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THE MINIMUM MAXIMAL-CUBE COVERING APPROACH TO SWITCHING THEORY

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THE MINIMUM MAXIMAL-CUBE COVERING
APPROACH TO SWITCHING THEORY

BY

YU-TSANG HWANG

A DISSERTATION

PRESENTED IN PARTIAL FULFILLMENT OF

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THE MINIMUM MAXIMAL-CUBE COVERING APPROACH TO SWITCHING THEORY

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YU-TSANG HWANG

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DEPARTMENT OF ELECTRICAL ENGINEERING

NEW JERSEY INSTITUTE OF TECHNOLOGY

BY

FACULTY COMMITTEE

APPROVED:______________________________CHAIRMAN

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MAY, 1980

ABSTRACT

This dissertation is devoted to the construction of a new algebraic structure for the K-map so that algorithms and theorems can be derived for the minimization of Boolean functions and at the same time provide for the manipulation of Boolean functions as freely as one could on the K-map.

Computer programs have been included to demonstrate the theory and numerous examples of various complexity have been worked out in the appendices.

PREFACE

With the advent of microprocessors and programmable logical arrays, low speed logic circuits can now be realized without performing the tedious and complicated process associated with switching-function minimization. However, as speeds and complexities of switching functions become ever faster and more complex, the classical two-level random logic approach, far from being obsolete, deserves serious consideration. Having the least propagation delay is sufficient advantage for the two-level AND/OR gating logic to be considered in future switching circuits. In an ultra-fast system, if propagation delays are of the same order of magnitude as pulse widths, as the clock period approaches twice the smallest pulse width, the system cycle time can actually become faster than the execution time of a microprocessor or the access time of a logic array. When this happens, two-level random logic is the only alternative. In the report of a study conducted by Rudolf E. Thun and Edward T. Lewis[1] at Raytheon Company, the impact of gigahertz logic on system design is discussed. Berthod G. Bosch[2] also did a study on the prospect of ultra-fast logic systems in the 1980's. These studies indicate that the coming of ultra-fast logic systems in the 1980's is clear and certain.

Research ranging from the hardware packaging to the new scope of the applications is in full swing; the Military VLSI ultra-fast speed logic program is also progressing swiftly.

The most recent attempts in the classical two-level logic minimizations are to unify the two steps of Quine's original concepts, i.e. to search the minimum set directly without finding the prime implicant tables. McKinney's[3] attempt with the direct search method, Curtis'[4] with the adjacent table method and Arevalo's[5] four-tuple method are all in this trend. The most natural and logical route is actually the algebraic equivalent to the Karnaugh-map method. However, the lack of an algebraic structure prevents one from manipulating the K-map analytically.

This dissertation is devoted to the construction of a new algebraic structure for the K-map so that algorithms and theorems can be derived for the minimization of Boolean functions and at the same time provide for the manipulation of Boolean functions as freely as one could on the K-map. We call it the minimum maximal-cube covering approach.[6]

In Chapter One, a brief historical survey and review of switching circuits, Boolean algebra and Boolean function minimization is presented. The comparison of various approaches to the minimization of switching circuits leads to a set of most desirable characteristics of the mechanics of minimization of switching circuits.

Chapter Two is devoted to the construction of the algebraic structure and the theoretical foundation of the minimum maximal-cube covering approach to switching theory. Starting from definitions and theorems of minimum maximal-cube covering approach, a deeper and richer insight of switching theory is shown. Applications and examples will be demonstrated in Chapters Three, Four and Five.

Chapter Three covers the algorithms of a cube graph approach with demonstrated example problems.

Chapter Four covers a different and more computer-oriented algorithm, the algebraic-geometric approach. Examples, problems and demonstrative computer programs are also shown in Chapter Four.

In Chapter Five, applications in other areas are explored - the decomposition of Boolean functions, the symmetry of Boolean functions, the sequential circuits, the coding theory, et cetera.

In Chapter Six, the concluding chapter, we summarize the results and also look beyond this dessertation to the prospective of minimum maximal cube covering approach.

[1] Thun, E. Rudolf and Edward T. Lewis, "Advanced Packaging Techniques, Final Report on Task I" AFAL-TR-77-165, part I, Raytheon Company, Missile Systems Division, Hartwell Road, Bedford, Massachusetts, 01730 pp. 10-16, August 1977.

[2] Bosch, B. G., "Gigabit Electronics", Proc. 1977 Europ. Microwave Conference, Copenhagen. (invited paper) pp.1-6.

[3] McKinney, M. H., "A directed search algorithm for the canonical minimization of switching functions", Ph.D. dissertation, Texas A&M University, College Station, TX, 1974.

[4] Curtis, H. A., "Adjacency table Method of Deriving Minimal Sums," IEEE Transaction on Computers Vol. C-26, No. 11, Nov. 1977, pp. 1136-1141.

[5] Arevalo, Z and Bredeson, J. G. "A Method to Simplify a Boolean Function into a near minimal sum-of-products for programmable logic arrays", IEEE Transaction of Computers Vol C-27, No. 11, Nov. 1978, pp. 1026-1039.

[6] For detailed explanation of the meaning of Minimum Maximal-Cube Covering, please refer to Chapter 1 and Chapter 2.

DEDICATION

To my wife, Rene, for her patience, her understanding, and her encouragement.

ACKNOWLEDGEMENTS

The author gratefully acknowledges the guidance and assistance of Professor Robert E. Anderson. He also expresses his gratitude to Drs. Kenneth Sohn, William J. Troop, and Simon Cohen.

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CHAPTER I

HISTORICAL REVIEW OF SWITCHING THEORY AND MINIMIZATION

1.1 Historical Review

At the beginning, primitive mathematical ideas tend to arise from the need of a better way to describe and analyze some special physical system; then through mathematician's hands it is further developed by deductive reasoning into a new branch of mathematics. Engineering tools for analysis and design of apparatus, starting from craftsman's rudimentary cut-and-try method to the borrowing of rigorous mathematics for refined and systematic synthesis, coincides with the progress of technological demands. In the 1680's, G.W. Leibniz' attempts to create a systematic mathematical logic led to the discovery of some of the basic Boolean algebra.[1] His contemplation of a calculating machine, although not rendered into hardware, was the starting point of modern digital systems and computers. During the following 200 years, little progress was made in mathematical logic. In 1854, British mathematician George Boole published his work, "An Investigation of the Laws of Thoughts"[2], which marked the beginning of modern symbolic logic and Boolean Algebra. Modern telegraphy and control systems have been using relay switching circuits since the late 19th century and engineers and designers depended mostly upon experience and the art of trial and error to the design of switching circuits. In 1938, C. E. Shannon's introduction of the idea of using Boolean algebra in the analysis and design of relay contact circuits started the rapid development of switching theory and digital systems.[4]

Thus engineers in digital or switching circuits acquired the proper tools to handle the analysis and synthesis of hardware; however, as the circuits and systems became more and more complex, the basic Boolean algebra manipulation became awkward and inefficient in handling the Boolean functions encountered in real systems. The first systematic attempt at more efficient synthesis of logic circuits was made by Harvard University's Computation Laboratory.[5] The next important advancement was made by W. V. Quine in the early 1950's.[6] In 1955, E. J. McCluskey generalized Quine's idea and also systematically developed the engineering notation and procedures for the synthesis of switching functions.

Another technique, the graphical method, also evolved. Veitch and Karnaugh , in slightly different manners independently derived a map from the Venn diagram which enables one to do the synthesis by visual inspection of the geometric patterns. Since then, there have been numerous improvements in these two basic approaches. However, the covering problem in Quine-McCluskey's method and the limitation in the number of variables in the map method were always problems. The most recent papers offer some improvements and refinements in these areas. More detailed comparisons of various methods will be made in the next section.

In the late 1950's, R. P. Roth[8] introduced the topological and simplex concept of treating Boolean functions, furthering the understanding of the interrelationship between the Boolean expressions and Boolean functions. Roth also introduced the extraction algorithms which were the basis of the early computer algorithms developed at the University of Wisconsin[9] and the computer programs at IBM.

As the technology in hardware improved, new devices and new circuits altered the cost value functions in minimizing hardware. Also, multi-valued logics, threshold logic, and sequential logic demands more generalized mathematical tools. With the coming of integrated circuits, the search for a minimum library of basic gates for all logic functions began. Thus a different approach to the synthesis of switching circuits developed. New technology like MOS also required a different class of algorithms. Different approaches to switching theory also arose from the increase in size of the integrated circuits and systems. The decomposition of Boolean functions, the symmetry of Boolean functions, and the multiple-output problem also became of more practical than theoretical interest.[10]

With the increasingly realizable hardware devices for threshold logic and multi-valued logic, newer and more generalized mathematical tools are in order for the practical engineer and designer.

By the late 1960's, Tison's Generalization of Consensus Theory and the computer algorithms derived from it dominated the industry; however, in the 1970's the need to refine the computer algorithms directed the synthesis of switching functions into a different horizon, that of faster and more efficient computation. The algorithms developed in Auburn University by C.C. Carrol's group started to look for relationships between minterms of a Boolean function, so that the covering problem could be avoided in the algorithmized methods. This approach conceptually coincides with the notion of realizing that synthesis can be done with a Karnaugh Map by recognizing geometric patterns. McKinney's directed search algorithm and Arevalo's Four-tuples Methods are recent contributions in this direction. More detailed comparisons will be presented in the next section. A more rational approach to the synthesis of switching functions is discussed in Section 3.

Footnotes: 1.1 Historical Review

[1] Bell, E. T., The Development of Mathematics (New York: McGraw Hill Company, 1962), pp. 553-554

[2] Boole, George, An Investigation of the Law of Thought (London: 1854).

[3] Bell, pp. 553-577, has a very good account of the historical development of mathematical logics.

[4] Shannon, C. E., A Symbolic Analysis of Relay and Switching Circuits, Transcript AIEE, Vol. 57 (1938) pp. 713-723. Other people in Russia and Japan had similar concepts, noteably, P. S. Ehrenfist of Russia in 1910, and Nakashima and Hanzawa of Japan in 1938. For more historical events, refer to G. C. Moisil, The Algebraic Theory of Switching Circuits (New York: Pergamon Press, 1969), and International Series of Monograps in Pure and Applied Mathematics, Vol. 41, pp. 11-20.

[5] See Book ref. (12) which stemmed from simplifying vacuum tube flip-flop circuits, which procedure can be applied to all switching functions.

[6] See ref. Q1, Q2, Q3

[7] See ref. K1, B7

[8] See ref. B6, S4, S5, S16, S17

[9] See ref. C10, S11, S15

[10] See ref. A6, B1, C10

[11] See ref. T1

1.2 Comparison of Various Approaches

The basic concern of engineering design is to save cost and to improve performance. When we say cost, we have to consider both the software cost, hardware cost, as well as the difference in labor cost and the yield of constructing or manufacturing an item. When we say performance, we mean not only the performance of the specified parameters but also the reliability.

To increase reliability, we have to increase redundancy which is the opposite of minimizing the circuits; to improve the tolerance of faulty hardware we also have to sacrifice minimization of circuits. Therefore, in search of an optimal mathematical tool for the analysis and synthesis of switching circuits, mathematical completeness may not yield the most economical hardware, minimized hardware may not be worth the software cost of generating the design of hardware, and the most economical circuits may not have the desirable reliability.

(1) The Harvard Computation Laboratory Method[1]

Listing all the minterms and all the possibilities of the basic functions which contain them is thorough and systematic; however, it is too bulky and cumbersome to handle in any large size practical problem.

(2) The Standard Quine-McCluskey Method[2]

Using every minterm to build up a prime implicant table is also systematic and thorough, but suffers from the long process of enumeration and the very difficult problem of finding a minimum covering for the non-irredundant prime implicants.[3] Many other tabular methods[4] including R. P. Roth's cubical method and Tison's generalized consensus methods are also influenced by this method. In fact, this method presently represents the main stream of tabular or algorithmized methods.

(3) The Karnaugh and Veitch Map Method.

Using grid squares to represent minterms and the relationship between minterms has both a graphical and a algebraic advantage; however, the graphical aspect of the method so dominates the significancy of the method, that its deeper algebraic implications are not fully developed. In this paper, some of the algebraic aspects of the method will be presented in a different manner.[5]

(4) Boolean Algebraic Methods.

R. T. Nelson's method[6] of alternatively changing back and forth from products-of-sums to a-sum-of-products during the algebraic cancellation or simplification of the Boolean expression has the advantage of not needing to convert Boolean expression into minterms. But this method has little usage because of the number of variables it can handle. Nevertheless, the characteristic of not requiring expansion of the Boolean expression into minterms is certainly a desirable feature. There are other algorithms which have this property[7]

(5) Topological and Simplex Cubical Method.

Urbano, Mueller, and R. P. Roth, in a series of papers, applied the topological simularity of a n-cube and the Boolean minterms, and advocated the cubical complex methods. Subsequently, Tison extended it to a generalized consensus method.[9] At the University, of Wisconsin a series of computer programs were generated based on this method. This method contributes a better representation and notation for Boolean functions and a better visualization of the interelationship between minterms. The n-cube notation and concept is adopted by most authors in the field[10] and we owe our notation and terminology to this approach.

(6) Graphical Methods

There have been several instructive graphical methods proposed in the last twenty years, noteably T. M. Booth's Vertex-Frame Method, M.E. Arthur's Geometric Mapping Method, and G.E. Marihugh and R.E. Anderson's H - Diagram Methods.[11] The H-Diagram, especially, has also proved to be fruitful in applications to other areas.[12] In this dissertation we offer a new graphical representation. This approach illustrates the concept of graphical depiction of Boolean functions.

(7) Mechanical Devices Method

A. Swoboda's contact grids (S22), F. B. Hall's Binary Sieve (H1), and A. K. Choudbury and M. S. Basu's Mechanical Charts represent attempts to use mechanical devices to achieve the synthesis of switching functions (C5). This approach demonstrates the feasibility of constructing a slide rule like device to synthesize switching functions. Combined with the characteristic of algorithm orientation, we believe a small hand-held portable calculator is also highly feasible.

(8) Computer Algorithms

Originally, the computer programs all stemmed from computerizing the Quine-McCluskey Method. There was limited success in programming Karnaugh's Map Method. The following group of computer programs have been reported in the literature:

(a) IBM's R. P. Roth Group computer program based on R.P. Roth's Algebraic Topological approach.(R5-R11)

(b) C. C. Carrol, S. E. Jorden, W. A. Hornfeck, S. G. Shiva, and H. T. Nagle's Fast Algorithms at Auburn University.(C13), (C14)

(c) R. Slage, C. Chang, and R. C. Lee's sprouting algorithm from NIH.(C13)

(d) D. L. Dietmeyer and Y-H Su's Consensus Methods from Wisconsin University.(D3), (S16), (S17)

(e) M. H. McKinney's direct-search method algorithms at Texas A&M University.(Mc5)

(f) Z. Arevalo, J. G. Bredeson's Four-Tuble Algorithm.(A8)

(g) Harmonic Analysis to generate prime implicant.[14]

(h) Bantee, and Mott have some older computer programming which is of historical interest.

(i) Other Notable Methods

H. R. Hwa(H12), A. H. Sheinman(S9), S. K. Das and N.S. Khabra(D2), . J. Mott(M15), A. K. Cloudbury

and S. K. Das[C12], D. M. Y. Chang and T. H. Mott[C2],
Angelo R. Meo[M6], J. F. Gimpel[G3], N.N. Biswas[B11]
S. C. DeSarkar, A. K. Basu, A. K. Choudbury[D7],
A. Natapoff[N1], O. L. Ostapko and S. J. Hong[O2].

In this dissertation, computer programs are given to illustrate the new general algorithm. One general characteristic is to economize computer memory and computing time.

(1) For detail procedure refer to ref. Book 12

(2) Refer to reference Mc1, Mc2, Mc3, Mc4, or any standard logic design and switching theory text book.

(3) Many good methods of finding the minimum set are theoretically interesting but not practical. References C6, C12, C15, C2, C3, P7, P8, G3.

(4) See reference H12, S9, D2.

(5) For original paper, refer to reference K1, V2, For examples and method, refer to a textbook, e.g. reference 4, 15, 36, 55, 39.

(6) Refer to reference N4.

(7) Refer to reference S13.

(8) Refer to reference V1, R6, R7, R8, R9, R10.

(9) Refer to reference T1

(10) A good exposition of this notation and terminology can be found in reference Ref. Book 19.

(11) Refer to reference B7, A5, M3

(12) To Walch functions

(13) Refer to reference C5, S22, H1

(14) Refer to reference Book 41

1.3 Desireable Characteristics and Discussion

A comparison of the approaches in synthesisizing Boolean functions and switching circuits indicates the following characteristics to be desirable:

1. Algorithm Oriented Characteristic

Because of the increasing use of computing machines and size, any approach to the synthesis of switching circuits has to be computerizable. F. Mileto and G. Putzolu(M8),(M9) did some initial work in estimating the merit and efficiency of computer algorithms for the minimization of Boolean functions based on the computing time and storage. However, because of the different criteria and concepts used in the different approaches, it was difficult to make an accurate and fair estimation. The size of the memory and the speed of calculation varies with different problems and settings. The skill of implementation into actual programs is also important. Although it is a very tedious job to evaluate the merits of the algorithms, it is mandatory to measure their effectiveness and efficiency. In this aspect, Roth's cube-array method is an improvement over Quine-McCluskey's, and the consensus algorithm is another step forward. Each of the newer algorithms claim some order of improvement on time and storage saving.

2. Limitation of the Size Characteristic

Most of the approaches do not differ much in efficiency when the size of the problem is both small in number of variables and small in the number of elements of the on-set and don't cares. Therefore, the real measure of the effectiveness is on the size of problem that a specific approach can handle. The comparative merits have to be measured against each other with respect to a reference frame for size. The basic limitation of the map method is the difficulty of programming on the computer. Therefore it basically can be worked efficiently only with graphical methods, and the difficulty of depicting 8 variables limits its usefulness.

3. Visual Depictable Characteristic

The synthesis of switching circuits is not only to find a minimum circuit for a Boolean function, but also to observe the influence of changes of one part of the circuit on the remainder. It is not desirable to have to minimize the whole circuit every time we perturb the system. In this aspect, the map method offers the best opportunity. In fact, most of the graphical methods try to gain this freedom and at the same time increase the number of usable variables. The H-Diagram is the most useful improvement since the original Karnaugh Map and can handle more variables. The fact that H-Diagrams have been applied to the Welch function indicates that this is important, and the wide use of the Karnaugh Map in industry for smaller problems shows the desireability of this feature.

4. Algebraical Structure Characteristic

Two-valued Boolean logic being the base for other logics, and logic design being the core of digital system and switching circuits, the algebraic structure of an approach would not only enrich the switching theory of Boolean functions, but could also be useful in other areas of switching theory.
Table 1.1 shows a comparison of the major approaches mentioned in Section 1.2 with these four characteristics. In Chapter 2, we shall develop a structure so that we can maximize all the advantages of different approaches.

CHARACTERISTICS APPROACHES	CHAR. 1 (ALGORITHM)	CHAR. 2 (SIZE)	CHAR. 3 (VISUAL)	CHAR. 4 (STRUCTURE)
AKERS (A1)			X	X
AREVALO (A3)	X	X		X
ARTHUR (A5)			X	
BATNI (B4)				X
BEATSON (B5)				X
BOOTH (B7)	X			X
BOWMAN (B8)				X
BREUER (B9,10)				X
BUBENIK (B11)				X
CHANG (C2 & 3)	X	X		
CHOUDHURY (C4)			X	
CHOUDHURY (C5)			X	
CHU (C6)				X
COBHAM (C7)				X
CURTIS (C11)			X	X
DAS (D1)			X	X
DAS (D2)			X	X
DUNHAM (D6)				
GILL (G1)				X
GIMPEL (G2,G3,G4)	X	X		X
GNAZALA (G5)	X			X
GOODRICH (G6)				X
GORDON (G7)				X
HALL (H1)				X
HARRIS (H1)				X

Table 1 Comparision of Different Approaches

CHARACTERISTICS APPROACHES	CHAR.1 (ALGORITHM)	CHAR. 2 (SIZE)	CHAR. 3 (VISUAL)	CHAR.4 (STRUCTURE)
POLANSKY (P4)				X
PYNE (P7,P8)				X
QUINE (Q1,Q2,Q3)				X
REUSH (R1)				X
RHYNE (R2)	X			X
ROBINSON (R3)				X
RODIN (R4)				X
ROTH (R5,R11)	X	X		X
ROY (R12)				X
SCHKOLNIK (S2)				X
SCHNEIDER (S4,S5)				X
SHANNON (S6,S7,S8)	X			X
SHEINMAN (S9)				X
SHEN (S10)				X
SHIRA (S12)				X
SLAGLE (S13)	X			X
STOFFERS (S15)				X
SURESCHANDER (S18)				X
SVOBODA (S20,21,22)			X	X
TISON (T1)	X			X
URBANO (U1)			X	X
VANDLING (V1)	X			X
VEITCH (V2)			X	X
VINK (V3)				X
ZISSOS (Z1)				X

Table 1 (continued)

CHARACTERISTICS APPROACHES	CHAR. 1 (ALGO-RITHM)	CHAR. 2 (SIZE)	CHAR. 3 (VISUAL)	CHAR. 4 (STRUC-TURE)
HARVARD (H13)	X	X		X
HIRSCHORN (H6)				X
HONG (H7)	X			X
HOUSE (H8)	X			X
HULME (H10)			X	X
HWA (H12)				X
IKRAM (I1)				X
KARNAUGH (k1)			X	X
KARP (K3)	X			X
LAWLER (L1,L2)	X			X
LIN (L4)			X	X
LUCCIO (L6)			X	X
MC CLUSKEY (M3)		X		X
MARCOVITS (M3)				X
MARIHUGH (M3)				X
MEO (M6)				X
MORREALE (M10,M11)				X
MOTT (M14,M15)			X	X
NATAPOFF (N1)	X			X
NECULA (N2)				X
NELSON (N4,N5,N6)	X			X
OKADA (01)				X
OSTAPKO (02)				X
PARKHOMENKO (P1)				X
PETRICK (P2)				X

Table 1 (continued)

CHAPTER II

ALGEBRAIC STRUCTURE AND THEORY

2.1 THE MINIMUM MAXIMAL-CUBE COVERING

The basic notation and terminology of the switching theory in this dissertation can be found in the textbook "Mathematical Theory of Switching and Automata" by S. T. Hu, a well executed attempt to organize all the known results in switching theory into a simple branch of pure mathematics with unified notation and terminology.(1) To develop our concepts properly, we found further structure had to be constructed to facilitate the progress in some areas. However, our main interest is in the hardware used in computer and digital systems rather than in the pure mathematics. The format chosen is not of a pure mathematical symbolic-logic appraoch, nor of the engineering journal's approach, which goes directly from results to examples placing definitions and proofs in the appendix. We chose to integrate the two. In this section, we will first discuss the properties of Karnaugh maps. Some of them have been utilized by other authors in recent algorithms independent of our development.(2) In following sections, we shall construct an algebraic structure to facilitate the use of those properties.

1. Adjacency:

Quine-McCloskey's method demonstrates the importance of this property. Roth's and Tison's studies of the general consensus appraoch, expanded the use of this adjacency property,(3) and most of the later authors also recognized the importance of this characteristic.(4) Recently, H. C. Curtis's adjacency table method C11 , M. H. McKinney's directed search method, Mc8, Mc9 and Z. Arevalo's 4-tuple- Method A3 all made much use of the property. Our appraoch is to use the number of adjacent minterms to any particular minterm as one of its characteristic numbers.(5) In the graphical methods, one of the advantages is that the adjacency can be determined by visual observation. In the analytical method, however, another kind of measure must be used to determine the adjacencies. The Hamming distance is the most logical metric and thus we shall adopt it.(6) There are two parts of information pertaining to the adjacency. One is the number of cubes adjacent to a given cube; the second is the coordinates or directions of the adjacencies. In graphical methods, both are easily preserved except where there is a miscount due to human error. In our approach, we choose only to list the first part, the number of adjacent cubes, rather than identify each one of them individually, because we claim that the second part of the information need only to be generated as needed. This saves labor of enumeration considerably.

2. Tops and Bottoms

Another important characteristic is the tops and bottoms of a cube. All the conventional approaches recognize this characteristic for any k-cube that may be chosen. However, their identification is still more or less a trial-and-error method as in the graphical approaches. We recognize the importance of this characteristic and will construct our algebraic structure in such a way that we can deduce the tops and bottoms analytically. Our investigation shows that the tops and bottoms not only need the information of adjacency, but also the two new concepts that we introduced in the investigation, the degree of coverablity, and the order of coverablity for a specific minterm point in the on-set. These will be discussed in detail in the section. These concepts have been implied in some of the recent results in a different form,[S12] that is, to identify the largest cube covering for a group of on-set elements. According to Hu's terminology, this is the maximal cube covering for a specific vertex in the on-set. Consequently, the minimum number of these maximal cube-coverings consists of the minimum set of prime implicants for any Boolean function. Our approach especially emphasizes this point of finding the minimum maximal cube covering directly. At the start, it seems that we must identify the maximal cube expended from any two vertices. The difficulty arises from the lack of analytic ways to prove that it is indeed the maximal cube covering. Our effort of constructing an algebraic structure is due to this demand. After we achieved this goal, we found the algebraic structure we built has more implication to switching theory than was thought. This shall be treated in later chapters.

3. Interrelationship Between Vertices

One of the most important advantages of a graphical approach is that the detailed relationship of one particular vertex with respect to all the rest of vertices of any on-set can be seen. In our investigation, it was found that from the concept of distance matrix and distance frequency matrix for the on-set, we were able to deduce most of these relationships easily. This new concept is a process similar to working with the map without drawing it. Additionally, we can employ most of the operation of the map method analytically.

4. Relationship Between Cubes

The prime implicant table covering problem stemmed from the intrisic properties of the relationship between cubes.

The analytical approaches center their efforts in finding efficient transformation of Boolean expressions(7) while graphical approaches focus their attention on a closer inspection of the graphical representation of the cubes.(8) There seems lacking a fundamental treatment of this aspect of the cubes. The cyclic cubes, the less-than cubes has haunted most of the researchers in switching theory since the day it was born. Our first approach to it is the theory of cube-graphs and branching graphs which will be fully developed in Section 2.3. Our second attack on it is in Section 2.4, where the geometric meanings of the cube-covering in the metric space (Q^n, P) are introduced. In graphical approaches, the covering is drawn so that a minimum number of covering cubes can be found by trial-and-error, but no known theory has been developed to prove the validity of the existance theorem of a minimum set. We offer an attempt to prove the existance of such a minimum set with respect to a specific value function.

5. Beyond the Cube-Covering

We constructed the metric space (Q^n, d) out of the conveniency for synthesising Boolean switching functions. It may well be worthwhile to look for more meaningful properties of Boolean switching functions direct from this discrete space. Further work in generalizing it into other kinds of switching functions is feasible and will be discussed in Chapter V.

Applications with illustrative examples for the Branching graphs and the geometric approach will be given in Chapters III and IV.

Footnotes for 2.1

(1) Hu's book represented the kind of effort by an eminent pure mathematician we referenced in the opening statements of this dissertation (refer to Chapter I, paragraph 1.1, page 1 of this dissertation). Reference Book 19.

(2) Our initial results were submitted to IEEE Transaction on computer in December 1974. See repositore on Computer magazine. (Reference H13, H14)

(3) Adjacency is actually the basis for consensus.

(4) Refer to Reference (H12)

(5) We use column two of our characteristic matrix to represent this characteristic.

(6) We use (Q^n, d) metric space. The definitions for (Q^n, d) and adjacency refer to Chapter II, para 2.2.

(7) For example, the work of R. T. Nelson (Reference N4), M. J. Gnzala (Reference G5), F. Luccio (Reference L6), Petrick (Reference P2,P3), I.B. Byne and E. J. McCluskey (References P7, P8) and J. F. Gimpel (Reference G3).

(8) Our deep conviction that the process of finding an optimal covering by inspection on Karnaugh Map must be proveable led us to devise a way of gaging the distance between two cubes and then to determine how close they really are.

2.2. Algebric Structure of Minimum Maximal-Cube Covering Approach

Definition 1: Q^n is the cartesian product of Q sets where $Q = \{0.1\}$. Q^n is called a n-cube.

Definition 2: (Q^n,d) is defined as a metric space with metric function d being defined as follows: If $x = (x_1, x_2, \ldots x_n)$, and $y=(y_1,y_2,\ldots y_n)$ x &y ϵ Q^n, then: $d(x,y) = \sum_{i=1}^{n} |x_i-y_i|$ is the distance between x and y. (The Hamming distance = the number of positions in which x and y differ)

Definition 2 and the definitions and theorems derived from it will suffice to render all the necessary information of a Karnaugh map. Although the main application of this approach is for the minimization of AND-OR (NAND) logic circuits. It can be shown that the theory developed in chapter 2 can be applied in many other areas.

Definition 3: Let $F= \{a_1,a_2, \ldots a_m\}$ be the on-set of a switching function F. The m x m matrix$\{A=[a_{ij}\}]$ is defined as $a_{ij}=d(a_i,a_j)$. The i-th row of A will be denoted by $d(a_i,F)$. The m x m matrix $B= [b_{ij}]$the distance frequency distribution matrix of F, that is, b_{ij} is equal to the number of points in F that have distance j to a_i. We denote by $R(a_i)$ the i-th row of B and by $C(j)$ the j^{th}column of B.

Definition 4: $Q^* = \{1, 0, *\}$, where * is the sign of don't care. If N= {1,2,...n} is the first N positive integer, then $\phi : N \rightarrow Q^*$. The coordinate function defines all the coordinates of a k-cube in Q^n. Furthermore d in (k)= the dimension of a k-cube = the cardinality of the subset $\phi^{-1}(*)$ of N. and Cod(k)=n- dim(k) = the cardinality of the subset $\phi^{-1}(Q)$ of N. [2]

Definition 5: A cube is covered by B cube if $A \subseteq B$.

Definition 6: A maximal cube is defined as the largest cube covering a subset.

When A cube in definition 5 reduces to a 0-cube, and B cube is taken from the union of on-set and don't care set, the definitions 5 and 6 are equivalent to the prime implicants in the Quine-McCluskey's minimization technique.

Example 2.2.1: $f(x) = \Sigma\ (0,2,3,4,5,6.7,13)$ with on-set F=(0,2,3,4,5,6,7,13) then: d(0.2) = 1, d(0,3)=2. etc.

Thus Matrix A and Matrix B for F are obtained in Figures 2.1 and 2.2 respectively. Figure 2.3 is the corresponding Karnaugh map.

We notice from the Karnaugh Map in Figure 2.3 that Vertex 5 has 3 adjacent Vertices, 4, 7 and 13, and they are on the left hand, right hand and above Vertex 5 respectively. Similarly, from Row Five and Column 2 of Matrix B we know Vertex 5 has 3 adjacent vertices, and from Row 5 of

$$\begin{array}{c|cccccccc}
 & 0 & 2 & 3 & 4 & 5 & 6 & 7 & 13 \\
\hline
0 & 0 & 1 & 2 & 1 & 2 & 2 & 3 & 3 \\
2 & 1 & 0 & 1 & 2 & 3 & 1 & 2 & 4 \\
3 & 2 & 1 & 0 & 3 & 2 & 2 & 1 & 3 \\
4 & 1 & 2 & 3 & 0 & 1 & 1 & 2 & 2 \\
5 & 2 & 3 & 2 & 1 & 0 & 2 & 1 & 1 \\
6 & 2 & 1 & 2 & 1 & 2 & 0 & 1 & 3 \\
7 & 3 & 2 & 1 & 2 & 1 & 1 & 0 & 2 \\
13 & 3 & 4 & 3 & 2 & 1 & 3 & 2 & 0
\end{array}$$

Figure 2.1 Matrix A

Distance Matrix for Example 2.2.1.

$$\begin{array}{cc} \begin{array}{ccccc} P_o & P_1 & P_2 & P_3 & P_4 \end{array} & \\ \left(\begin{array}{ccccc} 1 & 2 & 3 & 2 & 0 \\ 1 & 3 & 2 & 1 & 1 \\ 1 & 2 & 3 & 2 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 1 & 2 & 3 & 1 \end{array}\right) & \begin{array}{c} 0 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 13 \end{array} \end{array}$$

Figure 2.2 Matrix B

Distance Matrix for Example 2.2.1.

8	9	11	10
12	13 1	15	14
4 1	5 1	7 1	6 1
0 1	1	3 1	2 1

FIGURE 2.3. Karnaugh Map for Example 2.2.1.

Matrix A we know exactly they are 4, 7 and 13, because Column 4, 7 and 13 of Row 5 have values of 1. It is not known whether they are on its left hand side or right hand side. To find this, we have to compute them from 5 = (0101), 4 = (0100), 7 = (0111) and 13 = (1101), 4 differs at the fourth coordinate, 7 at the 3rd, and 13 at the 1st respectively. the relationship of 5 to any other vertices can also be derived similarly.

Definition 7: $\langle a,b\rangle_k$ is defined as the k-cube having a and b as its diametrical extreme points. It is also called a k-cube spanned by a and b.

Theorem 1: Let $a \varepsilon F$ and K F be a k-cube covering a. then there exists a unique point $b \varepsilon k$ such that $d(a,b)=k$ and $K-\langle a,b\rangle_k = \{x \varepsilon F: \ d(a,s) + d(b,x)=k\}$

Proof: 1) Let b be the point in k obtained from a by complementing the varying coordinates of a. then $d(a,b)=k$ and no other point in k can be at a distance k from a!

2) a: clearly $\langle a,b\rangle_k = \{x \varepsilon F: d(a,x) + d(b,x)=k\}$

b: In computing the integer $d(a,x)+d(b,x)$ each of the k positions where a and b differ makes a contribution of 1 to this number. Therefore if $d(a,x)+d(b,x)=k$, x must agree with a and b in the constant coordinate positions. Thus $\chi \varepsilon \langle a,b\rangle_k$.

Corollary 1 $\# \{\chi\varepsilon F: d(z,x)+d(b,x)\} = 2^k$ (trivial)

Corollary 2: If a is a point in a k-cube in F, then the sum of the distance of x to any pair of diametrical extreme points is k.

Corollary 3: If a point $a \varepsilon F$ is covered by a k-cube in F, then there exists a k-cube in F with a as a diametrical extreme point.

Now we can examine example 2.2.1 again. Suppose the vertex 5 is chosen, d(5,F)= (2,3,2,1,0,2,1,1). Since #(F) $=8=2^3$ and since at least one of the points (13), is not 3-cube coverable, we consider a 2-cube covering. Let's look for a 2-cube in F of the form $<5.b>_2$ that will cover 5. By theorem 1, we consider vertices that are 2-distance away. In this case, vertices 0,3. and 6 are all candidates for b. By corollary 1 of theorem 1, we need $2^2=4$

where b=0,3,or 6. We find d(5,F)+d(0.F) = (2,4,4,2,2, 4,4,4). There are only three 2's, less than $2^k=2^3=4$. Therefore, we know $<5.0>_2$ is not the 2-cube we are seeking nor is $<5,3>_2$. However, $<5,6>_2$ does satisfy the requirements. The only 2-cube existing is $<5,6>_2$. Thus, the cubes can be found without the use of the Karnaugh maps. We trade off the immediate identifiable missing vertices of a cube to gain the advantage of algorithmizing the process.

<u>Theorem 2</u>: (Q^n,d) and Distance Matrix A are equivalent to the Karnaugh Map with on-set F.

Proof: 1) By definition 2. matrix A gives us all the information about adjacency between elements of an on-set F.

2) By theorem 1, we are able to identify all the cubes on the map.

3) The physical relation of the map can be deduced from matrix A, and the n-cube coordinates representation.

from 1) to 3), we conclude that we can generate the same kind of information from (Q^n,d) and matrix A as the K-map. Therefore, they are equvivalent. Q.E.D.

The minimization of Boolean functions will be demonstrated. Instead of using visual inspection, we can now use deduction. The disadvantage of this approach is that m^2 units of storage is necessary to replace n squares of the map. Although matrix A is symmetrical, it still is necessary to use $m^2/2$ storage, where m is the number of minterms, and n is number of variables. However, in practice the memory is not the only factor, the calculation time is also important and matrix A does not have to be random accessable, it can be disc file .

In the next section, the theory for the synthesis of switching functions will be developed. In Section 2.4, the geometric interpretation and additional theory for more efficient mechanization of the algorithms will be developed. Chapters III and IV will implement the algorithms for the results developed in Sections 2.3 and 2.4 respectively. Chapter V will extend theory into other areas.

Definition 8: If there exists a cube that covers a point in the on-set F of dimension j, this point is said to be j-coverable by a j-cube. If the point can be covered by an i number of j-cubes, then it is j-coverable of i-th order.

Definition 9: The subset F_j^i of F is the set of all the points in F that are j-coverable of order i.

Lemma 1: If a point is j-coverable, it is automatically (j-1) - coverable. $(j \geq 1)$

Proof: If a is j-coverable, there exists a j-cube $\langle a,b\rangle_j$ which covers a. Let c be a point in $\langle a,b\rangle_j$ such that $d(a,c) = j-1$. Since $(c\langle a,b\rangle_j)$ C's coordinates differ from a at j-1 places while the remaining place it differs from b, there $\langle a,c\rangle_{j-1}\ C\langle a,b\rangle_j$: $\langle a,c\rangle_{j-1}$ is a cube covering for a. Therefore, a is j-1 coverable. Q.E.D.

Theorem 3: Let $\ell_{ij} = \operatorname*{Min}_{k=o}^{k=j} \left(\frac{b_{ik}}{\binom{j}{k}}\right)$. Then ℓ_{ij} is the maximal number of j-cubes that may exist to cover the point a_i of the on-set F which is represented by the i-th row in matrix A and matrix B. The matrix $L = [\ell_{ij}]$ represents the upper limit of the number of all maximal cubes that cover the points in the on-set F.

Proof: Since a cube is always symmetrical, the distance function is preserved under any rotation. Without loss of generality, the point I can be considered as the origin of the cube. If the cube is of dimension j, then by calculating the distance from every point in the cube to the origin and determining the number of points with equal distance, a distance frequency distribution pattern similar to the coefficient of binomial expansion (or Pascal triangle) is found. Thus, if there is a j-cube that covers the point I, then the number b_{ik} must be greater than or equal to the corresponding binomial coefficient $\binom{j}{k}$ L3, and it is obvious that the most it can have is $\left(\frac{b_{jk}}{\binom{j}{k}}\right)$ of them.

Therefore, for the range, $k = o$ to $k = j$, the smallest number represents the maximum number of j-cubes that may exist to cover a_i. Q.E.D.

Lemma 2: If a j-cube covering exists for point a_i then the number of points in F with $\ell_{ij} \geq 1$ is at least 2^j.

Proof: Since it taken 2^j points to form a cube, there must be 2^j points in the on-set F satisfying this condition.

Q.E.D.

Theorem 4: $<a_i a_j>_i$ is a cube of dimension j that cover a_i and a_j if and only if, the i-th row added to the j-th row in the distance matrix A produces exactly 2^j entries of j's in the resultant row. Furthermore, these 2^j entries correspond to the points contained in $<a_i,a_j>_j$ and $a_i a_j$ are the diametrical extremes.

Proof: 1. "If part":

Since there are exactly 2^j points which have the property of $d(a_i,a_k) + d(a_k,a_j) = j$, it suffices to prove that any point k having this property is contained in the j-cube, $<a_i,a_j>_j$. For any point ^{a}k having this property, ^{a}k differs from ^{a}i or ^{a}j by $d(a_i,a_k)$ or $d(a_k,a_j)$ in the corresponding coordinates respectively. These places are all disjoint since any overlapping would imply that the distance between ^{a}i and ^{a}j is less than j, contradicting the fact that ^{a}i and ^{a}j are exactly the j-places in which ^{a}i and ^{a}j differ. Therefore, a_k differs from ^{a}j at exactly those $d(a_i,a_k)$ places where a_i and a_j differ, and a_k differs from a_j at exactly those $d(a_i,a_j)$ places where the remaining ^{a}i and ^{a}j differ. Thus, ^{a}k is one of the points belonging to the j-cube with ^{a}i and ^{a}j as the diametrical extremes and $d(a_i,a_k) + d(a_j,a_k) = j$. Since there are 2^j

of them, it must contain every point of the j-cube, $<a_i,a_j>_j$.

2. <u>"only if part"</u>:

If $<a_i,a_j>_j$ is a j-cube covering $^a i,^a j$, then (a) $d(^a i,^a j) = j$, and (b) there are 2^j points in $<^a i,^a j>$. including $^a i,^a j$. Now, if any point $^a k \varepsilon <^a i,^a j>_j$, differs from $^a i$ in x places, then $d(a_i,a_k) = x$; if $^a k$ differs from $^a j$ in y places, then $d(a_k,a_j) = y$. $^a k \varepsilon <^a i,^a j>$, $^a k$ can not differ from $^a i$ or $^a j$ other than where $^a i$ and $^a j$ differ. Otherwise $^a k$ is not in $<a_i,a_j>_j$. $\therefore x+y = j = d(a_i,a_j)$, $..d(a_i a_k) = j$ and there are 2^j points in $<I,J>$. Thus, there must be exactly 2^jentries of j's in the resultant row formed by adding the rows a_i and a_j.

<u>Corollary</u>: Given $^a i,^a j$ and $^a k$ of the on-set F: If $d(a_i,a_k) + d(a_k,a_j) = d(a_i,a_j) = j$, and if there exists any cube-covering that includes $^a i$, $^a j$ and $^a k$, then the smallest cube is of dimension j.

<u>Proof</u>: If there is a covering, then $d(a_i,a_k) + d(a_k,a_j) =d(a_i,a_j) = j$ implies that the cube I,J covering K is of dimension j. Any cube smaller than dimension j cannot cover I and J. Therefore $<a_i,a_j>$ of j dimension is the smallest cube that covers $^a i,^a j$ and $^a k$.

<u>Definition 10</u>: A maximal cube which has to be included in any covering is essential and is called a core cube.

Lemma 3: Any maximal cube that can be included in the union of essential maximal cubes is redundant (trivial).

Definition 11: A maximal cube of a cover for a cube that can be included in the union of lower dimensioned cubes is a less than cube, thus redundant.

Definition 12: A point is said to be a j-coverable core point, if, and only if, it has one and only one maximal cube covering of dimension j, with all smaller cube covering included.

Theorem 5: If x is a j-coverable core point, then there are exactly j points in the on-set having distance of 1 from x.

Proof: Trivial (By definition 12)

The inverse of this theorem is not necessarily true.

Footnotes for 2.2

1. Reference book 18, 19.
2. Reference book 19, page 63-65

2.3 CUBE GRAPH AND BRANCHING GRAPH

To facilitate the analysis and synthesis, the concept of j-coverable is introduced so that the on-set F of a switching function can be divided into disjoint subsets of j-coverables. To further improve the ability to identify and select the cyclic cubes and the less-than cubes, the concept of a candidate for a core point, and the core point as well as the cube graph and a branching graph are used.

Definition 13: Two points are connected, if and only if they are covered by a common point.

Definition 14: Two cubes are connected, if and only if they both cover at least one common point.

Definition 15: A j-coverable branching graph is a linear graph representing all the connected j-coverable points.

The nodes in a j-coverable branching graph represent the diametrical extremes of the connecting cubes and the branching representing the connecting cubes.

The initial node can be any point in any set of connected j-coverable points. The next nodes are those points forming a connecting j-cube with the present node such that the diametrical extremes are the present node and the next node.

The terminal nodes are: (1) the nodes that have only one branch connected to them; (2) the nodes whose remaining connecting branches going to the new nodes consist of cubes already represented by existing branches; (3) the nodes belonging to don't care or higher coverable sets.

Definition 16: A j-coverable cube-graph is a linear graph representing the j-cubes covering the connected j-coverable points.

Definition 17: Two j-coverable branching graphs are said to be equivalent if and only if they have the same j-coverable cube-graph.

Lemma 4: For every j-coverable branching-graph, there is one unique j-coverable cube-graph.

Proof: Let G be a j-coverable branching graph with C and C' being cube-graphs of G. If C is not coincident with C', then there is at least one of the cubes which is different. Since every branch in a branching graph represents a distinct cube, the C must coincide with C'.

Lemma 5: A set of connected j-coverable points has a unique cube-graph.

Proof: Let F_j be a set of connected j-coverable points with C and C' being the cube graphs of F_j. Let cube $c \in C$ and $c \notin C'$. There is then at least one point x in c that is not in any cube of C'. Since x is not in any of the cubes in C', x is not connected to F_j. This contradicts the fact that x is a member of F_j.

Theorem 6: A set of connected j-coverable points must have 2^{j-1} different but equivalent branching graphs.

Proof : Since there are 2^{j-1} different pairs of diametrical extremes to a j-cube, there are 2^{j-1} different pairs of end nodes for each branch of a branching graph. When joined, there are 2^{j-1} distinct branching graphs. By Lemma 4 and Lemma 5, all are equivalent.

Theorem 7 : The minimum set of non-redundant maximal cubes is the union of minimum covering for the branching graph.

Proof: Since each branch of a branching graph represents a distinct maximal cube, the union of the minimum covering for each branching graph of a j-coverable subset ($j = 0$ to k) is the minimum set of non-redundant maximal cubes.

This theorem transforms the process of finding a minimum covering for a table of prime implicants into the finding of a minimum covering for each branching graph of a j-coverable subset of the on-set, taking the union over $j = 0$ to k, the highest dimension of the maximal cubes.

The advantages are: 1) No redundant maximal cubes are generated; 2) The on-set is paritioned into subsets of j-coverable connected points so that the covering problem can be treated separately for different subsets; and 3) The branching graphs are easier to work with.

The following theorems relate to the covering of branching graphs.

Theorem 8 : A minimum covering of a branching graph is the sum of the least number of branches needed to cover every circuit in the branching graph.

Proof: Since a branching graph represents the maximal cubes covering connected j-coverable points, and each loop in the branching graph represents a set of cyclic maximal cubes, therefore the sum of the least number of branches needed to cover each loop is the minimum covering of a branching graph. An open loop is a set of incomplete cyclic maximal cubes.

Theorem 9 : The least number of branches needed to cover a loop of n branches is $\frac{n}{2}$ if n is even or is ($\frac{n}{2}$ + 1) if n is odd.

Proof: Let the loop have n branches, then there are n nodes. Therefore, at least one half of these branches is needed to cover all those nodes because a loop is a closed plane curve and, within a loop, each node has exactly two branches connected to it. If n is odd, since a half-branch can not be used, an additional branch must be used.

Corollary 1: If the number of branches in a loop is even, the minimal covering of the loop is found by choosing every other branch.

Proof: Since in any loop, a cube represented by a branch can be divided into two parts: 1) the common part of this cube and the cube represented by the preceding branch; 2) The other is the common part of this cube and the cube represented by the following branch. Therefore if the preceding and the following branches are chosen, then the cube represented by this branch is covered and thus is redundant. Q.E.D.

Corollary 2: If the number of branches in a loop is odd, then except for the starting or finishing of the covering which has to be two consecutive branches, every other branch is the minimum covering.

Proof: Since the number of branches is odd, every other branch can not be chosen to cover a loop. There must be one part of a cube covered by two consecutive branches. Thereafter, it is the same as Corollary 1.

Theorem 10: If there are terminal nodes in a branching graph of the second kind, then these nodes have to be treated first for the elimination of the higher-dimensioned less-than cube.

Proof: If a terminal node is of the third kind, then it must be a higher-dimensioned cube coverable point. Since it is a terminal node of a branching graph, it has only one branch connected to it and that branch is necessary unless it is a less-than cube of some lower dimensioned cubes. Therefore, the higher dimensioned cube which also covers this node may not be necessary, if the rest of the cube is covered. Thus, the set of all terminal nodes of the second kind must be tested for redundancy of the higher dimensioned maximal cubes in a higher coverable branching graph.

Theorem11: If a terminal node of a branching graph is of the first kind, then it is a core point, and the only branch connected to it represents an essential maximal cube.

Proof: Since a terminal node of the first kind by definition has one and only one maximal cube covering it and since no higher cube covers it, therefore it meets the definition of a core point. The only branch connected to the node represents the maximal cube mentioned above. Therefore, it is essential.

Theorem12: The more loops sharing a branch, the higher is the priority for choosing this branch.

Proof: The more loops sharing a branch, the more this branch can be used to cover different loops. Since this branch represents a partial covering for each loop, a smaller number of branches is needed. Q.E.D.

If there is more than one branch connected to a node, and no branch is shared by two or more loops, then the priority for choosing a branch for this node is inversely proportional to the number of branches connected to the node.

Definition 18: If two branching graphs of different dimensions are linked such that the branches connected to the linking node in the higher dimensioned branching graph can be omitted in a minimum covering, the linkage is inseparable.

2.4 Geometric Structure of Maximal-Cube Covering

Since the use of Venn Diagrams, graphic depicting and geometric representation has been widely used in minimization of switching functions. In the author's earlier work, the geometric structure of the Karnaugh-Map and generalized finite discrete point representation of Boolean functions had been discussed.[H14,H15] The basic aim of using the geometric structure to represent functions is to seek visual recognition of the complex interrelationship. The analysis by the geometric structure of Maximal-cube covering approach is different than the graphical means.

(a) Diametric Extremes

The importance of using diametric extreme terminology is that cubes can be treated as if they were circles or spheres. Thus the drawing of circles tangent to each other would be a way of finding the minimum covering. Another advantage of this terminology is that the sense of interior points, boundary points, and exterior points can be introduced. In the case of Maximal-cubes of Boolean function, there are boundary points and exterior points, but no interior points. If x, y are a pair of diametric extremes of cube <x, y>, then any point z is either on the boundary of the cube or outside the cube.

Another feature of the diametric extreme is that the absolute origin of corrdinates is not needed, because any pair of diametric extremes can be used as the reference.

(b) Awayness

The major advantage of graphical methods is that the distribution of cubes (a minterm point can be treated as 0-cube) can be visually observed. However, pattern recognition is always needed. The purpose of introducing the concept of awayness and aloofness in the maximal-cube covering approach is to simulate the kind of pattern recognition used in the map method.

To arrive at a numerical measure of the awayness or closeness of a cube to the existing cube covering, the concept of distance function is expanded.

Definition 9 : A k-cube is re-defined as a pseudo-circle with diameter of k distance units.

Definition 20: The diameter of a pseudo-circle is the longest span of a pseudo-circle[equal to the dimension of a cube].

Definition 21: The two end-points of a diameter of a pseudo-circle is called the pair of diametric extreme points, or the diametric opposite points.

Definition 22: d(x, A) is defined as the distance from vertex x to any pseudo-circle A (cube A). $d(x, A) = \mathrm{Min}\ (d(x, a): a \varepsilon A)$

Definition 23: D(A) is defined as the distance units of diameters of pseudo-circle A (cube A) $D(A) = \mathrm{Max}\, \{d(a_1, a_2): a_1, a_2 \varepsilon A\}$

Definition 24: d(A, B) is defined as the distance between two cubes A and B.

$d(A,B) = \mathrm{Min}\, \{d(a_1, a_2) : a_1, \varepsilon A, a_2 \varepsilon B\}$

Theorem 13: The distance of a point x, the exterior point of a pseudo-circle (cube) to the pseudo-circle is the number of changes of coordinates between x and any point inside the pseudo-circle excluding those places where the points of the pseudo-circle vary their coordinates among themselves.

Proof: By Definition 18, $d(x, A) = \mathrm{Min}\, \{d(s, a) : a \varepsilon A\}$

Let $L = m + n = d(s, a)$ for all $a \varepsilon A$.

m = the number of changes of coordinates between x and a at those places where the pseudo-circle (or cube) A varying coordinates among its points.

n = the number of changes of coordinates between x and a, where the coordinates for all points in the pseudo-circle (or cube) A are invariant.

Then n is constant for all points in A. Only m varies.

Therefore: $d(x,A) = \mathrm{Min}\, \{d(x, a) : a \varepsilon A\}$

$$= \underset{m = 0}{\overset{m = k}{\mathrm{Min}}}\, (m + n) = n.$$

Q.E.D.

Corollary 1: The distance from interior points of pseudo-circle to the pseudo-circle is zero.

Proof: As in the proof of Theorem 13, when $n = 0$.

Definition 25: The smallest cube or pseudo-circle having a point of a cube A (or pseudo-circle A) and any point x is defined as the connecting cube (or pseudo-circle) between A and x.

Definition 26: The smallest pseudo-circle (or cube) having a pair of diametric extreme points where one is from pseudo-circle A (or cube A) and the other is from pseudo-circle B (or cube B) is called the connecting pseudo-circle (or cube) between pseudo-circle A (or cube A) and pseudo-circle B (or cube B).

Theorem 14: $d(x, A)$ = the dimension of the connecting cube between x and A,

$d(A, B)$ = The dimension of the connecting cube between cube A and cube B. (Since pseudo-circle = cube, we will use cube from now on.)

Definition 27: The foot of a connecting cube between a cube A and a point χ is a point in A such that $C = <\chi,a>$

Theorem 15: The distance between a point to a cube is equal to the distance between the point to the foot of the connecting cube.

Proof: Let a be the foot of the connecting cube for point x to cube A. By Definition 27, d(a, x) = dimension of connecting cube. By Definition 25, $d(a, x) = \text{Min}\,\{d(x, y) : y \varepsilon A\} = d\ (\dot{x}, A)$ Q.E.D.

Theorem 16: If a and b are a pair of diametric extremes, then all the places of the coordinates where the points of the cube vary their coordinates among themselves are complementary.

Proof: Let <a, b> be a cube of k dimension, then there are k places among the coordinates where the points of the cube vary among themselves. Since d(a, b) = k and only k place can vary, therefore a and b have to be complementary at these places. Q.E.D.

Lemma 6: If a_1, a_2 are diametric extremes of a k-cube, then $d(a_1, a_2) = D(A) = k$ = dimension of the cube = the diameter of the sphere or circle. (Trivial)

Theorem 17: If a_1, a_2 are diametric extremes of a k-cube and $x \varepsilon Q^n$, then

(1) $d(a_1, x) + d(a_2, x) = d(a_1, a_2) = k \rightarrow x \varepsilon \ <a_1, a_2>_k$.

(2) $d(a_1\ x) + d(a_2, x) < d(a_1, a_2) \rightarrow$ x is outside of $<a_1\ a_2>_k$

(3) $d(a_1, x) + d(a_2, x) < \ d(a_1, a_2)$ does not exist.

Proof: . (d, Q^n) is a metric space, implies

$$d(a_1, x) + d(a_2, x) \geq d(a_1, a_2).$$

by Theorem 1, Corollary 1 (1) holds.

(2) implies that point x has some coordinates differing from a_1, or a_2 which are not among the places a_1 and a_2 are different. Therefore x is not included in $<a_1, a_2>$. Thus x is outside of $<a_1, a_2>$. Q.E.D.

Theorem 18: If $A = <a_1, a_2>_k$, then for any $\chi \varepsilon Q^n$

$d(a_1, x) + d(a_2, x) - k = 2d$

where $d = d(x, A)$ = the distance from x to A.

Proof: If $\chi=\phi$ (Trivial)

If $\chi\neq\phi$, $d(a_1, x) + d(a_2, X) \geq d(a_1, a_2) = k$

(by definition of metric space)

1) If $\chi \varepsilon <a_1, a_2>_k$ then $d(\chi , A) = 0$

By Theorem 1, Corollary 1;

$d(a_1, x) + d(a_2, x) = k$

$d(a_1, x) + d(a_2, x) - k = 0 = d(X,A)$

2) If $\chi\varepsilon < a_1, a_2>_k$, then

By Theorem 13;

$d(a_1,x) = M_1 + d(X,A)$ ....(1)

$d(a_2,x) = M_2 + d(X,A)$ ....(2)

Where $d(x,A)$ = The number of coordinate positions where x differs from A, among the subset of coordinate positions where A does not vary its coordinates among its elements.

M_1 = The number of coordinate position x differ from a_1, among the subset of coordinate positions where A varying its coordinates.

M_2 = The number of coordinate position x differ from a_2, among the subset of coordinate positions where A varying its coordinates.

Since a_1 and a_2 are complementary at the varying coordinate positions, M_1 and M_2 must also be complementary. Thus $M_1 + M_2 = k$.

(1)+(2) resulted in

$$d(a_1, x) + d(a_2, x) = M_1 + M_2 + 2d(X,A)$$

$$d(a_1,x) + d(a_2, x) - k = 2d.$$

Q.E.D.

Since $<a_1, a_2>$ is an arbitary pair of diametrical extremes, and χ is any point in Q^n, therefore, this is true for any pair of diametrical extremes.

Thus, $d(a_1, x) + d(a_2, x) - k$ is an invariant.

Corollary 1: For all $\chi \varepsilon Q^n$, $a \varepsilon A$.

$$d\ (x_1\ a) = M + d(x, A)$$

Where M is the number of positions x differ from a, among those coordinate positions A is varying. and d(x,A) is the distance of x to A.

Proof: See main theorem.

Corollary 2: For all $\chi\varepsilon\ Q^n$, $a_1 \varepsilon A$, $a_2 \varepsilon A$. if $\{d(a_1,x) - d(x,A)\} + \{d(a_2,x) - d(x,A)\} = k$. a_1, a_2 form a pair of diametrical extreme points of A.

Proof: See theorem 18 (gives an invariant which is going to be useful in mimization. It will be called awayness.)

Definition 28: Awayness is defined as the distance of a point to cube, $W_{ab}(s) = d(a,x) + d(b,x) - k = 2d$ Definition 28 can also be extended to the distance between two cubes A, B.

Definition 29: Awayness of two cubes A, B is defined as

$W_A(B) = d(a_1, B) + d(a_2, B) - k_A + \#(R)$

where k_A is the dimension of cube A, and R is the subset of coordinate positions when both A, and B are varying its coordinates. We will define #(R) as the degree of skewness.

Definition 30: The skewness is defined as the number of coordinate positions both cube A and cube B are varying. We will denote it by $SK_A(B) = \#(R) = SK_B(A) = SK(AB)$

Theorem 19: Awayness is independent of drametric extremes

Proof: (1) If one of the cube is a 0-cube then theorem 19 reduces to theorem 18.

(2) If cube A and cube B are not 0-cubes, then:

$$W_A(B) = d(a_1, B) + d(a_2,B) - K_A = 2d - \#(R)$$
$$= d(b_1,A) + d(b_2,A) - K_B = W_B(A)$$

where $\langle a_1, a_2 \rangle_{KA} = A$, $\langle b_1, b_2 \rangle_{KB} = B$.

d is the distance between A and B. #(R) is the number of common parts in the varying coordinates of cubes A, B.

There are four different categories of coordinate positions for cubes A,B denoted as follows.

p = those positions only cube A varying its coordinates;

Q = those positions only cube B varying its coordinates:

R = both A and B varying its coordinates;

S = neither A or B varying its coordinates

It is obvious $\#(P+R) = K_A$

$\#(Q+R) = K_B$

$\#(P+Q+R+S) = n$, the dimension of Q^n.

Let $d(x,y) = \{x \in A, \ y \in B\}$.

Then $d(x,y) = M_p + M_q + m_r + M_s$

where M_p = The number of the positions X and y differ in values among P.

M_q = The number of the positions x and y differ in values among Q.

M_r = The number of the positions x and y differ in values among R.

M_s = The number of the positions x and y differ in values among S.

We observe the following facts.

M_s = Constant, because both cubes A, B are not varying.

M_p varies from 0 to $K_A - \#(R)$

M_q varies from 0 to $K_B - \#(R)$

M_r varies from 0 to $\#(R)$

Thus $d\ (a,b) = d(A,B) = \min d(x,y)$

$= M_s$

That is, the closest points a, b in A and B only differ in those places neither A and B are varying. If we complement a, and b in A and B, we obtain c in A, and d in B. and by definition $\langle c,a\rangle_{KA} = A$, $\langle b,d\rangle_{KB} = B$. Furthermore $d(c,d) = K_A+K_B-2A(R)+M_s$, because in the process of complementing a, and b,we change only those P and R values for a, and Q and R values for b and none in S. Since we are simultaneously complementing R for a and b, there is no difference in the resultant c, and d. Therefore none contribute to distance number; while in S subset; c and d differ the same as a and b, thus contribute M_s. However in the P and Q subsets, c and d differ #P and #Q as against no difference for a and b. Totalizing all the differences,

$d(c,d) = \#(P) + \#(Q)+M_s$ .. $d(c,d) = K_A-\#(R)+K_B-\#(R)+M_s$

$= K_A+K_B - 2\ \#(R)+M_s$

(These are the most distance points among A and B.)

i.e. $d(c.d)=\text{Max}\ \{d(x,y)\ |\ \chi\varepsilon A,\ y\varepsilon B\}$

$d(a,b) = M_s=\text{Min}\ d(x\ y)$ . . . . . (1)

$d(c,d)=K_A+K_B - 2\#(R) + M_s$ . . . . (2)

for any pair of diametric extremes a_1, a_2 of A.

Thus, $d(a_1,b)+d(a_2,b) = K_A+2d(b\ B)$ . . . . . (3)

$d(a_1,d) + d(a_2,d) = K_A + 2d(d,B)$ . . . . (4)

Since $2d(d_1B) = d(c_1b) + d(a_1b) - K_A$. . . . . (5)

$2d(d_1B) = d(c_1d) + d(a_1d) - K_A$ . . . . . (6)

and Since c, and d were obtained from a, and b,

$d(c,b) = \#(P) + \#(R) + M_s = K_A + M_s$ . . . . . (7)

$d(d,a) = \#(Q) + \#(R) + M_s = K_B + M_s$ . . . . . (8)

Substituting (1), (2), (7), (8) into (5), (6)

$2\ d(b,B) = K_A + M_s - K_A = 2\ M_s$ . . . . . (9)

$2\ d(d,B) = K_A+K_B - 2\ \#(R) + M_s+K_B + M_s - K_A$

$= 2\ [K_B+M_s - \#(R)]$ . . . . . (10)

Substituting (9) and (10) into (3), and (4).

$d(a_1,b) + d(a_2,b) = K_A + 2\ M_s$ . . . . . (11)

$d(a_1,d) + d(a_2,d) = K_A + 2\ [K_B+M_s - \#(R)]$ (12)

add (11) (12) and rearrange the left hand side to obtain.

$d(a_1,b) + d(a_1, d) + d(a_2,b) + d(a_2,d)$

$= K_B + 2d(a_1,B) + K_B + 2d(a_2,B)$

$= 2\ [K_B + d(a_1,B) + d(a_2, B)]$

While the right hand side is

$2[K_A + K_B + 2\ M_s - \#(R)]$

$\therefore K_B + d\ (a_1,B) + d(a_2, B)$

$= K_A + K_B + 2\ M_s - \#(R)$

$\therefore d(a_1,B) + d(a_2,B) - K_A$

$= 2M_s - \#(R) = 2d(a_1b) - \#(R)$

$= 2d - \#(R)$

Similarily $W_B(A) = 2d - \#(R)$.

Q.E.D.

Remark: If one of the cubes reduces to a point (i.e, 0-cube), then $\#(R) = 0$. Otherwise, the 0-cube becomes a higher dimension cubes. And from theorem 19, a new parameter appears, that is the $\#(R)$ which gives the relative direction of two cubes, which can be observed on the map. Theorem 18, 19 are analogous to plane geometry theorems shown in Figure 2.4.1, and Figure 2.4.2. Other examples and demonstrative figures are given at the end of this section and in Chapters IV and V.

For Figure 2.4 We have:

$\overline{Bx}.\overline{Ax} = \overline{Dx}.\overline{Cx}$

$= (D+d) \quad (d)$

$= Dd + d^2 =$ invariant

for any $\overline{DCx}$ line

through x which

intersects the

circle

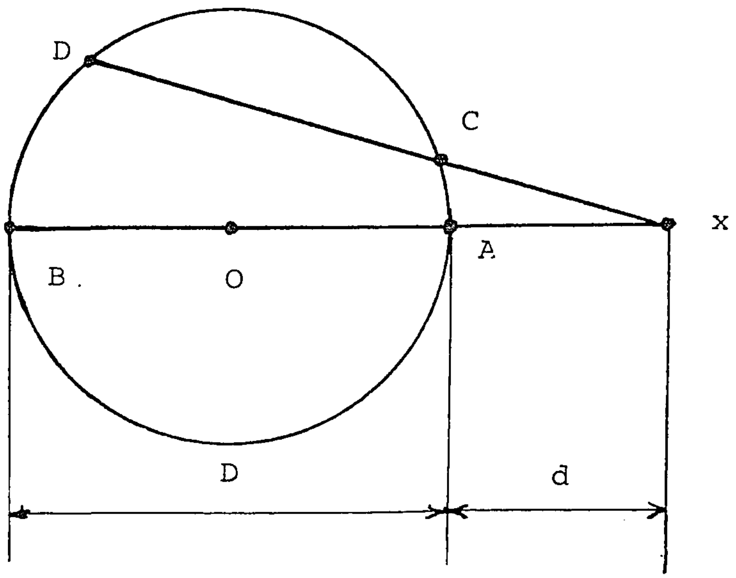


Figure 2.4 The Analogy of the Awayness of a Point to a Cube in Plane Geometry

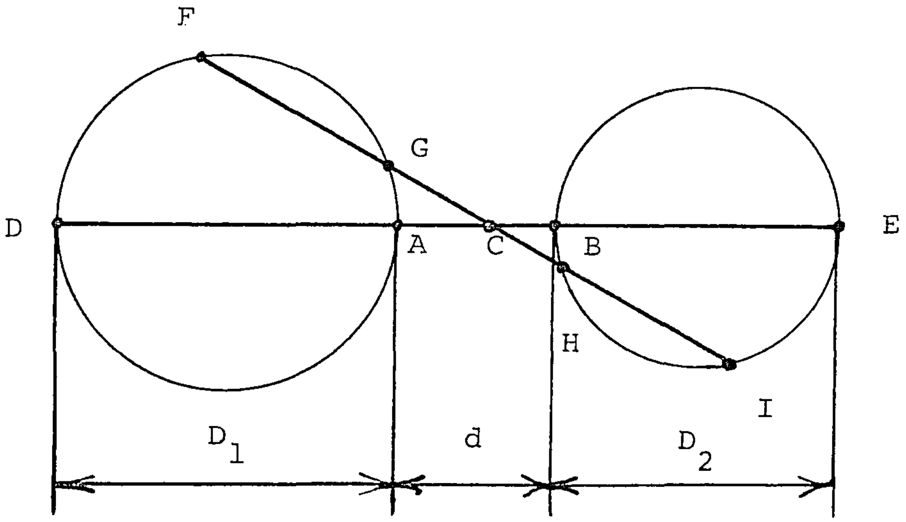


Figure 2.5 The Analogy of the Awayness of a Cube to a Cube in Plane Geometry

For Figure 2.5.

If, Let $AC = BC = \frac{1}{2} d$

Then $GC.FC = \frac{1}{2}D_1 d + \frac{1}{4} d^2 \ldots\ldots\ldots\ldots (1)$

$IC.HC = \frac{1}{2}D_2 d + \frac{1}{4} d^2 \ldots\ldots\ldots\ldots (2)$

$Gc.Fc + Ic.Hc = \frac{d}{2} \; (D_1+D_2) + \frac{d^2}{2}$

$GC.FC.IC.HC = (D_1 + \frac{d}{2}) \; (D_2+\frac{d}{2}) \; \frac{d^2}{4}$

For minimizaing minterms of a switching function we have the following theorems.

Theorem 20: If a point is j coverable and having awayness of W_{ab} to cube <a,b> of D dimension, then the number of the common points between the cube covering of the point and the original cube is zero if W_{ab} is greater than j. If W_{ab} is smaller than j, then there are at most $2^{j-W_{ab}}$ points in common between the old cube and the new one.

Proof: If W_{ab} is greater than k, then x is aloof from the old cube; thus no common points exist. If W_{ab} is smaller than j, then there may be at most $2^{j-W_{ab}}$ common points.

It is clear that W_{ab} is a very important factor in determining the priority of the Multiple-coverable points.

Theorem 21: $j-W_{ab}$ is alwasy smaller than D, the dimension of the chosen cube.

Proof: If $j-W_{ab}$ is larger than D, it is possible to find a cube that covers x and the whole cube <a,b> of dimension

j. But $<a,b>$ is the largest cube to cover a and b. If the j cube does not intersect $<a,b>$ then they are separated and there is no relation. This can also be extended to higher ordered cubes. Let us define the maximal distance of two cubes as the envelope of two cubes.

Definition 32: The envelope of two cubes A and B, [e(A,B)] is defined as $e(A,B) = K_A+K_B - \#(R) + d(A,B)$

Theorem 22: The smallest cube covering both A and B has the dimension of $e(A,B) + \#(R)$.

Proof: From the proof of Theorem 19.

$$\begin{aligned} & e(A,B) + \#(R) \\ & = K_A+K_B - 2\#(R) + d(A,B) + \#(R) \\ & = \{(K_A - \#(R)\} + \{K_B-\#(R)\} + d(A,B) + \#(R) \end{aligned}$$

where $K_A - \#(R)$ = the number of coordinate positions only cube A is varying.

$K_B - \#(R)$ = the number of coordinate positions only cube B is varying

$\#(R)$ = The number of coordinate positions both A.B are varying

$d(A,B)$ = The number of coordinate positions neither A nor B are varying but where A and B differ.

Q.E.D.

Theorem 23: If cube A and cube B are of the same dimension R, and d (A,B) = e(A,B) then all varying coordinates coincide.

Proof: $e(A,B) = K+K-2\#(R)+d(A,B)$ (By Theorem 22)

$$\therefore\ 2K - 2\#(R) = e(A.B) - d(A,B) = 0$$

$\therefore$ K = #(R) Q.E.D.

<u>Corollary</u>: Of e(A,B) = d(A,B) = 0, A=B.

<u>Theorem 24</u>: If cube A and cube B have the same dimension, d(A,B) = 1, e(A,B) = K+1 then A and B form a(k+1)-cube.

<u>Proof</u>: e(A,B) = K+K - 2#(R) +d(A,B)

= 2K-2#(R) + d(A,B)

By Theorem 22, the smallest cube covering A and B has a dimension of

e(A,B) + #(R) = 2K - 2#(R) + 1 + #(R) = 2k-#(R)+1

#(R)= 2K+1 - e(A,B) = 2K+1 - K - 1

= K.

$\therefore$ e(A,B) + #(R) = 2K - #(R) +1 = 2K-K+1

=K+1. Q.E.D.

<u>Theorem 25</u>: Let f(A,B) = max $\{d(x,y):\chi\varepsilon A, y\varepsilon B\}$.

Then (A,B) = e(a,b) iff#(R) = 0

1) if part,

<u>Proof</u>: f(A,B) = Max $(d(x,y) : \chi\varepsilon A, y\varepsilon B)$.

= Max $\{M_p + M_q + M_r + M_s.\}$

Maximum of $m_p = K_A - \#(R)$ Maximum of $M_q = K_B - \#(R)$,

$M_r = \#(R)$, and $M_s = d(A,B)$

Thus $f(A,B) = K_A - \#(R) + K_B - \#(R) + \#(R) + d(A,B)$

$= K_A + K_B - 2\#(R) + d(A,B) + \#(R)$

$= e(A,B) + \#(R)$

$\therefore$ #(R) = 0

2) only if part.

$\#(R) = 0$. implies $M_p=K_A, M_q=K_B$ $\quad M_r = 0$

$$M_s = d(A,B)$$

$\therefore$ $f(a.b) = K_A + K_B + d(A,B)$.

$e(A,B) = K_A + K_B - 0 + d(A,B) = K_A+K_B+d(A,B)$

$\therefore$ $f(a,b) = e(A,B)$

Corollary: $f(a,b) = e(A,B) + SK(A,B)$

Theorem 25: Given cubes A, B, and $d(A,B) = 0$, then the number of common points $= 2^{\#(R)} = 2^{SK_A(B)}$

Proof: (1) If $d(A,B) = 0$, there are no coordinate positions difference outside of those position where either A, or B are varying. Thus the intersection of cube A and cube B has a dimension of $\#(R)=SK(AB)$. and their varying coordinate positions are exactly those $\#(R)$ positions where both A and B are varying.

Theorem 26: Given cubes A and B. and $d(A,B)=1$. $e(A,B)=1$. Then $SK(AB)=\frac{KA+KB}{2}$, $f(a.b) = \frac{K_A+K_B+2}{2}$

Proof. $e(A,B)=K_A+K_B - 2SK(AB)+d(A,B)$

$\therefore$ $K_A+K_B - 2SK(A,B) = 0$

$\therefore$ $K_A+K_B = 2SK$

$$SK(AB) = \frac{K_A + K_B}{2}$$

$$f(a,b) = e(a,b) + SK(A,B) = 1 + \frac{K_A+K_B}{2} = \frac{K_A+K_B+2}{2}$$

Q.E.D.

(c) Aloofness

Another measure of the merit in finding the minimum covering is how evenly can the largest cubes be arranged. Ideally, we would like to find a maximal-cube adjacent to the existing cube covering. For this, we shall introduce the concept of aloofness, which turns out to be a way of gauging how evenly we can arrange the maximal cubes. In order to define aloofness, we shall generalize the definition of adjacency among 0-cubes of any dimensions.

Definition 33: The "aloofness" is defined as the frequency that a point's awayness exceeds the highest dimensions, that is, the number of times when $W_{ab}(x) - d_x \geq 0$, d_x being the highest dimension of cubes that x can have for maximal-cube covering.

Definition 34: $W_A(B) - d_x \geq 0$ is the aloofness for cube B with regard to cube A.

It us evident that the selection of the next point should be based on smaller awayness and larger aloofness so that we can evenly spread out as will be demonstrated in chapter IV.

(d) Adjacency

This property is commonly used in literature, and refers to finding as many adjacent sub-cubes to the existing cube covering as possible.

Definition 35: Adjacency is defined between two cubes only if cubes A and B have a distance of 1, i.e. $d(A,B) = 1$. It is obvious that we want to choose the maximal cubes that are adjacent. The reason the map method is so powerful is that one can observe this by visual investigation. In the method given here, it is analytical, not a trial and error method. To achieve the indicated results, a merit value is assigned to each uncovered point or cube.

Definition 36: The degree of adjacency is defined as the number of distinct sub-cubes that are adjacent to the existing maximal cube covering.

e) Probability Overlapping.

In the process of choosing maximal cube covering, the existing maximal cube covering should be overlapped as little as possible.

Definition 37: The probability of sharing common parts of a new cube with the existing cube covering is: (share) $= D$, if the highest coverable maximal cube has a dimension of $J \geq D + W_{ab}$ or $= J - W_{ab}$, if $J \leq D + W_{ab}$.

If the awayness and the aloofness are the same, then the next criteria is the relative probability of sharing. The last check is to compare the real over-lap of common parts.

Definition 38: If d(A,B) > 1 for all B C F, A is isolated.

Definition 39: Let * = 1/2 to be the don't care value Q* = {0, 1, *} for the value of the coordinates of the center of pseudo-circles.

f) Other related structure.

Lemma 7: If d(A,B) = 0, then A,B are coincident

Lemma 8: If d(A,B) > 1, then A,B are separated

CHAPTER III

BRANCHING GRAPH APPLICATION AND ILLUSTRATIVE EXAMPLES.

3.1 THE ALGORITHM

The minimization of two-level sum-of-products logic can be summarized as follows:

Step 1: The distance Matrix A is constructed;

Step 2: From Matrix A, the distance-frequency distribution Matrix B is obtained;

Step 3: From Matrix A and Matrix B, the limit matrix L is obtained;

Step 4: From the theorems and matrices A, B, and L and with possible core points as candidates, the essential maximal cubes (i.e, essential prime implicants) $\bigcup_{j=o}^{P} M_j^1$ are selected by starting with the highest dimensioned j-cubes down to the lowest, i.e., finding $\bigcup_{j=0}^{P} M_j^1(f) \supset \bigcup_{j=0} F_j^1$ (All of the essential maximal cubes have been found);

Step 5: The core points are used as the initial nodes to find the branching graph for each set of connected j-coverable points, where j=k, k-1, k-2, ... 3,2,1 and k is the highest dimension of the maximal cube covering;

Step 6: If there are terminal points of higher coverable points or branches containing higher coverable points, then they are compared with the cube graph of higher dimension, to see if any less-than cubes exist;

Step 7: A minimum covering of the branching graph, which the minimum non-redundant set of maximal cube covering, i.e., $\bigcup_{\substack{i=1\\ j=0}}^{p,q} MMF_j^i$ is found.

3.2 Demonstrative Electives:

Example 1: To illustrate that a non-essential prime implicant will not be generated if it is already included in the set of essential cubes:

$$f(x) = \Sigma\,(0, 2, 3, 4, 5, 6, 7, 13).$$

Step 1: Construct Matrix A (Fig. 1). (Matrix A is symmetrical).

Step 2: Construct Matrix B (Fig. 2) from Matrix A.

Step 3: Construct Matrix L (Fig. 3) from Matrix B .

1) Since only four points (4, 5, 6, and 7) satisfy the necessary conditions for a 3-cube covering, no 3-cube covering exists. Multiple 2-covering may exist for these points.

2) Only points 0 and 3 are possible 2-coverable core points.

3) Point 13 is a possible 1-coverable core point.

Step 4: Since 0 and 3 are of same order and degree, point 0 is arbitrarily chosen first. Only 2-cubes are being dealt with in this case, and therefore any distance over 3 is immaterial. All distances 3 or over are replaced by X. The table in Fig. 4 shows the distances for 0, 3, 5, and 6 taken from Matrix A. Since 3, 5, and 6 are points having a distance 2 to 0, they are possible candidates for diametrical extremes. Theorem 2 shows that only the <0,6> cube exists. (<0,6> = 0,2,4,6). Now L is updated to L_1, by changing the row representing 0 in L to (-,-,e), indicating that point $0 \epsilon F_2^1$, and 2, 4, and 6 belong to the don't care conditions (Fig. 5) .

Step 5: Continue to point 3, which is another candidate for a 2-coverable core point and find the <3,6> which is an essential 2-cube. Update L_1 to L_2 (Figs. 6 and 7).

Step 6: Having exhausted the 2-coverable core points, the possible 1-coverable core points are considered. Thus, point 13 is to be covered by <13, 5> . Update L_2 to L_3 (Figs. 8 and 9). Since all points are covered, the process is complete.

$$\begin{array}{c|cccccccc} & 0 & 2 & 3 & 4 & 5 & 6 & 7 & 13 \\ \hline 0 & 0 & 1 & 2 & 1 & 2 & 2 & 3 & 3 \\ 2 & 1 & 0 & 1 & 2 & 3 & 1 & 2 & 4 \\ 3 & 2 & 1 & 0 & 3 & 2 & 2 & 1 & 3 \\ 4 & 1 & 2 & 3 & 0 & 1 & 1 & 2 & 2 \\ 5 & 2 & 3 & 2 & 1 & 0 & 2 & 1 & 1 \\ 6 & 2 & 1 & 2 & 1 & 2 & 0 & 1 & 3 \\ 7 & 3 & 2 & 1 & 2 & 1 & 1 & 0 & 2 \\ 13 & 3 & 4 & 3 & 2 & 1 & 3 & 2 & 0 \end{array}$$

Figure 3.1 Distance Matrix A of Example 1

$$\begin{array}{cc}
\begin{array}{ccccc} \rho_0 & \rho_1 & \rho_2 & \rho_3 & \rho_4 \end{array} & \\
\left(\begin{array}{ccccc}
1 & 2 & 3 & 2 & 0 \\
1 & 3 & 2 & 1 & 1 \\
1 & 2 & 3 & 2 & 0 \\
1 & 3 & 3 & 1 & 0 \\
1 & 3 & 3 & 1 & 0 \\
1 & 3 & 3 & 1 & 0 \\
1 & 3 & 3 & 1 & 0 \\
1 & 1 & 2 & 3 & 1
\end{array}\right) &
\begin{array}{c} 0 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 13 \end{array}
\end{array}$$

Figure 3.2 Distance Frequency Matrix B of Example 1

$$\begin{array}{cc} & \begin{array}{ccccc} C_0 & C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{array}{c} 0 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 13 \end{array} & \left(\begin{array}{ccccc} - & - & c & 0 & 0 \\ - & - & - & 0 & 0 \\ - & - & c & 0 & 0 \\ - & - & - & c & 0 \\ - & - & - & c & 0 \\ - & - & - & c & 0 \\ - & - & - & c & 0 \\ - & c & 0 & 0 & 0 \end{array}\right) \end{array}$$

Figure 3.3 Matrix L of Example 1

	0	2	3	4	5	6	7	13
0	0	1	2	1	2	2	X	X
3	2	1	0	X	2	2	1	X
5	2	X	2	1	0	2	1	1
6	2	1	2	1	2	0	1	X

									N_2	C_2
0 + 3	2	2	2	X	X	X	X	X	3	NO
0 + 5	2	X	X	2	2	X	X	X	3	NO
0 + 6	2	2	X	2	X	2	X	X	4	YES

Figure 3.4 Table for Finding 2-Cubes Covering Point 0 in Example 1

$$\begin{array}{cc} & \begin{array}{ccc} C_0 & C_1 & C_2 \end{array} \\ \begin{array}{c} 0 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 13 \end{array} & \left(\begin{array}{ccc} - & - & e \\ - & - & d \\ - & - & c \\ - & - & d \\ - & - & - \\ - & - & d \\ - & - & - \\ - & c & 0 \end{array}\right) \end{array}$$

Figure 3.5 Matrix L_1 of Example 1

	0	2	3	4	5	6	7	13
3	2	1	0	X	2	2	1	X
5	2	X	2	1	0	2	1	1
6	2	1	2	1	2	0	1	X

									N_2	C_2
3 + 5	X	X	2	X	2	X	2	X	3	NO
3 + 6	X	2	2	X	X	2	2	X	4	YES

Figure 3.6 Table for Finding 2-Cubes Covering Point 3 in Example 1

$$\begin{array}{c|ccc|}
 & C_0 & C_1 & C_2 \\
0 & - & - & e \\
2 & - & - & d \\
3 & - & - & e \\
4 & - & - & d \\
5 & - & - & - \\
6 & - & - & d \\
7 & - & - & d \\
13 & - & c & 0
\end{array}$$

Figure 3.7 Matrix L_2 of Example 1

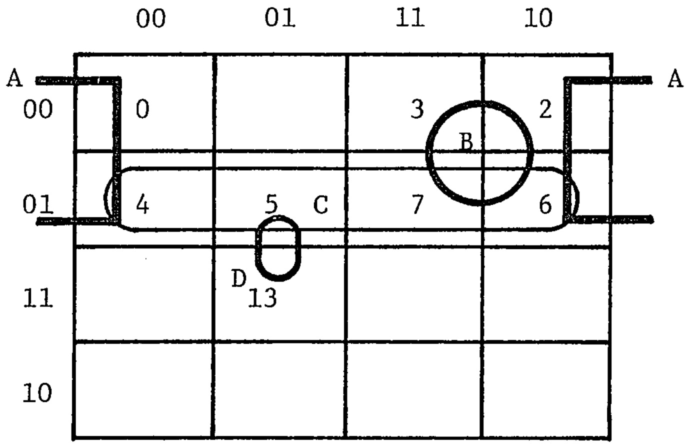
	0	2	3	4	5	6	7	13
13	X	X	X	X	1	X	X	0
5	X	X	X	1	0	X	1	1

									N_1	C_1
13 + 5	X	X	X	X	1	X	X	1	2	YES

Figure 3.8 Table for Finding 1-Cubes Covering Point 13 in Example 1

$$\begin{array}{cccc}
 & C_0 & C_1 & C_2 \\
0 & - & - & e \\
2 & - & - & d \\
3 & - & - & e \\
4 & - & - & d \\
5 & - & - & d \\
6 & - & - & d \\
7 & - & - & d \\
13 & - & e & 0
\end{array}$$

Figure 3.9 Matrix L_3 of Example 1



Example 3.10 Karnaugh Map for Example 1

Remarks: Comparing with the Karnaugh map method in Fig. 10, the result is obtained algebraically. The so-called "less than cube" (4,5,6,7) was not generated.

Example 2: Branching:

$$F = (0, 2, 3, 4, 5, 7).$$

Step 1: After obtaining matrices A, B, and L (Figs. 11, 12, and 13), it is observed that none of the points are possible core points, and that each is multiple 1-coverable. Starting arbitrarily with point 0, <0,2>, and <0,4> are found. Points 2 and 4 are the next nodes, and L is updated to L_j (Figs. 14 and 15).

Step 2: From point 2, point 3 is the next node with <2, 3> the connecting cube. From 4, point 5 is the next node with <4,5> the connecting cube. L_1 is updated to L_2 (Figs. 16, 17 and 18).

Step 3: Point 3 goes to point 7 by cube <3,7> and point 5 goes to point 7 by cube <5,7>. The next node, point 7, in both cases, is a terminal node (Figs. 19, 20 and 21).

Step 4: The branching graph for the connected j-coverable points and the cube-graph corresponding to it are developed in Fig. 22 (a)

$$\begin{array}{c|cccccc|}
 & 0 & 2 & 3 & 4 & 5 & 7 \\
\hline
0 & 0 & 1 & 2 & 1 & 2 & 3 \\
2 & 1 & 0 & 1 & 2 & 3 & 2 \\
3 & 2 & 1 & 0 & 3 & 2 & 1 \\
4 & 1 & 2 & 3 & 0 & 1 & 2 \\
5 & 2 & 3 & 2 & 1 & 0 & 1 \\
7 & 3 & 2 & 1 & 2 & 1 & 0
\end{array}$$

Figure 3.11 Matrix A of Example 2

$$\begin{array}{c|cccc|} & \rho_0 & \rho_1 & \rho_2 & \rho_3 \\ \hline 0 & 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 2 & 1 \\ 3 & 1 & 2 & 2 & 1 \\ 4 & 1 & 2 & 2 & 1 \\ 5 & 1 & 2 & 2 & 1 \\ 7 & 1 & 2 & 2 & 1 \end{array}$$

Figure 3.12 Matrix B of Example 2

$$\begin{array}{cc} & \begin{array}{ccc} C_0 & C_1 & C_2 \end{array} \\ \begin{array}{c} 0 \\ 2 \\ 3 \\ 4 \\ 5 \\ 7 \end{array} & \left(\begin{array}{ccc} - & - & 0 \\ - & - & 0 \\ - & - & 0 \\ - & - & 0 \\ - & - & 0 \\ - & - & 0 \end{array}\right) \end{array}$$

Figure 3.13 Matrix L of Example 2

	0	2	3	4	5	7
0	0	1	X	1	X	X
2	1	0	1	X	X	X
4	1	X	X	0	1	X

							N_2	C_2
0 + 2	1	1	X	X	X	X	2	YES
0 + 4	1	X	X	1	X	X	2	YES

Figure 3.14 Table for Finding 1-Cubes Covering Point 0 in Example 2

$$\begin{array}{c|cc|} & C_0 & C_1 \\ 0 & - & d \\ 2 & - & n \\ 3 & - & - \\ 4 & - & n \\ 5 & - & - \\ 7 & - & - \end{array}$$

Figure 3.15 Matrix L_1 of Example 2

	0	2	3	4	5	7
2	1	0	1	X	X	X
3	X	1	0	X	X	1

							N_1	C_1
2 + 3	X	1	1	X	X	X	2	YES

Figure 3.16 Table for Finding 1-Cubes Covering Point 2 of Example 2

	0	2	3	4	5	7
4	1	X	X	0	1	X
5	X	X	X	1	0	1

							N_1	C_1
4 + 5	X	X	X	1	1	X	2	YES

Figure 3.17 Table for Finding 1-Cubes Covering Point 4 of Example 2

$$\begin{array}{c|cc|} & C_0 & C_1 \\ 0 & - & d \\ 2 & - & d \\ 3 & - & d \\ 4 & - & d \\ 5 & - & d \\ 7 & - & - \end{array}$$

Figure 3.18 Matrix L_2 of Example 2

	0	2	3	4	5	7
5	X	X	X	1	0	1
7	X	X	1	X	1	0

							N_1	C_1
5 + 7	X	X	X	X	1	1	2	YES

	0	2	3	4	5	7
3	X	1	0	X	X	1
7	X	X	1	X	1	0

							N_1	C_1
3 + 7	X	X	1	X	X	1	2	YES

Figure 3.19 Table for Finding 1-Cubes Covering Points 3 and 5 of Example 2

$$\begin{array}{c|cc|}
 & c_0 & c_1 \\
\hline
0 & - & d \\
2 & - & d \\
3 & - & d \\
4 & - & d \\
5 & - & d \\
7 & - & d \\
\hline
\end{array}$$

Figure 3.20 Matrix L_3 of Example 2

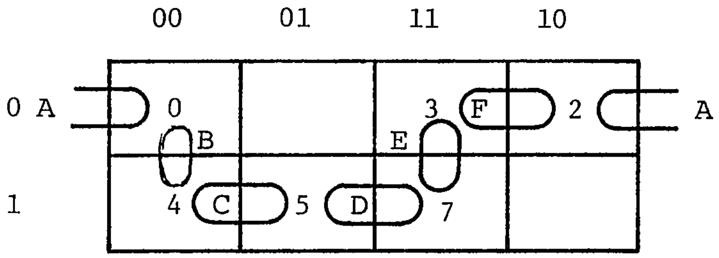
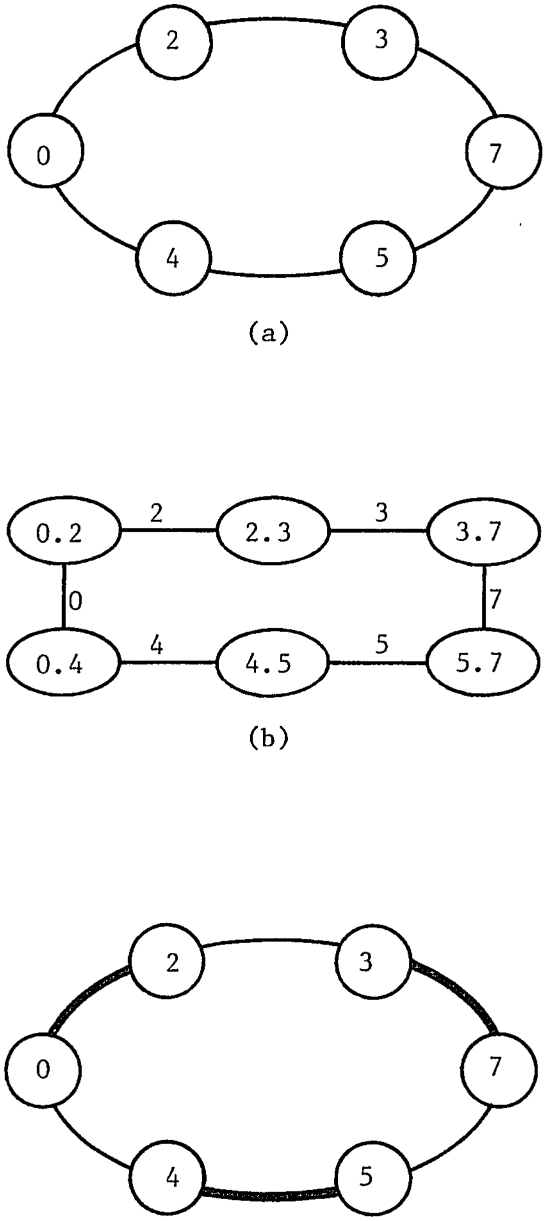


Figure 3.21 Karnaugh Map of Example 2



(a)

(b)

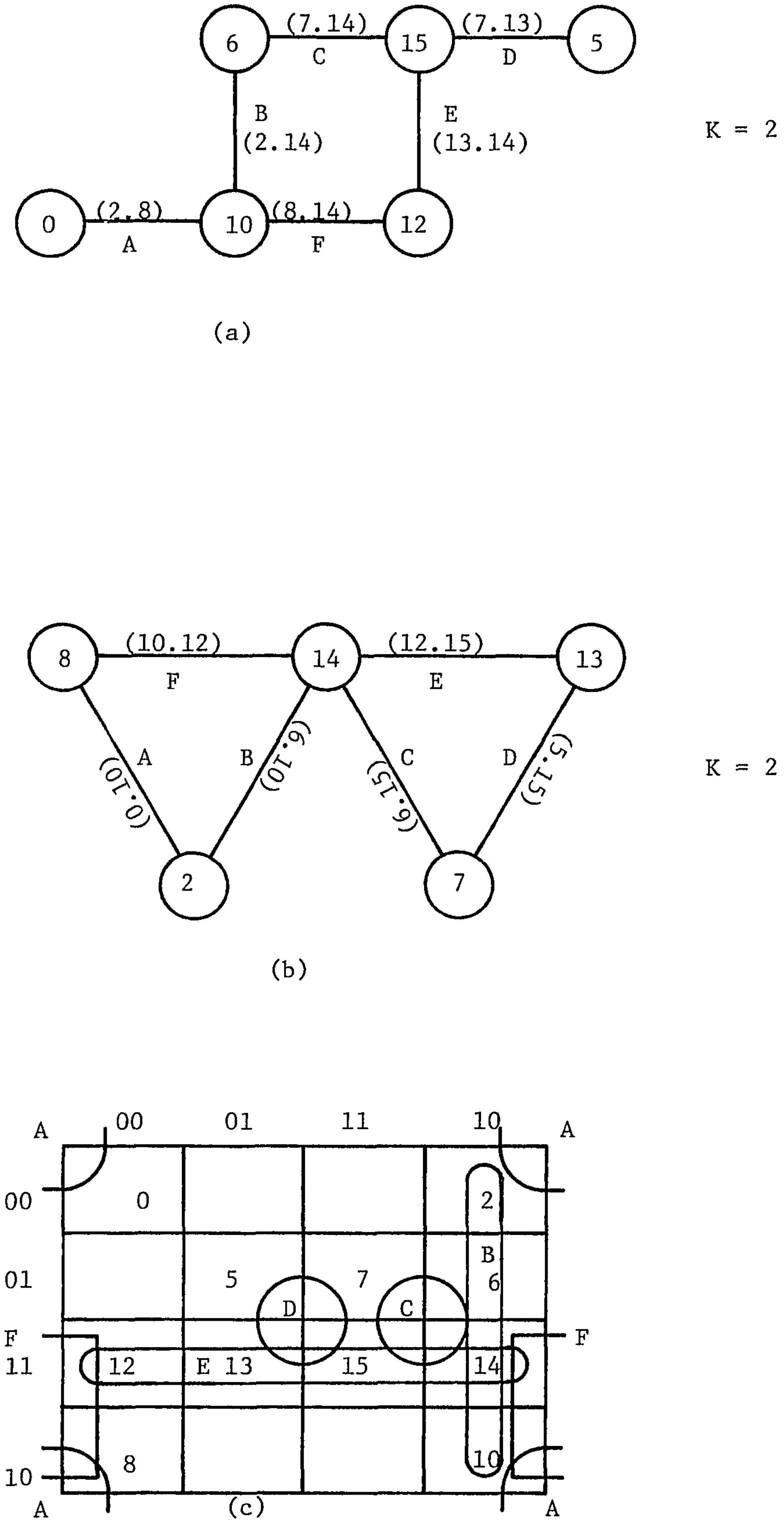
(c)

Figure 3.22 Branching Graph and Cube Graph of Example 2 and 22 (b) respectively. The minimum set includes every other branch in the branching graph (Fig. 22 (c)). (Theorem 7, Corollary 1).

Example 3: Covering of simple branching graph:

$$f(x) = \Sigma(0, 2, 5, 6, 7, 8, 10, 12, 13, 14, 15)$$

The two equivalent branching graphs for k=2 are shown in figures 23(a) and (b). Each of these will give the same answer. However, figure 23(a) is preferred because it has two terminal nodes of the first kind. From figure 23(a), it is clear that branches A and D are essential in any minimal set of non-redundant maximal cubes. As for the loop consisting of branches B, C, E, and F, either B and E, or C and F can be selected. If the branching graph shown in figure 23(b) is used, then there are two loops. For the loop on the left side which consists of branches A, B and F, A is chosen first. For the loop on the right side which consists of branches C, D and E, D is chosen first. Because node 14 has four branches connected to it while nodes 2,8,7 and 13 each have two branches connected to it, therefore either B or F in the left side loop is chosen. The last branch needed to complete the covering the loop on the right side depends on the previous selection of B or F. If B were chosen, E is required. If F were chosen, C is required. There is



(a)

(b)

(c)

Figure 3.23 Branching Graphs and Karnaugh Map for Example 3 more chance for making erroneous selections in using the branching graph of figure 23(b). This does not pose any difficulty because if a core point is used as a starting node, a branching graph similar to that in figure 23(a) will result. The solution using figure 23(b) illustrates the equivalency between branching graphs. Figure 23(c) is the Karnaugh map for Example 3.

Example 4: Branching graph of different dimensions with inseparable linkage:

$$f(x) = \Sigma(0, 1,2,5,6,7,8,10,12,13,14,15)$$

The branching graph for 2-coverable points is the same as in Example 3. In Example 4, an additional branching graph for 1-coverable points is shown in Figure 24(a). Figure 24(b) depicts the linking of the branching graph for the 1-coverables to the branching graph for the 2-coverables. The thick lines indicate the chosen branches, i.e., branches A, C, E, and H. Branches C and E are chosen for the circuit B, C, E, and F. Applying Definition 15 to eliminate branch D, A is essential. Therefore we find B and F are not necessary for the circuit of B, C, E, and F. (Figure 24(c) is the Karnaugh map for Example 4).

Example 5: Branching graph covering with linking for different k but separable:

$f(x) = (o,1,4,5,8,9,12, 13,16,17,18,19,20,21,$

22,23,24,25,26,27,28,29,30,31,32,33,36,37,
40,41,43,44,45,50,51,52,53,54,55,58,59,60,
61,62,63)

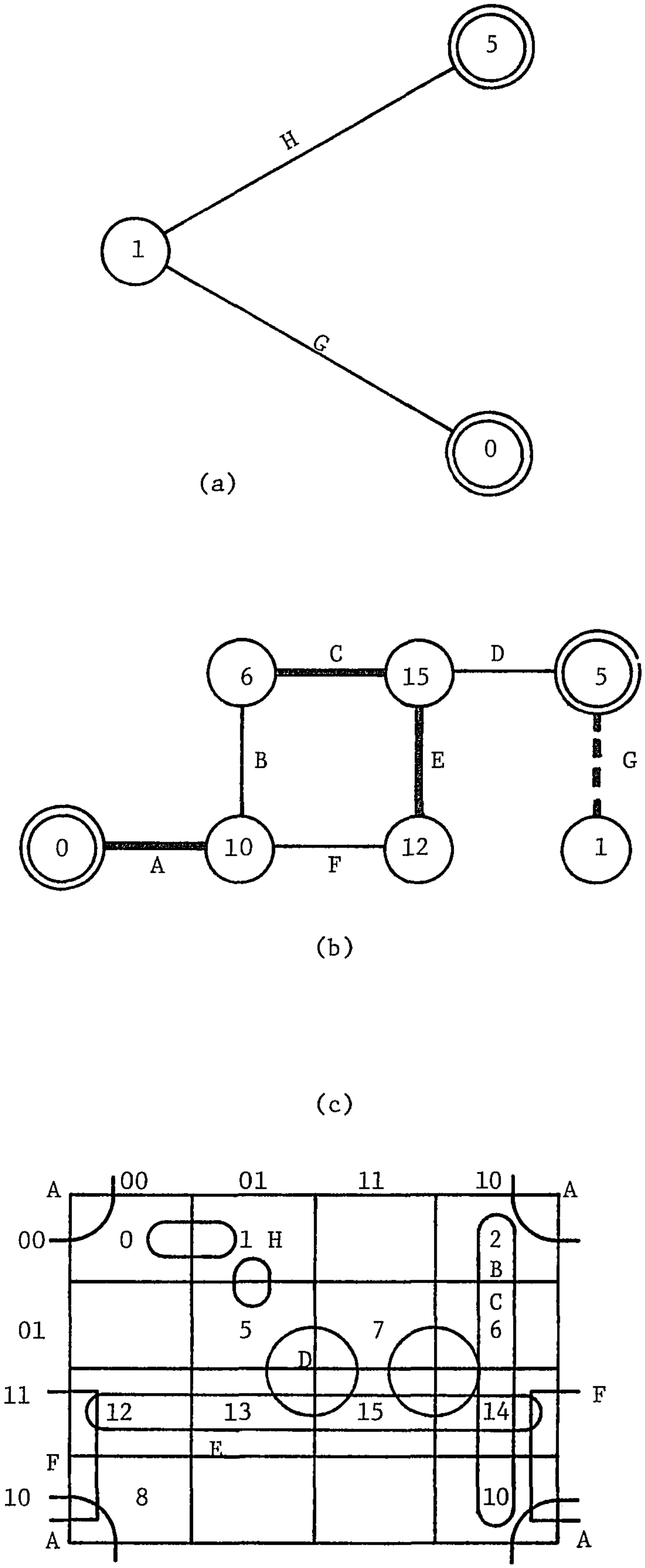
The branching graphs for k = 4 and k = 1 are shown in Figures 25(a) and (b). Although the branching graph is linkable, the linkage of the branching graph does not meet the requirement of Definition 15. Therefore, they are separate branching graphs. (Figure 26 is the linked branching graph. Figure 27 is the Karnaugh map for Example 5. Figure 28 is one of the equivalent branching graphs for 4-coverables in Example 5.)

Example 6: Having unspecified minterms:

$$f(x) = \Sigma(2,10,12) + \Sigma_d\ (0,1,3,5,7,8,9,11,14,15)$$

Matrices A, B, and L are shown in Figures 29(a), (b) and (c) respectively. Since only the points in the on-set (2,10,12) are necessary, these representing don't cares do not appear in matrices A, B and L. These matrices are therefore rectangular. From Figures 29(d) and 29(g), cubes <2, 9> and <12,10> were found, and Figure 29(g) indicates that all on-set points are covered.

The differences in having don't cares are: (1) The rows representing don't cares do not appear in matrices



(a)

(b)

(c)

K = 1

Figure 3.24 Branching Graphs and Karnaugh Map for Example 4

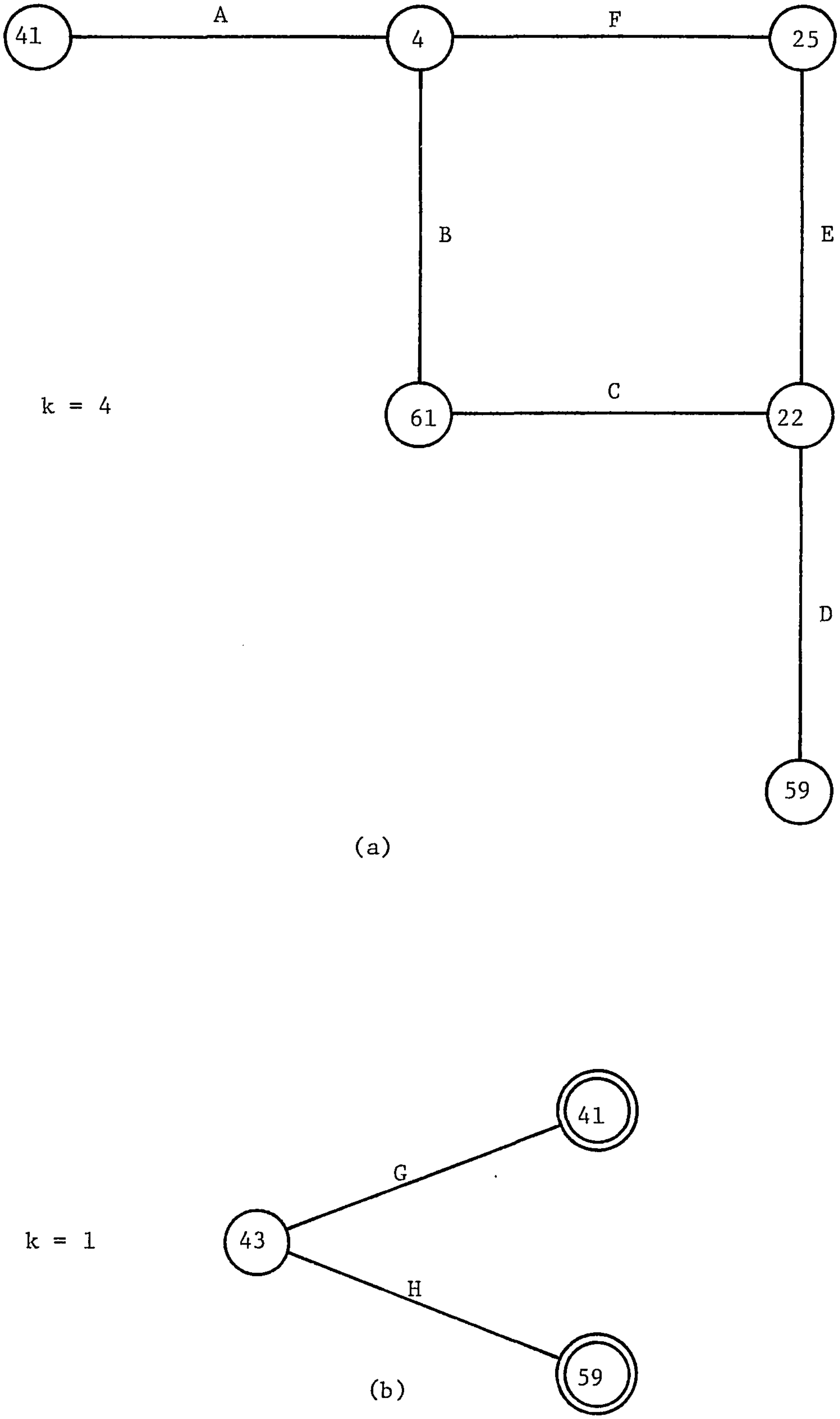


Figure 3.25 Branching Graph for Example 5

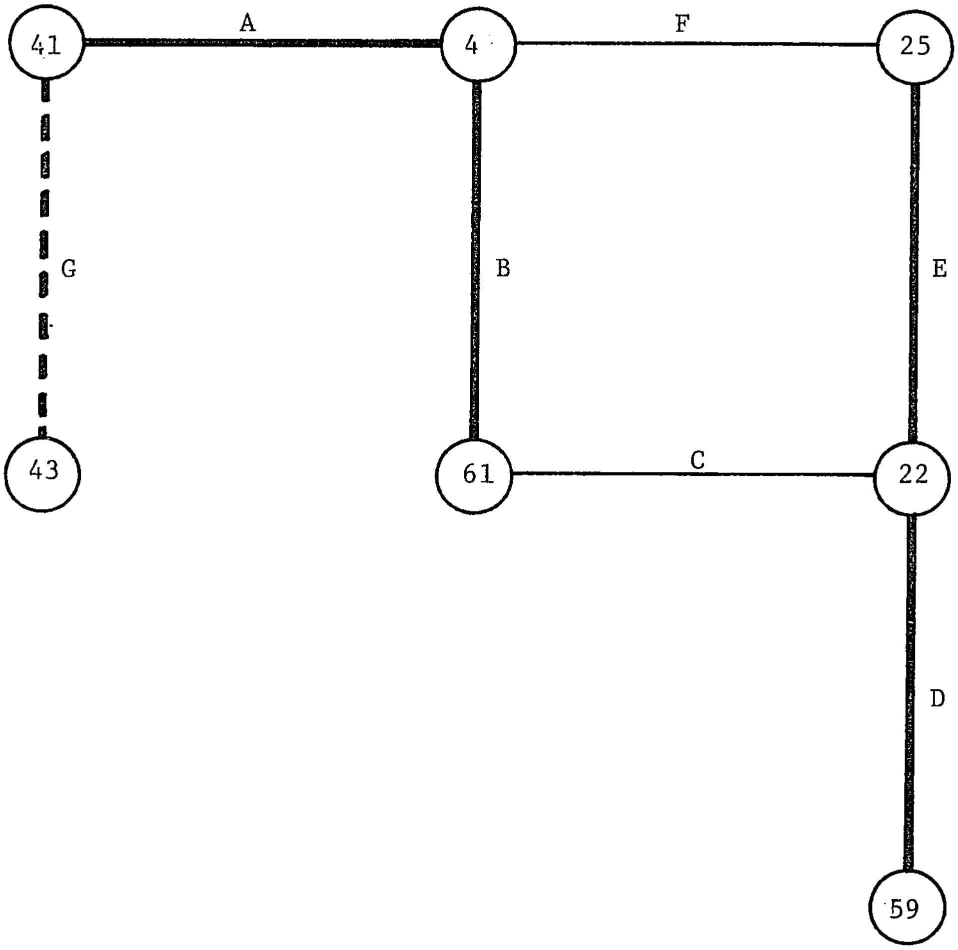


Figure 3.26 Branching Graph (Jointed and Covered) for Example 5

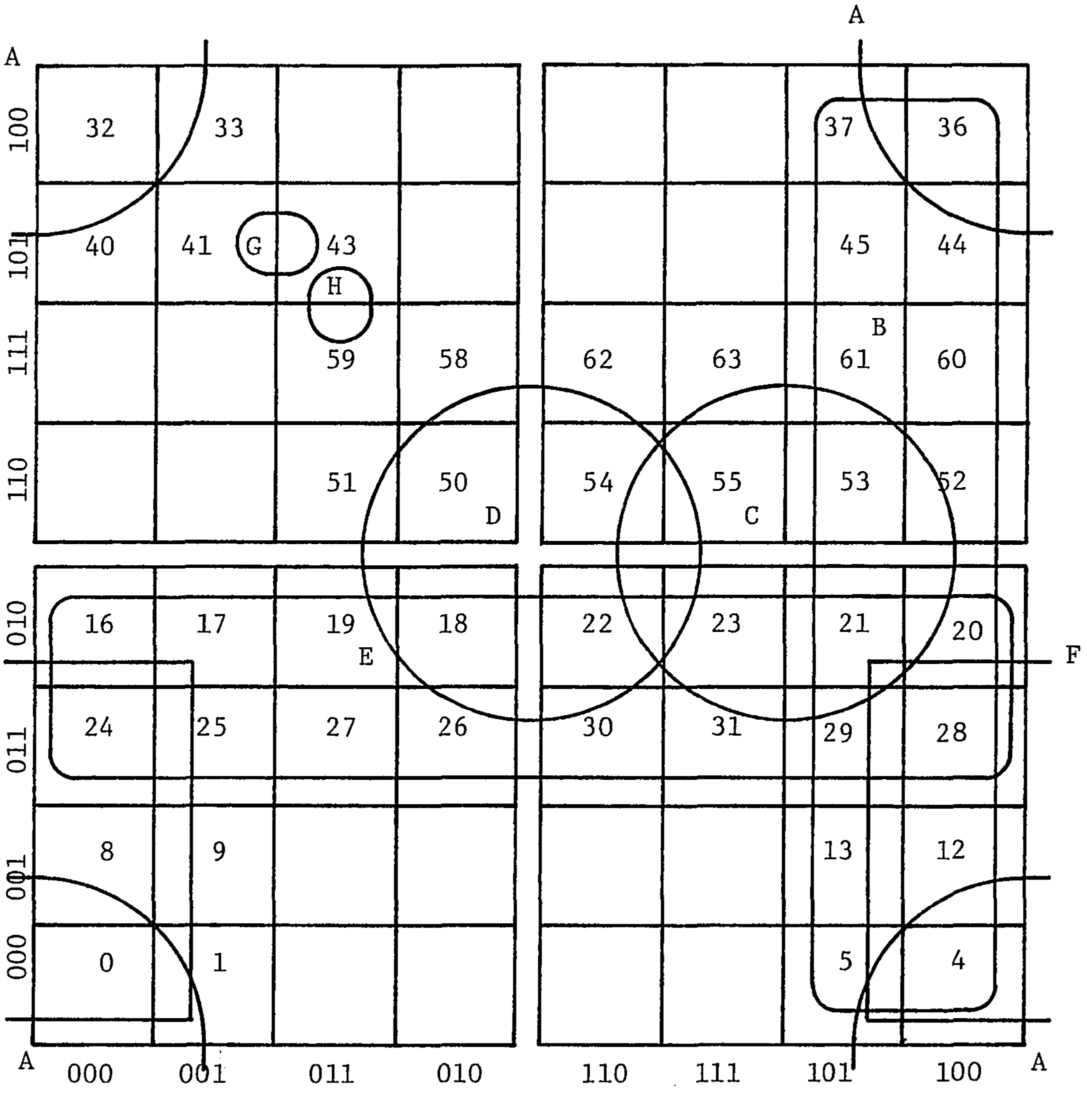


Figure 3.27 Karnaugh Map for Example 5

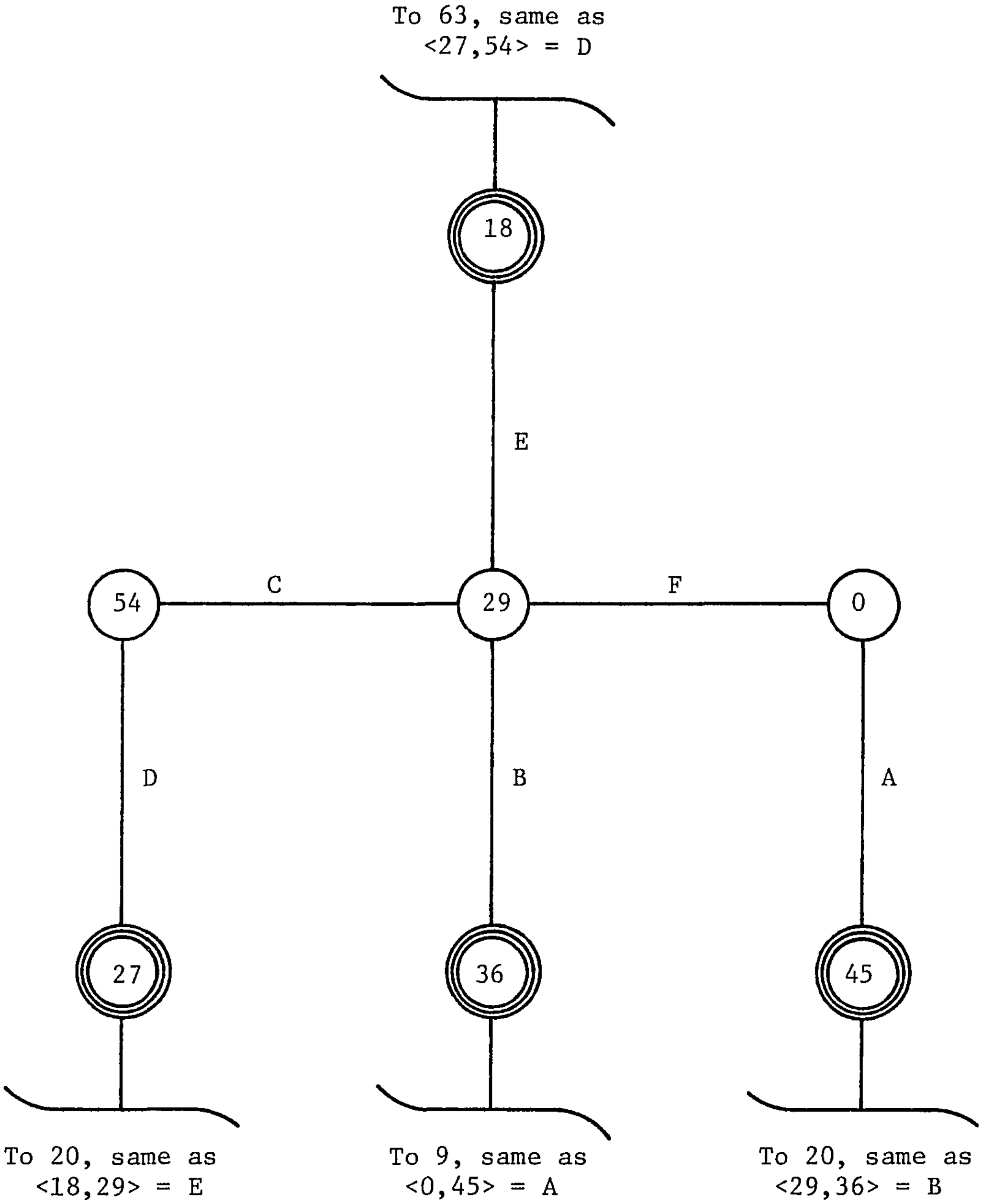


Figure 3.28 Branching Graph for Example 5
(Different but Isomorphic to Figure 3.25)

$$\begin{array}{c|ccccccccccccc|}
 & 2 & 10 & 12 & 0 & 1 & 3 & 5 & 7 & 8 & 9 & 11 & 14 & 15 \\
\hline
2 & 0 & 1 & 3 & 1 & 2 & 1 & 3 & 2 & 2 & 3 & 2 & 2 & 3 \\
10 & 1 & 0 & 2 & 2 & 3 & 2 & 4 & 3 & 1 & 2 & 1 & 1 & 2 \\
12 & 3 & 2 & 0 & 2 & 3 & 4 & 2 & 3 & 1 & 2 & 3 & 1 & 2
\end{array}$$

(a)

$$\begin{array}{c|ccccc|}
 & \rho_0 & \rho_1 & \rho_2 & \rho_3 & \rho_4 \\
\hline
2 & 1 & 3 & 5 & 4 & 0 \\
10 & 1 & 4 & 5 & 2 & 1 \\
12 & 1 & 2 & 5 & 4 & 1
\end{array}$$

(b)

$$\begin{array}{c|ccccc|}
 & C_0 & C_1 & C_2 & C_3 & C_4 \\
\hline
2 & - & - & - & c & 0 \\
10 & - & - & - & - & 0 \\
12 & - & - & c & 0 & 0
\end{array}$$

(c)

Figure 3.29 Diagrams for Example 6
(a) Matrix A
(b) Matrix B
(c) Matrix L

	2	10	12	0	1	3	5	7	8	9	11	14	15
2	0	1	3	1	2	1	3	2	2	3	2	2	3
12	3	2	0	2	3	X	2	3	1	2	3	1	2
5	3	X	2	2	1	2	0	1	3	2	3	3	2
9	3	2	2	2	1	2	2	3	1	0	1	3	2
15	3	2	2	X	3	2	2	1	3	2	1	1	0

														N_3	C_3
2 + 12	3	3	3	3	X	X	X	X	3	X	X	3	X	6	NO
2 + 5	3	X	X	3	3	3	3	3	X	X	X	X	X	6	NO
2 + 9	3	3	X	3	3	3	X	X	3	3	3	X	X	8	YES
2 + 15	3	3	X	X	X	3	X	3	X	X	3	3	3	7	NO

(d)

$$\begin{array}{c|cccc} & C_0 & C_1 & C_2 & C_3 \\ \hline 2 & - & - & - & e \\ 10 & - & - & - & d \\ 12 & - & - & c & 0 \end{array}$$

(e)

Figure 3.29 (cont.)
(d) Table for Finding 3-Cubes
(e) Matrix L_1

	2	10	12	0	1	3	5	7	8	9	11	14	15
12	X	2	0	2	X	X	2	X	1	2	X	1	2
10	X	0	2	2	X	X	X	X	1	2	X	1	2
0	X	2	2	0	X	X	2	X	1	2	X	X	X
5	X	X	2	2	X	X	0	X	X	2	X	X	2
9	X	2	2	2	X	X	2	X	1	0	X	X	2
15	X	2	2	X	X	X	2	X	X	2	X	1	0

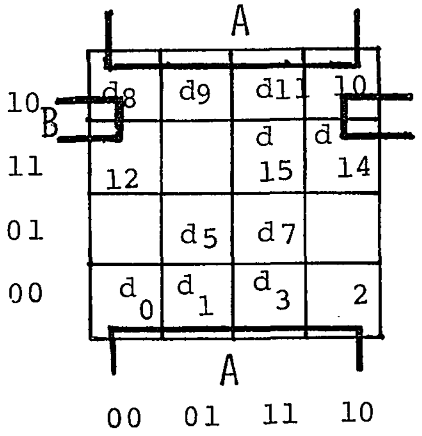
														N_2	C_2
12 + 10	X	2	2	X	X	X	X	X	2	X	X	2	X	4	YES
12 + 0	X	X	2	2	X	X	X	X	2	X	X	X	X	3	NO
12 + 5	X	X	2	X	X	X	2	X	X	X	X	X	X	2	NO
12 + 9	X	X	2	X	X	X	X	X	2	2	X	X	X	3	NO
12 + 15	X	X	2	X	X	X	X	X	X	X	X	2	2	3	NO

(f) Table for Finding 2-Cubes

$$\begin{array}{c|cccc} & C_0 & C_1 & C_2 & C_3 \\ \hline 2 & - & - & - & e \\ 10 & - & - & - & d \\ 12 & - & - & e & 0 \end{array}$$

(g)

(g) Matrix L_2



(h) Karnaugh Map

Figure 3.29 (continued)

A, B and L; (2) The distance row has to be established, only when the don't care point is a potential candidate for a diametrical extreme of a cube that covers an on-set point. Even then, only the relevant entries appear in the row. For example, in Figure 29(d), only the rows representing don't care points 5, 9 and 15 are needed. In Figure 29(f), although the don't care points 0,5,9 and 15 are needed, the entries of these rows under the columns of 2,1,3,7 and 11 are irrelevant. In this case, we are considering a 2-cube covering for point 12, yet all the distances from the points (2,1,3,7 and 11 to 12) are greater than 2. Therefore, a 3-cube covering cannot be formed. In the case where the majority of points are don't cares, then a covering for the on-set and the off-set would each be found and then combined by Nelson's method (N3) or McCluskey's method (Mc2).

3.3 CONCLUSION

The concept of distance matrices and distance-distribution matrices leads to the algorithm of finding the maximal cube covering starting with the largest possible cube first. This is the reverse procedure to those using conventional tabular methods. Obviously, when larger cubes exist, it is thus more effective. However, when there are numerous small cubes, the algorithm tends to increase the labor while the conventional methods tend to decrease it.

Initial work has not been effective in ascertaining the break even point. Presently, computer programming to minimize the effort is being studied.

Although similar in some respects to the map method, this method is more powerful and effective. Firstly, it does not depend upon the human faculty of recognizing geometric patterns; and secondly, it is systematic and precise and can be readily computerized.

The concept of the branching graph not only leads to solving the PI table covering problem more effectively, but also gives a definite way of finding all the maximal cubes.

Since the new approach breaks down the on-set of a switching function into connected j-coverable subsets, the maximal cube table (or prime-implicants table) evolves into an easier soluction. In fact, the branching graph shows more interrelationships between the maximal cubes and the elements of the on-set of a switching function than the row dominance and the column dominance in a prime-implicant table. Such relations include the diametrical opposite points of a cube, the elimination of less-than cubes, the true non-essential maximal cubes, and the connectivity of the maximal cubes.

Because of the corresponding characteristics between the Karnaugh map and the new approach, it is conceivable that many problems in switching theory and logic design can be solved with the new approach. These may include the multi-output problems, the switching functions with feedback and sequential digital circuits. The full impact of this method cannot be realized until more types of problems are studied.

CHAPTER IV

THE APPLICATION OF GEOMETRIC APPROACH

4.1 Algorithm for two level Logic Minimization.

Following the analysis in Chapter 2 Section 4, we can define the characteristics function for an array of the on-set and don't cares of a switching function as following:

Characteristic 1: The highest coverable dimension.

Characteristic 2: Core point candidacy.

Characteristic 3: Probability of sharing common parts.

Characteristic 4: The accumulative awayness.

The general algorithms as follows:

MAIN PROGRAM

(1) Read in number of variables and number of don't care minterms, minterm zero is included or not, data card of all minterms in the on-set and don't care set.

(2) Calculate the maximal cube dimension and binomial coefficients for that dimension.

(3) Calculate the distance Matrix A for all the minterms.

(4) Write matrix A into disc file.

(5) Calculate the distance frequency distribution for each minterm. Then immediately calculate Char. 1 and Char. 2.

(6) If $d(I,X) > 1$ for all X in the array, I is isolated.

(7) Print out all isolated minterms.

(8) Char (I,1) = Characteristic 1 of minterm I = the highest coverable dimension for satisfying the following two criteria.

(i) $$l_{ij} = \underset{k=o}{\overset{k=j}{\text{Min}}} \begin{pmatrix} bik \\ \binom{j}{k} \end{pmatrix} = \text{Char}(I,1) \quad \text{(Theorem 3)}$$

(ii) The number of minterms in the array meeting condition 1 is more than $2^{l_{ij}}$.

(9) If Frequency of distance = 1 or 2^j where j is the highest coverable cube covering, then the minterm in examination is a candidate of core point. Char. (I,2) = 1 or 2^j, otherwise Char. (I.2) = 5 or 999999, which means multiple covering.

(10) Check if Maximal Cube dimension is valid or not. If not, lower Maximal-Cube dimension by 1. The criterion is $\log_2$ (number of maximal cube coverable minterms) $\geq$ maximal cube dimension.

(11) Characteristic 3 = Char (I,3) = 0 for on-set
= 999999 for don't care.

at the start of the program. It will change as the process of finding cube covering progresses. At the start the on-set are not common with any cube covering and the don't care minterms are in common with every cube covering. Therefore the probability of sharing any common part for the on-set minterms is zero, for don't care minterms is 1, for which we use $999999/10^6$ to approximate and then conveniently drop the common denominator, 10^6.

(12) Test all the core point candidates as to whether there is a cube or not. If the cube exists, then this minterm is covered and all the minterms in this cube are also covered. Therefore their Characteristic 3 all become 99999. The probability of sharing is certain for future cube covering. If not, then this minterm being examined becomes a j-1 coverable minterm.

After a core point and core cube is established, the accumulative awayness is computed for the rest of the minterms in the array.

(13) Use the criteria that a new cube should have the least awayness from and the least intersection with the don't care set or covered set. The rest of the array is gradually covered with the minimum maximal-cube covering for the whole array.

SUBROUTINE CORE CUBES

(1) Pick any minterm that is a core point candidate; if all possible candiates have been checked stop.

(2) Check characteristic 3 to single out don't cares.

(3) Use characteristic 1 for dimension. Call subroutine CUBE to verify the existence of the core cube.

(4) If core point candidate does not have a cube covering, lower the dimension and charge the char (3.2) to non core candidate the minterm being processed.

(5) If indeed the cube exists, then print out the data for this cube.

(6) If not finished, return to step 1.

SUBROUTINE MULTIPLE COVERABLE POINTS.

1. Select starting point.
    a. Highest coverable by Char (I,1)
    b. Smallest awayness
    c. Largest aloofness.
    d. Least probability to share common parts with existing maximal-cube covering.
2. Call subrountine cubes to verify the existance of all the highest maximal cubes.
3. Check the actual number of common parts between the cubes found in 2. and choose the one that has the fewest common parts.
4. Checking less-than-cubes by characteristic 1 to check actual coverable dimension.
5. Starting dimension 1 to dimension maximal.
6. Go through 1 through 3.

SUBROUTINE CUBES

1. Called by main program minterm = MTH, Dimension = D, Type = ZNTH.

2. Read from file the distance function for Mth.
3. If ZNTH = 1, core point candidate verify the existing of a cube and print out otherwise, go to 5.
4. Calculate accumulated awayness, relative probability of showing common parts go to 7.
5. Calculate probability of share verigy all D-cubes covering MTH. If awayness are the same, then compare the probability of share.
6. Calculate accumulative awayness and aloofness.
7. Return to main program. The detailed flow diagrams are shown in Figure 4.1 for the main program. Figure 4.2 shows the sub-routine of finding the core cubes. 4.3 is program for finding the remainder of the minimum maximal cube covering, and Figure 4.3 shows the sub-routine cube identifying or cube. The listing of the Fortran IV programs is given in the appendix. A detailed example is worked out in the next section to demonstrate the mechanism of the program.

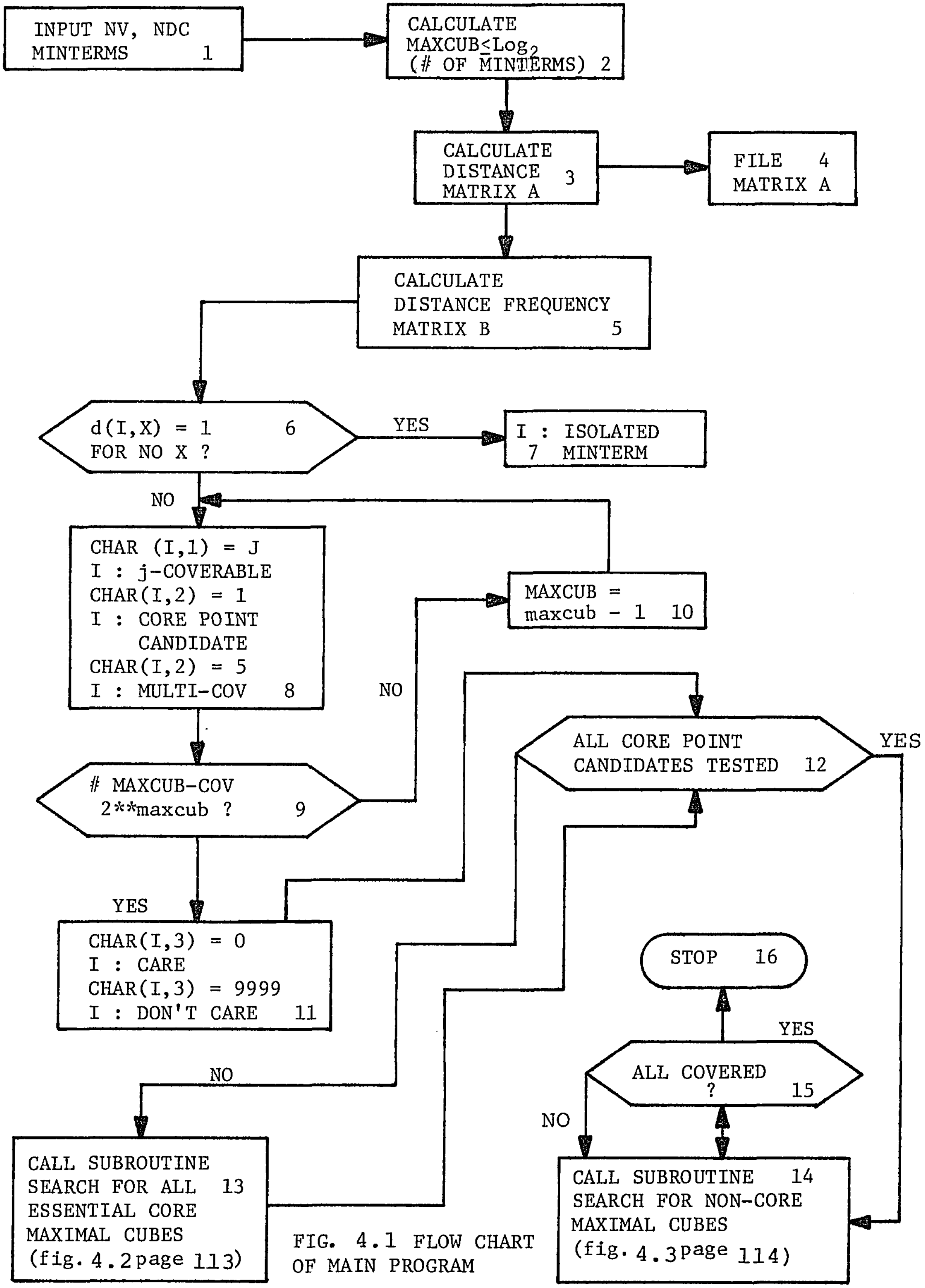


FIG. 4.1 FLOW CHART OF MAIN PROGRAM

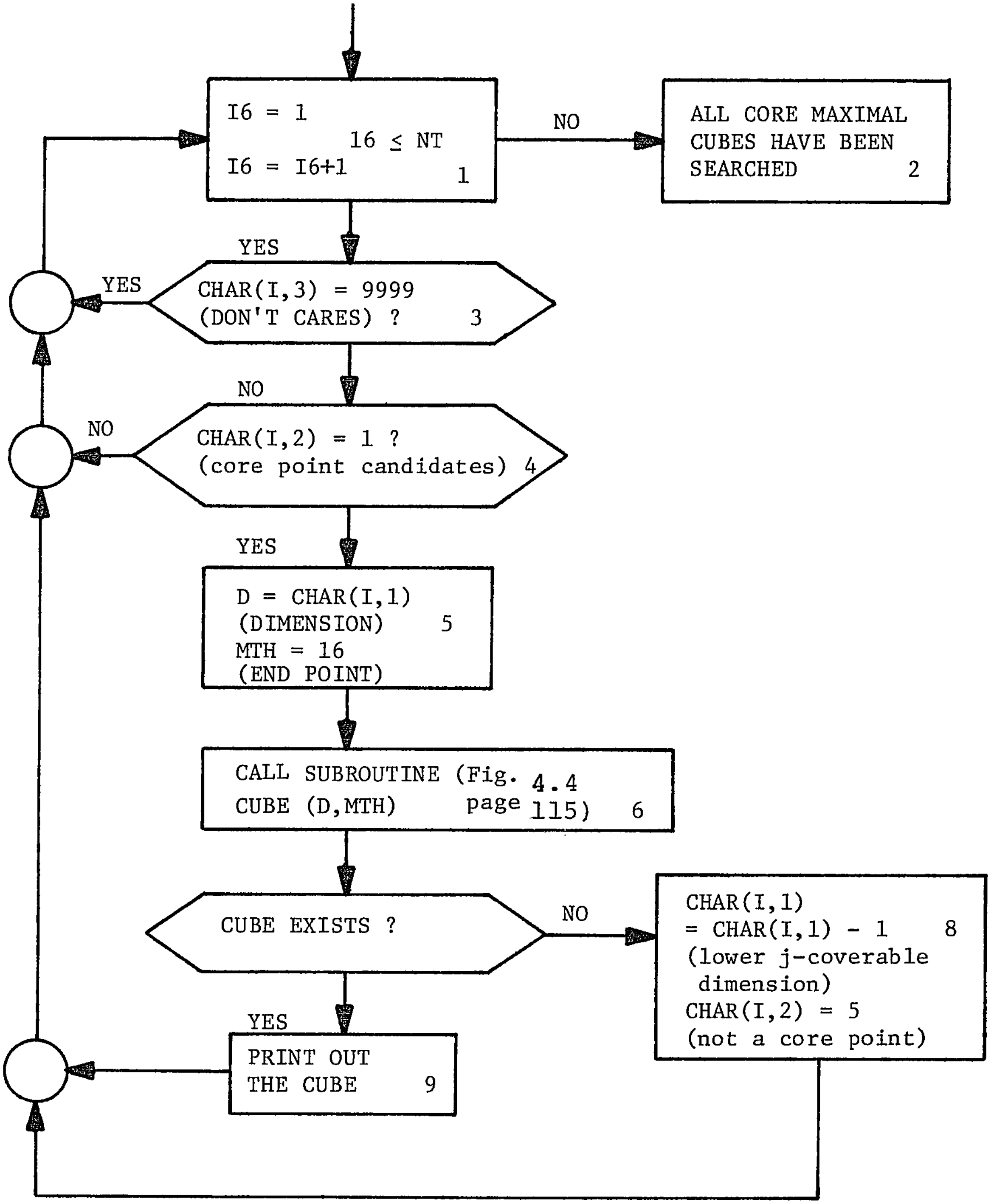


FIGURE 4.2 Flow Chart of Subroutine Search for All Essential Core Maximal Cubes (called by main program in Figure 4.1, Block 13)

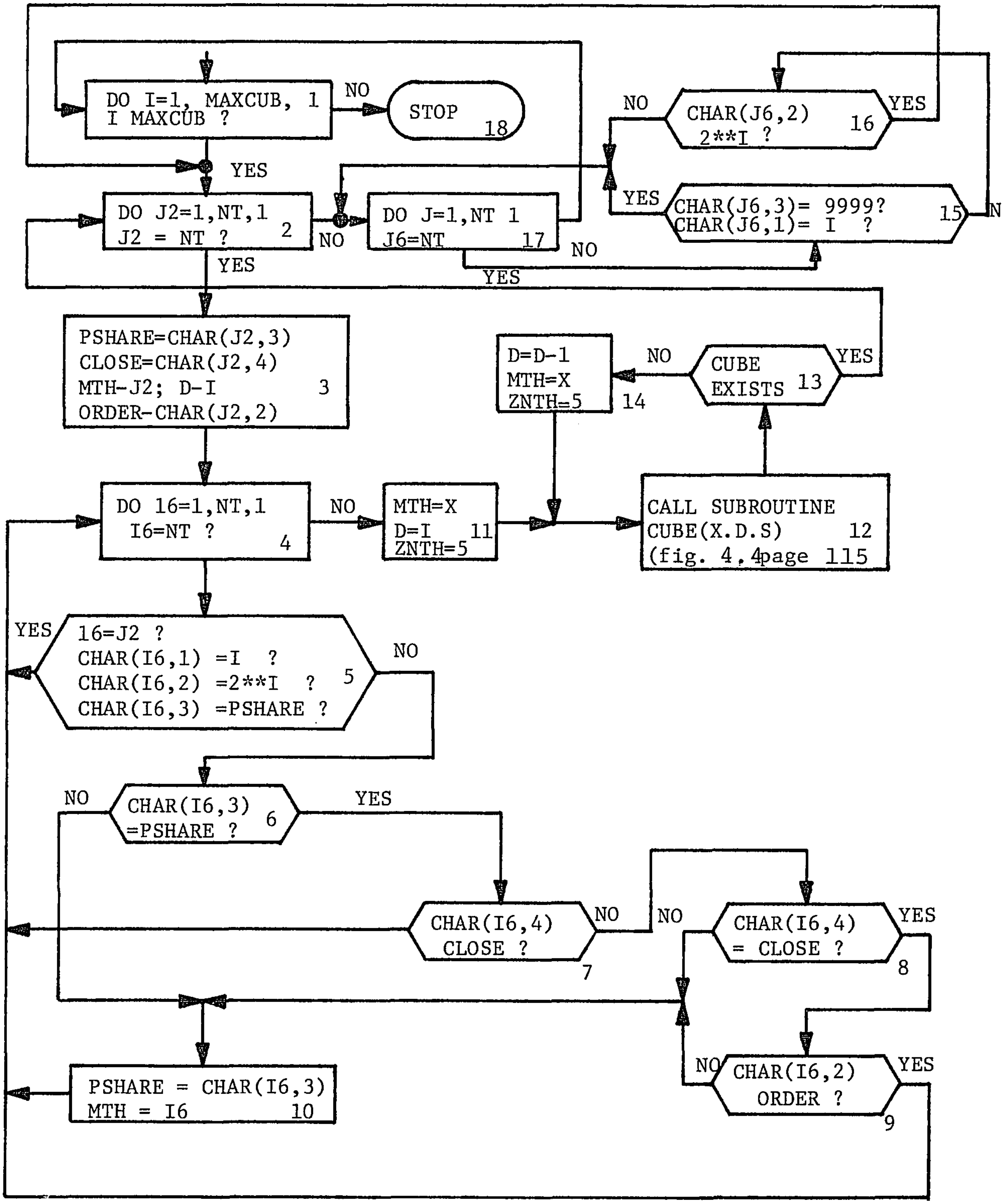


FIGURE 4.3 SUBROUTINE SEARCHING FOR NON-CORE MAXIMAL CUBES

(Called by main program fig.4.4 block 14, page 112)

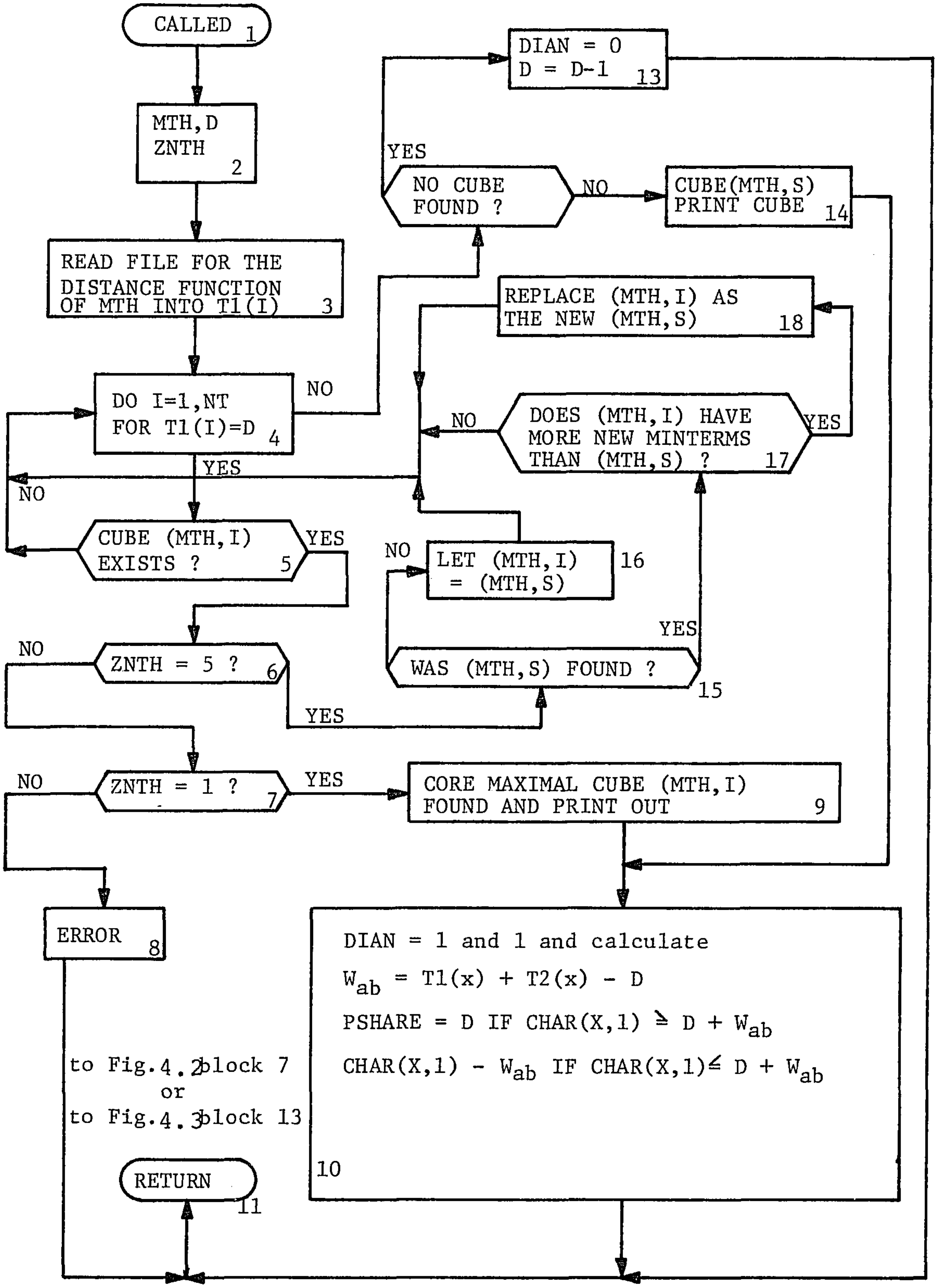


FIGURE 4.4 FLOW CHART OF SUBROUTINE CUBE
(Called by Fig.4.2 block 6, or Called by Fig 4.3 block 12)

4.2 Computer format and mechanism illustration. The computer program implemented in the Univac VMOS SYSTEM at New Jersey Institute of Technology can be illustrated as following:

1) Step 1. entering

the number of don't cares, and the number of variables.

Then enter all the minterms (on-set plus don't cares with don't cares after on-set as follow:

```
* 5    5    5
* 1    2    3    4    5    6    7    9    11    12    13
*14    15   17   20   23   24   25   27   28
*9
```

Step 2: The computer will calculate the distance function and store them in a disc file. Then calculate all the charateristic and point out

```
MAXIMUM CUBE DIMENSION    =    4
    1    0    0    0    2    1    0    0    3    3    1    0    4
    6    4    1

THE CHARACTERISTICS:
    4    1    2    1    4    1    4    1    4    1    4    1    4
    5    4    1    4    1    4    1    4    1    3    1    4    1
    2    1    2    1    1    1    2    1    3    5    2    1    3
    1
```

Step 3, the computer will start to search all the core cubes first; and point out, the relative probability of sharing (checking points), the accumulated awayness, the cube found and cube selected. When all the core cubes has been reached, it will then point out a statement of the fact and the maximum cube dimension of all the core cubes. A sample is as following:

```
CHECKING POINTS

      2 9999 9999     2     2 9999 9999     0     2     0     0     2
      2    0    0  9999  9999 9999 9999  9999

ACCUMULATED AWAYNESS:

      2    0    0     2     2    0    0     4     2     4     4     2
      2    4    4     0     0    0    0     0

CUBE   (  2   .   7  )  IS SELECTED.
CUBE   ( 14   .   5  )  HAS BEEN FOUND.

CHECKING POINTS

      4 9999 9999  9999  9999 9999 9999     2     4 9999 9999 9999
   9999    0    1  9999  9999 9999 9999  9999

ACCUMULATED AWAYNESS:

      4    0    0     2     2    0    0     6     4     4     4     2
      2    8    6     0     0    0    0     0

CUBE   ( 14   ,   5  )  IS SELECTED
CUBE   ( 17   ,   9  )  HAS BEEN FOUND.

CHECKING POINTS

   9999 9999 9999  9999  9999 9999 9999  9999     6 9999 9999 9999
   9999 9999    1  9999  9999 9999 9999  9999
```

```
ACCUMULATED AWAYNESS :

      4     0     0     2     2     0     0     6     6     4     4     2
      2     8    10     0     0     0     0     0

CUBE    (    17    ,    9    )  IS SELECTED.
CUBE    (    20    ,   12    )  HAS BEEN FOUND.


CHECKING POINTS

      9999 9999 9999 9999 9999 9999 9999 9999    6 9999 9999 9999
      9999 9999 9999 9999 9999 9999 9999 9999
```

```
ACCUMULATED AWAYNESS:

     4     0     0     2     2     0     0     6    12     4     4
     2     2     8    10     0     0     0     0     0

CUBE      (    20      ,     12      )  IS SELECTED

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS:  3
```

Step 4: after all the core cubes has been found, then it start to look for the rest of the minimum-set of maximal cube covering and prints out like this:

```
CUBE      (    11      ,      5      )  HAS BEEN FOUND

CHECKING POINTS

     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS:

     4     0     0     2     2     0     0     6    12     4     4
     2     2     8    10     0     0     0     0     0

CUBE      (    11      ,      5      )  IS SELECTED
**FORTRAN ** STOP
```

Notice the computer also prints out two up-dated value of characteristic. CHECKING POINTS are the relative probability of sharing. Accumulated Awayness is also printed out. This is for de-bugging and applications other than just the minimum set. For minimization only all this can be omitted. At this point, all the calculation is completed. And the results is output as follow:

```
Total Number of On-set and Dont*Cares =   20
ON-SET = 15   DONT-CARES =    5
NUMBERS OF VARIABLES = 5
```

```
      MINT.#  1  =  1  MINT.#  2  =  2  MINT.#  3  =  3
      MINT.#  4  =  4  MINT.#  5  =  5  MINT.#  6  =  6
      MINT.#  7  =  7  MINT.#  8  =  9  MINT.#  9  = 11
      MINT.# 10  = 12  MINT.# 11  = 13  MINT.# 12  = 14
      MINT.# 13  = 15  MINT.# 14  = 17  MINT.# 15  = 20
      MINT.# 16  = 23  MINT.# 17  = 24  MINT.# 18  = 25
      MINT.# 19  = 27  MINT.# 20  = 28  MINT.#

THE CUBE COVERING LIST:

The  1-TH CUBE COVERING IS:
    2-CUBE   =  (   2 ., 7.  )  =   00-1-

THIS CUBE COVERS THESE MINTERMS :
      2   7   3   6

THIS MINTERM MT(    2    )  =   2   IS A CORE POINT

THE  2-TH CUBE COVERING IS:
    3-CUBE   =  (  14  ,  5   )  =   0-1--

THIS CUBE COVERS THESE MINTERMS:

    14   5   4   6   7  12  13  15

THIS MINTERM MT(   12   )  =   14  IS A CORE POINT

THE 3-TH CUBE COVERING IS:
   2-CUBE   =  (  17  , 9    )  =  --001

THIS CUBE COVERS THESE MINTERMS:

    17   9   1   25

THIS MINTERM MT(   14   )  =   17  IS A CORE POINT

THE 4-TH CUBE COVERING IS:
   2-CUBE   =  (   20   , 12 )  =  --100

THIS CUBE COVERS THESE MINTERMS:

    20  12   4   28

THIS MINTERM MT(   15   )  =   20  IS A CORE POINT

THE 5-TH CUBE COVERING IS:
   3-CUBE   =   (  11   ,  5 )  =  0---1

THIS CUBE COVERS THESE MINTERMS:
    11   5   1   3    7  9  13  15
```

```
THE FINAL CHECKING

      205  101  205  205  205  205  205  205  105  205 205  101
      205  101  101  109  109  209  109  209
```

The final checking indicates what status of the maximal cube coverings and also checks whether all on-set is covered.

A second completed illustrative example is reproduced as following to show other features.

In the appendices, a collection of various example problems has been selected with the out-put result print out. For reference purpose and better understanding of the mechanics of the computer program. Those checking and debug out-put print out are also enclosed following some of the examples results out-put print.

Illustrative Examples 2.

```
FORTRAN IV PROGRAM LARGEE STARTED ---

6  9
  0     1     2     5     8
 13    18    19    28    34     37     39     47     56     57     61
  4    12    16    20    23     38     49     59     629

MAXIMUM CUBE DIMENSION  =   4

  1   0   0   0   2   1   0   0   3   3   1   0   4   6
  4   1

THE CHARACTERISTICS:
  3 5 2 1 3 1 3 5 2 1 2 1 3 1 2 1 2 1 2 1 2 1
  3 1 1 1 1 1 1 5 1 1 4 1 3 5 3 1 3 1 1 1 2 1
  1 1 1 1 0 0

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:    3

CUBE   (   1    ,    4    )  HAS BEEN FOUND.
```

```
CHECKING POINTS

      9999 9999    2 9999    1    1    0    0    0    0    1    0
         0    0    0    0 9999 9999 9999 9999 9999 9999 9999 9999
      9999

ACCUMULATED AWAYNESS:

         0    0    2    0    2    2    4    4    4    4    2    4
         6    6    6    6    0    0    0    0    0    0    0    0
         0

CUBE    (   1   ,    4   )  IS SELECTED.

CUBE    (   8   ,    4   )  HAS BEEN FOUND.

CHECKING POINTS

      9999 9999    3 9999 9999    2    0    0    1    0    1    0
         0    0    0    0 9999 9999 9999 9999 9999 9999 9999 9999
      9999

ACCUMULATED AWAYNESS:

         0    0    4    0    2    4    8   10    6    8    6   10
        12   10   12   12    0    0    0    0    0    0    0    0
         0

CUBE    (   8   ,    4   )  IS SELECTED.

CUBE    (  13   ,    4   )  HAS BEEN FOUND.

CHECKING POINTS

      9999 9999    3 9999 9999 9999    0    0    2    0    2    0
         0    0    0    0 9999 9999 9999 9999 9999 9999 9999 9999
      9999

ACCUMULATED AWAYNESS:

         0    0    8    0    2    4   14   16    8   14    8   14
        16   16   18   16    0    0    0    0    0    0    0    0
         0

CUBE    (   13   ,    4   )  IS SELECTED.

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:      2

CUBE    (   28   ,    4   )  HAS BEEN FOUND
```

```
 CHECKING POINTS
       9999 9999    3 9999 9999 9999    0    0 9999    0    2
          0    0    0    0    0 9999 9999 9999 9999 9999 9999
       9999 9999 9999
 ACCUMULATED AWAYNESS:
          0    0   12    0    2    4   18   22    8   20   12
         20   22   20   24   20    0    0    0    0    0    0
          0    0    0
 CUBE    (  28  ,   4  )    IS SELECTED.
 CUBE    (  47  ,  39  )    HAS BEEN FOUND.
 CHECKING POINTS
       9999 9999    3 9999 9999 9999    0    0 9999    0    2
       9999 9999    0    0    0 9999 9999 9999 9999 9999 9999
       9999 9999 9999
 ACCUMULATED AWAYNESS:
          0    0   18    0    2    4   26   28    8   24   14
         20   22   28   30   24    0    0    0    0    0    0
          0    0    0
 CUBE    (  47  ,  39  )    IS SELECTED.
 CUBE    (  56  ,  57  )    HAS BEEN FOUND.
 CHECKING POINTS
       9999 9999    3 9999 9999 9999    0    0 9999    0    2
       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
       9999 9999 9999
 ACCUMULATED AWAYNESS:
```

```
         0    0   36    0    2    4   40   40    8   38   24
        20   22   28   30   26    0    0    0    0    0    0
         0    0    0
CHECKING POINTS
      9999 9999    3 9999 9999 9999    0    0 9999    0    2
      9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
      9999 9999 9999
ACCUMULATED AWAYNESS:
         0    0   36    0    2    4   40   40    8   38   24
        20   22   28   30   26    0    0    0    0    0    0
         0    0    0
CUBE    (  61   ,  57   )    IS SELECTED.
ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :    2
CUBE    (  18   ,  0    )    HAS BEEN FOUND.
```

```
CHECKING POINTS

     9999 9999 9999 9999 9999 9999 9999    0 9999    0    2 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999

ACCUMULATED AWAYNESS:

        0    0   36    0    2    4   40   42    8   40    8   40
       22   28   30   26    0    0    0    0    0    0    0    0
       30   20
        0    0
        0

CUBE    (  18    ,    0    )  IS SELECTED
CUBE    (  34    ,    2    )  HAS BEEN FOUND.
CUBE    (  34    ,   38    )  HAS BEEN FOUND.
WATCH FOR    2    AND    38

CHECKING POINTS

     9999 9999 9999 9999 9999 9999 9999    0 9999 9999    2 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999

ACCUMULATED AWAYNESS:

        0    0   36    0    2    4   40   48    8   40   40   20
       22   28   30   26    0    0    0    0    0    0    0    0

CUBE    (  19    ,   23    )  IS SELECTED
CUBE    (  37    ,    5    )  HAS BEEN FOUND.

CUBE    (  37    ,   39    )  HAS BEEN FOUND.

CHECKING POINTS

     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999

ACCUMULATED AWAYNESS:

        0    0   36    0    2    4   40   48    8   40   40   20
       22   28   30   26    0    0    0    0    0    0    0    0
        0
```

```
CUBE    (  37    ,    5    )  IS SELECTED.

**FORTRAN ** STOP


TOTAL NUMBER OF ON-SET AND DONT*CARES =    25
ON-SET  =    16    DONT-CARES   =    9
NUMBERS OF VARIABLES - 6

      MINT.#  1   =  0  MINT.#  2   =   1  MINT.#  3   =  2
      MINT.#  4   =  5  MINT.#  5   =   8  MINT.#  6   = 13
      MINT.#  7   = 18  MINT.#  8   =  19  MINT.#  9   = 28
      MINT.# 10   = 34  MINT.# 11   =  37  MINT.# 12   = 39
      MINT.# 13   = 47  MINT.# 14   =  56  MINT.# 15   = 57
      MINT.# 16   = 61  MINT.# 17   =   4  MINT.# 18   = 12
      MINT.# 19   = 16  MINT.# 20   =  20  MINT.# 21   = 23
      MINT.# 22   = 38  MINT.# 23   =  49  MINT.# 24   = 59
      MINT.# 25   = 62  MINT.#

MINTERM #   25   =   62 IS ISOLATED AND ESSENTIAL
```

```
THE CUBE COVERING LIST:


THE    1-TH CUBE CORERING IS :
     2-CUBE = (  1   ,    4   ) =    000-0-

THIS CUBE COVERS THESE MINTERMS :
      1      4      0      5

THIS MINTERM MT(  2  )  =    1   IS A CORE POINT

THE    2-TH CUBE COVERING IS :
     2-CUBE = (  8   ,    4   ) =    00--00

THIS CUBE COVERS THESE MINTERMS :
      8      4      0     12

THIS MINTERM MT(  5  )  =    8   IS A CORE POINT

THE    3-TH CUBE COVERING IS :
     2-CUBE = ( 13   ,    4   ) =    00-10-

THIS CUBE COVERS THESE MINTERMS :
     13      4      5     12

THIS MINTERM MT(  6  )  =   13   IS A CORE POINT


THE    4-TH CUBE COVERING IS :
     2-CUBE = ( 28   ,    4   ) =    0--100

THIS CUBE COVERS THESE MINTERMS :
     28      4     12     20

THIS MINTERM MT(  9  )  =   28   IS A CORE POINT

THE    5-TH CUBE COVERING IS :
     1-CUBE = ( 47   ,   39   ) =    10-111

THIS CUBE COVERS THESE MINTERMS :
     47    39

THIS MINTERM MT( 13  )  -   47   IS A CORE POINT

THE    6-TH CUBE COVERING IS :
     1-CUBE = (  56   ,   57   )   = 11100-
```

```
THIS CUBE COVERS THESE MINTERMS :
     56   57

THIS MINTERM MT(  14  )  =   56   IS A CORE POINT


THE   7-TH CUBE COVERING IS:
     1-CUBE =  (  61   ,  57  )  =   111-01

THIS CUBE COVERS THESE MINTERMS :
     61   57

THIS MINTERM MT(  16  )  =   61   IS A CORE POINT


THE   8-TH CUBE COVERING IS :
     2-CUBE =  (  18  ,    0  )  =   0-00-0


THIS CUBE COVERS THESE MINTERMS :
     18   0   2   16


THE   9-TH CUBE COVERING IS :
     1-CUBE =  (  34   ,  38  )  =   100-10


THIS CUBE COVERS THESE MINTERMS :
     34   38


THE  10-TH CUBE COVERING IS :
     1-CUBE =  (  19   ,  23  )  =   010-11


THIS CUBE COVERS THESE MINTERMS :
     19   23


THE  11-TH CUBE COVERING IS :
     1-CUBE =  (  37   ,  5  )   =   00101


THIS CUBE COVERS THESE MINTERMS :
     37   5
```

```
THE FINAL CHECKING

   105  101  105  105  101  101  105  105  101  105  105  105
   101  101  205  101  105  105  105  105  209  209  109  109
   109
```

I) The tracing of cyclic cubes:

The computer is capable to trace out the cyclic cubes as shown in the follwing example.

```
TOTAL NUMBER OF ON-SET AND DONT*CARES =    14
ON-SET =    14   DONT=CARES =    0
NUMBERS OF VARIABLES =    5


MINT.#  1  =  1  MINT.#  2  =  4  MINT.#  3  =  9
MINT.#  4  = 11  MINT.#  5  = 12  MINT.#  6  = 27
MINT.#  7  = 26  MINT.#  8  = 29  MINT.#  9  = 28
MINT.# 10  = 18  MINT.# 11  = 22  MINT.# 12  = 23
MINT.# 13  = 21  MINT.# 14  =  0  MINT.#
```

```
THE CUBE COVERING LIST:


THE   1-TH CUBE COVERING IS :
     1-CUBE =  (  0   ,  4  )  =   00-00


THIS CUBE COVERS THESE MINTERMS :
     4   0


THE   2-TH CUBE COVERING IS :
     1-CUBE =  (  12   ,  28  )= --1100


THIS CUBE COVERS THESE MINTERMS :
    12  28


THE   3-TH CUBE COVERING IS :
     1-CUBE =  (  21   ,  29  )  = 1-101


THE   4-TH CUBE COVERING IS :
     1-CUBE =  (  22   ,  23  )  = 1011-


THIS CUBE COVERS THESE MINTERMS :
    22  23


THE   5-TH CUBE COVERING IS :
     1-CUBE =  (  18   ,  26  )  = 1-010


THIS CUBE COVERS THESE MINTERMS :
    26  18


THE   6-TH CUBE COVERING IS :
     1-CUBE =  (  27   ,  11  )  = -1011


THIS CUBE COVERS THESE MINTERMS :
    11  27
```

```
THE   7-TH CUBE COVERING IS :
      1-CUBE =  (  1   ,  9  )  =   0-001


THIS CUBE COVERS THESE MINTERMS :
      1   9


THE FINAL CHECKING

   105  105  105  105  105  105  105  105  105  105  105  105
   105  105
```

The way it trace out of the cyclic cubes is shown in the checking print out of the computer as follow:

```
* 5
*    1    4    9    11    12    27    26    29    28    18    22    23    21
*9


MAXIMUM CUBE DIMENSION =    3


     1    0    0   2    1    0    3    3    1

THE CHARACTERISTICS :

     2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2 1
     2  1  2  1  2  1


MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:   2

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:   1

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :   1

CUBE    (   0    ,    1    )    HAS BEEN FOUND.

CUBE    (   0    ,    4    )    HAS BEEN FOUND.
```

```
WATCH FOR   1   AND   4


CHECKING POINTS

     0 9999    0    0    0    0    0    0    0    0    0    0
     0 9999



ACCUMULATED AWAYNESS :
     2    0    4    6    2    8    6    6    4    4    4    6
     4    0

CUBE   (  0   ,  4   )  IS SELECTED.
CUBE   ( 12   ,  4   )  HAS BEEN FOUND.
CUBE   ( 12   , 28   )  HAS BEEN FOUND.

CHECKING POINTS
     0 9999    0    0 9999    0    0    0 9999    0    0    0
     0 9999

ACCUMULATED AWAYNESS :
     8    2    8   12    2   14   10    8    4   10    8   12
     8    4

CUBE   ( 12   , 28   )  IS SELECTED.
CUBE   ( 21   , 29   )  HAS BEEN FOUND.
CUBE   ( 21   , 23   )  HAS BEEN FOUND.

WATCH FOR   29   AND   23

CHECKING POINTS

     0 0000    0    0 9999    0    0 9999 9999    0    0    0
  9999 9999
```

```
ACCUMULATED AWAYNESS :

     12    6   12   18    6   18   16    8    6   16   12   14
      8   10

CUBE   (  21   ,  29   )  IS SELECTED.
CUBE   (  22   ,  18   )  HAS BEEN FOUND.
CUBE   (  22   ,  23   )  HAS BEEN FOUND.

WATCH FOR   18   AND   23

CHECKING POINTS

     0 9999    0    0 9999    0    0 9999 9999    0 9999 9999
  9999 9999

ACCUMULATED AWAYNESS :

     24   16   26   28   18   24   20   18   14   18   14   18
     16   20

CUBE   (  18   ,  26   )  IS SELECTED.

CUBE   (  27   ,  11   )  HAS BEEN FOUND.

CUBE   (  27   ,  26   )  HAS BEEN FOUND.
```

```
CHECKING POINTS

      0 9999    0 9999 9999 9999 9999 9999 9999 9999 9999 9999
   9999 9999

ACCUMULATED AWAYNESS :

      28   24   28   28   24   24   22   22   20   22   20   22

      22   26

CUBE   (  27   ,  11   )   IS SELECTED.

CUBE   (   1   ,   9   )   HAS BEEN FOUND.

CUBE   (   1   ,   0   )   HAS BEEN FOUND.

CHECKING POINTS

      9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
      9999 9999


ACCUMULATED AWAYNESS :

      28   28   28   30   28   28   28   26   26   28   28   28
      26   28


CUBE   (   1   ,   9   )  IS SELECTED.
```

2) To find out isolated minterms :

```
 -4
  0   3   5   6  12  15   9  109


MAXIMUM CUBE DIMENSION =   3

  1   0   0   2   1   0   3   3   1

THE CHARACTERISTICS :

  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:   2

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:   1

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:   1

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:   0

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :   0

*9
 **FORTRAN ** STOP


TOTAL NUMBER OF ON-SET AND DONT*CARES =   8
ON-SET =   8   DONT=CARES =   0
NUMBERS OF VARIABLES =   4

     MINT.#  1  =  0  MINT.#  2  =  3  MINT.#  3  =  5
     MINT.#  4  =  6  MINT.#  5  = 12  MINT.#  6  = 15
     MINT.#  7  =  9  MINT.#  8  = 10  MINT.#

MINTERM # 1  =  0 IS ISOLATED AND ESSENTIAL

MINTERM # 2  =  3 IS ISOLATED AND ESSENTIAL

MINTERM # 3  =  5 IS ISOLATED AND ESSENTIAL

MINTERM # 4  =  6 IS ISOLATED AND ESSENTIAL

MINTERM # 5  = 12 IS ISOLATED AND ESSENTIAL
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MINTERM # 6  =  15 IS ISOLATED AND ESSENTIAL

MINTERM # 7  =   9 IS ISOLATED AND ESSENTIAL

MINTERM # 8  =  10 IS ISOLATED AND ESSENTIAL
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4.3 CONCLUSION:

The computer programming was done on a relatively slow and limited storage teaching computer to demonstrate the theory of minimal maximal cube coverings. It is possible to improve the efficiency and enlarge the scope of the programs with a larger and faster computer. It is also better implemented with fortran 77, in which case the two assembly subroutines can be replaced with fortran 77 subroutines.

The many extra features in the programming demonstrate the applicability in other areas of switching theory. The examples selected in appendix II, are typical to its size of on-set number of don't cares, and number of variables. The time used on CPU is not optimized because of the need of checking out all the steps and understanding the computer with various real examples. Further research is directed to develop a whole series of canned programs to do all the logic design on computers by optimizing the programming.

CHAPTER V

EXTENSION OF THE MINIMUM MAXIMAL-CUBE THEORY TO HIGHER CUBES AND OTHER KIND OF SWITCHING FUNCTIONS

5.1 EXTENSION TO HIGHER DIMENSIONED CUBES

In chapter II, section 4, we had discussed the distance of cubes higher than 0.(Ref. to definition 24 and theorem 19). It is natural for one try to generalizing the theory into higher dimensioned cubes and to try the minimization or decomposition of functions having different dimensioned cubes. In chapter II, we studied the interrelationship between cubes and found that not only the distance between cubes played an important part but also the skewness of cubes. The reason we did not use the skewness in setting up distance matrix for 0-cubes is because the awayness theorems(theorem 18 and theorem 19) indicated that skewness is always zero for 0-cubes. To handle higher cubes, one has to set up a complex matrix C with entries of a pair (d,s) instead of a single number, where d = the distance between cubes, and s = the skewness between cubes. This was followed by a complex matrix D with entries of a pair (f_d,f_s) where f_d = the frequency of distances and f_s = the frequency of skewness. After setting up matrices C and D, one thencan establish the kind of charateristics one needs for minimization or decomposition. It appears to be very promising for further research work.

5.2 EXTENSION TO OTHER KIND OF SWITCHING FUNCTIONS

Since the whole theory started from a metric space, it is obvious to consider whatever the space and metric of a particular switching functions would able to render a similar kind of theory like what we have developed in chapter II. In other words, the space Q^n can be the cartesian product of a prime number or the power of a prime number, thus almost all kind of multi-valued logic or Galois switching fuctions are included. However, the metric of the space other than olean is usually not unique and simple, and therefore quite cumbersome to work with. Recent research indicates that ternarylogics and Boolean Galois switching functions are among the most promising kind other than binary (or Boolean) logic. The metric of these functions has been explored extensively by coding theoreticians. We speculat from this vantage point that the maximal-cube covering approach will play important role in the coding theory. It is conceivable to discover some new codes or to re-interpret some old codes. In any event, the only limitation of extending the theory of maximal-cube covering is ones own imagination.

5.3 DISCUSSION

It is interesting to note that although the original purpose of developing the algebraic structure of maximal-cube coveringwas to manipulate Boolean fuctions, however it approaches its own mathematical form and structure that warrants the study of its own. Further study in the interest of switching theory or coding theory or just abstract algebra are being pursued by the author of this dissertation; the result reported in this dissertation representing the original goal of the authors research subject.

CHAPTER VI

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary

In this dissertation, we have systematically developed the Minimum Maximal-Cube Covering approach to handle switching circuits and its extended application into other areas. The work can be summarized as follows:

(a) We have looked into the intrinsic characteristics of switching circuits by chronologically surveying previous work and comparing and analyzing various approaches thoroughly. The results have led to a defined set of most desirable characteristics for simplifying and realizing switching functions with the most economical circuits.

(b) We have developed the algebraic structure to facilitate the Minimum Maximal-Cube Covering approach to switching theory.

(c) The application of a branching graph to two-level minimization of switching circuits was fully illustrated with detailed examples. Other switching problems were also treated with the branching graph method.

(d) An alternative method of using algebraic-geometric properties of the Minimum Maximum-Cube Covering was also developed. The application of this method into switching circuits was illustrated with various examples. Since this method turns out to be better to implement with a computer program, demonstrative computer programming was carried out in detail to verify the validity of the intrinsic merits of computerization. Because of limited resources, we were able to use only the Univac Spectra 70 VMOS at New Jersey Institute of Technology. No attempt was made to optimize programming.

(e) Exploratory work has been carried into other fields of interest in Chapter Five.

6.2 Recommendations

(a) Due to lack of resources, we were not able to fully develop the complete set of computer programs for digital system usage in industry and for educational purposes in universities. We would urgently recommend this development using the theory and technique developed in this dissertation.

(b) The extension of the Minimum Maximal-Cube Covering into other fields of interest was only lightly explored. Further work should also be attempted.

6.3 Conclusions

(a) The mathematical properties of the Minimum Maximal-Cube Covering, in addition to their original purpose of facilitating handling of switching circuits and digital systems, are quite interesting and rich in their own right. In the area of Boolean algebra and the area of lattice theory, Minimum Maximal-Cube Covering seems to offer a very fresh, powerful new tool.

(b) With the ever-increasing availability of microprocessors and LSI's, hardware realized instruments and machines using Minimum Maximal-Covering principle are quite conceivable.

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APPENDIX A COMPUTER PROGRAMS

1. PROGRAM LARGE
2. SUBROUTINE CUBESD
3. PROGRAM DISTBL
4. PROGRAM DIGBIT
5. ASSEMBLY SUBROUTINE NDIST
6. ASSEMBLY SUBROUTINE BICHA

FORTRAN IV (VER 45) SOURCE LISTING:

```
      PROGRAM LARGE
      COMMON CHAR(520,4),MT(520),T1(520),T2(520),OUTA(8),NV4,NT,RW
     %,CUBEN,DIAN,MTH,ZNTH
      DIMENSION B(9),BINO(9,9),TEMT(1040)
      EQUIVALENCE (TEMT(1),T1(1))
      DEFINE FILE 28(520,520,E,RW)
      IMPLICIT  INTEGER (A-Z)
    1 READ(5,2,END=1000)NV,DONT,ZERO
    2 FORMAT(3I3)
      IF(NV.GE.900)GO TO 1000
      NV4=NV/4.0+0.8
      CALL NVARIS(NV)
      CALL BICHA(NV)
      DO 502 I=1,1040
  502 TEMT(I)=0
      DO 4 U=1,65
      VI=(U-1)*16
      VA=VI+1
      VB=VI+16
      READ(5,3)(TEMT(V),V=VA,VB)
    3 FORMAT(8I10)
      DO 4 V=VA,VB
      IF(TEMT(V).LT.2147483640)GO TO 4
      NTEM=V
      GO TO 5
    4 CONTINUE
    5 K=1
      DO 9 J=1,NTEM
      MT(K)=TEMT(J)
      IF(ZERO.EQ.0)GO TO 6
      IF(MT(K).EQ.0)GO TO 9
    6 J6=K-1
      IF(K.EQ.1)GO TO 8
      DO 7 I=1,J6
      IF(MT(I).EQ.MT(K))GO TO 9
    7 CONTINUE
    8 K=K+1
    9 CONTINUE
   10 NT=K-2
      CT=NT-DONT
      CUBEN=0
      PRINT 11,NT,CT,DONT,NV
   11 FORMAT('1TOATAL NUMBER OF ON-SET AND DONT-CARES = '
     %,I5,/,' ON-SET = ',I5,5X,' DONT-CARES = ',I5
     %,/,' NUMBERS OF VARIABLES = ',I5)
      IF(CT.LE.0)GO TO 1
      RW=1
      MAXCUB=1
      DO 12 I=1,10
      MAXCUB=MAXCUB*2
```

```
      IF(MAXCUB.GT.NT)GO TO 14
   12 CONTINUE
   14 MAXCUB=I-1
      BINO(1,1)=1
      BINO(2,1)=2
      BINO(2,2)=1
      DO 142 I=2,MAXCUB
      I1=I-1
      BINO(I,1)=I
      DO 141 J=2,I1
      J1=J-1
  141 BINO(I,J)=BINO(I1,J1)+BINO(I1,J)
  142 BINO(I,I)=1
      WRITE(2,145)MAXCUB,((BINO(I,J),J=1,MAXCUB),I=1,MAXCUB)
  145 FORMAT('1MAXIMUM CUBE DIMENSION = ',I3,//,(14I5))
      PRINT 30,((I,MT(I)),I=1,NT)
   30 FORMAT(/,2(5X,'MINT.#',I4,'  =  ',I10))
      DO 45 I=1,NT
      CHAR(I,1)=0
      CHAR(I,2)=0
      CHAR(I,3)=0
      CHAR(I,4)=0
   45 CONTINUE
      ZEN=0
      DO 100 I2=1,NT
      DO 51 I=1,9
   51 B(I)=0
      DO 60 I1=1,NT
      IF(I1.NE.I2)GO TO 56
      MTD=0
      GO TO 60
   56 CALL NDIST(MTD,MT(I1),MT(I2))
      IF(MTD.LE.MAXCUB)B(MTD)=B(MTD)+1
      IF(MTD.LE.9)GO TO 60
      MTD=9
   60 T1(I1)=MTD
      WRITE(28'RW,61)(T1(I),I=1,NT)
   61 FORMAT(200I1,200I1,120I1)
      IF(B(1).EQ.0)GO TO 85
      B1=MAXCUB
      B2=B1+1
      DO 80 J11=1,B1
      J1=B2-J11
      DO 70 J2=1,J1
      IF(B(J2).LT.BINO(J1,J2))GO TO 80
   70 CONTINUE
      CHAR(I2,1)=J1
      IF(B(1).NE.BINO(J1,1))GO TO 72
      CHAR(I2,2)=1
      GO TO 100
```

```
 72 CHAR(I2,2)=5
    GO TO 100
 80 CONTINUE
 85 MINT=MT(I2)
    PRINT 90,I2,MINT
 90 FORMAT('0MINTERM #',I4,5X,'=',I10,' IS ISOLATED AND ESSEN
100 CONTINUE
    PRINT 98
 98 FORMAT('1THE CUBE COVERING LIST :',//)
    WRITE(2,101)((CHAR(I,J),J=1,2),I=1,NT)
101 FORMAT('0THE CHARACTERISTICS :',/,(5X,22I3))
    CHECK=0
    IF(CT.GE.NT)GO TO 102
    DO 103 I=1,DONT
    X=CT+I
    CHAR(X,3)=9999
    CHAR(X,2)=109
103 CONTINUE
102 MAX=2**MAXCUB
    KOUNT=0
    DO 105 I=1,NT
    IF(CHAR(I,1).NE.MAXCUB)GO TO 105
    KOUNT=KOUNT+1
105 CONTINUE
    IF(KOUNT.GE.MAX)GO TO 107
    DO 106 I=1,NT
    IF(CHAR(I,1).NE.MAXCUB)GO TO 106
    CHAR(I,1)=CHAR(I,1)-1
    CHAR(I,2)=5
106 CONTINUE
    MAXCUB=MAXCUB-1
    WRITE(2,108)MAXCUB
108 FORMAT('0MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:',I10
    GO TO 102
107 CONTINUE
    IF(ZEN.EQ.1)GO TO 125
    DO 120 I6=1,NT
    IF(CHAR(I6,2).NE.1)GO TO 120
    D=CHAR(I6,1)
    MTH=I6
    ZNTH=1
    CALL CUBESD(ZNTH,MTH,D)
    IF(DIAN.NE.0)GO TO 119
    CHAR(MTH,1)=D-1
    CHAR(MTH,2)=5
    IF(D.NE.MAXCUB)GO TO 120
    IF(CHECK.NE.0)GO TO 120
    GO TO 102
119 IF(D.NE.MAXCUB)GO TO 120
    IF(CHECK.NE.0)GO TO 120
```

FORTRAN IV (VER 45) SOURCE LISTING: LARGE   PROGRAM

```
      CHECK=1
  120 CONTINUE
      ZEN=1
      WRITE(2,122)MAXCUB
  122 FORMAT('0ALL CORE CUBES HAVE BEEN SEARCHED.'
     %,/,' AND THE MAXIMUM CUBE DIMENSION IS :',I5)
  125 B1=MAXCUB
      B2=B1+1
      DO 280 J11=1,B1
  140 J1=B2-J11
      DO 180 J2=1,NT
      IF(CHAR(J2,1).NE.J1)GO TO 180
      IF(CHAR(J2,2).NE.5)GO TO 180
      PSHARE=CHAR(J2,3)
      CLOSE=CHAR(J2,4)
      MTH=J2
      D=J1
      DO 170 I6=1,NT
      IF(I6.EQ.J2)GO TO 170
      IF(CHAR(I6,1).NE.J1)GO TO 170
      IF(CHAR(I6,2).NE.5)GO TO 170
      IF(CHAR(I6,3).GT.PSHARE)GO TO 170
      IF(CHAR(I6,3).EQ.PSHARE)GO TO 150
 160  PSHARE=CHAR(I6,3)
      MTH=I6
      GO TO 170
  150 CONTINUE
      IF(CHAR(I6,4).GT.CLOSE)GO TO 170
      CLOSE=CHAR(I6,4)
      GO TO 160
  170 CONTINUE
      ZNTH=5
      CALL CUBESD(ZNTH,MTH,D)
      IF(DIAN.NE.0)GO TO 175
      CHAR(MTH,1)=D-1
      IF(CHECK.NE.0)GO TO 180
      IF(D.NE.MAXCUB)GO TO 180
      CHECK=CHECK+1
      GO TO 102
  175 IF(D.NE.MAXCUB)GO TO 180
      IF(CHECK.NE.0)GO TO 180
      CHECK=1
  180 CONTINUE
      DO 280 J6=1,NT
      IF(CHAR(J6,1).NE.J1)GO TO 280
      IF(CHAR(J6,2).EQ.5)GO TO 140
  280 CONTINUE
      PRINT 300,(CHAR(I,2),I=1,NT)
  300 FORMAT('0THE FINAL CHECKING ',/,(10X,12I5))
      GO TO 1
```

FORTRAN IV (VER 45) SOURCE LISTING: LARGE PROGRAM

```
 201  1000 STOP
 202       END
```

```
      SUBROUTINE CUBESD(ZNTH,MTH,D)
      IMPLICIT  INTEGER (A-Z)
      COMMON CHAR(520,4),MT(520),T1(520),T2(520),OUTA(8),NV4,NT,RW
     %,CUBEN,DIAN,MTH,ZNTH
      DIMENSION T(520),S(520)
      DEFINE FILE 28(520,520,E,RW)
      CHK=2**D
      RW=MTH
      READ(28'RW,125)(T1(I),I=1,NT)
  125 FORMAT(200I1,200I1,120I1)
      MINT=MT(MTH)
      IF(D.NE.9)GO TO 140
      DO 130 I=1,NT
      IF(T1(I).LT.9)GO TO 130
      CALL NDIST(MTD,MINT,MT(I))
      T1(I)=MTD
  130 CONTINUE
  140 IR=MTH
      DIAN=0
      RAWMT=0
      S3=NT+1
      CHAR(S3,4)=9000
      T(1)=IR
      DO 200 J3=1,NT
      IF(T1(J3).NE.D)GO TO 200
      IF(CHAR(J3,1).LT.D)GO TO 200
      RW=J3
      READ(28'RW,125)(T2(I),I=1,NT)
      T(2)=J3
      IT=2
      DO 160 J4=1,NT
      IF(T1(J4).GE.D)GO TO 160
      IF(T2(J4).GE.D)GO TO 160
      LD=T1(J4)+T2(J4)
      IF(LD.NE.D)GO TO 160
      IT=IT+1
      T(IT)=J4
  160 CONTINUE
      IF(CHK.NE.IT)GO TO 200
      WRITE(2,162)MT(IR),MT(J3)
  162 FORMAT('0CUBE   (',I10,'  ,',I10,'  )   HAS BEEN FOUND.')
      IS=IT
      DIAN=1
      IF(ZNTH.EQ.5)GO TO 165
      IF(ZNTH.EQ.1)S3=J3
      GO TO 300
  165 COUNT=0
      DO 170 I=1,IS
      TE=T(I)
      IF(CHAR(TE,2).GT.10)GO TO 170
```

```
      COUNT=COUNT+1
170   CONTINUE
      IF(COUNT.LT.RAWMT)GO TO  200
      IF(COUNT.EQ.RAWMT)GO TO  175
172   S3=J3
      RAWMT=COUNT
      DO 180 I=1,IS
      S(I)=T(I)
180   CONTINUE
      GO TO 200
175   CONTINUE
      WRITE(2,173)MT(S3),MT(J3)
173   FORMAT('0WATCH FOR ',I10,' AND ',I10)
      IF(CHAR(J3,4).GT.CHAR(S3,4))GO TO 200
      GO TO 172
200   CONTINUE
      IF(DIAN.EQ.0)GO TO 700
205   RW=S3
      READ(28'RW,125)(T2(I),I=1,NT)
      DO 210 I=1,IS
      TF=S(I)
      CHAR(TF,2)=CHAR(TF,2)+100
      CHAR(TF,3)=9999
210   CONTINUE
      DO 220 I=1,NT
      IF(CHAR(I,3).EQ.9999)GO TO 220
      WAB=T1(I)+T2(I)-D
      CHAR(I,4)=WAB+CHAR(I,4)
      DA=CHAR(I,1)
      IF(DA.LT.WAB)GO TO 220
      G=DA-WAB
      G1=G
      IF(G.GE.D)G1=D
      SHARE=2**G1
      CHAR(I,3)=CHAR(I,3)+SHARE
220   CONTINUE
      WRITE(2,212)(CHAR(I,3),I=1,NT)
212   FORMAT('0CHECKING POINTS',/,(10X,12I5))
      WRITE(2,216)(CHAR(I,4),I=1,NT)
216   FORMAT('0ACCUMULATED AWAYNESS :',//,(10X,12I5))
      CUBEN=CUBEN+1
      PRINT 228,CUBEN
228   FORMAT(///,'0THE ',I5,'-TH CUBE COVERING IS :')
      CALL BICHA2(MT(IR),MT(S3),OUTA)
      PRINT 230,D,MT(IR),MT(S3),(OUTA(O),O=1,NV4)
230   FORMAT(5X,I4,'-CUBE = (',I10,' ,',I10,'  ) =',5X,8A4)
      WRITE(2,222)MT(IR),MT(S3)
222   FORMAT('0CUBE   (',I10,'  ,',I10,'  )   IS SELECTED.')
      DO 232 I=1,IS
      STEM=S(I)
```

```
FORTRAN IV (VER 45 ) SOURCE LISTING: CUBESD  SUBROUTINE

          S(I)=MT(STEM)
      232 CONTINUE
          PRINT 235,(S(I),I=1,IS)
      235 FORMAT('0THIS CUBE COVERS THESE MINTERMS : ',/,(5X,6I10))
          IF(ZNTH.EQ.5)GO TO 700
          IF(ZNTH.NE.1)GO TO 680
          PRINT 240,IR,MT(IR)
      240 FORMAT('0THIS MINTERM MT(',I4,'  ) = ',I10,5X,'IS A CORE PO
          GO TO 700
      300 DO 320 I=1,IS
          S(I)=T(I)
      320 CONTINUE
          GO TO 205
      680 PRINT 685
      685 FORMAT('0THERE IS AN ERROR IN THE PROGRAM!!!')
      700 RETURN
          END
```

FORTRAN IV (VER 45) SOURCE LISTING:

```
        PROGRAM DISTBL
        COMMON MT(520),T1(520),T2(520),OUTA(8),NV4,NT,RW
       %,CUBEN,DIAN,MTH,ZNTH
        DIMENSION B(9),TEMT(1040)
        EQUIVALENCE (TEMT(1),T1(1))
        IMPLICIT  INTEGER (A-Z)
      1 READ(5,2,END=1000)NV,DONT,ZERO
      2 FORMAT(3I3)
        IF(NV.GE.900)GO TO 1000
        NV4=NV/4.0+0.8
        CALL NVARIS(NV)
        CALL BICHA(NV)
        DO 502 I=1,1040
    502 TEMT(I)=0
        DO 4 U=1,65
        VI=(U-1)*16
        VA=VI+1
        VB=VI+16
        READ(5,3)(TEMT(V),V=VA,VB)
      3 FORMAT(16I5)
        DO 4 V=VA,VB
        IF(TEMT(V).LT.90000)GO TO 4
        NTEM=V
        GO TO 5
      4 CONTINUE
      5 K=1
        DO 9 J=1,NTEM
        MT(K)=TEMT(J)
        IF(ZERO.EQ.0)GO TO 6
        IF(MT(K).EQ.0)GO TO 9
     6  J6=K-1
        IF(K.EQ.1)GO TO 8
        DO 7 I=1,J6
        IF(MT(I).EQ.MT(K))GO TO 9
      7 CONTINUE
      8 K=K+1
      9 CONTINUE
     10 NT=K-2
        CT=NT-DONT
        CUBEN=0
        PRINT 11,NT,CT,DONT,NV
     11 FORMAT('1TOATAL NUMBER OF ON-SET AND DONT*CARES = '
       %,I5,/,' ON-SET = ',I5,5X,' DONT-CARES = ',I5
       %,/,' NUMBERS OF VARIABLES = ',I5)
        IF(CT.LE.0)GO TO 1
        DO 100 I2=1,NT
        PRINT 51,I2,MT(I2)
     51 FORMAT('0THE DISTANCE FUNCTION FOR MINTERM #',I5,' = ',I5
       %,' IS :',//)
        DO 52 I=1,9
```

FORTRAN IV (VER 45) SOURCE LISTING: DISTBL PROGRAM

```
   52 B(I)=0
      DO 60 I1=1,NT
      IF(I1.NE.I2)GO TO 56
      MTD=0
      GO TO 60
   56 CALL NDIST(MTD,MT(I1),MT(I2))
      B(MTD)=B(MTD)+1
      IF(MTD.LE.9)GO TO 60
      MTD=9
   60 T1(I1)=MTD
      PRINT 61,((I,T1(I)),I=I2,NT)
   61 FORMAT(5(2X,'T(',I4,') = 'I1),/)
      PRINT 62,(B(I),I=1,9)
   62 FORMAT(//,' THE DISTANCE DISTRIBUTION ARE :',/,9I5,/////)
  100 CONTINUE
      GO TO 1
 1000 STOP
      END
```

FORTRAN IV (VER 45) SOURCE LISTING:

```
      PROGRAM DIGBIT
      COMMON CHAR(520,4),MT(520),T1(520),T2(520),OUTA(8),NV4,NT,PW
     %,CUBEN,DIAN,MTH,ZNTH
      DIMENSION B(9),BIND(9,9),TEMT(1040)
      EQUIVALENCE (TEMT(1),T1(1))
      IMPLICIT  INTEGER (A-Z)
    1 READ(5,2,END=1000)NV,DONT,ZERO
    2 FORMAT(3I3)
      IF(NV.GE.900)GO TO 1000
      NV4=NV/4.0+0.8
      CALL NVARIS(NV)
      CALL BICHA(NV)
      DO 502 I=1,1040
  502 TEMT(I)=0
      DO 4 U=1,65
      VI=(U-1)*16
      VA=VI+1
      VB=VI+16
      READ(5,3)(TEMT(V),V=VA,VB)
    3 FORMAT(16I5)
      DO 4 V=VA,VB
      IF(TEMT(V).LT.90000)GO TO 4
      NTEM=V
      GO TO 5
    4 CONTINUE
    5 K=1
      DO 9 J=1,NTEM
      MT(K)=TEMT(J)
      IF(ZERO.EQ.0)GO TO 6
      IF(MT(K).EQ.0)GO TO 9
    6 J6=K-1
      IF(K.EQ.1)GO TO 8
      DO 7 I=1,J6
      IF(MT(I).EQ.MT(K))GO TO 9
    7 CONTINUE
    8 K=K+1
    9 CONTINUE
   10 NT=K-2
      CT=NT-DONT
      CUBEN=0
      PRINT 11,NT,CT,DONT,NV
   11 FORMAT('1TOATAL NUMBER OF ON-SET AND DONT*CARES = '
     %,I5,/,' ON-SET = ',I5,5X,' DONT-CARES = ',I5
     %,/,' NUMBERS OF VARIABLES = ',I5)
      IF(CT.LE.0)GO TO 1
      DO 21 I=1,NT
      CALL BICHA1(MT(I),OUTA)
      PRINT 20,I,MT(I),(OUTA(J),J=1,NV4)
   20 FORMAT(5X,'MINTERM #',I4,' = ',I4,' IS ',5X,8A4)
   21 CONTINUE
```

FORTRAN IV (VER 45) SOURCE LISTING: DIGBIT PROGRAM

```
    51          GO TO 1
    52   1000   STOP
    53          END
```

```
          TITLE 'NDIST--A BINARY DISTANCE FINDING FUNCTION--OCT. 75'
*
*         FOR FORTRAN PROGRAM. FOLLOW THE FOLLOWING TWO STEPS:
*    1. GIVING THE # OF VARIABLES,N, BY CALL NVARIS(N)
*       (NOTE: THIS N WILL BE USED UNTIL NEXT CALL NVARIS. DEFAULT N=3
*    2. FINDING THE DISTANCE OF 2 VECTORS,I,J, BY FUNCTION NDIST(I,J)
*
          SPACE     3
NVARIS    CSECT
          USING     NVARIS,15
          L         1,0(0,1)
          L         1,0(0,1)
          ST        1,NV
          LNR       1,1
          A         1,N32
          STH       1,PRESHF+2
          MVI       12(13),X'FF'
          BR        14
          SPACE     3
          ENTRY     NDIST
          USING     NDIST,15
NDIST     STM       1,5,24(13)
          LM        3,4,4(1)
          L         5,0(0,1)
          L         2,0(0,3)
          L         3,0(0,4)
          XR        3,2
PRESHF    SLL       3,0
          SLR       1,1
          L         4,NV
LP        SLR       2,2
          SLDL      2,1
          AR        1,2
          BCT       4,LP
          ST        1,0(0,5)
          LM        1,5,24(13)
          MVI       12(13),X'FF'
          BR        14
          SPACE     3
N32       DC        F'32'
NV        DC        F'32'
          END
/EDN
```

```
         TITLE 'BICHA -- BITS TO CHARACTERS (FOR OUTPUT) '
BICHA    CSECT
         USING      BICHA,15
         L          1,0(0,1)
         L          1,0(0,1)
         ST         1,NV
         STC        1,SETN1+1
         STC        1,SETN2+1
         LNR        1,1
         A          1,N32
         STH        1,PRESHF1+2
         STH        1,PRESHF2+2
         STH        1,PRESHF3+2
         MVI        12(13),X'FF'
         BR         14
         SPACE      3
         ENTRY      BICHA1
         USING      BICHA1,15
BICHA1   STM        1,4,24(13)
         L          3,0(0,1)
         L          3,0(0,3)
         L          1,4(0,1)
SETN1    MVC        0(32,1),BLANK
         L          4,NV
PRESHF1  SLL        3,0
LP1      SLR        2,2
         SLDL       2,1
         AH         2,ZERO
         STC        2,0(0,1)
         A          1,ONE
         BCT        4,LP1
         MVI        12(13),X'FF'
         LM         1,4,24(13)
         BR         14
         SPACE      3
         ENTRY      BICHA2
         USING      BICHA2,15
BICHA2   STM        1,6,24(13)
         LM         3,4,0(1)
         L          3,0(0,3)
         L          5,0(0,4)
         L          1,8(0,1)
         L          6,NV
SETN2    MVC        0(32,1),BLANK
         XR         5,3
PRESHF2  SLL        3,0
PRESHF3  SLL        5,0
LP2      SLR        2,2
         SLDL       2,1
         SLR        4,4
         SLDL       4,1
         ALR        4,4
         BC         5,DASH
         AH         2,ZERO
```

```
         STC       2,0(0,1)
         B         ADDR
DASH     MVI       0(1),C'-'
ADDR     A         1,ONE
         BCT       6,LP2
         LM        1,6,24(13)
         MVI       12(13),X'FF'
         BR        14
ZERO     DC        X'00F0'
ONE      DC        F'1'
NV       DC        F'32'
N32      DC        F'32'
BLANK    DC        CL32' '
         END
```

APPENDIX B

EXAMPLE OF 4 VARIABLES

```
TOATAL NUMBER OF ON-SET AND DONT*CARES =      8
ON-SET =      8       DONT-CARES =      0
NUMBERS OF VARIABLES =      4

    MINT.#   1  =     0    MINT.#   2  =     1    MINT.#   3  =     5
    MINT.#   4  =     7    MINT.#   5  =    15    MINT.#   6  =    14
    MINT.#   7  =    10    MINT.#   8  =     8    MINT.#


  THE CUBE COVERING LIST :

  THE      1-TH CUBE COVERING IS :
         1-CUBE = (     8 ,    10   ) =      10-0

  THIS CUBE COVERS THESE MINTERMS :
           8   10

  THE      2-TH CUBE COVERING IS :
         1-CUBE = (    14 ,    15   ) =      111-

  THIS CUBE COVERS THESE MINTERMS :
         14   15

  THE      3-TH CUBE COVERING IS :
         1-CUBE = (     7 ,     5   ) =      01-1

  THIS CUBE COVERS THESE MINTERMS :
           7    5

  THE      4-TH CUBE COVERING IS :
         1-CUBE = (     1 ,     0   ) =      000-

  THIS CUBE COVERS THESE MINTERMS :
           1    0

  THE FINAL CHECKING
             105  105  105  105  105  105  105  105


TOATAL NUMBER OF ON-SET AND DONT*CARES =      9
ON-SET =      9       DONT-CARES =      0
NUMBERS OF VARIABLES =      4

    MINT.#   1  =     0    MINT.#   2  =     1    MINT.#   3  =     4
    MINT.#   4  =     5    MINT.#   5  =     7    MINT.#   6  =     8
    MINT.#   7  =    10    MINT.#   8  =    14    MINT.#   9  =    15
```

```
THE CUBE COVERING LIST :

THE      1-TH CUBE COVERING IS :
       2-CUBE = (     1 ,     4   ) =      0-0-

THIS CUBE COVERS THESE MINTERMS :
       1     4     0     5

THIS MINTERM MT(    2   ) =      1       IS A CORE POINT  .

THE      2-TH CUBE COVERING IS :
       1-CUBE = (    10 ,     8   ) =      10-0

THIS CUBE COVERS THESE MINTERMS :
      10     8

THE      3-TH CUBE COVERING IS :
       1-CUBE = (    14 ,    15   ) =      111-

THIS CUBE COVERS THESE MINTERMS :
      14    15

THE      4-TH CUBE COVERING IS :
       1-CUBE = (     7 ,     5   ) =      01-1

THIS CUBE COVERS THESE MINTERMS :
       7     5
THE FINAL CHECKING
             105   101   101   205   105   105   105   105   105
```

```
TOATAL NUMBER OF ON-SET AND DONT*CARES =     14
ON-SET =     14        DONT-CARES =        0
NUMBERS OF VARIABLES =        4

    MINT.#   1  =      1     MINT.#   2  =      2     MINT.#   3  =      3
    MINT.#   4  =      4     MINT.#   5  =      5     MINT.#   6  =      6
    MINT.#   7  =      7     MINT.#   8  =      8     MINT.#   9  =      9
    MINT.#  10  =     10     MINT.#  11  =     11     MINT.#  12  =     12
    MINT.#  13  =     13     MINT.#  14  =     14     MINT.#


THE CUBE COVERING LIST :



THE       1-TH CUBE COVERING IS :
        2-CUBE = (    14 ,      8   ) =      1--0

THIS CUBE COVERS THESE MINTERMS :
        14     8    10    12


THE       2-TH CUBE COVERING IS :
        2-CUBE = (     1 ,     13   ) =      --01

THIS CUBE COVERS THESE MINTERMS :
         1    13     5     9


THE       3-TH CUBE COVERING IS :
        2-CUBE = (     7 ,      2   ) =      0-1-

THIS CUBE COVERS THESE MINTERMS :
         7     2     3     6


THE       4-TH CUBE COVERING IS :
        2-CUBE = (    11 ,      8   ) =      10--

THIS CUBE COVERS THESE MINTERMS :
        11     8     9    10
```

```
THE      5-TH CUBE COVERING IS :
       2-CUBE = (     4 ,    14   ) =       -1-0

THIS CUBE COVERS THESE MINTERMS :
        4    14     6    12

THE FINAL CHECKING
             105   105   105   105   105   205   105   205   205   205   105   20
             105   205




TOATAL NUMBER OF ON-SET AND DONT*CARES =        9
ON_SET =       9       DONT_CARES =        0
NUMBERS OF VARIABLES =       4

    MINT.#    1   =       1     MINT.#    2   =       2     MINT.#    3   =     3
    MINT.#    4   =       5     MINT.#    5   =      15     MINT.#    6   =     1
    MINT.#    7   =       8     MINT.#    8   =       9     MINT.#    9   =     11


THE CUBE COVERING LIST :



THE      1-TH CUBE COVERING IS :
       1-CUBE = (     2 ,     3   ) =       001-

THIS CUBE COVERS THESE MINTERMS :
        2     3

THIS MINTERM MT(    2   ) =       2       IS A CORE POINT



THE      2-TH CUBE COVERING IS :
       1-CUBE = (     5 ,     1   ) =       0-01

THIS CUBE COVERS THESE MINTERMS :
        5     1

THIS MINTERM MT(    4   ) =       5       IS A CORE POINT
```

```
THE     3-TH CUBE COVERING IS :
        1-CUBE = (    14 ,    15   ) =      111-

THIS CUBE COVERS THESE MINTERMS :
        14   15

THIS MINTERM MT(    6  ) =      14       IS A CORE POINT


THE     4-TH CUBE COVERING IS :
        1-CUBE = (     8 ,     9   ) =      100-

THIS CUBE COVERS THESE MINTERMS :
         8    9

THIS MINTERM MT(    7  ) =       8       IS A CORE POINT


THE     5-TH CUBE COVERING IS :
        2-CUBE = (    11 ,     1   ) =      -0-1

THIS CUBE COVERS THESE MINTERMS :
        11    1    3    9

THE FINAL CHECKING
             205  101  205  101  105  101  101  205  105



TOATAL NUMBER OF ON-SET AND DONT*CARES =      12
ON-SET =      12       DONT-CARES =      0
NUMBERS OF VARIABLES =      5

    MINT.#   1  =     0    MINT.#   2  =     1    MINT.#   3  =
    MINT.#   4  =     5    MINT.#   5  =     6    MINT.#   6  =
    MINT.#   7  =    24    MINT.#   8  =    25    MINT.#   9  =     2
    MINT.#  10  =    28    MINT.#  11  =    30    MINT.#  12  =     3
```

```
THE CUBE COVERING LIST :


THE     1-TH CUBE COVERING IS :
     1-CUBE = (    31 ,    30   ) =      1111-

THIS CUBE COVERS THESE MINTERMS :
     31    30


THE     2-TH CUBE COVERING IS :
     1-CUBE = (    28 ,    24   ) =      11-00

THIS CUBE COVERS THESE MINTERMS :
     28    24


THE     3-TH CUBE COVERING IS :
     1-CUBE = (    27 ,    25   ) =      110-1

THIS CUBE COVERS THESE MINTERMS :
     27    25


THE     4-TH CUBE COVERING IS :
     1-CUBE = (     7 ,     6   ) =      0011-

THIS CUBE COVERS THESE MINTERMS :
      7     6


THE     5-TH CUBE COVERING IS :
     1-CUBE = (     5 ,     1   ) =      00-01

THIS CUBE COVERS THESE MINTERMS :
      5     1
```

```
THE       6-TH CUBE COVERING IS :
       1-CUBE = (     0  ,     2   ) =       000-0

THIS CUBE COVERS THESE MINTERMS :
        0      2

THE FINAL CHECKING
             105  105  105  105  105  105  105  105  105  105  105  10



TOATAL NUMBER OF ON-SET AND DONT*CARES =      16
ON-SET =       4       DONT-CARES =      12
NUMBERS OF VARIABLES =       5

    MINT.#   1   =      8      MINT.#   2   =     14      MINT.#   3   =     16
    MINT.#   4   =     19      MINT.#   5   =     20      MINT.#   6   =     21
    MINT.#   7   =     22      MINT.#   8   =     23      MINT.#   9   =     24
    MINT.#  10   =     25      MINT.#  11   =     26      MINT.#  12   =     27
    MINT.#  13   =     28      MINT.#  14   =     29      MINT.#  15   =     30
    MINT.#  16   =     31      MINT.#


THE CUBE COVERING LIST :



THE       1-TH CUBE COVERING IS :
       1-CUBE = (     8  ,    24   ) =       -1000

THIS CUBE COVERS THESE MINTERMS :
        8     24

THIS MINTERM MT(    1   ) =      8       IS A CORE POINT



THE       2-TH CUBE COVERING IS :
       1-CUBE = (    14  ,    30   ) =       -1110

THIS CUBE COVERS THESE MINTERMS :
       14     30

THIS MINTERM MT(    2   ) =     14       IS A CORE POINT
```

```
THE     3-TH CUBE COVERING IS :
         2-CUBE = (    16 ,    28   ) =      1--00

THIS CUBE COVERS THESE MINTERMS :
         16    28    20    24

THIS MINTERM MT(    3   ) =     16       IS A CORE POINT


THE      4-TH CUBE COVERING IS :
         2-CUBE = (    19 ,    31   ) =      1--11

THIS CUBE COVERS THESE MINTERMS :
         19    31    23    27

THIS MINTERM MT(    4   ) =     19       IS A CORE POINT

THE FINAL CHECKING
               101  101  101  101   209   109   109   209   309   109   109   2
               209  109  209  209



TOATAL NUMBER OF ON-SET AND DONT*CARES =      17
ON-SET =      10         DONT-CARES =       7
NUMBERS OF VARIABLES =       5

MINT.#   1   =      0    MINT.#   2   =      3    MINT.#   3   =      4
MINT.#   4   =      6    MINT.#   5   =      7    MINT.#   6   =     15
MINT.#   7   =     21    MINT.#   8   =     23    MINT.#   9   =     26
MINT.#  10   =     28    MINT.#  11   =      2    MINT.#  12   =      8
MINT.#  13   =     12    MINT.#  14   =     14    MINT.#  15   =     24
MINT.#  16   =     31    MINT.#  17   =     17    MINT.#

 THE CUBE COVERING LIST :


 THE      1-TH CUBE COVERING IS :
         2-CUBE = (     3 ,     6   ) =      00-1-

 THIS CUBE COVERS THESE MINTERMS :
          3     6     7     2

 THIS MINTERM MT(    2   ) =      3       IS A CORE POINT
```

```
THE       2-TH CUBE COVERING IS :
       1-CUBE = (    26 ,    24   ) =       110-0

THIS CUBE COVERS THESE MINTERMS :
       26    24

THIS MINTERM MT(    9   ) =     26       IS A CORE POINT


THE       3-TH CUBE COVERING IS :
       2-CUBE = (    28 ,     8   ) =       -1-00

THIS CUBE COVERS THESE MINTERMS :
       28     8    12    24

THIS MINTERM MT(  10   ) =     28       IS A CORE POINT


THE       4-TH CUBE COVERING IS :
       2-CUBE = (     0 ,     6   ) =       00--0

THIS CUBE COVERS THESE MINTERMS :
        0     6     4     2


THE       5-TH CUBE COVERING IS :
       2-CUBE = (    15 ,     6   ) =       0-11-

THIS CUBE COVERS THESE MINTERMS :
       15     6     7    14


THE       6-TH CUBE COVERING IS :
       2-CUBE = (    23 ,    15   ) =       --111

THIS CUBE COVERS THESE MINTERMS :
       23    15     7    31
THE       7-TH CUBE COVERING IS :
       1-CUBE = (    21 ,    17   ) =       10-01

THIS CUBE COVERS THESE MINTERMS :
       21    17

THE FINAL CHECKING
              105  101  105  205  205  205  105  105  101  101  105  10
              105  105  205  209  209
```

```
TOATAL NUMBER OF ON-SET AND DONT*CARES =     14
ON-SET =     14       DONT-CARES =       0
NUMBERS OF VARIABLES =      5

MINT.#    1   =        0     MINT.#    2   =        2     MINT.#    3   =        3
MINT.#    4   =        4     MINT.#    5   =        8     MINT.#    6   =       10
MINT.#    7   =       11     MINT.#    8   =       12     MINT.#    9   =       16
MINT.#   10   =       20     MINT.#   11   =       24     MINT.#   12   =       26
MINT.#   13   =       27     MINT.#   14   =       28     MINT.#


THE CUBE COVERING LIST :



THE      1-TH CUBE COVERING IS :
        2-CUBE = (      3 ,    10    ) =      0-01-

THIS CUBE COVERS THESE MINTERMS :
        3    10    2    11

THIS MINTERM MT(    3   ) =      3       IS A CORE POINT



THE      2-TH CUBE COVERING IS :
        3-CUBE = (      4 ,    24    ) =      ---00

THIS CUBE COVERS THESE MINTERMS :
        4    24     0     8    12    16    20    28

THIS MINTERM MT(    4   ) =      4       IS A CORE POINT



THE      3-TH CUBE COVERING IS :
        2-CUBE = (     27 ,    10    ) =      -101-

THIS CUBE COVERS THESE MINTERMS :
       27    10    11    26

THIS MINTERM MT(   13   ) =     27       IS A CORE POINT

THE FINAL CHECKING
             105  105  101  101  105  205  201  101  101  101  105  10
             101  101
```

```
MAXIMUM CUBE DIMENSION =    3

   1    0    0    2    1    0    3    3    1

THE CHARACTERISTICS :
     2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:         2

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:         1

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :     1

CUBE   (    8   ,     0   )   HAS BEEN FOUND.

CUBE   (    8   ,    10   )   HAS BEEN FOUND.

WATCH FOR      0 AND     10

CHECKING POINTS
            0    0    0    0    0    0 9999 9999

ACCUMULATED AWAYNESS :

            2    4    6    6    4    2    0    0

CUBE   (    8   ,    10   )   IS SELECTED.

CUBE   (   14   ,    15   )   HAS BEEN FOUND.

CUBE   (   14   ,    10   )   HAS BEEN FOUND.

CHECKING POINTS
            0    0    0    0 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            8   10   10    8    4    2    0    0

CUBE   (   14   ,    15   )   IS SELECTED.

CUBE   (    7   ,     5   )   HAS BEEN FOUND.

CUBE   (    7   ,    15   )   HAS BEEN FOUND.

CHECKING POINTS
            0    0 9999 9999 9999 9999 9999 9999
```

```
ACCUMULATED AWAYNESS :

            12    12    10     8     4     2     0     0

CUBE    (     7   ,     5   )    IS SELECTED.

CUBE    (     1   ,     0   )    HAS BEEN FOUND.

CUBE    (     1   ,     5   )    HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            12    12    10     8     4     2     0     0

CUBE    (     1   ,     0   )    IS SELECTED.
```

```
MAXIMUM CUBE DIMENSION =    3

     1    0    0    2    1    0    3    3    1

THE CHARACTERISTICS :
        2  5  2  1  2  1  2  5  2  1  2  1  2  1  2  1  2  1

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:           2

CUBE    (      1   ,      4   )    HAS BEEN FOUND.

CHECKING POINTS
               9999 9999 9999 9999     1     1     0     0     0

ACCUMULATED AWAYNESS :

                  0     0     0     0     2     2     4     4     4

CUBE    (      1   ,      4   )    IS SELECTED.

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :     2

CUBE    (     10   ,      8   )    HAS BEEN FOUND.

CUBE    (     10   ,     14   )    HAS BEEN FOUND.

WATCH FOR       8 AND     14

CHECKING POINTS
               9999 9999 9999 9999     1 9999 9999     0     0

ACCUMULATED AWAYNESS :

                  0     0     0     0     8     2     4     6     8

CUBE    (     10   ,      8   )    IS SELECTED.

CUBE    (     14   ,     10   )    HAS BEEN FOUND.

CUBE    (     14   ,     15   )    HAS BEEN FOUND.

CHECKING POINTS
               9999 9999 9999 9999     1 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                  0     0     0     0    10     2     4     6     8

CUBE    (     14   ,     15   )    IS SELECTED.

CUBE    (      7   ,      5   )    HAS BEEN FOUND.

CUBE    (      7   ,     15   )    HAS BEEN FOUND.
```

```
WATCH FOR      5 AND     15

CHECKING POINTS
            9999 9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                    0     0     0     0    10     2     4     6     8

CUBE    (     7   ,     5   )    IS SELECTED.



MAXIMUM CUBE DIMENSION =    3

     1          0     2     1     0     3     3     1

THE CHARACTERISTICS :
          3  1  3  1  3  5  3  1  3  5  3  5  3  1  3  1  3  5  3  5  3  1
          3  5  3  1  3  1

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:         2

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :     2

CUBE    (    14   ,     2   )    HAS BEEN FOUND,

CUBE    (    14   ,     4   )    HAS BEEN FOUND,

WATCH FOR      2 AND      4

CUBE    (    14   ,     8   )    HAS BEEN FOUND,

WATCH FOR      4 AND      8

CHECKING POINTS
                    0     1     0     1     0     1     0 9999     1 9999     1 9999
                    1 9999

ACCUMULATED AWAYNESS :

                    4     2     4     2     4     2     4     0     2     0     2     0
                    2     0

CUBE    (    14   ,     8   )    IS SELECTED,

CUBE    (     1   ,     7   )    HAS BEEN FOUND,

CUBE    (     1   ,    11   )    HAS BEEN FOUND,
```

```
WATCH FOR      7 AND    11

CUBE    (     1   ,     13   )    HAS BEEN FOUND.

WATCH FOR     11 AND    13

CUBE    (     1   ,     13   )    IS SELECTED.

CUBE    (     7   ,      1   )    HAS BEEN FOUND.

CUBE    (     7   ,      2   )    HAS BEEN FOUND.

CUBE    (     7   ,      4   )    HAS BEEN FOUND.

CHECKING POINTS
               9999 9999 9999    3 9999 9999 9999 9999 9999 9999    3 9999
               9999 9999

ACCUMULATED AWAYNESS :

                   4     6     6     6     4     6     6     0     2     0     6     0
                   2     0

CUBE    (     7   ,      2   )    IS SELECTED.

CUBE    (    11   ,      1   )    HAS BEEN FOUND.

CUBE    (    11   ,      2   )    HAS BEEN FOUND.

WATCH FOR      1 AND     2

CUBE    (    11   ,      8   )    HAS BEEN FOUND.

WATCH FOR      1 AND     8

CHECKING POINTS
               9999 9999 9999    3 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999

ACCUMULATED AWAYNESS :

                   4     6     6    10     4     6     6     0     2     0     6     0
                   2     0

CUBE    (    11   ,      8   )    IS SELECTED.

CUBE    (     4   ,      7   )    HAS BEEN FOUND.

CUBE    (     4   ,     13   )    HAS BEEN FOUND.
```

```
WATCH FOR      7 AND     13

CUBE    (      4   ,     14   )    HAS BEEN FOUND.

WATCH FOR     13 AND     14

CHECKING POINTS
              9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
              9999 9999

ACCUMULATED AWAYNESS :

                  4    6    6   10    4    6    6    0    2    0    6    0
                  2    0

CUBE    (      4   ,     14   )    IS SELECTED.



MAXIMUM CUBE DIMENSION =    3

      1    0    3    2    1    0    3    3    1

THE CHARACTERISTICS :
           3  1  1  1  3  1  1  1  2  1  1  1  1  1  3  1  3  1

MAXIMUM CUBE DIMENSION,HAS BEEN LOWERED TO:          2

CUBE    (      2   ,      3   )    HAS BEEN FOUND.

CHECKING POINTS
                 1 9999 9999    0    0    0    0    0    1

ACCUMULATED AWAYNESS :

                 2    0    0    4    4    4    4    4    2

CUBE    (      2   ,      3   )    IS SELECTED.

CUBE    (      5   ,      1   )    HAS BEEN FOUND.

CHECKING POINTS
              9999 9999 9999 9999    0    0    0    1    1

ACCUMULATED AWAYNESS :

                 2    0    0    4    8   10    8    6    6

CUBE    (      5   ,      1   )    IS SELECTED.
```

```
CHECKING POINTS
                  9999    1    1    2 9999    1    1 9999 9999 9999    2 9999
                  9999 9999

ACCUMULATED AWAYNESS :

                    4    6    6    4    4    6    6    0    2    0    4    0
                    2    0

CUBE    (     1  ,    13  )   IS SELECTED.

CUBE    (     7  ,     1  )   HAS BEEN FOUND.

CUBE    (     7  ,     2  )   HAS BEEN FOUND.

CUBE    (     7  ,     4  )   HAS BEEN FOUND.

CHECKING POINTS
                  9999 9999 9999    3 9999 9999 9999 9999 9999 9999    3 9999
                  9999 9999

ACCUMULATED AWAYNESS :

                    4    6    6    6    4    6    6    0    2    0    6    0
                    2    0

CUBE    (     7  ,     2  )   IS SELECTED.

CUBE    (    11  ,     1  )   HAS BEEN FOUND.

CUBE    (    11  ,     2  )   HAS BEEN FOUND.

WATCH FOR       1 AND       2

CUBE    (    11  ,     5  )   HAS BEEN FOUND.

WATCH FOR       1 AND       8

CHECKING POINTS
                  9999 9999 9999    3 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999

ACCUMULATED AWAYNESS :

                    4    6    6   10    4    6    6    0    2    0    6    0
                    2    0

CUBE    (    11  ,     8  )   IS SELECTED.

CUBE    (     4  ,     7  )   HAS BEEN FOUND.

CUBE    (     4  ,    13  )   HAS BEEN FOUND.
```

```
CUBE    (    14   ,    15   )    HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999 9999 9999    0    1    2

ACCUMULATED AWAYNESS :

            2    0    0    4    8   10   12   10    8

CUBE    (    14   ,    15   )    IS SELECTED.

CUBE    (     8   ,     9   )    HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999 9999 9999 9999 9999    3

ACCUMULATED AWAYNESS :

            2    0    0    4    8   10   12   10   10

CUBE    (     8   ,     9   )    IS SELECTED.

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :    2

CUBE    (    11   ,     1   )    HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            2    0    0    4    8   10   12   10   10

CUBE    (    11   ,     1   )    IS SELECTED.

 MAXIMUM CUBE DIMENSION =    3

     1    1    0    2    1    0    3    3    1

 THE CHARACTERISTICS :
        2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1
        2  1

 MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:        2

 MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:        1

 ALL CORE CUBES HAVE BEEN SEARCHED,
 AND THE MAXIMUM CUBE DIMENSION IS :    1
```

```
CUBE   (    31   ,    27   )   HAS BEEN FOUND.
CUBE   (    31   ,    30   )   HAS BEEN FOUND.
WATCH FOR     27 AND     30
CHECKING POINTS
                0    0    0    0    0    0    0    0    0    0 9999 9999
ACCUMULATED AWAYNESS :
                8    8    6    6    4    4    4    4    2    2    0    0
CUBE   (    31   ,    30   )   IS SELECTED.
CUBE   (    28   ,    24   )   HAS BEEN FOUND.
CUBE   (    28   ,    30   )   HAS BEEN FOUND.
CHECKING POINTS
                0    0    0    0    0    0 9999    0    0 9999 9999 9999
ACCUMULATED AWAYNESS :
               12   14   12   12   10   12    4    6    6    2    0    0
CUBE   (    28   ,    24   )   IS SELECTED.
CUBE   (    27   ,    25   )   HAS BEEN FOUND.
CUBE   (    27   ,    31   )   HAS BEEN FOUND.
CHECKING POINTS
                0    0    0    0    0    0 9999 9999 9999 9999 9999 9999
ACCUMULATED AWAYNESS :
               18   18   18   18   18   18    4    6    6    2    0    0
CUBE   (    27   ,    25   )   IS SELECTED.
CUBE   (     7   ,     5   )   HAS BEEN FOUND.
CUBE   (     7   ,     6   )   HAS BEEN FOUND.
WATCH FOR      5 AND      6
CHECKING POINTS
                0    0    0    0 9999 9999 9999 9999 9999 9999 9999 9999
```

```
ACCUMULATED AWAYNESS :
                 22   22   20   20   18   18    4    6    6    2    0    0
CUBE   (    7  ,     6  )   IS SELECTED.
CUBE   (    5  ,     1  )   HAS BEEN FOUND.
CUBE   (    5  ,     7  )   HAS BEEN FOUND.
CHECKING POINTS
                  0 9999    0 9999 9999 9999 9999 9999 9999 9999 9999 9999
ACCUMULATED AWAYNESS :
                 24   22   24   20   18   18    4    6    6    2    0    0
CUBE   (    5  ,     1  )   IS SELECTED.
CUBE   (    0  ,     1  )   HAS BEEN FOUND.
CUBE   (    0  ,     2  )   HAS BEEN FOUND.
CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
ACCUMULATED AWAYNESS :
                 24   22   24   20   18   18    4    6    6    2    0    0
CUBE   (    0  ,     2  )   IS SELECTED.


MAXIMUM CUBE DIMENSION =    4
       1    0    0    0    2    1    0    0    3    3    1    0    4    6
       4    1

THE CHARACTERISTICS :
            1  1  1  1  2  1  2  1  3  5  3  1  3  1  3  5  3  5  3  1  3  1
            3  5  3  5  3  5  3  5  3  5
MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:          3
CUBE   (    6  ,    24  )   HAS BEEN FOUND.
```

```
CHECKING POINTS
          9999    0    1    0 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999

ACCUMULATED AWAYNESS :

             0    4    2    6    0    0    0    0    0    0    0    0
             0    0    0    0

CUBE   (    8  ,   24  )   IS SELECTED.

CUBE   (   14  ,   30  )   HAS BEEN FOUND.

CHECKING POINTS
          9999 9999    1    0 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999

ACCUMULATED AWAYNESS :

             0    4    8   12    0    0    0    0    0    0    0    0
             0    0    0    0

CUBE   (   14  ,   30  )   IS SELECTED.

CUBE   (   16  ,   25  )   HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999    0 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999

ACCUMULATED AWAYNESS :

             0    4    8   16    0    0    0    0    0    0    0    0
             0    0    0    0

CUBE   (   16  ,   25  )   IS SELECTED.

CUBE   (   19  ,   31  )   HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999

ACCUMULATED AWAYNESS :

             0    4    8   16    0    0    0    0    0    0    0    0
             0    0    0    0

CUBE   (   19  ,   31  )   IS SELECTED.
```

```
ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :     3

MAXIMUM CUBE DIMENSION =    4

   1    0    0    0    2    1    0    0    3    3    1    0    4    6
   4    1

THE CHARACTERISTICS :
      3  1  2  1  3  1  3  5  3  5  3  1  2  1  3  1  1  1  2  1  3  1
      3  1  3  5  3  1  3  1  2  1  1  1

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:          3

CUBE    (     3   ,      6   )    HAS BEEN FOUND.

CHECKING POINTS
             1 9999    2 9999 9999    2    0    2    0    0 9999 9999
          9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

             2    0    2    0    0    2    4    2    4    6    0    0
             0    0    0    0    0

CUBE    (     3   ,      6   )    IS SELECTED.

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:          2

CUBE    (    26   ,     24   )    HAS BEEN FOUND.

CHECKING POINTS
             1 9999    2 9999 9999    2    0    2 9999    1 9999 9999
          9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

             6    0    8    0    0    8   10    8    4    8    0    0
             0    0    0    0    0

CUBE    (    26   ,     24   )    IS SELECTED.

CUBE    (    28   ,      8   )    HAS BEEN FOUND.

CHECKING POINTS
             2 9999    3 9999 9999    2    0    2 9999 9999 9999 9999
          9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

             8    0   10    0    0   12   14   14    4    8    0    0
             0    0    0    0    0
```

```
CUBE   (   28  ,     8  )   IS SELECTED.

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :     2

CUBE   (    0  ,     6  )   HAS BEEN FOUND.

CUBE   (    0  ,    12  )   HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999 9999    2    0    2 9999 9999 9999 9999
          9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

               8    0   10    0    0   16   18   18    4    8    0    0
               0    0    0    0    0

CUBE   (    0  ,     6  )   IS SELECTED.

CUBE   (   15  ,     6  )   HAS BEEN FOUND.

CUBE   (   15  ,    23  )   HAS BEEN FOUND.

WATCH FOR     6 AND    23

CHECKING POINTS
          9999 9999 9999 9999 9999 9999    0    3 9999 9999 9999 9999
          9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

               8    0   10    0    0   16   22   20    4    8    0    0
               0    0    0    0    0

CUBE   (   15  ,     6  )   IS SELECTED.

CUBE   (   23  ,    15  )   HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999 9999 9999    0 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

               8    0   10    0    0   16   24   20    4    8    0    0
               0    0    0    0    0

CUBE   (   23  ,    15  )   IS SELECTED.
```

```
 CUBE    (     21   ,    23   )    HAS BEEN FOUND.

 CUBE    (     21   ,    17   )    HAS BEEN FOUND.

 WATCH FOR      23 AND      17

 CHECKING POINTS
              9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999

 ACCUMULATED AWAYNESS :

                 8     0    10     0     0    16    24    20     4     8     0     0
                 0     0     0     0     0

 CUBE    (     21   ,    17   )    IS SELECTED.




 MAXIMUM CUBE DIMENSION =     3

     1     0     0     2     1     0     3     3     1

 THE CHARACTERISTICS :
           3  5  3  1  2  1  3  1  3  5  3  5  3  1  3  1  3  1  3  1  3  5
           3  1  2  1  3  1

 CUBE    (      3   ,    10   )    HAS BEEN FOUND.

 CHECKING POINTS
                 2 9999 9999     0     2 9999 9999     0     0     0     0     2
                 1     0

 ACCUMULATED AWAYNESS :

                 2     0     0     4     2     0     0     4     4     6     4     2
                 2     6

 CUBE    (      3   ,    10   )    IS SELECTED.

 CUBE    (      4   ,    24   )    HAS BEEN FOUND.

 CHECKING POINTS
              9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999     4
                 1 9999
```

```
ACCUMULATED AWAYNESS :

                    2    0    0    4    2    0    0    4    4    6    4    4
                    6    6

CUBE    (    4  ,    24  )   IS SELECTED.

CUBE    (   27  ,    10  )   HAS BEEN FOUND.

CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999

ACCUMULATED AWAYNESS :

                    2    0    0    4    2    0    0    4    4    6    4    4
                    6    6

CUBE    (   27  ,    10  )   IS SELECTED.

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :     3


**FORTRAN ** STOP
/LOGOFF

C E420 LOGOFF AT 2318 ON 11/30      FOR TSN 2390,

C E421 CPU TIME USED:  000090.4681 SECONDS.
```

APPENDIX C

EXAMPLE OF 6 VARIABLES, 18 MINTERMS

```
TOATAL NUMBER OF ON-SET AND DONT*CARES =     18
ON-SET =     18        DONT-CARES =      0
NUMBERS OF VARIABLES =      6

MINT.#   1  =    6      MINT.#   2  =    4      MINT.#   3  =    8
MINT.#   4  =    9      MINT.#   5  =   11      MINT.#   6  =   14
MINT.#   7  =   27      MINT.#   8  =   30      MINT.#   9  =   29
MINT.#  10  =   28      MINT.#  11  =   16      MINT.#  12  =   17
MINT.#  13  =   19      MINT.#  14  =   61      MINT.#  15  =   45
MINT.#  16  =    0      MINT.#  17  =   44      MINT.#  18  =   36


THE CUBE COVERING LIST :



THE      1-TH CUBE COVERING IS :
       1-CUBE = (  36 ,  44  ) =      10-100

THIS CUBE COVERS THESE MINTERMS :
       44  36



THE      2-TH CUBE COVERING IS :
       1-CUBE = (  45 ,  61  ) =      1-1101

THIS CUBE COVERS THESE MINTERMS :
       61  45



THE      3-TH CUBE COVERING IS :
       1-CUBE = (  28 ,  29  ) =      01110-

THIS CUBE COVERS THESE MINTERMS :
       29  28



THE      4-TH CUBE COVERING IS :
       1-CUBE = (   4 ,   0  ) =      000-00

THIS CUBE COVERS THESE MINTERMS :
        4   0
```

```
THE      5-TH CUBE COVERING IS :
      1-CUBE = (    8 ,    9  ) =      00100-

THIS CUBE COVERS THESE MINTERMS :
      8    9


THE      6-TH CUBE COVERING IS :
      1-CUBE = (  14 ,   30  ) =      0-1110

THIS CUBE COVERS THESE MINTERMS :
     14   30


THE      7-TH CUBE COVERING IS :
      1-CUBE = (    6 ,    4  ) =      0001-0

THIS CUBE COVERS THESE MINTERMS :
      6    4


THE      8-TH CUBE COVERING IS :
      1-CUBE = (  16 ,   17  ) =      01000-

THIS CUBE COVERS THESE MINTERMS :
     16   17


THE      9-TH CUBE COVERING IS :
      1-CUBE = (  11 ,   27  ) =      0-1011

THIS CUBE COVERS THESE MINTERMS :
     11   27


THE     10-TH CUBE COVERING IS :
      1-CUBE = (  19 ,   17  ) =      0100-1

THIS CUBE COVERS THESE MINTERMS :
     17   19

THE FINAL CHECKING
              105  205  105  105  105  105  105  105  105  105  105  20
              105  105  105  105  105  105
```

```
 MAXIMUM CUBE DIMENSION =    4

      1     0     0     0     2     1     0     0     3     3     1     0     4     6
      4     1

 THE CHARACTERISTICS :
          2  1  3  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1
          2  1  2  1  2  1  2  1  3  1  2  1  2  1

 MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:          3

 MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:          2

 MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:          1

 ALL CORE CUBES HAVE BEEN SEARCHED,
 AND THE MAXIMUM CUBE DIMENSION IS :     1

 CUBE    (  36   ,    4  )    HAS BEEN FOUND.

 CUBE    (  36   ,   44  )    HAS BEEN FOUND.

 WATCH FOR      4 AND     44

 CHECKING POINTS
                 0     0     0     0     0     0     0     0     0     0     0     0
                 0     0     0     0  9999  9999

 ACCUMULATED AWAYNESS :

                 4     2     4     6     8     4    10     6     6     4     6     8
                10     4     2     4     0     0

 CUBE    (  36   ,   44  )    IS SELECTED.

 CUBE    (  45   ,   61  )    HAS BEEN FOUND.

 CUBE    (  45   ,   44  )    HAS BEEN FOUND.

 CHECKING POINTS
                 0     0     0     0     0     0     0     0     0     0     0     0
                 0  9999  9999     0  9999  9999

 ACCUMULATED AWAYNESS :

                12     8    10    10    14    10    16    12     8     8    14    14
                18     4     2    12     2     4

 CUBE    (  45   ,   61  )    IS SELECTED.

 CUBE    (  28   ,   30  )    HAS BEEN FOUND.

 CUBE    (  28   ,   29  )    HAS BEEN FOUND.
```

```
WATCH FOR     30 AND     29

CHECKING POINTS
                0     0     0     0     0     0     0     0 9999 9999     0     0
                0 9999 9999     0 9999 9999

ACCUMULATED AWAYNESS :

               18    12    14    14    20    14    20    14     8     8    18    18
               24     6     6    18     6    10

CUBE   (  28  ,  29  )   IS SELECTED.

CUBE   (   4  ,   6  )   HAS BEEN FOUND.

CUBE   (   4  ,   0  )   HAS BEEN FOUND.

WATCH FOR      6 AND      0

CUBE   (   4  ,  36  )   HAS BEEN FOUND.

CHECKING POINTS
                0 9999     0     0     0     0     0     0 9999 9999     0     0
                0 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

               20    12    16    18    26    18    28    20    14    12    20    22
               30    14    12    18    10    12

CUBE   (   4  ,   0  )   IS SELECTED.

CUBE   (   8  ,   9  )   HAS BEEN FOUND.

CUBE   (   8  ,   0  )   HAS BEEN FOUND.

CHECKING POINTS
                0 9999 9999 9999     0     0     0     0 9999 9999     0     0
                0 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

               26    16    16    18    28    22    32    26    18    16    24    26
               36    20    16    20    14    18

CUBE   (   8  ,   9  )   IS SELECTED.

CUBE   (  14  ,   6  )   HAS BEEN FOUND.

CUBE   (  14  ,  30  )   HAS BEEN FOUND.

WATCH FOR      6 AND     30
```

```
CHECKING POINTS
             0 9999 9999 9999    0 9999    0 9999 9999 9999    0    0
             0 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            28    20    20    24    32    22    36    26    22    18    30    34
            42    26    22    26    18    24

CUBE    (   14   ,   30   )    IS SELECTED.

CUBE    (    6   ,    4   )    HAS BEEN FOUND.

CUBE    (    6   ,   14   )    HAS BEEN FOUND.

WATCH FOR      4 AND     14

CHECKING POINTS
          9999 9999 9999 9999    0 9999    0 9999 9999 9999    0    0
             0 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            28    20    24    30    38    24    44    30    28    22    34    40
            48    34    28    28    22    26

CUBE    (    6   ,    4   )    IS SELECTED.

CUBE    (   16   ,   17   )    HAS BEEN FOUND.

CUBE    (   16   ,    0   )    HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999    0 9999    0 9999 9999 9999 9999 9999
             0 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            34    24    28    34    44    32    48    36    32    26    34    40
            50    40    36    30    30    32

CUBE    (   16   ,   17   )    IS SELECTED.

CUBE    (   11   ,    9   )    HAS BEEN FOUND.

CUBE    (   11   ,   27   )    HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             0 9999 9999 9999 9999 9999
```

```
ACCUMULATED AWAYNESS :

                 40    32    32    36    44    36    48    40    36    32    40    44
                 52    46    42    36    38    42

CUBE    (  11   ,   27   )    IS SELECTED.

CUBE    (  19   ,   27   )    HAS BEEN FOUND.

CUBE    (  19   ,   17   )    HAS BEEN FOUND.

WATCH FOR    27 AND    17

CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                 46    38    38    40    48    44    50    46    40    38    42    44
                 52    52    50    40    48    50

CUBE    (  19   ,   17   )    IS SELECTED.
```

APPENDIX D

EXAMPLE OF 5 VARIABLES, 20 MINTERMS

```
TOATAL NUMBER OF ON-SET AND DONT*CARES =     20
ON-SET =     20       DONT*CARES =       0
NUMBERS OF VARIABLES =      5

MINT.#   1  =    1       MINT.#   2  =    3       MINT.#   3  =    5
MINT.#   4  =    4       MINT.#   5  =    8       MINT.#   6  =    9
MINT.#   7  =   11       MINT.#   8  =   10       MINT.#   9  =   25
MINT.#  10  =   27       MINT.#  11  =   26       MINT.#  12  =   30
MINT.#  13  =   29       MINT.#  14  =   28       MINT.#  15  =   19
MINT.#  16  =    0       MINT.#  17  =   18       MINT.#  18  =   22
MINT.#  19  =   21       MINT.#  20  =   20       MINT.#


THE CUBE COVERING LIST :



THE      1-TH CUBE COVERING IS !
      2-CUBE = (  20 ,  29  ) =       1-10-

THIS CUBE COVERS THESE MINTERMS :
      29  28  21  20



THE      2-TH CUBE COVERING IS !
      2-CUBE = (  18 ,  30  ) =       1--10

THIS CUBE COVERS THESE MINTERMS :
      26  30  18  22



THE      3-TH CUBE COVERING IS !
      2-CUBE = (   0 ,   5  ) =       00-0-

THIS CUBE COVERS THESE MINTERMS :
      1   5   4   0



THE      4-TH CUBE COVERING IS !
      2-CUBE = (  11 ,  19  ) =       --011

THIS CUBE COVERS THESE MINTERMS :
      3  11  27  19
```

```
THE      5-TH CUBE COVERING IS :
        2-CUBE = (    8 ,   11   ) =      010--

THIS CUBE COVERS THESE MINTERMS :
        8   9  11  10


THE      6-TH CUBE COVERING IS :
        2-CUBE = (  25  ,  11   ) =      -10-1

THIS CUBE COVERS THESE MINTERMS :
        9  11  25  27

THE FINAL CHECKING
             105  105  105  105  105  205  305  105  105  205  105  105
             105  105  105  105  105  105  105  105
```

```
*   5
*     1    3    5    4    8    9   11   10   25   27   26   30   29   28   19
*    18   22   21  209

 MAXIMUM CUBE DIMENSION =    4

    1    0    0    0    2    1    0    0    3    3    1    0    4    6
    4    1

 THE CHARACTERISTICS :
       4  1  3  1  3  1  3  1  3  1  4  1  3  5  3  1  3  1  4  1  4  1
       3  1  3  1  3  1  3  1  3  1  3  1  3  1  3  1  3  5

 MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:         3

 MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:         2

 ALL CORE CUBES HAVE BEEN SEARCHED,
 AND THE MAXIMUM CUBE DIMENSION IS :     2

 CUBE   (  20  ,    5  )   HAS BEEN FOUND.

 CUBE   (  20  ,   30  )   HAS BEEN FOUND.

 WATCH FOR     5 AND    30

 CUBE   (  20  ,   29  )   HAS BEEN FOUND.

 WATCH FOR    30 AND    29

 CHECKING POINTS
              0    0    1    1    0    0    0    0    1    0    0    1
           9999 9999    0    0    0    1 9999 9999

 ACCUMULATED AWAYNESS :

              4    6    2    2    4    4    6    6    2    4    4    2
              0    0    4    4    4    2    0    0

 CUBE   (  20  ,   29  )   IS SELECTED.

 CUBE   (  18  ,   27  )   HAS BEEN FOUND.

 CUBE   (  18  ,   30  )   HAS BEEN FOUND.

 WATCH FOR    27 AND    30

 CHECKING POINTS
              0    0    1    1    0    0    0    1    1    1 9999 9999
           9999 9999    1    0 9999 9999 9999 9999
```

```
ACCUMULATED AWAYNESS :

             10   10    8    6    8   10   10    8    6    6    4
              4    2    6    8    4    2    4    2

CUBE    (  18  ,  30  )   IS SELECTED.

CUBE    (   0  ,   5  )   HAS BEEN FOUND.

CUBE    (   0  ,   9  )   HAS BEEN FOUND.

WATCH FOR      5 AND      9

CHECKING POINTS
           9999    1 9999 9999    1    1    0    1    1    1 9999 999
           9999 9999    1 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

             10   12    8    6   10   12   14   12   10   12   10    8
              8    6   10    8    8    6    6    4

CUBE    (   0  ,   5  )   IS SELECTED.

CUBE    (  11  ,   1  )   HAS BEEN FOUND.

CUBE    (  11  ,   8  )   HAS BEEN FOUND.

CUBE    (  11  ,  25  )   HAS BEEN FOUND.

WATCH FOR      8 AND     25

CUBE    (  11  ,  26  )   HAS BEEN FOUND.

CUBE    (  11  ,  19  )   HAS BEEN FOUND.

WATCH FOR     25 AND     19

CHECKING POINTS
           9999 9999 9999 9999    1    2 9999    2    2 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

             12   12   12   12   14   14   14   14   12   12   12   12
             12   12   10   12   10   10   10   10

CUBE    (  11  ,  19  )   IS SELECTED.

CUBE    (   8  ,   1  )   HAS BEEN FOUND.

CUBE    (   8  ,  11  )   HAS BEEN FOUND.
```

```
CHECKING POINTS
          9999 9999 9999 9999 9999 9999 9999 9999    3 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            14    14    16    16    14    14    14    14    14    14    14    16
            16    16    14    14    14    16    16    16

CUBE    (   8  ,   11   )    IS SELECTED.

CUBE    (  25  ,   11   )    HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            16    16    20    22    16    14    14    16    14    14    16    20
            18    20    16    18    18    22    20    22

CUBE    (  25  ,   11   )    IS SELECTED.
```

APPENDIX E

EXAMPLE OF 7 VARIABLES, 53 MINTERMS

```
TOATAL NUMBER OF ON-SET AND DONT*CARES =      63
ON-SET =     53         DONT-CARES =      10
NUMBERS OF VARIABLES =       7

        MINT.#   1  =     5        MINT.#   2  =    10        MINT.#   3  =    13
        MINT.#   4  =    14        MINT.#   5  =    21        MINT.#   6  =    22
        MINT.#   7  =    29        MINT.#   8  =    30        MINT.#   9  =    34
        MINT.#  10  =    37        MINT.#  11  =    42        MINT.#  12  =    45
        MINT.#  13  =    50        MINT.#  14  =    53        MINT.#  15  =    61
        MINT.#  16  =    64        MINT.#  17  =    65        MINT.#  18  =    66
        MINT.#  19  =    67        MINT.#  20  =    68        MINT.#  21  =    69
        MINT.#  22  =    70        MINT.#  23  =    71        MINT.#  24  =    72
        MINT.#  25  =    74        MINT.#  26  =    80        MINT.#  27  =    81
        MINT.#  28  =    82        MINT.#  29  =    83        MINT.#  30  =    84
        MINT.#  31  =    85        MINT.#  32  =    86        MINT.#  33  =    87
        MINT.#  34  =    88        MINT.#  35  =    89        MINT.#  36  =    90
        MINT.#  37  =    91        MINT.#  38  =    92        MINT.#  39  =    93
        MINT.#  40  =    94        MINT.#  41  =    95        MINT.#  42  =    96
        MINT.#  43  =    97        MINT.#  44  =    98        MINT.#  45  =    99
        MINT.#  46  =   100        MINT.#  47  =   101        MINT.#  48  =   102
        MINT.#  49  =   103        MINT.#  50  =   104        MINT.#  51  =   105
        MINT.#  52  =   106        MINT.#  53  =   107        MINT.#  54  =   108
        MINT.#  55  =   109        MINT.#  56  =   110        MINT.#  57  =   111
        MINT.#  58  =   112        MINT.#  59  =   113        MINT.#  60  =   114
        MINT.#  61  =   115        MINT.#  62  =   116        MINT.#  63  =   117



THE CUBE COVERING LIST :



THE      1-TH CUBE COVERING IS :
        3-CUBE = (   13 ,   53   ) =       0---101

THIS CUBE COVERS THESE MINTERMS :
      13   53    5   21   29   37   45   61

THIS MINTERM MT(    3   ) =    13      IS A CORE POINT



THE      2-TH CUBE COVERING IS :
        2-CUBE = (   22 ,   94   ) =       -01-110

THIS CUBE COVERS THESE MINTERMS :
      22   94   30   86

THIS MINTERM MT(    6   ) =    22      IS A CORE POINT
```

```
THE      3-TH CUBE COVERING IS :
      2-CUBE = (  50 ,  98  ) =      -1-0010

THIS CUBE COVERS THESE MINTERMS :
     50  98  34 114

THIS MINTERM MT(  13  ) =   50      IS A CORE POINT


THE      4-TH CUBE COVERING IS :
      4-CUBE = (  89 ,  86  ) =      101----

THIS CUBE COVERS THESE MINTERMS :
     89  86  80  81  82  83  84  85  87  88  90  91  92  93  94  95

THIS MINTERM MT(  35  ) =   89      IS A CORE POINT


THE      5-TH CUBE COVERING IS :
      4-CUBE = ( 105 , 102  ) =      110----

THIS CUBE COVERS THESE MINTERMS :
    105 102  96  97  98  99 100 101 103 104 106 107 108 109 110 111

THIS MINTERM MT(  51  ) =  105      IS A CORE POINT


THE      6-TH CUBE COVERING IS :
      4-CUBE = (  67 ,  84  ) =      10-0---

THIS CUBE COVERS THESE MINTERMS :
     67  84  64  65  66  68  69  70  71  80  81  82  83  85  86  87


THE      7-TH CUBE COVERING IS :
      4-CUBE = ( 117 ,  64  ) =      1--0-0-

THIS CUBE COVERS THESE MINTERMS :
    117  64  65  68  69  80  81  84  85  96  97 100 101 112 113 116


THE      8-TH CUBE COVERING IS :
      4-CUBE = ( 114 ,  65  ) =      1--00--

THIS CUBE COVERS THESE MINTERMS :
    114  65  64  66  67  80  81  82  83  96  97  98  99 112 113 115
```

```
THE      9-TH CUBE COVERING IS :
         3-CUBE = (   72 ,   82   ) =      10--0-0

THIS CUBE COVERS THESE MINTERMS :
        72  82  64  66  74  80  88  90


THE      10-TH CUBE COVERING IS :
         2-CUBE = (   10 , 106   ) =      --01010

THIS CUBE COVERS THESE MINTERMS :
        10 106  42  74


THE      11-TH CUBE COVERING IS :
         1-CUBE = (   14 ,   30   ) =      00-1110

THIS CUBE COVERS THESE MINTERMS :
        14  30

THE FINAL CHECKING
             105  105  101  105  101  101  101  201  105  101  105  101
             101  101  101  405  305  305  205  205  205  105  105  105
             205  505  405  405  305  305  305  205  205  205  101  205
             101  101  105  105  101  305  305  205  205  205  205  105
             105  105  101  205  101  209  105  209  209  205  205  105
             209  209  105
```

```
MAXIMUM CUBE DIMENSION =    5

   1               0       0       2       1       0       0       0       3       3       1       0
   0       4       6       4       1       0       5      10      10       5       1

THE CHARACTERISTICS :
        4    1    3    1    3    1    2    1    4    1    2    1    4    1    3    1    3    1    4    1    3    1
        4    1    2    1    4    1    3    1    5    5    5    1    5    5    5    1    5    1    5    5    5    1
        5    1    4    1    5    1    5    5    5    5    5    5    5    5    5    5    5    5    5    5    5    1
        5    1    4    1    5    1    4    1    4    1    5    1    5    1    4    1    5    5    5    5    5    5
        5    5    5    5    5    5    5    1    5    1    5    1    4    1    5    5    4    1    4    1    5    1
        4    1    4    1    5    1    5    1    5    1    4    1    4    1    5    1

CUBE    (  13  ,   53  )    HAS BEEN FOUND.

CHECKING POINTS
              9999    0 9999    0 9999    0 9999    0    0 9999    0 9999
                 0 9999 9999    0    2    0    0    2    8    0    2    0
                 0    0    2    0    0    2    8    0    2    0    1    0
                 0    1    8    0    1    0    2    0    0    2    8    0
                 2    0    1    0    0 9999 9999 9999 9999 9999 9999 9999
              9999 9999 9999

ACCUMULATED AWAYNESS :

                 0    6    0    4    0    4    0    4    6    0    6    0
                 6    0    0    6    4    8    6    4    2    6    4    6
                 8    6    4    8    6    4    2    6    4    6    4    8
                 6    4    2    6    4    6    4    8    6    4    2    6
                 4    6    4    8    6    0    0    0    0    0    0    0
                 0    0    0

CUBE    (  13  ,   53  )    IS SELECTED.

CUBE    (  22  ,   94  )'   HAS BEEN FOUND.

CHECKING POINTS
              9999    0 9999    0 9999 9999 9999 9999    0 9999    0 9999
                 0 9999 9999    0    2    2    0    4    8    4    4    0
                 2    2    2    4    2    6   10 9999    6    2    1    4
                 1    5   10 9999    5    0    2    0    0    2    8    2
                 2    0    1    0    0 9999 9999 9999 9999 9999 9999 9999
              9999 9999 9999

ACCUMULATED AWAYNESS :

                 0   10    0    6    0    4    0    4   12    0   12    0
                10    0    0   12   12   12   12    8    8    8    8   12
                12   10   10   10   10    6    6    6    6   10   10   10
                10    6    6    6    6   14   14   14   14   10   10   10
                10   14   14   14   14    0    0    0    0    0    0    0
                 0    0    0
```

```
CUBE    (  22  ,  94  )   IS SELECTED.

CUBE    (  53  ,  98  )   HAS BEEN FOUND.

CHECKING POINTS
                 9999    0 9999    0 9999 9999 9999 9999 9999 9999    1 9999
                 9999 9999 9999    2    2    6    2    4    8    6    4    0
                    4    4    2    0    4    6   10 9999    6    2    1    6
                    1    5   10 9999    5    4    4 9999    4    4    8    6
                    4    2    1    4    1 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999

ACCUMULATED AWAYNESS :

                    0   14    0   12    0    4    0    4   12    0   14    0
                   10    0    0   16   18   14   16   14   16   12   14   18
                   16   14   16   12   14   12   14    6   12   16   18   14
                   16   14   16    6   14   16   18   14   16   14   16   12
                   14   18   20   16   18    0    0    0    0    0    0    0
                    0    0    0

CUBE    (  53  ,  96  )   IS SELECTED.

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:          4

CUBE    (  89  ,  86  )   HAS BEEN FOUND.

CHECKING POINTS
                 9999    0 9999    0 9999 9999 9999 9999 9999 9999    1 9999
                 9999 9999 9999    6    6   10    6    8   12   10    8    2
                    8 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999    5    5 9999    5    5    9    7
                    5    3    2    5    2 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999

ACCUMULATED AWAYNESS :

                    0   18    0   16    0    4    0    4   12    0   20    0
                   10    0    0   18   20   16   18   16   18   14   16   20
                   18   14   16   12   14   12   14    6   12   16   18   14
                   16   14   16    6   14   20   22   14   20   18   20   16
                   18   22   24   20   22    0    0    0    0    0    0    0
                    0    0    0

CUBE    (  89  ,  86  )   IS SELECTED.

CUBE    ( 105  , 102  )   HAS BEEN FOUND.
```

```
CHECKING POINTS
                    9999    0 9999    0 9999 9999 9999 9999 9999 9999    2 9999
                    9999 9999 9999   10   10   14   10   12   16   14   12    4
                      12 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999

ACCUMULATED AWAYNESS :

                       0   22    0   20    0    4    0    4   12    0   22    0
                      10    0    0   20   22   18   20   18   20   16   18   22
                      20   14   16   12   14   12   14    6   12   16   18   14
                      16   14   16    6   14   20   22   14   20   18   20   16
                      15   22   24   20   22    0    0    0    0    0    0    0
                       0    0    0

CUBE    ( 105   , 102   )    IS SELECTED.

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :     4

CUBE    (  67   ,  84   )    HAS BEEN FOUND.

CUBE    (  67   , 100   )    HAS BEEN FOUND.

WATCH FOR     84 AND    100

CUBE    (  67   , 112   )    HAS BEEN FOUND.

CHECKING POINTS
                    9999    0 9999    0 9999 9999 9999 9999 9999 9999    2 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999    6
                      16 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999

ACCUMULATED AWAYNESS :

                       0   26    0   24    0    4    0    4   12    0   28    0
                      10    0    0   20   22   18   20   18   20   16   18   24
                      22   14   16   12   14   12   14    6   12   16   18   14
                      16   14   16    6   14   20   22   14   20   18   20   16
                      18   22   24   20   22    0    0    0    0    0    0    0
                       0    0    0

CUBE    (  67   ,  84   )    IS SELECTED.

CUBE    ( 117   ,  64   )    HAS BEEN FOUND.
```

```
CHECKING POINTS
                9999    0 9999    0 9999 9999 9999 9999 9999 9999    2 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999    8
                  16 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999

ACCUMULATED AWAYNESS :

                   0   32    0   30    0    4    0    4   12    0   34    0
                  10    0    0   20   22   18   20   18   20   16   18   26
                  26   14   16   12   14   12   14    6   12   16   18   14
                  16   14   16    6   14   20   22   14   20   18   20   16
                  18   22   24   20   22    0    0    0    0    0    0    0
                   0    0    0

CUBE    ( 117   ,   64   )    IS SELECTED.

CUBE    ( 114   ,   65   )    HAS BEEN FOUND.

CHECKING POINTS
                9999    0 9999    0 9999 9999 9999 9999 9999 9999    2 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   10
                  18 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999

ACCUMULATED AWAYNESS :

                   0   36    0   36    0    4    0    4   12    0   38    0
                  10    0    0   20   22   18   20   18   20   16   18   28
                  28   14   16   12   14   12   14    6   12   16   18   14
                  16   14   16    6   14   20   22   14   20   18   20   16
                  18   22   24   20   22    0    0    0    0    0    0    0
                   0    0    0

CUBE    ( 114   ,   65   )    IS SELECTED.

CUBE    (  72   ,   82   )    HAS BEEN FOUND.

CUBE    (  72   ,   98   )    HAS BEEN FOUND.

WATCH FOR     82 AND     98

CHECKING POINTS
                9999    1 9999    0 9999 9999 9999 9999 9999 9999    2 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999
```

```
ACCUMULATED AWAYNESS :
                    0    38     0    40     0     4     0     4    12     0    42     0
                   10     0     0    20    22    18    20    18    20    16    18    28
                   28    14    16    12    14    12    14     6    12    16    18    14
                   16    14    16     6    14    20    22    14    20    18    20    16
                   18    22    24    20    22     0     0     0     0     0     0     0
                    0     0     0

CUBE    (   72   ,   82   )    IS SELECTED.

CUBE    (   10   ,  106   )    HAS BEEN FOUND.

CHECKING POINTS
               9999 9999 9999    0 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999

ACCUMULATED AWAYNESS :
                    0    38     0    42     0     4     0     4    12     0    42     0
                   10     0     0    20    22    18    20    18    20    16    18    28
                   28    14    16    12    14    12    14     6    12    16    18    14
                   16    14    16     6    14    20    22    14    20    18    20    16
                   18    22    24    20    22     0     0     0     0     0     0     0
                    0     0     0

CUBE    (   10   ,  106   )    IS SELECTED.

CUBE    (   14   ,   10   )    HAS BEEN FOUND.

CUBE    (   14   ,   30   )    HAS BEEN FOUND.

WATCH FOR    10 AND    30

CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999
```

```
ACCUMULATED AWAYNESS :

                    0    38     0    42     0     4     0     4    12     0    42     0
                   10     0     0    20    22    18    20    18    20    16    18    28
                   28    14    16    12    14    12    14     6    12    16    18    14
                   16    14    16     6    14    20    22    14    20    18    20    16
                   18    22    24    20    22     0     0     0     0     0     0     0
                    0     0     0

CUBE   (  14   ,   30   )   IS SELECTED.


**FORTRAN ** STOP
/LOGOFF

C E420 LOGOFF AT 1129 ON 11/14   , FOR TSN 4173.
C E421 CPU TIME USED:  000101.8603 SECONDS.
```

APPENDIX F

EXAMPLE OF 8 VARIABLES, 126 MINTERMS

```
TOATAL NUMBER OF ON-SET AND DONT*CARES =    126
ON-SET =   126       DONT-CARES =      0
NUMBERS OF VARIABLES =       8

       MINT.#   1  =    0      MINT.#   2  =    1      MINT.#   3  =    2
       MINT.#   4  =    3      MINT.#   5  =    4      MINT.#   6  =    5
       MINT.#   7  =    6      MINT.#   8  =    7      MINT.#   9  =    8
       MINT.#  10  =    9      MINT.#  11  =   10      MINT.#  12  =   11
       MINT.#  13  =   12      MINT.#  14  =   13      MINT.#  15  =   14
       MINT.#  16  =   15      MINT.#  17  =   16      MINT.#  18  =   17
       MINT.#  19  =   18      MINT.#  20  =   19      MINT.#  21  =   20
       MINT.#  22  =   21      MINT.#  23  =   22      MINT.#  24  =   23
       MINT.#  25  =   24      MINT.#  26  =   25      MINT.#  27  =   26
       MINT.#  28  =   27      MINT.#  29  =   28      MINT.#  30  =   29
       MINT.#  31  =   30      MINT.#  32  =   31      MINT.#  33  =   32
       MINT.#  34  =   33      MINT.#  35  =   34      MINT.#  36  =   35
       MINT.#  37  =   36      MINT.#  38  =   37      MINT.#  39  =   38
       MINT.#  40  =   39      MINT.#  41  =   40      MINT.#  42  =   41
       MINT.#  43  =   42      MINT.#  44  =   43      MINT.#  45  =   44
       MINT.#  46  =   45      MINT.#  47  =   46      MINT.#  48  =   47
       MINT.#  49  =   48      MINT.#  50  =   49      MINT.#  51  =   50
       MINT.#  52  =   51      MINT.#  53  =   52      MINT.#  54  =   53
       MINT.#  55  =   54      MINT.#  56  =   55      MINT.#  57  =   56
       MINT.#  58  =   57      MINT.#  59  =   58      MINT.#  60  =   59
       MINT.#  61  =   60      MINT.#  62  =   61      MINT.#  63  =   62
       MINT.#  64  =   63      MINT.#  65  =   64      MINT.#  66  =   65
       MINT.#  67  =   66      MINT.#  68  =   67      MINT.#  69  =   68
       MINT.#  70  =   69      MINT.#  71  =   70      MINT.#  72  =   71
       MINT.#  73  =   72      MINT.#  74  =   74      MINT.#  75  =   75
       MINT.#  76  =   76      MINT.#  77  =   77      MINT.#  78  =   78
       MINT.#  79  =   79      MINT.#  80  =   80      MINT.#  81  =   81
       MINT.#  82  =   82      MINT.#  83  =   83      MINT.#  84  =   84
       MINT.#  85  =   85      MINT.#  86  =   86      MINT.#  87  =   87
       MINT.#  88  =   88      MINT.#  89  =   89      MINT.#  90  =   90
       MINT.#  91  =   91      MINT.#  92  =   92      MINT.#  93  =   93
       MINT.#  94  =   94      MINT.#  95  =   95      MINT.#  96  =   96
       MINT.#  97  =   97      MINT.#  98  =   98      MINT.#  99  =   99
       MINT.# 100  =  100      MINT.# 101  =  101      MINT.# 102  =  103
       MINT.# 103  =  104      MINT.# 104  =  105      MINT.# 105  =  106
       MINT.# 106  =  107      MINT.# 107  =  108      MINT.# 108  =  109
       MINT.# 109  =  110      MINT.# 110  =  111      MINT.# 111  =  112
       MINT.# 112  =  113      MINT.# 113  =  114      MINT.# 114  =  115
       MINT.# 115  =  116      MINT.# 116  =  117      MINT.# 117  =  118
       MINT.# 118  =  119      MINT.# 119  =  120      MINT.# 120  =  121
       MINT.# 121  =  122      MINT.# 122  =  123      MINT.# 123  =  124
       MINT.# 124  =  125      MINT.# 125  =  126      MINT.# 126  =  127
```

```
THE CUBE COVERING LIST :


THE      1-TH CUBE COVERING IS :
         6-CUBE = (    9 ,   54   ) =      00------

THIS CUBE COVERS THESE MINTERMS :
          0    1    2    3    4    5    6    7    8    9   10   11   12   13   14   15
         16   17   18   19   20   21   22   23   24   25   26   27   28   29   30   31
         32   33   34   35   36   37   38   39   40   41   42   43   44   45   46   47
         48   49   50   51   52   53   54   55   56   57   58   59   60   61   62   63

THIS MINTERM MT(   10   ) =      9       IS A CORE POINT


THE      2-TH CUBE COVERING IS :
         6-CUBE = (   89 ,   54   ) =      0--1----

THIS CUBE COVERS THESE MINTERMS :
         16   17   18   19   20   21   22   23   24   25   26   27   28   29   30   31
         48   49   50   51   52   53   54   55   56   57   58   59   60   61   62   63
         80   81   82   83   84   85   86   87   88   89   90   91   92   93   94   95
        112  113  114  115  116  117  118  119  120  121  122  123  124  125  126  127

THIS MINTERM MT(   89   ) =     89       IS A CORE POINT


THE      3-TH CUBE COVERING IS :
         5-CUBE = (   77 ,   62   ) =      0---11--

THIS CUBE COVERS THESE MINTERMS :
         12   13   14   15   28   29   30   31   44   45   46   47   60   61   62   63
         76   77   78   79   92   93   94   95  108  109  110  111  124  125  126  127


THE      4-TH CUBE COVERING IS :
         5-CUBE = (   65 ,   55   ) =      0---0--1

THIS CUBE COVERS THESE MINTERMS :
          1    3    5    7   17   19   21   23   33   35   37   39   49   51   53   55
         65   67   69   71   81   83   85   87   97   99  101  103  113  115  117  119
```

```
THE      5-TH CUBE COVERING IS :
        5-CUBE = (   72 ,   50   ) =        0----0-0

THIS CUBE COVERS THESE MINTERMS :
         0   2   8  10  16  18  24  26  32  34  40  42  48  50  56  58
        64  66  72  74  80  82  88  90  96  98 104 106 112 114 120 122

----------------------------------------------------------------------

THE      6-TH CUBE COVERING IS :
        5-CUBE = (   75 ,   62   ) =        0---1-1-

THIS CUBE COVERS THESE MINTERMS :
        10  11  14  15  26  27  30  31  42  43  46  47  58  59  62  63
        74  75  78  79  90  91  94  95 106 107 110 111 122 123 126 127


THE      7-TH CUBE COVERING IS :
        5-CUBE = (   70 ,   29   ) =        0-0--1--

THIS CUBE COVERS THESE MINTERMS :
         4   5   6   7  12  13  14  15  20  21  22  23  28  29  30  31
        68  69  70  71  76  77  78  79  84  85  86  87  92  93  94  95


THE      8-TH CUBE COVERING IS :
        5-CUBE = ( 105 ,   52   ) =        0-1---0-

THIS CUBE COVERS THESE MINTERMS :
        32  33  36  37  40  41  44  45  48  49  52  53  56  57  60  61
        96  97 100 101 104 105 108 109 112 113 116 117 120 121 124 125

THE FINAL CHECKING
            205  205  205  205  205  305  205  305  205  101  305  205
            305  305  405  405  305  305  305  305  305  405  305  405
            305  205  405  305  405  405  505  505  305  305  205  205
            205  305  101  205  305  205  305  205  305  305  305  305
            405  405  305  305  305  405  205  305  405  305  405  305
            405  405  405  405  105  105  105  105  105  205  105  205
            105  205  105  205  205  305  305  205  205  205  205  205
            305  205  305  205  101  305  205  305  305  405  405  205
            205  105  105  105  205  105  205  105  205  105  205  205
            205  205  305  305  205  205  205  305  101  205  305  205
            305  205  305  305  305  305
```

```
MAXIMUM CUBE DIMENSION =    6

    1     0     0     0     0     0     2     1     0     0     0     0     3     3
    1     0     0     0     4     6     4     1     0     0     5    10    10     5
    1     0     6    15    20    15     6     1

THE CHARACTERISTICS :
          6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  1  6  5
          6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5
          6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5
          6  5  6  5  6  5  6  5  6  5  6  1  6  5  6  5  6  5  6  5  6  5
          6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5
          6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  1
          6  5  6  5  6  5  6  5  6  1  6  5  6  1  6  5  6  1  6  5  6  1
          6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5
          6  1  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  1  6  5
          6  1  6  5  6  1  6  5  6  1  6  5  6  5  6  5  6  5  6  1  6  5
          6  5  6  5  6  5  6  5  6  5  6  5  6  1  6  5  6  5  6  5  6  5
          6  5  6  5  6  5  6  5  6  5

CUBE    (    9  ,   54   )    HAS BEEN FOUND.

CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999   16   16   16   16   16   16   16   16
                 16   16   16   16   16   16   16   16   16   16   16   16
                 16   16   16   16   16   16   16   16   16   16   16   16
                 16   16   16   16   16   16   16   16   16   16   16   16
                 16   16   16   16   16   16   16   16   16   16   16   16
                 16   16   16   16   16   16

ACCUMULATED AWAYNESS :

                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    2    2    2    2    2    2    2    2
                  2    2    2    2    2    2    2    2    2    2    2    2
                  2    2    2    2    2    2    2    2    2    2    2    2
                  2    2    2    2    2    2    2    2    2    2    2    2
                  2    2    2    2    2    2    2    2    2    2    2    2
                  2    2    2    2    2    2

CUBE    (    9  ,   54   )    IS SELECTED.

CUBE    (   89  ,   54   )    HAS BEEN FOUND.
```

```
CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999   32   24   32   32   32   32   24   32
                 24   32   24   32   24   32   32 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   32
                 32   32   32   32   32   32   32   32   32   32   32   32
                 32   32 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    4    4    4    4    4    4    4    4
                  4    4    4    4    4    4    4    2    2    2    2    2
                  2    2    2    2    2    2    2    2    2    2    2    4
                  4    4    4    4    4    4    4    4    4    4    4    4
                  4    4    2    2    2    2    2    2    2    2    2    2
                  2    2    2    2    2    2

CUBE    (   89   ,   54   )    IS SELECTED.

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :      6

CUBE    (   77   ,   22   )    HAS BEEN FOUND.

CUBE    (   77   ,   52   )    HAS BEEN FOUND.

WATCH FOR      22 AND      52

CUBE    (   77   ,   55   )    HAS BEEN FOUND.

WATCH FOR      52 AND      55

CUBE    (   77   ,   62   )    HAS BEEN FOUND.

WATCH FOR      55 AND      62

CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
```

```
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999   34   26   34   34   40   40   32   40
       32   40   32 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   34
       34   34   34   40   40   40   40   40   40   40 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

          0    0    0    0    0    0    0    0    0    0    0    0
          0    0    0    0    0    0    0    0    0    0    0    0
          0    0    0    0    0    0    0    0    0    0    0    0
          0    0    0    0    0    0    0    0    0    0    0    0
          0    0    0    0    0    0    0    0    0    0    0    0
          0    0    0    0    8    8    8    8    6    6    6    6
          6    6    6    4    4    4    4    2    2    2    2    2
          2    2    2    2    2    2    2    2    2    2    2    8
          8    8    8    6    6    6    6    6    6    6    4    4
          4    4    2    2    2    2    2    2    2    2    2    2
          2    2    2    2    2    2

CUBE    (   77  ,   62  )    IS SELECTED.

CUBE    (   65  ,   22  )    HAS BEEN FOUND.

CUBE    (   65  ,   50  )    HAS BEEN FOUND.

WATCH FOR     22 AND     50

CUBE    (   65  ,   52  )    HAS BEEN FOUND.

WATCH FOR     50 AND     52

CUBE    (   65  ,   55  )    HAS BEEN FOUND.

WATCH FOR     52 AND     55

CHECKING POINTS
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999   42 9999   42 9999   48 9999   40 9999
       34   42   40 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   42
     9999   42 9999   48 9999 9999   42   48   42   48 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999
```

```
ACCUMULATED AWAYNESS :

           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0    10     8    10     8     8     6     8     6
          10    10     8     4     4     4     4     2     2     2     2     2
           2     2     2     2     2     2     2     2     2     2     2    10
           8    10     8     8     6     6    10     8    10     8     4     4
           4     4     2     2     2     2     2     2     2     2     2     2
           2     2     2     2     2     2

CUBE    (  65   ,   55   )    IS SELECTED.

CUBE    (  72   ,   22   )    HAS BEEN FOUND.

CUBE    (  72   ,   50   )    HAS BEEN FOUND.

CUBE    (  72   ,   52   )    HAS BEEN FOUND.

CUBE    (  72   ,   62   )    HAS BEEN FOUND.

CHECKING POINTS
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999   56 9999   48 9999
        9999 9999   48 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999   56 9999 9999 9999   56 9999   56 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0    10     8    10     8    10     6    10     6
          10    10    10     4     4     4     4     2     2     2     2     2
           2     2     2     2     2     2     2     2     2     2     2    10
           8    10     8    10     6     6    10    10    10    10     4     4
           4     4     2     2     2     2     2     2     2     2     2     2
           2     2     2     2     2     2

CUBE    (  72   ,   50   )    IS SELECTED.
```

```
CUBE    (  75   ,   22   )    HAS BEEN FOUND.

CUBE    (  75   ,   50   )    HAS BEEN FOUND.

WATCH FOR     22 AND     50

CUBE    (  75   ,   55   )    HAS BEEN FOUND.

WATCH FOR     50 AND     55

CUBE    (  75   ,   62   )    HAS BEEN FOUND.

WATCH FOR     55 AND     62

CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999   58 9999   56 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999   58 9999 9999 9999   64 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0   10    8   10    8   14    6   12    6
                 10   10   10    4    4    4    4    2    2    2    2    2
                  2    2    2    2    2    2    2    2    2    2    2   10
                  8   10    8   14    6    6   10   12   10   10    4    4
                  4    4    2    2    2    2    2    2    2    2    2    2
                  2    2    2    2    2    2

CUBE    (  75   ,   62   )    IS SELECTED.

CUBE    (  70   ,   17   )    HAS BEEN FOUND.

CUBE    (  70   ,   24   )    HAS BEEN FOUND.

WATCH FOR     17 AND     24

CUBE    (  70   ,   27   )    HAS BEEN FOUND.

CUBE    (  70   ,   29   )    HAS BEEN FOUND.
```

```
WATCH FOR     24 AND     29

CHECKING POINTS
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999   66 9999 9999 9999   66 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                       0    0    0    0    0    0    0    0    0    0    0    0
                       0    0    0    0    0    0    0    0    0    0    0    0
                       0    0    0    0    0    0    0    0    0    0    0    0
                       0    0    0    0    0    0    0    0    0    0    0    0
                       0    0    0    0    0    0    0    0    0    0    0    0
                       0    0    0    0   10    8   10    8   14    6   12    6
                      10   10   10    4    4    4    4    2    2    2    2    2
                       2    2    2    2    2    2    2    2    2    2    2   10
                       8   10    8   16    6    6   10   16   10   10    4    4
                       4    4    2    2    2    2    2    2    2    2    2    2
                       2    2    2    2    2    2

CUBE     (   70   ,   29   )    IS SELECTED.

CUBE     ( 105    ,   50   )    HAS BEEN FOUND.

CUBE     ( 105    ,   52   )    HAS BEEN FOUND.

CUBE     ( 105    ,   55   )    HAS BEEN FOUND.

CUBE     ( 105    ,   62   )    HAS BEEN FOUND.

CHECKING POINTS
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                    9999 9999 9999 9999 9999 9999
```

```
 ACCUMULATED AWAYNESS :

                0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0   10    8   10    8   14    6   12    6
               10   10   10    4    4    4    4    2    2    2    2    2
                2    2    2    2    2    2    2    2    2    2    2   10
                8   10    8   16    6    6   10   16   10   10    4    4
                4    4    2    2    2    2    2    2    2    2    2    2
                2    2    2    2    2    2

 CUBE    ( 105   ,   52   )    IS SELECTED.
*9
 **FORTRAN ** STOP
/LOGOFF BUT
%C E420 LOGOFF AT 1125 ON 11/09/77, FOR TSN 0951.
%C E421 CPU TIME USED:  000212.3728 SECONDS.
```

APPENDIX G

EXAMPLE OF 7 VARIABLES, 92 MINTERMS

```
TOATAL NUMBER OF ON-SET AND DONT*CARES =     92
ON-SET =     92       DONT-CARES =       0
NUMBERS OF VARIABLES =     7

    MINT,#   1  =     34    MINT,#   2  =     35    MINT,#   3  =    37
    MINT,#   4  =     39    MINT,#   5  =     44    MINT,#   6  =    47
    MINT,#   7  =     48    MINT,#   8  =     49    MINT,#   9  =    50
    MINT,#  10  =     51    MINT,#  11  =     52    MINT,#  12  =    53
    MINT,#  13  =     54    MINT,#  14  =     55    MINT,#  15  =    56
    MINT,#  16  =     57    MINT,#  17  =     58    MINT,#  18  =    61
    MINT,#  19  =     62    MINT,#  20  =     63    MINT,#  21  =    64
    MINT,#  22  =     83    MINT,#  23  =     84    MINT,#  24  =    85
    MINT,#  25  =     86    MINT,#  26  =     87    MINT,#  27  =    88
    MINT,#  28  =     89    MINT,#  29  =     90    MINT,#  30  =    95
    MINT,#  31  =      1    MINT,#  32  =      2    MINT,#  33  =     3
    MINT,#  34  =      4    MINT,#  35  =      5    MINT,#  36  =     6
    MINT,#  37  =      7    MINT,#  38  =      8    MINT,#  39  =     9
    MINT,#  40  =     10    MINT,#  41  =     11    MINT,#  42  =    14
    MINT,#  43  =     15    MINT,#  44  =     16    MINT,#  45  =    17
    MINT,#  46  =     18    MINT,#  47  =     19    MINT,#  48  =    20
    MINT,#  49  =     21    MINT,#  50  =     22    MINT,#  51  =    23
    MINT,#  52  =     24    MINT,#  53  =     25    MINT,#  54  =    26
    MINT,#  55  =     27    MINT,#  56  =     28    MINT,#  57  =    29
    MINT,#  58  =     30    MINT,#  59  =     31    MINT,#  60  =    32
    MINT,#  61  =     96    MINT,#  62  =     97    MINT,#  63  =    98
    MINT,#  64  =     99    MINT,#  65  =    100    MINT,#  66  =   101
    MINT,#  67  =    102    MINT,#  68  =    103    MINT,#  69  =   104
    MINT,#  70  =    105    MINT,#  71  =    106    MINT,#  72  =   107
    MINT,#  73  =    108    MINT,#  74  =    109    MINT,#  75  =   110
    MINT,#  76  =    111    MINT,#  77  =    112    MINT,#  78  =   113
    MINT,#  79  =    114    MINT,#  80  =    115    MINT,#  81  =   116
    MINT,#  82  =    117    MINT,#  83  =    118    MINT,#  84  =   119
    MINT,#  85  =    120    MINT,#  86  =    121    MINT,#  87  =   122
    MINT,#  88  =    123    MINT,#  89  =    124    MINT,#  90  =   125
    MINT,#  91  =    126    MINT,#  92  =    127    MINT,#

THE CUBE COVERING LIST :



THE      1-TH CUBE COVERING IS :
       1-CUBE = (    44 ,   108   ) =       -101100

THIS CUBE COVERS THESE MINTERMS :
       44  108

THIS MINTERM MT(    5   ) =     44      IS A CORE POINT
```

```
THE      2-TH CUBE COVERING IS :
        1-CUBE = (    64 ,    96  ) =      1-00000

THIS CUBE COVERS THESE MINTERMS :
        64   96

THIS MINTERM MT(  21  ) =     64      IS A CORE POINT



THE      3-TH CUBE COVERING IS :
        3-CUBE = (    83 ,    55  ) =      --10-11

THIS CUBE COVERS THESE MINTERMS :
        83   55   51   87   19   23  115  119

THIS MINTERM MT(  22  ) =     83      IS A CORE POINT



THE      4-TH CUBE COVERING IS :
        4-CUBE = (    84 ,    55  ) =      --101--

THIS CUBE COVERS THESE MINTERMS :
        84   55   52   53   54   85   86   87   20   21   22   23
       116  117  118  119

THIS MINTERM MT(  23  ) =     84      IS A CORE POINT



THE      5-TH CUBE COVERING IS :
        3-CUBE = (    89 ,    56  ) =      --1100-

THIS CUBE COVERS THESE MINTERMS :
        89   56   57   88   24   25  120  121

THIS MINTERM MT(  28  ) =     89      IS A CORE POINT



THE      6-TH CUBE COVERING IS :
        3-CUBE = (    90 ,    56  ) =      --110-0

THIS CUBE COVERS THESE MINTERMS :
        90   56   58   88   24   26  120  122

THIS MINTERM MT(  29  ) =     90      IS A CORE POINT
```

```
THE      7-TH CUBE COVERING IS :
       3-CUBE = (    95 ,    55   ) =       --1-111

THIS CUBE COVERS THESE MINTERMS :
        95    55    63    87    23    31   119   127

THIS MINTERM MT(  30   ) =     95      IS A CORE POINT


THE      8-TH CUBE COVERING IS :
       3-CUBE = (     4 ,    23   ) =       00-01--

THIS CUBE COVERS THESE MINTERMS :
         4    23     5     6     7    20    21    22

THIS MINTERM MT(  34   ) =      4      IS A CORE POINT


THE      9-TH CUBE COVERING IS :
       3-CUBE = (     8 ,    27   ) =       00-10--

THIS CUBE COVERS THESE MINTERMS :
         8    27     9    10    11    24    25    26

THIS MINTERM MT(  38   ) =      8      IS A CORE POINT


THE     10-TH CUBE COVERING IS :
       4-CUBE = (    14 ,    19   ) =       00---1-

THIS CUBE COVERS THESE MINTERMS :
        14    19     2     3     6     7    10    11    15    18    22    23
        26    27    30    31

THIS MINTERM MT(  42   ) =     14      IS A CORE POINT


THE     11-TH CUBE COVERING IS :
       4-CUBE = (    28 ,    19   ) =       001----

THIS CUBE COVERS THESE MINTERMS :
        28    19    16    17    18    20    21    22    23    24    25    26
        27    29    30    31

THIS MINTERM MT(  56   ) =     28      IS A CORE POINT
```

```
THE     12-TH CUBE COVERING IS :
        3-CUBE = (    32 ,   114   ) =       -1-00-0

THIS CUBE COVERS THESE MINTERMS :
        32   114    34    48    50    96    98   112

THIS MINTERM MT(  60   ) =    32      IS A CORE POINT



THE     13-TH CUBE COVERING IS :
        5-CUBE = (    97 ,   126   ) =       11-----

THIS CUBE COVERS THESE MINTERMS :
        97   126    96    98    99   100   101   102   103   104   105   106
       107   108   109   110   111   112   113   114   115   116   117   118
       119   120   121   122   123   124   125   127

THIS MINTERM MT(  62   ) =    97      IS A CORE POINT



THE     14-TH CUBE COVERING IS :
        4-CUBE = (    49 ,   118   ) =       -110---

THIS CUBE COVERS THESE MINTERMS :
        49   118    48    50    51    52    53    54    55   112   113   114
       115   116   117   119



THE     15-TH CUBE COVERING IS :
        3-CUBE = (    47 ,   119   ) =       -1--111

THIS CUBE COVERS THESE MINTERMS :
        47   119    39    55    63   103   111   127



THE     16-TH CUBE COVERING IS :
        3-CUBE = (    37 ,   119   ) =       -1-01-1

THIS CUBE COVERS THESE MINTERMS :
        37   119    39    53    55   101   103   117
```

```
THE     17-TH CUBE COVERING IS :
        3-CUBE = (     1 ,    23    ) =      00-0--1

THIS CUBE COVERS THESE MINTERMS :
          1    23    3    5    7    17    19    21



THE     18-TH CUBE COVERING IS :
        3-CUBE = (    35 ,   119    ) =      -1-0-11

THIS CUBE COVERS THESE MINTERMS :
        35   119    39    51    55    99   103   115



THE     19-TH CUBE COVERING IS :
        3-CUBE = (    61 ,   119    ) =      -11-1-1

THIS CUBE COVERS THESE MINTERMS :
        61   119    53    55    63   117   125   127



THE     20-TH CUBE COVERING IS :
        3-CUBE = (    62 ,   119    ) =      -11-11-

THIS CUBE COVERS THESE MINTERMS :
        62   119    54    55    63   118   126   127

THE FINAL CHECKING
            105  105  105  305  101  105  205  105  205  305  205  405
            305  905  205  105  105  105  105  405  101  101  101  101
            101  305  205  101  101  101  105  105  205  101  201  201
            305  101  101  201  201  101  101  105  205  205  405  305
            405  405  705  405  305  405  301  101  101  205  305  101
            305  101  205  205  101  205  101  405  101  101  101  101
            205  101  101  205  305  205  305  405  305  505  405 1005
            305  205  205  101  101  205  205  505
```

```
                               0       0       0       0       0       0       0       1       0       0       0       2
                            9999       2       2       2       4       4       4       0       4       4       4       0
                            9999       2       2       2       2       2       2       0       4       0       0       0
                               4       0       0       0       2       2       2       0

ACCUMULATED AWAYNESS :

                              10      14      10      14       0      14      10      14      14      18      10      14
                              14      18      10      14      14      14      14      18       6      18      10      14
                              14      18      10      14      14      18      12      12      16       8      12      12
                              16       8      12      12      16      12      16      12      16      16      20      12
                              16      16      20      12      16      16      20      12      16      16      20       6
                               4       8       8      12       4       8       8      12       4       8       8      12
                               0       8       8      12       8      12      12      16       8      12      12      16
                               8      12      12      16       8      12      12      16

CUBE    (   64   ,    96   )    IS SELECTED.

CUBE    (   83   ,    55   )    HAS BEEN FOUND.

CHECKING POINTS
                               2       4       0       8    9999       1       4       8       8    9999       3       8
                               8    9999       2       1       1       3       3       8    9999    9999       2       4
                               4    9999       1       0       0       2       2       4       8       0       2       2
                               8       0       0       0       2       1       2       4       8       8    9999       2
                               8       8    9999       0       2       2       8       1       2       2       8       2
                            9999       4       4      10       4       6       6       8       4       4       4       2
                            9999       2       2       4       4      10      10    9999       6       8       8    9999
                               4       2       2       8       2       4       4       8

ACCUMULATED AWAYNESS :

                              14      16      14      16       0      18      14      16      16      18      14      16
                              16      18      16      18      18      18      18      20       6      18      14      16
                              16      18      16      18      18      20      16      16      18      14      16      16
                              18      16      18      18      20      18      20      16      18      18      20      16
                              18      18      20      18      20      20      22      18      20      20      22      12
                               4      12      12      14      10      12      12      14      12      14      14      16
                               0      14      14      16      12      14      14      16      12      14      14      16
                              14      16      16      18      14      16      16      18

CUBE    (   83   ,    55   )    IS SELECTED.

CUBE    (   84   ,    55   )    HAS BEEN FOUND.

CHECKING POINTS
                               3       5       2      16    9999       1      12      16      16    9999    9999    9999
                            9999    9999       3       2       2       7       7      16    9999    9999    9999    9999
                            9999    9999       2       0       0       4       3       6      10       2      10      10
                              16       0       0       0       2       2       4      12      16      16    9999    9999
                            9999    9999    9999       2       4       4      10       5      10      10      16       2
```

```
MAXIMUM CUBE DIMENSION -    6

   1    0    0    0    0    0    2    1    0    0    0    0    3    3
   1    0    0    0    4    6    4    1    0    0    5   10   10    5
   1    0    6   15   20   15    6    1

THE CHARACTERISTICS :
      5  1  5  1  4  1  6  1  1  1  4  1  6  5  6  1  6  5  6  1  5  1
      6  5  6  1  6  5  5  1  5  1  5  1  5  1  5  1  6  1  1  1  3  1
      4  1  4  1  4  1  5  5  4  1  3  1  3  1  3  1  4  1  5  1  5  5
      3  1  5  1  5  1  6  1  3  1  4  1  5  1  5  1  4  1  5  1  5  1
      6  1  6  1  6  5  5  5  6  5  6  5  6  5  5  5  5  5  6  5  5  1
      4  1  5  1  6  1  6  5  3  1  5  5  5  1  6  1  6  1  5  1  5  5
      5  1  6  1  5  1  5  1  5  1  5  1  5  5  5  1  5  1  5  5  6  1
      6  1  6  1  6  5  6  5  6  5  6  5  6  5  6  5  6  5  6  5  5  1
      5  1  6  1  6  1  6  5

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:          5

CUBE   (    44   ,   108   )   HAS BEEN FOUND.

CHECKING POINTS
                 0    0    0    0 9999    1    0    0    0    0    2    0
                 0    0    2    0    0    2    2    0    0    0    0    0
                 0    0    0    0    0    0    0    0    0    0    0    0
                 0    0    0    0    0    1    0    0    0    0    0    0
                 0    0    0    0    0    0    0    1    0    0    0    0
                 2    0    0    0    2    2    2    0    2    2    2    0
              9999    2    2    2    0    0    0    0    2    0    0    0
                 2    0    0    0    2    2    2    0

ACCUMULATED AWAYNESS :

                 6    8    4    6    0    4    6    8    8   10    4    6
                 6    8    4    6    6    4    4    6    6   12    6    8
                 8   10    6    8    8    8    8    8   10    4    6    6
                 8    4    6    6    8    4    6    8   10   10   12    6
                 8    8   10    6    8    8   10    4    6    6    8    4
                 4    6    6    8    2    4    4    6    2    4    4    6
                 0    2    2    4    6    8    8   10    4    6    6    8
                 4    6    6    8    2    4    4    6

CUBE   (    44   ,   108   )   IS SELECTED.

CUBE   (    64   ,    96   )   HAS BEEN FOUND.

CHECKING POINTS
                 1    0    0    0 9999    1    2    0    0    0    2    0
                 0    0    2    0    0    2    2    0 9999    0    1    0
                 0    0    1    0    0    0    1    2    0    0    0    0
                 0    0    0    0    0    1    0    2    0    0    0    0
```

```
          9999    6    6   12   12   14   14   16    4    4    4    2
          9999    4    4    6   12   18   18 9999 9999 9999 9999 9999
             6    4    4   10   10   12   12   16

ACCUMULATED AWAYNESS :

            18   20   16   18    0   22   16   18   18   18   14   16
            16   18   20   22   22   20   20   22    6   18   14   16
            16   18   20   22   22   22   20   20   22   16   18   18
            20   22   24   24   26   22   24   18   20   20   20   16
            18   18   20   22   24   24   26   20   22   22   24   16
             4   16   16   18   12   14   14   16   18   20   20   22
             0   18   18   20   14   16   16   16   12   14   14   16
            18   20   20   22   16   18   18   20

CUBE    (    84  ,    55  )    IS SELECTED.

CUBE    (    89  ,    56  )    HAS BEEN FOUND.

CHECKING POINTS
             3    5    2   16 9999    1   20   24   18 9999 9999 9999
          9999 9999 9999 9999    6   11    8   18 9999 9999 9999 9999
          9999 9999 9999 9999    2    4    4    6   10    2   10   10
            16    2    4    2    4    2    4   20   24   18 9999 9999
          9999 9999 9999 9999 9999   12   18    9   18   12   18    2
          9999    8    6   12   12   14   14   16   12   12    6    4
          9999    6    4    6   20   26   20 9999 9999 9999 9999 9999
          9999 9999   12   18   18   20   14   18

ACCUMULATED AWAYNESS :

            24   26   22   26    0   28   18   20   22   18   14   16
            16   18   20   22   24   22   24   26    6   18   14   16
            16   18   20   22   24   26   24   26   28   22   24   26
            28   24   26   28   30   28   30   20   22   24   20   16
            18   18   20   22   24   26   28   22   24   26   28   20
             4   20   22   24   18   20   22   24   20   22   24   26
             0   22   24   26   16   18   20   16   12   14   14   16
            18   20   22   24   18   20   22   24

CUBE    (    89  ,    56  )    IS SELECTED.

CUBE    (    90  ,    56  )    HAS BEEN FOUND.

CHECKING POINTS
             4    5    2   16 9999    1   28   26   26 9999 9999 9999
          9999 9999 9999 9999 9999   12   12   20 9999 9999 9999 9999
          9999 9999 9999 9999 9999    4    4    8   10    2   10   10
            16    4    5   10    6    3    4   28   26   26 9999 9999
          9999 9999 9999 9999 9999 9999   26   13   20   20   20    2
          9999    8    8   12   12   14   14   16   20   14   14    6
```

```
             9999    6    6    6   28   28   28 9999 9999 9999 9999 9999
             9999 9999 9999   26   26   22   22   20

ACCUMULATED AWAYNESS :

               28   32   30   34    0   34   20   24   24   18   14   16
               16   18   20   22   24   26   26   30    6   18   14   16
               16   18   20   22   24   30   30   30   34   28   32   32
               36   26   30   30   34   32   36   22   26   26   20   16
               18   18   20   22   24   26   30   24   28   28   32   24
                4   26   26   30   24   28   28   32   22   26   26   30
                0   28   28   32   18   22   22   16   12   14   14   16
               18   20   22   26   20   24   24   28

CUBE    (   90  ,   56  )   IS SELECTED.

CUBE    (   95  ,   55  )   HAS BEEN FOUND.

CHECKING POINTS
                4    6    2   24 9999    3   28   28   28 9999 9999 9999
             9999 9999 9999 9999 9999   16   16 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999    4    8   12    2   12   12
               24    4    5   10    8    4   12   28   28   28 9999 9999
             9999 9999 9999 9999 9999 9999   34   14   28   28 9999    2
             9999    8    8   14   12   16   16   24   20   14   14    8
             9999    8    8   14   28   30   30 9999 9999 9999 9999 9999
             9999 9999 9999   34   28   30   30 9999

ACCUMULATED AWAYNESS :

               34   36   34   36    0   36   26   28   28   18   14   16
               16   18   20   22   24   28   28   30    6   18   14   16
               16   18   20   22   24   30   36   36   38   34   36   36
               38   34   36   36   38   36   38   28   30   30   20   16
               18   18   20   22   24   26   32   28   30   30   32   32
                4   32   32   34   30   32   32   34   30   32   32   34
                0   32   32   34   24   26   26   16   12   14   14   16
               18   20   22   28   24   26   26   28

CUBE    (   95  ,   55  )   IS SELECTED.

CUBE    (    4  ,   23  )   HAS BEEN FOUND.

CHECKING POINTS
                5    7    4   32 9999    3   30   30   30 9999 9999 9999
             9999 9999 9999 9999 9999   17   17 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999    6   12   20 9999 9999 9999
             9999    4    6   12   10    8   20   36   36   36 9999 9999
             9999 9999 9999 9999 9999 9999   36   18   36   36 9999    2
             9999    8    8   14   14   18   18   26   20   14   14    8
             9999    8    8   14   28   30   30 9999 9999 9999 9999 9999
             9999 9999 9999   34   28   30   30 9999
```

```
ACCUMULATED AWAYNESS :

               38      40      36      38       0      40      30      32      32      18      14      16
               16      18      20      22      24      32      32      30       6      18      14      16
               16      18      20      22      24      30      38      38      40      34      36      36
               38      38      40      40      42      38      40      30      32      32      20      16
               18      18      20      22      24      26      36      30      32      32      32      36
                4      38      38      40      34      36      36      38      38      40      40      42
                0      38      38      40      30      32      32      16      12      14      14      16
               18      20      22      36      30      32      32      28

CUBE    (      4   ,    23   )    IS SELECTED.

CUBE    (      8   ,    27   )    HAS BEEN FOUND.

CHECKING POINTS
                   6    8    4   32 9999    3   32   32   32 9999 9999 9999
               9999 9999 9999 9999 9999   18   18 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999    8   16   28 9999 9999 9999
               9999 9999 9999 9999 9999   12   28   44   44   44 9999 9999
               9999 9999 9999 9999 9999 9999 9999   22   44   44 9999    2
               9999    8    8   14   14   18   18   26   22   16   16   10
               9999    8    8   14   28   30   30 9999 9999 9999 9999 9999
               9999 9999 9999   36   28   30   30 9999

ACCUMULATED AWAYNESS :

               42      44      42      44       0      44      34      36      36      18      14      16
               16      18      20      22      24      36      36      30       6      18      14      16
               16      18      20      22      24      30      40      40      42      34      36      36
               38      38      40      40      42      40      42      32      34      34      20      16
               18      18      20      22      24      26      36      32      34      34      32      40
                4      44      44      46      42      44      44      46      42      44      44      46
                0      44      44      46      36      38      38      16      12      14      14      16
               18      20      22      40      36      38      38      28

CUBE    (      8   ,    27   )    IS SELECTED.

CUBE    (     14   ,    19   )    HAS BEEN FOUND.

CHECKING POINTS
                  10   12    4   40 9999    3   34   34   40 9999 9999 9999
               9999 9999 9999 9999 9999   19   22 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999   10 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999   32   32 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999   26   32 9999 9999    2
               9999    8   10   16   14   18   20   28   22   16   18   12
               9999    8   10   16   28   30   32 9999 9999 9999 9999 9999
               9999 9999 9999   38   28   30   32 9999
```

```
ACCUMULATED AWAYNESS :

                44    46    46    46     0    46    38    40    38    18    14    16
                16    18    20    22    24    40    38    30     6    18    14    16
                16    18    20    22    24    30    42    40    42    34    36    36
                38    38    40    40    42    40    42    34    36    34    20    16
                18    18    20    22    24    26    36    34    36    34    32    44
                 4    50    48    50    48    50    48    50    48    50    48    50
                 0    50    48    50    42    44    42    16    12    14    14    16
                18    20    22    44    42    44    42    28

CUBE    (    14   ,    19   )    IS SELECTED.

CUBE    (    28   ,    19   )    HAS BEEN FOUND.

CHECKING POINTS
                11    13     4    42  9999     5    42    42    48  9999  9999  9999
              9999  9999  9999  9999  9999    23    26  9999  9999  9999  9999  9999
              9999  9999  9999  9999  9999  9999    12  9999  9999  9999  9999  9999
              9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
              9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999     2
              9999     8    10    16    14    18    20    28    22    16    18    12
              9999     8    10    16    30    32    34  9999  9999  9999  9999  9999
              9999  9999  9999    40    30    32    34  9999

ACCUMULATED AWAYNESS :

                48    50    50    50     0    50    40    42    40    18    14    16
                16    18    20    22    24    42    40    30     6    18    14    16
                16    18    20    22    24    30    44    40    42    34    36    36
                38    38    40    40    42    40    42    34    36    34    20    16
                18    18    20    22    24    26    36    34    36    34    32    48
                 4    56    54    56    54    56    54    56    54    56    54    56
                 0    56    54    56    46    48    46    16    12    14    14    16
                18    20    22    48    46    48    46    28

CUBE    (    28   ,    19   )    IS SELECTED.

CUBE    (    32   ,   114   )    HAS BEEN FOUND.

CHECKING POINTS
              9999    17     4    44  9999     5  9999    50  9999  9999  9999  9999
              9999  9999  9999  9999  9999    23    27  9999  9999  9999  9999  9999
              9999  9999  9999  9999  9999  9999    12  9999  9999  9999  9999  9999
              9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
              9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
              9999    16  9999    24    22    20    28    30    30    18    26    14
              9999     8    12    16  9999    40  9999  9999  9999  9999  9999  9999
              9999  9999  9999    42    32    32    36  9999
```

```
ACCUMULATED AWAYNESS :

                48   52   54   54    0   56   40   44   40   18   14   16
                16   18   20   22   24   48   44   30    6   18   14   16
                16   18   20   22   24   30   48   40   42   34   36   36
                38   38   40   40   42   40   42   34   36   34   20   16
                18   18   20   22   24   26   36   34   36   34   32   48
                 4   58   54   58   56   60   56   60   56   60   56   60
                 0   62   58   62   46   50   46   16   12   14   14   16
                18   20   22   52   50   54   50   28

CUBE   (   32  ,  114  )   IS SELECTED.

CUBE   (   97  ,  126  )   HAS BEEN FOUND.

CHECKING POINTS
              9999   21    6   52 9999    7 9999   58 9999 9999 9999 9999
              9999 9999 9999 9999 9999   27   31 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999 9999   12 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                48   54   56   56    0   58   40   46   40   18   14   16
                16   18   20   22   24   50   46   30    6   18   14   16
                16   18   20   22   24   30   52   40   42   34   36   36
                38   38   40   40   42   40   42   34   36   34   20   16
                18   18   20   22   24   26   36   34   36   34   32   48
                 4   58   54   58   56   60   56   60   56   60   56   60
                 0   62   58   62   46   50   46   16   12   14   14   16
                18   20   22   52   50   54   50   28

CUBE   (   97  ,  126  )   IS SELECTED.

ALL CORE CUBES HAVE BEEN SEARCHED,
AND THE MAXIMUM CUBE DIMENSION IS :     5

CUBE   (   49  ,   22  )   HAS BEEN FOUND.

CUBE   (   49  ,  118  )   HAS BEEN FOUND.

WATCH FOR    22 AND   118

CHECKING POINTS
              9999   23    8   54 9999    7 9999 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999   29   33 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999 9999   12 9999 9999 9999 9999 9999
```

```
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            48    56    58    58     0    62    40    46    40    18    14    16
            16    18    20    22    24    52    48    30     6    18    14    16
            16    18    20    22    24    30    56    40    42    34    36    36
            38    38    40    40    42    40    42    34    36    34    20    16
            18    18    20    22    24    26    36    34    36    34    32    48
             4    58    54    58    56    60    56    60    56    60    56    60
             0    62    58    62    46    50    46    16    12    14    14    16
            18    20    22    52    50    54    50    28

CUBE    (    49   ,   118   )    IS SELECTED.

CUBE    (    47   ,    23   )    HAS BEEN FOUND.

CUBE    (    47   ,   119   )    HAS BEEN FOUND.

WATCH FOR     23 AND    119

CHECKING POINTS
          9999   25   10 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999   31   35 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999   12 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            48    58    60    58     0    62    40    46    40    18    14    16
            16    18    20    22    24    54    50    30     6    18    14    16
            16    18    20    22    24    30    62    40    42    34    36    36
            38    38    40    40    42    40    42    34    36    34    20    16
            18    18    20    22    24    26    36    34    36    34    32    48
             4    58    54    58    56    60    56    60    56    60    56    60
             0    62    58    62    46    50    46    16    12    14    14    16
            18    20    22    52    50    54    50    28

CUBE    (    47   ,   119   )    IS SELECTED.

CUBE    (    37   ,    23   )    HAS BEEN FOUND.

CUBE    (    37   ,   119   )    HAS BEEN FOUND.
```

```
WATCH FOR    23 AND   119

CHECKING POINTS
            9999   27 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999   33   35 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999   12 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

              48    60    60    58     0    62    40    46    40    18    14    16
              16    18    20    22    24    56    54    30     6    18    14    16
              16    18    20    22    24    30    66    40    42    34    36    36
              38    38    40    40    42    40    42    34    36    34    20    16
              18    18    20    22    24    26    36    34    36    34    32    48
               4    58    54    58    56    60    56    60    56    60    56    60
               0    62    58    62    46    50    46    16    12    14    14    16
              18    20    22    52    50    54    50    28

CUBE    (    37   ,   119   )    IS SELECTED.

CUBE    (     1   ,    23   )    HAS BEEN FOUND.

CUBE    (     1   ,    27   )   ,HAS BEEN FOUND.

WATCH FOR    23 AND    27

CHECKING POINTS
            9999   29 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999   33   35 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
            9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

              48    62    60    58     0    62    40    46    40    18    14    16
              16    18    20    22    24    60    60    30     6    18    14    16
              16    18    20    22    24    30    66    40    42    34    36    36
              38    38    40    40    42    40    42    34    36    34    20    16
              18    18    20    22    24    26    36    34    36    34    32    48
               4    58    54    58    56    60    56    60    56    60    56    60
               0    62    58    62    46    50    46    16    12    14    14    16
              18    20    22    52    50    54    50    28
```

```
CUBE    (     1   ,    23   )    IS SELECTED.

CUBE    (    35   ,    18   )    HAS BEEN FOUND.

CUBE    (    35   ,    23   )    HAS BEEN FOUND.

WATCH FOR     18 AND     23

CUBE    (    35   ,   114   )    HAS BEEN FOUND.

WATCH FOR     23 AND    114

CUBE    (    35   ,   119   )    HAS BEEN FOUND.

WATCH FOR     23 AND    119

CHECKING POINTS
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999   33   35 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                 48    62    60    58     0    62    40    46    40    18    14    16
                 16    18    20    22    24    64    64    30     6    18    14    16
                 16    18    20    22    24    30    66    40    42    34    36    36
                 38    38    40    40    42    40    42    34    36    34    20    16
                 18    18    20    22    24    26    36    34    36    34    32    48
                  4    58    54    58    56    60    56    60    56    60    56    60
                  0    62    58    62    46    50    46    16    12    14    14    16
                 18    20    22    52    50    54    50    28
CUBE    (    35   ,   119   )    IS SELECTED.

CUBE    (    61   ,    17   )    HAS BEEN FOUND.

CUBE    (    61   ,    23   )    HAS BEEN FOUND.

WATCH FOR     17 AND     23

CUBE    (    61   ,   113   )    HAS BEEN FOUND.

WATCH FOR     23 AND    113

CUBE    (    61   ,   119   )    HAS BEEN FOUND.
```

```
WATCH FOR     23 AND    119

CHECKING POINTS
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999   37 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

               48   62   60   58    0   62   40   46   40   18   14   16
               16   18   20   22   24   64   66   30    6   18   14   16
               16   18   20   22   24   30   66   40   42   34   36   36
               38   38   40   40   42   40   42   34   36   34   20   16
               18   18   20   22   24   26   36   34   36   34   32   48
                4   58   54   58   56   60   56   60   56   60   56   60
                0   62   58   62   46   50   46   16   12   14   14   16
               18   20   22   52   50   54   50   28

CUBE    (    61   ,   119   )    IS SELECTED.

CUBE    (    62   ,    18   )    HAS BEEN FOUND.

CUBE    (    62   ,    23   )    HAS BEEN FOUND.

WATCH FOR     18 AND     23

CUBE    (    62   ,   114   )    HAS BEEN FOUND.

WATCH FOR     23 AND    114

CUBE    (    62   ,   119   )    HAS BEEN FOUND.

WATCH FOR     23 AND    119

CHECKING POINTS
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999
```

```
 ACCUMULATED AWAYNESS :

                 48    62    60    58     0    62    40    46    40    18    14    16
                 16    18    20    22    24    64    66    30     6    18    14    16
                 16    18    20    22    24    30    66    40    42    34    36    36
                 38    38    40    40    42    40    42    34    36    34    20    16
                 18    18    20    22    24    26    36    34    36    34    32    48
                  4    58    54    58    56    60    56    60    56    60    56    60
                  0    62    58    62    46    50    46    16    12    14    14    16
                 18    20    22    52    50    54    50    28

 CUBE   (    62  ,   119   )   IS SELECTED.
*9
 **FORTRAN ** STOP        LOGOF
/F
%CAN
/LOGOFF BUT
%C E420 LOGOFF AT 2024 ON 08/09/78, FOR TSN 9003.
%C E421 CPU TIME USED:  000200.9825 SECONDS.
%C E222 PLEASE LOGON.
```

APPENDIX H

EXAMPLE OF 9 VARIABLES, 120 ON-SET MINTERMS, AND 40 DON'T CARE MINTERMS.

```
TOATAL NUMBER OF ON-SET AND DONT*CARES =    160                A96
ON_SET =    120       DONT_CARES =     40
NUMBERS OF VARIABLES =     10

    MINT.#   1  =    1      MINT.#   2  =    2      MINT.#   3  =    3
    MINT.#   4  =    4      MINT.#   5  =    5      MINT.#   6  =    6
    MINT.#   7  =    7      MINT.#   8  =    9      MINT.#   9  =   11
    MINT.#  10  =   12      MINT.#  11  =   13      MINT.#  12  =   14
    MINT.#  13  =   16      MINT.#  14  =   17      MINT.#  15  =   18
    MINT.#  16  =   19      MINT.#  17  =   21      MINT.#  18  =   22
    MINT.#  19  =   23      MINT.#  20  =   24      MINT.#  21  =   26
    MINT.#  22  =   27      MINT.#  23  =   28      MINT.#  24  =   29
    MINT.#  25  =   30      MINT.#  26  =   31      MINT.#  27  =   34
    MINT.#  28  =   35      MINT.#  29  =   36      MINT.#  30  =   37
    MINT.#  31  =   38      MINT.#  32  =   39      MINT.#  33  =   41
    MINT.#  34  =   42      MINT.#  35  =   43      MINT.#  36  =   45
    MINT.#  37  =   48      MINT.#  38  =   49      MINT.#  39  =   50
    MINT.#  40  =   51      MINT.#  41  =   52      MINT.#  42  =   53
    MINT.#  43  =   55      MINT.#  44  =   56      MINT.#  45  =   57
    MINT.#  46  =   58      MINT.#  47  =   59      MINT.#  48  =   60
    MINT.#  49  =   61      MINT.#  50  =   62      MINT.#  51  =   63
    MINT.#  52  =   64      MINT.#  53  =   68      MINT.#  54  =   69
    MINT.#  55  =   70      MINT.#  56  =   71      MINT.#  57  =   72
    MINT.#  58  =   75      MINT.#  59  =   76      MINT.#  60  =   77
    MINT.#  61  =   78      MINT.#  62  =   79      MINT.#  63  =   80
    MINT.#  64  =   81      MINT.#  65  =   90      MINT.#  66  =   91
    MINT.#  67  =   92      MINT.#  68  =   93      MINT.#  69  =   94
    MINT.#  70  =   95      MINT.#  71  =   96      MINT.#  72  =   97
    MINT.#  73  =   98      MINT.#  74  =  103      MINT.#  75  =  104
    MINT.#  76  =  105      MINT.#  77  =  106      MINT.#  78  =  107
    MINT.#  79  =  108      MINT.#  80  =  109      MINT.#  81  =  111
    MINT.#  82  =  112      MINT.#  83  =  113      MINT.#  84  =  114
    MINT.#  85  =  115      MINT.#  86  =  116      MINT.#  87  =  117
    MINT.#  88  =  121      MINT.#  89  =  122      MINT.#  90  =  123
    MINT.#  91  =  124      MINT.#  92  =  145      MINT.#  93  =  126
    MINT.#  94  =  127      MINT.#  95  =  128      MINT.#  96  =  129
    MINT.#  97  =  150      MINT.#  98  =  151      MINT.#  99  =  152
    MINT.# 100  =  153      MINT.# 101  =  156      MINT.# 102  =  157
    MINT.# 103  =  158      MINT.# 104  =  159      MINT.# 105  =  160
    MINT.# 106  =  161      MINT.# 107  =  162      MINT.# 108  =  163
    MINT.# 109  =  164      MINT.# 110  =  165      MINT.# 111  =  167
    MINT.# 112  =  168      MINT.# 113  =  170      MINT.# 114  =  172
    MINT.# 115  =  173      MINT.# 116  =  174      MINT.# 117  =  175
    MINT.# 118  =  176      MINT.# 119  =  178      MINT.# 120  =  179
    MINT.# 121  =  185      MINT.# 122  =  186      MINT.# 123  =  187
    MINT.# 124  =  188      MINT.# 125  =  189      MINT.# 126  =  190
    MINT.# 127  =  191      MINT.# 128  =  192      MINT.# 129  =  201
    MINT.# 130  =  202      MINT.# 131  =  203      MINT.# 132  =  204
    MINT.# 133  =  205      MINT.# 134  =  206      MINT.# 135  =  207
    MINT.# 136  =  213      MINT.# 137  =  214      MINT.# 138  =  215
    MINT.# 139  =  216      MINT.# 140  =  217      MINT.# 141  =  220
    MINT.# 142  =  223      MINT.# 143  =  224      MINT.# 144  =  225
    MINT.# 145  =  226      MINT.# 146  =  227      MINT.# 147  =  228
    MINT.# 148  =  229      MINT.# 149  =  230      MINT.# 150  =  245
    MINT.# 151  =  246      MINT.# 152  =  247      MINT.# 153  =  248
    MINT.# 154  =  249      MINT.# 155  =  256      MINT.# 156  =  257
    MINT.# 157  =  258      MINT.# 158  =  259      MINT.# 159  =  254
    MINT.# 160  =  260      MINT.#
```

```
THE CUBE COVERING LIST :


THE      1-TH CUBE COVERING IS :
         3-CUBE = (  81 ,  48  ) =       000--1000-

THIS CUBE COVERS THESE MINTERMS :
         81   48   16   17   49   80  112  113

THIS MINTERM MT(  64  ) =   81      IS A CORE POINT


THE      2-TH CUBE COVERING IS :
         4-CUBE = (  90 ,  63  ) =       000--11-1-

THIS CUBE COVERS THESE MINTERMS :
         90   63   26   27   30   31   58   59   62   91   94   95  122  123  126  127

THIS MINTERM MT(  65  ) =   90      IS A CORE POINT


THE      3-TH CUBE COVERING IS :
         4-CUBE = ( 156 ,  63  ) =       00-0-111--

THIS CUBE COVERS THESE MINTERMS :
       156   63   28   29   30   31   60   61   62  157  158  159  188  189  190  191


THE      4-TH CUBE COVERING IS :
         3-CUBE = (  38 ,   5  ) =       0000-001--

THIS CUBE COVERS THESE MINTERMS :
         38    5    4    6    7   36   37   39


THE      5-TH CUBE COVERING IS :
         3-CUBE = ( 162 ,  51  ) =       00-01-001-

THIS CUBE COVERS THESE MINTERMS :
       162   51   34   35   50  163  178  179
```

```
THE      6-TH CUBE COVERING IS :
        3-CUBE = (  41 , 123  ) =      000-1-10-1

THIS CUBE COVERS THESE MINTERMS :
      41 123  43  57  59 105 107 121

THE      7-TH CUBE COVERING IS :
        3-CUBE = (  69 ,  78  ) =      000100-1--

THIS CUBE COVERS THESE MINTERMS :
      69  78  68  70  71  76  77  79

THE      8-TH CUBE COVERING IS :
        3-CUBE = ( 164 , 225  ) =      001-100-0-

THIS CUBE COVERS THESE MINTERMS :
     164 225 160 161 165 224 228 229

THE      9-TH CUBE COVERING IS :
        3-CUBE = ( 150 ,  31  ) =      00-001-11-

THIS CUBE COVERS THESE MINTERMS :
     150  31  22  23  30 151 158 159

THE     10-TH CUBE COVERING IS :
        3-CUBE = ( 174 , 189  ) =      00101-11--

THIS CUBE COVERS THESE MINTERMS :
     174 189 172 173 175 188 190 191

THE     11-TH CUBE COVERING IS :
        3-CUBE = ( 116 ,  49  ) =      000-110-0-

THIS CUBE COVERS THESE MINTERMS :
     116  49  48  52  53 112 113 117
```

```
THE     12-TH CUBE COVERING IS :
        3-CUBE = (    2 ,   51   ) =      0000--001-

THIS CUBE COVERS THESE MINTERMS :
        2   51   3   18   19   34   35   50


THE     13-TH CUBE COVERING IS :
        3-CUBE = ( 170 ,    50   ) =      00-01--010

THIS CUBE COVERS THESE MINTERMS :
      170   50   34   42   58 162 178 186


THE     14-TH CUBE COVERING IS :
        3-CUBE = (   12 ,   93   ) =      000-0-110-

THIS CUBE COVERS THESE MINTERMS :
       12   93   13   28   29   76   77   92


THE     15-TH CUBE COVERING IS :
        3-CUBE = (   75 ,   59   ) =      000---1011

THIS CUBE COVERS THESE MINTERMS :
       75   59   11   27   43   91 107 123


THE     16-TH CUBE COVERING IS :
        3-CUBE = (   98 ,   58   ) =      000-1--010

THIS CUBE COVERS THESE MINTERMS :
       98   58   34   42   50 106 114 122


THE     17-TH CUBE COVERING IS :
        3-CUBE = (   45 ,   21   ) =      0000---101

THIS CUBE COVERS THESE MINTERMS :
       45   21    5   13   29   37   53   61
```

```
THE     18-TH CUBE COVERING IS :
        3-CUBE = (  14 ,   68   ) =      000-00-1-0

THIS CUBE COVERS THESE MINTERMS :
       14   68    4    6   12   70   76   78



THE     19-TH CUBE COVERING IS :
        3-CUBE = (  24 ,   62   ) =      0000-11--0

THIS CUBE COVERS THESE MINTERMS :
       24   62   26   28   30   56   58   60



THE     20-TH CUBE COVERING IS :
        3-CUBE = (   1 ,   23   ) =      00000-0--1

THIS CUBE COVERS THESE MINTERMS :
        1   23    3    5    7   17   19   21



THE     21-TH CUBE COVERING IS :
        3-CUBE = ( 111 ,   91   ) =      0001--1-11

THIS CUBE COVERS THESE MINTERMS :
      111   91   75   79   95  107  123  127



THE     22-TH CUBE COVERING IS :
        3-CUBE = ( 124 ,   30   ) =      000--111-0

THIS CUBE COVERS THESE MINTERMS :
      124   30   28   60   62   92   94  126



THE     23-TH CUBE COVERING IS :
        3-CUBE = ( 115 ,   48   ) =      000-1100--

THIS CUBE COVERS THESE MINTERMS :
      115   48   49   50   51  112  113  114
```

```
THE     24-TH CUBE COVERING IS :
        3-CUBE = (  55 ,  17  ) =      0000-10--1

THIS CUBE COVERS THESE MINTERMS :
        55  17  19  21  23  49  51  53


THE     25-TH CUBE COVERING IS :
        2-CUBE = ( 128 , 161  ) =      0010-0000-

THIS CUBE COVERS THESE MINTERMS :
      128 161 129 160


THE     26-TH CUBE COVERING IS :
        2-CUBE = ( 103 ,   7  ) =      000--00111

THIS CUBE COVERS THESE MINTERMS :
      103   7  39  71


THE     27-TH CUBE COVERING IS :
        2-CUBE = (  72 ,  96  ) =      0001-0-000

THIS CUBE COVERS THESE MINTERMS :
        72  96  64 104


THE     28-TH CUBE COVERING IS :
        2-CUBE = ( 168 , 162  ) =      001010-0-0

THIS CUBE COVERS THESE MINTERMS :
      168 162 160 170


THE     29-TH CUBE COVERING IS :
        2-CUBE = (   9 ,   5  ) =      000000--01

THIS CUBE COVERS THESE MINTERMS :
        9   5   1  13
```

```
THE     30-TH CUBE COVERING IS :
        2-CUBE = ( 145 ,   1  ) =      00-00-0001

THIS CUBE COVERS THESE MINTERMS :
     145   1  17 129


THE     31-TH CUBE COVERING IS :
        2-CUBE = ( 152 , 217  ) =      001-01100-

THIS CUBE COVERS THESE MINTERMS :
     152 217 153 216


THE     32-TH CUBE COVERING IS :
        2-CUBE = ( 108 , 105  ) =      0001101-0-

THIS CUBE COVERS THESE MINTERMS :
     108 105 104 109


THE     33-TH CUBE COVERING IS :
        2-CUBE = ( 176 ,  50  ) =      00-01100-0

THIS CUBE COVERS THESE MINTERMS :
     176  50  48 178


THE     34-TH CUBE COVERING IS :
        2-CUBE = ( 167 ,  37  ) =      00-01001-1

THIS CUBE COVERS THESE MINTERMS :
     167  37  39 165


THE     35-TH CUBE COVERING IS :
        2-CUBE = (  97 , 224  ) =      00-110000-

THIS CUBE COVERS THESE MINTERMS :
      97 224  96 225
```

THE FINAL CHECKING

```
305  105  205  205  405  205  305  105  105  205  305  105
105  405  105  305  305  105  305  105  205  205  405  305
505  305  405  205  105  305  105  305  105  205  205  105
405  405  605  405  105  305  105  105  105  405  305  305
205  405  205  105  205  105  205  205  105  205  305  205
205  205  105  101  101  301  205  105  205  205  205  105
105  105  205  205  105  305  105  105  105  305  305  205
105  105  105  105  205  405  105  105  205  201  105  205
105  105  105  105  105  105  205  205  305  205  305  105
105  205  105  105  205  105  105  105  105  105  305  105
109  209  109  309  205  205  205  109  109  109  109  109
109  109  109  109  109  109  209  209  109  109  205  309
109  109  209  209  109  109  109  109  109  109  109  109
109  109  109  109
```

```
MAXIMUM CUBE DIMENSION =   7

     1    0    0    0    0    0    0    2    1    0    0    0    0    0
     3    3    1    0    0    0    0    4    6    4    1    0    0    0
     5   10   10    5    1    0    0    6   15   20   15    6    1    0
     7   21   35   35   21    7    1

THE CHARACTERISTICS :
           6  1  5  1  6  5  6  1  6  5  6  5  6  1  4  1  5  1  5  1  6
           4  1  5  1  6  5  6  1  6  1  5  1  5  1  6  5  5  1  6  1  6
           6  5  6  5  6  5  6  5  6  5  6  1  5  1  6  1  4  1  6  5  5
           5  1  6  1  6  1  6  5  6  1  6  5  6  5  5  1  6  5  5  1  5
           6  5  6  5  6  5  6  5  6  5  6  1  7  1  5  1  5  1  4  1  4
           5  1  3  1  5  1  6  5  6  5  6  1  6  5  4  1  3  1  4  1  5
           6  1  4  1  6  1  6  5  6  1  4  1  5  1  3  1  5  1  6  1  5
           6  1  4  1  5  1  5  1  6  1  5  5  5  1  4  1  4  1  4  1  5
           6  1  6  5  5  1  3  1  6  1  5  1  3  1  4  1  4  1  4  1  4
           5  1  5  5  5  1  5  1  6  1  5  5  5  1  6  1  5  5  5  1  5
           4  1  3  1  5  1  5  1  5  1  4  1  4  1  3  1  5  1  4  1  5
           5  1  5  1  5  1  6  1  6  5  6  1  3  1  3  1  2  1  4  1  4
           4  1  4  1  5  1  2  1  3  1  4  5  4  1  4  1  4  1  4  1  6
           5  1  5  1  3  1  4  1  4  1  3  1  4  1  4  1  3  1  2  1  4
           3  1  3  1  3  1  3  1  3  1  2  1

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:          6

MAXIMUM CUBE DIMENSION HAS BEEN LOWERED TO:          5

CUBE    (  81  ,   48  )    HAS BEEN FOUND.

CHECKING POINTS
                 8    1    2    2    2    0    0    0    0    0    0    0
              9999 9999    8    8    4    1    2    4    2    2    2    2
                 0    0    2    2    1    2    0    0    1    0    0    0
              9999 9999    8    8    4    8    1    4    8    2    2    2
                 2    0    0    4    1    0    0    0    0    0    0    0
                 0    0 9999 9999    1    2    2    1    0    0    8    4
                 2    0    2    2    0    0    0    0    0 9999 9999    8
                 4    4    4    8    2    2    2    2    0    0    0    1
                 0    0    1    2    0    0    0    0    2    2    0    0
                 0    0    0    0    0    0    0    0    0    2    2    1
              9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
              9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
              9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                 2    4    4    4    4    6    6    4    6    6    6    8
                 0    0    2    2    2    4    4    2    4    4    4    4
                 6    6    4    4    4    4    6    6    4    6    6    6
```

```
                    0    0    2    2    2    2    4    2    2    4    4    4
                    4    6    6    2    4    4    6    6    4    6    6    6
                    8    8    0    0    4    4    4    4    6    6    2    2
                    4    6    4    4    6    6    6    6    8    0    0    2
                    2    2    2    2    4    4    4    2    6    6    4    4
                    6    6    4    4    6    6    8    8    4    4    6    6
                    6    6    8    6    8    8    8   10   10    2    4    4
                    0    0    0    0    0    0    0    0    0    0    0    0
                    0    0    0    0    0    0    0    0    0    0    0    0
                    0    0    0    0    0    0    0    0    0    0    0    0
                    0    0    0    0

   CUBE    (   81   ,   48   )    IS SELECTED.

   CUBE    (   90   ,   63   )    HAS BEEN FOUND.

   CHECKING POINTS
                    8    2    4    2    2    2    2    0    4    1    2    2
                 9999 9999   16   16    5    5   10    8 9999 9999   10   10
                 9999 9999    4    4    1    2    0    2    2    4    8    2
                 9999 9999   16   16    5   10    5    8   16 9999 9999   10
                   10 9999 9999    4    1    0    0    1    0    4    2    2
                    8    8 9999 9999 9999 9999   10    5 9999 9999    8    4
                    4    0    4    4    8    8    1    2    8 9999 9999   16
                    8    5    5   16 9999 9999   10    2 9999 9999    0    1
                    1    1    2    4    2    2    8    8    2    2    0    0
                    0    0    0    0    2    0    0    1    1    2    4    2
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999

   ACCUMULATED AWAYNESS :

                    8    8    8   10   10   10   10    8    8   10   10   10
                    0    0    4    4    6    6    6    4    4    4    6    6
                    6    6    8    8   10   10   10   10    8    8    8   10
                    0    0    4    4    6    6    6    4    4    4    4    6
                    6    6    6    8   10   10   10   10    8    8   10   10
                   10   10    0    0    4    4    6    6    6    6    8    8
                    8   10    8    8    8    8   10   10   10    0    0    4
                    4    6    6    4    4    4    6    8    6    6   12   12
                   10   10    8    8   10   10   10   10   12   12   12   12
                   14   14   14   12   12   14   14   14   14    8    8    8
                    0    0    0    0    0    0    0    0    0    0    0    0
                    0    0    0    0    0    0    0    0    0    0    0    0
                    0    0    0    0    0    0    0    0    0    0    0    0
                    0    0    0    0

   CUBE    (   90   ,   63   )    IS SELECTED.

   ALL CORE CUBES HAVE BEEN SEARCHED,
   AND THE MAXIMUM CUBE DIMENSION IS :     5
```

```
CUBE   ( 156  ,   63  )   HAS BEEN FOUND.

CHECKING POINTS
                     8    2    4    2    2    2    2    0    5    3    4    4
                  9999 9999   17   17    9    9   14   12 9999 9999 9999 9999
                  9999 9999    4    4    1    2    0    2    2    5    9    4
                  9999 9999   17   17    9   14    9   12   20 9999 9999 9999
                  9999 9999 9999    4    1    0    0    1    0    4    2    2
                     9    9 9999 9999 9999 9999   14    7 9999 9999    8    4
                     4    0    4    4    8    8    1    2    9 9999 9999   16
                     8    5    5   17 9999 9999   14    2 9999 9999    0    1
                     3    3    4    8 9999 9999 9999 9999    2    2    0    0
                     0    0    0    0    3    2    2    3    3    2    5    2
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                    14   14   14   14   14   14   14   12   12   12   12   12
                     0    0    8    8    8    8    8    6    4    4    6    6
                     6    6   14   14   14   14   14   14   12   12   12   12
                     0    0    8    8    8    8    8    6    6    4    4    6
                     6    6    6   16   16   16   16   16   14   14   14   14
                    14   14    0    0    4    4    8    8    6    6   16   16
                    16   16   14   14   14   14   14   14   14    0    0   10
                    10   10   10    8    4    4    8   12    6    6   18   18
                    12   12   10   10   10   10   10   10   18   18   18   18
                    18   18   18   16   16   16   16   16   16   12   12   12
                     0    0    0    0    0    0    0    0    0    0    0    0
                     0    0    0    0    0    0    0    0    0    0    0    0
                     0    0    0    0    0    0    0    0    0    0    0    0
                     0    0    0    0

CUBE   ( 156  ,   63  )   IS SELECTED.

CUBE   (  38  ,    3  )   HAS BEEN FOUND.

CUBE   (  38  ,    5  )   HAS BEEN FOUND.

WATCH FOR     3 AND     5

CHECKING POINTS
                    10    4    6 9999 9999 9999 9999    0    5    5    6    6
                  9999 9999   17   17   11   11   16   12 9999 9999 9999 9999
                  9999 9999    6    6 9999 9999 9999 9999    2    5    9    6
                  9999 9999   17   17   11   16   11   12   20 9999 9999 9999
                  9999 9999 9999    4    3    2    2    3    0    4    2    2
                     9    9 9999 9999 9999 9999   14    7 9999 9999    8    4
                     4    1    4    4    8    8    1    2    9 9999 9999   16
```

```
         8     5     5    17  9999  9999    14     2  9999  9999     0     1
         3     3     4     8  9999  9999  9999  9999     2     2     0     0
         2     2     2     0     3     2     2     3     3     2     5     2
      9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
      9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
      9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
      9999  9999  9999  9999

 ACCUMULATED AWAYNESS :

        16    16    16    14    14    14    14    16    16    14    14    14
         0     0    12    12    10    10    10    12     4     4     6     6
         6     6    16    16    14    14    14    14    16    16    16    14
         0     0    12    12    10    10    10    12    12     4     4     6
         6     6     6    20    18    18    18    18    20    20    18    18
        18    18     0     0     4     4    14    14     6     6    20    20
        20    18    20    20    20    20    18    18    18     0     0    16
        16    14    14    16     4     4    14    18     6     6    22    22
        16    16    18    18    10    10    10    10    22    22    22    22
        20    20    20    22    22    20    20    20    20    18    18    18
         0     0     0     0     0     0     0     0     0     0     0     0
         0     0     0     0     0     0     0     0     0     0     0     0
         0     0     0     0     0     0     0     0     0     0     0     0
         0     0     0     0

 CUBE    (   38   ,    5   )    IS SELECTED.

 CUBE    (  162   ,   51   )    HAS BEEN FOUND.

 CUBE    (  162   ,   58   )    HAS BEEN FOUND.

 CUBE    (  162   ,  225   )    HAS BEEN FOUND.

 CHECKING POINTS
        10     6     8  9999  9999  9999  9999     0     5     5     6     6
      9999  9999    19    19    11    11    16    12  9999  9999  9999  9999
      9999  9999  9999  9999  9999  9999  9999  9999     2     7    11     6
      9999  9999  9999  9999    11    16    13    12    20  9999  9999  9999
      9999  9999  9999     4     3     2     2     3     0     4     2     2
         9     9  9999  9999  9999  9999    14     7  9999  9999     8     4
         6     1     4     4     8     8     1     2     9  9999  9999    18
        10     5     5    17  9999  9999    14     2  9999  9999     0     1
         3     3     4     8  9999  9999  9999  9999     4     4  9999  9999
         2     2     4     0     5     2     2     3     3     3  9999  9999
      9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
      9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
      9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
      9999  9999  9999  9999

 ACCUMULATED AWAYNESS :
```

```
          20    18    18    14    14    14    14    22    20    22    22    20
           0     0    14    14    16    14    14    18     4     4     6     6
           6     6    16    16    14    14    14    14    20    18    18    20
           0     0    12    12    14    14    12    16    16     4     4     6
           6     6     6    26    26    26    24    24    28    26    28    28
          26    26     0     0     4     4    24    24     6     6    24    24
          22    22    26    26    24    24    26    26    24     0     0    18
          18    20    20    22     4     4    22    22     6     6    26    26
          20    20    24    24    10    10    10    10    24    24    22    22
          24    24    22    26    24    26    26    24    24    20    18    18
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0

 CUBE    ( 162   ,   51   )    IS SELECTED.

 CUBE    (  41   , 123   )    HAS BEEN FOUND.

 CHECKING POINTS
          10     6     8  9999  9999  9999  9999     1     7     5     6     6
        9999  9999    19    19    11    11    16    12  9999  9999  9999  9999
        9999  9999  9999  9999  9999  9999  9999  9999  9999     9  9999     8
        9999  9999  9999  9999    11    16    13    14  9999  9999  9999  9999
        9999  9999  9999     4     3     2     2     3     0     6     2     2
           9     9  9999  9999  9999  9999    14     7  9999  9999     8     6
           6     1     6  9999    10  9999     1     4    11  9999  9999    18
          12     5     5  9999  9999  9999    14     2  9999  9999     0     1
           3     3     4     8  9999  9999  9999  9999     4     4  9999  9999
           2     2     4     0     5     2     2     3     3     3  9999  9999
        9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
        9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
        9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
        9999  9999  9999  9999

 ACCUMULATED AWAYNESS :

          24    24    22    14    14    14    14    24    22    28    26    26
           0     0    20    18    22    22    20    22     4     4     6     6
           6     6    16    16    14    14    14    14    20    20    18    22
           0     0    12    12    20    18    16    18    16     4     4     6
           6     6     6    32    34    32    32    30    32    28    34    32
          32    30     0     0     4     4    30    28     6     6    28    26
          26    26    28    26    26    24    30    28    26     0     0    22
          20    26    24    22     4     4    26    28     6     6    34    32
          30    28    30    28    10    10    10    10    30    28    22    22
          32    30    28    30    28    32    30    30    28    26    18    18
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0     0     0     0     0     0     0     0     0
           0     0     0     0
```

```
CUBE    (  41  , 123  )    IS SELECTED.

CUBE    (  69  ,   6  )    HAS BEEN FOUND.

CUBE    (  69  ,  12  )    HAS BEEN FOUND.

CUBE    (  69  ,  78  )    HAS BEEN FOUND.

CHECKING POINTS
                       10    6    8 9999 9999 9999 9999    1    7    7    8    8
                     9999 9999   19   19   11   11   16   12 9999 9999 9999 9999
                     9999 9999 9999 9999 9999 9999 9999 9999 9999    9 9999    8
                     9999 9999 9999 9999   11   16   13   14 9999 9999 9999 9999
                     9999 9999 9999    6 9999 9999 9999 9999    1    8 9999 9999
                     9999 9999 9999 9999 9999 9999   16    9 9999 9999    8    6
                        6    2    6 9999   10 9999    2    6   13 9999 9999   18
                       12    5    5 9999 9999 9999   14    2 9999 9999    0    1
                        3    3    4    8 9999 9999 9999 9999    4    4 9999 9999
                        2    2    4    0    5    2    2    3    3    3 9999 9999
                     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                     9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                       28   28   26   14   14   14   14   28   26   30   28   28
                        0    0   26   24   26   26   24   28    4    4    6    6
                        6    6   16   16   14   14   14   14   20   26   18   26
                        0    0   12   12   26   24   22   26   16    4    4    6
                        6    6    6   34   34   32   32   30   34   30   34   32
                       32   30    0    0    4    4   32   30    6    6   32   30
                       30   28   32   26   30   24   32   30   28    0    0   28
                       26   30   28   22    4    4   30   36    6    6   40   38
                       36   34   38   36   10   10   10   10   38   36   22   22
                       38   36   34   38   36   38   36   36   34   36   18   18
                        0    0    0    0    0    0    0    0    0    0    0    0
                        0    0    0    0    0    0    0    0    0    0    0    0
                        0    0    0    0    0    0    0    0    0    0    0    0
                        0    0    0    0
CUBE    (  69  ,  78  )    IS SELECTED.

CUBE    ( 164  , 225  )    HAS BEEN FOUND.

CHECKING POINTS
                       10    6    8 9999 9999 9999 9999    1    7    7    8    8
                     9999 9999   19   19   11   11   16   12 9999 9999 9999 9999
                     9999 9999 9999 9999 9999 9999 9999 9999 9999    9 9999    8
                     9999 9999 9999 9999   11   16   13   14 9999 9999 9999 9999
```

```
          9999 9999 9999    6 9999 9999 9999 9999    1    8 9999 9999
          9999 9999 9999 9999 9999 9999   16    9 9999 9999   10    8
             6    2    6 9999   10 9999    2    6   13 9999 9999   18
            12    5    5 9999 9999 9999   14    2 9999 9999    1    2
             3    3    4    8 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999    6    1    5    4    4    3    3    4 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999

 ACCUMULATED AWAYNESS :

            32   34   32   14   14   14   14   34   34   36   34   36
             0    0   34   32   32   34   32   36    4    4    6    6
             6    6   16   16   14   14   14   14   20   32   18   30
             0    0   12   12   30   28   28   32   16    4    4    6
             6    6    6   38   34   32   32   30   40   38   34   32
            32   30    0    0    4    4   40   38    6    6   34   32
            34   32   36   26   36   24   36   34   34    0    0   34
            32   34   32   22    4    4   36   40    6    6   42   40
            42   40   44   42   10   10   10   10   38   36   22   22
            38   36   36   40   40   40   38   40   38   38   18   18
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0

 CUBE    ( 164   , 225   )    IS SELECTED.

 CUBE    ( 150   ,  31   )    HAS BEEN FOUND.

 CHECKING POINTS
            10    6    8 9999 9999 9999 9999    1    7    7    8   10
          9999 9999   21   21   13 9999 9999   12 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999    9 9999    8
          9999 9999 9999 9999   11   16   15   14 9999 9999 9999 9999
          9999 9999 9999    6 9999 9999 9999 9999    1    8 9999 9999
          9999 9999 9999 9999 9999 9999   16    9 9999 9999   10    8
             6    2    6 9999   10 9999    2    6   13 9999 9999   18
            12    5    5 9999 9999 9999   14    2 9999 9999    1    2
          9999 9999    4    8 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999    6    1    5    4    4    3    3    4 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999

 ACCUMULATED AWAYNESS :

            38   38   36   14   14   14   14   40   38   40   38   38
```

```
              0    0   36   34   34   34   32   40    4    4    6    6
              6    6   16   16   14   14   14   14   20   38   18   36
              0    0   12   12   34   32   30   38   16    4    4    6
              6    6    6   46   34   32   32   30   48   44   34   32
             32   30    0    0    4    4   44   42    6    6   44   42
             42   38   46   26   44   24   44   42   40    0    0   40
             38   40   38   22    4    4   42   44    6    6   48   46
             42   40   48   46   10   10   10   10   38   36   22   22
             38   36   40   48   46   46   44   44   42   44   18   18
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0

  CUBE    ( 150   ,  31   )   IS SELECTED.

  CUBE    ( 174   , 189   )   HAS BEEN FOUND.

  CHECKING POINTS
             10    6    8 9999 9999 9999 9999    1    7    7    8   10
           9999 9999   21   21   13 9999 9999   12 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999    9 9999   10
           9999 9999 9999 9999   11   16   15   14 9999 9999 9999 9999
           9999 9999 9999    6 9999 9999 9999 9999    1    8 9999 9999
           9999 9999 9999 9999 9999 9999   16    9 9999 9999   10    8
              6    2    6 9999   10 9999    2    6   13 9999 9999   18
             12    5    5 9999 9999 9999   14    2 9999 9999    1    2
           9999 9999    4    8 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999    8    2    7 9999 9999 9999 9999    4 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999

  ACCUMULATED AWAYNESS :

             46   46   44   14   14   14   14   46   44   44   42   42
              0    0   44   42   40   34   32   46    4    4    6    6
              6    6   16   16   14   14   14   14   20   42   18   38
              0    0   12   12   38   36   34   42   16    4    4    6
              6    6    6   56   34   32   32   30   56   52   34   32
             32   30    0    0    4    4   50   48    6    6   52   50
             50   44   52   26   50   24   48   46   44    0    0   48
             46   46   44   22    4    4   46   50    6    6   54   52
             42   40   52   50   10   10   10   10   38   36   22   22
             38   36   42   50   48   46   44   44   42   48   18   18
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0
```

```
CUBE   ( 174  , 189  )   IS SELECTED.

CUBE   ( 116  ,  49  )   HAS BEEN FOUND.

CHECKING POINTS
   10    6    8 9999 9999 9999 9999    1    7    7    8   10
 9999 9999   21   21   15 9999 9999   12 9999 9999 9999 9999
 9999 9999 9999 9999 9999 9999 9999 9999 9999    9 9999   10
 9999 9999 9999 9999 9999 9999   17   16 9999 9999 9999 9999
 9999 9999 9999    6 9999 9999 9999 9999    1    8 9999 9999
 9999 9999 9999 9999 9999 9999   16    9 9999 9999   12   10
    6    2    6 9999   10 9999    2    6   13 9999 9999   20
   14 9999 9999 9999 9999 9999   16    2 9999 9999    1    2
 9999 9999    4    8 9999 9999 9999 9999 9999 9999 9999 9999
 9999 9999    8    2    7 9999 9999 9999 9999    5 9999 9999
 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
 9999 9999 9999 9999

ACCUMULATED AWAYNESS :

   50   52   50   14   14   14   14   52   52   50   48   50
    0    0   48   46   42   34   32   50    4    4    6    6
    6    6   16   16   14   14   14   14   20   48   18   42
    0    0   12   12   38   36   36   44   16    4    4    6
    6    6    6   60   34   32   32   30   62   60   34   32
   32   30    0    0    4    4   54   52    6    6   54   52
   54   48   56   26   56   24   52   50   50    0    0   50
   48   46   44   22    4    4   48   54    6    6   60   58
   42   40   58   56   10   10   10   10   38   36   22   22
   38   36   48   56   56   46   44   44   42   50   18   18
    0    0    0    0    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0    0    0    0
    0    0    0    0

CUBE   ( 116  ,  49  )   IS SELECTED.

CUBE   (   2  ,  23  )   HAS BEEN FOUND.

CUBE   (   2  ,  39  )   HAS BEEN FOUND.

CUBE   (   2  ,  51  )   HAS BEEN FOUND.

WATCH FOR    23 AND    51

CHECKING POINTS
   12 9999 9999 9999 9999 9999 9999    1    9    7    8   10
 9999 9999 9999 9999   15 9999 9999   12 9999 9999 9999 9999
```

```
                      9999 9999 9999 9999 9999 9999 9999 9999 9999   11 9999   10
                      9999 9999 9999 9999 9999 9999   19   16 9999 9999 9999 9999
                      9999 9999 9999    6 9999 9999 9999 9999    1    8 9999 9999
                      9999 9999 9999 9999 9999 9999   16    9 9999 9999   12   10
                         8    2    6 9999   10 9999    2    6   13 9999 9999   22
                        16 9999 9999 9999 9999 9999   16    2 9999 9999    1    2
                      9999 9999    4    8 9999 9999 9999 9999 9999 9999 9999 9999
                      9999 9999    8    2    7 9999 9999 9999 9999    5 9999 9999
                      9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                      9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                      9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                      9999 9999 9999 9999

          ACCUMULATED AWAYNESS :

                        52   52   50   14   14   14   14   56   54   56   54   54
                         0    0   48   46   46   34   32   54    4    4    6    6
                         6    6   16   16   14   14   14   14   20   50   18   48
                         0    0   12   12   38   36   38   48   16    4    4    6
                         6    6    6   64   34   32   32   30   68   64   34   32
                        32   30    0    0    4    4   62   60    6    6   58   56
                        56   52   62   26   60   24   60   58   56    0    0   52
                        50   46   44   22    4    4   56   58    6    6   64   62
                        42   40   64   62   10   10   10   10   38   36   22   22
                        38   36   52   62   60   46   44   44   42   54   18   18
                         0    0    0    0    0    0    0    0    0    0    0    0
                         0    0    0    0    0    0    0    0    0    0    0    0
                         0    0    0    0    0    0    0    0    0    0    0    0
                         0    0    0    0

          CUBE    (    2   ,   51    )    IS SELECTED,

          CUBE    ( 170    ,   50    )    HAS BEEN FOUND.

          CHECKING POINTS
                        12 9999 9999 9999 9999 9999 9999    1    9    7    8   10
                      9999 9999 9999 9999   15 9999 9999   12 9999 9999 9999 9999
                      9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   10
                      9999 9999 9999 9999 9999 9999   19   18 9999 9999 9999 9999
                      9999 9999 9999    6 9999 9999 9999 9999    1    8 9999 9999
                      9999 9999 9999 9999 9999 9999   16    9 9999 9999   12   10
                        10    2    6 9999   12 9999    2    6   13 9999 9999   24
                        16 9999 9999 9999 9999 9999   16    2 9999 9999    1    2
                      9999 9999    4    8 9999 9999 9999 9999 9999 9999 9999 9999
                      9999 9999    8    3 9999 9999 9999 9999 9999    6 9999 9999
                      9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                      9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                      9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                      9999 9999 9999 9999

          ACCUMULATED AWAYNESS :
```

```
                  58    52    50    14    14    14    14    62    58    62    62    58
                   0     0    48    46    54    34    32    58     4     4     6     6
                   6     6    16    16    14    14    14    14    20    50    18    54
                   0     0    12    12    38    36    42    50    16     4     4     6
                   6     6     6    70    34    32    32    30    74    70    34    32
                  32    30     0     0     4     4    70    70     6     6    62    62
                  58    58    66    26    62    24    66    66    62     0     0    54
                  54    46    44    22     4     4    62    64     6     6    68    68
                  42    40    68    68    10    10    10    10    38    36    22    22
                  38    36    56    64    60    46    44    44    42    56    18    18
                   0     0     0     0     0     0     0     0     0     0     0     0
                   0     0     0     0     0     0     0     0     0     0     0     0
                   0     0     0     0     0     0     0     0     0     0     0     0
                   0     0     0     0

   CUBE    ( 170   ,   50   )    IS SELECTED.

   CUBE    (  12   ,   69   )    HAS BEEN FOUND.

   CUBE    (  12   ,   70   )    HAS BEEN FOUND.

   WATCH FOR     69 AND     70

   CUBE    (  12   ,   93   )    HAS BEEN FOUND.

   CUBE    (  12   ,   94   )    HAS BEEN FOUND.

   CHECKING POINTS
                  12 9999 9999 9999 9999 9999 9999    2    9 9999 9999   12
                9999 9999 9999 9999   17 9999 9999   14 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   12
                9999 9999 9999 9999 9999 9999   19   18 9999 9999 9999 9999
                9999 9999 9999    6 9999 9999 9999 9999    2    8 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   12   10
                  10    2    6 9999   12 9999    3    7   13 9999 9999   24
                  16 9999 9999 9999 9999 9999   18    2 9999 9999    1    2
                9999 9999    4    8 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999    8    3 9999 9999 9999 9999 9999    6 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999

   ACCUMULATED AWAYNESS :

                  62    52    50    14    14    14    14    64    62    62    62    60
                   0     0    48    46    56    34    32    60     4     4     6     6
                   6     6    16    16    14    14    14    14    20    50    18    56
                   0     0    12    12    38    36    48    54    16     4     4     6
                   6     6     6    74    34    32    32    30    76    74    34    32
                  32    30     0     0     4     4    70    70     6     6    68    68
```

```
      66      64      70      26      68      24      68      68      66       0       0      62
      62      46      44      22       4       4      64      70       6       6      74      74
      42      40      72      72      10      10      10      10      38      36      22      22
      38      36      64      70      60      46      44      44      42      64      18      18
       0       0       0       0       0       0       0       0       0       0       0       0
       0       0       0       0       0       0       0       0       0       0       0       0
       0       0       0       0       0       0       0       0       0       0       0       0
       0       0       0       0

  CUBE    (   12   ,   93   )    IS SELECTED.

  CUBE    (   75   ,   59   )    HAS BEEN FOUND.

  CUBE    (   75   ,  127   )    HAS BEEN FOUND.

  WATCH FOR     59 AND    127

  CHECKING POINTS
      12    9999    9999    9999    9999    9999    9999       3    9999    9999    9999      12
    9999    9999    9999    9999      17    9999    9999      14    9999    9999    9999    9999
    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999      12
    9999    9999    9999    9999    9999    9999      19      18    9999    9999    9999    9999
    9999    9999    9999       6    9999    9999    9999    9999       2    9999    9999    9999
    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999      12      10
      10       2       6    9999      14    9999       3       7      15    9999    9999      24
      18    9999    9999    9999    9999    9999      18       2    9999    9999       1       2
    9999    9999       4       8    9999    9999    9999    9999    9999    9999    9999    9999
    9999    9999       8       3    9999    9999    9999    9999    9999       6    9999    9999
    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999
    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999
    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999    9999
    9999    9999    9999    9999

  ACCUMULATED AWAYNESS :

      66      52      50      14      14      14      14      66      62      62      62      64
       0       0      48      46      62      34      32      64       4       4       6       6
       6       6      16      16      14      14      14      14      20      50      18      60
       0       0      12      12      38      36      52      58      16       4       4       6
       6       6       6      80      34      32      32      30      80      74      34      32
      32      30       0       0       4       4      70      70       6       6      74      72
      70      68      74      26      70      24      74      72      68       0       0      66
      64      46      44      22       4       4      70      76       6       6      82      80
      42      40      78      76      10      10      10      10      38      36      22      22
      38      36      70      76      60      46      44      44      42      72      18      18
       0       0       0       0       0       0       0       0       0       0       0       0
       0       0       0       0       0       0       0       0       0       0       0       0
       0       0       0       0       0       0       0       0       0       0       0       0
       0       0       0       0

  CUBE    (   75   ,   59   )    IS SELECTED.
```

```
CUBE    (   98   ,   58   )    HAS BEEN FOUND.

CHECKING POINTS
            12 9999 9999 9999 9999 9999 9999    3 9999 9999 9999   12
          9999 9999 9999 9999   17 9999 9999   14 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   12
          9999 9999 9999 9999 9999 9999   19   20 9999 9999 9999 9999
          9999 9999 9999    6 9999 9999 9999 9999    2 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   14   10
          9999    2    7 9999 9999 9999    3    7   15 9999 9999 9999
            20 9999 9999 9999 9999 9999   18    2 9999 9999    1    2
          9999 9999    4    8 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999    8    3 9999 9999 9999 9999 9999    6 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            72   52   50   14   14   14   14   72   62   62   62   68
             0    0   48   46   70   34   32   68    4    4    6    6
             6    6   16   16   14   14   14   14   20   50   18   66
             0    0   12   12   38   36   56   60   16    4    4    6
             6    6    6   84   34   32   32   30   84   74   34   32
            32   30    0    0    4    4   70   70    6    6   76   76
            70   72   76   26   70   24   78   78   72    0    0   66
            66   46   44   22    4    4   74   84    6    6   88   88
            42   40   84   84   10   10   10   10   38   36   22   22
            38   36   76   80   60   46   44   44   42   76   18   18
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0

CUBE    (   98   ,   58   )    IS SELECTED.

CUBE    (   45   ,   21   )    HAS BEEN FOUND.

CHECKING POINTS
            14 9999 9999 9999 9999 9999 9999    4 9999 9999 9999   12
          9999 9999 9999 9999 9999 9999 9999   14 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999   21   20 9999 9999 9999 9999
          9999 9999 9999    6 9999 9999 9999 9999    2 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   14   10
          9999    2    7 9999 9999 9999    3    8   15 9999 9999 9999
            20 9999 9999 9999 9999 9999   18    2 9999 9999    1    2
          9999 9999    4    8 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999    8    3 9999 9999 9999 9999 9999    6 9999 9999
```

```
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999

  ACCUMULATED AWAYNESS :

               74      52      50      14      14      14      14      74      62      62      62      72
                0       0      48      46      70      34      32      72       4       4       6       6
                6       6      16      16      14      14      14      14      20      50      18      66
                0       0      12      12      38      36      58      64      16       4       4       6
                6       6       6      90      34      32      32      30      90      74      34      32
               32      30       0       0       4       4      70      70       6       6      82      80
               70      76      62      26      70      24      82      80      76       0       0      66
               72      46      44      22       4       4      78      88       6       6      94      92
               42      40      90      88      10      10      10      10      38      36      22      22
               38      36      80      86      60      46      44      44      42      82      18      18
                0       0       0       0       0       0       0       0       0       0       0       0
                0       0       0       0       0       0       0       0       0       0       0       0
                0       0       0       0       0       0       0       0       0       0       0       0
                0       0       0       0

  CUBE    (   45   ,   21   )    IS SELECTED.

  CUBE    (   14   ,   68   )    HAS BEEN FOUND.

  CUBE    (   14   ,   92   )    HAS BEEN FOUND.

  WATCH FOR    68 AND     92

  CHECKING POINTS
               14 9999 9999 9999 9999 9999 9999    4 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999   14 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999   21   20 9999 9999 9999 9999
             9999 9999 9999    7 9999 9999 9999 9999    3 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   14   10
             9999    2    7 9999 9999 9999    4    8   15 9999 9999 9999
               20 9999 9999 9999 9999 9999   18    2 9999 9999    1    2
             9999 9999    4    8 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999    8    3 9999 9999 9999 9999 9999    6 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
             9999 9999 9999 9999

  ACCUMULATED AWAYNESS :

               78      52      50      14      14      14      14      78      62      62      62      72
                0       0      48      46      70      34      32      76       4       4       6       6
                6       6      16      16      14      14      14      14      20      50      18      66
```

```
          0       0      12      12      38      36      64      70      16       4       4       6
          6       6       6      92      34      32      32      30      92      74      34      32
         32      30       0       0       4       4      70      70       6       6      86      86
         70      80      86      26      70      24      84      84      80       0       0      66
         80      46      44      22       4       4      82      96       6       6      98      98
         42      40      96      96      10      10      10      10      38      36      22      22
         38      36      86      92      60      46      44      44      42      90      18      18
          0       0       0       0       0       0       0       0       0       0       0       0
          0       0       0       0       0       0       0       0       0       0       0       0
          0       0       0       0       0       0       0       0       0       0       0       0
          0       0       0       0

 CUBE   (  14  ,  68  )   IS SELECTED.

 CUBE   (  24  ,  50  )   HAS BEEN FOUND.

 CUBE   (  24  ,  62  )   HAS BEEN FOUND.

 WATCH FOR     50 AND     62

 CHECKING POINTS
       14 9999 9999 9999 9999 9999 9999    4 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999   21 9999 9999 9999 9999 9999
     9999 9999 9999    7 9999 9999 9999 9999    3 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   14   10
     9999    2    7 9999 9999 9999    4    8   15 9999 9999 9999
       20 9999 9999 9999 9999 9999   20    2 9999 9999    1    2
     9999 9999    5    8 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999    8    3 9999 9999 9999 9999 9999    6 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
     9999 9999 9999 9999

 ACCUMULATED AWAYNESS :

         84      52      50      14      14      14      14      82      62      62      62      72
          0       0      48      46      70      34      32      76       4       4       6       6
          6       6      16      16      14      14      14      14      20      50      18      66
          0       0      12      12      38      36      68      70      16       4       4       6
          6       6       6      98      34      32      32      30      96      74      34      32
         32      30       0       0       4       4      70      70       6       6      92      94
         70      88      90      26      70      24      88      90      86       0       0      66
         86      46      44      22       4       4      84     102       6       6     104     106
         42      40      98     100      10      10      10      10      38      36      22      22
         38      36      94      96      60      46      44      44      42      94      18      18
          0       0       0       0       0       0       0       0       0       0       0       0
          0       0       0       0       0       0       0       0       0       0       0       0
          0       0       0       0       0       0       0       0       0       0       0       0
          0       0       0       0
```

```
CUBE    (  24  ,  62  )   IS SELECTED.

CUBE    (   1  ,  23  )   HAS BEEN FOUND.

CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999   23 9999 9999 9999 9999 9999
               9999 9999 9999    7 9999 9999 9999 9999    3 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   14   10
               9999    2    7 9999 9999 9999    4    8   15 9999 9999 9999
                 20 9999 9999 9999 9999 9999   20    3 9999 9999    1    3
               9999 9999    5    8 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999    8    3 9999 9999 9999 9999 9999    6 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                 84   52   50   14   14   14   14   84   62   62   62   72
                  0    0   48   46   70   34   32   76    4    4    6    6
                  6    6   16   16   14   14   14   14   20   50   18   66
                  0    0   12   12   38   36   70   70   16    4    4    6
                  6    6    6  102   34   32   32   30  102   74   34   32
                 32   30    0    0    4    4   70   70    6    6   98   98
                 70   92   98   26   70   24   96   96   92    0    0   66
                 90   46   44   22    4    4   92  104    6    6  108  108
                 42   40  104  104   10   10   10   10   38   36   22   22
                 38   36   98  104   60   46   44   44   42  100   18   18
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0

CUBE    (   1  ,  23  )   IS SELECTED.

CUBE    ( 111  ,  91  )   HAS BEEN FOUND.

CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999   23 9999 9999 9999 9999 9999
               9999 9999 9999    7 9999 9999 9999 9999    3 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   14   10
               9999    3    7 9999 9999 9999    4    9 9999 9999 9999 9999
                 22 9999 9999 9999 9999 9999   20    3 9999 9999    1    3
```

```
          9999 9999    5    6 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999    8    3 9999 9999 9999 9999 9999    6 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            84   52   50   14   14   14   14   88   62   62   62   72
             0    0   48   46   70   34   32   76    4    4    6    6
             6    6   16   16   14   14   14   14   20   50   18   66
             0    0   12   12   38   36   74   70   16    4    4    6
             6    6    6  108   34   32   32   30  106   74   34   32
            32   30    0    0    4    4   70   70    6    6  104  102
            70   94  102   26   70   24  100   98   92    0    0   66
            92   46   44   22    4    4   96  112    6    6  118  116
            42   40  112  110   10   10   10   10   38   36   22   22
            38   36  104  112   60   46   44   44   42  110   18   18
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0

CUBE    ( 111  ,   91  )    IS SELECTED.

CUBE    ( 124  ,   30  )    HAS BEEN FOUND.

CHECKING POINTS
          9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999   23 9999 9999 9999 9999 9999
          9999 9999 9999    7 9999 9999 9999 9999    3 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   14   10
          9999    3    7 9999 9999 9999    5    9 9999 9999 9999 9999
            22 9999 9999 9999 9999 9999 9999    3 9999 9999    1    3
          9999 9999    5    8 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999    8    3 9999 9999 9999 9999 9999    6 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            84   52   50   14   14   14   14   94   62   62   62   72
             0    0   48   46   70   34   32   76    4    4    6    6
             6    6   16   16   14   14   14   14   20   50   18   66
             0    0   12   12   38   36   78   70   16    4    4    6
             6    6    6  114   34   32   32   30  110   74   34   32
```

```
                   32    30     0     0     4     4    70    70     6     6   110   110
                   70   100   106    26    70    24   102   102    92     0     0    66
                   98    46    44    22     4     4    96   120     6     6   126   126
                   42    40   116   116    10    10    10    10    38    36    22    22
                   38    36   112   118    60    46    44    44    42   116    18    18
                    0     0     0     0     0     0     0     0     0     0     0     0
                    0     0     0     0     0     0     0     0     0     0     0     0
                    0     0     0     0     0     0     0     0     0     0     0     0
                    0     0     0     0

  CUBE    ( 124   ,   30   )    IS SELECTED.

  CUBE    ( 115   ,   48   )    HAS BEEN FOUND.

  CUBE    ( 115   ,   57   )    HAS BEEN FOUND.

  WATCH FOR     48 AND     57

  CUBE    ( 115   ,   58   )    HAS BEEN FOUND.

  WATCH FOR     48 AND     58

  CHECKING POINTS
                 9999  9999  9999  9999  9999  9999  9999     5  9999  9999  9999  9999
                 9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
                 9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
                 9999  9999  9999  9999  9999  9999    25  9999  9999  9999  9999  9999
                 9999  9999  9999     7  9999  9999  9999  9999     3  9999  9999  9999
                 9999  9999  9999  9999  9999  9999  9999  9999  9999  9999    15    11
                 9999     3     7  9999  9999  9999     5     9  9999  9999  9999  9999
                 9999  9999  9999  9999  9999  9999  9999     3  9999  9999     1     3
                 9999  9999     5     8  9999  9999  9999  9999  9999  9999  9999  9999
                 9999  9999     8     3  9999  9999  9999  9999  9999     7  9999  9999
                 9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
                 9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
                 9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999  9999
                 9999  9999  9999  9999

  ACCUMULATED AWAYNESS :

                   84    52    50    14    14    14    14   100    62    62    62    72
                    0     0    48    46    70    34    32    76     4     4     6     6
                    6     6    16    16    14    14    14    14    20    50    18    66
                    0     0    12    12    38    36    80    70    16     4     4     6
                    6     6     6   118    34    32    32    30   116    74    34    32
                   32    30     0     0     4     4    70    70     6     6   112   112
                   70   104   110    26    70    24   108   108    92     0     0    66
                   98    46    44    22     4     4    96   124     6     6   132   132
                   42    40   122   122    10    10    10    10    38    36    22    22
                   38    36   118   124    60    46    44    44    42   118    18    18
```

```
                   0     0     0     0     0     0     0     0     0     0     0     0
                   0     0     0     0     0     0     0     0     0     0     0     0
                   0     0     0     0     0     0     0     0     0     0     0     0
                   0     0     0     0

     CUBE    ( 115   ,   48   )    IS SELECTED.

     CUBE    (  55   ,    3   )    HAS BEEN FOUND.

     CUBE    (  55   ,    5   )    HAS BEEN FOUND.

     WATCH FOR       3 AND       5

     CUBE    (  55   ,   17   )    HAS BEEN FOUND.

     WATCH FOR       5 AND      17

     CUBE    (  55   ,   27   )    HAS BEEN FOUND.

     WATCH FOR      17 AND      27

     CUBE    (  55   ,   29   )    HAS BEEN FOUND.

     WATCH FOR      17 AND      29

     CUBE    (  55   ,   57   )    HAS BEEN FOUND.

     WATCH FOR      17 AND      57

     CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999    7 9999 9999 9999 9999    3 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   15   11
               9999    3    7 9999 9999 9999    5    9 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999    4 9999 9999    1    3
               9999 9999    5    8 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999    8    3 9999 9999 9999 9999 9999    7 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
               9999 9999 9999 9999

     ACCUMULATED AWAYNESS :

                 84   52   50   14   14   14   14  104   62   62   62   72
                  0    0   48   46   70   34   32   76    4    4    6    6
                  6    6   16   16   14   14   14   14   20   50   18   66
                  0    0   12   12   38   36   80   70   16    4    4    6
```

```
            6    6    6  124   34   32   32   30  124   74   34   32
           32   30    0    0    4    4   70   70    6    6  118  116
           70  108  118   26   70   24  116  114   92    0    0   66
           98   46   44   22    4    4   96  126    6    6  138  136
           42   40  128  126   10   10   10   10   38   36   22   22
           38   36  122  132   60   46   44   44   42  122   18   18
            0    0    0    0    0    0    0    0    0    0    0    0
            0    0    0    0    0    0    0    0    0    0    0    0
            0    0    0    0    0    0    0    0    0    0    0    0
            0    0    0    0

   CUBE    (  55   ,  17   )    IS SELECTED.

   CUBE    ( 128   , 161   )    HAS BEEN FOUND.

   CUBE    ( 128   , 224   )    HAS BEEN FOUND.

   CHECKING POINTS
         9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
         9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
         9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
         9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
         9999 9999 9999    7 9999 9999 9999 9999    3 9999 9999 9999
         9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   15   11
         9999    3    7 9999 9999 9999    5    9 9999 9999 9999 9999
         9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
         9999 9999    5    8 9999 9999 9999 9999 9999 9999 9999 9999
         9999 9999    8    4 9999 9999 9999 9999 9999    8 9999 9999
         9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
         9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
         9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
         9999 9999 9999 9999

   ACCUMULATED AWAYNESS :.

           84   52   50   14   14   14   14  108   62   62   62   72
            0    0   48   46   70   34   32   76    4    4    6    6
            6    6   16   16   14   14   14   14   20   50   18   66
            0    0   12   12   38   36   80   70   16    4    4    6
            6    6    6  128   34   32   32   30  130   74   34   32
           32   30    0    0    4    4   70   70    6    6  122  120
           70  116  124   26   70   24  124  122   92    0    0   66
           98   46   44   22    4    4   96  128    6    6  138  136
           42   40  132  130   10   10   10   10   38   36   22   22
           38   36  126  134   60   46   44   44   42  124   18   18
            0    0    0    0    0    0    0    0    0    0    0    0
            0    0    0    0    0    0    0    0    0    0    0    0
            0    0    0    0    0    0    0    0    0    0    0    0
            0    0    0    0

   CUBE    ( 128   , 161   )    IS SELECTED.
```

```
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   12
                  9999 9999 9999 9999 9999 9999    6    9 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
                  9999 9999    5    8 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999    9    4 9999 9999 9999 9999 9999    8 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999 9999 9999

 ACCUMULATED AWAYNESS :

                    84   52   50   14   14   14   14  118   62   62   62   72
                     0    0   48   46   70   34   32   76    4    4    6    6
                     6    6   16   16   14   14   14   14   20   50   18   66
                     0    0   12   12   38   36   80   70   16    4    4    6
                     6    6    6  134   34   32   32   30  138   74   34   32
                    32   30    0    0    4    4   70   70    6    6  128  126
                    70  116  132   26   70   24  132  130   92    0    0   66
                    98   46   44   22    4    4   96  144    6    6  138  136
                    42   40  150  148   10   10   10   10   38   36   22   22
                    38   36  138  148   60   46   44   44   42  140   18   18
                     0    0    0    0    0    0    0    0    0    0    0    0
                     0    0    0    0    0    0    0    0    0    0    0    0
                     0    0    0    0    0    0    0    0    0    0    0    0
                     0    0    0    0

 CUBE    (  72   ,   96   ),  IS SELECTED.

 CUBE    ( 168   , 162   )    HAS BEEN FOUND.

 CUBE    ( 168   , 164   )    HAS BEEN FOUND.

 WATCH FOR   162 AND   164

 CUBE    ( 168   , 174   )    HAS BEEN FOUND.

 WATCH FOR   162 AND   174

 CHECKING POINTS
                  9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   12
                  9999 9999 9999 9999 9999 9999    6    9 9999 9999 9999 9999
                  9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
                  9999 9999    5    8 9999 9999 9999 9999 9999 9999 9999 9999
                  9999 9999    9 9999 9999 9999 9999 9999 9999    9 9999 9999
                  9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
```

```
CUBE   ( 103  ,   79  )   HAS BEEN FOUND.

WATCH FOR      7 AND     79

CHECKING POINTS
                       9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                       9999 9999 9999    7 9999 9999 9999 9999    3 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   15   11
                       9999 9999    7 9999 9999 9999    5    9 9999 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
                       9999 9999    5    8 9999 9999 9999 9999 9999 9999 9999 9999
                       9999 9999    9    4 9999 9999 9999 9999 9999    8 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                       9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                         84   52   50   14   14   14   14  114   62   62   62   72
                          0    0   48   46   70   34   32   76    4    4    6    6
                          6    6   16   16   14   14   14   14   20   50   18   66
                          0    0   12   12   38   36   80   70   16    4    4    6
                          6    6    6  134   34   32   32   30  138   74   34   32
                         32   30    0    0    4    4   70   70    6    6  128  124
                         70  116  132   26   70   24  130  126   92    0    0   66
                         98   46   44   22    4    4   96  136    6    6  138  136
                         42   40  144  140   10   10   10   10   38   36   22   22
                         38   36  128  144   60   46   44   44   42  134   18   18
                          0    0    0    0    0    0    0    0    0    0    0    0
                          0    0    0    0    0    0    0    0    0    0    0    0
                          0    0    0    0    0    0    0    0    0    0    0    0
                          0    0    0    0

CUBE   ( 103  ,    7  )   IS SELECTED,

CUBE   (  72  ,   68  )   HAS BEEN FOUND.

CUBE   (  72  ,   96  )   HAS BEEN FOUND.

CUBE   (  72  ,  108  )   HAS BEEN FOUND.

CHECKING POINTS
                       9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                       9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
```

```
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999

   ACCUMULATED AWAYNESS :

                  84   52   50   14   14   14   14  124   62   62   62   72
                   0    0   48   46   70   34   32   76    4    4    6    6
                   6    6   16   16   14   14   14   14   20   50   18   66
                   0    0   12   12   38   36   80   70   16    4    4    6
                   6    6    6  134   34   32   32   30  138   74   34   32
                  32   30    0    0    4    4   70   70    6    6  128  132
                  70  116  132   26   70   24  138  138   92    0    0   66
                  98   46   44   22    4    4   96  150    6    6  138  136
                  42   40  154  154   10   10   10   10   38   36   22   22
                  38   36  142  148   60   46   44   44   42  142   18   18
                   0    0    0    0    0    0    0    0    0    0    0    0
                   0    0    0    0    0    0    0    0    0    0    0    0
                   0    0    0    0    0    0    0    0    0    0    0    0
                   0    0    0    0

   CUBE    ( 168   , 162  )    IS SELECTED.

   CUBE    (    9  ,    3 )    HAS BEEN FOUND.

   CUBE    (    9  ,    5 )    HAS BEEN FOUND.

   WATCH FOR      3 AND      5

   CUBE    (    9  ,   43 )    HAS BEEN FOUND.

   WATCH FOR      5 AND     43

   CUBE    (    9  ,   45 )    HAS BEEN FOUND.

   WATCH FOR      5 AND     45

   CHECKING POINTS
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   12
                9999 9999 9999 9999 9999 9999    6    9 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999    5 9999 9999 9999 9999
                9999 9999    5    8 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999    9 9999 9999 9999 9999 9999 9999    9 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999
```

```
ACCUMULATED AWAYNESS :

             84   52   50   14   14   14   14  124   62   62   62   72
              0    0   48   46   70   34   32   76    4    4    6    6

              6    6   16   16   14   14   14   14   20   50   18   66
              0    0   12   12   38   36   80   70   16    4    4    6
              6    6    6  134   34   32   32   30  138   74   34   32
             32   30    0    0    4    4   70   70    6    6  128  136
             70  116  132   26   70   24  144  142   92    0    0   66
             98   46   44   22    4    4   96  154    6    6  138  136
             42   40  160  158   10   10   10   10   38   36   22   22
             38   36  148  148   60   46   44   44   42  150   18   16
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0

CUBE     (    9  ,    5  )    IS SELECTED.
CUBE     ( 145  ,    1  )    HAS BEEN FOUND.

CHECKING POINTS
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   12
           9999 9999 9999 9999 9999 9999    6    9 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999    5    9 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999    9 9999 9999 9999 9999 9999 9999    9 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999

ACCUMULATED AWAYNESS :

             84   52   50   14   14   14   14  124   62   62   62   72
              0    0   48   46   70   34   32   76    4    4    6    6
              6    6   16   16   14   14   14   14   20   50   18   66
              0    0   12   12   38   36   80   70   16    4    4    6
              6    6    6  134   34   32   32   30  138   74   34   32
             32   30    0    0    4    4   70   70    6    6  128  140
             70  116  132   26   70   24  154  150   92    0    0   66
             98   46   44   22    4    4   96  154    6    6  138  136
             42   40  164  160   10   10   10   10   38   36   22   22
             38   36  154  148   60   46   44   44   42  154   18   18
```

```
           0       0       0       0       0       0       0       0       0       0       0       0
           0       0       0       0       0       0       0       0       0       0       0       0
           0       0       0       0       0       0       0       0       0       0       0       0
           0       0       0       0

CUBE   ( 145  ,   1  )   IS SELECTED.

CUBE   ( 152  ,  28  )   HAS BEEN FOUND.

CUBE   ( 152  , 157  )   HAS BEEN FOUND.

CUBE   ( 152  , 217  )   HAS BEEN FOUND.

WATCH FOR   157 AND   217

CUBE   ( 152  , 220  )   HAS BEEN FOUND.

CHECKING POINTS
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   12
        9999 9999 9999 9999 9999 9999    6    9 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999    9 9999 9999 9999 9999 9999 9999    9 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999

ACCUMULATED AWAYNESS :

          84      52      50      14      14      14      14     124      62      62      62      72
           0       0      48      46      70      34      32      76       4       4       6       6
           6       6      16      16      14      14      14      14      20      50      18      66
           0       0      12      12      38      36      80      70      16       4       4       6
           6       6       6     134      34      32      32      30     138      74      34      32
          32      30       0       0       4       4      70      70       6       6     128     148
          70     116     132      26      70      24     162     158      92       0       0      66
          98      46      44      22       4       4      96     154       6       6     138     136
          42      40     164     160      10      10      10      10      38      36      22      22
          38      36     164     148      60      46      44      44      42     158      18      18
           0       0       0       0       0       0       0       0       0       0       0       0
           0       0       0       0       0       0       0       0       0       0       0       0
           0       0       0       0       0       0       0       0       0       0       0       0
           0       0       0       0

CUBE   ( 152  , 217  )   IS SELECTED.
```

```
CUBE    ( 108   ,   72  )    HAS BEEN FOUND.

CUBE    ( 108   ,   77  )    HAS BEEN FOUND.

CUBE    ( 108   ,   92  )    HAS BEEN FOUND.

CUBE    ( 108   , 105  )    HAS BEEN FOUND.

WATCH FOR     77 AND    105

CHECKING POINTS
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   13
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999    9 9999 9999 9999 9999 9999 9999    9 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                9999 9999 9999 9999

ACCUMULATED AWAYNESS :

                  84   52   50   14   14   14   14  124   62   62   62   72
                   0    0   48   46   70   34   32   76    4    4    6    6
                   6    6   16   16   14   14   14   14   20   50   18   66
                   0    0   12   12   38   36   80   70   16    4    4    6
                   6    6    6  134   34   32   32   30  138   74   34   32
                  32   30    0    0    4    4   70   70    6    6  128  150
                  70  116  132   26   70   24  162  158   92    0    0   66
                  98   46   44   22    4    4   96  154    6    6  138  136
                  42   40  164  160   10   10   10   10   38   36   22   22
                  38   36  172  148   60   46   44   44   42  166   18   18
                   0    0    0    0    0    0    0    0    0    0    0    0
                   0    0    0    0    0    0    0    0    0    0    0    0
                   0    0    0    0    0    0    0    0    0    0    0    0
                   0    0    0    0

CUBE    ( 108   , 105  )    IS SELECTED.

CUBE    ( 176   ,   50  )    HAS BEEN FOUND.

CUBE    ( 176   , 162  )    HAS BEEN FOUND.

WATCH FOR     50 AND    162
```

```
CHECKING POINTS
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   13
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999    9 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
        9999 9999 9999 9999

ACCUMULATED AWAYNESS :

          84   52   50   14   14   14   14  124   62   62   62   72
           0    0   48   46   70   34   32   76    4    4    6    6
           6    6   16   16   14   14   14   14   20   50   18   66
           0    0   12   12   38   36   80   70   16    4    4    6
           6    6    6  134   34   32   32   30  138   74   34   32
          32   30    0    0    4    4   70   70    6    6  128  156
          70  116  132   26   70   24  162  158   92    0    0   66
          98   46   44   22    4    4   96  154    6    6  138  136
          42   40  164  160   10   10   10   10   38   36   22   22
          38   36  178  148   60   46   44   44   42  166   18   18
           0    0    0    0    0    0    0    0    0    0    0    0
           0    0    0    0    0    0    0    0    0    0    0    0
           0    0    0    0    0    0    0    0    0    0    0    0
           0    0    0    0

CUBE    ( 176   ,   50   )    IS SELECTED.

CUBE    ( 167   ,   35   )    HAS BEEN FOUND.

CUBE    ( 167   ,   37   )    HAS BEEN FOUND.

WATCH FOR      35  AND      37

CUBE    ( 167   ,  161   )    HAS BEEN FOUND.

WATCH FOR      37  AND     161

CUBE    ( 167   ,  173   )    HAS BEEN FOUND.

WATCH FOR      37  AND     173

CHECKING POINTS
```

```
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999   13
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
          9999 9999 9999 9999

ACCUMULATED AWAYNESS :

            84   52   50   14   14   14   14  124   62   62   62   72
             0    0   48   46   70   34   32   76    4    4    6    6
             6    6   16   16   14   14   14   14   20   50   18   66
             0    0   12   12   38   36   80   70   16    4    4    6
             6    6    6  134   34   32   32   30  138   74   34   32
            32   30    0    0    4    4   70   70    6    6  128  160
            70  116  132   26   70   24  162  158   92    0    0   66
            98   46   44   22    4    4   96  154    6    6  138  136
            42   40  164  160   10   10   10   10   38   36   22   22
            38   36  178  148   60   46   44   44   42  166   18   16
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0

CUBE    (  167  ,   37  )    IS SELECTED.

CUBE    (   97  ,  104  )    HAS BEEN FOUND.

CUBE    (   97  ,  112  )    HAS BEEN FOUND.

WATCH FOR    104 AND    112

CUBE    (   97  ,  121  )    HAS BEEN FOUND.

WATCH FOR    112 AND    121

CUBE    (   97  ,  224  )    HAS BEEN FOUND.

WATCH FOR    112 AND    224

CHECKING POINTS
          9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
```

```
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
                 9999 9999 9999 9999

  ACCUMULATED AWAYNESS :

                   84   52   50   14   14   14   14  124   62   62   62   72
                    0    0   48   46   70   34   32   76    4    4    6    6
                    6    6   16   16   14   14   14   14   20   50   18   66
                    0    0   12   12   38   36   80   70   16    4    4    6
                    6    6    6  134   34   32   32   30  138   74   34   32
                   32   30    0    0    4    4   70   70    6    6  128  160
                   70  116  132   26   70   24  162  158   92    0    0   66
                   98   46   44   22    4    4   96  154    6    6  138  136
                   42   40  164  160   10   10   10   10   38   36   22   22
                   38   36  178  148   60   46   44   44   42  166   18   18
                    0    0    0    0    0    0    0    0    0    0    0    0
                    0    0    0    0    0    0    0    0    0    0    0    0
                    0    0    0    0    0    0    0    0    0    0    0    0
                    0    0    0    0

  CUBE    (  97   , 224   )    IS SELECTED.


  **FORTRAN ** STOP
  /LOGOFF

  C E420 LOGOFF AT 0949 ON 11/12 `, FOR TSN 3606.

  C E421 CPU TIME USED:  001328.9660 SECONDS.
```

APPENDIX I

EXAMPLE OF 9 VARIABLES, 5 ON-SET MINTERMS, AND 503 DON'T CARE MINTERMS.

```
TOATAL NUMBER OF ON-SET AND DONT*CARES =   508
ON-SET =     5     DONT-CARES =   503
NUMBERS OF VARIABLES =      9

        MINT,#   1  =    1     MINT,#   2  =    2     MINT.#   3  =    4
        MINT,#   4  =    8     MINT,#   5  =  256     MINT.#   6  =    0
        MINT,#   7  =    3     MINT,#   8  =    5     MINT,#   9  =    6
        MINT,#  10  =    7     MINT,#  11  =    9     MINT,#  12  =   10
        MINT.#  13  =   11     MINT.#  14  =   12     MINT.#  15  =   13
        MINT,#  16  =   14     MINT,#  17  =   15     MINT.#  18  =   17
        MINT.#  19  =   18     MINT.#  20  =   19     MINT.#  21  =   20
        MINT.#  22  =   21     MINT,#  23  =   22     MINT.#  24  =   23
        MINT.#  25  =   24     MINT,#  26  =   25     MINT.#  27  =   26
        MINT.#  28  =   27     MINT,#  29  =   28     MINT,#  30  =   29
        MINT.#  31  =   30     MINT.#  32  =   31     MINT.#  33  =   33
        MINT,#  34  =   34     MINT,#  35  =   35     MINT.#  36  =   36
        MINT.#  37  =   37     MINT.#  38  =   38     MINT.#  39  =   39
        MINT,#  40  =   40     MINT,#  41  =   41     MINT.#  42  =   42
        MINT.#  43  =   43     MINT.#  44  =   44     MINT.#  45  =   45
        MINT.#  46  =   46     MINT.#  47  =   47     MINT.#  48  =   48
        MINT.#  49  =   49     MINT.#  50  =   50     MINT.#  51  =   51
        MINT.#  52  =   52     MINT.#  53  =   53     MINT.#  54  =   54
        MINT.#  55  =   55     MINT,#  56  =   56     MINT.#  57  =   57
        MINT,#  58  =   58     MINT.#  59  =   59     MINT,#  60  =   60
        MINT.#  61  =   61     MINT,#  62  =   62     MINT.#  63  =   63
        MINT.#  64  =   65     MINT,#  65  =   66     MINT.#  66  =   67
        MINT.#  67  =   68     MINT.#  68  =   69     MINT.#  69  =   70
        MINT.#  70  =   71     MINT,#  71  =   72     MINT.#  72  =   73
        MINT.#  73  =   74     MINT.#  74  =   75     MINT.#  75  =   76
        MINT.#  76  =   77     MINT,#  77  =   78     MINT.#  78  =   79
        MINT.#  79  =   80     MINT.#  80  =   81     MINT.#  81  =   82
        MINT,#  82  =   83     MINT.#  83  =   84     MINT.#  84  =   85
        MINT.#  85  =   86     MINT.#  86  =   87     MINT.#  87  =   88
        MINT.#  88  =   89     MINT.#  89  =   90     MINT.#  90  =   91
        MINT.#  91  =   92     MINT.#  92  =   93     MINT.#  93  =   94
        MINT.#  94  =   95     MINT.#  95  =   96     MINT,#  96  =   97
        MINT.#  97  =   98     MINT.#  98  =   99     MINT.#  99  =  100
        MINT.# 100  =  101     MINT.# 101  =  102     MINT.# 102  =  103
        MINT.# 103  =  104     MINT.# 104  =  105     MINT.# 105  =  106
        MINT,# 106  =  107     MINT.# 107  =  108     MINT.# 108  =  109
        MINT.# 109  =  110     MINT,# 110  =  111     MINT.# 111  =  112
        MINT.# 112  =  113     MINT,# 113  =  114     MINT.# 114  =  115
        MINT.# 115  =  116     MINT,# 116  =  117     MINT.# 117  =  118
        MINT.# 118  =  119     MINT,# 119  =  120     MINT,# 120  =  121
        MINT.# 121  =  122     MINT.# 122  =  123     MINT.# 123  =  124
        MINT,# 124  =  125     MINT,# 125  =  126     MINT.# 126  =  127
        MINT,# 127  =  129     MINT,# 128  =  130     MINT.# 129  =  131
        MINT.# 130  =  132     MINT.# 131  =  133     MINT.# 132  =  134
        MINT.# 133  =  135     MINT.# 134  =  136     MINT.# 135  =  137
        MINT.# 136  =  138     MINT.# 137  =  139     MINT.# 138  =  140
        MINT,# 139  =  141     MINT.# 140  =  142     MINT.# 141  =  143
```

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MINT.# 142  =  144     MINT.# 143  =  145     MINT.# 144  =  146
MINT.# 145  =  147     MINT.# 146  =  148     MINT.# 147  =  149
MINT.# 148  =  150     MINT.# 149  =  151     MINT.# 150  =  152
MINT.# 151  =  153     MINT.# 152  =  154     MINT.# 153  =  155
MINT.# 154  =  156     MINT.# 155  =  157     MINT.# 156  =  158
MINT.# 157  =  159     MINT.# 158  =  160     MINT.# 159  =  161
MINT.# 160  =  162     MINT.# 161  =  163     MINT.# 162  =  164
MINT.# 163  =  165     MINT.# 164  =  166     MINT.# 165  =  167
MINT.# 166  =  168     MINT.# 167  =  169     MINT.# 168  =  170
MINT.# 169  =  171     MINT.# 170  =  172     MINT.# 171  =  173
MINT.# 172  =  174     MINT.# 173  =  175     MINT.# 174  =  176
MINT.# 175  =  177     MINT.# 176  =  178     MINT.# 177  =  179
MINT.# 178  =  180     MINT.# 179  =  181     MINT.# 180  =  182
MINT.# 181  =  183     MINT.# 182  =  184     MINT.# 183  =  185
MINT.# 184  =  186     MINT.# 185  =  187     MINT.# 186  =  188
MINT.# 187  =  189     MINT.# 188  =  190     MINT.# 189  =  191
MINT.# 190  =  192     MINT.# 191  =  193     MINT.# 192  =  194
MINT.# 193  =  195     MINT.# 194  =  196     MINT.# 195  =  197
MINT.# 196  =  198     MINT.# 197  =  199     MINT.# 198  =  200
MINT.# 199  =  201     MINT.# 200  =  202     MINT.# 201  =  203
MINT.# 202  =  204     MINT.# 203  =  205     MINT.# 204  =  206
MINT.# 205  =  207     MINT.# 206  =  208     MINT.# 207  =  209
MINT.# 208  =  210     MINT.# 209  =  211     MINT.# 210  =  212
MINT.# 211  =  213     MINT.# 212  =  214     MINT.# 213  =  215
MINT.# 214  =  216     MINT.# 215  =  217     MINT.# 216  =  218
MINT.# 217  =  219     MINT.# 218  =  220     MINT.# 219  =  221
MINT.# 220  =  222     MINT.# 221  =  223     MINT.# 222  =  224
MINT.# 223  =  225     MINT.# 224  =  226     MINT.# 225  =  227
MINT.# 226  =  228     MINT.# 227  =  229     MINT.# 228  =  230
MINT.# 229  =  231     MINT.# 230  =  232     MINT.# 231  =  233
MINT.# 232  =  234     MINT.# 233  =  235     MINT.# 234  =  236
MINT.# 235  =  237     MINT.# 236  =  238     MINT.# 237  =  239
MINT.# 238  =  240     MINT.# 239  =  241     MINT.# 240  =  242
MINT.# 241  =  243     MINT.# 242  =  244     MINT.# 243  =  245
MINT.# 244  =  246     MINT.# 245  =  247     MINT.# 246  =  248
MINT.# 247  =  249     MINT.# 248  =  250     MINT.# 249  =  251
MINT.# 250  =  252     MINT.# 251  =  253     MINT.# 252  =  254
MINT.# 253  =  255     MINT.# 254  =  257     MINT.# 255  =  258
MINT.# 256  =  259     MINT.# 257  =  260     MINT.# 258  =  261
MINT.# 259  =  262     MINT.# 260  =  263     MINT.# 261  =  264
MINT.# 262  =  265     MINT.# 263  =  266     MINT.# 264  =  267
MINT.# 265  =  268     MINT.# 266  =  269     MINT.# 267  =  270
MINT.# 268  =  271     MINT.# 269  =  272     MINT.# 270  =  273
MINT.# 271  =  274     MINT.# 272  =  275     MINT.# 273  =  276
MINT.# 274  =  277     MINT.# 275  =  278     MINT.# 276  =  279
MINT.# 277  =  280     MINT.# 278  =  281     MINT.# 279  =  282
MINT.# 280  =  283     MINT.# 281  =  284     MINT.# 282  =  285
MINT.# 283  =  286     MINT.# 284  =  287     MINT.# 285  =  288
MINT.# 286  =  289     MINT.# 287  =  290     MINT.# 288  =  291
MINT.# 289  =  292     MINT.# 290  =  293     MINT.# 291  =  294
MINT.# 292  =  295     MINT.# 293  =  296     MINT.# 294  =  297
MINT.# 295  =  298     MINT.# 296  =  299     MINT.# 297  =  300
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MINT.# 298  =  301      MINT.# 299  =  302      MINT.# 300  =  303
MINT.# 301  =  304      MINT.# 302  =  305      MINT.# 303  =  306
MINT.# 304  =  307      MINT.# 305  =  308      MINT.# 306  =  309
MINT.# 307  =  310      MINT.# 308  =  311      MINT.# 309  =  312
MINT.# 310  =  313      MINT.# 311  =  314      MINT.# 312  =  315
MINT.# 313  =  316      MINT.# 314  =  317      MINT.# 315  =  318
MINT.# 316  =  319      MINT.# 317  =  320      MINT.# 318  =  321
MINT.# 319  =  322      MINT.# 320  =  323      MINT.# 321  =  324
MINT.# 322  =  325      MINT.# 323  =  326      MINT.# 324  =  327
MINT.# 325  =  328      MINT.# 326  =  329      MINT.# 327  =  330
MINT.# 328  =  331      MINT.# 329  =  332      MINT.# 330  =  333
MINT.# 331  =  334      MINT.# 332  =  335      MINT.# 333  =  336
MINT.# 334  =  337      MINT.# 335  =  338      MINT.# 336  =  339
MINT.# 337  =  340      MINT.# 338  =  341      MINT.# 339  =  342
MINT.# 340  =  343      MINT.# 341  =  344      MINT.# 342  =  345
MINT.# 343  =  346      MINT.# 344  =  347      MINT.# 345  =  348
MINT.# 346  =  349      MINT.# 347  =  350      MINT.# 348  =  351
MINT.# 349  =  352      MINT.# 350  =  353      MINT.# 351  =  354
MINT.# 352  =  355      MINT.# 353  =  356      MINT.# 354  =  357
MINT.# 355  =  358      MINT.# 356  =  359      MINT.# 357  =  360
MINT.# 358  =  361      MINT.# 359  =  362      MINT.# 360  =  363
MINT.# 361  =  364      MINT.# 362  =  365      MINT.# 363  =  366
MINT.# 364  =  367      MINT.# 365  =  368      MINT.# 366  =  369
MINT.# 367  =  370      MINT.# 368  =  371      MINT.# 369  =  372
MINT.# 370  =  373      MINT.# 371  =  374      MINT.# 372  =  375
MINT.# 373  =  376      MINT.# 374  =  377      MINT.# 375  =  378
MINT.# 376  =  379      MINT.# 377  =  380      MINT.# 378  =  381
MINT.# 379  =  382      MINT.# 380  =  383      MINT.# 381  =  384
MINT.# 382  =  385      MINT.# 383  =  386      MINT.# 384  =  387
MINT.# 385  =  388      MINT.# 386  =  389      MINT.# 387  =  390
MINT.# 388  =  391      MINT.# 389  =  392      MINT.# 390  =  393
MINT.# 391  =  394      MINT.# 392  =  395      MINT.# 393  =  396
MINT.# 394  =  397      MINT.# 395  =  398      MINT.# 396  =  399
MINT.# 397  =  400      MINT.# 398  =  401      MINT.# 399  =  402
MINT.# 400  =  403      MINT.# 401  =  404      MINT.# 402  =  405
MINT.# 403  =  406      MINT.# 404  =  407      MINT.# 405  =  408
MINT.# 406  =  409      MINT.# 407  =  410      MINT.# 408  =  411
MINT.# 409  =  412      MINT.# 410  =  413      MINT.# 411  =  414
MINT.# 412  =  415      MINT.# 413  =  416      MINT.# 414  =  417
MINT.# 415  =  418      MINT.# 416  =  419      MINT.# 417  =  420
MINT.# 418  =  421      MINT.# 419  =  422      MINT.# 420  =  423
MINT.# 421  =  424      MINT.# 422  =  425      MINT.# 423  =  426
MINT.# 424  =  427      MINT.# 425  =  428      MINT.# 426  =  429
MINT.# 427  =  430      MINT.# 428  =  431      MINT.# 429  =  432
MINT.# 430  =  433      MINT.# 431  =  434      MINT.# 432  =  435
MINT.# 433  =  436      MINT.# 434  =  437      MINT.# 435  =  438
MINT.# 436  =  439      MINT.# 437  =  440      MINT.# 438  =  441
MINT.# 439  =  442      MINT.# 440  =  443      MINT.# 441  =  444
MINT.# 442  =  445      MINT.# 443  =  446      MINT.# 444  =  447
MINT.# 445  =  448      MINT.# 446  =  449      MINT.# 447  =  450
MINT.# 448  =  451      MINT.# 449  =  452      MINT.# 450  =  453
```

```
      MINT.# 451  =  454      MINT.# 452  =  455      MINT.# 453  =  456
      MINT.# 454  =  457      MINT.# 455  =  458      MINT.# 456  =  459
      MINT.# 457  =  460      MINT.# 458  =  461      MINT.# 459  =  462
      MINT.# 460  =  463      MINT.# 461  =  464      MINT.# 462  =  465
      MINT.# 463  =  466      MINT.# 464  =  467      MINT.# 465  =  468
      MINT.# 466  =  469      MINT.# 467  =  470      MINT.# 468  =  471
      MINT,# 469  =  472      MINT.# 470  =  473      MINT.# 471  =  474
      MINT.# 472  =  475      MINT.# 473  =  476      MINT.# 474  =  477
      MINT.# 475  =  478      MINT.# 476  =  479      MINT.# 477  =  480
      MINT.# 478  =  481      MINT.# 479  =  482      MINT.# 480  =  483
      MINT.# 481  =  484      MINT.# 482  =  485      MINT.# 483  =  486
      MINT,# 484  =  487      MINT.# 485  =  488      MINT.# 486  =  489
      MINT.# 487  =  490      MINT.# 488  =  491      MINT.# 489  =  492
      MINT.# 490  =  493      MINT.# 491  =  494      MINT.# 492  =  495
      MINT,# 493  =  496      MINT,# 494  =  497      MINT.# 495  =  498
      MINT.# 496  =  499      MINT.# 497  =  500      MINT.# 498  =  501
      MINT.# 499  =  502      MINT.# 500  =  503      MINT.# 501  =  504
      MINT.# 502  =  505      MINT.# 503  =  506      MINT.# 504  =  507
      MINT.# 505  =  508      MINT.# 506  =  509      MINT.# 507  =  510
      MINT.# 508  =  511      MINT.#


   THE CUBE COVERING LIST :



   THE      1-TH CUBE COVERING IS :
            8-CUBE = ( 256 , 511  ) =      1--------

   THIS CUBE COVERS THESE MINTERMS :
         256 511 257 258 259 260 261 262 263 264 265 266 267 268 269 270
         271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286
         287 288 289 290 291 292 293 294 295 296 297 298 299 300 301 302
         303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318
         319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334
         335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350
         351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366
         367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382
         383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398
         399 400 401 402 403 404 405 406 407 408 409 410 411 412 413 414
         415 416 417 418 419 420 421 422 423 424 425 426 427 428 429 430
         431 432 433 434 435 436 437 438 439 440 441 442 443 444 445 446
         447 448 449 450 451 452 453 454 455 456 457 458 459 460 461 462
         463 464 465 466 467 468 469 470 471 472 473 474 475 476 477 478
         479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494
         495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510
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THE      2-TH CUBE COVERING IS :
       8-CUBE = (    8 , 511   ) =        -----1---

THIS CUBE COVERS THESE MINTERMS :
         8  511    9   10   11   12   13   14   15   24   25   26   27   28   29   30
        31   40   41   42   43   44   45   46   47   56   57   58   59   60   61   62
        63   72   73   74   75   76   77   78   79   88   89   90   91   92   93   94
        95  104  105  106  107  108  109  110  111  120  121  122  123  124  125  126
       127  136  137  138  139  140  141  142  143  152  153  154  155  156  157  158
       159  168  169  170  171  172  173  174  175  184  185  186  187  188  189  190
       191  200  201  202  203  204  205  206  207  216  217  218  219  220  221  222
       223  232  233  234  235  236  237  238  239  248  249  250  251  252  253  254
       255  264  265  266  267  268  269  270  271  280  281  282  283  284  285  286
       287  296  297  298  299  300  301  302  303  312  313  314  315  316  317  318
       319  328  329  330  331  332  333  334  335  344  345  346  347  348  349  350
       351  360  361  362  363  364  365  366  367  376  377  378  379  380  381  382
       383  392  393  394  395  396  397  398  399  408  409  410  411  412  413  414
       415  424  425  426  427  428  429  430  431  440  441  442  443  444  445  446
       447  456  457  458  459  460  461  462  463  472  473  474  475  476  477  478
       479  488  489  490  491  492  493  494  495  504  505  506  507  508  509  510



THE      3-TH CUBE COVERING IS :
       8-CUBE = (    2 , 511   ) =        -------1-

THIS CUBE COVERS THESE MINTERMS :
         2  511    3    6    7   10   11   14   15   18   19   22   23   26   27   30
        31   34   35   38   39   42   43   46   47   50   51   54   55   58   59   62
        63   66   67   70   71   74   75   78   79   82   83   86   87   90   91   94
        95   98   99  102  103  106  107  110  111  114  115  118  119  122  123  126
       127  130  131  134  135  138  139  142  143  146  147  150  151  154  155  158
       159  162  163  166  167  170  171  174  175  178  179  182  183  186  187  190
       191  194  195  198  199  202  203  206  207  210  211  214  215  218  219  222
       223  226  227  230  231  234  235  238  239  242  243  246  247  250  251  254
       255  258  259  262  263  266  267  270  271  274  275  278  279  282  283  286
       287  290  291  294  295  298  299  302  303  306  307  310  311  314  315  318
       319  322  323  326  327  330  331  334  335  338  339  342  343  346  347  350
       351  354  355  358  359  362  363  366  367  370  371  374  375  378  379  382
       383  386  387  390  391  394  395  398  399  402  403  406  407  410  411  414
       415  418  419  422  423  426  427  430  431  434  435  438  439  442  443  446
       447  450  451  454  455  458  459  462  463  466  467  470  471  474  475  478
       479  482  483  486  487  490  491  494  495  498  499  502  503  506  507  510



THE      4-TH CUBE COVERING IS :
       8-CUBE = (    4 , 511   ) =        ------1--
```

```
THIS CUBE COVERS THESE MINTERMS :
         4 511   5   6   7  12  13  14  15  20  21  22  23  28  29  30
        31  36  37  38  39  44  45  46  47  52  53  54  55  60  61  62
        63  68  69  70  71  76  77  78  79  84  85  86  87  92  93  94
        95 100 101 102 103 108 109 110 111 116 117 118 119 124 125 126
       127 132 133 134 135 140 141 142 143 148 149 150 151 156 157 158
       159 164 165 166 167 172 173 174 175 180 181 182 183 188 189 190
       191 196 197 198 199 204 205 206 207 212 213 214 215 220 221 222
       223 228 229 230 231 236 237 238 239 244 245 246 247 252 253 254
       255 260 261 262 263 268 269 270 271 276 277 278 279 284 285 286
       287 292 293 294 295 300 301 302 303 308 309 310 311 316 317 318
       319 324 325 326 327 332 333 334 335 340 341 342 343 348 349 350
       351 356 357 358 359 364 365 366 367 372 373 374 375 380 381 382
       383 388 389 390 391 396 397 398 399 404 405 406 407 412 413 414
       415 420 421 422 423 428 429 430 431 436 437 438 439 444 445 446
       447 452 453 454 455 460 461 462 463 468 469 470 471 476 477 478
       479 484 485 486 487 492 493 494 495 500 501 502 503 508 509 510


THE      5-TH CUBE COVERING IS :
      8-CUBE = (   1 , 511   ) =       --------1

THIS CUBE COVERS THESE MINTERMS :
         1 511   3   5   7   9  11  13  15  17  19  21  23  25  27  29
        31  33  35  37  39  41  43  45  47  49  51  53  55  57  59  61
        63  65  67  69  71  73  75  77  79  81  83  85  87  89  91  93
        95  97  99 101 103 105 107 109 111 113 115 117 119 121 123 125
       127 129 131 133 135 137 139 141 143 145 147 149 151 153 155 157
       159 161 163 165 167 169 171 173 175 177 179 181 183 185 187 189
       191 193 195 197 199 201 203 205 207 209 211 213 215 217 219 221
       223 225 227 229 231 233 235 237 239 241 243 245 247 249 251 253
       255 257 259 261 263 265 267 269 271 273 275 277 279 281 283 285
       287 289 291 293 295 297 299 301 303 305 307 309 311 313 315 317
       319 321 323 325 327 329 331 333 335 337 339 341 343 345 347 349
       351 353 355 357 359 361 363 365 367 369 371 373 375 377 379 381
       383 385 387 389 391 393 395 397 399 401 403 405 407 409 411 413
       415 417 419 421 423 425 427 429 431 433 435 437 439 441 443 445
       447 449 451 453 455 457 459 461 463 465 467 469 471 473 475 477
       479 481 483 485 487 489 491 493 495 497 499 501 503 505 507 509

THE FINAL CHECKING
              105   105   105   105   105   109   309   309   309   409   309   309
              409   309   409   409   509   209   209   309   209   309   309   409
              209   309   309   409   309   409   409   509   209   209   309   209
              309   309   409   209   309   309   409   309   409   409   509   109
              209   209   309   209   309   309   409   209   309   309   409   309
              409   409   509   209   209   309   209   309   309   409   209   309
              309   409   309   409   409   509   109   209   209   309   209   309
              309   409   209   309   309   409   309   409   409   509   109   209
```

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209  309  209  309  309  409  209  309  309  409  309  409
409  509  109  209  209  309  209  309  309  409  209  309
309  409  309  409  409  509  209  209  309  209  309  309
409  209  309  309  409  309  409  409  509  109  209  209
309  209  309  309  409  209  309  309  409  309  409  409
509  109  209  209  309  209  309  309  409  209  309  309
409  309  409  409  509  109  209  209  309  209  309  309
409  209  309  309  409  309  409  409  509  109  209  209
309  209  309  309  409  209  309  309  409  309  409  409
509  109  209  209  309  209  309  309  409  209  309  309
409  309  409  409  509  109  209  209  309  209  309  309
409  209  309  309  409  309  409  409  509  109  209  209
309  209  309  309  409  209  309  309  409  309  409  409
509  309  309  409  309  409  409  509  309  409  409  509
409  509  509  609  209  309  309  409  309  409  409  509
309  409  409  509  409  509  509  609  209  309  309  409
309  409  409  509  309  409  409  509  409  509  509  609
209  309  309  409  309  409  409  509  309  409  409  509
409  509  509  609  209  309  309  409  309  409  409  509
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309  409  409  509  309  409  409  509  409  509  509  609
209  309  309  409  309  409  409  509  309  409  409  509
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309  409  409  509  409  509  509  609  209  309  309  409
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209  309  309  409  309  409  409  509  309  409  409  509
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309  409  409  509  309  409  409  509  409  509  509  609
209  309  309  409  309  409  409  509  309  409  409  509
409  509  509  609  209  309  309  409  309  409  409  509
309  409  409  509  409  509  509  609  209  309  309  409
309  409  409  509  309  409  409  509  409  509  509  609
209  309  309  409  309  409  409  509  309  409  409  509
409  509  509  609
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```
MAXIMUM CUBE DIMENSION =    8

      1     0     0     0     0     0     0     0     2     1     0     0     0     0
      0     0     3     3     1     0     0     0     0     0     4     6     4     1
      0     0     0     0     5    10    10     5     1     0     0     0     6    15
     20    15     6     1     0     0     7    21    35    35    21     7     1     0
      8    28    56    70    56    28     8     1

THE CHARACTERISTICS :
           8  5  8  5  8  5  8  5  8  5  5  1  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  1  8  1  8  5  8  1  8  5
           8  5  8  5  8  1  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  1
           8  1  8  5  8  1  8  5  8  5  8  5  8  1  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  7  1  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  1  8  1  8  5
           8  1  8  5  8  5  8  5  8  1  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  7  1  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  7  1  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  1  8  1  8  5  8  1  8  5  8  5
           8  5  8  1  8  5  8  5  8  5  8  5  8  5  8  5  8  5  7  1  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  7  1  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  7  1  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  1  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  1  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  1  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  1  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
           8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5  8  5
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         8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5
         8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5
         8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5
         8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5   8   5
         8   5   8   5

 ALL CORE CUBES HAVE BEEN SEARCHED,
 AND THE MAXIMUM CUBE DIMENSION IS :     8

 CUBE    ( 256   , 511   )   HAS BEEN FOUND.

 CHECKING POINTS
                64   64   64   64 9999 9999 9999 9999 9999 9999 9999 9999
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  ACCUMULATED AWAYNESS :
       2    2    2    2    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0
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```
CUBE    ( 256  , 511  )    IS SELECTED.

CUBE    (   8  , 511  )    HAS BEEN FOUND.

CHECKING POINTS
            128  128  128 9999 9999 9999 9999 9999 9999 9999 9999 9999
           9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
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ACCUMULATED AWAYNESS :
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             4    4    4    2    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
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             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0    0    0    0    0    0    0    0    0
             0    0    0    0

CUBE   (   8  , 511  )   IS SELECTED.

CUBE   (   2  , 511  )   HAS BEEN FOUND.

CHECKING POINTS
           192 9999  192 9999 9999 9999 9999 9999 9999 9999 9999 9999
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9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
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 ACCUMULATED AWAYNESS :

      6    4    6    2    0    0    0    0    0    0    0    0
      0    0    0    0    0    0    0    0    0    0    0    0
      0    0    0    0    0    0    0    0    0    0    0    0
      0    0    0    0    0    0    0    0    0    0    0    0
      0    0    0    0    0    0    0    0    0    0    0    0
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              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0    0    0    0    0    0    0    0    0
              0    0    0    0

CUBE   (    2  ,  511  )   IS SELECTED.

CUBE   (    4  ,  511  )   HAS BEEN FOUND.

CHECKING POINTS
          256 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
         9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
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9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999
9999 9999 9999 9999
ACCUMULATED AWAYNESS :
        8    4    6    2    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
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                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0    0    0    0    0    0    0    0
                  0    0    0    0

   CUBE    (    4  , 511  )   IS SELECTED.

   CUBE    (    1  , 511  )   HAS BEEN FOUND.

   CHECKING POINTS
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 999
               9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 9999 999
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9999 9999 9999 9999

     ACCUMULATED AWAYNESS :

       8    4    6    2    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
       0    0    0    0    0    0    0    0    0    0    0
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        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0    0    0    0    0    0    0    0    0
        0    0    0    0

 CUBE    (    1   , 511   )    IS SELECTED.


 **FORTRAN ** STOP
 /LOGOFF

 C E420 LOGOFF AT 0911 ON 11/23      FOR TSN 9977.

 C E421 CPU TIME USED:   000638.5355 SECONDS.
```