# ABSTRACT

Title of Thesis: An Equilibrium Theory of the pH Parametric Pump

Charles Omotayo Kerobo, Master of Science, 1979

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Parametric pumping, both batch and continuous, and cycling zone adsorption are theoretically scrutinized for various operating conditions by using the concept of equilibrium stages. The model system is Haemoglobin-Albumin-CM sepharose, a physical system in which a local equilibrium between liquid and sorbent in the layers of the separating medium in the column is characterized. For the batch process, the steady-state concentrations in both top and bottom reservoirs are found to be independent of the number of equilibrium stages, provided that $\alpha = 1$ (i.e., number of transfer steps/number of stages). For continuous parapump process, separation is enhanced with decreasing top feed ratio and/or decreasing bottom feed ratio. However, steadily degrading separation is found to exist when the top or bottom feed ratio exceeds the void volume of the column. The separation obtained for the cycling zone adsorption shows that an optimum number of stages is between 10 and 16. Below the lower limit, separation steadily decreases, and above the upper limit, the separation is found to be constant. Recycle ratio ($\beta$) is found to have an influence on the separation below the optimum number of stages and no influence above the optimum number of stages.

A computational algorithm for predicting equilibrium parametric pump performance (both batch and continuous) and cycling zone adsorption is developed. The algorithm is based on a set of exterior solute material balances and a linear adsorption isotherm for the solute-adsorbant system. The method of STOP-GO is combined with a finite difference approximation to solve the material balance equations.

# AN EQUILIBRIUM THEORY OF THE pH-PARAMETRIC PUMP

by
Charles Omotayo Kerobo

Thesis submitted to the Faculty of the Graduate School of
the New Jersey Institute of Technology in partial fulfillment
of the requirements for the degree of
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Having passed both parts of the doctoral qualifying examination in 1978, I am currently working towards the Doctor of Science degree.

*I would like to dedicate this thesis*
*to my wife, Faye Lea Kerobo,*
*and our baby, who is*
*due within a*
*few months*

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# Introduction

The initial theoretical analysis of parametric pumping was described by Wilhelm et al. in 1966. In this pioneering work, a recuperative mode of operation was applied; the fluid was heated in a heat exchanger before flowing up through the bed and cooled before flowing down. (See Figure 1(a).) This thermal recuperative mode was used for removing NaCl from water on a mixed bed of ion exchange resins. Wilhelm (1966) discussed the possibilities of applying the parapump idea to a variety of oscillating fields such as electrical, magnetic, or chemical potential. Wilhelm and Sweed (1968), and Wakao et al. (1968) independently developed the direct mode. (See Figure 1(b).) This direct mode of operation called for external application of heating and cooling source during up and down flow respectively (water jacket) on the stationary bed before upward flow of fluid, and then, the bed is cooled by an external source before downward flow of the fluid.

Sabadell and Sweed (1970) extended the parapump process to a recuperative pH mode to remove $K^+$ and $Na^+$ from water. In this recuperative mode of operation, the low pH end was closed, and the high pH end opened. To maintain the pH levels constant, HCl was added to the low end while fresh feed was introduced and product was withdrawn from the open end every half cycle. This setup was called a semibatch recuperative pH mode parapumping operation. The neutralization reaction which occurred in the

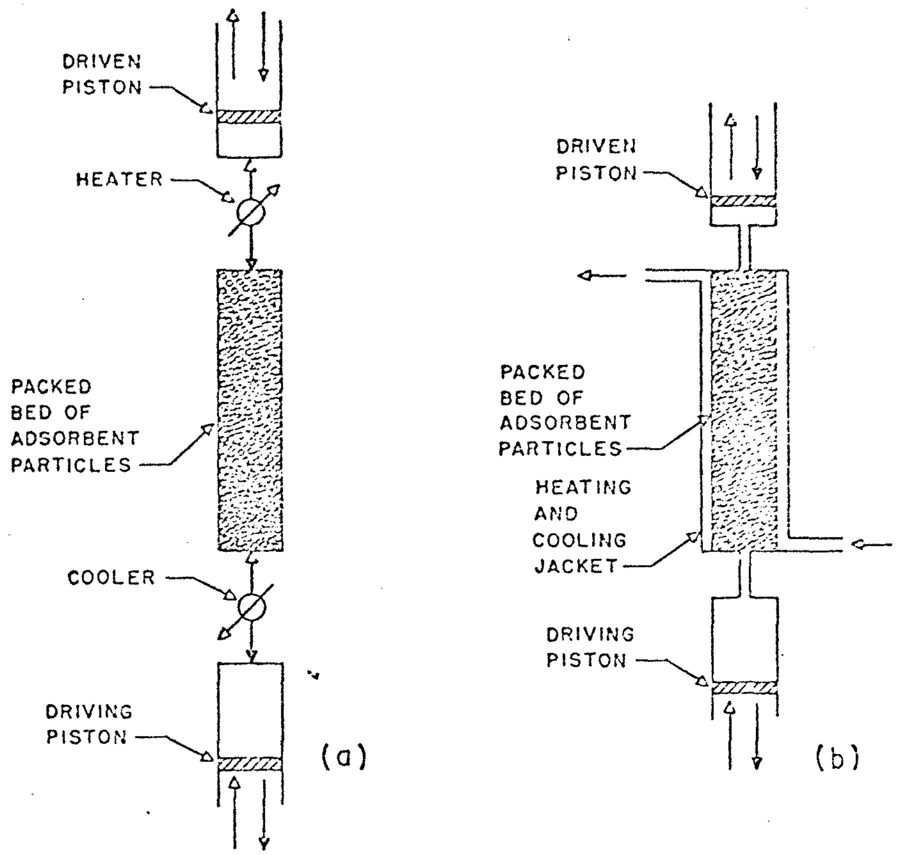


Fig. 1. Apparatus arrangement for thermal parametric pumping. (a) Recuperative mode. (b) Direct mode.

column was the source of energy used for the separation. Although this method of deriving energy source was not optimized, a separation factor of 1.84 was obtained. This value was better than that obtained by previous methods of parapumps.

## The Batch Pump

Included in the parapump methods of separation is the batch process. Various forms of energy, such as pH and temperature, have been harnessed to effect parametric pumping separations. Since the basic concepts of pH parametric pumping were developed from the thermal system, thermal parapumping operation will be used in illustrating the principle behind the batch process. There are two different methods of imposing temperature fluctuations in thermal parapumping operation: the "recuperative" thermal mode and the "direct" thermal mode. (See Figure 1.)

In the recuperative mode, the fluid flows back and forth through the bed and carries thermal energy into and out of the bed. The solid particles within the column come in contact alternately with fluid coming from the hot end and with fluid from the cold end. Thus, each particle experiences an alternating temperature field. The adsorption-desorption cycle produced by this field causes the alternating mass flux between particle and liquid phases (Rolke and Wilhelm, 1969). The face angle between the temperature, which varies with axial position within the column, limits the separation obtainable from this mode of operation.

The direct thermal mode was then developed to overcome the phase angle difficulty. Since the column is surrounded by heating and cooling jacket, temperatures can be changed uniformly throughout the bed. In both thermal modes, separation arises from the coupling of adsorption-desorption cycles with the reciprocating flow.

Many investigators have used different types of theoretical analysis in predicting the separations that can be obtained by batch process. Among these theories is the "equilibrium" theory promulgated by Pigford et al. (1969), and later generalized by Aris (1969). This equilibrium theory simplifies the transport equations by assuming that the solid and fluid are locally in equilibrium. Axial dispersion was neglected and the equilibrium relationship was assumed to be linear. The resulting equation after applying these assumptions is a hyperbolic partial differential equation solvable by method of characteristics. Pigford (1969) tested the validity of the analytical solution of the hyperbolic equation by applying a parameter curve fitting using Wilhelm and his co-worker's data (1968), but found no correlation. This lack of correlation, as one would suppose, is a result of the oversimplification of the transport equation obtained from the equilibrium theory. In any case, this paper (Pigford et al. 1969) served as the limelight behind the reason for separation. Pigford and his co-worker's paper (1969) raised considerable controversy amongst active and prominent investigators in this field because of difference of opinions.

Originally, Pigford et al. (1969) and Aris (1969) assumed that there was no dead volume in the reservoirs and that the reservoirs are perfectly mixed. Gregory and Sweed (1970), and Chen and Hill (1971) extended the equilibrium theory by taking into account the reservoir dead volume. In their papers, both open and batch systems were considered. For the batch case, their results agreed very well with Aris (1969) in the limiting case where the reservoirs have no dead volume. In the study made by Thompson and Bowen (1972), in which a perfectly mixed reservoir was assumed, it was theoretically shown that, with no mixing in the reservoir, a much greater separation could be achieved. A two-column arrangement operated back-to-back (to minimize mixing) was also discussed in this paper.

Butts et al. (1972) extended the equilibrium theory to the separations of multicomponent mixtures. In their study, a linear and noncompetitive equilibria was assumed; it was a batch parapump process with unsymmetric cycles (higher flow to one of the reservoirs). With such arrangement, the penetration distance into the column during each half cycle was different. A theory for separating multicomponent mixtures and a two-column arrangement, which could be used for isolating nth number of components, was also discussed. A batch process was probably not very suitable for multicomponent systems, since a large reservoir dead volume that would allow sufficient cycles for separation before the fluid was pumped to the bottom reservoir would be needed. A single column would be best suitable to separate a binary mixture or make a split within a series of solutes. Another method of approach was used by Butts et al. (1973) to separate a binary mixture of $K^+$ and $H^+$ on a Dowex 50 x 8 resin. In this process, the binary mixture was adjusted to build up the concentration of $K^+$ in the top reservoir and $H^+$ in the bottom reservoir. A high separation factor in the neighborhood of thousands were obtained. Butts and his co-workers noticed that electroneutrality must be preserved within the column so that separation of these ions would be possible, viz: when $K^+$ desorbs, $H^+$ must absorb. They also investigated the separation of ternary mixtures. In these experiments, $K^+$ ion was concentrated in the top reservoir, $H^+$ in the bottom reservoir, and $Na^+$ in the middle of the column. This paper brought to light the possibility of using a parametric pump to separate ionic mixtures.

Shortly after the promulgation of the equilibrium theory, Sweed and Wilhelm (1969) developed the STOP-GO algorithm. The STOP-GO method of solution is superior to that of characteristics in several respects. The number of ordinary differential equations to be solved are reduced by a factor of one-half. It also provided a very clear physical picture of what the calculations meant. The fluid is displaced a distance of one time step without interphase transfer (no axial mixing occurs). When the flow (GO) ceases, transfer between the phases begins (STOP). The computational STOP-GO algorithm is more realistic than the equilibrium model. The STOP-GO model was used to show that separation increased as a result of the following: (1) increase in mass transfer coefficient, (2) decrease in fluid velocity, (3) increase in cycle time, and (4) decrease in reservoir dead volume. The most intriguing part of this STOP-GO model is that it predicts that separation first increases exponentially and then levels off to some limiting value as the number of cycles increases. Recently, Grevillot and Tondeur (1976) studied equilibrium staged parametric pump with non-linear isotherms. One single equilibration step and discrete transfer was regarded as one-half cycle. Suggestive analogies similar to that of total reflux distillation were given.

## The Continuous Pump

Hung and Lee (1971) applied the equilibrium plate to parametric pumping. The model consisted of continuous contact columns in which the number of theoretical plates were adjusted to simulate experimental data. Unfortunately, this model did not agree with the equilibrium theory of Pigford et al. (1969). Wankat (1973) applied a two equilibrium stage theory for liquid-liquid extraction parapump (Figure 2) to test the hypothesis of Wilhelm et al. (1968), and Sweed and Wilhelm's (1969) (application of parapump to systems other than adsorption and ion exchange is possible).

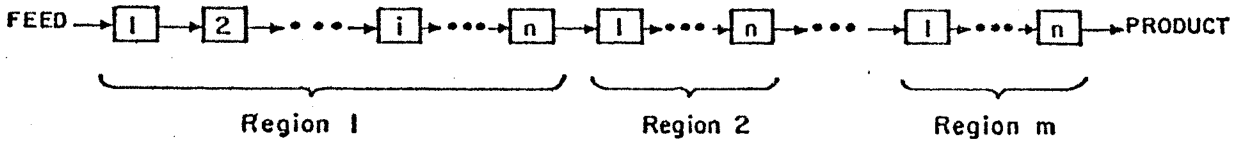


Fig. 2. Staged cycling zone extraction system (Wankat, 1973).

In this two equilibrium stage model, several methods for holding a liquid solvent stationary were presented. The system used was acetic acid and water using diethyl ether as the solvent in a horizontal helix and a test tube. This helical system consisted of a continuous-flow stage, the test modeled to fit a discrete transfer countercurrent distribution model similar to the process described in Craig and Craig's "Technique of Organic Chemistry," and the STOP-GO method (Sweed et al. in 1973). Comparatively, qualitive agreement was obtained, both experimentally and theoretically, but it was very poor. Hence, it was concluded that the application of the parapumping principle to extraction is inauspicious compared to adsorption and ion exchange.

In 1973, Gupta and Sweed used a mixing cell model to simulate non-equilibrium parapump. A "near equilibrium" approach to the analysis of parametric pumping, which is similar to the equilibrium staged model (Hung and Lee 1971, and Wankat 1973), was presented by the authors. The "near equilibrium" approach showed that significant deviations from equilibrium can exist and are, in fact, essential for improving conversion and concentration. In addition, the model requires considerably less computing time compared to other techniques, but it is rather very inaccurate. To obtain a good fit of experimental data by the mixing cell model, the number of cells has to be adjusted. A two-column paraphernalia for continuous separation of mixtures containing two solutes were also presented.

Application of parametric pumping process to the separation of liquids in open systems have been studied extensively, both continuously and semicontinuously, in the direct thermal mode. Sweed (1971) was the first to do a considerable amount of experimental work, while Horn and Lin (1969) were pioneers in presenting a theoretical calculation for such an open system. The experimental arrangement of Horn and Lin (1969) consisted of a two-column arrangement with a center feed, a center reservoir, and reservoirs at both ends of the column where products were withdrawn. The mathematical description of the apparatus was rigorous. The theme of Horn and Lin (1969) was two-fold. Firstly, a single solute system was used in which it was mathematically shown that the solute can be concentrated at one end of the reservoir (the "enrichment problem"). Secondly, the mathematical analysis of a two-component system (where these two components are to be separated from each other, by assuming that the components would be soluble in a suitable carrier liquid) was also presented (the "split problem").

Chen and Hill (1971), and Gregory and Sweed (1970) introduced the concept of "penetration distance," which was defined as the distance a concentration front will move into the column during a half cycle. Relative to this "penetration distance" concept, Chen and Hill (1971) used the equilibrium theory to study batch pump and open systems, while Gregory and Sweed (1970) applied the equilibrium theory in the study of the semicontinuous system. The point of reversal of the cycles were determined in terms of this so-called penetration distance. In addition to the theoretical paper by Chen and Hill (1971), Chen and his co-workers (1972, 1973a, 1973b) made extensive investigations, both experimentally and theoretically, on open systems.

Chen, Rak, Stokes, and Hill (1972) studied the continuous parapump operation experimentally with feed introduced at the top. The system used for this continuous parapump was toluene-n-heptane on silica gel. For this system, transfer rates were very high and compared very well with the equilibrium theory. A separation factor of over 600 was obtained for only 14 cycles in the region predicted by the equilibrium theory. In 1972, Chen, Jaferi and Stokes studied the multicomponent separation of fructose and glucose from water using fullers earth, and equilibrium isotherms of Langmuir type was obtained. The transfer rates for this system were very slow, requiring more than 2 hours to reach equilibrium. A semicontinuous parametric pump with top feed was studied experimentally by Chen, Reiss, Stokes, and Hill (1973). The system was toluene-n-heptane-silica gel and the results were compared with the continuous pump with top feed (Chen, et al. 1972). The bottom product concentration did not agree so well as that obtained previously. In 1973, Chen, Stokes and Lin studied another continuous parapump operation with top feed. This time the system was a mixture of toluene, aniline and n-heptane on silica gel. Equilibrium theory was applied by treating the multicomponent mixture as a series of pseudo-binary mixtures.

Thus far, the investigation of parapump operation by open, direct, and thermal mode has shown the possibility of obtaining continuous and semicontinuous separations. Large separation factors are readily obtainable when the shift of equilibrium with temperature is large and mass transfer is fast. Under these conditions, the equilibrium theory gives a good prediction, but if the mass transfer is slow, less separation is obtained and the equilibrium theory will not give a good prediction.

Shaffer and Hamrin (1975) combined affinity charomatography and pH parametric pumping to reduce trypsin concentration in an aqueous solution. Chen et al. (1977) experimentally investigated a semicontinuous pH parapump using haemoglobin and albumin on sephadex ion exchanger. The column had a center feed between an enriching column and a stripping column, and was operated batchwise during upflow and continuously during downflow. Chen et al. (1979) used a continuous pH parametric pumping for the separation of haemoglobin and albumin. The parapump had a feed, containing the protein mixture to be separated, introduced alternately to the column through the top and bottom of the column. The products were collected from the column during the top and bottom feed.

Stokes and Chen (1979) studied the design and scale-up of the separation of solutes from multicomponent systems on a commercial scale of a continuous thermal parapumping operation. The authors developed the design equations based on the analytical equations of dilute solution theory. Heat exchanger-type configuration to enable the authors to use direct thermal mode was adopted for the pilot plant and commercial systems. All the energy requirements for 0-100% solute separation were given, and auxiliary equipment was also included in this paper. It was shown that the energies required are of the same order of magnitude as that of distillation.

## Cycling Zone Adsorption

Cycling zone adsorption, first developed by Pigford and his co-workers (1969), is similar to parametric pumping, except that the fluid to be separated flows in a single direction through a series of columns. (See Figures 2 and 3.) The "direct or standing wave" mode of operation utilizes a water jacket which changes the temperature cyclically between the cold temperature and the hot temperature. The temperature of adjacent columns are out of phase with each other. In the first half of each cycle, the column sequence is hot, cold, hot, cold, etc., and in the second half of the cycle the column sequence is cold, hot, cold, hot, etc. Each region of constant temperature was regarded as a "zone" by Pigford et al. (1969). Another mode of operation is the "traveling wave." The

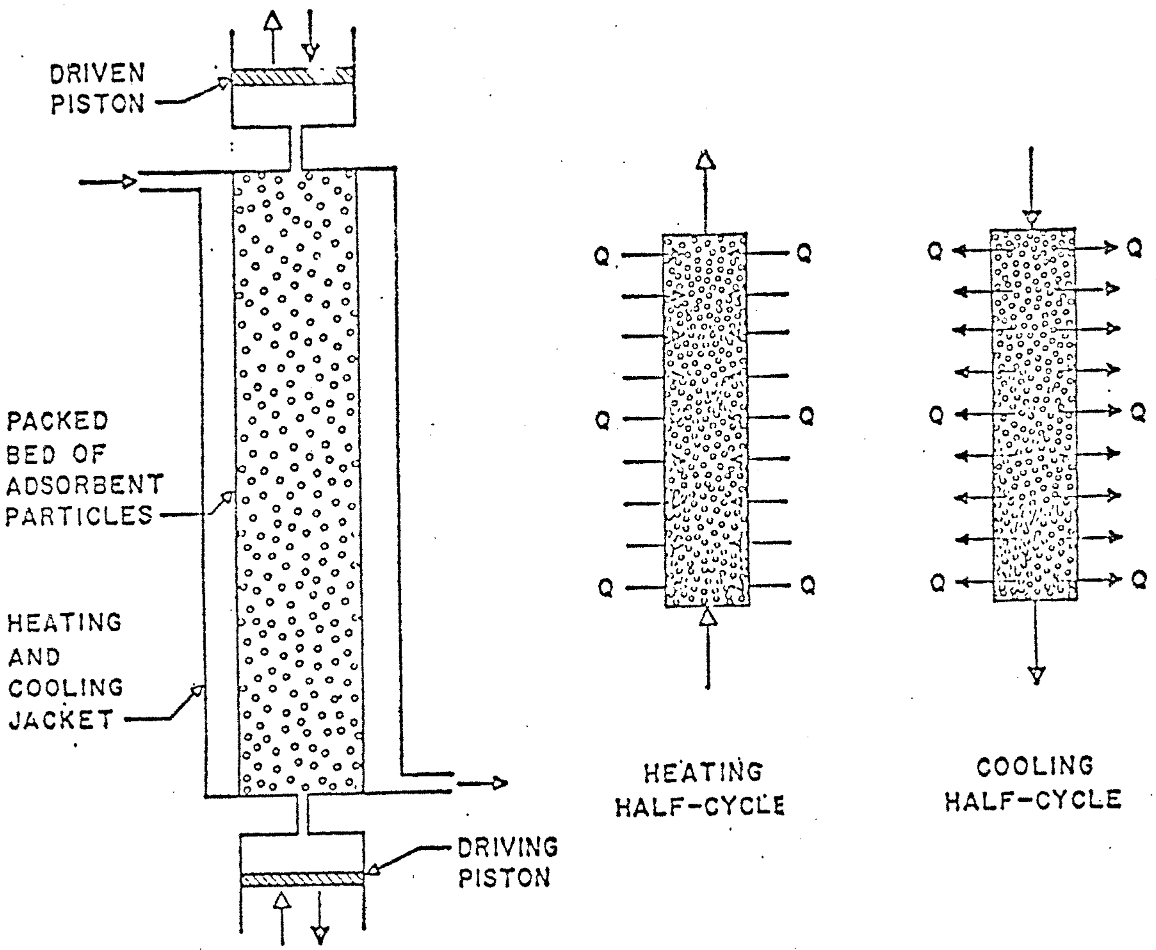


Fig. 3. Diagram of column for parametric pumping in direct mode (Pigford et al., 1969).

columns of the traveling wave are adiabatic and the fluid is alternately heated or cooled before entering the column. The temperatures are again out of phase in adjacent zones. In the standing wave mode of operation, experiment has shown that a two-zone system produced a larger separation than a single zone. It was also shown that a single-zone traveling wave separation was superior to the single-zone direct wave mode of separation. The reason for this difference in separation was attributed to the fact that the solid has better ability to remove solute from a fluid of low concentration by temporarily storing this solute and then give this solute up (when signal is given by increasing the temperature) to a fluid of high concentration. The author (Pigford et al. 1969) concluded that, since the use of several zones allow more opportunities to do this, the separation is enhanced when several zones are used.

Baker and Pigford (1969) did a detailed experimental and theoretical analysis on both the direct and traveling wave modes of operation. They also applied the local equilibrium theory by assuming that there is no axial dispersion and heat of adsorption as in parametric pumping. The solution of the resulting energy equation, after applying these assumptions, predicted that a temperature wave will pass through the column at a characteristic thermal wave velocity without changing shape or amplitude. The solution of the linear isotherm equation predicted that the separation factor would be infinite as the number of zones becomes very large, but for nonlinear isotherms, the separation would not be infinite.

Gupta and Sweed (1971) utilized the method of characteristics to develop an equilibrium theory for the direct wave of operation. They emphasized that the fluid displacement must be correctly chosen to get an increasing separation. Van der Vlist (1971) used the cycling zone adsorption to separate oxygen and nitrogen from air. A two-zone direct mode system was used and obtained a maximum oxygen separation factor of 10.6. Ginde and Chu (1972) used a mixed bed of ion exchange resins in a single-zone cycling zone adsorber to separate NaCl from water. This process was essentially an unsteady state cycling zone with total recycle, since products from the column were recycled until the desired separation was obtained. The parameters which affected separations were the amount of liquid in the system, the flow rate, and the cycle time.

In 1972, Rieke extensively studied the standing wave mode of cycling zone adsorbers with and without partial recycle. Experimental results showed that, for a single zone without recycle separating toluene from n-heptane on silica gel, optimization of separation could be achieved by switching temperature at an optimum frequency. Results showed that partial recycle of a product can increase the separation, but longitudinal mixing limited the amount of separation.

Wankat (1973, 1974) extended the cycling zone adsorption to extraction. (See Figure 2.) In both the direction and the traveling wave modes of operation, he used the counter-current distribution type system similar to that in Craig and Craig's "Technique of Organic Chemistry," with discrete transfer and equilibrium steps which were applied to keep one liquid phase stationary. The theoretical result obtained for the direct mode (Wankat, 1973) showed that if the cycle time is optimized, there would be an increase in separation from zone to zone. A qualitative agreement of experiment with theory was obtained when diethylamine-water-toluene in the test-tube system was used, but quantitative agreement was not so good. The theory similar to the direct mode (Wankat, 1973) was also derived for traveling wave (Wankat, 1974), but this time energy balance was added. Theoretical calculations were made, and the results showed great dependency of separation on the optimized thermal wave velocity. The separations obtained for traveling mode was lower than that obtained for direct mode because the thermal wave velocity was too low. Cycling zone separation process seems to be better for the separation of liquids than for large separations of gases. It also has an advantage over parametric pumping since it is inherently an open system and does not require flow reversal, but disadvantageous because it has one less degree of freedom.

This work was focused mainly on theoretical and computer simulation of various versions of pH-driven parametric pumps, viz: the batch parapump, the continuous parapump, and the cycling zone adsorption. A combination of STOP-GO algorithm and a simple equilibrium theory (Pigford et al., 1969; Jenczewski and Meyers, 1970; Wankat, 1974; Grevillot and Tondeur, 1976) and various performance characteristics were applied in predicting the protein separation.

# Chapter I

## PROCESS DESCRIPTION

Operational techniques were developed for continuous batch and cycling zone adsorption. In the simplest operational scheme, the separation was accomplished through dynamic adsorption of the mixture using a fixed bed. At all times, adsorption was followed by desorption of the mixture, provided the intensive variable (pH) is properly imposed on the column of ion exchange resins to effect the equilibrium distribution of the components being separated.

### Batch and Continuous Parapumping System

The one-column parametric pumping system in this study applies the dynamic adsorption principles for separation of components from a mixture (see Figure 4). It consists of a column packed with an ion exchanger (cation or anion) and reservoirs attached to each end. Axial displacement of the fluid in the column imposes a pH difference within the column which can be adjusted periodically to high and low. The adsorption resin adsorbs solute when the pH is low and releases solute when the pH is high.

In this recuperative pH mode, the reservoirs are maintained at two different pH values. As the solution oscillates through the bed, the particles of the resin

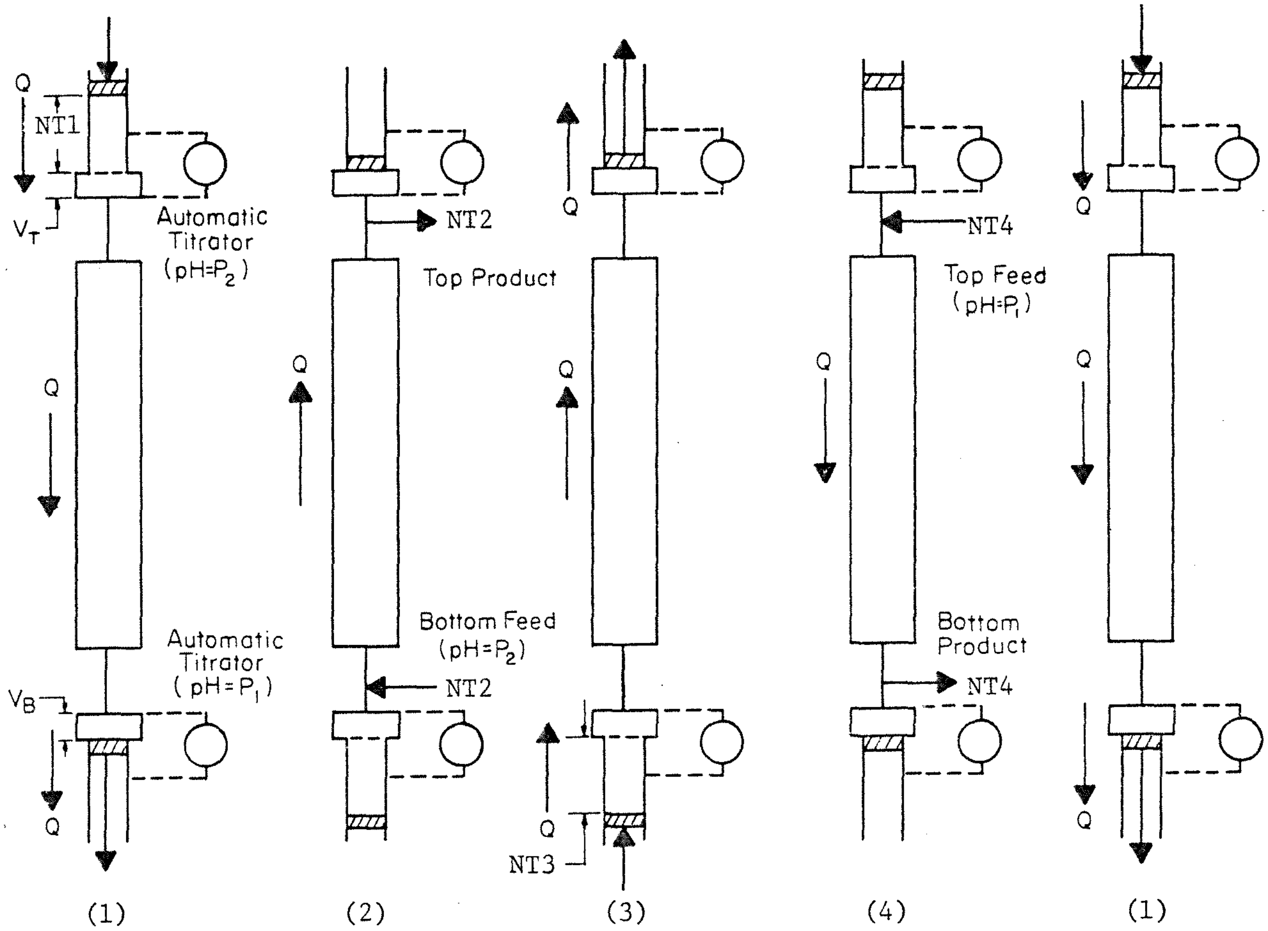


Fig. 4. Column diagram for continuous pH parametric pumping.

experience a periodic change in pH. During the first half-cycle, the fluid with low pH in the top reservoir is pumped into the top of the column. At the same time, the solution that emerges from the column fills the bottom reservoir. On the second half-cycle, the solution with high pH in the bottom reservoir flows back to the column. At the end of this half-cycle, the top reservoir is filled with a solution that comes forth from the top of the column, and one cycle is completed.

The top and bottom reservoirs of the parametric pumping system have dead volumes of $V_T$ and $V_B$ respectively. The top reservoir is maintained at a low pH level by an automatic titrator, while a second titrator is used to keep the bottom reservoir at a high pH level. The buffer ionic strengths for solutions in both top and bottom reservoirs are kept at $IS_2$ and $IS_1$ respectively, by means of two hollow fiber dialyzers manufactured by Amicon.

The mixture to be separated with concentration of $y_o(=1)$ (normalized) fills the column voids, the top reservoir, and the bottom reservoir dead volume at the start of the process. Each cycle of operation consists of four steps for a continuous process and two steps for a batch process, and the sequence of operation is as follows:

1. Transfer down (push down): The fluid in the top reservoir is transferred to the column, and the fluid in the column is transferred to the bottom reservoir. Since the column was saturated with haemoglobin concentration of

$y_o$(=1), the bottom reservoir concentration for the first cycle is $y_o$(=1).

2. Equilibration at pH (=6): During this equilibration step, low pH feed is introduced from the bottom of the column, while the emergent fluid from the top of the column is collected as the lean product.

3. Transfer up (push up): The solution formerly adsorbed by the adsorbent is now desorbed and brought to the top reservoir, while the solution in the bottom reservoir is returned to the column.

4. Equilibration at pH (=8): High pH feed is introduced from the top of the column, and the emergent fluid from the bottom of the column is the rich product. This step ends the cycle of operation. For subsequent cycles, steps 1 to 4 are repeated until steady state product concentrations are achieved. (Note that for the batch process, steps 2 and 4 are omitted.)

The flow rate within the column is always identical to the reservoir displacement rate Q. The volumes of the bottom and top feeds ( NT2 and NT4) are not necessarily equal to those of top and bottom products. For both the up and down flow, the reservoirs have the same displacement; that is, NT1 = NT2.

Since proteins carry both negatively and positively charged groups, which can normally bound to anionic or cationic exchangers, the net charge is dependent on the pH level. At low pH, the net charge is positive; at high pH, it is negative. At the isoelectric point (no net charge), the proteins are not bound to any type of ion exchangers.

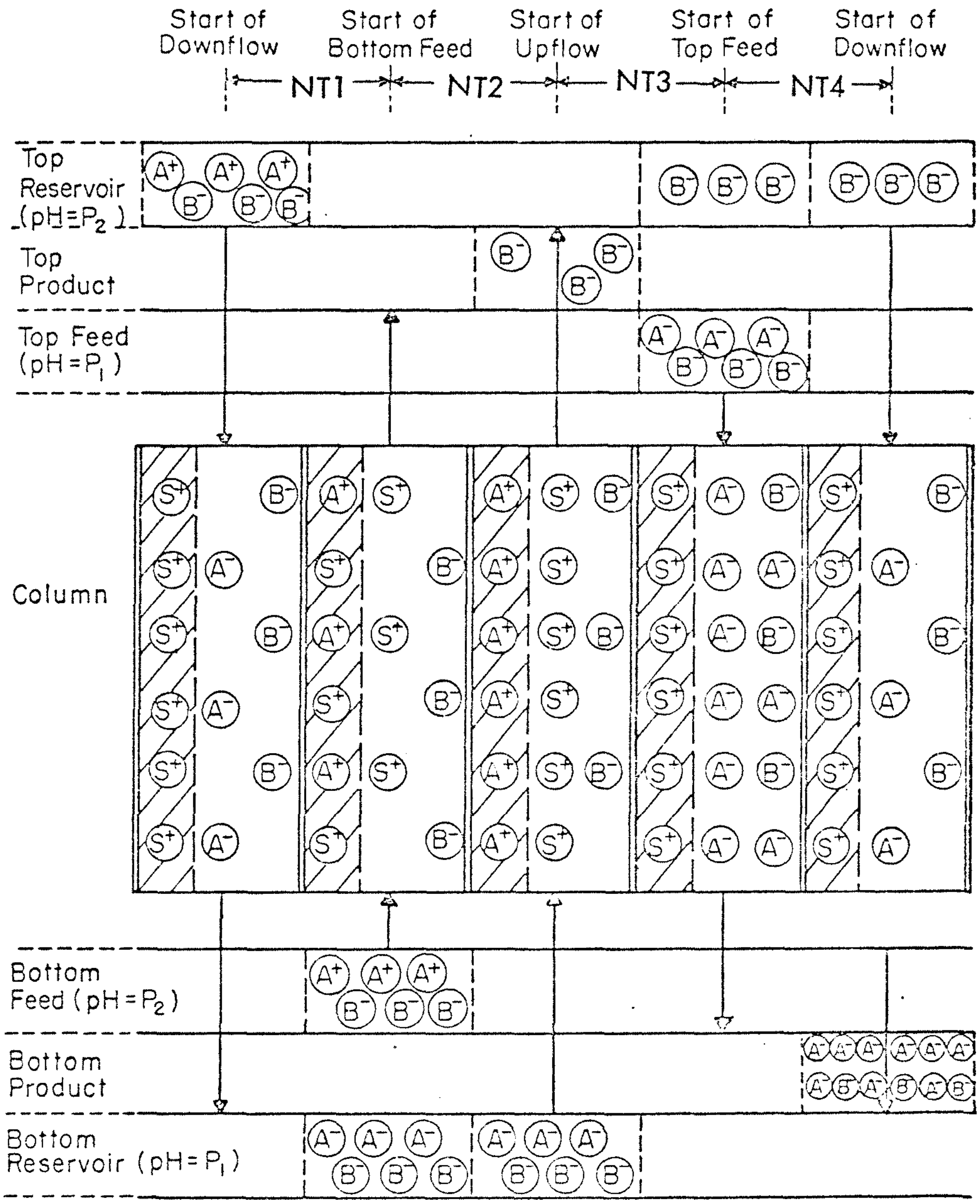
The schematic description of pH parametric pumping principle is shown in Figure 5. Suppose the protein mixtures to be separated contain species A and B with isoelectric points $I_A$ and $I_B$, whereas $I_A > I_B$. Two constant pH fields (that is, high and low pH, $P_1$, $P_2$) are imposed periodically on the systems, and $P_1 > I_A > P_2 > I_B$. The ion exchanger is assumed to be cationic with counter ions $S^+$. For illustrative purposes, assume the following (note that these assumptions are not necessarily true for the computational algorithm):

1. The displacement is equal to the void volume of the column $V_e$; that is, NT1 = NT3 = $V_e$.

2. The volume of either the top or the bottom feed is identical to $V_e$; that is, NT2 = NT4 = $V_e$.

3. The ionic exchange between the counter ions and the protein A is essentially complete at the end of each stage (Pharmacia Fine Chemicals, 1976).

At the start of the run, the void volume of the bed is filled with the high pH feed solution, and the top reservoir is filled with the solution containing a feed of pH = $P_2$. The net charges for A and B in the column are negative, and in the top reservoir are positive and negative, respectively. During the first downflow stage NT1 the low pH ($P_2$) fluid coming from the top reservoir enters the top of the column, while the solution emerged from the other end



A, B = substances with isoelectric points, $I_A$ and $I_B$ respectively,
$P_2 < I_A < P_1$ ; $I_B < P_2$ ; $P_1$ = high pH ; $P_2$ = low pH

$S^+$ = Counter ion

▨ = Cationic exchanger

Fig. 5. Schematic description of pH parametric pumping principle.

enters the bottom reservoir. The pH of the column is then changed from $P_1$ to $P_2$. As a result, $S^+$ is exchanged for the $A^+$ originated from the top reservoir. During NT2, a feed with pH = $P_2$ is introduced through the bottom, and simultaneously a top product containing only pure B is removed from the column at the same rate. In addition, the $A^+$ initially present in the bottom feed is exchanged for $S^+$. After this adsorption process, an upflow is followed, and the high pH enters the bottom of the column. The solution containing pure B flows out of the column to the top reservoir. Consequently, the pH in the column changes from $P_2$ to $P_1$, and desorption of A occurs. $S^+$ shifts back to the bed, and the ion exchanger is then regenerated. During NT4, a feed with pH = $P_1$ enters the top, while a product rich in A is withdrawn from the bottom of the column. One whole cycle is thus completed. All the solute A entering from either the top or bottom always moves toward the bottom product stream. Complete removal of A from the top product stream is achieved with one single complete cycle. Note that this result is based on the assumptions made above. In practice, it may not be possible to implement the operating conditions that satisfy the required assumptions.

## Cycling Zone Adsorption

The cycling zone adsorption is similar to parametric pumping, except that the fluid to be separated does not flow back and forth through a single bed, but instead flows in a single direction through a series of stages in the column. This process undertakes the pH recuperative mode of operation. The pH of the columns are changed cyclically between high pH (=8) and low pH (=6) (see Figure 6). In this figure, the column is represented as a series of equilibrium stages. Adjacent stages are out of phase with each other in that, when one stage has high pH (=8), the stages adjacent to it have low pH (=6).

During the first half of each cycle, the stage pH sequence is high, low, high, low, etc., and during the second half of the cycle, the stage pH sequence is low, high, low, high, etc. Each region of constant pH value is called a "zone;" hence, the name cycling zone adsorption. Various parameters such as number of stages and recycle ratios (B) were scrutinized in this study.

The product from the cycling zone adsorption is a function of time in that the pH and concentration of the product vary continually. However, a repeating state is reached where the product concentration repeats from cycle to cycle. The adsorption-desorption mechanism is similar to that of parametric pumping. When a zone is at high pH, the stationary phase rejects the solute and the moving phase becomes more concentrated. But, when a zone is at low pH, the equilibrium distribution coefficient changes and the stationary phase stores the solute. The separation is a function of the difference of the two pH levels. Therefore, for better separation, pH levels must be

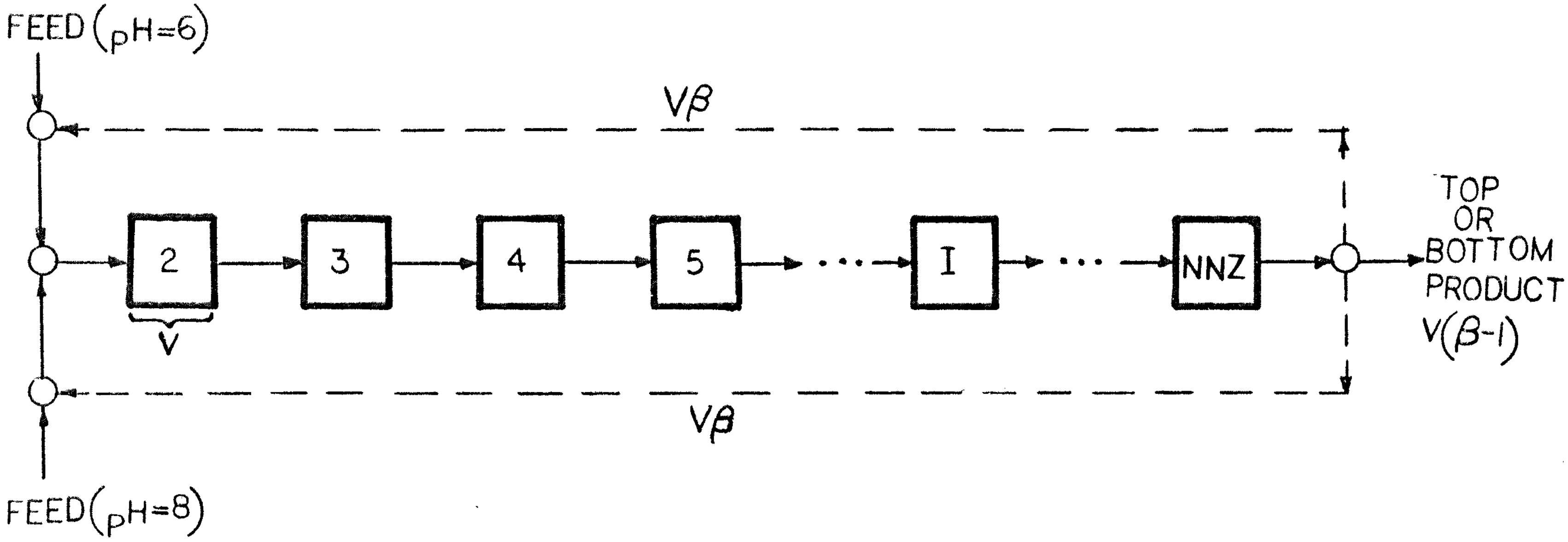


Fig. 6. Zone Diagram for pH Cycling Zone Adsorption with Recycle.

properly chosen to encompass the isoelectric points of the protein mixtures under consideration.

## Chapter II

## DERIVATION OF THE WORKING EQUATION

The working equation used for the simulation of the computer programming was derived by using two different methods of approach. The first method was the simplification of the general transport equation for a dilute binary system, and the second method called for a simple material balance of the extensive variables of the column.

To illustrate the first method of solution, consider the ion-exchange column with the initial conditions as shown in Figure 7.

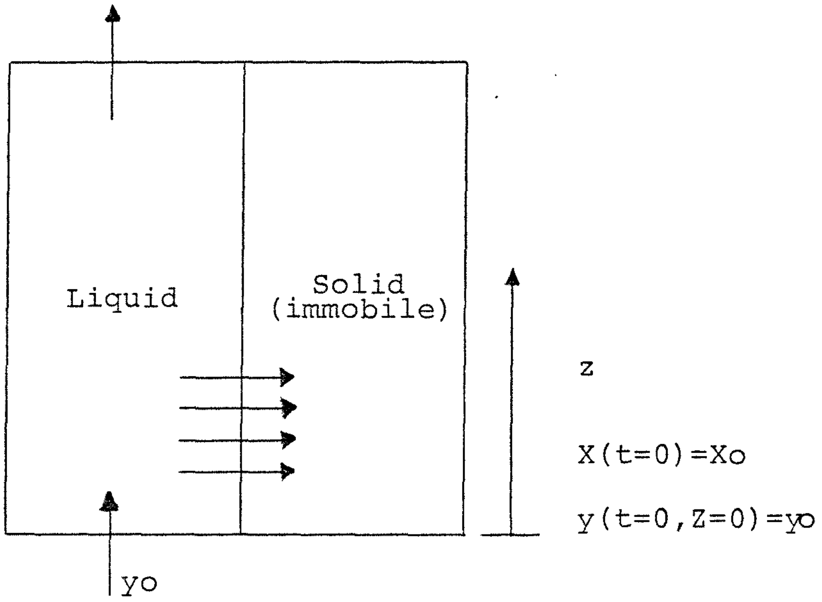


Fig. 7. Ion-exchange Column

Let y=moles of component A in the liquid phase per unit volume.

X = moles of component A in the solid phase (adsorbed on surfaces of course) per unit volume.

X = ky, the equilibrium relation for A between solid and liquid phases.

The transport equation for component A in the ion-exchange column (assuming dilute binary system) is:

$$\frac{\partial C_A}{\partial t} + Vx\,\frac{\partial C_A}{\partial x} + Vy\,\frac{\partial C_A}{\partial y} + Vz\,\frac{\partial C_A}{\partial z} \qquad (1)$$

$$= D_{AB}\left(\frac{\partial^2 C_A}{\partial x^2} \quad \frac{\partial^2 C_A}{\partial y^2} \quad \frac{\partial^2 C_A}{\partial z^2}\right) + R_A$$

Under the stratum of physiocochemical description, the principle of maximum gradient can be used for continuous flow system such as plug flow. This maximum gradient principle is a less detailed mode of description of flow equations, but it could be considered to be a simplified multiple-gradient model in which the dispersion terms are deleted, only one derivative retained in the bulk flow terms. Therefore, for a fixed bed adsorption column, only concentration gradients in the axial direction caused by the bulk flow are considered, and all radial gradients, dispersion, and the like are ignored. Applying the maximum gradient principle to equation 1, the following is obtained:

$$\frac{\partial C_A}{\partial t} \quad \frac{\partial}{\partial Z}(V_Z C_A) = R_A + m_A(t) \qquad (2)$$

Since there is no chemical reaction, $R_A = 0$ and $Vz$ = constant.

Therefore:

$$\frac{\partial C_A}{\partial t} + V_z \frac{\partial C_A}{\partial z} = m_A^{(t)} \qquad (3)$$

The interphase mass transfer term, $m_A^{(t)}$, for the fluid phase is identical to the mass transferred from the solid phase, and due to the sign convention for $m_A^{(t)}$,

$$m_{A,1}^{(t)} = -m_{A,s}^{(t)} \qquad (4)$$

The interfacial mass-transfer coefficient for the process is

$$m_{A,1}^{(t)} = -k_{A,1}(pH)a(C_{A,1}-C_{Ai,s}) \qquad (5)$$

where a = interfacial area per unit volume.

$C_A,1$ = bulk liquid phase concentration of A

$C_{Ai}$ = interfacial concentration of A

Combination of equations 3 and 5 give

$$\frac{\partial C_A}{\partial t} + V_z \frac{\partial C_A}{\partial z} = -k(pH)a(C_{A,1}-C_{Ai,s}) \qquad (6)$$

Using the notations depicted in Figure 1, and rewriting equation 6 for both liquid and solid phase, we get

Liquid: $$\frac{\partial y}{\partial t} + V_z \frac{\partial y}{\partial z} = -k(pH)a(y-y_e^*) \qquad (7)$$

Solid: $$\frac{\partial x}{\partial t} = k(pH)a(y-y_e^*) \qquad (8)$$

Combination of equations 7 and 8 gives

$$\frac{\partial y}{\partial t} + \frac{\partial x}{\partial t} + \frac{q}{A}\frac{\partial y}{\partial z} = 0 \quad \text{where } V_z = \frac{q}{A} \qquad (9)$$

Euler's single-step integration scheme written backward, but truncated before the first difference, is

$$\frac{dy}{dz} = f(y,z)$$

Therefore,

$$\text{if } \frac{dy}{dz} \text{ is replaced by } \frac{y_i - y_{i-1}}{z_i - z_{i-1}}$$

a difference equation of the first order is obtained,

$$\frac{y_i - y_{i-1}}{z_i - z_{i-1}} = f(y_i, z_i) \tag{10}$$

Rearranging equation 10.

$$y(i,j-1)=y(i-j)+f[y(i,j-1)]z(i,j-1)-z(i-1,j-1) \tag{11}$$

In general, difference equations does not require even spacing of the pivotal points, but assuming that equation 11 has evenly spaced pivot, $Z(i,j-1)-Z(i-1,j-1)$ is replaced by $\Delta z$ and we get

$$y(i,j-1) = y(i-1,j-1 + \Delta z f[y(i,j-1)] \tag{12}$$

upon rearrangement,

$$\frac{y(i,j-1)-y(i-1,j-1)}{\Delta z} = f[y(i,j-1)] = \frac{dy}{dz}$$

Similarly,

$$\frac{y(i,j)-y(i,j-1)}{\Delta t} = \frac{dy}{dt} \text{ and } \frac{x(i,j)-x(i,j-1)}{\Delta t} = \frac{dx}{dt} \tag{13}$$

Substituting equation 13 into equation 9, we get

$$\frac{y(i,j)-y(i,j-1)}{\Delta t} + \frac{x(i,j)-x(i,j-1)}{\Delta t} + \frac{q}{A}\,\frac{y(i,j-1)-y(i-1,j-1)}{\Delta t} \tag{14}$$

Multiply equation 14 by $\Delta t$ and $A\Delta z$ to get

$$\begin{aligned} &V[y(i,j)-y(i,j-1)] + \bar{V}[x(i,j)-X(i,j)-1)] + \\ &V[y(i,j-1)-y(i-1,j-1)] = 0 \end{aligned} \tag{15}$$

Where $V = \Delta tq = A\Delta z$,

$V$ = Volume of component A in the fluid phase

and $\bar{V}$ = Volume of component A in the solid phase rearranging equation 15, we obtain

$$Vy(i,j)-Vy(i,j-1)+\bar{V}x(i,j)-\bar{V}x(i,j-1)=Vy(i-1,j-1)-Vy(i,j-1)$$

$$Vy(i,j)+Vx(i,j)=Vy(i-1,j-1)+Vx(i,j-1) \qquad (16)$$

Let x-ky be a linear equilibrium relation for A between solid and liquid phases.

Expressing this equilibrium relation in finite difference form, we get

$$x(i,j-1) = k(i,j-1)y(i,j-1) \text{ and } x(i,j) = k(i,j)y(i,j) \qquad (17)$$

Substituting equation 17 into equation 16, we get

$$\begin{aligned} Vy(i,j)+\bar{V}k(i,j)y(i,j) &= Vy(i-1,j-1)+\bar{V}k(i,j-1)y(i,j-1) \\ y(i,j)[V + \bar{V}k(i,j)] &= Vy(i-1,j-1)+\bar{V}k(i,j-1)y(i,j-1) \end{aligned} \qquad (18)$$

Upon rearrangement,

$$y(i,j) = \frac{Vy(i-1,j-1) + \bar{V}k(i,j-1)y(i,j-1)}{V + \bar{V}k(i,j)} \qquad (19)$$

The second method of derivation which involves a simple mass balance of the extensive variables can be obtained from Figure 8.

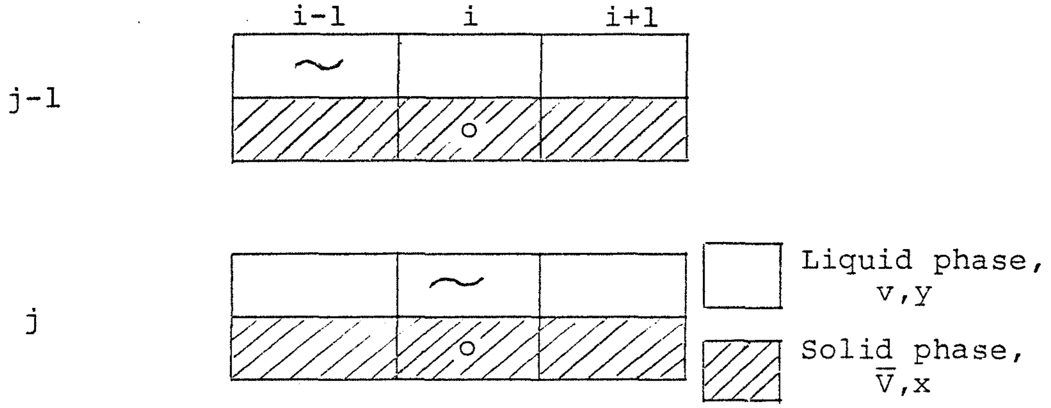


Fig. 8. Schematic of a One-Cell Model.

A mass balance for species A gives:

$$\frac{V[y(i,j)-y(i,j-1)]}{\Delta t} + \frac{\bar{V}[x(i,j)-x(i,j-1)]}{\Delta t} = q[y(i,j-1)-y(i-1,j-1)] \quad \ldots(20)$$

Since $\Delta tq=V$, combination of equation 20 and 17 gives equation 19, which was previously obtained via the transport equation. This equation is so general that it is good for batch, continuous parapump and cycling zone adsorption simulation. The inputs of the computer program simulation depends on the desired process, viz: batch or continuous, as the case may be, but for the cycling zone, a different algorithm is needed. The standard fortran lV language was used for the simulation.

As can be readily seen from equation 19, the concentration for the next transfer step can be solved in terms of the concentrations in the previous transfer step. Since the linear isotherm constant, k, is a function of pH, and the pH varies with the transfer step, it then becomes necessary to use the appropriate k(pH) value that corresponds to the pH of the transfer step under consideration. The isotherm slope depends on pH; therefore, the concentration is different for the two half-cycles. Usually, it is larger at the higher pH since k(pH) is smaller there. At low pH, the solute wave moves slow and it is held up by the stationary (solid) phase during the first half-cycle. During the second half-cycle, i.e., at the high pH, the solute wave tends to become more concentrated, since it moves faster. These two effects cause a separation with concentrated solute exiting when the pH is high.

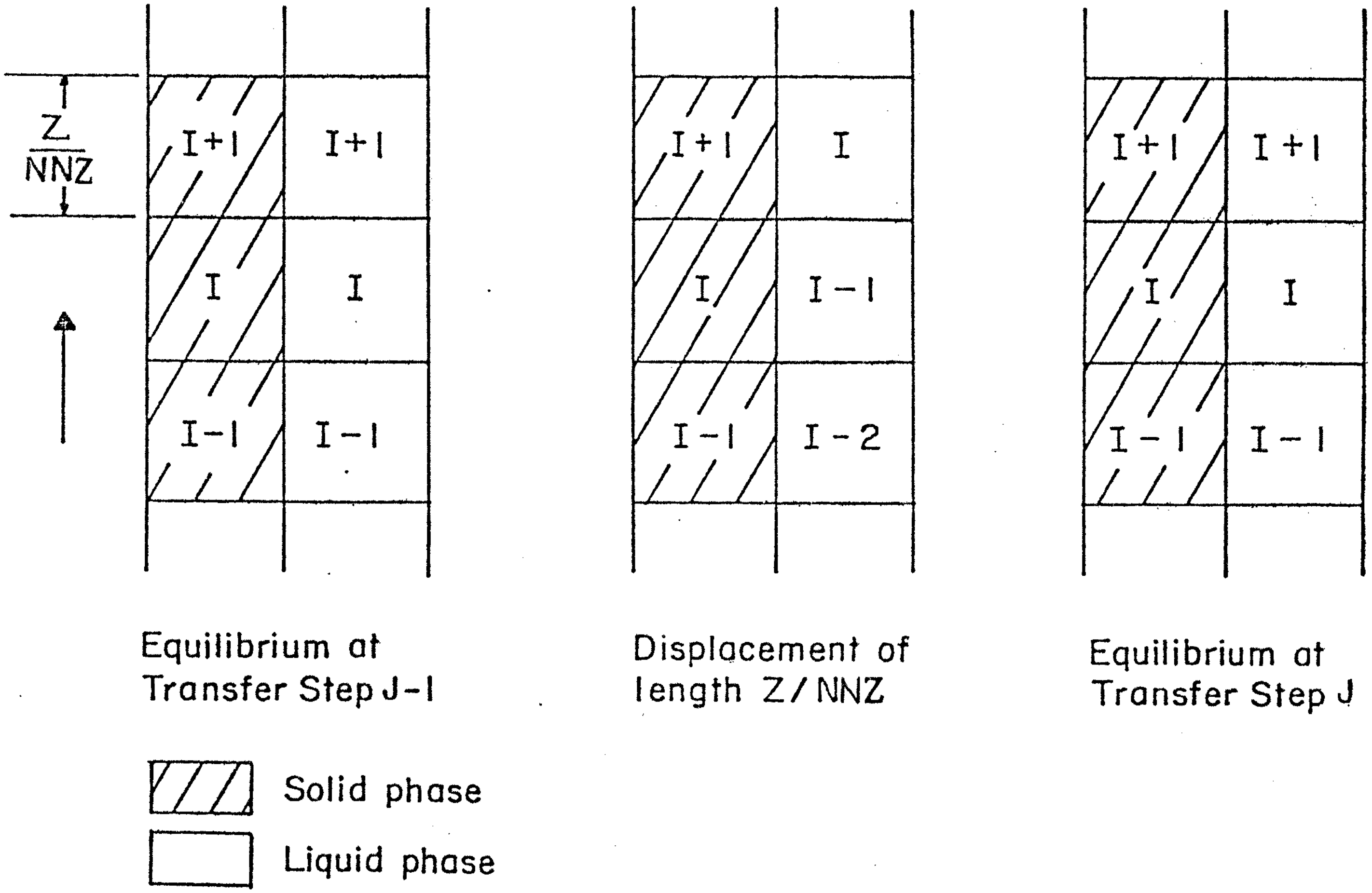
# Chapter III

# EQUILIBRIUM THEORY OF LINEAR ION-EXCHANGE ISOTHERMS

In the equilibrium theory for the linear isotherms used in this study, it is assumed that local equilibrium between liquid and sorbent in the layers of the separating medium in the column is attained. Deviations from local equilibrium can be accounted for in a semiempirical way by introducing the concept of "effective plates." In this discontinuous model, the solution in an effective plate of the bed attains equilibrium with the ion exchanger (or sorbent) before it moves on into the next plate. The effluent thus consists of a sequence of finite solution volumes, each of which is so large as to fill an effective plate. On their way through the column, these volumes are subjected to a series of equilibrations, one in each effective plate. These equilibrium theories substitute mixing in the plates for non-equilibrium as the cause of boundary spreading.

Theoretical calculations were based on batch, continuous flow through the column, and finite rates of ion-exchange or sorption. These calculations brought out the effect of the operating variables such as the number of effective plates or cells (for batch, continuous and cycling zone adsorption); rate of displacement for the batch process; rate of the feed from the bottom (or rate of the feed from the top); rate of product take-off for the continuous process; and the effect of recycle on the cycling zone adsorption. In any case, it has been shown both qualitatively and quantitatively that the column performance depends on these operating conditions.

The assumption of equilibrium theory is particularly very useful in simplifying the material balance made on the extensive variables of the column. The assumption of discrete transfer equilibrium stage model was used by Jenczewski and Meyer (1970); Wankat (1974); Grevillot and Tondeur (1976). The assumption of the equilibrium theory does enable us to have a clear insight of the pH-parametric pumping principle. The adsorbent bed is divided into N equal cells (plates or stages), each of length Z/NNZ, where Z is the length of the column, and each stage is represented as i, j. In this case, i will be the cell number and j, the transfer step. The schematic of this equilibrium theory is clearly depicted by Figure 9. Initially, the system is assumed to be in equilibrium at j-1, in which case each cell will have uniform concentrations in both the fluid and solid phases. If each fluid section is displaced exactly one step ahead in the transfer step, then the fluid y (i, j-1) originally opposite the solid section i will now be opposite i+1. After each transfer step, the operation is stopped, and all stages are allowed to reach equilibrium. Thereafter, equilibrium is immediately re-established and



Equilibrium at Transfer Step J-1

Displacement of length Z/NNZ

Equilibrium at Transfer Step J

Fig. 9. Schematic of Equilibrium Theory.

and the next transfer step (j) begins.

### Ion-Exchange Column

The framework of the ion-exchange is a positive or negative electric surplus charge, which is compensated by mobile counter ions of the opposite sign. Cation exchangers contain cations, and anion exchangers contain anions as counter ions. The counter ions can be exchanged for other ions of the same sign. Essentially, ion exchange is a diffusion process. The ion exchanger takes up certain counter ions in preference to others; i.e., it is selective. Ion-exchange equilibrium is attained when an ion exchanger is placed in an electrolyte solution containing a counter ion, which is different from that in the ion exchanger. Protein mixture is an amphoteric polyelectrolyte and can normally be bound to both anion and cation exchangers. Since they carry both negative (for example, carboxyl) and positive (for example, amino) groups, their net charge is dependent on pH. At low pH, the charge is positive; at high pH, it is negative. The point of zero charge is called the isoelectric point, and at this pH, the protein is not bound to either anion or cation exchangers.

At pH below their isoelectric points, proteins are bound to cation exchangers, and at pH above their isoelectric points, they are bound to anion exchangers. The binding is also dependent on the amount of charge carried by the protein. The greater the charge, the greater is the binding. The ionic strength of the environment (i.e., buffer concentration) has a pronounced influence on the binding. An increase in ionic strength of the eluant means increased competition for the binding sites of the ion exchanger; the electrical properties of the eluant are also changed, decreasing the electrostatic interaction between the ion exchanger and the counter ions. These combine to produce binding strength with the ion exchanger. The porosity of the ion exchanger does not influence the binding mechanism, but it influences the capacity, since much of the ion exchanger may be unavailable to large molecules.

The parameters which can be varied to carry out the fractionation of bound protein mixture are buffer pH and ionic strength. Towards the isoelectric point, a change in pH renders the protein mixture neutral, thereby causing it to be desorbed and eluted from the ion exchange. If the ionic strength is low, a minimum competition for the charged groups on the ion exchanger exists, but an increase of the ionic strength will increase the competition, thus reducing the interaction between the ion exchanger and the protein mixture.

The practicality of the elution process described above can easily be understood if one considers the following: suppose a protein mixture contains species A and B and that the corresponding isoelectric points are $I_A$ and $I_B$ respectively. Protein A can be separated from the rest of the protein species if $P_2 < I_A < P_1$, where $P_1$ and $P_2$ are the two constant pH fields ($P_1$=pH of the bottom reservoir and $P_2$=pH of the top reservoir). Species A will be negatively charged at $P_1$, and at $P_2$, it will be positively charged. Species A will be taken up by a cation exchanger, $R^-$ (with the counter ion $S^+$) at $P_2$ and released at $P_1$:

$$R^-S^+ + A^+ \longrightarrow R^-A^+ + S^+ \text{ at } P_2 \qquad (21)$$

$$R^-A^+ + S^+ \longrightarrow R^-S^+ + A^- \text{ at } P_1 \qquad (22)$$

The parapump with these pH levels will then remove species A from the mixture at the pH end of the column and concentrating it at the high pH end (see Figure 5).

# Chapter IV

# COMPUTATIONAL ALGORITHM

## Equilibrium Theory Cum STOP-GO Method

1. Divide the adsorbent bed into NNZ equal stages, each of length Z/NNZ and Z being the length of the bed.

2. Divide the time domain into NT increments. The time interval for introducing the feed from the bottom and product withdrawal from the top is NT2, and the time interval for introducing the feed from the top and product withdrawal from the bottom is NT4. The time interval for the downward fluid flow is NT1, and the time interval for the upward fluid flow is NT3. For continuous parapump process, NT2 and NT4 are greater than one; but for the batch parapump process, NT2=NT4=1 (i.e., no feed introduction or product withdrawal from either top or bottom of the column).

3. Initialize the fluid and solid compositions in the NNZ stages to some physically realizable values. Initial composition was assumed to be YINT=1 in this simulation. This initialization (equilibration) step is at j=1. (See Figure 10(a).)

The operational steps of the algorithm can now begin.

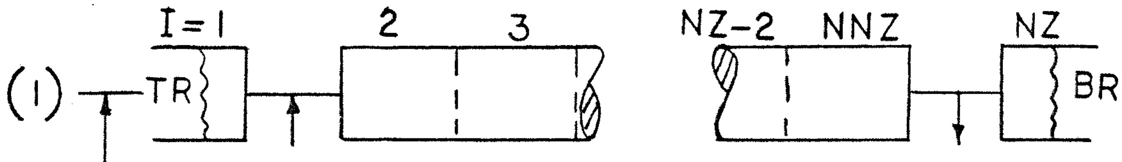
4. Push Down (Figure 10(b).): The time step NT1 for the downward flow of the fluid phase is divided into NNZ equal time increments of length NT1/NNZ. Each fluid section is now displaced one step ahead beginning at j=2 for

each time element NT1/NNZ. Equilibration is allowed to re-establish, the concentration profile Y in the column is determined and another displacement is made; this time at j=3. When j=NT1 time step is attained, the bottom reservoir concentration is calculated.

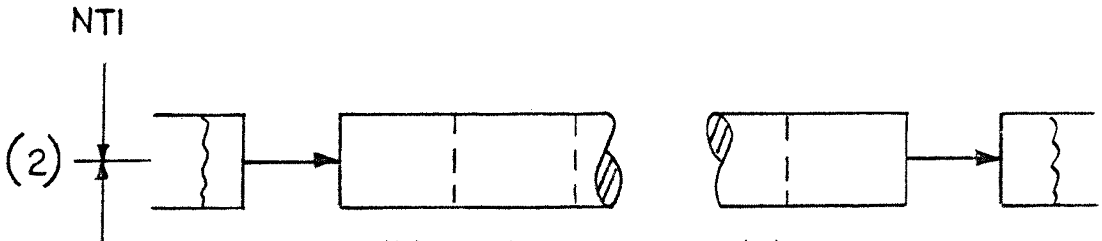
5. <u>Feed from Bottom and Product from Top (Figure 10(c).)</u>: The time step NT2 for the bottom feed and top product withdrawal is divided into NNZ equal time increments of length NT2/NNZ. A predetermined volume of feed (pH=6) equal to volume element NT2/NNZ is introduced beginning at j=2. Equilibration is re-established, the concentration profile YY in the column is determined, and the process is repeated until j=NT2, after which the top product concentration is calculated.

6. <u>Push Up (Figure 10(d).)</u>: The time step NT3 for the upward flow of the fluid phase is divided into NNZ equal time increments of length NT3/NNZ. Each fluid section is displaced one step ahead beginning at j=2 for each time element NT3/NNZ. Successive equilibration is allowed, and concentration profile YYY determined until a final displacement at j=NT3 is attained, after which the top reservoir concentration is calculated.

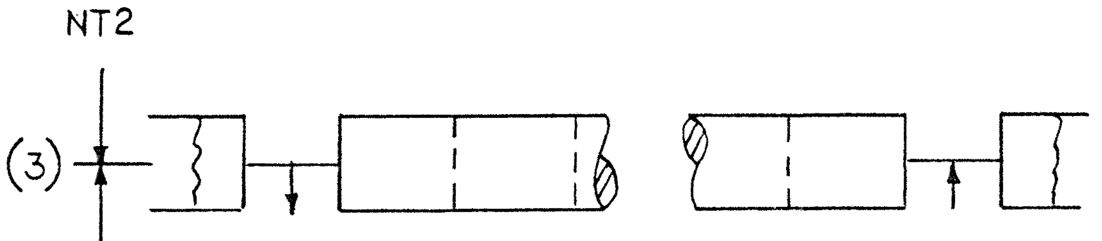
7. <u>Feed from Top and Product from Bottom (Figure 10(e).)</u>: The time step NT4 for the top feed and bottom product withdrawal is again divided into NNZ equal time increments of length NT4/NNZ. A predetermined volume of feed (pH=8) equal to volume element NT4/NNZ is introduced,



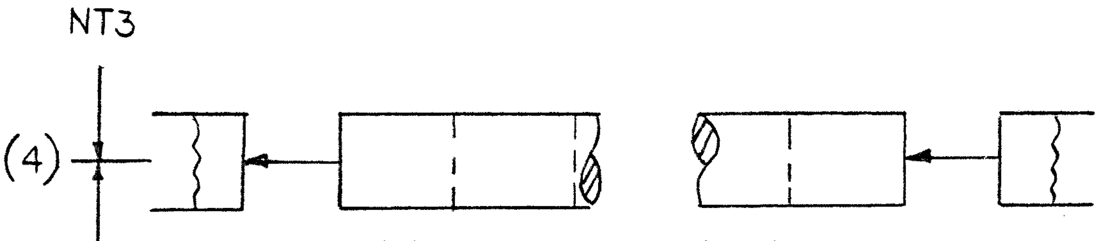
(a) Initialization Step



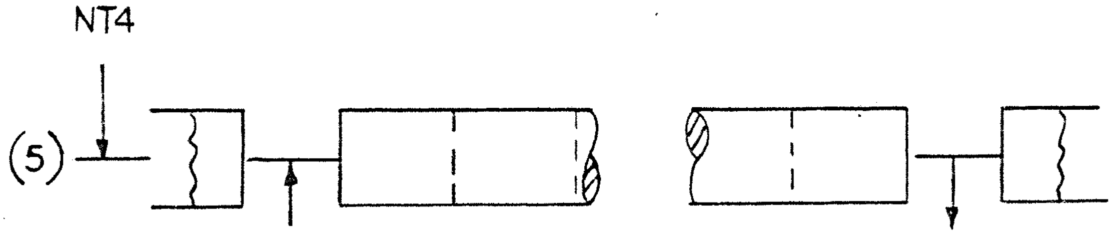
(b) Push Down Step (Y)



(c) Bottom Feed and Top Product Step (YY)



(d) Push Up Step (YYY)



(e) Top Feed and Bottom Product Step (YYYY)

Fig. 10. Diagram of Operational Steps used in Computer Simulation (batch and continuous).

beginning at j=2. Equilibration is re-established, the concentration profile YYYY determined, and the process repeated until j=NT4 is attained. The bottom product concentration is then calculated.

8. This sequence of operation ends the first cycle. For subsequent cycles, steps 4-7 have to be repeated.

Note that for batch parapumping calculation, steps 5 and 7 are omitted.

The simulation of this calculational algorithm assumes the following:

(a) That these NNZ increments (volume elements) are entirely independent of one another.

(b) That the volume elements represent batch reactors connected in series.

(c) That only partial equilibration between adjacent phases and full equilibration between opposite (solid and fluid) phases take place.

(d) That each volume element is treated individually for calculating concentrations.

Appendix A contains a listing of the FORTRAN IV digital computer program written to implement the EQUILIBRIUM THEORY CUM STOP-GO METHOD for the recuperative pH mode of parametric pumping.

### pH Driven Cycling-Zone Adsorption

1. Divide the adsorbent bed into NNZ equal stages each of Z/NNZ, where Z is a length of the bed. (See Figure 11.)

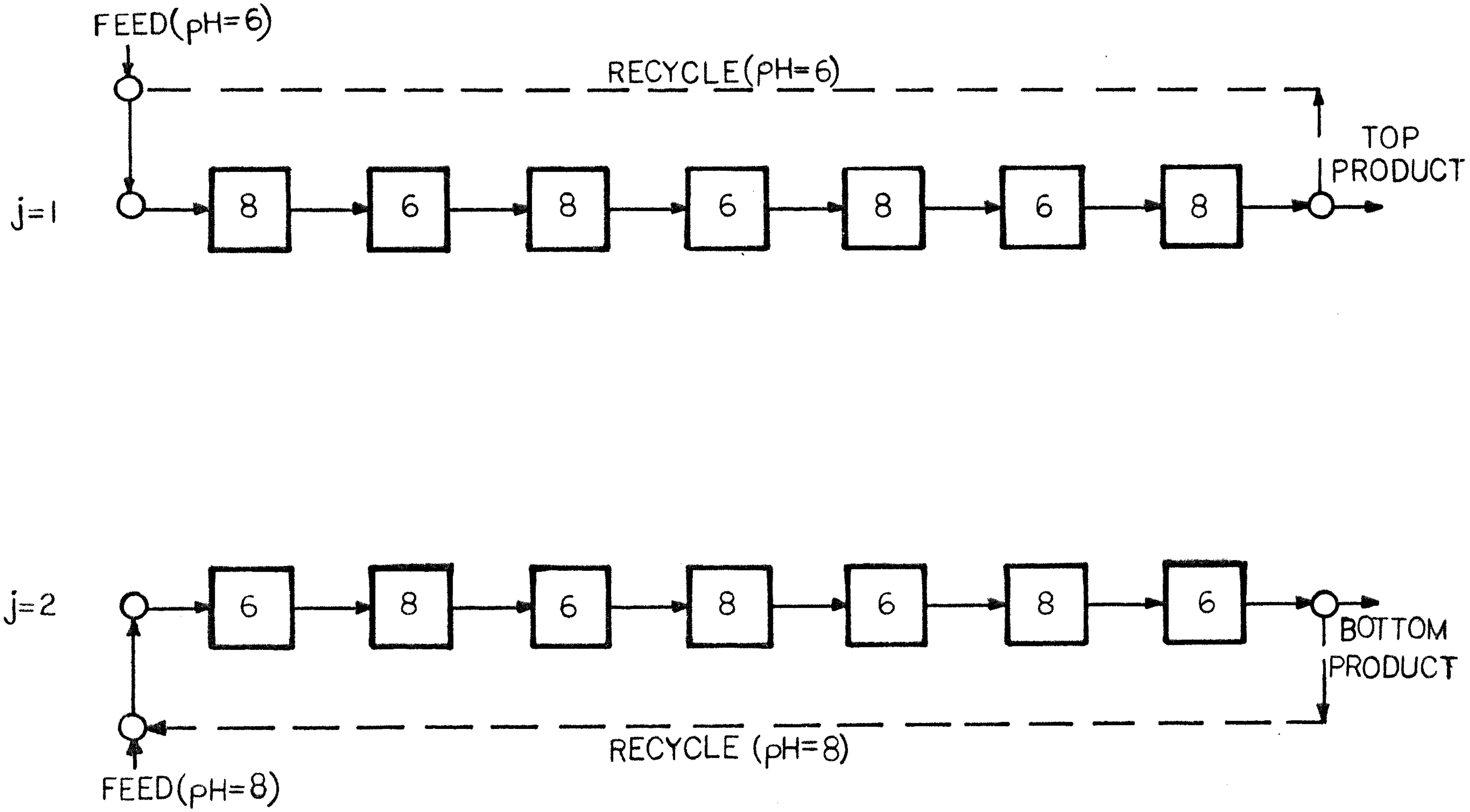


Fig. 11. Schematic Description of pH Cycling Zone Adsorption Principle.

2. Initialize the fluid and solid compositions in some fashion so that the pH of the NNZ stages vary alternately as a square-wave between high pH (=8) and low pH (=6). Initial composition was assumed equal for both high and low pH feed (YINT=1), while the starting cycle was taken as j=1.

3. Each fluid section is now displaced one step ahead with a volume Z/NNZ. This would mean that a high pH feed has been transferred to a stage originally occupied by flow of low pH, and this constitutes the first half cycle (j=2). Allow the equilibrium distribution coefficient to change so that the stationary phase now releases the solute. Bottom reservoir concentration can now be calculated.

4. Displace each of the fluid sections one step with a volume Z/NNZ. This time, the fluid of low pH is transferred to a stage originally occupied by fluid of high pH and the system is in the second half cycle (j=3). Equilibrium is again allowed to occur and the stationary phase would now store the solute. The top reservoir concentration is then calculated.

For subsequent cycles, steps 3-4 are repeated. These calculational steps assume no recycle (BETA=O). If recycle is assumed, the fresh feed is now used as a makeup solute to obtain a total displaceable volume of Z/NNZ. This algorithm is good for continuous separation with no recycle (BETA=O), and partial recycle (BETA< 1); and for batch separation; i.e., total recycle (BETA=1).

The following assumptions were made in the simulation of this algorithm:

(a) That the NNZ equal stages are independent of one another.

(b) That the volume element represent batch reactors connected in series.

(c) That partial equilibration between adjacent phases and full equilibration between moving and stationary phases take place.

(d) That each volume element is treated individually for calculating concentration values.

(e) That the pH wave is not affected by the solute concentration in the column.

(f) That the pH wave is time dependent.

(g) That the distribution between the stationary (solid) and mobile (fluid) phases obey linear equilibrium theory locally in each zone.

(h) That the cycle "halves" are equal (unequal halves could also be assumed).

Appendix B contains a listing of the FORTRAN IV digital computer program written to implement the calculational algorithm for the pH DRIVEN CYCLING ZONE ADSORPTION.

## Chapter V

# RESULTS AND DISCUSSION

### General

Computer solution of the mass balance equation (equation 19) was used to generate concentration curves for batch and continuous parametric pumps and cycling zone adsorption at various values of the operating parameters of interest in this study. These concentration curves give the variation in solute separation, i.e., top and bottom product concentrations for parametric pumping and cycling zone adsorption with number of cycles.

The major variables affecting the shape of the batch parametric pump concentration curves is $\alpha$, defined as the quotient of the reservoir displacement and the column void volume, while the concentration curves for the continuous was mostly affected by volume of top feed (NT4), and volume of bottom feed (NT2). It is also interesting to note that for large reservoir displacements, a large number of cycles (n) is needed to attain steady state. The cycling zone has a limited functional dependency on the number of stages (Nz) and recycle ratio ($\beta$). Above this limit, separation can no longer be improved upon, and below this limit, degrading separation is found to exist.

## Cycling Zone Adsorption

The conditions used for the simulation of the mass balance equation are summarized in Table 1. The theoretical results obtained for various number of stages and for different recycle ratio are shown in Tables 2 to 5. From these data, Figures 12 to 15, which show the average product concentration divided by the feed concentration for haemoglobin ($<y_B>_n/<y_T>_n$) are plotted against the number of cycles (n). Symmetric cycles have been used with 1, 4, 8 or 16 transfers per half cycle (stages). For the different stages used, a recycle ratio ($\beta$) of 0.0, 0.5, and 1.0 ($\beta$ = 0.0 means no recycle; $\beta$ = 0.5 partial recycle; and $\beta$ = 1.0, total recycle) were used. The results showed that recycle ratio only have some effect on the separation when the number of stages is small. But, when the number of stages becomes large, the effect of $\beta$ become negligibly small (see Figure 16).

Figure 12, which is the result of 16 stages, shows that recycle ratio has virtually no effect on the separation; and Figure 13, which is a plot for 8 stages, shows that the separation is equally as good as the separation obtained with 16 stages, but the top product has a small dependency on $\beta$. Figure 14 exhibits a plot for 4 stages, and it is found that separation is functionally dependent on $\beta$; the maximum separation obtained when $\beta$ = 1.0, and the minimum separation when $\beta$ = 0.0. Shown in Figure 15, is a plot of 1 stage versus n which gave inverse separation. This

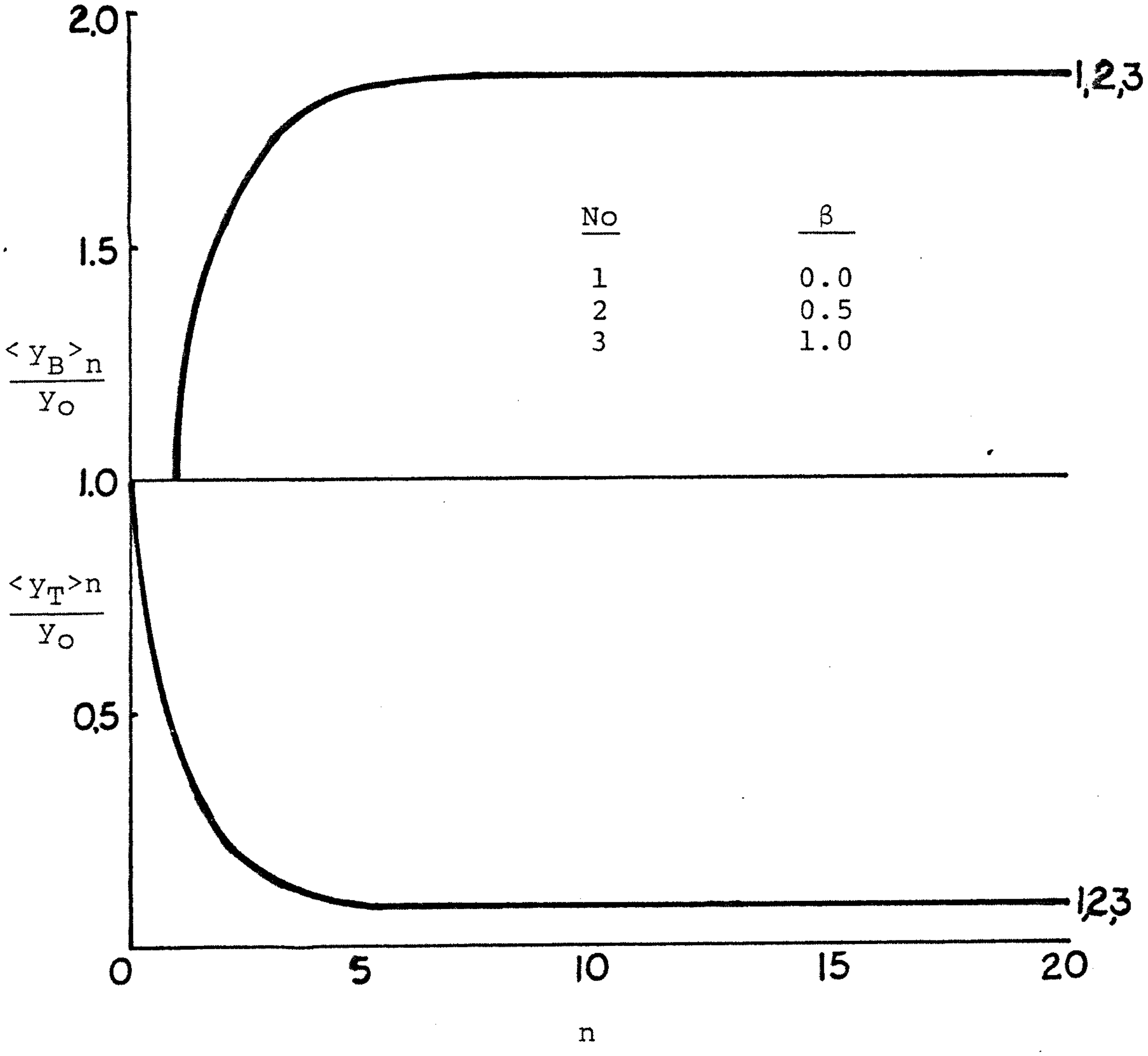


Fig. 12. Sixteen Stage Cycling Zone with Recycle.

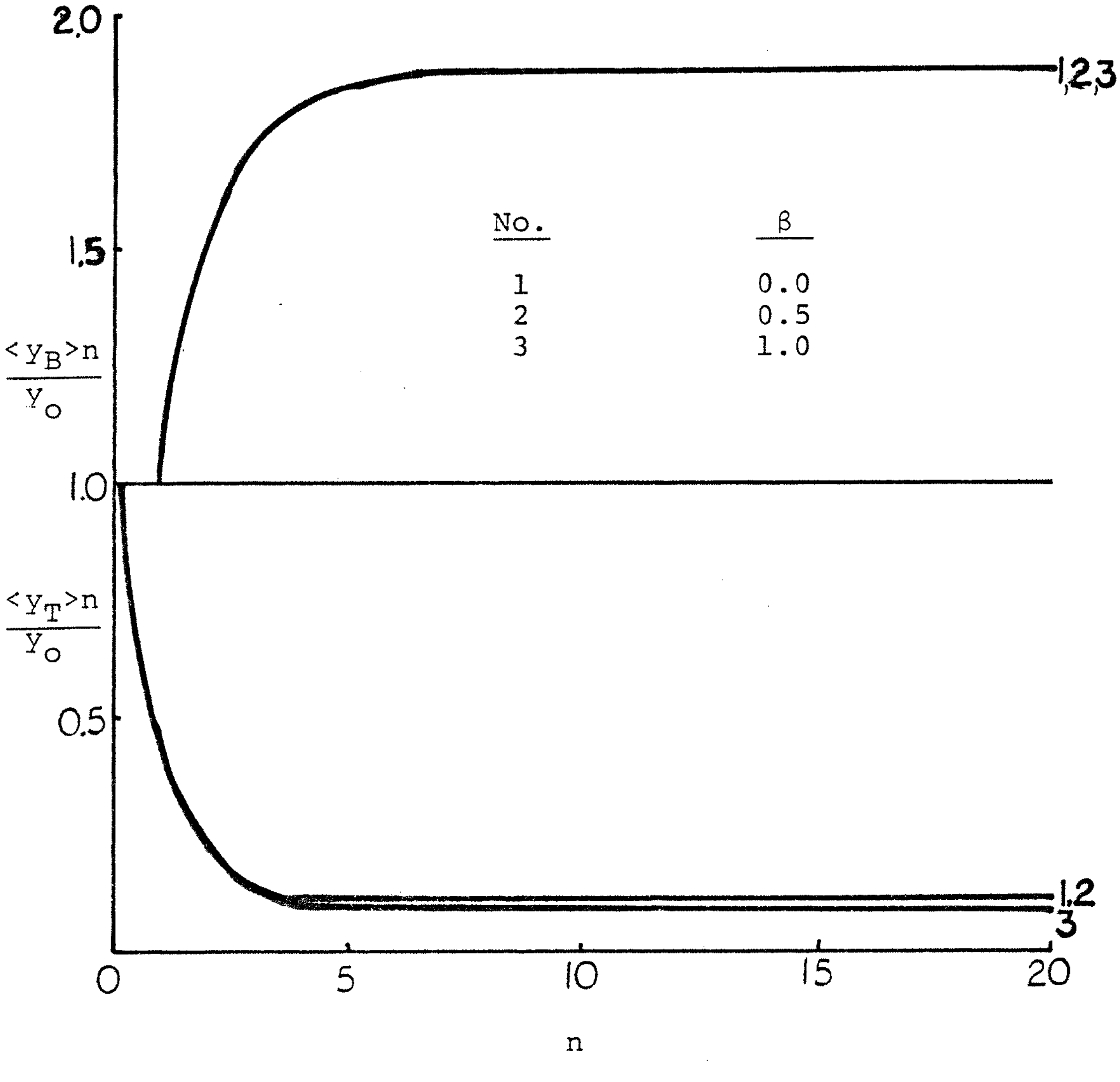


Fig. 13. Eight Stage Cycling Zone with Recycle.

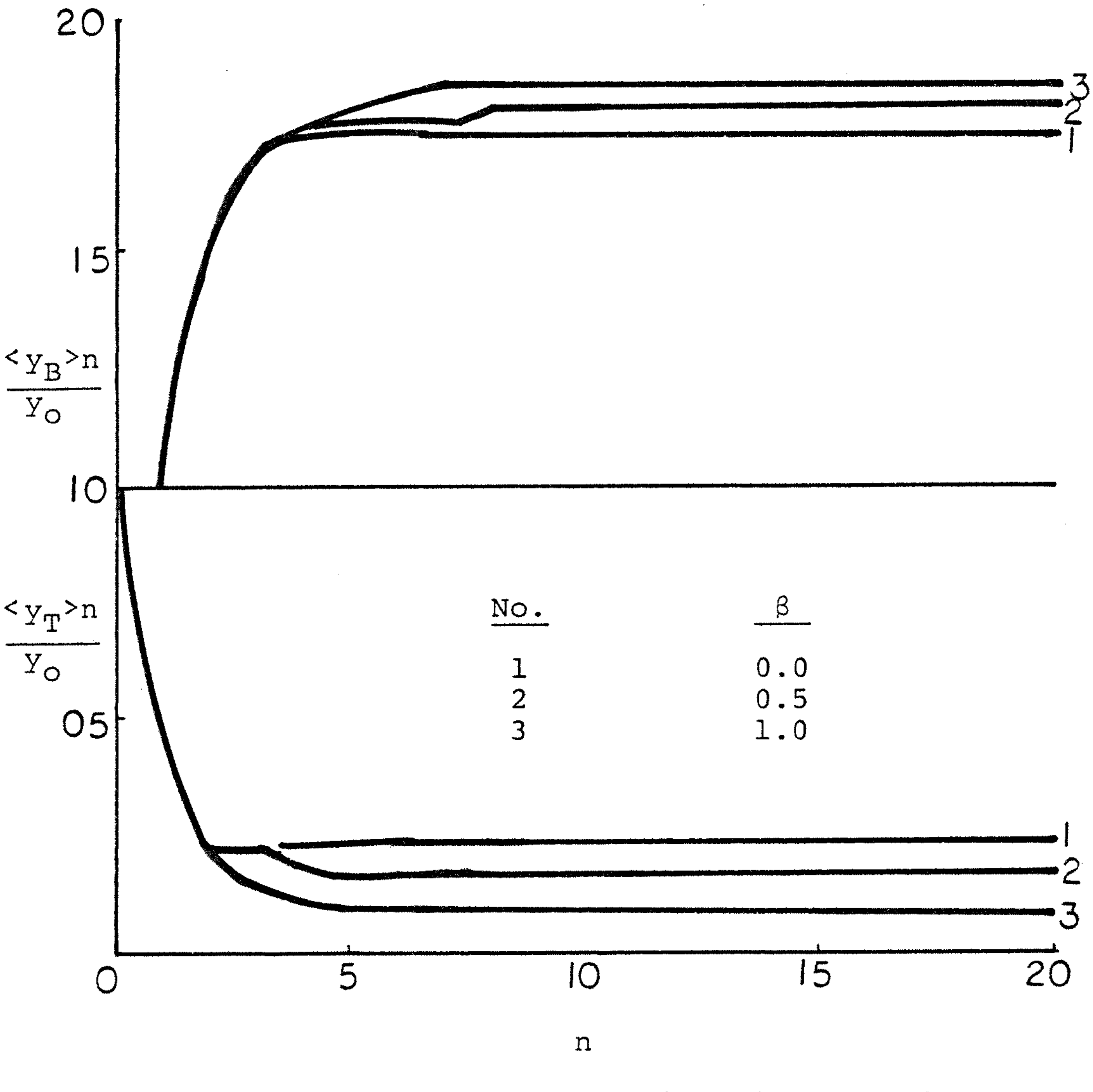


Fig. 14. Four Stage Cycling with Recycle.

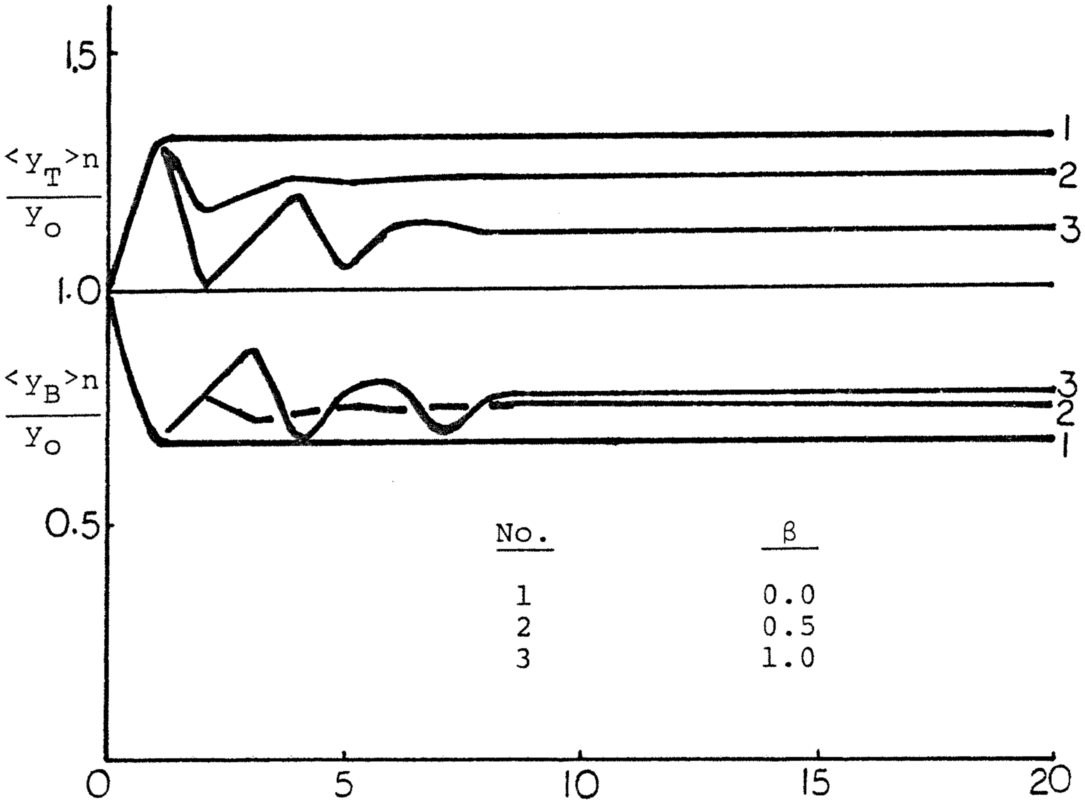


Fig. 15. One Stage Cycling Zone with Recycle.

phenomenon is consistent with previous reports by Wilhelm et al. (1966, 1968), Sweed and Rigaudeau (1975), and Chen et al. (1977) for small displacements. One can also see that separation is functionally dependent on β. Figure 16 is a plot of the separation factors ($<y_B>_\infty/<y_T>_\infty$) versus β (Table 6). β has the greatest effect on separation when the column is divided into 4 stages as the separation increases exponentially from $\beta = 0.0$ to $\beta = 1.0$. From this figure, the relative separation factors for all the simulated transfers per half cycle are easily discernable.

From the ongoing analysis, it can be seen that more separation would be obtained with a smaller number of theoretical stages if the adsorbent was operating near its saturation limit. With more stages, there is more adsorption between stages and more opportunities for equilibrium between the solid phase and the fluid phase. From the transient concentration plots, the values of the product concentrations approach a limit after few cycles. An increase in separation and decrease in the effect of β can be seen as the number of stages increases. However, as the number of stages becomes large, the effect of β on separation and the improvement on transient concentrations becomes non-existent.

## The Batch Parapump

The conditions for the simulation of the batch parametric pumping are given on Table 7, and calculated results obtained to demonstrate the effect of α on the transient

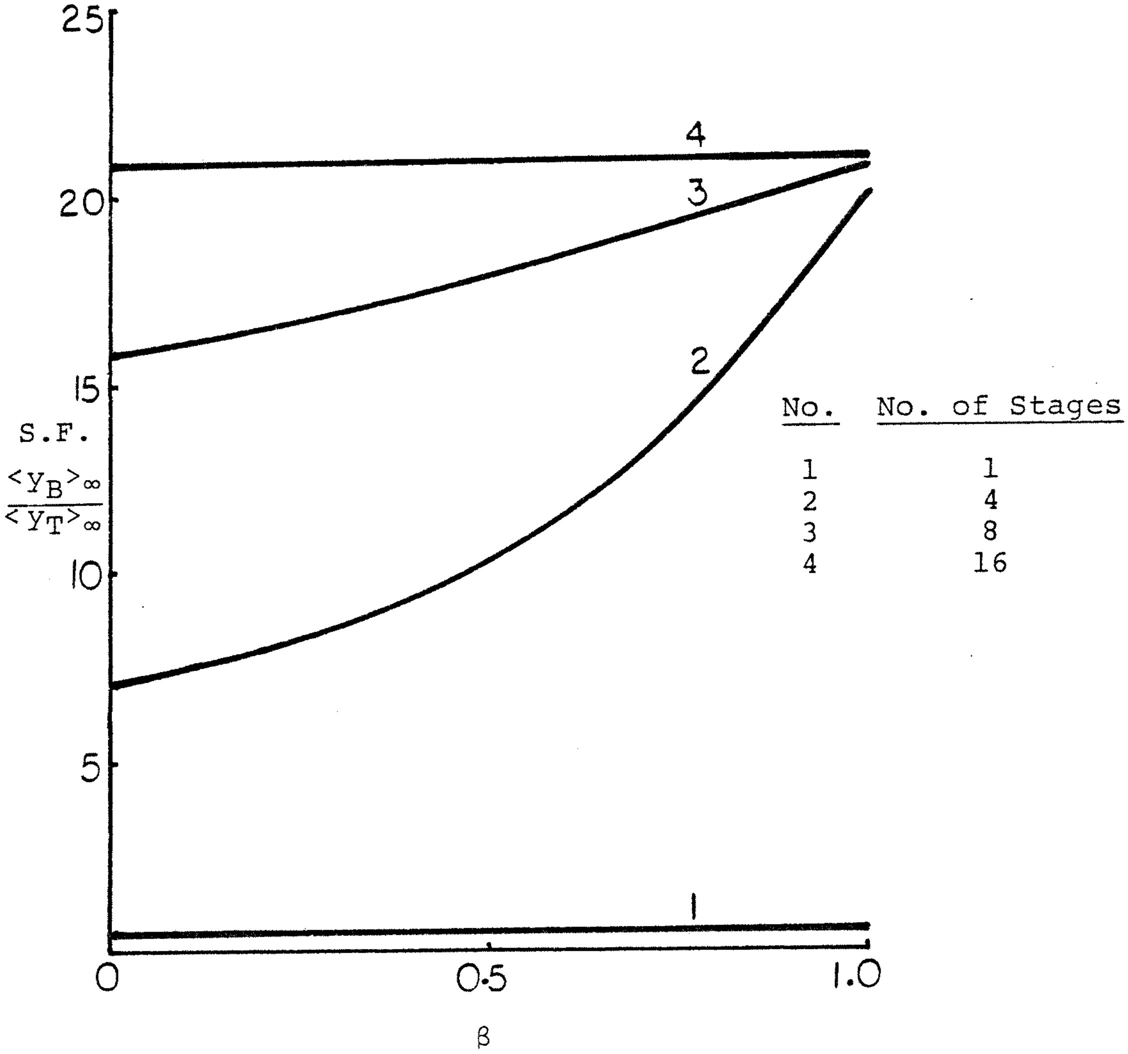


Fig. 16. Separation as a Function of Recycle Ratio (β).

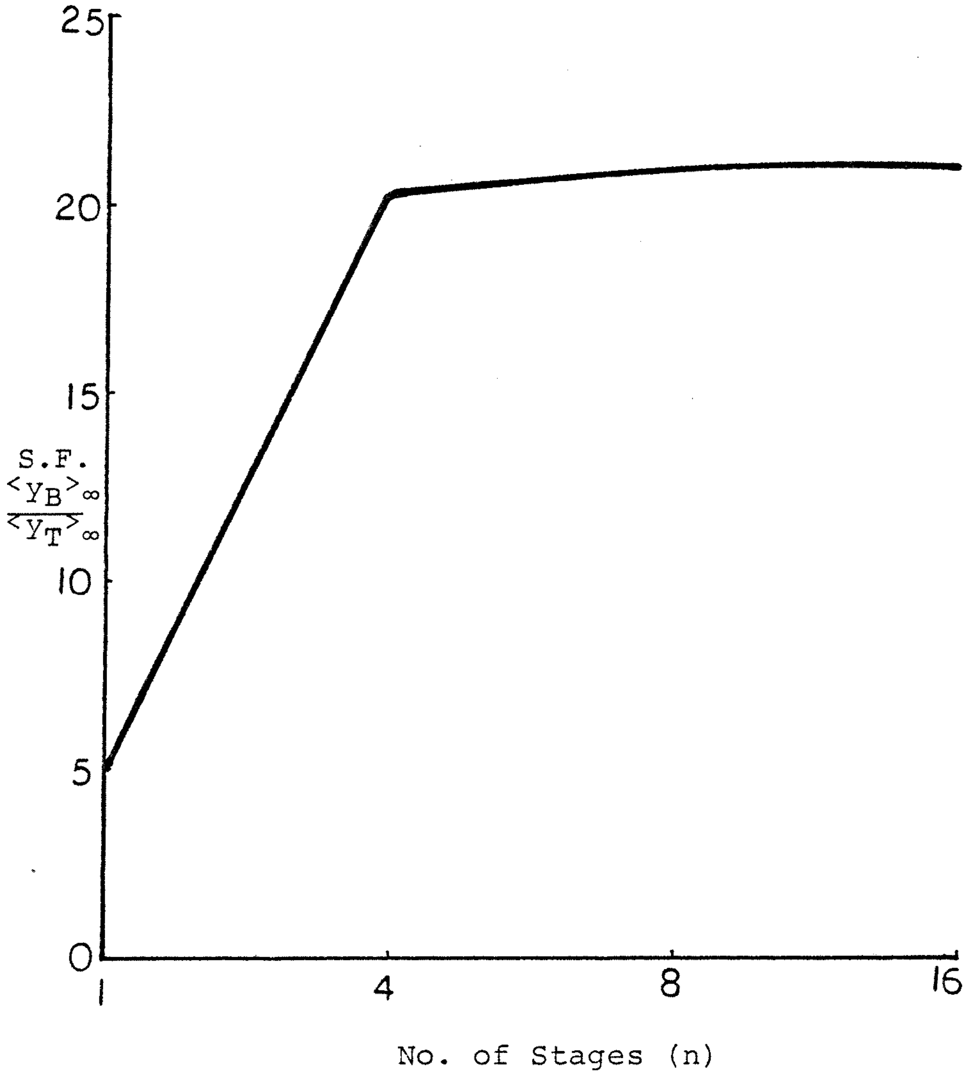


Fig. 17. Effect of Number of Stages on Separation for $\beta$=1.

| No. | No. of Stages | α |
|---|---|---|
| 1 | 4 | 0.5 |
| 2 | 1 | 1.0 |
| 3 | 8 | 1.0 |
| 4 | 4 | 2.0 |

—— Bottom Product
--- Top Product

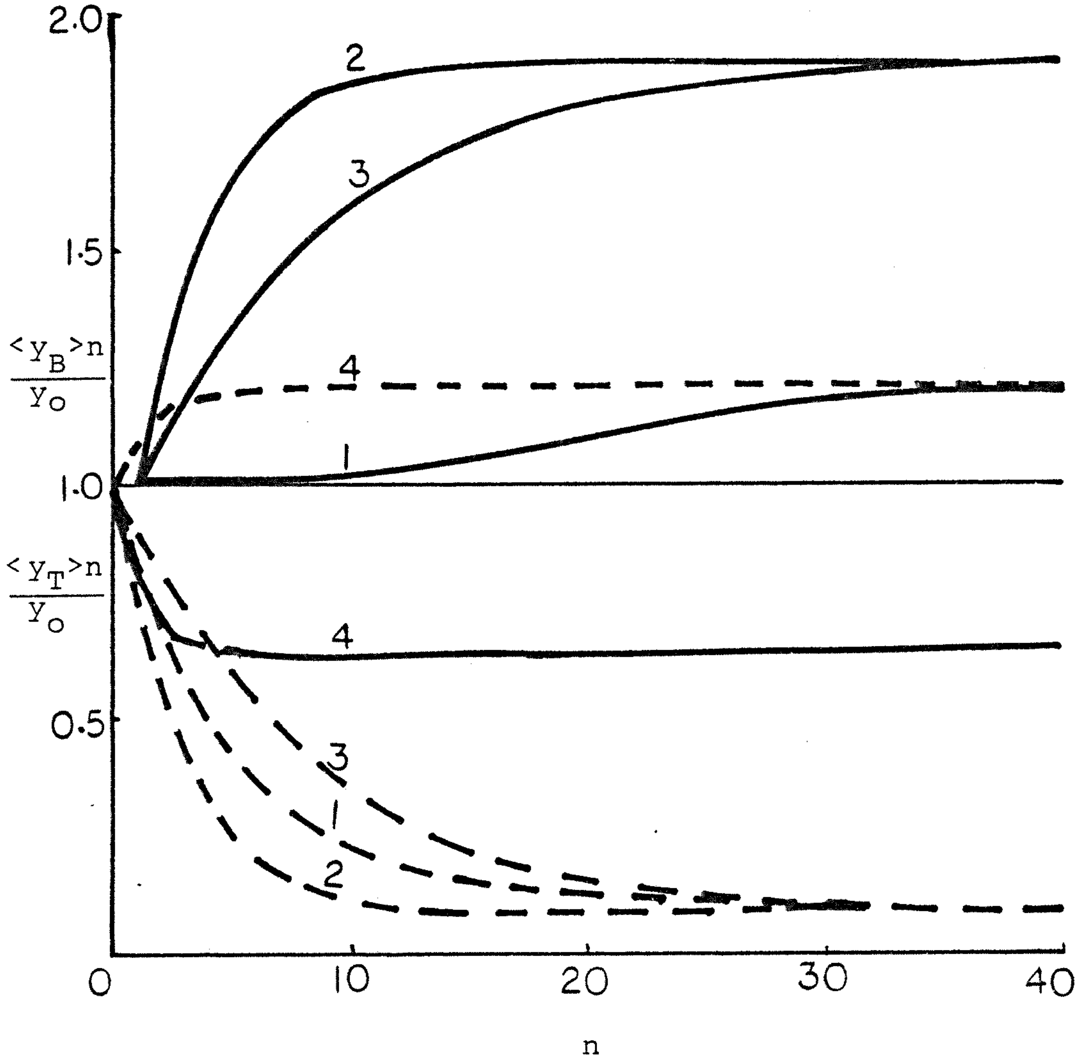


Fig. 18. Effect of α on Concentration Transients.

concentrations are shown on Tables 8 to 17. Following these data, the effect of $\alpha$ on concentration transients is shown in Figure 18. The ordinate is the average reservoir concentrations divided by the initial liquid phase concentration, while the abscissa is the number of cycles. For $\alpha = 1$, the separation first increases exponentially and then levels off to some limiting value as the number of cycles increases. $\alpha$ is defined as the quotient of the reservoir displacement and the column void volume (i.e., number of transfer steps/number of stages). This limiting value is the steady state concentration. The value is independent of N chosen. For $\alpha \lessgtr 1$, the steady state concentrations in both top and bottom reservoirs are functionally dependent on N chosen.

Figure 18 reveals three phenomenon, viz: Case I: $\alpha=1$ and $N = Q(\frac{\pi}{\omega})V$. In this case, the upward and downward flow fluid displacements corresponded to the column void volume, and maximum separation factor was obtained. The results obtained showed that the number of hypothetical cells or stages (N) chosen for the computational algorithm has no effect on the steady state average concentration values of the top and bottom reservoirs. In Case II: $\alpha < 1$ (the upward or downward fluid displacement is less than the column void volume), the number of cells (N) was found to have a marked influence on the separation. The steady state average concentration values of the top and bottom reservoirs were much less than those obtained with $\alpha = 1$.

The reason for this behavior is very clear, since the penetration distance is less than the column void volume, some of the solute are attached to the upper part of the column in upflow displacement and to the lower part of the column in the downflow displacement, thus resulting in a permanent loss of solute at steady state. Case III: $\alpha>1$ (the upward or downward fluid displacement is greater than the column void volume), some of the solute from the bottom reservoir is pushed to the top reservoir and vice versa. This resulted in inverse separation, which is consistent with the view that interface mass transfer is very slow, a basis on which equilibrium theory is based.

On Table 18, a summary of the effect of $\alpha$ on separation is given and plotted in Figure 19. The plot indicates that an increase in $\alpha$ will result in higher values of $\langle y_B \rangle_\infty / \langle y_T \rangle_\infty$ reaching 22:1 for $\alpha = 1$. Beyond that, the separation dropped sharply and becomes nonexistent because of intermixing between reservoirs. From this figure, it is obvious that the condition for maximum separation is $\alpha = 1$.

Figure 20 shows a one equilibrium stage recuperative pH batch parapumping system. From Figure 19, it is concluded that at $\alpha = 1$, the steady state separation is optimum and independent of the number of stages (N). Hence N=1 has been chosen to give a clearly describable diagram. In order to construct Figure 20, which is based on the mass balance equation (equation 19), two linear equilibrium isotherms with slopes $k_6=1.58$ and $k_8=0.07$ are drawn. These

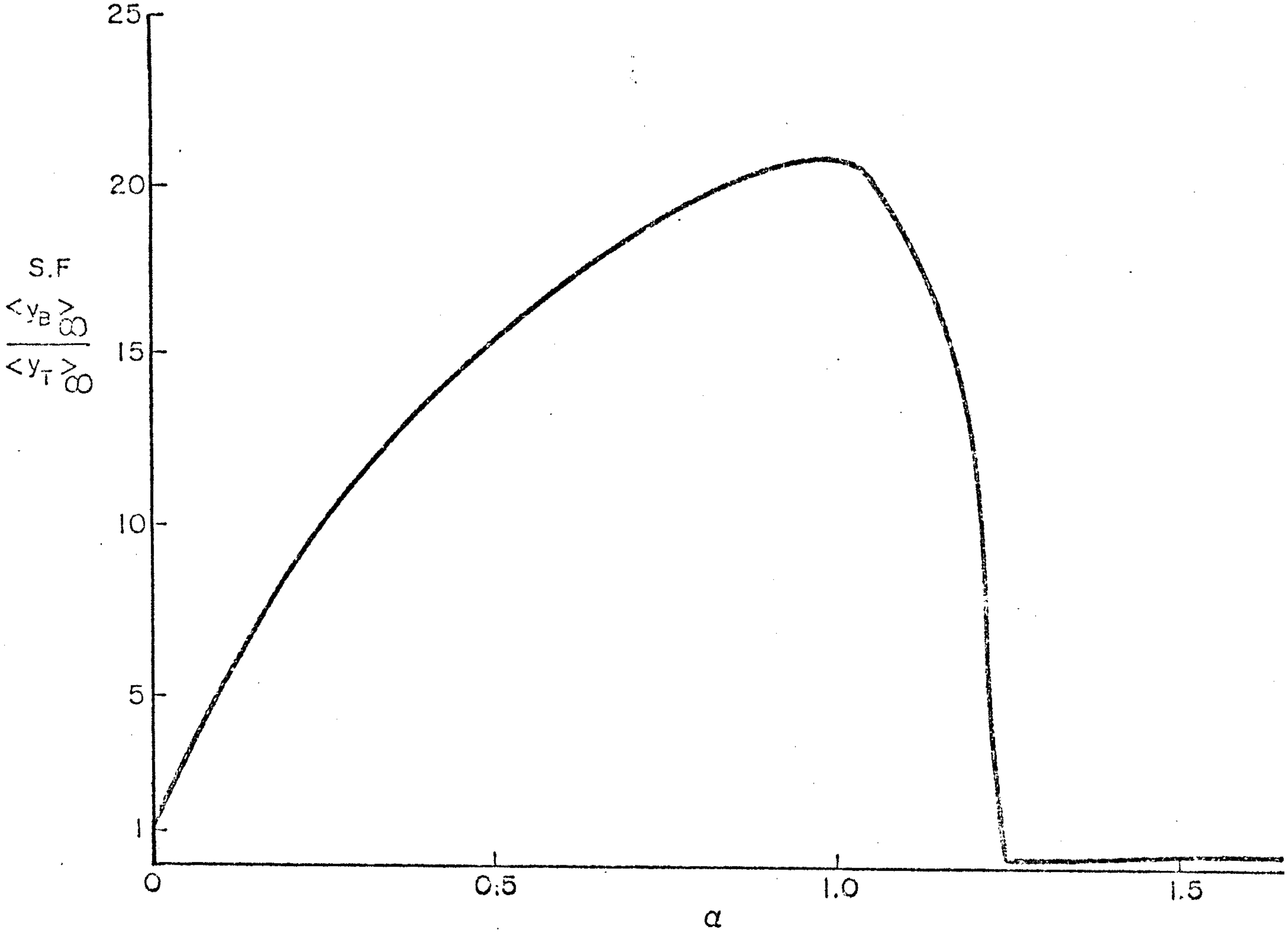


Fig. 19. Effect of α on Separation.

$A = \langle y_T \rangle_\infty$

$A^1 = \langle y_B \rangle_\infty$

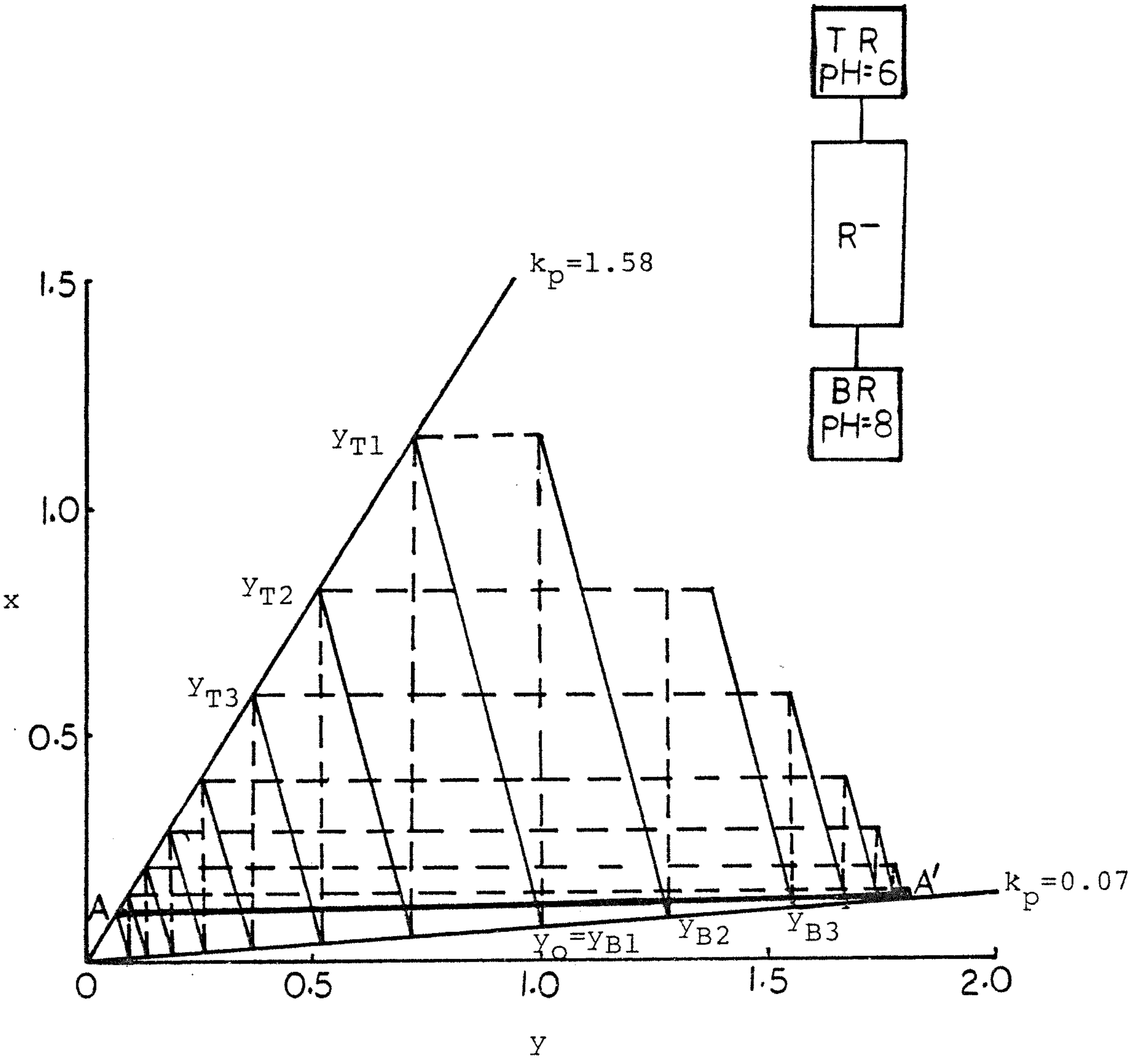


Fig. 20. One Equilibrium Stage of pH Batch Parapumping System.

equilibrium isotherms were obtained by trial and error procedure by correlating the experimental data obtained by Chen et al. (1978) semi-emperically.

The batch parapump apparatus consists of a column packed with cation exchanger and two reservoirs attached to each end. The pH values of the top and bottom reservoir were maintained at given levels $P_2$(=6) and $P_1$(=8) respectively. The operation began with column filled with mixture of concentration $y_o$(=1) and $P_1$ (=8), and was allowed to equilibrate with the solid. The bottom reservoir had a concentration of $y_o$(=1) and a pH of $P_2$(=6).

The first fluid motion was downward, with $V_T$=$V_B$=0. x and y here represent the concentrations of solute in the solid and fluid phases respectively (see equation 17). Based on the results of the simulation given in Table 12, an x-y diagram was constructed. The initial concentration in the column ($y_o$;$x_o$) is represented by the $y_o$(=1). A complete cycle of the operation included four steps. The procedure started as follows:

1. Transfer down: The fluid in the top reservoir is transferred to the column, while the fluid in the column is transferred to the bottom reservoir. The concentration of the bottom reservoir for the first cycle is $y_o$(=1).

2. Equilibrium at $P_2$ (=6): The column pH is changed from $P_1$(=8) to $P_2$(=6). The two phases (solid and liquid) are then allowed to equilibrate at $P_2$. A new composition in the column ($y_{T1}$; $x_{T1}$) is now the equilibrium line $k_6$ and

of the operating line passing through $(y_o;x_o)$. The slope of the operating line is $(-V/\overline{V})$ (see equation 16).

3. Transfer up: The solution in the bottom reservoir is returned to the column, while the solution in the column is brought to the top reservoir. The composition in the column is now $(y_{B1};x_{B1})$.

4. Equilibration at $P_1(=8)$: The column pH is shifted back to $P_1$ and a phase equilibrium is re-established. The new equilibrium point $(y_{B1};x_{B2})$, represented by $y_{B2}$, is located at the intersection of the equilibrium line $k_8$ and the operating line passing through $(y_{B1};x_{T1})$ and having a slope of $(-V/\overline{V})$. The first cycle is thus ended.

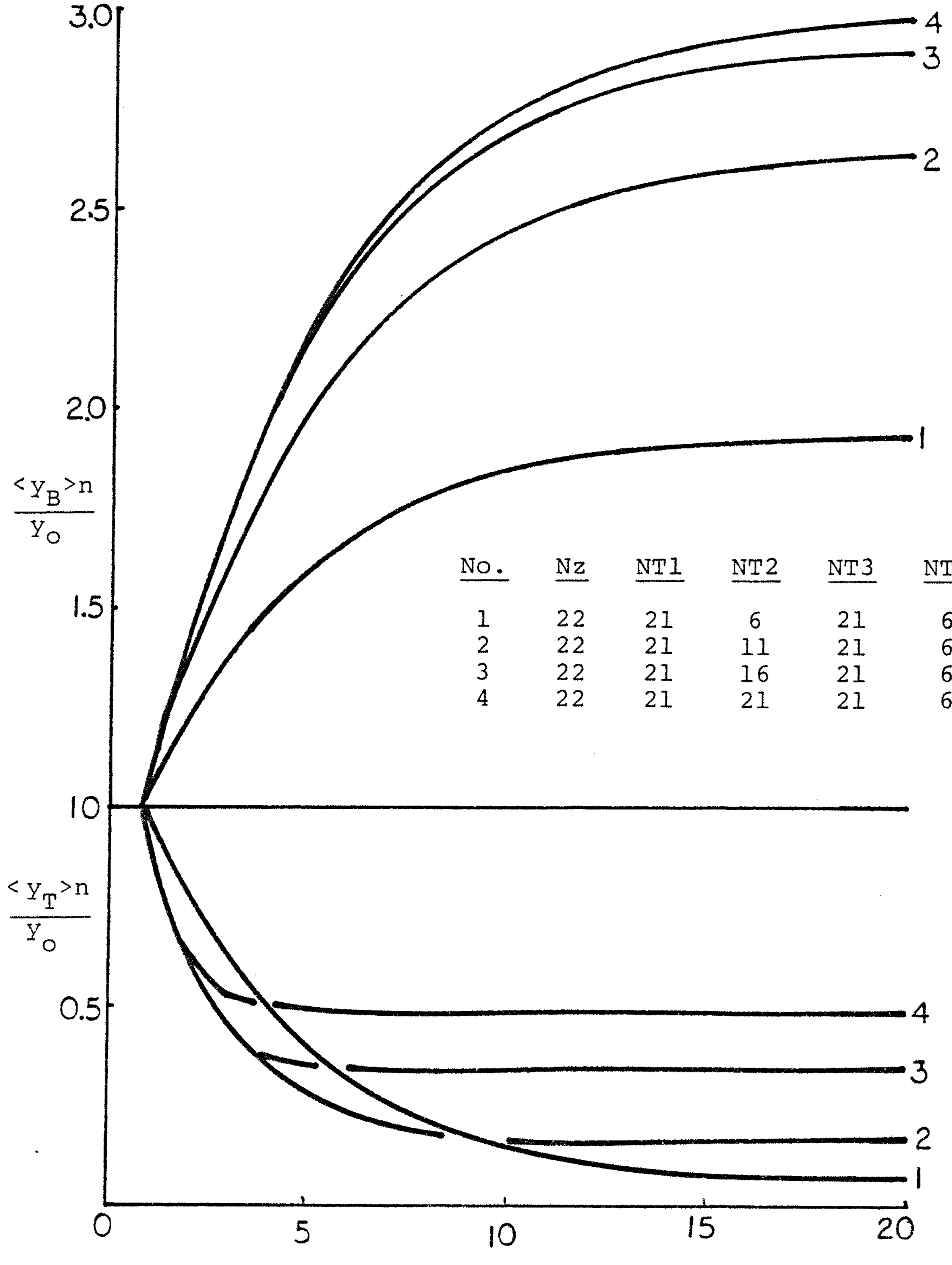
The second cycle is then started by transferring the fraction $y_{T1}$ from the top reservoir to the column and the fraction $y_{B2}$ to the bottom reservoir. To complete this cycle, steps 1 and 4 described above are repeated. Subsequent cycles are then constructed on the diagram using the results in Table 12. It can be seen that as the number of cycles (n) becomes large, the top and bottom reservoir concentration approach steady state values, $\langle y_T\rangle_\infty$ and $\langle y_B\rangle_\infty$ respectively. At steady state, the solid phase has a constant composition which is in equilibrium with both and $\langle y_B\rangle_\infty$, i.e., $x=k_8\langle y_B\rangle_\infty=k_6\langle y_T\rangle_\infty$, thereby leading to a line $\overline{\langle y_T\rangle_\infty\langle y_B\rangle_\infty}$ which is parallel to the y axis.

## The Continuous Parametric Pump

Presented on Table 19, is a summary of the conditions for runs with variable volume of bottom feed, and Tables 20 to 23 show the results obtained. Shown in Figures 21 to 29 are typical concentration transients and separation factors following the simulation of continuous parapumping operations for variable volume of bottom feed (NT2). For each value of top feed (NT4) used, four runs of NT2 were made, viz: 6, 11, 16 and 21. In Figures 21, 23, 25, and 27 plots of the average product concentrations divided by the initial liquid phase concentrations vs. the number of cycles (n) of operations are shown.

Figure 21 shows the concentration transients using different values of NT2 (=6, 11, 16, and 21) and a constant value of NT4 (=6). The results show that if a high separation is desired, NT4=6 and NT2=6 would be more preferable. For this condition, a separation factor of 28.167 was obtained for 20 cycles. On the other hand, if higher separation can be sacrificed for bottom product enrichment, NT4=6 and NT2=21 would give the best results. The plot of the separation factors ($<y_B>_\infty/<y_T>_\infty$) versus NT2 is shown in Figure 22. From this figure, one can see that, as NT2 increases, the separation factor decreases.

In Figure 23, a constant value of NT4 (=11) and NT2=6, 11, 16, and 21 were used. Each run consisted of NT2 and a constant NT4, NT2 being varied each time a run was made. The separation factors obtained for these set of runs are not as good as that obtained with NT4=6. Since NT4 is a high pH feed introduced from the top and bottom products withdrawn, one would expect that an increase in NT4 would result in



| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 22 | 21 | 6 | 21 | 6 |
| 2 | 22 | 21 | 11 | 21 | 6 |
| 3 | 22 | 21 | 16 | 21 | 6 |
| 4 | 22 | 21 | 21 | 21 | 6 |

Fig. 21. Effect of NT2 on Transient Concentrations for NT4=6.

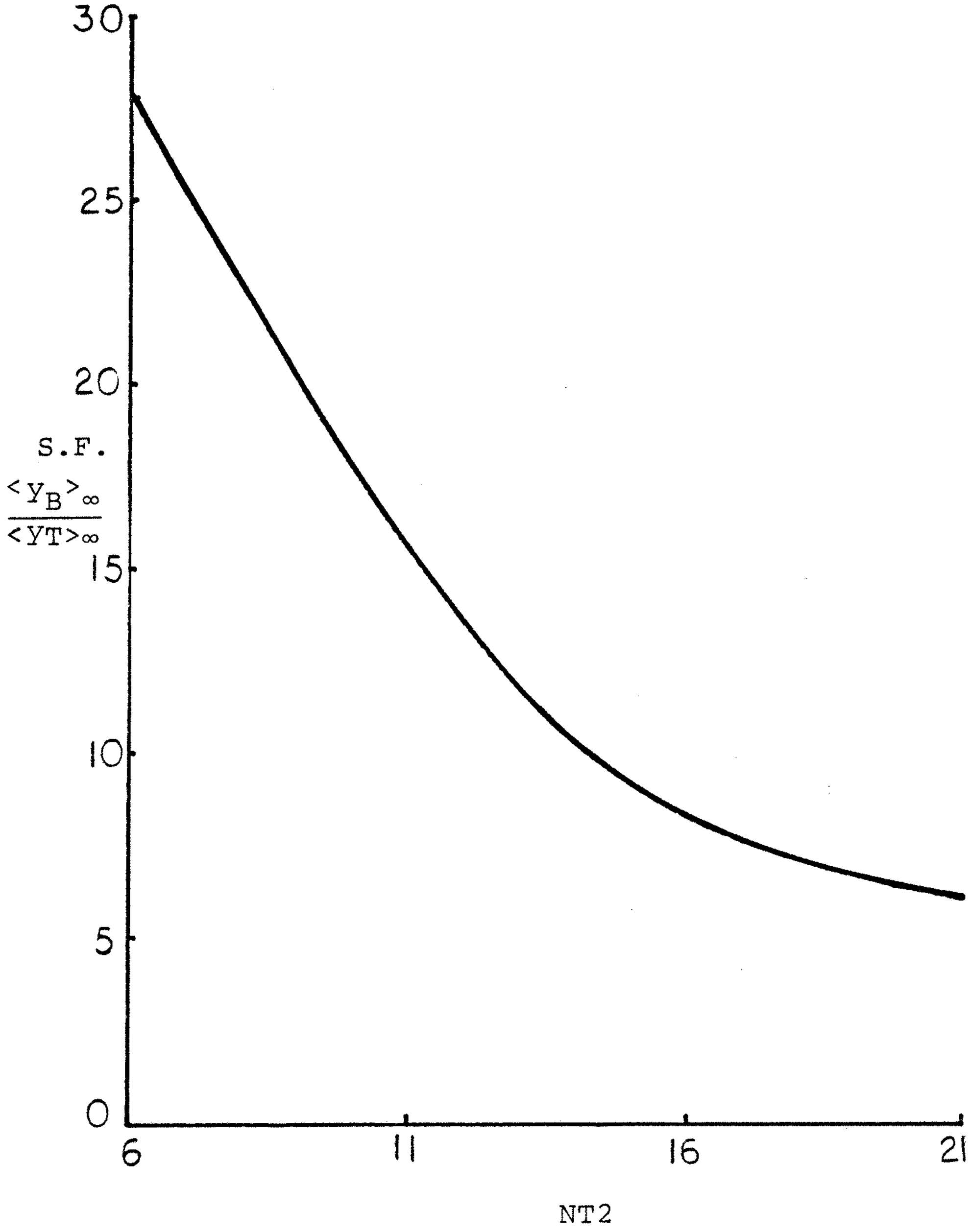
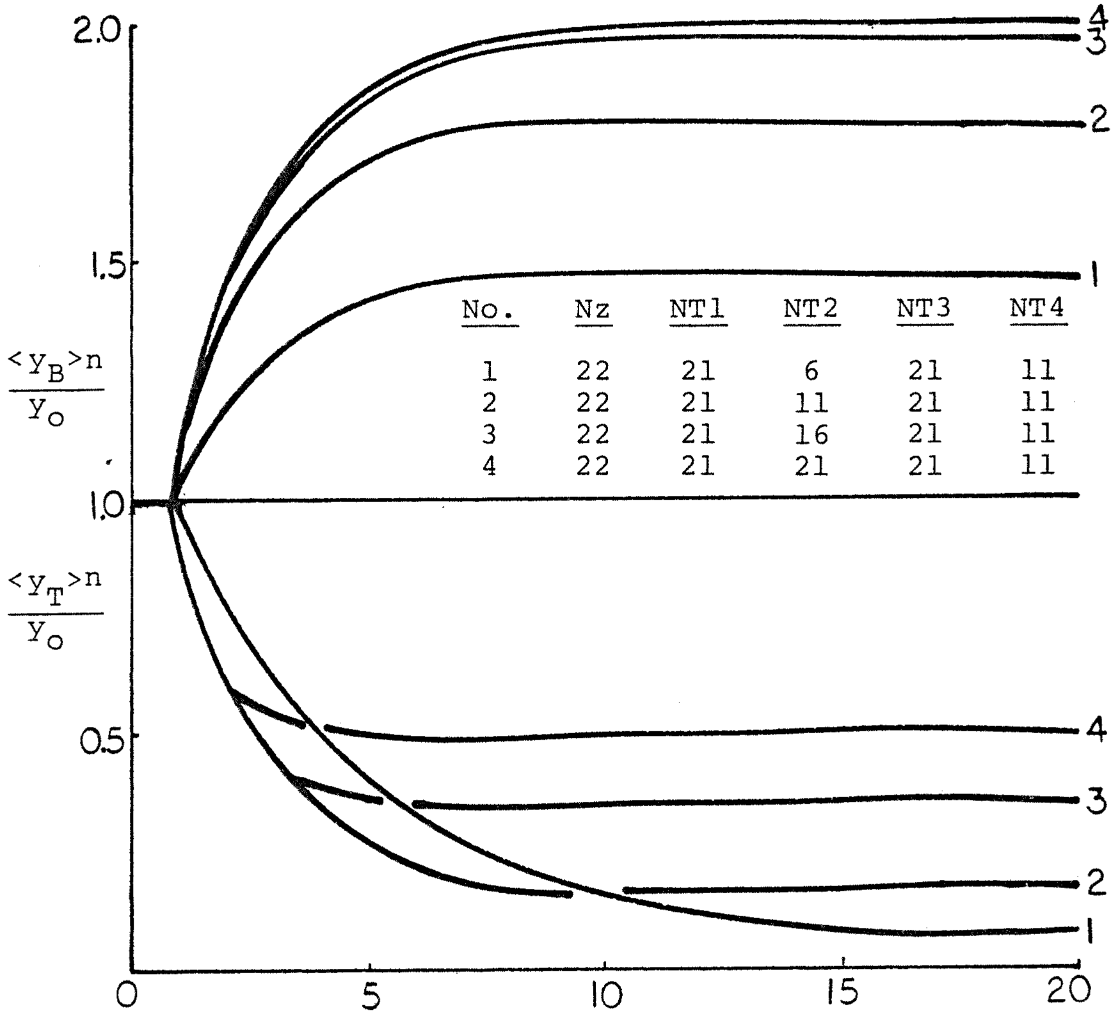


Fig. 22. Effect of NT2 on Separation Factor for NT4=6.



| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 22 | 21 | 6 | 21 | 11 |
| 2 | 22 | 21 | 11 | 21 | 11 |
| 3 | 22 | 21 | 16 | 21 | 11 |
| 4 | 22 | 21 | 21 | 21 | 11 |

Fig. 23. Effect of NT2 on Transient Concentration for NT4=11.

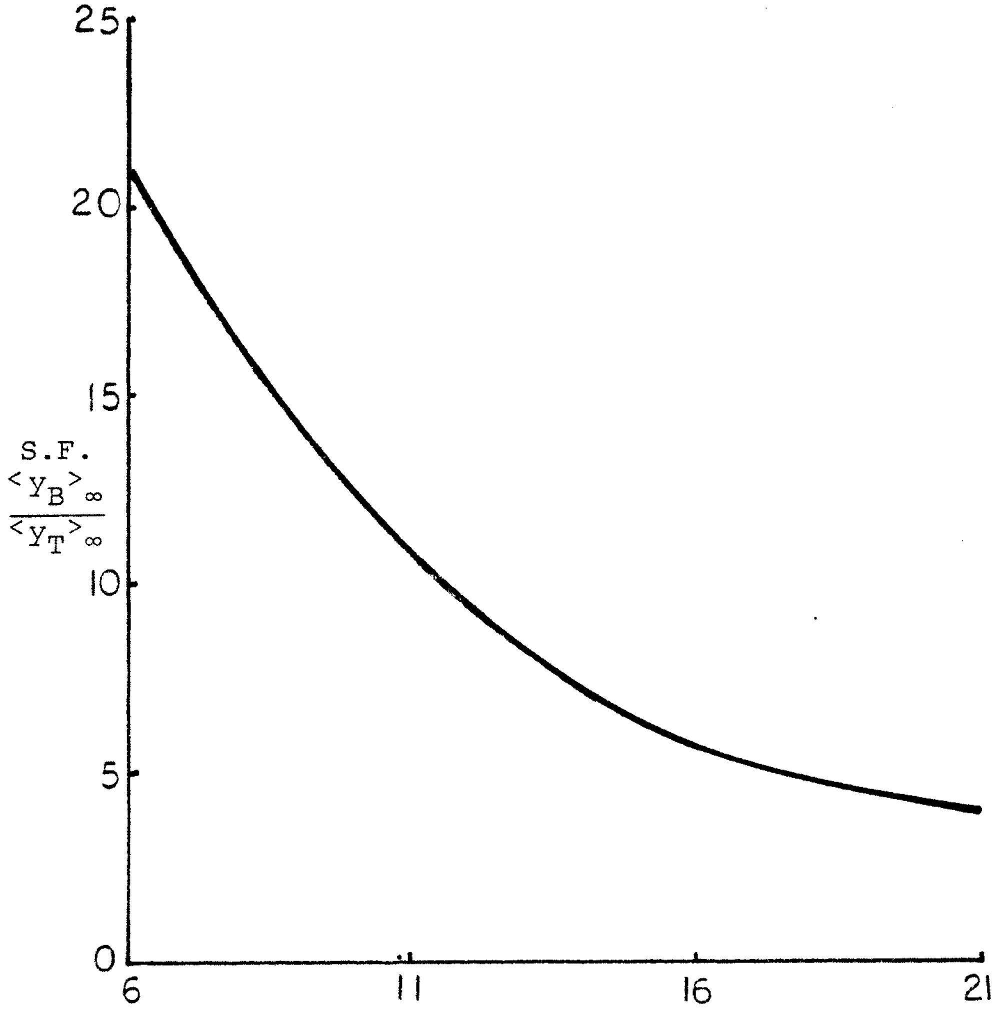
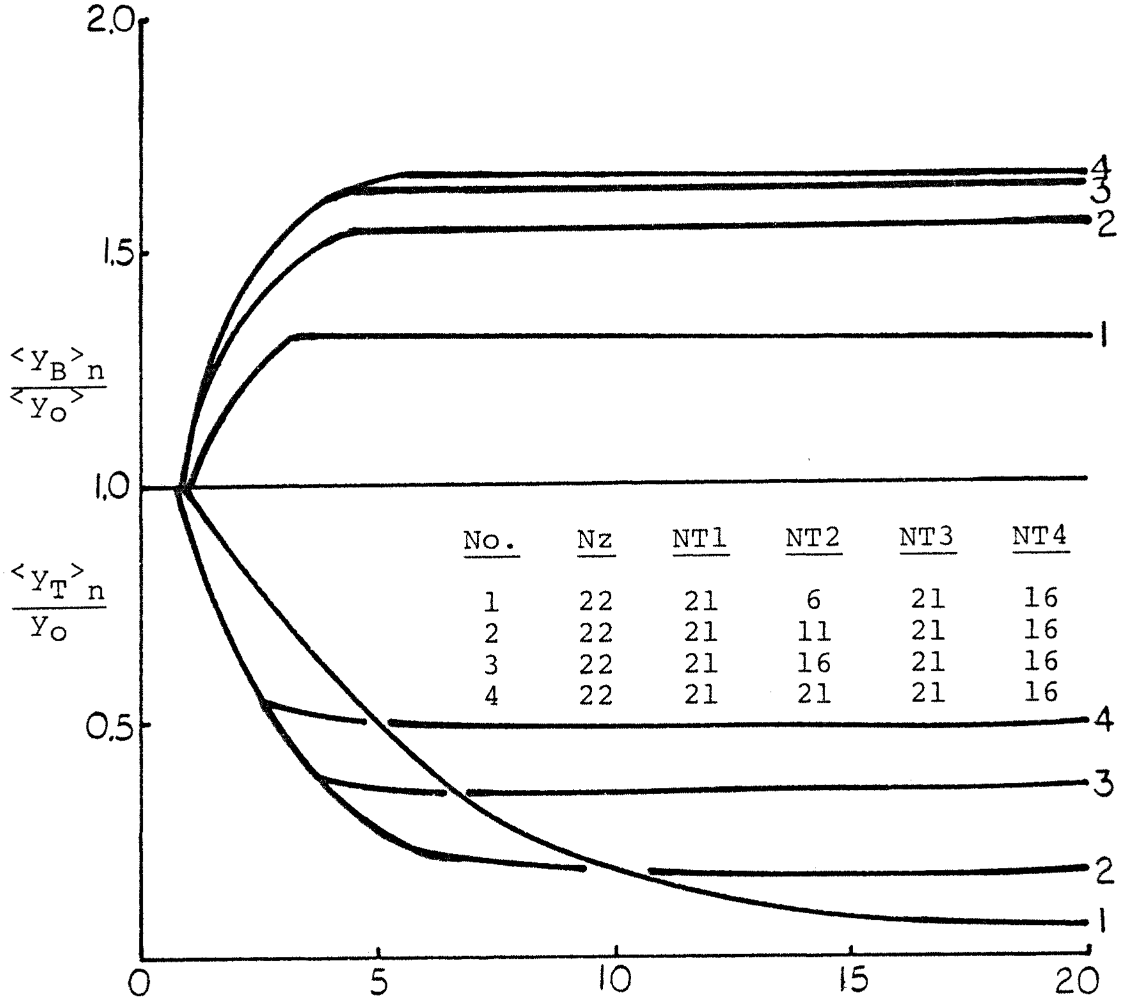


Fig. 24. Effect of NT2 on Separation Factor for NT4=11.

increased desorption of already adsorbed protein, and hence result in better separation. But, on the contrary, the separation was decreasing with increasing value of NT4. From this phenomenon, one would conclude that high pH feed is not the controlling factor in the separation. Comparing Figure 24 to Figure 22, the marked decrease in the separation factor can be seen.

Figure 25 is the plot for constant NT4 (=16) and various values of NT2 (=6, 11, 16, 21). The trend of decreasing separation factor for increasing values of NT4 can also be seen from this figure. Comparing the bottom product concentration ratios for NT2=16 and NT2=21, one can see that the difference in the separation is not very significant. The separation factors plotted against NT2 shown in Figure 26 decreases sharply as NT2 increases and tends to level off between NT2=16 and NT2=21. It seems obvious that NT2 greater than 16 for NT4=16 is of no practical value, since the separation factor is the lowest and does not give any appreciable improvement for enrichment purposes.

In Figure 27, a plot for NT4=21 and different values of NT2 (=6, 11, 16, and 21) is shown. During the first cycle, the top product concentration ratios remain essentially the same as the feed concentration ratio before decaying exponentially. As one can see, NT4 has the same value as NT1 and NT3 (downflow and upflow reservoir displacement). This seems to suggest that, when NT1, NT3 and



| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 22 | 21 | 6 | 21 | 16 |
| 2 | 22 | 21 | 11 | 21 | 16 |
| 3 | 22 | 21 | 16 | 21 | 16 |
| 4 | 22 | 21 | 21 | 21 | 16 |

Fig. 25. Effect of NT2 on Transient Concentrations for NT4=16.

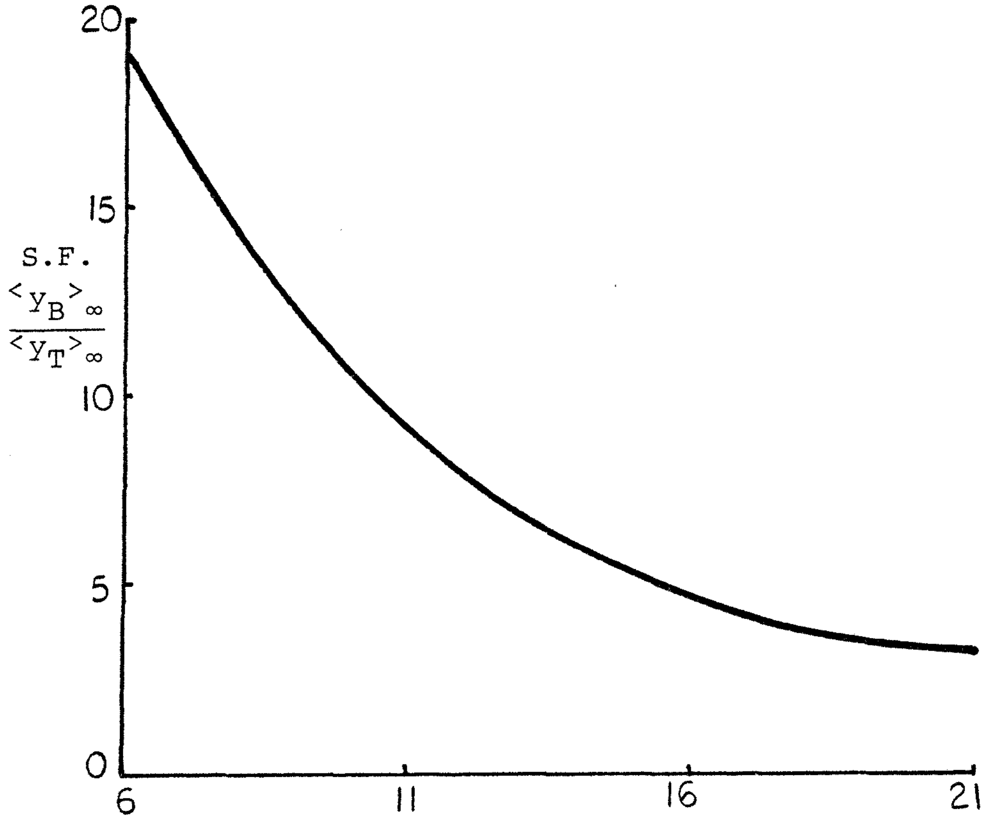
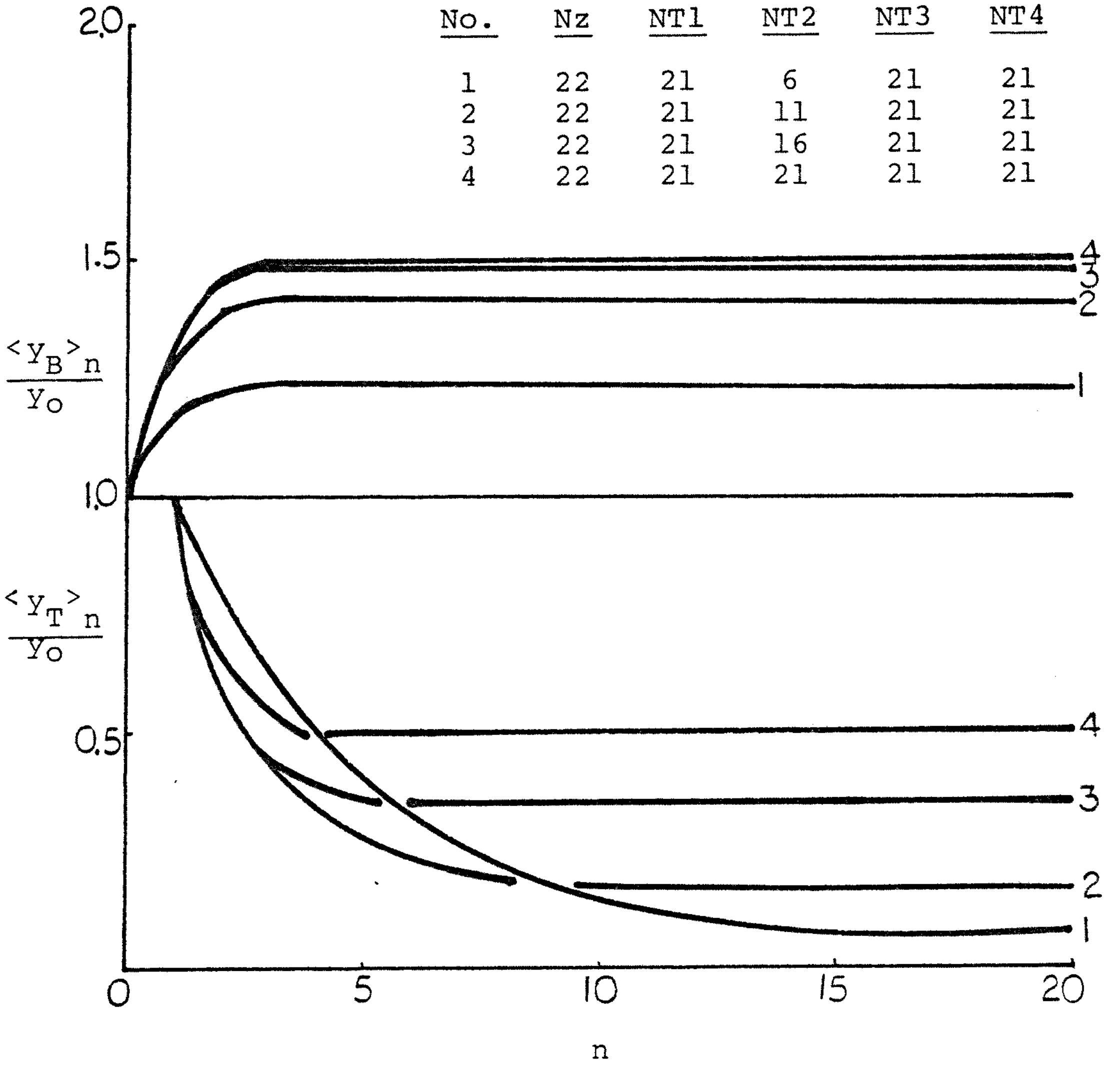


Fig. 26. Effect of NT2 on Separation Factor for NT4=16.

NT4 are equal, the concentration of the top product is essentially the same as the concentration of the fluid phase in the column for approximately the first cycle, since the fluid phase in the column is literally pushed out of the column and taken as sample, due to complete saturation of the column before the start of run. The separation factors for this set of runs is shown in Figure 28.

In Figure 29, a plot of the steady state separation factors vs. NT2 (Table 24) is shown for constant values of NT4. At NT4=6, the highest separation factors were obtained in both the high region (NT2=6) and low region (NT2=21), while at NT4=21, the lowest separation factors in both regions were obtained. The characteristic behavior of the separation factors with increasing values of NT4 in all four cases are the same. From the above results, it was evident that if a high degree of separation is desired, low values of NT2 and NT4 must be used, preferrably NT1=6 and NT4=6. If, on the other hand, enrichment of the rich section is desirable, then NT2=21 and NT4=6 would yield best results.

Since we have seen the effect variable NT2 for constant NT4 had on the separation, one is now in a position to scrutinize the effect of variable NT4 for constant value of NT2 has on transient concentrations. The theoretical results shown on Table 26-29 for this case using the operating conditions on Table 25 have been presented in Figures 30, 32, 34, and 36, while the corresponding



| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 22 | 21 | 6 | 21 | 21 |
| 2 | 22 | 21 | 11 | 21 | 21 |
| 3 | 22 | 21 | 16 | 21 | 21 |
| 4 | 22 | 21 | 21 | 21 | 21 |

Fig. 27. Effect of NT2 on Transient Concentrations for NT4=21.

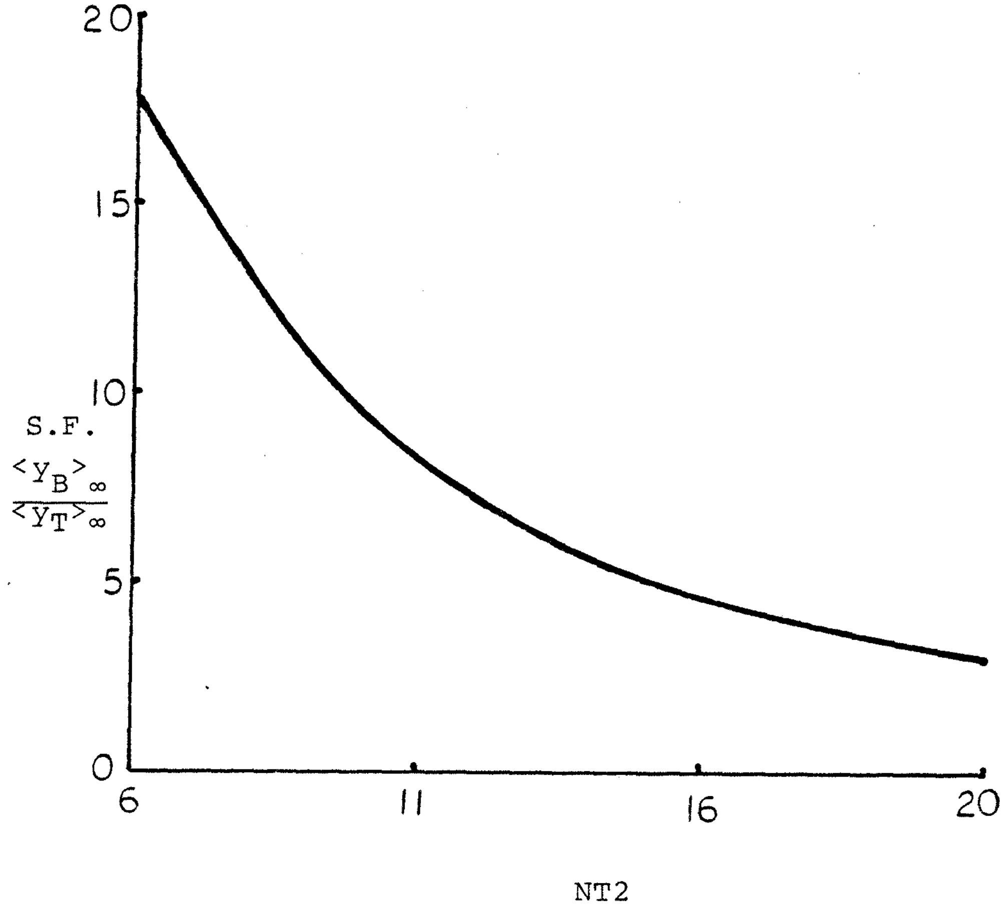


Fig. 28. Effect of NT2 on Separation Factor for NT4=21.

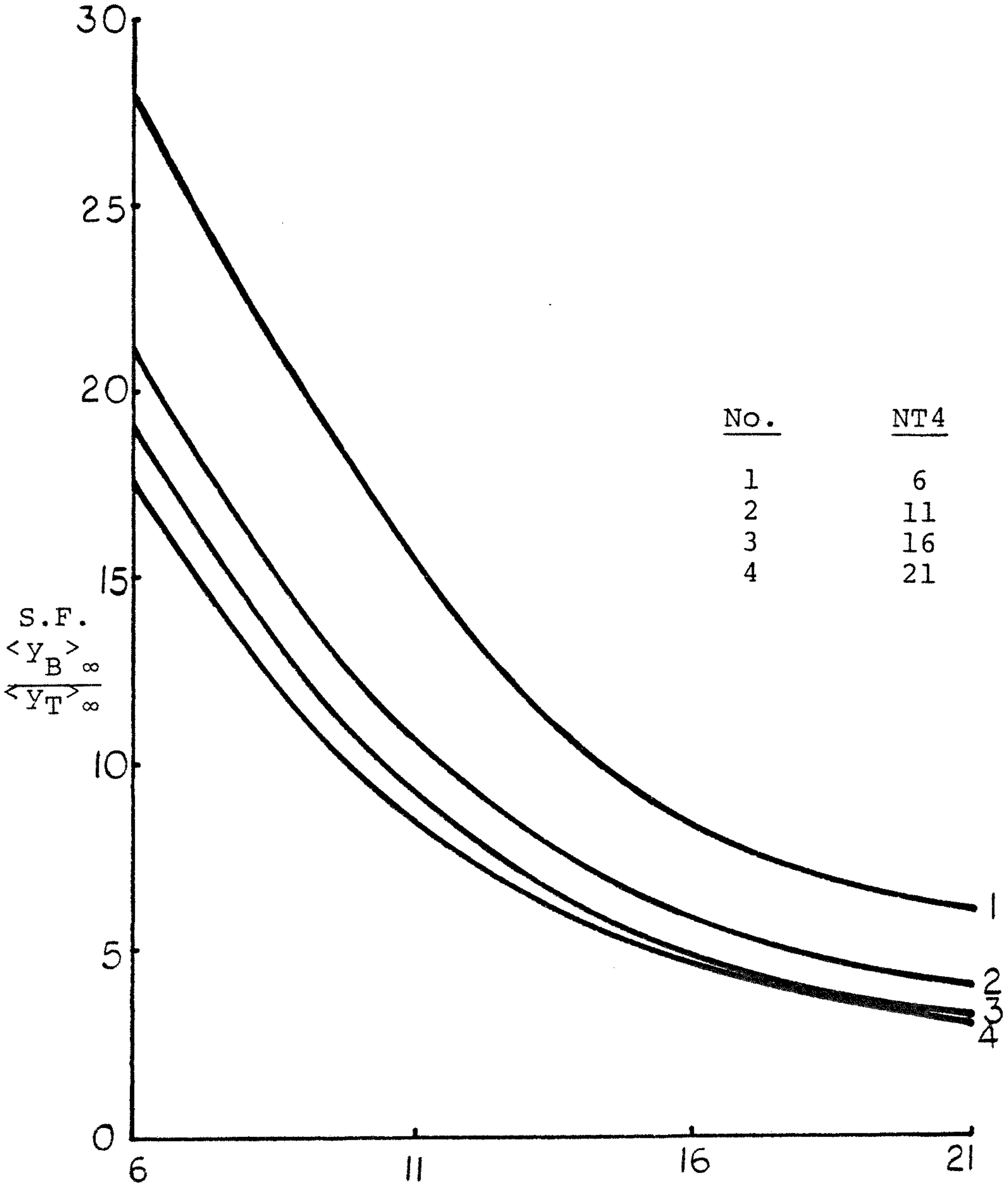
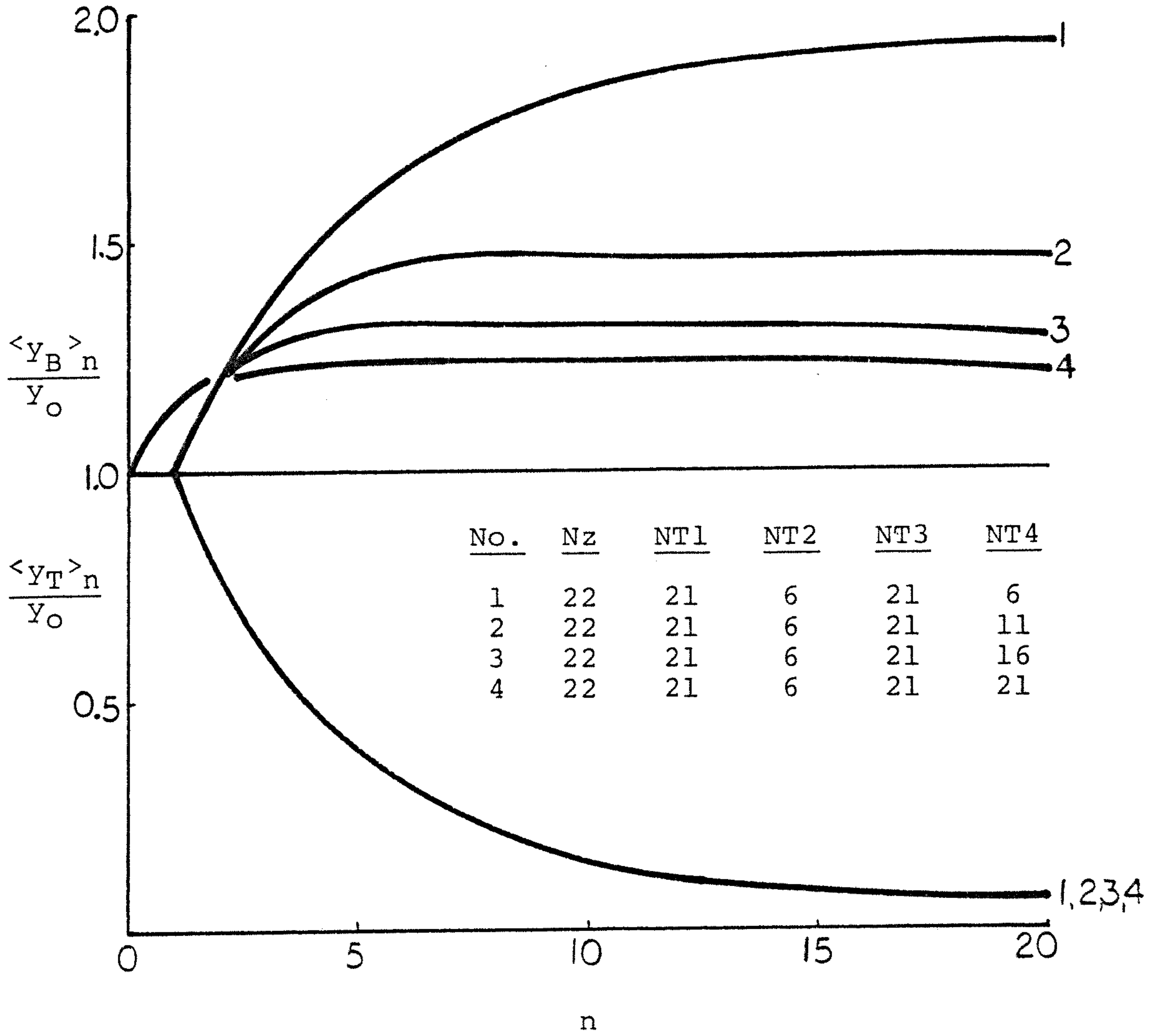


Fig. 29. Separation Factor vs. NT2.

separation factors have been respectively plotted against NT4 and are presented in Figures 31, 33, 35, and 37. For the purpose of comparison, the separation factors have been combined and presented in Figure 38. The results showed that when the volume of top products, NT2, is held constant, and the volume of bottom products varied, the lean product concentrations were found to be constant. In other words, if NT2 is held constant, and NT4 varied, the lean product concentrations are constant for the specific NT2 used. It should be emphasized that the amount of sample withdrawn as product (both top and bottom) must also be introduced as feed, and for a top product to be sampled, fresh feed must be introduced from the bottom and vice versa.

Figure 30 shows the plot for NT2(=6) and NT4(=6, 11, 16, and 21). The top product transient concentration ratio (lean product) for all values of NT4 have constant concentration value while the bottom product transient concentration ratio decreases as NT4 increases. In Figure 32, the plot for NT2(=11) for various values of NT4 (6, 11, 16 and 21) is shown, and NT2 (=16) for various values of NT4 (6, 11, 16 and 21) is shown in Figure 34. Figure 36 shows the plot for NT2 (=21) for various values of NT4 (6, 11, 16 and 21). The characteristic behavior of the separation for all of these three cases are the same, viz: constant top product concentration ratio is constant for specific value of NT2, while the bottom product transient concentration ratio decreases as NT4 increases. It is worth noticing that for constant



| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 22 | 21 | 6 | 21 | 6 |
| 2 | 22 | 21 | 6 | 21 | 11 |
| 3 | 22 | 21 | 6 | 21 | 16 |
| 4 | 22 | 21 | 6 | 21 | 21 |

Fig. 30. Effect of NT4 on Transient Concentrations for NT2=6.

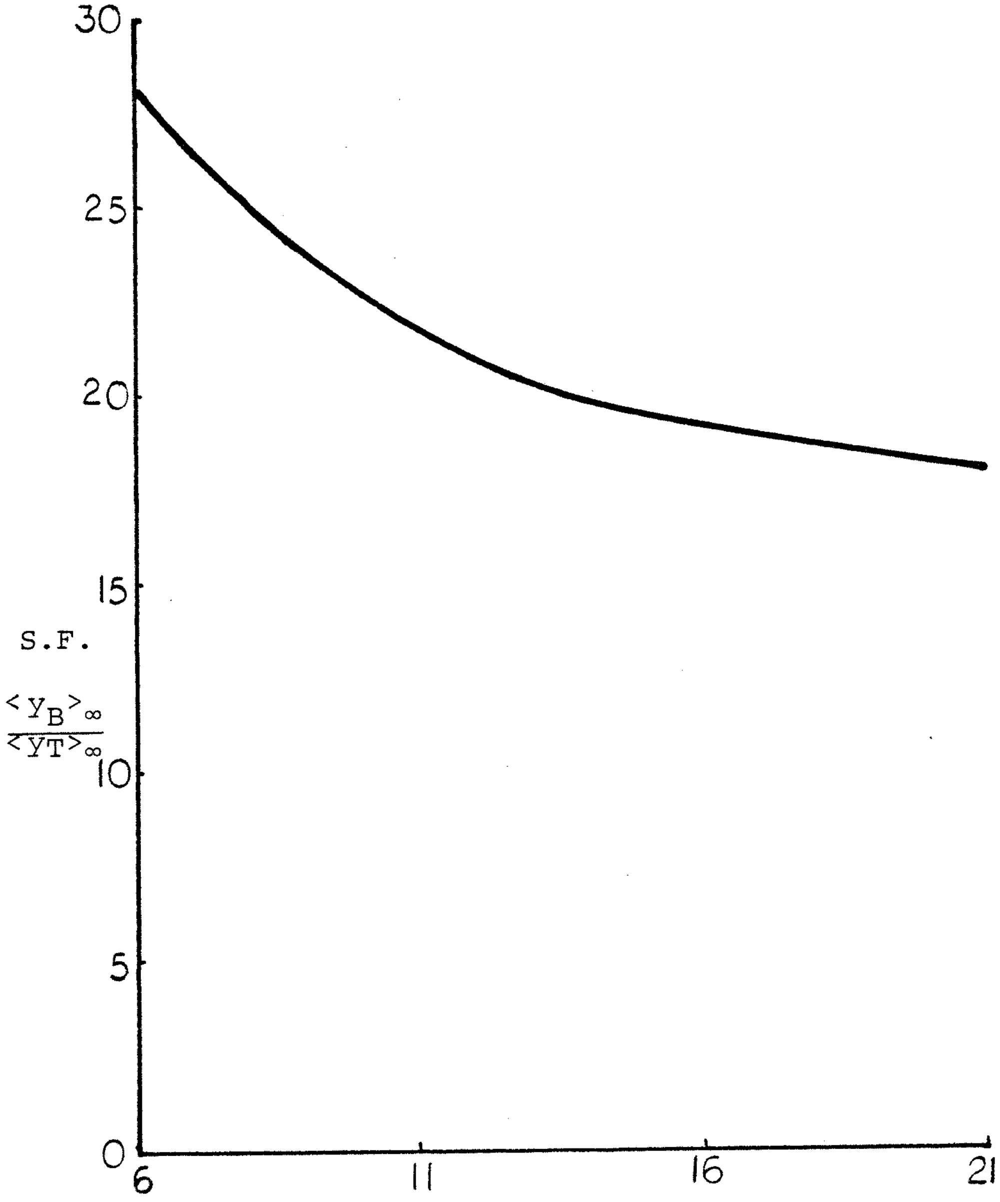
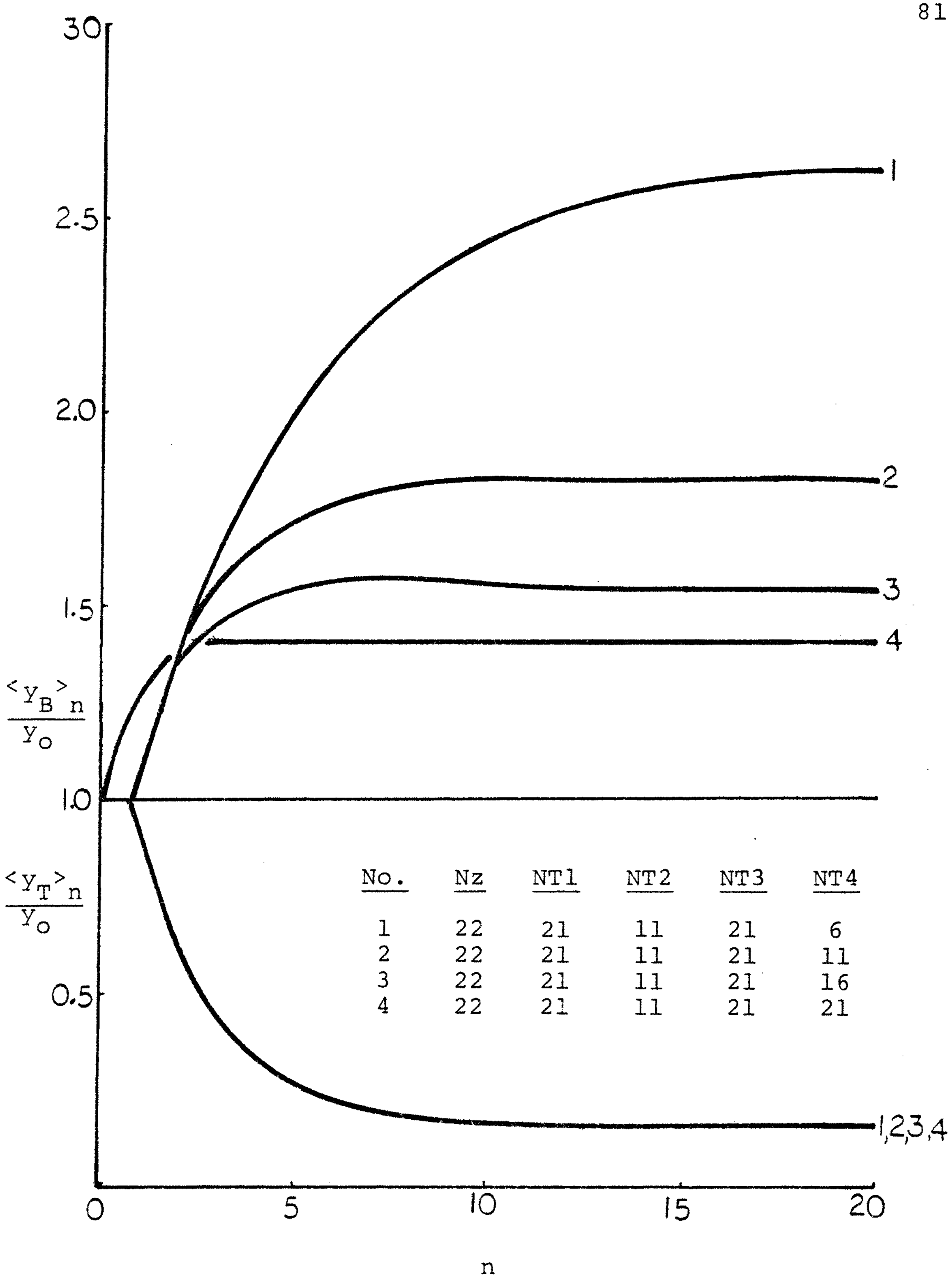


Fig. 31. Effect of NT4 on Separation for NT4=6.



| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 22 | 21 | 11 | 21 | 6 |
| 2 | 22 | 21 | 11 | 21 | 11 |
| 3 | 22 | 21 | 11 | 21 | 16 |
| 4 | 22 | 21 | 11 | 21 | 21 |

Fig. 32. Effect of NT4 on Transient Concentrations for NT2=11.

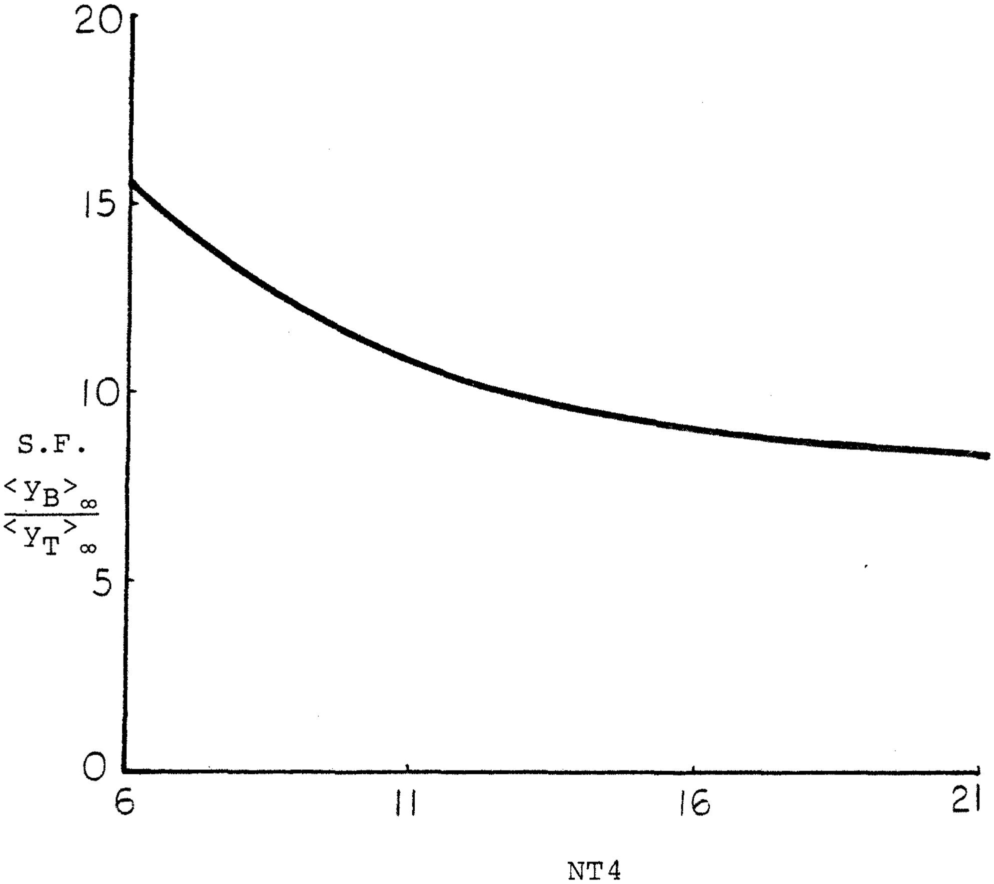
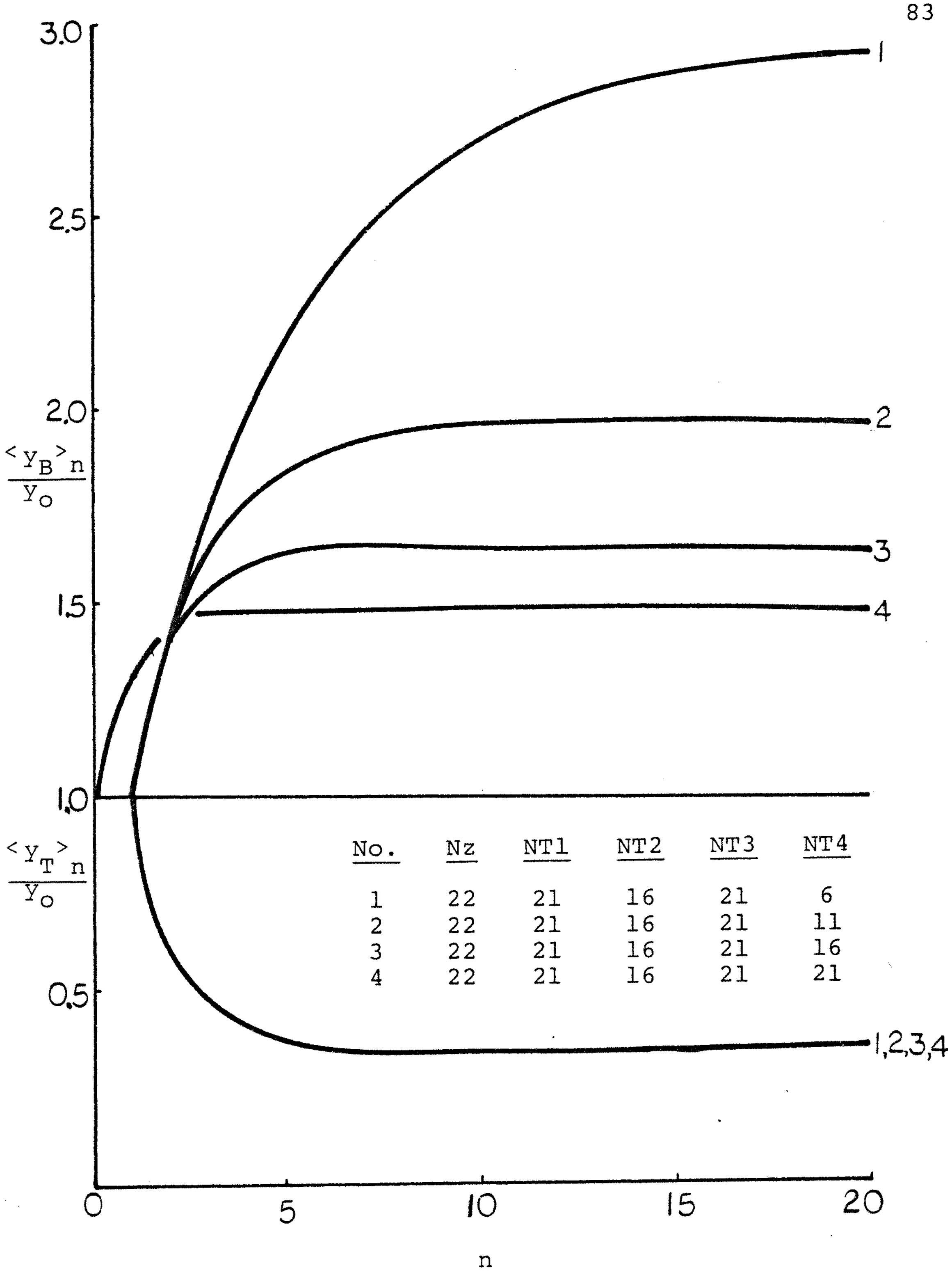


Fig. 33. Effect of NT4 on Separation Factors for NT2=11.



| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 22 | 21 | 16 | 21 | 6 |
| 2 | 22 | 21 | 16 | 21 | 11 |
| 3 | 22 | 21 | 16 | 21 | 16 |
| 4 | 22 | 21 | 16 | 21 | 21 |

Fig. 34. Effect of NT4 on Transient Concentrations for NT2=16.

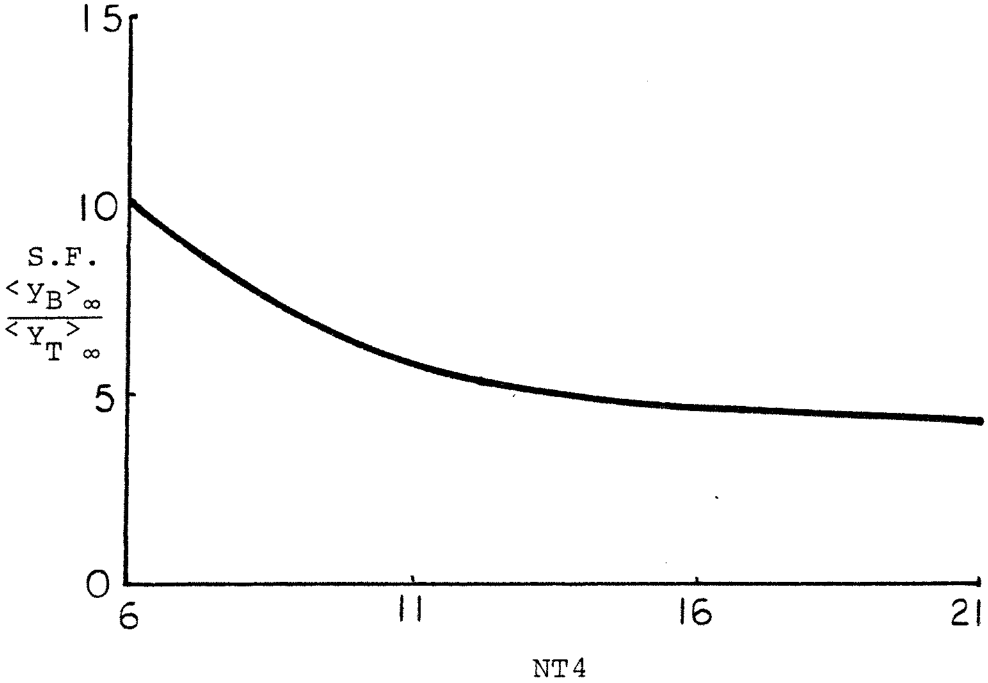
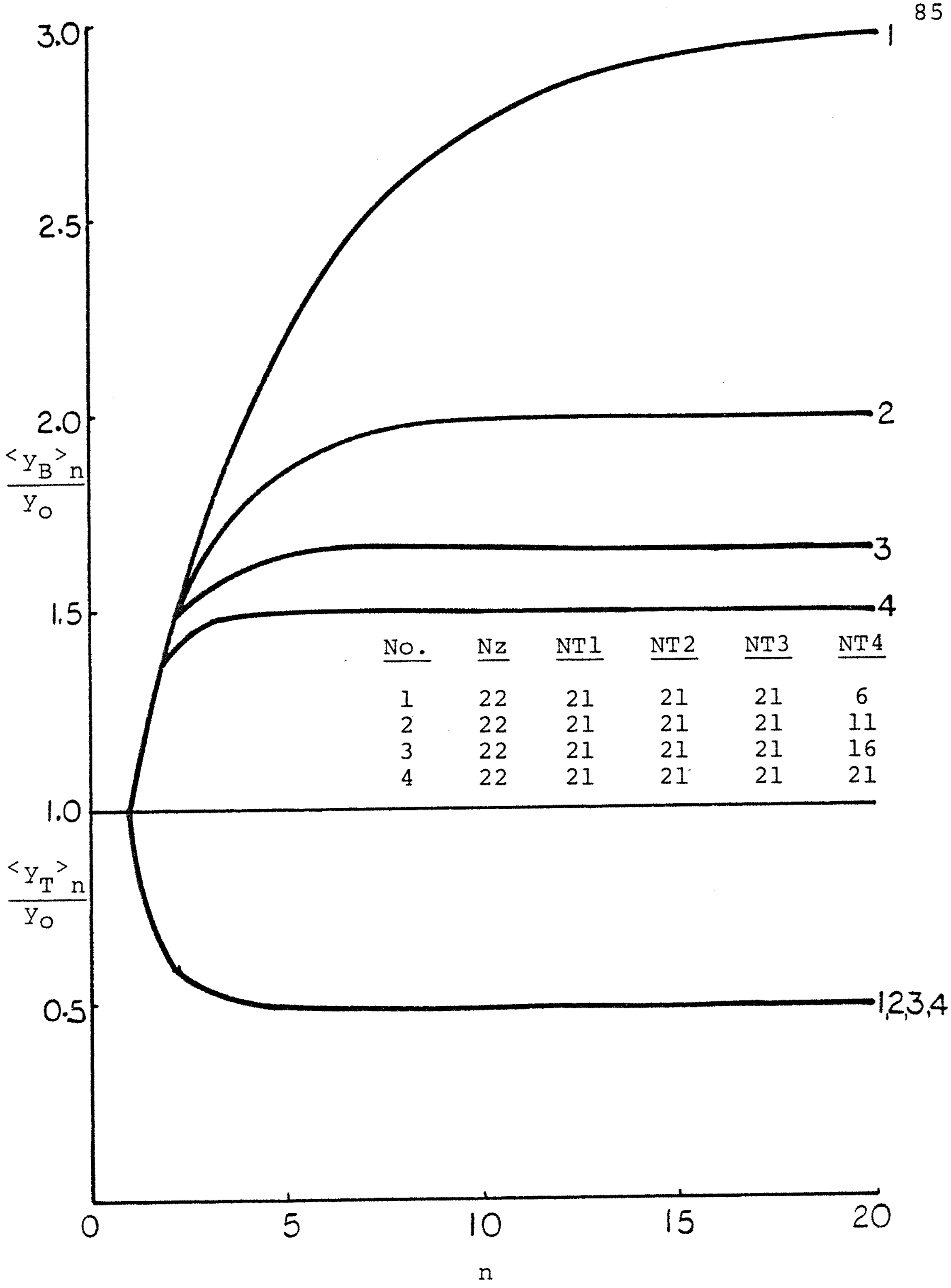


Fig. 35. Effect of NT4 on Separation Factor for NT2=16.



| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 22 | 21 | 21 | 21 | 6 |
| 2 | 22 | 21 | 21 | 21 | 11 |
| 3 | 22 | 21 | 21 | 21 | 16 |
| 4 | 22 | 21 | 21 | 21 | 21 |

Fig. 36. Effect of NT4 on Transient Concentrations for NT2=21.

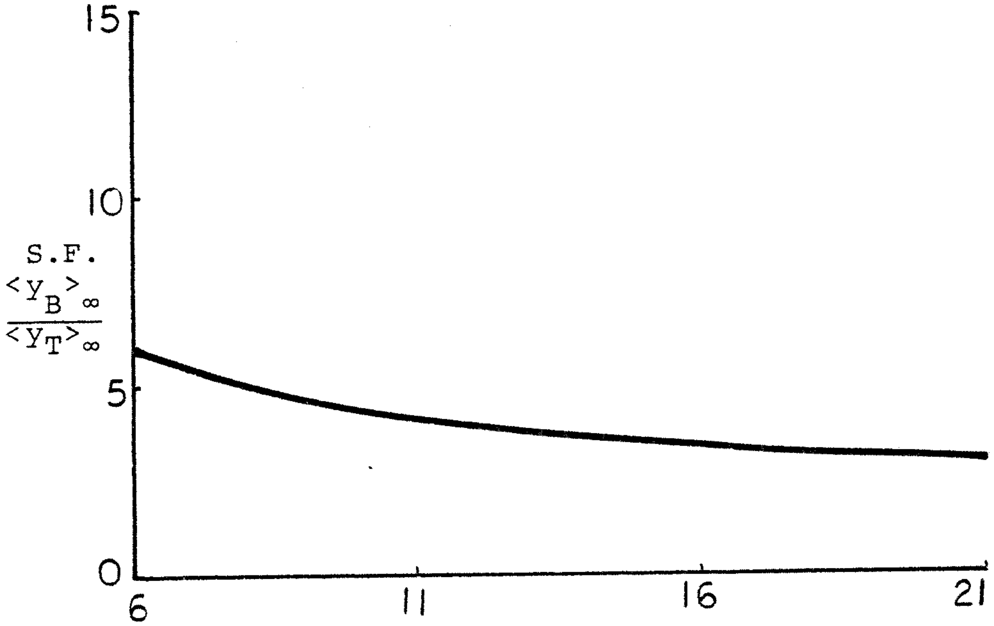


Fig. 37. Effect of NT4 on Separation Factors for NT2=21.

NT2, though the top product concentration ratio remain constant for various NT4 used, an increase of NT2 shifts the top product concentration ratio upward. Since we have seen from the ongoing analysis that increase in NT4 for constant value of NT2 decreases the separation factor (see Figures 31, 33 and 35), it is evident that, when NT4 becomes too excessive, haemoglobin from the top feed passes through the column and out as the bottom product. If NT4 is made even larger than the values used here (note that this also means large top feed), the top and bottom product will eventually have the same concentrations as the feed. NT4 and NT2 are not found to have the same effect on separation. However, constant NT4 for various values of NT2 described earlier do have a very good agreement with the experimental findings of Chen et al. (1979).

The steady state separation factors for the four values of NT2 used are shown on Table 30 and plotted against NT4 is presented in Figure 38. The slopes as can be seen from this graph for the four cases are somehow steeper between NT4=6 and NT4=11 as compared to the slopes between NT4=11 and NT4=21. These phenomenon seem to suggest that as NT4 increases, the separation decreases drastically after NT4=11 for the same reason discussed above.

The effect of NT2 and NT4 on the concentration transients is shown in Figure 39. The ordinate is the average product concentration (grams per cubic centi-

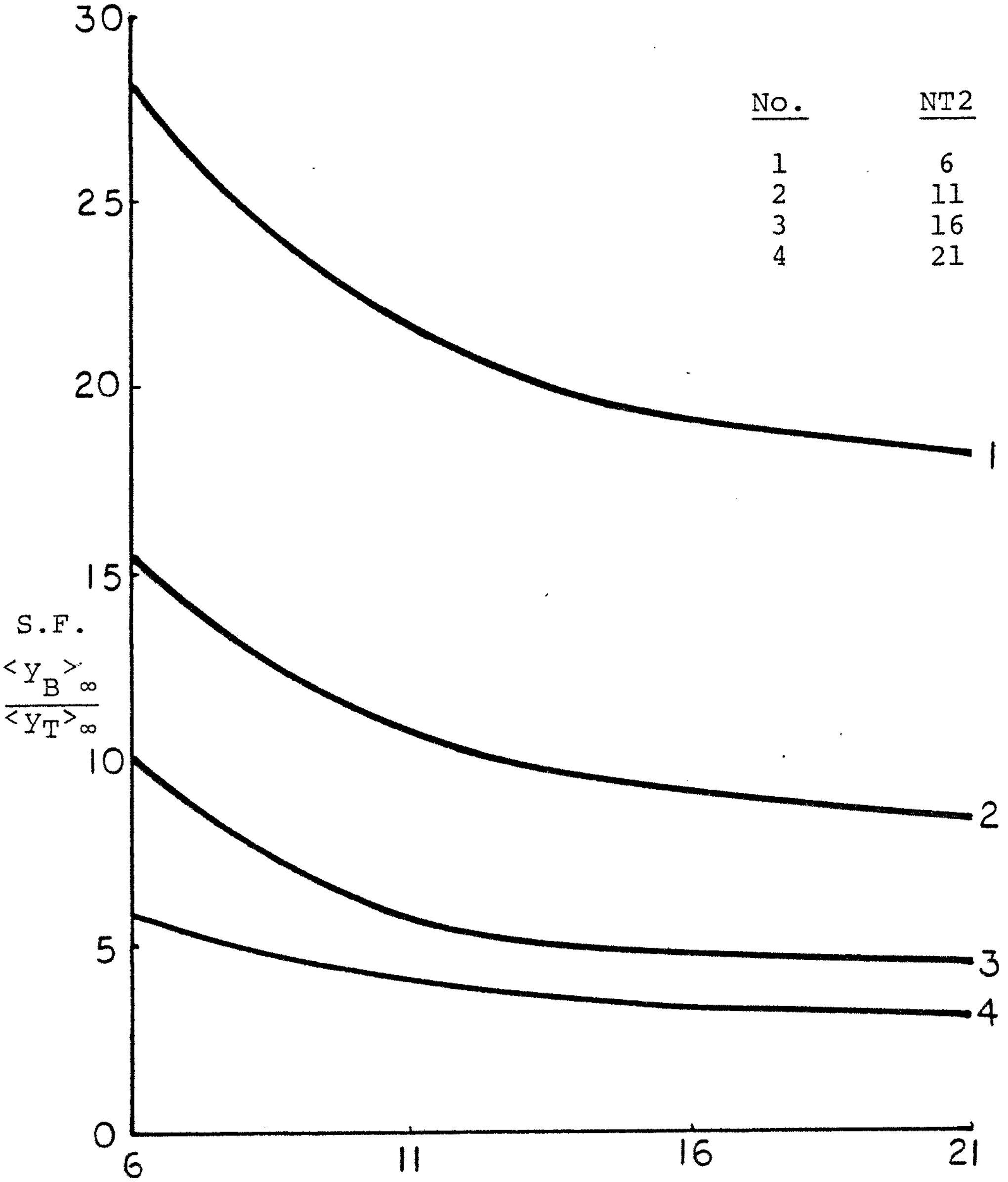


Fig. 38. Separation Factors vs. NT4.

| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 6 | 5 | 5 | 5 | 5 |
| 2 | 12 | 5 | 11 | 5 | 11 |

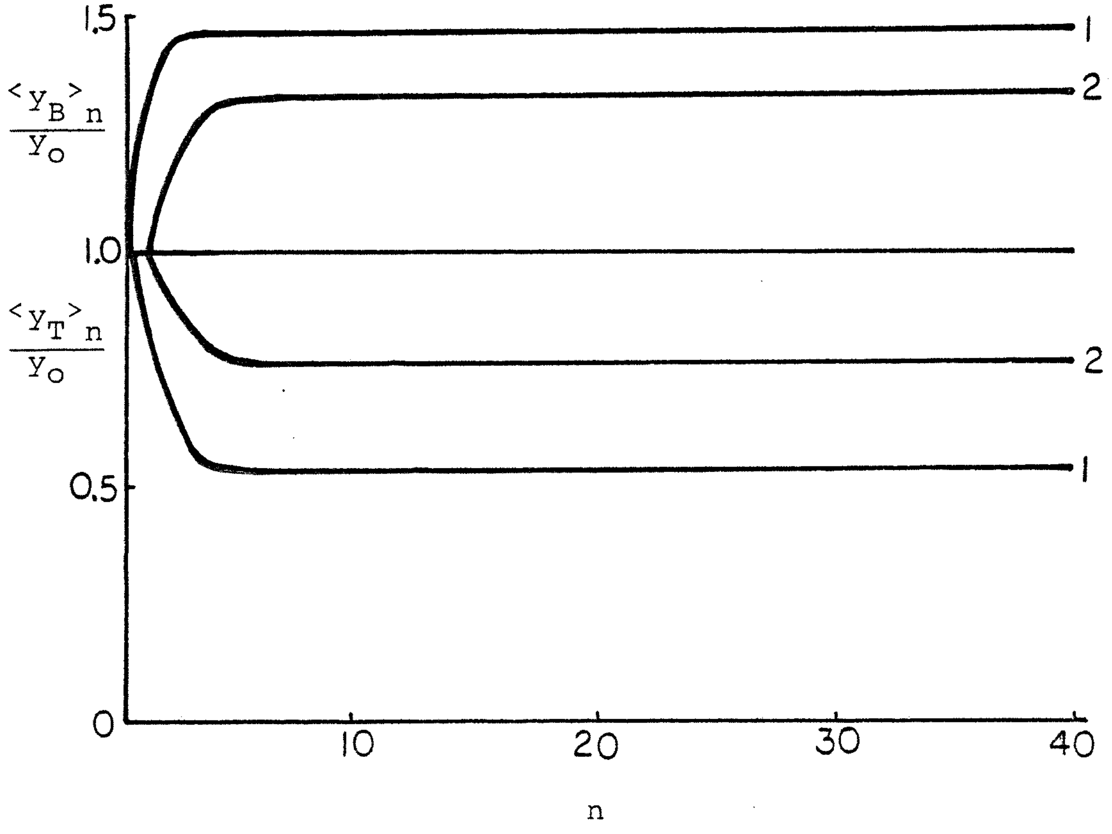


Fig. 39. Transient Concentrations as a Function of Number of Stages, Top and Bottom Feed.

meters) divided by the feed concentration for haemoglobin. The dimensionless concentration ratios for the bottom products are always greater than one, while those for the top product are always less than one. It can be seen from the graph that so long as the NT2 and NT4, which is the volume of top product and volume of bottom product respectively (volume of feed from bottom and volume of feed from the top), are equal or less than NT1 and NT3, upflow displacement and downflow displacement, that a reasonable separation can be obtained. The moment NT2 and NT4 are doubled, the top and bottom product concentration ratios both approach the dimensionless concentration ratio (i.e., $y_O=1$). From Figure 40, one can see that, if all the initial conditions are doubled including the number of theoretical plates, there is virtually no improvement in the separation obtained (see Tables 31 to 33). Since the time of operations are also doubled, it is probably not worthwhile doubling the initial conditions.

Table 34 gives the summary of the initial conditions for runs in which the volume of the top and bottom feed are varied simultaneously, and on Tables 35 to 39, the calculated results are given. On Table 40, and in Figure 41, it has been demonstrated that a considerable improvement in separation can be obtained if NT2<<NT1 and NT4<<NT3. The separation can be seen to increase for NT2=NT4=1 (batch) to a maximum when NT2=NT4=3 (approximately one-third the value of NT1 and NT3), and rapidly decreases exponentially. (See

| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 6 | 5 | 5 | 5 | 5 |
| 2 | 12 | 11 | 11 | 11 | 11 |

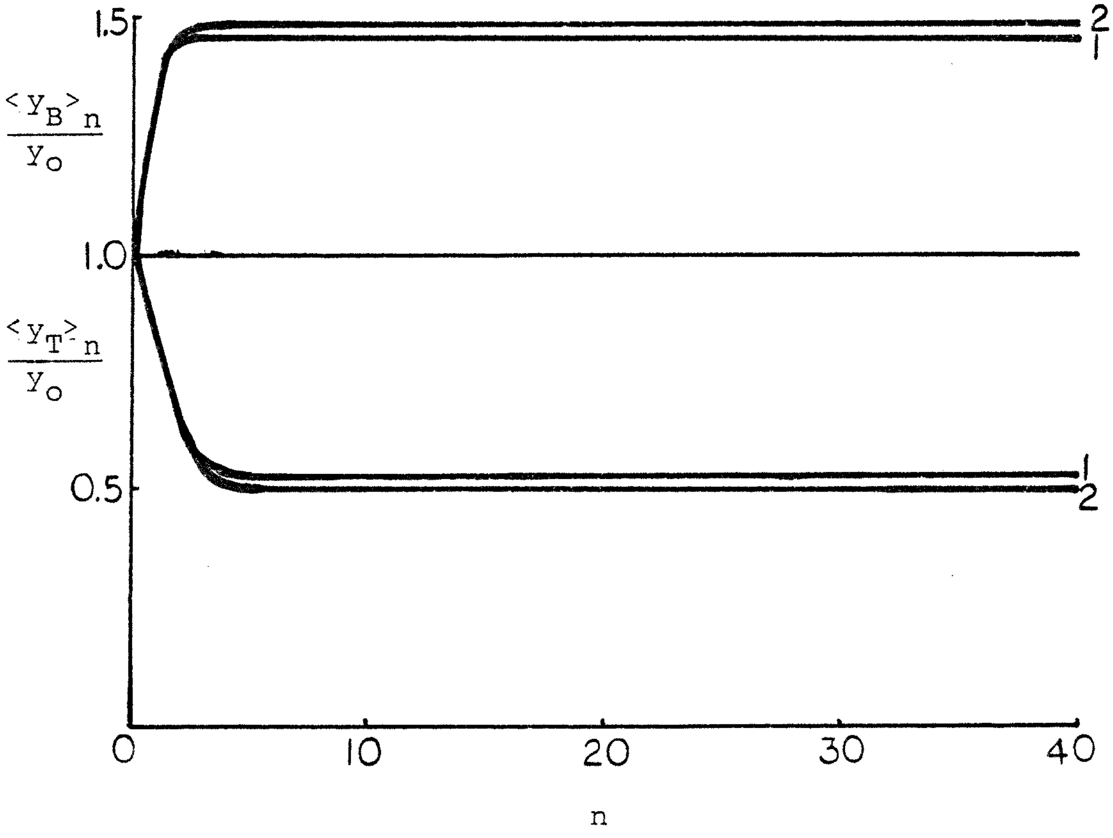


Fig. 40. Effect of Doubling Column Parameters.

| No. | Nz | NT1 | NT2 | NT3 | NT4 |
|---|---|---|---|---|---|
| 1 | 12 | 11 | 11 | 11 | 11 |
| 2 | 12 | 11 | 5 | 11 | 5 |
| 3 | 12 | 11 | 3 | 11 | 3 |
| 4 | 12 | 11 | 1 | 11 | 1 |

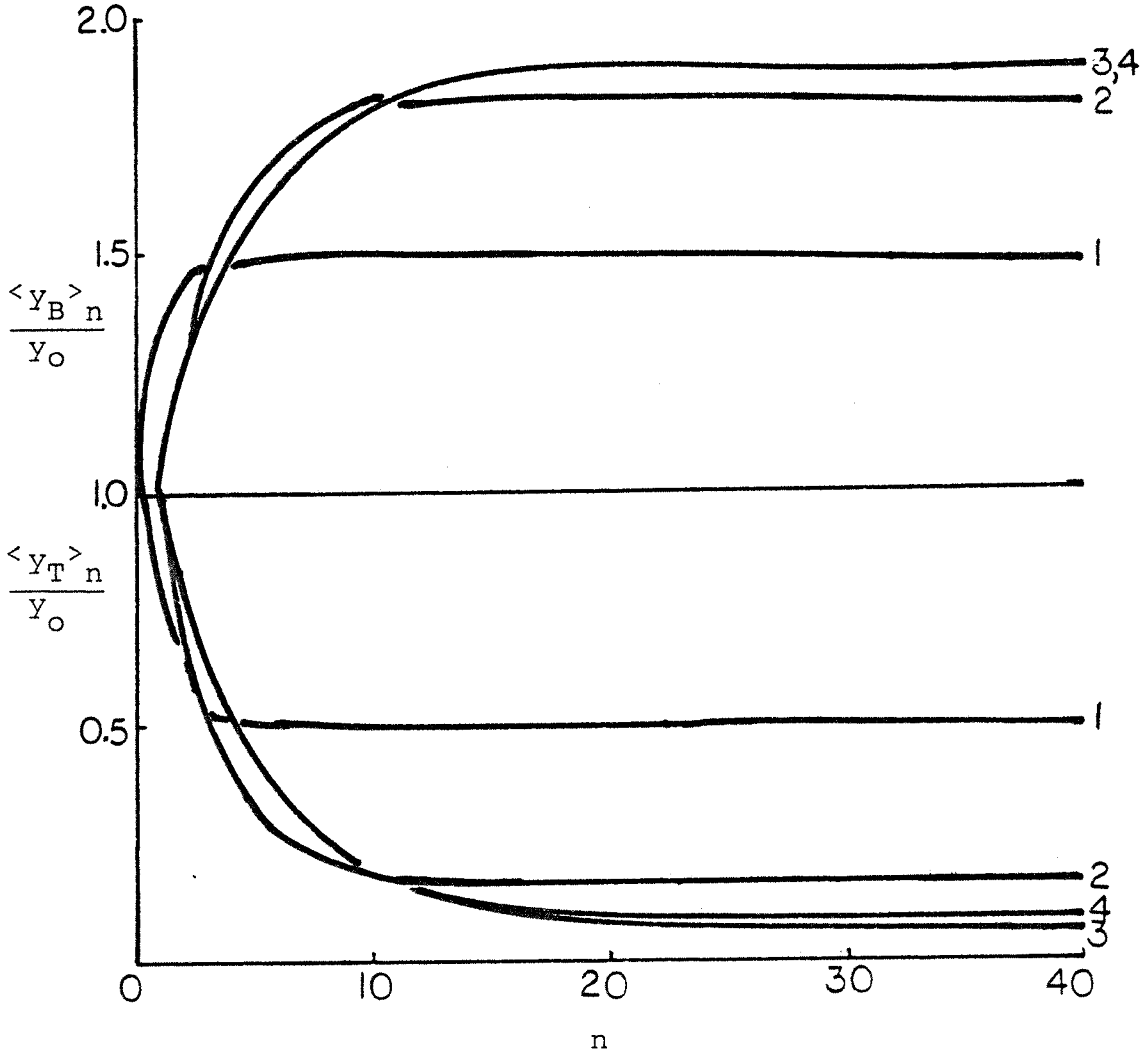


Fig. 41. Transient Concentrations as a Function of NT2 and NT4.

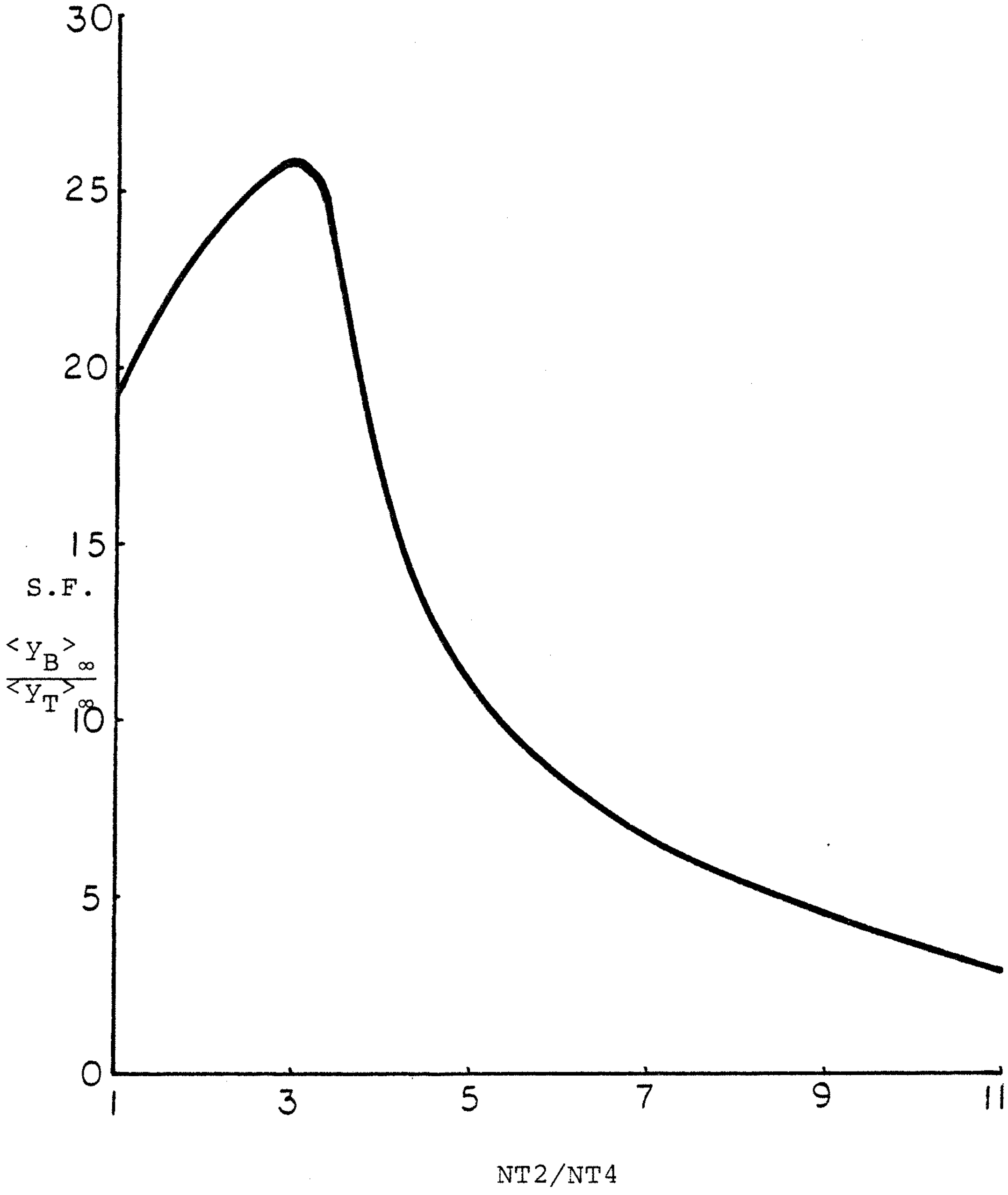


Fig. 42. Separation Factors as a Function of NT2 and NT4

Figure 42.) From the previous work in this area, batch process is known to offer the best separation. The reason for this behavior is yet to be substantiated theoretically and experimentally.

# Chapter VI

## SUMMARY OF CONCLUSIONS

This study further establishes the reliability of predicting column performance in terms of equilibrium theory. The simulated equation is based on the equations of change for the liquid-solid system with the diffusion term of negligible importance, and a linear equilibrium relation between the liquid and solid phases. The results show that a combination of equilibrium and STOP-GO algorithm give a good prediction of the concentration transients compared to previous methods of solution of the material balance relations of the extensive variables.

The results of the cycling zone adsorption simulation show very good agreement with results obtained by previous investigators, viz: limiting separation with increasing number of theoretical stages. This study, which extends the simulation to recycling of the product shows that increase in separation and decrease in the effect of $\beta$ can be seen as the number of stages increases.

The concentration transients obtained for the batch parapump system shows that the steady state concentrations in both top and bottom reservoirs are independent of the number of theoretical stages chosen provided $\alpha = 1$. For

$\alpha < 1$, both top and bottom reservoir concentrations are less than that obtained for $\alpha = 1$, and for $\alpha > 1$, an inverse separation is obtained. The optimum separation is also obtained when $\alpha$ is unity.

For the continuous parapump operation, the following were noted:

1. If high separation factor is desired, volume of top and bottom feed should be as small as possible, preferably approximately one-third the volume of top and bottom reservoir displacements.

2. If enrichment of rich product is desired, volume of bottom feed should be made as large as possible.

3. Literally, scaling up column parameters results in no improved separation.

## Nomenclature

$i$ = stage number

$I_i$ = isoelectric point of i

$IS_1$ = ionic strength in the bottom reservoir

$IS_2$ = ionic strength in the top reservoir

$J$ = transfer step

$k$ = x/y, equilibrium constant

$k_{p_1}$ = equilibrium constant at $pH = P_1$

$k_{p_2}$ = equilibrium constant at $pH = P_2$

$n$ = number of cycles of pump operation

$m_A^{(t)}$ = interface mass transfer

NNZ = number of stages or cells

NZ = number of stages or cells including top and bottom reservoir

$P_1$ = high pH level

$P_2$ = low pH level

$Q$ = reservoir displacement rate, $cm^3/s$

$V$ = volume of fluid phase per stage, $cm^3$

$\bar{V}$ = volume of solid phase per stage, $cm^3$

$V_B$ = bottom reservoir dead volume, $cm^3$

$V_T$ = top reservoir dead volume, $cm^3$

$x$ = concentration of solute in the solid phase, kg moles/$cm^3$

$y$ = concentration of solute in the fluid phase, kg moles/$cm^3$

$y_o$ = concentration of solute in the feed, kg moles/$cm^3$

$\langle y_B \rangle_n$ = average concentration of solute in the bottom reservoir at $n^{th}$ cycle, kg moles/$cm^3$

$<y_T>_n$ = average concentration of solute in the top reservoir at $n^{th}$ cycle, gm moles/$cm^3$

$<y_B>_\infty$ = steady state concentration of solute in the bottom reservoir, gm moles/$cm^3$

$<y_T>_\infty$ = steady state concentration of solute in the top reservoir, gm moles/$cm^3$

## Greek Letters

$\alpha$ = (reservoir displacement)/(column void volume)

$\beta$ = recycle ratio

## Other Symbols and Subscripts

< > = average value

n = cycle number

N = number of cells (used for analysis of batch parapump)

T = top

B = bottom

o = initial condition

## Nomenclature for Computer Program Input and Output Redefined

NT1 = Upflow displacement, cc.

NT2 = Volume of bottom feed

NT3 = Downflow displacement, cc.

NT4 = Volume of top feed, cc.

See Appendix A for other definitions.

# Appendix A

```
$JOB
C
C
C     EQUILIBRIUM THEORY CUM STOP-GO COMPUTER ALGORITHM FOR PH PARAPUM
C     SYSTEM WRITTEN IN FORTRAN IV FOR THE IBM 360 COMPUTER
C
C
C          DEPARTMENT OF CHEMICAL ENGINEERING
C                      N. J. I. T.
C                  NEWARK, NEW JERSEY
C
C
C     PROGRAM WRITTEN BY CHARLES O. KEROBO AND  H. T. CHEN
C
C
C
C     THIS PROGRAM CALCULATES THE SEPARATIONS ARISING FROM THE PH
C     RECUPERATIVE MODE OF PARAMETRIC PUMPING
C
C
C
C
C
C                 N O M E N C L A T U R E
C     NT1    =DOWNFLOW TIME INTERVAL
C     NT2    =BOTTOMFEED AND TOP PRODUCT WITHDRAWAL TIME INTERVAL
C     NT3    =UPFLOW TIME INTERVAL
C     NT4    =TOPFEED AND BOTTOM PRODUCT WITHDRAWAL TIME INTERVAL
C     Y      =CONCENTRATION PROFILE DURING DOWNFLOW
C     YY     =CONCENTRATION PROFILE DURING BOTTOMFEED AND TOP PRODUCT
C     YYY    =CONCENTRATION PROFILE DURING UPFLOW
C     YYYY   =CONCENTRATION PROFILE DURING TOPFEED AND BOTTOM PRODUCT
C     TP     =TOP PRODUCT
C     BP     =BOTTOM PRODUCT
C     BRES   =BOTTOM RESERVOIR CONCENTRATION
C     TRES   =TOP RESERVOIR CONCENTRATION
C     Q      =VOLUMETRIC FLOWRATE ,CC/SEC
C     AK8    =SPECIFIC RATE CONSTANT FOR PH=8
C     AK6    =SPECIFIC RATE CONSTANT FOR PH=6
C     YINT   =INITIAL SOLUTE CONCENTRATION
C     TVOL   =TOTAL VOLUME OF COLUMN
C     VOID   =COLUMN VOID VOLUME
C     HPH    =HIGH PH
C     LPH    =LOW PH
C     VTDEAD=DEAD VOLUME  OF TOP RESERVOIR
C     VBDEAD=DEAD VOLUME BOTTOM RESERVOIR
C     NFINL =NUMBER OF CYCLES
C
C
C
C
C
C
C
C
C
C     MAIN  PROGRAM
C
```

```
      REAL LPH
      DIMENSION Y(30,30),BP(50),BRES(50),YY(30,30),YYY(30,30),
     1YYYY(30,30),TP(50),TRES(50),PH(30,30),PPH(30,30)
      READ(5,65)D,AK8,AK6,YINT,TVOL,VOID,HPH,LPH,VTDEAD,VBDEAD
      READ(5,1001)NI
      DO 1000 IK=1,NI
 1001 FORMAT(I3)
      READ(5,66)NZ,NT1,NT2,NT3,NT4,NFINL
 65   FORMAT(12F5.2)
 66   FORMAT(12I5)
      NNZ=NZ-1
      AANZ=NZ-2
      V=(TVOL/AANZ)*VOID
      VB=(TVOL/AANZ)*(1.-VOID)
      PH(1,1)=LPH
      DO 560 I=2,NNZ
 560  PH(I,1)=HPH
      BRES(1)=YINT
      TRES(1)=YINT
      N=2
      YT=YINT
 500  J=1
      DO 10 I=1,NNZ
 10   Y(I,J)=YINT
      WRITE(6,102)
 102  FORMAT('1')
 550  WRITE(6,35)N
 35   FORMAT( // 5X,'CYCLE='I20)
 100  DO 30 J=2,NT1
      Y(1,J)=YT
      PH(1,J)=LPH
      PH(2,J)=LPH
      DO 800 I=3,NNZ
 800  PH(I,J)=PH(I-1,J-1)
      DO 32 I=2,NNZ
 32   Y(I,J)=ALPHA(V,VB,AK6,AK8,Y(I,J-1),Y(I-1,J-1),
     *PH(I,J),PH(I,J-1),LPH)
 30   CONTINUE
      SUM=0.
      NNT1=NT1-1
      ANNT1=NNT1
      DO 31 J=1,NNT1
 31   SUM=SUM+Y(NNZ,J)
      BRES(N)=((SUM/ANNT1)*(V*ANNT1)+VBDEAD*BRES(N-1))/(V*ANNT1+VBDEAD)
      WRITE(6,37)BRES(N)
 37   FORMAT( / 5X,'BOTTOM RESERVIOR CONCENTRATION=',E20.5)
      MM=1
      DO 50 I=2,NNZ
      M=NNZ-MM+1
      YY(I,1)=Y(M,NT1)
      PPH(I,1)=PH(M,NT1)
      MM=MM+1
 50   CONTINUE
      IF (NT2-1)552,552,562
  552 YY(1,1)=BRES(N)
      PPH(1,1)=HPH
      GO TO 570
  562 YY(1,1)=YINT
```

```
      PPH(1,1)=LPH
      DO 52 J=2,NT2
      PPH(1,J)=LPH
  52  YY(1,J)=YINT
      DO 43 J=2,NT2
      PPH(2,J)=LPH
      DO 810 I=2,NNZ
 810  PPH(I,J)=PPH(I-1,J-1)
      DO 42 I=2,NNZ
  42  YY(I,J)=ALPHA(V,VB,AK6,AK8,YY(I,J-1),YY(I-1,J-1),
     1PPH(I,J),PPH(I,J-1),LPH)
  43  CONTINUE
      SUM=0.
      NNT2=NT2-1
      ANNT2=NNT2
      DO 150 J=1,NNT2
 150  SUM=SUM+YY(NNZ,J)
      TP(N)=SUM/ANNT2
      WRITE(6,101)TP(N)
 101  FORMAT( // 5X,'TOP PRODUCT=',E20.5)
  570 DO 110 I=2,NNZ
      PPH(I,1)=PPH(I,NT2)
 110  YYY(I,1)=YY(I,NT2)
      DO 111 J=1,NT3
 111  YYY(1,J)=BRES(N)
      DO 120 J=2,NT3
      PPH(1,J)=HPH
      PPH(2,J)=HPH
      DO 820 I=3,NNZ
 820  PPH(I,J)=PPH(I-1,J-1)
      DO 122 I=2,NNZ
 122  YYY(I,J)=ALPHA(V,VB,AK6,AK8,YYY(I,J-1),YYY(I-1,J-1),
     *PPH(I,J),PPH(I,J-1),LPH)
 120  CONTINUE
      SUM=0.
      NNT3=NT3-1
      ANNT3=NNT3
      DO 431 J=1,NNT3
 431  SUM=SUM+YYY(NNZ,J)
      TRES(N)=((SUM/ANNT3)*(V*ANNT3)+VTDEAD*TRES(N-1))/(V*ANNT3+VTDEAD)
      WRITE(6,805)TRES(N)
 805  FORMAT( / 5X,'TOP RESERVIOR CONCENTRATION=',E20.5)
      MM=1
      DO 250 I=2,NNZ
      M=NNZ-MM+1
      YYYY(I,1)=YYY(M,NT3)
      PH(I,1)=PPH(M,NT3)
      MM=MM+1
 250  CONTINUE
      IF (NT4-1)580,580,590
  580 YYYY(1,1)=TRES(N)
      PPH(1,1)=LPH
      GO TO 595
  590 DO 252 J=1,NT4
 252  YYYY(1,J)=YINT
      DO 243 J=2,NT4
      PH(1,J)=HPH
      PH(2,J)=HPH
```

```
      DO 860 I=3,NNZ
  860 PH(I,J)=PH(I-1,J-1)
      DO 242 I=2,NNZ
  242 YYYY(I,J)=ALPHA(V,VB,AK6,AK8,YYYY(I,J-1),YYYY(I-1,J-1),
     *PH(I,J),PH(I,J-1),LPH)
  243 CONTINUE
      SUM=0.
      NNT4=NT4-1
      ANNT4=NNT4
      DO 300 J=1,NNT4
  300 SUM=SUM+YYYY(NNZ,J)
      BP(N)=SUM/ANNT4
      WRITE(6,301)BP(N)
  301 FORMAT( // 5X,'BOTTOM PRODUCT=',E20.5)
  595 YT=TRES(N)
      IF(N-NFINL)600,700,700
  600 N=N+1
      DO 520 I=2,NNZ
      Y(I,1)=YYYY(I,NT4)
  520 PH(I,1)=PH(I,NT4)
      PH(1,1)=LPH
      Y(1,1)=YT
      GO TO 550
  700 WRITE(6,5)
    5 FORMAT( 1H1,10X,'PH PARAMETRIC  PUMPING- BATCH PROCESS')
      WRITE(6,710)Q,AK8,AK6,YINT,TVOL,VOID,HPH,LPH,VTDEAD,VBDEAD
      WRITE(6,711)NZ,NT1,NT2,NT3,NT4,NFINL
  710 FORMAT('0','Q=',F5.2,5X,'AK8=',F5.2,5X,'AK6=',F5.2,//,
     11X,'YINT=',F5.2,5X,'TOTAL VOLUME=',F5.2,5X,'VOID VOLUME=',F5.2,
     2//,1X,'HIGH PH=',F5.2,5X,'LOW PH=',F5.2,//,
     31X,'DEAD VOLUME OF TOP RESERVOIR=',F5.2,5X,//,
     41X'DEAD VOLUME OF BOTTOM RESERVOIR=',F5.2)
  711 FORMAT('0','NZ=',I5,2X,'NT1=',I5,2X,'NT2=',I5,
     12X,'NT3=',I5,2X,'NT4=',I5,2X,//,'NFINAL=',I5)
      WRITE(6,740)
      DO 720 N=2,NFINL
      SF=BRES(N)/TRES(N)
      WRITE(6,730)N,TRES(N),BRES(N),SF
  740 FORMAT( // 4X,1HI,10X,4HTRES,11X,4HBRES,11X,2HSF)
  730 FORMAT( / I5,3F15.5)
  720 CONTINUE
      WRITE(6,1)
 1000 CONTINUE
    1 FORMAT('1')
      STOP
      END
C
C
C
C
C     FUNCTION FOR CALCULATING  THE CONCENTRATION PROFILE IN THE COLUM
C

      FUNCTION ALPHA(V,VB,AK6,AK8,YTP,YZP,PH,PHP,PHL)
      IF(PH-PHP)10,15,20
   10 A6=AK6
      A8=AK8
```

```
      GO TO 30
   20 A6=AK8
      A8=AK6
      GO TO 30
   15 IF(PH-PHL)40,40,50
   40 A6=AK6
      A8=AK6
      GO TO 30
   50 A6=AK8
      A8=AK8
   30 ALPHA=(1./(V+A6*VB))*(Y1P*A8*VB+V*YZP)
      RETURN
      END

$ENTRY
```

# Appendix B

```
$JOB
C
C
C      CYCLING ZONE ADSORPTION COMPUTER ALGORITHM FOR PH PARAPUMP
C      SYSTEM WRITTEN IN FORTRAN IV FOR THE IBM 360 COMPUTER
C
C
C           DEPARTMENT OF CHEMICAL ENGINEERING
C                    N. J. I. T.
C                 NEWARK, NEW JERSEY
C
C
C      PROGRAM WRITTEN BY CHARLES O. KERORO AND  H. T. CHEN
C
C
C
C      THIS PROGRAM CALCULATES THE SEPARATIONS ARISING FROM THE PH
C      RECUPERATIVE MODE OF CYCLING ZONE ADSORPTION
C
C
C
C
C
C                 N O M E N C L A T U R E
C      TP     =TOP PRODUCT
C      BP     =BOTTOM PRODUCT
C      BRES   =BOTTOM RESERVOIR CONCENTRATION
C      TRES   =TOP RESERVOTR CONCENTRATION
C      Q      =VOLUMETRIC FLOWRATE ,CC/SEC
C      AK8    =SPECIFIC RATE CONSTANT FOR PH=8
C      AK6    =SPECIFIC RATE CONSTANT FOR PH=6
C      YINT   =INITIAL SOLUTE CONCENTRATION
C      TVOL   =TOTAL VOLUME OF COLUMN
C      VOID   =COLUMN VOID VOLUME
C      HPH    =HIGH PH
C      LPH    =LOW PH
C      VTDEAD=DEAD VOLUME  OF TOP RESERVOIR
C      VBDEAD=DEAD VOLUME BOTTOM RESERVOIR
C      NFINL =NUMBER OF CYCLES
C
C
C
C
C
C
C
C
C
C      MAIN  PROGRAM
C
       REAL LPH
       DIMENSION Y(61,61),BP(20),BRES(61),YY(20,20),YYY(20,20),
      1YYYY(20,20),TP(20),TRES(61),PH(61,61),PPH(20,20)
       READ(5,65)Q,AK8,AK6,YINT,TVOL,VOID,HPH,LPH,VTDEAD,VBDEAD,BETA
       READ(5,66)NZ,NT1,NT2,NT3,NT4,NFINL
       WRITE(6,5)
     5 FORMAT( /// 10X,'*** PH PARAMETIC PUMPING -BATCH PROCESS ***' ///)
       WRITE(6,711)NZ,NT1,NT2,NT3,NT4,NFINL
```

```
       WRITE(6,710)Q,AK8,AK6,YINT,TVOL,VOID,HPH,LPH,VTDEAD,VBDEAD,BETA
 65    FORMAT(12F5.2) \
 66    FORMAT(12I5)
 710   FORMAT( // 5X,'Q=',F5.2,5X,'AK8=',F5.2,5X,'AK6=',F5.2,/,
      15X,'YINT=',F5.2,5X,'TOTAL VOLUME=',F5.2,5X,'VOID VOLUME=',F5.2,/
      25X,'HIGH PH=',F5.2,5X,'LOW PH=',F5.2,/,
      35X,'DEAD VOLUME OF THE FLUID PHASE=',F5.2,5X,
      4'DEAD VOLUME OF THE SOLID PHASE=',F5.2,
      55X,'RECYCLE RATIO=',F5.2)
 711   FORMAT( // 5X,'NZ=',I5,5X,'NT1=',I5,5X,'NT2=',I5,/,
      15X,'NT3=',I5,5X,'NT4=',I5,5X,'NFINAL=',I5)
       NNZ=NZ-1
       AANZ=NZ-2
       V=(TVOL/AANZ)*VOID
       VB=(TVOL/AANZ)*(1.-VOID)
       BRES(1)=YINT
       TRES(1)=YINT
       PH(1,1)=HPH
       Y(1,1)=YINT
       DO 1000 I=2,NNZ
       M=(-1)**I
       IF(M)1010,1020,1020
 1010  PH(I,1)=HPH
       Y(I,1)=YINT
       GO TO 1000
 1020  PH(I,1)=LPH
       Y(I,1)=YINT*(V+VB*AK8)/(V+VB*AK6)
 1000  CONTINUE
       PH(1,2)=LPH
       PH(1,3)=HPH
       Y(1,2)=YINT
       Y(1,3)=YINT
       J=2
 1070  DO 1800 I=2,NNZ
 1800  PH(I,J)=PH(I-1,J-1)
 350   WRITE(6,35)J
 35    FORMAT( // 5X,'CYCLE='I20)
       DO 32 I=2,NNZ
 32    Y(I,J)=ALPHA(V,VB,AK6,AK8,Y(I,J-1),Y(I-1,J-1),
      *PH(I,J),PH(I,J-1),LPH)
       Y(1,J)=Y(1,J-1)
       PH(1,J)=LPH
 210   WRITE(6,36)J,(I,PH(I,J),Y(I,J),I=1,NNZ)
 36    FORMAT( // 5X,'J=',I20/19X,'I',16X,'PH',16X,'Y'/(I20,2F20.5))
       BRES(J)=Y(NNZ,J-1)
       WRITE(6,37)BRES(J)
 37    FORMAT( / 5X,'BOTTOM RESERVIOR CONCENTRATION=',E20.5)
       PH(1,J)=LPH
       J=J+1
 1100  DO 1900 I=2,NNZ
 1900  PH(I,J)=PH(I-1,J-1)
       Y(1,J-1)=YINT*(1.-BETA)+BETA*TRES(J-2)
       DO 1032 I=2,NNZ
 1032  Y(I,J)=ALPHA(V,VB,AK6,AK8,Y(I,J-1),Y(I-1,J-1),
      *PH(I,J),PH(I,J-1),LPH)
       Y(1,J)=Y(1,J-1)
       PH(1,J)=HPH
       WRITE(6,36)J,(I,PH(I,J),Y(I,J),I=1,NNZ)
```

```
         TRES(J)=Y(NNZ,J-1)
         WRITE(6,805)TRES(J)
         IF(J-NFINL)1061,1065,1065
    1061 PH(1,J)=HPH
         J=J+1
    1060 Y(1,J-1)=YINT*(1.-BETA)+BETA*BRES(J-2)
         GO TO 1070
     805 FORMAT( / 5X,'TOP RESERVTOR CONCENTRATION=',F20.5)
    1065 STOP
         END


         FUNCTION ALPHA(V,VB,AK6,AK8,YTP,YZP,PH,PHP,PHL)
         IF(PH-PHP)10,15,20
      10 A6=AK6
         A8=AK8
         GO TO 30
      20 A6=AK8
         A8=AK6
         GO TO 30
      15 IF(PH-PHL)40,40,50
      40 A6=AK6
         A6=AK6
         GO TO 30
      50 A6=AK8
         A8=AK8
      30 ALPHA=(1./(V+A6*VB))*(YTP*A8*VB+V*YZP)
         RETURN
         END

 $ENTRY
```

## Appendix C

## Table 1

## CYCLING ZONE ADSORPTION

Conditions for Computational Algorithm with
Variable Number of Stages and Recycle Ratio

| Operating Variable | Value |
|---|---|
| Volumetric Flow Rate | 0.5 cc. |
| Specific Rate Constant for High pH | 0.07 |
| Specific Rate Constant for Low pH | 1.58 |
| Initial Feed Concentration (Normalized) | 1.00 gm. mole/cc. |
| Dead Volume of Top Reservoir | 5.00 cc. |
| Dead Volume of Bottom Reservoir | 5.00 cc. |
| Number of Cycles | 20 |

## Table 2

Recycle Ratio (β)

No. of Stages =16

| β | 0.00 | | 0.50 | | 1.00 | |
|---|---|---|---|---|---|---|
| n | Top Res.Conc | Bottom Res.Conc | Top Res.Conc | Bottom Res.Conc | Top Res.Conc | Bottom Res.Conc |
| 1 | 0.4561 | 1.0000 | 0.4561 | 1.0000 | 0.4561 | 1.0000 |
| 2 | 0.2348 | 1.5228 | 0.2348 | 1.5228 | 0.2348 | 1.5228 |
| 3 | 0.1467 | 1.7307 | 0.1467 | 1.7307 | 0.1467 | 1.7307 |
| 4 | 0.1117 | 1.8133 | 0.1117 | 1.8133 | 0.1117 | 1.8133 |
| 5 | 0.0978 | 1.8462 | 0.0978 | 1.8462 | 0.0978 | 1.8462 |
| 6 | 0.0923 | 1.8593 | 0.0923 | 1.8593 | 0.0923 | 1.8593 |
| 7 | 0.0912 | 1.8645 | 0.0912 | 1.8645 | 0.0901 | 1.8645 |
| 8 | 0.0892 | 1.8645 | 0.0895 | 1.8666 | 0.0892 | 1.8666 |
| 9 | 0.0895 | 1.8666 | 0.0907 | 1.8674 | 0.0895 | 1.8674 |
| 10 | 0.0910 | 1.8674 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |
| 11 | 0.0110 | 1.8995 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |
| 12 | 0.0110 | 1.8995 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |
| 13 | 0.0110 | 1.8995 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |
| 14 | 0.0110 | 1.8995 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |
| 15 | 0.0110 | 1.8995 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |
| 16 | 0.0110 | 1.8995 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |
| 17 | 0.0110 | 1.8995 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |
| 18 | 0.0110 | 1.8995 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |
| 19 | 0.0110 | 1.8995 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |
| 20 | 0.0110 | 1.8995 | 0.0907 | 1.8995 | 0.0904 | 1.8995 |

## Table 3

Recycle Ratio (β)

No. of Stages = 8

| β / n | 0.00 | | 0.50 | | 1.00 | |
|---|---|---|---|---|---|---|
| | Top Res.Conc | Bottom Res.Conc | Top Res.Conc | Bottom Res.Conc | Top Res.Conc | Bottom Res.Conc |
| 1 | 0.4561 | 1.0000 | 0.4561 | 1.0000 | 0.4561 | 1.0000 |
| 2 | 0.2348 | 1.5228 | 0.2348 | 1.5228 | 0.2348 | 1.5228 |
| 3 | 0.1467 | 1.7307 | 0.1467 | 1.7307 | 0.1467 | 1.7307 |
| 4 | 0.1117 | 1.8133 | 0.1179 | 1.8133 | 0.1117 | 1.8133 |
| 5 | 0.1162 | 1.8462 | 0.1083 | 1.8462 | 0.1162 | 1.8462 |
| 6 | 0.1176 | 1.8795 | 0.1050 | 1.8795 | 0.0991 | 1.8795 |
| 7 | 0.1176 | 1.8822 | 0.1039 | 1.8721 | 0.0924 | 1.8620 |
| 8 | 0.1177 | 1.8823 | 0.1034 | 1.8735 | 0.0901 | 1.8646 |
| 9 | 0.1170 | 1.8823 | 0.1036 | 1.8744 | 0.0892 | 1.8666 |
| 10 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |
| 11 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |
| 12 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |
| 13 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |
| 14 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |
| 15 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |
| 16 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |
| 17 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |
| 18 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |
| 19 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |
| 20 | 0.1170 | 1.8823 | 0.1036 | 1.8748 | 0.0895 | 1.8674 |

# Table 4

Recycle Ratio (β)

No. of Stages = 4

| β | 0.00 | | 0.50 | | 1.00 | |
|---|---|---|---|---|---|---|
| n | Top Res.Conc | Bottom Res.Conc | Top Res.Conc | Bottom Res.Conc | Top Res.Conc | Bottom Res.Conc |
| 1 | 0.4561 | 1.0000 | 0.4516 | 1.0000 | 0.4516 | 1.0000 |
| 2 | 0.2348 | 1.5228 | 0.2348 | 1.5228 | 0.2348 | 1.5228 |
| 3 | 0.2469 | 1.7307 | 0.2469 | 1.7307 | 0.2469 | 1.7307 |
| 4 | 0.2474 | 1.7519 | 0.1974 | 1.7519 | 0.1473 | 1.7519 |
| 5 | 0.2475 | 1.7525 | 1.1796 | 1.7832 | 0.1118 | 1.8139 |
| 6 | 0.2475 | 1.7525 | 0.1819 | 1.7994 | 0.1162 | 1.8462 |
| 7 | 0.2475 | 1.7525 | 0.1779 | 1.8160 | 0.0991 | 1.8795 |
| 8 | 0.2475 | 1.7525 | 0.1762 | 1.8123 | 0.0924 | 1.8620 |
| 9 | 0.2475 | 1.7525 | 0.1765 | 1.8130 | 0.0935 | 1.8646 |
| 10 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |
| 11 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |
| 12 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |
| 13 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |
| 14 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |
| 15 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |
| 16 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |
| 17 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |
| 18 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |
| 19 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |
| 20 | 0.2475 | 1.7525 | 0.1765 | 1.8215 | 0.0916 | 1.8646 |

# Table 5

Recycle Ratio (β)

No. of Stages = 1

| β | 0.00 | | 0.50 | | 1.00 | |
|---|---|---|---|---|---|---|
| n | Top Res.Conc | Bottom Res.Conc | Top Res.Conc | Bottom Res.Conc | Top Res.Conc | Bottom Res.Conc |
| 1 | 1.3206 | 0.6714 | 1.3206 | 0.6714 | 1.3206 | 0.6714 |
| 2 | 1.3233 | 0.6766 | 1.1630 | 0.6766 | 1.0027 | 0.6766 |
| 3 | 1.3233 | 0.6766 | 1.2182 | 0.7790 | 1.1130 | 0.8814 |
| 4 | 1.3233 | 0.6766 | 1.2421 | 0.7283 | 1.2068 | 0.6750 |
| 5 | 1.3233 | 0.6766 | 1.2268 | 0.7468 | 1.0433 | 0.7488 |
| 6 | 1.3233 | 0.6766 | 1.2397 | 0.7543 | 1.1455 | 0.8075 |
| 7 | 1.3233 | 0.6766 | 1.2410 | 0.7496 | 1.1486 | 0.7021 |
| 8 | 1.3233 | 0.6766 | 1.2408 | 0.7538 | 1.0802 | 0.7691 |
| 9 | 1.3233 | 0.6766 | 1.2431 | 0.7542 | 1.1461 | 0.7900 |
| 10 | 1.3233 | 0.6766 | 1.2433 | 0.7542 | 1.1245 | 0.7263 |
| 11 | 1.3233 | 0.6766 | 1.2422 | 0.7542 | 1.1245 | 0.7263 |
| 12 | 1.3233 | 0.6766 | 1.2422 | 0.7542 | 1.1245 | 0.7263 |
| 13 | 1.3233 | 0.6766 | 1.2422 | 0.7542 | 1.1245 | 0.7263 |
| 14 | 1.3233 | 0.6766 | 1.2422 | 0.7542 | 1.1245 | 0.7263 |
| 15 | 1.3233 | 0.6766 | 1.2422 | 0.7542 | 1.1245 | 0.7263 |
| 16 | 1.3233 | 0.6766 | 1.2422 | 0.7542 | 1.1245 | 0.7263 |
| 17 | 1.3233 | 0.6766 | 1.2422 | 0.7542 | 1.1245 | 0.7263 |
| 18 | 1.3233 | 0.6766 | 1.2422 | 0.7542 | 1.1245 | 0.7263 |
| 19 | 1.3233 | 0.6766 | 1.2422 | 0.7542 | 1.1245 | 0.7263 |
| 20 | 1.3233 | 0.6766 | 1.2422 | 0.7542 | 1.1245 | 0.7263 |

## Table 6

## CYCLING ZONE

Separation as a Function of Recycle Ratio (β)

| Number of Stages(n) | Recycle Ratio(β) | Steady State Product Concentration moles/cc. | | Separation Factor $(\langle y_B\rangle_\infty/\langle y_T\rangle_\infty)$ |
|---|---|---|---|---|
| | | Top | Bottom | |
| 16 | 0.00 | 0.09108 | 1.8995 | 20.8552 |
| | 0.50 | 0.09076 | 1.8995 | 20.9288 |
| | 1.00 | 0.0904 | 1.8995 | 21.0005 |
| 8 | 0.00 | 0.1170 | 1.8823 | 15.9923 |
| | 0.50 | 0.10361 | 1.8948 | 18.0947 |
| | 1.00 | 0.0895 | 1.8674 | 20.8648 |
| 4 | 0.00 | 0.24751 | 1.7525 | 7.0805 |
| | 0.50 | 0.17652 | 1.8215 | 10.3189 |
| | 1.00 | 0.09169 | 1.8988 | 20.3359 |
| 1 | 0.00 | 1.3233 | 0.6766 | 0.5112 |
| | 0.50 | 1.2433 | 0.7542 | 0.6066 |
| | 1.00 | 1.1245 | 0.7263 | 0.5833 |

## Table 7

## CONDITIONS FOR
## BATCH PARAMETRIC PUMPING

Effect of Reservoir Displacement/Number of Stages ($\alpha$) on Separation

| Operating Variable | Value |
|---|---|
| Volumetric Flow Rate | 0.5 cc. |
| Specific Rate Constant for High pH | 0.7 |
| Specific Rate Constant for Low pH | 1.58 |
| Feed Concentration (Normalized) | 1.00 gm. mole/cc. |
| Total Volume of Column | 30.00 cc. |
| Void Volume of Column | 0.75 |
| Dead Volume of Top Reservoir | 5.00 cc. |
| Dead Volume of Bottom Reservoir | 5.00 cc. |
| Volume of Bottom Feed (NT2) | 0.00 |
| Volume of Top Feed (NT4) | 0.00 |
| Numbers of Cycles | 40. |

## Table 8

$\alpha = 0.25$

Number of Stages=4

| I | TRES | BRES | SF |
|---|---|---|---|
| 2 | 0.82546 | 1.00000 | 1.21145 |
| 3 | 0.68540 | 1.00000 | 1.45901 |
| 4 | 0.57295 | 1.00000 | 1.74535 |
| 5 | 0.48262 | 1.00000 | 2.07203 |
| 6 | 0.41000 | 1.00000 | 2.43903 |
| 7 | 0.35157 | 1.00001 | 2.84443 |
| 8 | 0.30450 | 1.00002 | 3.28411 |
| 9 | 0.26655 | 1.00005 | 3.75183 |
| 10 | 0.23590 | 1.00010 | 4.23951 |
| 11 | 0.21110 | 1.00017 | 4.73778 |
| 12 | 0.19101 | 1.00026 | 5.23680 |
| 13 | 0.17468 | 1.00039 | 5.72707 |
| 14 | 0.16138 | 1.00055 | 6.20018 |
| 15 | 0.15050 | 1.00076 | 6.64939 |
| 16 | 0.14159 | 1.00100 | 7.06989 |
| 17 | 0.13424 | 1.00129 | 7.45885 |
| 18 | 0.12816 | 1.00163 | 7.81523 |
| 19 | 0.12311 | 1.00202 | 8.13944 |
| 20 | 0.11887 | 1.00246 | 8.43298 |
| 21 | 0.11531 | 1.00295 | 8.69808 |

## Table 8 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.11228 | 1.00350 | 8.93740 |
| 23 | 0.10969 | 1.00410 | 9.15378 |
| 24 | 0.10746 | 1.00476 | 9.35000 |
| 25 | 0.10552 | 1.00548 | 9.52871 |
| 26 | 0.10382 | 1.00624 | 9.69237 |
| 27 | 0.10231 | 1.00707 | 9.84315 |
| 28 | 0.10097 | 1.00795 | 9.98300 |
| 29 | 0.09976 | 1.00888 | 10.11357 |
| 30 | 0.09866 | 1.00987 | 10.23632 |
| 31 | 0.09765 | 1.01091 | 10.35245 |
| 32 | 0.09672 | 1.01200 | 10.46301 |
| 33 | 0.09586 | 1.01314 | 10.56885 |
| 34 | 0.09506 | 1.01434 | 10.67069 |
| 35 | 0.09430 | 1.01558 | 10.76916 |
| 36 | 0.09359 | 1.01686 | 10.86472 |
| 37 | 0.09292 | 1.01820 | 10.95783 |
| 38 | 0.09228 | 1.01957 | 11.04882 |
| 39 | 0.09167 | 1.02099 | 11.13797 |
| 40 | 0.09108 | 1.02245 | 11.22552 |

## Table 9

α = 0.5

No. of stages = 4

| I | TRES | BRES | SF |
|---|---|---|---|
| 2 | 0.82333 | 1.00000 | 1.21458 |
| 3 | 0.68332 | 1.00000 | 1.46344 |
| 4 | 0.57220 | 1.00015 | 1.74790 |
| 5 | 0.48385 | 1.00072 | 2.06824 |
| 6 | 0.41346 | 1.00188 | 2.42315 |
| 7 | 0.35725 | 1.00372 | 2.80958 |
| 8 | 0.31224 | 1.00629 | 3.22278 |
| 9 | 0.27610 | 1.00959 | 3.65666 |
| 10 | 0.24697 | 1.01361 | 4.10422 |
| 11 | 0.22340 | 1.01829 | 4.55817 |
| 12 | 0.20425 | 1.02360 | 5.01152 |
| 13 | 0.18861 | 1.02948 | 5.45815 |
| 14 | 0.17578 | 1.03587 | 5.89311 |
| 15 | 0.16517 | 1.04271 | 6.31276 |
| 16 | 0.15636 | 1.04993 | 6.71478 |
| 17 | 0.14898 | 1.05750 | 7.09803 |
| 18 | 0.14276 | 1.06534 | 7.46230 |
| 19 | 0.13747 | 1.07342 | 7.80812 |
| 20 | 0.13294 | 1.08169 | 8.13645 |
| 21 | 0.12903 | 1.09010 | 8.44862 |

Table 9 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.12561 | 1.09861 | 8.74603 |
| 23 | 0.12261 | 1.10721 | 9.03015 |
| 24 | 0.11995 | 1.11584 | 9.30237 |
| 25 | 0.11758 | 1.12450 | 9.56399 |
| 26 | 0.11544 | 1.13314 | 9.81620 |
| 27 | 0.11349 | 1.14175 | 10.06007 |
| 28 | 0.11172 | 1.15032 | 10.29650 |
| 29 | 0.11009 | 1.15882 | 10.52629 |
| 30 | 0.10858 | 1.16724 | 10.75014 |
| 31 | 0.10718 | 1.17557 | 10.96862 |
| 32 | 0.10586 | 1.18380 | 11.18222 |
| 33 | 0.10463 | 1.19192 | 11.39136 |
| 34 | 0.10347 | 1.19992 | 11.59639 |
| 35 | 0.10238 | 1.20780 | 11.79758 |
| 36 | 0.10134 | 1.21555 | 11.99520 |
| 37 | 0.10035 | 1.22317 | 12.18944 |
| 38 | 0.09940 | 1.23065 | 12.38049 |
| 39 | 0.09850 | 1.23798 | 12.56846 |
| 40 | 0.09763 | 1.24518 | 12.75350 |

## Table 10

$\alpha = 0.75$

No. of stages = 4

| I | TRES | BRES | SF |
|---|---|---|---|
| 2 | 0.83463 | 1.00000 | 1.19813 |
| 3 | 0.70253 | 1.00272 | 1.42730 |
| 4 | 0.59671 | 1.01456 | 1.70026 |
| 5 | 0.51169 | 1.03296 | 2.01871 |
| 6 | 0.44315 | 1.05592 | 2.38278 |
| 7 | 0.38766 | 1.08193 | 2.79090 |
| 8 | 0.34256 | 1.10983 | 3.23983 |
| 9 | 0.30571 | 1.13873 | 3.72490 |
| 10 | 0.27544 | 1.16798 | 4.24042 |
| 11 | 0.25043 | 1.19709 | 4.78009 |
| 12 | 0.22964 | 1.22569 | 5.33747 |
| 13 | 0.21223 | 1.25352 | 5.90641 |
| 14 | 0.19755 | 1.28041 | 6.48131 |
| 15 | 0.18509 | 1.30623 | 7.05733 |
| 16 | 0.17442 | 1.33092 | 7.63045 |
| 17 | 0.16523 | 1.35444 | 8.19746 |
| 18 | 0.15724 | 1.37677 | 8.75590 |
| 19 | 0.15025 | 1.39791 | 9.30396 |
| 20 | 0.14409 | 1.41790 | 9.84036 |
| 21 | 0.13863 | 1.43677 | 10.36419 |

## Table 10 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.13375 | 1.45455 | 10.87490 |
| 23 | 0.12938 | 1.47128 | 11.37211 |
| 24 | 0.12543 | 1.48701 | 11.85566 |
| 25 | 0.12185 | 1.50180 | 12.32542 |
| 26 | 0.11858 | 1.51568 | 12.78139 |
| 27 | 0.11560 | 1.52871 | 13.22357 |
| 28 | 0.11287 | 1.54093 | 13.65200 |
| 29 | 0.11036 | 1.55239 | 14.06677 |
| 30 | 0.10804 | 1.56313 | 14.46793 |
| 31 | 0.10590 | 1.57320 | 14.85555 |
| 32 | 0.10392 | 1.58263 | 15.22974 |
| 33 | 0.10208 | 1.59147 | 15.59059 |
| 34 | 0.10037 | 1.59974 | 15.93821 |
| 35 | 0.09878 | 1.60749 | 16.27275 |
| 36 | 0.09731 | 1.61475 | 16.59436 |
| 37 | 0.09593 | 1.62154 | 16.90321 |
| 38 | 0.09465 | 1.62790 | 17.19949 |
| 39 | 0.09345 | 1.63385 | 17.48337 |
| 40 | 0.09234 | 1.63942 | 17.75511 |

## Table 11

$\alpha = 1$

No. of stages = 8

| I | TRES | BRES | SF |
|---|---|---|---|
| 2 | 0.88896 | 1.00000 | 1.12491 |
| 3 | 0.79580 | 1.09311 | 1.37361 |
| 4 | 0.71384 | 1.18484 | 1.65981 |
| 5 | 0.64136 | 1.26690 | 1.97532 |
| 6 | 0.57723 | 1.33959 | 2.32072 |
| 7 | 0.52048 | 1.40393 | 2.69736 |
| 8 | 0.47026 | 1.46086 | 3.10648 |
| 9 | 0.42582 | 1.51124 | 3.54899 |
| 10 | 0.38649 | 1.55582 | 4.02545 |
| 11 | 0.35169 | 1.59527 | 4.53594 |
| 12 | 0.32090 | 1.63018 | 5.08001 |
| 13 | 0.29365 | 1.66107 | 5.65664 |
| 14 | 0.26953 | 1.68840 | 6.26414 |
| 15 | 0.24819 | 1.71259 | 6.90018 |
| 16 | 0.22931 | 1.73399 | 7.56174 |
| 17 | 0.21260 | 1.75293 | 8.24517 |
| 18 | 0.19781 | 1.76969 | 8.94624 |
| 19 | 0.18473 | 1.78452 | 9.66024 |
| 20 | 0.17315 | 1.79764 | 10.38206 |
| 21 | 0.16290 | 1.80925 | 11.10640 |

## Table 11 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.15383 | 1.81953 | 11.82785 |
| 23 | 0.14581 | 1.82862 | 12.54111 |
| 24 | 0.13871 | 1.83667 | 13.24111 |
| 25 | 0.13243 | 1.84378 | 13.92314 |
| 26 | 0.12687 | 1.85008 | 14.58302 |
| 27 | 0.12194 | 1.85566 | 15.21717 |
| 28 | 0.11759 | 1.86059 | 15.82259 |
| 29 | 0.11374 | 1.86495 | 16.39700 |
| 30 | 0.11033 | 1.86881 | 16.93877 |
| 31 | 0.10731 | 1.87222 | 17.44687 |
| 32 | 0.10464 | 1.87525 | 17.92097 |
| 33 | 0.10228 | 1.87792 | 18.36118 |
| 34 | 0.10019 | 1.88029 | 18.76801 |
| 35 | 0.09834 | 1.88238 | 19.14243 |
| 36 | 0.09670 | 1.88423 | 19.48570 |
| 37 | 0.09525 | 1.88587 | 19.79935 |
| 38 | 0.09397 | 1.88732 | 20.08495 |
| 39 | 0.09283 | 1.88860 | 20.34425 |
| 40 | 0.09183 | 1.88973 | 20.57910 |

## Table 12

$\alpha = 1$

No. of stages = 1

| I | TRES | BRES | SF |
|---|---|---|---|
| 2 | 0.73025 | 1.00000 | 1.36939 |
| 3 | 0.54067 | 1.26360 | 2.33711 |
| 4 | 0.40671 | 1.44995 | 3.56507 |
| 5 | 0.31206 | 1.58162 | 5.06836 |
| 6 | 0.24518 | 1.67466 | 6.83037 |
| 7 | 0.19792 | 1.74040 | 8.79337 |
| 8 | 0.16453 | 1.78665 | 10.86026 |
| 9 | 0.14094 | 1.81967 | 12.91120 |
| 10 | 0.12427 | 1.84286 | 14.82992 |
| 11 | 0.11249 | 1.85924 | 16.52856 |
| 12 | 0.10416 | 1.87082 | 17.96043 |
| 13 | 0.09828 | 1.87900 | 19.11841 |
| 14 | 0.09413 | 1.88478 | 20.02385 |
| 15 | 0.09119 | 1.88886 | 20.71339 |
| 16 | 0.08912 | 1.89175 | 21.22801 |
| 17 | 0.08765 | 1.89379 | 21.60634 |
| 18 | 0.08661 | 1.89522 | 21.88136 |
| 19 | 0.08588 | 1.89624 | 22.07970 |
| 20 | 0.08536 | 1.89696 | 22.22188 |
| 21 | 0.08500 | 1.89747 | 22.32339 |

## Table 12 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.08474 | 1.89782 | 22.39563 |
| 23 | 0.08456 | 1.89808 | 22.44696 |
| 24 | 0.08443 | 1.89825 | 22.48337 |
| 25 | 0.08434 | 1.89838 | 22.50916 |
| 26 | 0.08427 | 1.89847 | 22.52739 |
| 27 | 0.08423 | 1.89853 | 22.54030 |
| 28 | 0.08420 | 1.89857 | 22.54944 |
| 29 | 0.08417 | 1.89860 | 22.55539 |
| 30 | 0.08416 | 1.89862 | 22.56046 |
| 31 | 0.08415 | 1.89864 | 22.56369 |
| 32 | 0.08414 | 1.89865 | 22.56596 |
| 33 | 0.08413 | 1.89865 | 22.56758 |
| 34 | 0.08413 | 1.89866 | 22.56871 |
| 35 | 0.08412 | 1.89866 | 22.56952 |
| 36 | 0.08412 | 1.89866 | 22.57010 |
| 37 | 0.08412 | 1.89866 | 22.57050 |
| 38 | 0.08412 | 1.89866 | 22.57080 |
| 39 | 0.08412 | 1.89866 | 22.57100 |
| 40 | 0.08412 | 1.89866 | 22.57112 |

# Table 13

α = 1.25

No. of stages = 4

| I | TRES | BRES | SF |
|---|---|---|---|
| 2 | 1.10152 | 0.86759 | 0.78763 |
| 3 | 1.18294 | 0.76996 | 0.65089 |
| 4 | 1.24801 | 0.69196 | 0.55445 |
| 5 | 1.30001 | 0.62963 | 0.48433 |
| 6 | 1.34155 | 0.57982 | 0.43220 |
| 7 | 1.37476 | 0.54001 | 0.39281 |
| 8 | 1.40129 | 0.50820 | 0.36267 |
| 9 | 1.42248 | 0.48278 | 0.33939 |
| 10 | 1.43942 | 0.46247 | 0.32129 |
| 11 | 1.45296 | 0.44624 | 0.30712 |
| 12 | 1.46377 | 0.43327 | 0.29599 |
| 13 | 1.47241 | 0.42290 | 0.28722 |
| 14 | 1.47932 | 0.41462 | 0.28027 |
| 15 | 1.48483 | 0.40800 | 0.27478 |
| 16 | 1.48924 | 0.40270 | 0.27041 |
| 17 | 1.49276 | 0.39848 | 0.26694 |
| 18 | 1.49557 | 0.39510 | 0.26418 |
| 19 | 1.49782 | 0.39240 | 0.26198 |
| 20 | 1.49961 | 0.39024 | 0.26023 |
| 21 | 1.50104 | 0.38851 | 0.25883 |

## Table 13 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 1.50219 | 0.38713 | 0.25771 |
| 23 | 1.50310 | 0.38603 | 0.25682 |
| 24 | 1.50383 | 0.38515 | 0.25611 |
| 25 | 1.50441 | 0.38445 | 0.25555 |
| 26 | 1.50487 | 0.38388 | 0.25509 |
| 27 | 1.50524 | 0.38343 | 0.25473 |
| 28 | 1.50553 | 0.38307 | 0.25444 |
| 29 | 1.50576 | 0.38279 | 0.25421 |
| 30 | 1.50595 | 0.38255 | 0.25403 |
| 31 | 1.50609 | 0.38237 | 0.25388 |
| 32 | 1.50621 | 0.38222 | 0.25376 |
| 33 | 1.50630 | 0.38210 | 0.25367 |
| 34 | 1.50637 | 0.38201 | 0.25359 |
| 35 | 1.50643 | 0.38193 | 0.25354 |
| 36 | 1.50647 | 0.38187 | 0.25349 |
| 37 | 1.50651 | 0.38182 | 0.25345 |
| 38 | 1.50653 | 0.38178 | 0.25342 |
| 39 | 1.50655 | 0.38175 | 0.25339 |
| 40 | 1.50657 | 0.38172 | 0.25337 |

## Table 14

$\alpha = 1.5$

No. of stages = 4

| I | TRES | BRFS | SF |
|---|---|---|---|
| 2 | 1.13770 | 0.80886 | 0.71096 |
| 3 | 1.22758 | 0.68446 | 0.55757 |
| 4 | 1.28624 | 0.60327 | 0.46902 |
| 5 | 1.32452 | 0.55028 | 0.41546 |
| 6 | 1.34951 | 0.51570 | 0.38214 |
| 7 | 1.36581 | 0.49313 | 0.36105 |
| 8 | 1.37645 | 0.47839 | 0.34755 |
| 9 | 1.38339 | 0.46877 | 0.33886 |
| 10 | 1.38792 | 0.46250 | 0.33323 |
| 11 | 1.39087 | 0.45840 | 0.32958 |
| 12 | 1.39280 | 0.45572 | 0.32720 |
| 13 | 1.39405 | 0.45398 | 0.32565 |
| 14 | 1.39487 | 0.45284 | 0.32465 |
| 15 | 1.39540 | 0.45209 | 0.32399 |
| 16 | 1.39574 | 0.45160 | 0.32356 |
| 17 | 1.39596 | 0.45129 | 0.32328 |
| 18 | 1.39610 | 0.45108 | 0.32310 |
| 19 | 1.39620 | 0.45094 | 0.32298 |
| 20 | 1.39625 | 0.45085 | 0.32290 |
| 21 | 1.39629 | 0.45079 | 0.32285 |

## Table 14 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 1.39631 | 0.45075 | 0.32282 |
| 23 | 1.39632 | 0.45072 | 0.32279 |
| 24 | 1.39633 | 0.45071 | 0.32278 |
| 25 | 1.39633 | 0.45069 | 0.32277 |
| 26 | 1.39632 | 0.45068 | 0.32277 |
| 27 | 1.39632 | 0.45068 | 0.32276 |
| 28 | 1.39632 | 0.45067 | 0.32276 |
| 29 | 1.39631 | 0.45067 | 0.32276 |
| 30 | 1.39631 | 0.45067 | 0.32276 |
| 31 | 1.39630 | 0.45066 | 0.32275 |
| 32 | 1.39629 | 0.45066 | 0.32275 |
| 33 | 1.39629 | 0.45066 | 0.32275 |
| 34 | 1.39628 | 0.45065 | 0.32275 |
| 35 | 1.39627 | 0.45065 | 0.32275 |
| 36 | 1.39627 | 0.45065 | 0.32275 |
| 37 | 1.39626 | 0.45065 | 0.32275 |
| 38 | 1.39625 | 0.45065 | 0.32275 |
| 39 | 1.39625 | 0.45064 | 0.32275 |
| 40 | 1.39624 | 0.45064 | 0.32275 |

## Table 15

$\alpha = 1.75$

No. of stages = 4

| I | TRES | BRES | SF |
|---|---|---|---|
| 2 | 1.13161 | 0.79156 | 0.69949 |
| 3 | 1.20096 | 0.68173 | 0.56765 |
| 4 | 1.23750 | 0.62386 | 0.50413 |
| 5 | 1.25675 | 0.59336 | 0.47214 |
| 6 | 1.26689 | 0.57729 | 0.45567 |
| 7 | 1.27223 | 0.56882 | 0.44710 |
| 8 | 1.27505 | 0.56436 | 0.44262 |
| 9 | 1.27652 | 0.56200 | 0.44026 |
| 10 | 1.27730 | 0.56076 | 0.43902 |
| 11 | 1.27771 | 0.56011 | 0.43837 |
| 12 | 1.27792 | 0.55976 | 0.43802 |
| 13 | 1.27802 | 0.55957 | 0.43784 |
| 14 | 1.27808 | 0.55948 | 0.43775 |
| 15 | 1.27810 | 0.55942 | 0.43770 |
| 16 | 1.27811 | 0.55939 | 0.43767 |
| 17 | 1.27812 | 0.55938 | 0.43766 |
| 18 | 1.27811 | 0.55937 | 0.43765 |
| 19 | 1.27811 | 0.55936 | 0.43765 |
| 20 | 1.27811 | 0.55936 | 0.43764 |
| 21 | 1.27810 | 0.55935 | 0.43764 |

Table 15 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 1.27809 | 0.55935 | 0.43764 |
| 23 | 1.27809 | 0.55935 | 0.43764 |
| 24 | 1.27808 | 0.55934 | 0.43764 |
| 25 | 1.27807 | 0.55934 | 0.43764 |
| 26 | 1.27807 | 0.55934 | 0.43764 |
| 27 | 1.27806 | 0.55933 | 0.43764 |
| 28 | 1.27805 | 0.55933 | 0.43764 |
| 29 | 1.27805 | 0.55933 | 0.43764 |
| 30 | 1.27804 | 0.55932 | 0.43764 |
| 31 | 1.27803 | 0.55932 | 0.43764 |
| 32 | 1.27803 | 0.55932 | 0.43764 |
| 33 | 1.27802 | 0.55932 | 0.43764 |
| 34 | 1.27801 | 0.55931 | 0.43764 |
| 35 | 1.27801 | 0.55931 | 0.43764 |
| 36 | 1.27800 | 0.55931 | 0.43764 |
| 37 | 1.27799 | 0.55930 | 0.43764 |
| 38 | 1.27799 | 0.55930 | 0.43764 |
| 39 | 1.27798 | 0.55930 | 0.43764 |
| 40 | 1.27797 | 0.55930 | 0.43764 |

### Table 16

$\alpha = 2$

No. of stages = 4

| I | TRES | BRES | SF |
|---|---|---|---|
| 2 | 1.11523 | 0.79440 | 0.71232 |
| 3 | 1.16453 | 0.70643 | 0.60662 |
| 4 | 1.18563 | 0.66878 | 0.56407 |
| 5 | 1.19465 | 0.65267 | 0.54633 |
| 6 | 1.19850 | 0.64578 | 0.53882 |
| 7 | 1.20015 | 0.64282 | 0.53562 |
| 8 | 1.20085 | 0.64156 | 0.53425 |
| 9 | 1.20115 | 0.64101 | 0.53367 |
| 10 | 1.20127 | 0.64078 | 0.53342 |
| 11 | 1.20132 | 0.64068 | 0.53331 |
| 12 | 1.20133 | 0.64063 | 0.53327 |
| 13 | 1.20134 | 0.64061 | 0.53325 |
| 14 | 1.20134 | 0.64060 | 0.53324 |
| 15 | 1.20133 | 0.64059 | 0.53324 |
| 16 | 1.20133 | 0.64059 | 0.53323 |
| 17 | 1.20132 | 0.64058 | 0.53323 |
| 18 | 1.20131 | 0.64058 | 0.53323 |
| 19 | 1.20131 | 0.64058 | 0.53323 |
| 20 | 1.20130 | 0.64057 | 0.53323 |
| 21 | 1.20129 | 0.64057 | 0.53323 |

## Table 16 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 1.20129 | 0.64057 | 0.53323 |
| 23 | 1.20128 | 0.64056 | 0.53323 |
| 24 | 1.20127 | 0.64056 | 0.53323 |
| 25 | 1.20127 | 0.64055 | 0.53323 |
| 26 | 1.20126 | 0.64055 | 0.53323 |
| 27 | 1.20125 | 0.64055 | 0.53323 |
| 28 | 1.20125 | 0.64054 | 0.53323 |
| 29 | 1.20124 | 0.64054 | 0.53323 |
| 30 | 1.20123 | 0.64054 | 0.53323 |
| 31 | 1.20123 | 0.64053 | 0.53323 |
| 32 | 1.20122 | 0.64053 | 0.53323 |
| 33 | 1.20121 | 0.64053 | 0.53323 |
| 34 | 1.20121 | 0.64052 | 0.53323 |
| 35 | 1.20120 | 0.64052 | 0.53323 |
| 36 | 1.20119 | 0.64052 | 0.53323 |
| 37 | 1.20119 | 0.64051 | 0.53323 |
| 38 | 1.20118 | 0.64051 | 0.53323 |
| 39 | 1.20117 | 0.64050 | 0.53323 |
| 40 | 1.20116 | 0.64050 | 0.53323 |

## Table 17

$\alpha = 3$

No. of stages = 4

| I | TRES | BRES | SF |
|---|---|---|---|
| 2 | 1.06012 | 0.84445 | 0.79656 |
| 3 | 1.07271 | 0.81186 | 0.75683 |
| 4 | 1.07534 | 0.80503 | 0.74862 |
| 5 | 1.07589 | 0.80359 | 0.74691 |
| 6 | 1.07600 | 0.80329 | 0.74655 |
| 7 | 1.07602 | 0.80322 | 0.74647 |
| 8 | 1.07602 | 0.80320 | 0.74646 |
| 9 | 1.07601 | 0.80319 | 0.74646 |
| 10 | 1.07600 | 0.80319 | 0.74645 |
| 11 | 1.07600 | 0.80318 | 0.74645 |
| 12 | 1.07599 | 0.80318 | 0.74645 |
| 13 | 1.07596 | 0.80317 | 0.74645 |
| 14 | 1.07598 | 0.80317 | 0.74645 |
| 15 | 1.07597 | 0.80316 | 0.74645 |
| 16 | 1.07596 | 0.80316 | 0.74645 |
| 17 | 1.07596 | 0.80315 | 0.74645 |
| 18 | 1.07595 | 0.80315 | 0.74645 |
| 19 | 1.07594 | 0.80314 | 0.74645 |
| 20 | 1.07594 | 0.80314 | 0.74645 |
| 21 | 1.07593 | 0.80313 | 0.74645 |

## Table 17 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 1.07592 | 0.80313 | 0.74645 |
| 23 | 1.07592 | 0.80312 | 0.74645 |
| 24 | 1.07591 | 0.80312 | 0.74645 |
| 25 | 1.07590 | 0.80311 | 0.74645 |
| 26 | 1.07590 | 0.80311 | 0.74645 |
| 27 | 1.07589 | 0.80310 | 0.74645 |
| 28 | 1.07588 | 0.80310 | 0.74645 |
| 29 | 1.07588 | 0.80309 | 0.74645 |
| 30 | 1.07587 | 0.80309 | 0.74645 |
| 31 | 1.07587 | 0.80308 | 0.74645 |
| 32 | 1.07586 | 0.80308 | 0.74645 |
| 33 | 1.07585 | 0.80308 | 0.74645 |
| 34 | 1.07585 | 0.80307 | 0.74645 |
| 35 | 1.07584 | 0.80307 | 0.74645 |
| 36 | 1.07583 | 0.80306 | 0.74645 |
| 37 | 1.07583 | 0.80306 | 0.74645 |
| 38 | 1.07582 | 0.80305 | 0.74645 |
| 39 | 1.07581 | 0.80305 | 0.74645 |
| 40 | 1.07581 | 0.80304 | 0.74645 |

## Table 18

### Effect of Reservoir Displacement/Column Void Volume (α) on Separation

| $\alpha$ | $(\langle y_B \rangle_\infty / \langle y_T \rangle_\infty)$ |
|---|---|
| 0.25 | 10.6730 |
| 0.50 | 12.2700 |
| 1.00 | 20.8500 |
| 1.25 | 0.2580 |
| 1.50 | 0.3237 |
| 1.75 | 0.4384 |
| 2.00 | 0.5340 |
| 2.50 | 0.6670 |
| 3.00 | 0.7470 |

## Table 19

## CONTINUOUS PARAMETRIC PUMPING

### Conditions for Runs with Variable Volume of Bottom Feed (NT2)

| Operating Variable | Value |
|---|---|
| Volumetric Flow Rate | 0.5 cc. |
| Specific Rate Constant for High pH | 0.07 |
| Specific Rate Constant for Low pH | 1.58 |
| Feed Concentration (Normalized) | 1.00 gm. mole/cc. |
| Total Volume of Column | 30.00 cc. |
| Void Volume of Column | 0.75 |
| High pH | 8 |
| Low pH | 6 |
| Dead Volume of Top Reservoir | 5.00 cc. |
| Dead Volume of Bottom Reservoir | 5.00 cc. |
| Total Number of Stages (Nz) | 22 |
| Upflow Displacement (NT1) | 21 cc. |
| Downflow Displacement (NT3) | 21 cc. |
| Number of Cycles | 20 |

## Table 20

### Constant Volume of Top Feed (NT4 = 6 cc.)

### Volume of Bottom = 6 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99999 | 0.99999 | 0.99999 |
| 3 | 0.79271 | 1.20725 | 1.52294 |
| 4 | 0.63102 | 1.36891 | 2.16934 |
| 5 | 0.50492 | 1.49500 | 2.96090 |
| 6 | 0.40655 | 1.59335 | 3.91920 |
| 7 | 0.32983 | 1.67007 | 5.06347 |
| 8 | 0.26998 | 1.72990 | 6.40747 |
| 9 | 0.22330 | 1.77658 | 7.95590 |
| 10 | 0.18689 | 1.81298 | 9.70063 |
| 11 | 0.15849 | 1.84138 | 11.61802 |
| 12 | 0.13634 | 1.86352 | 13.66806 |
| 13 | 0.11906 | 1.88080 | 15.79667 |
| 14 | 0.10559 | 1.89428 | 17.94060 |
| 15 | 0.09507 | 1.90479 | 20.03487 |
| 16 | 0.08687 | 1.91299 | 22.02016 |
| 17 | 0.08048 | 1.91938 | 23.84953 |
| 18 | 0.07549 | 1.92437 | 25.49158 |
| 19 | 0.07160 | 1.92826 | 26.93120 |
| 20 | 0.06856 | 1.93130 | 28.16760 |

Table 20 (cont'd)

Volume of Bottom Feed = 11 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99932 | 0.99999 | 1.00067 |
| 3 | 0.65835 | 1.34144 | 2.03759 |
| 4 | 0.45745 | 1.61238 | 3.52471 |
| 5 | 0.33908 | 1.82750 | 5.38962 |
| 6 | 0.26933 | 1.99838 | 7.41983 |
| 7 | 0.22823 | 2.13417 | 9.35086 |
| 8 | 0.20402 | 2.24211 | 10.98977 |
| 9 | 0.18975 | 2.32792 | 12.26834 |
| 10 | 0.18134 | 2.39615 | 13.21334 |
| 11 | 0.17639 | 2.45040 | 13.89201 |
| 12 | 0.17347 | 2.49355 | 14.37448 |
| 13 | 0.17175 | 2.52788 | 14.71826 |
| 14 | 0.17074 | 2.55517 | 14.96549 |
| 15 | 0.17014 | 2.57688 | 15.14557 |
| 16 | 0.16979 | 2.59415 | 15.27864 |
| 17 | 0.16958 | 2.60788 | 15.37830 |
| 18 | 0.16946 | 2.61880 | 15.45385 |
| 19 | 0.16939 | 2.62749 | 15.51169 |
| 20 | 0.16935 | 2.63441 | 15.55640 |

## Table 20 (cont'd)

Volume of Bottom Feed = 16 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.98395 | 0.99999 | 1.01631 |
| 3 | 0.60162 | 1.39866 | 2.32483 |
| 4 | 0.45003 | 1.71572 | 3.81243 |
| 5 | 0.38993 | 1.96790 | 5.04680 |
| 6 | 0.36610 | 2.16848 | 5.92322 |
| 7 | 0.35665 | 2.32804 | 6.52751 |
| 8 | 0.35290 | 2.45495 | 6.95644 |
| 9 | 0.35142 | 2.55590 | 7.27311 |
| 10 | 0.35083 | 2.63621 | 7.51422 |
| 11 | 0.35060 | 2.70009 | 7.70143 |
| 12 | 0.35050 | 2.75091 | 7.84845 |
| 13 | 0.35047 | 2.79133 | 7.96461 |
| 14 | 0.35045 | 2.82348 | 8.05667 |
| 15 | 0.35045 | 2.84905 | 8.12978 |
| 16 | 0.35044 | 2.86939 | 8.18788 |
| 17 | 0.35044 | 2.88558 | 8.23410 |
| 18 | 0.35044 | 2.89846 | 8.27084 |
| 19 | 0.35044 | 2.90870 | 8.30006 |
| 20 | 0.35044 | 2.91684 | 8.32330 |

Table 20 (cont'd)

Volume of Bottom Feed = 21 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.91316 | 0.99999 | 1.09509 |
| 3 | 0.60466 | 1.41033 | 2.33242 |
| 4 | 0.52565 | 1.73673 | 3.30396 |
| 5 | 0.50541 | 1.99636 | 3.94998 |
| 6 | 0.50023 | 2.20289 | 4.40379 |
| 7 | 0.49890 | 2.36717 | 4.74479 |
| 8 | 0.49856 | 2.49785 | 5.01015 |
| 9 | 0.49847 | 2.60181 | 5.21957 |
| 10 | 0.49845 | 2.68450 | 5.38570 |
| 11 | 0.49844 | 2.75027 | 5.51772 |
| 12 | 0.49844 | 2.80259 | 5.62269 |
| 13 | 0.49844 | 2.84421 | 5.70619 |
| 14 | 0.49844 | 2.87732 | 5.77262 |
| 15 | 0.49844 | 2.90365 | 5.82545 |
| 16 | 0.49844 | 2.92460 | 5.86747 |
| 17 | 0.49844 | 2.94126 | 5.90090 |
| 18 | 0.49844 | 2.95451 | 5.92749 |
| 19 | 0.49844 | 2.96506 | 5.94864 |
| 20 | 0.49844 | 2.97344 | 5.96547 |

## Table 21

Constant Volume of Top Feed (NT4 = 11 cc.)

Volume of Bottom Feed = 6 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99999 | 0.99999 | 0.99999 |
| 3 | 0.79271 | 1.20724 | 1.52294 |
| 4 | 0.63102 | 1.32651 | 2.10216 |
| 5 | 0.50491 | 1.39449 | 2.76183 |
| 6 | 0.40655 | 1.43271 | 3.52406 |
| 7 | 0.32983 | 1.45377 | 4.40770 |
| 8 | 0.26998 | 1.46503 | 5.42645 |
| 9 | 0.22330 | 1.47076 | 6.58647 |
| 10 | 0.18689 | 1.47343 | 7.88388 |
| 11 | 0.15849 | 1.47444 | 9.30297 |
| 12 | 0.13634 | 1.47460 | 10.81563 |
| 13 | 0.11906 | 1.47435 | 12.38313 |
| 14 | 0.10558 | 1.47393 | 13.95987 |
| 15 | 0.09507 | 1.47348 | 15.49863 |
| 16 | 0.08687 | 1.47305 | 16.95654 |
| 17 | 0.08048 | 1.47267 | 18.29939 |
| 18 | 0.07549 | 1.47234 | 19.50441 |
| 19 | 0.07160 | 1.47208 | 20.56068 |
| 20 | 0.06856 | 1.47186 | 21.46761 |

## Table 21 (cont'd)

Volume of Bottom Feed = 11 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99932 | 0.99999 | 1.00067 |
| 3 | 0.65835 | 1.34144 | 2.03759 |
| 4 | 0.45745 | 1.54254 | 3.37205 |
| 5 | 0.33907 | 1.66097 | 4.89855 |
| 6 | 0.26932 | 1.73072 | 6.42616 |
| 7 | 0.22823 | 1.77179 | 7.76331 |
| 8 | 0.20401 | 1.79598 | 8.80336 |
| 9 | 0.18974 | 1.81023 | 9.54045 |
| 10 | 0.18134 | 1.81861 | 10.02902 |
| 11 | 0.17638 | 1.82356 | 10.33871 |
| 12 | 0.17346 | 1.82647 | 10.52944 |
| 13 | 0.17174 | 1.82819 | 10.64489 |
| 14 | 0.17073 | 1.82919 | 10.71396 |
| 15 | 0.17013 | 1.82979 | 10.75507 |
| 16 | 0.16978 | 1.83014 | 10.77939 |
| 17 | 0.16957 | 1.83034 | 10.79379 |
| 18 | 0.16945 | 1.83047 | 10.80236 |
| 19 | 0.16938 | 1.83054 | 10.80733 |
| 20 | 0.16934 | 1.83058 | 10.81027 |

## Table 21 (cont'd)

Volume of Bottom Feed = 16 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.98395 | 0.99999 | 1.01630 |
| 3 | 0.60162 | 1.39866 | 2.32483 |
| 4 | 0.45003 | 1.63417 | 3.63127 |
| 5 | 0.38992 | 1.77332 | 4.54786 |
| 6 | 0.36609 | 1.85553 | 5.06848 |
| 7 | 0.35664 | 1.90410 | 5.33897 |
| 8 | 0.35289 | 1.93280 | 5.47699 |
| 9 | 0.35141 | 1.94976 | 5.54841 |
| 10 | 0.35082 | 1.95978 | 5.58629 |
| 11 | 0.35059 | 1.96571 | 5.60691 |
| 12 | 0.35049 | 1.96920 | 5.61837 |
| 13 | 0.35046 | 1.97127 | 5.62486 |
| 14 | 0.35044 | 1.97250 | 5.62859 |
| 15 | 0.35044 | 1.97322 | 5.63075 |
| 16 | 0.35043 | 1.97365 | 5.63202 |
| 17 | 0.35043 | 1.97391 | 5.63276 |
| 18 | 0.35043 | 1.97405 | 5.63318 |
| 19 | 0.35043 | 1.97414 | 5.63342 |
| 20 | 0.35043 | 1.97420 | 5.63360 |

## Table 21 (cont'd)

Volume of Bottom Feed = 21 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.91316 | 0.99999 | 1.09509 |
| 3 | 0.60466 | 1.41033 | 2.33242 |
| 4 | 0.52565 | 1.65280 | 3.14432 |
| 5 | 0.50540 | 1.79608 | 3.55374 |
| 6 | 0.50022 | 1.88074 | 3.75982 |
| 7 | 0.49889 | 1.93077 | 3.87011 |
| 8 | 0.49855 | 1.96033 | 3.93205 |
| 9 | 0.49846 | 1.97780 | 3.96778 |
| 10 | 0.49844 | 1.98812 | 3.98867 |
| 11 | 0.49844 | 1.99422 | 4.00096 |
| 12 | 0.49843 | 1.99783 | 4.00820 |
| 13 | 0.49843 | 1.99996 | 4.01247 |
| 14 | 0.49843 | 2.00121 | 4.01499 |
| 15 | 0.49843 | 2.00196 | 4.01650 |
| 16 | 0.49843 | 2.00240 | 4.01737 |
| 17 | 0.49843 | 2.00266 | 4.01789 |
| 18 | 0.49843 | 2.00281 | 4.01820 |
| 19 | 0.49843 | 2.00290 | 4.01839 |
| 20 | 0.49843 | 2.00296 | 4.01849 |

## Table 22

Constant Volume of Top Feed (NT4 =16 cc.)

Volume of Bottom = 6 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99999 | 0.99999 | 0.99999 |
| 3 | 0.79271 | 1.20724 | 1.52293 |
| 4 | 0.63102 | 1.28411 | 2.03497 |
| 5 | 0.50491 | 1.31132 | 2.59711 |
| 6 | 0.40655 | 1.31988 | 3.24654 |
| 7 | 0.32983 | 1.32167 | 4.00717 |
| 8 | 0.26998 | 1.32117 | 4.89359 |
| 9 | 0.22330 | 1.32007 | 5.91162 |
| 10 | 0.18689 | 1.31890 | 7.05706 |
| 11 | 0.15849 | 1.31789 | 8.31524 |
| 12 | 0.13634 | 1.31707 | 9.66020 |
| 13 | 0.11906 | 1.31641 | 11.05659 |
| 14 | 0.10558 | 1.31588 | 12.46292 |
| 15 | 0.09507 | 1.31547 | 13.83666 |
| 16 | 0.08687 | 1.31515 | 15.13897 |
| 17 | 0.08048 | 1.31491 | 16.33914 |
| 18 | 0.07549 | 1.31470 | 17.41612 |
| 19 | 0.07160 | 1.31455 | 18.36044 |
| 20 | 0.06856 | 1.31444 | 19.17157 |

## Table 22 (cont'd)

Volume of Bottom Feed = 11 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99932 | 0.99999 | 1.00067 |
| 3 | 0.65835 | 1.34144 | 2.03758 |
| 4 | 0.45745 | 1.47270 | 3.21938 |
| 5 | 0.33907 | 1.52301 | 4.49170 |
| 6 | 0.26932 | 1.54221 | 5.72623 |
| 7 | 0.22823 | 1.54949 | 6.78928 |
| 8 | 0.20401 | 1.55222 | 7.60854 |
| 9 | 0.18974 | 1.55324 | 8.18605 |
| 10 | 0.18134 | 1.55359 | 8.56751 |
| 11 | 0.17638 | 1.55372 | 8.80884 |
| 12 | 0.17346 | 1.55375 | 8.95726 |
| 13 | 0.17174 | 1.55376 | 9.04700 |
| 14 | 0.17073 | 1.55376 | 9.10070 |
| 15 | 0.17013 | 1.55376 | 9.13262 |
| 16 | 0.16978 | 1.55375 | 9.15152 |
| 17 | 0.16957 | 1.55375 | 9.16272 |
| 18 | 0.16945 | 1.55375 | 9.16932 |
| 19 | 0.16938 | 1.55375 | 9.17321 |
| 20 | 0.16934 | 1.55375 | 9.17550 |

## Table 22 (cont'd)

Volume of Bottom Feed = 16 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.98395 | 0.99999 | 1.01630 |
| 3 | 0.60162 | 1.39866 | 2.32483 |
| 4 | 0.45003 | 1.55263 | 3.45006 |
| 5 | 0.38992 | 1.61209 | 4.13438 |
| 6 | 0.36609 | 1.63506 | 4.46627 |
| 7 | 0.35664 | 1.64392 | 4.60946 |
| 8 | 0.35289 | 1.64734 | 4.66809 |
| 9 | 0.35141 | 1.64867 | 4.69159 |
| 10 | 0.35082 | 1.64919 | 4.70095 |
| 11 | 0.35059 | 1.64938 | 4.70462 |
| 12 | 0.35049 | 1.64945 | 4.70607 |
| 13 | 0.35046 | 1.64948 | 4.70665 |
| 14 | 0.35044 | 1.64949 | 4.70688 |
| 15 | 0.35044 | 1.64949 | 4.70697 |
| 16 | 0.35043 | 1.64949 | 4.70700 |
| 17 | 0.35043 | 1.64949 | 4.70701 |
| 18 | 0.35043 | 1.64949 | 4.70702 |
| 19 | 0.35043 | 1.64949 | 4.70702 |
| 20 | 0.35043 | 1.64949 | 4.70702 |

## Table 22 (cont'd)

Volume of Bottom Feed = 21 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.91316 | 0.99999 | 1.09509 |
| 3 | 0.60466 | 1.41033 | 2.33242 |
| 4 | 0.52565 | 1.56887 | 2.98465 |
| 5 | 0.50540 | 1.63012 | 3.22538 |
| 6 | 0.50022 | 1.65378 | 3.30611 |
| 7 | 0.49889 | 1.66293 | 3.33324 |
| 8 | 0.49855 | 1.66646 | 3.34260 |
| 9 | 0.49846 | 1.66783 | 3.34593 |
| 10 | 0.49844 | 1.66835 | 3.34713 |
| 11 | 0.49844 | 1.66855 | 3.34758 |
| 12 | 0.49843 | 1.66863 | 3.34775 |
| 13 | 0.49843 | 1.66866 | 3.34781 |
| 14 | 0.49843 | 1.66867 | 3.34783 |
| 15 | 0.49843 | 1.66867 | 3.34783 |
| 16 | 0.49843 | 1.66868 | 3.34784 |
| 17 | 0.49843 | 1.66868 | 3.34784 |
| 18 | 0.49843 | 1.66868 | 3.34784 |
| 19 | 0.49843 | 1.66868 | 3.34784 |
| 20 | 0.49843 | 1.66868 | 3.34784 |

## Table 23

Constant Volume of Top Feed (NT4 = 21 cc.)

Volume of Bottom Feed = 6 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99999 | 1.17898 | 1.17899 |
| 3 | 0.79271 | 1.23628 | 1.55956 |
| 4 | 0.63102 | 1.24485 | 1.97275 |
| 5 | 0.50491 | 1.24450 | 2.46478 |
| 6 | 0.40655 | 1.24286 | 3.05708 |
| 7 | 0.32983 | 1.24130 | 3.76350 |
| 8 | 0.26998 | 1.24004 | 4.59308 |
| 9 | 0.22330 | 1.23905 | 5.54877 |
| 10 | 0.18689 | 1.23827 | 6.62561 |
| 11 | 0.15849 | 1.23766 | 7.80901 |
| 12 | 0.13634 | 1.23719 | 9.07431 |
| 13 | 0.11906 | 1.23682 | 10.38811 |
| 14 | 0.10558 | 1.23653 | 11.71138 |
| 15 | 0.09507 | 1.23630 | 13.00396 |
| 16 | 0.08687 | 1.23613 | 14.22935 |
| 17 | 0.08048 | 1.23599 | 15.35847 |
| 18 | 0.07549 | 1.23589 | 16.37201 |
| 19 | 0.07160 | 1.23580 | 17.26057 |
| 20 | 0.06856 | 1.23574 | 18.02365 |

## Table 23 (cont'd)

Volume of Bottom Feed = 11 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99932 | 1.28995 | 1.29083 |
| 3 | 0.65835 | 1.39196 | 2.11432 |
| 4 | 0.45745 | 1.41141 | 3.08541 |
| 5 | 0.33907 | 1.41493 | 4.17294 |
| 6 | 0.26932 | 1.41546 | 5.25562 |
| 7 | 0.22823 | 1.41547 | 6.20207 |
| 8 | 0.20401 | 1.41542 | 6.93800 |
| 9 | 0.18974 | 1.41538 | 7.45949 |
| 10 | 0.18134 | 1.41536 | 7.80519 |
| 11 | 0.17638 | 1.41534 | 8.02432 |
| 12 | 0.17346 | 1.41534 | 8.15930 |
| 13 | 0.17174 | 1.41533 | 8.24098 |
| 14 | 0.17073 | 1.41533 | 8.28988 |
| 15 | 0.17013 | 1.41533 | 8.31895 |
| 16 | 0.16978 | 1.41533 | 8.33618 |
| 17 | 0.16957 | 1.41533 | 8.34638 |
| 18 | 0.16945 | 1.41533 | 8.35239 |
| 19 | 0.16938 | 1.41533 | 8.35595 |
| 20 | 0.16934 | 1.41533 | 8.35803 |

## Table 23 (cont'd)

Volume of Bottom Feed = 16 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.98395 | 1.33584 | 1.35763 |
| 3 | 0.60162 | 1.45768 | 2.42294 |
| 4 | 0.45003 | 1.48141 | 3.29181 |
| 5 | 0.38992 | 1.48602 | 3.81107 |
| 6 | 0.36609 | 1.48691 | 4.06160 |
| 7 | 0.35664 | 1.48709 | 4.16970 |
| 8 | 0.35289 | 1.48712 | 4.21406 |
| 9 | 0.35141 | 1.48712 | 4.23189 |
| 10 | 0.35082 | 1.48713 | 4.23900 |
| 11 | 0.35059 | 1.48713 | 4.24182 |
| 12 | 0.35049 | 1.48713 | 4.24294 |
| 13 | 0.35046 | 1.48713 | 4.24339 |
| 14 | 0.35044 | 1.48713 | 4.24357 |
| 15 | 0.35044 | 1.48713 | 4.24364 |
| 16 | 0.35043 | 1.48713 | 4.24366 |
| 17 | 0.35043 | 1.48713 | 4.24367 |
| 18 | 0.35043 | 1.48713 | 4.24368 |
| 19 | 0.35043 | 1.48713 | 4.24368 |
| 20 | 0.35043 | 1.48713 | 4.24368 |

## Table 23 (cont'd)

Volume of Bottom Feed = 21 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.91316 | 1.34506 | 1.47298 |
| 3 | 0.60466 | 1.47102 | 2.43279 |
| 4 | 0.52565 | 1.49557 | 2.84520 |
| 5 | 0.50540 | 1.50035 | 2.96862 |
| 6 | 0.50022 | 1.50129 | 3.00125 |
| 7 | 0.49889 | 1.50147 | 3.00962 |
| 8 | 0.49855 | 1.50150 | 3.01173 |
| 9 | 0.49846 | 1.50151 | 3.01227 |
| 10 | 0.49844 | 1.50151 | 3.01241 |
| 11 | 0.49844 | 1.50151 | 3.01245 |
| 12 | 0.49843 | 1.50151 | 3.01246 |
| 13 | 0.49843 | 1.50151 | 3.01246 |
| 14 | 0.49843 | 1.50151 | 3.01246 |
| 15 | 0.49843 | 1.50151 | 3.01246 |
| 16 | 0.49843 | 1.50151 | 3.01246 |
| 17 | 0.49843 | 1.50151 | 3.01246 |
| 18 | 0.49843 | 1.50151 | 3.01246 |
| 19 | 0.49843 | 1.50151 | 3.01246 |
| 20 | 0.49843 | 1.50151 | 3.01246 |

## Table 24

## CONTINUOUS PARAPUMP

### Separation as a Function of NT2

| NT4 (cc) | NT2 (cc) | Steady State Product Concentration moles/cc. | | Separation Factor ($\langle y_B \rangle_\infty / \langle y_T \rangle_\infty$) |
|---|---|---|---|---|
| | | Top | Bottom | |
| 6 | 6 | 0.06856 | 1.9313 | 28.1676 |
| | 11 | 0.1693 | 2.6344 | 15.5564 |
| | 16 | 0.3504 | 2.9168 | 8.3233 |
| | 21 | 0.4984 | 2.9734 | 5.9654 |
| 11 | 6 | 0.0685 | 1.4718 | 21.4676 |
| | 11 | 0.1693 | 1.8305 | 10.8102 |
| | 16 | 0.3504 | 1.9742 | 5.6336 |
| | 21 | 0.4984 | 2.0029 | 4.0184 |
| 16 | 6 | 0.0685 | 1.3144 | 19.1715 |
| | 11 | 0.1693 | 1.5537 | 9.1755 |
| | 16 | 0.3504 | 1.6494 | 4.7070 |
| | 21 | 0.4984 | 1.6686 | 3.3478 |
| 21 | 6 | 0.0685 | 1.2357 | 18.0236 |
| | 11 | 0.1693 | 1.4153 | 8.3580 |
| | 16 | 0.3504 | 1.4871 | 4.2436 |
| | 21 | 0.4984 | 1.5015 | 3.0124 |

## Table 25

## CONTINUOUS PARAMETRIC PUMPING

### Conditions for Runs with Variable Volume of Top Feed (NT4)

| Operating Variable | Value |
|---|---|
| Volumetric Flow Rate | 0.5 cc/s |
| Specific Rate Constant for High pH | 0.07 |
| Specific Rate Constant for Low pH | 1.58 |
| Feed Concentration (Normalized) | 1.00 gm. mole/cc. |
| Total Volume of Column | 30.00 cc. |
| Void Volume of Column | 0.75 |
| High pH | 8 |
| Low pH | 6 |
| Dead Volume of Top Reservoir | 5.00 cc. |
| Dead Volume of Bottom Reservoir | 5.00 cc. |
| Total Number of Stages (Nz) | 22 |
| Upflow displacement (NT1) | 21 cc. |
| Downflow Displacement (NT3) | 21 cc. |
| Number of Cycles | 20 |

## Table 26

Constant Volume of Bottom Feed (NT2 = 6 cc.)

Volume of Top Feed = 6 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99999 | 0.99999 | 0.99999 |
| 3 | 0.79271 | 1.20725 | 1.52294 |
| 4 | 0.63102 | 1.36891 | 2.16934 |
| 5 | 0.50492 | 1.49500 | 2.96090 |
| 6 | 0.40655 | 1.59335 | 3.91920 |
| 7 | 0.32983 | 1.67007 | 5.06347 |
| 8 | 0.26998 | 1.72990 | 6.40747 |
| 9 | 0.22330 | 1.77658 | 7.95590 |
| 10 | 0.18689 | 1.81298 | 9.70063 |
| 11 | 0.15849 | 1.84138 | 11.61802 |
| 12 | 0.13634 | 1.86352 | 13.66806 |
| 13 | 0.11906 | 1.88080 | 15.79667 |
| 14 | 0.10559 | 1.89028 | 17.94060 |
| 15 | 0.09507 | 1.90479 | 20.03487 |
| 16 | 0.08687 | 1.91299 | 22.02016 |
| 17 | 0.08048 | 1.91938 | 23.84953 |
| 18 | 0.07549 | 1.92437 | 25.49158 |
| 19 | 0.07160 | 1.92826 | 26.93120 |
| 20 | 0.06856 | 1.93130 | 28.16760 |

# Table 26 (cont'd)

Volume of Top Feed = 11 cc.

| T | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99999 | 0.99999 | 0.99999 |
| 3 | 0.79271 | 1.20724 | 1.52294 |
| 4 | 0.63102 | 1.32651 | 2.10216 |
| 5 | 0.50491 | 1.39449 | 2.76183 |
| 6 | 0.40655 | 1.43271 | 3.52406 |
| 7 | 0.32983 | 1.45377 | 4.40770 |
| 8 | 0.26998 | 1.46503 | 5.42645 |
| 9 | 0.22330 | 1.47076 | 6.58647 |
| 10 | 0.18689 | 1.47343 | 7.88388 |
| 11 | 0.15849 | 1.47444 | 9.30297 |
| 12 | 0.13634 | 1.47460 | 10.81563 |
| 13 | 0.11906 | 1.47435 | 12.38313 |
| 14 | 0.10558 | 1.47393 | 13.95987 |
| 15 | 0.09507 | 1.47348 | 15.49863 |
| 16 | 0.08687 | 1.47305 | 16.95654 |
| 17 | 0.08048 | 1.47267 | 18.29939 |
| 18 | 0.07549 | 1.47234 | 19.50441 |
| 19 | 0.07160 | 1.47208 | 20.56068 |
| 20 | 0.06856 | 1.47186 | 21.46761 |

## Table 26 (cont'd)

Volume of Top Feed = 16 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99999 | 0.99999 | 0.99999 |
| 3 | 0.79271 | 1.20724 | 1.52293 |
| 4 | 0.63102 | 1.28411 | 2.03497 |
| 5 | 0.50491 | 1.31132 | 2.59711 |
| 6 | 0.40655 | 1.31988 | 3.24654 |
| 7 | 0.32983 | 1.32167 | 4.00717 |
| 8 | 0.26998 | 1.32117 | 4.89359 |
| 9 | 0.22330 | 1.32007 | 5.91162 |
| 10 | 0.18669 | 1.31890 | 7.05706 |
| 11 | 0.15849 | 1.31789 | 8.31524 |
| 12 | 0.13634 | 1.31707 | 9.66020 |
| 13 | 0.11906 | 1.31641 | 11.05659 |
| 14 | 0.10558 | 1.31588 | 12.46292 |
| 15 | 0.09507 | 1.31547 | 13.83666 |
| 16 | 0.08687 | 1.31515 | 15.13897 |
| 17 | 0.08048 | 1.31491 | 16.33914 |
| 18 | 0.07549 | 1.31470 | 17.41612 |
| 19 | 0.07160 | 1.31455 | 18.36044 |
| 20 | 0.06856 | 1.31444 | 19.17157 |

## Table 26 (cont'd)

Volume of Top Feed = 21 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99999 | 1.17898 | 1.17899 |
| 3 | 0.79271 | 1.23628 | 1.55956 |
| 4 | 0.63102 | 1.24485 | 1.97275 |
| 5 | 0.50491 | 1.24450 | 2.46478 |
| 6 | 0.40655 | 1.24266 | 3.05708 |
| 7 | 0.32983 | 1.24130 | 3.76350 |
| 8 | 0.26998 | 1.24004 | 4.59308 |
| 9 | 0.22330 | 1.23905 | 5.54877 |
| 10 | 0.18689 | 1.23827 | 6.62561 |
| 11 | 0.15849 | 1.23766 | 7.80901 |
| 12 | 0.13634 | 1.23719 | 9.07431 |
| 13 | 0.11906 | 1.23682 | 10.38811 |
| 14 | 0.10558 | 1.23653 | 11.71138 |
| 15 | 0.09507 | 1.23630 | 13.00396 |
| 16 | 0.08687 | 1.23613 | 14.22935 |
| 17 | 0.08048 | 1.23599 | 15.35847 |
| 18 | 0.07549 | 1.23589 | 16.37201 |
| 19 | 0.07160 | 1.23580 | 17.26057 |
| 20 | 0.06856 | 1.23574 | 18.02365 |

## Table 27

Constant Volume of Bottom Feed (NT2 = 11 cc.)

Volume of Top Feed = 6 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99932 | 0.99999 | 1.00067 |
| 3 | 0.65835 | 1.34144 | 2.03759 |
| 4 | 0.45745 | 1.61238 | 3.52471 |
| 5 | 0.33908 | 1.82750 | 5.38962 |
| 6 | 0.26933 | 1.99838 | 7.41983 |
| 7 | 0.22823 | 2.13417 | 9.35086 |
| 8 | 0.20402 | 2.24211 | 10.98977 |
| 9 | 0.18975 | 2.32792 | 12.26834 |
| 10 | 0.18134 | 2.39615 | 13.21334 |
| 11 | 0.17639 | 2.45040 | 13.89201 |
| 12 | 0.17347 | 2.49355 | 14.37443 |
| 13 | 0.17175 | 2.52788 | 14.71826 |
| 14 | 0.17074 | 2.55517 | 14.96549 |
| 15 | 0.17014 | 2.57688 | 15.14557 |
| 16 | 0.16979 | 2.59415 | 15.27864 |
| 17 | 0.16958 | 2.60788 | 15.37830 |
| 18 | 0.16946 | 2.61880 | 15.45385 |
| 19 | 0.16939 | 2.62749 | 15.51169 |
| 20 | 0.16935 | 2.63441 | 15.55640 |

## Table 27 (cont'd)

Volume of Top Feed = 11 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99932 | 0.99999 | 1.00067 |
| 3 | 0.65835 | 1.34144 | 2.03759 |
| 4 | 0.45745 | 1.54254 | 3.37205 |
| 5 | 0.33907 | 1.66097 | 4.89855 |
| 6 | 0.26932 | 1.73072 | 6.42616 |
| 7 | 0.22823 | 1.77179 | 7.76331 |
| 8 | 0.20401 | 1.79598 | 8.80336 |
| 9 | 0.18974 | 1.81023 | 9.54045 |
| 10 | 0.18134 | 1.81861 | 10.02902 |
| 11 | 0.17638 | 1.82356 | 10.33871 |
| 12 | 0.17346 | 1.82647 | 10.52944 |
| 13 | 0.17174 | 1.82819 | 10.64489 |
| 14 | 0.17073 | 1.82919 | 10.71396 |
| 15 | 0.17013 | 1.82979 | 10.75507 |
| 16 | 0.16978 | 1.83014 | 10.77939 |
| 17 | 0.16957 | 1.83034 | 10.79379 |
| 18 | 0.16945 | 1.83047 | 10.80236 |
| 19 | 0.16938 | 1.83054 | 10.80733 |
| 20 | 0.16934 | 1.83058 | 10.81027 |

## Table 27 (cont'd)

Volume of Top Feed = 16 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99932 | 0.99999 | 1.00067 |
| 3 | 0.65835 | 1.34144 | 2.03758 |
| 4 | 0.45745 | 1.47270 | 3.21938 |
| 5 | 0.33907 | 1.52301 | 4.49170 |
| 6 | 0.26932 | 1.54221 | 5.72623 |
| 7 | 0.22823 | 1.54949 | 6.78928 |
| 8 | 0.20401 | 1.55222 | 7.60854 |
| 9 | 0.18974 | 1.55324 | 8.18605 |
| 10 | 0.18134 | 1.55359 | 8.56751 |
| 11 | 0.17638 | 1.55372 | 8.80884 |
| 12 | 0.17346 | 1.55375 | 8.95726 |
| 13 | 0.17174 | 1.55376 | 9.04700 |
| 14 | 0.17073 | 1.55376 | 9.10070 |
| 15 | 0.17013 | 1.55376 | 9.13262 |
| 16 | 0.16978 | 1.55375 | 9.15152 |
| 17 | 0.16957 | 1.55375 | 9.16272 |
| 18 | 0.16945 | 1.55375 | 9.16932 |
| 19 | 0.16938 | 1.55375 | 9.17321 |
| 20 | 0.16934 | 1.55375 | 9.17550 |

## Table 27 (cont'd)

Volume of Top Feed = 21 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99932 | 1.28995 | 1.29083 |
| 3 | 0.65835 | 1.39196 | 2.11432 |
| 4 | 0.45745 | 1.41141 | 3.08541 |
| 5 | 0.33907 | 1.41493 | 4.17294 |
| 6 | 0.26932 | 1.41546 | 5.25562 |
| 7 | 0.22823 | 1.41547 | 6.20207 |
| 8 | 0.20401 | 1.41542 | 6.93800 |
| 9 | 0.18974 | 1.41538 | 7.45949 |
| 10 | 0.18134 | 1.41536 | 7.80519 |
| 11 | 0.17638 | 1.41534 | 8.02432 |
| 12 | 0.17346 | 1.41534 | 8.15930 |
| 13 | 0.17174 | 1.41533 | 8.24098 |
| 14 | 0.17073 | 1.41533 | 8.28983 |
| 15 | 0.17013 | 1.41533 | 8.31895 |
| 16 | 0.16978 | 1.41533 | 8.33618 |
| 17 | 0.16957 | 1.41533 | 8.34638 |
| 18 | 0.16945 | 1.41533 | 8.35239 |
| 19 | 0.16938 | 1.41533 | 8.35595 |
| 20 | 0.16934 | 1.41533 | 8.35803 |

## Table 28

## CONSTANT VOLUME OF BOTTOM FEED (NT2 = 16 cc.)

Volume of Top Feed = 6 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.98395 | 0.99999 | 1.01631 |
| 3 | 0.60162 | 1.39866 | 2.32483 |
| 4 | 0.45003 | 1.71572 | 3.81243 |
| 5 | 0.38993 | 1.96790 | 5.04680 |
| 6 | 0.36610 | 2.16848 | 5.92322 |
| 7 | 0.35665 | 2.32804 | 6.52751 |
| 8 | 0.35290 | 2.45495 | 6.95644 |
| 9 | 0.35142 | 2.55590 | 7.27311 |
| 10 | 0.35083 | 2.63621 | 7.51422 |
| 11 | 0.35060 | 2.70009 | 7.70143 |
| 12 | 0.35050 | 2.75091 | 7.84845 |
| 13 | 0.35047 | 2.79133 | 7.96461 |
| 14 | 0.35045 | 2.82348 | 8.05667 |
| 15 | 0.35045 | 2.84905 | 8.12978 |
| 16 | 0.35044 | 2.86939 | 8.18788 |
| 17 | 0.35044 | 2.88558 | 8.23410 |
| 18 | 0.35044 | 2.89846 | 8.27084 |
| 19 | 0.35044 | 2.90870 | 8.30006 |
| 20 | 0.35044 | 2.91664 | 8.32330 |

## Table 28 (cont'd)

Volume of Top Feed = 11 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.98395 | 0.99999 | 1.01630 |
| 3 | 0.60162 | 1.39866 | 2.32483 |
| 4 | 0.45003 | 1.63417 | 3.63127 |
| 5 | 0.38992 | 1.77332 | 4.54786 |
| 6 | 0.36609 | 1.85553 | 5.06848 |
| 7 | 0.35664 | 1.90410 | 5.33897 |
| 8 | 0.35289 | 1.93280 | 5.47699 |
| 9 | 0.35141 | 1.94976 | 5.54841 |
| 10 | 0.35082 | 1.95978 | 5.58629 |
| 11 | 0.35059 | 1.96571 | 5.60691 |
| 12 | 0.35049 | 1.96920 | 5.61837 |
| 13 | 0.35046 | 1.97127 | 5.62486 |
| 14 | 0.35044 | 1.97250 | 5.62859 |
| 15 | 0.35044 | 1.97322 | 5.63075 |
| 16 | 0.35043 | 1.97365 | 5.63202 |
| 17 | 0.35043 | 1.97391 | 5.63276 |
| 18 | 0.35043 | 1.97405 | 5.63318 |
| 19 | 0.35043 | 1.97414 | 5.63342 |
| 20 | 0.35043 | 1.97420 | 5.63360 |

## Table 28 (cont'd)

Volume of Top Feed = 16 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.98395 | 0.99999 | 1.01630 |
| 3 | 0.60162 | 1.39866 | 2.32483 |
| 4 | 0.45003 | 1.55263 | 3.45006 |
| 5 | 0.38992 | 1.61209 | 4.13438 |
| 6 | 0.36609 | 1.63506 | 4.46627 |
| 7 | 0.35664 | 1.64392 | 4.60946 |
| 8 | 0.35289 | 1.64734 | 4.66809 |
| 9 | 0.35141 | 1.64867 | 4.69159 |
| 10 | 0.35082 | 1.64919 | 4.70095 |
| 11 | 0.35059 | 1.64938 | 4.70462 |
| 12 | 0.35049 | 1.64945 | 4.70607 |
| 13 | 0.35046 | 1.64948 | 4.70665 |
| 14 | 0.35044 | 1.64949 | 4.70688 |
| 15 | 0.35044 | 1.64949 | 4.70697 |
| 16 | 0.35043 | 1.64949 | 4.70700 |
| 17 | 0.35043 | 1.64949 | 4.70701 |
| 18 | 0.35043 | 1.64949 | 4.70702 |
| 19 | 0.35043 | 1.64949 | 4.70702 |
| 20 | 0.35043 | 1.64949 | 4.70702 |

## Table 28 (cont'd)

Volume of Top Feed = 22 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.98395 | 1.33584 | 1.35763 |
| 3 | 0.60162 | 1.45768 | 2.42294 |
| 4 | 0.45003 | 1.48141 | 3.29181 |
| 5 | 0.38992 | 1.48602 | 3.81107 |
| 6 | 0.36609 | 1.48691 | 4.06160 |
| 7 | 0.35664 | 1.48709 | 4.16970 |
| 8 | 0.35289 | 1.48712 | 4.21406 |
| 9 | 0.35141 | 1.48712 | 4.23189 |
| 10 | 0.35082 | 1.48713 | 4.23900 |
| 11 | 0.35059 | 1.48713 | 4.24182 |
| 12 | 0.35049 | 1.48713 | 4.24294 |
| 13 | 0.35046 | 1.48713 | 4.24339 |
| 14 | 0.35044 | 1.48713 | 4.24357 |
| 15 | 0.35044 | 1.48713 | 4.24364 |
| 16 | 0.35043 | 1.48713 | 4.24366 |
| 17 | 0.35043 | 1.48713 | 4.24367 |
| 18 | 0.35043 | 1.48713 | 4.24368 |
| 19 | 0.35043 | 1.48713 | 4.24368 |
| 20 | 0.35043 | 1.48713 | 4.24368 |

## Table 29

### CONSTANT VOLUME OF BOTTOM FEED (NT2 = 21 cc.)

### Volume of Top Feed = 6 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.91316 | 0.99999 | 1.09569 |
| 3 | 0.60466 | 1.41033 | 2.33242 |
| 4 | 0.52565 | 1.73673 | 3.30396 |
| 5 | 0.50541 | 1.99636 | 3.94998 |
| 6 | 0.50023 | 2.20289 | 4.40379 |
| 7 | 0.49890 | 2.36717 | 4.74479 |
| 8 | 0.49856 | 2.49785 | 5.01015 |
| 9 | 0.49847 | 2.60181 | 5.21957 |
| 10 | 0.49845 | 2.68450 | 5.38570 |
| 11 | 0.49844 | 2.75027 | 5.51772 |
| 12 | 0.49844 | 2.80259 | 5.62269 |
| 13 | 0.49844 | 2.84421 | 5.70619 |
| 14 | 0.49844 | 2.87732 | 5.77262 |
| 15 | 0.49844 | 2.90365 | 5.82545 |
| 16 | 0.49844 | 2.92460 | 5.86747 |
| 17 | 0.49844 | 2.94126 | 5.90090 |
| 18 | 0.49844 | 2.95451 | 5.92749 |
| 19 | 0.49844 | 2.96506 | 5.94864 |
| 20 | 0.49844 | 2.97344 | 5.96547 |

## Table 29 (cont'd)

Volume of Top Feed = 11 cc.

| T | YT | YB | SF |
|---|---|---|---|
| 2 | 0.91316 | 0.99999 | 1.09569 |
| 3 | 0.60466 | 1.41033 | 2.33242 |
| 4 | 0.52565 | 1.65280 | 3.14432 |
| 5 | 0.50540 | 1.79608 | 3.55374 |
| 6 | 0.50022 | 1.88074 | 3.75962 |
| 7 | 0.49889 | 1.93077 | 3.87011 |
| 8 | 0.49855 | 1.96033 | 3.93205 |
| 9 | 0.49846 | 1.97780 | 3.96778 |
| 10 | 0.49844 | 1.98812 | 3.98867 |
| 11 | 0.49844 | 1.99422 | 4.00096 |
| 12 | 0.49843 | 1.99783 | 4.00820 |
| 13 | 0.49843 | 1.99996 | 4.01247 |
| 14 | 0.49843 | 2.00121 | 4.01499 |
| 15 | 0.49843 | 2.00196 | 4.01650 |
| 16 | 0.49843 | 2.00240 | 4.01737 |
| 17 | 0.49843 | 2.00266 | 4.01789 |
| 18 | 0.49843 | 2.00281 | 4.01820 |
| 19 | 0.49843 | 2.00290 | 4.01839 |
| 20 | 0.49843 | 2.00296 | 4.01849 |

## Table 29 (cont'd)

Volume of Top Feed = 16 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.91316 | 0.99999 | 1.09509 |
| 3 | 0.60466 | 1.41033 | 2.33242 |
| 4 | 0.52565 | 1.56887 | 2.98465 |
| 5 | 0.50540 | 1.63012 | 3.22538 |
| 6 | 0.50022 | 1.65378 | 3.30611 |
| 7 | 0.49889 | 1.66293 | 3.33324 |
| 8 | 0.49855 | 1.66646 | 3.34260 |
| 9 | 0.49846 | 1.66783 | 3.34593 |
| 10 | 0.49844 | 1.66835 | 3.34713 |
| 11 | 0.49844 | 1.66855 | 3.34758 |
| 12 | 0.49843 | 1.66863 | 3.34775 |
| 13 | 0.49843 | 1.66866 | 3.34781 |
| 14 | 0.49843 | 1.66867 | 3.34783 |
| 15 | 0.49843 | 1.66867 | 3.34783 |
| 16 | 0.49843 | 1.66868 | 3.34784 |
| 17 | 0.49843 | 1.66868 | 3.34784 |
| 18 | 0.49843 | 1.66868 | 3.34784 |
| 19 | 0.49843 | 1.66868 | 3.34784 |
| 20 | 0.49843 | 1.66868 | 3.34784 |

## Table 29 (cont'd)

Volume of Top Feed = 21 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.91316 | 1.34506 | 1.47298 |
| 3 | 0.60466 | 1.47102 | 2.43279 |
| 4 | 0.52565 | 1.49557 | 2.84520 |
| 5 | 0.50540 | 1.50035 | 2.96662 |
| 6 | 0.50022 | 1.50129 | 3.00125 |
| 7 | 0.49889 | 1.50147 | 3.00962 |
| 8 | 0.49855 | 1.50150 | 3.01173 |
| 9 | 0.49846 | 1.50151 | 3.01227 |
| 10 | 0.49844 | 1.50151 | 3.01241 |
| 11 | 0.49844 | 1.50151 | 3.01245 |
| 12 | 0.49843 | 1.50151 | 3.01246 |
| 13 | 0.49843 | 1.50151 | 3.01246 |
| 14 | 0.49843 | 1.50151 | 3.01246 |
| 15 | 0.49843 | 1.50151 | 3.01246 |
| 16 | 0.49843 | 1.50151 | 3.01246 |
| 17 | 0.49843 | 1.50151 | 3.01246 |
| 18 | 0.49843 | 1.50151 | 3.01246 |
| 19 | 0.49843 | 1.50151 | 3.01246 |
| 20 | 0.49843 | 1.50151 | 3.01246 |

# Table 30

## CONTINUOUS PARAPUMP

### Separation as a Function of NT4

| NT2 (cc) | NT4 (cc) | Steady State Product Concentration moles/cc. Top | Bottom | Separation Factor ($\langle y_B \rangle_\infty / \langle y_T \rangle_\infty$) |
|---|---|---|---|---|
| 6 | 6 | 0.0685 | 1.9313 | 28.1676 |
| | 11 | 0.0685 | 1.4718 | 21.4676 |
| | 16 | 0.0685 | 1.3144 | 19.1715 |
| | 21 | 0.0685 | 1.2357 | 18.0236 |
| 11 | 6 | 0.1693 | 2.6344 | 15.5564 |
| | 11 | 0.1693 | 1.8305 | 10.8102 |
| | 16 | 0.1693 | 1.5537 | 9.1755 |
| | 21 | 0.1693 | 1.4153 | 8.3580 |
| 16 | 6 | 0.3504 | 2.9168 | 8.3233 |
| | 11 | 0.3504 | 1.9742 | 5.6336 |
| | 16 | 0.3504 | 1.6494 | 4.7070 |
| | 21 | 0.3504 | 1.4871 | 4.2436 |
| 21 | 6 | 0.4984 | 2.9734 | 5.9654 |
| | 11 | 0.4984 | 2.0029 | 4.0184 |
| | 16 | 0.4984 | 1.6686 | 3.3478 |
| | 21 | 0.4984 | 1.5015 | 3.0124 |

## Table 31

## CONTINUOUS PARAMETRIC PUMPING

### Conditions for Runs to Show the Effect of Doubling Column Conditions

| Operating Variable | Value |
|---|---|
| Volumetric Flow Rate | 0.5 cc/sec |
| Specific Rate Constant for High pH | 0.07 |
| Specific Rate Constant for Low pH | 1.58 |
| Feed Concentration (Normalized) | 1.00 gm. mole/cc. |
| Total Volume of Column | 30.00 cc. |
| Void Volume of Column | 0.75 |
| Dead Volume of Top Reservoir | 5.00 cc. |
| Dead Volume of Bottom Reservoir | 5.00 cc. |
| Number of Cycles | 40 |

## Table 32

Upflow Displacement (NT1) = 11 cc.
Downflow Displacement (NT3) = 11 cc.
Volume of Top Feed (NT4) = 11 cc.
Volume of Bottom Feed (NT2) = 11 cc.

Number of Stages-12

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.87814 | 1.40082 | 1.59522 |
| 3 | 0.61286 | 1.47500 | 2.40676 |
| 4 | 0.53710 | 1.48959 | 2.77337 |
| 5 | 0.51547 | 1.49246 | 2.89532 |
| 6 | 0.50929 | 1.49302 | 2.93154 |
| 7 | 0.50753 | 1.49312 | 2.94194 |
| 8 | 0.50703 | 1.49315 | 2.94491 |
| 9 | 0.50688 | 1.49315 | 2.94575 |
| 10 | 0.50684 | 1.49315 | 2.94599 |
| 11 | 0.50683 | 1.49315 | 2.94606 |
| 12 | 0.50683 | 1.49315 | 2.94608 |
| 13 | 0.50683 | 1.49315 | 2.94609 |
| 14 | 0.50683 | 1.49315 | 2.94609 |
| 15 | 0.50682 | 1.49315 | 2.94609 |
| 16 | 0.50682 | 1.49315 | 2.94609 |
| 17 | 0.50682 | 1.49315 | 2.94609 |
| 18 | 0.50682 | 1.49315 | 2.94609 |
| 19 | 0.50682 | 1.49315 | 2.94609 |
| 20 | 0.50682 | 1.49315 | 2.94609 |
| 21 | 0.50682 | 1.49315 | 2.94609 |

# Table 32 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.50682 | 1.49315 | 2.94609 |
| 23 | 0.50682 | 1.49315 | 2.94609 |
| 24 | 0.50682 | 1.49315 | 2.94609 |
| 25 | 0.50682 | 1.49315 | 2.94609 |
| 26 | 0.50682 | 1.49315 | 2.94609 |
| 27 | 0.50682 | 1.49315 | 2.94609 |
| 28 | 0.50682 | 1.49315 | 2.94609 |
| 29 | 0.50682 | 1.49315 | 2.94609 |
| 30 | 0.50682 | 1.49315 | 2.94609 |
| 31 | 0.50682 | 1.49315 | 2.94609 |
| 32 | 0.50682 | 1.49315 | 2.94609 |
| 33 | 0.50682 | 1.49315 | 2.94609 |
| 34 | 0.50682 | 1.49315 | 2.94609 |
| 35 | 0.50682 | 1.49315 | 2.94609 |
| 36 | 0.50682 | 1.49315 | 2.94609 |
| 37 | 0.50682 | 1.49315 | 2.94609 |
| 38 | 0.50682 | 1.49315 | 2.94609 |
| 39 | 0.50682 | 1.49315 | 2.94609 |
| 40 | 0.50682 | 1.49315 | 2.94609 |

## Table 33

Upflow Displacement (NT1) = 5 cc.
Downflow Displacement (NT3) = 5 cc.
Volume of Top Feed (NT4) = 5 cc.
Volume of Bottom Feed (NT2) = 5 cc.
Number of Stages - 6

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.81182 | 1.42964 | 1.76104 |
| 3 | 0.62687 | 1.45885 | 2.32719 |
| 4 | 0.56547 | 1.46409 | 2.58914 |
| 5 | 0.54509 | 1.46494 | 2.68752 |
| 6 | 0.53832 | 1.46504 | 2.72151 |
| 7 | 0.53607 | 1.46504 | 2.73291 |
| 8 | 0.53533 | 1.46503 | 2.73671 |
| 9 | 0.53508 | 1.46503 | 2.73797 |
| 10 | 0.53500 | 1.46503 | 2.73839 |
| 11 | 0.53497 | 1.46503 | 2.73852 |
| 12 | 0.53496 | 1.46503 | 2.73857 |
| 13 | 0.53496 | 1.46503 | 2.73858 |
| 14 | 0.53496 | 1.46503 | 2.73859 |
| 15 | 0.53496 | 1.46503 | 2.73859 |
| 16 | 0.53496 | 1.46503 | 2.73859 |
| 17 | 0.53496 | 1.46503 | 2.73859 |
| 18 | 0.53496 | 1.46503 | 2.73859 |
| 19 | 0.53496 | 1.46503 | 2.73859 |
| 20 | 0.53496 | 1.46503 | 2.73859 |
| 21 | 0.53496 | 1.46503 | 2.73859 |

### Table 33 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.53496 | 1.46503 | 2.73859 |
| 23 | 0.53496 | 1.46503 | 2.73859 |
| 24 | 0.53496 | 1.46503 | 2.73859 |
| 25 | 0.53496 | 1.46503 | 2.73859 |
| 26 | 0.53496 | 1.46503 | 2.73859 |
| 27 | 0.53496 | 1.46503 | 2.73859 |
| 28 | 0.53496 | 1.46503 | 2.73859 |
| 29 | 0.53496 | 1.46503 | 2.73859 |
| 30 | 0.53496 | 1.46503 | 2.73859 |
| 31 | 0.53496 | 1.46503 | 2.73859 |
| 32 | 0.53496 | 1.46503 | 2.73859 |
| 33 | 0.53496 | 1.46503 | 2.73859 |
| 34 | 0.53496 | 1.46503 | 2.73859 |
| 35 | 0.53496 | 1.46503 | 2.73859 |
| 36 | 0.53496 | 1.46503 | 2.73859 |
| 37 | 0.53496 | 1.46503 | 2.73859 |
| 38 | 0.53496 | 1.46503 | 2.73859 |
| 39 | 0.53496 | 1.46503 | 2.73859 |
| 40 | 0.53496 | 1.46503 | 2.73859 |

## Table 34

## CONTINUOUS PARAMETRIC PUMPING

Conditions for Runs to Show the Effect of Simultaneously Varying Volume of Top and Bottom Feed (NT2 and NT4)

| Operating Variable | Value |
|---|---|
| Volumetric Flow Rate | 0.5 cc/sec |
| Specific Rate Constant for High pH | 0.07 |
| Specific Rate Constant for Low pH | 1.58 |
| Feed Concentration (Normalized) | 1.00 gm. mole/cc. |
| Total Volume of Column | 30.00 cc. |
| Void Volume of Column | 0.75 |
| Deat Volume of Top Reservoir | 5.00 cc. |
| Dead Volume of Bottom Reservoir | 5.00 cc. |
| Upflow Displacement (NT1) | 11 |
| Downflow Displacement (NT3) | 11 |
| Number of Cycles | 40 |

# Table 35

Volume of Top Feed (NT4) = 11 cc.

Volume of Bottom Feed (NT2) = 11 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99732 | 1.00000 | 1.00268 |
| 3 | 0.71024 | 1.28874 | 1.81451 |
| 4 | 0.52210 | 1.47773 | 2.83039 |
| 5 | 0.39679 | 1.60143 | 4.01572 |
| 6 | 0.31798 | 1.68238 | 5.29087 |
| 7 | 0.26502 | 1.73536 | 6.54815 |
| 8 | 0.23031 | 1.77003 | 7.68558 |
| 9 | 0.20756 | 1.79272 | 8.63726 |
| 10 | 0.19265 | 1.80757 | 9.38277 |
| 11 | 0.18288 | 1.81728 | 9.93720 |
| 12 | 0.17647 | 1.82364 | 10.33562 |
| 13 | 0.17228 | 1.82780 | 10.60971 |
| 14 | 0.16953 | 1.83052 | 10.79790 |
| 15 | 0.16772 | 1.83230 | 10.92457 |
| 16 | 0.16654 | 1.83346 | 11.00905 |
| 17 | 0.16577 | 1.83422 | 11.06507 |
| 18 | 0.16526 | 1.83472 | 11.10207 |
| 19 | 0.16493 | 1.83505 | 11.12643 |
| 20 | 0.16471 | 1.83526 | 11.14244 |
| 21 | 0.16457 | 1.83540 | 11.15295 |

# Table 35 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.16447 | 1.83549 | 11.15967 |
| 23 | 0.16441 | 1.83555 | 11.16439 |
| 24 | 0.16437 | 1.83559 | 11.16736 |
| 25 | 0.16434 | 1.83562 | 11.16932 |
| 26 | 0.16433 | 1.83563 | 11.17058 |
| 27 | 0.16432 | 1.83564 | 11.17141 |
| 28 | 0.16431 | 1.83565 | 11.17197 |
| 29 | 0.16430 | 1.83565 | 11.17232 |
| 30 | 0.16430 | 1.83566 | 11.17255 |
| 31 | 0.16430 | 1.83566 | 11.17271 |
| 32 | 0.16430 | 1.83566 | 11.17281 |
| 33 | 0.16430 | 1.83566 | 11.17288 |
| 34 | 0.16430 | 1.83566 | 11.17292 |
| 35 | 0.16430 | 1.83566 | 11.17295 |
| 36 | 0.16429 | 1.83566 | 11.17296 |
| 37 | 0.16429 | 1.83566 | 11.17297 |
| 38 | 0.16429 | 1.83566 | 11.17298 |
| 39 | 0.16429 | 1.83566 | 11.17298 |
| 40 | 0.16429 | 1.83566 | 11.17298 |

## Table 36

Volume of Top Feed (NT4) = 5 cc.

Volume of Bottom Feed (NT2) = 5 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 1.00000 | 0.98250 | 0.98250 |
| 3 | 0.78684 | 1.20680 | 1.53374 |
| 4 | 0.71070 | 1.28698 | 1.81085 |
| 5 | 0.68352 | 1.31563 | 1.92481 |
| 6 | 0.67380 | 1.32588 | 1.96775 |
| 7 | 0.67034 | 1.32954 | 1.98339 |
| 8 | 0.66910 | 1.33085 | 1.98902 |
| 9 | 0.66866 | 1.33131 | 1.99103 |
| 10 | 0.66850 | 1.33148 | 1.99175 |
| 11 | 0.66844 | 1.33154 | 1.99201 |
| 12 | 0.66842 | 1.33156 | 1.99210 |
| 13 | 0.66841 | 1.33157 | 1.99214 |
| 14 | 0.66841 | 1.33157 | 1.99215 |
| 15 | 0.66841 | 1.33157 | 1.99215 |
| 16 | 0.66841 | 1.33157 | 1.99215 |
| 17 | 0.66841 | 1.33157 | 1.99215 |
| 18 | 0.66841 | 1.33157 | 1.99215 |
| 19 | 0.66841 | 1.33157 | 1.99215 |
| 20 | 0.66841 | 1.33157 | 1.99215 |
| 21 | 0.66841 | 1.33157 | 1.99215 |

## Table 36 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.66841 | 1.33157 | 1.99215 |
| 23 | 0.66841 | 1.33157 | 1.99215 |
| 24 | 0.66841 | 1.33157 | 1.99215 |
| 25 | 0.66841 | 1.33157 | 1.99215 |
| 26 | 0.66841 | 1.33157 | 1.99215 |
| 27 | 0.66841 | 1.33157 | 1.99215 |
| 28 | 0.66841 | 1.33157 | 1.99215 |
| 29 | 0.66841 | 1.33157 | 1.99215 |
| 30 | 0.66841 | 1.33157 | 1.99215 |
| 31 | 0.66841 | 1.33157 | 1.99215 |
| 32 | 0.66841 | 1.33157 | 1.99215 |
| 33 | 0.66841 | 1.33157 | 1.99215 |
| 34 | 0.66841 | 1.33157 | 1.99215 |
| 35 | 0.66841 | 1.33157 | 1.99215 |
| 36 | 0.66841 | 1.33157 | 1.99215 |
| 37 | 0.66841 | 1.33157 | 1.99215 |
| 38 | 0.66841 | 1.33157 | 1.99215 |
| 39 | 0.66841 | 1.33157 | 1.99215 |
| 40 | 0.66841 | 1.33157 | 1.99215 |

## Table 37

Volume of Top Feed (NT4) = 4 cc.

Volume of Bottom Feed (NT2 = 4 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99922 | 1.00000 | 1.00077 |
| 3 | 0.75538 | 1.24418 | 1.64708 |
| 4 | 0.57859 | 1.42120 | 2.45630 |
| 5 | 0.45041 | 1.54953 | 3.44029 |
| 6 | 0.35746 | 1.64255 | 4.59505 |
| 7 | 0.29007 | 1.70998 | 5.89511 |
| 8 | 0.24120 | 1.75886 | 7.29205 |
| 9 | 0.20577 | 1.79429 | 8.71985 |
| 10 | 0.18008 | 1.81998 | 10.10650 |
| 11 | 0.16145 | 1.83860 | 11.38790 |
| 12 | 0.14794 | 1.85209 | 12.51879 |
| 13 | 0.13815 | 1.86187 | 13.47704 |
| 14 | 0.13105 | 1.86896 | 14.26144 |
| 15 | 0.12590 | 1.87410 | 14.88549 |
| 16 | 0.12217 | 1.87782 | 15.37089 |
| 17 | 0.11946 | 1.88052 | 15.74179 |
| 18 | 0.11750 | 1.88248 | 16.02141 |
| 19 | 0.11607 | 1.88390 | 16.23010 |
| 20 | 0.11504 | 1.88492 | 16.38853 |
| 21 | 0.11429 | 1.88567 | 16.49837 |

## Table 37 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.11375 | 1.88621 | 16.58179 |
| 23 | 0.11336 | 1.88660 | 16.64278 |
| 24 | 0.11307 | 1.88688 | 16.68727 |
| 25 | 0.11287 | 1.88709 | 16.71968 |
| 26 | 0.11272 | 1.88723 | 16.74324 |
| 27 | 0.11261 | 1.88734 | 16.76036 |
| 28 | 0.11253 | 1.88742 | 16.77281 |
| 29 | 0.11247 | 1.88748 | 16.78186 |
| 30 | 0.11243 | 1.88752 | 16.78841 |
| 31 | 0.11240 | 1.88755 | 16.79317 |
| 32 | 0.11238 | 1.88757 | 16.79662 |
| 33 | 0.11236 | 1.88759 | 16.79913 |
| 34 | 0.11235 | 1.88760 | 16.80099 |
| 35 | 0.11234 | 1.88761 | 16.80229 |
| 36 | 0.11234 | 1.88762 | 16.80322 |
| 37 | 0.11233 | 1.88762 | 16.80396 |
| 38 | 0.11233 | 1.88762 | 16.80444 |
| 39 | 0.11233 | 1.88763 | 16.80478 |
| 40 | 0.11233 | 1.88763 | 16.80505 |

## Table 38

Volume of Top Feed (NT4) = 3 cc.

Volume of Bottom Feed (NT2) = 3 cc.

| I | YT | YB | SF |
|---|---|---|---|
| 2 | 0.99983 | 1.00000 | 1.00016 |
| 3 | 0.80510 | 1.19439 | 1.48353 |
| 4 | 0.65144 | 1.34814 | 2.06946 |
| 5 | 0.53010 | 1.46955 | 2.77222 |
| 6 | 0.43427 | 1.56543 | 3.60471 |
| 7 | 0.35860 | 1.64115 | 4.57652 |
| 8 | 0.29884 | 1.70094 | 5.69176 |
| 9 | 0.25165 | 1.74816 | 6.94674 |
| 10 | 0.21439 | 1.78545 | 8.32821 |
| 11 | 0.18496 | 1.81490 | 9.81256 |
| 12 | 0.16172 | 1.83815 | 11.36654 |
| 13 | 0.14336 | 1.85652 | 12.94975 |
| 14 | 0.12887 | 1.87102 | 14.51667 |
| 15 | 0.11742 | 1.88247 | 16.03130 |
| 16 | 0.10839 | 1.89152 | 17.45158 |
| 17 | 0.10125 | 1.89866 | 18.75235 |
| 18 | 0.09561 | 1.90430 | 19.91682 |
| 19 | 0.09116 | 1.90875 | 20.93816 |
| 20 | 0.08765 | 1.91227 | 21.81799 |
| 21 | 0.08487 | 1.91505 | 22.56433 |

## Table 38 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.08268 | 1.91724 | 23.18913 |
| 23 | 0.08095 | 1.91898 | 23.70644 |
| 24 | 0.07958 | 1.92035 | 24.13087 |
| 25 | 0.07850 | 1.92143 | 24.47647 |
| 26 | 0.07765 | 1.92228 | 24.75620 |
| 27 | 0.07698 | 1.92295 | 24.98146 |
| 28 | 0.07644 | 1.92348 | 25.16217 |
| 29 | 0.07602 | 1.92390 | 25.30667 |
| 30 | 0.07569 | 1.92424 | 25.42189 |
| 31 | 0.07543 | 1.92450 | 25.51364 |
| 32 | 0.07522 | 1.92471 | 25.58649 |
| 33 | 0.07506 | 1.92487 | 25.64433 |
| 34 | 0.07493 | 1.92500 | 25.69017 |
| 35 | 0.07483 | 1.92510 | 25.72650 |
| 36 | 0.07475 | 1.92518 | 25.75529 |
| 37 | 0.07469 | 1.92525 | 25.77802 |
| 38 | 0.07464 | 1.92530 | 25.79601 |
| 39 | 0.07460 | 1.92534 | 25.81024 |
| 40 | 0.07456 | 1.92537 | 25.82153 |

## Table 39

Volume of Top Feed (NT4) = 1 cc.

Volume of Bottom Feed (NT2) = 1 cc.

| I | TRES | BRES | SF |
|---|---|---|---|
| 2 | 0.90029 | 1.00000 | 1.11075 |
| 3 | 0.81592 | 1.08016 | 1.32385 |
| 4 | 0.74083 | 1.16260 | 1.56934 |
| 5 | 0.67350 | 1.23787 | 1.83797 |
| 6 | 0.61308 | 1.30557 | 2.12952 |
| 7 | 0.55886 | 1.36636 | 2.44489 |
| 8 | 0.51020 | 1.42091 | 2.78502 |
| 9 | 0.46652 | 1.46987 | 3.15079 |
| 10 | 0.42732 | 1.51381 | 3.54255 |
| 11 | 0.39214 | 1.55325 | 3.96093 |
| 12 | 0.36057 | 1.58864 | 4.40594 |
| 13 | 0.33223 | 1.62040 | 4.87735 |
| 14 | 0.30680 | 1.64891 | 5.37459 |
| 15 | 0.28397 | 1.67449 | 5.89671 |
| 16 | 0.26348 | 1.69746 | 6.44233 |
| 17 | 0.24510 | 1.71806 | 7.00969 |
| 18 | 0.22860 | 1.73656 | 7.59659 |
| 19 | 0.21379 | 1.75315 | 8.20049 |
| 20 | 0.20049 | 1.76805 | 8.81845 |
| 21 | 0.18855 | 1.78142 | 9.44725 |

## Table 39 (cont'd)

| | | | |
|---|---|---|---|
| 22 | 0.17786 | 1.79341 | 10.08393 |
| 23 | 0.16825 | 1.80418 | 10.72333 |
| 24 | 0.15962 | 1.81384 | 11.36325 |
| 25 | 0.15188 | 1.82251 | 11.99947 |
| 26 | 0.14494 | 1.83029 | 12.62832 |
| 27 | 0.13870 | 1.83727 | 13.24636 |
| 28 | 0.13310 | 1.84354 | 13.85037 |
| 29 | 0.12808 | 1.84916 | 14.43741 |
| 30 | 0.12357 | 1.85421 | 15.00487 |
| 31 | 0.11953 | 1.85873 | 15.55062 |
| 32 | 0.11590 | 1.86279 | 16.07286 |
| 33 | 0.11264 | 1.86644 | 16.57027 |
| 34 | 0.10971 | 1.86971 | 17.04184 |
| 35 | 0.10709 | 1.87264 | 17.46697 |
| 36 | 0.10473 | 1.87528 | 17.90592 |
| 37 | 0.10262 | 1.87764 | 18.29752 |
| 38 | 0.10072 | 1.87976 | 18.66333 |
| 39 | 0.09902 | 1.88166 | 19.00365 |
| 40 | 0.09749 | 1.88337 | 19.31914 |

## Table 40

### Separation Factors as a Function of Top and Bottom Feed (NT2 and NT4)

| NT2 and or NT4 (cc) | Type of Parapump | Steady State Product Concentration moles/cc. |  | Separation Factor ($<y_B>_\infty / <y_T>_\infty$) |
|---|---|---|---|---|
|  |  | Top | Bottom |  |
| 1 | Batch | 0.0975 | 1.8834 | 19.3169 |
| 3 | Continuous | 0.0746 | 1.9254 | 25.8215 |
| 4 | Continuous | 0.1123 | 1.8876 | 16.8050 |
| 5 | Continuous | 1.1643 | 1.8357 | 11.1730 |
| 11 | Continuous | 0.5068 | 1.4932 | 2.9461 |

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