldots}^{n=0} W_{m n} \bar{Z}_{m n}\left[m^{2}-1\right.
$$

$$
\left.+\frac{v \pi^{2} \pi^{2}}{\alpha^{2}}\right] \sin m \phi \cdot \sin \frac{\lambda x}{R_{m}}
$$

$$
N_{x}=-\frac{6 E T \alpha^{2} M_{c}}{c_{o}^{3}-\frac{c_{i}^{4}}{c_{0}}} \sum_{m=0,1,2, \ldots n=1,3,5 \ldots}^{m=0} \sum_{m n}^{n=0} \bar{Z}_{m n}
$$

$$
\begin{equation*}
\frac{m^{2} n^{2}}{\left(m^{2} \alpha^{2}+n^{2} \pi^{2}\right)^{2}} \cdot \sin m \phi \cdot \sin \frac{\lambda}{R_{m}} x \tag{34}
\end{equation*}
$$

$N_{0}=-\frac{6 E T \pi^{2} m_{c}}{c_{0}^{3}-\frac{c_{i}^{4}}{c_{0}}} \sum_{m=0,1,2 \ldots, \ldots}^{m=\infty} \sum_{n=1,3.5 . . .}^{n=\infty} W_{m n} \bar{Z}_{m n} \frac{n^{4}}{\left(m^{2} \alpha^{2}+n^{2} \pi^{2}\right)^{2}}$ $\sin m \phi \cdot \sin \frac{\lambda x}{R_{m}} \quad$,
and

$$
\begin{aligned}
w & =\sum_{m=1,2,3, \ldots n=1,3,5 \ldots}^{m=0} \sum_{m}^{n-\infty} \cdot W_{m n} Z_{m n} \sin m \beta_{o} \sin \frac{n \pi}{2} \\
& =\frac{8 a}{\pi^{2}} \cdot\left\{\frac{3 \cdot M_{c}}{4\left[c_{0}^{3}-\frac{c_{1}^{4}}{c_{\bullet}}\right]}\right\} \sum_{m=1,2,3 \ldots n=1,3,5 \ldots}^{m=0} \sum_{m n}^{n=0} \bar{Z}_{m n} \sin m \beta_{0} \sin \frac{n \pi}{2},
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{Z}_{m \Omega}= & \frac{1}{m^{2} n} \cdot\left\{\frac{(-1)^{\frac{0-1}{2}}}{\beta_{0}} \cdot\left(\sin m \beta_{0}-m \beta_{0} \cos m \beta_{0}\right) \cdot \sin \frac{n \pi \beta_{0}}{\alpha}+\frac{(-1)^{\frac{0.1}{2}}}{\beta_{1}} \cdot\left(\sin m \beta_{1}\right.\right. \\
& \left.\left.\quad-m \beta_{\imath} \cos m \beta_{\imath}\right) \cdot \sin \frac{n \pi \beta_{i}}{\alpha}\right\}
\end{aligned}
$$

$$
\begin{equation*}
\text { when } m=1,2,3, \ldots ; \quad n=1,3,5, \ldots \tag{37}
\end{equation*}
$$

By definition of the circumferential spring constant,

$$
\begin{equation*}
K_{\mathrm{c}}=\frac{M_{c}}{\phi}=\frac{M_{c}}{\left(\frac{w}{c_{\mathrm{c}}}\right)} \tag{38}
\end{equation*}
$$

therefore, the circumferential spring constant is:

$$
\begin{equation*}
K_{c}=\frac{\pi^{2} \cdot\left[c_{0}^{3}-\frac{c_{1}^{4}}{c_{0}}\right] \cdot \beta_{0}}{6 \cdot \sum_{m=1,2,3, \ldots, \ldots=1,3,5, \ldots}^{m=0} W_{m n}^{n=0} \cdot \bar{Z}_{m n} \cdot \sin \left(m \beta_{0}\right) \cdot \sin \left(\frac{n \pi}{2}\right)} \tag{39}
\end{equation*}
$$

## c) Longitudinal moment:

From Bijlaard [3] again, the radial deflection of the cylinder due to a longitudinal moment can be expressed as:

$$
\begin{equation*}
w=\sum_{m=0,1,2, \ldots n=1,2,3, \ldots}^{m=0} \sum_{m}^{n=\infty} \cdot W_{m n} Z_{m n} \cos m \phi \sin \frac{\lambda^{\prime} x}{R_{m}} \tag{40}
\end{equation*}
$$

where

$$
\begin{align*}
& W_{m n}= \frac{\left(m^{2}+4 n^{2} \pi \pi^{2} / \alpha^{2}\right)^{2}}{} \\
&\left\{E /\left(12\left(1-v^{2}\right) y^{3}\right)\left\{\left(m^{2}+4 \pi^{2} \pi^{2} / \alpha^{2}\right)^{4}+12\left(1-v^{2}\right) \cdot\left(16 n^{4} \pi^{4} / \alpha^{4}\right) \cdot y^{2}-m^{2}\left[2 m^{4}+\left(6+v-v^{2}\right)\right.\right.\right. \\
&\left.\left.\left.\cdot\left(16 n^{4} \pi^{4} / \alpha^{4}\right)+(7+v) 4 m^{2} n^{2} \pi^{2} / \alpha^{2}\right]\right\}+\left(m^{2}-1+2 n^{2} \pi^{2} / \alpha^{2}\right) \cdot\left(m^{2}+4 n^{2} \pi^{2} / \alpha^{2}\right)^{2} \cdot q\right\}
\end{align*}
$$

and,

$$
\begin{align*}
Z_{m n}= & \frac{2(-1)^{n} P_{0} \alpha^{\prime}}{\pi^{3} n^{2}} \cdot\left(\sin \frac{n \pi \beta_{0}}{\alpha^{\prime}}-\frac{n \pi \beta_{0}}{\alpha^{\prime}} \cos \frac{n \pi \beta_{0}}{\alpha^{\prime}}\right) \\
& \text { when } m=0 ; n=1,2,3, \ldots \\
& =\frac{4(-1)^{n} P_{0} \alpha^{\prime}}{\pi^{3} m n^{2} \beta_{0}} \cdot\left(\sin \frac{n \pi \beta_{0}}{\alpha^{\prime}}-\frac{n \pi \beta_{0}}{\alpha^{\prime}} \cos \frac{n \pi \beta_{0}}{\alpha^{\prime}}\right) \cdot \sin m \beta_{0} \\
& \text { when } m=1,2,3, \ldots ; n=1,2,3, \ldots \tag{42}
\end{align*}
$$

Let, $\phi=0, x=l / 2+c_{0}$
and superimposing two mutually opposing longitudinal moments [15] as shown in Figure 3 (c), the net amount of longitudinal moment is the one applied to this square tubing attachment connection:

$$
\begin{align*}
& w=\sum_{m=0,1,2, \ldots n=1,2,3 \ldots}^{m=\infty} R_{m}^{n} \cdot W_{m n} Z_{m \Omega} \cos m \phi \sin \frac{\lambda^{\prime} x}{R_{m}} \\
& =\frac{6 R_{m} \alpha M_{L}}{8 \pi^{3}\left[c_{0}^{3}-\frac{c_{1}^{4}}{c_{0}}\right]} \sum_{m=0,1,2, \ldots n=1,2,3 \ldots}^{m=\infty} W_{m n} \bar{Z}_{m n} \sin \left(n \pi+\frac{2 n \pi \beta_{0}}{\alpha}\right)  \tag{43}\\
& M_{x}=\frac{3 D \alpha}{4 R_{m} \pi^{3}} \cdot \frac{M_{L}}{c_{0}^{3}-\frac{c_{i}^{4}}{c_{0}}} \sum_{m=0,1,2 \ldots n=1,2,3 \ldots}^{m=\infty} W_{m \beta}^{n=\infty} \bar{Z}_{m n}\left[\frac{n^{2} \pi t^{2}}{\alpha^{2}}\right. \\
& \left.+v\left(m^{2}-1\right)\right] \sin m \phi \cdot \sin \frac{\lambda x}{R_{m}} \\
& M_{\phi}=\frac{3 D \alpha}{4 R_{m} \pi^{3}} \cdot \frac{M_{L}}{c_{0}^{3}-\frac{c_{i}^{4}}{c_{0}}} \sum_{m=0,1,2, \ldots n}^{m=\infty} \sum_{1,2,3 \ldots}^{n=\infty} W_{m n} \bar{Z}_{m n}\left[m^{2}-1\right. \\
& \left.+\frac{v n^{2} \pi^{2}}{\alpha^{2}}\right] \cos m \phi \cdot \sin \frac{\lambda x}{R_{m}}
\end{align*}
$$

$$
\begin{align*}
& N_{x}=-\frac{3 E T \alpha^{3} M_{L}}{4 \pi\left(c_{0}^{3}-\frac{c_{i}^{4}}{c_{0}}\right)} \sum_{m=0,1,2, \ldots n=1,2,3, \ldots}^{m=\infty} W_{m n} \bar{Z}_{m n} \\
& \cdot \frac{m^{2} n^{2}}{\left(m^{2} \alpha^{2}+n^{2} \pi^{2}\right)^{2}} \cdot \cos m \phi \cdot \sin \frac{\lambda}{R_{m}} x  \tag{46}\\
& N_{\phi}=-\frac{3 E T \pi \alpha M_{L}}{4\left(c_{0}^{3}-\frac{c_{i}^{i}}{c_{0}}\right)} \sum_{m=0,1,2, \ldots n=1,2,3, \ldots}^{m=0} \sum_{m n}^{n=0} \bar{Z}_{m n} \\
& \cdot \frac{n^{4}}{\left(m^{2} \alpha^{2}+n^{2} \pi^{2}\right)^{2}} \cdot \cos m \phi \cdot \sin \frac{\lambda}{R_{m}} x \tag{47}
\end{align*}
$$

where

$$
\begin{align*}
& \bar{Z}_{m n}=\frac{(-1)^{n}}{n^{2}} \cdot\left(\sin \frac{n \pi \beta_{0}}{\alpha^{\prime}}-\frac{n \pi \beta_{0}}{\alpha^{\prime}} \cos \frac{n \pi \beta_{0}}{\alpha^{\prime}}\right) \\
& \quad+\frac{(-1)^{n}}{m n^{2}} \cdot\left(\sin \frac{n \pi \beta_{i}}{\alpha^{\prime}}-\frac{n \pi \beta_{i}}{\alpha^{\prime}} \cdot \cos \frac{n \pi \beta_{i}}{\alpha^{\prime}}\right) \\
& \text { when } \quad \mathrm{m}=0 ; \mathrm{n}=1,2,3, \ldots \\
& = \\
& \quad \begin{array}{l}
(-1)^{n} \\
m n^{2} \beta_{0}
\end{array}\left(\sin \frac{n \pi \beta_{0}}{\alpha^{\prime}}-\frac{n \pi \beta_{0}}{\alpha^{\prime}} \cos \frac{n \pi \beta_{o}}{\alpha^{\prime}}\right) \cdot \sin m \beta_{0} \\
& \quad+\frac{2(-1)^{n}}{m n^{2} \beta_{i}} \cdot\left(\sin \frac{n \pi \beta_{i}}{\alpha^{\prime}}-\frac{n \pi \beta_{i}}{\alpha^{\prime}} \cdot \cos \frac{n \pi \beta_{i}}{\alpha^{\prime}}\right) \cdot \sin m \beta_{i}  \tag{48}\\
& \text { when } \quad \mathrm{m}=1,2,3, \ldots ; \quad \mathrm{n}=1,2,3, \ldots
\end{align*}
$$

By the definition of the rotational longitudinal spring constant,

$$
\begin{equation*}
K_{L}=\frac{M_{L}}{\phi}=\frac{M_{L}}{\left(\frac{w}{c_{0}}\right)} \tag{49}
\end{equation*}
$$

then,
$K_{\iota}=\frac{4 \pi^{3} \cdot\left[c_{0}^{3}-\frac{c_{i}^{4}}{c_{0}}\right] \cdot \beta_{0}}{3 \alpha \cdot \sum_{m=0,1,2 \ldots n=1}^{m=0} \sum_{1,2,3, \ldots}^{n 00} W_{m \beta} \cdot \bar{Z}_{m n} \cdot \sin \left(n \pi+\frac{2 n \pi \beta_{0}}{\alpha}\right)}$

A Fortran program written for calculating above local stresses and spring coefficients due to the radial force and two overturning moment is included in Appendix D. This program is applicable for square tubing attached on the pipe.

## d) Shear forces:

Bijlaard has proposed that circumferential shear forces, $V_{C}$, and longitudinal shear force, $V_{L}$, can be assumed to be transmitted to the shell entirely by membrane shear forces [1][3]. Therefore the stresses in the shell at the nozzle-to-shell juncture can be approximated by WRC 107 as

# $\tau_{x \phi}=\frac{V_{c}}{\pi c T} \cdot \cos \theta$ (max. at $A$ and $B$ in Figure 4) 

$\tau_{d x}=\frac{V_{L}}{\pi c T} \cdot \sin \theta$ (max. at $C$ and $D$ in Figure 4)

If the attachment is rectangular in shape, then
$\tau_{x}=\frac{V_{c}}{4 c_{1} T}$
$\tau_{\phi x}=\frac{V_{L}}{4 c_{2} T}$
where $c_{1}$ and $c_{2}$ represent half length of rectangular loading in the circumferential and longitudinal directions respectively.

The equations above can only estimate the maximum shear stress without taking into account the induced membrane and bending stresses in the shear force direction. Due to the curvature of this pipe-nozzle geometry, these stresses caused by external shear forces are sometimes very significant and should never be ignored. In a later chapter, the membrane and bending stresses in the shear direction are investigated using the finite element method. Results and discussions are presented.


Figure 4. Shear forces and torsional moment acting on the pipe-nozzle model.

## e) Torsional moment:

WRC 107 presents an approximate equation for calculating the shear stress due to an external torsional moment on a round attachment to the shell connection. For this round attachment, the torsional moment is assumed to induce only shear stresses, so that shear stress in the shell at the pipe-nozzle juncture is given by:

$$
\tau_{\phi x}=\tau_{x \phi}=\frac{M_{T}}{2 \pi c^{2} T}
$$

### 3.3 Stress factors

According to WRC 107 [1], all the stresses are represented in dimensionless form as stress factors. Attention should be drawn to the beta value, which defines the dimensionless ratio of nozzle radius to cylindrical vessel radius. The derivations in this study are based on a uniform load in the square region acting on a closed cylindrical vessel. According to WRC 107, nozzle attachments can be estimated by using an equivalent square having an equivalent width which is 0.875 of the nozzle diameter.
a) The stress factors in WRC 107 are presented as follows: the local stress induced by the external radial force, $P$, is separated into membrane and bending stress components, i.e.
$\sigma_{\imath}=K_{n} \frac{N_{\imath}}{T} \pm K_{0} \frac{6 M_{i}}{T^{2}}$
where

$$
\frac{N_{i}}{T}=\left[\frac{N_{i}}{P / R_{m}}\right] \cdot\left[\frac{P}{R_{m} \cdot T}\right] \text { and } \frac{6 M_{i}}{T^{2}}=\left[\frac{M_{i}}{P}\right] \cdot\left[\frac{6 P}{T^{2}}\right]
$$

The stress factors are: $\left[\frac{N_{1}}{P / R_{m}}\right]$, and $\left[\frac{M_{i}}{P}\right]$ where $i=x$ or $\phi$ as shown in Figure 5 (a).

The membrane and bending stresses due to external moment loadings are similarly separated:
$\frac{N_{l}}{T}=\left[\frac{N_{l}}{M /\left(R_{m}^{2} \beta\right)}\right] \cdot\left[\frac{M}{R_{m}^{2} T \beta}\right] \quad$ and $\quad \frac{6 M_{l}}{T^{2}}=\left[\frac{M_{i}}{M /\left(R_{m} \beta\right)}\right] \cdot\left[\frac{6 M}{R_{m} T^{2} \beta}\right]$

The stress factors are: $\left[\frac{N_{l}}{M /\left(R_{m}^{2} \beta\right)}\right]$, and $\left[\frac{M_{1}}{M /\left(R_{m} T^{2} \beta\right)}\right]$
where $i=x$ or $\phi$ as shown in Figure 5 (a).

$$
\begin{align*}
M & =M_{c} \text { circumferential moment, lb-in } \\
& =M_{\iota} \text { longitudinal moment, lb-in } \tag{57}
\end{align*}
$$



Figure $5(a)$ stress direction for pipe-nozzle connection model.


Figure 5(b) Juncture points of nozzle-pipe connection.
b) The localized shear stresses due to a circumferential or a longitudinal shear force may be treated in the same fashion as other loadings:
(1) Circumferential shear force loading produces a maximum shear stress at points, $A$ and $B$, and a maximum membrane stress and a bending stress at points, $C$ and $D$, in both the circumferential and longitudinal directions as shown in Figure 4.
$\frac{N_{\phi x}}{T}=\left[\frac{N_{\phi x}}{V_{c} / R_{m}}\right] \cdot\left[\frac{V_{c}}{R_{m} \cdot T}\right]$
(58)
$\frac{N_{i}}{T}=\left[\frac{N_{t}}{V_{c} / R_{m}}\right] \cdot\left[\frac{V_{c}}{R_{m} \cdot T}\right] \quad$ and $\quad \frac{\sigma M_{i}}{T^{2}}=\left[\frac{M_{i}}{V_{c}}\right] \cdot\left[\frac{\sigma V_{c}}{T^{2}}\right]$

The stress factors are: $\left[\frac{N_{\phi x}}{V_{c} / R_{m}}\right],\left[\frac{N_{i}}{V_{c} / R_{m}}\right]$, and $\left[\frac{M_{i}}{V_{c}}\right]$
where $i=x$ or $\phi$ as shown in Figure 5 (a).
(2) Longitudinal shear force loading produces a maximum shear stress at points, $C$ and $D$, and a maximum membrane stress and a bending stress at points $A$ and $B$ in both the circumferential and longitudinal directions as shown in Figure 4 again.
$\frac{N_{x \phi}}{T}=\left[\frac{N_{x \phi}}{V_{L} / R_{m}}\right] \cdot\left[\frac{V_{L}}{R_{m} \cdot T}\right]$
$\frac{N_{t}}{T}=\left[\frac{N_{l}}{V_{L} / R_{m}}\right] \cdot\left[\frac{V_{L}}{R_{m} \cdot T}\right] \quad$ and $\quad \frac{6 M_{i}}{T^{2}}=\left[\frac{M_{i}}{V_{L}}\right] \cdot\left[\frac{6 V_{L}}{T^{2}}\right]$

The stress factors are: $\left[\frac{N_{x \phi}}{V_{L} / R_{m}}\right],\left[\frac{N_{i}}{V_{L} / R_{m}}\right]$, and $\left[\frac{M_{i}}{V_{L}}\right]$ where $i=x$ or $\phi$
c) Torsional moment loading on the nozzle-pipe connection only produces a shear stress along the intersection zone in the tangential direction for each point.
$\frac{N_{\phi x}}{T}=\left[\frac{N_{\phi x}}{M_{T} /\left(R_{m}^{2} \beta\right)}\right] \cdot\left[\frac{M_{T}}{R_{m}^{2} T \beta}\right]$
The shear stress factor is: $\left[\frac{N_{\phi x}}{M_{T} /\left(R_{m}^{2} \beta\right)}\right]$

It is noted that the finite element study shows, due to the variation in curvature around the conjuncture, that this local shear stress is not uniformly distributed.

### 3.4 Spring Constants

There are three basic equations from which the spring constants are derived due to the radial or imposed moment, i.e.

$$
\begin{align*}
& P=K_{R} \cdot w \\
& M_{c}=K_{C} \cdot \phi \\
& M_{\iota}=K_{\iota} \cdot \phi \tag{63}
\end{align*}
$$


#### Abstract

where $\mathrm{K}_{\mathrm{R}}=$ Spring constant caused by a radial loading. $\mathrm{K}_{\mathrm{C}}=$ Spring constant caused by a circumferential moment. $\mathrm{K}_{\mathrm{L}}=$ Spring constant caused by a longitudinal moment.

Based on the above definitions, the radial spring constant is defined as the radial force in pounds that would produce one inch of radial deflection, and the rotational spring constant in either the longitudinal or circumferential direction is defined as the moment in pound-inch that would produce one radian of rotation in its respective direction.


### 3.5 Stress Factors Summary

Using the finite element method, all the stress factors due to various external loading are summarized in Table 1. Data presented in various plots are shown as follows: Figure $A-1 A$ to $A-4 A$ are for circumferential moment; Figure $A-1 B$ to A-4B are for longitudinal moment; Figure A-1C to A-4C are for radial force; Figure $A-1 D$ to $A-5 D$ are for circumferential shear force; Figure $A-1 E$ to $A-5 E$ are for longitudinal shear force; finally, Figure $A-1 F$ is for torsional moment loading. Sign notations for stresses due to each loading for those eight critical points, as shown in Figure 5 (b), follow WRC 107's definitions and is shown in Table 2 . In general, all applied external loading should first be resolved into six independent components. Stress factors read from these plots for each loading case would be calculated as components of a biaxial state of stress, from which the principal stresses and the stress intensity can be calculated.

Table 1-Computation Sheet for Local Stresses in Cylindrical Shells

| From | Read Curves | Compute absolute values of stress |
| :---: | :---: | :---: |
| Fig. | for | stress and enter result direction |

Radial Load $P$

| $A-3 C$ | $\frac{N_{0}}{P / R_{m}}$ | $K_{n}\left[\frac{N_{0}}{P / R_{m}}\right] \cdot \frac{P}{R_{m} T}$ | $\phi$ |
| :---: | :--- | :--- | :--- |
| $A-1 C$ | $\frac{M_{0}}{P}$ | $K_{0}\left[\frac{M_{0}}{P}\right] \cdot \frac{6 P}{T^{2}}$ | $\phi$ |
| $A-4 C$ | $\frac{N_{x}}{P / R_{m}}$ | $K_{n}\left[\frac{N_{x}}{P / R_{m}}\right] \cdot \frac{P}{R_{m} T}$ | $x$ |
| $A-2 C$ | $\frac{M_{x}}{P}$ | $K_{0}\left[\frac{M_{x}}{P}\right] \cdot \frac{6 P}{T^{2}}$ | $x$ |

## Circumferential Moment $M_{c}$

$A-3 A \quad \frac{N_{1}}{M_{c} /\left(R_{m}^{2} \beta\right)} \quad K_{n}\left[\frac{N_{t}}{M_{c} \|\left(R_{m}^{2} \beta\right)}\right] \cdot \frac{M_{c}}{R_{m}^{2} \beta T}$
$A-1 A \quad \frac{M_{+}}{M_{c} /\left(R_{m} \beta\right)} \quad K_{b}\left[\frac{M_{t}}{M_{c} /\left(R_{m} \beta\right)}\right] \cdot \frac{6 M_{c}}{R_{m} \beta T^{2}} \quad \phi$
$A-4 A \quad \frac{N_{x}}{M_{c} /\left(R_{m}^{2} \beta\right)} \quad K_{n}\left[\frac{N_{x}}{M_{c} /\left(R_{m}^{2} \beta\right)}\right] \cdot \frac{M_{c}}{R_{m}^{2} \beta T}$
$A-2 A \quad \frac{M_{x}}{M_{c} /\left(R_{m} \beta\right)} \quad K_{b}\left[\frac{M_{x}}{M_{c} /\left(R_{m} \beta\right)}\right] \cdot \frac{6 M_{c}}{R_{m} \beta T^{2}}$

From Read Curves Fig.

tress factor

Compute absolute values of
stress stress and enter result direction

## Longitudinal Moment $M_{L}$

| $A-3 B$ | $\frac{N_{0}}{M_{L} /\left(R_{m}^{2} \beta\right)}$ | $K_{n}\left[\frac{N_{0}}{M_{L} /\left(R_{m}^{2} \beta\right)}\right] \cdot \frac{M_{L}}{R_{m}^{2} \beta T}$ |
| :---: | :---: | :---: |
| $A-1 B$ | $\frac{M_{0}}{M_{L} /\left(R_{m} \beta\right)}$ | $K_{b}\left[\frac{M_{0}}{M_{L} /\left(R_{m} \beta\right)}\right] \cdot \frac{6 M_{L}}{R_{m} \beta T^{2}}$ |
| $A-4 B$ | $\frac{N_{x}}{M_{L} /\left(R_{m}^{2} \beta\right)}$ | $K_{n}\left[\frac{N_{x}}{M_{L} /\left(R_{m}^{2} \beta\right)}\right] \cdot \frac{M_{L}}{R_{m}^{2} \beta T}$ |
| $A-2 B$ | $\frac{M_{x}}{M_{L} /\left(R_{m} \beta\right)}$ | $K_{0}\left[\frac{M_{x}}{M_{L} /\left(R_{m} \beta\right)}\right] \cdot \frac{6 M_{L}}{R_{m} \beta T^{2}}$ |

## Circumferential Shear Force $V_{c}$

| $A-3 D$ | $\frac{N}{V_{c} / R_{m}}$ | $K_{n}\left[\frac{N_{*}}{V_{c} / R_{m}}\right] \cdot \frac{V_{c}}{R_{m} T}$ | $\phi$ |
| :---: | :---: | :---: | :---: |
| $A-1 D$ | $\frac{M_{1}}{V_{c}}$ | $K_{b}\left[\frac{M_{0}}{V_{c}}\right] \cdot \frac{6 V_{c}}{T^{2}}$ | $\phi$ |
| $A-4 D$ | $\frac{N_{x}}{V_{c} / R_{m}}$ | $K_{n}\left[\frac{N_{x}}{V_{c} / R_{m}}\right] \cdot \frac{V_{c}}{R_{m} T}$ | $x$ |
| $A-2 D$ | $\frac{M_{x}}{V_{c}}$ | $K_{b}\left[\frac{M_{x}}{V_{c}}\right] \cdot \frac{\sigma V_{c}}{T^{2}}$ | $x$ |
| $A-5 D$ | $\frac{N_{0 x}}{V_{c} / R_{m}}$ | $K_{n}\left[\frac{N_{4 x}}{V_{c} / R_{m}}\right] \cdot \frac{V_{c}}{R_{m} T}$ | $\phi x$ |


| $\begin{aligned} & \text { From } \\ & \text { Fig. } \end{aligned}$ | Read Curves for Stress factor | Compute absolut stress and en | values of result | $\begin{aligned} & \text { stress } \\ & \text { direction } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| Longitudinal Shear Force $V_{L}$ |  |  |  |  |
| $A-3 E$ | $\frac{N_{1}}{V_{L} / R_{n}}$ | $K_{a}\left[\frac{N_{1}}{V_{L} / R_{m}}\right] \cdot \frac{V_{l}}{R_{m} T}$ | (negligible) | $\phi$ |
| $A-1 E$ | $\frac{M_{6}}{V_{6}}$ | $K_{0}\left[\frac{M_{0}}{V_{L}}\right] \cdot \frac{6 V_{L}}{T^{2}}$ | (negligible) | ) $\phi$ |
| $A-4 E$ | $\frac{N_{x}}{V_{L} / R_{m}}$ | $K_{n}\left[\frac{N_{x}}{V_{L} / R_{m}}\right] \cdot \frac{V_{L}}{R_{m} T}$ |  | $x$ |
| $A-2 E$ | $\frac{M_{x}}{V_{L}}$ | $K_{0}\left[\frac{M_{x}}{V_{L}}\right] \cdot \frac{6 V_{L}}{T^{2}}$ |  | $x$ |
| $A-5 E$ | $\frac{N_{x t}}{V_{l} / R_{m}}$ | $K_{n}\left[\frac{N_{x_{0}}}{V_{L} / R_{m}}\right] \cdot \frac{V_{L}}{R_{m} T}$ |  | $\phi x$ |


Torsional Moment $M_{T}$
A-1F $\frac{N_{* x}}{M_{T} \prime\left(R_{m}^{2} \beta\right)} \quad K_{n}\left[\frac{N_{0 x}}{M_{T} /\left(R_{m}^{2} \beta\right)}\right] \cdot \frac{M_{T}}{R_{m}^{2} \beta T} \quad \phi x$


Nda Nlgebraically for Summation of streasen, $\sigma$. $=$


Add Algebraically for Summation of $x$ Stresses, $\sigma_{x}=$

| A-1r | $\frac{N_{n}}{M_{1} /\left(\frac{\left.R_{m}^{2} \beta\right)}{}\right.}$ | $\left[\frac{N_{0 r}}{A_{r} /\left(R_{m}^{2} \beta\right)}\right] \cdot \frac{A_{r}}{R_{m}^{2} B T}=$ | + | $\dagger$ | + | + | - | - | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A-50 | $\frac{N_{o r}}{V_{c} / R_{m}}$ | $\left[\frac{N_{0}}{V_{c} / R_{m}}\right] \cdot \frac{V_{c}}{R_{m} T}=$ | $\pm$ | 4 | - | - |  |  |  |  |
| 1-5E | $\frac{N_{n t}}{V_{A} / R_{m}}$ | $\left[\frac{N_{n}}{V_{i} / R_{m}}\right] \cdot \frac{V_{l}}{R_{m} T}=$ |  |  |  |  | + | - | - | - |

Add Algebraically for Summation of Shear Strenses, $\boldsymbol{r}_{n \in}=$

There are several deficiencies in simulating the real pipe-nozzle model by the analytical method. Firstly, due to convergence difficulties in the Double Fourier series method [1] [14] [15], stress factors and stiffness coefficients are not obtainable for beta values larger than 0.55. Secondly, the original work of Bijlaard [3] for the cylindrical vessel is based on a square uniform load acting on a cylinder without an opening. Bijlaard's study does not represent the real problem at hand.

The computer has been playing an increasingly important role in engineering design and has become an indispensable tool for the analysts during last three decades. The finite element method for stress analysis fully utilizes the advantages of computer capability in performing speedy calculations. With sophisticated and well developed software packages, such as ANSYS, the finite element method provides very reliable solutions to a wide range of engineering design problems. This is particularly true when the problem is difficult to approach by a traditional mathematical model or when the geometrical model is too complex. In such cases, the finite element method provides a comprehensive, accurate, and efficient procedure for the stress analysis. A number of studies related to the
nozzle-pipe connection problem using the finite element approach have been published recently. Invariably most of them are restricted to some specific cases and provide only limited information for dat.a proofing purposes. None of the previous work provides a full range of data for this pipe-nozzle connection. This is due to the amount of work necessary to meet convergence requirements for each finite element model in preparation for preprocessing mode. A parametric model in terms of beta, gamma, and alpha has been established in this study to save repetitive preprocessing work.

In the study of the problem of the pipe-nozzle connection, a gamma value (radius to thickness ratio of pipe) of 10 is often considered as a lower bound for the applicability of thin shell theory. Both gamma and beta values are required to determine whether the "three-dimensional solid element model" or "thin shell element model" is appropriate in the analysis. Since the pipe is considered as thin shell for most cases, the latter model is appropriate for the pipe portion. Also, the gamma is defined by pipe geometry alone and it is further assumed that the nozzle and pipe have the same thickness, and therefore the local stresses and spring coefficients results are applicable only for the pipe portion. However, the three-dimensional solid element model is necessary if the nozzle portion data is considered to be more prominent than
that of the cylindrical vessel. Since the primary concern of this study is on the pipe, the majority of the data provided in this thesis will be based on the quadrilateral thin shell element model. Three-dimensional iso-parametric solid element models are only used for verification.

In this thesis, by using the ANSYS general finite element program from Swansion Analysis, Inc. [23], the nozzle-pipe connection is modeled by a quadrilateral thin shell element (STIF 63, shell element) and a three-dimensional iso-parametric solid element (STIF 45, 3D solid element) respectively for a wide range of beta and gamma. The wide range insures representative values for pressure vessel design. The detailed formulas from finite element theory are referenced from various sources [24] [25] [26] [27].

### 4.1 Geometry of the Finite Element Model

Since the geometry of the model, elastic properties, and support conditions are symmetric to the $x-y$ plane and the $y-z$ planes (See Figure 1), only one quarter of the geometry is necessary if a loading applied to the model is symmetric or uniformly distributed. A symmetric structure can carry either symmetric or anti-symmetric loads [see Figure 6(a) ].

(a) Quarter model of the pipe-nozzle juncture, the boundary conditions are either symmetric or anti-symmetric depending on the loading conditions as shown in (b), (c), and. (d).


- symmetrical B.C.
- anti-symmetrical
BC.
- symmetrical B.C.
$x$ symmetrical B.C. $x$ symmetrical B.C. $x$ anti-symmetrical BC.
(b) Radial force
(c) Circumferential moment
(d) Longitudinal moment

Figure 6. Loadings and boundary conditions for shell model due to radial force, circumferential and longitudinal moments.

The basic nomenclature of the nozzle-pipe connection model is defined as shown in Figure 1. For the analysis of this model, the following assumptions are used:

1. The material is assumed to be homogeneous and isotropic, and obeys Hooke's law. The resulting stresses and strains are within the proportional limit of the material.
2. The influences of self-weight and temperature are neglected.
3. In the pipe-nozzle connection model, both ends of the running pipe are assumed to be either fixed, or "built-in", which means there are zero degrees of freedom there. However, since the alpha (pipe length/pipe radius) is given as 8, which implies that the pipe is a "long cylinder", the boundary constraints do not significantly contribute to the results of the computation.
4. There are no transitions, fillets, or reinforcing at the juncture.

To satisfy the convergence requirement, many models with different element and node numbers have been studied. For optimum accuracy within the framework of the software, the finite element model of quadrilateral thin shell is adopted, it contains 981 nodes and 912 shell elements. This model requires 2294 seconds of CPU time to run a given
loading case. The three dimensional solid model has 1960 nodes and 1326 solid elements, and needs 4519 seconds of CPU time to solve a given loading case. All the computations were performed on a $3 / 60$ SUN workstation with 8 Megabytes of RAM. A computer graphic representation for both finite element models (quadrilateral thin shell element and three dimensional iso-parametric solid element) under different loading conditions is presented as shown in the Appendix $C$ in this thesis.

Since the purpose of this dissertation is to study the stresses and spring constants of the pipe portion, the thickness of the nozzle is assumed to be identical with the piping, therefore only the dimensionless parameters of beta and gamma are required to specify the pipe-nozzle geometry. In this study, eight beta values of $0.1,0.2$, to 0.8 are combined with eight gamma values of $5,10,15,25,35,50$, 75, and 100, 64 runs are executed to study each loading case.

### 4.2 Boundary Conditions and Loadings for Quadrilateral Shell Models

A. Radial force. Since the radial force is applied from top of the nozzle, the radial force is uniformly distributed in the negative $Y$ direction as shown in Figure 6(b). The
geometry of the structure and applied loading are symmetric to the $x-y$ plane and the $y-z$ plane, therefore the displacement in $z$ direction for all nodes on the $x-y$ plane and the displacement in $x$ direction for all nodes on the $y-z$ plane are restrained. Correspondingly, rotations around the $x$-axis and the $y$-axis for all nodes on the $x-y$ plane and rotations around the $y$-axis and the $z$-axis for all nodes on the $y-z$ plane are zero.

In a quadrilateral thin shell element model, there are 25 nodes (24 elements) on the top edge of the nozzle. In studying the radial loading case, the radial force is distributed equally at these nodes, except for two nodes (points $A^{\prime}$ and $B^{\prime}$, see Figure 6(b)) located on the planes of symmetry at each end. At these two points the value of the nodal force should be half of the value elsewhere to satisfy the symmetry condition. In the actual computations, an external radial loading of 1000 pounds is applied at the top of the entire nozzle which is equivalent to a 250 pound force applied to the quarter model. Therefore, each node supports 10.416667 pounds of nodal force in the negative $Y$ direction while the nodes at each ends support 5.208333 pounds.
B. Bending Moment. For the purpose of simulating real moment loading, a linearly distributed nodal force is
applied at top of the nozzle as shown in Figure $6(c)$ and 6(d) for circumferential and longitudinal moments respectively.

1) Boundary conditions for circumferential moment: Boundary conditions at the symmetric plane for circumferential moment are as following: Nodes on the $x-y$ plane Nodes on the $y-z$ plane (anti-symmetric boundary) (symmetric boundary) DX,DY,ROTZ are zero DX,ROTY,ROTZ are zero

The boundary conditions specified here are associated with a quadrilateral thin shell element, which has six degrees of freedom for each node.
2) Boundary conditions for longitudinal moment: A linearly distributed nodal force is again applied at the top of the nozzle in order to simulate a longitudinal moment as shown in Figure 6(d). Boundary conditions at the symmetric planes for longitudinal moment loading are as following: Nodes on the $x-y$ plane Nodes on the $y-z$ plane (symmetric boundary) (anti-symmetric boundary) DZ,ROTX,ROTY are zero DY,DZ,ROTX are zero
3) Loadings for bending moment loadings: When applying the external rotational moment at the top of nozzle to obtain the local stresses and the rotational spring coefficients, the linearly distributed nodal forces must be
applied at the nodes around the top edge of the nozzle. Since a moment of $1000 \mathrm{lb}-\mathrm{in}$ is assumed for the entire pipe-nozzle connection, a 250 lb-in moment is applied to the quarter model. Since the moment is the product of force and distance, the following equations are required for calculating the actual force at each nodal point.

For the circumferential moment loading:
$\frac{M_{c}}{4}=\sum_{\theta_{1}=0^{\circ} \cdot 3.75^{\circ} \ldots . .}^{90^{\circ}} f \cos \theta_{\imath} \cdot c \cdot \cos \theta_{\imath} \quad$.
or,
$f=\frac{M_{c}}{48 \cdot c}$
where $f$ is maximum nodal force at the node located at zero degree position and $c$ is the radius of the nozzle.

For the longitudinal moment loading:
$\frac{M_{L}}{4}=\sum_{\theta_{i}=0^{0}, 3.75^{\circ} \ldots \ldots}^{90^{\circ}} f \sin \theta_{i} \cdot c \cdot \sin \theta_{i}$
or,
$f=\frac{M_{L}}{48 \cdot c}$
where $f$ is maximum nodal force at the node located at 90 degree. Since there are 25 nodes equally distributed along a quarter circumference of the nozzle, the nodes are at angular increment of $3.75^{\circ}$.
C. Shear Force. The boundary conditions for the model subjected to circumferential shear force loading is identical to the model subjected to circumferential moment. A model subjected to longitudinal shear force has the same boundary conditions as the model subjected to longitudinal moment.

In the quadrilateral thin shell element model, there are 25 nodes ( 24 elements) on the intersecting edge of nozzle and pipe. In studying the shear force loading cases, the shear force is distributed equally at those nodes located at the pipe-nozzle juncture as shown in Figure 7(a), $7(\mathrm{~b}), 7(\mathrm{c})$, and $7(\mathrm{~d})$, except at the two nodes (points $A$ and C) located on the planes of symmetry at each end. The value of the nodal force applied at these two nodes should be half of the value elsewhere so as to satisfy the symmetry condition. In the computations, an external shear force of 1000 pound is applied. Therefore, each node sustains 10.416667 lb . of nodal force in the negative $Z$ direction which represents the circumferential shear force and the two

(a) Quarter model subjected to shear forces and torsion moment, the boundary conditions are either symmetric or anti-symmetric depending on the loading conditions as shown in (b), (c), and (d).


- anti-symmetrical
B. C.
x symmetrical B.C.


C

- anti-symmetrical B. C.
$x$ anti-symmetrical B.C.
(d) Torsional moment
- symmetrical B.C.
x anti-symmetrical B.C.

$$
\theta=0^{\circ}
$$

(c) Longitudinal shear force
(b) Circumferential
shear force
Figure 7. Loadings and boundary conditions for shell model due to circumferential and longitudinal shear forces and torsional moment.
nodes at the ends each sustain 5.208333 lb. If the shear is longitudinal, all nodal forces directions are changed to positive X.
D. Torsional moment. The torsional moment can be simulated by a series of tangential force on the cross section of the nozzle, as shown in Figure $7(\mathrm{~d})$. Boundary conditions on the symmetric plane are all anti-symmetric, therefore the nodal degree of freedom on the symmetric planes are defined as: Nodes on the $x-y$ plane Nodes on the $y-z$ plane (anti-symmetric boundary) (anti-symmetric boundary) DX,DY,ROTZ are zero DY,DZ,ROTX are zero

Since a 1000 in-lb torsional moment is applied, a 250 in-lb torsional moment is distributed on the 25 nodes for the quarter model. Hence, each node supports 10.416667 in-lb of moment except the two nodes at the end, $A$ and $C$ in Figure $7(d)$ located on the symmetric planes, which only support half of the moment effect. Finally, each node sustains $10.416667 / \mathrm{c}$ nodal force at its respective tangential direction where $c$ represents nozzle radius.

### 4.3 Boundary Conditions and Loadings for Three Dimensional Iso-parametric solid model.

For the three dimensional iso-parametric solid element, which has three degrees of freedom in translation for each node and no degree of freedom in rotation, one need only to restraint the translational degrees of freedom as specified in the quadrilateral thin-shell element model. There are 19 nodes on the circumference of each quarter nozzle, and five nodes (four elements) across the thickness as shown in Figure 8 (a).
a) Radial load. When applying the radial load of 1000 pounds to the entire top edge, only 250 pounds of radial force is uniformly distributed at the 95 nodes (19 X 5) on the top of the quarter model. This means that 1.38889 pounds are applied at the ten nodes on the sector boundary, and 2.77778 pounds of nodal force are applied to the remaining 85 nodes of the model.
b) Circumferential and longitudinal moments. When applying the rotational moments to the three dimensional iso-parametric solid elements model shown in Figure 8 (b), the nodal forces applied at the top of the nozzle must be linearly distributed along both the circumferential direction and across the thickness of the model. A 250 lb -in moment is used for the quarter shell-element model. In order to account for both the circumferential and longitudinal moments, the following equations apply:


Figure 8. Loading for three dimensional iso-parametric solid element.

$$
\begin{align*}
& \frac{M_{c}}{4}=\sum_{\theta_{1}=0^{\circ}, 5^{\circ} \cdot \ldots .}^{90^{\circ}} f \cos \theta_{i} \cdot c \cdot \cos \theta_{i} \text { or } f=\frac{M_{c}}{36 \cdot c}  \tag{66}\\
& \frac{M_{L}}{4}=\sum_{\theta_{1}=0^{\circ}, 5^{\cdot} \cdot \ldots .}^{90^{\circ}} f \sin \theta_{l} \cdot c \cdot \sin \theta_{i} \text { or } f=\frac{M_{L}}{36 \cdot c} \tag{67}
\end{align*}
$$

where $\overline{\mathrm{f}}$ represents the maximum value of the nodal force. This maximum occurs at zero degree for the circumferential moment and at 90 degrees for the longitudinal moment along the center line of the thickness. In addition to the nodal forces across the thickness of the nozzle being linearly distributed, the nodal forces along the circumference of the nozzle are distributed sinusoidally in order to achieve the actual circumferential or longitudinal moment effect as shown in Figure $8(b)$, i.e.,

$$
\begin{aligned}
f \cdot \cos \theta_{i} \cdot c \cdot \cos \theta_{i}= & \left(c-\frac{t}{2}\right) \cos \theta_{i} \cdot(f-2 \delta) \cos \theta_{i}+\left(c-\frac{t}{4}\right) \cos \theta_{i} \cdot(f-\delta) \cos \theta_{i} \\
+c \cdot \cos \theta_{i} \cdot f \cos \theta_{i} & +\left(c+\frac{t}{4}\right) \cos \theta_{i} \cdot(f+\delta) \cos \theta_{i} \\
& +\left(c+\frac{t}{2}\right) \cos \theta_{i} \cdot(f+2 \delta) \cos \theta_{i}
\end{aligned}
$$

or,
$f \cdot c=5 \cdot f \cdot c+\frac{5 \cdot t \cdot \delta}{2}$

Due to the linearity of nodal forces, the relationship of
these forces at each node across the thickness is:
$\frac{f+\delta}{f}=\frac{c+\frac{t}{4}}{c}$
$\delta=\frac{t \cdot f}{4 \cdot c}$
and

$$
f \cdot c=\left(5 c+\frac{5 \cdot t^{2}}{8 \cdot c}\right) \cdot f
$$

so that,

$$
f=\frac{8 \cdot 7 \cdot c^{2}}{40 c^{2}+5 t^{2}}
$$

and

$$
\delta=\frac{2 \cdot t \cdot \bar{f} \cdot c}{40 c^{2}+5 t^{2}}
$$

For a detail description of the three dimensional iso-parametric solid element model with various loading cases, one may refer to Appendix C.

## CHAPTER V

DATA COMPARISON
A. Comparison of stress factors: For moment loadings, the stress factors induced by the longitudinal bending moment are compared with previously published data from WRC 107, revised in 1974 [1], and the experimental data from Decock in 1980 [21]. Decock's experiment has four models with gamma values of 25 and beta values of $0.19,0.37,0.69$, and 1.0 respectively. For comparison, the bending and membrane stress factors in both longitudinal and transversal directions are plotted in Figures 9 through 12 respectively. Note that Decock's data provides only four discrete points. From these plots, one concludes that, in general, the finite element results are smaller than those of WRC No. 107, but are within the general range of both WRC No. 107 and Decock's experimental results. It is noted from Fig. 9 and 10 that extrapolation of data from WRC No. 107 when beta exceeds 0.55 is not conservative for certain cases. For radial force, longitudinal and circumferential moments, the stress factors are compared with model ORNL-3 from Gwaltney et al. [19], and model $C-1$ from Mershon (1981). Models ORNL-3 and $C-1$ are cited in steele's publication [6]. The geometry of these models is listed in Table 3 and the stress factors are shown in Table 4. One further notes, from Table 4, that there are four sets of data for each listed stress factor. The first set is from steele's work which is derived from the stress function approach; the second set is
$\gamma=25$

$\gamma=25$

| $*$ WRC 107 |
| :--- |
| * Shell |
| J. Decock |

$\gamma=25$

Fig. 11 Comparison of membrane stress factors in the $\varnothing$ direction due to longitudinal moment from 1. WRC 107 [1], $\beta$ due to longitudinal moment from 1. WRC 107 [1],
$\stackrel{N}{11}$

Fig. 12 Comparison of membrane stress factors in the $x$ direction 2. FEM shell, 3. Decock [21]


Table 3. Pipe-nozzle geonetry for model ORML-3[19] and model [-1 [6],

| Ref. Model | Vessel Type | L | 0 | $T$ | $d$ | $t$ | beta | gaana |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 19. ORNL-3 | Clamped End | 39 | 10 | 0.2 | 1.29 | 0.17 | 0.132 | 25 |
| 6. C-1 | Ring End | 69 | 24 | 0.104 | 12. | 0.102 | 0.5 | 114.5 |

Table 4. Comparison of stress factors vith Steele [6], ORNL-3, [-1, and URC 107 [1].

| Fig. Ho. | Stress factor | Steele [6] | $\begin{gathered} \text { ORNL-3 } \\ {[19]} \end{gathered}$ | FEH | $\begin{aligned} & \text { WRC107 } \\ & \text { [1] } \end{aligned}$ | Stecle [6] | $\begin{aligned} & \mathrm{C}-1 \\ & {[6]} \end{aligned}$ | FEM | $\begin{aligned} & \text { HRC107 } \\ & \text { [1] } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A-1C | $\mathrm{H}_{0} / \mathrm{P}$ | 0.159 | 0.14 | 0.098 | 0.138 | 0.0823 | 0.025 | 0.0158 | 0.009 |
| A-2C | $H_{x} / \mathrm{P}$ | 0.056 | 0.09 | 0.05 | 0.099 | 0.0258 | 0.008 | 0.0055 | 0.0023 |
| A-3C | $N_{0} /\left(P / R_{n}\right)$ | 3.8 | 2.75 | 2.7 | 3.9 | 2.405 | 1.145 | 1.2 | 2.1 |
| A-4C | $N_{x} /\left(P / R_{n}\right)$ | 4.25 | 3.25 | 3.5 | 4.5 | 18.32 | 0.8015 | 5.01 | 6.7 |
| A-1A | $\mathrm{H}_{0} /\left(\mathrm{H}_{\mathrm{c}} / \mathrm{R}^{1} \mathrm{~B}\right)$ | 0.1177 | 0.105 | 0.13 | 0.095 | 0.07875 | 0.0459 | 0.011 | 0.053 |
| A-2A | $M_{X} /\left(M_{C} / R_{B} \beta\right)$ | 0.0503 | 0.0744 | 0.055 | 0.056 | 0.02625 | 0.0175 | 0.0055 | 0.0256 |
| A-3A | $N_{0} /\left(H_{c} / H_{n}{ }^{2} \beta\right)$ | 2.056 | 1.3125 | 1.4 | 0.72 | 1.7034 | 2.0038 | 2.19 | 2.01 |
| A-4A | $N_{x} /\left(H_{c} / H_{n}{ }^{2}{ }^{\prime}\right)$ | 0.8203 | 0.9844 | 0.8 | 1.01 | 4.61 | 3.51 | 6.8 | 7.8 |
| A-18 | $\mathrm{H}_{0} /\left(\mathrm{H}_{L} / R_{1} \mathrm{~B}\right)$ | . 0381 | . 0486 | . 0425 | . 048 | . 0035 | . 0022 | . 0042 | 0.0028 |
| A-2B | $H_{x} /\left(H_{L} / R_{0} \beta\right)$ | . 0866 | . 06125 | . 055 | . 078 | . 0118 | . 0066 | . 0082 | 0.0055 |
| A-3B | $N_{0} /\left(H_{L} / A_{m}^{2} \beta\right)$ | . 9297 | 2.078 | 1.6 | 2.5 | 3.557 | 2.705 | 2.1 | 3.3 |
| A-4B | $N_{x} /\left(H_{L} / R_{n}{ }^{2} \beta\right)$ | 1.4766 | . 5469 | 1.2 | . 69 | . 351 | . 5511 | . 7 | 1.89 |

Table 5. Data comparison for HRC297, FEM, and WRC 107

| $\begin{array}{\|l\|} \text { Fig. } \\ \text { No. } \end{array}$ | Stress factor | Gamas 100 |  |  | 75 |  |  | 35 |  |  | 15 |  |  | 10 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | beta 0.05 |  |  | 0.0667 |  |  | 0.143 |  |  | 0.333 |  |  | 0.5 |  |  |
|  |  | \$297 | FEM | \$107 | H297 | FEH | W107 | W297 | FEM | H107 | H297 | FEM | W107 | W297 | FEM | H107 |
| A-1C | $\mathrm{H}_{0} / \mathrm{P}$ | . 235 | . 401 | . 21 | .213 .2 | . 211 | . 122 | . 17 | . 09 | . 11 | . 125 | . 045 | . 06 | . 108 | . 123 | . 040 |
| A-3C | $\mathrm{N}_{0} /\left(\mathrm{P} / \mathrm{R}_{\mathrm{s}}\right)$ | 53. | 61.7 | 15.5 | 32.25 | 42. 1 | 13. |  | 4.1 | 5. | 1.395 | . 99 | 1.45 | . 8 | . 65 | . 77 |
| A-2C | $H_{x} / \mathrm{P}$ | . 0815 | . 11 | . 12 | . 075. | . 14 | . 16 | . 057 | . 05 | . 51 | . 043 | . 02 | . 048 | . 035 | . 027 | . 033 |
| A-4C | $\mathrm{N}_{\mathrm{x}} /\left(\mathrm{P} / \mathrm{R}_{\mathrm{n}}\right)$ |  | 18. | 19. |  | 12. 1 | 13.5 |  |  | 6.1 |  | 1.93 | 2.2 |  | . 55 | 1.3 |
| A-1A | $H_{0} /\left(R_{c} / R_{B} \beta\right)$ | . 1264 | . 09 | . 087 | . 1247 | 7.11 | . 09 | . 0919 | . 091 | . 092 | . 08312 | . 082 | . 083 | . 0788 | . 075 | . 065 |
| A-3A | $N_{0} /\left(H_{C} / H_{B}{ }^{2} \mathrm{~B}\right)$ | 6.344 | 5.5 | 2.0 | 4.856 | 4.4 | 1.8 | 2.297 | 2.1 | 1.3 | . 958 | 1.1 | .67 | . 6038 | . 78 | . 465 |
| A-2A | $H_{x} /\left(H_{c} / R_{8} \beta\right)$ | . 0494 | . 046 | . 048 | . 0481 | . 05 | . 06 | . 046 | . 047 | . 05 | . 0394 | . 04 | . 043 | . 0376 | . 0273 | . 04 |
| A-4A | $\mathrm{H}_{\mathrm{x}} /\left(\mathrm{H}_{6} / \mathrm{R}^{2} \mathrm{~B}\right)$ | 3.413 | 3.8 | 4.05 | 2.166 | 2.6 | 2.7 | 1.103 | 1.15 | 1.7 | . 361 | .61 | 1.2 | . 245 | . 65 | .5 |
| A-28 | $H_{x} /\left(H_{L} / R_{n} \beta\right)$ | 1.028 | . 0573 | . 06 | . 0976 | . 062 | . 063 | . 0788 | . 041 | .043 | . 0556 | . 043 | . 042 | . 0459 | . 034 | . 02 |
| A-4B | $N_{x} /\left(H_{l} / R_{\text {m }}{ }^{2} \beta\right)$ | 5.6875 | 4.4 | 2.3 | 4.102 | 3.34 | 1.5 | 1.608 | 1.7 | 1.15 | . 525 | . 7 | . 65 | . 2844 | . 44 | . 46 |
| A-1B | $H_{0} /\left(H_{L} / R_{m} m^{(2)}\right.$ | . 0398 | . 0392 | . 06 | . 103 | . 043 | . 059 | . 0293 | . 035 | . 042 | . 02013 | . 036 | . 027 | . 0166 | . 031 | . 02 |
| A-38 | $\mathrm{N}_{0} /\left(\mathrm{H}_{\mathrm{L}} / R_{p}^{2} \beta\right)$ | 2.975 | 6.03 | 5.6 | 2.89 | 3.9 | 5.2 | 1.975 | 1.95 | 3.6 | . 906 | . 84 | 1.65 | . 525 | . 9 | 1.08 |

from the experimental work of the ORNL-3 or the $C-1$ model; the third set is from the quadrilateral thin shell model results and finite element method and the fourth set is interpolated from WRC No. 107. One may conclude from Table 4 that the stress factors from FEM are within reasonable range of those from previous work. For design purposes, the finite element solutions should furnish very reasonable results over an extended range of geometry. In addition, comparison of results with WRC No. 297 and WRC No. 107 are as shown in Table 5. This tabulation contains five different sets of beta and gamma combinations.

Since the definitions of stress factors in WRC 297 are different from those used in this thesis, the data from WRC 297 must be multiplied by a specific coefficient in order to arrive at the same stress factors.
a) Radial force loadings:
(1) for the bending stress, the factor in WRC 297 is multiplied by unity.
(2) for the membrane stress, the factor is multiplied by gamma.
b) Moment loadings:
(1) for the bending stress, the factor is multiplied by 0.4375 .
(2) for the membrane stress, the factor is multiplied by 0.4375 gamma.

In WRC 107 [1], the authors made these assumptions: shear forces are transmitted to the shell entirely by membrane shear force according to Bijlaard's paper [3]; the shear stresses due to the circumferential and longitudinal shear force loadings are expressed simply by equations (51) (52); and shear stress due to torsional moment are presented by equation (53). By normalizing these shear stresses due to shear forces and torsional moment loadings into dimensionless stress factors, one may derive the following:
using equations (51) \& (52),

$$
\begin{array}{ll}
\tau_{x \phi}=\frac{V_{c}}{\pi c T} & \text { (max. at } A \text { and } B, \text { see Figure 4) } \\
\tau_{\theta x}=\frac{V_{L}}{\pi c T} & \text { (max, at } C \text { and } D, \text { see Figure 4) }
\end{array}
$$

so that, $\frac{N_{x \phi}}{V_{c} / R_{m}}$ or $\frac{N_{\phi x}}{V_{L} / R_{m}}=\frac{1}{\pi \beta}$
and

$$
\begin{equation*}
\frac{N_{\phi x}}{M_{T} /\left(R_{m}^{2} \beta\right)}=\frac{1}{2 \pi \beta} \tag{71}
\end{equation*}
$$

The above normalization of shear stress factors is based on the assumptions from WRC 107 [1]. The FEM data are shown in Figure $A-1 D$ to $A-5 D, A-2 E, A-4 E, A-5 E$, and $A-1 F$. In comparing the FEM results with the results of WRC 107
obtained through equation (71), one concludes that WRC 107 shows more conservative shear stress factors. One would attribute this to the fact that in WRC 107 all shear forces are assumed to be transmitted to pipe shell and expressed in shear stresses only. For torsional moment loading, the FEM results are in good agreement with the data from WRC 107. Those data are obtained from equation (72) with beta less than 0.3. When beta is larger than 0.3 , the FEM results show a upper bound limit for data from equation (72).

In the finite element results, one finds that the shear force induces not only local shear stress but also induces local membrane and bending stresses at the front end of the nozzle. This explains the reason why Equation (71) from WRC 107 produces more conservative results than the FEM. Hence, when a pipe-nozzle model is subjected to a circumferential shear force, the force would induce a maximum shear stress at the two nodes on the pipe's longitudinal plane (points A and $B$ of Figure 4). That shear stress can be compared directly to values calculated from Equation (71). The other two nodes, on the pipe's transversal plane (point $C$ and $D$ of Figure 4), will show significant local membrane and bending stresses in both transversal and longitudinal directions. Similar situations are apparent when the pipe-nozzle is subjected to a longitudinal shear force, except that the normal stress in the transversal direction is small and hence can be neglected. The normal stress condition is


#### Abstract

reasonable since the pipe-nozzle connection model exhibits symmetric displacements when an external force is applied in the longitudinal direction. The local stress factors induced by the longitudinal shear force are plotted in Figure 2E, 4E, and 5E respectively. The local stresses in the transversal direction due to longitudinal shear force are small and can be neglected. For the torsional moment, the induced maximum local shear stress factors are plotted in Figure $\mathrm{A}-1 \mathrm{~F}$.


Again, one concludes that the FEM results are in good agreement with the results from other literature.
B. Comparison of spring coefficient: To verify the spring coefficient results, the spring coefficients computed from the three dimensional iso-parametric solid element model and the quadrilateral thin shell model are compared with data from Sun \& Sun [15], and Murad \& Sun [14]. The comparisons are shown in Figures 13 to 15 . These graphs indicate that the three dimensional iso-parametric solid element model and the quadrilateral thin shell model both generate very similar results for all the spring coefficients computed in this study. In addition, the results are seen to be in very close agreement to those published by Murad \& Sun [14]. However, the results from Sun \& Sun [15] are larger than the FEM results for the $K_{L} / R_{m}{ }^{3}$ and the $K_{R} / C$ cases. The reason for this discrepancy is explained by the fact that a double

| II |
| :--- |
| $\stackrel{11}{2}$ |



TempleGraph 2.2.1, Origin: hxs3286@mesunw7 - Wed Mar 14 20:20:26 1990 (data file was krcomfin)
$\gamma=25$


$\gamma=25$


Fourier series was used on a closed shell in the Sun \& Sun [15] study, while the attachment is, in reality, a square tube. The FEM solution utilizes more realistic geometry in representing the pipe-nozzle connection. Table 6 illustrates further comparisons of the finite elements solutions with previous published results from WRC 297 [2]. In addition, Fig. 13 - 15 and Table 6 indicate that the FEM solution shows larger flexibility in the radial spring coefficient; less flexibility in the circumferential spring coefficients; and agrees well with WRC 297 for flexibility in the longitudinal spring coefficient.

Zienkiewics [25] has stated that in three-dimensional element analysis it is possible, in theory, to achieve absolute convergence to the true solution of the elasticity problem. With a straight forward use of the three-dimensional concept, however, certain difficulties will be encountered. Firstly, the retention of the three degrees of freedom at each node leads to large stiffness coefficients for relative displacements along an edge corresponding to the shell thickness. These coefficients present numerical problems and may lead to ill-conditioned equations when the shell thickness become small compared with the other dimensions in the element. Secondly, the matter of cost effectiveness in computation. It is well-known that even for thick shell, the "normal" to the middle surface remains practically straight after deformation. Hence the use of

Table 6. Spring Coefficients Conparison of WRC 297 and FEM

## Geometries

| gasea | 10 | 15 | 25 | 35 | 50 | 75 | 100 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| beta | 0.1 | 0.2 | 0.3 | 0.3 | 0.1 | 0.2 | 0.1 |
| $K_{R} / \mathrm{c}$ Radial Spring Corffitients |  |  |  |  |  |  |  |
| HRC 297 [2] | 2.195E6 | 6.69855 | 2.978E5 | 1.554E5 | 8.939E4 | 3.018 E 4 | 1.6778 E 4 |
| FEM | $1.901 \mathrm{E6}$ | 4.841 E5 | 1.406E5 | 0.750 E 5 | $6.941 E 4$ | $1.945 E 4$ | 1.5020E4 |
| $K_{C} / \mathbb{R}_{n}{ }^{3}$ Circuaferential Rotational Spring Coefficients |  |  |  |  |  |  |  |
| HRC 297 [2] | 1.770E4 | 8.001 E3 | 2.880E3 | 1.259E3 | 2.064 E 2 | 1.208 E 2 | 3.330E1 |
| FEH | 1.650E4 | 10.650E3 | 5.540 E 3 | 2.488E3 | 2.220E2 | 2.040E2 | 3.880 El |
| $K_{L} / R_{a}{ }^{3}$ Longitudinal Rotational Spring Coeffirients |  |  |  |  |  |  |  |
| HRC 297 [2] | 3.150E4 | 2.401E4 | $1.824 E 4$ | 7.697E3 | $6.001 E 2$ | 7.110E2 | 1.350E2 |
| FE月 | 2.950E4 | 2.240 E4 | $1.680{ }^{4}$ | 9.138E3 | 5.890E2 | 9.450E2 | 1.390E2 |

several nodes across the shell thickness creates an unnecessarily high number of degrees of freedom and the additional computation time associated with them.

In studying the stress associated with the quadrilateral thin shell element and three-dimensional iso-parametric solid element models, it is found that when the shell is relatively thick ( gamma is small ) both models yield very similar results. However, in very thin shell, only the shell model yields accurate results, especially when bending stresses make a significant contribution to the total stress. The three-dimensional iso-parametric solid element model, which might be expected to yield better accuracy, actually leads to substantial error. The error is attributable the fact that the iso-parametric model does not treat the relatively large rotations that occur in thin shells. This phenomenon agrees well with Zienkiewicz's statement [25]. However, with respect to spring coefficients, the three-dimensional iso-parametric slid element model and the quadrilateral thin shell element model yield results which are in agreement.

## NUMERICAL EXAMPLE

## A) Spring Coefficients:

To calculate $K_{R}, K_{C}$, and $K_{L}$, the following example is given: A 12 in. sch std. pipe is intersected by a 8 inch nozzle which is a sch 40 API pipe. In this model, $R_{m}=$ 6.1875 in., $T=t=0.375$ in., $c=4.3125$ in. As a result: beta $\left(C / R_{m}\right)$ is 0.697 and gamma $\left(R_{m} / t\right)$ is 16.5. Assume alpha (l/ $R_{m}$ ) is 8.0 (i.e., a second nozzle, pipe bend, or trunnion is at least 49.5 inch away from the center line of the nozzle).

Figure $B-1$ gives $K_{R} / C=0.28 \times 10^{6}$, then

$$
\mathrm{K}_{\mathrm{R}}=0.12 \times 10^{7} \mathrm{lb} . / \mathrm{in} .
$$

Figure $B-2$ gives $K_{C} / R_{m}^{3}=0.92 \times 10^{5}$, then

$$
\mathrm{K}_{\mathrm{C}}=0.218 \times 10^{8} \mathrm{in} .-1 \mathrm{~b} . / \mathrm{rad} .
$$

Figure $B-3$ gives $K_{I} / R_{m}{ }^{3}=0.21 \times 10^{6}$, then

$$
\mathrm{K}_{\mathrm{L}}=0.497 \times 10^{8} \mathrm{in},-1 \mathrm{~b} . / \mathrm{rad} .
$$

## B) Local Stresses:

For the same pipe-nozzle model subjected to the following external loadings:

$$
\begin{aligned}
& P=400 \mathrm{lb} . \\
& M_{C}=500 \mathrm{lb} .-\mathrm{in} . \\
& M_{L}=500 \mathrm{lb} .-\mathrm{in} . \\
& M_{T}=500 \mathrm{lb} .-\mathrm{in} . \\
& V_{C}=300 \mathrm{lb} . \\
& V_{L}=-400 \mathrm{lb} . \\
& \text { the stresses are calculated by reading the dimensionless } \\
& \text { stress factors from various figures in Appendix A and } \\
& \text { multiplying the corresponding loadings by these factors as } \\
& \text { shown in the following table: }
\end{aligned}
$$

Table 7. Computing stresses

| Figure Value from |  |
| :---: | :---: | :---: |
| Figure | stress |

## Radial force, $P=400 \mathrm{lb}$.


membrane A-3C $\frac{N}{P / R_{m}}=0.34 \quad \sigma_{\theta}=K_{n} \cdot(0.34) \cdot \frac{P}{R_{m} T}=58.61 \mathrm{psi}$
membrane $\mathrm{A}-4 \mathrm{C} \quad \frac{N_{x}}{P / R_{m}}=1.15 \quad \sigma_{x}=K_{n} \cdot(1.15) \cdot \frac{P}{R_{n} T}=198.25 \mathrm{psi}$

| bending | $\mathrm{A}-1 \mathrm{C}$ | $\frac{M_{0}}{P}=0.0155$ | $\sigma_{0}=K_{b} \cdot(0.0155) \cdot \frac{6 P}{T^{2}}=264.53 \mathrm{psi}$ |
| :--- | :--- | :--- | :--- |
| bending | $\mathrm{A}-2 \mathrm{C}$ | $\frac{M_{x}}{P}=0.010$ | $\sigma_{x}=K_{0} \cdot(0.01) \cdot \frac{6 P}{T^{2}}=170.67 \mathrm{psi}$ |

## Circumferential moment, $M_{C}=500$ lb.-in.


membrane $\mathrm{A}-3 \mathrm{~A} \quad \frac{N_{1}}{M_{c} / R_{m}^{2} \beta}=0.88 \quad \sigma_{0}=K_{n} \cdot(0.88) \cdot \frac{M_{c}}{R_{m}^{2} \beta T}=44.1 \mathrm{psi}$
membrane $\mathrm{A}-4 \mathrm{~A} \quad \frac{N_{x}}{M_{c} / R_{m}^{2} \beta}=1.20 \quad \sigma_{x}=K_{n} \cdot(1.2) \cdot \frac{M_{c}}{R_{m}^{2} \beta T}=60.1 \mathrm{psi}$
bending A-1A $\frac{M_{0}}{M_{c} /\left(R_{m} \beta\right)}=0.038 \quad \sigma_{0}=K_{b} \cdot(0.038) \cdot \frac{6 M_{c}}{R_{m} \beta 7^{2}}=188.5 \mathrm{psi}$
bending $\mathrm{A}-2 \mathrm{~A} \quad \frac{M_{x}}{M_{c} /\left(R_{m} \beta\right)}=0.016 \quad \sigma_{x}=K_{b} \cdot(0.016) \cdot \frac{6 M_{c}}{R_{m} \beta T^{2}}=79.4 \mathrm{psi}$

Longitudinal moment, $\mathrm{M}_{\mathrm{L}}=500 \mathrm{lb} .-\mathrm{in}$.

membrane $\mathrm{A}-3 \mathrm{~B} \quad \frac{N_{1}}{M_{t} / R_{m}^{2} \beta}=0.35 \quad \sigma_{1}=K_{n} \cdot(0.35) \cdot \frac{V_{1}}{R_{m}^{2} B}=17.5 \mathrm{psi}$
membrane $\overline{A-4 B} \quad \frac{N_{x}}{M_{L} / R_{m}^{2} \beta}=0.335 \quad \sigma_{x}=K_{n} \cdot(0.335) \cdot \frac{M_{L}}{R_{n}^{2} \beta T}=16.8 \mathrm{psi}$

| bending | A-18 | $\frac{M_{0}}{M_{c} /\left(R_{m} \beta\right)}=0.016$ | $\sigma_{0}=K_{\mathrm{b}} \cdot(0.016) \frac{6 M_{l}}{R_{m} \beta T^{2}}=79.4 \mathrm{psi}$ |
| :---: | :---: | :---: | :---: |
| bending | A-2B | $\frac{M_{x}}{M_{\ell} J\left(R_{m} \beta\right)}=0.0165$ | $\sigma_{x}=K_{b} \cdot(0.0165) \frac{6 M_{L}}{R_{n} \beta T^{2}}=81.9 \mathrm{psi}$ |


Circumferential shear force, $V_{C}=3001 \mathrm{~b}$.
 membrane $\mathrm{A}-3 \mathrm{D} \quad \frac{N_{0}}{V_{c} / R_{m}}=0.0 \quad \sigma_{0}=K_{a} \cdot(0.0) \frac{1}{R_{m} I}-0.0 \mathrm{psi}$
membrane $\overline{A-4 D} \quad \frac{N_{x}}{V_{c} / R_{m}}=0.4 \quad \sigma_{x}=K_{n} \cdot(0.4) \cdot \frac{1_{c}^{\prime}}{R_{m} T}=51.72 \mathrm{psi}$
bending A-1D $\quad \frac{M_{t}}{V_{t}}=0.027 \quad \sigma_{t}=K_{4} \cdot(0.027) \frac{6 V_{c}}{T^{2}}=345.6 \mathrm{psi}$

| bending | $\mathrm{A}-2 \mathrm{D}$ | $\frac{M_{x}}{V_{c}}=0.028$ |
| :--- | :--- | :--- |
| shear | $\sigma_{x}=K_{0} \cdot(0.028) \frac{6 V_{c}}{T^{2}}=358.4 \mathrm{psi}$ |  |
|  | $\frac{N_{\theta x}}{V_{c} / R_{m}}=0.44$ | $\left.\sigma_{\theta x}=10.44\right) \frac{V_{c}}{R_{m} T}=56.89 \mathrm{psi}$ |

## Longitudinal shear force, $\mathrm{V}_{\mathrm{L}}=-400 \mathrm{lb}$.



| bending | $\mathrm{A}-2 \mathrm{E}$ | $\frac{M_{x}}{V_{L}}=0.013$ | $\sigma_{x}=K_{b} \cdot(0.013) \frac{6 V_{L}}{T^{2}}=-83.2 \mathrm{psi}$ |
| :--- | :--- | :--- | :--- |
| shear | $\mathrm{A}-5 \mathrm{E}$ | $\frac{N_{x t}}{V_{L} / R_{m}}=0.2$ | $\sigma_{x t}=$ |
|  | $(0.2) \frac{V_{l}}{R_{m} T}=-34.48 \mathrm{psi}$ |  |  |


Torsional moment, $M T=500$ 1b. - in.
 shear $\left.\quad \mathrm{A}-1 \mathrm{~F} \quad \frac{N_{\text {ox }}}{M_{T} / R_{m}^{2} \beta}=0.62 \quad \sigma_{\phi \times}=10.62\right)_{R_{0,}^{2} \beta T}^{M_{T}}=31.1 \mathrm{psi}$


The total stresses due to each of six loadings for $\lambda_{U}, A_{L}$, $B_{U}, B_{L}, C_{U}, C_{L}, D_{U}$, and $D_{L}$ points are summarized and placed in Table 2. By following sign notations specified in Table 2 , total stresses can be calculated.

| $\begin{aligned} & \text { From } \\ & \text { Fit. } \end{aligned}$ | $\begin{gathered} \text { Read Curves } \\ \text { for } \\ \text { Stress Factor } \end{gathered}$ |  | Compute Absolute Values of Stress and Enter Result | $A_{v}$ | $A_{1}$ | $B_{v}$ | $B 1$ | E. | $C$, | $D_{0}$ | D. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1-3C | $\frac{V_{1}}{\Gamma / R_{m}}$ |  | $K_{n}\left[\frac{N_{0}}{P / R_{m}}\right] \frac{P}{R_{m} T} \quad 38.61$ | - | - | - | - | - | - | - | - |
| A-1C | $\frac{11}{r}$ |  | K. $\int_{-} \frac{6}{P} \int \frac{g P}{T^{2}}=264.53$ | - | + | - | + | - | + | - | + |
| A-3A | $\frac{N_{0}}{M_{+} d\left(R_{m}^{3} \beta\right)}$ |  | $K_{0}\left[\frac{N_{0}}{H_{0} /\left(R_{m}^{2} \beta\right)}\right] \cdot \frac{M_{c}}{R_{4}^{2} \beta T}=44.1$ |  |  |  |  | - | - | + | $+$ |
| A-1才 | $\cdots=\frac{M}{M} /\left(R_{m} B \mid\right.$ |  | $K_{\bullet}\left[\frac{M_{4}}{M_{C}\left(R_{m} \beta\right)}\right] \cdot \frac{S A M_{e}}{R_{m} \beta T^{2}}=188.5$ |  |  |  |  | - | + | $+$ | - |
| A-3B | $\frac{N}{N / \lambda R_{m}^{j} \beta \mid}$ |  | $K_{n}\left[\frac{N_{0}}{M_{4} \lambda\left(R_{m}^{2} \beta \mid\right.}\right] \cdot \frac{M_{1}}{R_{n}^{2} \beta T}=17.5$ | - | $\checkmark$ | + | $+$ |  |  |  |  |
| A-IB | $\frac{N_{0}}{M_{4} /\left(R_{A} B \mid\right.}$ |  | $K_{*}\left[\frac{N_{1}}{M_{1} /\left(R_{m} B\right)}\right] \cdot \frac{6 M_{t}}{R_{m} B T^{3}}-79.4$ | - | + | $+$ |  |  |  |  |  |
| $A-3 D$ | $\frac{N_{1}}{v_{c} / R_{m}}$ |  | $K_{.}\left[\frac{N_{l}}{V_{c}^{\prime} R_{m}}\right] \cdot \frac{V_{c}}{R_{m} T} \quad=0.0$ |  |  |  |  | - | - | + | $+$ |
| A-10 | $\frac{M_{4}}{V_{c}}$ |  | $K_{0}\left[\frac{M_{0}}{V_{e}}\right] \cdot \frac{6 V_{0}}{T^{2}}=345.6$ |  |  |  |  | + | - | - | $+$ |
| A 3 F |  |  | $K \cdot\left[\frac{\bar{N}_{1}}{V R_{m}}\right] \cdot \frac{V_{i}}{R_{m} \bar{I}} \quad \because 0.0$ |  |  |  |  |  |  |  |  |
| 1-1F | $\frac{11}{16}$ |  | $K \cdot\left[\frac{N_{1}}{V_{t}}\right] \frac{6 V_{2}}{T^{3}}=0.0$ |  |  |  |  | $\pi$ |  |  |  |

Ndd lanbraically for Sumation 0 stresscs, $\theta_{1}=$


Add ilmebraically for Summation of $x$ stresses, $\sigma_{r}=$

| A-1F |  | $\left.\dagger \frac{V_{n}}{M \cdot /\left\|R_{n}^{j} \beta\right\|}\right] \frac{M_{r}}{R_{n}^{2} \beta T}=31.1$ | + | $+$ | + | + | - | - | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A-6D | $\frac{V \cdot 1}{-1 / R_{n}}$ | $\left\lfloor\frac{N_{i}}{V_{c} / R_{m}}\right\rfloor \frac{1}{R_{m} \bar{T}} \quad=56.89$ | + | + | - | - |  |  |  |  |
| A - 5 E | $\frac{N_{n}}{V_{1} / R_{n}}$ | $\left[\frac{N_{0}}{V_{i} / R_{m}}\right] \cdot \frac{V_{i}}{R_{m} T}=-34.48$ |  |  |  |  | - | - |  | + |

idd ilacbraically for summation of shear stresses, r. =

## CHAPTER VII CONCLUSIONS

## A) Local Stresses:

Since the finite element technique treats the true geometry of the pipe-nozzle connection and simulates the true loading conditions on the nozzle, this thesis presents a major improvement over all the previously published studies on local stresses. In addition, for the application of a radial force and two overturning moments, this thesis extends the maximum beta value from 0.55 to a previously unavailable value of 0.9. The stress factors due to the radial force and two overturning moments developed in this thesis show good agreement with those published in WRC No. 107 for beta values from 0.1 to 0.55 . Note, however, the values from WRC 107 are considered conservative since there is no real shell opening to simulate the true geometry. For beta larger than 0.55 , our data agrees well with published experimental work.

Figures $A-1 A$ to Figure $A-4 C$ presented in Appendix $A$ provide a direct replacement of the plots provided in WRC No. 107. These graphs are not only more accurate than those in WRC No. 107, but also extend beta's upper limit from 0.55 to 0.9.

For the longitudinal and circumferential shear forces and the torsional moment, this thesis presents the new, previously unavailable, shear stress factors. These stress factors are plotted in the $D, E$, and $F$ figure series in Appendix A. By studying these figures, one makes the following observations:

1. Shear stress factors due to circumferential shear force are larger than those factors due to longitudinal shear force. One would thus conclude that the pipe-nozzle model is more vulnerable to circumferential shear force than to longitudinal shear force.
2. The membrane and bending stress factors due to circumferential shear forces are larger than those due to longitudinal shear force. Hence the nodes on the circumferential plane of the pipe-nozzle connection would yield higher local membrane and bending stresses than nodes on the longitudinal plane. The elevated stress is due to the curvature of the cylindrical shell in which the longitudinal plane has more symmetric geometry than circumferential plane.
3. By considering the shear stress factors in Figures $A-5 D$ and $A-5 E$ due to shear forces, one concludes that the shear stress factor is independent of gamma. This fact is in agreement with the assumption in WRC 107 and is further illustrated in equations (71) and (72), both of which are functions of beta only.
4. When the model is subjected to a torsional moment, the curvature effect does not contribute much to the shear stress when beta is small. When beta increases, the nodes on the circumferential plane show higher shear stress factors than the nodes on the longitudinal plane.

## B) Spring Coefficients:

Spring constants are presented in coefficient forms as $K_{R} / C, K_{C} / R_{m}{ }^{3}$, and $K_{L} / R_{m}{ }^{3}$ respectively as shown in Figures B-1 to B-3 in Appendix B. From these Figures, one makes the following observations:

1. When the beta value is less than 0.5 , the radial spring coefficient $K_{R} / c$ decreases in value when beta increases. When beta is larger than $0.5, K_{R} / c$ essentially remain constant.
2. When the beta value increases, the rotational spring coefficients $\mathrm{K}_{\mathrm{C}} / \mathrm{R}_{\mathrm{m}}{ }^{3}$ and $\mathrm{K}_{\mathrm{L}} / \mathrm{R}_{\mathrm{m}}{ }^{3}$ increase smoothly.
3. The longitudinal rotational spring coefficients show higher values than the circumferential rotational spring coefficients. This is not surprising in view of the fact that, due to pipe geometry, the longitudinal direction is stiffer than the circumferential direction.
4. When the gamma value increases, all three spring coefficients decrease in value as expected, since the thinner shells are more flexible than the thicker ones.

Appendix A
Fig. A-1A

Fig. A-1A Moment $M_{\phi} /\left(M_{d} / R_{m} \beta\right)$ due to an external circumferential moment $M_{c}$ on a nozzle-to-pipe connection


Fig. A-3A Membrane force $N_{\phi} /\left(M_{c} / R_{m}{ }^{2} \beta\right)$ due to an external circumferential moment $M_{c}$ on a nozzle-to-pipe connection
Fig. A-4A

Fig. A-4A membrane force $N_{x} /\left(M_{d} / R_{m}{ }^{2} \beta\right)$ due to an external circumferential moment $M_{c}$ on a nozzle-to-pipe connection
Fig.A-1B

Fig. A-1B Moment $M_{\downarrow} /\left(M_{\nu} / R_{m} \beta\right)$ due to an external longitudinal moment $M_{L}$ on a nozzle-to-pipe connection

## $A-2 B$

 $\beta$
Fig. A-2B Moment $M_{x} /\left(M_{L} / R_{m} \beta\right)$ due to an external longitudinal moment $M_{L}$ on a nozzle-to-pipe connection
Fig. A-3B

Fig. A-3B Membrane force $N_{\phi} /\left(M_{L} / R_{m}{ }^{2} \beta\right)$ due to an external longitudinal moment $M_{L}$ on a nozzle-to-pipe connection
Fig. A-4B

Fig. A-4B Membrane force $N_{x} /\left(M_{\perp} / R_{m}{ }^{2} \beta\right)$ due to an external longitudinal moment $M_{L}$ on a nozzle-to-pipe connection
Fig. A-1C

Fig. A-1C Bending moment $\mathrm{M}_{\phi} / \mathrm{P}$ due to an external radial load P on a nozzle-to-pipe connection
Fig.A-2C

|  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  | $t=10$ |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  | 3 | $\cdots$ | $\cdots$ | $\underline{1}$ | - |  | - |  |  | 皆 |
|  | $\cdots$ |  |  |  | - |  |  |  |  |  |
|  |  | $\cdots$ | - +3 |  | 2. |  | - |  |  | $\underset{\substack{x=100}}{\substack{e-1}}$ |
|  | - |  | : |  | $\bigcirc$ |  | $\cdots$ |  | + |  |
|  |  |  |  |  | $\cdots$ |  | - |  |  |  |
|  |  |  |  |  |  |  | $\bigcirc$ |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  | - | + | - |  |  |  | - |  |  |  |
|  | - | - | - |  | : |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

Fig. A-2C Bending moment $\mathrm{M}_{x} / \mathrm{P}$ due to an external radial load P on a nozzle-to-pipe connection

## Fig. A-3C


Fig. A-3C Membrane force $N_{\phi} /\left(\mathrm{P} / \mathrm{R}_{\mathrm{m}}\right)$ due to an external radial load P on a nozzle-to-pipe connection
Fig. A-4C

Fig. A-1D
 Fig. A-1D Bending force $M_{\phi} / V_{c}$ due to a circumferetial shear force $V_{c}$

Fig. A-3D

Fig. A-3D Membrane force $N_{\phi} /\left(V_{c} / R_{m}\right)$ due to a circumferential shear force $V_{c}$
Fig. A-4D

Fig. A-5D
Fig. A-2E

Fig. A-2E Bending force $M_{x} / V_{L}$ due to a longitudinal shear force $V_{L}$
Fig. A-4E


Fig. A-1F


Appendix $B$
 $\beta$
0.40
$-$
$+$


1.00
$\beta$
Fig B-2 Circumferential spring coefficient of nozzle-to-pipe connection with $\mathrm{t} / \mathrm{T}=1, \mathrm{q}=0 \mathrm{psi}$

Fig B-3 Longitudinal spring coefficient of nozzle-to-pipe connection with $\mathrm{t} / \mathrm{T}=1, \mathrm{q}=0 \mathrm{psi}$

Appendix $C$

## APPENDIX C

## ANSYS FINITE ELEMENT MODEL AND PROGRAM


#### Abstract

The nozzle-piping connection geometries are modeled by the quadrilateral thin shell element (ET 63) and the three-dimensional isoparametric solid element (ET 45) respectively in the ANSYS program. General descriptions of these two types of element [23], and programs for establishing the nozzle-piping models are included in this appendix.


Finite element analysis is broadly defined as a group of numerical methods for approximating the governing equations of any continuous system. The theory of finite element is sometimes called the "theory of piecewise continuous approximation." A finite element is a subregion of a discretized continuum. It is of finite size and usually has a simpler geometry than that of the continuum. The finite element model is a geometrical representation of the actual physical structure being analyzed. The mathematically difficult problem of analyzing the opening of the vessel in the nozzle-piping connection model is solved by the finite element method in that it simulates the real geometry. In addition, the method checks results from previous investigations as well
as extending various design ranges. The finite element method is a computer-oriented method that must be implemented with appropriate digital computer programs, such as ANSYS from Swanson Analysis, Inc., etc. In this ANSYS finite element program, the matrix displacement method of analysis is based on the finite element idealization. The structural regions (called elements) are connected at a finite number of points (called nodal points). If the force-displacement relationship for each of these discrete structural element is known (the element "stiffness" matrix), then the force-displacement relationship for the entire structure can be assembled using standard matrix methods.
A) Quadrilateral thin shell element: This element has both bending and membrane capabilities. It permits both in-plane and normal loads. The element has all six degrees of freedom at each node. The geometry, nodal point locations, loading, and the coordinate system are shown in Fig $C-1$. The membrane stiffness is the same for the membrane shell element including the extra shapes. The bending stiffness is formed from the bending stiffness of four triangular shell elements. Two triangles have one diagonal of the element as a common side and two triangles have the other diagonal of the element as a common side. The stiffness is obtained from the sum of the four stiffnesses divided by two. The element is defined by


Pig. C-1 Coordinate system for quadrilateral thin shell elewent


Fig. C-2 Coordinate system for three dimensional solid slement
four nodal points with thickness, an elastic foundation stiffness, and the material properties. The material's X-direction corresponds to the element's X-direction. Since the material is isotropic, only $X$-direction properties need to be specified. In this ANSYS program, the thickness may vary smoothly over the area of the element, with different thickness input at the four nodal points. In this thesis the element has a constant thickness, therefore only $T K(1)$ need be specified.

Several models have been created with the same loading condition. One model in rough mesh, with 391 nodes and 342 (STIF 63) elements, needs 689 seconds of "CPU time to run a loading case. A second model in fine mesh (to insure convergence) has 981 nodes and 912 (STIF 63) elements, needs 2294 seconds of CPU time to run a loading case. Results from both models are in close agreement, and satisfy the convergence requirement.
B) Three-dimensional isoparametric solid element: The assumptions in three-dimensional isoparametric solid model are similar to those in quadrilateral thin shell model. The element in this model is defined by eight nodal points, each having three degrees of freedom: translations in the nodal $x, y$, and $z$ directions. The geometry, nodal point locations, loading, and the coordinate system for
this element are shown in Figure $\mathrm{C}-2$. The directions for stress output are parallel to the global cartesian coordinate directions.

Since this problem deals not only with membrane stresses but also with bending stresses along the intersection portion of nozzle-piping connections, it requires at least three elements along the thickness direction to furnish bending stress information. The model in this thesis is created with 1960 nodes and 1326 (STIF 45) elements and needs 4519 seconds of CPU time to solve one loading case. For the quarter nozzle-piping geometric model, five nodes (four elements) representing the thickness direction are analyzed.
A. Quadrilateral thin shell element model program:


```
L,5,6,4,0.3
TYPE,1
REAL,2
A,1,5,6,3
A,5,2,4,6
AMESH, ALL
CSYS,1 *** Global Coordinate ***
K,7,RPIP,90,MECE *** Pipe portion Key Point Define ***
K,8,RPIP,THED,MECE
K,9,RPIP,THED
K,10,RPIP, -90,
K,11,RPIP, -90,MECE
K,12,RPIP,-90,LORT
K,13,RPIP,THED, LORT
K,14,RPIP,90,LORT
K,15,RPIP,MIDD
K,16,RPIP,MIDD,MECE
K,17, RPIP,MIDD, LORT
L,3,9,12,1.5 *** Pipe Portion Meshing Define ***
L,6,8,12,1.5
L,4,7,12,1.5
REAL,1
TYPE,1
L, 8,9,12
L,7,8,12
A,3,6,8,9
A,6,4,7,8
TYPE,1
L,8,16,6
L,16,15,12
L,15,9,6
L,16,11,6
L,11,10,12
L,10,15,6
A,9,8,16,15
A,15,16,11,10
L,7,14,16
L, 14, 13, 12
L,13,8,16
A,8,7,14,13
L, 13,17,6
L,17,16,16
A,8,13,17,16
L,17,12,6
L,12,11,16
A,16,17,12,11
AMESH,ALL *** Automatical Meshing ***
CSYS,0
SYMBC,0,3,0,0.005 *** Symmetric Bounclary ***
SYMBC,0,1,0,0.005 *** Symmetric Boundary ***
MERGE,0.001
NALL
EALL
F,6,FY,PLB2,,16,1 *** Apply Loading ***
```

```
F,70,FY,PLB2, ,80,1
F,4,FY,PLB1,
F,69,FY, PLB1
F,5,FY, PLB2
NSEL,Z,LORT
D,ALL,ALL
NAL工
EALL
/VIEW,1,1,1,1
KNUM,1
KPLOT
/VIEW, 1,1,1
WFRONT
WSTART,ALL
WAVES
APLOT, ALL
/PBC, FORCE,1
/PBC,TDIS,I
/PBC,RDIS,1
/PBC, PRES,1
NPLOT
EPLOT
ITER,1,1,1
AFWRITE,,1
FINISH
/EXEC
/INPUT, 27
FINISH
/POST1 *** Post-Processing ***
/CUTPUT,35
/TITLE CASE-1 1/4 MODEL CIRCUMFERENTIAL DIRECTION
/COM RADIAL FORCE
/COM BETA=0.3 GAMMA=75 T=.2 T=0.2
/AUTO
STORE,STRES,DISP
/NOPR
/NOLIST
STRESS,SXCT,63,9
STRESS,SYCT,63,10
STRESS,SXYT,63,11
STRESS,SXCM,63,13
STREESS,SYCM,63,14
STRESS,SXYM,63,15
STRESS,MXC , 63,6
STRESS,MYC,63,7
STRRESS,MXYC,63,8
STRESS,NXIT,63,21
STRESS,NYIT,63,22
STRESS,NXIM,63,37
STRESS,NYIM,63,38
STRESS, PXCT,63,129
STRESS, PYCT,63,130
STRESS, PZCT,63,131
STRESS,PSIT,63,132
```

```
STRESS,SGET,63,133
STRESS, PXCM, 63,134
STRESS, PYCM, 63,135
STRESS,PZCM,63,136
STRESS,PSIM,63,137
STRESS,SGEM,63,138
STRESS,PSST,63,151
STRESS, PSSB,63,152
SET
NSEL,NODE, 81
ENODE
NASEL,NODE, 21
ENODE
PRELEM
PRSTRS,SXCT,SYCT,SXYT,SZCI',NXIT,NYIT, PXCI, PYCT, PZCT,PSIT
PRSTRS,SXCM, SYCM,SXYM,SZCM,NXIM,NYIM, PXCM, PYCM, PZCM, PSIM
PRSTRS,MXC,MYC,MXYC, PSST, PSSB, SEGT, SEGM
NELEM
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
NASEL,NODE, 22, 32,1
NASEL,NODE, 81,95,1
NASEL,NODE, 17
ENODE
NSORT,SY,,,6
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
NUSORT
ESORT,SYCT, , ,20
PRSTRS,SXCT,SYCT,SXYT,SZCT,NXIT,NYIT,PXCT, PYCT,PZCT,PSIT
PRSTRS,SXCM,SYCM,SXYM,SZCM,NXIM,NYIM,PXCM, PYCM, PZCM, PSIM
PRSTRS,MXC,MYC ,MXYC, PSST, PSSB, SEGT, SEGM
NUSORT
NSORT,DISP,, ,10
PRDISP
AVPRIN
PRNSTR,ALI.
NALL
EALL
/VIEW,,1,1,1
PLDISP
PLNSTR,SIGE,SZ
PLNSTR,SIGE,SX
PLNSTR,SX
PLNSTR,SY
SET,1,1
SAVE
FINISH
/PREP7
/OUTPUT,40
```

```
RESUME *** Start Second Loading ***
/TITLE CASE-2 *** For Circumferential Moment ***
/COM MOMENT LOADING IN CIRCUMFERENTIAL DIRECTION
/COM BETA=0.3 GAMMA=75 T=.2 T=0.2
/AUTO
*SET,RY,PLBS/(48*RTRU) *** Parametric Method ***
*SET,M1,RY*0.5
*SET,M2,RY*0.997859
*SET,M3,RY*0.991445
*SET,M4,RY*0.98079
*SET,M5,RY*0.965926
*SET,M6,RY*0.94693
*SET,M7,RY*0.92388
*SET,M8,RY*0.896873
*SET,M9,RY*0.86603
*SET,M10,RY*0.83147
*SET,M11,RY*0.79335
*SET,M12,RY*0.75184
*SET,M13,RY*0.70711
*SET,M14,RY*0.659346
*SET,M15,RY*0.608761
*SET,M16,RY*0.55557
*SET,M17,RY*0.5
*SET,M18,RY*0.442289
*SET,M19,RY*0.390731
*SET,M20,RY*0.3214395
*SET,M21,RY*0.258819
*SET,M22,RY*0.19509
*SET,M23,RY*0.13053
*SET,M24,RY*0.065403
NALL
EALL
DDELE,ALL,ALL *** Remove Previous Loalding ***
FDELE,ALL
ASYMBC,0,1,0,0.005 *** Redefine Boundary Condition ***
SYMBC,0,3,0,0.005
NSEL, Z, LORT
*** For Nodes on Symmetric Plane ***
*** Define Boundary Condition ***
D,ALL,ALL
NALL
EALL
F,80,FY,M24 *** ApplY Loading ***
F,79,FY,M23
F,78,FY,M22
F,77,FY,M21
F,76,FY,M20
F,75,FY,M19
F,74,FY,M18
F,73,FY,M17
F,72,FY,M116
F,71,FY,M15
F,70,FY,M14
F,5,FY,M13
F,16,FY,M12
F,15,FY,M11
```

```
F,14,FY,M1O
F,13,FY,M9
F,12,FY,M8
F,I1,FY,M7
F,10,FY,M6
F,9,FY,M5
F,8,FY,M4
F,7,FY,M3
F,6,FY,M2
F,4,FY,M1
/VIEW,,1,1,1
/PBC, FORCE, 1
/PBC,TDIS, 1
/PBC,RDIS,I
/PBC, PRES, 1
NPLOT
ITER,1,1,1
AFWRITE, ,1
FINISH
/EXEC *** Execute Program ***
/INPUT,27
FINISH
/POST1 *** Post-Processing ***
/OUTPUT,36
/TITLE CASE-2 1/4 MODEL CIRCUMFERENTIAL DIRECTION
/COM MOMENT IN CIRCUMFERENTIAL DIRECTION
/COM BETA=0.3 GAMMA=75 T=0.2 T=0.2
/AUTO
STORE,STRES,DISP
/NOPR
/NOLIST
STRESS,SXCT,63,9
STRESS,SYCT,63,10
STRESS,SXYT,63,11
STRESS,SXCM,63,13
STRESS,SYCM,63,14
STRESS,SXYM,63,15
STRESS,MXC,63,6
STRESS,MYC ,63,7
STRESS,MXYC,63,8
STRESS,NXIT,63,21
STRESS,NYIT,63,22
STRESS,NXIM,63,37
STRESS,NYIM,63,38
STRESS, PXCT,63,129
STRESS, PYCT,63,130
STRESS,PZCT,63,131
STRESS,PSIT,63,132
STRESS,SGET,63,133
STRESS,PXCM,63,134
STRESS, PYCM,63,135
STRESS,PZCM, 63,136
STRESS,PSIM,63,137
STRESS,SGEM,63,138
```

```
STRESS, PSST, 63,151
STRESS, PSSB,63,152
SET
NSEL,NODE, 81
ENODE
NASEL, NODE, 21
ENODE
PRELEM
PRSTRS,SXCT,SYCT,SXYT,SZCT,NXIT,NYIT, PXCT, PYCT,PZCT,PSIT
PRSTRS,SXCM,SYCM,SXYM,SZCM,NXIM,NYIM, PXCM, PYCM,PZCM, PSIM
PRSTRS,MXC,MYC,MXYC, PSST, PSSB,SEGT,SEGM
NELEM
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
NASEL,NODE, 22,32,1
NASEL,NODE, 81,95,1
NASEL,NODE, 17
ENODE
NSORT,SY, ,,6
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
NUSORT
ESORT,SYCT,,,20
PRSTRS,SXCT,SYCT,SXYT,SZCT,NXIT,NYIT, PXC', PYCI, PZCT, PSIT
PRSTRS,SXCM,SYCM, SXYM,SZCM,NXIM,NYIM, FXCM, PYCM, PZCM, PSIM
PRSTRS,MXC,MYC,MXYC, PSST, PSSB,SEGT,SEGM
NUSORT
NSORT,DISP, , ,10
PRDISP
AVPRIN
PRNSTR,ALL
NALL
EALL
/VIEW, 1,1,1
PLDISP
PLNSTR,SIGE,SZ
PLNSTR,SIGE,SX
PLNSTR,SX
PLNSTR,SY
SET,1,1
SAVE
FINISH
/PREP7 *** Third Loading ***
/OUTPUT,40 *** For Longitudinal Moment ***
RESUME
/TITLE CASE-3 1/4 MODEL LONGITUDINAL IIRECTION
/COM MOMENT LOADING LONGITUDINAL DIRECIION
/COM BETA=0.3 GAMMA=75, T=0.2 T=0.2
NALL
EALL
```

```
DDELE,ALL,ALL
FDELE,ALL
SYMBC,0,1,0,0.005
ASYMBC,0,3,0,0.005
NALL
EALI
F,69,FY,MI
F,80,FY,M2
F,79,FY,M3
F,78,FY,M4
F,77,FY,M5
F,76,FY,M6
F,75,FY,M7
F,74,FY,M8
F,73,FY,M9
F,72,FY,M10
F,71,FY,M11
F,70,FY,M12
F,5,FY,M13
F,16,FY,M14
F,15,FY,M15
F,14,FY,M16
F,13,FY,M17
F,12,FY,M18
F,iI,FY,M19
F,10,FY,M20
F,9,FY,M21
F,8,FY,M22
F,7,FY,M23
F,6,FY,M24
NSEL, Z,LORT
D,ALL, ALL
NALL
EALL
/VIEW, 1,1,1
NPLOT
EPLOT
ITER,1,1,1
AFWRITE, , 1
FINISH
/EXEC
/INPUT,27
FINISH
/POSTI
/OUTPUT,37
/TITLE CASE-3 1/4 MODEL LONGITUDINAL DIRECTION
/COM MOMENT IN LONGITUDINAL DIRECTION
/COM BETA=0.3 GAMMA=75 'T=0.2 'T}=0.
AUTO
STORE,STRES,DISP
/NOPR
/NOLIST
STRESS,SXCT,63,9
STRESS,SYCT,63,10
```

```
STRESS,SXYT,63,11
STRESS,SXCM,63,13
STRESS,SYCM,63,14
STRESS,SXYM,63,15
STRESS,MXC,63,6
STRESS,MYC,63,7
STRESS,MXYC,63,8
STRESS,NXIT,63,21
STRESS,NYIT,63,22
STRESS,NXIM,63,37
STRESS,NYIM,63,38
STRESS,PXCT,63,129
STRESS,PYCT,63,130
STRESS,PZCT,63,131
STRESS,PSIT,63,132
STRESS,SGET,63,133
STRESS,PXCM,63,134
STRESS, PYCM,63,135
STRESS,PZCM,63,136
STRESS,PSIM,63,137
STRESS,SGEM,63,138
STRESS, PSST,63,151
STRESS,PSSB,63,152
SET
NSEL,NODE,81
ENODE
NASEL,NODE, 21
ENODE
PRELEM
PRSTRS,SXCT,SYCT,SXYT,SZCT,NXIT,NYIT,PXCT,PYCT,PZCP, PSIT
PRSTRS,SXCM,SYCM,SXYM,SZCM,NXIM,NYIM, PXCM, PYCM, PZCM, PSIM
PRSTRS,MXC,MYC,MXYC, PSST, PSSB,SEGT,SEGM
NELEM
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
NASEL,NODE, 22,32,1
NASEL,NODE, 81,95,1
NASEL,NODE, 17
ENODE
NSORT,SY,,,6
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
NUSORT
ESORT,SYCT,, , 20
PRSTRS,SXCT,SYCT,SXYT,SZCT,NXIT,NYIT, FXCI, PYCI, PZC', PSIT
PRSTRS,SXCM,SYCM,SXYM,SZCM,NXIM,NYIM, PXCM, PYCM, PZCM, PSIM
PRSTRS,MXC,MYC,MXYC, PSST, PSSB,SEGT, SEGM
NUSORT
NSORT,DISP, , , 10
PRDISP
```

```
AVPRIN
PRNSTR,ALL
NALL
EALL
/VIEW, 1,1,1
PLDISP
PLNSTR,SIGE,SZ
PLNSTR,SIGE,SX
PLNSTR,SX
PLNSTR,SY
SET,1,1
SAVE
FINISH
/PREP7 *** Forth Loading ***
/OUTPUT,40 *** For Circumferential Shear Force ***
RESUME
/TITLE CASE-4 IN 1/4 MODEL CIRCUMFEREN'IIAL DIRECTION
/COM SHEAR FORCE IN CIRCUMFERENTAL DIREC'IION
/COM BETA=0.3 GAMMA=75 T=0.2 T=0.2
/AUTO
NALL
EALL
DDELE,ALL,ALL
FDELE,ALL
F,22,FX,PLB2, ,32,1
F,85,FX,PLB2,,95,1
F,21,FX,PLB1,
F,81,FX,PLB1
F,17, FX, PLB2
ASYMBC,0,1,0,0.005
SYMBC,0,3,0,0.005
NSEL,Z,LORT
D,ALL,ALL
NALL
EALL
/VIEW, 1,1,1
/PBC,FORCE,1
/PBC,TDIS, 1
/PBC,RDIS,1
/PBC, PRES,1
NPLOT
ITER, 1,1,1
AFWRITE, ,1
FINISH
/EXEC
/INPUT, 27
FINISH
/POST1
/OUTPUT,38
/TITLE CASE-4 1/4 MODEL CIRCUMFERENTIAL DIRECTION
/COM SHEAR FORCE IN CIRCUMFERENTIAL DIRECIION
/COM BETA=0.3 GAMMA=75 'T=0.2 T=0.2 WL'IHOU'I INTERNAL
PRESSURE/AUTO
STORE,STRES,DISP
```

```
/NOPR
/NOLIST
STRESS,SXCT,63,9
STRESS,SYCT,63,10
STRESS,SXYT,63,11
STRESS,SXCM,63,13
STRESS,SYCM,63,14
STRRESS,SXYM,63,15
STRESS,MXC,63,6
STRESS,MYC,63,7
STRESS,MXYC,63,8
STRESS,NXIT, 63,21
STRESS,NYIT,63,22
STRESS,NXIM,63,37
STREESS,NYIM, 63,38
STRESS, PXCT, 63,129
STRESS, PYCT,63,130
STRESS, PZCT,63,131
STRESS, PSIT,63,132
STRESS,SGET,63,133
STRESS, PXCM,63,134
STRESS, PYCM,63,135
STRESS,PZCM,63,136
STRESS, PSIM,63,137
STRESS,SGEM,63,138
STRESS,PSST,63,151
STRESS, PSSB,63,152
SET
NSEL,NODE,81
ENODE
NASEL,NODE, 21
ENODE
PRELEM
PRSTRS,SXCT,SYCT,SXYT,SZCT,NXIT,NYIT, FXC', PYCT, PZCT, PSIT
PRSTRS,SXCM,SYCM,SXYM,SZCM,NXIM,NYIM, PXCM, PYCM, PZCM, PSIM
PRSTRS ,MXC,MYC, MXYC , PSST, PSSB, SEG'T, SEGM
NELEM
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
NASEL,NODE, 22, 32,1
NASEL,NODE,81,95,1
NASEL,NODE,17
ENODE
NSORT,SY, , ,6
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
NUSORT
ESORT,SYCT,, ,20
PRSTRS,SXCT,SYCT,SXYT,SZCT,NXIT,NYIT, FXCTP, EYCT, PZCT,PSIT
PRSTRS,SXCM,SYCM,SXYM,SZCM,NXIM,NYIM, PXCM, PYCM, PZCM, PSIM
```

```
PRSTRS,MXC,MYC,MXYC, PSST, PSSB,SEGT, SEGM
NUSORT
NSORT,DISP,,,10
PRDISP
AVPRIN
PRNSTR,ALL
NALL
EALL
/VIEW, 1,1,1
PLDISP
PLNSTR,SIGE,SZ
PLNSTR,SIGE,SX
PLNSTR,SX
PLNSTR,SY
SET,1,1
SAVE
FINISH
/PREP7 *** Fifth Loading ***
/OUTPUT,40 *** For Longitudinal Shear Force ***
RESUME
/TITLE CASE-5 1/4 MODEL LONGITUDINAL DIRECTION
/COM SHEAR FORCE LOADING IN LONGITUDINAL DIRECTION
/COM BETA=0.3 GAMMA=75, T=0.2 T=0.2
NALL
EALL
DDELE,ALL,ALL
FDELE,ALL
SYMBC,0,1,0,0.005
ASYMBC,0,3,0,0.005
NALL
EALL
F,22,FZ,PLB2, , 32,1
F,85,FZ,PLB2, ,95,1
F,21,FZ,PLB1,
F,81,FZ,PLB1
F,17,FZ,PLB2
NSEL,Z,LORT
D,ALL,ALL
NALL
EALL
/VIEW , 1,1,1
NPLOT
EPLOT
ITER,1,1,1
AFWRITE,,1
FINISH
/EXEC
/INPUT,27
FINISH
/ POST1
/OUTPUT,39
/TITLE CASE-5 1/4 MODEL LONGITUDINAL ILRECTION
/COM SHEAR FORCE IN LONGITUDINAL DIRECTION
/COM BETA=0.3 GAMMA=75 T=0.2 T=0.2
```

```
/AUTO
STORE,STRES,DISP
/NOPR
/NOLIST
STRESS,SXCT,63,9
STRESS,SYCT,63,10
STRESS,SXYT,63,11
STRESS,SXCM,63,13
STRESS,SYCM,63,14
STRESS,SXYM,63,15
STRESS,MXC,63,6
STRESS,MYC,63,7
STRESS,MXYC,63,8
STRESS,NXIT,63,21
STRESS,NYIT,63,22
STRESS,NXIM,63,37
STRESS,NYIM,63,38
STRESS,PXCT,63,129
STRESS,PYCT,63,130
STRESS,PZCT,63,131
STRESS,PSIT,63,132
STRESS,SGET,63,133
STRESS, PXCM, 63,134
STRESS, PYCM,63,135
STRESS,PZCM,63,136
STRESS, PSIM,63,137
STRESS,SGEM,63,138
STRESS,PSST,63,151
STRRESS, PSSB,63,152
SET
NSEL,NODE,81
ENODE
NASEL,NODE, 21
ENODE
PRELEM
PRSTRS,SXCT,SYCT,SXYT,SZCT,NXIT,NYIT, PXC'I, EYCI,FZCT, PSIT
PRSTRS,SXCM,SYCM,SXYM,SZCM,NXIM,NYIM, IXCM, FYCM, IZCM, PSIM
PRSTRS, MXC,MYC,MXYC, PSST,PSSB,SEGT,SEGM
NELEM
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
NASEL,NODE, 22,32,1
NASEL,NODE, 81,95,1
NASEL, NODE, 17
ENODE
NSORT,SY,,,6
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
NUSORT
ESORT,SYCT, ,,20
```

```
PRSTRS,SXCT,SYCT,SXYT,SZCT,NXIT,NYIT,IXC'I,FYC'I,PZCT,PSIT
PRSTRS,SXCM,SYCM, SXYM,SZCM,NXIM,'NYIM, PXCM, FYCM, PZCM, PSIM
PRSTRS,MXC,MYC,MXYC, PSST, PSSB,SEGT,SEGM
NUSORT
NSORT,DISP,,,10
PRDISP
AVPRIN
PRNSTR,ALL
NALL
EALL
/VIEW,,1,1,1
PLDISP
PLNSTR,SIGE,SZ
PLNSTR,SIGE,SX
PLNSTR,SX
PLNSTR,SY
SET,1,1
SAVE
FINISH
/PREP7 *** Sixth Loading ***
/OUTPUT,40 *** For Torsional Moment ***
RESUME
/NOPRINT
/NOPRINT
/TITLE CASE-6 TORSION MOMENT
/COM BETA=0.3 GAMMA=75
*SET,TY,PLBS/(48*RTRU)
*SET,TC1,TY*0.5
*SET,TC2,TY*COS(3.75/57.2958)
*SET,TC3,TY*COS(7.5/57.2958)
*SET,TC4,TY*COS(11.25/57.2958)
*SET,TC5,TY*COS(15/57.2958)
*SET,TC6,TY*COS(18.75/57.2958)
*SET,TC7,TY*COS(22.5/57.2958)
*SET,TC8,TY*COS(26.25/57.2958)
*SET,TC9,TY*COS(30/57.2958)
*SET,TC10,TY*COS (33.75/57.2958)
*SET,TC11,TY*COS(37.5/57.2958)
*SET,TC12,TY*COS(41.25/57.2958)
*SET,TC13,TY*COS (45/57.2958)
*SET,TC14,TY*COS(48.75/57.2958)
*SET,TC15,TY*COS(52.5/57.2958)
*SET,TC16,TY*COS(56.25/57.2958)
*SET,TC17,TY*COS(60/57.2958)
*SET,TC18,TY*COS(63.75/57.2958)
*SET,TC19,TY*COS(67.5/57.2958)
*SET,TC20,TY*COS(71.25/57.2958)
*SET,TC21,TY*COS(75/57.2958)
*SET,TC22,TY*COS(78.75/57.2958)
*SET,TC23,TY*COS(82.5/57.2958)
*SET,TC24,TY*COS(86.25/57.2958)
*SET,TSI,TY*SIN(3.75/57.2958)
*SET,TS2,TY*SIN(7.5/57.2958)
*SET,TS3,TY*SIN(11.25/57.2958)
```

```
*SET,TS4,TY*SIN(15/57.2958)
*SET,TS5,TY*SIN(18.75/57.2958)
*SET,TS6,TY*SIN(22.5/57.2958)
*SET,TS7,TY*SIN(26.25/57.2958)
*SET,TS8,TY*SIN(30/57.2958)
*SET,TS9,TY*SIN(33.75/57.2958)
*SET,TSIO,TY*SIN(37.5/57.2958)
*SET,TS11,TY*SIN(41.25/57.2958)
*SET,TS12,TY*SIN (45/57.2958)
*SET,TS13,TY*SIN(48.75/57.2958)
*SET,TS14,TY*SIN(52.5/57.2958)
*SET,TS15,TY*SIN(56.25/57.2958)
*SET,TS16,TY*SIN(60/57.2958)
*SET,TS17,TY*SIN(63.75/57.2958)
*SET,TS18,TY*SIN(67.5/57.2958)
*SET,TS19,TY*SIN(71.25/57.2958)
*SET,TS20,TY*SIN(75/57.2958)
*SET,TS21,TY*SIN(78.75/57.2958)
*SET,TS22,TY*SIN(82.5/57.2958)
*SET,TS23,TY*SIN(86.25/57.2958)
*SET,TS24,TY*0.5
NALL
EALL
DDELE,ALL,ALL
FDELE, ALL,ALL
ASYMBC,0,1,0,0.005
ASYMBC,0,3,0,0.005
NSEL,Z, LORT
D, ALL, ALL
NALL
EALL
F,21,FX,TS24
F,32,FX,TS23
F,31,FX,TS22
F,30,FX,TS21
F,29,FX,TS20
F,28,FX,TS19
F,27,FX,TS18
F,26,FX,TS17
F,25,FX,TS16
F,24,FX,TS15
F,23,FX,TS14
F,22,FX,TS13
F,17,FX,TS12
F,95,FX,TS11
F,94,FX,TS10
F,93,FX,TS9
F,92,FX,TS8
F,91,FX,TS7
F,90,FX,TS6
F,89,FX,TS5
F,88,FX,TS4
F,87,FX,TS3
F,86,FX,TS2
```

```
F,85,FX,TS1
F,32,FX,TC24
F,31,FX,TC23
F,30,FX,TC22
F,29,FX,TC21
F,28,FX,TC20
F,27,FX,TC19
F,26,FX,TC18
F,25,FX,TC17
F,24,FX,TC16
F,23,FX,TC15
F,22,FX,TC14
F,17,FX,TC13
F,95,FX,TC12
F,94,FX,TCI1
F,93,FX,TC10
F,92,FX,TC9
F,91,FX, TC8
F,90,FX,TC7
F,89,FX, TC6
F,88,FX,TC5
F,87,FX,TC4
F,86,FX,TC3
F,85,FX,TC2
F,81,FX,TC1
NALL
EALL
/VIEW, ,1,1,-1
/PBCS,ALL,I
NPLOT
EPLOT
SLOAD
AFWRIT
FINISH
/INPUT,27
FINISH
/POST1
/OUTPUT,40
/COM BETA=0.3 GAMMA=75 TORSION MOMENT
STORE,STRES, DISP
STRESS,SXCT,63,9
STRESS,SYCT,63,10
STRESS,SXYT,63,11
STRESS,SZCT,63,12
STRESS,SXCM,63,13
STRESS,SYCM,63,14
STRESS,SXYM,63,15
STRESS,SZCM, 63,16
STRRESS,SXCB,63,17
STRESS,SYCB,63,18
STRESS,SXYB,63,19
STRESS,SZCB,63,20
STRESS,MXC,63,6
STRESS,MYC, 63,7
```

```
STRESS,MXYC,63,8
STRESS,PXCT,63,129
STRESS, PYCT, 63,130
STRESS,PZCT,63,131
STRESS,PSIT,63,132
STREESS,SGET,63,133
STRESS, PXCM,63,134
STRESS, PYCM, 63,135
STRESS,PZCM,63,136
STRESS, PSIM,63,137
STRESS,SGEM,63,138
STRESS, PXCB,63,139
STRESS, PYCB,63,140
STRESS, PZCB,63,141
STRESS, PSIB,63,142
STRESS,SGEB,63,143
STRESS,PSST,63,151
STRESS , PSSB, 63,152
SET
NSEL,NODE,65
ENODE
NASEL,NODE, }8
ENODE
PRSTRS,SXCT,SYCT,SXYT,SZCT,PXCT, PYCT,PZCT,PSIT
PRSTRS,SXCM,SYCM,SXYM,SZCM, PXCM, PYCM, PZCM,FSIM
PRSTRR, SXCB,SYCB, SXYB,SZCB, PXCB, PYCB, PZCB, PSIB
PRSTRS,MXC,MYC ,MXYC, PSST, PSSB, SEGT, SEGM
NELEM
TOP
PRNSTR,ALL
MID
PRNSTR,AIL
NASEL,NODE , 21,81,60
ENODE
NELEM
ENODE
NSORT,SY, , ,6
TOP
PRNSTR,ALL
MID
PRNSTR,ALL
BOT
PRNSTR,ALL
NUSORT
ESORT,SXCT,, ,10
PRSTRS,SXCT,SYCT,SXYT,SZCT,PXCT, PYCT,PZCT,PSIT
PRSTRS,SXCM,SYCM,SXYM,SZCM, PXCM, PYCM, PZCM,PSIM
PRSTRS,SXCB,SYCB,SXYB,SZCB, PXCB, PYCB, PZCB,PSIB
PRSTRS, MXC , MYC , MXYC , PSST, PSSB, SEGT , SEGM
NUSORT
NALL
EALL
ESORT,SXCT,,,5
PRSTRS,SXCT,SYCT,SXYT,SZCT, PXCT, PYCT,PZCT,PSIT
```

```
PRSTRS,SXCM,SYCM,SXYM,SZCM, PXCM, PYCM, PZCM, PS IM
PRSTRS,SXCB,SYCB,SXYB,SZCB, PXCB, PYCB, PZCB,PSIB
PRSTRS, MXC , MYC , MXYC , PSST, PSSB, SEGT, SEGM
NALL
EALL
NSORT,DISP, , ,10
PRDISP
SET,1,1
SAVE
FINISH
```

B. Three-dimensional isoparametria solid element model:

```
/PREP7
*** Radial Force ***
*** PreProcessing Mode Begin ***
/TITLE THREE-DIMENSIONAL ISOPARAMETRIC SOLLD ELEMENT
/COM RADIAL FORCE IS APPLIED AT TOP. OF NOZZLE
/COM BETA=0.4 GAMMA=5 AND T=0.2 AND t=0.2
/SHOW
*SET,BETA,0.4
*SET,GAMA,5
*SET,THNP,0.2
*SET,THNT,0.2
*SET,PRSS,0
*SET,PLBS,-1000
*SET, RPIP,THNP*GAMA
*SET,RTRU, BETA*RPIP
*SET,THP,THNP*0.5
*SET,THT,THNT*0.5
*SET,PVRI,RPIP-THP
*SET,PVRO,RPIP+THP
*SET,NORI, RTRU-THT
*SET, NORO, RTRU+THT
*SET, LENT, PVRO*0.2
*SET,LORT,RPIP*4
*SET, ANG, ACOS (NORO/PVRO)
*SET,THED,(ANG-0.5236)*57.296
*SET,MIDD,(THED-90)/2
*SET,RPME,RPIP+LENT
*SET,MMEE, (2.1-ANG)
*SET,MECE,MMEE*PVRO
*SET, RY, PLBS*0.002778
*SET,RRY, PI_BS*0.0013889
/NOPRINT
ET,1,45,,,,,,1,1 *** Element Type Define ***
EX,1,30E6 *** Material Define ***
NUXY,1,0.3
N,1 $,2,1 $,3,,1
CS,11,1,1,3,2 *** Local Coordinate ***
/VIEW,,1,1,1
CSYS,11
K,1,NORI,90,RPME *** Key Point Define ***
K,2,NORO,90,RPME
K, 3,NORI,45,RPME
K,4,NORO,45,RPME
K,5,NORI, 0, RPME
K,6,NORO,0,RPME
K,7,NORI,90,PVRI
K,8,NORO,90, PVRO
K,9,NORI,45,PVRI
K,10,NORO,45,PVRO
K,11,NORI,0,PVRI
K,12,NORO,0, PVRO
KMOVE,7,11,NORI ,90,999,1,PVRI , 999,0
KMOVE,8,11,NORO , 90, 999,1, PVRO, 999,0
```

```
KMOVE, 9, 11,NORI, 45,999,1, PVRI, 999,999
KMOVE, 10, 11, NORO; 45, 999, 1, PVRO, 999, 999
KMOVE, 11, 11, NORI, ,999, 1, PVRI, 90 , NORI
KMOVE, 12, 11, NORO, ,999,1, PVRO, 90 , NORO
\(L, 1,3,9\)
L, 3, 4, 4
L, 4, 2, 9
L, 2, 1, 4
L, 3,5,9
L, 5, 6, 4
L, 6, 4, 9
L, 1, 7, 4, 0. 3
L, 7, 8, 4
\(L, 2,8,4,0.3\)
\(L, 3,9,4,0.3\)
L, 4, 10, 4, 0.3
L, 9, 10, 4
L, 7, 9, 9
L, 10, 8,9
L, 11, 9,9
L, 12, 10,9
\(L, 5,11,4,0.3\)
L, 6, 12, 4, 0.3
L, 11, 12,4
CSYS,1 \(1 * *\) Global Coordinate \(* * *\)
K, 13, PVRI,THED \(\quad * * *\) Key Point For Pipe Portion \(* * *\)
K, 14, PVRO, THED
K, 15, PVRI, -45,
K, 16, PVRO, -45,
K, 17, PVRI, -90,
K, 18, PVRO, -90,
K, 19, PVRI, 90 , MECE
K, 20, PVRO, 90 , MECE
K, 21, PVRI, THED, MECE
K, 22, PVRO, THED, MECE
K, 23, PVRI, -45, MECE
K, 24, PVRO, -45, MECE
K, 25, PVRI, -90, MECE
K, 26 , PVRO, -90, MECE
K, 27, PVRI, 90, LORT
K, 28, PVRO, 90, LORT
K, 29 , PVRI , THED, LORT
K, 30 , PVRO, THED, LORT
K, 31 , PVRI, 45 , LORT
K, 32, PVRO, -45 , LORT
K, 33, PVRI, -90, LORT
K, 34 , PVRO, -90, LORT
L, 8, 14, 7
L, 10, 22, 7
L, 12, 20, 7
L, 7, 13, 7
L, 9, 21,7
L, 11, 19, 7
L, 14, 22,9
```

```
L, 13,21,9
L,22,20,9
L, 21, 19,9
L, 14, 13,4
L,22,21,4
L,20,19,4
L,20,28,13
L,19,27,13
L,22,30,13
L,21,29,13
L,24,32,13
L,23,31, 13
L,26,34,13
L, 25,33,13
L, 28,27,4
L, 30,29,4
L, 32, 31,4
L,34,33,4
L, 24,23,4
L,26,25,4
L, 16,15,4
L, 18,17,4
L,14,16,7
L, 13,15,7
L, 22, 24,7
L,21,23,7
L,30,32,7
L, 29,31,7
L, 28,30,9
L,27,29,9
L, 16,24,9
L, 15,23,9
L, 16,18,7
L, 15,17,7
L,24,26,7
L,23,25,7
L, 32,34,7
L, 31,33,7
L, 18, 26,9
L,17,25,9
V,1,3,9,7,2,4,10,8 *** Volume Define For Meshing ***
V,3,5,11,9,4,6,12,10
V,7,9,21,13,8,10,22,14
V,9,11,19,21,10,12,20,22
V,21,19,27,29,22,20,28,30
V,15,13,21,23,16,14,22,24
V,23,21,29,31,24,22,30,32
V,17,15,23,25,18,16,24,26
V,25,23,31,33,26,24,32,34
TYPE,I
VMESH,ALL *** Meshing Begin ***
MERGE
CSYS,0
SYMBC,0,1,0,0.05 *** Symmetric Plane Define ***
```

```
SYMBC,0,3,0,0.05.
NALL
EALL
F,5,FY,RY *** Loading Apply ***
F,54,FY,RY, 57,1
F,6,FY,RY, ,13
F,70,FY,RY, ,93
F,59,FY,RY,,66
F,255,FY,RY,,262
F,303,FY,RY,,334
F,4,FY,RRY, 58,54
F,67,FY,RRY, ,69,1
F,254,FY,RRY, , 299,45
F, 300,FY,RRY, , 302,1
NSEL,Z,LORT *** Boundary Define ***
D,ALL,ALL
NALL
EALL
ARALL
APSF,ALL,PRSS
PTPLOT
WFRONT
WSTART,ALL
WAVES
/PBC,TDIS,1
/PBC,RDIS,1
/PBC, FORCE, 1
/PBC, PRES,1
NPLOT
EPLOT
ITER,1,,1
AFWRITE,,1
FINISH
/EXEC *** Execute Program ***
/INPUT,27
FINISH
/POST1 *** PostProcessing Mode Begin ***
/OUTPUT,35 *** Typical One ***
/COM THREE-DIMENSIONAL ISOPARAMETRIC SOLID ELEMENT
/COM RADIAL LOAD ON TOP OF NOZZLE NODAL FORCE
/COM WITH BETA=0.4 GAMMA=5
STORE,STRES,DISP
STRESS,SXC,45,1
STRESS,SYC,45,2
STRESS,SZC,45,3
STRESS,SXYC,45,4
STRESS,SYZC,45,5
STRESS,SXZC,45,6
STRESS,NSXI,45,13
STRESS,NSYI, 45,14
STRESS,NSZI,45,15
STRESS,NSXY,45,16
STRESS,NSYZ,45,17
STRESS ,NSXZ,45,18
```

```
STRESS,ESG1,45,101
STRESS,ESG2,45,102
STRESS,ESG3,45,103
STRESS,ESI,45,104
STRESS,ESIG,45,105
SET
NALI
EALL
NSORT,SX
PRNSTR,ALL
PRSTRS,SXC,SYC,SZC,SXYC,SYZC,SXZC,ESG1,ESG2,ESG3
PRSTRS,NSXI,NSYI,NSZI,NSXY,NSYZ,NSXZ,ESI,ESIG
NALL
EALL
ESORT,SXC, , ,20
PRSTRS,SXC,SYC,SZC,SXYC,SYZC,SXZC,ESG1,ESG2,ESG3
ESORT,SYC, , ,20
PRSTRS,SXC,SYC,SZC,SXYC,SYZC,SXZC,ESG1,ESG2,ESG3
ESORT,SZC,,,20
PRSTRS,SXC,SYC,SZC,SXYC,SYZC,SXZC,ESG1,ESG2,ESG3
NALL
EALL
NSORT,DISP,,,20
PRDISP
/OUTPUT,34
SET,1,1
SAVE
FINISH
/PREP7 *** Circumferential Moment Begin ***
RESUME
/TITLE THREE-DIMENSIONAL ISOPARAMETRIC SOLID ELEMEIJT
/COM CIRCUMFERENTIAL MOMENT IS APPLIED AI' TOP. OF NOZZLE
/COM BETA=0.4 GAMMA=5 AND T=0.2 AND t=0.2
/OUTPUT,34
/NOPRINT
CSYS,O
NALL
EALL
DDELE,ALL,ALL
FDELE,ALL,ALL
*SET,RY, PLBS/(RTRU*36)
*SET,DEMI,(RTRU**2)*40
*SET,DEM2, (THNT**2)*5
*SET, DEMO, DEM1+DEM2
*SET,NUN1, (8*RY) * (RTRU**2)
*SET,NUN2,(RY*RTRU) * (2*THNT)
*SET,NUN3,NUN2 *2
*SET,FF5,(NUN1-NUN3)/DEMO
*SET,FF4,(NUN1-NUN2)/DEMO
*SET, FF1,(NUN1/DEMO)
*SET,FF2,(NUN1 +NUN2)/DEMO
*SET,FF3,(NUN1+INUN3)/DEMO
*SET,FB1, (FF1/2)
*SET,F12,FF1*COS(5/57.2958)
```

```
*SET,F13,FF1*COS(10/57.2958)
*SET, F14, FF1*COS (15/57. 2958)
*SET,F15, FF1*COS (20/57. 2958)
*SET,F16,FFI*COS (25/57.2958)
*SET,F17,FF1*COS (30/57.2958)
*SET,F18,FF1*COS (35/57.2958)
#SET,F19,FF1*COS(40/57.2958)
*SET,F110,FF1*COS (45/57.2958)
*SET,F111,FF1*COS (50/57.2958)
*SET,F112,FFI*COS (55/57.2958)
*SET,F113,FF1*COS (60/57.2958)
*SET,F114,FF1*COS (65/57.2958)
*SET,F115,FF1*COS (70/57.2958)
*SET,F116,FF1*COS (75/57.2958)
*SET,F117,FF1*COS(80/57.2958)
*SET,F118,FF1*COS (85/57.2958)
*SET,FB2, (FF2/2)
*SET,F22,FF2*COS (5/57. 2958)
*SET , F23, FF2*COS (10/57.2958)
*SET,F24,FF2*COS (15/57.2958)
*SET,F25,FF2*COS (20/57.2958)
*SET,F26,FF2*COS (25/57. 2958)
*SET,F27,FF2*COS (30/57. 2958)
*SET, F28, FF2*COS (35/57.2958)
*SET,F29,FF2*COS(40/57.2958)
*SET,F210,FF2*COS (45/57.2958)
*SET,F211,FF2*COS (50/57.2958)
*SET,F212,FF2*COS (55/57.2958)
*SET,F213,FF2*COS (60/57.2958)
*SET,F214,FF2*COS (65/57.2958)
*SET, F2 15,FF2*COS (70/57.2958)
*SET,F216,FF2*COS (75/57.2958)
*SET,F217, FF2*COS (80/57. 2958)
*SET,F218,FF2*COS (85/57.2958)
*SET,FB3, (FF3/2)
*SET,F32,FF3*COS(5/57. 2958)
*SET,F33,FF3*COS(10/57.2958)
*SET,F34,FF3*COS (15/57.2958)
*SET,F35,FF3*COS (20/57. 2958)
*SET,F36,FF3*COS (25/57. 2958)
*SET,F37,FF3*COS (30/57. 2958)
*SET,F38,FF3*COS(35/57. 2958)
*SET, F39,FF3*COS (40/57. 2958)
*SET,F310,FF3*COS (45/57.2958)
*SET,F311,FF3*COS (50/57. 2958)
*SET,F312,FF3*COS (55/57. 2958)
*SET,F313,FF3*COS (60/57. 2958)
*SET,F314,FF3*COS (65/57.2958)
*SET, F315,FF3*COS (70/57.2958)
*SET,F316,FF3*COS (75/57.2958)
*SET,F317, FF3*COS (80/57.2958)
*SET,F318,FF3*COS (85/57.2958)
*SET,FB4, (FF4/2)
*SET,F42,FF4*COS (5/57.2958)
```

```
*SET,F43,FF4*COS(10/57.2958)
*SET,F44,FF4*COS(15/57. 2958)
*SET, F45,FF4*COS (20/57.2958)
*SET,F46,FF4*COS (25/57.2958)
*SET,F47,FF4*COS(30/57.2958)
*SET, F48,FF4*COS (35/57.2958)
*SET, F49,FF4*COS (40/57.2958)
*SET, F410,FF4*COS (45/57. 2958)
*SET, F411, FF4*COS (50/57. 2958)
*SET, F4 12,FF4*COS (55/57.2958)
*SET,F413,FF4*COS (60/57.2958)
*SET, F414,FF4*COS (65/57.2958)
*SET, F415,FF4*COS(70/57.2958)
*SET, F416,FF4*COS(75/57.2958)
*SET, F417,FF4*COS (80/57.2958)
*SET,F418,FF4*COS (85/57. 2958)
*SET,FB5,(FF5/2)
*SET, F52,FF5*COS (5/57.2958)
*SET, F53,FF5*COS (10/57.2958)
*SET, F54,FF5*COS (15/57. 2958)
*SET, F55,FF5*COS (20/57.2958)
*SET,F56,FF5*COS (25/57. 2958)
*SET, F57,FF5*COS (30/57.2958)
*SET, F58,FF5*COS (35/57.2958)
*SET,F59,FF5*COS (40/57.2958)
*SET,F510,FF5*COS (45/57.2958)
*SET,F511,FF5*COS (50/57.2958)
*SET,F512,FF5*COS (55/57.2958)
*SET,F513,FF5*COS (60/57.2958)
*SET, F514,FF5*COS (65/57.2958)
*SET, F515,FF5*COS(70/57.2958)
*SET, F516, FF5*COS (75/57.2958)
*SET,F517,FF5*COS (80/57.2958)
*SET,F518,FF5*COS (85/57.2958)
ASYMBC,0,1,0,0.05
SYMBC,0,3,0,0.05
NALL
EALL
F,4,FY,FB5
F,6,FY,F52
F,7,FY,F53
F,8,FY,F54
F,9,FY,F55
F,10,FY,F56
F,11,FY,F57
F,12,FY,F58
F,13,FY,F59
F,5,FY,F510
F,255,FY,F511
F,256,FY,F512
F,257,FY,F513
F,258,FY,F514
F,259,FY,F515
F,260,FY,F516
```

$$
\begin{aligned}
& \text { F, } 261, F Y, F 517 \\
& F, 262, F Y, F 518 \\
& F, 69, F Y, F B 4 \\
& F, 70, F Y, F 42 \\
& F, 71, F Y, F 43 \\
& F, 72, F Y, F 44 \\
& F, 73, F Y, F 45 \\
& F, 74, F Y, F 46 \\
& F, 75, F Y, F 47 \\
& F, 76, F Y, F 48 \\
& F, 77, F Y, F 49 \\
& F, 55, F Y, F 410 \\
& F, 311, F Y, F 411 \\
& F, 312, F Y, F 412 \\
& F, 313, F Y, F 413 \\
& F, 314, F Y, F 414 \\
& F, 315, F Y, F 415 \\
& F, 316, F Y, F 416 \\
& F, 317, F Y, F 417 \\
& F, 318, F Y, F 418 \\
& F, 68, F Y, F B 1 \\
& F, 78, F Y, F 12 \\
& F, 79, F Y, F 13 \\
& F, 80, F Y, F 14 \\
& F, 81, F Y, F 15 \\
& F, 82, F Y, F 16 \\
& F, 83, F Y, F 17 \\
& F, 84, F Y, F 18 \\
& F, 85, F Y, F 19 \\
& F, 56, F Y, F 110 \\
& F, 319, F Y, F 111 \\
& F, 320, F Y, F 112 \\
& F, 321, F Y, F 113 \\
& F, 322, F Y, F 114 \\
& F, 323, F Y, F 115 \\
& F, 324, F Y, F 116 \\
& F, 325, F Y, F 117 \\
& F, 326, F Y, F 118 \\
& F, 67, F Y, F B 2 \\
& F, 86, F Y, F 22 \\
& F
\end{aligned}
$$

```
F,333,FY,F217
F,334,FY,F218
F,58,FY,FB3
F,66,FY,F32
F,65,FY,F33
F,64,FY,F34
F,63,FY,F35
F,62,FY,F36
F,61,FY,F37
F,60,FY,F38
F,59,FY,F39
F,54,FY,F310
F,310,FY,F311
F,309,FY, F312
F,308,FY,F313
F,307,FY,F314
F,306,FY,F315
F,305,FY,F316
F,304,FX,F317
F,303,FY,F318
NSEL,Z,LORT
D, ALL, ALL
NALL
EALL
ARALL
APSF,ALL, PRSS
NPLOT
EPLOT
ITER,1,1,1
AFWRITE, ,1
FINISH
/INPUT,27
FINISH
/POST1
FINISH
/PTEP7 *** Longitudinal Moment Begin ***
RESUME
/TITLE THREE-DIMENSIONAL ISOPARAMETRIC SOLID ELEMENT
/CON LONGITUDINAL MOMENT LOADING
/CON WITH BETA=0.4 GAMMA=5
/NOPRINT
NALL
EALL
DDELE,ALL,ALL
FDELE,ALL,ALL
SYMBC,0,1,0,0.05
ASYMBC,0,3,0,0.05
NALL
EALL
F,6,FY,F518
F,7,FY,F517
F,8,FY,F516
F,9,FY,F515
```

F, 10, FY, F514
F, 11,FY, F513
F, 12, FY, F512
F, 1.3, FY, F511
F, 5, FY, F5110
F, 255, FY, F59
F, 256, FY, F58
F, 257, FY, F57
F, 258, FY, F5 6
F, 259, FY,F55
F, 260, FY, F54
F, $261, F Y, F 53$
F, 262, FY, F52
F, 254, FY, FB5
F, 300, FY, FB4
F, 70,FY, F4 18
F, 71, FY, F4 17
F, 72,FY, F4 16
F, 73,FY,F415
F, 74, FY, F414
F, 75,FY,F413
F, 76, FY, F412
F, 77,FY,F411
F, 55,FY,F410
F, 311, FY, F49
F, 312, FY, F4 8
F, 313,FY,F47
F, 314, FY, F46
F, 315,FY,F45
F, 316, FY, F44
F, 317, FY, F43
F, 318, FY,F42
F, 301, FY, FB1
F, 78, FY, F118
F, 79, FY, F117
F, 80,FY,F116
F, 81,FY,F115
F, 82, FY, F114
F, 83, FY, F113
F, 84, FY, F112
F, 85, FY, F111
F, 56,FY, F110
F, 319, FY, F19
F, 320, FY, F18
F, 321,FY,F17
F, 322,FY, F16
F, 323, FY, F15
F, 324, FY, FI4
F, 325,FY,F13
F, 326,FY, F12
F, 302, FY, FB2
F, 86, FY, F218
F, 87, FY, F217
F, 88,FY,F216

```
F,89,FY,F215
F,90,FY,F214
F,91,FY,F213
F,92,FY,F212
F,93,FY,F211
F,57,FY,F210
F,327,FY,F29
F,328,FY,F28
F,329,FY,F27
F,330,FY,F26
F,331,FY,F25
F,332,FY,F24
F,333,FY,F23
F,334,FY,F22
F,299,FY,FB3
F,66,FY,F318
F,65,FY,F317
F,64,FY,F316
F,63,FY,F315
F,62,FY,F314
F,61,FY,F313
F,60, FY,F312
F,59,FY,F311
F,54,FY,F310
F,310,FY,F39
F,309,FY,F38
F,308,FY,F37
F,307,FY,F36
F,306,FY,F35
F,305,FY,F34
F,304,FY,F33
F,303,FY,F32
NSEL,Z,LORT
D,ALL,ALL
NALL
EALL
ARALL
APSF,ALI, PRSS
NPLOT
EPLOT
ITER,1,,1
AFWRITE,,1
FINISH
/INPUT,27
FINISH
/POST1
FINISH
/PREP7 *** Circumferential Shear Force Begin ***
RESUME
/OUTPUT,34
/TITLE CIRCUMFERENTIAL SHEAR LOADING AT IH'IERSECPION
/COM SHEAR LOADING AT CIRCUMFERENTIAL DIRECTION
/CON WITH BETA=0.4 GAMMA=5
```

```
/COM CSYS,O
NALL
EALL
DDELE,ALL,ALL
FDELE,ALL,ALL
ASYMBC,0,1,0,0.05
SYMBC,0,3,0,0.05
NALL
EALL
F,5,FX,RY
F,54,FX,RY, ,57,1
F,6,FX,RY, ,13
F,70,FX,RY,,93
F,59,FX,RY,,66
F,255,FX,RY,,262
F,303,FX,RY,,334
F,4,FX,RRY,,58,54
F,67,FX,RRY, , 69,1
F, 254,FX,RRY, , 299,45
F,300,FX,RRY, 302,1
NSEL,Z,IORT
D,ALL,ALI
NALL
EALL
ARALL
APSF,ALL,PRSS
NPLOT
EPLOT
ITER,1,,1
AFWRITE, ,1
FINISH
/INPUT,27
FINISH
/POSTI
FIIJISH
/FREP7 *** Longitudinal Shear force Begin ***
RESUME
/OUTPUT,34
/TITLE LONGITUDINAL SHEAR LOADING AT INTERSECTION
/NOPRINT
CSYS,0
NALL
EALL
DDELE, ALL, ALL
FDELE,ALL,ALL
SYMBC,0,1,0,0.05
ASYMBC,0,3,0,0.05
F,5,FZ,RY
F,54,FZ,RY, ,57,1
F,6,FZ,RY, ,13
F,70,FZ,RY,,93
F,59,FZ,RY,,66
F,255,FZ,RY,,262
```

F, 303, FZ, RY, , 334
F, 4, FZ, RRY, ,58,54
F, 67, FZ, RRY, ,69,1
F, 254, FZ, RRY, , 299, 45
F, 300,FZ, RRY, , 302, 1
NSEL, Z, LORT
D, ALL, ALL
NALL
EALL
ARALL
APSF, ALL, PRSS
NPLOT
EPLOT
ITER, 1, ,1,
AFWRITE, , 1
FINISH
/INPUT, 27
FINISH
/POST1
*** PostProcessing Program is omitted ***
FINISH






## Appendix D

IMPLICIT REAL* 8 ( $\mathrm{A}-\mathrm{H}, \mathrm{O}-\mathrm{Z}$ )
DOUBLE PRECCISION BBETA(10), GGAMMA(12), PPRESS(3)
DOUBLE PRECISION BI,BI2,BI3,BI4, $\mathrm{CO}, \mathrm{CO} 2, \mathrm{CO} 3, \mathrm{CO} 4, \mathrm{TBIPI}$
\%CMXB, CMPB, CNXM, CNPM, CCMX, CCMP, CI, CI2, CI4, P, DDT
DOUBLE PRECISION X,XMU,PI,PI2,PI3,PI4,PIX,TPIX,AL-
PHA, TH,
\%A2, A3, A4, A5, A6, GAMMA, BETA, B2, B3, B4, B5, G3, C1, C2, C3, EE \%, C4, CG, CC, CL, BOA, PIBOA, TPIBOA, PHIZMN, R, RI, XN
DOUBLE PRECISION AMNP,AMNP2,DOWN, PIIIMN, XMB, PIBOAN, ZMN
\%, PHIZ, SPRC, VA, C11, C22, C33, C44,VB, VC, SPRR, V, XM2,TPIBAN
\%, CMX, CMP, CNX, CNP, CMXC, CMXX, CMPH, CMPP, CNXC, CNXX, CNPH, CNPP DOUBLE

PRECISION
AMX , AMP, ANX, ANP , AMXC , AMXX, AMPH, AMPP, ANXC,
\%ANXX, ANPH, ANPP, AMXB, AMPB, ANXM, ANPM, PHIZA, SPRRA DOUBLE

PRECISION
BMX, BMP, BNX, BNP, BMXC, BMXX, BMPH, BMPP, BNXC,
\%BNXX, BNPH, BNPP, BMXB, BMPB, BNXM, BNFM, FHIZB, SPRRB
OPEN (6,FILE='CIRCUM.OUT', STATUS='NEW')
OPEN (7,FILE='LONGIT.OUT',STATUS='NEW')
OPEN (8,FILE='RAD.OUT' ,STATUS='NEW')
OPEN (9,FILE='RADB.OUT',STATUS='NEW')
OPEN (10,FILE='RADA.OUT' ,STATUS='NEW')
C TO COMPUTE PIPE-TRUNNION FLEXIBILITY IN BOTH
IONGITUDINAL AND
C CIRCUMFERENTIAL DIRECTIONS AND RADIAL DIPECIION.
BBETA (1) $=0.05$
$\operatorname{BBETA}(2)=0.1$
$\operatorname{BBETA}(3)=0.2$
BBETA (4) $=0.3$
BBETA (5) $=0.4$
BBETA ( 6 ) $=0.5$
$\operatorname{BBETA}(7)=0.0$
BBETA (8) $=0.7$
BBETA (9) $=0.8$
BBETA (10) $=0.9$
GGAMMA (1) $=5$
GGAMMA (2) $=10$
GGAMMA (3) $=15$
GGAMMA (4) $=25$
GGAMMA (5) $=35$
GGAMMA (6) $=50$
GGAMMA (7) $=75$
GGAMMA (8) $=100$
GGAMMA (9) $=150$
GGAMMA (10) $=200$
GGAMMA (11) $=300$
PPRESS (1) $=0$.
PPRESS (2) $=500$.
PRRESS (3)=1000.
$\mathrm{X}=0.5$
$\mathrm{XMU}=0.3$
EE $\quad=30000000$.
PI $=3.1415926535$

```
    PI2 = PI * PI
    PI3 = PI * PI2
    PI4 = PI2 * PI2
    PIX = PI * X
    TPIX = 2. * FIX
1200 FORMAT ('1',//////////3X,'ALGHA(L/A) =',F4.1)
    1600 FORMAT (/3X,'A = OUTSIDE RADIUS OF CYLINDRICAL
SHELL, (IN.).',/3X,
    2 'C= HALF LENGTH OF LOADING SURFACE, (IN.).',/3X,
    3'L = LENG'TH OF SHELL,(IN.).',/3X,
    4'T = THICKNESS OF SHELL,(IN.).',/3X,
    5 'KC = CIRCUMFERENTIAL SERING COEFFI-
CIENT, (IN-LBS/RAD.).',/3X,
    6 }\mp@subsup{}{}{\circ}\textrm{KL}= LONGITUDINAL SPRING COEFFI-
CIENT, (IN-LBS/RAD.).',/3X,
    7'KR = RADIAL SPRING COEFFICIENT,(LBS/IN.).')
1601 FORMAT(/1X,'SQ.TUBE', 2X,' THICK', 2X,'PIPE SIZE',4X,
    %'GAMMA', 8X,'SPRC',7X,'NO.LOOP',5X,'SPRL',7X,'NO.LOOP'
    %,5X,'SPRL', 6X,'NO.LOOP',5X,'SPRR', 6X,'NO.LOOP')
        DO 300 IP=1,3
        P=PPRESS(IP)
        WRITE(6,1031) P
        WRITE(7,1031) P
        WRITE (8,1031) P
1031 FORMAT(/3X,'P =',F10.0,' PSI.')
    WRITE (*,911)
    911 FORMAT(2X,' Please specify gamma value: from: to:')
    READ (*,*) IA1, IA2
    WRITE(*,912)
    912 FORMAT(2X,' Please specify beta value from: to:')
    READ (* * *) IB1, IB2
    WRITE(*,913)
913 FORMAT(2X,'If the nozzle is solid, please enter: 1',
    %/,' if the nozzle is hollow, please enter: 2')
    READ (*,*)NOZ
    IF(NOZ.EQ.1) GO TO 928
    WRITE (6,929)
    WRITE (7,929)
    WRITE (8,929)
929 FORMAT(2X,' NOZZLE IS HPLLOW TYPE')
    GO TO 930
928 WRITE (6,931)
    WRITE (7,931)
    WRITE (8,931)
931 FORMAT(2X,' NOZZLE IS SOLID TYPE')
930 WRITE(*,926)
926 FORMAT(2X,' Please input pipe thicfness: ')
    READ(*,*)TH
    WRITE(*,925)
925 FORMAT(2X, 'Please enter the alpha value:')
    READ (*,*)ALPHA
    WRITE (6,927) ALPHA
    WRITE(7,927)ALPHA
    WRITE (8,927)ALPHA
```

```
    WRITE (9,941)
    941 FORMAT(2X,', This is B point (circumf. plane) data:')
    WRITE(10,942)
    942 FORMAT(2X, ' This is A point (longit. plane) data:')
    927 FORMAT(2X,' ALPHA VALUE IS : ',F4.1)
    DO 300 J=IA1,IA2
    GAMMA=GGAMMA (J)
    WRITE (6,811)
    WRITE (6,812)
    WRITE(7,821)
    WRITE(7,812)
    WRITE (8,813)
    WRITE (8,812)
    WRITE (9,813)
    WRITE (9,812)
    WRITE(10,813)
    WRITE(10,812)
    DO 300 K=IB1,IB2
    BETA=BBETA (K)
822 FORMAT(2X,' Please specify nozzle diameter:')
    AA=TH*GAMMA
    CO=BETA*AA
    IF(NOZ.EQ.1) GO TO 921
    IF(NOZ.EQ.2) GO TO 922
921 CI=0.0000001
    GO TO }92
922 DDT=TH
    CI=CO-DDT
924 CO2=CO*CO
    CO3=CO2*CO
    CO4=CO2*CO2
    A2 = ALPHA * ALPHA
    A3 = ALPHA * ALPHA * ALPHA
    A4 = A2 * A2
    A5 = A4 * ALPHA
    A6 = A5 * ALPHA
    B2 = BETA * BETA
    B3 = BETA * BETA * BETA
    B4 = B2 * B2
    B5 = B2 * B3
    BI = CI / AA
    BI2 = BI * BI
    BI3 = BI2 * BI
    BI4 = BI2 * BI2
    BI5 = BI3 * BI2
    CI2 = CI * CI
    CI3 = CI2 * CI
    CI4 = CI2 * CI2
    G3 = GAMMA * GAMMA * GAMMA
    C1 = (12. * ( 1. - XMU * XMU ) * PI4 * GAMMA * GAMMA
    /A4
        C2 = (6. + XMU -(XMU * XMU ))* FI4 /A4
    C3 = ((7. + XMU ) * PI2 ) / A2
    C4 =30000000./(12.*(1.-XMU*XMU)*G3)
```

```
        CG = 6./(PI2*B5)
        CL = (3.*ALPHA) / (PI3*4.*B5)
        BOA = BETA / ALPHA
        PIBOA = PI * BOA
        TPIBOA = 2. * PIBOA
C CIRCUMFERENTIAL SPRING COEFFICIENT
    PHIZMN = 0.
    CMXX = 0.
    CMPP = 0.
    CNXX = 0.
    CNPP = 0.
    II1 = 1
    DO 90 NPM =2,100
    NPMM1 = NPM - 1
    DO 80 M = 1, NPMMI
    N = NPM - M
    R = N/2.
    II = R
    R1 = (R-II)
    IF (R1) 10,80,10
    10 XN = N
    XM = M
    XN2 = XN * XN
    XM2 = XM * XM
    XN4 = XN2 * XN2
    XM4 = XM2 * XM2
    AMNP }=(XM2 + (XN2 * PI2 / A2 ))
    AMNP2 = AMNP * AMNP
    CMX = (XN2*PI2/A2) +(XMU* (XM2-1))
    CMP = (XM2 - 1) + ( XMU * XN2 * PI2/A2-)
    CNX = (XM2*XN2)/((XM2*A2+XN2*PI2)**2)
    CNP = XN4/((XM2*A2+XN2*PI2)**2)
    DOWN =C4*(AMNP2*AMNP2+C1*XN4-XM2*(2.*XM4
    1 +C2*XN4+C3*XN2*XM2))+(XM2-1.+(XN2*PI2)/(2.*A2))
    2 * (XM2 + (XN2*PI2/A2))* (XM2 + (XN2*PI2/A2 ))* P
    PHIMN = AMNP2 / DOWN
    XMB = XM * BETA
    XMBI = XM*BI
    PIBOAN = PIBOA * XN
    ZMN = (((DSIN (XMB)-XMB*DCOS (XMB))*DSIN(PI-
BOAN) * ((-1.)
    %**((N-1)/2))/BETA+(DSIN(XMBI)-XMBI*DCOS (XMBI))
    %*DSIN (XN*PI*BI/ALPHA)* (-1.)**((N+1)/2)/BI))/(XM2*XN)
    PHIZ = PHIMN* ZMN * DSIN(XMB) * DSIN(XN*PI/2.)
    CMXC = CMX * PHIZ
    CMXX = CMXX + CMXC
    CMPH = CMP * PHIZ
    CMPP = CMPP + CMPH
    CNXC = CNX * PHIZ
    CNXX = CNXX + CNXC
    CNPH = CNP * PHIZ
    CNPP = CNPP + CNPH
    PHIZMN = PHIZ + PHIZMN
    80 CONTINUE
```

```
            CALI_ DEFINE (PHIZMN,II2)
            IF(III .EQ. II2) GO TO 113
            II1 = II2
            90 CONTINUE
    113 SPRC= PI2*(CO3-(CI4/CO))*BETA/(6.*PHIZMN*(AA**3))
    CMXB = (6./PI2)*BE-
TA*CMXX*C4*(AA**3)/(CO3-(CI4/CO))
                            CMPB = (6./PI2)*BE-
TA*CMPP*C4* (AA**3)/(CO3-(CI4/CO))
                                    CNXM = 6.*TH*A2*AA*AA*BETA*CNXX*EE/(CO3-(CI4/CO))
                            CNPM = 6.*PI2*EE*TH*AA*AA*BETA*CNPP/(CO3-(CI4/CO))
                    812 FORMAT (2X,', BETA',4X,'GAM-
MA', 6X,'MX', 5X,'MPH', 6X,'NX', 5X,
            %'NPH', 6X,'SPRING', 5X,'LOOP')
    831 FORMAT(2X,F4.2,2X,F5.1,2X,4(F8.4,1X),E14.8,2X,I4)
            WRITE (6,831) BETA , GAMMA, CMXB, CMPB, CNXM, CNPM, SPRC,III
    811 FORMAT(2X,' CIRCUMFERENTIAL MOMENT LOADING: ')
C LONGITUDINAL SPRING COEFFICIENT
        C11 = CI * 16.
        C22 = C2 * 16.
        C33 = C3 * 4.
        C44 = 30000000./(12.*(1.-XMU*XMU)*G3)
        PHIZMN = 0.
        CMXX =0.
        CMPP =0.
        CNXX = 0.
        CNPP = 0.
        II1 = 1
        DO 200 MPN =1,100
        DO 180 N = 1, MPN
        M = MPN - N
        XN = N
        XM =
        XN2 = XN * XN
        XM2 = XM * XM
        XN4 = XN2 * XN2
        XM4 = XM2 * XM2
        AMNP = (XM2 + (4.* XN2*PI2/A2 ))
        AMNP2 = AMNP * AMNP
        CMX = (4.*XN2*PI2/A2)+(XMU* (XM2-1))
        CMP =(XM2 - 1) + (4. * XMU * XN2 * PI2/A2 )
        CNX = (XM2*XN2)/((XM2*A2/4.+XN2*PI2)**2)
        CNP = XN4/((XM2*A2/4.+XN2*PI2)**2)
        DOWN = C44*(AMNP2*AMNP2+C11*XN4-XM2*(2. *XM4
        1 +C22*XN4+C33*XN2*XM2))+(XM2-1.+(2.*XN2*PI2/A2))
        2 *(XM2+(4.*XN2*PI2/A2))*(XM2+(4.*XN2*PI2/A2))*P
        PHIMN = AMNP2 / DOWN
        XMB = (XM*BETA)
        XMBI = XM*BI
        TPIBAN = TPIBOA * XN
        TBIPI = 2.*XN*PI*BI/ALPHA
        IF (M) 128,120, 128
120 ZMN1 = (1./XN2)*(DSIN(TPIBAN)-TPIBAN*DCOS(
    % TPIBAN)) * (-1.)**N
```

```
            ZMN2 = (1./XN2)*(DSIN(TBIPI) -TBIPI*DCOS(TBIPI))
    %*(-1.)**(N+1)
        GO TO 130
128 ZMN1=(2./(XM*XN2))*(DSIN(TPIBAN) -TPIBAN*DCOS (TPIBAN))
    1 *DSIN (XMB)*(-1.)**N/BETA
    ZMN2 = (2./(XN2*XM))*(DSIN(TBIPI) -TBIPI*DCOS (
    %TBIPI)) *DSIN(XM*BI)* (-1.)** (N+I)/BI
    130 CONTINUE
    ZMN=ZMNI+ZMN2
    PHIZ = PHIMN * ZMN * DSIN(PI*XN+TPIBAN)
    CMXC = CMX * PHIZ
    CMXX = CMXX + CMXC
    CMPH = CMP * PHIZ
    CMPP = CMPP + CMPH
    CNXC = CNX * PHIZ
    CNXX = CNXX + CNXC
    CNPH = CNP * PHIZ
    CNPP = CNPP + CNPH
    PHIZMN = PHIZ + PHIZMN
    180 CONTINUE
    CALL DEFINE (PHIZMN,II2)
    IF (III .EQ. II2) GO TO 213
    III = II2
    200 CONTINUE
        213 SPRI = 4./3.*PI3*(CO3-(CI4/CO))/(AL-
PHA*PHIZMN*(AA**3))*BETA
    CMXB = 0.75 * C4*(AA**3)*ALPHA*BE-
TA/(PI3*(CO3-(CI4/CO)))
    %*CMXX
        CMPB = 0.75 * C4*(AA**3)*ALPHA*BE-
TA/(PI3*(CO3-(CI4/CO)))
    % * CMPP
        CNXM = 0.75/4.*TH*A2*AA*AA*ALPHA*BETA*EE*CNXX/
    %(PI*(CO3-(CI4/CO)))
        CNPM = 0.75*EE*TH*PI*ALPHA*AA*AA*BETA*CNPP/
    %(CO3-(CI4/CO))
        WRITE (7, 831) BETA, GAMMA, CMXB, CMPB, CNXM, CNPM, SPRL, III
    821 FORMAT(2X,' LONGITUDINAL MOMENT LOADING: ')
C RADIAL SPRING COEFFICLENT
    Cl11 =(12.*(1-XMU*XMU) *PI4*GAMMA*GAMMA)/A4
    C222 = (6.+XMU-(XMU*XMU))*PI4/A4
    C333 = ((7.+XMU)*PI2)/A2
    C444 = 30000000./(12.*(1.-XMU*XMU)*G3)
    PHIZMN= 0.
    CMXX = 0.
    CMPP = 0.
    CNXX = 0.
    CNPP = 0.
    II1 = 1
    DO 201 MPN = 1,100
    DO 181 N= 1,MPN
    M = MPN-N
    R3 =N/2.
    I3 = R3
```

```
        R2 =(I3 - R3 )
        IF (R2) 122,181,122
    122 XN = N
        XM = M
        XN2 = XN * XN
        XN4 = XN2 * XN2
        XM2 = XM * XM
        XM4 = XM2 * XM2
        AMNP = (XM2 +(XN2*PI2/A2))
        AMNP2 = AMNP * AMNP
        CMX = (XN2*PI2/A2)+(XMU*(XM2-1))
        CMP = (XM2 - 1) + ( XMU * XN2 * PI2/A2 )
        CNX = (XM2*XN2)/((XM2*A2+XN2*PI2)**2)
        CNP = XN4/((XM2*A2+XN2*PI2)**2)
        DOWN = C444*(AMNP2*AMNP2 +C111*XN4-XM2*(2.*XM4
    1
+C222*XN4+C333*XM2*XN2)) +(XM2-1. +(XN2*PI2)/(2.*A2))
    2 *(XM2+(XN2*PI2/A2))*(XM2+(XN2*PI2/A2))*P
        PHIMN = AMNP2/DOWN
        PIBOAN = PIBOA * XN
        XMBI = XM*BI
        IF (M) 129,121,129
    121 ZMN1 =(BETA/XN)*(DSIN(PIBOAN))
    1 * (-1.)**((N-1)/2)
        GO TO 131
    129 ZMN1 = (2./(XM*XN))*(DSIN(XM*BETA))*DSIN(PIBOAN)
    1 * (-1.)**((N-1)/2)
        GO TO 141
131 ZMN2 =(BI/XN)*DSIN (XN*PI*BI/ALPHA)* (-1.)**((
    %N+1)/2)
        GO TO 151
141 ZMN2 = (2./(XM*XN))*(DSIN (XM*BI)*DSIN(XN*PI
    %*BI/ALPHA))*(-1.)**((N+1)/2)
151 CONTINUE
        ZMN=ZMN1+ZMN2
        PHIZ = PHIMN * ZMN * DSIN(XN*PI/2.)
        PHIZA = PHIMN * ZMN * DSIN(XN*PI/2 - XN*PI*BETA/AL-
PHA)
        PHIZB = PHIMN * ZMN * DCOS(XM*BETA)*DSIN(XN*PI/2)
        CMXC = CMX * PHIZ
        CMXX = CMXX + CMXC
        CMPH = CMP * PHIZ
        CMPP = CMPP + CMPH
        CNXC = CNX * PHIZ
        CNXX = CNXX + CNXC
        CNPH = CNP * PHIZ
        CNPP = CNPP + CNPH
        AMXC = CMX * PHIZA
        AMXX = AMXX + AMXC
        AMPH = CMP * PHIZA
        AMPP = AMPP + AMPH
        ANXC = CNX * PHIZA
        ANXX = ANXX + ANXC
        ANPH = CNP * PHIZA
```

```
    ANPP = ANPP + ANPH
    BMXC = CMX * PHIZB
    BMXX = BMXX + BMXC
    BMPH = CMP * PHIZB
    BMPP = BMPP + BMPH
    BNXC = CNX * PHIZB
    BNXX = BNXX + BNXC
    BNPH = CNP * PHIZB
    BNPP = BNPP + BNPH
    PHIZMN = PHIZ + PHIZMN
    PHAMN = PHIZA + PHAMN
    PHBMN = PHIZB + PHBMN
    181 CONTINUE
    CALL DEFINE(PHIZMN, II2)
    IF(III .EQ. II2) GO TO 203
    III=II2
    201 CONTINUE
203 SPRR = PI2* (CO2-CI2)/(AA*CO*PHIZMN)
    CMXB = (AA**3)*C4*CMXX /(AA*PI2*(CO2-CI2))
    CMPB = (AA**3)*C4*CMPP /(AA*PI2*(CO2-CI2))
    CNXM = EE*TH*A2*AA*CNXX /(CO2-CI2)
    CNPM = EE*TH*PI2*AA*CNPP /(CO2-CI2)
    SPRRA = PI2* (CO2-CI2)/(AA*CO*PHAMN)
    AMXB = (AA**3)*C4*AMXX /(AA*PI2* (CO2-CI2))
    AMPB = (AA**3)*C4*AMPP /(AA*FI2*(CO2-CI2))
    ANXM = 1.*EE*TH*A2*AA*ANXX / (CO2-Cl2)
    ANPM = 1.*EE*TH*PI2*AA*ANPP /(CO2-CI2)
    SPRRB = PI2*(CO2-CI2)/(AA*CO*PHBMN)
    BMXB = (AA**3)*C4*BMXX /(AA*PI2*(CO2-CI2))
    BMPB = (AA**3)*C4*BMPP /(AA*PI2*(CO2-CI2))
    BNXM = 1.*EE*TH*A2*AA*BNXX / (CO2-CI2)
    BNPM = 1.*EE*TH*PI2*AA*BNPP /(CO2-CI2)
    WRITE (8,831) BETA, GAMMA, CMXB, CMPB, CNXM, CNPM,SPRR,III
    WRITE (9,831) BETA,GAMMA, BMXB, BMPB,BNXM, BNPM,SPRRB,III
    WRITE (10, 831) BETA, GAMMA, AMXB, AMPB, ANXM, ANPM, SPRRA, III
813 FORMAT(2X,' RADIAL FORCE LOADING: ')
    450 CONTINUE
    7 0 7 \text { CONTINUE}
    505 CONTINUE
    4 0 0 ~ C O N T I N U E ~
    300 CONTINUE
    CLOSE (10)
    CLOSE (9)
    CLOSE (8)
    CLOSE (7)
    CLOSE (6)
    STOP
    END
    SUBROUTINE DEFINE(A,II2)
    IMPLICIT REAL*8(A-H,O-Z)
    V=A*1.OE+4
    B=1.0
    DO 100 ID=1,20
    B=B/10.
```

IF (A.GT. B) GO TO 311
$V=V * 10$.
100 CONTINUE
311 II2=V
RETURN
END

## REFERENCES

1. Wichman, K. R., A. G. Hopper and J. L. Mershon, 1965, "Local Stresses in Spherical and Cylindrical Shells Due to External Loading," Welding Research Council Bulletin No. 107.
2. Mershon, J.L., Mokhtarian, K., Ranjan, G.V.,Rodabaựh, E.C., 1984, "Local Stresses in Cylinderical shells due to external loadings on nozzles-supplement to WRC No.107" Welding Research Council Bulletin No. 297.
3. Bijlaard, P. P., 1955," Stresses from Local Loading In Cylindrical Pressure Vessels," Trans ASME 77, pp.805-816.
4. Bijlaard, P. P., 1954, "Stresses from Radial Loads in Cylindrical Pressure Vessels." Welding Journal.33 (12), Research Supplement, pp.615-623.
5. Bijlaard, P. P., 1955, " Stresses from Radial Loads and External Moments in Cylindrical Pressure Vessel." Ibid.,34 (12). Research Supplement, pp.608-617.
6. Steele, C. R, Steele, M. L., 1983, "Stress Analysis of Nozzles in Cylinderical Vessels with External Load." ASME, Journal of Pressure Vessel Technology, Vol. 105, pp.191-199.
7. Rodabaugh, E. C., Dodge, W. G., Moore, S. E., 1974, "Stress Indices at Lug Supports on Piping Systems.", Welding Research Council Bulletin No. 198.
8. Mirza, S., Gupgupoglu, K., 1983, "Stress Analysis of Cylindrical Pressure Vessels Subject to Band Loading." Proceedings of the and PEDAC Conference, Egypt, pp. 701-713.
9. Mirza, S., Gupgupoglu, K.,1988, "Stress Analysis of Pressure Vessels With Uniformly Spaced Lugs." Tran. of ASME, Vol.110, pp. 70-75.
10. Sadd, M. H., Avent, R. R.,1982, "Stress Analysis and Stress Index Development for a Trunnion Pipe Support." ASME, Journal of Pressure Vessel Technology, Vol.104, pp.73-78.
11. Tabone, C. J., Mallett, R. H.,1987, "Pressure-Plus-Moment Limit-Load Analysis for a Cylindrical Shell Nozzle." ASME, Journal of Pressure Vessel Tech. Vol.109, pp.297-301.
12. Brooks, G. N., 1988, "A Boundary Integral Analysis of Lugs Attached to Cylindrical Pressure Vessels." ASME, Journal of Pressure Vessel Tech., Vol. 110, pp.355-360.
13. Brooks, G. N., 1990, "Spring Constants for Rectangular Attachments to Cylindrical Shells." Proceeding Vol. 194, ASME, Pressure Vessels \& Piping Conference, Nashville, TN., pp.65-70.
14. Murad, F. P. and Sun, B. C., 1984, " On Radial and Rotational Spring Constants of Piping-Nozzles." Proceedings of Fifth International Conference on Pressure Vessel Technology, Vol. II, San Francisco, California, pp.236-250.
15. Sun, H. C., Sun, B. C., 1987, " On Radial and Rotational Spring Constants of Piping with Square Tubing Attachment." Vol. 120, ASME, Pressure Vessel and Piping Design Conference, San Diego, California, pp.51-63.
16. Sun, B. C., Sun, H. C., 1987, "Radial Rotational Spring Constants of Piping-Lug Attachments." Vol. 120, ASME, Pressure Vessel and Piping Design Conference, San Diego, California, pp.1-12.
17. Chiou, S. C., Sun, B. C., 1987, " On Torsional Spring Constants at Juncture of Piping-Nozzle." Vol. 120, ASME, Pressure Vessel and Piping Design Conference, San Diego, California, pp.83-86.
18. Chen, C. D., 1987, "On Longitudinal and Transverse Shear Spring Constants at the Juncture of Piping-Nozzle." Master Thesis, NJIT, Newark, NJ.
19. Gwaltney, R. C., Corum, J. M., Bolt, S. E., and Bryson, J. W., 1976, "Experimental Stress Analysis of Cylinder-to-Cylinder Shell Models and Comparison with Theoretical Predictions." Trans ASME 76, pp.283-290.
20. Brown, S. J., Jr., Haizlip, L. D., Nielsen, J. M., Reed, S. E. 1977, " Analytical and Experimental Stress Analysis of a Cylinder-to-Cylinder Structure." ASME, Journal of Pressure Vessel Tech., November, pp. 575-583.
21. Decock,J., 1980, "External Loadings on Nozzle in Cylindrical Shells." Proceedings, Forth International Conference on Pressure Vessel Technology, pp.127-134.
22. Timoshenko, S., Woinowsky-Krieger, S., Theory of Plates and Shells, Second Edition, McGraw-Hill, Inc. 1959, pp. 507-514.
23. ANSYS User Manual I \& II, Swanson Analysis Systems, Inc. 1983.
24. Cook, R. D., 1981, Concepts and Applications of Finite Element Analysis, Second Edition, John Wiley \& Sons, Inc., pp. 165-167.
25. Zienkiewicz, O. C., 1977, The Finite Element Method, Third Edition, McGraw-Hill Book Co. 1977, pp. 398-400.
26. Weaver, W. Jr., Johnston, P. R., 1984, Finite Elements for Structural Analysis, Prentice-Hall, Inc.
27. Bathe, K. J., 1982, Finite Element Procedures in Engineering Analysis, Prentice-Hall, Inc.
