ABSTRACT

Title of Thesis: On Analysis of Small Bore Piping Vibrations

Nitinkumar Shroff, Master of Science in Mechanical Engrg., 1984

Thesis directed by: Dr. Benedict C. Sun

This thesis investigates the natural frequency and vibration induced stress of small bore power plant piping. Straight piping and piping with concentrated load, such as a valve, located anywhere along the span are studied. Four different boundary conditions of fixed-free, simple-fixed, fixed-fixed and simple-simple are investigated, as any single span piping may be approximated by one of these boundary conditions.

The results otained for frequency and stress coefficients in the case of straight piping are plotted. Plots of frequency factor and stress ratio for the piping with concentrated load are also presented.

The results obtained for the natural frequencies are compared with the available literature, and it is observed that they are in good agreement. In the case of single span piping with concentrated load, the natural frequency asymptotically approaches that of uniform piping as the concentrated load approaches zero.

ON ANALYSIS OF

SMALL BORE PIPING VIBRATIONS

by

NITINKUMAR SHROFF

Thesis submitted to the Faculty of the Graduate School of
the New Jersey Institute of Technology in partial fulfillment of
the requirements for the degree of
Master of Science in Mechanical Engineering

APPROVAL SHEET

Title of Thesis: On Analysis of Small Bore Piping Vibrations

Name of Candidate: Nitinkumar Shroff

Thesis and Abstract
Approved :

Benedict C. Sun, Ph.D
Associate Professor,
Mechanical Engrg. Dept.

Date

Signatures of other
members of the
thesis committee

Date

Date

VITA

Name: Nitinkumar Shroff

Degree and Date to be conferred: Master of Science in Mechanical Engineering, 1984

Collegiate Institutions attended	Dates	Degree	Date of Degree
New Jersey Institute of Technology, Newark, NJ	1982-1984	M.S.M.E.	May, 1984
University of Bombay, Bombay, India	1977-1981	B.E.(Mech)	February, 1982
Parle College, Bombay, India	1976-1977	H.S.C.	August, 1977

Major: Mechanical Engineering

Positions held: Teaching Assistant, Research Assistant
New Jersey Institute of Technology

ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to his advisor, Dr. Benedict C. Sun, for his constant encouragement and guidance during the author's graduate research at New Jersey Institute of Technology.

The author would also like to thank Drs. B. Koplik and M. Pappas for their advice and comments.

TABLE OF CONTENTS

LIST OF SYMBOLS

a	-	cross sectional area, inches^2
b_n	-	n th mode frequency factor, non dimensional
f_n	-	n th mode frequency, cps
m	-	mass of valve, slugs
t	-	time, seconds
x_1, x_2	-	distance, ft.
$\bar{x}_1$, $\bar{x}_2$	-	normalized distances
x_{max}	-	location of maximum static deflection under concentrated load, ft.
y_n	-	deflection, inches
A_n	-	n th mode Amplitude, inches
A	-	Deflection Coefficient
$B_{i,n}$	-	constant
$C_{i,n}$	-	constant
D_n	-	constant
D_o	-	outside diameter of pipe, inches
DLF	-	Dynamic Loading Factor, non-dimensional
E	-	Modulus of Elasticity, psi
E_1	-	Modulus of Elasticity, psi
$\bar{F}$	-	Frequency Coefficient
F_y	-	force in 'y' direction, lbs.
H	-	constant
I	-	area moment of inertia, inch^4
$K_{i,n}$	-	constant

L	–	length of pipe, ft.
M	–	bending moment at 'x', lb-inch
P	–	weight of valve, lbs.
R_1	–	ratio $[P/(W \cdot L)]$, dimensionless
$\bar{S}$	–	Stress Coefficient
S_{el}	–	endurance limit, psi
W	–	weight of pipe per unit length, lb/ft.
Y_n	–	n th mode deflection, inches
$\bar{Y}_n$	–	n th mode characteristic shape
$\hat{Y}_n$	–	transformed deflection (Laplace transform)
Z	–	section modulus, $inch^3$
ρ	–	mass per unit volume, $slug/in^3$
λ_n	–	n th mode frequency constant, in ft^{-1} units, unless otherwise mentioned
σ_n	–	n th mode stress, psi
$\bar{\sigma}_n$	–	n th mode characteristic stress
σ_r	–	first mode stress ratio
ω_n	–	n th mode frequency, rad/sec
Δ_0	–	maximum static deflection due to uniform weight of pipe, inch
Δ_1	–	maximum static deflection due to concentrated load, inch
Δ_2	–	static deflection due to uniform weight of pipe, at location of maximum deflection due to concentrated load, inch
$(\ )_{xxxx}$	–	represents fourth order partial differential of the quantity in parenthesis, with respect to x
$(\ )_{tt}$	–	represents second order partial differential of the quantity in parenthesis, with respect to t

( )' - represents ordinary differential of the quantity in parenthesis, with respect to x

( )" - represents second order differential of the quantity in parenthesis, with respect to x

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Vibrations in Piping Systems are cause for concern in many ways. The foremost among them are resonance and the possibility of fatigue fracture due to vibration induced stresses. Some of the other undesirable effects are:

i) flow pulsations

ii) higher heat transfer rates due to increased flow turbulence

iii) damage or leakage at critical joints due to fatigue.

The vibration design of piping can be handled with two aspects:

i) fundamental frequency of the system

ii) vibration induced stress.

The fundamental frequency is very significant because it concerns the phenomenon of resonance. A state of resonance is imminent when a mechanical system is excited with a frequency that is very close to the fundamental frequency of the system. In absence of damping or in cases of light damping, the system responds to excitation even by the smallest impulse, with large amplitude of vibration accompanied by large fluctuating stresses that are likely to cause substantial fatigue damage and eventually catastrophic failure.

The piping vibrations may be of two types, free vibrations and forced vibrations. Free vibrations are caused by an impulse which persists for a very short time. Forced vibrations of the harmonic type can be caused by inadequately balanced rotary equipment (e.g. pumps).

The vibration induced stress is directly proportional to the

amplitude of vibration. The ratio between the induced amplitude due to a harmonic forcing input and the static deflection is the 'Dynamic Loading Factor' or 'Dynamic Magnification Factor' [1]*. The vibration induced stress can be evaluated if this factor is known. Evaluation of the fatigue life (in number of cycles to fatigue failure) follows the determination of the vibration induced stress.

The piping system is usually a three dimensional configuration. However, by applying various boundary conditions to single span piping the whole piping system can be simulated with good approximation.

The four boundary conditions considered here are:

1) Fixed-Free
2) Simple-Fixed
3) Fixed-Fixed
4) Simple-Simple

In addition to uniform span analysis, a single span with concentrated load, such as a valve, located anywhere along the span is also studied. The eigen value formulation of the latter is done by the method prescribed by Prof. Yu Chen [2]. The numerical values to the equations obtained by analysis are determined making use of computer-based Numerical Techniques.

In industrial practice, complex vibration problems are analyzed using available software. However, vibration analysis for small bore piping (4 inches and under) is frequently done by hand calculations.

---

* - [ ] denotes references at the end of this thesis

In this thesis an attempt is made to simplify the vibration analysis of such small bore piping by the use of graphs to immediately evaluate to an acceptable degree of accuracy the fundamental frequency and the fluctuating stress. Interactive FORTRAN programs are also presented.

CHAPTER II

GENERAL EQUATION AND SOLUTION

The equation of motion for a pipe is developed based on the equation of motion for flexural vibrations of a beam [1]. It is based on the assumption that plane cross-sections of a pipe remain plane during the vibrations. Moreover, it is assumed that the materials used in piping are homogenous and isotropic, the piping spans are of constant cross-section, and the effects of rotary inertia and shear deformation are negligible even in cases of spans having a concentrated load. Only free undamped vibrations are considered.

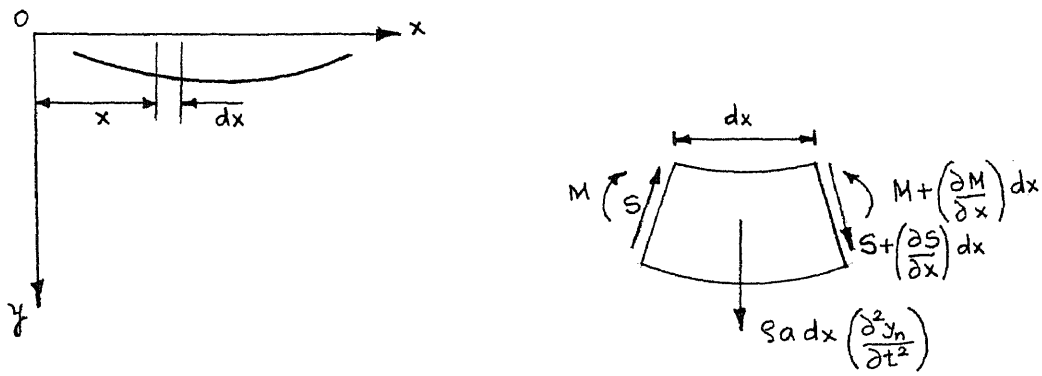


Fig. 1 Free body diagram of pipe element and coordinate axes

Consider an element of length 'dx' bounded by the plane faces perpendicular to the axis and located at 'x' from the left hand support. For equilibrium, the two conditions that must be satisfied are

i) The summation of shear forces in 'y' direction should be zero

ii) The summation of moments about any point should be zero.

From condition (i),

$$\Sigma F_y = 0 \qquad \text{....(1)}$$

$$-S - \rho a dx \frac{\partial^2 y_n}{\partial t^2} + (S + (\partial S/\partial x)dx) = 0,$$

or $$\partial S/\partial x = \rho a(\partial^2 y_n/\partial t^2) \qquad \text{....(2)}$$

From condition (ii),

$$\Sigma M = 0 \text{ (about center line of element)} \qquad \text{....(3)}$$

$$S(dx/2) + M + [S + (\partial S/\partial x)dx](dx/2) - [M + (M/x)dx)] = 0$$

or $$S = \frac{\partial M}{\partial x}, \text{ by neglecting higher order products} \qquad \text{....(4)}$$

Since, $$M = -EI\frac{\partial^2 y_n}{\partial x^2} \qquad \text{....(5)}$$

Equations (2) and (4) yield:

$$\frac{\partial S}{\partial x} = \frac{\partial^2 M}{\partial x^2} = \rho a \frac{\partial^2 y_n}{\partial t^2},$$

then $$\frac{\partial^2}{\partial x^2}\left(-EI\frac{\partial^2 y_n}{\partial x^2}\right) = \rho a \frac{\partial^2 y_n}{\partial t^2},$$

and $$EI\frac{\partial^4 y_n}{\partial x^4} + \rho a \frac{\partial^2 y_n}{\partial t^2} = 0,$$

or $$EI(y_n)_{xxxx} + \rho a(y_n)_{tt} = 0 \qquad \text{....(6)}$$

In order to solve the above equation for natural frequency, we assume harmonic motion, i.e.

$$y_n(x,t) = Y_n(x)\sin {}_n t \qquad \text{....(7)}$$

thus, $$\left(\frac{d^4 Y_n}{dx^4}\right) - \left(\frac{\rho a \omega_n^2}{EI}\right) Y_n = 0 \qquad \text{....(8)}$$

or $$Y_n^{iv} - \lambda_n^4 Y_n = 0 \qquad \text{....(9)}$$

where the differentiation is with respect to x and

$$\lambda_n^4 = \frac{\rho a \omega_n^2}{EI} \qquad \text{....(10)}$$

where λ_n is in inch^{-1} units

or $$\omega_n = \lambda_n^2 \sqrt{\frac{EI}{\rho a}}$$

and $$f_n = \frac{1}{2\pi} \cdot \frac{\lambda_n^2}{144} \cdot \sqrt{\frac{EI}{\rho a}} \quad \text{cps}$$

where λ_n is in ft^{-1} units

or $$f_n = 0.0753 \cdot \lambda_n^2 \cdot \sqrt{\frac{EI}{W}} \quad \text{cps} \qquad \text{....(10a)}$$

where W is in lb/ft.

The general solution to equation (9) is:

$$Y_n(x) = B_{1,n}\sin\lambda_n x + B_{2,n}\cos\lambda_n x + B_{3,n}\cosh\lambda_n x + B_{4,n}\sinh\lambda_n x \qquad \text{....(11)}$$

thus,

$$Y_n'(x) = \lambda_n(B_{1,n}\cos\lambda_n x - B_{2,n}\sin\lambda_n x + B_{3,n}\sinh\lambda_n x + B_{4,n}\cosh\lambda_n x) \qquad \text{....(12)}$$

thus,

$$Y_n''(x) = \lambda_n^2(-B_{1,n}\sin\lambda_n x - B_{2,n}\cos\lambda_n x + B_{3,n}\cosh\lambda_n x + B_{4,n}\sinh\lambda_n x) \qquad \text{....(13)}$$

Evaluation of $B_{2,n}$, $B_{3,n}$, $B_{4,n}$ in terms of $B_{1,n}$ using appropriate boundary conditions leads to the eigen value equation. On calculating the eigen value the deflection can be expressed as

$$Y_n(x) = B_{1,n}\overline{Y}_n(x) \qquad \text{....(14)}$$

and,

$$Y''_n(x) = B_{1,n}\lambda_n^2\bar{Y}''_n(x) = B_{1,n}\lambda_n^2\bar{\sigma}_n(x) \qquad \text{....(15)}$$

where $\bar{Y}_n(x)$, $\bar{Y}''_n(x)$ and $\bar{\sigma}_n(x)$ are all functions of λ_n and x only, and $\bar{Y}''_n(x) = \bar{\sigma}_n(x)$

Equation (14) yields:

$$(Y_n(x))_{max} \qquad B_{1,n} \cdot (\bar{Y}_n(x))_{max} \qquad \text{....(16)}$$

Also, from equation (5),

$$\sigma_n(x) = \frac{M}{Z} = \frac{M \cdot D_o}{2I} = \frac{-E \cdot Y''(x) \cdot D_o}{2} \qquad \text{....(17)}$$

thus,

$$\sigma_n(x) = \frac{-E\ B_{1,n}\ \lambda_n^2\ \bar{\sigma}_n(x)\ D_o}{(2)\ (144)} \text{ psi}$$

$$(\sigma_n(x))_{max} = \frac{-E\ B_{1,n}\ \lambda_n^2\ (\bar{\sigma}_n(x))_{max}\ D_o}{(2)\ (144)} \text{ psi} \qquad \text{....(18)}$$

Equations (18) and (16) yield:

$$\frac{(\sigma_n(x))_{max}}{(Y_n(x))_{max}} = \frac{-E\ \lambda_n^2\ (\bar{\sigma}_n(x))_{max}\ D_o}{(2)(144)\ (\bar{Y}_n(x))_{max}} \qquad \text{....(19)}$$

For the first mode of vibration (fundamental mode), the characteristic shape can be assumed to be the static deflection shape. Thus, $(Y_1(x))_{max}$ can be assumed to be a multiple of the maximum static

deflection, the multiplier being the Dynamic Loading Factor (DLF). Thus,

$$(Y_1(x))_{max} = A_1 = (\text{DLF}) \cdot (\text{MAX. STATIC DEFLECTION}) \qquad ....(20)$$

The stress corresponding to this amplitude 'A_1' can be calculated as

$$(\sigma_1(x))_{max} = \frac{-E \cdot A_1 \cdot D_o \cdot \lambda_1^2 \, (\bar{\sigma}_1(x))_{max}}{(2)(144) \cdot (\bar{Y}_1(x))_{max}} \qquad ....(21)$$

λ_1, $(\bar{Y}_1(x))_{max}$, and $(\bar{\sigma}_1(x))_{max}$ are determined numerically by making use of FORTRAN programs.

For spans with concentrated load the equation of motion (6) gets modified as [2]:

$$EIy_{xxxx} + [\rho a + m\delta(x-x_1)]y_{tt} = 0 \qquad ....(22)$$

where $$\int_0^L \delta(x-x_1)dx = 1 \qquad ....(23)$$

Again, assuming $y_n(x,t) = Y_n(x) \sin\omega_n t$ ....(7)

$$Y_n^{iv} - \lambda_n^4 Y_n - (m\omega_n^2/EI)\cdot(\delta(x-x_1)) = 0 \qquad ....(24)$$

where $\lambda_n^4 = \dfrac{\rho a \omega_n^2}{EI}$

Equation (24) can be solved by Laplace Transforms, applying appropriate boundary conditions to obtain the eigen value equation. Also, expressions for $Y_n(x)$, $Y_n'(x)$ and $Y_n''(x)$ can be obtained once the eigen value is obtained.

CHAPTER III

SOLUTION FOR DIFFERENT BOUNDARY CONDITIONS

The equations derived in Chapter II are now applied to different boundary conditions. The four boundary conditions considered here are:

1) Fixed-Free
2) Simple-Fixed
3) Fixed-Fixed
4) Simple-Simple

Case # 1: Fixed-Free

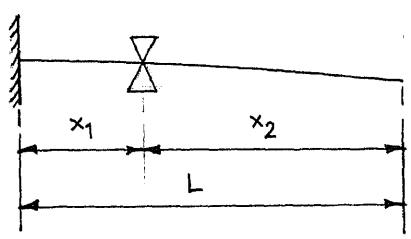


Fig. 2 Fixed-free piping with concentrated load

The boundary conditions for this case are:

$$Y_n(0) = Y_n'(0) = Y_n''(L) = Y_n'''(L) = 0$$

Uniform span

The eigen value equation is [1,3,4,5]:

$$\cos\lambda_n L \cosh\lambda_n L + 1 = 0 \qquad \text{....(9.1)}$$

thus,

$$Y_n(x) = B_{1,n}\left((\sin\lambda_n x - \sinh\lambda_n x) - K_{1,n}(\cos\lambda_n x - \cosh\lambda_n x)\right) \qquad \text{....(14.1)}$$

where $K_{1,n} = (\sin\lambda_n L + \sinh\lambda_n L)/(\cos\lambda_n L + \cosh\lambda_n L)$

i.e. $Y_n(x) = B_{1,n}\bar{Y}_n(x)$

and $Y''_n(x) = B_{1,n}\lambda_n^2\ ((-\sin\lambda_n x - \sinh\lambda_n x) + K_{1,n}(\cos\lambda_n x + \cosh\lambda_n x))$

i.e. $Y''_n(x) = B_{1,n}\lambda_n^2(\bar{\sigma}_n(x))$ ....(15.1)

Now, using equation (21), the maximum stress in the fundamental mode

is $$(\sigma_1(x))_{max} = \frac{-E\ A_1 D_o \lambda_1^2\ (\bar{\sigma}_1(x))_{max}}{(2)(144)\ (\bar{Y}_1(x))_{max}} \quad \text{psi}$$

<u>Span with Concentrated Load (Valve)</u>

The transformed equation of motion is [2,6]:

$$\hat{Y}_n(s) = Y''_n(0)\frac{s}{s^4 - \lambda^4} + Y'''_n(0)\frac{1}{s^4 - \lambda^4} + \frac{m\omega_n^2 Y_n(x_1)}{EI} \cdot \frac{e^{-x_1 s}}{s^4 - \lambda^4} \quad ....(24.1.1)$$

On using partial fractions and taking Laplace inverse [6], we obtain:

$$\begin{aligned} Y_n(x) = {} & (1/(2\lambda_n^2)) \cdot Y''_n(0) \cdot (\cosh\lambda_n x - \cos\lambda_n x) \\ & + (1/(2\lambda_n^3)) \cdot Y'''_n(0) \cdot (\sinh\lambda_n x - \sin\lambda_n x) \\ & + \frac{m\omega_n^2 Y_n(x_1)}{2EI\,\lambda_n^3}\ U(x-x_1)\ \ (\sinh\lambda_n(x-x_1) - \sin\lambda_n(x-x_1)) \end{aligned} \quad ....(24.1.2)$$

where $U(x-x_1)$ represents the unit step function at x_1

such that,

$$U(x-x_1) = \begin{cases} 0 & , x < x_1 \\ 1 & , x \geqslant x_1 \end{cases}$$

Constants $Y_n''(0)$ and $Y_n'''(0)$ can be determined from the two remaining boundary conditions, i.e. $Y_n''(L) = Y_n'''(L) = 0$

Thus,

$$Y_n''(0) = \frac{-m\omega_n^2 Y_n(x_1)\ K_{1,n}}{2\ EI\ \lambda_n D_n} \qquad \text{....(24.1.3)}$$

and,

$$Y_n'''(0) = \frac{-m\omega_n^2 Y_n(x_1)\ K_{2,n}}{2\ EI\ D_n} \qquad \text{....(24.1.4)}$$

where
$$\begin{aligned} K_{1,n} = {} & \sinh\lambda_n x_2 \cosh\lambda_n L + \sin\lambda_n x_2 \cosh\lambda_n L + \sinh\lambda_n x_2 \cos\lambda_n L \\ & + \sin\lambda_n x_2 \cos\lambda_n L - \cosh\lambda_n x_2 \sinh\lambda_n L - \cos\lambda_n x_2 \sinh\lambda_n L \\ & - \cosh\lambda_n x_2 \sin\lambda_n L - \cos\lambda_n x_2 \sin\lambda_n L \end{aligned} \qquad \text{....(24.1.5)}$$

and
$$\begin{aligned} K_{2,n} = {} & \cosh\lambda_n x_2 \cosh\lambda_n L + \cos\lambda_n x_2 \cosh\lambda_n L + \cosh\lambda_n x_2 \cos\lambda_n L \\ & + \cos\lambda_n x_2 \cos\lambda_2 L - \sinh\lambda_n x_2 \sinh\lambda_n L - \sin\lambda_n x_2 \sinh\lambda_n L \\ & + \sinh\lambda_n x_2 \sin\lambda_n L + \sin\lambda_n x_2 \sin\lambda_n L \end{aligned} \qquad \text{....(24.1.6)}$$

and
$$D_n = 1 + \cosh\lambda_n L \cos\lambda_n L \qquad \text{....(24.1.7)}$$

Letting $x=x_1$ in 24.1.2 results in the eigen value equation:

$$4 + \left(\frac{m}{\rho a}\right)\cdot\left(\frac{C_{1,n}\lambda_n + C_{2,n}\lambda_n}{D_n}\right) = 0$$

where λ_n is in inch^{-1} units,

or,

$$4 + \left(\frac{P}{W}\right)\left(\frac{C_{1,n}\lambda_n + C_{2,n}\lambda_n}{D_n}\right) = 0 \qquad \text{....(24.1.8)}$$

where λ_n is in ft^{-1} units,

and,
$$C_{1,n} = K_{1,n}\ (\cosh\lambda_n x_1 - \cos\lambda_n x_1) \qquad \text{....(24.1.9)}$$

and $$C_{2,n} = K_{2,n}\,(\sinh\lambda_n x_1 - \sin\lambda_n x_1) \qquad \text{....(24.1.10)}$$

Now, from 24.1.2, 24.1.3, and 24.1.4,

$$Y_n(x) = H_n \overline{Y}_n(x), \qquad \text{....(24.1.11)}$$

where $$H_n = \frac{m\,\omega_n^2\, Y_n(x_1)}{4EI\,\lambda_n^3} \qquad \text{....(24.1.12)}$$

and $$\overline{Y}_n(x) = (1/D_n)(-K_{1,n}(\cosh\lambda_n x + \cos\lambda_n x) - K_{2,n}(\sinh\lambda_n x - \sin\lambda_n x))$$

$$+ 2\,U(x-x_1)\,(\sinh\lambda_n(x-x_1) - \sin\lambda_n(x-x_1)) \qquad \text{....(24.1.13)}$$

and, $$Y_n''(x) = H_n \lambda_n^2\, \overline{\sigma}_n(x), \text{ where} \qquad \text{....(24.1.14)}$$

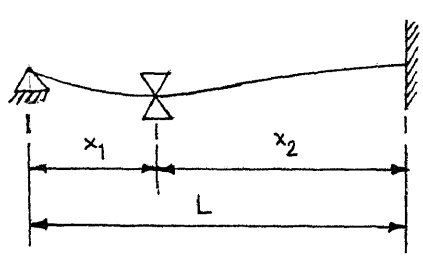
$$\overline{\sigma}_n(x) = (1/D_n)(-K_{1,n}(\cosh\lambda_n x + \cos\lambda_n x) - K_{2,n}(\sinh\lambda_n x + \sin\lambda_n x))$$

$$+ 2\,U(x-x_1)\,(\sinh\lambda_n(x-x_1) + \sin\lambda_n(x-x_1)) \qquad \text{....(24.1.15)}$$

Now, using equation (21), the maximum stress in the fundamental mode

is $$(\sigma_1(x))_{max} = \frac{-E\,A_1 D_o\,\lambda_1^2\,(\overline{\sigma}_1(x))_{max}}{(2)(144)\,(\overline{Y}_1(x))_{max}} \quad \text{psi}$$

<u>Case # 2</u>: Simple-Fixed



<u>Fig. 3 Simple-fixed piping with concentrated load</u>

The boundary conditions for this case are:

$$Y_n(0) = Y_n''(0) = Y_n(L) = Y_n'(L) = 0$$

Uniform span

The eigen value equation is [1,3,4,5]:

$$\sin\lambda_n L\cosh\lambda_n L - \sinh\lambda_n L\cos\lambda_n L = 0 \qquad \ldots(9.2)$$

thus,

$$Y_n(x) = B_{1,n}\,(\sin\lambda_n x - K_{1,n}\sinh\lambda_n x) \qquad \ldots.(14.2)$$

where $K_{1,n} = \cos\lambda_n L/\cosh\lambda_n L$

i.e. $Y_n(x) = B_{1,n}\bar{Y}_n(x)$

and $$Y_n''(x) = B_{1,n}\lambda_n^2(-\sin\lambda_n x - K_{1,n}\sinh\lambda_n x) \qquad \ldots.(15.2)$$

i.e. $Y_n''(x) = B_{1,n}\lambda_n^2(\bar{\sigma}_n(x))$

Again equation (21) can be used to determine the maximum stress in the fundamental mode.

Span with Concentrated Load (Valve)

The transformed equation of motion is [2,6]:

$$\hat{Y}_n(s) = Y_n'(0)\frac{s^2}{s^4-\lambda^4} + Y_n'''(0)\frac{1}{s^4-\lambda^4} + \frac{m\omega_n^2 Y_n(x_1)}{EI}\cdot\frac{e^{-x_1 s}}{s^4-\lambda^4} \qquad \ldots.(24.2.1)$$

On using partial fractions and taking Laplace inverse [6], we obtain:

$$\begin{aligned} Y_n(x) = {} & (1/(2\lambda_n))\cdot Y_n'(0)\cdot(\sin\lambda_n x + \sinh\lambda_n x) \\ & + (1/(2\lambda_n^3))\;\; Y_n'''(0)\;\; (\sinh\lambda_n x - \sin\lambda_n x) \\ & + \frac{m\omega_n^2 Y_n(x_1)}{2EI\lambda_n^3}\;\; U(x-x_1)\;\; (\sinh\lambda_n(x-x_1) - \sin\lambda_n(x-x_1)) \end{aligned} \qquad \ldots.(24.2.2)$$

Constants $Y_n'(0)$ and $Y_n'''(0)$ can be determined from the two remaining boundary conditions, i.e. $Y_n(L) = Y_n'(L) = 0$

Thus,

$$Y_n'(0) = \frac{m\omega_n^2 Y_n(x_1) K_{1,n}}{2 EI \lambda_n^2 D_n} \qquad \text{....(24.2.3)}$$

and

$$Y_n'''(0) = \frac{m\omega_n^2 Y_n(x_1) K_{2,n}}{2 EI D_n} \qquad \text{....(24.2.4)}$$

where
$$\begin{aligned} K_{1,n} = & -\sinh\lambda_n x_2 \cosh\lambda_n L + \sinh\lambda_n x_2 \cos\lambda_n L + \sin\lambda_n x_2 \cosh\lambda_n L \\ & - \sin\lambda_n x_2 \cos\lambda_n L + \cosh\lambda_n x_2 \sinh\lambda_n L - \cosh\lambda_n x_2 \sin\lambda_n L \\ & - \cos\lambda_n x_2 \sinh\lambda_n L + \cos\lambda_n x_2 \sin\lambda_n L \end{aligned} \qquad \text{....(24.2.5)}$$

and
$$\begin{aligned} K_{2,n} = & -\cosh\lambda_n x_2 \sin\lambda_n L + \cos\lambda_n x_2 \sin\lambda_n L - \cosh\lambda_n x_2 \sinh\lambda_n L \\ & + \cos\lambda_n x_2 \sinh\lambda_2 L + \sinh\lambda_n x_2 \cos\lambda_n L - \sin\lambda_n x_2 \cos\lambda_n L \\ & + \sinh\lambda_n x_2 \cosh\lambda_n L - \sin\lambda_n x_2 \cosh\lambda_n L \end{aligned} \qquad \text{....(24.2.6)}$$

and
$$D_n = \sin\lambda_n L \cosh\lambda_n L - \sinh\lambda_n L \cos\lambda_n L \qquad \text{....(24.2.7)}$$

Letting $x=x_1$ in equation 24.2.2 results in the eigen value equation:

$$4 - \left(\frac{P}{W}\right)\left(\frac{C_{1,n}\lambda_n + C_{2,n}\lambda_n}{D_n}\right) = 0 \qquad \text{....(24.2.8)}$$

where,
$$C_{1,n} = K_{1,n} (\sin\lambda_n x_1 + \sinh\lambda_n x_1) \qquad \text{....(24.2.9)}$$

and
$$C_{2,n} = K_{2,n} (\sinh\lambda_n x_1 - \sin\lambda_n x_1) \qquad \text{....(24.2.10)}$$

Again, using constant H_n as defined earlier,

$$Y_n(x) = H_n \bar{Y}_n(x), \qquad \text{....(24.2.11)}$$

where,

$$\bar{Y}_n(x) = (1/D_n)(K_{1,n}(\sin\lambda_n x + \sinh\lambda_n x) + K_{2,n}(\sinh\lambda_n x - \sin\lambda_n x))$$

$$+ 2\,U(x-x_1)\,(\sinh\lambda_n(x-x_1) - \sin\lambda_n(x-x_1)) \qquad \text{....(24.2.12)}$$

and, $Y''_n(x) = H_n\lambda_n^2\,\bar{\sigma}_n(x)$, where ....(24.2.13)

$$\bar{\sigma}_n(x) = (1/D_n)(K_{1,n}(-\sin\lambda_n x + \sinh\lambda_n x) - K_{2,n}(\sinh\lambda_n x + \sin\lambda_n x))$$

$$+ 2\,U(x-x_1)\,(\sinh\lambda_n(x-x_1) + \sin\lambda_n(x-x_1)) \qquad \text{....(24.2.14)}$$

Using equation (21), the maximum stress in the fundamental mode can now be determined.

Case # 3: Fixed-Fixed

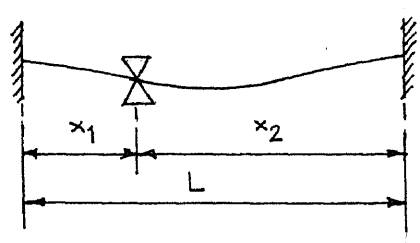


Fig. 4 Fixed-fixed piping with concentrated load

The boundary conditions for this case are:

$$Y_n(0) = Y'_n(0) = Y_n(L) = Y'_n(L) = 0$$

Uniform span

The eigen value equation is [1,3,4,5]:

$$\cos\lambda_n L\cosh\lambda_n L - 1 = 0 \qquad \text{....(9.3)}$$

thus,

$$Y_n(x) = B_{1,n}\,((\sin\lambda_n x - \sinh\lambda_n x) + K_{1,n}(\cos\lambda_n x - \cosh\lambda_n x)) \qquad \text{....(14.3)}$$

where $K_{1,n} = (\cos\lambda_n L - \cosh\lambda_n L)/(\sin\lambda_n L + \sinh\lambda_n L)$

i.e. $Y_n(x) = B_{1,n}\bar{Y}_n(x)$

and $$Y_n''(x) = B_{1,n}\lambda_n^2\left((-\sin\lambda_n x - \sinh\lambda_n x) - K_{1,n}(\cos\lambda_n x + \cosh\lambda_n x)\right) \quad \text{....(15.3)}$$

i.e. $Y_n''(x) = B_{1,n}\lambda_n^2(\bar{\sigma}_n(x))$

Equation (21) can now be used to determine the maximum stress in the fundamental mode.

Span with Concentrated Load (Valve)

The transformed equation of motion is [2,6]:

$$\hat{Y}_n(s) = Y_n''(0)\frac{s}{s^4 - \lambda^4} + Y_n'''(0)\frac{1}{s^4 - \lambda^4} + \frac{m\omega_n^2 Y_n(x_1)}{EI}\,\frac{e^{-x_1 s}}{s^4 - \lambda^4} \quad \text{....(24.3.1)}$$

On using partial fractions and taking Laplace inverse 6 , we obtain:

$$\begin{aligned} Y_n(x) = {} & (1/(2\lambda_n^2)) \cdot Y_n''(0) \cdot (\cosh\lambda_n x - \cos\lambda_n x) \\ & + (1/(2\lambda_n^3)) \cdot Y_n'''(0) \cdot (\sinh\lambda_n x - \sin\lambda_n x) \\ & + \frac{m\omega_n^2 Y_n(x_1)}{2EI\,\lambda_n^3}\, U(x-x_1)\ (\sinh\lambda_n(x-x_1) - \sin\lambda_n(x-x_1)) \end{aligned} \quad \text{....(24.3.2)}$$

Constants $Y_n''(0)$ and $Y_n'''(0)$ can be determined from the two remaining boundary conditions, i.e. $Y_n(L) = Y_n'(L) = 0$

Thus,

$$Y_n''(0) = \frac{-m\omega_n^2\, Y_n(x_1)\, K_{1,n}}{2\, EI\, \lambda_n D_n} \quad \text{....(24.3.3)}$$

and $$Y_n'''(0) = \frac{-m\omega_n^2\, Y_n(x_1)\, K_{2,n}}{2\, EI\, D_n} \quad \text{....(24.3.4)}$$

where $K_{1,n} = \sinh\lambda_n x_2 \cosh\lambda_n L - \sin\lambda_n x_2 \cosh\lambda_n L - \sinh\lambda_n x_2 \cos\lambda_n L$
$+ \sin\lambda_n x_2 \cos\lambda_n L - \cosh\lambda_n x_2 \sinh\lambda_n L + \cos\lambda_n x_2 \sinh\lambda_n L$
$+ \cosh\lambda_n x_2 \sin\lambda_n L - \cos\lambda_n x_2 \sin\lambda_n L$ ....(24.3.5)

and $K_{2,n} = \cosh\lambda_n x_2 \cosh\lambda_n L - \cos\lambda_n x_2 \cosh\lambda_n L - \cosh\lambda_n x_2 \cos\lambda_n L$
$+ \cos\lambda_n x_2 \cos\lambda_2 L - \sinh\lambda_n x_2 \sinh\lambda_n L + \sin\lambda_n x_2 \sinh\lambda_n L$
$- \sinh\lambda_n x_2 \sin\lambda_n L + \sin\lambda_n x_2 \sin\lambda_n L$ ....(24.3.6)

and $D_n = 1 - \cosh\lambda_n L \cos\lambda_n L$ ....(24.3.7)

Letting $x=x_1$ in 24.3.2 results in the eigen value equation:

$$4 + \left(\frac{P}{W}\right)\left(\frac{C_{1,n}\lambda_n + C_{2,n}\lambda_n}{D_n}\right) = 0 \qquad \text{....(24.3.8)}$$

where, $C_{1,n} = K_{1,n}\,(\cosh\lambda_n x_1 - \cos\lambda_n x_1)$ ....(24.3.9)

and $C_{2,n} = K_{2,n}\,(\sinh\lambda_n x_1 - \sin\lambda_n x_1)$ ....(24.3.10)

Again, using constant H_n as defined earlier,

$$Y_n(x) = H_n \overline{Y}_n(x), \qquad \text{....(24.3.11)}$$

where,

$$\overline{Y}_n(x) = (1/D_n)(-K_{1,n}(\cosh\lambda_n x - \cos\lambda_n x) - K_{2,n}(\sinh\lambda_n x - \sin\lambda_n x))$$
$$+ 2\,U(x-x_1)\,(\sinh\lambda_n(x-x_1) - \sin\lambda_n(x-x_1)) \qquad \text{....(24.3.12)}$$

and, $Y''_n(x) = H_n \lambda_n^2\, \overline{\sigma}_n(x)$, where ....(24.3.13)

$$\overline{\sigma}_n(x) = (1/D_n)(-K_{1,n}(\cosh\lambda_n x + \cos\lambda_n x) - K_{2,n}(\sinh\lambda_n x + \sin\lambda_n x))$$
$$+ 2\,U(x-x_1)\,(\sinh\lambda_n(x-x_1) + \sin\lambda_n(x-x_1)) \qquad \text{....(24.3.14)}$$

Equation (21) can now be used to determine the maximum stress in the fundamental mode.

Case # 4: Simple-Simple

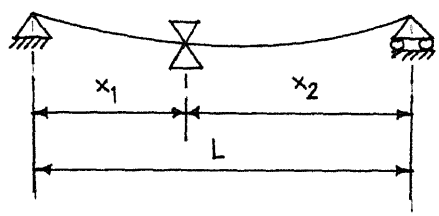


Fig. 5 Simple-simple piping with concentrated load

The boundary conditions for this case are:

$$Y_n(0) = Y_n''(0) = Y_n(L) = Y_n''(L) = 0$$

Uniform span

The eigen value equation is [1,3,4,5]:

$$\sin \lambda_n L = 0 \qquad \text{....(9.4)}$$

thus,

$$Y_n(x) = B_{1,n} \sin \lambda_n x \qquad \text{....(14.4)}$$

i.e. $Y_n(x) = B_{1,n} \overline{Y}_n(x)$

and $Y_n''(x) = B_{1,n} \lambda_n^2 (-\sin {}_n x)$ ....(15.4)

i.e. $Y_n''(x) = B_{1,n} \lambda_n^2 (\overline{\sigma}_n(x))$

Again equation (21) can be used to determine the maximum stress in the fundamental mode.

Span with Concentrated Load (Valve)

The transformed equation of motion is [2,6]:

$$\hat{Y}_n(s) = Y_n'(0)\frac{s^2}{s^4 - \lambda^4} + Y_n'''(0)\frac{1}{s^4 - \lambda^4} + \frac{m\omega_n^2 Y_n(x_1)}{EI} \cdot \frac{e^{-x_1 s}}{s^4 - \lambda^4} \qquad \text{....(24.4.1)}$$

On using partial fractions and taking Laplace inverse [6], we obtain:

$$\begin{aligned} Y_n(x) = {} & (1/(2\lambda_n)) \cdot Y_n'(0) \cdot (\sin\lambda_n x + \sinh\lambda_n x) \\ & + (1/(2\lambda_n^3)) \; Y_n'''(0) \; (\sinh\lambda_n x - \sin\lambda_n x) \\ & + \frac{m\omega_n^2 Y_n(x_1)}{2EI\,\lambda_n^3} \; U(x-x_1) \; (\sinh\lambda_n(x-x_1) - \sin\lambda_n(x-x_1)) \end{aligned} \qquad \text{....(24.4.2)}$$

Constants $Y_n'(0)$ and $Y_n'''(0)$ can be determined from the two remaining boundary conditions, i.e. $Y_n(L) = Y_n''(L) = 0$

Thus,

$$Y_n'(0) = \frac{-m\omega_n^2 Y_n(x_1) K_{1,n}}{2\,EI\,\lambda_n^2 D_n} \qquad \text{....(24.4.3)}$$

and

$$Y_n'''(0) = \frac{-m\omega_n^2 Y_n(x_1) K_{2,n}}{2\,EI\,D_n} \qquad \text{....(24.4.4)}$$

where $K_{1,n} = \sinh\lambda_n x_2 \sin\lambda_n L - \sin\lambda_n x_2 \sinh\lambda_n L$ ....(24.4.5)

and $K_{2,n} = \sinh\lambda_n x_2 \sin\lambda_n L + \sin\lambda_n x_2 \sinh\lambda_n L$ ....(24.4.6)

and $D_n = \sin\lambda_n L \sinh\lambda_n L$ ....(24.4.7)

Letting $x=x_1$ in 24.4.2 results in the eigen value equation:

$$2 + \left(\frac{P}{W}\right)\left(\frac{\sinh\lambda_n x_2 \sinh\lambda_n x_1}{\sinh\lambda_n L} - \frac{\sin\lambda_n x_2 \sin\lambda_n x_1}{\sin\lambda_n L}\right) = 0 \qquad \text{....(24.4.8)}$$

Again, using constant H_n as defined earlier,

$$Y_n(x) = H_n \bar{Y}_n(x), \qquad \text{....(24.4.9)}$$

where,

$$\bar{Y}_n(x) = (1/D_n)(-K_{1,n}(\sinh\lambda_n x + \sin\lambda_n x) - K_{2,n}(\sinh\lambda_n x - \sin\lambda_n x))$$

$$+ 2\,U(x-x_1)\,(\sinh\lambda_n(x-x_1) - \sin\lambda_n(x-x_1)) \qquad \text{....(24.4.10)}$$

and, $Y''_n(x) = H_n\lambda_n^2\,\bar{\sigma}_n(x)$, where ....(24.4.11)

$$\bar{\sigma}_n(x) = (1/D_n)(-K_{1,n}(\sinh\lambda_n x - \sin\lambda_n x) - K_{2,n}(\sinh\lambda_n x + \sin\lambda_n x))$$

$$+ 2\,U(x-x_1)\,(\sinh\lambda_n(x-x_1) + \sin\lambda_n(x-x_1)) \qquad \text{....(24.4.12)}$$

Equation (21) can now be used to determine the maximum stress in the fundamental mode.

Modifications in equations for the purpose of plotting graphs

The equations obtained are modified to suit the plotting of graphs. For uniform spans, graphs are plotted for unit values of I, w and D_o Thus, the user of the graphs only has to scale the graph readings appropriately during calculations. Three graphs are presented for the uniform spans, viz. fundamental frequency, maximum static deflection and maximum bending stress. The amplitude used in obtaining the graphs for maximum bending stress is the amplitude obtained from the maximum static deflection graph (which is by assuming DLF=1). However, it is known that DLF could be theoretically as large as 2.0 for a suddenly applied load.

For spans with a concentrated load, a non-dimensional frequency factor b_n is defined as follows:

$$b_n = \lambda_n L \qquad \text{....(25)}$$

and this non-dimensional frequency factor is used in all equations in

place of the frequency factor, λ_n. Thus, the eigen value equation, the equation for the characteristic shape and the equation for characteristic stress are all obtained in terms of b_n. Graphs are then presented with $R_1 = \frac{P}{WL}$ as the abcissa and b_n and the ratio $\sigma_r = \frac{(\bar{\sigma}_1(x))_{max}}{(\bar{Y}_1(x))_{max}}$ as the ordinate respectively.

Determination of the Maximum Static Deflection

The Amplitude to be used in determining the stresses is given by equation (20) as:

$$A_1 = (\text{DLF}) \cdot (\text{MAXIMUM STATIC DEFLECTION}) \qquad \ldots.(20)$$

Thus, it is necessary to determine the maximum static deflection in order to determine the stresses. The formulas for maximum static deflection are obtained from Ref. 7 and are tabulated in Appendix - F.

CHAPTER IV

DESCRIPTION OF THE COMPUTER PROGRAMS

Two different programs are presented for analysis, one for uniform spans and one for spans with a concentrated load. FORTRAN-IV has been used for the programs. In both cases the method followed in the development of the program is the same and is as follows:

1) Main program reads the input. The input is interactive and format-free. Following this, the main program calculates the maximum static deflection for the case chosen. Subroutines LAMDA, MAXY, and MAXSIG are then called and the fundamental frequency and vibration induced stress determined according to the chosen Dynamic Loading Factor. The main program then prints out the results.

2) Subroutine LAMDA uses the bisection algorithm to determine the fundamental frequency function, λ_1. The number of iterations, 'N' required for convergence of the algorithm is calculated as follows [8]:

$$N > \frac{\log_{10}(b-a) - \log_{10}(\epsilon)}{\log_{10} 2}, \text{ where}$$

'a' - lower limit, 'b' - upper limit and 'ϵ' is the allowable tolerance. For instance, consider a = 0.00015, b = 0.00015 + 0.005 and ϵ = 0.00001, then $N > 9$ iterations. Since the fundamental frequency is desired, 'a' has to be very close to zero. However, when 'a' is very small, the algorithm sometimes converges to the trivial solution (zero). This is due to truncation errors. In such cases, 'a' has to be incremented by a very small value.

3) Subroutine MAXY and MAXSIG calculate the maximum deflection

function and the maximum stress function respectively. This calculation is done by first dividing the entire span into small divisions and then making a numerical comparison between the value of the function at, for instance, 'x' and 'x + delta'. This comparison is begun from the left hand support, and is continued until a value is reached which is higher than its preceeding value.

The equations used for calculating the maximum static deflection are mentioned in Appendix - F. The symbols and variable names used in the programs are listed in Appendix - G. A sample run of the programs is presented in Appendix - B and Appendix - D.

The programs used for generating the data for graphs are similar to the above programs, with slight modifications. In the case of uniform spans, the program is run repetitively for different lengths so as to cover a practical range of length (2' - 14'). The input data to this program is unit quantities for W, I and D_o. The Modulus of Elasticity is chosen as 30×10^6 psi. For spans with a concentrated load, the equations are first modified as suggested earlier (p. 20) and the program is run repetitively to cover a practical range of values of R_1 and $\bar{x}_1$. These programs are developed separately for each different boundary condition. The only difference in these programs is the value of constants like $K_{1,1}$, $K_{2,1}$, $C_{1,1}$, $C_{2,1}$, the eigen value equation, and the formulas for $\bar{Y}_1$ and $\bar{\sigma}_1$. Two such programs are presented in Appendix - E. A sample of the output is also presented along with the programs.

CHAPTER V

GRAPHS

Results obtained by repeated application of the computer programs are presented in graphical form. For uniform spans, three graphs are presented for each case:

i) Natural frequency for unit values of I and W, ($\bar{F}$)

ii) Maximum Static Deflection for unit values of I and W, ($\bar{A}$)

iii) Maximum Bending Stress for unit values of I, W and D_o, ($\bar{S}$)

The graphs are plotted for the range of length from 2 to 14 ft.

For the spans with concentrated load, two graphs are presented for each case:

i) Natural frequency factor, b_1

ii) Maximum Stress ratio, σ_r

The ranges for the values are:

R_1: 0.1 to 10 (logarithmic)

$\bar{x}_1$: covering entire span at intervals of L/10

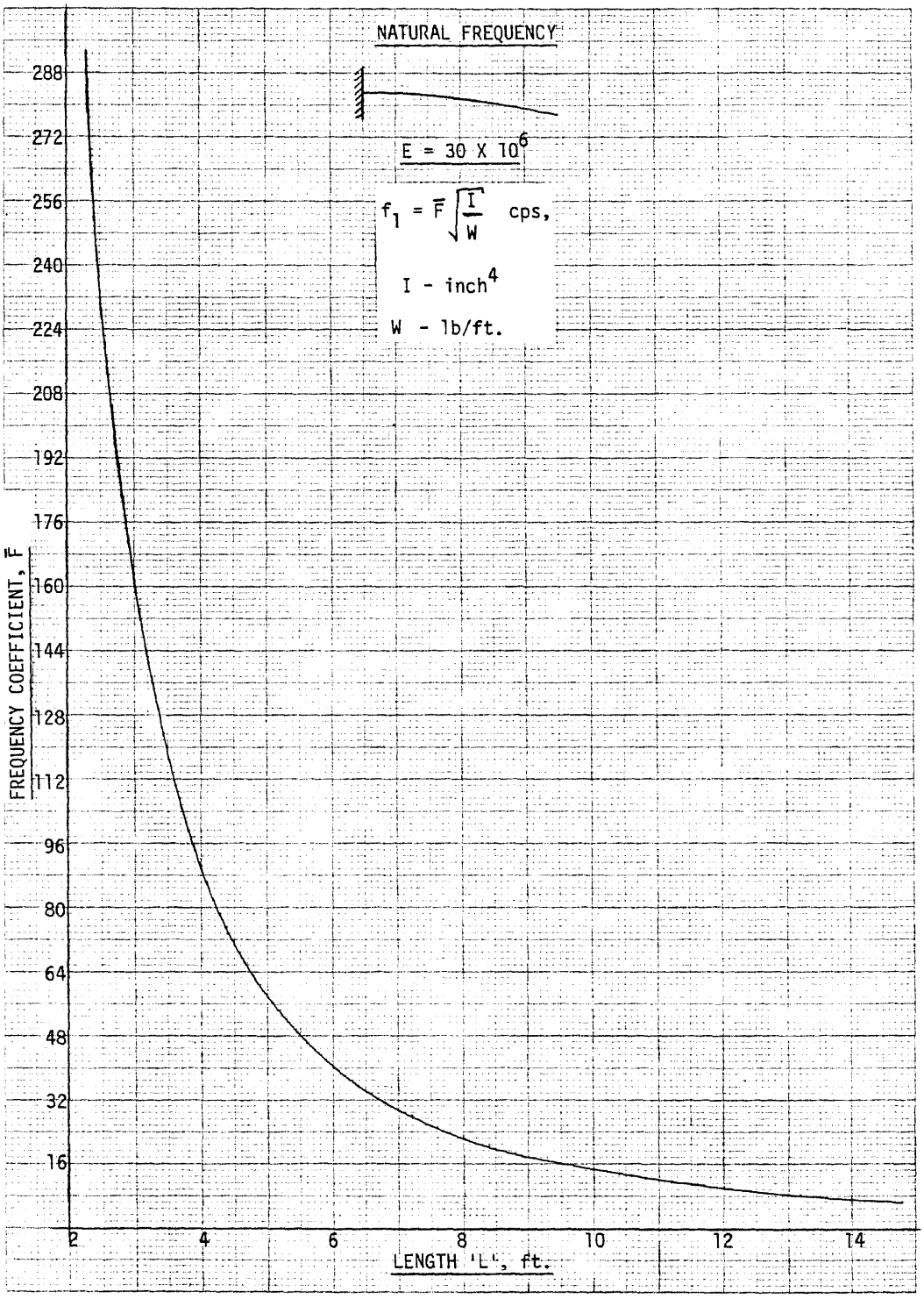


Fig. 6 Frequency Coefficient, $\bar{F}$, for Cantilever Pipe

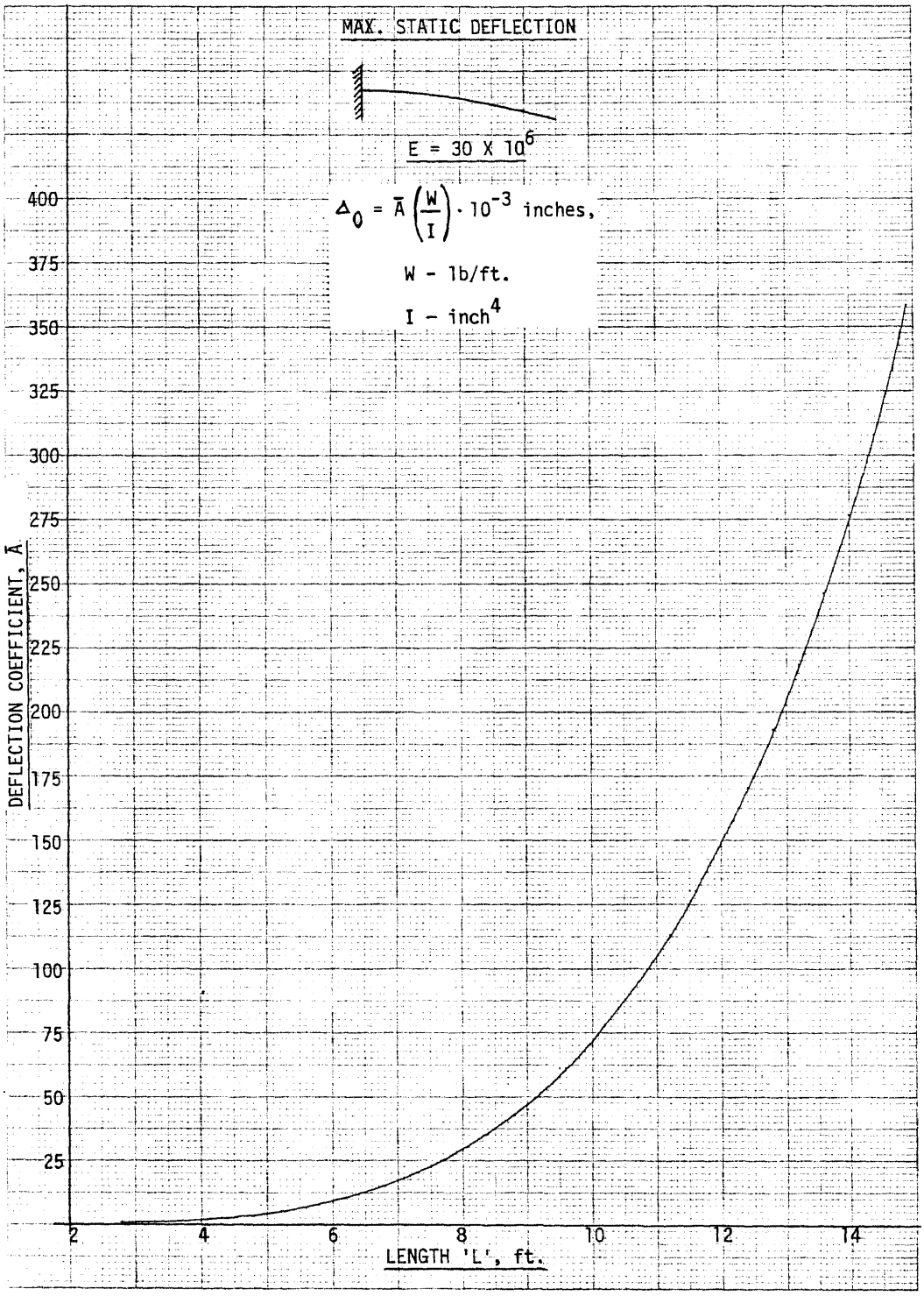


Fig. 7 Deflection Coefficient, $\bar{A}$, for Cantilever Pipe

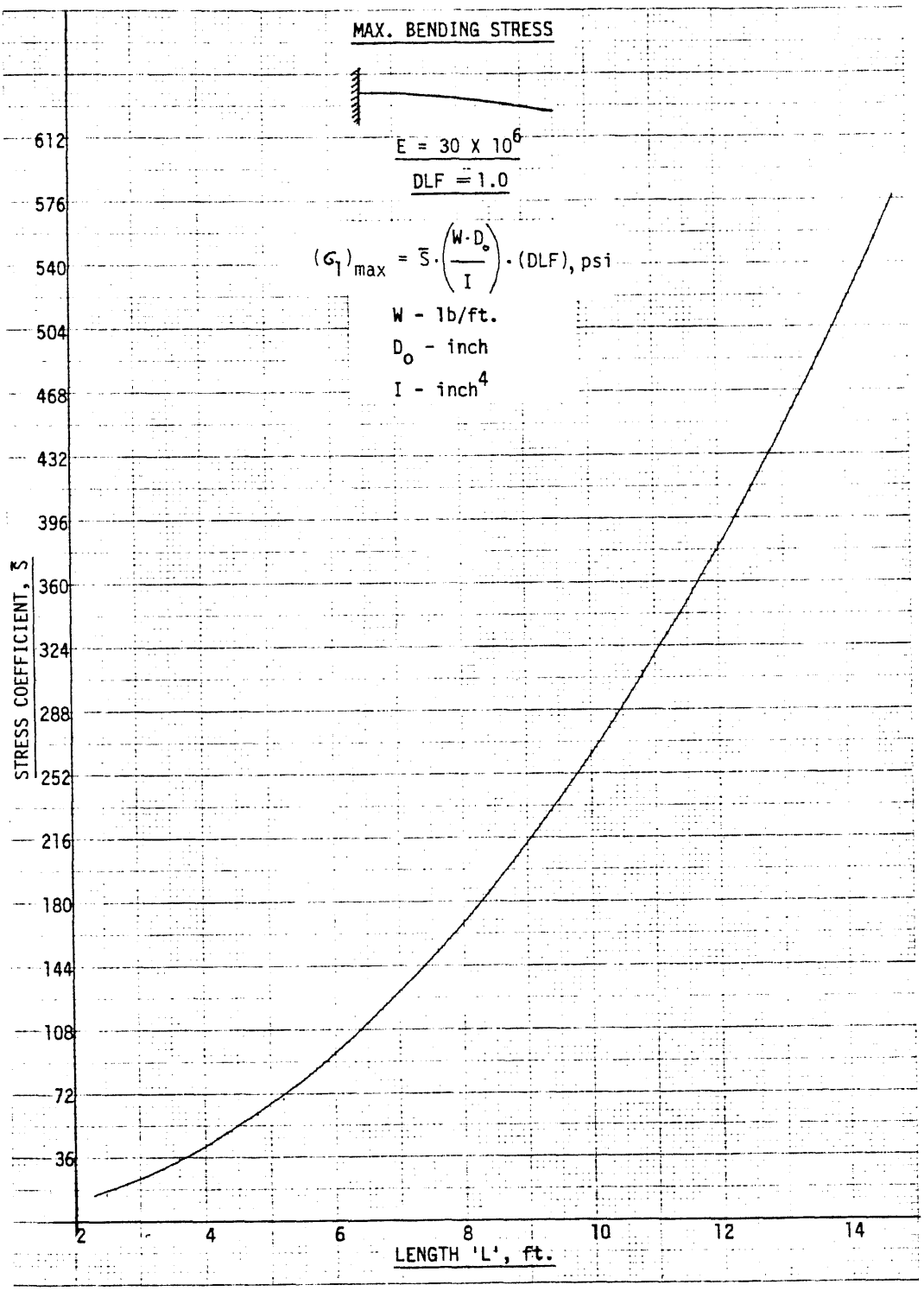


Fig. 8 Stress Coefficient, $\bar{S}$, for Cantilever Pipe

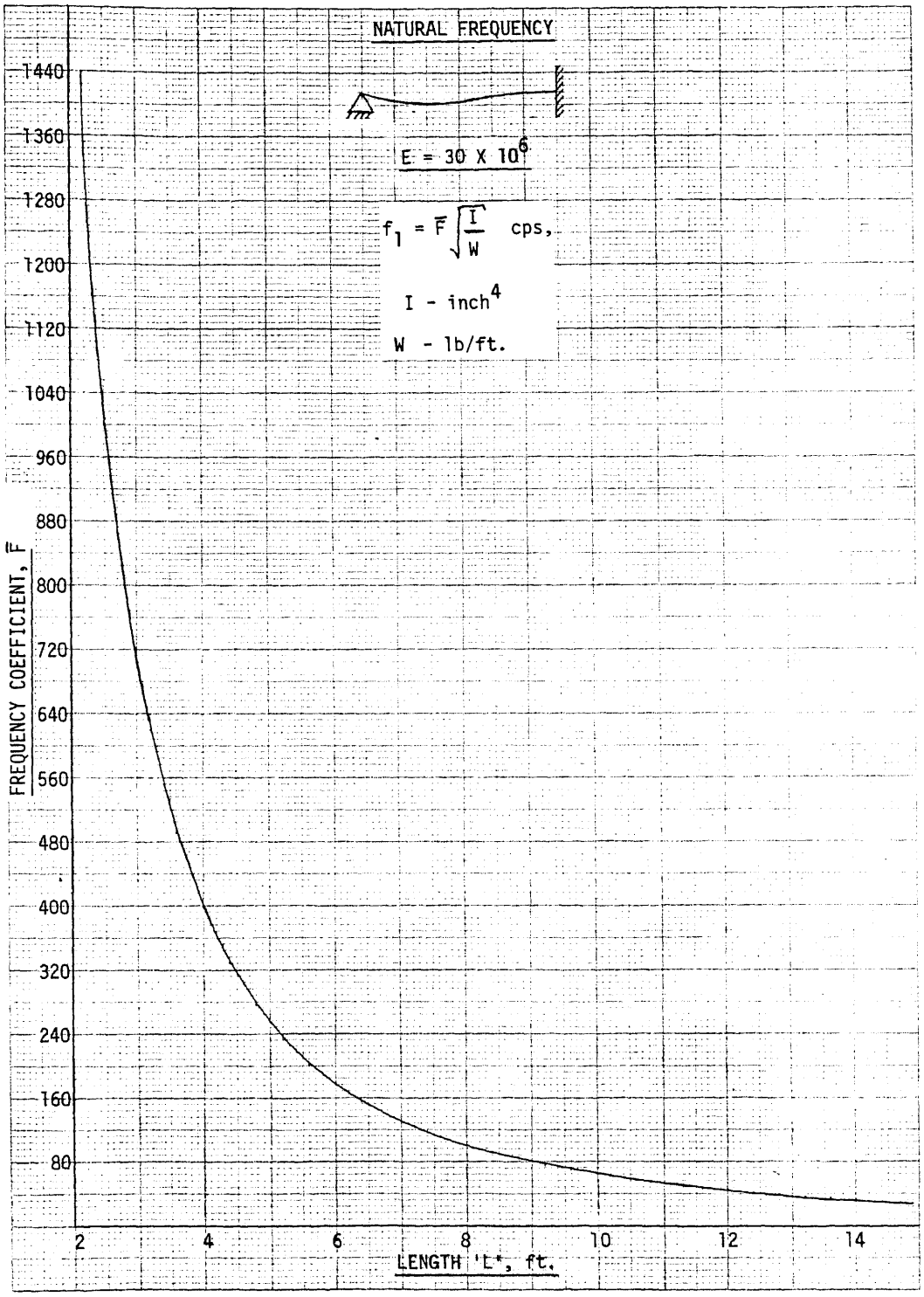


Fig. 9 Frequency Coefficient, $\bar{F}$, for Simple-Fixed Pipe

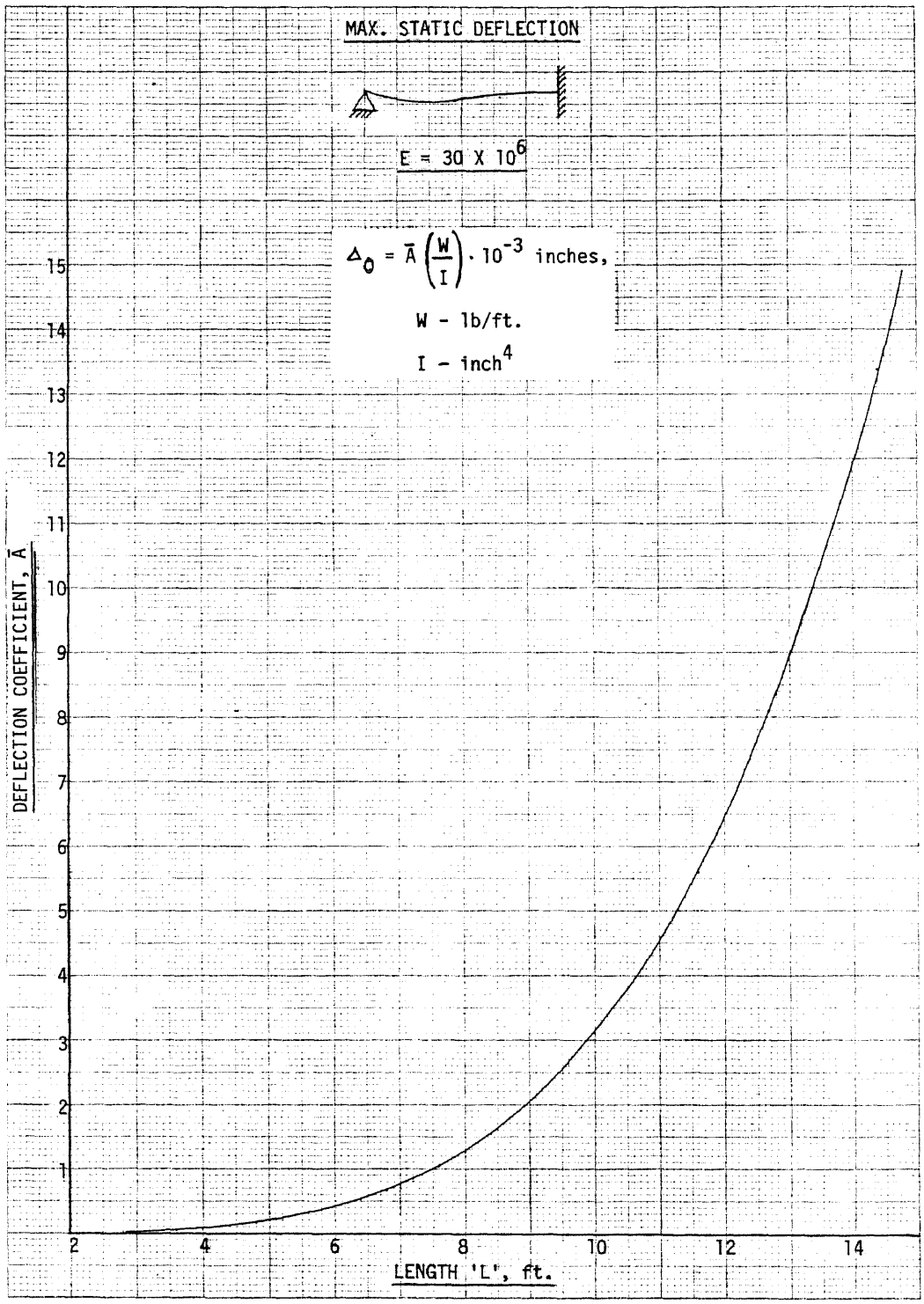


Fig. 10 Deflection Coefficient, $\bar{A}$, for Simple-Fixed Pipe

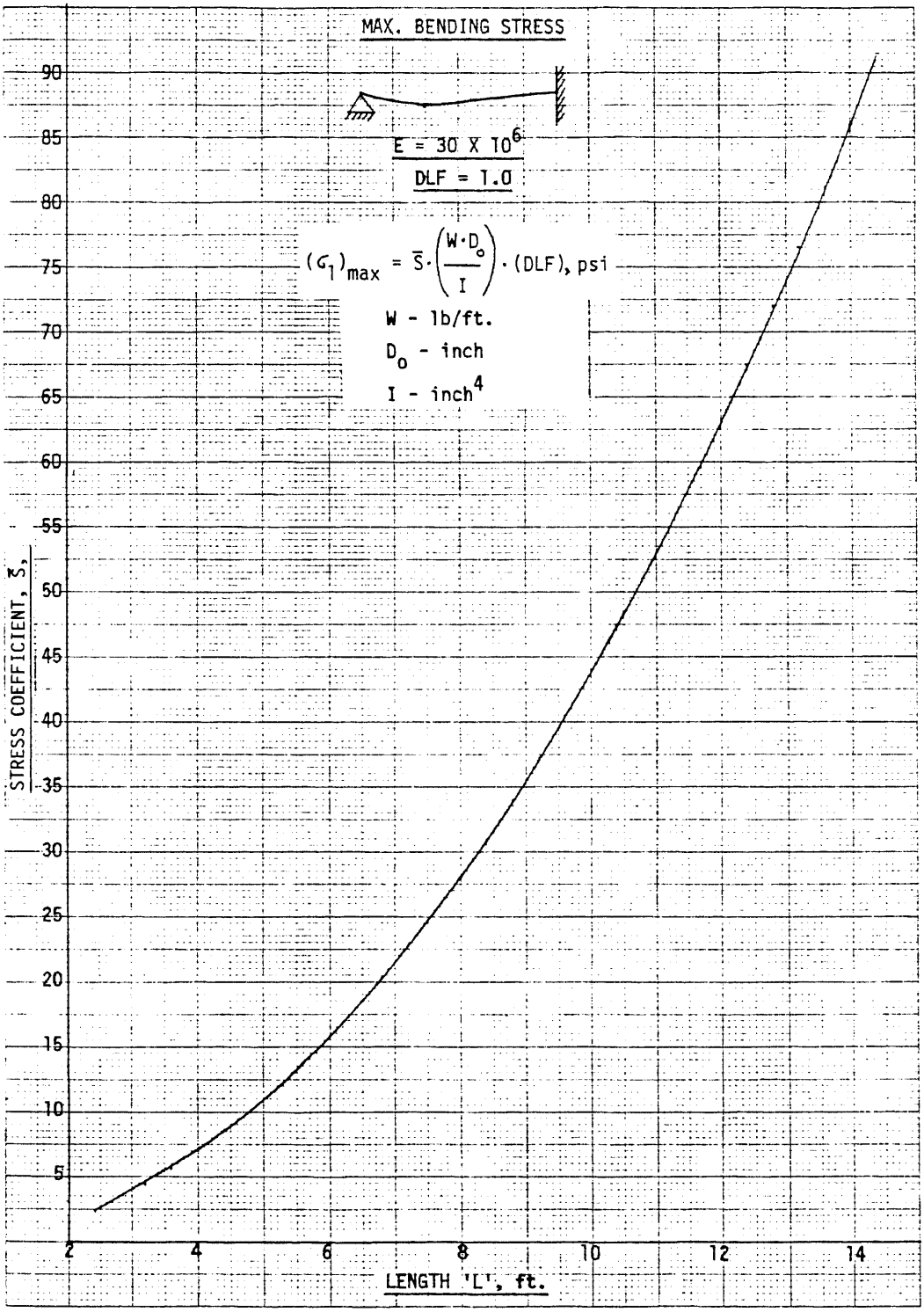


Fig. 11 Stress Coefficient, $\bar{S}$, for Simple-Fixed Pipe

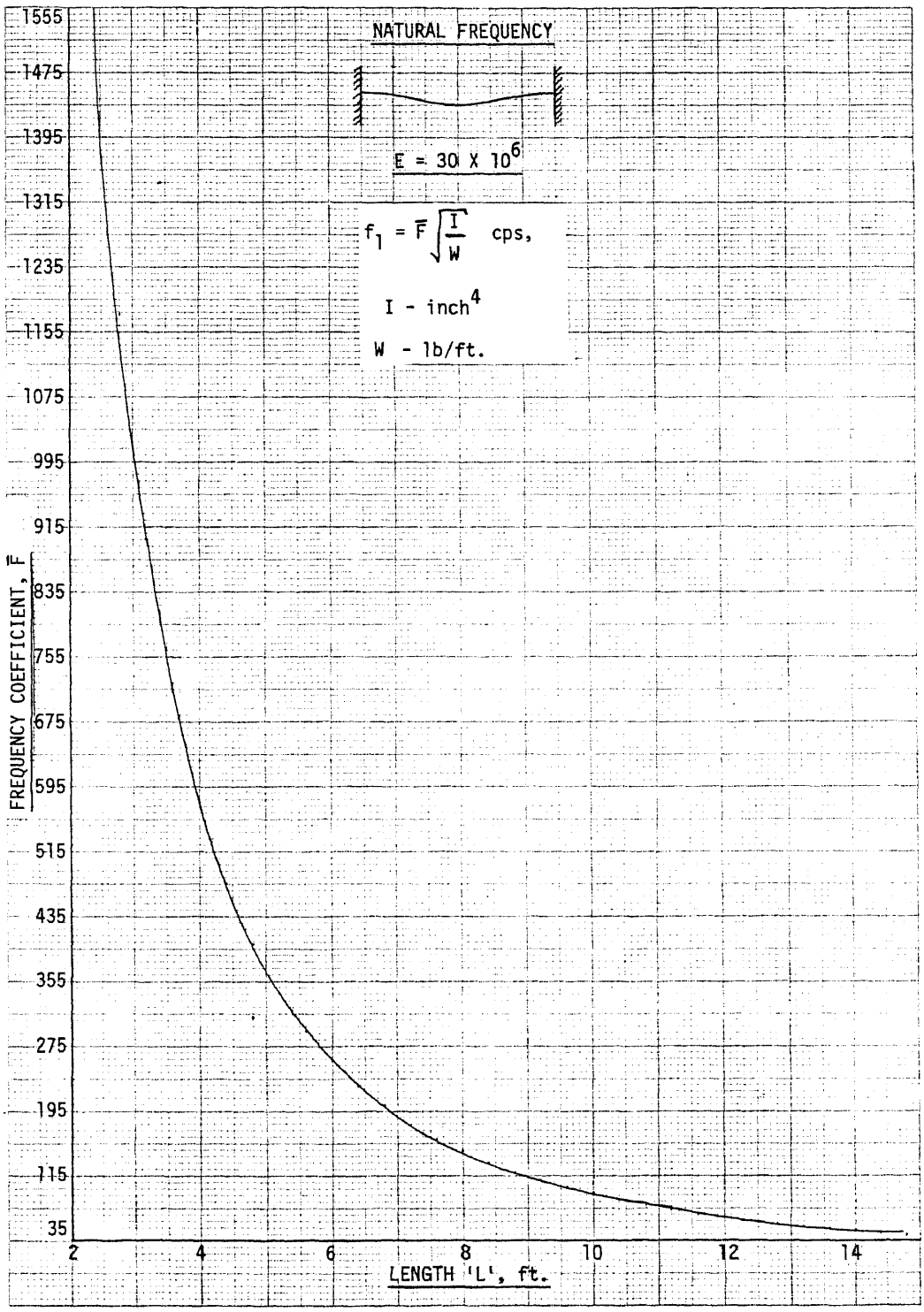


Fig. 12 Frequency Coefficient, $\bar{F}$, for Fixed-Fixed Pipe

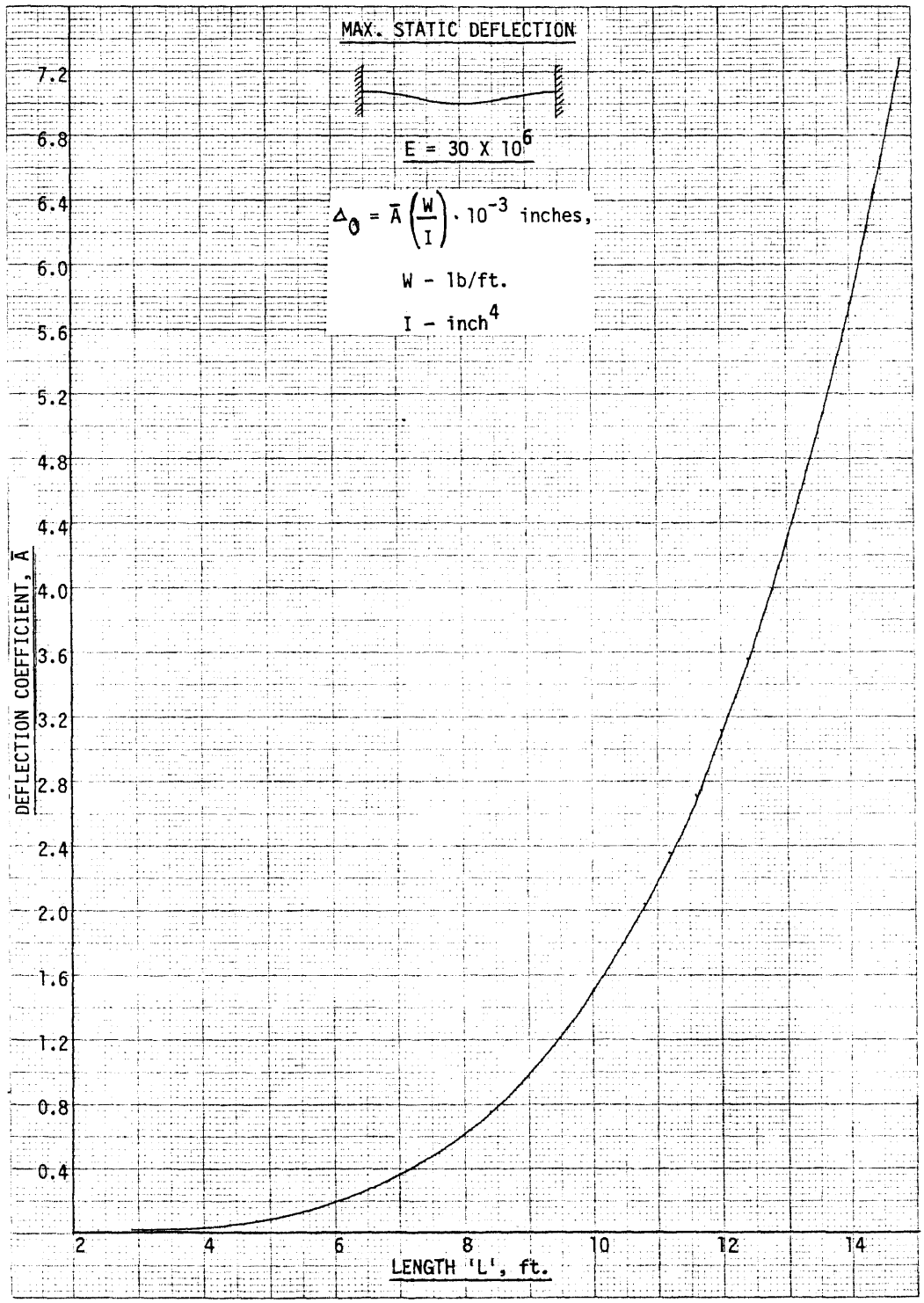


Fig. 13 Deflection Coefficient, $\bar{A}$, for Fixed-Fixed Pipe

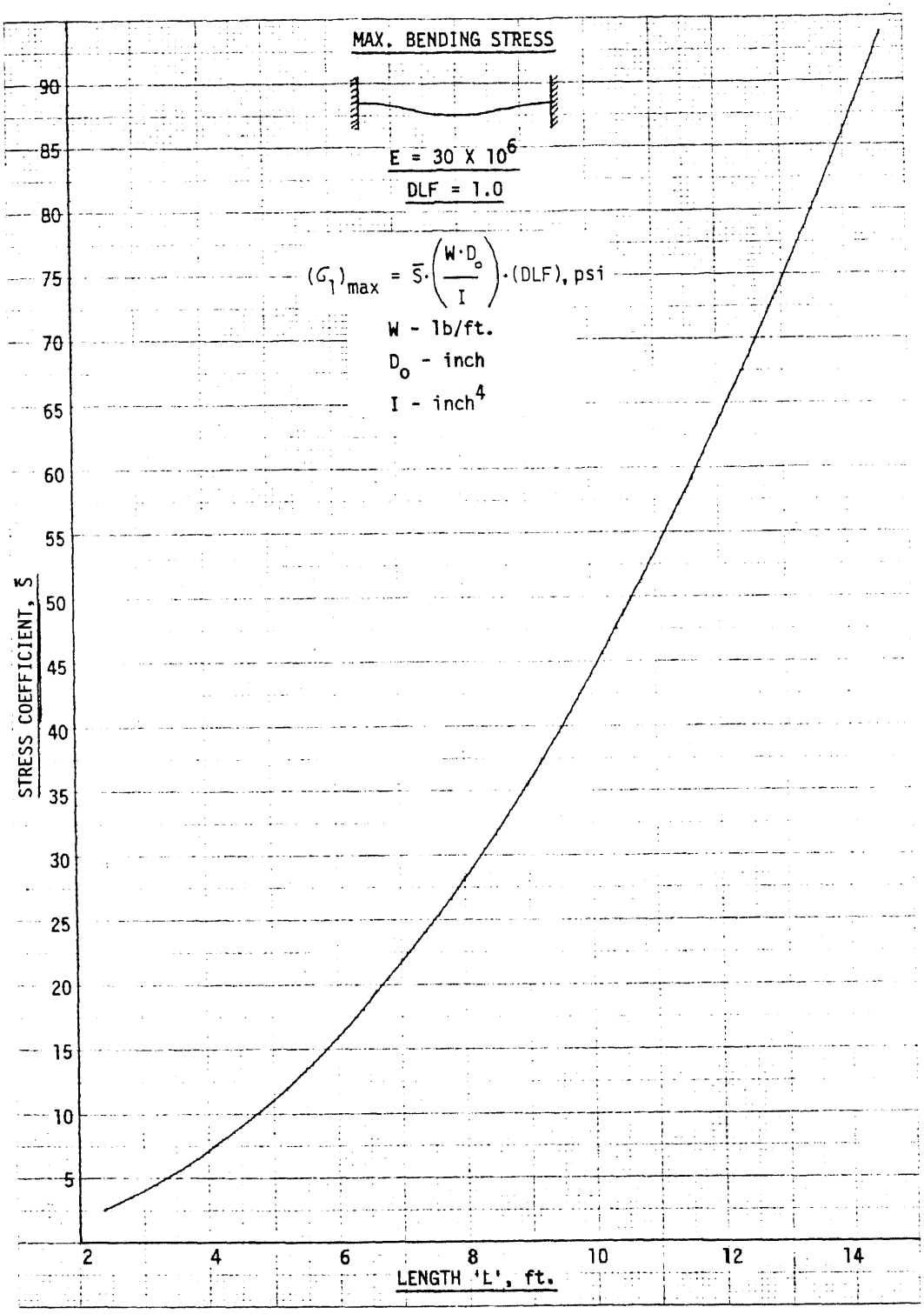


Fig. 14 Stress Coefficient, $\bar{S}$, for Fixed-Fixed Pipe

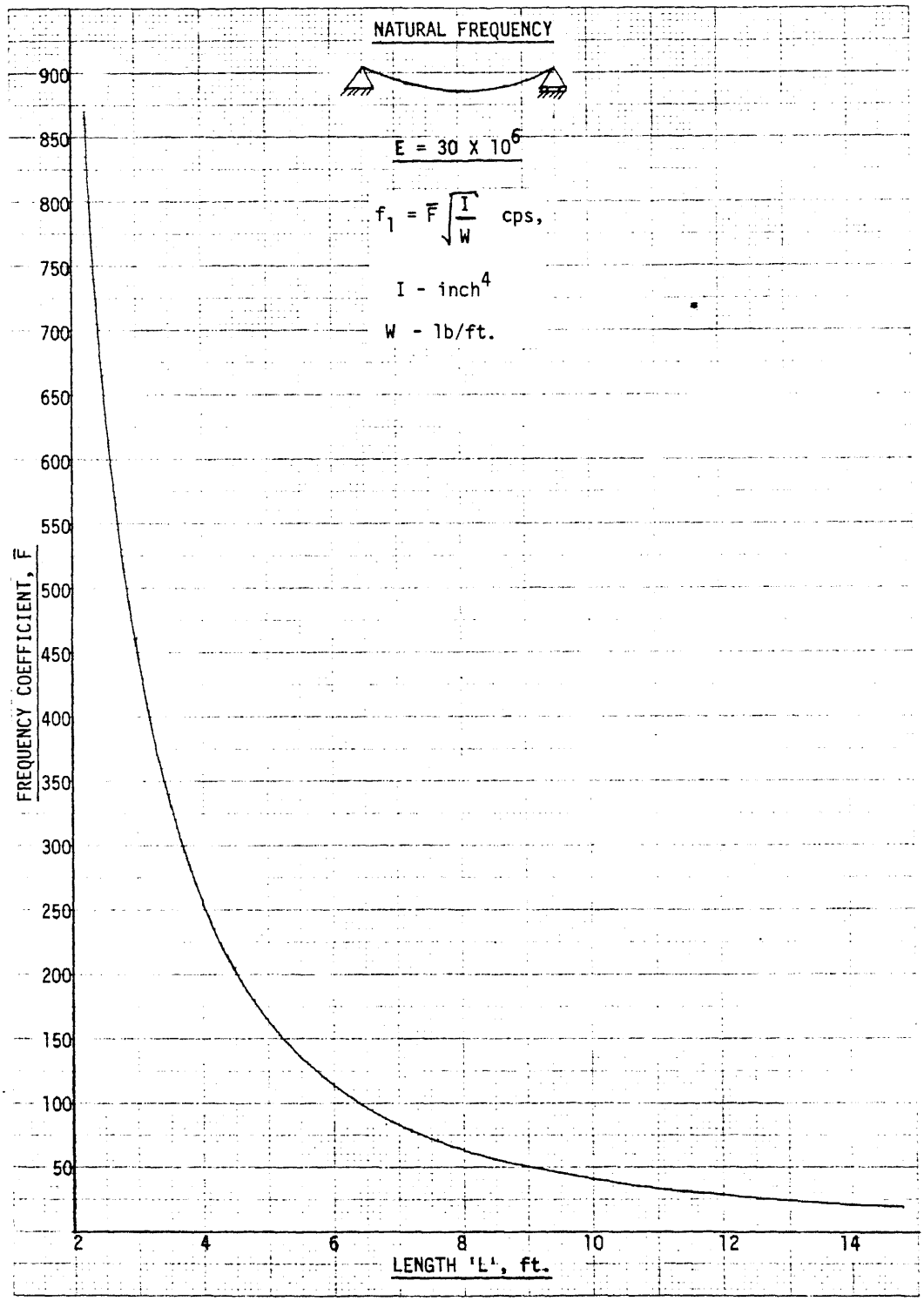


Fig. 15 Frequency Coefficient, $\bar{F}$, for Simple-Simple Pipe

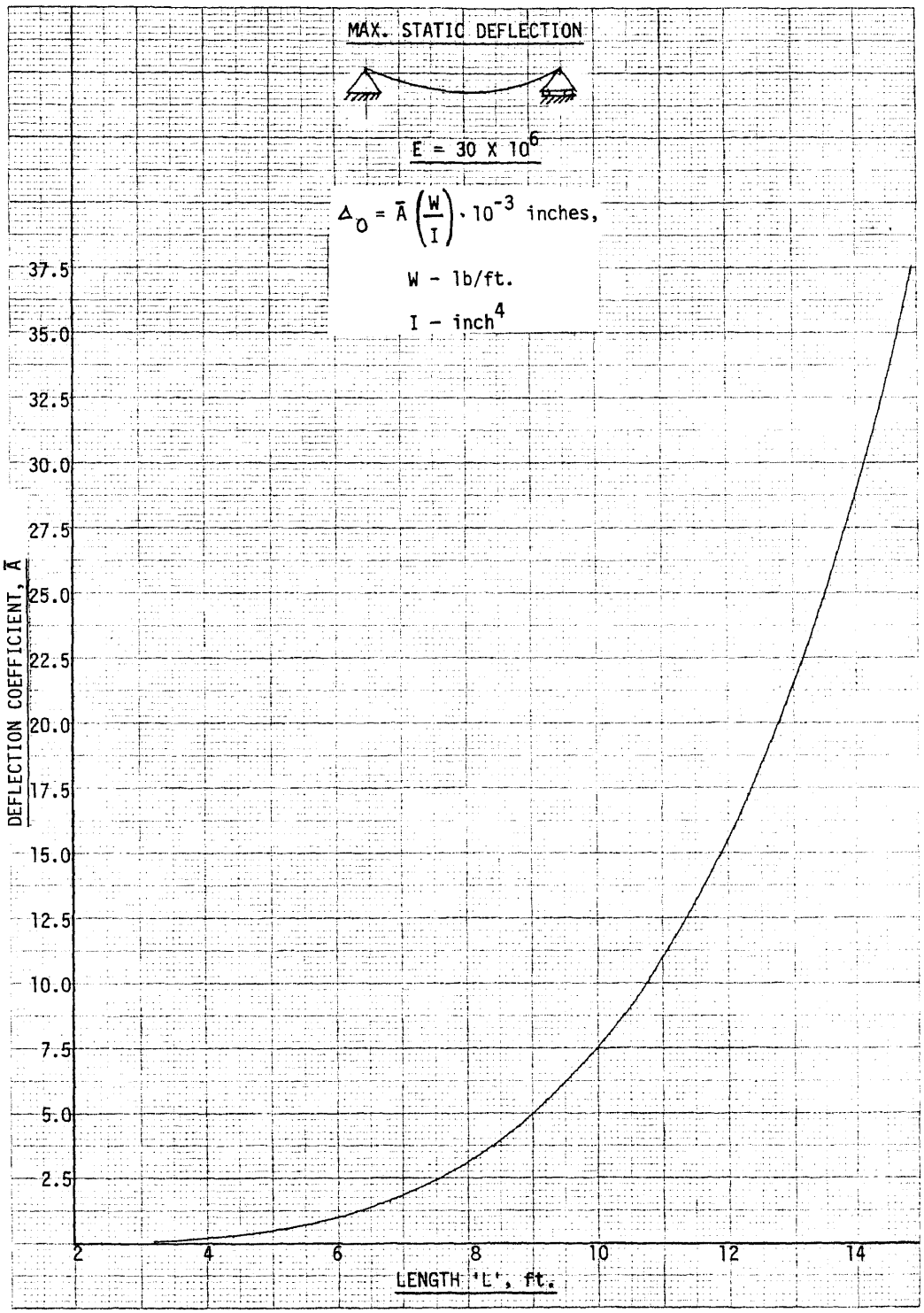


Fig. 16 Deflection Coefficient, $\bar{A}$, for Simple-Simple Pipe

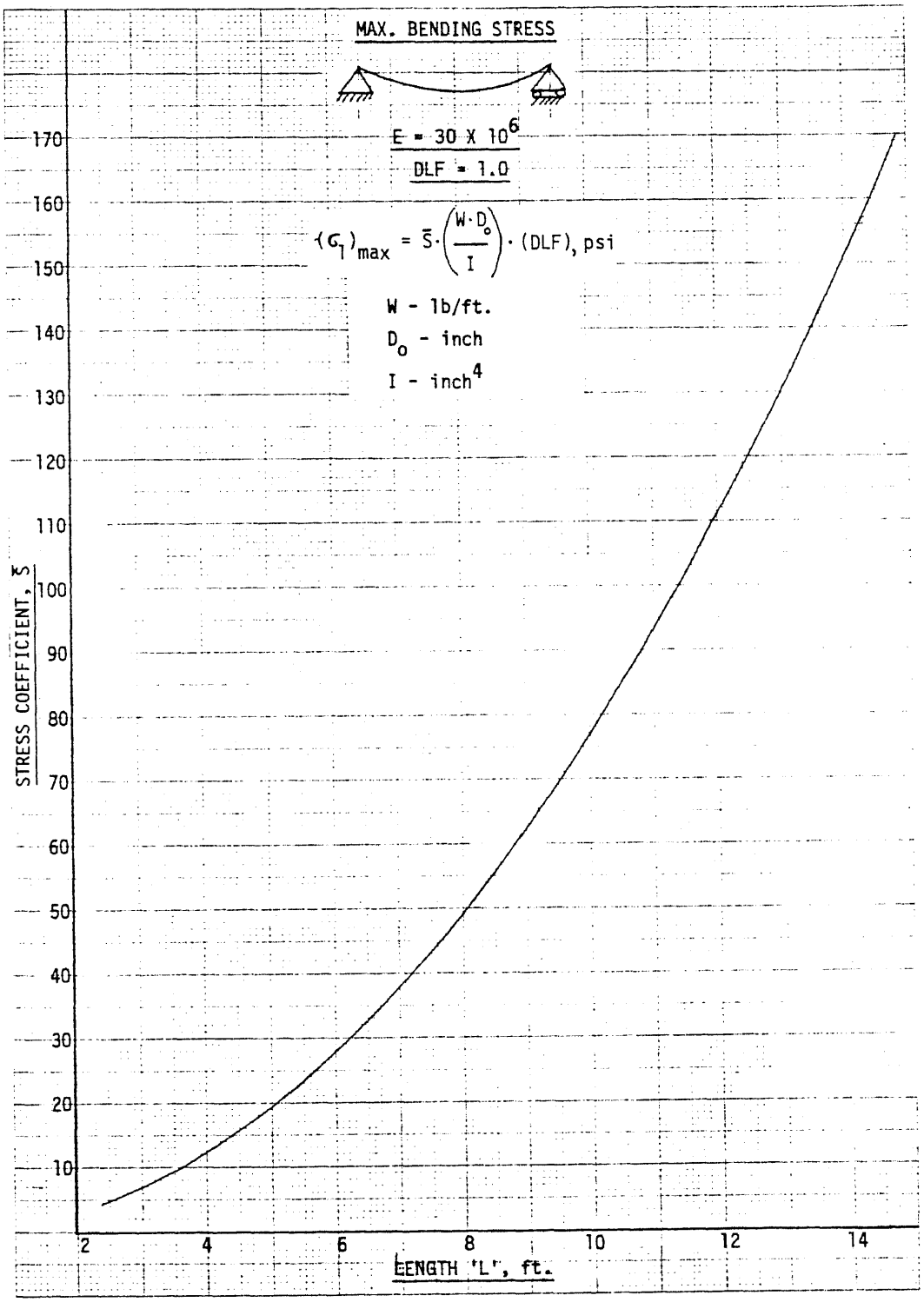


Fig. 17 Stress Coefficient, $\bar{S}$, for Simple-Simple Pipe

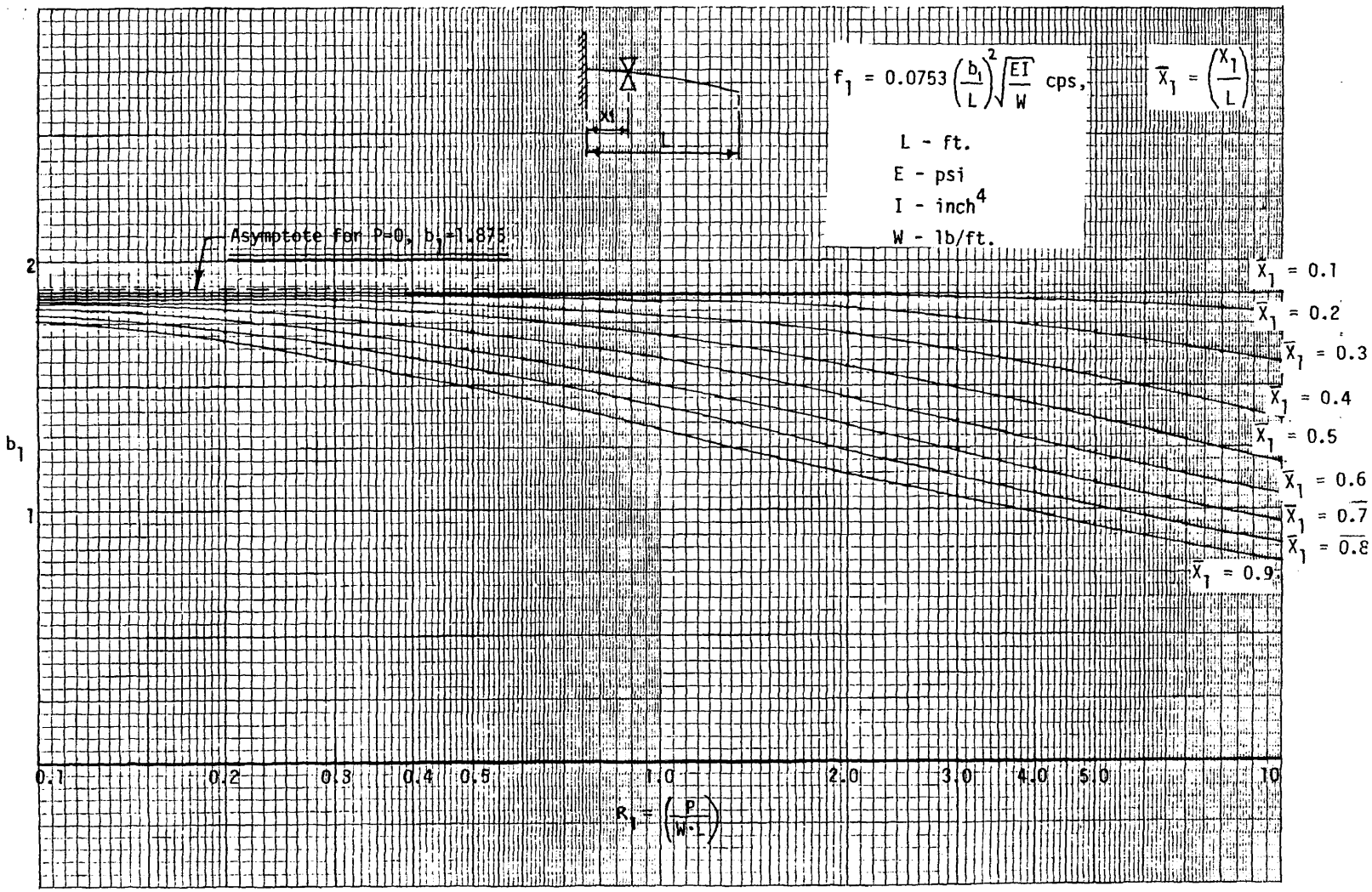


Fig. 18 Frequency Factor, b_1, for Cantilever Pipe with Concentrated Load

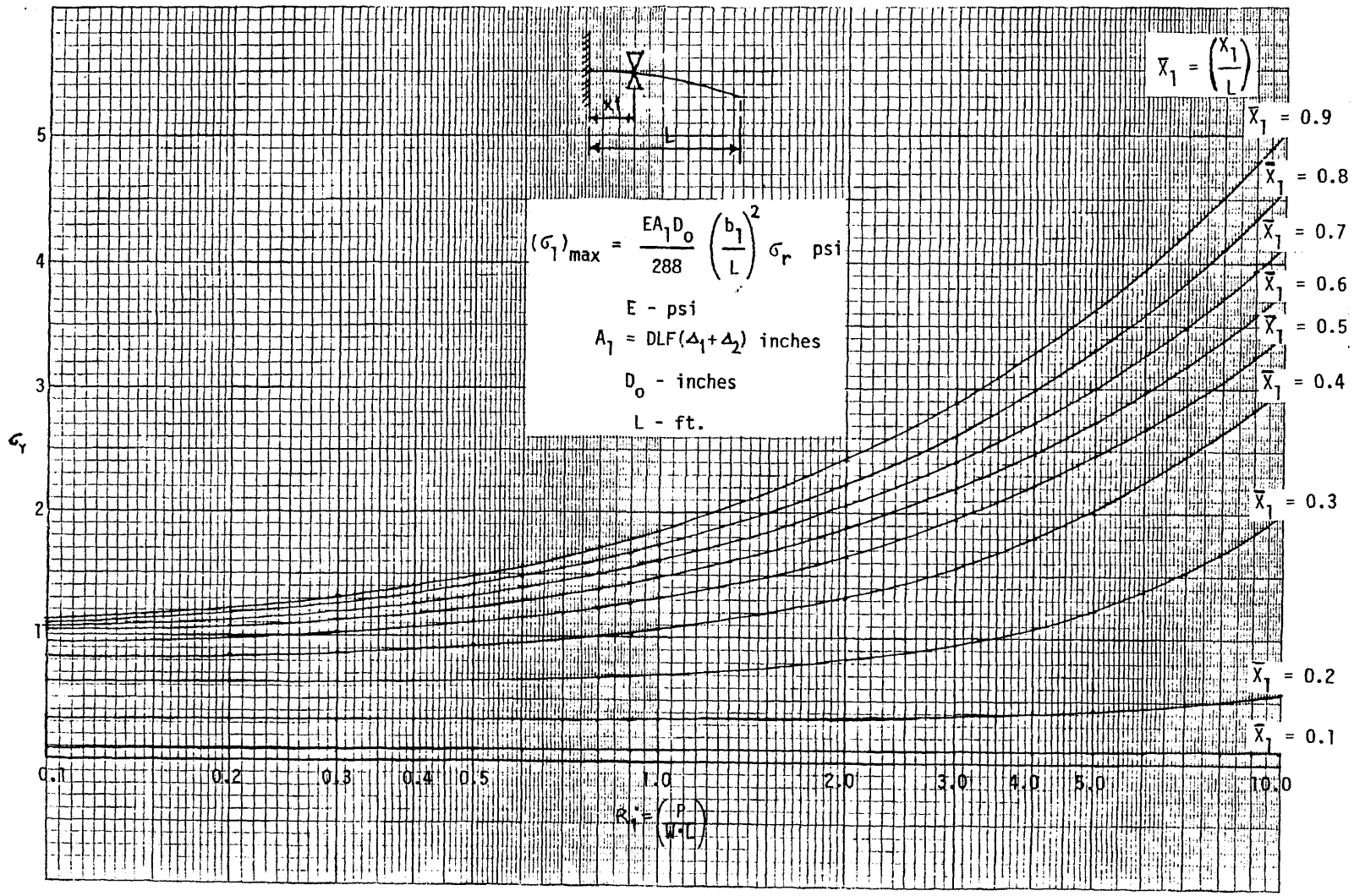


Fig. 19 Stress Ratio, σ_r, for Cantilever Pipe with Concentrated Load

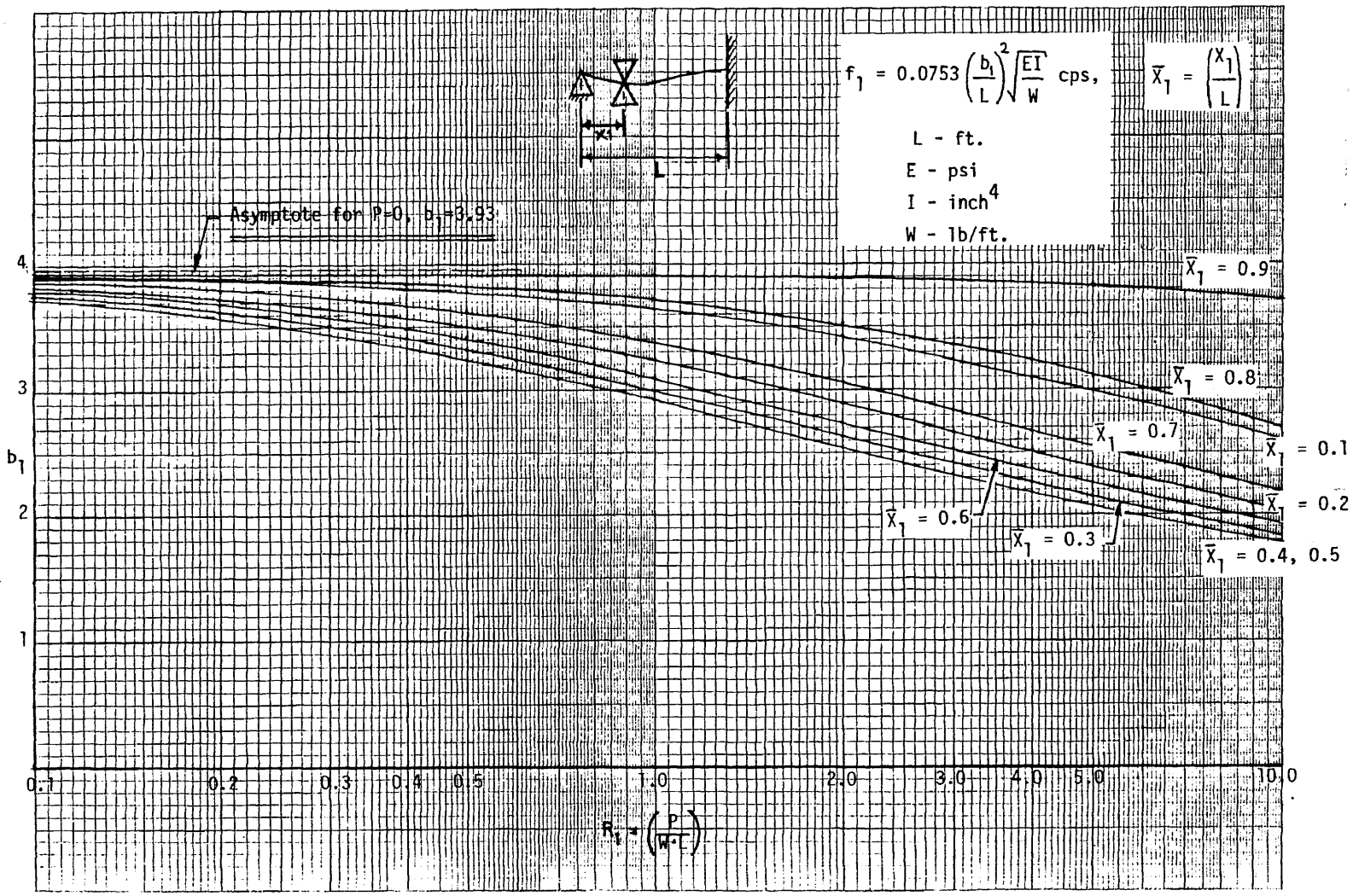


Fig. 20 Frequency Factor, b_1, for Simple-Fixed Pipe with Concentrated Load

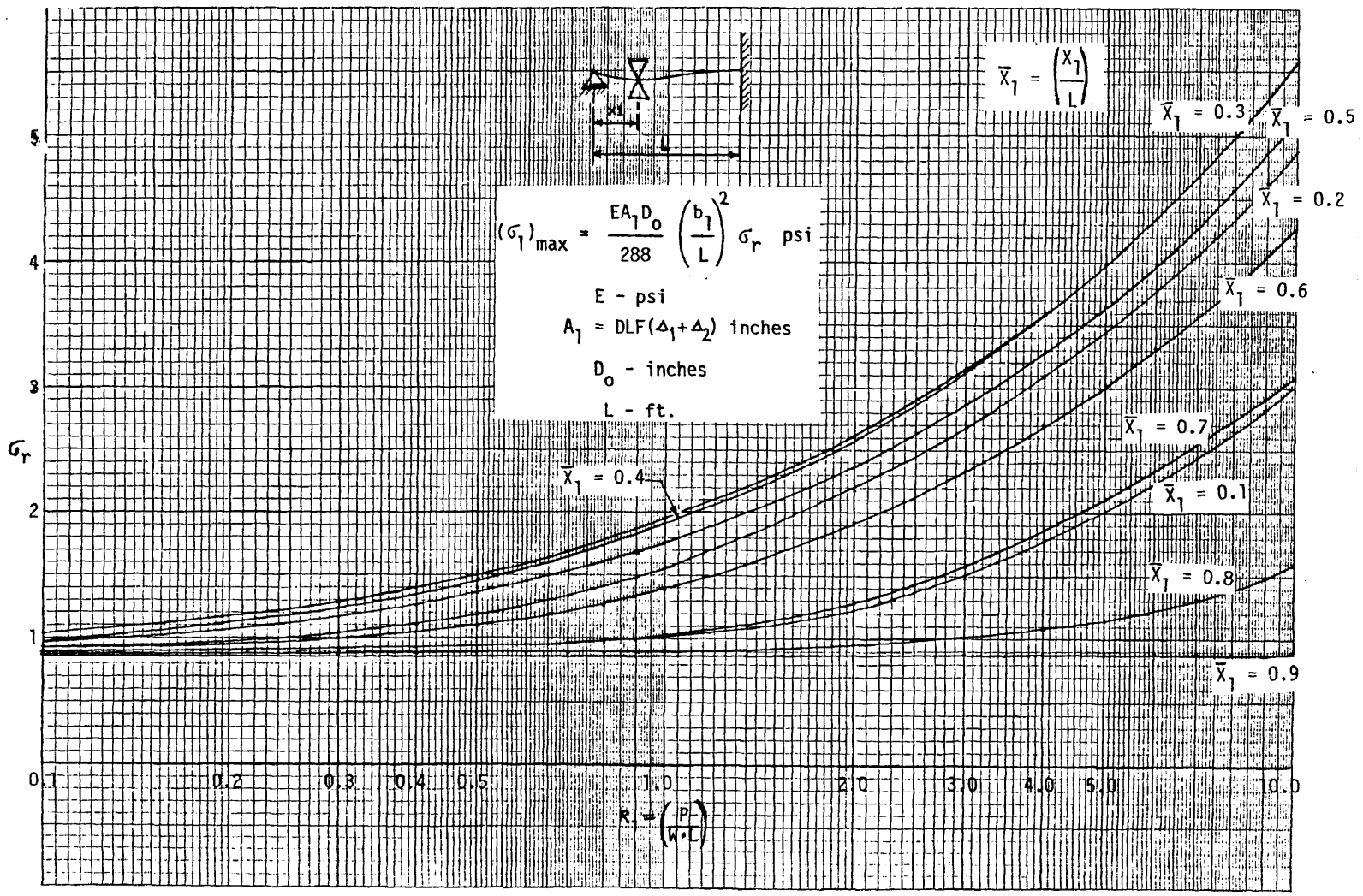


Fig. 21 Stress Ratio, σ_r, for Simple-Fixed Pipe with Concentrated Load

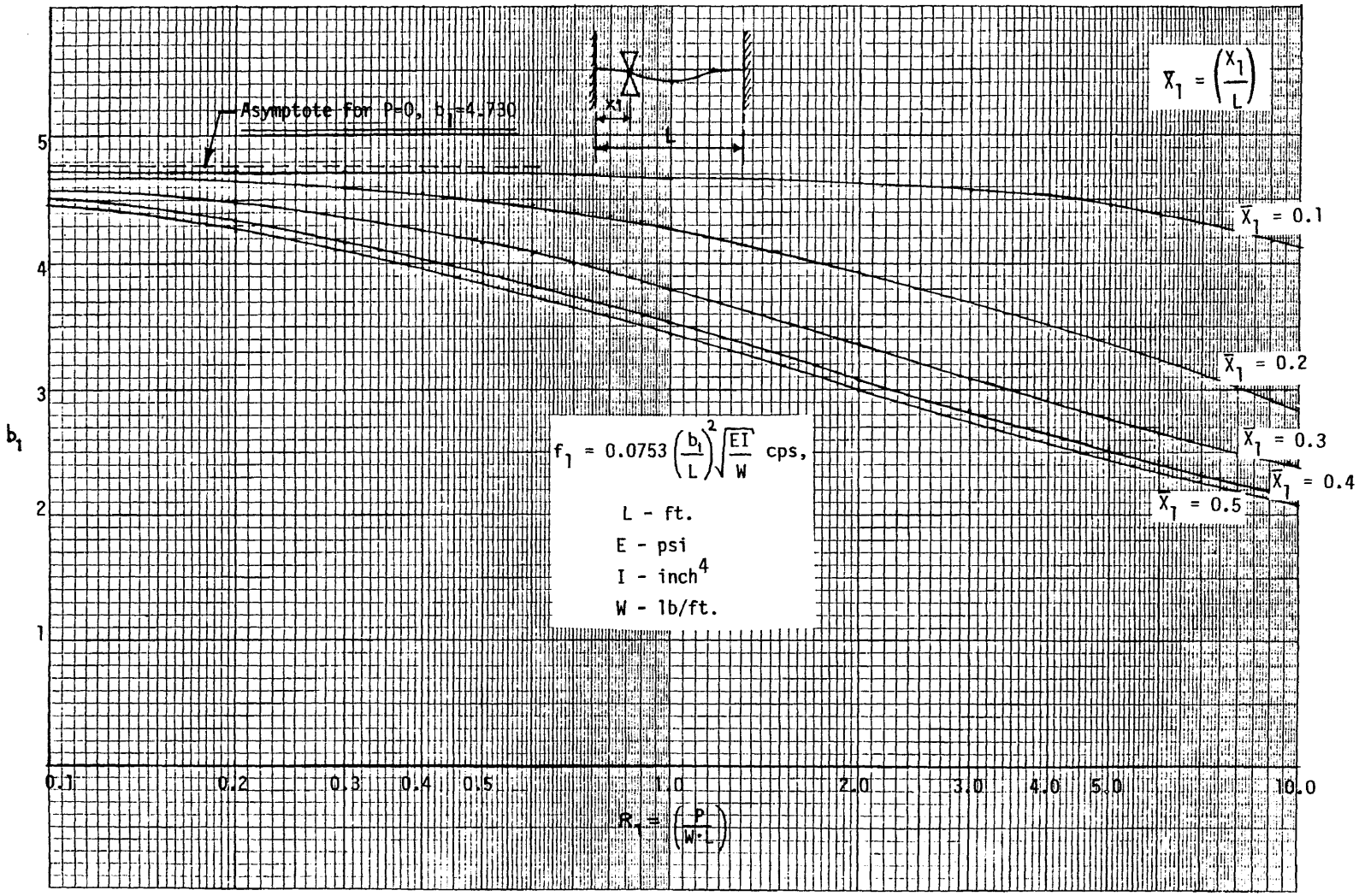


Fig. 22 Frequency Factor, b_1, for Fixed-Fixed Pipe with Concentrated Load

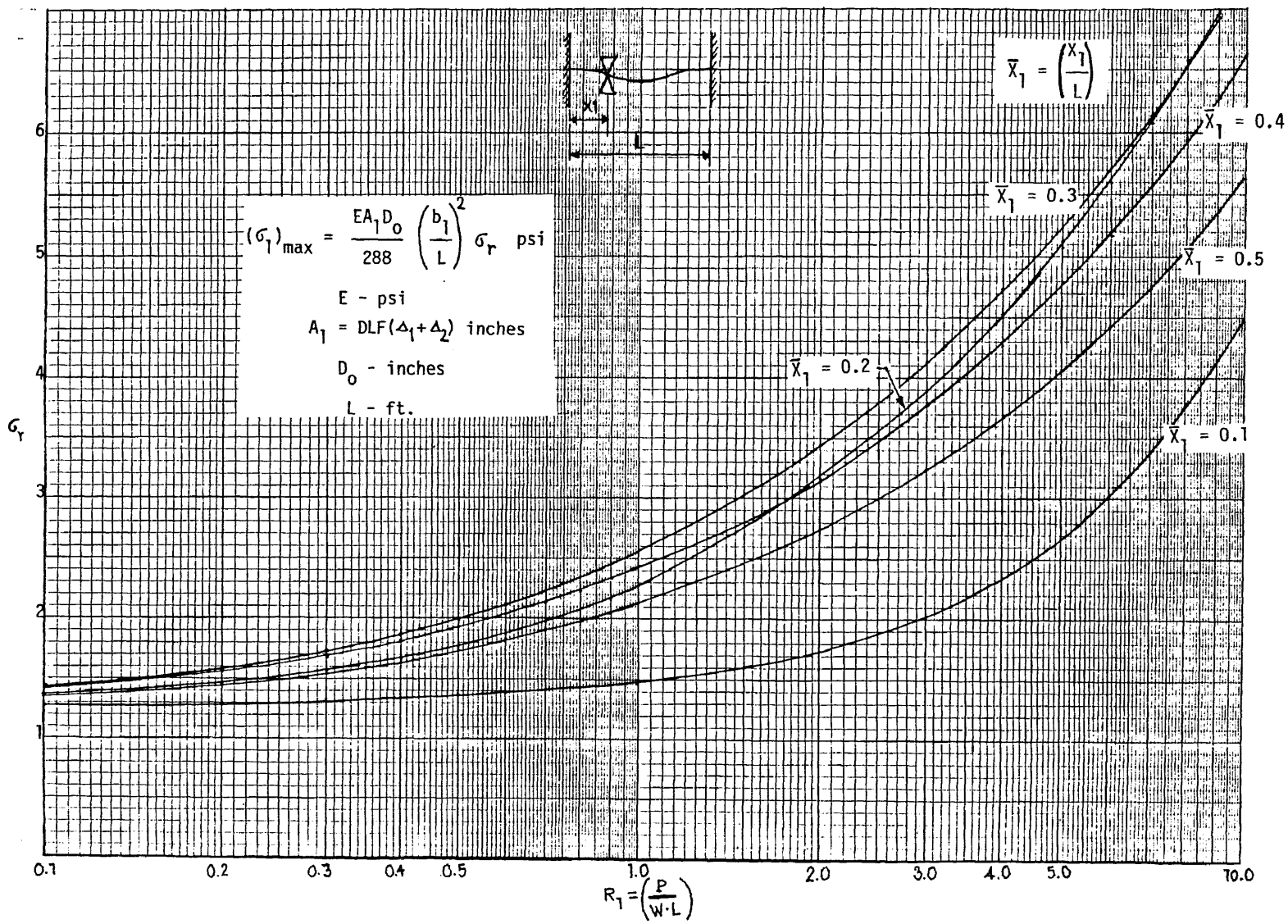


Fig. 23 Stress Ratio, σ_r, for Fixed-Fixed Pipe with Concentrated Load

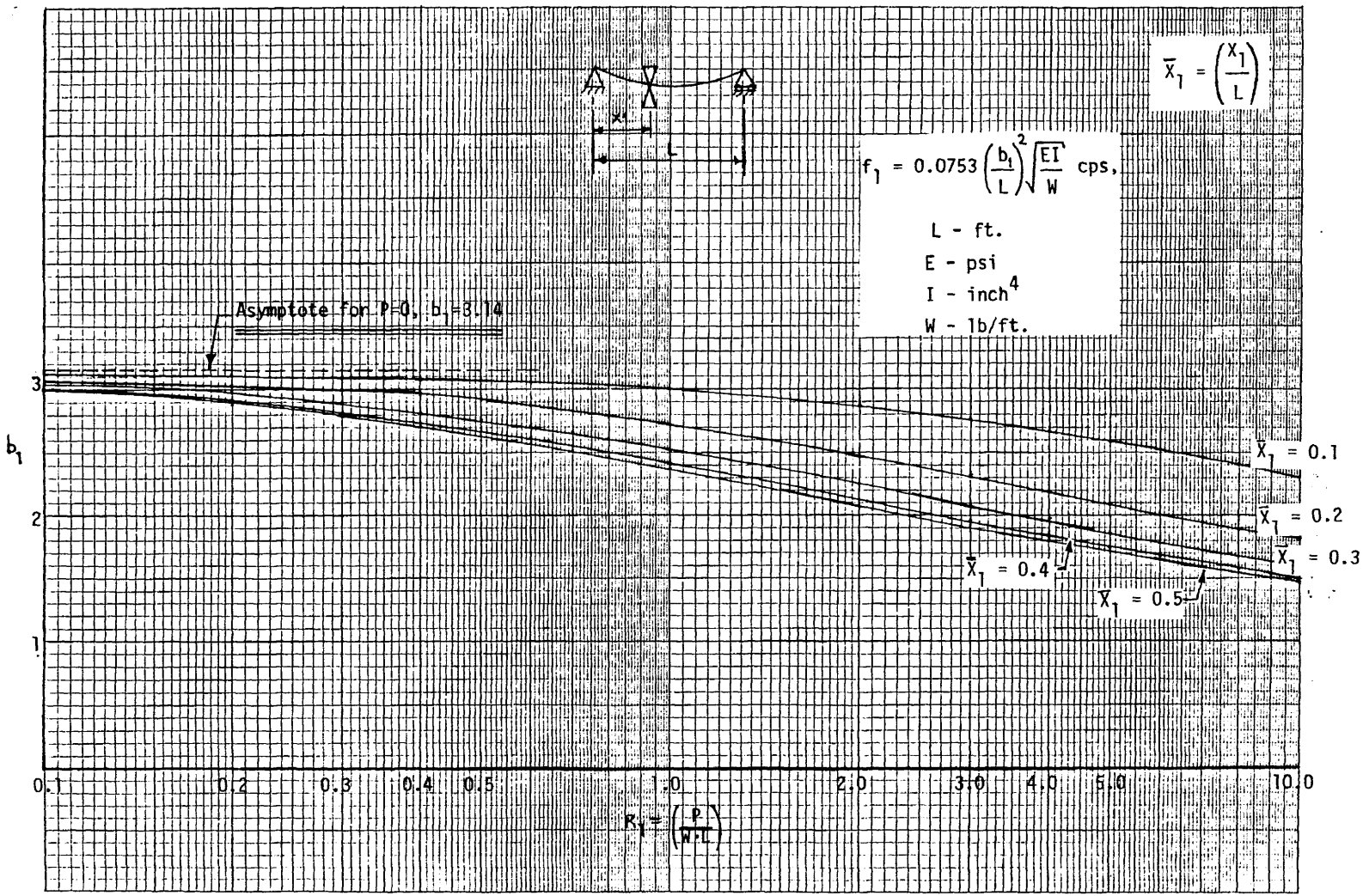


Fig. 24 Frequency Factor, b_1, for Simple-Simple Pipe with Concentrated Load

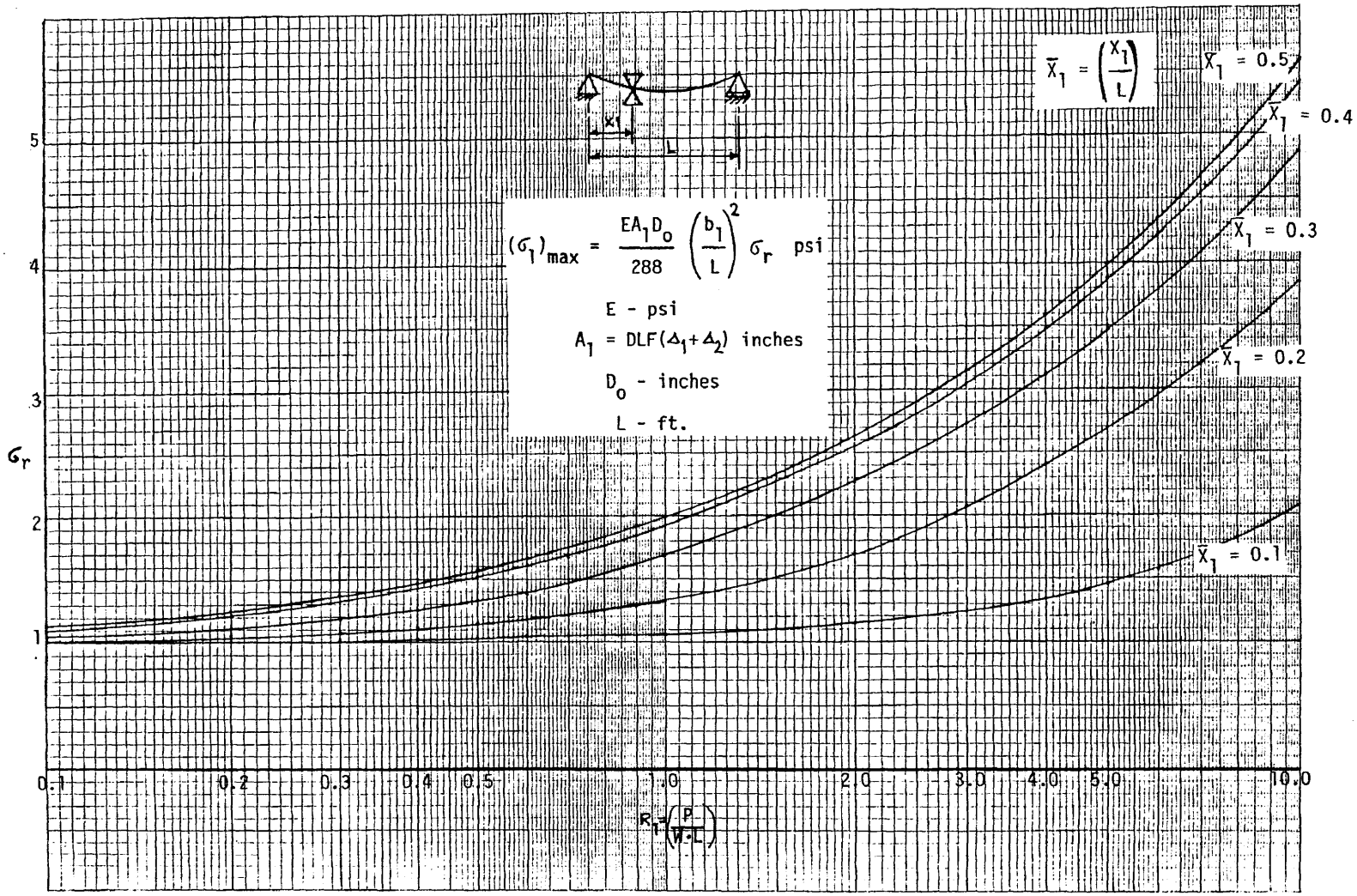


Fig. 25 Stress Ratio, σ_r, for Simple-Simple Pipe with Concentrated Load

CHAPTER VI

COMPUTATION AND DISCUSSION OF RESULTS

Computation of Results using Graphs

The procedure for using the graphs is as follows:

1) Uniform Spans: All graphs based upon $E = 30 \times 10^6$, psi

   a) Frequency: $f_1 = \bar{F}\sqrt{\frac{I}{W}}$, cps

   where $\bar{F}$ is frequency coefficient read from the graphs

   For cases with different 'E_1', multiply by $\sqrt{\frac{E_1}{E}}$, where

   E_1 is the new Modulus of Elasticity

   b) Maximum Static Deflection: $\Delta_0 = \bar{A} \times \left(\frac{W}{I}\right) \times 10^{-3}$ inches

   where $\bar{A}$ is the deflection coefficient read from the graphs.

   For cases with different 'E_1', multiply by $\frac{E}{E_1}$, where

   E_1 is the new Modulus of Elasticity.

   c) Amplitude: $A_1 = (DLF) \cdot \Delta_0$, inches

   d) Maximum Bending Stress: $(\sigma_1)_{max} = \bar{S} \times \left(\frac{WD_0}{I}\right)(DLF)$, psi

   where $\bar{S}$ is the stress coefficient read from the graphs.

   For cases with different 'E_1', multiply by $\frac{E_1}{E}$, where

   E_1 is the new Modulus of Elasticity.

2) Span with Concentrated Load

   a) Frequency:

   First determine: R_1 and $\bar{x}_1$

   then, from the appropriate graph (depending upon the end

conditions) determine corresponding b_1

thus, $f_1 = 0.0753 \times \left(\frac{b_1}{L}\right)^2 \sqrt{\frac{EI}{W}}$, cps

b) Maximum Static Deflection: Use equations for Δ_1 and Δ_2, as listed in Appendix - F.

c) Amplitude: $A_1 = (DLF) \cdot (\Delta_1 + \Delta_2)$, inches

d) Maximum Bending Stress:

Find σ_r, from appropriate graph (p.37 - p.44), corresponding to the ratios R_1 and $\bar{x}_1$

then, $(\sigma_1)_{max} = \frac{EA_1D_o}{288} \times \left(\frac{b_1}{L}\right)^2 \times \sigma_r$, psi

Illustrative Examples

1) Uniform Spans:

Consider a pipe span having the following:

Nominal pipe diameter: 1",

Schedule Number: 40,

Pipe Length: 4.9',

End Conditions: Simple-Fixed (Case # 2),

Dynamic Loading Factor: 1,

Now, using Ref. 9 , we obtain the following:

Outside diameter: 1.315"

Moment of Inertia, I : 0.0874 in^4

Wt. per unit length of empty pipe, W : 1.68 lb/ft.

Now, using Fig. 9, the frequency coefficient, F, corresponding to a length of 4.9' is $\bar{F} = 264$

thus, $f_1 = \bar{F}\sqrt{\frac{I}{W}} = 264\sqrt{\frac{0.0874}{1.68}} = 60.2$ cps.

and, the deflection coefficient, $\bar{A}$, (using Fig. 10, length = 4.9'), is $\bar{A} = 2$

thus, $\Delta_0 = \bar{A} \frac{W}{I} 10^{-3}$ inches,

$$= 2 \frac{1.68}{0.0874} 10^{-3} = 0.0038 \text{ inches}$$

thus, $A_1 = DLF \quad \Delta_0 = (1)(0.0038) = 0.0038$ inches

Finally, the stress coefficient, $\bar{S}$, (using Fig. 11, length = 4.9'), is $\bar{S} = 10.125$

thus, $$(\sigma_1)_{max} = \bar{S}\left(\frac{W}{I}\right) D_o \cdot DLF$$

$$= 10.125 \cdot \left(\frac{1.68}{0.0874}\right) \cdot 1.315 = 255.9 \text{ psi}$$

For purposes of comparison, the interactive program is run with the same data, and the results are as follows:

Quantity	Graph result	Program result
Frequency	60.2, cps	60.4, cps
Amplitude	0.0038, in	0.0035, in
Maximum bending stress	255.9, psi	266.5, psi

2) Span with Concentrated Load

Consider the same pipe span with the following modifications:

Weight of valve, P: 20 lbs (standard flanged bonnet gate valve, ref. 9, p. 178)

Valve location, x_1 : 1.5',

End Conditions: Simple-Simple (Case # 4)

Now, $R_1 = \frac{P}{WL} = \frac{20}{(1.68)(4.9)} = 2.43$

and, $\bar{x}_1 = \frac{x_1}{L} = \frac{1.5}{4.9} = 0.306$

Now, using Fig. 24, the frequency factor, b_1, is found as $b_1 = 2.17$; and thus,

frequency, $f_1 = 0.0753 \left(\frac{b_1}{L}\right)^2 \sqrt{\frac{EI}{W}}$, cps

i.e. $f_1 = 0.0753 \left(\frac{2.17}{4.9}\right)^2 \sqrt{\frac{30 \times 10^6 \times 0.0874}{1.68}} = 18.45$ cps

Also, using Appendix - F, the maximum static deflection is calculated as:

$$\Delta_1 = \frac{(12)^3 \cdot P x_1 (L^2 - x_1^2)^{1.5}}{9\sqrt{3}\, EIL} = \frac{(12)^3 \times 20 \times 1.5\, (4.9^2 - 1.5^2)^{1.5}}{9\sqrt{3} \cdot 30 \times 10^6 \times 4.9 \times 0.0874}$$

$$= 0.02627 \text{ inches}$$

and, $x_{max} = L - \sqrt{\frac{L^2 - x_1^2}{3}} = 4.9 - \sqrt{\frac{4.9^2 - 1.5^2}{3}} = 2.21$ ft.

and, $\Delta_2 = \frac{W x_{max}}{24EI} \left\{ L^3 - 2L x_{max}^2 + x_{max}^3 \right\} \times (12)^3 = 0.0082$ inches.

Thus, $A_1 = (DLF) \quad (\Delta_1 + \Delta_2)$, inches

i.e. $A_1 = 0.03447$ inches

Finally, using Fig.25, the maximum stress ratio, σ_r is $\sigma_r = 2.48$

$$(\sigma_1)_{max} = \frac{E A_1 D_o}{288} \cdot \left(\frac{b_1}{L}\right)^2 \cdot \sigma_r \quad \text{psi}$$

$$= \frac{30\times10^6\times0.03447\times1.315}{288}\cdot\left(\frac{2.17}{4.9}\right)^2\cdot 2.48$$

i.e. $(\sigma_1)_{max} = 2296.5$ psi

Again, the results from the interactive program and the graphs are compared as follows:

Quantity	Graph result	Program result
Frequency	18.45 cps	18.23 cps
Amplitude	0.0345 in	0.0345 in
Maximum bending stress	2296.5 psi	2275.5 psi

Verification of Results:

The results for fundamental frequencies for uniform spans are compared with those given in Ref. 10, p. 262. For purposes of comparison, let L = 3.0 ft., E = 30 10^6 psi, I = 0.0874 in^4 and W = 1.68 lb/ft. (Nominal pipe: 1", Sch. 40, empty pipe).

Fundamental frequency comparison, (cps)

Case	Calculated result	Result from Kellogg [10]
Cantilever	36.85	36.78
Simple-Fixed	160.5	161.0
Fixed-Fixed	231.5	234.5
Simple-Simple	102.6	103.1

Thus, it can be seen that the results for frequency of uniform spans are very close to that of the quoted reference.

The results for the fundamental frequencies for spans with concentrated load can be compared with those given in Ref. 11. However, Ref. 11 provides values for particular cases only (Simple-Simple and Fixed-Fixed, $\bar{x}_1 = 0.5$). On comparison, it can be observed that the results agree very closely. Another check for the results is the asymptotic value of b_1, i.e. the value when $R_1 = 0$. These values are obtained from equations for uniform spans. Also, since the method applied is an exact method, the results are authentic.

The calculation of maximum bending stress is very much dependent upon the calculation of the eigen value. The eigen value is used in determining the characteristic shape and then the characteristic stress. Since the eigen values are verified, the characteristic shape and characteristic stress are also verified. However, the algorithm used to determine the maxima of the characteristic shape and the characteristic stress function is found to calculate only one maxima even in the cases where more than one may be present. For instance, in the case of the Simple-Fixed end conditions, the maximum bending stress is calculated based upon the first maxima obtained while numerically comparing the characteristic stress function from the left hand support. It can be observed that this maxima happens to be the stress corresponding to the maximum positive bending moment, whereas there is a possibility of the negative bending moment at the fixed support being numerically greater than this positive bending moment. Thus, in this case only maximum stress in the pipe is found, excluding the stress at the fixed support.

CHAPTER VII

CONCLUSION

The graphs and computer programs presented in this thesis can be used as good vibration analysis tools. The natural frequencies estimated can be used as a guide to avoid resonance and subsequent fatigue damage. The maximum calculated alternating stress intensity due to repetitive vibrations should be limited for ASME Class 1 piping systems as follows:

$$C_2 \, K_2 \cdot \left(\frac{M}{Z}\right) \leqslant 0.8 \cdot S_{el}$$

in which C_2 and K_2 are the secondary and local stress indices, respectively, as specified in the ASME Boiler and Pressure Vessel Code, Section III, M is the maximum dynamic moment due to vibration only or in combination with other loads as required by the system design specification, and S_{el} is the endurance limit of the material. In this thesis, the maximum stress due to vibrations has been determined. Thus $\left(\frac{M}{Z}\right)$ in this equation can be replaced by $(\sigma_1)_{max}$ or a combination of $(\sigma_1)_{max}$ and the other stresses as appropriate, and the system can be checked for the maximum alternating stress criterion.

CHAPTER VIII

APPENDIX - A

LISTING OF FORTRAN PROGRAM FOR UNIFORM SPANS

```
C
C     INTERACTIVE PROGRAM FOR ANALYSIS OF PIPING VIBRATIONS
C     PROGRAM # 1 - UNIFORM SPANS
C
      COMMON K
      DIMENSION M3(20)
      DATA IN/1/,IO/2/
C
 1    WRITE (IO,2)
 2    FORMAT(5X,'ENTER THE DATA: D1,D2,D3,D4,D5,D6,D7,D8',/)
      WRITE (IO,3)
 3    FORMAT(10X,'D1 - NOMINAL PIPE DIAMETER, INCHES',
     %/,10X,'D2 - SCHEDULE NO.',
     %/,10X,'D3 - OUTSIDE DIAMETER, INCHES',
     %/,10X,'D4 - LENGTH OF THE PIPE, FEET' )
      WRITE(IO,4)
 4    FORMAT(10X,'D5 - MOMENT OF INERTIA, (INCHES)**4',
     %/,10X,'D6 - YOUNG"S MODULUS, (LBS/SQUARE INCHES)',
     %/,10X,'D7 - WEIGHT PER UNIT LENGTH, (LBS/FT.)',
     %/,10X,'D8 - DYNAMIC LOADING FACTOR, (NON-DIMENSIONAL)',/)
C
      READ(IN,*) PN, NS, OD, XL, XI, E, P, DF
C
 5    WRITE (IO,6)
 6    FORMAT(/,5X,'ENTER THE CASE NUMBER',/)
      WRITE(IO,7)
 7    FORMAT(10X,'CANTILEVER, ENTER 1',/
     %10X,'SIMPLE-FIXED, ENTER 2',/
     %10X,'FIXED-FIXED, ENTER 3',/
     %10X,'SIMPLY SUPPORTED, ENTER 4',/)
C
      READ (IN,*)K
C
      GO TO (8,10,12,14) ,K
 0    GO TO 58
C
 8    WRITE (IO,9)
 9    FORMAT(6X,'*** UNIFORM CANTILEVER ***',/)
C
      GO TO 16
C
 10   WRITE (IO,11)
 11   FORMAT(6X,'*** UNIFORM SIMPLE-FIXED ***',/)
C
      GO TO 16
C
 12   WRITE (IO,13)
 13   FORMAT(6X,'*** UNIFORM FIXED-FIXED ***',/)
C
      GO TO 16
```

```
C
 14   WRITE (IO,15)
 15   FORMAT(6X,'*** UNIFORM SIMPLY SUPPORTED ***',/)
C
 16   WRITE (IO,17) PN, NS
 17   FORMAT(5X,' NOMINAL PIPE DIAMETER =',F6.3,5X,' SCHEDULE NO. =',
     %I3,/)
      WRITE (IO,18) DF
 18   FORMAT(6X,'DYNAMIC FACTOR=',F5.2,/)
      WRITE (IO,19)
 19   FORMAT(9X,'LENGTH',5X,'FREQUENCY',4X,'AMPLITUDE',4X,
     %'MAXIMUM STRESS')
      WRITE (IO,20)
 20   FORMAT(9X,'(FEET)',7X,'(CPS)',7X,'(INCHES)',8X,'(PSI)',/)
C
      GO TO (21,22,23,24) ,K
C
 21   A = DF*216.0*P*XL**4/(E*XI)
      GO TO 25
C
 22   A = DF*0.0054*P*XL**4*12.0**3/(E*XI)
      GO TO 25
C
 23   A = DF*P*XL**4*12.0**3/(384.0*E*XI)
      GO TO 25
C
 24   A = DF*5.0*12.0**3*P*XL**4/(384.0*E*XI)
C
 25   CALL LAMDA(XL,XLAMDA)
C
      GO TO (26,27,28,29) ,K
C
 26   AK = (SIN(XLAMDA*XL) + SINH(XLAMDA*XL))/(COS(XLAMDA*XL) +
     %COSH(XLAMDA*XL))
      GO TO 30
C
27    AK = COS(XLAMDA*XL)/COSH(XLAMDA*XL)
      GO TO 30
C
 28   AK = (COS(XLAMDA*XL) -COSH(XLAMDA*XL))/(SIN(XLAMDA*XL) + SINH
     %(XLAMDA*XL))
      GO TO 30
C
 29   AK = 0.0
C
 30   CALL MAXY(XLAMDA,XL,AK,YMAX,XMAX)
      CALL MAXSIG(XLAMDA,XL,AK,SIGMAX,XSMAX)
C
      FR = 0.0753*XLAMDA**2*SQRT(E*XI/P)
      STRESS = (A*E*OD*XLAMDA**2*SIGMAX)/(144.0*2.0*YMAX)
```

```
C
      WRITE (IO,31) XL, FR, A, STRESS
 31   FORMAT(5X,F10.4,4X,F10.5,4X,F8.5,7X,F9.3,/)
      WRITE (IO,32) XMAX
 32   FORMAT(6X,'AMPLITUDE(MAXIMUM DEFLECTION) AT X=',F5.2,1X,'FT.',/)
      WRITE (IO,33) XSMAX
 33   FORMAT(6X,'MAXIMUM STRESS AT X=',F5.2,1X,'FT.',//)
C
      WRITE(IO,34)
 34   FORMAT(6X,'REPEAT CALCULATION?, YES=1, NO=0',//)
C
      READ (IN,*) M
C
      IF(M.EQ.0) GO TO 58
C
      WRITE (IO,35)
 35   FORMAT(/,6X,'SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0',/)
C
      READ (IN,*) M4
C
      IF(M4.NE.0) GO TO 5
C
      WRITE (IO,36)
 36   FORMAT(/,6X,'FRESH DATA OR MODIFICATION?, ENTER 1 OR 0',/)
C
      READ (IN,*) M1
C
      IF(M1.NE.0) GO TO 1
C
      WRITE (IO,37)
 37   FORMAT(6X,'LIST OF DATA WITH SERIAL NUMBERS:',//
     %,8X,'# 1 - NOMINAL DIAMETER \ # 2 - SCHEDULE NO.',/
     %,8X,'# 3 - OUTSIDE DIAMETER \ # 4 - PIPE LENGTH',/
     %,8X,'# 5 - MOM. OF INERTIA  \ # 6 - YOUNG"S MO.',/
     %,8X,'# 7 - WT./LENGTH       \ # 8 - DYNAMIC FACTOR',/)
      WRITE (IO,38)
 38   FORMAT(6X,'ENTER NUMBER OF CHANGES DESIRED',/)
C
      READ (IN,*) M2
C
      IF(M2.LE.0) GO TO 5
C
      WRITE (IO,39)
 39   FORMAT(6X,'ENTER DATA NUMBER(S) WHICH ARE TO BE CHANGED',/)
C
      READ (IN,*) (M3(I1), I1=1,M2)
C
      K1 =1
 40   M5 = M3(K1)
C
```

```
      GO TO (42,44,46,48,50,52,54,56), M5
 41   K1 = K1 + 1
      IF(K1.GT.M2) GO TO 5
      GO TO 40
C
 42    WRITE (IO,43)
 43    FORMAT(/,6X,'ENTER NEW NOMINAL PIPE DIAMETER',/)
       READ (IN,*) PN
      GO TO 41
C
 44    WRITE (IO,45)
 45    FORMAT(/,6X,'ENTER NEW SCHEDULE NO.',/)
       READ (IN,*) NS
      GO TO 41
C
 46    WRITE (IO,47)
 47    FORMAT(/,6X,'ENTER NEW OUTSIDE DIAMETER',/)
       READ (IN,*) OD
      GO TO 41
C
 48    WRITE (IO,49)
 49    FORMAT(/,6X,'ENTER NEW PIPE LENGTH',/)
       READ (IN,*) XL
      GO TO 41
C
 50    WRITE (IO,51)
 51    FORMAT(/,6X,'ENTER NEW MOM. OF INERTIA',/)
       READ (IN,*) XI
      GO TO 41
C
 52    WRITE (IO,53)
 53    FORMAT(/,6X,'ENTER NEW YOUNG"S MODULUS',/)
       READ (IN,*) E
      GO TO 41
 54    WRITE (IO,55)
 55    FORMAT(/,6X,'ENTER NEW WT. PER UNIT LENGTH',/)
       READ (IN,*) P
      GO TO 41
C
 56    WRITE (IO,57)
 57    FORMAT(/,6X,'ENTERNEW DYNAMIC FACTOR',/)
       READ (IN,*) DF
      GO TO 41
C
 58   STOP
      END
```

```
C
C     ////////////////////////////////////////////////////////////
C
      SUBROUTINE LAMDA(XL,XLAMDA)
C
C     SUBROUTINE FOR FINDING ROOT OF EQATION BY BISECTION ALGORITHM
C
C
      EPS = 0.00001
      A=0.00010
 1    B=A+0.005
      IF(FUN(A,XL)*FUN(B,XL).LT.0.0) GO TO 2
      A=B
      GO TO 1
C
 2     DO 4 I=1,15
          P=A+(B-A)/2.0
          IF(FUN(P,XL).EQ.0.0.OR.(B-A).LT.EPS) GO TO 5
          IF((FUN(A,XL)*FUN(P,XL)).GT.0.0) GO TO 3
          B=P
          GO TO 4
 3        A=P
 4     CONTINUE
C
 5    XLAMDA=P
      RETURN
      END
```

```
C
C     ////////////////////////////////////////////////////////////
C
      SUBROUTINE MAXY(XLAMDA,XL,AK,YMAX,X)
C
C     SUBROUTINE TO CALCULATE MAXIMUM DEFLECTION FUNCTION
C
      X=0.0
      DELTA=XL/100.0
C
       DO 1 I=1,100
          XNEW=X + DELTA
          IF(X.GT.XL) GO TO 3
          Y2=Y(XLAMDA,XNEW,AK)
          Y1=Y(XLAMDA,X,AK)
          IF((Y2-Y1).LT.0.0) GO TO 2
          X=XNEW
 1     CONTINUE
C
 2    YMAX=Y1
 3    RETURN
      END
```

```
C
C     ////////////////////////////////////////////////////////////
C
      SUBROUTINE MAXSIG(XLAMDA,XL,AK,SIGMAX,X)
C
C     SUBROUTINE TO CALCULATE MAXIMUM STRESS FUNCTION
C
      X=0.0
      DELTA=XL/100.0
C
       DO 1 I=1,100
          XNEW=X + DELTA
          IF(X.GT.XL) GO TO 3
          SIG2=SIGMA(XLAMDA,XNEW,AK)
          SIG1=SIGMA(XLAMDA,X,AK)
          IF((SIG2-SIG1).LT.0.0) GO TO 2
          X=XNEW
 1     CONTINUE
C
 2    SIGMAX=SIG1
 3    RETURN
      END
```

```
C
C     \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
C
      FUNCTION FUN(X,XL)
      COMMON K
C
      GO TO (1,2,3,4) ,K
C
 1    FUN = COS(X*XL)*COSH(X*XL) + 1.0
      GO TO 5
C
 2    FUN =  SIN(X*XL)*COSH(X*XL) -
     % SINH(X*XL)*COS(X*XL)
      GO TO 5
C
 3    FUN = COS(X*XL)*COSH(X*XL) - 1.0
      GO TO 5
C
 4    FUN = SIN(X*XL)
C
 5    RETURN
      END
```

```
C
C     \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
C
      FUNCTION Y(XLAMDA,X,AK)
      COMMON K
C
      GO TO (1,2,3,4) ,K
C
 1    Y =ABS(   SIN(XLAMDA*X) - SINH(XLAMDA*X) - AK* (COS(XLAMDA*X)
     %- COSH(XLAMDA*X)))
      GO TO 5
C
2     Y = ABS(SIN(XLAMDA*X) - AK*(SINH(XLAMDA*X)))
      GO TO 5
C
 3    Y = ABS( SIN(XLAMDA*X) - SINH(XLAMDA*X) + AK* (COS(XLAMDA*X)
     %- COSH(XLAMDA*X)))
      GO TO 5
C
 4    Y = ABS( SIN(XLAMDA*X) + AK )
C
 5    RETURN
      END
```

```
C
C     \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
C
      FUNCTION SIGMA(XLAMDA,X,AK)
      COMMON K
C
      GO TO (1,2,3,4) ,K
C
 1    SIGMA = ABS(  -SIN(XLAMDA*X) - SINH(XLAMDA*X) +
     %AK * (COS(XLAMDA*X) + COSH(XLAMDA*X)))
      GO TO 5
C
2     SIGMA = ABS(-SIN(XLAMDA*X) - AK*(SINH(XLAMDA*X)))
      GO TO 5
C
 3    SIGMA = ABS( -SIN(XLAMDA*X) - SINH(XLAMDA*X) -
     %AK * (COS(XLAMDA*X) + COSH(XLAMDA*X)))
      GO TO 5
C
 4    SIGMA = ABS( SIN(XLAMDA*X) + AK )
 5    RETURN
C
      END
```

CHAPTER IX

APPENDIX - B

SAMPLE RUN OF FORTRAN PROGRAM FOR UNIFORM SPANS

```
@ru


**FASTFOR (CONVERSATIONAL VER 10)**


     ENTER THE DATA: D1,D2,D3,D4,D5,D6,D7,D8

          D1 - NOMINAL PIPE DIAMETER, INCHES
          D2 - SCHEDULE NO.
          D3 - OUTSIDE DIAMETER, INCHES
          D4 - LENGTH OF THE PIPE, FEET
          D5 - MOMENT OF INERTIA, (INCHES)**4
          D6 - YOUNG"S MODULUS, (LBS/SQUARE INCHES)
          D7 - WEIGHT PER UNIT LENGTH, (LBS/FT.)
          D8 - DYNAMIC LOADING FACTOR, (NON-DIMENSIONAL)

*1.0,40,1.315,4.9,0.0874,30e06,1.68,1.0

     ENTER THE CASE NUMBER

          CANTILEVER, ENTER 1
          SIMPLE-FIXED, ENTER 2
          FIXED-FIXED, ENTER 3
          SIMPLY SUPPORTED, ENTER 4

*1
      *** UNIFORM CANTILEVER ***

      NOMINAL PIPE DIAMETER = 1.000       SCHEDULE NO. = 40

      DYNAMIC FACTOR= 1.00

         LENGTH      FREQUENCY     AMPLITUDE     MAXIMUM STRESS
         (FEET)        (CPS)        (INCHES)         (PSI)

         4.9000      13.77563      0.07978         1622.735

      AMPLITUDE(MAXIMUM DEFLECTION) AT X= 4.90 FT.

      MAXIMUM STRESS AT X= 0.00 FT.


      REPEAT CALCULATION?, YES=1, NO=0


*1
```

```
 SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*1

ENTER THE CASE NUMBER

     CANTILEVER, ENTER 1
     SIMPLE-FIXED, ENTER 2
     FIXED-FIXED, ENTER 3
     SIMPLY SUPPORTED, ENTER 4

*2
 *** UNIFORM SIMPLE-FIXED ***

 NOMINAL PIPE DIAMETER = 1.000      SCHEDULE NO. = 40

 DYNAMIC FACTOR= 1.00

    LENGTH      FREQUENCY     AMPLITUDE     MAXIMUM STRESS
    (FEET)        (CPS)        (INCHES)         (PSI)

    4.9000      60.40805      0.00345          266.597

 AMPLITUDE(MAXIMUM DEFLECTION) AT X= 2.06 FT.

 MAXIMUM STRESS AT X= 1.86 FT.


 REPEAT CALCULATION?, YES=1, NO=0


*1

 SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*1

ENTER THE CASE NUMBER

     CANTILEVER, ENTER 1
     SIMPLE-FIXED, ENTER 2
     FIXED-FIXED, ENTER 3
     SIMPLY SUPPORTED, ENTER 4

*3
```

```
*** UNIFORM FIXED-FIXED ***

NOMINAL PIPE DIAMETER = 1.000      SCHEDULE NO. = 40

DYNAMIC FACTOR= 1.00

   LENGTH      FREQUENCY     AMPLITUDE     MAXIMUM STRESS
   (FEET)        (CPS)        (INCHES)         (PSI)

   4.9000      87.65927      0.00166          267.184

AMPLITUDE(MAXIMUM DEFLECTION) AT X= 2.45 FT.

MAXIMUM STRESS AT X= 0.00 FT.


REPEAT CALCULATION?, YES=1, NO=0


*1


SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*1

ENTER THE CASE NUMBER

    CANTILEVER, ENTER 1
    SIMPLE-FIXED, ENTER 2
    FIXED-FIXED, ENTER 3
    SIMPLY SUPPORTED, ENTER 4

*4
*** UNIFORM SIMPLY SUPPORTED ***

NOMINAL PIPE DIAMETER = 1.000      SCHEDULE NO. = 40

DYNAMIC FACTOR= 1.00

   LENGTH      FREQUENCY     AMPLITUDE     MAXIMUM STRESS
   (FEET)        (CPS)        (INCHES)         (PSI)

   4.9000      38.66884      0.00831          467.953

AMPLITUDE(MAXIMUM DEFLECTION) AT X= 2.45 FT.

MAXIMUM STRESS AT X= 2.45 FT.


REPEAT CALCULATION?, YES=1, NO=0


*1
```

```
SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*0

FRESH DATA OR MODIFICATION?, ENTER 1 OR 0

*0
LIST OF DATA WITH SERIAL NUMBERS:

  # 1 - NOMINAL DIAMETER \ # 2 - SCHEDULE NO.
  # 3 - OUTSIDE DIAMETER \ # 4 - PIPE LENGTH
  # 5 - MOM. OF INERTIA  \ # 6 - YOUNG'S MO.
  # 7 - WT./LENGTH       \ # 8 - DYNAMIC FACTOR

ENTER NUMBER OF CHANGES DESIRED

*1
ENTER DATA NUMBER(S) WHICH ARE TO BE CHANGED

*4

ENTER NEW PIPE LENGTH

*4.6

ENTER THE CASE NUMBER

    CANTILEVER, ENTER 1
    SIMPLE-FIXED, ENTER 2
    FIXED-FIXED, ENTER 3
    SIMPLY SUPPORTED, ENTER 4

*1
*** UNIFORM CANTILEVER ***

NOMINAL PIPE DIAMETER = 1.000      SCHEDULE NO. = 40

DYNAMIC FACTOR= 1.00

   LENGTH      FREQUENCY    AMPLITUDE    MAXIMUM STRESS
   (FEET)        (CPS)       (INCHES)        (PSI)

   4.6000      15.63136     0.06197        1430.133

AMPLITUDE(MAXIMUM DEFLECTION) AT X= 4.60 FT.

MAXIMUM STRESS AT X= 0.00 FT.


REPEAT CALCULATION?, YES=1, NO=0


*1
```

```
SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*1

ENTER THE CASE NUMBER

    CANTILEVER, ENTER 1
    SIMPLE-FIXED, ENTER 2
    FIXED-FIXED, ENTER 3
    SIMPLY SUPPORTED, ENTER 4

*3
*** UNIFORM FIXED-FIXED ***

NOMINAL PIPE DIAMETER = 1.000      SCHEDULE NO. = 40

DYNAMIC FACTOR= 1.00

   LENGTH      FREQUENCY     AMPLITUDE     MAXIMUM STRESS
   (FEET)        (CPS)        (INCHES)         (PSI)

   4.6000      99.46519      0.00129          235.467

AMPLITUDE(MAXIMUM DEFLECTION) AT X= 2.30 FT.

MAXIMUM STRESS AT X= 0.00 FT.

REPEAT CALCULATION?, YES=1, NO=0

*1

SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*0

FRESH DATA OR MODIFICATION?, ENTER 1 OR 0

*0
LIST OF DATA WITH SERIAL NUMBERS:

  # 1 - NOMINAL DIAMETER \ # 2 - SCHEDULE NO.
  # 3 - OUTSIDE DIAMETER \ # 4 - PIPE LENGTH
  # 5 - MOM. OF INERTIA  \ # 6 - YOUNG"S MO.
  # 7 - WT./LENGTH       \ # 8 - DYNAMIC FACTOR

ENTER NUMBER OF CHANGES DESIRED

*2
```

```
      ENTER DATA NUMBER(S) WHICH ARE TO BE CHANGED

(6,8

      ENTER NEW YOUNG"S MODULUS

(29.6e06

      ENTERNEW DYNAMIC FACTOR

(1.1

     ENTER THE CASE NUMBER

          CANTILEVER, ENTER 1
          SIMPLE-FIXED, ENTER 2
          FIXED-FIXED, ENTER 3
          SIMPLY SUPPORTED, ENTER 4

*3
      *** UNIFORM FIXED-FIXED ***

      NOMINAL PIPE DIAMETER = 1.000       SCHEDULE NO. = 40

      DYNAMIC FACTOR= 1.10

         LENGTH      FREQUENCY     AMPLITUDE     MAXIMUM STRESS
         (FEET)        (CPS)        (INCHES)         (PSI)

         4.6000       98.79984      0.00144          259.013

      AMPLITUDE(MAXIMUM DEFLECTION) AT X= 2.30 FT.

      MAXIMUM STRESS AT X= 0.00 FT.


      REPEAT CALCULATION?, YES=1, NO=0


*0

 336.
```

CHAPTER X

APPENDIX - C

LISTING OF FORTRAN PROGRAM FOR SPANS WITH A CONCENTRATED LOAD (VALVE)

```
C
C     INTERACTIVE PROGRAM FOR ANALYSIS OF PIPING VIBRATIONS
C     PROGRAM # 2 - SPANS WITH VALVE AT ANY LOCATION
C
      COMMON K
      DIMENSION M3(20)
      DATA IN/1/,IO/2/
C
 1    WRITE (IO,2)
 2    FORMAT(5X,'ENTER THE DATA: D1,D2,D3,D4,D5,D6,D7,D8,D9,D10',/)
      WRITE (IO,3)
 3    FORMAT(10X,'D1 - NOMINAL PIPE DIAMETER, INCHES',
     %/,10X,'D2 - SCHEDULE NO.',
     %/,10X,'D3 - OUTSIDE DIAMETER, INCHES',
     %/,10X,'D4 - LENGTH OF THE PIPE, FEET' )
      WRITE(IO,4)
 4    FORMAT(10X,'D5 - MOMENT OF INERTIA, (INCHES)**4',
     %/,10X,'D6 - YOUNG"S MODULUS, (LBS/SQUARE INCHES)',
     %/,10X,'D7 - WEIGHT PER UNIT LENGTH, (LBS/FT.)',
     %/,10X,'D8 - DYNAMIC LOADING FACTOR, (NON-DIMENSIONAL)')
      WRITE (IO,5)
 5    FORMAT(10X,'D9 - WEIGHT OF VALVE, (LBS.)',/
     %,10X,'D10 - LOCATION OF VALVE (FROM L.H.S.), (FT.)',/)
C
      READ(IN,*) PN, NS, OD, XL, XI, E, P, DF, W, X1
C
 6    IF(X1.GT.XL) GO TO 67
      R = X1/XL
      X2 = XL-X1
      AM = W/P
C
      WRITE (IO,7)
 7    FORMAT(/,5X,'ENTER THE CASE NUMBER',/)
      WRITE(IO,8)
 8    FORMAT(10X,'CANTILEVER, ENTER 1',/
     %10X,'SIMPLE-FIXED, ENTER 2',/
     %10X,'FIXED-FIXED, ENTER 3',/
     %10X,'SIMPLY SUPPORTED, ENTER 4',/)
C
      READ (IN,*)K
C
      GO TO (9,11,13,15) ,K
 0    GO TO 67
C
 9    WRITE (IO,10)
 10   FORMAT(6X,'*** CANTILEVER WITH VALVE ***',/)
      GO TO 17
C
 11   WRITE (IO,12)
 12   FORMAT(6X,'** SIMPLE-FIXED WITH VALVE **',/)
```

```
      GO TO 17
C
 13   WRITE (IO,14)
 14   FORMAT(6X,'** FIXED - FIXED WITH VALVE **',/)
      GO TO 17
C
 15   WRITE (IO,16)
 16   FORMAT(6X,'*** SIMPLY SUPPORTED WITH VALVE ***',/)
C
 17   WRITE(IO,18) W,X1
 18   FORMAT(6X,'WT. OF VALVE=',F5.2,1X,'LBS.',4X,
     %'VALVE LOCATION=',F5.2,1X,'FT.',/)
      WRITE (IO,19) PN, NS
 19   FORMAT(5X,' NOMINAL PIPE DIAMETER =',F6.3,5X,' SCHEDULE NO. =',
     %I3,/)
      WRITE (IO,20) DF
 20   FORMAT(6X,'DYNAMIC FACTOR=',F5.2,//)
      WRITE (IO,21)
 21   FORMAT(9X,'LENGTH',4X,'FREQUENCY',4X,'AMPLITUDE',4X,
     %'MAXIMUM STRESS')
      WRITE (IO,22)
 22   FORMAT(9X,'(FEET)',6X,'(CPS)',7X,'(INCHES)',8X,'(PSI)',//)
C
      GO TO (23,24,29,34) ,K
C
 23    SD1 = (W*X1**2*(3.0*XL - X1)*12.0**3)/(6.0*E*XI)
       SD2 = (216.0*P*XL**4)/(E*XI)
       A = DF*(SD1 + SD2)
      GO TO 35
C
 24   IF((R-0.414)) 25,26,27
 25    XNUM = ABS(  W*X1*(XL**2 - X1**2)**3*12.0**3   )
       DENOM = ABS(  3.0*E*XI*(3.0*XL**2 - X1**2)**2   )
       SD1 = XNUM/DENOM
       XMAX =  XL*(XL**2+X1**2)/(3.0*XL**2-X1**2)
       SD2 = ABS( P * (3.0*XL*(XMAX)**3 - 2.0*(XMAX)**4 -
     % XL**3*(XMAX))*12.0**3/(48.0*E*XI))
      GO TO 28
C
 26    SD1 = 0.0098*W*XL**3*12.0**3/(E*XI)
       XMAX = R*XL
       SD2 = ABS( P * (3.0*XL*(XMAX)**3 - 2.0*(XMAX)**4 -
     % XL**3*(XMAX))*12.0**3/(48.0*E*XI))
      GO TO 28
C
 27    SD1 = ABS(  W*12.0**3*X1*X2**2*SQRT(X1/(2.0*XL+X1))/(6.*E*XI))
       XMAX = XL*SQRT(1-(2.0*XL/(3.0*XL-X2)))
       SD2 = ABS( P * (3.0*XL*(XMAX)**3 - 2.0*(XMAX)**4 -
     % XL**3*(XMAX))*12.0**3/(48.0*E*XI))
C
```

```
 28    A = DF*(SD1 + SD2)
       GO TO 35
C
 29    IF((R-0.5)) 30,31,32
 30     SD1 = ABS(  2.0*W*X1**2*X2**3*12.0**3/(3.0*E*XI*(XL+2.0*X2)**2))
        XMAX = XL - (2.0*X2*XL/(XL+2.0*X2))
        SD2 = ABS(  P*XMAX**2*(2.0*XL*XMAX - XL**2 - XMAX**2)/
      % (24.0*E*XI)   )
       GO TO 33
C
 31     SD1 = ABS(  W*XL**3*12.0**3/(192.0*E*XI) )
        SD2 = ABS(  P*XL**4*12.0**3/(384.0*E*XI)   )
       GO TO 33
 32     SD1 = ABS(  2.0*W*X1**3*X2**2*12.0**3/(3.0*E*XI*(XL+2.0*X1)**2))
        XMAX = 2.0*X1*XL/(XL+2.0*X1)
        SD2 = ABS(  P*XMAX**2*(2.0*XL*XMAX - XL**2 - XMAX**2)/
      % (24.0*E*XI)   )
C
 33    A = DF*(SD1 + SD2)
C
       GO TO 35
C
34     IF((R-0.5)) 341,341,342
 341     SD1 = W*12.0**3*X1*((XL**2-X1**2)**1.5)/(9.0*1.73205*XL*E*XI)
        XMAX = XL - SQRT((XL**2-X1**2)/3.0)
        SD2 = ABS(  P*XMAX*((XL**3)-(2.0*XL*XMAX**2)+XMAX**3)*12.0**3
      % /(24.0*E*XI)   )
       GO TO 343
 342     SD1 = W*12.0**3*X2*((XL**2-X2**2)**1.5)/(9.0*1.73205*XL*E*XI)
        XMAX = XL - SQRT((XL**2-X2**2)/3.0)
        SD2 = ABS(  P*XMAX*((XL**3)-(2.0*XL*XMAX**2)+XMAX**3)*12.0**3
      % /(24.0*E*XI)   )
 343   A = DF*(SD1 + SD2)
C
 35    CALL LAMDA(XL,AM,X1,X2,XLAMDA,CONST1,CONST2,CONST3)
       CALL MAXY(CONST1,CONST2,CONST3,XLAMDA,XL,X1,YMAX,XYMAX)
       CALL MAXSIG(CONST1,CONST2,CONST3,XLAMDA,XL,X1,SIGMAX,XSMAX)
C
       FR = 0.0753*XLAMDA**2*SQRT(E*XI/P)
       STRESS = (A*E*OD*XLAMDA**2*SIGMAX)/(144.0*2.0*YMAX)
       WRITE (IO,36) XL, FR, A, STRESS
 36    FORMAT(5X,F10.4,4X,F9.5,4X,F8.4,7X,F9.3,/)
       WRITE (IO,37) XYMAX
 37    FORMAT(6X,'AMPLITUDE(MAXIMUM DEFLECTION) AT X=',F5.2,1X,'FT.',/)
       WRITE (IO,38) XSMAX
 38    FORMAT(6X,'MAXIMUM STRESS AT X=',F5.2,1X,'FT.',//)
       WRITE(IO,39)
 39    FORMAT(6X,'REPEAT CALCULATION?, YES=1, NO=0',//)
       READ (IN,*) M
       IF(M.EQ.0) GO TO 67
```

```
C
      WRITE (IO,40)
 40   FORMAT(/,6X,'SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0',/)
C
      READ (IN,*) M4
C
      IF(M4.NE.0) GO TO 6
C
      WRITE (IO,41)
 41   FORMAT(/,6X,'FRESH DATA OR MODIFICATION?, ENTER 1 OR 0',/)
C
      READ (IN,*) M1
C
      IF(M1.NE.0) GO TO 1
C
      WRITE (IO,42)
 42   FORMAT(6X,'LIST OF DATA WITH SERIAL NUMBERS:',//
     %,8X,'# 1 - NOMINAL DIAMETER \ # 2 - SCHEDULE NO.',/
     %,8X,'# 3 - OUTSIDE DIAMETER \ # 4 - PIPE LENGTH',/
     %,8X,'# 5 - MOM. OF INERTIA  \ # 6 - YOUNG'S MO.',/
     %,8X,'# 7 - WT./LENGTH       \ # 8 - DYNAMIC FACTOR',/
     %,8X,'# 9 - WEIGHT OF VALVE  \ # 10 - VALVE LOCATION',/)
      WRITE (IO,43)
 43   FORMAT(6X,'ENTER NUMBER OF CHANGES DESIRED',/)
C
      READ (IN,*) M2
C
      IF(M2.LE.0) GO TO 6
C
      WRITE (IO,44)
 44   FORMAT(6X,'ENTER DATA NUMBER(S) WHICH ARE TO BE CHANGED',/)
C
      READ (IN,*) (M3(I1), I1=1,M2)
C
      K1 =1
 45    M5 = M3(K1)
C
       GO TO (47,49,51,53,55,57,59,61,63,65), M5
C
 46   K1 = K1 + 1
C
      IF(K1.GT.M2) GO TO 6
C
      GO TO 45
C
 47    WRITE (IO,48)
 48    FORMAT(/,6X,'ENTER NEW NOMINAL PIPE DIAMETER',/)
       READ (IN,*) PN
      GO TO 46
C
```

```
 49     WRITE (IO,50)
 50     FORMAT(/,6X,'ENTER NEW SCHEDULE NO.',/)
        READ (IN,*) NS
       GO TO 46
C
 51     WRITE (IO,52)
 52     FORMAT(/,6X,'ENTER NEW OUTSIDE DIAMETER',/)
        READ (IN,*) OD
       GO TO 46
C
 53     WRITE (IO,54)
 54     FORMAT(/,6X,'ENTER NEW PIPE LENGTH',/)
        READ (IN,*) XL
       GO TO 46
C
 55     WRITE (IO,56)
 56     FORMAT(/,6X,'ENTER NEW MOM. OF INERTIA',/)
        READ (IN,*) XI
       GO TO 46
C
 57     WRITE (IO,58)
 58     FORMAT(/,6X,'ENTER NEW YOUNG"S MODULUS',/)
        READ (IN,*) E
       GO TO 46
C
 59     WRITE (IO,60)
 60     FORMAT(/,6X,'ENTER NEW WT. PER UNIT LENGTH',/)
        READ (IN,*) P
       GO TO 46
C
 61     WRITE (IO,62)
 62     FORMAT(/,6X,'ENTER NEW DYNAMIC FACTOR',/)
        READ (IN,*) DF
       GO TO 46
C
 63     WRITE (IO,64)
 64     FORMAT(/,6X,'ENTER NEW VALVE WEIGHT',/)
        READ (IN,*) W
       GO TO 46
C
 65     WRITE (IO,66)
 66     FORMAT(/,6X,'ENTER NEW VALVE LOCATION',/)
        READ (IN,*) X1
       GO TO 46
C
 67    STOP
       END
```

```
C
C     ////////////////////////////////////////////////////////////
C
      SUBROUTINE LAMDA(XL,AM,X1,X2,XLAMDA,AK1XL,AK2XL,DXL)
C
C     SUBROUTINE FOR FINDING ROOT OF EQUATION BY BISECTION ALGORITHM
C
C
      EPS = 0.00001
      A=0.00009
 1    B=A+0.005
       CALL FUN(XL,X1,X2,A,AM,FA,AK1A,AK2A,DA)
       CALL FUN(XL,X1,X2,B,AM,FB,AK1B,AK2B,DB)
      IF((FA*FB).LT.0.0) GO TO 2
      A=B
      GO TO 1
C
 2    DO 4 I=1,15
         P=A+(B-A)/2.0
         CALL FUN(XL,X1,X2,P,AM,FP,AK1P,AK2P,DP)
         IF(FP.EQ.0.0.OR.(B-A).LT.EPS) GO TO 5
         IF((FA*FP).GT.0.0) GO TO 3
         B=P
         GO TO 4
 3       A=P
 4    CONTINUE
C
 5    XLAMDA=P
C
      CALL FUN(XL,X1,X2,XLAMDA,AM,FXL,AK1XL,AK2XL,DXL)
C
      RETURN
      END
```

```
C
C     ////////////////////////////////////////////////////////////
C
      SUBROUTINE FUN(XL,X1,X2,XLAMDA,AM,F,AK1,AK2,D)
C
      COMMON K
C
      GO TO (1,2,3,4) ,K
C
 1    D = 1.0+COSH(XLAMDA*XL)*COS(XLAMDA*XL)
C
        AK1 = SINH(XLAMDA*X2)*COSH(XLAMDA*XL)
     % + SIN(XLAMDA*X2)*COSH(XLAMDA*XL)+SINH(XLAMDA*X2)*
     % COS(XLAMDA*XL)+SIN(XLAMDA*X2)*COS(XLAMDA*XL)-COSH(XLAMDA*
     % X2)*SINH(XLAMDA*XL)-COS(XLAMDA*X2)*SINH(XLAMDA*XL)-
     % COSH(XLAMDA*X2)*SIN(XLAMDA*XL)-COS(XLAMDA*X2)*
     % SIN(XLAMDA*XL)
        AK2 = COSH(XLAMDA*X2)*COSH(XLAMDA*XL)+
     % COS(XLAMDA*X2)*COSH(XLAMDA*XL)+COSH(XLAMDA*X2)*
     % COS(XLAMDA*XL)+COS(XLAMDA*X2)*COS(XLAMDA*XL)-
     % SINH(XLAMDA*X2)*SINH(XLAMDA*XL)-SIN(XLAMDA*X2)*
     % SINH(XLAMDA*XL)+SINH(XLAMDA*X2)*SIN(XLAMDA*XL)+
     % SIN(XLAMDA*X2)*SIN(XLAMDA*XL)
C
        C1 = AK1*(COSH(XLAMDA*X1)-COS(XLAMDA*X1))
        C2 = AK2*(SINH(XLAMDA*X1)-SIN(XLAMDA*X1))
C
        F = 4.0 + AM*((C1*XLAMDA+C2*XLAMDA)/D)
      GO TO 5
C
 2    D = SIN(XLAMDA*XL)*COSH(XLAMDA*XL) -
     % SINH(XLAMDA*XL)*COS(XLAMDA*XL)
C
        AK1 = -SINH(XLAMDA*X2)*COSH(XLAMDA*XL) +SINH(XLAMDA*X2)*
     % COS(XLAMDA*XL) +SIN(XLAMDA*X2)*COSH(XLAMDA*XL)- SIN(XLAMDA*X2)*
     % COS(XLAMDA*XL) +COSH(XLAMDA*X2)*SINH(XLAMDA*XL) -COSH(XLAMDA*X2)*
     % SIN(XLAMDA*XL) -COS(XLAMDA*X2)*SINH(XLAMDA*XL) +COS(XLAMDA*X2)*
     % SIN(XLAMDA*XL)
        AK2 = -COSH(XLAMDA*X2)*SIN(XLAMDA*XL) +COS(XLAMDA*X2)*
     % SIN(XLAMDA*XL) -COSH(XLAMDA*X2)*SINH(XLAMDA*XL) +COS(XLAMDA*X2)*
     % SINH(XLAMDA*XL) +SINH(XLAMDA*X2)*COS(XLAMDA*XL) -SIN(XLAMDA*X2)*
     % COS(XLAMDA*XL)+ SINH(XLAMDA*X2)*COSH(XLAMDA*XL) -SIN(XLAMDA*X2)*
     % COSH(XLAMDA*XL)
C
        C1 = AK1*(SIN(XLAMDA*X1) + SINH(XLAMDA*X1))
        C2 = AK2*(SINH(XLAMDA*X1) - SIN(XLAMDA*X1))
C
        F = AM*XLAMDA*((C1+C2)/D) -4.0
      GO TO 5
C
```

```
 3     D = 1.0 - COSH(XLAMDA*XL)*COS(XLAMDA*XL)
C
       AK1 = SINH(XLAMDA*X2)*COSH(XLAMDA*XL) -
     % SIN(XLAMDA*X2)*COSH(XLAMDA*XL) - SINH(XLAMDA*X2)*COS(XLAMDA*XL)
     % + SIN(XLAMDA*X2)*COS(XLAMDA*XL) - COSH(XLAMDA*X2)*SINH(XLAMDA*XL)
     % + COS(XLAMDA*X2)*SINH(XLAMDA*XL) + COSH(XLAMDA*X2)*SIN(XLAMDA*XL)
     % - COS(XLAMDA*X2)*SIN(XLAMDA*XL)
       AK2 = COSH(XLAMDA*X2)*COSH(XLAMDA*XL) -
     % COSH(XLAMDA*X2)*COS(XLAMDA*XL) - COS(XLAMDA*X2)*COSH(XLAMDA*XL)
     % + COS(XLAMDA*X2)*COS(XLAMDA*XL) - SINH(XLAMDA*X2)*SINH(XLAMDA*XL)
     % + SIN(XLAMDA*X2)*SINH(XLAMDA*XL) - SINH(XLAMDA*X2)*SIN(XLAMDA*XL)
     % + SIN(XLAMDA*X2)*SIN(XLAMDA*XL)
C
       C1 = AK1*(COSH(XLAMDA*X1) - COS(XLAMDA*X1))
       C2 = AK2*(SINH(XLAMDA*X1) - SIN(XLAMDA*X1))
C
       F = 4.0 + AM*((C1*XLAMDA + C2*XLAMDA)/(D))
      GO TO 5
C
 4    D = SINH(XLAMDA*XL)*SIN(XLAMDA*XL)
C
       AK1 = SINH(XLAMDA*X2)*SIN(XLAMDA*XL)
     % - SIN(XLAMDA*X2)*SINH(XLAMDA*XL)
       AK2 = SINH(XLAMDA*X2)*SIN(XLAMDA*XL)
     % + SIN(XLAMDA*X2)*SINH(XLAMDA*XL)
C
       F = 2.0 + AM*XLAMDA * ((SINH(XLAMDA*X2)*SINH(XLAMDA*X1)/
     % SINH(XLAMDA*XL)) - (SIN(XLAMDA*X2)*SIN(XLAMDA*X1)/
     % SIN(XLAMDA*XL)))
C
 5    RETURN
      END
```

```
C
C     ////////////////////////////////////////////////////////////////
C
      SUBROUTINE MAXY(AK1,AK2,D,XLAMDA,XL,X1,YMAX,XYMAX)
C
C     SUBROUTINE TO CALCULATE MAXIMUM DEFLECTION FUNCTION
C
      X=0.0
      DELTA=XL/100.0
C
      DO 1 I=1,100
         XNEW=X + DELTA
         IF(X.GT.XL) GO TO 3
         CALL UNIT(XNEW,X1,UY2)
         Y2=Y(AK1,AK2,D,XLAMDA,XNEW,X1,UY2)
         CALL UNIT(X,X1,UY1)
         Y1=Y(AK1,AK2,D,XLAMDA,X,X1,UY1)
         IF((Y2-Y1).LT.0.0) GO TO 2
         X=XNEW
 1    CONTINUE
C
 2    YMAX = Y1
      XYMAX=X
C
 3    RETURN
      END
```

```
C
C     ////////////////////////////////////////////////////////////
C
      SUBROUTINE MAXSIG(AK1,AK2,D,XLAMDA,XL,X1,SIGMAX,XSMAX)
C
C     SUBROUTINE TO CALCULATE MAXIMUM STRESS FUNCTION
C
      X=0.0
      DELTA=XL/100.0
C
      DO 1 I=1,100
         XNEW=X + DELTA
         IF(X.GT.XL) GO TO 3
         CALL UNIT(XNEW,X1,US2)
         SIG2=SIGMA(AK1,AK2,D,XLAMDA,XNEW,X1,US2)
         CALL UNIT(X,X1,US1)
         SIG1=SIGMA(AK1,AK2,D,XLAMDA,X,X1,US1)
         IF((SIG2-SIG1).LT.0.0) GO TO 2
         X=XNEW
 1    CONTINUE
C
 2    SIGMAX = SIG1
      XSMAX = X
C
 3    RETURN
      END
```

```
C     ////////////////////////////////////////////////////////////////
C
      SUBROUTINE UNIT(ARGU,X1,U)
C
C     SUBROUTINE TO CALCULATE UNIT FUNCTION
C
      IF(ARGU-X1.GE.0.0) GO TO 1
       U=0.0
       GO TO 2
 1    U=1.0
C
 2    RETURN
      END
```

```
C
C     \\\\
```

```
C
C     \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
C
      FUNCTION SIGMA(AK1,AK2,D,XLAMDA,X,X1,U)
C
      COMMON K
C
      GO TO (1,2,3,4) ,K
C
 1    SIGMA = ABS(1.0/D
     % *(-AK1*(COSH(XLAMDA*X)+COS(XLAMDA*X)) - AK2*
     % (SINH(XLAMDA*X)+ SIN(XLAMDA*X))
     %  + 2.0*U*(SINH(XLAMDA*(X-X1))+SIN(XLAMDA*(X-X1)))))
      GO TO 5
C
 2    SIGMA = ABS( (AK1/D)*(-SIN(XLAMDA*X) + SINH(XLAMDA*X)) +
     % (AK2/D)*(SINH(XLAMDA*X) + SIN(XLAMDA*X))
     % + 2.0*U*(SINH(XLAMDA*(X-X1)) + SIN(XLAMDA*(X-X1))))
      GO TO 5
C
 3    SIGMA = ABS(   (-AK1*(COSH(XLAMDA*X)+COS(XLAMDA*X)) - AK2*
     % (SINH(XLAMDA*X)+SIN(XLAMDA*X)))/(D)
     %  + 2.0*U*(SINH(XLAMDA*(X-X1)) +
     % SIN(XLAMDA*(X-X1)))   )
      GO TO 5
C
 4    SIGMA = ABS(   (-AK1*(SINH(XLAMDA*X)-SIN(XLAMDA*X))
     %  - AK2*(SINH(XLAMDA*X)+SIN(XLAMDA*X)))/(D)
     %  + 2.0*U*(SINH(XLAMDA*(X-X1))
     % + SIN(XLAMDA*(X-X1))) )
C
 5    RETURN
      END
```

CHAPTER XI

APPENDIX - D

SAMPLE RUN OF FORTRAN PROGRAM FOR SPANS WITH A CONCENTRATED LOAD (VALVE)

```
@ru


**FASTFOR (CONVERSATIONAL VER 10)**


     ENTER THE DATA: D1,D2,D3,D4,D5,D6,D7,D8,D9,D10

          D1 - NOMINAL PIPE DIAMETER, INCHES
          D2 - SCHEDULE NO.
          D3 - OUTSIDE DIAMETER, INCHES
          D4 - LENGTH OF THE PIPE, FEET
          D5 - MOMENT OF INERTIA, (INCHES)**4
          D6 - YOUNG"S MODULUS, (LBS/SQUARE INCHES)
          D7 - WEIGHT PER UNIT LENGTH, (LBS/FT.)
          D8 - DYNAMIC LOADING FACTOR, (NON-DIMENSIONAL)
          D9 - WEIGHT OF VALVE, (LBS.)
          D10 - LOCATION OF VALVE (FROM L.H.S.), (FT.)

*1.0,40,1.315,4.9,0.0874,30e06,1.68,1.0,20,1.5

     ENTER THE CASE NUMBER

          CANTILEVER, ENTER 1
          SIMPLE-FIXED, ENTER 2
          FIXED-FIXED, ENTER 3
          SIMPLY SUPPORTED, ENTER 4

*1
      *** CANTILEVER WITH VALVE ***

      WT. OF VALVE=20.00 LBS.     VALVE LOCATION= 1.50 FT.

      NOMINAL PIPE DIAMETER = 1.000       SCHEDULE NO. = 40

      DYNAMIC FACTOR= 1.00


         LENGTH     FREQUENCY     AMPLITUDE     MAXIMUM STRESS
         (FEET)       (CPS)        (INCHES)         (PSI)


         4.9000      12.51632       0.1450         2467.984

      AMPLITUDE(MAXIMUM DEFLECTION) AT X= 4.90 FT.

      MAXIMUM STRESS AT X= 0.00 FT.


      REPEAT CALCULATION?, YES=1, NO=0


*1
```

```
 SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*1

ENTER THE CASE NUMBER

     CANTILEVER, ENTER 1
     SIMPLE-FIXED, ENTER 2
     FIXED-FIXED, ENTER 3
     SIMPLY SUPPORTED, ENTER 4

*2
 ** SIMPLE-FIXED WITH VALVE **

 WT. OF VALVE=20.00 LBS.    VALVE LOCATION= 1.50 FT.

 NOMINAL PIPE DIAMETER = 1.000      SCHEDULE NO. = 40

 DYNAMIC FACTOR= 1.00

    LENGTH     FREQUENCY     AMPLITUDE     MAXIMUM STRESS
    (FEET)       (CPS)       (INCHES)         (PSI)

    4.9000     24.81645      0.0174        1764.598

 AMPLITUDE(MAXIMUM DEFLECTION) AT X= 1.86 FT.

 MAXIMUM STRESS AT X= 1.52 FT.

 REPEAT CALCULATION?, YES=1, NO=0

*1

 SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*1

ENTER THE CASE NUMBER

     CANTILEVER, ENTER 1
     SIMPLE-FIXED, ENTER 2
     FIXED-FIXED, ENTER 3
     SIMPLY SUPPORTED, ENTER 4

*3
```

```
** FIXED - FIXED WITH VALVE **

WT. OF VALVE=20.00 LBS.     VALVE LOCATION= 1.50 FT.

NOMINAL PIPE DIAMETER = 1.000       SCHEDULE NO. = 40

DYNAMIC FACTOR= 1.00

   LENGTH     FREQUENCY     AMPLITUDE     MAXIMUM STRESS
   (FEET)       (CPS)        (INCHES)         (PSI)

   4.9000      40.31512       0.0057         1242.856

AMPLITUDE(MAXIMUM DEFLECTION) AT X= 2.11 FT.

MAXIMUM STRESS AT X= 0.00 FT.

REPEAT CALCULATION?, YES=1, NO=0

*1

SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*1

ENTER THE CASE NUMBER

     CANTILEVER, ENTER 1
     SIMPLE-FIXED, ENTER 2
     FIXED-FIXED, ENTER 3
     SIMPLY SUPPORTED, ENTER 4

*4
```

```
*** SIMPLY SUPPORTED WITH VALVE ***

WT. OF VALVE=20.00 LBS.     VALVE LOCATION= 1.50 FT.

NOMINAL PIPE DIAMETER = 1.000       SCHEDULE NO. = 40

DYNAMIC FACTOR= 1.00

   LENGTH     FREQUENCY     AMPLITUDE     MAXIMUM STRESS
   (FEET)       (CPS)        (INCHES)         (PSI)

   4.9000     18.23372       0.0345         2275.517

AMPLITUDE(MAXIMUM DEFLECTION) AT X= 2.25 FT.

MAXIMUM STRESS AT X= 1.52 FT.

REPEAT CALCULATION?, YES=1, NO=0

*1

SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*0

FRESH DATA OR MODIFICATION?, ENTER 1 OR 0

*0
LIST OF DATA WITH SERIAL NUMBERS:

  # 1 - NOMINAL DIAMETER \ # 2 - SCHEDULE NO.
  # 3 - OUTSIDE DIAMETER \ # 4 - PIPE LENGTH
  # 5 - MOM. OF INERTIA  \ # 6 - YOUNG'S MO.
  # 7 - WT./LENGTH       \ # 8 - DYNAMIC FACTOR
  # 9 - WEIGHT OF VALVE  \ # 10 - VALVE LOCATION

ENTER NUMBER OF CHANGES DESIRED

*2
ENTER DATA NUMBER(S) WHICH ARE TO BE CHANGED

*4,10

ENTER NEW PIPE LENGTH

*3.5
```

```
      ENTER NEW VALVE LOCATION

*2.0

     ENTER THE CASE NUMBER

          CANTILEVER, ENTER 1
          SIMPLE-FIXED, ENTER 2
          FIXED-FIXED, ENTER 3
          SIMPLY SUPPORTED, ENTER 4

*1
      *** CANTILEVER WITH VALVE ***

      WT. OF VALVE=20.00 LBS.    VALVE LOCATION= 2.00 FT.

      NOMINAL PIPE DIAMETER = 1.000      SCHEDULE NO. = 40

      DYNAMIC FACTOR= 1.00

         LENGTH     FREQUENCY    AMPLITUDE    MAXIMUM STRESS
         (FEET)       (CPS)       (INCHES)        (PSI)

         3.5000     14.28370      0.0955        4519.121

      AMPLITUDE(MAXIMUM DEFLECTION) AT X= 3.50 FT.

      MAXIMUM STRESS AT X= 0.00 FT.

      REPEAT CALCULATION?, YES=1, NO=0

*1

      SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*1

     ENTER THE CASE NUMBER

          CANTILEVER, ENTER 1
          SIMPLE-FIXED, ENTER 2
          FIXED-FIXED, ENTER 3
          SIMPLY SUPPORTED, ENTER 4

*2
```

```
** SIMPLE-FIXED WITH VALVE **

WT. OF VALVE=20.00 LBS.     VALVE LOCATION= 2.00 FT.

NOMINAL PIPE DIAMETER = 1.000       SCHEDULE NO. = 40

DYNAMIC FACTOR= 1.00

   LENGTH     FREQUENCY     AMPLITUDE     MAXIMUM STRESS
   (FEET)       (CPS)        (INCHES)         (PSI)

   3.5000      44.38217       0.0055          957.381

AMPLITUDE(MAXIMUM DEFLECTION) AT X= 1.61 FT.

MAXIMUM STRESS AT X= 1.99 FT.

REPEAT CALCULATION?, YES=1, NO=0

*1

SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*1

ENTER THE CASE NUMBER

     CANTILEVER, ENTER 1
     SIMPLE-FIXED, ENTER 2
     FIXED-FIXED, ENTER 3
     SIMPLY SUPPORTED, ENTER 4

*3
```

```
** FIXED - FIXED WITH VALVE **

WT. OF VALVE=20.00 LBS.     VALVE LOCATION= 2.00 FT.

NOMINAL PIPE DIAMETER = 1.000       SCHEDULE NO. = 40

DYNAMIC FACTOR= 1.00

   LENGTH     FREQUENCY     AMPLITUDE     MAXIMUM STRESS
   (FEET)       (CPS)        (INCHES)         (PSI)

   3.5000     56.37585       0.0028          684.767

AMPLITUDE(MAXIMUM DEFLECTION) AT X= 1.85 FT.

MAXIMUM STRESS AT X= 0.00 FT.

REPEAT CALCULATION?, YES=1, NO=0

*1

SAME DATA WITH DIFFERENT CASE?, YES =1, NO=0

*1

ENTER THE CASE NUMBER

     CANTILEVER, ENTER 1
     SIMPLE-FIXED, ENTER 2
     FIXED-FIXED, ENTER 3
     SIMPLY SUPPORTED, ENTER 4

*4
```

```
*** SIMPLY SUPPORTED WITH VALVE ***

WT. OF VALVE=20.00 LBS.     VALVE LOCATION= 2.00 FT.

NOMINAL PIPE DIAMETER = 1.000       SCHEDULE NO. = 40

DYNAMIC FACTOR= 1.00

   LENGTH     FREQUENCY     AMPLITUDE     MAXIMUM STRESS
   (FEET)       (CPS)        (INCHES)         (PSI)

   3.5000     27.48547       0.0136         1787.732

AMPLITUDE(MAXIMUM DEFLECTION) AT X= 1.82 FT.

MAXIMUM STRESS AT X= 1.99 FT.

REPEAT CALCULATION?, YES=1, NO=0

*0

 495.
```

CHAPTER XII

APPENDIX - E

LISTING AND SAMPLE RUN OF FORTRAN PROGRAMS USED TO GENERATE DATA FOR PLOTTING GRAPHS

```
C     PROGRAM TO GENERATE DATA FOR CURVES FOR CANTILEVER
C     PIPING WITH CONCENTRATED LOAD
      R1 = 0.00
      DO 1000 J=1,9
      R1 = R1+0.1
      R2 = 1.0 - R1
      WRITE(2,5)
5     FORMAT(5X,'CANTILEVER WITH VALVE',//)
      WRITE(2,6) R1
6     FORMAT(5X,'VALVE LOCATED AT X=',F5.3,'*LENGTH',/)
      WRITE(2,7)
7     FORMAT(4X,'************',2X,'************',
     %3X,'************')
      WRITE(2,8)
8     FORMAT(4X,'  RATIO R1  ',2X,'  FACTOR B1 ',
     %3X,'STRESS RATIO')
      WRITE(2,9)
9     FORMAT(4X,'************',2X,'************',
     %3X,'************',/)
      R = 0.0
      DO 100 I=1,10
      R = R + 0.1
      CALL LAMDA(R,R1,R2,BL,CONST1,CONST2,CONST3)
      CALL MAXY(CONST1,CONST2,CONST3,BL,R1,YMAX,XOC)
      CALL MAXSIG(CONST1,CONST2,CONST3,BL,R1,SIGMAX)
      RATIO = SIGMAX/YMAX
      WRITE(2,10)R,BL,RATIO
10    FORMAT(3X,3(F12.7,2X),/)
100   CONTINUE
1000  CONTINUE
      STOP
      END
```

```
C
C     ////////////////////////////////////////////////////////////////
C
      SUBROUTINE LAMDA(R,R1,R2,BL,CONST1,CONST2,CONST3)
C
C     SUBROUTINE FOR FINDING ROOT OF EQUATION BY BISECTION ALGORITHM
C
      D(BL) = 1.0+COSH(BL)*COS(BL)
      AK1(BL) = SINH(BL*R2)*COSH(BL)
     %+ SIN(BL*R2)*COSH(BL)+SINH(BL*R2)*
     %COS(BL)+SIN(BL*R2)*COS(BL)-COSH(BL*
     %R2)*SINH(BL)-COS(BL*R2)*SINH(BL)-
     %COSH(BL*R2)*SIN(BL)-COS(BL*R2)*
     %SIN(BL)
      AK2(BL) = COSH(BL*R2)*COSH(BL)+
     %COS(BL*R2)*COSH(BL)+COSH(BL*R2)*
     %COS(BL)+COS(BL*R2)*COS(BL)-
     %SINH(BL*R2)*SINH(BL)-SIN(BL*R2)*
     %SINH(BL)+SINH(BL*R2)*SIN(BL)+
     %SIN(BL*R2)*SIN(BL)
      C1(BL) = AK1(BL)*(COSH(BL*R1)-COS(BL*R1))
      C2(BL) = AK2(BL)*(SINH(BL*R1)-SIN(BL*R1))
C
      F(BL) = 4.0 + R * ((C1(BL)*BL + C2(BL)*BL)/
     %D(BL))
C
C     ----------------------------------------------------------------
C     F(X) = 0.0 IS THE FUNCTION
C     A AND B ARE THE LIMITS WITHIN WHICH THE PROGRAM SEARCHES THE
C     SOLUTION OF THE EQUATION, F(X) = 0
C     EPS IS THE ACCEPTABLE TOLERANCE
C     ----------------------------------------------------------------
      EPS = 0.00001
      A = 0.00010
600   B=A+0.015
      IF(F(A)*F(B).LE.0.0) GO TO 700
      A=B
      GO TO 600
700   DO 100 I=1,15
      P=A+(B-A)/2.0
      IF(F(P).EQ.0.0.OR.(B-A).LT.EPS) GO TO 200
      IF((F(A)*F(P)).GT.0.0) GO TO 300
      B=P
      GO TO 100
300   A=P
100   CONTINUE
200   BL=P
      CONST1 = AK1(BL)
      CONST2 = AK2(BL)
      CONST3 = D(BL)
      RETURN
      END
```

```
C
C     ////////////////////////////////////////////////////////////////
C
      SUBROUTINE MAXY(AK1,AK2,D,BL,R1,YMAX,RA)
C
C     SUBROUTINE TO CALCULATE MAXIMUM DEFLECTION FUNCTION
C
      Y(RA) = ABS(1.0/(D)*
     %(-AK1*(COSH(BL*RA)-COS(BL*RA)) - AK2*
     %(SINH(BL*RA) - SIN(BL*RA))
     % + U*2.0*(SINH(BL*(RA-R1))-SIN(BL*
     %(RA-R1))))))
      RA=0.0
      DELTA = 0.01
      DO 100 I=1,100
      RANEW=RA + DELTA
      IF(RA.GT.1.0) GO TO 1000
      CALL UNIT(RANEW,R1,U)
      Y2=Y(RANEW)
      CALL UNIT(RA,R1,U)
      Y1=Y(RA)
      IF((Y2-Y1).LT.0.0) GO TO 200
      RA=RANEW
100   CONTINUE
200   YMAX=Y(RA)
1000  RETURN
      END
```

```
C
C     ////////////////////////////////////////////////////////////////
C
      SUBROUTINE UNIT(ARGU,X1,U)
C
C     SUBROUTINE TO CALCULATE UNIT FUNCTION
C
      IF(ARGU-X1.GE.0.0) GO TO 300
      U=0.0
      GO TO 400
300   U=1.0
400   RETURN
      END
```

```
C
C     ////////////////////////////////////////////////////////////
C
      SUBROUTINE MAXSIG(AK1,AK2,D,BL,R1,SIGMAX)
C
C     SUBROUTINE TO CALCULATE MAXIMUM STRESS FUNCTION
C
      SIGMA(RA) = ABS(1.0/D
     %*(-AK1*(COSH(BL*RA)+COS(BL*RA)) - AK2*
     %(SINH(BL*RA)+ SIN(BL*RA))
     % + 2.0*U*(SINH(BL*(RA-R1))+SIN(BL*(RA-R1)))))
      RA=0.0
      DELTA = 0.01
      DO 100 I=1,100
      RANEW=RA + DELTA
      IF(RA.GT.1.0) GO TO 1000
      CALL UNIT(RANEW,R1,U)
      SIG2=SIGMA(RANEW)
      CALL UNIT(RA,R1,U)
      SIG1=SIGMA(RA)
      IF((SIG2-SIG1).LT.0.0) GO TO 200
      RA=RANEW
100   CONTINUE
200   SIGMAX=SIGMA(RA)
1000  RETURN
      END
```

FASTFOR (CONVERSATIONAL VER 10)

CANTILEVER WITH VALVE

VALVE LOCATED AT X=0.100*LENGTH

RATIO R1	FACTOR B1	STRESS RATIO
0.1000000	1.8750510	0.0790985
0.2000000	1.8750000	0.0791572
0.3000001	1.8749480	0.0792135
0.4000001	1.8748890	0.0792783
0.5000001	1.8748380	0.0793357
0.6000001	1.8747870	0.0793931
0.7000002	1.8747350	0.0794514
0.8000002	1.8746770	0.0795150
0.9000002	1.8746250	0.0795742
1.0000000	1.8745740	0.0796303

CANTILEVER WITH VALVE

VALVE LOCATED AT X=0.200*LENGTH

RATIO R1	FACTOR B1	STRESS RATIO
0.1000000	1.8743400	0.3202305
0.2000000	1.8735700	0.3221532
0.3000001	1.8728020	0.3240785
0.4000001	1.8720330	0.3260180
0.5000001	1.8712640	0.3279622

```
C     PROGRAM TO GENERATE DATA FOR CURVES FOR
C     SIMPLE-FIXED UNIFORM PIPING
      DATA OD,XI,E,P,DF/1.0,1.0,30.0E06,1.0,1.0/
      XL=1.6
      WRITE (2,10)
10    FORMAT(9X,'SIMPLE-FIXED UNIFORM PIPING',/)
      WRITE (2,20)
      DO 100 I=1,36
      XL=XL+0.4
      A = DF*0.0054*P*XL**4*12.0**3/(E*XI)
      CALL LAMDA(XL,XLAMDA)
      AK = COS(XLAMDA*XL)/COSH(XLAMDA*XL)
      CALL MAXY(XLAMDA,XL,AK,YMAX,XLOC)
      CALL MAXSIG(XLAMDA,XL,AK,SIGMAX)
      F = 0.0753*XLAMDA**2*SQRT(E*XI/P)
      STRESS = (A*E*OD*XLAMDA**2*SIGMAX)/(144.0*2.0*YMAX)
      WRITE (2,40) XL, F, A, STRESS
100   CONTINUE
40    FORMAT(5X,F10.4,5X,F9.4,5X,F8.4,5X,F9.3,/)
20    FORMAT(9X,'LENGTH',4X,'FREQ. COEFF',4X,'DEF. COEFF',4X,
     %'STRESS COEFF',/)
      STOP
      END
```

```
C
C     ////////////////////////////////////////////////////////////
C
      SUBROUTINE LAMDA(XL,XLAMDA)
C
C     SUBROUTINE FOR FINDING ROOT OF EQUATION BY BISECTION ALGORITHM
C
      F(XLAMDA) = SIN(XLAMDA*XL)*COSH(XLAMDA*XL) -
     % SINH(XLAMDA*XL)*COS(XLAMDA*XL)
C
C     ------------------------------------------------------------
C     F(X) = 0.0 IS THE FUNCTION
C     A AND B ARE THE LIMITS WITHIN WHICH THE PROGRAM SEARCHES THE
C     SOLUTION OF THE EQUATION, F(X) = 0
C     EPS IS THE ACCEPTABLE TOLERANCE
C     ------------------------------------------------------------
C
      EPS = 0.00001
      A=0.00015
600   B=A+0.005
      IF(F(A)*F(B).LT.0.0) GO TO 700
      A=B
      GO TO 600
700   DO 100 I=1,15
      P=A+(B-A)/2.0
      IF(F(P).EQ.0.0.OR.(B-A).LT.EPS) GO TO 200
      IF((F(A)*F(P)).GT.0.0) GO TO 300
      B=P
      GO TO 100
300   A=P
100   CONTINUE
200   XLAMDA=P
      RETURN
      END
```

```
C
C     ////////////////////////////////////////////////////////////////
C
      SUBROUTINE MAXY(XLAMDA,XL,AK,YMAX,X)
C
C     SUBROUTINE TO CALCULATE MAXIMUM DEFLECTION FUNCTION
C
      Y(X) = ABS(SIN(XLAMDA*X)-AK*(SINH(XLAMDA*X)))
      X=0.0
      DELTA=XL/100.0
      DO 100 I=1,100
      XNEW=X + DELTA
      IF(X.GT.XL) GO TO 1000
      Y2=Y(XNEW)
      Y1=Y(X)
      IF((Y2-Y1).LT.0.0) GO TO 200
      X=XNEW
100   CONTINUE
200   YMAX=Y(X)
1000  RETURN
      END
```

```
C
C     ////////////////////////////////////////////////////////////
C
      SUBROUTINE MAXSIG(XLAMDA,XL,AK,SIGMAX)
C
C     SUBROUTINE TO CALCULATE MAXIMUM STRESS FUNCTION
C
      SIGMA(X) = ABS(-SIN(XLAMDA*X) - AK*(SINH(XLAMDA*X)))
      X=0.0
      DELTA=XL/100.0
      DO 100 I=1,100
      XNEW=X + DELTA
      IF(X.GT.XL) GO TO 1000
      SIG2=SIGMA(XNEW)
      SIG1=SIGMA(X)
      IF((SIG2-SIG1).LT.0.0) GO TO 200
      X=XNEW
100   CONTINUE
200   SIGMAX=SIGMA(X)
1000  RETURN
      END
```

@RU

FASTFOR (CONVERSATIONAL VER 10)

SIMPLE-FIXED UNIFORM PIPING

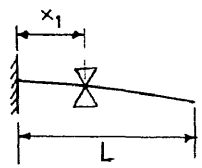
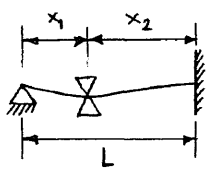
LENGTH	FREQ. COEFF	DEF. COEFF	STRESS COEFF
2.0000	1589.7540	0.0000	1.757
2.4000	1103.9880	0.0000	2.530
2.8000	811.1003	0.0000	3.444
3.2000	621.0010	0.0000	4.498
3.6000	490.6646	0.0001	5.693
4.0000	397.4353	0.0001	7.028
4.4000	328.4609	0.0001	8.505
4.8000	276.0000	0.0002	10.121
5.2000	235.1698	0.0002	11.878
5.6000	202.7746	0.0003	13.776
6.0000	176.6378	0.0004	15.814
6.4000	155.2498	0.0005	17.993
6.8000	137.5207	0.0007	20.312
7.2000	122.6650	0.0008	22.772
7.6000	110.0935	0.0010	25.373
8.0000	99.3606	0.0013	28.115
8.4000	90.1206	0.0015	30.995
8.8000	82.1168	0.0019	34.019
9.2000	75.1300	0.0022	37.181
9.6000	68.9998	0.0026	40.484
10.0000	63.5913	0.0031	43.929

CHAPTER XIII

APPENDIX - F

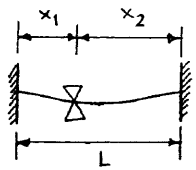
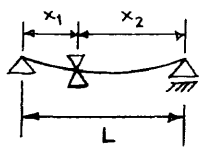
TABLE FOR MAXIMUM STATIC DEFLECTION FORMULAE

TABLE FOR MAXIMUM STATIC DEFLECTION [7]

END CONDITIONS	Δ_1^*	x_{max}	Δ_2^*
	$\frac{P\,x_1^2\,(3L-x_1)}{6EI}$	L	$\frac{W\,L^4}{8EI}$
	$x_1 < (0.414)\,L$ $\frac{P\,x_1\,(L^2-x_1^2)^3}{3EI(3L^2-x_1^2)^2}$ $x_1 = (0.414)\,L$ $\frac{0.0098PL^3}{EI}$ $x_1 > (0.414)\,L$ $\frac{Px_1x_2^2}{6EI}\cdot\sqrt{\frac{x_1}{2L+x_1}}$	$\frac{L(L^2+x_1^2)}{3L^2-x_1^2}$ $(0.414)\,L$ $L\sqrt{\frac{x_1}{2L+x_1}}$	$\frac{W}{48EI}\,(3Lx_{max}^3 - 2x_{max}^4 - L^3x_{max})$

* - Multiply by $(12)^3$ to obtain deflections in inches

TABLE FOR MAXIMUM STATIC DEFLECTION [7]

END CONDITIONS	Δ_1^*	x_{max}	Δ_2^*
	$x_1 < (0.5)\ L$ $\left(\frac{2}{3}\right)\cdot\left(\frac{P}{EI}\right)\cdot\left(\frac{x_1^2\ x_2^3}{(L+2x_2)^2}\right)$ $x_1 = (0.5)\ L$ $\frac{PL^3}{192EI}$ $x_1 > (0.5)\ L$ $\left(\frac{2}{3}\right)\cdot\left(\frac{P}{EI}\right)\left(\frac{x_1^3\ x_2^2}{(L+2x_1)^2}\right)$	$L - \frac{2x_2L}{(L+2x_2)}$ $(0.5)\ L$ $\frac{2x_1L}{(L+2x_1)}$	$\frac{Wx_{max}}{24EI}\ (2Lx_{max} - L^2 - x_{max}^2)$
	$x_1 \leqslant (0.5)\ L$ $\frac{P\ x_1\ (L^2-x_1^2)^{1.5}}{9\sqrt{3}\ EIL}$ $x_1 > (0.5)\ L$ $\frac{P\ x_2\ (L^2-x_2^2)^{1.5}}{9\sqrt{3}\ EIL}$	$L - \sqrt{\frac{L^2-x_1^2}{3}}$ $L - \sqrt{\frac{L^2-x_2^2}{3}}$	$\frac{Wx_{max}}{24EI}\ (L^3 - 2Lx_{max}^2 + x_{max}^3)$

* - Multiply by $(12)^3$ to obtain deflections in inches

CHAPTER XIV

APPENDIX - G

SYMBOLS USED IN COMPUTER PROGRAMS

The following symbols are used in the computer programs presented in this thesis for the various arbitrary constants and variables appearing in the theory.

Symbol/Name used in Computer Program	Variable/Constant appearing in the theory
A	A_1
AK	$K_{1,1}$ (Uniform span)
AK1	$K_{1,1}$
AK2	$K_{2,1}$
AM	Ratio ($m/\rho a$) or (P/W)
C1	$C_{1,1}$
C2	$C_{2,1}$
D	D_1
DF	DLF
E	E
FR	f_1
NS	Schedule Number
OD	D_o
P	W
PN	Nominal pipe dia.
R	R_1
R1	$\bar{x}_1$

Symbol/Name used in Computer Program	Variable/Constant appearing in the theory
R2	x_2
SD1	Δ_1
SD2	Δ_2
SIGMA	$\bar{\sigma}_1$
SIGMAX	$(\sigma_1)_{max}$
STRESS	$(\sigma_1)_{max}$
U	U
W	P
X	x
X1	x_1
X2	x_2
XI	I
XL	L
XLAMDA	λ_1
XMAX	x_{max}
XSMAX	location of $(\sigma_1)_{max}$
XYMAX	location of $(Y_1)_{max}$
Y	$\bar{Y}_1$
YMAX	$(\bar{Y}_1)_{max}$

CHAPTER XV

REFERENCES

(1) Warburton, G. B. - "THE DYNAMICAL BEHAVIOUR OF STRUCTURES" - Pergamon Press, 1964, pp. 12, 79-120

(2) Chen, Yu - "ON THE VIBRATIONS OF BEAMS OR RODS CARRYING A CONCENTRATED MASS" - Transactions of the ASME, June 1963, pp. 310-311

(3) Bishop, R. E. D. - "THE MECHANICS OF VIBRATION" - Cambridge University Press, 1960, pp.282-406

(4) Timoshenko, S. - "VIBRATION PROBLEMS IN ENGINEERING" - 4th Edition, John Wiley and Sons, Inc., 1974, pp. 415-431

(5) Thomson, W. T. - "THEORY OF VIBRATIONS WITH APPLICATIONS" - 2nd Edition, Prentice-Hall Inc., pp. 218-221

(6) Kaplan, W. - "ADVANCED MATHEMATICS FOR ENGINEERS" - Addison-Wesley Publishing Company, 1981, pp. 179-216

(7) Roark, R. J. - "FORMULAS FOR STRESS AND STRAIN" - 5th Edition, McGraw Hill Book Company, Inc., pp. 96-97

(8) Burden, R. L. - "NUMERICAL ANALYSIS" - 2nd Edition, Prindle, Weber and Schmidt, 1981, pp. 21-25

(9) ITT, GRINNELL - "PIPING DESIGN AND ENGINEERING" - ITT Grinnell Industrial Piping, Inc., pp. 10-11, 177-185

(10) THE M. W. KELLOGG COMPANY - "DESIGN OF PIPING SYSTEMS" - John Wiley and Sons, 1956, pp. 261-194

(11) Baker, W. E. - "VIBRATION FREQUENCIES FOR UNIFORM BEAMS WITH CENTRAL MASSES" - Transactions of the ASME, June 1964, pp. 335-337