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ABSTRACT

A basic theory for the size reduction operation by jet milling has been developed.

Typical samples of feedstock have been analyzed using screen analysis to determine the initial distribution of polymeric particulate sizes prior to jet milling.

A statistical theory was developed using probabilities to represent the likelihood that specific size reductions would occur. These probabilities include the effects of various equipment.

The theory was then used to predict the distribution of polymeric particulate sizes after jet milling. The agreement between the predictions and actual results was within 5 %.

Encouraged by the successful work of the simulation of the process, the simulation for the energy consumed will be expected.

By finishing the energy portion, screen analysis-operation-energy unit will comprise a core unit in the Computer Integrated Manufacturing (CIM).

2) SIZE REDUCTION OF POLYMERIC PARTICULATES

BY JET MILLING

by

1) CHENG HUA
//

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Title of Thesis : **SIZE REDUCTION OF POLYMERIC
PARTICULATES BY JET MILLING**

Name of Candidate: Hua Cheng

Master of Science, 1991

Thesis and Abstract Approved:

Dr. Keith O'Brien

Date

Professor of Mechanical
Engineering

Signatures of other members of the thesis committee.

Date

Date

VITA

Name : Hua Cheng

Degree and date to
be conferred : Master of Science, 1991

Secondary education: Yanan Middle School,
Shanghai, P. R. China
Graduated in July, 1984

Colleges attended	Dates	Degree	Date of Degree
Shanghai University of Technology	Sep., 1984- Jul., 1988	BS	Jul., 1988
New Jersey Insti- tute of Tech.	Sep., 1989- May, 1991	MS	May, 1991

Major : Manufacturing Engineering

Positions held : Application Engineer
Liquiflo Equipment Co.
140 Mt. Bethel Rd.,
Warren, NJ 07060

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CHAPTER I
OVERVIEW OF SIZE REDUCTION TECHNIQUES

1.1 The Purpose Of Size Reduction

The term size reduction is applied to all the ways in which particles of solids are cut, or broken into smaller particles. Throughout the process industries the size of particles are reduced by many different methods for many different purposes. Chunks of crude ore are crushed to workable size; synthetic chemicals are ground into powder; and sheets of plastic are cut into tiny cubes or diamonds. Commercial products must often meet stringent specifications regarding the size, and sometimes the shape of the particles they contain. Reducing the particle size also increases the reactivity of solids; it permits separation of unwanted ingredients by mechanical methods; and it reduces the bulk of fibrous materials for easier handling.

1.2 Characteristics Of Polymeric Particulates

The grinding characteristic of various plastics, such as resins and gums depend greatly upon their softening temperatures. When a finely divided product is required, it is often necessary to use a water-jacketed mill or a pulverizer with an air classifier in which cooled air is introduced into the system. Not all plastics can be ground, in as much as some of them are soft at the temperatures encountered during the operation. However a great many of them can be powdered if precautions are taken to prevent overheating. Some low softening temperature resins can be ground by mixing with 15-50% by weight of dry ice before grinding. Refrigerated air is sometimes introduced into the hammer mill to prevent softening & agglomeration.

1.3 Technique Classification

There are four commonly used techniques in size reduction:

- 1) Compression: gives relatively few fines for coarse reduction of hard solids.
- 2) Impact: gives coarse, medium, or fine products.
- 3) Attrition: yields very fine products from soft, nonabrasive materials.
- 4) Cutting: gives a definite particle size and sometimes a definite shape, with few or no fines.

Feedstocks and finished products from size reduction operations are defined in terms of the sizes involved. It is also desirable to know whether the ultimate individual particle size is being measured, or, if any aggregation or agglomeration of particles exists, and whether or not this has been created by the size-reduction operation.

CHAPTER II
PARTICLE SIZE DISTRIBUTION

2.1 Statistical Representation

The fullest description of a powder is given by its particle-size distribution. This can be plotted in terms of cumulative percent oversize or undersize in relation to the diameters of particles or it can be plotted as a distribution of the amounts present in each unit of diameter against the several diameters. It is common to employ a weight basis for percentage, but there are some data in the literature in which frequency, or number of particles is used. The basis of percentage, whether weight, frequency, or some less commonly used factor, should be specified, as should also be stated the diameter, units, and preferably whether it is determined by sieve, settling velocity, or otherwise.

The figures 2.1 & 2.2 present two sets of distributions, one cumulative and the other in unit intervals. The slope of the 5-um intervals of the

cumulative curves are converted to percentage per micrometer and plotted as a block, or histogram, from which smooth curves are derived.

Powder A has a narrower or tighter size range for the bulk of its weight than powder B. Both of them have the same weights below and above the size marked by the arrow.

Complete particle-size reduction analysis to show distribution is essential to not only most comparisons and calculations but also for practical usage in the factory. One of the important usage is through a detailed particle-size distribution as shown in figures 2.1 & 2.2 a work process can be simulated. During the simulation different-sized particles are processed thus produce the product of a certain particle size distribution.

Revisions

can also be applied to the process to get a narrower or tighter distribution. Sometimes product which is not met the requirement can even be returned as the raw material thus get a better or more desirable product.

Here a little bit more can be dwelled into the derailed of the principle how we can simulate the whole production. First a particle-size distribution can be obtained by any sample selected from thousands of the raw material the factory supposed to process. The first graph of figure 2.1 & 2.2 is a typical distribution represent. Just as powder B histogram is applied thus a thorough separation of the whole distribution is obtained. The frequency or percentage of the size distribution of the whole raw material is also obtained. The purpose of this process is when we put the raw material into the working process we treat them separately . Also when the product which has not met the requirement is put back to the

starting point and product can be divided in this method too. Thus through every step an understanding and control of the whole process can be achieved.

After the detailed particle-size distribution is obtained. The continuum of the whole simulation is to set the working process into two categories. The first is the grinding-rate function of the particles during the machine operation, S_u . The other is the breakage function of the particle breakage if they meet a collision, $B_{n,u}$. Through this simulation can be approximately known the particle-size distribution of the product. Usually we can apply different S_u or $B_{n,u}$ for various frequency or percentage of even one product. But in some case this may cause tremendous increase of cost because of requirement of more work stage and equipment. So sometimes the first or second made product is simply put back and treated as raw material again to get the desirable particle size distribution.

The key portion of the whole process is to get a nearly perfect approximation of P_c and P_{cb} . This portion is fundamentally laid on the input, which means a well defined and mathematical separated particle-size distributing of the raw material or re-feed material.

2.2 Mathematical Expressions

A number of equations have been proposed to correlate the quantity of a particulate material with its particle size to obtain a distribution relationship. A lot of literature often assumed that a powder must follow some distribution, such as the ROSIN-RAMMLER-BENNET: [13]

$$Y=1-[\exp-(X/X')^n] \quad 2.1$$

or the GATES-GAUDIN-SCHUMANN distribution: [15]

$$Y=(X/k)^m \quad 2.2$$

or the LOGRITHMATIC-PROBABILITY distribution: [19]

$$Y = \text{erf}\left(\frac{\ln X/X'}{\sigma}\right) \quad 2.3$$

or the GAUDIN-MELOY distribution: [20]

$$Y = 1 - (1 - X/X')^r \quad 2.4$$

where

Y=cumulative fraction by weight undersize;

X=size;

k,X'=parameters with dimension of size;

m,n,r=dimensionless exponents;

erf=error function;

σ =standard deviation parameter;

There is no fundamental reason why a particular powder must obey one of these empirical laws; forcing it to do so will result in error. Furthermore, it is difficult to tell whether the fit is good because any cumulative-size plot will give the appearance of a good fit; random numbers appear to fit the ROSIN-RAMMLER curve 9 times out of 10. Differential plots show size deviation more clear.

In special cases a few size-data points can be plotted and the rest of the curve assumed to follow trends previously established. This may be performed in the application of the mill to a particular mineral when many previous runs have been made for similar materials and conditions.

Several of these laws are useful simply for curve-fitting purposes. The ROSIN-RAMMLER can represent a distribution with a peak in the differential curve;

the GATES-GAUDIN-SCHUMANN has the advantage of simplicity; the GAUDIN -MELOY has the advantage that it can fit a variety of curves found in practice. Size data may also be represented in tabular form, thus avoiding the problem of curve-fitting.

The assumption this thesis is going to employ and represent is to use screen analysis which based on mass fraction.

MASS FRACTION, means tabulated method to show the mass fraction in each size increment as a function of the average particle size in the increment. The first graph in figure 2.1 is a kind of analysis called differential analysis. The second graph of figure one called cumulative analysis which is just simply presenting but by adding, consecutively, the individual increments, starting with that containing the smallest particles, and tabulating or plotting the cumulative sums against the maximum particle diameter in the increment.

Per cent by weight
(for 5-micron increments)

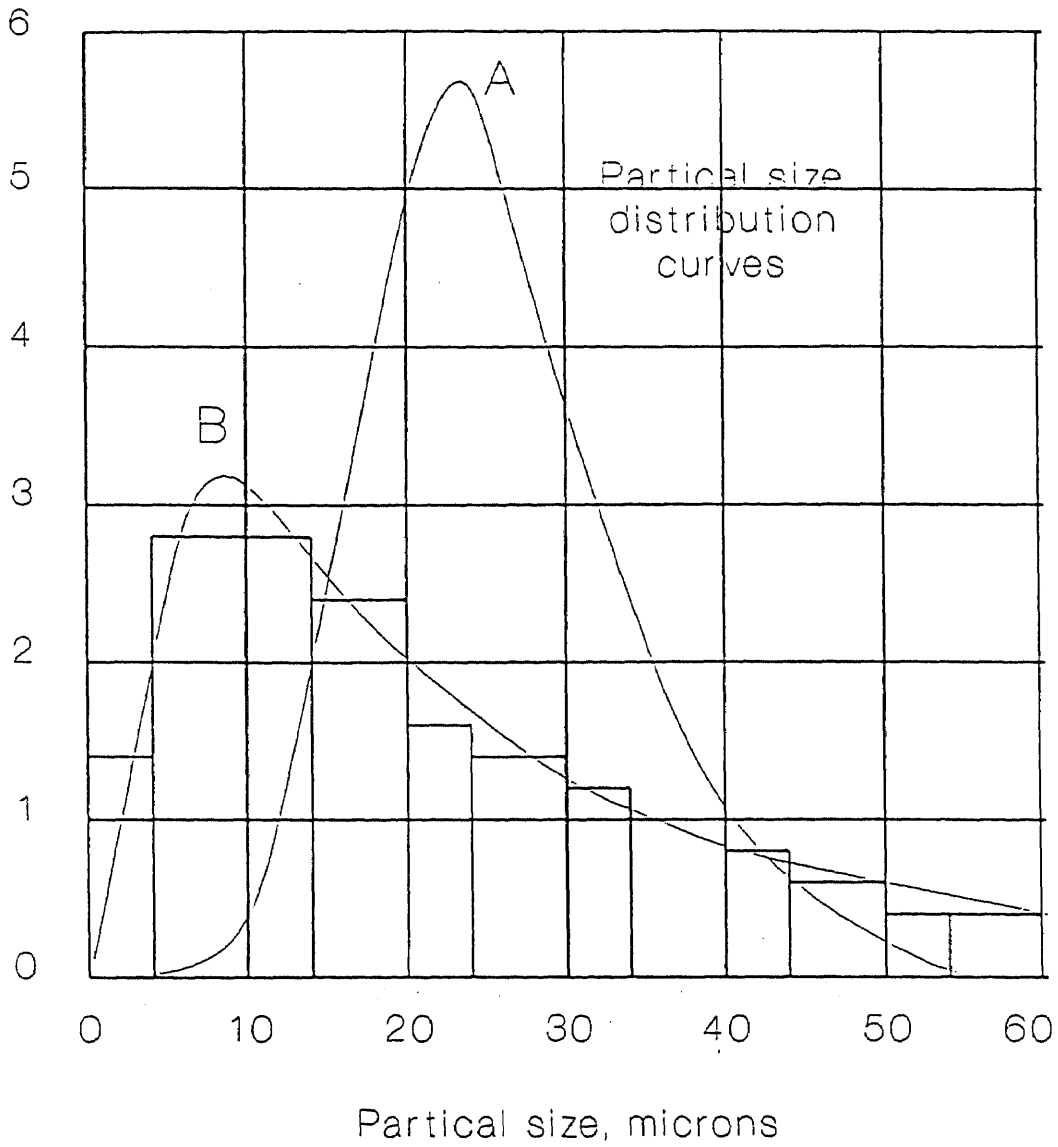
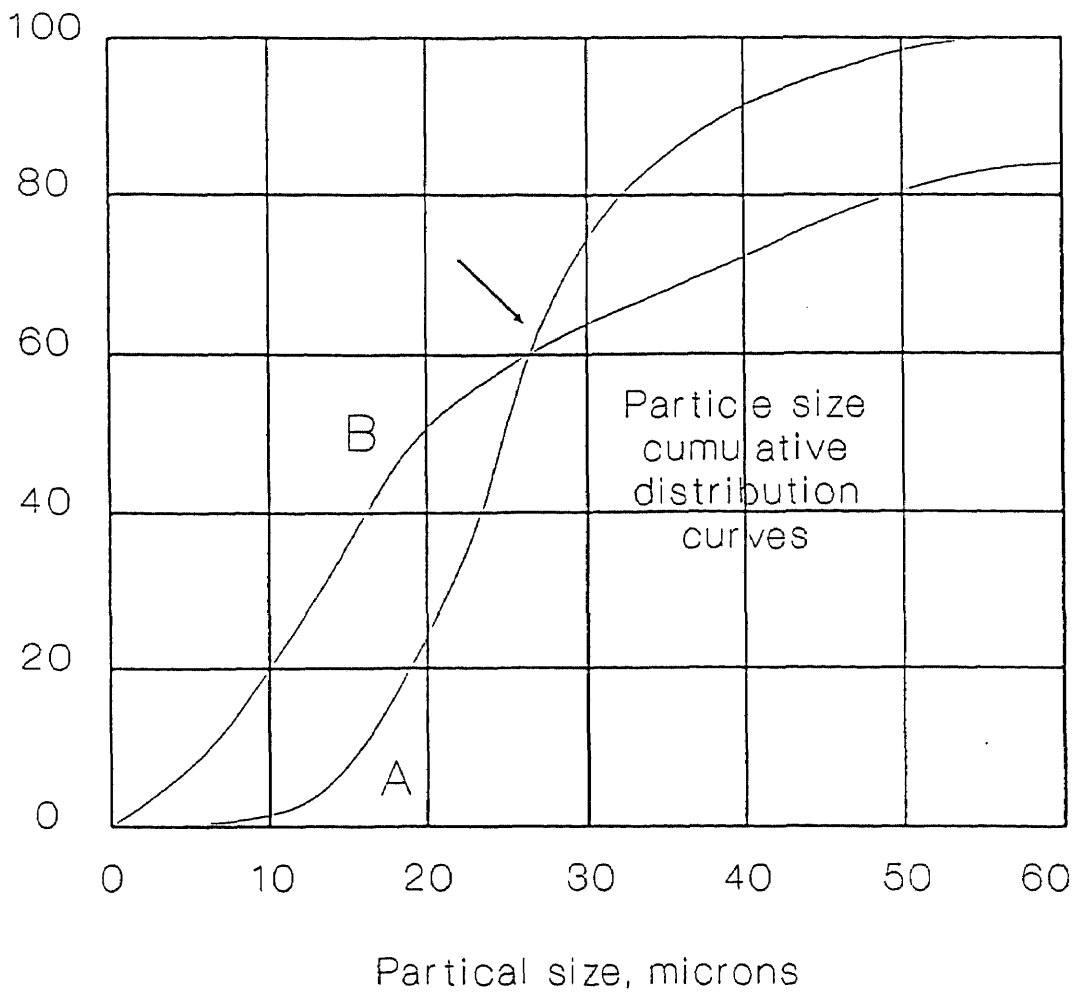


Fig 2.1A Particle-size distribution curve
for simple powders [11]

Fig 2.2 Particle-size distribution curves for simple powers [11]

Cumulation amount smaller than stated size, per cent



CHAPTER III
MEASUREMENT TECHNIQUES

3.1 Size Measurement

Screen Analysis, is used to measure the size of particles in the size range about 3 and 0.0015 in. (76mm and 38um) [In the standard cases]. Testing sieves are made of woven wire screens, the mesh and dimension of which are carefully standardized. The openings are square. Each screen is identified in meshes per inch. The actual openings are smaller than those corresponding to the mesh numbers, however, because of the thickness of the wires.

The characteristics of common series are usually given in standard tables. The area of the openings in any one screen in the series is exactly twice that of the openings in the next smaller screen. The ratio of the actual mesh dimension of any screen to that of the next smaller screen is the square root of 2, i.e. 1.414. For closer sizing, intermediate screens are also available, the ratio can be 1.189, but ordinarily these intermediate screens are not used.

In making an analysis a set of standard screen is arranged in series in a stack, with the smallest mesh at the bottom and the largest at the top. The sample is placed on the top screen and the stack shaken mechanically for a definite time. The particles retained on each screen are removed and weighed, and the masses of the individual screen increments are converted to mass fraction or mass percentage of the total sample. Any particles that pass the finest screen are caught in a pan at the bottom of the stack. The results of a screen analyses are tabulated to show the mass fraction of each screen increment. Since the particles on any one screen are passed by the screen immediately ahead of it, two numbers are needed specify the size range of an increment, one for the screen through which the fraction passes and the other on which it is retained. Thus, the notation 14/20 means "through 14 mesh and on 20 mesh".

A typical screen analysis is shown in Table 3.1. The first two columns give the mesh size and width of opening of the screens; the third column is the mass fraction of the total sample which is retained on the designated screen. This is X_i , where i is the number of the screen starting at the bottom of the stack; thus $i=1$ for the pan, and screen $i+1$ is the screen immediately above screen i . The symbol D_{pi} means the particle diameter equal to the mesh opening of screen i . The last two columns in Table 3.1 show the average particle diameter D_{pia} in each increment and the cumulative fraction smaller than each value of D_{pi} . In screen analyses cumulative fractions are sometimes written starting at the top of the stack and are expressed as the fraction "larger than" a given size.

A differential plot of the data in columns 2 and 3 of Table 3.1 gives a false impression of the particle size distribution because the range of particle sizes covered differs from increment to increment. Less material is retained in an increment when the particle size range is narrow than when it is wide. In figure 3.1 the ranges were all equal and the data could be plotted of $X_i/(D_{pi+1}-D_{pi})$, where $D_{pi+1}-D_{pi}$ is the particle size range in increment i . This is illustrated by figure 3.1 a and b, which are direct and adjusted differential plots for the 20/28-mesh and smaller particle size in Table 3.1.

Cumulative plots are made from results like those in column 2 and 5 of Table 3.1. When the overall range of particle size is large, such plots often show the diameter on a logarithmic scale. A semilogarithmic-probability paper on which the abscissa scale is divided in accordance with a Gaussian probability distribution. Size analysis of the product from a size reduction machine often give linear plots on such paper, at least over much of the particle size range. Plots of this kind were formerly used for extrapolation to small particle sizes below the range of testing sieves, but with the present availability of methods for measuring extremely small particles this is no longer necessary.

3.2 Size Measurement With Extremely Fine Particles

The size of particles too fine for screen analysis are measured by a variety of methods, including differential sedimentation, porosity measurements on settled beds, light absorption of gases on the particle surface, and by visual counting using a microscope.

3.3 Particle-size Distribution

Through different way (mainly screen analysis), the percentage or frequency of the particle size distribution can be obtained in the form of mass distribution, as shown in figure 3.1, 3.2 and 3.3. From this point we can continue our work on the computer simulation of the working process. A very important note is that the particle-size distribution is measured rather than assumed. Thus a more accurate percentage or frequency can always be employed to the work forward. Only one work shop is needed for the factory to get all the particle-size distribution of each kind of the raw material and the re-feed material. To do so also provide a very good chance to get the future manufacturing more predictable and controllable which finally lead to a totally Computer Integrated Manufacturing (CIM).

Table 3.1 Standard Screen Analysis Result [21]

Mesh	Screen Opening Du, mm	Mass Function Retained Xi	Average Particle Diameter in Increment	Cumulati- ve func- tion Dpi
4	4.699	0.0000	--	1.0000
6	3.327	0.0251	4.013	0.9749
8	2.362	0.1250	2.845	0.8499
10	1.651	0.3207	2.007	0.5292
14	1.168	0.2570	1.409	0.2722
20	0.833	0.1590	1.001	0.1132
28	0.589	0.0538	0.711	0.0594
35	0.417	0.0210	0.503	0.0384
48	0.295	0.0102	0.356	0.0282
65	0.208	0.0077	0.252	0.0205
100	0.147	0.0058	0.178	0.0147
150	0.104	0.0041	0.126	0.0106
200	0.074	0.0031	0.089	0.0075
Pan	--	0.0075	0.037	0.0000

Fig 3.1 Differential Screen analyses
plot as for size range of increment [21]

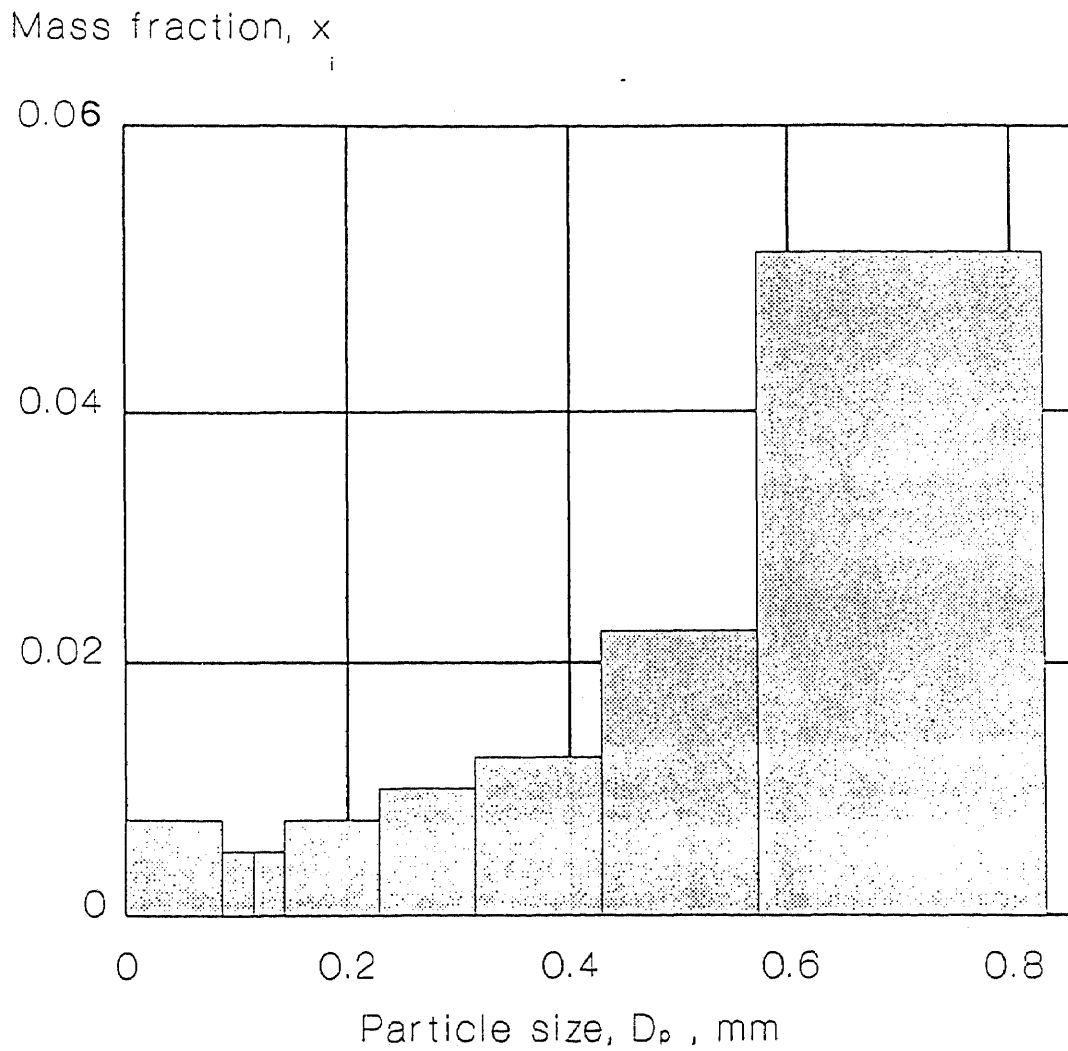
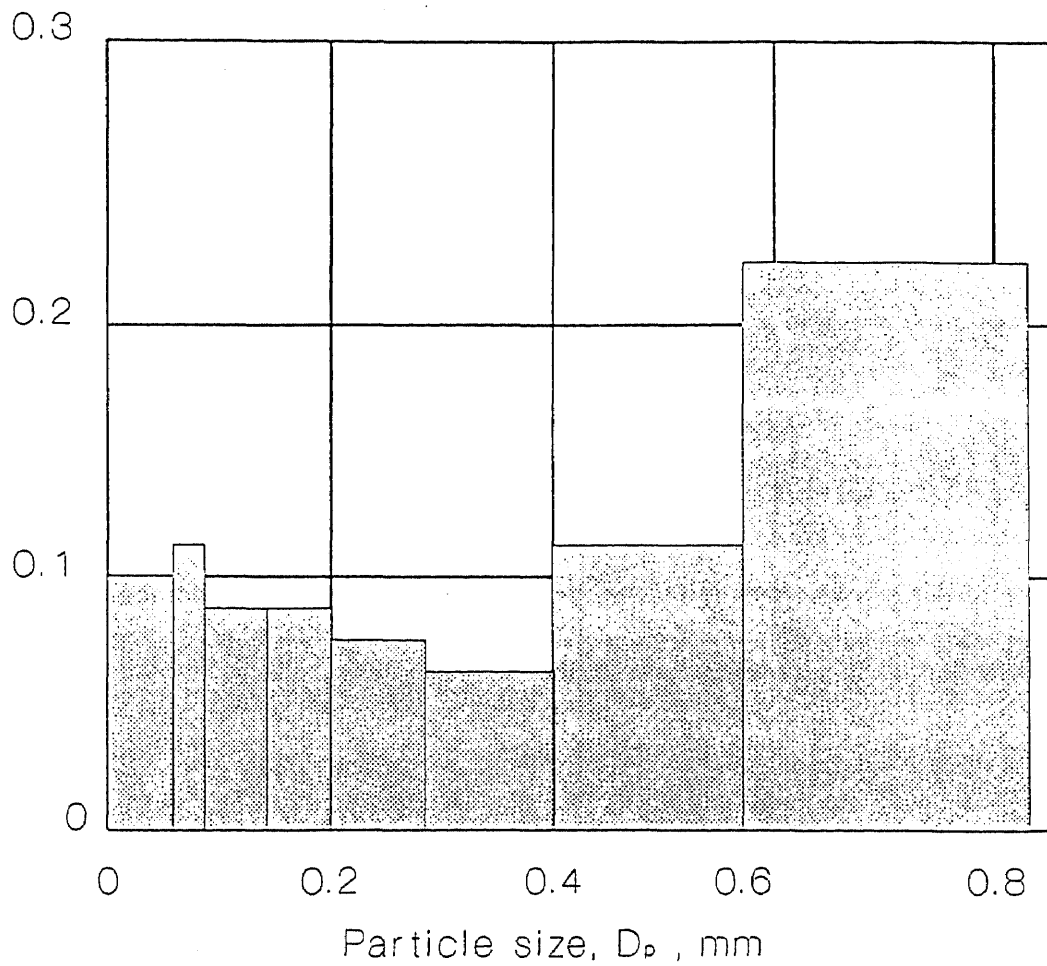


Fig 3.2 Differential Screen analyses
plot as for size range of increment [21]

Mass fraction per mm
 $x_i / (D_{pi+1} - D_{pi})$



CHAPTER IV
COMMINUTION PROCESS & COMPUTER SIMULATION

4.1 Principles Of Comminution

A. Criteria For Comminution

Comminution is a generic term for size reduction; crushers and grinders are types of comminuting equipment. An ideal crusher or grinder would have the following characteristics:

1. Have a large capacity
2. Require a small power input per unit of product
3. Yield a product of the single size or the size distribution desired.

B. Process Performance

The usual method of studying the performance of process equipment is to set up an ideal operation as a standard, compare the characteristics of the actual

equipment with those of the ideal unit, and account for the difference between the two. When this method is applied to crushing and grinding equipment, the differences between the ideal and actual are very great, and despite extensive study the gaps have not been completely accounted for. On the other hand, useful empirical equations for predicting equipment performance have been developed from the incomplete theory now at hand.

The capacities of comminution machines will be best discussed when the individual types of equipment are described. The fundamentals of product size and shape and of energy requirements are, however, common to most machines and can be discussed more generally.

4.2 Characteristics Of Comminuted Products

The objective of crushing and grinding is to produce small particles from larger ones. Smaller particles are desired either because of their large surface or because of their shape, size, and number. One measure of the efficiency of the operation is based on the energy required to create new surface, as known, the surface area of a unit mass of particles increases greatly as the particle size is reduced.

Unlike an ideal crusher or grinder, an actual unit does not yield a uniform product, whether the feed is uniformly sized or not. The product always consists of a mixture of particles, ranging in size from a definite maximum to a submicroscopic minimum. Some machines, especially in the grinder class, are designed to control the magnitude of the largest particles in their products, but the fine sizes are not under control. In some types of grinders fines are minimized, but they are not eliminated. If the feed is homogeneous, both in the shapes of the particles and in the chemical and physical structure, the shapes of the individual units in the product may be quite uniform; otherwise, the grains in the various sizes of a single product may vary considerably in proportions.

The ratio of the diameters of the smallest and the largest particles in a comminuted product is of order of 10^4 . Because of this extreme variation in the sizes of the individual particles, relationships adequate for uniform size must be modified when applied to such mixtures. The term "average size", for example, is meaningless until the method of averaging is defined, and, several different average sizes can be calculated.

Unless they are smoothed by abrasion after crushing, comminuted particles resemble polyhedrons with nearly plane faces and sharp edges and corners. The particles may be compact, with length, breadth, and thickness nearly equal, or they may be platelike or needlelike. For compact grains, the largest dimension or apparent diameter is generally taken as the particle size. For particles that are platelike, two dimensions should be given to characterize their size.

4.3 Computer Simulation of Milling Operations

The size distribution of products from various types of size reduction equipment can be predicted by a computer simulation of the comminution process. This makes use of two basic concepts, that a grinding-rate function S_u and a breakage function $\Delta B_{n,u}$. The material in a mill or crusher at any time is made up of particles of many different sizes, and they all interact with one another during the size-reduction process, but for purposes of computer simulation the material is imagined to be divided into a number of discrete fractions (such as the ones retained on the various standard screens) and that particle breakage occurs in each fraction more or less independently of the other fractions.

Consider a stack of N_t standard screens, and let N be the number of a particular screen in the stack. Here it

is convenient to number the screens from the top down, beginning with the coarsest screen. For any given value of N , let the upper screens, coarser than screen N , be designated by the subscript u . (Assume $u < N$). The grinding-rate function S_u is the fraction of the material of a given size, coarser than that on screen N , which is broken in a given time. If X_u is the mass fraction retained on one of the upper screens, its rate of change by breakage to smaller sizes is: [21]

$$dX_u/dt = -S_u X_u$$

4.1

Suppose, for example, that the coarsest material in the charge to a grinding mill is 4/6 mesh, that the mass fraction of this material X_1 is 0.05, and that one-hundredth of this material is broken every second. Then S_u would be 0.01 s^{-1} . and the X_1 would diminish at the rate of $0.01 * 0.05 = 0.0005 \text{ s}^{-1}$.

The breakage function $\delta_{Bn,u}$ gives the size distribution resulting from the breakage of the upper material. Some of the 4/6 mesh material, after breaking, would be fairly coarse, some very small, and some in between. Probably very little would be as large as 6/8 mesh, and only a small amount as small as 200 mesh. One would expect sizes in the intermediate range to be favored. Consequently $\delta_{Bn,u}$ varies with both n and u . Furthermore it varies with the composition of the material in the mill, since coarse particles may break differently in the presence of large amounts of fines than they do in the absence of fines. In a batch mill, therefore, $\delta_{Bn,u}$ would be expected to vary with time as well as with all the other milling variable.

If $\delta_{Bn,u}$ and S_u are known or can be assumed, the rate of change of any given fraction can be found as follows. For any fraction except the coarsest, the

initial amount is diminished by breakage to smaller sizes and simultaneously augmented by the creation of new particles from breakage of all coarser fractions. If input and outgo to a given screen are at equal rates, the fraction retained on that screen remains constant. Usually, however, this is not the case, and the mass fraction retained on screen N changes according to the equation: [21]

$$dX_n/dT = -S_n X_n + \sum_u X_u S_u \Delta_{Bn,u} \quad 4.2$$

The equation can be simplified if it is assumed that S_u and $\Delta_{Bn,u}$ are constant, and analytical and matrix solutions are available for this case, but these assumptions are highly unrealistic. In crushing coal, for particles larger than about 28 mesh, S_u has been found to vary with the cube of the particle size and the breakage function to depend on the reduction ratio D_n/D_u according to the equation: [21]

$$B_{n,u} = (D_n/D_u)^b$$

4.3

where the exponent b may be constant or may vary with the value of B .

In the equation above, $B_{n,u}$ is the total mass fraction smaller than size D_n . It is cumulative mass fraction, in contrast with $\Delta B_{n,u}$, which is the fraction of size D_n (retained between screen N and $N=1$) resulting from breakage of particles of size D_u .

If B in the equation above is constant, this equation says that the particle size distribution of the crushed material is the same for all sizes of the initial material. The value of ΔB_u in crushing 4/6 mesh material to 8/10 mesh will be the same as in crushing 6/8 mesh particles to 10/14 mesh, since the size-reduction ratio is the same.

Usually the equation is solved by the Euler method of numerical approximation in which the changes in all fractions during successively short time intervals, Δt , say 30 seconds, are calculated by the approximation $\frac{dX_n}{dt} = \frac{\Delta X_n}{\Delta t}$. Changes in S_u and $\Delta B_{n,u}$ with screen size and with time can be incorporated. A computer is needed to make the lengthy calculations. The method is illustrated in the example in Chapter VIII.

CHAPTER V

ENERGY CONSUMED IN SIZE REDUCTION OPERATIONS

5.1 Energy And Power Requirements In Comminution

The cost of power is major expense in crushing and grinding, so the factors that control this cost are important. During size reduction, the particles of feed material are first distorted and strained. The work necessary to strain them is stored in a coiled spring. As additional force is applied to the stressed particles, they are distorted beyond their ultimate strength and suddenly rupture into fragments. New surface is generated. Since a unit area of solid has a definite amount of surface energy, the creation of new surface requires work, which is supplied by the release of energy of stress when the particle breaks by conservation of energy, all energy of stress in excess of the new surface energy created must appear as heat.

5.2 Relationship between final product size and energy required

The fineness to which a material is ground has a marked effect on its production rate. Figure 5.1 is an example showing how the capacity decreases and the specific energy and cost increases as the product is ground finer.

Because of the rising cost of energy, more concentration had been drawn to the point. The United States industries uses approximately 32 billion kw of electrical energy per annum in size reduction operations. (From National Material Advisory Board, Comminution and energy consumption, Publ. NMAB-364, National Academy Press, Washington, 1981; available National Technical Information Service, Springfield, Va. 22151)

More than half of the energy is consumed in the crushing and grinding of minerals, one-quarter in the production of cement, one-eighth in coal, and one-eighth in agriculture products. Five areas were recommended to save the energy involved:

Classification-device design

Mill design

Control

activities to resist wear

material to resist wear

The report reviews these areas with an extensive bibliography.

It is always very important to determine the energy required in relation to different size reduction

operation. Many factors, or variables will be involved and thus make the calculation become very complex. Developing a set of laws for this is desirable.

5.3 Energy Laws

Several laws have been proposed to relate size reduction to a single to a single variable, the energy input to the mill. These laws are encompassed in a general differential equation: [22]

$$dE = -C \frac{dX}{X^n} \quad 5.1$$

where E is the work done, X is the particle size, and C and n are constants. For n=1 the solution is Kick's Law.

The law can be written: [23]

$$E = C \log(X_f/X_p) \quad 5.2$$

X_f is the feed particle size. X_p is the product size, and X_f/X_p is the reduction ratio. for $n > 1$ the solution is

$$E = (C/(n-1)) / (1/(X_p^{n-1}) - 1/(X_f^{n-1})) \quad 5.3$$

For $n=2$ this becomes Rittinger's Law, which states that the energy is proportional to the new surface produced.

The Bond Law corresponds to the case in which $n=1.5$:

[24]

$$E = 100 E_t (1/(X_p)^{0.5} - 1/(X_f)^{0.5}) \quad 5.4$$

where E_t is the Bond work index, in other word, work required to reduce a unit weight from a theoretical

infinite size to 80 percent passing 100 um. Extensive data on the work index have made this law useful for rough mill sizing . Summery data are given in Table 5.1.

The work index may be found experimentally from laboratory crushing and grinding tests or from commercial mill operations. Some rules of thumb for extrapolating the work index to conditions different from those measured are that for dry grinding the index must be increased by a factor of 1.34 over that measured in wet grinding; for open-circuit operations another factor of 1,34 is required over the measured in closed circuit; if the product size X_p is extrapolated below 70um, an additional correction factor is $(10.03+X_p)/1.145X_p$.

Also for a jaw or gyratory crusher the work index may be estimated from

$$E_t = 2.59C_s/ps$$

5.5

where C_s is impact crushing resistance, (ft-lb)/in if the thickness required to break; ps is specific gravity; and E_t is expressed in kwh/ton.

None of the energy laws apply well in practice, and they have failed to yield a starting point for further development of understanding of milling. Most of the early papers supporting one law or another were based on extrapolations of size distributions to finer sizes on the assumption of one or another size-distribution law. With present particle size analysis techniques applicable to the finest sizes, such confusion is no longer necessary. The relation of energy expenditure to the size distribution produced has been thoroughly examined.

5.4 Energy Coefficient

This usually based on the Rittinger's law. For example new surface produced per unit of energy input with the indirect expression of grinding time as an experimental variable is usually applied. The energy coefficient may also be expressed as tons per horsepower-hour passing a certain size.

There are two other very important terms concerns with this area:

Grinding Efficiency: the energy efficiency of a grinding operation is defined as the energy consumed compared with some ideal energy requirement.

Practical energy efficiency is defined as the efficiency of technical grinding compared with that laboratory crushing experiments.

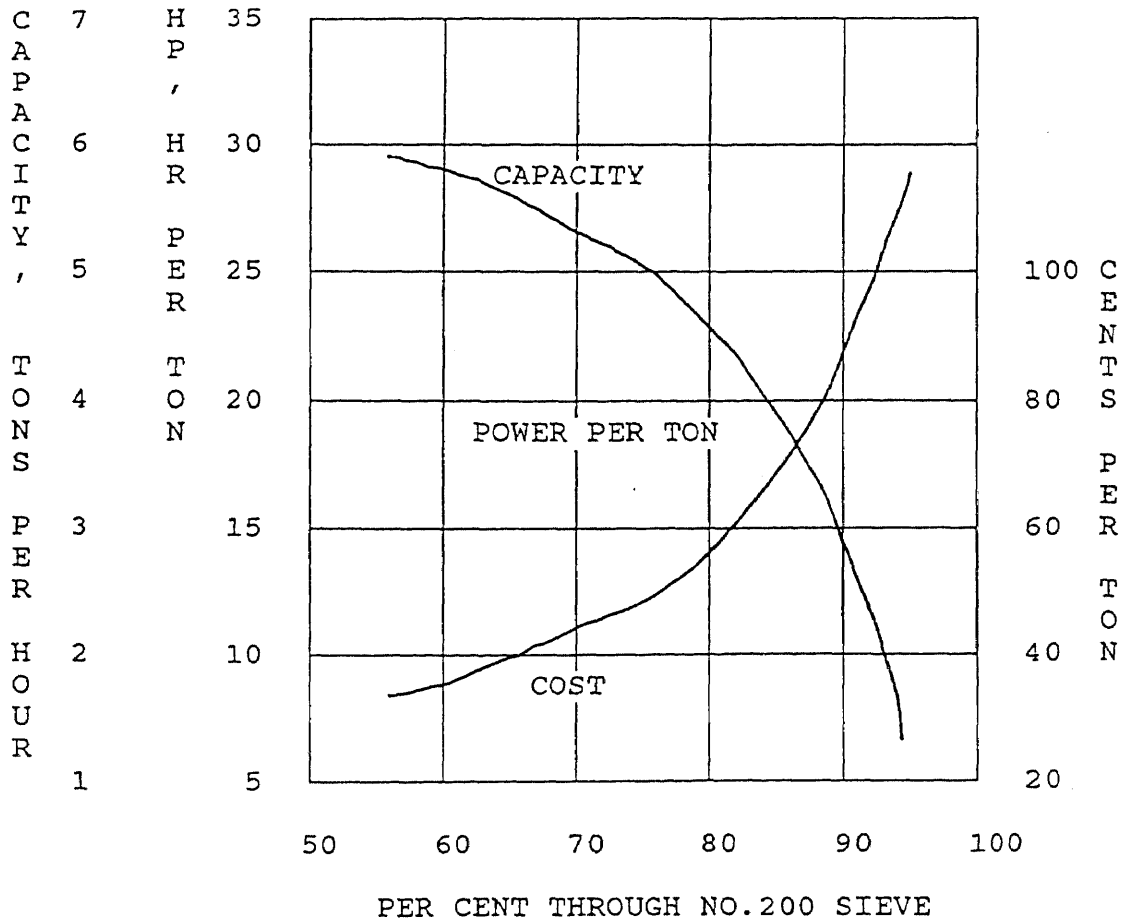


FIG. 5.1 Variation in capacity, power, and cost of grinding relative to fineness of product. [11]

TABLE 5.1 Average Work Indices For Various Materials

Material	No. of tests	Specific gravity	Work index
All Materials Tested	2088	--	13.81
Andesite	6	2.84	22.13
Barite	11	4.28	6.24
Basalt	10	2.89	20.41
Bauxite	11	2.38	9.45
Cement Clinker	60	3.09	13.49
Cement Raw Material	87	2.67	10.57
Chrome Ore	4	4.06	9.60
Clay	9	2.23	7.10
Clay, Calcined	7	2.32	1.43
Coal	10	1.63	11.37
Coke	12	1.51	20.70
Coke, Fluid Petroleum	2	1.63	11.37
Coke, Petroleum	2	1.78	73.80
Copper Ore	308	3.02	13.13
Coral	5	2.70	10.16
Diorite	6	2.78	19.40
Dolomite	18	2.82	11.31
Emery	4	3.48	58.18
Feldspar	8	2.59	11.67
Ferrochrome	18	6.75	8.87
Ferromanganese	10	5.91	7.77
Ferrosilicon	15	4.91	12.83
Flint	5	2.65	26.16
Fluorspar	8	2.98	9.76
Gabbro	4	2.83	18.45
Galena	7	5.39	10.19
Garnet	3	3.30	12.37
Glass	5	2.58	3.08
Gneiss	3	2.71	20.13
Gold Ore	209	2.86	14.83
Granite	74	2.68	14.39
Graphite	6	1.75	45.03
Gravel	42	2.07	25.17
Gypsum Rock	5	2.69	8.16
Ilmenite	7	4.27	13.11
Iron Ore	8	3.96	15.44
Hematite	79	3.76	12.68
Hematite -- Specular	74	3.29	15.40
Oolitic	6	3.32	11.33
Limanite	2	2.53	8.45
Magnetite	83	3.88	10.21

CHAPTER VI
SIZE REDUCTION EQUIPMENT

6.1 Classification There are also a lot of size reduction machine we can use, the principle type are:

A. Jaw Crushers

1. blake
2. Overhead eccentric
3. Dodge

B. Gyratory crushers

1. Primary
2. Secondary
3. Cone

C. Heavy-duty impact-mills

1. Rotor breakers
2. Hammer Mills
3. Cage Impactors

D. Roll Crushers

1. Smooth rolls (double)
2. Toothed Rolls (single and double)

- E. Dry pans and chaser mills
- F. Shredders
 - 1. Toothed shredders
 - 2. Cage disintegrators
 - 3. Disk mills
- G. Rotary cutters and dicers
- H. Media mills
 - 1. Ball, pebble, rod, and compartment mills
 - a. Batch
 - b. Continuous
 - 2. Autogenous tumbling mills
 - 3. Stirred ball and sand mills
 - 4. Vibratory mills
- I. Medium peripheral-speed mills
 - 1. Ring-roll and bowl mills
 - 2. Roll mills, cereal type
 - 3. Roll mills, paint and rubber types
 - 4. Buhrstones

J. High peripheral-speed mills

1. Fine grinding hammer mills
2. Pin mills
3. Colloid mills
4. Wood-pulp beaters

K. Fluid energy superfine mills

1. Centrifugal jet
2. Opposed jet
3. Jet with anvil

6.2 Fluid Energy Mills

A typical fluid-energy mill is illustrated in Figure 6.1.

In those mills the solid particles are suspended in a gas stream and convoyed at high velocity in a circular or elliptical path. Some reduction occurs when the particles strike or rub against the walls of the confining chamber, but most of the reduction is believed to be caused by interparticle attrition. Internal classification keeps the larger particles in the mill until they are reduced to the desired size.

The suspending gas is usually compressed air or superheated steam, admitted at a pressure of $100 \text{ lb}_f/\text{in.}^2$ (6.9 atm) through energizing nozzles. In the mills shown in the figure 6.1. the grinding chamber is an oval loop of pipe 1 to 8 in. (25 to 200 mm) in diameter and 4 to 8ft (1.2 to 2.4m) high. Feed enters near the bottom of the loop through a venturi injector. Classification of the ground particles takes place at the upper bend of the loop. As the gas stream flows around this bend at high speed, the coarser particles are thrown outward against the outer wall while the fines congregate at the inner wall. A discharge opening in the inner wall at this point leads to a cyclone separator and a bag collector for the product. The classification is aided by the complex pattern of swirl generated in the gas stream at the bend in the loop of pipe. Fluid-energy mills can accept feed particles as large as 1/2 in. (13 mm) but are more effective when the feed particles are no larger than 100-mesh. They reduce up to 1 ton/h of nonsticky solid to particles averaging 1/2 to 10 μm in diameter, using 1 to 4 lb (or kg) of steam or 6 to 9 lb (or kg) of air per pound (or kilogram) of product.

Air-mills, which is a special kind of fluid-energy mills. The figure 6.2 show the Trost air mill from Colt Industries are available in five size. The smallest is a research unit and can be used for fine-grinding studies. Capacities of 1 to 2300 kg/hr are available. Air flow rates vary from 0.2 to 28 m³/min. Encapsulation of particles is possible by the injection of coating material into the feed.

6.3 Equipment Operation

For the proper selection and economical operation of size-reduction machinery , attention must be given to many details of procedure and of auxiliary equipment. A crusher, grinder, or cutter cannot be expected to perform satisfactorily unless

1. The feed is of suitable size and enters at a uniform rate.
2. The product is removed as soon as possible after the particles are of the desired size.
3. Unbreakable material is kept out of the machine
4. In the reduction of low-melting or heat-sensitive products, the heat generated in the mills is removed.

Thus heaters and coolers, metal separators, pumps, and blowers, and constant-rate feeders are important adjuncts to the size reduction unit.

A. Open-circuit And Closed-circuit Operation

In many mills the feed is broken into particles of satisfactory size by passing it once through the mill. When no attempt is made to return oversize particles to the machine for further reduction, the mill is said to be operating in open circuit. This may require excessive amount of power, for much energy is wasted in regrinding particles that are already fine enough. If a 50-mesh product is desired, it is obviously wasteful to continue grinding 100- or 2000-mesh material. Thus it is often economical to remove partially ground material from the mill and pass it through a size-separation device. The undersize becomes the product and the oversize is returned to be ground. The separation device is sometimes inside the mill, as in ultrafine grinders; or, as is more common, it is outside the mill. Closed-circuit operation is the term applied to the action of a mill and separator connected so that oversize particles are returned to the mill.

For coarse particles the separation device is a screen or grizzly; for fine powders it is some form of classifier. A typical set of size-reduction machines and separators operating in closed circuit can be diagrammed clearly. The product from a gyratory crusher is screened into three fractions, fines, intermediate, and oversize. The oversize is sent back to the gyratory; the fines are fed directly to the final reduction unit, a ball mill. Intermediate particles are broken in a rod mill before they enter the ball mill. In the arrangement shown in the diagram the ball mill is grinding wet; i.e., water is pumped through the mill with the solid to carry the broken particles to a centrifugal classifier. The classifier throws down the oversize into a sludge, which is repulped with more water and returned to the mill. The undersize, or the product, emerges from the classifier as a slurry containing particles of acceptable size. Although screens are simpler to operate than classifiers, they cannot economically make separations when the particles are smaller than about 150- to 2000 mesh.

It is the overgrinding of precisely these fine particles that results in excessive consumption of energy. Close-circuit operation is therefore of most value in reduction to fine and ultrafine sizes, which demand that the separation be done by wet classifier or air separators. Energy must of course be supplied to drive the conveyors and separators in a closed-circuit system, but despite this, the reduction in total energy requirement over open-circuit grinding often reaches 25 percent.

B. Feed Control

Of the operations auxiliary to the size reduction itself, control of the feed to the mill is the most important. The particles in the feed must be of appropriate size. Obviously they must not be so large that they cannot be broken by the mill; if too many of the particles are very fine, the effectiveness of many machines, especially intermediate crushers and grinders, is seriously reduced.

With some solids precompression or chilling of the feed before it enters the mill greatly increases the ease with which it can be ground. In continuous mills the feed rate must be controlled within close limits to avoid choking or erratic variations in load and yet make full use of the capacity of the machine. In cutting sheet material into precise squares or thread into uniform lengths for flock, exact control of the feed rate obviously essential.

C. Mill Discharge

To avoid buildup in a continuous mill the rate of discharge must equal the rate of feed. Furthermore, the discharge rate must be such that the working parts of the mill can operate most effectively on the material to be reduced. In a jaw crusher, for example, particles may collect in the discharge opening and be crushed many times before they drop out. As mentioned before, this is wasteful of energy if many of the particles are crushed more necessary.

Operation of a crusher in this way is sometimes deliberate; it is known as choke crushing. Usually, however, the crusher is designed and operated so that the crushed particles readily drop out, perhaps carrying some oversize particles, which are separated and returned. This kind of operation is called free-discharge crushing or free crushing. Choke crushing is used only in unusual problems, for it requires large amounts of power and may damage the mill.

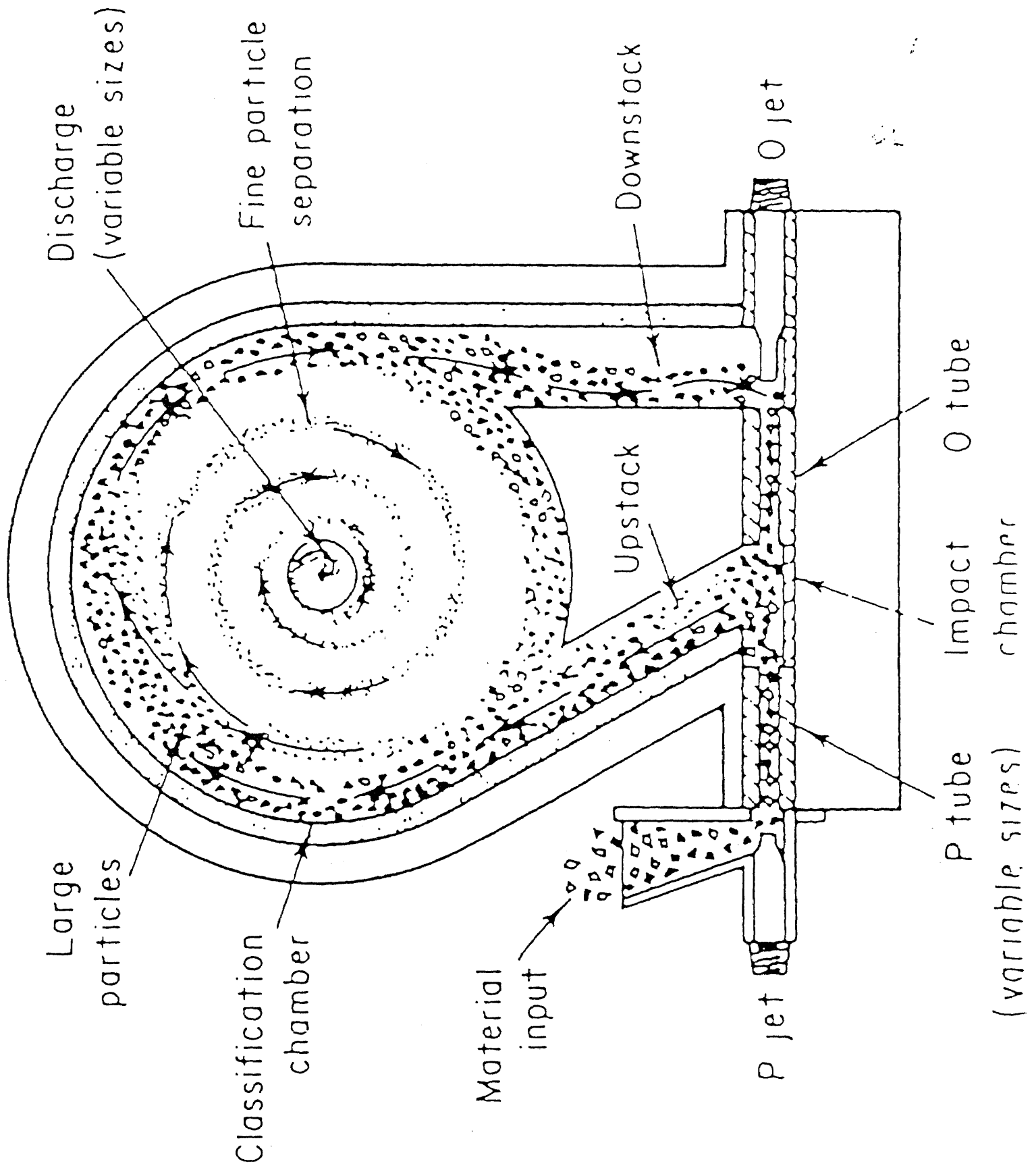
With fairly coarse comminuted products, as from a crusher, intermediate grinder, or cutter, the force of gravity is sufficient to give free discharge. The product usually drops out the bottom of the mill. In a revolving mill it escapes through openings in the chamber wall at one end of the cylinder; or it is lifted by scoops and dropped into a cone which directs it out through a hollow trunnion. A slotted grate or a diaphragm keeps the grinding medium from leaving with the product. Peripheral discharge is common in rod mills, trunnion discharge in ball mills and tube mills.

In discharging mills for fine and ultrafine grinding the force of gravity is replaced by the drag of a fluid carrier. The fluid may be a liquid or a gas. Wet grinding with a liquid carrier is common in revolving mills. It causes more wear on the chamber wall and on the grinding medium than dry grinding, but it saves energy, increases capacity, and simplifies handling and classification of the product. A sweep of air, steam, or inert gas removes the product from attrition mills, fluid-energy mills, and many hammer mills. The powder is taken out of the gas stream by cyclone separators or bag filters.

6.4 Removal Or Supply Of Heat

Since only a very small fraction of the energy supplied to the solid is used in creating new surface, the bulk of the energy is converted to heat, which may raise the temperature of the solid by degrees. The solid may melt, decompose, or explode unless this heat is removed.

For this reason cooling water or refrigerated brine is often circulated through coils or jackets in the mill. Sometimes the air blown through the mill is refrigerated, or solid carbon dioxide (dry ice) is admitted with the feed. Still more drastic temperature reduction is achieved with liquid nitrogen to give grinding temperatures below -75°C , the purpose of such low temperatures is to alter the breaking characteristics of the solid, usually by making it more friable. In this way substances like lard and beeswax become hard enough to shatter hammer mill; tough plastics, which stall a mill at ordinary temperatures, become brittle enough to be ground without difficulty.



CHAPTER VII
THEORETICAL AND EXPERIMENTAL RESULTS

7.1 Experimental Results

The following data is obtained from Wedco Technology, Inc through 8-inch diameter machine with an air manifold around the periphery. This air manifold has multiple holes in it which act as nozzles or jets to direct the air streams at an angle in order to form a tangent circle. The laboratory-size mill was operated with 100 cfm of compressed air at 100 psi. The percentage calculated for each size band is in Table 7.1. All the data is examined through screen analysis.

The mill acts as an air classifier as well in that the finer particles will exit the mill through the center discharge and the heavier particles will return to the periphery by centrifugal force and be again picked up by the air stream for additional grinding.

7.2 Theoretical Results

Applying the theory which is developed in the computer simulation, the following assumption is been held:

Starting grinding rate	$S_1 = 0.015$
Exponent	1.3
Time	$t = 3 \text{ min}$

Put the same input as experimental result, run through the program which is written in PASCAL. The output result corresponds to the experimental result almost perfectly. Figure 7.1 is the comparison of the results.

Input the same data as above, the starting grinding rate, exponent (indicates the combined effect of speed and equipment) and operation time are varied to get a set of generalized curve to demonstrate the different effect of each variable. The curves are shown in Fig. 7.2-7.4.

Table 7.1 Experimental Size Distribution

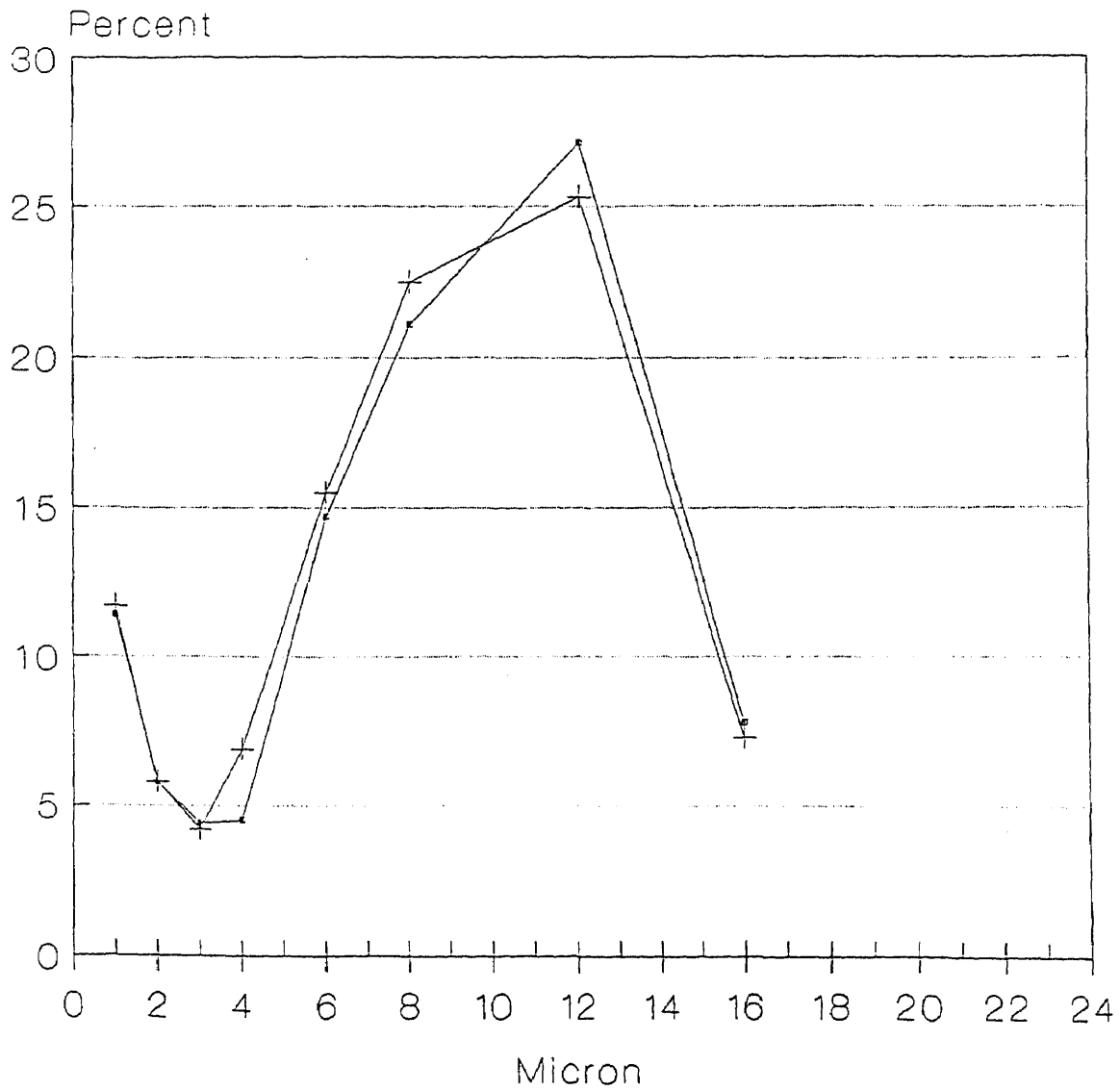
Raw feed		Finished	
micron	percentage	micron	percentage
1	10.9%	1	11.4%
1.5	2.4%	1.5	3.0%
2	3.9%	2	5.8%
3	2.1%	3	4.4%
4	2.4%	4	4.3%
6	11.4%	6	14.7%
8	15.6%	8	21.1%
12	25.1%	12	27.1%
16	14.1%	16	7.8%
24	11.6%	24	0.0%

Fig 7.1 Results Simulation

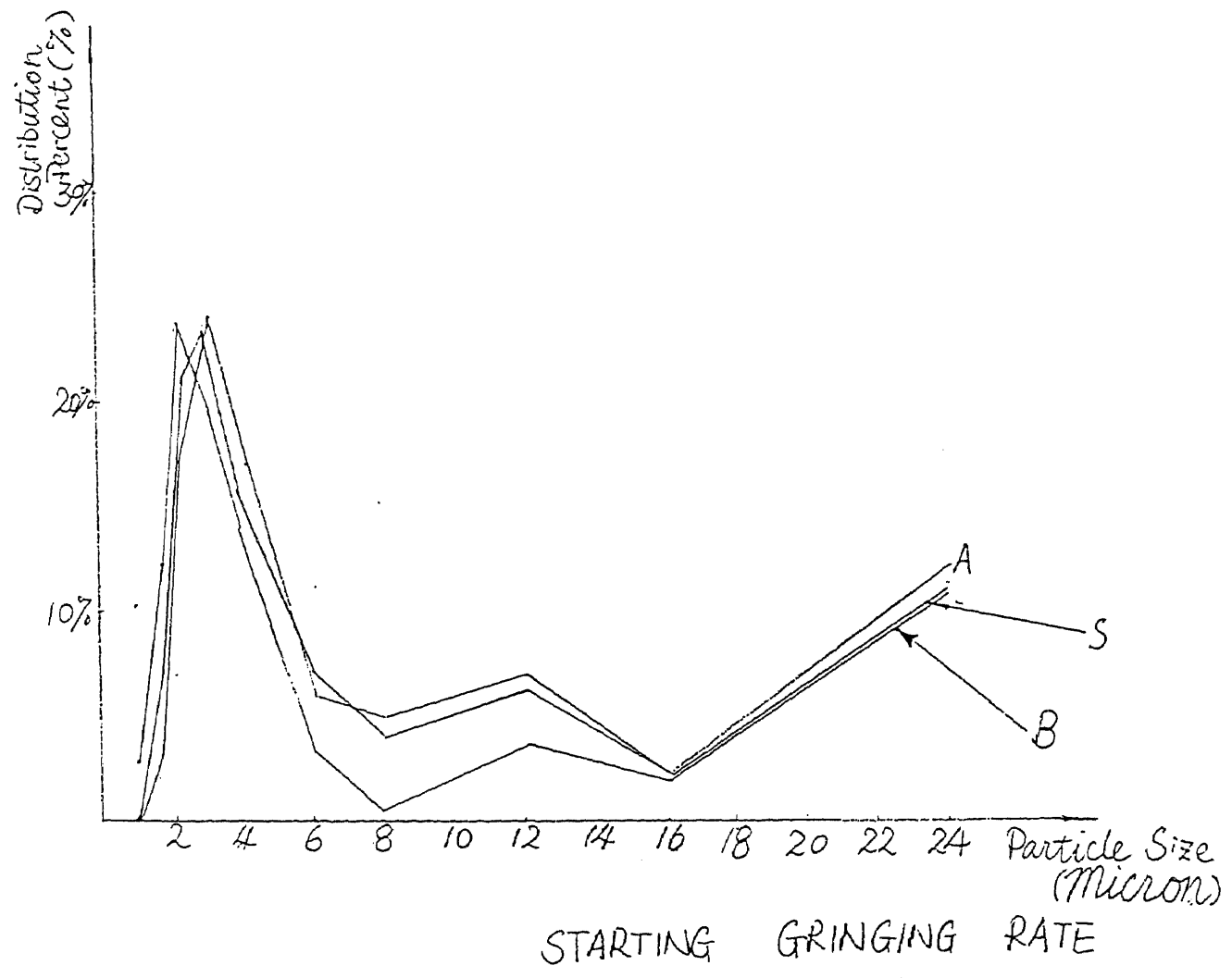
63

Series 1 ---- Experimental data

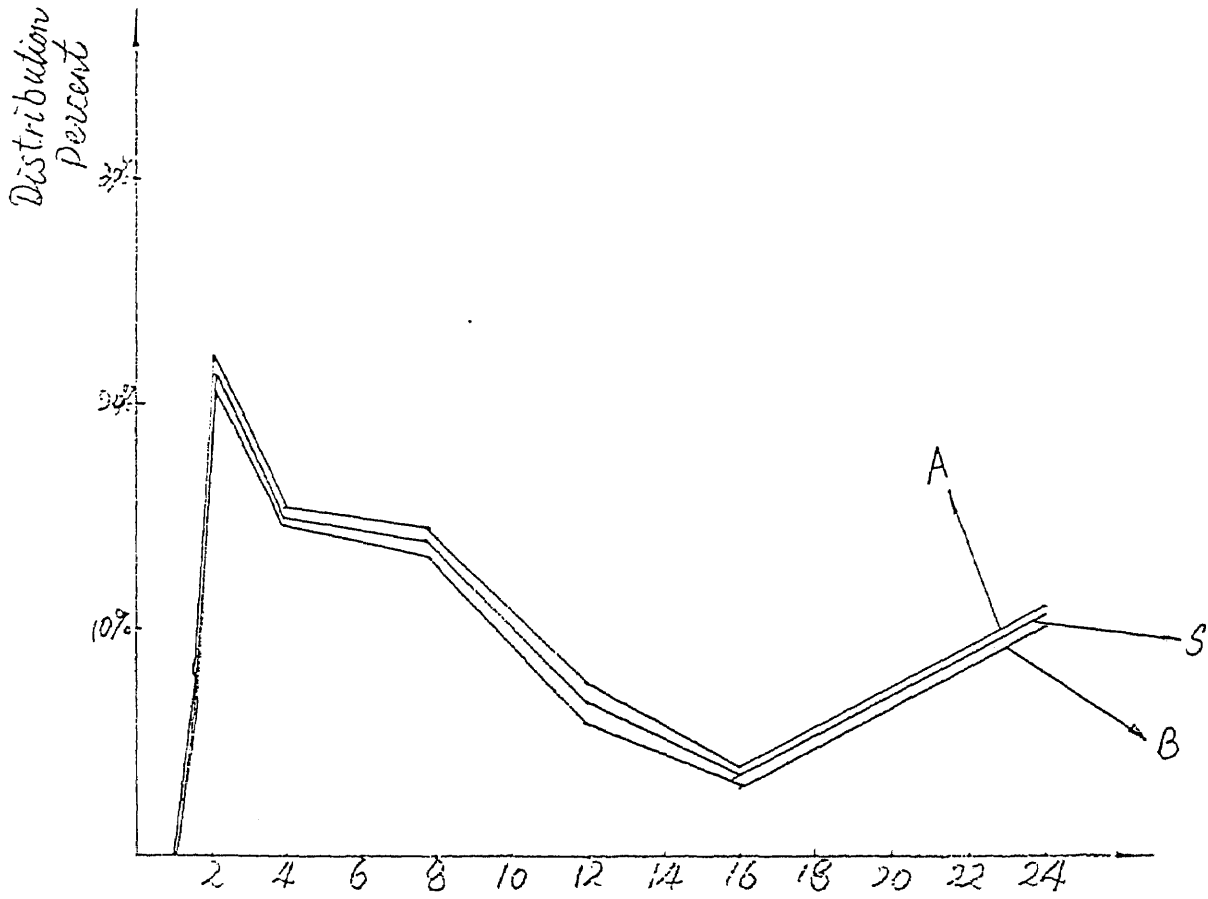
Series 2 ---- Computer data



—•— Series 1 —+— Series 2

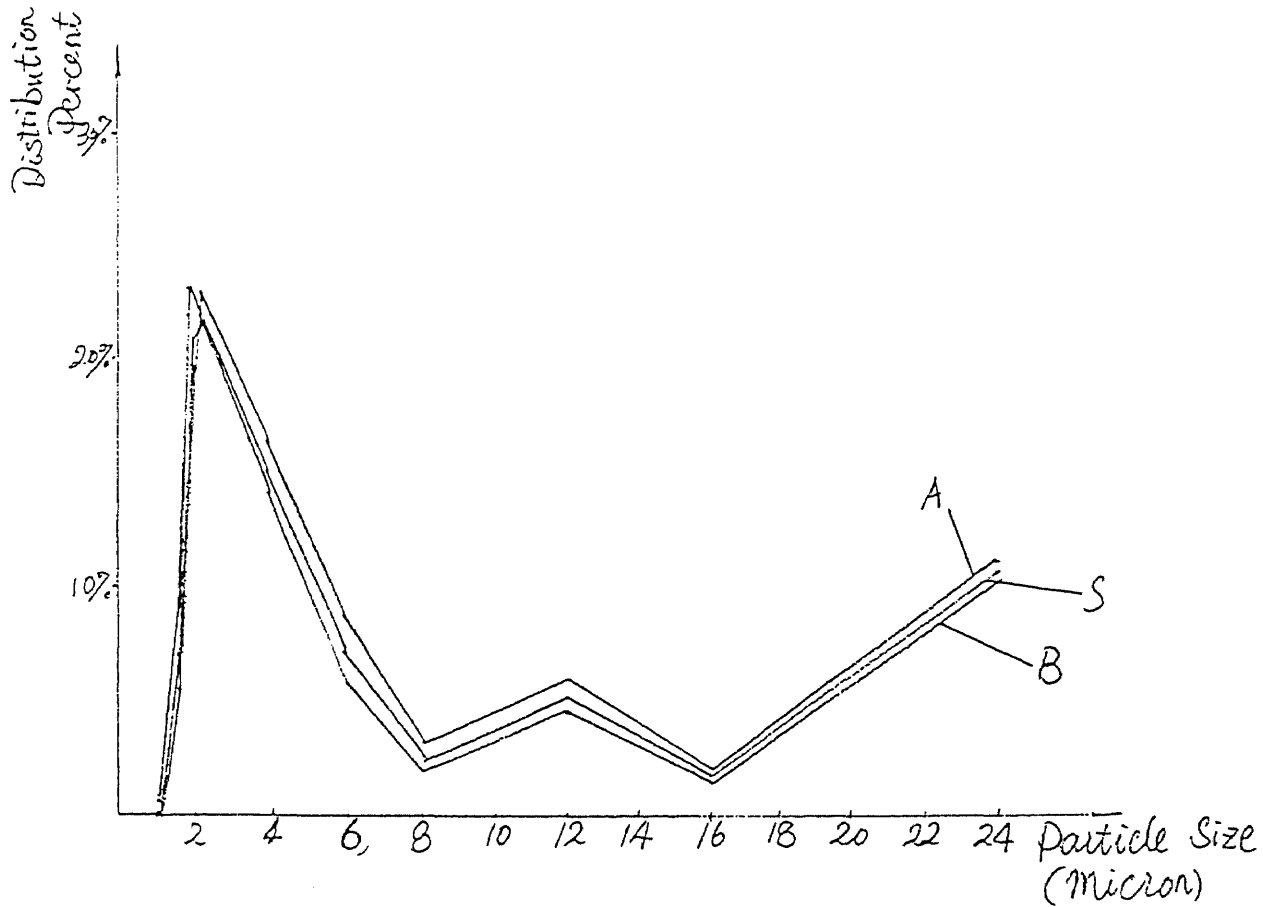


Curve	A:	0.02
Curve	S:	STANDARD (0.013)
Curve	B:	0.005



Exponential

Curve A	1.5
Curve S	1.3 (STANDARD)
Curve B	1.1



		Time	
Curve	A	4 (min)	
Curve	S	3 (min)	STANDARD
Curve	B	2 (min)	

CHAPTER VIII

DISCUSSION

One important aspect of the theory is that since none of the pure theoretical size distributions of the feedstock fit the actual distribution. Screen analysis has been employed here. This optimizes the latter computer simulation and also eliminates the narrow applicable range of each theory for different particles.

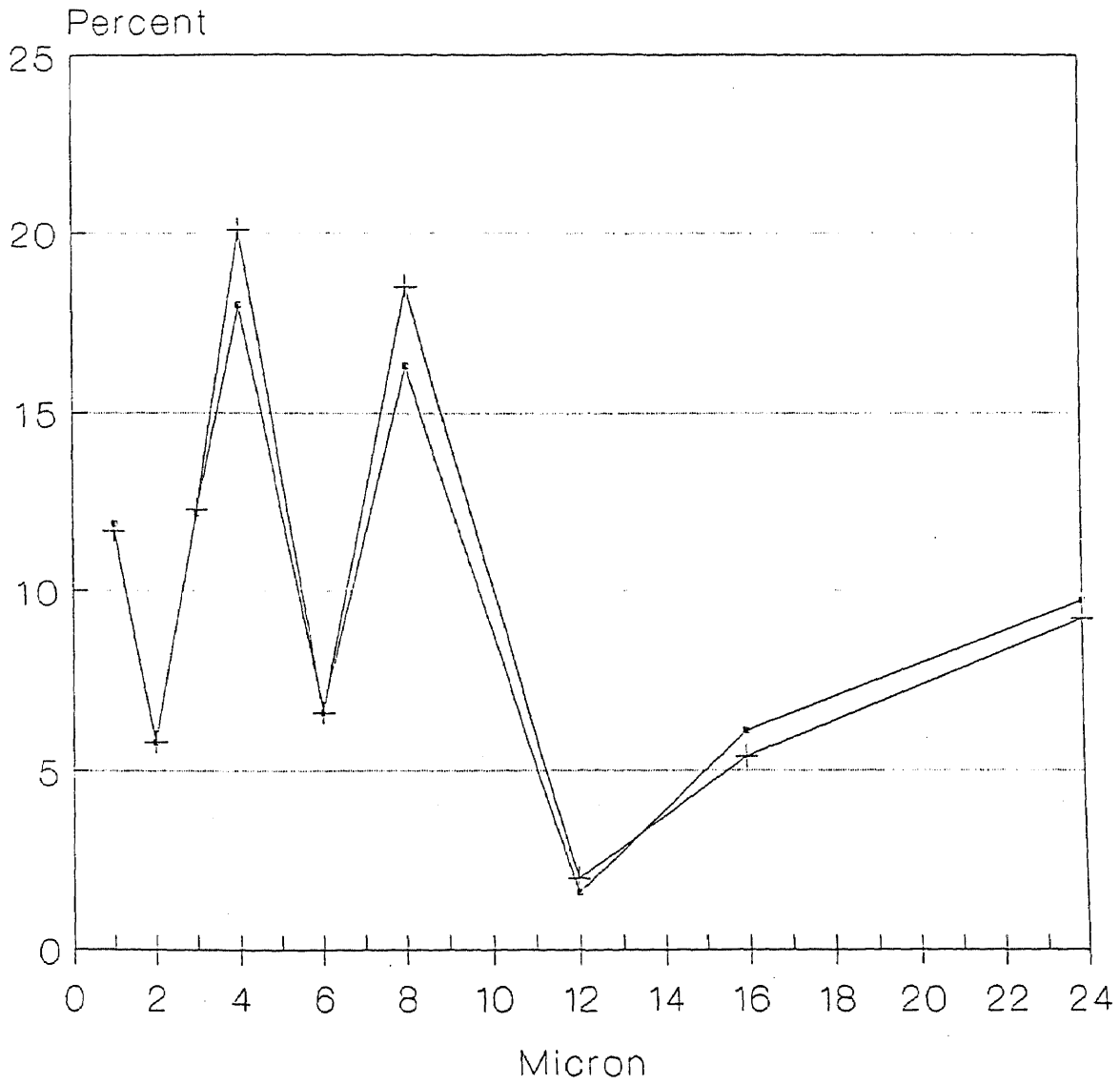
During the whole simulation process, the value of the starting grinding rate is the key variable. Thorough calculations have to be carried to find the suitable value for different kinds of equipment.

Another value t is calculated to be 3 minutes to perfectly fit the experimental result. Since the practical operation is a continuous process of air manifold around the periphery, this variable is not meant to be extremely accurate. As mentioned in the previous chapter, if continuous mills supplied, time can even become a constant.

Fig 8.1 Results comparison

Series 1 ---- Experimental data

Series 2 ---- Computer data

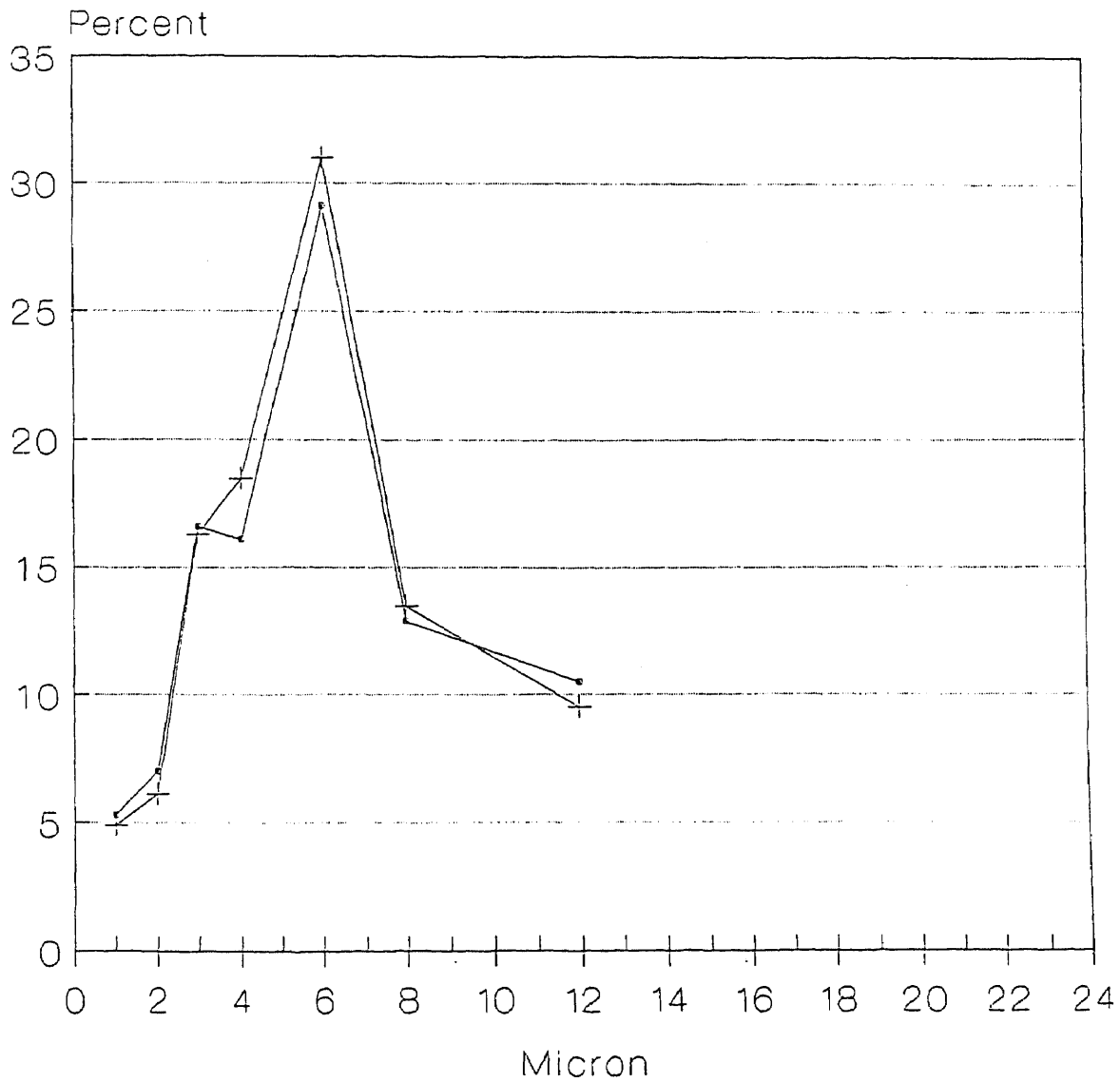


Series 1 Series 2

Fig 8.2 Results comparison

Series 1 ---- Experimental data

Series 2 ---- Computer data



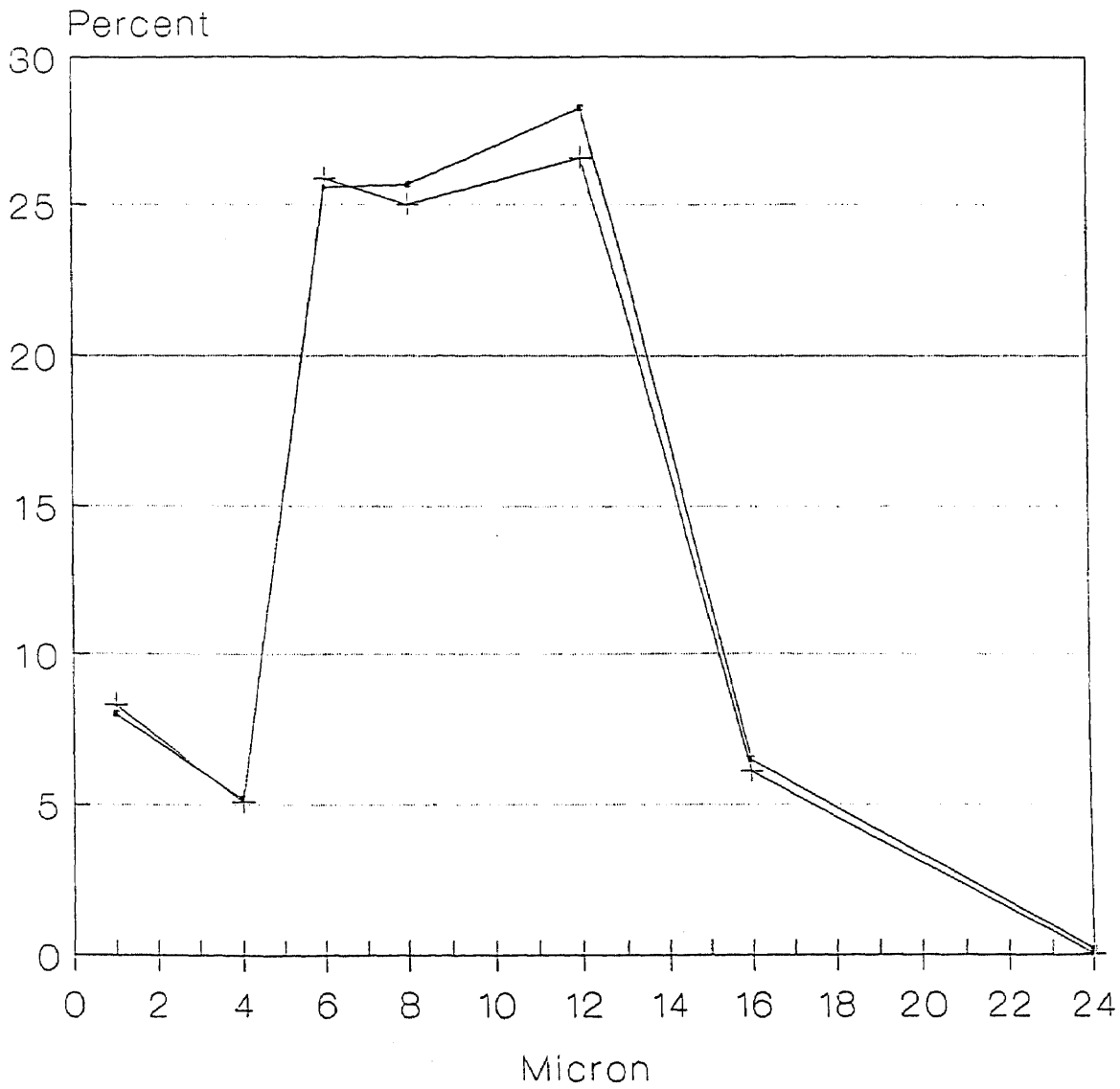
—•— Series 1 —+— Series 2

Fig 8.3 Results comparison

70

Series 1 ---- Experimental data

Series 2 ---- Computer data



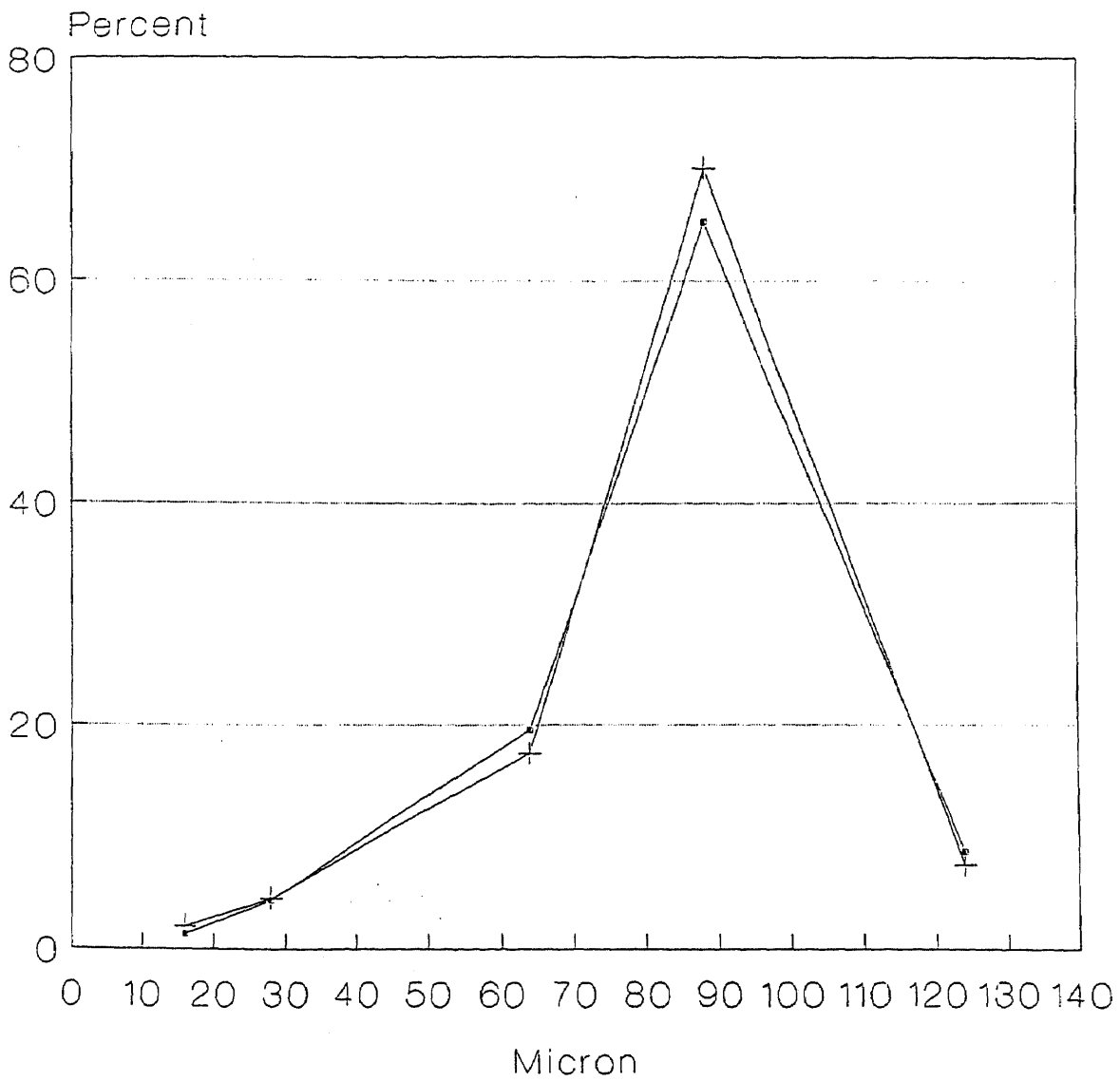
—•— Series 1 —+— Series 2

Fig 8.4 Results comparison

74

Series 1 ---- Experimental data

Series 2 ---- Computer data



Series 1 Series 2

CHAPTER IX
CONCLUSIONS

It is concluded in the work of the previous chapters that a computer simulation system for the size-reduction equipment fits almost perfect with the experimental data. Through the change of different variables in the system various equipment can be simulated. The particle size distribution of the final product can be predicted for different feedstock size through a variety of machines.

It is also concluded that the whole operation of various size-reduction requirement can be perfectly predicted through the theory presented here. The computer simulation for particle size distribution before and after the operation has shown great success. Almost all of the existing equipment can simulated through different variables in the program in the Appendix A.

The effect of each process effect can also be explicitly observed in the curves shown in Fig. 7.2-7.4.

CHAPTER X

FUTURE WORK

As stated in the abstract in the thesis, the final purpose is to establish a Computer Integrated Manufacturing system for the size-reduction operation. The work had already been finished here are simulations for feedstock and final product particle size distributions. To furnish the whole purpose work needed to be done is the simulation for the energy requirement for size reduction operation and the parallel or series operation systems.

Future focus should also be on the integration of the simulations. The optimization of the system should also be furnished to achieve the goal description in the abstract.

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Simulation Program

Program

```

Program simulation;
uses crt;
var
  diameter      : array [1..30] of real;
  percent       : array[1..30] of real;
  s             : array[1..30] of real;
  break        : array[1..30,1..30] of real;
  diffbreak    : array[1..30,1..30] of real;
  mass         : array [1..30,0..30] of real;
  out          : text;
Function power(base,expo:real):real;
var
  temp:real;
begin
  temp:=expo*ln(base);
  power:=exp(temp);
end;

-----
Procedure getdata;
var
  i:integer; infile:text;k:real;
begin
  for i:=1 to 30 do
    begin
      diameter[i]:=0.0; percent[i]:=0.0; s[i]:=0.0;
    end;
  assign(infile,'a:\onfo.txt');
  reset(infile);
  for i:=1 to 10 do
    begin
      read(infile,diameter[i]);
    end;
  readln(infile);
  for i:=1 to 10 do
    read(infile,percent[i]);
    for i:=1 to 10 do
      mass[i,0]:=percent[i];
    for i:=1 to 10 do
      begin
        k:=diameter[i]/diameter[1];
        s[i]:=0.013*(k*k*k);
        write(s[i]);
      end;
    end;
end;

```

```
Procedure calcul;  
var  
  m,n,k:integer;  
  {m,n,k only used for counter}  
  null:char;  
begin  
  
  null:=' '  
  for n:=1 to 30 do  
    begin  
      for m:=1 to 30 do  
        begin  
          break[n,m]:=0;  
          diffbreak[n,m]:=0;  
        end;  
      end;  
    end;  
  for n:=1 to 10 do  
    begin  
      m:=1;  
      while m<=10 do  
        begin  
          if m<n then break[m,n]:=0  
          else  
            begin  
              break[m,n]:=power((diameter[m]/diameter[n]),1.3);  
            end;  
          m:=m+1;  
        end;  
      end;  
    end;  
  n:=1; m:=2;  
  while (m>n) and (m<=10) do  
    begin  
      for k:=m to 10 do  
        diffbreak[k,n]:=break[k-1,n]-break[k,n];  
      n:=n+1;  
      m:=m+1;  
    end;  
    for k:=2 to 10 do  
      diffbreak[k,k]:=1;  
    clrscr;  
    for n:=1 to 10 do  
      begin  
        for m:=1 to 10 do  
          begin  
            write(diffbreak[m,n]:1:4, null:2);  
          end;  
          writeln;  
        end;  
      end;  
    end;  
end;
```

```

Procedure  calcul2;
var
  u,t,m,k : integer; sum:real; null:char;
begin
  assign(out,'a:\out.dat');
  null:=' ';
  sum:=0;
  for m:=1 to 10 do
  begin
    for t:=1 to 15 do
    begin
      sum:=0;
      for u:=1 to m-1 do
      sum:=sum+(mass[u,t-1]*s[u]*diffbreak[m,u]);
      mass[m,t]:=mass[m,t-1]*(1-s[m]*60)+60*sum;
    end;
  end;
  clrscr;
  rewrite(out);
  for t:=1 to 4 do
  begin
    for m:=1 to 10 do
      write(out,mass[m,t]:1:3,null:1);
      writeln(out);
    end;
    close(out);
  end;
end;

```

```

-----
begin { of main }
  getdata;
  calcul;
  calcul2;
end. { of main }
-----

```

Input data 1

```

24 16 12 8 6 4 3 2 1.5 1
0.116 0.141 0.251 0.156 0.114 0.024 0.021 0.039 0.024 0.109
-----

```

Output data 1

```

0.026 0.145 0.253 0.186 0.129 0.040 0.028 0.045 0.027 0.112
0.006 0.120 0.243 0.203 0.140 0.051 0.033 0.050 0.029 0.114
0.001 0.094 0.229 0.216 0.148 0.061 0.038 0.054 0.031 0.115
0.000 0.073 0.213 0.225 0.155 0.069 0.042 0.058 0.032 0.117

```