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Title of Thesis: One-Dimensional Compressible Flow Analysis - Isentropic And Normal Shock.

Su-Bo Wong, Master of Science in Mechanical Engineering 1983.

Thesis Directed by: Rong-yaw Chen
Professor of Mechanical Engineering

A computer program to compute and analyse one-dimensional isentropic compressible flow through variable cross-sectional area with or without a normal shock is developed. The program is written in the "FORTRAN LANGUAGE".

In this work, the area change is the predominant cause of change of flow condition. One of the advantage of this program is set on general uses for isentropic flow. In common practice, the values of the isentropic flow property ratios were tabulated or graphically presented as function of the Mach number with a specified specific heat ratio, $K$ (normally $K=1.40$ was presented). With todays technology, the most versatile method is by implementation of computer programming method.

The computer program presented can solve all the onedimensional isentropic flow problems and to analyse the flow characteristic and the flow patterns in converging nozzle and
converging-diverging nozzle. The value of $K$ can be assigned as any value as one's requirement. All the solutions are computed within $0.1 \%$ error. For solving Mach number and location of normal shock inside the nozzle, ITERATION method is employed instead of numerical method. In most cases, a few iterations (less than ten) may arise a reasonable solutions.

by<br>Su-Bo Wong

Thesis submitted to the Faculty of the Graduate School of the New Jersey Institute of Technology in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering 1983

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## TAELE OF CONTENTS

Chapter Fage

1. INTRODUCTION ..... 1
2. ANALYSIS ..... 6
2.1 Fundamental Concepts ..... 6
2.2 Classification of Compressible Flow. ..... 7
2.3 Assumptions on The Flow Through a Normal
Shoc: Wave ..... 8
2.4 Governing Equations. ..... 8
(A) For Isentropic Flow ..... 8
(B) For Normal Shock Wave ..... 11
2.5 Ferformance of Converging Nozzle ..... 13
2.6 Ferformance of Converging-Diverging Nozzle ..... 16
3. METHOD OF SDLUTIONS ..... 22
उ. 1 Introduction ..... 22
4. 2 Techniques of Frogramming the Frogram ..... 23
3.3.1 Frogram (1) ..... 28
3.3. 2 Frogram (2) ..... 29
3.3.3 Frogram (3) ..... 29
5. CONCLUSION ..... 33
6. FECOMMENDATIONS ..... 35
AFFENDIX A Fortran IV Compliter Frogram. ..... 38
AFFENDIX B Sample of Example ..... 60
REFERENCES ..... 64

| a | Speed of sound |
| :--- | :--- |
| A | Cross-sectional area of duct |
| h | Specific enthalpy |
| K | Specific gas ratio |
| M | Mach number |
| $\dot{\mathrm{M}}$ | Mass flow rate |
| MF | Mass flux $=\dot{\mathrm{M}} / \mathrm{A}$ |
| P | Static pressure |
| R | Specific heat constant |
| s | Specific entropy |
| T | Static temperature |
| V | Velocity of the fluid flow |
| V | Specific volume |
| X | Position coordinate |
| P | Density |
| max | Maximum |
| O | Stagnation condition |
| * | Condition at M = 1 |

## 1. INTRODUCTION

Fluid mechanics is that study of fluid motion involving a rational method of approach based on general physical laws and consistent with the results of modern experimental study. There is hardly a branch of engineering that is not concermed with fluids or does not make use of them. Real economy and value are achieved in studying at one time the same principles underlying the flow of different fluids. Such a study tends to develop a sound background and to make one versatile in approaching new problems.


#### Abstract

A fluid may be considered as compressible or incompressible. As an ekample in the compressible flow, when the relative velocity of the fluid with respect to the immersed body became high. the results of the analysis for the pressure coefficient began to depart from that based on incompressible flow. Also, when the speed of the fluid relative to the immersed body approached the speed of sound in the flowing medium, owing to the compressibility of the fluid, the deviation from incompressible flow analysis become pronounced.


The fact that the flow is compressible indicates that the density, $\rho$ of the flowing medium is a sensitive function of the pressure. The introduction of this new variable $\rho$ into the equations of motion will necessitate the study of its
relationship to the other properties of the medium. This need then invokes the principles of thermodynamics which represent a separate and independent approach from the dynamic equations of motion. Since a new variable $\rho$ has been added; this independent approach is necessary for the solution of the problem, and the concepts of thermodynamics will play an important role in the theory of compressible flow.

Friction, heat transfersarea change and electromagnatic fields have a great effect on compressible flow. In most physical situations: more than one of these effects occur simultaneously; for example, flow in a rocket nozzle involves area change, friction, and heat transfer. However, one of the effects is usually predominant: in the rocket nozzle, area change is the factor having greatest influence on the flow. The frequent predominance of one factor provides a justification for separating the effects, including them one at a time in the equations of motiong and studying the resultant property variations.

Whereas a certain loss of generality is incurred by treating each of the effects individually, this procedure does simplify the equations of motion so that the results of each of the effects can be easily appreciated. Further, this simplification enables approximate solutions to be derived for a wide range of problems in compressible flow: such


#### Abstract

solutions are sufficiently accurate for many engineering applications. Attempts to include all the effects simultaneously in the equations of motion lead to mathematical complexities that mask the physical situation. In many cases exact solutions to these generalized equations of motion are impossible.


This study is concerned with compressible, isentropic flow through varying area ducts. such as nozzles passages. Friction and heat transfer are negligible for this isentropic flow: variation in properties are brought about by area change. One-dimensional steady flow of a perfact gas is assumed in order to reduce the equetions to a workable form. Since this is a study of gas flows, changes in potential energy and gravitational forces are neglected.

The intention on this study has been to provide a good understanding of the physical behaviour of compressibe fluid flow and an adequate appreciation of the principles behind the design of modern engines such as nozzles and their equivalent form a very important item of all turbine and jet devices. In this work, a computer program is implemented to solve most of the engineering problems arising from isentropic compressible flow.

In general, the changing variable properties are To,fo, T,F:
$\mathcal{P O}_{0}, \mathcal{P}, \mathrm{M}, \mathrm{V}, \mathrm{m} / \mathrm{A}(\mathrm{MF}), \mathrm{K}, \mathrm{F}$. Since $K$ and F are the properties of the gas and usually listed in most of the science or engineering data booklets. The remainder nine properties at which three properties must be given in order to obtain the rest of the variable properties, this gives the initial cross-section (station 1) properties. The program may proceed further to consider any other sections of the duct with no discontimuities or shock wave occurs inbetween. since the stagnation properties are remains constant at any sections of the duct, by giving one properties from T,F, $M_{3} \cup, \rho, A R(A 1 / A 2$ - area ratio), the other properties can be obtained.


#### Abstract

To shows the great effect of mach number and wisely needed in engineering applications: converging nozzle and converging-diverging nozzle were chosen to demonstrate on this study. The program is constructed to consider a fluid stored in a large reservoir and is discharged through a converging nozzle; by varying the back pressure to analyse the characteristics and compute the properties of the fluid flow at the exit plane. For converging-diverging nozzle, it is desired to consider the pressure distribution in the nozzle over a range of values of Fb/Po with Po maintained constant (except across the shock wave). The phenomenon of the choked or unchoked, shock wave occurs inside or outside the nozzle, overexpansion, underexpansion or designed condition: all those situations will be detected and printed


at the output of the solutions:

The main fundamental concepts are discussed in Chapter 2 . which include the analysis: governing equations for isentropic flow and flow across the normal shoct wave and the features of the fluid flows through the converging nozzle and converging-diverging nozzle.

Chapter 3 is the method of solutions which explain the techniques of writting this program and the procedures of inputing the data.

Conclusions are presented in Chapter 4 and suggested recommendations are in Chapter 5 . Appendix - As is the computer program and Appendix - $E_{9}$ is the sample of examples. Finally, the References.

## 2. ANALYSIS

### 2.1 Fundamental Concepts

Four basic laws can be readily applied in studying the flow of compressible fluids. These fundamental laws or principles upon which the analysis presented in this study depend directly or indirectly are :
(A) The law of conservation of mass
$\rho \vee A=$ constant
(B) Newton's second law of motion
$1 / \rho d p+V d V=0$
(C) The first law of thermodynamics
$n+v^{2} / 2=$ conatant
(D) The second law of thermodynamics
$T d s=d h-v d p$

In any flow analysis some information about the properties of the fluid must be known. This information (such as equation of state of a perfact gas ) is used in connection with the basic laws to provide maximum knowledge of the flow. Compressible flow can be treated at various levels of complexity. In this work, the most elementary level possible was taken and the results by virtue of their simplicity, are among the most instructive.

The basic approximations are that the flow is steady and one-dimensional or: more precisely: steady and quasi-onedimensional. By this we mean that the flow can be treated according to a one-dimensional model even though the "real" flow is in fact three-dimensional.
2.2 Classification of Compressible Flows

There are large differences in flow patterns with compressible flows. General behaviour of the flow depends on whether the fluid velocity is greater or less than the local velocity of sound. Thus, the compressible flows may be classified as follows:
(1) $M<1$---- Subsonic flow:
(2) $M=1 \quad-\infty-$ Sonic fiow
(3) $M>1$---- Supersonic flow.

A transonic flow is defined as a flow having regions in which the flow speed changes from subsonic to supersonic. For example, transonic flows can occur in convergingdiverging nozzles and in flow over bodies.

A hypersonic flow is a supersonic flow at high mach number Soften defined as a flow whose mach number is greater than 5) . Hypersonic flows are so called because they require treatment somewhat different from low mach number supersonic flows.
2. 3 Assumptions on The Flow Through a Normal Shock Wave

The assumptions are made as follows:
(1) The boundary surface forming the stream tube is far removed from the boundary layers adjacent to any solid surface. Since all friction forces may be assumed to be confined to the shearing stresses in the boundary layer: the configuration under discussion is a frictionless duct.
(2) The shock process takes place at constant area; that is, the streamlines forming the boundary of the stream tube are parallel.
(3) The shock wave is perpendicular to the streamlines.
(4) The flow process, including the shock wave is adiabatic: no external work is performed, and the effects of body forces are negligible.

### 2.4 Governing Equations

(A) For Isentropic Flow:

The ratio of the speed of the fluid at a point to the local speed of sound at that point is a useful index for identifying the flow. this ratio is called the Mach number, M:

$$
\begin{equation*}
M=\frac{V}{a} \tag{1}
\end{equation*}
$$

For isentropic flow,

$$
a^{2}=\left(\frac{\partial \mathrm{p}}{\partial \rho}\right)_{\mathrm{s}}
$$

Where subscript 5 denotes constant entropy.
For an ideal gas under isentropic process $F^{-\mathrm{K}}=$ const.
Thus,

$$
\begin{equation*}
a=\sqrt{\left(\frac{\partial P}{\partial \rho}\right)_{s}}=\sqrt{\frac{K P}{\rho}}=\sqrt{K R T} \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
\because \quad M=\frac{V}{\sqrt{\mathrm{KRT}}} \tag{3}
\end{equation*}
$$

The stagnation temperature, To is the temperature where the speed of the fluid is zero. With the energy equation for isentropic flow, we get

$$
\begin{equation*}
T_{0}=T\left(1+\frac{K-1}{2} M^{2}\right) \tag{4}
\end{equation*}
$$

The stagnation pressure Fo and the stagnation temperature To are related by

$$
\begin{equation*}
\frac{P_{0}}{P}=\left(\frac{T_{0}}{T}\right)^{\frac{K}{K-1}} \tag{5a}
\end{equation*}
$$

$$
\begin{equation*}
\text { or } \quad \frac{P_{0}}{P}=\left(1+\frac{K-1}{2} m^{2}\right)^{\frac{K}{K-1}} \tag{5b}
\end{equation*}
$$

The stagnation or total density, $\rho$ is the density corresponding to the stagnation temperature and pressure.

From the thermal equation of state.

$$
\begin{align*}
P o & =\rho_{0 R T}  \tag{6a}\\
\text { or } \quad P & =\rho_{R T} \tag{6b}
\end{align*}
$$

Substituting equations (4) \& (5) into equation (6) yields

$$
\begin{equation*}
\frac{\rho_{0}}{\rho}=\left(1+\frac{K-1}{2} M^{2}\right)^{\frac{1}{(K-1)}} \tag{7}
\end{equation*}
$$

The continuity equation may be transformed to read

$$
\begin{equation*}
\frac{\dot{\mathrm{M}}}{\mathrm{~A}}=\rho_{\mathrm{V}}=\frac{\mathrm{PV}}{\mathrm{RT}} \tag{Ba}
\end{equation*}
$$

Substituting equation (S) into equation (Ba) yields

$$
\begin{align*}
\frac{\dot{\mathrm{M}}}{\mathrm{~A}}=\frac{\mathrm{PV}}{\mathrm{RT}} & =\frac{\mathrm{PV}}{(\mathrm{KRT})^{1 / 2}}\left(\frac{\mathrm{~K}}{\mathrm{RT}}\right)^{1 / 2} \\
& =P M\left(\frac{\mathrm{~K}}{\mathrm{RT}}\right)^{1 / 2} \tag{Bb}
\end{align*}
$$

From equation (4), we have

$$
T=\frac{T_{0}}{\left(1+\frac{K-1}{2} M^{2}\right)}
$$

Hence in terms of the stagnation temperature To equation (Bb) becomes

$$
\begin{equation*}
\frac{\dot{M}}{A}=P M\left[\frac{K}{R T_{0}}\left(1+\frac{K-1}{2} M^{2}\right)\right]^{1 / 2} \tag{8c}
\end{equation*}
$$

From equation (Sb), it follows that

$$
P=P O\left[1+\frac{K-1}{2} M^{2}\right]^{-\frac{K}{K-1}}
$$

Hence, the continuity equation expressed in terms of the stagnation pressure Fo for the actual flow becomes

$$
\begin{equation*}
\frac{\dot{M}}{A}=\operatorname{MPo}\left(\frac{K}{R T o}\right)^{1 / 2} \cdot\left[1+\frac{K-1}{2} M^{2}\right]^{\frac{-(K+1)}{2(K-1)}} \tag{id}
\end{equation*}
$$

or in terms of $T$, we have

$$
\begin{equation*}
\frac{\dot{M}}{A}=\operatorname{MPo}\left(\frac{K}{R T}\right)^{1 / 2} \cdot\left(1+\frac{K-1}{2} M^{2}\right)^{\frac{K}{1-K}} \tag{Be}
\end{equation*}
$$

(B) For Normal Shock Wave:

In below is the application of the equations developed in the above to normal shock wave.

Since To is a constant, therefore equation (4) gives

$$
\begin{equation*}
\frac{T_{2}}{T_{1}}=\frac{1+\frac{k-1}{2} M_{1}^{2}}{1+\frac{K-1}{2} M_{2}^{2}} \tag{9}
\end{equation*}
$$

From equation (ga) ${ }^{2}$, as $\dot{M} / A=$ Const. : we can derived that

$$
\begin{equation*}
\frac{\mathrm{V}_{2}}{\mathrm{~V}_{1}}=\frac{\rho_{1}}{\rho_{2}}=\frac{\mathrm{P}_{1} \mathrm{~T}_{2}}{\mathrm{P}_{2} \mathrm{~T}_{1}} \tag{10}
\end{equation*}
$$

Substituting equation (3) into equation (10) gives

$$
\begin{equation*}
\frac{P_{2}}{P_{1}}=\frac{M_{1}}{M_{2}}\left(\frac{1+\frac{K-1}{2} M_{1}^{2}}{1+\frac{K-1}{2} M_{2}^{2}}\right)^{1 / 2} \tag{11}
\end{equation*}
$$

Introducing the momentum equation,

$$
\begin{equation*}
P_{1}+\rho_{1} v_{1}^{2}=P_{2}+\rho_{2} v_{2}^{2}=\text { constant } \tag{12a}
\end{equation*}
$$

or $\quad P_{1}\left(1+\frac{1_{1} V_{1}^{2}}{P_{1}}\right)=P_{2}\left(1+\frac{\rho_{2} V_{2}^{2}}{P_{2}}\right)$
From equation (2), noting that $a^{2}=K P / P$ equation (12b) yields

$$
\begin{equation*}
\frac{\mathrm{P}_{2}}{\mathrm{P}_{1}}=\frac{1+\mathrm{KM}_{1}^{2}}{1+\mathrm{KM}_{2}^{2}} \tag{13}
\end{equation*}
$$

Equating the right-hand sides of equation (11) and (13), We can obtained that

$$
\begin{equation*}
\frac{M_{1}\left(1+\frac{K-1}{2} M_{1}^{2}\right)^{1 / 2}}{1+K M_{1}^{2}}=\frac{M_{2}\left(1+\frac{K-1}{2} M_{2}^{2}\right)^{1 / 2}}{1+K M_{2}^{2}} \tag{14}
\end{equation*}
$$

Squaring both sides of equation (14) and solving for M2 yields

$$
\begin{align*}
M_{2} & =M_{1}  \tag{15a}\\
\text { or } \quad M_{2}^{2} & =\frac{M_{1}^{2}+\frac{2}{K-1}}{\frac{2 K}{K-1} M_{1}^{2}-1} \tag{15b}
\end{align*}
$$

Equation (15a) expresses the trival result that no change occurs across the normal shock wave: that iss the latter has infinitestimal strength. Equation (15b) gives the solution for a normal shock wave of finite strength.

For a normal shoct: wave to occur, the mach number, M1 must be greater than unity and equation (15b) shows that as M1 in front of the normal shoct wave is increased infinitely, the mach number, M2 in back: of the normal shock: wave continually decreases but approaches the limiting value $\sqrt{(k-1) / 2 k}$. The limiting value for M2 depends only on the specific heat ratio for the gas. For air and diatomic gases. $k=1.40$ and $\sqrt{(1:-1) / 2 \mathrm{G}}=0.378$. (Note that the computer program presented is permitted to put any value of $k$ as one"s necessity).

The property ratios across a normal shock wave may be expressed in terms of the mach number ahead of the shock wave M1 by substituting equation (15b) into equations (9), (10) and (13) to eliminate M2. Thus

$$
\begin{equation*}
\frac{T_{2}}{T_{1}}=\left(\frac{2 K}{K+1} M_{1}^{2}-\frac{K-1}{K+1}\right)\left[\frac{K-1}{K+1}+\frac{2}{(K+1) M_{1}^{2}}\right] \tag{16}
\end{equation*}
$$

$$
\begin{align*}
& \frac{\rho_{2}}{\rho_{1}}=\frac{(K+1) M_{1}^{2}}{2+(K-1) M_{1}^{2}}  \tag{17}\\
& \frac{P_{2}}{P_{1}}=\frac{2}{K+1} M_{1}^{2}-\frac{K-1}{K+1}  \tag{18}\\
& \frac{P_{0}}{P_{0}}=\left\{\left(\frac{K-1}{K+1}+\frac{2}{(K+1)_{M_{1}^{2}}^{2}}\right)^{K} \cdot\left[\frac{2 K}{K+1} M_{1}^{2}-\frac{K-1}{K+1}\right]\right\} \frac{-1}{K-1} \tag{19}
\end{align*}
$$

It follows from equation (17) that as the mach number M1 approaches infinity, the density ratio $\rho / \rho_{0}$ approaches $(k+1) /(k-1)$ as a limiting valuen (i.e. $=$ G. for $k=1.40$ ). On the other hand, the pressure ratio F2/F1 (see equation (18) ) increases contimuously with M1.

The flow conditions immediately in front of and in back of the normal shock wave are isentropic, thus all the governing equations derived for isentropic flow can be applied to those sections.

### 2.5 Ferformance of Converging Nozzle

Consider the configuration shown in fig.2.1., where a simple converging duct discharges into a region where the back pressure Fb is controlled by a valve. Let fe be the pressure in the exit plane of the nozzle and fo the reservoir pressure $($ stagnation pressure for isentropic flow).

fig.2.1. Flow Through a Converging nozzle

If $\mathrm{Fb} / \mathrm{Fo}=1$ (at condition 1 , $\mathrm{fig}: 2.1$ ) then the pressure i 5 constant throughout the nozzle and there is no flow. If fb is slightly reduced from this value (condition 2, fig.2.1) a subsonic flow will be established with the exit pressure Fe equal to the bact: pressure Fb. If Fb is reduced further to condition 3 , the flow remains subsonic with fb=Pe, but the mass flow rate increases. This increase continues until Fb/Fo reaches the critical pressure ratio (Fb/Fo = F'Fo = Fe/Po, where Me $=1$, i.e. condition 4 . If Fb is reduced further: (condition 5 ), the pressure fe can not become less than $F$ since Me stays at 1. Therefore the flow rate remains constant and pressure distribution inside the nozzle remains the same as for condition $4 . \quad$ The pressure distribution in the chamber outside the mozale for $\mathrm{Fb} / \mathrm{Fe}$ can not be predicted accurately by a one-dimensional model and is indicated by a wavy lime.

The maximum mass flus: (MF)max can be found from equation (8c) with Me = 1, gives

$$
\begin{equation*}
\left(\frac{\dot{M}}{A}\right)_{\max }=\operatorname{Po} \sqrt{\frac{K}{\operatorname{RTo}}\left(\frac{2}{K+1}\right)^{\frac{K+1}{K-1}}} \tag{20}
\end{equation*}
$$

and from equation (5b),

$$
\begin{equation*}
\frac{P^{*}}{P_{0}}=\left(\frac{2}{K+1}\right)^{\frac{K}{K-1}} \tag{21}
\end{equation*}
$$

For $k=1.40, \quad F * / F O=0.528 Z$
2.6 Ferformance of Converging-Diverging Nozzle

Consider the case in which a fluid stored in a large reservoir is to be discharged through a converging-diverging nozzle. The experiment can be carried out similar to the above case (converging nozzle) by controlling the valve to vary the back pressure. Fb. It was pointed out that for certain ratios of back pressure to supply pressume. isentropic, one-dimensional solutions to the equations of motion are not possible. However, it is sufficient to analyee the normal shock occured inside or at the exit plane and studied in details. It is desired to find the pressure distribution in the nozzle over a range of values of fosfoy with Fo maintained constant. (see fig.2. ふa)


## fig. 2.3 a

With Fb Fo, there is no flow in the nozzle. As Fb is reduced below Fo: subsonic flow is induced through the nozzle with pressure decreasing to the throat then increasing in the diverging portion of the nozzle. When the back pressure is lowered to that of curve 4 sonic flow
occurs at the nozzle throat. Further reductions in back: pressure can induced no more flow through the mozzle. As the bact: pressure is reduced below that of curve 4; a normal shock appears in the nozzle just downstream of the throat (curve a ). Further reductions in back pressure cause the shock to move downstream (curve b), until for a low enough back pressure, the normal shock positions itself at the nozzle exit plane (curvec ) Consider in detail a curve of $F$ versus $X$ with a shock in the nozzle (fig.2.3b). the static pressure decreases in the converging portion of the nozzle: with $M=1$ at the throat. In the diverging portion. with the flow supersonic, the pressure continues to decrease up to the normal shock: After the shock: flow in the diverging part of the nozzle is subsonic: the static pressure increasing to the exit plane pressure. With subsonic flow at the exit. the exit plane pressure is equal to the back pressure.

fig. 2. 3 b

As the back pressure is lowered below that of curve c: a shock wave inclined at an angle to the flow appears at the exit plane of the nozzle (fig.2. Zc ). This shoct wave,
weaker than a normal shoct: is called an oblique shocki. Further reductions in bacti pressure cause the angle between the shock and the flow to decrease, thus decreasing the shock: strength (fig.2. $3 d$ ) until eventually the isentropic case, curve 5 is reached. Curve 5 corresponds to the design condition in which the flow is perfectly expanded in the nozzle to the bact: pressure.


Po

fig.2.3c

fig.2.3d

For back pressure below that of curve 5 , exit plane pressure is greater than the back: pressure. A pressure decrease occurs outside the nozzle in the form of expansion waves (fig.2.Je), bulique shock waves and expansion waves represent flows which are not one-dimensional and can not be treated directly with the equations derived before.


Po

fig.2.3e
It is important to realize that for all back pressures below that of curve $c$ the flow adjusts to the back pressure outside the nozzle. over this range of back pressures
(below c): flow inside the nozzle remains unchanged as the back pressure is varied. For example, the exit plane pressure and exit velocity are the same for all back pressures below c. If a rocket nozzle is designed to operate isentropically at sea level, the rocket exhaust velocity and exit plane pressure do not change as the rocket moves upwards through the atmosphere (assuming constant chamber temperature and pressure).

Fig.2.3f depicts the variation of exit plane pressure with back pressure. For subsonic flow at the exit plane (curves $1,2,3,4, a, b, c$ ), and for the design condition (curve 5), the exit plane pressure is equal to the back pressure. For supersonic flow at the exit plane (curve d, 5 , e) the exit Plane pressure is equal to that for the design condition. For back pressure between $c$ and 5 , the exit plane pressure is less than the back pressure, so that the nozzle is termed overexpanded. For back pressure below 5, with the exit plane pressure greater than the back pressure, the nozzle is termed underexpanded.

Fig.2.3g shows the variation of pressure at the throat with back pressure, when back pressure is in between 1 and 4 , the pressure at throat varies with back pressure (i.e. unchoked condition). Till back pressure equals or less than 4 , choked condition, Ft always remain at F (i.e. critical stage).

fig. 2. $3 f$


Summarising, there are four regimes of flow:
(I) Subsonic flow throughout the duct, maximum velocity is reached at the throat.
(II) Subsonic flow at the throat, then supersonic up to the normal shock; following by subsonic compression.
(III) Subsonic flow to the throat, followed by supersonic flow to the exit plane. Non-isentropic re-compression outside the duct though oblique shock waves.
(IV) Flow in the duct identical to (III), supersonic jet expanding out of the nozzle exit.

The nozzle is choked in regimes (II), (III) \& (IV). The mass flow rates $M$ is independent of the back pressure and is a maximum. Only in regime (I) can the mass flow rate be changed by variation in the back pressure.

## 3. METHOD OF SOLUTIONS

## 3. 1 Introduction

By use of the governing equations developed in Chapter 2 we can express the flow property ratios (T/To. F/Fo, etc.) in a steady one-dimensional isentropic flow as a function of the locel flow mach number, M for a given value of the specific heat ratioy $k$. In most of the engineering compressible text books, the values of the isentropic flow property ratios are tabulated or grapically presented as function of the mach number for $k=1.40$. It has been a common practice of using table or graphss whenever possible, in solving steady one-dimensional flow problems.

In most cases, the initial conditions at the inlet cross-section of the passage are specified. Using those
 may be read directly from tables or graphs. The reference condition $T o, F O 』 \rho_{0}$, A may than be calculated.

Dne condition at the exit cross-section of the flow passage must be knowns so that tables or graphs may be employed for determine the property ratios, and hence the flow properties at the ewit cross-section. Many problems of the practical


#### Abstract

interest are not that straight-forwardy and may be too complicated for a closed-form mathematical solution. A numerical solution is required and the most versatile method is by implementation of computer programming method. With present-days technology: computer is common and wisely used. It may solves the problem in shortest time with great efficient and high merit of accuracy. In this work: a computer program is developed to solve the isentropic flow problems instead of traditional method by using tables or graphs to compute and analyse the flow pattern. Also; it can be used for any different values of specific heat ratio, k:

Dne of the advantages of setting the program on general purpose is that many of the solution steps are common to all application area. For instant, the procudure for solving protalems in converging or Eonverging-diverging nozale involve many of the same steps found in general program: ine. for given three properties and compute the remainder properties.


## Z. 2 Techniques of Frogramming the Frogram

A genaral computer flow diagram is presented in fig. $\mathrm{I}_{\mathrm{g}}$. 1 . This flow diagram is specifically for the one-dimensional isentropic flow with variable cross-sectional area and is

fig.3.1. A General Flow Diagram
valid for all of the application areas discussed in Chapter 2. Each major step of the flow chart will be discussed in genaral terms rather than with respect to a specific example.

Firstly, the general program was established. In general, the changing variable flow properties are To,Fo,T,F: $\rho, \rho_{0}$ My $V_{9} M F$ and $F i$ Since $K$ and $F$ are the properties of the ges and usually listed in most of the text books: The remainder nine properties at which three must be given to make the solution possible.

From the knowledge of mathematical statistics, if one is interested only in what particular objects are selected when $r$ objects are chosen from $n$ objects (where $r$ and $n$ are any arbitrary numbers , without regard to their arrangement in a line, then the unloaded selection is called a "COMEINATIDN ".

Employing the formula with $r=3$ and $n=9$ :
Combination formula:

$$
\begin{aligned}
&\binom{n}{r} \\
&=\frac{n}{r!(n-r)!} \\
& \text { i.e. } \quad\binom{9}{3}=\frac{9!}{3!(9-3)!}=84 .
\end{aligned}
$$

From the above, it is understood that there are 84 ways of setting the problems.

Mass flux, MF is a most common property that either known or unknown value. This property is taken as a reference and divide the program into two parts. i.e. (1) Given two properties with mass flux, MF: or (2) Given three properties without MF.

Using the same combination knowledge, we get in case (I) is 28 ways and 56 ways in case (II). Due to certain properties gives in the combination is exactly the terms of the governing equation, it is unable or should say not enough information to solve the problem. Such as occured in case (I) has two cases and eight cases in case (II). When this situation appears, the output will print " NOT ENOUGH INFORMATION".

The logical If statement is used as a transfer control for the given condition. A pattern of entering the code number is fixed by entering the code number in ascending order.

For example, The coding as follows:

| TO-1 | $T-3$ | $D N-5$ | $M-7$ |
| :--- | :--- | :--- | :--- |
| FO-2 | $F-4$ | $D N O-6$ | $V-8$ |

For given To, Fo, $F$ then enter 124 without any punctuation in between the numbers.

One of the most tedious part in the program is to solve for
$M$ by trial and error. Here, ITEFATION method is used instead of other numerical method, life Newton-Fiaphson method, because numerical method may easily diverged and mat:e the solution impossible.

For instant, given MF, Fo and To from equation (Bc), we have

$$
M F=M \operatorname{Po}\left(\frac{K}{R T_{0}}\right)^{1 / 2}\left(1+\frac{K-1}{2} M^{2}\right)^{-\frac{(K+1)}{2(K-1)}}
$$

Solving for $M$,

$$
M=\frac{M F\left(1+\frac{K-1}{2} M^{2}\right)^{\frac{(K-1)}{2(K-1)}}}{\operatorname{Po}\left(\frac{k}{R T_{0}}\right)^{1 / 2}}
$$

By iteration, the DO-LOOF being employed. The Fortran statement may be written as follows:

DLMMY=FO*SQFT (K/R/TD)
$M=M F *(1+C 2) * * F W 5 / D U M M Y$

DO $111 \mathrm{I}=1,30$
$M F=M F *(1+C 2 * M * M) * * F W 5 / D U M M Y$
$E F=A B S(1-M F / M)$

IF (EF.LT. O. OOOO1) GO TO 1000
$M=M F$

111 CONTINUE

1000

--..--.-.-.

In most casesy only a few iterations ( normally less than 10) may gives the reasonable solution within 0.001 $\%$ error.

Due to many solution steps are common in most of the cases. in preparing the program. one must solve all the 84 cases at one time to search where the solution steps are common. By grouping those steps together to form the SUBROUTINE. In this program, eleven subroutines were employed.

Z. 3 Comments on the Computer Frogram<br>When the program get startedy it prints the NOTATION of the symbols used then follows by select program:<br>(1) Given three properties of a flow in a duct, find the other properties of the flow:<br>(2) Converging nozzle flow from reservoir:<br>(3) Converging-Diverging nozzle.

Sn $\underset{\sim}{ } 1$ Frogram (1)

1. Select either (a) Given mass flux with two properties or (b) Given three properties without mass flue.
2. Input $K$ and $R$.

उ. Input the known properties with the coding specified in ascending order.
4. Input the values of the known properties.
5. The computer may compute the unknown properties at this initial section and print out the solutions.
6. It may proceed further to compute any section of the duct by entering one known properties.
7. The program may feep on repeated until all the section of the required properties are obtained
З. $\because 2$ Frogram (2)

1. Input K and F
2. Input To,Fo and Fb ( Eack pressure)
S. The computer may detect the characteristic of the flow pattern through the nozzle. Fefer to Chapter two. fig.2.1.
(i) $\quad \mathrm{FO}=\mathrm{Fb}$
(ii) Fiegime $I(F o<F b<F(i v)$ )
(iii) Fegime II (Fb \& $F(i v)$ )
$(i v) F B=F(i v)$
3. Frint the properties at the exit plame.

Zus Frogram ( 3 )

1. Input $K$ and $F i$
2. Input To, Fo and $F b$ ( Bact: pressure)
3. Input area ratio between exit and throat (Ae/At)
4. The program is able to detect the charecteristic of the flow pattern. Refer to fig. 3.2 in below :

fig. 3.2
(i) Find Me? and Me4
(ii) Determine Fe?, Fez and Fe4
(iii) Using Logical IF to classify the flow pattern. Generally, it consists of the following casesn ( Note : All the details had been included in Chapter 2 ).
(a) $\mathrm{FO}=\mathrm{Fb}$
(b) Fiegime I (Fo >Fb >Fea)
(c) $\mathrm{Fb}=\mathrm{Fe} 2$
(d) Regime II ( Fe ( $>\mathrm{Fb}$ > Fe ( )
(e) Fb = Pes
(f) Fiegime III (Fes > Fb > Fe4 )
(g) $\mathrm{Fb}=\mathrm{Pe} 4$
(h) Fiegime IV (Fe4 > Fb )

In the above eight cases, themost difficult part of the program is in Fegime II (i.e. to locate normal shock inside
the nozzle). There are many ways of solving this problem. Here, the iteration method is employed to find the area ratio $A R_{\text {: }}$ where the normal shock occurs. The program is constructed as follows:
(i) By averaging the area ratios AR at throat and exit to assume the normal shoct: wave occurs half way between the throat and the exit plane.
(ii) Fromthe throat condition with $M=1$ and the known Fo, To. Using subroutine with estimated AFi to compute the properties before the normal shock occurs.
(iii) Compute the properties after normal shock: wave occurs. (iv) Using subroutine to determine the properties at the exit planex
(v) To compare is the computed exit pressure Fe = Fb?
(vi) (a) If Fe $>\mathrm{Fb}$; take AR as the average of the estimated $A F$ and area ratio at the throat.
(b) If Fe $\& F b$ take $A R$ as the average of the estimated $A F$ and the area ratio at the exit.
(vii) Repeating the procedures by taking the average until Fe within $0.1 \%$ error equals to Fb. In most cases, within ten iterations the solution may arise.
(5) Frint all the necessary solutions. i.e
(i) Throat properties:
(ii) exit plane properties:
(iiii) normal shock, before and after.

The program constructed in the above are three programs
that are commonly needed for studying one-dimensinal isentropic compressible flow with variable cross-sectional area. The program may be modified or extended to fit others requirement.

## 4. CONCLUSION

A computer program to aid the analysis of one-dimensional
isentropic compressible flow through variable cross-
sectional area, and discharges of gas from a reservoir
through a converging or converging-diverging nozzle has been
developed. The program was written in FORTRAN LANGUAGE.

In this work, the area change is the predominant cause of change of flow condition. One of the advantage of this program is set on general uses for isentropic cases. Three application solutions on fluid flow through a variable cross-sectional area duct are described and had demonstrated how those problems being solved. The three applications are :
(1) Three properties of the initial conditions at the inlet cross-section or any particular section of the flow passage are specified, the corresponding properties can be computed and proceed further with one condition given at any section or at the exit cross-section of the flow passage, all other required properties are computed.

A fluid stored in a large reservoir with given fo and To is discharged through a converging nozzle and by varying the back pressure to analyse the features and the patterns of the flow, choked or unchoked and compute the properties at the exit plane. To is discharged through a converging-diverging nozzle. The fluid flow has been carefully considered in all flow patterns,i.e.subsonic or supersonic. It can detect the shock: occured inside, outside or at the exit plane. Furthermore it analyses the flow at design conditiong overexpansion or underexpansion, and choked or unchoked conditions. It also computes all the necessary properties at throat, exit plane, before and after the normal shock wave. Especially, it can locate the position of the normal shock wave occured inside the nozzle.

The computer program presented in Appendex $A$ is not limited to solving these three types of applications; instead it can be easily extended to fit all other applications or any section of the isentropic compressible flow. In genaral, graphs or tables for $K=1.40$ is attached at the textbook for solving isentropic flow. In this program, value of $K$ can be assigned as any value as one's requirement.

## 5. FECOMMENDATIONS

 Nozzle design and operation have been studied up to this point by means of a one-dimensional flow analysis. Although this method of analysis is adequate for the solution of many engineering problems: certain limitations become apparent. For example, in the design of a supersonic nozzle, area ratios can be determined for a given supersonic. Mach number: But the length of the nozzle or the rate of change of area with axial distance cannot be prescribed from one-dimensional flow considerations. Further. due to presence of boundary layers on the nozale walls: the area available to the main flow is somewhat reducedy the areas calculated from a one-dimensional flow analysis may have to be enlarged to account for boundary layers. For an advanced and complete analysis of the operation and design of a converging or converging-diverging nozzle, a study of twoand three-dimensional analysis is required. A good engineering approximations may then be obtained for the solution of a wide range of compressible flow problems.(2) Compressible flow in ducts was analyzed for the case in which changes in flow properties were brought about solely by area change. In a real flow situation, however: frictional forces are present and may have a decisive effect on the resultant flow Eharacteristic. Naturally the inclusion of friction terms in the equations of motion makes
the resultant analysis more complex. For this reasony to study the effect of friction on compressible flow in durts: certain restrictions may be placed on the flow. (at By considering compressible flow with friction in constant-area insulated ducts, which eliminate the effects of area change and heat addition. In a practical sense, these restrictions limit the applicability of the resultant analysis: however, certain problems such as flow in short ducts can be handleds and furthermore an insight is provided into the general effects of friction on a compressible flow. (b) Dealing with the flow with friction in constant-area ducts, in which the fluid temperature is assumed comstant, this approximates the flow of a gas through a long uninsulated pipe line. Thus these two cases cover a wide range of frictional flows and are consequently of great significance.
(3) Flow assumed to be adiabatic and study the effects on a gas flow of area change and friction. Heres the effect of heat addition or loss on a gas flow may be investigated. Flows with heat transfer occur in a wide variety of situations, for example, combustion chambers, in which the heat addition is supplied internally by a chemical reaction, or heat exchangers, in which heat flow occurs the system boundaries.
(4) Furthermore, the investigation may be ventured into the case of flow with applied electric and magnetic fields. Under certain conditions: however, a gas can be made into an electrically conducting fluid, possessing electrical
properties similar to a solid conductor. If such a
conducting gas stream is allowed to pass through a magnetic
field aligned perpendicular to the flow, an electrical field
is induced normal to the flow direction and the magnetic
field. Since the gas is able to conduct electrically, the
electrif field can be used to generate a current between
electrodes placed in the fluid, and this current is able to
produce work through an external load.

## APPENDIX A

## FORTRAN IV COMPUTER PROGRAM

|  |  |
| :---: | :---: |
| C． | MAIN FROGRAM |
| C．．－．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．． |  |
| C |  |
| $C$ | ＊FROGFAM FOF SOLUING ONE－ITMENSIONAL ISENTEOFTC＊ |
| C． | ＊COMFFESSTELE FLOW＊NCLULING FLOW THFOUGH＊ |
| C． | ＊CONUEFGTNG NOZZLEE ANG CONVEFGTNG－TIUEFGTNG＊ |
| 0 | ＊NOZZLI．．E゙＊ |
| C |  |
|  |  |
|  |  |
| Write（2y 1 ） |  |
| d | FOFMAT（IX，TO LEFINE THE：SYMBOLS USEH＊＇） |
|  |  |
| 2 | FOFMAT $/ / /$ y $7 \times$ ，MFF $=$ MASS FLUX＇，／／\％ |
|  |  |
|  |  |
|  | \＄6Xy，M＝MACH NO＊＇y／／\％ |
|  | \＄ $6 \times, \mathrm{T}=\mathrm{STATIC}$ TEMFEFATUFE，／／／\％ |
|  |  |
|  | ¢ $6 X_{y}$ ，LIN＝STATTC MENSTTY＇，／／， |
|  |  |
|  |  |
|  | \＄ $6 \times$ ， $\mathrm{FO}=\mathrm{STAGNATION} \mathrm{FFESSUFE*}, \mathrm{//} \mathrm{\%}$ |
|  | \＄6X，MNO＝STAGNATTON LENSITY＇） |
|  | WFITE（2， 3 L ） |
| 31. | FOFMAT（／／y $2 X$, SEIEET FROGFAM＊＇， |
|  |  |
|  | \＄／y 1 GX，FINX THE OTHEF FFOFEFTIES OF THE FLOW＇y |
|  | \＄／／， $7 X_{*} / 2-$ CONUEFGING NOZZLE FLLOW FFOM FESEFUOTR \％ |
|  |  |
|  | FEEALI ，J |
| 9999 | JFF（JJ，EQ＋1）WFITE（2，10） |
| 10 |  |
|  | \＄6X， 1 －GIUEN MASS FLUX WTTH TWO FFOFEETIES＇，／\％， |
|  |  |
|  | J，$=0$ |
|  | IFF（J1＋E（d＋1）FEEALI y JJ |
|  | WRTTE（2，6） |
| 6 | FOFMAT（／／y $3 \times$ ，ENTEFK K゙ッド＇） |
|  | FEAM，ッドッド |
|  | WFITTE（2，17）バッド |
| 1.7 | FOFMAT（／／， $3 \times$, SFECIFTC HEAT FATIO＝，FI2， $4 . / \%$ ， |
|  | \＄3X，SFECTFIC GAS CONSTANT＝＇，F12，4y／／） |
|  |  |

    FOFMAT(//,3X, ENTEF MF')
    IF (JJ, EQ.1.) FEACI MF
    Clle2/(K- 1)
    \(C 2=(k-1) / 2\)
    \(C 3=(\) k゙-1)/(k゙+1)
    C4 \(2=\) (K゙ +1 )
    FWI: = K- -
    FW2 \(=1 /(k-1)\)
    FWア M K K (
    FW4 \(=(\mathbb{O}-1) / K\)
    FW5=(K+1)/(2*(K゙-1))
    KNOR:=0
    IF ( (J1.EQ.2).OR. (J1.EQ.3)) GO TO 2001
    WFITE (2, 3)
    FOFMAT(//y IX, SELECT THE CONDITIONS :',//,
    क \(3 \times\), ENTERING THE COME OF THE GIUEN CONOITIONS IN ASCENAI
    क NG, OFEEEF:
    



REEAI $N$
WFITE (2,4)N


IF ((J., EQ.1).ANI. (N.GT.100)) G0 T0 9996
TF ((JJ.EQ+2) + ANM + (N+GT.100)) G0 TO 9998
IF ((JJ.EQ.2) +ANI. (N.LT.100)) G0 T0 9996

| IF | （N＋EQ＋12） | G0 | TO | 100 |
| :---: | :---: | :---: | :---: | :---: |
| IF | （N＋EQ＋1．4） | 90 | TO | 150 |
| ］F | （ $N+E Q+23$ ） | G0 | T0 | 200 |
| IF | （N＋EQ＋ 34 ） | G0 | TO | 250 |
| IF | （ $N+E \mathrm{E}+35$ ） | G0 | TO | 300 |
| TF | （N，EQ＋4\％） | G0 | T0 | 350 |
| ］．F | （ $N+E Q+38$ ） | G0 | T0 | 400 |
| IF | （N．EQ．48） | 60 | T0 | 450 |
| IF | （ $N+E Q \cdot 13$ ） | G0 | T0 | 500 |
| IF | （N＋EQ＋1 5 ） | 60 | T0 | 520 |
| IF | （ $N+E$ C +18 ） | G0 | TO | 540 |
| IF | （N＋EQ．78） | G0 | To | 560 |
| IF | （N，EQ ，57） | G0 | T0 | 580 |
| I．F | （N＋EQ，1．7） | G0 | T0 | 600 |
| IF | （N＋EQ＋ 37 ） | G0 | TO | 620 |
| IF＇ | （N，EQ，28） | G0 | T0 | 640 |
| J．F | （N＋EC，25） | G0 | T0 | 660 |
| I．F | （ $N+E Q+24$ ） | G0 | T0 | 680 |
| I．${ }^{-}$ | （N，EQ，27） | GO | TO | 700 |

```
    IF (N+EQ.47) GO TO 720
    IF (N+EQ+67) GO TO 740
    IF (N*EQ*W6) GO TO 760
    IF (N+EQ.68) GO TO 780
    IF (N+EQ.16) GO TO 782
    IF (N+E(N+26) ©O TO 78E
    IF (N+EQ.36) G0 T0 787
    TF (N+EQ.46) GO TO 1111
    IF (N,EQ+EB) GO TO 1111
O FOFMAT(1X,'TNFUT TO,FO')
    FEAMy TO,FO
    CALL TSEN7
    00 T0 222%
150 WFITE(2y15)
1.% FOFMAT(IX,'TNFUT TO,F')
    FEAIM yTOyF'
    M=NF/SORT(1+CN)/(F*SQFT(K゙/F/TO))
    n0 15G I=1, 30
    MF=NF/SQRT(1+C2*M*M)/(F*SOFT(K゙/R/TO))
    EF=ABS(1-NF/M)
    IF (EFF+LT.O.000OI) GO TO 1500
    M=NMF
15% CONTINUE
1500 T=TO/(1+C2*M*M)
    CMLLL ISENTI.
    FO=F*(1+C2*M*M)**FW3
    GOTO 22`2
200 WFTTE (2y20)
OO FOFMAT(IX,'TNFUT FO,T')
    FEAN y FOyT
    M=MF*(1+C2)**FW3/(FO*SQRT (K゙/R/T))
    M0 20G T=1. 30
    MF:=MF**(1+C2*M*M)**FW3/(FO*SQFT(K゙/F゙/T))
    EFF:=ABS(1-MF/M)
    IF(EF+L.T,0.00001) GO TO 2000
    M=:MF'
2OS CONTINUE
2000 TO=T*(1+C,2*M*M)
    CAl..L ISENTI
    F=FO/(1+CO*M隹)**FW3
    G0 TO 2222
2#0 WFITE(2y25)
SF FOFMAT(IX,'TNFUT T,F'^)
    FEAII,T,F
    M=MF/(F*SQFT(N゙/F゙/T))
    CALIL IGENTI.
    FO=FF*(1+C2*M*M)**FW3
    T0=T*(1+C2*M*M)
```

60 TO 2222

2,40
FOFMAT（IX，＇INFUT TyU＇）
FEEAK！y TッU
MN：MF／V
$\mathrm{F}=\mathrm{KIN} * \mathrm{~F} *$ T
M＝U／（ド＊F゙＊T）＊＊O． G

$\mathrm{T} 0=\mathrm{T} *(1+\mathrm{C} 2 * M * M)$
UNO＝WN＊（1＋C2＊M＊M）＊＊FW2
00 TO 222
WFITTE（2，45）
FOFMAT（IX，＇TNFUT F＇，U＇）
FEAII，Fy，V
IN：MMF／V
$T=F /(\operatorname{IN} * F)$
GO TO 46

FOFMMAT（IX，＇INFUT TOyT＇）
FE：ACI y TO：T
TOT＝TO／T
$M=((2 *(T O T-1)) /($ ド… 1．））＊＊0．

LIN二MF／V
CALLI ISENT2
GO T0 2222

FORMAT（IX．＇INFUT TO．IIN＇）
FEAII，TO．DN
V＝MF／TIN
M＝SQFT（U＊U／（（TO＊R゙＊Fi）－（U＊U＊C⿳）））
$\mathrm{T}=\mathrm{TO} /(1+\mathrm{C} 2 * M * M)$
CALL ISENT2
GO TO 2222

```
540 WFTTE(2,54)
F4 FOFMMT(IX*'INFUT TOyU')
    EEAT:,TO,V
    IIN=MF/N
    00 T0 5%
#60 WKTTE(2,56)
FG FOFMAT(1Xy'JNFUT M%U')
    FEAII yM,V
    INN=MF/N
F9 T=U*V/(M*M**゙*F゙)
    T0=T*(1+CO2*隹M)
    CAL..... TSENTב
    60 T0 222%
580 WFTTE (2y50)
GO FORMAT(IX,'TNFUT IN,M')
    FE:AL, TINYM
    V=MF/WIN
    60 T0 5%
600 WFTTE (2.60)
60 FOFMAT(IX, INFUT TOYM')
    FEEALI yTO&M
    CAL.LL TSENS
    60 T0 2人22
    WFITE(2,62)
    FOFEMAT(1X,'INFUT T, M')
    FEAM,T,M
790 T0:T*(1+C2*M*M)
    CALLL TSENB
    G0 TO 2222
    WFITE:(2,64I.)
640
64. EDFMAT(IX,'INFUT FO%V')
    MN=NF//V
662 M=U*(1+C2)**(O.5*FW3)*SQFT(NN/FO/K゙)
    10 644 I=1.30
    MF:=U*(1+C2*M*M)**(O.F*FWZ)*SQFT(INN/FO/KK)
    EF=ABS(1.-MF/M)
    ITF (EFF,LT.0.00001) GO TO 64%
    M=#MF
644 CONTTNUE
645 F=FO/(1+C2*M*M)**FWZ
    T=F/(INN*F)
    TO=T*(1+CD*M*M)
    INO=IIN*(1+C2*M*M)**FW"
    GO TO 2`22
660 WRITE(2y66J)
661 FOFMAT(JX,'INFUT FOO%LIN')
    FEAII, FOO,IN
    V=NF/DIN
    GO TO 662
```

```
680 WFITTE(2.681)
681. FOFMAAT(1X,'INFUT FO,F')
    READ,FO,F
    M=SQRT(C1*((FO/F)**FW4-1))
```




```
    TO=T*(1+C2*M*M)
    MN=N作/N
    DNO=IN*(1+C2*M*M)**FW2
    GO T0 2222
700 WFTTE(2.701)
701. FONMAT(1XX'INFUT FO,M')
FEAIM,FO,M
F=FO/(1+C2*M*M)**FW3
G0 T0 688
720 WFITE(2.721)
721. FOFMAT(IX,INFUT F:M')
    FEALI,F:M
    FG=F*(1+C2*M*M)**F'W3
    00 T0 680
740 WFITE(2,741)
74.1. FOKMAT(IX,INNFUT DNO,M')
    FEADI, LNNOM
    IN=INO/(1+C2*M*M)**FW2
766 V=MF/INN
    T=U*U/(M*位に゙*F)
    TOWT*(1+C2*M*M)
    F==\N*):*
    FOWF*(1+C2*M*M)**FW3
    G0 ro 2222
760 WFITE(2,761)
76.1 FOFMMT(1X,'INFUT IIN, DNNO')
    REAIM YIN,DNO
767 [OU=(LNO/LNN)**(K゙-1)--1.
    M=SQRT(2*IU/(K゙-1))
    60 ro 766
780 WFTTE(2,781)
78. FOFMAT(1X:INFUT LNNO,V')
    FEAL, KNOMO
    INN=MF/V
    00 T0 767
78% WFTTE(2,783)
783 FOFMAT(1X,'INFUT TO,LNO')
    FEAD, TO, LINO
    FO=INO*F*TO
    CALLE TSEN7
    g0 T0 2222
785 WFITE(2.786)
786 FOFMAT(1X,'INFUUT FO,HNO')
    REALI,FO, IINO
```



```
    IF (N.EQ.235) G0 TO 860
    IF (N.EQ.238) GO TO 863
    IF (N.EQ.237) G0 10 866
    TF (N.EO.158) GO TO 868
    IF (N.EQ.157) GO TO 871
    IF (N,EC.258) G0 TO 873
    IF (N.EQ.257) GO TO 878
    IF (N+EQ.268) GO TO 880
    IF (N.E(N,267) GO TO 882
    IF (N+E(Q+358) GO T0 885
    IF (N+E(N.368) 60 T0 889
    IF (N.EQ.237) G0 T0 892
    IF (N.EQ.458) G0 TO 894
    IF (N.EQ.45%) GO rO 896
    IF (N.EE(468) GO TO 898
    IF (N.EQ.467) 60 TO 903
    IF (N.EQ.1.78) G0 T0 1.11.
    IF (N.EQ+378) GO T0 1111.
    IF (N.EQ.478) G0 TO 905
    IF (N+EQ.278) G0 TO 907
    IFF (N.EQ.E78) GO TO 913
    IF (N+EQ.67B) GO TO 910
800 WFITE(2,801)
80% FOFMAT(1X,'INFUT TO.FO.T')
    REALI,TOYFO,T
    M=SQRT(C1*(TO/T-1))
    F=FO/(1+C2*M*M)**FW3
    CALLE ISENJ.
    GO T0 2222
802 WFITE(2.803)
803 FORMAT(1X,'INFUT TO,T,F')
    FEAGO,TOyT,F
    M=SRNT(CH.*(TO/T-\cdots1))
    FO=F*(1+C2*M*M)**FW3
    CALI TSENJ
    G0 T0 2222
    WRITE(2.805)
80S FORIMAT(IX,'TNFUT TO,T,INN')
    READ, TC,T,RN
920 M=SQFT(C1*(T0/T-1))
    F==IN*R*T
    FO=FF*(1+C2*M*M)**F'W3
    CALL ISENI
    G0 TO 2222
806 WFITE(2,807)
807 FORMAT(1X,'INFUT TO,T, INO')
    FEADI, TO,T,MNO
    M=SQNT(C1*(TO/T--1))
    LN=LINO/(1+C2*M*M)**FW2
```

|  | $F=\mathrm{LIN} * \mathrm{~F} * \mathrm{~T}$ |
| :---: | :---: |
|  | CALL ISEN？ |
|  | 60 TO 2222 |
| 808 | WFITE（2，809） |
| 809 | FOFMAT（1Xy ${ }^{\prime}$ INFUT FO，Fy LINO＇） |
|  | FEALI， $\mathrm{FO} \mathrm{O}, \mathrm{F}, \mathrm{LNO}$ |
|  | $M=5 Q F T(C 1 *((F O / F) * * F W 4-1))$ |
|  | CAIIL TSENE |
|  | 00 T0 2222 |
| 810 | WFTTE（2，811） |
| 8 IL | FOFMAT（1X，TNFUT FO，F\％U＇） |
|  | FEALI y FO y $\mathrm{F}, \cup$ |
|  |  |
|  | $\mathrm{T}=\mathrm{U} *$ U／（M＊M＊ば＊F゙） |
|  | $\mathrm{TG}=\mathrm{T} *(1+\mathrm{C} 2 * M * M)$ |
|  | CALI．TSEN1． |
|  | 60 TO 2 se |
| 812 | WFTTE（2，8I马） |
| 813 |  |
|  | FEAII， $\mathrm{FO}, \mathrm{T}, \mathrm{F}$ |
|  | $M=S \mathrm{CFT}(\mathrm{CJ} *($（ $\mathrm{F} \mathrm{O} / \mathrm{F}) * * F W 4-1)$ ） |
|  | TO：$=$ T＊（ $1+\mathrm{CO}$＊M＊M） |
|  | CALL ISENI |
|  | 60 T0 2222 |
| 81.4 | WFTTE：（2，815） |
| 815 | FOFMAT（1X，＇INPUT FO，FyN＇） |
|  | FEAM， FO |
|  | $\left.M=S Q R T\left(C 1 *\left(F^{\prime} \mathrm{C} / \mathrm{F}\right) * * F W 4-1\right)\right)$ |
| 972 | LINO＝IN＊（1＋C2＊M＊M）＊＊FW？ |
|  | CALL ISENS |
|  | 60 T0 2222 |
| 816 | WFETTE（2，817） |
| 81.7 | FOFMAT（1X，＇INFUT F9XINMNO＇） |
|  | FEALI FFMESMO |
|  | M＝SQFT（Ci＊（ 1 INO／LN）＊＊FWI－1） |
|  | $\mathrm{FO} \mathrm{O}=\mathrm{FF} *(1+\mathrm{C} 2 * \cdots * M) * * F W 3$ |
|  | CALL I SENE |
|  | G0 TO 2222 |
| 81.8 | WFITE（2，819） |
| 81.9 | FOFMAT＇（1X，＇INFUT FO，DIN，INNO＇） |
|  | FEACI \％ FO ，LINy LINO |
|  |  |
|  |  |
|  | CALL TSENE |
|  | GO TO 2222 |
| 820 | WFITE（2，821） |
| 821 | FOFMAT（1X，＇INFUT T，LIN，LINO＇） |
|  | FEALI Y T LIN，LINO |
|  | $M=$ SQFTT（C1＊（（LNO／LN）＊＊FWI－1）） |


|  | TO=T* (1+CO*M*M) |
| :---: | :---: |
|  | CALL ISEN2 |
|  | G0 T0 222a |
| 822 | WFITE (2,823) |
| 823 | FOFMAT (IX. INFUT TO. LINy INO') |
|  | FEEATI TO. MIN MINO |
| 922 | $M=S Q F T(C) *((L N O / L N) * * F W 1-1)$ ) |
|  | $\mathrm{T}=\mathrm{TO} \mathrm{T} /$ (1+C)*M*M) |
|  | CALL ISEN2 |
|  | GO TO 2222 |
| 824 |  |
| 82 | FOFMAT (IX, 'INFUT LIN, MNO, U') |
|  | FEAM , MNy MNOy |
|  |  |
|  |  |
|  | $\mathrm{T} 0=\mathrm{Y} *(1+\mathrm{C} 2 *$ 隹 M$)$ |
|  | CALI TSEN2 |
|  | GO TO 2222 |
| 826 | WFTTE (2,827) |
| 827 | FCOFMAT (IX, J.NFUT TOyFOy LiN') |
|  | FEATH YO, FOO, LIN |
|  | CMO=FO/(F*TO) |
|  |  |
|  | CAI..... ISEN3 |
|  | G0 TO 2222 |
| 828 | WRTTE (29829) |
| 829 | FORMAT (JX:'INFUT TOyFOy ${ }^{\prime}$ () |
|  |  |
|  | TNO=FFO/E/TO |
| 841. | CAL..... ISENM |
| 831 | $\mathrm{LN}=\mathrm{LNO}$ / (I+C2*M*M)**FW2 |
|  | CALE TSEN3 |
|  | 00 TO 2222 |
| 832 | WFITE (2,833) |
| 833 | FOFMAT (JX, TNFUT TO,FO,M') |
|  | FEAD, TO,FOMM |
|  | CALL TSENG |
|  | 60 TO 2222 |
| 834 | WKITE (2,835) |
| 835 | FOFMAT(1X, ${ }^{\prime}$ NFUT TO,FOyF') |
|  | FEAL, TO,FO,F' |
| 2004 | CONTINUE |
|  | $M=S Q K T(C 1 *((F O / F) * * F W 4 \cdots 1))$ |
|  | $\mathrm{T}=\mathrm{TO} /\left(1+\mathrm{C}\right.$ * $\mathrm{M}^{(1)}$ M) |
|  | CALL ISENJ |
|  | IF (JI +EQ.2) G0 T0 2005 |
|  | IF (JI+EQ.3) GO TO 2042 |
|  | G0 T0 2223 |
| 836 | WFTTE (2,837) |
| 837 | FOFMAT (IX, 'INFUT T,FP, IINO') |

```
    REAII ,TyFyMINO
    LIN:WF/F/T
    M=SQFT(C1*((INO/LIN)**FW!-L))
    CALLI TSENA
    GO TO 2222
    WFITE(2.839)
    FOFMAT(JX:'TNFUT T,FyU')
    FEAIM yTyFgV
    MN:WF/F゙T
    M=:=U/SQFT(K゙*F゙*T)
840 INNO=LN*(1+C2*M*M)**FW%
    CAl..... ISENA
    Q0 ro 2222
842 WFITE(2.843)
8A3 FOFMAT(IX,'INFUT T,F'M')
    &EAO!TyFyM
    lNN:=F/F/T
    00 T0 840
844 WKTTE(2.845)
84G FOFMAT(1X,'INFUT TOyFyNNO')
    FEATI yO,FymNO
92. FO=\NO*F*TO
    M=SQFT(CJ東((FO/F)**FWA-1))
    T=TO/(1+CO*M*M)
    CALI... TSENL
    GO TO 2222
846 WFITE (2,847)
847 FOFMAT(IX,'INFUT TOyMNOyU')
    FEEALI,TO,INO,V
    FG=#NO*F*TO
    00 T0 84!
848 WWTTE(2,649)
849 FOFMAT(1X,'INFUT TO,INO,M')
    FEAII YO,TNO,M
923 FO=LNNO*F*TO
    INN=TNO/(1+C2*M*M)**FW?
    CALLLISENO
    GO T0 2022
8%O WFTTE(2,851)
8%1 FOFMAT(1X,'TNFUT TO,FgRN')
    FEAIN,TO,F,RN
    T=F/(F*NN)
    M=5QFT(C1*((TO/T)-1))
    FO=F**(1+CO*M*M)**FW3
    CALILL ISENL
    GO TO 222N
852 WFTTE(2,853)
8G% FOFMAT(IX,'INFUT TO,F,V')
    FEAL ,TO,F'V
    CALI. TSENMM
```

```
8#4 FO=F**(1+C2*M*M)**FW3
    r== TO/(1+C2*M*M)
    CALI. ISEN1
    G0 ro 22"2
85% WFTTE(2,856)
8GG FOFMAT(1X,'TNFUT TOyFyM')
    FEEAM y TO,F,M
2OIO CONTTNUF
    T=T0/(1.tC2*M*M)
    GO TO 8:34
857 WFTTE(2,858)
8G8 FOFMAT(IX,'INFUT FO,T,MNO')
    FEAM, y'OY'Y,GNO
    TO=FOO/F/GNO
    M=:SQET(CJ*(TO/T--1))
    F:%FO/(I+CO*M*M)**FW`
    CAL.... TSENI.
    G0 T0 2222
    WFTT'TE: (2,86I)
    FOFMAT(JX,'INFUT FO,T,MN')
    FE:AX,FOyT,GN
    F=:F**IN*T
    M=:SNFT(C1*((FO/F)**FW4\cdots1))
    T0=T*(1.+C2*M*M)
    CALL.ISEND
    O0 T0 2222
    WFTTTE (2,864)
    FOFMAT(1X,'INFUT FO,TyU')
    Fil:=All, FOy,T,U
    M=U/SQFT(K゙*F゙*T)
86G TO=T*(1+CO*M*M)
    00 TO 85%
866 WFETE:(2,867)
86% FOFMAT(IX,'INFUT FO,T,M')
    FEAG,FO,T,M
    GO T0 865
    WFITE(2.869)
    FOFMAT(1X,'INFUT TO,ONyU')
    FEACI ,TO,MNyU
    CAL..L ISENM1
    T=:U*U/(ば*F**M*M)
870 INO=IN*(1+C2*M*M)**FW2
    CALIL ISEN?
    60 T0 2222
    WFITE(2,872)
    FOKMAT(IX,'INFUT TO,INN,M')
    FEAC MG,MNyM
    T=TC)(1+C%*M*M)
    G0 T0 870
873
    WFTTE(2,874)
```

```
874 FOFMMAT(1X,'INFUT FO,MN,V')
    FEAN FOQ,MN&U
    M=:U*(1+C2)**(0.E*FW3)*SQRT(IN/K゙/FO)
    [10 875 T=1.30
    MF=V*(1+CO*M*M)**(O+5*FWW)*SQFT(IN/K゙/FO)
    FF:-ABS(1.-MF/M)
    TFF(EF+LT.O,0000I) GO TO 876
    M:=MF
8%G CONTINUE
876 F==F口O/(1+C2*M*M)**FW3
877 [INO=\NN*(1+C2*M*M)**FW2
    CALLL TSENG
    00 T0 2222
    WFITE::2,879)
    FOFMAT(IX,'INFUT FO, NIN,M')
    FEAII,FOy,INyM
    G0 ro 8%6
    WFTTE (2,881)
    FORMAT(JX,'INFUT FO, LNNO,V')
    FEALI,FO, MNO,V
    TO=FO/F/INNO
    CALL ISENMI
    F=-FO/(1+C2*M*M)**FW3
    IIN=INNO/(I+C2*M*M)**FW2
    CALLL ISENZ
    60 T0 2222
882 WFETE:(2, 883)
8日3 FOFMAT(IX,'INFUT FO&INO,M')
    FEAD,FO, पINO,M
    FWOW/(1+C2*M*M)**FW3
8日4 IM=[NO/(1+C口*M*M)**FW2
    CALLL ISENS
    GO TO 2222
    WFTTE(2,886)
    FOFMMAT (IX,'INFUT T,MNyU')
    FEAN,T, IIN%U
    M=U/SQFT(K゙*F*T)
    T0=T**(1+C工*M*M)
    GO T0 870
887 WFITE:2,888)
888 FOFMAT(IX,INFUT T,IINgM')
    FEALI,T,INNM
    TO=T*(1+C2*M*M)
    GO TO 870
889 WFTTE(2y890)
890 FOFMAT(1X,'INFUT T,INNO,U')
    FEEATI,T,IINO%V
    M=U/SQFTT (ド*FF*T)
891.TO=U*V*(1+C2*M*M)/(N゙*F*M**)
    IN=INO/(1+C2*M*M)**FW2
```

```
    CALL ISEN"
    G0 T0 2222
892 WFTTE (2,893)
893 FOFMAT(1X.'INFUT FO,T,M')
    FEAN, T,FO,M
    GO TO 891
894 WFTTE(2,895)
89% FOFMAT(1X,'INFUT FF,ONyV')
    FEAI, F, INNV
    T=F/F/TMN
    M=:U/SQFT(ド*F゙*T)
    INO=ON*(1+C2*M*M)**FW2
    CALIL ISENA
    00 T0 2222
896 WFTTE(2,897)
897 FOFMAT(IXy'INFUT F,GINyM')
    FEAN,F,HIN,M
    FO=F**(1+C2*M*M)**FW3
    G0 T0 972
898 WFITE(2,899)
899 FORMMT(1X,'INFUT F'INNO,U')
    FEACI,FF,LNOYU
    M=SOFT (1NO)*U*U/(K゙*F*(1+C2)**FFW2))
    MO 900 I=1. y 30
    MF=GQFT(INNO*U*U/(K゙*F*(1+C2*M*M)**FW2))
    EF=ABS(1-MF/M)
    T.F (EF.EO+0.00001) GO TO 901.
    M=N隹:
900 CONTINUE
901. T=U*U/(M*M*F**)
    TO=T*(1+C工*M*M)
902 FO=F*(1+C2*M*M)**FW3
    CAlLL ISENI
    60 TO 2222
903 WFITE(2.904)
904 FOFMAT(IX,'INFUT F',IND,M')
    FEAI, F, LINO,M
    FCl=F**(1+C工*M*M)**FWZ
    GO TO 884
905 WFITTE(2.906)
906 FOFMAT (1X,'INFUT F'gM,V')
    FEALI FFMM,V
    T=U*U/(M*M*F゙*バ)
    T0=T*(1+C2*M*M)
    GOTO 854
907 WFITTE(2.908)
908 FOFIMAT(IX,'INFUT FO,M,U')
    FEAD,FOyM,U
    T=U*V/(M*M*F*ド)
    TO=T*(1+C2*M*M)
```

```
    F=FO/(1+C2***M)**FW3
    CALL ISENI
    GO ro 2222
9.3 WFTTE(2,909)
909 FOFMAT(IX,'JNFUT LIN,M,V')
    FEEM, CNNM,U
    T=U*V/(M*陎下**)
    T0=T*(1+C2*M*M)
    00 ro 870
910 WFTTE(2.911)
911 FOFMAT(IX,INFUT GNO,M,V')
    FEAB y MNOyM, (V
    T=|*V/(M*很に*N)
    go T0 89%.
C - .-................................................................................
C FROGRAM FOR CONUEFGTNG NOZZLEE ANM
C CONVEFGTNG-mIVERGTNO NOZZLE
C
2001 K゙バ=0
    WFITE (2,2003)
2003 FOFMAT(1Xy INFUT TO,FO,FB(BACK FRESSURE)')
    FEAM,TO,FOyFB
    IF (FO,EQ,FB) WFTTTE (2,2002)
2002 FORMAT(//,4X,'㮩 FLUTH IS FEMAIN STATIONAFY ###:'//)
2007 FOFMAT(//,3X,'## UNCHOKEI CONLITION (SUBSONTC FLOW) #
    #*)
2008 FOFMAT(//,3X,'## CRETICAL CONOTTION (SONTC FLOW) ##
    F
2009 FOFMAT(//y 3X,'## CHOKEL CONNTTION ##')
    IF (FO.EQ.FB) GO TO 9997
    IF (FO.LT.FB) 60 T0 9996
    IF (J., E(Q.3) GO TO 2020
    M=1
    CAILI. ISENG
    WFITE: (2,2006)
2006 FOFMAT(//y3X,'### FROFEFTIES AT THE EXIT FLANE: ##',//)
    IF (FE,EQ,F) WFTTE (2,2008)
    TF (FE,LT,F) WFITE (2,2009)
    IF (FB+LE,F) GO TO 2222
    F=FB
    GO TO 2004
200E CONTINUE
    IF ((M.GT.0.9995).ANN.(M+LT+1.0005)) M=1.
    TFF (M,NE,1) GO TO 204O
    CALL TSENG
    WFETTE (2,2008)
    F=FB
    GO TO 2222
2040 WFITE (2,2007)
    00 T0 2222
2020 WFITE(2,202I)
2021 FORMAT(//,IX:INFUT AREA FATIO EETWEEN EXIT % THROAT (AE
/AT)':/()
```

```
        FEALI,AFTO
        AF=ARTO
C
C TO FINI MACH NUMBEFS,MES & MEA AT THE EXIT FLANE
C
    ME2=:((C4+C3)**FWS)/AR
    H0 2030 I=1, %30
    MF:=((C4+MEO*ME2*C3)**FW以)/AR
    EF=#ABS(1-mF/MEO)
    IF (EFF+LT+0+00001) GO TO 2031,
    ME S=MF
2030 CONTINUE
2031. ME4=SQFT((AF***(1/FW5)-C4)*(K゙+1)/(K゙-1))
    10 2032 T=1.30
    MF=SQRT(((MEA&AR)**(1/FWF)-C4)*(N゙+1)/(K゙- I.))
    EF:=ABS(I.-MF/MEA)
    IF*(EFF+LT+O.00001) GO TO 2033
    ME4=:MF:
2032 CONTINUE
2033 CONTINUE
    IF (MES+GT,I) ME:A=MEZ
    IF (ME:4.LT,1) MEO=MEA
    M=MEO
C
C TO METEFMINE FEO,FES,FEA
O
```



```
    IF ((FEZ,GT,FE), ANH, (FE,GT,FE4)) WFITE (2,2O48)
2OB5 FOFMAT(//, 3X, ## SHOCK WAUE OCCUFEG OUYSTME THE NOZZLE
```



```
2O36 FOFMMAT(3X,'非 SHOCK AT EXIT F'INANE ###:,1/)
2O23 FOFMAT(1X,'## THE CFITICAL. FROFEFTIES AT THE THFOAT AFE
    F'{\mp@code{M/人)}
    1/)
2027 FOFmAT(//,3X,*## IESIGN CONIITION ###,//)
2OZ7 FOFMAT(//,3X'## SHOCK INETME ##')
    IF (FE,LT,FEZ) FE=FFEA
```

```
    F=FE
    G0 T0 2004
2O42 CONTINUE
    TF (FB,NE,FE3) GO TO 2047
    WFTTE (2,2036)
    WFTTE (2,2043)
2O43 FOFMAT (//,3X,'## FROFERTIES BEFOFE THE NOFMAL. SHOCK WAV
    E ##:'//)
    WFITE (2y7) MF゙,TOyFOyTyF, LINy,LNO,MyU
    M=SQFT((M*M+CL)/(N゙*C1*M*M-1))
```



```
    F=F1.*CA*&゙*M*M-C3
    IUM=:(K゙+I)*M*M/(2+(K゙-1)*M*M)
    V=\U/TWM
    MNWMNEIWTUM
    T=T*(K゙*C.4*M*M\cdotsC3)*(C3+C4/M/M)
    WFITE (2,2O4A)
2O4A FGFMAT(//, SX,'## FFOFEFTTES AFTEF THE NOFMAL SHOCK WAUE
    ###'y//)
    00 T0 2222
2OA% TF ((FE2+GT+FE) &NM, (FE,GT,FEZ)) GO TO 2049
```



```
    TF (FW, LF:FEO) WRTTE (2, 2O23)
    IF (FE,LE,FFO) M=1
    IF (FB+LEFFEO) CALLN. ISENG
    IF (FG+LE,FEO) 60 ro 2222
    WFITE (2,2O24)
    MF:=MF***F
    CALIN TSEN7
    GO TO 2222
2049 WFTTE (2,2037)
    FF=FO
    AFFF=AFTO
    ARLI..=:I+O
    (10)2050 I=1.40
    M=1
    FO=:FF'
    CAI...I. ISENG
    AK==(AFF+ARL...)/2
    M=SOFT((AF゙**(1/FW5)-CA)*(N゙+1)/(K゙-1))
    10 2056 IL=1.,30
    MF=SQFT(((M*AF゙)**(1/FW5)-C.4)*(K゙+1)/(K゙-1))
    EF=AES (1 -MF/M)
    IF (EFF,LT,O,OOOOI) GOTO 2OG%
    M=MF
2O5G CONTTNUE
205% CONTINUE
    CALII.. ISENG
    MBI=M
    MNOEI =MNO
    TOBI=TO
```

```
    MFBy=MF
    FF=FO
    F1=F
    UBI=U
    LINBI=IIN
    TB1=T
    IF (M.LTT.1) G0 T0 2070
    M=SQFT((M*M+C1)/(K*CIN*M*M-1))
```



```
    F=FF1*C4*R゙*MB1*MB1.-C3
    muM=(K゙+1)*MB1*MB1/(2+(K゙...1)*MB1*MB1)
    V=VBL/DUM
    MN:=MNBL*IUM
    T=TB1*(K*C4*MB1*MBI-C3)*(C3+C4/MB1/MB1)
    FO2=FO
    MB2=#M
    MFB2=推
    F2=F
    UB2=0
    IINE2%INN
    TB2#T
    MF:=MF**AR/ARTO
    CALIL ISEN7
    EF=ABS(1-F/FB)
    TF (EF+LT.0.001) GO TO 2051
2070 TF (F+GT+FE) AFL=AF;
    TF (F,LT,FB) ARE=AR
    M=1
    FO=FF
2050 CONTINUE
    F=FR
2051 WFTTE(2,2071) AFi
2071. FOFMAT(//,3X:'NORMAL SHOCK WAUE OCCURS AT AREA RATIO =',
    F6.3:1/2
    WRITE (2,2043)
    WFITE (2,7)MFB1,TO,FF',TBI,F1,WNEI,WNO,ME1,UB1
    WFITE (2,2044)
    WFITE (2,7) MFBQ,TO,FO2,TB2,F2, LINED, DNOyMB2,UB2
    WFITE (2,2006)
    F=FE
    WFITE(2,7) MF,TO,FO,T,F,GN, LINO,M,U
    WFITE (2,2023)
    M=1
    FO=FF
    CALL ISENG
2222 CONTINUE
    IF (K゙K.E(Q.1) WFITE(2,1200)
    IF (KK.GE,2) WFITE(2,1201) KK゙
1200 FORMAT(//,3X, THE RESULTS FOR INITIAL SECTION ## STATION
    FORMAT(///)}3\times,'RESULTS FOR OTHEFE SECTION. ##STATION',I2,'
    **)
```

```
    WFITEE(2,7) MF,TO,FO,T,F, LIN, MNO,M,U
    FOFMAT(//,GX,'MASS FLLUX =',F12+3,//,
    $6X,'STAGNATTON TEMFEFATUFE=`, F12, 3,//,
```



```
    $6X,'STATTC TEMFEFATUFE=',F12.3,//,
    $6X;'STATTC F'KESSUFE:N', F12.3.//
    $6X,'STATTE MENSTTY:=',FI2.3.//%
    $6X;'STAGNATION LENSTTY=',F12.3.//,
    $6X,'MACH NUMEEF:=',F12,3,//,
    $6X, 'LOCAL FLUTH UELOCTTY=',F12,3,//)
    IF ((JI,EQ,2),OF+(J1,EQ.3)) GO TO 9997
    GO TO 998
1111 WFTTE(2,1112)
I112 FOFMAT(/, 2X,'NOT ENOUGH INFOFMATION TO GET ALI.. THE FFOFE
    RTIES')
    GO TO 9997
    WFITE(2,999)
```



```
        ')
    FEARI y, J
    TF (J.EQ.O) GO TO 9997
C
C
C
C
1.91.9
#
998
999
7
```



```
    GTUEN ONE FROFEFTY FFOM ANY CFOSS--GECTION TO
        COMFUTE THE OTHEF UNKNOWN FFOFERTIES
    WFITE (2,1919)
    FOFMAT(//, 2X,'FINO FFOFEFTIES AT OTHEF SEETTON',//)
    K゙K゙#ドN゙な1
    WFITEE(2,5J.)
    FOFMAT(IX,'THE GTUEN FROFERTY IS',//,
    $6X,'1 - T2',6X,4 - M2',//
    $6X,'2 - F2',6X,'5 ---V2',//
    $6X,'3 - INN2',5X,'6 -- AF(AJ/A2)',//)
    FEAM y,N
    TF (.J2+EQ+I) GO TO 1203
    TF (J2+EQ+2) GO TO 120G
    IF (J2+EQ+3) GO TO 1207
    TF (J2+EQ.4) GO TO 12O9
    TF (JN+EQ.G) GO TO 1225
    IF (J2+E(Q.6) GO TO 1212
1.203 WFTTE (2.53)
53 FOFMAT(IX,'INFUT T2')
    FEAIN T
    GO TO 920
1205 WFITE (2,1206)
1206 FOFMAT(1X,'INFUT F2')
    FEALI %F
    GO TO 921
1207 WFITE (2,57)
57 FOFMAT(IX,'INFUT IINO')
    FEAII MIN
    00 T0 922
```

```
1209 WFITTE (2,1210)
1210 FOFMAT(1X.'INFUT M2')
    FEAI! y M
    G0 T0 92%
1225 WFTTE(2y61)
61. FOFIMAT(1X:'INFUT V2')
    FEAD, Y
    GOTO 841
1212 WFTTE(2,1213)
1213 FOFMAT(IX,'INFUT AR(A1/A2)')
    FEAI!,AF
    MF=MF**AF
    CAl.L. IGEN7
    GO TO 2222
9996 WFITE (2.9995)
999G FOFMAT (//,3X,'WFONG INFOFMATION: CHECK THE INFUT IAATA A
    GAIN.')
9997 CONTINUE
        STOF:
        END
C.-..................................................................................................
C SUBFOUTTNE FFOCRAMS
C...........-.................................................................--........-----
    SURFOUTINE ISENTI.
    COMMON UyIN, INO,K゙,F,MyMF,TO,T,F,FO,C2,FW2,FW3,FWE
    FEEAL..*4 MF:MgK゙作'
    U=M*SQRT(ド*F゙*T)
    INN=MF/V
    INO=LN*(1+C2*M*M)**FW2
    FETTUFIN
    ENTI
    SUEROUTINE ISENT2
    COMMON V,LN,LINO,K゙,Fi,MyMF,TO,T,FFFO,C2,FW2,FW3,FWE
    FEAL*4 MFyMッドッMF
    F=IN*F*T
    FC]=F*(1+C2*M**)**FWZ
    INO=LIN*(1+C2*M*M)**FW2
    FETUFIN
    ENII
    SUEFOUTTNE ISENMI
    COMMON U,IN, INO,K゙,F゙,M,HF,TO,T,F,FO,C2,FW2,FW3,FWE
    FEAL_*A MF,Mッド,MF
    M=U*SQFT((1+C2)/(*゙*F*TO))
    IO 830 I=1.30
    MF=U*SQFTT((1+C2*M*M)/(ド*F**TO))
    EF:=AES(1-MF/M)
    IF(EF.LT.0.00001) G0 TO 912
    M=MF
830 CONTINUE
91.2 CONTINUE
    FETUFIN
```

```
ENO
SUBFOUTINE ISEND
COMMON U,IN,MNO,K゙ッF,M,MF,TO,T,F,FOO,C2,FW2,FW3,FWE
FEAL*A M:K゙,MF
INN=F/(F゙水T)
INO=FO/(F*TO)
V=H*SQFT(バ*F゙*T)
MF==GN*V
FETUFN
ENO
SUBFOUTTNE TSENO
COMMON U,IN,GNO,K,F,M,MF,TOyT,F,FO,C2,FW2,FWZ,FWS
FEAL..*4 MッK゙ッMF
F:=WN*F;*T
FO=INO*FFTO
U=M゙&SQFT(バ*F゙*T)
MF=:IMN*U
FETUF:N
EN\I
SUEROUTINE ISENB
COMMON U,LN,INN,N゙,F,M,MF,TO,T,F,FO,C2,FW2,FW3,FWF
FEEAL*& MッK゙ッMF
T=TO/(1+C2*M*M)
F:=[以N*F*T
V=W*SQFT(N゙*F*T)
MF=mN*U
FETUFN
ENLI
SUBFOUTINE ISENA
COMMON V,NN,TINO,K,F,M,MF,TO,T,F,FO,CO,FW2,FW3,FWG
FEAL_*A MッドッMF
TO=T*(1+C2*M*M)
FO=INO*F*TO
U=M*SQFT(N゙*F*T)
MF=IN*V
FETUFIN
ENII
SUEFOUTINE ISEN:S
COMMON U,IN, LNO,N゙ッF,MッMF,TO,T,F,FO,C2,FW2,FW3,FWG
FEEAL*& MッK゙ッMF
T=F%/(F**IN)
TO=FO/(F*INNO)
V=M*SQFTT(K゙*Fi*T)
MF=INN*V
FETURN
ENII
SUBFOUTINE ISENG
```



```
FEAL.*4 MFyMyK゙yMF
MNC=FO/F/TO
```

```
    MN=MNO/(1+C2***M)**FW2
    CALI. TSENZ
    FE:TUFN
    ENO
    GUBFOUTTNE ISEN7
```



```
    FEALN*A NF,MッドッMF
    IUMMY=FO*SQRT (N゙/F/TO)
    M=MF**(I+CD)**FWG/HUM件Y
    M011. T=1,30
    MF:=MF**(1+CD*M*M)**FW5/GUMMY
    EF\cdots=ASS(1.-MF/M)
    TF(EF.LT.O.0000N) GO TO 1000
    H二贝刂゙
111 CONTINUE
1000 T=TO/(1+C2*M*M)
    CALL TSENT1
    F=FO/(1+C2*M*M)**FW3
    FETUFN
    ENI
    SUBROUTTNE TSENB
    COMMON U,MN, CNO,N゙ッF゙ッMッMF,TO,T,F゙,FO,CO,FW%,FWZ,FWE
    FEALI*4 MF゙yMッドッMF
    T=T0/(1+C2*M*M)
63 V="M*SQFT(バ*F*T)
    MN=MF/V
    CALIL TSENT?
    FETUMRN
ENII
```

Sample of Example
Problem : Air flows through a frictionless adiabatic converging-diverging nozzle. The air stagnation temperature and pressure are 500 K and $7.0 \times 10^{5} \mathrm{~N} / \mathrm{M}^{2}$ respectively. The diverging portion of the nozzle has an area ratio between the exit plane and the throat ( $\left.A_{e} / A_{t}\right)$ of 11.91. The back pressure at the exit plane is controlled to be at $2.2623 \times 10^{5} \mathrm{~N} / \mathrm{M}^{2}$. Analyse the flow characteristic and calculate all the properties of the flow. Assume $K=1.40$ and $R=287.04 \mathrm{~J} / \mathrm{KG}-\mathrm{K}$.

Results obtained from computer: (All in SI UNIT)

```
**FASTFOR (CONUFRGATIONAL, UEF 9)**
```

TO REFTNE THE SYMBOLS USEG :

```
MF= MASS FLuX
F = SFEC+HEAT CONST
K = SFEC.GAS RATIO.
M = MACH NO.
r = STATTC TEMFEFATURE
F= STATIC FRESSURE
INN = STATIC MENSITY
v= LOCAL FLUTR UELOCTTY
TO = STAGNATION TEMFERATURE
FO= STAGNATJON FRESGURE
LNO= STAGNATION DENSTTY
```

```
    SELECT FFOGFAM :
                            1. -. GUVN THFEE FROFERTIES OF A FLOW IN A TUCT,
                FINIM THE OTHEF FFOFEFTIES OF THE FL.OW
            2 -- CONUEKGTNG NOZZILE FLOW FROM FESEFUOTR
            3-CONUEFGTNG\cdotsMDEFGTNG NOZZIE
    *3
    ENTEF K゙y隹
*1.4,287.04
    SFECIFIC HEAT FATTO=: 1.4000
    SFECIFIC GAS CONSTANT= 287.0400
    INFUT TOyFO,FE(BACK FFESGUFE:
*500,700000.226230
INFUT AFEA FAOTO BETWEEN EXIT & THFOAT (AE/AT)
*11.9%
```


## \＃\＃SHOCK゙ INSTME \＃：\＃

```
NOFMAL SHOCK WAVF：OCCUFS AT AFEA FATTO＝ 4.239
\＃\＃：FFOFEFTTES EEFORE THE NOKMAL．SHOCK WAUE ：\＃：
```

```
MASS FLUX =: 298+482
```

MASS FLUX =: 298+482
STAGNATION TEMFEFATUFE= = FOO.000
STAGNATION TEMFEFATUFE= = FOO.000
STAGNATION FRESSURE=700000.000
STAGNATION FRESSURE=700000.000
STATIC TEMFEFATUFE:=- 1.78.491
STATIC TEMFEFATUFE:=- 1.78.491
STATIC FFESSUFE= 1.9026.570
STATIC FFESSUFE= 1.9026.570
STATIC LHNSTYY= 0.371
STATIC LHNSTYY= 0.371
STAGNATION MENSITY=
STAGNATION MENSITY=
1. }60

```
                                1. }60
```

MACH NUMBEF:= $\quad 3.001$
LOCAL FLUIX VELOCITY= 803.741
\#: FROFERTIES AFTER THE NORMAL GHOCK WAUE \#\#

```
MASS FLUX =: 298.482
```

STAGNATION TEMFEEATUREE =: $\quad 500.000$
GTAGNATTON FFESSURK: $=229633.900$
STATIC TEMFEFATURE: $\quad 478.400$
STATIC FFESSURE= 199918.500
STATHC MENSTTY = 1.433
STAGNATION RENSTTY= $\quad 1.600$
MACH NUMBER= $\quad 0+475$
IOCAL. FLUTM UFIOCTTY= 208. 32 E
\#\# FROFEFTTES AT THE EXIT FLANE \#\#
MASS FLUX $=\quad 106.233$
STAGNATTON TEMPERATURE $=\quad 500.000$
STAGNATION FRESGURE: $229633+900$
STATIC TEMFERATURE= 497.756
STATIC FFEGSURE= 226230.000
STATC DENGITY= 1.582
STAGNATION DENSITY= 1. 600
MACH NUMEER= 0.150
LOCAL FLUTH VELOCTTY=
\#\# THE CRITICAL FROFERTEES AT THE THROAT ARE :

```
MASS FLUX = 1265.208
STAGNATION TEMFEFATURE == FO0.000
STAGNATION FRESSUFE:" 700000.000
STATIO TEMFEFATURE= 416.667
STATIC FRESSURE== 369797.000
STATTC DENSTTY= 3.092
STAGNATTON DENGTTY=
    4.8%7
MACH NUMEFF= 1.000
LOCAL..FLUTM VFLGOTYY%:
    409+194
```


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