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ABSTRACT

Title of Thesis: Statistical Multiplexing of Video Sources
For Packet Switching Networks

Krishna Kancherla, Master of Science in Electrical Engineering, 1989

Thesis Directed by: Dr. Constantine Manikopoulos, Associate Professor

Communication networks are fast evolving towards truly integrated networks handling all types of traffic. They employ integrated switching technologies for voice, video and data. Statistical or asynchronous time division multiplexing of full motion video sources is an initial step towards packetized video networks. The main goal is to utilize the common communication channel efficiently, without losing quality at the receiver. This work discusses the concept of using statistical multiplexing for packet video communications. The topology of a single internal packet network to support ISDN services has been adopted. Simulations have been carried out to demonstrate the statistical smoothing effect of packetized video in the networks having high speed links. Results indicate that the channel rate per source decreased in an exponential manner as the number of sources increased. An expression for the average usage time $t$ of the channel has been derived in terms of channel rate per source and the number of sources multiplexed. Also the average usage time of the channel is higher for buffered data than that of the multiplexed data. The high speed communication links in the internal network are lightly loaded, which indicates that these links can accommodate more data.
STATISTICAL MULTIPLEXING OF VIDEO SOURCES FOR PACKET SWITCHING NETWORKS

by
Krishna Kancherla

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For Packet Switching Networks

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Chapter 1

Introduction

The increasing popularity of packet switching to handle data, voice and video integrated services has resulted in great concern for high network congestion levels and packet losses due to buffer delays. As video is expected to be a major traffic component on Integrated Services Digital Networks (ISDN's), it is very important to transmit image of constant quality with variable transmission rate. High capacities have been made possible by the increasing use of optical fiber links and the development of highly sophisticated VLSI communication devices. But as more bandwidth becomes available due to the use of optical fiber links, more services are designed to use them up. Besides loading the communication lines, switched broad-band services impose large loads on the switches; this is of great concern, if the switching in networks of future will use an integrated flexible technology such as fast packet switching for voice, video and data alike. The high traffic congestion is expected in the context of a network serving a large number
This work discusses the concept of using statistical multiplexing for packet switching networks to support ISDN services, particularly for video sources. Simulation studies has been done to study the network performance under increasing load conditions. Also the transmission efficiency of the high speed links has been studied as the number of sources increases.

Packet switching of variable bit-rate real-time video sources is a means for the efficient sharing of communication sources, while maintaining uniform picture quality. Variable bit-rate video coding exhibits the type of statistical variations that made packet switching attractive for bursty traffic. Advantages of packet switching over conventional circuit switching, and the transmission efficiency of packet switching due to statistical multiplexing for video communication are explained in detail in chapter II. Also a network architecture that support integrated services using high speed transmission links and high performance packet switches has been described.

Chapter III explains two performance models of statistical multiplexing in packet video communications. The coding bit rate of a single video source as a function of time based on experimental data [1] of a video telephone. Two correlated Markov process models (one in discrete time and one in continuous time) are explained to fit the experimental data that are used to model the input rates of several independent video sources into a statistical multiplexer.

The model of variable bit rate sources derived from chapter III are used for
our simulation purposes. The simulation model, the bit rate data and the network under consideration are explained in detail in chapter IV, and finally the results are presented. Our results show that as the number of sources increase from 1 to 100 at the statistical multiplexer, the channel rate per source decreased exponentially from 7.5 Mbits per second to 1.9 Mbits per second, while maintaining the channel utilization at 85%. An approximate expression for the average usage time \( t \) has been derived in terms of channel rate per source \( \lambda(n) \), and the number of sources multiplexed \( n \). Also it has been noticed that the average usage time decreased as the number of sources multiplexed increased.

For the same data rate we have to use a buffer of size 2Mbits to get the equivalent effect to statistical multiplexing. The buffering effect has been found to be negligible as the number of sources increase beyond 50. The average usage time of the channel is higher for the buffered data than that of the multiplexed data. The links and the packet switches of the internal packet network are lightly loaded even for large number of sources, indicating that the performance is good.
Chapter 2

Packet Switching Networks

2.1 Introduction

Packet switching technology has achieved a great success since it was introduced in the 1970's mainly for bursty computer communications. Packet switching is a technique widely used today in specialized data communication networks with a growing number of actual networks working in the world today. Packet switching networks aim at offering data transmission services for multiple applications. The integration of different types of information over such media becomes attractive for large scale economy and for better utilization of these powerful resources. The popularity of packet switching integrated services for an increasing number of applications is cause for concern in terms of offering good performance. Traffic congestion has been a real problem in achieving good network performance.

Data from all users are statistically multiplexed on a shared network resources; the moment-to-moment bandwidth requirements of individual users often vary dramatically. Today's networks are designed for the applications of modest traffic
level fluctuations. But whenever it exceeds the limit, the network traffic congestion level rises resulting in long transmission delays, causing heavy data losses and even sometimes blocking up the network altogether.

In the mean time, more and more new services are being introduced into packet switching networks, such as packet voice, image data transmission and teleconferencing, etc. These services demand the network to provide truly guaranteed type-of-service (TOS) transmissions that should meet diverse and stringent performance requirements. Adequate network resources are a necessary condition for good performance, but the resources alone do not automatically ensure a satisfactory performance. The poor performance we see today is more due to lack of traffic control than due to resource limitations. Theoretically, a packet switching network can allow any single user an unlimited share of the network resources. This is not feasible in practice due to performance criteria. Even though more bandwidth is available due to the use of fibre optic links, these new services load the communication lines, resulting in high congestion levels. Some aspects of designing a new architecture for packet switching communication networks have been discussed in [4].

2.2 Attractive Features of Packet Switching

Besides all of the above mentioned drawbacks of packet switching technology, there are some factors that make it an attractive method of transmission for integrated services.
1. *Transmission Efficiency:*

Many data services are characterized by bursty communications patterns which make poor use of conventional circuit switched facilities. For example, interactive data users typically use only a few percent of the bandwidth available to them. Although it is less widely recognized, voice is also quite bursty. In the average telephone conversation, less than 40% of the available bandwidth is actually used. Packet switching can exploit this burstiness to allow many users to share the same transmission facility.

2. *Adaptability to new coding techniques:*

Packet switching also makes it attractive to consider new coding techniques for information transmission. Most information coding techniques in use today attempt to maintain a smooth flow of information across the transmission facility in order to use it efficiently. With packet switching one can exploit coding techniques which produce bursty information streams.

3. *Adaptability to changing traffic:*

Packet switching naturally provides each user with exactly the bandwidth required. As new services are developed with different bandwidth requirements, packet switching can adapt to the changing conditions much more easily than conventional circuit switching can.

4. *Integrated internal architecture:*
Most proposals for ISDN services provide integrated access but require separate switching networks for different types of information. Packet switching can provide both an integrated customer interface and a single network solution for a wide range of different information transport needs, including voice, data and video signals. Substantial cost savings could be made possible both in switching and terminal equipment. The details of such an architecture [2] are given in section 2.4.

### 2.3 Statistical Multiplexing

#### 2.3.1 Introduction:

The transmission efficiency of packet switching is due to statistical multiplexing. The packets of all traffic streams are merged into a single queue and transmitted on a *first-come first-serve* basis in sequence, one packet at a time. However, if the queue of a traffic stream is empty, the next traffic stream is served and no communication resource is wasted. Since the entire transmission capacity $C$ (bits/second) is allocated to a single packet at a time, it takes $L/C$ seconds to transmit a packet that is $L$ bits long.

In *time-division (TDM)* and *frequency-division multiplexing (FDM)* with $m$ traffic streams, the link capacity is essentially subdivided into $m$ portions, one per traffic stream. In *FDM*, the channel bandwidth $W$ is subdivided into $m$ channels each with bandwidth $W/m$. The transmission capacity of each channel is roughly $C/m$, where $C$ is the capacity that would be obtained if the entire bandwidth were
allocated to a single channel. The transmission time of a packet that is $L$ bits long is $Lm/C$, or $m$ times longer than in the corresponding statistical multiplexing scheme. In TDM, allocation is done by dividing the time axis into slots of fixed length (e.g., one bit or one byte long, or perhaps one packet long for fixed length packets). Again, conceptually, the communication link can be viewed as having $m$ separate links with capacity $C/m$. In the case where the slots are short relative to packet length, the transmission time of a packet $L$ bits long is $Lm/C$. In the case where the slots are of packet length, the transmission time of an $L$ bit packet is $L/C$, but there is a wait of $(m-1)$ packet transmission times between packets of the same stream. One of the themes that will emerge from queueing analysis is that the statistical multiplexing has a smaller average delay per packet than either TDM or FDM. The main reason for the poor delay performance of TDM and FDM is that the communication resources are wasted when allocated to a traffic stream with a momentarily empty queue, while other traffic streams have packets waiting in their queue.

In a conventional circuit switched network, each user is provided with a communication channel that is dedicated to that user. If the user does not make use of the available bandwidth, it is lost; there is no opportunity for making that bandwidth available to another user. Statistical multiplexing provides a way for customers to share transmission facilities on demand basis. This is shown in Figure 2.1. User information enters at the left, where it is broken into blocks with a header added. This yields a packet. Packets from different sources are then
Fig 2.1 Statistical Multiplexing
placed in a queue and sent across the transmission facility. Upon receipt, packets are sent to proper destinations and the original data streams are reconstructed. Packet switching extends this idea to include the switching function, which leads to further cost savings.

In conventional circuit switched networks, the principal source of delay is the finite propagation speed of signals. Packet switching introduces a variable delay due to the queueing at each outgoing transmission link. The average delay \([7]\) can be approximated by

$$q = \frac{bn}{s(1 - \rho)}$$

where \(b\) is the packet length, \(n\) is the number of tandem links, \(s\) is the speed of the transmission facility in bits per second, \(\rho\) is the average utilization of the facility and \(q\) is the queueing delay in seconds. The delay can be decreased by decreasing the packet length, decreasing the number of links, increasing the speed of the transmission facility or decreasing the occupancy of the transmission facility. Decreasing the packet length is helpful up to a point, but can lead to inefficient use of the transmission link since as packets become shorter, a larger percentage of the available bandwidth is used to transmit the header information rather than the original data. Decreasing the link occupancy sacrifices the efficiency of statistical multiplexing for speed. The number of links is a function of the size of the network and the way that the packet switches are interconnected. The remaining variable in the equation is the transmission speed \(s\), which can be varied over a wide range.
Figure 2.2. Transmission Speed Vs. Delay

5000 mile circuit
1000 bit packets
10 links
90% occupancy
Reproducing the curve reported in [2] (fig 2, p 2.1.2), Figure 2.2 shows how the total delay varies with transmission speed for a ten link connection spanning a distance of 5000 miles. The minimum delay shown in the figure is due to fixed transmission speeds that are present even in circuit switched networks.

2.3.2 Statistical multiplexing compared with TDM and FDM

Assume that $m$ statistically identical and independent Poisson packet streams each with an arrival rate of $\lambda/m$ packets/sec are to be transmitted over a communication line [5]. The average transmission time is $1/\mu$. If the streams are merged into a single Poisson stream, with rate $\lambda$, like statistical multiplexing, the average delay per packet is

$$T = \frac{1}{\mu - \lambda} \quad (2.2)$$

If, instead the transmission capacity is divided into $m$ equal portions, one per packet stream as in time- and frequency-division multiplexing, each portion behaves like an $M/M/1$ queue with arrival rate $\lambda/m$ and average service rate $\mu/m$. Therefore, the average delay per packet is

$$T = \frac{m}{\mu - \lambda} \quad (2.3)$$

i.e., $m$ times larger than for statistical multiplexing.

The preceding argument indicates that multiplexing a large number of traffic streams on separate channels in a transmission line performs very poorly in terms
of delay. The performance is even poorer if the capacity of the channels is not allocated in direct proportion to the arrival rates of the corresponding streams. This shows that statistical multiplexing performs well when compared to TDM and FDM.

2.3.3 Using one vs. multiple channels in statistical multiplexing

Consider a communication link serving $m$ independent Poisson traffic streams with rate $\lambda/m$ each. Suppose that link is divided into $m$ separate channels with one channel assigned to each traffic stream. However, if a traffic stream has no packet awaiting transmission, its corresponding channel is used to transmit a packet of another stream. The transmission times of packets on each of the channels are exponentially distributed with mean $1/\mu$. The system can be modeled by the same Markov chain as the $M/M/m$ queue. Let us compare the average delays per packet of this system, and an $M/M/1$ system with the same arrival rate $\lambda$ and service rate $m\mu$ (statistical multiplexing with one channel having $m$ times larger capacity). In the former case, the average delay per packet is given by

$$T = \frac{1}{\mu} + \frac{P_Q}{m\mu - \lambda} \quad (2.4)$$

where $P_Q$ is the probability that an arrival will find all servers busy and will be forced to wait in queue and is given by

$$P_Q = P(\text{Queueing}) = \frac{p_0(m\rho)^m}{m! (1 - \rho)} \quad (2.5)$$

This equation is known as the Erlang C formula and is in wide use in telephony.
While in the case of \( M/M/1 \) system, the average delay per packet is

\[
\hat{T} = \frac{1}{\mu} + \frac{\hat{P}_Q}{m\mu - \lambda}
\]  

(2.6)

where \( P_Q \) and \( \hat{P}_Q \) denote the queueing probability in each case. When \( \rho \ll 1 \) (lightly loaded system) we have \( P_Q \approx 0, \hat{P}_Q \approx 0 \)

and

\[
T/\hat{T} \approx m
\]  

(2.7)

When \( \rho \) is only slightly less than 1, we have \( P_Q \approx 1, 1/\mu \ll 1/(m\mu - \lambda) \) and

\[
T/\hat{T} \approx 1
\]  

(2.8)

Therefore, for a light load, statistical multiplexing with \( m \) channels produces a delay almost \( m \) times larger than the delay of statistical multiplexing with \( m \) channels combined in one (about the same as time- and frequency-division multiplexing). For a heavy load, the ratio of the delays is close to one.

2.4 Network Architecture

As we discussed in section 2.1, although higher transmission speeds can reduce queueing delay to acceptable levels, their exploitation will require a new generation of packet switching equipment. The improvements in transmission efficiency obtainable with packet switching are dependent on the characteristics of the particular application. Packet switching is best suited to the applications that are
Figure 2.3 Basic Process of Packet Data
bursty, that is, applications that have a peak bandwidth requirement that is substantially higher than their average bandwidth requirement. Typical example is real time video signals.

Figure 2.3 illustrates some basic processes in packetization of a signal. The signal is first converted to digital form using an A/D converter. It is then coded using an appropriate coding scheme depending on cost, quality and bandwidth required. After the signal is coded, it is broken into blocks, a header is added, and it is sent through the network. At the destination the coding process is reversed, the signal is converted to analog and played back. Because the network introduces a variable delay, different packets within the same signal may experience different delays in crossing the network. Hence, care must be taken to ensure that this effect does not degrade the quality of the received signal.

Reviewing the motivation for constructing an integrated services network using packet switching, a network architecture was proposed in [2]. The basic components of this architecture are shown in Figure 2.4. They are Packet Switches (PS), Packet Network Interfaces (PNI), High Speed Links (HSL), Customer Interfaces (CI) and Network Administration Centers (NAC).

There may be different kinds of PNIs depending on the nature of the access method and services. Customers may communicate with the PNI using Digital Subscriber Lines (DSL) having a 64 Kbs voice channel and a separate packet mode signalling channel for voice sources. In this case the PNI would provide voice coding and packetizing, but the far end customer would be given a voice
Figure 2.4. Network Architecture
signal as reconstructed by the far end PNI. Signalling would be handled through messages exchanged with the CI. Another possibility is a pure Packet Line (PL) (used in this course of work for simulation purposes). In this case the coding and packetizing would be provided in the CI and the PNI would provide statistical multiplexing. A third possibility is a high speed packet mode line using a protocol similar to the internal network protocol.

The above described internal packet network architecture must have the following characteristics:

1. **High speed digital transmission:**
Modern digital transmission facilities offer higher speed and better error performance. The simplest way to solve the delay problem is to increase the basic transmission speed to the order of several Mbits depending on the data rate.

2. **Simple internal network protocols:**
The use of high speed digital transmission facilities creates opportunities for a greatly simplified packet protocols with in the network. Simple protocols may also be needed to ensure that fast and inexpensive protocol processors can be built.

3. **Hardware switching:**
Conventional packet switches rely on general purpose computers or collection of
microprocessors. Novel packet switch architectures based on VLSI components will be needed to attain high speed operation and economic implementation.

4. Separation of services from transport:
As the networks are expected to handle the frequently changing traffic, the internal network should provide basic transport without changing the parameters of itself. The above architecture accommodates different service needs by equipment at the access points to the internal packet network, and by special service nodes attached to but separate from this network.

2.5 Conclusions

This chapter can be summarized as
1. Several attractive features of packet switching technology were discussed.
2. Statistical multiplexing was described in detail in comparison with TDM and FDM.
3. The concept of using a single internal packet network to support ISDN services was discussed.
Chapter 3

Packet Video Communications

3.1 Introduction

A PCM coded video signal sequence results in a large bit rate (hundreds of Mbits/sec). In a limited bandwidth communication networks (circuit switching) a few video sources can easily fill all line capacities. Advanced data compression techniques are employed to transmit coded full motion video within digital lines of lower capacity. There is a complicated tradeoff between the minimum achievable coding rate and distortion of the decoded images. To tailor variable rate codes into a channel of lower speed than the coding rate at very high activity, multimode encoders equipped with buffers are used. The buffer absorbs statistical peaks of the coding rate by temporarily storing data in excess a certain threshold, the encoder is instructed to switch into a coding mode that has lower rate but worse quality to avoid buffer over flow. Similarly, when buffer underflow is approached, the channel speed is maintained by increasing the bit rate, thus improving the quality. The efficiency of the multimode scheme increases with lower fixed speed
on the output line; unfortunately the quality of the received image will start suffering from highly visible variations.

Variable bit rate video coding exhibits the type of statistical variations that made statistical multiplexing and packet switching attractive for bursty communications. These network architectures can dynamically support variable bit rate sources by smoothing out the aggregate of several independent streams in common buffers within the network. Hopefully, such technologies may provide efficient video transmission without the unpleasant variations in quality of multimode coders.

As explained in previous chapter, in statistical or asynchronous multiplexing, several independent sources share a line of capacity less than the sum of their peak rates. Instead of using individual buffers all sources feed a common buffer and their cumulative bit rate tends to smooth out around the average rate as indicated by the law of large numbers. We can see that packet switching is a network extension of statistical multiplexing, with data from individual sources segmented into small packets that are stored and forwarded from switch to switch towards their destination. The statistical smoothing of variable rate sources is achieved as packets are buffered in the network switches. But again, both statistical multiplexing and packet switching introduce variable delays in delivering data due to the buffering stages. In addition, they may introduce data losses due to buffer overflow. Thus, it is very important to select parameters and line speeds
to minimize these effects while maintaining the efficiency of statistical averaging.

Statistical Multiplexing of video was proposed and simulated at Bell Laboratories in 1972 for picturephone interframe coders [3]. Applications that require timely and synchronous delivery, such as telephone voice and real-time video, currently use dedicated circuit switching. This may change with new concept of fast packet switching [2], explained in previous chapter.

The asynchronous packetized transmission of conversational voice requires very stringent end-to-end delay; voice packets exceeding a given time threshold may not be used to synthesize the continuous voice stream at the receiver. Late packets thus result in voice losses that can be tolerated up to a certain degree. A similar problem is considered in full motion video in [1]. The complexity in assessing the buffering statistics of packetized voice results from the correlated nature of the stochastic process that models the rate of a voice source. A video source generates correlated bit rates but of a different statistical nature. The output of a digital voice phone alternates between talkspurts and silent periods. This corresponds to two states for the source output rate; the state corresponding to zero bit rate occurs 60 percent of the time. In contrast, the output rate of a variable bit rate video coder exhibits continuous variations. Hence new models and analytic techniques are developed in [1] to evaluate the queueing statistics in packetized video. In this course of work we have used the data they generated using conditional replenishment interframe coding scheme for our network simulation purposes. Hence a statistical analysis of the data and two markov
models that match their basic statistics are presented in the next sections. The first model (Model A) is a continuous-state autoregressive discrete-time Markov process which is used for simulation purposes in [1] and for ours too. The second model (Model B) is a discrete-state, continuous-time Markov process which is used in queueing analysis. The analytic model is used to analyze the queueing behavior of statistical multiplexing of several independent, identically distributed video sources. Numerical results are presented in Appendix 1.

3.2 Modeling and Analysis Assumptions

Video sources are assumed to be generating 30 frames/second. Each frame consists of approximately 250,000 pixels that are digitally coded. Assume that \( N \) independent video sources are multiplexed [3] into a high-speed trunk. The unbuffered coded bits from each source are first stored in separate prebuffers and then join a common multiplexer as in Fig. 3.1. The multiplexer assembles the data into blocks that are transmitted over the high speed communication line. The blocks may be asynchronous time frames that combine portions of data from each source. The frame length depends on the instantaneous amount of data from the sources. Frame delimiters and source identification information must be included to enable the demultiplexing at the destination. A block may also be a packet of data assembled from a single source. Packets are stored and forwarded in a FIFO (first-in-first-out) mode in the same order as they are assembled. The packet length may be variable due to temporal changes in the source bit rate, or fixed, in
Figure 3.1. Statistical Multiplexer of N Sources
which case the variable source rate will cause varying packet arrival rates. Each packet needs a header identifying the source, the destination, its sequence number, and possibly a time stamp to alert the network in case of excessive buffering delays. Error detection and correction overhead bits are added.

The statistical multiplexer was modeled as a queue that receives the encoded bit streams from the prebuffers; its service rate is determined by the speed of the high speed channel. The analysis is based on continuous fluid flow [7] approximation of the traffic that does not take into account the discrete nature of the packets or frames. The queue behaves like a reservoir of water that is fed from water supplies of time-varying rates and empties through a fixed-rate sink. The analysis agrees with simulation model, in which bit streams from individual sources are assembled into packets of fixed length.

In order to analyze the common buffer statistics, the coded source rate needs a model. The rate depends on the compression algorithm and the nature of the video scene. For a scene with out abrupt movement, such as the head of a talking person in a picturephone, it is expected that the rate will have a bell-shaped stationary probability density and to exhibit significant correlations for an interval of several frames. This behavior has been verified experimentally in [1].

The sequence had a duration of 10 \textit{seconds} or 300 frames. The instantaneous bit rate $\lambda(t)$ was measured in bits/pixel. Although the measured rate is fixed for the duration of a frame (1/30 sec), it was treated as a continuous-time function.
since the frame period is very small compared to the time scale. Recall that there are about 250,000 pixels per frame and 30 frames per second, thus 1 bit/pixel corresponds to 7.5 Mbits/sec. The measured bit rate over all 300 frames (10 sec) has a average rate $\mu = 0.52$ bits/pixel, standard deviation $\sigma = 0.23$ bits/pixel. The maximum value of the bit rate was 1.41 bits/pixel and the minimum 0.08 bits/pixel. Fig 3.2 shows a histogram of the values for the bit rate, which is an indication for normal probability density function. The autocovariance

$$C(\tau) = E[\lambda(t) \lambda(t + \tau)] - \mu^2$$

of the sequence was evaluated and is proved to fit the exponential form, which was recently verified by Verbiest [9] in experiments involving a variety of moving scenes. The best fit exponent of the autocovariance exponent was found to be $\alpha = 3.9 \, \text{s}^{-1}$.

As two models of the encoded bit rate of a video source are presented, in both cases the first- and second-order statistical (mean and variance) properties of the measured data, as well as some features of the steady-state distribution are matched. In particular, the models the same mean and variance as the experimental data. The steady-state distributions of the models are unimodal and bell-shaped, which reflects the nature of the experimental data shown in Fig. 3.2. The queueing behavior at the multiplexer is not very sensitive to the specific nature of the distribution as the results (Appendix 1) show. The results in [3] supports this present assumption of an exponential auto covariance.
Figure 3.2. Bit rate histogram
3.3 Source Models

3.3.1 Model A: Continuous-State, Discrete-Time Markov Model

In Model A the coder rate was modeled as a continuous state, discrete-time stochastic process. Let $\lambda(n)$ represent the bit rate of single source during the $n$th frame. A first-order autoregressive Markov process $\lambda(n)$ is generated by the recursive relation

$$\lambda(n) = a\lambda(n-1) + bw(n)$$  \hspace{1cm} (3.1)

where $w(n)$ is a sequence of independent Gaussian random variables and $a$ and $b$ are constants. Assume that $w(n)$ has mean $\eta$ and variance 1. Further, assume that $a < 1$; thus, the process achieves steady-state average $E(\lambda)$ and discrete autocovariance $C(n)$ are given by [8]

$$E(\lambda) = \frac{b}{(1-a)}\eta$$  \hspace{1cm} (3.2)

$$C(n) = \frac{b^2}{1-a^2}a^n \hspace{1cm} n \geq 0$$  \hspace{1cm} (3.3)

The autocovariance is exponential and can fit the experimental data. The steady-state distribution of $\lambda$ is Gaussian with mean $E(\lambda)$ and variance $C(0)$.  

28
The histogram in Fig. 3.2 shows a bell-shaped density, truncated to zero. Assuming that the negative tail of the density of $\lambda$ is very small, there is a reasonable matching of experimental data and the autoregressive model. From the measured data

$$E(\lambda) = 0.52 \text{ bits/pixel}$$

$$C(n) \simeq 0.0536 \times (e^{-0.13})^n \quad (\text{bits/pixel})^2$$

The discrete autocovariance $C(n)$ is obtained from the experimental fit $\hat{C}(\tau) = 0.0536 \times e^{-3.9\tau}$ by sampling at $n/\tau = 30$ frames/sec. Matching equations (3.2) and (3.3) with the measured data,

$$a \simeq 0.8781, \quad b \simeq 0.1108, \quad n \simeq 0.572 \quad (3.4)$$

It is believed that the continuous-state autoregressive model provides a rather accurate approximation of the bit rate.

### 3.3.2 Queueing Simulation Using Model A

The model in eq.(3.1) with the parameters in eq.(3.4) was used to generate the bit rates of each source in queueing simulation experiments. $N$ identical sources generate independent bit streams with rate $\lambda(n)$ for the duration of their $n$th frame. It is assumed that the sources need not be synchronized in their frame sequences. Thus, the first frame occurrence of the $N$ sources is randomized over...
the interval of a frame. Once initialized, the sources keep their individual frame synchronizations and their rate is generated according to the autoregressive formula (3.1). Bits generated from a single source over a period join a prebuffer as in Fig 3.1. At the end of the frame, the bits in the prebuffer are packetized into fixed length packets that proceed into the common queue. Leftover bits that could not fill a full packet remain in the prebuffer, and are packetized within the next frame period. Packets in the multiplexer are served in a FIFO order. Packet arrivals are assumed to be poisson. Statistics are collected and the results are explained in Appendix 1.

3.3.3 Model B: Discrete-State, Continuous-Time Markov Process

In Model B the bit rate is quantized into finite discrete levels. Transitions between the levels are assumed to occur with exponential transition rates that may depend on the current level. Thus, unlike in Model A, the bit rate was approximated by a continuous-time process $\lambda(t)$ with discrete jumps at random poisson times. The rate of these poisson times and the probability of the jump size may change depending on the level of the bit rate. Model B is obtained from the continuous-state bit rate by sampling the at random poisson time instances and quantizing the state at these points as shown in Fig 3.3. The approximation in Model B is improved by decreasing the quantization step and increasing the sampling rate.

Model B is a discrete finite-state, continuous-time Markov process. Its state
Poisson sampling and quantization of the source rate.

Figure 3.3.
space is the set of the quantized levels up to a maximum level. The quantization step, the number of states, and the transition rates are tuned to fit the average variance and auto covariance function of the measured data as before. Model B is used to analyze the statistical multiplexer as a continuous-state queue that is filled from $N$ variable rate sources each with rate $\lambda(t)$. Thus, the aggregate input source rate will be $\lambda_N(t)$, instead of $\lambda(t)$. The total rate is the sum of $N$ independent random processes each with mean $E(\lambda)$ and autocovariance $C(\tau) \simeq C(0)e^{-\alpha \tau}$ at steady state. The steady state mean and autocovariance of $\lambda_N(t)$ will then be

$$E(\lambda_N) = N \times E(\lambda) \simeq 0.52 \times N \text{bits/pxel} \quad (3.5)$$

$$C_N(\tau) = N \times C(0) \times e^{-\alpha \tau} \simeq 0.0536 N \times e^{-3.9\tau} \quad (3.6)$$

There are infinite number of choices of Markov models that can fit the parameters in (3.5) and (3.6). It was concluded that a birth-death Markov model will accurately describe the aggregate source bit rate (a birth-death process allows only transitions between neighboring states or quantization levels [8]). The tendency of the bit rate toward higher levels to decrease at high levels, and inversely, the tendency of the bit rate towards lower levels to increase at high levels was expected. This results in a normal stationary distribution of the state as in Fig 3.2. A simple birth-death process that exhibits this behavior, and has exponential autocovariance, is given by the state transition diagram in Fig 3.4. The state $\lambda_N(t)$ of the process in Fig. 3.4 represents the quantized level of the aggregate bit rate of $N$
State transition diagram—Model $B$. 
sources. An uniform quantization step $A$ bits/pixel, and $M+1$ possible levels, $(0, A, \ldots, MA)$ were assumed. The exponential transition rates $r_{i,j}$ from state $iA$ to state $jA$ are given by

$$
\begin{align*}
    r_{i,i+1} &= (M - i)\alpha & i < M \\
    r_{i,i-1} &= i\beta & i > 0 \\
    r_{i,i} &= 0 \\
    r_{i,j} &= 0 & |i-j| > 1
\end{align*}
$$

(3.7)

It can be shown [7] that $\lambda_N(t)$ at steady state will have a binomial distribution with mean $E(\lambda_N)$, variance $C_N(0)$, and exponential autocovariance $\tilde{C}_N$

$$
P[\lambda_N(t) = kA] = M e_k \ p^k(1 - p)^{M-k}
$$

where $p = \frac{\alpha}{\alpha + \beta}$

$$
E(\lambda_N) = MAp
$$

(3.9)

$$
\tilde{C}_N(0) = MA^2 p(1 - p)
$$

(3.10)

$$
\tilde{C}_N(\tau) = C_N(0)e^{-(\alpha + \beta)\tau}
$$

(3.11)
The parameters of the model $M$, $A$, $\alpha$ and $\beta$ are obtained by matching equations (3.8)-(3.11) with measured values in (3.5) and (3.6). With the number of multiplexed levels $M$ as a parameter, and for a given number of multiplexed sources $N$, they are

\[
\beta = a \left(1 + \frac{N \times E^2(\lambda_N)}{M \times C_N(0)} \right) \\
= 3.9 \left(1 + \frac{0.04458N}{M} \right) \\
\alpha = a - \beta = 3.9 - \beta \quad (3.12)
\]

\[
A = \frac{C_N(0)}{E(\lambda_N)} + \frac{E(\lambda_N)}{M} \\
= 0.1 + 0.52 \frac{N}{M} \quad (3.13)
\]

Interestingly, $\lim_{M \to \infty} = \frac{C_N(0)}{E(\lambda_N)} = 0.1$ bits/pixel. Thus for a finite number of sources $N$, the binomial model does not converge to the continuous-state Model $A$ by decreasing the step size. Nevertheless, it was shown that the analytic results using Model $B$ are in close agreement with simulations that use Model $A$ (see Appendix 1). In the binomial model, the rate $\lambda_N(t)$ can be assumed as the aggregate rate from $M$ independent minisources, each alternating between transmitting 0 bits/pixel (called the OFF state) and $A$ bits/pixel (the ON state) according to a Bernoulli distribution. As shown in Fig 3.5, a minisource turns ON with exponential rate $\alpha$ and OFF with rate $\beta$. The aggregate rate out of $M$
Figure 3.5. Minisource Model
M sources correspond exactly to quantizing the aggregate bit rate of $N$ video sources into $M$ levels.

### 3.3.4 Queueing Analysis Using Model B

The general continuous-time, discrete-state Markov process approximation of the aggregate input bit rate follows. The binomial source model is a special case. Let a continuous-state queue be fed by an input source with rate $\lambda_N(t)$ units of flow (bits/sec). The input flow rate can assume discrete levels (0, $A$, 2$A$, ..., $MA$). Let $r_{i,j}$ denote the exponential transition rate from level $i$ to level $j$. The queue empties with fixed rate $c$ units of flow per time unit. Let $q(t)$ denote the size of the queue. A complete description of the queueing system requires a two dimensional state $\{q(t), \lambda_N(t)\}$. The first component is a continuous variable, while the second is discrete, assuming finite values. The joint statistics of the state at a time instance $t$ can be described in terms of

$$P_i(t, x) = P\{\lambda_N(t) = iA, q(t) \leq x\} \quad (3.14)$$

The forward transition equations from time $t$ to $t + \Delta t$ are

$$P_i(t + \Delta t, x) = \sum_{j \neq i} r_{j,i} \Delta t P_j(t, x) + (1 - \Delta t \sum_{j \neq i} r_{i,j})$$

$$\cdot P_i(t, x - (iA - c)\Delta t) + O(\Delta t^2) \quad (3.15)$$
As $\Delta t \to 0$, and ignoring second-order terms $\Delta t^2$, the evolution of the process is governed by the system of linear differential equations,

$$\frac{\partial P_i(t, x)}{\partial t} + (iA - c)\frac{\partial P_i(t, x)}{\partial x} = \sum_{j \neq i} r_{j,i} P_j(t, x) - P_i(t, x) \sum_{j \neq i} r_{i,j}, \quad 0 \leq i \leq M \quad (3.16)$$

If the utilization $\rho$ is less than 1

$$\rho = \frac{E(\lambda_N)}{c} < 1$$

the process achieves steady state with limiting distribution $\lim_{t \to \infty} P_i(t, x) = F_i(x)$. Then, the system is described by a system of ordinary differential equations,

$$(iA - c)\frac{dF_i(x)}{dx} = \sum_{j \neq i} r_{j,i} F_j(x) - F_i(x) \sum_{j \neq i} r_{i,j}, \quad 0 \leq i \leq M$$

$$F_i(x) = 0 \quad x < 0$$

$$F_i(0) = 0 \quad \text{for } iA > c \quad (3.17)$$

The initial conditions follow from the observation that the buffer cannot be empty if the instantaneous rate $iA$ is larger than the service rate $c$. An additional condition is obtained as $x \to \infty$

$$F_i(\infty) = P(\lambda_N(t) = iA)$$

The set of equations (3.17) can be written in matrix form by using the vector

$$\mathbf{F}(x) = (F_0(x), ..., F_M(x))^T$$
\[ \mathbf{D} \dot{\mathbf{F}}(x) = \mathbf{R} \mathbf{F}(x) \] 

(3.18)

where \( \mathbf{D} \) is a diagonal matrix with elements \((iA - c)\) and \( \mathbf{R} \) is the rate transition matrix from (3.17). Assuming that the multiplexer rate \( c \) is not equal to any of the input levels \( iA \), \( \mathbf{D} \) is nonsingular. Let \( \Phi_i \) and \( z_i \) denote the eigenvectors and eigenvalues of \( \mathbf{D}^{-1} \mathbf{R} \). The solution of (3.18) is given in terms of its known value at \( x = \infty \), the eigenvalues and corresponding eigenvectors of \( \mathbf{D}^{-1} \mathbf{R} \) as

\[ \mathbf{F}(x) = \mathbf{F}(\infty) + \sum_i k_i \Phi_i e^{z_i x} \] 

(3.19)

The sum in (3.19) is taken over all eigenvalues in the strict left half-plane for the solution to be a steady-state probability distribution. The constants \( k_i \) in the above equation are determined from the initial conditions

\[ \mathbf{F}_j(0) = 0 = \mathbf{F}_j(\infty) + \sum_i k_i \phi_{ij} \]

where \( \phi_{ij} \) denotes the \( j \)th element of \( \Phi_i \). The steady state distribution of the buffer length \( x \) is given by

\[ F(x) = P\{q(t) \leq x\} = \sum_{i=0}^{M} F_i(x) \] 

(3.20)

and the probability that the buffer size exceeds a certain length, referred to as survivor function, is \( \overline{F}(x) = 1 - F(x) \)

For the special case of the source model in Fig 3.4, the rates in (3.7) simplify (3.17) to
The matrix $R$ becomes a tridiagonal matrix. The initial conditions in (3.17) provide enough equations to solve for the constants $k_i$ in (3.19). Results indicate a close agreement between the two models, even though Model $A$ uses Gaussian distribution for source rates, whereas Model $B$ uses Binomial distribution. The results are presented in Appendix 1.

3.4 Conclusions

Several criteria to be considered while transmitting video packets using statistical multiplexing were discussed. Variable bit rate video coding can take advantage of dynamic bandwidth sharing via statistical multiplexing and packet switching. Two queueing models of variable bit rate video sources were discussed in detail. The probability of delaying video data beyond an acceptable limit drops dramatically as the number of multiplexed sources increases beyond one. Statistical or asynchronous time division multiplexing also efficiently absorb temporal variations of the bit rate of individual sources without the significant variations in reception quality.
Chapter 4
Simulation Model

In this chapter we will present our simulation model of the network and various other numerical parameters used for the high speed transmission links and finally results.

The data rates of sources are taken from [1], the experimental values of the source model explained in the previous chapter. We know that these values are generated using modified version of conditional replenishment scheme. Using the network architecture explained in chapter II and with source model from chapter III we have simulated a network shown in Fig 4.1.

4.1 System Characteristics

We assumed all sources 1, 2, ..., \( N \) are homogeneous generating the same data rates. Assume video sources are generating 30 frames per second. Each frame consists of approximately 250,000 pixels. Thus 1 bit per pixel corresponds
PNI: Packet Network Interface
HSL: High Speed Link
PS1: Packet Switch 1
PS2: Packet Switch 2
PS3: Packet Switch 3
PS4: Packet Switch 4

Figure 4.1. Simulation Model of Network
to 7.5 Mbits per second. $N$ independent video sources are multiplexed into a high speed channel (HSL). The statistical multiplexing is provided by Packet Network Interface. Packets are stored and forwarded on a first-come first-serve basis. The test sequence measured from [1] for 10 seconds will have 300 frames. The bit rate is normally distributed according to Fig 3.2, as follows

\[
\begin{align*}
\text{Mean } \mu &= 0.52 \text{ bits/pixel} \\
\text{Standard deviation } \sigma &= 0.23 \text{ bits/pixel} \\
\text{Lower bound } &= 0.08 \text{ bits/pixel} \\
\text{Upper bound } &= 1.40 \text{ bits/pixel}
\end{align*}
\]

Each source is having an input/output set-up time of 0.1 microsecond which it will take for adding header information and packetization. Transmission links are modeled for high speed transmission purposes. Each packet 1024 bits long, including 1000 bits of original data and 24 bits of overhead. The links operate on first come first serve protocol. Since the packet length is fixed, the variable source rates will cause varying packet arrival rates. Hence Poisson arrival rates are assumed.

The statistical smoothing of the variable bit rate sources is achieved when the data from several sources is averaged together in PNI prior to the transmission on to the channel and subsequently to the network. As the packets arrive at the packet switch 1 (PS1), then are placed on transmission links link12 & link13 each sharing 50% of the data from PS1. The packets are stored and forwarded from
switch to switch on the network towards their destination, where they achieve a little more statistical averaging. Hence we will notice the links are moderately loaded even when 100 sources are generating data simultaneously.

4.2 Simulation Results

Simulations are carried out using *NETWORK II.5* simulation package [10], [11]. Fig 4.2 shows the percentage utilization of the high speed channel in 1000 microseconds which is around 85%. This utilization is maintained constant, and as the number of sources increased the channel rate was increased accordingly. Hence we have,

\[
\text{Channel rate/source} = \frac{\text{Aggregate channel rate}}{\text{Number of sources}}
\]

Thus, Fig 4.3a shows the number of sources versus channel rate per source. It shows that the channel rate per source \(\lambda(n)\) decreases exponentially from 7.5 Mbits per second to 1.9 Mbits per second as the sources increase from 1 to 100. This effect demonstrates the statistical averaging of variable bit sources. To verify the exponential fit of \(\lambda(n)\), we have approximated \(\log \lambda(n)\) as a straight line shown in Fig 4.3b. We have

\[
\log \lambda(n) = 15.77 - 0.0141n \tag{4.1}
\]

\[
\log \lambda(n) - 15.77 = -0.0141n \tag{4.2}
\]

\[
\log \lambda(n) - \log(7.060 \times 10^6) = -0.0141n \tag{4.3}
\]
Figure 4.2. % Utilization of High Speed Channel
Considering \( a = 7.060 \times 10^6 \), we have

\[
\log \frac{\lambda(n)}{a} = -0.0141n \tag{4.4}
\]

\[
\frac{\lambda(n)}{a} = e^{-0.014n} \tag{4.5}
\]

\[
\lambda(n) = ae^{-0.0141n} \tag{4.6}
\]

Therefore,

\[
\lambda(n) = ae^{-\alpha n} \tag{4.7}
\]

where \( a = 7.060 \times 10^6 \) and \( \alpha = 0.0141 \).

Fig 4.3c shows the channel rate per source derived from above equation versus the number of sources multiplexed.

Fig.4.4a shows the number of sources versus the average usage time of the high speed channel. We noticed that the average usage time, \( t \) decreased as the number of sources increased. We have observed that the function \( f = \lambda(n)e^{-t} \) is also of exponential form. To verify this, we have Fig 4.4b, which shows that \( \log \lambda(n)e^{-t} \) versus number of sources is approximated to a straight line. Hence we have,

\[
\log \{\lambda(n)e^{-t}\} = mn + b \tag{4.8}
\]

\[
\log \{\lambda(n)\} = t + mn + b \tag{4.9}
\]

\[
\lambda(n) = e^t \times e^{(mn+b)} \tag{4.10}
\]

Therefore,

\[
e^t = \frac{\lambda(n)}{e^{(mn+b)}} \tag{4.11}
\]
Ave. Usage Time of HSL Vs. # of Sources

Figure 4.4a

No. of Sources

Ave. Usage Time of HSL in Micro sec
Log $\lambda (n) e^{-t}$ Vs. No. of Sources

Figure 4.4b
where $\lambda(n)$ is from equation (4.6), $m \approx 15.9$ and $b \approx -0.0143$. Hence we have an approximate expression for $t$ in terms of $n$ and $\lambda(n)$. The results indicate that the average usage time from simulations is approximately equal to that is derived from equation (4.11). However they are not exactly equal because of approximate values of $m$ and $b$.

For the same data rates we have to use a buffer of 2 Mbits for each source to get the same effect as that of the statistical multiplexing. Fig 4.5a & b shows that the effect on the channel rate with buffer and without buffer. The effect of multiplexing seems to be negligible as the sources increase beyond 50. Also the average usage time of the channel is higher for the buffered data than that of the multiplexed data. This is shown on Fig 4.6.

Coming to the internal packet network, the simulation results are summarized in Table 4.1, which shows the percentage utilizations of the links and Table 4.2 shows the percentage utilization of packet switches for different number of sources. The links are moderately loaded, indicating that they are capable of handling more data coming in from $PS2$, $PS3$ and $PS4$ as well. Also the average usage is decreased for higher number of sources, hence we can say that the performance is good. In Table 4.2 we see that $PS1$ is heavily utilized because the data from sources is entering the network through $PS1$, whereas the other switches are moderately utilized. This shows that the network still can handle the data coming in from
Channel rate/source Vs. No. of Sources

Figure 4.5b

Channel rate/source in Mbits/sec

No. of Sources

Multiplexed Data

3.9 Mbit Buffer
Ave. Usage Time of HSL Vs. # of Sources

Figure 4.6

Ave. Usage Time of HSL in Micro sec

No. of Sources

Multiplexed Data

Buffered Data
Table 4.1 % Utilizations of links in the Network

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<th>link13</th>
<th>link24</th>
<th>link34</th>
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other packet switches.

4.3 Conclusions

The results presented in this work demonstrate that the statistically multiplexed video data can be transmitted very efficiently on a packet switched network having high speed transmission links and high performance packet switches. Also as the load on the network increases the channel rate decreases exponentially for multiplexed data. An approximate expression for the average usage time of the channel has been derived in terms of channel rate per source and the number of sources multiplexed. Also the average usage time of the channel is higher for the buffered data than that of the multiplexed data. The high speed links of the internal packet network are lightly loaded indicating that they can accommodate more data.
Variable bit rate video coding can take advantage of dynamic bandwidth sharing via \textit{statistical multiplexing} and \textit{packet switching}. In this course of work we have demonstrated the effect of statistical multiplexing on sharing the common communication channel. When video is packetized and many sources are present, then the buffering can be provided by the network itself by multiplexer. Due to statistical smoothing in the packet network interface, peak rates can be transmitted.

Simulations are done to show the effect of statistical smoothing of packetized video in the packet switching networks having high speed channels. Results indicate that the channel rate per source decreased in an exponential manner as the number of sources increased. An approximate expression for the average usage time of the channel has been derived in terms of the channel rate per source used and the number of sources multiplexed on the channel. For the same data rates
we used 2Mbit buffer to get the same effect as that obtained with statistical multiplexing. Also, the average usage time of the channel is higher for the buffered data than that of the multiplexed data.

This work also discusses the simulation results of the performance of an internal packet network, which encourages to convey large volumes of packetized video data offering good performance. One of the attractive features of this high speed packet network is that it makes it very easy to provide services without embedding them within the internal portions of the network. Services that are embedded in the internals of a communication network can be difficult to change because they effect all customers, often in unexpected ways. This creates obstacles that must be overcome when introducing new services or changing existing services. Separating the services from the network reduces the interactions among different services. This can reduce development costs which are becoming a major portion of the cost of communication services. Of all these advantages, packet switching introduces variable packet losses, both due to transmission errors and variable delays.

These simulation results have been obtained by utilizing a particular coding scheme as well as data taken from[1], for moving image sequences. It would be quite interesting to study the transmission of still image data bases. All transmission links are considered to be working in unidirectional (simplex) manner. Further interesting work that can be done is having links in full or half duplex links.
Appendix 1

Numerical Results of Models

Model A is difficult to analyze but straightforward to simulate. Thus queueing dynamics of Model A is simulated, queueing analysis of Model was presented. In both cases, we are interested in the mean and variance of the queue size at the multiplexer, and the survivor function $F(x)$. The survivor function $F(x) = P\{q(t) \geq x\}$ at steady state is an appropriate figure of merit because it represents the fraction of data that join the multiplexer when its queue size exceeds a threshold $x$. For real time video, there is a maximum allowable queue size $x_0$; any packets that join the queue with more than $x_0$ data ahead of them will arrive at their destination too late for resynthesizing the transmitted image sequence. For this reason, the survivor function $F(x_0)$ can be viewed as loss probability. The authors have chosen the queue size $x$ in time units, $x=30$ ms denotes that it would require 30 ms to empty a queue of this size at the output speed of the multiplexer.

Fig. 1 shows the relative insensitivity of the survivor function $F(x)$ to packet length for the simulation results using Model A, when the queue size is measured in ms. In the simulation runs for Fig. 1, the nominal utilization was $\rho = 80\%$. However, due to the variation in the generated random sequences driving the runs, the actual utilization varied slightly. The simulation was carried out for the situation of one video source followed by a prebuffer feeding a packet queue.

Fig.2 and Fig.3 exhibit the close agreement between the survivor functions for the two models for the case of one and five video sources. The nominal utilization was 80% but the analysis with Model B was carried out for the actual utilization
observed in Model A. The agreement between two models is encouraging. Model A uses Gaussian distribution for source rates, while Model B uses a discrete binomial distribution. The agreement thus shows the insensitivity of the queueing behavior to the specific steady-state distribution used and supports the approach of matching the first and second order statistical properties. The difference in survivor function in Fig. 3 at near zero queue size level are due to the basic differences in the models. In Model A, the queue is a single server queue, while in Model B because of flow approximation the queue behaves like a infinite server queue.

The insensitivity of the survivor function to the number of quantization levels in the analysis using Model B is the subject of Fig. 4. The results presented are for one video source, but the trend carries over for more than one source. Figs. 5-7 show the results of applying the analysis to the multiplexing of more than one video source. Fig. 5 shows the dramatic reduction in the survivor function due to the multiplexing of several video sources, for a constant utilization of $\rho - 80\%$. Even as $N$ increases from 1 to 2 sources, the loss probabilities drop by an order of magnitude. Fig.6 shows the same for a constant buffer size and varying $\rho$.

The loss probabilities shown, demonstrate that statistical multiplexing of variable bit rate video coders will not exhibit perceptible quality variations in video reception. Thus, it is a viable networking alternative to the use of multimode video coders which maintain a synchronous transmission at the expense of variable reception quality.
Figure 1

Variation of loss probability with packet size Model A—simulations.

Figure 2

Comparisons of loss probabilities—simulations, analysis, Model A and Model B.
Figure 3

Comparisons of loss probabilities—five sources simulations (Model A) and analysis (Model B).

Figure 4

Sensitivity of loss probabilities to the number of video flow levels—Model B, analysis.
Figure 5

Variation of loss probabilities with the queue size parametrized by the number of sources—utilization 80 percent.

Figure 6

Variation of loss probabilities with utilization parametrized by the number of sources—queue size 30 ms.
BIBLIOGRAPHY


