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THIXOTROPIC PROPERTIES OF WHOLE HUMAN BLOOD

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THIXOTROPIC PROPERTIES OF WHOLE HUMAN BLOOD

BY

JEN AN SU

A DISSERTATION

PRESENTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE

OF

DOCTOR OF ENGINEERING SCIENCE IN CHEMICAL ENGINEERING

AT

NEW JERSEY INSTITUTE OF TECHNOLOGY

Newark, New Jersey
1980

APPROVAL OF DISSERTATION

THIXOTROPIC PROPERTIES OF WHOLE HUMAN BLOOD

BY

JEN AN SU

FOR

DEPARTMENT OF CHEMICAL ENGINEERING
AND
CHEMISTRY

NEW JERSEY INSTITUTE OF TECHNOLOGY

BY

FACULTY COMMITTEE

APPROVED:__________________________CHAIRMAN

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NEWWARK, NEW JERSEY

MAY, 1980

ABSTRACT

The rheological property of whole blood from various human subjects was studied with a Weissenberg Rheogoniometer, modified with a continuously variable speed drive. Experimental data showed a hysteresis loop in the shear stress versus the shear rate plot and a torque-decay in the shear stress versus the shearing time plot which is under a constant shear rate. The rheological equation previously developed by Huang was employed to define the thixotropic parameters of each whole blood sample based upon the recorded rheograms.

The altered thixotropic parameters for the blood from patients with open heart surgery at different clinical stages were quantitatively determined. The viscosity by non-Newtonian contribution during the stage of cardiopulmonary bypass showed tremendously high values for the expired patients. Effect of temperature on the blood from normal healthy adults may imply a particular thixotropic temperature existing in a thixotropic system, at which the thixotropic properties reach minimum. It also reveals that $37^{o}C$ is the optimal temperature for human subjects at normal physiological conditions. The rheological behaviors of blood affected by normal linear alkanols were mainly determined

by the solubilities of alkanols in water and chemical speciality of the red blood cell. Both hydrophilic and hydrophobic alkanols tended to increase blood thixotropic properties. Amphiphilic alkanols increased blood thixotropy at low concentration and, hemolysed blood to a Newtonian fluid at high concentration.

A theoretical analysis of the artifacts of the torsion head to the experimentally obtained rheograms of torque-decay curve and hysteresis loop demonstrated the dynamic behaviors of the torsion head as well predicted the real rheograms of the tested fluid. The experimental data which have been proved from the study are true hemorheological properties and involve no artifacts.

This investigation has shown the significance of the rheological test of whole human blood. It can be developed as a clinical test, which will supply diagnostic information beyond the standard clinical tests available at this time.

DEDICATION

To my wife, Shuh, whose inspiration, encouragement and understanding made it possible; and to my son, Hansen, who has made it worthwhile; and to my parents who taught me the value of education.

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ACKNOWLEDGMENT

The auther wishes to express his appreciation to Professor Ching-Rong Huang as my principal advisor for sponsoring and directing this study; and to Professor D. Kristol, my second advisor for numerous suggestions.

Many thanks are due to St. Barnabas Medical Center at Livingston, New Jersey and North New Jersey Blood at East Orange, New Jersey, which have provided the auther blood samples for this work.

LIST OF FIGURES

CHAPTER I

INTRODUCTION

To induce flow of a fluid, a force must be exerted on the fluid so that the viscous forces of mutual attraction between molecules are overcome and the molecules are displaced relative to each other. Rheology is the study of deformation and flow of matter. The physical property that characterizes the flow of simple fluids is the viscosity. The equation that describes the relationship between shear stress and shear rate is called the rheological equation of the fluid at the particular state.

Numerous empirical equations, or models have been proposed to express the steady-state relation between shear stress and shear rate depending on the rheological properties of the fluid at the state. The simplest one which describes the rheological properties of fluids is the Newtonian model.

$$\tau_{r\theta} = -\eta \dot{\gamma}_{r\theta} \qquad (I.1)$$

where $\tau_{r\theta}$ is the shear stress, $\dot{\gamma}_{r\theta}$ is the shear rate and, η the viscosity, a constant.

However, there are many materials that flow but for which the viscosity is not a constant and these are called non-Newtonian fluids. They need two or more rheological parameters to describe their rheological behaviors such as the Bingham model, the Ostwald-de Waele model, the Eyring model (all being two parameter models) and the Ellis model, the Reiner-Philippoff model (all being three parameter models) (1), of which the apparent viscosities are not constants as those of Newtonian fluids but a function of shear rate or shear stress. All these models are mainly these dealing with fluids with a monotonic rheological properties, for which there is one definite shear stress associated with each value of the shear rate.

Among the non-Newtonian fluids, there are certain materials which have more complicated rheological behaviors. Their viscosities are not just a function of shear rate but also a function of time. This usually indicates that the fluid processes a structure transformation while a mechanical disturbance is induced to the system. The amount of structure change is dependent on the energy (shear rate) that is forced into the system and how long a time the mechanical disturbance acts on the system. The thixotropic material is characterized by an isothermal, reversible structural breakdown due to the action of a mechanical disturbance on the material. The thixotropic system exhibits the following rheological properties (2):

1. A torque-decay curve (Fig.I.1) will be generated if a suitable single step change shear rate is induced to the system.

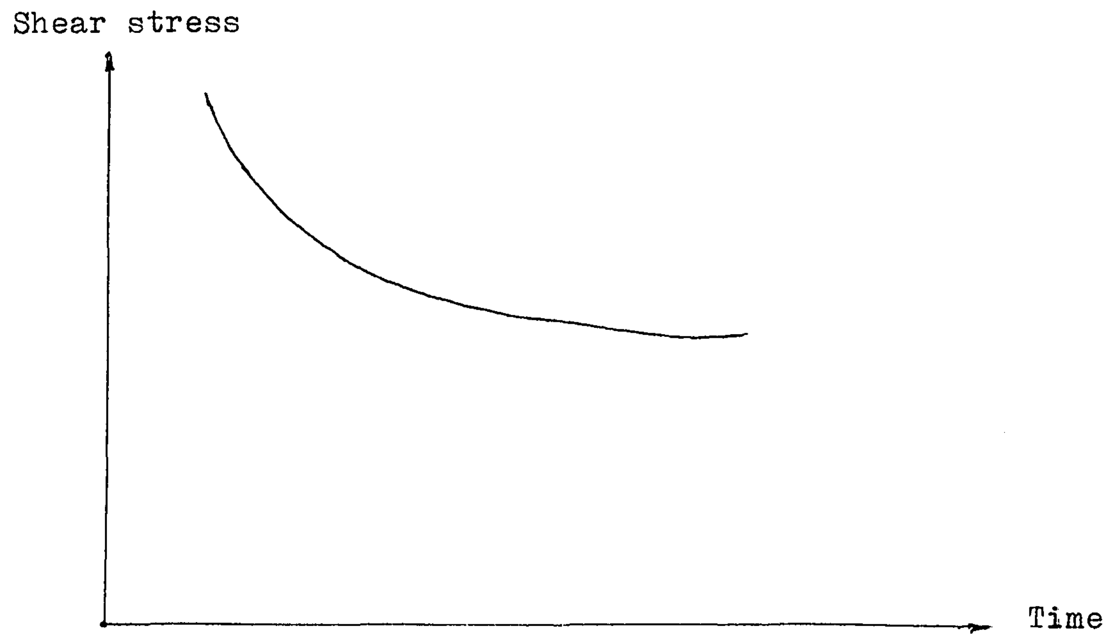


Fig.I.1 Torque-decay curve

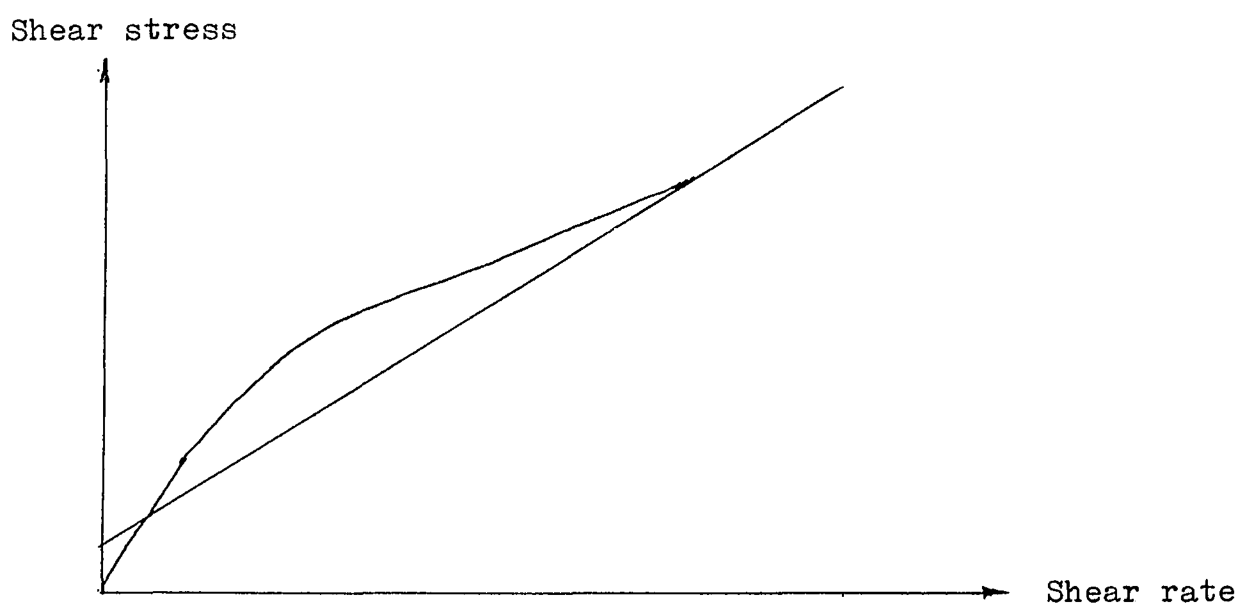


Fig.I.2 Hysteresis loop

2. A hystersis loop will happen (Fig.1.2.) if a triangular step change shear rate is induced to the system.

3. A shear-thinning phenomenon will be brought up if the mechnical disturbance continues as the 2 above. That means the hystersis loop will become smaller and smaller and, finally turn to pseudoplastic behaviors.

4. Once the mechanical disturbance is removed, the system will recover to its original structure after certain time. This means that the thixotropic material has memory.

5. The system may have or may not have yield stress.

Since Freundlich introduced thixotropy in 1928, many investigators (47 to 53) have been attracted to study it both empirically and theoretically. Most of equations available in the literature are unable to explain the various thixotropic phenomena. Based on irrevesible thermodynamic principles,using a molecular arrangement parameter to describe the structure breakdown of thixotropic materials during shearing Huang (2) derived an equation containing five parameters which is suitable to describe the rheological behaviors of thixotropic materials. The five parameters can be used to characterize the flow properties of the whole thixotropic system and, simultaneously to avoid the variations in results due to the varying conditions of tests at different shear rates.

From a macroscopic point of view, human blood is a concentrated suspension of formed elements (primarily red cells, white cells, and platelets) in plasma (3). The plasma in turn is a colloidal suspension of the plasma proteins (mainly serum albumin, serum globulins, and fibrinogen) in a weak electrolyte of composition. The predominant formed elements are the red cells, which are typically in the form of biocave disc about 8μ in (42) diameter when relaxed, but which can undergo very severe deformations. They typically make up about 93% by number of the formed elements or about 40 to 45% of the blood volume, and they have very pronounced effect on blood rheology. The platelets are considerably smaller and are present to the extent of only about 5% of the red-cell volume. They have little direct effect on rheology, but play an important role in clotting. The white cells or leucocytes occur in a relatively small numbers, and are of little direct importance in rheology. The major plasma protein, fibrinogen that make red cells tend to aggregate, also play an important effect on the rheological properties of blood.

Recently it was confirmed that blood is a nearly Newtonian fluid at sufficiently high shear rates which can usually be considered so in arterial flow. However at low shear rates which are of most clinical importance in the microcirculation where shear rates tend to low, blood is a complex thixotropic system (37).

Basically, the bulk of thixotropic properties of the blood appear to be due to the reversible aggregation of the red cells. The degree of aggregation of the red cells may be affected due to a number of causes such as physical (shear rate, time, force fields, etc.), chemical (hematocrit, fibrinogen, chemicals, etc.), and pathological (heart diseases, diabetes, etc.) reasons. One of these investigations which has been done in this study is relative to open heart surgery. An extensive research on its pathogenesis attracted many able investigators, and the biochemistry and histology have been studied. Yet at the same time, little attention has been paid to the role of the fluid which has been driven through the heart, and particularly the studies of the altered rheological properties of the blood from patients at various clinical stages were also neglected. Effects of temperature and chemicals on blood rheology are also very significant. Based on the Huang model, the rheological parameters generated from these investigations on the blood under different pathological, chemical, and physical conditions appear to be truly meaningful and valuable.

Theoretical study showed (9) that a Newtonian fluid would generate a hysteresis loop provided that a triangular step shear rate change was induced to the system. The artifact of rheograms due to the influence of torsion head should be eliminated, or at least minimized in order to obtain most accurate experimental results from the system. Thus, to understand the dynamic behaviors of torsion head, a theoretical analysis of the system is necessary, which will provide some critical information in the arrangement of torsion head, and in determining the accuracy of experiments.

The significance of this theoretical analysis is its ability to predict the real rheograms of the tested fluid. The experimentally rheological data for whole blood from this investigation, which have been proved by this theoretical analysis are true hemorheological properties without any artifacts.

CHAPTER II

THEORY

1. The Huang Model - Thixotropic Fluids

Since Freundlich first introduced the term thixotropy in 1928, various attempts have been made, over the years, to define " thixotropy ". Most of the model available in the literature (47, 48, 49, 50, 51) are only specific with respect to a particular thixotropic property. Others (52, 53) which are more general require too many constants to be evaluated, are not quantitative, or have not been rigorously tested.

Recently it has been confirmed that blood, mainly due to the rouleaux formation of red cells, is a thixotropic fluid. Much of the work on blood and its components involves quite advanced mathematics as well a considerable knowledge of haematological terms (3). Casson's equation could be only applied at low shear-rates, and especially for bovine blood which will not form rouleax. Based on an unsuitable assumption that the red cell was a rigid spherical particle and the rouleaux were broken into two equal parts by shearing, Murata (4) theoretically studied the effect of rouleaux on the non-Newtonian viscosity of blood at low shear rates. His derivation finally resulted in an equation much the same as Casson's equation.

More recently, Thurston (5) used a generalized Maxwell model containing N relaxing spring-dashpot combinations to describe the viscoelastic behaviors of blood. This model would induce uncertain rheological parameters for different blood under same experimental conditions, and brought difficulties to explain the physical meaning of blood thixotropy. Bureau et al. (6) only chose the acceleration constant and the time at the maximum shear rate as parameters from a triangular step change shear rate to correleate the hysteresis shape of blood. Again, these two experimental parameters do not reveal any thixotropic meaning of blood.

Based on irreversible thermodynamic principle as mentioned red cell rouleaux dissociation which is the basis of the model Huang (2), who introduced a molecular arrangement parameter, generalized a rheological equation for time-dependent and time-independent non-Newtonian fluids. This equation containing five parameters with their suitable physical meanings is adapted to be used in quantitative analysis of the hysteresis (including single and multiple) and torque-decay phenomena of thixotropic materials.

The Huang equation (Appendix I.2) is:

$$\tau_{r\theta} = \tau_0 + \mu \dot{\gamma}_{r\theta} + C A \dot{\gamma}_{r\theta}^{n} e^{-\int_0^t C |\dot{\gamma}_{r\theta}|^n dt} \qquad \text{(II.1-1)}$$

<u>For a single hysteresis loop:</u> the shear rate linearly increases from zero to a maximum value (at time t_1) and then decreases again toward zero (at time $2t_1$).

when $0 \leq t \leq t_1$

$$\tau_{r\theta} = \tau_0 + \mu \dot{\gamma}_{r\theta} + CA\, \dot{\gamma}_{r\theta}^{n} \exp\left(-\frac{C\dot{\gamma}_{r\theta}^{n+1}}{\alpha(n+1)}\right)$$

when $t_1 \leq t \leq 2t_1$ (II.1-2)

$$\tau_{r\theta} = \tau_0 + \mu \dot{\gamma}_{r\theta} + CA\, \dot{\gamma}_{r\theta}^{n} \exp\left(-\frac{C}{\alpha(n+1)}\left[2\dot{\gamma}_{r\theta}^{n+1}(t_1) - \dot{\gamma}_{r\theta}^{n+1}\right]\right)$$

(II.1-3)

For a torque-decay curve: a single step change shear rate.

$$\tau_{r\theta} = \tau_0 + \mu \dot{\gamma}_{r\theta} + CA\, \dot{\gamma}_{r\theta}^{n} \exp\left(-C\dot{\gamma}_{r\theta}^{n} t\right)$$

(II.1-4)

Where

$\tau_{r\theta}$ = $r\theta$-component of shear stress, dyne-cm^{-2}

$\dot{\gamma}_{r\theta}$ = $r\theta$-component of shear rate, sec^{-1}

t = time of shear, sec

τ_0 = yield stress, dyne-cm^{-2}

μ = Newtonian contribution of blood viscosity, dyne-sec-cm^{2}

C = kinetic rate constant of structural breakdown of rouleaux to individual erythrocytes, sec^{n-1}

A = equilibrium value of structural arrangement parameter, dyne-sec-cm^{-2}

n = reaction order of structural breakdown of rouleaux to individual erythrocytes

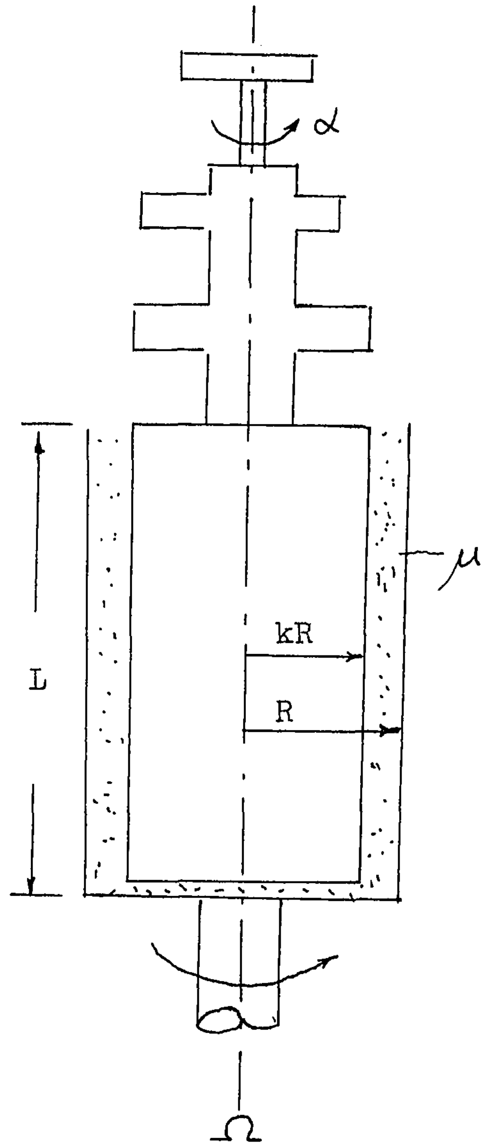
Theoretically this model can be applied to any ranges of shear rates and time. As to the steady state viscosity at a constant shear rate, which most investigators used to test the rheological properties of blood, this too can be obtained from the above model as time goes to infinity. The non-Newtonian contribution of viscosity at a constant shear rate is determined by $\eta_s - \mu$ where η_s is the steady state viscosity at a constant shear rate.

Most investigators in rheology have only considered that the apparent viscosities of thixotropic materials (blood)are a function of shear-stress or, shear-rate, and have ignored the importance of time factor. To emphasize the time dependency which will affect the rheological properties of thixotropic materials under a certain shear rate is one of the particular points in the Huang model. The separation of mechanical disturbance (shear-rate), time, and other factors which will influence the thixotropic behaviors of blood is another speciality of the model. So to characterize the thixotropic properties of blood quantitatively and qualitatively, the Huang model is able to provide some simple and meaningful index through its rheological parameters which actually are the functions of blood under various physical, chemical, and pathological conditions. It is possible that the rheological parameters introduced through the Huang equation could be applied in the various branches of science related to technologies of thixotropic materials.

2. Dynamic Behaviors of the Torsion Head

In most conventional viscometers, such as two parallel plates, two coaxial cylinders, cone and plate system, etc., the torque is transmitted by the measured fluid to the torsion bar due to the angular movement of the rotating part of the viscometer. Calibration of the torsion bar constant is usually done experimentally, and it is assumed that the angular deflection of the torsion bar is proportional to the supplied torque. Van de Ven (8) analyzed the dynamic behavior of viscometer by assuming the hollow cylinder or bob is suspended by a torsion wire which was given a forced oscillation at the top. He made a flate plate approximation in which he failed to consider the curvature of the bob. Also a little attention has been paid to the artefact of rheograms due to the influence of the mechanical properties of the torsion head. In this analysis, the angular displacement, angular velocity, and the angular acceleration of the whole torsion head induced by the rotating cup all are considered. In the following derivation, both a single step rotation and a triangular step rotation have been forced on the system.

Fig.II.1-1 Single couette cell



α = Angular deflection of torsion bar

G = Torsion bar constant

η = Torsion head damping coefficient

I = Moment of Inertia of torsion head

μ = Viscosity of tested fluid

R = Radius of rotating cylinder

L = Length of inner cylinder

Ω = RPM of rotating cylinder

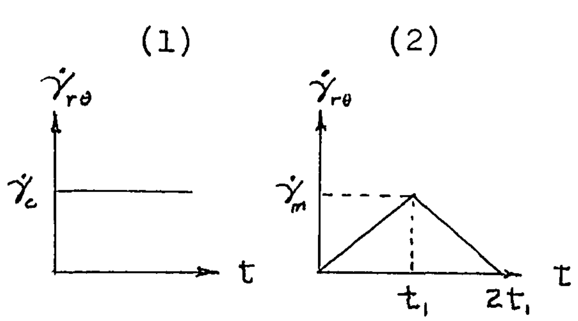
a = Acceleration (deceleration) constant for a triangular step change

t = Time

$\dot{\gamma}_{r\theta}$ = Shear rate

(1) = Single step change

(2) = Triangular step change



Equation of motion of the torsion head

$$I\frac{d^2\alpha}{dt^2} + \eta\frac{d\alpha}{dt} + G\alpha = T(t) \qquad \text{(II.2-1)}$$

$$\text{B.C.} \quad \alpha(0) = 0 \quad ; \quad \alpha'(0) = 0$$

Assume an incompressible Newtonian fluid in the viscometer.

Equation of motion of the fluid

$$\rho\frac{\partial V_\theta}{\partial t} = \mu\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(rV_\theta)}{\partial r}\right] \qquad \text{(II.2-2)}$$

For a step change

$$V_\theta(r,0) = 0 \qquad t = 0$$

$$V_\theta(KR,t) = 0 \qquad t \geqslant 0 \qquad \text{(II.2-3)}$$

$$V_\theta(R,t) = R\Omega \qquad t \geqslant 0$$

Fabisiak and Huang (9) have found that the transient terms are very small, and can be neglected. The torque applied to the inner cylinder is:

$$T(t) = 2\pi K^2R^2L\mu\frac{2\Omega}{1-K^2} = A_0 = \text{Constant} \qquad \text{(II.2-4)}$$

Similarly for a triangular step change (hysteresis loop), the boundary conditions for the fluid will be:

$$V_\theta(r,0) = 0$$

$$V_\theta(KR,t) = 0 \qquad \text{(II.2-5)}$$

$$V_\theta(R,t) = F(t) = \begin{cases} aRt & ; \quad 0 \le t \le t_1 \\ 2aRt_1 - aRt & ; \quad t_1 \le t \le 2t_1 \end{cases}$$

where a = angular acceleration constant of outer cylinder.

It was shown (9) that the dimensionless group, N_{HF} may be defined:

$$N_{HF} = \frac{(1-K^2)R}{2t_1}\sqrt{\frac{\rho}{\mu}} \quad \frac{E_1}{\beta_1} Z_0 \left(\sqrt{\frac{\rho}{\mu}} \beta_1 KR \right) << 1 \qquad \text{(II.2-6)}$$

The torque applied to the inner cylinder by an incompressible Newtonian fluid can be simplified as follow:

$$T(t) = \frac{4\pi a L K^2 R^2 \mu t}{1-K^2} = \beta t \qquad ; \quad 0 \le t \le t_1$$

$$\qquad\qquad \text{(II.2-7)}$$

$$= \beta(2t_1 - t) \qquad ; \quad t_1 \le t \le 2t_1$$

Equation (II.2-1) can be rewritten as follow:

$$\tau^2 \frac{d^2\alpha}{dt^2} + 2\xi\tau\frac{d\alpha}{dt} + \alpha = \frac{1}{G}T(t) \qquad \text{(II.2-8)}$$

Boundary conditions: $\alpha(0) = 0$

$\alpha'(0) = 0$

Where $\tau = (I/G)^{1/2}$ Time constant

$\xi = \eta / 2(IG)^{1/2}$ Damping factor

Substitute Eq. II.2-4 for a single step change, (40) and Eq. II.2-7 for a triangular step change (hysteresis loop) into Eq. II.2-8. Then, apply the method of Laplace transformation to solve Eq. II.2-8. The following solutions were obtained:

Case 1: For small τ and ξ, or $\tau \to 0$, $\xi \to 0$

Step change

$$\alpha = \frac{4\pi K^2 R^2 L \mu \Omega}{(1-K^2)\,G} = \frac{A_0}{G} \qquad \text{(II.2-9)}$$

Triangular step change

$$\alpha(t) = \begin{cases} \dfrac{B}{G}\,t & ; \quad 0 \le t \le t_1 \\ \dfrac{B}{G}(2t_1 - t) & ; \quad t_1 \le t \le 2t_1 \end{cases} \qquad \text{(II.2-10)}$$

Case 2: For small ξ, or $\xi \to 0$

Step change

$$\alpha(t) = \frac{A_0}{G}\left(1 - \cos\frac{t}{\tau}\right) \qquad \text{(II.2-11)}$$

Triangular step change

$$\alpha(t) = \begin{cases} \dfrac{B}{G}\left(t - \tau \sin\dfrac{t}{\tau}\right) & ; \quad 0 \le t \le t_1 \\[2ex] \dfrac{B}{G}\left[2t_1\left(1 - \cos\dfrac{t}{\tau}\right) - \left(t - \tau \sin\dfrac{t}{\tau}\right)\right] & ; \quad t_1 \le t \le 2t_1 \end{cases} \qquad \text{(II.2-12)}$$

Case 3. Overdamped, $\xi > 1$

Step change

$$\alpha(t) = \frac{A_0}{G}\left[1 - \frac{1}{2\sqrt{\xi^2-1}}\left\{\left(\xi + \sqrt{\xi^2-1}\right) e^{-\left(\xi - \sqrt{\xi^2-1}\right)\frac{t}{\tau}} - \left(\xi - \sqrt{\xi^2-1}\right) e^{-\left(\xi + \sqrt{\xi^2-1}\right)\frac{t}{\tau}}\right\}\right] \qquad \text{(II.2-13)}$$

Triangular step change

$$\alpha(t) = \frac{B}{G}\left[t - 2\xi\tau + \frac{\tau}{2\sqrt{\xi^2-1}}\left\{\left(2\xi^2-1 + 2\xi\sqrt{\xi^2-1}\right) \times e^{-\left(\xi + \sqrt{\xi^2-1}\right)\frac{t}{\tau}} - \left(2\xi^2-1 - 2\xi\sqrt{\xi^2-1}\right) e^{-\left(\xi - \sqrt{\xi^2-1}\right)\frac{t}{\tau}}\right\}\right] ;$$

$$0 \le t \le t_1 \qquad \text{(II.2-14)}$$

$$\alpha(t) = \frac{2t_1 B}{G}\Bigg[\ 1 + \frac{1}{2\sqrt{\xi^2-1}}\Big\{ (\xi - \sqrt{\xi^2-1})\, e^{-(\xi+\sqrt{\xi^2-1})\frac{t}{\tau}} -$$

$$(\xi + \sqrt{\xi^2-1})\, e^{-(\xi-\sqrt{\xi^2-1})\frac{t}{\tau}}\Big\}\Bigg] - \frac{B}{G}\Bigg[t - 2\xi\tau$$

$$+ \frac{\tau}{2\sqrt{\xi^2-1}}\Big\{(2\xi^2 - 1 + 2\xi\sqrt{\xi^2-1})\, e^{-(\xi+\sqrt{\xi^2-1})\frac{t}{\tau}}$$

$$(2\xi^2 - 1 - 2\xi\sqrt{\xi^2-1})\, e^{-(\xi-\sqrt{\xi^2-1})\frac{t}{\tau}}\Big\}\Bigg] \quad ;$$

$$t_1 \le t \le 2t_1$$

<u>Case 4</u>. Underdamping, $\xi < 1$

<u>Step change</u>

$$\alpha(t) = \frac{A_0}{G}\left[1 - \frac{e^{-\xi\frac{t}{\tau}} \sin\left(\sqrt{1-\xi^2}\,\frac{t}{\tau} - \phi\right)}{\sqrt{1-\xi^2}}\right]$$

(II.2-15)

$$\text{where} \quad \phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{-\xi}$$

<u>Triangular step change</u>

$$\alpha(t) = \frac{B}{G}\left[t - 2\xi\tau + \frac{\tau e^{-\xi\frac{t}{\tau}}}{\sqrt{1-\xi^2}} \sin\left\{\sqrt{1-\xi^2}\,\frac{t}{\tau} - 2\tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{-\xi}\right)\right\}\right] \quad ; \quad 0 \le t \le t_1$$

(II.2-16)

$$\alpha(t) = \frac{2t_1 B}{G}\left[1 + \frac{e^{-\xi\frac{t}{\tau}} \sin\left\{\sqrt{1-\xi^2}\,\frac{t}{\tau} - \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{-\xi}\right)\right\}}{\sqrt{1-\xi^2}}\right]$$

$$- \frac{B}{G}\left[t - 2\xi\tau + \frac{\tau e^{-\xi\frac{t}{\tau}}}{\sqrt{1-\xi^2}} \sin\left\{\sqrt{1-\xi^2}\,\frac{t}{\tau} - 2\tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{-\xi}\right)\right\}\right] \quad ; \quad t_1 \leq t \leq 2t_1$$

<u>Case 5</u>. Critical damping, $\xi = 1$

<u>Step change</u>

$$\alpha(t) = \frac{A_0}{G}\left[1 - \left(1 + \frac{t}{\tau}\right) e^{-t/\tau}\right] \tag{II.2-17}$$

<u>Triangular step change</u>

$$\alpha(t) = \frac{B}{G}\left(t e^{-t/\tau} + \frac{2}{\tau} e^{-t/\tau} + t - \frac{2}{\tau}\right)$$

$$= \frac{B}{G} F_1(t) \quad ; \quad 0 \leq t \leq t_1$$

$$\alpha(t) = \frac{B}{G}\left[2t_1\left\{1 - \left(1 + \frac{t}{\tau}\right) e^{-t/\tau}\right\} - F_1(t)\right] \quad ; \tag{II.2-18}$$

$$t_1 \leq t \leq 2t_1$$

Comparison of the theoretical shear stress and the artificial shear stress

The theoretical shear stress is directly from the solution of Eq.II.2-2. with some suitable boundary conditions (Appendix I.1).

Single step change

$$(\tau_{r\theta})_{T.S} = -\mu r \frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)\Big|_{r=KR} = -\mu \frac{2\Omega}{1-K^2} \qquad \text{(II.2-19)}$$

Triangular step change

The transient terms have been found very small (9), so

$$(\tau_{r\theta})_{T.t} = -\mu r \frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)\Big|_{r=KR} = \begin{cases} -\mu \dfrac{2at}{1-K^2} ; & 0 \le t \le t_1 \\ -\mu \dfrac{2a(2t_1-t)}{1-K^2} ; & t_1 \le t \le 2t_1 \end{cases} \qquad \text{(II.2-20)}$$

At the same single step change and the same triangular step change, the real shear stress due to the effect of the torsion head will be:

$$(\tau_{r\theta})_{j,i} = -\frac{T}{2\pi L (KR)^2} = -\frac{K_f \cdot f \cdot \alpha_{j,i}(\tau,\xi)}{2\pi L (KR)^2} \qquad \text{(II.2-21)}$$

Where i is s (single step change) or t (triangular step change), j is the real Case No., f is a scale adjusting factor of instrument for clear readings on recorder. The artifact will be:

$$(\Delta \tau_{r\theta})_{j,i} = (\tau_{r\theta})_{j,i} - (\tau_{r\theta})_{T,i} \qquad \text{(II.2-22)}$$

It is obviously that Case 1., in which the influence of the torsion head (τ, ξ) is almost zero is an ideal case. The shear stress derived from Case 2. with very small damping factor ($\xi \to 0$) is in a continuous oscillation with respect to time. This can be corrected by adjusting the factor.

The following typical figures coming from a computer program (Appendix I.2) exhibit how the damping factor combined with a time constant influences the rheogram. This involves three cases - underdamping ($\xi < 1$), critical damping ($\xi = 1$), and overdamping ($\xi > 1$).

In case τ is very small, the system even with a large overdamped coefficient ξ (but never for critical damping) can provide very well responses closing to the ones in ideal case for both a single step change and a triangular step change (see Fig.III.1-2 at τ = 0.01 sec^{-1}).

Explanation of Fig.III.1-2:

1. Upper figure : System responses due to a single step change.
2. Lower figure : System responses due to a triangular step change.
3. Solid lines : Not ideal cases.
4. Dashed lines : Ideal cases.

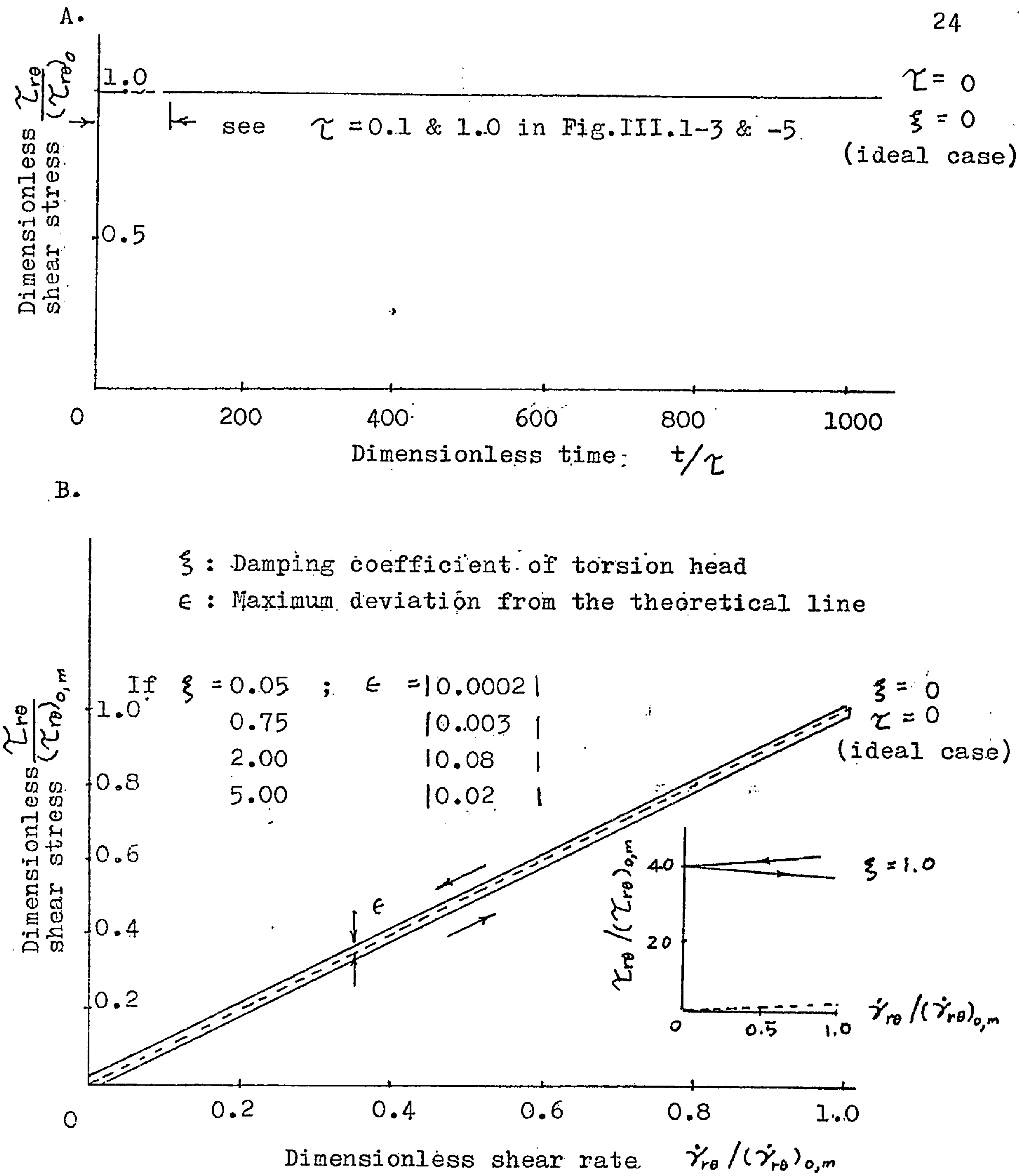


Fig. III.1-2 Dynamic behaviors of torsion head at τ = 0.01 sec.(A. System response to a single step change, B. System response to a triangular step change; dashed lines for ideal cases, solid lines for no ideal cases due to the artifacts of torsion head; $(\tau_{r\theta})_{o,m}$ and $(\dot{\gamma})_{o,m}$ theoretical maximum shear stress and shear rate for ideal cases during a triangular step change; $(\tau_{r\theta})_o$ theoretical shear stress at a single step change). τ is time constant of torsion head.

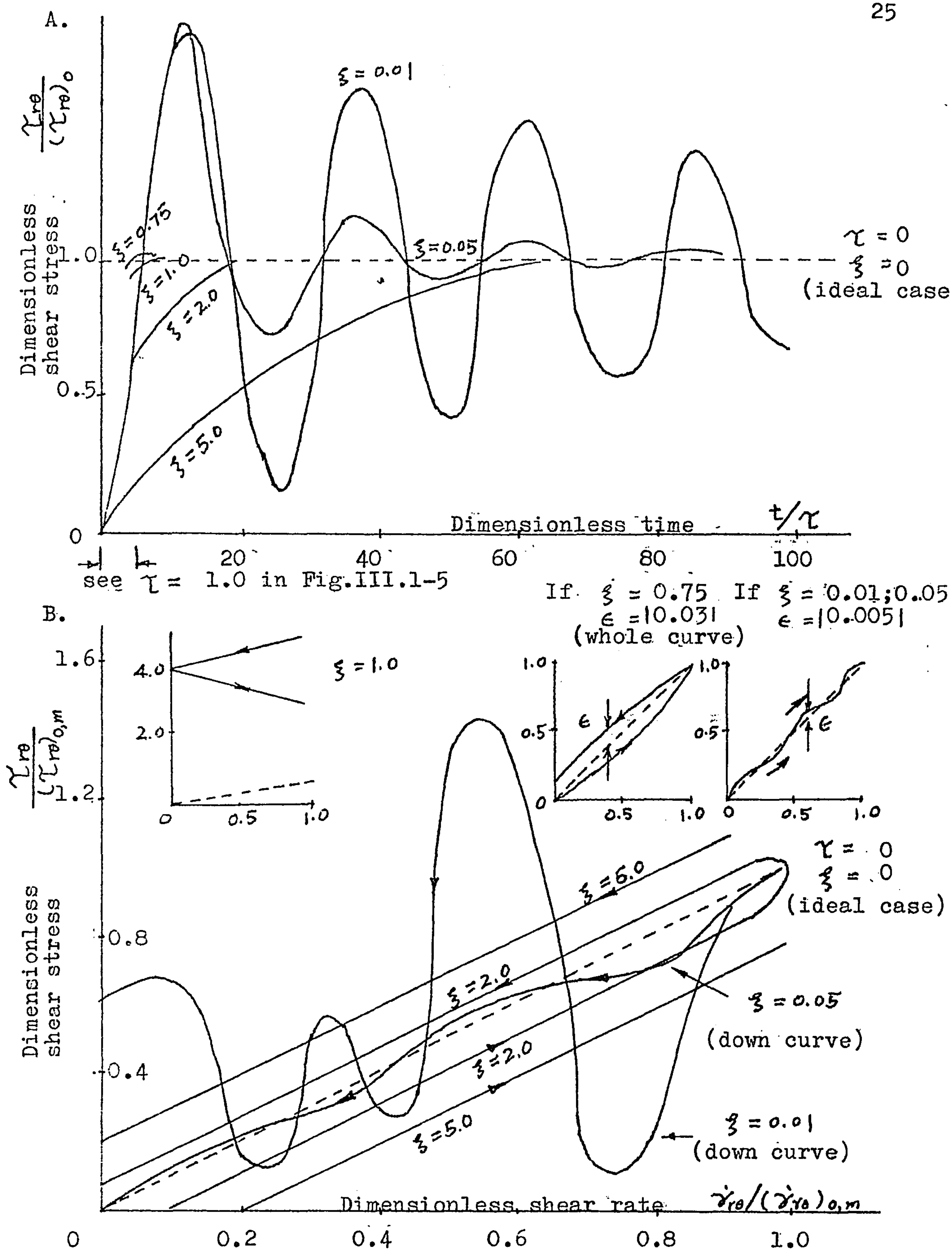


Fig.III.1-3 Dynamic behaviors of torsion head at time constant $\tau = 0.1$sec. (ξ, damping coefficients of torsion head; ϵ, maximum deviation from theoretical line; the other descriptions for this figure are same as Fig.III.1-2).

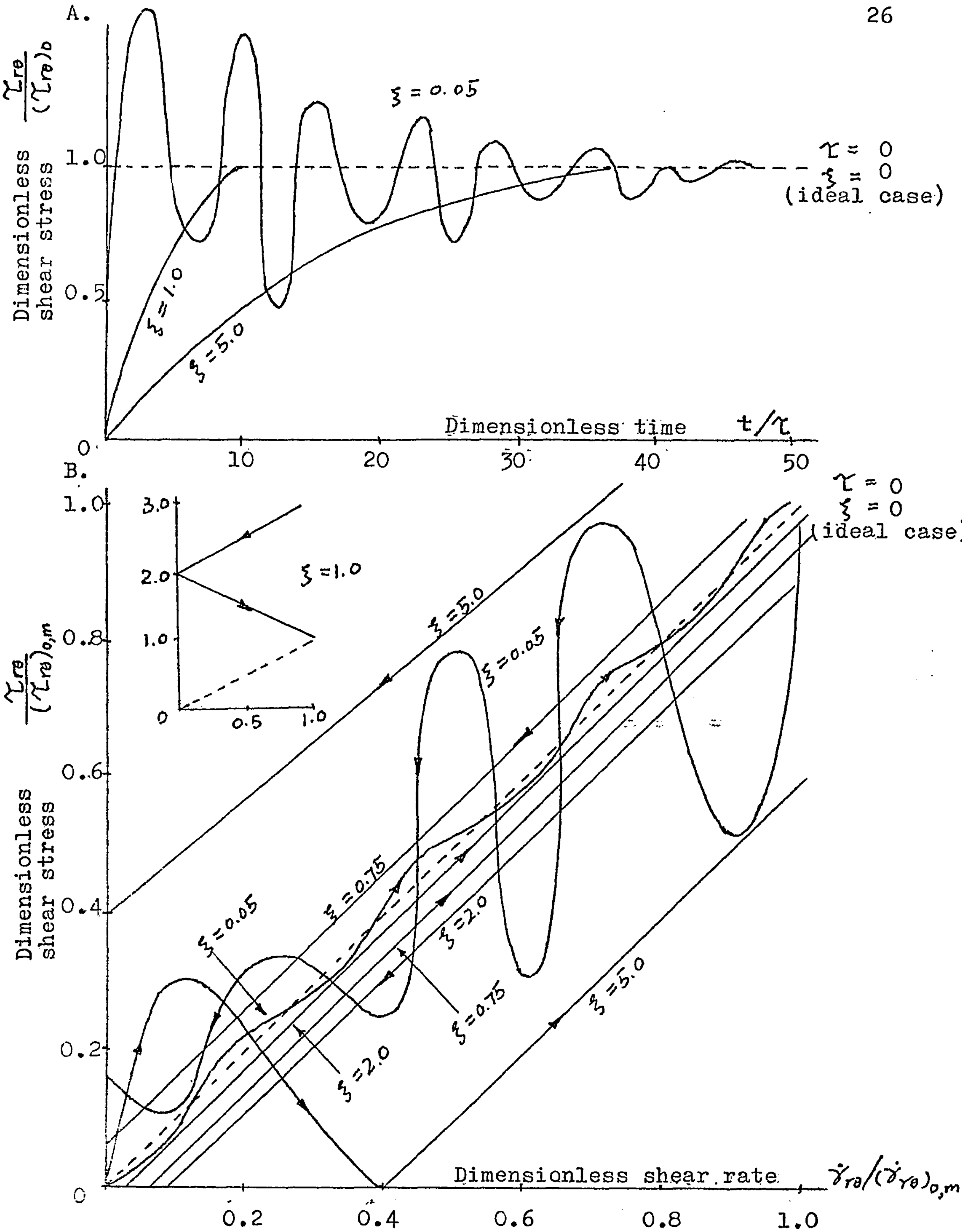
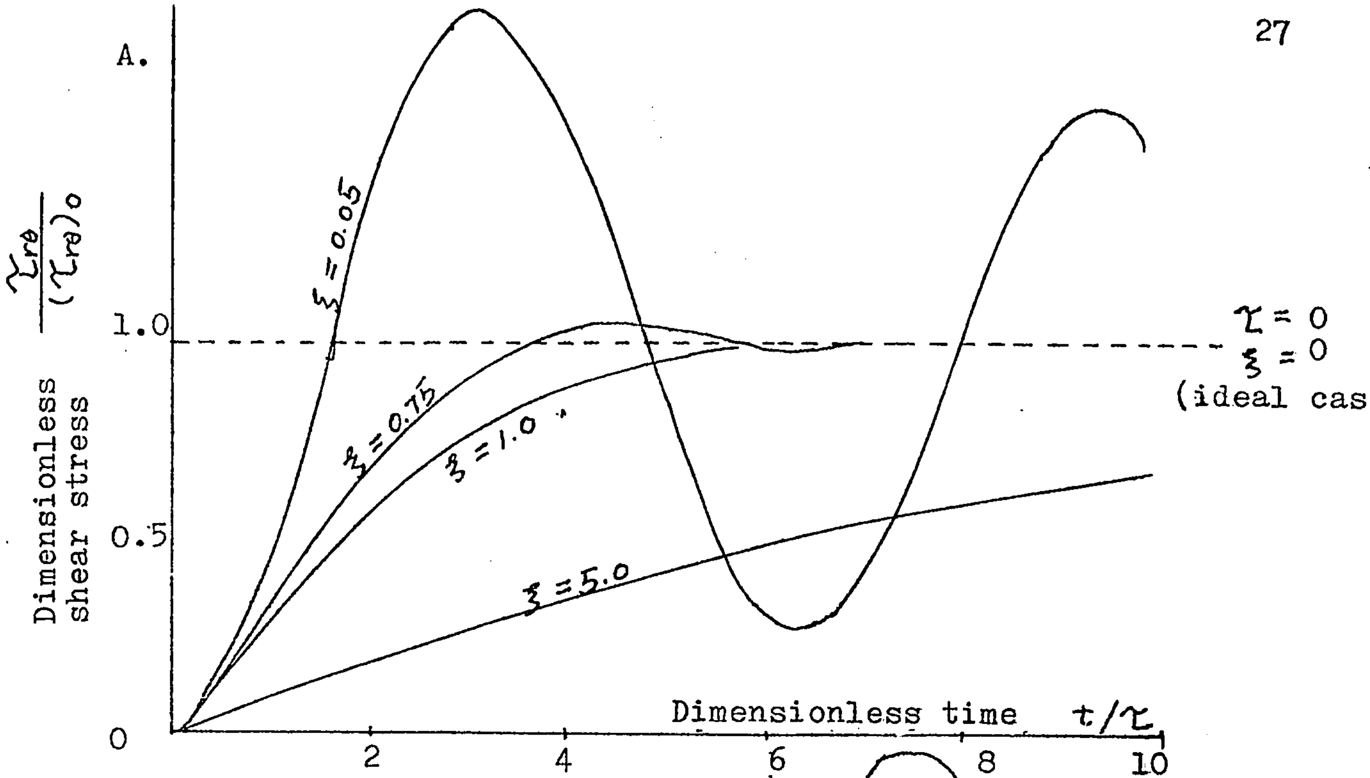


Fig.III.1-4 Dynamic behaviors of torsion head at time constant $\tau = 0.2$ sec. (ξ, damping coefficients of torsion head; the other descriptions for this figure are same as Fig.III.1-2).



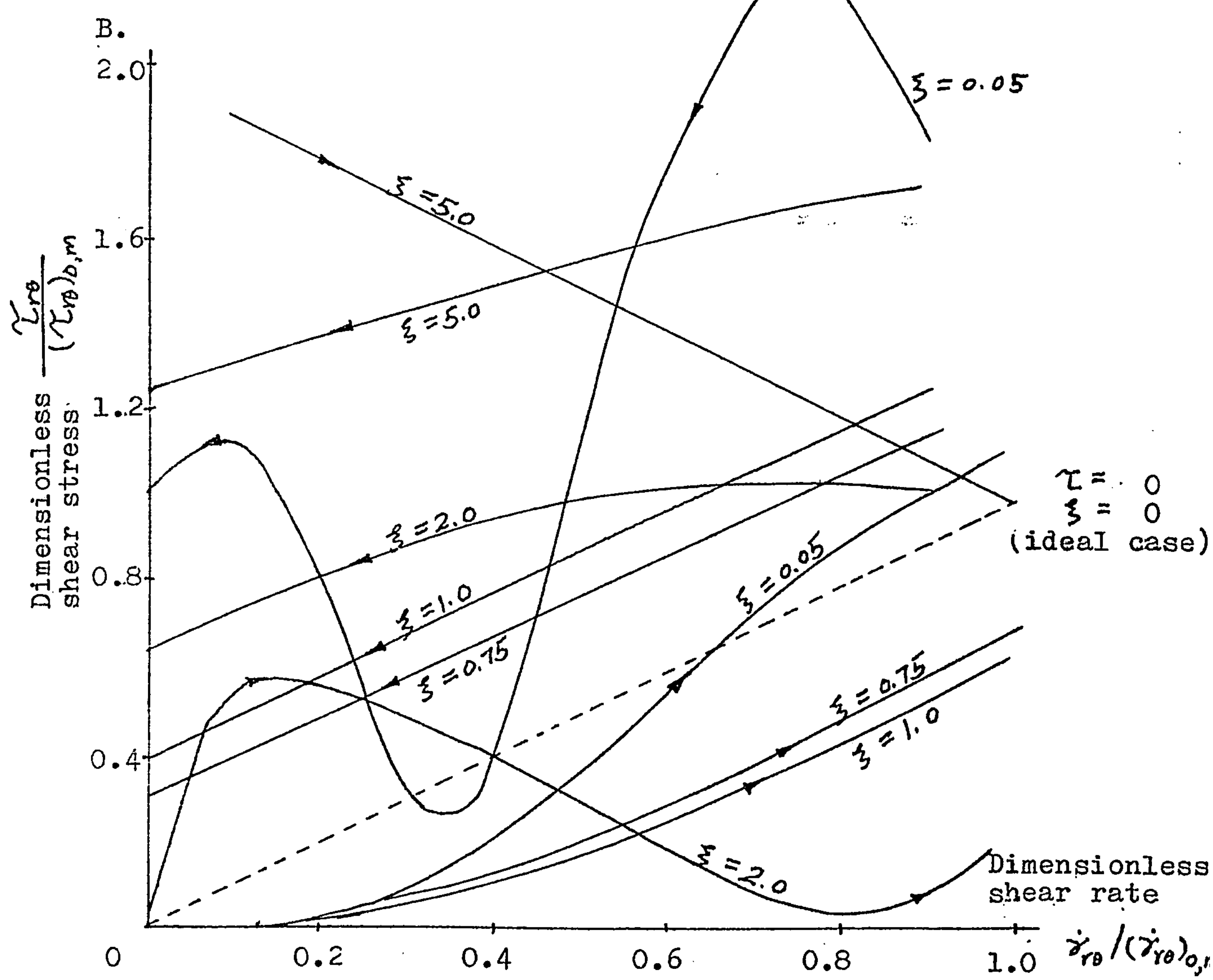


Fig.III.1-5 Dynamic behaviors of torsion head at time constant $\tau = 1.0$ sec. (for details, see Fig.III.1-4).

Discussion

Case 1 is an ideal case for both a single step change and a triangular step change.

Case 2. Sine waves are generated for both cases, they can be corrected by the installation of a damping plate placed in a viscous fluid in order to adjust the damping factor.

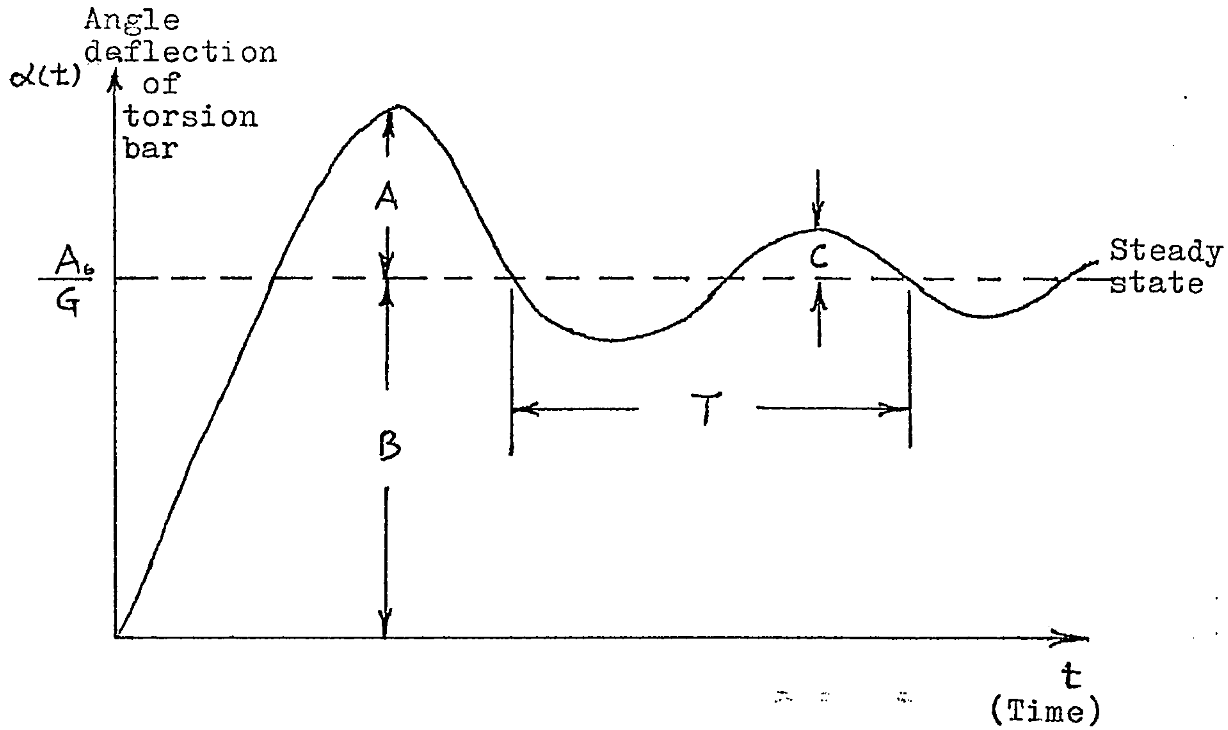
Case 3. The response due to a single step change will be slow. For the triangular step change, artificial hysteresis loops will happen, their sizes and shapes are dependent on the combination of time constant and damping factor.

Case 4. If a suitable combination of time constant and damping factor are selected, the artifacts for both a single step change and a triangular step change would be very small and can be neglected.

Case 5. The artifact may be very small for a step change, but always generate tremendous artifact for a triangular step change.

So, to minimize the artifacts for both cases, the torsion head must be at underdamping. And, a particular set of time constant and damping factor of the system is only suitable for a specific range of shear rates without obvious errors happened in the rheograms generated by the viscometer with a specific damping fluid.

And, from (40), page 88, system parameters τ and ξ for the underdamping case can be calculated from experimental response of $\alpha(t)$ in a single step change.



$$A/B = \text{overshot} = \exp\left(-\pi\xi\sqrt{1-\xi^2}\right)$$

$$C/A = \text{decay ratio} = \exp\left(-2\pi\xi\sqrt{1-\xi^2}\right) = (A/B)^2$$

$$f = \text{frequency} = 1/T$$

These relationships can be further applied to modify the torsion head system.

CHAPTER III

APPARATUS AND EXPERIMENTAL PROCEDURES

1. Apparatus

A modified Weissenberg Rheogoniometer Model R-18 was used in the generation of rheograms including a hysteresis loop and a torque-decay curve. The whole experimental system is shown in a schematic diagram as Fig.III.1-1. The main functions of each part are described as follow:

(a). The double couette cell

In order to minimize the unnecessary chemical reactions between blood and the cell, a gold platted double couvette cell was designed as shown in Fig.III.1-2. All the rings have the same working height as L 2.003 inches, but their inner and outer radii differ. The inner and outer radii of the suspended hollow (bob) were 1.137 inches and 1.218 inches, and those of the annulus (rotating ring or cup) were 1.104 inches and 1.252 inches. The dimensions were selected in such manner that the shear rate developed at the inner and outer surface of the susponded hollow is always the same (Appendix III). The actual gap between the surfaces of bob and of cup is 0.017 inches. The gap between the bottoms of the two rings is 500 micrometer. Thus the errors induced by the end effect can be neglected due to the

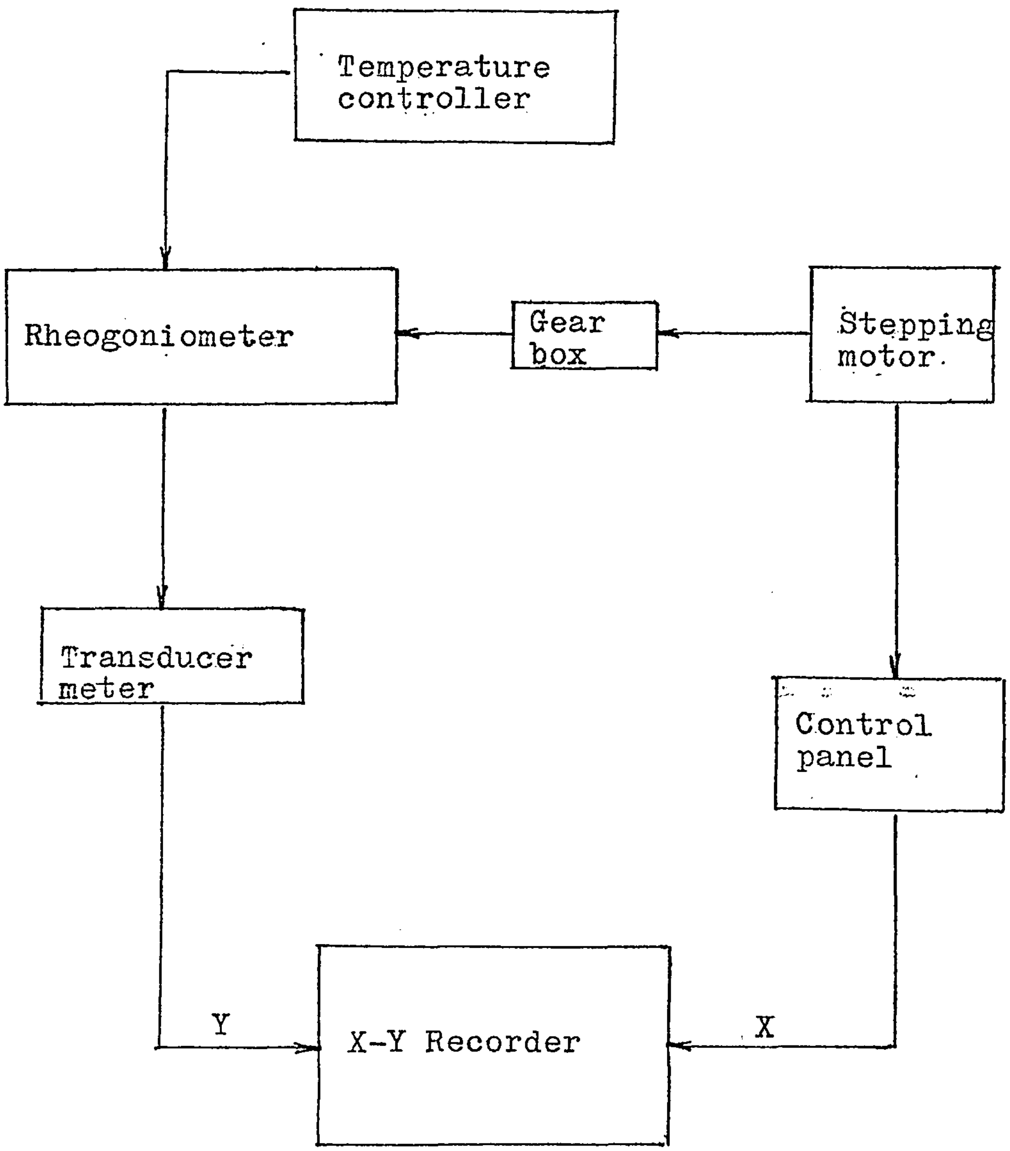


Fig.III.1-1 Block diagram of the modified Weissenberg Rheogoniometer

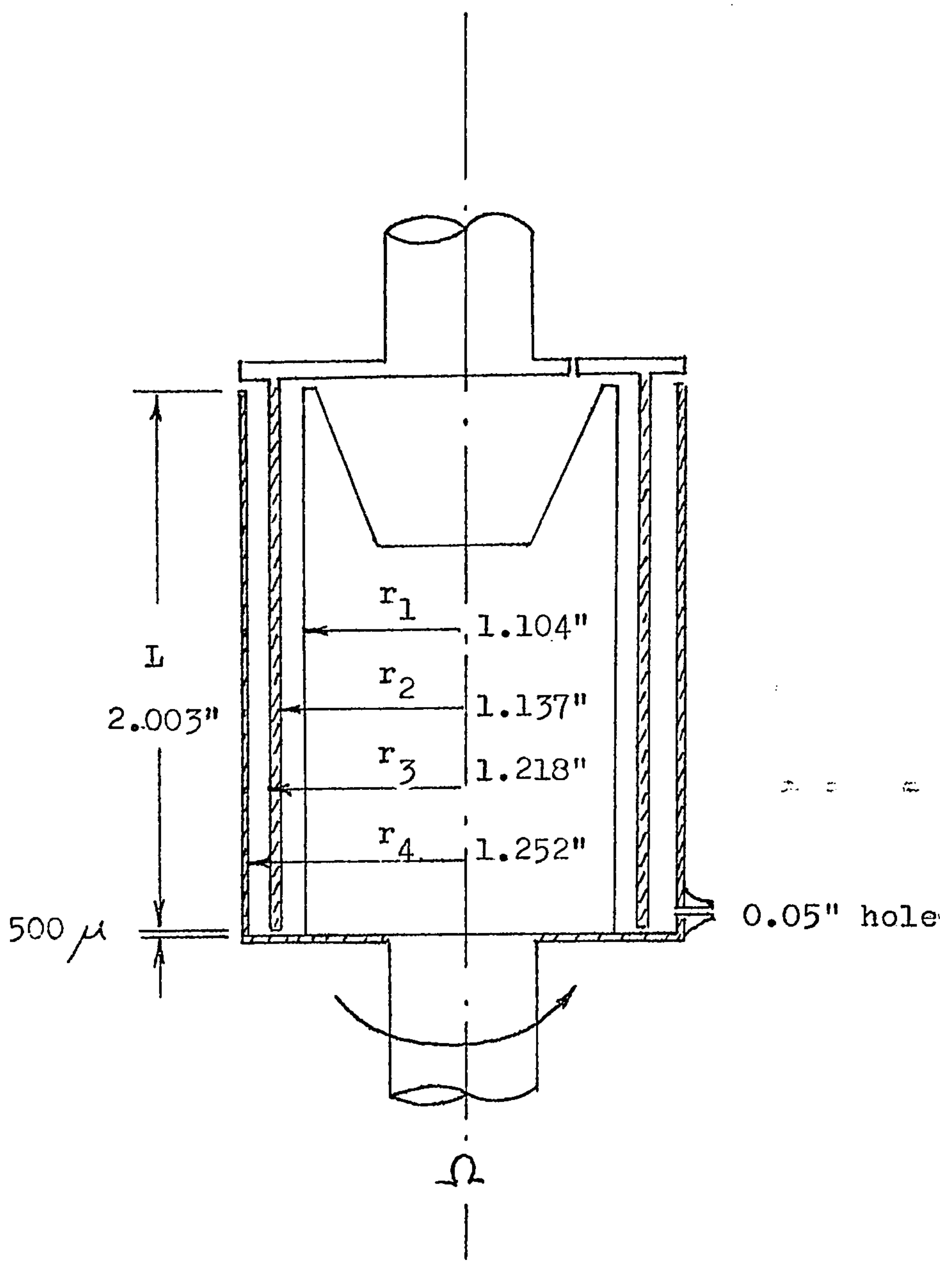


Fig.III.1-2 The double couette cell

large ratios of the ring height to the gaps between bob and cup, and of the shearing area of the vertical surfaces to the shearing surfaces of the cell ends.

A guard ring was also extended from the top of the hollow to reduce the reaction between blood and air. The hollow is attached through a universal joint to the torsion bar, and is floated by a 20 psi. dry compressed air in order to eliminate any mechanical friction. The volume of sample needed is at least 4.20 ml. It must be stressed that the shear rates given by this rotational viscometer are homogenous ones. The equations used to calculate the shear rate and the shear stress are adhered in Appendix III.

(b). Direct reading transducer meter

A type EP597A Sangamo Controls Limited (England) transducer was used. Amplifing ranges contain 5, 20, 50, 200, 500, and 2000 times. Output of the torque from cell can directly be read out in this meter, and can also be transferred to the X-Y recorder.

(c). X-Y recorder

Hewlett Packard 7045A X-Y recorder was connected to the system as shown in Fig.III.1-1. The surface area which can be utilized is 15 inches by 10 inches (X by Y).

(d). Temperature control chamber

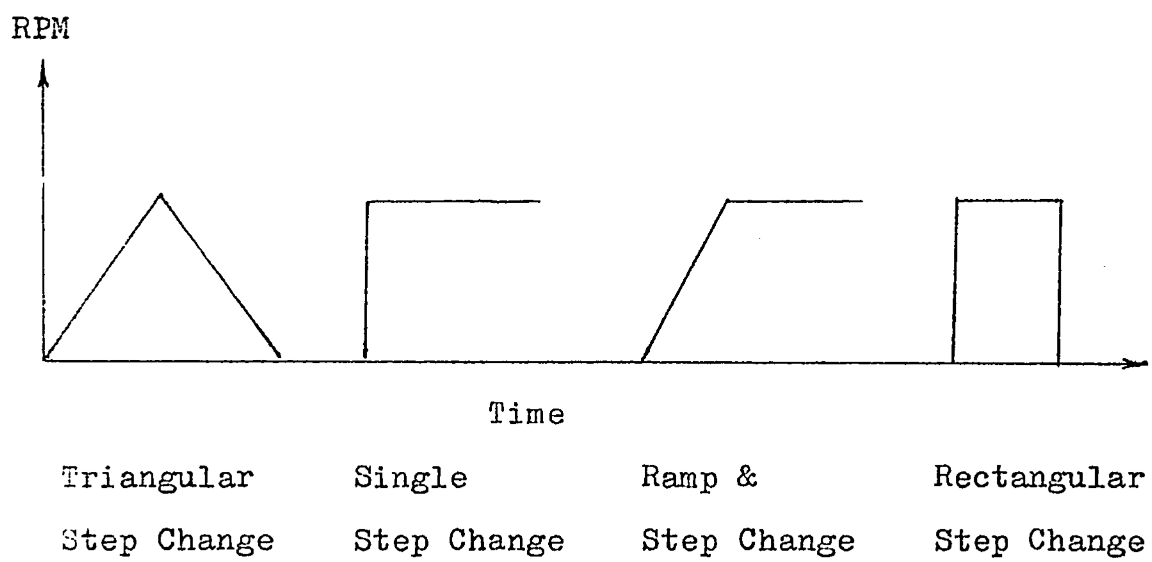
The whole rheogoniometer was enveloped by a highly heat-resistant plastic chamber. Eight 60 watt electric bulbs as heat source were installed around the bottom inside the chamber. Temperature controller can control the viscometer temperature from room temperature up to 45°C with an accuracy of ± 0.1°C

(e). Original motor, gear box, and drive unit

A SLO-SYN Synchronous motor with 1500 rpm was installed, and controlled by a home-made control panel. Gear box has 60 step settings. Drive unit is controlled by a clutch including "Drive", "Brake", and "Off".

(f). Control panel

A home-made electronic device combining with (e) can control the rotating speed of cup from 0 to 146.5 rpm differentially. It also included a time setting. By adjusting the setting in the panel and using the clutch, the following functions for the rotating part of viscometer can be generated:



(g). Refrigerator was kept at $4^{o}C$ for the storage of blood samples. Water bath was used to warm the blood sample at the designed degree and time.

(h). Syringe: 5 c.c. sterile single-use Plastipak B-D syringe was utilized to inject the blood sample into the double couette.

2. Experimental procedures

(a). Set the desired experimental temperature, let the rheogoniometer (vicometer) warm overnight in order that its temperature reaches a steady state.

(b). Choose a suitable setting at gear box. For blood, the setting at 2.3218 rpm for the stepping motor is desired, Since it will give the maximum shear rate at 8.0361 sec^{-1} for the doùble couvette (Appendix IV). This (40) low shear rate would not cause any damage to the blood sùbphases, and would generate a clear hysteresis loop for whole blood.

(c). Adjust transducer meter and X-Y recorder to a suitable scale.

(d). Gently shake and heat the blood sample in water bath at the desired temperature and time, then gently inject it into the viscometer.

(e). For hysteresis loop, use control panel to make a triangular step change at cup (shear rate linearly accelerates from 0 to 8.0361 sec^{-1}; then linearly decelerates to 0).

(f). For torque-decay curve, use control panel to set the shear rate at 3.2144 sec^{-1}, which will generally make clear torque-decay curve for whole blood, then suddenly switch the clutch which originally set at "Brake" to "Drive". A shear stress at constant shear rate 3.2144 sec^{-1} can also be obtained.

To check if the rheograms can be reproduced, several runs for (e) and (f) should be carried out.

(g). The experimental data from both rheograms have been transferred to the modified Marquardt computer program. Eventually, the five thixotropic parameters in the Huang model have been evaluated. Because of its importance for the main topic of the thesis and its complexity, the actual computer program has been included into the thesis as an Appendix I.3.

CHAPTER IV

EXPERIMENTAL RESULTS

1. Altered thixotropic properties of blood during cardiopulmonary bypass

Variations of blood rheological properties are always accompanied by changes in physiological, psychological, and pathological factors. Ehrly (10) exhibited circadian rhythm of young female blood viscosity. Dintenfass (11,12) correlated between bio-chemical and rheological parameters in patients with myocardial infarction, haemophilia and thyroid diseases; also described the influences of ABO blood groups and fibrinogen on thrombus formation and aggregation of red cells in cardiovascular and malignant diseases(13). Schönbein , et al. had found that pathological red cell aggregation in myeloma patients presented higher abnormal shear resistance. Again Dintenfass (14,15) introduced the psychological score index related to the elevation of blood viscosity, and assumed a hypothetical viscoreceptor mechanism. Elevation of any of the blood viscosity factors is a risk factor and a warning sign, especially in the cardiovascular disorder.

The orthodox studies on open heart surgery were mainly with the pathogenetic pathways via abnormalities of blood pressure, metabolism, dietary regime, formation of atherosclerotic plaques, cholesterol level, and so on; but the intrinsic role of blood rheology was paid little attention, or just was partially investigated only at steady state shear rates. Its thixotropic properties including hysteresis loop and torque-decay curve were rather neglected.

This investigation was concerned with the thixotropic properties of blood during cardiopulmonary bypass. The blood was collected at different clinical stages from patients from the time of entering hospital to the time of leaving hospital (Dr. J. Cohn, St. Barnabas Medical Center, Livingston, New Jersey, 1977-79). For each stage, haemoanalysis was done by routine hospital work. For rheological analysis, each sample was collected as 7 ml. alioquots and anti-coagulated with 10.5 mg. EDTA (ethylene diaminetetraacetic acid). Then rheograms of hysteresis loop and torque-decay curve were obtained through the modified Weissenberg rheogoniometer at room temperature. The thixotropic parameters were evaluated by a method of non-linear least square parameter estimation based upon a modified Marquardt program on a Univac Spectra 70 digital computer (Appendix 1.3).

The data of hematological evaluation have proved that there are no constituents in blood from all stages from all blood samples that present any abnormality (Appendix II.1-A).

However, the rheological evaluation indicates that the rheological results have shown a significant difference between patients and normal healthy people, and the results at each clinical stage on patients are also different. Fig.IV.1-1 summarizes their rheological data (Appendix II.1-B).

The parameter A, the equilibrium value of structural arrangement is proportional to the number of erythrocytes in the form of ordered rouleaux formation. The decrease in the value for the structural arrangement parameter A, indicates that more individual erythrocytes or fewer rouleaux formations are present within the whole blood sample at the applied shear rate.

The decrease in the value for C implies that the rate constant for obtaining equilibrium between individual erythrocytes and rouleaux arrangement will shift toward the formation of rouleaux forms.

τ_o is a physical property of blood flow and is directly related or associated with the formation of rouleaux. Variations in this value follow same patterns as A. μ, η_s and $\eta_s - \mu$ follow same patterns of variation as A. n reflects the order of reaction of rouleaux breakdown, which implies a certain reaction mechanism.

Observations have been made that the high blood viscosity in non-Newtonian contribution from the expired patients may be due to an excessive aggregation of the red cells. However, the hematological evaluation can not provide any information in this respect. It appear that there could be other causes which easily induce the excessive aggregation of the red cells when some certain shear rates are applied on the blood for specific time intervals.

This investigation implies that the rheological parameters from the Huang model are not only fundamental in indications of pathogenesis and consequences of heart diseases, but that these parameters might supply a solution for prediction and diagnosis. It is also suggested that such rheological tests will allow, in some cases, a more rapid determination of disease when the current laboratory tests are inadequate.

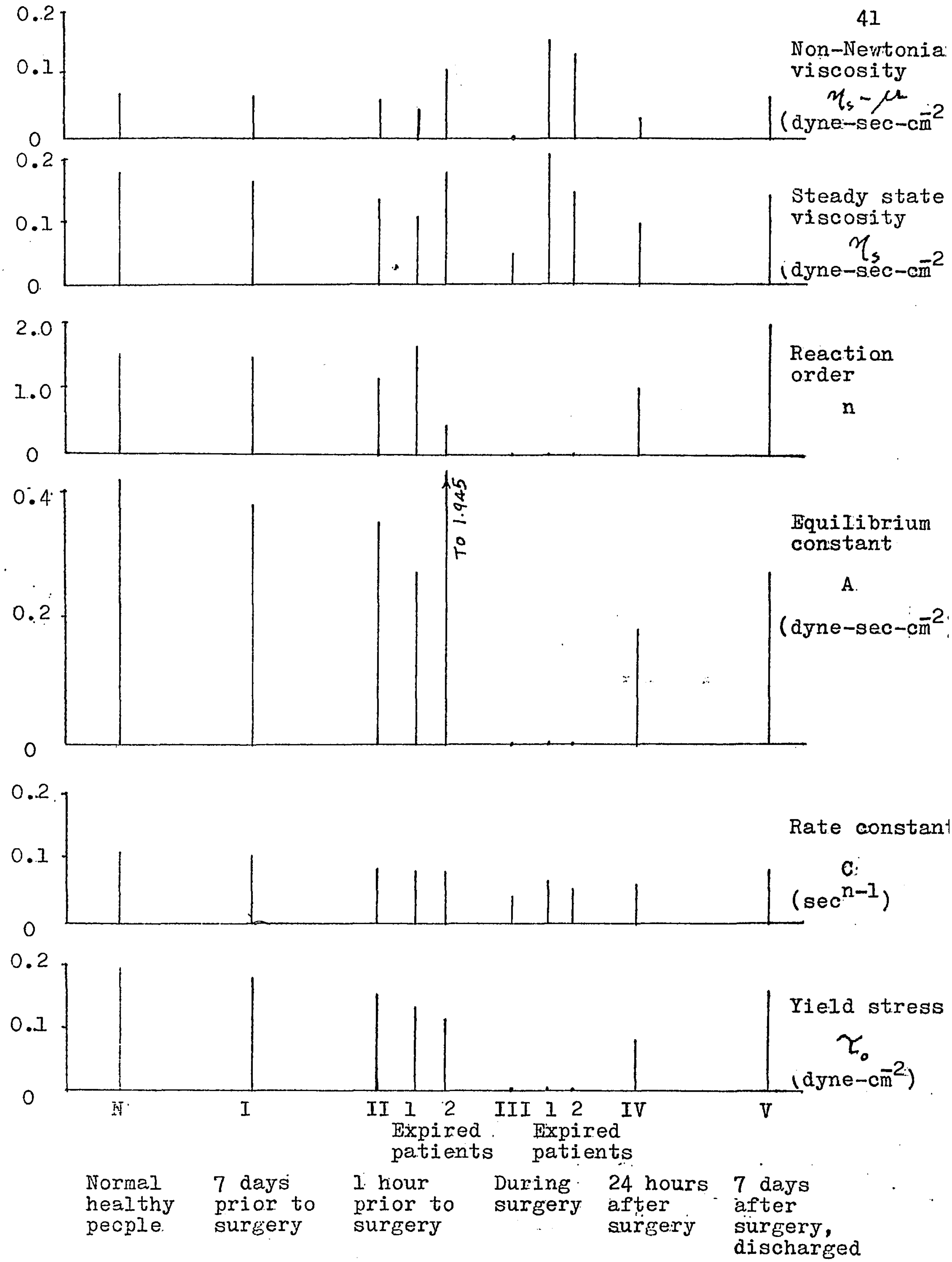


Fig.IV.1-1 Rheological parameters of patients during cardiopulmonary bypass

2. Effect of temperature on thixotropic properties of blood

There are numerous factors that affect the rheological properties of human blood. Temperature changes may be regarded as a thermal disturbance. In 1963, Cokelet (16) found that the apparent viscosities of plasma and water had same temperature dependence i.e. they followed Arrhenius law with the same active energy but with a different rate constant. The relative viscosity of blood (relative to water) was independent of temperature between 10°C and 37°C at shear rates larger than 1 sec^{-1}, but the relative viscosity increased with temperature by about 20% at very low shear rates (less than 1 sec^{-1}). Chien, et al. (17) and Barbee (18) also discovered that the relative viscosity of blood (relative to plasma) was independent of temperature between 20°C and 37°C at high shear rates over 50 sec^{-1}, but had an exponential relationship with hematocrit as follows:

$$\eta = \eta_o \text{ Exp } (bH) \qquad \text{(IV.2-1)}$$

Where η_o is the apparent viscosity of plasma, η is the apparent viscosity of blood, H is the hematocrit, and b is a constant which is inversely proportional to the shear rate.

All the previous results were considering only one rheological parameter (apparent viscosity), and which was evaluated at high steady-state shear rate. Due to their ignorance of the historical significance and hysteresis phenomenon in blood, this brings much difficulty to explain its thixotropic properties. Based on the Huang model, an investigation in the effect of temperature on the whole blood from healthy adults was attempted. The statistical results were plotted in Fig.IV.2-1 and Fig.IV.2-2. The detailed data were listed in Appendix II.2.

From the two figures, one may find that n is constantly independent of temperature, and C has a slightly exponential relationship with temperature. However, the other parameters vs. temperature almost present same shape with the lowest values at 37°C. Once a slight degree deviation from 37°C happens in the whole blood, i.e. the blood has absorbed or released a small thermal energy, the yield stress, Newtonian viscosity, equilibrium constant, steady-state viscosity, and non-Newtonian viscosity will increase tremendously. These phenomena may be explained as follow:

Since blood is a solution containing high molecular weight sustances, such as macroglobulins, albumins, cells, and especially fibrinogen, etc., which are sensitive to temperature and are greatly affected by temperature changes, the temperature changes may cause a certain orientation and

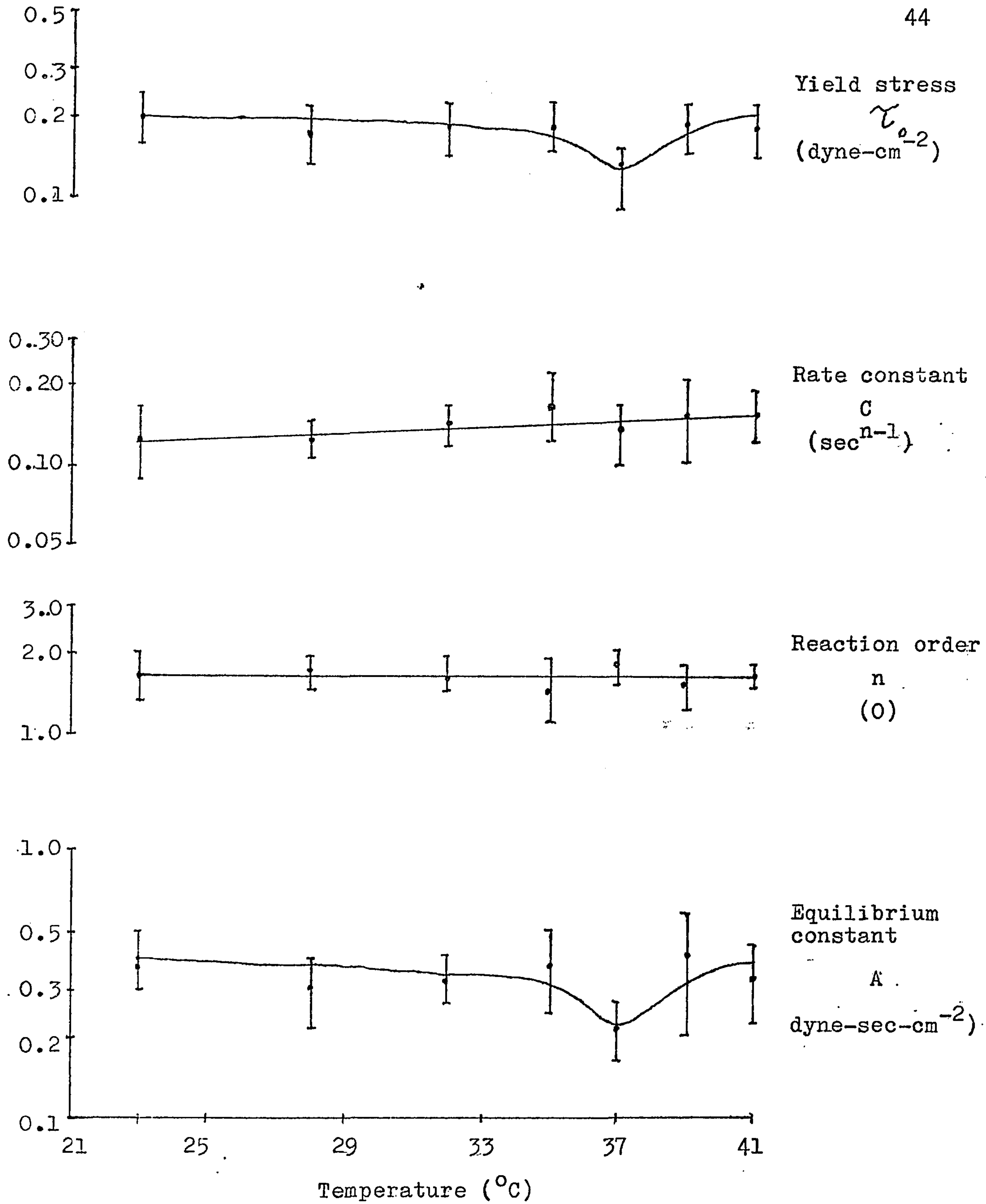


Fig.IV.2-1 Rheological parameters vs. temperature

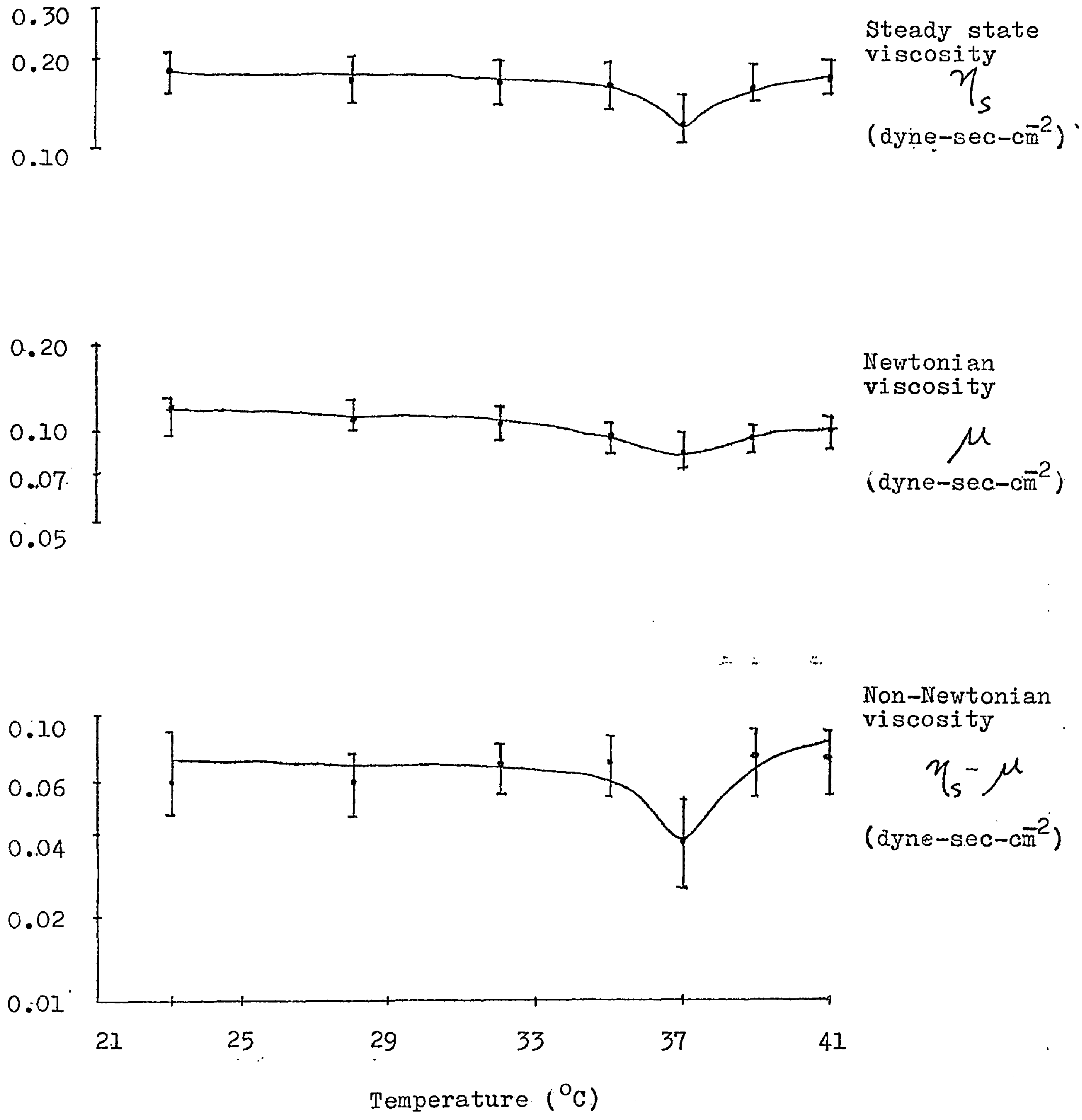


Fig.IV.2-2 Rheological parameters vs. temperature morphological changes of the macromelecules, erythrocytes and their rouleaux. At 37°C, one may say as a transition point, the flexibility and deformability of erythrocytes arrive at their highest points, and the intramolecular forces reach their lowest points. Thus the rheological resistance of blood drops to its minimum. Once a couple of degree offsetting occurred from the turning point (37°C), the rheological resistance will increase quickly, and then reach to a constant over certain range of temperature.

If the temperature is beyond 37°C, the elevation of the thixotropic properties of blood may be due to the orientation changes of high molecules such as fibrinogen, thus changes will make the aggregation of erythrocytes more firmly. In case the temperature of blood drops below 37°C, the flexibility and deformability of red cells will increase, i.e. RBC will become much stiffer, thus the intercellular friction among red cells in rouleaux will increase. Therefore it needs more mechanical energy to shear the blood and to break its rouleaux. This is probably why the thixotropic properties of blood also increase.

It is worthwhile in a further study to model the temperature dependence of rheological properties of blood mathematically. In general, in gases at low density, the viscosity dependence on temperature follows the following equation:

$$\eta = a\sqrt{T} + b \qquad \text{(IV.2-2)}$$

where a, and b are empirical constants. For the pure Newtonian fluid, the most commonly used expression relating viscosity to temperature (in normal range) is the Arrhenius equation (19).

$$\eta = A_o \, \mathrm{Exp} \, (E/RT) \qquad \text{(IV.2-3)}$$

where R is the gas constant, E the energy of activation for flow, and A_o a coefficient depending upon the nature of the liquid. For non-Newtonian fluids, such as certain polymers, another equation sometimes useful in correlating viscosity-temperature data in the region of Newtonian behavior is suggested (19):

$$\eta = a \, \mathrm{Exp} \, (-bT) \qquad \text{(IV.2-4)}$$

where a, and b are empirical constants.

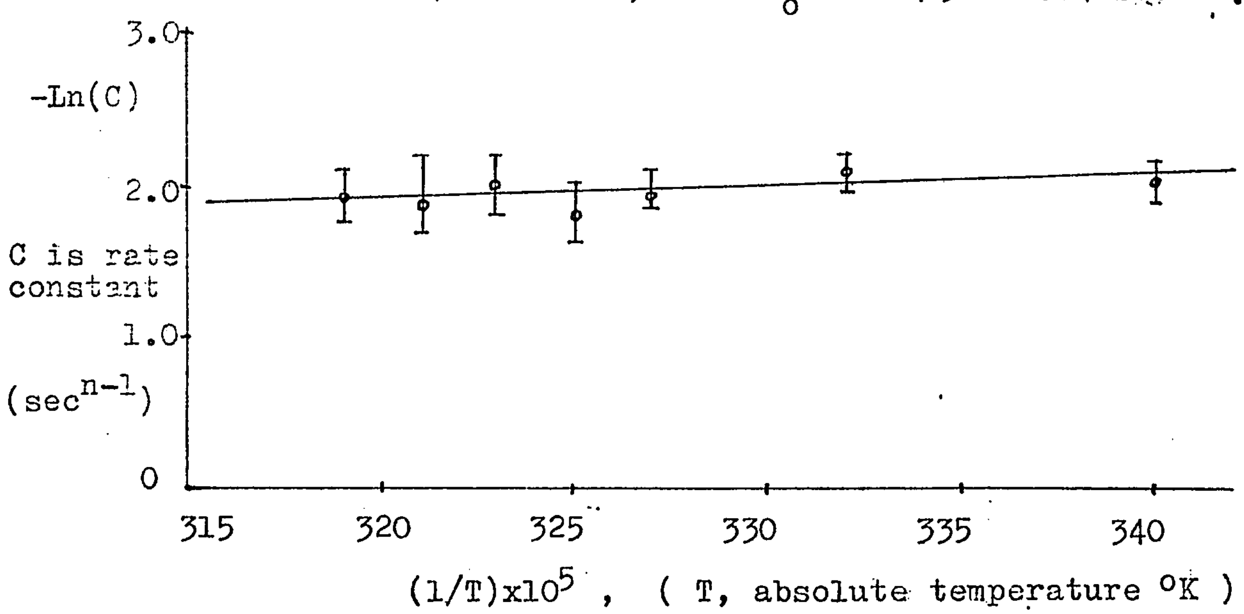
However, theoretically it will be very difficult to establish the temperature dependence of the rheological parameters of blood, since blood is a very complex, thixotropic fluid with suspended particles about which there there is relatively little knowledge available. While a certain physicochemical disturbance is induced in blood, the suspended particles may undergo rotation, translation, deformation, aggregation, disaggregation, and other interaction or chemical reaction and so on. But how and why? it will be a worthwhile subject for a further study.

The statistical results show that n is always a constant, independent of temperature; the temperature dependence of other oarameters τ_0 , A , μ , η_s , and $\eta_s - \mu$ all present as a valley-shape. It will not be easy to develop an equation to fit the experimental data. It ia also a worthwhile subject for a further attack.

The parameter C follows the exponential form as the Arrhenius equation:

$$C = A_o \text{ Exp } (-E/RT) \qquad \text{(IV.2-5)}$$

where E the activitive energy, A_o the coefficient depending on the nature of whole blood, R the gas constant. E and A_o can be obtained by plotting ln(C) versus 1/T. Their values are E = 192.2 cal./mole , and A_o = 0.123 0.04 sec^{n-1}.



All the previous investigations were based upon the normal healthy blood. For the pathological blood, Reis (43) discovered that for rising temperature from $0^{o}C$ to $50^{o}C$, the viscosity of blood serum decreased progressively. There was a temperature existing, above which the viscosity started to increase due to certain physico-chemical, and pathological factors. Stoltz, el at. (44) also showed the abnormal relation between plasma viscosity and temperature for the blood containing extra macromolecules. The temperature dependence of pathological blood may have different proriles of the rheological properties.

3. <u>Correlation of thixotropic properties and chemical tests of whole human blood</u>

There is too much unknown in blood. From the molecular level, the whole structure of blood is still mysterious. The change in part or in whole blood induced by altered physico-chemical environment has attracted many investigators.

Arellie, et al. (20) discovered platelet aggregation in platet-rich plasma induced by catecholamines (including adrenaline, nonadrenaline, dopamine, and 5-hydroxytryptamine). Newman (21) found that the viscosity of whole blood increased as a result of an increase in cholesterol level, especially more remarkably at the lower shear rates. Also dextran, a plasma expander which influenced the whole blood viscosity was exhibited by Singh (22). Anticoagulant heparin was showed to reduce the storage component of the elastic modulus and to increase the clotting time (24). Using erythrocytes suspended in buffer and morphology altering agents (2,4,6-trinitrobenzene, 2,4-dinitrobenzene, chloropromazine.HCl, and sodium salicylate), Meiselman (25) exhibited that the rheological effects of the discocyte-echinocyte shape transformation existed at the lower shear rates. Houbouyan, et al. (26) indicated that some antibiotics affected the rheological properties of blood and platelet aggregation.

To deal with the field of molecular rheology, subphases of blood, morphological structure of cells, various amphiphilic agents may be employed to transfer the normal biconcave shape (discocyte) into either the crenated (echinocyte) or cupped (stomatocyte) form (25,26). These agents appear to act as true antagonists, although at high concentrations there is an irreversible process of smooth sphere to haemolysis (27). Sheetz and Singer (28) have proposed a theory for these shape transformations based on an asymetric bilayer model of the RBC (red cells) membrane; echinocyte agents intercalate preferentially into the exterior half of the bilayer whereas stomatocytic agents are suggested to act mainly on the interior half.

The rheological properties of whole blood are mainly determined by the situation of erythrocytes which present complicated response to various chemicals. Motais (29) found that organic anion (mainly acids) transport in red blood cells was determined by the membrane specificity, Missirlis, et al.(30) used micropipette analysis to estmate the haemolytic stress of hypotonic erythrocytes under the influence of lipid-soluble compounds. Jain, et al. (31) exhibited that the difference of intrinsic perturbing ability of alkanols in lipid bilayers arised from a specificity of interaction between alkanols and lipid bilayer.

Systems are studied generally as a whole. An attempt to isolate certain rheological processes from individual fragments of more complex blood and then combine them in order to explain the functions of the whole blood can not be expected to supply the whole truth In this study, first around seventy chemicals which were considered having certain important effect in blood have been selected on a screening test to see if they can cause any influence in the thixotropic properties of blood. Finally a series of normal alkanols including one (C_1) to eleven (C_{11}) carbons which were found obviously causing the thixotropic changes in blood have been chosen as a typical example in this study. The other reason is that these alkanols have systematically exhibited different molecular sizes and solubilities in water and lipids, which are main compositions in blood and red cells respectively. (32,35,36)

To each 5.0 ml. of blood which was obtained from Northen New Jersey Blood Bank, East Orange, New Jersey, a certain amount (not exceeding 0.1 ml.) of alkanol was added; then the blood was gently shaked and warmed at 37°C for 30 minutes. Then, the sample was ready for rheological tests at 37°C. The results from this investigation were plotted in Fig.IV.3-1 to Fig.IV.3-7. The detailed data have been shown in Appendix II.3.

The A, τ_0, C, and n were plotted, on a linear scale, against the number of carbon molecules (or molecular size). The η_s and μ were plotted, on semilog scale, against the number of carbon. One may observe the following phenomena:

(a). Different alkanols have different effects in the rheological properties of the same blood.

(b). Different bloods have different responses to the same alkanol.

(c). In this investigation, all blood, two normal and two abnormal samples follow a similar thixotropic pattern with respect to the eleven alkanols.

(d). Alkanols with C_5 to C_9 can change the thixotropic blood to a Newtonian blood, in which C_7 drops the Newtonian viscosity to the lowest point.

It is also obviously that the results can be grouped according to the solubility of alkanols in water (Fig.IV.3-5). The first group, hydrophilic alkanols, containing methanol (C_1), ethyl alcohol (C_2), n-propyl alcohol (C_3), and n-butanol (C_4), shows extreme solubility in water; the second group, amphiphilic alkanols containing normal alkanols with five to nine carbons, shows partial solubility in water and partial solubility in lipids; while the third group, hydrophobic alkanols, composed of 1-decanol and 1-undecanol, exhibits a complete insolubility in water, but is soluble in lipids.

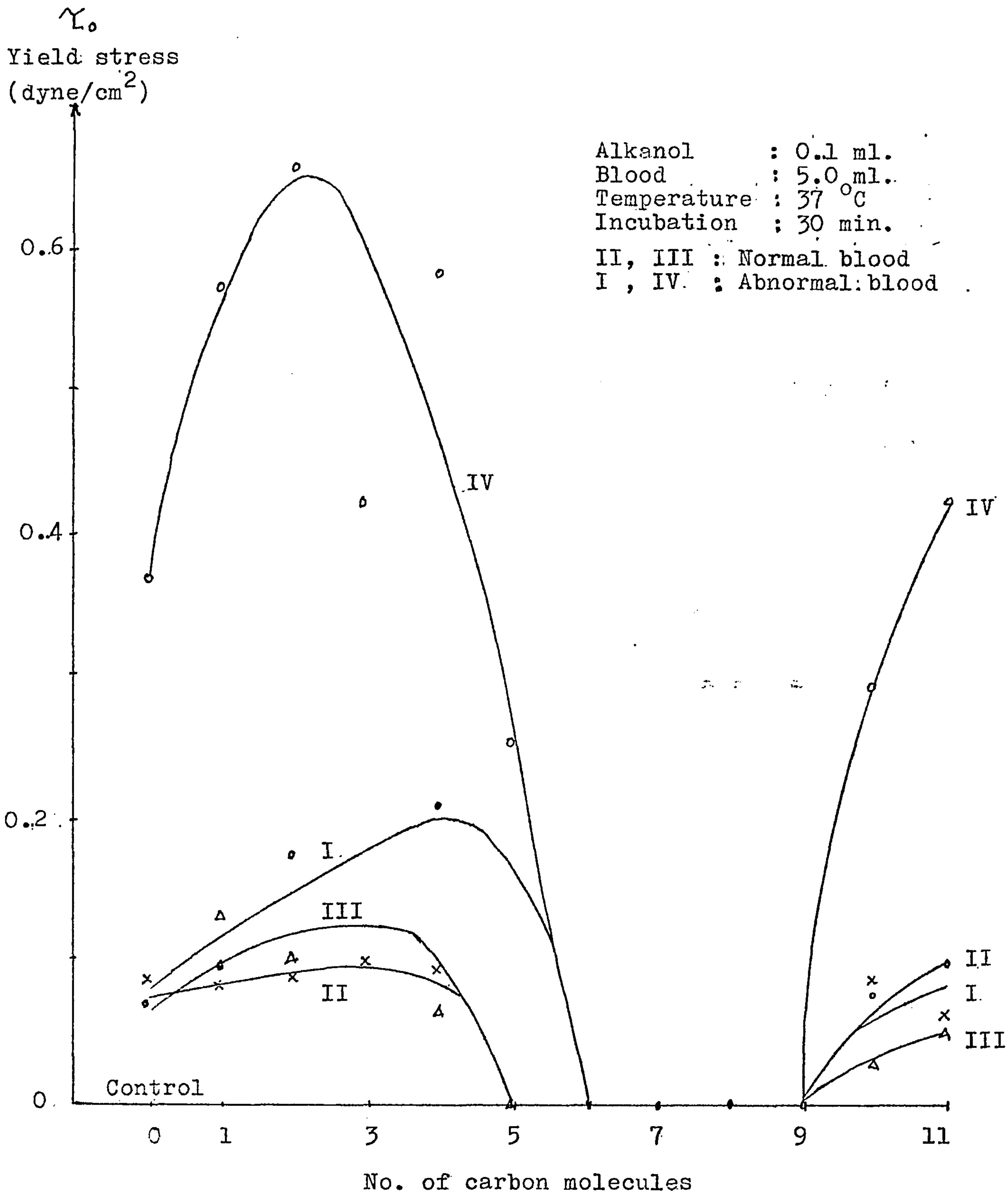


Fig.IV.3-1 τ_o vs. no. of carbon molecules in linear normal alkanols (blood rheological properties affected by normal alkanols)

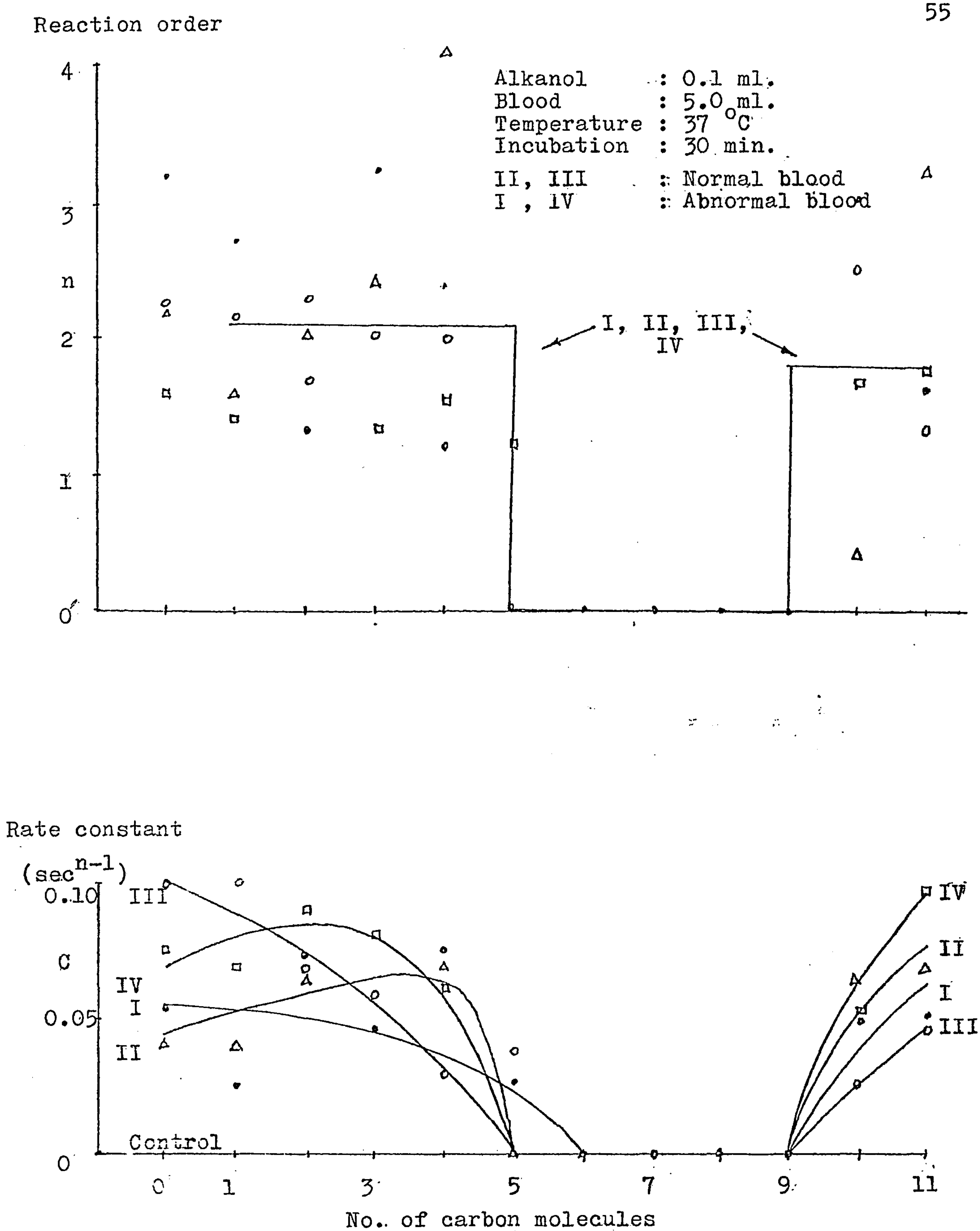


Fig.IV.3-2 C, n vs. no. of carbon molecules in linear normal alkanols (blood rheological properties affected by normal alkanols)

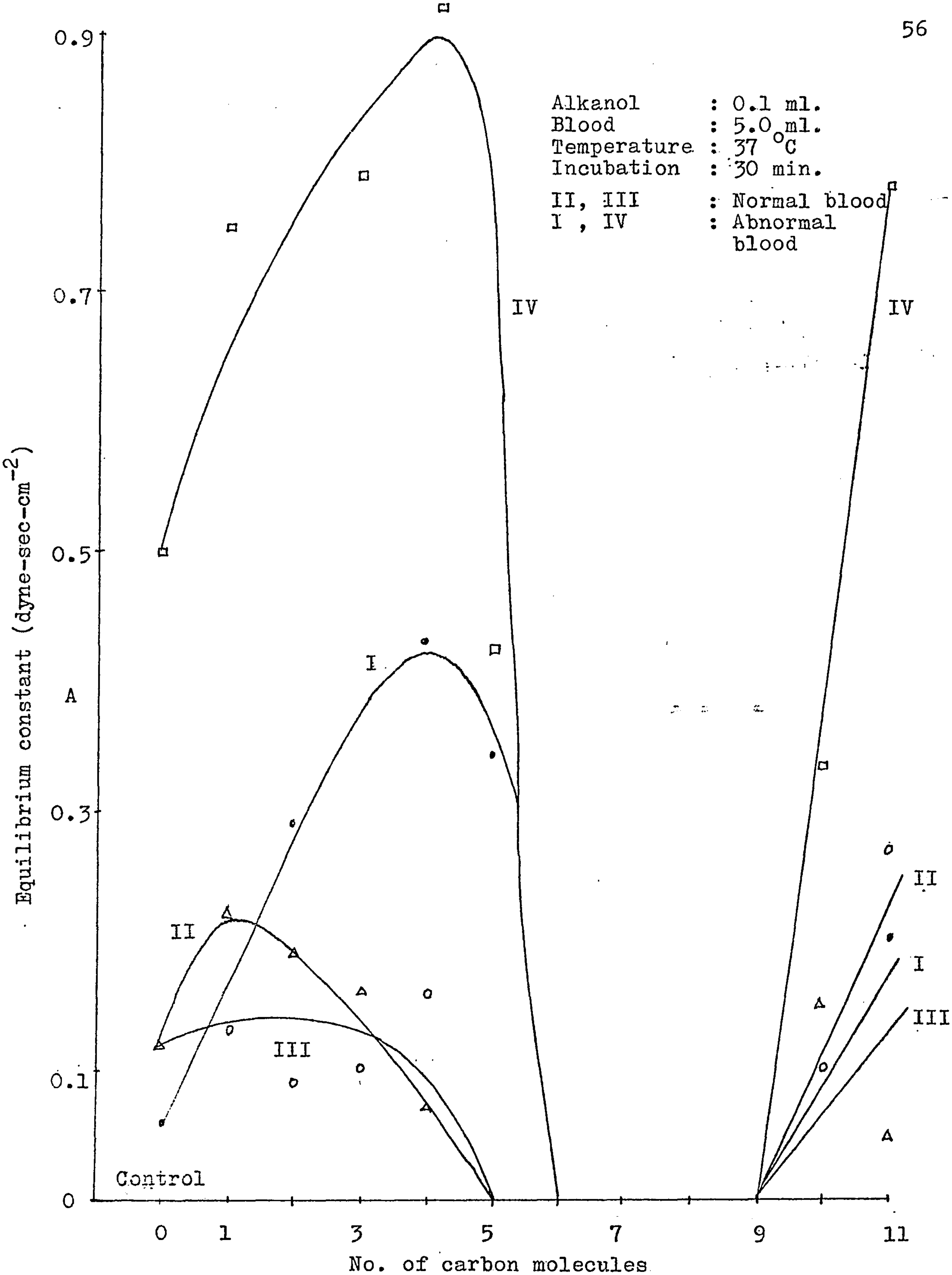


Fig.IV.3-3 A vs. no. of carbon molecules in linear normal alkanols (blood rheological properties affected by normal alkanols)

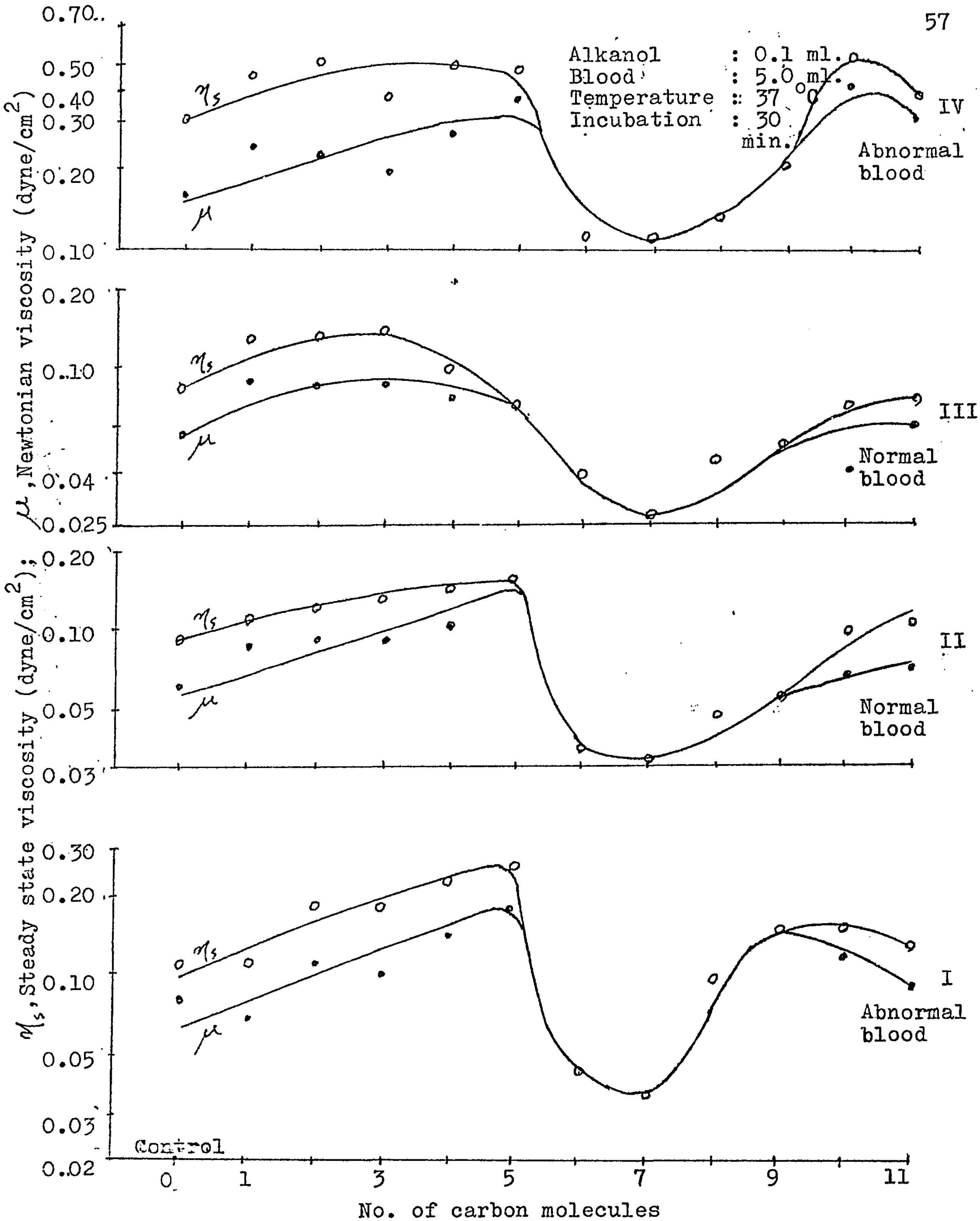
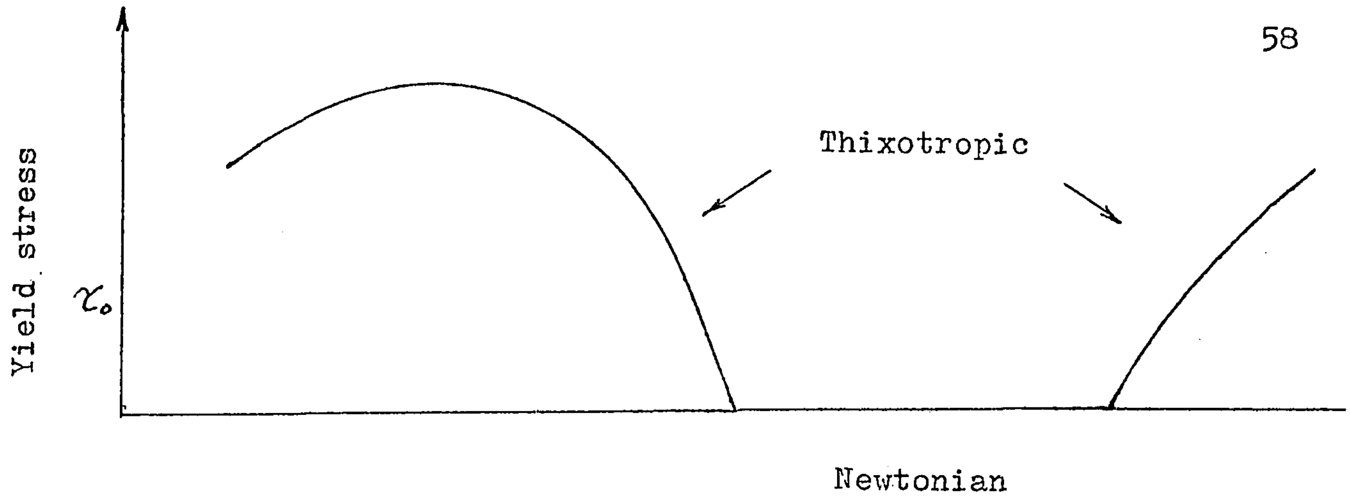


Fig.IV.3-4 η_s, μ vs. no. of carbon molecules in linear normal alkanols (blood rheological properties affected by normal alkanols)



Polarity: ← Hydrophilic | Amphiphilic | Hydrophobic →

Solubility ($g/100g\ H_2O$)	∞	∞	∞	9.15	2.78	0.60	0.18	0.054	0^+	0	0
Alkanol in plasma (g/3g)	0.1	0.1	0.1	0.1	0.083	0.054	0.0054	0.0017	0^+	0	0
Alkanol in RBC (g/2g)	0	0	0	0	0.017	0.046	0.095	0.098	0.1^-	0.1	0.1
Estimated partition coefficient	0	0	0	0	0.30	3.83	2.628	30.34	∞^-	∞	∞

Molecular weight (size) ——— Increase ———→

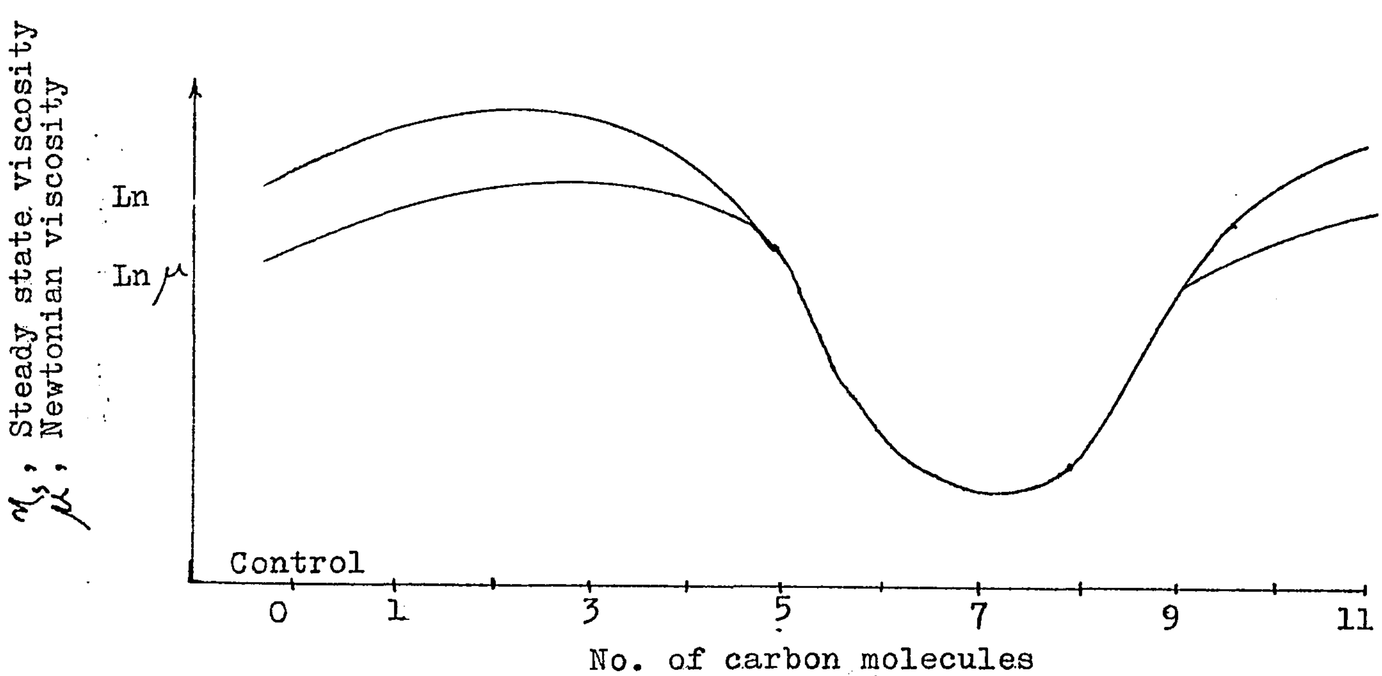


Fig.IV.3-5 Blood rheological properties vs. physical data of alkanols (blood rheological properties affected by normal alkanols)

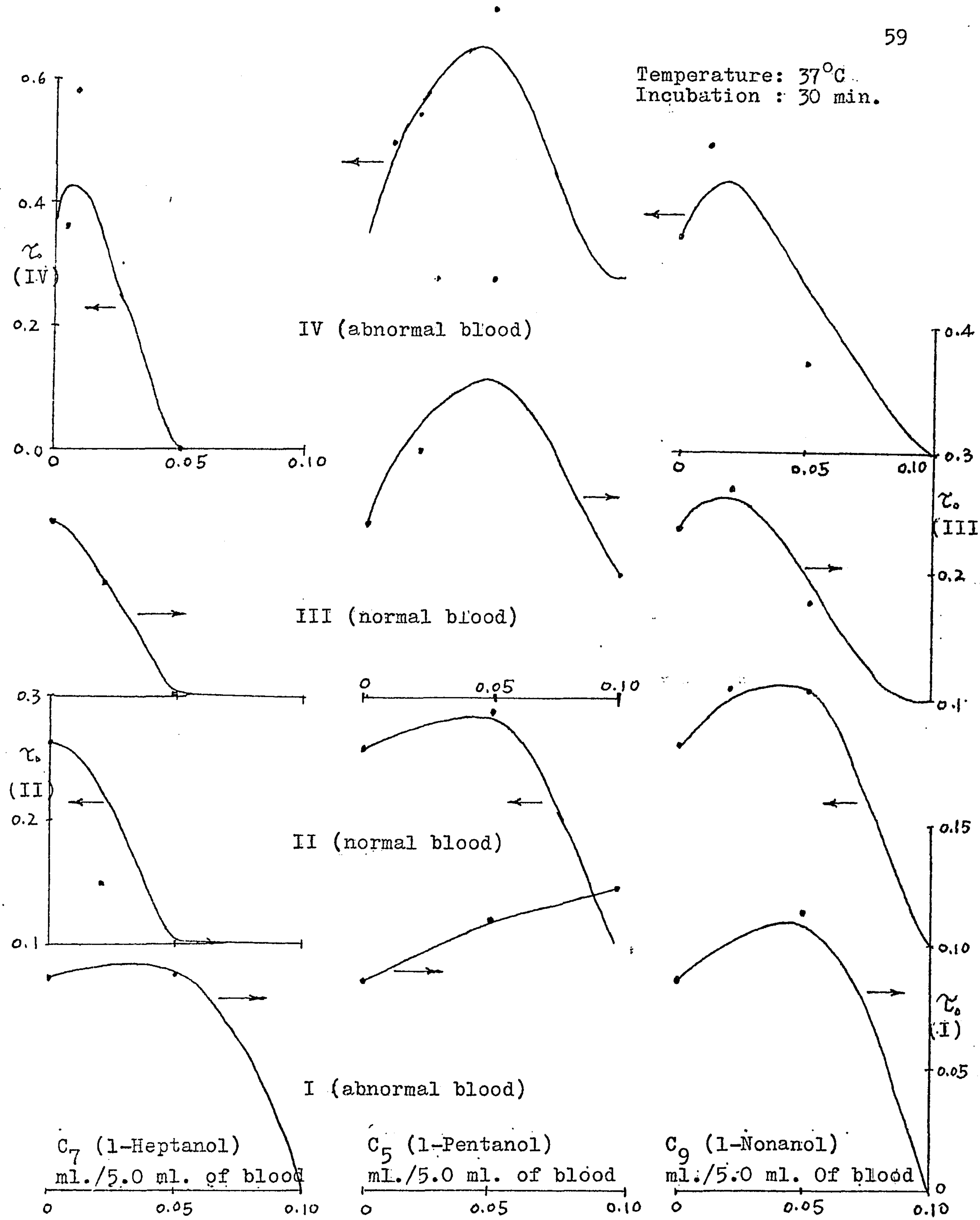
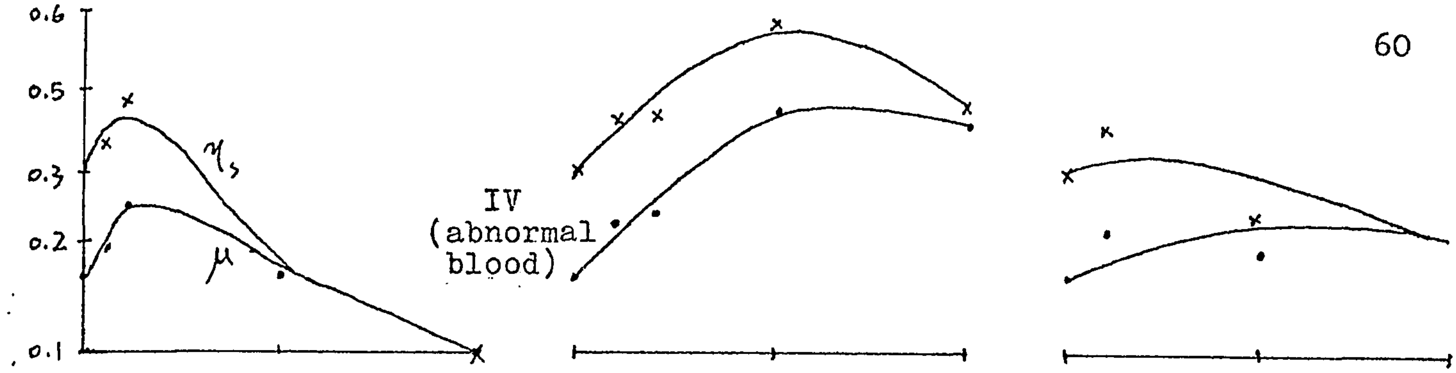


Fig.IV.3-6 τ_o vs. C_5, C_7, C_9-alkanol concentration (x5)
(⟶ indicates the coordinate should be used)



η_s, Steady state viscosity; μ, Newtonian viscosity (dyne-sec-cm^{-2})
Temperature - 37°C; Incubation - 30 min.

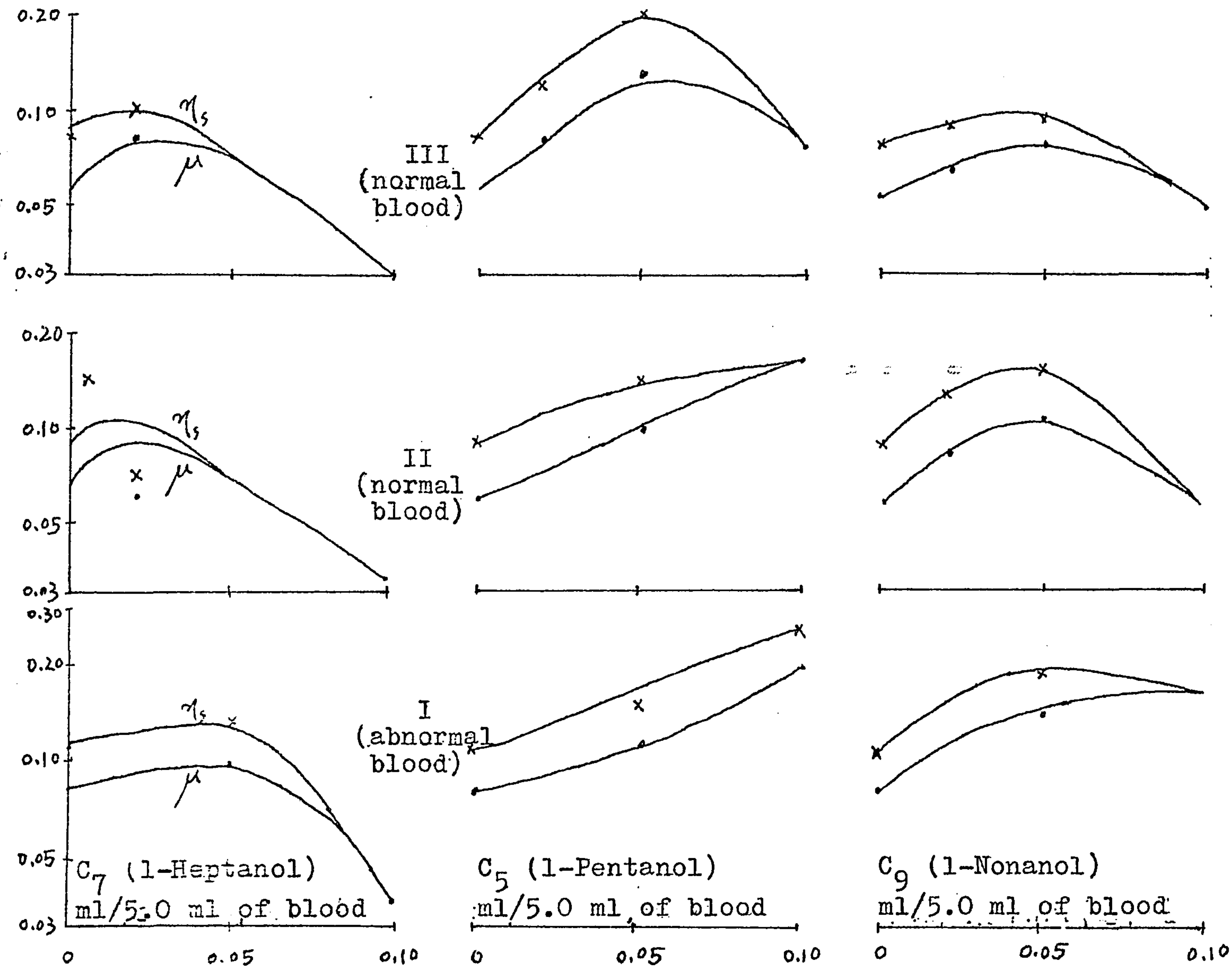


Fig.IV.3-7 η_s, μ vs. C_5, C_7, C_9-alkanol concentration (x5)

With regard to the effect of concentration of alkanols in the blood rheology, three alkanols (C_5, C_7, and C_9) which were found more active to change the rhological properties of blood than others have been choosen as examples. The relationship of τ_o, μ, and η_s vs. the concentration of the alkanols were plotted in Fig.IV.3-6 and Fig.IV.3-7, which have showed the two normal blood samples (II and III) have similar responses to the amphiphilic alkanols, but not for the abnormal ones (I and IV). Furthermore, if looking in detail, even the two similar ones (normal blood II and III) have showed different responses to the same alkanol C_5 (Fig.IV.3-7). And, no matter whether normal or abnormal, all blood samples are much more sensitive to C_7 alkanol than to others.

To explain these phenomena it will be very difficult from the microscopic viewpoint of blood, since there is too much knowledge indeed needed to develop and to understand the erythrocyte structure. For simplification, let's see the blood from a macroscopic point of view. Blood can be considered as a part water soluble (plasma), and a part water insoluble (erythrocytes) as shown in Fig.IV.3-8.

(a) When the hydrophilic substances (C_1 to C_4) dissolve in plasma, they may modify the exterior surface of the erythrocyte membrane in some way to strengthen the rouleaux formation. Thus it will cause the increase in

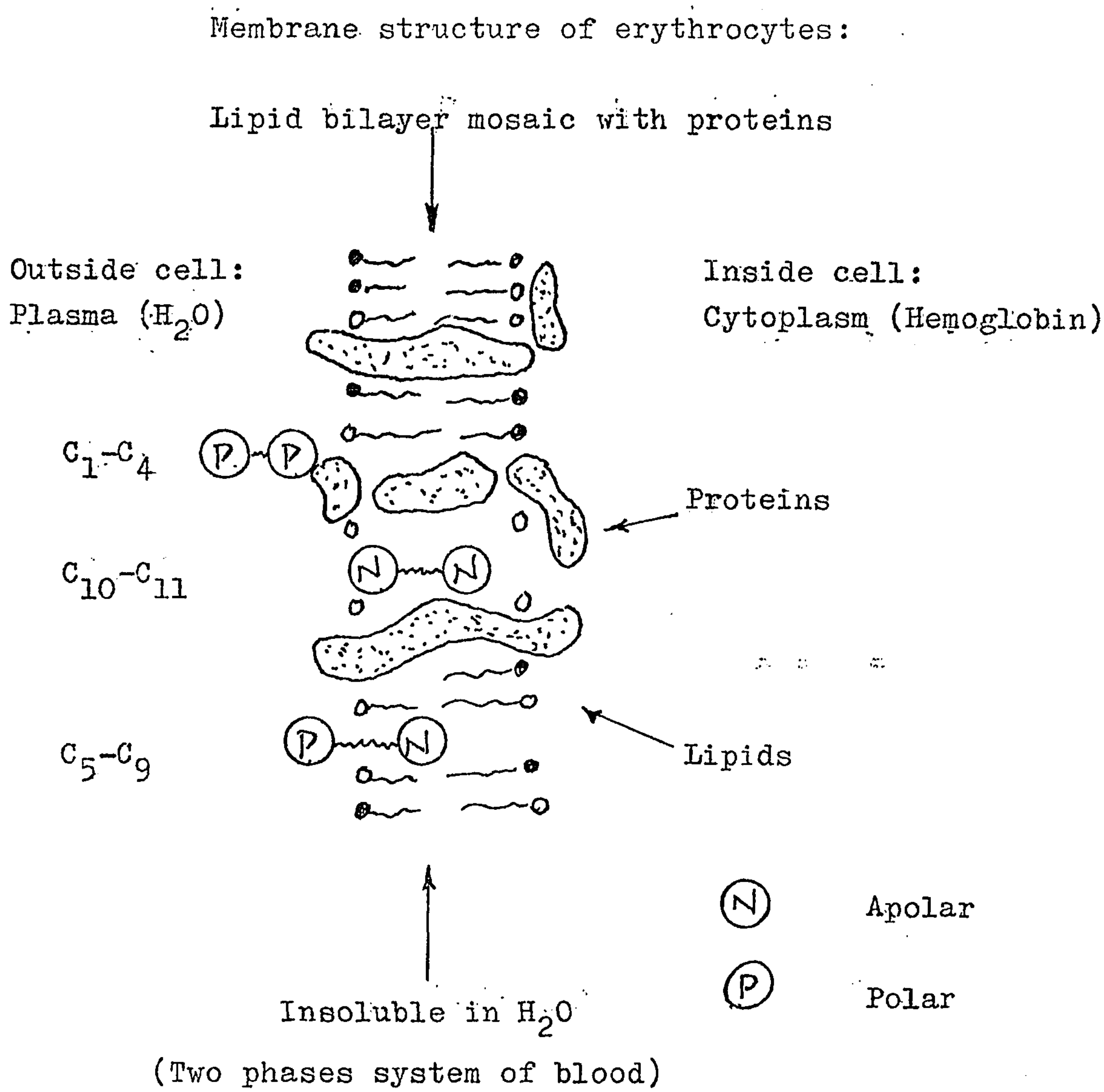


Fig.IV.3-8 Mechanisms of linear normal alkanols reacting with erythrocytes; C_1 to C_{11} indicate the alkanols which have 1 to 11 carbon molecules thixotropic properties of whole blood.

(b). While the hydrophobic substances (C_{10} to C_{11}) favorably join in the erythrocyte membrane in loose part to reinforce its strength, and make the red cells much stiffer. In other words, the deformability or flexibility of erythrocytes and rouleaux will decrease, that means the thixotropic properties of blood will increase.

(c). While the amphiphilic substances (C_5 to C_9) dissolve in blood, their apolar tails will insert in the erythrocyte membrane, and their polar heads will suspend on the exterior surface of the membrane. At low concentrations, they act as a combination of hydrophilic and hydrophobic substances. Once their concentration presented in blood is beyond a certain level at which the membrane is already saturated by the substances. The residual amphiphilic molecules in plasma will continuously attack the membrane. The haemolysis will occur until the intramolecular force between the molecules in plasma and the molecules in the saturated erythrocytes overcomes the intermolecular force which holds the whole membrane as a unit.

(d). The different responses from different bloods to the same alkanol may imply that each individual blood has its own signature (intrinsic ultrastructure of the erythrocyte) and chemical selectivity which were found most favorably to amphiphilic C_7. The different Newtonian behaviors from the haemolysis caused by the attack of C_5 to C_9 may reveal the different size of fragments broken from erythrocytes.

CHAPTER V

CONCLUSIONS

Traditionally, most investigators used apparent viscosities at various constant shear rates to evaluate the rheological property of fluid, it would be okay if the fluid were just shear -rate dependent, or a monotonic system. However, to a multitonic system such as blood- a thixotropic fluid, the traditional method would not be suitable to evaluate its rheological properties, since such a system is just dependent upon the shear rate but also the time.

To correct the traditional method, the Huang model which can be applied to very wide ranges of shear rates and time, emphasizes to use rheological parameters to characterize the flow properties of thixotropic system from its hysteresis loop and torque-decay curve. Once the parameters are determined, the apparent viscosity can be evaluated at any shear rate and at any time of shearing. Thus, the use of the parameters will also eliminate the uncertainty of the apparent viscosity of a sensitive thixotropic system such as blood tested at very low shear rate, obtained by different investigators.

Physiologically, low velocities of blood flow occuring in the microcirculation in capillary or small vessels would be mainly influenced by the presence of aggregates of the red cells, which in turn depend on the shear rate and, on the history of the blood and the originate of the blood. Low shear rates also exist particularly in pathological conditions such as circulatory shock and thrombosis. The rheological parameters are relevant to the practice of clinical medicine.

The study of open heart surgery during cardiopulmonary bypass is a typical example, which brings together information essential to clinicians and medical workers studying the causes for circulatory diseases, and for engineers who want to apply concepts of the more developed sciences to the so complex problems of medicine. If a simulation study could be made prior to the operation, it might probably have saved the lives of patients who were expired after open heart surgery. In other words, the highly abnormal rheological syndrome which might be of clinical and diagnostic importance could be predicted. A series of tests might help to discover early or silent conditions of hematological disorders or malignancy. The complex clinical tests could be supplemented, and sometimes replaced , by the new rheological methods.

Although changes in rheological parameters may play an important role in clinical medicine or pathophysiology that exists within blood flow, the diagnostic value is sometime difficult to be specific since different causes of illness may lead to same changes in rheological properties. However, the rheological parameters, in combination with other changes in the blood such as changes in temperature, inducement of chemicals or other physio-chemical factors, serve to narrow the range of diagnosis required.

The rheological study of blood added with different chemicals also imply its applications to other sciences such as pharmocology, toxicology, and ultrastructures of blood, and so on. The effect of temperature on blood thixotropy shows, at least that a living thing containing a thixotropic blood circulating through its organs has its own body temperature, at which the resistance for blood flow is going to be minimum, and at which its physiological functions are operating normally. It further indicates that each individual thixotropic system may have its own thixotropic temperature, at which its flow properties turn to minimum. The temperature study also paves a way to solve certain thermodynamic problems in the thixotropic system.

The study of dynamic behaviors of the torsion head indicates that the rheogram is controlled by the time constant and damping factor of the head, which in turn are determined by the torsion bar constant and the geometrical and physical properties of the head. This provides an information in its installation, of which the damping factor can be adjustable by changing the viscosity of damping fluid. To minimize the artefact of rheograms, which is mainly dominated by the torsion head provided that the inertia constant and damping coefficient of the tested fluid comparing with those of the head can be neglected, a suitable underdamping factor combining with a particular time constant is necessary if both a single step change and a triangular step change in the rotating part of the viscometer are assigned to a specific rpm range.

CHAPTER VI

RECOMMENDATION

1. Some unusual lumps in hysteresis loop and torque-decay curve were occasionally found from the pathological blood during cardiopulmonary bypass. A simulation approach by assuming the increase of catchamines in blood owing to certain physical or psychological stress that occurred in the patient, was attempted, but failed to produce results. These lumps may imply some very important pathological factors. A further investigation for their solution would probably bring about significant improvements in preventive medicine.

2. Blood, especially for the particular pathological conditions, usually is not easy to obtain. In order to facilitate the research, using the simulated blood from normal people or other living subjects is recommended.

3. It takes almost 5 ml. of blood sample in the present cell for each experiment. A specific pathological blood is hard to acquire. In order to expand its usefulness, to reduce the size of cell (microviscometer) is therefore recommended.

4. The chemicals in blood in vitro are more stable at 4°C than at 37°C. The latter temperature may cause their denaturalization and affect the blood rheology. Also

Reis (43) and Stoltz (44) indicated that the viscosity of blood was much higher at 4°C than that at 37°C. Then the rheological parameters obtained at 4°C in place of 37°C would provide clear and better results. This, in turn automatically involves that a cooling system installed in the rheogoniometer is necessary.

5. There is need for more research to find out if there are any chemicals which are widely involved in our ordinary lives, such as enviromental polluters, pharmaceuticals, and food additives and etc., that present any instantaneous or potential danger to the human blood or our health (38).

6. Though the recovery of hysteresis phenomena in blood is fast, it has been observed that the speed sometimes is different due to different blood samples. This may indicate the memorial property of blood. How to define it? How to find it? What is its relation with respect to other variables existing in blood? All these problems are worhty of a future investigation.

7. To accelerate to obtain the experimental results, an automation of the whole system is absolutely necessary.

8. Systematic dictionaries of the rheological parameter vs. various physical, chemical, and pathological factors which affect blood will bring a great advantage to life sciences.

Appendix O. Nomenclature

A : Equilibrium value of structural arrangement parameter, dyne-sec-cm^{-2}.

a : Acceleration constant for a triangular step change, cm-sec^{-2}.

C : Kinetic rate constant of structural breakdown of rouleaux to individual erythrocytes, sec^{n-1}.

G : Torsion bar constant, dyne-cm/micrometer.

I : Moment of inertia of torsion head, dyne-cm-sec^{2}/micrometer.

L : Length of the inner cylinder of the single couette cell, cm.

n : Reaction order of structural breakdown of rouleaux to individual erythrocytes.

R : Radius of the rotating cylinder of the single couette cell, cm.

t : Time, sec.

α : Angular deflection of torsion bar, micrometer.

$\dot{\gamma}_{r\theta}$: rθ-component of shear rate, sec^{-1}.

$\tau_{r\theta}$: rθ-component of shear stress, dyne-cm^{-2}.

τ_0 : Yield stress, dyne-cm^{-2}.

μ : Newtonian viscosity, dyne-sec-cm^{-2}.

η : Torsion head damping coefficient.

Ω : RPM of the rotating cylinder in the single couette cell.

Appendix I.1 - Mathematical Derivation for the Dynamic Behavior of Torsion Head

In a rotating viscometer such as couette, cone and plate and etc., a torque T(t) is transmitted by the tested fluid to the torsion bar due to the angular movement of the bottom shaft (Fig.II.1). For most viscometer design, it is assumed that the torque is proportional to the angular deflection of the torsion bar $\alpha(t)$.

In this derivation, the angular velocity and angular acceleration are also being considered. From the derivation, we can established under what contions, the assumption of T(t) is linear with respect to $\alpha(t)$ is held for both a single step change and a triangular step change.

Equation of motion of the torsion head

$$I\frac{d^2\alpha}{dt^2} + \eta\frac{d\alpha}{dt} + G\alpha = T(t)$$

$$\text{B.C.} \quad \alpha(0) = 0 \;, \quad \alpha'(0) = 0$$

Define new parameters:

$$\tau = (I/G)^{1/2} = \quad \text{--- characteristic time constant}$$

$$\xi = \eta/2(IG)^{1/2} = \quad \text{--- damping factor}$$

Then the equation can be rearranged as follow:

$$\tau \frac{d^2\alpha}{dt^2} + 2\xi\tau\frac{d\alpha}{dt} + \alpha = \frac{1}{G}T(t) \qquad \text{(A.I.1-1)}$$

Assume an incompressible Newtonian fluid in the viscometer.

Equation of motion of the fluid

$$\rho \frac{\partial V_\theta}{\partial t} = \mu \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rV_\theta)}{\partial r}\right] \qquad \text{(A.I.1-2)}$$

(A). For a single step change

$$\text{B.C.} \quad \begin{aligned} V_\theta(r, 0) &= 0 \quad ; \quad t = 0 \\ V_\theta(KR, t) &= 0 \quad ; \quad t \geq 0 \\ V_\theta(R, t) &= R\Omega \quad ; \quad t \geq 0 \end{aligned} \qquad \text{(A.I.1-3)}$$

The solution of Eq.AI.1-2 for (A) is shown in (9):

$$\frac{V_\theta}{R\Omega} = \left(\frac{1}{1-K^2}\right)\frac{r}{R} - \left(\frac{K^2}{1-K^2}\right)\frac{R}{r} + \sum_{n=1}^{\infty} E_n e^{-\beta_n^2 t} Z_1(\theta_n r)$$

(B). For a triangular step change

$$\text{B.C.} \quad V_\theta(r, 0) = 0 \qquad \text{(A.I.1-4)}$$

$$V_\theta(KR, t) = 0$$

$$V_\theta(R, t) = R\Omega(t) = \begin{cases} aRt & ; \ 0 \leq t \leq t_1 \\ 2aRt_1 - aRt & ; \ t_1 \leq t \leq 2t_1 \end{cases}$$

where a is angular acceleration constant of rotating cylinder.

The solution of Eq.AI.1-2 for (B) is also shown in (9):

When $0 \leq t \leq t_1$

$$V_\theta = a\left(\frac{r}{1-K^2} - \frac{K^2R^2}{1-K^2}\frac{1}{r}\right)t + \sum_{n=1}^{\infty} \frac{aR}{\beta_n^2} E_n \left(1 - e^{-\beta_n^2 t}\right) Z_1(\theta_n r)$$

When $t_1 \leq t \leq 2t_1$

$$V_\theta = a\left(\frac{r}{1-K^2} - \frac{K^2R^2}{1-K^2}\frac{1}{r}\right)(2t_1 - t)$$

$$+ \sum_{n=1}^{\infty} \frac{aR}{\beta_n^2} E_n \left(1 - e^{-\beta_n^2 t_1} + e^{-\beta_n^2 t}\right) Z_1(\theta_n r)$$

Where $g = (\rho/\mu)^{1/2}$; $\theta_n = g\beta_n$

$$Z_1(\theta_n r) = J_1(\theta_n r)Y_1(\theta_n KR) - Y_1(\theta_n r)J_1(\theta_n KR)$$

with $J_1(\theta_n r)$ and $Y_1(\theta_n r)$ being Bessel's functions of the first and second kind respectively.

The eigenvalue satisfy the following relationship:

$$J_1(\theta_n KR)Y_1(\theta_n R) = J_1(\theta_n R)Y_1(\theta_n KR)$$

The coefficient of E_n:

$$E_n = \frac{\frac{1}{1-K^2}\left[Z_o(\theta_n R) - K^2 Z_o(\theta_n KR)\right] - \frac{K^2}{1-K^2}\left[Z_o(\theta_n R) - Z_o(\theta_n KR)\right]}{\frac{1}{2}\theta_n R\left[Z_o^2(\theta_n R) - K^2 Z_o^2(\theta_n KR)\right]}$$

The shear stress distribution and the torque applied to the inner cylinder :

$$\tau_{r\theta} = -\mu r \frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right) = -\mu \dot{\gamma}_{r\theta} \; ; \quad T(t) = 2\pi KRL(-\tau_{r\theta})\Big|_{r=KR} \cdot KR$$

(A). For a single step change

$$T(t) = 2\pi K^2 R^2 L\mu\left[\frac{2\Omega}{1-K^2} - \sum_{n=1}^{\infty}\theta_n E_n e^{-\beta_n^2 t} Z_2(\theta_n KR)\right]$$

The transient term may be neglected in most viscometer calculation (9), therefore

$$T(t) = 2\pi K^2 R^2 L\mu \frac{2\Omega}{1-K^2} = A_o = \text{Constant} \qquad \text{(A.I.1-5)}$$

(B). For a triangular step change

When $o \leq t \leq t_1$

$$T(t) = \frac{4\pi a L K^2 R^2 \mu t}{1-K^2} +$$

$$2\pi a L K^2 R^3 \sqrt{\rho\mu} \sum_{n=1}^{\infty} \frac{E_n}{\beta_n} Z_o(\theta_n KR)\left(1 - e^{-\beta_n^2 t}\right)$$

When $t_1 \leq t \leq 2t_1$

$$T(t) = \frac{4\pi a L K^2 R^2 \mu (2t_1 - t)}{1 - K^2} +$$

$$2\pi a L K^2 R^3 \sqrt{\rho\mu} \sum_{n=1}^{\infty} \frac{E_n}{\beta_n} Z_0(\theta_n K R)\left(1 - 2e^{-\beta_n^2 t_1} + e^{-\beta_n^2 t}\right)$$

It was shown (9), if the dimensionless group

$$N_{HF} = \frac{(1-K^2)R}{2t_1} g \frac{E_1}{\beta_1} Z_0(\theta_1 K R) << 1 \quad ,$$

the summation terms of T(t) can be neglected, and since for most viscometers, $N_{HF} << 1$, so

$$T(t) = \begin{cases} Bt & ; \quad 0 \leq t \leq t_1 \\ \\ B(2t_1 - t) & ; \quad t_1 \leq t \leq 2t_1 \end{cases} \qquad \text{(A.I.1-6)}$$

where $B = \dfrac{4\pi a L K^2 R^2 \mu}{1 - K^2}$

Substitue T(t) (Eq.AI.1-5, AI.1-6) into Eq.AI.1-1, and using the method of Laplace transformation to solve Eq.AI.1-1, the following solutions have been obtained:

Case 1. For small τ and ξ , or $\tau \to 0$, $\xi \to 0$

Single step change

$$\bar{\alpha}(s) = \frac{A_0}{G\,s} \quad ; \quad \alpha(t) = \frac{A_0}{G}$$

Triangular step change

$$\bar{\alpha}(s) = \frac{B}{G}\,\bar{T}(s) = \frac{B}{G} \times \begin{cases} \dfrac{1}{s^2} & ; \quad 0 \leq t \leq t_1 \\ \dfrac{2t_1}{s} - \dfrac{1}{s^2} & ; \quad t_1 \leq t \leq 2t_1 \end{cases}$$

$$\alpha(t) = \frac{B}{G} \times \begin{cases} t & ; \quad 0 \leq t \leq t_1 \\ (2t_1 - t) & ; \quad t_1 \leq t \leq 2t_1 \end{cases}$$

Case 2. For small ξ or $\xi \to 0$

Single step change

$$\bar{\alpha}(s) = \frac{A_0/G}{s(\tau^2 s^2 + 1)} = \frac{A_0/G}{s} - \frac{(A_0\tau^2/G)\,s}{\tau^2 s^2 + 1}$$

$$\alpha(t) = \frac{A_0}{G}\,(1 - \cos(t/\tau))$$

Triangular step change

$$\bar{\alpha}(s) = \frac{B/G}{\tau^2 s^2 + 1} \times \begin{cases} \dfrac{1}{s^2} & ; \quad 0 \leqslant t \leqslant t_1 \\[2ex] \dfrac{2t_1}{s} - \dfrac{1}{s^2} & ; \quad t_1 \leqslant t \leqslant 2t_1 \end{cases}$$

$$\alpha(t) = \frac{B}{G} \times \begin{cases} \left(t - \tau \sin \dfrac{t}{\tau}\right) \\[4ex] \left[2t_1\left(1 - \cos \dfrac{t}{\tau}\right) - \left(t - \tau \sin \dfrac{t}{\tau}\right)\right] \end{cases}$$

Case 3. Overdamped, $\xi > 1$

Single step change

$$\bar{\alpha}(s) = \frac{A_0/G}{s(\tau^2 s^2 + 2\xi\tau s + 1)} = \frac{A_0}{G}\left(\frac{1}{s} - \frac{\tau^2 s + 2\xi\tau}{\tau^2 s^2 + 2\xi\tau s + 1}\right)$$

From (45), page 556

$$\alpha(t) = \frac{A_0}{G}\left[1 - \frac{(\xi + \sqrt{\xi^2 - 1})\, e^{-(\xi - \sqrt{\xi^2-1})\frac{t}{\tau}} - (\xi - \sqrt{\xi^2 - 1})\, e^{-(\xi + \sqrt{\xi^2-1})\frac{t}{\tau}}}{2\sqrt{\xi^2 - 1}}\right]$$

Triangular step change

When $0 \leqslant t \leqslant t_1$

$$\bar{\alpha}(s) = \frac{B/G\tau^2}{s^2\left(s^2 + 2\frac{\xi}{\tau}s + \frac{1}{\tau^2}\right)} = \frac{B/G\tau^2}{s^2\left[s + \frac{1}{\tau}(\xi + \sqrt{\xi^2 - 1})\right]\left[s + \frac{1}{\tau}(\xi - \sqrt{\xi^2 - 1})\right]}$$

From (46), page 203

$$\alpha(t) = \frac{B}{G}\left[t - 2\xi\tau - \frac{\tau}{2\sqrt{\xi^2-1}}\left(2\xi^2-1 + 2\xi\sqrt{\xi^2-1}\right)e^{-(\xi+\sqrt{\xi^2-1})\frac{t}{\tau}} - \left(2\xi^2-1-2\xi\sqrt{\xi^2-1}\right)e^{-(\xi-\sqrt{\xi^2-1})\frac{t}{\tau}}\right]$$

When $t_1 \leq t \leq 2t_1$

$$\bar{\alpha}(s) = \frac{B/G}{\tau^2 s^2 + 2\xi\tau s + 1}\left(\frac{2t_1}{s} - \frac{1}{s^2}\right)$$

$$= \frac{2t_1 B/G\tau^2}{s\left[s + \frac{1}{\tau}(\xi - \sqrt{\xi^2-1})\right]\left[s + \frac{1}{\tau}(\xi - \sqrt{\xi^2-1})\right]} - \frac{B/G}{s^2(\tau^2 s^2 + 2\xi\tau s + 1)}$$

From (46), page 195

$$\alpha(t) = \frac{2t_1 B}{G}\left\{1 + \frac{1}{2\sqrt{\xi^2-1}}\left[(\xi - \sqrt{\xi^2-1})e^{-(\xi+\sqrt{\xi^2-1})\frac{t}{\tau}} - (\xi + \sqrt{\xi^2-1})e^{-(\xi-\sqrt{\xi^2-1})\frac{t}{\tau}}\right]\right.$$

$$\left. - \frac{B}{G}\left\{t - 2\xi\tau + \frac{\tau}{2\sqrt{\xi^2-1}}\left[\left(2\xi^2-1+2\xi\sqrt{\xi^2-1}\right)e^{-(\xi+\sqrt{\xi^2-1})\frac{t}{\tau}} - \left(2\xi^2-1-2\xi\sqrt{\xi^2-1}\right)e^{-(\xi-\sqrt{\xi^2-1})\frac{t}{\tau}}\right]\right\}\right.$$

Case 4. Underdamping, $\xi < 1$

Single step change

$$\bar{\alpha}(s) = \frac{A_o/G}{s(\tau^2 s^2 + 2\xi\tau s + 1)}$$

From (46), page 192

$$\alpha(t) = \frac{A_o}{G}\left[1 - \frac{e^{-\xi\frac{t}{\tau}} \sin\left(\sqrt{1-\xi^2}\;\frac{t}{\tau} - \phi\right)}{\sqrt{1-\xi^2}}\right]$$

$$\text{with} \quad \phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{-\xi}$$

Triangular step change

When $o \leq t \leq t_1$

$$\bar{\alpha}(s) = \frac{B/G}{s^2(\tau^2 s^2 + 2\xi\tau s + 1)}$$

From (46), page 201

$$\alpha(t) = \frac{B}{G}\left[t - 2\xi\tau + \frac{\tau e^{-\xi\frac{t}{\tau}}}{\sqrt{1-\xi^2}} \sin\left(\sqrt{1-\xi^2}\;\frac{t}{\tau} - 2\phi\right)\right] = \frac{B}{G}F_1(t)$$

When $t_1 \leq t \leq 2t_1$

$$\bar{\alpha}(s) = \frac{B/G}{\tau^2 s^2 + 2\xi\tau s + 1}\left(\frac{2t_1}{s} - \frac{1}{s^2}\right)$$

From (46), page 192

$$\alpha(t) = \frac{2t_1 B}{G}\left[1 + \frac{e^{-\xi\frac{t}{\tau}} \sin(\sqrt{1-\xi^2}\ \frac{t}{\tau} - \phi)}{\sqrt{1-\xi^2}}\right] - \frac{B}{G} F_1(t)$$

Case 5. Critical damping, $\xi = 1$

Single step change

$$\bar{\alpha}(s) = \frac{A_0/G}{s(\tau^2 s^2 + 2\tau s + 1)}$$

$$\alpha(t) = \frac{A_0}{G}\left[1 - (1 + \frac{t}{\tau})\, e^{-t/\tau}\right]$$

Triangular step change

When $0 \leqslant t \leqslant t_1$

$$\alpha(s) = \frac{B/G}{s^2(\tau^2 s^2 + 2\tau s + 1)}$$

$$\alpha(t) = \frac{B}{G}\left(t\, e^{-t/\tau} + \frac{2}{\tau} e^{-t/\tau} + t - \frac{2}{\tau}\right) = \frac{B}{G} F_2(t)$$

When $t_1 \leq t \leq 2t_1$

$$\bar{\alpha}(s) = \frac{B/G}{\tau^2 s^2 + 2\tau s + 1}\left(\frac{2t_1}{s} - \frac{1}{s^2}\right)$$

$$\alpha(t) = \frac{B}{G}\left\{2t_1\left[1 - \left(1 + \frac{t}{\tau}\right) e^{-t/\tau}\right] - F_2(t)\right\}$$

Appendix I.2 - Mathematical Derivation of the Huang Equation

A brief mathematical derivation of the Huang equation is described as follows(2):

Although the system of a time-dependent, homogeneous, and non-Newtonian fluid is under non-equilibrium conditions during shearing at isothermal state; based on irreversible thermodynamics, it can be assumed that there exists within small mass a state of local equilibrium.

Therefore, the rate of generation of entropy due to the shear stress for a fluid with structural change as modeled by Huang is:

$$\sigma = -\frac{1}{T}\left[\tau^{ij}\frac{d\gamma_{ij}}{dt} + \tau^{ij}\frac{d\beta_{ij}}{dt}\right] \qquad \text{(AI.2-1)}$$

where γ_{ij} is the strain tensor, β_{ij} is the Huang's molecular rearrangement parameter, τ^{ij} is the stress tensor, t is the time, and T is the absolute temperature.

Huang then relates the contravarient tensor of first and second order in the above equation to the rate of strain, and the rate of molecular rearrangement parameter by the following phenomenological equations:

$$\tau^{ij} = \mu\frac{d\gamma^{ij}}{dt}$$

$$\tau^{ij} = -\xi\frac{d\beta^{ij}}{dt} \qquad \text{(AI.2-2)}$$

where μ is the apparent viscosity, and ξ is the molecular rearrangement coefficient. For a thixotropic fluid, Huang then assumed:

$$\frac{d\beta^{ij}}{dt} = -C_1 \beta^{ij} |\dot{\gamma}^{ij}|^n \qquad \text{for} \quad |\dot{\gamma}^{ij}| > 0 \qquad \text{(AI.2-3)}$$

$$\frac{d\beta^{ij}}{dt} = C_2 (\beta_e^{ij} - \beta^{ij}) \qquad \text{for} \quad |\dot{\gamma}^{ij}| = 0 \qquad \text{(AI.2-4)}$$

where β_e^{ij} is the equilibrium value of β^{ij} at t = 0, and C_1 and C_2 are rate constants, and n is the order of the rate equation.

An overall apparent viscosity η can be defined which will relate the shear stress to the rate of strain by considering both the rate of strain effect and the rate of molecular rearrangement effect as follow:

$$\tau^{ij} = \eta \frac{d\gamma^{ij}}{dt} = \mu \frac{d\gamma^{ij}}{dt} - \xi \frac{d\beta^{ij}}{dt}$$

$$\text{or} \quad \eta = \mu - \xi \frac{d\beta^{ij}/dt}{d\dot{\gamma}^{ij}/dt} \qquad \text{(AI.2-5)}$$

If the fluid has a yield stress τ_0^{ij} ; combining the equations (AI.2-2), (AI.2-3), and (AI.2-5), the Huang equation is obtained:

$$\tau^{ij} = \tau_0^{ij} + \mu \dot{\gamma}^{ij} + CA |\dot{\gamma}^{ij}|^n e^{-C\int_0^t |\dot{\gamma}^{ij}|^n dt} \qquad \text{(AI.2-6)}$$

where C = C_1, the rate constant; $A = \xi \beta_e^{ij}$, the molecular rearrangement parameter; $\dot{\gamma}^{ij} = \frac{d\gamma^{ij}}{dt}$, the shear rate.

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FORTRAN IV (VER 45 ) SOURCE LISTING:                    05/26/78  15:35:56  P

            PROGRAM BLOOD
      C*************************************************************************
      C     THIS PROGRAM WAS WRITTEN BY WALTER FABISIAK.  THIS PROGRAM
      C     UTILIZES A NON-LINEAR REGRESSION ALGORITHM DEVELOPED BY
      C     D.W. MARQUARDT(MARQUARDT, D.W., J. SOC. INDUST. AND APPL.
      C     MATH., 11, NO.2, (1963) 431-441).  THIS PROGRAM WAS SPECIFICALLY
      C     DESIGNED TO CALCULATE THE BEST ESTIMATES OF THE ADJUSTABLE
      C     PARAMETERS FOUND IN THE HUANG RHEOLOGICAL EQUATION OF STATE
      C     FOR THIXOTROPIC FLUIDS(HUANG, C.R., THE CHEM. ENG. JOURNAL,
      C     3, 100(1972).
      C*************************************************************************
            COMMON Y(100),X(100,5),PARAM(10),PRNT(5),CONST(4)
            DIMENSION INFO(20)
      124   READ(5,890,END=123)INFO
      890   FORMAT(20A4)
      C               READING IN THE DATA SET INFORMATION
      C               INFO IS THE NAME OF THE DATA SET BEING USED
            WRITE(6,891)INFO
      891   FORMAT(1H1,35X,20A4)
        5   READ(5,892)CONST(1),CONST(2),CONST(3)
       892  FORMAT(3F10.3)
      C               CONST(I) IS A CONSTANT USED IN THE MODEL BEING TESTED.
      C.........................................................................
            CALL FITIT
      C               FITIT IS THE NONLINEAR REGRESSION SUBROUTINE
      C.........................................................................
            WRITE(6,893)INFO
       893  FORMAT(/30X,6HEND OF,20A4)
            GO TO 124
      123   STOP
            END
```

Appendix I.3. Modified Marquardt computer program for Huang's parameters

(Since this program is very complicated, it is better to keep the original program in place of a flow chart)

```
      SUBROUTINE FITIT
C*****************************************************************************
C                   NONLINEAR REGRESSION SUBROUTINE
C                   THIS SUBROUTINE IS A MODIFICATION OF THE SUBROUTINE
C                   SNOWJO WRITTEN BY R. ROBERTSON(M.S.CHE.,1972,N.C.E.).
C                   MODIFIED VERSION PROGRAMMED BY WALTER FABISIAK
C                   THE FOLLOWING COMMENTS ILLUSTRATE THE OPERATIONAL
C                   SEQUENCE OF THE NONLINEAR REGRESSION SUBROUTINE.
C.............................................................................
C     CALL SUBZ(Y,X,PARAM,PRNT,NPRNT,NDATA)
C                   CODING FOR CASE INITIALIZING GOES HERE
C                   NPRNT IS THE NUMBER OF OTHER TERMS TO BE PRINTED
C                   THE TERMS TO BE PRINTED ARE IN PRNT(1)...PRNT(5)
C     CALL MODEL(Y,X,PARAM,PRNT,FCN,I,RESDUE)
C                   CODING TO MAKE FCN GOES HERE
C                   THE EQUATION TO BE TESTED IS WRITTEN HERE
C                   IT IS SET EQUAL TO FCN(Y HAT)
C     CALL DERIV(PARTL,X,PARAM,PRNT,FCN,I)
C                   THIS SUBROUTINE IS FOR THE USE OF
C                   ANALYTIC PARTIAL DERIVATIVES
C                   CODING TO MAKE (PARTIAL FCN/PARTIAL PARAM) GOES HERE
C                   MAKE NPARAM OF THEM AND CALL THEM PARTL(J)
C.............................................................................
C                   READ FIRST CARD OF THE NEXT CASE
C*****************************************************************************
      COMMON Y(100),X(100,5),PARAM(10),PRNT(5),CONST(4)
      INTEGER IDATA(5)/' ','O','P','Y','X'/
      EQUIVALENCE (IBCH,IDATA(1)),(IOCH,IDATA(2)),(IPCH,IDATA(3)),
     1(IYCH,IDATA(4)),(IXCH,IDATA(5))
      DIMENSION SPARAM(10),DPARAM(10),BPARAM(10),G(10),IPARAM(9),
     1 SA(10),PARTL(10),A(10,11),PMAX(10),SPRNT(5),PMIN(10)
      REAL LAMBDA,LENGTH
      NPRNT=0
  651 ICOUNT=0
      IBOUT=0
    1 READ(5,900,END=660)NDATA,NPARAM,NFIXED,NVAR,IFPLOT
  900 FORMAT(10I3)
C                   READING IN THE PROGRAM CONTROLS
C                   NDATA IS THE NUMBER OF DATA POINTS.  THE MAXIMUM
C                   NUMBER OF DATA POINTS IS 100.
C                   NPARAM IS THE TOTAL NUMBER OF PARAMETERS IN THE MODEL
C                   TO BE TESTED.  THE MAXIMUM NUMBER OF PARAMETERS IS 10.
C                   NFIXED IS THE NUMBER OF PARAMETERS WITH FIXED VALUES.
C                   THE MAXIMUM NUMBER OF FIXED PARAMETERS IS 9.  NFIXED
C                   MUST ALWAYS BE LESS THAN NPARAM.
C                   NVAR IS THE NUMBER OF INDEPENDENT VARIABLES IN THE
C                   MODEL TO BE TESTED.  THE MAXIMUM NUMBER OF INDEPENDENT
C                   VARIABLES IS 5.
C                   IFPLOT IS AN OUTPUT CONTROL VARIABLE.  A VALUE OF ZERO
C                   GIVES TABULATED RESULTS.  A VALUE OF ONE GIVES PLOTTED
```

```
C                     RESULTS.
            IF(NDATA.GT.0) GO TO 2
C                     THE PROGRAM WILL TERMINATE THE PRESENT CASE IF NO DATA
C                     POINTS HAVE BEEN SUPPLIED
            WRITE(6,940)
      940   FORMAT(//20X,40HCASE TERMINATED: NO DATA POINTS SUPPLIED)
            GO TO 660
    2       READ(5,900)NSW1,NSW2,NSW3,NSW4,NSW5,NSW6
C                     READING IN THE SENSE SWITCH CONTROLS
C                     SETTING OF THE SENSE SWITCHES(NSW)
C*****************************************************************************
C           NSW         EQUAL TO ZERO              NOT EQUAL TO ZERO
C.............................................................................
C            1        DETAILED OUTPUT ON         NO DETAILED OUTPUT ON
C                     ONLINE PRINTER             ONLINE PRINTER
C            2        ANALYTIC DERIVATIVES       ESTIMATED DERIVATIVES
C            3        DETAILED PRINTOUTS         NSW3 ABBREVIATED PRINTOUTS
C                     ON OUTPUT UNIT             ON OUTPUT UNIT
C            4        FORCED BRANCH TO
C                     CONFIDENCE REGION
C                     CALCULATIONS
C            5        FORCED BRANCH TO
C                     NEXT CASE
C            6        CONFIDENCE REGION           CONFIDENCE REGION
C                       DESIRED                     NOT DESIRED(=1)
C*****************************************************************************
C                     TESTING FOR PLOTTING OR TABULATING OPTIONS
            IF(IFPLOT.LE.0) GO TO 22
            READ(5,930)YMIN,SPREAD
C                     READING IN THE PLOTTING CONTROLS
C                     THE PLOTTING CONTROLS ARE REQUIRED ONLY IF IFPLOT IS SET
C                     EQUAL TO ONE.
C                     YMIN IS THE VALUE OF THE LEFT SIDE OF THE PLOT.
C                     SPREAD IS THE SPREAD OF THE PLOT.
        930 FORMAT(2F10.0)
C                     TESTING FOR MODEL PARAMETERS WITH FIXED VALUES
         22 IF(NFIXED.LE.0) GO TO 32
 24         READ(5,900)(IPARAM(I),I=1,NFIXED)
C                     READING IN THE SUBSCRIPTS OF THE MODEL PARAMETERS
C                     HAVING FIXED VALUES.
C                     IPARAM(I) IS THE SUBSCRIPT OF THE MODEL PARAMETER
C                     HAVING A FIXED VALUE
            DO 26 I=1,NFIXED
            IF(IPARAM(I).GT.0) GO TO 26
     25     WRITE(6,926)
    926     FORMAT(//10X,47HBAD DATA; FIXED PARAMETERS HAVE ZERO SUBSCRIPTS)
            IBOUT=1
 26         CONTINUE
C*****************************************************************************
C                     THE FOLLOWING ARE INPUT CONSTANTS SUPPLIED BY THE PROGRAM
```

```
   32 FF=4.
C                 FF IS A VARIANCE RATIO STATISTIC.
   36 E=.0000005
C                 E IS A CONVERGENCE CRITERION.
   38 TAU=.001
C                 TAU IS A CONVERGENCE CRITERION
   40 T=2.
C                 T IS THE STUDENT'S T.
   51 GAMCR=45.
C                 GAMCR IS THE CRITICAL ANGLE.
      DELTA=.00001
C                 DELTA IS A MULTIPLIER USED IN THE FINITE DIFFERENCE
C                 DERIVATIVES.
      ZETA=.1E-30
C                 ZETA IS A SINGULARITY CRITERION FOR MATRIX INVERSION.
      LAMBDA=0.01
C                 LAMBDA IS A PROGRAM PARAMETER
 53   XKDB=1.0
C                 XKDB IS A MULTIPLIER USED TO INCREMENT THE PARAMETERS
C*************************************************************************
      READ(5,901)(PARAM(I),I=1,NPARAM)
C                 READING IN THE INITIAL VALUES OF THE MODEL PARAMETERS.
C                 THEY ARE READ SEVEN TO THE CARD.
C                 PARAM(I) IS THE VALUE OF THE MODEL PARAMETER.
  901 FORMAT(7F10.0)
      READ(5,901)(PMIN(I),I=1,NPARAM)
C                 READING IN THE MINIMUM VALUES OF THE MODEL PARAMETERS
C                 PMIN(I) IS THE MINIMUM VALUE OF THE PARAMETER
      READ(5,901)(PMAX(I),I=1,NPARAM)
C                 READING IN THE MAXIMUM VALUES OF THE MODEL PARAMETERS
C                 PMAX(I) IS THE MAXIMUM VALUE OF THE PARAMETER
      DO 56 I=1,NDATA
 56   READ(5,901)Y(I),(X(I,L),L=1,NVAR)
C                 READING IN THE DATA POINTS
C                 Y(I) IS THE VALUE OF THE INDEPENDENT VARIABLE
C                 X(I,L) IS THE VALUE OF THE DEPENDENT VARIABLE
C*************************************************************************
C                 STARTING THE NONLINEAR REGRESSION SEQUENCE
      CALL SUBZ(Y,X,PARAM,PRNT,NPRNT,NDATA)
C.........................................................................
 9999 NSW33=NSW3
      NTILDA=NDATA
      XNT=NTILDA
      NSW44=NSW4
      NNDATA=NDATA
      ICOUNT=0
      IBKT=1
      NSW11=NSW1
      NSW22=NSW2
      NSW55=NSW5
```

```
          IFPP=IFPLOT
          IF(IBOUT.NE.0) GO TO 660
       59 IBKA=1
          IF(IFPLOT.LE.0) GO TO 61
          WRITE(6,907)NDATA,NPARAM,NFIXED,NVAR,IFPLOT,GAMCR,DELTA,
         1 FF,T,E,TAU,LAMBDA,ZETA
  907     FORMAT(/5X,8HNDATA = ,I3,4X,9HNPARAM = ,I2,4X,9HNFIXED = ,
         1 I1,4X,7HNVAR = ,I1,4X,9HIFPLOT = ,I1,4X,13HGAMMA CRIT = ,
         2 1PE10.3,4X,8HDELTA = ,1PE10.3,/5X,5HFF = ,1PE10.3,4X,4HT = ,
         3 1PE10.3,4X,4HE = ,1PE10.3,4X,6HTAU = ,1PE10.3,4X,9HLAMBDA = ,
         4 1PE10.3,4X,7HZETA = ,1PE10.3)
   60     NSW3=NSW3-1
          NSW3=MAX0(NSW3,0)
C                   START THE CALCULATION OF THE PTP MATRIX
   61     DO 62 I=1,NPARAM
          G(I)=0.
          DO 62 J=1,NPARAM
       62 A(I,J)=0.
          WRITE(6,941)(PMIN(I),I=1,NPARAM)
  941     FORMAT(/5X,18HPARAMETER MINIMUMS,1P5E18.8/(23X,1P5E18.8))
          WRITE(6,942)(PMAX(I),I=1,NPARAM)
  942     FORMAT(5X,18HPARAMETER MAXIMUMS,1P5E18.8/(23X,1P5E18.8))
   70     WRITE(6,908)ICOUNT,(PARAM(J),J=1,NPARAM)
  908     FORMAT(/5X,1H(,I2,19H) MODEL PARAMETERS ,1P5E18.8/(25X,1P5E18.8))
          IF(NSW3.EQ.0) GO TO 73
   71     IF(IFPLOT.LE.0) GO TO 68
C                   THE FOLLOWING STATEMENTS INITIALIZE THE PLOT
       67 WS=YMIN+SPREAD
          WRITE( 6,906)YMIN,WS
  906     FORMAT(/7X,1PE9.2,90X,1PE9.2,/10X,1H+,99X,1H+)
          GO TO 73
       68 WRITE( 6,910)
  910     FORMAT(/10X,8HOBSERVED,9X,9HPREDICTED,8X,10HDIFFERENCE)
       73 I=1
          PHI=0.
C                   PHI IS THE SUM OF THE SQUARES OF THE RESIDUALS
          PHIN=0.
C                   TESTING FOR ANALYTIC OR ESTIMATED PARTIAL DERIVATIVE
C                   OPTIONS
   72     IF(NSW2.EQ.1) GO TO 602
C.....................................................................
          CALL MODEL(Y,X,PARAM,PRNT,FCN,I,RESDUE)
C                   THIS IS THE ANALYTIC PARTIALS OPTION
C                   GET PARTIALS AND FUNCTION
C.....................................................................
          CALL DERIV(PARTL,X,PARAM,PRNT,FCN,I)
C.....................................................................
   75     IF(NFIXED.LE.0) GO TO 80
       76 DO 77 II=1,NFIXED
          IWS=IPARAM(II)
```

```
    77 PARTL(IWS)=0.
 C                  THIS IS THE END OF THE ANALYTIC PARTIALS OPTION
 C*****************************************************************************
       GO TO 80
 C*****************************************************************************
 C                  THIS IS THE ESTIMATED PARTIALS OPTION
 C                  MAKE NPARAM OF THEM AND CALL THEM PARTL(J)
 C                  THEY ARE MADE FROM X(I,L) AND PARAM(J)
 602   CALL MODEL(Y,X,PARAM,PRNT,FCN,I,RESDUE)
 C.............................................................................
 606   RWS=RESDUE
       FSAVE=FCN
       DO 607 II=1,NPRNT
   607 SPRNT(II)=PRNT(II)
       J=1
  608  IF(NFIXED.LE.0) GO TO 618
   610 DO 612 II=1,NFIXED
       IF((J-IPARAM(II)).EQ.0) GO TO 621
   612 CONTINUE
   618 DBW=PARAM(J)*DELTA
       TWS=PARAM(J)
       PARAM(J)=PARAM(J)+DBW
 C.............................................................................
       CALL MODEL(Y,X,PARAM,PRNT,FCN,I,RESDUE)
 C.............................................................................
   620 PARAM(J)=TWS
       IF(DBW.EQ.0)DBW=DELTA
       PARTL(J)=-(RESDUE-RWS)/DBW
       GO TO 622
   621 PARTL(J)=0.
   622 J=J+1
       IF((J-NPARAM).LE.0) GO TO 608
  624  RESDUE=RWS
       FCN=FSAVE
       DO 625 II=1,NPRNT
   625 PRNT(II)=SPRNT(II)
 C                  THIS IS THE END OF THE ESTIMATED PARTIALS ROUTINE
 C*****************************************************************************
 C                  NOW USE THE PARTIALS TO MAKE THE PARTIALS MATRIX
    80 DO 82 JJ=1,NPARAM
       G(JJ)=G(JJ)+RESDUE*PARTL(JJ)
       DO 82 II=JJ,NPARAM
       A(II,JJ)=A(II,JJ)+PARTL(II)*PARTL(JJ)
    82 A(JJ,II)=A(II,JJ)
       IF(IFPLOT.LE.0) GO TO 318
       IF(NSW3.EQ.0.OR.1.GT.NDATA) GO TO 314
 C*****************************************************************************
 C                  STARTING THE PLOTTING SEQUENCE
 C                  PLOTTING Y(I), FCN
   802 IO=((Y(I)-YMIN)*100./SPREAD)+10
```

```
      IPP=((FCN-YMIN)*100./SPREAD)+10
      IF(IO.EQ.IPP) GO TO 808
      IF(IO.GT. IPP) GO TO 812
C                    Y(I) OUT FIRST
  804 IP1=IOCH
      IP2=IPCH
      I1=IO
      I2=IPP
      GO TO 816
C                    ONLY ONE CHARACTER
  808 IP1=IYCH
      IP2=IBCH
      I1=IO
      I2=IPP
      GO TO 816
C                    FCN OUT FIRST
  812 IP1=IPCH
      IP2=IOCH
      I1=IPP
      I2=IO
C                    ZERO PLOTS IN THE LEFT HAND COLUMN
C                    I1 IS ITS OWN BLANK COUNTER
C                    OVERFLOWS PLOT X IN COLUMN 112
C                    UNDERFLOWS ALSO PLOT X IN COLUMN TEN
  816 IF(I2.LE.111) GO TO 819
  817 I2=111
      IP2=IXCH
      IF(I1.LT.111) GO TO 819
  818 I1=111
      IP1=IXCH
      IP2=IBCH
      GO TO 825
  819 IF(I1.GE.10) GO TO 825
  822 I1=10
      IP1=IXCH
      IF(I2.GT.10) GO TO 825
  823 I2=1
      IP2=IBCH
  825 I1M1=I1
      I1M2=I2-I1-1
      IF(I1M1.GT.0) GO TO 832
  820 IF(I1M2.GT.0) GO TO 828
  824 WRITE(6,928)IP1,IP2
  928 FORMAT(1H ,120A1)
      GO TO 844
  828 WRITE(6,928)IP1,(IBCH,II=1,I1M2),IP2
      GO TO 844
  832 IF(I1M2.GT.0) GO TO 840
  836 WRITE(6,928)(IBCH,II=1,I1M1),IP1,IP2
      GO TO 844
```

```
  840 WRITE(6,928)(IBCH,II=1,I1M1),IP1,(IBCH,II=1,I1M2),IP2
C                  END OF PLOTTING SEQUENCE
C*************************************************************************
  844 GO TO 314
 318  WS=RESDUE
      IF(NSW3.EQ.0.OR.I.GT.NDATA) GO TO 314
  308 IF(NPRNT.GT.0) GO TO 312
 310  WRITE(6,925)Y(I),FCN,WS
 925  FORMAT(5X,1P6E18.8/59X,1P2E18.8)
      GO TO 314
 312  WRITE(6,925)Y(I),FCN,WS,(PRNT(JJ),JJ=1,NPRNT)
 314  WS=RESDUE
      PHI=PHI+WS*WS
      IF(I.GT.NDATA) GO TO 313
      PHIN=PHIN+WS*WS
      GO TO 315
 313  CONTINUE
315   I=I+1
      IF(I.LE.NTILDA) GO TO 72
 84   IF(NFIXED.LE.0) GO TO 88
 85   DO 87 JJ=1,NFIXED
      IWS=IPARAM(JJ)
      DO 86 II=1,NPARAM
      A(IWS,II)=0.
   86 A(II,IWS)=0.
   87 A(IWS,IWS)=1.
 88   GO TO (90,704,703),IBKA
   90 DO 92 I=1,NPARAM
C                  SAVE SQUARE ROOTS OF DIAGONAL ELEMENTS
 92   SA(I)=SQRT (A(I,I))
      DO 106 I=1,NPARAM
      DO 100 J=1,NPARAM
      WS=SA(I)*SA(J)
      IF(WS.GT.0.) GO TO 98
   96 A(I,J) =0.
      GO TO 100
   98 A(I,J)=A(I,J)/WS
  100 CONTINUE
      IF(SA(I).GT.0.) GO TO 104
  102 G(I)=0.
      GO TO 106
  104 G(I)=G(I)/SA(I)
  106 CONTINUE
      DO 110 I=1,NPARAM
  110 A(I,I)=1.
 120  PHIZ=PHI
C                  WE NOW HAVE PHI(ZERO)
 1132 DO 1133 II=1,NPARAM
      III=II+25
      DO 1133 JJ=1,NPARAM
```

```
 1133 A(III,JJ)=A(II,JJ)
 1134 CONTINUE
      IF(ICOUNT.NE.0) GO TO 163
C***********************************************************************
C                  STARTING THE FIRST ITERATION
 152  LAMBDA=0.01
 154  DO 161 J=1,NPARAM
  161 SPARAM(J)=PARAM(J)
C                  SPARAM(J) CORRESPONDS TO PHIZ
  163 IBK1=1
      WS=NDATA-NPARAM+NFIXED
      ICOUNT=ICOUNT+1
      SE=SQRT(PHIN/WS)
C                  SE IS THE STANDARD ERROR OF THE ESTIMATE
 160  IF(NSW3.GT.0) GO TO 165
  162 IF(NSW2.EQ.0) GO TO 168
 167  WRITE(6,911)PHIZ,SE,LENGTH,GAMMA,LAMBDA
 911  FORMAT(/14X,3HPHI,15X,3HS E,12X,6HLENGTH,7X,5HGAMMA,7X,
     1 6HLAMBDA,10X,24HESTIMATED PARTIALS USED/5X,1P2E18.8,1P3E13.3)
      GO TO 169
 168  WRITE(6,912)PHIZ,SE,LENGTH,GAMMA,LAMBDA
 912  FORMAT(/14X,3HPHI,15X,3HS E,12X,6HLENGTH,7X,5HGAMMA,7X,
     1 6HLAMBDA,10X,22HANALYTIC PARTIALS USED/5X,1P2E18.8,1P3E13.3)
      GO TO 169
 165  CONTINUE
 166  WRITE(6,939)
 939  FORMAT(/34X,22HPTP CORRELATION MATRIX)
 111  DO 114 I=1,NPARAM
 114  WRITE(6,937) I, (A(I,J),J=1,NPARAM)
 937  FORMAT (10X,I5,2X,10F10.4/(10X,10F10.4))
      IF(NSW2.EQ.0) GO TO 1161
      WRITE(6,903)PHIZ,SE,LAMBDA
 903  FORMAT(/14X,3HPHI,15X,3HS E,12X,6HLAMBDA,10X,
     1 24HESTIMATED PARTIALS USED/5X,1P2E18.8,1PE13.3)
      GO TO 169
 1161 WRITE(6,909)PHIZ,SE,LAMBDA
 909  FORMAT(/14X,3HPHI,15X,3HS E,12X,6HLAMBDA,10X,
     1 22HANALYTIC PARTIALS USED,/5X,1P2E18.8,1PE13.3)
  169 GO TO 200
  164 PHIL=PHI
C                  WE NOW HAVE PHI(LAMBDA)
      DO 170 J=1,NPARAM
      IF(ABS(DPARAM(J)/(ABS(PARAM(J))+TAU)).GE.E) GO TO 172
  170 CONTINUE
      WRITE(6,923)
 923  FORMAT(1H1,50X,19HPASSED EPSILON TEST)
      GO TO 700
 172  IF(NSW5.EQ.0) GO TO 1720
 1720 IF(NSW4.EQ.0) GO TO 173
      IF(NSW4.EQ.1) GO TO 171
```

```
           NSW4=NSW4-1
           GO TO 173
     171   WRITE(6,924)
     924   FORMAT(1H1,30X,40HCASE TERMINATED: REQUIRE MORE ITERATIONS)
           GO TO 700
      173 XKDB=1.0
           IF(PHIL.GT.PHIZ) GO TO 190
      174 XLS=LAMBDA
           DO 176 J=1,NPARAM
           BPARAM(J)=PARAM(J)
      176 PARAM(J)=SPARAM(J)
           IF(LAMBDA.GT..00000001) GO TO 175
     1175 DO 1176 J=1,NPARAM
           PARAM(J)=BPARAM(J)
     1176 SPARAM(J)=PARAM(J)
           GO TO 60
      175 LAMBDA=LAMBDA/10.
           IBK1=2
           GO TO 200
      177 PHL4=PHI
 C                    WE NOW HAVE PHI(LAMBDA/10)
           IF(PHL4.GT.PHIZ) GO TO 184
      182 DO 183 J=1,NPARAM
      183 SPARAM(J)=PARAM(J)
           GO TO 60
      184 LAMBDA=XLS
           DO 186 J=1,NPARAM
           SPARAM(J)=BPARAM(J)
      186 PARAM(J)=BPARAM(J)
           GO TO 60
      190 IBK1=4
           XLS=LAMBDA
           LAMBDA=LAMBDA/10.
           DO 185 J=1,NPARAM
      185 PARAM(J)=SPARAM(J)
           GO TO 200
     187   IF(PHI.LE.PHIZ) GO TO 196
      191 LAMBDA=XLS
           IBK1=3
      192 LAMBDA=LAMBDA*10.
      195 DO 193 J=1,NPARAM
      193 PARAM(J)=SPARAM(J)
           GO TO 200
      194 PHIT4=PHI
 C                    WE NOW HAVE PHI(LAMBDA*10)
     180   IF(PHIT4.GT.PHIZ) GO TO 198
      196 DO 197 J=1,NPARAM
      197 SPARAM(J)=PARAM(J)
           GO TO 60
      198 IF(GAMMA.GE.GAMCR) GO TO 192
```

```
  199 XKDB=XKDB/2.
      DO 1199 J=1,NPARAM
      IF(ABS(DPARAM(J)/(ABS(PARAM(J))+TAU)).GE.E) GO TO 195
 1199 CONTINUE
      DO 1200 J=1,NPARAM
 1200 PARAM(J)=SPARAM(J)
      WRITE(6,934)
 934  FORMAT(1H1,50X,25HPASSED GAMMA EPSILON TEST)
      GO TO 700
C                SET UP FOR MATRIX INVERSION
 200  CONTINUE
 1102 DO 1103 II=1,NPARAM
      III=II+25
      DO 1103 JJ=1,NPARAM
 1103 A(II,JJ)=A(III,JJ)
 1104 DO 202 I=1,NPARAM
  202 A(I,I)=A(I,I)+LAMBDA
      IBKM=1
C.........................................................................
  404 CALL GJR(A,NPARAM,ZETA,MSING)
C                GET INVERSE OF A AND SOLVE FOR DPARAM(J)'S
C                THIS IS THE MATRIX INVERSION ROUTINE
C                NPARAM IS THE SIZE OF THE MATRIX
C.........................................................................
      GO TO(415,660),MSING
  415 GO TO (416,710), IBKM
C                THIS IS THE END OF THE MATRIX INVERSION.
C************************************************************************
C                SOLVE FOR DPARAM(J)
  416 DO 420 I=1,NPARAM
      DPARAM(I)=0.
      DO 421 J=1,NPARAM
  421 DPARAM(I)=A(I,J)*G(J)+DPARAM(I)
  420 DPARAM(I)=XKDB*DPARAM(I)
C                DPARAM IS THE INCREMENT OF THE PARAMETER
      LENGTH=0.
      DTG=0.
      GTG=0.
      DO 250 J=1,NPARAM
      LENGTH=LENGTH+DPARAM(J)*DPARAM(J)
      DTG=DTG+DPARAM(J)*G(J)
      GTG=GTG+G(J)**2
      IF(SA(J).EQ.0.) GO TO 699
      DPARAM(J)=DPARAM(J)/SA(J)
      IF(PARAM(J)+DPARAM(J).LT.PMIN(J)) DPARAM(J)=ABS(DPARAM(J))
      IF(PARAM(J)+DPARAM(J).GT.PMAX(J)) DPARAM(J)=-DPARAM(J)
      PARAM(J)=PARAM(J)+DPARAM(J)
 250  CONTINUE
      KIP=NPARAM-NFIXED
      IF(LENGTH*GTG.LE.0.) GO TO 1257
```

```
      IF(KIP.EQ.1) GO TO 1257
      CGAM=DTG/SQRT(LENGTH*GTG)
      JGAM=1
      IF(CGAM.GT..0) GO TO 253
  251 CGAM=ABS(CGAM)
      JGAM=2
253   GAMMA=57.2957795*(1.5707288+CGAM*(-0.2121144+CGAM*(0.074261
     1-CGAM*.0187293)))*SQRT(1.-CGAM)
      GO TO (257,255), JGAM
  255 GAMMA=180.-GAMMA
      IF(LAMBDA.LT.1.0) GO TO 257
 1255 WRITE(6,922)XL,GAMMA
 922  FORMAT(1H1,30X,24HPASSED GAMMA LAMBDA TEST,5X,1P2E13.3)
      GO TO 700
 1257 GAMMA=0.
 257  LENGTH=SQRT(LENGTH)
      IBK2=1
      GO TO 300
 252  IF(NSW3 .EQ.0) GO TO 256
      WRITE(6,905)PHI,LAMBDA,GAMMA,LENGTH
 905  FORMAT(/15X,3HPHI,12X,6HLAMBDA,7X,5HGAMMA,8X,6HLENGTH,/6X,
     1 1PE18.8,1P3E13.3)
 254  WRITE(6,904)(DPARAM(J),J=1,NPARAM)
 904  FORMAT(/5X,20HPARAMETER INCREMENTS,1P5E18.8,/(25X,1P5E18.8))
  256 GO TO (164,177,194,187),IBK1
C                   CALCULATE PHI
  300 I=1
      PHI=0.
      PHIN=0.
C.............................................................................
800   CALL MODEL(Y,X,PARAM,PRNT,FCN,I,RESDUE)
C.............................................................................
      IF(RESDUE.GE.1.E33) GO TO 699
      IF(I.GT.NDATA) GO TO 305
      PHIN=PHIN+RESDUE*RESDUE
 305  I=I+1
      IF(I.LE.NTILDA) GO TO 800
      PHI=PHIN
  316 GO TO (252,780,704,762,766,772),IBK2
C******************************************************************************
C                   THIS IS THE CONFIDENCE LIMIT CALCULATION
699   WRITE(6,943)
 943  FORMAT(//30X,38HCASE TERMINATED: RESULTS HAVE BLOWN UP)
      GO TO 660
  700 DO 702 J=1,NPARAM
  702 PARAM(J)=SPARAM(J)
      WRITE(6,933)NDATA,NPARAM,NFIXED,NVAR,FF,T,E,TAU
 933  FORMAT(/5X,8HNDATA = ,I3,4X,9HNPARAM = ,I2,4X,9HNFIXED = ,
     1 I1,4X,7HNVAR = ,I1,/5X,5HFF = ,1PE10.3,4X,4HT = ,1PE10.3,
     2 4X,4HE = ,1PE10.3,4X,6HTAU = ,1PE10.3)
```

```
             IBKA=2
             NTILDA=NDATA
      C              THIS WILL PRINT THE Y, YHAT, DELTA Y
             ICOUNT=ICOUNT-1
             NSW3=1
             GO TO 61
       704   IF(IFPLOT.LE.0) GO TO 703
       705   IBKA=3
             IFPLOT=0
             GO TO 61
       703   CONTINUE
      706    WS=NDATA-NPARAM+NFIXED
             IF(WS.LE.0) GO TO 660
             SE=SQRT(PHI/WS)
             PHIZ=PHI
             IF(NSW2.EQ.0) GO TO 709
       707   WRITE(6,903)PHIZ,SE,LAMBDA
             GO TO 708
       709   WRITE(6,909) PHIZ,SE,LAMBDA
       708   DO 1123 II=1,NPARAM
             III=II+25
             DO 1123 JJ=1,NPARAM
      1123   A(III,JJ)=A(II,JJ)
      C              WE NOW HAVE MATRIX A
      1124   IBKM=2
             GO TO 404
      C              WE NOW HAVE C=A INVERSE
       710   DO 711 J=1,NPARAM
             IF(A(J,J).LE.0) GO TO 713
       711   SA(J)=SQRT(A(J,J))
             GO TO 715
       713   IBOUT=1
       715   KST=-4
             WRITE(6,916)
       916   FORMAT(/40X,11HPTP INVERSE)
       234   KST=KST+5
             KEND=KST+4
             IF(KEND.LT.NPARAM) GO TO 719
             KEND=NPARAM
       719   DO 712 I=1,NPARAM
       712   WRITE(6,918)I,(A(I,J),J=KST,KEND)
       918   FORMAT(5X,I3,1P5E18.8)
             IF(KEND.LT.NPARAM) GO TO 234
             IF(IBOUT.EQ.0) GO TO 717
             WRITE(6,936)
       936   FORMAT(//25X,25HNEGATIVE DIAGONAL ELEMENT)
             GO TO 660
       717   DO 718 I=1,NPARAM
             DO 718 J=1,NPARAM
             WS=SA(I)*SA(J)
```

```
          IF(WS.GT.0.) GO TO 716
      714 A(I,J)=0.
          GO TO 718
      716 A(I,J)=A(I,J)/WS
      718 CONTINUE
          DO 720 J=1,NPARAM
      720 A(J,J)=1.
          WRITE(6,917)
      917 FORMAT(/23X,28HPARAMETER CORRELATION MATRIX)
          KST=-9
      721 KST=KST+10
          KEND=KST+9
          IF(KEND.LT.NPARAM) GO TO 722
          KEND=NPARAM
      722 DO 724 I=1,NPARAM
      724 WRITE(6,935)I,(A(I,J),J=KST,KEND)
      935 FORMAT(5X,I3,2X,10F10.4)
          IF(KEND.LT.NPARAM) GO TO 721
C                   GET T*SE*SQRT(C(I,I))
          DO 726 J=1,NPARAM
      726 SA(J)=  SE*SA(J)
     1112 DO 1113 II=1,NPARAM
          III=II+25
          DO 1113 JJ=1,NPARAM
     1113 A(II,JJ)=A(III,JJ)
     1114 CONTINUE
      740 WRITE(6,919)
      919 FORMAT(/16X,3HSTD,19X,13HONE-PARAMETER,23X,13HSUPPORT PLANE/4X,
         1 4HPARA,7X,5HERROR,13X,5HLOWER,13X,5HUPPER,13X,5HLOWER,13X,
         2 5HUPPER)
          WS=NPARAM-NFIXED
          DO 750 J=1,NPARAM
          IF(NFIXED.LE.0) GO TO 743
      741 DO 742 I=1,NFIXED
          IF(J.EQ.IPARAM(I)) GO TO 746
      742 CONTINUE
      743 HJTD=SQRT(WS*FF)*SA(J)
          STE=SA(J)
          OPL=SPARAM(J)-SA(J)*T
          OPU=SPARAM(J)+SA(J)*T
          SPL=SPARAM(J)-HJTD
          SPU=SPARAM(J)+HJTD
          WRITE(6,927)J,STE,OPL,OPU,SPL,SPU
      927 FORMAT(5X,I2,1P5E18.8)
          GO TO 750
      746 WRITE(6,913)J
      913 FORMAT(5X,I2,10X,22HPARAMETER WAS NOT USED)
      750 CONTINUE
C*****************************************************************************
C                   NON-LINEAR CONFIDENCE LIMIT CALCULATION
```

```
      IF(NSW6.EQ.1) GO TO 660
      WS=NPARAM-NFIXED
      WS1=NDATA-NPARAM+NFIXED
      PKN=WS/WS1
      PC=PHIZ*(1.+FF*PKN)
      WRITE(6,920)PC
  920 FORMAT(/15X,27HNONLINEAR CONFIDENCE LIMITS,10X,
     1 15HPHI CRITICAL = ,1PE15.8)
      WRITE(6,921)
  921 FORMAT(/5X,4HPARA,6X,7HLOWER B,11X,9HLOWER PHI,9X,
     1 7HUPPER B,11X,9HUPPER PHI)
      IFSS3=1
      DO 790 J=1,NPARAM
      IBKP=1
      DO 752 JJ=1,NPARAM
  752 PARAM(JJ)=SPARAM(JJ)
      IF(NFIXED.LE.0) GO TO 758
  754 DO 756 JJ=1,NFIXED
      IF(J.EQ.IPARAM(JJ)) GO TO 787
  756 CONTINUE
  758 DD=-1.
      IBKN=1
  760 D=DD
      PARAM(J)=SPARAM(J)+D*SA(J)
      IBK2=4
      GO TO 300
  762 PHI1=PHI
      IF(PHI1.GE.PC) GO TO 770
  764 D=D+DD
      IF(D/DD.GE.5.) GO TO 788
  765 PARAM(J)=SPARAM(J)+D*SA(J)
      IBK2=5
      GO TO 300
  766 PHID=PHI
      IF(PHID.LT.PC) GO TO 764
      IF(PHID.GE.PC) GO TO 778
  770 D=D/2.
      IF(D/DD.LE..001) GO TO 788
  771 PARAM(J)=SPARAM(J)+D*SA(J)
      IBK2=6
      GO TO 300
  772 PHID=PHI
      IF(PHID.GT.PC) GO TO 770
  778 XK1=PHIZ/D+PHI1/(1.-D)+PHID/(D*(D-1.))
      XK2=-(PHIZ*(1.+D)/D+D/(1.-D)*PHI1+PHID/(D*(D-1.)))
      XK3=PHIZ-PC
      BC=(SQRT(XK2*XK2-4.*XK1*XK3)-XK2)/(2.*XK1)
      GO TO (779,784),IBKN
  779 PARAM(J)=SPARAM(J)-SA(J)*BC
      GO TO 781
```

```
  784 PARAM(J)=SPARAM(J)+SA(J)*BC
 781  IBK2=2
      GO TO 300
  780 GO TO (782,786),IBKN
  782 IBKN=2
      DD=1.
      BL=PARAM(J)
      PL=PHI
      GO TO 760
  786 BU=PARAM(J)
      PU=PHI
      GO TO (783,795,785,789),IBKP
 783  WRITE(6,918)J,BL,PL,BU,PU
      GO TO 790
 795  WRITE(6,915)J,BU,PU
 915  FORMAT(2X,I2,36X,1P2E18.8)
      GO TO 790
 785  WRITE(6,918)J,BL, PL
      GO TO 790
 787  WRITE(6,913)J
      GO TO 790
 789  WRITE(6,914)J
 914  FORMAT(2X,I2,10X,10HNONE FOUND)
      GO TO 790
  788 GO TO (791,792),IBKN
C                   DELETE LOWER PRINT
  791 IBKP=2
      GO TO 780
  792 GO TO (793,794),IBKP
C                   DELETE UPPER PRINT
  793 IBKP=3
      GO TO 780
C                   LOWER IS ALREADY DELETED, SO DELETE BOTH
  794 IBKP=4
      GO TO 780
  790 CONTINUE
C                   THIS THE END OF THE NONLINEAR REGRESSION SEQUENCE
C*************************************************************************
660   CONTINUE
      NSW3=NSW33
      ICOUNT=0
      NSW4=NSW44
      IFPLOT=IFPP
      NDATA=NNDATA
      NSW1=NSW11
      NSW2=NSW22
      NSW5=NSW55
      IBOUT=0
      READ(5,900)ININ
C                   READING IN THE PROGRAM CONTROL VARIABLE
```

```
      GO TO(662,9998,651),ININ
9998  READ(5,901)(PARAM(JJ),JJ=1,NPARAM)
C                   READING IN NEW VALUES OF THE MODEL PARAMETERS
      GO TO 9999
662   RETURN
      END
```

```
      SUBROUTINE GJR(A,N,EPS,MSING)
C***********************************************************************
C                    GAUSS-JORDAN RUTISHAUSER MATRIX INVERSION
C                    WITH DOUBLE PIVOTING
C***********************************************************************
      DIMENSION A(10,10),B(10),C(10),P(10),Q(10)
      INTEGER P,Q
      MSING=1
      DO 10 K=1,N
      PIVOT=0.
      DO 20 I=K,N
      DO 20 J=K,N
      IF(ABS(A(I,J))-ABS(PIVOT))20,20,30
C                    DETERMINATION OF THE PIVOT ELEMENT
   30 PIVOT=A(I,J)
      P(K)=I
      Q(K)=J
   20 CONTINUE
      IF(ABS(PIVOT)-EPS)40,40,50
   50 IF(PEKN-K)60,80,60
   60 DO 70 J=1,N
      L=P(K)
      Z=A(L,J)
      A(L,J)=A(K,J)
   70 A(K,J)=Z
   80 IF(Q(K)-K)85,90,85
   85 DO 100 I=1,N
      L=Q(K)
      Z=A(I,L)
C                    EXCHANGE OF THE PIVOTAL COLUMN WITH THE KTH COLUMN
      A(I,L)=A(I,K)
  100 A(I,K)=Z
   90 CONTINUE
      DO 110 J=1,N
C                    JORDAN STEP
      IF(J-K)130,120,130
  120 B(J)=1./PIVOT
      C(J)=1.
      GO TO 140
  130 B(J)=-A(K,J)/PIVOT
      C(J)=A(J,K)
  140 A(K,J)=0.
  110 A(J,K)=0.
      DO 10 I=1,N
      DO 10 J=1,N
   10 A(I,J)=A(I,J)+C(I)*B(J)
      DO 155 M=1,N
C                    REORDERING THE MATRIX
      K=N-M+1
      IF(P(K)-K)160,170,160
```

```
 160 DO 180 I=1,N
     L=P(K)
     Z=A(I,L)
     A(I,L)=A(I,K)
 180 A(I,K)=Z
 170 IF(Q(K)-K)190,155,190
 190 DO 150 J=1,N
     L=Q(K)
     Z=A(L,J)
     A(L,J)=A(K,J)
 150 A(K,J)=Z
155  CONTINUE
 151 RETURN
  40 WRITE(2,45)P(K),Q(K),PIVOT
  45 FORMAT(16H0SINGULAR MATRIX3H I=I3,3H J=I3,7H PIVOT=E16.8/)
     MSING=2
     GO TO 151
     END
```

```
      SUBROUTINE SUBZ(Y,X,PARAM,PRNT,NPRNT,NDATA)
C*************************************************************************
      COMMON Y(100),X(100,5),PARAM(10),PRNT(5),CONST(4)
      NPRNT=1
      RETURN
      END
```

```
      SUBROUTINE MODEL(Y,X,PARAM,PRNT,FCN,I,RESDUE)
C*********************************************************************
      COMMON Y(100),X(100,5),PARAM(10),PRNT(5),CONST(4)
C             TESTING OF THE THIXOTROPIC MODEL
C             THIS SUBROUTINE WILL TEST A HYSTERESIS LOOP AND A
C             TORQUE-DECAY CURVE
C             PARAM(1) REPRESENTS THE YIELD STRESS
C             PARAM(2) REPRESENTS THE VISCOSITY
C             PARAM(3) REPRESENTS A RATE CONSTANT
C             PARAM(4) REPRESENTS A LUMPED PARAMETER
C             PARAM(5) REPRESENTS THE ORDER OF THE RATE EQUATION
C             X(I,1) IS THE SHEAR RATE AND POSITIVE WHEN USED
C             IN THE UPCURVE OR DOWNCURVE
C             X(I,1) IS THE TIME WHEN USED IN THE TORQUE-DECAY CURVE
C             X(I,2) IS A VARIABLE WHICH TESTS THE DATA POINT TO
C             DETERMINE WHETHER IT IS REPRESENTATIVE OF THE UPCURVE,
C             DOWNCURVE, OR THE TORQUE-DECAY CURVE
C             CONST(1) IS THE MAXIMUM SHEAR RATE FOR THE UPCURVE OR DOWN
C             CONST(2) IS THE CONSTANT SHEAR RATE FOR THE TORQUE-DEACY C
C             CONST(3) IS A PROPORTIONALITY CONSTANT BETWEEN THE
C             SHEAR RATE AND TIME
      T=X(I,1)
      PRNT(1)=T
      T1=CONST(1)
      T2=CONST(2)
      ALPHA=CONST(3)
C             THE FOLLOWING VARIABLES ARE DEFINED TO SIMPLIFY THE
C             MODEL TO BE TESTED
      AONE=PARAM(5)+1.0
      ATWO=ALPHA*AONE
      ATHREE=T**AONE
      AFOUR=T**PARAM(5)
      AFIVE=T1**AONE
      ASIX=T2**PARAM(5)
      ASEVEN=PARAM(3)*PARAM(4)*AFOUR
      AEIGHT=PARAM(3)*PARAM(4)*ASIX
      ANINE=PARAM(3)*ASIX*X(I,1)
      ATEN=PARAM(3)/ATWO
      BONE=ATEN*ATHREE
      BTWO=2.0*AFIVE-ATHREE
      BTHREE=ATEN*BTWO
      BFOUR=PARAM(4)*AFOUR
      BFIVE=PARAM(3)*AFOUR
      BSIX=BONE*ALOG(T)
      BSEVEN=BONE/AONE
      BEIGHT=2.0*AFIVE*ALOG(T1)
      BNINE=ATHREE*ALOG(T)
      BTEN=ATEN*(BEIGHT-BNINE)
      CONE=BTHREE/AONE
      CTWO=PARAM(4)*ASIX
```

```
      CTHREE=PARAM(3)*ASIX
      CFOUR=ALOG(T2)*(1.0-ANINE)
C             THE FOLLOWING STATEMENTS TEST THE DATA POINT TO
C             DETERMINE WHETHER IT IS REPRESENTATIVE OF THE UPCURVE,
C             DOWNCURVE, OR THE TORQUE-DECAY CURVE
      IF(X(I,2).GT.10.0) GO TO 40
      IF(X(I,2).GT.2.0.AND.X(I,2).LT.3.0) GO TO 20
C             THIS EQUATION REPRESENTS THE UPCURVE
      FCN=PARAM(1)+PARAM(2)*T+ASEVEN*EXP(-BONE)
      GO TO 30
C             THIS EQUATION REPRESENTS THE DOWNCURVE
 20   FCN=PARAM(1)+PARAM(2)*T+ASEVEN*EXP(-BTHREE)
      GO TO 30
C             THIS EQUATION REPRESENTS THE TORQUE-DECAY CURVE
 40   FCN=PARAM(1)+PARAM(2)*T2+AEIGHT*EXP(-ANINE)
 30   RESDUE=Y(I)-FCN
      RETURN
      END
```

```
      SUBROUTINE DERIV(PARTL,X,PARAM,PRNT,FCN,I)
C*************************************************************************
      COMMON Y(100),X(100,5),PARAM(10),PRNT(5),CONST(4)
C               INSERT THE VARIOUS PARTIAL DERIVATIVES IN THIS
C               SUBROUTINE ONLY IF ANALYTIC PARTIALS ARE TO BE USED
      DIMENSION PARTL(10)
      T=X(I,1)
      T1=CONST(1)
      T2=CONST(2)
      ALPHA=CONST(3)
C               THE FOLLOWING VARIABLES ARE DEFINED TO SIMPLIFY THE
C               MODEL TO BE TESTED
      AONE=PARAM(5)+1.0
      ATWO=ALPHA*AONE
      ATHREE=T**AONE
      AFOUR=T**PARAM(5)
      AFIVE=T1**AONE
      ASIX=T2**PARAM(5)
      ASEVEN=PARAM(3)*PARAM(4)*AFOUR
      AEIGHT=PARAM(3)*PARAM(4)*ASIX
      ANINE=PARAM(3)*ASIX*X(I,1)
      ATEN=PARAM(3)/ATWO
      BONE=ATEN*ATHREE
      BTWO=2.0*AFIVE-ATHREE
      BTHREE=ATEN*BTWO
      BFOUR=PARAM(4)*AFOUR
      BFIVE=PARAM(3)*AFOUR
      BSIX=BONE*ALOG(T)
      BSEVEN=BONE/AONE
      BEIGHT=2.0*AFIVE*ALOG(T1)
      BNINE=ATHREE*ALOG(T)
      BTEN=ATEN*(BEIGHT-BNINE)
      CONE=BTHREE/AONE
      CTWO=PARAM(4)*ASIX
      CTHREE=PARAM(3)*ASIX
      CFOUR=ALOG(T2)*(1.0-ANINE)
      AONE=PARAM(5)+1.0
      ATWO=ALPHA*AONE
      ATHREE=T**AONE
      AFOUR=T**PARAM(5)
      AFIVE=T1**AONE
      ASIX=T2**PARAM(5)
      ASEVEN=PARAM(3)*PARAM(4)*AFOUR
      AEIGHT=PARAM(3)*PARAM(4)*ASIX
      ANINE=PARAM(3)*ASIX*X(I,1)
      ATEN=PARAM(3)/ATWO
      BONE=ATEN*ATHREE
      BTWO=2.0*AFIVE-ATHREE
      BTHREE=ATEN*BTWO
      BFOUR=PARAM(4)*AFOUR
```

```
      BFIVE=PARAM(3)*AFOUR
      BSIX=BONE*ALOG(T)
      BSEVEN=BONE/AONE
      BEIGHT=2.0*AFIVE*ALOG(T1)
      BNINE=ATHREE*ALOG(T)
      BNINE=ATHREE*ALOG(T)
      BTEN=ATEN*(BEIGHT-BNINE)
      CONE=BTHREE/AONE
      CTWO=PARAM(4)*ASIX
      CTHREE=PARAM(3)*ASIX
      CFOUR=ALOG(T2)*(1.0-ANINE)
C              THE FOLLOWING STATEMENTS TEST THE DATA POINT TO
C              DETERMINE WHETHER IT IS REPRESENTATIVE OF THE UPCURVE,
C              DOWNCURVE, OR THE TORQUE-DECAY CURVE
      IF(X(I,2).GT.10.0) GO TO 40
      IF(X(I,2).GT.2.0.AND.X(I,2).LT.3.0) GO TO 20
C              THE FOLLOWING PARTIAL DERIVATIVES ARE FOR THE UPCURVE
      PARTL(1)=1.0
      PARTL(2)=T
      PARTL(3)=BFOUR*EXP(-BONE)*(1.0-BONE)
      PARTL(4)=BFIVE*EXP(-BONE)
      PARTL(5)=ASEVEN*EXP(-BONE)*(ALOG(T)-BSIX+BSEVEN)
      GO TO 30
C              THE FOLLOWING PARTIAL DERIVATIVES ARE FOR THE DOWNCURVE
   20 PARTL(1)=1.0
      PARTL(2)=T
      PARTL(3)=BFOUR*EXP(-BTHREE)*(1.0-BTHREE)
      PARTL(4)=BFIVE*EXP(-BTHREE)
      PARTL(5)=ASEVEN*EXP(-BTHREE)*(ALOG(T)-BTEN+CONE)
      PARTL(5)=-BSEVEN*ATWO*EXP(-ATHREE)
      GO TO 30
C              THE FOLLOWING PARTIAL DERIVATIVES ARE FOR THE TORQUE-
C              DECAY CURVE
   40 PARTL(1)=1.0
      PARTL(2)=T2
      PARTL(3)=CTWO*EXP(-ANINE)*(1.0-ANINE)
      PARTL(4)=CTHREE*EXP(-ANINE)
      PARTL(5)=AEIGHT*EXP(-ANINE)*CFOUR
   30 CONTINUE
      RETURN
      END
```

```
      DIMENSION TAV(50),FN(50),EV(50),ASS(50),ASR(50),GS(50),TTS(50),
     1TSS(50),GA(50,50),GD(50,50),GR(50,50),TTR(50,50),TSR(50),QQ(500),
     2GC(50),GV(50),W(50),WW(100),YW(100),WS1(100),WS2(100),WR1(100),
     3AS2(50),AR1(50),AR2(50),AS1(50),WR2(100),DLGC(50),
     4DLTSS(50),DLASS(50),DLTSR(50),DLASR(50),DLQQ(50),DLGS(50),DLGV(50)
     5,DLW(100),DLWW(100),DLYW(100) ,
     6DLAS1(50),DLAS2(50),DLAR1(50),DLAR2(50)
      EQUIVALENCE (W(1),TSS),(W(22),ASS)
      EQUIVALENCE (WW(1),TSR),(WW(22),ASR)
      EQUIVALENCE (YW(1),GC),(YW(22),GV)
      EQUIVALENCE (DLW(1),DLTSS),(DLW(22),DLASS)
      EQUIVALENCE (DLWW(1),DLTSR),(DLWW(22),DLASR)
      EQUIVALENCE (DLYW(1),DLGC),(DLYW(22),DLGV)
      EQUIVALENCE (WS1(1),DLTSS),(WS1(22),DLAS1)
      EQUIVALENCE (WR1(1),DLTSR),(WR1(22),DLAR1)
      EQUIVALENCE (WS2(1),DLTSS),(WS2(22),DLAS2)
      EQUIVALENCE (WR2(1),DLTSR),(WR2(22),DLAR2)
  633 FORMAT (///,30X,'RELATIONSHIP OF INPUT SHEAR RATES VS. TIME'/)
  634 FORMAT (15X,'TIME(SEC)',10X,'DL.T.',8X,'DL.SH.R.(STEP CHANGE)',4X,
     1'DL.SH.R.(TRIANGULAR STEP CHANGE)'/)
 3050 FORMAT (13X,'FOR TA=0.0,E=0.0,DL.T.=TIME/2*T1')
    1 FORMAT (6F10.4)
   11 FORMAT (2I10,3F10.4)
   30 FORMAT ('1')
  983 FORMAT (I10,1F10.4)
  982 FORMAT (I8,9F8.4)
  100 FORMAT (I10,3F10.4)
  301 FORMAT ('0',/////,10X,'R1='F10.4,4X,'R2='F10.4,4X,'B='F10.4,4X,
     1'TK='F10.4///)
  302 FORMAT (10X,'NR='I10,5X,'NI='I10,5X,'S='F10.4,5X,'MM='I10,5X,
     1'NN='I10///)
  303 FORMAT (10X,'U='F10.4,4X,'A='F10.4,4X,'WO='F10.4,4X,'T1='F10.4)
  405 FORMAT ('1',30X,'TA=0.0',1X,'SEC',10X,'E=0.0'//)
  406 FORMAT ('1',30X,'TA='F10.4,1X,'SEC',10X,'E=0.0'//)
  201 FORMAT ('1',20X,'DEFINITION:')
   66 FORMAT ('0',10X,'TT1='F10.4,1X,'SEC',10X,'TSS(1)='F10.4,'DYNE/CM-
     1CM'10X,'TSR(MAX)='F10.4,'DYNE/CM-CM'/)
  666 FORMAT ('0',10X,'TA='F10.4,1X,'SEC',5X,'E='F10.4, 5X,'GS(1)='F9.4
     1,'/SEC',5X,'GV(MAX)='F9.4,'/SEC'/)
  202 FORMAT (10X,'R1=RADIUS OF THE STATIONARY CYLINDER,CM')
  203 FORMAT (10X,'R2=RADIUS OF THE ROTATING CYLINDER,CM')
  205 FORMAT (10X,'R =ARBITRARY RADIUS BETWEEN R1 AND R2, CM')
  208 FORMAT (10X,'A=ACCELERATION CONSTANT OF THE ROTATING CYLINDER')
  209 FORMAT (10X,'B=LENGTH OF THE COUVETTE')
  210 FORMAT (10X,'Q=TIME,SEC')
  219 FORMAT (10X,'T1=TIME AT MAXIMUM SHEAR RATE,SEC.')
  220 FORMAT (10X,'NI,S=SECTIONAL CONSTANT FOR TIME,0')
  221 FORMAT (10X,'NR=SECTIONAL CONSTANT FOR RADIUS,0')
  222 FORMAT (10X,'UA,UB,UC=DELTA FUNCTIONS IF (UA,ETC.) 0,0,1')
  211 FORMAT (10X,'G=SHEAR RATE,1/SEC')
```

Appendix I. 4. Computer program for the dynamic behavior of torsion head

```
 2121 FORMAT (10X,'AS=DEFLECTION OF THE TORSION BAR AT A CONSTANT',1X,
     1'ROTATING RATE WO,MICRO')
 2122 FORMAT (9X,'AR=DEFLECTION OF THE TORSION BAR DURING A LINEAR',1X,
     1'ACCELERATION OF THE'/,12X,'ROTATING CYLINDER,MICRO')
  214 FORMAT (10X,'U=NEWTONIAN VISCOSITY,POISE')
  215 FORMAT (10X,'WO=CONSTANT REVOLUTION RATE OF THE ROTATING CYLINDER,
     11/SEC')
  216 FORMAT (10X,'TK=TORSION BAR CONSTANT,DYNE-CM/MICRO')
 2171 FORMAT (9X,'TSR=THEORETICAL SHEAR STRESS,DYNE/CM,CM')
 2172 FORMAT (10X,'ASR=ARTIFICIAL  SHEAR STRESS,DYNE/CM,CM')
  313 FORMAT (10X,'TA='F10.4,10X,'E=0,0'/)
  402 FORMAT (30X,'STEADY STATE FLOW AT CONSTANT ROTATING RATE',1X,
     1'AFTER A STEP CHANGE'/)
 4031 FORMAT (15X,'TIME',13X,'SHEAR RATE',10X,'THEO-SHEAR STRESS',10X,
     1'ARTI-SHEAR STRESS'/)
 5031 FORMAT (/,15X,'DL.T.',12X,'DL.SH.RATE',9X,'DL.THEO.SH.STRESS',9X,
     1'DL.ARTI.SH.STRESS'/)
 4032 FORMAT (10X,F10.4,10X,F10.4,15X,F10.4,15X,F10.4)
 4033 FORMAT (10X,I10,  10X,F10.4,15X,F10.4,15X,F10.4)
  404 FORMAT ( / ,30X,'UNSTEADY STATE FLOW DURING A LINEAR'1X,
     1'ACCELERATION AND DECELERATION'/)
  501 FORMAT ('0',10X,'PLOT OF SHEAR STRESSES (Y-AXIS) VS. TIME
     1AFTER A STEP CHANGE')
  504 FORMAT ('0',10X,'PLOT OF SHEAR STRESSES (Y-AXIS) VS. SHEAR RATE
     1DURING A LINEAR ACCELARATION AND DECELERATION')
  505 FORMAT ('0',10X,'PLOT OF SHEAR RATES (Y-AXIS) VS. TIME (X-AXIS)')
  600 FORMAT (50X,'UNDERDAMPING'/)
  601 FORMAT (50X,'CRITICAL DAMPING'/)
  602 FORMAT (50X,'OVERDAMPING'/)
  206 FORMAT (10X,'TA=TIME CONSTANT OF THE TORSION HEAD,SEC')
  207 FORMAT (10X,'E=DAMPING COEFFICIENT,0')
 3051 FORMAT (10X,'DL.T.(DIMENSIONLESS TIME=TIME(10SEC)/TIME CONST.TA')
 3052 FORMAT (10X,'DL.S.R.(DIMENSIONLESS SHEAR RATE=SHEAR RATES (STEP',/
     1,13X,'CHANGE/TRIANGULAR STEP)/(CONST./MAX.) SHEAR RATE (AFTER A',/
     2,13X,'STEP CHANGE/DURING A TRIANGULAR STEP CHANGE)')
 3053 FORMAT(10X,'DL.(THEO/ARTE) S.S.(DIMENSIONLESS SHEAR STRESS',/,
     113X,'(THEORETICAL/ARTEFACT)=SHEAR STRESS(THEORETICAL/ARTEFACT)',/,
     213X,'/THEORETICAL SHEAR STRESS AT MAX. SHEAR RATE')
  901 FORMAT (10X,'GS(1)=SHEAR RATE FOR A STEP CHANGE')
  902 FORMAT (10X,'GV(MAX)=MAX. SHEAR RATE DURING A LINEAR ACCLERATION
     1AND DECELERATION'/,12X,'TRIANGULAR STEP CHANGE')
  903 FORMAT (10X,'TT1=TOTAL TIME FOR A TRIANGULAR STEP CHANGE')
  904 FORMAT (10X,'TSS(1)=THEORETICAL SHEAR STRESS FOR A STEP CHANGE')
  905 FORMAT (10X,'TSR(MAX)=THEORETICAL MAX. SHEAR STRESS FOR A TRIANGUL
     1AR STEP CHANGE')
 1234 FORMAT (10X,'MM=NO. OF TA')
 1235 FORMAT (10X,'NN=NO. OF E')
      WRITE (6,201)
      WRITE (6,202)
      WRITE (6,203)
```

```
      WRITE (6,205)
      WRITE (6,208)
      WRITE (6,209)
      WRITE (6,210)
      WRITE (6,211)
      WRITE (6,2121)
      WRITE (6,2122)
      WRITE (6,214)
      WRITE (6,215)
      WRITE (6,216)
      WRITE (6,2171)
      WRITE (6,2172)
      WRITE (6,219)
      WRITE (6,220)
      WRITE (6,221)
      WRITE (6,222)
      WRITE (6,206)
      WRITE (6,207)
      WRITE (6,1234)
      WRITE (6,1235)
      WRITE (6,901)
      WRITE (6,902)
      WRITE (6,903)
      WRITE (6,904)
      WRITE (6,905)
      WRITE (6,3051)
      WRITE (6,3050)
      WRITE (6,3052)
      WRITE (6,3053)
 1980 READ (5,1) U,R1,R2,B,TK,A
      READ (5,11) NR,NI,WO,S,T1
 9876 READ (5,100) MM, (TAV(MTA),MTA=1,MM)
  999 READ (5,982) NN,(EV(NE),NE=1,NN)
      WRITE (6,301) R1,R2,B,TK
      WRITE (6,302) NR,NI,S,MM,NN
      WRITE (6,303) U,A,WO,T1
      RK=R1/R2
      PI=3.1416
      CS=4*PI*R1*R1*B*U*WO/((1.0-RK*RK)*TK)
      CR=4*PI*R1*R1*B*U*A/((1.0-RK*RK)*TK)
      CT=-TK/(2*PI*R1*R1*B)
      DO 1000 MTA=1,MM
      TA=TAV(MTA)
      DO 1000 NE=1,NN
      E=EV(NE)
      DO 2 I=1,NI
      Q=S*(I-1.0)
      QT=Q/TA
      QA=Q
      QB=QA-T1
```

```
      QC=QA-2*T1
      TT1=2*T1
      J=(NI+1)/2
      UA=QA
      UB=QB
      UC=QC
      IF (UA-0.0) 40,40,41
   40 UA=0.0
      GO TO 42
   41 UA=1.0
   42 IF (UB-0.0) 50,50,51
   50 UB=0.0
      GO TO 52
   51 UB=1.0
   52 IF (UC-0.0) 60,60,61
   60 UC=0.0
      GO TO 62
   61 UC=1.0
   62 E=EV(NE)
      IF (E-1.0) 70,71,72
   70 X=SQRT(1.0-E*E)
      XT=X*QT
      P=ATAN(-X/E)
      EC=EXP(-E*QT)
C     CASE THREE,TA IS NOT EQUAL TO ZERO,E<1.0,UNDERDAMPING
      AS=CS*(1.0-(EC*SIN(XT-P))/X)
      AA=CR*(Q-2*E*TA+(TA*EC/X)*SIN(XT-2*P))
      AD=CR*2*T1*(1.0+(EC*SIN(XT-P))/X)-
     1CR*(Q-2*E*TA+(TA*EC/X)*SIN(XT-2*P))
      AR=AA*(UA-UB)+AD*(UB-UC)
      ATS=CT*AS
      ATR=CT*AR
      GO TO 80
   71 EC=EXP(-Q/TA)
C     CASE FOUR,TA IS NOT EQUAL TO ZERO,E=1.0,CRITICAL DAMPING
      AS=CS*(1.0-(1.0+Q/TA)*EC)
      FA=Q*EC+2*EC/TA+Q-2/TA
      FD=2*T1*(1.0-(1.0+Q/TA)*EC)
      AA=CR*FA
      AD=CR*(FD-FA)
      AR=AA*(UA-UB)+AD*(UB-UC)
      ATS=CT*AS
      ATR=CT*AR
      GO TO 80
   72 X=SQRT(E*E-1.0)
      XP=E+X
      XN=E-X
      EP=EXP(-XP*QT)
      EN=EXP(-XN*QT)
C     CASE FIVE,TA IS NOT EQUAL TO ZERO,E>1.0
```

```
      AS=CS*(1.0-(XP*EN-XN*EP)/(2*X))
      QE=Q-2*E*TA
      TE=TA/(X*2)
      DP=2*E*E-1.0+2*E*X
      DN=2*E*E-1.0-2*E*X
      AA=CR*(QE+TE*(DP*EP-DN*EN))
      AD=2*T1*CR*(1.0+(1/(2*X))*(XN*EP-XP*EN))-
     1CR*(QE+TE*(DP*EP-DN*EN))
      AR=AA*(UA-UB)+AD*(UB-UC)
      ATS=CT*AS
      ATR=CT*AR
   80 ASS(I)=ABS(ATS)
      ASR(I)=ABS(ATR)
C     THEORETICAL SHEAR STRESS AND SHEAR RATE
      DO 2 M=1,NR
      R=R1+(M-1)*0.1*(R2-R1)
      SUMS=0.0
      GS(M)=2*R1*R1*WO/((1.0-RK*RK)*R*R)+SUMS
      TTS(M)=-U*GS(M)
      TSS(I)=ABS(TTS(1))
      SUMA=0.0
      GA(M,I)=(2*R1*R1*A/((1.0-RK*RK)*R*R))*Q+SUMA
      SUMD=0.0
      GD(M,I)=(2*R1*R1*A/((1.0-RK*RK)*R*R))*(-QC)+SUMD
      GR(M,I)=GA(M,I)*(UA-UB)+GD(M,I)*(UB-UC)
      GV(I)=GR(1,I)
      K=I+NI
      GV(K)=GR(1,I)
      TTR(M,I)=-U*GR(M,I)
      TSR(I)=ABS(TTR(1,I))
      QQ(I)=QT
      L=I+NI
      QQ(L)=QQ(I)
    2 CONTINUE
      DO 266 I=1,NI
      J=(NI+1)/2
      TT1=2*T1
      DLTSS(I)=TSS(I)/TSS(1)
      DLASS(I)=ASS(I)/TSS(1)
      DLTSR(I)=TSR(I)/TSR(J)
      DLASR(I)=ASR(I)/TSR(J)
      DLQQ(I)=QQ(I)/TT1
      DLGS(I)=GS(1)/GS(1)
      DLGC(I)=DLGS(I)
      DLGV(I)=GV(I)/GV(J)
  266 CONTINUE
      WRITE (6,30)
      WRITE (6,666) TA,E,GS(1),GV(J)
      WRITE (6,66) TT1,TSS(1),TSR(J)
      IF (E-1.0) 700,711,712
```

```
  700 WRITE (6,600)
      GO TO 677
  711 WRITE (6,601)
      GO TO 677
  712 WRITE (6,602)
  677 WRITE (6,402)
C     WRITE (6,4031)
C     DO 3 I=1,NI
C     Q=S*(I-1,0)
C     QQ(I)=Q
C     L=I+NI
C     QQ(L)=QQ(I)
C     GC(I)=GS(1)
C     WRITE (6,4032) QQ(I),GC(I),TSS(I),ASS(I)
C   3 CONTINUE
      WRITE (6,5031)
      DO 363 I=1,NI
      Q=S*(I-1.0)
      QQ(I)=Q/TA
      L=I+NI
      DLGC(L)=DLGC(I)
      WRITE (6,4032)    QQ(I),DLGC(I),DLTSS(I),DLASS(I)
  363 CONTINUE
      WRITE (6,404)
C     DO 4 I=1,NI
C     Q=S*(I-1.0)
C     QQ(I)=Q
C     L=I+NI
C     QQ(L)=QQ(I)
C     GV(I)=GR(1,I)
C     K=I+NI
C     GV(K)=GR(1,I)
C     WRITE (6,4032) QQ(I),GV(I),TSR(I),ASR(I)
C   4 CONTINUE
      DO 464 I=1,NI
      Q=S*(I-1.0)
      QQ(I)=Q/TA
      DLGV(I)=GV(I)/GV(J)
      L=I+NI
      DLGV(L)=DLGV(I)
      WRITE (6,4032)    QQ(I),DLGV(I),DLTSR(I),DLASR(I)
  464 CONTINUE
      NP=2*NI
      WRITE (6,30)
C     CALL XYPLOT (NP,QQ,W)
C     WRITE (6,30)
      CALL XYPLOT (NP,QQ,DLW)
      WRITE (6,501)
      WRITE (6,30)
C     CALL XYPLOT (NP,GV,WW)
```

```
C       WRITE (6,30)
        CALL XYPLOT (NP,DLGV,DLWW)
        WRITE (6,504)
        WRITE (6,30)
        CALL XYPLOT (NP,QQ,DLWW)
        WRITE (6,504)
 1000   CONTINUE
 6351   WRITE (6,30)
        WRITE (6,633)
        WRITE (6,634)
 6352   DO 351 I=1,NI
        Q=S*(I-1,0)
        QQ(I)=Q/TT1
        DLGC(I)=GS(1)/GS(1)
        DLGV(I)=GV(I)/GV(J)
        L=I+NI
        QQ(L)=QQ(I)
        WRITE (6,4032) Q,QQ(I),DLGC(I),DLGV(I)
  351   CONTINUE
        WRITE (6,30)
 3522   NP=2*NI
        Q=S*(I-1,0)
        QQ(I)=Q/TT1
        CALL XYPLOT (NP,QQ,DLYW)
        WRITE (6,505)
        READ (5,100) MM, (TAV(MTA),MTA=1,MM)
        DO 1001 MTA=1,MM
        DO 1001 I=1,NI
        Q=S*(I-1,0)
        QA=Q
        QB=QA-T1
        QC=QA-2*T1
        UA=QA
        UB=QB
        UC=QC
        IF (UA-0,0) 43,43,44
   43   UA=0.0
        GO TO 45
   44   UA=1.0
   45   IF (UB-0,0) 53,53,54
   53   UB=0.0
        GO TO 55
   54   UB=1.0
   55   IF (UC-0.0) 63,63,64
   63   UC=0.0
        GO TO 65
   64   UC=1.0
C       CASE ONE TA=0.0,E=0.0
   65   AS=CS
        AA=CR*QA
```

```
      AD=CR*(-QC)
      AR=AA*(UA-UB)+AD*(UB-UC)
      ATS=CT*AS
      ATR=CT*AR
      AS1(I)=ABS(ATS)
      AR1(I)=ABS(ATR)
C     CASE TWO,TA IS NOT EQUAL TO ZERO,E=0.0
      TA=TAV(MTA)
      QT=Q/TA
      AS=CS*(1.0-COS(QT))
      AA=CR*(Q-TA*SIN(QT))
      AD=CR*(2*T1*(1.0-COS(QT))-(Q-TA*SIN(QT)))
      AR=AA*(UA-UB)+AD*(UB-UC)
C     NATURAL FREQUENCY
      FN(MTA)=1.0/(2*PI*TA)
      ATS=CT*AS
      ATR=CT*AR
      AS2(I)=ABS(ATS)
      AR2(I)=ABS(ATR)
 1001 CONTINUE
      WRITE (6,405)
      WRITE (6,402)
      WRITE (6,5031)
      DO 703 I=1,NI
      Q=S*(I-1.0)
      QQ(I)=Q/TT1
      L=I+NI
      QQ(L)=QQ(I)
      DLGS(I)=GS(1)/GS(1)
      DLGC(I)=DLGS(I)
      DLGC(L)=DLGC(I)
      DLAS1(I)=AS1(I)/TSS(1)
      WRITE (6,4032) QQ(I),DLGC(I),DLTSS(I),DLAS1(I)
  703 CONTINUE
      WRITE (6,404)
      DO 704 I=1,NI
      Q=S*(I-1.0)
      QQ(I)=Q/TT1
      L=I+NI
      QQ(L)=QQ(I)
      DLGV(I)=GV(I)/GV(J)
      DLGV(L)=DLGV(I)
      DLAR1(I)=AR1(I)/TSR(J)
      WRITE (6,4032) QQ(I),DLGV(I),DLTSR(I),DLAR1(I)
  704 CONTINUE
      WRITE (6,30)
      CALL XYPLOT (NP,QQ,WS1)
      WRITE (6,501)
      WRITE (6,30)
      CALL XYPLOT (NP,DLGV,WR1)
```

```
      WRITE (6,505)
      DO 709 MTA=1,MM
      TA=TAV(MTA)
      WRITE (6,406) TA
      WRITE (6,402)
      WRITE (6,5031)
      DO 705 I=1,NI
      Q=S*(I-1,0)
      QQ(I)=Q/TA
      L=I+NI
      QQ(L)=QQ(I)
      DLGC(I)=DLGS(I)
      DLGC(L)=DLGC(I)
      DLAS2(I)=AS2(2)/TSS(1)
      WRITE (6,4032) QQ(I),DLGC(I),DLTSS(I),DLAS2(I)
  705 CONTINUE
      WRITE (6,404)
      DO 706 I=1,NI
      Q=S*(I-1,0)
      QQ(I)=Q/TA
      L=I+NI
      QQ(L)=QQ(I)
      DLGV(I)=GV(I)/GV(J)
      DLGV(L)=DLGV(I)
      DLAR2(I)=AR2(I)/TSR(J)
      WRITE (6,4032) QQ(I),DLGV(I),DLTSR(I),DLAR2(I)
  706 CONTINUE
      WRITE (6,30)
      CALL XYPLOT (NP,QQ,WS2)
      WRITE (6,501)
      WRITE (6,30)
      CALL XYPLOT (NP,DLGV,WR2)
      WRITE (6,505)
      WRITE (6,30)
      CALL XYPLOT (NP,QQ,WR2)
      WRITE (6,505)
  709 CONTINUE
      GO TO 1980
 9666 STOP
      END
```

DEFINITION:

R1=RADIUS OF THE STATIONARY CYLINDER,CM
R2=RADIUS OF THE ROTATING CYLINDER,CM
R =ARBITRARY RADIUS BETWEEN R1 AND R2, CM
A=ACCELERATION CONSTANT OF THE ROTATING CYLINDER
B=LENGTH OF THE COUVETTE
O=TIME,SEC
G=SHEAR RATE,1/SEC
AS=DEFLECTION OF THE TORSION BAR AT A CONSTANT ROTATING RATE WO,MICRO
AR=DEFLECTION OF THE TORSION BAR DURING A LINEAR ACCELERATION OF THE ROTATING CYLINDER,MICRO
U=NEWTONIAN VISCOSITY,POISE
WO=CONSTANT REVOLUTION RATE OF THE ROTATING CYLINDER,1/SEC
TK=TORSION BAR CONSTANT,DYNE-CM/MICRO
TSR=THEORETICAL SHEAR STRESS,DYNE/CM.CM
ASR=ARTIFICIAL SHEAR STRESS,DYNE/CM.CM
T1=TIME AT MAXIMUM SHEAR RATE,SEC.
NI,S=SECTIONAL CONSTANT FOR TIME,0
NR=SECTIONAL CONSTANT FOR RADIUS,0
UA,UB,UC=DELTA FUNCTIONS IF (UA,ETC.) 0,0,1
TA=TIME CONSTANT OF THE TORSION HEAD,SEC
E=DAMPING COEFFICIENT,0
MM=NO. OF TA
NN=NO. OF E
GS(1)=SHEAR RATE FOR A STEP CHANGE
GV(MAX)=MAX. SHEAR RATE DURING A LINEAR ACCLERATION AND DECELERATION TRIANGULAR STEP CHANGE
TT1=TOTAL TIME FOR A TRIANGULAR STEP CHANGE
TSS(1)=THEORETICAL SHEAR STRESS FOR A STEP CHANGE
TSR(MAX)=THEORETICAL MAX. SHEAR STRESS FOR A TRIANGULAR STEP CHANGE
DL.T.(DIMENSIONLESS TIME=TIME(10SEC)/TIME CONST.TA
FOR TA=?.0,E=0.0,DL.T.=TIME/2*T1
DL.S.R.(DIMENSIONLESS SHEAR RATE=SHEAR RATES (STEP CHANGE/TRIANGULAR STEP)/(CONST./MAX.) SHEAR RATE (AFTER A STEP CHANGE/DURING A TRIANGULAR STEP CHANGE)
DL.(THEO/ARTE) S.S.(DIMENSIONLESS SHEAR STRESS (THEORETICAL/ARTEFACT)=SHEAR STRESS(THEORETICAL/ARTEFACT) /THEORETICAL SHEAR STRESS AT MAX. SHEAR RATE

Appendix II.1-A Hematological Parameters During Cardiopulmonary Bypass

Parameter	Sample	N	$\bar{x}$	S.D.	P
hemoglobin	N	18	15.2	1.2	-
	1	6	14.7	1.3	-
	2	13	13.0	1.2	0.01
	3	11	8.1	0.9	< 0.01
	4	8	10.8	1.9	< 0.01
	5	4	12.0	0.5	< 0.01
hematocrit	N	18	45.7	3.5	-
	1	6	43.3	3.3	-
	2	13	38.1	3.3	< 0.01
	3	11	25.6	2.7	< 0.01
	4	8	31.7	5.3	< 0.01
	5	4	35.6	1.2	< 0.01
red blood count	N	18	4.80	0.54	-
	1	6	4.91	0.27	-
	2	13	4.47	0.48	< 0.05
	3	11	2.66	0.35	< 0.01
	4	8	3.51	0.67	< 0.01
	5	4	4.09	0.10	< 0.02
fibrinogen	N	18	216	58	-
	1	6	297	92	< 0.02
	2	13	298	53	< 0.01
	3	11	154	34	< 0.01
	4	8	293	63	< 0.01
	5	4	489	71	< 0.01
total protein	N	18	7.4	0.5	-
	1	6	7.7	0.6	-
	2	13	6.5	0.4	< 0.01
	3	11	4.2	1.0	< 0.01
	4	8	5.4	0.4	< 0.01
	5	4	7.4	0.4	-

Appendix II.1-A - Continued

Parameter	Sample	N	$\bar{x}$	S.D.	P
albumin	N	18	58.9	5.0	-
	1	6	54.8	1.8	-
	2	13	55.9	3.6	-
	3	11	56.0	5.6	-
	4	8	61.9	6.7	-
	5	4	49.8	3.0	<0.01
α_1 globulin	N	17	2.7	0.8	-
	1	6	3.4	0.4	<0.05
	2	13	3.3	0.7	<0.05
	3	11	3.2	0.7	<0.05
	4	8	4.2	1.1	<0.01
	5	4	4.8	0.8	<0.01
α_2 globulin	N	17	8.5	1.3	-
	1	6	10.7	10.0	<0.01
	2	13	10.0	2.1	<0.02
	3	11	11.4	6.8	-
	4	8	7.3	1.8	-
	5	4	12.1	3.4	<0.01
β globulin	N	17	11.9	1.1	-
	1	6	12.7	0.5	-
	2	13	12.7	1.3	-
	3	11	11.6	1.8	-
	4	8	11.1	2.3	-
	5	4	13.5	0.8	<0.01
γ globulin	N	17	17.7	4.4	-
	1	6	18.4	1.7	-
	2	13	18.1	3.7	-
	3	11	17.3	3.9	-
	4	8	15.5	2.3	-
	5	4	19.8	2.6	-

Appendix II.1-B Rheological Parameters During Cardiopulmonary Bypass

Rheological Parameter	Sample	N	$\bar{x}$	S.D.	P
τ_0	N	21	0.196	0.042	-
	1	6	0.168	0.054	-
	2	13	0.141	0.040	<0.001
	3	12	-	-	-
	4	8	0.069	0.035	<0.001
	5	4	0.127	0.017	<0.01
μ	N	21	0.116	0.016	-
	1	6	0.116	0.025	-
	2	13	0.091	0.015	-
	3	12	0.047	0.010	<0.001
	4	8	0.066	0.015	<0.010
	5	4	0.093	0.007	-
C	N	21	0.125	0.038	-
	1	6	0.111	0.021	-
	2	13	0.111	0.034	-
	3	12	-	-	-
	4	8	0.107	0.041	-
	5	4	0.080	0.025	<0.05
A	N	21	0.407	0.101	-
	1	6	0.377	0.152	-
	2	13	0.350	0.143	-
	3	12	-	-	-
	4	8	0.182	0.047	<0.001
	5	4	0.276	0.114	<0.05

Appendix II.1-B - Continued

Rheological Parameter	Sample	N	$\bar{x}$	S.D.	P
η	N	21	1.661	0.340	-
	1	6	1.646	0.137	-
	2	13	1.578	0.275	-
	3	12	-	-	-
	4	8	1.526	0.155	-
	5	4	2.048	0.627	-
η_s	N	21	0.184	0.039	-
	1	6	0.174	0.043	-
	2	13	0.138	0.029	< 0.001
	3	12	0.048	0.009	< 0.001
	4	8	0.090	0.025	< 0.001
	5	4	0.136	0.014	-
$\eta_s - \mu$	N	21	0.068	0.022	-
	1	6	0.058	0.018	-
	2	13	0.048	0.016	< 0.01
	3	12	0.0004	0.0007	< 0.001
	4	8	0.024	0.010	< 0.001
	5	4	0.044	0.008	< 0.05

Appendix II.1-B Comparison of Rheological Parameters Between 13 Patients Who Survived and Two Patients Who Expired After Cardic Surgery

Rh.P.	Pre-Operative Study			Post-Cardiopulmonary Bypass study		
	$\bar{X}$	1	2	$\bar{X}$	1	2
τ_0	0.141	0.137	0.133	-	-	-
μ	0.091	0.090	0.097	0.047	0.070	0.041
C	0.111	0.084	0.162	-	-	-
A	0.350	0.293	1.945	-	-	-
n	1.578	1.786	0.403	-	-	-
η_s	0.138	0.129	0.194	0.048	0.204	0.153
$\eta_s - \mu$	0.048	0.039	0.097	0.0004	0.134	0.112

Rh.P. : Rheological parameters.

$\bar{X}$: Mean value of 13 survivors.

Appendix II.2 <u>Rheological Parameters from the Effect of Temperature on Blood</u>

Rheological Parameter	Temperature	N	X	S.D.	P
τ_o	22.8°C	21	0.196	0.042	-
	28.0	10	0.172	0.044	0.0026
	32.0	10	0.178	0.041	0.0004
	35.0	10	0.183	0.038	-
	37.0	10	0.120	0.033	-
	39.0	10	0.183	0.038	-
	41.0	10	0.181	0.039	0.0002
μ	22.8	21	0.116	0.016	0.05
	28.0	10	0.112	0.019	0.0002
	32.0	10	0.106	0.015	0.0001
	35.0	10	0.097	0.016	0.052
	37.0	10	0.084	0.013	-
	39.0	10	0.093	0.010	0.12
	41.0	10	0.097	0.013	0.052
C	22.8	21	0.125	0.038	0.529
	28.0	10	0.122	0.020	0.347
	32.0	10	0.139	0.026	0.720
	35.0	10	0.167	0.048	0.081
	37.0	10	0.134	0.034	-
	39.0	10	0.153	0.053	0.343
	41.0	10	0.153	0.033	0.215

Appendix II.2 - Continued

Rheological Parameter	Temperature	N	X	S.D.	P
A	22.8	21	0.407	0.101	-
	28.0	10	0.300	0.090	0.009
	32.0	10	0.333	0.066	-
	35.0	10	0.374	0.128	0.0002
	37.0	10	0.213	0.056	-
	39.0	10	0.399	0.196	0.004
	41.0	10	0.336	0.109	0.0014
n	22.8	21	1.661	0.340	0.303
	28.0	10	1.701	0.262	0.478
	32.0	10	1.623	0.223	0.153
	35.0	10	1.490	0.369	0.045
	37.0	10	1.790	0.295	-
	39.0	10	1.514	0.298	0.184
	41.0	10	1.593	0.158	0.033
η_s	22.8	21	0.184	0.030	-
	28.0	10	0.171	0.033	0.0002
	32.0	10	0.171	0.027	-
	35.0	10	0.166	0.029	-
	37.0	10	0.121	0.023	-
	39.0	10	0.163	0.024	-
	41.0	10	0.169	0.025	-
$\eta_s - \mu$	22.8	21	0.068	0.022	-
	28.0	10	0.059	0.015	0.0002
	32.0	10	0.065	0.013	-
	35.0	10	0.068	0.017	-
	37.0	10	0.037	0.012	-
	39.0	10	0.071	0.017	-
	41.0	10	0.072	0.016	-

Appendix II.3 Rheological Parameters from the Effect of Alknols on Blood

Note:

C_1 to C_4 : No. of Pure Blood Sample (Control).

1 to 11 : The number indicates the carbon number of the alkanol which has been added to 5.0 ml. of the pure blood sample.

0.1 to 0.005 : ml. of the pure alkanol added to 5.0 ml. of the pure blood sample.

Incubation : at 37°C for 30 minutes.

S : Sample.

V : ml. of alkanol added to the control.

Sample & Alkanols		Rheological Parameters						
S	V	τ_0	μ	c	A	n	η_s	$\eta_s - \mu$
C_1	-	0.0844	0.0800	0.0547	0.0622	3.1840	0.1059	0.0259
1	0.10	0.0963	0.0682	0.0206	0.1258	2.6984	0.1059	0.0377
2	0.10	0.1702	0.1145	0.0738	0.2885	1.3385	0.1853	0.0708
	0.05	0.0913	0.0867	0.0805	0.0521	3.1817	0.1147	0.0280
3	0.10	0.1377	0.1037	0.0456	0.0915	3.2345	0.1815	0.0481
	0.05	0.0974	0.0860	0.0809	0.1354	1.9355	0.1235	0.0375
4	0.10	0.2072	0.1355	0.0759	0.4315	1.2900	0.2250	0.0895
	0.05	0.1033	0.0891	0.0361	0.0634	3.6344	0.1253	0.0344
5	0.10	0.1536	0.1904	0.0269	0.3440	1.9676	0.2558	0.0684
	0.05	0.1169	0.1055	0.0412	0.1628	1.7969	0.1500	0.0445
6	0.10	0.0000	0.0423	0.0000	0.0000	0.0000	0.0423	0.0000
	0.05	0.1261	0.1190	0.0091	0.1427	2.7400	0.1676	0.0480
7	0.10	0.0000	0.0353	0.0000	0.0000	0.0000	0.0353	0.0000
	0.05	0.0896	0.0945	0.0040	0.0631	3.9301	0.1235	0.0290
8	0.10	0.0000	0.0988	0.0000	0.0000	0.0000	0.0988	0.0000
	0.05	0.1046	0.1208	0.0344	0.2595	1.7266	0.1676	0.0468
9	0.10	0.0000	0.1588	0.0000	0.0000	0.0000	0.1588	0.0000
	0.05	0.1308	0.1409	0.0301	0.0706	3.1957	0.1853	0.0444
10	0.10	0.0764	0.1268	0.0502	0.0881	3.0723	0.1544	0.0276
	0.05	0.0873	0.0887	0.0592	0.5127	0.8494	0.1368	0.0481
11	0.10	0.0999	0.0903	0.0497	0.2085	1.6492	0.1332	0.0429
	0.05	0.1271	0.0963	0.0487	0.2712	1.5707	0.1500	0.0537

Appendix II.3 - Continued

S	V	τ_0	μ	c	A	n	η_s	$\eta_s-\mu$
C_2	-	0.0806	0.0600	0.0425	0.1196	2.2536	0.0900	0.0300
1	0.10	0.0828	0.0837	0.0401	0.1260	2.1390	0.1164	0.0327
	0.05	0.0969	0.0711	0.0394	0.1081	2.5765	0.1094	0.0383
2	0.10	0.0865	0.0900	0.0647	0.0910	2.2829	0.1217	0.0317
	0.05	0.0736	0.0772	0.0773	0.1028	2.1026	0.1058	0.0286
3	0.10	0.0969	0.0901	0.0581	0.1089	2.0269	0.1252	0.0351
	0.05	0.0926	0.0812	0.0564	0.0856	2.5970	0.1147	0.0335
4	0.10	0.0942	0.1001	0.0707	0.1615	1.9805	0.1393	0.0392
	0.05	0.1152	0.1049	0.0721	0.2189	1.4041	0.1464	0.0415
	0.02	0.0956	0.0818	0.0725	0.1533	1.6082	0.1200	0.0382
5	0.10	0.0000	0.1588	0.0000	0.0000	0.0000	0.1588	0.0000
	0.05	0.0945	0.1018	0.0521	0.0810	2.7056	0.1368	0.0250
6	0.10	0.0000	0.0353	0.0000	0.0000	0.0000	0.0353	0.0000
	0.05	0.0000	0.1199	0.0000	0.0000	0.0000	0.1199	0.0000
	0.02	0.1178	0.1135	0.0185	0.0586	3.9991	0.1500	0.0365
7	0.10	0.0000	0.0325	0.0000	0.0000	0.0000	0.0325	0.0000
	0.05	0.0000	0.0649	0.0000	0.0000	0.0000	0.0649	0.0000
	0.02	0.0257	0.0596	0.0155	0.0491	2.4563	0.0706	0.0110
	0.005	0.1118	0.1028	0.1233	0.1464	1.7576	0.1412	0.0384
8	0.10	0.0000	0.0466	0.0000	0.0000	0.0000	0.0466	0.0000
	0.05	0.0398	0.0912	0.0620	0.3445	1.4685	0.1111	0.0199
9	0.10	0.0000	0.0564	0.0000	0.0000	0.0000	0.0564	0.0000
	0.05	0.1104	0.1105	0.0627	0.2897	1.5793	0.1640	0.0535
	0.02	0.1098	0.0885	0.0468	0.1288	2.5020	0.1279	0.0394
10	0.10	0.0860	0.0705	0.0645	0.1081	2.4780	0.1023	0.0318
11	0.10	0.0608	0.0743	0.0670	0.2697	1.3094	0.1094	0.0351

Appendix II.3 - Continued

S	V	τ_0	μ	C	A	n	η_s	$\eta_s - \mu$
C_3	-	0.0716	0.0563	0.1078	0.1237	2.1768	0.0811	0.0248
1	0.10	0.1266	0.0881	0.1033	0.2184	1.6263	0.1341	0.0460
	0.05	0.0565	0.0527	0.0905	0.0899	2.4550	0.0776	0.0249
2	0.10	0.1088	0.0840	0.0667	0.1902	2.0729	0.1288	0.0448
	0.05	0.0369	0.0382	0.0301	2.2631	0.3562	0.0829	0.0447
3	0.10	0.1364	0.0839	0.0585	0.1627	2.3690	0.1376	0.0540
	0.05	0.0720	0.0741	0.0827	0.1451	1.9083	0.1058	0.0317
4	0.10	0.0625	0.0750	0.0294	0.0777	4.0822	0.1023	0.0273
	0.05	0.0379	0.0503	0.0681	0.0765	2.0545	0.0642	0.0139
5	0.10	0.0000	0.0706	0.0000	0.0000	0.0000	0.0706	0.0000
	0.05	0.1702	0.1337	0.0269	0.2685	2.0898	0.2029	0.0692
	0.02	0.1002	0.0822	0.0603	0.1945	1.9441	0.1200	0.0378
6	0.10	0.0000	0.0396	0.0000	0.0000	0.0000	0.0396	0.0000
	0.05	0.0000	0.0635	0.0000	0.0000	0.0000	0.0635	0.0000
	0.02	0.0804	0.0829	0.0396	0.0896	2.8835	0.1129	0.0300
7	0.10	0.0000	0.0282	0.0000	0.0000	0.0000	0.0282	0.0000
	0.05	0.0000	0.0706	0.0000	0.0000	0.0000	0.0706	0.0000
	0.02	0.0472	0.0796	0.0294	0.0762	2.1098	0.0970	0.0174
8	0.10	0.0000	0.0466	0.0000	0.0000	0.0000	0.0466	0.0000
	0.05	0.0351	0.0679	0.0795	0.0918	2.0166	0.0819	0.0140
9	0.10	0.0000	0.0522	0.0000	0.0000	0.0000	0.0522	0.0000
	0.05	0.0449	0.0846	0.0532	0.1351	1.6320	0.1041	0.1095
	0.02	0.0658	0.0645	0.0263	0.0822	2.8202	0.0924	0.0279
10	0.10	0.0276	0.0415	0.0243	1.5484	0.4731	0.0758	0.0343
11	0.10	0.0446	0.0607	0.0476	0.0461	3.1744	0.0758	0.0151

Appendix II.3 - Continued

S	V	τ_o	μ	C	A	n	η_s	$\eta_s - \mu$
C_4	-	0.3736	0.1614	0.0773	0.5050	1.5982	0.3088	0.2474
1	0.10	0.5743	0.2362	0.0685	0.7531	1.3840	0.4588	0.2226
	0.05	0.4371	0.1900	0.0778	0.5327	1.6269	0.3617	0.1717
2	0.10	0.7409	0.2343	0.0903	0.6852	1.6463	0.5029	0.2686
	0.05	0.4698	0.1840	0.0632	0.5478	1.9426	0.3617	0.1717
3	0.10	0.4173	0.1950	0.0810	0.7860	1.3358	0.3706	0.1756
	0.05	0.3640	0.1835	0.0838	0.6778	1.3747	0.3353	0.1518
4	0.10	0.5785	0.2684	0.0611	0.9183	1.5300	0.4941	0.2257
	0.05	0.4048	0.1961	0.0858	0.8291	1.4055	0.3706	0.1745
5	0.10	0.2492	0.3695	0.0838	0.4254	1.2335	0.4658	0.0963
	0.05	0.7213	0.4439	0.0351	1.1456	1.7890	0.7411	0.2972
	0.02	0.5434	0.2331	0.0563	0.8345	1.6153	0.4367	0.2036
	0.01	0.4930	0.2186	0.0833	0.8777	1.3885	0.4235	0.2049
6	0.10	0.0000	0.1059	0.0000	0.0000	0.0000	0.1059	0.0000
	0.05	0.6983	0.1955	0.0137	0.1225	2.4727	0.2206	0.0351
	0.02	0.4144	0.1991	0.0406	0.4955	1.9768	0.3573	0.1582
7	0.10	0.0000	0.1059	0.0000	0.0000	0.0000	0.1059	0.0000
	0.05	0.0000	0.1588	0.0000	0.0000	0.0000	0.1588	0.0000
	0.01	0.5913	0.2503	0.0640	0.5408	1.6445	0.4588	0.2085
	0.005	0.3928	0.1928	0.0827	1.0362	1.1058	0.3661	0.1633
8	0.10	0.0000	0.1306	0.0000	0.0000	0.0000	0.1306	0.0000
	0.05	0.0000	0.2117	0.0000	0.0000	0.0000	0.2177	0.0000
	0.01	0.5619	0.2158	0.0624	0.5662	1.7319	0.4235	0.2077
	0.005	0.4419	0.1914	0.0794	0.6882	1.3571	0.3723	0.1809
9	0.10	0.0000	0.2117	0.0000	0.0000	0.0000	0.2117	0.0000
	0.05	0.1481	0.1881	0.0047	0.0279	0.0014	0.2294	0.0413
	0.01	0.4894	0.2136	0.0657	0.7343	1.7782	0.4147	0.2011
10	0.10	0.2884	0.4213	0.0522	0.3481	1.6234	0.5294	0.1081
	0.05	0.4621	0.2469	0.0977	0.9640	1.4437	0.4367	0.1901
11	0.10	0.4182	0.2142	0.0963	0.7802	1.7356	0.3810	0.1668
	0.05	0.3611	0.1915	0.0916	0.5394	1.8906	0.3309	0.1394

Appendix III. Equations for the Calculation of Shear Rate and Shear Stress on a Double Couette

The double couette was shown in Fig.III.1-2.

Assuming an incompressible Newtonian fluid flows at steady state in θ -direction only i.e. $V_r = V_z = 0$

Equation of motion of the fluid in θ - direction

$$0 = \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rV_\theta)\right] \qquad \text{(A.III-1)}$$

(A) Inner couvette ; $r_1 \leq r \leq r_2$

B.C. $V_\theta(r_1) = 2\pi r_1 \Omega$ with Ω [=] rad/sec

$V_\theta(r_2) = 0$

The solution of Eq.AIII-1 is

$$V_\theta(r) = \frac{2\pi r_1^2 \Omega}{r_2^2 - r_1^2}\left(\frac{r_2^2}{r} - r\right)$$

$$\dot{\gamma}_{r\theta}\Big|_{r=r_2} = r\frac{d}{dr}\left(\frac{V_\theta}{r}\right)\Big|_{r=r_2} = -\frac{4\pi r_1^2 \Omega}{r_2^2 - r_1^2} \ \sec^{-1}$$

$$\tau_{r\theta}\Big|_{r=r_2} = -\mu\,\dot{\gamma}_{r\theta}\Big|_{r=r_2}$$

$$T_2 = 2\pi r_2 L\, \tau_{r\theta}\Big|_{r=r_2} \cdot r_2 = 4\pi L \mu \Omega \frac{r_1^2 r_2^2}{r_2^2 - r_1^2}$$

(B). Outter couette ; $r_3 \leqslant r \leqslant r_4$

B.C. $V_\theta(r_3) = 0$

$V_\theta(r_4) = 2\pi r_4 \Omega$

The solution of Eq.AIII-1 is

$$V_\theta(r) = \frac{2\pi r_4^2}{r_4^2 - r_3^2}\left(r - \frac{r_3^2}{r}\right)$$

$$\dot{\gamma}_{r\theta}\Big|_{r=r_3} = r\frac{d}{dr}\left(\frac{V_\theta}{r}\right)\Big|_{r=r_3} = \frac{4\pi r_4^2 \Omega}{r_4^2 - r_3^2} \ \sec^{-1}$$

$$\tau_{r\theta}\Big|_{r=r_3} = -\mu\dot{\gamma}_{r\theta}\Big|_{r=r_3}$$

$$T_3 = 2\pi r_3 L(-\tau_{r\theta})\Big|_{r=r_3} \cdot r_3 = 4\pi L\mu\Omega \frac{r_3^2 r_4^2}{r_4^2 - r_3^2}$$

(C). Design condition

In order to have the same $\dot{\gamma}_{r\theta}$ and same $\tau_{r\theta}$ on the surfaces at r_2 and r_3, the double couette should have :

$$\dot{\gamma}_{r\theta}\Big|_{r=r_2} = -\dot{\gamma}_{r\theta}\Big|_{r=r_3} \quad \text{so that} \quad \tau_{r\theta}\Big|_{r=r_2} = -\tau_{r\theta}\Big|_{r=r_3}$$

or $$\frac{r_1^2}{r_2^2 - r_1^2} = \frac{r_4^2}{r_4^2 - r_3^2}$$

(D). Calculation of shear stress from total torque

$$T = T_2 + T_3 = \text{Total torque applied to both } r_2 \text{ \& } r_3 \text{ surfaces}$$

$$= 4\pi\mu L\Omega\left(\frac{r_1^2 r_2^2}{r_2^2 - r_1^2} + \frac{r_3^2 r_4^2}{r_4^2 - r_3^2}\right)$$

$$\frac{T_2}{T} = \frac{r_2^2}{r_2^2 + r_3^2}$$

or

$$T_2 = T\left(\frac{r_2^2}{r_2^2 + r_3^2}\right) = 2\pi L r_2^2 (\tau_{r\theta})\Big|_{r=r_2}$$

$$\therefore (\tau_{r\theta})\Big|_{r=r_2} = (-\tau_{r\theta})\Big|_{r=r_3} = \frac{T}{2\pi L(r_2^2 + r_3^2)}$$

(E). Calculation of total torque from the measurement of Y in X-Y recorder

Let R = setting of range of the maximum deflection in micro of the transducer meter.

Y_m = the reading in Y of the X-Y recorder corresponding to the maximum angle deflection of the torsion bar.

Y = reading of Y at the X-Y recorder.

G = torsion bar constant.

α = angle deflection of torsion bar.

$$T = G \cdot \alpha = G \cdot R \cdot \frac{Y}{Y_m}$$

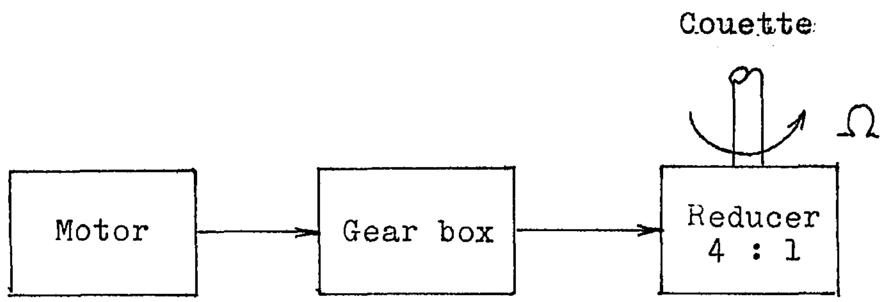
so,

$$(\tau_{r\theta})\Big|_{r=r_2} = (-\tau_{r\theta})\Big|_{r=r_3} = \frac{G \cdot R \cdot Y / Y_m}{2\pi L(r_2^2 + r_3^2)}$$

$$(\dot{\gamma}_{r\theta})\Big|_{r=r_2} = (-\dot{\gamma}_{r\theta})\Big|_{r=r_3} = -\frac{4\pi r_1^2 \Omega}{r_2^2 - r_1^2}$$

For Ω , see next Appendix.

Appendix IV. Calculation of the RPM of viscometer in Weissenberg Rheogoniometer, Model 18



Input shaft		Couette
1500 rpm (motor)		Ω_o rpm
$\frac{99999 \times 60}{256 \times 200} \times 5$	585.93 rpm (control panel)	Ω ?

Note:

1. Constant speed of motor is 1500 rpm.
2. Control panel sets at 99999 with high switch 5 (low switch 1) giving the new impulse driven motor at the above speed (585.93 rpm).
3. Ω_o is determined by the gear box setting (see Weissenberg manual, Appendix 1).

so,

$$\Omega = \frac{585.93}{1500}\,\Omega_o \ (rpm)$$

$$= \frac{585.93}{1500}\,\Omega_o \times \frac{2\pi}{60} \quad sec^{-1}$$

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