A THREE-DIMENSIONAL KINEMATIC MODEL OF THE HUMAN KNEE

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Thesis submitted to the Faculty of the Graduate School of the New Jersey Institute of Technology in partial fulfillmant of the requirements for the degree of

Master of Science in Mechanical Engineering

APPROVAL OF THESIS

Title of Thesis: A THREE-DIMENSIONAL KINEMATIC MODEL OF THE HUMAN KNEE

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Publication: "Conjugate Beam Method for Finding Deflection of Beams" — Published in Tamkang Mechanical Engineering Periodical, Vol. II, pp. 36, Sept. 1975.

ABSTRACT

This thesis describes two kinematic models of human knee which may be used to study the motion of the knee and the forces in the ligaments of the knee. In one model, the ligaments are simulated by rigid links connected to links representing the articulating bones at the centroids of the areas of insertion of the ligaments. The condylar higher pair connections are simulated by equivalent linkages connected by revolutes. In an advanced model, the rigid ligament links are replaced by springs.

The IMP procedure, a powerful kinematic linkage analysis method, is employed to study the motion and forces in this model. This procedure contains elements for position, velocity, acceleration, force, and equilibrium analysis of spatial linkages.

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CHAPTER I INTRODUCTION

BASIC ANATOMY

The knee joint provides the articulation between the distal (far) femur and proximal (near) tibia and the articulation between the patella, a sesamoid bone embedded in the patella tendon and the anterior region of the distal femur. The structures are shown in Figures 1 and 2. Articular surfaces consisting of hyaline cartilage cover the contact surfaces between the bones. The joint is enclosed by a membrane called the joint capsule which contains a fluid, called synovial fluid, which lubricates and nourishes the articular cartilage. (1), (2).

The distal femur terminates in two bony projections. The two projections are the medial (inside) and lateral (outside) condyles. The proximal, or upper end of the tibia, consists of the medial and lateral plateau or condyles. The condyles are covered with a layer of hyaline articular cartilage. Two cartilagonous endarthroidal plates, in the form of the lateral and medial menisci, interpose between the condyles of the femur and the tibial plateau. These menisci are roughly semi-circular (C-shaped) and cover about two-thirds of the tibial plateau. The menisci serve as load distributing agents over the articular surface of the knee joint and protect the wall of the joint capsule against impingement between the articulating bones.

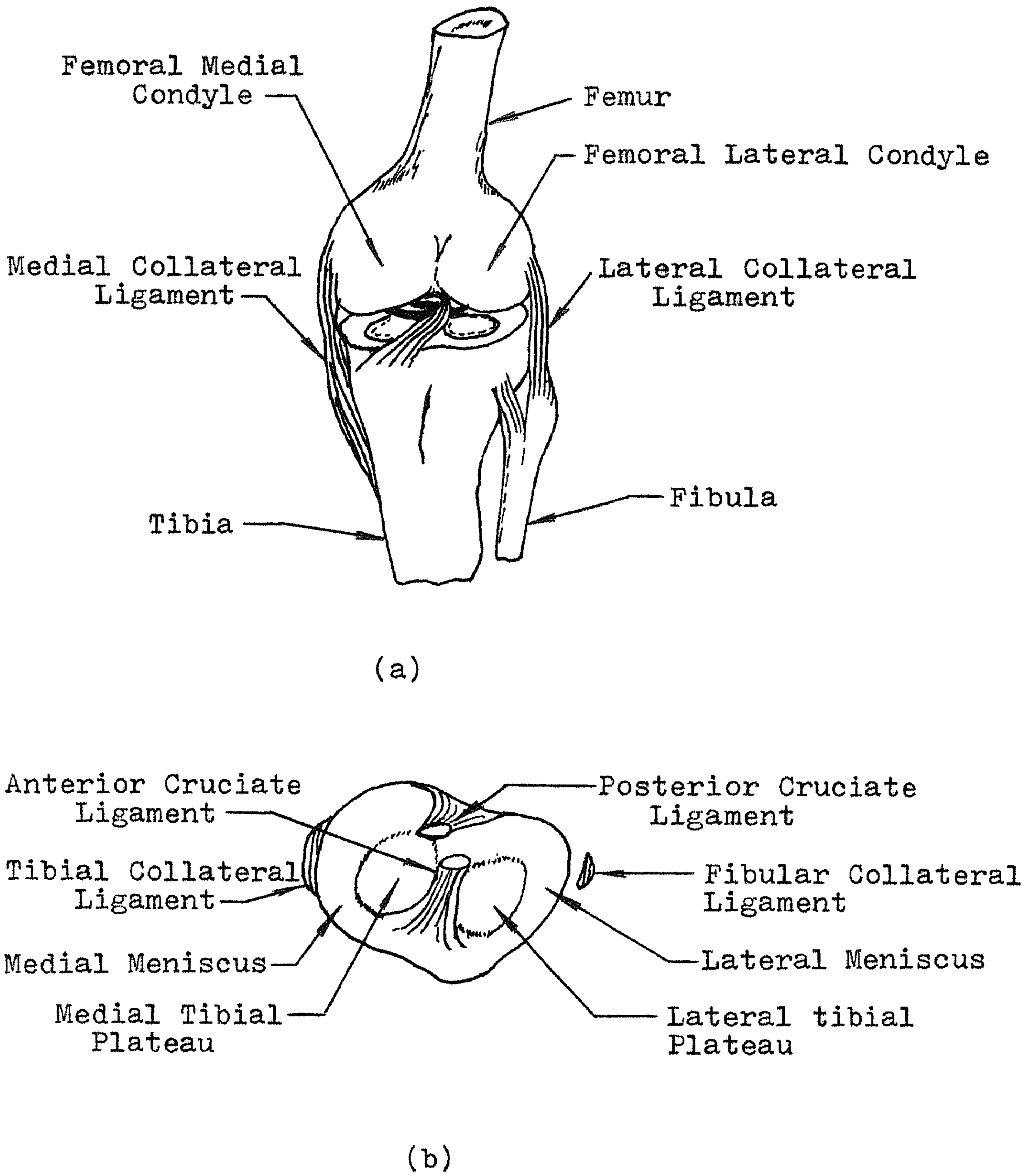

Fig. 1 Principal Structures of the Knee Joint:(a) view from the front, (b) view from above with femur removed (1).

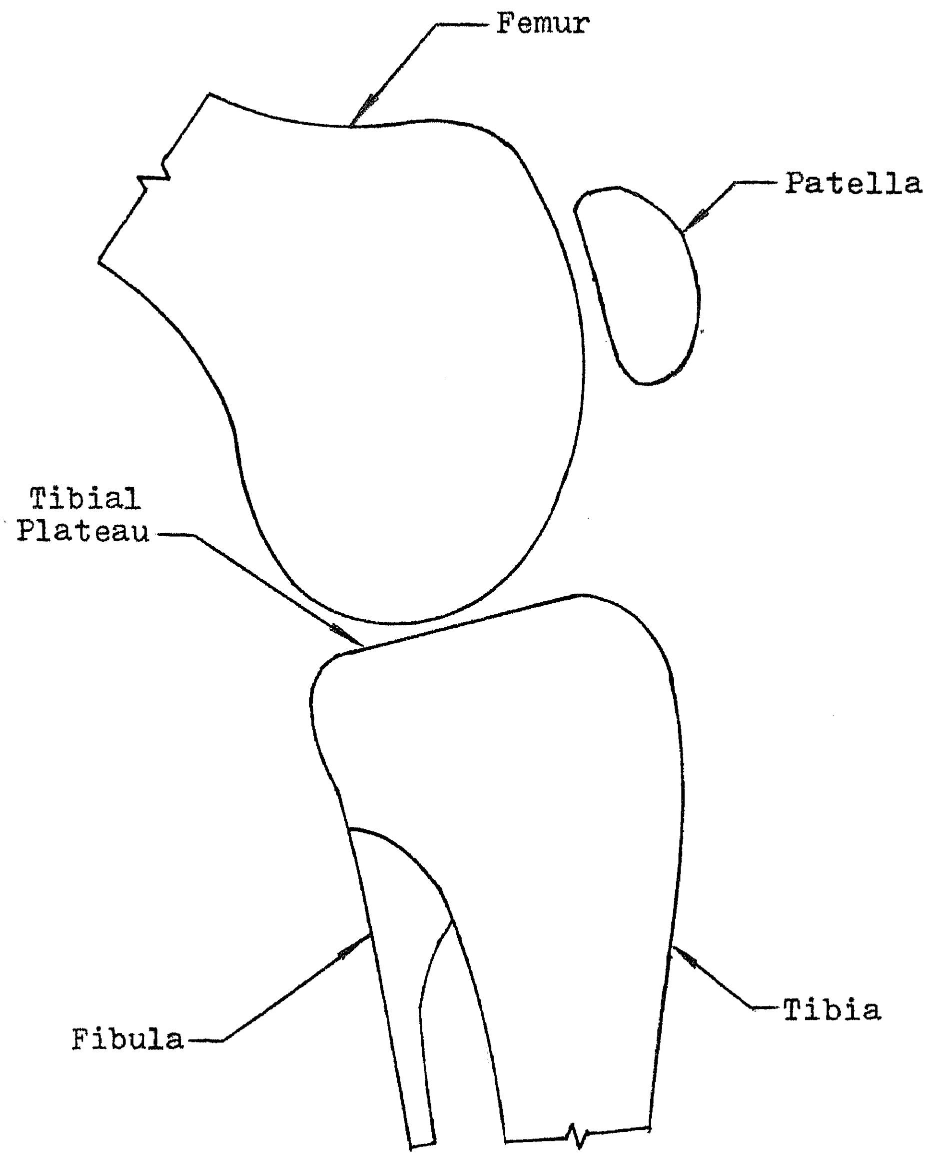

Fig. 2 Saggital Plane View of Tibia and Femur (2).

The patella is a sesamoid bone in the tendinous attachment of the quadriceps extensor femoris muscle. The patella rides over the front surface of the lower end of femur. The quadriceps muscle is attached below by the patella ligament to the front of the tibia.

The integirity of the joint is maintained by means of four ligaments, two collateral ligaments, one on the medial side and one on the lateral side, and two cruciate ligaments crisscrossed in the center of the joint between the condylar surfaces. As shown in Fig. 1, the medial collateral ligament is in the form of a broad band which is connected to the medial epicondyle of the femur and the medial surface of the upper end of the tibia. The lateral collateral ligament attaches on the lateral epicondyle of the femur and the head of the fibula. The anterior and posterior cruciate ligaments cross each other at the center of the knee joint, and each is attached to both the femur and the tibia.

The action of the cruciate ligaments is quite complex. Basically they control anterior-posterior movement of the proximal end of the tibia relative to the distal end of the femur. Specifically, a forward movement of the tibia is restricted or produced by tension in the anterior cruciate ligament, while a backward movement is restricted by or, results from tension in the posterior cruciate ligament. These ligaments in conjunction with the condylar shapes produce posterior (rearward) translation of the femur relative to the tibia during flexion and anterior (forward) translation or shift during extension. This shift produces more efficient muscle action.(3)

BIOMECHANICS

The biomechanics of the knee joint has been studied from many different vantage-points. In 1939, Elftman (4) used a force plate and motion picture study to perform a two-dimensional analysis of the resultant force acting at the ankle, knee and hip joints. There were no actual muscle or ligament forces calculated, only resultant forces. In 1945, Bresler and Frankel (5) calculated resultant forces and moments at the ankle, knee, and hip joint for four subjects during normal walking. Although the analysis was three-dimensional, no muscle, ligament forces were found. In 1950, Cunningham (6) analyzed the forces acting on limb segments of normal and amputee subjects. However, the actual loads in the muscles or ligaments were not found. In 1967, Morrison (7-9) made a substantial advance toward developing a more accurate picture of the knee joint motion and forces during normal walking. In 1970, more specific studies were done by Mcleish and Charnley (10) who investigated the abduction forces for a non-legged static stance position, and Reilly and Marten (11) who studied the quadriceps force and patella reaction force during various activities. In 1972, two-dimensional studies of the forces on the tibial plateaus in static loading conditions have been done by Ford, Perry and Atonelli (12) for flexed knee stance and by Kettelkamp and Chao (13) for lateral compressive forces during standing. In 1974, Seireg (14) built upon Morrison's work to produce an analysis of normal and pathological giant. Although there were many interesting results produced in these studies, a substantial amount of simplications and assumptions were made which affect the usability of the results from a prosthesis design standpoint.

In 1975, Seireg and Arvikar (15) developed a mathematical model which they solved using linear programming optimization methods to find muscle energy solution to the forces on the lower leg. Crowinshield and Pope (16) used mathematical model to predict the stability of the knee after injury to one or several ligaments. In 1976, Cappozzo, Figura, and Marchetti (17) utilized an "energy analysis" to compute muscular, gravitational and inertial forces during level walking at natural speed. Tansey (2) found that the spatial linkage developed by Kinzel (18) can be used to describe the three-dimensional kinematics of a cadaver joint. This spatial linkage is placed on a fresh cadaver specimen and loading forces which simulate the conditions of the knee during walking are applied. It was shown that the cruciate ligament forces, the muscle forces and the plateau forces contribute stability to the knee joint during normal level walking, while the collateral ligaments were found to be of much less significance. The medial plateau force was found to average approximately three times the lateral plateau force, while the maximum total plateau force was found to be 3.3 times body weight. In 1977, Piziali, Rastegar and Nagel (19) used the stiffness influence coefficient method to determine the structural characteristics of the human knee. The non-linear loading displacement data is approximated by a least squares polynomial and differentiated to determine the stiffness coefficient as a function of displacement. Cross axes coupling produces a full matrix and individual curves are seen to be a function of the initial displacement state of the joint. The resulting data provides a complex description of joint stiffness. In 1977, P. G. Maquet and G. A. Pelzer (20) analyzed the effect on the maximum stress (a) of a varus, or valgus deformity, (b) of a change of the magnitude of the muscular force, (c) of a change of the force due to the partial body-mass, (d) of a horizontal displacement of the partial center of gravity in the coronal plane, in Osteo-Arthritis of the knee. They found that any change of a single parameter from normal increases the maximum stress. In 1977, the functional anatomy of human knee joint was studied by Harding and Goodfellow (21). Their study described a simple rig and demonstrated its use in illustrating 3 aspects of human knee functional anatomy. In 1978, a technique for calculating in VIVO ligament lengths with application to the human knee joint was studied by Lew and Lewis (22). The basis of their method is an anthropometric scaling procedure whereby locations of soft tissue origins and insertions in a human subject body segment are determined by mathematically transforming their locations from a dissected cadaver specimen to their respective positions in the human subject. A mathematical development of the technique is presented with the procedure being adapted to the knee joint.

BIOKINEMATICS

Several investigations such as Townsend (23) and Kettelkamp (24) have studied angular tibio-femoral motion. Here the concern is for a more complete motion analysis including in particular translations as well. One simple way of determining the character of the relative motion between the condyles of femur and tibia is to study the contact points on the sagital plane when the full range of the knee motion is executed. This study is well documented by Steindler (25) in 1955.

The knee motion is considered as a combination of rolling and sliding in his study. For the pure rolling, there should be a one-to-one correspondence between the contact points of the tibia and femur. For the sliding motion, a point on the tibia sweeps many points of the femur during contact. Since the femoral contact curve is larger than the tibial curve, the relative motion must include a sliding component. At the beginning of flexion and the final phase of extension, the motion is rolling. When the flexion increases, a sliding component joints the motion. In addition, because the femoral condyles are not quite parallel and difer in size, the relative motion between the femur and tibia ceases to be a planar motion due to a rotation about the tibial axis during the final phase of extension.

There is another technique using instant centers of rotation to describe the knee motion. This technique is based on the notion that for a body executing a general plane motion, one can show that at any given instant, the velocities of the various points of the body are the same as if the body were rotating about a certain axis perpendicular to the plane motion, which is called the instantaneous axis of rotation. The intersection of this axis with the plane of motion is called the instant center of rotation. As the motion of the body proceeds, the instant center moves on the plane. If the centers are marked each time on the body, they describe a curve, which is called body centrode. If the same thing is done with respect to a fixed reference, another curve is obtained, which is called space centrode. (26) At any instant these two curves are tangent at the corresponding instant center. When the body moves, the body centrode rolls on the space centrodes. The application of instant center to the kinematics of the knee joint was well described in (27-30).

In 1969, Gardner and Clippinger (27) developed a method to locate the instant center based on the dimensional relationships. In 1971, Frankel, Burstein and Brooks (28) used the technique of instant center of motion by method of Rouleaux to study 25 normal knees and the instant centers were determined for the range of motion between full extension and 90 degrees of flexion. They found that in all cases, the contacting surfaces rotated about the instant centers which produced motions whose velocities at the joint surface were tangent to the surface. They also investigated 30 knees which had some injury or disorder and found that all knees demonstrated some irregularity in dislocation of instant centers from the normal positions.

In 1972, Walker, Shoji and Erkman (29) developed a "grid method" to locate the instant centers of rotation.

This method used the principle that the axis of rotation for an angular movement of the joint was the point on successive views of femur which remained stationary relative to the tibia. In this graphical technique, the instant center was found by locating a point that remained in the same position between two successive plates using grip paper.

In order to increase the accuracy of finding instant center, hand drawn procedure was computerized by Mayott (30) in 1975. Nine male knees were radiographed for the range of motion between full extension and full flexion. The radiographs were then processed so that five points were marked on each that were in the same position relative to the bones. The center of the X-ray beam was then located using a collimator and the data entered into a PDP-15 computer. The glometric data was then rotated and translated so that three sets of points representing the motion of the femur with respect to the tibia were located. The tangent method was used to locate the instant centers of rotation and the Rouleaux method was performed as a check.

In 1976, Tansey (2) used a seven-bar spatial linkage to determine the relative motion of tibio-femoral in space and to do forces analysis. In the basic configuration of his three-dimensional spatial linkage, there are two ends of the linkage to which are attached reference blocks which are in turn rigidly attached to the tibia and femur. The movements of the links are described by the outputs of the six potentiometers. Knowing the output voltages, one can compute the matrices which transform from one end of the linkage to the other , that is, from the tibia to the femur and vice-versa.

Thus, the literature although providing estimates of the restraining effects and forces produced by the ligaments does not provide a detailed quantitative estimate of the tension in the ligament themselves for various activities and motion phases. Furthermore, most complete motion studies of the knee are deficient since they attempt to use two-dimensional concepts to describe three-dimensional relative motion.

The spatial motion of the knee and forces in the knee ligaments may be studied by use of a suitable kinematic model of the knee. Fortunately, recent developments in kinematic and dynamic analysis of mechanism (31) has provided a means for the analysis of the motion and forces in a relatively realistic and complex three-dimensional spatial mechanism model. This work presents mechanical models of the knee which are compatible with the powerful new solution procedures developed for the kinematic, static and dynamic analysis of linkages.

CHAPTER II KNEE MODELING

The tibio-femoral articulation may be simulated by a chain of kinematic links. The kinematic, static and dynamic properties of this linkage may then be determined by using an appropriate mathematical analysis.

PRELIMINARY RIGID LIGAMENT MODEL

For the first knee model, assume that:

1. All links are rigid bodies connected by kinematic pairs. This implies that the flexibilities of the bones and articular cartilage are ignored and that ligaments a rigid in tension. Since these structures are in fact flexible, some error is introduced by this assumption. Still the stiffness of these structures is such that errors are likely to be small compared to the gross motion of the joint. In any event the validity of this assumption can be checked by using a more sophisticated model to be described later.

2. All ligaments may be represented by links connected at two points, the centroid of the areas of insertion of the actual ligaments. There are two

primary types of errors introduced by this assumption. First, ligaments attach over a fairly substantial area rather than a point. The action of the ligament can be quite complex with various strands being subjected to varying tension during activity. Thus, the location of the resultant ligament force varies during the activity. Secondly, the effect of possible contact between ligaments and bone or between ligaments is ignored. This effect may be of particular importance in the action of the cruciate ligaments which partially wrap each other. The importance of these effects is not clear at this time, but it is felt that any error introduced although perhaps significant would not invalidate results of this preliminary study obtained using this assumption.

With the above assumptions, the knee joint may be modeled as follows: (Refer to Fig. 3)

(A) The contact between the femur and the tibia may be modeled as two higher pairs. This of course approximates condylar contact under the assumption of rigidity.

(B) Each ligament may be modeled as a link with one globular and one universal connection. The links must act only as tensile elements which cannot

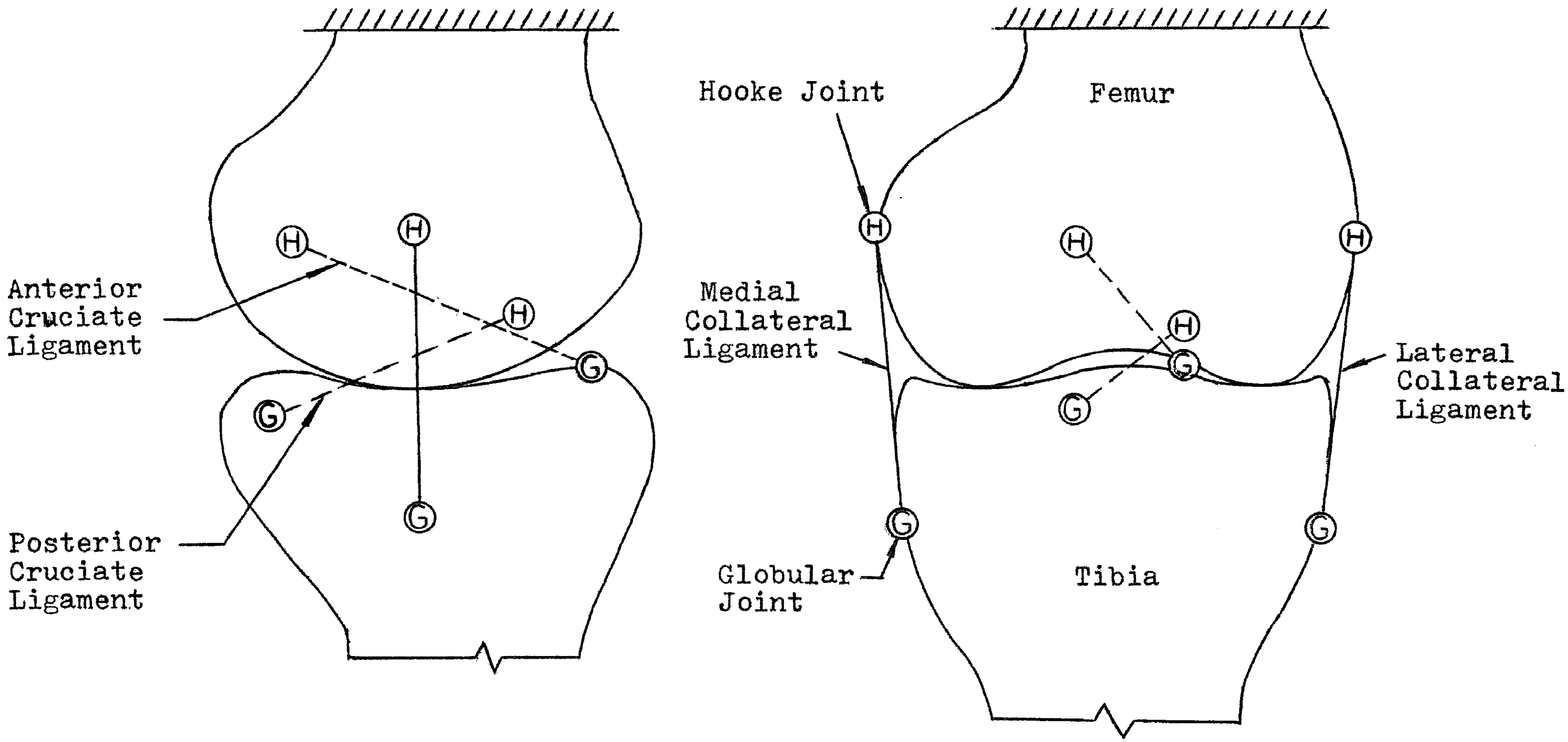

Fig. 3 Preliminary Rigid Ligament Knee Model

resist compressive force. The metnod of connection will not admit torque around the line between the connection. This closely simulates ligaments since they are essentially tensile elements, and have only minor twisting or bending resistance under load. A universal joint is used rather than two globular pairs, to prevent undefined and unimportant axial rotation of the link.

MOVABILITY ANALYSIS

Consider the criterion of movability for a kinematic chain (32). Suppose that there is a closed kinematic chain of N links, connected by G globular pairs, H higher pairs, R revolute joints, and U universal joints. Each link initially possesses six degrees of freedom before connection to any other link. Therefore one has a total of 6N degrees of freedom. On choosing one link as reference for all others, that is, fixing one link, N-1 moving links remain, and one has 6(N-1) degrees of freedom. Each globular connection means the loss of 3 degrees of freedom; with G connections there is a loss of 3G degrees of freedom. Similarly, each higher pair, revolute joint, universal joint means the loss of 1. 5. 4 degrees of freedom, respectively. Hence, the suitable movability equation for a chain with these connections is

$$F = 6(N-1)-3G-H-5R-4U \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where: F: degrees of freedom

N: number of links

G: number of globular pairs

H: number of higher pairs

R: number of revolute joints

U: number of universal joints

For the knee mechanism as shown in Fig. 3, N=6, G=4, H=2, R=0, U=4
Substitution these into equation (1), one obtains
F=0
F equals to zero means that one has a statically determinated structure. This occurs, for example, at full extension, where the knee is locked against further motion. At other phase, however, one ligament must be slack for motion to occur. In this case, one link and associated connections must be eliminated.

If one ligament is eliminated, the N, G and U are reduced by one, that is,
N=5, G=3, H=2, R=0, U=3.
Then, F equals one. This infers that although knee motion is three dimensional consisting of flexion-extension, ad-abduction, and internal-external rotation

these three motions are coupled, since one has a one degree of freedom system.

To do the force analysis, assume that forces or moments are applied to the bones of the leg. First, consider all ligaments as tension-compression elements and solve for ligament forces. Remove any ligaments found to be in compression and repeat the analysis until the resulting system contains only ligaments in tension.

LOWER PAIR MODEL

Since current analytical methods are directly applicable to linkages containing only lower pairs, each higher pair must be replaced by an appropriate equivalent linkage with lower pair connections.

Consider the replacement of a higher pair by an equivalent linkage with lower pair connections in two-dimensions (33). Figure 4 shows a mechanism with its equivalent linkages shown with dashed lines. Note that the floating link of the equivalent linkage is drawn along the common normal of the two contacting surfaces and extends to the center of curvature of

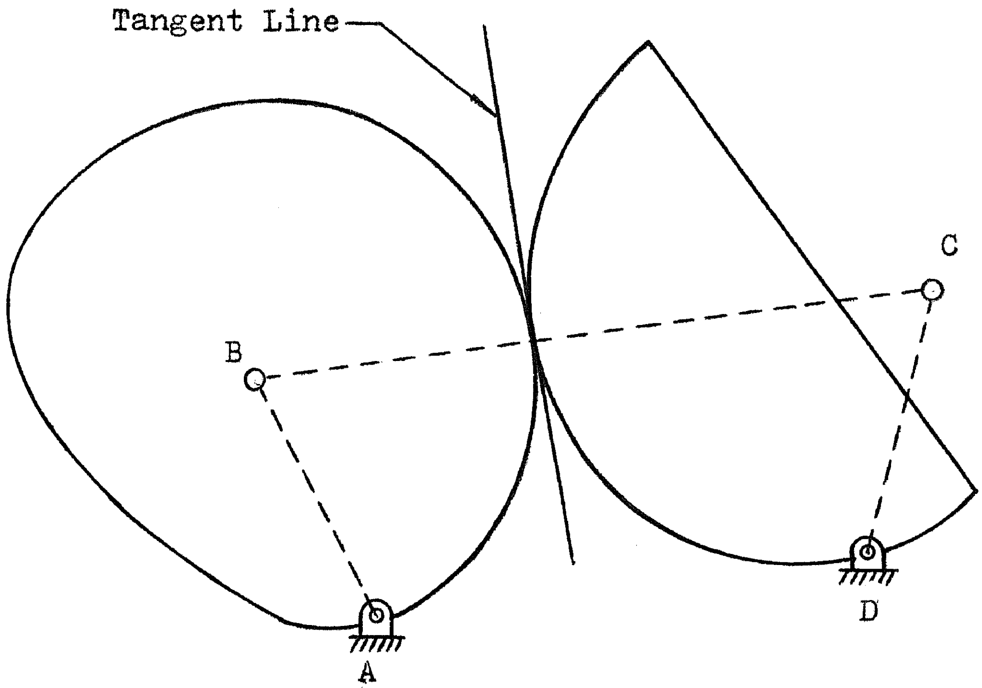

Fig. 4 H. P. Equivalent Linkage in 2-D

each of the surfaces. This ensures that the two pivoted links of the equivalent mechanism will have the same angular velocity ratio as the two pivoted links of the original mechanism and can, therefore, be substituted for purposes of velocity and acceleration analysis.

In three dimensions, higher pairs as shown in Figure 5, may be replaced by lower pairs. These rigid bodies are in contact at a point. Sections of the boundaries of the bodies at a point of contact are considered to be smooth curves, with the maximum and minimum principal radii of curvature of the surface of the upper body at the point of contact R_1' and R_1' respectively, and R_2' and R_2' the maximum and minimum radii of curvature respectively, for the lower body. The axis A-A passes through the centers of the curvature of the bodies and through the point of contact, and is perpendicular to a plane which tangent to both bodies at the point of contact.

Because the motion of high pair is constrained only by not permitting motion along axis A-A, higher pair connection, restricts only one degree of freedom, that is, 5 degrees of freedom exists in three dimensions.

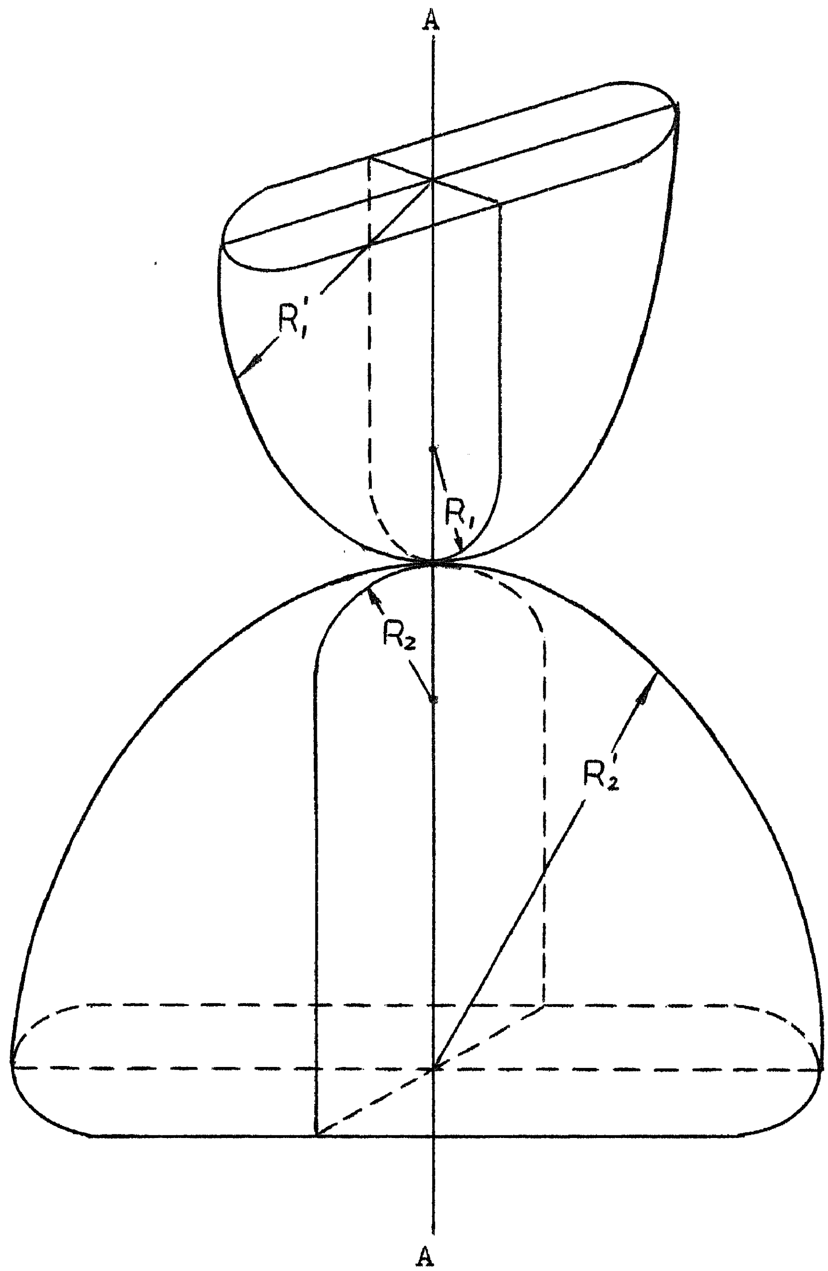

Fig. 5 Higher Pair Connection

Consider the motion of the lower body relative to upper body. The following motions are permitted.

1. Rotation of lower body about axis A-A.
2. Rotation of lower body about center of R_1 ; that is, sliding along trace of R_1.
3. Rotation of lower body about center of R_1' ; that is, sliding along trace of R_1'.
4. Rotation of lower body about center of R_2 ; that is, sliding along trace of R_2.
5. Rotation of lower body about center of R_2' ; that is, sliding along trace of R_2'.

These rotations are basic motions associated with 5 degrees of freedom, and may be considered the coordinates describing the position of lower body relative to upper body. Any other motion may be considered to consist of combination of these motions. For example:

6. Rotation of lower body about center of R_2 while sliding along trace of R_1.
7. Rotation of lower body about center of R_2 while sliding along trace of R_1'
8. Rotation of lower body about center of R_2' while sliding along trace of R_1 etc.

are combinations of basic motions.

Now, consider the motion of equivalent linkage shown in Fig. 6, a higher pair is replaced by the following:

1. A revolute at each of the four centers of the principal radii of curvature of the two bodies in contact directly, where the axis of the revolute is normal to the plane of its associated radius.
2. Two links colinear with axis A-A of Fig. 5. These links are connected by a revolute, whose axis is colinear with the links.

Thus, a higher pair is replaced by four links and five revolutes. Note that four links are colinear. This equivalent linkage allows:

(a) Rotation of lower body about Z_1 which is equivalent to motion 1 of the higher pair connection.

(b) Rotation of lower body about Z_2 which is equivalent to motion 2 of the higher pair connection.

(c) Rotation of lower body about Z_3 which is equivalent to motion 3 of the higher pair connection.

(d) Rotation of lower body about Z_4 which is equivalent to motion 4 of the higher pair connection.

(e) Rotation of lower body about Z_5 which is equivalent to motion 5 of the higher pair connection.

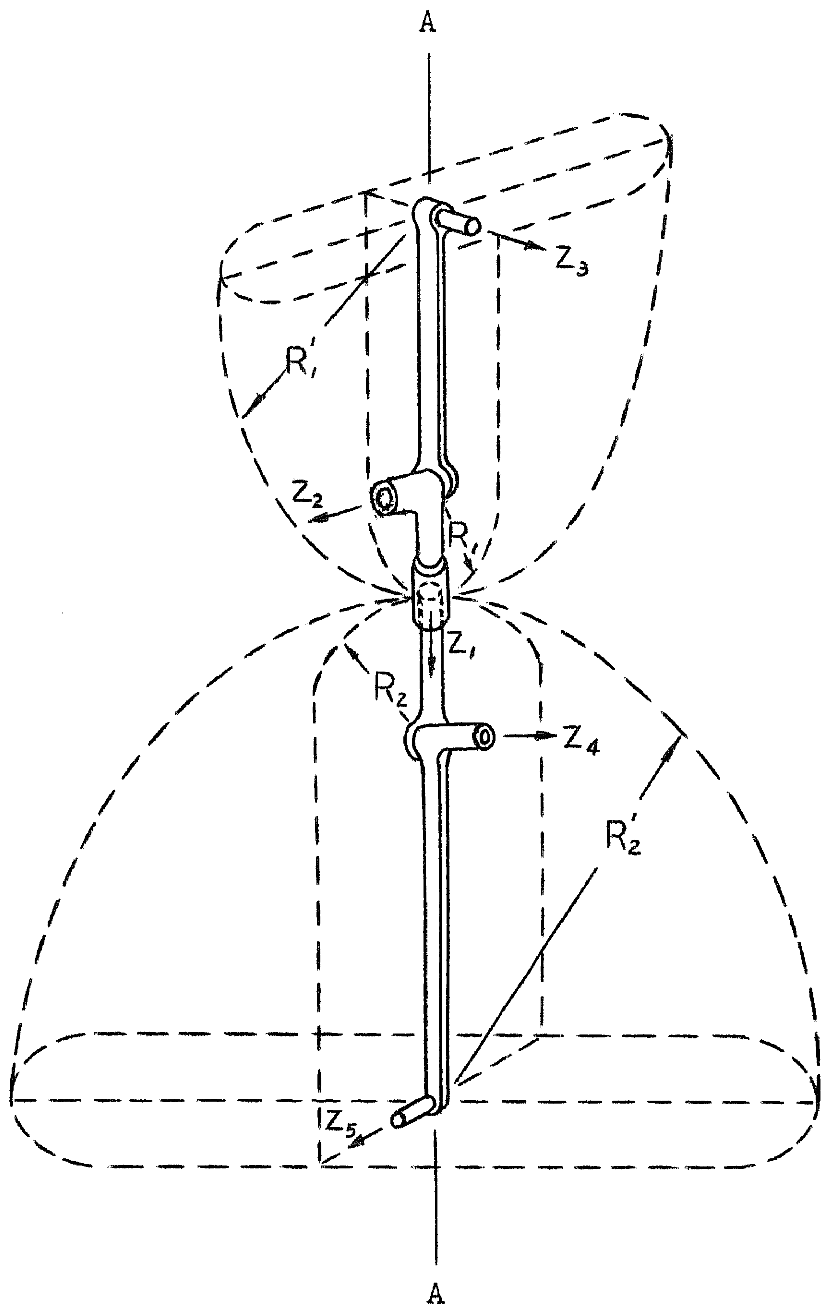

Fig. 6 Equivalent Linkage for Higher Pairs

Thus, it may be seen that the same rotation angles may be used to describe the relative position of bodies 1 and 2 as in the higher pair connection. Now, consider the movability equation for the equivalent linkage. Here,

$$F = 6(N-1)-3G-H-5R-4U$$

Where: $N=6$, $G=0$, $H=0$, $R=5$, $U=0$

Thus, $F=5$, as in the higher pair connection. Therefore, Fig. 6 is the equivalent linkage for a higher pair connection.

Now, the movability equation for the new knee model as shown in Fig. 7 yields

$N=14$, $G=4$, $H=0$, $R=10$, $U=4$

$F=0$

and if one ligament is slack,

$N=13$, $G=3$, $H=0$, $R=10$, $U=3$

$F=1$

as before.

FLEXIBLE LIGAMENT MODEL

Since the ligaments are not perfectly rigid bodies, an alternate model may be formulated, where the ligaments are replaced by springs. Such a model allows the motions associated with ligament flexibility and allows comparison

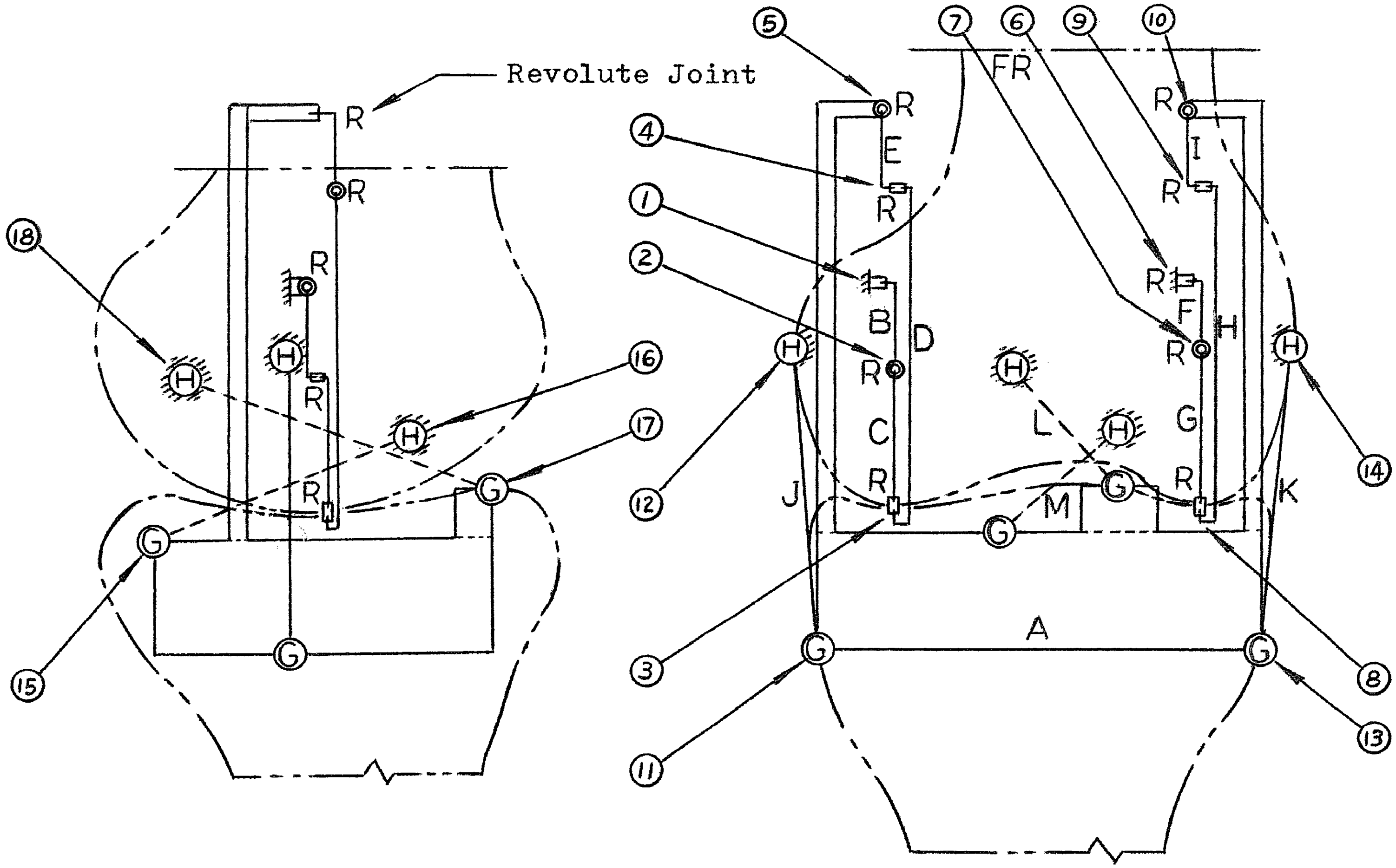

Fig. 7 Lower Pair Knee Model

of motions and forces with different ligament properties, that is, different spring constants.

Consider the movability equation for this model. Here,

$$F = 6(N+S-1)-3G-H-5R-4U+S \quad \text{...............(2)}$$

Where: S=Number of springs = 4

N=10, G=4, H=0, R=10, U=4

F=4

These four degrees of freedom are associated with the deformations of the springs. The position of equilibrium may be found by the method given by Livermore (34).

More advanced models may be developed to consider dynamic models using dashpots to account for the viscoelastic behavior of ligaments and perhaps even the viscoelastic properties of the bones and cartilage.

PROBLEM FORMULATION

The rigid ligament model may be used to solve the following problems:

1. Given the location of all of the points of connection of the ligament links and given the geometry of the condylar surfaces in terms of their principal radii of curvature at the contact point and the location of the contact point for one phase of motion and given a force system applied to the tibia, find the forces in all the links.

2. Given the above and the assumption that condylar geometry is unchanging and given a small change in the position variable for the tibial link, find the new orientation of the tibia and all remaining links.

The flexible ligament model may be used to solve the problem.

Given that in problem 1 above and the assumption that condylar geometry is unchanging, find the change in orientation of the tibia relative to the femur and the loads in the ligaments resulting from the application of a load system to the tibia.

These problems may be solved by the above models using the procedure described in Chapter III.

CHAPTER III DESCRIPTION OF SOLUTION PROCEDURE

A newly developed method, the IMP (Integrated Mechanisms Program), is a very powerful and straightforward method for static, kinematic and dynamic analysis. This program consists of several analysis elements coupled so as to produce an analysis capability. Topology analysis (35), is used to define an unscaled representation of the linkage. A generalized symbolic notation (36, 37), is utilized to define the linkage configuration and properties in terms of lower pair connections for position. Differentiation of the loop equations and their solution by similar technique yields the velocity and acceleration analysis (38), which are additional program elements. Static force analysis is obtained in another program element by a method (38) formulated from the principle of virtual work and Largrange equation. Equilibrium position of a spring restrained system is found by a procedure similar to that of Livermore (34). Other elements determine dynamic response or natural frequency.

TOPOLOGY ANALYSIS

In IMP system, the user is required to specify enough information to completely define the topology of each mechanism. The topological analysis of a mechanism includes the recognition of the number of links, the number and types of joints, the order in which the links and joints are arranged, the number and the order of kinematic loops and other such characteristics which are solely determined by the connectivity of the mechanism. The topological network of a mechanism thus represents an unscaled picture of the mechanism. Figure 8 shows the topological network of the knee mechanism.

IMP uses an algorithm based on network theory (35) to automate the recognition of the loops. The algorithm follows a search procedure which identifies an "optimum" set of loops satisfying the following conditions:

1. The set must contain λ independent loops, where λ is given by $\lambda = J-L+1$ in which J is the total number of joints, and L is the total number of links.

2. Each loop must "leave" the fixed link through a common joint.

3. Any two loops passing through the same joint must do so in the same direction.

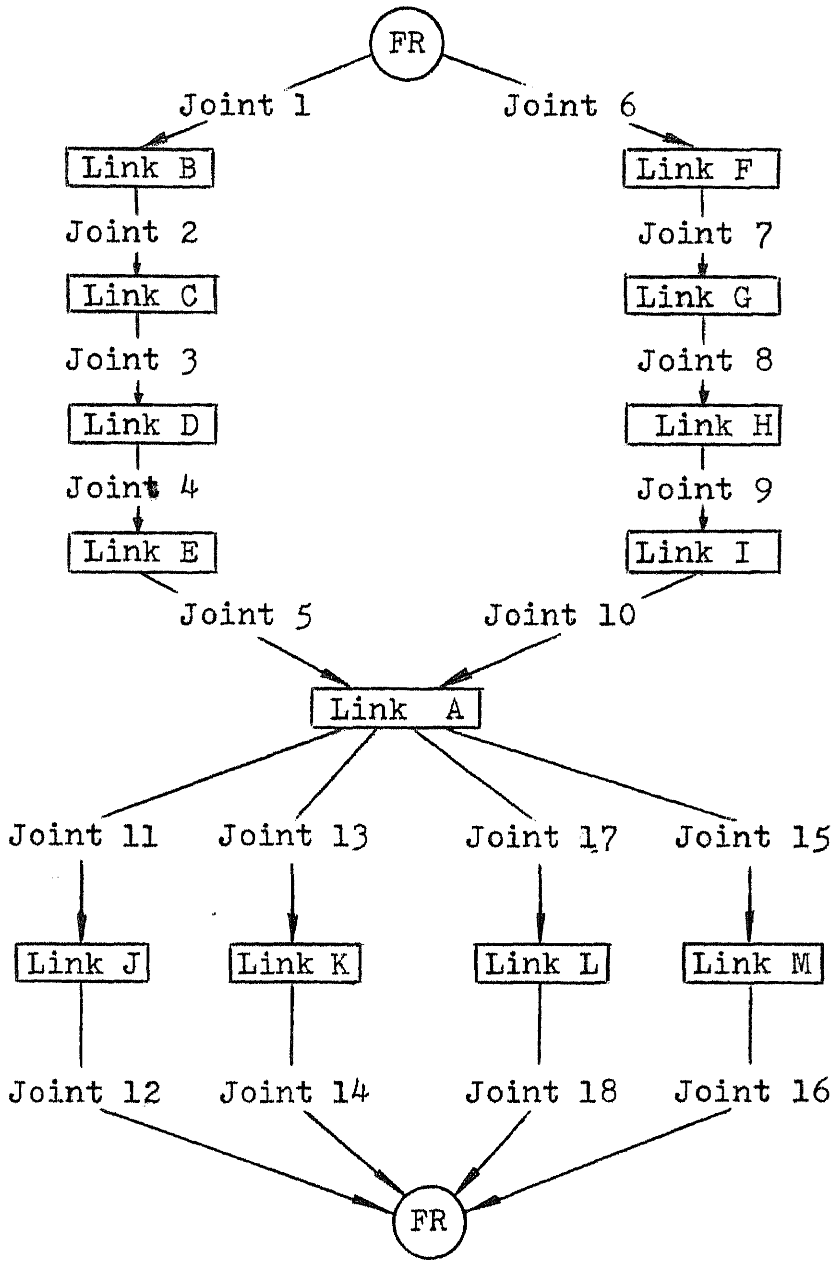

Fig. 8 The Topological Network of Fig. 7.

4. The number of joints in each loop must be the minimum achievable under the above conditions.

In Fig. 8, J=18, L=14, and condition 1 tells one that λ = J-L+1 = 5. That is, 5 independent loops for the knee mechanism. The computer determined loops for the knee mechanism shown in Fig. 7 are also indicated by the arrows shown in Fig. 8.

SYMBOLIC NOTATION

The next step is dimensional analysis which is the collection and reduction of the dimensional data supplied by the user to describe the particular mechanism to be analyzed. The notation contains the specification of variables and constants of the system. Denavit-Hartenberg (36), used a symbolic description of spatial linkages and their displacement analysis based on purely algebraic methods making use of matrix algebra. The four parameters, a, α, θ and s in the Denavit-Hartenberg notation, are the essential parameters which describe the "shape" of each link and its motion with respect to the previous link in the kinematic loop. The main difficulty in the Denavit-Hartenberg notation is that the four parameters depend not only the shape of the link, but also on the shape of the previous link in the kinematic loop. Moreover, the four parameters simultaneously contain the constant dimensions of the rigid link as well as the pair variable describing the joint motion. Hence, a modified symbolic notation was made by P. N. Sheth and J. J. Uicker Jr. (37) to provide a clear separation of the pair variables and constant parameters of a mechanism, and thus provides a framework in which higher pairs can be systematically modeled. In the modified symbolic notation, six parameters, a_{jk}, α_{jk}, b_{jk}, β_{jk}, c_{jk}, and r_{jk} are required to define the shape of the link.

A shape transformation matrix is used to convert the convenient six parameters to four necessary parameters. For every column of these six parameters in the modified notation, there is a transformation matrix which relates the coordinate systems at the two ends of the corresponding link. The shape of a link can be completely described by a transformation matrix.

The action of a kinematic pair can be represented by the functional relationship describing the relative motion of the joint. Appropriate transformation pair matrices for revolute pair, prismatic pair, cylindric pair, screw pair, spheric pair, plat pair and gear pair, are given in (37).

MATRIX-LOOP EQUATION

there are five independent loops in the mechanism as shown in Fig. 8. Each loop represents a closed sequence of coordinate reference frames with constant and variable parameters between them. The five loops of the mechanism can be represented symbolically as follows: (Note that the subscripts i and j of $X_{ij}Y_{ij}Z_{ij}$ represent the joint numbers and link names, in lower case symbols, respectively, as shown in Fig. 7.)

Loop 1:

$$X_{1fr}Y_{1fr}Z_{1fr}\underbrace{\longrightarrow}_{\text{joint 1}}X_{1b}Y_{1b}Z_{1b}\underbrace{\longrightarrow}_{\text{link b}}X_{2b}Y_{2b}Z_{2b}\underbrace{\longrightarrow}_{\text{joint 2}}X_{2c}Y_{2c}Z_{2c}\underbrace{\longrightarrow}_{\text{link c}}X_{3c}Y_{3c}Z_{3c}\underbrace{\longrightarrow}_{\text{joint 3}}\cdots$$

$$X_{3d}Y_{3d}Z_{3d}\underbrace{\longrightarrow}_{\text{link d}}X_{4d}Y_{4d}Z_{4d}\underbrace{\longrightarrow}_{\text{joint 4}}X_{4e}Y_{4e}Z_{4e}\underbrace{\longrightarrow}_{\text{link e}}X_{5e}Y_{5e}Z_{5e}\underbrace{\longrightarrow}_{\text{joint 5}}X_{5a}Y_{5a}Z_{5a}\underbrace{\longrightarrow}_{\text{link a}}\cdots$$

$$X_{11a}Y_{11a}Z_{11a}\underbrace{\longrightarrow}_{\text{joint 11}}X_{11j}Y_{11j}Z_{11j}\underbrace{\longrightarrow}_{\text{link j}}X_{12j}Y_{12j}Z_{12j}\underbrace{\longrightarrow}_{\text{joint 12}}X_{12fr}Y_{12fr}Z_{12fr}\underbrace{\longrightarrow}_{\text{frame}}\cdots$$

$$X_{1fr}Y_{1fr}Z_{1fr}$$

Loop 2:

$$X_{1fr}Y_{1fr}Z_{1fr}\underbrace{\longrightarrow}_{\text{joint 1}}X_{1b}Y_{1b}Z_{1b}\underbrace{\longrightarrow}_{\text{link b}}X_{2b}Y_{2b}Z_{2b}\underbrace{\longrightarrow}_{\text{joint 2}}X_{2c}Y_{2c}Z_{2c}\underbrace{\longrightarrow}_{\text{link c}}X_{3c}Y_{3c}Z_{3c}\underbrace{\longrightarrow}_{\text{joint 3}}\cdots$$

$$X_{3d}Y_{3d}Z_{3d}\underbrace{\longrightarrow}_{\text{link d}}X_{4d}Y_{4d}Z_{4d}\underbrace{\longrightarrow}_{\text{joint 4}}X_{4e}Y_{4e}Z_{4e}\underbrace{\longrightarrow}_{\text{link e}}X_{5e}Y_{5e}Z_{5e}\underbrace{\longrightarrow}_{\text{joint 5}}X_{5a}Y_{5a}Z_{5a}\underbrace{\longrightarrow}_{\text{link a}}\cdots$$

$$X_{13a}Y_{13a}Z_{13a}\underbrace{\longrightarrow}_{\text{joint 13}}X_{13k}Y_{13k}Z_{13k}\underbrace{\longrightarrow}_{\text{link k}}X_{14k}Y_{14k}Z_{14k}\underbrace{\longrightarrow}_{\text{joint 14}}X_{14fr}Y_{14fr}Z_{14fr}\underbrace{\longrightarrow}_{\text{frame}}\cdots$$

$$X_{1fr}Y_{1fr}Z_{1fr}$$

Loop 3:

$$X_{1fr}Y_{1fr}Z_{1fr} \rightarrow X_{1b}Y_{1b}Z_{1b} \rightarrow X_{2b}Y_{2b}Z_{2b} \rightarrow X_{2c}Y_{2c}Z_{2c} \rightarrow X_{3c}Y_{3c}Z_{3c} \rightarrow$$

joint 1 — link b — joint 2 — link c — joint 3 ...

$$X_{3d}Y_{3d}Z_{3d} \rightarrow X_{4d}Y_{4d}Z_{4d} \rightarrow X_{4e}Y_{4e}Z_{4e} \rightarrow X_{5e}Y_{5e}Z_{5e} \rightarrow X_{5a}Y_{5a}Z_{5a} \rightarrow$$

link d — joint 4 — link e — joint 5 — link a ...

$$X_{17a}Y_{17a}Z_{17a} \rightarrow X_{17L}Y_{17L}Z_{17L} \rightarrow X_{18L}Y_{18L}Z_{18L} \rightarrow X_{18fr}Y_{18fr}Z_{18fr} \rightarrow$$

joint 17 — link L — Joint 18 — frame ...

$$X_{1fr}Y_{1fr}Z_{1fr}$$

Loop 4:

$$X_{1fr}Y_{1fr}Z_{1fr} \rightarrow X_{1b}Y_{1b}Z_{1b} \rightarrow X_{2b}Y_{2b}Z_{2b} \rightarrow X_{2c}Y_{2c}Z_{2c} \rightarrow X_{3c}Y_{3c}Z_{3c} \rightarrow$$

joint 1 — link b — joint 2 — link c — joint 3 ...

$$X_{3d}Y_{3d}Z_{3d} \rightarrow X_{4d}Y_{4d}Z_{4d} \rightarrow X_{4e}Y_{4e}Z_{4e} \rightarrow X_{5e}Y_{5e}Z_{5e} \rightarrow X_{5a}Y_{5a}Z_{5a} \rightarrow$$

link d — joint 4 — link e — joint 5 — link a ...

$$X_{15a}Y_{15a}Z_{15a} \rightarrow X_{15m}Y_{15m}Z_{15m} \rightarrow X_{16m}Y_{16m}Z_{16m} \rightarrow X_{16fr}Y_{16fr}Z_{16fr} \rightarrow$$

joint 15 — link m — joint 16 — frame ...

$$X_{1fr}Y_{1fr}Z_{1fr}$$

Loop 5:

$$X_{6fr}Y_{6fr}Z_{6fr} \rightarrow X_{6f}Y_{6f}Z_{6f} \rightarrow X_{7f}Y_{7f}Z_{7f} \rightarrow X_{7g}Y_{7g}Z_{7g} \rightarrow X_{8g}Y_{8g}Z_{8g} \rightarrow \cdots$$

joint 6   link f   joint 7   link g   joint 8

$$X_{8h}Y_{8h}Z_{8h} \rightarrow X_{9h}Y_{9h}Z_{9h} \rightarrow X_{9i}Y_{9i}Z_{9i} \rightarrow X_{10i}Y_{10i}Z_{10i} \rightarrow \cdots$$

link h   joint 9   link i   joint 10

$$X_{10a}Y_{10a}Z_{10a} \rightarrow X_{15a}Y_{15a}Z_{15a} \rightarrow X_{15m}Y_{15m}Z_{15m} \rightarrow X_{16m}Y_{16m}Z_{16m} \rightarrow \cdots$$

link a   joint 15   link m   joint 16

$$X_{16fr}Y_{16fr}Z_{16fr} \rightarrow X_{6fr}Y_{6fr}Z_{6fr}$$

frame

The relationship between the coordinate frames of each loop can be formulated by appropriate Transformation matrices. If ϕ_j stands for a variable transformation pair matrix corresponding to the jth joint motion and if T_{jk} denotes a constant transformation shape matrix corresponding to the rigid link joining the jth and kth coordinate frames, the following five matrix loop equations written.

Loop 1:

$$\phi_1 T_{1,2}\phi_2 T_{23}\phi_3 T_{3,4}\phi_4 T_{4,5}\phi_5 T_{5,11}\phi_{11} T_{11,12}\phi_{12} T_{12,1} = I \quad .....(3)$$

Loop 2:

$$\phi_1 T_{1,2}\phi_2 T_{2,3}\phi_3 T_{3,4}\phi_4 T_{4,5}\phi_5 T_{5,13}\phi_{13} T_{13,14}\phi_{14} T_{14,1} = I \quad .....(4)$$

Loop 3:

$$\phi_1 T_{1,2}\phi_2 T_{2,3}\phi_3 T_{3,4}\phi_4 T_{4,5}\phi_5 T_{5,17}\phi_{17} T_{17,18}\phi_{18} T_{18,1} = I \quad .....(5)$$

Loop 4:

$$\emptyset_1 T_{1,2} \emptyset_2 T_{2,3} \emptyset_3 T_{3,4} \emptyset_4 T_{4,5} \emptyset_5 T_{5,15} \emptyset_{15} T_{15,16} \emptyset_{16} T_{16,1} = I \quad \ldots\ldots(6)$$

Loop 5:

$$\emptyset_6 T_{6,7} \emptyset_7 T_{7,8} \emptyset_8 T_{8,9} \emptyset_9 T_{9,10} \emptyset_{10} T_{10,15} \emptyset_{15} T_{15,16} \emptyset_{16} T_{16,6} = I \quad \ldots\ldots(7)$$

The symbol I on the right hand side of equations (3-7) is the unit matrix, indicating that the transformations close on themselves.

POSITION ANALYSIS

Equations (3-7) completely describes the geometry of the knee mechanism, and its solution will yield a complete displacement analysis, that is, the values of all pair variables of the linkage in terms of the constant parameters and input variables. The mechanism will be analyzed at a series of instantaneous positions. At any one of these positions, the pair variables will be determined by an iterative technique. (39) This technique, makes some initial estimates of the values of the unknown pair variables, then evaluates the error between the estimated and the computed values. When the error terms have been found, they must be added to the initial estimate value to give an improved approximation of the exact values of the unknown pair variables. The iteration process may be continued until the error terms are smaller than the desired accuracy.

FORCE ANALYSIS

Static forces are found by combining the method of virtual work and the matrix-loop equations to relate the virtual displacement of the load to the given virtual deformations of the links. The method applies to any single-degree-of-freedom, simple-closed linkage consisting of revolute and prismatic connections (38). Other lower pairs and certain higher pairs may be included in this analysis if they are replaced by their equivalent combinations of revolute and prismatic pairs. For example, a circular cylinder pair is equivalent to a coaxial revolute and a prismatic pair. A spheric pair is equivalent to a combination of three three revolutes. A plane pair is equivalent to a combination of a revolute and two prisms. The dynamic forces produced by the acceleration of the moving links can be found by using Lagrange equations with a varying constraint (40).

EQUILIBRIUM ANALYSIS

For the flexible ligament model, which is a spring restrained mechanisms, the equilibrium position may be found by a search procedure which seeks a minimum potential energy state for the mechanism. The procedure is similar to that developed by Livermore (34), where the number of free generalized coordinates (FGC) are corrected by the following equation:

$$[K]\{\Delta q\} = \{\Delta F\} \qquad \text{..........................(8)}$$

until the mechanism is brought sufficiently close to a static equilibrium position, that is, $\{\Delta F\} = 0$. Here, $[K]$ is the symmetric "stiffness matrix", whose elements K_{ij} are a function of the geometry of the mechanism, and involve the first and second partial derivatives of the pair variables with respect to the n FGC, $\{\Delta q\}$ is the vector of first order corrections to the FGC and $\{\Delta F\}$ is the vector of resultant generalized forces at the FGC.

CHAPTER IV CONCLUSION

The matrix methods for the above analysis are well suited to provide the required kinematic relations because of their generality. The analytical approach is independent of visualization and may be programmed for the digital computation with ease.

Unfortunately, the size of the current implementation of IMP is insufficient to allow the analysis of the model described above. Work on expanding storage capacity to allow this analysis is now in progress. Testing of the model must await this program modification.

REFERENCES

1. Engin, A. E., Mechanics of the Knee Joint, Ohio State University, Reston Pub. Co., 1976.

2. Tansey , H. H.,III ,"A Three-Dimensional Kinematic and Force Analysis of the Human Tibio-Femoral Joint During Normal Walking", Master Thesis, Purdue University, Aug., 1976.

3. Goodfellow, J., and O'Connor J., "The Mechanics of the Knee and Prosthesis Design", Journal of Bone and Joint Surgery, British Volume 60-B No. 3, pp. 358-369, Aug., 1978.

4. Elftman, H., "The Measurement of External Force in Walking", Science, Vol. 88, pp. 152, 1938.

5. Bresler, B. and Frankel, J. P. "The Forces and Moments in the Leg During Level Walking", Transactions of the American Society of Mechanical Engineers, 72, 27, (Paper No. 48-A-62), 1950.

6. Cunningham, D. M., and Brown, G. W., "Two Devices for Measuring the Forces Acting on the Human Body During Walking", Proceedings of the Society of Experimental Stress Analysis, IX, Vol. 2, pp. 75, 1952.

7. Morrison, J. B., "Bioengineering Analysis of Force Actions Transmitted by the Knee Joint", Bio-Medical Engineering, Vol. 3, No. 4, pp. 164-170, April 1968.

8. Morrison, J. B., "The Mechanics of Muscle Function in Locomotion", Journal of Biomechanics, Vol. 3, pp. 431-451, 1970.

9. Morrison, J. B., "The Mechanics of the Knee Joint in Relation to Normal Walking", Journal of Biomechanics, Vol. 3, pp. 51-56, 1970.

10. McLeish, R. D., and Charnley, J., "Abduction Forces in the One-Legged Stance", Journal of Biomechanics, Vol. 3, pp. 191-209, 1970.

11. Reilly, D. T., and Martens, M., "Experimental Analysis of the Quadriceps Muscle Force and Patello-Femoral Joint Reaction Force for Various Activities", Acta Orthopaedica Scandinavica, 43, pp. 126-137, 1972.

12. Ford, W. R., Perry, J., and Antonelli, D., "Analysis of Knee Joint Forces During Flexed Knee Stance, Orthopaedic Research Society Meeting, Washington, D.C., January 27, 1972.

13. Kettelkamp, D. B., and Chao, E. Y., "A Method for Quantitative Analysis of Medical and Lateral Compression Forces at the Knee During Standing", Clinical Orthopaedics, No. 83, pp. 202-213, March-April, 1972.

14. Seireg, A., and Arvikar, R. J., "A Mathematical Model for Evaluation of Forces in Lower Extremeties of the Musculo-Skeletal System", Journal of Biomechanics, Vol. 6, pp. 313-326, 1973.

15. Seireg, A., and Arvikar, R. J., "The Prediction of Muscular Load Sharing and Joint Forces in the Lower Extremeties During Walking", Journal of Biomechanics, Vol. 8, pp. 89-102, 1975.

16. Crowinshield, R. D., and Pope, M. H., "An Analytical Model of the Knee", Applied Mechanics Conference, American Society of Mechanical Engineers, Troy, New York, June 1975.

17. Cappozzo, A., Figura, F., Marchetti, M., and Pedotti, A., "The Interplay of Muscular and External Forces in Human Ambulation", Journal of Biomechanics, Vol. 9, pp. 35-43, 1976.

18. Kinzel, G. L., Hall, A. S., Hillberry, B. M., Vansickle, D. G., and Harvey, W. M., "Measurement of the Total Motion Between Two Body Segments - II, Description of Application", Journal of Biomechanics, Vol. 5, pp. 283-293, 1972.

19. Piziali, R. L., Rastegar, J. C., and Nagel, D. A., "Measurement of the nonlinear, coupled stiffness characteristics of the human knee", Journal of Biomechanics, Vol. 10, No. 1, pp. 45, 1977.

20. Maquet, P. G., and Pelzer, G. A., "Evolution of the Maximum Syress in Osteo-arthritis of the Knee", Journal of Biomechanics, Vol. 10, No. 2, pp. 107, 1977.

21. Harding, M. L., and Goodfellow, J. W., " A Preliminary Report of a Simple Rig to Aid Study of the Functional Anatomy of the Cadaver Human Knee Joint", Journal of Biomechanics, Vol. 10, No. 8, pp. 517, 1977.

22. Lew, W. D., and Lewis, J. L., "A Technique for Calculating in Vivo Ligament Lengths with Application to the Human Knee Joint", Journal of Biomechanics, Vol. II, No. 8, pp. 365, 1978.

23. Townsend, J. L., "Kinematics of the Human Knee Joint", Bulletin of Mathematical Biophysics, Vol. 31, pp. 215-232, 1969.

24. Kettelkamp, D. B., "Tibiofemoral Contact Area - Determination and Implications", Journal of Bone and Joint Surgery, Vol. 54-A, No. 2, March 1972.

25. Steidler, A., Kinesiology, Charles C. Thomas Publishers, Springfield, Illinois, 1955.

26. Greenwood, D. T., Principles of Dynamics, Prentice-Hall Inc., Englewood Cliffs, New Jersey.

27. Gardener, H. F., and Clippinger, F. W., "A Method for Location of Prosthetic and Orthodotic Knee Joints", Artificial Limbs, Vol. 13, No. 2, pp. 31-35, 1969.

28. Frankel, V. H., Burstein, A. H., and Brooks, D. B., "Biomechanics of Internal Derangement of the Knee", Journal of Joint and Bone Surgery, 53A2, pp. 945-962, 1971.

29. Walker, P. S., Shoji, H., and Erkman, M. J., "The Rotational Axis of the Knee and Its Significance to Prothesis Design", Clin. Orthoped. and Rela. Res., No. 89, pp. 150-167, 1972.

30. Mayott III, C. W., "Computerized Instant Centers of Rotation of the Human Knee", Master Thesis, Clemson University, Dec., 1975.

31. Uicker, J. J., Jr., "IMP User's Guide, A Problem Oriented Language for the Computer-Aided and Analysis of Mechanisms", The University of Wisconsin, Sept., 1974.

32. Hartenberg, R. S., and Denavit, J., Kinematic Synthesis of Linkages, McGraw-Hill Book Company, 1964.

33. Kepler, B. H., Basic Graphical Kinematics, McGraw-Hill Book Company, 1973.

34. Livermore, D. F., "The Determination of Equilibrium Configurations of Spring-Restrained Mechanisms Using (4x4) Matrix Methods, Journal of Engineering for Industry, Trans. ASME, Series B, Vol. 89, No. 1, Feb., 1967.

35. Hu, T. C., Integer Programming and Network Flows, Addison-Wesley Publishing Company, Inc., 1969.

36. Denavit, J., and Hartenberg, R. S., "A Kinematic Notation for the Low-Pair Mechanisms Based on Matrices", Journal of Applied Mechanics, Vol. 22, Trans. ASME, Vol. 77, Series E, 1955, pp. 215-221.

37. Sheth, P. N., and Uicker, J. J., "A Generalized Symbolic Notation for Mechanisms" Journal of Engineering for Industry, Trans. ASME, Series B Vol. 93, No. 1, Feb., 1971.

38. Denavit, J., Hartenberg, R. S., Razi, R., and Uicker, J. J. Jr. "Velocity, Acceleration and Static Force Analysis of Spatial Linkage", Journal of Applied Mechanics, Vol. 32, Trans. ASME, Series E, Vol. 87, No. 4, Dec., 1965.

39. Uicker, J. J., Jr., Denavit, J., and Hartenberg, R. S., "An Iterative Method for the Displacement Analysis of Spatial Mechanisms," Journal of App-Mechanics,Vol. 31, Trans. ASME, Series E, Vol. 86, No. 2, June 1964.

40. Uicker, J. J., Jr., "Dynamic Force Analysis of Spatial Linkages," Journal of Applied Mechanics, Vol. 34, Trans. ASME, Series E, Vol. 89, No. 2, June 1967.

A P P E N D I X

IMP-75 Computer Program

```
************************************************************************
*                              IMP-75                                  *
*                THE INTEGRATED MECHANISMS PROGRAM                     *
************************************************************************

      SYSTEM=KNEE FORCE ANALYSIS

      REMARK/DEFINITION STATEMENTS

      ZERO(SYSTEM)=0.0001

      ZERO(POSITION)=0.0005

      ZERO(DATA)=0.001

      GROUND=FR

      REVLUT(FR,B)=JT1

      REVLUT(B,C)=JT2

      REVLUT(C,D)=JT3

      REVLUT(D,E)=JT4

      REVLUT(E,A)=JT5

      REVLUT(FR,F)=JT6

      REVLUT(F,G)=JT7

      REVLUT(G,H)=JT8

      REVLUT(H,I)=JT9

      REVLUT(I,A)=JT10

      SPHERE(A,J)=JT11

      UJOINT(J,FR)=JT12

      SPHERE(A,K)=JT13

      UJOINT(K,FR)=JT14

      SPHERE(A,M)=JT15

      UJOINT(M,FR)=JT16

      SPHERE(A,L)=JT17

      UJOINT(L,FR)=JT18

      POINT(A)=PT1,PT2,PT3,PT4

      FORCE(PT1/PT1,PT2)=F1

      FORCE(PT3/PT3,PT4)=F2

      REMARK/DATA STATEMNTS
```

```
DATA/LINK(FR,JT1)=0,1.38,0/1,1.38,0/0,0.38,0
DATA/LINK(B,JT1)=0,1.38,0/1,1.38,0/0,0.38,0
DATA/LINK(B,JT2)=0,1,0/0,1,1/0,0,0
DATA/LINK(C,JT2)=0,1,0/0,1,1/0,0,0
DATA/LINK(C,JT3)=0,0,0/0,-1,0/1,0,0
DATA/LINK(D,JT3)=0,0,0/0,-1,0/1,0,0
DATA/LINK(D,JT4)=0,1.75,0/1,1.75,0/0,0.75,0
DATA/LINK(E,JT4)=0,1.75,0/1,1.75,0/0,0.75,0
DATA/LINK(E,JT5)=0,1.8,0/0,1.8,1/0,0.8,0
DATA/LINK(A,JT5)=0,1.8,0/0,1.8,1/0,0.8,0
DATA/LINK(FR,JT6)=2,1.38,0/3,1.38,0/2,0.38,0
DATA/LINK(F,JT6)=2,1.38,0/3,1.38,0/2,0.38,0
DATA/LINK(F,JT7)=2,1.25,0/2,1.25,1/2,0.25,0
DATA/LINK(G,JT7)=2,1.25,0/2,1.25,1/2,0.25,0
DATA/LINK(G,JT8)=2,0,0/2,-1,0/3,0,0
DATA/LINK(H,JT8)=2,0,0/2,-1,0/3,0,0
DATA/LINK(H,JT9)=2,1.75,0/3,1.75,0/2,0.75,0
DATA/LINK(I,JT9)=2,1.75,0/3,1.75,0/2,0.75,0
DATA/LINK(I,JT10)=2,1.8,0/2,1.8,1/2,0.8,0
DATA/LINK(A,JT10)=2,1.8,0/2,1.8,1/2,0.8,0
DATA/LINK(A,JT11)=-0.5,-0.9,0/-0.5,-0.9,1/0.5,-0.9,1
DATA/LINK(J,JT11)=-0.5,-0.9,0/0.5,-0.86,0/-0.55,0.1,0
DATA/LINK(J,JT12)=-0.6,1.1,0/0.4,1.16,0/-0.6,1.1,1
DATA/LINK(FR,JT12)=-0.6,1.1,0/-0.6,1.1,1/0.4,1.1,0
DATA/LINK(A,JT13)=2.5,-0.9,0/2.5,-0.9,1/3.5,-0.9,0
DATA/LINK(K,JT13)=2.5,-0.9,0/3.5,-0.95,0/2.55,0.1,0
DATA/LINK(K,JT14)=2.6,1.1,0/3.59,1.06,0/2.6,1.1,1
DATA/LINK(FR,JT14)=2.6,1.1,0/2.6,1.1,1/3.6,1.1,0
DATA/LINK(A,JT15)=0.7,-0.1,-0.9/0.7,-0.1,0.1/1.7,-0.1,-0.9
DATA/LINK(M,JT15)=0.7,-0.1,-0.9/-0.2,0.027,-0.46/1.16,0.2,-0.064
```

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DATA/LINK(N,JT16)=1.3,0.3,0.2/2.143,-0.147,-0.098/0,1.744,0.384

DATA/LINK(FR,JT16)=1.3,0.3,0.2/0,1.744,0.384/1.9,0.7,1.3

DATA/LINK(A,JT17)=1.3,0.1,0.9/1.3,0.1,1.9/2.3,0.1,0.9

DATA/LINK(L,JT17)=1.3,0.1,0.9/3.7,0,0/0.7,1,-0.8

DATA/LINK(L,JT18)=0.7,1,-0.8/0,2.04,0/0,0.53,-0.8

DATA/LINK(FR,JT18)=0.7,1,-0.8/0,0.53,-0.8/0.1,1.9,-2.5

DATA/POINT(PT1,JT5)=2.4,1,-1

DATA/POINT(PT2,JT5)=1.3,1,-1.3

DATA/POINT(PT3,JT5)=3.8,1,1.4

DATA/POINT(PT4,JT5)=1.6,1,2.2

DATA/FORCE(F1)=160

DATA/FORCE(F2)=250

PRINT/FORCE(ALL)

EXECUTE
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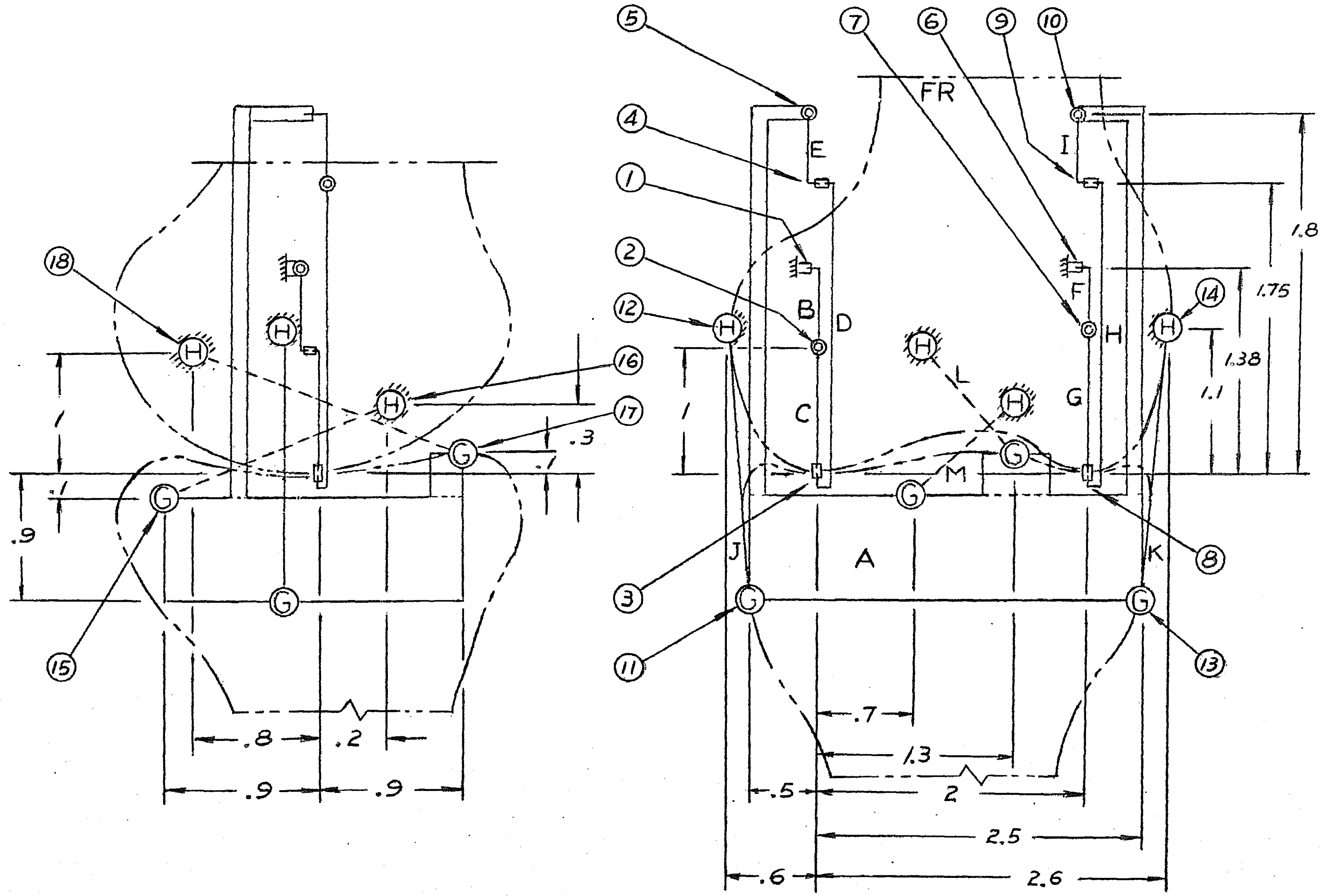

Fig. A1 Lower Pair Knee Mode' mate Dimensions, Inches