Competitive Bidding Strategy in the Construction Industry

Game Theoretic Approach

by

Hongbin Chen

Thesis submitted to the Faculty of the Graduate School of

the New Jersey Institute of Technology in partial fulfillment of

the requirements for the degree of

Master of Science in Civil Engineering

May, 1989

APPROVAL SHEET

Title of Thesis: Competitive Bidding Strategy in the Construction Industry --- Game Theoretic Approach

Name of Candidate: Hongbin Chen
Master of Science in Civil Engineering

Thesis and Abstract Approved by:

Charles H. Gould, P.E. — Date
Associate Professor
of Civil and Environmental Engineering
Department of Civil and Environmental Engineering

Members of Thesis Advisory Committee:

Walter Konon, P.E. — Date
Associate Professor
of Civil and Environmental Engineering
Associate Chairperson of the Department
Department of Civil and Environmental Engineering

Alfred Levinson, Ph.D., P.E. — Date
Assistant Professor of Economics
Department of Organizational and Social Science

VITA

Name: Hongbin Chen.

Permanent address:

Degree and date to be conferred: M.S.C.E., 1989.

Date of birth:

Place of birth:

Secondary education: Sampu Town High School(China), 1978.

Collegiate institutions attended	Dates	Degree	Date of Degree
New Jersey Institute of Technology	1987-1989	M.S.C.E.	1989
Tongji University(P. R. China)	1978-1982	B.Eng.	1982

Major: Construction Management

ABSTRACT

Title of Thesis: Competitive Bidding Strategy in the Construction Industry --- Game Theoretic Approach

Hongbin Chen, Master of Science in Civil Engineering, 1989

Thesis directed by:

Charles H. Gould, P.E.
Associate Professor
of Civil and Environmental Engineering
Department of Civil and Environmental Engineering

A game theoretic approach is applied to analyze competitive bidding in the construction industry because previous models do not consider the conflict of interest that exists among competitors. The game theoretic model improves corporate performance when compared to previous Bayesian analyses.

The game theoretic model is discussed in conjunction with construction contracting practice. Competitive bidding is formulated as a game theoretic model in which a contractor optimizes his bid price to maximize his utility or corporate performance. Using available historical data, order

statistics are employed to access the distribution of estimated costs among bidders for a project. The winner's curse problem related to biased estimated cost is also solved by means of order statistics. An empirical approach is proposed to define the degree of the winner's curse in a local market.

A basic model is derived using complex mathematics. This is followed by a simplified solution that enhances the understanding and application of game theory in the construction industry. The simplified model is in a linear form that makes it practical for use in a business environment.

The historical bidding data of two contractors engaged in the construction industry are used to evaluate the proposed simplified model. The results show that, even in its linear form, the model improves the contractors' performance significantly when compared to previous Bayesian analyses.

Future research directions in game theoretic modelling for competitive bidding are suggested.

Blank Page

To my beloved parents

Acknowledgments

I wish to express my deep appreciation to Prof. Charles H. Gould, under whose direction and advice this research was developed. Without his considerable efforts, this thesis could not have been carried through to completion.

I am grateful to the members of my Thesis Advisory Committee: Prof. Edward G. Dauenheimer, who has reviewed the manuscript and offered valuable comments; Prof. Alfred Levinson, who has encouraged my interest in economics theory and applications; and Prof. Walter Konon, who has motivated and helped me in the graduate program.

Thanks are also due to Ms. Juliet Zheng for her "computerized" assistance.

Last but not least I express my great appreciation to my dear parents, brothers and sisters, for their encouragements and supports

Any residual oversights and errors in the thesis are my sole responsibility.

Contents

List of Figures

List of Tables

Chapter I

INTRODUCTION

Competitive bidding has long been used as method for allocating and procuring contracts in the construction industry. Especially in the public sectors, it is a legal requirement to use competitive bidding to award most contracts. With a slowdown in the global economic growth, firms must take business away from competitors if they are to sustain their growth rate. Globalization of the market, and rapidly changing technology are producing new sources of competition. In order to be successful in the competitive economy some contractors have applied strategies in the competitive bidding situation.

One of the approaches is to use bidding models to predict the behavior of the competition. Modeling the competitive bidding was initiated by Friedman [7,8] in 1956. Since then many scholars and practitioners have contributed to this topic. The bidding models, however, have not been widely used in the construction industry.

A Significance of Research

The critiques and arguments about the previous competitive bidding models center on their applicability and calculability. Many studies indicate a number of different points of view such as winning probability assessment, cost distribution assessment, and bid value assessment. The current applications and literature in the construction industry show that Bayesian analysis is an approach employed most frequently in the competitive bidding modeling, and the game theoretic approach is rarely discussed.

The Bayesian approach presupposes the existence of a (possibly subjective) prior probability distribution, and of a gain (loss) function developed through utility considerations. Given this approach, one is forced, by the nature of utility theory, to use overall expected gains (loss) in evaluating decision rules. Bayesian decision theorist, accepting the foundation of subjective probability, do not generally make a distinction between decisions under risk and decisions under uncertainty.

The game theoretic approach was developed as a decision-making tool to be used in situations where chance and the user's choice are not the only factors operating. One additional significant factor considered in game theory is the conflict among the competitors. In other words, while decision makers are trying to manipulate their environment, their environment is trying to manipulate them. The competitive bidding process is a type of game in which the bidders make decisions which take into account not only the nature of the project (i.e. size, specification and site condition etc.), but the competitors' possible bid price as well.

The mathematical complexity of most existing bidding models precludes their application, even though the models themselves are theoretically sound. Any proposed model must not only be realistic enough that it will derive meaningful synthetic data, but it must also be computationally tractable. That is, the model must be both useful and usable. These two requirements, along with the problems involved in the integration of unique characteristics of construction industry, are the driving forces behind a new research direction.

To adequately model competitive bidding, it is necessary to rely more and more on recent advances in the operations research / economics. The game theory has been used to successfully model the competitors' behavior in other industries such as the manufacturing industry and the petroleum industry, because its structure can be set up to be compatible with the real market structure. The use of the game theoretic approach to model the competition in the construction industry is certainly feasible and worthy of being explored.

B Plan of Study

The study starts with a review of relevant literature in chapter II. The major models are selected from the ample literature which is discussed in detail. These models are Friedman's [8], Gate's [10], Broemser's [4], LOMARK [22], and Carr's [5]. The significant innovations of the other's models are also covered. The competitive bidding literature is reviewed separately; first in the field of operations research/economics, and then the construction industry.

Prior to a detailed discussion of the game theoretic approach application, the game theory is introduced as to it's concept, structure, constraints and solutions. The vital theory for a game theoretic bidding model is discussed more substantially in chapter III.

The bidding model is proposed based on the n-player game theoretic approach. The general bidding model is introduced at the first stage of the chapter IV. The order statistics technique is applied to assess the potential cost distribution among the bidders in a project. The winner's curse may be an important factor in the competitive bidding. Therefore, the winner's curse coefficient is included in the model with the purpose of reflecting a more realistic situation. Due to the mathematical complexity of the general model, a simplified bidding model is developed in a linear form.

The empirical testing of the proposed bidding model is proceeded by comparing the results of the proposed model and the actual outcome in the light of same data set. The data set is divided into two parts. The first part is considered as the historical data and the balance as the actual outcome.

The bidding game-theoretic model is the most respected simulation of the real competitive situation. It has not received enough attention in construction industry. Even in its simplified form it appears to produce better results than the Bayesian approach. Although simplified game theoretic model is proposed, the possible modifications and further research directions are suggested in chapter VI.

This paper is not intended to be a complete methodology for the game theory application in the competitive bidding, but it is believed that the more realistic assumption, which reflects the conflict of interests among bidders, is a significant start for the future exploration. Also, no matter how good a methodology is for a particular firm, there can be no substitute for people with experience, good judgement, and motivation. If a project team already has such qualifications, then this methodology should provide a perfect complement.

Chapter II

LITERATURE REVIEW

The subject of competitive bidding strategies crosses many and varied fields of study. In fact, the subject embraces engineering, economics, statistics, and operations research. The operations research is a systematic and scientific approach to solving complex business and organizational problems, and is certainly the most significant literature to be searched.

The construction industry is a unique industry in the general economy. The publicly funded sector is one of the major market in the construction industry. The public sector awards a substantial number of contracts through the mechanism of publicly advertised competitive bidding. Many contracting firms place their primary emphasis on competing in this market in order to sustain the stable growth in their business. The demand for the winning strategies in this environment has promoted the development of competitive bidding modelling.

Therefore, it is logical to sequentially review the relevant literature in operations research and the construction industry . The major existing models in the construction industry are discussed in order to present the state-of-art in this field. The review of operations research literature is relatively succinct. In 1980, Engelbrecht-Wiggans found that there were more than five hundred papers dealing with the auction and bidding models [6].

A Operations Research Literature

The first formal bidding model was developed by Friedman in 1956 [8]. In his article published in the "Operations Research" he proposed the probabilistic model to simulate the competitive bidding circumstance. He used the Bayesian analysis approach although the game theoretic approach was discussed in his subsequently published doctoral dissertation [7].

1 Friedman's Model

Let $P(b)$ be the probability that a bid of b will be lowest and will win the contract. Then the expected profit, $E(\pi)$, if a bid b is made, will be

$$E(\pi) = \int_0^\infty P(b)(b - sc)h(s)ds \tag{1}$$

Where c - estimated cost, s - ratio of true cost to estimated cost, $h(s)$ - distribution of s, π - profit gained from the contract.

Friedman argues that $P(b)$ is independent of s, and $\int_0^\infty h(s)ds = 1$, therefore equation 1 becomes

$$E(\pi) = P(b)(b - c')$$

where $c' = c\int_0^\infty sh(s)ds$, is called the estimated cost corrected.

In general, $E(\pi)$ will take on values similar to those shown in Figure 1.

Once the expected profit curve is determined, it is relatively straightforward procedure to determine the bid that maximizes profit.

The probability of winning is determined from historical data. If the identity of all the historical competitors and the identity of the competitors participating in next competition are known then all competitor's bidding pattern may be studied. The distributions of the ratios of known competitor's cost to contractor's cost are shown in Figure 2. The winning probability of the subject contractor is shown in the shaded area. The distribution of s (competitor's bid to firm's cost ratio) can

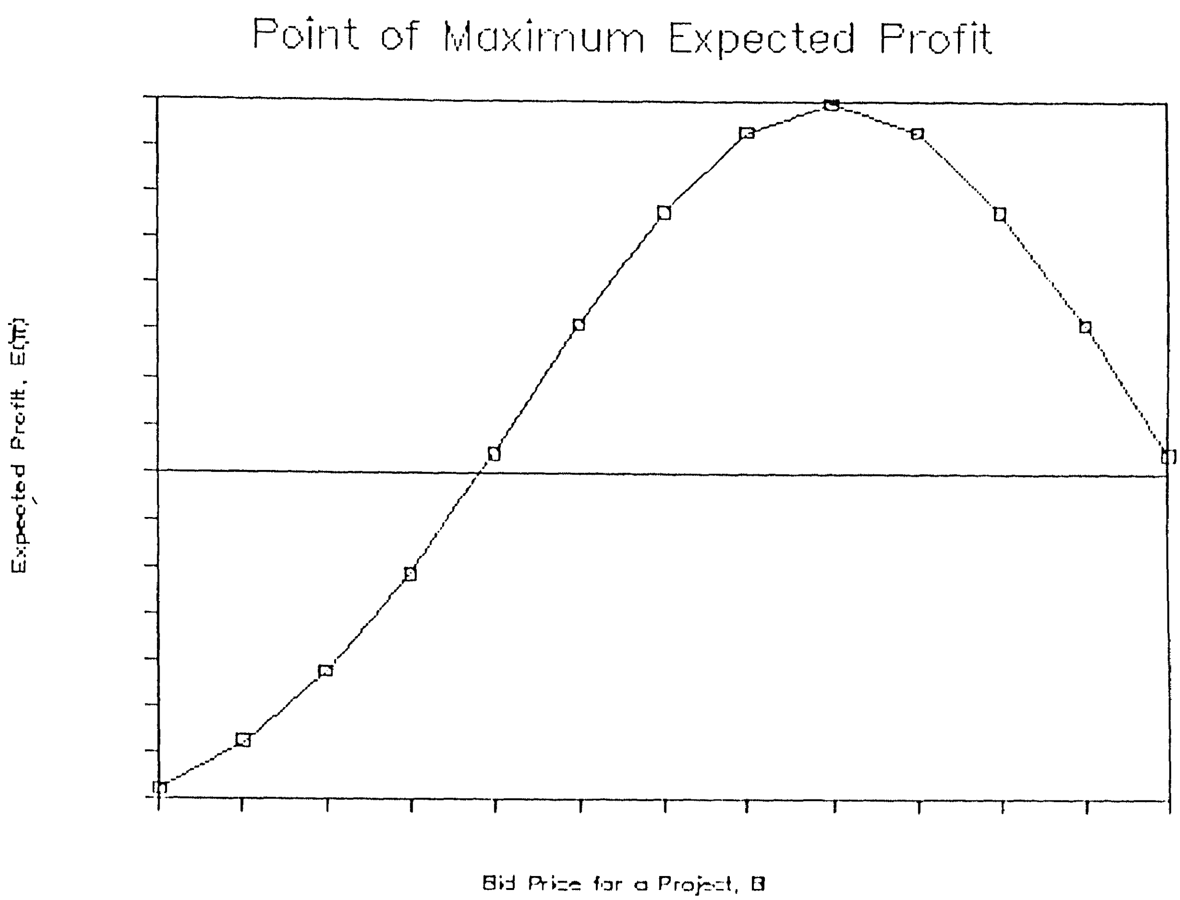


Figure 1: Expected Profit vs Bid Price

be predicted as

$$P(b) = \sum_{i=1}^{n} P(\frac{b}{c} < \frac{b_i}{c})$$

If it is not known exactly how many competitors will submit bids, the concept of *average bidder* is used. The bidding distribution of the *average bidder* is found by combining all previous ratios of an opposing bid to the firm's cost estimate to one distribution function

$$P(b) = \left(\int_{b/c}^{\infty} f(r)dr\right)^k$$

where k is the number of average bidders. $f(r)$ is the winning bid cost ratio distribution function against an individual *average bidder*. $P(b)$ is shown in the shaded area of Figure 3.

If one can determine the probability of k bidders submitting bids and if this probability is $p(k)$, the probability $P(b)$ of a bid b being the lowest bid becomes

$$P(b) = \sum_{k=0}^{\infty} p(k) \left(\int_{b/c}^{\infty} f(r)dr\right)^k$$

$f(r)$ can be found by fitting a curve to the data available. A gamma distribution will frequently be a good fit to data of this type.

$$f(r) = \frac{\alpha^{\beta+1}}{\beta!} r^{\beta} e^{-\alpha r}$$

where α and β are constants obtained from curve fitting the frequency data of the gamma distribution.

It is also reasonable to assume that the number of bidders might have a Poison distribution. That is, if λ is the estimated number of bidders then $p(k) = \frac{\lambda^k e^{-\lambda}}{k!}$.

Based on the above assumptions, Friedman found the winning probability against *average bidders*

$$\begin{aligned} P(b) &= e^{-\lambda} \sum_{k=0}^{\infty} \frac{1}{k!} [\lambda \int_{b/c}^{\infty} \frac{\alpha^{\beta+1}}{\beta!} r^{\beta} e^{-\alpha r} dr)]^k \\ &= e^{-\lambda} [1 - \sum_{i=0}^{b} \frac{1}{i!} (\frac{\alpha b}{c})^i e^{-\alpha b/c}] \end{aligned}$$

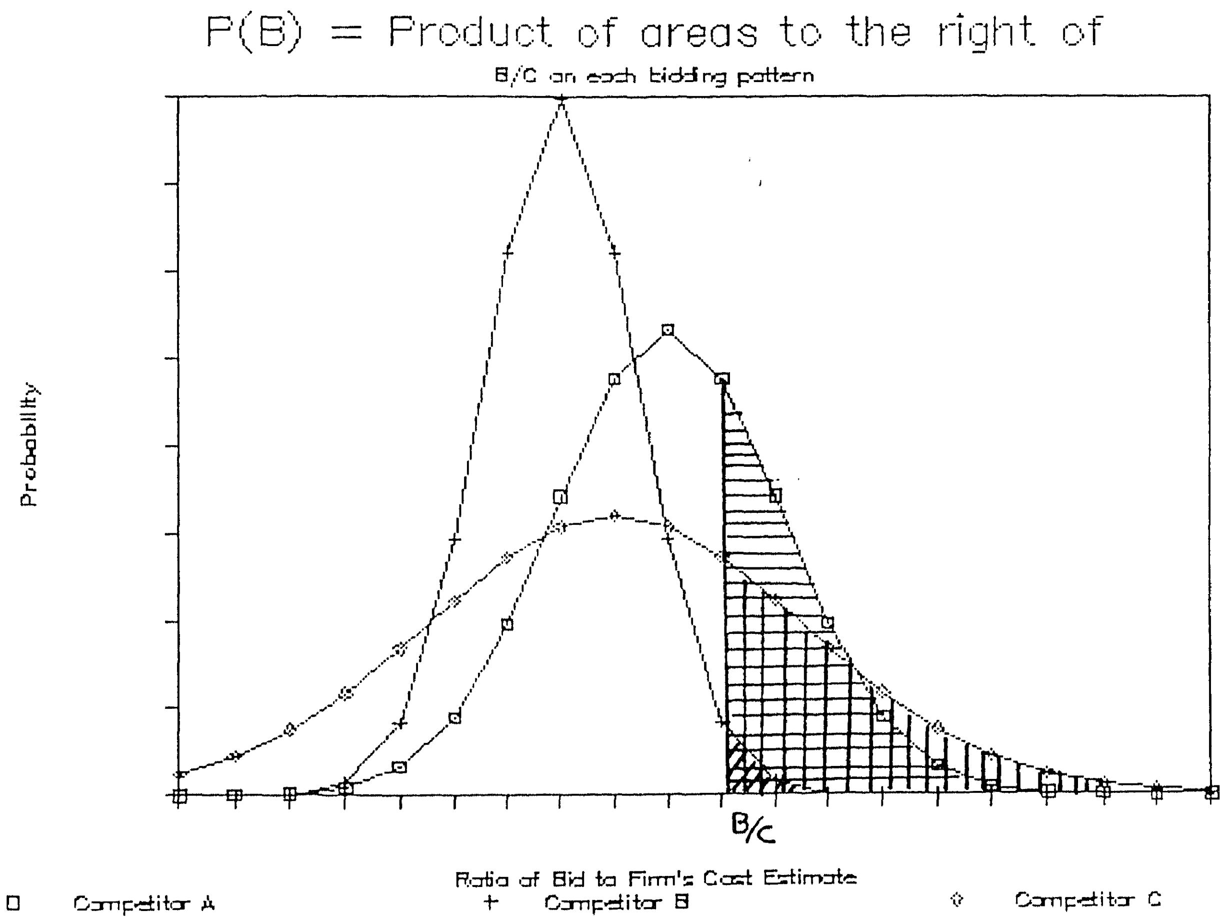


Figure 2: Winning Probability When Competitors' Identity Known

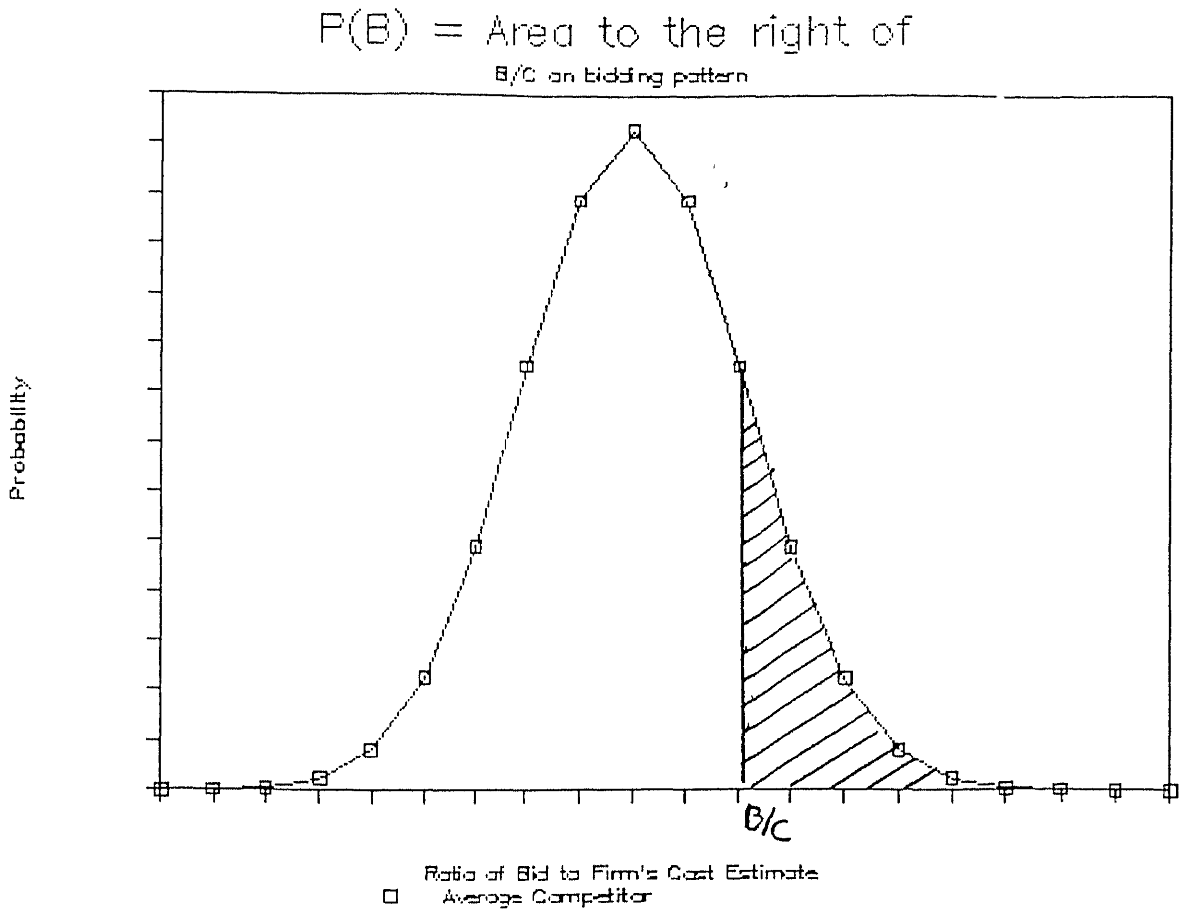


Figure 3: Winning Probability When Competitors' Identity Unknown

2 Broemser's Model

Broemser [4] generalized the value of a bid for a project, given the bid and the special state of information, S, represented by its total a priori knowledge, as

$$E(M_x/B_xS) = (B_x - C_x)P(B_x < B_w/S)$$

where M_x - the gain of the x contractor from a project, B_x - the bid of the contractor, C_x - the cost of the project, B_w - the winning bid, and S - state of information.

The relationship of actual cost to estimated cost is not taken into consideration, so that he disregards bias and errors in the estimated cost. Normalizing the foregoing equation by dividing by his contractor's estimated cost, C_x, he has

$$E(m_x/b_xS) = (b_x - 1)P(b_x, b_w/S)$$

By setting the derivative of the expected value, given his bidder's bid and its total a priori knowledge, with the respect to b_x equal to zero, and after manipulating the equation, he arrives at the optimum condition, as follows:

$$\begin{aligned}\frac{\partial(b_x - 1)P(b_x < b_w/S)}{\partial b_x} &= (b_x - 1)\frac{\partial P(b_x < b_w/S)}{\partial b_x} + P(b_x < b_w/S) \\ &= 0 \qquad (2)\end{aligned}$$

Because $P(b_x < b_w/S)$ is the right tail of the complementary cumulative probability distribution of $P(b_w/S)$, then

$$\frac{\partial P(b_x < b_w/S)}{\partial b_x} = -P(b_w/S)$$

By substituting this in equation 2

$$-(b_x - 1)P(b_w/S) + P(b_x < b_w/S) = 0$$

therefore

$$\frac{P(b_w/S)}{P(b_x < b_w/S)} = \frac{1}{b_x - 1}$$

Broemser's model has significant contribution to the subject of winning probability assessment, as mentioned by Alpert [1]. Broemser determined the probability distribution of the lowest competitive bid by setting up a linear regression model for 76 projects bid by his contractor over a period of about one year. He argues that the percentage markup on the cost of a competitor is determined by several readily ascertainable conditions with respect to a particular job. Each of these conditions or variables, after weighting would affect a competitor's markup percentage applied to his estimated cost of a project. Broemser assumed that each competitor assigned the same weight to the same characteristics for any project he proposed to bid. Therefore, each competitor's percentage markup on cost is the sum of the products of each condition or variable and the related weight. Consequently the lowest bid on any job is

$$B_w = C_w(1 + \sum_{i=1}^{k} g_{w_i} x_{w_i})$$

where g_{w_i} is i^{th} weight used by the lowest competitor for its i^{th} variables, x_{w_i}, $i = 1, 2, ..., k$.

By allowing g_{w_0} and x_{w_0} each to be equal 1, the equation becomes

$$B_w = C_w \sum_{i=0}^{k} g_{w_i} x_{w_i}$$

If the ratio of the lowest competitor's cost to the subject bidder's cost is c_w (a random variable), then $C_w = c_w C_x$. Substituting $c_w C_x$ for C_w in preceding equation and dividing throughout by C_x, the result is

$$\frac{B_w}{C_x} = b_w = c_w \sum_{i=0}^{k} g_{w_i} x_{w_i} \qquad (3)$$

When β_{w_i} is set equal to $c_w g_{w_i}$, he has in regression form,

$$b_w = \sum_{i=0}^{k} \beta_{w_i} x_{w_i} + \varepsilon_i$$

In his model the dependant variable is the ratio of the bid of the lowest competitor to his contractor's estimated cost, and the parameters are considered to be the product of the applicable weight that the lowest competitor attaches to each of the independent variables and the ratio of the lowest competitor's estimated cost to subject contractor's estimated cost. The first term of the multiple regression,

$\beta_{w_0} x_{w_0}$, *or* $c_w g_{w_0} x_{w_0}$, is simply the ratio of the lowest bidder's estimated cost to the subject bidder's estimated cost, since g_{w_0} *and* x_{w_0} have each been set equal to 1. The other variables are factors such as percentage of estimated cost not subcontracted, estimated project duration, and estimated project duration divided by estimated cost.

These variables may be used one or more times by raising them to different powers to reflect their effects on the bid if it is determined that the effect is not linear.

Broemser next proceeded to apply his model to 76 projects. He found it convenient to try succeedingly larger values of b_x until $E(m_x/b_x)$ in his normalized general equation reaches a maximum. The optimum normalized bid indicates a markup of 4.64% on his contractor's cost estimate and a probability of winning of 0.317.

He then compasses this model with one from which all the independent variables except x_{w_0} are omitted and recommends the more complicated model because it would have result in a 18% increase in his contractor's profit comparable to the simple model. In this simple model, only costs are considered, i.e. $b_w = kc_w$, where k is a constant determined by the regression.

3 Game Theoretic Models

The essence of competition is interdependence and conflicts of interest among the interdependent firms. Game theory is the dominant conceptual paradigm employed in operations research for the modeling of competitive bidding since it deals with the methodical solution to conflicts.

Game theoretic models rely on a set of assumptions about the behavior of competing firms and information available to them. On the basis of these assumptions, the bids submitted by firms in competitive bidding and the bidding outcomes (profit, market share, etc.) are derived analytically based on a state of equilibrium among the bidders. Equilibrium is defined as a list of strategies, one for each firm, with the property that no firm would like to unilaterally change its strategy.

Noncooperative game theory seeks to predict the behavior of rational, intelligent firms competing independently [16]. Both "rationality" and "intelligence" have special meaning in this theory. Firms are rational if they make decisions by maximizing their subjective expected "utility". Firms are intelligent if they recognize that other firms are rational.

Among the vast amount of work dealing with the game theory application in competitive bidding, Ortega-Reichert's research is the most extensive and complete [18].

In the operations research field the game theoretic model was developed for an auction, that is a competition where the highest bid wins . The general auction model for a first-price sealed-bid auction is found as follows [14].

$$B(v_i) = v_i - \frac{\int_{v_l}^{v_i}[F(\xi)]^{n-1}d\xi}{[F(v_i)]^{n-1}} \qquad i = 1, 2, ..., n. \tag{4}$$

where

$B(.)$ – optimal bidding function
v_l – lowest possible valuation in an auction
v_i – valuation of i^{th} participant on the auction object
n – the number of participants
$F(.)$ –the cumulative distribution function of the other participants' valuation on the object from the i^{th} participant point of view.

Two assumptions were made for this model:

(1) The participants are rational and have the same objective function
(2) The participants' valuation are distributed identically and independently

The game theoretic models are usually in more complex forms than the Bayesian bidding model. Perhaps this is one reason why these models have had little use as tools to aid top management in the industry.

B Construction Industry Literature

William R. Park is credited as being the pioneer who introduced the formal competitive bidding model to the construction industry [19]. His book , "The Strategy of Contracting for Profit", published in 1966, is an extensive study of the competitive bidding process in the industry. The basic approach he proposes for solving the decision-making problem in competitive bidding circumstances are mostly adapted from Friedman's model.

Following Park's book, scholars and practitioners have contributed significantly to this subject. The significant and innovative models developed since 1966 have been the Gates's model [10], the LOMARK model [22] and Carr's model [5].

All the above mentioned models have as their basis the Bayesian theory and are based on the same assumption that the firms are maximizing the expected profit, which is the basic object function of Friedman's model.

$$E(\pi) = P(b)\pi$$

where $E(\pi)$ is expected value

$P(b)$ is the probability of winning a bid

π is the profit generated from the project if the contractor win the bid

The innovation of the different models is found in the means for assessing the winning probability.

1 Gates's Model

The significant aspect of Gates's model lies in the assessment of the winning probability [10].

In case where many bidders are involved, Gates presented a general relationship

$$P(b) = \frac{t}{T}$$

where t is any order number of the project which is listed ascending order based on the ratio of competitor's bid to contractor's bid, and T is the greatest order number.

If all the identities of the competitors are known to the contractor, and the historical winning probability distributions are available, then

$$P(b) = \frac{1}{\left(\sum_{i=1}^{n} \frac{1-P_i(b)}{P_i(b)}\right) + 1} \tag{5}$$

where $P_i(b)$ $(i = 1, 2, ..., n)$ is the winning probability of the contractor over the i^{th} competitor.

When the number of competitors can be predicted with confidence but their identities are unknown, the above formula can be simplified by assuming that the probability of winning over each competitor is the same, then

$$P(b) = \frac{1}{n\frac{1-P_{avg}(b)}{P_{avg}(b)} + 1} \tag{6}$$

in which $P_{avg}(b)$ is the probability of winning over the "average competitors" , and n is the total number of competitors

2 LOMARK Model

LOMARK model was proposed by Wade and Harris in 1976 [22]. The model represented a new approach , which is a simple and inexpensive method for a small to medium-sized contractor to assess his competition and relate his assessment to his future bidding strategies.

The essence of the model is that only major competitors in the local market are considered in the probability of winning assessment. The winning probability is the product of the probability that the contractor's empirical winning probability over the major competitors, and the probability that the anticipated competitors will submit bids.

$$P(BC_0 < LBC/X, Y, Z) = P(BC_0 < LBC)P(X, Y, Z) \tag{7}$$

where BC_0 - ratio of contractor's bid to it's estimated cost; LBC - lowest ratio of bid to contractor's estimated cost, among the competitors X, Y and Z; $P(X, Y, Z)$ -probability that X, Y and Z will submit the bids.

The $P(BC_0 < LBC)$ is determined by the historical data using an approach similar to Gates's, i.e.

$$P(BC_0 < LBC) = \frac{T - t + 1}{T}$$

where t is any order number of the project which is listed descending based on the magnitude of the ratio between competitor's bid and contractor's bid, and T is the greatest order number [11].

The probability that the major competitors will bid the future job is subject to the contractor's own ad-hoc judgement based on the available information.

3 Carr's Model

Carr generalized a competitive bidding model so that it would not be limited by the assumptions on which Friedman's and Gates's models depend [5]. It is applicable to the situation in which a contractor's cost and competitor's bid distribution can be estimated.

If contractor i has a standardized cost (i.e. the ratio of the estimated cost to the mean of a group of estimated costs for a project) on project k of C'_{ik}, the probability that $(B/C)_{ijk}$ (ie. $(B_j/C_i)_k$) will exceed a value b is given by

$$P\left[(\frac{B}{C})_{ijk} > b/C'_{ik}\right] = P(B'_{jk} > bC'_{ik}) = \int_{bC'_{ik}}^{\infty} f(B'_j)dx \qquad (8)$$

where

$B'_{jk} = \frac{B_{jk}}{C_k}$

$f(.)$ is the distribution function of bid cost ratio or standardized cost.

The probability that $(B/C)_{ijk}$ will exceed b when the value of C'_{ik} is not known is

$$\begin{aligned} P[(\frac{B}{C})_{ijk} > b] &= \int_b^{\infty} f(\frac{B_j}{C_i})dx \\ &= \int_{-\infty}^{\infty} f(C'_i)P(B'_{jk} > bc'_{ik})dx \\ &= \int_{-\infty}^{\infty} f(C'_i)\int_{bC'_{ik}}^{\infty} f(B'_j)dxdx \end{aligned}$$

If more than one competitor is involved, a contractor's bid must be lower than the lowest competing bid in order to win a project. The probability that the lowest $(B/C)_{ijk}$ of n_k competitor j in project k will be described by

$$P(LBC_{ik} > b) = \int_{-\infty}^{\infty} f(C_i')\{\prod_{j=1}^{n_k}[\int_{bC_i'}^{\infty} f(B_j')dx]\}dx \tag{9}$$

Against n_k competitors who can all be described by the same distribution, then

$$P(LBC_{ik} > b) = \int_{-\infty}^{\infty} f(C_i') \left[\int_{bC_i'}^{\infty} f(B_a')dx\right]^{n_k} dx \tag{10}$$

in which B_a is standardized bid of an average competitor.

C Review Summary

A review of the construction industry literature shows that the game theoretic bidding model has not been used. Some of the proposed Bayesian expected value models seem too complex to be applied in the daily practise. On the other hand, the simplifying assumptions required to make them usable in practise generates unsatisfactory results in some models.

In contrast to the construction industry literature, the game theory has played an important role in the modelling of competitive bidding. A model may serve one of two major purposes: either descriptive, for explaining and/or understanding; or prescriptive, by predicting and/or duplicating behavioral characteristics. The bidding models used in operations research literature usually fall in the category of descriptive, which is not useful in direct application.

More and more intensive competition in the construction industry forces firms to compete rationally by means of optimized strategies in order to keep their positions in the market. The demand for a better model becomes more and more serious. The "goodness" of a model depends on its approximation and calculability.

The more realistic the assumptions in a model, the more closer approximation the result. The Bayesian approach has a major weakness in that it ignores the matter of conflict among competitors. The seemingly realistic nature of competitive reactions and the solution concepts used in game theory has lead to the need of study in the application of game theoretic model in construction industry.

Applicability is an important factor in modelling. A differential function (equation 4) in the existing game solution of the bidding models prevents practitioners from utilizing the theoretically sound models. The conflicting aspects of approximation and applicability must be traded off.

Chapter III

GAME THEORY

More recently the social and behavioral sciences have been making great progress in developing mathematical description of human behavior, by replacing or supplementing the pure verbal descriptions with more precise mathematical ones. This effort goes on over a broad range of interests, but one area in particular is business decision-making. How do, or how should, people choose among alternative courses of action? There are many different mathematical formulations which deal with different aspects of this problem. One approach has led to a mathematical theory of fundamental importance – the game theory. Its particular concern is with certain activity in which several people participate, each having some power of choice that would affect the outcomes of activity, and each having somewhat conflicting desires for the outcomes of the activity. The decision-making problem in competitive bidding is a perfect application for game theory.

The game theory became a serious mathematical tool for examining certain aspects of human behavior in 1944 with the publication of "Theory of Games and Economic Behavior" by John von Neumann and Oskar Morgenstern [21]. The theory has made substantial contributions to the social and behavioral sciences by providing a conceptual framework for viewing decision-making in the presence of conflict of interest.

There is a simple example indicating the distinction between the game theory and the Bayesian theory. When trying to forecast the weather, one would make statistical analysis on the available historical weather data to obtain the distribution of the possible outcomes. It is certain that nature never acts against human intentionally. This is the Bayesian theory application in which no conflict is present. If, however, one were in competitive bidding situation, the participants have their own interests, which are very often conflicting . Each participant has to take into account the opponents' possible courses of action in their decision-making in order to be successful in the competition. Game theory considers the conflict and is the appropriate model of competitive bidding.

To fully understand game theory would require a thorough grounding in higher mathematics. A sense of what is involved, however, can be illustrated with simple mathematics. This understanding can throw a much more informed light on how to go about making the "right" decision with help from the game theoretic bidding model.

The basic concepts in the formation of a bidding model are: Players; Objects; Payoff function; and Strategies [6]. These elements are subsequently described. In the discussion the terms "known", "identical", and "symmetric" have special meanings. The values are " known" if there is no uncertainty about them. The values would be "identical" if they are all equal to the single outcome of the random variable and "symmetric" if they are equal to the outcomes of independent identically distributed random variables.

A Players

A player, or strategic bidder, is anyone whose bidding strategy is unspecified by the model. In a competitive bidding game, there may one or more contractors that are considered a player. The number of participants or players in each contract bidding may be known or unknown. If it is unknown, the number of prospective participants can be considered as a random number drawn from a population. Friedman suggested that the bidder number distribution fits a Poison distribution [8]. In the construction industry the number of the bidders is usually known prior to submitting a bid.

Each contractor has his own interest in a project. In other words, every player is assumed to have a cardinal utility function which is linear or nonlinear. The attitude of a contractor toward the risk associated with the performing a project, if he wins it, is the dominant factor in the assumption and formation of the utility function. The assumption that the object of all bidders is to maximize expected profit is the most popular due to its linear form of function.

B Objects

A contractor may face one, or more than one project, available for bidding at a point in time. If a model is developed for considering more than one project, it is called a multiple project model. On the other hand, the contractor may consider the available projects one at a time over a period of time. This requires sequential modelling in order to consider the interdependence of the sequential bidding activity. Multiple and sequential modelling usually generates complex models.

The true state of nature is the combination of the characteristics of a project. The characteristics of a project can be known or uncertain. The gross floor area of a building is known to all bidders, but the geological condition of the site is uncertain due to the fact that each bidder has different access to and perception of the available information. One bidder may perform his own subsurface investigation in order to obtain more accurate geological state but the other bidders may just make the judgements based on only the subsurface data provided by the owner.

A project cost is the true state of a project. The estimated cost can be considered as a random number drawn from a probability population which is the pool population consisting of a number of random variables such as materials price, geological condition and project duration etc. McCaffer made an extensive statistical analysis on hundreds of building and road contract bidding data and came up with the result that the distribution of bids very closely approximates the normal distribution [15].

C Payoff Function

The payoff function of a game determines who gets what on the basis of the strategies chosen by the players and the true state of nature. In competitive bidding the payoff function can be different among the bidders. Fundamental to any analysis of correct bidding strategy must be a clear understanding of the objectives of all competitors.

As a first approximation, it is generally assumed that firms attempt to maximize profits. The simple and popular payoff function is the profit gained which is the difference between the bid and the true cost, if the bid is the lowest one.

$$Gain = Bid - Cost$$

The profit maximization assumption has limitations. One limitation is that the making of profits requires time and energy, and if the owners of the firm are the managers as well, they may decide that it is preferable to sacrifice profits for leisure. Other limitation may be resources such as capital or trained personnel. In a case of this sort, it is more accurate to assume that the firm is maximizing a utility. Generally speaking, the utility payoff function requires more complex study and more information.

The market share is also a vital payoff to a firm in the business world. The market share can be applied in the sequential bidding model in which the firm is trying to maximize the total number of won projects in a period of time.

The another alternative payoff function is related to the spread of bid, which is the difference between the low bid and the second low bid [19]. Several reasons make it significant:

- the spread indicates, to some extent, the intensity of competition for a project
- the spread measures the amount of money left "on the table", and tells how much higher the low bidder could have been and still have won the project
- a wide spread is probably the indication of an estimating error on the low bidder's part, especially if the second and higher bids are grouped closely together.

The spread of a bid is considered a real loss to the winner of the competition. In this case the payoff function is the profit minus the spread of bid.

$$Gain = Profit - Spread\ of\ Bid$$

D Strategies

A pure strategy for a firm is a plan of action [16]. It specifies what the firm will do as a function of what the firm knows. A mixed strategy is a probability distribution on the firm's feasible set of pure strategies. In other words, by choosing a mixed strategy a firm is really choosing a randomization device and the strategy played will depend on the outcome of the randomization.

A mixed strategy appears not to be acceptable to industry since no firm would make decisions as the result of randomization.

A firm may be assumed to select their bidding strategy according to any one of a number of criteria. In *min-max* models, each firm chooses a strategy which maximize the minimum possible utility of the final outcome over all possible combinations of bidding strategies of the remaining opponent firm, However easy to calculate, such strategies appear to have little practical use [6].

Nash equilibrium is the central concept of the most multi-bidder models. Strategies are in equilibrium if each firm uses a strategy which, for the particular strategies used by the remaining firms, maximizes the utility of outcome. Equilibrium is a list of strategies (pure or mixed), one for each firm, with the property that no firm would like to unilaterally change its strategy. Nash equilibrium is the solution to the noncooperative game with the assumptions that all the firms are rational and intelligent [17]. "Noncooperative" means that all the firms act independently and there is no collusion among the firms presents.

Chapter IV

BIDDING MODEL—SINGLE BID CASE

In the contract bidding process the project cost is a variable for the different bidders because the individual bidder has different access to the information about the project condition and has a different cost function in term of the project scope. All bidders know their own costs, but not their opponents'.

The potential estimated costs for a prospective project can be considered as a random sample drawn from a cost population. Since bid rigging is illegal , collusion is unlikely to occur. Furthermore, the intensive competition in the contract market prevents the exchange of bidding information from competitors. Therefore, it is reasonable to assume the individual bidder's estimated cost is independent with respect to the other bidders.

The characteristics of a cost population are determined by several factors such as project and market. The project factors include the type, size, and scope features. The market factors are subject to the location, competitors and time period, etc. In general, similar projects in the same markets will have the same cost population.

Based upon the historical data, a firm can estimate the parameters of the potential cost population for an upcoming project. The *order statistics* would be the proper tool for the historical cost data analysis in a competitive bidding situation.

A Order Statistics

Order statistics is a branch of statistics, which is employed for the analysis of the property of the extreme of random variables [2,12,9,13]

When the values of a sequence $x_1, x_2, ..., x_n$ of random variables are arranged in an increasing order $x_{1;n} \leq x_{2;n} \leq ... \leq x_{n;n}$ of magnitude, then the r^{th} member $x_{r;n}$ of this new sequence is called the r^{th} order statistic of the $x_j, 1 \leq j \leq n$. The two terms $x_{1;n} = min(x_1, x_2, ..., x_n)$ and $x_{n;n} = max(x_1, x_2, ..., x_n)$ are called extremes.

Assume $x_1, x_2, ..., x_n$ are independent and identically distributed (i.e. random samples are from the same population) . The common distribution function is denoted by $F(x) = P(x_j < x)$.

$$F_{r;n}(x) = P(x_{r;n} < x) = \sum_{k=r}^{n} \binom{n}{k} [1 - F(x)]^{n-k} = r \binom{n}{r} \int_0^{F(x)} t^{r-1}(1-t)^{n-r}\, dt$$

In particular

$$F_{n;n}(x) = P(x_{n;n} < x) = F^n(x)$$

$$F_{1;n}(x) = P(x_{1;n} < x) = 1 - [1 - F(x)]^n$$

$$P(x_{1;n} > x) = 1 - F_{1;n}(x) = [1 - F(x)]^n \tag{11}$$

If $F(x)$ has a density function $F'(x) = f(x)$ then

$$F'_{r;n}(x) = r \binom{n}{r} F^{r-1}(x)[1 - F(x)]^{n-r} f(x)$$

$$F'_{n;n}(x) = nF^{n-1}(x)f(x)$$

$$F'_{1;n}(x) = n[1 - F(x)]^{n-1} f(x) \tag{12}$$

If the common distribution has a location parameter μ(or in engineering terms, project parameter), and a spread parameter σ(or in engineering terms, market parameter), the expected value of r^{th} order statistic, $E(x_{r;n}) = \mu + \sigma E(Z_{r;n})$ where

$Z_{r;n} = \frac{x_{r;n}-\mu}{\sigma}$ (standardized random value). The $E(Z_{r;n}) = n \begin{pmatrix} n-1 \\ r-1 \end{pmatrix} \int tF(t)^{r-1}[1-F(t)]^{n-r}dF(t)$. Values may be calculated by using a computer program or obtained from the order statistics tables in which appear some statistics handbooks [13] (see Appendix G).

In particular, the expected value of the minimum order statistic

$$x_{1;n} = \mu + \sigma Z_{1;n} \tag{13}$$

$$E(x_{1;n}) = \mu + \sigma E(Z_{1;n}) \tag{14}$$

B Cost Distribution

If a project has a real cost, μ, which is the value of a function of the project factors and market factors, then the $i^{th}(i = 1, 2, ..., n)$ bidder's estimated cost is a random number c_i $(i = 1, 2, ..., n)$ drawn from a cost population with location(project) parameter μ and spread parameter σ. The spread(market) parameter σ indicates the variance of the bidders' access to information about the project and the dispersion of the bidders' cost function, etc. Therefore, σ is a market-specific and project-specific factor. Similar projects in the same type of market would have the same spread parameter σ.

As part of determining an optimal bidding strategy, it is necessary to know the cost distribution of a population of bidders, and that requires estimating the location parameter and the spread parameter. In competitive bidding, the most frequently available historical bidding information is the lowest bid (winning bid) and the number of bidders. The *order statistics* can be applied to estimate the population parameters(i.e. location and spread parameters) based upon the available information of extreme value and the size of population.

Projects may have similar distributions of potential bids, thus having the same spread parameter, but they may have different location parameters. For example, a two-story office building and a four-story office building project may be considered as similar since they would attract same group of contractors in the market to submit the bids. It is reasonable to assume that the population's spread parameters of these two projects are same because the bids are from the same group of bidders, however, the location parameters (real costs of the projects) are obviously not same.

If there is historical bidding data available, it is possible to group the data based on the characteristics of the projects and form several different bidder populations. The bids from one bidder population would be expected to have same spread parameter since the physical and behavioral differences among the bidders are nearly the same. The grouping criteria may be the features of the projects, such as the gross floor area , order of magnitude , type of construction, bid price, etc. The criteria are subject to the manager's ad-hoc judgement and depends on the information available for identifying project features.

Based on the grouped historical data, it is possible to estimate the future expected lowest bid. Similar to Broemser's regression model (equation 3), the regression formula may be

$$b_{1;n} = c_0 \sum_{i=0}^{k} \beta_i x_i + \varepsilon_i \tag{15}$$

where

$b_{1;n}$ – –– the lowest bid (winning bid in a project with n bidders)

c_0 – – – – the subject firm's own estimated cost

β_i – – – – the regression coefficient (the weight to the i^{th} feature of a project)

x_i – – – – the i^{th} feature of a project

ε_i – – – – regression variance on i^{th} feature

Applying the least square regression method on equation 15, the expected lowest bid $E(b_{1;n})$ and the variance of the lowest bid $Var(b_{1;n})$ can be obtained.

From equation 14, the variance of the first order statistic is

$$Var(b_{1;n}) = \sigma_b^2 Var(Z_{1;n}) \tag{16}$$

Similar to the $E(Z_{1;n})$, the value of $Var(Z_{1;n})$ depends only on the form of a distribution (e.g. standard normal), and the sample size n. Assuming that the bid distribution is normal [15], the variance of standard normal variable $Var(Z_{1;n})$ is available in the statistics handbook [13] (see Appendix G). Given the regression result of $Var(b_{1;n})$, and tabulated $Var(Z_{1;n})$; substituting into the equation 16, then

$$\sigma_b = \sqrt{\frac{Var(b_{1;n})}{Var(Z_{1;n})}} \tag{17}$$

Further, by assuming that the bid price to estimate cost relationship is linear, then the standard deviation of the cost, σ_c (spread parameter), is equal to the standard deviation of the bid, σ_b.

C Game Theoretic Bidding Model

In a competitive bidding game an individual competitor's action is influenced by the other competitors' action [14]. Assuming the objective of all competitors is to maximize the expected profit gained from the project, then a Nash equilibrium for this game is found as follows [14]. Consider the decision of bidder i, whose estimated cost is c_i. He assumes that the other bidders are following a decision rule given by a bidding function $B(.)$: that is , he predicts that any other bidder j will bid an amount $B(c_j)$ if his estimated cost is c_j (although bidder i does not know his competitor's estimated cost). Assume that $B(.)$ is a monotonously increasing function(i.e. higher estimated cost, higher bid price). If bidder i bids an amount b_i and wins, he earns a profit of $b_i - c_i$. The probability that all of the other bidders $(n-1)$ have estimated costs c_j $(j = 1, 2, ..., n.\ j \neq i)$ such that $B(c_j) > b_i$; this probability is $\{1 - F[B^{-1}(b_i)]\}^{n-1}$, based upon the *order statistics* equation 11, where, as before, $F(.)$ represents the distribution of estimated costs. Bidder i chooses his bid b_i to maximize his expected profit:

$$\pi_i = (b_i - c_i)\{1 - F[B^{-1}(b_i)]\}^{n-1} \tag{18}$$

Thus he chooses b_i such that $\frac{\partial \pi_i}{\partial b_i} = 0$, which is the extreme condition. By differentiating π_i with respect to c_i, we obtain

$$\frac{d\pi_i}{dc_i} = \frac{\partial \pi_i}{\partial c_i} + (\frac{\partial \pi_i}{\partial b_i})(\frac{db_i}{dc_i})$$

Therefor, by differentiating equation 18, an optimally chosen bid b_i must satisfy

$$\frac{d\pi_i}{dc_i} = \frac{\partial \pi_i}{\partial c_i} = -\{1 - F[B^{-1}(b_i)]\}^{n-1} \tag{19}$$

In this game setting, it is assumed all bidders act rationally, in other words, use the same decision rule, such as maximizing the expected profits. Therefore, any two bidders with the same estimated cost will submit the same bid, in mathematical terms, $b_i = B(c_i)$, at Nash equilibrium. By substituting this Nash condition into equation 19, bidder i's expected profit at a Nash equilibrium is defined as

$$\frac{d\pi_i}{dc_i} = -[1 - F(c_i)]^{n-1} \tag{20}$$
$$(i = 1, 2, ..., n)$$

Solving the differential equation 20 for π_i by integration (use the boundary condition, that a bidder has the lowest possible estimated cost $c_i = 0$), and substituting π_i in equation 18, and the Nash condition $b_i = B(c_i)$, each bidder's decision rule is determined to be

$$B(c_i) = c_i - \frac{\int_0^{c_i} [1 - F(t)]^{n-1} dt}{[1 - F(c_i)]^{n-1}} \tag{21}$$

Note that $\int_0^{c_i} [1 - F(t)]^{n-1} dt \leq 0$, because $[1 - F(t)]^{n-1}$ is a monotonously decreasing function. Therefore, the last part ($-\frac{\int_0^{c_i} [1-F(t)]^{n-1} dt}{[1-F(c_i)]^{n-1}} > 0$) of equation 21 is the amount which bidder i markups his estimated cost c_i.

From calculus, the following is established.

$$\begin{aligned} \frac{d}{dc_i} \int_0^{c_i} [1 - F(t)]^{n-1} dt &= [1 - F(c_i)]^{n-1} \\ \frac{d}{dc_i} [1 - F(c_i)]^{n-1} &= (n-1)[1 - F(c_i)]^{n-2}[-f(c_i)] \end{aligned}$$

By differentiating equation 21 with respect to c_i

$$\begin{aligned} B'(c_i) &= 1 - \frac{[1 - F(c_i)]^{2(n-1)} + (n-1)[1 - F(c_i)]^{n-2} f(c_i) \int_0^{c_i} [1 - F(t)]^{n-1} dt}{[1 - F(c_i)]^{(n-1)*2}} \\ &= -(n-1) \frac{\int_0^{c_i} [1 - F(t)]^{n-1} dt}{[1 - F(c_i)]^n} f(c_i) \end{aligned}$$

From the above equation

$$\int_0^{c_i} [1 - F(t)]^{n-1} dt = -B'(c_i) \frac{[1 - F(c_i)]^n}{(n-1)f(c_i)} \tag{22}$$

Substitute equation 22 into equation 21 to obtain the most important and significant equation:

$$B(c_i) = c_i + \frac{1 - F(c_i)}{f(c_i)(n-1)} B'(c_i) \tag{23}$$

D Winner's Curse

In a competitive bidding environment a bidder may have a biased estimate of project cost. If a bidder has a biased estimated cost that is lower than all other bidders' estimated costs, then it is likely that he will win the bid consistent with the decision rule condition:$b_i = B(c_i)$, at Nash equilibrium. This is an unexpected and undesirable situation from the bidder's point of view, because the project would result in a loss . This phenomenon is defined as the *winner's curse.*

Whether or not the winner's curse exists in reality depends on many factors such as the type of industry, market, project, and point in time. The degree of the winner's curse varies, too.

In order to preclude the winner's curse from the bidding decision, a bidder should use a statistical methodology such as order statistics to adjust the initial estimated cost.

From *order statistics* equations 13 and 14 we have

$$c_{1;n} = \mu + \sigma Z_{1;n}$$

$$E(c_{1;n}) = \mu + \sigma E(Z_{1;n})$$

Assuming that the winner's cost adjustment function $C_{adj}(c_{1;n}) = c_{1;n} + \Delta$, then he can eliminate the winner's curse by adjusting the expected value of the adjusted estimated cost $E[C_{adj}(c_{1;n})] = \mu$ [20], the actual cost of the project. Thus,

$$E[C_{adj}(c_{1;n})] = E(c_{1;n}) + \Delta = \mu + \sigma E(Z_{1;n}) + \Delta$$

When $\Delta = -\sigma E(Z_{1;n})$, the $E[C_{adj}(c_{1;n})] = \mu$. Therefore the estimated cost adjustment function for all bidders is

$$C_{adj}(c_i) = c_i - \sigma E(Z_{1;n}) \tag{24}$$

$$(i = 1, 2, ..., n)$$

Note that $c_i = c_{1;n}$ if bidder i wins the project.

The condition described above is for when a perfect winner's curse exists. In reality, it is unlikely that it will occur universally since many bidders are aware of the winner's curse and try to avoid it. In recognizing this situation a coefficient α ($\alpha = [0, 1]$) is used to modify the estimated cost function (equation 24).

$$C_{adj}(c_i) = c_i - \alpha\sigma E(Z_{1;n}) \tag{25}$$

α is an empirical modification factor that indicates the extent of winner's curse in a industry, market and type of project etc.. When $\alpha = 1$, the perfect winner's curse exists. When $\alpha = 0$, no winner's curse exists.

Substituting equation 25 into the equation 23, the bidding strategy subject to winner's curse is

$$B(c_i) = c_i - \alpha\sigma E(Z_{1;n}) + \frac{1 - F(c_i)}{f(c_i)(n-1)} B'(c_i) \tag{26}$$

where

$B(c_i)$ – – – – the strategy function (decision rule) with respect to estimated cost

$B'(c_i)$ – – – – the derivative of the strategy function

c_i – – – – the estimated cost of i^{th} bidder

$F(.)$ – – – – the common cumulative distribution of the estimated costs

$f(.)$ – – – – the common density function of the estimated costs

n – – – – number of bidders

σ – – – – the standard deviation of the estimated cost distribution

$E(Z_{1;n})----$ the expected value of the standardized variable. If it is standard normal then it can be obtained from the available order statistic table [13] (see Appendix G).

$\alpha ----$ the degree of winner's curse. It can be determined by empirical study.

E Simplified Bidding Strategy

In order to apply the game theoretic strategy that has been developed in the last sections, the mathematical model is simplified by removing the differential equation of the optimal bid(equation 26). The simplification is aimed at developing a linear form of the bidding model.

The decision rule $B(c_i)$ is linear, if $B'(c_i) = 1$. Therefore, from equation 26

$$B(c_i) = c_i - \alpha\sigma E(Z_{1;n}) + \frac{1 - F(c_i)}{f(c_i)(n-1)} \tag{27}$$

For the purpose of simplification, the common cost distribution function of a potential bidder population may be considered to be normally distributed with a location parameter, μ, and a spread parameter, σ, [15].

$$F(c_i) = \int_{-\infty}^{c_i} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-\mu)^2}{4\sigma^2}} dt \tag{28}$$

Let $\xi = \frac{t-\mu}{\sigma}$, then

$$\begin{aligned} F(c_i) &= \int_{-\infty}^{\frac{c_i-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{\xi^2}{2}} d\xi \\ &= G(\frac{c_i-\mu}{\sigma}) \\ f(c_i) &= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(c_i-\mu)^2}{4\sigma^2}} \\ &= \frac{1}{\sigma} g(\frac{c_i-\mu}{\sigma}) \end{aligned}$$

where G(.) and g(.) are the distribution function and density function of the standard normal distribution, respectively.

Since bidder i tries his best to estimate the project cost c_i, it is reasonable to make an assumption that $c_i = \mu$ [20]. Therefore,

$$F(c_i) = G(\frac{c_i - \mu}{\sigma}) = G(0) = \frac{1}{2} \tag{29}$$

$$f(c_i) = \frac{1}{\sigma} g(\frac{c_i - \mu}{\sigma}) = \frac{1}{\sigma} g(0) = \frac{1}{\sqrt{2\pi}\sigma} \tag{30}$$

Substituting equations 29 and 30 into the linear optimal strategy equation 27, results in :

$$\begin{aligned} B(c_i) &= c_i - \alpha\sigma E(Z_{1;n}) + \frac{1 - F(c_i)}{f(c_i)(n-1)} \\ &= c_i - \alpha\sigma E(Z_{1;n}) + \sqrt{\frac{\pi}{2}}\,\frac{\sigma}{n-1} \end{aligned} \tag{31}$$

This is the simplified linear bidding strategy under the assumption that the potential cost distribution is normal.

A corresponding formula may be derived for different forms of the cost distribution, such as lognormal distribution, left side truncated normal distribution, etc.

Chapter V

MODEL EVALUATION

In this chapter the simplified bidding model developed in chapter 4 is tested using the actual bidding data from two contractors. A flow chart for processing model information is presented. The model is then applied using two sets of data . The results of processing the data are measured in terms of profit (P), dollar volume of projects won(V_w) and the ratio of profit to volume won (P/V_w) are compared to actual outcomes involving the same data . The model is tested to determined the increase of the contractors' wealth if it were used.

A Framework of Information Processing

Contracting firms can improve the efficiency and effectiveness of bid preparation by adopting a systematic approach to bid preparation. As part of a systematic approach, they can use the above mentioned bidding model to improve their performance in such areas as profitability, market share and rate of return etc.. A general procedure for preparing competitive bids is outlined in Figure 4. The segments of the model are presented in Figure 5 and Figure 6.

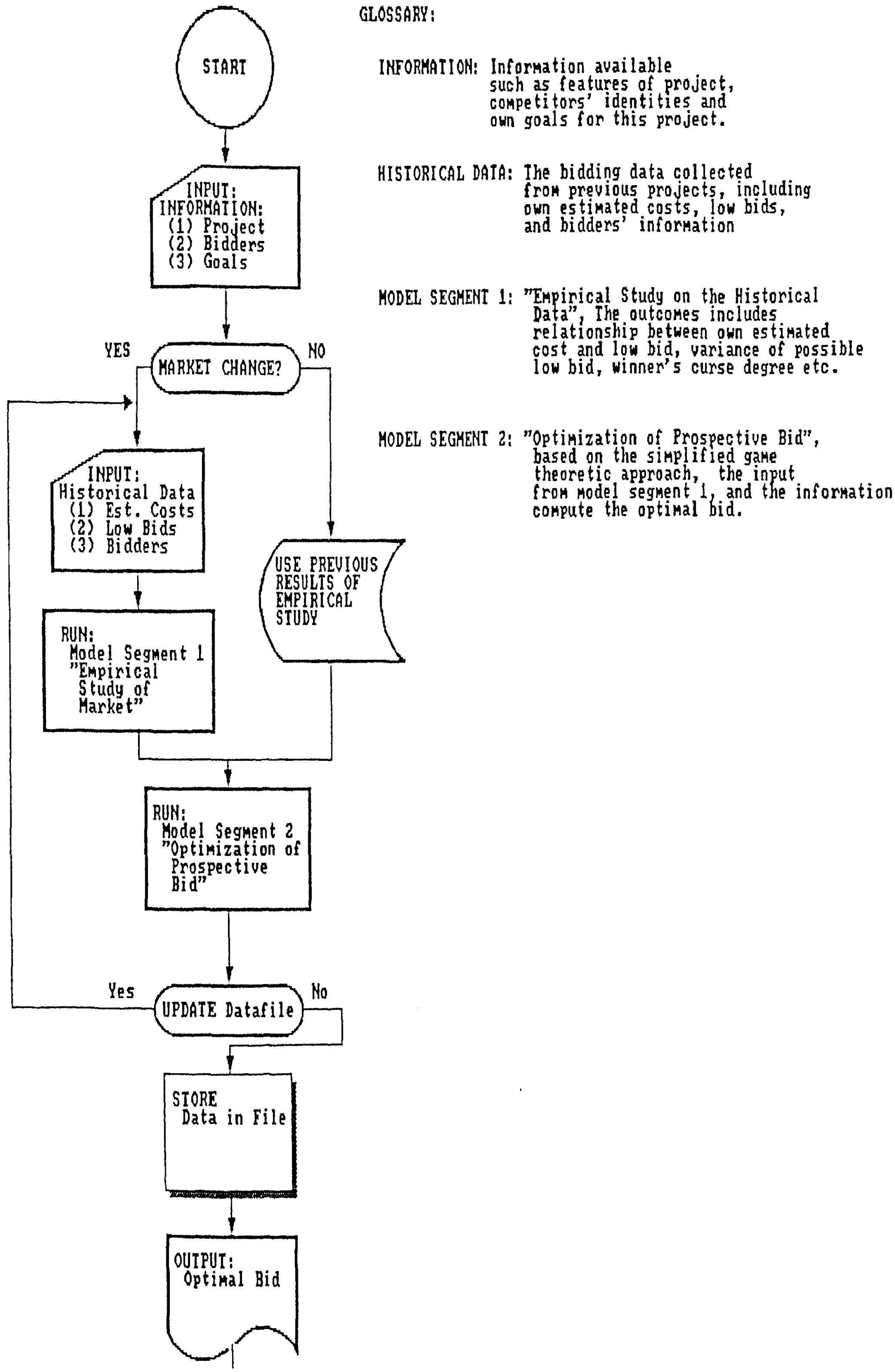
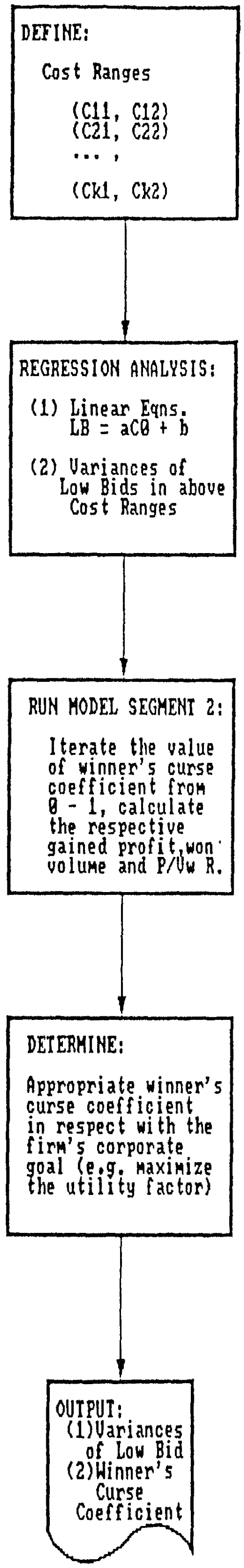


Figure 4: Flow Chart of the Bidding Model



GLOSSARY:

Cost Ranges: cost ranges in that the variances of the low bids are considered the same

Cij: i = 1, 2, ..., k; j= 1, 2
the lower bounds and upper bounds of the cost ranges

k: number of the cost ranges

LB: low bid (winning bid)

C0: firm's estimated cost

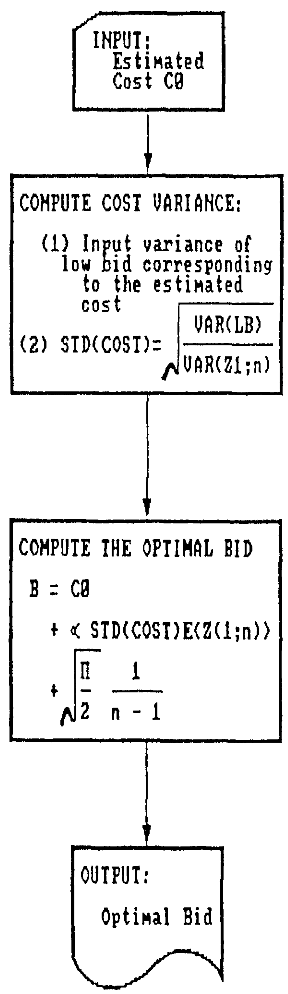
a, b: coefficient of regression analysis

P/Uw R.: ratio of gained profit to won volume

Figure 5: Model Segment 1

MODEL SEGMENT 2

Optimization of Prospective Bid



GLOSSARY:

STD(COST):	standard deviation of the estimated costs among the potential bidders
VAR(LB):	variance of the low bid in a specific cost range
VAR(Z1;N):	variance of first order statistic of standard normal distribution
B:	optimal bid
C0:	firm's estimated cost
n:	number of bidders
α:	winner's curse coefficient

Figure 6: Model Segment 2

B Evaluation Methodology

The bidding data are collected from two sources. One set of data is obtained from Benjamin's PhD dissertation [3]. The another set of data is from Broemser's PhD dissertation [4]. Both sets of data were collected from two different construction contractors in California in the period of 1965 - 1968. These data were used because of the difficulty of obtaining historical cost data from contractors or from private owners. A portion of the data is used to evaluate the model proposed in this thesis. Most contractors in the competitive bidding market keep their cost data from the public to sustain competitiveness. A secondary benefit of reusing existing data is that the model proposed in this thesis can be compared to some existing models that are based on the same data.

The computations for the model are performed in a microcomputer using Lotus 1-2-3. The order statistic value of a standard normal distribution are obtained from Krishnaiah's book [13], see appendix G.

Three performance measures are evaluated; profit(P), volume won(V_w) and ratio of profit to volume won(P/V_w). Generally speaking, the profit and P/V_w ratio are expected to be as high as possible, but the optimal level of won volume is governed by the contractor's capacity(resources) and desires. Extremely small and large values of won volume are not usually expected or accepted.

C Benjamin's Data Set

There are 130 construction projects in Benjamin's data set. The project bids range of $10,000 to $2,500,000 [3]. The data is reprinted in Appendix C. To evaluate the proposed model the 130 projects is separated into two groups. Projects 1 through 66 are treated as historical data, and 67 through 130 are viewed as upcoming projects to be bid. The cost variance among the bids on the same project depends on the scale of the estimated costs. The higher the estimated cost, the larger the variance, because there are more or larger cost items involved. Every project has a unique estimated cost variance. However, the variances among a specific range of bids can be considered to be the same for the purpose of simplification in calculation and tractability in the data collection. In the light of this assumption, the bid ranges, among which the cost variance is considered to be the same, are defined as:

- \$0 - \$100,000
- \$100,000 - \$500,000
- \$500,000 - \$1,000,000
- \$1,000,000 - Up

Based on equation 32, the results of the least square regression analysis for each range of data for projects 1 through 66 are shown in Table 1. Details of the regression analysis are developed in Appendix D.

$$LB = aC_0 + b \tag{32}$$

where LB — the lowest bid price, which is assumed to be the winning bid, C_0 —contractor's estimated cost.

Table 1: Results of Regression Analysis on Historical Data(Projects 1-66)

	Cost Range (1,000)			
	\$0-\$100	\$100-\$500	\$500-\$1,000	\$1,000-Up
Standard Deviation σ_{LB}	2.69	9.80	28.31	60.16
Coefficient a	.95	1.00	0.96	0.98
Standard Deviation σ_a	0.03	0.02	0.05	0.18
Constant b	1.85	1.86	22.60	14.88
r Square	0.99	0.99	0.96	0.78
No. of Observations	13	29	14	10

As proved in Chapter IV, the cost variance among the potential bidders are equal to the bid variance under the assumption that there is a linear relationship between bid price and estimated cost. Using the order statistics theory, the bid variance is computed as

$$\sigma_b = \sqrt{\frac{Var(b_{1;n})}{Var(Z_{1;n})}} = \sqrt{\frac{\sigma_{LB}}{Var(Z_{1;n})}} \tag{33}$$

where $Var(Z_{1;n})$ is tabulated from Appendix G.

For example, a contractor has a cost estimate of $C_0 = \$303,300$ on a project, and knows that there will be 6 bidders (including itself). From Table 1, $\sigma_{LB} = 9.80$. From Appendix G, the $Var(Z_{1;6}) = 0.4159$. Accordingly, from equation 33 and the assumption of linearity, then, the standard deviation of estimated costs is

$$\sigma_c = \sigma_b = \sqrt{\frac{9.80}{0.4159}} = 15.2$$

The proposed simplified bidding model, equation 31, is then applied to determine the optimal bid price which will maximize the expected profit. By assuming a winner's curse coefficient $\alpha = 0.5$, knowing the estimated cost $C_0 = \$303,300$, standard cost deviation $\sigma_c = 15.2$, number of bidders $n = 6$, and tabulated value of $E(Z_{1;6}) = -1.2672$ from Appendix G, the optimal bid price is

$$\begin{aligned} B &= C_0 - \alpha\sigma_c E(Z_{1;6}) + \sqrt{\frac{\pi}{2}}\frac{\sigma_c}{n-1} \\ &= 303,300 - 0.5 * 15.2 * (-1.2676) + \sqrt{\frac{\pi}{2}}\frac{15.2}{6-1} \\ &= \$316,740 \end{aligned}$$

The winner's curse coefficient α is determined empirically from historical data of projects 1 through 66. Applying the proposed simplified bidding model (equation 31) with iteration values of $\alpha = 0,\ 0.1,\ 0.2,\ ...,\ 1.0$ respectively to these projects results in the data shown in Table 2:

The curves in the figures 7,8 and 9 indicate the patterns of the total profit, volume won and ratio of profit to volume won. As the winner's curse coefficient α increases, the won volume decreases and the ratio of profit to volume increases. This is an expected phenomenon, since the higher the winner's curse, the more cautious the bidder is. The result is that the markup on estimated cost rises to defend against the winner's curse.

To determine the optimal strategy, one should not only consider the total profit, but the projects scope and the rate of return as well. The weighting method which

ANALYSIS OF WINNER'S CURSE COEFFICIENT: ALPHA
(DATA SET : PROJECTS # 1 - 66)

ALPHA	GAINED PROFIT	UTILITY W(P)	WON VOLUME	P/VwRATIO (%)	UTILITY W(P/Vw)	UTILITY W
0.0	159.04	0.15	9646	1.65	0.03	0.18
0.1	115.03	0.11	5471	2.10	0.03	0.14
0.2	138.86	0.13	4807	2.89	0.05	0.18
0.3	137.46	0.13	3710	3.71	0.06	0.19
0.4	132.17	0.13	2753	4.80	0.08	0.20
0.5	99.83	0.10	1779	5.61	0.09	0.19
0.6	113.93	0.11	1779	6.41	0.10	0.21
0.7	56.43	0.05	982	5.74	0.09	0.15
0.8	39.42	0.04	553	7.13	0.11	0.15
0.9	22.32	0.02	208	10.74	0.17	0.19
1.0	24.44	0.02	208	11.75	0.19	0.21
SUM:	1038.93	1		62.53	1	

NOTE:

ALPHA: WINNER'S CURSE COEFFICIENT
P: GAINED PROFITS
Vw: WON VOLUME
P/Vw: RATIO OF PROFIT TO WON VOLUME
W(P): UTILITY WEIGHT OF PROFIT
= PROFIT/SUM OF PROFIT

W(P/Vw): UTILITY WEIGHT OF P/Vw RATIO
=[(P/Vw) RATIO]/[SUM OF (P/Vw) RATIO]

W: UTILITY FACTOR
= W(P) + W(P/Vw)

Table 2: Analysis of Winner's Curse Coefficient: α(Projects 1-66)

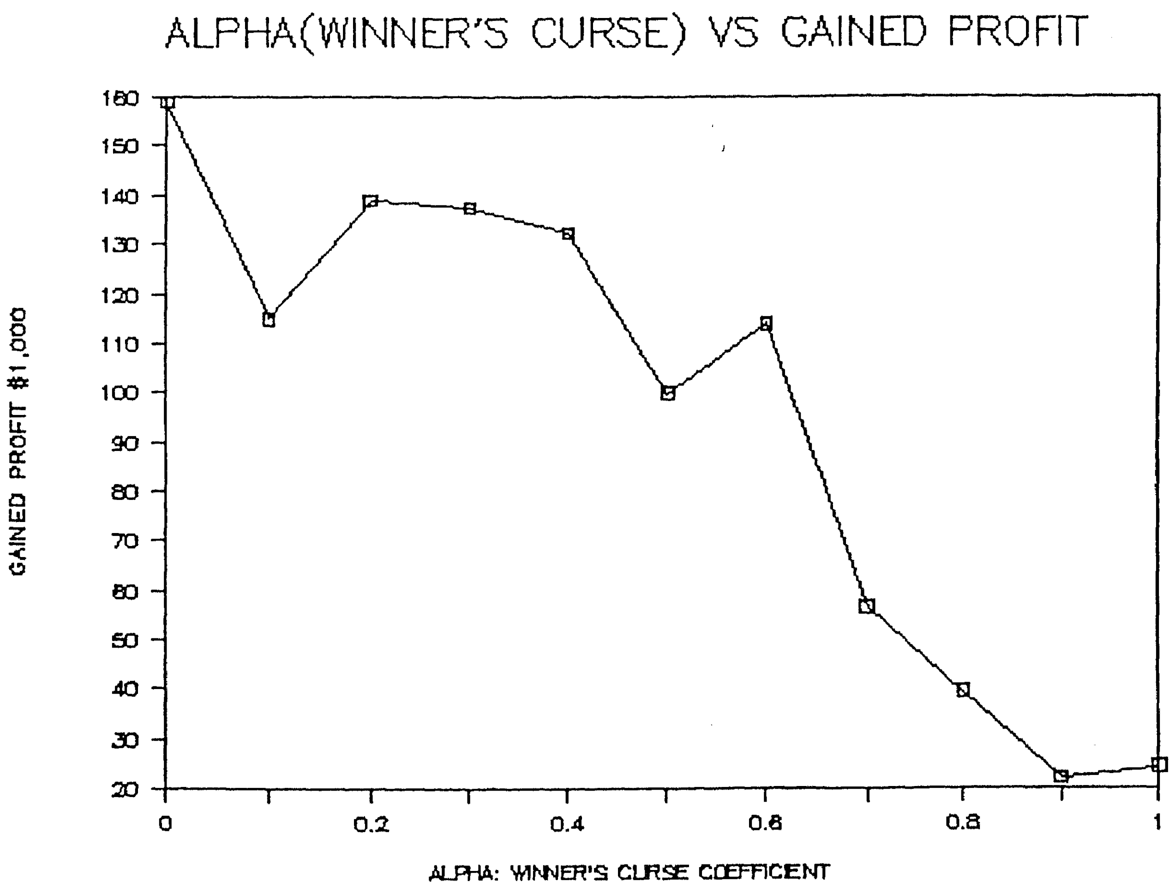


Figure 7: α (Winner's Curse) vs Gained Profit(Projects 1-66)

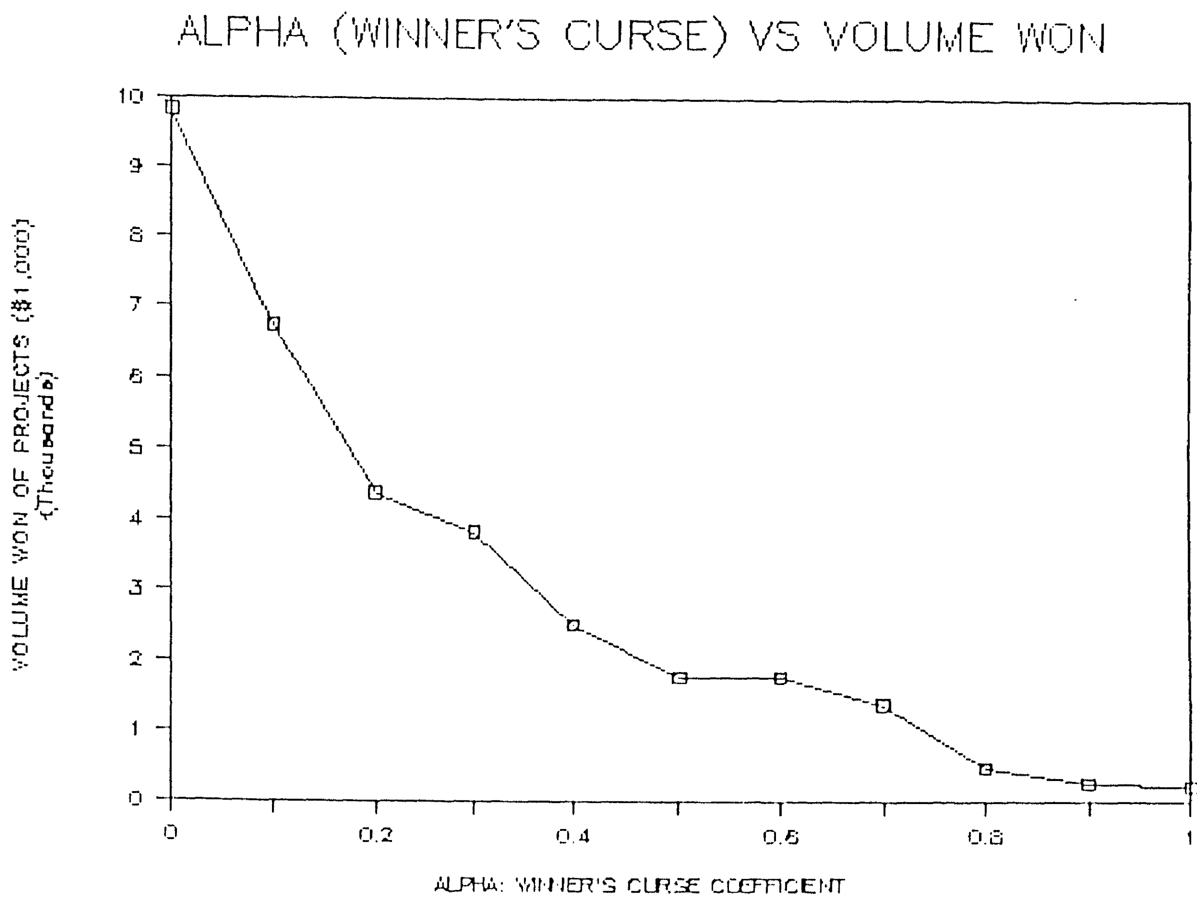


Figure 8: α (Winner's Curse) vs Won Volume(Projects 1-66)

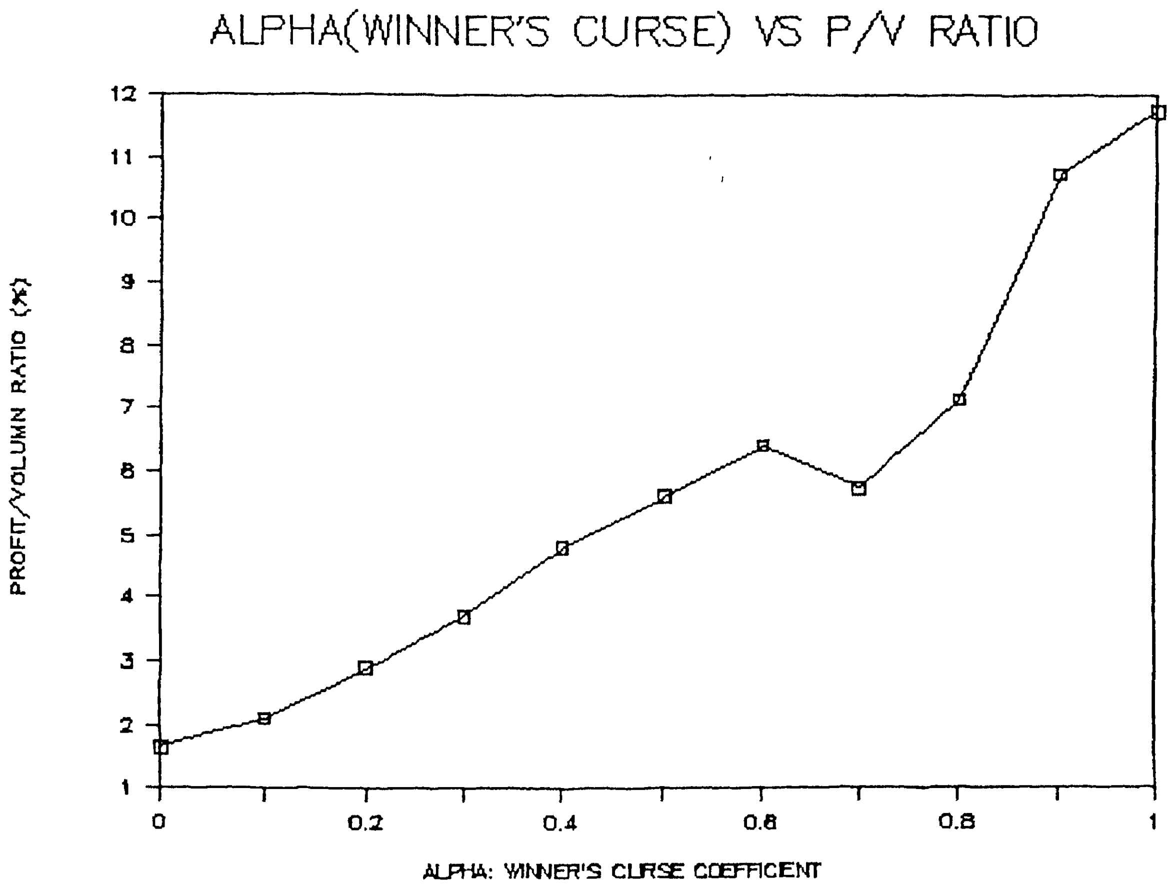


Figure 9: α (Winner's Curse) vs P/V_w Ratio(Projects 1-66)

defines the contractor's utility as the sum of the weight factors of volume and P/V ratio in Table 2 is used to evaluate a strategy. The results are shown in Figure 10. It is obvious that the winner's curse coefficient $\alpha = 0.5$ is the point where the contractor's utility curve reaches its crown.

Based upon previous analysis of historical data the model is applied to a series of future projects 67 through 130, which produces the values shown in Table 3. For detailed computation process see Appendix D.

Table 3: Comparison of Corporate Performances(Benjamin's Data)

	Projects 1-66		Projects 67-130	
	Contractor's Actual Results Intuitive Model	Results Using the Model	Contractor's Actual Results Intuitive Model	Results Using the Model
No. of Jobs Won	8	7	12	10
Total Profit	$82,540	$99,830	$213,960	$205,350
Total Volume Won	$2,039,700	$1,778,700	$4,210,100	$3,820,700
Ratio of $P/V_w(\%)$	4.05	5.61	5.08	5.37

From the above results it appears that the model behaves reasonably and produces better performance than the original performance of the contractor.

If using different α values (winner's curse coefficient), the model outcomes based on the inputs of this set of data (projects 67 - 130) is listed in table 4. Graphing the profits vs α produces Figure 11. It appears that $\alpha = 0.5$ is also the point corresponding to the maximum profits, which verifies the prior assumption that the $\alpha = 0.5$ in this market.

D Broemser's Data Set

In order to further test the model, another set of data which is adapted from Broemser's PhD dissertation [4] is used (see Appendix E). There are 76 projects in

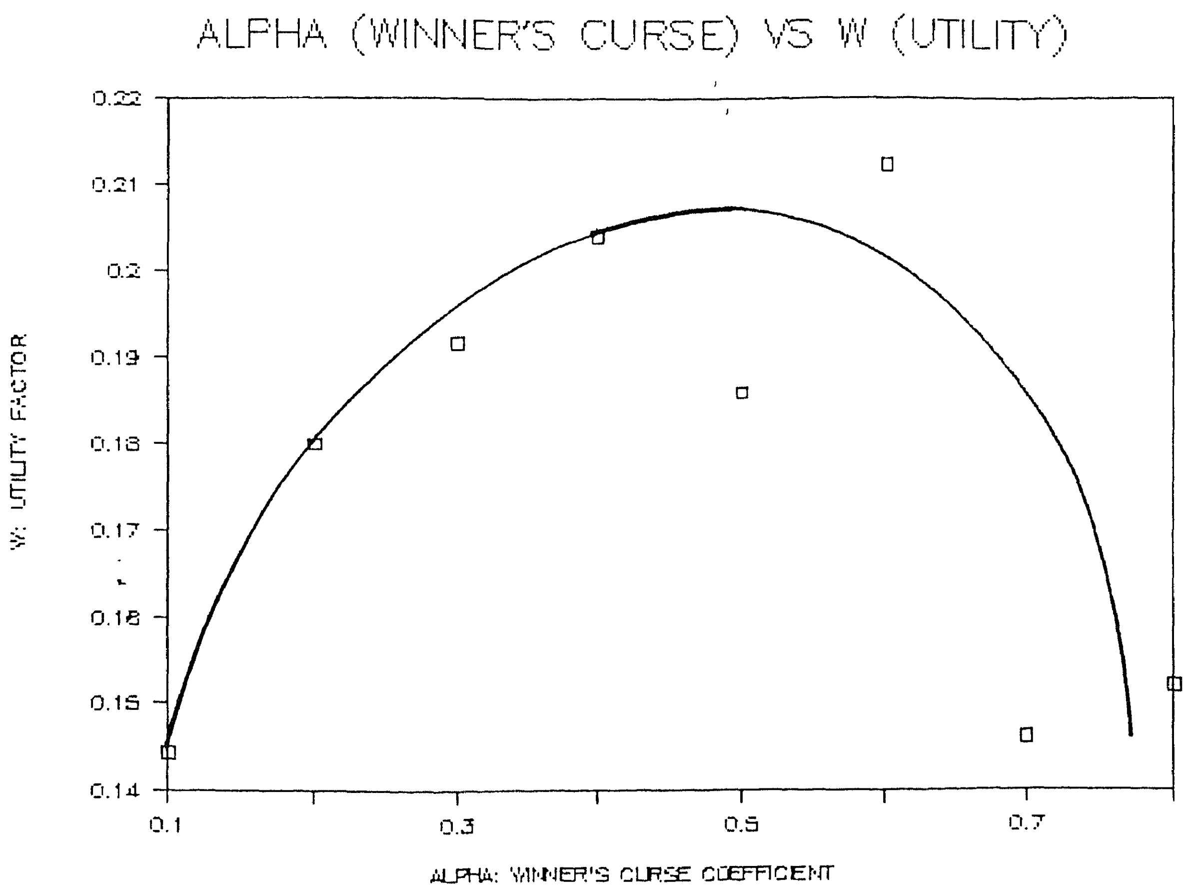


Figure 10: α (Winner's Curse) vs Utility(Projects 1-66)

ANALYSIS OF WINNER'S CURSE COEFFICIENT: ALPHA
(DATA SET : PROJECTS # 67 - 130)

ALPHA	GAINED PROFIT	UTILITY W(P)	WON VOLUME	P/VwRATIO (%)	UTILITY W(P/Vw)	UTILITY W
0.0	122.74	0.09	8334	1.53	0.03	0.12
0.1	157.61	0.11	6525	2.42	0.05	0.16
0.2	143.43	0.10	4436	3.23	0.06	0.17
0.3	164.75	0.12	4105	4.01	0.08	0.20
0.4	177.53	0.13	3821	4.65	0.09	0.22
0.5	205.35	0.15	3821	5.37	0.10	0.25
0.6	85.34	0.06	1634	5.22	0.10	0.16
0.7	94.45	0.07	1634	5.78	0.11	0.18
0.8	84.35	0.06	1380	6.11	0.12	0.18
0.9	73.35	0.05	1109	6.61	0.13	0.18
1.0	73.49	0.05	1044	7.04	0.14	0.19
SUM:	1382.39	1		51.97	1	

NOTE:

ALPHA: WINNER'S CURSE COEFFICIENT
P: GAINED PROFITS
Vw: WON VOLUME
P/Vw: RATIO OF PROFIT TO WON VOLUME
W(P): UTILITY WEIGHT OF PROFIT
= PROFIT/SUM OF PROFIT

W(P/Vw): UTILITY WEIGHT OF P/Vw RATIO
=[(P/Vw) RATIO]/[SUM OF (P/Vw) RATIO]

W: UTILITY FACTOR
= W(P) + W(P/Vw)

Table 4: Analysis of Winner's Curse Coefficient: α(Projects 67-130)

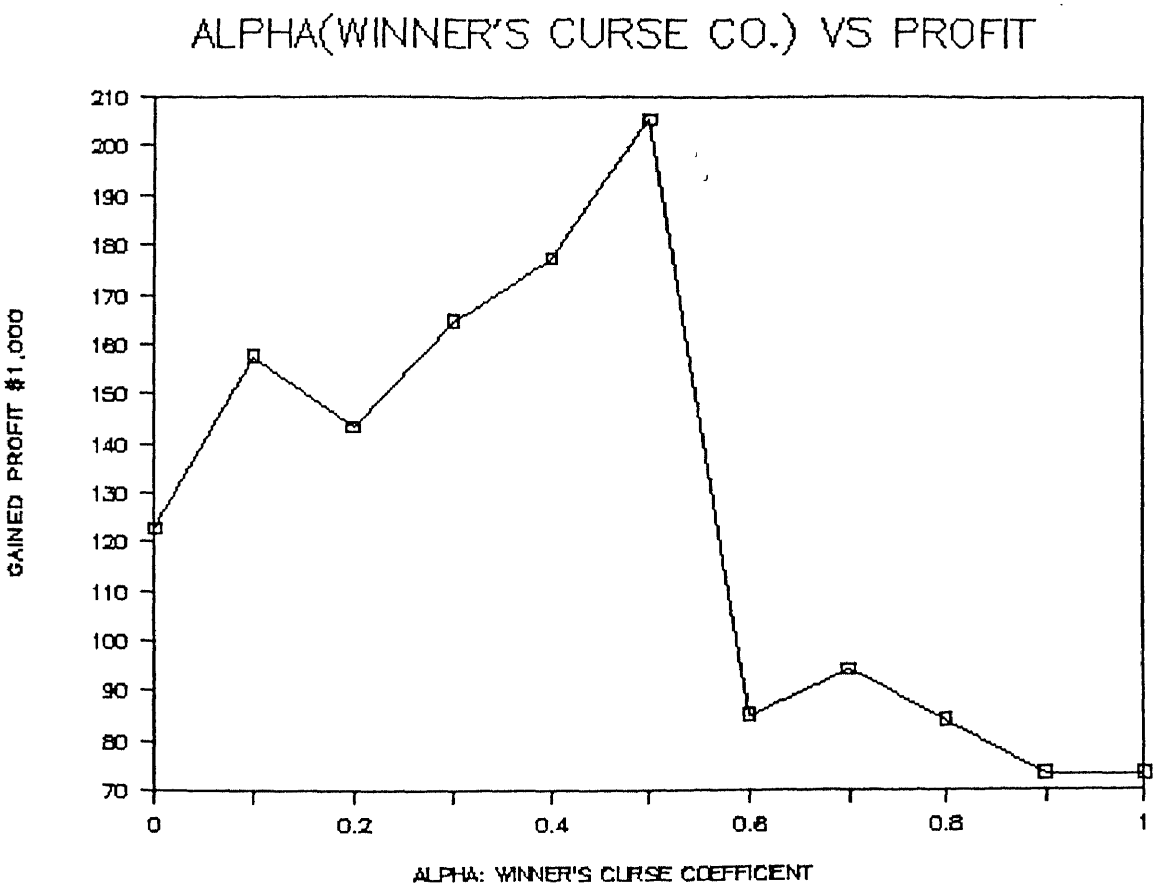


Figure 11: α (Winner's Curse) vs Gained Profit(Projects 67-130)

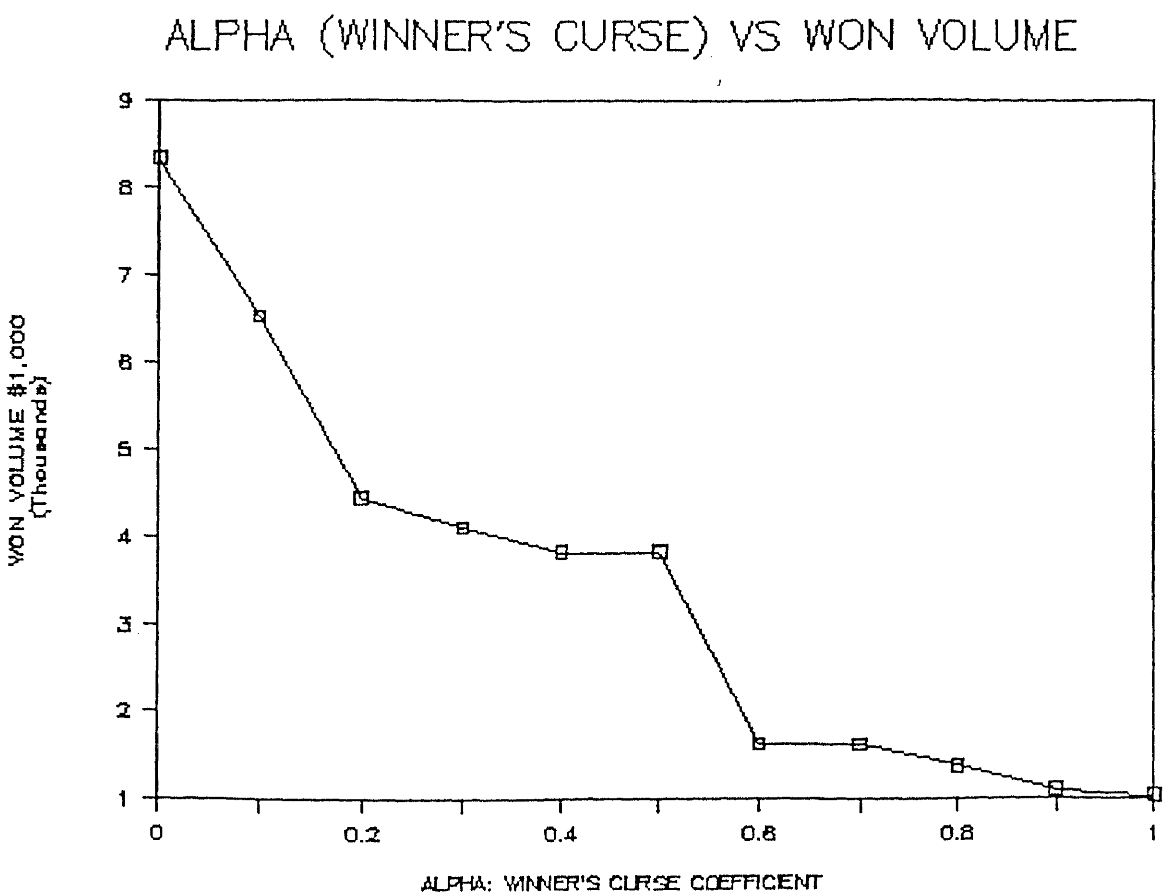


Figure 12: α (Winner's Curse) vs Won Volume(Projects 67-130)

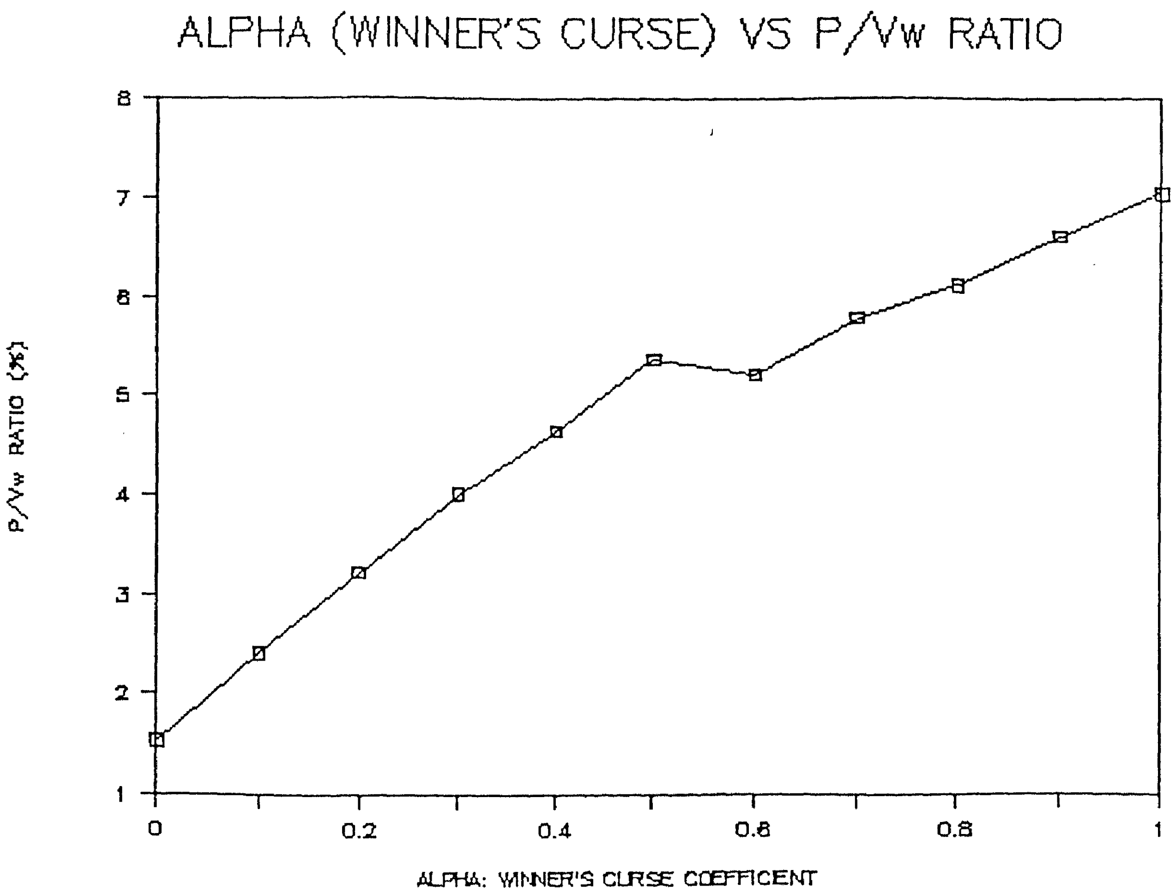


Figure 13: α (Winner's Curse) vs P/V_w Ratio(Projects 67-130)

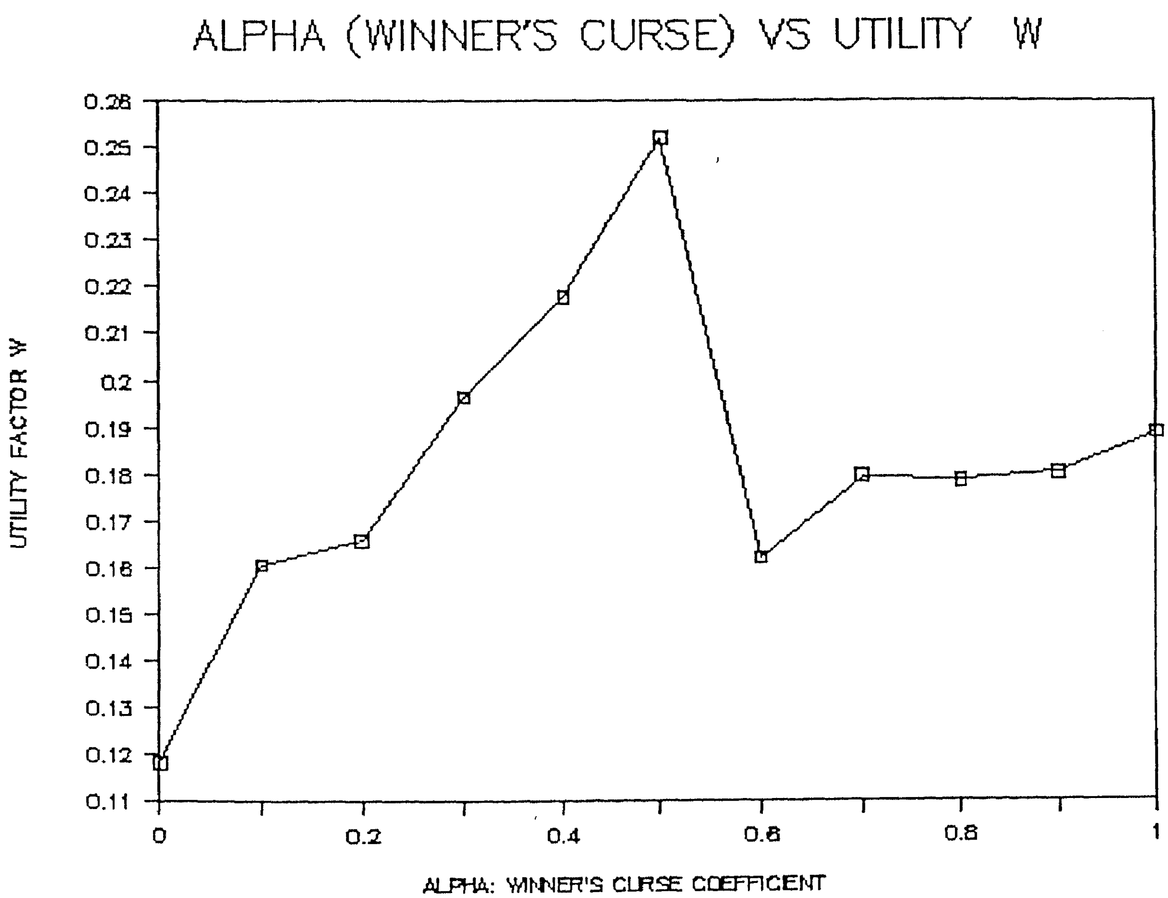


Figure 14: α (Winner's Curse) vs Utility(Projects 67-130)

this set of data. The data was furnished by a northern California general contractor during the period of 1965-1968. Coincidentally, Benjamin's bidding data was provided by the another contractor also located in northern California during the same period of time [3]. Therefore, it is logical to assume the degree of the winner's curse, encountered by these two contractors, was not different due to the same location and same period of time. Therefore, $\alpha = 0.5$ is used in the model computation.

Broemser performed a regression analysis on his data and found that the low bid cost ratios (lowest bid/contractor's estimated cost) were distributed normally with a mean of 1.021 and a standard deviation of 0.0518 [4]. Therefore, the standard deviation of a lowest bid is equal to the product of 0.0518 and the contractor's estimated cost. Performing the cost variance analysis similar to the method used for Benjamin's data will determine the standard deviation of the costs among the potential bidders.

The same model computation as used for the Benjamin's data was performed in a microcomputer using Lotus 1-2-3 to obtain the following results: Table 5 (for detail see appendix F):

Table 5: Comparison of Corporate Performances(Broemser's Data)

	Contractor's Actual Results Intuitive Model	Results Using the Proposed Model	Results Using Broemser's Simple Model	Results Using Broemser's Optimal Model
No. of Jobs Won	7	4	11	12
Total Profit	\$478,100	\$547,350	\$589,400	\$610,200
Volume Won	\$10,000,000	\$7,635,500	\$14,800,000	\$15,500,000
Ratio of $P/V_w(\%)$	4.78	7.17	3.98	3.93

The result shows that the proposed model out-performed Broemser's model in the sense that it attained nearly the same profit level with only half the volume of work and only a third as many projects. The model also has the one practical advantage compared to Broemser's model, in that it is vastly simpler to calculate.

Chapter VI

CONCLUSIONS AND RECOMMENDATIONS

It has been shown in this thesis that the game-theoretic model provides a better understanding of the competitive bidding situation in which conflict among the bidders is present. The simplified bidding model is useful in giving a more reliable quantitative method for the decision making process. Since the business world is very complex, further exploration in game theoretic bidding models is certainly warranted.

A Conclusions

Game theoretic bidding models provide a closer approximation of the actual competitive bidding situations. In contrast to the Bayesian Analysis, the game approach takes into account the conflict of interest among the competing firms. The current state of game theory offers wider fields of consideration in modeling the more complex situations.

The perception that the game theoretic approach is much more complicated than the Bayesian approach needs to be corrected. The differences in all the available approaches depend mainly upon the assumptions made with respect to the actual situation. A model is considered to be a good one if the assumptions simulate the actual conditions closely and the model outcomes are optimal so far as the objectives are concerned. Game approach has the advantage in both these evaluation criteria.

The simplified game theoretic bidding strategy is proposed for application in the construction industry. The model has been tested by real data and proved to be quite acceptable. One advantage of the simplified model is the lack of complicated computation which makes it easier to be implemented in day to day practice. Manual calculations are sufficient for operating the model although a microcomputer with an electronic spreadsheet software is more efficient.

There are evidences that the winner's curse exists in competitive bidding in the construction industry. Under estimating or over estimating this factor would either result in a loss to the contractor with a low biased cost estimate or weaken it's competitiveness if the estimated cost is high biased. Empirical analysis is one of the approaches to assess the degree of local winner's curse .

B Recommendations for Future Research

For any bidding models the cost distribution form is the crucial factor to be defined. In order to simplify the bidding model proposed in this thesis, an assumption of normal distribution was made. However, lognormal distribution seems to more closely replicate the real distribution than the normal. Furthermore, for both the normal and lognormal distribution, the possible negative value in the distributions is unrealistic in the practice since there are no negative costs for a construction project. One type of distribution that provides only a realistic range of values is the truncated normal distribution. It can be single side truncated or both sides truncated since there is neither a negative cost nor an infinite cost.

The empirical analysis for the assessment of competitors' behavior patterns requires moving pattern adjustment. The market is changing, the competitors are growing up over time. Simply using the "one shot" regression analysis can not indicate the impact from the changes of opponents' behavior and the physical environments. Markov Chain is a good approach to predicting and modeling changing states, but it needs to be simplified for practical purposes.

The complete game theoretic bidding formula consists of a rather complex differential equation. Numerical method could be applied in order to calculate values by the means of a computer. The computing method is worthy of being researched.

The firm's objective is a vital issue in game theoretic modeling. A firm usually considers not only the short term profit of the operation but other corporate objectives as well. How to define the utility of a firm is an area of great potential for the improvement of game theoretic modeling.

Sub-optimization is a serious problem in business. Single bid bidding model has a possibility that creates an optimal strategy for a "one shot" deal, but it may not be optimal from the systematic and sequential point of view. Although the winner's curse coefficient α in the proposed model has implicit consideration in sequential optimization through empirical optimization in terms of the corporate utility, a better understanding of the sequential bidding process is expected from a better model.

Decision making in a competitive bidding process is a form of information processing work. The more significant information the decision maker has, the better the decision will be made. However, perfect information is costliest. The tradeoff between the cost of information and the benefits derived from information should be studied from the viewpoint of game theory.

Appendix A

NOTATION

LIST OF SYMBOLS:

α:	Degree of winner's curse (0 - 1)
π:	Profit
b_i:	Bid price of i^{th} bidder
c_i:	Estimated cost of i^{th} bidder
n:	Number of bidders for a project
B:	Bid price
$P(B)$:	Probability of winning with bid price B
$B(.)$:	Optimal bid function
$B'(.)$:	Derivative of optimal bid function
LB:	Low bid (winning bid)
LBC:	Low bid cost ratio (ratio of low bid to firm's cost estimate)
$F(.)$:	Cumulative probability function
μ:	Location parameter of population (i.e. Mean)
σ:	Spread parameter of a population (i.e. Variance)
$x_{r;n}$:	r^{th} order statistic
$Z_{r;n}$:	standardized value of $x_{r;n}$ $(= \frac{x_{r;n}-\mu}{\sigma})$
$E(x_{r;n})$:	Expected value of r^{th} order statistic
$E(Z_{r;n})$:	Expected value of $Z_{r;n}$
$VAR(Z_{r;n})$:	Variance of $Z_{r;n}$
$b_{1;n}$:	Low bid among n bidders for a project
$c_{1;n}$:	Low estimated cost among n bidders for a project
$C_{adj}(.)$:	Estimated cost adjustment function for winner's curse
$G(.)$:	Cumulative probability function of standard normal distribution
$g(.)$:	Density function of standard normal distribution
P:	Profit
V_w:	Won volume of projects
P/V_w:	Ratio of gained profit to won volume
σ_c:	Standard deviation of estimated costs for a project
σ_B:	Standard deviation of bid prices for a project

σ_{LB}: Standard deviation of low bid prices in a specific group of projects (e.g. a cost range)

$W(P)$: Utility weight of gained profit

$W(P/V_w)$: Utility weight of P/V_w ratio

W: Utility factor $[= W(P) + W(P/V_w)]$

$STD(COST)$: Standard deviation of estimated cost for a project

$STD(LB)$: σ_{LB}

$VAR(LB)$: σ^2_{LB}

$OPBID$: Optimal bid price

$OPM0$: Optimal markup

Appendix B

MODEL CALCULATION FLOW

The model test computation performed on the two sets of bidding data (Benjamin's and Broemser's) follows the following notation and formula.

NOTATION:

n:	number of bidders for a project
C0:	Firm's estimated cost of a project
LB:	lowest bid for a project (winning bid)
$STD(LB)$:	
σ_{LB}:	standard deviation of the possible lowest bid in a project
$E < Z(1;n) >$:	expected value of the first order statistic of a standard normal distribution. It's value depends only on the value of n, and can be obtained from appendix G.
$VAR < Z(1;n) >$:	variance of the first order statistic of a standard normal distribution. It's value depends only on the value of n, and can be obtained from appendix G.
$STD(COST)$:	

σ_c:	standard deviation of the estimated costs among the bidders in a project.
$W(n)$:	function value of n.
α:	winner's curse coefficeient from empirical study
$OPBID$:	optimal bid computed by the model
$OPM0$:	optimal markup on the estimated cost

FORMULA:

$$\sigma_c = \sqrt{\frac{\sigma_{LB}^2}{VAR < Z(1;n) >}}$$

or in the another notation:

$$STD(COST) = \sqrt{\frac{STD(LB)^2}{VAR < Z(1;n) >}}$$

$$W(n) = \sqrt{\frac{\pi}{2}}\,\frac{1}{n-1}$$

$$OPBID = C0 - \alpha\ \sigma_c\ E < Z(1;n) > + \sqrt{\frac{\pi}{2}}\,\frac{1}{n-1}$$

$$OPM0 = \frac{OPBID}{C0} - 1$$

$$PROFIT = OPBID - C0$$

Appendix C

BENJAMIN'S DATA

DATA ON CONTRACTOR'S BIDDING HISTORY

JOB NUMBER	NO. OF BIDDERS	ESTIMATED COST C0($1000)	MARKUP MU0	LOWEST COMPETING LBC(%)	FIRM' BID B0	LOW BID LB
1	5	16.8	15.34	81.97	19.38	13.77
2	7	309.5	5.20	101.77	325.59	314.98
3	9	239.2	5.88	96.16	253.26	230.01
4	3	824.2	5.05	99.50	865.82	820.08
5	5	37.9	9.33	112.90	41.44	41.44
6	5	11.0	19.35	105.90	13.13	11.65
7	4	1377.7	7.99	99.15	1487.78	1365.99
8	5	218.7	4.33	103.72	228.17	226.84
9	8	147.6	5.56	98.83	155.81	145.87
10	5	391.1	7.06	104.43	418.71	408.43
11	9	689.1	4.25	97.37	718.39	670.98
12	6	851.2	4.53	97.51	889.76	830.01
13	5	298.2	5.05	102.37	313.26	305.27
14	4	214.4	5.03	96.03	225.18	205.89
15	6	556.2	4.54	108.30	581.45	581.45
16	6	236.9	6.54	101.30	252.39	239.98
17	6	272.5	3.94	96.58	283.24	263.18
18	6	303.3	7.81	98.86	326.99	299.84
19	13	365.9	3.76	94.01	379.66	343.98
20	5	417.9	4.90	99.05	438.38	413.93
21	11	802.2	5.13	90.57	843.35	726.55
22	7	1188.2	4.34	102.75	1239.77	1220.88
23	7	505.8	3.92	100.37	525.63	507.67
24	7	493.0	12.70	100.96	555.61	497.73
25	8	637.7	4.81	96.29	668.37	614.04
26	3	75.3	5.97	101.22	79.80	76.22
27	6	226.6	3.43	101.43	234.37	229.84
28	6	217.4	5.28	107.61	228.88	228.88
29	9	154.7	4.85	97.84	162.20	151.36
30	6	55.7	4.36	102.91	58.13	57.32
31	5	34.5	9.25	102.80	37.69	35.47
32	8	1066.5	5.37	97.40	1123.77	1038.77
33	2	50.5	6.08	101.97	53.57	51.49
34	3	1246.8	5.50	103.46	1315.37	1289.94
35	5	712.3	4.83	98.13	746.70	698.98
36	10	570.0	4.13	100.13	593.54	570.74
37	4	1307.5	4.97	103.17	1372.48	1348.95
38	7	169.3	3.61	101.84	175.41	172.42
39	4	22.9	12.71	118.18	25.81	25.81
40	6	1053.0	7.19	102.73	1128.71	1081.75
41	7	56.9	9.64	95.73	62.39	54.47
42	7	484.9	4.82	96.88	508.27	469.77

DATA ON CONTRACTOR'S BIDDING HISTORY

JOB NUMBER	NO. OF BIDDERS	ESTIMATED COST C0($1000)	MARKUP MU0	LOWEST COMPETING LBC(%)	FIRM' BID B0	LOW BID LB
43	6	529.9	-0.41	100.01	527.73	527.73
44	6	345.3	5.79	105.90	365.29	365.29
45	10	898.0	7.13	98.77	962.03	886.95
46	9	690.2	4.93	100.98	724.23	696.96
47	4	1043.6	4.99	100.61	1095.68	1049.97
48	5	1189.2	5.65	96.28	1256.39	1144.96
49	7	185.4	5.08	101.38	194.82	187.96
50	5	1219.4	4.42	88.73	1273.30	1081.97
51	6	437.2	3.94	102.07	454.43	446.25
52	6	127.4	6.89	101.21	136.18	128.94
53	4	10.6	14.00	107.46	12.08	11.39
54	6	319.3	4.93	102.63	335.04	327.70
55	8	956.9	5.26	103.35	1007.23	988.96
56	7	207.9	6.90	112.12	222.25	222.25
57	7	359.9	4.11	103.48	374.69	372.42
58	2	40.7	14.72	92.25	46.69	37.55
59	8	614.3	10.15	105.77	676.65	649.75
60	9	99.4	6.36	96.34	105.72	95.76
61	8	312.2	5.61	100.59	329.71	314.04
62	7	129.1	5.86	92.55	136.67	119.48
63	8	133.7	7.48	103.34	143.70	138.17
64	10	1232.7	7.75	95.30	1328.23	1174.76
65	4	87.4	10.79	92.61	96.83	80.94
66	7	122.2	5.89	106.29	129.40	129.40
67	5	1215.4	6.41	106.63	1293.31	1293.31
68	3	333.0	3.20	93.38	343.66	310.96
69	7	1572.0	5.32	97.56	1655.63	1533.64
70	7	1534.3	6.38	95.70	1632.19	1468.33
71	4	12.3	29.32	122.96	15.91	15.12
72	8	685.1	5.90	102.57	725.52	702.71
73	3	271.0	8.39	106.58	293.74	288.83
74	5	69.9	6.73	105.77	74.60	73.93
75	6	310.0	6.11	100.98	328.94	313.04
76	8	644.8	2.91	101.57	663.56	654.92
77	2	33.5	18.61	137.68	39.73	39.73
78	5	137.3	4.66	98.98	143.70	135.90
79	5	199.3	4.07	98.82	207.41	196.95
80	6	38.5	6.01	106.11	40.81	40.81
81	6	489.3	3.95	97.94	508.63	479.22
82	4	417.4	6.75	93.16	445.57	388.85
83	8	409.7	5.25	100.38	431.21	411.26
84	7	6.7	17.02	93.18	7.84	6.24

DATA ON CONTRACTOR'S BIDDING HISTORY

JOB NUMBER	NO. OF BIDDERS	ESTIMATED COST C0($1000)	MARKUP MU0	LOWEST COMPETING LBC(%)	FIRM' BID B0	LOW BID LB
85	5	65.0	7.32	108.74	69.76	69.76
86	9	15.4	10.02	93.07	16.94	14.33
87	8	128.0	4.60	101.21	133.89	129.55
88	8	692.0	4.72	101.61	724.66	703.14
89	5	288.0	5.99	101.72	305.25	292.95
90	3	305.8	5.15	84.57	321.55	258.62
91	7	187.1	3.76	94.54	194.13	176.88
92	9	9.9	9.55	90.94	10.85	9.00
93	11	901.3	5.13	105.65	947.54	947.54
94	12	184.0	4.75	102.43	192.74	188.47
95	5	120.9	4.53	102.43	126.38	123.84
96	9	440.9	4.05	99.29	458.76	437.77
97	10	18.5	7.56	97.70	19.90	18.07
98	2	399.8	6.31	100.05	425.03	400.00
99	8	300.5	6.27	102.96	319.34	309.39
100	7	38.0	2.86	105.15	39.09	39.09
101	9	100.2	9.56	93.42	109.78	93.61
102	8	803.2	3.65	93.38	832.52	750.03
103	6	273.1	6.31	87.88	290.33	240.00
104	5	40.4	8.37	91.60	43.78	37.01
105	7	136.1	3.87	100.27	141.37	136.47
106	6	254.2	5.57	107.54	268.36	268.36
107	7	247.4	4.55	99.74	258.66	246.76
108	2	38.6	9.44	106.24	42.24	41.01
109	4	22.7	18.06	95.54	26.80	21.69
110	4	9.4	10.82	87.78	10.42	8.25
111	6	739.8	3.49	102.40	765.62	757.56
112	8	1230.0	2.30	98.56	1258.29	1212.29
113	10	227.5	4.29	93.46	237.26	212.62
114	9	899.5	7.14	99.33	963.72	893.47
115	7	236.6	6.25	104.38	251.39	246.96
116	5	172.2	4.83	95.62	180.52	164.66
117	5	1235.9	3.92	101.79	1284.35	1258.02
118	5	265.1	3.43	92.40	274.19	244.95
119	10	1501.1	5.10	96.39	1577.66	1446.91
120	8	2005.5	0.22	93.43	2009.91	1873.74
121	4	339.0	4.95	107.18	355.78	355.78
122	4	9.9	14.62	89.72	11.35	8.88
123	8	618.8	2.15	102.75	632.10	632.10
124	10	469.0	4.38	99.78	489.54	467.97
125	9	575.2	4.00	100.14	598.21	576.01
126	3	38.6	6.72	84.73	41.19	32.71

DATA ON CONTRACTOR'S BIDDING HISTORY

JOB NUMBER	NO. OF BIDDERS	ESTIMATED COST CO($1000)	MARKUP MUO	LOWEST COMPETING LBC(%)	FIRM' BID BO	LOW BID LB
127	5	420.7	4.25	106.72	438.58	438.58
128	7	1307.3	5.36	100.20	1377.37	1309.91
129	8	250.7	4.38	110.86	261.68	261.68
130	6	35.0	6.62	108.20	37.32	37.32

Appendix D

EMPIRICAL STUDY 1

REGRESSION ANALYSIS OF EXPECTED LOWEST BID
(COST RANGE: 0 - 100)
($1,000)

ESTIMATED COST CO	LOW BID LB
10.6	11.39
11.0	11.65
16.8	13.77
22.9	25.81
34.5	35.47
37.9	41.44
40.7	37.55
50.5	51.49
55.7	57.32
56.9	54.47
75.3	76.22
87.4	80.94
99.4	95.76

Regression Output:

Constant		1.85
Std Err of Y Est		2.69
R Squared		0.99
No. of Observations		13.00
Degrees of Freedom		11.00
X Coefficient(s)	0.95	
Std Err of Coef.	0.03	

REGRESSION ANALYSIS OF EXPECTED LOWEST BID
(COST RANGE: 100 - 500)
($1,000)

ESTIMATED COST CO	LOW BID LB
122.2	129.4
127.4	128.94
129.1	119.48
133.7	138.17
147.6	145.87
154.7	151.36
169.3	172.42
185.4	187.96
207.9	222.25
214.4	205.89
217.4	228.88
218.7	226.84
226.6	229.84
236.9	239.98
239.2	230.01
272.5	263.18
298.2	305.27
303.3	299.84
309.5	314.98
312.2	314.04
319.3	327.70
345.3	365.29
359.9	372.42
365.9	343.98
391.1	408.43
417.9	413.93
437.2	446.25
484.9	469.77
493.0	497.73

Regression Output:

Constant		1.86
Std Err of Y Est		9.80
R Squared		0.99
No. of Observations		29.00
Degrees of Freedom		27.00
X Coefficient(s)	1.00	
Std Err of Coef.	0.02	

REGRESSION ANALYSIS OF EXPECTED LOWEST BID
(COST RANGE: 500 - 1000)
($1,000)

ESTIMATED COST CO	LOW BID LB
505.8	507.67
529.9	527.73
556.2	581.45
570.0	570.74
614.3	649.75
637.7	614.04
689.1	670.98
690.2	696.96
712.3	698.98
802.2	726.55
824.2	820.08
851.2	830.01
898.0	886.95
956.9	988.96

Regression Output:

Constant		22.66
Std Err of Y Est		28.31
R Squared		0.96
No. of Observations		14.00
Degrees of Freedom		12.00
X Coefficient(s)	0.96	
Std Err of Coef.	0.05	

REGRESSION ANALYSIS OF EXPECTED LOWEST BID
(COST RANGE: 1,000- up)
($1,000)

ESTIMATED COST CO	LOW BID LB
1043.6	1049.97
1053.0	1081.75
1066.5	1038.77
1188.2	1220.88
1189.2	1144.96
1219.4	1081.97
1232.7	1174.76
1246.8	1289.94
1307.5	1348.95
1377.7	1365.99

Regression Output:

Constant		14.88
Std Err of Y Est		60.16
R Squared		0.78
No. of Observations		10.00
Degrees of Freedom		8.00
X Coefficient(s)	0.98	
Std Err of Coef.	0.18	

COEFFICIENTS AND PARAMETERS CALCULATION

JOB NUMBER	STD(LB)	E<Z(1:N)>	VAR<Z1;N>	STD(COST) STD(BID)	W(N)
1	2.69	-1.1630	0.4475	4.02	0.3133
2	9.80	-1.3522	0.3919	15.65	0.2089
3	9.80	-1.4850	0.3574	16.39	0.1567
4	28.31	-0.8463	0.5595	37.85	0.6267
5	2.69	-1.1630	0.4475	4.02	0.3133
6	2.69	-1.1630	0.4475	4.02	0.3133
7	60.16	-1.0290	0.4917	85.79	0.4178
8	9.80	-1.1630	0.4475	14.65	0.3133
9	9.80	-1.4236	0.3729	16.05	0.1790
10	9.80	-1.1630	0.4475	14.65	0.3133
11	28.31	-1.4850	0.3574	47.35	0.1567
12	28.31	-1.2672	0.4159	43.90	0.2507
13	9.80	-1.1630	0.4475	14.65	0.3133
14	9.80	-1.0290	0.4917	13.98	0.4178
15	28.31	-1.2672	0.4159	43.90	0.2507
16	9.80	-1.2672	0.4159	15.20	0.2507
17	9.80	-1.2672	0.4159	15.20	0.2507
18	9.80	-1.2672	0.4159	15.20	0.2507
19	9.80	-1.6680	0.3152	17.46	0.1044
20	9.80	-1.1630	0.4475	14.65	0.3133
21	28.31	-1.5864	0.3332	49.04	0.1253
22	60.16	-1.3522	0.3919	96.10	0.2089
23	28.31	-1.3522	0.3919	45.22	0.2089
24	9.80	-1.3522	0.3919	15.65	0.2089
25	28.31	-1.4236	0.3729	46.36	0.1790
26	2.69	-0.8463	0.5595	3.60	0.6267
27	9.80	-1.2672	0.4159	15.20	0.2507
28	9.80	-1.2672	0.4159	15.20	0.2507
29	9.80	-1.4850	0.3574	16.39	0.1567
30	2.69	-1.2672	0.4159	4.17	0.2507
31	2.69	-1.1630	0.4475	4.02	0.3133
32	60.16	-1.4236	0.3729	98.52	0.1790
33	2.69	-0.5642	0.6817	3.26	1.2533
34	60.16	-0.8463	0.5595	80.43	0.6267
35	28.31	-1.1630	0.4475	42.32	0.3133
36	28.31	-1.5388	0.3443	48.25	0.1393
37	60.16	-1.0290	0.4917	85.79	0.4178
38	9.80	-1.3522	0.3919	15.65	0.2089
39	2.69	-1.0290	0.4917	3.84	0.4178
40	60.16	-1.2672	0.4159	93.29	0.2507
41	2.69	-1.3522	0.3919	4.30	0.2089
42	9.80	-1.3522	0.3919	15.65	0.2089

COEFFICIENTS AND PARAMETERS CALCULATION

JOB NUMBER	STD(LB)	E<Z(1:N)>	VAR<Z1:N>	STD(COST) STD(BID)	W(N)
43	28.31	-1.2672	0.4159	43.90	0.2507
44	9.80	-1.2672	0.4159	15.20	0.2507
45	28.31	-1.5388	0.3443	48.25	0.1393
46	28.31	-1.4850	0.3574	47.35	0.1567
47	60.16	-1.0290	0.4917	85.79	0.4178
48	60.16	-1.1630	0.4475	89.93	0.3133
49	9.80	-1.3522	0.3919	15.65	0.2089
50	60.16	-1.1630	0.4475	89.93	0.3133
51	9.80	-1.2672	0.4159	15.20	0.2507
52	9.80	-1.2672	0.4159	15.20	0.2507
53	2.69	-1.0290	0.4917	3.84	0.4178
54	9.80	-1.2672	0.4159	15.20	0.2507
55	28.31	-1.4236	0.3729	46.36	0.1790
56	9.80	-1.3522	0.3919	15.65	0.2089
57	9.80	-1.3522	0.3919	15.65	0.2089
58	2.69	-0.5642	0.6817	3.26	1.2533
59	28.31	-1.4236	0.3729	46.36	0.1790
60	2.69	-1.4850	0.3574	4.50	0.1567
61	9.80	-1.4236	0.3729	16.05	0.1790
62	9.80	-1.3522	0.3919	15.65	0.2089
63	9.80	-1.4236	0.3729	16.05	0.1790
64	60.16	-1.5388	0.3443	102.53	0.1393
65	2.69	-1.0290	0.4917	3.84	0.4178
66	9.80	-1.3522	0.3919	15.65	0.2089

RESULTS OF GAME AND INTUITIVE MODEL

JOB NUMBER	GAME MODEL OPMO	OPBID	PROFIT	WIN VOL	INTUITIVE MODEL PROFIT	WIN VOL
1	0.2142	20.40	0.00	0.00	0.00	0.00
2	0.0448	323.35	0.00	0.00	0.00	0.00
3	0.0616	253.94	0.00	0.00	0.00	0.00
4	0.0482	863.93	0.00	0.00	0.00	0.00
5	0.0949	41.50	3.60	37.90	3.54	37.90
6	0.3271	14.60	0.00	0.00	0.00	0.00
7	0.0581	1457.68	0.00	0.00	0.00	0.00
8	0.0599	231.81	0.00	0.00	0.00	0.00
9	0.0969	161.90	0.00	0.00	0.00	0.00
10	0.0335	404.21	13.11	391.10	0.00	0.00
11	0.0618	731.68	0.00	0.00	0.00	0.00
12	0.0456	890.02	0.00	0.00	0.00	0.00
13	0.0440	311.31	0.00	0.00	0.00	0.00
14	0.0608	227.43	0.00	0.00	0.00	0.00
15	0.0698	595.02	38.82	556.20	25.25	556.20
16	0.0567	250.34	0.00	0.00	0.00	0.00
17	0.0493	285.94	0.00	0.00	0.00	0.00
18	0.0443	316.74	0.00	0.00	0.00	0.00
19	0.0448	382.28	0.00	0.00	0.00	0.00
20	0.0314	431.01	0.00	0.00	0.00	0.00
21	0.0562	847.25	0.00	0.00	0.00	0.00
22	0.0716	1273.25	0.00	0.00	0.00	0.00
23	0.0791	545.82	0.00	0.00	0.00	0.00
24	0.0281	506.85	0.00	0.00	0.00	0.00
25	0.0648	679.00	0.00	0.00	0.00	0.00
26	0.0501	79.08	0.00	0.00	0.00	0.00
27	0.0593	240.04	0.00	0.00	0.00	0.00
28	0.0618	230.84	13.44	217.40	11.48	217.40
29	0.0953	169.44	0.00	0.00	0.00	0.00
30	0.0662	59.39	0.00	0.00	0.00	0.00
31	0.1043	38.10	0.00	0.00	0.00	0.00
32	0.0823	1154.26	0.00	0.00	0.00	0.00
33	0.0991	55.50	0.00	0.00	0.00	0.00
34	0.0677	1331.23	0.00	0.00	0.00	0.00
35	0.0532	750.17	0.00	0.00	0.00	0.00
36	0.0769	613.84	0.00	0.00	0.00	0.00
37	0.0612	1387.48	0.00	0.00	0.00	0.00
38	0.0818	183.15	0.00	0.00	0.00	0.00
39	0.1562	26.48	3.58	22.90	2.91	22.90
40	0.0783	1135.49	0.00	0.00	0.00	0.00
41	0.0668	60.70	0.00	0.00	0.00	0.00
42	0.0286	498.75	0.00	0.00	0.00	0.00

RESULTS OF GAME AND INTUITIVE MODEL

JOB NUMBER	GAME MODEL OPMO	OPBID	PROFIT	WIN VOL	INTUITIVE MODEL PROFIT	WIN VOL
43	0.0733	568.72	0.00	0.00	-2.17	529.90
44	0.0389	358.74	13.44	345.30	19.99	345.30
45	0.0488	941.84	0.00	0.00	0.00	0.00
46	0.0617	732.78	0.00	0.00	0.00	0.00
47	0.0766	1123.58	0.00	0.00	0.00	0.00
48	0.0677	1269.67	0.00	0.00	0.00	0.00
49	0.0747	199.25	0.00	0.00	0.00	0.00
50	0.0660	1299.87	0.00	0.00	0.00	0.00
51	0.0307	450.64	0.00	0.00	0.00	0.00
52	0.1055	140.84	0.00	0.00	0.00	0.00
53	0.3374	14.18	0.00	0.00	0.00	0.00
54	0.0421	332.74	0.00	0.00	0.00	0.00
55	0.0432	998.20	0.00	0.00	0.00	0.00
56	0.0666	221.75	13.85	207.90	14.35	207.90
57	0.0385	373.75	0.00	0.00	0.00	0.00
58	0.1229	45.70	0.00	0.00	0.00	0.00
59	0.0672	655.60	0.00	0.00	0.00	0.00
60	0.0407	103.45	0.00	0.00	0.00	0.00
61	0.0458	326.50	0.00	0.00	0.00	0.00
62	0.1073	142.95	0.00	0.00	0.00	0.00
63	0.1069	148.00	0.00	0.00	0.00	0.00
64	0.0756	1325.86	0.00	0.00	0.00	0.00
65	0.0409	90.98	0.00	0.00	0.00	0.00
66	0.1134	136.05	0.00	0.00	7.20	122.20
$\alpha =$	0.5000	TOTAL:	99.83	1778.70	82.54	2039.70
	1.0000	PROFIT/VOLUME =		0.0561		0.0405

COEFFICIENTS AND PARAMETERS CALCULATION

JOB NUMBER	STD(LB)	E<Z(1:N)>	VAR<Z1:N>	STD(COST) STD(BID)	W(N)
67	60.16	-1.1630	0.4475	89.93	0.3133
68	9.80	-0.8463	0.5595	13.10	0.6267
69	60.16	-1.3522	0.3919	96.10	0.2089
70	60.16	-1.3522	0.3919	96.10	0.2089
71	2.69	-1.0290	0.4917	3.84	0.4178
72	28.31	-1.4236	0.3729	46.36	0.1790
73	9.80	-0.8463	0.5595	13.10	0.6267
74	2.69	-1.1630	0.4475	4.02	0.3133
75	9.80	-1.2672	0.4159	15.20	0.2507
76	28.31	-1.4236	0.3729	46.36	0.1790
77	2.69	-0.5642	0.6817	3.26	1.2533
78	9.80	-1.1630	0.4475	14.65	0.3133
79	9.80	-1.1630	0.4475	14.65	0.3133
80	2.69	-1.2672	0.4159	4.17	0.2507
81	9.80	-1.2672	0.4159	15.20	0.2507
82	9.80	-1.0290	0.4917	13.98	0.4178
83	9.80	-1.4236	0.3729	16.05	0.1790
84	2.69	-1.3522	0.3919	4.30	0.2089
85	2.69	-1.1630	0.4475	4.02	0.3133
86	2.69	-1.4850	0.3574	4.50	0.1567
87	9.80	-1.4236	0.3729	16.05	0.1790
88	28.31	-1.4236	0.3729	46.36	0.1790
89	9.80	-1.1630	0.4475	14.65	0.3133
90	9.80	-0.8463	0.5595	13.10	0.6267
91	9.80	-1.3522	0.3919	15.65	0.2089
92	2.69	-1.4850	0.3574	4.50	0.1567
93	28.31	-1.5864	0.3332	49.04	0.1253
94	9.80	-1.6292	0.3236	17.23	0.1139
95	9.80	-1.1630	0.4475	14.65	0.3133
96	9.80	-1.4850	0.3574	16.39	0.1567
97	2.69	-1.5388	0.3443	4.58	0.1393
98	9.80	-0.5642	0.6817	11.87	1.2533
99	9.80	-1.4236	0.3729	16.05	0.1790
100	2.69	-1.3522	0.3919	4.30	0.2089
101	9.80	-1.4850	0.3574	16.39	0.1567
102	28.31	-1.4236	0.3729	46.36	0.1790
103	9.80	-1.2672	0.4159	15.20	0.2507
104	2.69	-1.1630	0.4475	4.02	0.3133
105	9.80	-1.3522	0.3919	15.65	0.2089
106	9.80	-1.2672	0.4159	15.20	0.2507
107	9.80	-1.3522	0.3919	15.65	0.2089
108	2.69	-0.5642	0.6817	3.26	1.2533

COEFFICIENTS AND PARAMETERS CALCULATION

JOB NUMBER	STD(LB)	E<Z(1:N)>	VAR<Z1:N>	STD(COST) STD(BID)	W(N)
109	2.69	-1.0290	0.4917	3.84	0.4178
110	2.69	-1.0290	0.4917	3.84	0.4178
111	28.31	-1.2672	0.4159	43.90	0.2507
112	60.16	-1.4236	0.3729	98.52	0.1790
113	9.80	-1.5388	0.3443	16.70	0.1393
114	28.31	-1.4850	0.3574	47.35	0.1567
115	9.80	-1.3522	0.3919	15.65	0.2089
116	9.80	-1.1630	0.4475	14.65	0.3133
117	60.16	-1.1630	0.4475	89.93	0.3133
118	9.80	-1.1630	0.4475	14.65	0.3133
119	60.16	-1.5388	0.3443	102.53	0.1393
120	60.16	-1.4236	0.3729	98.52	0.1790
121	9.80	-1.0290	0.4917	13.98	0.4178
122	2.69	-1.0290	0.4917	3.84	0.4178
123	28.31	-1.4236	0.3729	46.36	0.1790
124	9.80	-1.5388	0.3443	16.70	0.1393
125	28.31	-1.4850	0.3574	47.35	0.1567
126	2.69	-0.8463	0.5595	3.60	0.6267
127	9.80	-1.1630	0.4475	14.65	0.3133
128	60.16	-1.3522	0.3919	96.10	0.2089
129	9.80	-1.4236	0.3729	16.05	0.1790
130	2.69	-1.2672	0.4159	4.17	0.2507

RESULTS OF GAME AND INTUITIVE MODEL

JOB	GAME MODEL				INTUITIVE MODEL	
NUMBER	OPMO	OPBID	PROFIT	WIN VOL	PROFIT	WIN VOL
67	0.0662	1295.87	80.47	1215.40	77.91	1215.40
68	0.0413	346.75	0.00	0.00	0.00	0.00
69	0.0541	1657.05	0.00	0.00	0.00	0.00
70	0.0554	1619.35	0.00	0.00	0.00	0.00
71	0.2908	15.88	0.00	0.00	0.00	0.00
72	0.0603	726.40	0.00	0.00	0.00	0.00
73	0.0508	284.75	13.75	271.00	0.00	0.00
74	0.0515	73.50	3.60	69.90	0.00	0.00
75	0.0433	323.44	0.00	0.00	0.00	0.00
76	0.0641	686.10	0.00	0.00	0.00	0.00
77	0.1493	38.50	5.00	33.50	6.23	33.50
78	0.0955	150.41	0.00	0.00	0.00	0.00
79	0.0658	212.41	0.00	0.00	0.00	0.00
80	0.0958	42.19	0.00	0.00	2.31	38.50
81	0.0275	502.74	0.00	0.00	0.00	0.00
82	0.0312	430.43	0.00	0.00	0.00	0.00
83	0.0349	424.00	0.00	0.00	0.00	0.00
84	0.5676	10.50	0.00	0.00	0.00	0.00
85	0.0554	68.60	3.60	65.00	4.76	65.00
86	0.2627	19.45	0.00	0.00	0.00	0.00
87	0.1117	142.30	0.00	0.00	0.00	0.00
88	0.0597	733.30	0.00	0.00	0.00	0.00
89	0.0455	301.11	0.00	0.00	0.00	0.00
90	0.0450	319.55	0.00	0.00	0.00	0.00
91	0.0740	200.95	0.00	0.00	0.00	0.00
92	0.4087	13.95	0.00	0.00	0.00	0.00
93	0.0500	946.35	45.05	901.30	46.24	901.30
94	0.0869	200.00	0.00	0.00	0.00	0.00
95	0.1084	134.01	0.00	0.00	0.00	0.00
96	0.0334	455.64	0.00	0.00	0.00	0.00
97	0.2252	22.67	0.00	0.00	0.00	0.00
98	0.0456	418.02	0.00	0.00	0.00	0.00
99	0.0476	314.80	0.00	0.00	0.00	0.00
100	0.1001	41.80	0.00	0.00	1.09	38.00
101	0.1471	114.94	0.00	0.00	0.00	0.00
102	0.0514	844.50	0.00	0.00	0.00	0.00
103	0.0492	286.54	0.00	0.00	0.00	0.00
104	0.0891	44.00	0.00	0.00	0.00	0.00
105	0.1018	149.95	0.00	0.00	0.00	0.00
106	0.0529	267.64	13.44	254.20	14.16	254.20
107	0.0560	261.25	0.00	0.00	0.00	0.00
108	0.1296	43.60	0.00	0.00	0.00	0.00

RESULTS OF GAME AND INTUITIVE MODEL

JOB NUMBER	GAME MODEL OPMO	OPBID	PROFIT	WIN VOL	INTUITIVE MODEL PROFIT	WIN VOL
109	0.1576	26.28	0.00	0.00	0.00	0.00
110	0.3805	12.98	0.00	0.00	0.00	0.00
111	0.0525	778.62	0.00	0.00	0.00	0.00
112	0.0714	1317.76	0.00	0.00	0.00	0.00
113	0.0667	242.68	0.00	0.00	0.00	0.00
114	0.0473	942.08	0.00	0.00	0.00	0.00
115	0.0586	250.45	0.00	0.00	0.00	0.00
116	0.0761	185.31	0.00	0.00	0.00	0.00
117	0.0651	1316.37	0.00	0.00	0.00	0.00
118	0.0494	278.21	0.00	0.00	0.00	0.00
119	0.0621	1594.26	0.00	0.00	0.00	0.00
120	0.0438	2093.26	0.00	0.00	0.00	0.00
121	0.0384	352.03	13.03	339.00	16.78	339.00
122	0.3613	13.48	0.00	0.00	0.00	0.00
123	0.0667	660.10	0.00	0.00	13.30	618.80
124	0.0324	484.18	0.00	0.00	0.00	0.00
125	0.0740	617.78	0.00	0.00	0.00	0.00
126	0.0978	42.38	0.00	0.00	0.00	0.00
127	0.0312	433.81	13.11	420.70	17.88	420.70
128	0.0651	1392.35	0.00	0.00	0.00	0.00
129	0.0570	265.00	14.30	250.70	10.98	250.70
130	0.1054	38.69	0.00	0.00	2.32	35.00
$\alpha =$	0.5000	TOTAL:	205.35	3820.70	213.96	4210.10
	1.0000	PROFIT/VOLUMN =		0.0537		0.0508

Appendix E

BROEMSER'S DATA

DATA ON CONTRACTOR'S BIDDING HISTORY
(BROEMSER'S DATA SET)

JOB NO.	NO. OF BIDDERS	ESTIMATED COST CO($1000)	LOWEST COM. MU. LMU	LOWEST COM. BID LB
1	5	3191.0	100.90	3219.72
2	8	380.9	99.93	380.63
3	4	288.9	91.11	263.22
4	10	1039.4	100.95	1049.27
5	4	1302.5	100.46	1308.49
6	11	659.4	102.18	673.77
7	12	1394.0	97.17	1354.55
8	4	1217.8	85.82	1045.12
9	8	239.9	101.43	243.33
10	6	98.8	102.96	101.72
11	9	270.9	95.91	259.82
12	9	2105.5	102.27	2153.29
13	8	676.7	103.32	699.17
14	8	1444.8	94.44	1364.47
15	7	1422.8	96.49	1372.86
16	13	3097.0	102.09	3161.73
17	10	829.5	103.37	857.45
18	2	559.3	102.38	572.61
19	7	2391.9	99.24	2373.72
20	5	4228.7	107.25	4535.28
21	10	5257.3	102.57	5392.41
22	10	1566.8	91.48	1433.31
23	10	81.6	89.25	72.83
24	8	1234.0	104.29	1286.94
25	9	129.0	96.58	124.59
26	7	115.3	91.38	105.36
27	11	1120.2	91.15	1021.06
28	3	2493.1	109.30	2724.96
29	10	1707.7	103.29	1763.88
30	7	925.8	106.44	985.42
31	9	503.7	99.03	498.81
32	10	824.6	110.80	913.66
33	9	592.5	93.33	552.98
34	7	222.3	95.91	213.21
35	6	3449.1	99.88	3444.96
36	8	2254.7	103.30	2329.11
37	14	1073.9	95.23	1022.67
38	4	1802.7	101.68	1832.99
39	6	1298.0	103.31	1340.96
40	4	3731.1	96.03	3582.98
41	7	789.4	106.46	840.40
42	4	1560.8	98.54	1538.01

DATA ON CONTRACTOR'S BIDDING HISTORY
(BROEMSER'S DATA SET)

JOB NO.	NO. OF BIDDERS	ESTIMATED COST CO($1000)	LOWEST COM. MU. LMU	LOWEST COM. BID LB
43	5	223.3	100.43	224.26
44	5	233.4	102.42	239.05
45	9	89.1	110.93	98.84
46	5	155.6	100.25	155.99
47	9	3892.5	96.65	3762.10
48	7	405.6	97.42	395.14
49	10	493.3	86.70	427.69
50	4	1726.6	91.39	1577.94
51	6	2105.4	102.31	2154.03
52	5	3721.8	91.47	3404.33
53	4	2035.6	102.18	2079.98
54	4	207.4	105.30	218.39
55	5	150.7	96.57	145.53
56	5	168.6	103.10	173.83
57	7	140.2	86.04	120.63
58	5	137.1	101.19	138.73
59	6	822.0	101.15	831.45
60	8	2545.5	104.17	2651.65
61	7	655.8	99.36	651.60
62	4	575.7	103.85	597.86
63	5	1015.1	101.65	1031.85
64	3	224.1	99.53	223.05
65	7	1020.5	101.20	1032.75
66	4	310.8	102.51	318.60
67	6	250.0	105.48	263.70
68	5	1320.4	87.21	1151.52
69	6	1126.7	98.84	1113.63
70	3	2684.3	99.26	2664.44
71	6	2790.1	97.71	2726.21
72	5	237.6	97.52	231.71
73	7	1291.4	91.84	1186.02
74	10	1920.5	98.37	1889.20
75	7	3409.0	93.12	3174.46
76	6	1257.7	105.35	1324.99

Appendix F

EMPIRICAL STUDY 2

COEFFICIENTS PARAMETERS CALCULATION

JOB NO.	STD(B1)	E<Z(1:N)>	VAR<Z1:N>	STD(COST) STD(BID)	W(N)
1	165.29	-1.1630	0.4475	247.09	0.3133
2	19.73	-1.4236	0.3729	32.31	0.1790
3	14.97	-1.0290	0.4917	21.34	0.4178
4	53.84	-1.5388	0.3443	91.76	0.1393
5	67.47	-1.0290	0.4917	96.22	0.4178
6	34.16	-1.5864	0.3332	59.17	0.1253
7	72.21	-1.6292	0.3236	126.94	0.1139
8	63.08	-1.0290	0.4917	89.96	0.4178
9	12.43	-1.4236	0.3729	20.35	0.1790
10	5.12	-1.2672	0.4159	7.94	0.2507
11	14.03	-1.4850	0.3574	23.47	0.1567
12	109.06	-1.4850	0.3574	182.43	0.1567
13	35.05	-1.4236	0.3729	57.40	0.1790
14	74.84	-1.4236	0.3729	122.56	0.1790
15	73.70	-1.3522	0.3919	117.73	0.2089
16	160.42	-1.6680	0.3152	285.74	0.1044
17	42.97	-1.5388	0.3443	73.23	0.1393
18	28.97	-0.5642	0.6817	35.09	1.2533
19	123.90	-1.3522	0.3919	197.92	0.2089
20	219.05	-1.1630	0.4475	327.45	0.3133
21	272.33	-1.5388	0.3443	464.11	0.1393
22	81.16	-1.5388	0.3443	138.32	0.1393
23	4.23	-1.5388	0.3443	7.20	0.1393
24	63.92	-1.4236	0.3729	104.68	0.1790
25	6.68	-1.4850	0.3574	11.18	0.1567
26	5.97	-1.3522	0.3919	9.54	0.2089
27	58.03	-1.5864	0.3332	100.52	0.1253
28	129.14	-0.8463	0.5595	172.65	0.6267
29	88.46	-1.5388	0.3443	150.76	0.1393
30	47.96	-1.3522	0.3919	76.61	0.2089
31	26.09	-1.4850	0.3574	43.64	0.1567
32	42.71	-1.5388	0.3443	72.80	0.1393
33	30.69	-1.4850	0.3574	51.34	0.1567
34	11.52	-1.3522	0.3919	18.39	0.2089
35	178.66	-1.2672	0.4159	277.04	0.2507
36	116.79	-1.4236	0.3729	191.26	0.1790
37	55.63	-1.6680	0.3152	99.08	0.0964
38	93.38	-1.0290	0.4917	133.17	0.4178
39	67.24	-1.2672	0.4159	104.26	0.2507
40	193.27	-1.0290	0.4917	275.62	0.4178
41	40.89	-1.3522	0.3919	65.32	0.2089
42	80.85	-1.0290	0.4917	115.30	0.4178

COEFFICIENTS PARAMETERS CALCULATION

JOB NO.	STD(B1)	E<Z(1:N)>	VAR<Z1:N>	STD(COST) STD(BID)	W(N)
43	11.57	-1.1630	0.4475	17.29	0.3133
44	12.09	-1.1630	0.4475	18.07	0.3133
45	4.62	-1.4850	0.3574	7.72	0.1567
46	8.06	-1.1630	0.4475	12.05	0.3133
47	201.63	-1.4850	0.3574	337.27	0.1567
48	21.01	-1.3522	0.3919	33.56	0.2089
49	25.55	-1.5388	0.3443	43.55	0.1393
50	89.44	-1.0290	0.4917	127.55	0.4178
51	109.06	-1.2672	0.4159	169.11	0.2507
52	192.79	-1.1630	0.4475	288.19	0.3133
53	105.44	-1.0290	0.4917	150.37	0.4178
54	10.74	-1.0290	0.4917	15.32	0.4178
55	7.81	-1.1630	0.4475	11.67	0.3133
56	8.73	-1.1630	0.4475	13.06	0.3133
57	7.26	-1.3522	0.3919	11.60	0.2089
58	7.10	-1.1630	0.4475	10.62	0.3133
59	42.58	-1.2672	0.4159	66.02	0.2507
60	131.86	-1.4236	0.3729	215.93	0.1790
61	33.97	-1.3522	0.3919	54.26	0.2089
62	29.82	-1.0290	0.4917	42.53	0.4178
63	52.58	-1.1630	0.4475	78.60	0.3133
64	11.61	-0.8463	0.5595	15.52	0.6267
65	52.86	-1.3522	0.3919	84.44	0.2089
66	16.10	-1.0290	0.4917	22.96	0.4178
67	12.95	-1.2672	0.4159	20.08	0.2507
68	68.40	-1.1630	0.4475	102.24	0.3133
69	58.36	-1.2672	0.4159	90.50	0.2507
70	139.05	-0.8463	0.5595	185.89	0.6267
71	144.53	-1.2672	0.4159	224.11	0.2507
72	12.31	-1.1630	0.4475	18.40	0.3133
73	66.89	-1.3522	0.3919	106.86	0.2089
74	99.48	-1.5388	0.3443	169.54	0.1393
75	176.59	-1.3522	0.3919	282.08	0.2089
76	65.15	-1.2672	0.4159	101.02	0.2507

RESULTS OF GAME MODEL

JOB NO.	OPMO	OPBID	PROFIT	WIN VOL
1	0.0693	3412.11	0.00	0.00
2	0.0756	409.68	0.00	0.00
3	0.0689	308.80	0.00	0.00
4	0.0802	1122.78	0.00	0.00
5	0.0689	1392.20	0.00	0.00
6	0.0824	713.75	0.00	0.00
7	0.0846	1511.87	0.00	0.00
8	0.0689	1301.67	0.00	0.00
9	0.0756	258.03	0.00	0.00
10	0.0710	105.82	0.00	0.00
11	0.0779	292.01	0.00	0.00
12	0.0779	2269.54	0.00	0.00
13	0.0756	727.84	0.00	0.00
14	0.0756	1553.98	0.00	0.00
15	0.0732	1526.99	0.00	0.00
16	0.0866	3365.15	0.00	0.00
17	0.0802	896.04	0.00	0.00
18	0.0963	613.18	0.00	0.00
19	0.0732	2567.05	0.00	0.00
20	0.0693	4521.71	293.01	4228.70
21	0.0802	5679.02	0.00	0.00
22	0.0802	1692.48	0.00	0.00
23	0.0802	88.15	0.00	0.00
24	0.0756	1327.25	0.00	0.00
25	0.0779	139.05	0.00	0.00
26	0.0732	123.74	0.00	0.00
27	0.0824	1212.54	0.00	0.00
28	0.0727	2674.35	181.25	2493.10
29	0.0802	1844.68	0.00	0.00
30	0.0732	993.59	0.00	0.00
31	0.0779	542.94	0.00	0.00
32	0.0802	890.75	66.15	824.60
33	0.0779	638.66	0.00	0.00
34	0.0732	238.58	0.00	0.00
35	0.0710	3694.08	0.00	0.00
36	0.0756	2425.08	0.00	0.00
37	0.0858	1166.09	0.00	0.00
38	0.0689	1926.85	0.00	0.00
39	0.0710	1390.19	0.00	0.00
40	0.0689	3988.06	0.00	0.00
41	0.0732	847.21	0.00	0.00
42	0.0689	1668.29	0.00	0.00

RESULTS OF GAME MODEL

JOB NO.	OPMO	OPBID	PROFIT	WIN VOL
43	0.0693	238.77	0.00	0.00
44	0.0693	249.57	0.00	0.00
45	0.0779	96.04	6.94	89.10
46	0.0693	166.38	0.00	0.00
47	0.0779	4195.76	0.00	0.00
48	0.0732	435.30	0.00	0.00
49	0.0802	532.87	0.00	0.00
50	0.0689	1845.51	0.00	0.00
51	0.0710	2254.94	0.00	0.00
52	0.0693	3979.69	0.00	0.00
53	0.0689	2175.79	0.00	0.00
54	0.0689	221.68	0.00	0.00
55	0.0693	161.14	0.00	0.00
56	0.0693	180.28	0.00	0.00
57	0.0732	150.47	0.00	0.00
58	0.0693	146.60	0.00	0.00
59	0.0710	880.38	0.00	0.00
60	0.0756	2737.86	0.00	0.00
61	0.0732	703.82	0.00	0.00
62	0.0689	615.35	0.00	0.00
63	0.0693	1085.44	0.00	0.00
64	0.0727	240.39	0.00	0.00
65	0.0732	1095.23	0.00	0.00
66	0.0689	332.20	0.00	0.00
67	0.0710	267.76	0.00	0.00
68	0.0693	1411.89	0.00	0.00
69	0.0710	1206.72	0.00	0.00
70	0.0727	2879.45	0.00	0.00
71	0.0710	2988.27	0.00	0.00
72	0.0693	254.06	0.00	0.00
73	0.0732	1385.97	0.00	0.00
74	0.0802	2074.55	0.00	0.00
75	0.0732	3658.63	0.00	0.00
76	0.0710	1347.03	0.00	0.00
		$\alpha =$ 0.50	TOTAL:	
		1.00		
			547.35	7635.50

P/V RATIO 0.0717

Appendix G

ORDER STATISTICS TABLE

EXPECTED VALUES AND VARIANCES OF FIRST ORDER STATISTICS
FROM STANDARD NORMAL POPULATION

n	E<Z(1;n)>	VAR<Z(1;n)>
2	-0.56419	0.68169
3	-0.84628	0.55947
4	-1.02938	0.49172
5	-1.16296	0.44753
6	-1.26721	0.41593
7	-1.35218	0.39192
8	-1.42360	0.37290
9	-1.48501	0.35735
10	-1.53875	0.34434
11	-1.58644	0.33325
12	-1.62923	0.32364
13	-1.66799	0.31521
14	-1.70338	0.30773
15	-1.73591	0.30104
16	-1.76599	0.29501
17	-1.79394	0.28953
18	-1.82003	0.28453
19	-1.84448	0.27994
20	-1.86748	0.27570

SOURCE: P. R. Krishnaiah, P. K. Sen
"Handbook of Statistics -- Nonparametric Methods"
Volume 4, North Holland, 1984

Bibliography

[1] Irving Alpert. *Competitive Bidding on Publicly - Advertised Construction Projects.* PhD thesis, New York University, Graduate School of Business Administration, 1973.

[2] Lee J. Bain and Max Engelhardt. *Introduction to Probability and Mathematical Statistics.* Volume of , Duxbury Press, edition, 1987.

[3] Neal Bernard Hugh Benjamin. *Competitive Bidding for Building Construction Contracts.* PhD thesis, Stanford University, 1969.

[4] Gary Milton Broemser. *Competitive Bidding in the Construction Industry.* PhD thesis, Stanford University, 1968.

[5] Robert I. Carr. General bidding model. *Journal of the Construction Division, ASCE*, 108(CO4):639–650, December 1982.

[6] Richard Engelbrecht-Wiggans. Auctions and bidding models: a survey. *Management Science*, 26(2):119–142, February 1980.

[7] Lawrence Friedman. *Competitive Bidding Strategies.* PhD thesis, Case Institute of Technology, 1957.

[8] Lawrence Friedman. A competitive-bidding strategy. *Operations Research*, 4():104–112, 1956.

[9] Janos Galambos. *The Asymptotic Theory of Extreme Order Statistics.* Volume of , John Wiley, edition, 1978.

[10] Marvin Gates. Bidding strategies and probabilities. *Journal of the Construction Division*, 93(CO1):75–107, March 1967.

[11] Charles H. Gould. *Fundamentals of Construction Cost Estimating.* Unpublished manuscript, New Jersey Institute of Technology, 1988.

[12] E. J. Gumbel. *Statistics of Extremes.* Volume of , Columbia University Press, edition, 1959.

[13] P. R. Krishnaiah and P. K. Sen. *Handbook of Statistics – Nonparametric Methods.* Volume 4, North Holland, 1984.

[14] R. Preston McAfee and John Mcmillan. Auction and bidding. *Journal of Economic Literature*, XXV():699–738, June 1987.

[15] R. McCaffer and A. N. Pettitt. Distribution of bids for building and roads contracts. *Operational Research Quarterly*, 27(4.i):835–43, 1976.

[16] K. Sridhar Moorthy. Using game theory to model competition. *Journal of Marketing Research*, XXII():262–82, August 1985.

[17] J. Nash. The bargaining problem. *Econometrica*, 18():155–162, 1950.

[18] Armando Ortega-Reichert. *Models for Competitive Bidding under Uncertainty.* PhD thesis, Stanford University, 1968.

[19] W. R. Park. *The Strategy of Contracting for Profit.* Volume of , Prentice-Hall,inc., Englewood Cliffs, N.J., edition, 1966.

[20] Stuart E. Thiel. Some evidence on the winner's curse. *The American Economic Review*, 78(5):884–895, December 1988.

[21] J. Von Neumann and O. Morgenstern. *Theory of Games and Economics Behavior.* Volume of , Princeton, edition, 1944.

[22] Richard Louis Wade and Robert B. Harris. Lomark: a bidding strategy. *Journal of the Construction Division, ASCE*, 102(CO1):197–211, March 1976.