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Zivotofsky, Bernard

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THE EFFECT OF A HIGHER ORDER POLE ON THE OPTIMUM DESIGN OF MINIMUM THRESHOLD PHASELOCKED DETECTORS

by

Bernard Zivotofsky

Dissertation submitted to the Faculty of the Graduate School of the New Jersey Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Engineering Science 1985

APPROVAL SHEET

Title of Thesis:	The Effect of a Higher Order Pole on the Optimum Design of Minimum Threshold Phaselocked Detectors.
Name of Candidate:	Bernard Zivotofsky Doctor of Engineering Science, 1985
Thesis and Abstract Approved:	Dr. Jacob Klapper, Professor Electrical Engineering

Signatures of other members of the thesis committee

Date

Date

Date

ABSTRACT

Title of Thesis: The Effect of a Higher Order Pole on the Optimum Design Of Minimum Threshold Phaselocked Detectors

Bernard Zivotofsky, Doctor of Engineering Science, 1985

Thesis Directed by Professor Jacob Klapper

The phase lockedloop is widely used as an FM demodulator known as a phaselocked detector (PLD). It is particularly useful in the detection of weak signals because lower threshold is possible with a PLD than with a conventional limiter discriminator. An enhanced version PLD known as an extended range phaselocked detector (ERPLD) is capable of further threshold reduction. The most popular PLD is a second order type I system employing a loop filter having one real pole and one real zero.

One implementation of an ERPLD calls for the addition of a differentiator to the standard loop filter which replaces the real zero by a pair of complex zeros. Previous studies have assumed an ideal differentiator which results in an unbounded noise bandwidth and therefore requires the use of a predetection filter. But this is an idealization which is not physically realizable because there will always be an additional pole due either to design or to stray effects. This dissertation is a study of the effect of an

additional pole upon the threshold performance of these detectors.

A number of different PLDs and ERPLDs were selected for study. They are characterized by the presence or absence of an additional pole, the presence or absence of a predetection filter, and either test tone or voice modulation. In all, ten systems are analyzed and each is optimized with respect to threshold by a determination of the set of loop parameters which results in the minimum threshold carrier-to-noise-ratio, $(CNR)_{TH}$. Mathematical models are developed for $(CNR)_{TH}$ in each case and the optimization is performed by digital computer algorithm.

The calculations reveal that for a PLD with no predetection filter and for an ERPLD with a predetection filter, threshold decreases with increasing pole frequency, becoming optimum as the pole location approaches infinity (i.e. no second pole). However, only a small penalty is incurred if the pole exists at a sufficiently high frequency. Thus for 1 KHz test tone modulation and 10 KHz frequency deviation, a second pole at 140 KHz increases the minimum $(CNR)_{TH}$ of the PLD by only about 0.1 dB.

With the second pole added, the ERPLD can be optimized for use with no predetection filter and its minimum $(CNR)_{TH}$ is about 0.5 dB below that for the optimum standard PLD. In this design the real zero in the loop filter is located at infinity (i.e. eliminated).

The model is shown to be inapplicable to the case of a standard loop when used with a predetection filter and therefore no optimization is achieved for this case.

Practical optimum loop filter designs with components having ordinary tolerances are feasible because the minimum threshold levels are not very sensitive to small variations in parameter values.

Experimental verification of the computer results is given for several systems with test tone modulation. The theoretically derived optimum systems were implemented in the laboratory with high gain active filters and discrete phaselocked loops and exhibited measured threshold levels consistent with the calculated values.

VITA

Name: Bernard Zivotofsky

Degree and Date to be conferred: D. Eng. Sc. 1985

Secondary Education: Brooklyn Technical H.S., 1953.

Collegiate Institutions attended	Dates	Degree	Date
Cooper Union	1954-7	B.E.E.	1957
Polytechnic Inst. of N.Y.	1958-61	M.E.E.	1961
New Jersey Inst. of Technology	1973-85	D.Eng.Sc.	1985

Major: Electrical Engineering.

Positions Held: Associate Professor, Essex County College Newark, N.J.

Engineer, Kollsman Instrument Corporation Syosset, N.Y.

Engineer, Polytechnic Research and Development Corporation, Brooklyn, N.Y.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER I

BACKGROUND INFORMATION AND MOTIVATION

1.1 Introduction

Among its many other applications [Ref. 4, sec. 9.4] the phaselocked loop (PLL) may be employed as a frequency demodulator and as such it is superior to the conventional discriminator in weak signal service. Like the discriminator, the PLL exhibits a noise threshold which defines the signal quality that is required for successful detection, but the PLL is capable of achieving lower threshold levels than the discriminator.

A specific loop configuration has emerged as the standard for PLD applications and a modified version known as an extended range phaselocked detector [Ref. 2, 3, 4] (ERPLD) has been studied and used. The ERPLD is capable of achieving lower threshold than is the standard PLD.

This dissertation is a study of the effect upon the threshold performance of both the PLD and the ERPLD of an additional pole in the PLL. An optimum design procedure is used to achieve minimum threshold for each of the configurations under study.

In this chapter some of the basic theory of PLL operation is reviewed. A linear model is developed to facilitate analysis of the loop response to an input frequency -modulated signal in the presence of additive noise. Although the threshold phenomenon results from a non-linearity in the loop, the linear model is useful because it is an adequate descriptor of loop performace above threshold and a good predictor of the onset of the nonlinearity and the consequent threshold. In chapters 2 and 3, these models are the bases for the derivations of expressions for threshold Carrier-to-Noise-Ratio $(CNR)_{TH}$ for the various detectors. These expressions are minimized in chapter 4, resulting in optimum designs with respect to threshold level.

1.2 The Linear Model

A phaselocked loop consists of three component sections arranged in a closed loop configuration as shown in figure 1-1 [Ref. 1, P. 2, Ref. 4, P. 77]

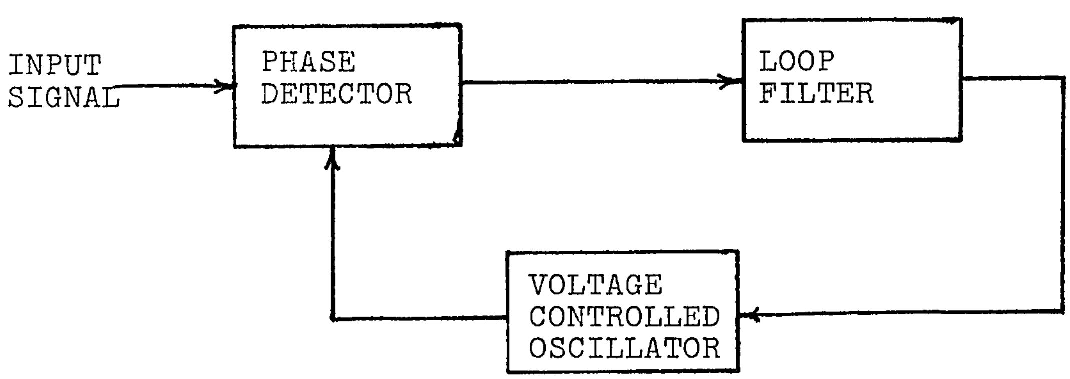


FIGURE 1-1 GENERAL PHASELOCKED LOOP

When the loop is locked, the cycle to cycle count of the voltage controlled oscillator (VCO) is identical to that of the input signal. The Phase detector compares the phase of the VCO output with the phase of the input signal and develops an output voltage which depends upon the phase difference. This error voltage is fed back to the control input of the VCO via the loop filter with the appropriate polarity to reduce the phase difference and thus perpetuate the locked condition.

For purposes of analysis consider the input signal to be an angle modulated sinewave which can be described by:

$$v_i(t) = V_i \cos [w_i t + \theta_i(t)] \tag{1-1}$$

where w_i is a carrier frequency and $\theta_i(t)$ is a time dependent phase angle which usually bears any encoded information. The VCO output may be described by:

$$v_o(t) = V_o \sin [w_o t + \theta_o(t)] \tag{1-2}$$

which has an instantaneous frequency

$$w_c(t) = w_o + \frac{d}{dt} \theta_o(t)$$

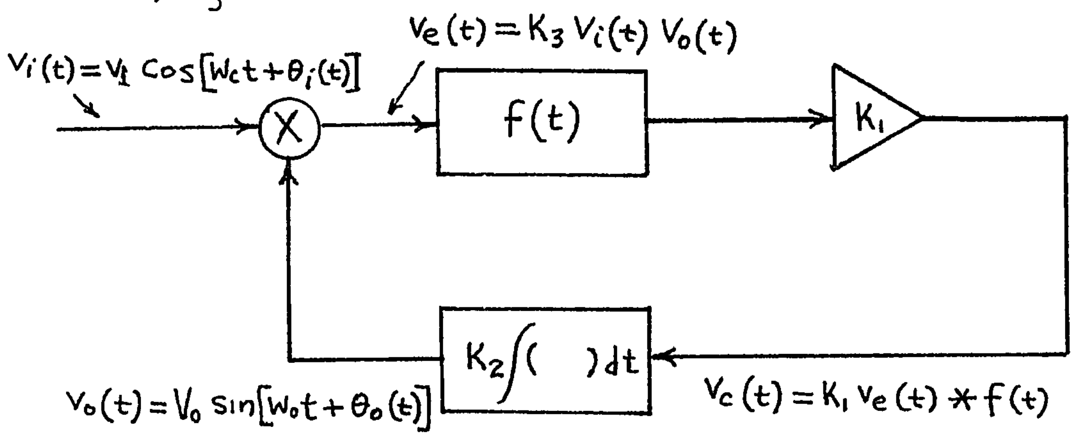
Here w_o is the resting frequency of the VCO and the time dependent term is determined by the control voltage. If K_2(Rads/Sec/Volt) is the sensitivity of the VCO ,then

$$K_2 v_c(t) = \frac{d}{dt}\ \theta_o(t), \tag{1-3}$$

which may be written in transform notation as

$$\theta_o(s) = \frac{K_2}{s}\ V_c(s)\ . \tag{1-4}$$

The loop filter is a linear network with transfer function F(s) and d.c gain, K_1. These notions are represented in figure 1-2 where the phase detector is shown as a multiplier which is characterized by a multiplicative constant, K_3.



<u>FIGURE</u> <u>1-2</u> PLL SIGNALS

At the detector output an error voltage $v_e(t)$ is developed.

$$v_e(t) = K_3 v_i(t) v_o(t)$$

$$= V_i V_o \cos[w_i t + \theta_i(t)] \sin[w_o t + \theta_o(t)]$$

$$= (K_3V_iV_o/2)\sin[(w_i+w_o)t + \theta_i(t) + \theta_o(t)] + \sin[(w_i-w_o)t + \theta_i(t)-\theta_o(t)].$$

If the VCO resting frequency agrees with the signal carrier frequency then $w_i = w_o$ and

$$v_e(t) = \frac{K_3V_iV_o}{2}\left\{\sin[2w_it+\theta_i(t)+\theta_o(t)] + \sin[\theta_i(t)-\theta_o(t)]\right\} \quad (1-5)$$

which contains a low frequency term and a term at twice the carrier frequency.

Assuming that the high frequency term will be rejected by the loop filter and letting $\frac{K_3V_iV_o}{2} = \delta$ (1-6)

results in

$$v_e(t) = \delta\sin[\theta_i(t)-\theta_o(t)] .$$

For small phase differences

$$v_e(t) \approx \delta[\theta_i(t)-\theta_o(t)] = \delta\,\theta_e(t)$$

where $\theta_e(t)$ is defined as the phase error, $\theta_i(t)-\theta_o(t)$. This completes the linear modeling of the loop and enables the use of transfer function techniques in its analysis.

figure 1-3 shows the linear model of a PLL in response to phase.

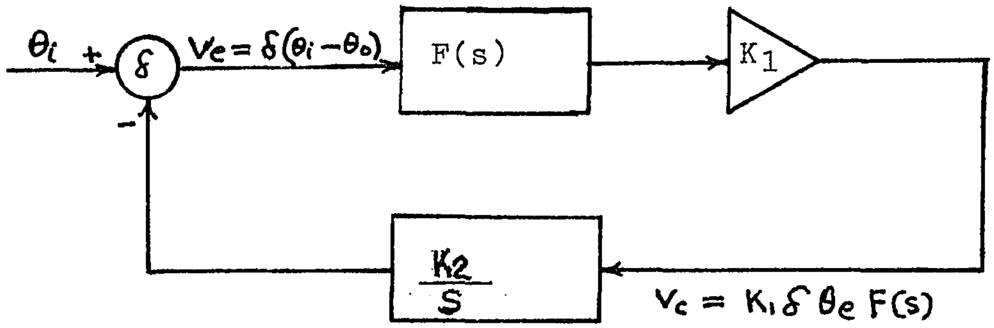


FIGURE 1-3 LINEAR MODEL

In the Laplace transform domain an expression for $\theta_o(s)$ may be derived from the diagram.

$$\theta_o(s) = \delta[\theta_i(s)-\theta_o(s)]F(s)K_1K_2/s$$

From the following algebraic manipulation of this equation there emerges an expression for the closed loop transfer function

$$H(s) = \frac{\theta_o(s)}{\theta_i(s)} \quad .$$

$$\theta_o + \delta K_1\theta_oF(s)K_2/s = \delta K_1\theta_iF(s)K_2/s$$

or $\quad \theta_o[1+\delta K_1K_2F(s)/s] = \delta K_1\theta_iF(s)K_2/s$

From which

$$H(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{\delta K_1 K_2 F(s)}{s + \delta K_1 K_2 F(s)}$$

The notation is simplified if we define $\delta K_1 K_2 = K$. Then

$$H(s) = \frac{KF(s)}{s+KF(s)} \qquad (1\text{-}7)$$

Also, the error voltage transfer function can be written as

$$\frac{V_e(s)}{\theta_i(s)} = \frac{(\theta_i - \theta_o)}{\theta_i} = (1-\theta_o/\theta_i) = [1-H(s)]$$

or $$\frac{\theta_e}{\theta_i} = 1-H(s) \qquad (1\text{-}8)$$

where $\theta_e = \theta_i - \theta_o$

Equation (1-7) and (1-8) apply to all linearized PLL models and will be used repeatedly in the development that follows.

1.2.1 Incorporation of Noise into the Linear Model

For purposes of investigating the influence of noise on the performance of the loop it is convenient to incorporate the noise into the linear model and render it subject to analysis by transfer function methods. If stationary, additive, band limited gaussian noise is assumed to accompany a sinusoidal signal at the input to the loop then an equivalent noise component injected into the output of the phase detector (PD) can be formulated.

There are two waveforms driving the PD. One is the input, $v_i(t)$, consisting of the sinusoidal signal plus the additive noise and the other is the output from the VCO, $v_o(t)$. These are described respectively by

$$v_i(t) = V_i \cos [w_i t + \theta_i] + N(t) \text{ and}$$

$$v_o(t) = -V_o \sin [w_o t + \theta_o]$$

The noise component may be represented by $N(t) = X(t) \cos w_i t - Y(t) \sin w_i t$, where $X(t)$ and $Y(t)$ are zero mean, gaussian functions of time[Ref. 6, P.165]. Assume θ_i and θ_o are constants and again let the PD be an ideal multiplier. Then its output, $v_e(t) = K_3 v_i(t) v_o(t)$. As in equation (1-6) let $K_3 V_i V_o/2 = \delta$ and assume a locked loop so that $w_o = w_i$ to get

$$v_e(t) = \delta \sin (\theta_i - \theta_o) + \frac{X(t)\delta}{V_i} \sin \theta_o - \frac{Y(t)\delta}{V_i} \cos \theta_o$$

$$+ \sin (2w_i t + \theta_i + \theta_o) + \frac{X(t)\delta}{V_i} \sin (2w_i t + \theta_o)$$

$$+ \frac{Y(t)\delta}{V_i} \cos (2w_i t + \theta_o)$$

The last three terms are at twice the carrier frequency and may be assumed rejected by the loop filter. The low frequency terms contain a signal term and two noise terms.

$$\text{Let } \frac{X(t)}{V_i} \sin \theta_o - \frac{Y(t)}{V_i} \cos \theta_o = N'(t) \qquad (1\text{-}9)$$

and rewrite the error voltage for small signal-related phase difference as:

$$v_e(t) = \delta(\theta_i - \theta_o) + \delta N'(t) \qquad (1\text{-}10)$$

This suggests the linearized model of the loop including the effects of noise shown in figure 1-4 [Ref. 4,P.83,Ref. 1, p.7]

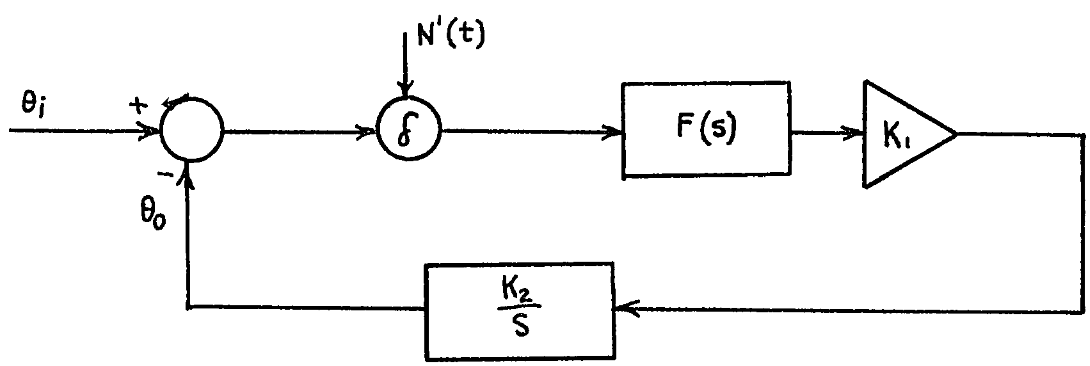


FIGURE 1-4 LINEAR NOISE MODEL

For the linear model to be useful in executing noise analyses the power spectral density (PSD) of N'(t) must be known.

If θ_o in equation (1-9) is treated as a constant,the variance of N'(t) is

$$\overline{N'^2}(t) = \frac{1}{V_i^2}\left(\overline{X^2}\cos^2\theta_o+\overline{Y^2}\sin^2\theta_o+2\overline{XY}\sin\theta_o\cos\theta_o\right)$$

But $\overline{X^2} = \overline{Y^2} = \overline{N^2}$, $\overline{XY} = 0$ [Ref. 1, Appendix A]

and $\cos^2\theta_o+\sin^2\theta_o=1$. Thus

$$\overline{N'^2(t)} = \frac{1}{V_i^2}\,\overline{N^2}$$

The autocorrelation of N'(t),$R_{n'}(t_1,t_2)$, may be written as a function of the autocorrelation of N(t), and its fourier transform yields the desired PSD.

$$R_{N'}(t_1,t_2)= \overline{N(t_1)\ N'(t_2)} =$$

$$\frac{1}{V_i^2}\left\{\cos^2\theta_o\ \overline{X(t_1)X(t_2)}+\sin^2\theta_o\ \overline{Y(t_1)Y(t_2)}\right.$$

$$\left.+\sin\theta_o\cos\theta_o[\overline{X(t_1)Y(t_2)}+\overline{Y(t_1)X(t_2)}]\right\}$$

But $\overline{X(t_1)Y(t_2)} = \overline{-Y(t_1)X(t_2)}$ and $R(t_1,t_2)= R(\tau)$

where $\tau = t_2-t_1$, so

$$N'(\tau)= \frac{1}{V_i^2} \left[R_x(\tau)\cos^2\theta_o+Ry(\tau)\sin^2\theta_o\right] \qquad (1\text{-}11)$$

Transforming both sides of 1-11 yields the spectral density relation.

$$\phi_{N'}(f)= \frac{1}{V_i^2} \left[\phi_x(f)\cos^2\theta_o+\phi_y(f)\sin^2\theta_o\right]$$

But $\phi_x(f)=\phi_y(f)=\phi_N(f_i-f)+\phi_N(f_i+f)$

where $\phi_\alpha(f)$ represents the PSD of a random variable, $\alpha(t)$.

Thus $\phi_{N'}(f) = \frac{1}{V_i^2} \left[\phi_N(f_i-f)+\phi_N(f_i+f)\right]$

If the input noise is flat over the frequency range of interest and in that range $\phi_N(f)=\eta\,(V^2/Hz)$ then

$$\phi_{N'}(f) = 2\,\frac{\eta}{V_i^2} \quad (V^2/Hz) \qquad (1\text{-}11a)$$

Equation (1-11a) gives the PSD of the equivalent noise source injected into the phase detector output of the linear model as shown in FIGURE 1-4.

As an alternative to viewing the noise as a component of error voltage injected into the output of the phase detector as shown in figure 1-4, one can view it as a component of input phase equal to N'(t) additively combined with Θ_i. Both approaches are based upon equation (1-9). The response of the loop to this input noise phase is to produce a noise related phase in the VCO output that has a PSD

$$\phi_{no}(f) = \phi'_n(f) \left|H(jw)\right|^2 (Rad^2/Hz)$$

H (jw) is the closed loop frequency response given by equation (1-7) with s=jw.

The variance of the output noise phase may be calculated from

$$\overline{\Theta_{no}{}^2} = \int_0^\infty \phi_n'(f) \left|H(jw)\right|^2 df (Rad^2) \qquad (1\text{-}12)$$

This is also the variance of the error phase due to the noise, $\overline{\Theta_{ne}{}^2}$.

1.3 The Phaselocked Loop as an FM Demodulator

In angle modulation systems the information is encoded as a phase variation in the carrier waveform. This is represented in equation (1-1) as $\Theta_i(t)$. For frequency modulation it is the derivative of the phase variation that corresponds to the signal waveform. Thus if m(t) is the instantaneous modulation in an FM signal then

$$m(t) = \frac{d\theta_i(t)}{dt}$$

In the transform domain

$$M(s) = s\theta_i(s) \tag{1-13}$$

With reference to figure 1-3 the following relationship may be written:

$$\theta_o(s) = \frac{K_2}{s} V_c(s)$$

$$\text{or} \quad V_c(s) = \frac{s\theta_o(s)}{K_2} \tag{1-14}$$

$$\text{But } \theta_o(s) = H(s)\theta_i(s) \tag{1-15}$$

Substitution of equation (1-13) and (1-15) into (1-14) yields

$$V_c(s) = \frac{H(s)}{K_2} M(s) \tag{1-16}$$

Equation 1-16 shows that the VCO control voltage is the modulation signal processed by the closed loop transfer function of the PLL divided by the sensitivity of the VCO.

If H(jw) is wider than the modulation bandwidth then $v_c(t)$ will be a replica of the modulation waveform and is the output of the PLD.

1.3.1 FM Improvement in Phaselock Detectors

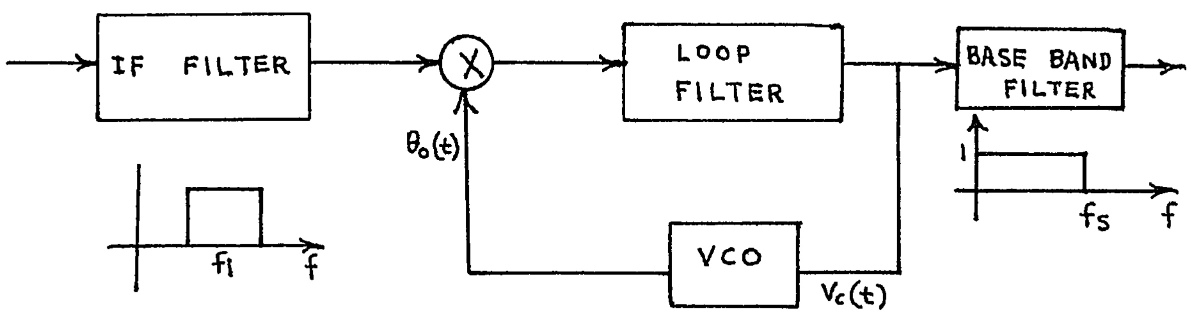


FIGURE 1-4a PLD WITH PRE AND POST DETECTION FILTERS

Figure 1-4a illustrates the linearized PLD which will be analyzed to exhibit the phenomenon known as FM improvement. Ideal rectangular filters are assumed to precede and follow the detector as shown. The closed loop response of the linear model is taken as unity so that

$$H(s) = \frac{\theta_o(s)}{\theta_i(s)} = 1$$

for all frequencies of interest and it follows that $\theta_o(t)=\theta_i(t)$. The input to the IF filter is a tone modulated

FM wave corrupted by additive white gaussian noise of power spectral density η (V^2/Hz).

$\theta_{ni}(t)$ represents the equivalent noise phase input that was described in section 1.2.1. $\theta_i(t)$ is the signal phase which for tone modulation may be written as

$$\theta_i(t) = \beta \cos w_m t$$

where, β ,the peak signal phase, is known as the modulation index.

Assuming linearity, we calculate separately the signal power and the noise power at the output of the post-detection baseband filter of bandwidth f_s (Hz) and then the output signal-to-noise ratio, $(SNR)_o$.

The input carrier-to-noise ratio, $(CNR)_i$ is calculated at the output of the IF filter.

Consider the action of the VCO. Its output frequency deviation, $w_d(t)$, is related to its input control voltage, $v_c(t)$, by the constant K_2. Thus

$$w_d(t) = K_2 v_c(t).$$

But because $\theta_o = \theta_i$ and frequency is defined as the derivative of phase,

$$w_d(t) = \frac{d}{dt}\,\theta_o(t) = \frac{d}{dt}\,\theta_i(t) = \beta\, w_m \cos w_m t.$$

Thus, the peak frequency deviation is $\beta\, w_m$ and the peak signal output is $V_c = \dfrac{\beta\, w_m}{K_2}$. It follows that the output signal power is

$$\frac{\beta^2 w_m^2}{2K_2^2}\,. \tag{1-17}$$

To obtain an expression for output noise power one invokes the equivalent input noise phase that was derived in section 1.2.1, applies linear system transform methods to get the noise PSD, and then integrates over the baseband.

The equivalent noise input or input phase jitter has a rectangular PSD of $2\eta/V_i^2$ (rad^2/Hz) over a bandwidth $B_p/2$. If the loop remains locked in the presence of this noise then the phase jitter in the VCO output tracks the input and has the same PSD. The transfer function of the VCO is K_2/jw so the PSD of the noise component of the VCO control voltage is $2\eta\, w^2/(V_i^2 K_2^2)$. At the output of the post detection filter the noise power is

$$P_n = \frac{2\eta}{V_i^2 K_2^2}\int_0^{f_s} w^2\, df = \frac{\eta}{V_i^2 K_2\; 2\pi}\int_0^{f_s} w^2\, dw = \frac{\eta\, w_s^3}{3\pi V_i^2 K_2^2} \tag{1-17a}$$

where f_s is the basebandwidth and $w_s = 2\pi f_s$.

The ratio of (1-17) to (1-17a) is the output signal-to-noise ratio.

$$(SNR)_o = \frac{\beta^2 w_m^2}{2K_2^2} \cdot \frac{3\pi V_i^2 K_2^2}{w_s^3}$$

$$= 3\beta^2 \frac{V_i^2 w_m^2}{2\eta w_s^3} \qquad (1\text{-}17b)$$

V_i is the amplitude of the carrier at the IF output so the carrier-to-noise ratio at the detector input is

$$(CNR)_i = \frac{V_i^2}{2\eta B_p} = \frac{\pi V_i^2}{\eta w_B} \qquad (1\text{-}17c)$$

where $w_B = 2\pi B_p$

From (1-17b) and (1-17c) we get

$$\frac{(SNR)_o}{(CNR)_i} = 3\beta^2 \frac{V_i^2 w_m^2}{2\eta w_s^3} \cdot \frac{\eta w_B}{\pi V_i^2}$$

$$= 3\beta^2 \frac{w_m^2 w_B}{2w_s^3}$$

$$= 3\beta^2 \frac{f_m{}^2 B_p}{2f_s{}^3} \tag{1-17d}$$

For the case where the $(CNR)_i$ is calculated in twice the basebandwidth and the test tone is at the upper limit of the baseband we have

$f_m = f_B$ and

$B_p = 2f_B$,whence,

$(SNR)_o = 3\ \beta^2 (CNR)_i$, which is the familiar form [Ref. 4, Pg. 29] and describes the upper segment of figure (1-5). This result is identical to that for the classical limiter discriminator type of FM demodulator and accounts for the advantage of FM over AM. Output signal purity may be improved by increasing the modulation index at the implicit expense of rf bandwidth. The PLD has the additional feature of not requiring that a limiter precede the detector.

In practice, however, the FM advantage suggested by equation (1-17d) can only be realized for CNR above a limiting value known as threshold. For CNR below the threshold level the relationship does not apply and the FM advantage is lost.

1.3.2 FM Threshold Effect

Figure 1-5 is a Log-Log plot of output (SNR) Vs. input (CNR) which illustrates the threshold effect. For large CNR the curve is linear with unity slope, but below threshold the CNR decreases rapidly with declining CNR.

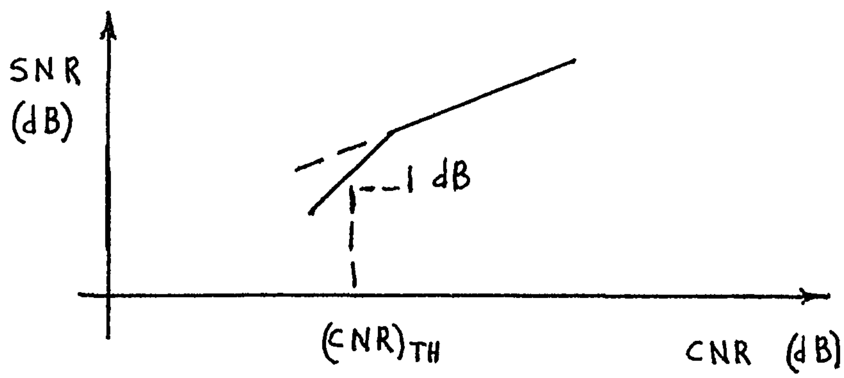


FIGURE 1-5 THRESHOLD EFFECT

It is customary to define threshold as the CNR for which the actual SNR output is 1 dB below the value predicted by equation (1-17d) shown as a broken line extension. Because of the practical significance of the threshold phenomenon, much effort has been expended in the search for methods to calculate threshold CNR, and on attempts to develop types of demodulators that will exhibit reduced threshold levels. It has long been known that below-threshold demodulator performance is characterized by large, short duration pulses in the output waveform and it has also been recognized that the onset of threshold is due to nonlinearities in the detectors. An exact nonlinear analysis of a PLL due to Viterbi [Ref. 7] is based upon the solution of a Fokker-Planck equation but is limited to a first-order loop; that is, a loop with $F(s)=1$ in Figure 1-4, and no modulation. A variety [Ref. 1, Pg. 39] of approximate methods of analyses have been devised for the more important second-order loops which have resulted in predictions that are in close agreement with experimentally obtained statistics of phase error and pulse occurrence. Summaries of some of these works and bibliographies are given by Gardner [Ref. 1, Chap.3], Klapper and Frankle [Ref. 4, Chap. 5], and Lindsey [Ref. 12].

For both Limiter Discriminators (LD) and PLDs, threshold may be explained in terms of the sharp pulses or spikes that occur only rarely above threshold but with a rapidly increasing average rate as threshold is approached.

Consider the phasor representation of an FM waveform shown in figure 1-6.

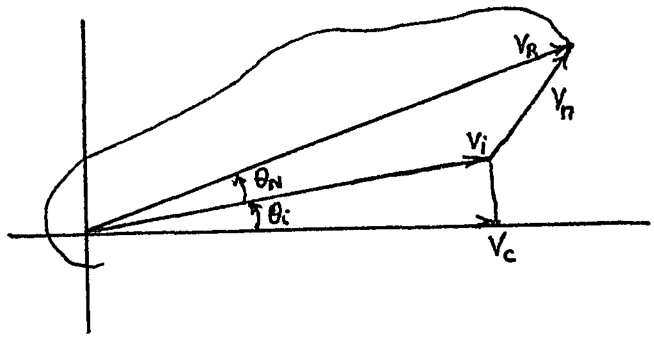


FIGURE 1-6 PHASOR DIAGRAM FOR SPIKES

The carrier phasor ,V_c, is shown as a reference and V_i represents the modulated input waveform. When V_i is combined with additive input noise, V_n,the resultant, V_R, is produced. Θ_i is the signal input phase and Θ_N is the noise related phase which accounts for the noise component in the detected wave. Normally the resultant fluctuates randomly above and below the reference; but occasionally an event of note takes place when due to the noise, the resultant follows a trajectory like that of figure 1-6 and encircles the origin. Such an encirclement in either direction will introduce a phase disturbance of 2π radians. In the detector output, where a waveform related to the time derivative of phase is developed, the encirclements are manifested as sharp pulses or spikes. Because these spikes represent significant energy within the baseband of the signal, they account for much added noise and can be related to the onset of threshold.

If the spike rate is known,the additional noise may be calculated and the threshold level determined. A method due to Rice [Ref. 8] enables the calculation of click (spike) rate for an ideal discriminator in terms of input noise spectrum, CNR, and type of modulation. Good predictions of threshold levels for LDs can thus be made. But these thresholds do not represent the minimum attainable levels for FM detection. Frankle [Ref. 11] presents a graphical comparative summary of the threshold performance of various FM demodulators which is reproduced here in figure 1-7 from Klapper and Frankle [Ref. 4, Pg. 39].

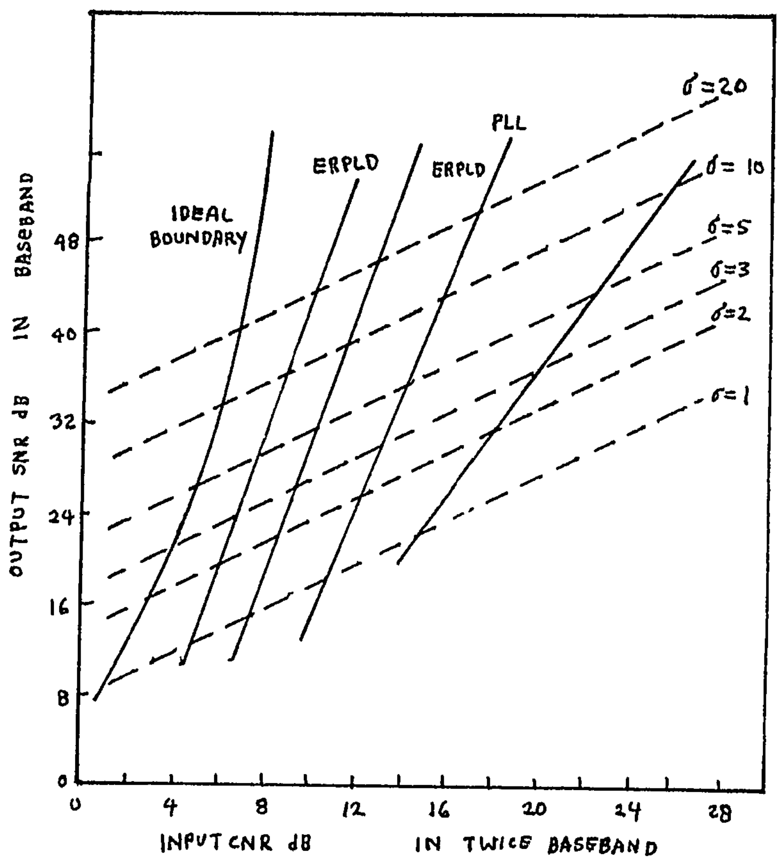


FIGURE 1-7 - THRESHOLD PERFORMANCE OF FM DEMODULATION

The dashed lines characterize the performance of a well designed demodulator of any type and illustrate the FM improvement expressed by equation (1-17d). In this model the modulation is assumed to be voice with RMS modulation index, σ ,in the presence of white gaussian noise. The solid lines are the loci of threshold levels for the various systems at different modulation indices. (Thus for a PLL detector with a modulation index of 5, the threshold CNR is approximately 14 dB). The rightmost line is for a conventional detector and the next is for an angular feedback type detector of which a PLL is representative. The next two are for improved angle feedback types known as extended range phaselock detectors which are discussed in the next section. Finally, the leftmost curve represents an information theoretic limit based on Shannon's limit for "errorless" transmission as derived by Klapper and Frankle [Ref. 4, Appendix C].

This dissertation deals with optimization of phaselocked detectors with respect to threshold reduction. Specifically, it is a study of the effect of a higher order pole in the loop filter upon the minimum achievable threshold level. In general the click rates in PLDs cannot be calculated [Ref. 1, Pg. 183], but it is known, as illustrated in Figure 1-7, that a PLD with the same input filter as an LD can have a lower threshold. The reduction in threshold is probably due to the inability of the loop to follow all of the clicks.

When the loop is operating near threshold the noise phasor of figure 1-6 does not overwhelm the signal phasor, but rather barely overcomes it, and thus the encirclements occur rapidly and with small amplitude. Because no limiter precedes the loop, and because the gain of the loop depends upon the signal amplitude, the PLD will be unable to track all of the encirclements and some clicks will be neglected. Two interesting consequences emerge from this argument.First,the improved threshold characteristics of a PLD derive in part from a weakness in its performance (i.e. limited tracking capability) and second, that a limiter would adversely affect threshold performance somewhat.

Klapper and Frankle provide additional insight into the threshold phenomenon in a PLL which clarifies several attempts at optimization. Spikes resulting from two distinct phenomena are identified. The first, called threshold impulses (ThI),are similar to the spikes in an LD, and occur whenever the resultant of signal plus noise causes an origin encirclement. The second type of spike is due to the periodic nature of the phase discriminator and is known as loss of lock impulse or LLI. [Ref. 4, Pg. 109]

For a multiplier type discriminator the error signal developed by the PD is given by equation (1-6) as

$v_e(t) = \mathcal{S}\sin[\theta_i(t) - \theta_o(t)]$. Should the phase difference exceed $\pm\pi/2$ radians, the slope of the error signal would be inverted causing regeneration which nearly always results in a 2π radian rapid phase change and an LLI. The LLIs are essentially indistinguishable from the ThIs and it is the sum of their rates of occurrence which determines the threshold in a PLL FM detector. A design objective would therefore be the minimization of this sum.

The ThI rate in a PLL is less than that of an ideal discriminator because of the inability of the PLL to track all of the ThI. One might optimize this effect by decreasing the loop bandwidth but would thereby incur two undesirable effects. For one, the ability of the loop to track the modulation would be impaired; and the second is that larger errors would develop, resulting in more frequent loss of lock impulses. It would seem that the loop response is critical and a careful design is required in order to minimize the total click rate and thus reduce the threshold level. Any optimum design procedure would require a means of formulating the click rate in terms of the loop parameters; a feat which has only been achieved for a first order loop with unmodulated carrier and white gaussian additive input noise [Ref. 7]. In addition, some experimental results are also available for the second order loop under similar operating conditions [Ref. 4, Pg. 133].The general guidelines that have been gleaned from these efforts and successfully applied in design procedures will be incorporated into the threshold models to be used in this study. Two models are described in chapter 2; one for use with test-tone modulation and the second for voice modulation. For the test-tone model, threshold is associated with a particular probability of occurrence of LLIs. This approach presumes that for threshold reduction, the LLIs are the dominant source of spikes and it neglects ThIs. For voice modulation, the model is based upon total mean squared phase error, $\overline{\theta_e^2}$, and presupposes that threshold may be identified with a critical value of $\overline{\theta_e^2}$.

1.4 - <u>EXTENDED RANGE PHASELOCKED DETECTOR (ERPLD)</u>

Recognition of the causal relationships between the phase detector range limitation and LLIs, and between LLIs and the onset of threshold,has led to the study of extended range phase detectors. It was reasoned that a loop which had been optimized with a standard phase detector of sinusoidal characteristic could be improved by extending the dynamic phase error range of the detector. This would reduce the rate of LLIs without materially affecting the ThIs which are due to a different mechanism [Ref. 4, Pg. 248]. A new optimum loop with a lower combined spike rate would then be possible and a reduction in threshold could be achieved.

Several techniques were proposed to accomplish the extension of the monotonic range of the phase detectors. Among these proposals were the following:

1 - Carrier signal waveshaping - This method requires that the carrier waveshape and/or the VCO waveshape which are the two inputs to the phase detector be other than the usual sinewaves.

2 - Postdetection linear synthesis - Here the input waveforms are standard but the same effect is achieved by synthesizing the various harmonic terms that would have resulted from method (1) in the detector output, and summing all the terms.

3 - Postdetection non-linear synthesis - In this method, in response to a phase difference $\theta_e = \theta_i - \theta_o$ the voltages $\cos\theta_e$ and $\sin\theta_e$ are generated. Then a voltage

$$v_e(\theta_e) \propto \frac{(1+K)\sin\theta_e}{(1+K)\cos\theta_e} \quad , \quad 0 \leqslant K \leqslant 1 \qquad \text{is}$$

developed. The monotonic range of the function $v_e(\theta_e)$ depends upon the value of K, approaching $\pm\ \pi/2$ radians as K approaches unity.

These rather elaborate schemes for achieving range extension are discussed by Klapper and Frankle in Ref. 4 , pages 247-257. The inherent difficulties and the limitations of each of these techniques are described in some detail. Yet another method has been devised to create an extended range phase discriminator from a sinusoidal type device. It is a historical antecedent of the subject of this dissertation and will be described here.

1.4.1 Phase Feedback ERPLD

Acampora and Newton [Ref. 9, PP. 577-599] conceived, built, and tested a PLL with an internal loop that performs the function of phase subtraction. They achieved a reduction in threshold with respect to that achievable with a standard PLL and attributed the reduction to phase extension. Figure 1-8 is the block diagram of an almost linearized phase subtraction ERPLD. $K_1F_2(s)$ is the response of the loop filter and the VCO is represented as an ideal integrator with sensitivity K_2 Rad/Sec-Volt. The section enclosed by the broken line is functionally an extended range phase detector which compares θ_o with θ_i and developes an output voltage, e, which is a function of the error phase $\theta_e=\theta_i-\theta_o$. The feedback loop with scale factor K_3 Radians/Volt includes a phase modulator that develops an output having a phase θ_o equal to the phase sum, θ_r+K_3e. The phase detector output is given by

$$\begin{aligned} v_e &= \mathcal{S}\sin[\theta_i-(\theta_r+K_3v_e)] \\ &= \mathcal{S}\sin(\theta_e-K_3v_e) \end{aligned} \tag{1-18}$$

which is a nonlinear function of the phase error and has a characteristic which depends upon the unitless product $K_3\mathcal{S}$.

Figure 1-9 is a plot of $|v_e|$ Vs. $|\Theta_e|$ for several different values of parameter $K_3\delta$. When this parameter is zero, the characteristic of the discriminator reduces to the usual sine function having monotonic range of $|\Theta_e| \ll \pi/2$. With $K_3 \delta = 1$, the range is extended to $(\pi/2+1)$ rads before slope inversion and regeneration takes place. The investigators operated their ERPLD with a 1 KHz test tone, 10 KHz peak deviation and a 34 KHz predetection filter. They measured a threshold reduction of 4 dB compared to a PLD, which agrees very well with the curves of figure 1.7. Their improvement with respect to an LD is about 8 dB and agrees with figure 1.7 for lower modulation index.

It might appear that Viterbi's method for calculating cycle slipping rates in terms of phase-error limits could be applied here by simply using the extended range in his formulations. But that would lead to an unjustified optimistic expectation of reduced spike rates because it neglects the effect of noise which is fed back with the phase error signal [Ref. 4, Pg. 259]. Klapper and Frankle suggested an alternative analytical approach to explain the threshold reduction that makes use of a remarkable identity between the phase feedback ERPLD and a loop known as an equivalent filter ERPLD which uses no phase feedback.

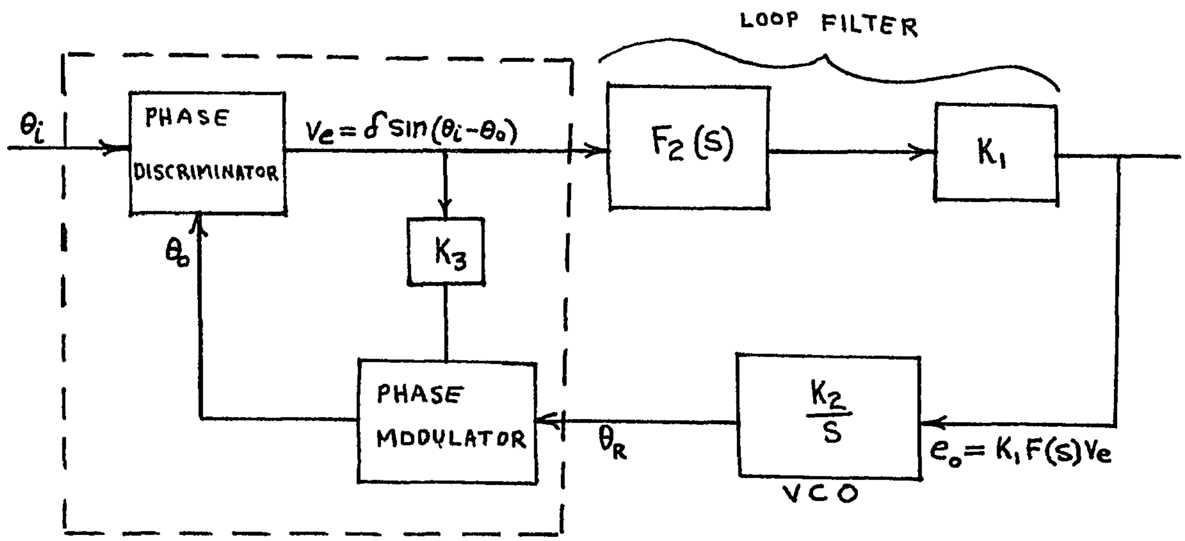


FIGURE 1-8 MODEL OF PHASE FEEDBACK ERPLD

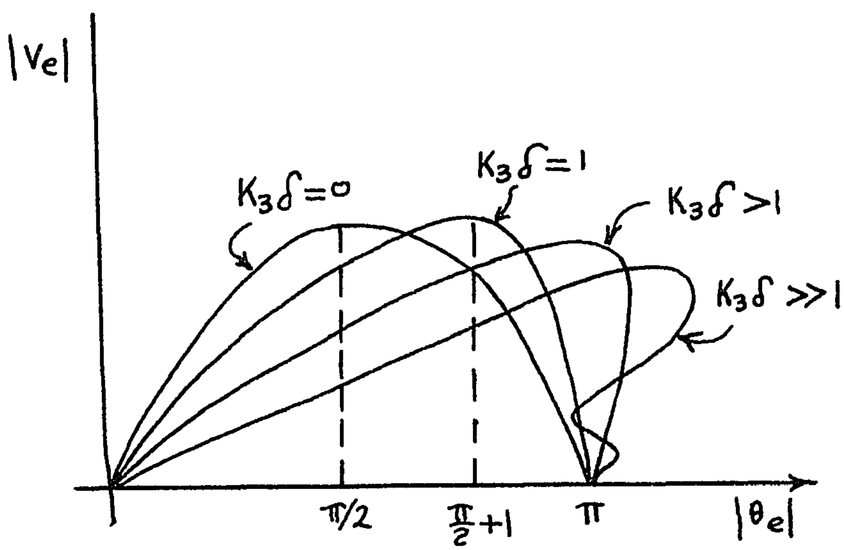


FIGURE 1-9 $|V_e|$ Vs. $|\theta_e|$

1.4.2 Equivalent Filter ERPLD

The equivalence may be observed by comparing the differential equation for a standard PLL with that for one with phase feedback.

For the phase feedback system the equation is derived with reference to Figure 1-8.

$$\theta_o = \theta_R + K_3 v_e = \theta_R + K_3\,\delta \sin(\theta_i - \theta_o)$$

Call $(\theta_i - \theta_o)$ the phase error, θ_e,

then $\theta_o = \theta_R + K_3\,\delta \sin\theta_e$

Differentiating the phase error yields

$$\dot{\theta}_e = \dot{\theta}_i - \dot{\theta}_o = \dot{\theta}_i - \dot{\theta}_R - K_3\,\delta\, d/dt(\sin\theta_e)$$

In terms of the operator $P \equiv d/dt$ and the linear-filter response, $F_2(p)$

$$\dot{\theta}_R = K_2\, e_o = K_2 K_1 F_2(P)\,\delta \sin\theta_e \text{ and}$$

$$\dot{\theta}_e = \dot{\theta}_i - K_1 K_2\,\delta\, F_2(P) \sin\theta_e - K_3\,\delta\, P \sin\theta_e$$

Letting $K_1K_2\delta = K$ results in

$$\dot{\theta}_e=\dot{\theta}_i-K\left[F_2(P)+\frac{K_3}{K_1K_2}P\right]\sin\theta_e \qquad (1\text{-}19)$$

For the PLL we refer to Figure 1-3 but retain the sinusoidal characteristic for the phase discriminator.

$$\theta_e=\theta_i-\theta_o$$

$$\dot{\theta}_e=\dot{\theta}_i-\dot{\theta}_o \qquad (1\text{-}20)$$

But $\dot{\theta}_o=K_1K_2\ F(P)V_e=KF(P)\sin\theta_e$

which ,upon substitution into equation (1-20) ,yields

$$\dot{\theta}_e=\dot{\theta}_i-KF(P)\sin\theta_e\quad . \qquad (1\text{-}21)$$

Equation (1-19) is identical in form to equation (1-21), if F(P) in (1-21) is replaced by F'(P) where

$$F'(P)=F_2(P)+\frac{K_3}{K_1K_2}P \qquad (1\text{-}22)$$

This suggests that the threshold improvement attributed to phase feedback might be realized in a standard configured PLD if the loop filter response is given by equation (1-22). The additive term corresponds to a perfect differentiator so that F'(P) may be implemented as in figure 1-10.

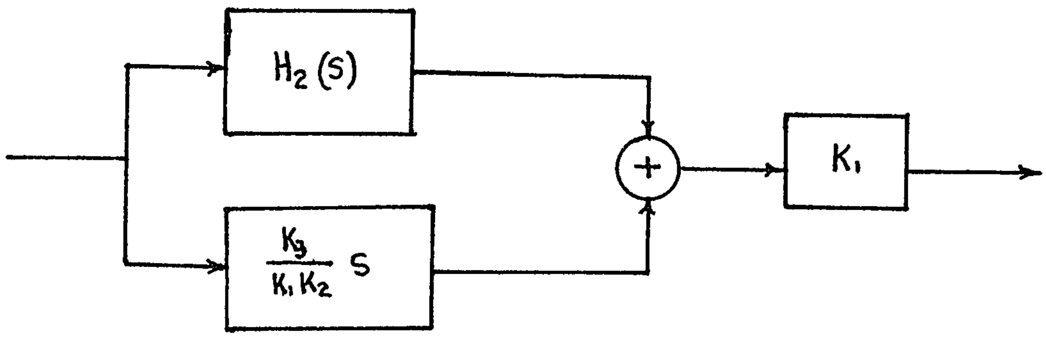


FIGURE 1-10 LOOP FILTER FOR EQUIVALENT FILTER ERPLD

If $H_2(s)$ is the lag-lead network of a typical second order type 1 PLL then

$$H_2(s) = \frac{s/a+1}{s/b+1} \qquad \text{and}$$

$$F'(s) = \frac{s/a+1}{s/b+1} + \frac{K_3}{K_1 K_2} s$$

$$= \frac{s^2\,[K_3/(K_1K_2b)]+s[1/a+K_3/(K_1K_2)]+1}{s/b+1} \qquad 1\text{-}23$$

which is characterized by the single pole at -b and a pair of complex zeros.

1.4.3 Optimization of the ERPLD

Novick [Ref. 2] addresses the problem of minimizing threshold for the generalized second order PLD which is characterized by the filter of equation (1-23), with the expectation of improvement over the standard second order PLD because the advantage of extended range is incorporated. His method for executing the optimization is an extension of the method used by Klapper and Frankle for the PLD [Ref. 4, P. 139]. It requires that the onset of threshold be identified with a critical value of total error phase variance, ν Rad^2. An expression for $(CNR)_{TH}$ is written in terms of ν, the four system parameters, a, b, K_3, and $K=K_1K_2\delta$, and the phase PSD for the modulation, and minimized with respect to the system parameters by a computer algorithm based on Powell's method [Ref. 10].

The total error phase consists of two components; one due to the equivalent noise source injected into the loop, and the second due to the modulation signal phase, θ_i. For gaussian distributed independent modulation and noise the combined RMS phase error is given by

$$\overline{\theta_e^2} = \overline{\theta_{ne}^2} + \overline{\theta_{se}^2}$$

Let the PSD of the modulation phase, θ_i, be represented by $\phi_i(f)$ Rad^2/Hz. Then

$$\overline{\theta_{se}^2} = \int_0^\infty \phi_i(f) \; |1-H(jw)|^2 \; df \text{ Rad}^2$$

The noise term related to N' (t) of figure 1-4 is given by equation (1-12) as

$$\overline{\theta_{ne}^{2}} = \int_{0}^{\infty} \phi_{n}'(f) \; \left|H(jw)\right|^{2} \, df \; \text{Rad}^{2}$$

In chapter 2, the threshold level is formulated for the case of a gaussian signal as

$$(CNR)_{TH} = \frac{\displaystyle\int_{0}^{\infty} \left|H(jw)\right|^{2} \, df}{B_{P}\left[\gamma - \displaystyle\int_{0}^{\infty} \phi_{i}(f) \; \left|1-H(jw)\right|^{2} \, df\right]} \qquad (1\text{-}24)$$

where B_P is the bandwidth of the predetection filter and H(jw) is the closed loop response of the PLL.

For several types of modulation (i.e. specific functional forms of $\phi_i(f)$ Novick obtained closed form solutions for the integrals of equation (1-24) and by means of his minimization algorithm proceded to optimize the loop. Taking γ as 0.25 [Ref. 2, Pg. 52] and using the voice model described in section 2.4 he determines $(CNR)_{TH}$ for the optimum system to be 0.378 dB. This is slightly better than the 0.573 dB threshold level that he calculates [Ref. 2, Pg. 117] for the system parameters used by Acampora and Newton [Ref. 9, Pg. 588] in their experimental system.

It should be noted that Acampora and Newton's laboratory measurements of threshold were only performed with test tone modulation and Novick's numerical optimization was carried out only for the gaussian voice modulation model. In each case the alternative would have been more difficult.

1.5 Motivation For This Study

Novick's analysis and optimization were executed for the ERPLD whose loop filter contains an ideal differentiator and was shown to have the transfer function given by equation (1-23). A comparison of that equation with the standard form,

$$F'(s) = \frac{s^2/w_n^2+2\xi s/w_n+1}{s/b+1} , \qquad (1\text{-}25)$$

identifies the natural frequency and damping factor by the relationships

$$w_n^2 = \frac{K_1K_2b}{K_3} \qquad \text{and}$$

$$\frac{2\xi}{w_n} = \frac{1}{a} + \frac{K_3}{K_1K_2}$$

For the complete loop, including the VCO as an ideal integrator as shown in Figure 1-4 , the open loop response is given by

$$G(s) = \frac{s^2[K_3/(K_1b)]+s(K_2/a+K_3/K_1)+K_2}{s(s/b+1)} \quad 1\text{-}26$$

Its asymptotic logarithmic response has the form shown in figure 1-11.

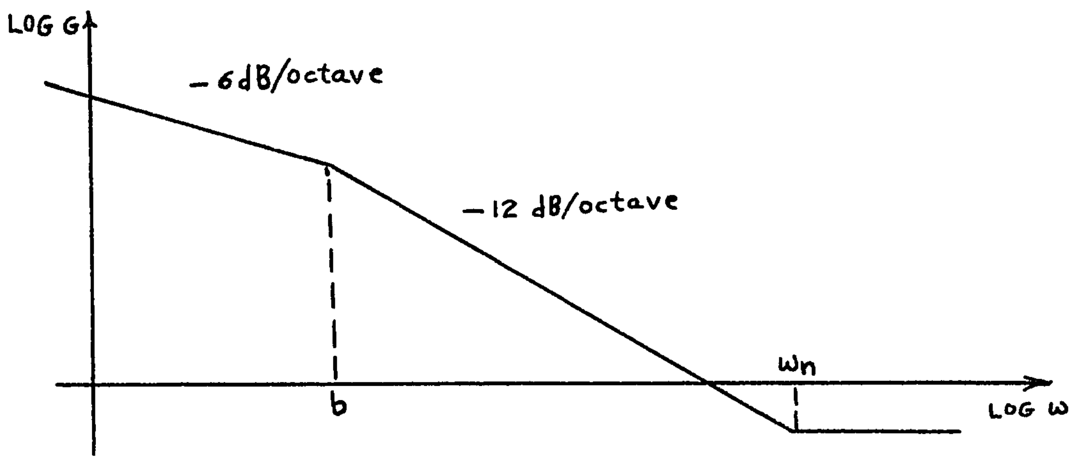


FIGURE 1-11 OPEN LOOP RESPONSE OF ERPLD

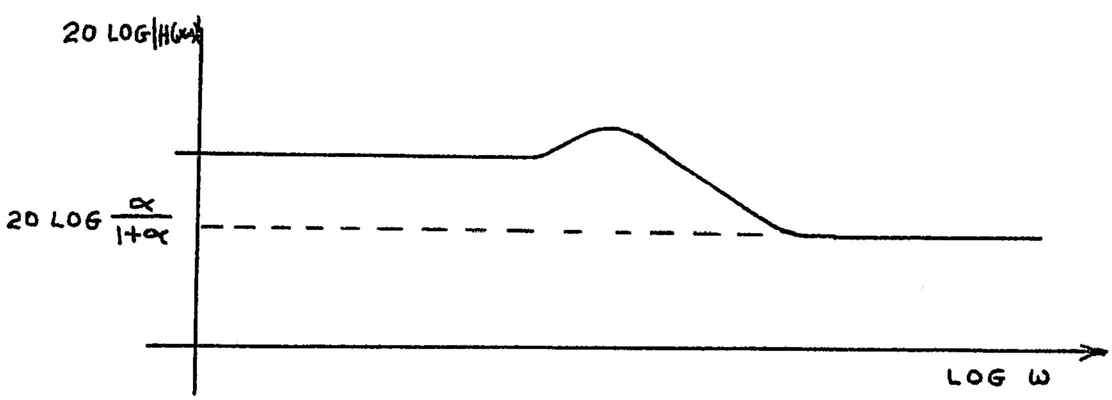


FIGURE 1-12 CLOSED LOOP AMPLITUDE RESPONSE FOR ERPLD

The closed loop response for this loop is obtained by substituting equation (1-23) into equation (1 - 7) and replacing s by jw to yield

$$|H(jw)|^2 = \frac{w^4\left(\frac{\alpha}{Kb}\right)^2 + w^2\left[\left(\frac{1}{a} + \left(\frac{\alpha}{K}\right)\right)^2 - 2\ \frac{\alpha}{Kb}\right] + 1}{w^4\left(\frac{1+\alpha}{Kb}\right)^2 + w^2\left[\left(\frac{1+\alpha}{K} + \frac{1}{\alpha}\right)^2 - 2\left(\frac{1+\alpha}{Kb}\right)\right] + 1} \qquad (1\text{-}27)$$

where $K=K_1K_2\delta$ and $\alpha=K_3\delta$.

The form of the logarithmic amplitude response of this loop is sketched in figure 1-12. The degree of peaking depends upon the amount of damping that is present in the loop. A noteworthy feature of figure 1-12 is the flat response of this loop at high frequencies. This represents an unbounded loop noise bandwidth which imposes the requirement that this form of ERPLD can only be used in conjunction with a predetection IF filter. The filter provides an upper frequency bound for the noise spectrum, which the loop cannot do. Novick, in his analysis [Ref. 2, Pg. 48], and Acampora and Newton in their experimental work [Ref. 9, Pg. 590], include a predetection filter in their detection systems, as indeed they must, and the loop threshold performance is critically dependent upon the filter bandwidth [Ref. 13].

But the foregoing system description and the consequent unbounded closed loop response is an idealization which is not physically realizable. In practice an ideal differentiator cannot be implemented. There will inevitably be present a higher frequency pole which will introduce a high frequency rolloff in the response of figure 1-12. A loop filter with a realizable differentiator will take the form shown in figure 1-13(a) rather than that of figure 1-10. A real pole at -d Rad/sec is assumed for the differentiator, either as a design parameter or as a result of stray effects, and it is shown in the figure.

The open loop response for the realizable filter is

$$F(s)=H_2(s) + \frac{K_3}{K_1K_2} \quad \frac{s}{s/d+1}$$

$$= \frac{s^2\left(\frac{1}{ad} + \frac{K_3}{K_1K_2b}\right) + s\left(\frac{1}{a} + \frac{1}{d} + \frac{K_3}{k_1K_2}\right) + 1}{(s/b+1)\quad(s/b+1)} \qquad (1\text{-}28)$$

By comparing equation (1-28) with equation (1-25) one identifies the loop natural frequency as

$$w_n = \left(\frac{abdK_1K_2}{bK_1K_2+adK_3}\right)^{1/2}$$

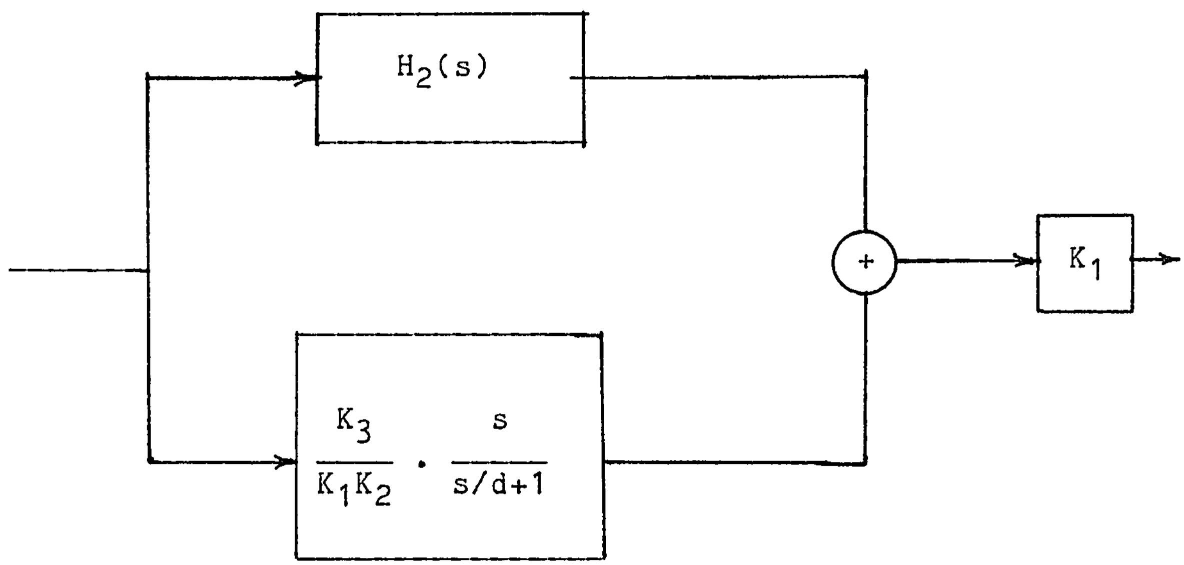


FIGURE 1-13(a) LOOP FILTER WITH REALIZABLE DIFFERENTIATOR

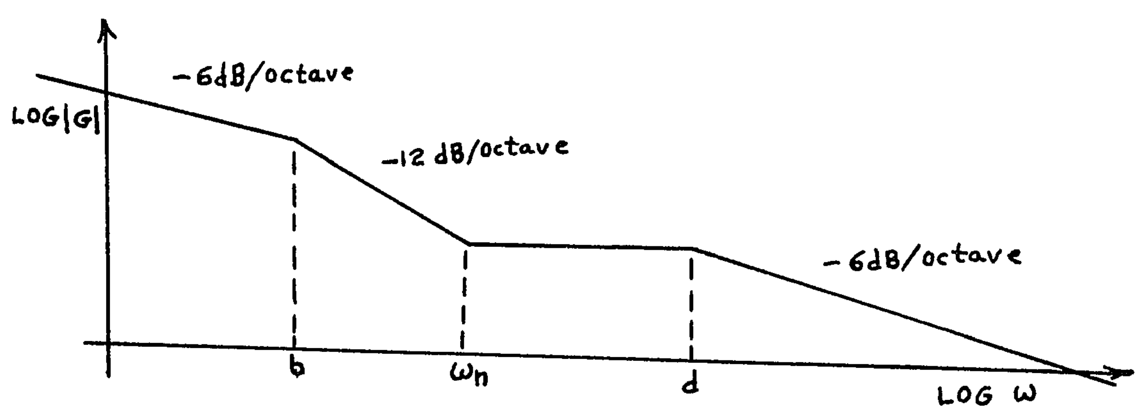


FIGURE 1-13(b) OPEN LOOP RESPONSE OF REALIZABLE LOOP FILTER

Figure 1-13(b) is a plot of the open loop response including the effect of the VCO.

Substituting equation (1-28) into equation (1-7) yields for the closed loop response (details are shown in section 3.1.3),

$$H(s)=\frac{s^2(Kb/a+\alpha d)+s(Kbd/a+Kb+\alpha bd)+Kbd}{s^3+s^2(d+b+Kb/a+\alpha d)+s(bd+Kbd/a+Kb+\alpha bd)+Kbd} \qquad 1\text{-}29$$

At low frequencies $|H(jw)| \approx 1$

and at high frequencies $|H(jw)| \approx (1/w)(Kb/a+\alpha d)$,

a rolloff of 6 dB/octave.

The general character of the closed loop response is sketched in figure 1-13(c). It suggests that this ERPLD should be functional even in a system with broadband input noise.

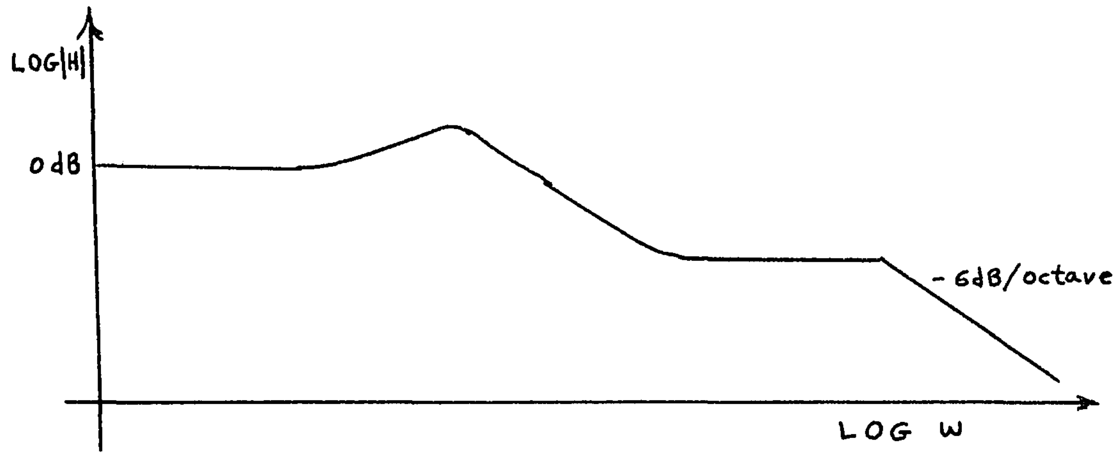


FIGURE 1-13(c) CLOSED LOOP RESPONSE OF REALIZABLE ERPLD

It seems reasonable to inquire about the impact of the higher frequency pole upon the threshold performance of the ERPLD. The pursuit of that inquiry constitutes the substance of the succeeding chapters and is the essence of this dissertation.

The realizable loop presents five design parameters a,b,d,α, and K which can be chosen so as to achieve minimum $(CNR)_{TH}$. To establish meaningful comparisons, the same minimization technique is applied to a variety of systems having different combinations of features. Along with the ERPLD we also optimize the standard second order type 1 PLD and a variation arrived at by adding an additional pole to that loop filter. In all, ten systems are generated which are differentiated by the presence or absence of a predtection filter, the presence or absence of a higher order pole and the use of test-tone or voice modulation.

The various systems are identified as cases 1 through 10 and are organized by features in the form of a tree diagram in figure 1-14.

Cases 1 and 2 are conventional second order Phaselock detectors. An additional pole in the loop filter leads to cases 3 and 4. The other six cases all include a differentiator in the loop filter to effectively extend the phase detector range and are therefore classified as ERPLDs. Of these, the first two employ ideal differentiators and are the type that was studied by Novick. Case 6 corresponds exactly to one of the systems that he optimized and it will serve as a convenient confirmation of the correct implementation of the optimization algorithm. As previously noted these loops which require ideal differentiation can only be used in conjunction with predetection filtering. To eliminate this restriction, and to render the filter realizable, an additional pole is added to the transfer function of the differentiator, which generates the last four cases for consideration.

Our objective is to investigate the effect of the additional pole upon the minimum achievable threshold for the PLD and ERPLD configurations that make up the tree diagram.

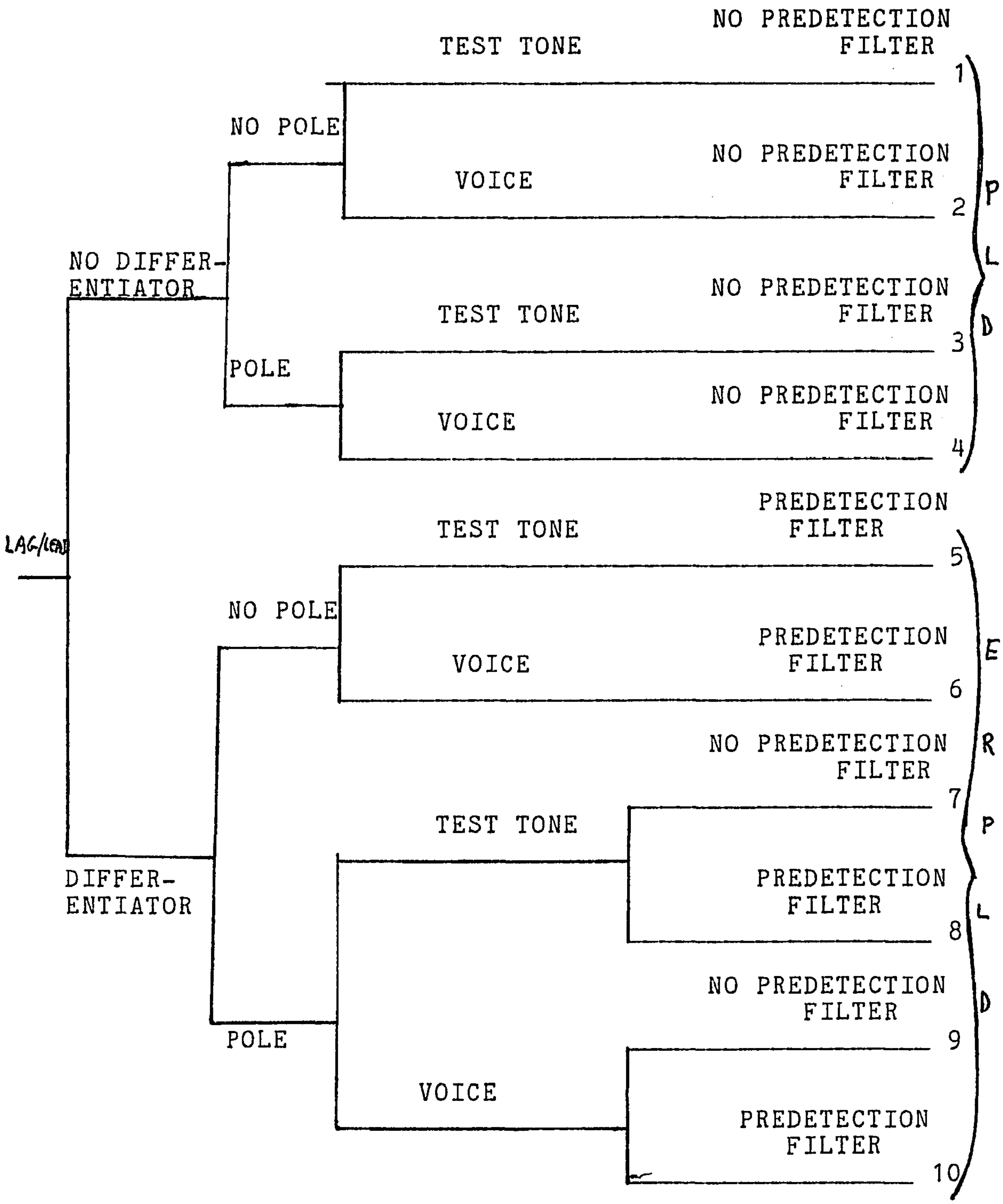


FIGURE 1-14 TREE DIAGRAM OF CASES

In chapter 2, different mathematical models of the threshold mechanism are developed for test-tone modulation and voice modulation. These models are then used in the derivation of expressions for $(CNR)_{TH}$ in the various PLDs. Chapter 3 continues with these formulations to include the six cases of ERPLDs. Chapter 4 discusses the method of loop optimization and presents the results of the process in each of the 10 cases. Several selected cases were implemented in the laboratory and tested with test-tone modulation. Details of the test procedures and the results are reported in chapter 5.

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CHAPTER II

FORMULATIONS FOR ANALYSIS OF THE EFFECT OF A HIGHER ORDER POLE ON THE OPTIMUM THRESHOLD OF A PHASELOCKED DETECTOR (PLD)

2.1 Introduction

All of the loops to be considered in this study are configured as in figure 2-1. They differ from one another in respect to the filter in the forward path having transfer function, F(s).

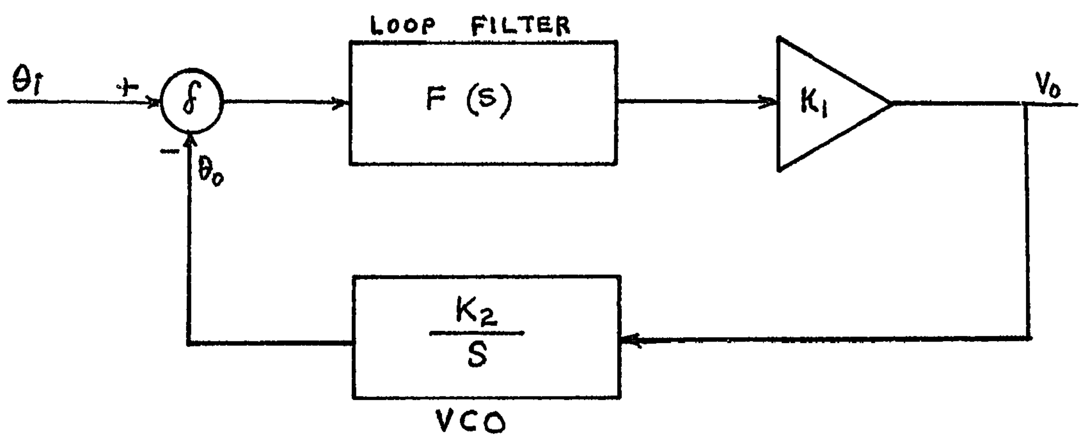


FIGURE 2-1 LOOP CONFIGURATION

Each of the loops employs one of the four filters whose transfer functions are indicated in the diagrams of figure 2-2. Filters I and II result in standard PLDs, III and IV in ERPLDs. Filter I leads to a second order loop and is commonly used in practice. Filter II includes an additional pole and leads to a type of third order loop. These two filters are incorporated into the systems of cases (1) through (4) on the tree diagram in figure 1-14. Cases (5) and (6) utilize loop filter III, which results in the Extended Range Phase Locked Detector (ERPLD). Here, an ideal differentiator has been added to the standard second order loop filter to form the equivalent filter ERPLD discussed in section 1.4.2. Adding a pole, or accounting for the inevitable presence of another pole in a realizable differentiator leads to filter IV and cases (7) through (10).

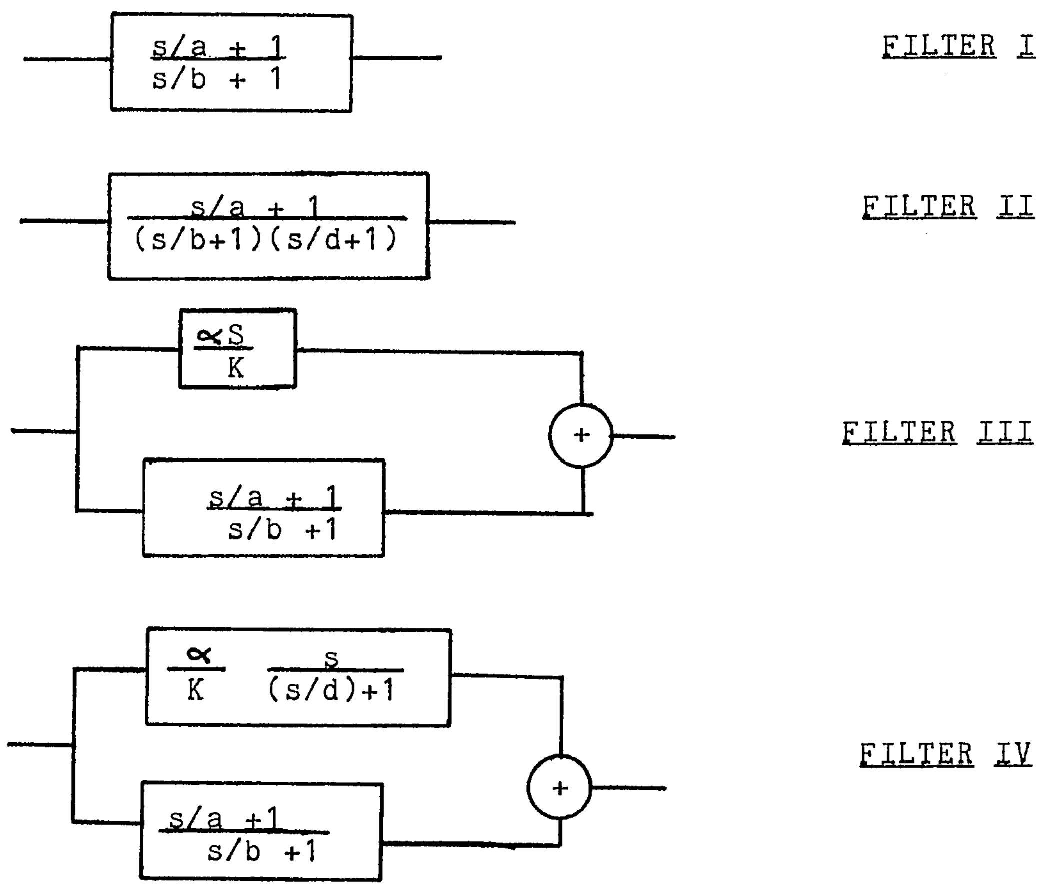


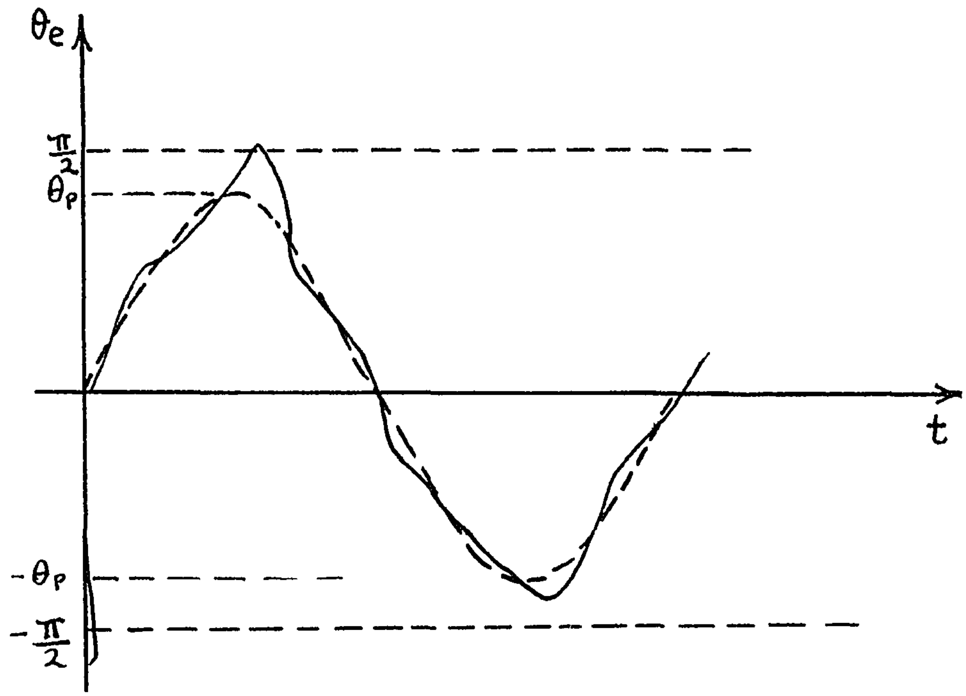
FIGURE 2-2 LOOP FILTERS

In this chapter, the analytical expressions for threshold carrier to noise ratio, $(CNR)_{TH}$, for filters I and II, cases (1) through (4) will be developed. Filters III and IV and their associated cases will be dealt with in the following chapter. Filter optimization for each case is then discussed in chapter IV.

2.2 Development of a Threshold Model for a PLD with Test-Tone Modulation

Threshold in phase-locked-detectors was shown in section 1.2.2 to be related to the "click" rate. The clicks, or frequency spikes were shown to be of two types: LLI and THI. Because this study was in some measure, motivated by previous work in the development of extended range detectors [Ref. 1, 2] which depend upon reduction in the rate of LLI for their reduced threshold, a model is developed which is based upon LLI.

Assume that threshold onset is determined by some rate of loss-of-lock impulses and that these may be attributed to the phase discriminator nonlinearity. In a multiplier type discriminator, when the absolute instantaneous phase error exceeds $\pi/2$ radians an LLI will nearly always be generated [Sec. 1.1.2].



-------------- Test - Tone Signal

______________ Signal Plus noise

FIGURE 2-3 PHASE ERROR DUE TO TEST-TONE SIGNAL AND GAUSSIAN NOISE

Consider a carrier with sinusoidal modulation and assume linear operation of the loop. The phase error due to the modulation will be a pure sine wave of peak value, θ_p. Let there also be a component of phase error due to the additive gaussian noise as shown in figure 2-3. Whenever the composite phase error waveform exceeds $\pi/2$ radians a spike is produced. Measurements indicate that threshold will occur when the absolute value of the total phase error exceeds $\pi/2$ radians with probability 0.0015 [Ref.3, Pg. 136]. We assume therefore that a criterion for threshold is:

$$P(|\theta_e| > \pi/2) = 0.0015$$

A gaussian distributed variable of variance σ^2 exceeds a level $a\sigma$ in absolute value with probability .0015 if $a \approx \pi$ [Ref. 4, Pg. 1117].

Because a sinusoidal signal is near its peak value for a considerable fraction of the time [i.e. $P(|\theta_s| > 0.7\theta_p) > 0.5$] and the variance of the noise related error phase is $\overline{\theta_n^2}$, we take as the condition on the noise for the onset of threshold:

$$\pi/2 - \theta_p = \pi(\overline{\theta_n^2})^{1/2}$$

From which, $$(\overline{\theta_n^2})^{1/2} = \frac{1}{2} - \frac{\theta_p}{\pi} \qquad (2\text{-}1)$$

The carrier-to-noise ratio may be written as:

$$(CNR) = \frac{A^2/2}{\eta B_p} = \frac{A^2}{2\eta B_p} \qquad (2\text{-}2)$$

where: A is the carrier amplitude

η is the noise power spectral density

B_p is the noise bandwidth (Baseband width)

For the linear model, based on section 1.1.1

$$\overline{(\theta_n^2)}^{1/2} = \left[\frac{1}{A^2} \int_0^\infty W_{\theta n} \left| H(jw) \right|^2 df \right]^{1/2}$$

where $W_{\theta n}$ is the noise power spectral density accompanying the modulated signal. In all the cases to be considered, only two noise spectrums will be involved. The input to the PLD will be combined with either additive white gaussian noise or with such noise having passed through a band pass filter. In the former instance the noise spectrum is flat and

$W_{\theta n} = 2\eta$ Rad^2/Hz, and in the latter

$W_{\theta n} = 2\eta$ Rad^2/Hz for $f_a < f < f_b$

0 elsewhere.

In either case

$$\left(\overline{\theta_n^2} \right)^{1/2} = \left[\frac{2\eta}{A^2} \int_0^\infty \left| H(jw) \right|^2 df \right]^{1/2} \qquad (2\text{-}3)$$

with the integration limits properly assigned. But from equation (2-2): $\dfrac{2\eta}{A^2} = \dfrac{1}{B_p(CNR)}$

$$\text{and } \left(\overline{\theta_n^2} \right)^{1/2} = \left(\frac{1}{B_p(CNR)} \int_0^\infty \left| H(jw) \right|^2 df \right)^{1/2} \qquad (2\text{-}4)$$

Upon substituting (2-4) into (2-1) we get:

$$\left(\overline{\theta_n^2}\right)^{1/2} = \frac{1}{2} - \frac{\theta_p}{\pi}$$

$$= \left(\frac{1}{B_p(CNR)_{TH}} \int_0^\infty |H(jw)|^2 df\right)^{1/2}$$

or $$\left(\frac{\pi - 2\theta_p}{2\pi}\right)^2 = \frac{1}{B_p(CNR)_{TH}} \int_0^\infty |H(jw)|^2\, df$$

which is solved for $(CNR)_{TH}$ yielding

$$(CNR)_{TH} = \frac{4\pi^2}{B_p(\pi - 2\theta_p)^2} \int_0^\infty |H(jw)|^2\, df \qquad (2\text{-}5)$$

Our objective is to optimize the loop for minimum threshold; that is to choose the set of values for the loop filter parameters that results in a minimum $(CNR)_{TH}$. These parameters are implicit in H(jw) and θ_p. H(jw) is the closed loop transfer function $\theta_o/\theta_i(jw)$ of figure 2-1. θ_p is the peak phase error response to a test tone signal and may be formulated as follows:

From equation (1-8), $\theta_e=\theta_i[1-H(jw)]$, (2-6)

where θ_i is the input signal phase. For FM the input signal voltage may be written as $v(t)=A\sin(w_i t+\beta\cos w_m t)$

where the modulation index, $\beta = (\Delta w_p / w_m)$.

$\theta_i(t) = \beta \cos w_m t$ and the peak input phase $\theta_{ip} = \beta$.

The peak error phase is the response to the peak input phase and is given by equation (2-6) as:

$$\theta_p = \theta_{ep} = \beta \left| 1 - H(jw_{TT}) \right| \qquad (2\text{-}7)$$

where $w_{TT} = 2\pi f_{TT}$ and f_{TT} is the test tone frequency.

We will optimize the various systems for test tone modulation by minimizing $(CNR)_{TH}$ in equation (2-5) with respect to the loop filter parameters.

2.3 Development of a Threshold Model for a PLD With Voice Modulation

Unlike the phase error due to test-tone modulation, that due to voice modulation is not a deterministic waveform but rather a noiselike random fluctuation. For the purpose of constructing a threshold model we assume that the signal is characterized by a normal distribution. This assumption and the resulting model have been successfully employed by Novick [Ref. 2, Pg. 47] for the same purpose. But if the modulating signal is gaussian, then so too is the modulated signal frequency deviation, $\Delta w(t)$. The input signal phase, $\theta_i(t)$ is related to w(t) by $\theta_i(t) = \int_0^t \Delta w(\tau)\, d\tau$, a linear process, and is therefore also gaussian [Ref. 6, Pg. 156].

From the linear model of figure 2-1 θ_{se} , the signal induced phase error is also a normally distributed variable and is independent of θ_{ne}, the noise induced error phase.

The total mean square error phase can then be written as

$$\overline{\theta_e^2} = \overline{\theta_{ne}^2} + \overline{\theta_{se}^2} \qquad (2\text{-}9)$$

$\overline{\theta_e^2}$ is used as the criterion for threshold and we assume that threshold will occur when $\overline{\theta_e^2}$ is equal to a critical value, γ [Ref. 3, Pg. 136]. Klapper and Novick take γ =0.25 [Ref. 5, Pg. 32D-1]. Novick finds that in his study [our case (6)] the results of the optimization are relatively insensitive to small changes in γ .

Using the PSD for the equivalent noise source given by equation (1-12), we can write for the noise induced error phase mean square value ,

$$\overline{\theta_{ne}^2} = \int_0^\infty \phi_n'(f) \left|H(jw)\right|^2 df, \qquad (2\text{-}10)$$

where $\phi_n'(f)$ is the noise-phase power spectral density of the equivalent noise source, $n_i(t)$, injected into the loop as shown in figure (1-4). From equation (1-8) we have

$$\theta_{se} = [1-H(jw)]\theta_i$$

whence
$$\overline{\theta_{se}^2} = \int_0^\infty \phi_i(f) \left|1-H(jw)\right|^2 df \qquad (2\text{-}11)$$

with $\phi_i(f)$, the power spectral density of the input phase, θ_i. For white gaussian input noise of power spectral density η Volts^2/Hz it was shown in equation (1-12) that

$$\phi_n'(f) = 2\eta/V_i^2 \quad \text{Rad}^2/\text{Hz} \tag{2-12}$$

We define (CNR) as above in section (2-1) for noise contained in a bandwidth B_p Hz and get equation (2-2)

$$(CNR) = \frac{A^2}{2\eta B_p}$$

and $$\phi_n'(f) = 2\eta/A^2 = \frac{1}{(CNR)B_p} \tag{2-13}$$

Substituting (2-13) in (2-10)

$$\overline{\theta_{ne}^2} = \frac{1}{(CNR)B_p}\int_0^\infty \left|H(jw)\right|^2 df$$

and the criterion for threshold may be written

$$\nu = \overline{\theta_{se}^2} + \overline{\theta_{ne}^2}$$

$$= \int_0^\infty \phi_i(f)\left|1-H(jw)\right|^2 df + \frac{1}{B_p(CNR)_{TH}}\int_0^\infty \left|H(JW)\right|^2 df$$

from which we obtain an expression for threshold carrier to noise ratio,

$$(CNR)_{TH} = \frac{\int_0^\infty \left|H(jw)\right|^2 df}{B_p\left[\nu - \int_0^\infty \phi_i(f)\left|1-H(jw)\right|^2 df\right]}. \tag{2-14}$$

H(jw) which appears in the integrands of the numerator and the denominator is a function of the loop design parameters. $\phi_i(f)$ depends upon the modulation model which will be described presently. $(CNR)_{TH}$ as formulated here will be minimized by the optimization procedure to be presented in chapter 4.

The power spectral density (PSD) of voice is taken to be an inverse square function of frequency and confined within a well defined range of frequencies. Thus:

$$W_v = \frac{N_v}{w^2} \qquad \text{for} \quad f_a \leqslant f \leqslant f_b$$

$$W_v = 0 \quad \text{elsewhere.}$$

In a frequency modulated wave, the PSD of frequency deviation corresponds to that of the modulation so the PSD of (Δw) is as shown in figure 2-4.

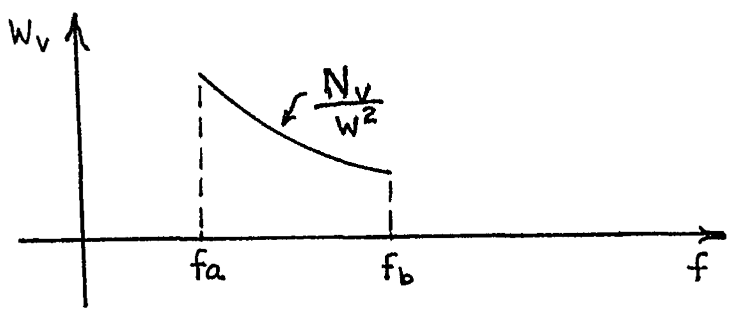


FIGURE 2-4 VOICE MODEL PSD

But the signal phase, θ_i, is the integral of (Δw) and in the transform domain

$$\theta_i(s) = \frac{1}{s} \Delta w(s)$$

Thus the signal phase power $\overline{\theta_i^2}(s) = \frac{1}{s^2} [\Delta W(s)]^2$

and the PSD of signal phase is given by:

$$\phi_i(w) = \frac{\eta_m}{w^4} \quad \text{Rad}^2/\text{Rad/sec} \tag{2-15}$$

for $w_a \ll w \ll w_b$

and 0, elsewhere.

Where η_m is a constant relating to the modulation. Equation (2-14) with (2-15) will be used in the analysis for all the cases involving voice modulation.

2.4 Derivation of Expressions for $(CNR)_{TH}$ for the Various PLDs Identified in the Table of Cases

For each of the identified cases it will be necessary to evaluate $\int \left|H(jw)\right|^2 df$ with appropriate limits of integration and to evaluate

$$\theta_p = \beta \sqrt{\left|1-H(jw_{TT})\right|^2} \tag{2-16}$$

The model and the method will first be applied to case (1), a standard second order PLD with test tone modulation. This system has been analyzed by Klapper and Frankle [Ref. 3, Pg. 139] and optimized for minimum $(CNR)_{TH}$. Their conclusions will be used for comparison with those achieved here.

2.4.1 Second Order PLD With No Predetection Filter

Case (1) employs in its for ward loop a filter I which has a Laplace domain transfer function $F(s) = \dfrac{s/a+1}{s/b+1}$ (2-17)

The loop, repeated in figure (2-5)

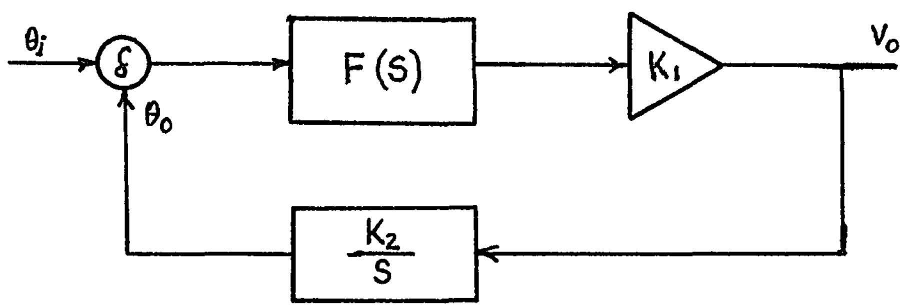


FIGURE 2-5 PLL

has an open loop response, $G(s)=K_1K_2\,\delta\;\dfrac{s/a+1}{s(s/b+1)}$

which is sketched as an assymptotic logarithmic Bode plot in figure 2-6.

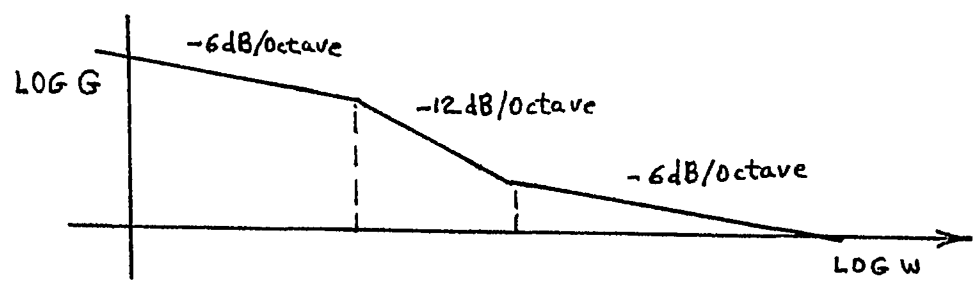


FIGURE 2-6 SECOND ORDER OPEN LOOP RESPONSE

Its closed loop transfer function $H(s) = \dfrac{\theta_o(s)}{\theta_i(s)}$ is derived as follows:

$$\theta_o(s) = \frac{v_e K_1 K_2 F(s)}{s} = K_1 K_2 \delta\, (\theta_i - \theta_o)[F(s)/s]$$

$$\theta_o(s)\left[1 + \frac{K_1 K_2 \delta F(s)}{s}\right] = K_1 K_2\, \delta\, \theta_i [F(s)/s]$$

and $$H(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{K_1 K_2 \delta F(s)/s}{1+[K_1 K_2 \delta F(s)/s]} = \frac{K_1 K_2 \delta F(s)}{s + K_1 K_2}$$

Letting $K = K_1 K_2 \delta$ we get

$$H(s) = \frac{KF(s)}{s + KF(s)} \tag{2-18}$$

Eq. (2-18) is independent of the particular loop filter and will apply to all the cases to be considered.

Substituting (2-17) into (2-18) we get

$$H(s) = \frac{K(s/a+1)/(s/b+1)}{s + K(s/a+1)/(s/b+1)} = \frac{K(s/a+1)}{s(s/a+1) + K(s/b+1)}$$

$$= \frac{\frac{s}{a} + 1}{S\left(\frac{s}{kb} + \frac{1}{k}\right) + \frac{s}{a} + 1} = \frac{\frac{s}{a} + 1}{\frac{1}{Kb} s^2 + \left(\frac{1}{k} + \frac{1}{a}\right) s + 1} \tag{2-19}$$

H(s) may be written in the standard form

$$H(s) = \frac{(s/a_o)w_n+1}{(s/w_n)^2+2\xi s/w_n+1}$$

with $w_n = \sqrt{Kb}$

and $\xi = 1/2\sqrt{b/K} + \sqrt{Kb}/(2a)$

where as usual w_n and ξ represent respectively the loop natural frequency and its damping factor.

By replacing s with jw one rewrites the transfer function in terms of frequency.

$$H(jw) = \frac{(1/a)(jw)}{(1/Kb)(jw)^2+(1/k+1/a)jw+1} \qquad (2\text{-}20)$$

This expression will be used in the process of minimizing equation (2-5) and it will become apparent in chapter 4 that the particular form of 2-20 is most convenient. In case (1) there is no predetection filtering and the integration is over all frequencies. Thus we will evaluate

$$\int_0^\infty |H(jw)|^2 \, df.$$

We also need Θ_p which is from equation (2-16),

$$\phi \sqrt{|1-H(jw)|^2}$$

Using (2-18) $1-H(s) = 1- \dfrac{KF(s)}{s+KF(s)} = \dfrac{s}{s+KF(s)}$

Substituting for F(s) from (2-9)

$$1-H(s)=\frac{s}{s+\dfrac{K(s/a+1)}{s/b+1}}=\frac{s(s/b+1)}{s(s/b+1)+s(K/a)+K}$$

$$=\frac{(1/b)s^2+s}{(1/b)s^2+(1+K/a)s+K}=\frac{s(s+b)}{s^2+(b+Kb/a)s+Kb}\ .$$

Again, replacing s by jw we get:

$$1-H(jw)=\frac{(jw)^2+b(jw)}{(jw)^2+(b+Kb/a)jw+Kb}=\frac{-w^2+jbw}{Kb-w^2+jwb(1+K/a)}$$

To obtain $\left|1-H(jw)\right|^2$ we use the fact that in general

$$\left|\frac{a+jb}{c+jd}\right|^2=\frac{a^2+b^2}{c^2+d^2} \tag{2-21}$$

and get:

$$\left|1-H(jw)\right|^2=\frac{w^4+w^2b^2}{(Kb-w^2)^2+[wb(1+K/a)]^2}$$

$$=\frac{w^2(w^2+b^2)}{w^4+w^2(K^2b^2/a^2+2Kb^2/a+b^2-2Kb)+K^2b^2} \tag{2-22}$$

and finally

$$\theta_p=\beta\left|1-H(jw)\right|$$

$$= \beta \sqrt{\frac{w^2(w^2+b^2)}{w^4+w^2(K^2b^2/a^2+2Kb^2/a+b^2-2Kb)+K^2b^2}} \qquad (2\text{-}23)$$

Equation (2-20) and (2-23) will be used in chapter 4 in the optimization procedure.

2.4.2 Second Order PLD With No Predetection Filter and Voice Modulation

Loop filter I is used in this system as in the previous one so the transfer functions are identical.

From equation (2-11)

$$H(jw) = \frac{(1/a)\ (jw)}{(1/Kb)(jw)^2+1/k+(1/a)jw+1}$$

From equation (2-12)

$$\left|1-H(jw)\right|^2 = \frac{w^2(w^2+b^2)}{w^4+w^2(K^2b^2/a^2+2Kb^2/a+b^2-2Kb)+K^2b^2}$$

In order to minimize the expression in equation (2-14) we will need a numerical value for η_m of equation (2-15) and this value depends upon the paramaters of the modulation. We have chosen the same model for the modulation as that used by Acampora and Newton [Ref. 1, Pg. 586] in their experimental work and by Novick [Ref.2,Pg.105] in his optimization procedure. These are typical parameters

and will enable a meaningful comparison of our results with theirs. Thus the following description of a voice channel is assumed:

Baseband voice channel: 300 Hz - 3300 Hz

Peak to RMS Amplitude ratio: 10 dB

RMS deviation: 10 KHz

IF bandwidth: 35 KHz

To obtain a value for η_m we use the fact that $(\Delta f)_{RMS} = 1/(2\pi)(\Delta w)_{RMS} = \sqrt{10}$ KHz. $(\Delta w)_{RMS}{}^2$ is the deviation power and is related to its PSD, $W_\Delta(f)$, by

$$(\Delta w)_{RMS}{}^2 = \int_0^\infty W_\Delta(f)df$$

But the power spectral density of the frequency deviation is related to the PSD of phase $\phi(f)$ by

$$W_\Delta(f) = w^2\phi(f) = w^2\eta_m/w^4$$

Thus $$[(\Delta w)_{RMS}]^2 = \int_{f_a}^{f_b} w^2\phi_i(f)df = \frac{1}{2\pi}\int_{w_a}^{w_b}\frac{\eta_m}{w^2}dw$$

$$= \frac{\eta_m}{2\pi}[1/w]\Big|_{w_a}^{w_b} = \eta_m/(2\pi)\ (1/w_a - 1/w_b)$$

and $$\eta_m = 2\pi(\Delta w)_{RMS}{}^2\ \frac{w_a w_b}{w_b - w_a}$$

For the voice model, $(\Delta w)_{RMS}{}^2 = (2\pi\Delta f)_{RMS}{}^2$

$$=(2\pi\sqrt{10}\times 10^3)^2 = 4\pi^2\times 10^7$$

From which $\eta_m = 8\pi^3 \times 10^7 \dfrac{w_a w_b}{w_b - w_a}$

This will be used in all of the cases involving voice modulation.

2.4.3 Case (3). Additional Pole, No Predetection Filter, Test-Tone Modulation

It is useful to know the impact of a higher order pole on the threshold performance of a PLD. The pole may be due to stray effects or, if it proves to be beneficial, may be designed into the detector. In either event, the addition of a pole to a standard second order PLD results in a loop filter like filter II of figure 2-2 which has a response

$$F(s) = \frac{s/a + 1}{(s/b+1)(s/d+1)}$$

The open loop assymptotic Bode diagram including the VCO is shown in figure 2-7.

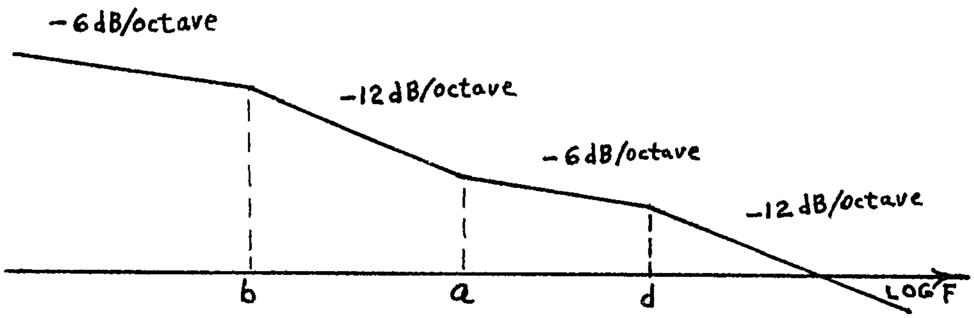


FIGURE 2-7 BODE DIAGRAM FOR FILTER II

Using equation (2-10) for the closed loop response we write

$$H(s) = \frac{KF(s)}{s+KF(s)} = \frac{\dfrac{K\,(s/a+1)}{(s/b+1)(s/d+1)}}{s+\dfrac{K(s/a+1)}{(s/b+1)(s/d+1)}}$$

$$= \frac{K(s/a+1)}{s(s/b+1)(s/d+1)+K(s/a+1)}$$

$$= \frac{s/a+1}{s^3\,\dfrac{1}{Kbd}+s^2\left(\dfrac{1}{Kb}+\dfrac{1}{Kd}\right)+s\left(\dfrac{1}{a}+\dfrac{1}{k}\right)+1} \qquad (2\text{-}24)$$

Letting $A=\dfrac{1}{a}$, $B=\dfrac{1}{Kbd}$, $C=\dfrac{1}{Kb}+\dfrac{1}{Kd}$, $D=\dfrac{1}{a}+\dfrac{1}{K}$

equation (2-24) can be written

$$H(s) = \frac{1+AS}{BS^3+CS^2+DS+1} \qquad (2\text{-}25)$$

With a change of variable we get

$$H(jw) = \frac{1+A(jw)}{B(jw)^3+C(jw)^2+D(jw)+1} \quad , \qquad (2\text{-}26)$$

a form which is suitable for evaluating

$$\int_0^\infty \left|H(jw)\right|^2 df \quad .$$

We will also need an expression for $\left|1-H(jw)\right|^2$.

From equation (2-25)

$$1-H(s) = \frac{BS^3+CS^2+(D-A)S}{BS^3+CS^2+DS+1}$$

and $$1-H(jw) = \frac{B(jw)^3+C(jw)^2+(D-A)jw}{B(jw)^3+C(jw)^2+D(jw)+1}$$

$$= \frac{-Cw^2+j\left[(1/K)w-Bw^3\right]}{1-Cw^2+j(Dw-Bw^3)}$$

where 1/K has been substituted for D-A.

Applying equation (2-21) results in

$$\left|1-H(jw)\right|^2 = \frac{C^2w^4+[(1/K)w-Bw^3]^2}{(1-Cw^2)^2+(Dw-Bw^3)^2}$$

which can be expanded and written as a ratio of polynomials in w^2 as:

$$\left|1-H(jw)\right|^2 = \frac{w^2[B^2w^4+(C^2-2B/K)w^2+1/K^2}{B^2w^6+(C^2-2DB)w^4+(D^2-2C)w^2+1} \qquad (2-27)$$

Finally the expression for the maximum modulation related phase error due to a test-tone of radian frequency W_T Rad/sec may be written as

$$\theta_P=\beta\sqrt{\frac{w_T{}^2[B^2w_T{}^4+(C^2-2B/K)w_T{}^2+(1/K^2)}{B^2w_T{}^6+(C^2-2DB)w_T{}^4+(D^2-2C)w_T{}^2+1}} \qquad (2-28)$$

Equations (2-26) and (2-28) will be used in chapter 4 in the minimization of $(CNR)_{TH}$ as given by equation (2-5).

2.4.4 Case (4). Additional Pole, No Predetection Filter, Voice Modulation

For this system the loop configuration is identical to that in the previous case but the modulation is voice rather than test-tone and thus the model of section 2.4 applies as does equation (2-14) which derives from it. For substitution into this expression for threshold level, the numerator remains unchanged and is therefore given by equation (2-26).

Thus $$H(jw) = \frac{Ajw + 1}{B(jw)^3 + C(jw)^2 + D(jw) + 1}$$

with $A = \frac{1}{a}$, $B = \frac{1}{Kbd}$, $C = \frac{1}{Kb} + \frac{1}{Kd}$, $D = \frac{1}{a} + \frac{1}{K}$

For use in the denominator we must evaluate

$$\int_0^\infty \phi_i(f) \left|1 - H(jw)\right|^2 df$$

From equation (2-27) we have

$$\left|1 - H(jw)\right|^2 = \frac{w^2[B^2w^4 + (C^2 - 2B/K)w^2 + (1/K^2)]}{B^2w^6 + (C^2 - 2DB)w^4 + (D^2 - 2C)w^2 + 1}$$

and from equation (2-15), $$\phi_i = \frac{\eta_m}{w^4} \qquad f_a < f < f_b$$

$$0, \text{ elsewhere.}$$

Upon combining these two expressions and substituting $w = 2\pi f$

we get

$$\int_0^\infty \phi_i(f) \left|1 - H(jw)\right|^2 df$$

$$= \frac{\eta_m}{2\pi} \int_{w_a}^{w_b} \frac{w^4 + C^2B^2 - \frac{2}{KB} w^2 + \frac{1}{B^2K^2}}{w^2\left[w^6 + \left(\frac{C^2}{B^2} - \frac{2D}{B}\right) w^4 + \left(\frac{D^2}{B^2} - \frac{2C}{B^2}\right) w^2 + \frac{1}{B^2}\right]} dw \qquad (2\text{-}29)$$

In order to optimize this system, equation (2-26) and (2-29) must be substituted into the expression for $(CNR)_{TH}$ given by equation (2-14) and the result minimized. Details of the optimization procedure and its results are presented in chapter 4.

Chapter 3 continues with the development of mathematical formulations for the ERPLDs.

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CHAPTER III

FORMULATIONS FOR THE ANALYSIS OF THE EFFECTS OF A HIGHER ORDER POLE ON THRESHOLD PERFORMANCE OF AN ERPLD

3.0 Background

Previous efforts [Ref. 1,2,3] have established that Phaselocked loops with extended range (beyond $\pi/2$ radians) phase comparators can perform FM demodulation and achieve even greater threshold reduction than the standard PLD. One such method, described in chapter I, employs phase subtraction via a feedback path to reduce the error phase and thus extend the effective range of the phase comparator. By way of explanation it was argued [Ref.2 ,P. 577-78] that the extended dynamic range of lock retards the onset of LLI spikes and thus reduces the threshold level. This explanation has been called into question because it does not take into account the effect of noise that is fed back along with phase [Ref. 4, P. 258] . Analysis of the phase feedback ERPLD has shown it to be equivalent to a PLD with a variant form of loop filter [Ref. 4, P. 260], filter III of figure 2-2. An alternative explanation of the success of the ERPLD in reducing threshold has been sought through analysis of the linear model which incorporates the equivalent filter. This approach has led to the work of Novick. But the "equivalent filter ERPLD" requires the use of an ideal differentiator which is, of course, physically unrealizable and is also restrictive because it requires that an I F filter precede the PLD to limit the noise spectrum which would otherwise be unbounded. We will carry out an optimization of the ERPLD for two types of modulation. This is in part a duplication of the work done by Novick [Ref. 5] and will confirm his results as well as establish confidence in the procedure. Then we will proceed to investigate and optimize the ERPLD with an added pole in the differentiator.

3.1 Derivation of Expressions for $(CNR)_{TH}$ For the Various ERPLDs Identified in the Table of Cases

Expressions for $(CNR)_{TH}$ will be formulated for each of cases (5) through (10). All of these loops employ either filter III or filter IV with or without predetection filtering and with either test-tone modulation or voice modulation. In this section appropriate mathematical forms for evaluating $|H(jw)|^2$ and $|1-H(jw)|^2$ for these six systems will be derived. Use of these forms in equation (2-5) and (2-19) for minimizing threshold will be undertaken in chapter 4.

We begin with an analysis of Filter III for case (4).

3.1.1 ERPLD with Ideal Differentiator and Test-Tone Modulation

In filter III the response of the ideal differentiator is added to that of the standard filter yielding a combined transfer function

$$F(s) = \frac{s/a + 1}{s/b + 1} + \frac{\alpha}{K} S \qquad (3\text{-}1)$$

The differentiator constant α/K retains the significance assigned by Novick and relates to the phase subtractor shown in figure (1-8). There K_3 is the gain of the feedback path and δ is the sensitivity of the phase discriminator so $\alpha = K_3\,\delta$ represents the loop gain of the phase feedback circuit.

Combining the terms above we get

$$F(s) = \frac{s/a + 1 + (\alpha/K)s(s/b + 1)}{s/b + 1}$$

$$= \frac{s^2\,\alpha/(Kb) + s(\alpha/K + 1/a) + 1}{s/b + 1} \qquad (3\text{-}2)$$

The open loop response including the VCO as an ideal integrator with constant K_2 is

$$G(s) = \frac{[s^2\,\alpha/(Kb) + s(\alpha/K + 1/a) + 1]K_2K_1\delta}{s(s/b + 1)}$$

$$= \frac{s^2\,\alpha/(Kb) + s(\alpha/K + 1/a) + 1}{s[s/(Kb) + 1/K]}$$

where again $K = K_1K_2\,\delta$

This form can be identified with the standard open loop response having a pair of complex poles which is written as

$$G(s) = \frac{s^2/w_n^2 + 2\,\xi\, s/w_n + 1}{s(s/b + 1)}$$

with $w_n = Kb/\alpha$ and $\xi = b/2\ (1 + K/a\ )$.

Figure 3-1 is a sketch of this open loop response.

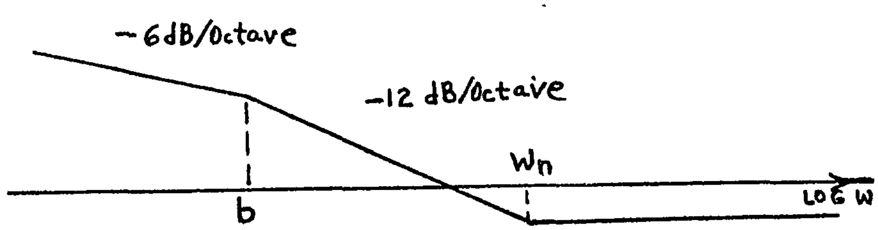

FIGURE 3-1 ERPLD OPEN LOOP RESPONSE

From equations (3-2) and (1-7)

$$H(s) = \frac{KF(s)}{s+KF(s)} = \frac{s^2\,\alpha/b + s(\alpha + K/a) + K}{s(s/b + 1) + s^2\,\alpha/b + s(\alpha + K/a) + K}$$

which can be rearranged as

$$H(s) = \frac{s^2\,\alpha/(Kb) + s(\alpha/K + 1/a) + 1}{s^2(1/(Kb) + \alpha/Kb) + s(\alpha/K + 1/a + 1/K) + 1} \qquad (3\text{-}3)$$

By substituting jw for s and defining convenient constants:

$$A' = \frac{\alpha}{Kb}\ ,\quad B' = \frac{\alpha}{K} + \frac{1}{a}\ ,\quad C' = \frac{1+\alpha}{Kb}\ ,\quad D' = \frac{1+\alpha}{K} + \frac{1}{a}$$

we get $$H(jw) = \frac{A'(jw)^2 + B'(jw)+1}{C'(jw)^2+D'(jw)+1}$$

$$= \frac{1-A'w^2+jB'w}{1-C'w^2+jD'w}$$ and upon squaring

$$\left|H(jw)\right|^2 = H(jw)xH(jw)^* = \frac{(1-A'w^2)^2+B'^2w^2}{(1-C'w^2)^2+D'^2w^2}$$

$$= \frac{A'^2w^4+(B'^2-2A')w^2+1}{C'^2w^4+(D'^2-2C)w^2+1}$$

For further compactness choose a new set of constants as follows:

$$A=A'^2 = \left(\frac{\alpha}{Kb}\right)^2\ ,\quad B=B'^2-2A'=(1/a + \alpha/K)^2-2\alpha/(Kb)$$

$$C=C'^2= 1+\left(\frac{\alpha}{kb}\right)^2\ ,\quad D=D'^2-2C' = \left(\frac{1+\alpha}{k} + \frac{1}{a}\right)^2-2\ \frac{1+\alpha}{kb}$$

and get

$$\left|H(jw)\right|^2 = \frac{Aw^4+Bw^2+1}{Cw^4+Dw^2+1} \qquad (3\text{-}4)$$

The requirement for a predetection filter is implicit in equation (3-4) because as w becomes large, $|H(jw)|^2$ becomes constant.

$$\lim_{w \to \infty} |H(jw)|^2 = A/C = \left(\frac{\alpha}{Kb}\right)^2 \left(\frac{Kb}{1+\alpha}\right)^2 = \left(\frac{\alpha}{1+\alpha}\right)^2$$

and were it not for the filter, the loop noise

$\eta/A^2 \int_0^\infty |H(jw)|^2 \, df$ would be unbounded.

Here we assume an ideal bandpass predetection filter of half bandwidth $B_p/2$ and for use in equation (2-5) must evaluate

$$\int_0^{\pi B_p} |H(jw)|^2 \, df = 1/(2\pi) \int_0^{\pi B_p} \frac{Aw^4+Bw^2+1}{Cw^4+Dw^2+1} \, dw \; . \qquad (3\text{-}5)$$

Now we formulate $|1-H(jw)|^2$. From equation (3-2)

$$1-H(s) = \frac{s^2 \, 1/Kb + s\,(1/K)}{s^2\left(\frac{1}{Kb} + \frac{\alpha}{kb}\right) + s\left(\frac{\alpha}{K} + \frac{1}{a} + \frac{1}{K}\right) + 1}$$

Let: $E = 1/(Kb)$, $F = 1/k$, $G = 1+\alpha/(Kb)$, $H = (1+\alpha)/K + 1/a$

then $1-H(s) = \dfrac{Es^2+Fs}{Gs^2+Hs+}$ so that

$$1-H(jw) = \frac{E(jw)^2+F(jw)}{G(jw)^2+H(jw)+1} = \frac{-Ew^2+jFw}{1-Gw^2+jHw}$$

and by squaring , $$\left|1-H(jw)\right|^2 = \frac{E^2w^4+F^2w^2}{(1-Gw^2)^2+H^2w^2} \qquad (3\text{-}6)$$

3.1.2 Case (6), ERPLD With Ideal Differentiator and Voice Modulation

This is one of the systems that was optimized by Novick [Ref. 3, P. 117]. It differs from the previous system only with respect to the type of modulation and therefore we can draw on some of the results of section 3.1.1.

A threshold model for voice modulation was developed in section 2.2 and may be applied here. An expression for threshold carrier-to-noise ratio is given by equation (2-19) and only the limits on the integration will have to be adapted for the current system.

Equation (2-14) gives for the threshold condition,

$$(CNR)_{TH} = \frac{\left|H(jw)\right|^2 df}{B_p[\nu - \int_0^\infty \phi_i(f)\left|1-H(jw)\right|^2 df]} \qquad (3\text{-}7)$$

For the numerator we have equation (3-5) where the constants are defined as in the previous section.

For the denominator start with equation (3-6),

$$\left|1-H(jw)\right|^2 = \frac{E^2w^4+F^2w^2}{(1-G'w^2)^2+H^2w^2}$$, which upon expansion

becomes $$\left|1-H(jw)\right|^2 = \frac{w^2(E^2w^2+F^2)}{G'^2w^4+(H'^2-2G')w^2+1}, \tag{3-8}$$

where the primed quantities are given by: $G' = (1+\alpha)/Kb$, $H' = (1+\alpha)/K + 1/a$

Now define: $G = G'^2 = [(1+\alpha)/(Kb)]^2$ and $H = H'^2 - 2G' = [(1+\alpha)/K + 1/a]^2 - 2(1+\alpha)/(Kb)$. The constants G and H are seen to be respectively C and D of the previous section so that (3-8) can be written as

$$\left|1-H(jw)\right|^2 = \frac{w^2(E^2w^2+F^2)}{Cw^4+Dw^2+1} \tag{3-9}$$

We assume here the same voice model that was adopted in section 2.2. There it emerged that the signal phase power spectral density, $\phi_i(f)$ of equation (3-7), can be written as

$$\phi_i(w) = \eta_m/w^4 \text{ Rad}^2/\text{Rad/sec for } w_a \leq w \leq w_b$$
$$= 0 \quad \text{elsewhere} \tag{3-10}$$

and the constant

$$\eta_m = 8\pi^3 \times 10^7 \, w_a w_b/(w_b - w_a) \tag{3-11}$$

Using (3-9), (3-10), and (3-11) and changing the variable from f to w, one is able to write the integral in the denominator of (3-7) as

$$4\pi^2 x 10^7 \; w_a w_b/(w_b - w_a) \int_{w_a}^{w_b} \frac{E^2 w^2 + F^2}{w^2(Cw^4 + Dw^2 + 1)} dw \qquad (3\text{-}12)$$

3.1.3 Case (7), ERPLD with Additional Pole, No Predetection Filter, Test-Tone Modulation

With the addition of a pole to the differentiator of the equivalent filter ERPLD of the previous section two improvements are introduced. The differentiator is rendered realizable and it becomes feasible to operate the PLD without a predetection filter.

In practice there inevitably exists a higher frequency rolloff. If it is not included by design then it will be the result of stray effects, especially in wideband systems such as TV and FDM. We now investigate the impact of that higher pole upon the threshold of the detector.

To model the innovation we add a pole to the differentiator and obtain filter IV. In the last four cases to be investigated we examine the performance of a PLD with filter IV as loop filter, with and without prefiltering and for both test-tone modulation and voice modulation. Figure (3-2) shows the loop with the new filter. The loop filter response is

$$F(s) = \frac{s/a + 1}{s/b + 1} + \frac{\alpha}{K} \, \frac{s}{s/d + 1}$$

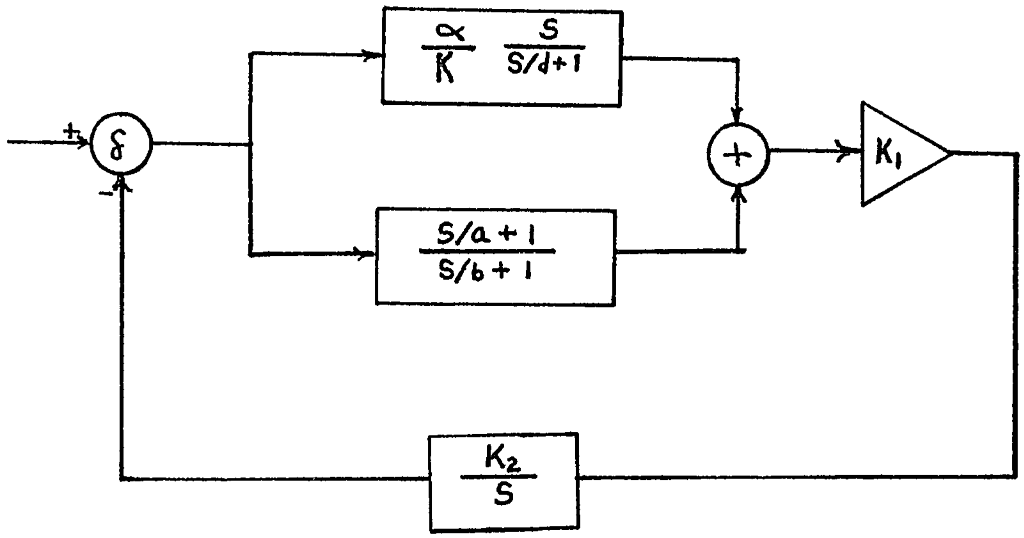

FIGURE 3-2 PLD WITH FILTER IV

As previously, α is the part of the error phase that is fed-back in the phase subtraction configuration, which was a precursor of the equivalent filter ERPLD, and $K = K_1 K_2 \delta$.

Combining terms over a common denominator and simplifying we get

$$F(s) = \frac{(s/d + 1)(s/a + 1) + \alpha/K(s/b + 1)s}{(s/b + 1)(s/d + 1)}$$

$$= \frac{s^2\left(\frac{1}{ad} + \frac{\alpha}{Kb}\right) + s\left(\frac{1}{a} + \frac{1}{d} + \frac{\alpha}{K}\right) + 1}{(s/b + 1)(s/d + 1)} \qquad (3\text{-}13)$$

The open loop response may now be written as

$$G(s) = \frac{s^2\left(\frac{1}{ad} + \frac{\propto}{Kb}\right) + s\left(\frac{1}{a} + \frac{1}{d} + \frac{\propto}{K}\right) + 1}{s\left(\frac{s}{Kb} + \frac{1}{K}\right)\left(\frac{s}{d} + 1\right)}$$

which again has a pair of complex zeros. If the numerator is identified with the standard form

$$s^2/w_n^2 + 2\,\xi\, s/w_n + 1$$

then $w_n = \left(\frac{Kb + \propto ad}{abdK}\right)^{-\frac{1}{2}}$ and $\xi = \frac{\sqrt{b}\,[dK + a(K + \propto d)]}{2\sqrt{Kb + \propto ad}\sqrt{adK}}$ (3-14)

This response is sketched in Figure 3-3

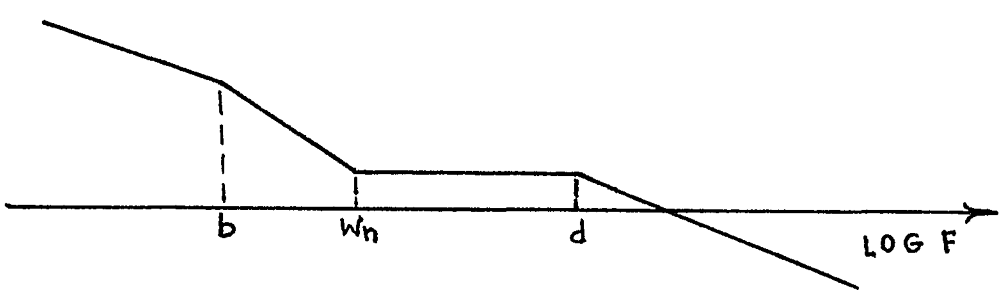

FIGURE 3-3 OPEN LOOP RESPONSE-ERPLD WITH FILTER IV

To derive the expression for the closed loop response, let N(s) and D(s) represent the numerator and denominator respectively of equation (3-13) and use (1-7) to get

$$H(s) = \frac{KF(s)}{s+KF(s)} = \frac{K\ N(s)/D(s)}{s+K\ N(s)/D(s)} = \frac{KN(s)}{sD(s)+KN(s)}$$

$$= \frac{s^2(K/ad + \alpha/b)+s(K/a + K/d+\alpha)+K}{s[\ s^2 \frac{1}{bd} +s\ (\frac{1}{b} + \frac{1}{d} +1)]+ s^2(\frac{K}{ad} + \frac{\alpha}{b})+s(\frac{K}{a}+ \frac{K}{d}+\alpha)+K}$$

and upon multiplying through by bd and collecting terms

$$H(s) = \frac{s^2(Kb/a+\alpha d)+s(Kbd/a+Kb+\alpha bd)+Kbd}{s^3+s^2(d+b+Kb/a+\alpha d)+s(bd+Kbd/a+Kb+\alpha bd)+Kbd} \quad . \quad (3\text{-}15)$$

Letting $A = Kb/a + \alpha d$, $B = Kbd/a+Kb + \alpha bd$

$C = Kbd$, $D = d+b+Kb/a + \alpha d$ and

$E = bd + Kbd/a + Kb + \alpha bd$ we get

$$H(s) = \frac{As^2+Bs+C}{s^3+Ds^2+ES+C} \qquad (3\text{-}16)$$

Replacing s by jw yields

$$H(jw) = \frac{A(jw)^2+B(jw)+C}{(jw)^3+D(jw)^2+E(jw)+C} \qquad (3\text{-}17)$$

which is in a convenient form for substitution into the expression for threshold (CNR) of equation (2-5). We will require

$$\int_0^\infty |H(jw)|^2\ df.$$

We will also require an expression for $|1-H(jw)|^2$ in order to evaluate the peak signal induced phase error, θ_p.

From equation (3-16) one gets

$$1-H(s) = \frac{s^3+(D-A)s^2+(E-B)s}{s^3+Ds^2+Es+C}$$

$$\text{and } 1-H(jw) = \frac{(jw)^3+(D-A)(jw)^2+(E-B)jw}{(jw)^3+D(jw)^2+Ejw+C}$$

$$= \frac{(A-D)w^2+j[(E-B)w-w^3]}{C-Dw^2+j(Ew-w^3)}$$

After squaring ,

$$\left|1-H(jw)\right|^2 = \frac{(A-D)^2w^4+(E-B)^2w^2+w^6-2(E-B)w^4}{C^2+D^2w^4-2CDw^2+E^2w^2+w^6-2Ew^4}$$

and collecting terms ,

$$\left|1-H(jw)\right|^2 = \frac{w^6+[(A-D)^2-2(E-B)]w^4+(E-B)^2w^2}{w^6+(D^2-2E)w^4+(E^2-2CD)w^2+C^2} \qquad (3\text{-}18)$$

Equations (3-17) and (3-18) will be used to evaluate and minimize the threshold (CNR).

3.1.4 Case (8), ERPLD With Pole, Predetection Filter, Test-Tone Modulation

This system is similar to that of the previous section except that a predetection filter is assumed. The filter response referred to the base band is taken to be

$$H_f(f) = 1 \quad \text{for } 0 < f < B_p/2$$

$$0 \quad \text{elsewhere}$$

We need $\int_0^{B_p/2} \left|H(jw)\right|^2 df$. The loop filter is filter IV, the same as for the previous section so that from equation (3-17) we have

$$H(jw) = \frac{A(jw)^2+B(jw)+C}{(jw)^3+D(jw)^2+E(jw)+C}$$

$$= \frac{C-Aw^2+jBw}{C-DW^2+j(Ew-w^3)}$$

where A,B,C, and D are given in section 3.1.3.

$$\left|H(jw)\right|^2 = H(jw)xH^*(jw) = \frac{(C-Aw^2)^2+B^2w^2}{(C-Dw^2)^2+(Ew-w^3)^2}$$

and upon expanding we get

$$\left|H(jw)\right|^2 = \frac{A^2w^4+(B^2-2AC)w^2+C^2}{w^6+(D^2-2E)w^4+(E^2-2CD)w^2+C^2} \qquad (3\text{-}19)$$

This rather formidable ratio of polynomials will have to be integrated between finite limits by a method to be developed in chapter 4.

To evaluate the peak signal induced error phase it will be necessary to formulate $\left|1-H(jw)\right|^2$. But because the loop and the modulation are identical to those of case (7), the expression was generated in section 3.1.3 as

$$\left|1-H(jw)\right|^2 = \frac{w^6+[(a-D)^2(E-B)]w^4+(E-B)^2w^2}{w^6+(D^2-2E)w^4+(E^2-2CD)w^2+C^2} \qquad (3\text{-}20)$$

These formulations will be used to evaluate and minimize $(CNR)_{TH}$ as given by equation (2-5).

3.1.5 Case (9), ERPLD With Pole, No Predetection Filter, Voice Modulation

As in the previous two cases, filter IV is used here. This system is similar to that of case (7), differing only with respect to the type of modulation. We consider voice modulation and employ the models that were adopted in section 2.2 for the modulation and for the threshold criterion. The signal is said to be band limited and gaussian with inverse square PSD, and threshold is assumed to occur at some critical value of total mean square error phase, ν . Thus for threshold (CNR) we have:

$$(CNR)_{TH} = \frac{\int_0^\infty \left|H(jw)\right|^2 df}{B_p[\nu - \int_0^\infty \phi_i(f)\left|1-H(jw)\right|^2 df]}$$

Because there is no predetection filtering, the noise spectrum is unlimited, and using equation (3-14) we get

$$\int_0^\infty \left|H(jw)\right|^2 df = \frac{1}{2\pi}\int_0^\infty \frac{A(jw)^2+B(jw)+C}{(jw)^3+D(jw)^2+E(jw)+C}\, dw \qquad (3\text{-}21)$$

For the integral in the denominator, equation (3-15) was derived for the loop with filter IV.

$$\left|1-H(jw)\right|^{2} = \frac{w^6+[(A-D)^2-2(E-B)]w^4+(E-B)^2w^2}{w^6+(D^2-2E)w^4+(E^2-2CD)w^4+C^2} \qquad (3\text{-}18)$$

The constants A,B,C,D and E are given in section 3.1.3 as functions of the system parameters a,b,d, α , and K.

From section 2.2 we have for the signal phase PSD

$$\phi_i(w) = \eta_m/w^4$$

with $\eta_m = 8\,\pi^3 \mathrm{x} 10^7 \; w_a w_b/(w_b-w_a)$.

Substitution of these quantities into equation (2-19) yields for the integral in the denominator

$$4\,\pi^2 \mathrm{x} 10^7 \, \frac{w_a w_b}{w_b-w_a} \int_{w_a}^{w_b} \frac{w^2\left\{w^4+[(A-D)^2-2(E-B)]w^2+(E-B)^2\right\}}{w^4[w^6+(D^2-2E)w^4+(E^2-2CD)w^2+C^2]}\,dw \qquad (3\text{-}22)$$

Again the expression for $(\mathrm{CNR})_{TH}$ is complete in terms of the system paramaters and can be minimized for system optimization.

3.1.6 <u>Case (10), ERPLD With Pole, With Predetection Filter, Voice Modulation</u>

This is the fourth loop incorporating filter IV and the last system to be considered in this study. It differs from the previous case only in that a predetection filter precedes the PLD. This system is a combination of aspects of systems (8) and (9) and therefore the function to be minimized may be constructed from the applicable parts of the two previous formulations.

The numerator is borrowed from case (8), equation (3-19) and may be written as

$$\int_0^\infty \left|H(jw)\right|^2 df = \frac{1}{2\pi}\int_0^{\pi B_p} \frac{Aw^4+(B^2-2AC)w^2+C^2}{w^6+(D^2-2E)w^4+(E^2-2CD)w^2+C^2}\, dw \quad (3\text{-}23)$$

The denominator is identical to that of case (9) and includes equation (3-22).

This completes the preparation of the quantities to be minimized in the optimization procedure for the ten systems under study. The results of this preparation are presented in TABLE 3-1.

TABLE 3-1 SUMMARY OF RESULTS OF CHAPTERS 2 AND 3

Threshold Model I For Test-Tone Mod. $(CNR)_{TH} = \dfrac{4\pi^2 \int_0^\infty |H(j\omega)|^2\, df}{B_P(\pi - 2\theta_P)^2}$

Threshold Model II For Voice Mod. $(CNR)_{TH} = \dfrac{\int_0^\infty |H(j\omega)|^2\, df}{B_P\left[\nu - \int \Phi_i(f)\,|1-H(j\omega)|^2 df\right]}$

CASE	LOOP FILTER	THRESHOLD MODEL	NUMERATOR INTEGRAL	DENOMINATOR INTEGRAL OR θ_P, PEAK PHASE ERROR	CONSTANTS
1	I	I	$\int_0^\infty \left\lvert \dfrac{\frac{1}{a}(jW)+1}{\frac{1}{Kb}(jW)^2+\left(\frac{1}{K}+\frac{1}{a}\right)jW+1}\right\rvert^2 df$	$\beta\sqrt{\dfrac{W_T^2(W_T^2+b^2)}{W_T^4+W_T^2\left(\frac{K^2b^2}{a^2}+2\frac{Kb^2}{a}+b^2-2Kb\right)+K^2b^2}}$	β = Modulation Index $W_{TT} = 2\pi f_{TT}$ f_{TT} = Test-Tone freq.
2	I	II	$\int_0^\infty \left\lvert \dfrac{\frac{1}{a}(j\omega)+1}{\frac{1}{Kb}(j\omega)^2+\left(\frac{1}{K}+\frac{1}{a}\right)j\omega+1}\right\rvert^2 df$	$\dfrac{\eta_m}{2\pi}\int_{\omega_a}^{\omega_b} \dfrac{\omega^2+b^2}{\omega^2\left[\omega^4+D\omega^2+(Kb)^2\right]}\, d\omega$	$\eta_m = 8\pi^3\times10^7 \dfrac{\omega_a\omega_b}{\omega_b-\omega_a}$ $D=\left(\frac{Kb}{a}\right)^2+b^2+\frac{2Kb}{a}-2K$
3	II	I	$\int_0^\infty \left\lvert \dfrac{A(j\omega)+1}{B(j\omega)^3+C(j\omega)^2+D(j\omega)+1}\right\rvert^2 df$	$\beta\sqrt{\dfrac{W_T^2\left[B^2W_T^4+\left(C^2-2\frac{B}{K}\right)W_T^2+\frac{1}{K^2}\right]}{B^2W_T^6+(C^2-2DB)W_T^4+(D-2C)W_T^2+1}}$	$A=\frac{1}{a}$ $B=\frac{1}{Kbd}$ $C=\frac{1}{Kb}+\frac{1}{Kd}$ $D=\frac{1}{a}+\frac{1}{K}$
4	II	II	$\int_0^\infty \left\lvert \dfrac{A(j\omega)+1}{B(j\omega)^3+C(j\omega)^2+D(j\omega)+1}\right\rvert^2 df$	$\dfrac{\eta_m}{2\pi}\int_{\omega_a}^{\omega_b} \dfrac{\omega^4+\left(\frac{C^2}{B^2}-\frac{2}{KB}\right)\omega^2+\frac{1}{B^2K^2}}{\omega^2\left[\omega^6+\left(\frac{C^2}{B^2}-\frac{2D}{B}\right)\omega^4+\left(\frac{D^2}{B^2}-\frac{2C}{B^2}\right)\omega^2+\frac{1}{B^2}\right]}\, d\omega$	

TABLE 3-1 CONTINUED

CASE	LOOP FILTER	THRESH-HOLD MODEL	NUMERATOR INTEGRAL	DENOMINATOR INTEGRAL OR θ_p, PEAK PHASE ERROR	CONSTANTS
5	III	I	$\frac{1}{2\pi}\int_0^{\pi B_p} \frac{AW^4+BW^2+1}{CW^4+DW^2+1}\,dw$	$\beta\sqrt{\frac{E^2W_T^4+F^2W_T^2}{(1-GW_T)^2+H^2W_T^2}}$	$A=\left(\frac{\alpha}{Kb}\right)^2$, $B=\left(\frac{1}{a}+\frac{\alpha}{K}\right)^2-2\frac{\alpha}{Kb}$ $C=\left(\frac{1+\alpha}{Kb}\right)^2$, $D=\left(\frac{1+\alpha}{K}+\frac{1}{a}\right)^2-2\left(\frac{1+\alpha}{Kb}\right)$ $E=\frac{1}{Kb}$, $F=\frac{1}{K}$, $G=\frac{1+\alpha}{Kb}$, $H=\frac{1+\alpha}{K}+\frac{1}{a}$
6	III	II	$\frac{1}{2\pi}\int_0^{\pi B_p} \frac{AW^4+BW^2+1}{CW^4+DW^2+1}$	$\frac{\eta_m}{2\pi}\int_{W_a}^{W_b} \frac{E^2W^2+F^2}{W^2(CW^4+DW^2+1)}\,dW$	
7	IV	I	$\int_0^{\infty}\left\lvert\frac{A(jw)^2+B(jw)+C}{(jw)^3+D(jw)^2+E(jw)+C}\right\rvert^2 df$	$\beta\sqrt{\frac{W_T^6+\left[(A-D)^2-2(E-B)\right]W_T^4+(E-B)^2W_T^2}{W_T^6+(D^2-2E)W_T^4+(E^2-2CD)W_T^2+C^2}}$	$A=\frac{Kb}{a}+\alpha d$ $B=\frac{Kbd}{a}+Kb+\alpha bd$ $C=Kbd$ $D=d+b+\frac{Kb}{a}+\alpha d$ $E=bd+\frac{Kbd}{a}+Kb+\alpha bd$ $\eta_m=8\pi^3\times10^7\frac{W_aW_b}{W_b-W_a}$
8	IV	I	$\frac{1}{2\pi}\int_0^{\pi B_p}\frac{A^2\omega^4+(B^2-2AC)W^2+C^2}{w^6+(D^2-2E)\omega^4+(E^2-2CD)\omega^2+C^2}\,dw$	$\beta\sqrt{\frac{W_T^6+\left[(A-D)^2-2(E-B)\right]W_T^4+(E-B)^2W_T^2}{W_T^6+(D^2-2E)W_T^4+(E^2-2CD)W_T^2+C^2}}$	
9	IV	II	$\int_0^{\infty}\left\lvert\frac{A(jw)^2+B(jw)+C}{(jw)^3+D(jw)^2+E(jw)+C}\right\rvert^2 df$	$\frac{\eta_m}{2\pi}\int_{W_a}^{W_b}\frac{W^4+\left[(A-D)^2-2(E-B)\right]W^2+(E-B)^2}{W^2\left[W^6+(D^2-2E)W^4+(E^2-2CD)W^2+C^2\right]}\,dW$	
10	IV	II	$\frac{1}{2\pi}\int_0^{\pi B_p}\frac{A^2w^4+(B^2-2AC)W^2+C^2}{w^6+(D^2-2E)w^4+(E^2-2CD)\omega^2+C^2}\,dw$	$\frac{\eta_m}{2\pi}\int_{W_a}^{W_b}\frac{w^4+\left[(A-D)^2-2(E-B)\right]W^2+(E-B)^2}{W^2\left[W^6+(D^2-2E)W^4+(E^2-2CD)W^2+C^2\right]}\,dW$	

References- Chapter 3

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2. A. Acampora and A Newton,"Use of Phase Subtraction to Extend the Range of a Phaselocked Demodulator", R.C.A. Rev. 27,No. 4, pp. 577-599, 1966.

3. W.A. Novick, "Investigation and Optimum Design of the Generalized Second-Order Phase-Locked Loop ", Doctoral Dissertation, New Jersey Institute of Technology, 1976

4. J. Klapper and J. Frankle, Phase-Locked and Frequency Feedback Systems: Principles and Techniques, Academic Press, New York, 1972.

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CHAPTER IV

OPTIMIZATION PROCEDURE AND RESULTS

4.1 - Introduction

In this chapter, the various systems that were chosen for study will be optimized for minimum threshold. In each case, the appropriate threshold model from Table 3-1 will be identified and the indicated integrations will be performed. Where necessary, algebraic manipulations of the integrand will be performed to enable the integration in closed form and avoid the need for numerical integration. Some of the forms will be recurrent among the different cases and solutions once derived will be applied repeatedly.

An efficient computer numerical search technique will be employed to locate a point in system-parameter space where the threshold is a minimum.

4.2 - Powell's Method for Locating a Function Minimum

A method due to Powell [Ref. 1] and subsequently improved by Sangwell [Ref. 2] has previously been used successfully to minimize functions similar to those of eq (2-5) and (2-14). In fact, Novick [Ref. 3] used the method to optimize the system here identified as case (6). The algorithm may not be recognizable as that employed by Novick because it was originally written for BASIC and later translated into FORTRAN for faster execution.

In the Powell method, an arbitrary starting point is selected in N-dimension space and a search is then conducted to locate the function minimum along each coordinate direction in turn. The search procedure is then repeated with one of the N directions altered to improve the efficiency of the search. The new direction is chosen parallel to the line in N space that joins the initialization point to the final point of the previous iteration. Each final point becomes the next starting point and the iteration is repeated until the coordinate change during the course of an iteration becomes less than some predetermined criterion distance difference.

For each of the ten systems that are listed in TABLE 3-1 there will be either three, four, or five coordinate dimensions corresponding to the number of loop parameters that must be chosen for an optimum design. A standard second order loop (filter I) has three variable parameters; the pole frequency, the zero frequency, and the loop gain. With the addition of either another pole (filter II) or an ideal differentiator (filter III) the number of variables increases to four. If the differentiator is made realizable by the inclusion of a pole (filter IV) the number of variables becomes five.

The search for a function minimum proceeds along each separate coordinate direction in two stages. First the point is stepped in the direction of decreasing function value by ever increasing increments until a change in slope is detected. Having thus passed the local minimum, the point is then stepped back and forth in a sequence of decreasing, overlapping increments according to a Fibernacci [Ref. 4] pattern until the local minimum is located within a predetermined error.

A more complete description of the search logic is included in Appendix D.

4.3 - Derivation of Closed Forms for the Integrals Appearing in Table 3-1.

For each PLD under study we will apply Powell's method to search for a set of system parameters that results in the minimum threshold. In the course of the Powell search there will be repeated calculations of $(CNR)_{TH}$ based on the appropriate one of threshold models I and II. But all of these calculations involve evaluations of integrals as summarized in Table 3-1, which can be performed by trapezoidal integration at the cost of much computer CPU time. Instead we will derive closed form solutions for all of the integrals which will enable computer evaluation by numerical substitution rather than the lengthier process of iterative summation.

With reference to Table 3-1 we proceed now to prepare the required closed forms on a case by case basis. For each system the function to be evaluated will be presented and then the integrations will be performed.

4.3.1 - CASE (1) - STANDARD SYSTEM

$$(CNR)_{TH} = \frac{4\pi^2 \int_0^\infty \left| H(jw) \right|^2 df}{Bp\,(\pi - 2\theta p)^2} \tag{4-1}$$

$$\int_0^\infty \left| H(jw) \right|^2 df = \int_0^\infty \left| \frac{\frac{1}{a}\,(jw) + 1}{\frac{1}{Kb}\,(jw)^2 + \left(\frac{1}{K} + \frac{1}{a}\right)(jw) + 1} \right|^2 df$$

This infinite integral with an integrand that is a ratio of polynomials in (jw) is a commonly occuring form in linear network noise computation and its solution is readily available [Ref. 5, P. 21]. The particular integrand is of the form

$$I = \int_0^\infty \left| \frac{c_1 \, (jw) + c_o}{d_2 \, (jw)^2 + d_1 \, (jw) + d_o} \right|^2 df \qquad \text{and its}$$

solution is given as $I = \dfrac{c_1^2 \, d_o + c_o^2 \, d_2}{4 d_o d_1 d_2}$.

We identify the coefficients as:

$$c_1 = \frac{1}{a} \quad , \quad c_o = 1 \quad , \quad d_2 = \frac{1}{Kb} \quad , \quad d_1 = \frac{1}{K} + \frac{1}{a} \quad , \quad d_o = 1$$

and get $\displaystyle\int_0^\infty \left| H(jw) \right|^2 df = \frac{\dfrac{1}{a^2} + \dfrac{1}{Kb}}{4\left(\dfrac{1}{K} + \dfrac{1}{a}\right) \dfrac{1}{Kb}}$.

Multiplying numerator and denominator by a^2k^2b we have

$$\int_0^\infty \left| H(jw) \right|^2 df = \frac{K^2b + a^2K}{4(aK + a^2)} = \frac{K \, (Kb + a^2)}{4a \, (a + K)} \tag{4-2}$$

In the denominator we use the expression for θp which is listed in the table.

$$\theta_p = \beta \sqrt{\frac{w_T^2 \, (w_T^2 + b^2)}{w_T^{4} + w_T^{2}\left(\frac{K^2b^2}{a^2} + 2\,\frac{Kb^2}{a} + b^2 - 2Kb\right) + K^2b^2}}$$

Where θ_p is the peak phase error due to signal,

β is the modulation index and w_T is the test-tone radian frequency.

4.3.2 - Standard System With Voice Modulation

$$(CNR)_{TH} = \frac{\int_0^\infty \left| H(jw)\right|^2 df}{B_p \, [\nu - \int_0^\infty \phi_\lambda(f) \left| 1 - H(jw)\right|^2 df]} \qquad (4\text{-}3)$$

From Table 3-1 we observe that the numerator integral is the same as for the previous case and therefore its value is given above in eq. (4-2).

In the denominator we have

$$\frac{\eta_m}{2\pi} \int_{w_a}^{w_b} \frac{w^2 \, (w^2 + b^2)}{w^4 \, [w^4 + w^2 \, (\frac{K^2b^2}{a^2} + 2\,\frac{Kb^2}{a} + b^2 - 2Kb) + K^2b^2]} \, dw \qquad (4\text{-}4)$$

$$\text{Let } D = \frac{K^2b^2}{a^2} + 2\,\frac{Kb^2}{a} + b^2 - 2Kb$$

and write $$I = \frac{\eta_m}{2\pi} \int_{w_a}^{w_b} \frac{w^2 + b^2}{w^2\ (w^4 + Dw^2 + K^2b^2)}\ dw \qquad (4\text{-}5)$$

To evaluate I, we replace w^2 by X for convenience and expand the integrand, $\dfrac{x + b^2}{x\ (x^2 + Dx + K^2b^2)}$, by the method of partial fractions.

The form of this expansion and consequently of the integration depends upon the nature of the roots of the quadratic factor in the denominator which is determined by the discriminant, $D^2 - 4K^2\ b^2$. If $D^2 > 4K^2\ b^2$ then both roots of the quadratic factor are real but if $D^2 < 4K^2\ b^2$ then the roots comprise a complex conjugate pair.

We consider first the situation when the discriminant is positive and the roots are real. The denominator may be expanded as

$$\frac{x + b^2}{x\ (x^2 + Dx + K^2b^2} = \frac{K_o}{x} + \frac{K_1}{x - R_1} + \frac{K_2}{x - R_2} \qquad (4\text{-}6)$$

where the K_is are constants and R_1 and R_2 are the roots which are given by:

$$R_1 = -\frac{D}{2} + \sqrt{\left(\frac{D}{2}\right)^2 - K^2b^2}$$

$$R_2 = -\frac{D}{2} - \sqrt{\left(\frac{D}{2}\right)^2 - K^2b^2}$$

The constants in eq. (4-6) are evaluated by application of

$$K_i = \lim_{x \to R_i} \frac{(x+b^2)(x-R_i)}{x(x-R_1)(x-R_2)} \qquad \text{where } R_0 = 0,\ i = 0,1,2.$$

This yields:

$$K_0 = \frac{b^2}{R_1 R_2}$$

$$K_1 = \frac{R_1 + b^2}{R_1 (R_1 - R_2)}$$

$$K_2 = \frac{b^2 + R_2}{R_2 (R_2 - R_1)} \quad .$$

We can now rewrite the integral of eg. (4-5) as

$$I = \frac{\eta_m}{2\pi}\left[K_0\int_{w_a}^{w_b}\frac{dw}{w^2} + K_1\int_{w_a}^{w_b}\frac{dw}{w^2-R_1} + K_2\int_{w_a}^{w_b}\frac{dw}{w^2 - R_2}\right] \qquad (4\text{-}7)$$

and perform each of the three indicated integrations.

$$I_0 = \int_{w_a}^{w_b}\frac{dw}{w^2} = \left[-\frac{1}{w}\right]_{w_a}^{w_b} = \frac{1}{w_a} - \frac{1}{w_b} \qquad (4\text{-}8a)$$

The latter two integrals are similar and have solutions which are derived in APPENDIX A. They are found to take one of two possible forms depending upon whether the root R_i is greater or less than 0.

For $R_i > 0$ we have from eq. (A-10)

$$I_{1,2} = \int_{w_a}^{w_b} \frac{dw}{w^2 - R_i} = \frac{1}{2\sqrt{R_i}} \ln \frac{w_b - \sqrt{R_i}}{w_b + \sqrt{R_i}} \cdot \frac{w_a + \sqrt{R_i}}{w_a - \sqrt{R_i}} \quad (4\text{-}8b)$$

and for $R_i < 0$ we have from eq. (A-13)

$$I_{1,2} = \int_{w_a}^{w_b} \frac{dw}{w^2 - R_i} = \frac{1}{\sqrt{-R_i}} \left[\mathrm{Tan}^{-1} \frac{w_b}{\sqrt{-R_i}} - \mathrm{Tan}^{-1} \frac{w_a}{\sqrt{-R_i}} \right] \quad (4\text{-}8c)$$

Combining the results of eqs. (4-8) we have for the integral of eq. (4-5)

$$I = \frac{\eta_m}{2\pi} \left(K_0 I_0 + K_1 I_1 + K_2 I_2 \right)$$

Now consider the alternative condition; the discriminant is negative and the roots are complex.

Again letting $w^2 = x$ we expand the integrand of eq. (4-5) as

$$\frac{x + b^2}{x\,(x^2 + Tx + U)} = \frac{V_1}{x} + \frac{V_2 x + V_3}{x^2 + Tx + U}$$

For purposes of uniformity which will become clear presently we have replaced D by T and let $U = K^2 b^2$. V_1, V_2 and V_3 are constants which will be evaluated by a method similar to that used for the Ks.

$$V_1 = \lim_{x \to 0} \frac{x(x+b^2)}{x(x^2 + Tx + U)} = \frac{b^2}{U} .$$

Now to evaluate V_2 and V_3, write

$$\frac{x + b^2}{x(x^2 + Tx + U)} - \frac{V_1}{x} = \frac{V_2 x + V_3}{x^2 + Tx + U} .$$

This may be rearranged as

$$\frac{x + b^2 - V_1(x^2 + Tx + U)}{x^2 + Tx + U} = \frac{x(V_2x + V_3)}{x^2 + Tx + U} .$$

Collecting like terms and equating the numerators gives

$-V_1x^2 + (1 - TV_1)x + b^2 - V_1U = V_2\ x^2 + V_3x.$

Equating coefficients of like powers of x yields

$$V_2 = -V_1 = -\frac{b^2}{U}$$

$$V_3 = 1 - TV_1 = 1 - \frac{Tb^2}{U} .$$

We can now write the integral of eq. (4-5) as

$$I = \frac{\eta_m}{2\pi}\left[V_1\int_{w_a}^{w_b}\frac{dw}{w^2} + V_3\int_{w_a}^{w_b}\frac{\frac{V_2}{V_3}\ w^2 + 1}{w^4 + Tw^2 + U}\ dw\right]$$

The first term integrates directly as

$$I_1 = \int_{w_a}^{w_b}\frac{dw}{w^2} = \frac{1}{w_b} - \frac{1}{w_a} \tag{4-9}$$

The second term was purposefully organized so that it might easily be identified with equation (A-7) of APPENDIX A with $\Gamma = V_2/V_3$ and the complex quantities G and G* as defined there are respectively $-R_1$ and $-R_2$; where R_1 and R_2 are the roots of the quadratic.

$$G = \frac{T}{2} - j\sqrt{U - \left(\frac{T}{2}\right)^2}\ , \quad G^* = \frac{T}{2} + j\sqrt{U - \left(\frac{T}{2}\right)^2}$$

and the solution for the second term is taken from eq. (A-7) as

$$I_2 = \int_{w_a}^{w_b} \frac{\frac{V_2}{V_3} w^2 + 1}{w^4 + Tw^2 + U} dw = Re\left[\frac{1 - \frac{V_2}{V_3} G}{\sqrt{-G}\,(G^*-G)} \ln \frac{w_b - \sqrt{-G}}{w_b + \sqrt{-G}} \cdot \frac{w_a + \sqrt{-G}}{w_a - \sqrt{-G}}\right] \quad (4\text{-}10)$$

Combining (4-9) and (4-10) we get for the integral of (4-5)

$$I = V_1 I_1 + V_3 I_2 \ .$$

4.3.3 - Additional Pole and No Prefilter, Test-Tone Modulation

As a result of the added pole in the loop filter, the transfer function for case (3) has a third order denominator in (jw) as given in TABLE 3-1.

$$I = \int_0^\infty \left|H(jw)\right|^2 df = \int_0^\infty \left|\frac{A\,(jw) + 1}{B(jw)^3 + C(jw)^2 + D(jw)}\right|^2 df \ . \quad (4\text{-}11)$$

Having infinite limits this integral is of the standard form

$$I_3 = \int_0^\infty \frac{c_2(jw)^2 + c_1(jw) + c_0}{d_3(jw)^3 + d_2(jw)^2 + d_1(jw) + c} dw \qquad \text{[Ref. 5, P. 21]}$$

With $c_2 = 0$, $C_1 = A$, $c_o = 1$, $d_3 = B$, $d_2 = C$, $d_1 = D$, $d_o = 0$

where in general, $I_3 = \dfrac{c_2^2 d_0 d_1 + (c_1^2 - 2c_0 c_2)\, d_0 d_3 + c_0^2 d_2 d_3}{4 d_0 d_3\, (d_1 d_2 - d_0 d_3)}$,

and for eq. (4-11)

$$I = \frac{A^2 + C^2}{4(DC-B)} ,$$

where, from TABLE 3-1,

$$A = \frac{1}{a} \quad , \quad B = \frac{1}{Kbd} \quad , \quad C = \frac{1}{Kb} + \frac{1}{Kd} \quad , \quad D = \frac{1}{a} + \frac{1}{K} \quad .$$

For the denominator of eq. (2-5) we have θ_p from the table as

$$\theta_p = \beta \sqrt{\frac{w_T^2 \left[B w_T^4 + \left(C^2 - \frac{2B}{K} \right) w_T^2 + \frac{1}{K} \right]}{B^2 w_T^6 + (C^2 - 2DB)\, w_T^4 + (D^2 - 2C)\, w_T^2 + 1}}$$

Where β is the modulation index, w_T is the Test-Tone frequency, and the constants are as above.

4.3.4 - Additional Pole and No Prefilter, Voice Modulation

The circuit configuration here is the same as that of the previous section, but the modulation is voice and therefore threshold model II is used. From the table we observe that the numerator is identical to that of the previous case and its evaluation has been performed above for eq. (4-11).

The denominator integral is

$$\int_{\omega_a}^{\omega_b} \frac{w^4 + \left(\frac{C^2}{B^2} - \frac{2}{KB}\right) w^2 + \frac{1}{B^2K^2}}{w^2\left[\ w^6 + \left(\frac{C^2}{B^2} - \frac{2D}{B}\right) w^4 + \left(\frac{D^2}{B^2} - \frac{2C}{B^2}\right) w^2 + \frac{1}{B^2}\right]}\ dw\ .$$

If we substitute: $P = \frac{C^2}{B^2} - \frac{2}{KB}$, $Q = \frac{1}{B^2K^2}$,

$L = \frac{C^2}{B^2} - \frac{2D}{B}$, $M = \frac{D^2}{B^2} - \frac{2C}{B^2}$, and x for w^2, then the

integrand becomes $I_2(x) = \dfrac{x^2 + Px + Q}{x(x^3 + Lx^2 + Mx + \frac{1}{B^2})}$, which, but for the

last constant in the denominator is identical to equation (4-24) of Sec. 4.3.9 and its integration is dealt with in that section.

4.3.5 - Ideal Differentiator With Test-Tone Modulation

In this, the fifth situation for study, the necessary predetection filter imposed a non-infinite upper limit on the numerator integral which is shown in the table as

$$I = \int_0^\infty |H(jw)|^2 df = \frac{1}{2\pi}\int_0^{\pi B_p} \frac{Aw^4 + Bw^2 + 1}{cw^4 + Dw^2 + 1}\ dw \qquad (4\text{-}12)$$

To facilitate the integration we algebraically manipulate the integrand and re-express it as a sum of recognizable, integrable terms.

First we perform the indicated division to reduce the order of the numerator polynomial and obtain

$$\left|H(jw)\right|^2 = \frac{A}{C} + \frac{\left(B - \frac{AD}{C}\right) w^2 + 1 - \frac{A}{C}}{Cw^4 + Dw^2 + 1}$$

After some re-arranging of constants this becomes

$$\frac{A}{C} + \left(\frac{C - A}{C^2}\right) \frac{\frac{BC - AD}{C - A} w^2 + 1}{w^4 + \frac{D}{C} w^2 + \frac{1}{C}} \tag{4-13}$$

Integration of the first term is straightforward.

$$I_1 = \frac{1}{2\pi} \int_0^{\pi B_p} \frac{A}{C}\, dw = \frac{A}{2\pi C} \pi B_p = \frac{A \pi B_p}{2\pi C} \tag{4-14}$$

Integration of the second term depends upon the roots of its denominator. If the roots are complex, corresponding to a negative discriminant, then this term is similar to the integrand of eq. (A-7) of

APPENDIX A with a premultiplier $\frac{C - A}{C2}$.

$$\Gamma = \frac{BC-AD}{C - A} \quad , \quad T = \frac{D}{C} \quad , \text{ and } U = \frac{1}{C} \quad .$$

As in the previous application of this solution in Section 4.2.2 we define the complex quantities

$$G = \frac{T}{2} - j\sqrt{U - \left(\frac{T}{2}\right)^2}\ , \qquad G = \frac{T}{2} + j\sqrt{U - \left(\frac{T}{2}\right)^2}\ ,$$

$$J = \sqrt{-G}$$

and use the result of eq. (A-7). For use in the right side of (A-7) we write

$$I - \Gamma G = 1 - \frac{C}{C-A}\left(B - \frac{AD}{C}\,G\right) \text{ and}$$

$$I_2 = \frac{C - A}{2\pi C2}\int_0^{\pi B_p} \frac{\left(\frac{BC-AD}{C - A}\right) w^2 + 1}{w4 + Tw2 + U}\, dw$$

$$= \frac{1}{C}\,\mathrm{Re}\left[\frac{1-BG-\frac{A}{C}(1-DG)}{J(G^* - G)}\ \ln\frac{J - \pi B_p}{J + \pi B_p}\right] \qquad (4\text{-}15)$$

Combining eqs. (4-14) and (4-15) we get for the total integral of eq. (4-12)

$$I = I_1 + I_2 = \int_0^{\infty} |H(jw)|^2\, df = \frac{1}{2\pi C}\left\{\pi AB_p + \mathrm{Re}\left[\frac{1-BG-\frac{A}{C}(1-DG)}{J(G^* - G)}\ \ln\frac{J - \pi B_p}{J + \pi B_p}\right]\right\}$$

Should the denominator of the integrand have real roots, indicated by a positive discriminant, then the second term in eq. (4-13) may be expanded as

$$\frac{\frac{C - A}{C^2}\left[\frac{BC - AD}{C - A} \; w^2 + 1\right]}{w^4 + \frac{D}{C} \; w^2 + \frac{1}{C}} = L \; \frac{MX + 1}{x^2 + Tx + U} = L\left[\frac{K_1{}'}{x - R_1} + \frac{K_2{}'}{x - R_2}\right]$$

where the following substitutions were made:

$$L = \frac{C - A}{C^2} \quad , \quad M = \frac{BC - AD}{C - A} \quad , \quad x = w^2$$

$$T = \frac{D}{C} \quad , \quad U = \frac{1}{C}$$

$$R_1 = -\frac{T}{2} + \sqrt{\left(\frac{T}{2}\right)^2 - U} \;\; , \;\; R_2 = -\frac{T}{2} - \sqrt{\left(\frac{T}{2}\right)^2 - U}$$

and the constants $K_1{}'$ and $K_2{}'$ are found from

$$K_i{}' = \lim_{x \to R_i} \frac{(Mx + 1)\,(x - R_i)}{(x - R_1)(x - R_2)} \; . \qquad \text{Thus}$$

$$K_1{}' = \frac{MR_1 + 1}{R_1 - R_2} \quad , \quad K_2{}' = \frac{MR_2 - 1}{R_2 - R_1}$$

Letting $K_1 = LK_1{}'$ and $K_2 = LK_2{}'$ we can write the entire second term of the integral as

$$I_2 = \frac{K_1}{2} \int_0^{\pi B_p} \frac{dw}{w^2 - R_1} + \frac{K_2}{2} \int_0^{\pi B_p} \frac{dw}{w^2 - R_2} \; .$$

Each of these two terms is integrated by the results of section A.2 of APPENDIX A.

If $Ri > 0$, then from eq. (A-9)

$$\frac{K_i}{2\pi}\int_0^{\pi B_p}\frac{dw}{w^2 - R_i} = \frac{K_i}{4\pi\sqrt{R_i}}\ \ln\frac{\sqrt{R_i} - \pi B_p}{\sqrt{R_i} + \pi B_p}$$

and if $R_i < 0$ then from eq. (A-12)

$$\frac{K_i}{2\pi}\int_0^{\pi B_p}\frac{dw}{w^2 - R_i} = \frac{K_i}{2\pi\sqrt{-R_i}}\ \mathrm{Tan}^{-1}\frac{\pi B_p}{\sqrt{-R_i}}$$

To construct the denominator for eq. (2-5) in this instance we simply substitute the appropriate constants into the expression for θ_p in TABLE 3-1.

$$\theta_p = \beta\sqrt{\frac{E^2 w_T^{\ 4} + F^2 w_T^{\ 2}}{(1 - Gw_T^{\ 2})^2 + H^2 w_T^{\ 2}}}\ .$$

Again β is the modulation index and w_T, the Test-Tone frequency. The constants A through H are expressed in the table in terms of the loop parameters a, b, K, and α .

4.3.6 - <u>Ideal Differentiator With Voice Modulation.</u>

Because case (6) involves voice modulation we use threshold model II and must minimize

$$(CNR)_{TH} = \frac{\int_0^{\infty}\left|H(jw)\right|^2\ df}{B_p\ [\nu - \int_0^{\infty}\phi_i\ (f)\ \left|1 - H(jw)\right|^2\ df]}\ .$$

But the filter is the same as that for the previous case and therefore the numerators are identical as shown in the table. Our only remaining task, therefore, is to carry out the integration in the denominator which is given in the table as

$$I = \frac{\eta_m}{2\pi} \int_{w_a}^{w_b} \frac{E^2 w^2 + F^2}{w^2 (Cw^4 + Dw^2 + 1)} \, dw \qquad (4\text{-}17)$$

This is similar to the integration for case (2) which was carried out in section 4.2.2; differing only with respect to the constants. To facilitate the integration we make the following substitutions:

$$E' = \frac{E^2}{C} \;, \quad F' = \frac{F^2}{C} \;, \quad T = \frac{D}{C} \;, \quad U = \frac{1}{C} \;, \text{ and } x = w^2 \,.$$

Then the integrand may be conveniently rewritten as

$$\frac{E'x + F'}{x\,(x^2 + Tx + U)} \quad .$$

Once again we treat separately the two likely possibilities for the roots of the quadratic factor.

If $T^2 > 4U$ then the roots are real and the integrand may be expanded into three partial fraction terms as

$$\frac{E'x + F'}{x\,(x^2 + Tx + U)} = \frac{E'x + F'}{x\,(x-R_2)(x-R_3)} = \frac{K_1}{x} + \frac{K_2}{x-R_2} + \frac{K_3}{x-R_3} \qquad (4\text{-}18)$$

$$R_2 = -\frac{T}{2} + \sqrt{\frac{T^2}{4} - U}$$

$$R_3 = -\frac{T}{2} - \sqrt{\frac{T^2}{4} - U}$$

and the constants are evaluated from

$$K_i = \lim_{x \to R_i} \frac{(x-R_i)(E'x+F')}{x\ (x-R_2)(x-R_3)} \qquad \text{for } i = 1, 2, 3 \text{ and } R_1 = 0$$

Thus:

$$K_1 = \lim_{x \to 0} \frac{x(E'x + F')}{x\ (x^2 + Tx + U)} = \frac{F'}{U}$$

$$K_2 = \lim_{x \to R_2} \frac{(x-R_2)(E'x + F')}{x(x-R_2)(x-R_3)} = \frac{E'R_2 + F'}{R_2(R_2-R_3)}$$

$$K_3 = \lim_{x \to R_3} \frac{(x-R_3)(E'x + F')}{x(x-R_2)(x-R_3)} = \frac{E'R_3 + F'}{R_3(R_3-R_2)}$$

$$\text{and } I = \frac{\eta_m}{2\pi} \left[K_1 \int_{w_a}^{w_b} \frac{dw}{w^2} + K_2 \int_{w_a}^{w_b} \frac{dw}{w^2-R_1} + K_3 \int_{w_a}^{w_b} \frac{dw}{w^2-R_2} \right]$$

This is identical to eq. (4.7) and the solutions are given by eq. (4-8).

The first term, $K_1 I_1 = K_1 \int_{w_a}^{w_b} \frac{dw}{w^2} = K_1 \left(\frac{1}{w_b} - \frac{1}{w_a} \right)$ (4-19)

The next two terms for i = 2, 3 are:

$$K_i I_i = K_i \int_{w_a}^{w_b} \frac{dw}{w^2-R_i} = \frac{1}{2\sqrt{R_i}} \ln \frac{w_b - \sqrt{R_i}}{w_b + \sqrt{R_i}} \cdot \frac{w_a + \sqrt{R_i}}{w_a - \sqrt{R_i}} \quad \text{for } R_i > 0$$

$$= \frac{1}{\sqrt{-R_i}} \quad \mathrm{Tan}^{-1} \frac{w_b}{\sqrt{-R_i}} - \mathrm{Tan}^{-1} \frac{w_a}{\sqrt{-R_i}} \quad \text{for } R_i < 0$$

and $I = \frac{\eta_m}{2\pi}(K_1 I_1 + K_2 I_2 + K_3 I_3)$

On the other hand if $T^2 < 4U$ the expansion takes the form

$$\frac{E'x + F'}{x(x^2 + Tx + U)} = \frac{V_1}{x} + \frac{V_6 x + V_7}{x^2 + Tx + U}$$

Combining the right side over a common denominator and equating its numerator to that of the left side we obtain an equation that can be solved for the constants.

$$(V_1 + V_6)x^2 + (V_1 T + V_7)x + V_1 U = E'x + F'$$

leads to:

$V_1 + V_6 = 0$

$V_1 T + V_7 = E$

$V_1 U = F,$

from which

$$V_1 = \frac{F'}{U} = \frac{1}{K^2}$$

$$V_6 = -V_1 = -\frac{F'}{U} = -\frac{1}{K^2}$$

$$V_7 = E' - V_1 T = E' - \frac{T}{K^2}$$

Eq. (4-17) takes the form

$$I = \frac{\eta_m}{2\pi}\int_{w_a}^{w_b} \frac{V_1\, dw}{w^2} + \int_{w_a}^{w_b} \frac{V_6 w^2 + V_7}{w^4 + Tw^2 + U}\, dw$$

The first term is identical to the first term in eq. (4-18) including the constant V_1 which is the same as K_1. Its integral is thus given by eq. (4-19). The second term is similar to eq. (4-10) which was integrated by application of eq. (A-7).

We have for the complete integral of eq. (4-17) in this instance:

$$I = \int_0^{\infty} \phi_i \left|1 - H(jw)\right|^2 df$$

$$= \frac{\eta_m}{2\pi}\left\{V_1\left(\frac{1}{w_b} - \frac{1}{w_a}\right) + V_7\left[\frac{1-\frac{V_6}{V_7}G}{J(G^*-G)}\left(\ln\frac{w_b-J}{w_b+J} - \ln\frac{w_a-J}{w_a+J}\right)\right]\right\}$$

where G, G* and J are as previously defined in 4.3.5.

4.3.7 - Differentiator With Pole Added, Test-Tone Modulation, No Predetection Filter.

Although case (7) incorporates the most complex of the loop filters, the other system conditions are such that the threshold calculations are relatively simple. The absence of a predetection filter implies an unbounded input noise spectrum and the simpler numerator integration. Test-tone modulation requires the use of threshold model II of Table 3-1 and no integration in the denominator of the threshold calculation. From the table the numerator is

$$\int_0^{\infty}\left|H(jw)\right|^2 df = \int_0^{\infty}\left|\frac{A(jw)^2 + B(jw) + C}{(jw)^3 + D(jw)^2 + E(jw) + C}\right|^2 df \quad .$$

which is of the form

$$\int_0^\infty \left| \frac{c_2(jw)^2 + c_1(jw) + c_o}{d_3(jw)^3 + d_2(jw)^2 + d_1(jw) + d_o} \right|^2 df = \frac{c_2^{\,2} d_o d_1 + (c_1^{\,2} - 2c_o c_2) d_2 d_3 + c_o^{\,2} d_2 d_3}{4 d_o d_3 (d_1 d_2 - d_o d_3)}$$

with $c_2 = A$, $c_1 = B$, $c_o = C$

$d_3 = 1$, $d_2 = D$, $d_1 = E$, $d_o = 0$ [Ref. 5 P. 21]

$$\int_0^\infty \left| H(jw) \right|^2 df = \frac{A^2EC + (B^2 - 2CA) + C^2D}{4C\ (ED-C)} \quad .$$

From section 3.1.3 and the table we have for the peak signal induced error phase

$$\theta_p = \beta \sqrt{\frac{w_T^{\,6} + [(A-D)^2 - 2(E-B)]w^4 + (E-B)^2\ w_T^{\,2}}{w_T^{\,6} + (D^2-2E)\ w_T^{\,4} + (E-2CD)\ w_T^{\,2} + C^2}} \quad .$$

In these expressions and in those of the next three sections the constants are related to the five system parameters as follows:

$$A = \frac{Kb}{a} + \alpha d$$

$$B = \frac{Kbd}{a} + Kb + \alpha bd$$

$$C = Kbd$$

$$D = d + b + \frac{Kb}{a} + \alpha d$$

$$E = bd + \frac{Kbd}{a} + Kb + \alpha bd$$

4.3.8 - Differentiator with Pole, Predetection Filter, Test-Tone Modulation

With the predetection filter in place the evaluation of $\int_0^\infty |H(jw)|^2 df$ becomes more complicated than for the previous case, even though the same closed loop response is involved. We assume an ideal rectangular predetection filter response referred to the baseband as

$$H_f\ (f) = 1\ ;\quad 0 \leqslant f < \frac{B_p}{2}$$

$$H_f\ (f) = 0\ ;\quad \text{elsewhere}$$

and therefore require

$$\int_0^{B_P/2} \left| H(jw) \right|^2 df$$

which is from TABLE 3-1

$$\frac{1}{2\pi} \int_0^{\pi B_p} \frac{A^2w^4 + (B^2-2AC)\ w^2 + C^2}{w^6 + (D^2-2E)\ w^4 + (E^2 - 2CD)\ w^2 + C^2}\ dw\ . \qquad (4\text{-}20)$$

An integrand of this type has not yet been dealt with because the denominator polynomial is of sixth order in w. Actually it is a third order in w^2 and for notational simplification we make the following substitutions:

Let $B^2 - 2AC = J$, $D^2 - 2E = L$, $E^2 - 2CD = M$, and $w^2 = x$.

Then

$$\left| H(jw) \right|^2 = \frac{A^2x^2 + Jx + C^2}{x^3 + Lx^2 + Mx + C^2} \qquad (4\text{-}21)$$

The denominator is a cubic in x^2 with real coefficients and therefore has at least one real root, R_1. It can be written in factored form as $(x-R_1)(x^2 + Tx + U)$ and the quadratic factor will have a pair of real equal roots, real unequal roots, or complex conjugate roots depending upon the discriminant $T^2 - 4U$ being respectively equal to, greater than, or less than zero.

Let $y = x^3 + Lx^2 + Mx + C$

then $y' = 3x^2 + 2Lx + M$

The first, real root, is found by a Newton Raphson [Ref. 6, P. 34] iteration process. An arbitrary value, x_0 is selected, for which

$$y_0 = x_0^3 + Lx_0^2 + Mx_0 + C \text{ and } y_0' = \left.\frac{dy}{dx}\right|_{x = x_0}$$

$= 3x_0^2 + 2Lx_0 + M$ are calculated.

A new value x_1 is then computed as

$$x_1 = x_0 - \frac{y_0}{y_0'}$$ and the process is repeated. The search

is ended when $|y_i| < \epsilon\left(|x_i^3| + |Lx_i^2| + |Mx_i| + C^2\right)$ where ϵ is an arbitrarily small number, and then x_i is taken as R_1. T and U are determined by equating

$$x^3 + Lx^2 + Mx + C^2 = (x-R_1)(x^2 + Tx + U)$$

and expanding the right side to get

$$x^3 + Lx^2 + Mx + C^2 = x^3 + (T - R_1)x^2 + (U - TR_1)x - R_1U.$$

By equating coefficients of like powers of x we get:

$$T - R_1 = L$$

$$U - TR_1 = M$$

$$- R_1 U = c^2 \ ,$$

from which

$$T = L + R_1$$

$$U = M + TR_1 \quad \text{or} \quad U = - \frac{C}{R_1} \quad .$$

To obtain (4-20) in closed form we consider each of the two likely conditions. First, suppose the discriminant, $T^2 - 4U > 0$. Then eq. (4-21) becomes

$$\frac{A^2x^2 + Jx + C^2}{x^3 + Lx^2 + Mx + C^2} = \frac{A^2x^2 + Jx + c^2}{(x-R_1)(x-R_2)(x-R_3)} = \frac{K_1}{x-R_1} + \frac{K_2}{x-R_2} + \frac{K_3}{x-R_3} \quad .$$

where $R_2 = - \dfrac{T}{2} + \sqrt{\left(\dfrac{T}{2}\right)^2 - U}$

and $R_3 = - \dfrac{T}{2} - \sqrt{\left(\dfrac{T}{2}\right)^2 - U} \quad .$

The three constants are calculated from

$$K_i = \lim_{x \to R_i} (x-R_i) \quad I(x) = \frac{A^2R_i^{\,2} + JR_i + c^2}{(x-R_j)(x-R_K)} \qquad \begin{matrix} i,j,K = 1,2,3 \\ j \neq K \neq i \end{matrix}$$

and found to be:

$$K_1 = \frac{A^2R_1^{\,2} + JR_1 + c^2}{(R_1-R_2)(R_1-R_3)}$$

$$K_2 = \frac{A^2R_2^{\,2} + JR_2 + c^2}{(R_2-R_1)(R_2-R_3)}$$

$$K_3 = \frac{A^2R_3{}^2 + JR_3 + C^2}{(R_3-R_1)(R_3-R_2)} \quad .$$

Eq. (4-20) can now be evaluated in closed form.

$$I = \frac{1}{2\pi} \sum_{i=1}^{3} K_i \int_0^{\pi B_p} \frac{dw}{w^2-R_i} \tag{4-22}$$

$$\text{where} \int_0^{\pi B_p} \frac{dw}{w^2-R_i} = \frac{1}{2\sqrt{R_i}} \ln \frac{\sqrt{R_i} - B_p}{\sqrt{R_i} + B_p} \quad \text{for } R_i > 0$$

$$\text{and} \int_0^{\pi B_p} \frac{dw}{w^2-R_i} = \frac{1}{\sqrt{-R_i}} \text{Tan}^{-1} \frac{B_p}{\sqrt{-R_i}} \quad \text{for } R_i < 0.$$

We note in passing that there will always be an odd number of negative roots (i.e. 1 or 3), for upon expansion of the denominator product

$\prod_{i=1}^{3} (x-R_i)$ the constant term is $- R_1R_2R_3$.

But the constant term in the denominator of I(x) is C^2, a positive number.

Next we turn to the case of complex roots resulting from $T^2 < 4U$. Eq. (4-21) must now be expanded as

$$\frac{A^2x^2 + Jx + C^2}{x^3 + Lx^2 + Mx + C^2} = \frac{A^2x^2 + Jx + C^2}{(x-R_1)(x^2 + Tx + U)} = \frac{V_1}{x-R_1} + \frac{V_2x + V_3}{x^2 + Tx + U}$$

By methods similar to those employed in section 4.3.2 we evaluate the three constants. Thus,

$$V_1 = \lim_{x \to R_1} \frac{(A^2x^2 + Jx + C^2)(x-R_1)}{(x-R_1)(x^2 + Tx + U)} = \frac{A^2R_1^{\,2} + JR_1 + C^2}{R_1^{\,2} + TR_1 + U} .$$

Then, from $\dfrac{A^2x^2 + Jx + C^2}{(x-R_1)(x^2+Tx+U)} - \dfrac{V_1}{x-R_1} = \dfrac{V_2x + V_3}{x^2 + Tx + U}$

$$V_2 = \frac{A^2U - JR_1 - C^2 + R_1TA^2}{R_1^{\,2} + TR_1 + U}$$

$$V_3 = \frac{JU - R_1c^2 + A^2UR_1 - TC^2}{R_1^{\,2} + TR_1 + U}$$

Note that the residue of the real pole, V_1, is the same as K_1 .

The integral of the first term is identical to that for the case of all real roots and is given by eq. (4-22) with $i = 1$.

The second term may be written as

$$I_2 = \frac{V_3}{2\pi} \int_0^{\pi B_p} \frac{\frac{V_2}{V_3} w^2 + 1}{w^4 + Tw^2 + U} \, dw$$ which is

recognized as eq. (A-7) with $\Gamma = \dfrac{V_2}{V_3}$, and premultiplied by $\dfrac{V_3}{2\pi}$. Upon substituting the indicated limits we get

$$I_2 = \frac{V_3}{2\pi} \quad \mathrm{Re} \left\{ \frac{1 - \frac{V_2}{V_3} G}{J\ (G^* - G)} \left[\ln \frac{\pi B_p - J}{\pi B_p + J} - \ln (-1) \right] \right\}$$

$$= \frac{V_3}{2\pi} \quad \mathrm{Re} \left[\frac{1 - \frac{V_2}{V_3} G}{J\ (G^* - G)} \quad \ln \frac{J - \pi B_p}{J + \pi B_p} \right] \tag{4-23}$$

where, as before, $G = \frac{T}{2} - j \sqrt{U - \left(\frac{T}{2}\right)^2}$

and $J = \sqrt{G}$.

To complete the expression for $(CNR)_{TH}$ we need the peak signal related error phase which is presented in TABLE 3-1.

$$\theta_p = \frac{\eta_m}{2\pi} \sqrt{\frac{W_T^6 + [(A-D)^2 - 2(E-B)]\ W_T^4 + (E-B)^2\ W_T^2}{W_T^6 + (D^2 - 2E)\ W_T^4 + (E^2 - 2CD)\ W_T^2 + C^2}}$$

4.3.9 - Differentiator With Pole, No Predetection Filter, Voice Modulation.

From the entries in TABLE 3-1 it is apparent that the numerator for case (9) is identical to that for case (7) which has been evaluated in section 4.2.7. Thus, here too

$$\int_0^{\infty} \left| H\ (jw) \right|^2 df = \frac{A^2 ED + (B^2 - 2CA) + C^2 D}{4C\ (ED - C)} \quad .$$

The constants A through E are specified in that section as well as in the table.

For the denominator we must seek a closed form solution for

$$I = \frac{\eta_m}{2\pi} \int_{\omega_a}^{\omega_b} \frac{w^4 + [(A-D)^2 - 2\,(E-B)]\,w^2 + (E-B)^2}{w^2\,[w^6 + (D^2-2E)\,w^4 + (E^2 - 2CD)\,w^2 + C^2]}\,dw \tag{4-24}$$

To simplify the writing of the above integrand we define the following new constants:

$L = D^2 - 2E$, $M = E^2 - 2CD$

$P = (A-D)^2 - 2(E-B)$, $Q = (E-B)^2$

The integrand can now be written

$$I_2 = \frac{w^4 + Pw^2 + Q}{w^2\,(w^6 + Lw^4 + Mw^2 + C^2)}$$

, a ratio of polynomials of even powers of w. Replace w^2 by x and get

$$I_2(x) = \frac{x^2 + Px + Q}{x(x^3 + Lx^2 + Mx + C^2)} \tag{4-25}$$

We write the denominator in factored form and expand $I_2(x)$ by partial fractions. There are four factors here, including x. The cubic is handled the same way as for the numerator integral in Section 4.2.8. In fact (see the table) it is the same cubic and the roots are determined by the same process. First, the real root is found by Newton-Raphson, and then the quadratic quotient is factored by the quadratic formula yielding either a pair of real roots or a pair of complex roots.

In the former case, the integrand is expanded as

$$I_2(x) = \frac{x^2 + Px + Q}{x(x-R_1)(x-R_2)(x-R_3)} = \frac{K_0}{x} + \frac{K_1}{x-R_1} + \frac{K_2}{x-R_2} + \frac{K_3}{x-R_3}$$

As in previous cases, we determine the K_i from

$$K_i = \lim_{x \to R_i} (x-R_i)I \quad \text{for } i = 0 \text{ to } 3 \tag{4-26}$$

and $R_0 = 0$

Thus $$K_0 = \lim_{x \to 0} x\, I_2(x) = \left.\frac{x^2 + Px + Q}{(x-R_1)(x-R_2)(x-R_3)}\right|_{x=0} = \frac{Q}{-R_1R_2R_3} = \frac{Q}{C^2}$$

$$K_1 = \lim_{x \to R_1} (x-R_1)I_2(x) = \left.\frac{x^2 + Px + Q}{x(x-R_1)(x-R_2)}\right|_{x=R_1} = \frac{R_1^2 + PR_1 + Q}{R_1(R_1-R_2)(R_1-R_3)}$$

$$K_2 = \lim_{x \to R_2} (x-R_2)I_2(x) = \left.\frac{x^2 + Px + Q}{x(x-R_1)(x-R_3)}\right|_{x=R_2} = \frac{R_2^2 + PR_2 + Q}{R_2(R_2-R_1)(R_2-R_3)}$$

$$K_3 = \lim_{x \to R_3} (x-R_3)I_2(x) = \left.\frac{x^2 + Px + Q}{x(x-R_1)(x-R_3)}\right|_{x=R_3} = \frac{R_3^2 + PR_3 + Q}{R_3(R_3-R_1)(R_3-R_2)}$$

In these expressions $$R_2 = -\frac{T}{2} + \sqrt{\left(\frac{T}{2}\right)^2 - U}$$

and $$R_1 = -\frac{T}{2} - \sqrt{\left(\frac{T}{2}\right)^2 - U}$$

Eq. (4-24), when expanded, takes the form

$$I = \frac{\eta_m}{2\pi} \left[K_0 \int_{w_a}^{w_b} \frac{dw}{w^2} + K_1 \int_{w_a}^{w_b} \frac{dw}{w^2 - R_1} + K_2 \int_{w_a}^{w_b} \frac{dw}{w^2 - R_2} + K_3 \int_{w_a}^{w_b} \frac{dw}{w^2 - R_3} \right]$$

$$= \frac{\eta_m}{2\pi} \sum_{i=0}^{3} K_i \int_{w_a}^{w_b} \frac{dw}{w^2 - R_i} \qquad \text{and } R_0 = 0 \qquad (4\text{-}27)$$

$$\text{For } i = 0 \; , \quad \int_{w_a}^{w_b} \frac{K_0}{w^2}\, dw = K_0 \left(\frac{1}{w_a} - \frac{1}{w_b} \right)$$

and for i = 1, 2, or 3, from (A-10) and (A-13)

$$\int_{w_a}^{w_b} \frac{K_i}{w^2 - R_i}\, dw = \frac{K_i}{2\sqrt{R_i}} \ln \left[\frac{w_b - \sqrt{R_i}}{w_b + \sqrt{R_i}} \cdot \frac{w_a + \sqrt{R_i}}{w_a - \sqrt{R_i}} \right] \qquad \text{for } R_i > 0$$

$$\int_{w_a}^{w_b} \frac{K_i}{w^2 - R_i}\, dw = \frac{K_i}{\sqrt{-R_i}} \left[\operatorname{Tan}^{-1} \frac{w_b}{\sqrt{-R_i}} - \operatorname{Tan}^{-1} \frac{w_a}{\sqrt{-R_i}} \right] \qquad \text{for } R_i < 0$$

In the alternative situation, when $T^2 < 4U$, the integrand must be expanded as

$$I_2(x) = \frac{x^2 + Px + Q}{x(x-R_1)(x-R_2)(x-R_3)} = \frac{V_0}{x} + \frac{V_1}{x-R_1} + \frac{V_2 x + V_3}{x^2 + Tx + U} \qquad (4\text{-}28)$$

and once again there are four constants to be evaluated.

By applying eq. (4-26) to (4-28) we get

$$V_0 = \lim_{x \to 0} \frac{x^2 + Px + Q}{(x-R_1)(x-R_2)(x-R_3)} = - \frac{Q}{R_1 R_2 R_3} = - \frac{Q}{U R_1} = \frac{Q}{C^2} \qquad (4\text{-}29)$$

and

$$V_1 = \lim_{x \to R_1} \frac{x^2 + Px + Q}{x(x-R_2)(x-R_3)} = \frac{R_1{}^2 + PR_1 + Q}{R_1(R_1-R_2)(R_1-R_3)} = \frac{R_1{}^2+R_1P+Q}{R_1(R_1{}^2+U+TR_1)} .$$

Then, from eq. (4-28)

$$\frac{x^2 + Px + Q}{x(x-R_1)(x^2+Tx+U)} - \frac{V_0}{x} - \frac{V_1}{x-R_1} = \frac{V_2x + V_3}{x^2 + Tx + U} .$$

Combining the left side, we get

$$\frac{x^2+Px+Q-(x-R_1)(x^2+Tx+U)V_0-V_1x(x^2+Tx+U)}{x^2 + Tx + U} = \frac{V_2X + V_3}{x^2 + Tx + U} . \qquad (4\text{-}30)$$

After rearranging eq. (4-29) and collecting terms we have

$$\frac{-x^3(V_0+V_1)+x^2(V_0R_1+1-TV_0-TV_1)+x(P+TV_0R_1-UV_0-UV_1)+Q+UV_0R_1}{x^2 + Tx + U}$$

$$= \frac{V_2x^3 + x^2(V_3-V_2R_1) - V_3R_1x}{x^2 + Tx + u} . \qquad (4\text{-}31)$$

V_2 and V_3 are extracted by equating coefficients of like terms in the numerators of eq. (4-31).

Note that with the second form in eq. (4-29), the constant term on the left of (4-31) is zero, as it should be.

$$V_2 = -(V_0+V_1) = \frac{Q}{UR_1} - \frac{R_1P + R_1{}^2 + Q}{R_1\ (R_1{}^2+U+TR_1)}$$

$$= \frac{QR_1 + TQ - UP - UR_1}{U(R_1{}^2 + U + TR_1} .$$

Also $V_3R_1 = U(V_0+V_1)-P-TV_0R_1 = \dfrac{U(R_1P+R_1{}^2+Q) - Q(R_1{}^2+U+TR_1)}{R_1(R_1{}^2 + U + TR_1)}$

$$-P + \frac{QT}{U}$$

After some algebraic manipulation we get

$$V_3 = \frac{U^2 - PU(R_1 + T) + Q(TR_1 + T^2 - U)}{U(R_1{}^2 + U + TR_1)}$$

Not surprisingly V_0 and V_1 are the same as K_0 and K_1.

For this condition, eq. (4-24) takes the form

$$I = \frac{\eta_m}{2\pi}\int_{w_a}^{w_b}\left(\frac{V_0}{w^2} + \frac{V_1}{w^2-R_1} + \frac{V_2w^2+V_3}{w^4+Tw^2+U}\right) dw \text{ ,}$$

The first two terms are identical to those for the case of a positive discriminant and have been evaluated in that context for i = 0 and 1.

The last term is identified as eq. (A-7) with $\Gamma = \dfrac{V_2}{V_3}$

and integrates as

$$V_3\frac{\eta_m}{2\pi}\int_{w_a}^{w_b}\frac{\frac{V_2}{V_3} + 1}{w^4+Tw^2+U}\ dw =$$

$$\frac{V_3 \eta_m}{2\pi} \operatorname{Re}\left[\frac{1 - \frac{V_2}{V_3} G}{J\,(G^* - G)} \ln\left(\frac{w_b - J}{w_b + J} \cdot \frac{w_a + J}{w_a - J}\right)\right] \qquad (4\text{-}32)$$

where the complex numbers G and J are

$$G = \frac{T}{2} - j\sqrt{U - \left(\frac{T}{2}\right)^2} \quad \text{and} \quad J = \sqrt{G}$$

4.3.10 - Differentiator With Pole, Predetection Filter, Voice Modulation.

This is the last case to be considered. From the relevant entries in Table 3-1 we observe that both required integrations have already been dealt with in separate previous cases. The numerator integral is identical to that for case (8) and was evaluated in closed form as either eq. (4-22) for all terms having real poles or as eq. (4-23) for those having complex poles.

The denominator integral is identical to that for case (9) and was evaluated immediately above as eq. (4-27) for the partial fraction terms having real poles and as eq. (4-30) for the complex-pair poles.

This completes the preparation of analytical expressions to be minimized.

4.4 - Implementation of the Optimization.

For each of the ten cases under study a FORTRAN program was written to search for and to identify the set of loop parameters for which the $(CNR)_{TH}$ as formulated in chapters 2 and 3 is a minimum. Although the various programs are all based on the same search algorithim, they nevertheless differ from one another in several respects. Depending upon the loop filter involved, the dimension of the search space can be three, four, or five as specified in section 4.1. In several instances it proved necessary to hold one variable fixed and to proceed with the search in N-1 dimensions. This will be described presently. And, of course, in each case the function to be minimized is unique. APPENDIX B contains the listings of all the programs that were used in this study. Among them are the optimization procedures for the ten systems. Each of these is identified by case number and a statement setting forth the salient features of the system.

Before executing the optimization for each system, we ran a test of the closed form integral for that case as developed in Sections 4.2.1 through 4.2.10.

For comparison with the closed form of the integral, a numerical integration was performed by a rectangular method [Ref. 6, Page 138]. The domain of the variable of integration, w, is partitioned into a large number of small intervals and all of the incremental areas under the curve of the function are summed. The functions for numerical integration were written in terms of the original system parameters so that agreement between the values obtained by the two methods established the veracity of all the various substitutions and determinations of residues at the poles, as well as of the closed forms. A sample FORTRAN routine for the numerical integration is included in APPENDIX B. The agreement between the two values obtained was quite good, often extending to six or seven significant figures.

It was necessary to perform the test integrations in different areas of parameter space so that the different closed forms corresponding to complex, real positive, and real negative roots could be subjected to the verification test procedure.

The method of numerical testing also proved to be useful as a troubleshooting tool during the course of developing the programs.

Wherever the modulation waveform is a test-tone, a 1 KHz tone frequency is assumed and the modulation index is taken as 10 (i.e., peak frequency deviation is 10 KHz). The predetection filter is assumed to be a rectangular band pass filter of 35 KHz bandwidth. Where the modulation is voice, the model of Section 2.3 is used with lower and upper cutoff frequencies of 300 Hz and 3300 Hz respectively. These values correspond to those used by Novick [Ref. 3] and by Acampara and Newton [Ref. 7, Page 586].

4.4.1 - Results of the Optimization

CASE 1 - Standard System with no Prefilter and Test-Tone Modulation.

In this as in all of the cases, the search does not result in a single global minimum. Rather, the terminus point of the optimization process depends upon the initial point. However, the functions are such that the threshold levels are almost identical for all of the local minima. TABLE 4-1 presents a sample of local minima for the standard second order loop having a lag-lead loop filter with transfer function $F(S) = \frac{s/a + 1}{s/b + 1}$ and loop gain, K.

a (Rad/Sec)	b (Rad/Sec)	K (sec^{-1})	$(CNR)_{TH}$	Kb
3.77×10^4	1.96×10^3	6.70×10^5	3.09 dB	1.31×10^9
3.81×10^4	2.35×10^3	5.58×10^5	3.09 dB	1.31×10^9
3.83×10^4	2.58×10^3	5.08×10^5	3.09 dB	1.31×10^9
3.85×10^4	2.83×10^3	4.63×10^5	3.10 dB	1.31×10^9
3.88×10^4	3.22×10^3	4.05×10^5	3.11 dB	1.31×10^9

TABLE 4-1 - LOCAL MINIMA FOR CASE (1)

The product, Kb, is included as the last column of the table. Note that although the pole location, b, varies by more than 50% over the sample, the product, Kb, remains constant. This accords well with the analytical results obtained by Klapper and Frankle [Ref. 5, Page 139] in their optimization of this loop. They find that for a $\ll$ K, the noise bandwidth is a function of a and Kb only.

Thus $B_n = \left(\frac{Kb}{a} + a\right) /4.$

Upon minimizing the noise by letting $\frac{\partial B_n}{\partial a} = 0$ they get $a = \sqrt{Kb} = w_n$.

From the table, $\sqrt{Kb} = \sqrt{13.1 \times 10^8} = 3.62 \times 10^4 \approx a$. The approximation is accurate to better than 7% for all the entries in the table. The damping factor was identified in eq (2-11) as

$$\zeta = \frac{1}{2}\left(\frac{b}{K}\right)^{1/2} + \frac{1}{2a}(bK)^{1/2} \quad .$$

For $b \ll K$, $\zeta = \frac{1}{2}\frac{(bK)^{1/2}}{a} = 0.5$

Closed loop amplitude response curves for all of the optimized systems are presented in this section. The programs for calculaing these curves are called "RESPONSE" and are included in APPENDIX B.

Figure 4-1 is the response for the optimization of case (1). It is that of a second order type 1 system with $w_n = 3.62 \times 10^4$ and $\zeta = 1/2$.

The minimum $(CNR)_{TH}$ is 4.9 dB.

CASE 2 - Standard System with Voice Modulation.

Using the same loop filter configuration and optimizing for the voice modulation model we arrived at:

$a = 24000$ Rad/Sec

$b = 3655$ Rad/Sec

$K = 170{,}000$ Sec^{-1}

and $(CNR)_{TH}$ minimum = 2.1 dB.

For this system $W_n = \sqrt{Kb} = 2.5 \times 10^4$ Rad/Sec which is again approximately equal to a.

Figure 4-2 is the plot of the closed loop response for this set of system parameters.

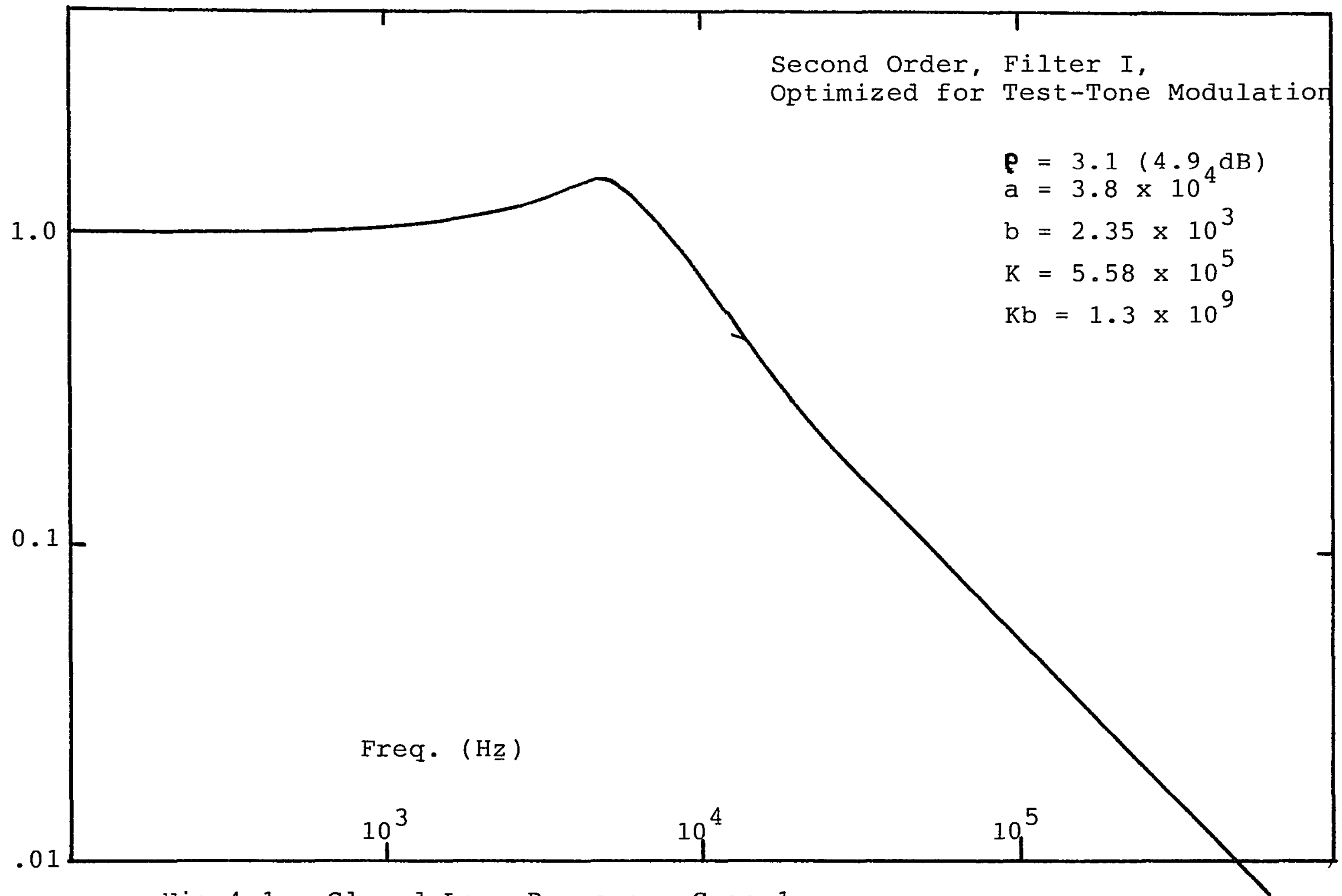


Fig 4-1   Closed Loop Response, Case 1.

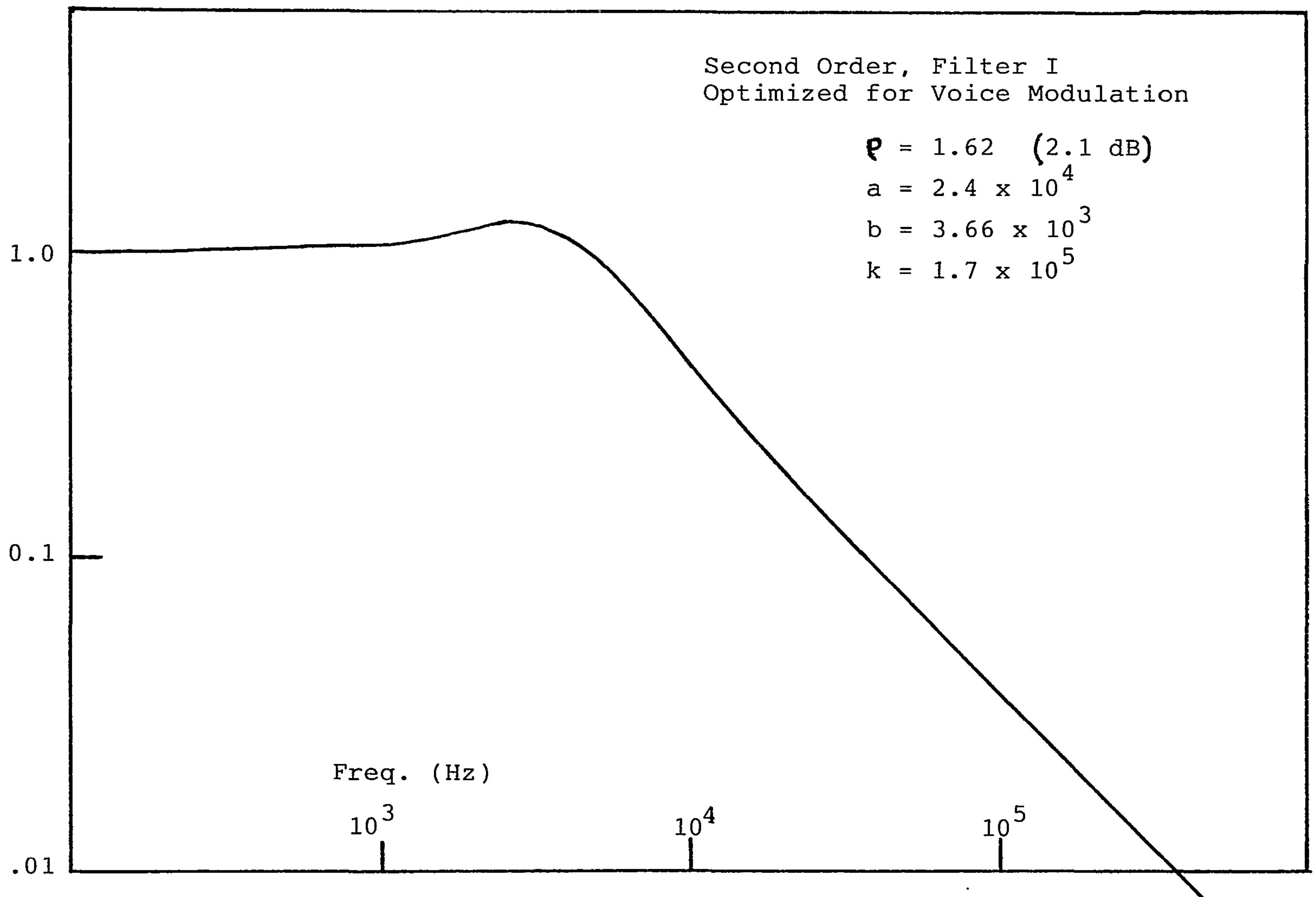


Fig. 4-2. Closed Loop Response, Case 2

These two basic systems provide standards against which the performances of the more complex systems may be compared.

CASE 3

An additional pole in the loop filter proves to be of no advantage with respect to threshold reduction. The search procedure had to be modified because as originally written, the program moves the pole out towards infinity and fails to locate a minimum. We overcame the difficulty by fixing the pole location and allowing the search to proceed in three rather than in four dimensions. As the pole is moved further out, the threshold decreases and asymptotically approaches the optimum that was found for case (1).

Figure 4-3 presents threshold CNR as a function of pole location. For d $>10^6$ Rad/Sec (i.e. beyond about 150 KHz) the threshold is within 0.1 dB of the minimum and for practical purposes the loop may be considered to have been optimized.

In Figure 4-4 the closed loop responses for several optimized systems are plotted. Each curve is based upon a different fixed value of d. For sufficiently large d, the response is indistinguishable from that resulting from the optimization of case (1). Clearly, no threshold reduction can be achieved by adding the additional pole. But should a higher frequency pole be present as a result of strays, an optimum system can be designed around it. Furthermore, if the pole frequency is sufficiently high then the threshold will be only slightly higher than for the ideal situation.

CASE 4

Just as for the previous case involving test tone modulation, we find that the additional pole does not contribute to threshold

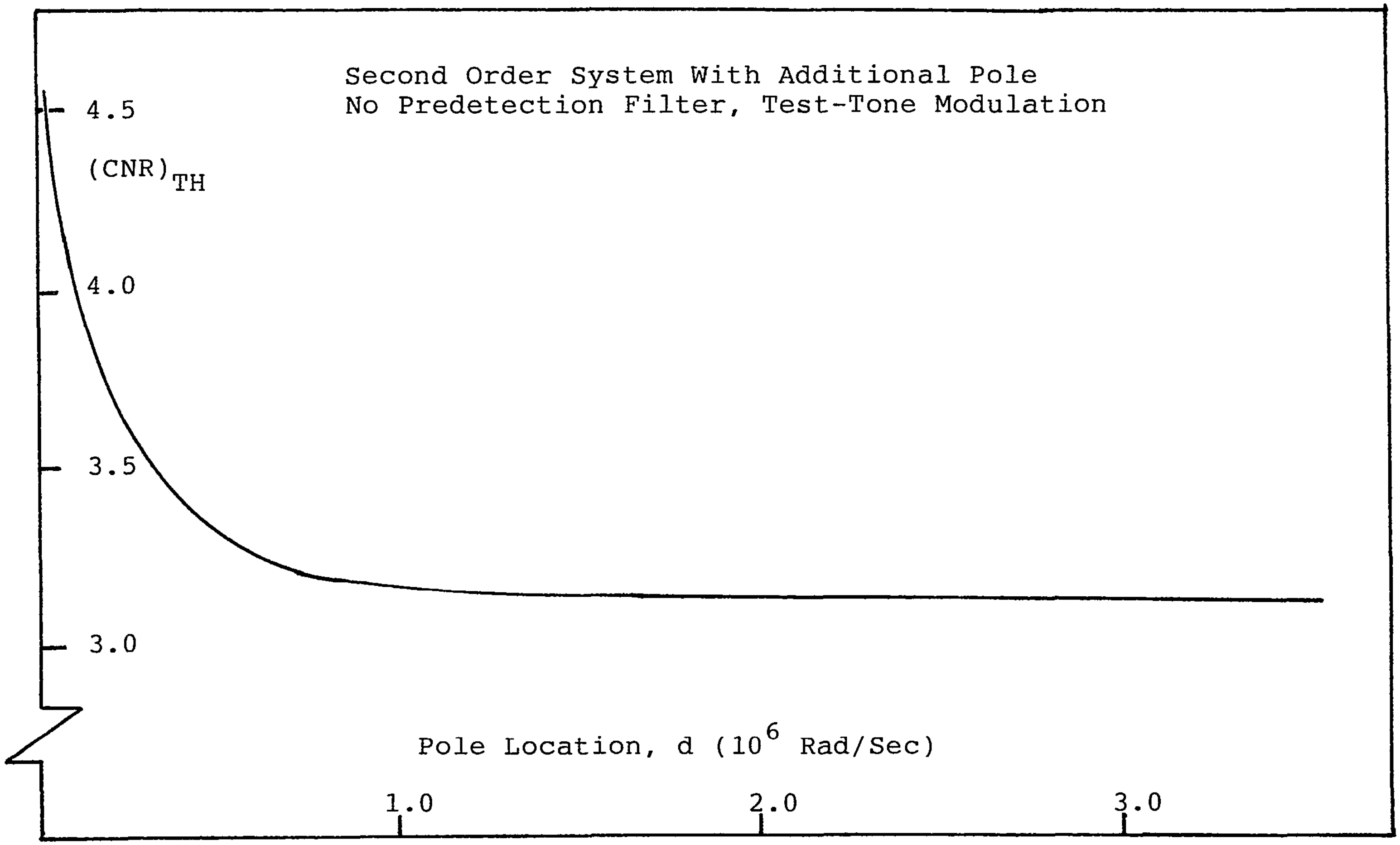


Fig. 4-3. $(CNR)_{TH}$ Vs. Pole Location. Case 3

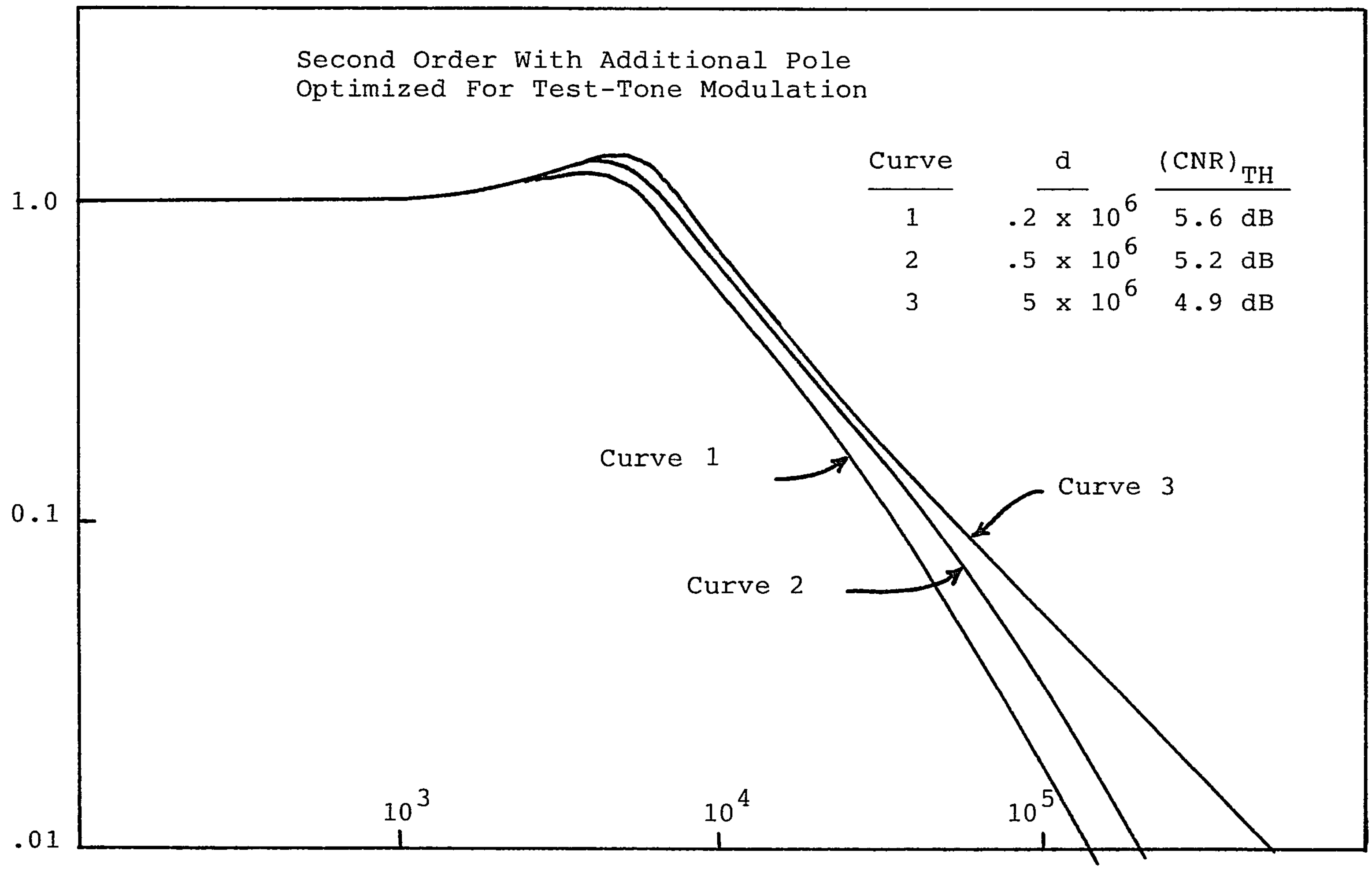


Fig. 4-4. Closed Loop Response - Cases (1) and (3)

reduction when the modulation is voice. As above, a modified optimization algorithm had to be adopted because the optimum extra pole location tends to infinity. Minimums were found for different pole locations and the results are plotted in figure 4-5.

CASE 5

Addition of an ideal differentiator to the filter introduced a fourth variable and generated a pair of complex zeros in the numerator of the transfer function without affecting the order of the system [See eqs. (3-1) and (3-2)]. Optimization of this ERPLD for test-tone modulation leads to the following set of system parameters:

$$a = 74{,}600 \text{ Rad/Sec}$$

$$b = 2840 \text{ Rad/Sec}$$

$$\alpha = 1.79$$

$$K = 1 \times 10^6 \text{ Sec}^{-1}$$

and a minimum threshold of 2.1 dB.

Figure 4-6 is a plot of the closed loop response for this minimum threshold system. The optimization was performed for a system that includes a predetection bandpass filter and it is apparent from the response curve that the filter is essential. Were it not for this filter, the loop noise would be unbounded and the detector unusable.

Utilizing the code that was developed for optimizing this system but treating the bandwidth of the predetection filter as a variable we were able to investigate the significance of the bandwidth for optimum threshold. Of course, in a practical design, there are other considerations in the choice of predetection bandwidth. It must be broad enough to pass the IF without causing excessive signal distortion. Carson's rule [Ref. 9, Page 77] is a commonly applied rule

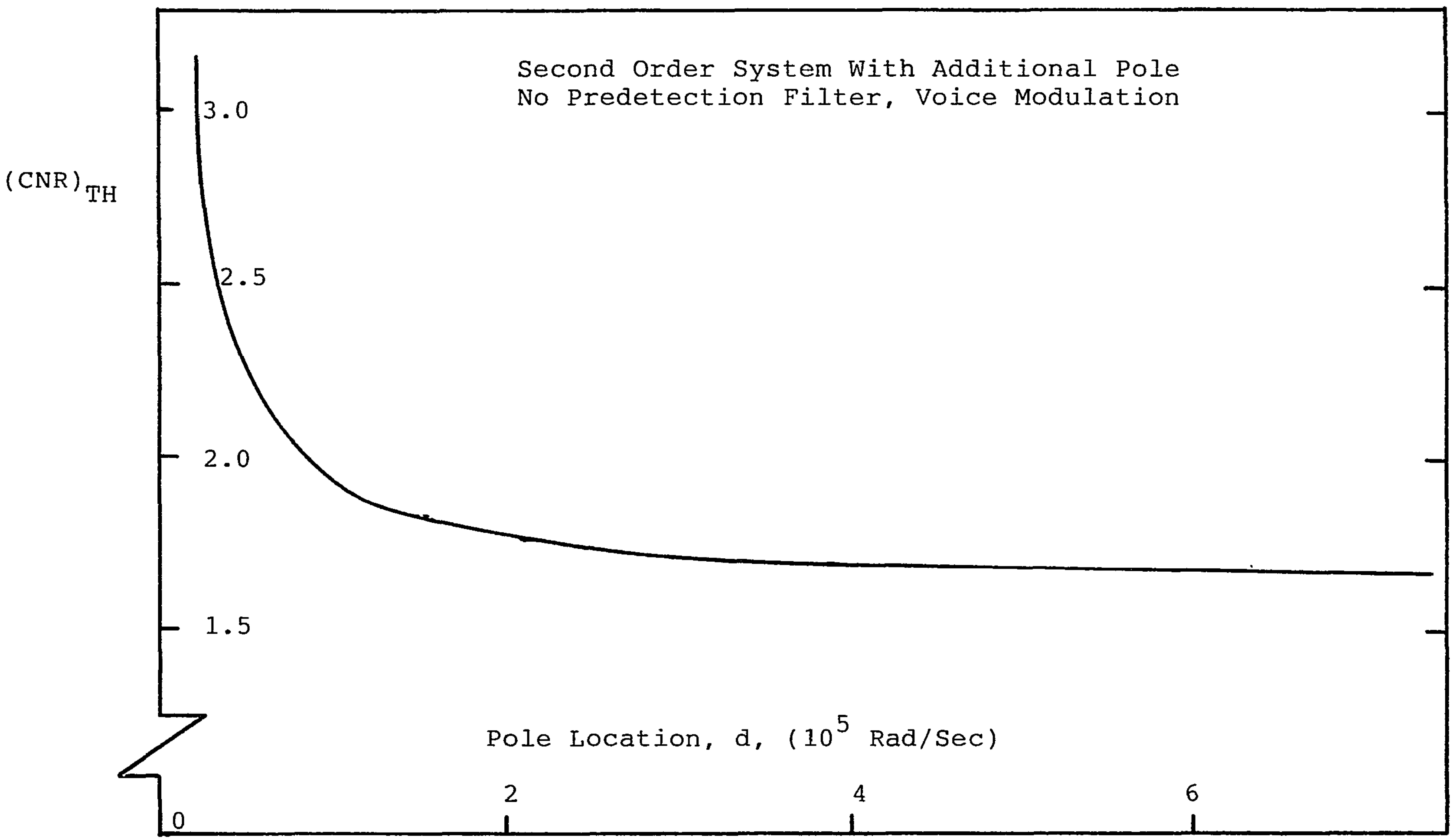


Fig. 4-5. $(CNR)_{TH}$ Vs. Pole Location Case 4

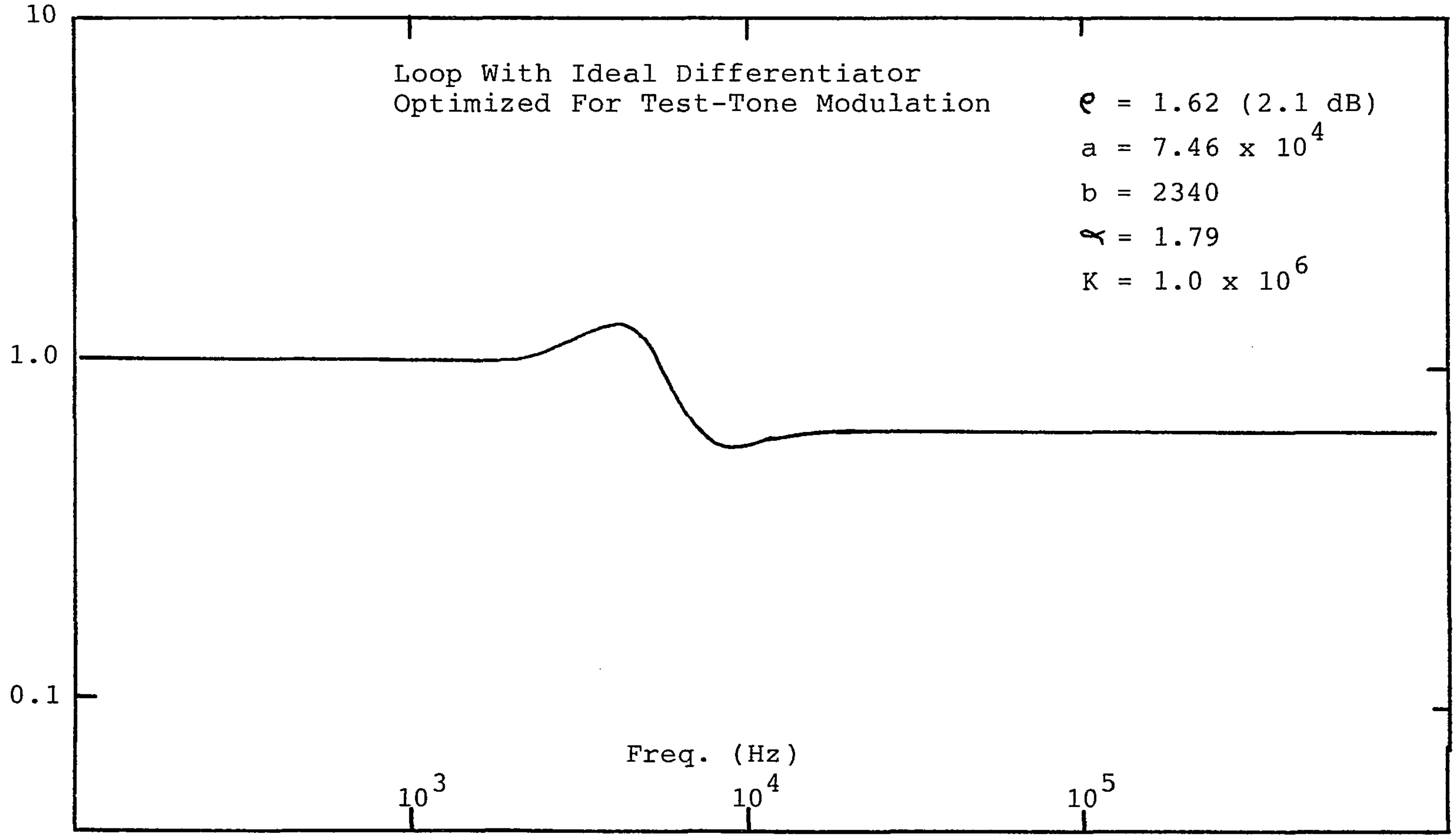


Fig. 4-6. Closed Loop Response, Case 5

of thumb. It states that the minimun bandwith should be twice the sum of the maximun deviation and the highest modulation frequency. To enable meaningful comparisons, the noise power for the calculation of $(CNR)_{TH}$ was based upon the same 35 KHz bandwidth in all cases regardless of the actual predetection bandwidth being considered.

Not unexpectedly, a narrower predetection filter results in an improved threshold; but as the filter is opened up, the optimum system moves asymptotically towards a limiting condition and approaches the optimum standard second order system of case (1). Figure 4-6(a) is a plot of optimum $(CNR)_{TH}$ Vs. prefilter bandwidth for the system. For large bandwidths, the threshold approaches 4.9 db which is the minimum $(CNR)_{TH}$ that was determined for case (1).

Each predetection filter bandwidth results in a different set of loop parameters. The set of closed loop response curves for a selection of optimum systems is shown in figure 4-6 (b). Note that as the predetection filter becomes wider, the location of an apparent high frequency pole moves out and results in a reduced response at higher frequencies. A comparison of figure 4-6(b) with figure 4-1 for case (1) reveals that in the limit the response curves for the two systems become almost identical.

Actually it is the influence of the differentiator that diminishes with increasing bandwidth; its nullification of the rolloff of the lag-lead does not become effective until higher frequencies. Figure 4-6(c) is a plot of the differentiator gain, α , as a function of IF bandwidth, B_p, for the optimized loops. The constant, α , corresponds to the gain of the internal loop of the phase feedback ERPLD [Ref. 7]. For large IF bandwidths, α approaches zero and the

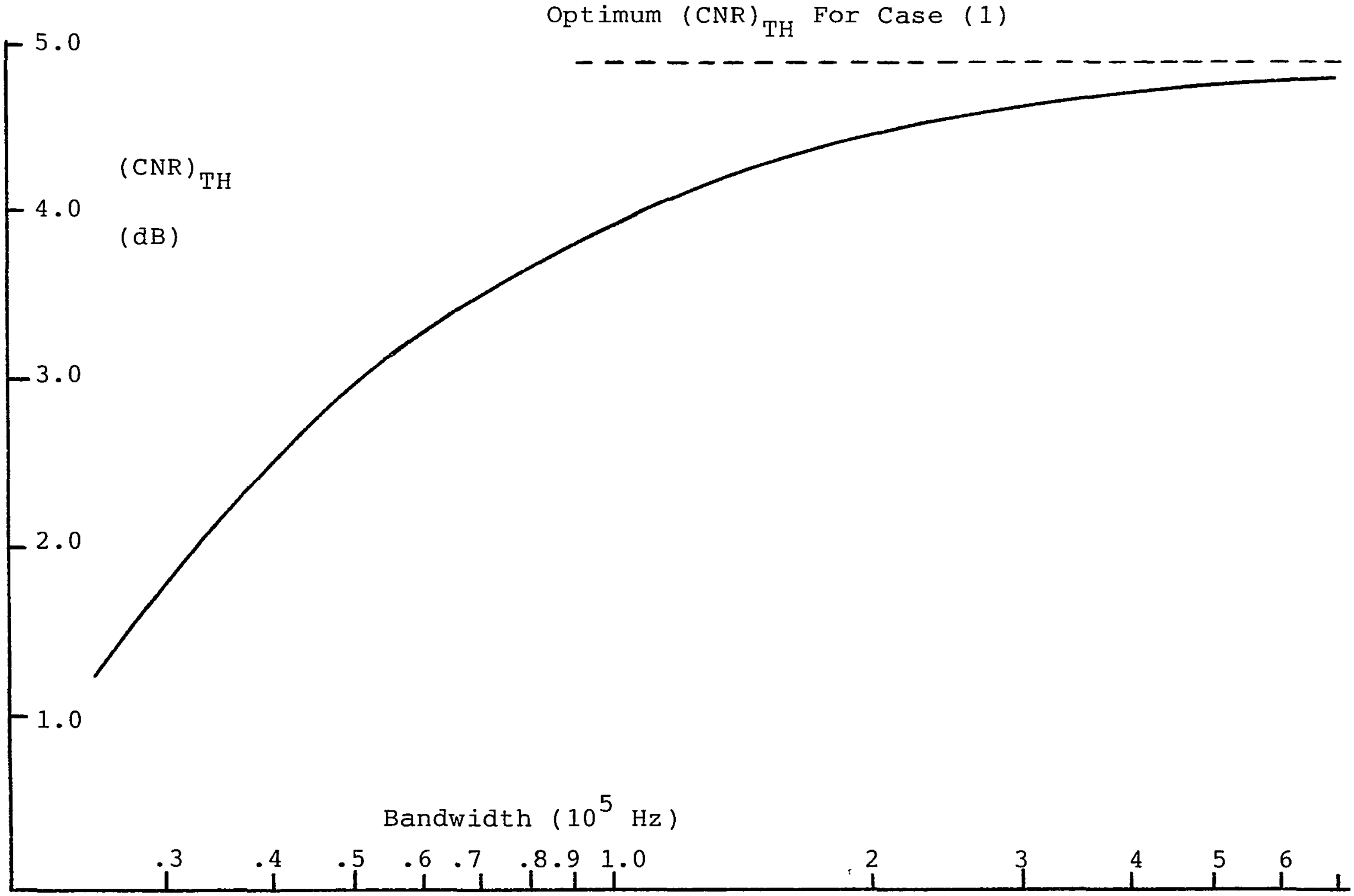


Fig. 4-6(a). Optimum Threshold Vs. IF Bandwidth For ERPLD
With Ideal Differentiator

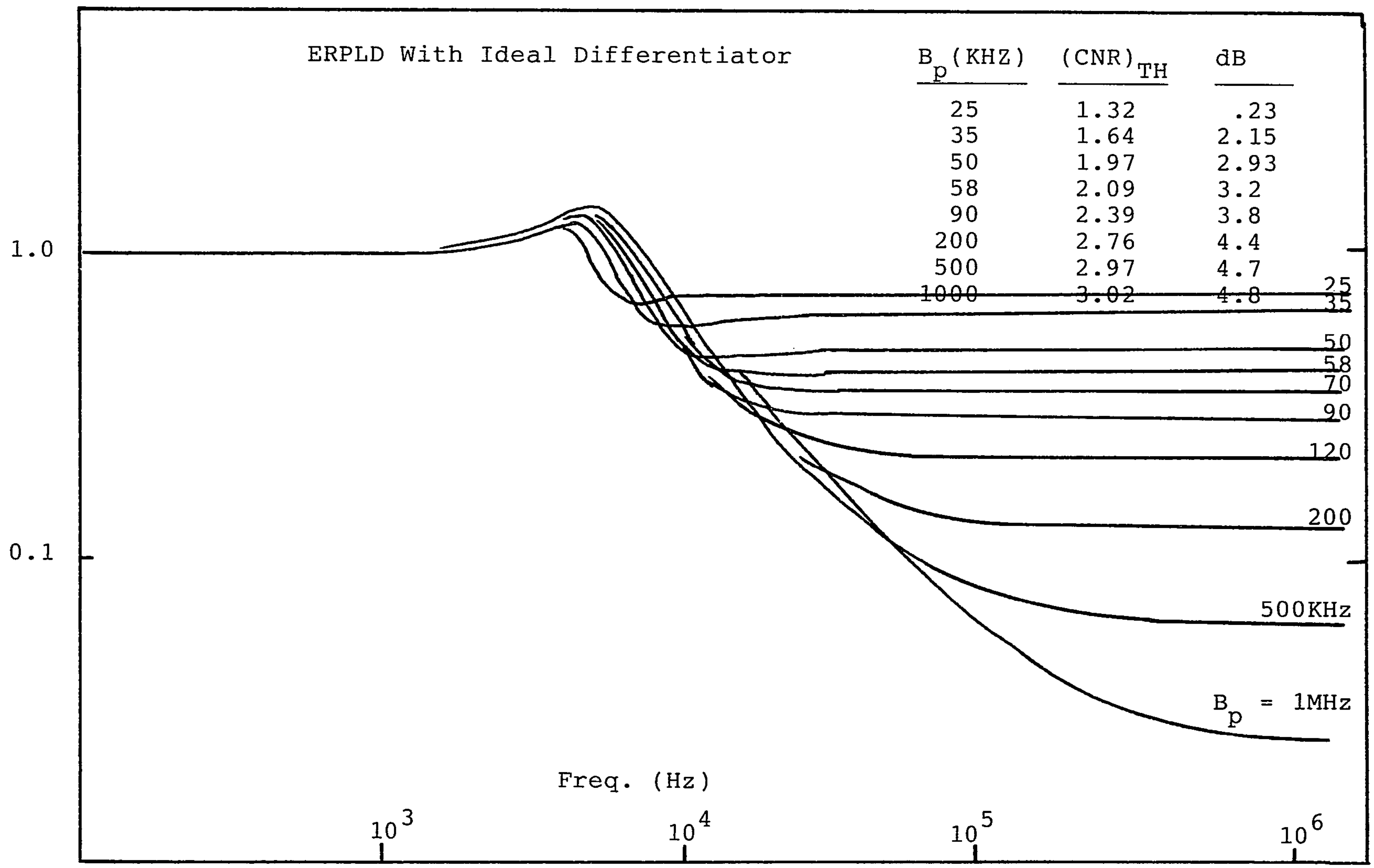
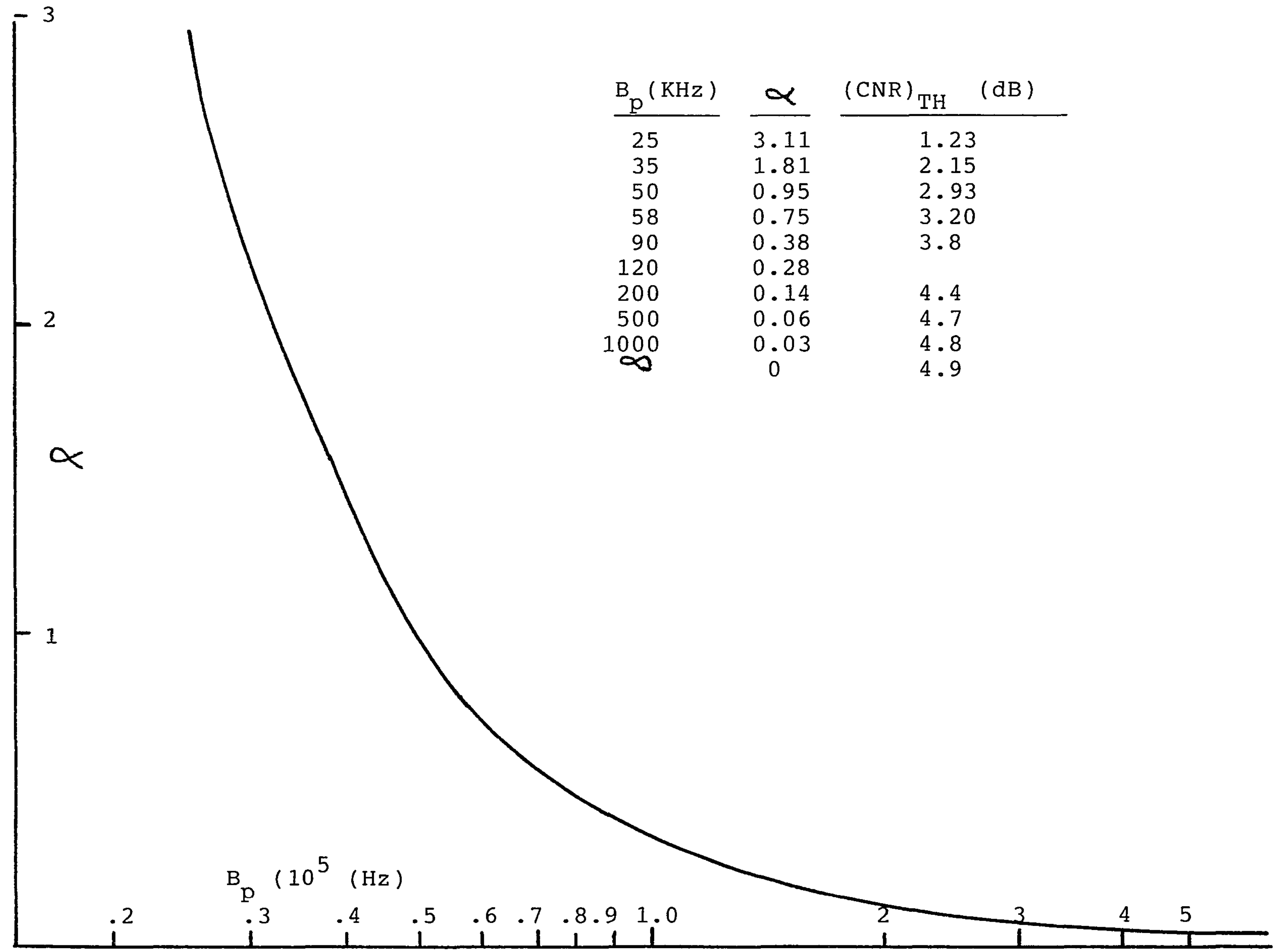


Fig. 4-6(b). Closed Loop Response, Case (5)



B_p (KHz)	α	$(CNR)_{TH}$ (dB)
25	3.11	1.23
35	1.81	2.15
50	0.95	2.93
58	0.75	3.20
90	0.38	3.8
120	0.28	
200	0.14	4.4
500	0.06	4.7
1000	0.03	4.8
∞	0	4.9

Fig. 4-6(c) α opt Vs. Predetection Filter Bandwidth

ERPLD reverts to the standard system.

CASE 6

When optimized, the system employing an ideal differentiator, a predetection filter and voice modulation results in the following parameters:

$$a = 39{,}700 \text{ Rad/Sec}$$

$$b = 5{,}760 \text{ Rad/Sec}$$

$$\alpha = .718$$

$$K = 163{,}453 \text{ Sec}^{-1}$$

and threshold (CNR) of 0.38 dB. This system is identical to the one analyzed by Novick as an Equivalent Filter ERPLD and our results confirm his with regard to optimum point and threshold CNR. [Ref. 3, Page 111].

The closed loop response for the optimized loop is plotted in figure 4-7. Again it is apparent that predetection filtering of the noise is essential if the loop is to be usable.

CASE 7

With the addition of a pole to the differentiator, the differentiator becomes realizable and the detector usable even without predetection filtering. The loop now employs filter IV and the optimization procedure requires a search in five dimensions. Here, as in previous instances, the terminus of the search depends upon its initialization but the threshold level remains nearly constant. TABLE 4-2 lists a set of four optimums that resulted from four arbitrarily selected starting points for the search procedure. The table indicates the initialization (in parentheses), the optimum point, the threshold level, and the Kb product for each optimum.

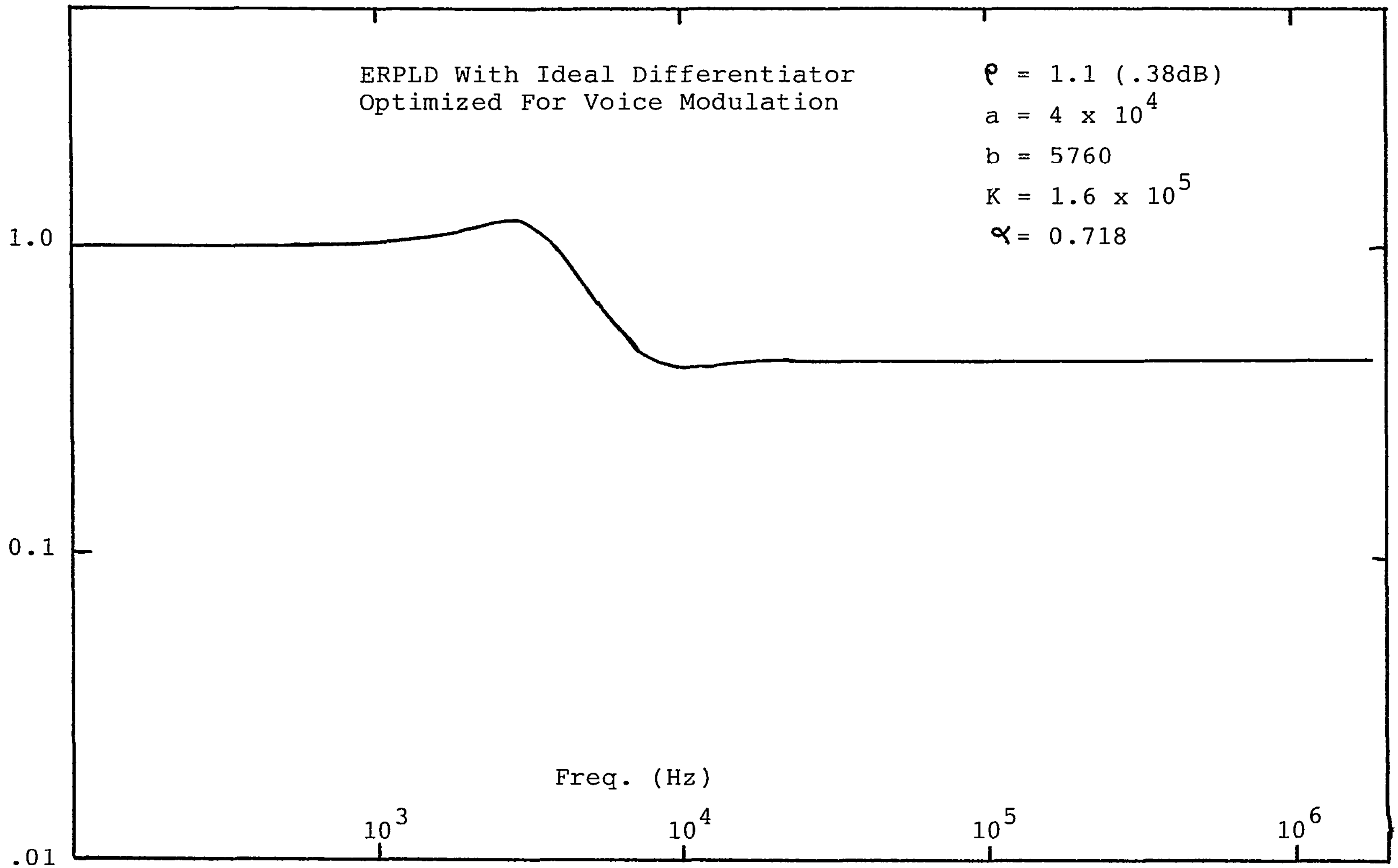


Fig. 4-7. Closed Loop Response, Case 6

We find that over this set of local minima which result from significantly different starting points, the first pole location, b, varies by 15% but the threshold CNR varies by only 0.04 dB. This is not surprising because we note that in the same set the product Kb varies by only 2%. Referring to the expression for the closed loop transfer function with loop filter IV, we observe that for $K \gg d \gg b$ the function does not depend upon either K or b alone but only upon their product.

We note also that "a" varies over a wide range and we will discuss this fact presently.

$(CNR)_{TH}$	a	b	d		K	Kb
(2.79) 4.45 dB	(8×10^5) 5.65×10^5	(3000) 2295	(3×10^4) 2.75×10^4	(1.0) 1.44	(5×10^5) 6.22×10^5	1.42×10^9
(2.80) 4.47 dB	(3×10^5) 3.39×10^5	(4000) 2621	(5×10^4) 2.87×10^4	(1.0) 1.31	(5×10^5) 5.37×10^5	1.41×10^9
(2.77) 4.43 dB	(6×10^5) 10^6	(4000) 2634	(5×10^4) 2.63×10^4	(0.8) 1.63	(5×10^5) 5.47×10^5	1.44×10^9
(2.79) 4.45 dB	(4×10^5) 7.30×10^5	(6000) 2310	(5×10^4) 2.72×10^4	(1.1) 1.47	(7×10^5) 6.18×10^5	1.43×10^9

TABLE 4-2 (a) - MINIMA FOR CASE 7

In order to ascertain that the terminus point is indeed a minimum, the $(CNR)_{TH}$ is plotted in figures (4-8) through (4-12) as a function of each variable in turn, as the other four are held constant. These curves are based on the last row of TABLE 4-2(a).

In each instance we observe that the minimum is rather broad and that the threshold is not sensitive to small variations in parameter values. TABLE 4-2(b) is based on the curves and indicates the acceptable tolerance for each parameter if it is not to cause a change in threshold of more than 0.1 dB by itself.

Parameter	Tolerance
b	+ 22%, -30%
d	+ 28%, -22%
α	+ 25%, -22%
K	+ 29%, -19%

TABLE 4-2(b)- PARAMETER TOLERANCE

The "flatness" in the neighborhood of the minimum is a common property of optima [Ref. 8, Page 140] which enables practical designs for best loop performance without the need for critical adjustments of the parameter values.

With respect to "a" we note that the curve assymptotically approaches a minimum threshold with increasing "a", achieving 0.1 dB above the minimum for $a = 2 \times 10^5$ Rad/Sec. This suggests that perhaps "a" can be moved out to infinity, in effect eliminating the zero and reducing the number of parameters to four. The loop filter

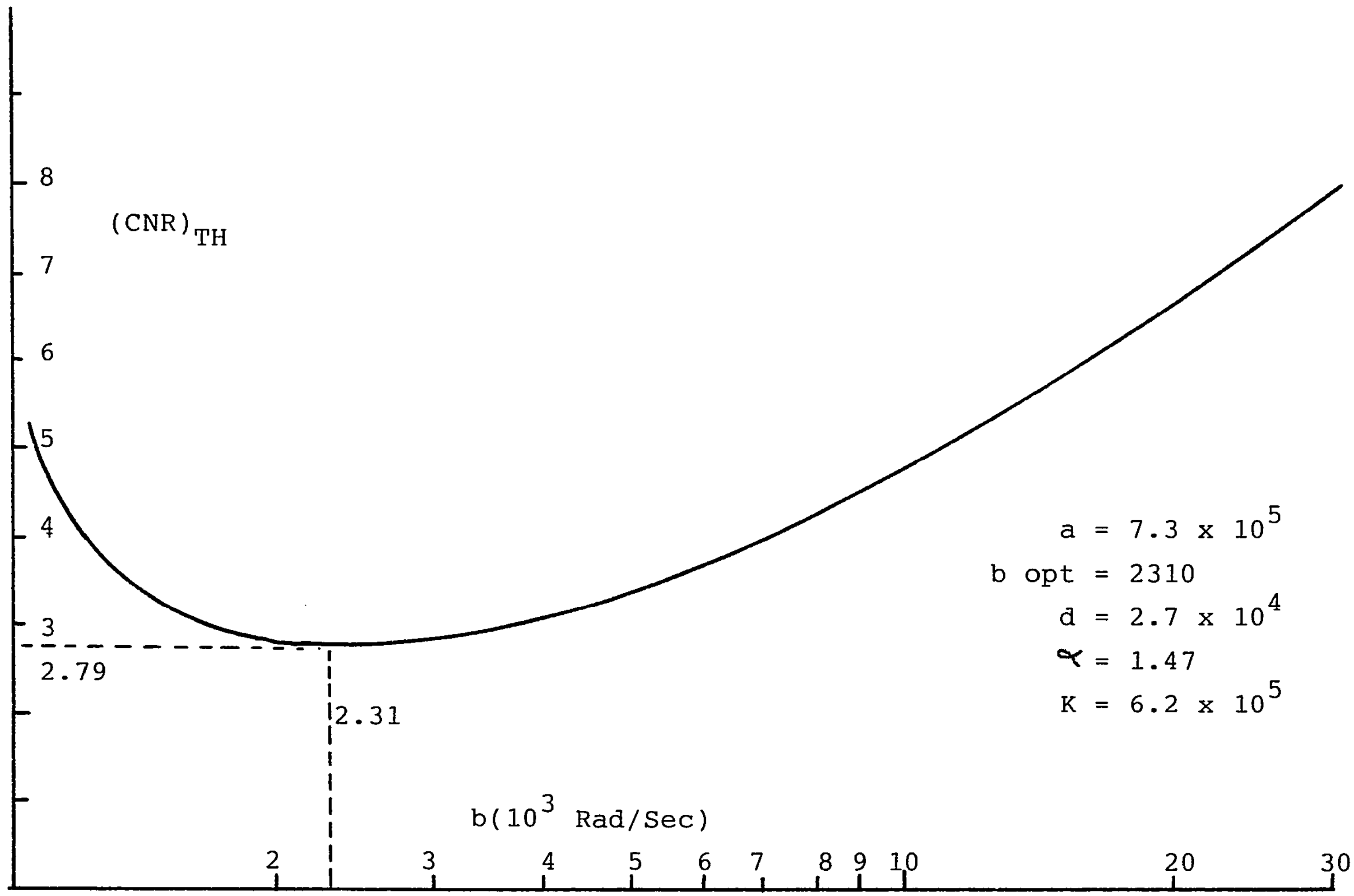


Fig. 4-8. $(CNR)_{TH}$ Vs. b, Case (7)

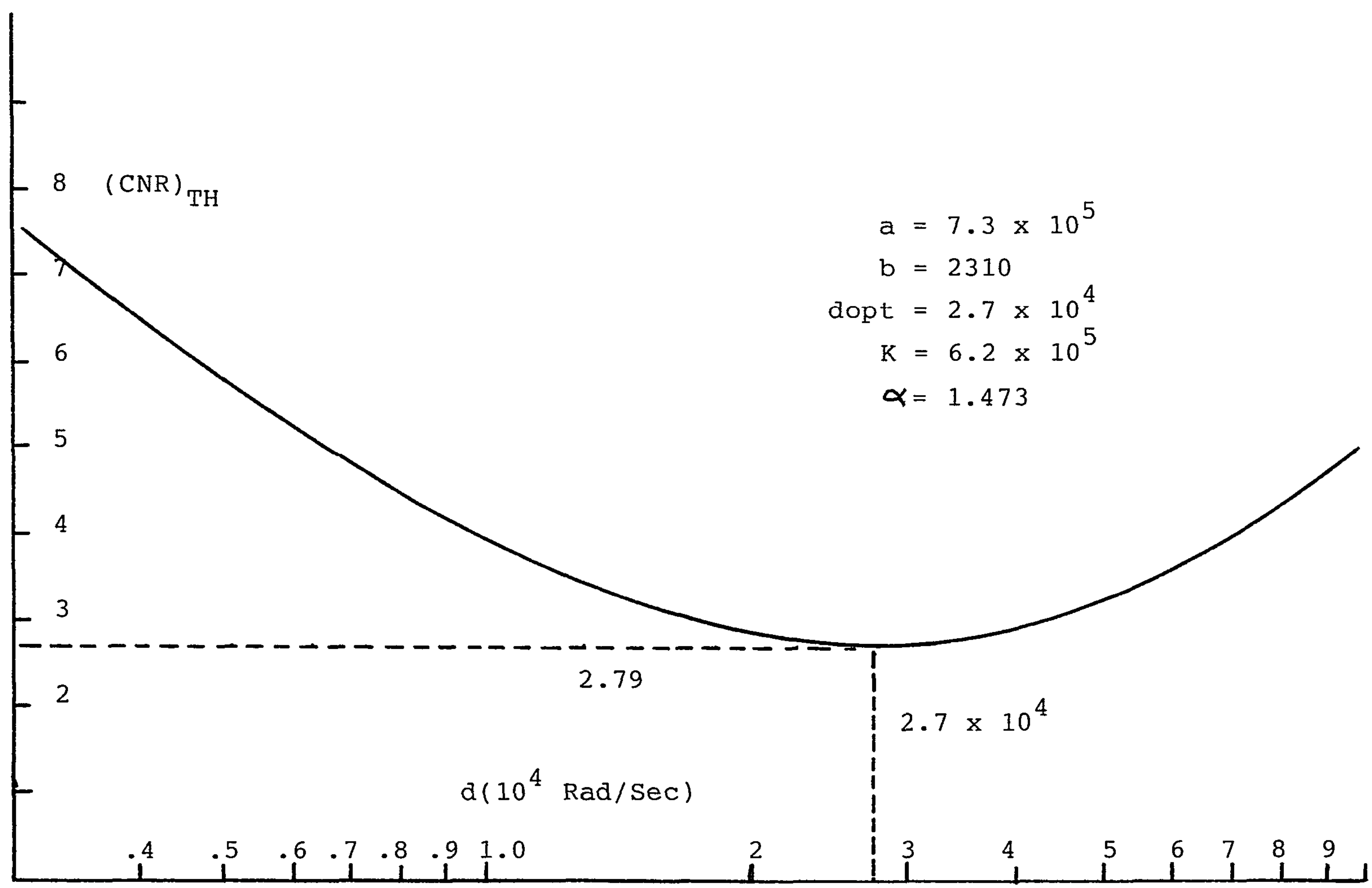


Fig. 4-9 $(CNR)_{Th}$ Vs. d, Case (7)

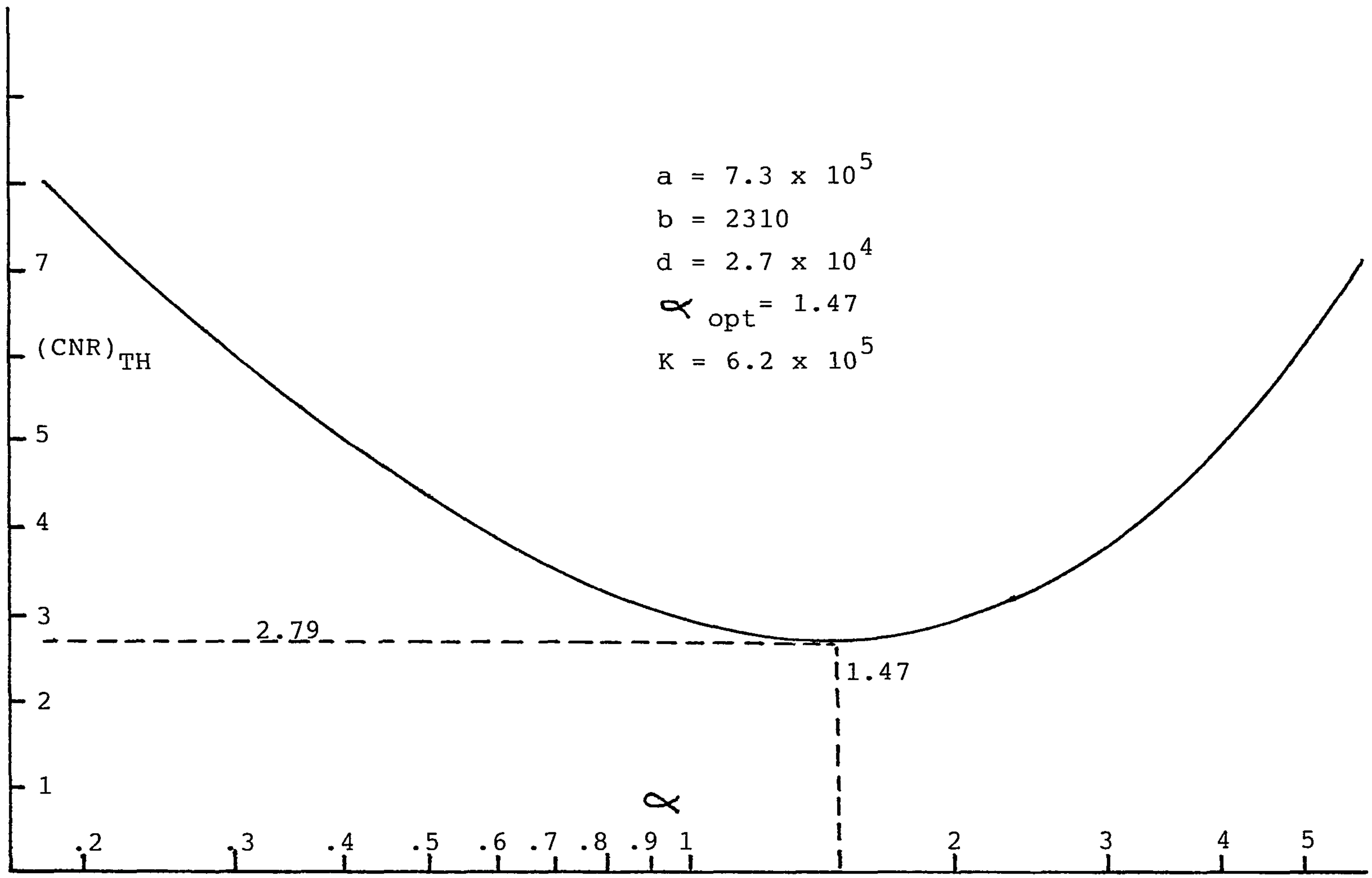


Fig. 4-10. $(CNR)_{TH}$ Vs. α , Case (7)

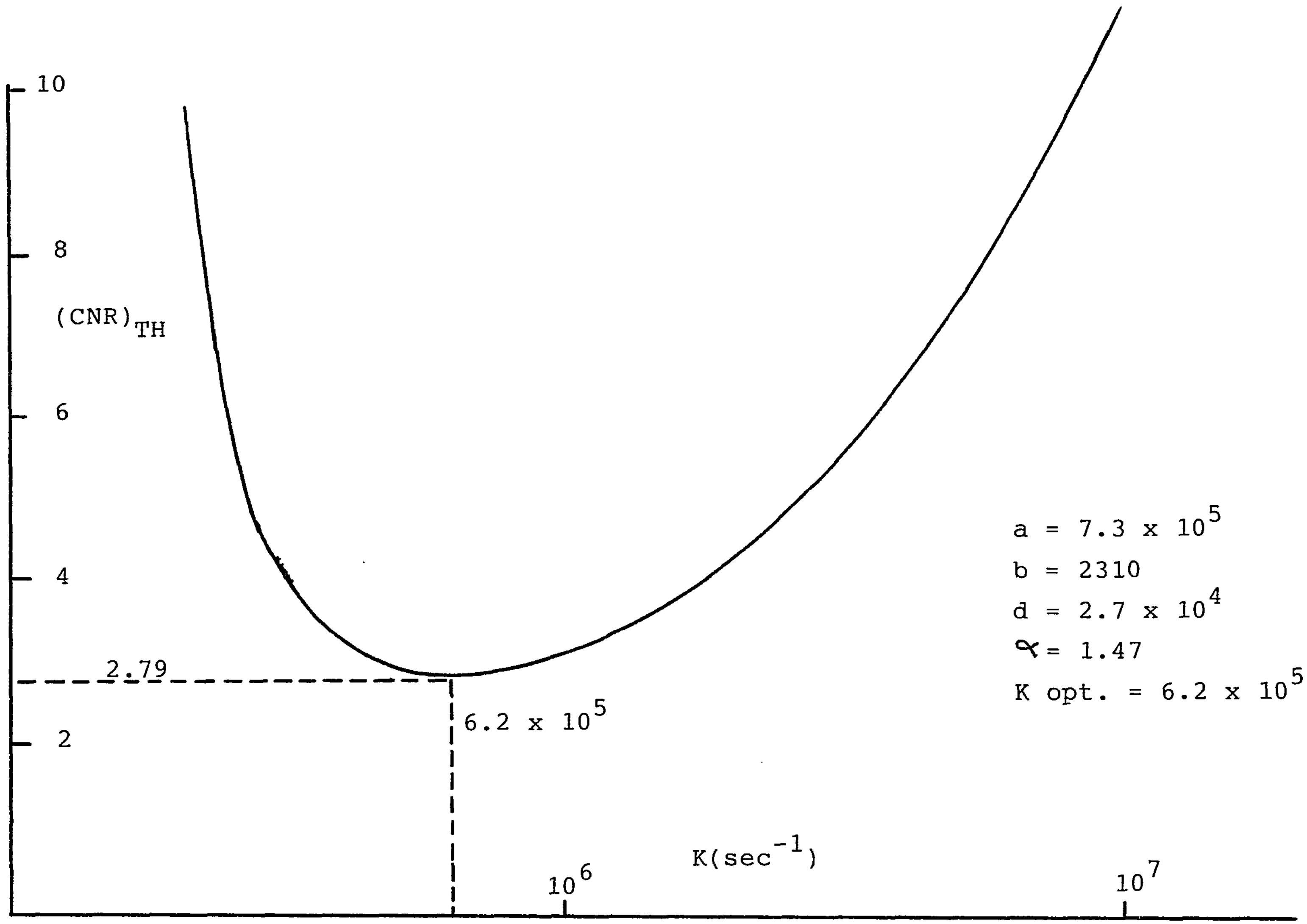


Fig. 4-11. $(CNR)_{TH}$ Vs. K, Case (7)

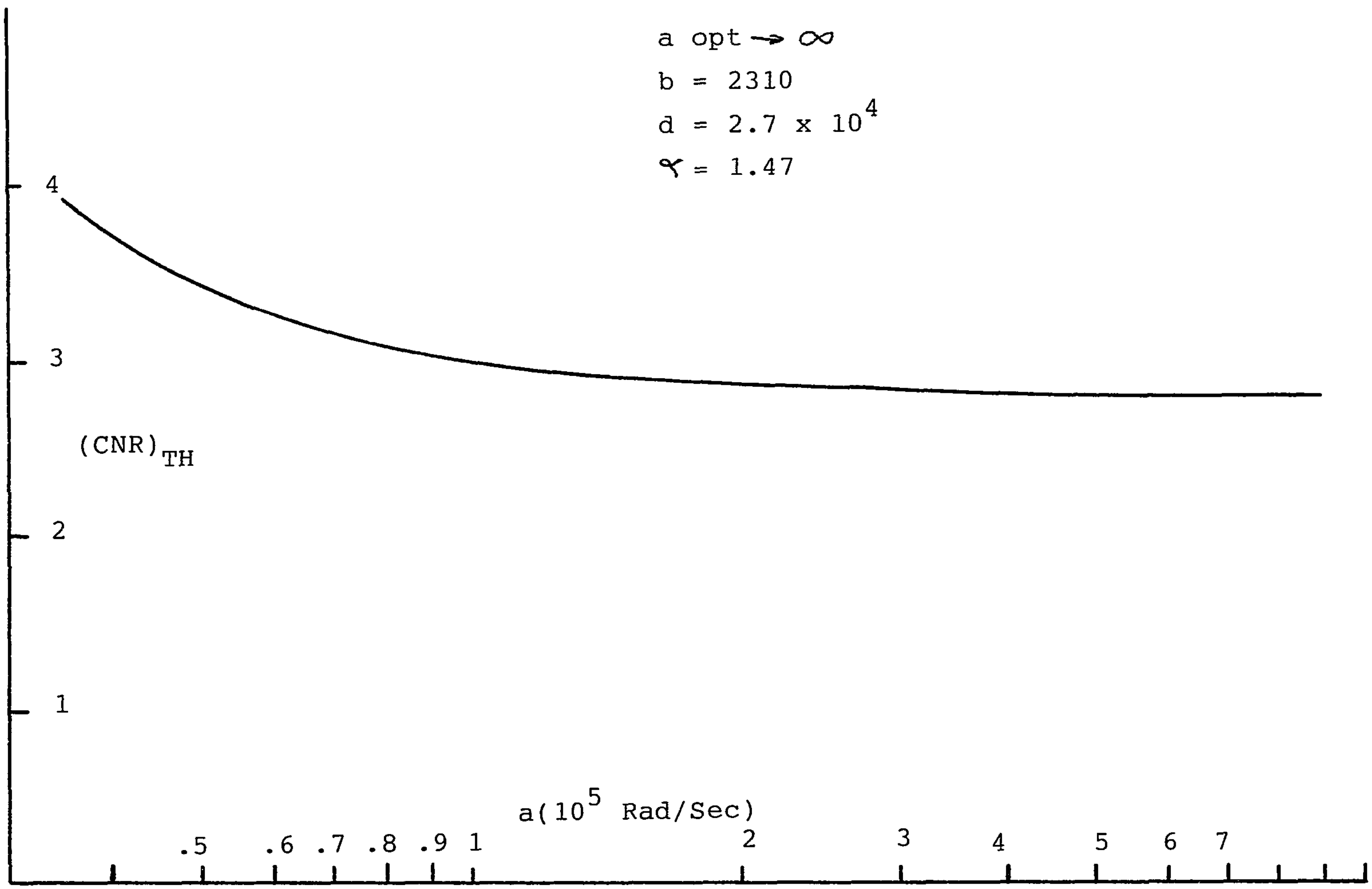
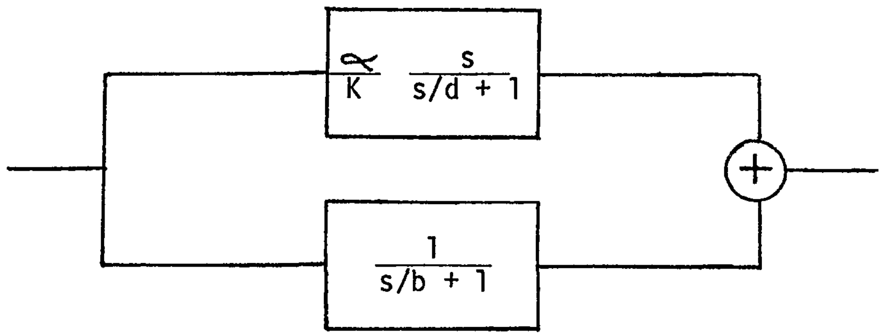


Fig. 4-12 $(CNR)_{TH}$ Vs. a, Case (7)

would then reduce to:



with transfer function

$$F(S) = \frac{\alpha}{K} \frac{s}{s/d+1} + \frac{1}{s/b+1} = \frac{(\alpha/K)s(s/b+1)+s/d+1}{(s/d+1)\ (s/b+1)}$$

$$= \frac{s^2 \ \alpha/(Kb) + s\ (\alpha/K + 1/d) + 1}{(s/b + 1)\ (s/d + 1)}$$

which is of the same form as eq (3-10) except that the terms having "a" in the denominator do not appear.

To test the idea we allow $a \rightarrow \infty$, rewrite the constants in eq. (3-13) as:

$A = \alpha d$, $B = Kb + \alpha bd$, $C = Kbd$, $D = d + b + \alpha d$, and $E = bd + Kb + \alpha bd$, and run the optimization program as CASE (7A).

The result is a detector with the same threshold CNR of 4.4 dB and a Kb product as before equal to 1.44×10^9 Rad/Sec^2.

TABLE 4-2(c) lists the results of three optimization runs from different initialization points.

$(CNR)_{TH}$	b	d	α	K	Kb
(2.78)	(1600)	(4.0×10^4)	(1.0)	(6×10^5)	
4.43 dB	2403	2.73×10^4	1.55	6.01×10^5	1.44×10^9
(2.78)	(3000)	(4×10^4)	(0.9)	(6×10^5)	
4.43 dB	2213	2.63×10^4	1.60	6.51×10^5	1.44×10^9
(2.83)	(3000)	(4×10^4)	(0.9)	(3×10^5)	
4.52 dB	4281	3.01×10^4	1.36	3.31×10^5	1.42×10^9

TABLE 4-2 (c) - MINIMA FOR CASE 7A (a = infinity)

Figure 4-13 is a plot of the closed loop response of the optimum detector based on the first row of TABLE 4-2(a) but for three different values of "a" to illustrate the effect of the "Zero" on the response.

This system may be compared with the standard system of case (1). Both were optimized for test tone modulation and operate with no predetection filter. A modest improvement of approximately 0.5 dB is achieved at a cost of a slight increase in loop complexity.

To understand why the "zero" may be omitted and why the more complex system is similar in performance to the standard second order system, we study the forward loop transfer function for Filter IV.

$$F(S) = \frac{s/a + 1}{s/b + 1} + \frac{\alpha}{K} \frac{s}{s/d + 1}$$

At low frequencies the first term is dominant and the loop responds like a standard second order system. At higher frequencies the second term (due to the differentiator) with its lead/lag characteristic

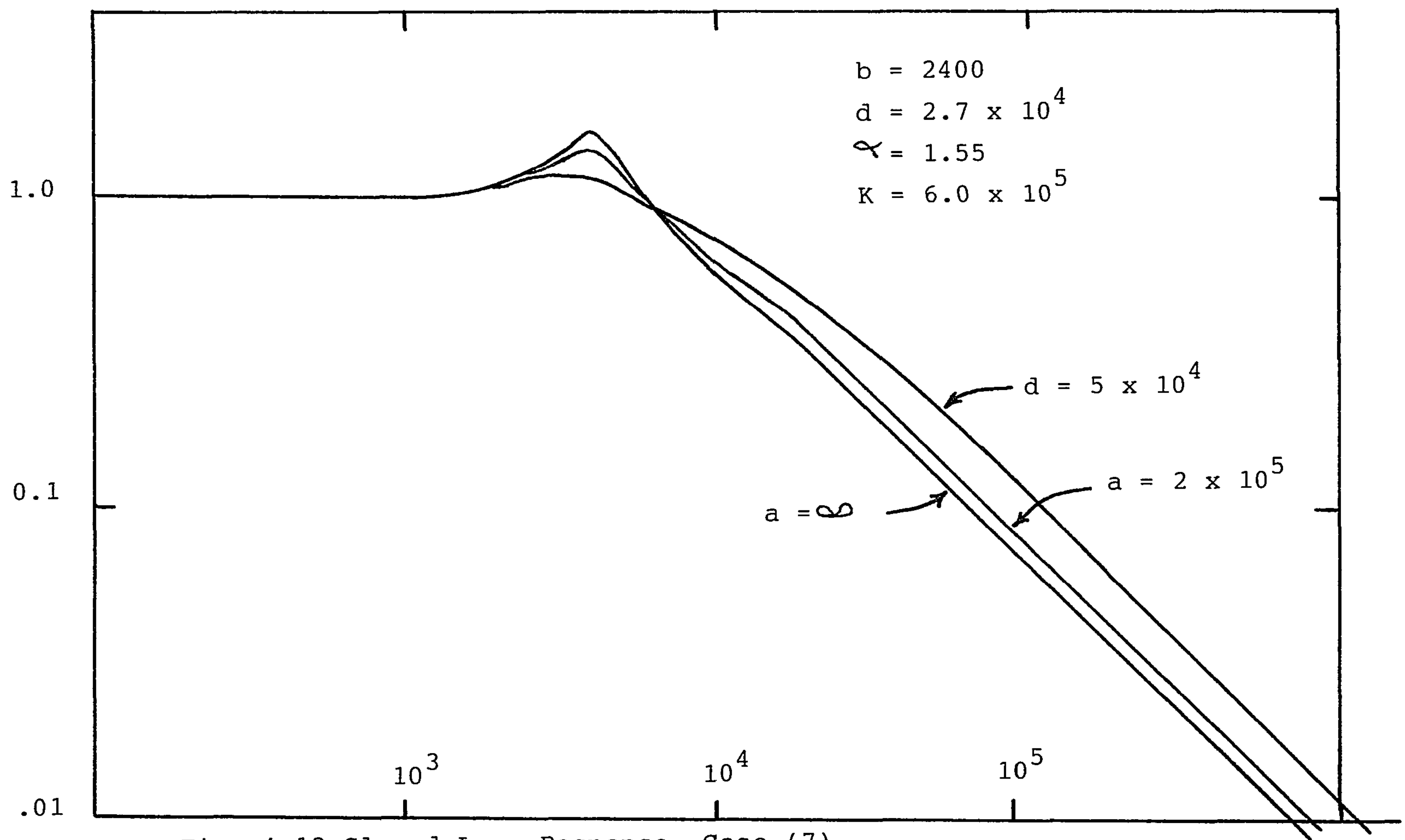


Fig. 4-13 Closed Loop Response, Case (7)

becomes more important. Figure 14(a) exhibits the Log magnitude response for each of the two terms separately and (b), an approximation of the composite Log magnitude.

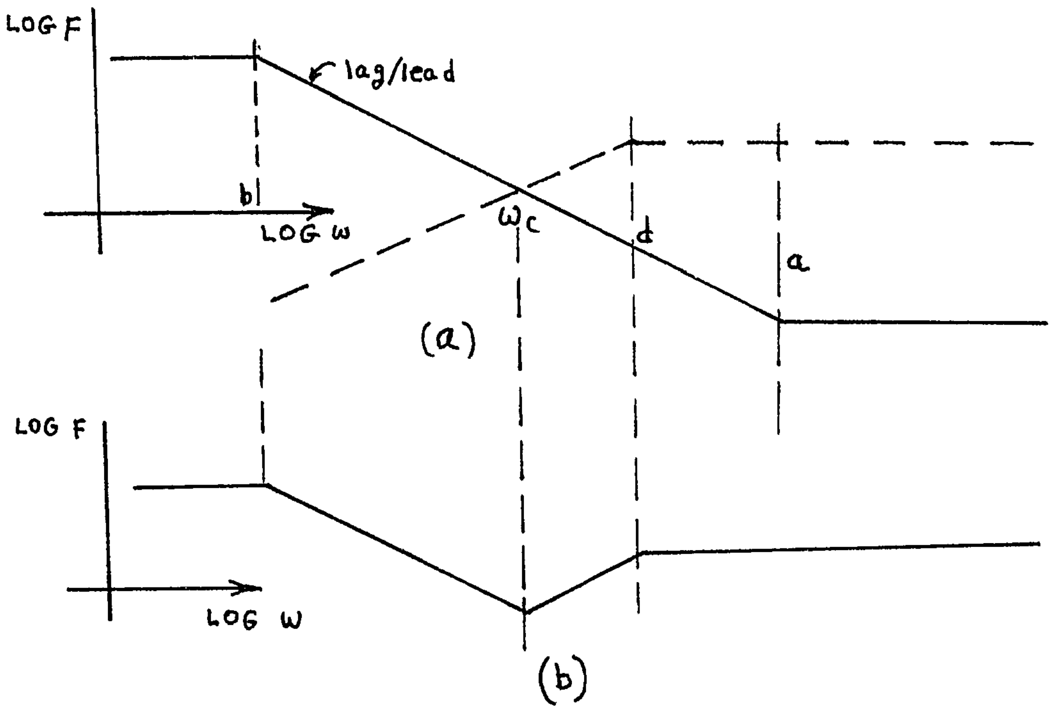


FIGURE 4-14 Log Amplitude Response of Filter IV

a) Individual Terms

b) Composite Function

Crossover occurs at a frequency, w_c, where the two terms have equal magnitudes. The composite approximation is least accurate in the neighborhood of w_c where the individual magnitudes are nearly equal and their phase relationship, which depends upon the value of "d", is most significant.

If $a \gg d \gg w_c$, then

$$\frac{b}{w_c} \approx \frac{w_c}{K} \quad \text{and} \quad w_c \approx \sqrt{\frac{bK}{\alpha}} \; .$$

w_c is identifiable as w_n of equations (3-10) and (3-11) for large "a". At this frequency the magnitude of each term is approximately

$$\frac{b}{w_c} = \sqrt{\frac{\alpha b}{K}}$$

We observe that for large "a" the specific value of "a" has little effect upon the composite characteristic.

The search algorithm yielded the following optimum set of parameters [The last row in Table 4-2 (a)]:

$a \rightarrow \infty$

$b = 2310$ Rad/sec

$d = 2.72 \times 10^4$ Rad/sec

$\alpha = 1.473$

$K = 6.2 \times 10^5 \text{ sec}^{-1}$

For these values, $w_c = 3.1 \times 10^4 \approx d$ and the response is approximately as shown in Figure 4-15.

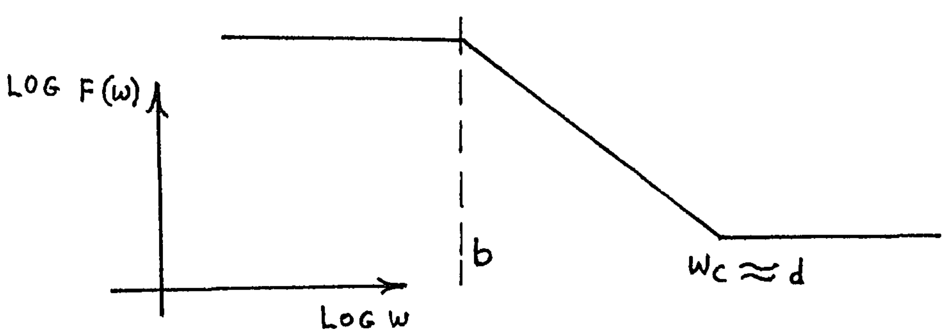


FIGURE 4-15 - Approximate Open Loop Response with w_c = d

But this is similar to the response of Filter I, the lag/lead filter of the second order system, with $a = w_c$. The response of Filter I in combination with an ideal integrator representing the VCO is shown in figure 2-5. In fact, the optimization of CASE (1) resulted in $a = 3.8 \times 10^4$ (See above in this section) which is not very different from the value of w_c for the optimized case (7).

The slight reduction in minimum threshold achieved by the realizable ERPLD may be attributed to the fine tuning of Filter IV (less its zero), made possible by the pole which introduces one additional degree of freedom.

A plot of the calculated log amplitude response for the optimized system is given in Figure 4-13 for three values of "a".

CASE 8

The additional pole that distinguishes loop filter IV from loop filter III contributes no threshold reduction when the loop is preceded by the bandpass filter. In fact the minimization procedure tends to increase the pole frequency ad infinitum just as it did in case 3 and it does not achieve an optimum. We modified the algorithim to treat the pole location as a constant and to search for an optimum in only four dimensions and obtained the results that are indicated by the curve of figure 4-16. This is a plot of optimum $(CNR)_{TH}$ as a function of "higher pole" location. In the limit, as $d \rightarrow \infty$, the threshold approaches the level achievable with an ERPLD employing an ideal differentiator. We observe, however, that for a realizable differentiator having a break frequency of about 160 KHz (i.e. $d = 10^6$) the minimum achievable threshold is within 0.1 dB of the ideal optimum.

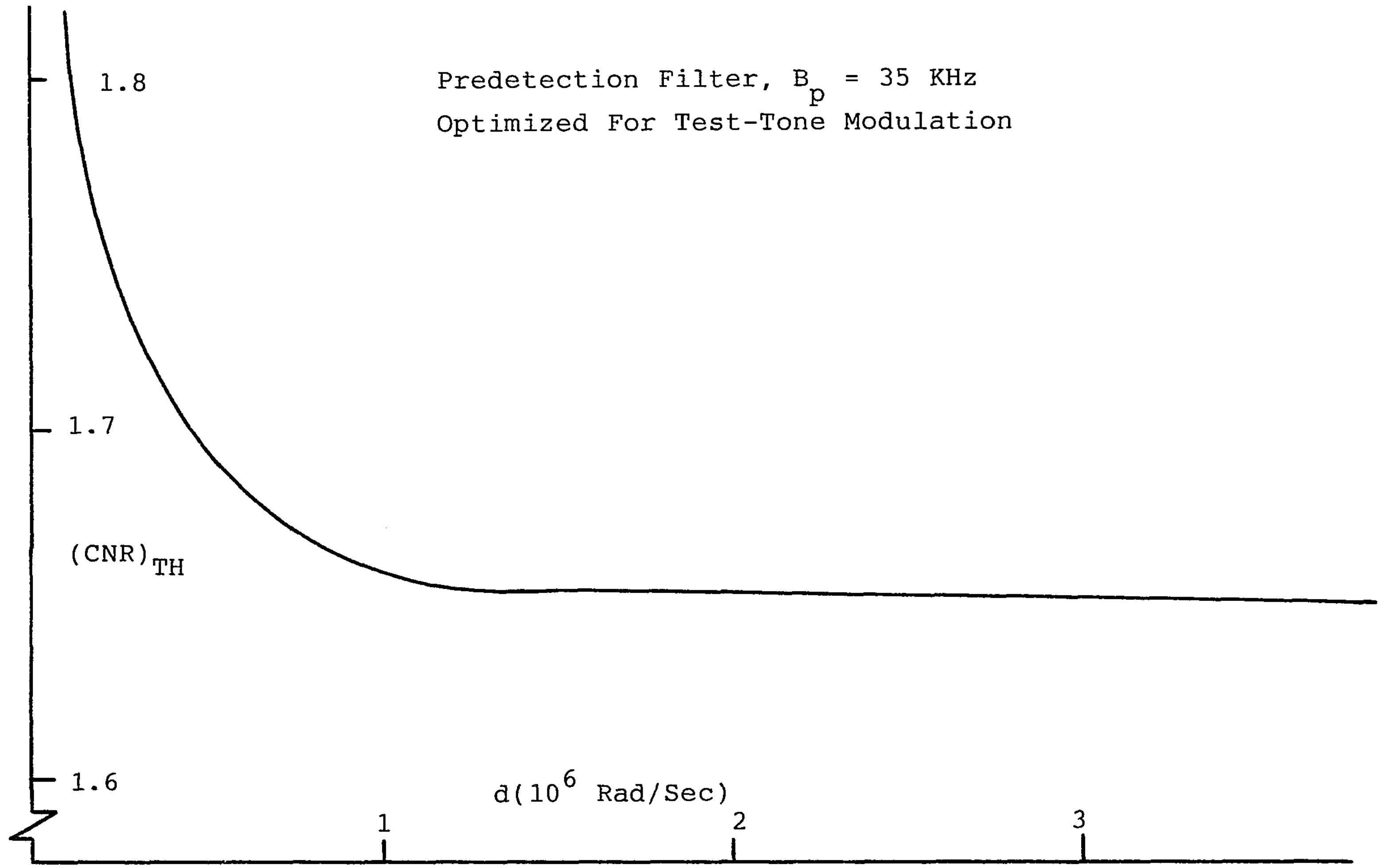


Fig. 4-16 $(CNR)_{TH}$ Vs. Pole Location, d, Case (8)

Figure 4-17 shows the closed loop response curves for several optimized systems. Each one is optimized for a different fixed pole location. As the pole moves out to higher frequencies, the optimum system approaches that of case 5 as may be noted by comparing figure 4-17 with figure 4-6.

We note here as we did in connection with case 4, that the additional pole contributes no threshold reduction beyond that achievable with the simpler system. However, the pole may be incorporated into the loop filter without significantly affecting the threshold level. This latter statement is more significant here than it was for the earlier system because in this case, the simpler system is not physically realizable.

CASE 9

The final two applications of loop Filter IV are in systems that are optimized for voice modulation; in the first instance without a predetection filter and in the second, with one.

The optimization with no prefilter leads to a minimum at:

$$a = 8.0 \times 10^4 \text{ Rad/Sec}$$

$$b = 4.80 \times 10^3 \text{ Rad/Sec}$$

$$d = 1.43 \times 10^4 \text{ Rad/Sec}$$

$$\alpha = 1.27$$

$$K = 1.57 \times 10^5 \text{ sec}^{-1}$$

and $(CNR)_{TH} = 2.0$ dB

But once again the optimum design with respect to minimum threshold is not unique. Other designs yielding the same threshold level can be achieved by either starting the search process from different initial

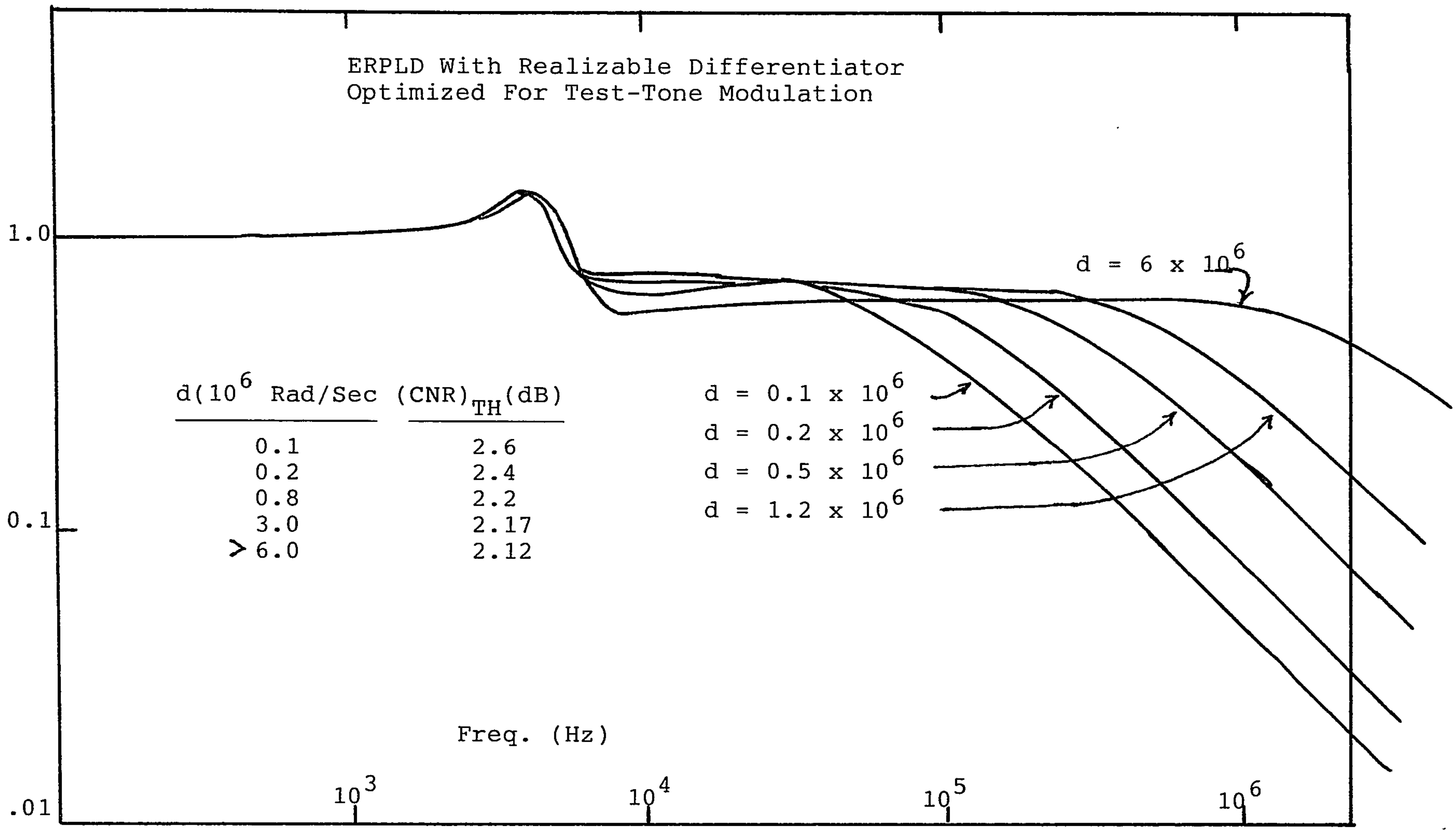


Fig. 4-17. Closed Loop Response, Case (8)

points or by fixing one of the design parameters and searching the other four. These are not really different designs because they all result in the same closed loop response characteristic which is plotted in figure 4-18.

This curve is plotted from eq (3-14),

$$H(jw) = \frac{A(jw)^2 + B\,(jw) + C}{jw^3 + D\,(jw)^2 + E(jw) + C}$$

where:

$$A = \frac{Kb}{a} + \alpha d, \quad B = \frac{Kbd}{a} + Kb + \alpha bd\ ,$$

$$C = Kbd\ , \quad D = d + b + \frac{Kb}{a} + \alpha d \quad \text{and } E = bd + \frac{Kbd}{a} + Kb + \alpha bd.$$

TABLE 4-3 was prepared in order to illustrate the similarity among the several systems.

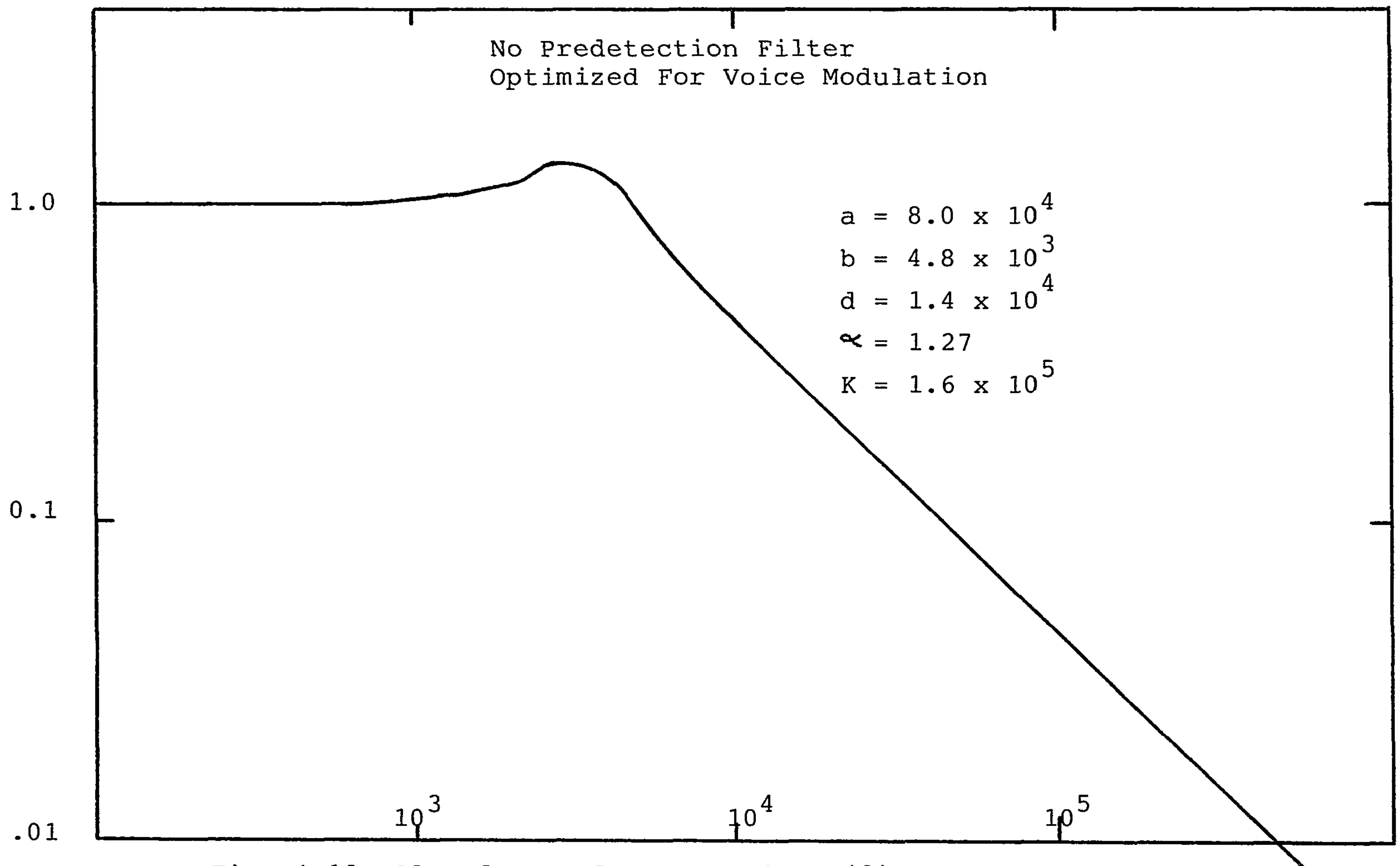


Fig. 4-18. Closed Loop Response, Case (9)

Optimum No.	1	2	3	Variation
a	4.737×10^4	6.60×10^4	8.03×10^4	70%
b	4.072×10^3	4.306×10^3	4.80×10^3	18%
d	1.709×10^4	1.27×10^4	1.43×10^4	14%
α	.734	1.27	1.27	74%
K	1.743×10^5	1.66×10^5	1.57×10^5	11%
A	2.75×10^4	2.71×10^4	2.75×10^4	1.4%
B	1.017×10^4	0.93×10^9	0.98×10^9	3.9%
C	1.213×10^{13}	0.917×10^{13}	1.08×10^{13}	24%
D	4.87×10^4	4.41×10^4	4.66×10^4	10%
E	1.08×10^9	0.99×10^9	1.04×10^9	8.3%
ρ	1.6 (2.0 dB)	1.6 (2.0 dB)	1.6 (2.0 dB)	

TABLE 4-3 - PARAMETERS AND CONSTANTS FOR CASE 9

A study of the table reveals that among the three minima there is much less variation of the values of the coefficients than of the values of the actual parameters.

A more noteworthy similarity of results may be observed by comparing the minimum threshold levels and frequency response curves for the optimums of cases 2 and 9. By superimposing figures 4-2 and 4-18 we discover that the curves are nearly congruent.

At high frequencies, for the conventional system of case (2) we have from eq. (2-11)

$$\left|H\ (jw)\right|_{\text{large } w} = \frac{1/a\ (jw)}{1/Kb\ (jw)^2} = \frac{Kb}{aw} \ ,$$

and for case (9) from eq (3-16)

$$\left|H\ (jw)\right|_{\text{large } w} = \frac{A}{w} = \frac{Kb}{aw} + \frac{d}{w}$$

By substituting values from each optimum point we get

for case (2) $$\left|H(jw)\right|_{\text{large } w} = \frac{1.7 \times 10^5 \quad \times 3.66 \times 10^3}{2.4 \times 10^4\ w}$$

$$= \frac{2.59 \times 10^4}{w} \quad ;$$

and for case (9)

$$\left|H\ (jw)\right|_{\text{large } w} = \frac{2.75 \times 10^4}{w} \quad .$$

Thus, along their skirts, the curves differ by only 6%. In fact over the entire frequency range the responses differ by no more than 6%. Therefore, it is not surprising that the calculated thresholds for these two systems agree to within 0.1 dB.

From the point of view of threshold reduction, not much can be gained by using the more complex Filter IV in place of the standard filter I. For test-tone modulation the improvement was about 0.5 dB and for voice, even less.

CASE 10

Finally, for the system that includes a predetection filter, we find that for voice modulation no improvement results from the addition of another pole. Just as for the test-tone modulation case (8) the optimization algorithim locates the pole at infinity (i.e. no pole) and finds the minimum threshold to be $(CNR)_{TH}$ = 1.1 (.380 dB).

As we did for test-tone modulation, we investigated the effect of the inevitable high frequency pole for this case of voice modulation by fixing the pole location and optimizing with respect to the other parameters. The results are tabulated in TABLE 4-4 and plotted in figure 4-19 where threshold is shown as a function of high frequency pole location.

Pole Location	Threshold Level
$.05 \times 10^6$	1.299 (1.14 dB)
$.1 \times 10^6$	1.232 (.906 dB)
$.2 \times 10^6$	1.175 (.70 dB)
$.5 \times 10^6$	1.128 (.523 dB)
1.0×10^6	1.11 (.454 dB)
2.0×10^6	1.101 (.418 dB)
3.0×10^6	1.098 (.406 dB)

TABLE 4-4 - EFFECT OF HIGH FREQUENCY POLE IN CASE 10

The curve assymptotically approaches a minimum as d moves to infinity; but for reasonably large values of d, the threshold becomes only imperceptably larger than the minimum. For d = 0.6×10^6 Rad/Sec the threshold exceeds the minimum by about 0.1 dB. Thus with a pole located at 100 KHz, which is less than three times the predetection filter bandwidth, the loop design is essentially optimum.

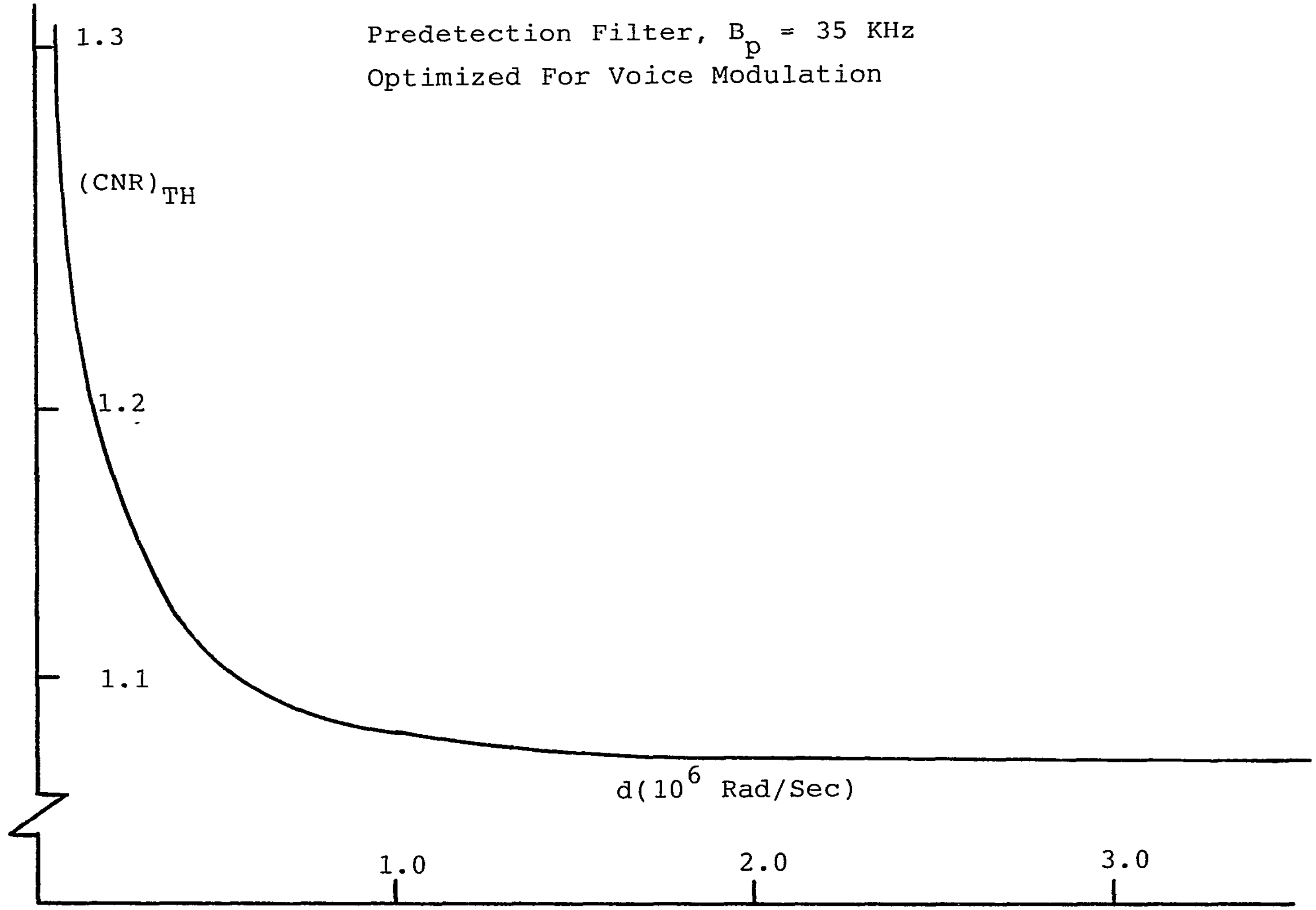


Fig. 4-19 $(CNR)_{TH}$ Vs. Pole Location, d, Case (10)

4.4.2 - Summary of Results

All of the results of the foregoing calculations are collected and presented in TABLE 4-5. It must be noted that the indicated optimum points are not unique but are rather representative sets of loop parameters that result in optimum closed loop responses. The arrows indicate pairs of systems that exhibit equivalent threshold performance. In each pair, the threshold level of the loop containing an additional pole approaches that of the simpler loop as the pole moves towards infinity.

CASE	LOOP FILTER	PRE-FILTER	MODULATION	OPTIMUM POINT	$(CNR)_{TH}$
1	I LAG/LEAD	NO	T.T.	a = 38000 b = 2350 K = 560,000	3.1 (4.9 dB)
2	I	NO	VOICE	a = 24,000 b = 3655 K = 170,000	1.61 (2.1 dB)
3	II EXTRA POLE	NO	T.T.	a = 38,000 b = 2550 d 2×10^7 k = 520,000	3.1 (4.9 dB)
4	II	NO	VOICE	a = 23,250 b = 3432 d = 5×10^6 K = 175,200	1.62 (2.1 dB)
5	III IDEAL DIFFERENTIATOR	YES	T.T.	a = 74,600 b = 2840 x = 1.79 K = 1.0×10^6	1.62 (2.1 dB)
6	III	YES	VOICE	a = 39,700 b = 5760 x = .718 K = 163453	1.1 (.380 dB)
7	IV REALIZABLE DIFFERENTIATOR	NO	T.T.	a 7×10^5 b = 2403 d = 27311 x = 1.55 K = 6×10^5	2.78 (4.4 dB)
8	IV	YES	T.T.	a = 68000 b = 2460 d 6×10^6 x = 1.45 K = 9.5×10^5	1.62 (2.1 dB)
9	IV	NO	VOICE	a = 80,349 b = 4797 d = 14,300 x = 1.266 K = 157,144	1.6 (2.0 dB)
10	IV	YES	VOICE	a = 38,000 b = 4558 d 3×10^6 x = .715 K = 170,000	1.1 (.380 dB)

TABLE 4-5 - SUMMARY OF CALCULATED RESULTS

References - Chapter 4

1. M.J. Powell, "An Efficient Method for Finding the Minimum of a Function of Several Variables Without Calculating Derivatives", Computer Journal, No. 7, 1964.

2. W.I. Zangwill, "Minimizing a Function Without Calculating Derivatives', Computer Journal 10, 1967, pp. 293-296.

3. W.A. Novick, "Investigation and Optimum Design of the Generalized Second-Order Phase-Locked Loop", Doctoral Dissertation, New Jersey Institute of Technology, 1976.

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6. R. Burden and J.D. Faires and A.C. Reynolds, Numerical Analysis Second Edition, P.W.S. Publishers, Boston, 1981.

7. A. Acampora and A. Newton, "Use of Phase Subtraction to Extend the Range of a Phase-Locked Demodulator", RCA Review, Vol. XXVII, No. 4, December, 1966.

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CHAPTER V - EXPERIMENTAL VERIFICATION

5.1 - INTRODUCTION

In the previous sections optimum PLDs were sought and found for specific loop configurations and system conditions. The optimizations are based upon models that have been generated to account for the threshold phenomenon and upon assumptions that were made to facilitate the analysis. These models have previously been used in various studies which produced conclusions that were confirmed by test.

Novick [Ref. 1] invoked the voice modulation model and threshold criterion to optimize several ERPLD systems including our CASE 6. He confirmed the results by means of digital computer simulations. With less justification [Ref. 1, Page 98] he applied the same threshold criterion to the ERPLD with test tone modulation and obtained results that were in reasonable agreement with the measured values obtained by Acampora and Newton [Ref. 2]. Klapper and Frankle [Ref. 3 Page 136] have successfully used the test tone threshold model to establish design criteria for PLDs.

To confirm the results obtained by computation in this study we have gone into the laboratory to build the circuits, simulate the conditions and determine the threshold levels by measurement. We have adopted the commonly accepted definition for the onset of threshold and identify it on the diagram of Figure 1-5 [Ref. 3, Page 24]. $(CNR)_{TH}$ is the abscissa value for which the SNR is 1 dB below the extrapolation of the FM improvement line. There is no simple, direct measurement that can be made of the threshold point. One must obtain the necessary data for plotting a curve of detected SNR Vs. input CNR and then identify the threshold on the curve in accordance with the definition given above.

Noise measurements are made at the input to the detector and also at its output. These measurements must be made in bandwidths which correspond to those that are assumed in the calculations.The calculated input CNR is based on a flat noise spectrum of specified width whether or not a predetection filter precedes the PLD, and the same bandwidth must be used for measuring CNR.

5.2 - TEST PROCEDURE

The test setup for performing these measurements is shown in figure 5-2.

Sinusoidal modulation was chosen for the threshold measurements because of the difficulties inherent in generating a Voice-like signal and in distinguishing that signal from noise. A carrier wave of 455 KHz is frequency modulated by a 1KHz sinewave to the desired modulation index and the resulting modulated wave is combined with additive white gaussian noise in a summing amplifier. The resulting waveform is fed to the Phase locked detector either directly or via the IF band-pass filter. At the detector output, a low pass filter eliminates the high frequency content of the detected wave and allows the audio to be measured by a true RMS voltmeter. With the noise source either strongly attenuated or completely disconnected from the summing amplifier, the output signal voltage can be measured by itself. A signal cancelling circuit is included in the setup so that the output noise can be measured by itself even when it is produced in the presence of signal. The Wave Analyzer is used to determine the noise spectrum and the noise power at the input to the detector.

A measurement run for a particular PLD proceeds as follows: The loop filter parameters are chosen to correspond to the optimization

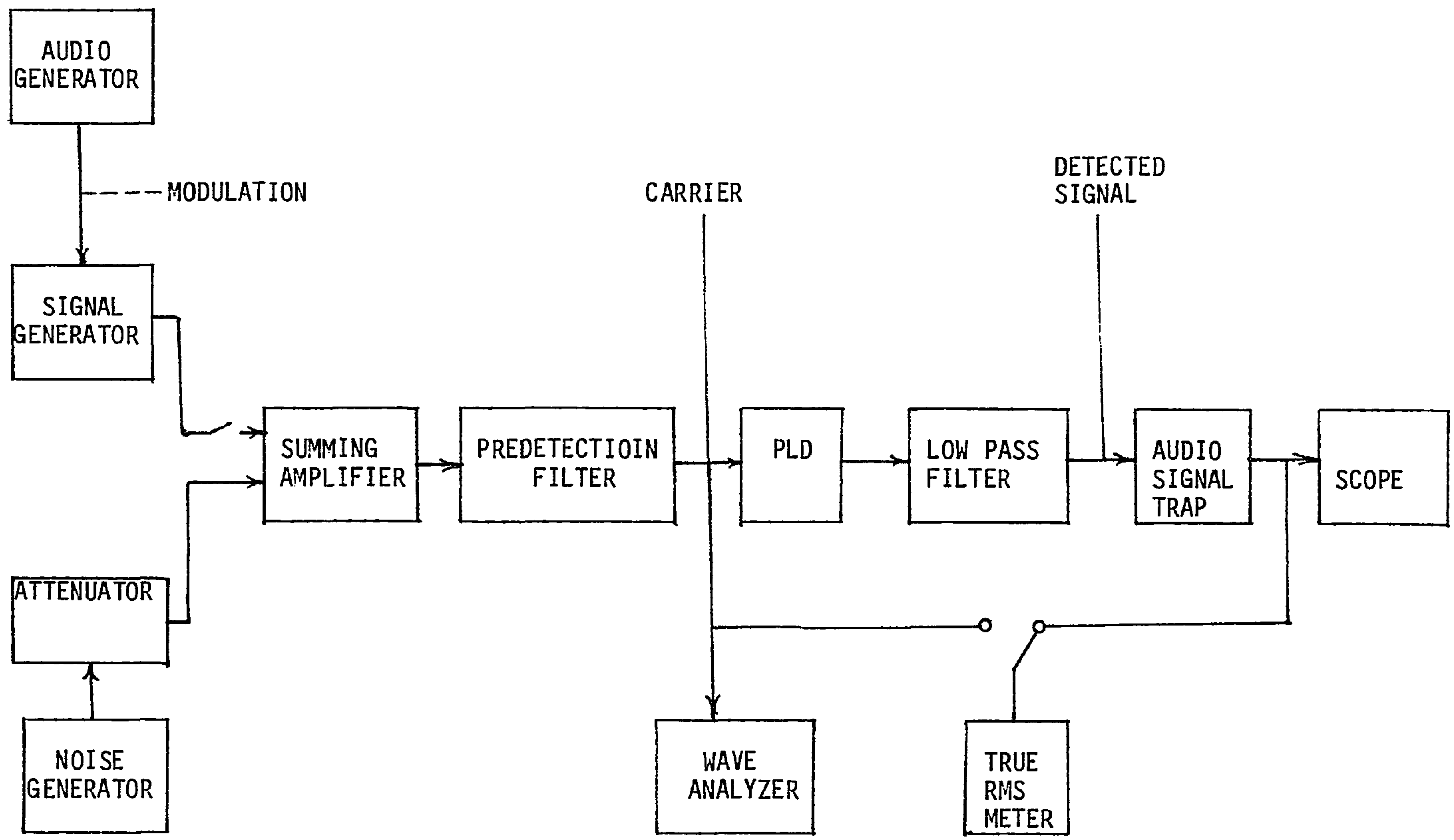


FIGURE 5-2 TEST SET UP

determined in Chapter 4. This includes a proper choice for carrier amplitude because the sensitivity of the phase detector depends upon the input level. The audio voltage is then adjusted to produce the desired modulation index and the output signal is read on the voltmeter. The input noise level is adjusted so that the input CNR measured in bandwidth Bp is well above threshold. The signal cancellation circuit is switched in and the output noise is read by the voltmeter. This establishes a point on the SNR Vs. CNR curve. The procedure is then repeated for different settings of the attenuator and thus for different noise levels, yielding additional points on the curve.

A detailed discussion about the measurements and the results will be presented following a description of the components that comprise the test setup.

5.3 - THE COMPONENTS

The test arrangement of figure 5-2 was realized by a combination of standard test equipment and specially designed circuits. These components are identified and their salient features are described in the following two sections.

5.3.1 - SPECIALLY DESIGNED CIRCUITS

At the core of the laboratory arrangement is the PLD that is undergoing test. It consists of the four components illustrated in figure 5-3.

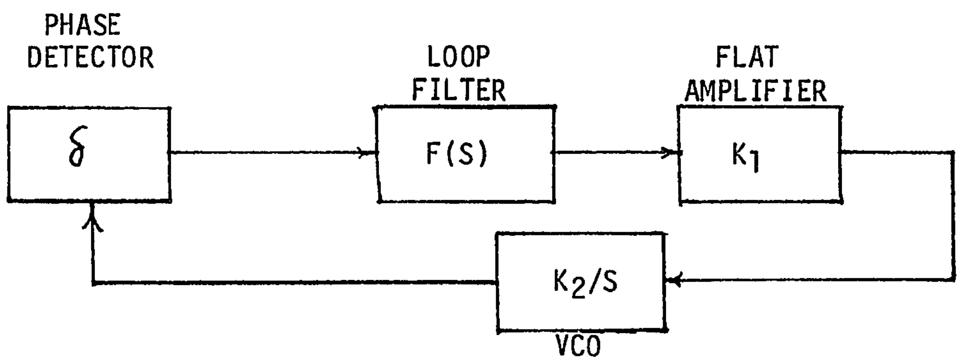


Figure 5-3 PLD

The flat amplifier of gain K_1 is isolated from the loop filter only for analytical convenience but is actually implemented as part of the loop filter having response $K_1F(S)$ which will be described presently. The phase detector and VCO are commercial units whose performance characteristics were measured and will be described.

The phase detector, a Hewlett Packard double balanced mixer model 10534A is driven by two waveforms which may or may not be at the same frequency. When the loop is closed and locked, the frequencies are identical and an output is developed which depends upon the phase difference between the two inputs. In the linear model the output was assumed to be $Ve = \delta(\theta_i - \theta_r)$ and in the lab we determine the value of the constant, δ (Volts/Rad) by test. A double frequency component accompanies the error signal at the detector output and must be filtered out before the measurement can be made. Therefore the error signal is taken at the output of a low pass filter as shown in the diagram of figure 5-4. One of two methods can be used to measure the detector sensitivity. With the carrier frequency offset from the VCO resting frequency and the loop locked and stressed, a direct measurement of phase difference and corresponding output error voltage leads to a characteristic curve of v_e Vs. $\Delta\theta$. The slope of this curve is the sensitivity of the detector.

Alternatively a more convenient technique can be employed with the loop open and the carrier offset from the VCO.

Let the carrier be represented by $v_c = V_c \sin w_c t$ and the VCO output by $V_{co} \cos w_{co} t$. An instantaneous phase difference is developed which may be represented as $\Theta(t) = (w_c - w_{co})t$.

The filter output appears as a sinewave at the beat frequency w_c-w_{co} and can be described by $v_e = V_e \sin (w_c - w_{co})t = V_e \sin \theta(t)$

But for the linear model:

$$= \left.\frac{d\, v_e}{d\theta}\right|_{\theta=0} = \left. V_e \cos (w_c - w_{co})t \right|_{\theta=0} = V_e \,\frac{\text{Volts}}{\text{Rad}}$$

Thus the value of δ is the peak value of the observed beat frequency signal. It is found to be independent of frequency and insensitive to small changes in a relatively large VCO signal (i.e. $V_{co} \gg Vc$). However, it does depend upon the carrier amplitude and is measured by means of the test configuration shown in figure 5-4.

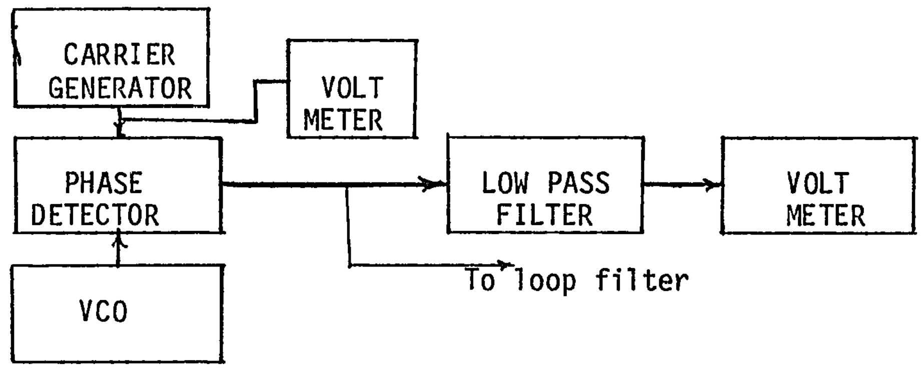


FIGURE 5-4- - MEASUREMENT

When loaded, the VCO drives the detector with an almost square waveform. Its amplitude is variable and was adjusted to approximately 1.2 Vp.p. With the phase detector loaded by the loop filter, but the loop open, the beat signal amplitude was measured as a function of carrier level. The results are plotted in figure 5-5. For carrier levels below $400mv_{rms}$ the curve is linear and the detector sensitivity may be expressed as a function of carrier voltage as

$$\delta = 0.6\ V_c\ \text{mv/Rad} \qquad (5\text{-}1)$$

The simplicity of the beat frequency method makes it feasible to dynamically adjust the value of δ to a desired level by controlling the carrier amplitude.

VOLTAGE CONTROLLED AMPLIFIER (VCO)

The VCO section of integrated circuit monolithic function generator XR-2206 is employed as the VCO in the PLL. Its sensitivity, designated as K_2 (KHz/Volt), may be determined by test in the laboratory by direct measurement, or by an indirect method. In the direct method a variable dc voltage is applied to the control input of the VCO and

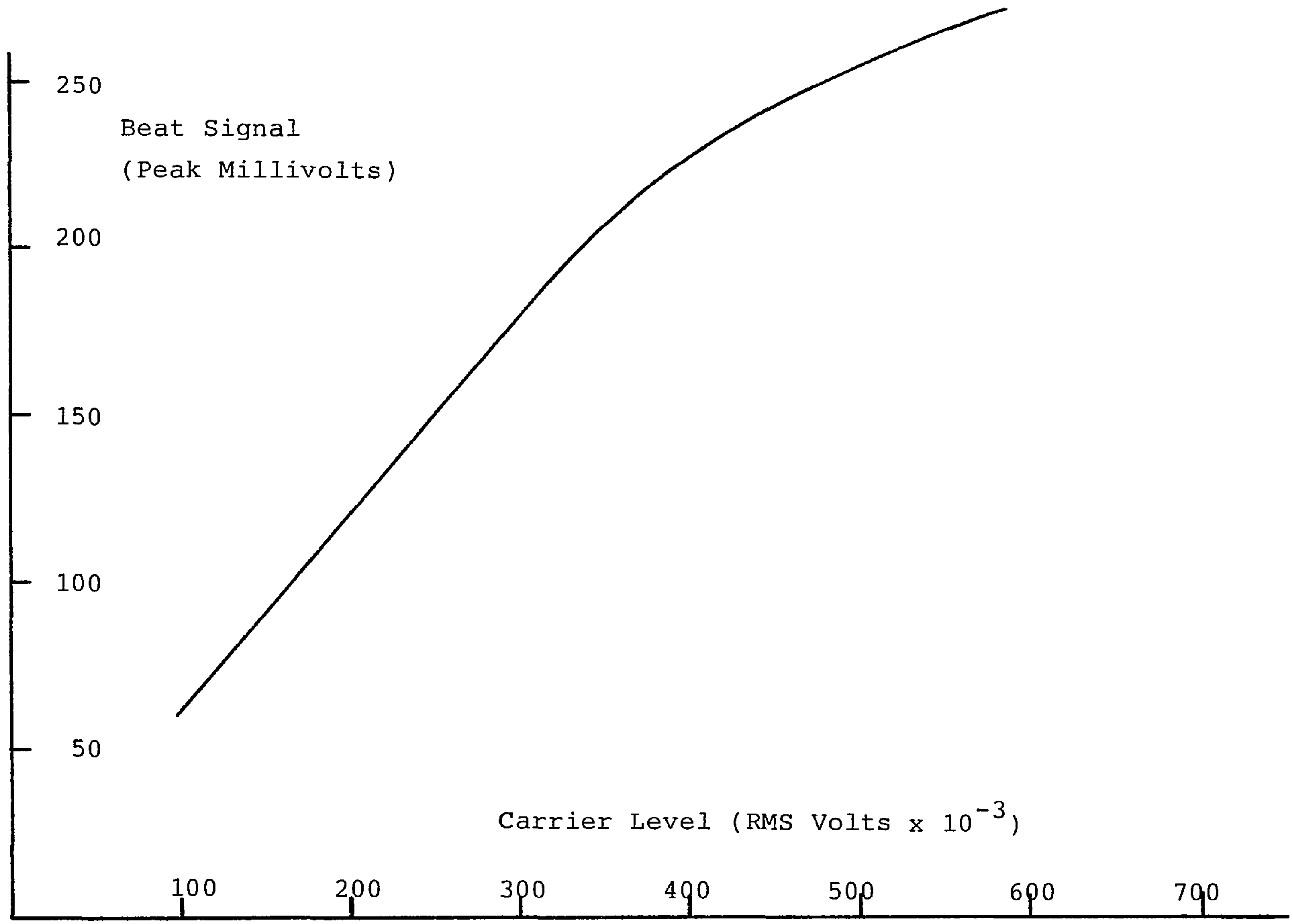


Fig. 5-5. Phase Detector Sensitivity

the corresponding output frequency is measured. One obtains a plot of VCO frequency vs control voltage and its slope is the desired constant, K_2. The shortcoming of this method is that it only reveals the dc value of the constant but doesn't indicate any frequency dependence. A dynamic method utilizing the test setup shown in Figure 5-6 was used instead.

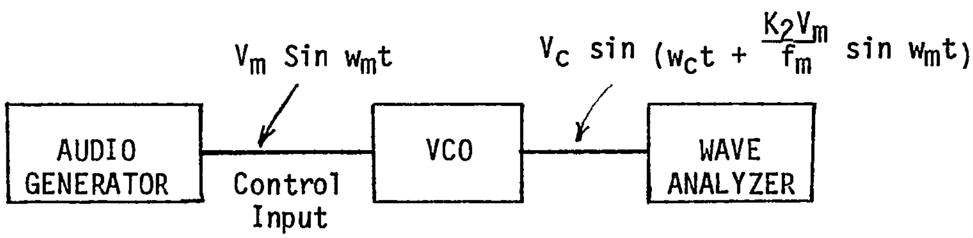


FIGURE 5-6 MEASUREMENT OF K_2

The audio signal generator applies a voltage, $v_m = V_m \text{ Sin } w_m t$ to the control input of the VCO and frequency modulates the output to a maximum deviation, Δf (Hz).

$$\Delta f = K_2 V_m$$

The modulated waveform is described by:

$$V_o = V_c \cos (w_o t + \beta \cos w_m t) \,, \qquad (5\text{-}2)$$

where the modulation index, β, is given by

$$\beta = \frac{\Delta f}{f_m} = \frac{K_2 V_m}{f_m} \qquad (5\text{-}3)$$

Equation 5-2 may be expanded to emphasize the spectral content of the FM waveform as [Ref. 4, Pages 534-35]:

$$\begin{aligned} v_o = J_o(\beta)\cos w_c t & - J_1(\beta)\sin(w_c + w_m)t + \sin(w_c - w_m)t \\ & - J_2(\beta)\cos(w_c + 2w_m)t + \cos(w_c - 2w_m)t \\ & + J_3(\beta)\sin(w_c + 3w_m)t + \sin(w_c - 3w_m)t \end{aligned} \tag{5-4}$$

Here, $J_n(\beta)$ is the Bessel function of the first kind and n^{th} order with argument, β.

A graphical representation of the frequency spectrum of the modulated wave is shown in figure 5-7.

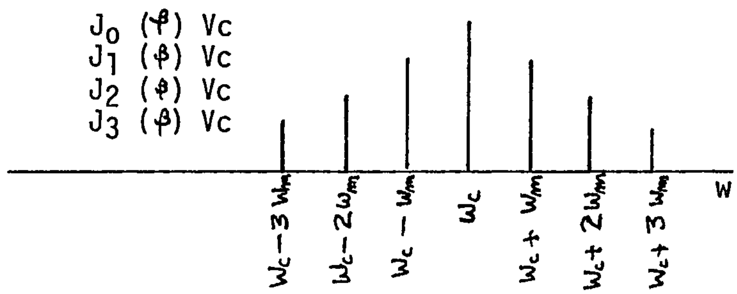


FIGURE 5-7 - SPECTRUM OF SINEWAVE MODULATED FM WAVE

If the passband of the wave analyzer is narrower than twice the modulation frequency (i.e. $Bp < 2 f_m$), then the analyzer will read out the magnitude of one and only one frequency component. We utilize the fact that $J_0(2.4) = 0$, center the analyzer passband on the carrier frequency, adjust V_m until a null is indicated and then apply eq. (5-3)to calculate K_2.

$$K_2 = \frac{f_m \beta}{V_m} = 2.4 \frac{f_m}{V_m} \text{ (KHz/Volt)} \tag{5-5}$$

Applying this method we obtain for the VCO:

$$K_2 = 135 \text{ KHz/Volt}$$

For nulling the carrier frequency term we have

$$\Delta f = \beta f_m = 2.4 f_m.$$

At higher frequencies (i.e. large f_m), the method becomes impractical because the frequency deviation, Δf, becomes excessive. A variant of the method that requires less frequency deviation and measurement of the first side frequency component rather than of the carrier was used. Because $J_1(.5) = .24$ and $J_1(.2) = .1$, we adjust the modulation amplitude, V_m, until the first side frequency amplitude is 0.24 V_c or 0.1 V_c and then use $K_2 = \frac{.5f_m}{V_m}$ or $\frac{.1f_m}{V_m}$ respectively.

From these tests we conclude that K_2 is constant for modulating frequencies up to 200 KHz.

Because of an impedance mismatch the VCO is unable to drive the phase detector directly and had to be coupled through an emitter follower driver.

LOOP FILTER AND AMPLIFIER

Active filters based on high gain broad band differential input operational amplifiers are used to implement the optimized loop filters [Ref. 6,7]. Figure 5-8 shows a schematic diagram of the circuit arrangement for filter IV and the foreward loop gain K_1.

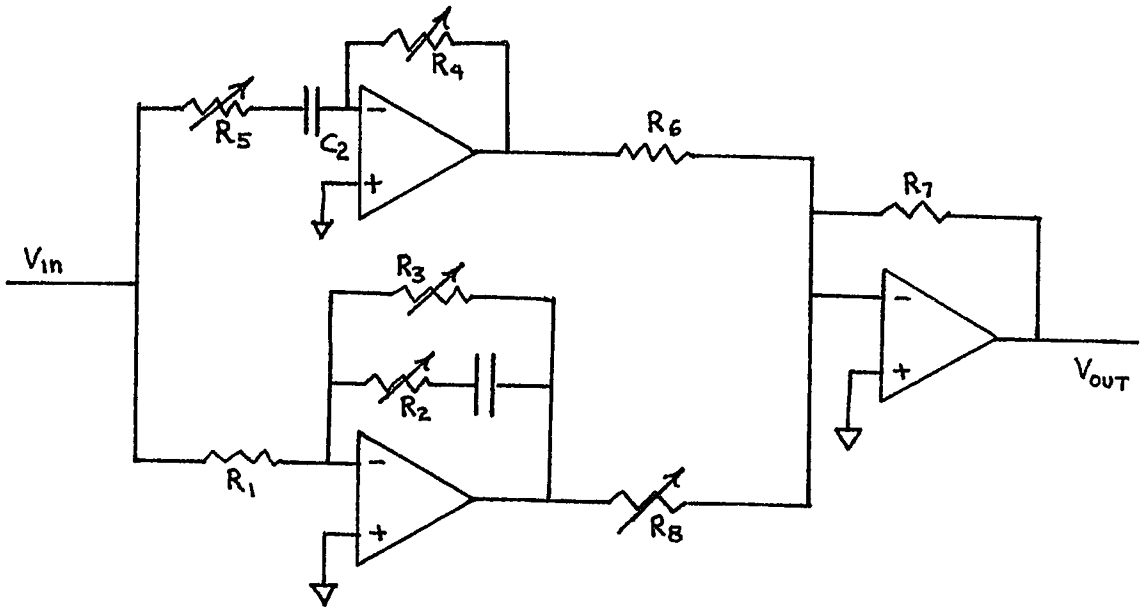


FIGURE 5-8 - LOOP FILTER IMPLEMENTATION

The op amps are type LF357 which were chosen for their large gain bandwidth product. For these devices this figure of merit is 20 MHz, which enables realization of the required filter characteristics of gain and frequency response without regard to any "chip" imposed limitation.

The function to be implemented is

$$K_1F(S) = K_1 \quad \frac{s/a + 1}{s/b + 1} + \frac{\alpha}{K}\,\frac{s}{s/d + 1} \tag{5-6}$$

where $K_1 = \dfrac{K}{\delta K_2}$ is the overall dc gain of the circuit , $\left.\dfrac{V_{out}}{V_{in}}\right|_{dc}$.

The gain constants in eq (5-6) carry the significance indicated by figure 5-3.

Five variable resistances are shown in Figure 5-8 and these will be shown to provide the five degrees of freedom implied by the five system parameters in loop filter IV.

For a high gain op amp with feedback path impedance $Z_2(s)$ and source impedance $Z_1(s)$ as shown in FIGURE 5-9 the response may be calculated from

$$F(S) = \frac{V_{out}(S)}{V_{in}(S)} = \frac{Z_2(S)}{Z_1(S)} \qquad \text{[Ref. 6,7]}$$

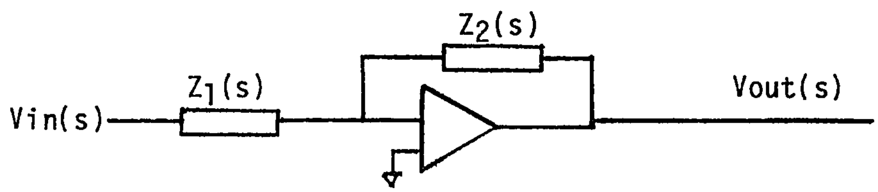


FIGURE 5 - 9 - GENERAL ACTIVE FILTER

We formulate the filter response of figure 5-8 as the sum of two functions. First consider the lower signal path and write

$$K_1\ F_L(S) = \frac{R_3\left(R_2 + \frac{1}{SC_1}\right)}{\left(R_3 + R_2 + \frac{1}{SC_1}\right.} \cdot \frac{1}{R_1} \cdot \frac{R_7}{R_8} \qquad (5\text{-}7)$$

$$= \frac{R_7R_3}{R_8R_1} \quad \frac{SC_1R_2 + 1}{SC_1(R_3 + R_2) + 1}$$

Then consider the upper signal path and write

$$K_1\ F_u(S) = \frac{R_4}{R_5 + 1/(SC_2)} \cdot \frac{R_7}{R_6} = \frac{R_7R_4C_2}{R_6} \quad \frac{S}{SC_2R_5+1} \qquad (5\text{-}8)$$

Each branch is considered as though the other did not exist because any attenuation due to loading is exactly compensated by the increased gain of the third op-amp due to the same loading. See APPENDIX C. Combining eqs (5-7) and (5-8) we obtain an expression for the total transfer function of the circuit.

$$K_1\ F(S) = K_1F_L(S) + K_1F_u(S) =$$

$$\frac{R_7R_3}{R_8R_1}\ \frac{SC_1R_2+1}{SC_1(R_c+R_3)+1} + \frac{R_7R_4C_2}{R_6}\ \frac{S}{SC_2R_5+1}$$

$$= \frac{R_7R_3}{R_8R_1}\ \frac{SC_1R_2+1}{SC_1(R_2+R_3)+1} + \frac{R_4R_8R_1C_2}{R_6R_3}\ \frac{S}{SC_2R_5+1} \tag{5-9}$$

Identifying eq (5-9) with eq (5-6) leads to the following set of equations which relate the system parameters to the circuit component values:

$$a = \frac{1}{R_2\ C_1}$$

$$b = \frac{1}{(R_3+R_2)\ C_1}$$

$$d = \frac{1}{R_5C_2}$$

(5-10)

$$K = K_1K_2\delta = \frac{R_7R_3}{R_8R_1}\quad K_2$$

$$\alpha = \delta\ K_1K_2\ \frac{R_4R_8R_1C_2}{R_6R_3} = \frac{R_7R_3}{R_8R_1}\ \frac{R_4R_8R_1C_2}{R_6R_3}\ K_2\,\delta$$

$$\propto = \frac{R_7 R_4 C_2 K_2}{R_6}$$

These five relations translate into a design procedure for implementing any of the optimum systems involving loop filter IV.

K_2 and δ are known from eqs (5-1) and (5-5). After choosing convenient values for the five fixed components: C_1, C_2, R_1, R_6, and R_7, one then calculates values for the remaining five variable components in order to realize the optimum design according to eqs.(5-10) as follows:

Given	Choose	
a	$R_2 = \dfrac{1}{ac_1}$	(5-11)
b	$R_3 = \dfrac{1}{bc_1} - R_2$	(5-12)
d	$R_5 = \dfrac{1}{dc_2}$	(5-13)
K	$R_8 = \dfrac{R_7 R_3 K_2}{K\ R_1}$	(5-14)
$\propto$	$R_4 = \dfrac{R_6}{R_7 C_2 K_2}$	(5-15)

Of the five calculated resistances only two [eqs (5-12) and (5-14)] depend upon other calculated values. But because the zero break-frequency, "a", will be much greater than the pole break-frequency, b, R_3 is essentially independent of R_2, and therefore of "a". Thus four of the five system parameters may each be independently adjusted by means of a single variable resistance. In order to change the value of b alone it is necessary to vary both R_3 and R_8.

Summer and Predetection Filter

These two functions are implemented on a single chassis that was designed and built for use in an experimental procedure that had previously been performed in the same lab. It is a Butterworth filter with a nominal passband of 60 KHz centered at 455 KHz. Its transfer characteristic was measured and is shown as $H_{if}(f)$ in figure 5-10.

By means of graphical integration based upon the curve we calculate the bandwidth of the filter to be

$$BW = \frac{\sum H_{if}\ (f_n)\ ^2 \Delta f}{H_{if}\ (455\ \text{KHz})^2} = 58\ \text{KHz} \qquad (5\text{-}16)$$

Cases 5, 6, 8 and 10 which require IF filtering were optimized in chapter 4 for use with a 35 KHz filter bandwidth. That value was chosen to enable comparisons with the results of earlier investigations. With the computer programs written and in place it is a straightforward, simple matter to change the bandwidth to conform to that of the filter

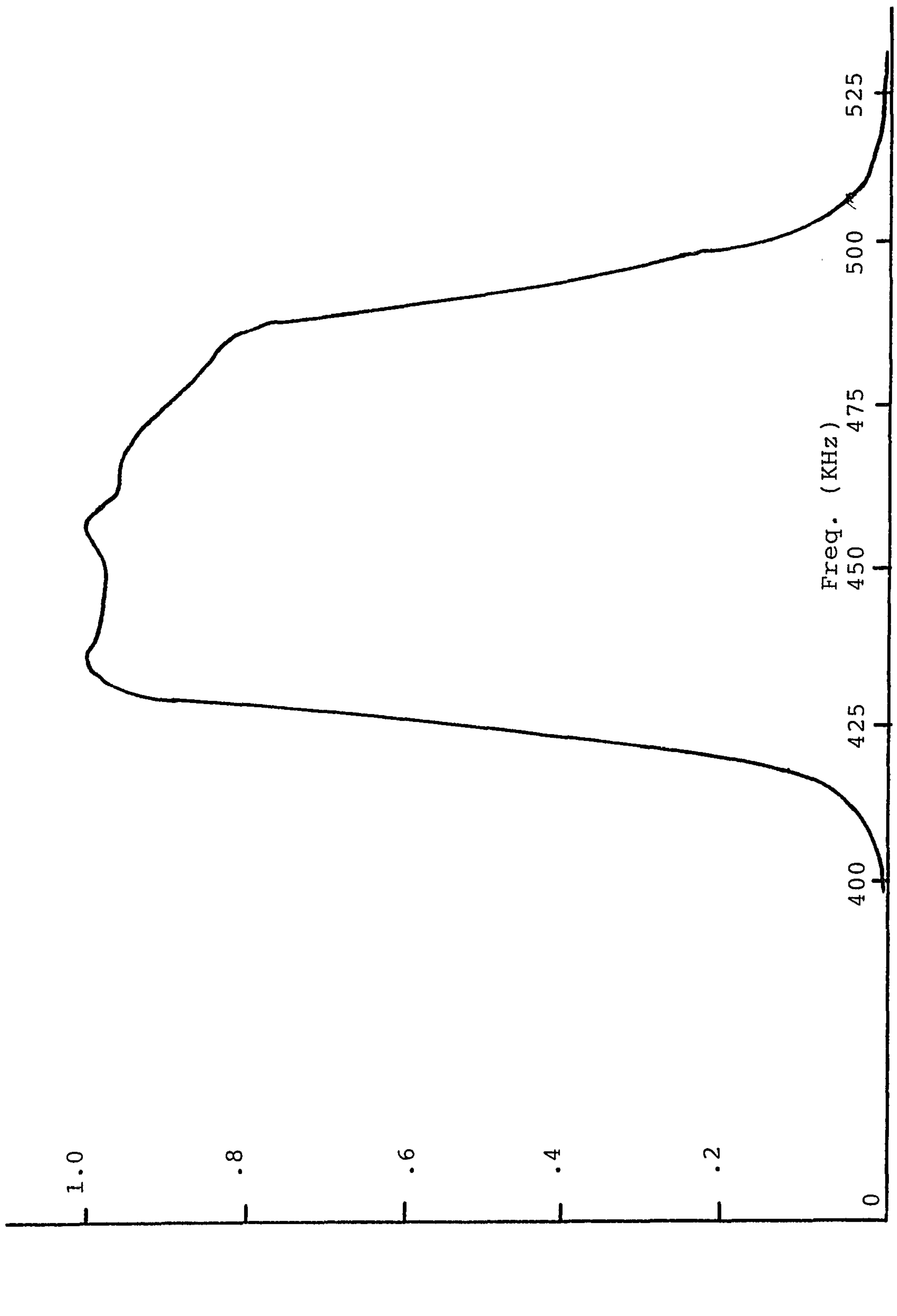


Fig. 5-10. Normalized Power Spectral Response of Predetection Filter

which is available in the lab and obtain a new set of optimum parameters. Case(8)was optimized for a prefilter bandwidth of 58 KHz and the results are presented in the next section where they are compared with the experimental results for the same system.

Audio Signal Trap

The signal-to-noise-ratio measurement requires independent measurements of output signal and output noise in the presence of signal. The latter is facilitated by the audio signal trap which eliminates the signal while passing the noise. It is a notch filter that was designed for this application by S. K. Lee [Ref. 5] and achieves 60 dB signal suppression at 1 KHz. The 3 dB notch width is only 16 Hz and therefore affects the noise measurement only negligibly.

5.3.2 - Commercial Test Equipment

Standard laboratory test equipment was used for the remainder of the functions illustrated in figure 5-2. These units are identified here as follows:

Audio Generator and Signal Generator

Both of these functions are served by Wavetek Model 800 signal generators. This unit features a "VCG Input and can thus be used as an FM signal generator. The sensitivity of the modulator section was measured by the method that was outlined in section 5.3.1 in connection with the V.C.O. and found to be 1 KHz/mv for low frequency modulation and its frequency response is shown in figure 5-11. This characteristic must be known if the modulation index is to be correctly set and for measuring the closed loop response which will be described in section 5-4.

Noise Generator

General Radio Model 1390-B Noise Generator outputs spectrally flat, gaussian noise distributed over either 500 KHz or 5 MHz. Its flatness was confirmed by scanning with the 1 KHz and 3 KHz filters of the Hewlett Packard wave analyzer model 310A.

Attenuator

This is a Leeds and Northup W-10367 Decade Attenuator. It features a broad bandwidth and is adjusted by decade dials which are calibrated in dB. With the attenuator it is possible to vary the level of the additive noise without modifying the setting of the noise generator and thus avoid a source of drift during the measurement.

Low Pass Filter

The output of the PLD is filtered to eliminate any residual higher order multiplier products in the phase discriminator output. This is most important for those cases wherein the loop filter does not roll off significantly at high frequencies.

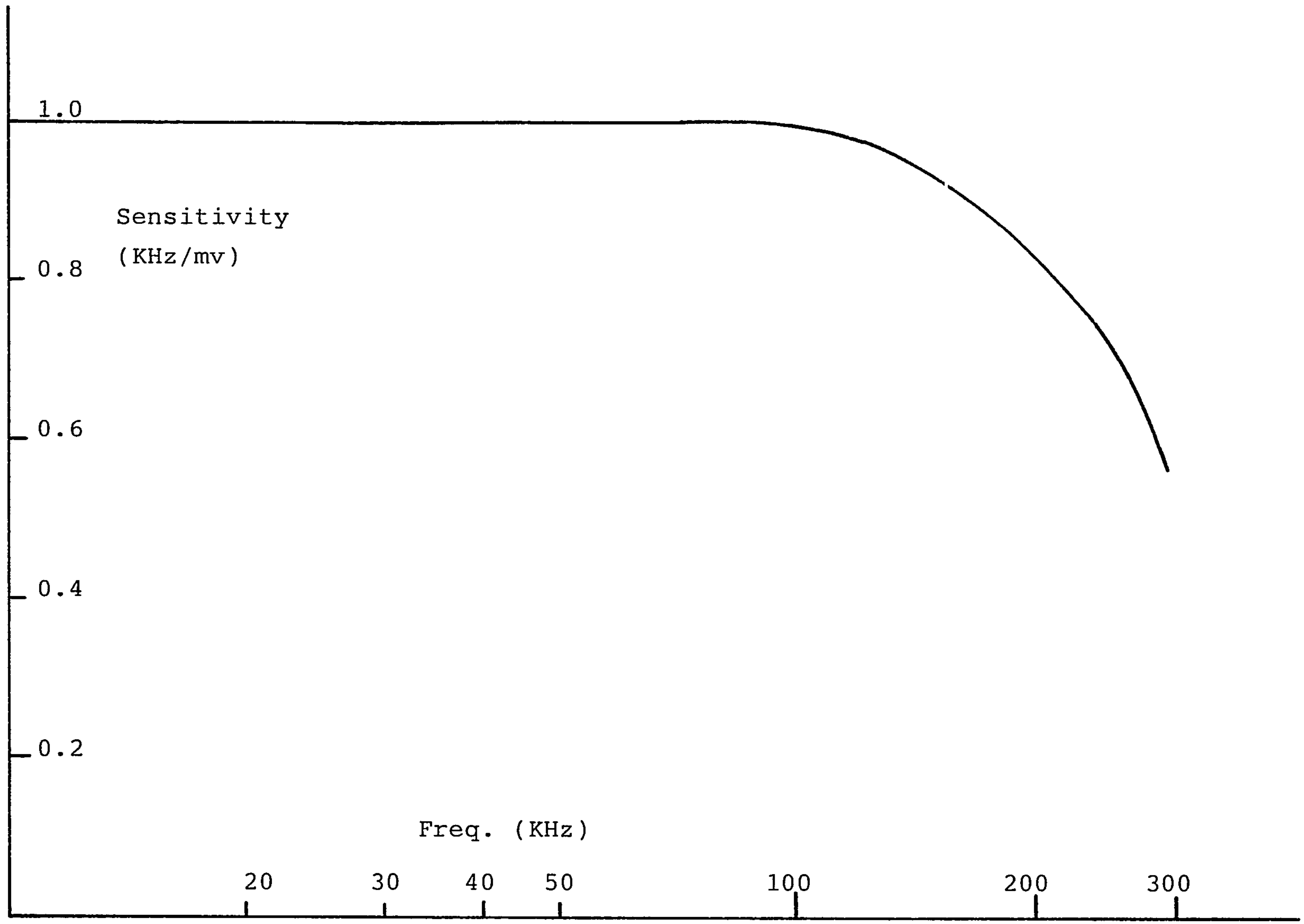


Fig. 5-11. Modulation Sensitivity of Wavetek Model 180 VCG

A Krohn Hite model 3550 adjustable filter is used in its low pass mode. It has a cutoff frequency, adjustable up to 200 KHz and features a rapid falloff beyond cutoff of 80 dB/decade.

Figure 5-12 is a plot of the measured frequency response of the filter.

By adjusting the critical frequency to 3.3 KHz we were able to measure the signal-to-noise ratio in the same bandwidth as that used by Acampora and Newton [Ref. 2].

True RMS Meter.

All the voltage measurements, whether of periodic signals or of noise waveforms, were made with the same instrument. It is a Ballantine model 323 true RMS meter. This unit has a wide frequency range which encompasses the total spectrum of interest in these experiments. It also features a variable time constant which is particularly effective for noise measurements in the vicinity of threshold.

Oscilloscope

A scope is used to display the output noise and the noise spikes which occur near and below threshold. Observation of these spikes is used for dynamically adjusting the VCO resting frequency with the loop in the locked condition.

The output of the final op-amp of the active filter, V_{out} in figure 5-8, is directly coupled to the VCO control input as shown in

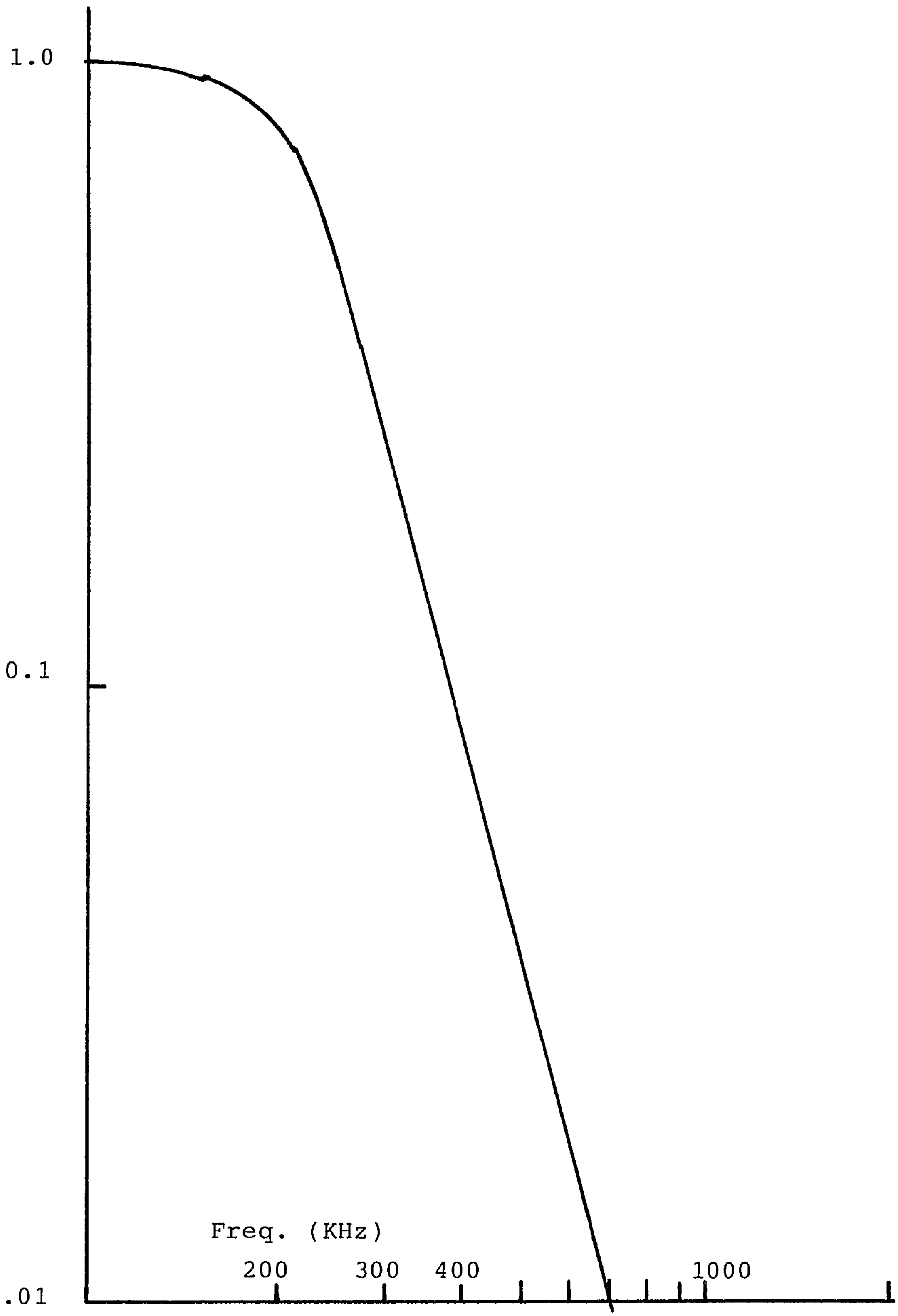


Fig. 5-12. Response of Krohn Hite Filter

figure 5-3. A dc offset in V_{out} will cause a shift in the VCO frequency and a concomitant dc component of error voltage in the output of the phase detector. This would be manifest as a vertical shift of the error voltage waveform represented in figure 2-3 and a predominance of either positive or negative spikes. While observing the output noise waveform, one adjusts the VCO frequency to establish approximate balance between positive and negative spikes.

5.4 - Experimental Results

Cases (1) and (8) were selected for experimental confirmation of the computed optimum designs of chapter 4. For each case the circuit component values were calculated with the aid of equations (5-11) through (5-15) based on the system parameters specified by the methods of chapter 4.

5.4.1 - Case (1)

To implement loop filter I for case 1, we let $\alpha = 0$ and from eq. (5-15) obtain $R_4 = 0$. This is equivalent to eliminating the differentiator section in figure 5-8 which renders eq. (5-13) for R_5 irrelevant and results in the simpler configuration with the values shown in figure 5-13.

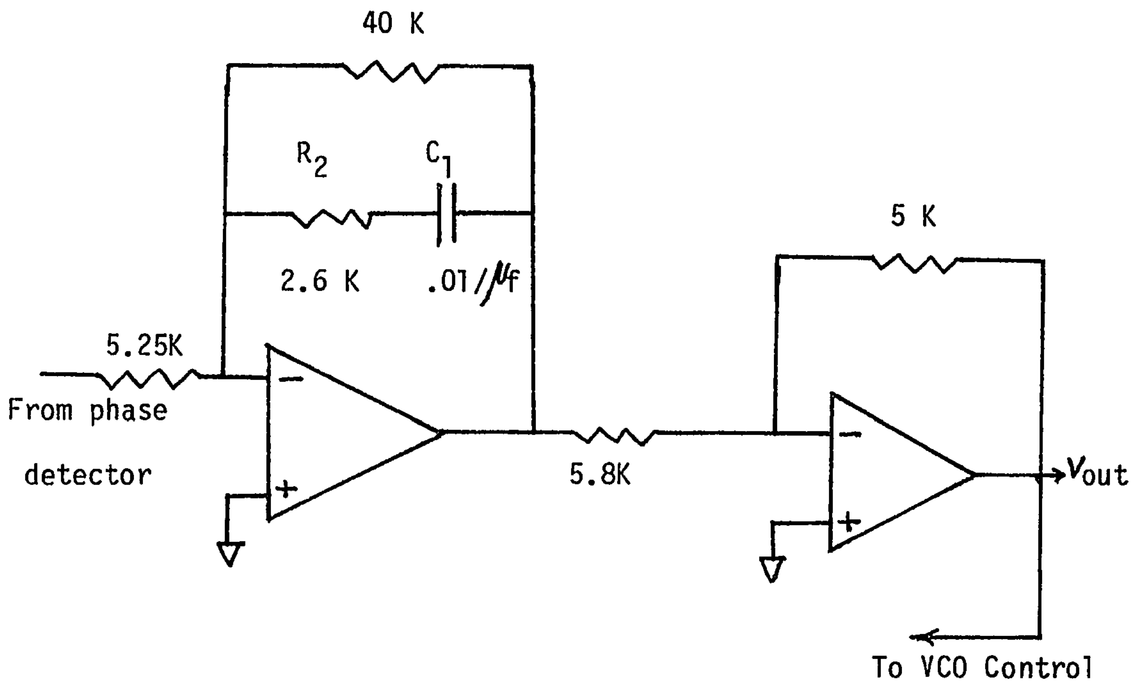


FIGURE 5-13 ACTIVE FILTER FOR SECOND ORDER LOOP

This loop corresponds to the first row entry in Table 4-5 where:

$$a = 38{,}000 \text{ (Rad/Sec)}$$

$$b = 2350 \text{ (Rad/Sec)}$$

$$K = 560000 \text{ (Sec}^{-1}\text{)}$$

and ρ = 4.9 dB.

The implementation is verified by measuring the closed loop response and comparing it to the calculated response which is plotted as Figure

4-1. The technique for measuring the closed loop response is based upon an analysis of the linear model which is reproduced here in figure 5-14.

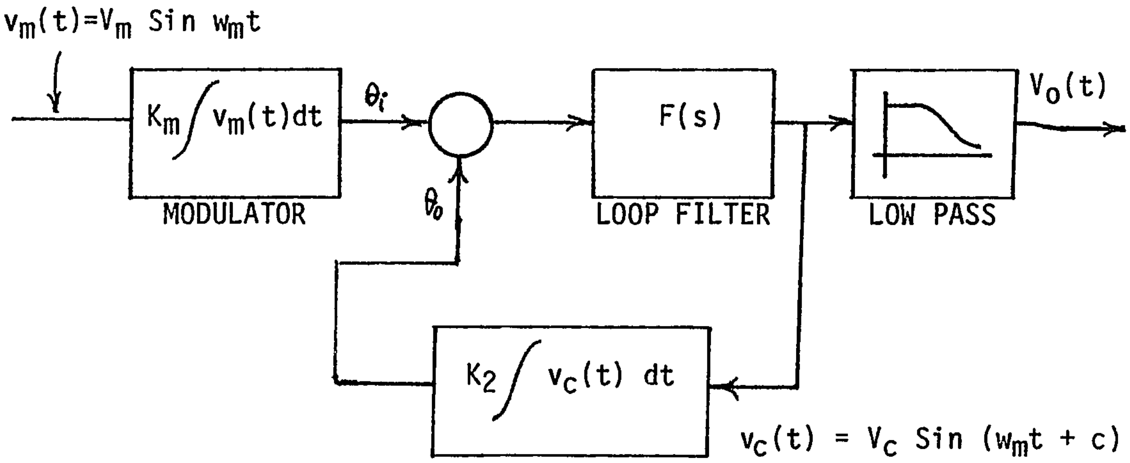


FIGURE 5-14 LINEAR MODEL FOR RESPONSE TEST

Let the modulator sensitivity be K_m and let $V_m(t)$ be a sinusoidal modulating signal. The modulator and the VCO are perfect integrators so we have

$$V_c(t) = \frac{1}{K_2} \dot{\theta}_o(t)$$

$$V_m(t) = \frac{1}{K_m} \dot{\theta}_i(t) .$$

$$\dot{\theta}_o(t) = K_2 \; v_c(t) = K_2 \; V_c \sin (W_m t + \theta c)$$

$$\text{and } \theta_o(t) = - \frac{K_2 \; V_c}{W_m} \cos (W_m t + \theta_c) . \tag{5-17}$$

similarly

$$\dot{\theta}_i(t) = K_m V_m \sin w_m t$$

$$\text{and} \qquad \theta_i(t) = -\frac{K_m V_m}{W_m} \cos w_m t \quad . \tag{5-18}$$

The magnitude response of the loop is the ratio of the magnitudes of the expressions in eqs. (5-17) and (5-18). Thus

$$H(f) = \left| \frac{\theta_o(t)}{\theta_i(t)} \right|_f = \frac{K_2 V_c/w_m}{K_m V_m/w_m} = \frac{K_2}{K_m} \frac{V_c(f)}{V_m(f)} \tag{5-19}$$

By scanning through the frequency range and measuring the ratio of loop output to modulation input one obtains a measure of the closed loop response. The results obtained in this way are indistinguishable from the curve of figure 4-1.

For convenience the summer and predetection filter are used in the threshold determination even though case 1 involves no predetection filtering. The justification of this approach and an estimation of the expected error are made with respect to figure 4-1.

Recall that the filter bandwidth, B_p, is 58 KHz and therefore the equivalent input noise PSD referred to the baseband is $(1/2)B_p$ [Sec. 1.2.1] = 29 KHz. But from figure 4-1 we find that the bandwidth of the loop is only about 10 KHz. To estimate the error due to the use of the filter we cite eq. (2-3) for the noise related error phase.

$$\overline{\theta_n^{\,2}} = \frac{2\eta}{A^2}\int_0^\infty \left| H(jw)\right|^2 df$$

and divide the integral into the domain which is included within the filter bandwidth and one which is beyond it. Thus,

$$\int_0^\infty \left|H(jw)\right|^2 df = \int_0^{Bp/2} \left|H(jw)\right|^2 df + \int_{Bp/2}^\infty \left| H(jw)\right|^2 df \qquad (5\text{-}20)$$

and evaluate the two integrals on the right side of the equation. From Figure 4-1 we note that for high frequencies (i.e. $f > Bp/2$) $H(jw)$ decreases at a constant rate of -20 dB/decade.

$$\left|H(f)\right| = \frac{A}{f}$$ and the constant, A, is found from the curve

to be A = 6000. Thus

$$\int_{Bp/2}^\infty \left|H(jw)\right|^2 df = (6 \times 10^3)^2 \int_{2.9 \times 10^3}^\infty \frac{df}{f^2} = (6 \times 10^3)^2 \cdot \left.\frac{-1}{f}\right|_{29000}^\infty$$

$$= \frac{3.6 \times 10^7}{2.9 \times 10^4} = 1.25 \times 10^3$$

Also from the curve we calculate

$$\int_0^{B_p/2} \left| H(jw) \right|^2 df \approx 12 \times 10^3$$ and estimate the error due to

neglecting the last integral in eq. (5-20) to be 0.4 dB. That is, the measured threshold with the filter is 0.4 dB less than it would be without the filter.

At the filter output we measure $(CNR)_i$ in a 58 KHz bandwidth. For a meaningful comparison with the predicted threshold this value must be corrected to that which would be measured in a 35 KHz bandwidth.

$$dB(CNR)_{35} = 10 \log (CNR)_{35} = 10 \log \frac{58}{35} (CNR)_{58} .$$ Whence

$$\rho_{35} = \rho_{58} + 2.2 \text{ dB}$$

where ρ is the threshold level expressed in dB.

At the output, the measured noise includes a component which results from nonlinear distortion in the loop and imperfection in the signal trap. This component, for all the measurements, ranged between 0.1 dB and 0.5 dB at threshold and was mathematically corrected for each measured point.

The carrier amplitude is experimentally set to yield the required phase detector sensitivity, δ , and the modulation amplitude is set to produce a modulation index, β = 10. These values are: Carrier in = 115 mv and Signal out = 540 mv.

TABLE 5-1 presents the raw data in the first two columns and the corresponding calculated values in the last two.

Raw Data		Calculated Values	
Measured Noise in	Measured Noise out	Corrected $(CNR)_{35}$	Corrected $(SNR)_o$
0	4 mv		
22.5 mv	8.5	16.4 dB	36.9 dB
28.2	10.4	14.4	34.9
34.8	12.9	12.6	32.8
43.7	16.2	10.6	30.6
49.0	18.2	9.6	29.6
54.5	20.2	8.7	28.6
62.0	23.0	7.6	27.5
69.5	26.5	6.6	26.2
79.0	30	5.5	25.1
82.5	35	5.1	23.7
87.5	42	4.6	22.1
92.5	52	4.1	20.3

TABLE 5-1 THRESHOLD MEASUREMENT FOR CASE (1)

Figure 5-15 is a plot of $(SNR)_o$ Vs. $(CNR)_{in}$ based on the last two columns of the table. From the curve, after adjusting for the use of a predetection filter, we have as the measured value of threshold, $\rho = 5.4$ dB, a value that agrees quite well with the predicted threshold at $\rho = 4.9$ dB.

It is instructive to compare the above-threshold segment of figure 5-15 with that predicted by the development in Section 1.3.1. From eq. (1-17d) we get an expression for dB $(SNR)_o$ as a function of dB $(CNR)_{in}$.

$$dB(SNR)_o = 10 \log \frac{3\beta^2 f_m^2 B_p}{2 f_s^3} + dB(CNR)_{in}$$

which may be plotted as a straight line that intercepts the vertical axis $[(CNR)_{in} = 1]$ at

$$10 \log \frac{3\beta^2 f_m^2 B_p}{2 f_s^3} .$$

For : $f_m = 1000$ Hz

$B_p = 35000$ Hz

and $f_s = 3300$ Hz

the intercept is at 10 log 146 = 21.6 dB which is 2.6 dB above the experimental value of figure 5-15. Most of the discrepancy is due to the assumptions involved in the derivation of eq. (1-17d).

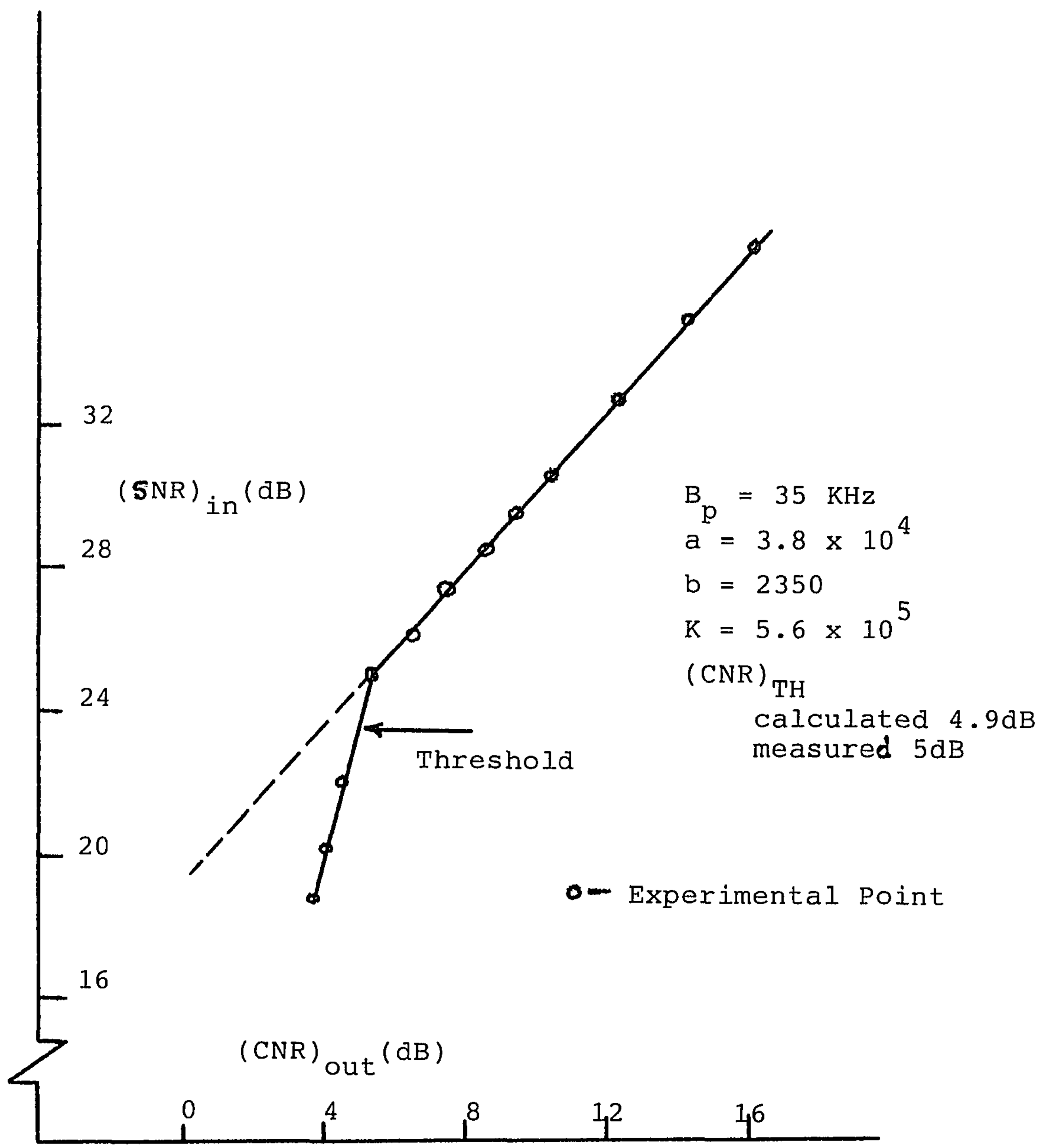


Fig. 5-15. Measured Threshold, Case (1)

Implicit in eq. (1-17a) for output power is the assumption of a flat closed loop response within the passband of the audio filter. Assume rather, that for frequencies up to 3.3 KHz, the response is as shown in figure 5-16 as a linear plot. This is based upon the calculated (and also measured) plot of figure 4-1.

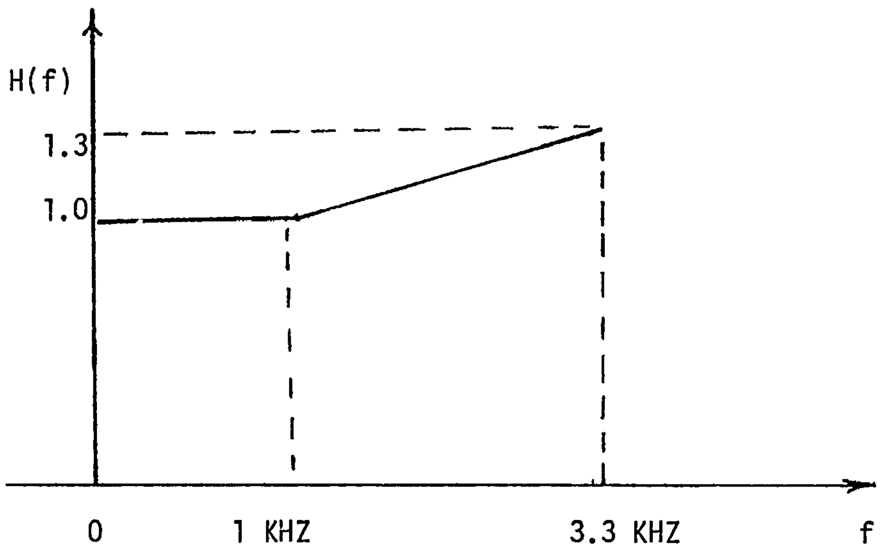


FIGURE 5-16 LINEAR APPROXIMATION OF H(f)

Expressed algebraically,

$$H(f) = \begin{cases} 1 & ; 0 \leqslant f \leqslant 1 \text{ KHZ} \\ Af + B & ; 1 \text{ KHZ} < f \leqslant 3.3 \text{ KHZ.} \end{cases}$$

where $A = 1.3 \times 10^{-4}$ and $B = 0.87$.

The integration in eq. (1-17a) may be more correctly written as

$$\int_0^{w_c} w^2 \left|H(jw)\right|^2 dw$$

$$= \int_0^{2\pi \times 10^3} w^2\, dw + \int_{2\pi \times 10^3}^{2\pi \times 3.3 \times 10^3} \left(\frac{A}{2} w + B\right)^2 w^2\, dw = 4.24 \times 10^{12} .$$

But $\int_0^{w_s} w^2\, dw = 2.9 \times 10^{12}$, a difference of 1.65 dB.

The correction reduces the vertical displacement of the measured FM improvement curve from the calculated curve to less than 1 dB. Part of the residual discrepancy is due to the non-ideal characteristic of the output low pass filter.

Continuing with the experimental verification we confirm that the designed system is in fact an optimum by varying the location of the pole, b, and repeating the threshold measurement procedure. The change is introduced by altering the value of C_1. R_2 is varied appropriately in order to maintain a constant value of "a" according to the first of equations (5-10).

Figure 5-17 shows the curves resulting from these measurements for the two sets of conditions. The Vertical displacement between the curves is due to the different equivalent noise bandwidths of the two loops.

Table 5-2 summarizes these data including threshold level, which is a minimum for the designed system as predicted.

C_1	R_2	b	TH	
.01 μf	2.6 K	2350	5.4 dB	← Design Value
.005 μf	5.2 K	4440	5.7 dB	
.02 μf	1.3 K	1210	8.6 dB	

TABLE 5-2 CASE (1) MINIMIZATION CONFIRMATION

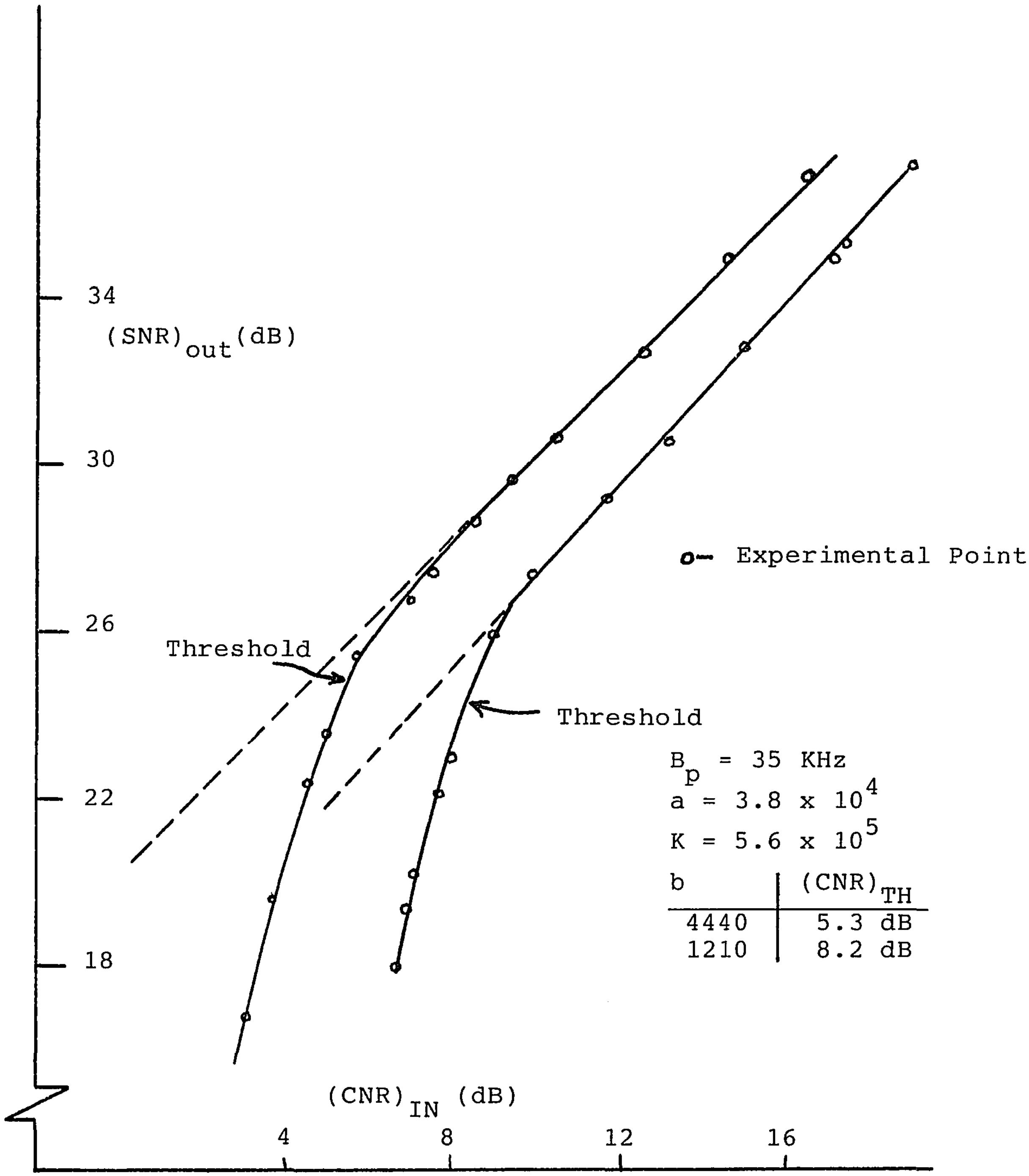


Fig. 5-17 Measured Threshold, Case (1), Perturbed b

5.4.2 - Case (8)

Case (8) was selected for test because it is our most general system for test-tone modulation. Its loop filter includes a realizable differentiator and the loop is preceded by an IF filter. For test purposes the available 58 KHz Butterworth filter was used and that necessitated redesigning the optimum loop by the methods of Chapter 4. In fact, several different systems were designed, each for a different value of d, so that a variety of optimum solutions could be subjected to laboratory confirmation, as will be detailed below.

As previously, it was discovered that the optimum points are not unique but rather depend upon the initialization of the algorithmic search. But once again, the optimum threshold level and the Kb product are unique.

Recall from Chapter 4, figure 4-14, that the minimum achievable threshold decreases as the value of d increases without bound, becoming optimum as case (8) approaches case (5). On the other hand, the best threshold increases as the predetection bandwidth becomes wider and case (5) approaches case (1). From the curve of figure 4-6(a) we observe that the best threshold for B_p = 58 KHz is 3.2 dB with the CNR calculated in a 35 KHz bandwidth. When an adjustment is made for measurement in 58 KHz as in our test procedure, this figure becomes 1.0 dB. Most of the testing was done on a system that was optimized for a differentiator pole located at d = 1.0 x 10^4 Rad/Sec. Its threshold level is 2.2 dB which is just 1.2 dB above that for $d = \infty$. A solution set was selected having the following parameter values:

$$a = 2.42 \times 10^5 \ \text{Rad/Sec}$$

$$b = 2363 \ \text{Rad/Sec}$$

$$d = 1.0 \times 10^4 \ \text{Rad/Sec}$$

$$\alpha = 3.86$$

$$K = 6.25 \times 10^5 \ \text{sec}^{-1}$$

$$Kb = 1.48 \times 10^9 \ \text{Rad/Sec}^2$$

Figure 5-18 shows calculated response curves for the open and closed loop which were calculated with the aid of the program called RESPONSE4 of APPENDIX B. The loop implementation is accomplished by means of eqs. (5-11) through (5-15) with the following specific results:

$$C_1 = .0057 \ \text{Nf}$$

$$C_4 = .01 \ \text{Nf}$$

$$R_1 = 24.8 \ \text{K}$$

$$R_2 = 732 \ \Omega$$

$$R_3 = 73.5 \ \text{K}$$

$$R_4 = 4.49 \ \text{K}$$

$$R_5 = 9.62 \ \text{K}$$

$$R_6 = 4.99 \ \text{K}$$

$$R_8 = 1.96 \ \text{K}$$

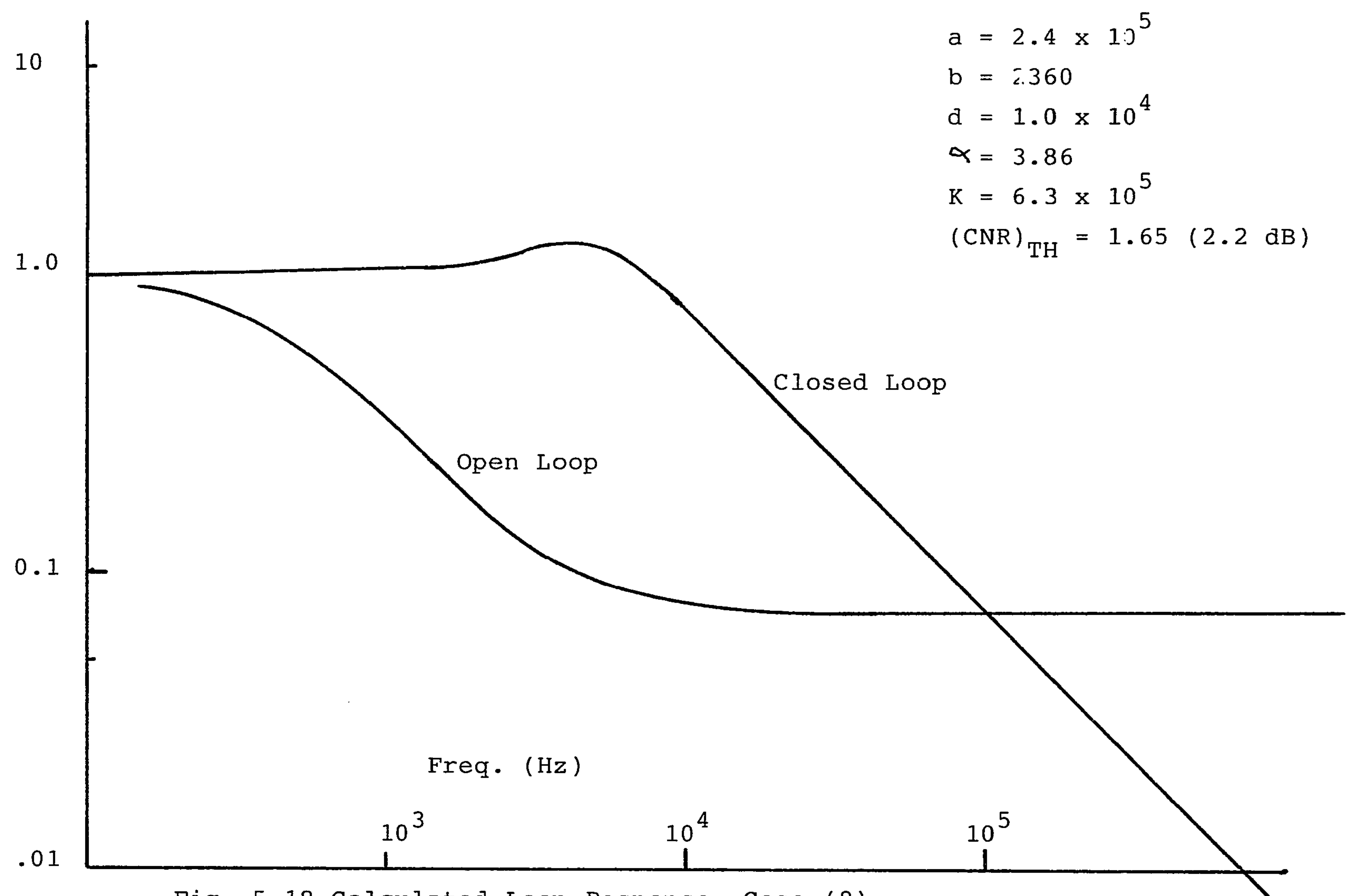


Fig. 5-18 Calculated Loop Response, Case (8)

The closed loop response for this detector was measured and is indistinguishable from that of figure 5-18.

Proceeding as above we obtain data for the threshold determination and these are plotted in figure 5-19. From the curve the measured threshold is found to be 2.4 dB, just 0.2 dB above the calculated optimum. To confirm that this is indeed a minimum, we alter the parameter values in both directions by varying the appropriate circuit values and determine the threshold in each case. Variations of b, α, and K were introduced in turn and threshold measurements were made. b is controlled by the value of C_1 as above and in each instance R_2 is selected so that a remains unchanged. α is controlled by R_4 in accordance with eq. (5-15) and K, by R_8 in accordance with (5-14).

We begin by perturbing b above and below the design value and obtain the three curves shown in figure 5-20. The various values of C_1, R_2, b, and the corresponding measured threshold levels are given in the table on the diagram. These results are summarized graphically in figure 5-21 where the calculated curve for $(CNR)_{TH}$ as a function of b is shown along with the measured threshold levels.

Similar tests were run with α as the variable and the results are presented in figures (5-22) and (5-23). The first includes the threshold curves and the relevant resulting data in tabulated format and the second is a calculated curve of $(CNR)_{TH}$ Vs. α with the experimental points added.

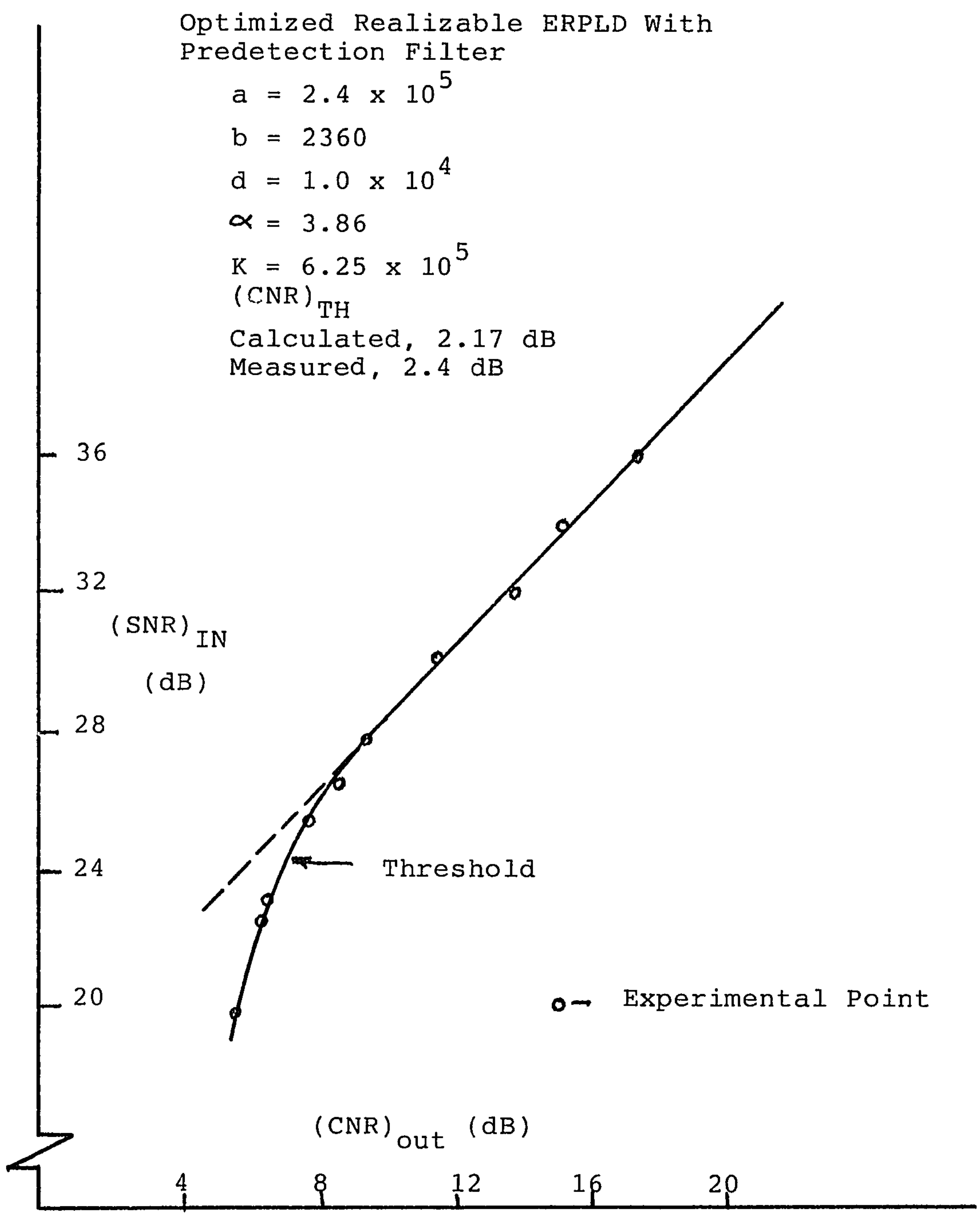
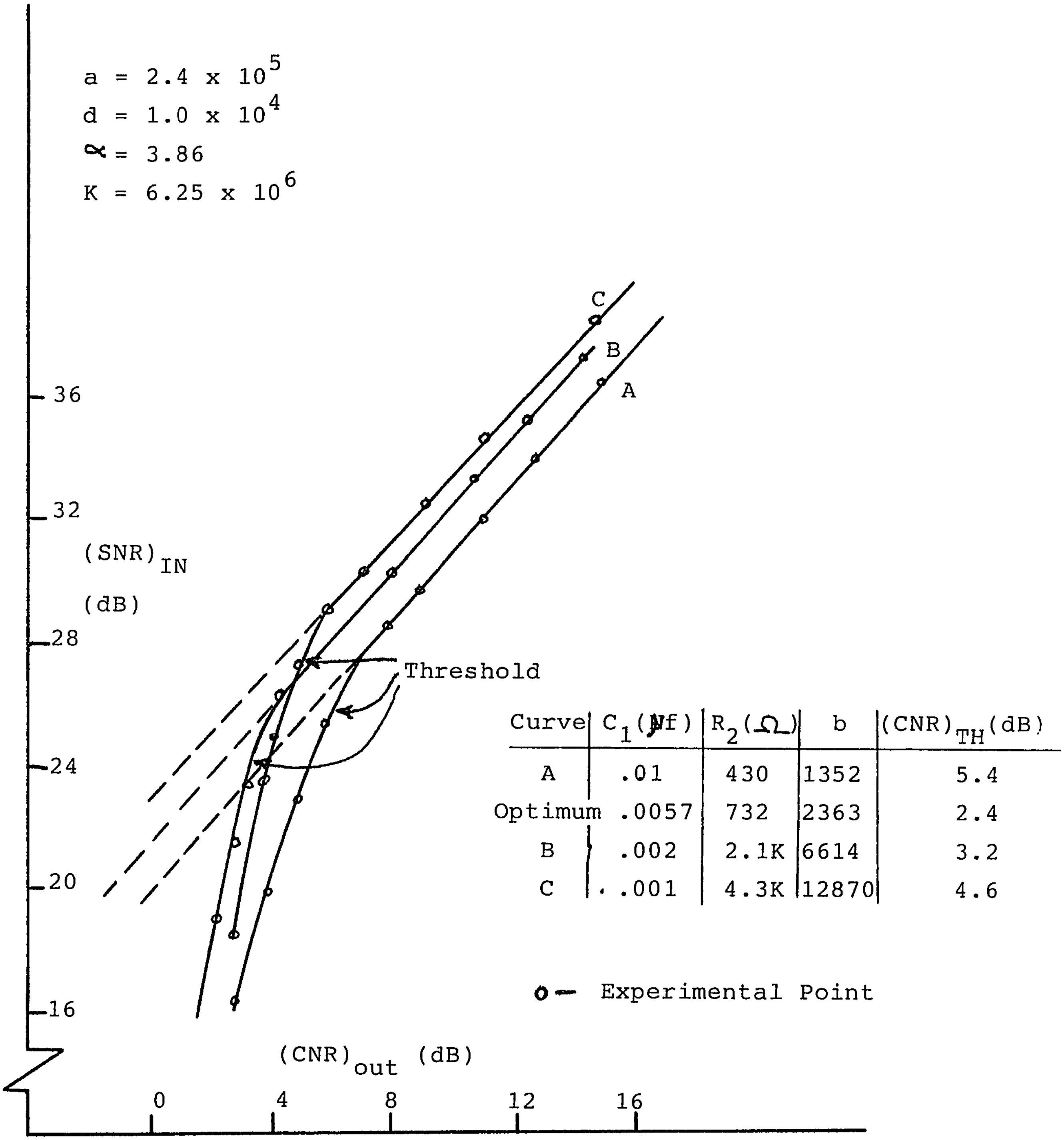


Fig. 5-19 Measured Threshold, Case (8)



Curve	C_1 (μf)	R_2 (Ω)	b	$(CNR)_{TH}$ (dB)
A	.01	430	1352	5.4
Optimum	.0057	732	2363	2.4
B	.002	2.1K	6614	3.2
C	.001	4.3K	12870	4.6

Fig. 5-20. Measured Threshold, Case (8), Perturbed b

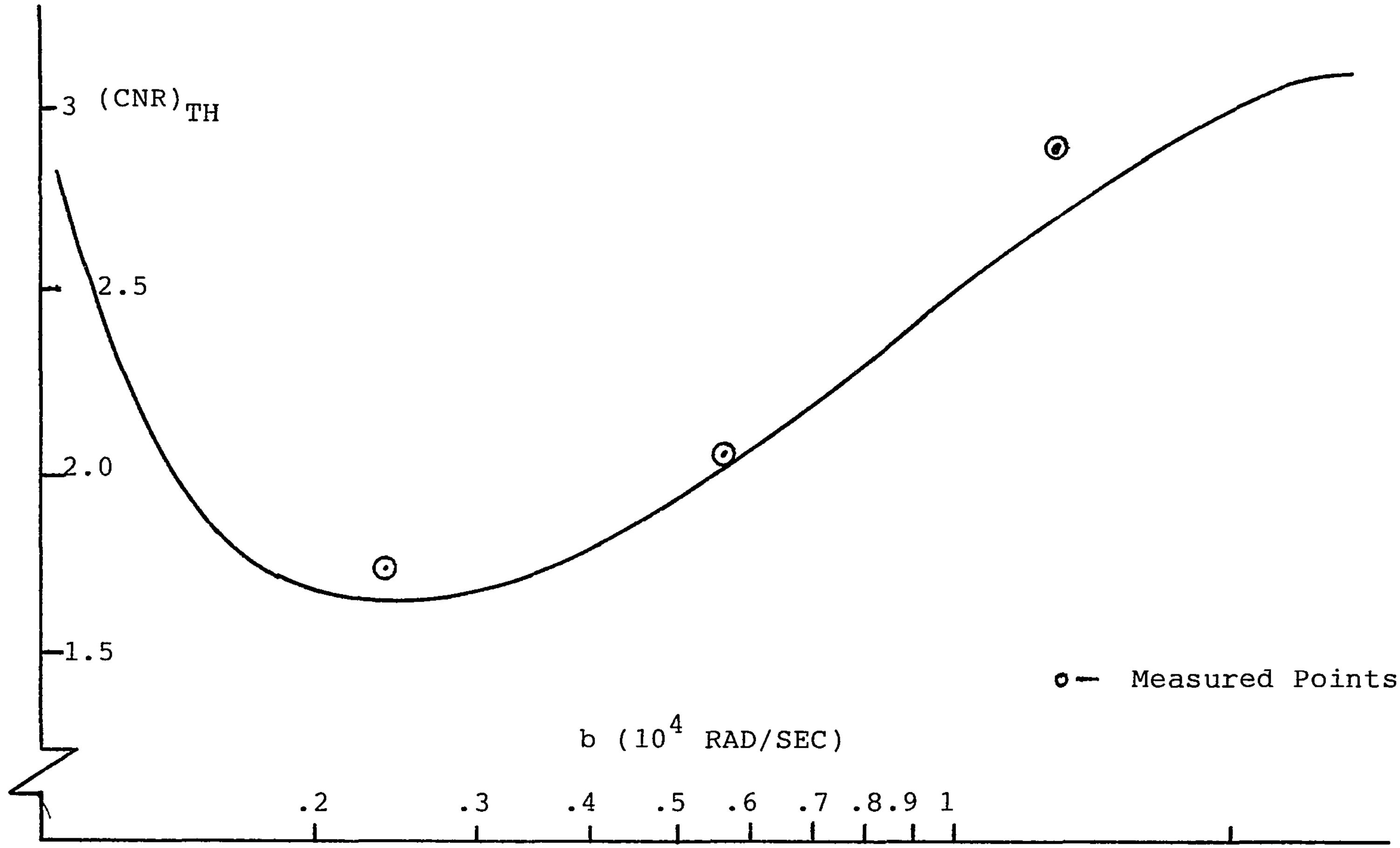
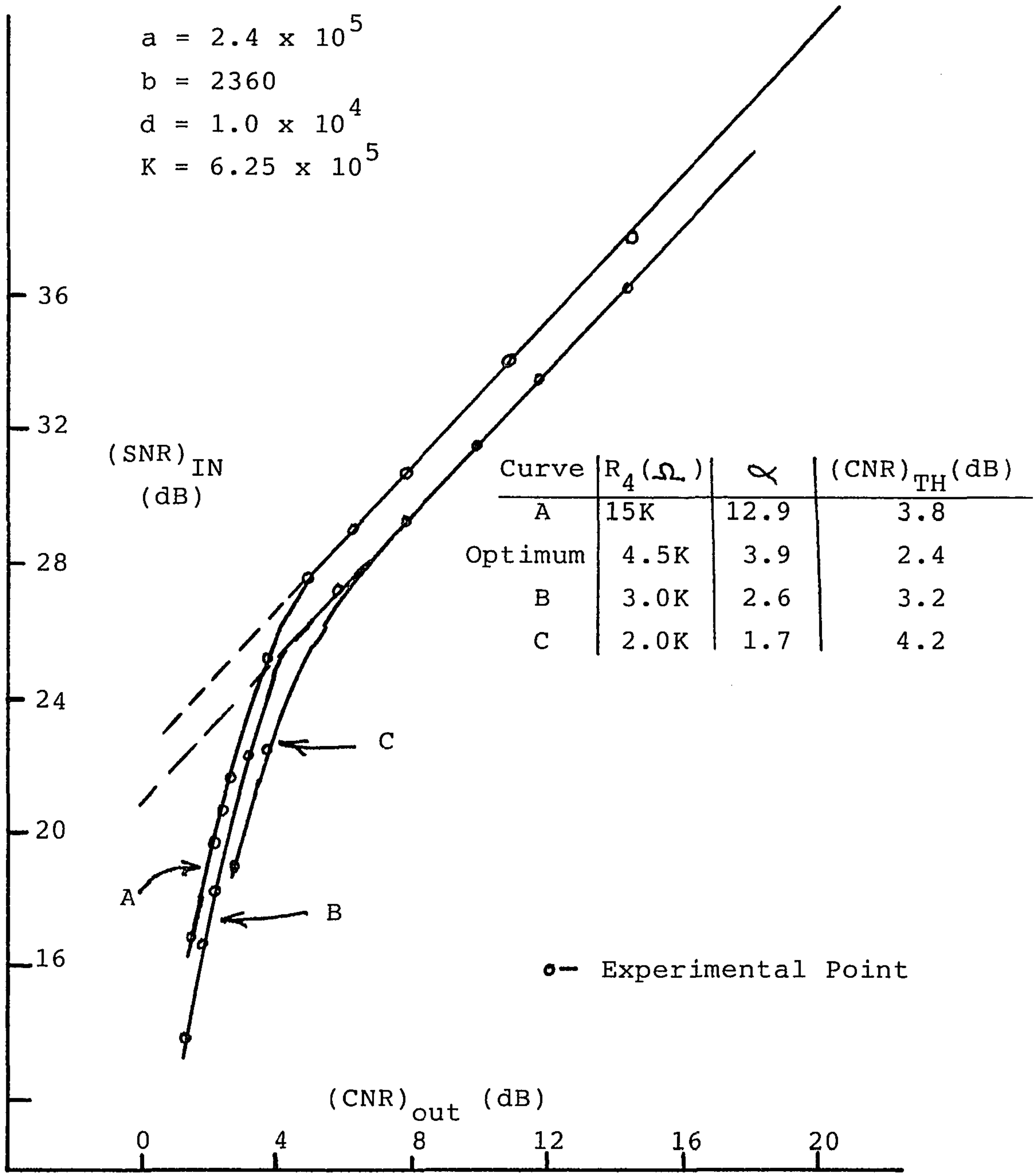


Fig. 5-21. $(CNR)_{TH}$ Vs. b, Case (8)



Curve	$R_4(\Omega)$	α	$(CNR)_{TH}$(dB)
A	15K	12.9	3.8
Optimum	4.5K	3.9	2.4
B	3.0K	2.6	3.2
C	2.0K	1.7	4.2

Fig. 5-22. Measured $(CNR)_{TH}$, Case (8), Perturbed (α)

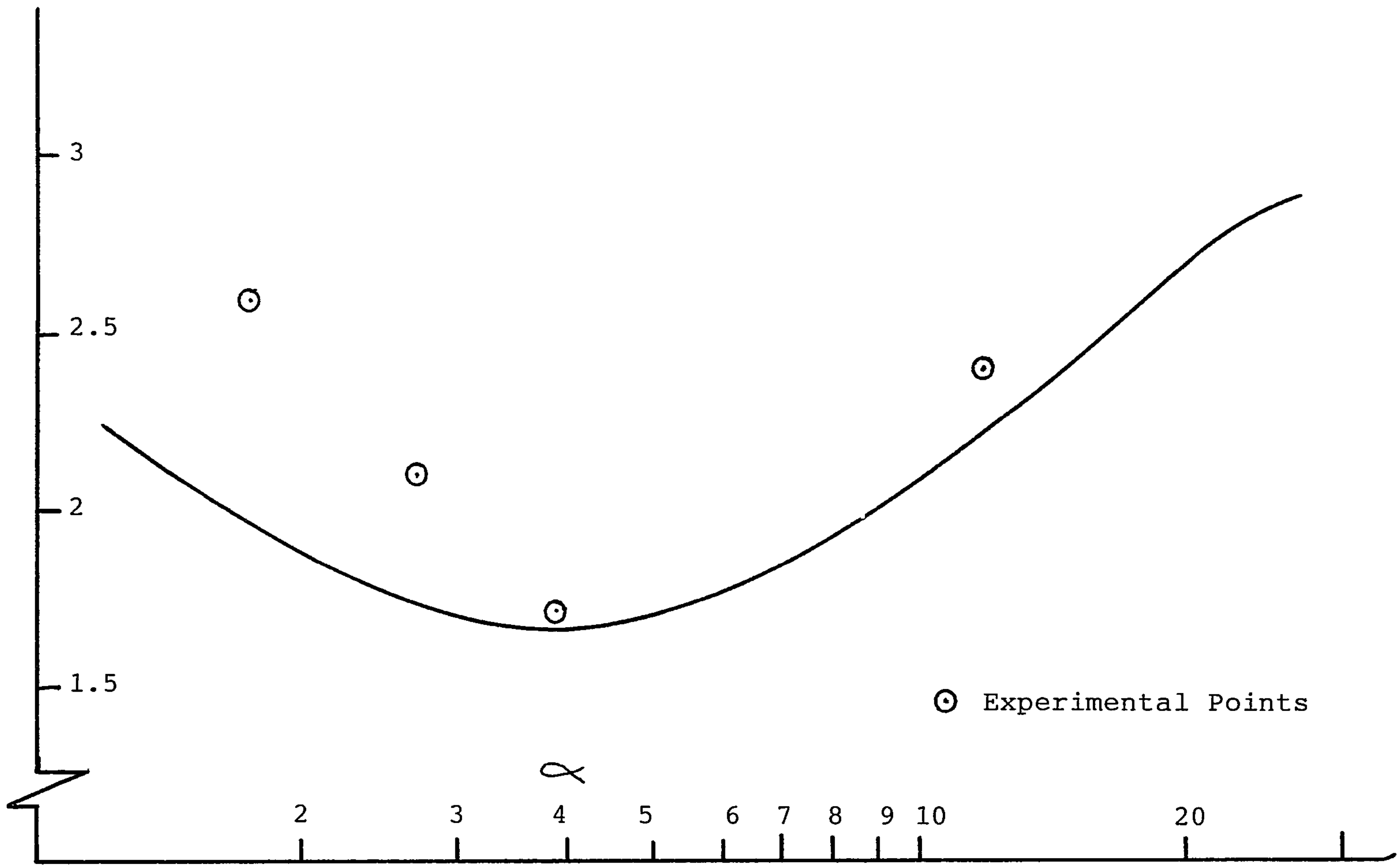


Fig. 5-23. $(CNR)_{TH}$ Vs. α, Case (8)

Treating K as an experimental variable produces results that are presented in these same formats in figures 5-24 and 5-25.

As each parameter value departs increasingly from the design value the measured threshold becomes larger; thus confirming that the calculated design is indeed an optimum.

The program called CASE 8 was again employed to find optimum designs for other values of differentiator pole location and these too were subjected to laboratory verifications. For $d = 1.0 \times 10^3$ and 1.0×10^5 the design parameter values and corresponding calculated thresholds are:

for $d = 1.0 \times 10^3$ Rad/Sec,

$a = 3.35 \times 10^4$ Rad/Sec

$b = 2295$ Rad/Sec

$\alpha = 1.13$

$k = 6.3 \times 10^5\ \text{Sec}^{-1}$

$\rho = 1.72$ (2.38 dB)

and for $d = 1.0 \times 10^5$ Rad/Sec ,

$a = 1.21 \times 10^5$ Rad/Sec

$b = 2210$ Rad/Sec

$\alpha = 1.12$

$k = 9.15 \times 10^5\ \text{Sec}^{-1}$

$\rho = 1.43$ (1.55 dB).

These threshold data and that for $d = 1.0 \times 10^4$ are plotted in figure 5-26. The limit value as d increases without bound is extracted from figure 4-6(a) which was calculated with CASE 5, and adjusted for

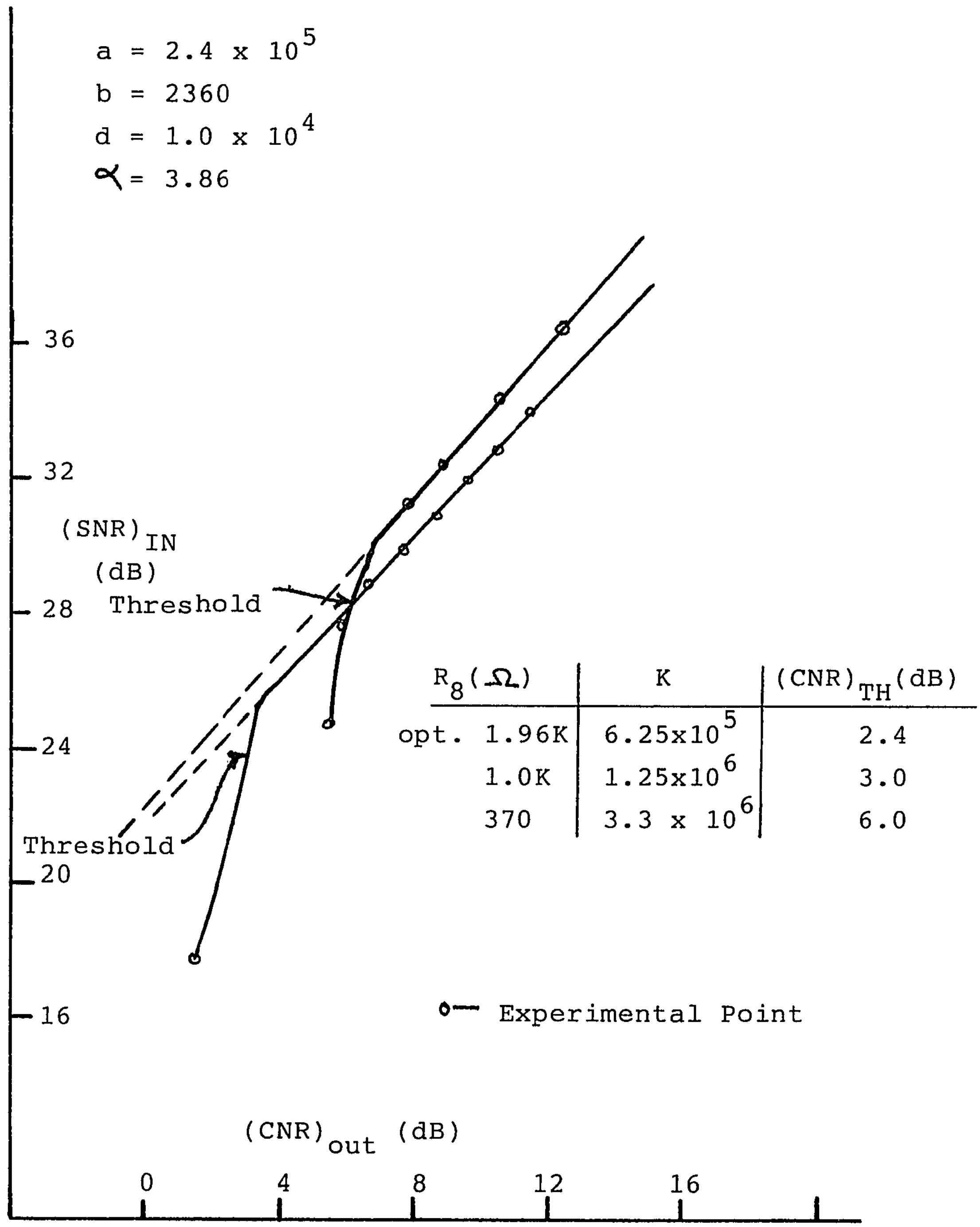


Fig. 5-24. Measured Threshold, Case (8), Perturbed K

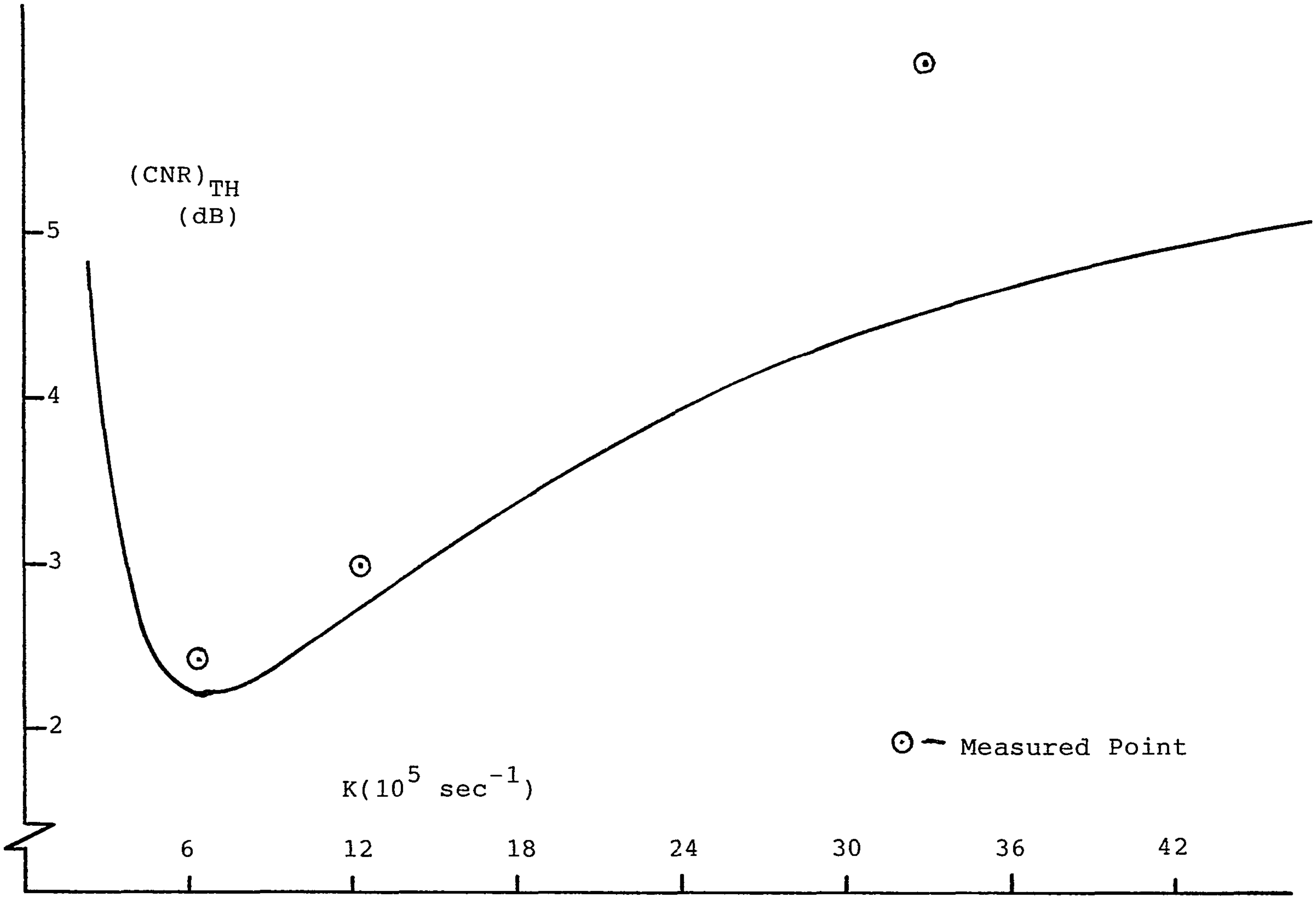


Fig. 5-25. $(CNR)_{TH}$ Vs. K, Case (8)

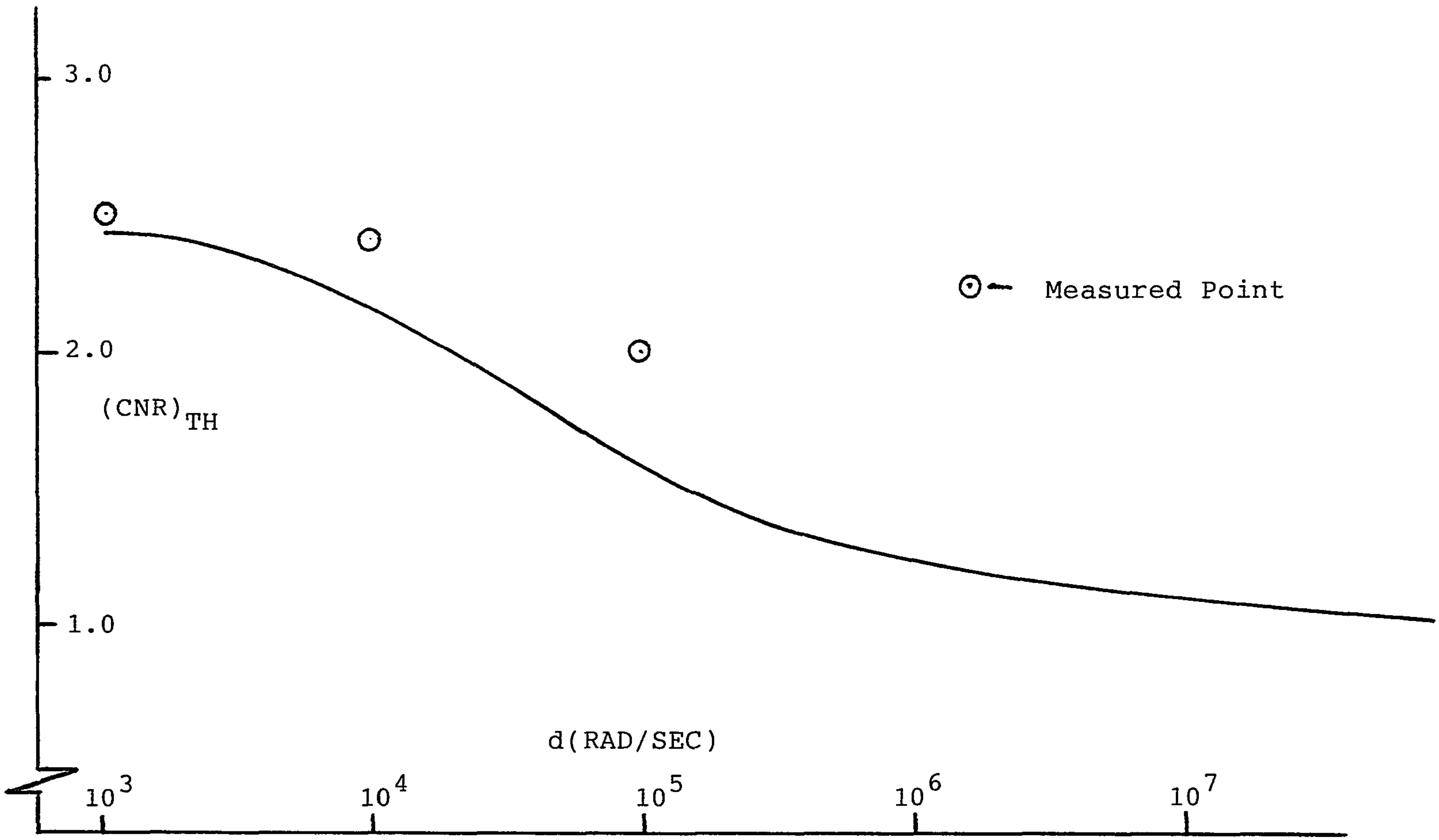


Fig. 5-26. Optimum Threshold Vs. Pole Location, d, Case (8)

the 58 KHz measurement bandwidth as mentioned above.

With the aid of design equations 5-11 through 5-15 the two loops were implemented. Threshold measurements were made and the results are shown in figures 5-27 and 5-28. The measured threshold values for these two and also for the $d = 10^4$ optimum loop (figure 5-19) are plotted on figure 5-26. They are all slightly above but within 0.5 dB of the calculated values.

As a further check on the accuracy of the laboratory procedure closed loop response characteristics were obtained for the $d = 10^5$ optimum loop by calculation and also by measurement. The results, plotted in figure 5-29 for purposes of comparison, are in reasonable agreement. The slightly smaller damping of the implemented circuit might be due to component tolerances. In order to confirm that the design is a true minimum we again varied the value of b about the design value and ran threshold tests. The resulting curves are shown in figures 5-30 and 5-31. In each case the threshold is larger than for the optimum.

In all of the foregoing instances the experimental results strongly support the contention that at least the test-tone model that was described in Chapter 2 and the computer algorithm of Chapter 4 do yield a loop design that exhibits the minimum threshold. Therefore we express confidence in the information that is summarized in Table 4-5 and in the statements that were made in Chapter 4 with respect to the impact of an additional pole on the threshold performance of PLDs and ERPLDs.

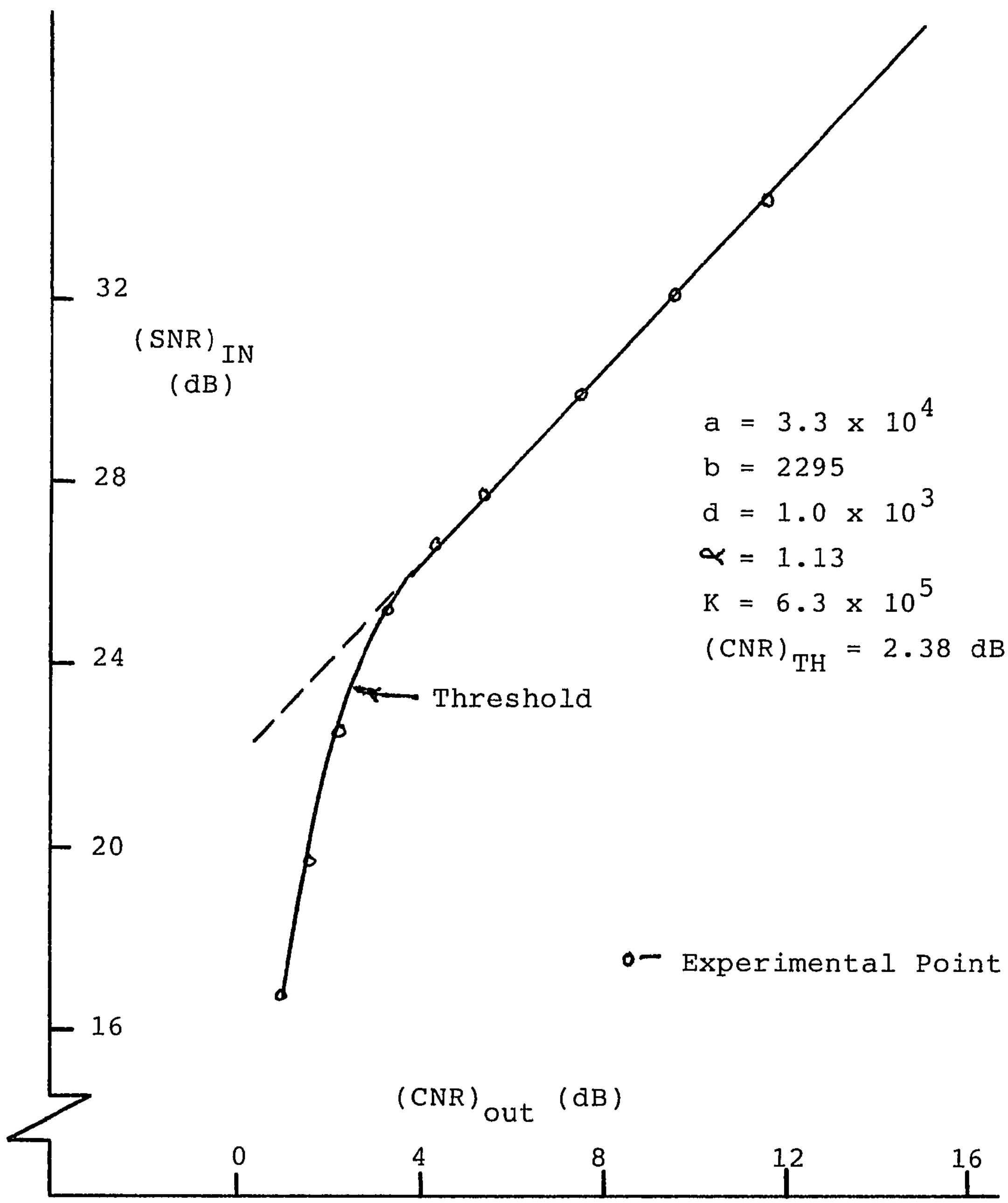


Fig. 5-27. Measured Threshold, Case (8), Optimized for d = 1.0 x 10^3

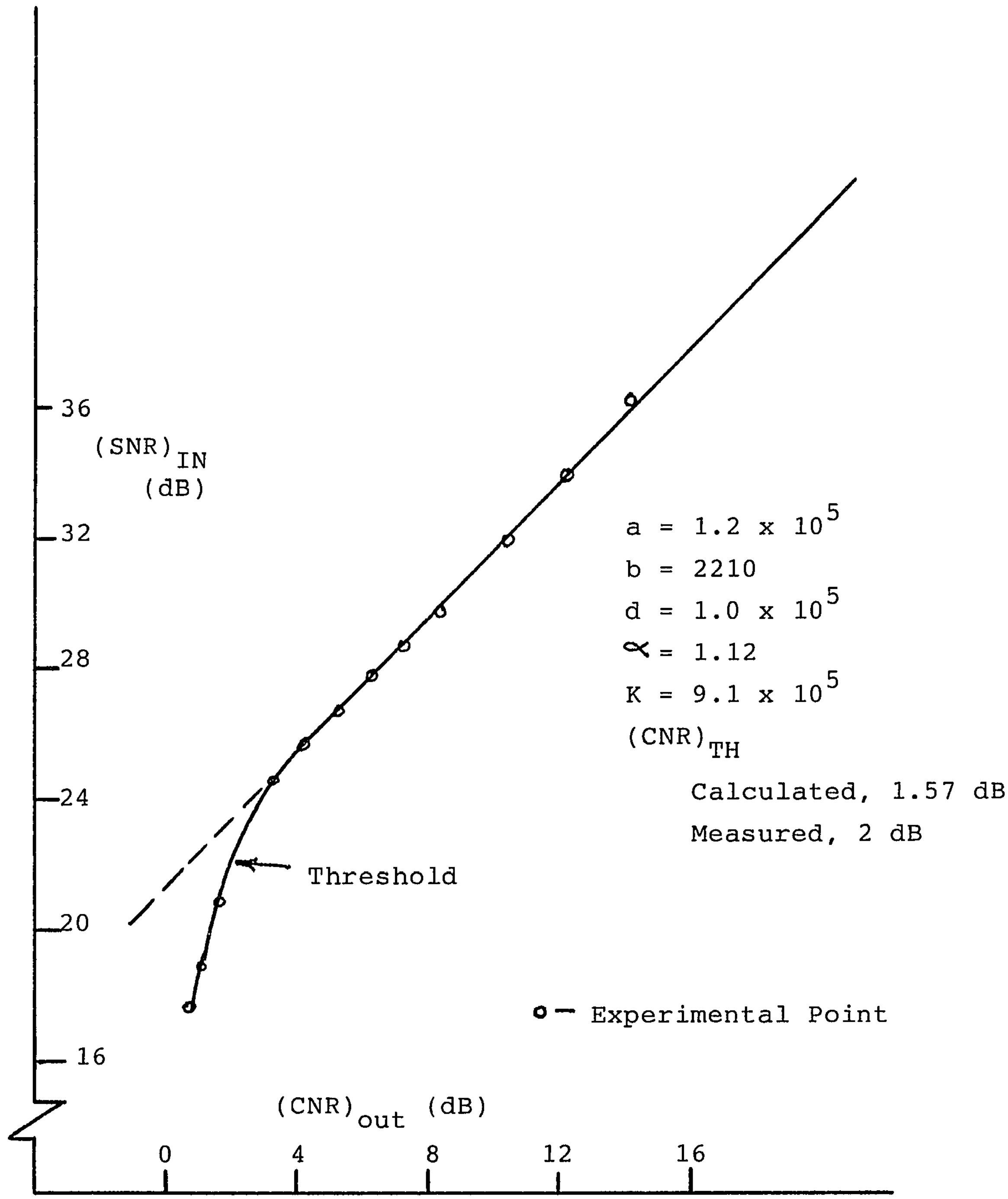


Fig. 5-28 - Measured Threshold, Case (8), Optimized for d = 1.0×10^5

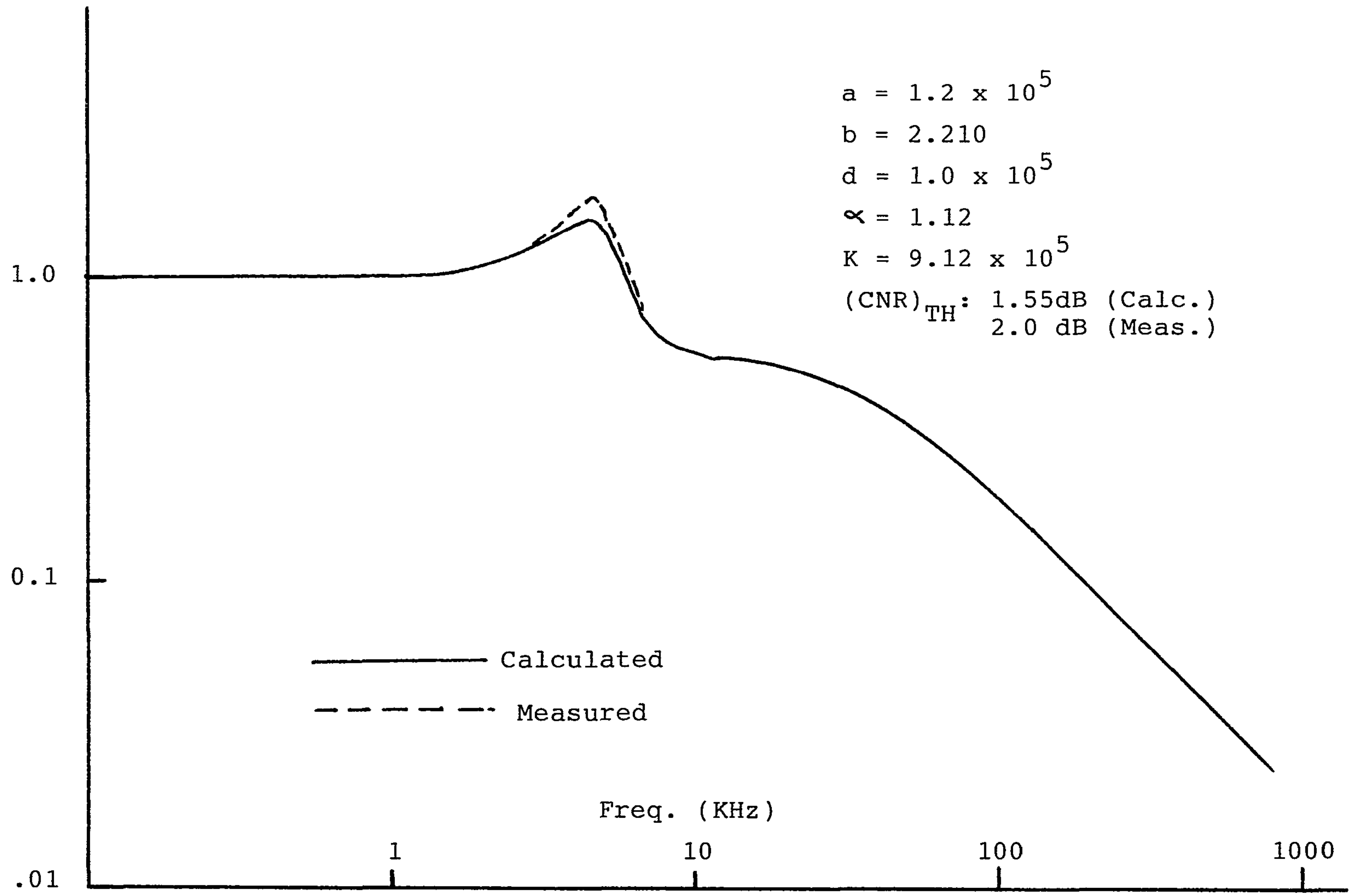


Fig. 5-29. Frequency Response, Case (8), $d = 1.0 \times 10^5$

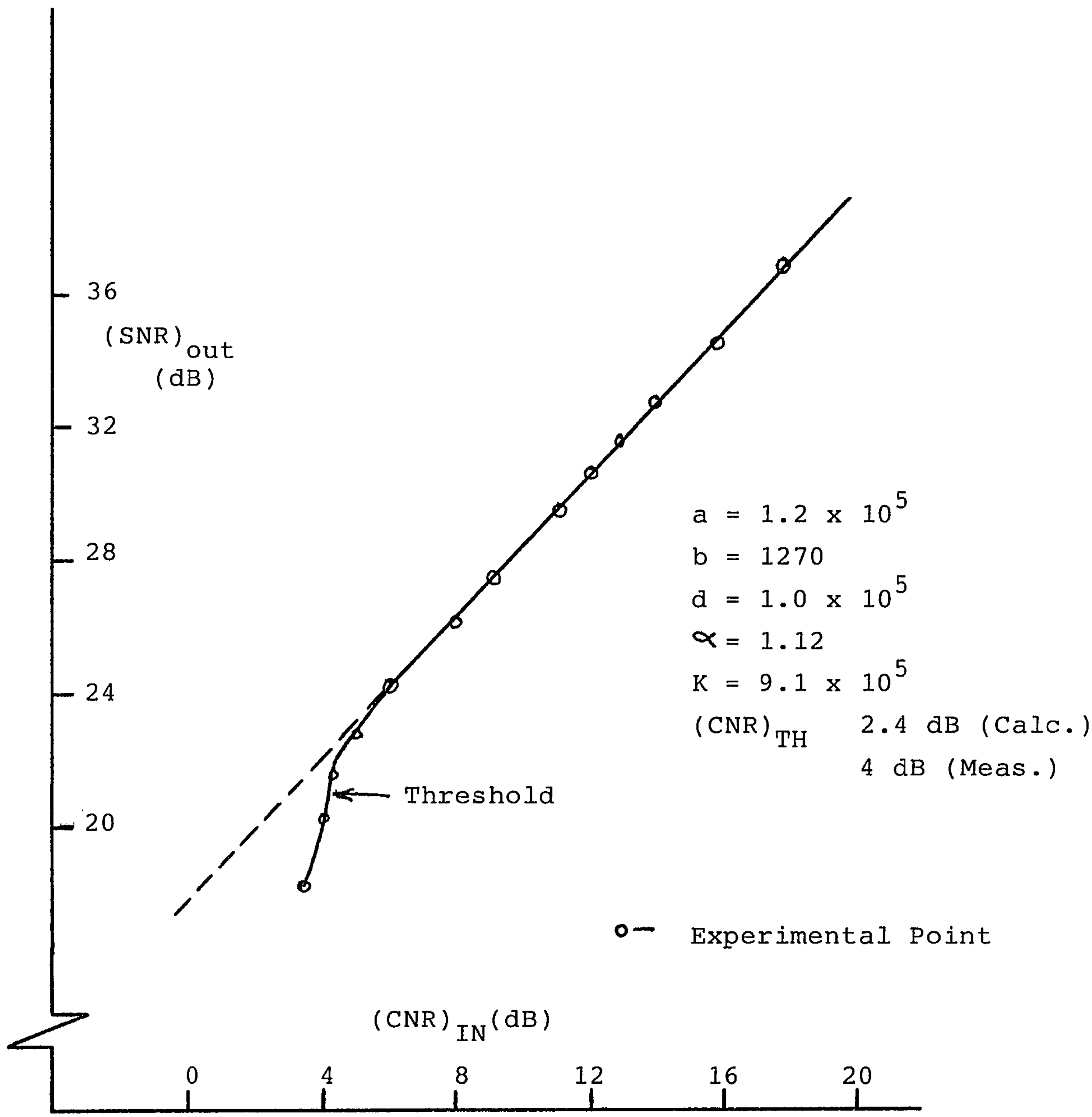


Fig. 5-30. Measured Threshold, Case (8), Decreased b

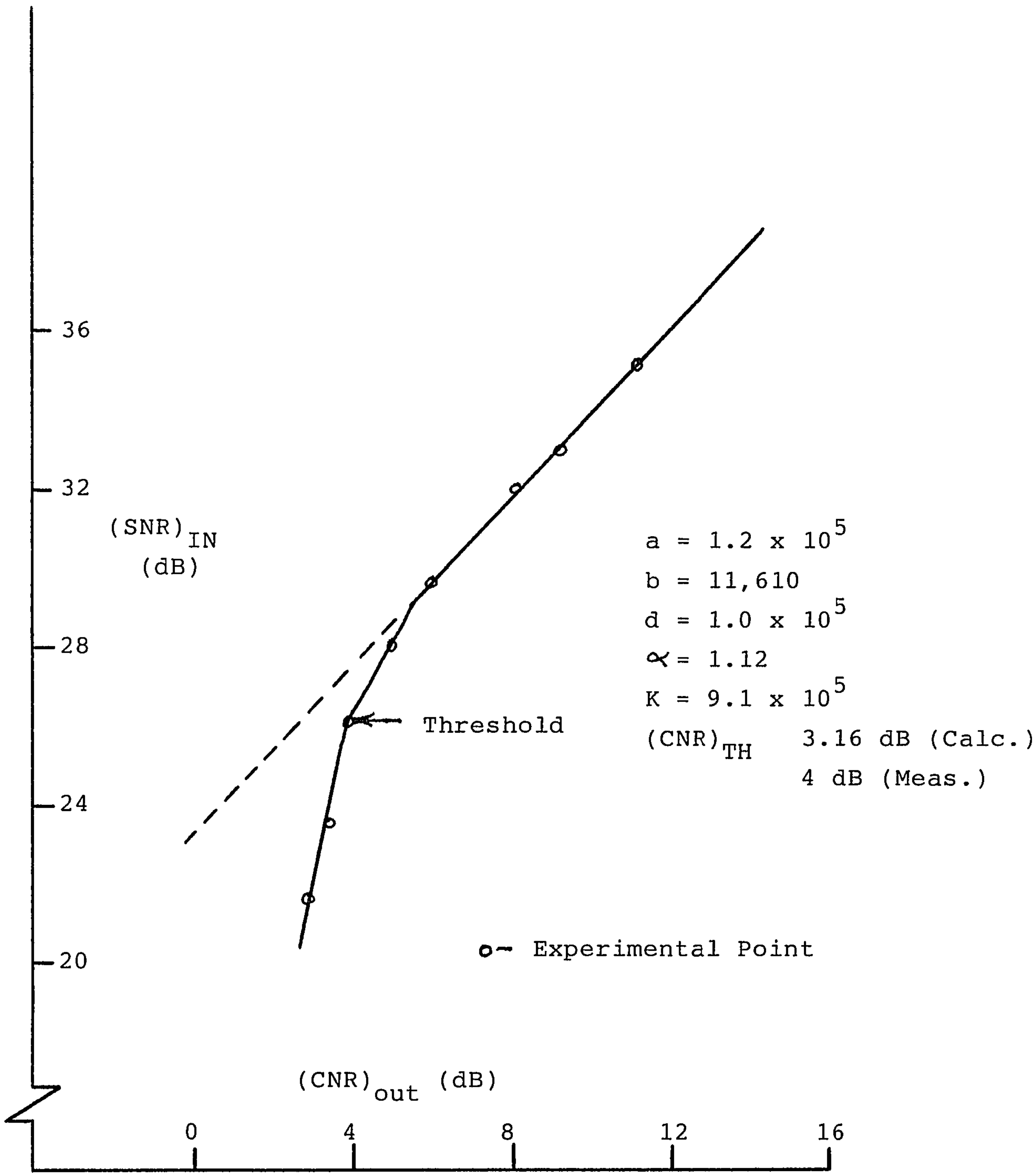


Fig. 5-31. Measured Threshold, Case (8), Increased b

References - Chapter 5

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CHAPTER VI CONCLUSIONS AND RECOMMENDATIONS

6.1 - Summary

Ten different phaselock loop FM detectors, distinguished by their loop filters, type of modulating signal, and presence or absence of a predetection filter were studied. The specific configurations were chosen to enable a study of the effect of an additional pole in the loop filter upon the threshold performance of the detectors, as an aid in optimum design.

We found that the addition of another pole to the loop filter of a standard second order PLD results in an increased threshold level for both test-tone and voice modulation. Similarly, a pole added to the differentiator section of an equivalent filter ERPLD which is preceded by an IF filter works to the detriment of the threshold for both types of modulation. In both of these instances, as the pole location is moved out towards infinity, the minimum achievable threshold decreases and approaches that of the simpler loop. Thus the best pole for minimum threshold is "no pole". However, should there be no alternative to the higher frequency pole (i.e. where the pole results from stray effects), an optimum detector can be designed which incorporates the given pole. In that case, if the pole frequency is sufficiently beyond the upper limit of the baseband, an optimally designed detector will have a threshold only slightly higher than the ideal (no pole) detector. Thus a second order loop that is modulated by a 1 KHz test tone to a peak deviation of 10 KHz will suffer a 0.5 dB penalty if the additional pole is at 55 KHz and similarly for voice modulation if the pole is located at 20 KHz.

In the case of the ERPLD, although the added pole cannot reduce the threshold of a detector system which includes a predetection filter, it does have merit in that it enables the detector to be used without a predetection filter. It also renders the zero in the lag-lead network superfluous which results in a loop filter consisting of a lag network plus a realizable differentiator. When optimized for a 1 KHz test-tone, this configuration has a threshold level comparable to that achievable with an IF filter of 180 KHz bandwidth, which is approximately 0.5 dB below that for the optimized second order system. Narrower IF bandwidths result in reduced threshold. As the bandwidth is increased, the optimum system approaches the standard second order system with no predetection filter.

These results are given credence by the affirmative experimental verification for several cases of test-tone modulation as reported in Chapter 5.

6.2 - Limitations of the Model

One of the objectives of this study was to compare the threshold performance of different phaselock detectors with and without predetection filters. Two detector configurations were studied with and without prefiltering and in both instances the use of prefiltering resulted in threshold reduction. Case 10 which includes the effect of prefiltering achieves a minimum threshold 1.6 dB below that achievable in case 9 which does not; and case 8 with prefiltering achieves a threshold 2.3 dB below that achievable by case 7 which has no prefiltering.

As cases 1 and 3 we optimized respectively a standard second order system and one with an additional pole; each with test-tone modulation and no predetection filtering. Neither of these two systems has ever been optimized for use with predetection filtering and so the attempt was made to do so.

In pursuit of the minimizations, the optimization algorithm was applied to these two cases and the formulations and results are presented here.

The model for test-tone modulation was employed and the expression for threshold is from eq. (2-5),

$$(CNR)_{TH} = \frac{4\pi^2 \int_0^\infty \left|H(jw)\right|^2 df}{B_p \, (\pi - 2\theta_p)^2} \tag{6-1}$$

The integrand in the numerator is the closed loop response of the PLL with the limits defined by the predetection band pass filter. Thus for the first case which uses loop filter I

$$H(jw) = \frac{\frac{1}{a}(jw) + 1}{\frac{1}{Kb}(jw)^2 + \left(\frac{1}{a} + \frac{1}{K}\right) jw + 1}$$

(6-2)

and $$\left|H(jw)\right|^2 = \frac{1 + \frac{1}{a^2} w^2}{\left(1 - \frac{1}{Kb} w^2\right)^2 + \left(\frac{1}{a} + \frac{1}{K}\right) w^2}$$

The peak signal related phase error, θp, that appears in the denominator of eq. (6-1) is the same as that for case 1 and is given in TABLE 3-1 as

$$\theta p = \beta \sqrt{\frac{W_T{}^2 (W_T{}^2 + b^2)}{W_T{}^4 + W_T{}^2\left(\frac{K^2b^2}{a^2} + 2\frac{Kb^2}{a} + b^2 - 2Kb\right) + K^2b^2}} \quad (6\text{-}3)$$

Where w_T is the modulation frequency and β is the modulation index.

For the second case being considered filter II of figure 2-1 is used as the loop filter for which the closed loop response is found from equation (2-25) to be given by

$$H(S) = \frac{s\frac{1}{a} + 1}{s^3\frac{1}{Kbd} + s^2\left(\frac{1}{Kb} + \frac{1}{Kd}\right) + s\left(\frac{1}{a} + \frac{1}{K}\right) + 1}, \quad (6\text{-}4)$$

and for use in evaluating θ_p we have

$$1 - H(S) = \frac{\frac{1}{Kbd}s^3 + \left(\frac{1}{Kb} + \frac{1}{Kd}\right)s^2 + \frac{1}{K}s}{\frac{1}{Kbd}s^3 + \left(\frac{1}{Kb} + \frac{1}{Kd}\right)s^2 + \left(\frac{1}{K} + \frac{1}{a}\right)s + 1} \cdot (6\text{-}5)$$

Equations (6-2) through (6-5) are used in eq. (6-1) for these two cases. The details of the algebraic manipulations and the derivations of closed form integrals in the preparation of manageable expressions for the minimization of $(CNR)_{TH}$ are presented in APPENDIX E.

These expressions for threshold were encoded into the optimization algorithm and are included among the listings of APPEMDIX B as CASE 11 and CASE 12. But for these cases the algorithm produces erroneous conclusions.

Reminiscent of the result of optimizing case (3), wherein d tends to infinity and the optimum is the same as for case (1), the optimization of case (12) moves d towards infinity and the system approaches that of case (11). This is a second order system for which

$$H(S) = \frac{S\,\frac{1}{a} + 1}{S^2\,\frac{1}{Kb} + S\left(\frac{1}{a} + \frac{1}{K}\right) + 1}\,. \tag{6-6}$$

The algorithm moves "a" towards zero, adjusts the Kb product to a "sufficiently" large value and finds $(CNR)_{TH}(min)$ to be approximately 2. Furthermore these results are grossly insensitive to the specific values of the system parameters. Thus the optimization search may be initialized any-place in a large area of parameter space and the algorithm will quickly find an optimum in the same neighborhood. If the initial point satisfies the very general conditions mentioned above for "a" and Kb then the program will run to completion in only one or two iterations.

By identifying eq (6-6) with the standard form of eq. (2-19a) we can develop some appreciation of these performance features.

$W_n = \sqrt{Kb}$ and

$$= \frac{1}{2}\left(\frac{1}{a} + \frac{1}{K}\right)\sqrt{Kb} \qquad \sqrt{\frac{Kb}{2a}} \qquad \text{for } a \ll K\,.$$

For $Wn \gg Bp/2$ and large damping, the closed loop response is as shown in figure 6-1 Superimposed on the predetection filter response.

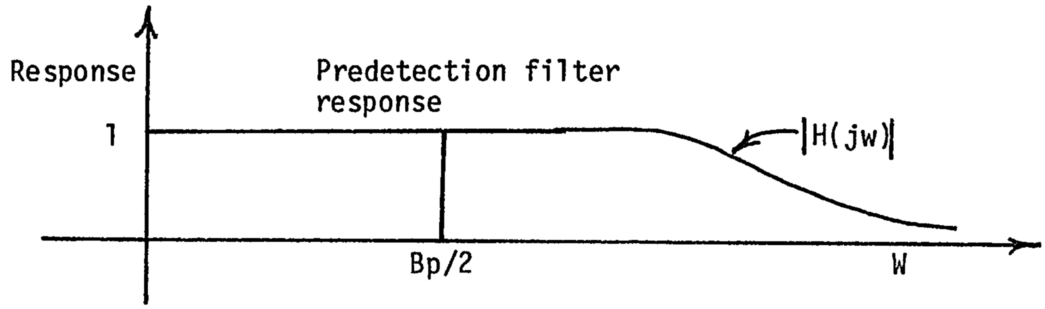


FIGURE 6-1 - RESPONSE OF WIDE BAND LOOP

Consider the threshold model for test-tone modulation,

$$(CNR)_{TH} = \frac{4\pi^2}{Bp\,(\pi - \theta p)^2} \int_0^{\infty} \left| H(jw) \right|^2 df \tag{6-7}$$

and suppose that H(jW) equals unity and remains flat to frequencies beyond Bp/2 as shown in the sketch.

From eq. (1-8) we have

$$\theta p = \beta \left| 1 - H(jw) \right|$$

and for $W_T < Bp/2$ $\qquad \theta p \approx 0$

$$\text{Then} \quad (CNR)_{TH} = \frac{4\pi^2}{\pi Bp} \int_0^{Bp/2} df = \frac{4\pi^2\, Bp}{2\pi^2\, Bp} = 2 \quad ,$$

the value predicted by the computer search. This result suggests that the optimum loop for a second order PLD is a wide band filter with $w_n \gg w_p$.

This is an erroneous conclusion stemming from an approximation in the development of the model which does not apply in this case. The model neglects the THI and attributes threshold to the LLI only [Sec 2-2]. But for a wide band loop the THI actually become the dominant source of spikes and further broadening of the response would work to the detriment of the threshold level.

It is clear that our model is useless here as it will not enable an optimum design of a standard PLD preceded by a predetection filter. But it is equally clear that the optimization would be useful.

6.2 - Recommendations for Further Activity

We recommend that as an extension of this research effort the optimization of a second order PLD preceded by a predetection filter ought to be attempted. A suitable alternative model to those of Sections 2.2 and 2.3 should be sought.

Also, additional applications of the numerical optimizations based upon our models should be undertaken in order to shed more light on the threshold character of these loops. With only minor modifications of the software, the effect upon the optimum design of the detectors due to variations in other parameters could be investigated. Modulation index, predetection bandwidth and test-tone frequency are candidates.

We further recommend that an attempt be made to support the conclusions resulting from this study by alternative modeling and/or analytic methods. A careful study of our results and of the results of additional optimizations might yield the necessary insight to encourage successful analysis.

APPENDIX A

EVALUATION OF INTEGRALS

Two forms of integrals are recurrent throughout this study and they will be evaluated here. A third form will also be evaluated but a case will be made for not employing it in the study.

A.1 $\displaystyle\int \frac{\Gamma w^2 + 1}{w^4 + Tw^2 + U}\, dw$ where , T, and U are constants and $T^2 < 4U$.

Because only even powers of w are present, for notational convenience we substitute $X = w^2$ in the integrand and write

$$\text{INTEGRAND} = \frac{\Gamma X + 1}{X^2 + TX + U}$$

The roots of the denominator are a complex conjugate pair given by

$$R_1 = -\frac{T}{2} + \sqrt{\left(\frac{T}{2}\right)^2 - U} = -\frac{T}{2} + j\sqrt{U - \left(\frac{T}{2}\right)^2}$$

$$R_2 = -\frac{T}{2} - \sqrt{\left(\frac{T}{2}\right)^2 - U} = -\frac{T}{2} - j\sqrt{U - \left(\frac{T}{2}\right)^2}$$

To facilitate the integration we expand the integrand by partial fractions.

$$\frac{\Gamma X + 1}{X^2 + TX + U} = \frac{A}{X-R_1} + \frac{B}{X-R_2} = \frac{\Gamma X + 1}{(X-R_1)(X-R_2)}$$

$$A = \lim_{X \to R_1} \frac{\Gamma X + 1}{X-R_2} = \frac{\Gamma R_1 + 1}{R_1 - R_2} = \frac{\Gamma R_1 + 1}{2j\sqrt{U - (T/2)^2}}$$

$$B = \lim_{X \to R_2} \frac{\Gamma X + 1}{X - R_1} = \frac{\Gamma R_2 + 1}{R_2 - R_1} = \frac{R_2 + 1}{-2j\sqrt{U - (T/2)^2}}$$

Let $G = -R_1$ and $G^* = -R_2$

$$\text{Then } A = \frac{1 - \Gamma G}{G^* - G}$$

$$\text{and } B = \frac{1 - \Gamma G^*}{G - G^*} \tag{A-1}$$

The integral can now be written as

$$I = A \int \frac{dw}{w^2 - R_1} + B \int \frac{dw}{w^2 - R_2} \tag{A-2}$$

Both terms are of the form

$$\int \frac{dw}{w^2 - a^2} \quad \text{where } a^2 \text{ is a complex number.} \tag{A-3}$$

let $a = \alpha e^{j\beta}$ and $a^2 = \alpha^2 e^{j2\beta}$

$$\int \frac{dw}{w^2 - \alpha^2 e^{j2\beta}} = \int \frac{dw}{(w + \alpha e^{j\beta})(w - \alpha e^{j\beta})}$$

By partial fraction expansion one gets

$$\frac{\gamma}{w + \alpha e^{j\beta}} + \frac{\delta}{w - \alpha e^{j\beta}} = \frac{1}{(w + \alpha e^{j\beta})(w - \alpha e^{j\beta})}$$

where γ and δ are constants which are evaluated as above and found to to be

$$\gamma = \frac{-1}{2\alpha e^{j\beta}} \quad , \quad \delta = \frac{1}{2\alpha e^{j\beta}}$$

Upon substituting these values we get for the integral of A-3

$$\int \frac{dw}{w^2 - a^2} = \frac{1}{2\alpha e^{j\beta}} \left[\int \frac{dw}{w - \alpha e^{j\beta}} - \int \frac{dw}{w + \alpha e^{j\beta}} \right]$$

$$= \frac{1}{2\alpha e^{j\beta}} \left[\ln (w + \alpha e^{j\beta}) - \ln (w - \alpha e^{j\beta}) \right]$$

$$= \frac{1}{2a} \ln \frac{w + a}{w - a} \qquad \text{(A-4)}$$

We now use A-4 in each term of A-2 to obtain an expression for the integral, I.

let $\epsilon = \sqrt{-G}$ and $\upsilon = \sqrt{G}$

then $$\int \frac{dw}{w^2 + G} = \frac{1}{2\epsilon} \ln \frac{W - \epsilon}{W + \epsilon} \qquad \text{(A-5)}$$

and $$\int \frac{dw}{w^2 + G^*} = \frac{1}{2\upsilon} \ln \frac{w - \upsilon}{w + \upsilon}$$

but $\upsilon = \epsilon^*$, so $$\int \frac{dw}{w^2 + G^*} = \frac{1}{2\epsilon^*} \ln \frac{w - \epsilon^*}{w + \epsilon^*} \qquad \text{(A-6)}$$

and noting from A-1 that B = A* we substitute A-5 and A-6 into A-2 and obtain for the integral

$$I = \frac{A}{2\epsilon} \ln \frac{w - \epsilon}{w + \epsilon} + \frac{A^*}{2\epsilon^*} \ln \frac{w - \epsilon^*}{w + \epsilon^*}$$

But the sum of a complex quantity and its conjugate is twice the real part of the quantity. Thus

$$I = \int \frac{\Gamma w^2 + 1}{w^4 + Tw^2 + U} dw = 2 \operatorname{Re} \left[\frac{A}{2} \ln \frac{w - \epsilon}{w + \epsilon} \right]$$

$$= \operatorname{Re} \left[\frac{1 - G}{\sqrt{-G}\,(G^* - G)} \ln \frac{w - \sqrt{-G}}{w + \sqrt{-G}} \right] \qquad \text{(A-7)}$$

where $G = -R_1 = \frac{T}{2} - j\sqrt{U - \left(\frac{T}{2}\right)^2} = |G| \angle \theta_G$

Thus $|G| = \sqrt{\left(\frac{T}{2}\right)^2 + U - \left(\frac{T}{2}\right)^2} = \sqrt{U}$

$$\theta_G = \mathrm{Tan}^{-1}\sqrt{\frac{U - (T/2)^2}{T/2}} = \mathrm{Tan}^{-1}\sqrt{\frac{4U}{T^2} - 1}$$

If we let $J = \sqrt{-G} = |J| \angle \theta_J$

Then $J = U^{1/4}$

$$\theta_J = \frac{1}{2}\ \mathrm{Tan}^{-1}\sqrt{\frac{4U}{T^2} - 1}$$

A.2 $\displaystyle\int \frac{dw}{w^2-R}$; R is real

The form of this integral depends upon whether the real number R is greater or less than zero.

For $R > 0$ let $c^2 = R$.

Then $\displaystyle\int \frac{dw}{w^2 - R} = \int \frac{dw}{w^2 - c^2} = \frac{1}{2c} \ln \frac{w-c}{w+c}$ [Ref 1, P. 1068](A-8)

We will be evaluating A-8 between two sets of limits and the solutions from A-8 are:

$$\int_0^{\pi B_p} \frac{dw}{w^2-R} = \frac{1}{2\sqrt{R}} \quad \ln \frac{\pi Bp - \sqrt{R}}{\pi Bp + \sqrt{R}} - \ln \frac{-\sqrt{R}}{\sqrt{R}}$$

$$= \frac{1}{2\sqrt{R}} \ln \frac{\sqrt{R} - \pi Bp}{\sqrt{R} + \pi Bp} \qquad \text{(A-9)}$$

and

$$\int_{w_a}^{w_b} \frac{dw}{w^2 - R} = \frac{1}{2\sqrt{R}} \left[\ln \frac{w_b - \sqrt{R}}{w_b + \sqrt{R}} - \ln \frac{w_a - \sqrt{R}}{w_a + \sqrt{R}} \right]$$

$$= \frac{1}{2\sqrt{R}} \ln \frac{w_b - \sqrt{R}}{w_b + \sqrt{R}} \cdot \frac{w_a + \sqrt{R}}{w_a - \sqrt{R}} \qquad \text{(A-10)}$$

For $R < 0$ let $a^2 = -R > 0$

$$\text{Then} \int \frac{dw}{w^2 - R} = \int \frac{dw}{w^2 + a^2} = \frac{1}{a} \operatorname{Tan}^{-1} \frac{w}{a} \quad \text{[Ref. 1, P. 1068]} \qquad \text{(A-11)}$$

Again we shall have occasion to evaluate this integral between two pairs of limits and will use the following results:

$$\int_0^{\pi Bp} \frac{dw}{w^2 - R} = \frac{1}{\sqrt{-R}} \operatorname{Tan}^{-1} \frac{\pi Bp}{\sqrt{-R}} \qquad \text{(A-12)}$$

and

$$\int_{w_a}^{w_b} \frac{dw}{w^2 - R} = \frac{1}{\sqrt{-R}} \left[\operatorname{Tan}^{-1} \frac{w_b}{\sqrt{-R}} - \operatorname{Tan}^{-1} \frac{w_a}{\sqrt{-R}} \right] \qquad \text{(A-13)}$$

A.3 <u>Consider the integral of A.1 when $T^2 = 4U$.</u>

The denominator of $\frac{\Gamma X + 1}{X^2 + TX + U}$ now has a pair of equal, real roots; $R_1 = R_2 = R = -\frac{T}{2}$.

$$\frac{\Gamma X + 1}{X^2 + TX + U} = \frac{\Gamma X + 1}{(X - R)^2} \qquad \text{(A-14)}$$

Expand A-14 by partial fractions and get

$$\frac{\Gamma X + 1}{(X - R)^2} = \frac{A}{X - R} + \frac{B}{(X - R)^2} \quad .$$

Combining the right side over a common denominator and equating the numerator to the left side yields

$$A(X-R) + B = AX + B - AR = \quad X + 1 \quad .$$

From which, $A = \Gamma$ and $B = 1 + \Gamma R$.

$$I = \frac{\Gamma w^2 + 1}{w^4 + Tw^2 + U} \, dw = \int \frac{\Gamma \, dw}{w^2 - R} + \int \frac{\Omega \, dw}{(w^2 - R)^2} \qquad \text{(A-15)}$$

Where $\Omega = 1 + \Gamma R$.

Let $a = -R$ and evaluate the last term. [Ref. 2, form 51]

$$\int \frac{\Omega \, dw}{(a + w^2)^2} = \frac{w \, \Omega}{2a \, (a + w^2)} + \frac{\Omega}{2a} \int \frac{dw}{w^2 + a}$$

$$= \frac{\Omega \; w}{2R \, (w^2 - R)} - \frac{\Omega}{2R} \int \frac{dw}{w^2 - R}$$

Substituting this result into A-15 yields

$$I = \frac{(1 + \Gamma R) \; w}{2R \, (w^2 - R)} + \left[\Gamma - \frac{1 + \Gamma R}{2R} \right] \int \frac{dw}{w^2 - R}$$

$$= \frac{(1 + \Gamma R) \; w}{2R \, (w^2 - R)} + \frac{1}{2} \left(\Gamma - \frac{1}{R} \right) \int \frac{dw}{w^2 - R} \quad . \qquad \text{(A-16)}$$

This last integral was treated in A-2 and is given by eq (A-8) or (A-11) depending upon R greater or less than zero respectively.

These solutions are developed for use in optimization algorithms. Eq (A-16) will not be incorporated into these algorithms because it only applies in the rare case of $T^2 = 4U$ which is extremely unlikely to occur within the numerical accuracy of the computer. Rather than extend the code to include A-16 in all the programs where it might apply, we simply halt the running of those programs and indicate the occurrence of the equality.

APPENDIX B-LISTING OF FORTRAN PROGRAMS

```
C  *****CASE1******* STANDARD SECOND SECOND ORDER SYSTEM
C    THREE VARIABLES A,B,K
       DIMENSION X(50,3),E2(4,4),Y4(40),B4(3),X0(3),X1(3)
       DIMENSION X2(3),D2(3),U2(3),X4(3),X3(3),X7(3),A1(3)
       DIMENSION X8(3)
       B6=10.0
       BK8=3.14159
       N2=3
       K2=30
       S2=0.01
       T2=0.001
       F2=0.001
       X(1,1)=10000
       X(1,2)=5000
       X(1,3)=300000
       DO400 L=1,N2
       DO410 N=1,N2
410    E2(L,N)=0
400    CONTINUE
       DO420 L=1,N2
420    E2(L,L)=1
       I2=1
300    M2=0
       M3=0
       D0=1
       J2=1
       DO10 JQ=1,N2
10       X3(JQ)=X(I2,JQ)
       DO430 I=1,N2
430    A1(I)=X3(I)
       AL2=0
       DO440 I=1,N2
440    X4(I)=X3(I)
       Y4(I2)=F3(X4)
       B4(J2)=Y4(I2)
       B2=B4(J2)
       DO20 JQ=1,N2
20       X0(JQ)=X(I2,JQ)
C        SEARCH BEGINS
195    K3=1
       DO30 JQ2=1,K2
```

```
      S3=(2**(JQ2-1))*S2
      DO40 JZ=1,N2
40    X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
      DO 450 I=1,N2
450   X4(I)=X0(I)
      Y2=F3(X4)
      IF((B2-Y2).GT..00001)  GO TO 50
      IF((Y2-B2).GT..00001)  GO TO 60
      IF((Y2-B2).LT..00001)  GO TO 30
      GO TO 30
60    IF(JQ1.EQ.0) GO TO 70
      IF(K3.EQ.2) GO TO 80
      DO460 M=1,N2
460   X1(M)=X0(M)
      DO 90 JZ=1,N2
90     X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
      GOTO80
50    JQ1=1
      K3=1
      GO TO30
70      DO470 JK=1,N2
470   X1(JK)=X0(JK)
      S2=-S2
      K3=2
      JQ1=1
30    B2=Y2
      GO TO 100
C       BEGIN FIBERNACI SEARCH
80    R2=3*((2**(JQ2-2))*ABS(S2))
      JQ1=0
      N4=INT(1-2.1*ALOG(F2/R2))
      DO480 JL=1,N2
480   X7(JL)=X1(JL)-X0(JL)
      DO490 KL=1,N2
490   D2(KL)=.618*X7(KL)
      DO 500 JM=1,N2
500   X8(JM)=X0(JM)+D2(JM)
      DO 510 JN=1,N2
510   X2(JN)=X1(JN)-D2(JN)
      DO 520 KM=1,N2
520   X4(KM)=X2(KM)
      V2=F3(X4)
      DO530 KN=1,N2
530   X4(KN)=X8(KN)
      W2=F3(X4)
      DO130 K9=1,N4
      DO540 LL=1,N2
540   D2(LL)=.618*D2(LL)
      IF (ABS(W2-V2).LT..00001) GO TO 110
      IF(W2.GT.V2) GO TO 120
      DO550 JJ=1,N2
550   X0(JJ)=X2(JJ)
      DO560 KK=1,N2
560   X2(KK)=X8(KK)
      DO 570 MM=1,N2
570   X8(MM)=X0(MM)+D2(MM)
      V2=W2
      DO580 NN=1,N2
580   X4(NN)=X8(NN)
      W2=F3(X4)
      GO TO 140
120      DO590 II=1,N2
590   X1(II)=X8(II)
      DO595 IJ=1,N2
595   X8(IJ)=X2(IJ)
      DO600 JJ=1,N2
600   X2(JJ)=X1(JJ)-D2(JJ)
      W2=V2
      DO610 KK=1,N2
610   X4(KK)=X2(KK)
130   V2=F3(X4)
140   CONTINUE
110      DO620 LL=1,N2
620   X7(LL)=X2(LL)+X8(LL)
      DO630 MM=1,N2
630   X0(MM)=(.5)*X7(MM)
      IF(J2.EQ.N2+1) GO TO 150
      A3=E2(1,J2)
      JZ1=1
      DO 160 JZ=2,N2
      IF (E2(JZ,J2).GT.A3) GO TO 170
      GO TO 160
170   A3=E2(JZ,J2)
      JZ1=JZ
160   CONTINUE
      AL1=(X0(JZ1)-A1(JZ1))/A3
      IF (ABS(AL1).GT.ABS(AL2 ))GO TO 180
      GO TO 190
180   AL2=AL1
      JS=J2
190   DO640 NN=1,N2
640   A1(NN)=X0(NN)
      J2=J2+1
      DO650 JJ=1,N2
650   X4(JJ)=X0(JJ)
      B2=F3(X4)
      IF (J2.LT.(N2+1)) GO TO 195
      DO660 KK=1,N2
660   U2(KK)=X0(KK)-X3(KK)
      U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2)
      DO670 LL=1,N2
670   U2(LL)=(1/U1)*U2(LL)
      DO200 JZ=1,N2
```

```
200   E2(JZ,N2+1)=U2(JZ)
      GO TO 195
150      I2=I2+1
      WRITE(2,1100)I2
1100  FORMAT('     I2=',I3)
      DO 210 JZ=1,N2
210   X(I2,JZ)=X0(JZ)
      DO 220 JZ=1,N2
220      IF (ABS((X0(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
      GO TO 240
230   IF(I2.EQ.50) GO TO 270
      AL3=AL2*D0/U1
      IF (ABS(AL3).LT..000001) GO TO 280
      D0=AL3
      DO 290 JZ=1,N2
290   E2(JZ,JS)=U2(JZ)
280   GO TO 300
240   DO700 KK=1,N2
700   X4(KK)=X0(KK)
      Y3=F3(X4)
270   CONTINUE
      WRITE(2,310)Y3
      WRITE(6,310)Y3
310   FORMAT('MINIMUM (CNR)TH ='F10.7)
      WRITE(2,320)X0(1),X0(2),X0(3)
      WRITE(6,320)X0(1),X0(2),X0(3)
320   FORMAT('  A=',F9.2,'   B=',F8.2,'   K= ',F10.1)
      WRITE(2,325)X(1,1),X(1,2),X(1,3)
325   FORMAT(' STARTED FROM A=',F7.1,'  B= ',F6.1,' K= ',F8.0)
      IF (I2.LT.50) GO TO 330
      WRITE(2,340)
340      FORMAT('SOLUTION DID NOT CONVERGE')
      GO TO 330
100      WRITE(2,350)
350   FORMAT('THERE IS NO MINIMUM')
330   STOP
      END
      FUNCTION F3(X4)
      DIMENSION X4(3)
      A=X4(1)
      B=X4(2)
      C=X4(3)
      PI=3.1415927
      BP=35000.0
      W=2*PI*1000
      BETA=10.0
      D=C**2*B**2/A**2+2*C*B**2/A**2+B**2-2*C*B
      W2=W**2
      PHI=BETA*SQRT((W2*(W2+B**2))/(W2**2+W2*D+(C*B)**2))
      T4=C*(C*B+A**2)/4/A/(A+C)
      F3=4*PI**2*T4/BP/(PI-2*PHI)**2
      RETURN
      END
```

```
C  *****CASE2******* STANDARD SECOND SECOND ORDER SYSTEM
C    NO PREFILTER   VOICE MODULATION
C    THREE VARIABLES A,B,K
      DIMENSION X(50,3),E2(4,4),Y4(40),B4(3),X0(3),X1(3)
      DIMENSION X2(3),D2(3),U2(3),X4(3),X3(3),X7(3),A1(3),X8(3)
      B6=10.0
      BK8=3.14159
      N2=3
      K2=30
      S2=0.01
      T2=0.001
      F2=0.001
      X(1,1)=15000
      X(1,2)=4000
      X(1,3)=200000
      DO400 L=1,N2
      DO410 N=1,N2
410   E2(L,N)=0
400   CONTINUE
      DO420 L=1,N2
420   E2(L,L)=1
      I2=1
300   M2=0
      M3=0
      D0=1
      J2=1
      DO10 JQ=1,N2
10      X3(JQ)=X(I2,JQ)
      DO430 I=1,N2
430   A1(I)=X3(I)
      AL2=0
      DO440 I=1,N2
440   X4(I)=X3(I)
      Y4(I2)=F3(X4)
      B4(J2)=Y4(I2)
      B2=B4(J2)
      DO20 JQ=1,N2
20      X0(JQ)=X(I2,JQ)
C       SEARCH BEGINS
195   K3=1
      JQ1=0
      DO30 JQ2=1,K2
      S3=(2**(JQ2-1))*S2
      DO40 JZ=1,N2
40    X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
      DO 450 I=1,N2
450   X4(I)=X0(I)
      Y2=F3(X4)
      IF((B2-Y2).GT..00001)  GO TO 50
      IF((Y2-B2).GT..00001)  GO TO 60
      IF((Y2-B2).LT..00001)  GO TO 30
      GO TO 30
60    IF(JQ1.EQ.0) GO TO 70
      IF(K3.EQ.2) GO TO 80
      DO460 M=1,N2
460   X1(M)=X0(M)
      DO 90 JZ=1,N2
90     X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
      GOTO80
50    JQ1=1
      K3=1
      GO TO30
70      DO470 JK=1,N2
470   X1(JK)=X0(JK)
      S2=-S2
      K3=2
      JQ1=1
30    B2=Y2
      GO TO 100
80    R2=3*((2**(JQ2-2))*ABS(S2))
C     BEGIN FIBERNACI SEARCH
      JQ1=0
      N4=INT(1-2.1*ALOG(F2/R2))
      DO480 JL=1,N2
480   X7(JL)=X1(JL)-X0(JL)
      DO490 KL=1,N2
490   D2(KL)=.618*X7(KL)
      DO 500 JM=1,N2
500   X8(JM)=X0(JM)+D2(JM)
      DO 510 JN=1,N2
510   X2(JN)=X1(JN)-D2(JN)
      DO 520 KM=1,N2
520   X4(KM)=X2(KM)
      V2=F3(X4)
      DO530 KN=1,N2
530   X4(KN)=X8(KN)
      W2=F3(X4)
      DO130 K9=1,N4
      DO540 LL=1,N2
540   D2(LL)=.618*D2(LL)
      IF (ABS(W2-V2).LT..00001) GO TO 110
      IF(W2.GT.V2) GO TO 120
      DO550 JJ=1,N2
550   X0(JJ)=X2(JJ)
      DO560 KK=1,N2
560   X2(KK)=X8(KK)
      DO 570 MM=1,N2
570   X8(MM)=X0(MM)+D2(MM)
      V2=W2
```

```
      DO580 NN=1,N2
580   X4(NN)=X8(NN)
      W2=F3(X4)
      GO TO 140
120      DO590 II=1,N2
590   X1(II)=X8(II)
      DO595 IJ=1,N2
595   X8(IJ)=X2(IJ)
      DO600 JJ=1,N2
600   X2(JJ)=X1(JJ)-D2(JJ)
      W2=V2
      DO610 KK=1,N2
610   X4(KK)=X2(KK)
130   V2=F3(X4)
140   CONTINUE
110      DO620 LL=1,N2
620   X7(LL)=X2(LL)+X8(LL)
      DO630 MM=1,N2
630   X0(MM)=(.5)*X7(MM)
      IF(J2.EQ.N2+1) GO TO 150
      A3=E2(1,J2)
      JZ1=1
      DO 160 JZ=2,N2
      IF (E2(JZ,J2).GT.A3) GO TO 170
      GO TO 160
170   A3=E2(JZ,J2)
      JZ1=JZ
160   CONTINUE
      AL1=(X0(JZ1)-A1(JZ1))/A3
      IF (ABS(AL1).GT.ABS(AL2 ))GO TO 180
      GO TO 190
180   AL2=AL1
      JS=J2
190   DO640 NN=1,N2
640   A1(NN)=X0(NN)
      J2=J2+1
      DO650 JJ=1,N2
650   X4(JJ)=X0(JJ)
      B2=F3(X4)
      IF (J2.LT.(N2+1)) GO TO 195
      DO660 KK=1,N2
660   U2(KK)=X0(KK)-X3(KK)
      U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2
      DO670 LL=1,N2
670   U2(LL)=(1/U1)*U2(LL)
      DO200 JZ=1,N2
200   E2(JZ,N2+1)=U2(JZ)
      GO TO 195
150      I2=I2+1
      WRITE(2,1100)I2
1100  FORMAT('    I2=',I3)
      DO 210 JZ=1,N2
210   X(I2,JZ)=X0(JZ)
      DO 220 JZ=1,N2
220      IF (ABS((X0(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
      GO TO 240
230   IF(I2.EQ.50) GO TO 270
      AL3=AL2*D0/U1
      IF (ABS(AL3).LT..000001) GO TO 280
      D0=AL3
      DO 290 JZ=1,N2
290   E2(JZ,JS)=U2(JZ)
280   GO TO 300
240   DO700 KK=1,N2
700   X4(KK)=X0(KK)
      Y3=F3(X4)
270   CONTINUE
      WRITE(2,310)Y3
      WRITE(6,310)Y3
310   FORMAT('MINIMUM (CNR)TH ='F10.7)
      WRITE(2,320)X0(1),X0(2),X0(3)
      WRITE(6,320)X0(1),X0(2),X0(3)
320   FORMAT('  A=',F9.2,'   B=',F8.2,'   K= ',F10.1)
      WRITE(2,325)X(1,1),X(1,2),X(1,3)
325   FORMAT(' STARTED FROM A=',F7.1,'  B= ',F6.1,' K= ',F8.0)
      IF (I2.LT.50) GO TO 330
      WRITE(2,340)
340      FORMAT('SOLUTION DID NOT CONVERGE')
      GO TO 330
100      WRITE(2,350)
350   FORMAT('THERE IS NO MINIMUM')
330   STOP
      END
      FUNCTION F3(X4)
      DIMENSION X4(3),V(7),FK(3),R(3),H(2),H1(2)
      A=X4(1)
      B=X4(2)
      C=X4(3)
      PI=3.1415927
      BP=35000.0
      D=C*C*B*B/A/A+2.0*C*B*B/A+B*B-2.0*C*B
      T4=C*(C*B+A**2)/4.0/A/(A+C)
      PI=3.141593
      T=D
      U=C*C*B*B
      WA=1884.96
      WB=20734.5
      D5=T**2/4-U
      COND=222.
      IF(D5.LT.0)GOTO47
      COND=111.
```

```
      R(1)=-T/2+SQRT(D5)
      R(2)=-T/2-SQRT(D5)
      FK0=B*B/R(1)/R(2)
      FK(1)=(R(1)+B*B)/R(1)/(R(1)-R(2))
      FK(2)=(R(2)+B*B)/R(2)/(R(2)-R(1))
      DO10 J6=1,2
      IF(R(J6).GT.0)GO TO     39
      IF(R(J6).LT.0) GO TO 42
      GO TO 36
39      S5=SQRT(R(J6))
      H1(J6)=0.5/S5*ALOG((WB-S5)*(WA+S5)/(WB+S5)/(WA-S5))
      GO TO 10
42      S6=SQRT(-R(J6))
      H1(J6)=1.0/S6*(ATAN(WB/S6)-ATAN(WA/S6))
10    CONTINUE
      HA1=(FK(1)*H1(1)+FK(2)*H1(2)+FK0*(1.0/WA-1.0/WB))/2./PI
      GO TO88
36      WRITE(2,250)J6
250   FORMAT('  R(',I1,')=0')
      STOP
47    G7=T/2.0
      JK1=2
      JK2=2
      JK3=2
      JK4=2
      JK5=2
      JK6=2
      JK7=2
      G8=-SQRT(U-T*T/4)
      FJ6=U**0.25
      FJ9=0.5*ATAN((-SQRT(C*C*B*B-(T/2.0)**2))/(T/2.0))
      IF(T.GT.0.)FJ9=FJ9+PI/2.0
      FJ7=FJ6*COS(FJ9)
      FJ8=FJ6*SIN(FJ9)
      V(1)=B*B/U
      V(6)=-V(1)
      V(7)=1.0-T*V(1)
      RT17=WB-FJ7
      RT18=-FJ8
      RT16=SQRT(RT17**2+RT18**2)
      RT19=ATAN(RT18/RT17)
      IF(RT17.LT.0.0)JK1=1
      IF(RT17.LT.0.)      RT19=RT19+PI
      RT27=WB+FJ7
      RT28=FJ8
      RT26=SQRT(RT27**2+RT28**2)
      RT29=ATAN(RT28/RT27)
      IF(RT27.LT.0.0)JK2=1
      IF(RT27.LT.0.) RT29=RT29+PI
      RT37=WA+FJ7
      RT38=FJ8
      RT36=SQRT(RT37**2+RT38**2)
      RT39=ATAN(RT38/RT37)
      IF(RT37.LT.0.0)JK3=1
      IF(RT37.LT.0.0) RT39=RT39+PI
      RT47=WA-FJ7
      RT48=-FJ8
      RT46=SQRT(RT47**2+RT48**2)
      RT49=ATAN(RT48/RT47)
      IF(RT47.LT.0.0)JK4=1
      IF(RT47.LT.0.0)RT49=RT49+PI
      RT6=RT16*RT36/RT26/RT46
      RT9=RT19+RT39-RT29-RT49
      IF(RT9.LT.0.0)JK5=1
      IF(RT9.LT.0.0)RT9=RT9+2.*PI
      RT7=RT6*COS(RT9)
      RT8=RT6*SIN(RT9)
      RIGT7=ALOG(RT6)
      RIGT8=RT9
      RIGT6=SQRT(RIGT7**2+RIGT8**2)
      RIGT9=ATAN(RIGT8/RIGT7)
      IF(RIGT7.LT.0.0)JK6=1
      IF(RIGT7.LT.0.0)RIGT9=RIGT9+PI
      UPP7=1.0-V(6)/V(7)*G7
      UPP8=-V(6)/V(7)*G8
      UPP6=SQRT(UPP7**2+UPP8**2)
      UPP9=ATAN(UPP8/UPP7)
      IF(UPP7.LT.0.0)JK7=1
      IF(UPP7.LT.0.0)UPP9=UPP9+PI
      DOWN6=2.0*FJ6*G8
      DOWN9=FJ9-1.570796
      RATIO6=UPP6/DOWN6
      RATIO9=UPP9-DOWN9
      ROD6=RIGT6*RATIO6
      ROD9=RATIO9+RIGT9
      H2=ROD6*COS(ROD9)
      HA1=(V(1)*(1.0/WA-1.0/WB)+V(7)*H2)/2.0/PI
88      GNU=0.25
      ETA=8.*PI**3*1.E7*WA*WB/(WB-WA)
      F3=T4/BP/(GNU-ETA*HA1)
34    RETURN
      END
```

```
C  **CASE3 ** STANDARD SYSTEM PLUS ADDITIONAL POLE 4 VARIABLES
NO FILTER 4 VARIABLES D IS HELD CONSTANT
C  RECIPROCAL CONSTANTS
      DIMENSION X(40,4),E2(5,5),Y4(40),B4(4),X0(4),X1(4),X5(4)
      DIMENSION X2(4),D2(4),U2(4),X3(4),X4(4),X7(4),A1(4),X8(4)
      PI=3.141593
      N2=4
      K2=50
      S2=8
      T2=0.01
      F2=0.001
      X(1,1)=35000
      X(1,2)=2400
      X(1,3)=200000
      X(1,4)=500000
      DO400 L=1,N2
      DO410 N=1,N2
410   E2(L,N)=0
400   CONTINUE
      DO420 L=1,N2
420   E2(L,L)=1
      I2=1
300   M2=0
      M3=0
      D0=1
      J2=1
      DO10 JQ=1,N2
10      X3(JQ)=X(I2,JQ)
      DO430 I=1,N2
430   A1(I)=X3(I)
      AL2=0
      DO440 I=1,N2
440   X4(I)=X3(I)
      Y4(I2)=F3(X4)
      B4(J2)=Y4(I2)
      B2=B4(J2)
      DO20 JQ=1,N2
20      X0(JQ)=X(I2,JQ)
C       SEARCH BEGINS
      JQ1=0
195   K3=1
      DO30 JQ2=1,K2
      S3=(2**(JQ2-1))*S2
      DO45 K=1,N2
45    X5(K)=X0(K)
      DO40 JZ=1,N2
      IF(JZ.EQ.3)GOTO40
      X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
40    CONTINUE
      DO48 K=1,N2
48    IF(X0(K).NE.X5(K))GOTO49
```

```
      GOTO128
49    DO450 I=1,N2
450   X4(I)=X0(I)
      Y2=F3(X4)
      IF((B2-Y2).GT..0005)  GO TO 50
      IF((Y2-B2).GT..0005)  GO TO 60
      IF((Y2-B2).LT..0005)  GO TO 30
      GO TO 30
60    IF(JQ1.EQ.0) GO TO 70
      IF(K3.EQ.2) GO TO 80
      DO460 M=1,N2
460   X1(M)=X0(M)
      DO 90 JZ=1,N2
      IF(JZ.EQ.3)GOTO90
      X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
90    CONTINUE
      GOTO80
50    JQ1=1
      K3=1
      GO TO30
70      DO470 JK=1,N2
470   X1(JK)=X0(JK)
      S2=-S2
      K3=2
      JQ1=1
30    B2=Y2
      GO TO 100
80    R2=3*((2**(JQ2-2))*ABS(S2))
C     BEGIN FIBERNACI SEARCH
      JQ1=0
      N4=INT(1-2.1*ALOG(F2/R2))
      DO480 JL=1,N2
480   X7(JL)=X1(JL)-X0(JL)
      DO490 KL=1,N2
490   D2(KL)=.618*X7(KL)
      DO 500 JM=1,N2
500   X8(JM)=X0(JM)+D2(JM)
      DO 510 JN=1,N2
510   X2(JN)=X1(JN)-D2(JN)
      DO 520 KM=1,N2
520   X4(KM)=X2(KM)
      V2=F3(X4)
      DO530 KN=1,N2
530   X4(KN)=X8(KN)
      W2=F3(X4)
      DO140 K9=1,N4
      DO540 LL=1,N2
540   D2(LL)=.618*D2(LL)
      IF (ABS(W2-V2).LT..001) GO TO 110
      IF(W2.GT.V2) GO TO 120
      DO550 JJ=1,N2
550   X0(JJ)=X2(JJ)
      DO560 KK=1,N2
560   X2(KK)=X8(KK)
      DO 570 MM=1,N2
570   X8(MM)=X0(MM)+D2(MM)
      V2=W2
      DO580 NN=1,N2
580   X4(NN)=X8(NN)
      W2=F3(X4)
      GO TO 140
120      DO590 II=1,N2
590   X1(II)=X8(II)
      DO595 IJ=1,N2
595   X8(IJ)=X2(IJ)
      DO600 JJ=1,N2
600   X2(JJ)=X1(JJ)-D2(JJ)
      W2=V2
      DO610 KK=1,N2
610   X4(KK)=X2(KK)
130   V2=F3(X4)
140   CONTINUE
110      DO620 LL=1,N2
620   X7(LL)=X2(LL)+X8(LL)
      DO630 MM=1,N2
630   X0(MM)=(.5)*X7(MM)
128   IF(J2.EQ.N2+1) GO TO 150
      J2=J2+1
      IF(J2.EQ.3)J2=4
      IF(J2.LT.N2+1)GOTO710
      DO660 KK=1,N2
660   U2(KK)=X0(KK)-X3(KK)
      U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2+U2(4)**2)
      DO670 LL=1,N2
670   U2(LL)=(1/U1)*U2(LL)
      DO695 JZ=1,N2
695   E2(JZ,N2+1)=U2(JZ)
      DO680 KL=1,N2
      DO690 LK=1,N2
690   E2(LK,KL)=E2(LK,KL+1)
680   CONTINUE
      GOTO195
710   DO720 KM=1,N2
720   X4(KM)=X0(KM)
      B2=F3(X4)
      GOTO195
150      I2=I2+1
      WRITE(2,1100)I2
1100  FORMAT('    I2=',I3)
      DO161KL=1,N2
```

```
161      X4(KL)=X0(KL)
      WRITE(2,1150)F3(X4)
1150  FORMAT('     F3=',F12.5)
      WRITE(2,1160)X4(1),X4(2),X4(3),X4(4)
1160  FORMAT(5F13.4)
      DO 210 JZ=1,N2
210   X(I2,JZ)=X0(JZ)
      DO 220 JZ=1,N2
220      IF (ABS((X0(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
      GO TO 240
230   IF(I2.EQ.50) GO TO 270
      AL3=AL2*D0/U1
      IF (ABS(AL3).LT..000001) GO TO 280
      D0=AL3
      DO 290 JZ=1,N2
290   E2(JZ,JS)=U2(JZ)
280   GO TO 300
240   DO700 KK=1,N2
700   X4(KK)=X0(KK)
      Y3=F3(X4)
270   CONTINUE
      WRITE(2,310)Y3
310   FORMAT('MINIMUM (CNR)TH ='F10.7)
      WRITE(2,320)X0(1),X0(2),X0(3),X0(4)
320   FORMAT(' A=',F8.0,' B=',F8.0,'D='F8.0,'K=',F8.0)
      WRITE(2,370)
      WRITE(2,360)X(1,1),X(1,2),X(1,3),X(1,4)
370   FORMAT('  THE INITIAL POINT WAS  ')
360   FORMAT(' A=',F8.0,'  B=',F6.0,'  D=',F7.0,
     C '  K=',F8.0)
      IF (I2.LT.50) GO TO 330
      WRITE(2,340)
340      FORMAT('SOLUTION DID NOT CONVERGE')
      GO TO 330
100      WRITE(2,350)
350   FORMAT('THERE IS NO MINIMUM')
330   STOP
      END
      FUNCTION F3(X4)
      DIMENSION X4(4),R(3),FK(7),H(3),V(7)
      DOUBLE PRECISION A,B,C,D,E,FJ,FL,FM
      PI=3.141593
      A1=1.0/X4(1)
      B1=1.0/X4(2)/X4(3)/X4(4)
      C1=1.0/X4(2)/X4(4)+1.0/X4(3)/X4(4)
      D1=1.0/X4(1)+1.0/X4(4)
      A=A1*A1/B1/B1
      B=(X4(2)*X4(3)*X4(4))**2
      C=(C1*C1-2.0*D1*B1)/B1/B1
      D=(D1*D1-2.0*C1)*B
      BP=35000.0
      HA=(A1*A1+C1)/(4.*(D1*C1-B1))
88    T4=HA
      W=1000*6.2832
      T6=(C1*C1*W**4+(1.0/X4(4)*W-B1*W**3)**2)/((1.0-C1*W*W)
     C**2+(D1*W-B1*W**3)**2)
      BETA=10.0
      BP=35000.0
      F3=4.0*PI**2*T4/BP/(PI-2.0*BETA*SQRT(T6))**2
      GOTO34
111      S=0.
      FN=100.0
      DELT=PI*BP/FN
      Z=DELT/2.
      NFN=INT(FN)
      DO345N=1,NFN
      S=S+(1.0+A1*Z*Z*A1)/((1.0-C1*Z*Z)**2+(D1*Z-B1*Z**3)
     C**2)*DELT/6.28319
      Z=Z+DELT
345   CONTINUE
      WRITE(2,346)S,T4
346   FORMAT('  S=',E13.4,'  T4=',E13.4)
34    RETURN
      END
```

```
C  **CASE4 ** STANDARD SYSTEM PLUS ADDITIONAL POLE 4 VARIABLES
 NO FILTER TEST TONE MOD D IS HELD CONSTANT
C  RECIPROCAL CONSTANTS
        DIMENSION X(40,4),E2(5,5),Y4(40),B4(4),X0(4),X1(4),X5(4)
        DIMENSION X2(4),D2(4),U2(4),X3(4),X4(4),X7(4),A1(4),X8(4)
        PI=3.141593
        N2=4
        K2=50
        S2=8
        T2=0.01
        F2=0.001
        X(1,1)=20000
        X(1,2)=5000
        X(1,3)=1000000
        X(1,4)=120000
        DO400 L=1,N2
        DO410 N=1,N2
410     E2(L,N)=0
400     CONTINUE
        DO420 L=1,N2
420     E2(L,L)=1
        I2=1
300     M2=0
        M3=0
        DO=1
        J2=1
        DO10 JQ=1,N2
10        X3(JQ)=X(I2,JQ)
        DO430 I=1,N2
430     A1(I)=X3(I)
        AL2=0
        DO440 I=1,N2
440     X4(I)=X3(I)
        Y4(I2)=F3(X4)
        B4(J2)=Y4(I2)
        B2=B4(J2)
        DO20 JQ=1,N2
20        X0(JQ)=X(I2,JQ)
C         SEARCH BEGINS
        JQ1=0
195     K3=1
        DO30 JQ2=1,K2
        S3=(2**(JQ2-1))*S2
        DO45 K=1,N2
45      X5(K)=X0(K)
        DO40 JZ=1,N2
        IF(JZ.EQ.3)GOTO40
        X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
40      CONTINUE
        DO48 K=1,N2
48      IF(X0(K).NE.X5(K))GOTO49
```

```
      GOTO128
49    DO450 I=1,N2
450   X4(I)=X0(I)
      Y2=F3(X4)
      IF((B2-Y2).GT..0005)  GO TO 50
      IF((Y2-B2).GT..0005)  GO TO 60
      IF((Y2-B2).LT..0005)  GO TO 30
      GO TO 30
60    IF(JQ1.EQ.0) GO TO 70
      IF(K3.EQ.2) GO TO 80
      DO460 M=1,N2
460   X1(M)=X0(M)
      DO 90 JZ=1,N2
      IF(JZ.EQ.3)GOTO90
      X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
90    CONTINUE
      GOTO80
50    JQ1=1
      K3=1
      GO TO30
70      DO470 JK=1,N2
470   X1(JK)=X0(JK)
      S2=-S2
      K3=2
      JQ1=1
30    B2=Y2
      GO TO 100
80    R2=3*((2**(JQ2-2))*ABS(S2))
C     BEGIN FIBERNACI SEARCH
Y
      JQ1=0
      N4=INT(1-2.1*ALOG(F2/R2))
      DO480 JL=1,N2
480   X7(JL)=X1(JL)-X0(JL)
      DO490 KL=1,N2
490   D2(KL)=.618*X7(KL)
      DO 500 JM=1,N2
500   X8(JM)=X0(JM)+D2(JM)
      DO 510 JN=1,N2
510   X2(JN)=X1(JN)-D2(JN)
      DO 520 KM=1,N2
520   X4(KM)=X2(KM)
      V2=F3(X4)
      DO530 KN=1,N2
530   X4(KN)=X8(KN)
      W2=F3(X4)
      DO140 K9=1,N4
      DO540 LL=1,N2
540   D2(LL)=.618*D2(LL)
      IF (ABS(W2-V2).LT..001) GO TO 110
      IF(W2.GT.V2) GO TO 120
      DO550 JJ=1,N2
550   X0(JJ)=X2(JJ)
      DO560 KK=1,N2
560   X2(KK)=X8(KK)
      DO 570 MM=1,N2
570   X8(MM)=X0(MM)+D2(MM)
      V2=W2
      DO580 NN=1,N2
580   X4(NN)=X8(NN)
      W2=F3(X4)
      GO TO 140
120     DO590 II=1,N2
590   X1(II)=X8(II)
      DO595 IJ=1,N2
595   X8(IJ)=X2(IJ)
      DO600 JJ=1,N2
600   X2(JJ)=X1(JJ)-D2(JJ)
      W2=V2
      DO610 KK=1,N2
610   X4(KK)=X2(KK)
130   V2=F3(X4)
140   CONTINUE
110     DO620 LL=1,N2
620   X7(LL)=X2(LL)+X8(LL)
      DO630 MM=1,N2
630   X0(MM)=(.5)*X7(MM)
128   IF(J2.EQ.N2+1) GO TO 150
      J2=J2+1
      IF(J2.EQ.3)J2=4
      IF(J2.LT.N2+1)GOTO710
      DO660 KK=1,N2
660   U2(KK)=X0(KK)-X3(KK)
      U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2+U2(4)**2)
      DO670 LL=1,N2
670   U2(LL)=(1/U1)*U2(LL)
      DO695 JZ=1,N2
695   E2(JZ,N2+1)=U2(JZ)
      DO680 KL=1,N2
      DO690 LK=1,N2
690   E2(LK,KL)=E2(LK,KL+1)
680   CONTINUE
      GOTO195
710   DO720 KM=1,N2
720   X4(KM)=X0(KM)
      B2=F3(X4)
      GOTO195
150     I2=I2+1
      WRITE(2,1100)I2
1100  FORMAT('    I2=',I3)
```

```
      DO161KL=1,N2
161     X4(KL)=X0(KL)
      WRITE(2,1150)F3(X4)
1150  FORMAT('     F3=',E13.4)
      WRITE(2,1160)X4(1),X4(2),X4(3),X4(4)
1160  FORMAT(5F13.4)
      DO 210 JZ=1,N2
210   X(I2,JZ)=X0(JZ)
      DO 220 JZ=1,N2
220     IF (ABS((X0(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
      GO TO 240
230   IF(I2.EQ.50) GO TO 270
      AL3=AL2*D0/U1
      IF (ABS(AL3).LT..000001) GO TO 280
      D0=AL3
      DO 290 JZ=1,N2
290   E2(JZ,JS)=U2(JZ)
280   GO TO 300
240   DO700 KK=1,N2
700   X4(KK)=X0(KK)
      Y3=F3(X4)
270   CONTINUE
      WRITE(2,310)Y3
310   FORMAT(' MINIMUM (CNR)TH ='E13.4)
      WRITE(2,320)X0(1),X0(2),X0(3),X0(4)
320   FORMAT(' A=',F8.0,' B=',F8.0,'D='F8.0,'K=',F8.0)
      WRITE(2,370)
      WRITE(2,360)X(1,1),X(1,2),X(1,3),X(1,4)
370   FORMAT('  THE INITIAL POINT WAS  ')
360   FORMAT(' A=',F8.0,'  B=',F6.0,'  D=',F7.0,
     C '  K=',F8.0)
      IF (I2.LT.50) GO TO 330
      WRITE(2,340)
340     FORMAT('SOLUTION DID NOT CONVERGE')
      GO TO 330
100     WRITE(2,350)
350   FORMAT('THERE IS NO MINIMUM')
330   STOP
      END
      FUNCTION F3(X4)
      DIMENSION X4(4),R(3),FK(7),H(7),V(7),H1(4)
      DOUBLE PRECISION A,B,C,D,E,FJ,FL,FM
      PI=3.141593
      A=1.0/X4(1)
      B=1.0/X4(2)/X4(3)/X4(4)
      C=1.0/X4(2)/X4(4)+1.0/X4(3)/X4(4)
      D=1.0/X4(1)+1.0/X4(4)
      BP=35000.0
      HA=(A*A+C)/(4.*(D*C-B))
      P=C*C/B/B-2.0/X4(4)/B
      Q=1.0/B/B/X4(4)/X4(4)
      FL=C*C/B/B-2.0*D/B
      FM=D*D/B/B-2.0*C/B/B
      WA=300.*2.*PI
      WB=3300.*2.*PI
C    FIND THE ROOT OF X**3+LX**2+MX+1/(B**2)
      DOUBLE PRECISION X,XFIRST,Y,Y5,YA,T,U
      XFIRST=-1.0E13
      ACC=1.0E-13
      LH=0
      GOTO31
32    XFIRST=10*XFIRST
      LH=LH+1
      GOTO31
38    IF(LH.NE.5) GO TO 32
      IF(ACC.LT.1.0E-6)ACC=ACC*10
      PRINT,ACC
      IF(ACC.LT.1.0E-6) GO TO 31
      WRITE(2,35)X4(3)
35    FORMAT('  NO ROOT FOUND FOR X4( )=',F9.1)
      GO TO 34
31    LF=0
      X=XFIRST
33    NU=0
      GOTO17
37    IF(LF.EQ.1) GO TO 38
      X=-X
      NU=0
      LF=1
17      Y=X**3+FL*X**2+FM*X+1.0/B/B
      Y5=3*X**2+2*FL*X+FM
      AB1=X**3
      AB2=FL*X**2
      AB3=FM*X
      YA=(ABS(AB1)+ABS(AB2)+ABS(AB3)+1.0/B/B)*ACC
      ABY=Y
      IF(ABS(ABY).LT.YA) GOTO24
      X=X-Y/Y5
      NU=NU+1
      IF(NU.EQ.100)GOTO37
      GOTO 17
24      T=FL+X
      U=FM+X*T
      R(1)=X
      WA=1884.96
      WB=20734.5
      H1(4)=1.0/WA-1.0/WB
      D5=T**2/4-U
      IF(D5.LT.0)GOTO47
      R(2)=-T/2+SQRT(D5)
```

```
      R(3)=-T/2-SQRT(D5)
      FK(4)=Q*B*B
      FK(5)=(R(1)**2+P*R(1)+Q)/R(1)/(R(1)-R(2))/(R(1)-R(3))
      FK(6)=(R(2)**2+P*R(2)+Q)/R(2)/(R(2)-R(3))/(R(2)-R(1))
      FK(7)=(R(3)**2+P*R(3)+Q)/R(3)/(R(3)-R(1))/(R(3)-R(2))
      DO10 J6=1,3
      IF(R(J6).GT.0)GO TO     39
      IF(R(J6).LT.0) GO TO 42
      GO TO 36
39      S5=SQRT(R(J6))
      H1(J6)=0.5/S5*ALOG((WB-S5)*(WA+S5)/(WB+S5)/(WA-
      GO TO 10
42      S6=SQRT(-R(J6))
      H1(J6)=1.0/S6*(ATAN(WB/S6)-ATAN(WA/S6))
10    CONTINUE
      HA1=(FK(5)*H1(1)+FK(6)*H1(2)+FK(7)*H1(3)+FK(4)*H1(4))/2.
/3.141593
      GO TO88
47      D6=R(1)**2+U+T*R(1)
      D7=R(1)*U**2+R(1)**2*T*U+R(1)**3*U
      V(4)=Q*B*B
      DEN=U*R(1)*(R(1)**2+U+T*R(1))
      TOP5=U*(R(1)*P+R(1)**2+Q)
      TOP6=R(1)*Q*(T+R(1))-U*R(1)*(R(1)+P)
      TOP7=U*U*R(1)-P*R(1)*U*(R(1)+T)+Q*R(1)*(T*R(1)-U+T*T)
      V(5)=TOP5/DEN
      V(6)=TOP6/DEN
      V(7)=TOP7/DEN
      IF(R(1).GT.0) GO TO 55
      IF(R(1).LT.0) GO TO 65
36      WRITE(2,200)J6
200   FORMAT('  R(',I1,')=0')
      STOP
55      S5=SQRT(R(1))
      H1(1)=0.5/S5*ALOG((WB-S5)*(WA+S5)/(WB+S5)/(WA-S5))
      GO TO58
65      S6=SQRT(-R(1))
      H1(1)=1.0/S6*(ATAN(WB/S6)-ATAN(WA/S6))
C  SUBSCRIPTS  6(MAG), 7(REAL),  8 (IMAG) , 9 (ANGLE)
58    G7=T/2
      G8=-SQRT(-D5)
      G6=SQRT(G8**2+G7**2)
      FJ6=SQRT(G6)
      FJ9=ATAN(G8/G7)/2
      IF(G7.GT.0) FJ9=FJ9+1.57079
      FJ7=FJ6*COS(FJ9)
      FJ8=FJ6*SIN(FJ9)
      RT17=WB-FJ7
      RT18=-FJ8
      RT16=SQRT(RT17**2+RT18**2)
      RT19=ATAN(RT18/RT17)
      IF(RT17.LT.0.)      RT19=RT19+PI
      RT27=WB+FJ7
      RT28=FJ8
      RT26=SQRT(RT27**2+RT28**2)
      RT29=ATAN(RT28/RT27)
      IF(RT27.LT.0.) RT29=RT29+PI
      RT37=WA+FJ7
      RT38=FJ8
      RT36=SQRT(RT37**2+RT38**2)
      RT39=ATAN(RT38/RT37)
      IF(RT37.LT.0.0) RT39=RT39+PI
      RT47=WA-FJ7
      RT48=-FJ8
      RT46=SQRT(RT47**2+RT48**2)
      RT49=ATAN(RT48/RT47)
      IF(RT47.LT.0.0)RT49=RT49+PI
      RT6=RT16*RT36/RT26/RT46
      RT9=RT19+RT39-RT29-RT49
      IF(RT9.LT.0.0)RT9=RT9+2.*PI
      RT8=RT6*COS(RT9)
      RT7=RT6*SIN(RT9)
      RIGT7=ALOG(RT6)
      RIGT8=RT9
      RIGT6=SQRT(RIGT7**2+RIGT8**2)
      RIGT9=ATAN(RIGT8/RIGT7)
      IF(RIGT7.LT.0.0)RIGT9=RIGT9+PI
      UPP7=1.0-V(6)/V(7)*G7
      UPP8=-V(6)/V(7)*G8
      UPP6=SQRT(UPP7**2+UPP8**2)
      UPP9=ATAN(UPP8/UPP7)
      IF(UPP7.LT.0.0)UPP9=UPP9+PI
      BOT6=2.0*FJ6*G8
      BOT9=FJ9-1.570796
      FRACT6=UPP6/BOT6
      FRACT9=UPP9-BOT9
      PROD6=RIGT6*FRACT6
      PROD9=FRACT9+RIGT9
      H1(2)=PROD6*COS(PROD9)
      HA1=(V(4)*H1(4)+V(5)*H1(1)+V(7)*H1(2))/2.0/PI
88    CONTINUE
      GNU=0.25
      ETA=8.*PI**3*1.E7*WA*WB/(WB-WA)
      F3=HA/BP/(GNU-ETA*HA1)
34    RETURN
      END
```

```
C    ****  CASE5  ***  D IS INFINITE   MINIMIZING WITH
C        FOUR  VARIABLES     RECIPROCAL CONSTANTS
C     WITH PREDETECTION FILTER     TEST TONE CASE
      DIMENSION X(40,4),E2(5,5),Y4(40),B4(4),X0(4),X1(4),X5(4)
      DIMENSION X2(4),D2(4),U2(4),X3(4),X4(4),X7(4),A1(4),X8(4)
      N2=4
      K2=50
      S2=8
      T2=0.01
      F2=0.001
      X(1,1)=40000
      X(1,2)=1500
      X(1,3)=10000.
      X(1,4)=500000.
      DO400 L=1,N2
      DO410 N=1,N2
410   E2(L,N)=0
400   CONTINUE
      DO420 L=1,N2
420   E2(L,L)=1
      I2=1
300   M2=0
      M3=0
      D0=1
      J2=1
      DO10 JQ=1,N2
10      X3(JQ)=X(I2,JQ)
      DO430 I=1,N2
430   A1(I)=X3(I)
      AL2=0
      DO440 I=1,N2
440   X4(I)=X3(I)
      Y4(I2)=F3(X4)
      B4(J2)=Y4(I2)
      B2=B4(J2)
      DO20 JQ=1,N2
20      X0(JQ)=X(I2,JQ)
C       SEARCH BEGINS
      JQ1=0
195   K3=1
      DO30 JQ2=1,K2
      S3=(2**(JQ2-1))*S2
      DO45 K=1,N2
45    X5(K)=X0(K)
      DO40 JZ=1,N2
      X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
40    CONTINUE
      DO48 K=1,N2
48    IF(X0(K).NE.X5(K))GOTO49
      GOTO128
```

```
49      DO450 I=1,N2
450     X4(I)=X0(I)
        Y2=F3(X4)
        IF((B2-Y2).GT..0005)  GO TO 50
        IF((Y2-B2).GT..0005)  GO TO 60
        IF((Y2-B2).LT..0005)  GO TO 30
        GO TO 30
60      IF(JQ1.EQ.0) GO TO 70
        IF(K3.EQ.2) GO TO 80
        DO460 M=1,N2
460     X1(M)=X0(M)
        DO 90 JZ=1,N2
        X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
90      CONTINUE
        GOTO80
50      JQ1=1
        K3=1
        GO TO30
70        DO470 JK=1,N2
470     X1(JK)=X0(JK)
        S2=-S2
        K3=2
        JQ1=1
30      B2=Y2
        GO TO 100
80      R2=3*((2**(JQ2-2))*ABS(S2))
C       BEGIN FIBERNACI SEARCH
        JQ1=0
        N4=INT(1-2.1*ALOG(F2/R2))
        DO480 JL=1,N2
480     X7(JL)=X1(JL)-X0(JL)
        DO490 KL=1,N2
490     D2(KL)=.618*X7(KL)
        DO 500 JM=1,N2
500     X8(JM)=X0(JM)+D2(JM)
        DO 510 JN=1,N2
510     X2(JN)=X1(JN)-D2(JN)
        DO 520 KM=1,N2
520     X4(KM)=X2(KM)
        V2=F3(X4)
        DO530 KN=1,N2
530     X4(KN)=X8(KN)
        W2=F3(X4)
        DO140 K9=1,N4
        DO540 LL=1,N2
540     D2(LL)=.618*D2(LL)
        IF (ABS(W2-V2).LT..001) GO TO 110
        IF(W2.GT.V2) GO TO 120
        DO550 JJ=1,N2
550     X0(JJ)=X2(JJ)
        DO560 KK=1,N2
560     X2(KK)=X8(KK)
        DO 570 MM=1,N2
570     X8(MM)=X0(MM)+D2(MM)
        V2=W2
        DO580 NN=1,N2
580     X4(NN)=X8(NN)
        W2=F3(X4)
        GO TO 140
120       DO590 II=1,N2
590     X1(II)=X8(II)
        DO595 IJ=1,N2
595     X8(IJ)=X2(IJ)
        DO600 JJ=1,N2
600     X2(JJ)=X1(JJ)-D2(JJ)
        W2=V2
        DO610 KK=1,N2
610     X4(KK)=X2(KK)
130     V2=F3(X4)
140     CONTINUE
110       DO620 LL=1,N2
620     X7(LL)=X2(LL)+X8(LL)
        DO630 MM=1,N2
630     X0(MM)=(.5)*X7(MM)
        IF(J2.EQ.N2+1) GO TO 150
128     J2=J2+1
        IF(J2.LT.N2+1)GOTO710
        DO660 KK=1,N2
660     U2(KK)=X0(KK)-X3(KK)
        U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2+U2(4)**2)
        DO670 LL=1,N2
670     U2(LL)=(1/U1)*U2(LL)
        DO695 JZ=1,N2
695     E2(JZ,N2+1)=U2(JZ)
        DO680 KL=1,N2
        DO690 LK=1,N2
690     E2(LK,KL)=E2(LK,KL+1)
680     CONTINUE
        GOTO195
710     DO720 KM=1,N2
720     X4(KM)=X0(KM)
        B2=F3(X4)
        GOTO195
150       I2=I2+1
        WRITE(2,1100)I2
1100    FORMAT('    I2=',I3)
        DO161KL=1,4
161       X4(KL)=X0(KL)
        WRITE(2,1150)F3(X4)
1150    FORMAT('     F3=',F12.5)
```

```
      WRITE(2,1160)X4(1),X4(2),X4(3),X4(4)
 1160 FORMAT(5F13.4)
      DO 210 JZ=1,N2
  210 X(I2,JZ)=X0(JZ)
      DO 220 JZ=1,N2
  220    IF (ABS((X0(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
      GO TO 240
  230 IF(I2.EQ.50) GO TO 270
      AL3=AL2*D0/U1
      IF (ABS(AL3).LT..000001) GO TO 280
      D0=AL3
      DO 290 JZ=1,N2
  290 E2(JZ,JS)=U2(JZ)
  280 GO TO 300
  240 DO700 KK=1,N2
  700 X4(KK)=X0(KK)
      Y3=F3(X4)
  270 CONTINUE
      WRITE(2,310)Y3
  310 FORMAT(' MINIMUM (CNR)TH ='F10.7)
      WRITE(2,320)X0(1),X0(2),X0(3)/10000,X0(4)
  320 FORMAT(' A=',F8.0,' B=',F8.0,'ALFA=',F8.4,'K=',F8.0)
      WRITE(2,370)
      WRITE(2,360)X(1,1),X(1,2),X(1,3)/10000,X(1,4)
  370 FORMAT('  THE INITIAL POINT WAS  ')
  360 FORMAT(' A=',F8.0,'  B=',F6.0,'  ALF=',F8.4,' K='
     C F8.0)
      IF (I2.LT.40) GO TO 330
      WRITE(2,340)
  340    FORMAT('SOLUTION DID NOT CONVERGE')
      GO TO 330
  100    WRITE(2,350)
  350 FORMAT('THERE IS NO MINIMUM')
  330 STOP
      END
      FUNCTION F3(X4)
      DIMENSIONX4(4)
      X4(3)=X4(3)/10000.
      A1=X4(1)
      B1=X4(2)
      AL=X4(3)
      K1=X4(4)
      PI=3.141593
      BP=58000
      REALJ7,J8,J9,J6
      A=(AL/K1/B1)**2
      B=(1./A1+AL/K1)**2-2.*AL/K1/B1
      C=((1+AL)/K1/B1)**2
      D=((1.+AL)/K1+1./A1)**2-2.*((1.+AL)/K1/B1)
      G7=D/2./C
      G8=-SQRT(1./C-(D/2/C)**2)
      J6=(1./C)**0.25
      J9=1.0/2.0*ATAN((-SQRT(1/C-(D/2/C)**2))/(D/2/C))
      IF(D.GT.0)J9=J9+PI/2
      J7=J6*COS(J9)
      J8=J6*SIN(J9)
      TOP7=J7-PI*BP
      TOP8=J8
      TOP6=SQRT(TOP7**2+TOP8**2)
      TOP9=ATAN(TOP8/TOP7)
      IF(TOP7.LT.0.0)TOP9=TOP9+PI
      BOT7=J7+PI*BP
      BOT8=J8
      BOT6=SQRT(BOT7**2+BOT8**2)
      BOT9=ATAN(BOT8/BOT7)
      IF(BOT7.LT.0.0)BOT9=BOT9+PI
      RT6=TOP6/BOT6
      RT9=TOP9-BOT9
      WRT7=ALOG(RT6)
      WRT8=RT9
      WRT6=SQRT(WRT7**2+WRT8**2)
      WRT9=ATAN(WRT8/WRT7)
      IF(WRT7.LT.0.0)WRT9=WRT9+PI
      FNUM7=1.-A/C+G7*(A*D/C-B)
      FNUM8=G8*(A*D/C-B)
      FNUM6=SQRT(FNUM7**2+FNUM8**2)
      FNUM9=ATAN(FNUM8/FNUM7)
      IF(FNUM7.LT.0.0)FNUM9=FNUM9+PI
      DEN7=2.*G8*J8
      DEN8=-2.*G8*J7
      DEN6=SQRT(DEN7**2+DEN8**2)
      DEN9=ATAN(DEN8/DEN7)
      IF(DEN7.LT.0.0)DEN9=DEN9+PI
      FRACT6=FNUM6/DEN6
      FRACT9=FNUM9-DEN9
      PROD6=FRACT6*WRT6
      PROD9=FRACT9+WRT9
      PROD7=PROD6*COS(PROD9)
      ANSWER=0.5/PI/C*(PI*A*BP+PROD7)
      BETA=10.0
      E=1.0/K1/B1
      F=1.0/K1
      G=(1.0+AL)/K1/B1
      H=(1.0+AL)/K1+1.0/A1
      WTT=2000.0*PI
      ETA=BETA*SQRT(E*E*WTT**4+F*F*WTT*WTT)
      UND=SQRT((1.-G*WTT*WTT)**2+H*H*WTT*WTT)
      THETA=ETA/UND
      F3=4.0*PI*PI/35000./(PI-2.*THETA)**2*ANSWER
      RETURN
      END
```

```
C    ****  CASE6  ***  D IS INFINITE   MINIMIZING WI
C         FOUR  VARIABLES     RECIPROCAL CONSTANTS
C     WITH PREDETECTION FILTER      VOICE MODULATION
      DIMENSION X(40,4),E2(5,5),Y4(40),B4(4),X0(4),X1(4),X5(4)
      DIMENSION X2(4),D2(4),U2(4),X3(4),X4(4),X7(4),A1(4),X8(4)
      N2=4
      K2=50
      S2=8
      T2=0.01
      F2=0.001
      X(1,1)=48000
      X(1,2)=5800
      X(1,3)=7200
      X(1,4)=160000.
      DO400 L=1,N2
      DO410 N=1,N2
410   E2(L,N)=0
400   CONTINUE
      DO420 L=1,N2
420   E2(L,L)=1
      I2=1
300   M2=0
      M3=0
      D0=1
      J2=1
      DO10 JQ=1,N2
10      X3(JQ)=X(I2,JQ)
      DO430 I=1,N2
430   A1(I)=X3(I)
      AL2=0
      DO440 I=1,N2
440   X4(I)=X3(I)
      Y4(I2)=F3(X4)
      B4(J2)=Y4(I2)
      B2=B4(J2)
      DO20 JQ=1,N2
20      X0(JQ)=X(I2,JQ)
C       SEARCH BEGINS
      JQ1=0
195   K3=1
      DO30 JQ2=1,K2
      S3=(2**(JQ2-1))*S2
      DO45 K=1,N2
45    X5(K)=X0(K)
      DO40 JZ=1,N2
      X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
40    CONTINUE
      DO48 K=1,N2
48    IF(X0(K).NE.X5(K))GOTO49
      GOTO128
49    DO450 I=1,N2
450   X4(I)=X0(I)
      Y2=F3(X4)
      IF((B2-Y2).GT..0005)  GO TO 50
      IF((Y2-B2).GT..0005)  GO TO 60
      IF((Y2-B2).LT..0005)  GO TO 30
      GO TO 30
60    IF(JQ1.EQ.0) GO TO 70
      IF(K3.EQ.2) GO TO 80
      DO460 M=1,N2
460   X1(M)=X0(M)
      DO 90 JZ=1,N2
      X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
90    CONTINUE
      GOTO80
50    JQ1=1
      K3=1
      GO TO30
70      DO470 JK=1,N2
470   X1(JK)=X0(JK)
      S2=-S2
      K3=2
      JQ1=1
30    B2=Y2
      GO TO 100
80    R2=3*((2**(JQ2-2))*ABS(S2))
C     BEGIN FIBERNACI SEARCH
      JQ1=0
      N4=INT(1-2.1*ALOG(F2/R2))
      DO480 JL=1,N2
480   X7(JL)=X1(JL)-X0(JL)
      DO490 KL=1,N2
490   D2(KL)=.618*X7(KL)
      DO 500 JM=1,N2
500   X8(JM)=X0(JM)+D2(JM)
      DO 510 JN=1,N2
510   X2(JN)=X1(JN)-D2(JN)
      DO 520 KM=1,N2
520   X4(KM)=X2(KM)
      V2=F3(X4)
      DO530 KN=1,N2
530   X4(KN)=X8(KN)
      W2=F3(X4)
      DO140 K9=1,N4
      DO540 LL=1,N2
540   D2(LL)=.618*D2(LL)
      IF (ABS(W2-V2).LT..001) GO TO 110
      IF(W2.GT.V2) GO TO 120
      DO550 JJ=1,N2
550   X0(JJ)=X2(JJ)
```

```
      DO560 KK=1,N2
560   X2(KK)=X8(KK)
      DO 570 MM=1,N2
570   X8(MM)=X0(MM)+D2(MM)
      V2=W2
      DO580 NN=1,N2
580   X4(NN)=X8(NN)
      W2=F3(X4)
      GO TO 140
120      DO590 II=1,N2
590   X1(II)=X8(II)
      DO595 IJ=1,N2
595   X8(IJ)=X2(IJ)
      DO600 JJ=1,N2
600   X2(JJ)=X1(JJ)-D2(JJ)
      W2=V2
      DO610 KK=1,N2
610   X4(KK)=X2(KK)
130   V2=F3(X4)
140   CONTINUE
110      DO620 LL=1,N2
620   X7(LL)=X2(LL)+X8(LL)
      DO630 MM=1,N2
630   X0(MM)=(.5)*X7(MM)
      IF(J2.EQ.N2+1) GO TO 150
128   J2=J2+1
      IF(J2.LT.N2+1)GOTO710
      DO660 KK=1,N2
660   U2(KK)=X0(KK)-X3(KK)
      U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2+U2(4)**2)
      DO670 LL=1,N2
670   U2(LL)=(1/U1)*U2(LL)
      DO695 JZ=1,N2
695   E2(JZ,N2+1)=U2(JZ)
      DO680 KL=1,N2
      DO690 LK=1,N2
690   E2(LK,KL)=E2(LK,KL+1)
680   CONTINUE
      GOTO195
710   DO720 KM=1,N2
720   X4(KM)=X0(KM)
      B2=F3(X4)
      GOTO195
150      I2=I2+1
      WRITE(2,1100)I2
1100  FORMAT('      I2=',I3)
      DO161KL=1,4
161      X4(KL)=X0(KL)
      WRITE(2,1150)F3(X4)
1150  FORMAT('      F3=',F12.5)
      WRITE(2,1160)X4(1),X4(2),X4(3),X4(4)
1160  FORMAT(5F13.4)
      DO 210 JZ=1,N2
210   X(I2,JZ)=X0(JZ)
      DO 220 JZ=1,N2
220      IF (ABS((X0(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
      GO TO 240
230   IF(I2.EQ.50) GO TO 270
      AL3=AL2*D0/U1
      IF (ABS(AL3).LT..000001) GO TO 280
      D0=AL3
      DO 290 JZ=1,N2
290   E2(JZ,JS)=U2(JZ)
280   GO TO 300
240   DO700 KK=1,N2
700   X4(KK)=X0(KK)
      Y3=F3(X4)
270   CONTINUE
      WRITE(2,310)Y3
310   FORMAT(' MINIMUM (CNR)TH ='F10.7)
      WRITE(2,320)X0(1),X0(2),X0(3),X0(4)
320   FORMAT(' A=',F8.0,' B=',F8.0,'ALFA='F8.0,'K=',F8.0)
      WRITE(2,370)
      WRITE(2,360)X(1,1),X(1,2),X(1,3),X(1,4)
370   FORMAT('  THE INITIAL POINT WAS  ')
360   FORMAT(' A=',F8.0,'  B=',F6.0,'  ALF=',F7.0,' K=',
     C F8.0)
      IF (I2.LT.40) GO TO 330
      WRITE(2,340)
340      FORMAT('SOLUTION DID NOT CONVERGE')
      GO TO 330
100      WRITE(2,350)
350   FORMAT('THERE IS NO MINIMUM')
330   STOP
      END
      FUNCTION F3(X4)
      DIMENSIONX4(4),V(7),H(2)
      X4(3)=X4(3)/10000.
      A1=X4(1)
      B1=X4(2)
      AL=X4(3)
      K1=X4(4)
      PI=3.141593
      BP=35000.0
      REAL J7,J8,J9,J6
      WA=300.0*2.0*PI
      WB=3300.0*2.0*PI
      A=(AL/K1/B1)**2
      B=(1./A1+AL/K1)**2-2.*AL/K1/B1
      C=((1+AL)/K1/B1)**2
```

```
      D=((1.+AL)/K1+1./A1)**2-2.*((1.+AL)/K1/B1)
      G7=D/2./C
      G8=-SQRT(1./C-(D/2/C)**2)
      J6=(1./C)**0.25
      J9=1.0/2.0*ATAN((-SQRT(1.0/C-(D/2./C)**2))/(D/2./C))
      IF(D.GT.0)J9=J9+PI/2
      J7=J6*COS(J9)
      J8=J6*SIN(J9)
      TOP7=J7-PI*BP
      TOP8=J8
      TOP6=SQRT(TOP7**2+TOP8**2)
      TOP9=ATAN(TOP8/TOP7)
      IF(TOP7.LT.0.0)TOP9=TOP9+PI
      BOT7=J7+PI*BP
      BOT8=J8
      BOT6=SQRT(BOT7**2+BOT8**2)
      BOT9=ATAN(BOT8/BOT7)
      IF(BOT7.LT.0.0)BOT9=BOT9+PI
      RT6=TOP6/BOT6
      RT9=TOP9-BOT9
      WRT7=ALOG(RT6)
      WRT8=RT9
      WRT6=SQRT(WRT7**2+WRT8**2)
      WRT9=ATAN(WRT8/WRT7)
      IF(WRT7.LT.0.0)WRT9=WRT9+PI
      FNUM7=1.-A/C+G7*(A*D/C-B)
      FNUM8=G8*(A*D/C-B)
      FNUM6=SQRT(FNUM7**2+FNUM8**2)
      FNUM9=ATAN(FNUM8/FNUM7)
      IF(FNUM7.LT.0.0)FNUM9=FNUM9+PI
      DEN7=2.*G8*J8
      DEN8=-2.*G8*J7
      DEN6=SQRT(DEN7**2+DEN8**2)
      DEN9=ATAN(DEN8/DEN7)
      IF(DEN7.LT.0.0)DEN9=DEN9+PI
      FRACT6=FNUM6/DEN6
      FRACT9=FNUM9-DEN9
      PROD6=FRACT6*WRT6
      PROD9=FRACT9+WRT9
      PROD7=PROD6*COS(PROD9)
      HA=0.5/PI/C*(PI*A*BP+PROD7)
      V(1)=1.0/K1/K1
      V(6)=-V(1)
      V(7)=1.0/C/K1/K1*(1.0/B1/B1-D)
      RT17=WB-J7
      RT18=-J8
      RT16=SQRT(RT17**2+RT18**2)
      RT19=ATAN(RT18/RT17)
      IF(RT17.LT.0.)      RT19=RT19+PI
      RT27=WB+J7
      RT28=J8
      RT26=SQRT(RT27**2+RT28**2)
      RT29=ATAN(RT28/RT27)
      IF(RT27.LT.0.) RT29=RT29+PI
      RT37=WA+J7
      RT38=J8
      RT36=SQRT(RT37**2+RT38**2)
      RT39=ATAN(RT38/RT37)
      IF(RT37.LT.0.0) RT39=RT39+PI
      RT47=WA-J7
      RT48=-J8
      RT46=SQRT(RT47**2+RT48**2)
      RT49=ATAN(RT48/RT47)
      IF(RT47.LT.0.0)RT49=RT49+PI
      RT6=RT16*RT36/RT26/RT46
      RT9=RT19+RT39-RT29-RT49
      IF(RT9.LT.0.0)RT9=RT9+2.*PI
      RT8=RT6*COS(RT9)
      RT7=RT6*SIN(RT9)
      RIGT7=ALOG(RT6)
      RIGT8=RT9
      RIGT6=SQRT(RIGT7**2+RIGT8**2)
      RIGT9=ATAN(RIGT8/RIGT7)
      IF(RIGT7.LT.0.0)RIGT9=RIGT9+PI
      UPP7=1.0-V(6)/V(7)*G7
      UPP8=-V(6)/V(7)*G8
      UPP6=SQRT(UPP7**2+UPP8**2)
      UPP9=ATAN(UPP8/UPP7)
      IF(UPP7.LT.0.0)UPP9=UPP9+PI
      DOWN6=2.0*J6*G8
      DOWN9=J9-1.570796
      RATIO6=UPP6/DOWN6
      RATIO9=UPP9-DOWN9
      ROD6=RIGT6*RATIO6
      ROD9=RATIO9+RIGT9
      H1=ROD6*COS(ROD9)
      HA1=(V(1)*(1.0/WA-1.0/WB)+V(7)*H1)/2.0/PI
      GNU=0.25
      ETA=8.*PI**3*1.E7*WA*WB/(WB-WA)
      F3=HA/BP/(GNU-ETA*HA1)
 34   RETURN
      END
```

```
C "CASE7" TEST TONE CASE 5 VARIABLES NO PREDETECTION FILTER
C  DIRECT CONSTANTS
      DIMENSION X(50,5),E2(6,6),Y4(50),B4(5),X0(5),X1(5)
      DIMENSION X2(5),D2(5),U2(5),X3(5),X4(5),X7(5),A1(5),X8(5)
      B6=10.0
      BK8=3.14159
      N2=5
      K2=50
      S2=0.01
      T2=0.001
      F2=0.001
      X(1,1)=800000
      X(1,2)=3000
      X(1,3)=30000
      X(1,4)=1.0
      X(1,5)=500000
      DO400 L=1,N2
      DO410 N=1,N2
410   E2(L,N)=0
400   CONTINUE
      DO420 L=1,N2
420   E2(L,L)=1
      I2=1
300   M2=0
      M3=0
      D0=1
      J2=1
      DO10 JQ=1,N2
10      X3(JQ)=X(I2,JQ)
      DO430 I=1,N2
430   A1(I)=X3(I)
      AL2=0
      DO440 I=1,N2
440   X4(I)=X3(I)
      Y4(I2)=F3(X4)
      B4(J2)=Y4(I2)
      B2=B4(J2)
      DO20 JQ=1,N2
20      X0(JQ)=X(I2,JQ)
C       SEARCH BEGINS
      JQ1=0
195   K3=1
      DO30 JQ2=1,K2
      S3=(2**(JQ2-1))*S2
      DO40 JZ=1,N2
40    X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
      DO 450 I=1,N2
450   X4(I)=X0(I)
      Y2=F3(X4)
      IF((B2-Y2).GT..00001)  GO TO 50
```

```
       IF((Y2-B2).GT..00001)  GO TO 60
       IF((Y2-B2).LT..00001)  GO TO 30
       GO TO 30
60     IF(JQ1.EQ.0) GO TO 70
       IF(K3.EQ.2) GO TO 80
       DO460 M=1,N2
460    X1(M)=X0(M)
       DO 90 JZ=1,N2
90      X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)  )
       GOTO80
50     JQ1=1
       K3=1
       GO TO30
70       DO470 JK=1,N2
470    X1(JK)=X0(JK)
       S2=-S2
       K3=2
       JQ1=1
30     B2=Y2
       GO TO 100
80     R2=3*((2**(JQ2-2))*ABS(S2))
C      BEGIN FIBERNACI SEARCH
       JQ1=0
       N4=INT(1-2.1*ALOG(F2/R2))
       DO480 JL=1,N2
480    X7(JL)=X1(JL)-X0(JL)
       DO490 KL=1,N2
490    D2(KL)=.618*X7(KL)
       DO 500 JM=1,N2
500    X8(JM)=X0(JM)+D2(JM)
       DO 510 JN=1,N2
510    X2(JN)=X1(JN)-D2(JN)
       DO 520 KM=1,N2
520    X4(KM)=X2(KM)
       V2=F3(X4)
       DO530 KN=1,N2
530    X4(KN)=X8(KN)
       W2=F3(X4)
       DO140 K9=1,N4
       DO540 LL=1,N2
540    D2(LL)=.618*D2(LL)
       IF (ABS(W2-V2).LT..00001) GO TO 110
       IF(W2.GT.V2) GO TO 120
       DO550 JJ=1,N2
550    X0(JJ)=X2(JJ)
       DO560 KK=1,N2
560    X2(KK)=X8(KK)
       DO 570 MM=1,N2
570    X8(MM)=X0(MM)+D2(MM)
       V2=W2
       DO580 NN=1,N2
580    X4(NN)=X8(NN)
       W2=F3(X4)
       GO TO 140
120       DO590 II=1,N2
590    X1(II)=X8(II)
       DO595 IJ=1,N2
595    X8(IJ)=X2(IJ)
       DO600 JJ=1,N2
600    X2(JJ)=X1(JJ)-D2(JJ)
       W2=V2
       DO610 KK=1,N2
610    X4(KK)=X2(KK)
130    V2=F3(X4)
140    CONTINUE
110       DO620 LL=1,N2
620    X7(LL)=X2(LL)+X8(LL)
       DO630 MM=1,N2
630    X0(MM)=(.5)*X7(MM)
       IF(J2.EQ.N2+1) GO TO 150
       A3=E2(1,J2)
       JZ1=1
       DO 160 JZ=2,N2
       IF (E2(JZ,J2).GT.A3) GO TO 170
       GO TO 160
170    A3=E2(JZ,J2)
       JZ1=JZ
160    CONTINUE
       AL1=(X0(JZ1)-A1(JZ1))/A3
       IF (ABS(AL1).GT.ABS(AL2 ))GO TO 180
       GO TO 190
180    AL2=AL1
       JS=J2
190    DO640 NN=1,N2
640    A1(NN)=X0(NN)
       J2=J2+1
       DO650 JJ=1,N2
650    X4(JJ)=X0(JJ)
       B2=F3(X4)
       IF (J2.LT.(N2+1)) GO TO 195
       DO660 KK=1,N2
660    U2(KK)=X0(KK)-X3(KK)
       U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2+U2(4)**2+U2(5)**2)
       DO670 LL=1,N2
670    U2(LL)=(1/U1)*U2(LL)
       DO200 JZ=1,N2
200    E2(JZ,N2+1)=U2(JZ)
       GO TO 195
150       I2=I2+1
       WRITE(2,1100)I2
```

```
1100   FORMAT('    I2=',I3)
       DO 210 JZ=1,N2
210    X(I2,JZ)=X0(JZ)
       DO 220 JZ=1,N2
220       IF (ABS((X0(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
       GO TO 240
230    IF(I2.EQ.50) GO TO 270
       AL3=AL2*D0/U1
       IF (ABS(AL3).LT..000001) GO TO 280
       D0=AL3
       DO 290 JZ=1,N2
290    E2(JZ,JS)=U2(JZ)
280    GO TO 300
240    DO700 KK=1,N2
700    X4(KK)=X0(KK)
       Y3=F3(X4)
270    CONTINUE
       WRITE(2,310)Y3
310    FORMAT(' MINIMUM (CNR)TH ='F10.7)
       WRITE(2,320)X0(1),X0(2),X0(3),X0(4),X0(5)
320    FORMAT(' A=',F8.0,' B=',F8.0,'D='F8.0,'ALFA=',F8.5,'K=',
F8.0)
       WRITE(2,370)
       WRITE(2,360)X(1,1),X(1,2),X(1,3),X(1,4),X(1,5)
370    FORMAT('  THE INITIAL POINT WAS  ')
360    FORMAT(' A=',F8.0,'  B=',F6.0,'  D=',F7.0,'  ALF=',
      C F5.3,'  K=',F8.0)
       IF (I2.LT.50) GO TO 330
       WRITE(2,340)
340       FORMAT('SOLUTION DID NOT CONVERGE')
       GO TO 330
100       WRITE(2,350)
350    FORMAT('THERE IS NO MINIMUM')
330    STOP
       END
       FUNCTION F3(X4)
       DIMENSION X4(5)
       A=X4(5)*X4(2)/X4(1)+X4(4)*X4(3)
       B=X4(5)*X4(2)*X4(3)/X4(1)+X4(5)*X4(2)+X4(4)*X4(2)*X4(3)
       C=X4(5)*X4(3)*X4(2)
       D=X4(3)+X4(2)+X4(5)*X4(2)/X4(1)+X4(4)*X4(3)
       E=X4(3)*X4(2)+X4(5)*X4(2)*X4(3)/X4(1)+X4(5)*X4(2)
       E=E+X4(4)*X4(2)*X4(3)
       T4=(A**2*E*C+(B**2-2*C*A)*C+C**2*D)/(4*C*(E*D-C))
       W6=1000*6.2832
       T6=(W6**6+((A-D)**2-2*(E-B))*W6**4+(E-B)**2*W6**2)
       T6=T6/(W6**6+(D**2-2*E)*W6**4+(E**2-2*C*D)*W6**2+C**2)
       B6=10.0
       B8=35000.0
       BK8=3.1415927
       F3=4*BK8**2*T4/B8/(3.1415927-2*B6*SQRT(T6))**2
       RETURN
       END
```

```
C 'CASE8' TEST TONE CASE 5 VARIABLES *WITH* PREDETECTION FILTER
C  'DOUBLE PRECISION IN --NEWTON--  ROOT'
C  DIRECT CONSTANTS   ** D IS HELD CONSTANT DURING SEARCH **
      DIMENSION X(50,5),E2(6,6),Y4(50),B4(5),X0(5),X1(5),X5(5)
      DIMENSION X2(5),D2(5),U2(5),X3(5),X4(5),X7(5),A1(5),X8(5)
      B6=10.0
      PI=3.14159
      N2=5
      K2=50
      S2=8
      T2=0.01
      F2=0.001
      X(1,1)=800000
      X(1,2)=2000.
      X(1,3)=1.0E4
      X(1,4)=30000
      X(1,5)=9.00E5
      DO400 L=1,N2
      DO410 N=1,N2
410   E2(L,N)=0
400   CONTINUE
      DO420 L=1,N2
420   E2(L,L)=1
      I2=1
300   M2=0
      M3=0
      D0=1
      J2=1
      DO10 JQ=1,N2
10      X3(JQ)=X(I2,JQ)
      DO430 I=1,N2
430   A1(I)=X3(I)
      AL2=0
      DO440 I=1,N2
440   X4(I)=X3(I)
      Y4(I2)=F3(X4)
      B4(J2)=Y4(I2)
      B2=B4(J2)
      DO20 JQ=1,N2
20      X0(JQ)=X(I2,JQ)
C       SEARCH BEGINS
      JQ1=0
195   K3=1
      DO30 JQ2=1,K2
      S3=(2**(JQ2-1))*S2
      DO45 K=1,N2
45    X5(K)=X0(K)
      DO40 JZ=1,N2
      IF(JZ.EQ.3)GOTO40
      X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
```

```
40     CONTINUE
       DO48 K=1,N2
48     IF(XO(K).NE.X5(K))GOTO49
       GOTO128
49     DO450 I=1,N2
450    X4(I)=XO(I)
       Y2=F3(X4)
       IF((B2-Y2).GT..0005)  GO TO 50
       IF((Y2-B2).GT..0005)  GO TO 60
       IF((Y2-B2).LT..0005)  GO TO 30
       GO TO 30
60     IF(JQ1.EQ.0) GO TO 70
       IF(K3.EQ.2) GO TO 80
       DO460 M=1,N2
460    X1(M)=XO(M)
       DO 90 JZ=1,N2
       IF(JZ.EQ.3)GOTO90
       XO(JZ)=XO(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
90     CONTINUE
       GOTO80
50     JQ1=1
       K3=1
       GO TO30
70       DO470 JK=1,N2
470    X1(JK)=XO(JK)
       S2=-S2
       K3=2
       JQ1=1
30     B2=Y2
       GO TO 100
80     R2=3*((2**(JQ2-2))*ABS(S2))
C      BEGIN FIBERNACI SEARCH
       JQ1=0
       N4=INT(1-2.1*ALOG(F2/R2))
       DO480 JL=1,N2
480    X7(JL)=X1(JL)-XO(JL)
       DO490 KL=1,N2
490    D2(KL)=.618*X7(KL)
       DO 500 JM=1,N2
500    X8(JM)=XO(JM)+D2(JM)
       DO 510 JN=1,N2
510    X2(JN)=X1(JN)-D2(JN)
       DO 520 KM=1,N2
520    X4(KM)=X2(KM)
       V2=F3(X4)
       DO530 KN=1,N2
530    X4(KN)=X8(KN)
       W2=F3(X4)
       DO140 K9=1,N4
       DO540 LL=1,N2
540    D2(LL)=.618*D2(LL)
       IF (ABS(W2-V2).LT..001) GO TO 110
       IF(W2.GT.V2) GO TO 120
       DO550 JJ=1,N2
550    XO(JJ)=X2(JJ)
       DO560 KK=1,N2
560    X2(KK)=X8(KK)
       DO 570 MM=1,N2
570    X8(MM)=XO(MM)+D2(MM)
       V2=W2
       DO580 NN=1,N2
580    X4(NN)=X8(NN)
       W2=F3(X4)
       GO TO 140
120      DO590 II=1,N2
590    X1(II)=X8(II)
       DO595 IJ=1,N2
595    X8(IJ)=X2(IJ)
       DO600 JJ=1,N2
600    X2(JJ)=X1(JJ)-D2(JJ)
       W2=V2
       DO610 KK=1,N2
610    X4(KK)=X2(KK)
130    V2=F3(X4)
140    CONTINUE
110      DO620 LL=1,N2
620    X7(LL)=X2(LL)+X8(LL)
       DO630 MM=1,N2
630    XO(MM)=(.5)*X7(MM)
       IF(J2.EQ.N2+1) GO TO 150
128    J2=J2+1
       IF(J2.EQ.3)J2=4
       IF(J2.LT.N2+1)GOTO710
       DO660 KK=1,N2
660    U2(KK)=XO(KK)-X3(KK)
       U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2+U2(4)**2+U2(5)**2)
       DO670 LL=1,N2
670    U2(LL)=(1/U1)*U2(LL)
       DO695 JZ=1,N2
695    E2(JZ,N2+1)=U2(JZ)
       DO680 KL=1,N2
       DO690 LK=1,N2
690    E2(LK,KL)=E2(LK,KL+1)
680    CONTINUE
       GOTO195
710    DO720 KM=1,N2
720    X4(KM)=XO(KM)
       B2=F3(X4)
       GOTO195
150      I2=I2+1
```

```
      WRITE(2,1100)I2
 1100 FORMAT('    I2=',I3)
      DO161KL=1,5
 161     X4(KL)=X0(KL)
      WRITE(2,1150)F3(X4)
 1150 FORMAT('     F3=',F12.5)
      WRITE(2,1160)X4(1),X4(2),X4(3),X4(4),X4(5)
 1160 FORMAT(5F13.4)
      DO 210 JZ=1,N2
 210  X(I2,JZ)=X0(JZ)
      DO 220 JZ=1,N2
 220     IF (ABS((X0(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
      GO TO 240
 230  IF(I2.EQ.50) GO TO 270
      AL3=AL2*D0/U1
      IF (ABS(AL3).LT..000001) GO TO 280
      D0=AL3
      DO 290 JZ=1,N2
 290  E2(JZ,JS)=U2(JZ)
 280  GO TO 300
 240  DO700 KK=1,N2
 700  X4(KK)=X0(KK)
      Y3=F3(X4)
 270  CONTINUE
      WRITE(2,310)Y3
 310  FORMAT(' MINIMUM (CNR)TH ='F10.7)
      WRITE(2,320)X0(1),X0(2),X0(3),X0(4)/1.E4,X0(5)
 320  FORMAT(' A=',F8.0,' B=',F8.0,'D='F8.0,'ALFA=',F8.4,' K='
,E13.3)
      WRITE(2,370)
      WRITE(2,360)X(1,1),X(1,2),X(1,3),X(1,4),X(1,5)
 370  FORMAT('  THE INITIAL POINT WAS  ')
 360  FORMAT(' A=',F8.0,'  B=',F6.0,'  D=',E13.4,'  ALF=',
     C F8.0,'  K=',F8.0)
      IF (I2.LT.50) GO TO 330
      WRITE(2,340)
 340     FORMAT('SOLUTION DID NOT CONVERGE')
      GO TO 330
 100     WRITE(2,350)
 350  FORMAT('THERE IS NO MINIMUM')
 330  STOP
      END
      FUNCTION F3(X4)
      DIMENSION X4(5),R(3),FK(7),H(3),V(7)
      X4(4)=X4(4)/10000
      DOUBLE PRECISION A,B,C,D,E,FJ,FL,FM
      A=X4(5)*X4(2)/X4(1)+X4(4)*X4(3)
      B=X4(5)*X4(2)*X4(3)/X4(1)+X4(5)*X4(2)+X4(4)*X4(2)*X4(3)
      C=X4(5)*X4(3)*X4(2)
      D=X4(3)+X4(2)+X4(5)*X4(2)/X4(1)+X4(4)*X4(3)
      E=X4(3)*X4(2)+X4(5)*X4(2)*X4(3)/X4(1)+X4(5)*X4(2)
```

```
      E=E+X4(4)*X4(2)*X4(3)
      FJ=B**2-2*A*C
      FL=D**2-2*E
      FM=E**2-2*C*D
      BP=58000.0
C    FIND THE ROOT OF X**3+LX**2+MX+C**2
      DOUBLE PRECISION X,XFIRST,Y,Y5,YA,T,U
      XFIRST=-1.0E12
      ACC=1.0E-13
      LH=0
      GOTO31
32    XFIRST=10*XFIRST
      LH=LH+1
      GOTO31
38    IF(LH.NE.5) GO TO 32
      IF(ACC.LT.1.0E-6)ACC=ACC*10
      PRINT,ACC
      IF(ACC.LT.1.0E-6) GO TO 31
      WRITE(2,35)X4(3)
35    FORMAT('  NO ROOT FOUND FOR X4( )=',F9.1)
      GO TO 34
31    LF=0
      X=XFIRST
33    NU=0
      GOTO17
37    IF(LF.EQ.1) GO TO 38
      X=-X
      NU=0
      LF=1
17      Y=X**3+FL*X**2+FM*X+C**2
      Y5=3*X**2+2*FL*X+FM
      AB1=X**3
      AB2=FL*X**2
      AB3=FM*X
      YA=(ABS(AB1)+ABS(AB2)+ABS(AB3)+C*C)*ACC
      ABY=Y
      IF(ABS(ABY).LT.YA) GOTO24
      X=X-Y/Y5
      NU=NU+1
      IF(NU.EQ.100)GOTO37
      GOTO 17
24      T=FL+X
      U=FM+X*T
      R(1)=X
      D5=T**2/4-U
      IF(D5.LT.0)GOTO47
      TEST=12.
      R(2)=-T/2+SQRT(D5)
      R(3)=-T/2-SQRT(D5)
      FK(1)=(A**2*R(1)**2+FJ*R(1)+C**2)/(R(1)-R(2))/(R(1)-R(3))
```

```
      FK(2)=(A**2*R(2)**2+FJ*R(2)+C**2)/(R(2)-R(3))/(R(2)-R(1))
      FK(3)=(A**2*R(3)**2+FJ*R(3)+C**2)/(R(3)-R(1))/(R(3)-R(2))
      DO10 J6=1,3
      IF(R(J6).GT.0)GO TO     39
      IF(R(J6).LT.0) GO TO 42
      GO TO 36
39      S5=SQRT(R(J6))
      H(J6)=1/2/S5*ALOG((S5-3.14159*BP)/(S5+3.14159*BP))
      GO TO 10
42      S6=SQRT(-R(J6))
      H(J6)=1/S6*ATAN(3.14159*BP/S6)
10    CONTINUE
      HA=(FK(1)*H(1)+FK(2)*H(2)+FK(3)*H(3))/2/3.1415927
      GO TO88
47      D6=R(1)**2+U+T*R(1)
      TEST=78.
      V(1)=(A**2*R(1)**2+C**2+FJ*R(1))/D6
      D7=R(1)*U**2+R(1)**2*T*U+R(1)**3*U
      V(2)=(A**2*U-FJ*R(1)-C**2+R(1)*T*A**2)/D6
      V(3)=(FJ*U-R(1)*C**2+A**2*U*R(1)-C**2*T)/D6
      IF(R(1).GT.0) GO TO 55
      IF(R(1).LT.0) GO TO 65
36      WRITE(2,200)J6
200   FORMAT('  R(',I1,')=0')
      STOP
55      S5=SQRT(R(1))
      RS=55
      H(1)=1/2/S5*ALOG((S5-3.1415927*BP)/(S5+3.1415927*BP))
      GO TO58
65      S6=SQRT(-R(1))
      RS=65
      H(1)=1/S6*ATAN(3.14159*BP/S6)
C  SUBSCRIPTS  6(MAG), 7(REAL),  8 (IMAG) , 9 (ANGLE)
58      G7=T/2
      G8=-SQRT(-D5)
      G6=SQRT(G8**2+G7**2)
      FJ6=SQRT(G6)
      FJ9=ATAN(G8/G7)/2
      IF(G7.GT.0) FJ9=FJ9+1.57079
      FJ7=FJ6*COS(FJ9)
      FJ8=FJ6*SIN(FJ9)
      FN7=FJ7-BP*3.1415927
      FN6=SQRT(FN7**2+FJ8**2)
      FN9=ATAN(FJ8/FN7)
      IF(FN7.LT.0) FN9=FN9+3.1415927
      E7=FJ7+BP*3.1415927
      E6=SQRT(E7**2+FJ8**2)
      E9=ATAN(FJ8/E7)
      IF(E7.LT.0) E9=E9+3.1415927
      F6=FN6/E6
```

```
      F9=FN9-E9
      IF(F9.GT.3.1415927) F9=F9-3.1415927
      IF(F9.LT.(-(3.1415927))) F9=F9+3.1415927
      FL7=ALOG(F6)
      FL6=SQRT(FL7**2+F9**2)
      FL9=ATAN(F9/FL7)
      FM7=1-V(2)/V(3)*G7
      FM8=-V(2)/V(3)*G8
      FM6=SQRT(FM7**2+FM8**2)
      FM9=ATAN(FM8/FM7)
      IF(FM7.LT.0)FM9=FM9+3.1415927
      B6=-4*FJ6*G8
      B9=FJ9-1.57079
      P6=FM6/B6
      P9=FM9-B9
      R6=P6*FL6
      R9=P9+FL9
      H(2)=2*R6*COS(R9)*V(3)
      HA=(V(1)*H(1)+H(2))/2/3.1415927
88    T4=HA
      W6=1000*6.2832
      T6=(W6**6+((A-D)**2-2*(E-B))*W6**4+(E-B)**2*W6**2)
      T6=T6/(W6**6+(D**2-2*E)*W6**4+(E**2-2*C*D)*W6**2+C**2)
      B6=10.0
      BP=58000.0
      PI=3.1415927
      F3=4*PI**2*T4/BP/(3.1415927-2*B6*SQRT(T6))**2
      GOTO34
111      S=0.
      FN=100.0
      DELT=3.141593*BP/FN
      Z=DELT/2.
      NFN=INT(FN)
      DO345N=1,NFN
      S=S+(A*A*Z**4+(B*B-2.*A*C)*Z*Z+C*C)/(Z**6+(D*D-2.*E)*Z**4
     C+(E*E-2.*C*D)*Z*Z+C*C)*DELT/2./3.141593
      Z=Z+DELT
345   CONTINUE
      WRITE(2,346)S,T4,TEST
346   FORMAT('  S=',E13.4,'  T4=',E13.4,'TEST=',F5.0)
34    RETURN
      END
```

```
C  *** CASE9 ***   VOICE MODULATION  FIVE VARIABLES WITH **NO*
* PREDETECTION FILTER
C  'DOUBLE PRECISION IN --NEWTON--  ROOT'
C  DIRECT CONSTANTS
      DIMENSION X(50,6),E2(6,6),Y4(50),B4(5),X0(5),X1(5),X5(5)
      DIMENSION X2(5),D2(5),U2(5),X3(5),X4(5),X7(5),A1(5),X8(5)
      N2=5
      K2=50
      S2=8
      T2=0.01
      F2=0.001
      X(1,1)=60000
      X(1,2)=3000
      X(1,3)=20000
      X(1,4)=3000
      X(1,5)=300000
      DO400 L=1,N2
      DO410 N=1,N2
410   E2(L,N)=0
400   CONTINUE
      DO420 L=1,N2
420   E2(L,L)=1
      I2=1
300   M2=0
      M3=0
      D0=1
      J2=1
      DO10 JQ=1,N2
10      X3(JQ)=X(I2,JQ)
      DO430 I=1,N2
430   A1(I)=X3(I)
      AL2=0
      DO440 I=1,N2
440   X4(I)=X3(I)
      Y4(I2)=F3(X4)
      B4(J2)=Y4(I2)
      B2=B4(J2)
      DO20 JQ=1,N2
20      X0(JQ)=X(I2,JQ)
C       SEARCH BEGINS
      JQ1=0
195   K3=1
      DO30 JQ2=1,K2
      S3=(2**(JQ2-1))*S2
      DO45 K=1,N2
45    X5(K)=X0(K)
      DO40 JZ=1,N2
      X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
40    CONTINUE
      DO48 K=1,N2
48    IF(X0(K).NE.X5(K))GOTO49
```

```
      GOTO128
49    DO450 I=1,N2
450   X4(I)=X0(I)
      Y2=F3(X4)
      IF((B2-Y2).GT..0005)  GO TO 50
      IF((Y2-B2).GT..0005)  GO TO 60
      IF((Y2-B2).LT..0005)  GO TO 30
      GO TO 30
60    IF(JQ1.EQ.0) GO TO 70
      IF(K3.EQ.2) GO TO 80
      DO460 M=1,N2
460   X1(M)=X0(M)
      DO 90 JZ=1,N2
      X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
90    CONTINUE
      GOTO80
50    JQ1=1
      K3=1
      GO TO30
70      DO470 JK=1,N2
470   X1(JK)=X0(JK)
      S2=-S2
      K3=2
      JQ1=1
30    B2=Y2
      GO TO 100
80    R2=3*((2**(JQ2-2))*ABS(S2))
C     BEGIN FIBERNACI SEARCH
      JQ1=0
      N4=INT(1-2.1*ALOG(F2/R2))
      DO480 JL=1,N2
480   X7(JL)=X1(JL)-X0(JL)
      DO490 KL=1,N2
490   D2(KL)=.618*X7(KL)
      DO 500 JM=1,N2
500   X8(JM)=X0(JM)+D2(JM)
      DO 510 JN=1,N2
510   X2(JN)=X1(JN)-D2(JN)
      DO 520 KM=1,N2
520   X4(KM)=X2(KM)
      V2=F3(X4)
      DO530 KN=1,N2
530   X4(KN)=X8(KN)
      W2=F3(X4)
      DO140 K9=1,N4
      DO540 LL=1,N2
540   D2(LL)=.618*D2(LL)
      IF (ABS(W2-V2).LT..001) GO TO 110
      IF(W2.GT.V2) GO TO 120
      DO550 JJ=1,N2
550   X0(JJ)=X2(JJ)
      DO560 KK=1,N2
560   X2(KK)=X8(KK)
      DO 570 MM=1,N2
570   X8(MM)=X0(MM)+D2(MM)
      V2=W2
      DO580 NN=1,N2
580   X4(NN)=X8(NN)
      W2=F3(X4)
      GO TO 140
120     DO590 II=1,N2
590   X1(II)=X8(II)
      DO595 IJ=1,N2
595   X8(IJ)=X2(IJ)
      DO600 JJ=1,N2
600   X2(JJ)=X1(JJ)-D2(JJ)
      W2=V2
      DO610 KK=1,N2
610   X4(KK)=X2(KK)
130   V2=F3(X4)
140   CONTINUE
110     DO620 LL=1,N2
620   X7(LL)=X2(LL)+X8(LL)
      DO630 MM=1,N2
630   X0(MM)=(.5)*X7(MM)
      IF(J2.EQ.N2+1) GO TO 150
128   J2=J2+1
      IF(J2.LT.N2+1)GOTO710
      DO660 KK=1,N2
660   U2(KK)=X0(KK)-X3(KK)
      U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2+U2(4)**2+U2(5)**2)
      DO670 LL=1,N2
670   U2(LL)=(1/U1)*U2(LL)
      DO695 JZ=1,N2
695   E2(JZ,N2+1)=U2(JZ)
      DO680 KL=1,N2
      DO690 LK=1,N2
690   E2(LK,KL)=E2(LK,KL+1)
680   CONTINUE
      GOTO195
710   DO720 KM=1,N2
720   X4(KM)=X0(KM)
      B2=F3(X4)
      GOTO195
150     I2=I2+1
      WRITE(2,1100)I2
1100  FORMAT('    I2=',I3)
      DO161KL=1,5
161     X4(KL)=X0(KL)
      WRITE(2,1150)F3(X4)
```

```
1150  FORMAT('     F3=',F12.5)
      WRITE(2,1160)X4(1),X4(2),X4(3),X4(4),X4(5)
1160  FORMAT(5F13.4)
      DO 210 JZ=1,N2
210   X(I2,JZ)=X0(JZ)
      X(I2,N2+1)=F3(X4)
      DO 220 JZ=1,N2
220      IF (ABS((X0(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
      GO TO 240
230   IF(I2.EQ.50) GO TO 270
      GOTO300
240   DO700 KK=1,N2
700   X4(KK)=X0(KK)
      Y3=F3(X4)
270   CONTINUE
      WRITE(2,310)Y3
310   FORMAT(' MINIMUM (CNR)TH ='F10.7)
      WRITE(2,320)X0(1),X0(2),X0(3),X0(4),X0(5)
320   FORMAT(' A=',F8.0,' B=',F8.0,'D='E12.6,'ALFA=',F8.0,'K='
F8.0)
      WRITE(2,370)
      WRITE(2,360)X(1,1),X(1,2),X(1,3),X(1,4)/10000,X(1,5)
370   FORMAT('  THE INITIAL POINT WAS  ')
360   FORMAT(' A=',F8.0,'  B=',F6.0,'  D=',E8.2,'  ALF=',
     C F8.4,'  K=',F8.0)
      IF (I2.LT.50) GO TO 330
      WRITE(2,340)
340      FORMAT('SOLUTION DID NOT CONVERGE')
      GO TO 330
100      WRITE(2,350)
350   FORMAT('THERE IS NO MINIMUM')
330   STOP
      END
      FUNCTION F3(X4)
      DIMENSION X4(5),R(3),FK(7),H(7),V(7),H1(4)
      X4(4)=X4(4)/10000
      DOUBLE PRECISION A,B,C,D,E,FJ,FL,FM
      PI=3.141593
      WA=300.*2.*PI
      WB=3300.*2.*PI
      A=X4(5)*X4(2)/X4(1)+X4(4)*X4(3)
      B=X4(5)*X4(2)*X4(3)/X4(1)+X4(5)*X4(2)+X4(4)*X4(2)*X4(3)
      C=X4(5)*X4(3)*X4(2)
      D=X4(3)+X4(2)+X4(5)*X4(2)/X4(1)+X4(4)*X4(3)
      E=X4(3)*X4(2)+B
      FJ=B**2-2*A*C
      FL=D**2-2*E
      FM=E**2-2*C*D
      P=(A-D)**2-2.*(E-B)
      Q=(E-B)**2
      BP=35000.0
```

```
C     FIND THE ROOT OF X**3+LX**2+MX+C**2
        DOUBLE PRECISION X,XFIRST,Y,Y5,YA,T,U
        XFIRST=-1.0E11
        ACC=1.0E-13
        LH=0
        GOTO31
   32   XFIRST=10*XFIRST
        LH=LH+1
        GOTO31
   38   IF(LH.NE.5) GO TO 32
        IF(ACC.LT.1.0E-6)ACC=ACC*10
        PRINT,ACC
        IF(ACC.LT.1.0E-6) GO TO 31
        WRITE(2,35)X4(3)
   35   FORMAT('  NO ROOT FOUND FOR X4( )=',F9.1)
        GO TO 34
   31   LF=0
        X=XFIRST
   33   NU=0
        GOTO17
   37   IF(LF.EQ.1) GO TO 38
        X=-X
        NU=0
        LF=1
   17     Y=X**3+FL*X**2+FM*X+C**2
        Y5=3*X**2+2*FL*X+FM
        AB1=X**3
        AB2=FL*X**2
        AB3=FM*X
        YA=(ABS(AB1)+ABS(AB2)+ABS(AB3)+C*C)*ACC
        ABY=Y
        IF(ABS(ABY).LT.YA) GOTO24
        X=X-Y/Y5
        NU=NU+1
        IF(NU.EQ.100)GOTO37
        GOTO 17
   24     T=FL+X
        U=FM+X*T
        R(1)=X
        WA=1884.96
        WB=20734.5
        H1(4)=1.0/WA-1.0/WB
        D5=T**2/4-U
        IF(D5.LT.0)GOTO47
        R(2)=-T/2+SQRT(D5)
        R(3)=-T/2-SQRT(D5)
        FK(4)=Q/C/C
        FK(5)=(R(1)**2+P*R(1)+Q)/R(1)/(R(1)-R(2))/(R(1)-R(3))
        FK(6)=(R(2)**2+P*R(2)+Q)/R(2)/(R(2)-R(3))/(R(2)-R(1))
        FK(7)=(R(3)**2+P*R(3)+Q)/R(3)/(R(3)-R(1))/(R(3)-R(2))
```

```
      DO10 J6=1,3
      IF(R(J6).GT.0)GO TO     39
      IF(R(J6).LT.0) GO TO 42
      GO TO 36
39      S5=SQRT(R(J6))
      H1(J6)=0.5/S5*ALOG((WB-S5)*(WA+S5)/(WB+S5)/(WA-S5))
      GO TO 10
42      S6=SQRT(-R(J6))
      H1(J6)=1.0/S6*(ATAN(WB/S6)-ATAN(WA/S6))
10    CONTINUE
      HA1=(FK(5)*H1(1)+FK(6)*H1(2)+FK(7)*H1(3)+FK(4)*H1(4))/2.
'3.141593
      GO TO88
47      D6=R(1)**2+U+T*R(1)
      D7=R(1)*U**2+R(1)**2*T*U+R(1)**3*U
      V(4)=Q/C/C
      DEN=U*R(1)*(R(1)**2+U+T*R(1))
      TOP5=U*(R(1)*P+R(1)**2+Q)
      TOP6=R(1)*Q*(T+R(1))-U*R(1)*(R(1)+P)
      TOP7=U*U*R(1)-P*R(1)*U*(R(1)+T)+Q*R(1)*(T*R(1)-U+T*T)
      V(5)=TOP5/DEN
      V(6)=TOP6/DEN
      V(7)=TOP7/DEN
      IF(R(1).GT.0)  GO TO 55
      IF(R(1).LT.0)  GO TO 65
36      WRITE(2,200)J6
200   FORMAT('  R(',I1,')=0')
      STOP
55      S5=SQRT(R(1))
      H1(1)=0.5/S5*ALOG((WB-S5)*(WA+S5)/(WB+S5)/(WA-S5))
      GO TO58
65      S6=SQRT(-R(1))
      H1(1)=1.0/S6*(ATAN(WB/S6)-ATAN(WA/S6))
C  SUBSCRIPTS  6(MAG), 7(REAL),  8 (IMAG) , 9 (ANGLE)
58    G7=T/2
      G8=-SQRT(-D5)
      G6=SQRT(G8**2+G7**2)
      FJ6=SQRT(G6)
      FJ9=ATAN(G8/G7)/2
      IF(G7.GT.0) FJ9=FJ9+1.57079
      FJ7=FJ6*COS(FJ9)
      FJ8=FJ6*SIN(FJ9)
      RT17=WB-FJ7
      RT18=-FJ8
      RT16=SQRT(RT17**2+RT18**2)
      RT19=ATAN(RT18/RT17)
      IF(RT17.LT.0.)      RT19=RT19+PI
      RT27=WB+FJ7
      RT28=FJ8
      RT26=SQRT(RT27**2+RT28**2)
      RT29=ATAN(RT28/RT27)
      IF(RT27.LT.0.) RT29=RT29+PI
      RT37=WA+FJ7
      RT38=FJ8
      RT36=SQRT(RT37**2+RT38**2)
      RT39=ATAN(RT38/RT37)
      IF(RT37.LT.0.0) RT39=RT39+PI
      RT47=WA-FJ7
      RT48=-FJ8
      RT46=SQRT(RT47**2+RT48**2)
      RT49=ATAN(RT48/RT47)
      IF(RT47.LT.0.0)RT49=RT49+PI
      RT6=RT16*RT36/RT26/RT46
      RT9=RT19+RT39-RT29-RT49
      IF(RT9.LT.0.0)RT9=RT9+2.*PI
      RT8=RT6*COS(RT9)
      RT7=RT6*SIN(RT9)
      RIGT7=ALOG(RT6)
      RIGT8=RT9
      RIGT6=SQRT(RIGT7**2+RIGT8**2)
      RIGT9=ATAN(RIGT8/RIGT7)
      IF(RIGT7.LT.0.0)RIGT9=RIGT9+PI
      UPP7=1.0-V(6)/V(7)*G7
      UPP8=-V(6)/V(7)*G8
      UPP6=SQRT(UPP7**2+UPP8**2)
      UPP9=ATAN(UPP8/UPP7)
      IF(UPP7.LT.0.0)UPP9=UPP9+PI
      BOT6=2.0*FJ6*G8
      BOT9=FJ9-1.570796
      FRACT6=UPP6/BOT6
      FRACT9=UPP9-BOT9
      PROD6=RIGT6*FRACT6
      PROD9=FRACT9+RIGT9
      H1(2)=PROD6*COS(PROD9)
      HA1=(V(4)*H1(4)+V(5)*H1(1)+V(7)*H1(2))
88    CONTINUE                                   /2.0/PI
      HA=(A*A*E+B*B-2.0*C*A+C*D)/4.0/(E*D-C)
      GNU=0.25
      ETA=8.*PI**3*1.E7*WA*WB/(WB-WA)
      F3=HA/BP/(GNU-ETA*HA1)
34    RETURN
      END
```

```
C  *** CASE10 ***   VOICE MODULATION  FIVE VARIABLES WITH PRED
ETECTION FILTER
C  'DOUBLE PRECISION IN --NEWTON--  ROOT'
C  DIRECT CONSTANTS
      DIMENSION X(50,6),E2(6,6),Y4(50),B4(5),X0(5),X1(5),X5(5)
      DIMENSION X2(5),D2(5),U2(5),X3(5),X4(5),X7(5),A1(5),X8(5)
      N2=5
      K2=50
      S2=8
      T2=0.01
      F2=0.001
      X(1,1)=40000
      X(1,2)=4000
      X(1,3)=3.0E6
      X(1,4)=7000
      X(1,5)=160000
      DO400 L=1,N2
      DO410 N=1,N2
410   E2(L,N)=0
400   CONTINUE
      DO420 L=1,N2
420   E2(L,L)=1
      I2=1
300   M2=0
      M3=0
      D0=1
      J2=1
      DO10 JQ=1,N2
10      X3(JQ)=X(I2,JQ)
      DO430 I=1,N2
430   A1(I)=X3(I)
      AL2=0
      DO440 I=1,N2
440   X4(I)=X3(I)
      Y4(I2)=F3(X4)
      B4(J2)=Y4(I2)
      B2=B4(J2)
      DO20 JQ=1,N2
20      X0(JQ)=X(I2,JQ)
C       SEARCH BEGINS
      JQ1=0
195   K3=1
      DO30 JQ2=1,K2
      S3=(2**(JQ2-1))*S2
      DO45 K=1,N2
45    X5(K)=X0(K)
      DO40 JZ=1,N2
      IF(JZ.EQ.3)GOTO40
      X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
40    CONTINUE
      DO48 K=1,N2
```

```
48    IF(X0(K).NE.X5(K))GOTO49
      GOTO128
49    DO450 I=1,N2
450   X4(I)=X0(I)
      Y2=F3(X4)
      IF((B2-Y2).GT..0005)  GO TO 50
      IF((Y2-B2).GT..0005)  GO TO 60
      IF((Y2-B2).LT..0005)  GO TO 30
      GO TO 30
60    IF(JQ1.EQ.0) GO TO 70
      IF(K3.EQ.2) GO TO 80
      DO460 M=1,N2
460   X1(M)=X0(M)
      DO 90 JZ=1,N2
      IF(JZ.EQ.3)GOTO90
      X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
90    CONTINUE
      GOTO80
50    JQ1=1
      K3=1
      GO TO30
70      DO470 JK=1,N2
470   X1(JK)=X0(JK)
      S2=-S2
      K3=2
      JQ1=1
30    B2=Y2
      GO TO 100
80    R2=3*((2**(JQ2-2))*ABS(S2))
C     BEGIN FIBERNACI SEARCH
      JQ1=0
      N4=INT(1-2.1*ALOG(F2/R2))
      DO480 JL=1,N2
480   X7(JL)=X1(JL)-X0(JL)
      DO490 KL=1,N2
490   D2(KL)=.618*X7(KL)
      DO 500 JM=1,N2
500   X8(JM)=X0(JM)+D2(JM)
      DO 510 JN=1,N2
510   X2(JN)=X1(JN)-D2(JN)
      DO 520 KM=1,N2
520   X4(KM)=X2(KM)
      V2=F3(X4)
      DO530 KN=1,N2
530   X4(KN)=X8(KN)
      W2=F3(X4)
      DO140 K9=1,N4
      DO540 LL=1,N2
540   D2(LL)=.618*D2(LL)
      IF (ABS(W2-V2).LT..001) GO TO 110
      IF(W2.GT.V2) GO TO 120
      DO550 JJ=1,N2
550   X0(JJ)=X2(JJ)
      DO560 KK=1,N2
560   X2(KK)=X8(KK)
      DO 570 MM=1,N2
570   X8(MM)=X0(MM)+D2(MM)
      V2=W2
      DO580 NN=1,N2
580   X4(NN)=X8(NN)
      W2=F3(X4)
      GO TO 140
120     DO590 II=1,N2
590   X1(II)=X8(II)
      DO595 IJ=1,N2
595   X8(IJ)=X2(IJ)
      DO600 JJ=1,N2
600   X2(JJ)=X1(JJ)-D2(JJ)
      W2=V2
      DO610 KK=1,N2
610   X4(KK)=X2(KK)
130   V2=F3(X4)
140   CONTINUE
110     DO620 LL=1,N2
620   X7(LL)=X2(LL)+X8(LL)
      DO630 MM=1,N2
630   X0(MM)=(.5)*X7(MM)
      IF(J2.EQ.N2+1) GO TO 150
128   J2=J2+1
      IF(J2.EQ.3)J2=4
      IF(J2.LT.N2+1)GOTO710
      DO660 KK=1,N2
660   U2(KK)=X0(KK)-X3(KK)
      U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2+U2
     (4)**2+U2(5)**2)
      DO670 LL=1,N2
670   U2(LL)=(1/U1)*U2(LL)
      DO695 JZ=1,N2
695   E2(JZ,N2+1)=U2(JZ)
      DO680 KL=1,N2
      DO690 LK=1,N2
690   E2(LK,KL)=E2(LK,KL+1)
680   CONTINUE
      GOTO195
710   DO720 KM=1,N2
720   X4(KM)=X0(KM)
      B2=F3(X4)
      GOTO195
150     I2=I2+1
      WRITE(2,1100)I2
1100  FORMAT('    I2=',I3)
```

```
      DO161KL=1,5
161      X4(KL)=XO(KL)
      WRITE(2,1150)F3(X4)
1150  FORMAT('     F3=',F12.5)
      WRITE(2,1160)X4(1),X4(2),X4(3),X4(4),X4(5)
1160  FORMAT(5F13.4)
      DO 210 JZ=1,N2
210   X(I2,JZ)=XO(JZ)
      X(I2,N2+1)=F3(X4)
      DO 220 JZ=1,N2
220      IF (ABS((XO(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
      GO TO 240
230   IF(I2.EQ.50) GO TO 270
      GOTO300
240   DO700 KK=1,N2
700   X4(KK)=XO(KK)
      Y3=F3(X4)
270   CONTINUE
      WRITE(2,310)Y3
310   FORMAT(' MINIMUM (CNR)TH ='F10.7)
      WRITE(2,320)XO(1),XO(2),XO(3),XO(4),XO(5)
320   FORMAT(' A=',F8.0,' B=',F8.0,'D='E12.6,'ALFA=',F8.0,'K='
,F8.0)
      WRITE(2,370)
      WRITE(2,360)X(1,1),X(1,2),X(1,3),X(1,4),X(1,5)
370   FORMAT('  THE INITIAL POINT WAS  ')
360   FORMAT(' A=',F8.0,'  B=',F6.0,'  D=',E8.2,'  ALF=',
     C F8.0,'  K=',F8.0)
      IF (I2.LT.50) GO TO 330
      WRITE(2,340)
340      FORMAT('SOLUTION DID NOT CONVERGE')
      GO TO 330
100      WRITE(2,350)
350   FORMAT('THERE IS NO MINIMUM')
330   STOP
      END
      FUNCTION F3(X4)
      DIMENSION X4(5),R(3),FK(7),H(7),V(7),H1(4)
      X4(4)=X4(4)/10000
      DOUBLE PRECISION A,B,C,D,E,FJ,FL,FM
      PI=3.141593
      WA=300.*2.*PI
      WB=3300.*2.*PI
      A=X4(5)*X4(2)/X4(1)+X4(4)*X4(3)
      B=X4(5)*X4(2)*X4(3)/X4(1)+X4(5)*X4(2)+X4(4)*X4(2)*X4(3)
      C=X4(5)*X4(3)*X4(2)
      D=X4(3)+X4(2)+X4(5)*X4(2)/X4(1)+X4(4)*X4(3)
      E=X4(3)*X4(2)+B
      FJ=B**2-2*A*C
      FL=D**2-2*E
      FM=E**2-2*C*D
```

```
      P=(A-D)**2-2.*(E-B)
      Q=(E-B)**2
      BP=35000.0
C    FIND THE ROOT OF X**3+LX**2+MX+C**2
      DOUBLE PRECISION X,XFIRST,Y,Y5,YA,T,U
      XFIRST=-1.0E14
      ACC=1.0E-13
      LH=0
      GOTO31
32    XFIRST=10*XFIRST
      LH=LH+1
      GOTO31
38    IF(LH.NE.5) GO TO 32
      IF(ACC.LT.1.0E-6)ACC=ACC*10
      PRINT,ACC
      IF(ACC.LT.1.0E-6) GO TO 31
      WRITE(2,35)X4(3)
35    FORMAT('  NO ROOT FOUND FOR X4( )=',F9.1)
      GO TO 34
31    LF=0
      X=XFIRST
33    NU=0
      GOTO17
37    IF(LF.EQ.1) GO TO 38
      X=-X
      NU=0
      LF=1
17      Y=X**3+FL*X**2+FM*X+C**2
      Y5=3*X**2+2*FL*X+FM
      AB1=X**3
      AB2=FL*X**2
      AB3=FM*X
      YA=(ABS(AB1)+ABS(AB2)+ABS(AB3)+C*C)*ACC
      ABY=Y
      IF(ABS(ABY).LT.YA) GOTO24
      X=X-Y/Y5
      NU=NU+1
      IF(NU.EQ.100)GOTO37
      GOTO 17
24      T=FL+X
      U=FM+X*T
      R(1)=X
      WA=1884.96
      WB=20734.5
      H1(4)=1.0/WA-1.0/WB
      D5=T**2/4-U
      IF(D5.LT.0)GOTO47
      R(2)=-T/2+SQRT(D5)
      R(3)=-T/2-SQRT(D5)
      FK(1)=(A**2*R(1)**2+FJ*R(1)+C**2)/(R(1)-R(2))/(R(1)-R(3))
```

```
      FJ6=SQRT(G6)
      FJ9=ATAN(G8/G7)/2
      IF(G7.GT.0) FJ9=FJ9+1.57079
      FJ7=FJ6*COS(FJ9)
      FJ8=FJ6*SIN(FJ9)
      FN7=FJ7-BP*3.1415927
      FN6=SQRT(FN7**2+FJ8**2)
      FN9=ATAN(FJ8/FN7)
      IF(FN7.LT.0) FN9=FN9+3.1415927
      E7=FJ7+BP*3.1415927
      E6=SQRT(E7**2+FJ8**2)
      E9=ATAN(FJ8/E7)
      IF(E7.LT.0) E9=E9+3.1415927
      F6=FN6/E6
      F9=FN9-E9
      IF(F9.GT.3.1415927) F9=F9-3.1415927
      IF(F9.LT.(-(3.1415927))) F9=F9+3.1415927
      FL7=ALOG(F6)
      FL6=SQRT(FL7**2+F9**2)
      FL9=ATAN(F9/FL7)
      FM7=1-V(2)/V(3)*G7
      FM8=-V(2)/V(3)*G8
      FM6=SQRT(FM7**2+FM8**2)
      FM9=ATAN(FM8/FM7)
      IF(FM7.LT.0)FM9=FM9+3.1415927
      B6=-4*FJ6*G8
      B9=FJ9-1.57079
      P6=FM6/B6
      P9=FM9-B9
      R6=P6*FL6
      R9=P9+FL9
      H(2)=2*R6*COS(R9)*V(3)
      HA=(V(1)*H(1)+H(2))/2/3.1415927
      RT17=WB-FJ7
      RT18=-FJ8
      RT16=SQRT(RT17**2+RT18**2)
      RT19=ATAN(RT18/RT17)
      IF(RT17.LT.0.)      RT19=RT19+PI
      RT27=WB+FJ7
      RT28=FJ8
      RT26=SQRT(RT27**2+RT28**2)
      RT29=ATAN(RT28/RT27)
      IF(RT27.LT.0.) RT29=RT29+PI
      RT37=WA+FJ7
      RT38=FJ8
      RT36=SQRT(RT37**2+RT38**2)
      RT39=ATAN(RT38/RT37)
      IF(RT37.LT.0.0) RT39=RT39+PI
      RT47=WA-FJ7
      RT48=-FJ8
      RT46=SQRT(RT47**2+RT48**2)
      RT49=ATAN(RT48/RT47)
      IF(RT47.LT.0.0)RT49=RT49+PI
      RT6=RT16*RT36/RT26/RT46
      RT9=RT19+RT39-RT29-RT49
      IF(RT9.LT.0.0)RT9=RT9+2.*PI
      RT8=RT6*COS(RT9)
      RT7=RT6*SIN(RT9)
      RIGT7=ALOG(RT6)
      RIGT8=RT9
      RIGT6=SQRT(RIGT7**2+RIGT8**2)
      RIGT9=ATAN(RIGT8/RIGT7)
      IF(RIGT7.LT.0.0)RIGT9=RIGT9+PI
      UPP7=1.0-V(6)/V(7)*G7
      UPP8=-V(6)/V(7)*G8
      UPP6=SQRT(UPP7**2+UPP8**2)
      UPP9=ATAN(UPP8/UPP7)
      IF(UPP7.LT.0.0)UPP9=UPP9+PI
      BOT6=2.0*FJ6*G8
      BOT9=FJ9-1.570796
      FRACT6=UPP6/BOT6
      FRACT9=UPP9-BOT9
      PROD6=RIGT6*FRACT6
      PROD9=FRACT9+RIGT9
      H1(2)=PROD6*COS(PROD9)
      HA1=(V(4)*H1(4)+V(5)*H1(1)+V(7)*H1(2))/2.0/PI
88    T4=HA
      GNU=0.25
      ETA=8.*PI**3*1.E7*WA*WB/(WB-WA)
      F3=HA/BP/(GNU-ETA*HA1)
34    RETURN
      END
```

```
      FK(2)=(A**2*R(2)**2+FJ*R(2)+C**2)/(R(2)-R(3))/hR(2)-R(1))
      FK(3)=(A**2*R(3)**2+FJ*R(3)+C**2)/(R(3)-R(1))/(R(3)-R(2))
      FK(4)=Q/C/C
      FK(5)=(R(1)**2+P*R(1)+Q)/R(1)/(R(1)-R(2))/(R(1)-R(3))
      FK(6)=(R(2)**2+P*R(2)+Q)/R(2)/(R(2)-R(3))/(R(2)-R(1))
      FK(7)=(R(3)**2+P*R(3)+Q)/R(3)/(R(3)-R(1))/(R(3)-R(2))
      DO10 J6=1,3
      IF(R(J6).GT.0)GO TO     39
      IF(R(J6).LT.0) GO TO 42
      GO TO 36
39      S5=SQRT(R(J6))
      H(J6)=1/2/S5*ALOG((S5-3.14159*BP)o(S5+3.14159*BP))
      H1(J6)=0.5/S5*ALOG((WB-S5)*(WA+S5)/(WB+S5)/(WA-S5))
      GO TO 10
42      S6=SQRT(-R(J6))
      H(J6)=1/S6*ATAN(3.14159*BP/S6)
      H1(J6)=1.0/S6*(ATAN(WB/S6)-ATAN(WA/S6))
10    CONTINUE
      HA=(FK(1)*H(1)+FK(2)*H(2)+FK(3)*H(3))/2/3.1415927
      HA1=(FK(5)*H1(1)+FK(6)*H1(2)+FK(7)*H1(3)+FK(4)*H1(4))/2.
/3.141593
      GO TO88
47      D6=R(1)**2+U+T*R(1)
      V(1)=(A**2*R(1)**2+C**2+FJ*R(1))/D6
      D7=R(1)*U**2+R(1)**2*T*U+R(1)**3*U
      V(2)=(A**2*U-FJ*R(1)-C**2+R(1)*T*A**2)/D6
      V(3)=(FJ*U-R(1)*C**2+A**2*U*R(1)-C**2*T)/D6
      V(4)=Q/C/C
      DEN=U*R(1)*(R(1)**2+U+T*R(1))
      TOP5=U*(R(1)*P+R(1)**2+Q)
      TOP6=R(1)*Q*(T+R(1))-U*R(1)*(R(1)+P)
      TOP7=U*U*R(1)-P*R(1)*U*(R(1)+T)+Q*R(1)*(T*R(1)-U+T*T)
      V(5)=TOP5/DEN
      V(6)=TOP6/DEN
      V(7)=TOP7/DEN
      IF(R(1).GT.0) GO TO 55
      IF(R(1).LT.0) GO TO 65
36      WRITE(2,200)J6
200   FORMAT('  R(',I1,')=0')
      STOP
55      S5=SQRT(R(1))
      H(1)=1/2/S5*ALOG((S5-3.1415927*BP)/(S5+3.1415927*BP))
      H1(1)=0.5/S5*ALOG((WB-S5)*(WA+S5)/(WB+S5)/(WA-S5))
      GO TO58
65      S6=SQRT(-R(1))
      H(1)=1/S6*ATAN(3.14159*BP/S6)
      H1(1)=1.0/S6*(ATAN(WB/S6)-ATAN(WA/S6))
C  SUBSCRIPTS  6(MAG), 7(REAL),  8 (IMAG) , 9 (ANGLE)
58    G7=T/2
      G8=-SQRT(-D5)
      G6=SQRT(G8**2+G7**2)
```

```
C     INTEGRAL1 TO CHECK CLOSED FORM SOUTION
      DIMENSION X4(5),R(3),FK(7),H(3),V(7)
      X4(1)=800000000.0
      X4(2)=3000
      X4(3)=180000
      X4(4)=1.0
      X4(5)=500000
      A=X4(5)*X4(2)/X4(1)+X4(4)*X4(3)
      B=X4(5)*X4(2)*X4(3)/X4(1)+X4(5)*X4(2)+X4(4)*X4(2)*X4(3)
      C=X4(5)*X4(3)*X4(2)
      D=X4(3)+X4(2)+X4(5)*X4(2)/X4(1)+X4(4)*X4(3)
      E=X4(3)*X4(2)+X4(5)*X4(2)*X4(3)/X4(1)+X4(5)*X4(2)
      E=E+X4(4)*X4(2)*X4(3)
      FJ=B**2-2*A*C
      FL=D**2-2*E
      FM=E**2-2*C*D
      B8=35000.0
C    FIND THE ROOT OF X**3+LX**2+MX+C**2
      X=-1.0E12
17      Y=X**3+FL*X**2+FM*X+C**2
      Y5=3*X**2+2*FL*X+FM
      YA=(ABS(X**3)+ABS(FL*X**2)+ABS(FM*X)+C**2)*1.0E-7
      IF(ABS(Y).LT.YA) GOTO24
      X=X-Y/Y5
      GOTO 17
24      T=FL+X
      U=FM+X*T
      R(1)=X
      D5=T**2/4-U
      IF(D5.LT.0)GOTO47
      R(2)=-T/2+SQRT(D5)
      R(3)=-T/2-SQRT(D5)
      FK(1)=(A**2*R(1)**2+FJ*R(1)+C**2)/(R(1)-R(2))/(R(1)-R(3))
      FK(2)=(A**2*R(2)**2+FJ*R(2)+C**2)/(R(2)-R(3))/(R(2)-R(1))
      FK(3)=(A**2*R(3)**2+FJ*R(3)+C**2)/(R(3)-R(1))/(R(3)-R(2))
      DO10 J6=1,3
      IF(R(J6).GT.0)GO TO     39
      IF(R(J6).LT.0) GO TO 42
      GO TO 36
39      S5=SQRT(R(J6))
      H(J6)=1/2/S5*ALOG((S5-3.1415927*B8)/(S5+3.1415927*B8))
      GO TO 10
42      S6=SQRT(-R(J6))
      H(J6)=1/S6*ATAN(3.1415927*B8/S6)
10    CONTINUE
      HA=(FK(1)*H(1)+FK(2)*H(2)+FK(3)*H(3))/2/3.1415927
      GO TO 88
47      D6=R(1)**2+U+T*R(1)
```

```
V(1)=(A**2*R(1)**2+C**2+FJ*R(1))/D6
       D7=R(1)*U**2+R(1)**2*T*U+R(1)**3*U
       V(2)=(A**2*U-FJ*R(1)-C**2+R(1)*T*A**2)/D6
       V(3)=(FJ*U-R(1)*C**2+A**2*U*R(1)-C**2*T)/D6
       IF(R(1).GT.0) GO TO 55
       IF(R(1).LT.0) GO TO 65
36       WRITE(2,200)J6
200    FORMAT('R(',I1,')=0')
       STOP
55       S5=SQRT(R(1))
       H(1)=1/2/S5*ALOG((S5-3.1415927*B8)/(S5+3.1415927*B8))
       GO TO58
65       S6=SQRT(-R(1))
       H(1)=1/S6*ATAN(3.14159*B8/S6)
C   SUBSCRIPTS  6(MAG), 7(REAL),  8 (IMAG) , 9 (ANGLE)
58       G7=T/2
       G8=-SQRT(U-T**2/4)
       G6=SQRT(G8**2+G7**2)
       FJ6=SQRT(G6)
       FJ9=ATAN(G8/G7)/2
       SG7=G7
       IF(G7.GT.0) FJ9=FJ9+1.57079
       FJ7=FJ6*COS(FJ9)
       FJ8=FJ6*SIN(FJ9)
       FN7=FJ7-B8*3.1415927
       FN6=SQRT(FN7**2+FJ8**2)
       FN9=ATAN(FJ8/FN7)
       SFN7=FN7
       IF(FN7.LT.0) FN9=FN9+3.1415927
       E7=FJ7+B8*3.1415927
       E6=SQRT(E7**2+FJ8**2)
       E9=ATAN(FJ8/E7)
       SE7=E7
       IF(E7.LT.0) E9=E9+3.1415927
       F6=FN6/E6
       F9=FN9-E9
       SF9=F9
       IF(F9.GT.3.1415927) F9=F9-3.1415927
       IF(F9.LT.(-(3.1415927))) F9=F9+3.1415927
       FL7=ALOG(F6)
       FL6=SQRT(FL7**2+F9**2)
       FL9=ATAN(F9/FL7)
       FM7=1-V(2)/V(3)*G7
       FM8=-V(2)/V(3)*G8
       FM6=SQRT(FM7**2+FM8**2)
       FM9=ATAN(FM8/FM7)
       IF(FM7.LT.0)FM9=FM9+3.1415927
       B6=-4*FJ6*G8
```

```
    B9=FJ9-1.57079
          P6=FM6/B6
          P9=FM9-B9
          R6=P6*FL6
          R9=P9+FL9
          H(2)=2*R6*COS(R9)*V(3)
          HA=(V(1)*H(1)+H(2))/2/3.1415927
  88        IF(R(1).LT.0) WRITE(2,30)
  30      FORMAT( '   R(1) IS NEGETIVE')
          IF(R(1).GT.0) WRITE(2,40)
  40      FORMAT('  R(1) IS POSITIVE')
          IF(D5.LT.0) WRITE(2,50)
  50      FORMAT('  R(2) & R(3) ARE COMPLEX')
          IF(D5.GT.0) WRITE(2,60)
  60      FORMAT('  R(2) & R(3) ARE REAL')
          WRITE(2,70)
  70      FORMAT('  HOW MANY INTERVALS ?')
          READ(1,80)N
  80      FORMAT(I3)
          D1=3.1415927*B8/N
          S=0
          S1=0
          S2=0
          X5=D1/2
          DO90 J5=1,N
          SN=A**2*X5**4+FJ*X5**2+C**2
          FUN=SN/(X5**6+FL*X5**4+FM*X5**2+C**2)
          S1=S1+SN/(X5**6+FL*X5**4+FM*X5**2+C**2)*D1/2/3.1415927
          X5=X5+D1
  90      CONTINUE
          WRITE(2,100)S1
  100     FORMAT('  NUMERICAL INTEGRAL=',F12.3)
          WRITE(2,110)HA
  110     FORMAT('  CLOSED FORM=',F12.3)
          S2=(A**2*E*C+(B**2-2*C*A)*C+C**2*D)/(4*C*(E*D-C))
          WRITE(2,120)S2
  120     FORMAT('  INFINITE INTEGRAL=',F12.3)
          WRITE(2,128)R(1),V(1)*H(1),H(2)
  128     FORMAT('  R(1)=',E15.8,'  H(1)=',E15.8,'H(2)=',E15.8)
          WRITE(2,129)SG7 ,SFN7,SE7,SF9
  129     FORMAT(' G7=',E15.8,'FN7=',E15.7,'E7=',E15.9,'F9=',E15.9)
          STOP
          END
```

```
C ****RESPONSE3  ***  FILTER3  4 VARIABLES  IDEAL DIFFERENTIAT
OR
C    CALCULATES OPEN AND CLOSED LOOP RESPONSES
      A1=74600
      B1=2840
      AL=1.79
      FK1=1.E6
      A=(AL/FK1/B1)**2
      B=(1./A1+AL/FK1)**2-2.*(AL)/FK1/B1
      C=((1.+AL)/FK1/B1)**2
      D=((1.+AL)/FK1+1./A1)**2-2.*(1.+AL)/FK1/B1
      WRITE(2,10)
10    FORMAT(' OPEN AND CLOSED LOOP RESPONSE  IDEAL DIFFERENTI
ATOR')
      WRITE(2,20)
20    FORMAT('  ********************************************')
      WRITE(2,30)A1,B1,AL,FK1
30    FORMAT(' A=',F8.1,'   B=',F8.1,'  ALFA='F5.2,'  K=',F9.1)
      DO40N=1,30
      F=SQRT(2.0)**N*100
      W=2.*3.141593*F
      H=SQRT((A*W**4+B*W*W+1.)/(C*W**4+D*W*W+1.))
      F2=SQRT((1.-AL/FK1/B1*W*W)**2+(W*W*(1./A1+AL/FK1)**2))
     C/SQRT((1.+W*W/B1/B1))
      WRITE(2,35)F,H,F2
35    FORMAT('   F=',F10.1,'   H(JW)=',F8.4,'  F2=',F8.4)
40    CONTINUE
      WRITE(2,45)
45    FORMAT(' --------------------------------------------')
      F=2000
      DO50N=1,20
      W=2.*3.141593*F
      H=SQRT((A*W**4+B*W*W+1.)/(C*W**4+D*W*W+1.))
      F2=SQRT((1.-AL/FK1/B1*W*W)**2+(W*W*(1./A1+AL/FK1)**2))
     C/SQRT((1.+W*W/B1/B1))
      WRITE(2,35)F,H,F2
      F=1.0838*F
50    CONTINUE
      STOP
      END
```

```
C  ****RESPONSE4 *****  5  VARIABLES
C    CALCULATES OPEN AND CLOSED LOOP RESPONSES
      DIMENSION X(5)
      WRITE(2,10)
10    FORMAT('  OPEN & CLOSED LOOP   FILTER 4')
      X(1)=241556
      X(2)=2363
      X(3)=1.0E4
      X(4)=3.856
      X(5)=6.25E5
      A=X(5)*X(2)/X(1)+X(4)*X(3)
      B=X(5)*X(2)*X(3)/X(1)+X(5)*X(2)+X(4)*X(2)*X(3)
      C=X(5)*X(3)*X(2)
      D=X(3)+X(2)+X(5)*X(2)/X(1)+X(4)*X(3)
      E=X(3)*X(2)+B
      G=X(5)*(X(3)+X(2))
      WRITE(2,20)
20    FORMAT('  ********************************************')
      WRITE(2,30)X(1),X(2),X(3),X(4),X(5)
30    FORMAT(' A=',F9.1,'  B=',F8.1,'  D=',E14.4,'  ALF=',F6.3
     C,'  K=',F9.0)
      DO40N=1,41
      F=SQRT(2.0)**N*100
      W=2.*3.141593*F
      H=SQRT(((C-A*W*W)**2+(B*W)**2)/((C-D*W*W)**2
     C+(E*W-W**3)**2))
      F2=SQRT(((C-A*W*W)**2+(B*W)**2)/((C-X(5)*W*W)**2
     C+G*G*W*W))
      WRITE(2,35)F,H,F2
35    FORMAT('   F=',F11.0,'   H(JW)=',F8.4,'  F2=',F8.4)
40    CONTINUE
      WRITE(2,45)
45    FORMAT(' ---------------------------------------------')
      F=2000
      DO50N=1,20
      W=2.*3.141593*F
      H=SQRT(((C-A*W*W)**2+(B*W)**2)/((C-D*W*W)**2
     C+(E*W-W**3)**2))
      F2=SQRT(((C-A*W*W)**2+(B*W)**2)/((C-X(5)*W*W)**2
     C+G*G*W*W))
      WRITE(2,35)F,H,F2
      F=1.0838*F
50    CONTINUE
      STOP
      END
```

```
C *** CASE11 **  TEST-TONE   STANDARD SYSEM THREE VARIABLES
C  PRE DETECTION FILTER
      DIMENSION X(30,3),E2(4,4),Y4(30),B4(3),X0(3),X1(3),X5(3)
      DIMENSION X2(3),D2(3),U2(3),X3(3),X4(3),X7(3),A1(3),X8(3)
      B6=10.0
      BK8=3.14159
      N2=3
      K2=50
      S2=8
      T2=0.01
      F2=0.001
      X(1,1)=9000
      X(1,2)=6000
      X(1,3)=600000
      DO400 L=1,N2
      DO410 N=1,N2
410   E2(L,N)=0
400   CONTINUE
      DO420 L=1,N2
420   E2(L,L)=1
      I2=1
300   M2=0
      M3=0
      D0=1
      J2=1
      DO10 JQ=1,N2
10      X3(JQ)=X(I2,JQ)
      DO430 I=1,N2
430   A1(I)=X3(I)
      AL2=0
      DO440 I=1,N2
440   X4(I)=X3(I)
      Y4(I2)=F3(X4)
      B4(J2)=Y4(I2)
      B2=B4(J2)
      DO20 JQ=1,N2
20      X0(JQ)=X(I2,JQ)
C       UNRESTRICTED SEARCH BEGINS
      JQ1=0
195   K3=1
      DO30 JQ2=1,K2
      S3=(2**(JQ2-1))*S2
      DO45 K=1,N2
45    X5(K)=X0(K)
      DO40 JZ=1,N2
      X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
40    CONTINUE
      DO48 K=1,N2
```

```
48    IF(X0(K).NE.X5(K))GOTO49
      GOTO128
49    DO450 I=1,N2
450   X4(I)=X0(I)
      Y2=F3(X4)
      IF((B2-Y2).GT..0005)  GO TO 50
      IF((Y2-B2).GT..0005)  GO TO 60
      IF((Y2-B2).LT..0005)  GO TO 30
      GO TO 30
60    IF(JQ1.EQ.0) GO TO 70
      IF(K3.EQ.2) GO TO 80
      DO460 M=1,N2
460   X1(M)=X0(M)
      DO 90 JZ=1,N2
      X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
90    CONTINUE
      GOTO80
50    JQ1=1
      K3=1
      GO TO30
70      DO470 JK=1,N2
470   X1(JK)=X0(JK)
      S2=-S2
      K3=2
      JQ1=1
30    B2=Y2
      GO TO 100
80    R2=3*((2**(JQ2-2))*ABS(S2))
      JQ1=0
      N4=INT(1-2.1*ALOG(F2/R2))
      DO480 JL=1,N2
480   X7(JL)=X1(JL)-X0(JL)
      DO490 KL=1,N2
490   D2(KL)=.618*X7(KL)
      DO 500 JM=1,N2
500   X8(JM)=X0(JM)+D2(JM)
      DO 510 JN=1,N2
510   X2(JN)=X1(JN)-D2(JN)
      DO 520 KM=1,N2
520   X4(KM)=X2(KM)
      V2=F3(X4)
      DO530 KN=1,N2
530   X4(KN)=X8(KN)
      W2=F3(X4)
      DO140 K9=1,N4
      DO540 LL=1,N2
540   D2(LL)=.618*D2(LL)
      IF (ABS(W2-V2).LT..001) GO TO 110
```

274.

```
        IF(W2.GT.V2) GO TO 120
        DO550 JJ=1,N2
550     XO(JJ)=X2(JJ)
        DO560 KK=1,N2
560     X2(KK)=X8(KK)
        DO 570 MM=1,N2
570     X8(MM)=XO(MM)+D2(MM)
        V2=W2
        DO580 NN=1,N2
580     X4(NN)=X8(NN)
        W2=F3(X4)
        GO TO 140
120        DO590 II=1,N2
590     X1(II)=X8(II)
        DO595 IJ=1,N2
595     X8(IJ)=X2(IJ)
        DO600 JJ=1,N2
600     X2(JJ)=X1(JJ)-D2(JJ)
        W2=V2
        DO610 KK=1,N2
610     X4(KK)=X2(KK)
130     V2=F3(X4)
140     CONTINUE
110        DO620 LL=1,N2
620     X7(LL)=X2(LL)+X8(LL)
        DO630 MM=1,N2
630     XO(MM)=(.5)*X7(MM)
        IF(J2.EQ.N2+1) GO TO 150
128     J2=J2+1
        IF(J2.LT.N2+1)GOTO710
        DO660 KK=1,N2
660     U2(KK)=XO(KK)-X3(KK)
        U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2)
        DO670 LL=1,N2
670     U2(LL)=(1/U1)*U2(LL)
        DO695 JZ=1,N2
695     E2(JZ,N2+1)=U2(JZ)
        DO680 KL=1,N2
        DO690 LK=1,N2
690     E2(LK,KL)=E2(LK,KL+1)
680     CONTINUE
        GOTO195
710     DO720 KM=1,N2
720     X4(KM)=XO(KM)
        B2=F3(X4)
        GOTO195
150        I2=I2+1
        WRITE(2,1100)I2
```

```
1100 FORMAT('    I2=',I3)
     DO161KL=1,N2
161     X4(KL)=X0(KL)
     WRITE(2,1150)F3(X4)
1150 FORMAT('     F3=',F12.5)
     WRITE(2,1160)X4(1),X4(2),X4(3)
1160 FORMAT(3F13.4)
     DO 210 JZ=1,N2
210  X(I2,JZ)=X0(JZ)
     DO 220 JZ=1,N2
220     IF (ABS((X0(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
     GO TO 240
230  IF(I2.EQ.50) GO TO 270
     AL3=AL2*D0/U1
     IF (ABS(AL3).LT..000001) GO TO 280
     D0=AL3
     DO 290 JZ=1,N2
290  E2(JZ,JS)=U2(JZ)
280  GO TO 300
240  DO700 KK=1,N2
700  X4(KK)=X0(KK)
     Y3=F3(X4)
270  CONTINUE
     WRITE(2,310)Y3
310  FORMAT('MINIMUM (CNR)TH ='F10.7)
     WRITE(2,320)X0(1),X0(2),X0(3)
320  FORMAT(' A=',F8.0,' B=',F8.0,'K=',F8.0)
     WRITE(2,370)
     WRITE(2,360)X(1,1),X(1,2),X(1,3)
370  FORMAT('  THE INITIAL POINT WAS  ')
360  FORMAT(' A=',F8.0,'  B=',F6.0,  'K=',F8.0)
     IF (I2.LT.50) GO TO 330
     WRITE(2,340)
340     FORMAT('SOLUTION DID NOT CONVERGE')
     GO TO 330
100     WRITE(2,350)
350  FORMAT('THERE IS NO MINIMUM')
330  STOP
     END
     FUNCTION F3(X4)
     DIMENSION X4(3),R(3),FK(3),H(3)
     PI=3.141593
     A=1.0/X4(1)**2
     B=1.0/(X4(2)*X4(3))**2
     C=(1.0/X4(1)+1.0/X4(3))**2-2.0/X4(2)/X4(3)
     BP=35000.0
     D5=(C/2.0/B)**2-1.0/B
     IF(D5.LT.0)GOTO58
```

```
      R(2)=-C/2.0/B+SQRT(D5)
      R(3)=-C/2.0/B-SQRT(D5)
      FK(2)=(A/B*R(2)+1.0/B)/(R(2)-R(3))
      FK(3)=(A/B*R(3)+1.0/B)/(R(3)-R(2))
      DO10 J6=2,3
      IF(R(J6).GT.0)GO TO     39
      IF(R(J6).LT.0) GO TO 42
      GO TO 36
39      S5=SQRT(R(J6))
      H(J6)=0.5/S5*ALOG((S5-3.14159*BP)/(S5+3.14159*BP))
      GO TO 10
42      S6=SQRT(-R(J6))
      H(J6)=1.0/S6*ATAN(3.14159*BP/S6)
10    CONTINUE
      HA=(FK(2)*H(2)+FK(3)*H(3))/2.0/PI
      GO TO88
36      WRITE(2,200)J6
200   FORMAT('  R(',I1,')=0')
      STOP
C  SUBSCRIPTS  6(MAG), 7(REAL),  8 (IMAG) , 9 (ANGLE)
58    G7=C/2.0/B
      G8=-SQRT(-D5)
      G6=SQRT(G8**2+G7**2)
      FJ6=SQRT(G6)
      FJ9=ATAN(G8/G7)/2
      IF(G7.GT.0) FJ9=FJ9+1.57079
      FJ7=FJ6*COS(FJ9)
      FJ8=FJ6*SIN(FJ9)
      FN7=FJ7-BP*3.1415927
      FN6=SQRT(FN7**2+FJ8**2)
      FN9=ATAN(FJ8/FN7)
      IF(FN7.LT.0) FN9=FN9+3.1415927
      E7=FJ7+BP*3.1415927
      E6=SQRT(E7**2+FJ8**2)
      E9=ATAN(FJ8/E7)
      IF(E7.LT.0) E9=E9+3.1415927
      F6=FN6/E6
      F9=FN9-E9
      IF(F9.GT.3.1415927) F9=F9-3.1415927
      IF(F9.LT.(-(3.1415927))) F9=F9+3.1415927
      FL7=ALOG(F6)
      FL6=SQRT(FL7**2+F9**2)
      FL9=ATAN(F9/FL7)
      FM7=1.0-A*G7
      FM8=-A*G8
      FM6=SQRT(FM7**2+FM8**2)
      FM9=ATAN(FM8/FM7)
      IF(FM7.LT.0)FM9=FM9+3.1415927
```

```
          B6=-4.0*FJ6*G8
          B9=FJ9-1.57079
          F6=FM6/B6
          P9=FM9-B9
          R6=P6*FL6
          R9=P9+FL9
          HA=R6*COS(R9)/B/PI
 88       T4=HA
          W=1000.*2.*PI
          T6=(W*W*(B*W*W+1.0/X4(3)/X4(3)))/
         C(B*W**4+W*W*((1.0/X4(1)+1.0/X4(3))**2-2.0/X4(3)/X4(2))+1
.0)
          BETA=10.0
          BP=35000.0
          F3=4.0*PI**2*T4/BP/(PI-2.*BETA*SQRT(T6))**2
 111         S=0.
          S2=0.
          S3=0.
          FN=10
          DELT=PI*BP/FN
          Z=DELT/2.
          NFN=INT(FN)
          DO345N=1,NFN
 800         S=S+(A*Z*Z+1.)/(B*Z**4+C*Z*Z+1.)*DELT/2./PI
          S2=S2+1.0/(Z*Z-R(2))*DELT
          S3=S3+1.0/(Z*Z-R(3))*DELT
          Z=Z+DELT
 345      CONTINUE
          PRINT,S,S2,S3
          PRINT,T4,H(2),H(3)
          PRINT,FK(2)*H(2),FK(3)*H(3)
          RETURN
          END
 BP97-144
```

```
C  ** CASE12  ** STANDARD  SYSTEM WITH ADDITIONAL POLE 4 VARIA
BLES  WITH  FILTER
C  'DOUBLE PRECISION IN --NEWTON--  ROOT'
C  RECIPROCAL CONSTANTS
         DIMENSION X(40,4),E2(5,5),Y4(40),B4(4),X0(4),X1(4),X5(4)
         DIMENSION X2(4),D2(4),U2(4),X3(4),X4(4),X7(4),A1(4),X8(4)
         PI=3.141593
         N2=4
         K2=50
         S2=8
         T2=0.01
         F2=0.001
         X(1,1)=20000
         X(1,2)=500000
         X(1,3)=20000
         X(1,4)=200000
         DO400 L=1,N2
         DO410 N=1,N2
410      E2(L,N)=0
400      CONTINUE
         DO420 L=1,N2
420      E2(L,L)=1
         I2=1
300      M2=0
         M3=0
         D0=1
         J2=1
         DO10 JQ=1,N2
10         X3(JQ)=X(I2,JQ)
         DO430 I=1,N2
430      A1(I)=X3(I)
         AL2=0
         DO440 I=1,N2
440      X4(I)=X3(I)
         Y4(I2)=F3(X4)
         B4(J2)=Y4(I2)
         B2=B4(J2)
         DO20 JQ=1,N2
20         X0(JQ)=X(I2,JQ)
C         UNRESTRICTED SEARCH BEGINS
         JQ1=0
195      K3=1
         DO30 JQ2=1,K2
         S3=(2**(JQ2-1))*S2
         DO45 K=1,N2
45       X5(K)=X0(K)
         DO40 JZ=1,N2
         IF(JZ.EQ.3)GOTO40
         X0(JZ)=X0(JZ)+S3*E2(JZ,J2)
```

```
40    CONTINUE
      DO48 K=1,N2
48    IF(X0(K).NE.X5(K))GOTO49
      GOTO128
49    DO450 I=1,N2
450   X4(I)=X0(I)
      Y2=F3(X4)
      IF((B2-Y2).GT..0005)  GO TO 50
      IF((Y2-B2).GT..0005)  GO TO 60
      IF((Y2-B2).LT..0005)  GO TO 30
      GO TO 30
60    IF(JQ1.EQ.0) GO TO 70
      IF(K3.EQ.2) GO TO 80
      DO460 M=1,N2
460   X1(M)=X0(M)
      DO 90 JZ=1,N2
      IF(JZ.EQ.3)GOTO90
      X0(JZ)=X0(JZ)-(S3+((2**(JQ2-2))*S2))*E2(JZ,J2)
90    CONTINUE
      GOTO80
50    JQ1=1
      K3=1
      GO TO30
70      DO470 JK=1,N2
470   X1(JK)=X0(JK)
      S2=-S2
      K3=2
      JQ1=1
30    B2=Y2
      GO TO 100
80    R2=3*((2**(JQ2-2))*ABS(S2))
      JQ1=0
      N4=INT(1-2.1*ALOG(F2/R2))
      DO480 JL=1,N2
480   X7(JL)=X1(JL)-X0(JL)
      DO490 KL=1,N2
490   D2(KL)=.618*X7(KL)
      DO 500 JM=1,N2
500   X8(JM)=X0(JM)+D2(JM)
      DO 510 JN=1,N2
510   X2(JN)=X1(JN)-D2(JN)
      DO 520 KM=1,N2
520   X4(KM)=X2(KM)
      V2=F3(X4)
      DO530 KN=1,N2
530   X4(KN)=X8(KN)
      W2=F3(X4)
      DO140 K9=1,N4
```

```
      DO540 LL=1,N2
540   D2(LL)=.618*D2(LL)
      IF (ABS(W2-V2).LT..001) GO TO 110
      IF(W2.GT.V2) GO TO 120
      DO550 JJ=1,N2
550   XO(JJ)=X2(JJ)
      DO560 KK=1,N2
560   X2(KK)=X8(KK)
      DO 570 MM=1,N2
570   X8(MM)=XO(MM)+D2(MM)
      V2=W2
      DO580 NN=1,N2
580   X4(NN)=X8(NN)
      W2=F3(X4)
      GO TO 140
120      DO590 II=1,N2
590   X1(II)=X8(II)
      DO595 IJ=1,N2
595   X8(IJ)=X2(IJ)
      DO600 JJ=1,N2
600   X2(JJ)=X1(JJ)-D2(JJ)
      W2=V2
      DO610 KK=1,N2
610   X4(KK)=X2(KK)
130   V2=F3(X4)
140   CONTINUE
110      DO620 LL=1,N2
620   X7(LL)=X2(LL)+X8(LL)
      DO630 MM=1,N2
630   XO(MM)=(.5)*X7(MM)
128   IF(J2.EQ.N2+1) GO TO 150
      J2=J2+1
      IF(J2.EQ.3)J2=4
      IF(J2.LT.N2+1)GOTO710
      DO660 KK=1,N2
660   U2(KK)=XO(KK)-X3(KK)
      U1=SQRT(U2(1)**2+U2(2)**2+U2(3)**2+U2(4)**2)
      DO670 LL=1,N2
670   U2(LL)=(1/U1)*U2(LL)
      DO695 JZ=1,N2
695   E2(JZ,N2+1)=U2(JZ)
      DO680 KL=1,N2
      DO690 LK=1,N2
690   E2(LK,KL)=E2(LK,KL+1)
680   CONTINUE
      GOTO195
710   DO720 KM=1,N2
720   X4(KM)=XO(KM)
```

```
      B2=F3(X4)
      GOTO195
150      I2=I2+1
      WRITE(2,1100)I2
1100  FORMAT('     I2=',I3)
      DO161KL=1,N2
161      X4(KL)=XO(KL)
      WRITE(2,1150)F3(X4)
1150  FORMAT('      F3=',F12.5)
      WRITE(2,1160)X4(1),X4(2),X4(3),X4(4)
1160  FORMAT(5F13.4)
      DO 210 JZ=1,N2
210   X(I2,JZ)=XO(JZ)
      DO 220 JZ=1,N2
220      IF (ABS((XO(JZ)-X3(JZ))/X3(JZ)).GT.T2) GO TO 230
      GO TO 240
230   IF(I2.EQ.50) GO TO 270
      AL3=AL2*DO/U1
      IF (ABS(AL3).LT..000001) GO TO 280
      DO=AL3
      DO 290 JZ=1,N2
290   E2(JZ,JS)=U2(JZ)
280   GO TO 300
240   DO700 KK=1,N2
700   X4(KK)=XO(KK)
      Y3=F3(X4)
270   CONTINUE
      WRITE(2,310)Y3
310   FORMAT('MINIMUM (CNR)TH ='F10.7)
      WRITE(2,320)XO(1),XO(2),XO(3),XO(4)
320   FORMAT(' A=',F8.0,' B=',F8.0,'D='F9.0,'K=',F8.0)
      WRITE(2,370)
      WRITE(2,360)X(1,1),X(1,2),X(1,3),X(1,4)
370   FORMAT('  THE INITIAL POINT WAS  ')
360   FORMAT(' A=',F8.0,'  B=',F6.0,'  D=',F9.0,
     C '  K=',F8.0)
      IF (I2.LT.50) GO TO 330
      WRITE(2,340)
340      FORMAT('SOLUTION DID NOT CONVERGE')
      GO TO 330
100      WRITE(2,350)
350   FORMAT('THERE IS NO MINIMUM')
330   STOP
      END
      FUNCTION F3(X4)
      DIMENSION X4(4),R(3),FK(7),H(3),V(7)
      DOUBLE PRECISION A,B,C,D,E,FJ,FL,FM
      PI=3.141593
```

```
      A1=1.0/X4(1)
      B1=1.0/X4(2)/X4(3)/X4(4)
      C1=1.0/X4(2)/X4(4)+1.0/X4(3)/X4(4)
      D1=1.0/X4(1)+1.0/X4(4)
      A=A1*A1/B1/B1
      B=(X4(2)*X4(3)*X4(4))**2
      C=(C1*C1-2.0*D1*B1)/B1/B1
      D=(D1*D1-2.0*C1)*B
      BP=35000.0
C    FIND THE ROOT OF  X**3+CX**2+DX+B
      DOUBLE PRECISION X,XFIRST,Y,Y5,YA,T,U
205   XFIRST=-1.0E14
      ACC=1.0E-13
      LH=0
      GOTO31
32    XFIRST=10*XFIRST
      LH=LH+1
      GOTO31
38    IF(LH.NE.5) GO TO 32
      IF(ACC.LT.1.0E-6)ACC=ACC*10
      PRINT,ACC
      IF(ACC.LT.1.0E-6) GO TO 31
      WRITE(2,35)X4(3)
35    FORMAT('  NO ROOT FOUND FOR X4( )=',F9.1)
      GO TO 34
31    LF=0
      X=XFIRST
33    NU=0
      GOTO17
37    IF(LF.EQ.1) GO TO 38
      X=-X
      NU=0
      LF=1
17      Y=X**3+C*X**2+D*X+B
      Y5=3*X**2+2*C*X+D
      AB1=X**3
      AB2=C*X**2
      AB3=D*X
      AB4=B
      YA=(ABS(AB1)+ABS(AB2)+ABS(AB3)+ABS(AB4))*ACC
      ABY=Y
      IF(ABS(ABY).LT.YA) GOTO24
      X=X-Y/Y5
      NU=NU+1
      IF(NU.EQ.100)GOTO37
      GOTO 17
24      T=C+X
      U=D+X*T
```

```
      R(1)=X
      D5=T**2/4-U
      IF(D5.LT.0)GOTO47
      R(2)=-T/2+SQRT(D5)
      R(3)=-T/2-SQRT(D5)
      FK(1)=(A*R(1)+B)/(R(1)-R(2))/(R(1)-R(3))
      FK(2)=(A*R(2)+B)/(R(2)-R(1))/(R(2)-R(3))
      FK(3)=(A*R(3)+B)/(R(3)-R(1))/(R(3)-R(2))
      DO10 J6=1,3
      IF(R(J6).GT.0)GO TO     39
      IF(R(J6).LT.0) GO TO 42
      GO TO 36
39      S5=SQRT(R(J6))
      H(J6)=0.5/S5*ALOG((S5-PI*BP)/(S5+PI*BP))
      GO TO 10
42      S6=SQRT(-R(J6))
      H(J6)=1.0/S6*ATAN(PI*BP/S6)
10    CONTINUE
      HA=(FK(1)*H(1)+FK(2)*H(2)+FK(3)*H(3))/2/PI
      GO TO88
47      D6=R(1)**2+U+T*R(1)
      V(1)=(B+A*R(1))/D6
      V(2)=-V(1)
      V(3)=(A*U-B*(R(1)+T))/D6
      IF(R(1).GT.0) GO TO 55
      IF(R(1).LT.0) GO TO 65
36      WRITE(2,200)J6
200   FORMAT('  R(',I1,')=0')
      STOP
55      S5=SQRT(R(1))
      H(1)=0.5/S5*ALOG((S5-PI*BP)/(S5+PI*BP))
      GO TO58
65      S6=SQRT(-R(1))
      H(1)=1.0/S6*ATAN(PI*BP/S6)
C  SUBSCRIPTS  6(MAG), 7(REAL),  8 (IMAG) , 9 (ANGLE)
58      G7=T/2
      G8=-SQRT(-D5)
      G6=SQRT(G8**2+G7**2)
      FJ6=SQRT(G6)
      FJ9=ATAN(G8/G7)/2
      IF(G7.GT.0) FJ9=FJ9+1.57079
      FJ7=FJ6*COS(FJ9)
      FJ8=FJ6*SIN(FJ9)
      FN7=FJ7-BP*PI
      FN6=SQRT(FN7**2+FJ8**2)
      FN9=ATAN(FJ8/FN7)
      IF(FN7.LT.0) FN9=FN9+PI
      E7=FJ7+BP*PI
```

```
      E6=SQRT(E7**2+FJ8**2)
      E9=ATAN(FJ8/E7)
      IF(E7.LT.0) E9=E9+PI
      F6=FN6/E6
      F9=FN9-E9
      IF(F9.GT.PI) F9=F9-PI
      IF(F9.LT.(-(PI))) F9=F9+PI
      FL7=ALOG(F6)
      FL6=SQRT(FL7**2+F9**2)
      FL9=ATAN(F9/FL7)
      FM7=1-V(2)/V(3)*G7
      FM8=-V(2)/V(3)*G8
      FM6=SQRT(FM7**2+FM8**2)
      FM9=ATAN(FM8/FM7)
      IF(FM7.LT.0)FM9=FM9+PI
      B6=-4*FJ6*G8
      B9=FJ9-1.57079
      P6=FM6/B6
      P9=FM9-B9
      R6=P6*FL6
      R9=P9+FL9
      H(2)=2*R6*COS(R9)*V(3)
      HA=(V(1)*H(1)+H(2))/2/PI
88    T4=HA
      W=2.0*PI*1000.0
      T6=(C1*C1*W**4+(1.0/X4(4)*W-B1*W**3)**2)/((1.0-C1*W*W)
     C**2+(D1*W-B1*W**3)**2)
      BETA=10.0
      BP=35000.0
      F3=4.0*PI**2*T4/BP/(PI-2.0*BETA*SQRT(T6))**2
111      S=0.
      FN=100.0
      DELT=PI*BP/FN
      Z=DELT/2.
      NFN=INT(FN)
      DO345N=1,NFN
      S=S+(1.0+A1*Z*Z*A1)/((1.0-C1*Z*Z)**2+(D1*Z-B1*Z**3)
     C**2)*DELT/6.28319
      Z=Z+DELT
345   CONTINUE
      WRITE(2,346)S,T4
346   FORMAT('  S=',E13.4,'  T4=',E13.4)
34    RETURN
      END
```

APPENDIX C

DERIVATION OF THE RESPONSE OF THE ACTIVE FILTER

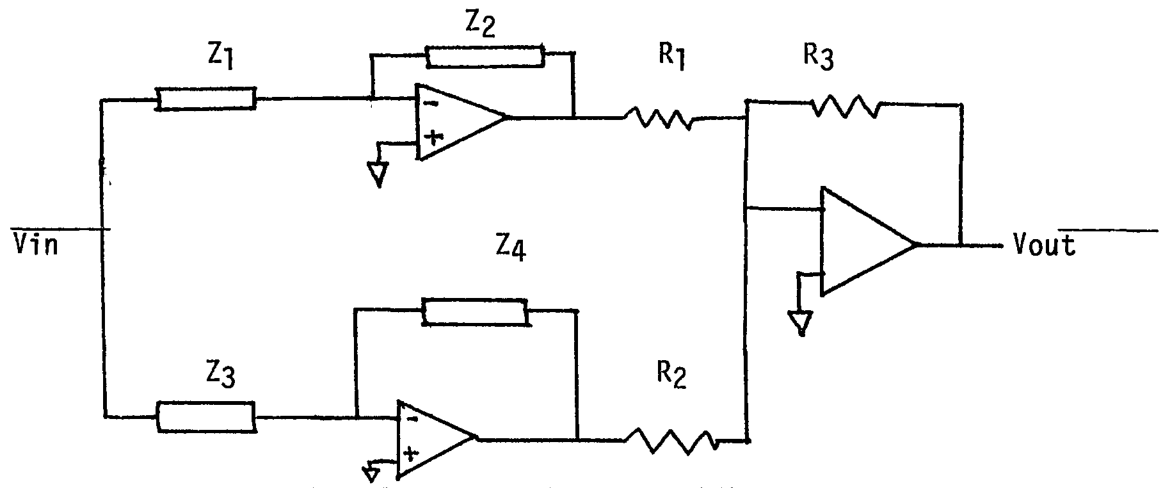
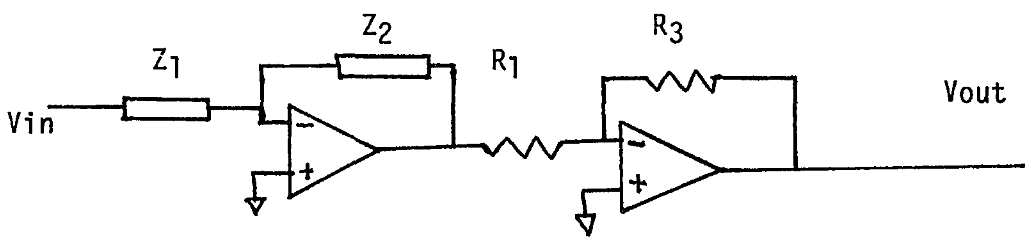
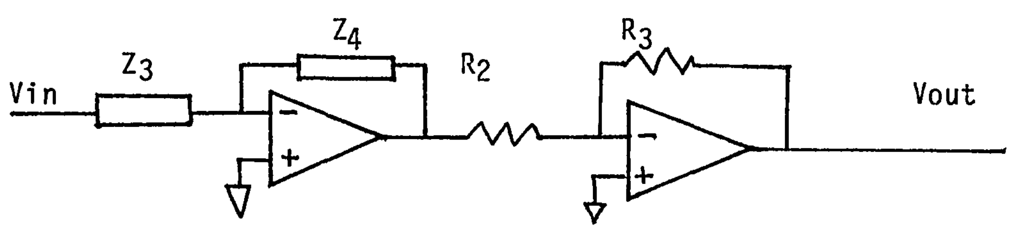


FIGURE C1 - COMPOSITE ACTIVE FILTER

We wish to show that the response of the circuit of FIGURE C1 is identical to the sum of the responses of the two circuits in FIGURE C2.



(a)



(b)

FIGURE C2 - SEPARATED ACTIVE FILTER

The response of C2(a) is

$$Fa = \frac{Z_2}{Z_1} \quad \frac{R_3}{R_1}$$

and of C2(b) it is

$$Fb = \frac{Z_4}{Z_3} \quad \frac{R_3}{R_2}$$

In figure C1, treat the input as though it was applied independently to the upper and lower paths and use superposition.

First consider the lower path and write

$$V1 = Vin \quad \frac{Z_4}{Z_3}$$

The output impedance of the op-amp is much smaller than R_2 and R_1 (i.e. $|Z_{out}| \ll R_2; |Z_{out}| \ll R_1$) so that V_1 may be considered an ideal voltage source and point (2) a ground point. The output op-amp can therefore be drawn as in FIGURE C3.

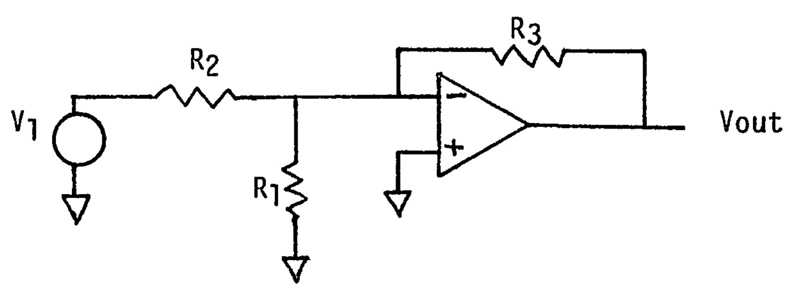
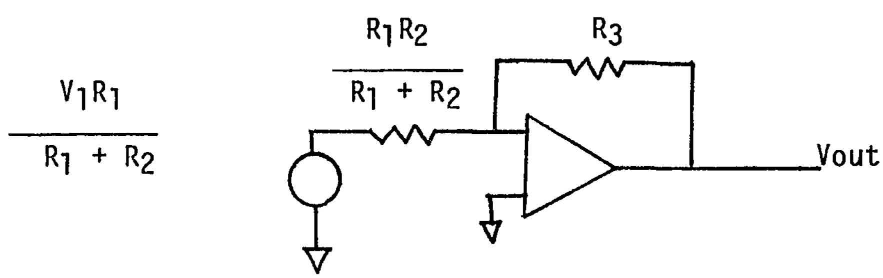


FIGURE C3 - SIMPLIFIED CIRCUIT

Thevenizing the input yields the equivalent circuit



and $V_{out}(b) = \dfrac{V_1 R_1}{R_1 + R_2} \quad \dfrac{R_3}{R_1 R_2/(R_1 + R_2)} = V_1 \dfrac{R_1 R_3}{R_1 R_2}$

$$= \text{Vin} \frac{Z4}{Z3} \cdot \frac{R3}{R2} = V_{in} F_b$$

similarly for the upper branch

$$\text{Vout(a)} = V_{in} \frac{Z_2}{Z_1} \frac{R_3}{R_1} = V_{in} F_a$$

and by superposition

$$\text{Vout} = V_{in} \text{ (Fa + Fb)} .$$

Q.E.D.

APPENDIX D

DETAILS OF THE SEARCH ALGORITHM

Assume a function of N real variables, F $(Y_1, Y_2 \ldots Y_N)$ and the requirement to search for the set of values of the Y_i for which F is a minimum. Let the variables correspond to the coordinates of an N-dimensional coordinate system so that a variable set may be represented as a point in the space.

The minimization search algorithm is based on a method due to Powell [Ref. 1] as modified by Sangwell [Ref. 2]. It consists of a sequence of iterations, each of which is comprised of N segments. In a search segment the function is repeatedly evaluated as the coordinate point is stepped along in a fixed direction in coordinate space until a function minimum is located. This is then repeated for the next direction. An iteration is ended when this process has been completed for each of the N directions, which will be specified presently. The terminal point of each iteration serves as the initialization for the next.

For the first iteration, a starting point is arbitrarily chosen and the N search directions are taken as the N coordinate directions. That is, each segment search is conducted with N-1 variables held constant and the function minimization is with respect to the remaining variable. Upon completion of the first, and all succeeding iterations, the search directions are altered as follows. Let the search directions for the k^{th} iteration be represented by the N unit vectors in N-space, d_1^k, $d_2^k, \ldots d_n^k$. The next set of search directions, d_i^{k+1}, are chosen as

$$d_i^{k+1} = d_{i+1}^{k} \quad \text{for } i = 1 \text{ to } N-1, \text{ and } d_n^{k+1} \text{ is a new unit vector}$$

parallel to the direction of a line drawn between the initial and final points of the k^{th} iteration.

After N iterations, in general, all of the search directions will have been modified and all subsequent iterations will involve directions that are not parallel to any of the coordinates. The directions are specified by an N x N square matrix whose columns are unit vectors in the N directions. Initially the matrix is the identity matrix having '1's along its main diagonal and '0's everywhere else.

Along each of the N directions, the search procedes in two phases. In the first phase, the algorithm evaluates the function at different points increasingly distant from the starting point in the direction of decreasing F. The increment between adjacent points is increased geometrically and the process continues until the slope of the function reverses and F begins to increase. Thus, if Figure D-1 represents the function in one direction, d, phase 1 will terminate at d_4.

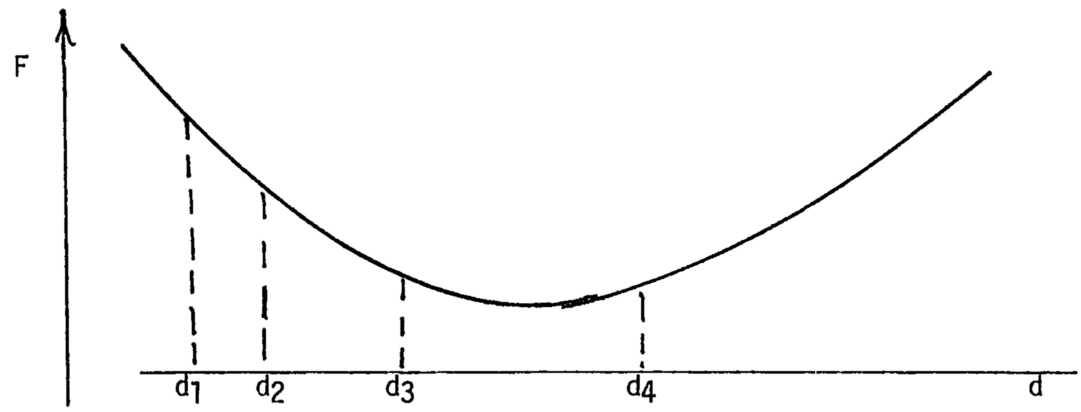


FIGURE D-1 FUNCTIONAL VARIATION IN ONE DIRECTION

Clearly the minimum lies between d_2 and d_4.

In phase 2 of the algorithm the domain of uncertainty is successively narrowed to an arbitrarily small value by a method which is described with reference to figure D-2. Let the lower and upper bounds of the interval of uncertainty be designated as X_o and X_1 respectively.

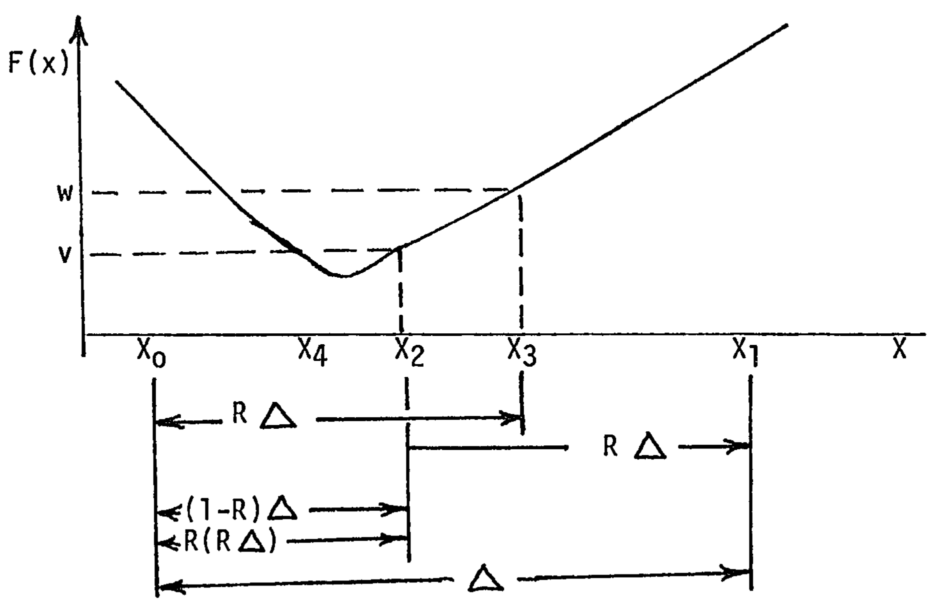


FIGURE D-2 - LOCATING A MINIMUM

Choose two additional points X_2 and X_3 within the interval and evaluate $V = F(X_2)$ and $W = F(X_3)$.

If $W > V$ as shown then the minimum lies between X_0 and X_3; X_3 is renamed X_1, X_2 is renamed X_3, a new X_2 is chosen at the point designated X_4 and the process is repeated.

If, on the other hand, $V > W$ then the minimum lies between X_2 and X_1 and the process moves to the right side of the interval. The logic would be equally sound if the minimum were located within the interval $X_2 - X_3$ rather than outside the interval as exemplified in the diagram.

An efficient method for locating the points X_2 and X_3 is established by requiring that in preparation for the second iteration X_2 should partition the interval $X_0 - X_3$ in the same ratio that X_3 partitions the interval $X_0 - X_1$.

Let the difference $X_1 - X_0 = \Delta$ and let $R < 1$ be such that the difference $X_3 - X_0 = R\Delta$. Symmetrically $X_1 - X_2 = R\Delta$ and therefore $X_2 - X_0 = \Delta - R\Delta = \Delta(1-R)$. But for the subsequent iteration $X_2 - X_0 = R \cdot \Delta R = R^2\Delta$. Equating these two expressions for $X_2 - X_0$ we get $R^2\Delta = (1-R)\Delta$

$$\text{or } R^2 + R - 1 = 0$$

and solving the quadratic equation for the positive root yields

$$R = \frac{1}{2} + \sqrt{\frac{5}{4}} = 0.618$$

Thus $X_3 = X_0 + 0.618\Delta$

$X_2 = X_1 - 0.618\Delta$.

For the second iteration Δ is redefined as $\Delta' = X_3 - X_0 = 0.618\Delta$

and $X_2 = X_4 = X_3 - 0.618\ \Delta' = X_3 - (0.618)^2\Delta$.

This process is repeated until the interval of uncertainty diminishes to a predetermined value,. It is known as a "search by golden section" [Ref. 3, Page 33].

After the first iteration the interval equals $R\Delta$ and after the $K^{\underline{th}}$ it is $R^K\Delta$.

To find K, the required number of iterations, write $R^K\Delta = \epsilon$ and solve for K.

$$K \ln R = \ln \frac{\epsilon}{\Delta}$$

from which $$K = \frac{1}{\ln R} \ln \frac{\epsilon}{\Delta} = -\frac{1}{\ln(0.618)} \ln \frac{\Delta}{\epsilon}$$

and $$K = 2.08 \ln \frac{\Delta}{\epsilon} \quad .$$

ϵ is the acceptable tolerance in the location of the optimum.

Δ is determined at the end of phase 1 as

$\Delta = d_4 - d_2$ on FIGURE D-1

References - Appendix D

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APPENDIX E

DETAILS OF CALCULATIONS FOR CASES 11 AND 12

For substitution into eq. (6-1) it is necessary to evaluate

$$\int \left| H\ (jw) \right|^{2} df$$

and to derive an expression for

$$\left| 1 - H\ (jw) \right|^{2}$$

for each of Cases 11 and 12.

For Case (1ī) we have from eq. (6-2)

$$H(jw) = \frac{\frac{1}{a}(jw) + 1}{\frac{1}{Kb}(jw)^2 + \left(\frac{1}{a} + \frac{1}{K}\right) jw + 1}$$

and

$$\left| H(jw) \right|^2 = \frac{1 + \frac{1}{a^2} w^2}{\left(1 - \frac{1}{Kb} w^2\right)^2 + \left(\frac{1}{a} + \frac{1}{K}\right) w^2} \qquad \text{(E-1)}$$

$$= \frac{1 + \frac{1}{a^2} w^2}{\frac{1}{K^2 b^2} W^4 + \left[\left(\frac{1}{a} + \frac{1}{K}\right)^2 - \frac{2}{Kb}\right] w^2 + 1}$$

For convenience let $\dfrac{1}{a^2} = A, \quad \dfrac{1}{(Kb)^2} = B$,

$$\text{and} \quad \left(\frac{1}{a} + \frac{1}{K}\right)^2 - \frac{2}{Kb} = C$$

The integral may now be written as

$$I = \int_0^{B_p/2} \left|H(jw)\right|^2 df = \frac{1}{2\pi}\int_0^{B_p/2} \frac{Aw^2 + 1}{Bw^4 + Cw^2 + 1} \, dw$$

$$\text{or} \quad I = \frac{1}{2\pi B} \int_0^{\pi B_p} \frac{Aw^2 + 1}{w^4 + \frac{C}{B} w^2 + \frac{1}{B}} \, dw \qquad \text{(E-2)}$$

Eq (E-2) is identified with eq (A-1) of the Appendix with $\Gamma = A$, $T = C/B$ and $U = 1/B$. The closed form solution of E-2 depends upon the discriminant of the quadratic in w^2.

For $C^2 < 4B$ which implies complex roots, the solution is, from eq (A-7)

$$I = \frac{1}{2\pi B} R_e\left[\frac{1 - AG}{J(2j)\sqrt{\frac{1}{B} - \frac{C}{2B}^{2}}} \ln \frac{\pi Bp - J}{\pi B_p + J}\right] \qquad \text{(E-3)}$$

Where G and J are the complex quantities

$$G = \frac{c}{2B} - j\sqrt{\frac{1}{B} - \left(\frac{c}{2B}\right)^2} = |G| \angle \theta_G$$

$$|G| = \frac{1}{\sqrt{B}} = \frac{1}{Kb}$$ and

$$J = |J| \angle \theta_j$$ where

$$J = \sqrt{|G|} = \left(\frac{1}{B}\right)^{1/4} = \frac{1}{\sqrt{Kb}}$$

and $$\theta_G = \mathrm{Tan}^{-1} \frac{\sqrt{\frac{1}{B} - \left(\frac{c}{2B}\right)^2}}{\frac{c}{2B}} = \sqrt{\frac{4B-C}{C}}$$

and $\theta_j = 1/2\,\theta_G$.

For $C^2 > 4B$ the roots are real and the integrand is expandable as

$$\frac{\frac{A}{B} w^2 + \frac{1}{B}}{w^4 + \frac{C}{B} w^2 + \frac{1}{B}} = \frac{\frac{A}{B} w^2 + \frac{1}{B}}{(w^2 - R_2)\,(w^2 - R_3)} = \frac{K_2}{w^2-R_2} + \frac{K_3}{w^2-R_3}$$

where $$R_2 = -\frac{C}{2B} + \sqrt{\left(\frac{C}{2B}\right)^2 - \frac{1}{B}} ,$$

$$R_3 = -\frac{C}{2B} - \sqrt{\left(\frac{C}{2B}\right)^2 - \frac{1}{B}} ,$$

and $$K_2 = \frac{\frac{A}{B} R_2 + \frac{1}{B}}{R_2 - R_3} = \frac{AR_2 + 1}{\sqrt{C^2 - 4B}}$$

and $$K_3 = \frac{\frac{A}{B} R_3 + \frac{1}{B}}{R_3 - R_2} = \frac{AR_3 + 1}{\sqrt{C^2 - 4B}} .$$

The integral of eq (6-2) is

$$I = \int_0^{\pi B_p} \left| H(jw) \right|^2 df = \frac{1}{2\pi} \left[\int_0^{\pi B_p} \frac{K_2}{w^2 - R_2} dw + \int_0^{\pi B_p} \frac{K_3}{w^2 - R_3} dw \right] \qquad \text{(E-4)}$$

These two integrals are identified with eq (A-3) of Appendix A and depend upon the sign of R_i.

$$\text{For } R_i < 0 , \int_0^{\pi B_p} \frac{dw}{w^2 - R_i} = \frac{1}{\sqrt{-R_i}} \, \mathrm{Tan}^{-1} \frac{\pi B_p}{\sqrt{-R_i}}$$

$$\text{and for } R_i > 0 , \int_0^{\pi B_p} \frac{dw}{w^2 - R_i} = \frac{1}{2\sqrt{R_i}} \ln \frac{\sqrt{R_i} - \pi B_p}{\sqrt{R_i} + \pi B_p} . \qquad \text{(E-5)}$$

The peak signal related phase error, θp, that appears in the denominator of eq (6-1) is the same as that for case 1 and is given in Table 3-1 as

$$\theta p = \beta \sqrt{\frac{w_T{}^2 (w_T{}^2 + b^2)}{w_T{}^4 + w_T{}^2 \left(\frac{K^2 b^2}{a^2} + 2 \frac{K b^2}{a} + b^2 - 2Kb \right) + K^2 b^2}} \qquad \text{(E-6)}$$

Where w_T is the modulation frequency and β is the modulation index.

For the second case being considered Filter II of Figure 2-1 is used as the loop filter for which the closed loop response was found to be given by equation (2-25) as

$$H(jw) = \frac{A'(jw) + 1}{B'(jw)^3 + C'(jw)^2 + D'(jw) + 1} \qquad \text{(E-7)}$$

where $A' = \frac{1}{a}$, $B' = \frac{1}{Kbd}$, $C' = \frac{1}{Kb} + \frac{1}{Kd}$

and $D' = \frac{1}{a} + \frac{1}{K}$.

These quantities are listed as unprimed constants for cases 3 and 4 in Table 3-1 because for those applications the algebraic development was complete with the derivation of eq. (E-7). Here we must evaluate

$\int_{f_a}^{f_b} |H(jw)|^2\, df$ for substitution into eq. (6-1).

Upon squaring eq. (E-7) we get

$$|H(jw)|^2 = \frac{1 + A'^2 w^2}{(1-C'w^2)^2 + (D'w - B'w^3)^2}$$

$$= \frac{\frac{A'}{B'} w^2 + \frac{1}{B'^2}}{w^6 + \frac{C'-2D'B'}{B'^2} w^4 + \frac{D'^2 - 2C'}{B'^2} w^2 + \frac{1}{B'^2}}$$

We simplify this expression by making the following substitutions:

$A = \frac{A'^2}{B'^2}$, $B = \frac{1}{B'^2}$, $C = \frac{C'^2 - 2D'B'}{B'^2}$, $D = \frac{D'^2 - 2C'}{B'}$

Now

$$|H(jw)|^2 = \frac{Aw^2 + B}{w^6 + Cw^4 + Dw^2 + B} , \qquad \text{(E-8)}$$

where, in terms of the original circuit parameters,

$$A = \left(\frac{Kbd}{a}\right)^2 \quad , \quad B = Kbd \quad , \quad C = (d+b)^2 + 2bd\ (\ 1 + \frac{K}{a}\ )$$

$$\text{and} \quad D = (Kbd)^{\ 2}\left[\frac{1}{a^2} + \frac{1}{K^2} + \frac{2}{K}\left(\frac{1}{a} - \frac{1}{b} - \frac{1}{d}\right)\right]$$

Combining the effect of the ideal predetection filter and the function of eq (E-8), the integral of eq (6-1) becomes

$$I = \int_0^{\pi B_p} \left|H(jw)\right|^2 df = \frac{1}{2\pi} \int_0^{\pi B_p} \frac{Aw^2 + B}{w^6 + Cw^4 + Dw^2 + B}\ dw \quad . \qquad \text{(E-9)}$$

This form has not been encountered in any previous case but the denominator, a cubic in W^2, did occur in cases 8 and 10 and a similar approach is used to evaluate the integral. The cubic is factored into the "certain" real factor and a quadratic. The form of the solution depends upon the nature of the roots of the quadratic.

Let the real root be R_1, replace w^2 by X, and expand the integrand by partial fractions. Thus

$$\left|H(X)\right|^2 = \frac{AX + B}{X^3 + CX^2 + DX + B} = \frac{AX + B}{(X - R_1)\ (X^2 + TX + U)}$$

For $T^2 > 4U$

$$H(X)^{\ 2} = \frac{K_1}{X-R_1} + \frac{K_2}{X-R_2} + \frac{K_3}{X-R_3} \qquad \text{(E-10)}$$

and for $T^2 < 4U$ the expansion is

$$H(X)^{\ 2} = \frac{V_1}{X-R_1} + \frac{V_2X + V_3}{X^2 + TX + U} \qquad \text{(E-11)}$$

The three constants in eq. (E-10) are evaluated by

$$K_i = \lim_{X \to R_i} \frac{(X-R_i)(AX + B)}{X^3 + CX^2 + DX + B}$$ with the following results:

$$K_1 = \frac{AR_1 + B}{(R_1-R_2)(R_1-R_3)}$$

$$K_2 = \frac{AR_2 + B}{(R_2-R_1)(R_2-R_3)}$$

$$K_3 = \frac{AR_3 + B}{(R_3-R_1)(R_3-R_2)}$$

These constants are then substituted into eq. (E-9) for evaluation of the integral.

$$I = \frac{1}{2\pi} \int_0^{\pi B_p} \left| H(jw) \right|^2 dw = \frac{1}{2\pi} \sum_{i=1}^{3} \int_0^{\pi B_p} \frac{K_i}{w^2-R_i} \quad dw.$$

Each term in the sum is treated as in eqs. (E-5) according to the sign of R_i.

To determine the constants V_i in eq. (E-11) we combine the right side over a common denominator and identify the resulting numerator with the numerator of eq. (6-8). Thus

$$(V_1+V_2)\ X^2 + (V_1T + V_3 - R_1V_2) + V_1U - R_1V_3 = AX + B$$

Equating coefficients of like powers of X leads to the system of linear equations:

$$V_1 + V_2 = 0$$

$$TV_1 - R_1V_2 + V_3 = A$$

$$UV_1 - R_1V_3 = B$$

which may be solved for the constants V_i with the following results:

$$V_1 = \frac{B + AR_1}{U + R_1 (R_1 + T)}$$

$$V_2 = - V_1$$

$$V_3 = \frac{AU - B (R_1 + T)}{U + R_1 (R_1 + T)}$$

These expressions for threshold were encoded into the optimization algorithm and are included among the listings of Appendix B as CASE 11 and CASE 12.

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