CIRCULAR PLATE ANALYSIS USING FINITE ELEMENT METHOD

by

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ABSTRACT

Title of Thesis: Circular Plate Analysis Using Finite Element Method

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The ANSYS computer program is the application of finite element methods to large-scale engineering problems. In this work the ANSYS program has been employed to solve the circular plate under uniformly loaded with various boundary conditions as follows: (1) clamped along entire edge, (2) simply supported along entire edge, (3) clamped at several points along the boundary, (4) simply supported at several points along the boundary, and (5) simply supported 36 points with 4 points clamped.

The results obtained from the ANSYS program illustrated that the finite element method has no boundary conditions constraints and presents good approximations to the exact solutions derived by differential equations. These examples also have clearly shown that the results converge to the exact solutions when the number of elements is increased.

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NOTATION

[ ]	A rectangular or square matrix or a row vector
{ }	A column vector
$[\ ]^T$	Matrix transpose
A	Area
a	Radius of circular plate
[B]	The strain-displacement matrix
D	Flexural rigidity
[D]	The stress-strain matrix
E	Modulus of elasticity
{f}	Forces applied by element to nodes(nodal element forces)
G	Modulus of elasticity in shear
h	Thickness
[k]	Element stiffness matrix
M_x , M_y	Bending moments per unit length on x and y planes
M_{xy}	Twisting moment per unit length on x plane
M_r , M_t	Radial and tangential moments per unit length
M_{rt}	Twisting moment per unit length on radial plane
[N]	Matrix of shape function
O	Origin of coordinates
q	Intensity of a continuously distributed load
Q	Shear force per unit length

Q_x , Q_y	Shear force per unit length on x and y planes
Q_r , Q_t	Radial and tangential shear forces per unit length
r	Radial distances of points in the middle plane of plate
r_n , r_t	Radii of curvature of midsurface
SX	Direct stress in x direction
SY	Direct stress in y direction
SXY	Shear stress
SIG1,SIG2,SIG3	Principal stresses
SI	Stress intensity, SI = MAX (|SIG1 - SIG2|, |SIG2 - SIG3|, |SIG3 - SIG1|)
SIGE	Equivalent stress, $SIGE = \{ \frac{1}{2} [(SIG1 - SIG2)^2 + (SIG2 - SIG3)^2 + (SIG3 - SIG1)^2] \}$
$\overline{U}$	Strain energy
{u}	Displacement matrix
{U}	Displacements at nodal points
u, v, w	Components of displacements in x, y, and z directions
V	Volume
W	Work done by external forces
w_c	Center deflection of circular plate
x, y, z	Rectangular coordinate
X, Y, Z	Components of body forces in the x, y, and z directions
$\overline{X}$, $\overline{Y}$, $\overline{Z}$	Components of surface tractions in the x, y, and z directions
θ	Angle

ϕ	Slope
ν	Poission's ratio
σ_x , σ_y , σ_z	Normal components of stresses on the x, y, and z planes
σ_r , σ_t	Radial and tangential normal stresses
ε_x , ε_y , ε_z	Normal strains in x, y, and z directions
γ_{xy} , γ_{yz} , γ_{zx}	Shear strains on the xy, yz, and zx planes
τ_{xy} , τ_{yz} , τ_{zx}	Shear stresses components on the x, y, and z direction
τ_{rt}	Shear stress on radial plane and parallel to the tangential plane
II	Total potential energy
$\{\sigma\}$	Stresses
$\{\varepsilon\}$	Strains

CHAPTER I

INTRODUCTION

The problem of mechanical strength is one of the most important features of the design of structures. Consequently, the objective of mechanical analysis is the determination of the stresses, strains, and deformations produced by the loads. In classical methods, a field problem is usually described by a set of differential equations with proper boundary conditions, or by the extremum of a variational principle, if it exists, or by some forms of variational statements (incomplete variational principle). The solution sought for in classical methods usually possesses high-order differentiablity, satisfies the differential equations everywhere, and satisfies all the boundary conditions.

In practice, many practical problems in engineering are either extremely difficult or impossible to solve by traditional mathematical method and has to rely on numerical analyses. The subject of numerical analysis is concerned with devising methods for approximating, in an efficient manner, the solutions to mathematically expressed problems. The finite element method is a powerful numerical analysis technique for obtaining approximate solutions to the mathematical problems of physics and engineerings that are much difficult to obtain by analytical methods. The finite element method not only overcomes the shortcoming of the traditional variation methods, it is also endowed with the features of an effective computational techniques.

Finite element methods were originated in the field of structural analysis and were widely developed and exploited in the aerospace industries during the '50s and '60s. Finite element methods are also widely used by mechanical engineers, particularly for the analysis of stress in solid components, plates and shells, vibrations, buckling of structures, elastic-plastic behavior, fluid mechanics and heat transfer. All finite element methods involve dividing the physical systems, such as structures, solid or fluid continua, into small subregions or elements. Each element is an essentially simple unit, the behavior of which can be readily analyzed.

The major objective of this thesis is to apply ANSYS program, which is the application of finite element methods to large-scale engineering problems, to analyze the stresses and displacements of circular plates under various boundary conditions. In addition, it includes the comparision of the approximate solutions of ANSYS program with the exact solutions of the governing differential equations. This type of problem has wide application in practical engineering systems which consists of cylinder with end plates.

The basic equations of uniformly loaded circular plate with various boundary constraints are discussed in Chapter

II. Chapter III considers the general formulation of the finite element displacement method of plane elasticity. It employs the potential energy method to formulate the element stiffness equations. The circular plate under various boundary conditions solved by ANSYS program will be developed in Chapter IV. The input data to ANSYS program and selected portions of the output for uniformly loaded circular plates are listed in Appendices.

CHAPTER II

BASIC EQUATIONS OF CIRCULAR PLATE-BENDING

In this chapter, the governing equations for the circular plate bending are discussed. The basic equation of plate theory is a differential equation of the fourth order linking the displacement of the middle plane w to the load q. The derivation of the governing equation for deflection of circular plate from the stress-strain relations and plane-stress is given in Appendix A.

A. Differential Equation for Symmetrical Bending of Laterally Loaded Circular Plates.

If a circular plate is loaded symmetrically distributed about the axis perpendicular to the plate through its center, the deflection surface of the plate is also symmetrical. At all points, equal distance from the center of the plate, the deflections will be the same and a consideration of a diametral section through this axis is sufficient for calculating deflections and stresses. Fig.1 represents such a diametral section with the axis of symmetry OZ. The deflection of the plate w in the Z direction will depend upon radial position r only when the applied load and end restraints are independent of the angle θ. The situation described is the axisymmetrical bending of the plate. Let ϕ denote the maximum slope of the deflection surface at any point A, which is then equal to

$$\phi = - \frac{dw}{dr} \tag{1}$$

and the curvature of the plate in the diametral section rZ is

$$\frac{1}{r_n} = - \frac{d^2w}{dr^2} = \frac{d\phi}{dr} \tag{2}$$

In determining the radius of curvature, which we denote by r_t, in the section through the normal AB and perpendicular to the rZ plane, it is necessary to note that after deflection of the plate, the normals, such as AB form a conical surface with apex B. Then the length of AB represents the radius of curvature r_t, and from (Fig. 1), we obtain

$$\frac{1}{r_t} = - \frac{1}{r}\frac{dw}{dr} = \frac{\phi}{r} \tag{3}$$

For axisymmetrical bending, we assume that the effect of shear on bending is negligible and that the relation between the bending moments and the curvatures is the same as in pure bending of a plate. The material of the plate is assumed to be linearly elastic with Young's modulus E and Poisson's ratio ν, and the flexural rigidity of the plate is given by

$$D = \frac{Eh^3}{12\ (1 - \nu^2)} \tag{4}$$

where h denotes the thickness of the plate. The moments and shear force in an axisymmetrically loaded circular plate can be obtained by using the above relations

$$M_r = - D\left(\frac{d^2w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right) = D\left(\frac{d}{dr} + \frac{\nu}{r} \phi \right) \tag{5}$$

$$M_t = - D\left(\frac{1}{r} \frac{dw}{dr} + \nu \frac{d^2w}{dr^2} \right) = D\left(\frac{\phi}{r} + \nu \frac{d\phi}{dr} \right) \tag{6}$$

$$Q = - D \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (r\phi) \right] \tag{7a}$$

$$Q = - D \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr}\right) \right] \tag{7b}$$

In these equations M_r and M_t denote bending moments per unit length, and Q represents the shearing force per unit length. The moment M_r acts along circumferential sections of the plate, such as the section made by the conical surface with the apex at B, and M_t acts along the diametral section rZ of the plate.

B. Uniformly Loaded Circular Plates.

If a circular plate with radius a under a load of intensity q uniformly distributed over the entire surface of the plate, the shearing force Q at a distance r from the center of the plate is determined from the equation

$$2\pi r Q = \pi r^2 q \tag{8}$$

from which

$$Q = \frac{qr}{2} \tag{9}$$

Substituting in Eq.(7b), we obtain

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{qr}{2D} \tag{10}$$

The deflection w is obtained by successive integrations when q is given

$$w = \frac{qr^4}{64D} + \frac{C_1 r^2}{4} + C_2 \log\frac{r}{a} + C_3 \tag{11}$$

where C_1, C_2, C_3, are constants of integration, and must be determined in each particular case from the conditions at the edge of the plate.

(1) Circular Plate with Clamped Edges (Fig. 2).

In this case the boundary conditions are

$$w = 0 \qquad \text{at} \qquad r = a$$

and

$$\frac{dw}{dr} = 0 \qquad \text{at} \qquad r = a \qquad \text{and} \qquad r = 0$$

Substituting these conditions in Eq.(11), we find

$$C_1 = -\frac{qa^2}{8D}$$

$$C_2 = 0$$

$$C_3 = \frac{qa^4}{64D}$$

The deflection of such a plate is then

$$w = \frac{q}{64D} \left(a^2 - r^2 \right)^2 \tag{12}$$

The maximum deflection occurs at the center ($r = 0$) of the plate and, from Eq.(12), is equal to

$$w_{max} = \frac{qa^4}{64D} \tag{13}$$

Substituting Eq.(12) into Eq.(5) and Eq.(6), we find

$$M_r = \frac{q}{16} [a^2(1 + \nu) - r^2(3 + \nu)] \tag{14}$$

$$M_t = \frac{q}{16} [a^2(1 + \nu) - r^2(1 + 3\nu)] \tag{15}$$

The maximum bending moments occur at the edge of the plate ($r = a$) and are

$$\begin{aligned} (M_r)_{r=a} &= -\frac{qa^2}{8} \\ (M_t)_{r=a} &= -\frac{qa^2}{8} \end{aligned} \tag{16}$$

From Eq.(16) it is seen that the maximum bending stress is at the edge of the plate where

$$(\sigma_r)_{max} = -\frac{6M_r}{h^2} = \frac{3qa^2}{4h^2} \tag{17}$$

(2) Circular Plate with Simply Supported Edges (Fig. 3).

In this case the boundary conditions are

$M_r = 0$ and $w = 0$ at $r = a$

Substituting these conditions in Eq.(5) and (11) yield the following respective expressions

$$C_1 = -\frac{qa^2}{8D}\frac{3+\nu}{1+\nu}$$

$$C_2 = 0$$

$$C_3 = \frac{qa^4}{64D}\frac{5+\nu}{1+\nu}$$

The plate deflection is then

$$w = \frac{q(a^2 - r^2)}{64D}\left(\frac{5+\nu}{1+\nu}a^2 - r^2\right) \qquad (18)$$

Substituting r = 0 in this expression we obtain the maximum deflection of the plate at the center (r = 0)

$$w_{max} = \frac{qa^4}{64D}\frac{5+\nu}{1+\nu} \qquad (19)$$

From the deflection curve w, Eq.(18), the distribution of moments can readily be obtained in the form

$$M_r = \frac{q}{16}(3+\nu)(a^2 - r^2)$$

$$(20)$$

$$M_t = \frac{q}{16}[a^2(3+\nu) - r^2(1+3\nu)]$$

Hence, the maximum bending moment is at the center ($r = 0$) of the plate where

$$(M_r)_{r=0} = (M_t)_{r=0} = \frac{qa^2}{16}(3 + \nu) \tag{21}$$

The corresponding maximum stress is

$$(\sigma_r)_{max} = \frac{6M_r}{h^2} = \frac{3qa^2}{8h^2}(3 + \nu) \tag{22}$$

In the foregoing discussion the effect of shearing strain on the deflection has been neglected. When the thickness of the plate is not small in comparision with its radius, this effect may be considerable and must be taken into account.

Several other cases of practical importance can also be treated on the basis of the mathematical analyses described in the preceding discussion.

CHAPTER III

FINITE ELEMENT ANALYSIS OF PLANE ELASTICITY

In the previous chapter, we discussed the governing equation for the circular plate bending and some fundamental equations under different boundary conditions.

The matrix displacement method of analysis based upon finite element idealization is employed throughout the ANSYS program. In this chapter, the basic steps of the finite element analysis and the application of the displacement method to the plane elasticity will be developed.

The solution of problems in the theory of elasticity can be obtained by two methods. One can solve the governing differential equations for the specified boundary conditions, or one can minimize the potential energy that relates to the strain energy and work done by external forces. The finite element formulation of elasticity problems utilizes the latter approach.

In the finite element displacement method, the displacement equations selected must satisfy the displacement boundary conditions, and the elements are assumed to be interconnected at a discrete number of nodal points situated on their boundaries. The displacements of these nodal points are taken as the basic unknown and the displacement field is defined in terms of these discrete variables. Once the discrete displacements are known, the strains are evaluated from the strain-displacement relations and, finally, the stresses are determined from the stress-strain relations. The general procedures involved in the finite element analysis are as follows:

(1) Discretization of the body into elements, i.e. selection of elements interconnected at certain nodal points.

(2) Derivation of element equations, i.e. evaluation of element stiffness and nodal force matrices.

(3) Assembly of element equations, i.e. assemblage of the stiffness and force matrices for the system of elements and nodes.

(4) Introduction of the boundary conditions.

(5) Solution of the assembled equations.

(6) Postprocessing of the solution, i.e. calculation of strains and stresses based on the nodal displacements.

(A) Matrix Formulation of Plane Elasticity Equations.

The governing equations of two-dimensional plane elasticity are summaried below.

(1) Equilibrium equations in terms of stresses.

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0 \qquad (23)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = 0$$

where X and Y denote the body forces along the x and y directions, σ_x and σ_y are the normal stresses, and τ_{xy} is the shear stress, respectively.

(2) Strain-displacement relations.

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \tag{24}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

where u and v are the displacement components in the x, y coordinate directions, ε_x and ε_y are the normal strains and γ_{xy} is the shear strain.

(3) Stress-strain relations.

$$\sigma_x = c_{11}\,\varepsilon_x + c_{12}\,\varepsilon_y$$

$$\sigma_y = c_{12}\,\varepsilon_x + c_{22}\,\varepsilon_y \tag{25}$$

$$\tau_{xy} = c_{33}\,\gamma_{xy}$$

where c_{ij} ($c_{ji} = c_{ij}$) are the elasticity (material) constants. For an isotropic elastic body, c_{ij} are the function of the modulus of elasticity E and the Poisson's ratio ν. For plane stress

$$C_{11} = C_{22} = \frac{E}{1 - \nu^2}$$

$$C_{12} = \frac{E}{1 - \nu^2} \qquad (26)$$

$$C_{33} = \frac{E}{2(1 + \nu)}$$

Eqs.(23) through (25) are rewritten in matrix form. To this end let

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial}{\partial x} & 0 \\ 0 & \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \qquad (27)$$

For a particular case of plane stress three components of stress corresponding to the strains already defined have to be considered, and can be expressed in the form

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1 - \nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad (28)$$

The displacement components can be written in terms of the nodal values as

$$\{ u \} = [N] \{ U \} \tag{29}$$

where $[N]$ is the matrix of shape functions and $\{ U \}$ is the nodal displacements.

From Hooke's law, the stress-strain relations and the strain-displacement relations to be given by

$$\begin{aligned} \{ \sigma \} &= [D] \{ \varepsilon \} \\ \{ \varepsilon \} &= [B] \{ U \} \end{aligned} \tag{30}$$

in which $[D]$ is material property matrix and $[B]$ is strain-displacement matrix based on the element shape functions.

(B) The Total Potential Energy Formulation.

The total potential energy, II, can be written as

$$II = \bar{U} - W \tag{31}$$

where $\bar{U}$ is the total strain energy stored in a deformed elastic body,

$$\bar{U} = \iiint_V \frac{1}{2} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz}) \, d(\text{Volume}) \tag{32}$$

and the work done, W, by the body forces and the forces applied at the boundary-surface of the body,

$$W = \iiint_V (Xu + Yv + Zw) \, d(\text{Volume}) + \iint_S (\bar{X}u + \bar{Y}v + \bar{Z}w) \, dA \tag{33}$$

where X, Y, Z be the x, y, z components of the body forces per unit volume and $\bar{X}$, $\bar{Y}$, $\bar{Z}$ be the x, y, z components of the surface tractions per unit area, respectively.

Utilizing above equations, the strain energy $\bar{U}$ and the work done W for a typical element e can be written as

$$\bar{U}^{(e)} = \frac{1}{2} \int_{V^{(e)}} \sigma^T \varepsilon \, dV$$

$$= \frac{1}{2} \int_{V^{(e)}} \{ U \}^T [B^{(e)}]^T [D^{(e)}] [B^{(e)}] \{ U \} dV \tag{34}$$

and

$$W^{(e)} = \int_{V^{(e)}} \{ U \}^T [N^{(e)}]^T \begin{Bmatrix} X^{(e)} \\ Y^{(e)} \end{Bmatrix} dV$$

$$+ \int_{S^{(e)}} \{ U \}^T [N^{(e)}]^T \begin{Bmatrix} \overline{X}^{(e)} \\ \overline{Y}^{(e)} \end{Bmatrix} dS \tag{35}$$

Substituting Eqs.(34) and (35) into Eq.(31), the total potential energy, II, is obtained

$$II = \frac{1}{2}\int_{V^{(e)}} \{ U \}^T [B^{(e)}]^T [D^{(e)}] [B^{(e)}] \{ U \} dV$$

$$- \int_{V^{(e)}} \{ U \}^T [N^{(e)}]^T \begin{Bmatrix} X^{(e)} \\ Y^{(e)} \end{Bmatrix} dV \tag{36}$$

$$- \int_{S^{(e)}} \{ U \}^T [N^{(e)}]^T \begin{Bmatrix} \overline{X}^{(e)} \\ \overline{Y}^{(e)} \end{Bmatrix} dS$$

To minimize the total potential energy, II, differentiating Eq.(36) with respect to $\{ U \}$ and setting it equal to zero, we obtain

$$[K^{(e)}] \{ U \} = \{ f^{(e)} \} \tag{37}$$

where an element stiffness matrix $[K^{(e)}]$ and an element force vector $\{ f^{(e)} \}$ are expressed as

$$[K^{(e)}] = \int_{V^{(e)}} [B^{(e)}]^T [D^{(e)}] [B^{(e)}] \, dV \tag{38}$$

and

$$\{ f^{(e)} \} = \int_{V^{(e)}} [N^{(e)}]^T \begin{Bmatrix} X^{(e)} \\ Y^{(e)} \end{Bmatrix} dV + \int_{S^{(e)}} [N^{(e)}]^T \begin{Bmatrix} \overline{X}^{(e)} \\ \overline{Y}^{(e)} \end{Bmatrix} dS \tag{39}$$

The next step is the determination of the solution to the unknown displacements. Once the displacement matrix is determined, the strains can be evaluated from the strain-displacement relations and the stresses can be obtained from the stress-strain relations.

The general procedure for the finite element displacement method in solving plane elasticity is discussed in the preceding section. In next chapter, the application of the ANSYS program to circular plate analysis will be developed.

CHAPTER IV

THE ANSYS PROGRAM FOR CIRCULAR PLATE ANALYSIS

The ANSYS program is the application of the finite element displacement method. The finite element displacement method was discussed in Chapter III. In this chapter, the basic concepts of the ANSYS program and the application of the ANSYS program to circular plate analysis will be developed. Most of the following materials are taken directly from the ANSYS Manual [17].

A. Organization of ANSYS

The ANSYS program is a self-contained general purpose finite element program which was developed and maintained by Swanson Analysis Systems, Inc. The program contains many routines for solving engineering problems. Analysis capabilities include (1) static and dynamic; (2) elastic, plastic, creep and swelling; (3) buckling; (4) small and large deflections; (5) steady state and transient heat transfer, fluid and current flow.

Loading on the structure may be forces, displacements, pressures, temperatures, or response spectra and may be arbitrary functions of time for linear and nonlinear analysis. Heat transfer analysis include all modes of heat transfer (i.e. conduction, convection, and radiation) and all types of boundary conditions (i.e. convection and radiation boundaries, and specified temperatures or heat flows). Internal heat generation is also allowed.

The ANSYS program use the wave-front (or " frontal ") direct solution method for the system of simultaneous linear equations developed with the matrix displacement method, and give results of high accuracy with a minimum amount of computer time. The number of elements used in an analysis has no limit. Also, there is no " band width " limitation in the analysis definition.

As in any other commercial available finite element program, an engineering problem is usually solved in three phases: 1) Preprocessing, 2) Solution, and 3) Postprocessing. Some of the operation in each phase are illustrated as follows:

PREPROCESSING PHASE

. Mesh generation
. Geometry definitions
. Material definitions
. Constraint definitions
. Load definitions
. Model ploting

SOLUTION PHASE

- Element matrix formulation
- Overall matrix triangularization
- Displacement, stress, etc., calculations

POSTPROCESSING PHASE

- Post solution operations
- Post data printout (for reports)
- Post data scanning
- Post data plots

B. The General Procedure of Input Data in ANSYS Program Analysis.

The following procedure is a guide for defining input data for a basic analysis.

PREPROCESSING PROCEDURE:

1) Define initial analysis data.
2) Select analysis options, if desired.
3) Define material property values.
4) Define real constant values.
5) Generate model geometry.
6) Begin load step data.
7) Define constraints and loads.
8) Merge coincident nodes, if necessary.
9) Write analysis file.

SOLUTION PROCEDURE:

1) Set check option, if desired.
2) Switch input to analysis file.
3) terminate load steps and solution phase.

POSTPROCESSING PROCEDURE:

1) Select postprocessor.
2) Enter postprocessing data.
3) Exit postprocessing routine.
4) Select another postprocessor, if desired, and repeat postprocessing steps.

The next examples illustrate the use of ANSYS program in the solution of circular plate that is subject to uniformly distributed load and various boundary constraints.

EXAMPLE 1: A solid circular steel plate, 0.3 in thick and 16 in diameter, is loaded with a uniformly distributed load of 10 lb/in^2. Determine the center deflection w_c under (1) clamped along the edge, and (2) simply supported along the edge. Given: E = 30 (10^6) lb/in^2 and ν = 0.3.

The model is generated with the use of the ANSYS program using axisymmetric conical shell element (STIF 11). The line element model (8 elements and 9 nodes) is used because of symmetry (Fig.4). The results obtained from the ANSYS program are compared with theoretical solutions are shown in Table I and II. The input data to ANSYS program in this problem and selected portions of the output are listed in Appendix B.

Table I Comparison of center deflection between ANSYS program and theory for circular plate under uniform loading with clamped entire edge.

	Center Deflection w_c, in
ANSYS	0.00862836
THEORY	0.008628148
DIFFERENCE	0.0025%

Table II Comparison of center deflection between ANSYS program and theory for circular plate under uniform loading with entire edge simply supported.

	Center Deflection w_c, in
ANSYS	0.0351765
THEORY	0.035176296
DIFFERENCE	0.00058%

EXAMPLE 2: Refer to Example 1 and find the solutions of the following various boundary constraints.

Case 1 Simply supported at several points along the boundary.

Case 2 Clamped at several points along the boundary.

Case 3 Clamped and simply supported at several points along the boundary.

The model is generated with the use of the ANSYS program using quadrilateral shell element (STIF 63). The one-quarter model is used because of symmetry. The one-fourth plate was divided into 80 elements as shown in (Fig.7). The results obtained from the ANSYS program in Case (1) and Case (2) are shown in Table III and IV. The explanations of the input data and selected parts of the output are represented in Appendix C.

Case 1:

Table III Center deflection for circular plate under uniform loading with simply supported 4, 8, 20, and 40 points along the edge.

Simply Supported Points	4	8	20	40
Center Deflection w_c, in	0.0479844	0.0372286	0.0361963	0.0361678

Case 2:

Table IV Center deflection for circular plate under uniform loading with clamped 4, 8, 20, and 40 points along the edge.

Clamped Points	4	8	20	40
Center Deflection w_c, in	0.0218783	0.0155845	0.0108028	0.00889159

The one-fourth circular plate was divided into 24, 40, 80, and 160 elements as shown in Figures 5, 6, 7, and 8 for circular plate under uniform loading with simply supported 4 points at equidistant along the edge. The results of the center deflection obtained from the ANSYS program are listed in Table V.

Table V The ANSYS program solution for the center deflection obtained by 24, 40, 80, and 160 elements of one-fourth circular plate under uniform loading with simply supported 4 points along the edge.

Elements	Nodes	Center deflection w_c, in
24	36	0.0481984
40	54	0.0480135
80	99	0.0479844
160	188	0.0479618

Case 3: The center deflection of circular plate under uniform loading simply supported 36 points with 4 clamped points. Though it is difficult to find the solution by using differential equations in this problem, the approximate solution (0.021111 in) can be easily obtained in the ANSYS program.

EXAMPLE 3: A circular plate, 0.3 in thick and 16 in diameter, is loaded with a uniformly distributed load of 10 lb/in^2. The plate was divided into 72, 96, 144, 192, and 288 elements as shown in Figures 9, 10, 11, 12, and 13. The results obtained from the ANSYS program solutions are compared with those obtained from a series solution [1] as shown in Table VI and Figures 14, 15. Stresses contours (top surface) for 96 elements are shown in Figures 16 to 23.

The circular plate under uniformly loaded with various boundary constraints has been solved from the ANSYS program in the previous section. The results and discussion will be developed in the Chapter V.

Table VI Comparision of center deflection between ANSYS program and theory for circular plate under uniform loading with simply supported 3 points.

Elements	Center Deflection w_c , in	Difference
72	0.063720	1.39%
96	0.0635079	1.13%
144	0.0633526	0.88%
192	0.0632830	0.77%
288	0.0632253	0.68%

Theory $$w_c = 0.0362 \frac{q \pi a^4}{D} = 0.062799474$$

CHAPTER V

RESULTS AND DISCUSSION

The purpose of this study is to construct a finite element model for a circular plate under uniform loading with various boundary constraints and determine maximum deflection and maximum stress using ANSYS General Purpose Finite Element Computer Program.

The results for all edge clamped is listed in Table I. The difference in center deflection between ANSYS program (using 8 elements) and theoretical solution is only 0.0025%. The difference for all other points are less than that at the center. It is clear that the ANSYS program gives very good results for all edge clamped, even with only 8 elements. Further refinement of the region is not necessary. Table II shows the center deflection for a simply supported plate. The error is only 0.00058% which is within the accuracy of the single precision used in the computation. The same conclusion for this case may also be drawn.

Table III and IV list the deflection of multiple-point simply supported and multiple-point clamped edge conditions. From Table III, the center deflection was reduced from 0.0479844 in to 0.0361678 in , when the simply supported points were increased from 4 to 40. The difference in center deflection between simply supported 40 points and simply supported entire edge was 2.8%. From Table IV, the center deflection was reduced from 0.0218783 in to 0.00889159 in , when the clamped points were increased from 4 to 40. The difference in center deflection between clamped 40 points and clamped entire edge was 3.05%. These results have shown that the deflection decreases with increasing number of points of constrained edge condition.

Case III of Example 2 studied simply supported 36 points with 4 clamped points mixed boundary condition. The approximate solution in center deflection (0.021111 in) can be easily obtained from the ANSYS program. There are no difficulties in handling mixed boundary conditions in finite element method.

For simply supported 3 points at equidistance along the edge, the center deflection is 0.063225% in by ANSYS program and 0.062799474 in by series solutions. The difference reduced from 1.39% to 0.68% when the number of elements was increased from 72 to 288. From Fig.15, it has clearly shown that the curve of difference in the center deflection is reduced when the number of elements is increased. However this difference is not linearly reduced. The result of the example has shown that the solution of the finite element method represent good approximation to the exact solution when the number of elements is 288. Consequently, if the computational costs have to be considered and the small difference can be tolerated, further mesh refinements are not necessary.

Similar comparison on the stress can also been done. For Example 1, the maximum stress intensity at top surface with clamped edge is occured at edge (4288.1) and that for simply supported edge is occured at center (8768.6). This shows that the simply supported constraint gives higher stress and deflection. Figures 16 to 23 show stress contours for 96 elements in Example 3.

The results demostrated that, in general, the accuracy of a finite element solution can be obtained by mesh refinement. Since most practical problems with geometric complexity are approximated in their engineering formulations (of the governing equations), one cannot be overconcerned with the numerical accuracy of the solution. Therefore, if the difference is negligibly small, further mesh refinements are not necessary. Obviously as the mesh is refined, the number of elements, data preparation, and computer CPU time are increased, and the computational costs are increased as well.

CHAPTER VI

CONCLUSION

Classical mathematical solutions have series limitations in practice because unusual geometries or boundary conditions lead to prohibitive complexities in their differential equations of equilibrium. The finite element method is nowadays the most general and one of the most powerful techniques of numerical analysis of structures. By applying the finite element method, no limitations are imposed with regard to the boundary conditions.

The ANSYS computer program is a large-scale, general purpose computer program for the solution of several classes of engineering analysis. The ANSYS program employed the matrix displacement method of the finite element idealization. The program has the capability of solving large structures. The number of elements and " band width " are no limitations in the analysis; however, there is " wave-front " restriction. The " wave-front " restriction depends on the amount of core storage available for a given problem.

The finite element method for circular plate analysis under uniformly loaded with simply supported and clamped along the edge has been presented in this thesis. In order to illustrate the compability and versatility of this finite element circular plate analysis procedure, analysis have been made of a wide range of circular plate systems involving many different boundary conditions. The examples presented herein demonstrated the versatility of the finite element procedure in treating uniform loading circular plate of arbitrary boundary conditions.

The results of the examples considered are seen to represent good approximations to the exact solutions derived by differential equations. For all edge clamped, the difference of center deflection between ANSYS program and theory solutions was 0.0025%. Similarly, the difference for all edge simply supported is 0.00058% only. The analysis of the circular plate under uniform loading with simply supported 3 points at equidistance along the edge demonstrates the convergence of the process as the finite element mesh is refined. Thus it seems reasonable to conclude that equivalent accuracy and convergence properties would be obtained in the analysis of other circular plate.

The application of the ANSYS General Purpose Finite Element Computer Program method has been presented as an analytical technique which not only can be applied to a circular plate but also a very broad class of all the physical problems that are governed by differential equations. Several advantageous properties of the finite element method can be drawn from this study. Some of the main ones include:

1. The number of elements can be varied. This property allows the element grid to be expanded or refined as the need exists. As the number of element is increased, the accuracy of the results is improved.

2. Boundary conditions such as discontinuous edge constraint present no difficulties for the method. Mixed boundary conditions can be easily handled.

3. As the number of points of constraint at the edge is increased, the deflection and stress approach to that of the edge entirely constrained.

The primary disadvantage of the finite element method is the need for computer programs and computer facilities. The computations involved in the finite element method are too numerous for hand calculations even when solving very small problems. The digital computer is a necessity , and computers with large memories are needed to solve large complicated problems.

APPENDIX A

THE GOVERNING EQUATION FOR DEFLECTION OF PLATES

The plane parallel to the faces of the plate and bisecting the thickness of the plate, in the undeformed state, is called the middle plane of the plate. Consider the coordinate axes, in which the x and y axes are in the middle plane of the plate and the z axis is perpendicular to the middle plane (Fig. A1). The components of displacement at a point, occurring in the x, y, and z directions, are denoted by u, v, and w, respectively (Fig. A2).

If a thin plate is bent with small deflection, i.e., when the deflection of the middle plane is small compared with the thickness h, the following fundamental assumptions can be made.

(1) The normals of the middle plane before bending are deformed into the normals of the middle plane after bending. This means the vertical shear strains γ_{yz} and γ_{xz} are zero.

(2) The normal stress, σ_z, is small compared with the other stress components and may be neglected.

(3) The middle plane remains unstrained after bending.

According to the strain-displacement relations, we have

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = 0$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \text{(A.1a - f)}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0$$

Integrating Eq.(A.1c), we obtain

$$w = w\ (x,\ y) \qquad \text{(A.2)}$$

In a like manner, integration of the expressions for γ_{xz} and γ_{yz} gives

$$u = -\ z\ \frac{\partial w}{\partial x} + u_o\ (x,\ y)$$

$$\text{(A.3)}$$

$$v = -\ z\ \frac{\partial w}{\partial y} + v_o\ (x,\ y)$$

It is clear that u_o (x, y) and v_o (x,y) represent, respectively, the values of u and v on the middle plane. Based on assumption (3), we conclude that $u_o = v_o = 0$. Thus

$$u = - z \frac{\partial w}{\partial x}$$

$$v = - z \frac{\partial w}{\partial y} \tag{A.4}$$

Substituting Eq.(A.4) into Eq.(A.1) yields

$$\varepsilon_x = - z \frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = - z \frac{\partial^2 w}{\partial y^2} \tag{A.5}$$

$$\gamma_{xy} = - 2z \frac{\partial^2 w}{\partial x \partial y}$$

According to assumption (2), the stress-strain relations for a thin plate in bending are

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu \sigma_y)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu \sigma_x) \tag{A.6}$$

$$\gamma_{xy} = \frac{1}{G} \tau_{xy}$$

from which we obtain

$$\sigma_x = \frac{E}{1 - \nu^2} (\varepsilon_x + \nu \varepsilon_y)$$

$$\sigma_y = \frac{E}{1 - \nu^2} (\varepsilon_y + \nu \varepsilon_x) \qquad \text{(A.7)}$$

$$\tau_{xy} = G \gamma_{xy} = \frac{E}{2(1+\nu)} \gamma_{xy}$$

where the constants E, ν, and G represent the modulus of elasticity, Poisson's ratio, and the shear modulus of elasticity, respectively.

Substitution of Eqs.(A.5) into Eqs.(A.7) yields

$$\sigma_x = - \frac{Ez}{1 - \nu^2} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

$$\sigma_y = - \frac{Ez}{1 - \nu^2} \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) \qquad \text{(A.8)}$$

$$\tau_{xy} = - \frac{Ez}{1+\nu} \frac{\partial^2 w}{\partial x \partial y}$$

With these relations, the bending and the twisting moments per unit length acting on any section of the plate parallel to the xz and yz planes can be obtained by integration. Thus

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z dz$$

$$= - \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{Ez}{1 - \nu^2} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) zdz$$

$$= - \frac{Ez}{1 - \nu^2} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \left(\frac{z^3}{3} \right) \Bigg|_{-\frac{h}{2}}^{\frac{h}{2}}$$

$$= - D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \tag{A.9}$$

Similarly, we have

$$M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y z dz = - D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right)$$

$$M_{xy} = - \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} z dz = D(1-\nu) \frac{\partial^2 w}{\partial x \partial y} \tag{A.10}$$

where

$$D = \frac{Eh^3}{12(1 - \nu^2)} \qquad (A.11)$$

and is called the flexural rigidity of the plate.

The stresses are found from Eqs.(A.8) by substituting Eq.(A.9) and Eqs.(A.10) and by employing Eq.(A.11). In this way we obtain

$$\sigma_x = \frac{12M_x z}{h^3}$$

$$\sigma_y = \frac{12M_y z}{h^3} \qquad (A.12)$$

$$\tau_{xy} = \frac{12M_{xy} z}{h^3}$$

The maximum stresses occur on the bottom and top surfaces (at $z = \pm \frac{h}{2}$) of the plate.

Since the middle plane is assumed unstrained, the summations of forces in the x and y directions are identically zero. The condition that the sum of the z components of the forces be zero becomes

$$\frac{\partial Q_x}{\partial x} dxdy + \frac{\partial Q_y}{\partial y} dydx + qdxdy = 0$$

or

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0 \qquad (A.13)$$

The equilibrium of moments with respect to the x axis is governed by

$$\frac{\partial M_{xy}}{\partial x} dxdy + \frac{\partial M_y}{\partial y} dxdy - Q_y dxdy = 0$$

or

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = 0 \qquad (A.14)$$

Similarly, from the equilibrium of moments with respect to the y axis gives

$$\frac{\partial M_{xy}}{\partial y} + \frac{\partial M_x}{\partial x} - Q_x = 0 \qquad (A.15)$$

Substitution of the expression for Q_x and Q_y from Eqs.(A.14) and (A.15) into Eq.(A.13) yields

$$\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = - q \qquad (A.16)$$

This is the differential equation of equilibrium for bending of thin plates.

From Eqs.(A.9), (A.10), (A.14), (A.15), we obtain

$$Q_x = - D \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = - D \frac{\partial}{\partial x} (\nabla^2 w)$$

$$Q_y = - D \frac{\partial}{\partial y} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = - D \frac{\partial}{\partial y} (\nabla^2 w) \qquad \text{(A.17)}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad \text{(A.18)}$$

is the Laplace operator.

Substituting (A.17) into (A.13), we obtain the differential equation which governs the small deflection of a thin plate with constant thickness under bending

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

$$\nabla^4 w = \frac{q}{D} \qquad \text{(A.19)}$$

In the discussion of bending of circular plates, polar coordinates are preferred over cartesian coordinates. The polar coordinate set (r, θ) and the cartesian coordinate set (x, y) are related by the equations (Fig. A.3).

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$\theta = \arctan \frac{y}{x}$$

referring to the above,

$$\frac{\partial r}{\partial x} = \frac{x}{r} = \cos\theta$$

$$\frac{\partial r}{\partial y} = \frac{y}{r} = \sin\theta$$

$$\frac{\partial \theta}{\partial x} = -\frac{y}{r^2} = -\frac{\sin\theta}{r}$$

$$\frac{\partial \theta}{\partial y} = \frac{x}{r^2} = \frac{\cos\theta}{r}$$

From these expressions, using the chain rule and considering w as a function of r and θ yield

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial w}{\partial \theta}\frac{\partial \theta}{\partial x}$$

$$= \frac{\partial w}{\partial r}\cos\theta - \frac{1}{r}\frac{\partial w}{\partial \theta}\sin\theta$$

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial w}{\partial \theta}\frac{\partial \theta}{\partial y}$$

$$= \frac{\partial w}{\partial r}\sin\theta - \frac{1}{r}\frac{\partial w}{\partial \theta}\cos\theta$$

To obtain the second derivatives, it is necessary only to repeat the above operation. Hence

$$\frac{\partial^2 w}{\partial x^2} = \cos\theta \frac{\partial}{\partial r}\left(\frac{\partial w}{\partial x}\right) - \frac{1}{r}\sin\theta \frac{\partial}{\partial \theta}\left(\frac{\partial w}{\partial x}\right)$$

$$= \frac{\partial^2 w}{\partial r^2}\cos^2\theta - 2\frac{\partial^2 w}{\partial \theta \partial r}\frac{\sin\theta\cos\theta}{r} + \frac{\partial w}{\partial r}\frac{\sin^2\theta}{r}$$

$$+ 2\frac{\partial w}{\partial \theta}\frac{\sin\theta\cos\theta}{r^2} + \frac{\partial^2 w}{\partial \theta^2}\frac{\sin^2\theta}{r^2} \tag{A.20}$$

Similarly,

$$\frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial r^2}\sin^2\theta + 2\frac{\partial^2 w}{\partial \theta \partial r}\frac{\sin\theta\cos\theta}{r} + \frac{\partial w}{\partial r}\frac{\cos^2\theta}{r}$$

$$- 2\frac{\partial w}{\partial \theta}\frac{\sin\theta\cos\theta}{r^2} + \frac{\partial^2 w}{\partial \theta^2}\frac{\cos^2\theta}{r^2} \tag{A.21}$$

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial r^2}\sin\theta\cos\theta + \frac{\partial^2 w}{\partial r \partial \theta}\frac{\cos^2\theta - \sin^2\theta}{r} + \frac{\partial w}{\partial \theta}\frac{\cos^2\theta - \sin^2\theta}{r^2}$$

$$- \frac{\partial w}{\partial r}\frac{\sin\theta\cos\theta}{r} - \frac{\partial^2 w}{\partial \theta^2}\frac{\sin\theta\cos\theta}{r^2} \tag{A.22}$$

Substituting above equations into Eq.(A.19), the Laplace operator becomes

$$\nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}$$

$$= \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} \tag{A.23}$$

To derive the fundamental equations for the moments and shearing forces in polar coordinates, we consider an infinitesimal element described in polar coordinates (Fig. A.4). Assume that the x axis coincides with the direction of radius r, i.e., $\theta = 0$. The moments M_r, M_t, M_{rt}, and the shearing forces Q_r, Q_t then have the same values as the moments M_x, M_y, M_{xy} and the shearing forces Q_x, Q_y at the same point in the plate. Thus, letting $\theta = 0$ in Eqs.(A.20), (A.21), and (A.22) and substituting the resulting expressions into Eqs.(A.9), (A.10), and (A.17), we obtain

$$M_r = -D\left[\frac{\partial^2 w}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right]$$

$$M_t = -D\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \nu\frac{\partial^2 w}{\partial r^2}\right)$$

$$M_{rt} = D(1-\nu)\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta}\right) \tag{A.24}$$

$$Q_r = -D \frac{\partial}{\partial r} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right)$$

$$= -D \frac{\partial}{\partial r} (\nabla^2 w)$$

$$Q_t = -D \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right)$$

$$= -D \frac{1}{r} \frac{\partial}{\partial \theta} (\nabla^2 w)$$

Similarly, formulas for the plane stress components, from Eqs.(A.12), are written in the following form

$$\sigma_r = \frac{12 M_r z}{h^3}$$

$$\sigma_t = \frac{12 M_t z}{h^3} \qquad \text{(A.25)}$$

$$\tau_{rt} = \frac{12 M_{rt} z}{h^3}$$

Substitution of Eqs.(A.20), (A.21), and (A.22) into Eq.(A.19), the governing differential equation for plate deflection in polar coordinates is derived.

$$\nabla^4 w = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right)$$

$$= \frac{P}{D} \qquad \text{(A.26)}$$

The general solution of the equation is expressed

$$w = w_h + w_p$$

where w_p is the particular solution of Eq.(A.26) and w_h is the solution of the homogeneous equation

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 w_h}{\partial r^2} + \frac{1}{r}\frac{\partial w_h}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w_h}{\partial \theta^2}\right) = 0 \qquad (A.27)$$

This homogeneous solution to be expressed by the following series

$$w_h = \sum_{n=0}^{\infty} P_n \cos n + \sum_{n=1}^{\infty} R_n \sin n \qquad (A.28)$$

where P_n and R_n are functions of r only. Substituting Eq.(A.28) in Eq.(A.27), we obtain for each of these functions an ordinary differential equation of the following kind:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{n^2}{r^2}\right)\left(\frac{d^2 P_n}{dr^2} + \frac{1}{r}\frac{dP_n}{dr} - \frac{n^2 P_n}{r^2}\right) = 0$$

Then, according to mathematical theorem, the general solution of this equation for $n > 1$ is

$$P_n = A_n r^n + B_n r^{-n} + C_n r^{2+n} + D_n r^{2-n}$$

For $n = 0$ and $n = 1$ the solutions are

$$P_o = A_o + B_o r^2 + C_o \log r + D_o r^2 \log r$$

and

$$P_1 = A_1 r + B_1 r^3 + C_1 r^{-1} + D_1 r \log r$$

Similar expressions can be written for the function R_n. The constants A_n, B_n, ..., D_n in each particular case must be determined so as to satisfy the boundary conditions.

APPENDIX B

INPUT AND OUTPUT OF ANSYS PROGRAM FOR EXAMPLE 1

Table B-1 Data input to the ANSYS program for circular plate problem in Example 1

```
/PREP7
/TITLE,CASE 1- CLAMPED ALONG ENTIRE EDGE
ET,1,11,,,,,1,1
EX,1,30E6
R,1,0.3
N,1
N,9,8
FILL
NPLOT,1
E,1,2
EGEN,8,1,1
ENUM,1
GLINE,1
EPLOT
ITER,1,1,1
KRF,1
D,1,UX,,,,,ROTZ
D,9,ALL
EP,1,1,-10,,8
AFWRITE
FINISH
```

Table B-1 (Continued)

```
/INPUT,27
FINISH
/POST1
SET,1,1,1
PRDISP
PLDISP
FINISH
/PREP7
RESUME
/TITLE,CASE 2- SIMPLY SUPPORTED ALONG ENTIRE EDGE
DDELE,9,ROTZ
AFWRITE
FINISH
/INPUT,27
FINISH
/POST1
SET,1,1,1
PRDISP
PLDISP
FINISH
```

*[EOB]

Table B-2 Selected portions of the output for circular plate problem in Example 1

CASE 1- CLAMPED ALONG ENTIRE EDGE

*** DISPLACEMENT SOLUTION *** IN NODAL COORDINATES

NODE	UY	ROTZ
1	0.862836E-02	0.000000E+00
2	0.836074E-02	-0.530740E-03
3	0.758343E-02	-0.101106E-02
4	0.637219E-02	-0.139025E-02
5	0.485340E-02	-0.161776E-02
6	0.320401E-02	-0.164304E-02
7	0.165151E-02	-0.141555E-02
8	0.473975E-03	-0.884719E-03
9	0.000000E+00	0.000000E+00

*** ELEMENT STRESSES ***

EL	MOM S	MOM TH	SBEND S	SBEND TH	S.I.T	UN
1	51.328	51.678	3421.9	3445.2	3435.2	-0.85609E-02
2	47.208	49.286	3147.2	3285.7	3275.7	-0.80321E-02
3	38.955	44.533	2957.0	2968.9	2958.9	-0.70252E-02
4	26.579	37.407	1771.9	2493.8	2483.8	-0.56412E-02
5	10.079	27.907	671.92	1860.5	1850.5	-0.40319E-02
6	-10.546	16.032	-703.09	1068.8	1771.9	-0.23993E-02
7	-35.296	1.7816	-2353.1	118.78	2471.9	-0.99639E-03
8	-64.172	-14.843	-4278.1	-989.56	4288.1	-0.12640E-03

Table B-2 (Continued)

CASE 2- SIMPLY SUPPORTED ALONG ENTIRE EDGE

*** DISPLACEMENT SOLUTION *** IN NODAL COORDINATES

NODE	UY	ROTZ
1	0.351765E-01	0.000000E+00
2	0.344941E-01	-0.136037E-02
3	0.324723E-01	-0.267032E-02
4	0.291870E-01	-0.387914E-02
5	0.247645E-01	-0.493628E-02
6	0.193818E-01	-0.579119E-02
7	0.132663E-01	-0.639333E-02
8	0.669620E-02	-0.669213E-02
9	0.000000E+00	-0.663704E-02

*** ELEMENT STRESSES ***

EL	MOM S	MOM TH	SBEND S	SBEND TH	S.I.T	UN
1	131.33	131.68	8755.2	8778.6	8768.6	-0.35005E-01
2	127.21	129.29	8480.5	8619.0	8609.0	-0.33647E-01
3	118.95	124.53	7930.3	8302.2	8292.2	-0.30981E-01
4	106.58	117.41	7105.3	7827.1	7817.1	-0.27108E-01
5	90.079	107.91	6005.3	7193.8	7183.8	-0.22180E-01
6	69.454	96.032	4630.2	6402.1	6392.1	-0.16399E-01
7	44.704	81.782	2980.2	5452.1	5442.1	-0.10019E-01
8	15.828	65.157	1055.2	4343.8	4333.8	-0.33412E-02

Table B-3 Element printout explanations for Example 1

Label	Explanation
MOM S	Meridional moment per unit length of circumference
MOM TH	Circumferential moment per unit axial length
SBEND S	Meridional bending stress
SBEND TH	Circumferential bending stress
S.I.T	Stress intensity at top surface
UN	Normal deflection at this location

APPENDIX C

INPUT AND OUTPUT OF ANSYS PROGRAM FOR EXAMPLE 2

Table C-1 Data input to ANSYS program for circular plate with clamped 40 points and simply supported 4 points problems in Example 2

```
/PREP7                 *BEGIN PREP7 PREPROCESSING

/TITLE,CASE 1- CLAMPED 40 POINTS ALONG EDGE

ET,1,63                *DEFINE ELEMENT TYPE FOR MODEL
                        GENERATION
EX,1,30E6              *DEFINE MODULUS OF ELASTICITY
                        MATERIAL
R,1,0.3                *DEFINE THICKNESS REAL CONSTANT

N,1                    *DEFINE NODE 1

N,2,1                  *DEFINE NODE 2

N,9,8                  *DEFINE NODE 9

FILL                   *FILL BETWEEN PREVIOUS TWO NODES

LOCAL,11,1             *DEFINE CYLINDRICAL

NGEN,11,9,1,9,,,9      *GENERATE 11 RADIAL LINES FROM
                        NODE 1 TO NODE 9 BY 9
E,2,11,1,1             *DEFINE ELEMENT 1

E,3,12,11,2            *DEFINE ELEMENT 2

EGEN,7,1,-1            *GENERATE 7 ELEMENTS FROM ELEMENT 1

EGEN,10,9,-8           *GENERATE 10 SETS OF ELEMENT FROM
                        ELEMENT 1 TO 8
MERGE                  *MERGE NODES ALONG COINCIDENT
                        REGION BOUNDARY
ITER,1,1,1             *DEFINE ITERATIONS,PRINT AND POST
                        CONTROLS
KRF,1                  *ACTIVATES THE NODAL AND REACTION
                        FORCE CALCULATION
D,1,UX,,,,,UY,ROTX,ROTZ

D,2,UY,,,8,,ROTX,ROTZ

D,9,ALL,,,99,9         *DEFINE DISPLACEMENT CONSTRAINTS
```

Table C-1 (Continued)

```
D,91,UX,,,98,,ROTY,ROTZ
EP,1,1,-10,,80        *DEFINE PRESSURE LOAD
TDBC,1                *INCLUDE DISPLACEMENT BOUNDARY
                       CONDITION ON PLOT
NPLOT,1               *INCLUDE NODE NUMBER ON PLOT
ENUM,1                *INCLUDE ELEMENT NUMBER ON PLOT
EPLOT                 *PRODUCE ELEMENT PLOT
AFWRITE               *WRITE ANALYSIS FILE
FINISH                *TERMINATE PREP7 FILE
/INPUT,27             *SUBMIT ANALYSIS FILE TO SOLUTION
                       PHASE
FINISH                *TERMINATE SOLUTION PHASE
/POST1                *BEGIN POSTPROCESSING PHASE
SET,1,1,1             *DEFINE DATA SET
PRDISP                *PRINTOUT THE NODAL DISPLACEMENTS
PRESTR                *PRINTOUT ELEMENT STRESSES
TOP                   *TOP SURFACE OF SHELL
PRNSTRS,ALL           *PRINTOUT NODAL STRESSES
PLDISP                *PLOT DISPLACEMENT SHAPE
PLNSTR                *PLOT STRESS CONTOURS
FINISH                *TERMINATE POST1 ROUTINE
/PREP7                *BEGIN PREP7 PREPROCESSING
RESUME                *RESETTING THE CORE DATA
/TITLE,CASE 2- SIMPLY SUPPORTED 4 POINTS ALONG EDGE
DDELE,18,ALL,90,9
DDELE,9,ROTX,99,90 *DELETE PREVIOUSLY DISPLACEMENT
                    CONSTRAINTS
DDELE,9,ROTY,99,90
```

Table C-1 (Continued)

```
DDELE,9,ROTZ,99,90
AFWRITE                *WRITE ANALYSIS FILE
FINISH                 *TERMINATE PREP7 FILE
/INPUT,27              *SUBMIT ANALYSIS FILE TO SOLUTION
                        PHASE
FINISH                 *TERMINATE SOLUTION PHASE
/POST1                 *BEGIN POSTPROCESSING PHASE
SET,1,1,1              *DEFINE DATA SET
PRDISP                 *PRINTOUT THE NODAL DISPLACEMENTS
PRESTR                 *PRINTOUT ELEMENT STRESSES
TOP                    *TOP FACE OF SHELL
PRNSTRS,ALL            *PRINTOUT NODAL STRESSES
PLDISP                 *PLOT DISPLACEMENT SHAPE
PLNSTR,SX              *PLOT STRESSES CONTOUR
FINISH                 *TERMINATE POST1 ROUTINE
```

*[EOB]

Table C-2 Selected portions of the output for circular plate problems in Example 2

CASE 1- CLAMPED 40 POINTS ALONG EDGE

*** DISPLACEMENT SOLUTION *** IN NODAL COORDINATES

NODE	UZ	NODE	UZ
1	-0.889159E-02	2	-0.846309E-02
3	-0.764079E-02	4	-0.640621E-02
5	-0.487541E-02	6	-0.321995E-02
7	-0.166362E-02	8	-0.481577E-03
9	0.000000E+00	11	-0.846423E-02
12	-0.764872E-02	13	-0.640695E-02
14	-0.487598E-02	15	-0.322032E-02
16	-0.166381E-02	17	-0.481633E-03
18	0.000000E+00	20	-0.846756E-02
21	-0.764440E-02	22	-0.640911E-02
23	-0.487761E-02	24	-0.322140E-02
25	-0.166437E-02	26	-0.481794E-02
27	0.000000E+00	29	-0.847283E-02
30	-0.764858E-02	31	-0.621247E-02
32	-0.488016E-02	33	-0.322309E-02
34	-0.166524E-02	35	-0.482047E-03
36	0.000000E+00	38	-0.847962E-02
39	-0.765382E-02	40	-0.641670E-02
41	-0.488339E-02	42	-0.322522E-02
43	-0.166635E-02	44	-0.482367E-03
45	0.000000E+00	47	-0.848739E-02

Table C-2 (Continued)

NODE	UZ	NODE	UZ
48	-0.765957E-02	49	-0.642141E-02
50	-0.488699E-02	51	-0.322761E-02
52	-0.166758E-02	53	-0.482724E-03
54	0.000000E+00	56	-0.849545E-02
57	-0.766525E-02	58	-0.642613E-02
59	-0.489061E-02	60	-0.323001E-02
61	-0.166882E-02	62	-0.483084E-03
63	0.000000E+00	65	-0.850307E-02
66	-0.767028E-02	67	-0.643045E-02
68	-0.489392E-02	69	-0.323219E-02
70	-0.166995E-02	71	-0.483422E-03
72	0.000000E+00	74	-0.850943E-02
75	-0.767420E-02	76	-0.643395E-02
77	-0.489656E-02	78	-0.323393E-02
79	-0.167085E-02	80	-0.483672E-03
81	0.000000E+00	83	-0.851378E-02
84	-0.767672E-02	85	-0.643627E-02
86	-0.489827E-02	87	-0.323506E-02
88	-0.167143E-02	89	-0.483841E-03
90	0.000000E+00	92	-0.851549E-02
93	-0.767758E-02	94	-0.643708E-02
95	-0.489887E-02	96	-0.323545E-02
97	-0.167163E-02	98	-0.483899E-03
99	0.000000E+00		

Table C-2 (Continued)

*** POST1 NODAL STRESS LISTING *** IN ELEMENT COORDINATES

NODE	SX	SY	SIG1	SIG2	SIG3	SI
1	-3395.	-2789.	-0.3390E-07	-2322.	-3861.	3861.
2	-2814.	-2645.	-0.3159E-07	-2632.	-2827.	2827.
3	-3116.	-2846.	-0.3449E-07	-2846.	-3116.	3116.
4	-2740.	-2200.	-0.2830E-07	-2199.	-2741.	2741.
5	-2196.	-1266.	-0.1888E-07	-1264.	-2197.	2197.
6	-1491.	-49.08	-0.9400E-06	-47.03	-1493.	1493.
7	-626.8	1448.	1450.	0.1328E-07	-629.4	2080.
8	396.5	3222.	3226.	393.3	-0.3640E-07	3226.
9	1641.	5480.	5483.	1637.	0.4532E-07	5483.
18	1641.	5480.	5484.	1638.	0.4532E-07	5484.
27	1642.	5482.	5486.	1638.	0.4534E-07	5486.
36	1643.	5485.	5489.	1639.	0.4536E-07	5489.
45	1644.	5489.	5492.	1640.	0.4539E-07	5492.
47	-3417.	-2990.	-0.3607E-07	-2697.	-3710.	3710.
48	-3208.	-2482.	-0.3493E-07	-2831.	-3281.	3281.
49	-2776.	-2208.	-0.2853E-07	-2206.	-2779.	2779.
50	-2214.	-1271.	-0.1900E-07	-2206.	-2779.	2779.
51	-1500.	-50.34	0.2519	-48.48	-1502.	1502.
52	-630.1	1451.	1453.	0.1332E-07	-632.8	2086.
53	397.3	3230.	3234.	394.0	0.3650E-07	3234.
54	1645.	5493.	5497.	1642.	0.4543E-07	5497.
63	1646.	5497.	5501.	2643.	0.4546E-07	5501.
72	1648.	5501.	5505.	1644.	0.4549E-07	5505.

Table C-2 (Continued)

NODE	SX	SY	SIG1	SIG2	SIG3	SI
81	1649.	5504.	5508.	1645.	0.4552E-07	5508.
90	1649.	5506.	5510.	1646.	0.4553E-07	5510.
92	-4577.	-3490.	-0.4448E-07	-3097.	-4917.	4917.
93	-3274.	-2485.	-0.3522E-07	-2809.	-3310.	3310.
94	-2820.	-2221.	-0.2882E-07	-2220.	-2821.	2821.
95	-2234.	-1278.	-0.1912E-07	-1276.	-2236.	2236.
96	-1510.	-51.54	0.7541	-50.21	-1512.	1513.
97	-633.7	1454.	1457.	0.1336E-07	-636.3	2093.
98	398.0	3238.	3242.	394.7	0.3661E-07	3242.
99	1649.	5507.	5510.	1646.	0.4554E-07	5510.

CASE 2- SIMPLY SUPPORTED 4 POINTS ALONG EDGE

*** DISPLACEMENT SOLUTION *** IN NODAL COORDINATES

NODE	UZ	NODE	UZ
1	-0.479844E-01	2	-0.468971E-01
3	-0.447226E-01	4	-0.411796E-01
5	-0.361864E-01	6	-0.296763E-01
7	-0.215754E-01	8	-0.117009E-01
9	0.000000E+00	11	-0.469006E-01
12	-0.447352E-01	13	-0.412336E-01
14	-0.363518E-01	15	-0.300801E-01
16	-0.224732E-01	17	-0.137765E-01
18	-0.462250E-02	20	-0.469107E-01
21	-0.447690E-01	22	-0.413750E-01

Table C-2 (Continued)

NODE	UZ	NODE	UZ
23	-0.367789E-01	24	-0.311082E-01
25	-0.245992E-01	26	-0.176405E-01
27	-0.106924E-01	29	-0.469261E-01
30	-0.448130E-01	31	-0.415505E-01
32	-0.372988E-01	33	-0.323068E-01
34	-0.269003E-01	35	-0.214420E-01
36	-0.162557E-01	38	-0.469451E-01
39	-0.448535E-01	40	-0.416958E-01
41	-0.377135E-01	42	-0.332294E-01
43	-0.285920E-01	44	-0.241179E-01
45	-0.200590E-01	47	-0.469655E-01
48	-0.448790E-01	49	-0.417601E-01
50	-0.378777E-01	51	-0.335782E-01
52	-0.292123E-01	53	-0.250783E-01
54	-0.214055E-01	56	-0.469855E-01
57	-0.448841E-01	58	-0.417233E-01
59	-0.377377E-01	60	-0.332498E-01
61	-0.286085E-01	62	-0.241308E-01
63	-0.200692E-01	65	-0.470034E-01
66	-0.448711E-01	67	-0.416082E-01
68	-0.373448E-01	69	-0.323451E-01
70	-0.269307E-01	71	-0.214648E-01
72	-0.162722E-01	74	-0.470176E-01
75	-0.448488E-01	76	-0.414473E-01
77	-0.368429E-01	78	-0.311602E-01

Table C-2 (Continued)

NODE	UZ	NODE	UZ
79	-0.246389E-01	80	-0.176679E-01
81	-0.107086E-01	83	-0.470271E-01
84	-0.448290E-01	85	-0.413188E-01
86	-0.364258E-01	87	-0.301404E-01
88	-0.225175E-01	89	-0.138034E-01
90	-0.463179E-02	92	-0.470309E-01
93	-0.448212E-01	94	-0.412693E-01
95	-0.362641E-01	96	-0.297394E-01
97	-0.216210E-01	98	-0.117256E-01
99	0.000000E+00		

*** POST1 NODAL STRESS LISTING *** IN NODAL COORDINATES

NODE	SX	SY	SIG1	SIG2	SIG3	SI
1	-8612.	-7133.	-0.8641E-07	-5947.	-9798.	9798.
2	-7031.	-6984.	-0.8117E-07	-6833.	-7182.	7182.
3	-7817.	-8591.	-0.9488E-07	-7813.	-8596.	8596.
4	-6803.	-8637.	-0.8824E-07	-6793.	-8646.	8646.
5	-5293.	-8511.	-0.7621E-07	-5274.	-8530.	8530.
6	-3360.	-8418.	-0.6099E-07	-3331.	-8447.	8447.
7	510.9	-8168.	609.9	0.1500E-06	-8266.	8876.
8	6660.	-3195.	9427.	0.8977E-07	-5962.	15390.
9	1131.	-3263.	8514.	0.8244E-07	10650.	19160.
18	-2157.	-1522.	3136.	0.3328E-07	-6816.	9952.
27	-5648.	-599.6	1501.	0.4020E-07	-7785.	9286.
36	-8061.	-250.4	557.2	0.1391E-06	-8844.	9421.

Table C-2 (Continued)

NODE	SX	SY	SIG1	SIG2	SIG3	SI
45	-9278.	-123.5	100.6	0.3073E-05	-9502.	9603.
47	-8840.	-7531.	-0.9202E-07	-6833.	-9538.	9538.
48	-9073.	-7392.	-0.9442E-07	-7371.	-9094.	9094.
49	-9233.	-6229.	-0.8651E-07	-6215.	-9246.	9246.
50	-9566.	-4771.	-0.7630E-07	-4747.	-9589.	9589.
51	-9920.	-3322.	-0.6892E-07	-3286.	-9955.	9955.
52	-10120.	-2072.	-0.7439E-07	-2027.	-10160.	10160.
53	-10040.	-1037.	-0.1139E-06	-988.6	-10090.	10090.
54	-9680.	-84.57	-0.2465E-05	-41.03	-9723.	9723.
63	-9291.	-123.1	103.6	0.2943E-05	-9517.	9621.
72	-8038.	-249.8	582.6	0.1387E-06	-8871.	9453.
81	-5710.	-599.4	1508.	0.4037E-07	-7817.	9325.
90	-2176.	-1523.	3145.	0.3339E-07	-6844.	9988.
92	-11500.	-9109.	-0.1140E-06	-8075.	-12530.	12530.
93	-8229.	-8582.	-0.9721E-07	-7976.	-8815.	8815.
94	-7016.	-8692.	-0.9000E-07	-7008.	-8700.	8700.
95	-5405.	-8551.	-0.7726E-07	-5387.	-8569.	8569.
96	-3424.	-8448.	-0.6154E-07	-3395.	-8477.	8477.
97	484.4	-8192.	577.5	0.1762E-06	-8291.	8868.
98	6658.	-3204.	9434.	0.8985E-07	-5980.	15410.
99	1129.	-3266.	8534.	0.8264E-07	-10670.	19210.

Table C-3 Element printout explanations for Example 2

Label	Explanation
SX,SY	Combined membrane and bending stresses
SIG1,SIG2,SIG3	Principal stresses
S.I.	Stress intensity

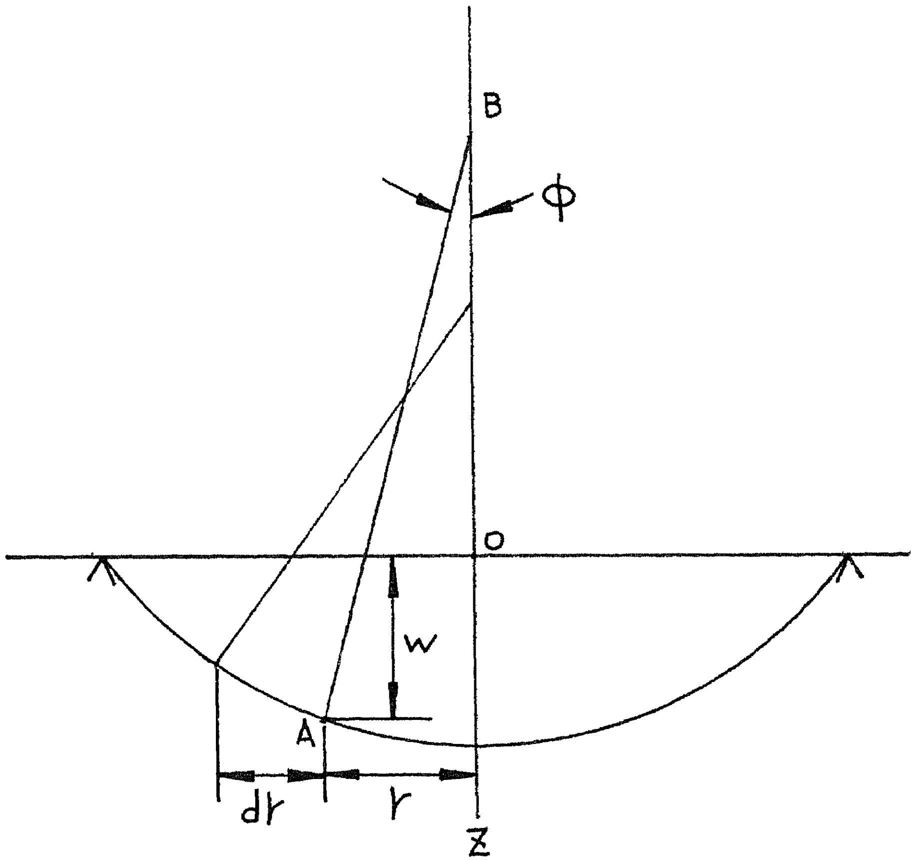

Figure 1 A Diametral Section of Plate

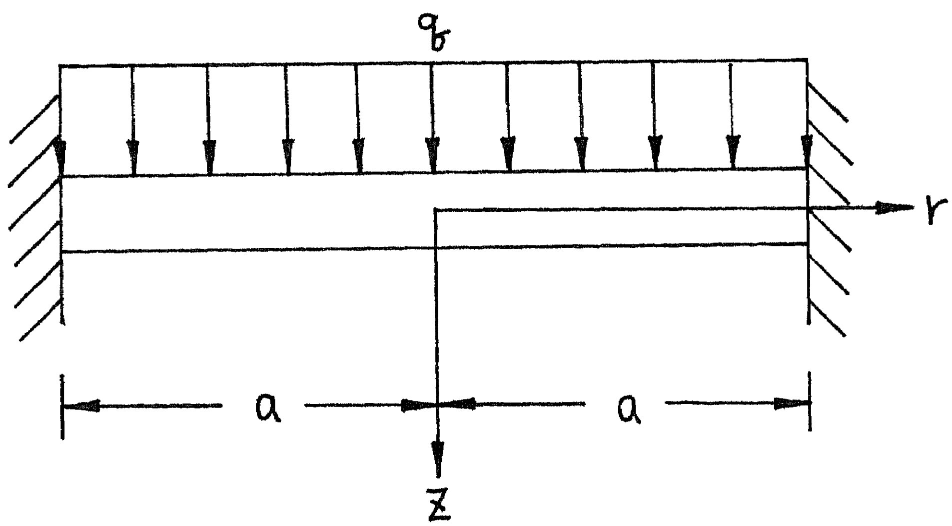

Figure 2 Circular Plate with Clamped Edge

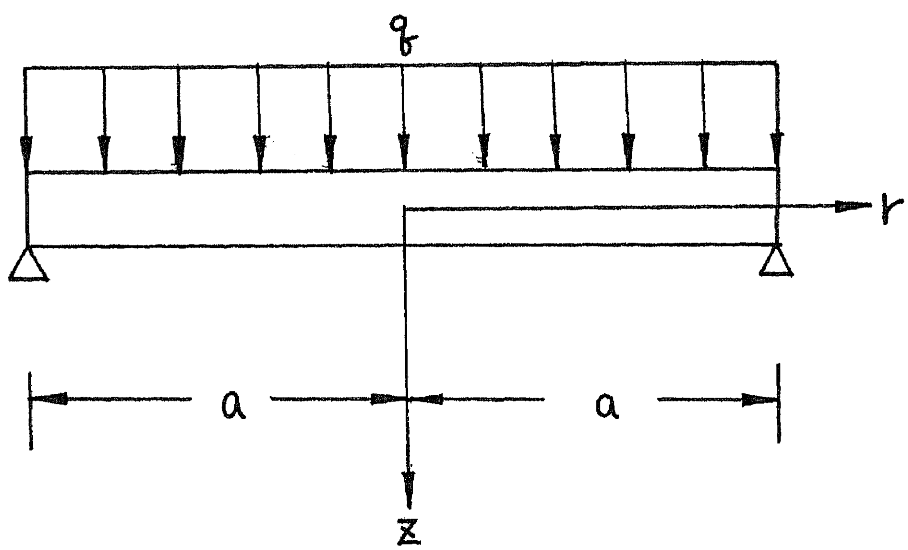

Figure 3 Circular Plate with Simply Supported Edge

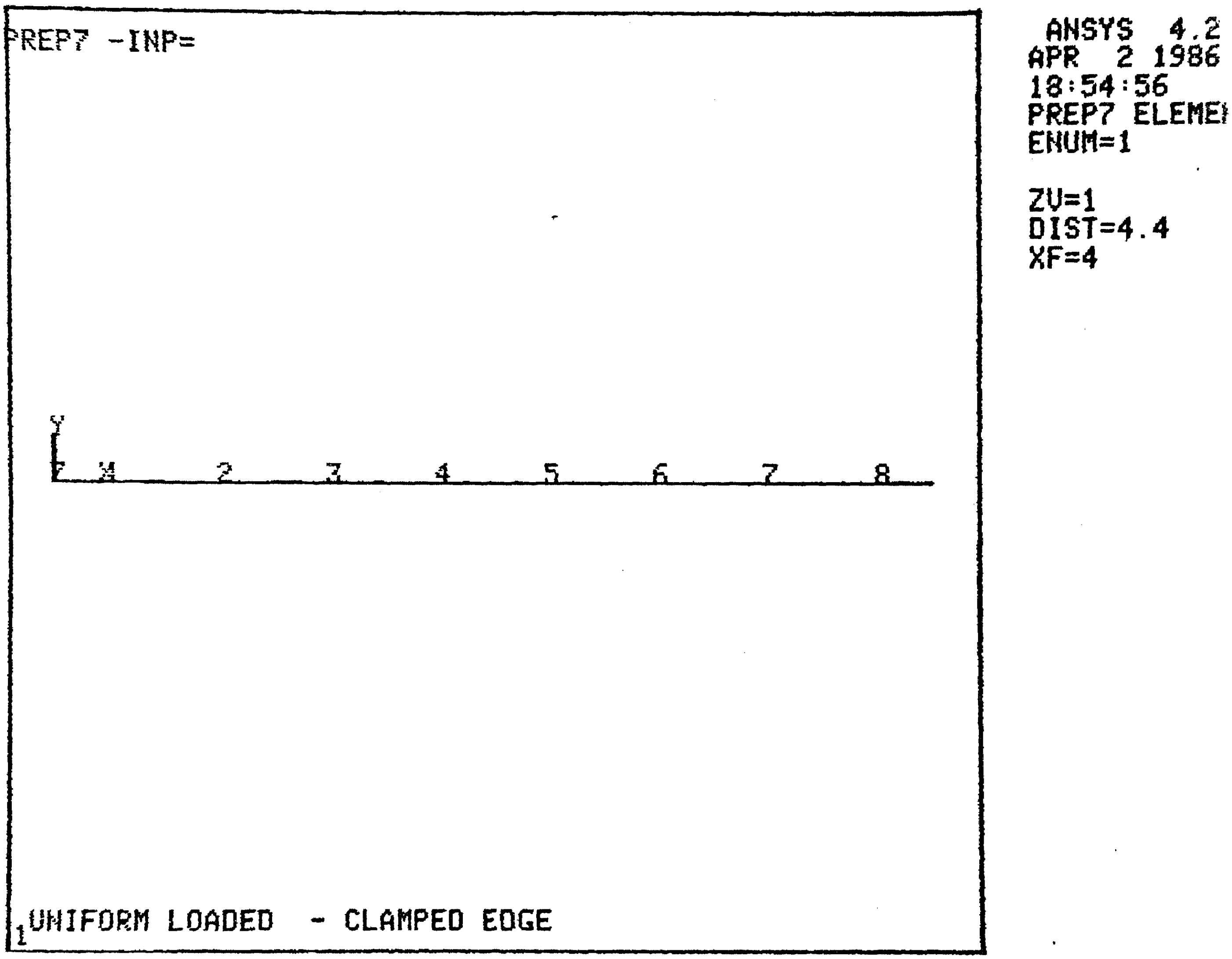

Figure 4 Finite Element Mesh for Line Model of Circular Plate (8 Elements and 9 Nodes)

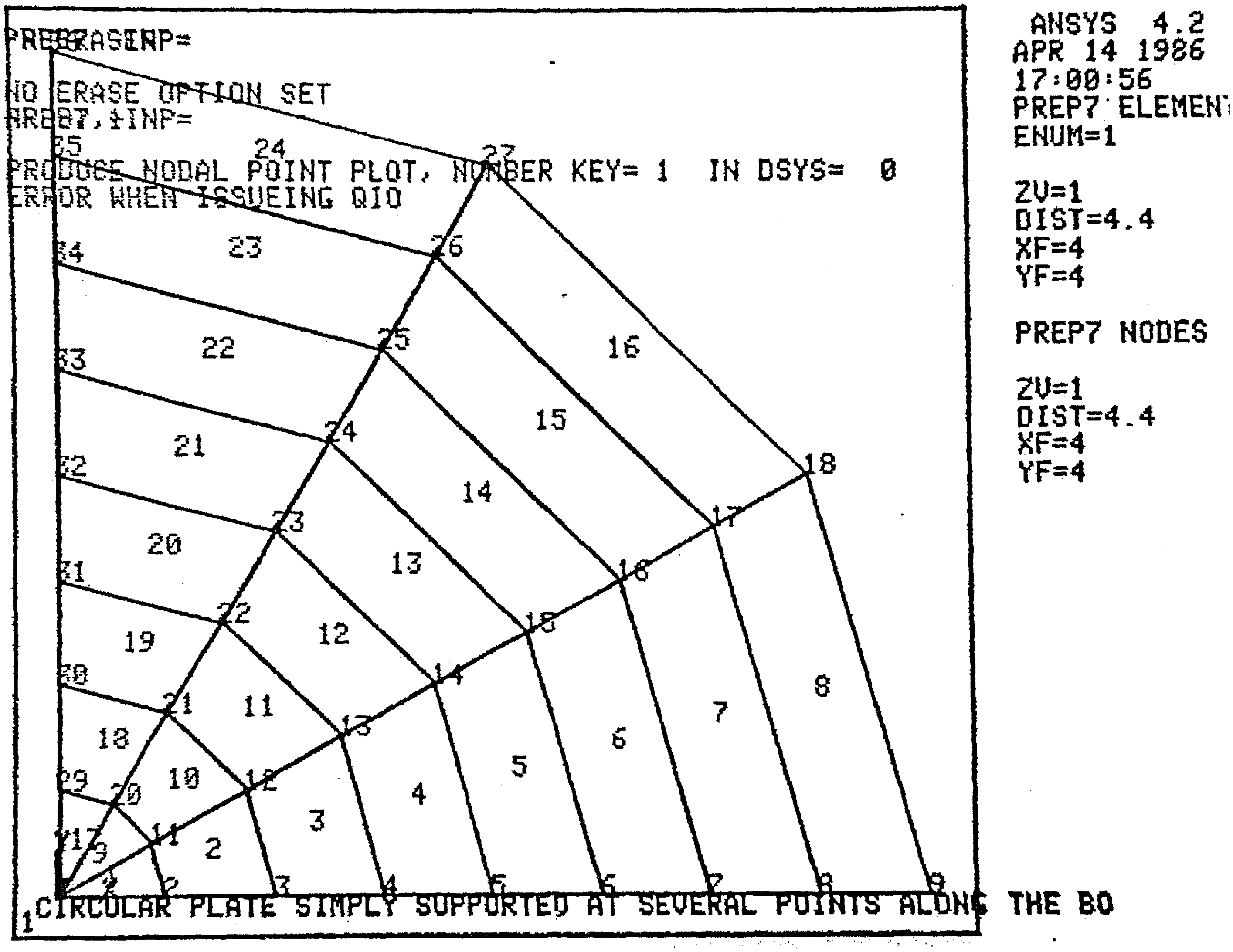

Figure 5 Finite Element Mesh for One-fourth of Circular Plate (24 Elements and 36 Nodes)

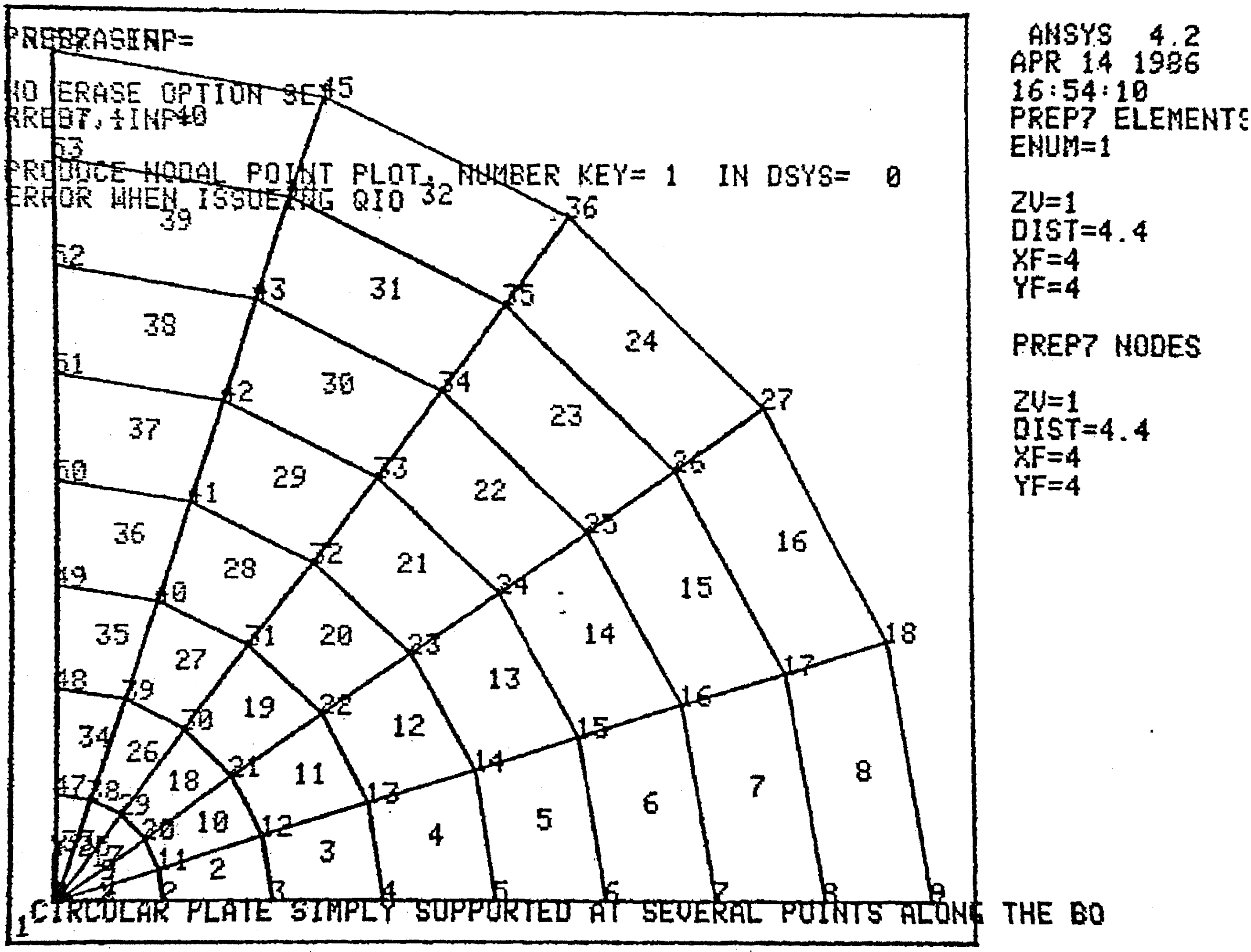

Figure 6 Finite Element Mesh for One-fourth of Circular Plate (40 Elements and 54 Nodes)

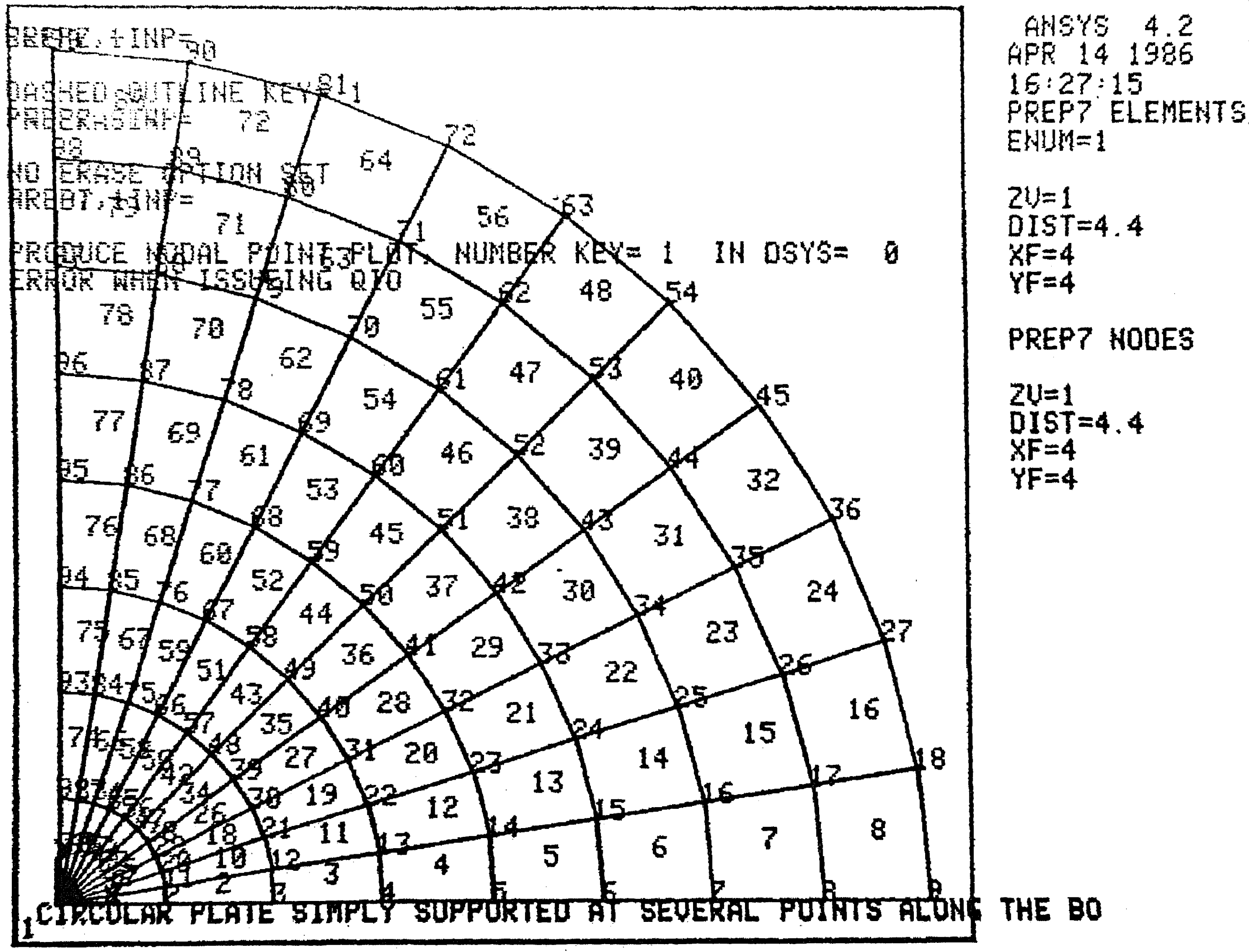

Figure 7 Finite Element Mesh for One-fourth of Circular Plate (80 Elements and 99 Nodes)

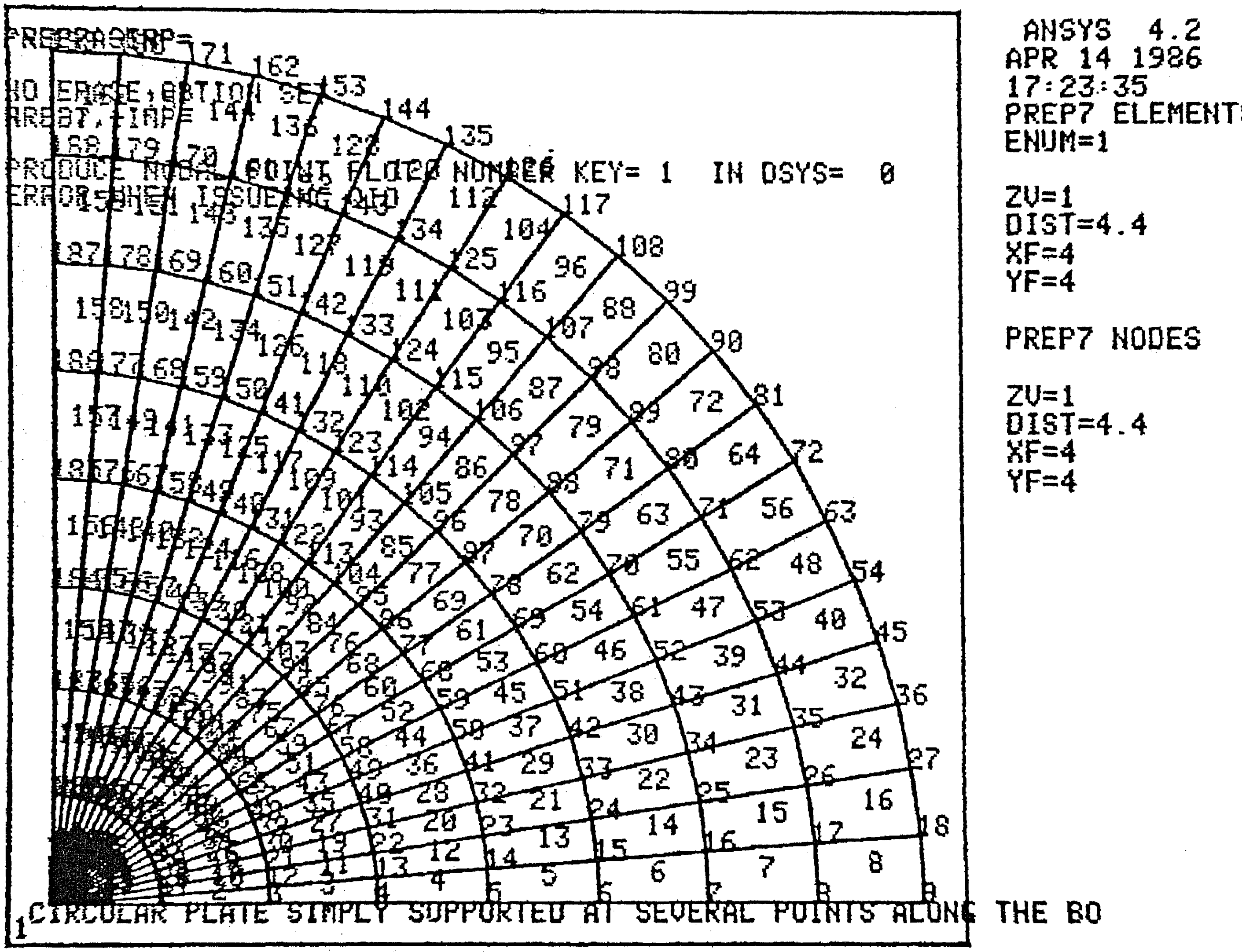

Figure 8 Finite Element Mesh for One-fourth of Circular Plate (160 Elements and188 Nodes)

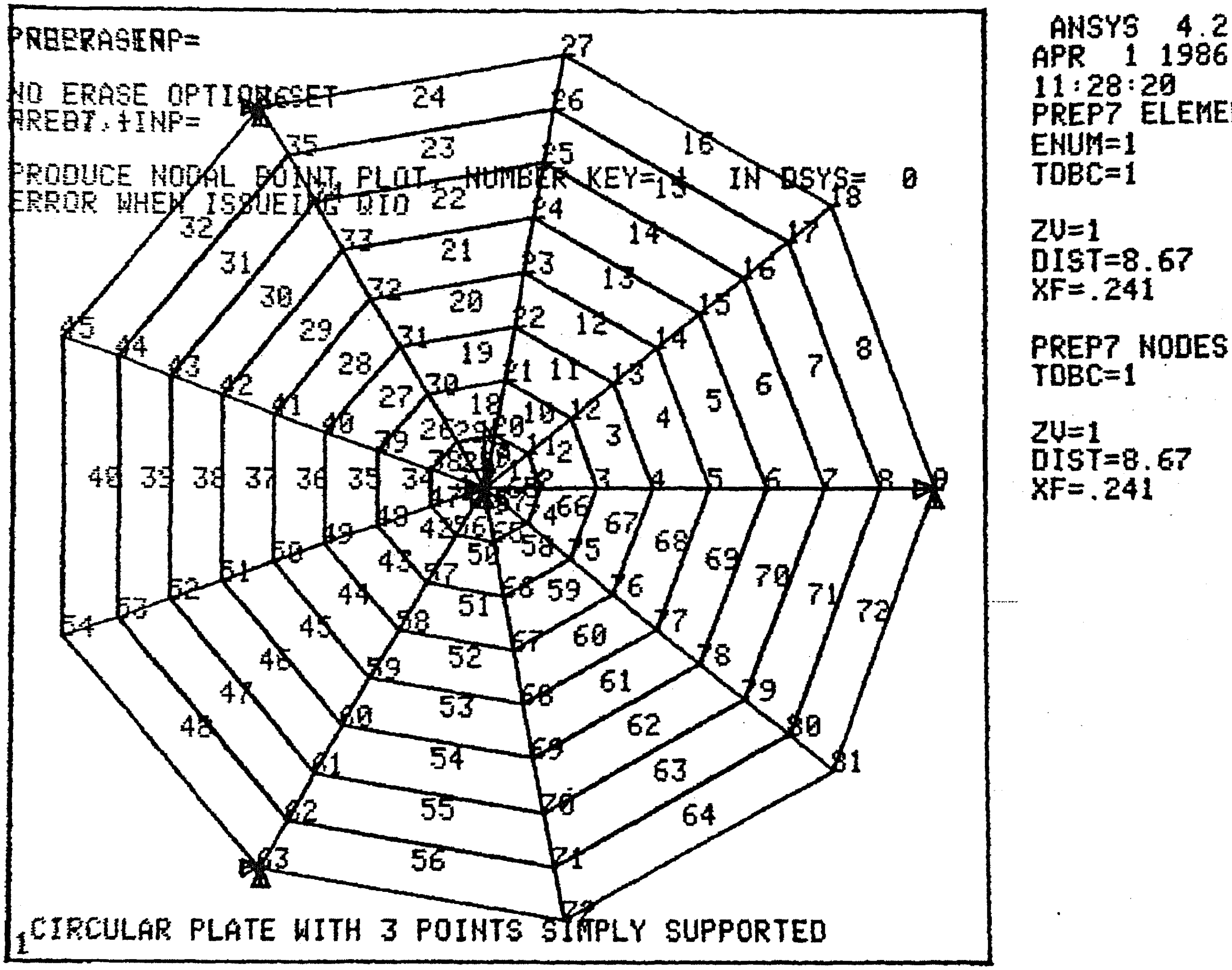

Figure 9 Finite Element Mesh for Circular Plate

(72 Elements and 81 Nodes)

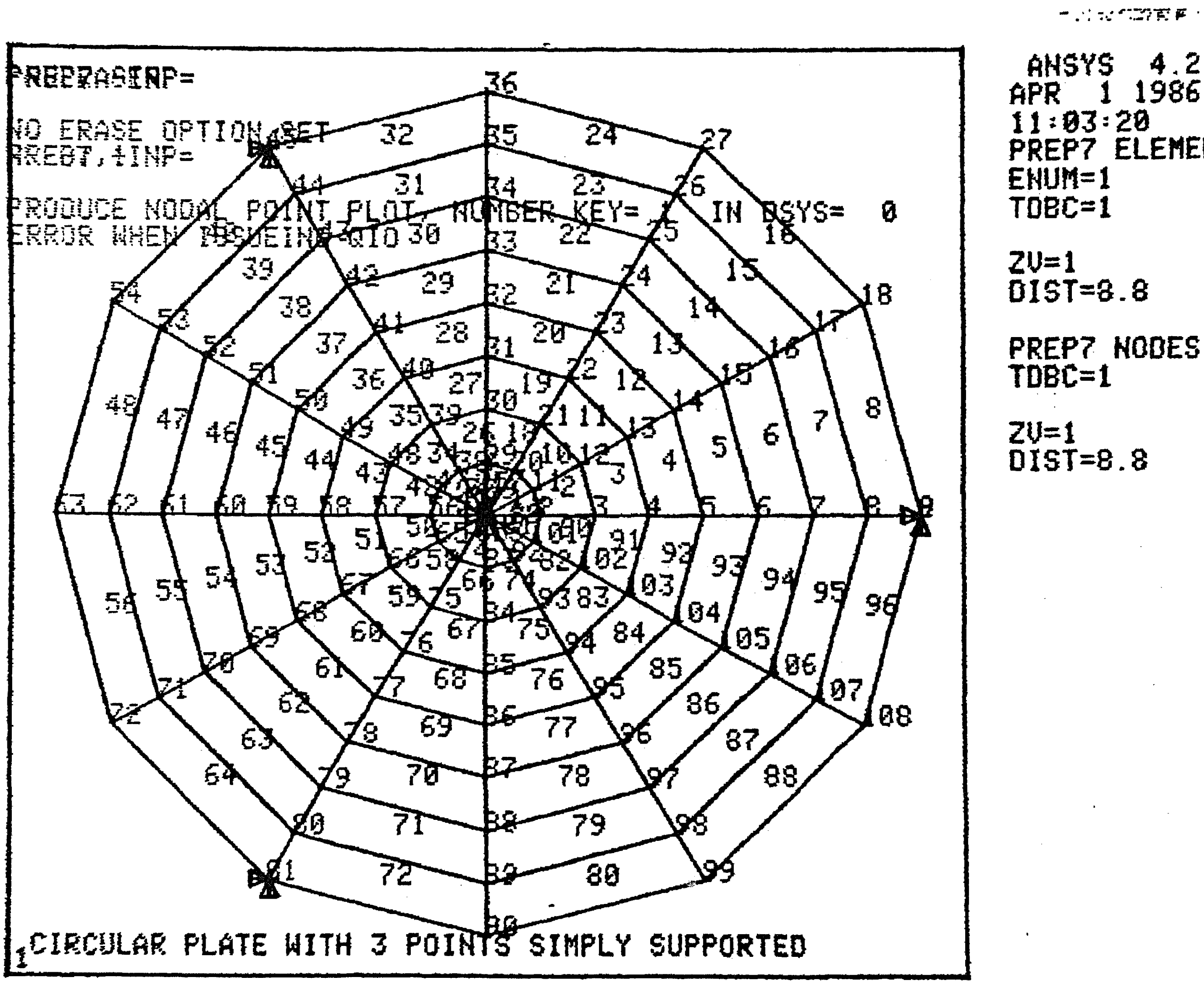

Figure 10 Finite Element Mesh for Circular Plate

(96 Elements and 108 Nodes)

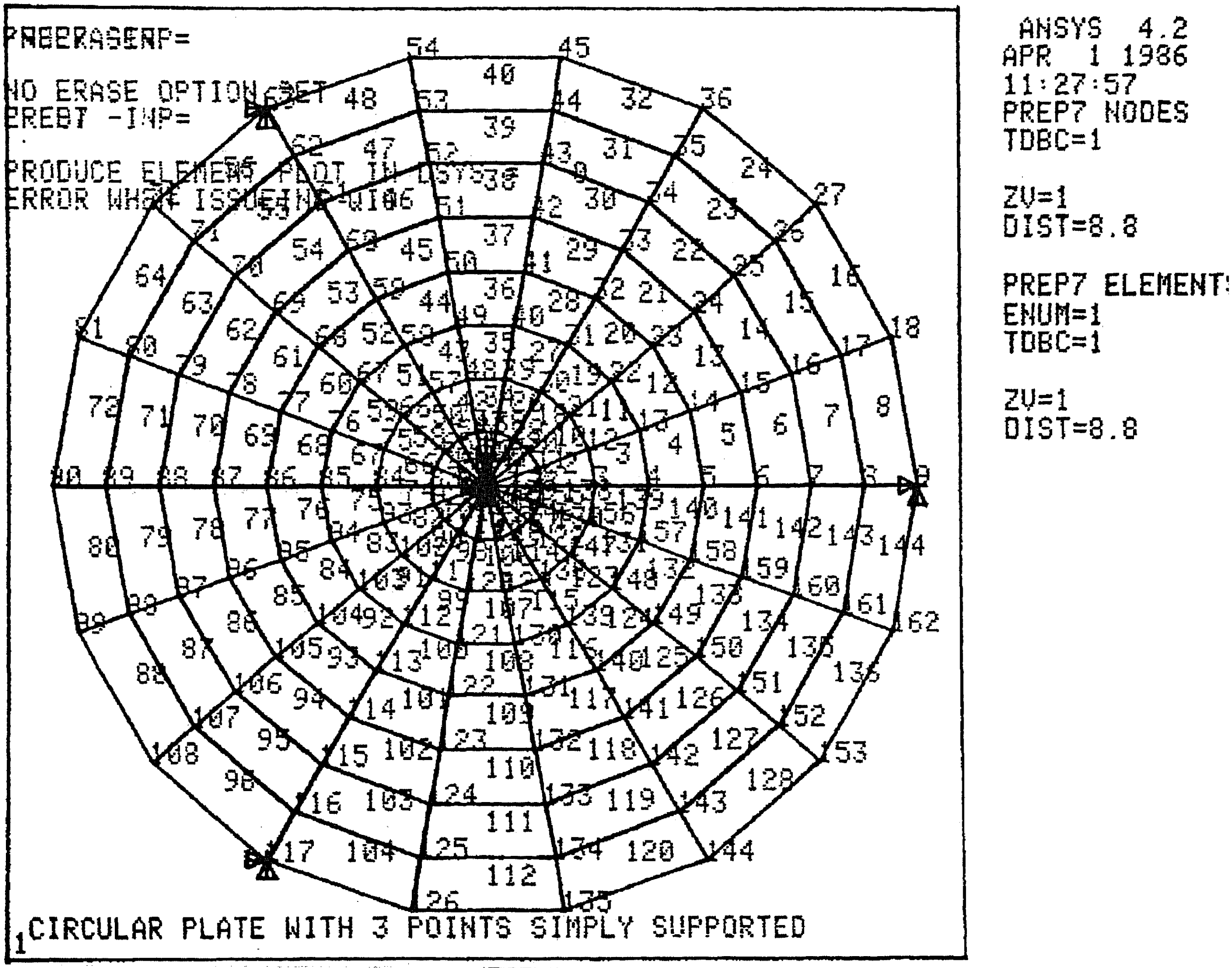

Figure 11 Finite Element Mesh For Circular Plate

(144 Elements and 162 Nodes)

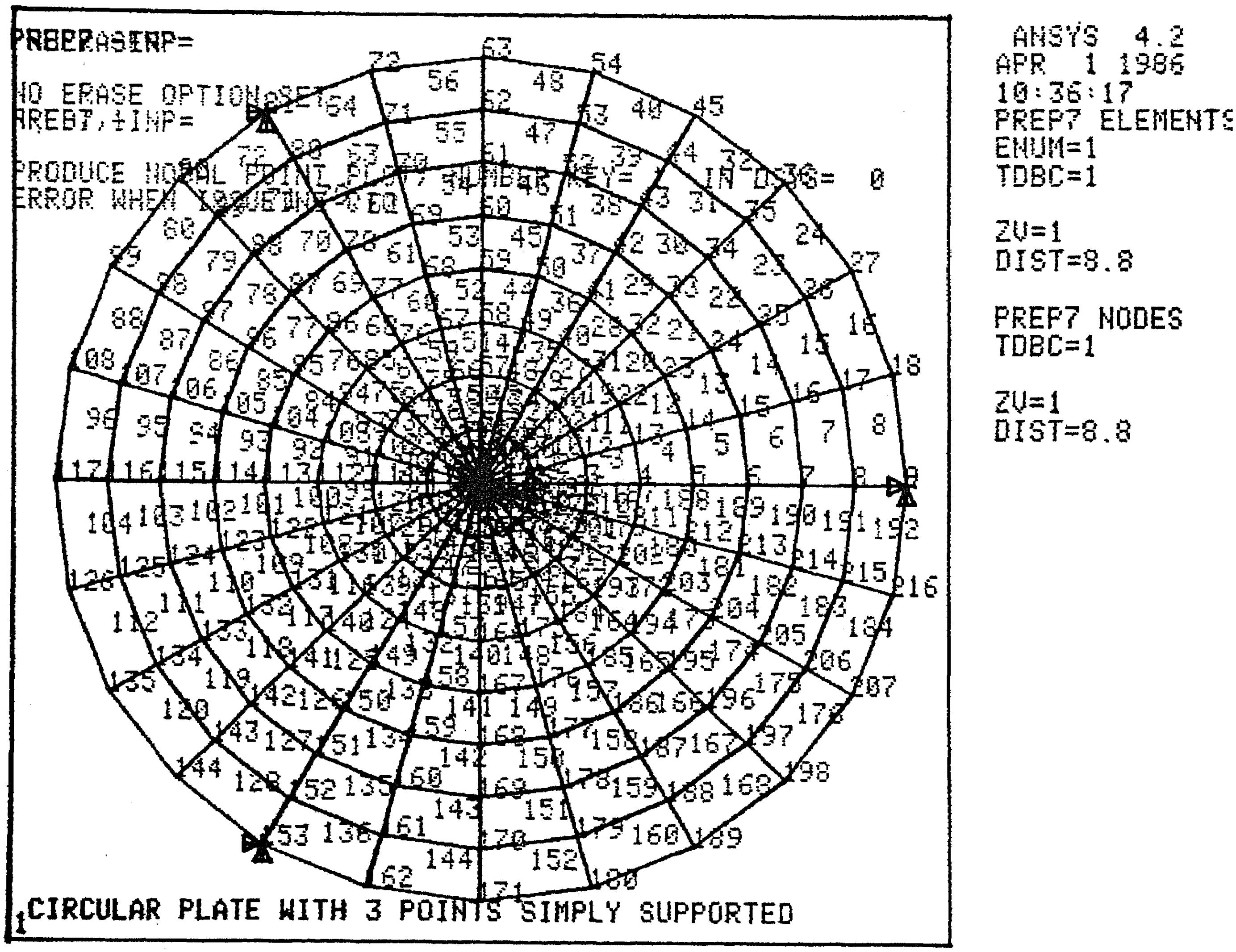

Figure 12 Finite Element Mesh For Circular Plate

(192 Elements and 216 Nodes)

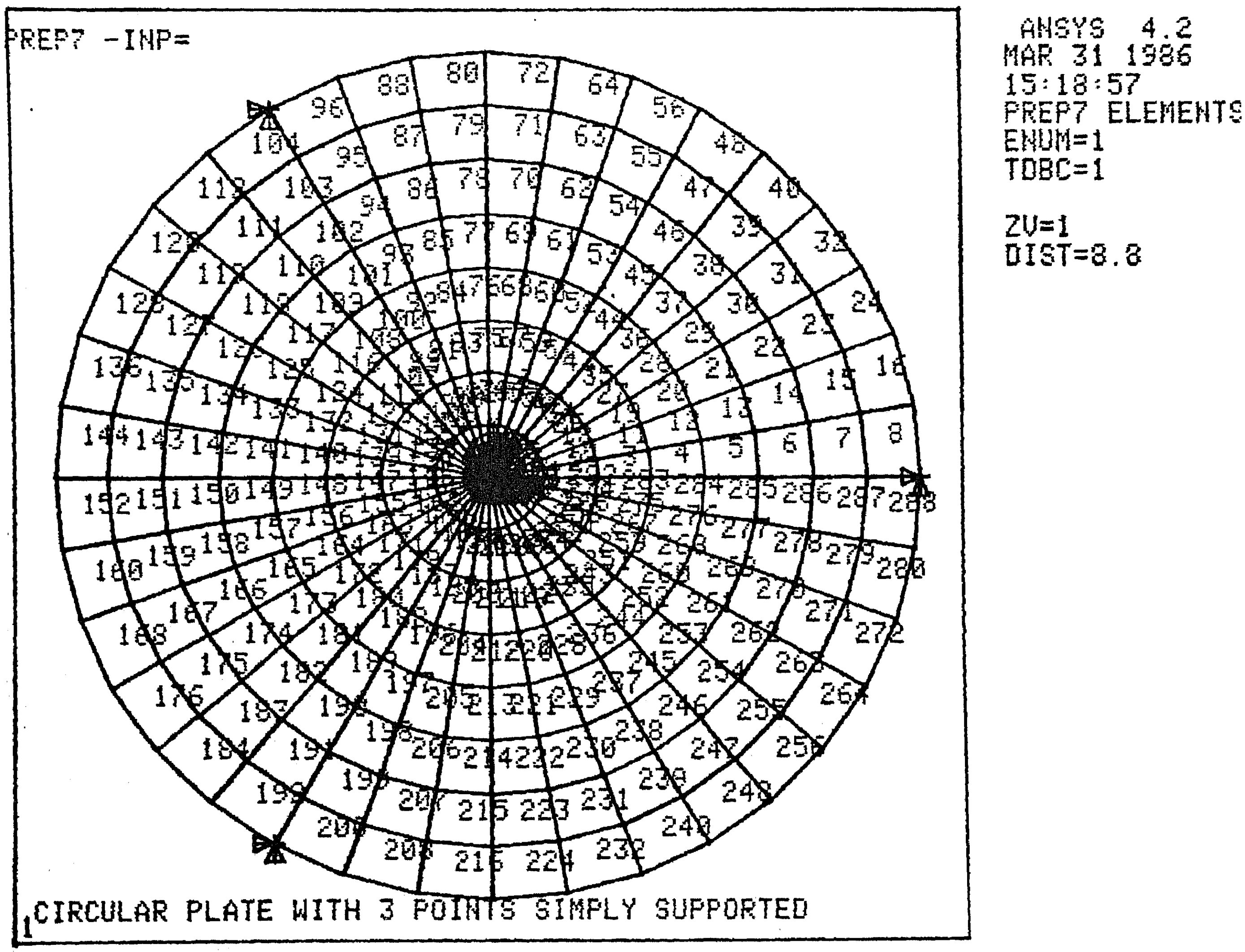

Figure 13 Finite Element Mesh for Circular Plate

(288 Elements and 324 Nodes)

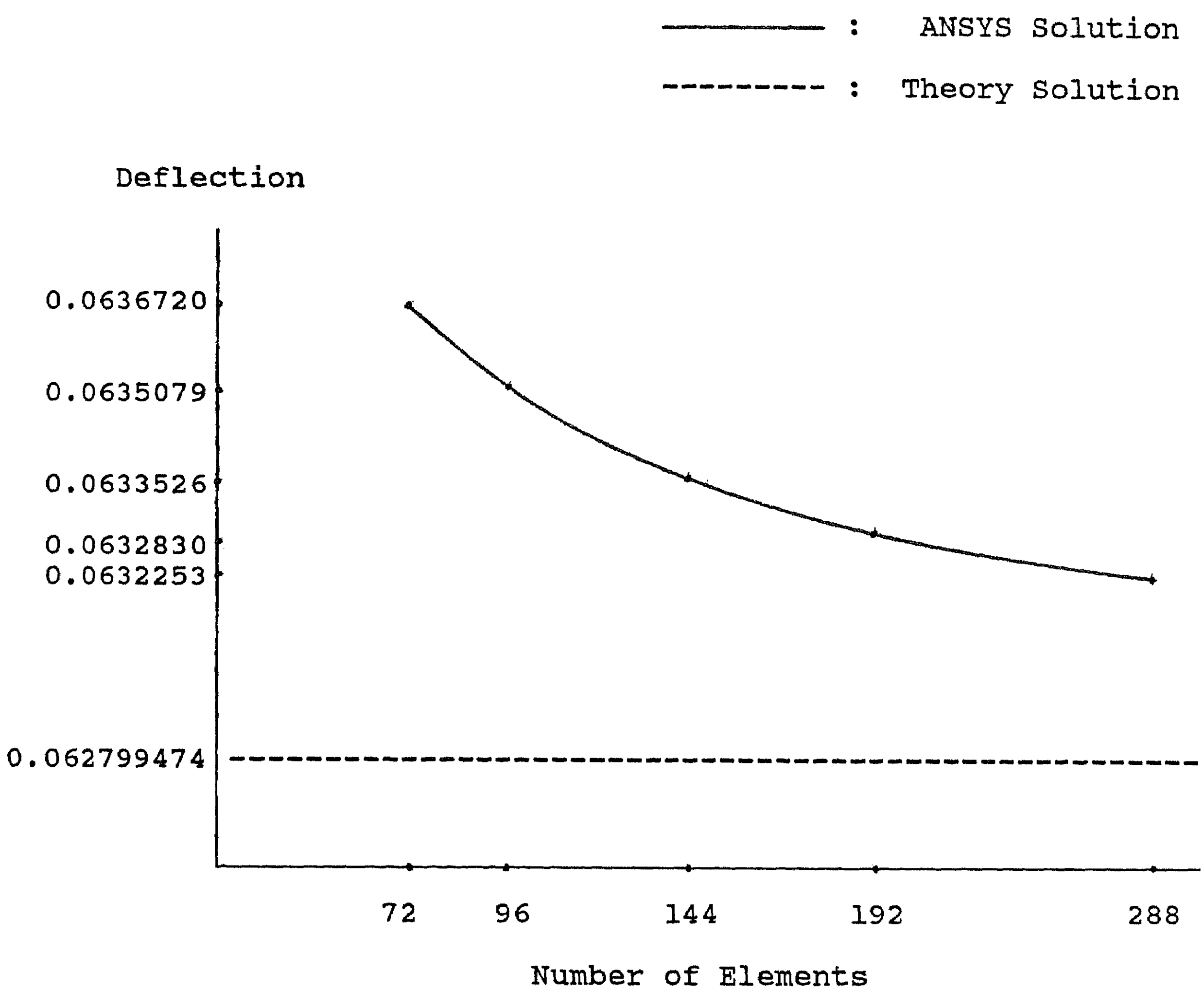

Figure 14 Comparision of the Finite Element Solution with the Series Solution for Center Deflection of a Circular Plate with Simply Supported 3 Points

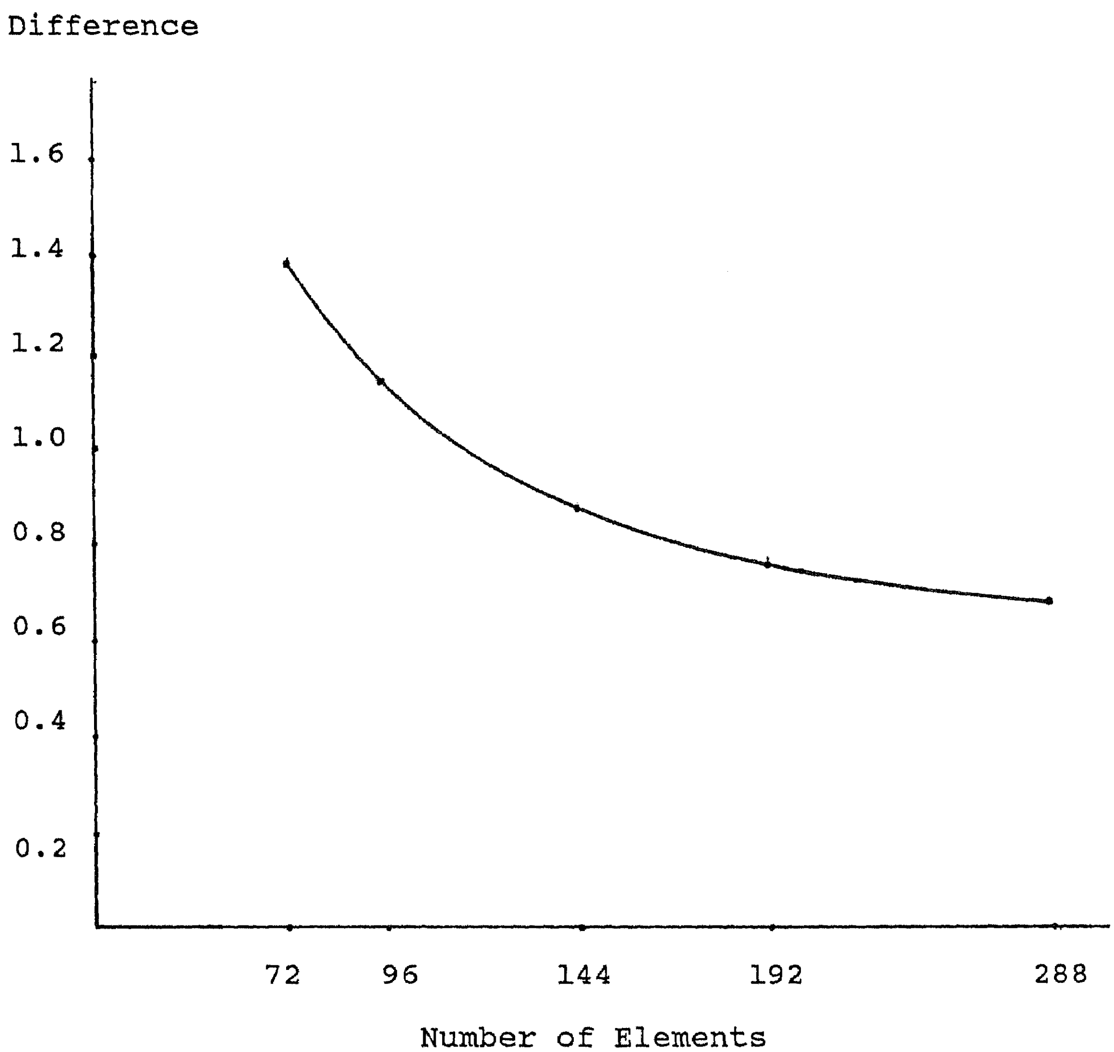

Figure 15 Number of Elements and Difference

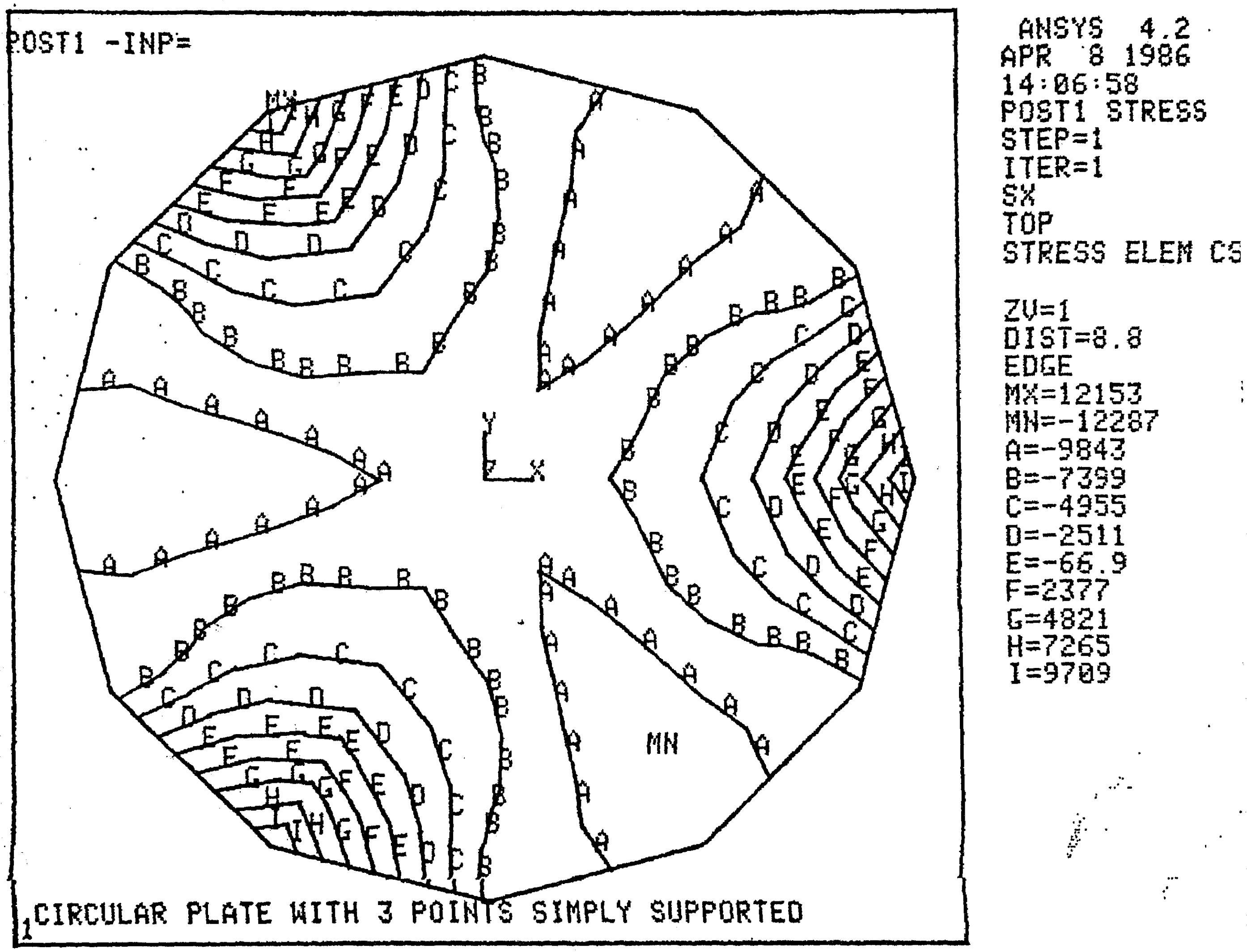

Figure 16 Stress Contour (SX) for 96 Elements in Example 3

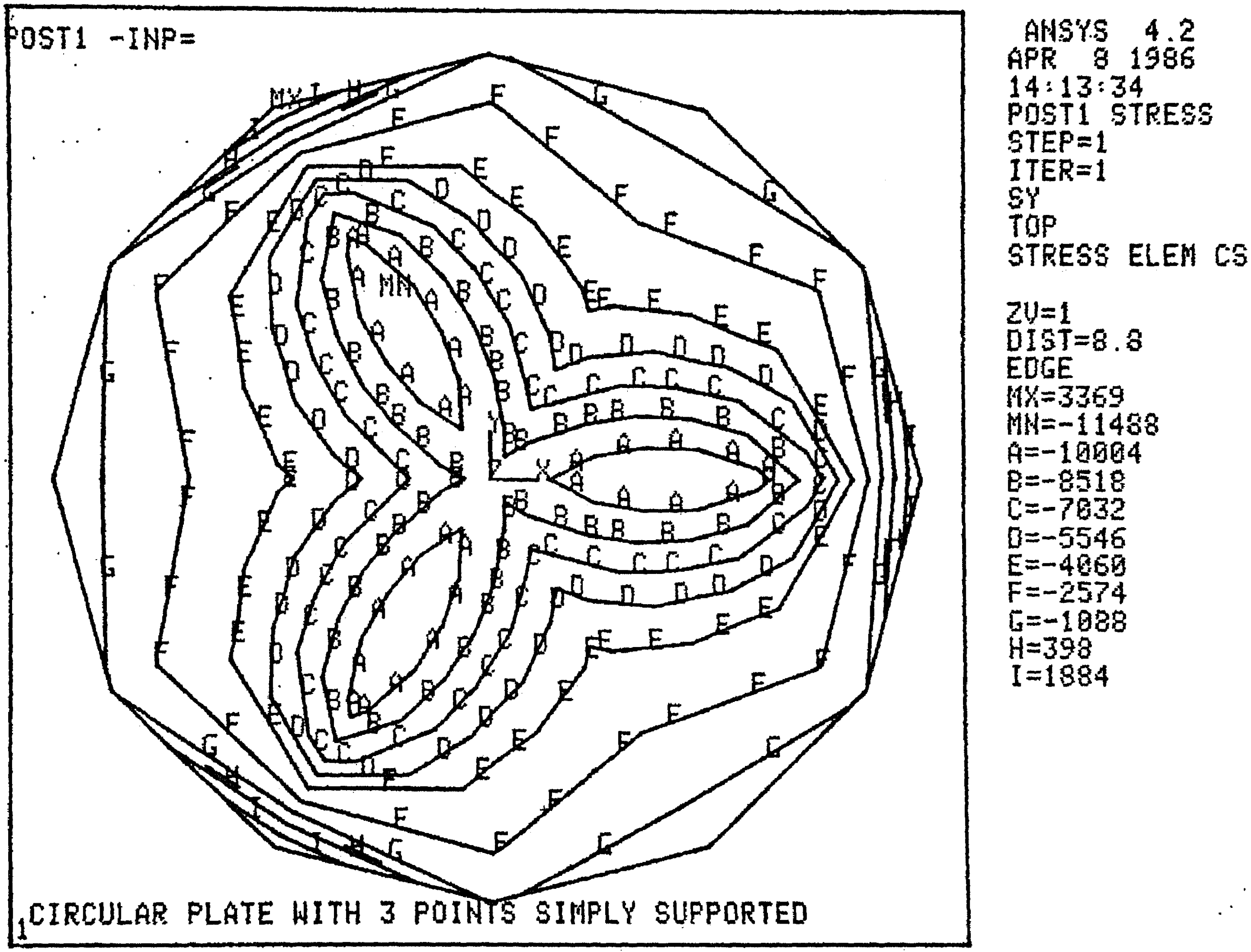

Figure 17 Stress Contour (SY) for 96 Elements in Example 3

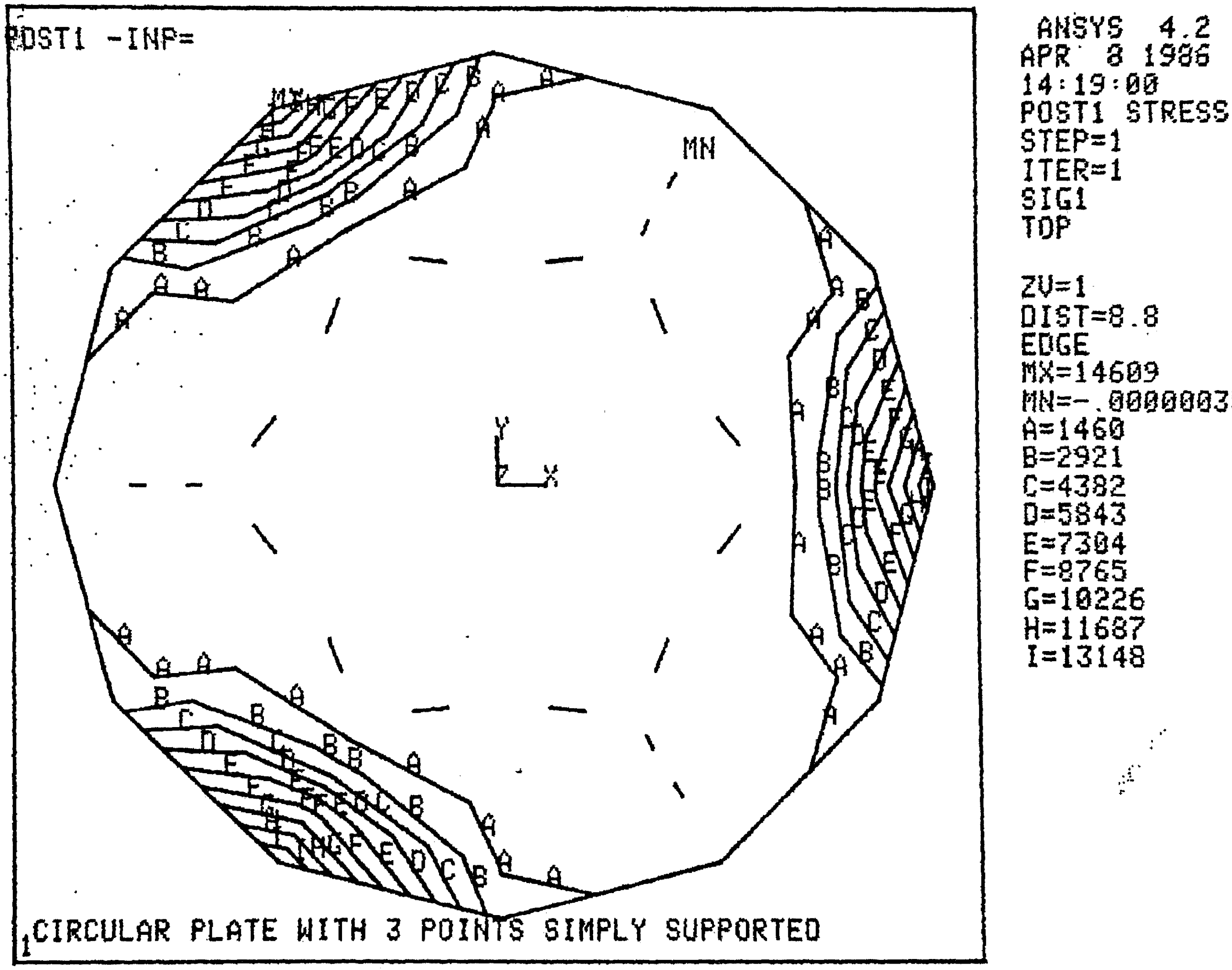

Figure 18 Stress Contour (SIG1) for 96 Elements in Example 3

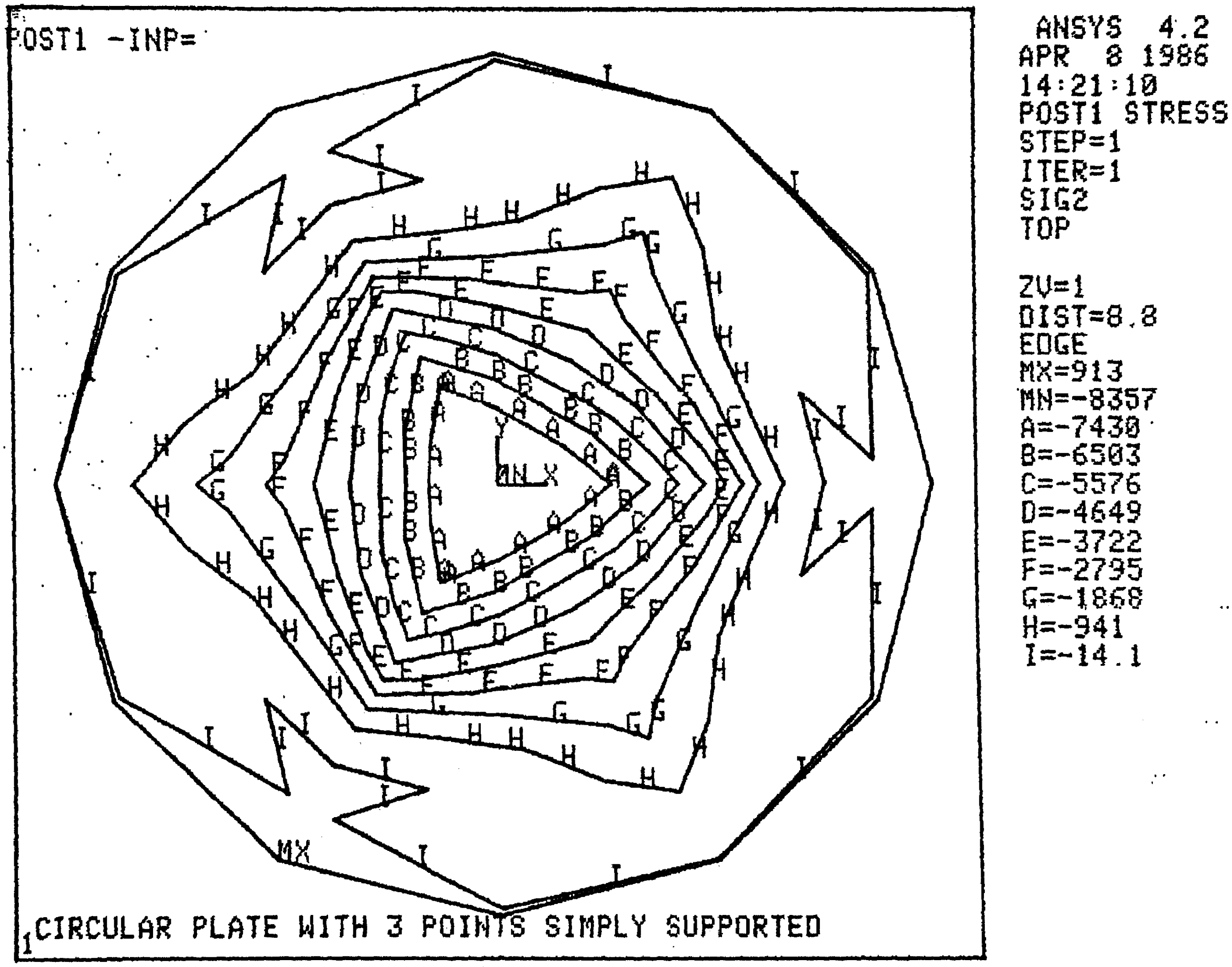

Figure 19 Stress Contour (SIG2) for 96 Elements in Example 3

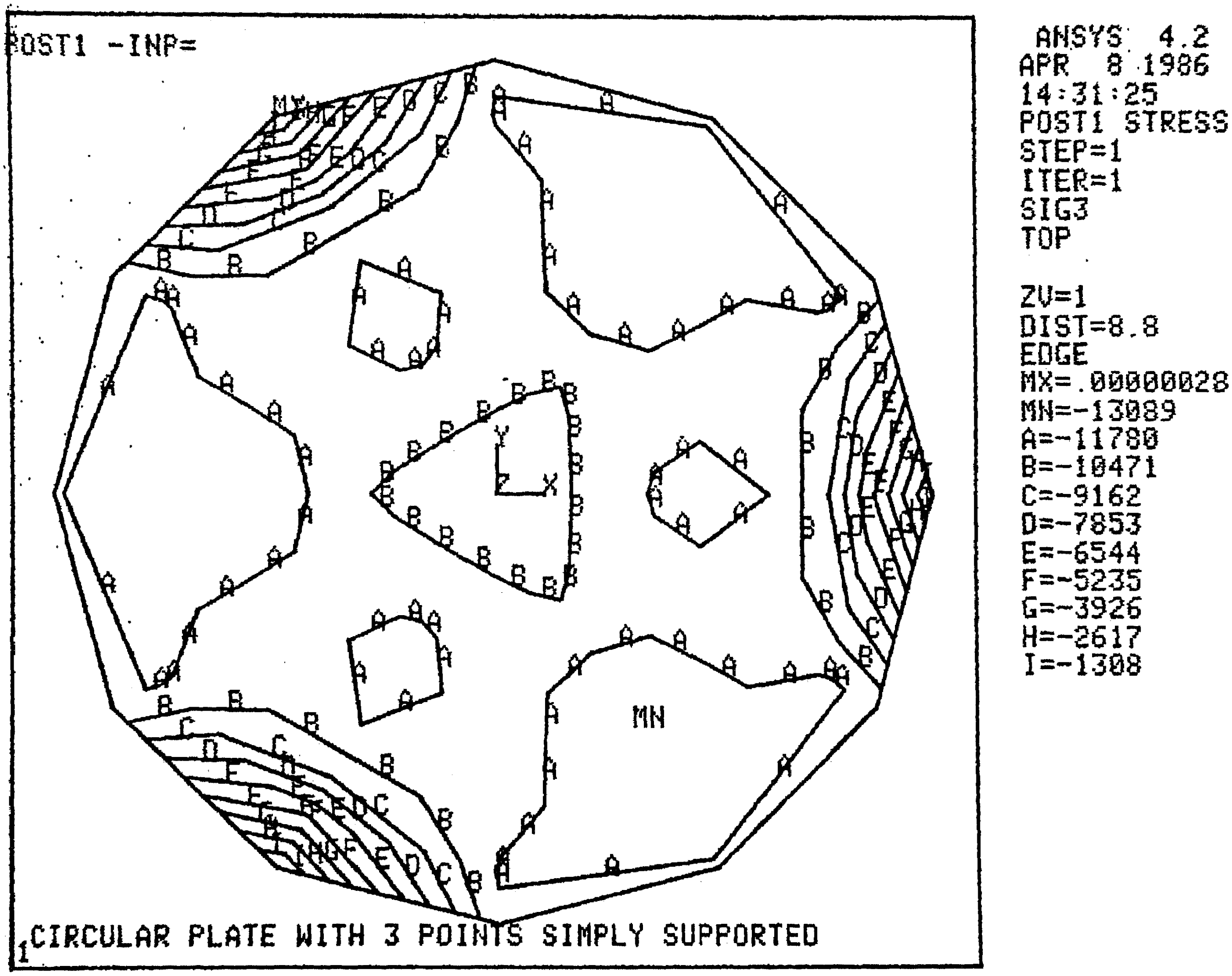

Figure 20 Stress Contour (SIG3) for 96 Elements in Example 3

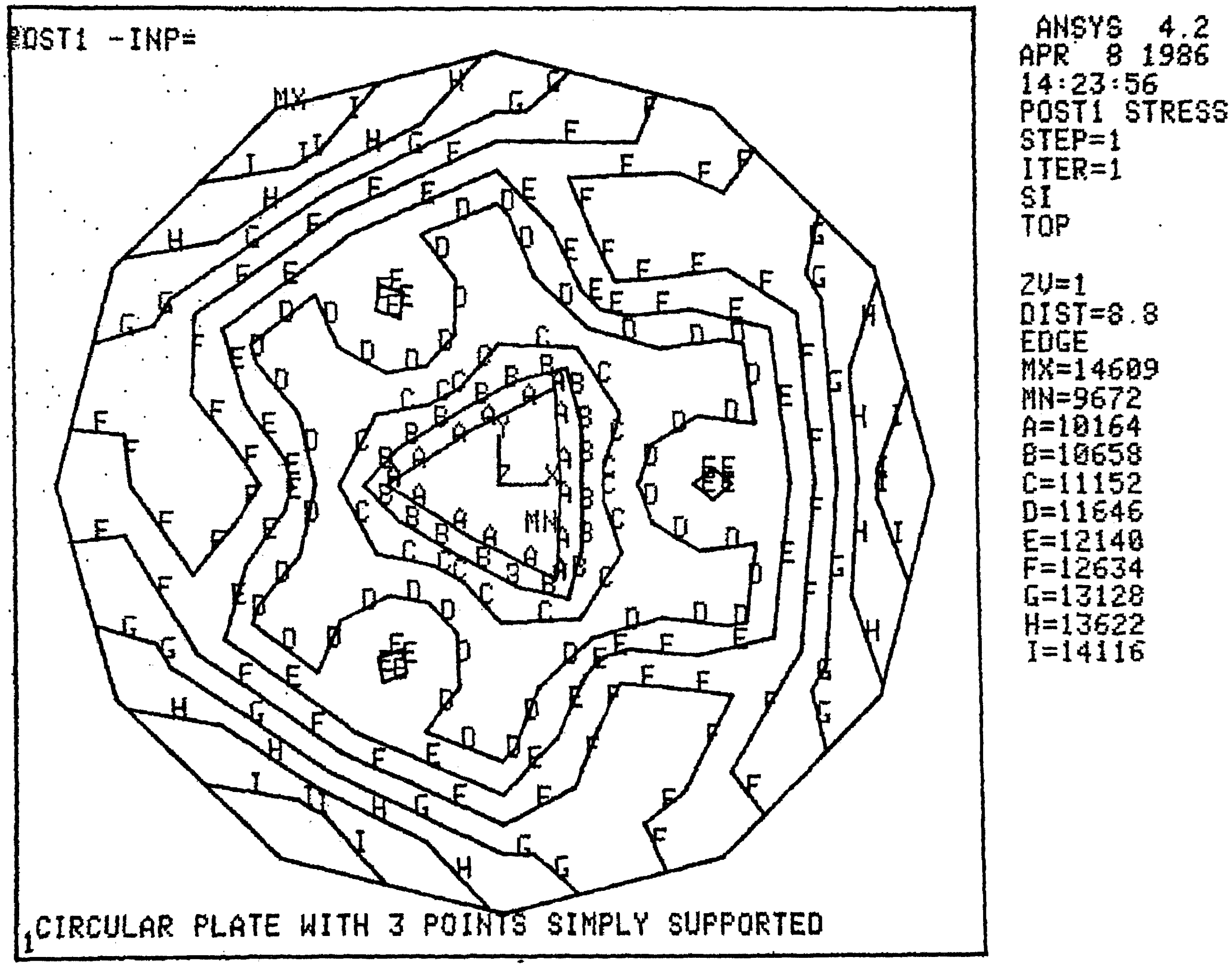

Figure 21 Stress Contour (SI) for 96 Elements in Example 3

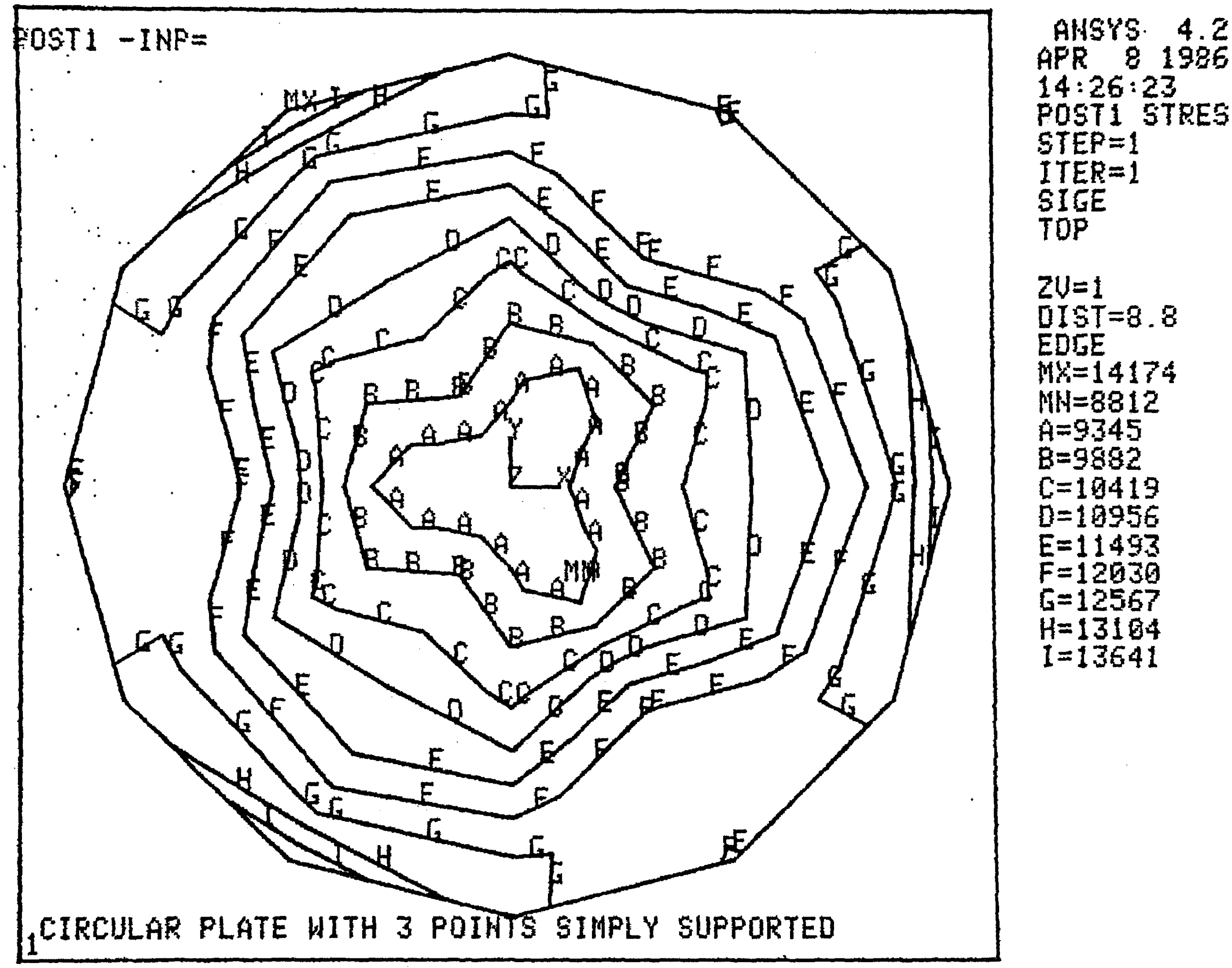

Figure 22 Stress Contour (SIGE) for 96 Elements in Example 3

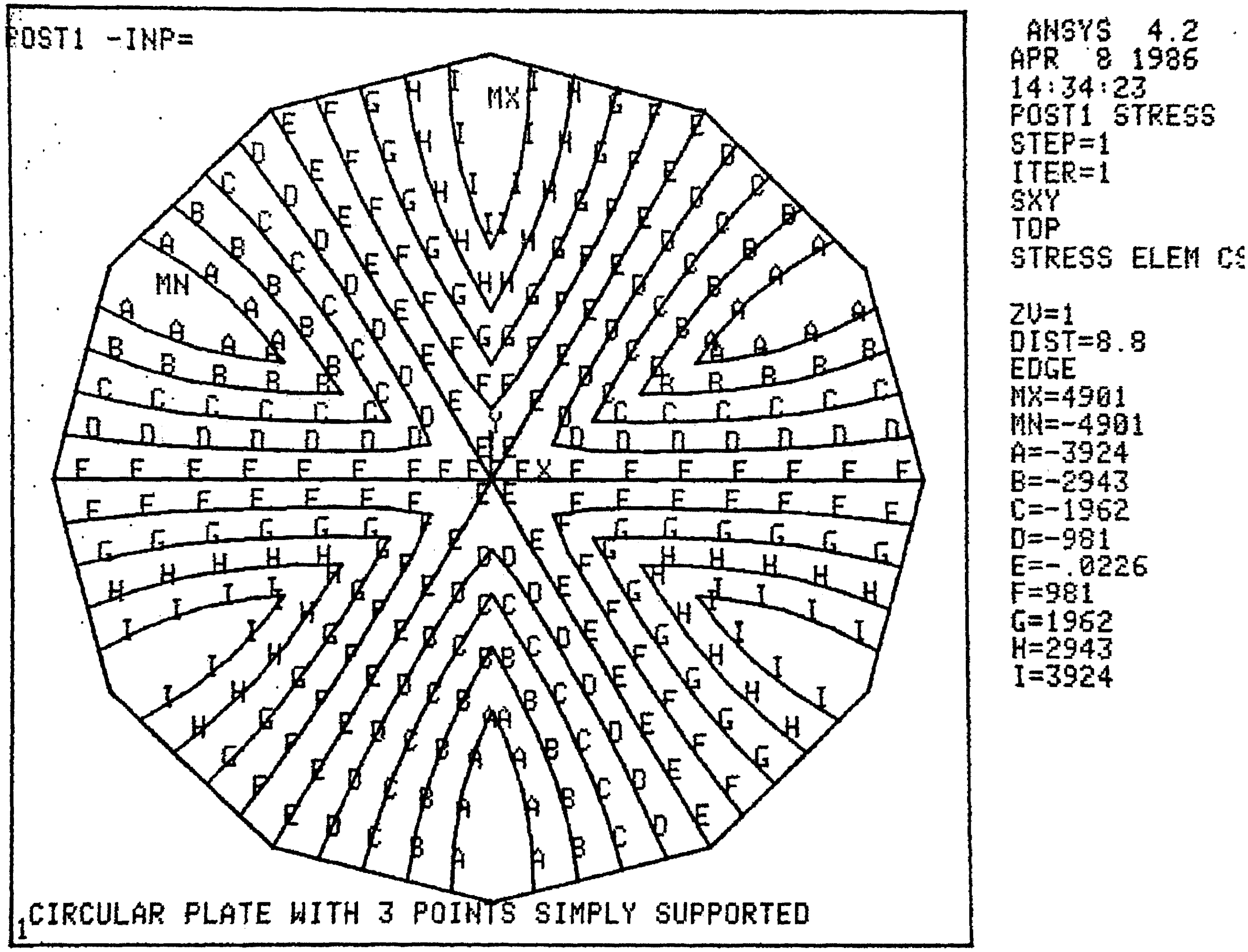

Figure 23 Stress Contour (SXY) for 96 Elements in Example 3

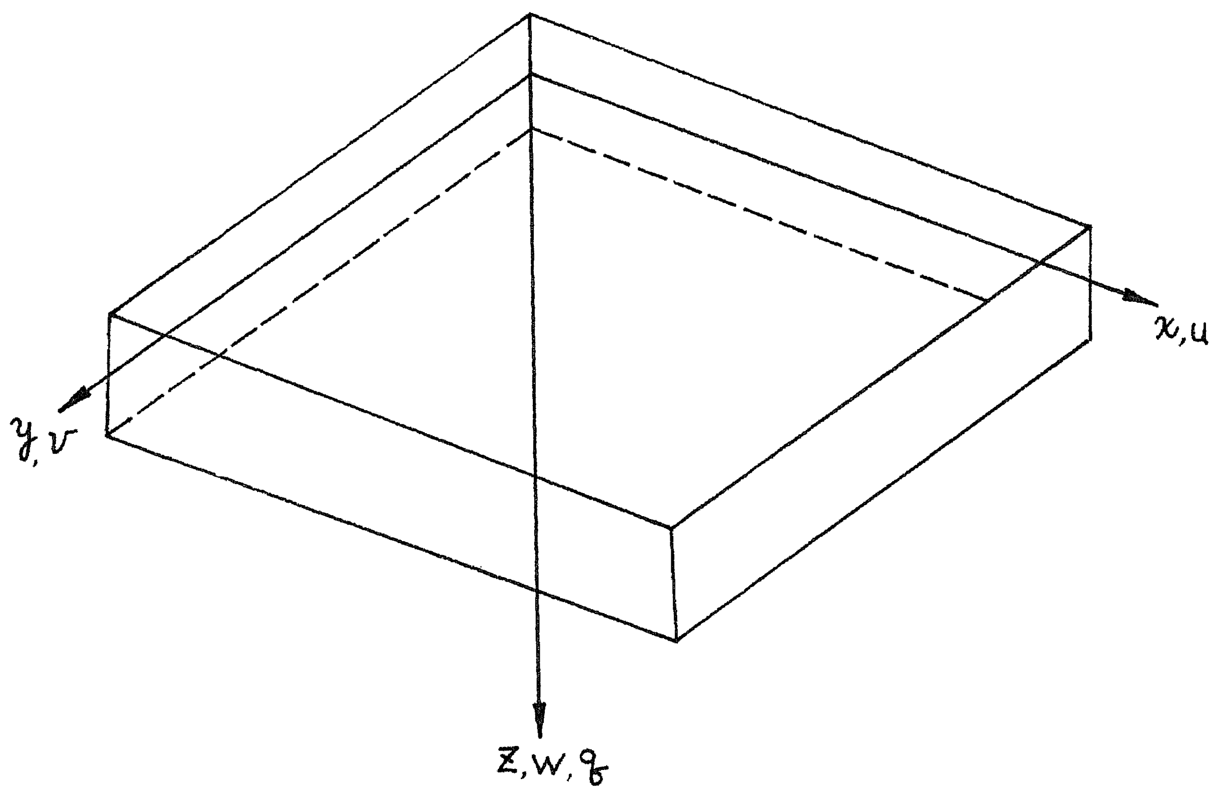

Figure A1 Displacement of Midsurface to the Load

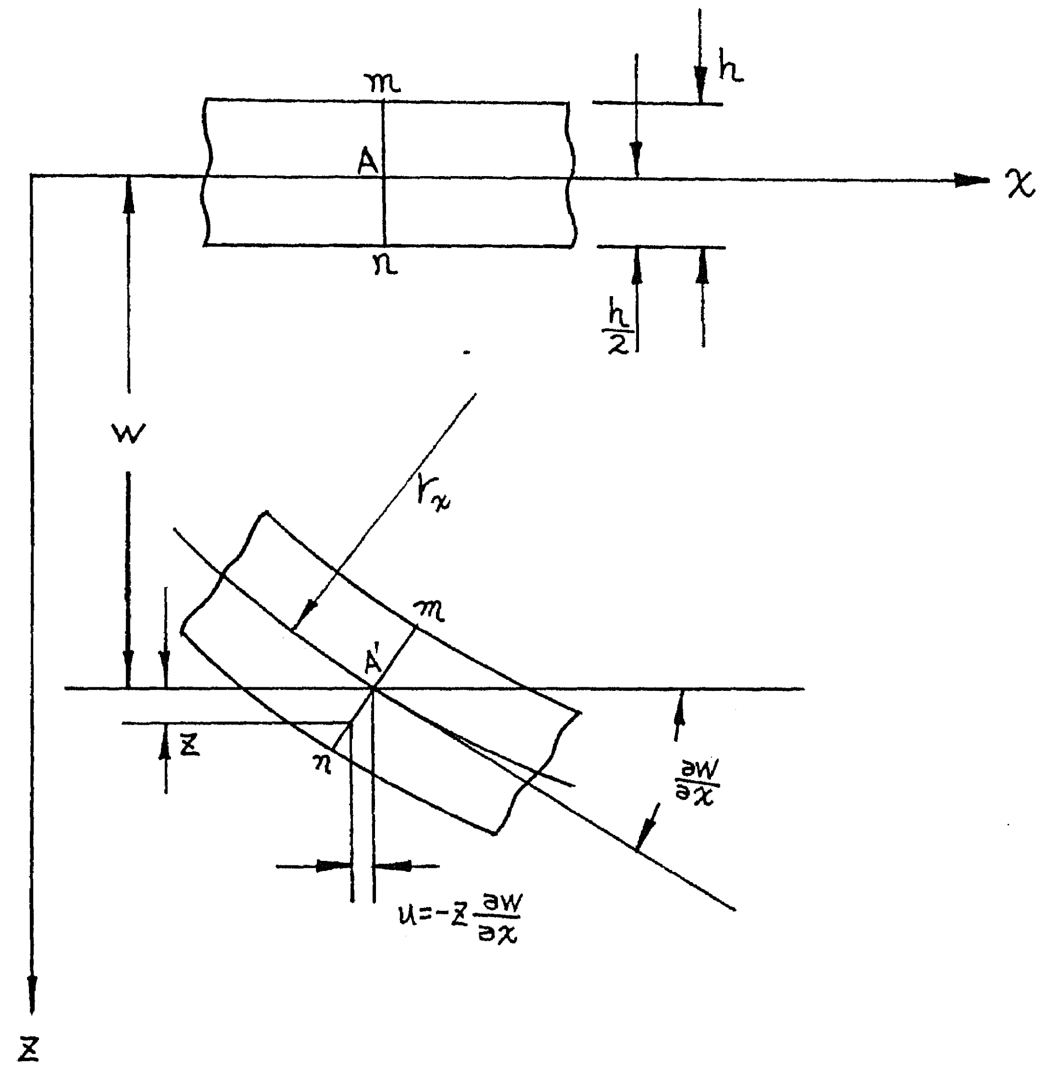

Figure A2 Deflection of Midsurface of Plate

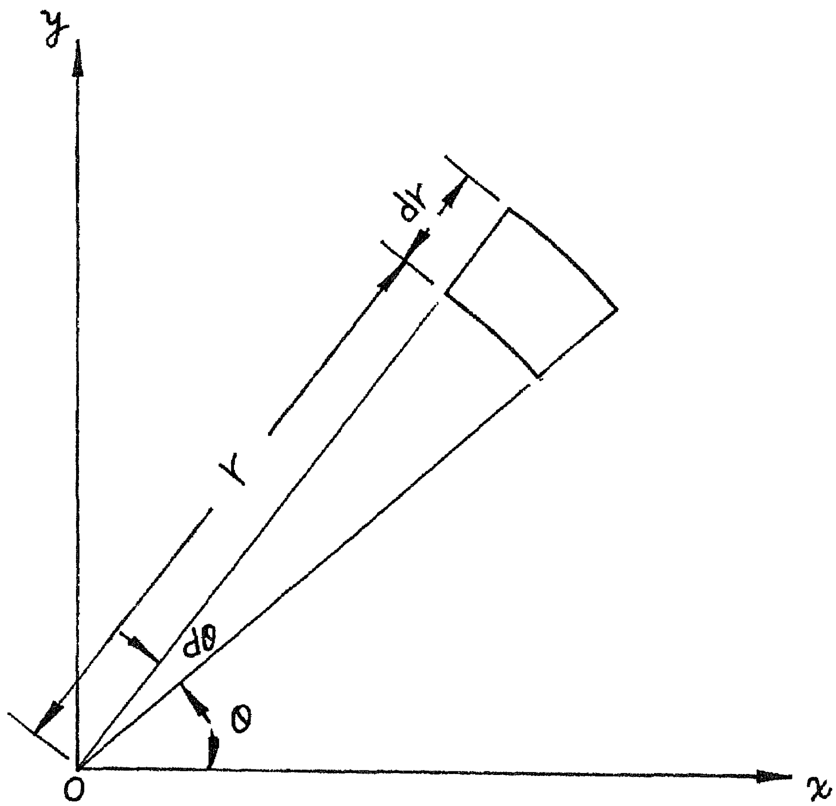

Figure A3 Relation Between Polar and Cartesian Coordinates

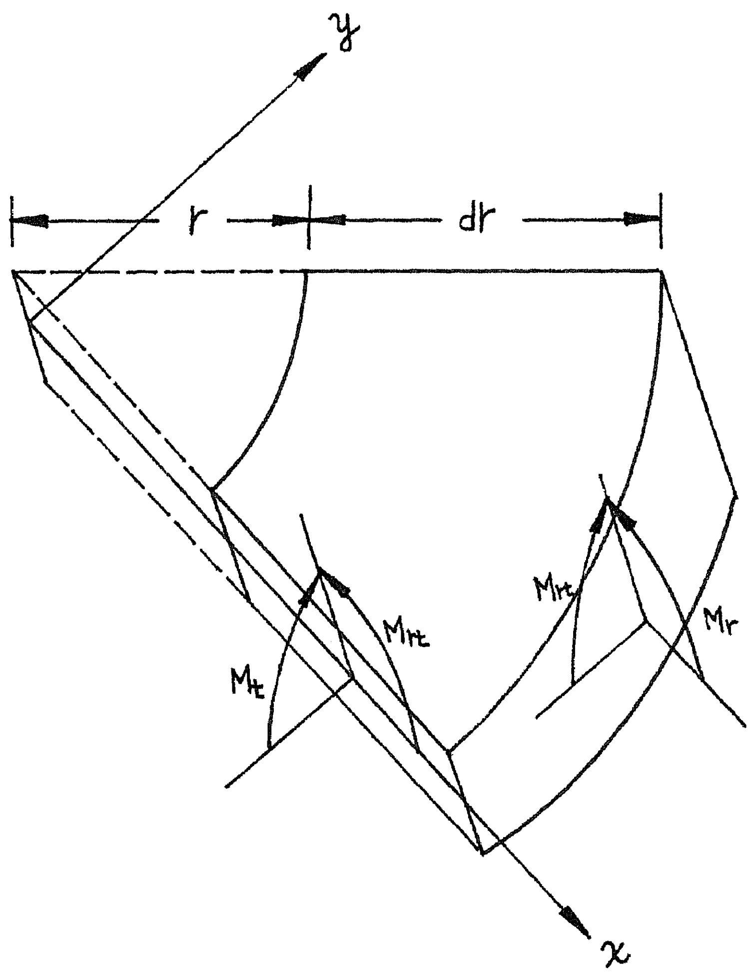

Figure A4 An Element of Circular Plate Bounded by Two Adjacent Axial Planes and by Two Cylindrical Surfaces

REFERENCE

1. Timoshenko, S. and Woinowsky-Krieger, S., Theory of Plates and Shells, 2nd Ed., McGraw-Hill Book Co., New York, 1959.

2. Timoshenko, S., Strength of Materials, Part II, 3rd Ed., D. Van Nostrand Co., 1956.

3. Ugural, A. C., Stresses in Plates and Shells, McGraw-Hill Book Co., 1981.

4. Wang, Chi-Teh, Applied Elascity, McGraw-Hill Book Co.,1953.

5. Marguerre, Karl and Woernle, Hans-Theo, Elastic PLates, Blaisell Publishing Co., 1969.

6. Timoshenko, S. P. and Goodier, J. N., Theory of Elasticity, 3rd Ed., McGraw-Hill Book Co., New York, 1970.

7. Roark, R. J. and Young, W. C., Formulas for Stress and Strain, 5th Ed., McGraw-Hill Book Co., New York, 1975.

8. Reddy, J. N., An Introduction to the Finite Element Method, McGraw-Hill Book Co., 1984.

9. Fenner, R. T., Finite Element Methods for Engineers, Macmillan, London, 1975.

10. Norrie, D. H. and Vries, G. de,The Finite Element Method: Fundamentals and Applications, Acadamic Press, New York, 1973.

11. Norrie, D. H. and Vries, G. de, An Introduction to Finite Element Analysis, Acadamic Press, New York, 1978.

12. Tong, P. and Rossetlos J. N., Finite Element Method: Basic Technique and Implementation, MIT Press, Cambridge, Mass.,1977.

13. Zienkiewicz, O. C., The Finite Element Method, 3rd Ed., McGraw-Hill Book Co., New York, 1977.

14. Bathe, J. K., Finite Element Procedures in Engineering Analysis, Prentice-Hall, N.J.,1982.

15. Brebbia, C. A. and Connor, J. J.,Fundamentials of Finite Element Techniques for Structure Engineers, John Wiley & Sons, Inc., New York, 1974.

16. Segerlind, Larry J., Applie Finite Element Analysis, Wiley, New York, 1976.

17. DeSalvo, Gabriel J. and Swanson, John A., ANSYS Engineering Analysis System User's Manual, Revision 4.1, Swanson Analysis System Inc., 1983.

18. DeSalvo, Gabriel J., ANSYS Engineering Analysis System Verification Manual, Revision 4.1, Swanson Analysis System Inc., 1983.