Copyright Warning & Restrictions

The copyright law of the United States (Title 17, United States Code) governs the making of photocopies or other reproductions of copyrighted material.

Under certain conditions specified in the law, libraries and archives are authorized to furnish a photocopy or other reproduction. One of these specified conditions is that the photocopy or reproduction is not to be "used for any purpose other than private study, scholarship, or research." If a, user makes a request for, or later uses, a photocopy or reproduction for purposes in excess of "fair use" that user may be liable for copyright infringement,

This institution reserves the right to refuse to accept a copying order if, in its judgment, fulfillment of the order would involve violation of copyright law.

Please Note: The author retains the copyright while the New Jersey Institute of Technology reserves the right to distribute this thesis or dissertation

Printing note: If you do not wish to print this page, then select "Pages from: first page # to: last page #" on the print dialog screen

MATHEMATICAL MODEL OF A CONTINUOUS STERILIZATION SYSTEM FOR FERMENTATION MEDIA CONTAINING SUSPENDED SOLIDS

by

Mark A. Leskowicz

Thesis submitted to the Faculty of the Graduate School of the New Jersey Institute of Technology in partial fulfillment of the requirements for the degree of Master of Science in Chemical Engineering

1989

APPROVAL SHEET

Title of Thesis: Mathematical Model of a Continuous Sterilization System for Fermentation Media Containing Suspended Solids

Name of Candidate: Mark A. Leskowicz

Master of Science in Chemical Engineering, 1989

Thesis and Abstract Approved: _____ _____

Dr. Piero M. Armenante Date

Assistant Professor

Chemistry and Chemical Engineering Department

Signatures of other members of the thesis committee

_____ _____

Dr. Gordon A. Lewandowski Date

Professor

Chemistry and Chemical Engineering Department

_____ _____

Dr. Dana E. Knox Date

Assistant Professor

Chemistry and Chemical Engineering Department

VITA

Name: Mark A. Leskowicz

Permanent address:

Degree and date to be conferred: Master of Science, 1989

Date of birth:

Place of birth:

Secondary education: Colonia, New Jersey 1979

Collegiate institutions attended	Dates	Degrees	Date of Degree
New Jersey Institute of Technology	1989	M.S,	1989
New Jersey Institute of Technology	1983	B.S.	1983

Major: Chemical Engineering

Minor: None

Publication: To be submitted for publication

Position held: Staff Development Engineer

Engelhard Corporation

Menlo Park CN28, Edison, New Jersey 08818

ABSTRACT

Title of Thesis: Mathematical Model of a Continuous Sterilization System for Fermentation Media Containing Suspended Solids

Mark A. Leskowicz, Master of Science, 1989

Thesis directed by: Dr. Piero M. Armenante
Assistant Professor of Chemistry and Chemical Engineering Department

A mathematical model has been developed for a continuous sterilization system where the role of suspended solids on the overall degree of sterilization achieved is taken into account. Evaluation of a "typical" commercial continuous sterilization system showed the role of suspended solids to have little effect on the overall sterilization obtained. For continuous sterilization systems operating in a total residence time of less than one minute, the role of solids may become important and should be evaluated.

DEDICATION

To my fiancee, Noreen

ACKNOWLEDGMENT

I would like to thank Dr. Piero M. Armenante for his great contribution in developing my confidence and for all his help. Thanks are also extended to Mr. Dinh Dang, Mr. Dave C. Bogert, Dr. Barry K. Speronello, and Mr. Larry B. Dight of Engelhard Corporation for the use of company facilities and for their encouragement.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION

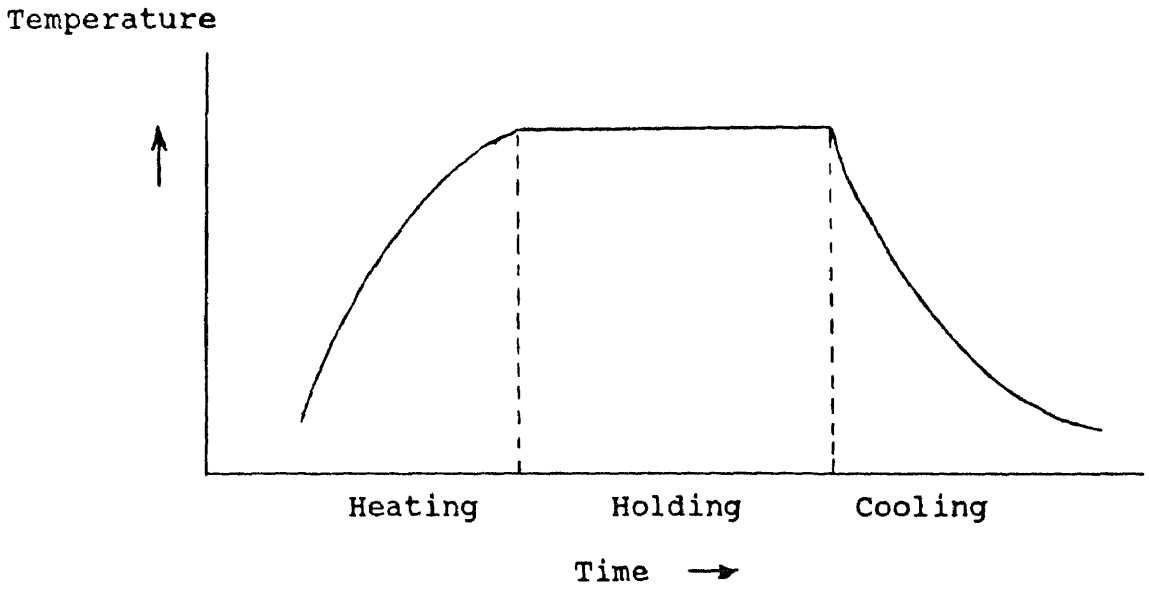
Sterilization is the process of eliminating undesirable contaminating organisms which would otherwise interfere with a subsequent process or lead to the devaluation of a product. For example, in biological processes, contaminating organisms might interfere with efficient fermentation, and in the food and canning industries, active organisms present in foods lead to spoilage and possibly illness or death of the consumer. Sterilization of liquids can be accomplished by several means, including radiation, sonication, filtration, heating, and chemical addition.[1] Only the last two are used in large scale processes. Heat treatment processing, or thermal sterilization, is probably the most widely practiced method of sterilization, and is used extensively in the food, drug, and fermentation industries.

The requirements for destruction of viable microbes and viruses vary widely depending upon the material and its intended use. In waste water treatment, for example, the required microorganisms are naturally present in the process fluid so that little or no sterilization is required.[1] In alcohol production, inhibitors for growth of unwanted organisms are rapidly evolved, so that again sterilization requirements are not extreme. Milk pasteurization kills most but not all actively growing microbes because more severe treatment leads to excessive degradation of desired components. The canning industry, however, requires extremely severe sterilization levels. Here, extinction is required because a

single surviving spore could cause lethal contamination. The typical design criterion is that the spore survival probability, $(1-P_0)$, be reduced to less than 10^{-12}. Physically, this can be interpreted to mean that only one batch in 10^{12} will fail to be contaminant free.[1]

The commonly practiced method of heat sterilization in the fermentation industry is that of batch sterilization. In the batch process, fermenter feed is charged to the fermenter reactor and the temperature is raised to the desired temperature for sterilization, held there for a specified amount of time for the degree of sterilization required, and then cooled to near ambient temperatures to allow proper fermentation to take place with desired, added cultures. The total degree of sterilization can be viewed as the sum of the sterilization achieved in each of the three periods; the heating, holding, and cooling periods (Figure 1).

Figure 1 - Schematic Diagram of Batch Sterilizer Temperature-Time Profile



There are several disadvantages to batch sterilization. It is a fairly labor intensive process. It is generally a fairly long process, and adds a considerable amount of time to the turn-around time of a fermenter. It also requires the fermenter to be designed with the ability to heat and cool (this usually means the vessel will be jacketed or have internal coils).

The batch process can also have a deleterious effect on desirable components in the feed. Many vitamins are destroyed by heating and proteins can be denatured at elevated temperatures.[1] The destruction of these elements generally follows the same first order kinetics as that of the death kinetics of the spores and microbes to be sterilized. However, the activation energies for these unwanted side reactions are much lower than those of the sterilization reaction. This means that at the lower temperatures

associated with batch sterilization (where prolonged heatup and cooldown of the feed and, generally, lower sterilization temperatures are employed, in comparison to continuous sterilization), the ratio of the unwanted side reactions to desired sterilization is higher than desired. A high temperature, short holding time condition (HTST), typically associated with continuous sterilization processes, would greatly reduce this ratio. Continuous sterilization is much better suited to achieving these conditions.

Continuous sterilization is, just as its name implies, the continuous sterilization of the fermenter feed as it is being charged to the fermenter vessel. In these systems, the temperature of the medium is raised and lowered quickly, with the holding time in between also being very short but at a much higher temperature than those considered in batch sterilization processes. For example, the degree of sterilization accomplished in a batch sterilizer in 30 minutes at 120^{o}C is reached in 8 seconds by a continuous sterilizer at 145^{o}C.

In general, continuous sterilizers rely either on direct steam injection to raise temperature and rapid expansion for fast cooling, or on heat exchangers with high surface to volume ratios to indirectly heat and cool the medium quickly. The drawbacks to continuous sterilization lie in these aspects of the process. The direct steam heating process adds excess water to the medium whereas the indirect heating process ulilizes heat exchangers that

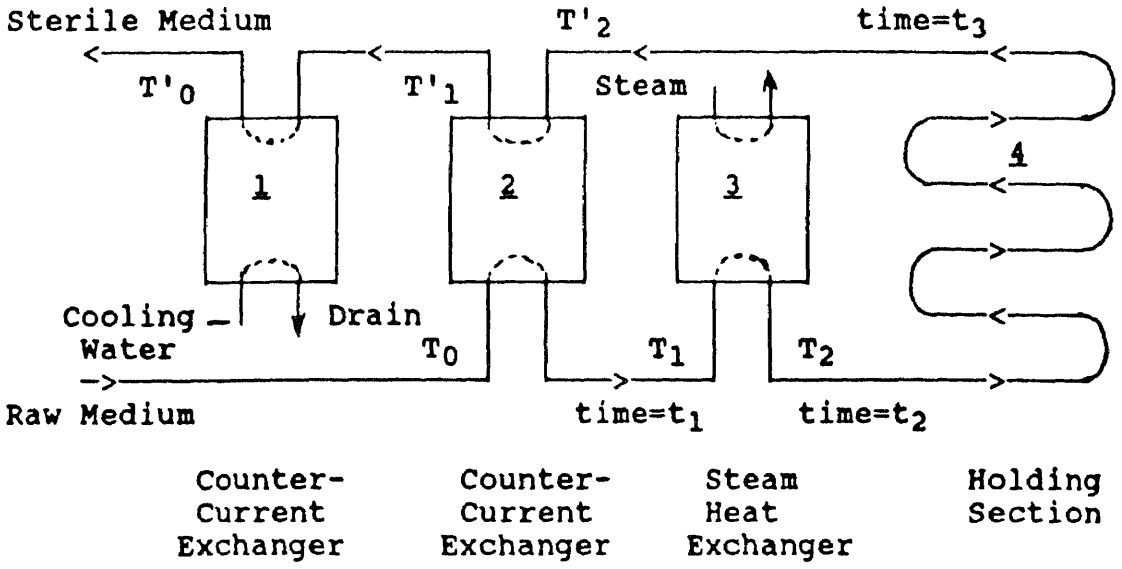
can be fouled by suspended solids, as well as by protein and sugar thermal degradation by-products adhering to the exchanger walls.

In addition, because of the very short residence time in a continuous sterilizer, small changes in performance conditions can have large effects on the degree of sterilization achieved. Some find it a prudent practice to ignore the contribution to the sterilization process provided by the heating and cooling sections of a continuous sterilizer and to consider only the holding section in the design of continuous sterilizers. Changes of only a few degrees in any of the sections of the sterilizer (especially the holding section, which operates at the maximum temperature) can result in differences in the remaining viable fraction of the organism of many orders of magnitude. Suspended solids, which will heat more slowly than the bulk of the fluid, may also cause the degree of sterilization to be inadequate, especially at the center of the solid. It is this aspect of the operation of continuous sterilization that will be addressed in this thesis.

2. OBJECTIVES OF THIS WORK

A schematic diagram of the continuous sterilizer considered in this work is shown in Figure 2. The system consists of a counter-current preheat exchanger, a steam heat exchanger, and an adiabatic holding section, and is typical of many industrial set-ups.

Figure 2 - Schematic Diagram of Continuous Sterilization System



The ultimate objective of this project is to develop a working mathematical model to predict the remaining viable fraction (N/N_0) of bacteria present in a nutrient stream after passing through a continuous sterilizer. A number of equations have been reported in the literature for this purpose. These equations are briefly reviewed below. However, none of the systems previously studied considers the influence on the continuous sterilization process of the suspended solid particles present in typical industrial

fermentation media. The problem originates from the fact that heat transfer through the particles is different than that in the bulk of the fluid, resulting in a different temperature profile with time inside the particle and, ultimately, in a different remaining viable fraction of bacteria.

In other terms, the solid particles may protect the bacteria lodged in them from the sterilization process, with potentially severe impact on the sterility of the subsequent operations.

This problem is more significant in continuous sterilization rather than batch processes since the available sterilization time in the former case is of the order of seconds, whereas in the latter case it is about 20-30 minutes.

It appears that some large-scale industrial continuous sterilization processes are over-designed precisely because of the (so far) unaccountable effect on sterility of the solid particles in the medium.

Therefore, the specific objective of this work is to develop design equations for continuous sterilization processes in which the presence of solid particles in the fermentation medium is accounted for in the design of the system.

3. APPROACH

The approach that will be taken in this work is to calculate temperature profiles for both the fluid and solid spheres as functions of time. Once these profiles are known, the remaining viable fraction of bacteria will be calculated by substituting these profiles into the temperature depended death rate equation (described in the following section).

The approach can be summarized as follows:

- Derivation of temperature-time profiles for fluid heated by
    - Electrical heating elements (constant flux)
    - Saturated Steam (constant external temperature)
    - Counter-Current heat exchange
    - Counter-Current (Pre-Heat) - exiting cooling fluid is used to preheat incoming fluid.
- Derivation of temperature-time profiles in suspended spherical solids
- Two cases considered:
    - Fluid profiles used as boundary conditions for surface
    - External heat transfer coefficients considered
- Profiles arranged in the order seen as fluid (spheres) move through the continuous exchanger system.
- Computer program calculates values for temperature profile of the particle at any time.

- Remaining viable fraction (N/N_0) calculated by numerically integrating the temperature profiles
    - Both fluid and solid particles evaluated

Equations describing the temperature profile of the nutrient fluid will be derived for four types of heat transfer: electrical heat, steam heat, counter-current fluid exchange (general), and counter-current exchange (pre-heat) where the heating fluid is of the same mass and physical properties of the cooling fluid.

Next comes the derivation of the equations describing the temperature profile in the solid spherical particle suspended in the fluids described above with an infinite external heat transfer coefficient (the surface temperature of the sphere is equal to the bulk fluid temperature). One additional case, the simplest case of the particle in a constant temperature bath, will also be derived. The derivation of these equations will provide the background for the consideration of the more complicated case of an external heat transfer coefficient limiting heat transfer into the particles. All of the equations will be rederived considering a film heat transfer coefficient.

With these equations in place, one analytical equation describing the temperature of the fluid through the sterilizer and one equation describing the temperature profile of a particle (as a

function of radius) as it travels with the fluid through the sterilizer will be written.

A computer program to calculate values for the temperature of the fluid and temperature profile of the particle is then enlisted to obtain numerical results. The program will be specific to the sterilizer considered in the respect that a counter-current exchanger is followed by a steam heat exchanger which is in turn followed by a holding section. However, any variables relating to the physical dimensions and properties of the fluid and solid particles can be altered to demonstrate differences between materials/exchangers.

Once the temperature profiles of the fluid and the particles are known, the computer model will be extended to calculate N/N_0 for the fluid and for the solid particle, which will have an average N/N_0 based on actual radial values integrated over the volume of the particle. This is the ultimate objective of the project. With a model to predict the remaining fractions of bacteria in both a fluid and particles suspended in that fluid, design specifications for heat exchanger dimensions including residence times and outlet temperatures can be predicting with some degree of accuracy. As an example, the design of an operational continuous sterilizer system is critically evaluated, and conclusions are drawn with regard to its performance.

4. PREVIOUS AND RELATED WORK

As already mentioned above, previous work on continuous sterilizers has been focused only on sterilization of homogeneous fluids without any reference to the role of suspended solids in the medium. The models for many different types of sterilization processes can be found in the literature.[1] In general, these models are the combinations of heat transfer and organism death rate expressions. The rate of thermal death for most organisms follows first order kinetics. As with most rate constants, the first order constant is a function of temperature and activation energy. Mathematically, the death rate can be expressed as:

$$\frac{dN}{dt} = -K_{d_0} e^{-E_d/RT(t)} N \qquad (1)$$

Separating variables in (1) and integrating results in the equation:

$$\ln \frac{N_f}{N_0} = \int_0^{t_f} -K_{d_0} e^{-E_d/RT(t)} dt \qquad (2)$$

Temperature is considered to be a function of time in this expression since the fluid is heated up as it moves through the exchanger train. The type of function depends on the design of the sterilizer. Common batch sterilization designs include one or more of the following heat sources: steam sparging, electrical heating, steam heating, and heating or cooling with a two-fluid heat exchanger[1]. Deindoerfer and Humphrey[2] have associated with each of these methods of heat transfer a time-temperature profile

which can be derived by simple energy balances. Each of these profiles can be substituted into the right hand of equation (2) and the integral evaluated to model each separate section of the complete sterilizer (Table I)[1]. Evaluation of each part of the sterilizer results in a logarithmic ratio of final to initial concentrations, $\ln(N_f/N_0)$.

For a sterilizer with only three sections, such as one that contains a heat-up section using steam, a holding section, and a cooling section (see Figure 1), the overall ratio of final to initial viable cell concentration can be expressed as:

$$\ln \frac{N_f}{N_0} = \ln \frac{N_f}{N_0(\text{coolant})} \frac{N_f(\text{holding})}{N_0(\text{holding})} \frac{N_f(\text{steam})}{N_0(\text{steam})} \quad (3)$$

since $N_0(\text{coolant}) = N_f(\text{holding})$ and $N_0(\text{holding}) = N_f(\text{steam})$. This can be rewritten as:

$$\ln \frac{N_f}{N_0} = \ln \frac{N_f(c)}{N_0(c)} + \ln \frac{N_f(h)}{N_0(h)} + \ln \frac{N_f(s)}{N_0(s)} \quad (4)$$

The overall result, $\ln(N_f/N_0)$, is obtained by adding the three individual solutions above, each evaluated for time-temperature distribution in that mode of operation.

For continuous sterilizers, similar models have been developed for fluid sterilization, with a range of complexities. If ideal plug flow assumptions are used, temperature-time profiles are very similar to those of the the batch sterilizer for the same type of heating. These are derived later in this paper. The modelling

ıle 1 · Temperature-Time Profiles and Remaining Viable Fractions (ln(Nf/No)) in Batch Sterilization

e of Heat Transfer	Temperature-Time Profile	Parameters	Remaining Viable Fraction (ln(Nf/No))
stant Temperature	Constant T		$Kdo*tf*Exp\ -(Ed/R*To)$
am Sparging	$T=To[1.0+at/(1+bt)]$	$a=hs/N*To*\rho*Cp$	$\frac{Kdo*a*(Ed/R*To)}{(a+b)} * Exp\ -[Ed/R*To]*(b/(a+b))$
	hyperbolic	$b=s/N$	$*\left[E_2\frac{Ed}{R*To}\left(\frac{1+b*tf}{1+(a+b)*tf}-\frac{b}{a+b}\right)-E_2\frac{Ed}{R*To}\frac{a}{a+b}\right]$
;trical Heating	$T=To(1.0+at)$	$a=q/N*To*\rho*Cp$	$\frac{Kdo*Ed}{R*To*a}\left(E_1\frac{Ed/R*To}{1+atf}-E_1\frac{Ed}{R*To}\right)$
	linear		
m (Heat Exchange)	$T=Ts(1+b*Exp\ -(at))$	$a=U*A/N*c$	$\frac{Kdo}{a}\left[E_1\left(\frac{Ed/R*Ts}{1+b}\right)-E_1\frac{E/R*Ts}{1+b*Exp\ -(a*tf)}\right]$
	exponential	$b=\frac{To-Ts}{Ts}$	$-\frac{Kdo}{a}Exp\ -(Ed/R*Ts)\left[-E_1\left(\frac{Ed}{R*Ts}\frac{b}{1+b}\right)-E_1\left(\frac{Ed}{R*Ts}\frac{b*Exp\ -(a*tf)}{1+b*Exp\ -(a*t}\right.\right.$
(ant (Heat Exchange)	$T=Tco(1+b*Exp\ -(at))$	$a=u*c'/N*\rho*Cp$ $*(1-Exp(-x))$	$\frac{Kdo}{a}\left[E_1\left(\frac{Ed/R*Ts}{1+b}\right)-E_1\frac{E/R*Ts}{1+b*Exp\ -(a*tf)}\right]$
	exponential	where $x=U*A/w*c'$	$-\frac{Kdo}{a}Exp\ -(Ed/R*Ts)\left[-E_1\left(\frac{Ed}{R*Ts}\frac{b}{1+b}\right)-E_1\left(\frac{Ed}{R*Ts}\frac{b*Exp\ -(a*tf}{1+b*Exp\ -(a*t}\right.\right.$
		$b=\frac{To-Tco}{Tco}$	where
			$E_n(z)=\int_z^{\infty}\frac{Exp\ -(w)\ dw}{w}$

h :halpy differences between steam sparger temperature and raw medium temperature
s :am mass flow rate
N: tial medium mass
T: nitial medium temperature
q: e of heat transfer, kcal per unit time
U: rall heat transfer coefficient, kcal/(m h C)
A: t transfer area, m
T: mperature of steam or heat source (Tco)
w: lant mass flow rate
c': olant specific heat
Tc: oolant inlet temperature '
ρ dium density
Cp: dium heat capacity

becomes much more complex if axial and backward diffusion are considered. Bailey and Ollis derive a sterilization model based on a dispersion model for the fluid motion[1]. This model, which is a modification of the ideal plug flow tubular reactor, (PFTR), is derived by considering an axial diffusion process which is superimposed on the convective flow through the tube. The dispersion model mass balance can be written:

$$u \frac{dc}{dz} = D_z \frac{d^2c}{dz^2} + r_{fc} \qquad (5)$$

where r_{fc} is the rate of formation term for component c, u is the velocity through the reactor, D_z is the effective axial dispersion coefficient, and z is the distance travelled through the tube. Since Equation (5) is a second-order differential equation, two boundary conditions are required. The generally accepted ones are [1]:

$$uc_f = uc - D_z \frac{dc}{dz}_{z=0} \qquad (6)$$

$$\left.\frac{dc}{dz}\right|_{z=L} = 0 \qquad (7)$$

In our case, c in Equations (6) and (7) are identified with N, and we take $r_{fN} = -k_d N$; the organism concentration throughout from the tubular reactor with dispersion can then be determined analyti-

cally. The resulting value of N at the sterilizer effluent is given by [1]:

$$\frac{N(L)}{N_0} = \frac{4y \exp[Pe/2]}{(1+y)^2\exp[(Pe)(y)/2]-(1-y)^2\exp[-(Pe)(y)/2]} \qquad (8)$$

with

$$y = 1 + \left[\frac{4\ Da}{Pe}\right]^{1/2} \qquad (9)$$

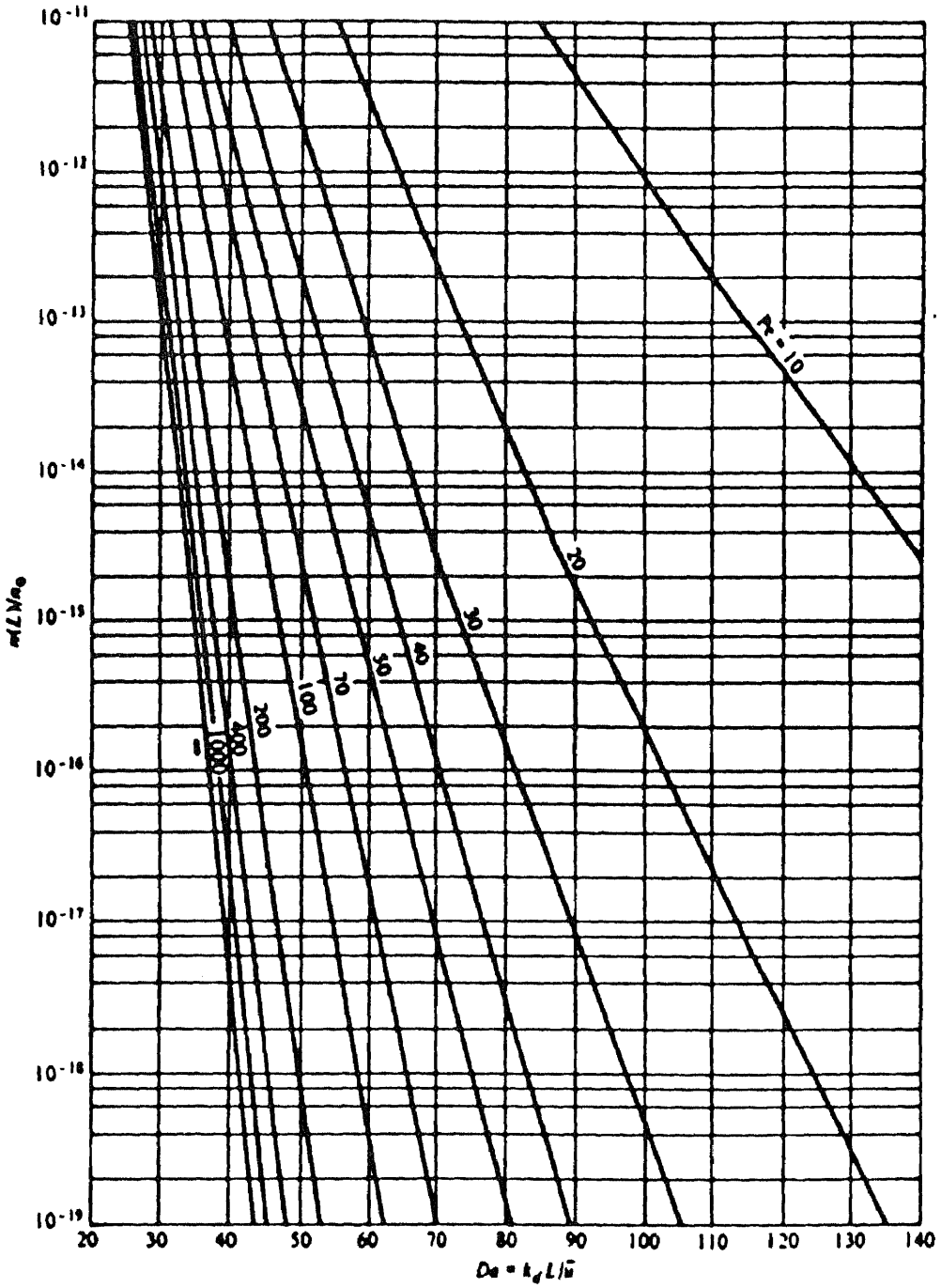
where the Damkohler number Da is defined by $Da = (k_d L/u)$. For small deviations from plug flow (Pe^{-1} small), Equation (8) reduces to the simpler form:

$$\frac{N(L)}{N_0} = \exp\left[-Da + \frac{Da^2}{Pe}\right] \qquad (10)$$

These solutions are displayed in Figure 3 as a plot of remaining viable fraction $N(L)/N_0$ vs. the dimensionless group Da for various values of the Peclet number. As Pe -> ∞ so that ideal plug flow is approximated, the desired degree of medium sterility can be achieved with the shortest possible sterilizer. Therefore, the flow system should be designed to minimize dispersion.

Residence time distribution theory can also be used to develop a model for continuous fluid sterilization.[1] Assuming that back

Figure 3 - Effect of Axial Dispersion on Organism Destruction in a Continuous Sterilizer



(Reprinted from S. Aiba, A.E. Humphrey, and N.F. Millis, Biochemical Engineering, 2nd ed., p.263, University of Tokyo Press, Tokyo, 1973.)

diffusion of organisms into the sterilizer feed line is negligible, the remaining viable fraction can be expressed:

$$\frac{N}{N_0} = \int_0^t E(t)\, e^{-k_d t}\, dt \qquad (11)$$

where E(t) is the RTD of the continuous sterilizer.

Another consideration in the modelling of sterilizers is that of suspended solids. This condition protects organisms contained in the interior of the solids, and more extensive heating is needed to obtain the required degree of sterilization.

Carslaw and Jaeger[3] derived a huge volume of analytical solutions describing transient heat conduction in solids under various conditions, and many of these solutions can also be found in J. Crank's text[4] on mass diffusion, which can be treated mathematically in a similar fashion. It is Crank's solutions that are used to confirm some of the cases derived later in this paper. While diffusion in a several geometric shapes, including infinite and semi-infinite media, plane sheets, and cylinders is considered by Crank, it is the solutions to diffusion in spheres that are of particular interest for application to sterilizers, as many particles are roughly of this shape.

Crank lists several cases which are derived from temperature-time profiles of the forms discussed in Table I for batch sterilization

and which will be derived in the next section and used in the continuous sterilization model. Among these are the case of

- a sphere at an initially uniform temperature exposed to a constant external surface temperature at time zero
- a linearly increasing surface temperature, $f(t)=mt$
- exponentionally increasing temperature, $f(t)=T_0(1-\exp(-Bt)$
- surface evaporation from the sphere

Each of these cases arise in sections of the continuous sterilizer being considered. The corresponding equations (see Nomenclature section for a description of variables) for the temperature profiles as functions of time and radial position are, respectively:

$$T(r,t)=T_1 + \frac{2R(T_1-T_0)}{\pi r} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi r}{R} \exp - \frac{k}{\rho Cp} \frac{n^2\pi^2 t}{R^2} \qquad (12)$$

$$T= m \left[t- \frac{R^2-r^2}{6\alpha} \right] - \frac{2mR^3}{\alpha\pi^3 r} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \exp(- \alpha\, n^2\pi^2 t/R^2) \sin \frac{n\pi r}{R} \qquad (13)$$

$$\frac{T}{T_0} = 1- \frac{R}{r} \exp(-Bt) \frac{\sin\{(BR^2/\alpha)^{1/2} r/R\}}{\sin\{BR^2/\alpha\}^{1/2}} - \frac{2BR^3}{\pi\alpha r} \sum_{n=1}^{\infty} (-1)^n \frac{\exp(-\alpha\, n^2\pi^2 t/R^2)}{n(n^2\pi^2-BR^2/\alpha)} \sin \frac{n\pi r}{R} \qquad (14)$$

If the sphere is initially at a uniform temperature T_1 and there is a surface condition (for r = R) where surface temperature is

dependant on an external heat transfer coefficient related to the fluid properties as well as internal conduction:

$$-k\,\partial T/\partial r = h(T(r) - T_0) \quad \text{at } r = R \tag{15}$$

where T_0 is the temperature in the bulk of the fluid outside the sphere, then

$$\frac{T-T_0}{T_1-T_0} = \frac{2LR}{r} \sum_{n=1}^{\infty} \frac{\exp(-\alpha\,\beta_n^2 t/R^2)}{(\beta_n^2 + L(L-1))} \frac{\sin \beta_n r/R}{\sin \beta_n} \tag{16}$$

The β_n's are the roots of

$$\beta_n \cot \beta_n + L - 1 = 0 \tag{17}$$

and $L = Rh/k$. The resulting destruction of microbes and spores can be calculated in each of these temperature-time profiles by several methods.

In the food industry, an often used practice is to consider only the organism concentration at the center of the sphere. Since the center heats most slowly, it is assumed that if the center is adequately sterilized, the remainder of the solid is sufficiently sterilized as well. In this case, Equations (12-16) can be simplified slightly to describe the center temperature, and then numerically integrated in the organism death function.

Another method is to determine the concentration of organisms at each point in the sphere. Then, by integrating over the volume of the sphere, the remaining viable concentration can be calculated. This method requires a good deal of calculation, and is best left to a computer program. In fact, this method is used later in this paper in the computer model of the continuous sterilizer.

Thus, the calculation of the remaining viable concentration of microorganisms in fluid processed in various sterilizers is obtained by the substitution of the temperature-time profile of the fluid (or solid) through the sterilizer into the organism death function. In the next section we will derive those temperature-time relationships describing our continuous sterilizer system.

5. THEORY (DERIVATION)

In the following sections, the equations describing the temperature-time relationships of both the fluid medium travelling through the heat exchanger and the temperature distribution (as a function of radius) of a sphere suspended in the fluid are derived, and, where possible, compared with solutions in the literature. Derivations will be in the following order:

- Temperature-time profile of the fluid moving through exchanger[5]
- Radial temperature profile of a sphere suspended in that fluid:
    - Infinite external heat transfer coefficient
        - Surface temperature of sphere equals fluid temperature
    - Finite external heat transfer coefficient[6]
        - Surface temperature dependant on relative rates of:
            - Heat transfer through the fluid
            - Conduction of heat into the solid
- Equations will be derived for the cases of:
    - Constant temperature fluid
    - Electrically heated fluid
    - Steam heated fluid
    - Fluid in counter-current heat exchanger

Several assumptions will be used in the derivation of the equations describing the time-temperature profiles for both the

fluid and the solids particles. The most important of these assumptions refer to the fluid. These are:

- Ideal plug flow (constant radial temperature)
- Heat conduction in direction of fluid insignificant

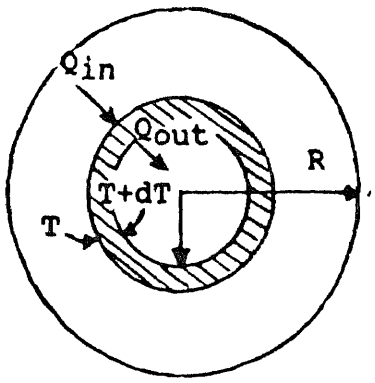
At the end of this section, the equation describing the temperature of a sphere moving through the entire continuous sterilizer train is derived. While completely describing the system, this expression is difficult to use in the calculation of numerical values, so that in subsequent sections, a computer program is enlisted to calculate numerical values out of each exchanger, and to use these values as the initial condition for the subsequent exchanger (equation).

6. TEMPERATURE DISTRIBUTION IN A SPHERE IMMERSED IN A CONSTANT TEMPERATURE FLUID

The following section considers the case of a spherical particle surrounded by a fluid at constant temperature. The temperature profile of the sphere is left as an unknown function in the derivations of equations, but is assumed as a constant temperature different from that of the fluid for the comparison with Crank's solution, which assumes this initial condition. Two equations are derived for two different boundary conditions. The first assumes an infinite external heat transfer coefficient. This means that the temperature of the surface of the sphere is the same as that of the bulk fluid. The second case considers a finite external heat transfer coefficient, meaning the temperature at the surface of the sphere now depends on both heat transfer characteristics of the fluid and the solid.

6.1 Infinite External Heat Transfer Coefficient

For the case of a sphere suddenly surrounded by a constant temperature fluid, derivation begins with an energy balance around a shell in the sphere. A general partial differential equation is obtained which describes the temperature profile in the sphere. Separation of variables is then used, along with the substitution of the proper boundary conditions for this case, to yield an analytical equation descibing temperaure of the sphere as a function of radius and time.



Energy into the shell is expressed:

$$Q_{in} = -kA\,\partial T/\partial r = -k\,(4\pi r^2)\,\partial T/\partial r$$

Energy out of the shell is:

$$Q_{out} = -k4\pi(r+dr)^2 * \partial(T+dT)/\partial r$$

$$= -k4\pi(r+dr)^2 \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\,dr\right)$$

The energy balance around the shell is then:

$$Q_{net} = Q_{in} + Q_{out}$$

$$= 4\pi k \quad (r+dr)^2 \quad \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr \quad - r^2 \frac{\partial T}{\partial r}$$

Expanding, simplifying, and neglecting smaller terms

$$Q_{net} = 4\pi k(r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r))$$

The rate of energy accumulated in the shell is also

$$E = 4\pi r^2 dr \rho C_p \partial T/\partial r$$

These terms are equal, so that

$$4\pi r^2 dr \rho C_p \partial T/\partial r = 4\pi k(r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r))$$

Collecting like terms and simplifying:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \quad \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r}$$

This is our basic equation describing the temperature of the sphere as a function of time and radius.

Initial and Boundary Conditions

Boundary Conditions

In this case, the surface temperature of the sphere is considered to be equal to the bulk temperature of the fluid:

BC # 1 : at $r = R$ $T = T_B$ at $t>0$

BC # 2 : at the center of the sphere (r=0), conduction is zero, or $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$ when $t>0$

Initial Condition

At t = 0, the sphere is initially at some temperature distribution $\phi(r/R)$: $T(r,0) = \phi(r/R)$

Let $\alpha = k/\rho C_p$ (thermal diffusivity constant)

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}$$

Letting $y = r(T-T_B)$, then $T = y/r + T_B$

Taking derivatives and simplifying

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{y}{t}$$

$$\frac{\partial T}{\partial r} = \frac{-1}{r^2}\, y + \frac{1}{r}\frac{y}{r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{1}{r}\frac{\partial^2 y}{\partial r^2} - \frac{2}{r^2}\frac{\partial y}{\partial r} + \frac{2}{r^3}\, y$$

Substitution into the PDE with some collection of terms yields:

$$\frac{1}{\alpha}\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2}$$

Substitution into the Boundary Conditions:

BC # 1 at r=R, $T = T_B$ $\quad y = r(T_B - T_B) = 0 \quad y=0$

BC # 2 at the center of the sphere,

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \quad \text{when } t>0$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 = \frac{-y}{r^2} \qquad \text{so } y = 0$$

IC For $t = 0$, $T(r,0) = \phi(r/R)$

The Boundary Conditions are linear and homogeneous, so that the separation of variables method can be used to solve the problem. Assume the solution can be written as the product of 2 functions:

$$y(r,t) = f(t) * g(r)$$

$$\partial y/\partial t = f'(t) * g(r)$$

$$\partial y/\partial r = f(t) * g'(r)$$

$$\partial^2 y/\partial r^2 = f(t) * g''(r)$$

Substituting into the PDE

$$1/\alpha \quad f'(t) * g(r) = f(t) * g''(r)$$

$$\frac{1}{\alpha}\frac{f'(t)}{f(t)} = \frac{g''(r)}{g(r)}$$

The only way this equation can hold is for the 2 functions to be constant. The constant cannot assume any value. It turns out it must be negative in order to get a solution, thus;

$$\frac{1}{\alpha}\frac{f'(t)}{f(t)} = \frac{g''(r)}{g(r)} = \text{constant} = -\lambda^2$$

$$g''(r)/g(r) = -\lambda^2$$

$$g''(r) + \lambda^2 g(r) = 0$$

This is a 2nd order homogeneous equation with constant coefficients. Trying the following solutions:

$$g(r) = e^{mr}$$

$$g'(r) = m*e^{mr}$$

$$g''(r) = m^2*e^{mr}$$

$$m^2*e^{mr} + \lambda^2 e^{mr} = 0$$

$$m^2 + \lambda^2 = 0$$

$$m = \pm \lambda i$$

$$g(r) = C_1 \cos \lambda r + C_2 \sin \lambda r$$

Imposing Boundary Conditions, at $r = 0$, $y = 0$

$$y(r,t) = f(t) * g(r)$$

$$0 = f(t) * g(0) \qquad \text{so, } g(0) = 0$$

$$g(0) = C_1 \cos \lambda (0) + C_2 \sin \lambda (0)$$

$$C_1 + 0 = 0$$

$$C_1 = 0$$

$$g(r) = C_2 \sin \lambda r$$

At $r = R$, $y = 0$

$$f(t) * g(R) = 0 \quad \text{...therefore...} g(R) = 0$$

$$C_2 \sin \lambda R = 0 \qquad \text{For non-trivial solution, } C_2 = 0$$

$$\sin \lambda R = 0$$

Sine function is zero for all integral multiples of π , thus

$$\lambda R = \pi \ , \ 2\pi \ , \ 3\pi \ , \ 4\pi \ ....$$

$$\lambda R = n\pi \quad \text{where } n = 1, 2, 3, 4 ....$$

$$\lambda = n\pi/R$$

$$g(r) = C_n \sin (n\pi/R)$$

$$\frac{1}{\alpha} \frac{f'(t)}{f(t)} = -\lambda^2$$

$$\frac{df(t)}{f(t)} = -\lambda^2\alpha \int dt$$

$$\ln f(t) = -\lambda^2\alpha\, t + \ln C_3$$

$$\ln f(t) - \ln C_3 = -\lambda^2\alpha\, t$$

$$\ln \{f(t)/C_3\} = -\lambda^2\alpha\, t$$

$$f(t)/C_3 = e^{-\lambda^2\alpha\, t} \qquad \text{where} \quad \lambda = n\pi/R$$

$$f(t) = C_3\, e^{-(n^2\pi^2\alpha\, t/R^2)}$$

The complete solution is:

$$y = f(t) * g(r) = A_n \exp\left(\frac{-\alpha\, n^2\pi^2}{R^2}\, t\right) \sin\frac{n\pi r}{R} \qquad \text{where } A_n = C_nC_3$$

No single value of A will satisfy the conditions; however, the original equation (in y) was linear. The boundary conditions will be satisfied by a solution which contains an infinite series of terms:

$$y(r,t) = \sum_{n=1}^{\infty} A_n \exp\left(\frac{-\alpha\, n^2\pi^2}{R^2}\, t\right) \sin\frac{n\pi r}{R}$$

Now applying the initial condition:

At $t = 0$, $y = \phi(r/R)$

Therefore, $y(r,0) = \sum_{n=1}^{\infty} A_n \sin(n\pi r/R) = \phi(r/R)$

$$\int_0^1 \phi\left(\frac{r}{R}\right) \sin m\pi\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = A_m \int_0^1 \sin^2\left(m\pi\left(\frac{r}{R}\right)\right) d\left(\frac{r}{R}\right) = \frac{1}{2} A_m$$

letting $E = r/R$,

$$A_m = 2 \int_0^1 \phi(E) \sin(m\pi E)\,dE$$

Finally,

$$y(r,t) = \sum_{n=1}^{\infty} 2 \exp\left(\frac{-\alpha\, n^2\pi^2}{R^2}\, t\right) \sin\frac{n\pi r}{R} \int_0^1 \phi(E) \sin(m\pi E)\,dE$$

For the case where the initial temperature distribution is constant throughout the sphere at t = 0:

$$y = \phi(E) = r(T_0 - T_B) = RE(T_0 - T_B)$$

$$\int_0^1 R(T_0-T_B)(E) \sin(m\pi E)\,dE$$

$$= R(T_0-T_B)\int_0^1 (E) \sin(m\pi E)\,dE$$

$$= R(T_0-T_B)\left[\frac{-E\cos(n\pi E)}{n\pi} + \frac{\sin(n\pi E)}{(n\pi)^2}\right]_0^1$$

$$= R(T_0-T_B)\left[\frac{-\cos(n\pi)}{n\pi}\right]_0^1$$

$$\cos(n\pi) = \begin{array}{ll} -1 & \text{for } n = 1, 3, 5, 7 \ldots \\ +1 & \text{for } n = 2, 4, 6, 8 \ldots \end{array}$$

So the integral equals

$$\frac{(T_0-T_B)}{(n\pi/R)} \cdot (-1)^{n+1}$$

Substituting back into the solution, and substituting $y = r(T-T_B)$

Or $y/r + T_B = T(r,t)$

$$T = T_B + \frac{2R}{r\pi}(T_0-T_B) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \exp\left(\frac{-\alpha\, n^2\pi^2}{R^2}\, t\right) \sin\frac{n\pi r}{R}$$

To determine the temperature at the center of the sphere,

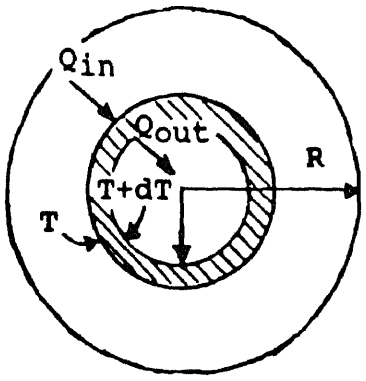
$$T = T_B + 2(T_0-T_B) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin(n\pi r/R)}{(n\pi r/R)} \exp \frac{- n^2\pi^2}{R^2} t$$

Taking the limit as $r \longrightarrow 0$

$$T(0,t) = T_B + 2(T_0-T_B) \sum_{n=1}^{\infty} (-1)^{n+1} \exp \frac{-\alpha\, n^2\pi^2}{R^2} t$$

6.2 Finite External Heat Transfer Coefficient

For the case of a sphere surrounded by a constant temperature fluid, derivation begins with an energy balance around a shell in the sphere, as before. In fact, the derivation is very similar, and only the boundary condition at r = R has changed. The solution is again obtained by separation of variable and substitution of boundary conditions, this time considering an external heat transfer coefficient limiting heat transfer into the sphere.



The energy into the shell is described as:

$$Q_{in} = -kA\, \partial T/\partial r = -k\,(4\pi r^2)\, \partial T/\partial r$$

The energy out of the sphere is:

$$Q_{out} = -k4\pi(r+dr)^2 * \partial(T+dT)/\partial r$$

$$= -k4\pi(r+dr)^2 \left[\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\, dr\right]$$

The energy balance around the shell is:

$$Q_{net} = Q_{in} + Q_{out}$$

$$= 4\pi k \quad (r+dr)^2 \quad \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr \quad - r^2 \frac{\partial T}{\partial r}$$

Expanding, simplifying, and neglecting smaller terms

$$Q_{net} = 4\pi k(r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r))$$

The rate of energy accumulated in the shell is also

$$E = 4\pi r^2 dr \rho C_p \partial T/\partial r$$

These terms are equal, so that

$$4\pi r^2 dr \rho C_p \partial T/\partial r = 4\pi k\{r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r)\}$$

Collecting like terms and simplifying:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \quad \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r}$$

This is our basic equation describing the temperature of the sphere as a function of time and radius.

Initial and Boundary Conditions

Boundary Conditions

In this case, the condition existing at the outer radius of the sphere is dependant on both fluid and solid heat transfer:

BC # 1 : at $r = R$ $\quad h(T_B - T) = k\ (dT/dr)$

BC # 2 : at the center of the sphere (r=0), conduction is zero, or $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$ when $t>0$

Initial Condition

At t = 0, the sphere is initially at some temperature distribution $\phi(r/R)$: $T(r,0) = \phi(r/R)$

Let $\alpha = k/\rho C_p$ (thermal diffusivity constant)

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}$$

Letting $y = r(T-T_B)$, then $T = y/r + T_B$

Taking derivatives and simplifying

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{y}{t}$$

$$\frac{\partial T}{\partial r} = \frac{-1}{r^2}\, y + \frac{1}{r}\frac{\partial y}{\partial r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{1}{r}\frac{\partial^2 y}{\partial r^2} - \frac{2}{r^2}\frac{\partial y}{\partial r} + \frac{2}{r^3}\, y$$

Substitution into the PDE with some collection of terms yields:

$$\frac{1}{\alpha}\frac{y}{t} = \frac{\partial^2 y}{\partial r^2}$$

Substitution into the Boundary Conditions:

BC # 1 $\quad y = r(T - T_B)$ then $T = T_B + y/r$

$$-h\,\frac{y}{r} = +k\left(-\frac{y}{r^2} + \frac{1}{r}\frac{y}{r}\right)$$

$$+hy = +k(y/r) - k(dy/dr)$$

$$k(dy/dr) - k(y/r) + hy = 0 \qquad \text{at } r = R$$

BC # 2 at the center of the sphere,

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \qquad \text{when } t>0$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 = \frac{-y}{r^2} \qquad \text{so } y = 0$$

IC For $t = 0$, $T(r,0) = \phi(r/R)$

Assume Boundary Conditions are linear and homogeneous, so that the separation of variables method can be used to solve the problem. Assume the solution can be written as the product of 2 functions:

$$y(r,t) = f(t) * g(r)$$

$$\partial y/\partial t = f'(t) * g(r)$$

$$\partial y/\partial r = f(t) * g'(r)$$

$$\partial^2 y/\partial r^2 = f(t) * g''(r)$$

Substituting into the PDE

$$1/\alpha \quad f'(t) * g(r) = f(t) * g''(r)$$

$$\frac{1}{\alpha}\frac{f'(t)}{f(t)} = \frac{g''(r)}{g(r)}$$

The only way this equation can hold is for the 2 functions to be constant. The constant cannot assume any value...it turns out it must be negative in order to get a solution, thus;

$$\frac{1}{\alpha}\frac{f'(t)}{f(t)} = \frac{g''(r)}{g(r)} = \text{constant} = -\lambda^2$$

$$g''(r)/g(r) = -\lambda^2$$

$$g''(r) + \lambda^2 g(r) = 0$$

This is a 2nd order homogeneous equation with constant coefficients. Trying the following solutions:

$$g(r) = e^{mr}$$

$$g'(r) = m*e^{mr}$$

$$g''(r) = m^2*e^{mr}$$

$$m^2*e^{mr} + \lambda^2 e^{mr} = 0$$

$$m^2 + \lambda^2 = 0$$

$$m = \pm \lambda i$$

$$g(r) = C_1 \cos \lambda r + C_2 \sin \lambda r$$

Imposing Boundary Conditions, at $r = 0$, $y = 0$

$$y(r,t) = f(t) * g(r)$$

$$0 = f(t) * g(0) \qquad \text{so, } g(0) = 0$$

$$g(0) = C_1 \cos\lambda (0) + C_2 \sin \lambda (0)$$

$$C_1 + 0 = 0$$

$$C_1 = 0$$

$$g(r) = C_2 \sin \lambda r$$

From the last case, we saw that $f(t) = C_3 \exp(-\alpha n^2\pi^2 t/R^2)$

Similarly, in this case, $f(t) = C_3 \exp(-\alpha \beta_n^2 t/R^2)$

At $r = R$, $\quad k \left.\frac{\partial y}{\partial r}\right|_{r=R} - k \frac{y}{r} + hy = 0$

$$y(r,t) = f(t) * g(r) = f(t) * C_2 \sin \lambda r$$

$$\partial y/\partial r \,|_{r=R} = f(t) * C_2 \lambda \cos \lambda R$$

Substituting back into the BC at r = R

$$k * f(t) * C_2 \lambda \cos \lambda R - k * f(t)/R * C_2 \sin \lambda R + h * f(t) * C_2 \sin \lambda r = 0$$

$$k \lambda [\cos \lambda R] = (k/R - h)[\sin \lambda R]$$

$$\lambda R \frac{\cos \lambda R}{\sin \lambda R} = 1 - \frac{hR}{k}$$

Let $\lambda R = \beta_n$ and $L = hR/k$, then

$$\beta_n \cot \beta_n + L - 1 = 0$$

Letting $C_2C_3 = A_n$ and $\lambda = \beta_n/R$, the solution for y is:

$$y = f(t) * g(r) = A_n \exp\left(\frac{-\alpha \beta_n^2}{R^2} t\right) \sin \frac{\beta_n r}{R} \quad \text{where } A_n = C_2C_3$$

No single value of A will satisfy the conditions; however, the original equation (in y) was linear. The boundary conditions will be satisfied by a solution which contains an infinite series of terms:

$$y(r,t) = \sum_{n=1}^{\infty} A_n \exp\left(\frac{-\alpha \beta_n^2}{R^2} t\right) \sin \frac{\beta_n r}{R}$$

Now applying the initial condition:

At $t = 0$, $y = \phi(r/R)$

Therefore, $y(r,0) = \sum_{n=1}^{\infty} A_n \sin(\beta_n r/R) = \phi(r/R)$

$$\int_0^1 \phi\left(\frac{r}{R}\right) \sin \beta_m \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = A_m \int_0^1 \sin^2 \left(\beta_m \left(\frac{r}{R}\right)\right) d\left(\frac{r}{R}\right)$$

letting $E = r/R$,

$$\int_0^1 \phi(E)\sin(\beta_m E)\,dE = A_m \frac{\beta_m - \sin\beta_m\cos\beta_m}{2\beta_m}$$

$$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)\,dE$$

Finally,

$$y(r,t) = \sum_{n=1}^{\infty} A_n \exp\left(\frac{-\alpha\,\beta_n^2}{R^2}\,t\right) \sin\frac{\beta_n r}{R}$$

where

$$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)\,dE$$

Substituting $y = r(T-T_B)$ or $T = y/r + T_B$

$$T(r,t) = T_B + \sum_{n=1}^{\infty} \frac{A_n}{r} \exp\left(\frac{-\alpha\,\beta_n^2}{R^2}\,t\right) \sin\frac{\beta_n r}{R}$$

where

$$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)\,dE$$

For the case where the initial temperature distribution is constant throughout the sphere at t = 0:

$$y = \phi\,(E) = r(T_0 - T_B) = RE(T_0 - T_B)$$

$$\int_0^1 R(T_0-T_B)(E)\,\sin(\beta_n E)\,dE$$

$$= R(T_0-T_B)\int_0^1 (E)\,\sin(\beta_n E)\,dE$$

$$= R(T_0-T_B)\left[\frac{-E\cos(\beta_n E)}{\beta_n} + \frac{\sin(\beta_n E)}{(\beta_n)^2}\right]_0^1$$

$$= R(T_0-T_B)\left[\frac{-\cos(\beta_n E)}{\beta_n} + \frac{\sin(\beta_n)}{(\beta_n)^2}\right]$$

Substituting back into A_n and simplifying:

$$A_n = \frac{-2\beta_n^2\cos\beta_n + 2\beta_n\sin\beta_n}{\beta_n^3 - \beta_n^2\sin\beta_n\cos\beta_n} R(T_0-T_B)$$

Substituting back into the solution,

$$T(r,t) = T_B + \sum_{n=1}^{\infty} \frac{A_n}{r} \exp\left(\frac{-\alpha\ \beta_n^2}{R^2}\ t\right) \sin\frac{\beta_n r}{R}$$

where

$$A_n = \frac{-2\beta_n^2\cos\beta_n + 2\beta_n\sin\beta_n}{\beta_n^3 - \beta_n^2\sin\beta_n\cos\beta_n} R(T_0-T_B)$$

To determine the temperature at the center of the sphere, multiply numerator and denominator by β_n and let $r = ER$

$$T(r,t) = T_B + \sum_{n=1}^{\infty} \frac{A_n}{R} \exp\left(\frac{-\alpha\ \beta_n^2}{R^2}\ t\right) \frac{\sin(\beta_n E)}{\beta_n E}$$

6.3 Comparison with Crank Solution

In Crank's "Mathematics of Diffusion", a similar problem is solved for evaporation from a sphere. The goal in this section is to show that the equation derived above is the same as that derived by Crank for the analogous mass diffusion problem.

Rearranging our answer:

$$\frac{\beta_n \cos\beta_n}{\sin\beta_n} + L - 1 = 0$$

$$\beta_n \cos\beta_n = (1-L)\sin\beta_n$$

$$\sin\beta_n = \beta_n \cos\beta_n/(1-L)$$

then $$-2\beta_n\cos\beta_n + 2\sin\beta_n = -2(1-L)\sin\beta_n + 2\sin\beta_n$$

$$\sin\beta_n*(2)*(-(1-L)+1) = 2L\sin\beta_n$$

$$A_n = \frac{2L\sin\beta_n}{\beta_n^2 - \beta_n\sin\beta_n\cos\beta_n} R(T_0-T_B)$$

$\beta_n^2 - \beta_n\sin\beta_n\cos\beta_n$ ——> Let $\sin\beta_n = \beta_n\cos\beta_n/(1-L)$

$\beta_n^2 - \beta_n^2\cos^2\beta_n/(1-L)$ ——> Let $\beta_n^2\cos^2\beta_n = (1-L)^2\sin^2\beta_n$

$$A_n = \frac{2L\sin\beta_n * R * (T_0-T_B)}{\beta_n^2 + (L-1)\sin^2\beta_n}$$

So our solution can be expressed:

$$\frac{T-T_B}{T_0-T_B} = \frac{2LR}{r} \sum_{n=1}^{\infty} \frac{\sin\beta_n}{\beta_n^2 + (L-1)\sin^2\beta_n} \exp\left(\frac{-\alpha\,\beta_n^2}{R^2}\,t\right)\sin(\beta_n E)$$

Compare with Crank Solution which has been rewritten for heat transfer:

$$\frac{T-T_B}{T_0-T_B} = \frac{2LR}{r} \sum_{n=1}^{\infty} \frac{1}{(\beta_n^2 + L(L-1))\sin\beta_n} \exp\left(\frac{-\alpha\,\beta_n^2}{R^2}\,t\right)\sin(\beta_n E)$$

For solutions to be equal,

$$\frac{1}{(\beta_n^2 + L(L-1))\sin\beta_n} \quad \text{must equal} \quad \frac{\sin\beta_n}{\beta_n^2 + (L-1)\sin^2\beta_n}$$

$$(\beta_n^2 + L(L-1))\sin^2\beta_n = \beta_n^2 + (L-1)\sin^2\beta_n$$

$$\beta_n^2\sin^2\beta_n + L^2\sin^2\beta_n - L\sin^2\beta_n = \beta_n^2 - L\sin^2\beta_n - \sin^2\beta_n$$

$$L^2\sin^2\beta_n - 2L\sin^2\beta_n + \sin^2\beta_n = \beta_n^2 - \beta_n^2\sin^2\beta_n$$

$$\sin^2\beta_n(L^2 - 2L + 1) = \beta_n^2(1 - \sin^2\beta_n)$$

$$\sin^2\beta_n(1-L)^2 = \beta_n^2(1 - \sin^2\beta_n)$$

$$(\sin\beta_n(1-L))^2 = \beta_n^2(1 - \sin^2\beta_n)$$

$$(\beta_n\cos B_n)^2 = \beta_n^2(1 - \sin^2\beta_n)$$

$$\cos^2\beta_n = 1 - \sin^2\beta_n \quad \text{or} \quad \sin^2\beta_n + \cos^2\beta_n = 1$$

This is an identity, therefore the solutions match.

7. TEMPERATURE DISTRIBUTION IN A SPHERE IMMERSED IN AN ELECTRICALLY HEATED FLUID

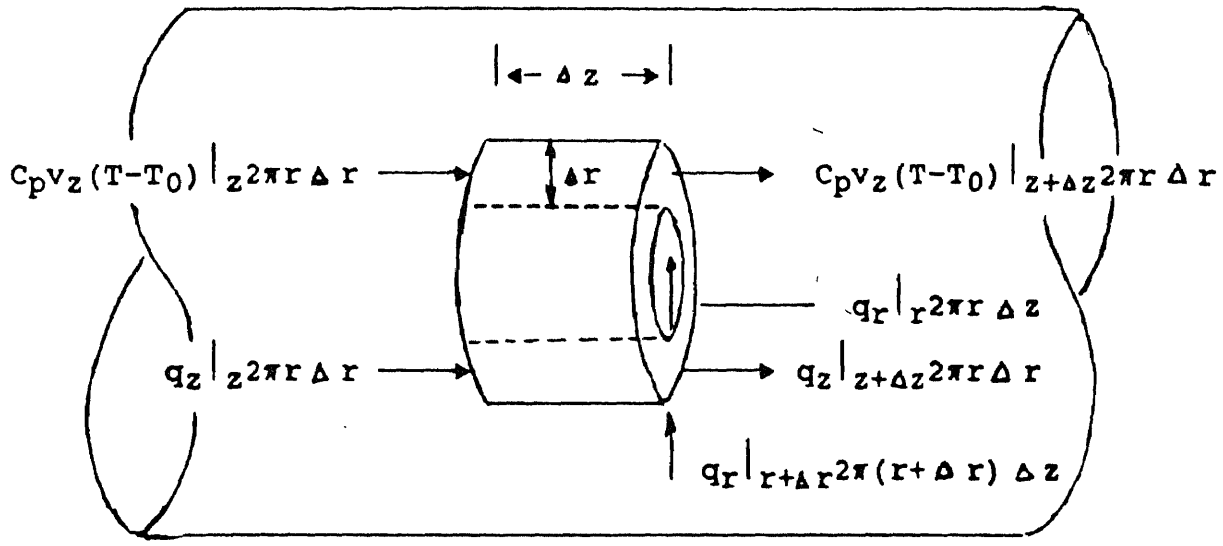
In the following sections, equations will be derived which express the:

- Time-temperature profile of an electrically heated fluid.
- Temperature distribution in a sphere immersed in the above fluid:
    - With an infinite external heat transfer coefficient.
    - With a finite external heat transfer coefficient.

A solution is available in the literature for the infinite external heat transfer coefficient case, and a comparison between our derived solution and that found in the literature is made.

7.1 Temperature Distribution in an Electrically Heated Fluid

In this section an equation describing the temperature-time profile of a fluid moving through an electical heater is derived. The derivation begins with an energy balance around a shell in a section of pipe. As mentioned previously, assumptions of ideal plug flow with no forward or backward conduction of heat are employed. For this case, a constant heat flux, associated with electrical heating, is assumed.



The energy balance around a shell of fluid moving through a section of pipe is written:

$$\text{Energy In} - \text{Energy Out} = \text{Energy Accumulated} = 0$$

$$q_z|_z 2\pi r\Delta r + q_r|_{r+\Delta r} 2\pi(r+\Delta r)\Delta z + \rho C_pv_z(T-T_0)|_z 2\pi r\Delta r$$
$$- q_z|_{z+\Delta z} 2\pi r\ r + q_r|_r 2\pi r\Delta z + \rho C_pv_z(T-T_0)|_{z+\Delta z} 2\pi r\Delta r = 0$$

Divide by $2\pi\Delta r\Delta z$ and combine terms

$$\frac{(rq_r)|_{r+\Delta r} - (rq_r)|_r}{\Delta r} + r\frac{q_z|_{z+\Delta z} - q_z|_z}{\Delta z} + r\rho C_pv_z\frac{T|_{z+\Delta z} - T|_z}{\Delta z} = 0$$

As Δr and Δz approach zero

$$\rho C_p v_z \frac{\partial T}{\partial z} = -\frac{1}{r}\frac{\partial}{\partial r}(rq_r) - \frac{\partial q_z}{\partial z}$$

Assume v_z is constant = dz/dt

q_r is constant for electrical heating = $-q_r$ (heat in)

$q_z = -k\,\partial T/\partial z$

$$\rho C_p v_z \frac{\partial T}{\partial z} = -\frac{1}{r}\frac{\partial}{\partial r}(rq_r) - \frac{\partial}{\partial z}\left(-k\frac{\partial T}{\partial z}\right)$$

$$\rho C_p v_z \frac{\partial T}{\partial z} = \frac{q_r}{r} + k\frac{\partial^2 T}{\partial z^2}$$

Assume $\partial^2 T/\partial z^2$ small compared to T/z term, then

$$\rho C_p v_z \frac{\partial T}{\partial z} = \frac{q_r}{r}$$

$$\frac{\partial T}{\partial z} = \frac{q_r}{r\rho C_p v_z}$$

$$dT = \frac{q_r\,dz}{r\rho C_p v_z} \qquad \text{let } P = q_r/(r\rho C_p v_z)$$

$$dT = Pdz$$

At $z = 0$, $T_B = T_0$ and at $z = z$, $T_B = T_B$

$$(T_B - T_0) = Pz$$

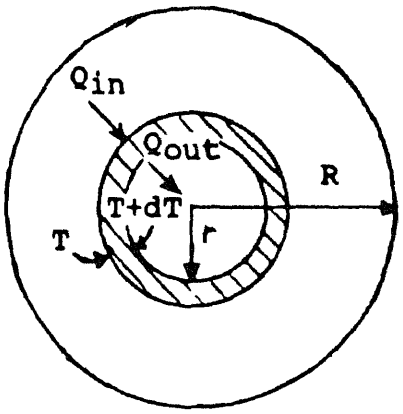
$$z = v_z t$$

$$T_B = T_0 + P't \quad \text{where} \quad P' = Pv_z$$

Thus, the temperature profile is a linear one.

7.2 Infinite External Heat Transfer Coefficient

In this section, an equation describing the temperature distribution in a sphere suspended in the electrically heated fluid above is derived. An energy balance around a shell in the sphere yields a general partial differential equation describing the radial temperature profile. However, in this case the boundary condition of the sphere must include the bulk temperature of the fluid changing with time. This leads to a non-homogeneous partial differential equation, which is solved by using both an Eigen function expansion of the non-homogeneous term and separation of variables as in the previous cases.



The energy into the shell:

$$Q_{in} = -kA\,\partial T/\partial r = -k\,(4\pi r^2)\,\partial T/\partial r$$

The energy out of the shell:

$$Q_{out} = -k4\pi(r+dr)^2 * \partial(T+dT)/\partial r$$

$$= -k4\pi(r+dr)^2 \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\,dr\right)$$

The overall energy balance around the shell:

$$Q_{net} = Q_{in} + Q_{out}$$

$$= 4\pi k \left[(r+dr)^2 \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr \right) - r^2 \frac{\partial T}{\partial r} \right]$$

Expanding, simplifying, and neglecting smaller terms

$$Q_{net} = 4\pi k\{r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r)\}$$

The rate of energy accumulated in the shell is also

$$E = 4\pi r^2 dr \rho C_p \partial T/\partial r$$

These terms are equal, so that

$$4\pi r^2 dr \rho C_p \partial T/\partial r = 4\pi k\{r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r)\}$$

Collecting like terms and simplifying:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right]$$

This is our basic equation describing the temperature of the sphere as a function of time and radius.

<u>Initial and Boundary Conditions</u>

Boundary Conditions

The surface of the sphere is in contact with and is at the same temperature as the bulk fluid. In this case, the temperature of the bulk fluid changes linearly with time.

BC # 1 : at $r = R$ $T = T_B = T_0 + P't$ at $t>0$

BC # 2 : at the center of the sphere (r=0), conduction is zero, or $\left. \frac{\partial T}{\partial r} \right|_{r=0} = 0$ when $t>0$

Initial Condition

At t = 0, the sphere is initially at some temperature distribution $\phi(r/R)$: $T(r,0) = \phi(r/R)$

Let $\alpha = k/\rho C_p$ (thermal diffusivity constant)

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}$$

Letting $y = r(T-T_B)$, then $T = y/r + T_B$

Taking derivatives and simplifying

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{y}{t} + P'$$

$$\frac{\partial T}{\partial r} = \frac{-1}{r^2}y + \frac{1}{r}\frac{\partial y}{\partial r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{1}{r}\frac{\partial^2 y}{\partial r^2} - \frac{2}{r^2}\frac{\partial y}{\partial r} + \frac{2}{r^3}y$$

Substitution into the PDE with some collection of terms yields:

$$\frac{1}{\alpha}\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} + \frac{-P'r}{\alpha}$$

$$\frac{y}{t} = -P'r + \alpha\frac{\partial^2 y}{\partial r^2}$$

Substitution into the Boundary Conditions:

BC # 1 at r=R, $T = T_B$ $\quad y = r(T_B - T_B) = 0 \quad y=0$

BC # 2 at the center of the sphere,

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \quad \text{when } t>0$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 = \frac{-y}{r^2} \qquad \text{so} \quad y = 0$$

IC For $t = 0$, $T(r,0) = \phi(r/R)$

The Boundary Conditions are homogeneous, but the PDE is not.

$\frac{\partial Y}{\partial t} = -P'r + \alpha \frac{\partial^2 Y}{\partial r^2}$ can be expressed $U_t = \alpha U_{rr} + f(r,t)$

Decomposing $f(r,t)$ into simple components

$f(r,t) = f_1(t)X_1(r) + f_2(t)X_2(r) + \ldots + f_n(t)X_n(r) + \ldots$

Find the response $U_n(r,t)$ to each of these individual components, $f_n(t)X_n(r)$. The solution to the problem is then

$$U(r,t) = \sum_{n=1}^{\infty} U_n(r,t)$$

The $X_n(r)$ factors are the eigen vectors of the Sturm-Liouville system we get when solving the associated homogeneous problem by separation of variables

$$U_t = \alpha U_{rr}$$

$$U_r(0,t) = 0$$

$$U(1,t) = 0$$

$$U(r,0) = \phi(r/R)$$

From the previous case of a sphere in a constant temperature fluid, we find that the solution is

$$X_n(r) = C_n \sin(n\pi r/R)$$

$f(r,t) = f_1(t)\sin(\pi r/R) + f_2(t)\sin(2\pi r/R) + \ldots + f_n(t)\sin(n\pi r/R)$

To find $f_n(t)$, multiply each side of the equation by $\sin(m\pi r/R)$ and integrate from 0 to R (with respect to r) or 0 to 1 (with respect to r/R):

$$\int_0^R f(r,t)\sin(\pi mr/R)\,dr = \sum_{n=1}^{\infty} f_n(t)\int_0^R \sin(\pi mr/R)\sin(\pi nr/R)\,dr$$

Let $r/R = E$ and $dr/R = dE$

$$\int_0^R f(r,t)\sin(\pi mE)\,dr = \sum_{n=1}^{\infty} f_n(t)\int_0^1 \sin(\pi mE)\sin(\pi nE)\,dE$$

$$f_n(t) = 2\int_0^1 f(r,t)\sin(n\pi E)\,dE$$

We now find the response $U_n(r,t) = T_n(t)X_n(r)$ to input $f_n(t)X_n(r)$

We replace f(r,t) by its decomposition

$$f(r,t) = \sum_{n=1}^{\infty} f_n(t)\sin(n\pi r/R)$$

and try to find individual responses

$$U(r,t) = \sum_{n=1}^{\infty} T_n(t)\sin(n\pi r/R)$$

Substituting into the PDE system:

$$U_t = \alpha\, U_{rr} + \sum_{n=1}^{\infty} f_n(t)\sin(n\pi r/R)$$

$$U(0,t) = 0$$

$$U(1,t) = 0$$

$$U(r,0) = \phi(r/R)$$

gives:

$$\sum_{n=1}^{\infty} T_n'(t)\sin(n\pi E) = -\alpha \sum_{n=1}^{\infty} (n\pi/R)^2 T_n(t)\sin(n\pi E) + \sum_{n=1}^{\infty} f_n(t)\sin(n\pi E)$$

BC#1 $\sum_{n=1}^{\infty} T_n'(t)\sin(0) = 0$ (says nothing: 0 = 0)

BC#2 $\sum_{n=1}^{\infty} T_n(t)\sin(n\pi R/R) = 0$ (0 = 0)

IC $\sum_{n=1}^{\infty} T_n(0)\sin(n\pi r/R) = \phi(r/R)$

$T_n'(t)$ will satisfy the initial value problem

$$T_n'(t) + \alpha (n\pi/R)^2 T_n(t) = f_n(t)$$

$$T_n(0) = 2\int_0^1 \phi(E)\sin(n\pi E)dE = A_n$$

This ordinary differential equation has the solution:

$$T_n(t) = A_n \exp\{-\alpha (n\pi/R)^2 t\} + \int_0^t \exp\{-\alpha (n\pi/R)^2 (t-\tau)\} fn(\tau)d\tau$$

$$\int_0^1 \exp\{-\alpha (n\pi/R)^2 (t-\tau)\} fn(\tau)d\tau =$$

$$\int_0^t \exp\{-\alpha (n\pi/R)^2 (t-\tau)\} * 2\int_0^1 f(E,t)\sin(n\pi E)dE\, d\tau$$

$$= 2\int_0^t \exp\{-\alpha (n\pi/R)^2 (t-\tau)\}\int_0^1 -P'RE\sin(n\pi E)dE\, d\tau$$

$$= -2P'R\int_0^t \exp\{-\alpha (n\pi/R)^2 (t-\tau)\}\int_0^1 E\sin(n\pi E)dE\, d\tau$$

$$\int_0^1 E\sin(n\pi E)dE = \left[\frac{-E\cos(n\pi E)}{n\pi} + \frac{\sin(n\pi E)}{(n\pi)^2}\right]_0^1$$

$$= \frac{(-1)^{n+1}}{(n\pi)}$$

$$\int_0^1 \exp\{-\alpha \frac{n\pi}{R}^2 (t-\tau)\} fn(\tau)d\tau = -2P'R \frac{(-1)^{n+1}}{(n\pi)} \int_0^t \exp\{-\alpha \frac{n\pi}{R}^2 (t-\tau)\}d\tau$$

$$U(r,t) = \sum_{n=1}^{\infty} T_n(t)\sin(n\pi E)$$

$$T_n(t) = A_n\exp\{-\alpha (n\pi/R)^2 t\} - 2P'R \frac{(-1)^{n+1}}{(n\pi)} \int_0^t \exp\{-\alpha \frac{n\pi}{R}^2 (t-\tau)\}d\tau$$

$$U(r,t) = \sum_{n=1}^{\infty} [A_n\exp\{-\alpha (n\pi/R)^2 t\}\sin(n\pi E)] -$$

$$\sum_{n=1}^{\infty} \left[\sin(n\pi E)\ 2P'R \frac{(-1)^{n+1}}{(n\pi)} \int_0^t \exp\{-\alpha \frac{n\pi}{R}^2 (t-\tau)\}d\tau\right.$$

where $$A_n = 2\int_0^1 \phi(E)\sin(n\pi E)dE$$

$$\int_0^1 \exp\{-\alpha (n\pi/R)^2 (t-\tau)\}d\tau = \frac{1 - \exp\{-\alpha (n\pi/R)^2 t\}}{\alpha(n\pi/R)^2}$$

$$U(r,t) = \sum_{n=1}^{\infty} [A_n\exp\{-\alpha (n\pi/R)^2 t\}\sin(n\pi E)] -$$

$$\sum_{n=1}^{\infty} \left[\sin(n\pi E) \frac{2P'R^3(-1)^{n+1}}{\alpha\ n^3\pi^3} \quad 1 - \exp\{-\alpha \frac{n\pi}{R}^2 t\}\right]$$

where $$A_n = 2\int_0^1 \phi(E)\sin(n\pi E)dE$$

For the case where the initial temperature profile $\phi(E)=0$, $A_n=0$

$$U(r,t) = -\sum_{n=1}^{\infty} \left[\sin(n\pi E) \frac{2P'R^3(-1)^{n+1}}{\alpha\ n^3\pi^3} \quad 1 - \exp\{-\alpha \frac{n\pi}{R}^2 t\}\right]$$

$$U(r,t) = y(r,t) = r(T-T_B)$$

$$T = y(r,t)/r + T_B$$

$$T(r,t)=T_0 + P't + \sum_{n=1}^{\infty}\left[\sin(n\pi E)\,\frac{2P'R^3(-1)^n}{\alpha\, n^3\pi^3 r}\left\{1 - \exp\left(-\alpha\left(\frac{n\pi}{R}\right)^2 t\right)\right\}\right]$$

The general solution is

$$T(r,t) = T_0 + P't + \sum_{n=1}^{\infty}\left[A_n/r\,\exp(-\alpha\,(n\pi/R)^2 t)\sin(n\pi E)\right] -$$

$$\sum_{n=1}^{\infty}\left[\sin(n\pi E)\,\frac{2P'R^3(-1)^{n+1}}{\alpha\, n^3\pi^3 r}\left\{1 - \exp\left(-\alpha\left(\frac{n\pi}{R}\right)^2 t\right)\right\}\right]$$

where

$$A_n = 2\int_0^1 \phi(E)\sin(n\pi E)\,dE$$

7.3 Comparison with Crank Solution

The case of a sphere immersed in an electrically heated fluid is similar to an equation found in Crank's text for mass diffusion. This offers the opportunity to confirm our expression with that found in the literature. Crank considers a case for mass transfer into a sphere where the surface concentration varies linearly with time, i.e. $Z(t)=P't$. His solution, rewritten for the analogous heat transfer problem, is:

$$T = T_0 + P't - P'/6\alpha\ (R^2-r^2) -$$

$$\frac{2P'R^3}{\pi^3 r\alpha} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \exp(-\alpha\ n^2\pi^2 t/R^2)\sin(n\pi r/R)$$

Our solution, expanded, is

$$T(r,t)=T_0 + P't + \sum_{n=1}^{\infty}\left[\sin(n\pi E)\ \frac{2P'R^3(-1)^n}{\alpha n^3\pi^3 r}\right] -$$

$$\frac{2P'R^3}{\pi^3 r\alpha} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \exp(-\alpha\ n^2\pi^2 t/R^2)\sin(n\pi r/R)$$

For solutions to match,

$$\sum_{n=1}^{\infty}\left[\sin(n\pi E)\ \frac{2P'R^3(-1)^n}{\alpha\ n^3\pi^3 r}\right] = \frac{-P'}{6\ \alpha}\ (R^2-r^2)$$

$$-\frac{R^3}{\pi^3 r} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}\sin(n\pi r/R) = \frac{1}{12}\ (R^2-r^2)$$

$$-\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}\sin(n\pi r/R) = \frac{\pi^3}{12R^3}(R^2-r^2)$$

Let $f(r) = \frac{\pi^3}{12R^3}\ (R^2r-r^3)$

and make a Fourrier Expansion of f(r)

$$f(r) = \sum_{n=1}^{\infty} b_n \sin(n\pi r/R)$$

where $$b_n = \frac{2}{R}\int_0^R f(r)\sin(n\pi r/R)\,dr$$

$$b_n = \frac{2}{R}\int_0^R \frac{\pi^3}{12R^3}(R^2r - r^3)\sin(n\pi r/R)\,dr$$

$$= \frac{\pi^3}{6R^4}\left[\int_0^R R^2 r\sin(n\pi r/R)\,dr - \int_0^R r^3\sin(n\pi r/R)\,dr\right]$$

The left integral has been solved before, and can be expressed,

$$\frac{-R^4}{n\pi}(-1)^n$$

The right integral can be expressed as

$$\left[\frac{3(n\pi/R)^2r^2 - 6}{(n\pi r/R)^4}\sin(n\pi r/R) - \frac{(n\pi/R)^2r^3 - 6r}{(n\pi r/R)^3}\cos(n\pi r/R)\right]_0^R$$

$$-\left[\frac{R^3}{(n\pi/R)} - \frac{6R}{(n\pi/R)^3}\right]\cos(n\pi)$$

$$-(R^4/n\pi)\cos(n\pi) + 6R^4/(n\pi)^3\cos(n\pi)$$

$$\frac{-R^4}{n\pi}(-1)^n + \frac{6R^4}{(n\pi)^3}(-1)^n$$

Now combining the two integrals

$$b_n = -\frac{\pi^3}{(n\pi)^3}(-1)^n$$

$$f(r) = \sum_{n=1}^{\infty} -\frac{\pi^3}{(n\pi)^3}(-1)^n\sin(n\pi r/R)$$

$$f(r) = - \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \sin(n\pi r/R)$$

Thus, from the Fourrier Expansion,

$$\frac{\pi^3}{12R^3}(R^2r-r^3) = - \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \sin(n\pi r/R)$$

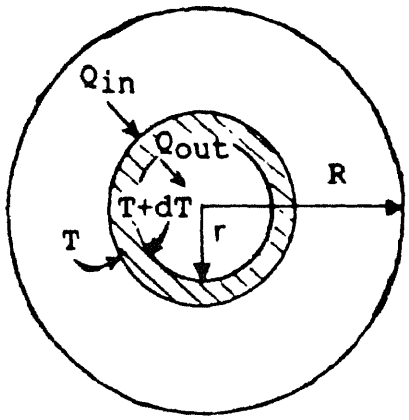
Multiplying both sides by $(-2R^3P')/(r\ \alpha\pi^3)$

$$\sum_{n=1}^{\infty} \left[\sin(n''E) \frac{2P'R^3(-1)^n}{\alpha n^3 \pi^3 r} \right] = \frac{-P'}{6\ \alpha}(R^2-r^2)$$

This is what we wanted to show originally, therefore, the solutions match.

7.4 Finite External Heat Transfer Coefficient

In this section, an equation describing the temperature distribution in a sphere suspended in the electrically heated fluid above is derived. An energy balance around a shell in the sphere yields a general partial differential equation describing the radial temperature profile. However, in this case the boundary condition of the sphere must include the bulk temperature of the fluid changing with time. This leads to a non-homogeneous partial differential equation, which is solved by using both an Eigen function expansion of the non-homogeneous term and separation of variables as in the previous cases. This case is different from the previous one in that the boundary condition at the surface of the sphere (r = R) considers an external heat transfer coefficient which results in a surface temperature that depends on both fluid and solid heat transfer characteristics.



The energy into and out of the shell:

$$Q_{in} = -kA\, \partial T/\partial r = -k\,(4\pi r^2)\, \partial T/\partial r$$

$$Q_{out} = -k4\pi(r+dr)^2 * \partial(T+dT)/\partial r$$

$$= -k4\pi(r+dr)^2 \quad \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr$$

The overall energy balance:

$$Q_{net} = Q_{in} + Q_{out}$$

$$= 4\pi k \quad (r+dr)^2 \quad \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr \quad - r^2 \frac{\partial T}{\partial r}$$

Expanding, simplifying, and neglecting smaller terms

$$Q_{net} = 4\pi k(r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r))$$

The rate of energy accumulated in the shell is also

$$E = 4\pi r^2 dr \rho C_p \partial T/\partial r$$

These terms are equal, so that

$$4\pi r^2 dr \rho C_p \partial T/\partial r = 4\pi k(r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r))$$

Collecting like terms and simplifying:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \quad \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r}$$

This is our basic equation describing the temperature of the sphere as a function of time and radius.

Initial and Boundary Conditions

Boundary Conditions

In this case, the bulk temperature of the fluid changes with time, and the temperature at the surface of the sphere depends on both fluid and solid heat transfer characteristics.

BC # 1 : at r = R $\quad h(T_B-T) = k\, dT/dr$

$T = T_B = T_0 + P't$ at $t>0$

BC # 2 : at the center of the sphere (r=0), conduction is zero, or $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$ when $t>0$

Initial Condition

At t = 0, the sphere is initially at some temperature distribution $\phi(r/R)$: $\quad T(r,0) = \phi(r/R)$

Let $\alpha = k/\ C_p$ (thermal diffusivity constant)

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}$$

Letting $y = r(T-T_B)$, then $T = y/r + T_B$

Taking derivatives and simplifying

$$\frac{\partial T}{\partial t} = \frac{1}{r}\ \frac{y}{t} + P'$$

$$\frac{\partial T}{\partial r} = \frac{-1}{r^2}\ y + \frac{1}{r}\ \frac{\partial y}{\partial r}$$

$$\frac{^2T}{r^2} = \frac{1}{r}\frac{\partial^2 y}{\partial r^2} - \frac{2}{r^2}\ \frac{\partial y}{\partial r} + \frac{2}{r^3}\ y$$

Substitution into the PDE with some collection of terms yields:

$$\frac{1}{\alpha}\ \frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} + \frac{-P'r}{\alpha}$$

$$\frac{\partial y}{\partial t} = -P'r + \alpha\ \frac{\partial^2 y}{\partial r^2}$$

Substitution into the Boundary Conditions:

BC # 1 $\quad y = r(T - T_B) \quad$ then $T = T_B + y/r$

$$-h\left(\frac{y}{r}\right) = +k\left(-\frac{y}{r^2} + \frac{1}{r}\frac{\partial y}{\partial r}\right)$$

$$+hy = +k(y/r) - k(dy/dr)$$

$$k(dy/dr) - k(y/r) + hy = 0 \qquad \text{at } r = R$$

BC # 2 at the center of the sphere,

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \qquad \text{when } t>0$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 = \frac{-y}{r^2} \qquad \text{so } y = 0$$

IC For $t = 0$, $T(r,0) = \phi(r/R)$

Assume the Boundary Conditions are homogeneous, but the PDE is not.

$$\frac{\partial y}{\partial t} = -P'r + \alpha\frac{\partial^2 y}{\partial r^2} \quad \text{can be expressed } U_t = \alpha U_{rr} + f(r,t)$$

Decomposing $f(r,t)$ into simple components

$$f(r,t) = f_1(t)X_1(r) + f_2(t)X_2(r) + \ldots + f_n(t)X_n(r) + \ldots$$

Find the response $U_n(r,t)$ to each of these individual components, $f_n(t)X_n(r)$. The solution to the problem is then

$$U(r,t) = \sum_{n=1}^{\infty} U_n(r,t)$$

The $X_n(r)$ factors are the eigen vectors of the Sturm-Liouville

system we get when solving the associated homogeneous problem by separation of variables

$$U_t = \alpha U_{rr}$$

$$U(0,t) = 0$$

$$U(1,t) = kU_r - kU/r + hU = 0$$

$$U(r,0) = \phi(r/R)$$

From the previous case of a sphere in a constant temperature fluid, we find that the solution is

$$X_n(r) = C_n \sin(\beta_n r/R)$$

where $\beta_n \cot\beta_n + L - 1 = 0$

$$f(r,t) = f_1(t)\sin(\beta_1 r/R) + f_2(t)\sin(\beta_2 r/R) + \ldots + f_n(t)\sin(\beta_n r/R)$$

To find $f_n(t)$, multiply each side of the equation by $\sin(\beta_m r/R)$ and integrate from 0 to R (with respect to r) or 0 to 1 (with respect to r/R):

$$\int_0^R f(r,t)\sin(\beta_m r/R)\,dr = \sum_{n=1}^{\infty} f_n(t)\int_0^R \sin(\beta_m r/R)\sin(\beta_n r/R)\,dr$$

Let $r/R = E$ and $dr/R = dE$

$$\int_0^R f(r,t)\sin(\beta_m E)\,dr = \sum_{n=1}^{\infty} f_n(t)\int_0^1 \sin(\beta_m E)\sin(\beta_n E)\,dE$$

$$f_n(t) = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n}\int_0^1 f(r,t)\sin(\beta_n E)\,dE$$

$$G = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n}$$

$$f_n(t) = G\int_0^1 f(r,t)\sin(\beta_n E)\,dE$$

We now find the response $U_n(r,t) = T_n(t)X_n(r)$ to input $f_n(t)X_n(r)$

We replace f(r,t) by its decomposition

$$f(r,t) = \sum_{n=1}^{\infty} f_n(t)\sin(\beta_n r/R)$$

and try to find individual responses

$$U(r,t) = \sum_{n=1}^{\infty} T_n(t)\sin(\beta_n r/R)$$

Substituting into the PDE system:

$$U_t = \alpha U_{rr} + \sum_{n=1}^{\infty} f_n(t)\sin(\beta_n r/R)$$

$$U_r(0,t) = 0$$

$$U(1,t): \quad kU_r - kU/R + hU = 0 \quad \text{at } r = R$$

$$U(r,0) = \phi(r/R)$$

gives:

$$\sum_{n=1}^{\infty} T_n'(t)\sin(\beta_n E) = -\sum_{n=1}^{\infty} \alpha(\beta_n/R)^2 T_n(t)\sin(\beta_n E) + \sum_{n=1}^{\infty} f_n(t)\sin(\beta_n E)$$

BC#2 $$\sum_{n=1}^{\infty} T_n(t)\sin(\beta_n(0)/R) = 0 \quad (0 = 0)$$

IC $$\sum_{n=1}^{\infty} T_n(0)\sin(\beta_n r/R) = \phi(r/R)$$

BC#1 $$k\sum_{n=1}^{\infty} Tn(t)(\beta_n/R)\cos(\beta_n E) - k/R \sum_{n=1}^{\infty} T_n(t)\sin(\beta_n E)$$

$$+ h \sum_{n=1}^{\infty} T_n(t)\sin(\beta_n E) = 0 \quad \text{at } r = R$$

At r = R, E = 1

$$\sum_{n=1}^{\infty} (k(\beta_n/R)\cos\beta_n - (k/R)\sin\beta_n + h\sin\beta_n)\, Tn(t) = 0$$

$$(k/R)\beta_n\cot\beta_n - (k/R - h) = 0$$

$$\beta_n\cot\beta_n - 1 + hR/k = 0$$

$$\beta_n\cot\beta_n + L - 1 = 0$$

This is our Eigen vector

$T_n'(t)$ will satisfy the initial value problem

$$T_n'(t) + \alpha (\beta_n/R)^2 T_n(t) = f_n(t)$$

$$T_n(0) = G\int_0^1 \phi(E)\sin(\beta_n E)\,dE = A_n$$

This ordinary differential equation has the solution:

$$T_n(t) = A_n\exp\{-\alpha (\beta_n/R)^2 t\} + \int_0^t \exp\{-\alpha (\beta_n/R)^2 (t-\tau)\} fn(\tau)\,d\tau$$

$$\int_0^1 \exp\{-\alpha (\beta_n/R)^2 (t-\tau)\} fn(\tau)\,d\tau =$$

$$\int_0^t \exp\{-\alpha (\beta_n/R)^2 (t-\tau)\} * G\int_0^1 f(E,t)\sin(\beta_n E)\,dE\,d\tau$$

$$= G\int_0^t \exp\{-\alpha (\beta_n/R)^2 (t-\tau)\}\int_0^1 -P'RE\sin(\beta_n E)\,dE\,d\tau$$

$$= -GP'R\int_0^t \exp\{-\alpha (\beta_n/R)^2 (t-\tau)\}\int_0^1 E\sin(\beta_n E)\,dE\,d\tau$$

$$\int_0^1 E\sin(\beta_n E)\,dE = \left[\frac{-E\cos(\beta_n E)}{\beta_n} + \frac{\sin(\beta_n E)}{(\beta_n)^2}\right]_0^1$$

$$= \frac{-\cos\beta_n}{(\beta_n)} + \frac{(\sin\beta_n)/\beta_n}{(\beta_n)}$$

$$f_n(t) = GP'R/\beta_n \{\cos\beta_n - (\sin\beta_n)/\beta_n\}$$

$$f_n(t) = \frac{2P'R\{\cos\beta_n - (\sin\beta_n)/\beta_n}{\beta_n - \sin\beta_n\cos\beta_n}$$

Substituting back into the integral,

$$\int_0^1 \exp\{-\alpha \frac{\beta_n}{R}^2 (t-\tau)\}fn(\tau)d\tau = \int_0^t \exp\{-\alpha \frac{\beta_n}{R}^2 (t-\tau)\}\frac{2P'R\{\cos\beta_n-\sin\beta_n/\beta_n\}}{\beta_n - \sin\beta_n\cos\beta_n}$$

$$= \frac{2P'R\{\cos\beta_n - (\sin\beta_n)/\beta_n\}}{\beta_n - \sin\beta_n\cos\beta_n}\int_0^t \exp\{-\alpha (\beta_n/R)^2(t-\tau)\}d\tau$$

$$= \frac{2P'R\{\cos\beta_n - (\sin\beta_n)/\beta_n\}}{\beta_n - \sin\beta_n\cos\beta_n}\left[\frac{1 - \exp\{-\alpha (\beta_n/R)^2t}{\alpha (\beta_n/R)^2}\right]$$

$$= \frac{2P'R^3\{\cos\beta_n - (\sin\beta_n)/\beta_n\}}{(\beta_n^3 - \beta_n^2\sin\beta_n\cos\beta_n)}[1 - \exp\{-\alpha (\beta_n/R)^2t]$$

$$T_n(t) = A_n e^{-\alpha(\beta_n/R)^2t} + \frac{2P'R^3\{\cos\beta_n-(\sin\beta_n)/\beta_n\}}{(\beta_n^3-\beta_n^2\sin\beta_n\cos\beta_n)}[1 - e^{-\alpha(\beta_n/R)^2t}]$$

$$U(r,t) = \sum_{n=1}^{\infty} T_n(t)\sin(\beta_n E)$$

$$U(r,t) = \sum_{n=1}^{\infty} [A_n\exp\{-\alpha (\beta_n/R)^2t\}\sin(\beta_n E)] +$$

$$\sum_{n=1}^{\infty}\left[\sin(\beta_n E)\frac{2P'R^3\{\cos\beta_n - (\sin\beta_n)/\beta_n\}}{(\beta_n^3 - \beta_n^2\sin\beta_n\cos\beta_n)}[1 - \exp\{-\alpha (\beta_n/R)^2t]\right]$$

$$U(r,t) = y(r,t) = r(T - T_B)$$

$$T = y(r,t)/r + T_B \quad \text{where} \quad T_B = T_0 + P't$$

$$T = T_0 + P't + \sum_{n=1}^{\infty} [A_n/r \exp\{-\alpha (\beta_n/R)^2 t\}\sin(\beta_n E)] +$$

$$\sum_{n=1}^{\infty} \left[\sin(\beta_n E) \frac{2P'R^3\{\cos\beta_n - (\sin\beta_n)/\beta_n\}}{r(\beta_n^3 - \beta_n^2 \sin\beta_n \cos\beta_n)} [1 - \exp\{-\alpha (\beta_n/R)^2 t\}]\right]$$

To find A_n, apply initial conditions

$$y(r,0) = \phi(E)$$

$$\phi(E) = \sum_{n=1}^{\infty} A_n \sin(\beta_n E)$$

Expand the IC as a sum of eigen functions

Multiply each side by $\sin(\beta_m E)$ and integrate from 0 to 1 with E

$$\int_0^1 \phi(E)\sin(\beta_m E)dE = \sum_{n=1}^{\infty} A_n \int_0^1 \sin(\beta_m E)\sin(\beta_n E)dE$$

$$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n \cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)dE$$

The final solution, then, is:

$$T = T_0 + P't + \sum_{n=1}^{\infty} [A_n/r \exp\{-\alpha (\beta_n/R)^2 t\}\sin(\beta_n E)] +$$

$$\sum_{n=1}^{\infty} \left[\sin(\beta_n E) \frac{2P'R^2\{\cos\beta_n - (\sin\beta_n)/\beta_n\}}{E(\beta_n^3 - \beta_n^2 \sin\beta_n \cos\beta_n)} [1 - \exp\{-\alpha (\beta_n/R)^2 t\}]\right]$$

where $$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n \cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)dE$$

For the case where the initial temperature profile $\phi(E)=0$, $A_n=0$

$$T(r,t)=T_0 + P't +$$

$$\sum_{n=1}^{\infty} \left[\sin(\beta_n E) \frac{2P'R^2\{\cos\beta_n - (\sin\beta_n)/\beta_n\}}{E(\beta_n^3 - \beta_n^2 \sin\beta_n \cos\beta_n)} [1 - \exp\{-\alpha (\beta_n/R)^2 t\}]\right]$$

And at the center of the sphere

$$T(r,t)=T_0 + P't +$$

$$\sum_{n=1}^{\infty}\left[\frac{2P'R^2(\cos\beta_n - (\sin\beta_n)/\beta_n)}{(\beta_n^2 - \beta_n\sin\beta_n\cos\beta_n)}\ [1 - \exp\{-\alpha\ (\beta_n/R)^2 t]\right]$$

8. TEMPERATURE DISTRIBUTION IN A SPHERE IMMERSED IN A STEAM HEATED FLUID

In the following sections, equations will be derived which express the:

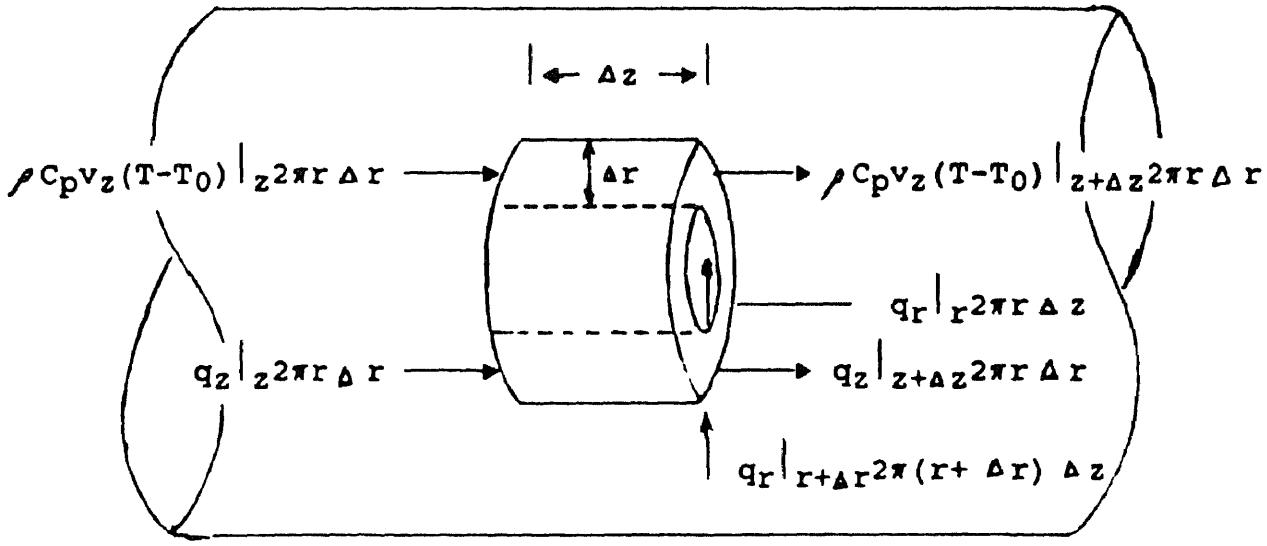
- Time-temperature profile of a steam heated fluid.
- Temperature distribution in a sphere immersed in the above fluid:
    - With an infinite external heat transfer coefficient.
    - With a finite external heat transfer coefficient.

A solution is available in the literature for the infinite external heat transfer coefficient case, and a comparison between our derived solution and that found in the literature is made.

8.1 Temperature Distribution in a Steam Heated Fluid

In this section an equation describing the temperature-time profile of a fluid moving through an electical heater is derived.

The derivation begins with an energy balance around a shell in a section of pipe. As mentioned previously, assumptions of ideal plug flow with no forward or backward conduction of heat are employed. For this case, a constant surface temperature for the pipe, associated with condensing steam, is assumed.



An energy balance is made around a shell of fluid moving through the section of pipe:

Energy In - Energy Out = Energy Accumulated = 0

$$q_z\big|_z 2\pi r\,\Delta r + q_r\big|_{r+\Delta r} 2\pi (r+\Delta r)\,\Delta z + \rho C_p v_z (T-T_0)\big|_z 2\pi r\,\Delta r$$
$$- q_z\big|_{z+\Delta z} 2\pi r\,\Delta r + q_r\big|_r 2\pi r\,\Delta z + \rho C_p v_z (T-T_0)\big|_{z+\Delta z} 2\pi r\,\Delta r = 0$$

Divide by $2\pi\,\Delta r\,\Delta z$ and combine terms

$$\frac{(rq_r)\big|_{r+\Delta r} - (rq_r)\big|_r}{\Delta r} + r\,\frac{q_z\big|_{z+\Delta z} - q_z\big|_z}{\Delta z} + r\rho C_p v_z\,\frac{T\big|_{z+\Delta z} - T\big|_z}{\Delta z} = 0$$

As Δr and Δz approach zero

$$\rho C_p v_z\,\frac{\partial T}{\partial z} = -\frac{1}{r}\,\frac{\partial}{\partial r}(rq_r) - \frac{\partial q_z}{\partial z}$$

Assume v_z is constant = dz/dt

The temperature of the pipe is constant and equal to T_w.

q_r steam heating $= -h_A(T_w-T)$

$q_z = -k\,\partial T/\partial z$

$$\rho C_p v_z\,\frac{\partial T}{\partial z} = -\frac{1}{r}\,\frac{\partial}{\partial r}(-rh_A(T_w-T)) - \frac{\partial}{\partial z} -k\,\frac{\partial T}{\partial z}$$

$$\rho C_p v_z\,\frac{\partial T}{\partial z} = \frac{h_A}{r}(T_w-T) + k\,\frac{\partial^2 T}{\partial z^2}$$

Assume $\partial^2 T/\partial z^2$ small compared to $T/\ z$ term, then

$$\rho C_p v_z\,\frac{\partial T}{\partial z} = \frac{h_A}{r}(T_w-T)$$

$$\frac{dT}{(T_w-T)} = \frac{h_A\,dz}{r\rho C_p v_z}$$

At $z = 0$, $T_B = T_0$ and at $z = z$, $T_B = T_B$

$$-\ln(T_w-T)\Big|_{T0}^{TB} = \frac{h_A\,dz}{r\rho C_p v_z}$$

Let $B = h_A/(r\rho C_p v_z)$

$$\ln\left[\frac{T_w-T_0}{T_w-T_B}\right] = Bz$$

Let $B' = h_A/(r\rho C_p)$

$$\ln\left[\frac{T_w-T_0}{T_w-T_B}\right] = B't$$

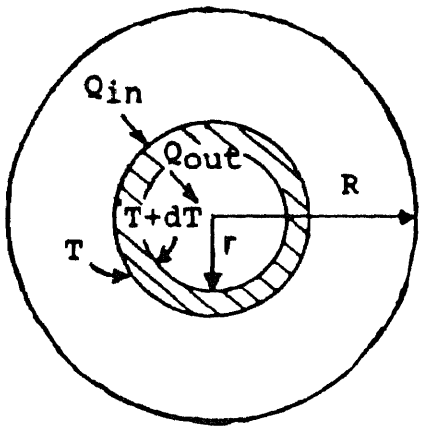
Taking the exponential of both sides and rearranging gives:

$$T_B = T_w + (T_0-T_w)\exp(-B't)$$

Thus, the temperature profile of a steam heated fluid is an exponential one.

8.2 Infinite External Heat Transfer Coefficient

In this section, an equation describing the temperature distribution in a sphere suspended in the steam heated fluid above is derived. An energy balance around a shell in the sphere yields a general partial differential equation describing the radial temperature profile. However, in this case the boundary condition of the sphere must include the bulk temperature of the fluid changing with time. This leads to a non-homogeneous partial differential equation, which is solved by using both an Eigen function expansion of the non-homogeneous term and separation of variables as in the previous cases.



The energy into and out of a shell in the sphere:

$$Q_{in} = -kA\,\partial T/\partial r = -k\,(4\pi r^2)\,\partial T/\partial r$$

$$Q_{out} = -k4\pi(r+dr)^2 * \partial(T+dT)/\partial r$$

$$= -k4\pi(r+dr)^2 \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\,dr\right)$$

The overall energy balance around the shell:

$$Q_{net} = Q_{in} + Q_{out}$$

$$= 4\pi k \left[(r+dr)^2 \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr \right) - r^2 \frac{\partial T}{\partial r} \right]$$

Expanding, simplifying, and neglecting smaller terms

$$Q_{net} = 4\pi k(r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r))$$

The rate of energy accumulated in the shell is also

$$E = 4\pi r^2 dr \rho C_p \partial T/\partial r$$

These terms are equal, so that

$$4\pi r^2 dr \rho C_p \partial T/\partial r = 4\pi k(r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r))$$

Collecting like terms and simplifying:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right]$$

This is our basic equation describing the temperature of the sphere as a function of time and radius.

Initial and Boundary Conditions

Boundary Conditions

In this case, the surface temperature of the sphere is equal to the bulk temperature of the fluid, which is changing exponentially with time.

BC # 1 : at $r = R$ $T = T_B = T_w + \exp(-B't)(T_0 - T_w)$ at $t>0$

BC # 2 : at the center of the sphere (r=0), conduction is zero, or $\left. \frac{\partial T}{\partial r} \right|_{r=0} = 0$ when $t>0$

Initial Condition

At t = 0, the sphere is initially at some temperature distribution $\phi(r/R)$: $T(r,0) = \phi(r/R)$

Let $\alpha = k/\rho C_p$ (thermal diffusivity constant)

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}$$

Letting $y = r(T-T_B)$, then $T = y/r + T_B$

Taking derivatives and simplifying

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{y}{t} + -B'\exp(-B't)(T_0-T_w)$$

$$\frac{\partial T}{\partial r} = \frac{-1}{r^2}y + \frac{1}{r}\frac{\partial y}{\partial r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{1}{r}\frac{\partial^2 y}{\partial r^2} - \frac{2}{r^2}\frac{\partial y}{\partial r} + \frac{2}{r^3}y$$

Substitution into the PDE with some collection of terms yields:

$$\frac{1}{\alpha}\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} + \frac{rB'(T_0-T_w)\exp(-B't)}{\alpha}$$

$$\frac{\partial y}{\partial t} = \alpha\frac{\partial^2 y}{\partial r^2} + rC'\exp(-B't)$$

where $C' = B'(T_0-T_w)$

Substitution into the Boundary Conditions:

BC # 1 at r=R, $T = T_B$ $\quad y = r(T_B - T_B) = 0 \quad y=0$

BC # 2 at the center of the sphere,

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \quad \text{when } t>0$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 = \frac{-y}{r^2} \qquad \text{so} \quad y = 0$$

IC For $t = 0$, $T(r,0) = \phi(r/R)$

The Boundary Conditions are homogeneous, but the PDE is not.

It can be expressed as $U_t = \alpha U_{rr} + f(r,t)$

Decomposing $f(r,t)$ into simple components

$$f(r,t) = f_1(t)X_1(r) + f_2(t)X_2(r) + \ldots + f_n(t)X_n(r) + \ldots$$

Find the response $U_n(r,t)$ to each of these individual components, $f_n(t)X_n(r)$. The solution to the problem is then

$$U(r,t) = \sum_{n=1}^{\infty} U_n(r,t)$$

The $X_n(r)$ factors are the eigen vectors of the Sturm-Liouville system we get when solving the associated homogeneous problem by separation of variables

$$U_t = \alpha\, U_{rr}$$

$$U_r(0,t) = 0$$

$$U(1,t) = 0$$

$$U(r,0) = \phi(r/R)$$

From the previous case of a sphere in a constant temperature fluid, we find that the solution is

$$X_n(r) = C_n \sin(n\pi r/R)$$

$$f(r,t) = f_1(t)\sin(\pi r/R) + f_2(t)\sin(2\pi r/R) + \ldots + f_n(t)\sin(n\pi r/R)$$

To find $f_n(t)$, multiply each side of the equation by $\sin(m\pi r/R)$

and integrate from 0 to R (with respect to r) or 0 to 1 (with respect to r/R):

$$\int_0^R f(r,t)\sin(\pi mr/R)dr = \sum_{n=1}^{\infty} f_n(t)\int_0^R \sin(\pi mr/R)\sin(\pi nr/R)dr$$

Let $r/R = E$ and $dr/R = dE$

$$\int_0^R f(r,t)\sin(\pi mE)dr = \sum_{n=1}^{\infty} f_n(t)\int_0^1 \sin(\pi mE)\sin(\pi nE)dE$$

$$f_n(t) = 2\int_0^1 f(r,t)\sin(n\pi E)dE$$

We now find the response $U_n(r,t) = T_n(t)X_n(r)$ to input $f_n(t)X_n(r)$

We replace $f(r,t)$ by its decomposition

$$f(r,t) = \sum_{n=1}^{\infty} f_n(t)\sin(n\pi r/R)$$

and try to find individual responses

$$U(r,t) = \sum_{n=1}^{\infty} T_n(t)\sin(n\pi r/R)$$

Substituting into the PDE system:

$$U_t = \alpha U_{rr} + \sum_{n=1}^{\infty} f_n(t)\sin(n\pi r/R)$$

$$U(0,t) = 0$$

$$U(1,t) = 0$$

$$U(r,0) = \phi(r/R)$$

gives:

$$\sum_{n=1}^{\infty} T_n'(t)\sin(n\pi E) = -\sum_{n=1}^{\infty} \alpha(n\pi/R)^2 T_n(t)\sin(n\pi E) + \sum_{n=1}^{\infty} f_n(t)\sin(n\pi E)$$

BC#1 $\sum_{n=1}^{\infty} T_n'(t)\sin(0) = 0$ (says nothing: 0 = 0)

BC#2 $\sum_{n=1}^{\infty} T_n(t)\sin(n\pi R/R) = 0$ (0 = 0)

IC $\sum_{n=1}^{\infty} T_n(0)\sin(n\pi r/R) = 0(r/R)$

$T_n'(t)$ will satisfy the initial value problem

$$T_n'(t) + \alpha (n\pi/R)^2 T_n(t) = f_n(t)$$

$$T_n(0) = 2\int_0^1 \phi(E)\sin(n\pi E)dE = A_n$$

This ordinary differential equation has the solution:

$$T_n(t) = A_n\exp\{-\alpha (n\pi/R)^2 t\} + \int_0^t \exp\{-\alpha (n\pi/R)^2 (t-\tau)\} f_n(\tau)d\tau$$

$$\int_0^1 \exp\{-\alpha (n\pi/R)^2 (t-\tau)\} f_n(\tau)d\tau =$$

$$\int_0^t \exp\{-\alpha (n\pi/R)^2 (t-\tau)\} * 2\int_0^1 f(E,t)\sin(n\pi E)dE\, d\tau$$

$$= 2\int_0^t \exp\{-\alpha (n\pi/R)^2 (t-\tau)\}\int_0^1 -C'RE\sin(n\pi E)\exp(-B't)dE\, d\tau$$

$$= -2C'R\int_0^t \exp\{-\alpha (n\pi/R)^2 (t-\tau)\}\int_0^1 \exp\{-B't)E\sin(n\pi E)dE\, d\tau$$

$$\int_0^1 E\sin(n\pi E)dE = \left[\frac{-E\cos(n\pi E)}{n\pi} + \frac{\sin(n\pi E)}{(n\pi)^2}\right]_0^1$$

$$= \frac{(-1)^{n+1}}{(n\pi)}$$

$$\int_0^1 \exp\{-\alpha \frac{n\pi}{R}^2 (t-\tau)\}fn(\tau)d\tau =$$

$$-2C'R \frac{(-1)^n}{(n\pi)} \int_0^t \exp\{-\alpha \frac{n\pi}{R}^2 (t-\tau)\} \exp\{-B'\tau\}\, d\tau$$

$$\int_0^t \exp\{-\alpha \frac{n\pi}{R}^2 (t-\tau)\} \exp\{-B'\tau\}\, d\tau =$$

$$\exp\{-\alpha\ (n\pi/R)^2 t\} \int_0^t \exp\{\alpha\ (n\pi/R)^2 - B')\tau\, d\tau$$

$$\exp\{-\alpha\ (n\pi/R)^2 t\} \left[\frac{\exp\{\ \alpha(n\pi/R)^2 - B')\tau}{(\ \alpha(n\pi/R)^2 - B')} \right]_0^t$$

$$\exp\{-\alpha\ (n\pi/R)^2 t\} \left[\frac{\exp\{\ \alpha(n\pi/R)^2 - B')\tau\ - 1}{\{\alpha\ (n\pi/R)^2 - B')} \right]_0^t$$

$$= \frac{\exp\{-B't\} - \exp\{-\alpha\ (n\pi/R)^2 t\}}{(n\pi/R)^2 - B'}$$

So now,

$$\int_0^1 \exp\{-\alpha \frac{n\pi}{R}^2 (t-\tau)\}fn(\tau)d\tau =$$

$$-2C'R \frac{(-1)^n}{(n\pi)} \left[\frac{\exp\{-B't\} - \exp\{-\alpha\ (n\pi/R)^2 t\}}{(n\pi/R)^2 - B'} \right]$$

$$U(r,t) = \sum_{n=1}^{\infty} T_n(t)\sin(n\pi E)$$

$$T_n(t) = A_n \exp\{-\alpha\ (n\pi/R)^2 t\}$$

$$- 2C'R \frac{(-1)^n}{(n\pi)} \left[\frac{\exp\{-B't\} - \exp\{-\alpha\ (n\pi/R)^2 t\}}{\alpha(n\pi/R)^2 - B'} \right]$$

$$U(r,t) = \sum_{n=1}^{\infty} [A_n \exp\{-\alpha\ (n\pi/R)^2 t\} \sin(n\pi E)]$$

$$- \sum_{n=1}^{\infty} \left[\sin(n\pi E)\ 2C'R\ \frac{(-1)^n}{(n\pi)} \left[\frac{\exp\{-B't\} - \exp\{-\alpha\ (n\pi/R)^2 t\}}{\alpha(n\pi/R)^2 - B'} \right] \right]$$

where $$A_n = 2\int_0^1 \phi(E) \sin(n\pi E)\, dE$$

$$U(r,t) = y(r,t) = r(T-T_B)$$

$$T = y(r,t)/r + T_B$$

$$T(r,t) = T_w + \exp\{-B't\}(T_0-T_w) + \sum_{n=1}^{\infty} [A_n/r \exp\{-\alpha(n\pi/R)^2 t\} \sin(n\pi E)]$$

$$- \sum_{n=1}^{\infty} \left[\sin(n\pi E)\ 2C'R\ \frac{(-1)^n}{r(n\pi)} \left[\frac{\exp\{-B't\} - \exp\{-\alpha\ (n\pi/R)^2 t\}}{\alpha(n\pi/R)^2 - B'} \right] \right]$$

where $$A_n = 2\int_0^1 \phi(E) \sin(n\pi E)\, dE$$

At the center of the sphere,

$$T(r,t) = T_w + \exp\{-B't\}(T_0-T_w) + \sum_{n=1}^{\infty} [A_n/r \exp\{-\alpha(n\pi/R)^2 t\} \sin(n\pi E)]$$

$$- \sum_{n=1}^{\infty} \left[2C'\ \frac{(-1)^n}{(n\pi)} \left[\frac{\exp\{-B't\} - \exp\{-\alpha\ (n\pi/R)^2 t\}}{\alpha(n\pi/R)^2 - B'} \right] \right]$$

For the case where the initial temperature profile $\phi=0$, $A_n=0$

$$T(r,t) = T_w + \exp\{-B't\}(T_0-T_w)$$

$$- \sum_{n=1}^{\infty} \left[\sin(n\pi E)\ 2C'R\ \frac{(-1)^n}{(n\pi)r} \left[\frac{\exp\{-B't\} - \exp\{-\alpha\ (n\pi/R)^2 t\}}{\alpha(n\pi/R)^2 - B'} \right] \right]$$

8.3 Comparison with Crank Solution

The literature affords us the opportunity to confirm this solution. Crank has considered the analogous case for mass transfer in which the surface concentration varies exponentially with time, $F(t) = T(1 - \exp(-B't))$. Then, with some rearrangement and using our nomenclature, his solution can be expressed as:

$$T = T_w - \frac{T_w R}{r} \exp(-B't) \frac{\sin((B'R^2/\alpha)^{1/2}\, r/R)}{\sin(B'R^2/\alpha)^{1/2}} -$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{2B'T_0R}{\pi rn(\alpha\,(n\pi/R)^2 - B')} \exp(-\alpha\,(n\pi/R)^2 t) \sin(n\pi r/R)$$

Our answer for the given initial condition is:

$$T(r,t) = T_w + \exp(-B't)(T_0 - T_w)$$

$$- \sum_{n=1}^{\infty} \left[\sin(n\pi E)\; 2C'R \frac{(-1)^n}{(n\pi)r} \left[\frac{\exp(-B't) - \exp(-\alpha\,(n\pi/R)^2 t)}{\alpha(n\pi/R)^2 - B'}\right]\right]$$

which can be rearranged to give

$$T(r,t) = T_w - T_w \exp(-B't) +$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{2B'T_wR}{\pi rn(\alpha\,(n\pi/R)^2 - B')} \exp(-B't) \sin(n\pi r/R) -$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{2B'T_wR}{\pi rn(\alpha\,(n\pi/R)^2 - B')} \exp(-\alpha\,(n\pi/R)^2 t) \sin(n\pi r/R)$$

For the two solutions to be equal, then

$$- \frac{T_w R}{r} \exp(-B't) \frac{\sin((B'R^2/\alpha)^{1/2}\, r/R)}{\sin(B'R^2/\alpha)^{1/2}} \text{ must equal}$$

$$- T_w \exp(-B't) + \sum_{n=1}^{\infty} (-1)^n \frac{2B'T_wR}{\pi rn(\alpha\ (n\pi/R)^2 - B')} \exp(-B't)\ \sin(n\pi r/R)$$

Assume these expressions are equal, then

$$1 - \frac{R\ \sin((B'R^2/\alpha)^{1/2}\ r/R)}{r\ \sin(B'R^2/\alpha)^{1/2}} = \sum_{n=1}^{\infty}(-1)^n \frac{2B'R}{\pi rn(\alpha\ (n\pi/R)^2 - B')} \sin(n\pi r/R)$$

$$r - R\ \frac{\sin((B'R^2/\alpha)^{1/2}\ r/R)}{\sin(B'R^2/\alpha)^{1/2}} = \sum_{n=1}^{\infty} (-1)^n \frac{2B'R}{\pi n(\alpha\ (n\pi/R)^2 - B')} \sin(n\pi r/R)$$

$$r - R\ \frac{\sin((B'R^2/\alpha)^{1/2}\ r/R)}{\sin(B'R^2/\alpha)^{1/2}} = \quad f(r)$$

Now make a Fourier Expansion of f(r)

$$f(r) = \sum_{n=1}^{\infty} b_n \sin(n\pi r/R)$$

where $$b_n = \frac{2}{R}\int_0^R f(r)\sin(n\pi r/R)dr$$

$$b_n = \frac{2}{R}\int_0^R r\ \sin(n\pi r/R)dr - \left[\frac{2}{R}\int_0^R R\ \frac{\sin((B'R^2/\alpha)^{1/2}\ r/R)}{\sin(B'R^2/\alpha)^{1/2}} \sin(n\pi r/R)dr\right]$$

The left hand side integral has been evaluated before, and is

$$-2R(-1)^n/n\pi$$

The right side integral can be expressed

$$-2\int_0^R \frac{\sin(a\ r/R)}{\sin(a)} = \sin(n\pi r/R)dr \qquad a = (B'R^2/\alpha)^{1/2}$$

This is of the form

$$\int(\sin mx)(\sin nx)dx = \frac{\sin(m-n)\ x}{2(m-n)} - \frac{\sin(m+n)\ x}{2(m+n)} \qquad m^2 \neq n^2$$

Let $a/R = m$ $\quad n\pi/R = n$

$m-n = (a-n\pi)/R$ $\quad m+n = (a+n\pi)/R$

$$\frac{-2}{\sin(a)}\left[\frac{\sin((a-n\pi)/R)r}{2((a-n\pi)/R)} - \frac{\sin((a+n\pi)/R)r}{2((a+n\pi)/R)}\right]_0^R$$

$$\frac{-R}{\sin(a)}\left[\frac{\sin(a-n\pi)}{(a-n\pi)} - \frac{\sin(a+n\pi)}{(a+n\pi)}\right]$$

Then

$$b_n = -2R(-1)^n/n\pi - \frac{R}{\sin(a)}\left[\frac{\sin(a-n\pi)}{(a-n\pi)} - \frac{\sin(a+n\pi)}{(a+n\pi)}\right]$$

$$f(r) = \sum_{n=1}^{\infty} b_n \sin(n\pi r/R)$$

$$= \sum_{n=1}^{\infty} -2R(-1)^n/n\pi - \frac{R}{\sin(a)}\left[\frac{\sin(a-n\pi)}{(a-n\pi)} - \frac{\sin(a+n\pi)}{(a+n\pi)}\right] \sin(n\pi r/R)$$

$$\sin(a-n\pi) = \begin{cases} -\sin a & n=1 \\ \sin a & n=2 \\ -\sin a & n=3 \end{cases} = (-1)^n \sin(a)$$

similarly, $\sin(a+n\pi) = (-1)^n \sin(a)$

Substituting and simplifying,

$$f(r) = \sum_{n=1}^{\infty}\left[-\frac{2R}{n\pi}(-1)^n - \frac{2n\pi R}{B'R^2\alpha - (n\pi)^2}(-1)^n\right] \sin(n\pi r/R)$$

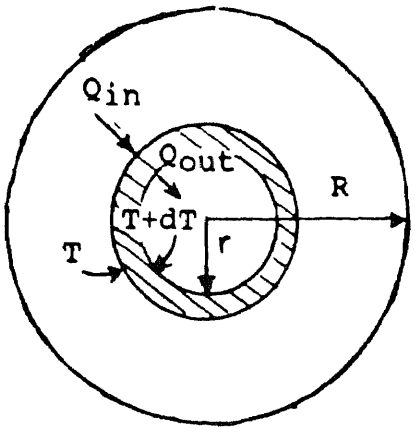
Multiplication by $-\alpha/R^2$ and further simplification yields

$$f(r) = \sum_{n=1}^{\infty} (-1)^n \frac{2B'R}{n\pi\{\alpha\ (n\pi/R)2 - B'\}} \sin(n\pi r/R)$$

This is what we set out to show, therefore, the solutions match.

8.4 Finite External Heat Transfer Coefficient

In this section, an equation describing the temperature distribution in a sphere suspended in the steam heated fluid above is derived. An energy balance around a shell in the sphere yields a general partial differential equation describing the radial temperature profile. However, in this case the boundary condition of the sphere must include the bulk temperature of the fluid changing with time. This leads to a non-homogeneous partial differential equation, which is solved by using both an Eigen function expansion of the non-homogeneous term and separation of variables as in the previous cases. This case is different from the previous one in that the boundary condition at the surface of the sphere ($r = R$) considers an external heat transfer coefficient which results in a surface temperature that depends on both fluid and solid heat transfer characteristics.



The energy into and out of a shell in the sphere:

$$Q_{in} = -kA\,\partial T/\partial r = -k\,(4\pi r^2)\,\partial T/\partial r$$

$$Q_{out} = -k4\pi(r+dr)^2 * \partial(T+dT)/\partial r$$

$$= -k4\pi(r+dr)^2 \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\,dr\right)$$

The overall energy balance around the shell:

$$Q_{net} = Q_{in} + Q_{out}$$

$$= 4\pi k \left[(r+dr)^2 \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\,dr\right) - r^2 \frac{\partial T}{\partial r}\right]$$

Expanding, simplifying, and neglecting smaller terms

$$Q_{net} = 4\pi k(r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r))$$

The rate of energy accumulated in the shell is also

$$E = 4\pi r^2 dr \rho C_p\,\partial T/\partial r$$

These terms are equal, so that

$$4\pi r^2 dr \rho C_p\,\partial T/\partial r = 4\pi k(r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r))$$

Collecting like terms and simplifying:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r}$$

This is our basic equation describing the temperature of the sphere as a function of time and radius.

Initial and Boundary Conditions

Boundary Conditions

In this case, the bulk temperature of the fluid changes with time, and the temperature at the surface of the sphere depends on both fluid and solid heat transfer characteristics.

BC # 1 : at $r = R$ $\quad h(T_B - T) = k\, dT/dr$

where $\quad T_B = T_w + \exp(-B't)(T_0 - T_w) \quad$ at $t>0$

BC # 2 : at the center of the sphere ($r=0$), conduction is zero, or $\quad \left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \quad$ when $t>0$

Initial Condition

At $t = 0$, the sphere is initially at some temperature distribution $\phi(r/R)$: $\quad T(r,0) = \phi(r/R)$

Let $\alpha = k/\rho C_p$ (thermal diffusivity constant)

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r}$$

Letting $y = r(T-T_B)$, then $T = y/r + T_B$

Taking derivatives and simplifying

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{y}{t} + -B'\exp(-B't)(T_0-T_w)$$

$$\frac{\partial T}{\partial r} = \frac{-1}{r^2}\,y + \frac{1}{r}\frac{\partial y}{\partial r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{1}{r}\frac{\partial^2 y}{\partial r^2} - \frac{2}{r^2}\frac{\partial y}{\partial r} + \frac{2}{r^3}\,y$$

Substitution into the PDE with some collection of terms yields:

$$\frac{1}{\alpha}\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} + \frac{rB'(T_0-T_w)\exp(-B't)}{\alpha}$$

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} + rC'\exp(-B't)$$

where $C' = B'(T_0-T_w)$

Substitution into the Boundary Conditions:

BC # 1 $\quad y = r(T - T_B)$ then $T = T_B + y/r$

$$-h\frac{y}{r} = +k\left(-\frac{y}{r^2} + \frac{1}{r}\frac{\partial y}{\partial r}\right)$$

$$+hy = +k(y/r) - k(dy/dr)$$

$$k(dy/dr) - k(y/r) + hy = 0 \qquad \text{at } r = R$$

BC # 2 at the center of the sphere,

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \qquad \text{when } t>0$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 = \frac{-y}{r^2} \qquad \text{so } y = 0$$

IC For $t = 0$, $T(r,0) = \phi(r/R)$

Assume the Boundary Conditions are homogeneous, but the PDE is not.

The PDE can be expressed $U_t = U_{rr} + f(r,t)$

Decomposing $f(r,t)$ into simple components

$$f(r,t) = f_1(t)X_1(r) + f_2(t)X_2(r) + \ldots + f_n(t)X_n(r) + \ldots$$

Find the response $U_n(r,t)$ to each of these individual components, $f_n(t)X_n(r)$. The solution to the problem is then

$$U(r,t) = \sum_{n=1}^{\infty} U_n(r,t)$$

The $X_n(r)$ factors are the eigen vectors of the Sturm-Liouville system we get when solving the associated homogeneous problem by separation of variables

$$U_t = \alpha\, U_{rr}$$

$$U(0,t) = 0$$

$$U(1,t) : kU_r - kU/r + hU = 0$$

$$U(r,0) = \phi(r/R)$$

From the previous case of a sphere in a constant temperature fluid, we find that the solution is

$$X_n(r) = C_n \sin(\beta_n r/R)$$

where $\beta_n \cot\beta_n + L - 1 = 0$

$$f(r,t) = f_1(t)\sin(\beta_1 r/R) + f_2(t)\sin(\beta_2 r/R) + \ldots + f_n(t)\sin(\beta_n r/R)$$

To find $f_n(t)$, multiply each side of the equation by $\sin(\beta_m r/R)$

and integrate from 0 to R (with respect to r) or 0 to 1 (with respect to r/R):

$$\int_0^R f(r,t)\sin(\beta_m r/R)\,dr = \sum_{n=1}^{\infty} f_n(t)\int_0^R \sin(\beta_m r/R)\sin(\beta_n r/R)\,dr$$

Let $r/R = E$ and $dr/R = dE$

$$\int_0^R f(r,t)\sin(\beta_m E)\,dr = \sum_{n=1}^{\infty} f_n(t)\int_0^1 \sin(\beta_m E)\sin(\beta_n E)\,dE$$

$$f_n(t) = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n}\int_0^1 f(r,t)\sin(\beta_n E)\,dE$$

$$G = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n}$$

$$f_n(t) = G\int_0^1 f(r,t)\sin(\beta_n E)\,dE$$

We now find the response $U_n(r,t) = T_n(t)X_n(r)$ to input $f_n(t)X_n(r)$

We replace $f(r,t)$ by its decomposition

$$f(r,t) = \sum_{n=1}^{\infty} f_n(t)\sin(\beta_n r/R)$$

and try to find individual responses

$$U(r,t) = \sum_{n=1}^{\infty} T_n(t)\sin(\beta_n r/R)$$

Substituting into the PDE system:

$$U_t = \alpha U_{rr} + \sum_{n=1}^{\infty} f_n(t)\sin(\beta_n r/R)$$

$$U_r(0,t) = 0$$

$$U(1,t): \quad kU_r - kU/R + hU = 0 \quad \text{at } r = R$$

$$U(r,0) = \phi(r/R)$$

gives:

$$\sum_{n=1}^{\infty} T_n'(t)\sin(\beta_n E) = -\sum_{n=1}^{\infty} \alpha(\beta_n/R)^2 T_n(t)\sin(\beta_n E) + \sum_{n=1}^{\infty} f_n(t)\sin(\beta_n E)$$

BC#2 $$\sum_{n=1}^{\infty} T_n(t)\sin(\beta_n(0)/R) = 0 \quad (0 = 0)$$

IC $$\sum_{n=1}^{\infty} T_n(0)\sin(\beta_n r/R) = \phi(r/R)$$

BC#1 $$k\sum_{n=1}^{\infty} Tn(t)(\beta_n/R)\cos(\beta_n E) - k/R \sum_{n=1}^{\infty} T_n(t)\sin(\beta_n E)$$

$$+ h \sum_{n=1}^{\infty} T_n(t)\sin(\beta_n E) = 0 \quad \text{at } r = R$$

At $r = R$, $E = 1$

$$\sum_{n=1}^{\infty} (k(\beta_n/R)\cos\beta_n - (k/R)\sin\beta_n + h\sin\beta_n)\, Tn(t) = 0$$

$$(k/R)\beta_n\cot\beta_n - (k/R - h) = 0$$

$$\beta_n\cot\beta_n - 1 + hR/k = 0$$

$$\beta_n\cot\beta_n + L - 1 = 0$$

This is our Eigen vector

$T_n'(t)$ will satisfy the initial value problem

$$T_n'(t) + \alpha (\beta_n/R)^2 T_n(t) = f_n(t)$$

$$T_n(0) = G\int_0^1 \phi(E)\sin(\beta_n E)dE = A_n$$

This ordinary differential equation has the solution:

$$T_n(t) = A_n\exp\{-\alpha (\beta_n/R)^2 t\} + \int_0^t \exp\{-\alpha (\beta_n/R)^2 (t-\tau)\}fn(\tau)d\tau$$

$$\int_0^1 \exp\{-\alpha (\beta_n/R)^2 (t-\tau)\}fn(\tau)d\tau =$$

$$\int_0^t \exp\{-\alpha (\beta_n/R)^2 (t-\tau)\} * G\int_0^1 f(E,t)\sin(\beta_n E)dE\, d\tau$$

$$= G\int_0^t \exp\{-\alpha (\beta_n/R)^2 (t-\tau)\}\int_0^1 \exp\{-B't\}C'RE\sin(\beta_n E)dE\, d\tau$$

$$= GC'R\exp\{-B't\}\int_0^t \exp\{-\alpha (\beta_n/R)^2 (t-\tau)\}\int_0^1 E\sin(\beta_n E)dE\, d\tau$$

$$\int_0^1 E\sin(\beta_n E)dE = \left[\frac{-E\cos(\beta_n E)}{\beta_n} + \frac{\sin(\beta_n E)}{(\beta_n)^2}\right]_0^1$$

$$= \frac{-\cos\beta_n}{(\beta_n)} + \frac{(\sin\beta_n)/\beta_n}{(\beta_n)}$$

$$f_n(t) = -(GC'R\exp\{-B't\}/\beta_n) * \{\cos\beta_n - (\sin\beta_n)/\beta_n\}$$

$$f_n(t) = \frac{-2C'R\{\cos\beta_n - (\sin\beta_n)/\beta_n}{\beta_n - \sin\beta_n\cos\beta_n}\exp\{-B't\}$$

Substituting back into the integral,

$$\int_0^t \exp\{-\alpha\left(\frac{\beta_n}{R}\right)^2 (t-\tau)\}fn(\tau)d\tau =$$

$$\int_0^t \exp\{-\alpha\left(\frac{\beta_n}{R}\right)^2 (t-\tau)\} \frac{-2C'R(\cos\beta_n - \sin\beta_n/\beta_n)}{\beta_n - \sin\beta_n\cos\beta_n} \exp\{-B't)$$

$$= \frac{-2C'R\{\cos\beta_n - (\sin\beta_n)/\beta_n\}}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^t \exp\{-\alpha\ (\beta_n/R)^2(t-\tau)\}\exp\{-B'\tau\}d\tau$$

$$\int_0^t \exp\{-\alpha\left(\frac{\beta_n}{R}\right)^2 (t-\tau)\}\ \exp\{-B'\tau\}\ d\tau \quad =$$

$$\exp\{-\alpha\ (\beta_n/R)^2 t\} \int_0^t \exp\{\alpha\ (\beta_n/R)^2 - B'\}\tau\ d\tau$$

$$\exp\{-\alpha\ (\beta_n/R)^2 t\} \left[\frac{\exp\{\ \alpha(\beta_n/R)^2 - B'\}\tau}{\{\ \alpha(\beta_n/R)^2 - B'\}}\right]_0^t$$

$$\exp\{-\alpha\ (\beta_n/R)^2 t\} \left[\frac{\exp\{\ \alpha(\beta_n/R)^2 - B'\}t\ -\ 1}{(\alpha\ (\beta_n/R)^2 - B')}\right]_0^t$$

$$= \frac{\exp\{-B't) - \exp\{-\alpha\ (\beta_n/R)^2 t)}{\alpha\ (\beta_n/R)^2 - B'}$$

So now,

$$\int_0^1 \exp\{-\alpha\left(\frac{\beta_n}{R}\right)^2 (t-\tau)\}fn(\tau)d\tau \quad =$$

$$-2C'R\ \frac{\cos\beta_n - \sin\beta_n/\beta_n}{\beta_n - \sin B_n\cos\beta_n} \left[\frac{\exp\{-B't) - \exp\{-\alpha\ (\beta_n/R)^2 t)}{\alpha\ (\beta_n/R)^2 - B'}\right]$$

$$U(r,t) = \sum_{n=1}^{\infty} T_n(t)\sin(\beta_n E)$$

$$T_n(t) = A_n \exp(-\alpha\ (\beta_n/R)^2 t)$$

$$= \frac{-2C'R(\cos\beta_n - (\sin\beta_n)/\beta_n)}{\beta_n - \sin\beta_n\cos\beta_n} \left[\frac{\exp(-B't) - \exp(-\alpha\ (\beta_n/R)^2 t}{\alpha\ (\beta_n/R)^2 - B'}\right]$$

$$U(r,t) = \sum_{n=1}^{\infty} T_n(t)\sin(\beta_n E)$$

$$U(r,t) = \sum_{n=1}^{\infty} [A_n \exp(-\alpha\ (\beta_n/R)^2 t)\sin(\beta_n E)] +$$

$$\frac{-2C'R(\cos\beta_n - (\sin\beta_n)/\beta_n)\ \sin(\beta_n E)}{(\beta_n - \sin\beta_n\cos\beta_n)(\alpha\ (\beta_n/R)^2 - B')} [\exp(-B't) - \exp(-\alpha\ (\beta_n/R)^2 t]$$

$$U(r,t) = y(r,t) = r(T - T_B)$$

$$T = y(r,t)/r + T_B \quad \text{where} \quad T_B = T_w + (T_0 - T_w)\exp(-B't)$$

$$T = T_w + (T_0 - T_w)\exp(-B't) + \sum_{n=1}^{\infty} [A_n/r\ \exp(-\alpha\ (\beta_n/R)^2 t)\sin(\beta_n E)] -$$

$$\frac{2C'R(\cos\beta_n - (\sin\beta_n)/\beta_n)\ \sin(\beta_n E)}{r(\beta_n - \sin\beta_n\cos\beta_n)(\alpha\ (\beta_n/R)^2 - B')} [\exp(-B't) - \exp(-\alpha\ (\beta_n/R)^2 t]$$

To find A_n, apply initial conditions

$$y(r,0) = \phi(E)$$

$$\phi(E) = \sum_{n=1}^{\infty} A_n \sin(\beta_n E)$$

Expand the IC as a sum of eigen functions

Multiply each side by $\sin(\beta_m E)$ and integrate from 0 to 1 with E

$$\int_0^1 \phi(E)\sin(\beta_m E)dE = \sum_{n=1}^{\infty} A_n \int_0^1 \sin(\beta_m E)\sin(\beta_n E)dE$$

$$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)dE$$

The final solution, then, is:

$$T = T_w + (T_0-T_w)\exp(-B't) + \sum_{n=1}^{\infty} [A_n/r \exp(-\alpha (\beta_n/R)^2 t)\sin(\beta_n E)] -$$

$$\frac{2C'R(\cos\beta_n - (\sin\beta_n)/\beta_n) \sin(\beta_n E)}{r(\beta_n - \sin\beta_n\cos\beta_n)(\alpha (\beta_n/R)^2 - B')} [\exp(-B't) - \exp(-\alpha (\beta_n/R)^2 t]$$

where $$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)dE$$

At the center of the sphere

$$T = T_w + (T_0-T_w)\exp(-B't) + \sum_{n=1}^{\infty} [A_n B_n/R \exp(-\alpha (\beta_n/R)^2 t)] -$$

$$\frac{2C'B_n(\cos\beta_n - (\sin\beta_n)/\beta_n)}{(\beta_n - \sin\beta_n\cos\beta_n)(\alpha (\beta_n/R)^2 - B')} [\exp(-B't) - \exp(-\alpha (\beta_n/R)^2 t]$$

where $$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)dE$$

9. TEMPERATURE DISTRIBUTION IN A SPHERE IMMERSED IN A COUNTER CURRENT HEAT EXCHANGER FLUID

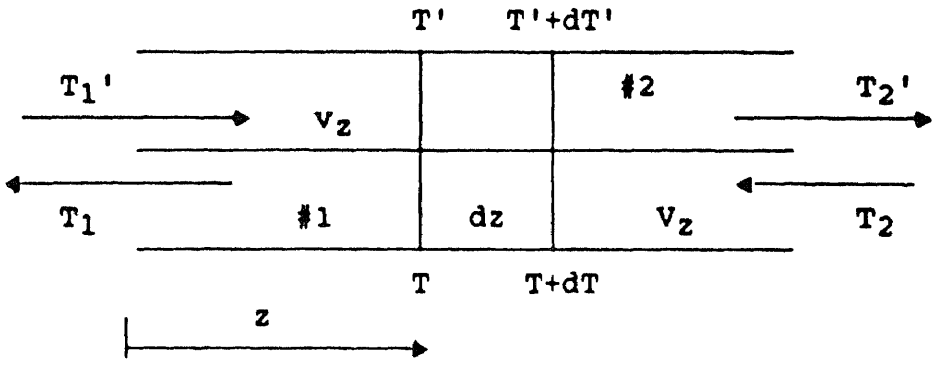
In the following sections, equations will be derived which express the:

- Time-temperature profile of a fluid in a counter-current heat exchanger
- Temperature distribution in a sphere immersed in the above fluid:
    - With an infinite external heat transfer coefficient.
    - With a finite external heat transfer coefficient.

9.1 Temperature Distribution of a Fluid in a Counter-Current Heat Exchanger

In this section an equation describing the temperature-time profile of a fluid moving through a counter-current heat exchanger is derived.

The derivation begins with two energy balances. One balance considers only one stream, while the other balance is an overall balance between both fluid streams. Solution of both equations simultaneously is required to achieve the desired temperature-time equation for the fluid. As mentioned previously, assumptions of ideal plug flow with no forward or backward conduction of heat are employed.



The energy balance around a single stream is:

$$\text{Energy In} = \text{Energy Out (\#1 stream)}$$

$$-(T-T')UdA + V_z \rho_1 (\pi r_1^2) Cp_1 T = V_z \rho_1 (\pi r_1^2) Cp_1 (T+dT)$$

$$-(T-T')UdA = V_z \rho_1 (\pi r_1^2) CpdT \qquad \text{(I)}$$

Energy In = Energy Out (Both streams)

$$Cp_2 V_z \rho_1 (\pi r_1^2)(T+dT) + Cp_2 v_z \rho_2 (\pi r_2^2) T' =$$

$$Cp_2 v_z \rho_2 (\pi r_2^2)(T'+dT') + Cp_1 V_z \rho_1 (\pi r_1^2) T$$

$$\int_{T1}^{T2} Cp_1 V_z \rho_1 (\pi r_1^2)\, dT = \int_{T1'}^{T2'} Cp_2 v_z \rho_2 (\pi r_2^2)\, dT'$$

$$[Cp_1 V_z \rho_1 (\pi r_1^2)](T_2 - T_1) = [Cp_2 v_z \rho_2 (\pi r_2^2)](T_2' - T_1')$$

$$T_2' - T_1' = \frac{Cp_1 V_z \rho_1 r_1^2}{Cp_2 v_z \rho_2 r_2^2} (T_2 - T_1)$$

let $a = Cp_1 V_z \rho_1 r_1^2 / (Cp_2 v_z \rho_2 r_2^2)$

$T_2' - T_1' = a(T_2 - T_1)$

$T_2' = aT_2 - aT_1 + T_1'$

Substituting into Equation (I)

$$-(T - aT + aT_1 - T_1')UdA = V_z \rho_1 (\pi r_1^2) Cp_1 dT$$

$$(-T + aT - aT_1 + T_1')UdA = [(a-1)T - aT_1 + T_1']UdA =$$

$$V_z \rho_1 (\pi r_1^2) Cp_1 dT$$

$$\int_{T1}^{T} \frac{dT}{(a-1)T + (T_1' - aT_1)} = \int_0^z \frac{UdA}{V_z \rho_1 (\pi r_1^2) Cp_1}$$

$A = 2\pi r_1 z \qquad dA = 2\pi r_1 dz$

$$= \int_0^z \frac{U(2\pi r_1)dz}{V_z \rho_1 (\pi r_1^2) Cp_1} = \int_0^z \frac{2U}{V_z \rho_1 r_1 Cp_1} dz$$

let $2U/(V_z \rho_1 r_1 Cp_1) = b$

$$\int_{T1}^{T} \frac{dT}{(a-1)T + (T_1' - aT_1)} = \int_0^z bdz$$

$$\int \frac{dx}{a+bx} = \frac{1}{b} \ln(a+bx)$$

$$\frac{1}{(a-1)} \ln \left[(a-1)T + (t_1-aT_1)\right]_{T1}^{T} = \left[bz \right]_0^z$$

$$\frac{1}{(a-1)} \ln \left[\frac{(a-1)T + (t_1-aT_1)}{(t_1-T_1)}\right] = bz$$

$$\left[\frac{(a-1)T + (t_1-aT_1)}{(T_1'-T_1)}\right] = \exp(b(a-1)z)$$

$$(a-1)T = (T_1'-T_1)\exp(b(a-1)z) - (T_1'-aT_1)$$

$$T = \frac{(T_1'-T_1)\exp(b(a-1)z) - (T_1'-aT_1)}{(a-1)}$$

where

$b = 2U/(V_z \rho_1 r_1 Cp_1)$ and $a = Cp_1 V_z \rho_1 r_1^2/(Cp_2 v_z \rho_2 r_2^2)$

$z = v_z t$ and $b' = 2U/(Cp_1 \rho_1 r_1)$

$$T = c\exp(b'(a-1))t - d$$

where

$$c = \frac{(T_1'- T_1)}{(a-1)} \quad d = \frac{(T_1'- aT_1)}{(a-1)} \quad \text{and} \quad b' = \frac{2U}{Cp_1 \rho_1 r_1}$$

Thus, the temperature profile of a fluid in a counter-current heat exchanger is an exponential one.

Special Case -

Heated fluid is used to preheat incoming fluid with equal flow rate and physical properties. In this case:

a=1 then $T_2'-T_1' = T_2-T_1$

$T_2' = T_2 - T_1 + T_1'$

Substituting into (I)

$$-(T - T + T_1 - T_1')UdA = V_z \rho_1 (\pi r_1^2) Cp_1 dT$$

$$-(T_1 - T_1')UdA = V_z \rho_1 (\pi r_1^2) Cp_1 dT$$

$$\int_{T1}^{T} \frac{dT}{(T_1 - T_1')} = \int_{0}^{z} \frac{UdA}{V_z \rho_1 (\pi r_1^2) Cp_1}$$

$$(T-T_1)/(T_1'-T_1) = bz \qquad \text{where } b = 2U/(V_z \rho_1 r_1^2 Cp_1)$$

$$T = T_1 + b(T_1'-T_1)z \qquad \text{where } b = 2U/(V_z \rho_1 r_1^2 Cp_1)$$

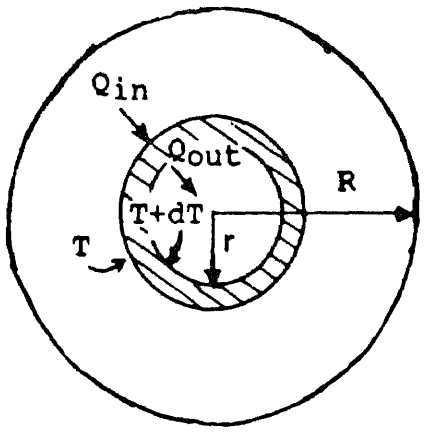
or, letting $z = v_z t$

$$T = T_1 + b'(T_1'-T_1)t \qquad \text{where } b = 2U/(\rho_1 r_1^2 Cp_1)$$

Thus, in the pre-heat case, the temperature profile is linear.

9.2 Infinite External Heat Transfer Coefficient

In this section, an equation describing the temperature distribution in a sphere suspended in the counter-current exchanger fluid above is derived. An energy balance around a shell in the sphere yields a general partial differential equation describing the radial temperature profile. However, in this case the boundary condition of the sphere must include the bulk temperature of the fluid changing with time. As it turns out, this case becomes equivalent to previous cases, so that once this becomes clear in the derivation, only the final solutions are documented.



The energy into and out of a shell in the sphere:

$$Q_{in} = -kA\,\partial T/\partial r = -k\,(4\pi r^2)\,\partial T/\partial r$$

$$Q_{out} = -k4\pi(r+dr)^2 * \partial(T+dT)/\partial r$$

$$= -k4\pi(r+dr)^2 \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\,dr\right)$$

The overall energy balance around the shell:

$$Q_{net} = Q_{in} + Q_{out}$$

$$= 4\pi k \quad (r+dr)^2 \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr \quad - r^2 \frac{\partial T}{\partial r}$$

Expanding, simplifying, and neglecting smaller terms

$$Q_{net} = 4\pi k\{r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r)\}$$

The rate of energy accumulated in the shell is also

$$E = 4\pi r^2 dr \rho C_p \partial T/\partial r$$

These terms are equal, so that

$$4\pi r^2 dr \rho C_p \partial T/\partial r = 4\pi k\{r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r)\}$$

Collecting like terms and simplifying:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r}$$

This is our basic equation describing the temperature of the sphere as a function of time and radius.

<u>Initial and Boundary Conditions</u>

Boundary Conditions

BC # 1 : at $r = R$ $T = T_B = c\exp(b'(a-1)t) - d$ at $t>0$

BC # 2 : at the center of the sphere (r=0), conduction is zero, or $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$ when $t>0$

Initial Condition

At t = 0, the sphere is initially at some temperature distribution $\phi(r/R)$: $T(r,0) = \phi(r/R)$

Let $\alpha = k/\rho C_p$ (thermal diffusivity constant)

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}$$

Letting $y = r(T-T_B)$, then $T = y/r + T_B$

Taking derivatives and simplifying

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{y}{t} + cb'(a-1)\exp(b'(a-1)t)$$

$$\frac{\partial T}{\partial r} = \frac{-1}{r^2}y + \frac{1}{r}\frac{\partial y}{\partial r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{1}{r}\frac{\partial^2 y}{\partial r^2} - \frac{2}{r^2}\frac{\partial y}{\partial r} + \frac{2}{r^3}y$$

Substitution into the PDE with some collection of terms yields:

$$\frac{\partial y}{\partial t} = \alpha\frac{\partial^2 y}{\partial r^2} - rcb'(a-1)\exp(b'(a-1)t)$$

Letting $cb'(a-1) = C'$ and $b'(a-1) = -B'$

$$\frac{\partial y}{\partial t} = \alpha\frac{\partial^2 y}{\partial r^2} + rC'\exp(-B't)$$

This case has already been solved in the case of a sphere in a steam heated fluid. The final solution is:

$$T(r,t) = T_w + \exp(-B't)(T_0-T_w) + \sum_{n=1}^{\infty}[A_n/r\ \exp(-\alpha(n\pi/R)^2 t)\sin(n\pi E)]$$

$$- \sum_{n=1}^{\infty}\left[\sin(n\pi E)\ 2C'R\ \frac{(-1)^n}{r(n\pi)}\left[\frac{\exp(-B't) - \exp(-\alpha\ (n\pi/R)^2 t)}{\alpha(n\pi/R)^2 - B'}\right]\right]$$

where $$A_n = 2\int_0^1 \phi(E)\sin(n\pi E)\,dE$$

At the center of the sphere,

$$T(r,t) = T_w + \exp(-B't)(T_0-T_w) + \sum_{n=1}^{\infty} [A_n/r \exp(-\alpha(n\pi/R)^2 t)\sin(n\pi E)]$$

$$- \sum_{n=1}^{\infty} \left[2C' \frac{(-1)^n}{(n\pi)} \left[\frac{\exp(-B't) - \exp(-\alpha (n\pi/R)^2 t)}{\alpha(n\pi/R)^2 - B'}\right]\right]$$

For the case where the initial temperature profile $\phi=0$, $A_n=0$

$$T(r,t) = T_w + \exp(-B't)(T_0-T_w)$$

$$- \sum_{n=1}^{\infty} \left[\sin(n\pi E)\ 2C'R \frac{(-1)^n}{(n\pi)r} \left[\frac{\exp(-B't) - \exp(-\alpha (n\pi/R)^2 t)}{\alpha(n\pi/R)^2 - B'}\right]\right]$$

Special Case -

Heated fluid is used to preheat incoming fluid with equal flow rate and physical properties. In this case:

<u>Initial and Boundary Conditions</u>

Boundary Conditions

BC # 1 : at $r = R$ $T = T_B = T_0 + b'(t_1-T_1)t$ at $t>0$

BC # 2 : at the center of the sphere ($r=0$), conduction is zero, or $$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \quad \text{when } t>0$$

Initial Condition

At $t = 0$, the sphere is initially at some temperature distribution $\phi(r/R)$: $T(r,0) = \phi(r/R)$

Let $\alpha = k/\rho C_p$ (thermal diffusivity constant)

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}$$

Letting $y = r(T-T_B)$, then $T = y/r + T_B$

Taking derivatives and simplifying

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{y}{t} + b'(t_1-T_1)$$

$$\frac{\partial T}{\partial r} = \frac{-1}{r^2}\, y + \frac{1}{r}\frac{\partial y}{\partial r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{1}{r}\frac{\partial^2 y}{\partial r^2} - \frac{2}{r^2}\frac{\partial y}{\partial r} + \frac{2}{r^3}\, y$$

Substitution into the PDE with some collection of terms yields:

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} - rb'(t_1-T_1)$$

Letting $S = b'(t_1-T_1)$

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} + rS$$

This case has already been solved in the case of a sphere in an electrically heated fluid. The final solution is:

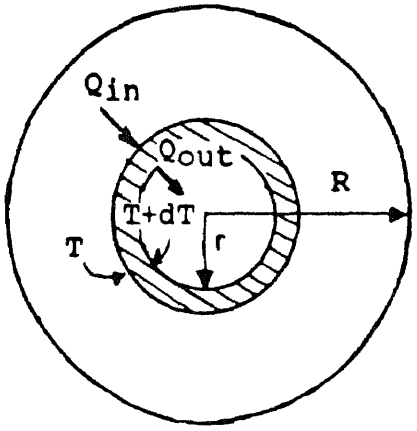
$$T(r,t) = T_0 + St + \sum_{n=1}^{\infty} [A_n/r \exp(-\alpha (n\pi/R)^2 t)\sin(n\pi E)] -$$

$$\sum_{n=1}^{\infty}\left[\sin(n\pi E)\,\frac{2SR^3(-1)^{n+1}}{n^3\pi^3 r}\left(1 - \exp\left(-\alpha\left(\frac{n\pi}{R}\right)^2 t\right)\right)\right]$$

where $$A_n = 2\int_0^1 \phi(E)\sin(n\pi E)\,dE$$

9.3 Finite External Heat Transfer Coefficient

In this section, an equation describing the temperature distribution in a sphere suspended in the counter-current exchanger fluid above is derived. An energy balance around a shell in the sphere yields a general partial differential equation describing the radial temperature profile. However, in this case the boundary condition of the sphere must include the bulk temperature of the fluid changing with time. Also, in this case, a finite external heat transfer coefficient is considered, such that the temperature at the surface of the sphere depends on both fluid and solid heat transfer characteristics. Again, it turns out that this case becomes equivalent to previous cases, so that once this becomes clear in the derivation, only the final solutions are documented.



The energy into and out of a shell in the sphere:

$$Q_{in} = -kA\,\partial T/\partial r = -k\,(4\pi r^2)\,\partial T/\partial r$$

$$Q_{out} = -k4\pi(r+dr)^2 * \partial(T+dT)/\partial r$$

$$= -k4\pi(r+dr)^2 \left[\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr \right]$$

The overall energy balance around the shell:

$$Q_{net} = Q_{in} + Q_{out}$$

$$= 4\pi k \left[(r+dr)^2 \left[\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr \right] - r^2 \frac{\partial T}{\partial r} \right]$$

Expanding, simplifying, and neglecting smaller terms

$$Q_{net} = 4\pi k\{r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r)\}$$

The rate of energy accumulated in the shell is also

$$E = 4\pi r^2 dr \rho C_p \partial T/\partial r$$

These terms are equal, so that

$$4\pi r^2 dr \rho C_p \partial T/\partial r = 4\pi k\{r^2(\partial^2 T/\partial r^2)dr + 2rdr(\partial T/\partial r)\}$$

Collecting like terms and simplifying:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right]$$

This is our basic equation describing the temperature of the sphere as a function of time and radius.

Initial and Boundary Conditions

Boundary Conditions

BC # 1 : at $r = R$ $\quad h(T_B - T) = k\, dT/dr$

where $\quad T_B = c\exp(b'(a-1)t) - d \qquad$ at $t>0$

BC # 2 : at the center of the sphere ($r=0$), conduction is zero, or $\quad \left. \frac{\partial T}{\partial r} \right|_{r=0} = 0 \quad$ when $t>0$

Initial Condition

At t = 0, the sphere is initially at some temperature distribution $\phi(r/R)$: $T(r,0) = \phi(r/R)$

Let $\alpha = k/\rho C_p$ (thermal diffusivity constant)

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}$$

Letting $y = r(T-T_B)$, then $T = y/r + T_B$

Taking derivatives and simplifying

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{y}{t} + cb'(a-1)\exp(b'(a-1)t)$$

$$\frac{\partial T}{\partial r} = \frac{-1}{r^2}y + \frac{1}{r}\frac{\partial y}{\partial r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{1}{r}\frac{\partial^2 y}{\partial r^2} - \frac{2}{r^2}\frac{\partial y}{\partial r} + \frac{2}{r^3}y$$

Substitution into the PDE with some collection of terms yields:

$$\frac{1}{\alpha}\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} + \frac{rcb'(a-1)\exp(b'(a-1)t)}{\alpha}$$

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} - rcb'(a-1)\exp(b'(a-1)t)$$

Letting $cb'(a-1) = C'$ and $b'(a-1) = -B'$

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} + rC'\exp(-B't)$$

This case has already been solved in the case of a sphere in a steam heated fluid. The final solution is:

$$T = T_w + (T_0-T_w)\exp(-B't) + \sum_{n=1}^{\infty} [A_n/r \exp\{-\alpha (\beta_n/R)^2 t\}\sin(\beta_n E)] -$$

$$\frac{2C'R(\cos\beta_n - (\sin\beta_n)/\beta_n)\sin(\beta_n E)}{r(\beta_n - \sin\beta_n\cos\beta_n)(\alpha (\beta_n/R)^2 - B')} [\exp(-B't) - \exp\{-\alpha (\beta_n/R)^2 t]$$

where $$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)dE$$

At the center of the sphere

$$T = T_w + (T_0-T_w)\exp(-B't) + \sum_{n=1}^{\infty} [A_nB_n/R \exp\{-\alpha (\beta_n/R)^2 t\}] -$$

$$\frac{2C'B_n\{\cos\beta_n - (\sin\beta_n)/\beta_n\}}{(\beta_n - \sin\beta_n\cos\beta_n)(\alpha (\beta_n/R)^2 - B')} [\exp(-B't) - \exp\{-\alpha (\beta_n/R)^2 t]$$

where $$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)dE$$

Special Case -

Heated fluid is used to preheat incoming fluid with equal flow rate and physical properties. In this case:

Initial and Boundary Conditions

Boundary Conditions

BC # 1 : at $r = R$ $\quad h(T_B-T) = k\, dT/dr$

where $\quad T_B = T_0 + b'(t_1-T_1)t$ at $t>0$

BC # 2 : at the center of the sphere ($r=0$), conduction is zero, or $\quad \left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$ $\quad$ when $t>0$

Initial Condition

At t = 0, the sphere is initially at some temperature distribution $\phi(r/R)$: $T(r,0) = \phi(r/R)$

Let $\alpha = k/\rho C_p$ (thermal diffusivity constant)

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}$$

Letting $y = r(T-T_B)$, then $T = y/r + T_B$

Taking derivatives and simplifying

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{y}{t} + b'(t_1-T_1)$$

$$\frac{\partial T}{\partial r} = \frac{-1}{r^2}y + \frac{1}{r}\frac{\partial y}{\partial r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{1}{r}\frac{^2y}{r^2} - \frac{2}{r^2}\frac{\partial y}{\partial r} + \frac{2}{r^3}y$$

Substitution into the PDE with some collection of terms yields:

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} - rb'(t_1-T_1)$$

Letting $S = b'(t_1-T_1)$

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial r^2} + rS$$

This case has already been solved in the case of a sphere in an electrically heated fluid. The final solution is:

$$T = T_0 + St + \sum_{n=1}^{\infty} [A_n/r \exp(-\alpha (\beta_n/R)^2 t)\sin(\beta_n E)] +$$

$$\sum_{n=1}^{\infty} \left[\sin(\beta_n E) \frac{2SR^2(\cos\beta_n - (\sin\beta_n)/\beta_n)}{E(\beta_n^3 - \beta_n^2 \sin\beta_n \cos\beta_n)} [1 - \exp(-\alpha (\beta_n/R)^2 t] \right]$$

where $$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)\, dE$$

For the case where the initial temperature profile $\phi(E)=0$, $A_n=0$

$$T(r,t)=T_0 + St +$$

$$\sum_{n=1}^{\infty} \left[\sin(\beta_n E) \frac{2SR^2(\cos\beta_n - (\sin\beta_n)/\beta_n)}{E(\beta_n^3 - \beta_n^2 \sin\beta_n \cos\beta_n)} [1 - \exp(-\alpha (\beta_n/R)^2 t] \right]$$

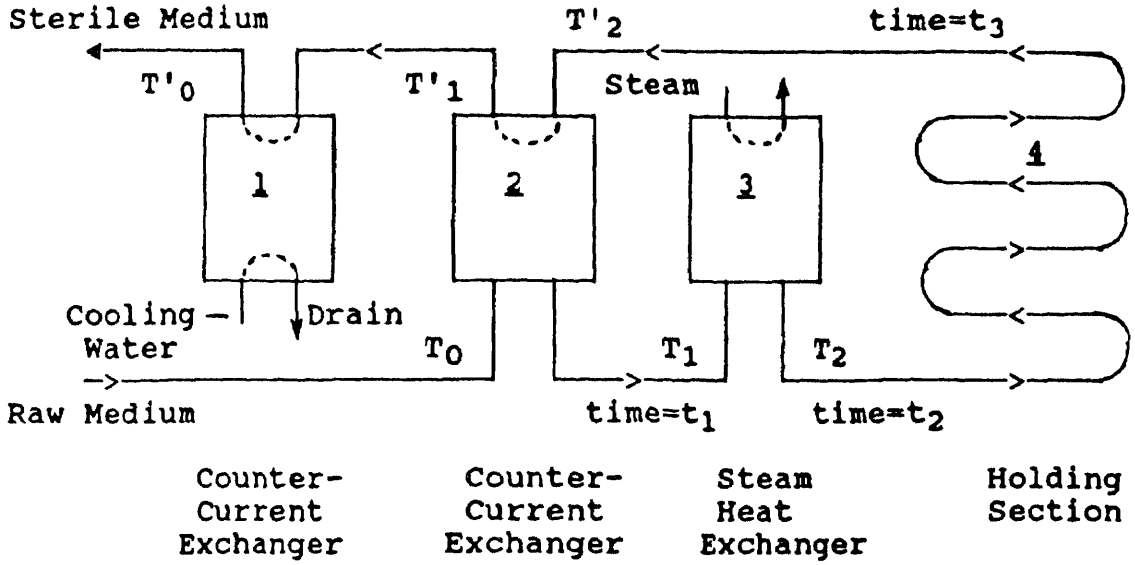
And at the center of the sphere

$$T(r,t)=T_0 + St +$$

$$\sum_{n=1}^{\infty} \left[\frac{2SR^2(\cos\beta_n - (\sin\beta_n)/\beta_n)}{(\beta_n^2 - \beta_n \sin\beta_n \cos\beta_n)} [1 - \exp(-\alpha (\beta_n/R)^2 t] \right]$$

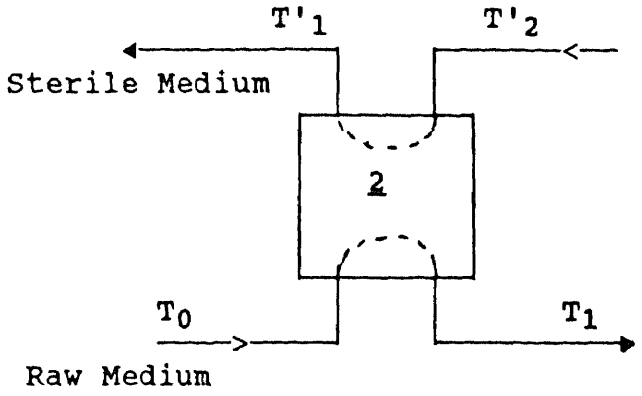
10. TEMPERATURE PROFILE IN A SPHERE MOVING THROUGH A CONTINUOUS STERILIZER

The flow diagram describing our continuous sterilization system, once again, is:



In this section, a single equation describing the temperature profile in a sphere moving through the above system through the holding section is derived. The contribution to sterilization of the cooling sections is not considered in subsequent sections, so that these sections are not considered here either. The approach taken in this section is to move through the exchangers as the sphere does. The temperature profile of the sphere out of one exchanger then becomes the initial temperature profile function for the subsequent exchanger. The initial condition of the sphere as it enters the first (pre-heat) exchanger is assumed constant and equal to the fluid temperature.

Counter-Current Exchanger #2 (Heat Recovery Exchanger)



This heat exchanger is counter-current, with both streams having the same flow rates and about the same physical properties. We therefore use the special case equation for counter-current exchange.

Bulk Fluid Temperature Profile is:

$$T = T_0 + b'(T'_1 - T_0)t \qquad \text{where } b' = 2U/(\rho_1 r_1 Cp_1)$$

Temperature Profile in the Spherical Particle is:

$$T(r,t)=T_0 + St + \sum_{n=1}^{\infty}\left[\sin(\beta_n E)\,\frac{2SR^2(\cos\beta_n - (\sin\beta_n)/\beta_n)}{E(\beta_n^3 - \beta_n^2\sin\beta_n\cos\beta_n)}\,[1 - \exp\{-\alpha\,(\beta_n/R)^2 t]\right]$$

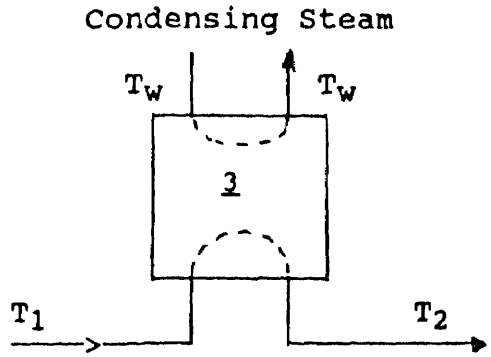
For residence time (t_1), the outlet temperature of the fluid is:

$$T = T_0 + b'(T'_1 - T_0)t_1 \qquad \text{where } b' = 2U/(\rho_1 r_1 Cp_1)$$

And that of the sphere is:

$$T(r,t)=T_0 + St_1 + \sum_{n=1}^{\infty}\left[\sin(\beta_n E)\,\frac{2SR^2(\cos\beta_n - (\sin\beta_n)/\beta_n)}{E(\beta_n^3 - \beta_n^2\sin\beta_n\cos\beta_n)}\,[1 - \exp\{-\alpha\,(\beta_n/R)^2 t_1]\right]$$

Steam Heat Exchanger #3 (Heater)



The exchanger is a steam heat exchanger.

Bulk fluid temperature profile is:

$$T = T_w + \exp(-B'(t-t_1))\ [T_1 - T_w]$$

The temperature profile of a spherical particle is:

$$T = T_w + (T_0-T_w)\exp(-B't) + \sum_{n=1}^{\infty} [A_n/r\ \exp(-\alpha\ (\beta_n/R)^2 t)\sin(\beta_n E)] -$$

$$\frac{2C'R(\cos\beta_n - (\sin\beta_n)/\beta_n)\ \sin(\beta_n E)}{r(\beta_n - \sin\beta_n\cos\beta_n)(\alpha\ (\beta_n/R)^2 - B')}\ [\exp(-B't) - \exp(-\alpha\ (\beta_n/R)^2 t]$$

where $$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)\,dE$$

Applying our initial conditions, the exit conditions from exchanger #2, yields:

Bulk fluid temperature profile is:

$$T_2 = T_w + \exp(-B'(t_2-t_1))\ [T_1 - T_w]$$

The temperature profile of a spherical particle is:

$$T = T_w + (T_1 - T_w)\exp(-B'(t_2 - t_1)) +$$

$$+ \sum_{n=1}^{\infty} [A_n/r \exp(-\alpha (\beta_n/R)^2 (t_2 - t_1))\sin(\beta_n E)] -$$

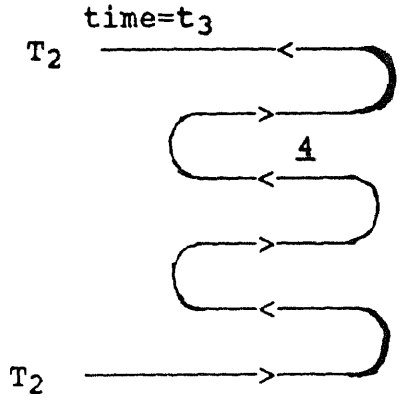
$$\frac{2C'R(\cos\beta_n - (\sin\beta_n)/\beta_n)\sin(\beta_n E)}{r(\beta_n - \sin\beta_n\cos\beta_n)(\alpha (\beta_n/R)^2 - B')}\left[e^{-B'(t_2 - t_1)} - e^{-\alpha(\beta_n/R)^2(t_2 - t_1)}\right]$$

where $$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n}\int_0^1 \phi(E)\sin(\beta_n E)\,dE$$

And $\phi(E)$ =

$$\sum_{n=1}^{\infty}\left[\sin(\beta_n E)\frac{2SR^3(\cos\beta_n - (\sin\beta_n)/\beta_n)}{(\beta_n^3 - \beta_n^2\sin\beta_n\cos\beta_n)}[1 - \exp(-\alpha (\beta_n/R)^2 t_1]\right]$$

Holding Section



Holding Section

The holding section is a constant temperature section of a length to provide a residence time $(t_3 - t_2)$.

The temperature profile of a spherical particle, in general:

$$T(r,t) = T_B + \sum_{n=1}^{\infty} \frac{A_n}{r} \exp\left(\frac{-\alpha\ \beta_n^2}{R^2}\ t\right) \sin\frac{\beta_n r}{R}$$

where

$$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)dE$$

For our initial condition, that of the particle leaving the steam exchanger, the solution is :

$$T(r,t) = T_2 + \sum_{n=1}^{\infty} \frac{A_n'}{r} \exp\left(\frac{-\alpha\ \beta_n^2}{R^2}\ (t_3-t_2)\right) \sin\frac{\beta_n r}{R}$$

where $$A_n' = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi'(E)\sin(\beta_n E)dE$$

and $$\phi'(E) = \sum_{n=1}^{\infty} [A_n \exp\{-\alpha\ (\beta_n/R)^2(t_2-t_1)\}\sin(\beta_n E)] -$$

$$\frac{2C'R\{\cos\beta_n - (\sin\beta_n)/\beta_n)\ \sin(\beta_n E)}{(\beta_n - \sin\beta_n\cos\beta_n)(\alpha\ (\beta_n/R)^2 - B')}[e^{-B'(t_2-t_1)} - e^{-\alpha(\beta_n/R)^2(t_2-t_1)}]$$

where $$A_n = \frac{2\beta_n}{\beta_n - \sin\beta_n\cos\beta_n} \int_0^1 \phi(E)\sin(\beta_n E)dE$$

and $\phi(E)$ =

$$\sum_{n=1}^{\infty}\left[\sin(\beta_n E)\ \frac{2SR^3(\cos\beta_n - (\sin\beta_n)/\beta_n)}{(\beta_n^3 - \beta_n^2\sin\beta_n\cos\beta_n)}\ [1 - \exp\{-\alpha\ (\beta_n/R)^2 t_1]\right]$$

This is our final equation describing the temperature profile of the sphere through the holding section of our continuous sterilizer.

11. ORGANISM DEATH RATE KINETICS

It has already been mentioned that most analytical modelling of sterilization systems combines a temperature-time profile for a fluid (or solid) based on heat transfer equations with an expression for organism death rate kinetics. This section briefly discusses the mathematics associated with organism thermal death rate kinetics.

The rate of thermal death for most organisms follows first order kinetics. The rate of organsim destruction, then, is directly proportional to the number of organisms present. This is expressed mathematically as:

$$\frac{dN}{dt} = -K_d N$$

Here, K_d is the death rate constant. As with most rate constants, this first order constant is a function of temperature and activation energy. This is expressed as:

$$\frac{dN}{dt} = -K_{d_0} e^{-E_d/RT(t)} N$$

Separating variables and integrating the above expression yields:

$$\ln \frac{N_f}{N_0} = \int_0^{t_f} -K_{d_0} e^{-E_d/RT(t)} dt$$

Temperature is considered to be a function of time in this expression since the fluid is heated up as it moves through the exchanger train. In the case of our continuous sterilization

system, the expression describing the temperature profile of a sphere could be substituted directly into the above equation for T(t). However, this would merely add to the complexity of an already unwieldy expression. It is apparant, however, that the above expression lends itself quite nicely to numerical integration of the temperature-time profile. It is this type of numerical approach that is discussed in the next section.

12. NUMERICAL EVALUATION OF ANALYTICAL SOLUTION

12.1 Approach

The complexity of the above analytical expression describing the temperature profile of a sphere passing through our continuous sterilizer greatly limits its usefulness in a numerical evaluation of an actual sterilization system. However, its summation terms and nested loops make it readily adaptable to evaluation by computer.

A Fortran program was written for the purpose of calculating values of temperature at various increments of time and, further, calculating the remaining organism concentration in both the liquid and solid fraction of the fermentation medium.

The way the program works is straight forward. It first generates numerical values in an array form for the fluid and solid temperature at chosen increments of time. For the spherical solid particle, temperature is calculated at eleven points inside the sphere, from the center to the outer surface.

When the entire time-temperature profile for a single exchanger has been generated, the temperature points associated with the time the sphere leaves the exchanger become the initial temperature profile for the subsequent exchanger. In this way,

excessive nesting of equations associated with initial conditions are eliminated.

After temperature profiles are generated for all three exchangers considered, the remaining organism concentration is calculated numerically from the array of temperature-time data. For the spheres, a second numerical integration over volume is performed so that one value for the remaining viable fraction is generated for the solid. A more detailed account of how the program works mechanically is described next.

Programming

The program written from the above equations describing time-temperature-radius relationships for our continuous sterilization system consists of the main program and six subroutines. Each subroutine is called by the main and returns to the main. The subroutines are called sequentially as they calculate values for liquid and suspended sphere in the first counter-current exchanger, then the steam exchanger, and finally the holding section. Because of the similarity between subroutines which calculate the values for the liquid condition, and those which calculate values for the sphere, discussion of these routines will be done as routines related to liquids and those related to solids. The degree of sterilization is calculated separately and is done in the last subroutine called. A complete listing of the program is included in the appendix.

Main Program

The main program is used merely to set values for residence times in each exchanger, initialize temperature into the first exchanger and steam temperature in the second, and to call the subroutines which perform all actual calculations. Subroutines are called sequentially as if moving through the sterilizer system.

Calculation of Eigenvalues

Subroutine EIGEN is the first called by the main, and calculates the eigenvalues for the equations to follow for a value of L (hR/k) specified in the main. The program moves by small increments through a range of values the size of one π. It either passes a value and then converges on it, or it concludes that the value in this range is π or a multiple of π. It then begins increments in the next range which is again one π in size. Twenty eigen values are calculated, and then sent to a data file. Values are also returned to main to be used in subsequent routines.

Liquid Related Subroutines

These routines, consisting of LIQUID1 and XLIQ2, calculate time-temperature profiles of the bulk fluid through the counter-current exchanger and steam exchanger respectively. These routines are

very short, and merely include the physical characteristics of the exchangers, and the equation to calculate temperature at a given time.

Solids Related Subroutines

The subroutines SPHERE1, XSPH2, and SPLIQ2 are all subroutines which calculate the radial temperature profiles of the sphere suspended in the sterilization fluid. They correspond to fluid passing through the counter-current, steam, and holding sections, respectively. The physical properties of the sphere are contained in the first of these routines, SPHERE1, and are passed as parameters into the subsequent subroutines.

The equations derived for the solid sphere in the previous section, in general, are in 2 parts; one which accounts for initial conditions, the temperature profile of the sphere as it enters the new condition, and one which is dependent on the boundary conditions, or the surrounding fluid temperature profile.

The initial condition of the sphere entering the first exchanger is a temperature profile which is constant and equal to fluid temperature. Entering subsequent exchangers, the initial profile is the same profile that exists at the discharge of the previous exchangers. By initializing values in this way, initial conditions are dependent only on the exchanger directly before the present exchanger. This greatly reduces the calculations involved

involved for this part of the equation (for example, see the complete analytical expression describing the temperature profile of the sphere out of the holding section in the derivation section of this report). These initializations are done at the beginning of each of the solids related subroutines.

The next section in each of these subroutines is the calculation of temperature as a function of time at the center of the sphere. This is done separately from the other values of radius because it requires its own, slightly modified, forms of the equations. Finally, the time-temperature profiles are generated for the other values of radius, and then written to a data file.

Sterilization Related Subroutine

The last subroutine, DEATH, inputs all the time-temperature data for both the liquid and solid fractions, and, for each heat exchanger, numerically integrates these profiles to determine the degree of sterilization achieved. In the case of the solid particle, a second numerical integration is done with respect to the volume of the sphere in order to determine a volume average of the degree of sterilization. The second half of this subroutine prints to a data file the temperature profiles throughout the entire system, and the degree of sterilization achieved for liquid and solid in each of the three exchangers.

11.2 Parameters for a Typical Commercial Continuous Sterilizer

In the last section the programming used in the numerical evaluation of the analytical solution was described in some detail. However, in order to actually evaluate a continuous sterilization system, physical parameters must be known or estimated. Among these parameters are the :

- physical dimensions of the exchanger
- physical properties of the liquid and solid
- heat transfer coefficients in the exchangers
- external heat transfer coefficient around the solid
- organism death rate kinetics constants

Some of these parameters are readily available in the literature. Properties of many types of liquids and solids, for example, are tabulated in a number of sources. For our analysis, the properties of water and asbestos (for its low heat conductivity) will be assumed for the properties of our liquid and solid fractions, respectively. The values are shown in Table II.

Table II - Physical Properties of Water(93°C) and Asbestos[5]

Property		Water (93°C)	Asbestos
ρ	(kg/m^3)	962.7	577
C_p	(J/kg*K)	4229	1050
k	(W/m*K)	0.6802	0.168
Pr	(---)	1.91	
u	(kg/ms)	0.3066×10^{-3}	

Typical dimensions for heat exchanger systems of this type are as follows[9]:

Table III - Physical Dimensions of Sterilizer

Recovery Exchanger (#2)

Volumetric Flow Rate (m3/hr)	40
Linear Velocity in Exchangers (m/s)	0.93
Effective Diameter (m)	0.012
Heat Transfer Surface (m^2)	600
Residence Time (s)	120
Heat Recovery (%)	60-70
Inlet Fluid Temperature (^oC)	25

Steam Exchanger (#3)

Volumetric Flow Rate (m3/hr)	40
Linear Velocity in Exchangers (m/s)	0.93
Effective Diameter (m)	0.012
Heat Transfer Surface (m^2)	600
Residence Time (s)	120
Steam Temperature (^oC)	140-150

Holding Section (#4)

Volumetric Flow Rate (m3/hr)	40
Linear Velocity in Exchangers (m/s)	0.93
Effective Diameter (m)	0.012
Residence Time (s)	60

From the information in Tables II and III, overall heat transfer coefficients can be estimated for each exchanger. In order to recover 70% of the heat required to raise the fluid temperature from 25 to 145^{o}C in the counter-current exchanger, we go back to the equation describing the temperature profile in this exchanger as a function of time:

$$T_1 = T_0 + \frac{U}{\rho C_p r} (T_1' - T_0) * t_1$$

Using an energy balance around the exchanger and the 70% recovery condition results in $T_1' = 61^oC$. All other values, except U, are known. Solving for U, $U = 475\ J/m^2sK$.

Using this value, along with a single side heat transfer coefficient $h = 3766\ W/m^2K$ [5,8,10] for a water/water exchanger, allows us to solve for a fouling factor (ff):

$$\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2} + ff \qquad ff = 0.00157$$

For the steam exchanger, the single side heat transfer coefficient for the water side is still $3766\ W/m^2K$, however, the heat transfer coefficient is assumed to be very large compared to the water side and is assumed to provide negligable resistance. Also, the fouling factor on the water side will be the same as before, but no fouling will be assumed on the steam side. The fouling factor, therefore, is half of 0.00157 for this calculation, and the overall heat transfer coefficient for the steam exchanger is calculated to give $U = 952\ J/m^2sK$.

If the fouling is a result of solids build-up on the walls of the exchangers, and the solids is still assumed to have the properties of asbestos, then the thickness of the solids can be estimated from:

$$\text{Resistance} = ff = \frac{x}{k}$$

Solving for x, the thickness of the build-up in each tube, results in x = 0.13 mm.

The final value required for a numerical evaluation of the system is the external heat transfer coefficient around the suspended particle. While limited data for this heat transfer coefficient is available, the related mass transfer coefficient is found in the literature[11,12,13,14] to be an expression involving the specific power group $(\epsilon^{1/3}Dp^{4/3}/v)$. Here, ϵ is the energy dissipation rate per unit mass of liquid, Dp is the diameter of the particle, and v is the kinematic viscosity of the liquid (m^2/s). The expression, altered for heat transfer, is:

$$Nu = hD/k = 2 + 0.59(\epsilon^{1/3}Dp^{4/3}/v)^{0.57}Pr^{1/3}$$

The energy dissipation rate is calculated by first determing the pressure drop through a section of exchanger, multiplying by the volumetric flow rate to get the power dissipated, and then dividing by the mass of the fluid in that section of exchanger to get energy dissipated per unit mass.

For our system:

$$\Delta p = 4f\rho \frac{\Delta L}{D} \frac{v^2}{2} \qquad \Delta p = 228{,}976\ N/m^2 \text{ for } L = 300m$$

The fanning friction factor (f) is dependant on the Reynolds Number. For our Reynolds Number of 35,000, f = 0.0055 for smooth pipe. Our volumetric flow rate is 40 m^3/hr = 0.0111 m^3/s, and the mass of fluid in a section of this size is 3414 kg. Solving for the energy dissipation rate per unit mass, ϵ = 0.745 J/kg*s. Substituting into our equation , Nu = 64.2.

Back calculation to find h around a 0.005 m particle:

$$\frac{hD}{k_1} = \frac{h(0.005)}{0.6802} = 64.2$$

$$h = 8734 \ W/m^2K$$

To find the value for L, for our Eigenvalues $\beta_n \cot\beta_n + L - 1 = 0$:

$$L = \frac{hR}{k_s} = \frac{8734(0.0025)}{0.168} = 130$$

Finally, constants must be estimated in the organism thermal death rate expression. From previous sections, the expression is first order:

$$\ln \frac{N_f}{N_0} = -\int_0^{t_f} K_{d_0} e^{-E_d/RT(t)} dt$$

In this expression, values for thermally resistant organisms, such as Bacillus stearothermophilus, will be assumed. These values are available in the literature[7], and are:

$$K_{d0} = 9.5 \times 10^{37} \ minutes^{-1}$$

$$E_d = 70 \ kcal/gmole$$

These are the last unknown values required for numerical evaluation of the sterilization system. The next section will provide the results of the evaluation.

13. RESULTS AND DISCUSSION

The temperature-time profile for both the fluid and solid moving through the sterilizer is a function of many parameters. Some of these have already been discussed, including the:

- physical dimensions of the exchanger
- physical properties of the liquid and solid
- heat transfer coefficients in the exchangers
- external heat transfer coefficient around the solid

Others, including design parameters such as:

- steam temperature in the heater
- flow rate through the exchangers

can also be varied to result in different temperature-time profiles. Also, for the solid, the size of the particle will change the temperature profile through the sphere.

The permutations possible with these parameters are enormous. However, some parameters will play a more important role than others. For demonstration purposes, only a few of these more

important parameters are evaluated in this section. These include:

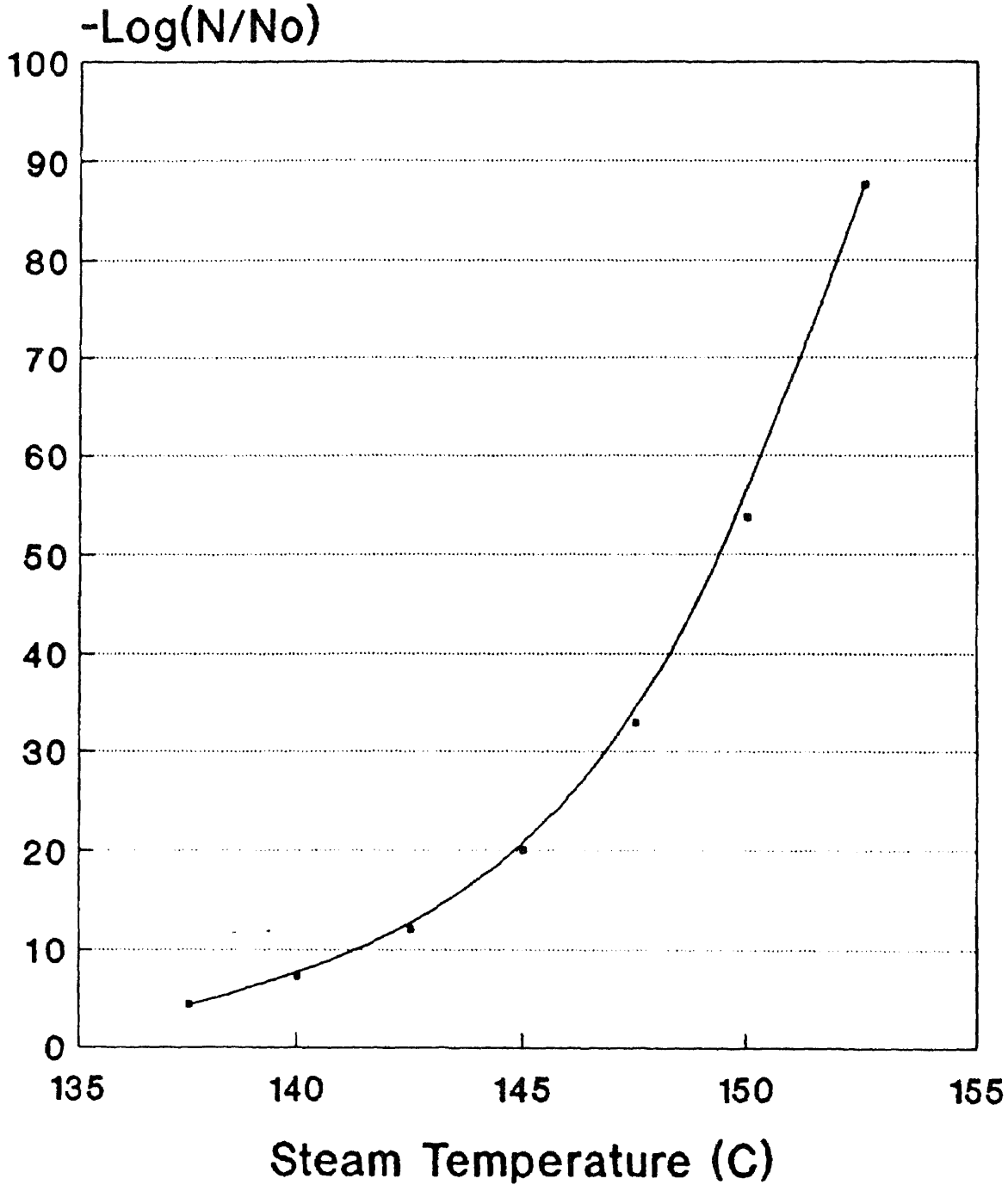
- Steam Temperature
- Heat Recovery Obtained
- External Heat Transfer Coefficient
- Residence Time

In this section, only selected data is presented to demonstrate the observations seen. Full tables of data can be found in Appendix B of this paper.

Steam Temperature - Changing the steam temperature while holding the residence time constant will obviously result in a change in the maximum temperaure achieved in the system. While this is important merely from a temperature profile change, it is seen to be critical in the degree of sterility achieved (Figure 4). In the short span of 15C°, the change in the degree of sterilization is 80 orders of magnitude. It should be noted in this section, and remembered in subsequent sections, that a typical value for what is considered a high degree of sterility is of the order of $N/N_0 = 10^{-20} - 10^{-25}$ (20 to 25 orders of magnitude).[7]

Heat Recovery in Counter-Current Exchanger - In the previous section it was shown that the % heat recovered determined the heat transfer coefficients in both exchangers. Evaluation of our system at two conditions, 60 and 70% recovery was made. Also,

Figure 4 - The Degree of Sterilization is Very Sensitive to Temperature



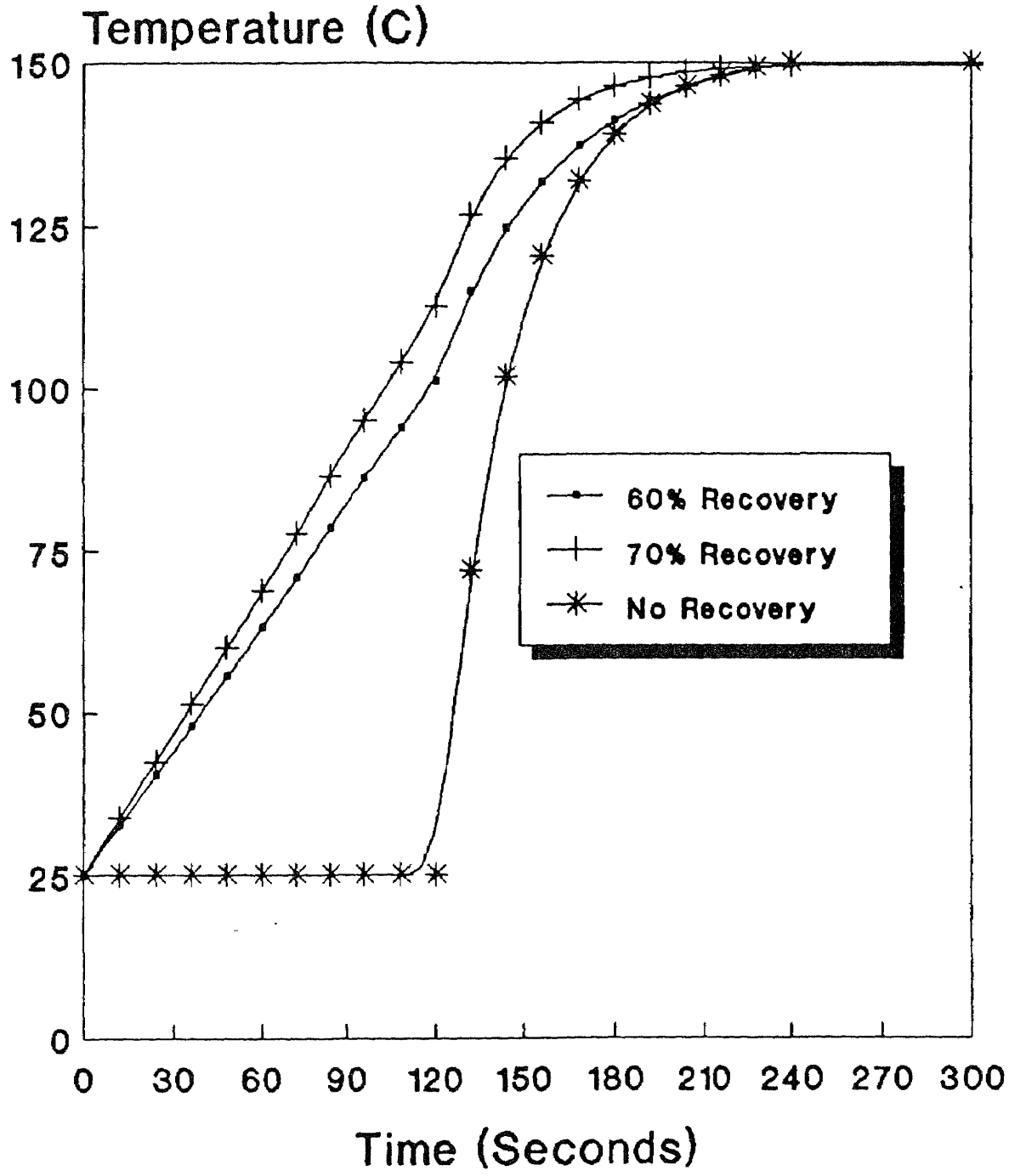
70% Recovery Case

using the values for the heat transfer coefficients determined at both these conditions, the system was evaluated assuming there was no heat recovery exchanger, and that the heating of the fluid takes place entirely in the steam exchanger. This is a situation that might take place if for some reason the recovery exchanger went down for repair, and the system was kept in operation by adjusting steam temperature to remain at the same holding temperature. This results in a much faster heat-up profile than either the 60% or 70% recovery cases, as shown in Figure 5.

It is seen in Figure 6 that the 70% heat recovery system provides more sterilization than the 60% heat recovery system. As one would expect, this is the result of the 70% system reaching holding temperature sooner, providing "extra" sterilization in the steam heater section. This is shown in Figure 7, where the sterilization in each of the two exchanger sections are shown separately. Note that the sterilization achieved in the holding section for both the 60% and 70% cases are equal.

Up to this point the sterilization achieved in the solid fraction has not been discussed. In each of the systems evaluated, 0%, 60%, and 70% recovery, there has been little difference in the degree of sterilization achieved between fluid and solid. This is shown in Figure 8, which is a plot of the ratio of the remaining organisms in the solid to that in the fluid. Even at the highest

Figure 5 - Temperature-Time Profiles For Various Amounts of Heat Recovery



Holding Temperature is 150 C

Figure 6 - At Equal Holding Temperature, the 70% Recovery Exchanger Achieves More Sterilization

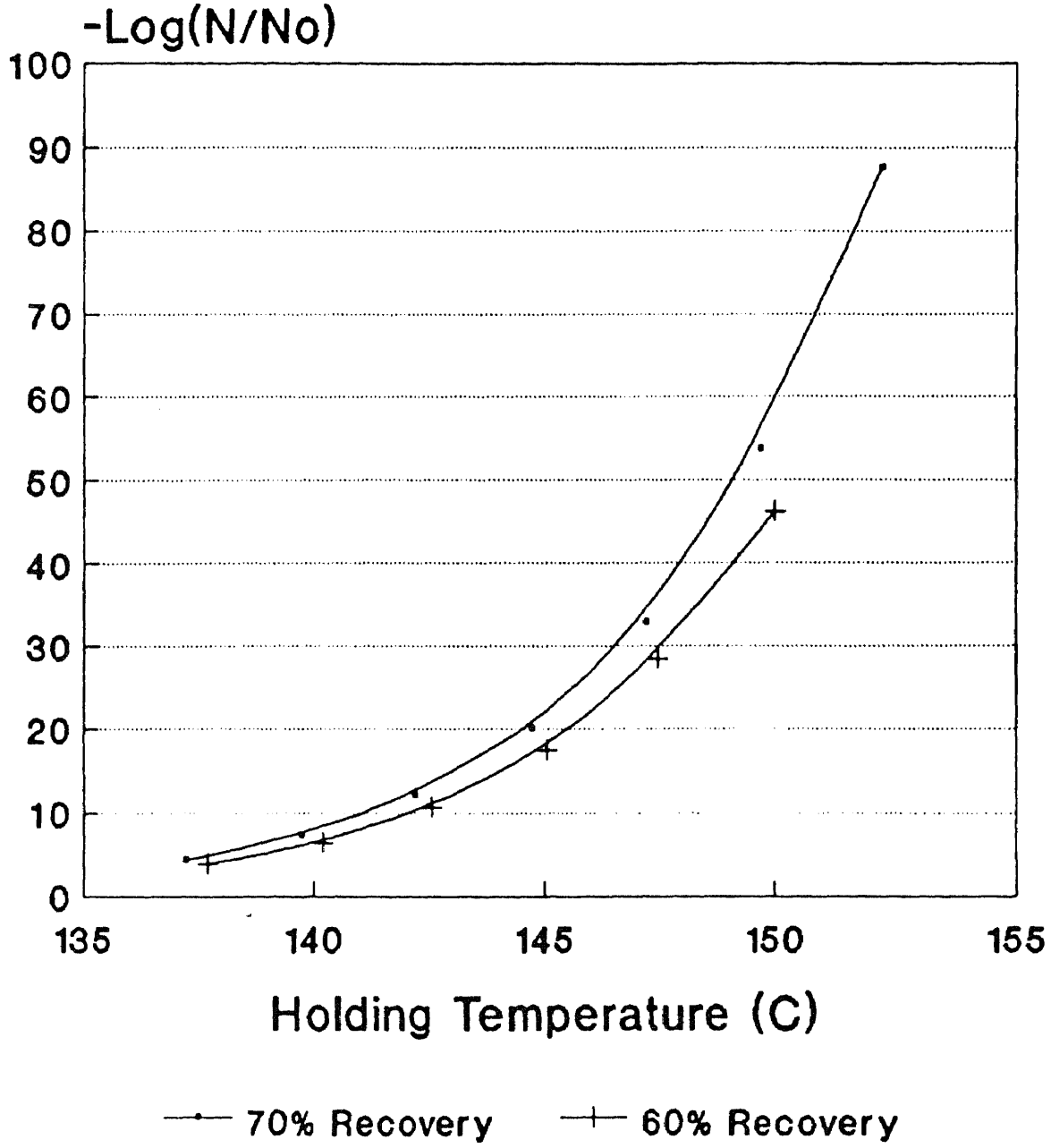


Figure 7 - The "Extra" Sterilization for the 70% Recovery System takes place in the Steam Exchanger

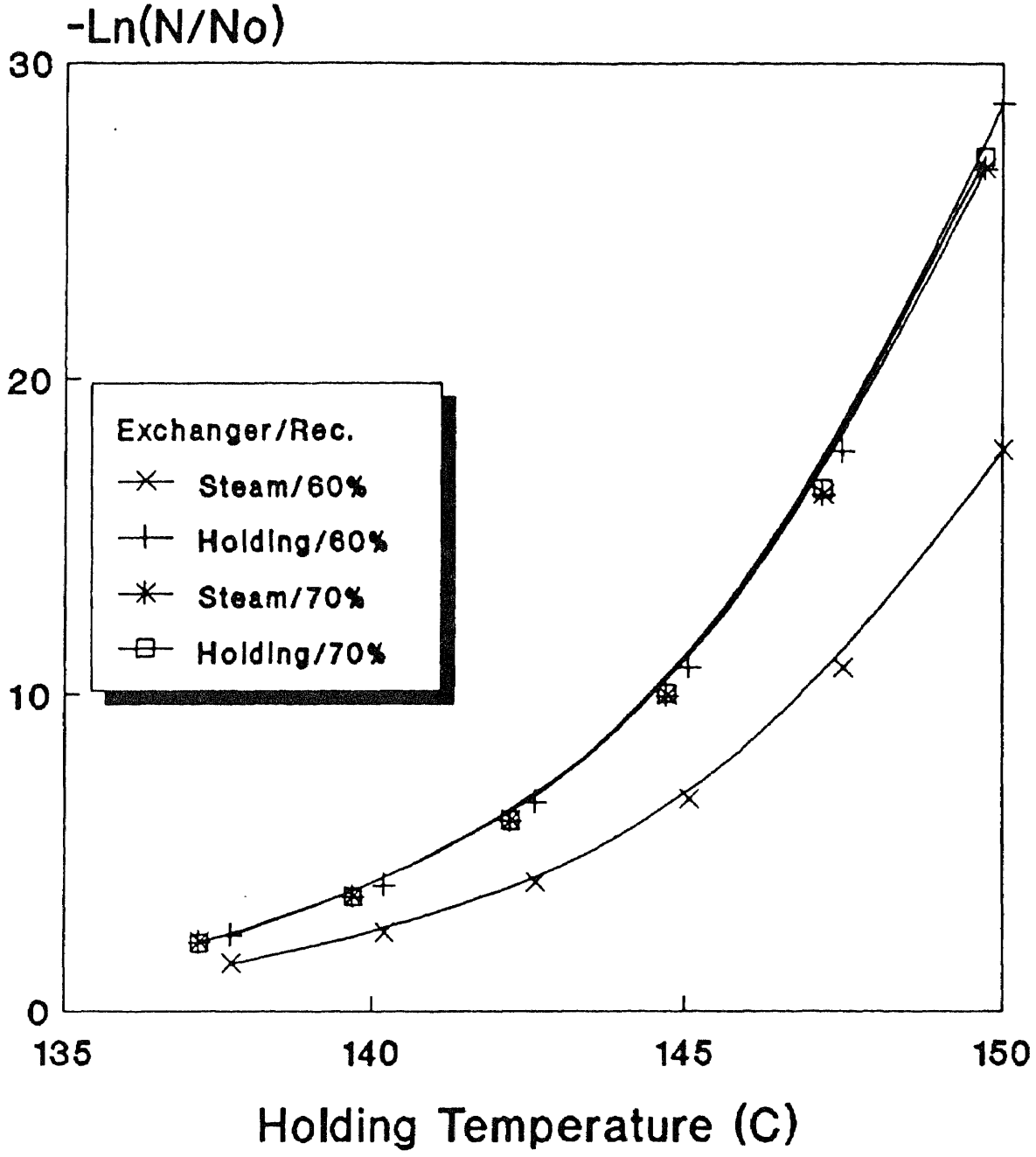
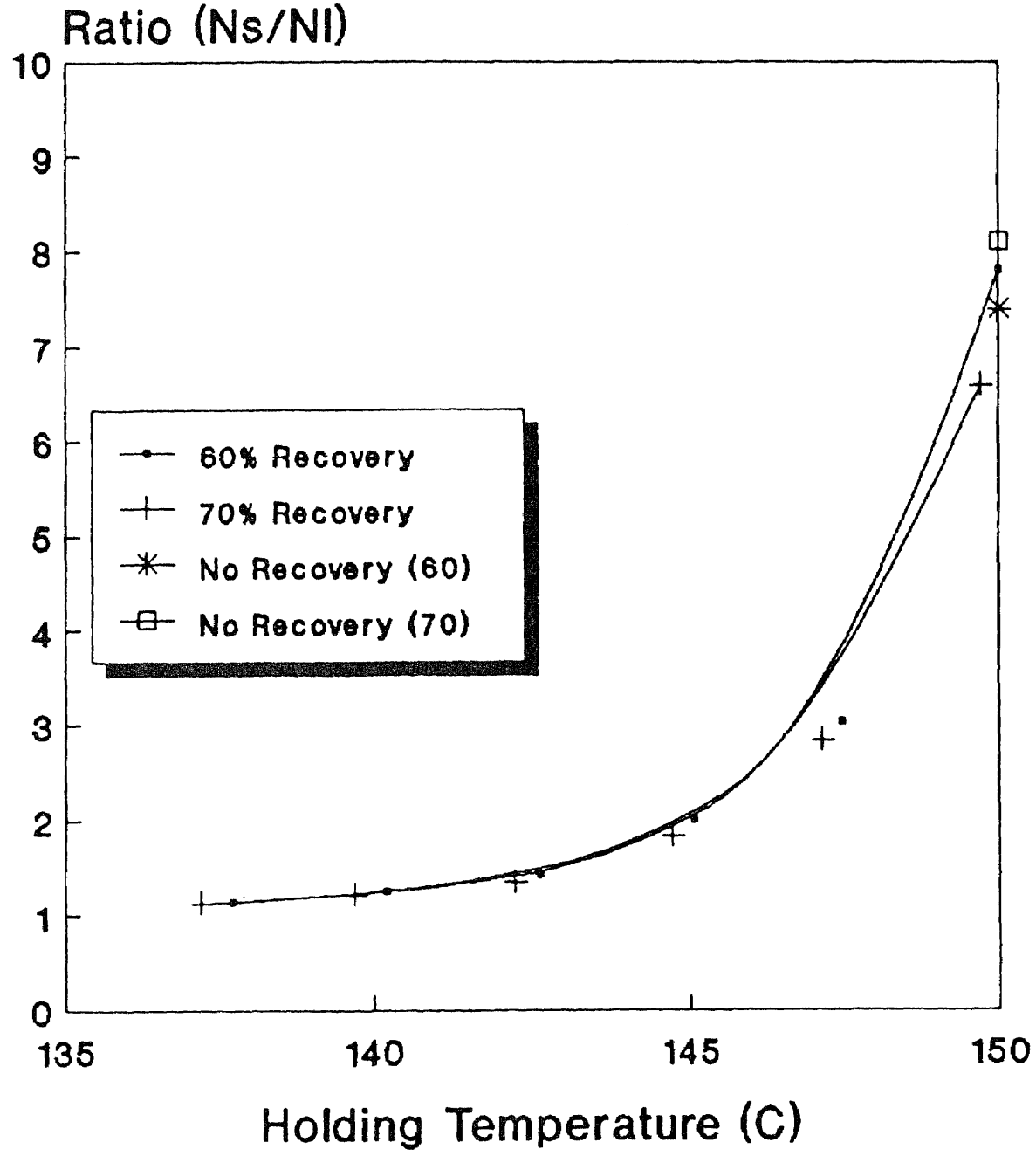


Figure 8 - Typical Continuous Sterilizer Shows Little Difference Between the Steriliization of Solid and Liquid



Ns/Nl = Solid(N/No)/Liquid(N/No)

holding temperature and fastest heat-up conditions, the difference between liquid and solid is less than one order of magnitude.

The reason for the lack of difference in shown in Figure 9, which shows that the temperature reached at the center of the sphere is only slightly below that of the fluid as it moves through the sterilizer. This plot uses the fastest heat-up case with no heat recovery, 70% recovery values for the overall heat transfer coefficient U, and a 150°C holding temperature. The largest delta between center and fluid temperature is still only 14°C, this occurring at the beginning of the heat-up, so that the contribution to sterilization is small.

External Heat Transfer Coefficient - While a value of about 9000 W/m^2K was obtained through calculation of our system, I was curious as to how critical this value was. Two values for L (hR/k) were evaluated at 60% recovery. The value of 130 obtained through calculation, and a tenth of this value, 13, which would correspond to an h of only about 900 W/m^2K. It is seen in Figure 10 that in any of the operating conditions evaluated, the external heat transfer coefficient is not a critical parameter. The delta between solid and liquid sterility is enhanced, at most, by a factor of 2.

Residence Time - Intuitively, the faster the heat-up of the medium, the greater the difference between the temperature inside suspended solids and the fluid temperature. This is the result of

Figure 9 - Temperature-Time Profile Shows Little Difference Between Fluid and Solid (at Center) Temperature

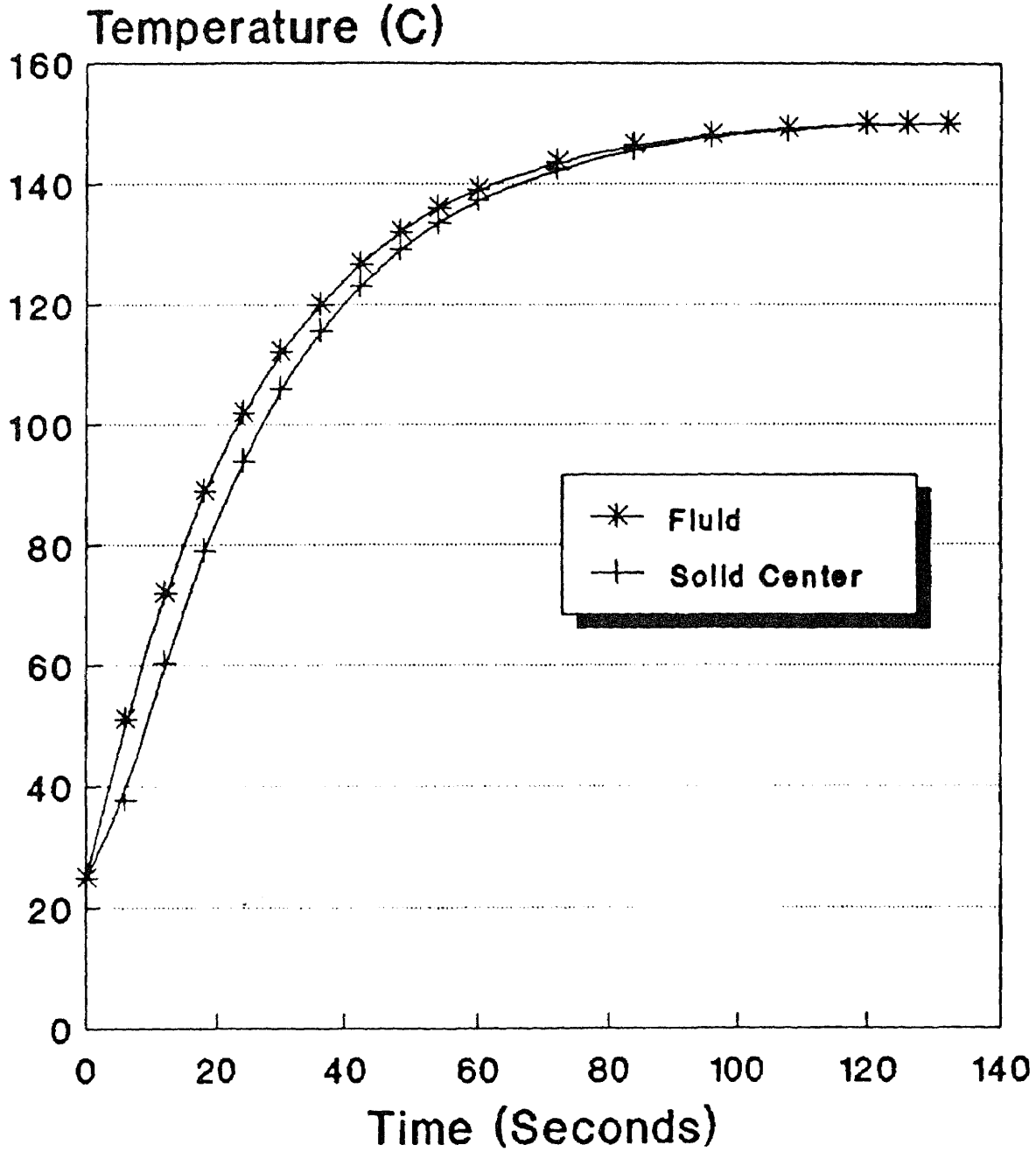
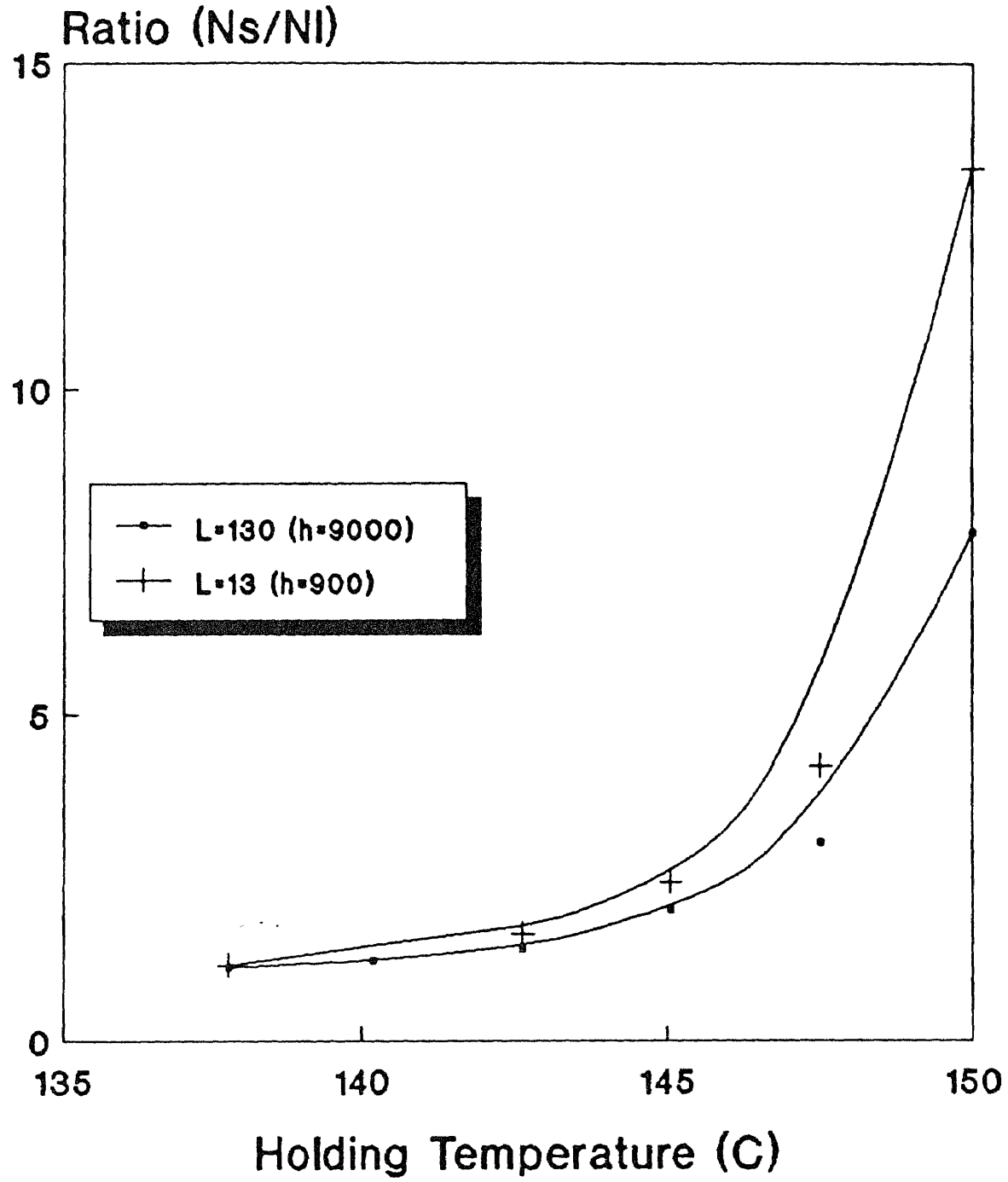


Figure 10 - External Heat Transfer Coefficient is Not a Critical Parameter



Ns/Nl = Solid(N/No)/Liquid(N/No)

the "lag" in heat transfer into the solid. Evaluation of this parameter was made in the following way:

- Assume no recovery section
- Adjust steam temperature
- Calculate Residence time required to achieve product sterilization temperature

The higher the steam temperature, the shorter the residence time in the exchanger in order to achieve the product temperature.

Figure 11 plots the residence time required to reach a 150°C holding temperature (for the fluid) as a function of the steam temperature used. While saturated steam temperatures of 200 and 300°C are not feasible in a typical steam exchanger, they are useful in demonstrating the trends in the data.

Figure 12, a plot of the degree of sterilization as a function of residence time, shows that the overall degree of sterilization decreases as residence time decreases. This, again, is due to a loss in sterilization achieved in the steam exchanger. The amount of sterilization in the holding section (for the fluid) remains constant, since residence time and temperature in this section are held constant.

The effect of residence time on the degree of sterilization achieved in the suspended solids is shown in Figure 13. Once again, this plot represents the ratio of organisms in the solid to

Figure 11 - Steam Heater Residence Time Required to Reach 150 C, as a Function of Steam Temperature

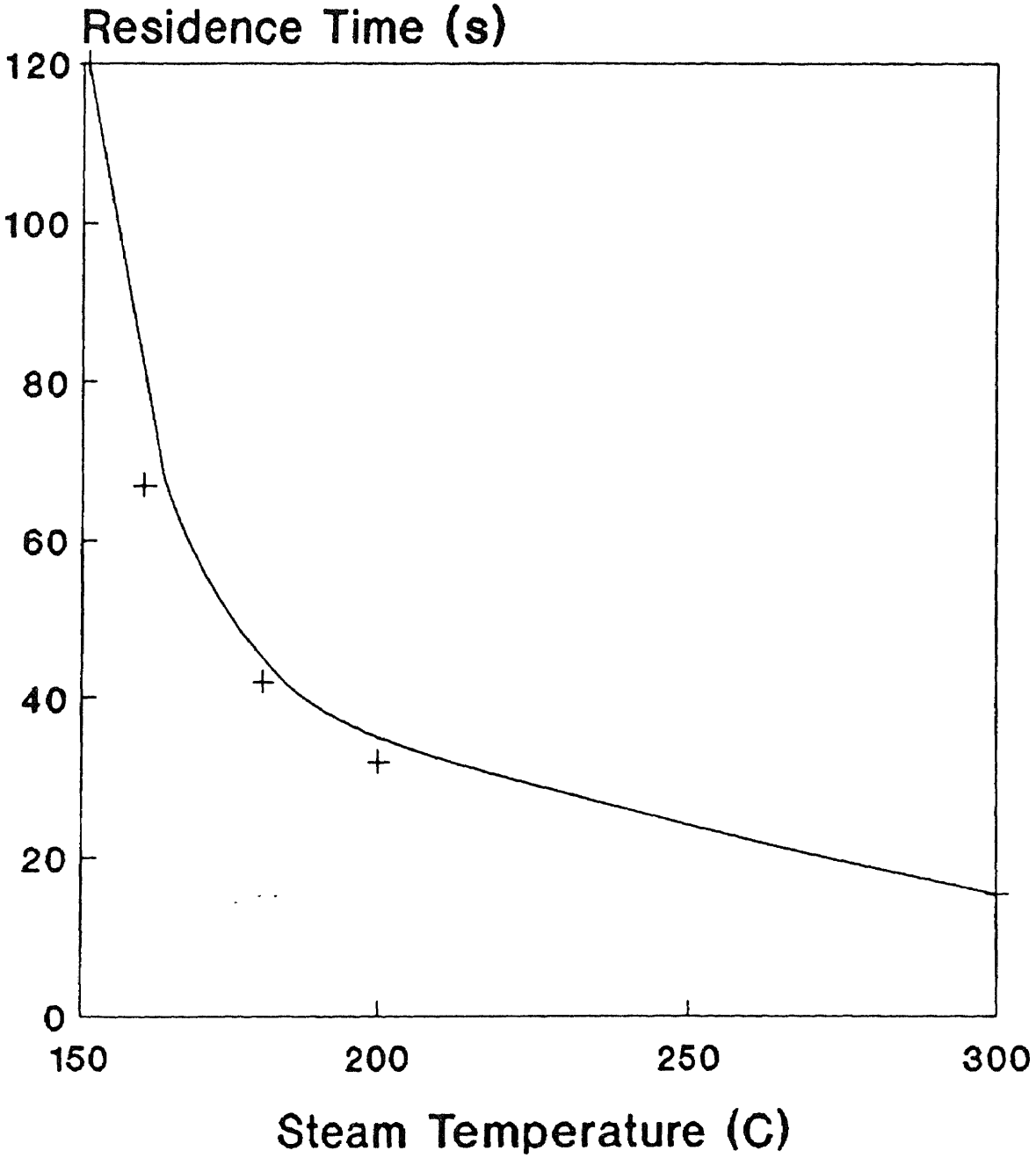


Figure 12 - The Overall Degree of Sterilization Decreases as Residence Time Decreases

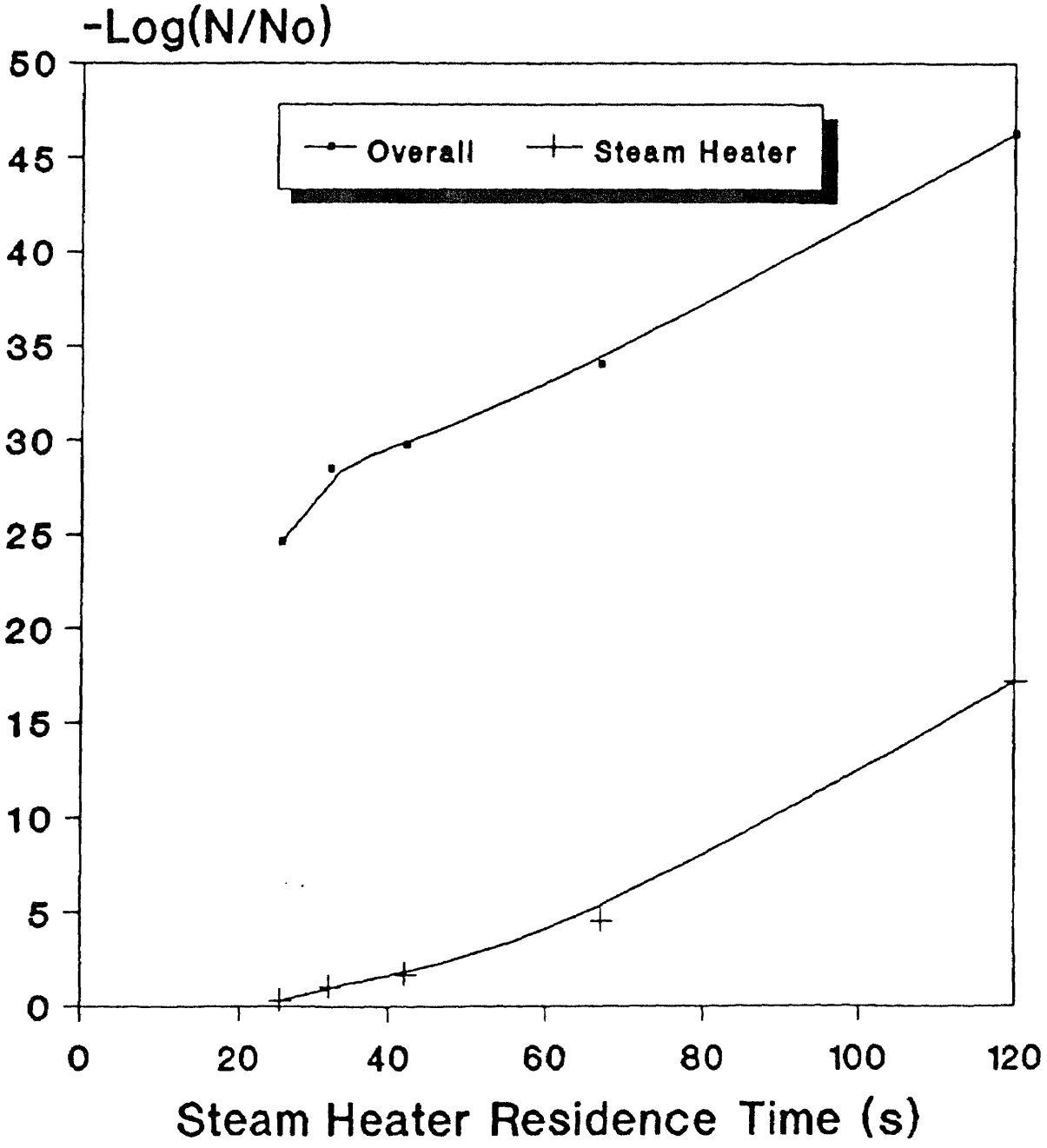
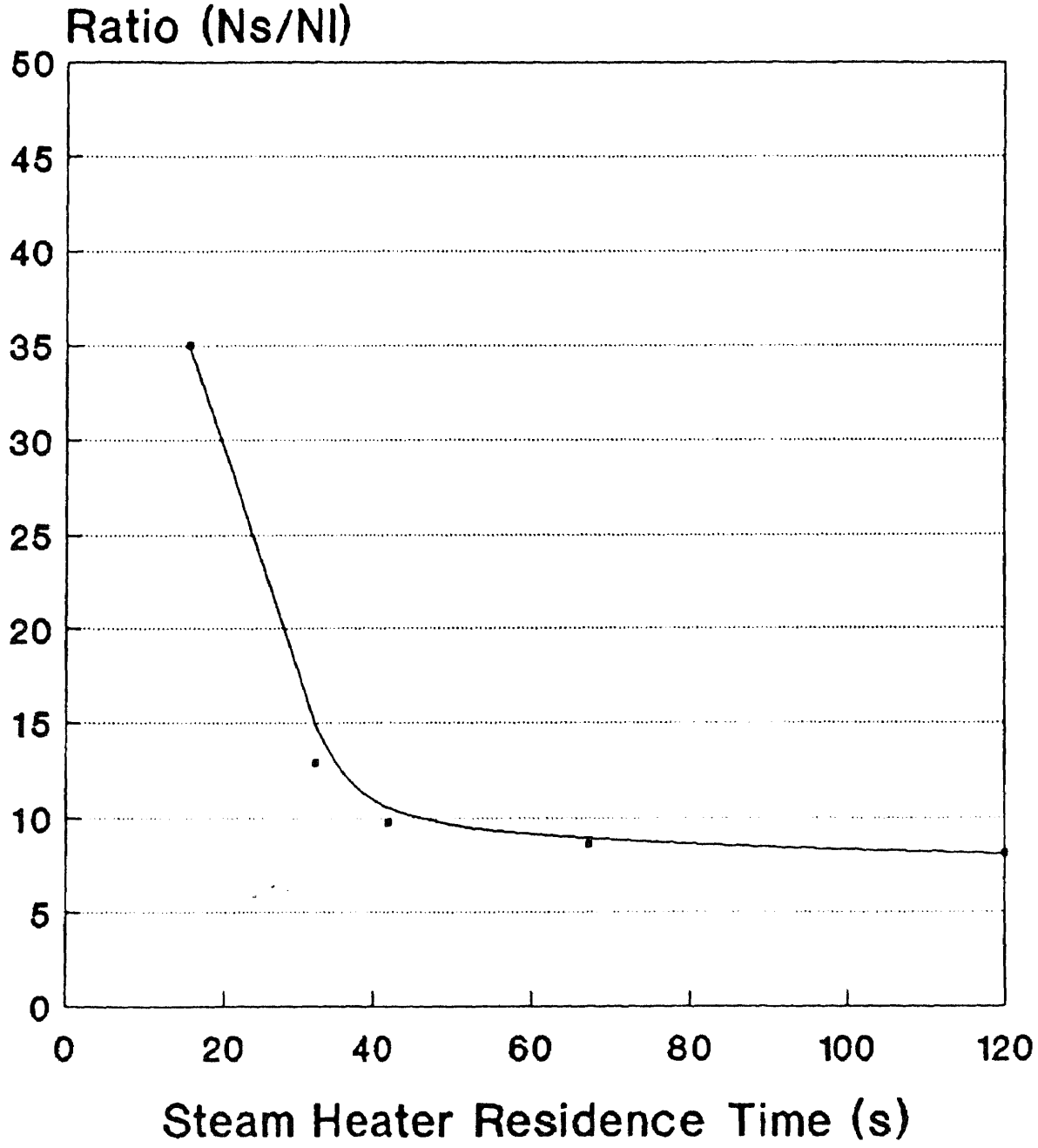


Figure 13 - Even With a Short Heat-up Time, Little Difference Between Liquid and Solid Sterilization is Apparant



Ns/Nl = Solid(N/No)/Liquid(N/No)

those in the fluid. Even at the shortest heat-up rate, 15 seconds, the degree of sterility between liquid and solids differs by less than a factor of 100 (2 orders of magnitude).

One final case was run in which no recovery section was used, the heat up section residence time was set at 35 seconds by adjusting steam temperature, the holding temperature was raised to 155°C, and the residence time in the holding section was reduced to 15 seconds. Under these somewhat extreme conditions, a difference in the degree of sterilization between solids and liquids approached 10^4 (4 orders of magnitude). It appears that for suspended solids of this particle diameter to deviate substantially from the liquid in the degree of sterility achieved, sterilization time must be in the order of tens of seconds, rather than minutes.

While the particle size of the suspended solid is obviously critical to the temperature profile achieved within it, it is intuitive that the larger the sphere, the greater the difference in core to surface temperature, and thus the greater the difference in sterility obtained between fluid and solid. The diameter of the sphere used in all evaluations is the largest possible for flow through the exchangers. For other systems, however, which may carry large suspended particles through very rapid sterilization system, the effect of these suspended solids on sterilization may be large.

14. CONCLUSIONS

A mathematical model has been developed for a continuous sterilization system where the role of suspended solids on the degree of sterilization achieved is taken into account.

Using physical parameters which are "typical" for commercial continuous sterilization systems, and assuming the properties of asbestos for our solid and water for our fluid, several parameters were evaluated for the effect they have on creating a delta in the degree of sterilization achieved between the liquid and solid fraction. These are listed in their order of importance:

- Residence Time
- Sterilization Temperature
- Heat Transfer Coefficients in the Exchangers
- External Heat Transfer Coefficient Around the Solid

Under our typical conditions, the role of the suspended solids did not have a large effect on the overall sterilization obtained (less than 2 orders of magnitude in N/N_0). However, for very fast, very high temperature systems, the effect of solids can become important, and should be evaluated.

15. NOMENCLATURE

T	:	Temperature (^oC)
T_B	:	Bulk fluid Temperature (^oC)
T_w	:	Steam Temperature (pipe wall temperature) (^oC)
v_z	:	Velocity of fluid in z direction (m/s)
V_z	:	Velocity of fluid in z direction (m/s)
ρ	:	Density (kg/m^3)
k	:	Thermal conductivity (W/m*K)
C_p	:	Heat capacity (J/kg*K)
z	:	Length (m)
A	:	Area (m^2)
V	:	Volume (m^3)
h	:	Heat transfer coefficient (W/m^2*C^o)
U	:	Overall heat transfer coefficient (W/m^2*C^o)
n		Integer values (-)
β_n	:	Eigenvalues satisfying $\beta_n \cot\beta_n + L - 1 = 0$
α	:	Thermal diffusivity constant $k/\ C_p$ (m^2/s)
R	:	Outer radius of sphere (m)
r	:	Radius (m)
Q	:	Energy (J/s)
E	:	Dimensionless radius r/R (-)
$\phi(E)$	:	Initial temperature profile of sphere (m^oC)

t	:	Time (s unless otherwise noted)
L	:	hR/k (-)
Nu	:	Nusselt Number hD/k (-)
Pr	:	Prandtl Number $C_p\ u/k$ (-)
N_f/N_0	:	Remaining viable fraction of organism
Δp	:	Pressure drop (N/m^2)
v	:	Kinematic viscosity (m^2/s)
ΔL	:	Length (m)
ϵ	:	Energy dissipation rate (m^2/s^2)
R	:	Universal gas constant = 1.987 cal/gmole^oK

APPENDIX A. PROGRAM

```
C     THIS IS THE MAIN PROGRAM
C
      DOUBLE PRECISION NUM,DENOM,PAR,EQUA,SUM
      DIMENSION BN(20),TS(5,30,20),TIME(5,30),TL(5,30)
      REAL L,W,XBN,ZBN,P,XCOTBN,XY,XF,ZCOTBN,ZY,ZF,Q,TIME1,TL0
      REAL TL1P,TL,TIME2,TW,TIME,TIME3
      OPEN (6,FILE='MODEL.DAT',STATUS='NEW')
C
C     THE PROGRAM ASKS FOR A REAL VALUE OF L TO BE SPECIFIED
C
      WRITE(*,19)
   19 FORMAT(1X,'ENTER REAL VALUE FOR L')
      READ(*,21) L
   21 FORMAT(F5.2)
      W=L-1.0
C
C     THE VALUE FOR L DETERMINES THE EIGEN VALUES USED IN THE
C     INFINITE SERIES SOLUTIONS TO ALL THE TEMPERATURE PROFILE
C     EQUATIONS.  THE CALCULATION OF THE EIGEN VALUES IS DONE
C     IN THE SUBROUTINE EIGEN
C
      CALL EIGEN (W,L,BN)
C
C     TEMPERATURES OUT OF THE FIRST EXCHANGER ARE CALCULATED
C     EXCHANGER RESIDENCE TIME AND TEMPERATURE DRIVING FORCE
C     ARE DEFINED, SUBROUTINE LIQUID1 CALCULATES THE OUTLET
C     LIQUID TEMPERATURE, SUBROUTINE SPHERE1 CALCULATES THE
C     TEMPERATURE PROFILE OF THE SPHERICAL PARTICLE
C
      TIME1=120.
      TL0=25.
      TL1P=73.0
      CALL LIQUID1 (TIME1,TL0,TL1P,TL,BP,TIME)
      CALL SPHERE1 (TIME1,TL0,TL1P,TL,BN,R,BP,TS,ALF,TIME)
C
C     TEMPERATURES OUT OF THE SECOND EXCHANGER ARE CALCULATED
C     EXCHANGER RESIDENCE TIME AND STEAM TEMPERATURE
C     ARE DEFINED, SUBROUTINE XLIQ2 CALCULATES THE OUTLET
C     LIQUID TEMPERATURE, SUBROUTINE XSPH2 CALCULATES THE
C     TEMPERATURE PROFILE OF THE SPHERICAL PARTICLE
C
      TIME2=120.
      TW=145.0
      CALL XLIQ2 (TIME2,TW,TL,BP2,TIME)
      CALL XSPH2 (TIME2,TL,TW,BN,R,BP2,TS,ALF,TIME)
C
C     TEMPERATURES OUT OF THE THIRD EXCHANGER ARE CALCULATED
C     THIS IS THE HOLDING SECTION.  RESIDENCE TIME IS DEFINED,
C     SUBROUTINE SPLIQ3 CALCULATES THE TEMPERATURE PROFILE OF
C     THE SPHERICAL PARTICLE
C
      TIME3=60.
      CALL SPLIQ3 (TIME3,TL,BN,R,TS,ALF,TL3,TIME)
C
```

```
C     FINALLY, THE REMAINING VIABLE FRACTION IS CALCULATED FOR
C     BOTH FRACTIONS AND IS PRINTED OUT
C
      CALL DEATH (TS,TIME,TL0,TL3,TL)
C
      STOP
      END
```

```
      SUBROUTINE EIGEN (W,L,BN)
C
C     THIS SUBROUTINE CALCULATES THE EIGEN VALUES FOR
C     THE SPECIFIED VALUE OF L
C
      REAL W,L,XBN,ZBN,P,XCOTBN,XY,XF,ZCOTBN,ZY,ZF,Q
      INTEGER J,M
      DIMENSION BN(20)
      OPEN (7,FILE='EIGEN.DAT',STATUS='NEW')
      XBN=0.00001
      J=1
   94 CONTINUE
C
C     INCREMENTS OF SIZE P ARE SEARCHED FOR A SOLUTION TO THE
C     EIGEN FUNCTION...THE SIZE OF P IS REDUCED ONCE A VALUE
C     HAS BEEN LOCATED...THE PROCESS IS REPEATED UNTIL AN
C     ACCURATE EIGEN VALUE HAS BEEN DETERMINED
C
      P=(3.141593-0.00002)/30.0
      M=1
   33 CONTINUE
C
C     THE FOLLOWING EQUATIONS RESULT IN 2 VALUES BEING TESTED
C     IN THE EIGEN FUNCTION
C
      XCOTBN=COS(XBN)/SIN(XBN)
      XY=XBN*XCOTBN
      XF=XY+W
      ZBN=XBN+P
      ZCOTBN=COS(ZBN)/SIN(ZBN)
      ZY=ZBN*ZCOTBN
      ZF=ZY+W
C
C     THE 2 VALUES FUNCTIONS ARE NOW MULTIPLIED TOGETHER AND THE
C     PRODUCT IS EVALUATED FOR ITS SIGN
C
      Q=XF*ZF
C
C     IF THE SIGN IS GREATER THAN ZERO, THE 2 VALUES ARE OF THE
C     SAME SIGN, AND AN EIGEN VALUE HAS NOT BEEN PASSED.
C     THE PROGRAM SEARCHES THE NEXT INCREMENT.
C
      IF(Q .GT. 0.0)GO TO 44
   75 CONTINUE
C
C     IF THE SIGN IS NEGATIVE, HOWEVER, AN EIGEN VALUE HAS BEEN
C     LOCATED.  THE NEW INCREMENT SIZE TO BE SEARCHED IS ONE
C     DECIMAL SPACE SMALLER
C
      P=P/10.0
C
C     IF THE EIGEN VALUE IS ACCURATE TO ENOUGH DECIMAL PLACES
C     THE VALUE IS RECORDED AS A SOLUTION
C
```

```
      IF(P .LE. 0.00002)GO TO 66
      ZBN=XBN
   44 CONTINUE
C
C     IF THE PROGRAM HAS GONE THROUGH AN ENTIRE RANGE OF PI
C     WITHOUT FINDING AN EIGEN VALUE, THE VALUE MUST BE THAT
C     MULTIPLE OF PI, SINCE THERE MUST BE ONE VALUE
C
      IF(M .GT. 30)GO TO 78
C
C     IF AN EIGEN VALUE HAS BEEN LOCATED AND P ADJUSTED, THE
C     PROGRAM MOVES BACK TO THE FIRST VALUE ABOVE TO BEGIN
C     THE NEW, SMALLER INCREMENT SEARCH
C
      IF(P. LE. 0.10)GO TO 43
      M=M+1
   43 CONTINUE
      XBN=ZBN
      GO TO 33
   66 CONTINUE
      GO TO 39
   78 CONTINUE
      BN(J)=J*3.141593
      GO TO 38
   39 CONTINUE
      BN(J)=ZBN
   38 CONTINUE
C
C     THE PROGRAM HAS FOUND AN EIGEN VALUE.  IT NOW SEARCHES
C     THE NEXT PI INCREMENT FOR THE NEXT VALUE
C
      J=J+1
C
C     UNLESS IT HAS ALREADY FOUND ENOUGH OF THEM.  IT WILL
C     THEN EXIT THE SUBOUTINE
C
      IF(J .GE. 21)GO TO 71
   73 CONTINUE
      XBN=3.141594*(J-1)+0.00001
      GO TO 94
   71 CONTINUE
      WRITE(7,16) L
      DO 31 J=1,20
      WRITE(7,17)J,BN(J)
   31 CONTINUE
      RETURN
   16 FORMAT(2X,'L EQUALS',F5.2)
   17 FORMAT(1X,I3,3X,F8.3)
      END
```

```
      SUBROUTINE LIQUID1 (TIME1,TL0,TL1P,TL,BP,TIME)
C
C     THIS SUBROUTINE CALCULATES THE FLUID TEMPERATURE AT 20
C     INTERVALS THROUGH THE FIRST EXCHANGER
C
      DIMENSION TL(5,30),TIME(5,30)
      REAL HL,RHOL,RL,CPL,BP,TL,T,TIME
      INTEGER IT,I
C
C     FLUID DATA IS ENTERED HERE.  RL IS THE RADIUS OF THE PIPE.
C
      HL=305.
      RHOL=962.7
      RL=0.006
      CPL=4229
      BP=(HL)/(RHOL*RL*CPL)
      T=TIME1/20.
C
C     THIS LOOP IS WHERE THE 20 VALUES OF TEMP ARE CALCULATED.
C
      DO 48 IT=1,20
      TIME(1,IT)=IT*T
      TL(1,IT)=TL0+BP*(TL1P-TL0)*TIME(1,IT)
   48 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SPHERE1 (TIME1,TL0,TL1P,TL,BN,R,BP,TS,ALF,TIME)
C
C     THIS SUBROUTINE CALCULATES THE TEMPERATURE PROFILE OF A
C     SPHERICAL PARTICLE AT THE SAME 20 INTERVALS THROUGH THE
C     FIRST EXCHANGER.  TEN PERCENT INCREMENTS OF THE PARTICLE
C     RADIUS ARE USED TO GENERATE THE PROFILE.
C
      DIMENSION BN(20),TS(5,30,20),TIME(5,30),TL(5,30)
      REAL TIME1,TL0,TL1P,TL,BN,E,SUM,PAR,NUM,R,S,DENOM,ALF,EQUA
      REAL TIME,T
      INTEGER M,N,IT,I
      OPEN (6,FILE='MODEL.DAT',STATUS='NEW')
C
C     HERE, DATA IS ENTERED FOR THE SOLID SPHERE.
C
      R=0.0025
      T=TIME1/20.
      S=BP*(TL1P-TL0)
      ALF=0.0000003
C
C     THIS LOOP GENERATES THE 20 INTERVALS THROUGH THE EXCHANGER
C
      DO 48 IT=1,20
      TIME(1,IT)=IT*T
      E=0.0
      SUM=0.0
      M=1
C
C     THIS FIRST SET OF EQUATIONS CALCULATES THE TEMPERATURE
C     VALUES AT THE CENTER OF THE SPHERE
C
C     THIS LOOP IS THE SUMMATION OF THE INFINITE SERIES SOLUTION
C
      DO 39 N=1,20
      NUM=2.0*S*R*R*(COS(BN(N))-(SIN(BN(N))/BN(N)))
      DENOM=ALF*(BN(N)**2.0-BN(N)*SIN(BN(N))*COS(BN(N)))
      PAR=1.0-EXP(-1.0*ALF*(BN(N)/R)**2.0*TIME(1,IT))
      EQUA=(NUM/DENOM)*PAR
      SUM=SUM+EQUA
   39 CONTINUE
C
C     THESE VALUES ARE RECORDED
C
      TS(1,IT,0)=TL(1,IT)+SUM
   37 CONTINUE
C
C     THESE NEXT SETS OF EQUATIONS CALCULATE THE REST OF THE
C     PROFILE POINTS THROUGH THE RADIUS.  E=INCREMENT/RADIUS.
C
      SUM=0.0
      E=M/10.
      DO 41 N=1,20
      NUM=2.0*S*R*R*(COS(BN(N))-(SIN(BN(N))/BN(N)))*SIN(BN(N)*E)
      DENOM=ALF*E*(BN(N)**3.0-BN(N)**2.0*SIN(BN(N))*COS(BN(N)))
```

```
      PAR=1.0-EXP(-1.0*ALF*(BN(N)/R)**2.0*TIME(1,IT))
      EQUA=(NUM/DENOM)*PAR
      SUM=SUM+EQUA
   41 CONTINUE
C
C     THESE VALUES ARE RECORDED
C
      TS(1,IT,M)=TL(1,IT)+SUM
      M=M+1
      IF(M .GT. 10)GO TO 38
      GO TO 37
   38 CONTINUE
   48 CONTINUE
C
C     VALUES ARE WRITTEN INTO A FILE
C
      DO 27 IT=1,20
      WRITE(6,92) TIME(1,IT)
      DO 17 J=1,11
      K=J-1
      WRITE(6,31) TS(1,IT,K)
   17 CONTINUE
   27 CONTINUE
      RETURN
   31 FORMAT(2X,'TS EQUALS',F8.2)
   92 FORMAT(1X,'TIME IN EXCHANGER 1 EQUALS',F8.2)
      END
```

```
      SUBROUTINE XLIQ2 (TIME2,TW,TL,BP2,TIME)
C
C     THIS SUBROUTINE CALCULATE FLUID TEMPERATURE VALUES FOR
C     20 INTERVALS THROUGH HEAT EXCHANGER 2.
C
      DIMENSION TL(5,30),TIME(5,30)
      REAL HL2,RHOL,RL2,CPL,BP2,TL,TW,TIME2,T,TIME
      INTEGER IT,I
C
C     FLUID DATA IS ENTERED HERE
C
      HL2=610.
      RHOL=962.7
      RL2=0.006
      CPL=4229.
      BP2=HL2/(RHOL*RL2*CPL)
      T=TIME2/20.
C
C     THIS LOOP CALCULATES THE 20 VALUES
C
      DO 48 IT=1,20
      TIME(2,IT)=IT*T
      TL(2,IT)=TW+EXP(-1.*BP2*TIME(2,IT))*(TL(1,20)-TW)
   48 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE XSPH2 (TIME2,TL,TW,BN,R,BP2,TS,ALF,TIME)
C
C     THIS SUBROUTINE CALCULATES THE TEMPERATURE PROFILE OF THE
C     SPHERE AT THE SAME 20 INTERVALS THROUGH EXCHANGER 2.
C
      DIMENSION BN(20),TS(5,30,20),TIME(5,30),TL(5,30)
      REAL TIME2,TL,TW,BN,R,BP2,TS,E,SUM,PART,NUM,DENO,ALF,T,TIME
      REAL EQUA,XICSUM,XIC,AN,AN1,E1,DE1,ASUMI1,ASUMI2,ASUMI,AVGTS
      REAL F,G
      INTEGER M,N,J,I,L,K,DI,IT,II
      OPEN (6,FILE='MOD.DAT',STATUS='NEW')
      CPP=BP2*(TL(1,20)-TW)
      T=TIME2/20.
C
C     THIS LOOP GENERATES THE 20 INTERVALS THROUGH THE EXCHANGER.
C
      DO 48 IT=1,20
      TIME(2,IT)=IT*T
      E=0.0
      SUM=0.0
      XICSUM=0.0
      J=1
C
C     THIS LOOP CALCULATES THE SUMMATION OF 20 TERMS IN THE
C     GENERATION OF THE EFFECTS OF INITIAL CONDITIONS ON THE
C     SOLUTION
C
      DO 54 M=1,20
      AN1=0.0
      I=0
      DI=1
      DE1=0.1
C
C     THESE VALUES ARE CALCULATED THROUGH THE SUMMATION OF TERMS
C     ASSOCIATED WITH RADIUS.
C
      DO 57 N=1,10
      E1=I/10.0
      AVGTS=((TS(1,20,I)+TS(1,20,I+1))/2.)
      ASUMI1=(E1+DE1/2.)*R*(AVGTS-TL(1,20))
      ASUMI2=SIN(BN(M)*(E1+DE1/2.))*DE1
      ASUMI=ASUMI1*ASUMI2
      AN1=AN1+ASUMI
      I=I+DI
   57 CONTINUE
      AN=(2.*BN(M))/(BN(M)-SIN(BN(M))*COS(BN(M)))*AN1
      XIC=(BN(M)/R)*EXP(-1.*ALF*(BN(M)/R)**2.*TIME(2,IT))*AN
      XICSUM=XICSUM+XIC
   54 CONTINUE
C
C     THIS LOOP IS THE SUMMATION OF 20 TERMS IN THE GENERATION
C     OF THE BOUNDARY CONDITION EFFECTS ON THE SOLUTION.
C
      DO 53 M=1,20
```

```
      NUM=2.*CPP*BN(M)*(COS(BN(M))-SIN(BN(M))/BN(M))
      DENO=(BN(M)-SIN(BN(M))*COS(BN(M)))*(ALF*(BN(M)/R)**2.-BP2)
      F=EXP(-1.*BP2*TIME(2,IT))
      PART=F-EXP(-1.*ALF*(BN(M)/R)**2.*TIME(2,IT))
      EQUA=(NUM/DENO)*PART
      SUM=SUM+EQUA
   53 CONTINUE
C
C     THE SOLUTION FOR THE 20 VALUES AT THE CENTER OF THE SPHERE
C     IS THE COMBINATION OF THE 2 PARTS ABOVE.
C
      TS(2,IT,0)=TL(2,IT)+XICSUM-SUM
   59 CONTINUE
C
C     THE PROCESS IS REPEATED FOR THE OTHER TEMPERATURE PROFILE
C     POINTS (VALUES OF RADIUS) IN THE SPHERE.
C
      SUM=0.0
      XICSUM=0.0
      E=J/10.
C
C     THIS LOOP CALCULATES THE SUMMATION OF 20 TERMS IN THE
C     GENERATION OF THE EFFECTS OF INITIAL CONDITIONS ON THE
C     SOLUTION
C
      DO 64 M=1,20
      AN1=0.0
      I=0
      DI=1
      DE1=0.1
C
C     THESE VALUES ARE CALCULATED THROUGH THE SUMMATION OF TERMS
C     ASSOCIATED WITH RADIUS.
C
      DO 67 N=1,10
      E1=I/10.
      AVGTS=((TS(1,20,I)+TS(1,20,I+1))/2.)
      ASUMI1=(E1+DE1/2.)*R*(AVGTS-TL(1,20))
      ASUMI2=SIN(BN(M)*(E1+DE1/2.))*DE1
      ASUMI=ASUMI1*ASUMI2
      AN1=AN1+ASUMI
      I=I+DI
   67 CONTINUE
      AN=(2.*BN(M))/(BN(M)-SIN(BN(M))*COS(BN(M)))*AN1
      G=SIN(BN(M)*E)/(E*R)
      XIC=G*EXP(-1.*ALF*(BN(M)/R)**2.*TIME(2,IT))*AN
      XICSUM=XICSUM+XIC
   64 CONTINUE
C
C     THIS LOOP IS THE SUMMATION OF 20 TERMS IN THE GENERATION
C     OF THE BOUNDARY CONDITION EFFECTS ON THE SOLUTION.
C
      DO 63 M=1,20
      NUM=2.*CPP*SIN(BN(M)*E)*(COS(BN(M))-SIN(BN(M))/BN(M))
```

```
      DENO=E*(BN(M)-SIN(BN(M))*COS(BN(M)))*(ALF*(BN(M)/R)**2.-BP2)
      F=EXP(-1.*BP2*TIME(2,IT))
      PART=F-EXP(-1.*ALF*(BN(M)/R)**2.*TIME(2,IT))
      EQUA=(NUM/DENO)*PART
      SUM=SUM+EQUA
   63 CONTINUE
C
C     THE SOLUTION FOR THE 20 VALUES AT EACH INCREMENT OF RADIUS
C     IS THE COMBINATION OF THE 2 PARTS ABOVE.
C
      TS(2,IT,J)=TL(2,IT)+XICSUM-SUM
      J=J+1
      IF(J .GT. 10)GO TO 68
      GO TO 59
   68 CONTINUE
   48 CONTINUE
C
C     THE DATA GENERATED IS PRINTED INTO A FILE.
C
      DO 27 IT=1,20
      WRITE(6,32) TIME(2,IT)
      DO 77 L=1,11
      K=L-1
      WRITE(6,31) K,TS(2,IT,K)
   77 CONTINUE
   27 CONTINUE
      RETURN
   31 FORMAT(2X,'E EQUALS',I4,2X,'TS EQUALS',F8.2)
   32 FORMAT(1X,'TIME IN EXCHANGER 2 EQUALS',F8.2)
      END
```

```
      SUBROUTINE SPLIQ3 (TIME3,TL,BN,R,TS,ALF,TL3,TIME)
C
C     THIS SUBROUTINE CALCULATES THE TEMPERATURE PROFILE OF THE
C     SPHERE AT 20 INTERVALS THROUGH THE HOLDING SECTION.  THE
C     FLUID TEMPERATURE IS CONSTANT IN THIS SECTION.
C
      DIMENSION BN(20),TS(5,30,20),TIME(5,30),TL(5,30)
      REAL TIME3,TL3,TL,BN,R,TS,E,XICSUM,XIC,AN,AN1,E1,DE1,DE
      REAL ASUMI,ASUMI1,ASUMI2,TIME,T,AVGTS,G
      INTEGER M,J,I,K,L,N,DI,IT,II
      OPEN (3,FILE='LAST.DAT',STATUS='NEW')
C
C     THE FLUID TEMPERATURE IS CONSTANT THROUGH THIS SECTION AND
C     EQUAL TO THE TEMPERATURE OF THE FLUID OUT OF EXCHANGER 2.
C
      TL3=TL(2,20)
      WRITE(3,13) TL3
   13 FORMAT(2X,'TL3 EQUALS',F8.3)
      T=TIME3/20.
C
C     THIS LOOP GENERATES THE 20 INTERVALS THROUGH THE EXCHANGER
C
      DO 48 IT=1,20
      TIME(3,IT)=IT*T
      E=0.0
      XICSUM=0.0
      J=1
C
C     THIS FIRST SET OF EQUATIONS DETERMINES THE VALUES AT THE
C     CENTER OF THE SPHERE.
C
C     THIS LOOP IS THE SUMMATION OF 20 TERMS IN THE INFINITE
C     SERIES SOLUTION.
C
      DO 54 M=1,20
      AN1=0.0
      I=0
      DI=1
      DE1=0.1
C
C     THESE VALUES ARE CALCULATED THROUGH THE SUMMATION OF TERMS
C     ASSOCIATED WITH RADIUS.
C
      DO 57 N=1,10
      E1=I/10.0
      AVGTS=((TS(2,20,I)+TS(2,20,I+1))/2.)
      ASUMI1=(E1+DE1/2.)*R*(AVGTS-TL(2,20))
      ASUMI2=SIN(BN(M)*(E1+DE1/2.))*DE1
      ASUMI=ASUMI1*ASUMI2
      AN1=AN1+ASUMI
      I=I+DI
   57 CONTINUE
      AN=(2.*BN(M))/(BN(M)-SIN(BN(M))*COS(BN(M)))*AN1
      XIC=(BN(M)/R)*EXP(-1.*ALF*(BN(M)/R)**2.*TIME(3,IT))*AN
```

```
      XICSUM=XICSUM+XIC
   54 CONTINUE
C
C     THE SOLUTION FOR THE 20 VALUES AT THE CENTER OF THE SPHERE
C     IN THIS CASE IS THE EXPRESSION BELOW, WHICH CONTAINS ONLY
C     AN INITIAL CONDITION TYPE EXPRESSION PLUS THE BULK FLUID
C     TEMPERATURE.
C
      TS(3,IT,0)=TL3+XICSUM
   59 CONTINUE
C
C     THE REST OF THE VALUES AT INTERVALS OF RADIUS ARE CALCULATED
C     BELOW.
C
      XICSUM=0.0
      E=J/10.
C
C     THIS LOOP CALCULATES THE SUMMATION OF 20 TERMS IN THE
C     INFINITE SERIES SOLUTION.
C
      DO 64 M=1,20
      AN1=0.0
      I=0
      DI=1
      DE1=0.1
C
C     THESE VALUES ARE CALCULATED THROUGH THE SUMMATION OF TERMS
C     ASSOCIATED WITH RADIUS.
C
      DO 67 N=1,10
      E1=I/10.
      AVGTS=((TS(2,20,I)+TS(2,20,I+1))/2.)
      ASUMI1=(E1+DE1/2.)*R*(AVGTS-TL(2,20))
      ASUMI2=SIN(BN(M)*(E1+DE1/2.))*DE1
      ASUMI=ASUMI1*ASUMI2
      AN1=AN1+ASUMI
      I=I+DI
   67 CONTINUE
      AN=(2.*BN(M))/(BN(M)-SIN(BN(M))*COS(BN(M)))*AN1
      G=SIN(BN(M)*E)/(E*R)
      XIC=G*EXP(-1.*ALF*(BN(M)/R)**2.*TIME(3,IT))*AN
      XICSUM=XICSUM+XIC
   64 CONTINUE
C
C     AGAIN, THE SOLUTION CONTAINS ONLY ONE OTHER TERM ASIDE FROM
C     THE FLUID BULK TEMPERATURE, AND THAT TERM IS ASSOCIATED
C     WITH THE INITIAL CONDITIONS ONLY.
C
      TS(3,IT,J)=TL3+XICSUM
      J=J+1
      IF(J .GT. 10)GO TO 68
      GO TO 59
   68 CONTINUE
   48 CONTINUE
```

```
C
C     THE DATA GENERATED IS PRINTED INTO A FILE.
C
      DO 27 IT=1,20
      WRITE(3,32) TIME(3,IT)
      DO 77 L=1,11
      K=L-1
      WRITE(3,31) K,TS(3,IT,K)
   77 CONTINUE
   27 CONTINUE
      RETURN
   31 FORMAT(2X,'E EQUALS',I4,2X,'TS EQUALS',F8.2)
   32 FORMAT(1X,'TIME IN EXCHANGER 3 EQUALS',F8.2)
      END
```

```
      SUBROUTINE DEATH (TS,TIME,TL0,TL3,TL)
C
C     THIS SUBROUTINE CALCULATES THE REMAINING VIABLE FRACTION
C     OF BACTERIA OUT OF EACH OF THE 3 SECTIONS OF EXCHANGER
C     FOR BOTH THE FLUID AND THE SOLID SPHERICAL PARTICLE.  IT
C     ALSO PRINTS OUT THE TEMPERATURE PROFILES OF BOTH FLUID AND
C     PARTICLE THROUGHOUT THE COURSE OF THE ENTIRE STERILIZER.
C
      DIMENSION TS(5,30,20),TIME(5,30),TL(5,30),XNL(5),XNS(5,30)
      DIMENSION AVGD(5),LPOWER(5),SPOWER(5)
      DOUBLE PRECISION XKD0,XLN,P1,PT,PART,ED,XNL,XNS,AVGD
      DOUBLE PRECISION LPOWER,SPOWER
      REAL TS,TIME,TL,TL0,TL3,R,E
      REAL DE,DELT
      INTEGER N,M,I,K,J
      OPEN (5,FILE='DEATH.DAT',STATUS='NEW')
      R=0.0019872
      TL(0,20)=TL0
      ED=70.0
      XKD0=1.583*10.**36.
C
C     THIS SMALL LOOP SETS THE FLUID TEMPERATURE OF EACH INTERVAL
C     IN EXCHANGER 3 AS THE CONSTANT VALUE TL3.
C
      DO 14 J=1,20
      TL(3,J)=TL3
   14 CONTINUE
      DE=0.1
C
C     THIS LOOP GENERATES DATA FOR ALL 3 EXCHANGERS.
C
      DO 99 N=1,3
      TL(N,0)=TL(N-1,20)
      XLN=0.0
C
C     THE INTEGRATION TO BE PERFORMED IS WITH RESPECT TO TIME.
C     EACH EXCHANGER HAS 20 INTERVALS OF TIME, BUT THE ARE NOT
C     THE SAME FROM EXCHANGER TO EXCHANGER.  THE INTERVAL OF TIME
C     USED IN THE INTEGRATION IS SET HERE.
C
      DELT=TIME(N,3)-TIME(N,2)
C
C     THIS LOOP PERFORMS THE INTEGRATION TO DETERMINE REMAINING
C     VIABLE FRACTION FOR THE FLUID.
C
      DO 89 M=1,20
      PART=DEXP(-1.*ED/(R*((TL(N,M-1)+TL(N,M))/2.+273.15)))
      PT=XKD0*PART*DELT
      XLN=PT+XLN
   89 CONTINUE
      IF(XLN. GT. 150.)XLN=150.
      LPOWER(N)=XLN
      XLN=DEXP(XLN)
      XNL(N)=1./XLN
```

```
C
C     THE INITIAL TEMPERATURE OF THE SPHERE IS ASSUMED CONSTANT
C     AND AT INITIAL FLUID TEMPERATURE TL0.
C
      DO 84 K=1,10
      TS(0,20,K)=TL0
   84 CONTINUE
C
C     THIS SECTION CALCULATES N/N0 FOR THE SOLID PARTICLE.
C
      AVGD(N)=0.
C
C     THE INITIAL TEMPERATURE AT EACH INCREMENT OF RADIUS INTO
C     AN EXCHANGER IS SET TO THE DEPARTING TEMPERATURE OF THE
C     PREVIOUS EXCHANGER.
C
      DO 79 I=1,10
      E=I/10.
      XLN=0.0
      TS(N,0,I)=TS(N-1,20,I)
C
C     THIS LOOP CALCULATES 20 TERMS IN THE INFINITE SERIES
C     SOLUTION.
C
      DO 69 M=1,20

      P1=DEXP(-1.*ED/(R*((TS(N,M-1,I)+TS(N,M,I))/2.+273.15)))
      PART=XKD0*P1*DELT
      XLN=PART+XLN
   69 CONTINUE
      IF(XLN. GT. 150.)XLN=150.
      XLN=DEXP(XLN)
      XNS(N,I)=1./XLN
C
C     THIS EXPRESSION VOLUME AVERAGES N/N0 FOR THE SPHERE.
C
      AVGD(N)=AVGD(N)+3.*E**2.*DE*XNS(N,I)
   79 CONTINUE
C
C     THERE IS A FINAL NORMALIZATION TO COMPENSATE FOR THE ERROR
C     GENERATED IN THE NUMERICAL INTERGRATION
C
      AVGD(N)=AVGD(N)/1.155
      SPOWER(N)=DLOG(1./AVGD(N))
   99 CONTINUE
C
C     THE FOLLOWING SECTION MERELY PRINTS OUT THE DATA GENERATED
C
      WRITE(5,31)
   31 FORMAT(1X,'EXCHANGER',2X,'TIME',2X,'LIQ TEMP',2X,'XNL')
      WRITE(5,32) TL0
   32 FORMAT(13X,'0',6X,F7.2)
      DO 13 I=1,3
      DO 15 J=1,20
```

```
      WRITE(5,33) I,TIME(I,J),TL(I,J)
   33 FORMAT(5X,I1,5X,F6.2,1X,F7.2)
   15 CONTINUE
      WRITE(5,34) XNL(I),LPOWER(I)
   34 FORMAT(28X,D15.6,3X,D15.6)
   13 CONTINUE
C
C
      WRITE(5,35)
   35 FORMAT(15X,'SOLIDS TEMPERATURE')
      WRITE(5,36)
   36 FORMAT(1X,'E = 0',5X,'1',6X,'2',6X,'3',6X,'4')
      WRITE(5,37) TL0,TL0,TL0,TL0,TL0
   37 FORMAT(1X,5(F6.2,1X))
      DO 16 I=1,3
      DO 17 J=1,20
      WRITE(5,8) TS(I,J,0),TS(I,J,1),TS(I,J,2),TS(I,J,3),TS(I,J,4)
    8 FORMAT(1X,5(F6.2,1X))
   17 CONTINUE
   16 CONTINUE
C
      WRITE(5,55)
   55 FORMAT(15X,'SOLIDS TEMPERATURE')
      WRITE(5,56)
   56 FORMAT(1X,'E = 5',5X,'6',6X,'7',6X,'8',6X,'9')
      WRITE(5,57) TL0,TL0,TL0,TL0,TL0
   57 FORMAT(1X,5(F6.2,1X))
      DO 18 I=1,3
      DO 19 J=1,20
      WRITE(5,9) TS(I,J,5),TS(I,J,6),TS(I,J,7),TS(I,J,8),TS(I,J,9)
    9 FORMAT(1X,5(F6.2,1X))
   19 CONTINUE
   18 CONTINUE
C
      WRITE(5,55)
      WRITE(5,58)
   58 FORMAT(1X,'E = 10',5X,'AVGD',7X,'SPOWER')
      WRITE(5,59) TL0
   59 FORMAT(1X,F6.2)
      DO 20 I=1,3
      WRITE(5,24) AVGD(I),SPOWER(I)
   24 FORMAT(12X,D15.6,7X,D15.6)
      DO 21 J=1,20
      WRITE(5,7)  TS(I,J,10)
    7 FORMAT(1X,F6.2)
   21 CONTINUE
   20 CONTINUE
      RETURN
      END
```

APPENDIX B. TABLES OF RESULTS

RECOVERY, 137.5 C STEAM, L = 130

R	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	6.00	28.94	26.80	26.82	26.88	26.98	27.12	27.31	27.54	27.81	28.13	28.50	28.91
	12.00	32.89	30.58	30.60	30.67	30.79	30.94	31.15	31.40	31.69	32.04	32.42	32.85
	18.00	36.83	34.52	34.54	34.61	34.72	34.88	35.09	35.34	35.64	35.98	36.37	36.80
	24.00	40.78	38.46	38.48	38.55	38.67	38.83	39.03	39.28	39.58	39.92	40.31	40.74
	30.00	44.72	42.41	42.43	42.50	42.61	42.77	42.98	43.23	43.52	43.87	44.25	44.69
	36.00	48.67	46.35	46.37	46.44	46.56	46.72	46.92	47.17	47.47	47.81	48.20	48.63
	42.00	52.61	50.30	50.32	50.39	50.50	50.66	50.87	51.12	51.41	51.76	52.14	52.58
	48.00	56.56	54.24	54.26	54.33	54.45	54.60	54.81	55.06	55.36	55.70	56.09	56.52
	54.00	60.50	58.19	58.21	58.28	58.39	58.55	58.75	59.01	59.30	59.65	60.03	60.47
	60.00	64.45	62.13	62.15	62.22	62.33	62.49	62.70	62.95	63.25	63.59	63.98	64.41
	66.00	68.39	66.08	66.10	66.16	66.28	66.44	66.64	66.90	67.19	67.53	67.92	68.36
	72.00	72.34	70.02	70.04	70.11	70.22	70.38	70.59	70.84	71.14	71.48	71.87	72.30
	78.00	76.28	73.97	73.99	74.05	74.17	74.33	74.53	74.78	75.08	75.42	75.81	76.25
	84.00	80.23	77.91	77.93	78.00	78.11	78.27	78.48	78.73	79.03	79.37	79.76	80.19
	90.00	84.17	81.86	81.88	81.94	82.06	82.22	82.42	82.67	82.97	83.31	83.70	84.14
	96.00	88.11	85.80	85.82	85.89	86.00	86.16	86.37	86.62	86.92	87.26	87.65	88.08
	102.00	92.06	89.74	89.76	89.83	89.95	90.11	90.31	90.56	90.86	91.20	91.59	92.02
	108.00	96.00	93.69	93.71	93.78	93.89	94.05	94.26	94.51	94.80	95.15	95.54	95.97
	114.00	99.95	97.63	97.65	97.72	97.84	98.00	98.20	98.45	98.75	99.09	99.48	99.91
	120.00	103.89	101.58	101.60	101.67	101.78	101.94	102.15	102.40	102.69	103.04	103.42	103.86
	6.00	110.90	107.07	107.10	107.22	107.41	107.67	108.02	108.44	108.93	109.50	110.13	110.84
	12.00	116.45	113.27	113.29	113.39	113.55	113.78	114.07	114.42	114.83	115.30	115.82	116.40
	18.00	120.84	118.31	118.33	118.41	118.54	118.72	118.95	119.23	119.55	119.93	120.34	120.80
	24.00	124.31	122.31	122.33	122.39	122.49	122.63	122.82	123.04	123.30	123.59	123.92	124.28
	30.00	127.06	125.48	125.49	125.54	125.62	125.73	125.88	126.05	126.26	126.49	126.75	127.04
	36.00	129.24	127.98	128.00	128.03	128.10	128.19	128.30	128.44	128.60	128.79	128.99	129.22
	42.00	130.96	129.97	129.98	130.01	130.06	130.13	130.22	130.33	130.46	130.60	130.77	130.95
	48.00	132.32	131.54	131.55	131.57	131.61	131.67	131.74	131.82	131.93	132.04	132.17	132.31
	54.00	133.40	132.78	132.79	132.81	132.84	132.88	132.94	133.01	133.09	133.18	133.28	133.39
	60.00	134.26	133.77	133.77	133.79	133.81	133.85	133.89	133.94	134.01	134.08	134.16	134.25
	66.00	134.93	134.54	134.55	134.56	134.58	134.61	134.64	134.69	134.74	134.79	134.86	134.93
	72.00	135.47	135.16	135.16	135.17	135.19	135.21	135.24	135.27	135.31	135.36	135.41	135.46
	78.00	135.89	135.65	135.65	135.66	135.67	135.69	135.71	135.74	135.77	135.80	135.84	135.89
	84.00	136.23	136.03	136.04	136.04	136.05	136.07	136.08	136.10	136.13	136.16	136.19	136.22
	90.00	136.49	136.34	136.34	136.35	136.35	136.36	136.38	136.40	136.42	136.44	136.46	136.49
	96.00	136.70	136.58	136.58	136.59	136.59	136.60	136.61	136.63	136.64	136.66	136.68	136.70
	102.00	136.87	136.77	136.77	136.78	136.78	136.79	136.80	136.81	136.82	136.83	136.85	136.87
	108.00	137.00	136.92	136.93	136.93	136.93	136.94	136.94	136.95	136.96	136.97	136.99	137.00
	114.00	137.10	137.04	137.05	137.05	137.05	137.05	137.06	137.07	137.07	137.08	137.09	137.10
	120.00	137.19	137.14	137.14	137.14	137.14	137.15	137.15	137.16	137.16	137.17	137.18	137.19
	3.00	137.19	137.17	137.17	137.17	137.18	137.18	137.18	137.18	137.18	137.18	137.19	137.19
	6.00	137.19	137.18	137.18	137.18	137.18	137.18	137.18	137.19	137.19	137.19	137.19	137.19
	9.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	12.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	15.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	18.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	21.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	24.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	27.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	30.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	33.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	36.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	39.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	42.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	45.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	48.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	51.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	54.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	57.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19
	60.00	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19	137.19

LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
997560+00	.2436290-03	.9997990+00	.2006130-03
901530-02	.497601D+01	.7703980-02	.486602D+01
006080-02	.4960980+01	.7010880-02	.4960290+01

FOR 70% RECOVERY, 140 C STEAM, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	29.03	26.84	26.86	26.92	27.02	27.17	27.36	27.59	27.87	28.20	28.57	29.00
1	12.00	33.06	30.71	30.73	30.80	30.91	31.08	31.29	31.54	31.84	32.19	32.59	33.03
1	18.00	37.10	34.73	34.75	34.82	34.94	35.10	35.31	35.57	35.87	36.22	36.62	37.06
1	24.00	41.13	38.76	38.78	38.85	38.97	39.13	39.34	39.60	39.90	40.25	40.65	41.09
1	30.00	45.16	42.79	42.82	42.89	43.00	43.17	43.38	43.63	43.94	44.29	44.68	45.13
1	36.00	49.19	46.83	46.85	46.92	47.03	47.20	47.41	47.66	47.97	48.32	48.71	49.16
1	42.00	53.23	50.86	50.88	50.95	51.07	51.23	51.44	51.70	52.00	52.35	52.75	53.19
1	48.00	57.26	54.89	54.91	54.98	55.10	55.26	55.47	55.73	56.03	56.38	56.78	57.22
1	54.00	61.29	58.92	58.94	59.01	59.13	59.29	59.50	59.76	60.06	60.41	60.81	61.25
1	60.00	65.32	62.96	62.98	63.05	63.16	63.33	63.54	63.79	64.10	64.45	64.84	65.29
1	66.00	69.35	66.99	67.01	67.08	67.19	67.36	67.57	67.82	68.13	68.48	68.87	69.32
1	72.00	73.39	71.02	71.04	71.11	71.23	71.39	71.60	71.86	72.16	72.51	72.91	73.35
1	78.00	77.42	75.05	75.07	75.14	75.26	75.42	75.63	75.89	76.19	76.54	76.94	77.38
1	84.00	81.45	79.08	79.10	79.17	79.29	79.45	79.66	79.92	80.22	80.57	80.97	81.42
1	90.00	85.48	83.12	83.14	83.21	83.32	83.49	83.70	83.95	84.26	84.61	85.00	85.45
1	96.00	89.51	87.15	87.17	87.24	87.36	87.52	87.73	87.99	88.29	88.64	89.04	89.48
1	102.00	93.55	91.18	91.20	91.27	91.39	91.55	91.76	92.02	92.32	92.67	93.07	93.51
1	108.00	97.58	95.21	95.23	95.30	95.42	95.58	95.79	96.05	96.35	96.70	97.10	97.54
1	114.00	101.61	99.25	99.27	99.34	99.45	99.62	99.83	100.08	100.39	100.74	101.13	101.58
1	120.00	105.64	103.28	103.30	103.37	103.48	103.65	103.86	104.11	104.42	104.77	105.16	105.61
2	6.00	112.81	108.89	108.92	109.04	109.24	109.51	109.86	110.29	110.79	111.37	112.02	112.75
2	12.00	118.48	115.22	115.25	115.35	115.52	115.75	116.05	116.41	116.82	117.30	117.84	118.43
2	18.00	122.96	120.38	120.40	120.48	120.62	120.80	121.04	121.32	121.65	122.03	122.46	122.93
2	24.00	126.52	124.47	124.49	124.55	124.66	124.80	124.99	125.21	125.48	125.78	126.12	126.49
2	30.00	129.33	127.71	127.72	127.77	127.86	127.97	128.12	128.30	128.51	128.74	129.01	129.30
2	36.00	131.55	130.27	130.28	130.32	130.39	130.48	130.60	130.74	130.90	131.09	131.30	131.53
2	42.00	133.31	132.30	132.31	132.34	132.39	132.46	132.56	132.67	132.80	132.95	133.12	133.30
2	48.00	134.71	133.91	133.91	133.94	133.98	134.04	134.11	134.20	134.30	134.42	134.55	134.70
2	54.00	135.81	135.18	135.18	135.20	135.23	135.28	135.34	135.41	135.49	135.58	135.69	135.80
2	60.00	136.69	136.18	136.19	136.20	136.23	136.26	136.31	136.36	136.43	136.50	136.59	136.68
2	66.00	137.38	136.98	136.98	136.99	137.01	137.04	137.08	137.12	137.17	137.23	137.30	137.37
2	72.00	137.92	137.61	137.61	137.62	137.64	137.66	137.69	137.72	137.76	137.81	137.86	137.92
2	78.00	138.36	138.11	138.11	138.12	138.13	138.15	138.17	138.20	138.23	138.27	138.31	138.35
2	84.00	138.70	138.50	138.50	138.51	138.52	138.53	138.55	138.57	138.60	138.63	138.66	138.70
2	90.00	138.97	138.81	138.82	138.82	138.83	138.84	138.85	138.87	138.89	138.91	138.94	138.97
2	96.00	139.18	139.06	139.06	139.07	139.07	139.08	139.09	139.11	139.12	139.14	139.16	139.18
2	102.00	139.35	139.26	139.26	139.26	139.27	139.27	139.28	139.29	139.31	139.32	139.34	139.35
2	108.00	139.49	139.41	139.41	139.42	139.42	139.42	139.43	139.44	139.45	139.46	139.47	139.49
2	114.00	139.60	139.53	139.54	139.54	139.54	139.54	139.55	139.56	139.56	139.57	139.58	139.60
2	120.00	139.68	139.63	139.63	139.63	139.64	139.64	139.64	139.65	139.66	139.66	139.67	139.68
3	3.00	139.68	139.67	139.67	139.67	139.67	139.67	139.67	139.67	139.67	139.68	139.68	139.68
3	6.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	9.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	12.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	15.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	18.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	21.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	24.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	27.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	30.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	33.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	36.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	39.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	42.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	45.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	48.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	51.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	54.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	57.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68
3	60.00	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68	139.68

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.9996300+00	.3699880-03	.9996960+00	.3039690-03
2	.2420690-03	.8326290+01	.2923860-03	.8137430+01
3	.2408840-03	.8331190+01	.2411640-03	.8330030+01

FOR 70% RECOVERY, 142.5 C, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	29.12	26.88	26.90	26.96	27.07	27.22	27.41	27.65	27.93	28.27	28.65	29.08
1	12.00	33.24	30.83	30.85	30.92	31.04	31.21	31.42	31.68	31.99	32.35	32.75	33.20
1	18.00	37.36	34.94	34.96	35.03	35.15	35.32	35.53	35.80	36.11	36.46	36.87	37.32
1	24.00	41.48	39.06	39.08	39.15	39.27	39.44	39.65	39.92	40.23	40.58	40.99	41.44
1	30.00	45.60	43.18	43.20	43.27	43.39	43.56	43.77	44.04	44.35	44.70	45.11	45.56
1	36.00	49.72	47.30	47.32	47.39	47.51	47.68	47.89	48.16	48.47	48.82	49.23	49.68
1	42.00	53.84	51.42	51.44	51.51	51.63	51.80	52.01	52.28	52.59	52.94	53.35	53.80
1	48.00	57.96	55.54	55.56	55.63	55.75	55.92	56.13	56.40	56.71	57.06	57.47	57.92
1	54.00	62.08	59.66	59.68	59.75	59.87	60.04	60.25	60.51	60.82	61.18	61.59	62.04
1	60.00	66.20	63.78	63.80	63.87	63.99	64.16	64.37	64.63	64.94	65.30	65.71	66.16
1	66.00	70.32	67.90	67.92	67.99	68.11	68.28	68.49	68.75	69.06	69.42	69.83	70.28
1	72.00	74.44	72.02	72.04	72.11	72.23	72.40	72.61	72.87	73.18	73.54	73.95	74.40
1	78.00	78.56	76.14	76.16	76.23	76.35	76.52	76.73	76.99	77.30	77.66	78.07	78.52
1	84.00	82.68	80.26	80.28	80.35	80.47	80.64	80.85	81.11	81.42	81.78	82.19	82.64
1	90.00	86.80	84.38	84.40	84.47	84.59	84.76	84.97	85.23	85.54	85.90	86.31	86.76
1	96.00	90.91	88.50	88.52	88.59	88.71	88.88	89.09	89.35	89.66	90.02	90.43	90.88
1	102.00	95.03	92.62	92.64	92.71	92.83	93.00	93.21	93.47	93.78	94.14	94.55	95.00
1	108.00	99.15	96.74	96.76	96.83	96.95	97.11	97.33	97.59	97.90	98.26	98.66	99.12
1	114.00	103.27	100.86	100.88	100.95	101.07	101.23	101.45	101.71	102.02	102.38	102.78	103.24
1	120.00	107.39	104.98	105.00	105.07	105.19	105.35	105.57	105.83	106.14	106.50	106.90	107.36
2	6.00	114.71	110.71	110.74	110.86	111.06	111.34	111.70	112.14	112.65	113.25	113.91	114.65
2	12.00	120.51	117.18	117.21	117.32	117.49	117.72	118.02	118.39	118.82	119.31	119.86	120.46
2	18.00	125.09	122.45	122.48	122.56	122.69	122.88	123.12	123.41	123.75	124.14	124.58	125.05
2	24.00	128.72	126.63	126.65	126.72	126.82	126.97	127.16	127.39	127.66	127.97	128.31	128.69
2	30.00	131.60	129.94	129.96	130.01	130.09	130.21	130.36	130.54	130.76	131.00	131.27	131.57
2	36.00	133.87	132.56	132.57	132.61	132.68	132.77	132.89	133.04	133.20	133.40	133.61	133.85
2	42.00	135.67	134.63	134.64	134.67	134.73	134.80	134.89	135.01	135.14	135.30	135.47	135.65
2	48.00	137.09	136.27	136.28	136.31	136.35	136.41	136.48	136.57	136.68	136.80	136.93	137.08
2	54.00	138.22	137.57	137.58	137.60	137.63	137.68	137.74	137.81	137.89	137.99	138.09	138.21
2	60.00	139.11	138.60	138.60	138.62	138.65	138.68	138.73	138.79	138.85	138.93	139.01	139.11
2	66.00	139.82	139.41	139.42	139.43	139.45	139.48	139.52	139.56	139.61	139.67	139.74	139.81
2	72.00	140.38	140.06	140.06	140.07	140.09	140.11	140.14	140.17	140.21	140.26	140.31	140.37
2	78.00	140.82	140.57	140.57	140.58	140.59	140.61	140.63	140.66	140.69	140.73	140.77	140.82
2	84.00	141.17	140.97	140.97	140.98	140.99	141.00	141.02	141.04	141.07	141.10	141.13	141.17
2	90.00	141.45	141.29	141.29	141.29	141.30	141.31	141.33	141.35	141.37	141.39	141.42	141.45
2	96.00	141.67	141.54	141.54	141.55	141.55	141.56	141.57	141.59	141.60	141.62	141.64	141.67
2	102.00	141.84	141.74	141.74	141.74	141.75	141.76	141.77	141.78	141.79	141.80	141.82	141.84
2	108.00	141.98	141.90	141.90	141.90	141.91	141.91	141.92	141.93	141.94	141.95	141.96	141.98
2	114.00	142.09	142.02	142.02	142.03	142.03	142.03	142.04	142.05	142.06	142.06	142.07	142.09
2	120.00	142.17	142.12	142.12	142.13	142.13	142.13	142.14	142.14	142.15	142.16	142.16	142.17
3	3.00	142.17	142.16	142.16	142.16	142.16	142.16	142.16	142.17	142.17	142.17	142.17	142.17
3	6.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	9.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	12.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	15.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	18.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	21.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	24.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	27.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	30.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	33.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	36.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	39.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	42.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	45.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	48.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	51.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	54.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	57.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17
3	60.00	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17	142.17

	LIQUID N/No	-ln(N/NO)	SOLID SPHERE N/No	-ln(N/NO)
1	.9994400+00	.5599050-03	.9995410+00	.4589630-03
2	.9683030-06	.1384770+02	.1342460-05	.1352100+02
3	.9151350-06	.1390420+02	.9169260-06	.1390220+02

FOR 70% RECOVERY, 145 C STEAM, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	29.21	26.92	26.94	27.00	27.11	27.26	27.46	27.71	28.00	28.34	28.73	29.17
1	12.00	33.41	30.96	30.98	31.05	31.17	31.34	31.56	31.82	32.14	32.50	32.92	33.38
1	18.00	37.62	35.15	35.17	35.25	35.37	35.54	35.76	36.03	36.34	36.71	37.12	37.58
1	24.00	41.83	39.36	39.38	39.45	39.58	39.75	39.97	40.23	40.55	40.92	41.33	41.79
1	30.00	46.04	43.57	43.59	43.66	43.78	43.95	44.17	44.44	44.76	45.12	45.54	46.00
1	36.00	50.24	47.77	47.80	47.87	47.99	48.16	48.38	48.65	48.96	49.33	49.74	50.21
1	42.00	54.45	51.98	52.00	52.08	52.20	52.37	52.59	52.85	53.17	53.54	53.95	54.41
1	48.00	58.66	56.19	56.21	56.28	56.40	56.57	56.79	57.06	57.38	57.74	58.16	58.62
1	54.00	62.86	60.40	60.42	60.49	60.61	60.78	61.00	61.27	61.59	61.95	62.36	62.83
1	60.00	67.07	64.60	64.62	64.70	64.82	64.99	65.21	65.48	65.79	66.16	66.57	67.03
1	66.00	71.28	68.81	68.83	68.90	69.03	69.20	69.42	69.68	70.00	70.37	70.78	71.24
1	72.00	75.49	73.02	73.04	73.11	73.23	73.40	73.62	73.89	74.21	74.57	74.99	75.45
1	78.00	79.69	77.22	77.25	77.32	77.44	77.61	77.83	78.10	78.41	78.78	79.19	79.66
1	84.00	83.90	81.43	81.45	81.53	81.65	81.82	82.04	82.31	82.62	82.99	83.40	83.86
1	90.00	88.11	85.64	85.66	85.73	85.85	86.03	86.24	86.51	86.83	87.19	87.61	88.07
1	96.00	92.31	89.85	89.87	89.94	90.06	90.23	90.45	90.72	91.04	91.40	91.82	92.28
1	102.00	96.52	94.05	94.07	94.15	94.27	94.44	94.66	94.93	95.24	95.61	96.02	96.49
1	108.00	100.73	98.26	98.28	98.35	98.48	98.65	98.87	99.13	99.45	99.82	100.23	100.69
1	114.00	104.94	102.47	102.49	102.56	102.68	102.85	103.07	103.34	103.66	104.02	104.44	104.90
1	120.00	109.14	106.67	106.70	106.77	106.89	107.06	107.28	107.55	107.86	108.23	108.64	109.11
2	6.00	116.62	112.53	112.57	112.69	112.89	113.18	113.54	113.99	114.52	115.12	115.80	116.56
2	12.00	122.54	119.14	119.17	119.28	119.45	119.69	120.00	120.37	120.81	121.31	121.87	122.49
2	18.00	127.22	124.52	124.55	124.63	124.77	124.96	125.21	125.50	125.85	126.25	126.69	127.18
2	24.00	130.93	128.79	128.81	128.88	128.99	129.14	129.33	129.57	129.84	130.16	130.51	130.90
2	30.00	133.86	132.17	132.19	132.24	132.33	132.45	132.60	132.79	133.00	133.25	133.53	133.84
2	36.00	136.18	134.85	134.86	134.90	134.97	135.06	135.19	135.33	135.51	135.70	135.92	136.17
2	42.00	138.02	136.96	136.97	137.01	137.06	137.14	137.23	137.35	137.49	137.64	137.82	138.01
2	48.00	139.48	138.64	138.65	138.67	138.72	138.78	138.85	138.94	139.05	139.18	139.31	139.47
2	54.00	140.63	139.97	139.97	139.99	140.03	140.07	140.13	140.21	140.29	140.39	140.50	140.62
2	60.00	141.54	141.02	141.02	141.04	141.06	141.10	141.15	141.21	141.27	141.35	141.44	141.53
2	66.00	142.26	141.85	141.85	141.86	141.88	141.91	141.95	142.00	142.05	142.11	142.18	142.26
2	72.00	142.83	142.50	142.51	142.52	142.53	142.56	142.59	142.62	142.67	142.71	142.77	142.83
2	78.00	143.28	143.02	143.03	143.03	143.05	143.07	143.09	143.12	143.15	143.19	143.23	143.28
2	84.00	143.64	143.44	143.44	143.44	143.46	143.47	143.49	143.51	143.54	143.57	143.60	143.64
2	90.00	143.93	143.76	143.76	143.77	143.78	143.79	143.80	143.82	143.84	143.87	143.89	143.92
2	96.00	144.15	144.02	144.02	144.03	144.03	144.04	144.05	144.07	144.08	144.10	144.12	144.15
2	102.00	144.33	144.22	144.23	144.23	144.23	144.24	144.25	144.26	144.27	144.29	144.31	144.33
2	108.00	144.47	144.39	144.39	144.39	144.39	144.40	144.41	144.42	144.43	144.44	144.45	144.47
2	114.00	144.58	144.51	144.51	144.52	144.52	144.52	144.53	144.54	144.55	144.56	144.57	144.58
2	120.00	144.67	144.62	144.62	144.62	144.62	144.62	144.63	144.63	144.64	144.65	144.66	144.67
3	3.00	144.67	144.65	144.65	144.65	144.65	144.65	144.66	144.66	144.66	144.66	144.66	144.67
3	6.00	144.67	144.66	144.66	144.66	144.66	144.66	144.66	144.66	144.66	144.67	144.67	144.67
3	9.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	12.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	15.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	18.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	21.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	24.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	27.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	30.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	33.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	36.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	39.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	42.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	45.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	48.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	51.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	54.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	57.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67
3	60.00	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67	144.67

	LIQUID N/No	LIQUID -ln(N/No)	SOLID SPHERE N/No	SOLID SPHERE -ln(N/No)
1	.9991560+00	.844349D-03	.9993100+00	.6905960-03
2	.1141790-09	.228933D+02	.2026550-09	.2231950+02
3	.9628770-10	.230637D+02	.9660360-10	.2306040+02

FOR 70% RECOVERY, 147.5 C STEAM, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	29.29	26.96	26.98	27.05	27.16	27.31	27.51	27.76	28.06	28.41	28.81	29.26
1	12.00	33.59	31.08	31.10	31.18	31.30	31.47	31.69	31.97	32.29	32.66	33.08	33.55
1	18.00	37.88	35.36	35.39	35.46	35.58	35.76	35.98	36.26	36.58	36.95	37.37	37.85
1	24.00	42.18	39.66	39.68	39.75	39.88	40.05	40.28	40.55	40.87	41.25	41.67	42.14
1	30.00	46.47	43.95	43.97	44.05	44.17	44.35	44.57	44.84	45.17	45.54	45.96	46.44
1	36.00	50.77	48.25	48.27	48.34	48.47	48.64	48.87	49.14	49.46	49.84	50.26	50.73
1	42.00	55.06	52.54	52.56	52.64	52.76	52.94	53.16	53.43	53.76	54.13	54.55	55.03
1	48.00	59.36	56.84	56.86	56.93	57.06	57.23	57.46	57.73	58.05	58.42	58.85	59.32
1	54.00	63.65	61.13	61.15	61.23	61.35	61.53	61.75	62.02	62.35	62.72	63.14	63.61
1	60.00	67.95	65.43	65.45	65.52	65.65	65.82	66.04	66.32	66.64	67.01	67.44	67.91
1	66.00	72.24	69.72	69.74	69.82	69.94	70.12	70.34	70.61	70.94	71.31	71.73	72.20
1	72.00	76.54	74.02	74.04	74.11	74.24	74.41	74.63	74.91	75.23	75.60	76.03	76.50
1	78.00	80.83	78.31	78.33	78.41	78.53	78.71	78.93	79.20	79.53	79.90	80.32	80.79
1	84.00	85.13	82.61	82.63	82.70	82.83	83.00	83.22	83.50	83.82	84.19	84.62	85.09
1	90.00	89.42	86.90	86.92	87.00	87.12	87.29	87.52	87.79	88.11	88.49	88.91	89.38
1	96.00	93.71	91.19	91.22	91.29	91.42	91.59	91.81	92.09	92.41	92.78	93.20	93.68
1	102.00	98.01	95.49	95.51	95.59	95.71	95.88	96.11	96.38	96.70	97.08	97.50	97.97
1	108.00	102.30	99.78	99.81	99.88	100.00	100.18	100.40	100.68	101.00	101.37	101.79	102.27
1	114.00	106.60	104.08	104.10	104.17	104.30	104.47	104.70	104.97	105.29	105.67	106.09	106.56
1	120.00	110.89	108.37	108.40	108.47	108.59	108.77	108.99	109.26	109.59	109.96	110.38	110.86
2	6.00	118.53	114.35	114.39	114.51	114.72	115.01	115.39	115.84	116.38	117.00	117.69	118.46
2	12.00	124.57	121.10	121.13	121.24	121.42	121.66	121.98	122.36	122.81	123.32	123.89	124.52
2	18.00	129.35	126.60	126.62	126.71	126.85	127.04	127.29	127.60	127.95	128.36	128.81	129.31
2	24.00	133.13	130.95	130.97	131.04	131.15	131.31	131.51	131.75	132.03	132.35	132.71	133.10
2	30.00	136.13	134.40	134.42	134.47	134.56	134.68	134.84	135.03	135.25	135.51	135.79	136.10
2	36.00	138.50	137.13	137.15	137.19	137.26	137.36	137.48	137.63	137.81	138.01	138.23	138.48
2	42.00	140.38	139.30	139.31	139.34	139.39	139.47	139.57	139.69	139.83	139.99	140.17	140.36
2	48.00	141.86	141.01	141.01	141.04	141.08	141.15	141.22	141.32	141.43	141.55	141.69	141.85
2	54.00	143.04	142.36	142.37	142.39	142.42	142.47	142.53	142.61	142.69	142.79	142.91	143.03
2	60.00	143.97	143.43	143.44	143.45	143.48	143.52	143.57	143.63	143.70	143.77	143.86	143.96
2	66.00	144.70	144.28	144.28	144.30	144.32	144.35	144.39	144.43	144.49	144.55	144.62	144.70
2	72.00	145.29	144.95	144.95	144.97	144.98	145.01	145.04	145.07	145.12	145.17	145.22	145.28
2	78.00	145.75	145.48	145.49	145.49	145.51	145.53	145.55	145.58	145.61	145.65	145.70	145.74
2	84.00	146.11	145.90	145.91	145.91	145.92	145.94	145.96	145.98	146.01	146.04	146.07	146.11
2	90.00	146.40	146.24	146.24	146.24	146.25	146.26	146.28	146.30	146.32	146.34	146.37	146.40
2	96.00	146.63	146.50	146.50	146.51	146.51	146.52	146.53	146.55	146.56	146.58	146.61	146.63
2	102.00	146.81	146.71	146.71	146.71	146.72	146.73	146.73	146.75	146.76	146.78	146.79	146.81
2	108.00	146.96	146.87	146.87	146.88	146.88	146.89	146.89	146.90	146.91	146.93	146.94	146.95
2	114.00	147.07	147.00	147.00	147.01	147.01	147.01	147.02	147.03	147.04	147.05	147.06	147.07
2	120.00	147.16	147.11	147.11	147.11	147.11	147.12	147.12	147.13	147.13	147.14	147.15	147.16
3	3.00	147.16	147.14	147.14	147.14	147.15	147.15	147.15	147.15	147.15	147.16	147.16	147.16
3	6.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	9.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	12.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	15.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	18.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	21.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	24.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	27.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	30.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	33.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	36.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	39.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	42.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	45.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	48.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	51.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	54.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	57.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16
3	60.00	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16	147.16

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.9987320+00	.1268900-02	.9989650+00	.1035580-02
2	.4564490-16	.3762560+02	.1280400-15	.3659420+02
3	.3052280-16	.3802810+02	.3068970-16	.3802260+02

FOR 70% RECOVERY, 150 C STEAM, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	29.38	27.00	27.02	27.09	27.20	27.36	27.56	27.82	28.12	28.48	28.88	29.35
1	12.00	33.76	31.20	31.23	31.30	31.43	31.60	31.83	32.11	32.44	32.82	33.25	33.73
1	18.00	38.15	35.58	35.60	35.67	35.80	35.98	36.21	36.49	36.81	37.20	37.63	38.11
1	24.00	42.53	39.96	39.98	40.06	40.18	40.36	40.59	40.87	41.20	41.58	42.01	42.49
1	30.00	46.91	44.34	44.36	44.44	44.56	44.74	44.97	45.25	45.58	45.96	46.39	46.87
1	36.00	51.29	48.72	48.74	48.82	48.95	49.12	49.35	49.63	49.96	50.34	50.77	51.25
1	42.00	55.68	53.10	53.13	53.20	53.33	53.51	53.73	54.01	54.34	54.72	55.15	55.64
1	48.00	60.06	57.49	57.51	57.58	57.71	57.89	58.12	58.40	58.73	59.11	59.54	60.02
1	54.00	64.44	61.87	61.89	61.97	62.09	62.27	62.50	62.78	63.11	63.49	63.92	64.40
1	60.00	68.82	66.25	66.27	66.35	66.48	66.65	66.88	67.16	67.49	67.87	68.30	68.78
1	66.00	73.20	70.63	70.65	70.73	70.86	71.03	71.26	71.54	71.87	72.25	72.68	73.17
1	72.00	77.59	75.01	75.04	75.11	75.24	75.42	75.65	75.92	76.25	76.63	77.07	77.55
1	78.00	81.97	79.40	79.42	79.50	79.62	79.80	80.03	80.31	80.64	81.02	81.45	81.93
1	84.00	86.35	83.78	83.80	83.88	84.00	84.18	84.41	84.69	85.02	85.40	85.83	86.31
1	90.00	90.73	88.16	88.18	88.26	88.39	88.56	88.79	89.07	89.40	89.78	90.21	90.69
1	96.00	95.11	92.54	92.57	92.64	92.77	92.95	93.17	93.45	93.78	94.16	94.59	95.08
1	102.00	99.50	96.93	96.95	97.02	97.15	97.33	97.56	97.84	98.16	98.55	98.98	99.46
1	108.00	103.88	101.31	101.33	101.41	101.53	101.71	101.94	102.22	102.55	102.93	103.36	103.84
1	114.00	108.26	105.69	105.71	105.79	105.91	106.09	106.32	106.60	106.93	107.31	107.74	108.22
1	120.00	112.64	110.07	110.09	110.17	110.30	110.47	110.70	110.98	111.31	111.69	112.12	112.61
2	6.00	120.43	116.17	116.21	116.34	116.55	116.85	117.23	117.69	118.24	118.87	119.58	120.37
2	12.00	126.60	123.06	123.09	123.20	123.38	123.63	123.96	124.34	124.80	125.32	125.90	126.55
2	18.00	131.48	128.67	128.69	128.78	128.92	129.12	129.38	129.69	130.05	130.46	130.93	131.44
2	24.00	135.34	133.12	133.14	133.20	133.32	133.48	133.68	133.92	134.21	134.54	134.90	135.31
2	30.00	138.40	136.64	136.65	136.71	136.80	136.92	137.08	137.28	137.50	137.76	138.05	138.37
2	36.00	140.82	139.42	139.44	139.48	139.55	139.65	139.78	139.93	140.11	140.31	140.54	140.80
2	42.00	142.73	141.63	141.64	141.67	141.73	141.81	141.91	142.03	142.17	142.33	142.51	142.71
2	48.00	144.25	143.37	143.38	143.41	143.45	143.52	143.59	143.69	143.80	143.93	144.08	144.23
2	54.00	145.45	144.76	144.76	144.78	144.82	144.87	144.93	145.01	145.10	145.20	145.31	145.44
2	60.00	146.40	145.85	145.85	145.87	145.90	145.94	145.99	146.05	146.12	146.20	146.29	146.39
2	66.00	147.15	146.71	146.72	146.73	146.75	146.78	146.82	146.87	146.93	146.99	147.06	147.14
2	72.00	147.74	147.40	147.40	147.41	147.43	147.46	147.49	147.52	147.57	147.62	147.67	147.74
2	78.00	148.21	147.94	147.94	147.95	147.97	147.99	148.01	148.04	148.08	148.12	148.16	148.21
2	84.00	148.59	148.37	148.37	148.38	148.39	148.41	148.43	148.45	148.48	148.51	148.54	148.58
2	90.00	148.88	148.71	148.71	148.72	148.73	148.74	148.75	148.77	148.79	148.82	148.85	148.88
2	96.00	149.11	148.98	148.98	148.98	148.99	149.00	149.01	149.03	149.05	149.07	149.09	149.11
2	102.00	149.30	149.19	149.19	149.20	149.20	149.21	149.22	149.23	149.24	149.26	149.28	149.30
2	108.00	149.44	149.36	149.36	149.36	149.37	149.37	149.38	149.39	149.40	149.41	149.43	149.44
2	114.00	149.56	149.49	149.49	149.50	149.50	149.50	149.51	149.52	149.53	149.54	149.55	149.56
2	120.00	149.65	149.60	149.60	149.60	149.60	149.61	149.61	149.62	149.63	149.63	149.64	149.65
3	3.00	149.65	149.64	149.64	149.64	149.64	149.64	149.64	149.64	149.65	149.65	149.65	149.65
3	6.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	9.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	12.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	15.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	18.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	21.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	24.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	27.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	30.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	33.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	36.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	39.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	42.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	45.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	48.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	51.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	54.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	57.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65
3	60.00	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65	149.65

LIQUID N/No	LIQUID -ln(N/No)	SOLID SPHERE N/No	SOLID SPHERE -ln(N/No)
.9981010+00	.1900390-02	.9984540+00	.1547650-02
.1988390-26	.6148250+02	.1337850-25	.5957610+02
.8491920-27	.6233330+02	.8568940-27	.6232420+02

FOR 70% RECOVERY, 152.5 C STEAM, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	29.47	27.04	27.06	27.13	27.24	27.41	27.61	27.87	28.18	28.55	28.96	29.43
1	12.00	33.94	31.33	31.35	31.43	31.56	31.74	31.97	32.25	32.59	32.97	33.41	33.90
1	18.00	38.41	35.79	35.81	35.89	36.02	36.20	36.43	36.71	37.05	37.44	37.88	38.37
1	24.00	42.88	40.26	40.28	40.36	40.49	40.67	40.90	41.18	41.52	41.91	42.35	42.84
1	30.00	47.35	44.73	44.75	44.83	44.96	45.14	45.37	45.65	45.99	46.38	46.82	47.31
1	36.00	51.82	49.20	49.22	49.30	49.42	49.61	49.84	50.12	50.46	50.85	51.29	51.78
1	42.00	56.29	53.66	53.69	53.77	53.89	54.08	54.31	54.59	54.93	55.32	55.76	56.25
1	48.00	60.76	58.13	58.16	58.23	58.36	58.55	58.78	59.06	59.40	59.79	60.23	60.72
1	54.00	65.23	62.60	62.63	62.70	62.83	63.01	63.25	63.53	63.87	64.26	64.70	65.19
1	60.00	69.70	67.07	67.10	67.17	67.30	67.48	67.72	68.00	68.34	68.73	69.17	69.66
1	66.00	74.17	71.54	71.57	71.64	71.77	71.95	72.19	72.47	72.81	73.20	73.64	74.13
1	72.00	78.64	76.01	76.04	76.11	76.24	76.42	76.66	76.94	77.28	77.67	78.11	78.60
1	78.00	83.11	80.48	80.51	80.58	80.71	80.89	81.13	81.41	81.75	82.14	82.58	83.07
1	84.00	87.58	84.95	84.98	85.05	85.18	85.36	85.60	85.88	86.22	86.60	87.04	87.54
1	90.00	92.05	89.42	89.44	89.52	89.65	89.83	90.07	90.35	90.69	91.07	91.51	92.01
1	96.00	96.51	93.89	93.91	93.99	94.12	94.30	94.54	94.82	95.16	95.54	95.98	96.48
1	102.00	100.98	98.36	98.38	98.46	98.59	98.77	99.01	99.29	99.63	100.01	100.45	100.95
1	108.00	105.45	102.83	102.85	102.93	103.06	103.24	103.47	103.76	104.10	104.48	104.92	105.42
1	114.00	109.92	107.30	107.32	107.40	107.53	107.71	107.94	108.23	108.57	108.95	109.39	109.88
1	120.00	114.39	111.77	111.79	111.87	112.00	112.18	112.41	112.70	113.04	113.42	113.86	114.35
2	6.00	122.34	117.99	118.03	118.16	118.38	118.68	119.07	119.54	120.10	120.75	121.47	122.28
2	12.00	128.63	125.02	125.05	125.16	125.35	125.60	125.93	126.33	126.80	127.33	127.92	128.58
2	18.00	133.61	130.74	130.77	130.85	131.00	131.20	131.47	131.78	132.15	132.57	133.04	133.56
2	24.00	137.55	135.28	135.30	135.37	135.48	135.64	135.85	136.10	136.39	136.73	137.10	137.51
2	30.00	140.66	138.87	138.88	138.94	139.03	139.16	139.32	139.52	139.75	140.02	140.31	140.64
2	36.00	143.13	141.71	141.72	141.77	141.84	141.94	142.07	142.23	142.41	142.62	142.85	143.11
2	42.00	145.08	143.96	143.97	144.00	144.06	144.14	144.24	144.37	144.51	144.68	144.86	145.07
2	48.00	146.63	145.74	145.75	145.78	145.82	145.89	145.97	146.06	146.18	146.31	146.46	146.62
2	54.00	147.85	147.15	147.16	147.18	147.21	147.26	147.33	147.41	147.50	147.60	147.72	147.84
2	60.00	148.82	148.27	148.27	148.29	148.32	148.36	148.41	148.47	148.54	148.62	148.71	148.82
2	66.00	149.59	149.15	149.15	149.17	149.19	149.22	149.26	149.31	149.37	149.43	149.50	149.58
2	72.00	150.20	149.85	149.85	149.86	149.88	149.90	149.94	149.97	150.02	150.07	150.13	150.19
2	78.00	150.68	150.40	150.40	150.41	150.43	150.45	150.47	150.50	150.54	150.58	150.62	150.67
2	84.00	151.06	150.84	150.84	150.85	150.86	150.87	150.89	150.92	150.95	150.98	151.01	151.05
2	90.00	151.36	151.18	151.19	151.19	151.20	151.21	151.23	151.25	151.27	151.30	151.32	151.36
2	96.00	151.60	151.46	151.46	151.46	151.47	151.48	151.49	151.51	151.53	151.55	151.57	151.59
2	102.00	151.78	151.68	151.68	151.68	151.69	151.69	151.70	151.72	151.73	151.75	151.76	151.78
2	108.00	151.93	151.85	151.85	151.85	151.86	151.86	151.87	151.88	151.89	151.90	151.92	151.93
2	114.00	152.05	151.98	151.98	151.99	151.99	151.99	152.00	152.01	152.02	152.03	152.04	152.05
2	120.00	152.15	152.09	152.09	152.09	152.10	152.10	152.11	152.11	152.12	152.13	152.13	152.14
3	3.00	152.15	152.13	152.13	152.13	152.13	152.13	152.14	152.14	152.14	152.14	152.14	152.15
3	6.00	152.15	152.14	152.14	152.14	152.14	152.14	152.14	152.14	152.14	152.14	152.14	152.15
3	9.00	152.15	152.14	152.14	152.14	152.14	152.14	152.14	152.14	152.14	152.15	152.15	152.15
3	12.00	152.15	152.14	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	15.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	18.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	21.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	24.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	27.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	30.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	33.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	36.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	39.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	42.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	45.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	48.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	51.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	54.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	57.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15
3	60.00	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15	152.15

LIQUID		SOLID SPHERE	
N/No	-ln(N/No)	N/No	-ln(N/No)
.9971670+00	.2836540-02	.9976970+00	.2305200-02
.4123500-43	.9989700+02	.1481740-41	.9631540+02
.7644470-44	.1015820+03	.7758890-44	.1015670+03

FOR 60% RECOVERY, 140 C STEAM, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	28.45	26.57	26.59	26.64	26.73	26.85	27.02	27.22	27.45	27.73	28.05	28.42
1	12.00	31.89	29.88	29.90	29.96	30.05	30.19	30.37	30.59	30.85	31.15	31.48	31.86
1	18.00	35.34	33.32	33.33	33.39	33.49	33.63	33.81	34.03	34.29	34.59	34.93	35.31
1	24.00	38.78	36.76	36.78	36.84	36.94	37.08	37.26	37.48	37.74	38.04	38.38	38.75
1	30.00	42.23	40.21	40.23	40.29	40.39	40.52	40.70	40.92	41.18	41.48	41.82	42.20
1	36.00	45.68	43.65	43.67	43.73	43.83	43.97	44.15	44.37	44.63	44.93	45.27	45.65
1	42.00	49.12	47.10	47.12	47.18	47.28	47.42	47.60	47.82	48.08	48.37	48.71	49.09
1	48.00	52.57	50.55	50.56	50.62	50.72	50.86	51.04	51.26	51.52	51.82	52.16	52.54
1	54.00	56.01	53.99	54.01	54.07	54.17	54.31	54.49	54.71	54.97	55.27	55.61	55.98
1	60.00	59.46	57.44	57.46	57.52	57.62	57.76	57.93	58.15	58.41	58.71	59.05	59.43
1	66.00	62.91	60.88	60.90	60.96	61.06	61.20	61.38	61.60	61.86	62.16	62.50	62.88
1	72.00	66.35	64.33	64.35	64.41	64.51	64.65	64.83	65.05	65.31	65.60	65.94	66.32
1	78.00	69.80	67.78	67.79	67.85	67.95	68.09	68.27	68.49	68.75	69.05	69.39	69.77
1	84.00	73.25	71.22	71.24	71.30	71.40	71.54	71.72	71.94	72.20	72.50	72.84	73.22
1	90.00	76.69	74.67	74.69	74.75	74.85	74.99	75.17	75.38	75.64	75.94	76.28	76.66
1	96.00	80.14	78.12	78.13	78.19	78.29	78.43	78.61	78.83	79.09	79.39	79.73	80.11
1	102.00	83.58	81.56	81.58	81.64	81.74	81.88	82.06	82.28	82.54	82.84	83.17	83.55
1	108.00	87.03	85.01	85.03	85.08	85.18	85.32	85.50	85.72	85.98	86.28	86.62	87.00
1	114.00	90.48	88.45	88.47	88.53	88.63	88.77	88.95	89.17	89.43	89.73	90.07	90.45
1	120.00	93.92	91.90	91.92	91.98	92.08	92.22	92.40	92.62	92.87	93.17	93.51	93.89
2	6.00	100.34	96.79	96.82	96.93	97.10	97.35	97.66	98.05	98.50	99.03	99.63	100.29
2	12.00	105.87	102.68	102.71	102.80	102.96	103.19	103.48	103.83	104.24	104.71	105.24	105.82
2	18.00	110.63	107.87	107.90	107.98	108.12	108.31	108.56	108.86	109.22	109.62	110.08	110.59
2	24.00	114.72	112.35	112.37	112.44	112.56	112.73	112.94	113.20	113.51	113.86	114.25	114.69
2	30.00	118.24	116.20	116.22	116.28	116.38	116.53	116.71	116.94	117.20	117.50	117.84	118.21
2	36.00	121.27	119.52	119.53	119.59	119.68	119.80	119.96	120.15	120.38	120.64	120.93	121.25
2	42.00	123.88	122.37	122.39	122.43	122.51	122.61	122.75	122.92	123.11	123.33	123.58	123.86
2	48.00	126.13	124.83	124.84	124.88	124.95	125.04	125.15	125.30	125.46	125.66	125.87	126.11
2	54.00	128.06	126.94	126.95	126.99	127.04	127.12	127.22	127.35	127.49	127.66	127.84	128.05
2	60.00	129.73	128.76	128.77	128.80	128.85	128.92	129.00	129.11	129.23	129.38	129.54	129.71
2	66.00	131.16	130.33	130.34	130.36	130.40	130.46	130.54	130.63	130.73	130.86	130.99	131.15
2	72.00	132.39	131.68	131.68	131.70	131.74	131.79	131.86	131.93	132.03	132.13	132.25	132.38
2	78.00	133.45	132.84	132.84	132.86	132.89	132.93	132.99	133.06	133.14	133.23	133.33	133.44
2	84.00	134.36	133.83	133.84	133.86	133.88	133.92	133.97	134.02	134.09	134.17	134.26	134.36
2	90.00	135.15	134.69	134.70	134.71	134.73	134.77	134.81	134.86	134.92	134.98	135.06	135.14
2	96.00	135.83	135.43	135.44	135.45	135.47	135.50	135.53	135.57	135.62	135.68	135.75	135.82
2	102.00	136.41	136.07	136.07	136.08	136.10	136.12	136.15	136.19	136.23	136.28	136.34	136.40
2	108.00	136.91	136.62	136.62	136.63	136.64	136.66	136.69	136.72	136.76	136.80	136.85	136.90
2	114.00	137.34	137.09	137.09	137.10	137.11	137.13	137.15	137.18	137.21	137.25	137.29	137.33
2	120.00	137.71	137.49	137.50	137.50	137.51	137.53	137.55	137.57	137.60	137.63	137.67	137.71
3	3.00	137.71	137.65	137.65	137.65	137.65	137.66	137.67	137.68	137.69	137.69	137.70	137.71
3	6.00	137.71	137.69	137.69	137.69	137.70	137.70	137.70	137.70	137.70	137.71	137.71	137.71
3	9.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	12.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	15.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	18.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	21.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	24.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	27.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	30.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	33.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	36.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	39.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	42.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	45.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	48.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	51.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	54.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	57.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	60.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71

	LIQUID N/No	LIQUID -ln (N/No)	SOLID SPHERE N/No	SOLID SPHERE -ln (N/No)
1	.9999790D+00	.2103040-04	.9999820+00	.1753300-04
2	.3176510-01	.3449390+01	.3581460-01	.3329400+01
3	.3954100-02	.5533000+01	.3967780-02	.5529550+01

60% RECOVERY, 142.5 C STEAM, L = 130

HANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	28.52	26.61	26.62	26.68	26.77	26.89	27.06	27.26	27.51	27.79	28.12	28.49
1	12.00	32.04	29.99	30.00	30.06	30.16	30.31	30.49	30.71	30.98	31.28	31.62	32.01
1	18.00	35.56	33.50	33.52	33.58	33.68	33.82	34.00	34.23	34.49	34.80	35.16	35.53
1	24.00	39.08	37.02	37.04	37.10	37.20	37.34	37.52	37.75	38.01	38.32	38.67	39.05
1	30.00	42.61	40.54	40.56	40.62	40.72	40.86	41.05	41.27	41.53	41.84	42.19	42.57
1	36.00	46.13	44.06	44.08	44.14	44.24	44.38	44.57	44.79	45.06	45.36	45.71	46.10
1	42.00	49.65	47.58	47.60	47.66	47.76	47.90	48.09	48.31	48.58	48.88	49.23	49.62
1	48.00	53.17	51.10	51.12	51.18	51.28	51.43	51.61	51.83	52.10	52.40	52.75	53.14
1	54.00	56.69	54.62	54.64	54.70	54.80	54.95	55.13	55.35	55.62	55.92	56.27	56.66
1	60.00	60.21	58.14	58.16	58.22	58.32	58.47	58.65	58.87	59.14	59.45	59.79	60.18
1	66.00	63.73	61.67	61.68	61.74	61.85	61.99	62.17	62.40	62.66	62.97	63.31	63.70
1	72.00	67.25	65.19	65.20	65.26	65.37	65.51	65.69	65.92	66.18	66.49	66.83	67.22
1	78.00	70.77	68.71	68.72	68.79	68.89	69.03	69.21	69.44	69.70	70.01	70.36	70.74
1	84.00	74.29	72.23	72.25	72.31	72.41	72.55	72.73	72.96	73.22	73.53	73.88	74.26
1	90.00	77.82	75.75	75.77	75.83	75.93	76.07	76.26	76.48	76.75	77.05	77.40	77.78
1	96.00	81.34	79.27	79.29	79.35	79.45	79.59	79.78	80.00	80.27	80.57	80.92	81.31
1	102.00	84.86	82.79	82.81	82.87	82.97	83.11	83.30	83.52	83.79	84.09	84.44	84.83
1	108.00	88.38	86.31	86.33	86.39	86.49	86.64	86.82	87.04	87.31	87.61	87.96	88.35
1	114.00	91.90	89.83	89.85	89.91	90.01	90.16	90.34	90.56	90.83	91.13	91.48	91.87
1	120.00	95.42	93.35	93.37	93.43	93.54	93.68	93.86	94.09	94.35	94.66	95.00	95.39
2	6.00	101.98	98.35	98.38	98.49	98.67	98.92	99.24	99.64	100.10	100.64	101.25	101.93
2	12.00	107.63	104.37	104.40	104.49	104.66	104.89	105.18	105.54	105.96	106.44	106.98	107.58
2	18.00	112.49	109.67	109.70	109.78	109.92	110.12	110.38	110.69	111.05	111.46	111.93	112.45
2	24.00	116.67	114.25	114.27	114.34	114.46	114.63	114.85	115.12	115.43	115.79	116.19	116.63
2	30.00	120.27	118.18	118.20	118.27	118.37	118.52	118.71	118.93	119.20	119.51	119.86	120.24
2	36.00	123.37	121.57	121.59	121.64	121.73	121.86	122.02	122.22	122.45	122.71	123.01	123.34
2	42.00	126.03	124.49	124.50	124.55	124.63	124.74	124.88	125.04	125.24	125.47	125.73	126.01
2	48.00	128.33	127.00	127.01	127.05	127.12	127.21	127.33	127.48	127.65	127.84	128.07	128.31
2	54.00	130.30	129.16	129.17	129.20	129.26	129.34	129.45	129.57	129.72	129.89	130.08	130.29
2	60.00	132.00	131.02	131.03	131.06	131.11	131.18	131.26	131.37	131.50	131.64	131.81	131.99
2	66.00	133.47	132.62	132.63	132.65	132.69	132.75	132.83	132.92	133.03	133.16	133.30	133.45
2	72.00	134.72	134.00	134.00	134.02	134.06	134.11	134.18	134.26	134.35	134.46	134.58	134.71
2	78.00	135.81	135.18	135.19	135.21	135.24	135.28	135.34	135.41	135.49	135.58	135.68	135.80
2	84.00	136.74	136.20	136.21	136.22	136.25	136.29	136.34	136.40	136.46	136.54	136.63	136.73
2	90.00	137.54	137.08	137.08	137.10	137.12	137.15	137.19	137.25	137.31	137.37	137.45	137.54
2	96.00	138.23	137.83	137.84	137.85	137.87	137.90	137.93	137.98	138.03	138.09	138.15	138.23
2	102.00	138.83	138.48	138.49	138.50	138.52	138.54	138.57	138.61	138.65	138.70	138.76	138.82
2	108.00	139.34	139.04	139.05	139.06	139.07	139.09	139.12	139.15	139.19	139.23	139.28	139.34
2	114.00	139.78	139.53	139.53	139.54	139.55	139.57	139.59	139.62	139.65	139.69	139.73	139.78
2	120.00	140.16	139.94	139.94	139.95	139.96	139.98	139.99	140.02	140.05	140.08	140.12	140.16
3	3.00	140.16	140.09	140.10	140.10	140.10	140.11	140.12	140.13	140.14	140.14	140.15	140.16
3	6.00	140.16	140.14	140.14	140.14	140.15	140.15	140.15	140.15	140.15	140.16	140.16	140.16
3	9.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	12.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	15.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	18.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	21.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	24.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	27.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	30.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	33.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	36.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	39.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	42.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	45.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	48.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	51.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	54.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	57.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16
3	60.00	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16	140.16

LIQUID		SOLID SPHERE	
N/No	-ln(N/No)	N/No	-ln(N/No)
.9999690+00	.3062170-04	.9999750+00	.2548880-04
.3339710-02	.5701870+01	.4096070-02	.5497730+01
.1012320-03	.9198100+01	.1018210-03	.9192300+01

FOR 60% RECOVERY, 145 C STEAM, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE										
			E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	28.60	26.64	26.66	26.71	26.80	26.93	27.10	27.31	27.56	27.85	28.19	28.57
1	12.00	32.19	30.09	30.11	30.17	30.27	30.42	30.61	30.83	31.10	31.41	31.77	32.16
1	18.00	35.79	33.68	33.70	33.76	33.86	34.01	34.20	34.42	34.69	35.01	35.36	35.76
1	24.00	39.38	37.27	37.29	37.35	37.46	37.60	37.79	38.02	38.29	38.60	38.96	39.35
1	30.00	42.98	40.87	40.89	40.95	41.05	41.20	41.39	41.62	41.89	42.20	42.55	42.95
1	36.00	46.58	44.47	44.48	44.55	44.65	44.80	44.98	45.21	45.48	45.79	46.15	46.54
1	42.00	50.17	48.06	48.08	48.14	48.25	48.39	48.58	48.81	49.08	49.39	49.74	50.14
1	48.00	53.77	51.66	51.68	51.74	51.84	51.99	52.17	52.40	52.67	52.99	53.34	53.74
1	54.00	57.36	55.25	55.27	55.33	55.44	55.58	55.77	56.00	56.27	56.58	56.94	57.33
1	60.00	60.96	58.85	58.87	58.93	59.03	59.18	59.37	59.60	59.87	60.18	60.53	60.93
1	66.00	64.56	62.45	62.46	62.53	62.63	62.78	62.96	63.19	63.46	63.77	64.13	64.52
1	72.00	68.15	66.04	66.06	66.12	66.23	66.37	66.56	66.79	67.06	67.37	67.72	68.12
1	78.00	71.75	69.64	69.66	69.72	69.82	69.97	70.15	70.38	70.65	70.97	71.32	71.72
1	84.00	75.34	73.23	73.25	73.31	73.42	73.56	73.75	73.98	74.25	74.56	74.92	75.31
1	90.00	78.94	76.83	76.85	76.91	77.01	77.16	77.35	77.58	77.85	78.16	78.51	78.91
1	96.00	82.54	80.42	80.44	80.51	80.61	80.76	80.94	81.17	81.44	81.75	82.11	82.50
1	102.00	86.13	84.02	84.04	84.10	84.21	84.35	84.54	84.77	85.04	85.35	85.70	86.10
1	108.00	89.73	87.62	87.63	87.70	87.80	87.95	88.13	88.36	88.63	88.95	89.30	89.70
1	114.00	93.32	91.21	91.23	91.29	91.40	91.54	91.73	91.96	92.23	92.54	92.90	93.29
1	120.00	96.92	94.81	94.83	94.89	94.99	95.14	95.33	95.56	95.83	96.14	96.49	96.89
2	6.00	103.62	99.91	99.94	100.05	100.24	100.49	100.82	101.22	101.70	102.25	102.87	103.56
2	12.00	109.39	106.05	106.08	106.19	106.35	106.59	106.89	107.25	107.68	108.17	108.73	109.34
2	18.00	114.35	111.47	111.50	111.59	111.73	111.93	112.19	112.51	112.88	113.30	113.78	114.31
2	24.00	118.62	116.15	116.17	116.24	116.37	116.54	116.76	117.04	117.35	117.72	118.13	118.58
2	30.00	122.30	120.17	120.19	120.25	120.36	120.51	120.70	120.93	121.21	121.52	121.87	122.27
2	36.00	125.46	123.63	123.64	123.70	123.79	123.92	124.09	124.29	124.52	124.79	125.10	125.43
2	42.00	128.18	126.61	126.62	126.67	126.75	126.86	127.00	127.17	127.38	127.61	127.87	128.16
2	48.00	130.53	129.17	129.18	129.22	129.29	129.39	129.51	129.66	129.83	130.03	130.26	130.51
2	54.00	132.54	131.38	131.39	131.42	131.48	131.56	131.67	131.80	131.95	132.12	132.31	132.53
2	60.00	134.28	133.27	133.28	133.31	133.36	133.44	133.53	133.64	133.77	133.91	134.08	134.27
2	66.00	135.77	134.91	134.92	134.94	134.99	135.05	135.12	135.22	135.33	135.46	135.60	135.76
2	72.00	137.06	136.31	136.32	136.34	136.38	136.43	136.50	136.58	136.68	136.79	136.91	137.05
2	78.00	138.17	137.52	137.53	137.55	137.58	137.63	137.69	137.76	137.84	137.93	138.04	138.16
2	84.00	139.12	138.57	138.57	138.59	138.62	138.65	138.70	138.77	138.84	138.92	139.01	139.11
2	90.00	139.94	139.46	139.47	139.48	139.51	139.54	139.58	139.63	139.70	139.77	139.84	139.93
2	96.00	140.64	140.23	140.24	140.25	140.27	140.30	140.34	140.38	140.43	140.49	140.56	140.64
2	102.00	141.25	140.90	140.90	140.91	140.93	140.96	140.99	141.03	141.07	141.12	141.18	141.25
2	108.00	141.77	141.47	141.47	141.48	141.50	141.52	141.55	141.58	141.62	141.66	141.71	141.77
2	114.00	142.22	141.96	141.96	141.97	141.99	142.00	142.03	142.06	142.09	142.13	142.17	142.22
2	120.00	142.61	142.39	142.39	142.39	142.41	142.42	142.44	142.47	142.50	142.53	142.57	142.61
3	3.00	142.61	142.54	142.54	142.55	142.55	142.56	142.57	142.58	142.58	142.59	142.60	142.61
3	6.00	142.61	142.59	142.59	142.59	142.60	142.60	142.60	142.60	142.60	142.61	142.61	142.61
3	9.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	12.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	15.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	18.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	21.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	24.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	27.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	30.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	33.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	36.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	39.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	42.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	45.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	48.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	51.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	54.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	57.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	60.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61

	LIQUID		SOLID SPHERE	
	N/No	-ln(N/No)	N/No	-ln(N/No)
1	.9999560+00	.4446560-04	.9999630+00	.3694670-04
2	.8516400-04	.9370930+01	.1208730-03	.9020770+01
3	.2505530-06	.1519960+02	.2529920-06	.1518990+02

FOR 60% RECOVERY, 147.5 C STEAM, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.0
1	6.00	28.67	26.68	26.69	26.75	26.84	26.98	27.15	27.36	27.61	27.91	28.25	28.6
1	12.00	32.34	30.20	30.22	30.28	30.38	30.53	30.72	30.95	31.23	31.55	31.91	32.3
1	18.00	36.01	33.86	33.88	33.94	34.05	34.20	34.39	34.62	34.90	35.22	35.58	35.9
1	24.00	39.68	37.53	37.55	37.61	37.72	37.87	38.06	38.29	38.57	38.89	39.25	39.6
1	30.00	43.35	41.20	41.22	41.28	41.39	41.54	41.73	41.96	42.24	42.56	42.92	43.3
1	36.00	47.03	44.87	44.89	44.95	45.06	45.21	45.40	45.63	45.91	46.23	46.59	46.9
1	42.00	50.70	48.54	48.56	48.62	48.73	48.88	49.07	49.30	49.58	49.90	50.26	50.6
1	48.00	54.37	52.21	52.23	52.29	52.40	52.55	52.74	52.97	53.25	53.57	53.93	54.33
1	54.00	58.04	55.88	55.90	55.97	56.07	56.22	56.41	56.65	56.92	57.24	57.60	58.01
1	60.00	61.71	59.55	59.57	59.64	59.74	59.89	60.08	60.32	60.59	60.91	61.27	61.6
1	66.00	65.38	63.23	63.24	63.31	63.41	63.56	63.75	63.99	64.26	64.58	64.94	65.35
1	72.00	69.05	66.90	66.91	66.98	67.08	67.23	67.42	67.66	67.93	68.25	68.61	69.02
1	78.00	72.72	70.57	70.59	70.65	70.76	70.90	71.10	71.33	71.61	71.92	72.29	72.69
1	84.00	76.39	74.24	74.26	74.32	74.43	74.58	74.77	75.00	75.28	75.59	75.96	76.36
1	90.00	80.06	77.91	77.93	77.99	78.10	78.25	78.44	78.67	78.95	79.27	79.63	80.03
1	96.00	83.73	81.58	81.60	81.66	81.77	81.92	82.11	82.34	82.62	82.94	83.30	83.70
1	102.00	87.40	85.25	85.27	85.33	85.44	85.59	85.78	86.01	86.29	86.61	86.97	87.37
1	108.00	91.08	88.92	88.94	89.00	89.11	89.26	89.45	89.68	89.96	90.28	90.64	91.04
1	114.00	94.75	92.59	92.61	92.67	92.78	92.93	93.12	93.35	93.63	93.95	94.31	94.71
1	120.00	98.42	96.26	96.28	96.35	96.45	96.60	96.79	97.03	97.30	97.62	97.98	98.38
2	6.00	105.26	101.47	101.51	101.62	101.80	102.06	102.40	102.81	103.30	103.86	104.49	105.20
2	12.00	111.14	107.74	107.77	107.88	108.05	108.29	108.59	108.97	109.40	109.91	110.47	111.09
2	18.00	116.21	113.28	113.30	113.39	113.54	113.75	114.01	114.33	114.71	115.14	115.63	116.17
2	24.00	120.57	118.04	118.07	118.14	118.27	118.45	118.68	118.95	119.28	119.65	120.07	120.53
2	30.00	124.32	122.15	122.17	122.23	122.34	122.50	122.69	122.93	123.21	123.53	123.89	124.29
2	36.00	127.55	125.68	125.70	125.76	125.85	125.98	126.15	126.36	126.60	126.87	127.18	127.53
2	42.00	130.33	128.72	128.74	128.79	128.87	128.98	129.13	129.30	129.51	129.75	130.01	130.31
2	48.00	132.73	131.34	131.35	131.39	131.46	131.56	131.69	131.84	132.02	132.22	132.45	132.71
2	54.00	134.78	133.59	133.60	133.64	133.70	133.78	133.89	134.02	134.17	134.35	134.55	134.77
2	60.00	136.56	135.53	135.54	135.57	135.62	135.69	135.79	135.90	136.03	136.18	136.35	136.54
2	66.00	138.08	137.20	137.21	137.23	137.28	137.34	137.42	137.52	137.63	137.76	137.91	138.07
2	72.00	139.39	138.63	138.64	138.66	138.70	138.76	138.82	138.91	139.01	139.12	139.24	139.38
2	78.00	140.52	139.87	139.87	139.89	139.93	139.97	140.03	140.10	140.19	140.29	140.39	140.51
2	84.00	141.50	140.93	140.94	140.95	140.98	141.02	141.07	141.14	141.21	141.29	141.38	141.49
2	90.00	142.33	141.85	141.85	141.87	141.89	141.93	141.97	142.02	142.08	142.16	142.24	142.33
2	96.00	143.05	142.64	142.64	142.65	142.67	142.70	142.74	142.79	142.84	142.90	142.97	143.05
2	102.00	143.67	143.31	143.32	143.33	143.35	143.37	143.40	143.44	143.49	143.54	143.60	143.67
2	108.00	144.21	143.90	143.90	143.91	143.92	143.95	143.97	144.01	144.05	144.09	144.14	144.20
2	114.00	144.66	144.40	144.40	144.41	144.42	144.44	144.47	144.49	144.53	144.57	144.61	144.66
2	120.00	145.06	144.83	144.83	144.84	144.85	144.87	144.89	144.91	144.94	144.98	145.01	145.06
3	3.00	145.06	144.99	144.99	145.00	145.00	145.01	145.02	145.03	145.03	145.04	145.05	145.06
3	6.00	145.06	145.04	145.04	145.04	145.05	145.05	145.05	145.05	145.05	145.06	145.06	145.06
3	9.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	12.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	15.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	18.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	21.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	24.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	27.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	30.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	33.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	36.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	39.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	42.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	45.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	48.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	51.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	54.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	57.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	60.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06

	LIQUID		SOLID SPHERE	
	N/No	-ln(N/No)	N/No	-ln(N/No)
1	.9999360+00	.6439310-04	.9999470+00	.5340440-04
2	.2235350-06	.1531370+02	.4113640-06	.1470380+02
3	.1431670-10	.2496960+02	.1454900-10	.2495350+02

FOR 60% RECOVERY, 150 C STEAM, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	28.75	26.71	26.73	26.78	26.88	27.02	27.19	27.41	27.67	27.97	28.32	28.71
1	12.00	32.49	30.30	30.32	30.39	30.49	30.65	30.84	31.08	31.36	31.68	32.05	32.46
1	18.00	36.24	34.04	34.06	34.12	34.23	34.38	34.58	34.82	35.10	35.42	35.79	36.20
1	24.00	39.98	37.79	37.80	37.87	37.98	38.13	38.32	38.56	38.84	39.17	39.54	39.95
1	30.00	43.73	41.53	41.55	41.61	41.72	41.87	42.07	42.31	42.59	42.92	43.28	43.70
1	36.00	47.47	45.28	45.30	45.36	45.47	45.62	45.82	46.05	46.34	46.66	47.03	47.44
1	42.00	51.22	49.02	49.04	49.11	49.21	49.37	49.56	49.80	50.08	50.41	50.78	51.19
1	48.00	54.97	52.77	52.79	52.85	52.96	53.11	53.31	53.55	53.83	54.15	54.52	54.93
1	54.00	58.71	56.51	56.53	56.60	56.71	56.86	57.05	57.29	57.57	57.90	58.27	58.68
1	60.00	62.46	60.26	60.28	60.34	60.45	60.60	60.80	61.04	61.32	61.64	62.01	62.42
1	66.00	66.20	64.01	64.02	64.09	64.20	64.35	64.54	64.78	65.06	65.39	65.76	66.17
1	72.00	69.95	67.75	67.77	67.84	67.94	68.10	68.29	68.53	68.81	69.14	69.50	69.92
1	78.00	73.70	71.50	71.52	71.58	71.69	71.84	72.04	72.27	72.56	72.88	73.25	73.66
1	84.00	77.44	75.24	75.26	75.33	75.44	75.59	75.78	76.02	76.30	76.63	77.00	77.41
1	90.00	81.19	78.99	79.01	79.07	79.18	79.33	79.53	79.77	80.05	80.37	80.74	81.15
1	96.00	84.93	82.73	82.75	82.82	82.93	83.08	83.27	83.51	83.79	84.12	84.49	84.90
1	102.00	88.68	86.48	86.50	86.56	86.67	86.82	87.02	87.26	87.54	87.86	88.23	88.65
1	108.00	92.42	90.23	90.24	90.31	90.42	90.57	90.76	91.00	91.29	91.61	91.98	92.39
1	114.00	96.17	93.97	93.99	94.06	94.16	94.32	94.51	94.75	95.03	95.36	95.72	96.14
1	120.00	99.92	97.72	97.74	97.80	97.91	98.06	98.26	98.49	98.78	99.10	99.47	99.88
2	6.00	106.90	103.03	103.07	103.18	103.37	103.64	103.98	104.40	104.90	105.47	106.12	106.84
2	12.00	112.90	109.43	109.46	109.57	109.74	109.99	110.30	110.68	111.13	111.64	112.21	112.85
2	18.00	118.07	115.08	115.10	115.19	115.35	115.56	115.83	116.15	116.54	116.98	117.48	118.03
2	24.00	122.52	119.94	119.97	120.04	120.17	120.36	120.59	120.87	121.20	121.58	122.01	122.48
2	30.00	126.35	124.13	124.15	124.22	124.33	124.49	124.69	124.93	125.22	125.54	125.91	126.32
2	36.00	129.65	127.74	127.75	127.81	127.91	128.04	128.21	128.42	128.67	128.95	129.27	129.62
2	42.00	132.48	130.84	130.85	130.90	130.99	131.10	131.25	131.43	131.64	131.88	132.16	132.46
2	48.00	134.92	133.51	133.52	133.56	133.64	133.74	133.86	134.02	134.20	134.41	134.64	134.90
2	54.00	137.02	135.81	135.82	135.86	135.92	136.00	136.11	136.25	136.40	136.58	136.78	137.01
2	60.00	138.83	137.79	137.79	137.83	137.88	137.95	138.05	138.16	138.30	138.45	138.63	138.82
2	66.00	140.39	139.49	139.50	139.52	139.57	139.63	139.71	139.81	139.93	140.06	140.21	140.38
2	72.00	141.73	140.95	140.96	140.98	141.02	141.08	141.15	141.23	141.33	141.45	141.57	141.72
2	78.00	142.88	142.21	142.22	142.24	142.27	142.32	142.38	142.45	142.54	142.64	142.75	142.87
2	84.00	143.87	143.30	143.30	143.32	143.35	143.39	143.44	143.51	143.58	143.66	143.76	143.87
2	90.00	144.73	144.23	144.24	144.25	144.28	144.31	144.36	144.41	144.47	144.55	144.63	144.72
2	96.00	145.46	145.04	145.04	145.05	145.07	145.10	145.14	145.19	145.24	145.31	145.38	145.46
2	102.00	146.09	145.73	145.73	145.74	145.76	145.79	145.82	145.86	145.91	145.96	146.02	146.09
2	108.00	146.64	146.32	146.33	146.34	146.35	146.37	146.40	146.44	146.48	146.52	146.58	146.63
2	114.00	147.11	146.84	146.84	146.85	146.86	146.88	146.90	146.93	146.97	147.01	147.05	147.10
2	120.00	147.51	147.28	147.28	147.29	147.30	147.31	147.34	147.36	147.39	147.43	147.46	147.51
3	3.00	147.51	147.44	147.44	147.45	147.45	147.46	147.47	147.47	147.48	147.49	147.50	147.51
3	6.00	147.51	147.49	147.49	147.49	147.50	147.50	147.50	147.50	147.50	147.51	147.51	147.51
3	9.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	12.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	15.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	18.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	21.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	24.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	27.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	30.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	33.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	36.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	39.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	42.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	45.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	48.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	51.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	54.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	57.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	60.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.9999070+00	.9300060-04	.9999230+00	.7698100-04
2	.1556890-10	.2488570+02	.4618380-10	.2379840+02
3	.1941830-17	.4078290+02	.1994200-17	.4075630+02

FOR 60% RECOVERY, 152.5 C STEAM, L = 130

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	28.82	26.74	26.76	26.82	26.92	27.06	27.24	27.46	27.72	28.03	28.39	28.79
1	12.00	32.64	30.41	30.43	30.49	30.60	30.76	30.96	31.20	31.48	31.81	32.19	32.61
1	18.00	36.46	34.22	34.24	34.31	34.42	34.57	34.77	35.01	35.30	35.63	36.01	36.43
1	24.00	40.28	38.04	38.06	38.13	38.24	38.39	38.59	38.83	39.12	39.45	39.83	40.25
1	30.00	44.10	41.86	41.88	41.95	42.06	42.21	42.41	42.65	42.94	43.27	43.65	44.07
1	36.00	47.92	45.68	45.70	45.77	45.88	46.03	46.23	46.48	46.76	47.09	47.47	47.89
1	42.00	51.74	49.50	49.52	49.59	49.70	49.85	50.05	50.30	50.58	50.92	51.29	51.71
1	48.00	55.57	53.32	53.34	53.41	53.52	53.67	53.87	54.12	54.40	54.74	55.11	55.53
1	54.00	59.39	57.14	57.16	57.23	57.34	57.50	57.69	57.94	58.22	58.56	58.93	59.35
1	60.00	63.21	60.96	60.98	61.05	61.16	61.32	61.51	61.76	62.05	62.38	62.75	63.17
1	66.00	67.03	64.79	64.80	64.87	64.98	65.14	65.34	65.58	65.87	66.20	66.57	66.99
1	72.00	70.85	68.61	68.63	68.69	68.80	68.96	69.16	69.40	69.69	70.02	70.39	70.81
1	78.00	74.67	72.43	72.45	72.51	72.62	72.78	72.98	73.22	73.51	73.84	74.22	74.64
1	84.00	78.49	76.25	76.27	76.33	76.44	76.60	76.80	77.04	77.33	77.66	78.04	78.46
1	90.00	82.31	80.07	80.09	80.15	80.26	80.42	80.62	80.86	81.15	81.48	81.86	82.28
1	96.00	86.13	83.89	83.91	83.97	84.09	84.24	84.44	84.68	84.97	85.30	85.68	86.10
1	102.00	89.95	87.71	87.73	87.80	87.91	88.06	88.26	88.50	88.79	89.12	89.50	89.92
1	108.00	93.77	91.53	91.55	91.62	91.73	91.88	92.08	92.32	92.61	92.94	93.32	93.74
1	114.00	97.59	95.35	95.37	95.44	95.55	95.70	95.90	96.14	96.43	96.76	97.14	97.56
1	120.00	101.41	99.17	99.19	99.26	99.37	99.52	99.72	99.96	100.25	100.58	100.96	101.38
2	6.00	108.53	104.59	104.63	104.74	104.94	105.21	105.56	105.99	106.49	107.08	107.74	108.47
2	12.00	114.66	111.12	111.15	111.26	111.44	111.69	112.01	112.39	112.85	113.37	113.96	114.61
2	18.00	119.93	116.88	116.91	117.00	117.15	117.37	117.64	117.98	118.37	118.82	119.33	119.89
2	24.00	124.47	121.84	121.87	121.95	122.08	122.26	122.50	122.79	123.13	123.51	123.95	124.43
2	30.00	128.38	126.11	126.13	126.20	126.32	126.48	126.68	126.93	127.22	127.55	127.93	128.34
2	36.00	131.74	129.79	129.81	129.87	129.97	130.10	130.28	130.49	130.74	131.03	131.35	131.71
2	42.00	134.63	132.96	132.97	133.02	133.11	133.22	133.38	133.56	133.78	134.02	134.30	134.61
2	48.00	137.12	135.68	135.69	135.74	135.81	135.91	136.04	136.20	136.38	136.60	136.84	137.10
2	54.00	139.27	138.02	138.03	138.07	138.14	138.22	138.33	138.47	138.63	138.81	139.02	139.25
2	60.00	141.11	140.04	140.05	140.08	140.14	140.21	140.31	140.43	140.56	140.72	140.90	141.09
2	66.00	142.70	141.78	141.79	141.81	141.86	141.92	142.01	142.11	142.23	142.36	142.51	142.68
2	72.00	144.06	143.27	143.28	143.30	143.34	143.40	143.47	143.56	143.66	143.78	143.91	144.05
2	78.00	145.24	144.56	144.56	144.58	144.62	144.67	144.73	144.80	144.89	144.99	145.10	145.23
2	84.00	146.25	145.66	145.67	145.69	145.72	145.76	145.81	145.88	145.95	146.04	146.13	146.24
2	90.00	147.12	146.62	146.62	146.64	146.66	146.70	146.74	146.80	146.86	146.94	147.02	147.11
2	96.00	147.87	147.44	147.44	147.45	147.48	147.51	147.55	147.59	147.65	147.71	147.79	147.86
2	102.00	148.52	148.14	148.15	148.16	148.18	148.20	148.24	148.28	148.33	148.38	148.44	148.51
2	108.00	149.07	148.75	148.75	148.76	148.78	148.80	148.83	148.87	148.91	148.95	149.01	149.07
2	114.00	149.55	149.27	149.27	149.28	149.30	149.32	149.34	149.37	149.41	149.45	149.49	149.55
2	120.00	149.96	149.72	149.72	149.73	149.74	149.76	149.78	149.81	149.84	149.87	149.91	149.96
3	3.00	149.96	149.89	149.89	149.89	149.90	149.91	149.92	149.92	149.93	149.94	149.95	149.96
3	6.00	149.96	149.94	149.94	149.94	149.95	149.95	149.95	149.95	149.95	149.96	149.96	149.96
3	9.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	12.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	15.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	18.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	21.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	24.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	27.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	30.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	33.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	36.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	39.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	42.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	45.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	48.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	51.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	54.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	57.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	60.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.9998660+00	.1339590-03	.9998890+00	.1106670-03
2	.3412090-17	.4021920+02	.2496810-16	.3822890+02
3	.1718340-28	.6623360+02	.1795330-28	.6618980+02

OR 60% RECOVERY, 140 C STEAM, L = 13

XCHANGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	28.45	26.39	26.41	26.46	26.54	26.66	26.81	27.00	27.23	27.50	27.81	28.16
1	12.00	31.89	29.61	29.63	29.69	29.79	29.93	30.10	30.32	30.58	30.87	31.21	31.59
1	18.00	35.34	33.04	33.06	33.12	33.22	33.36	33.54	33.76	34.02	34.31	34.65	35.03
1	24.00	38.78	36.48	36.50	36.56	36.66	36.80	36.98	37.20	37.46	37.76	38.10	38.48
1	30.00	42.23	39.93	39.95	40.01	40.11	40.25	40.43	40.65	40.91	41.21	41.54	41.92
1	36.00	45.68	43.38	43.40	43.46	43.56	43.69	43.87	44.09	44.35	44.65	44.99	45.37
1	42.00	49.12	46.82	46.84	46.90	47.00	47.14	47.32	47.54	47.80	48.10	48.44	48.82
1	48.00	52.57	50.27	50.29	50.35	50.45	50.59	50.77	50.99	51.25	51.54	51.88	52.26
1	54.00	56.01	53.71	53.73	53.79	53.89	54.03	54.21	54.43	54.69	54.99	55.33	55.71
1	60.00	59.46	57.16	57.18	57.24	57.34	57.48	57.66	57.88	58.14	58.44	58.78	59.15
1	66.00	62.91	60.61	60.63	60.69	60.79	60.93	61.10	61.32	61.58	61.88	62.22	62.60
1	72.00	66.35	64.05	64.07	64.13	64.23	64.37	64.55	64.77	65.03	65.33	65.67	66.05
1	78.00	69.80	67.50	67.52	67.58	67.68	67.82	68.00	68.22	68.48	68.77	69.11	69.49
1	84.00	73.25	70.95	70.96	71.02	71.12	71.26	71.44	71.66	71.92	72.22	72.56	72.94
1	90.00	76.69	74.39	74.41	74.47	74.57	74.71	74.89	75.11	75.37	75.67	76.01	76.39
1	96.00	80.14	77.84	77.86	77.92	78.02	78.16	78.34	78.55	78.81	79.11	79.45	79.83
1	102.00	83.58	81.28	81.30	81.36	81.46	81.60	81.78	82.00	82.26	82.56	82.90	83.28
1	108.00	87.03	84.73	84.75	84.81	84.91	85.05	85.23	85.45	85.71	86.01	86.34	86.72
1	114.00	90.48	88.18	88.19	88.25	88.35	88.49	88.67	88.89	89.15	89.45	89.79	90.17
1	120.00	93.92	91.62	91.64	91.70	91.80	91.94	92.12	92.34	92.60	92.90	93.24	93.62
2	6.00	100.34	96.35	96.38	96.48	96.66	96.90	97.21	97.59	98.04	98.56	99.15	99.81
2	12.00	105.87	102.22	102.25	102.35	102.51	102.73	103.02	103.38	103.79	104.27	104.80	105.39
2	18.00	110.63	107.46	107.49	107.58	107.72	107.91	108.16	108.47	108.83	109.24	109.70	110.21
2	24.00	114.72	112.00	112.02	112.09	112.21	112.38	112.60	112.86	113.17	113.53	113.92	114.36
2	30.00	118.24	115.90	115.92	115.98	116.09	116.23	116.42	116.64	116.91	117.22	117.56	117.94
2	36.00	121.27	119.26	119.27	119.33	119.42	119.54	119.70	119.90	120.13	120.39	120.69	121.01
2	42.00	123.88	122.15	122.16	122.21	122.29	122.39	122.53	122.70	122.90	123.12	123.38	123.66
2	48.00	126.13	124.64	124.65	124.69	124.76	124.85	124.97	125.11	125.28	125.48	125.69	125.94
2	54.00	128.06	126.78	126.79	126.82	126.88	126.96	127.06	127.19	127.33	127.50	127.69	127.90
2	60.00	129.73	128.62	128.63	128.66	128.71	128.78	128.86	128.97	129.10	129.24	129.40	129.58
2	66.00	131.16	130.21	130.21	130.24	130.28	130.34	130.42	130.51	130.62	130.74	130.88	131.03
2	72.00	132.39	131.57	131.58	131.60	131.64	131.69	131.75	131.83	131.92	132.03	132.15	132.28
2	78.00	133.45	132.75	132.75	132.77	132.80	132.85	132.90	132.97	133.05	133.14	133.24	133.36
2	84.00	134.36	133.76	133.76	133.78	133.80	133.84	133.89	133.95	134.02	134.10	134.19	134.28
2	90.00	135.15	134.63	134.63	134.64	134.67	134.70	134.74	134.79	134.85	134.92	135.00	135.08
2	96.00	135.83	135.38	135.38	135.39	135.41	135.44	135.47	135.52	135.57	135.63	135.69	135.77
2	102.00	136.41	136.02	136.02	136.03	136.05	136.07	136.11	136.14	136.19	136.24	136.29	136.36
2	108.00	136.91	136.57	136.58	136.59	136.60	136.62	136.65	136.68	136.72	136.76	136.81	136.86
2	114.00	137.34	137.05	137.05	137.06	137.07	137.09	137.12	137.14	137.18	137.21	137.25	137.30
2	120.00	137.71	137.46	137.46	137.47	137.48	137.50	137.52	137.54	137.57	137.60	137.64	137.68
3	3.00	137.71	137.62	137.62	137.63	137.63	137.64	137.65	137.66	137.67	137.68	137.69	137.70
3	6.00	137.71	137.68	137.68	137.68	137.69	137.69	137.69	137.69	137.70	137.70	137.70	137.71
3	9.00	137.71	137.70	137.70	137.70	137.70	137.70	137.70	137.71	137.71	137.71	137.71	137.71
3	12.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	15.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	18.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	21.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	24.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	27.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	30.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	33.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	36.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	39.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	42.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	45.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	48.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	51.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	54.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	57.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71
3	60.00	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71	137.71

LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
.9999790+00	.2103040-04	.9999840+00	.1627730-04
.3176510-01	.3449390+01	.3747700-01	.3284030+01
.3954100-02	.5533000+01	.3975420-02	.5527630+01

FOR 60% RECOVERY, 145 C STEAM, L = 13

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE										
			E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	28.60	26.45	26.47	26.52	26.61	26.73	26.89	27.09	27.33	27.61	27.93	28.30
1	12.00	32.19	29.81	29.83	29.90	30.00	30.14	30.33	30.55	30.82	31.13	31.48	31.87
1	18.00	35.79	33.39	33.41	33.47	33.58	33.72	33.91	34.14	34.41	34.72	35.07	35.47
1	24.00	39.38	36.98	37.00	37.07	37.17	37.32	37.50	37.73	38.00	38.31	38.67	39.06
1	30.00	42.98	40.58	40.60	40.66	40.77	40.91	41.10	41.33	41.60	41.91	42.26	42.66
1	36.00	46.58	44.18	44.20	44.26	44.36	44.51	44.69	44.92	45.19	45.51	45.86	46.26
1	42.00	50.17	47.77	47.79	47.85	47.96	48.10	48.29	48.52	48.79	49.10	49.46	49.85
1	48.00	53.77	51.37	51.39	51.45	51.55	51.70	51.89	52.12	52.39	52.70	53.05	53.45
1	54.00	57.36	54.96	54.98	55.05	55.15	55.30	55.48	55.71	55.98	56.29	56.65	57.04
1	60.00	60.96	58.56	58.58	58.64	58.75	58.89	59.08	59.31	59.58	59.89	60.24	60.64
1	66.00	64.56	62.15	62.18	62.24	62.34	62.49	62.67	62.90	63.17	63.49	63.84	64.24
1	72.00	68.15	65.75	65.77	65.83	65.94	66.08	66.27	66.50	66.77	67.08	67.44	67.83
1	78.00	71.75	69.35	69.37	69.43	69.53	69.68	69.87	70.10	70.37	70.68	71.03	71.43
1	84.00	75.34	72.94	72.96	73.03	73.13	73.28	73.46	73.69	73.96	74.27	74.63	75.02
1	90.00	78.94	76.54	76.56	76.62	76.73	76.87	77.06	77.29	77.56	77.87	78.22	78.62
1	96.00	82.54	80.13	80.15	80.22	80.32	80.47	80.65	80.88	81.15	81.47	81.82	82.22
1	102.00	86.13	83.73	83.75	83.81	83.92	84.06	84.25	84.48	84.75	85.06	85.42	85.81
1	108.00	89.73	87.33	87.35	87.41	87.51	87.66	87.85	88.07	88.35	88.66	89.01	89.41
1	114.00	93.32	90.92	90.94	91.01	91.11	91.25	91.44	91.67	91.94	92.25	92.61	93.00
1	120.00	96.92	94.52	94.54	94.60	94.71	94.85	95.04	95.27	95.54	95.85	96.20	96.60
2	6.00	103.62	99.45	99.49	99.59	99.77	100.02	100.35	100.74	101.21	101.76	102.37	103.06
2	12.00	109.39	105.57	105.61	105.71	105.88	106.11	106.42	106.78	107.22	107.71	108.27	108.89
2	18.00	114.35	111.05	111.08	111.17	111.31	111.52	111.78	112.10	112.47	112.90	113.38	113.92
2	24.00	118.62	115.78	115.80	115.88	116.01	116.18	116.41	116.68	117.01	117.37	117.79	118.25
2	30.00	122.30	119.85	119.87	119.94	120.05	120.20	120.39	120.63	120.91	121.22	121.58	121.98
2	36.00	125.46	123.36	123.37	123.43	123.52	123.65	123.82	124.03	124.26	124.54	124.85	125.19
2	42.00	128.18	126.37	126.39	126.44	126.52	126.63	126.77	126.95	127.15	127.39	127.65	127.95
2	48.00	130.53	128.97	128.98	129.02	129.09	129.19	129.31	129.46	129.66	129.84	130.07	130.32
2	54.00	132.54	131.20	131.21	131.25	131.31	131.39	131.50	131.63	131.78	131.96	132.15	132.37
2	60.00	134.28	133.12	133.13	133.17	133.22	133.29	133.38	133.49	133.62	133.77	133.94	134.13
2	66.00	135.77	134.78	134.79	134.81	134.86	134.92	135.00	135.10	135.21	135.34	135.48	135.64
2	72.00	137.06	136.20	136.21	136.23	136.27	136.33	136.39	136.48	136.57	136.68	136.81	136.95
2	78.00	138.17	137.43	137.44	137.46	137.49	137.53	137.59	137.66	137.75	137.84	137.95	138.07
2	84.00	139.12	138.48	138.49	138.51	138.54	138.57	138.63	138.69	138.76	138.84	138.93	139.04
2	90.00	139.94	139.39	139.40	139.41	139.44	139.47	139.51	139.57	139.63	139.70	139.78	139.87
2	96.00	140.64	140.17	140.18	140.19	140.21	140.24	140.28	140.32	140.38	140.44	140.51	140.58
2	102.00	141.25	140.85	140.85	140.86	140.88	140.90	140.94	140.98	141.02	141.07	141.13	141.20
2	108.00	141.77	141.43	141.43	141.44	141.45	141.47	141.50	141.54	141.58	141.62	141.67	141.73
2	114.00	142.22	141.92	141.93	141.93	141.95	141.97	141.99	142.02	142.05	142.09	142.14	142.18
2	120.00	142.61	142.35	142.35	142.36	142.37	142.39	142.41	142.43	142.46	142.50	142.53	142.58
3	3.00	142.61	142.52	142.52	142.52	142.53	142.54	142.55	142.56	142.57	142.58	142.59	142.60
3	6.00	142.61	142.58	142.58	142.58	142.59	142.59	142.59	142.59	142.60	142.60	142.60	142.61
3	9.00	142.61	142.60	142.60	142.60	142.60	142.60	142.60	142.61	142.61	142.61	142.61	142.61
3	12.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	15.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	18.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	21.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	24.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	27.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	30.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	33.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	36.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	39.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	42.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	45.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	48.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	51.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	54.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	57.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61
3	60.00	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61	142.61

	LIQUID		SOLID SPHERE	
	N/No	-ln(N/No)	N/No	-ln(N/No)
1	.9999560+00	.4446560-04	.9999660+00	.3423490-04
2	.8516400-04	.9370930+01	.1368750-03	.8896440+01
3	.2505530-06	.1519960+02	.2543590-06	.1518450+02

FOR 60% RECOVERY, 147.5 C STEAM, L = 13

EXCHANGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	28.67	26.48	26.50	26.55	26.64	26.76	26.93	27.13	27.38	27.66	27.99	28.37
1	12.00	32.34	29.91	29.93	30.00	30.10	30.25	30.44	30.67	30.94	31.26	31.62	32.02
1	18.00	36.01	33.56	33.58	33.65	33.75	33.90	34.09	34.33	34.60	34.92	35.28	35.69
1	24.00	39.68	37.23	37.25	37.32	37.42	37.57	37.76	38.00	38.27	38.59	38.95	39.36
1	30.00	43.35	40.90	40.92	40.99	41.09	41.24	41.43	41.67	41.94	42.26	42.62	43.03
1	36.00	47.03	44.57	44.60	44.66	44.77	44.91	45.11	45.34	45.61	45.93	46.29	46.70
1	42.00	50.70	48.25	48.27	48.33	48.44	48.58	48.78	49.01	49.29	49.60	49.97	50.37
1	48.00	54.37	51.92	51.94	52.00	52.11	52.26	52.45	52.68	52.96	53.28	53.64	54.04
1	54.00	58.04	55.59	55.61	55.67	55.78	55.93	56.12	56.35	56.63	56.95	57.31	57.71
1	60.00	61.71	59.26	59.28	59.34	59.45	59.60	59.79	60.02	60.30	60.62	60.98	61.38
1	66.00	65.38	62.93	62.95	63.01	63.12	63.27	63.46	63.69	63.97	64.29	64.65	65.05
1	72.00	69.05	66.60	66.62	66.68	66.79	66.94	67.13	67.36	67.64	67.96	68.32	68.72
1	78.00	72.72	70.27	70.29	70.35	70.46	70.61	70.80	71.03	71.31	71.63	71.99	72.39
1	84.00	76.39	73.94	73.96	74.03	74.13	74.28	74.47	74.71	74.98	75.30	75.66	76.07
1	90.00	80.06	77.61	77.63	77.70	77.80	77.95	78.14	78.38	78.65	78.97	79.33	79.74
1	96.00	83.73	81.28	81.30	81.37	81.47	81.62	81.81	82.05	82.32	82.64	83.00	83.41
1	102.00	87.40	84.95	84.97	85.04	85.14	85.29	85.48	85.72	85.99	86.31	86.67	87.08
1	108.00	91.08	88.62	88.65	88.71	88.82	88.96	89.16	89.39	89.67	89.98	90.35	90.75
1	114.00	94.75	92.30	92.32	92.38	92.49	92.64	92.83	93.06	93.34	93.65	94.02	94.42
1	120.00	98.42	95.97	95.99	96.05	96.16	96.31	96.50	96.73	97.01	97.33	97.69	98.09
2	6.00	105.26	101.00	101.04	101.15	101.33	101.59	101.92	102.32	102.80	103.36	103.98	104.69
2	12.00	111.14	107.25	107.29	107.39	107.56	107.80	108.11	108.49	108.93	109.44	110.00	110.64
2	18.00	116.21	112.84	112.87	112.96	113.11	113.32	113.59	113.91	114.30	114.73	115.23	115.77
2	24.00	120.57	117.67	117.69	117.77	117.90	118.08	118.31	118.59	118.92	119.30	119.72	120.19
2	30.00	124.32	121.83	121.85	121.92	122.03	122.18	122.38	122.62	122.90	123.23	123.59	124.00
2	36.00	127.55	125.40	125.42	125.48	125.58	125.71	125.88	126.09	126.33	126.61	126.93	127.27
2	42.00	130.33	128.48	128.50	128.55	128.63	128.75	128.89	129.07	129.28	129.52	129.79	130.09
2	48.00	132.73	131.13	131.15	131.19	131.26	131.36	131.49	131.64	131.82	132.03	132.26	132.52
2	54.00	134.78	133.41	133.43	133.46	133.52	133.61	133.72	133.85	134.01	134.18	134.38	134.61
2	60.00	136.56	135.38	135.39	135.42	135.47	135.54	135.64	135.75	135.89	136.04	136.21	136.40
2	66.00	138.08	137.07	137.08	137.10	137.15	137.21	137.29	137.39	137.50	137.64	137.79	137.95
2	72.00	139.39	138.52	138.53	138.55	138.59	138.64	138.71	138.80	138.90	139.01	139.14	139.28
2	78.00	140.52	139.77	139.78	139.80	139.83	139.88	139.94	140.01	140.10	140.19	140.30	140.43
2	84.00	141.50	140.85	140.85	140.87	140.90	140.94	140.99	141.05	141.13	141.21	141.31	141.41
2	90.00	142.33	141.78	141.78	141.80	141.82	141.85	141.90	141.95	142.02	142.09	142.17	142.26
2	96.00	143.05	142.57	142.58	142.59	142.61	142.64	142.68	142.73	142.78	142.84	142.91	142.99
2	102.00	143.67	143.26	143.26	143.27	143.29	143.32	143.35	143.39	143.44	143.49	143.55	143.62
2	108.00	144.21	143.85	143.85	143.86	143.88	143.90	143.93	143.96	144.00	144.05	144.10	144.16
2	114.00	144.66	144.36	144.36	144.37	144.38	144.40	144.43	144.46	144.49	144.53	144.58	144.63
2	120.00	145.06	144.80	144.80	144.81	144.82	144.83	144.86	144.88	144.91	144.94	144.98	145.03
3	3.00	145.06	144.97	144.97	144.97	144.98	144.99	145.00	145.01	145.02	145.03	145.04	145.05
3	6.00	145.06	145.03	145.03	145.03	145.04	145.04	145.04	145.04	145.05	145.05	145.05	145.06
3	9.00	145.06	145.05	145.05	145.05	145.05	145.05	145.05	145.06	145.06	145.06	145.06	145.06
3	12.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	15.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	18.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	21.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	24.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	27.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	30.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	33.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	36.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	39.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	42.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	45.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	48.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	51.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	54.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	57.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06
3	60.00	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06	145.06

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.9999360+00	.643931D-04	.999951D+00	.494377D-04
2	.2235350-06	.153137D+02	.504321D-06	.145001D+02
3	.143167D-10	.249696D+02	.146797D-10	.249446D+02

60% RECOVERY, 150 C STEAM, L = 13

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	28.75	26.51	26.53	26.58	26.67	26.80	26.97	27.18	27.42	27.72	28.05	28.43
1	12.00	32.49	30.01	30.04	30.10	30.21	30.36	30.55	30.78	31.06	31.39	31.75	32.16
1	18.00	36.24	33.74	33.76	33.82	33.93	34.08	34.28	34.52	34.80	35.12	35.49	35.90
1	24.00	39.98	37.48	37.50	37.57	37.68	37.83	38.02	38.26	38.54	38.87	39.24	39.65
1	30.00	43.73	41.23	41.25	41.31	41.42	41.57	41.77	42.01	42.29	42.62	42.98	43.40
1	36.00	47.47	44.97	45.00	45.06	45.17	45.32	45.52	45.75	46.04	46.36	46.73	47.14
1	42.00	51.22	48.72	48.74	48.81	48.91	49.07	49.26	49.50	49.78	50.11	50.48	50.89
1	48.00	54.97	52.47	52.49	52.55	52.66	52.81	53.01	53.25	53.53	53.85	54.22	54.63
1	54.00	58.71	56.21	56.23	56.30	56.41	56.56	56.75	56.99	57.27	57.60	57.97	58.38
1	60.00	62.46	59.96	59.98	60.04	60.15	60.30	60.50	60.74	61.02	61.34	61.71	62.12
1	66.00	66.20	63.70	63.72	63.79	63.90	64.05	64.24	64.48	64.76	65.09	65.46	65.87
1	72.00	69.95	67.45	67.47	67.53	67.64	67.79	67.99	68.23	68.51	68.84	69.20	69.62
1	78.00	73.70	71.19	71.22	71.28	71.39	71.54	71.74	71.97	72.26	72.58	72.95	73.36
1	84.00	77.44	74.94	74.96	75.03	75.13	75.29	75.48	75.72	76.00	76.33	76.70	77.11
1	90.00	81.19	78.69	78.71	78.77	78.88	79.03	79.23	79.47	79.75	80.07	80.44	80.85
1	96.00	84.93	82.43	82.45	82.52	82.63	82.78	82.97	83.21	83.49	83.82	84.19	84.60
1	102.00	88.68	86.18	86.20	86.26	86.37	86.52	86.72	86.96	87.24	87.56	87.93	88.34
1	108.00	92.42	89.92	89.94	90.01	90.12	90.27	90.46	90.70	90.98	91.31	91.68	92.09
1	114.00	96.17	93.67	93.69	93.76	93.86	94.02	94.21	94.45	94.73	95.06	95.42	95.84
1	120.00	99.92	97.41	97.44	97.50	97.61	97.76	97.96	98.19	98.48	98.80	99.17	99.58
2	6.00	106.90	102.55	102.59	102.70	102.89	103.15	103.49	103.90	104.39	104.95	105.60	106.31
2	12.00	112.90	108.93	108.97	109.07	109.25	109.49	109.81	110.19	110.64	111.16	111.74	112.38
2	18.00	118.07	114.63	114.66	114.76	114.91	115.12	115.40	115.73	116.12	116.57	117.07	117.62
2	24.00	122.52	119.56	119.59	119.67	119.80	119.98	120.22	120.50	120.84	121.22	121.66	122.14
2	30.00	126.35	123.80	123.82	123.89	124.01	124.16	124.37	124.61	124.90	125.23	125.61	126.02
2	36.00	129.65	127.45	127.47	127.53	127.63	127.76	127.94	128.15	128.40	128.69	129.01	129.36
2	42.00	132.48	130.60	130.61	130.66	130.75	130.86	131.01	131.20	131.41	131.66	131.93	132.24
2	48.00	134.92	133.30	133.31	133.36	133.43	133.53	133.66	133.82	134.00	134.21	134.45	134.71
2	54.00	137.02	135.63	135.64	135.68	135.74	135.83	135.94	136.07	136.23	136.41	136.62	136.84
2	60.00	138.83	137.63	137.64	137.67	137.73	137.80	137.90	138.01	138.15	138.31	138.48	138.68
2	66.00	140.39	139.35	139.36	139.39	139.44	139.50	139.58	139.68	139.80	139.94	140.09	140.25
2	72.00	141.73	140.84	140.85	140.87	140.91	140.96	141.03	141.12	141.22	141.34	141.47	141.61
2	78.00	142.88	142.11	142.12	142.14	142.18	142.22	142.28	142.36	142.45	142.55	142.66	142.78
2	84.00	143.87	143.21	143.22	143.24	143.27	143.31	143.36	143.42	143.50	143.58	143.68	143.79
2	90.00	144.73	144.16	144.16	144.18	144.20	144.24	144.28	144.34	144.40	144.48	144.56	144.65
2	96.00	145.46	144.97	144.98	144.99	145.01	145.04	145.08	145.13	145.18	145.25	145.32	145.40
2	102.00	146.09	145.67	145.68	145.69	145.71	145.73	145.77	145.81	145.86	145.91	145.97	146.04
2	108.00	146.64	146.28	146.28	146.29	146.31	146.33	146.36	146.39	146.43	146.48	146.53	146.59
2	114.00	147.11	146.80	146.80	146.81	146.82	146.84	146.86	146.89	146.93	146.97	147.02	147.07
2	120.00	147.51	147.24	147.24	147.25	147.26	147.28	147.30	147.33	147.36	147.39	147.43	147.48
3	3.00	147.51	147.42	147.42	147.42	147.43	147.44	147.45	147.46	147.47	147.48	147.49	147.50
3	6.00	147.51	147.48	147.48	147.48	147.49	147.49	147.49	147.49	147.50	147.50	147.50	147.51
3	9.00	147.51	147.50	147.50	147.50	147.50	147.50	147.50	147.51	147.51	147.51	147.51	147.51
3	12.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	15.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	18.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	21.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	24.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	27.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	30.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	33.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	36.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	39.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	42.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	45.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	48.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	51.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	54.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	57.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51
3	60.00	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51	147.51

LIQUID		SOLID SPHERE	
N/No	-ln(N/No)	N/No	-ln(N/No)
.9999070+00	.9300060-04	.9999290+00	.7119610-04
.1556890-10	.2488570+02	.6433690-10	.2346690+02
.1941830-17	.4078290+02	.2023860-17	.4074150+02

R 60% RECOVERY, 152.5 C STEAM, L = 13

CHANGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	28.82	26.54	26.56	26.61	26.71	26.84	27.01	27.22	27.47	27.77	28.11	28.50
1	12.00	32.64	30.11	30.14	30.20	30.31	30.46	30.66	30.90	31.18	31.51	31.89	32.30
1	18.00	36.46	33.91	33.94	34.00	34.11	34.27	34.47	34.71	35.00	35.33	35.70	36.12
1	24.00	40.28	37.73	37.75	37.82	37.93	38.09	38.28	38.53	38.82	39.15	39.52	39.94
1	30.00	44.10	41.55	41.57	41.64	41.75	41.91	42.11	42.35	42.64	42.97	43.34	43.76
1	36.00	47.92	45.37	45.40	45.46	45.57	45.73	45.93	46.17	46.46	46.79	47.16	47.58
1	42.00	51.74	49.19	49.22	49.28	49.39	49.55	49.75	49.99	50.28	50.61	50.98	51.40
1	48.00	55.57	53.02	53.04	53.10	53.21	53.37	53.57	53.81	54.10	54.43	54.81	55.23
1	54.00	59.39	56.84	56.86	56.92	57.03	57.19	57.39	57.63	57.92	58.25	58.63	59.05
1	60.00	63.21	60.66	60.68	60.74	60.85	61.01	61.21	61.45	61.74	62.07	62.45	62.87
1	66.00	67.03	64.48	64.50	64.56	64.68	64.83	65.03	65.27	65.56	65.89	66.27	66.69
1	72.00	70.85	68.30	68.32	68.39	68.50	68.65	68.85	69.09	69.38	69.71	70.09	70.51
1	78.00	74.67	72.12	72.14	72.21	72.32	72.47	72.67	72.91	73.20	73.53	73.91	74.33
1	84.00	78.49	75.94	75.96	76.03	76.14	76.29	76.49	76.73	77.02	77.35	77.73	78.15
1	90.00	82.31	79.76	79.78	79.85	79.96	80.11	80.31	80.56	80.84	81.17	81.55	81.97
1	96.00	86.13	83.58	83.60	83.67	83.78	83.93	84.13	84.38	84.66	84.99	85.37	85.79
1	102.00	89.95	87.40	87.42	87.49	87.60	87.75	87.95	88.20	88.48	88.82	89.19	89.61
1	108.00	93.77	91.22	91.24	91.31	91.42	91.57	91.77	92.02	92.30	92.64	93.01	93.43
1	114.00	97.59	95.04	95.06	95.13	95.24	95.40	95.59	95.84	96.13	96.46	96.83	97.25
1	120.00	101.41	98.86	98.88	98.95	99.06	99.22	99.42	99.66	99.95	100.28	100.65	101.07
2	6.00	108.53	104.10	104.14	104.26	104.45	104.71	105.06	105.48	105.98	106.55	107.21	107.94
2	12.00	114.66	110.61	110.65	110.75	110.93	111.18	111.50	111.90	112.36	112.88	113.47	114.13
2	18.00	119.93	116.43	116.46	116.55	116.71	116.92	117.20	117.54	117.94	118.40	118.91	119.48
2	24.00	124.47	121.45	121.48	121.56	121.69	121.88	122.12	122.41	122.76	123.15	123.59	124.08
2	30.00	128.38	125.78	125.80	125.87	125.99	126.15	126.35	126.61	126.90	127.24	127.62	128.04
2	36.00	131.74	129.50	129.52	129.58	129.68	129.82	130.00	130.21	130.47	130.76	131.09	131.45
2	42.00	134.63	132.71	132.72	132.78	132.86	132.98	133.13	133.32	133.54	133.79	134.07	134.38
2	48.00	137.12	135.47	135.48	135.52	135.60	135.70	135.83	135.99	136.18	136.40	136.64	136.91
2	54.00	139.27	137.84	137.85	137.89	137.95	138.04	138.16	138.29	138.46	138.64	138.85	139.08
2	60.00	141.11	139.88	139.89	139.93	139.98	140.06	140.15	140.27	140.41	140.57	140.75	140.95
2	66.00	142.70	141.64	141.65	141.68	141.73	141.79	141.87	141.98	142.10	142.23	142.39	142.56
2	72.00	144.06	143.15	143.16	143.19	143.23	143.28	143.36	143.44	143.55	143.66	143.80	143.94
2	78.00	145.24	144.46	144.46	144.48	144.52	144.57	144.63	144.71	144.79	144.90	145.01	145.14
2	84.00	146.25	145.58	145.58	145.60	145.63	145.67	145.73	145.79	145.87	145.96	146.05	146.16
2	90.00	147.12	146.54	146.55	146.56	146.59	146.62	146.67	146.73	146.79	146.87	146.95	147.05
2	96.00	147.87	147.37	147.38	147.39	147.41	147.44	147.48	147.53	147.59	147.65	147.73	147.81
2	102.00	148.52	148.09	148.09	148.10	148.12	148.15	148.18	148.22	148.27	148.33	148.39	148.46
2	108.00	149.07	148.70	148.71	148.72	148.73	148.75	148.78	148.82	148.86	148.91	148.96	149.02
2	114.00	149.55	149.23	149.23	149.24	149.26	149.28	149.30	149.33	149.37	149.41	149.46	149.51
2	120.00	149.96	149.69	149.69	149.70	149.71	149.73	149.75	149.77	149.80	149.84	149.88	149.92
3	3.00	149.96	149.86	149.87	149.87	149.88	149.88	149.89	149.91	149.92	149.93	149.94	149.95
3	6.00	149.96	149.93	149.93	149.93	149.94	149.94	149.94	149.94	149.95	149.95	149.95	149.96
3	9.00	149.96	149.95	149.95	149.95	149.95	149.95	149.95	149.96	149.96	149.96	149.96	149.96
	12.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	15.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	18.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	21.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	24.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	27.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	30.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	33.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	36.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	39.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	42.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	45.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	48.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	51.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	54.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	57.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
	60.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96

LIQUID		SOLID SPHERE	
N/No	-ln(N/No)	N/No	-ln(N/No)
.9998660+00	.1339590-03	.9998980+00	.1022550-03
.3412090-17	.4021920+02	.4265640-16	.3769340+02
.1718340-28	.6623360+02	.1839440-28	.6616550+02

X NO RECOVERY EXCHANGER, 70% RECOVERY VALUES FOR U, 141 C STEAM, L = 130

CHANGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2	6.00	49.19	36.55	36.65	37.02	37.63	38.50	39.61	40.98	42.60	44.48	46.61	49.00
2	12.00	68.33	57.39	57.48	57.82	58.38	59.15	60.15	61.35	62.77	64.38	66.18	68.17
2	18.00	83.48	74.76	74.84	75.11	75.56	76.18	76.97	77.93	79.06	80.34	81.78	83.36
2	24.00	95.48	88.57	88.63	88.85	89.20	89.69	90.32	91.08	91.97	92.99	94.12	95.38
2	30.00	104.97	99.50	99.55	99.72	100.00	100.39	100.89	101.49	102.19	103.00	103.90	104.89
2	36.00	112.48	108.15	108.19	108.33	108.55	108.86	109.25	109.73	110.28	110.92	111.63	112.42
2	42.00	118.43	115.00	115.03	115.14	115.31	115.56	115.87	116.25	116.69	117.19	117.76	118.38
2	48.00	123.13	120.42	120.45	120.53	120.67	120.86	121.11	121.41	121.76	122.16	122.60	123.09
2	54.00	126.86	124.71	124.73	124.80	124.91	125.06	125.26	125.49	125.77	126.09	126.44	126.83
2	60.00	129.81	128.11	128.13	128.18	128.26	128.39	128.54	128.73	128.95	129.20	129.48	129.78
2	66.00	132.14	130.80	130.81	130.85	130.92	131.02	131.14	131.29	131.46	131.66	131.88	132.12
2	72.00	133.99	132.92	132.93	132.97	133.02	133.10	133.19	133.31	133.45	133.61	133.78	133.97
2	78.00	135.45	134.61	134.62	134.64	134.69	134.75	134.82	134.91	135.02	135.15	135.29	135.44
2	84.00	136.61	135.94	135.95	135.97	136.00	136.05	136.11	136.18	136.27	136.37	136.48	136.60
2	90.00	137.52	137.00	137.00	137.02	137.04	137.08	137.13	137.19	137.26	137.33	137.42	137.52
2	96.00	138.25	137.83	137.83	137.85	137.87	137.90	137.94	137.98	138.04	138.10	138.17	138.24
2	102.00	138.82	138.49	138.49	138.50	138.52	138.55	138.58	138.61	138.65	138.70	138.76	138.82
2	108.00	139.28	139.01	139.02	139.03	139.04	139.06	139.08	139.11	139.14	139.18	139.22	139.27
2	114.00	139.64	139.43	139.43	139.44	139.45	139.46	139.48	139.50	139.53	139.56	139.60	139.63
2	120.00	139.92	139.76	139.76	139.76	139.77	139.78	139.80	139.82	139.84	139.86	139.89	139.92
3	3.00	139.92	139.87	139.87	139.87	139.88	139.88	139.89	139.90	139.90	139.91	139.91	139.92
3	6.00	139.92	139.91	139.91	139.91	139.91	139.91	139.91	139.91	139.92	139.92	139.92	139.92
3	9.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	12.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	15.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	18.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	21.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	24.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	27.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	30.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	33.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	36.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	39.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	42.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	45.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	48.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	51.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	54.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	57.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92
3	60.00	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92	139.92

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.1000000+01	.9293840-13	.1000000+01	-.3981700-07
2	.5273320-02	.5245100+01	.6424950-02	.5047570+01
3	.1577540-03	.8754480+01	.1584020-03	.8750370+01

OR NO RECOVERY EXCHANGER, 70% RECOVERY VALUES FOR U, 151 C STEAM, L = 130

XCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2	6.00	51.31	37.56	37.68	38.08	38.75	39.68	40.90	42.38	44.15	46.19	48.51	51.11
2	12.00	72.14	60.23	60.34	60.71	61.31	62.16	63.24	64.55	66.09	67.84	69.80	71.97
2	18.00	88.63	79.14	79.22	79.52	80.00	80.68	81.54	82.59	83.81	85.21	86.77	88.49
2	24.00	101.67	94.16	94.23	94.46	94.84	95.38	96.06	96.89	97.86	98.97	100.20	101.56
2	30.00	112.00	106.05	106.11	106.29	106.59	107.02	107.56	108.21	108.98	109.86	110.84	111.91
2	36.00	120.17	115.47	115.51	115.65	115.89	116.23	116.66	117.18	117.78	118.48	119.25	120.11
2	42.00	126.64	122.92	122.95	123.07	123.26	123.52	123.86	124.27	124.75	125.30	125.91	126.59
2	48.00	131.76	128.81	128.84	128.93	129.08	129.29	129.56	129.89	130.27	130.70	131.19	131.72
2	54.00	135.82	133.48	133.50	133.57	133.69	133.86	134.07	134.33	134.63	134.97	135.36	135.78
2	60.00	139.02	137.18	137.19	137.25	137.34	137.48	137.64	137.85	138.09	138.36	138.66	139.00
2	66.00	141.56	140.10	140.11	140.16	140.23	140.34	140.47	140.63	140.82	141.04	141.28	141.54
2	72.00	143.57	142.41	142.43	142.46	142.52	142.60	142.71	142.84	142.98	143.15	143.35	143.56
2	78.00	145.16	144.25	144.25	144.28	144.33	144.40	144.48	144.58	144.70	144.83	144.98	145.15
2	84.00	146.42	145.70	145.70	145.73	145.76	145.81	145.88	145.96	146.05	146.16	146.28	146.41
2	90.00	147.42	146.84	146.85	146.87	146.90	146.94	146.99	147.05	147.13	147.21	147.31	147.41
2	96.00	148.21	147.75	147.76	147.77	147.79	147.83	147.87	147.92	147.98	148.04	148.12	148.20
2	102.00	148.83	148.47	148.47	148.49	148.50	148.53	148.56	148.60	148.65	148.70	148.76	148.83
2	108.00	149.32	149.04	149.04	149.05	149.07	149.09	149.11	149.14	149.18	149.22	149.27	149.32
2	114.00	149.72	149.49	149.49	149.50	149.51	149.53	149.55	149.57	149.60	149.63	149.67	149.71
2	120.00	150.03	149.85	149.85	149.85	149.86	149.88	149.89	149.91	149.93	149.96	149.99	150.02
3	3.00	150.03	149.97	149.97	149.98	149.98	149.99	149.99	150.00	150.01	150.01	150.02	150.02
3	6.00	150.03	150.01	150.01	150.01	150.01	150.02	150.02	150.02	150.02	150.02	150.02	150.02
3	9.00	150.03	150.02	150.02	150.02	150.02	150.02	150.02	150.02	150.02	150.02	150.02	150.03
3	12.00	150.03	150.02	150.02	150.02	150.02	150.02	150.02	150.02	150.02	150.02	150.03	150.03
3	15.00	150.03	150.02	150.02	150.02	150.02	150.02	150.02	150.02	150.03	150.03	150.03	150.03
3	18.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	21.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	24.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	27.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	30.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	33.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	36.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	39.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	42.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	45.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	48.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	51.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	54.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	57.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03
3	60.00	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03	150.03

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.1000000+01	.9293840-13	.1000000+01	-.3981700-07
2	.7126130-17	.3948280+02	.5641940-16	.3741370+02
3	.7360450-29	.6708140+02	.7606920-29	.6704850+02

LNGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
)	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
I	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
:	6.00	41.86	32.93	33.00	33.26	33.69	34.29	35.07	36.03	37.18	38.51	40.02	41.73
:	12.00	56.37	48.02	48.09	48.34	48.76	49.35	50.10	51.02	52.09	53.33	54.71	56.25
:	18.00	68.86	61.63	61.69	61.91	62.28	62.79	63.44	64.23	65.16	66.23	67.43	68.76
:	24.00	79.61	73.39	73.44	73.63	73.94	74.38	74.94	75.63	76.43	77.35	78.38	79.52
:	30.00	88.87	83.50	83.55	83.71	83.99	84.36	84.85	85.43	86.12	86.91	87.80	88.79
:	36.00	96.83	92.21	92.25	92.39	92.63	92.95	93.37	93.87	94.47	95.15	95.91	96.76
:	42.00	103.68	99.71	99.74	99.87	100.07	100.35	100.70	101.14	101.65	102.24	102.89	103.62
:	48.00	109.58	106.16	106.19	106.29	106.47	106.71	107.02	107.39	107.83	108.33	108.90	109.53
:	54.00	114.65	111.71	111.74	111.83	111.98	112.18	112.45	112.77	113.15	113.58	114.07	114.61
:	60.00	119.02	116.49	116.51	116.59	116.72	116.90	117.12	117.40	117.73	118.10	118.52	118.99
:	66.00	122.78	120.60	120.62	120.69	120.80	120.95	121.15	121.39	121.67	121.99	122.35	122.75
:	72.00	126.02	124.14	124.16	124.22	124.31	124.44	124.61	124.82	125.06	125.33	125.65	125.99
:	78.00	128.80	127.19	127.20	127.25	127.33	127.45	127.59	127.77	127.98	128.21	128.48	128.78
:	84.00	131.20	129.81	129.82	129.86	129.93	130.03	130.16	130.31	130.49	130.69	130.92	131.18
:	90.00	133.26	132.07	132.08	132.11	132.17	132.26	132.37	132.50	132.65	132.83	133.02	133.24
:	96.00	135.04	134.01	134.02	134.05	134.10	134.17	134.27	134.38	134.51	134.66	134.83	135.02
:	102.00	136.56	135.68	135.69	135.71	135.76	135.82	135.90	136.00	136.11	136.24	136.39	136.55
:	108.00	137.88	137.12	137.12	137.15	137.19	137.24	137.31	137.39	137.49	137.60	137.73	137.87
:	114.00	139.01	138.36	138.36	138.38	138.41	138.46	138.52	138.59	138.68	138.77	138.88	139.00
:	120.00	139.98	139.42	139.43	139.44	139.47	139.51	139.56	139.62	139.70	139.78	139.87	139.98
:	3.00	139.98	139.82	139.82	139.83	139.84	139.86	139.88	139.90	139.92	139.94	139.97	139.98
:	6.00	139.98	139.94	139.94	139.95	139.95	139.95	139.96	139.96	139.97	139.97	139.98	139.98
:	9.00	139.98	139.97	139.97	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	12.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	15.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	18.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	21.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	24.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	27.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	30.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	33.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	36.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	39.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	42.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	45.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	48.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	51.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	54.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	57.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
:	60.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98

LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
.1000000D+01	.9293840-13	.1000000+01	-.3981700-07
.5104340-01	.2975080+01	.6183000-01	.2783370+01
.1401830-03	.8872560+01	.1422010-03	.8858270+01

NO RECOVERY EXCHANGER, 60% RECOVERY VALUES FOR U, 156 C STEAM, L = 130

CHANGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2	6.00	43.33	33.61	33.69	33.97	34.44	35.10	35.95	36.99	38.24	39.68	41.33	43.18
2	12.00	59.10	50.01	50.09	50.37	50.83	51.46	52.28	53.28	54.44	55.79	57.29	58.96
2	18.00	72.67	64.81	64.88	65.12	65.51	66.07	66.78	67.64	68.65	69.81	71.11	72.56
2	24.00	84.35	77.58	77.64	77.85	78.19	78.67	79.28	80.02	80.89	81.89	83.01	84.25
2	30.00	94.41	88.58	88.63	88.81	89.10	89.51	90.04	90.68	91.43	92.29	93.25	94.32
2	36.00	103.06	98.05	98.09	98.24	98.50	98.85	99.30	99.85	100.50	101.24	102.07	102.99
2	42.00	110.51	106.19	106.23	106.36	106.58	106.88	107.27	107.75	108.30	108.94	109.65	110.44
2	48.00	116.92	113.20	113.24	113.35	113.54	113.80	114.13	114.54	115.02	115.57	116.18	116.86
2	54.00	122.43	119.24	119.27	119.36	119.52	119.75	120.04	120.39	120.80	121.27	121.80	122.39
2	60.00	127.18	124.43	124.45	124.54	124.68	124.87	125.12	125.42	125.77	126.18	126.64	127.14
2	66.00	131.27	128.90	128.92	128.99	129.11	129.28	129.49	129.75	130.06	130.41	130.80	131.23
2	72.00	134.78	132.75	132.76	132.83	132.93	133.07	133.26	133.48	133.74	134.04	134.38	134.75
2	78.00	137.81	136.06	136.07	136.12	136.21	136.34	136.49	136.69	136.91	137.17	137.46	137.78
2	84.00	140.41	138.90	138.92	138.96	139.04	139.15	139.28	139.45	139.64	139.87	140.12	140.39
2	90.00	142.66	141.36	141.37	141.41	141.47	141.56	141.68	141.82	141.99	142.18	142.40	142.64
2	96.00	144.59	143.47	143.48	143.51	143.57	143.65	143.75	143.87	144.01	144.18	144.36	144.57
2	102.00	146.25	145.28	145.29	145.32	145.37	145.44	145.52	145.63	145.75	145.90	146.05	146.23
2	108.00	147.67	146.85	146.85	146.88	146.92	146.98	147.05	147.14	147.25	147.37	147.51	147.66
2	114.00	148.90	148.19	148.20	148.22	148.26	148.31	148.37	148.45	148.54	148.65	148.76	148.89
2	120.00	149.96	149.35	149.36	149.37	149.40	149.45	149.50	149.57	149.65	149.74	149.84	149.95
3	3.00	149.96	149.78	149.78	149.79	149.81	149.82	149.85	149.87	149.89	149.92	149.94	149.96
3	6.00	149.96	149.92	149.92	149.92	149.92	149.93	149.93	149.94	149.95	149.95	149.96	149.96
3	9.00	149.96	149.95	149.95	149.95	149.95	149.95	149.96	149.96	149.96	149.96	149.96	149.96
3	12.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	15.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	18.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	21.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	24.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	27.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	30.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	33.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	36.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	39.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	42.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	45.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	48.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	51.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	54.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	57.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	60.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.1000000+01	.9293840-13	.1000000+01	-.3981700-07
2	.3947140-09	.2165290+02	.2688000-08	.1973450+02
3	.1662200-28	.6626680+02	.1863600-28	.6615250+02

OR NO RECOVERY EXCHANGER, 60% RECOVERY VALUES FOR U, 146 C STEAM, L = 13

XCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2	6.00	41.86	32.01	32.09	32.33	32.74	33.32	34.06	34.99	36.09	37.38	38.85	40.51
2	12.00	56.37	46.84	46.92	47.17	47.60	48.18	48.94	49.86	50.94	52.18	53.58	55.13
2	18.00	68.86	60.56	60.64	60.86	61.23	61.74	62.40	63.20	64.15	65.23	66.44	67.78
2	24.00	79.61	72.46	72.52	72.72	73.03	73.48	74.05	74.74	75.55	76.48	77.52	78.68
2	30.00	88.87	82.71	82.76	82.93	83.20	83.58	84.07	84.67	85.37	86.17	87.07	88.06
2	36.00	96.83	91.53	91.58	91.72	91.95	92.28	92.70	93.22	93.82	94.51	95.28	96.14
2	42.00	103.68	99.12	99.16	99.28	99.48	99.77	100.13	100.57	101.09	101.68	102.35	103.09
2	48.00	109.58	105.65	105.69	105.79	105.97	106.21	106.52	106.90	107.35	107.86	108.43	109.07
2	54.00	114.65	111.28	111.31	111.40	111.55	111.76	112.02	112.35	112.73	113.17	113.67	114.21
2	60.00	119.02	116.11	116.14	116.22	116.35	116.53	116.76	117.04	117.37	117.75	118.17	118.64
2	66.00	122.78	120.28	120.30	120.37	120.48	120.64	120.83	121.08	121.36	121.69	122.05	122.46
2	72.00	126.02	123.86	123.88	123.94	124.04	124.17	124.34	124.55	124.79	125.07	125.39	125.74
2	78.00	128.80	126.95	126.96	127.01	127.10	127.21	127.36	127.54	127.75	127.99	128.26	128.56
2	84.00	131.20	129.60	129.62	129.66	129.73	129.83	129.96	130.11	130.29	130.50	130.73	130.99
2	90.00	133.26	131.89	131.90	131.94	132.00	132.08	132.19	132.33	132.48	132.66	132.86	133.08
2	96.00	135.04	133.86	133.87	133.90	133.95	134.02	134.12	134.23	134.37	134.52	134.69	134.88
2	102.00	136.56	135.55	135.56	135.58	135.63	135.69	135.77	135.87	135.99	136.12	136.27	136.43
2	108.00	137.88	137.00	137.01	137.04	137.07	137.13	137.20	137.28	137.38	137.50	137.62	137.77
2	114.00	139.01	138.26	138.26	138.28	138.32	138.36	138.42	138.50	138.58	138.68	138.79	138.91
2	120.00	139.98	139.34	139.34	139.36	139.39	139.43	139.48	139.54	139.62	139.70	139.80	139.90
3	3.00	139.98	139.76	139.76	139.77	139.78	139.80	139.83	139.86	139.88	139.91	139.94	139.97
3	6.00	139.98	139.92	139.92	139.92	139.93	139.93	139.94	139.95	139.96	139.96	139.97	139.98
3	9.00	139.98	139.96	139.97	139.97	139.97	139.97	139.97	139.97	139.98	139.98	139.98	139.98
3	12.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	15.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	18.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	21.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	24.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	27.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	30.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	33.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	36.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	39.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	42.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	45.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	48.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	51.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	54.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	57.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98
3	60.00	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98	139.98

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.1000000+01	.9293840-13	.1000000+01	-.3981700-07
2	.5104340-01	.2975080+01	.6632650-01	.271317D+01
3	.1401830-03	.8872560+01	.1433310-03	.885035D+01

NO RECOVERY EXCHANGER, 60% RECOVERY VALUES FOR U, 156 C STEAM, L = 13

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2	6.00	43.33	32.62	32.71	32.97	33.41	34.04	34.85	35.85	37.05	38.45	40.05	41.86
2	12.00	59.10	48.73	48.82	49.10	49.56	50.20	51.02	52.02	53.19	54.54	56.06	57.74
2	18.00	72.67	63.65	63.73	63.97	64.37	64.93	65.65	66.52	67.54	68.72	70.04	71.50
2	24.00	84.35	76.58	76.65	76.86	77.20	77.68	78.30	79.05	79.94	80.95	82.08	83.34
2	30.00	94.41	87.72	87.78	87.95	88.25	88.67	89.20	89.85	90.61	91.48	92.45	93.54
2	36.00	103.06	97.30	97.35	97.51	97.76	98.12	98.58	99.13	99.79	100.54	101.38	102.31
2	42.00	110.51	105.55	105.60	105.73	105.95	106.26	106.65	107.13	107.69	108.34	109.06	109.86
2	48.00	116.92	112.65	112.69	112.80	112.99	113.26	113.60	114.01	114.49	115.05	115.67	116.36
2	54.00	122.43	118.76	118.79	118.89	119.06	119.28	119.58	119.93	120.35	120.82	121.36	121.96
2	60.00	127.18	124.02	124.05	124.13	124.27	124.47	124.72	125.03	125.39	125.80	126.26	126.77
2	66.00	131.27	128.55	128.57	128.64	128.77	128.93	129.15	129.41	129.72	130.08	130.47	130.91
2	72.00	134.78	132.44	132.46	132.53	132.63	132.78	132.96	133.19	133.45	133.76	134.10	134.48
2	78.00	137.81	135.80	135.81	135.87	135.96	136.08	136.24	136.44	136.67	136.93	137.22	137.55
2	84.00	140.41	138.68	138.70	138.74	138.82	138.93	139.06	139.23	139.43	139.65	139.91	140.19
2	90.00	142.66	141.16	141.18	141.22	141.28	141.38	141.49	141.64	141.81	142.00	142.22	142.46
2	96.00	144.59	143.30	143.31	143.35	143.40	143.48	143.59	143.71	143.86	144.02	144.21	144.42
2	102.00	146.25	145.14	145.15	145.18	145.23	145.30	145.39	145.49	145.62	145.76	145.92	146.10
2	108.00	147.67	146.72	146.73	146.76	146.80	146.86	146.93	147.03	147.13	147.26	147.40	147.55
2	114.00	148.90	148.09	148.09	148.12	148.15	148.20	148.27	148.35	148.44	148.55	148.67	148.80
2	120.00	149.96	149.26	149.26	149.28	149.31	149.36	149.41	149.48	149.56	149.65	149.76	149.87
3	3.00	149.96	149.72	149.72	149.73	149.75	149.77	149.79	149.82	149.85	149.88	149.91	149.94
3	6.00	149.96	149.89	149.89	149.89	149.90	149.90	149.91	149.92	149.93	149.94	149.95	149.96
3	9.00	149.96	149.94	149.94	149.94	149.94	149.95	149.95	149.95	149.95	149.96	149.96	149.96
3	12.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	15.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	18.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	21.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	24.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	27.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	30.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	33.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	36.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	39.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	42.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	45.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	48.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	51.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	54.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	57.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96
3	60.00	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96	149.96

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.1000000+01	.9293840-13	.1000000+01	-.3981700-07
2	.3947140-09	.2165290+02	.4465350-08	.1922690+02
3	.1662200-28	.6626680+02	.1983800-28	.6609000+02

FOR NO RECOVERY EXCHANGER, 70% RECOVERY VALUES FOR U, 160 C STEAM, 67 S RESIDENCE TIME

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2	3.35	41.52	28.70	28.78	29.09	29.62	30.38	31.41	32.72	34.34	36.30	38.61	41.31
2	6.70	56.02	41.33	41.46	41.89	42.62	43.63	44.96	46.54	48.42	50.60	53.06	55.81
2	10.05	68.75	55.13	55.26	55.67	56.36	57.33	58.56	60.05	61.81	63.82	66.07	68.55
2	13.40	79.92	67.82	67.93	68.30	68.92	69.78	70.88	72.21	73.77	75.55	77.55	79.74
2	16.75	89.72	79.07	79.17	79.49	80.04	80.80	81.76	82.94	84.31	85.88	87.64	89.57
2	20.10	98.32	88.97	89.05	89.34	89.82	90.49	91.34	92.37	93.57	94.95	96.49	98.19
2	23.45	105.87	97.66	97.74	97.99	98.41	98.99	99.74	100.64	101.70	102.91	104.26	105.75
2	26.80	112.50	105.29	105.36	105.58	105.95	106.46	107.11	107.91	108.84	109.90	111.09	112.39
2	30.15	118.31	111.99	112.04	112.24	112.56	113.01	113.59	114.28	115.10	116.03	117.07	118.22
2	33.50	123.41	117.86	117.91	118.08	118.37	118.76	119.27	119.88	120.60	121.41	122.33	123.33
2	36.85	127.89	123.02	123.06	123.21	123.46	123.81	124.25	124.79	125.42	126.14	126.94	127.82
2	40.20	131.82	127.55	127.59	127.72	127.94	128.24	128.63	129.10	129.65	130.28	130.98	131.76
2	43.55	135.27	131.52	131.55	131.67	131.86	132.13	132.47	132.88	133.37	133.92	134.54	135.22
2	46.90	138.30	135.00	135.03	135.14	135.30	135.54	135.84	136.20	136.63	137.11	137.65	138.25
2	50.25	140.95	138.06	138.09	138.18	138.33	138.53	138.80	139.11	139.49	139.91	140.39	140.91
2	53.60	143.28	140.75	140.77	140.85	140.98	141.16	141.39	141.67	142.00	142.37	142.79	143.25
2	56.95	145.33	143.10	143.13	143.19	143.31	143.47	143.67	143.91	144.20	144.53	144.89	145.30
2	60.30	147.13	145.17	145.19	145.25	145.35	145.49	145.67	145.88	146.13	146.42	146.76	147.10
2	63.65	148.70	146.99	147.00	147.06	147.14	147.27	147.42	147.61	147.83	148.08	148.37	148.68
2	67.00	150.08	148.58	148.59	148.64	148.72	148.82	148.96	149.13	149.32	149.54	149.79	150.06
3	3.00	150.08	149.64	149.65	149.67	149.70	149.75	149.80	149.86	149.92	149.98	150.03	150.08
3	6.00	150.08	149.97	149.98	149.98	149.99	150.00	150.01	150.03	150.04	150.06	150.07	150.08
3	9.00	150.08	150.06	150.06	150.06	150.06	150.06	150.07	150.07	150.07	150.08	150.08	150.08
3	12.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	15.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	18.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	21.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	24.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	27.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	30.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	33.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	36.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	39.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	42.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	45.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	48.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	51.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	54.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	57.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08
3	60.00	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08	150.08

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.1000000+01	.9293840-13	.1000000+01	-.3981700-07
2	.2933520-04	.1043670+02	.1884310-03	.8576780+01
3	.3357730-29	.6786630+02	.4505070-29	.6757230+02

NO RECOVERY EXCHANGER, 70% RECOVERY VALUES FOR U, 180 C STEAM, 42 S RESIDENCE TIME

XCHANGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2	2.10	37.18	25.99	26.03	26.21	26.55	27.06	27.80	28.83	30.19	31.96	34.20	36.95
2	4.20	48.40	32.42	32.54	32.96	33.68	34.71	36.05	37.73	39.77	42.17	44.97	48.15
2	6.30	58.74	41.85	41.99	42.49	43.31	44.47	45.97	47.80	49.97	52.48	55.32	58.49
2	8.40	68.27	51.89	52.03	52.53	53.35	54.50	55.98	57.77	59.88	62.30	65.02	68.03
2	10.50	77.05	61.65	61.79	62.26	63.04	64.13	65.53	67.22	69.21	71.48	74.02	76.83
2	12.60	85.14	70.83	70.96	71.40	72.13	73.15	74.45	76.02	77.87	79.98	82.34	84.94
2	14.70	92.60	79.37	79.49	79.90	80.57	81.51	82.71	84.17	85.88	87.83	90.01	92.41
2	16.80	99.47	87.26	87.37	87.75	88.37	89.24	90.35	91.69	93.27	95.07	97.08	99.29
2	18.90	105.79	94.54	94.64	94.99	95.57	96.37	97.39	98.63	100.08	101.74	103.59	105.63
2	21.00	111.63	101.26	101.35	101.67	102.20	102.94	103.88	105.02	106.36	107.89	109.60	111.48
2	23.10	117.00	107.44	107.53	107.82	108.31	108.99	109.86	110.91	112.15	113.55	115.13	116.86
2	25.20	121.95	113.14	113.22	113.49	113.94	114.57	115.37	116.34	117.48	118.78	120.23	121.82
2	27.30	126.51	118.40	118.47	118.72	119.14	119.71	120.45	121.35	122.39	123.59	124.92	126.39
2	29.40	130.71	123.24	123.31	123.54	123.92	124.45	125.13	125.96	126.92	128.02	129.25	130.61
2	31.50	134.59	127.70	127.76	127.97	128.33	128.82	129.44	130.20	131.09	132.10	133.24	134.49
2	33.60	138.16	131.81	131.87	132.06	132.39	132.84	133.42	134.12	134.93	135.87	136.91	138.06
2	35.70	141.44	135.60	135.65	135.83	136.13	136.54	137.08	137.72	138.48	139.34	140.30	141.36
2	37.80	144.47	139.08	139.13	139.30	139.58	139.96	140.45	141.04	141.74	142.53	143.42	144.40
2	39.90	147.27	142.30	142.35	142.50	142.75	143.11	143.56	144.10	144.75	145.48	146.29	147.19
2	42.00	149.84	145.26	145.30	145.45	145.68	146.00	146.42	146.93	147.52	148.19	148.94	149.77
3	3.00	149.84	148.49	148.51	148.58	148.68	148.81	148.97	149.15	149.33	149.51	149.68	149.83
3	6.00	149.84	149.50	149.51	149.53	149.55	149.58	149.62	149.67	149.71	149.76	149.80	149.84
3	9.00	149.84	149.76	149.76	149.76	149.77	149.78	149.79	149.80	149.81	149.82	149.83	149.84
3	12.00	149.84	149.82	149.82	149.82	149.82	149.82	149.82	149.83	149.83	149.83	149.84	149.84
3	15.00	149.84	149.83	149.83	149.83	149.83	149.83	149.83	149.84	149.84	149.84	149.84	149.84
3	18.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	21.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	24.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	27.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	30.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	33.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	36.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	39.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	42.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	45.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	48.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	51.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	54.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	57.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84
3	60.00	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84	149.84

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.1000000+01	.9293840-13	.1000000+01	-.3981700-07
2	.2003800-01	.3910130+01	.7937230-01	.2533610+01
3	.8325850-28	.6465560+02	.2037440-27	.6376070+02

FOR NO RECOVERY EXCHANGER, 70% RECOVERY VALUES FOR U, 200 C STEAM, 32 S RESIDENCE TIME

EXCHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2	1.60	35.58	25.39	25.39	25.51	25.73	26.09	26.65	27.48	28.68	30.32	32.52	35.34
2	3.20	45.52	29.23	29.33	29.72	30.38	31.33	32.62	34.27	36.32	38.81	41.78	45.24
2	4.80	54.86	36.24	36.38	36.90	37.77	39.00	40.59	42.58	44.96	47.75	50.95	54.57
2	6.40	63.63	44.56	44.72	45.28	46.21	47.53	49.22	51.29	53.74	56.57	59.77	63.35
2	8.00	71.87	53.20	53.37	53.93	54.87	56.17	57.85	59.89	62.30	65.06	68.16	71.60
2	9.60	79.62	61.72	61.88	62.43	63.33	64.60	66.21	68.18	70.49	73.13	76.09	79.36
2	11.20	86.90	69.91	70.07	70.59	71.45	72.65	74.19	76.06	78.25	80.76	83.56	86.65
2	12.80	93.73	77.70	77.84	78.33	79.15	80.29	81.75	83.51	85.58	87.95	90.59	93.50
2	14.40	100.16	85.05	85.19	85.65	86.43	87.50	88.87	90.54	92.49	94.71	97.20	99.94
2	16.00	106.19	91.98	92.11	92.55	93.28	94.29	95.58	97.15	98.98	101.07	103.41	105.99
2	17.60	111.86	98.50	98.63	99.04	99.72	100.67	101.89	103.36	105.08	107.05	109.25	111.67
2	19.20	117.19	104.64	104.75	105.14	105.78	106.67	107.81	109.20	110.82	112.67	114.74	117.01
2	20.80	122.20	110.40	110.51	110.87	111.47	112.31	113.39	114.69	116.21	117.95	119.89	122.03
2	22.40	126.90	115.81	115.92	116.26	116.82	117.61	118.62	119.84	121.27	122.91	124.73	126.74
2	24.00	131.32	120.90	121.00	121.32	121.85	122.59	123.54	124.69	126.03	127.57	129.28	131.17
2	25.60	135.47	125.68	125.77	126.07	126.58	127.27	128.16	129.24	130.50	131.95	133.56	135.33
2	27.20	139.37	130.18	130.26	130.54	131.01	131.67	132.50	133.52	134.71	136.06	137.57	139.24
2	28.80	143.04	134.40	134.48	134.74	135.18	135.80	136.59	137.54	138.65	139.92	141.35	142.91
2	30.40	146.48	138.36	138.44	138.69	139.10	139.68	140.42	141.31	142.36	143.56	144.89	146.37
2	32.00	149.72	142.09	142.16	142.39	142.78	143.33	144.02	144.86	145.85	146.97	148.22	149.61
3	3.00	149.72	147.47	147.51	147.61	147.78	148.01	148.27	148.57	148.87	149.17	149.45	149.70
3	6.00	149.72	149.16	149.17	149.20	149.24	149.29	149.36	149.43	149.51	149.58	149.65	149.71
3	9.00	149.72	149.58	149.58	149.59	149.60	149.61	149.63	149.65	149.67	149.68	149.70	149.72
3	12.00	149.72	149.68	149.68	149.69	149.69	149.69	149.70	149.70	149.70	149.71	149.71	149.72
3	15.00	149.72	149.71	149.71	149.71	149.71	149.71	149.71	149.71	149.71	149.71	149.72	149.72
3	18.00	149.72	149.71	149.71	149.71	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	21.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	24.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	27.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	30.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	33.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	36.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	39.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	42.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	45.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	48.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	51.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	54.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	57.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72
3	60.00	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72	149.72

	LIQUID N/No	LIQUID -ln(N/No)	SOLID SPHERE N/No	SOLID SPHERE -ln(N/No)
1	.1000000+01	.9293840-13	.1000000+01	-.3981700-07
2	.9178600-01	.2388300+01	.2749680+00	.1291100+01
3	.3819230-27	.6313230+02	.1640400-26	.6167490+02

NO RECOVERY EXCHANGER, 70% RECOVERY VALUES FOR U, 300 C STEAM, 15.4 S RESIDENCE TIME

CHANGER	TIME	LIQ TEMP	SOLIDS TEMPERATURE E = 0	1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2	0.77	33.13	25.05	25.01	25.02	25.05	25.12	25.28	25.63	26.32	27.57	29.65	32.84
2	1.54	41.02	25.52	25.52	25.69	26.00	26.52	27.35	28.59	30.39	32.90	36.28	40.65
2	2.31	48.68	27.46	27.54	27.93	28.62	29.67	31.15	33.15	35.76	39.10	43.24	48.26
2	3.08	56.11	30.99	31.13	31.71	32.71	34.16	36.12	38.65	41.81	45.66	50.27	55.67
2	3.85	63.32	35.72	35.92	36.63	37.83	39.56	41.84	44.71	48.22	52.39	57.27	62.86
2	4.62	70.31	41.28	41.50	42.30	43.64	45.54	48.02	51.10	54.81	59.16	64.18	69.86
2	5.39	77.10	47.35	47.59	48.44	49.86	51.86	54.45	57.65	61.46	65.90	70.96	76.65
2	6.16	83.69	53.71	53.97	54.84	56.30	58.36	61.00	64.25	68.10	72.55	77.60	83.24
2	6.93	90.09	60.22	60.48	61.37	62.84	64.91	67.57	70.83	74.67	79.09	84.09	89.65
2	7.70	96.29	66.77	67.03	67.92	69.39	71.45	74.10	77.33	81.13	85.49	90.41	95.86
2	8.47	102.31	73.29	73.55	74.43	75.89	77.93	80.54	83.72	87.46	91.74	96.56	101.89
2	9.24	108.16	79.73	79.99	80.85	82.29	84.29	86.86	89.98	93.64	97.84	102.54	107.75
2	10.01	113.83	86.06	86.31	87.16	88.56	90.53	93.04	96.09	99.67	103.77	108.36	113.43
2	10.78	119.33	92.26	92.50	93.33	94.71	96.62	99.08	102.06	105.55	109.54	114.01	118.94
2	11.55	124.68	98.31	98.55	99.36	100.70	102.57	104.96	107.86	111.26	115.15	119.50	124.30
2	12.32	129.86	104.21	104.44	105.23	106.54	108.36	110.69	113.51	116.82	120.60	124.83	129.49
2	13.09	134.89	109.95	110.18	110.95	112.22	113.99	116.25	119.00	122.22	125.89	130.00	134.53
2	13.86	139.77	115.54	115.76	116.51	117.74	119.47	121.67	124.34	127.46	131.03	135.02	139.42
2	14.63	144.51	120.97	121.19	121.91	123.11	124.79	126.93	129.52	132.56	136.02	139.90	144.17
2	15.40	149.10	126.25	126.46	127.16	128.33	129.96	132.03	134.55	137.50	140.86	144.63	148.78
3	3.00	149.10	142.38	142.48	142.80	143.31	143.98	144.77	145.65	146.57	147.47	148.31	149.05
3	6.00	149.10	147.44	147.46	147.54	147.67	147.84	148.03	148.25	148.48	148.70	148.91	149.09
3	9.00	149.10	148.69	148.70	148.72	148.75	148.79	148.84	148.89	148.95	149.00	149.05	149.10
3	12.00	149.10	149.00	149.00	149.01	149.02	149.03	149.04	149.05	149.07	149.08	149.09	149.10
3	15.00	149.10	149.08	149.08	149.08	149.08	149.08	149.09	149.09	149.09	149.10	149.10	149.10
3	18.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	21.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	24.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	27.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	30.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	33.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	36.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	39.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	42.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	45.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	48.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	51.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	54.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	57.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10
3	60.00	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10	149.10

LIQUID N/No	LIQUID -ln(N/No)	SOLID SPHERE N/No	SOLID SPHERE -ln(N/No)
.1000000+01	.9293840-13	.1000000+01	-.3981700-07
.4812930+00	.7312790+00	.7875120+00	.2388770+00
.5109330-24	.5593360+02	.1091500-22	.5287190+02

RECOVERY EXCHANGER, 200 C STEAM, 35 S STEAM RESIDENCE TIME, 15 S HOLDING TIME

CHANGER	TIME	LIQ TEMP	SOLIDS E = 0	TEMPERATURE 1	2	3	4	5	6	7	8	9	10
0	0.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	6.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	12.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	18.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	24.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	30.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	36.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	42.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	48.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	54.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	60.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	66.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	72.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	78.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	84.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	90.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	96.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	102.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	108.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	114.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
1	120.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
2	1.74	36.49	25.55	25.56	25.71	25.97	26.40	27.04	27.97	29.27	31.03	33.33	36.25
2	3.48	47.22	30.32	30.43	30.84	31.55	32.58	33.94	35.68	37.82	40.39	43.43	46.94
2	5.23	57.26	38.39	38.54	39.07	39.97	41.23	42.87	44.89	47.31	50.13	53.35	56.97
2	6.97	66.63	47.63	47.80	48.36	49.30	50.62	52.31	54.38	56.82	59.64	62.81	66.35
2	8.71	75.38	57.03	57.19	57.75	58.67	59.96	61.62	63.63	66.00	68.70	71.75	75.12
2	10.45	83.57	66.15	66.30	66.84	67.72	68.95	70.53	72.45	74.69	77.26	80.14	83.31
2	12.20	91.21	74.82	74.96	75.47	76.30	77.47	78.95	80.76	82.87	85.29	87.99	90.97
2	13.94	98.35	82.98	83.12	83.59	84.38	85.47	86.87	88.56	90.54	92.81	95.34	98.13
2	15.68	105.03	90.64	90.77	91.21	91.95	92.97	94.28	95.87	97.72	99.84	102.21	104.82
2	17.42	111.26	97.81	97.93	98.35	99.03	99.99	101.21	102.70	104.43	106.41	108.63	111.07
2	19.17	117.09	104.52	104.63	105.02	105.66	106.55	107.70	109.08	110.71	112.56	114.63	116.91
2	20.91	122.53	110.78	110.89	111.25	111.85	112.69	113.76	115.05	116.57	118.30	120.23	122.36
2	22.65	127.62	116.64	116.74	117.08	117.64	118.42	119.42	120.63	122.05	123.66	125.47	127.46
2	24.39	132.37	122.11	122.20	122.52	123.04	123.78	124.71	125.84	127.16	128.67	130.36	132.22
2	26.14	136.81	127.23	127.31	127.61	128.10	128.78	129.65	130.71	131.95	133.36	134.93	136.67
2	27.88	140.96	132.00	132.08	132.36	132.82	133.46	134.27	135.26	136.41	137.73	139.21	140.83
2	29.62	144.84	136.47	136.54	136.80	137.23	137.82	138.59	139.51	140.59	141.82	143.20	144.72
2	31.36	148.46	140.64	140.71	140.95	141.35	141.91	142.62	143.48	144.49	145.64	146.93	148.34
2	33.11	151.84	144.54	144.60	144.83	145.20	145.72	146.39	147.19	148.13	149.21	150.41	151.74
2	34.85	155.00	148.18	148.24	148.45	148.80	149.28	149.91	150.66	151.54	152.54	153.67	154.90
3	0.75	155.00	149.68	149.75	149.95	150.28	150.74	151.32	152.00	152.74	153.52	154.27	154.95
3	1.50	155.00	151.06	151.12	151.29	151.57	151.95	152.40	152.91	153.46	154.00	154.51	154.97
3	2.25	155.00	152.17	152.22	152.35	152.56	152.84	153.17	153.54	153.92	154.31	154.66	154.98
3	3.00	155.00	152.99	153.03	153.12	153.27	153.47	153.71	153.97	154.24	154.51	154.77	154.99
3	3.75	155.00	153.58	153.61	153.67	153.78	153.92	154.09	154.28	154.47	154.66	154.84	154.99
3	4.50	155.00	154.00	154.02	154.07	154.14	154.24	154.36	154.49	154.63	154.76	154.89	155.00
3	5.25	155.00	154.30	154.31	154.34	154.40	154.47	154.55	154.64	154.74	154.83	154.92	155.00
3	6.00	155.00	154.51	154.51	154.54	154.57	154.62	154.68	154.75	154.82	154.88	154.94	155.00
3	6.75	155.00	154.65	154.66	154.67	154.70	154.74	154.78	154.82	154.87	154.92	154.96	155.00
3	7.50	155.00	154.76	154.76	154.77	154.79	154.82	154.84	154.88	154.91	154.94	154.97	155.00
3	8.25	155.00	154.83	154.83	154.84	154.85	154.87	154.89	154.91	154.94	154.96	154.98	155.00
3	9.00	155.00	154.88	154.88	154.89	154.90	154.91	154.92	154.94	154.96	154.97	154.99	155.00
3	9.75	155.00	154.92	154.92	154.92	154.93	154.94	154.95	154.96	154.97	154.98	154.99	155.00
3	10.50	155.00	154.94	154.94	154.95	154.95	154.96	154.96	154.97	154.98	154.99	155.00	155.00
3	11.25	155.00	154.96	154.96	154.96	154.97	154.97	154.98	154.98	154.99	154.99	155.00	155.00
3	12.00	155.00	154.97	154.97	154.97	154.98	154.98	154.98	154.99	154.99	155.00	155.00	155.00
3	12.75	155.00	154.98	154.98	154.98	154.98	154.99	154.99	154.99	154.99	155.00	155.00	155.00
3	13.50	155.00	154.99	154.99	154.99	154.99	154.99	154.99	155.00	155.00	155.00	155.00	155.00
3	14.25	155.00	154.99	154.99	154.99	154.99	154.99	155.00	155.00	155.00	155.00	155.00	155.00
3	15.00	155.00	155.00	155.00	155.00	155.00	155.00	155.00	155.00	155.00	155.00	155.00	155.00

	LIQUID N/No	-ln(N/No)	SOLID SPHERE N/No	-ln(N/No)
1	.1000000+01	.9293840-13	.1000000+01	-.3981700-07
2	.5127040-03	.7575810+01	.2730940-01	.3600520+01
3	.6758790-19	.4414090+02	.4098830-17	.4003580+02

REFERENCES

1. Bailey, J.E. and Ollis, D.F., Biochemical Eng. Fundamentals, 2nd Ed., McGraw-Hill Book Co., New York, 1986, pp.586-595.

2. Deindoerfer, F.H. and Humphrey, A.E., "Analytical Method for Calculating Heat Sterilization Times", Applied Microbiology, 7:256, 1959.

3. Carslaw, H.S. and Jaeger, J.C., Conduction of Heat in Solids, Second Ed., Claredon Press, Oxford, 1959.

4. Crank, J., The Mathematics of Diffusion, 2nd Ed., Oxford Science Publications, 1983, pp.89-105.

5. Geankoplis, C.J., Transport Processes and Unit Operations, Allyn and Bacon, Inc., Boston, 1978.

6. Farlow, S.J., Partial Differential Equations, John Wiley & Sons, New York, 1979.

7. Ashley, M.H.J. and Mooyman, J.G., "Continuous Sterilisation of Media", Presented to ACS (Meeting), New York, 1981.

8. Bird, R.B., Stewart, W.E., and Lightfoot, E.N., Transport Phenomena, John Wiley & Sons, New York, 1960.

9. Armenante, P., Conversation, February, 1989.

10. Peters, M.S., and Timmerhaus, K.D., Plant Design and Economics for Chemical Engineers, Second Ed., McGraw-Hill Book Company, New York, 1968, pp.640-652.

11. Ohashi, H. et al., "Correlation of Liquid-Side Mass Transfer Coefficient for Single Particles and Fixed Beds", Journal of Chemical Engineering of Japan, Vol.14, No.6, 1981.

12. Kikuchi, K. et al., "Correlation of Mass Transfer Coefficient Between Particles and Liquid in Liquid Fluidized Beds", Journal of Chemical Engineering of Japan, Vol.16, No.5, 1983.

13. Ohashi, H. et al., "Mass Transfer Between Particles and Liquid in Solid-Liquid Two-Phase Upflow in the Low-Velocity Region Through Vertical Tubes", Journal of Chemical Engineering of Japan, Vol.14, No.6, 1981.

14. Levins, D.M. et al., "Particle-Liquid Hydrodynamics and Mass Transfer in a Stirred Vessel, Part II-Mass Transfer", Transcripts Institute of Chemical Engineers, Vol.50, 1972.